DETERMINATION OF A MOBILE CRANE CHARACTERISTICS BY FLEXIBLE MULTIBODY ANALYSIS

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ABSTRACT

DETERMINATION OF A MOBILE CRANE CHARACTERISTICS BY FLEXIBLE MULTIBODY ANALYSIS

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In this thesis, mobile crane characteristics are determined by using flexible multibody analysis. The coupled rigid and elastic motion of the crane is formulated by using absolute coordinates and modal variables. In order to perform the analysis a computer code is produced which is capable of making the dynamic analysis of the crane. Using the computer code variation of piston force with respect to the boom angular position for different hook loads and for different piston velocity profiles are simulated and compared with the experimental results. Simulations are done for 30 seconds and 10 seconds boom motion. Moreover,transverse deflections of node 3, node 8 and node 13 are obtained with respect to boom angular positions for 32.4 kN hook load. Finally, load curves are generated for 30 seconds motion and compared to those of manufacturer.

Keywords: Mobile Crane Characteristics, Flexible Multibody Analysis, Boom, Hook Load, Transverse Deflection, Lifting Capacity

ÖZ

MOBİL KREYN KAREKTERİSTİKLERİNİN ESNEK ÇOK UZUVLU SİSTEM ANALİZİYLE SAPTANMASI

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Bu tezde, esnek çok uzuvlu sistem analizi kullanılarak mobil kreyn karakteristikleri saptanır. Kreynin birbirine bağımlı rijit ve elastik hareketi mutlak eksenler ve modal koordinatlar kullanılarak formulize edilir. Analizi yapmak için kreynin dinamik analizini yapma kapasitesine sahip bir bilgisayar kodu üretilmiştir. Bilgisayar kodu kullanılarak, farklı kanca yükleri ve farklı piston hız profilleri için piston kuvvetinin boom açısal pozisyonuna göre değişimi simule edilmiş ve deneysel sonuçlarla karşılaştırılmıştır. Simulasyonlar 30 saniyelik ve 10 saniyelik boom hareketi için yapılmıştır. Ayrıca düğüm 3, düğüm 8 ve düğüm 13 deki transvers çökmeler 32.4 kN luk kanca yükü için boom açısal pozisyonlarına göre elde edilmiştir. Son olarak da 30 saniyelik hareket için yük eğrisi çıkarılmış ve imalatçı firma tarafindan verilenle karşılaştırılmıştır.

Anahtar Kelimeler: Mobil Kreyn Karakteristikleri, Esnek Çok Uzuvlu Sistem Analizi, Boom, Kanca Yükü, Transvers Çökme, Kaldırma Kapasitesi

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CHAPTER 1

INTRODUCTION

Cranes as mechanical systems are in general closed-chain mechanisms with flexible members. For the problem of determination of safe loads as a function of boom angular position the solution of the dynamic equations is necessary.

In the literature, there are few studies related to the dynamics and control of mobile cranes for various applications and objectives. In these studies the body flexibilities are not taken into consideration. Among these the following may be cited. A dynamical model for the control of a flexible rotary crane which carries out three kinds of motion (rotation, load hoisting and boom hoisting) simultaneously is derived by Sato and Sakawa [1]. Only the joint between the boom and the jib is assumed to be flexible. The goal is to transfer a load to a desired place in such a way that at the end of the transfer the swing of the load decays as quickly as possible. First an open-loop control input is applied to the system such that the state of the system can be transferred to a neighbourhood of the equilibrium state. Then a feedback control signal is applied so that the state of the system approaches the equilibrium state as quickly as possible.

Posiadala, Skalmierski and Tomski [2] studied the kinematics of the crane telescopic boom for possible movements produced by its controls. The controls producting the change of boom length, rope length, angle of boom inclination and rotation angle are considered. For the description of kinematics of the boom end, where the rope with the lifted load passes, cartesian coordinate sysytems have been applied in such a way that their relative motions are translatory. This considerably simplifies the description. The equations for the lifted load motions, considered as a particle, have been derived.

The application of a hook load and safe load indicator and limiter for mobile cranes is presented by Balkan [3,4]. The microprocessor-based control system for the determination of current hook load is based on oil pressure and boom angle. Results are given and discussed for the application of the control system to a COLES Mobile 930, 10 ton mobile crane.

The aim of this study is to determine the mobile crane characteristics by using flexible multibody analysis and obtain more insight data to the previous studies which was done by Balkan [3,4].

In the analysis of the crane, only the boom is taken as flexible. Other bodies are assumed as rigid. The computer program that is written is capable of making dynamic an analysis of the crane.

In the flexible dynamic analysis, the coupled rigid and elastic motion of the system is formulated by using absolute coordinates and modal variables[5,6]. Then, joint connections and prescribed motions are imposed as constraint equations. The flexible body is modeled by finite element method and modal variables are used as the elastic variables by utilizing modal transformation.

Using the computer program, the variations of the piston force with respect to the boom angular positions are analyzed for different piston velocity profiles and different hook loads. These numerical simulations are done for 30 seconds boom motion. The numerical simulation results founds for the COLES Mobile 930, 10 ton crane is compared with the experimental results for the 30 seconds motion of the boom. Moreover, transverse deflections of node 3, node 8 and node 13 are obtained with respect to boom angular positions for 32.4 kN hook load. Finally, load curves are generated for 30 seconds motion and compared to those of manifacturer.

CHAPTER 2

MODELING OF FLEXIBLE MULTIBODY SYSTEMS

2.1 NOMENCLATURE AND PRELIMINARY CONSIDERATIONS

There are two basic approaches to model a mechanical system composed of rigid and flexible bodies. In the first one the motion of each body is formulated with respect to a fixed frame in terms of its absolute rigid body and elastic degrees of freedom. Then the interconnections of the bodies are defined through a set of geometrical constraint equations [7,8]. The second approach involves a recursive modeling in terms of the joint coordinates and elastic deformation variables of a tree-like system obtained by temporarily cutting the closed loops open. The closed loops are then imposed as a set of geometrical constraint equations [9,10]. The formulation using the second approach results in a fewer number of equations but these equations are relatively more complicated than those obtained using the first approach. In this study, the first approach is used.

A multibody system consisting of N flexible bodies is shown in Figure 2.1. Based on Shabana's and Yoo and Haug's studies [7,8] each body is modeled using the absolute coordinates seperately as if there are no joints. The joint connections and prescribed motions are then imposed as a set of constraint equations.

In order to specify the configuration of a body, it is necessary to define a set of generalized coordinates such that the global position and orientation of every infinitesimal volume in the body is determined by these generalized coordinates. As shown in Figure 2.2, let $\mathbf{n}$ represent general reference axis frame (inertal frame) and $\mathbf{n}^k$ represent body reference axis frame that is rigidly attached to some point on body k. Using finite elements, body k is divided into a number of interconnected elements.

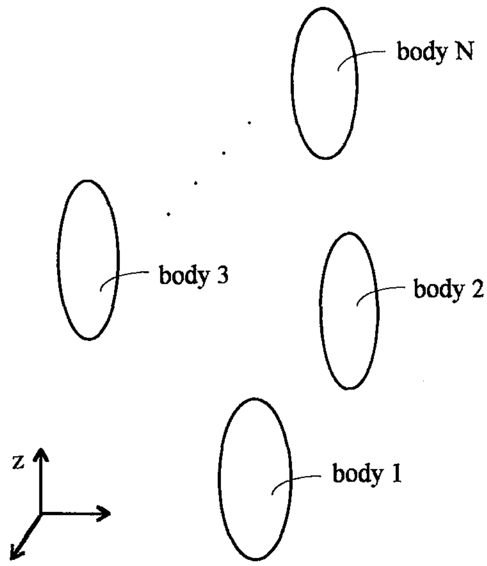

Figure 2.1 A Multibody System Consisting of N Flexible Bodies

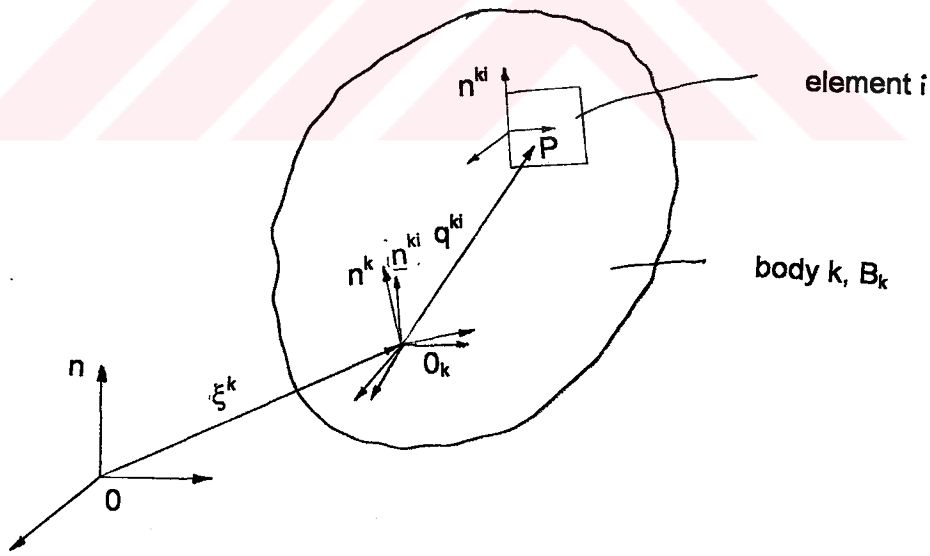

Figure 2.2 A Body with Rigid Body Motion and Deformation

The location of an arbitrary infinitesimal volume on the i th element of this body is defined by two sets of generalized coordinates. The first set consists of reference coordinates that locate the position of the body-fixed coordinate system relative to the inertial frame **n**. The second set consists of elastic coordinates that characterize deformation of the body. Elastic coordinates represent relative translational and angular displacement at nodal points on the body. The location of every point in each element can be approximated in terms of its elastic coordinates and its shape function.

Let $\mathbf{n}^{ki}$, as shown in Figure 2.2, be element axis frame with its origin fixed to some point on the i th element of body k. The location of an arbitrary point **P** on this element is determined by specifying the position and orientation of the $\mathbf{n}^{k}$ frame and the location of the point P with respect to the $\mathbf{n}^{k}$ frame. Let ξ^{k} and θ^{k} be, respectively, the translational and rotational coordinates of the $\mathbf{n}^{k}$ frame, with respect to the inertial frame **n**. And let $\underline{\mathbf{n}}^{ki}$ be an axis frame that is parallel to the element axis frame $\mathbf{n}^{ki}$ and located at the origin of the $\mathbf{n}^{k}$ frame. And let α be the nodal coordinates of lement i of body k, with respect to the $\mathbf{n}^{ki}$ frame. And let **u** denote deformation displacement of point P with respect to the $\mathbf{n}^{ki}$ frame. Then the vector **u** can be written as

$$\mathbf{u} = \mathbf{s}\,\alpha \tag{2.1}$$

where **s** is the shape function of element i of body k. Using transformation matrix, **u** and α can be written with respect to $\mathbf{n}^{k}$ frame as follows

$$\mathbf{u}^{ki} = \mathbf{T}^{ki}\,\mathbf{u} \tag{2.2}$$

$$\alpha^{ki} = \mathbf{R}^{ki}\,\alpha \tag{2.3}$$

where $\mathbf{u}^{ki}$ represents deformation displacement vector defined in $\mathbf{n}^{k}$, α^{ki} is the vector of element nodal variables defined with respect to the $\mathbf{n}^{k}$, $\mathbf{T}^{ki}$ and $\mathbf{R}^{ki}$ are coordinate transformation matrices that are determined according to element type [7,9].

Hence, in $\mathbf{n}^k$ frame, deformation displacement vector $\mathbf{u}^{ki}$ is represented by

$$\mathbf{u}^{ki} = \mathbf{T}^{ki}\, \mathbf{s}\; \mathbf{R}^{ki^T}\; \alpha^{ki} \tag{2.4}$$

Above equation can be written as

$$\mathbf{u}^{ki} = \phi^{ki}\, \alpha^{ki} \tag{2.5}$$

where ϕ^{ki} is the modified shape function of element i of body k, given by

$$\phi^{ki} = \mathbf{T}^{ki}\, \mathbf{s}\; \mathbf{R}^{ki^T} \tag{2.6}$$

If the element axis frame $\mathbf{n}^{ki}$ has the same orientation with the body axis frame $\mathbf{n}^k$, ϕ^{ki} is the same as $\mathbf{s}$.

Similarly, deformation rotation vector θ^{ki} of an arbitrary infinitesimal element at P in the finite element is represented in the following form

$$\theta^{ki} = \psi^{ki}\, \alpha^{ki} \tag{2.7}$$

where ψ^{ki} denotes the deformation rotation shape function matrix. The translation and rotational shape functions for beam elements are given in Appendix A.

The global position of point P is

$$\mathbf{R}^{ki} = \xi^k + \mathbf{T}^k\, \mathbf{q}^{ki} \tag{2.8}$$

where $\mathbf{T}^k$ is the transformation matrix from the $\mathbf{n}^k$ to the $\mathbf{n}$ coordinate system, and $\mathbf{q}^{ki}$ is the location of point P with respect to the $\mathbf{n}^k$, represented as

$$\mathbf{q}^{ki} = \mathbf{r}^{ki} + \mathbf{u}^{ki} \tag{2.9}$$

where $\mathbf{r}^{ki}$ denotes undeformed position vector to point P with respect to the $\mathbf{n}^k$ frame.

$\mathbf{r}^{ki}$ can be expressed as

$$\mathbf{r}^{ki} = \phi^{ki}\, \mathbf{e}_0^{ki} \tag{2.10}$$

where $\mathbf{e}_0^{ki}$ is obtained by using the position coordinates of the nodes instead of the deformation displacements in α^{ki}.

Sum of the α^{ki} and $\mathbf{e}_0^{ki}$ vectors can be represented as

$$\mathbf{e}^{ki} = \mathbf{e}_0^{ki} + \alpha^{ki} \tag{2.11}$$

where $\mathbf{e}^{ki}$ denotes the vector of nodal variables of element i with respect to the $\mathbf{n}^k$ frame and $\mathbf{q}^{ki}$ can be written as

$$\mathbf{q}^{ki} = \phi^{ki}\, \mathbf{e}^{ki} \tag{2.12}$$

Since the elements are connected to each other with nodes, some nodes are common for two or more elements. Using boolean matrices for each element, the element nodal coordinates in the vector of body nodal coordinates can be identified; that is to say, the connectivity of the elements are automatically taken into consideration. Therefore, $\mathbf{e}^{ki}$ can be written as

$$\mathbf{e}^{ki} = \mathbf{B}^{ki}\, \mathbf{e}^{k} \tag{2.13}$$

where $\mathbf{e}^k$ is the vector of nodal variables of body k and $\mathbf{B}^{ki}$ is the connectivity boolean matrix for the element i. Similarly, $\mathbf{e}_0{}^{ki}$ and α^{ki} can be represented as

$$\mathbf{e}_0{}^{ki} = \mathbf{B}^{ki}\,\mathbf{e}_0{}^{k} \qquad (2.14)$$

$$\alpha^{ki} = \mathbf{B}^{ki}\,\alpha^{k} \qquad (2.15)$$

where $\mathbf{e}_0{}^{k}$ and α^k are the aggregated vectors of all undeformed nodal coordinates and all element nodal deformation displacement coordinates in body k, respectively.

Hence, equation (2.8) takes the following form

$$\mathbf{R}^{ki} = \xi^k + \mathbf{T}^k\,\phi^{ki}\,\mathbf{B}^{ki}\,(\mathbf{e}_0{}^{k} + \alpha^k\,) \qquad (2.16)$$

2.2 DECREASING THE NUMBER OF ELASTIC VARIABLES

Adequate representation of large scale nonlinear mechanical systems using the finite element method may require a large number of nodal coordinates. It is necessary to reduce this number of coordinates if a solution is to be obtained with a reasonable amount of computer time. Substructuring and component mode synthesis techniques have been used extensively in structural dynamics to reduce problem dimensionality. In many applications, the number of elastic coordinates is much larger than the number of reference coordinates. The problem dimension can be significantly decreased if insignificant elastic coordinates are eliminated.

By imposing the appropriate boundary conditions one can identify a transformation from the space of system nodal coordinates to the space of system modal coordinates of lower dimension.

To this end, the free vibration problem of each body is considered as

$$\mathbf{M}^k \ddot{\alpha}^k + \mathbf{K}^k \alpha^k = 0 \tag{2.17}$$

where $\mathbf{M}^k$ and $\mathbf{K}^k$ are the structural mass and stiffness matrices obtained by imposing the reference (boundary) conditions at the connection points.

Equation (2.17) is the eigenvalue problem which can be solved for a set of eigenvalues ω_p^2 and the corresponding mode shapes $\mathbf{a}_p^k$, p = 1, 2, ... , n^k , where n^k is the number of elastic nodal coordinates of body k. A reduced order model can be achieved by using only m^k mode shapes, where $m^k < n^k$. Then, the coordinate transformation from the physical nodal coordinates to the modal elastic coordinates is written as

$$\alpha^k = \chi^k \eta^k \tag{2.18}$$

where χ^k is the (n^k x m^k) dimensional modal transformation matrix whose columns are the low frequency m^k mode shapes. The vector $\mathbf{n}^k$ is the (m^k x 1) dimensional vector of modal coordinates. The m^k mode shapes should be selected such that a good approximation for the displaced shape can be obtained.

2.3 VELOCITY AND ACCELERATION OF AN ARBITRARY POINT

In order to decrease the number of elastic variables equation (2.18) is substituted in equation (2.16). Thus, the global position of point P is given by

$$\mathbf{R}^{ki} = \xi^k + \mathbf{T}^k \phi^{ki} \mathbf{B}^{ki} (\mathbf{e}_0{}^k + \chi^k \eta^k) \tag{2.19}$$

Differentiating the above equation with respect to time yields the following velocity vector of point P

$$\mathbf{v}^{ki} = \dot{\xi}^{k} + \mathbf{T}^{k}\,\tilde{\mathbf{q}}^{ki}\,\bar{\omega}^{k} + \mathbf{T}^{k}\,\phi^{ki}\,\mathbf{B}^{ki}\,\chi^{k}\,\dot{\eta}^{k} \tag{2.20}$$

where $(\dot{\ })$ denotes differentiation with respect to time, $\bar{\omega}^{k}$ denotes absolute angular velocity of body k with components along $\mathbf{n}^{k}$, represented as

$$\bar{\omega}^{k} = \mathbf{T}^{k^{T}}\,\omega^{k} \tag{2.21}$$

and skew symmetric matrix $\tilde{\mathbf{q}}^{ki}$ is represented by

$$\tilde{\mathbf{q}}^{ki} = \begin{bmatrix} 0 & q_3^{ki} & -q_2^{ki} \\ -q_3^{ki} & 0 & q_1^{ki} \\ q_2^{ki} & -q_1^{ki} & 0 \end{bmatrix} \tag{2.22}$$

If $\dot{\xi}^{k}$, $\bar{\omega}^{k}$ and η^{k} are selected as the generalized speeds [11], $\mathbf{v}^{ki}$ can be written in the following form

$$\mathbf{v}^{ki} = \begin{bmatrix} \mathbf{I} & \mathbf{T}^{k}\tilde{\mathbf{q}}^{ki} & \mathbf{T}^{k}\phi^{ki}\mathbf{B}^{ki}\chi^{k} \end{bmatrix} \begin{bmatrix} \dot{\xi}^{k} \\ \bar{\omega}^{k} \\ \dot{\eta}^{k} \end{bmatrix} \tag{2.23}$$

This equation can also be expressed as

$$\mathbf{v}^{ki} = \mathbf{v}^{ki}\,\mathbf{y}^{k} \tag{2.24}$$

where $\mathbf{v}^{ki}$ is the influence coefficient matrix and $\mathbf{y}^{k}$ is the generalized speeds vector.

The accelaration vector of point P is obtained by differentiating the equation (2.24)

$$\mathbf{a}^{ki} = \mathbf{v}^{ki}\ \dot{\mathbf{y}}^{k} + \dot{\mathbf{v}}^{ki}\ \mathbf{y}^{k} \qquad (2.25)$$

where $\dot{\mathbf{v}}^{ki}\ \mathbf{y}^{k}$ is coriolis and centrifugal acceleration term which is given by

$$\dot{\mathbf{v}}^{ki}\ \mathbf{y}^{k} = \dot{\mathbf{T}}^{k}\ \tilde{\mathbf{q}}^{ki}\ \overline{\omega}^{k} + 2\,\dot{\mathbf{T}}^{k}\ \phi^{ki}\,\mathbf{B}^{ki}\,\chi^{k}\ \dot{\eta}^{k} \qquad (2.26)$$

and $\dot{\mathbf{y}}^{k}$ is the generalized accelerations vector, denoted by

$$\dot{\mathbf{y}}^{k} = \left[\ \ddot{\xi}^{k^{T}}\ \ \dot{\overline{\omega}}^{k^{T}}\ \ \ddot{\eta}^{k^{T}}\ \right] \qquad (2.27)$$

2.4 EQUATIONS OF MOTION

In this section, the equations of motion of the flexible multibody system as developed by İder [11,6] are given. Lagranges form of d'Alembert's principle for each body k can be expressed as

$$f_r^{*k} + f_r^{k} + f_r^{s^k} + f_r^{d^k} = 0 \qquad r = 1, \dots, 6+m^k \qquad (2.28)$$

where f_r^{*k} are the generalized inertia forces, f_r^{k} are the generalized external forces, $f_r^{s^k}$ are the generalized structural stiffness forces, and $f_r^{d^k}$ are the generalized structural damping forces. These equations are considered in detail in the following sections.

2.4.1 GENERALIZED INERTIA FORCES

The generalized inertia forces due to the inertias of the particles in body k are

$$f_r^{*k} = -\sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki} \frac{\partial \mathbf{v}^{ki}}{\partial y_r} \cdot \mathbf{a}^{ki} dV \qquad r = 1, \ldots, 6+m^k \tag{2.29}$$

where E_k denotes number of elements in body k, ρ^{ki} denotes the mass density of the element i of body k, and V_{ki} denotes the volume of element i of body k.

Above equation can be put into the following form

$$f^{*k} = -\mathbf{M}^k \dot{\mathbf{y}}^k + \mathbf{Q}^k \tag{2.30}$$

where $\mathbf{M}^k$ is the generalized mass matrix of body k and obtained as

$$\mathbf{M}^k = -\sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki} \mathbf{v}^{ki^T} \mathbf{v}^{ki} \, dV \tag{2.31}$$

and $\mathbf{Q}^k$ is the coriolis and centrifugal force matrix, given by

$$\mathbf{Q}^k = -\sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki} \mathbf{v}^{ki^T} \dot{\mathbf{v}}^{ki} \mathbf{y}^k \, dV \tag{2.32}$$

In order to deal with each term separately, the submatrices of $\mathbf{M}^k$ are labelled as below

$$\mathbf{M}^k = \begin{bmatrix} \mathbf{M}^k_{\xi\xi} & \mathbf{M}^k_{\xi\omega} & \mathbf{M}^k_{\xi\eta} \\ \mathbf{M}^{k\,T}_{\xi\omega} & \mathbf{M}^k_{\omega\omega} & \mathbf{M}^k_{\omega\eta} \\ \mathbf{M}^{k\,T}_{\xi\eta} & \mathbf{M}^{k\,T}_{\omega\eta} & \mathbf{M}^k_{\eta\eta} \end{bmatrix} \tag{2.33}$$

Similarly submatrices of $\mathbf{Q}^k$ are labelled as

$$\mathbf{Q}^k = \begin{bmatrix} \mathbf{Q}_\xi^k \\ \mathbf{Q}_\omega^k \\ \mathbf{Q}_\eta^k \end{bmatrix} \tag{2.34}$$

Notice that for the determination of the generalized inertia forces the generalized inertia force of the orbitrary point P is integrated over the element volume. Since it is not practical to perform the space integrations at each time step of the motion, the time and space dependent terms have to be seperated so that the spatial properties of the flexible bodies can be seperately calculated.

The mass submatrices can be defined as

$$\mathbf{M}_{\xi\xi}^k = \begin{bmatrix} m^k & 0 & 0 \\ 0 & m^k & 0 \\ 0 & 0 & m^k \end{bmatrix} \tag{2.35}$$

in which m^k is the mass of the body k and this submatrix $\mathbf{M}_{\xi\xi}^k$ is diagonal and time-invariant. The submatrix $\mathbf{M}_{\xi\omega}^k$ which represents the inertia coupling between the translation and rotation of the body reference is defined as

$$\mathbf{M}_{\xi\omega}^k = \mathbf{T}^k \,\tilde{\mathbf{d}}^k \tag{2.36}$$

where ($\tilde{}$) represents skew symmetric matrix of vector $\mathbf{d}^k$, and $\mathbf{d}^k$ is given by

$$\mathbf{d}^k = \sum_{i=1}^{E_k} \left(\int_{V_{ki}} \rho^{ki} \, \phi^{ki} \, dV \, \mathbf{B}^{ki} \right) \chi^k \tag{2.37}$$

This matrix is also required for evaluating the matrix $\mathbf{M}_{\xi\eta}^k$ which represents the inertia coupling between the translation of the body reference and the elastic deformation of the body. $\mathbf{M}_{\xi\eta}^k$ is represented as

$$\mathbf{M}^{k}_{\xi\eta} = \mathbf{T}^{k}\,\mathbf{d}^{k} \tag{2.38}$$

and the central term in the matrix of equation (2.33) is obtained as

$$\mathbf{M}^{k}_{\omega\omega} = \sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki}\ \tilde{\mathbf{q}}^{ki^T}\ \tilde{\mathbf{q}}^{ki}\ dV \tag{2.39}$$

$\mathbf{M}^{k}_{\omega\eta}$ describes the inertia coupling between the rotation of the body reference and the deformation of the body, given by

$$\mathbf{M}^{k}_{\omega\eta} = \sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki}\ \tilde{\mathbf{q}}^{ki^T}\ \boldsymbol{\phi}^{ki}\ \mathbf{B}^{ki}\ \boldsymbol{\chi}^{k}\ dV \tag{2.40}$$

Finally, the last term in equation (2.33) is denoted as $\mathbf{M}^{k}_{\eta\eta}$. This term is a constant matrix and can be written as

$$\mathbf{M}^{k}_{\eta\eta} = \boldsymbol{\chi}^{k^T}\ \mathbf{M}^{k}_{\alpha\alpha}\ \boldsymbol{\chi}^{k} \tag{2.41}$$

where $\mathbf{M}^{k}_{\alpha\alpha}$ denotes structural mass matrix of the body k which is expressed as

$$\mathbf{M}^{k}_{\alpha\alpha} = \sum_{i=1}^{E_k}\left(\mathbf{B}^{ki^T} \int_{V_{ki}} \rho^{ki}\ \boldsymbol{\phi}^{ki^T}\ \boldsymbol{\phi}^{ki}\ dV\,\mathbf{B}^{ki}\right) \tag{2.42}$$

and if eigenvectors $\boldsymbol{\chi}^{k}$ are normalized with respect to $\mathbf{M}^{k}_{\alpha\alpha}$, $\mathbf{M}^{k}_{\eta\eta}$ is identity matrix.

The submatrices of $\mathbf{Q}^{k}$ can be computed as

$$\mathbf{Q}_{\xi}^{k} = \mathbf{T}^{k}\,\tilde{\bar{\omega}}^{k} \sum_{i=1}^{E_k}\left(\int_{V_{ki}} \rho^{ki}\,\tilde{\mathbf{q}}^{ki}\,dV\right)\bar{\omega}^{k} +$$

$$2\,\mathbf{T}^{k}\,\tilde{\bar{\omega}}^{k} \sum_{i=1}^{E_k}\left(\int_{V_{ki}} \rho^{ki}\,\phi^{ki}\,dV\,\mathbf{B}^{ki}\right)\chi^{k}\,\dot{\eta}^{k} \tag{2.43}$$

$$\mathbf{Q}_{\omega}^{k} = \sum_{i=1}^{E_k}\left(\int_{V_{ki}} \rho^{ki}\,\tilde{\mathbf{q}}^{ki^{T}}\,\tilde{\bar{\omega}}^{k}\,\tilde{\mathbf{q}}^{ki}\,dV\right)\bar{\omega}^{k} +$$

$$2\sum_{i=1}^{E_k}\left(\int_{V_{ki}} \rho^{ki}\,\tilde{\mathbf{q}}^{ki^{T}}\,\tilde{\bar{\omega}}^{k}\,\phi^{ki}\,dV\,\mathbf{B}^{ki}\right)\chi^{k}\,\dot{\eta}^{k} \tag{2.44}$$

$$\mathbf{Q}_{\eta}^{k} = \sum_{i=1}^{E_k}\left(\int_{V_{ki}} \rho^{ki}\,\mathbf{B}^{ki^{T}}\,\phi^{ki^{T}}\,\tilde{\bar{\omega}}^{k}\,\tilde{\mathbf{q}}^{ki}\,dV\right)\bar{\omega}^{k} +$$

$$2\sum_{i=1}^{E_k}\left(\int_{V_{ki}} \rho^{ki}\,\mathbf{B}^{ki^{T}}\,\phi^{ki^{T}}\,\tilde{\bar{\omega}}^{k}\,\phi^{ki}\,dV\,\mathbf{B}^{ki}\right)\chi^{k}\,\dot{\eta}^{k} \tag{2.45}$$

where skew symmetric matrix $\tilde{\bar{\omega}}^{k}$ is given as

$$\tilde{\bar{\omega}}^{k} = \begin{bmatrix} 0 & \bar{\omega}_3^{k} & -\bar{\omega}_2^{k} \\ -\bar{\omega}_3^{k} & 0 & \bar{\omega}_1^{k} \\ \bar{\omega}_2^{k} & -\bar{\omega}_1^{k} & 0 \end{bmatrix} \tag{2.46}$$

The spatial mass properties of flexible bodies that are needed in the submatrices and subvectors of $\mathbf{M}^{k}$ and $\mathbf{Q}^{k}$ are $(3xn^{ki})$ matrix $\int_{V_{ki}} \rho^{ki}\,\phi^{ki}\,dV$ and a $(n^{ki}xn^{ki})$ matrices $\int_{V_{ki}} \rho^{ki}\,\phi_p^{ki^{T}}\,\phi_r^{ki}\,dV$ p,r = 1, 2, 3 where ϕ_i^{ki} is the i th row of ϕ^{ki} [11,6].

2.4.2 GENERALIZED EXTERNAL FORCES

The external forces applied to the body are classified in two groups:

a) Consider a concentrated force **F** applied at a point A of body k. The generalized external force on body k is then represented by

$$f_r^k = \frac{\partial \mathbf{v}^A}{\partial y_r} \cdot \mathbf{F} \qquad r = 1, \dots, 6+m^k \qquad (2.47)$$

This equation can be written as

$$\mathbf{f}^k = \mathbf{v}^{A^T} \mathbf{F} \qquad (2.48)$$

and submatrices of $\mathbf{f}^k$ are given by

$$\mathbf{f}^k = \begin{bmatrix} \mathbf{f}_\xi^k \\ \mathbf{f}_\omega^k \\ \mathbf{f}_\eta^k \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \tilde{\mathbf{q}}^{A^T} \mathbf{T}^{k^T} \mathbf{F} \\ \boldsymbol{\chi}^{k^T} \mathbf{B}^{ki} \boldsymbol{\phi}^{A^T} \mathbf{T}^{k^T} \mathbf{F} \end{bmatrix} \qquad (2.49)$$

b) Consider the gravitational force (body force). As the gravitational force applied to all elements of the body k, the total gravitational force on the body k is found as,

$$f_r^k = \sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki} \frac{\partial \mathbf{v}^{ki}}{\partial y_r} \cdot \mathbf{s}\, g\, dV \qquad r = 1, \dots, 6+m^k \qquad (2.50)$$

where **s** is the unit vector along gravitational acceleration in fixed frame **n**. In matrix notation, equation (2.50) can be obtained as

$$\mathbf{f}^{k} = \begin{bmatrix} \mathbf{f}_{\xi}^{k} \\ \mathbf{f}_{\omega}^{k} \\ \mathbf{f}_{\eta}^{k} \end{bmatrix} = \begin{bmatrix} g \sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki}\, \mathbf{s}\, dV \\ g \sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki}\, \tilde{\mathbf{q}}^{ki^T}\, \mathbf{T}^{k^T}\, \mathbf{s}\, dV \\ g \sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki}\, \chi^{k^T}\, \mathbf{B}^{ki}\, \phi^{ki^T}\, \mathbf{T}^{k^T}\, \mathbf{s}\, dV \end{bmatrix} \tag{2.51}$$

2.4.3 GENERALIZED STRUCTURAL STIFFNESS FORCES

Generalized structural stiffness forces are found from the work done by the stiffness forces which is equal to the minus sign of the strain energy of the body k. Hence, $\mathbf{f}^{s^k}$ takes the following form

$$\mathbf{f}^{s^k} = -\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \chi^{k^T}\, \mathbf{K}^k\, \chi^k\, \eta^k \end{bmatrix} \tag{2.52}$$

Since strain energy does not depend on rigid body coordinates first two submatrices of equation (2.52) are zero.

2.4.4 GENERALIZED STRUCTURAL DAMPING FORCES

Generalized structural damping forces are obtained from the work done by the viscoelastic forces and can be written as

$$\mathbf{f}_{d}^{k} = -\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \chi^{k^T}\, \mathbf{C}^k\, \chi^k\, \dot{\eta}^k \end{bmatrix} \tag{2.53}$$

where $\mathbf{C}^k$ is the damping matrix of the body k. In most of the practical analyses Rayleigh damping is assumed in the following form

$$\mathbf{C}^k = \alpha^k\, \mathbf{M}^k + \beta^k\, \mathbf{K}^k \tag{2.54}$$

where $\mathbf{M}^k$ and $\mathbf{K}^k$ are structural mass and stiffness matrices in nodal variables, and α^k and β^k are constants to be determined from the two given damping ratios that correspond to the two unequal frequencies of vibration. For the smallest two frequencies ω_1 and ω_2 , α^k and β^k are given as

$$\alpha^k = \frac{2\omega_1\omega_2(\zeta_1\omega_2 - \zeta_2\omega_1)}{\omega_2^2 - \omega_1^2} \tag{2.55}$$

$$\beta^k = \frac{2(\zeta_1\omega_1 - \zeta_2\omega_2)}{\omega_1^2 - \omega_2^2} \tag{2.56}$$

where ζ_1 and ζ_2 are the given two damping ratios. It is observed in practice that ζ_i are between 0.02 and 0.05.

2.5 EQUATIONS OF MOTION OF CONSTRAINED SYSTEM

When the system is subject to geometrical and prescribed motion constraints, the equation of the multibody system is altered such that,

$$\mathbf{f}^* + \mathbf{f} + \mathbf{f}^s + \mathbf{f}^d + \mathbf{f}^c = 0 \tag{2.57}$$

where $\mathbf{f}^c$ is the vector of generalized constraint forces. The constraint equations can be written in displacement level as

$$\varphi_i(\mathbf{x},t) = 0 \qquad i = 1, \dots, c \tag{2.58}$$

where $\mathbf{x}$ is the vector of generalized coordinates and c is the number of the constraint equations. When the time derivative of equation (2.58) is taken, the constraint equations at velocity level are obtained as

$$\dot{\varphi} = \mathbf{B}\,\mathbf{y} - \mathbf{g} = \mathbf{0} \tag{2.59}$$

The constraint forces applied to the system can be simply calculated by,

$$\mathbf{f}^{c} = - \mathbf{B}^{T} \boldsymbol{\lambda} \tag{2.60}$$

where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers representing the constraint reaction forces.

In order to generate the constraint equations and the constraint matrix **B**, the influence coefficient matrices can be utilized. In a multibody system, between bodies B_r and B_s there may be a pin joint, ball joint, prismatic joint, etc. Let the joint is labelled as A. Joint A is defined by points A_r and A_s and axes frames $\mathbf{n}^{A_r}$ and $\mathbf{n}^{A_s}$ that are fixed to B_r and B_s at A_r and A_s respectively. Depending on the joint type there are six possible constraint equations at velocity level:

The first possible three equations are obtained by equating the components of $\mathbf{v}^{A_r}$ and $\mathbf{v}^{A_s}$ to each other which are given as

$$\mathbf{v}^{A_r} = \dot{\xi}^{r} + \mathbf{T}^{r} \tilde{\mathbf{q}}^{A_r} \overline{\omega}^{r} + \mathbf{T}^{r} \phi^{A_r} \dot{\eta}^{r} \tag{2.61}$$

$$\mathbf{v}^{A_s} = \dot{\xi}^{s} + \mathbf{T}^{s} \tilde{\mathbf{q}}^{A_s} \overline{\omega}^{s} + \mathbf{T}^{s} \phi^{A_s} \dot{\eta}^{s} \tag{2.62}$$

where $\mathbf{q}^{A_r}$ and $\mathbf{q}^{A_s}$ are represented as

$$\mathbf{q}^{A_r} = \mathbf{r}^{A_r} + \phi^{A_r} \eta^{r} \tag{2.63}$$

$$\mathbf{q}^{A_s} = \mathbf{r}^{A_s} + \phi^{A_s} \eta^{s} \tag{2.64}$$

where ϕ^{A_r} denotes the shape function matrix ϕ^{r} evaluated at A_r and ϕ^{A_s} denotes the shape function matrix ϕ^{s} evaluated at A_s.

The last three possible equations are obtained by equating the components of the angular velocities of frames $\mathbf{n}^{A_r}$ and $\mathbf{n}^{A_s}$ to each other which are expressed in the following form

$$\omega^{A_r} = \mathbf{T}^r \, \bar{\omega}^r + \mathbf{T}^r \, \psi^{A_r} \, \dot{\eta}^r \tag{2.65}$$

$$\omega^{A_s} = \mathbf{T}^s \, \bar{\omega}^s + \mathbf{T}^s \, \psi^{A_s} \, \dot{\eta}^s \tag{2.66}$$

where ψ^{A_r} represents deformation rotation shape function matrix ψ^r evaluated at A_r and ψ^{A_s} represents deformation rotation shape function matrix ψ^s evaluated at A_s.

Differentiation of equation (2.59) yields the constraint equations at the acceleration level as

$$\ddot{\varphi} = \mathbf{B} \, \dot{\mathbf{y}} + \dot{\mathbf{B}} \, \mathbf{y} - \dot{\mathbf{g}} = \mathbf{0} \tag{2.67}$$

Equations (2.57) and (2.67) represent the governing dynamical equations of motion. Combining the two and making use of equations (2.30) and (2.60), augmented form of the dynamical equations are obtained as,

$$\begin{bmatrix} \mathbf{M} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{y}} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q} + \mathbf{f} + \mathbf{f}^s + \mathbf{f}^d \\ -\dot{\mathbf{B}} \, \mathbf{y} + \dot{\mathbf{g}} \end{bmatrix} \tag{2.68}$$

Here, $\mathbf{M}$, $\mathbf{Q}$, $\mathbf{f}$, $\mathbf{f}^s$, $\mathbf{f}^d$, and $\mathbf{y}$ are composite matrices and given by

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}^1 & & & \mathbf{0} \\ & \mathbf{M}^2 & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{M}^N \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \mathbf{Q}^1 \\ \mathbf{Q}^2 \\ \vdots \\ \mathbf{Q}^N \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}^1 \\ \mathbf{f}^2 \\ \vdots \\ \mathbf{f}^N \end{bmatrix}, \quad \mathbf{f}^s = \begin{bmatrix} \mathbf{f}^{s^1} \\ \mathbf{f}^{s^2} \\ \vdots \\ \mathbf{f}^{s^N} \end{bmatrix}, \quad \mathbf{f}^d = \begin{bmatrix} \mathbf{f}^{d^1} \\ \mathbf{f}^{d^2} \\ \vdots \\ \mathbf{f}^{d^N} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \mathbf{y}^1 \\ \mathbf{y}^2 \\ \vdots \\ \mathbf{y}^N \end{bmatrix} \tag{2.69}$$

where N denotes the number of bodies.

The accelerations $\dot{\mathbf{y}}$ and Lagrange multipliers $\boldsymbol{\lambda}$ are calculated from equations (2.68) at each time step. **y** and **x** are determined by numerical integration.

2.6 AUTOMATED ANALYSIS OF FLEXIBLE SYSTEM

This section is devoted to describing a sequence of computations for solving constrained flexible systems. It is observed from the development of this chapter that a set of time-invariant matrices are required for each deformable body in order to generate the inertia coupling between the reference motion and elastic deformation of the body. These matrices which are $\int_{V_{ki}} \rho^{ki}\, \phi^{ki}\, dV$, $\int_{V_{ki}} \rho^{ki}\, \phi_p^{ki^T}\, \phi_r^{ki}\, dV$, $\mathbf{M}_{\alpha\alpha}^{k}$, $\mathbf{K}^{k}$, χ^{k} can be evaluated only once in advance for the dynamic analysis using a structural analysis program. It is more computationally efficient to evaluate these matrices, once in advance for the dynamic analysis and store them to use whenever they are needed. Having determined the time invariant matrices of each deformable body, one can input these matrices to the dynamic analysis program along with the description of the rigid components in the multibody system. In the dynamic analysis program, firstly, generalized mass matrix, coriolis and centrifugal force matrix, and generalized external force matrix are evaluated for each rigid and flexible bodies. In addition to these, generalized structural stiffness force matrix and generalized structural damping force matrix are calculated for each flexible bodies. Then, constraint coefficient matrices are evaluated. Finally, constrained system differential equations of motion is constructed and they are numerically integrated to find the generalized acceleration vectors and Lagrange multipliers which are the negative of the constraint reaction forces at the points of contact. Dynamic analysis program part is repeated until the simulation time ends.

CHAPTER 3

ANALYSIS OF THE CRANE

There has been an experimental work on the tipping load control of a Coles Mobile 930 crane [3,4]. Therefore, for the application of developed software, the structure of the above mentioned crane and its parameters are used. However, the method of analysis can easily be applied to similar types of cranes with simple modifications in the assumptions. Firstly, kinematic model of the crane is constructed. Then, its flexible dynamic analysis is done. Finally, this analysis results are compared with the experimental results.

3.1 MODELING OF THE CRANE

The Coles Mobile 930 is a self propelled one man operated diesel hydraulic crane with full circle slew. The crane has a 4x2 wheel drive chassis, the steering and driving axles being solidly mounted to the chassis frame. The chassis houses four independently controlled teloscopic beams with vertical jacks. A 20 mm diameter x 92 meters spin resistant rope having a hook at its end pases through the fixed pulley at the end of the boom and is wound to the grooved barrel that is driven by a motor.

When the motion of the crane is considered, generally crane is operated under blocked conditions by the vertical jacks. Then, load is set up to the hook and the boom is hoisted. Since the excessive raising of the load is dangerous, height of the load is controlled (adjusted) by lengthening the rope. After that, crane is rotated if desired. And it is saved from the blocked position by descending the jacks. Finally, crane is blocked by the vertical jacks and load is lowered. During the hoisting, lowering and transportation of the load crane is not rotated, due to some restrictions such as, very huge and/or heavy loads, space problems etc.

In every angular position of the boom, there is a maximum load above which tipping might probably occur. Since angular position of the boom changes only during the up and down motion of the load, which is actually a 2-D motion, modelling and analysis are carried out in 2-D.

The schematic representation of COLES Mobile 930 crane is shown in Figure 3.1.

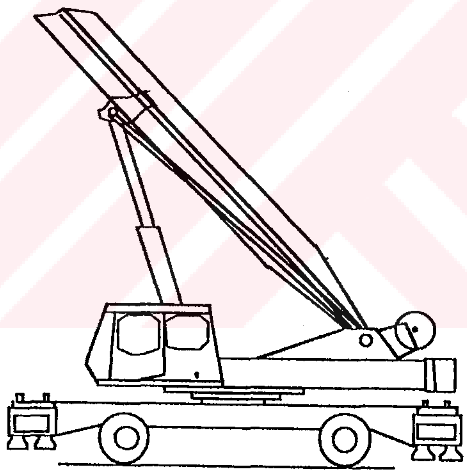

Figure 3.1 Schematic Representation of the COLES Mobile 930 Crane

The kinematic model of the crane can be represented with four bodies shown in Figure 3.2.

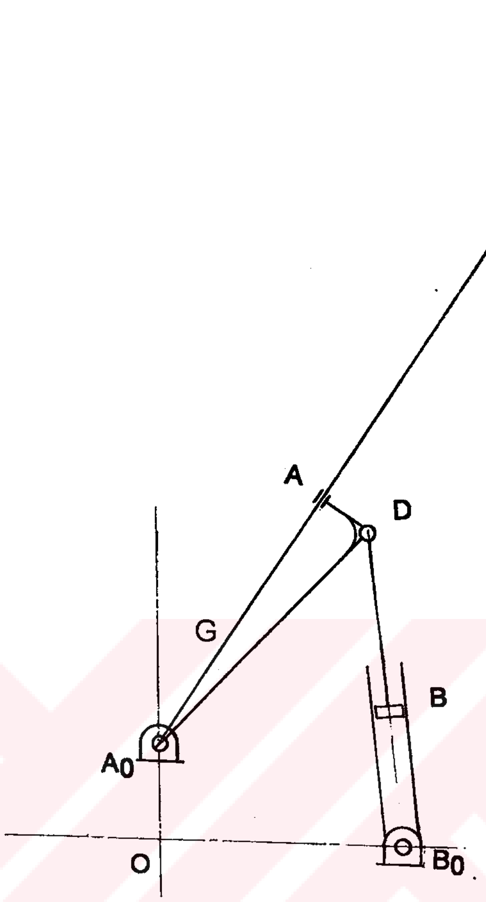

Figure 3.2. Kinematic Model of the COLES Mobile 930 Crane

3.2 DIMENSIONS OF THE CRANE

Cross section dimensions and lengths of each body are obtained from crane's technical data sheet [12] or measured directly from the test crane and given as follows

Body 1

The cross section of the body 1 is a hollow polygon of thickness, t, as shown in Figure 3.3. In the figure e_0, c_0, and f_0 denote the outer edges and e_i, c_i, and f_i denote the inner edges of the cross section, respectively.

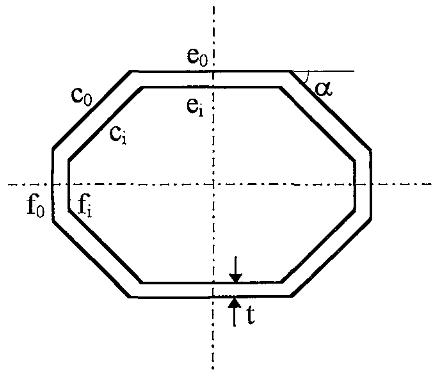

Figure 3.3 Cross Section of Body 1

The cross section dimensions, increases from A_0 to G and decreases from G to C linearly, and are given as follows at A_0, G and C.

at A_0

$e_{0_1} = 453.33$ mm $\quad c_{0_1} = 200$ mm $\quad f_{0_1} = 120$ mm

at G

$e_{0_3} = 160$ mm $\quad c_{0_3} = 440$ mm $\quad f_{0_3} = 120$ mm

at C

$e_{0_5} = 160$ mm $\quad c_{0_5} = 206.67$ mm $\quad f_{0_5} = 120$ mm

Using trigonometry, c_0, e_0, f_0, c_i, e_i, and f_i lengths of the cross section at any position can be represented as follows

Between A_0 - G

$$c_0 = c_{0_1} + \frac{(c_{0_3} - c_{01})}{A_0G}\ell$$

$$e_0 = e_{0_1} - \frac{(e_{0_1} - e_{0_3})}{A_0G}\ell$$

$$f_0 = 120 \text{ mm}$$

where length ℓ is measured from A_0

Between G - C

$$c_0 = c_{0_3} - \frac{(c_{0_3} - c_{0_3})}{GC}\ell$$

$$e_0 = 160 \text{ mm}$$

$$f_0 = 120 \text{ mm}$$

where length ℓ is measured from G

$$c_i = c_0 + t\left[\frac{(\sin\alpha - 1)}{\cos\alpha} - \frac{(\cos\alpha - 1)}{\sin\alpha}\right]$$

$$e_i = e_0 + 2t\left(\frac{\cos\alpha - 1}{\sin\alpha}\right)$$

$$f_i = f_0 + 2t\left(\frac{\sin\alpha - 1}{\sin\alpha}\right)$$

In addition,

$\overline{A_0G} = 2000$ mm $\quad \overline{GC} = 17500$ mm $\quad \overline{A_0A} = 5823$ mm and $t = 20$ mm

Body 2

Body 2 is a cylindrical rod with the following dimensions.

Diameter : 25 mm

$\overline{A_0D}$: 5850 mm

$\overline{AD}$: 565 mm

Body 3

Body 3 is a piston with the following dimensions.

$\overline{BD}$: 3455 mm

Spool thickness : 20 mm

Piston diameter : 180 mm

Body 4

Body 4 is a cylinder with the following dimensions.

Inner diameter : 230 mm

Outer diameter : 246 mm

Cylinder length : 3440 mm

Furthermore,

$\overline{OB_0} = 2350$ mm $\overline{OA_0} = 805$ mm

3.3 FLEXIBLE DYNAMIC ANALYSIS OF THE CRANE

For 3-D systems the reference coordinates of a body are three translation and three rotation coordinates which define the location and orientation of the body reference frame $\mathbf{n}^k$ relative to the inertial frame $\mathbf{n}$. The elastic generalized coordinates are defined with respect to the $\mathbf{n}^{ki}$ frame. For a 3-D beam element these are 12 nodal coordinates and represent the location of the nodes and slopes of the element axis at these nodes. In a 2-D system the reference coordinates are two translations and one rotation of the body reference frame. For a 2-D beam element 6 nodal coordinates which are elastic generalized coordinates relative to the $\mathbf{n}^{ki}$ frame denote the position of the nodes and slopes of the element axis at these nodes. Shape function matrix of 2-D beam element is given in Appendix A.

In the previous chapter, general three dimensional motion is considered. For the two dimensional motion, mass and coriolis and centrifugal force submatrices of the body k take the following form [11]

$$\mathbf{M}^k_{\xi\xi} = \begin{bmatrix} m^k & 0 \\ 0 & m^k \end{bmatrix} \tag{3.1}$$

$$\mathbf{M}^k_{\xi\theta} = \mathbf{T}^k_\theta \ \mathbf{A}^k \ \mathbf{e}^k \tag{3.2}$$

where $\mathbf{A}^k$ does not depend on time, given by

$$\mathbf{A}^k = \sum_{i=1}^{E_k} \int_{V_{ki}} \rho^{ki} \boldsymbol{\phi}^{ki} dV \mathbf{B}^{ki} \tag{3.3}$$

$$\mathbf{M}^k_{\xi\eta} = \mathbf{T}^k \ \mathbf{A}^k \ \boldsymbol{\chi}^k \tag{3.4}$$

$$\mathbf{M}^k_{\theta\theta} = \mathbf{e}^{k^T} \ \mathbf{M}^k_{\alpha\alpha} \ \mathbf{e}^k \tag{3.5}$$

$$\mathbf{M}_{\theta\eta}^{k} = \mathbf{e}^{k^T} \ \mathbf{AA}^{k} \ \chi^{k} \tag{3.6}$$

where time-invariant matrix $\mathbf{AA}^{k}$ is represented as

$$\mathbf{AA}^{k} = \sum_{i=1}^{E_k} \mathbf{B}^{ki^T} \int_{V_{ki}} \rho^{ki} \phi^{ki^T} \tilde{\mathbf{I}} \phi^{ki} dV \mathbf{B}^{ki} \tag{3.7}$$

where ($\tilde{}$) denotes skew symmetric matrix and **I** is identity matrix.

$$\mathbf{M}_{\eta\eta}^{k} = \chi^{k^T} \ \mathbf{M}_{\alpha\alpha}^{k} \ \chi^{k} \tag{3.8}$$

$$\mathbf{Q}_{\xi}^{k} = \mathbf{T}^{k} \ \mathbf{A}^{k} \ \mathbf{e}^{k} \ \dot{\theta}^{k^2} - 2 \ \mathbf{T}_{\theta}^{k} \ \mathbf{A}^{k} \ \chi^{k} \ \dot{\eta}^{k} \ \dot{\theta}^{k} \tag{3.9}$$

$$Q_{\theta}^{k} = \mathbf{e}^{k^T} \ \mathbf{AA}^{k} \ \mathbf{e}^{k} \ \dot{\theta}^{k^2} - 2 \ \mathbf{e}^{k^T} \ \mathbf{M}_{\alpha\alpha}^{k} \ \chi^{k} \ \dot{\eta}^{k} \ \dot{\theta}^{k} \tag{3.10}$$

$$\mathbf{Q}_{\eta}^{k} = \chi^{k^T} \ \mathbf{M}_{\alpha\alpha}^{k} \ \mathbf{e}^{k} \ \dot{\theta}^{k^2} + 2 \ \chi^{k^T} \ \mathbf{AA}^{k} \ \chi^{k} \ \dot{\eta}^{k} \ \dot{\theta}^{k} \tag{3.11}$$

In 2-D analysis the time independent mass properties are $\mathbf{A}^{k}$, $\mathbf{AA}^{k}$ and $\mathbf{M}_{\alpha\alpha}^{k}$.

When the body k is taken to be rigid, the terms coming from the elastic deformation drop off. Hence, generalized structural stiffness and damping matrices also drop off. In this case, mass and coriolis and centrifugal force matrices of the body k as a reduced submatrices are

$$\mathbf{M}_{\xi\xi}^{k} = \begin{bmatrix} m^{k} & 0 \\ 0 & m^{k} \end{bmatrix} \tag{3.12}$$

$$\mathbf{M}_{\xi\theta}^{k} = \mathbf{T}_{\theta}^{k}\left(\int_{V_k} \rho^k \mathbf{r}^k dV\right) \tag{3.13}$$

$$\mathbf{M}_{\theta\theta}^{k} = \int_{V_k} \rho^k \mathbf{r}^{k^T} \mathbf{r}^k dV \tag{3.14}$$

$$\mathbf{Q}_{\xi}^{k} = \mathbf{T}^{k}\left(\int_{V_k} \rho^k \mathbf{r}^k dV\right)\dot{\theta}^{k^2} \tag{3.15}$$

$$Q_{\theta}^{k} = 0 \tag{3.16}$$

When dimensions (lengths and cross sections) and elastic rigidities of the bodies of the crane are considered, it is sufficient to take only the body 1 (the boom) as flexible. In this case, other bodies are assumed to be rigid.

In the analysis of the crane, some assumptions are made. These are as follows

i) Mass of the hydraulic oil is included in the mass of the cylinder (body 4).
ii) Hydraulic oil is assumed as incompressible fluid.
iii) Varying mass of the cylinder due to varying amounts of hydraulic oil inside the cylinder is taken into consideration.
iv) Hook load is considered as a point mass and connected to the boom end with the rope which is taken as rigid rod. This rope is free for planar rotation about point C. In other words, body 5 is taken as rigid rod with point mass at the end and connected to the boom end with revolute joint.
v) Structural damping of the boom is taken into account.
vi) The distance between the load and base is assumed to be kept constant by varying the length of the rope during the up and down motion of the crane.
vii) Variation of the mass of the rope due to boom angle change is considered in the mass of the body 5.

3.3.1 CONSTRAINT EQUATIONS OF THE CRANE

In this section, firstly, joint connections of the crane is determined, then geometrical and prescribed motion constraints of the crane is written.

Reference frames of all the bodies and general reference frame (inertial frame) are shown on crane kinematic model in Figure 3.4.

Body 1 is connected to the base and to body 5 using revolute joints and to body 2 with slider joint. Body 2 is connected to the base and to body 3 using revolute joints. The connections of the body 4 with body 3 and base are slider and revolute joints respectively. This, 5 revolute joints and 2 slider joints exist in the crane.

Two scalar equations are written from the velocity components equality of the bodies at the connection points for each revolute joint. One equation comes from angular velocity equality and one from velocity components equality for each slider joint. Hence, total number of constraint equations are 14 for the crane. These are as follows.

a) Revolute joints at point A_0 and point B_0

These revolute joints are fixed to the base. Consequently, the velocities of A_0 and B_0 are equal to zero as given in the following form

$$\mathbf{v}^{A_0} = 0 \qquad \text{constraint equations (1), (2), (3), (4)} \qquad (3.17)$$

$$\mathbf{v}^{B_0} = 0 \qquad \text{constraint equations (5), (6)} \qquad (3.18)$$

Since between these points and inertial frame there are only reference coordinates, above equations reduces to the following form

$$\dot{\xi}_1^{(1)} = 0 \tag{3.19}$$

$$\dot{\xi}_2^{(1)} = 0 \tag{3.20}$$

$$\dot{\xi}_1^{(2)} = 0 \tag{3.21}$$

$$\dot{\xi}_2^{(2)} = 0 \tag{3.22}$$

$$\dot{\xi}_1^{(4)} = 0 \tag{3.23}$$

$$\dot{\xi}_2^{(4)} = 0 \tag{3.24}$$

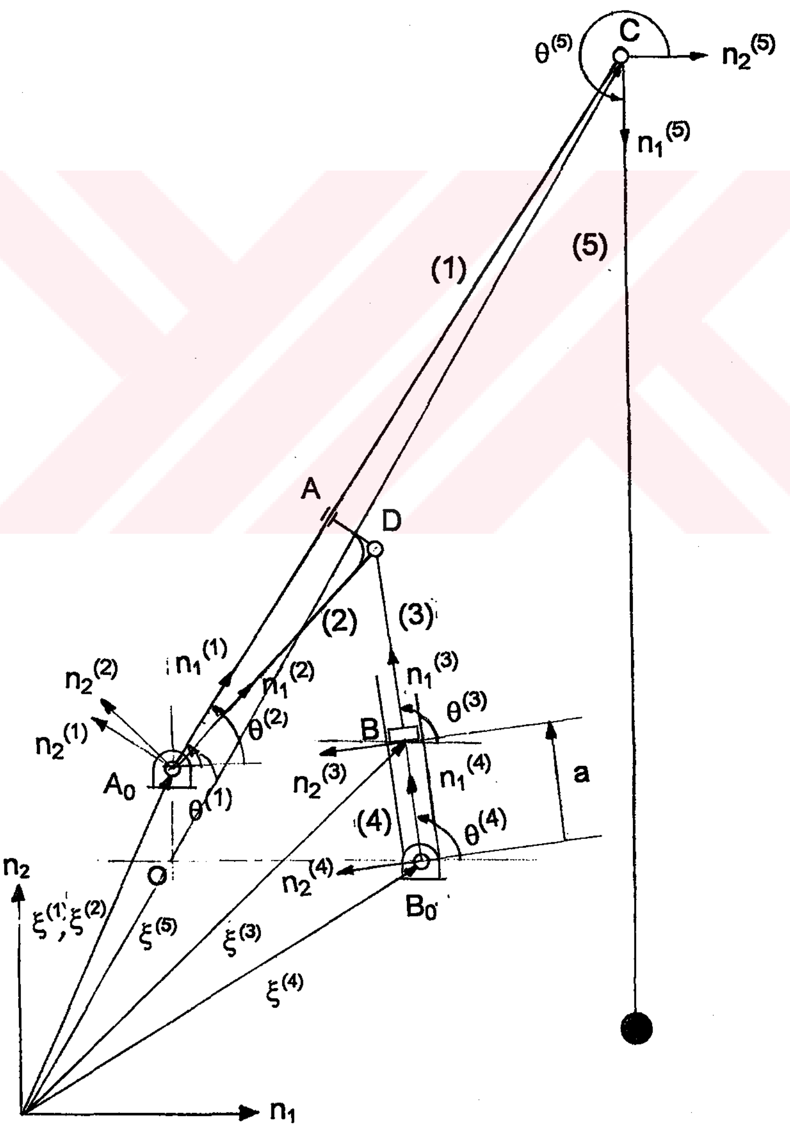

Figure 3.4 General and Body Reference Frames on Crane Kinematic Model

b) Slider joint at point A

In the normal direction with respect to the slider, the velocity of body 1 at point A equals the velocity of body 2 at point A, given by

$$v_n^{A_1} = v_n^{A_2} \qquad \text{constraint equation (7)} \qquad (3.25)$$

This velocity components can be written in terms of general reference axis frame velocity components as

$$v_n^{A_1} = -\sin\left(\theta^{(2)} + \gamma\right) v_1^{A_1} + \cos\left(\theta^{(2)} + \gamma\right) v_2^{A_1} \qquad (3.26)$$

$$v_n^{A_2} = -\sin\left(\theta^{(2)} + \gamma\right) v_1^{A_2} + \cos\left(\theta^{(2)} + \gamma\right) v_2^{A_2} \qquad (3.27)$$

where $v_1^{A_1}$, $v_2^{A_1}$ and $v_1^{A_2}$, $v_2^{A_2}$ denote the velocity components of body 1 and body 2 at point A respectively. γ represents the angle between body 1 and body 2 at undeformed state. $\mathbf{v}^{A_1}$ and $\mathbf{v}^{A_2}$ are given as

$$\mathbf{v}^{A_1} = \dot{\xi}^{(1)} + \mathbf{T}_\theta^{(1)} \, \phi^{1i} \, \mathbf{B}^{1i} \, \mathbf{e}^{(1)} \, \dot{\theta}^{(1)} + \mathbf{T}^{(1)} \, \phi^{1i} \, \mathbf{B}^{1i} \, \chi^{(1)} \, \dot{\eta}^{(1)} \qquad (3.28)$$

$$\mathbf{v}^{A_2} = \dot{\xi}^{(2)} + \mathbf{T}_\theta^{(2)} \, \mathbf{r}^{(2)} \, \dot{\theta}^{(2)} \qquad (3.29)$$

where superscript 1i represents the i th finite element of body 1.

Angular velocity of body 1 at point A is the same as the angular velocity of body 2. That is

$$\omega^{A_1} = \omega^{A_2} \qquad \text{constraint equation (8)} \qquad (3.30)$$

Above constraint equation becomes

$$\dot{\theta}^{(1)} + \psi^{(1i)} \mathbf{B}^{(1i)} \chi^{(1)} \dot{\eta}^{(1)} \tag{3.31}$$

c) Revolute joint at point D

The velocities of body 2 and body 3 at point D are equal to each other, represented by

$$\mathbf{v}^{D_2} = \mathbf{v}^{D_3} \qquad \text{constraint equation (9), (10)} \tag{3.32}$$

These are given as

$$\mathbf{v}^{D_2} = \dot{\xi}^{(2)} + \mathbf{T}_{\theta}^{(2)} \mathbf{r}^{D_2} \dot{\theta}^{(2)} \tag{3.33}$$

$$\mathbf{v}^{D_3} = \dot{\xi}^{(3)} + \mathbf{T}_{\theta}^{(3)} \mathbf{r}^{D_3} \dot{\theta}^{(3)} \tag{3.34}$$

d) Slider joint at point B

In the normal direction relative to the slider, the velocities of the body 3 and body 4 at point B are the same which is represented as

$$v_n^{B_3} = v_n^{B_4} \qquad \text{constraint equation (11)} \tag{3.35}$$

these components can be written as

$$v_n^{B_3} = -\sin\theta^{(3)} v_1^{B_3} + \cos\theta^{(3)} v_2^{B_3} \tag{3.36}$$

$$v_n^{B_4} = -\sin\theta^{(4)} v_1^{B_4} + \cos\theta^{(4)} v_2^{B_4} \tag{3.37}$$

where $v_1^{B_3}$, $v_2^{B_3}$ and $v_1^{B_4}$, $v_2^{B_4}$ are the velocity components of body 3 and body 4 at point B respectively. $\mathbf{v}^{B_3}$ and $\mathbf{v}^{B_4}$ are taken in the following form

$$\mathbf{v}^{B_3} = \dot{\xi}^{(3)} \tag{3.38}$$

$$\mathbf{v}^{B_4} = \dot{\xi}^{(4)} + \mathbf{T}_\theta^{(4)}\ \mathbf{r}^{(4)}\ \dot{\theta}^{(4)} + \dot{\mathbf{r}}^{(4)} \tag{3.39}$$

where $\mathbf{r}^{(4)}$ is given as

$$\mathbf{r}^{(4)} = \begin{bmatrix} a \\ 0 \end{bmatrix} \tag{3.40}$$

where a is the position of point B with respect to the point B_0, represented by

$$a = \frac{(A_0D)\ \sin\theta^{(2)} - (BD)\ \sin\theta^{(3)} + OA_0}{\sin\theta^{(4)}} \tag{3.41}$$

Angular velocities of the body 3 and body 4 are equal to each other which is given as

$$\omega^{(3)} = \omega^{(4)} \qquad \text{constraint equation (12)} \tag{3.42}$$

This constraint equation can be written as

$$\dot{\theta}^{(3)} = \dot{\theta}^{(4)} \tag{3.43}$$

e) Revolute joint at point C

The velocity of body 1 at point A is the same as the velocity of body 5 at point A which is given as

$$\mathbf{v}^{C_1} = \mathbf{v}^{C_5} \qquad \text{constraint equation (13), (14)} \qquad (3.44)$$

These velocities are represented as follows

$$\mathbf{v}^{C_1} = \dot{\xi}^{(1)} + \mathbf{T}_{\theta}^{(1)} \, \phi^{1i} \, \mathbf{B}^{1i} \, \mathbf{e}^{(1)} \, \dot{\theta}^{(1)} + \mathbf{T}^{(1)} \, \phi^{1i} \, \mathbf{B}^{1i} \, \chi^{(1)} \, \dot{\eta}^{(1)} \qquad (3.45)$$

$$\mathbf{v}^{C_5} = \dot{\xi}^{(5)} \qquad (3.46)$$

If body 1 and body 2 are fixed to the different points, the system composed of these bodies is a structure. But, if these bodies are fixed to the same point, then the special case of this structure is obtained and the system can move. Thus, the constraint equations (1), (2), (3), (4), (7) and (8) written for body 1 and body 2 are linearly dependent. One of the constraint equations dropped to remove the linear dependency. Since the body 1 has pin connection at A_0, fixed at A and free at C, all of which are the boundary conditions of the body 1, constraint equations (7) and (8) are automatically imposed to the system. Hence, it is the most appropriate to drop one of the constraint equations (7) and (8).

f) Prescribed motion

Boom is driven by the hydraulic actuator which is controlled by the operator. Throughout the motion, hydraulic actuator is driven with constant velocity v_0 so that the boom and piston force oscillation is to be in minimum level. Moreover, to avoid the impact loading, actuator velocity is increased from zero to v_0 at the beginning of the motion and decreased from v_0 to zero at the end of the motion cycloidally in appropriate time. This desired motion velocity profile and its expression can be represented as in Figure 3.5 and in equation (3.47) respectively.

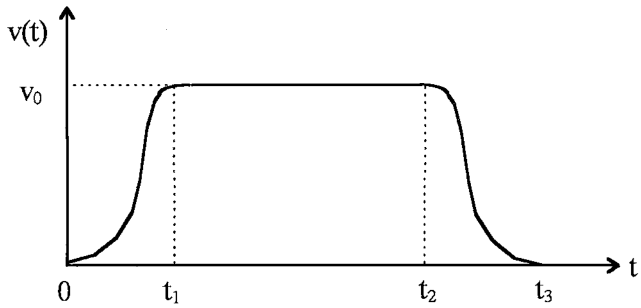

Figure 3.5 Velocity Profile with Cycloidal Acceleration and Deceleration

$$v(t)=\begin{cases} v_0\dfrac{1}{t_1}\left[t-\dfrac{t_1}{2\pi}\sin\dfrac{2\pi}{t_1}t\right] & \text{for } 0\le t\le t_1 \\ v_0 & \text{for } t_1<t<t_2 \\ v_0-\dfrac{v_0}{(t_3-t_2)}\left[(t-t_2)-\dfrac{(t_3-t_2)}{2\pi}\sin\dfrac{2\pi(t-t_2)}{(t_3-t_2)}\right] & \text{for } t_2\le t\le t_3 \end{cases} \quad (3.47)$$

It is obvious that obtaining the cycloidal acceleration and deceleration for the initial and final parts of the motion is desirable from the point of smooth motion of the load. However, this is not so applicable since it is dependent on the operator's skill and experience. So linear acceleration and deceleration profile that is more realistic than the cycloidal one is also considered during the analysis for the related parts of the motion. Graphical representation of this velocity profile and its expression can be seen in Figure 3.6 and equation (3.48) respectively.

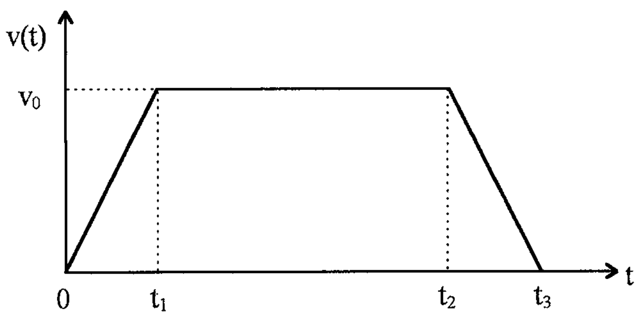

Figure 3.6 Velocity Profile with Linear Acceleration and Deceleration

$$v(t) = \begin{cases} \dfrac{v_0}{t_1} t & \text{for} \quad 0 \le t \le t_1 \\ v_0 & \text{for} \quad t_1 < t < t_2 \\ v_0 - v_0 \dfrac{(t - t_2)}{(t_3 - t_2)} & \text{for} \quad t_2 \le t \le t_3 \end{cases} \tag{3.48}$$

The prescribed motion represents the piston velocity which is the same as the velocity of the body 3 along its direction. This constraint can be represented as

$$v_t^{B_3} = v(t) \qquad \text{constraint equation (15)} \tag{3.49}$$

where $v_t^{B_3}$ denotes tangential velocity component of the body 3 with respect to the cylinder which is in the following form

$$v_t^{B_3} = v_1^{B_3} \cos\theta^{(3)} + v_2^{B_3} \sin\theta^{(3)} \tag{3.50}$$

Above constraint equations are put in equation (2.59) form which is at velocity level. Then differentiation of this form yields the constraint equations at acceleration level which is used in the dynamical equations of the crane.

CHAPTER 4

COMPUTER SIMULATION OF THE CRANE CHARACTERISTICS AND COMPARISON WITH THE EXPERIMENTAL RESULTS

In this chapter, firstly, the computer code used for numerical simulation and experimental method are briefly explained. Then variations of piston force with respect to the boom angular positions for the unloaded motion of the boom and for 32.4 kN hook load are simulated for the 30 seconds motion of the boom using the computer code for two different piston velocity profiles given in the previous chapter and compared with the experimental results.

In the experimental study, the boom was moved from the horizontal to the vertical position and vice versa in 30 seconds. This speed was selected in order to minimize the effects of flexibility.

Similarly variations of piston force with respect to boom angular positions for 32.4 kN hook load are simulated for 10 seconds motion of the boom by using the computer code in order to make the effects of flexibility more significant. This simulation is also performed for two different piston velocity profiles. Moreover, transverse deflections of node 3, which is between points A_0 and A of the boom, node 8, which is between points A and C of the boom and node 13, which is the tip point (point C) of the boom are obtained with respect to the boom angular positions for 32.4 kN hook load. Finally, load curves are generated for the 30 seconds motion and compared to those of manufacturer.

4.1 COMPUTER CODE USED FOR NUMERICAL SIMULATION

A computer program for the analysis of the COLES Mobile 930 crane is developed. As mentioned before, body 1 is taken flexible and other bodies are assumed to be rigid. In the program one can take any number of finite elements and modal variables for the body 1. The program consists of MAIN program and TIND, JACOBI, ELIM subroutines.

In the MAIN program, firstly, finite elements cross section dimensions and lengths of the body 1 are generated according to the given finite element numbers. Then structural mass, stiffness and damping matrices of body 1 are calculated. After applying the boundary conditions to the structural mass and stiffness matrices, eigenvalues and eigenvectors of body 1 are obtained by calling the subroutine JACOBI and modal matrix is constructed. In addition to the time invariant structural mass, stiffness and damping matrices, remaining time invariant matrices which are presented in the previous sections are calculated to decrease the computational time significantly. After that initial generalized position and velocity vectors are specified. After this stage, calling the subroutine TIND generalized mass matrix, coriolis and centrifugal force matrix, generalized external force matrix of each body and generalized structural stiffness force matrix, and generalized structural damping force matrix of body 1 are calculated using the generalized position and velocity vectors. Then constraint coefficient matrices are calculated and constrained system differential equations of motion constructed and solved using the subroutine ELIM for the generalized acceleration vectors and Lagrange multipliers. Finally, generalized acceleration vectors are integrated forward in time to find the updated generalized position and velocity vectors.

The MAIN program calls the above mentioned codes at each time step for the generation of the equations of motion and for integration forward in time so that updated position, velocity, acceleration vectors and Lagrange multipliers are calculated. Simulation of the 30 seconds motion of the boom takes approximately

24 hours in an IBM compatible PC having a 486 DX2-66 MHz type microprocessor and 8 MB RAM.

The source listing of the program is given in Appendix B.

4.2 EXPERIMENTAL METHOD

The pressures in the hydraulic actuator can be measured using pressure transducers and the angular position of the boom can be measured using a pendulum-type potentiometer.

The model for the hydraulic actuator of the test crane which is a double acting hydraulic ram is shown in Figure 4.1. Therefore, the piston force F can be related to the pressures p_1 in the piston side and p_2 in the shaft side as follows

$$F = p_1A - p_2(A - A_s) \tag{4.1}$$

where A and A_s represent the cross sectional areas of the piston and the shaft, respectively. Thus, piston force F can easily be obtained from equation (4.1).

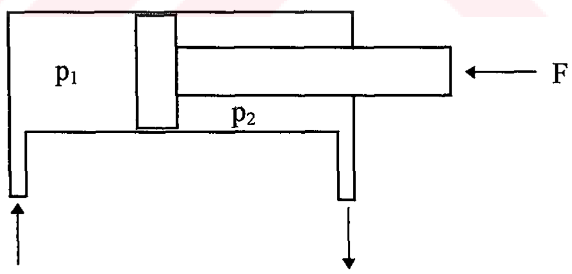

Figure 4.1 Model for the Hydraulic Actuator

A study using the above mentioned experimental method has been carried out by Balkan [4] for the 30 seconds motion of the boom. In that study, the pressures in the hydraulic actuator were measured by using two 20 MPa capacity pressure transducers with built-in amplifiers, and the angular positions of the boom from the horizontal plane was measured by using a pendulum-type potentiometer with oil based internal damping. Thus, a voltage that is proportional to boom angular position is generated. The oscillations in the pressures resulting from the boom oscillatins are filtered out in the control system, hence are not seen in the measured data. The test crane is moved without hook load and with 32.4 kN hook load in the upward and downward directions, and the variations of the pressures p_1 and p_2 with respect to the boom angular positions are given in Figure 4.2 and Figure 4.3 [4].

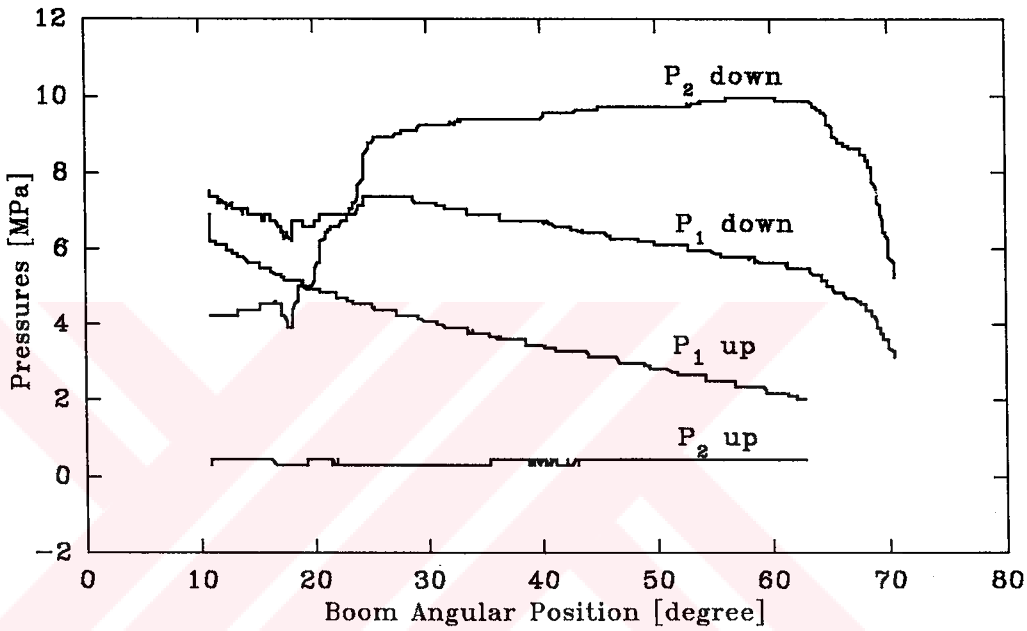

Figure 4.2 Variation of Measured Pressures for the test Crane without Hook Load

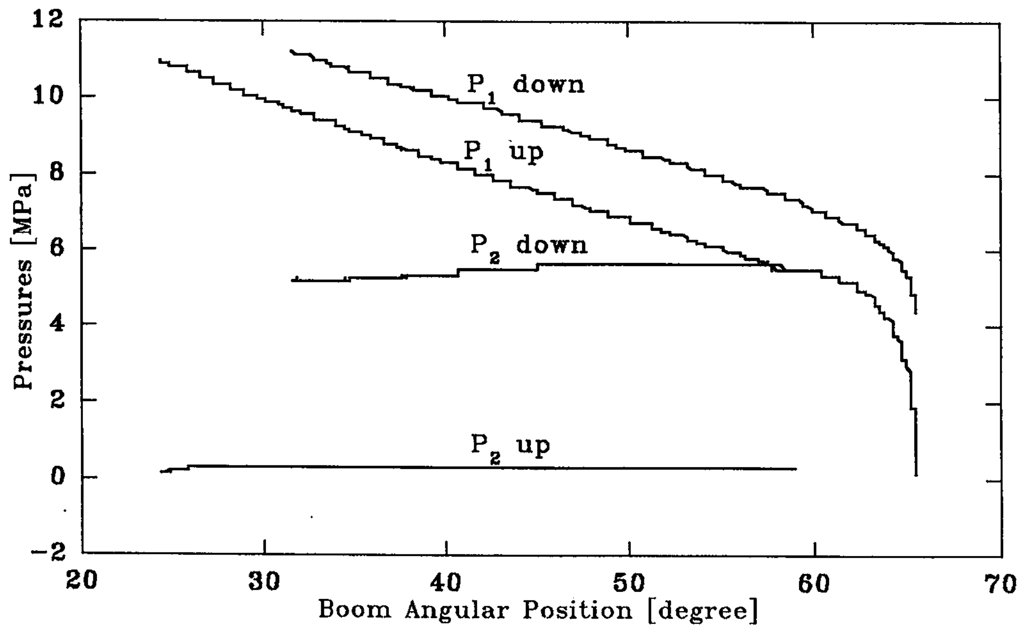

Figure 4.3 Variation of Measured Pressures with respect to Boom Angular Position for 32.4 kN Hook Load

4.3 CASE STUDIES

In all of the computer simulations, boom is divided into 12 finite elements. Two of them are taken in the $\overline{A_0G}$ length where cross sectional area is increasing from A_0 to G linearly and ten of them are taken in the $\overline{GC}$ length where cross sectional area is decreasing from G to C linearly. First longitidunal deformation mode, first bending mode of the $\overline{A_0A}$ part and first two bending modes of the $\overline{AC}$ part of body 1 are taken as the modal matrix. In addition to these, damping is included for body 1 by using 2 % damping ratio for the first two modes, i.e. $\alpha = \beta = 0.02$. It is assumed that first 1.5 seconds is used for the acceleration and last 1.5 seconds is used for the deceleration of the boom for the 30 seconds boom motion. In the case of 10 seconds boom motion acceleration and deceleration times are assumed to be 1 second.

4.3.1 SIMULATION OF PISTON FORCE AND LOCAL DEFLECTIONS

Variations of piston force with respect to the boom angular positions for the unloaded motion of the boom are shown in Figures 4.4 and 4.5.

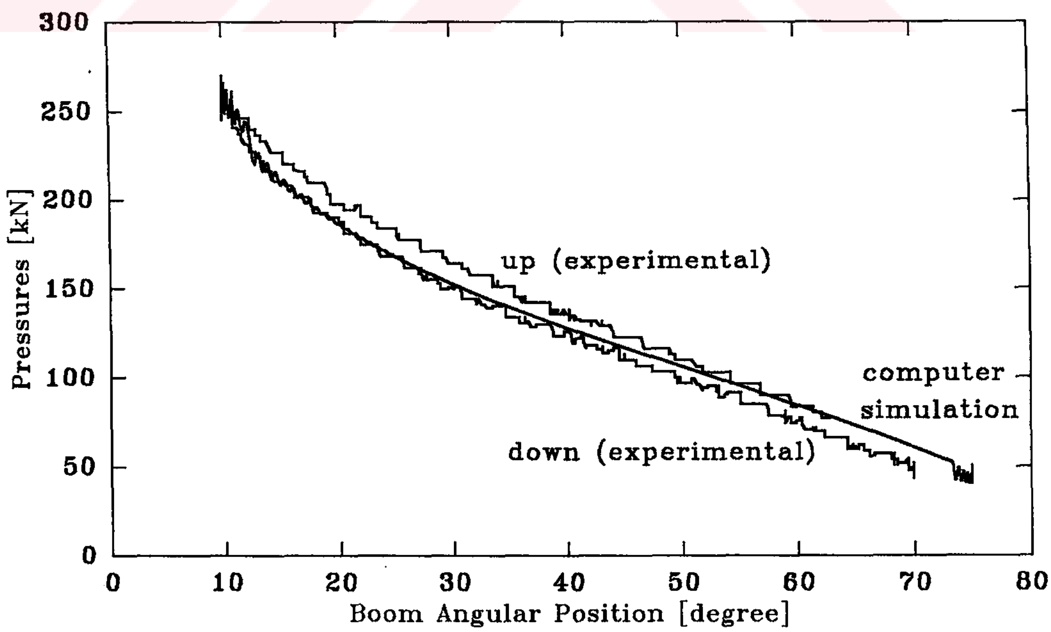

Figure 4.4 Variation of Piston Force with respect to Boom Angular Position for the Unloaded 30 Seconds Motion of the Boom and for the Linear Acceleration and Deceleration Case

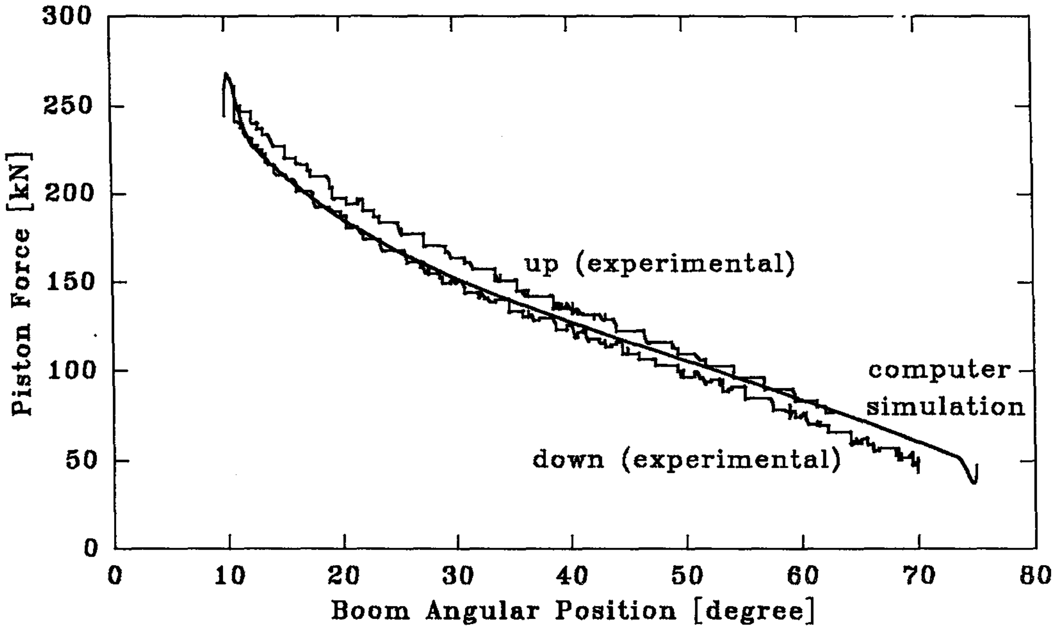

Figure 4.5 Variation of Piston Force with respect to Boom Angular Position for the Unloaded 30 Seconds Motion of the Boom and for the Cycloidal Acceleration and Deceleration Case

When Figure 4.4 is considered, it is seen that piston force oscillation amplitude is bigger in the linear acceleration part of the piston. Since slope of this velocity profile is not defined at $t = t_1$,which is the end of the acceleration period, there is a discontinuity in the piston acceleration where it suddenly drops to zero level. For this reason, a decrease in piston force is seen at $t = t_1$. An impact loading occurs just after the time $t = t_1$. Due to structural damping of the boom, oscillation of the piston force decreases and disappears in time. As the slope of this velocity profile is not defined yet at $t = t_2$,which is beginning of the deceleration period, a decrease is seen in the piston force at $t = t_2$. Similar to situation at $t = t_1$, just after the time $t = t_2$ an impact loading occurs and oscillations in the amplitude of piston force are seen.

In Figure 4.5, since slope of the velocity profile including cycloidal acceleration and deceleration parts is defined at every point, i.e. continuous acceleration of the piston, impact loading doesn't occur. Due to cycloidal acceleration and deceleration of the piston, the variation of the piston force at these regions are very smooth. Moreover, it is seen that there is no oscillation in the piston force coming from the boom's own weight for the whole simulation time.

Experimental results for the unloaded 30 seconds motion of the boom is shown both in Figure 4.4 and Figure 4.5. Since these graphs do not include the data related with the piston acceleration and deceleration parts, impact loadings are not seen. A hysteresis is seen in the motion of the boom in the upward and downward directions.

Computer simulations of piston force and experimental results are given in Figures 4.6, 4.8, 4.10 and 4.12 for 32.4 kN hook load and 30 seconds boom motion. Moreover, transverse deflections of nodes 3, 8 and 13 are given in Figures 4.7, 4.9, 4.11 and 4.13. Since the magnitude of transverse deflections of node 3 is in the order of 10^{-5} , deflection of this node is not seen in the figures.

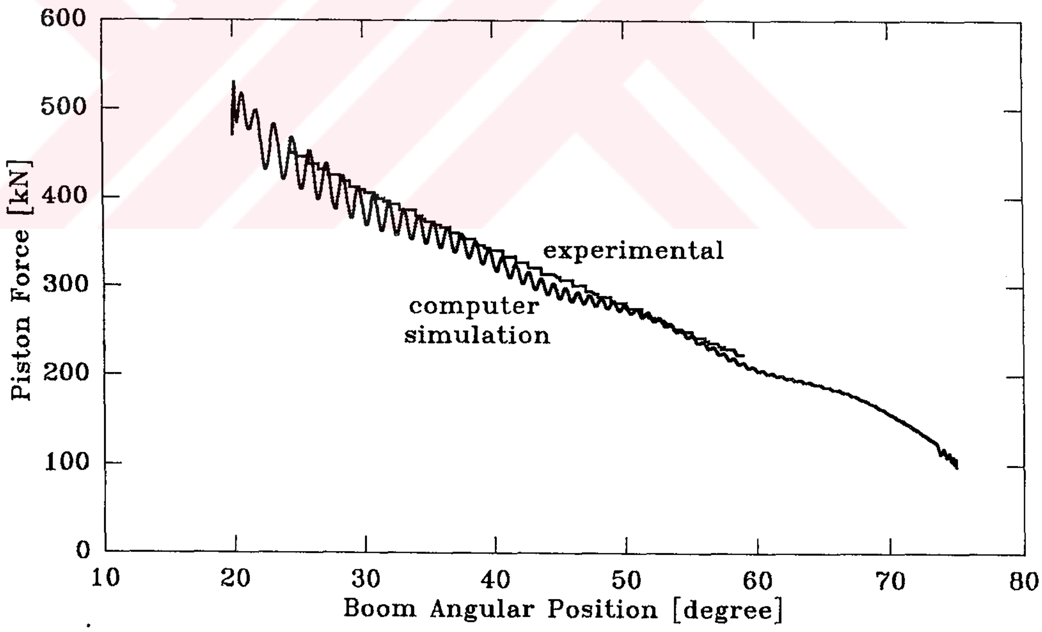

Figure 4.6 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 30 Seconds Boom Upward Direction Motion

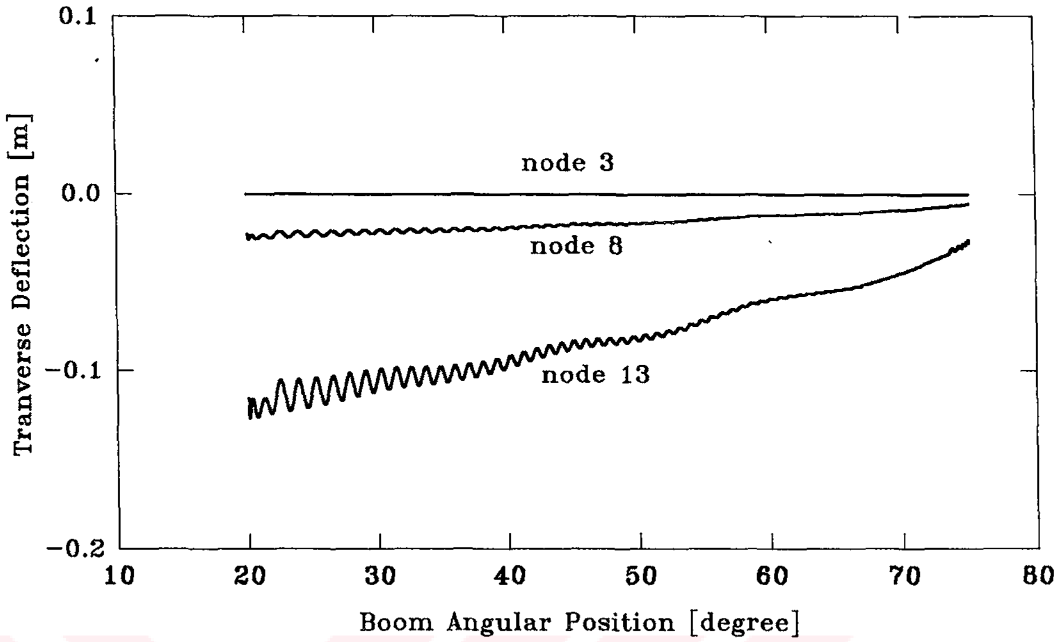

Figure 4.7 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 30 Seconds Boom Upward Direction Motion

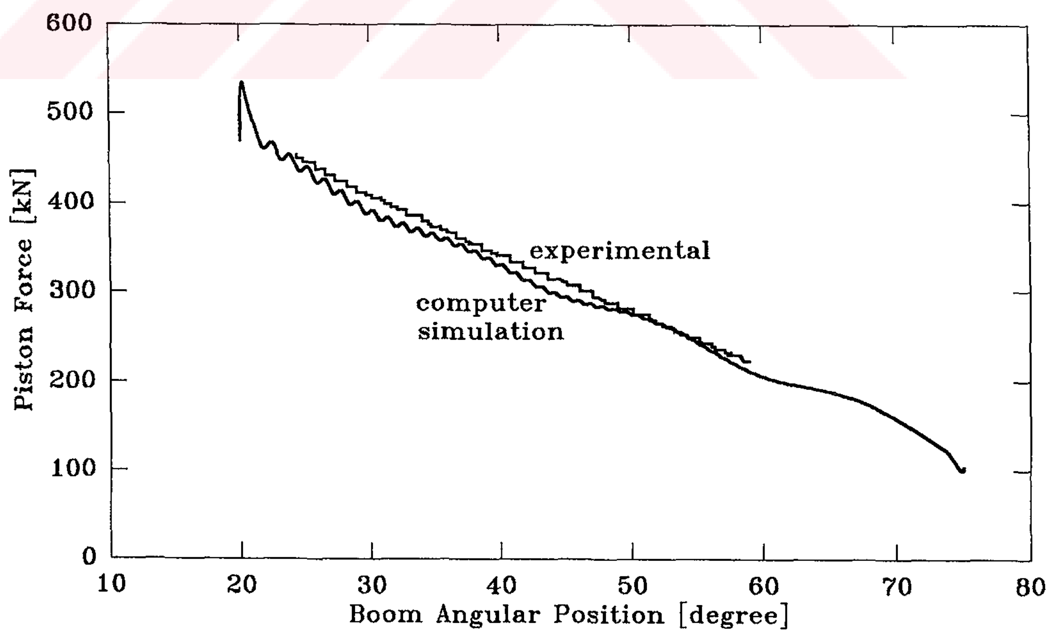

Figure 4.8 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 30 Seconds Boom Upward Direction Motion

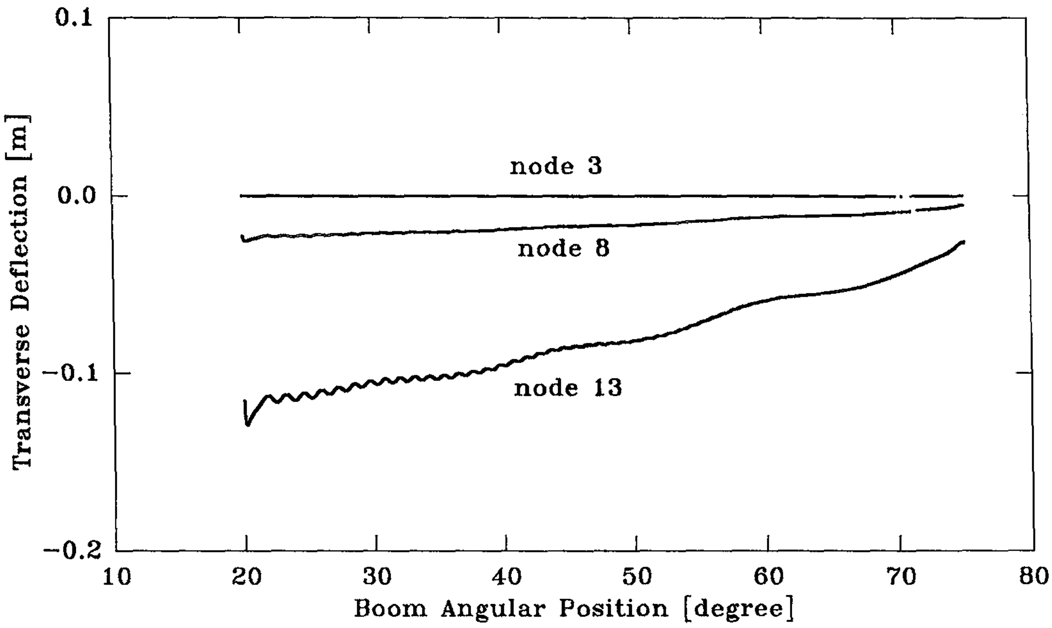

Figure 4.9 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 30 Seconds Boom Upward Direction Motion

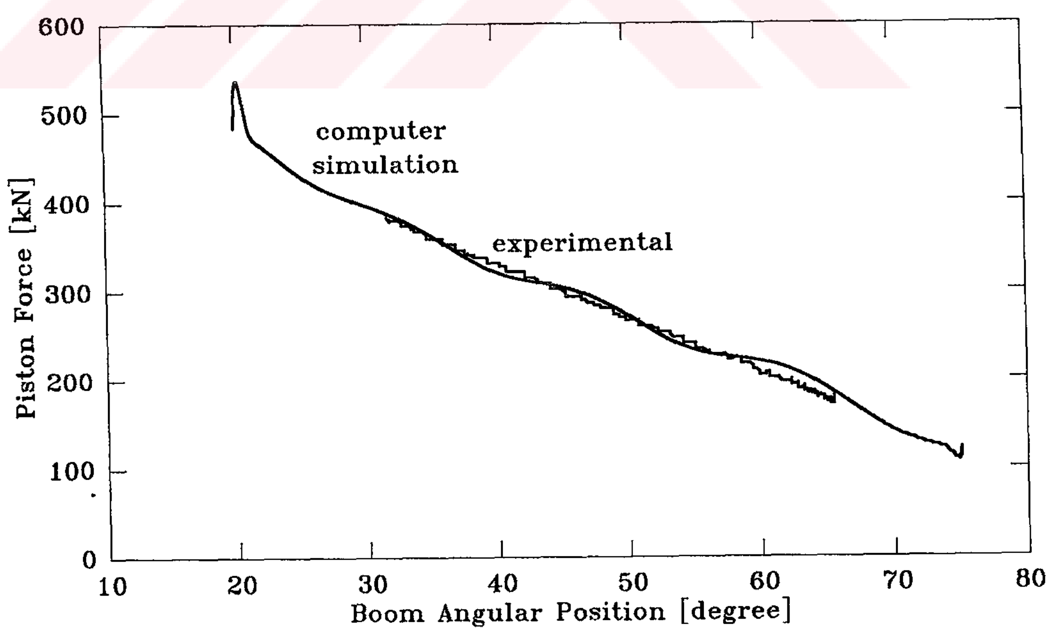

Figure 4.10 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 30 Seconds Boom Downward Direction Motion

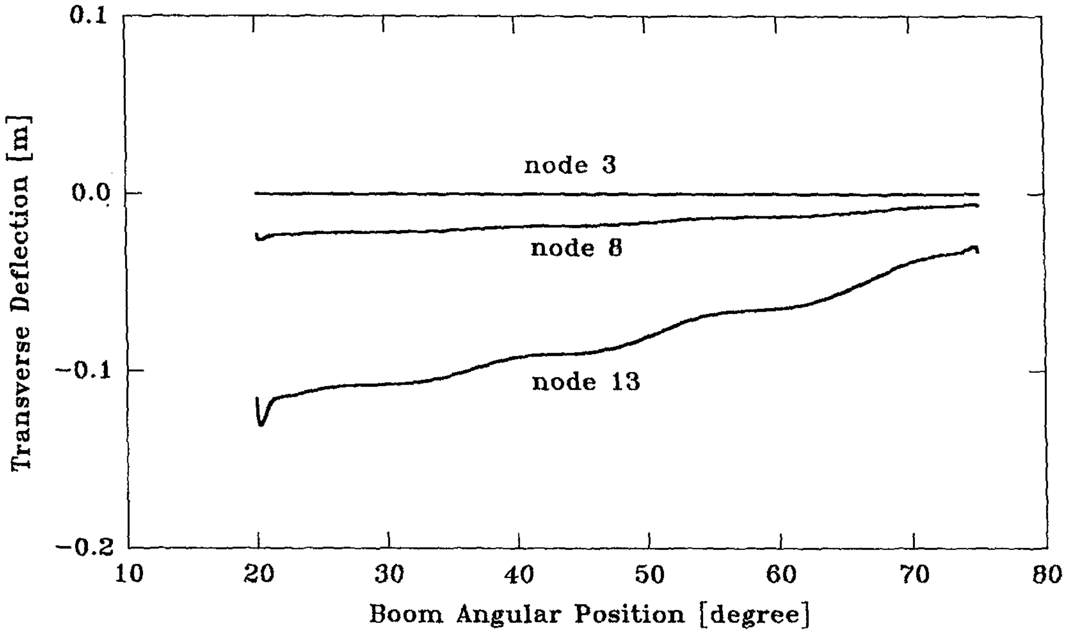

Figure 4.11 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 30 Seconds Boom Downward Direction Motion

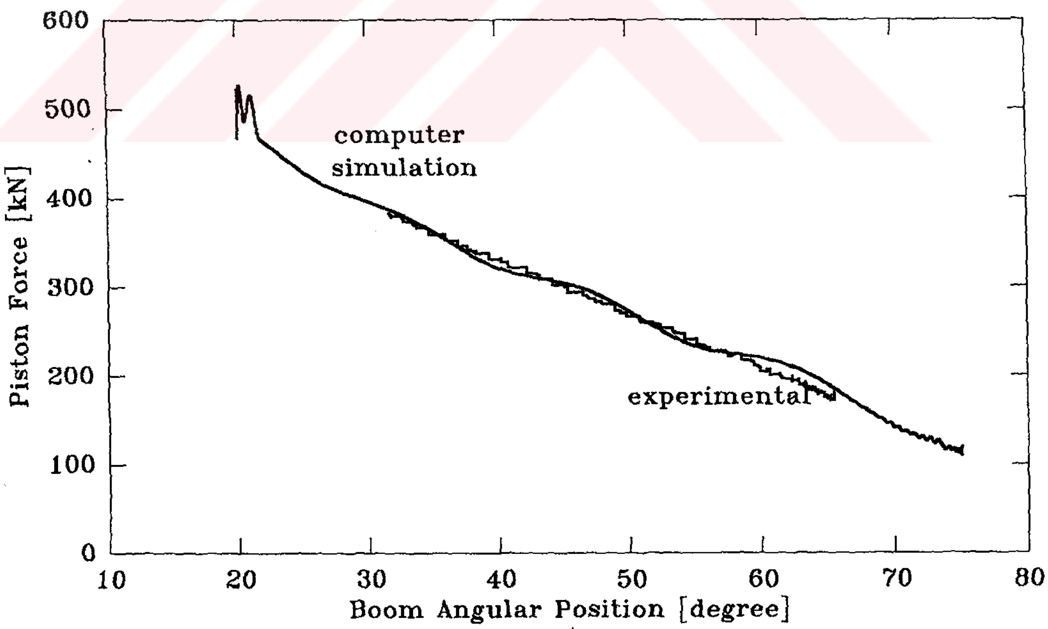

Figure 4.12 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 30 Seconds Boom Downward Direction Motion

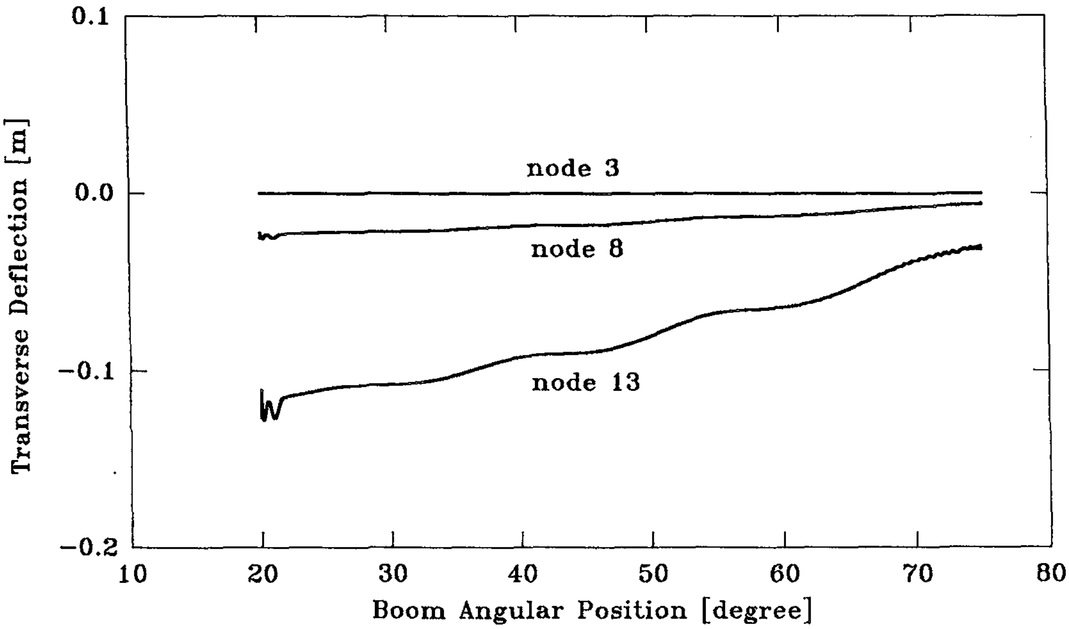

Figure 4.13 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 30 Seconds Boom Downward Direction Motion

From the above figures it is seen that although oscillation amplitudes of piston force and transverse deflections of nodes are bigger in the upward than the downward motion of the boom, oscillation amplitudes of both piston force and transverse deflections of nodes are small. Moreover, computer simulations and experimental results are closer to each other.

Computer simulation of piston force with respect to boom angular position for 10 seconds motions of the boom and for both piston velocity profiles are given in Figures 4.14, 4.16, 4.18 and 4.20 for 32.4 kN hook load and transverse deflections of nodes 3, 8 and 13 with respect to boom angular position are given in Figures 4.15, 4.17, 4.19 and 4.21.

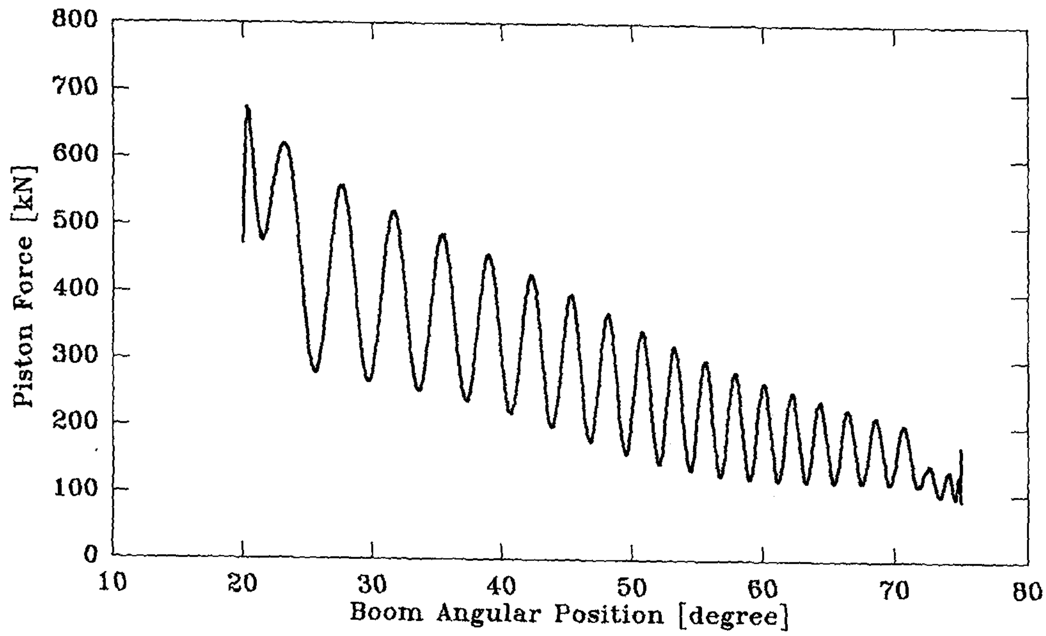

Figure 4.14 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 10 Seconds Boom Upward Direction Motion

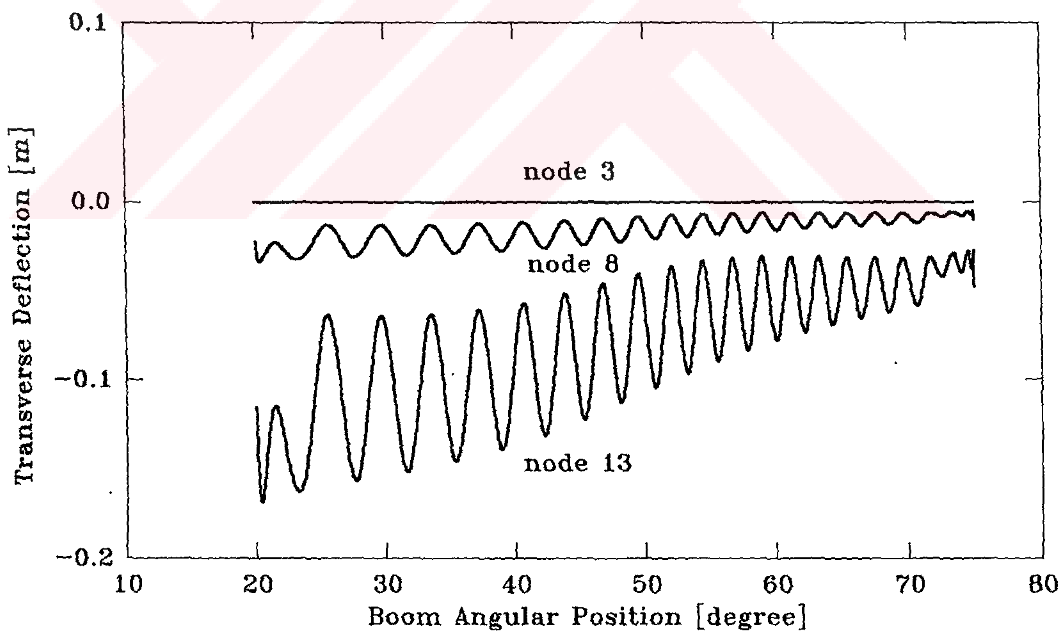

Figure 4.15 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 10 Seconds Boom Upward Direction Motion

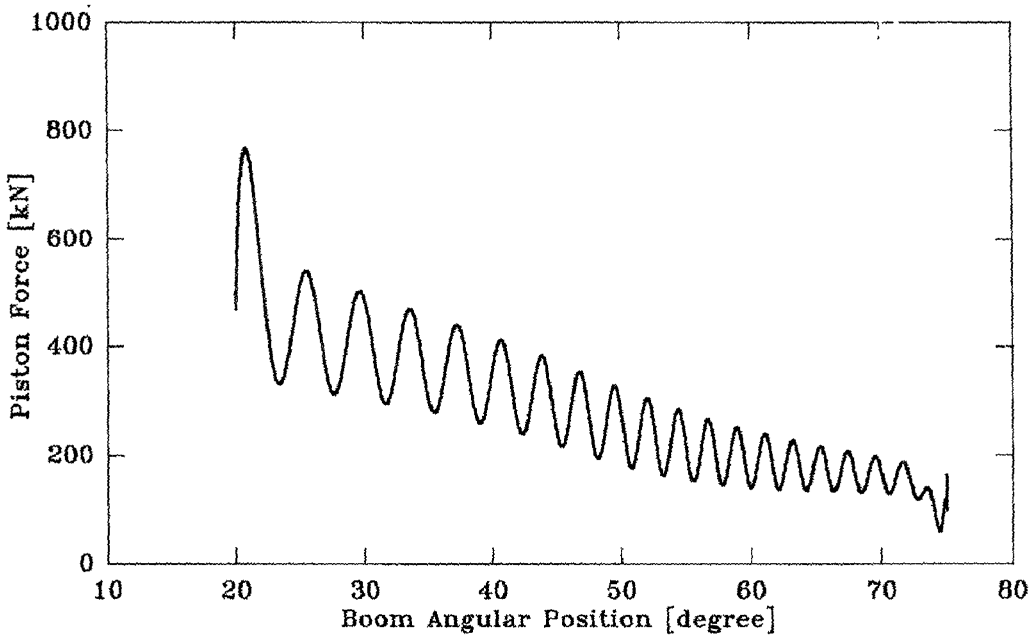

Figure 4.16 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 10 Seconds Boom Upward Direction Motion

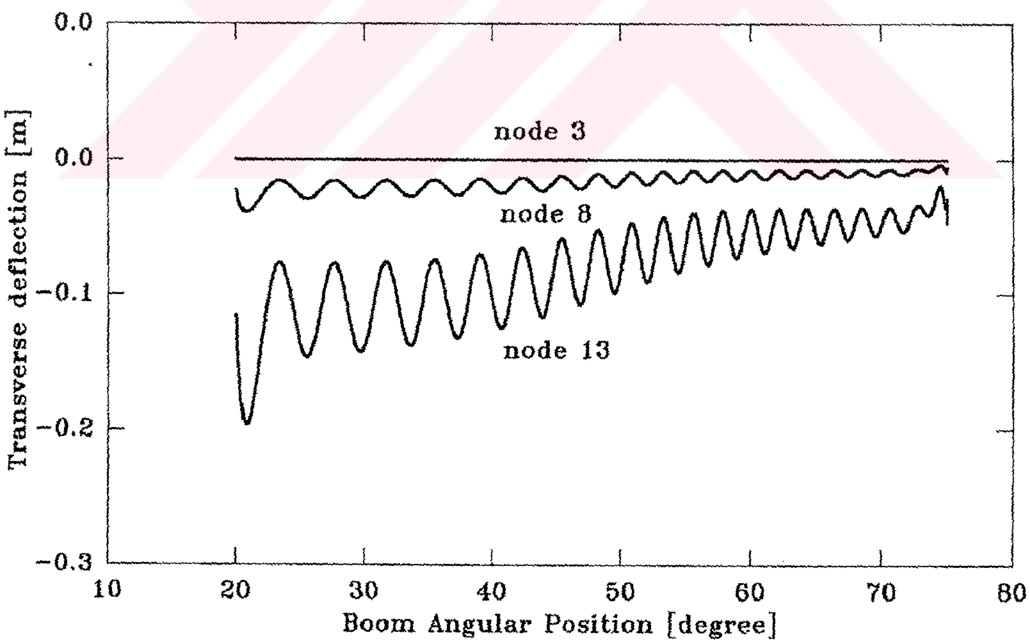

Figure 4.17 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 10 Seconds Boom Upward Direction Motion

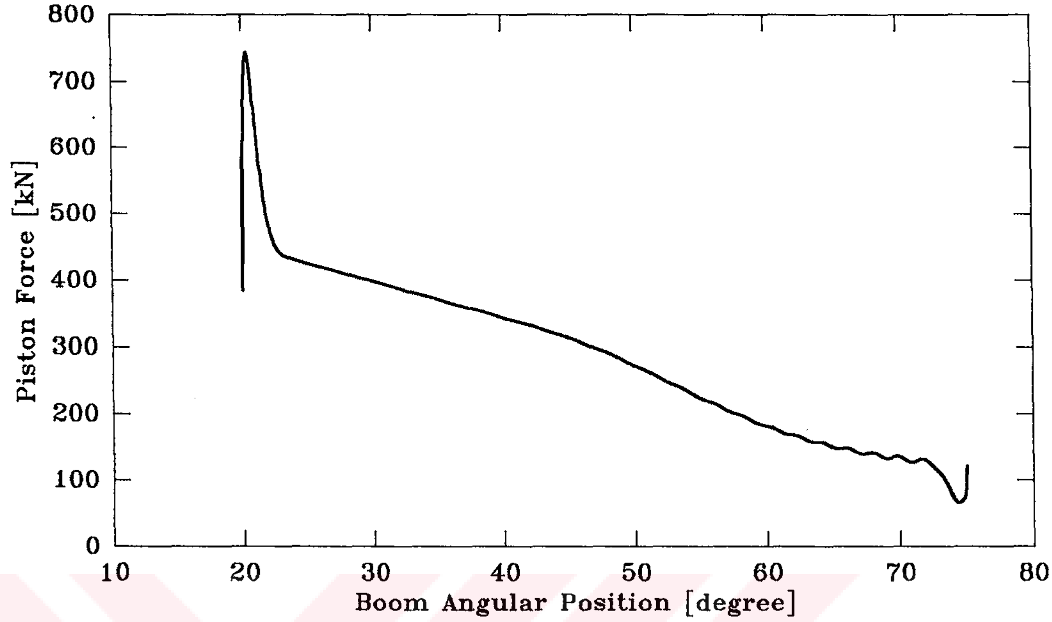

Figure 4.18 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 10 Seconds Boom Downward Direction Motion

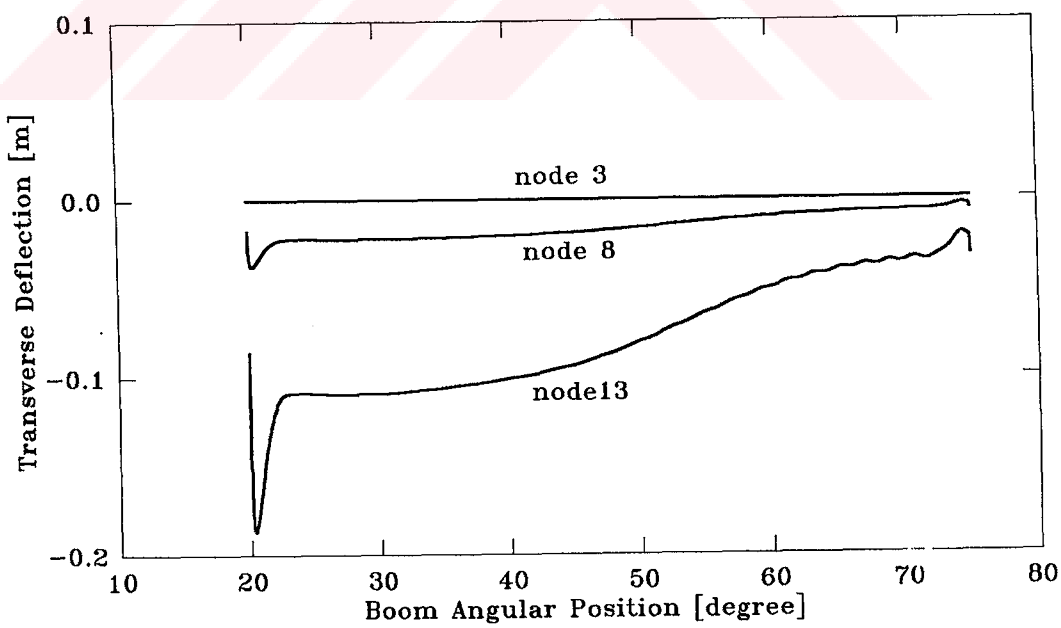

Figure 4.19 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Cycloidal Acceleration and Deceleration Case and for 10 Seconds Boom Downward Direction Motion

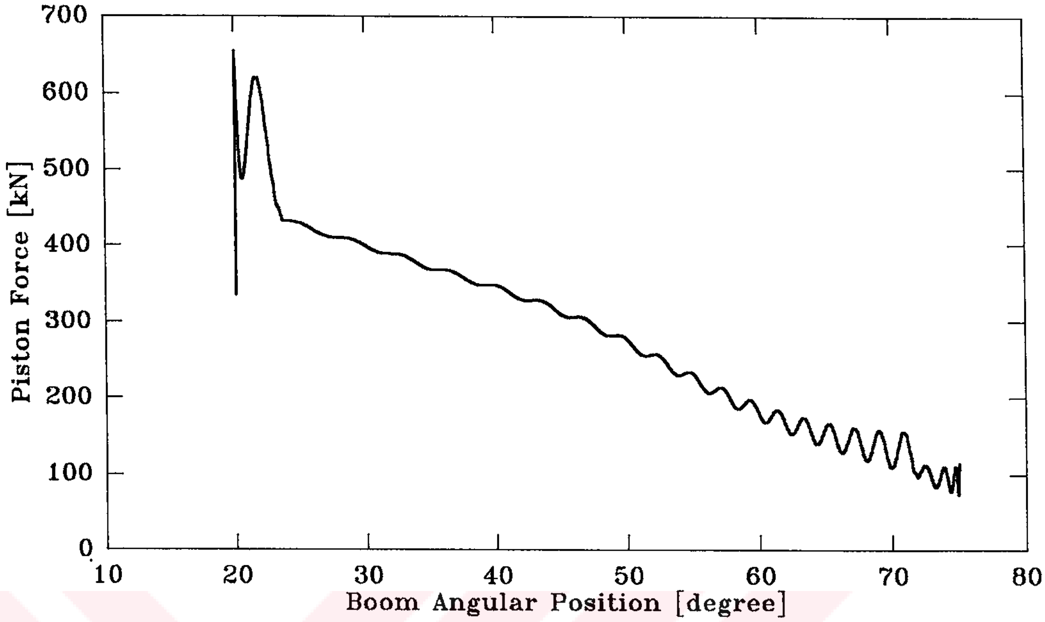

Figure 4.20 Variation of Piston Force with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 10 Seconds Boom Downward Direction Motion

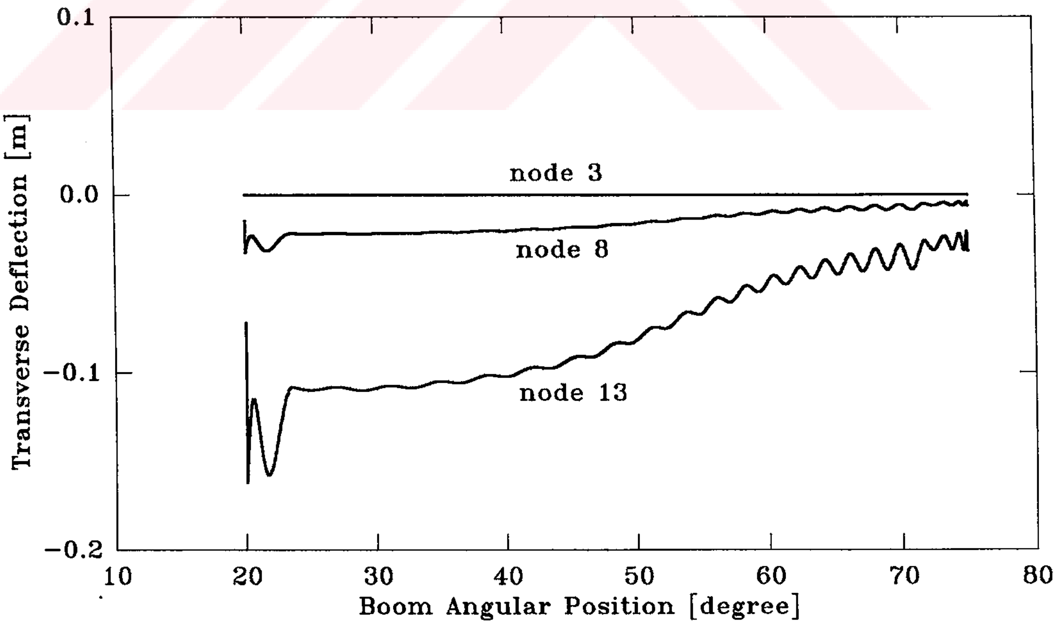

Figure 4.21 Variation of Transverse Deflections of Nodes 3, 8 and 13 with respect to Boom Angular Position for 32.4 kN Hook Load, for the Linear Acceleration and Deceleration Case and for 10 Seconds Boom Downward Direction Motion

When these figures are examined it is seen that oscillation amplitude of both piston force and transverse deflections of nodes are bigger than those related to 30 seconds motion of the boom.

4.3.2 SIMULATION OF THE LIFTING CAPACITY ON THE HOOK

As the boom makes motion in the upward and downward direction after the crane is blocked, tipping simulation is performed for the blocked case of the crane. When one of the reaction forces coming from the ground to the vertical jacks is zero, tipping condition occur. Using the free body diagram of the crane chasis, given in Figure 4.22, equation (4.2) is written for the tipping case

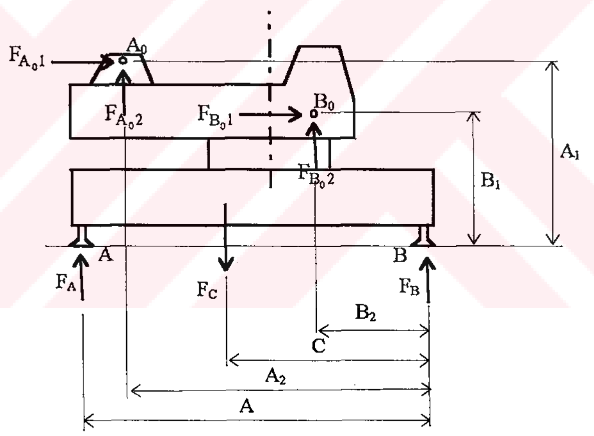

Figure 4.22 Free Body Diagram of the Crane Chasis

$$F_A = \frac{1}{A}\left(-F_{A_01}A_1 - F_{A_02}A_2 - F_{B_01}B_1 - F_{B_02}B_2 + F_C C\right) \qquad (4.2)$$

where F_{A_01} , F_{A_02} and F_{B_01} , F_{B_02} are the reaction forces components coming from the boom and cylinder to the crane chasis, F_A and F_B are the reaction forces coming from the ground to the jacks and F_C is the body force of the crane chasis.

When F_A is smaller than or equal to the zero, tipping condition occur.

Magnitudes of the reaction forces are dependent on the bodies angular positions and hook load. Using the computer code, boom angular positions are determined when the F_A is zero for different hook loads. Using these data the lifting capacity or allowable load with a safety factor taken as 1.5 on the tipping load is simulated. These simulation results and manufacturer's allowable load data with a safety factor of 1.5 for the test crane are given in Figure 4.23.

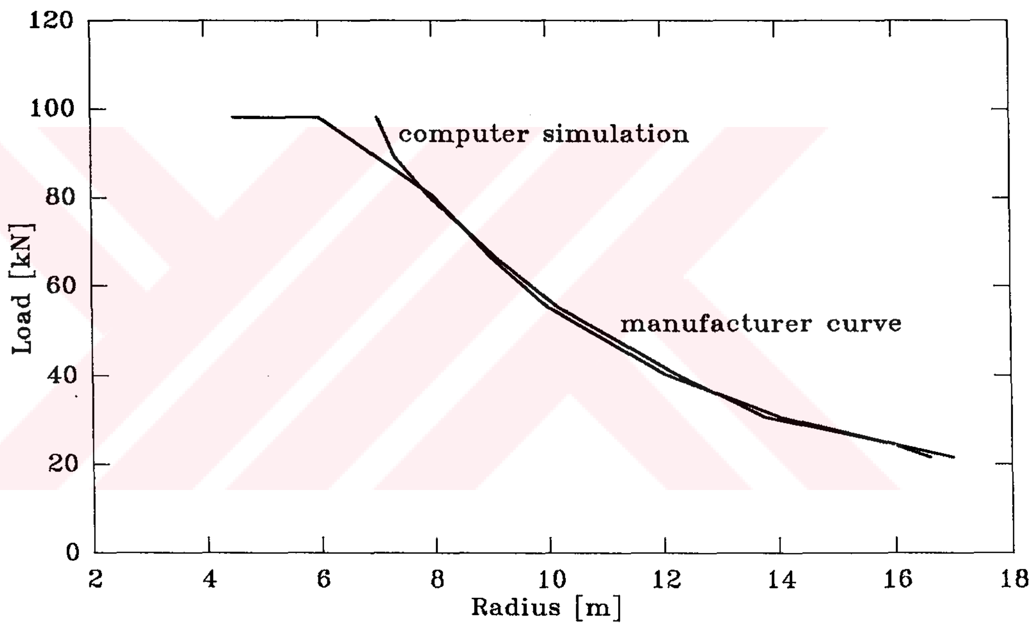

Figure 4.23 Lifting Capacity on the Hook

In Figure 4.23 radius is defined as the distance measured from the vertical axis of rotation of the crane to the tip of the hook in the horizontal plane and given as

$$R = 19.5 \cos\theta^{(1)} - 1.83 \qquad [m] \qquad (4.3)$$

It is seen in Figure 4.23 that when the hook load increases, radius decreases as expected.

CHAPTER 5

CONCLUSION

In this study mobile crane characteristics are determined by using flexible multibody analysis. In order to achieve this goal a computer program is developed which is capable of making dynamic analysis of the crane.

The coupled rigid and elastic motion of the system is formulated by using absolute coordinates and modal variables [5,6]. Then joint connections and prescribed motions are imposed as constraint equations. The flexible body is modeled by finite element method and modal variables are used as the elastic variables by utilizing modal transformation.

Variations of piston force with respect to the boom angular positions for the unloaded motion of the boom and for 32.4 kN hook load are simulated for the 30 seconds motion of the boom using the computer code for two different piston velocity profiles and compared with the experimental results. Similarly variations of piston force with respect to the boom angular positions for 32.4 kN hook load are simulated for the 10 seconds motion of the boom using the computer code. This simulation is also performed for two different piston velocity profiles. Moreover, transverse deflections of node 3, node 8 and node 13 are obtained with respect to the boom angular positions for 32.4 kN hook load. Finally, load curves are generated for the 30 seconds motion and compared to those of manufacturer.

In all of the computer simulations, boom is divided into 12 finite elements. First longitidunal deformation mode, first bending mode of the $\overline{A_0A}$ part and first two bending modes of the $\overline{AC}$ part of the boom are taken as the modal matrix. 2 % damping ratio for the boom is considered by assuming Rayleigh structural damping.

is used for the deceleration of the boom for the 30 seconds boom motion. In the case of 10 seconds boom motion acceleration and deceleration times are assumed to be 1 second.

When the boom motion times are considered, it is seen that in the small piston speeds (i.e. 30 seconds motion of the boom), the effect of flexibility approximately is not seen. However, when the piston speed is increased (i.e. 10 seconds motion of the boom), the effect of flexibility is dominant. Moreover, velocity profile applied to the piston is also important. This means that operator's skill and experience gain significance.

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APPENDIX A

MODE SHAPES OF A BEAM

Consider an element with nodes A and B. Let ν_i and θ_i , i = 1, 2, 3 denote the elastic displacements and rotations respectively of axis frames fixed along the centroidal line of the element, and u_i denote the elastic displacements of arbitrary points in the element.

ν_i and θ_i are expressed in terms of the nodal variables α_j by utilizing polynomials of appropriate order as

$$\nu_i = \Phi_{ij}\,\alpha_j \;;\; i = 1, 2, 3 \quad j = 1, \dots , 12 \tag{A-1}$$

and

$$\theta_i = \psi_{ij}\,\alpha_j \tag{A-2}$$

where α_j are the displacements of rotations at the nodes,

$$\alpha = \begin{bmatrix} \nu_1^A & \nu_2^A & \nu_3^A & \theta_1^A & \theta_2^A & \theta_3^A & \nu_1^B & \nu_2^B & \nu_3^B & \theta_1^B & \theta_2^B & \theta_3^B \end{bmatrix}^T \tag{A-3}$$

and the element shape functions neglecting shear deformation are given by

$$\Phi = \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_1 & 0 & 0 & 0 & b_2 & 0 & b_3 & 0 & 0 & 0 & b_4 \\ 0 & 0 & b_5 & 0 & b_6 & 0 & 0 & 0 & b_7 & 0 & b_8 & 0 \end{bmatrix} \tag{A-4}$$

and

$$\psi = \begin{bmatrix} 0 & 0 & 0 & a_1 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 \\ 0 & 0 & c_5 & 0 & c_6 & 0 & 0 & 0 & c_7 & 0 & c_8 & 0 \\ 0 & c_1 & 0 & 0 & 0 & c_2 & 0 & c_3 & 0 & 0 & 0 & c_4 \end{bmatrix} \quad \text{(A-5)}$$

where

$$a_1 = 1 - \xi \quad \text{(A-6)}$$

$$a_2 = \xi \quad \text{(A-7)}$$

$$b_1 = (1 - 3\xi^2 + 2\xi^3) \quad \text{(A-8)}$$

$$b_2 = L(\xi - 2\xi^2 + \xi^3) \quad \text{(A-9)}$$

$$b_3 = (3\xi^2 - 2\xi^3) \quad \text{(A-10)}$$

$$b_4 = L(-\xi^2 + \xi^3) \quad \text{(A-11)}$$

$$b_5 = (1 - 3\xi^2 + 2\xi^3) \quad \text{(A-12)}$$

$$b_6 = L(-\xi + \xi^2 - \xi^3) \quad \text{(A-13)}$$

$$b_7 = (3\xi^2 - 2\xi^3) \quad \text{(A-14)}$$

$$b_8 = L(-\xi^2 + \xi^3) \quad \text{(A-15)}$$

$$c_1 = \frac{6}{L}(-\xi + \xi^2) = -c_3 \quad \text{(A-16)}$$

$$c_2 = (1 - 4\xi + 3\xi^2) = -c_6 \quad \text{(A-17)}$$

$$c_4 = (-2\xi + 3\xi^2) \tag{A-18}$$

$$c_5 = \frac{6}{L}(-\xi + \xi^2) = -c_7 \tag{A-19}$$

$$c_8 = (-2\xi + 3\xi^2) \tag{A-20}$$

In equations (A-6) to (A-20), $\xi = \frac{x}{L}$ where x is measured from the element axis fixed at node A and L is the length of the element.

Using ν_i and θ_i, the displacement field for arbitrary points in the beam element, $u_i(x,y,z,t)$ can be derived for small rotations such that

$$u_1(x,y,z,t) = \nu_1(x,t) + z\theta_2(x,t) - y\theta_3(x,t) \tag{A-21}$$

$$u_2(x,y,z,t) = \nu_2(x,t) - z\theta_1(x,t) \tag{A-22}$$

$$u_3(x,y,z,t) = \nu_3(x,t) + y\theta_1(x,t) \tag{A-23}$$

Using equations (A-1) and (A-2) and the relations given by equations (A-21), (A-22) and (A-23), u_i is obtained as

$$u_i = \phi_{ij}\,\alpha_j \quad i = 1, 2, 3 \quad j = 1, \ldots, 12 \tag{A-24}$$

where the shape function matrix ϕ for including rotary inertia becomes

$$\phi = \begin{bmatrix} a_1 & -yc_1 & zc_5 & 0 & zc_6 & -yc_2 & a_2 & -yc_3 & zc_7 & 0 & zc_8 & -yc_4 \\ 0 & b_1 & 0 & -za_1 & 0 & b_2 & 0 & b_3 & 0 & -za_2 & 0 & b_4 \\ 0 & 0 & b_5 & ya_1 & b_6 & 0 & 0 & 0 & b_7 & ya_2 & b_8 & 0 \end{bmatrix} \tag{A-25}$$

The 2-D simplification of α, Φ, ψ and ϕ are

$$\alpha = \begin{bmatrix} v_1^A & v_2^A & \theta^A & v_1^B & v_2^B & \theta^B \end{bmatrix}^T \tag{A-26}$$

$$\Phi = \begin{bmatrix} a_1 & 0 & 0 & a_2 & 0 & 0 \\ 0 & b_1 & b_2 & 0 & b_3 & b_4 \end{bmatrix} \tag{A-27}$$

$$\psi = \begin{bmatrix} 0 & c_1 & c_2 & 0 & c_3 & c_4 \end{bmatrix} \tag{A-28}$$

$$\phi = \begin{bmatrix} a_1 & -yc_1 & -yc_2 & a_2 & -yc_3 & -yc_4 \\ 0 & b_1 & b_2 & 0 & b_3 & b_4 \end{bmatrix} \tag{A-29}$$

APPENDIX B

LISTING OF THE COMPUTER PROGRAM

```
C
      PROGRAM MAIN
C     THIS PROGRAM IS WRITTEN TO MAKE THE DYNAMIC ANALYSIS
C     OF THE CRANE USING FLEXIBLE MULTIBODY ANALYSIS
      IMPLICIT REAL*8(A-H,O-Z)
C     M=NUMBER OF ELEMENTS
C     N=NUMBER OF NODES
C     N=M+1
C*    NFEF=NUMBER OF FINITE ELEMENTS IN THE FIRST PART
C     NFEF=2
C*    NFES=NUMBER OF FINITE ELEMENTS IN THE SECOND PART
C     NFES=10
C     M=NFEF+NFES
C     NM=NUMBER OF NODAL VARIABLES
C     NM=(M+1)*3
C     MM=M*4
C     MN=NM-3
C     NMN=(M+1)*4
C     NNM=NUMBER OF MODAL VARIABLES

C     DIMENSION CO(M+1),AO(M+1),BO(M+1),EO(M+1),FO(M+1)
C     DIMENSION CI(M+1),AI(M+1),BI(M+1),EI(M+1),FI(M+1)
      DIMENSION CO(13),AO(13),BO(13),EO(13),FO(13)
      DIMENSION CI(13),AI(13),BI(13),EI(13),FI(13)

C     DIMENSION CRA(MM,4),CRE(NMN,4)
C     DIMENSION AQ1(M),A2(M),A3(M),A4(M),AR(M)
C     DIMENSION WI(M),WI1(M),WI2(M),WI3(M),WI4(M),WI5(M)
C     DIMENSION WI6(M),WI7(M),WI8(M)
C     DIMENSION TW(M),ES(M,NM,NM),SM(NM,NM)
C     DIMENSION WW(M),AK(M,NM,NM),SS(NM,NM)

      DIMENSION CRA(48,4),CRE(52,4)
      DIMENSION AQ1(12),A2(12),A3(12),A4(12),AR(12)
      DIMENSION WI(12),WI1(12),WI2(12),WI3(12),WI4(12),WI5(12)
      DIMENSION WI6(12),WI7(12),WI8(12)
      DIMENSION TW(12),ES(12,39,39),SM(39,39)
      DIMENSION WW(12),AK(12,39,39),SS(39,39)

C     DIMENSION A(NM-4,NM-4),B(NM-4,NM-4),XX(NM-4,NM-4)
C     DIMENSION EIGV(NM-4),D(NM-4),XXX(NM,NNM),XXC(NM-4,NM-4)
C     DIMENSION XXBL(NM-4,NM-4),XXBR(NM-4,NM-4),XXL(NM-4,NM-4)
C     DIMENSION EIGVBL(NM-4),EIGVBR(NM-4),EIGVL(NM-4),EVALUE(NNM)

      DIMENSION A(35,35),B(35,35),XX(35,35)
      DIMENSION EIGV(35),D(35),XXX(39,4),XXC(35,35)
      DIMENSION XXBL(35,35),XXBR(35,35),XXL(35,35)
      DIMENSION EIGVBL(35),EIGVBR(35),EIGVL(35),EVALUE(4)

C     DIMENSION CMOD(M,1),CMOE(M,1),CMOF(M,1),CMOG(M,1)
C     DIMENSION CMOH(NM,1),XXI(MN,MN),XXK(MN,NM),CMOJ(MN,1)
C     DIMENSION ABXX(NM-15,2*(NM-15))
      DIMENSION CMOD(12,1),CMOE(12,1),CMOF(12,1),CMOG(12,1)
      DIMENSION CMOH(39,1),XXI(36,36),XXK(36,39),CMOJ(36,1)
      DIMENSION ABXX(24,48)

C     DIMENSION FORCE(NM,1),FORCEN(NM-15,1),CMOT(NM-15,1),
C    + ANV(NM-15,NM-15),AU(NM-15,NM-15)
      DIMENSION FORCE(39,1),FORCEN(24,1),CMOT(24,1),
     + ANV(24,24),AU(24,24)

      DIMENSION FORCEL(11,1),CMOT1(24,1),ABXX1(11,22),
     + ANV1(11,11),AU1(11,11),CRAT(13,1)
```

```
C     DIMENSION XXXZ(4,4),BXX(4,8),XXXI(4,4)
C     DIMENSION XXXY(4,NM),CETA(4,1)
C     DIMENSION SD(NM,NM)
      DIMENSION XXXZ(4,4),BXX(4,8),XXXI(4,4)
      DIMENSION XXXY(4,39),CETA(4,1)
      DIMENSION SD(39,39)

C     DIMENSION Y(2*(NNM+15)),TS(J),YS(2*(NNM+15),J),IFL(J),
C    + SEQ(NNM+29,1),YDL(NNM+29,J),YP(2*(NNM+15))
C     DIMENSION YI(2*(NNM+15)),YV(2*(NNM+15))
      DIMENSION Y(2*(4+15)),TS(4),YS(2*(4+15),4),IFL(4),
     + SEQ(4+29,1),YDL(4+29,4),YP(2*(4+15))
      DIMENSION YI(2*(4+15)),YV(2*(4+15))

C     DIMENSION AT(M,2,NM),A1(2,NM),YOO1(2,NNM),AMZZ(2,2),
C    + AAT(M,NM,NM),AA1(NM,NM),YOO2(NM,NNM),XXXT(NNM,NM),
C    + YOO3(NNM,NM),AMNN(NNM,NNM),YOO4(NM,NNM),YOO5(NNM,NNM),
C    + A1T(NM,2),YOO6(NNM,2),YOO7(NNM,NM),YOO8(NNM,NNM),
C    + YOO9(NNM,NM),YOO10(NNM,NNM)
C     DIMENSION FAY(2,6),CSI(1,6),BOCE(6,NM),YOO11(2,NM),
C    + YOO12(2,NNM),YOO13(1,NM),YOO14(1,NNM),YOO15(2,NM),
C    + YOO16(2,NNM)
      DIMENSION AT(12,2,39),A1(2,39),AMZZ(2,2),
     + AAT(12,39,39),AA1(39,39),XXXT(4,39),AMNN(4,4),A1T(39,2)
      DIMENSION FAY(2,6),CSI(1,6),BOCE(6,39)

      COMMON /A2/B
      COMMON /A3/A
      COMMON /A4/XXX
      COMMON /A5/SM
      COMMON /A6/SS,SD
      COMMON /A7/AR,TW,AOA,AOG
      COMMON /A8/CRA,PI
      COMMON /A9/NFEF,NFES,M,NM,MM,MN,NMN,NNM
      COMMON /A10/ YOO1(2,4),YOO2(39,4),YOO3(4,39),YOO4(39,4),
     + YOO5(4,4),YOO6(4,2),YOO7(4,39),YOO8(4,4),YOO9(4,39),YOO10(4,4)
      COMMON /A12/ YOO11(2,39),YOO12(2,4),YOO13(1,39),YOO14(1,4),
     + YOO15(2,39),YOO16(2,4)
      COMMON /A13/ AMZZ,A1,AA1,AMNN,A1T,AMS
      COMMON /A14/ AT,AAT,XXXT,FAY,CSI,BOCE
      COMMON /A16/SEQ
      COMMON /A17/IIIIZ,NOO,NOOM,CODIS,RO,WF
      COMMON /A18/TIME1,TIME2,TIME3,VZERO

      IO=6
      IR=5
      OPEN(11,FILE='OUT11.FOR',STATUS='OLD')
      OPEN(IO,FILE='OUT12.FOR',STATUS='OLD')
      OPEN(13,FILE='OUT13.FOR',STATUS='OLD')
      OPEN(14,FILE='OUT14.FOR',STATUS='OLD')
      OPEN(15,FILE='OUT15.FOR',STATUS='OLD')
      OPEN(16,FILE='OUT16.FOR',STATUS='OLD')
      OPEN(17,FILE='OUT17.FOR',STATUS='OLD')
      OPEN(18,FILE='OUT18.FOR',STATUS='OLD')
      OPEN(19,FILE='OUT19.FOR',STATUS='OLD')
      OPEN(20,FILE='OUT20.FOR',STATUS='OLD')
      OPEN(21,FILE='OUT21.FOR',STATUS='OLD')
      OPEN(22,FILE='OUT22.FOR',STATUS='OLD')
      OPEN(23,FILE='OUT23.FOR',STATUS='OLD')
      OPEN(24,FILE='OUT24.FOR',STATUS='OLD')

C*
      NFEF=2
      NFES=10
      M=NFEF+NFES
      NM=(M+1)*3
      MM=M*4
      MN=NM-3
      NMN=(M+1)*4
      NNM=4

      DO 1 I=1,NMN
      DO 1 J=1,4
      CRE(I,J)=0.D0
    1 CONTINUE
      DO 2 I=1,MM
      DO 2 J=1,4
      CRA(I,J)=0.D0
    2 CONTINUE
C
C     CALCULATION OF THE NODE VALUES
```

```
C
C   AL=ANGLE ALFA OF THE BEAM
C
C    PIV=-1.D0
C    PI=DACOS(PIV)
    PI=3.141592653589793120
C*
    AL=PI/180.D0*45.D0
C   AOG=LENGTH OF THE BODY 1 IN THE FIRST PART
    AOG=2.D0
C   GC=LENGTH OF THE BODY 1 SECOND PART
    GC=17.5D0
C   AOA=DISTANCE BETWEEN Ao AND A
    AOA=5.822651887D0
C   CROSS SECTION 1 DIMENSIONS
    EO1=0.45333D0
    CO1=0.2D0
    FO1=0.12D0
C   CROSS SECTION 3 DIMENSIONS
    EO3=0.16D0
    CO3=0.44D0
    FO3=0.12D0
C   CROSS SECTION 5 DIMENSIONS
    EO5=0.16D0
    CO5=0.20667D0
    FO5=0.12D0
C   NFEF=NUMBER OF FINITE ELEMENTS IN THE FIRST PART
C   NFES=NUMBER OF FINITE ELEMENTS IN THE SECOND PART
C
C   CALCULATION OF THE NODE VALUES IN THE FIRST PART
C
    DO 7 N=1,NFEF+1
    CO(N)=CO1+(CO3-CO1)/NFEF*(N-1)
    AO(N)=CO(N)*SIN(AL)
    BO(N)=CO(N)*COS(AL)
    EO(N)=EO1-(EO1-EO3)/NFEF*(N-1)
    FO(N)=FO1
    CRE(N,1)=AOG/NFEF
    CRE(N,2)=AO(N)+FO(N)/2.D0
    CRE(N,3)=BO(N)+EO(N)/2.D0
    CRE(N,4)=FO(N)/2.D0
  7 CONTINUE
C
C   CALCULATION OF THE NODE VALUES IN THE SECOND PART
C
    L=0
    DO 9 N=NFEF+2,NFEF+NFES+1
    L=L+1
    EL=(AOA-AOG)/2*L
    IF(EL.GT.GC) EL=GC
    CO(N)=CO3-(CO3-CO5)/GC*EL
    AO(N)=CO(N)*SIN(AL)
    BO(N)=CO(N)*COS(AL)
    EO(N)=EO3
    FO(N)=FO3
    CRE(N,1)=(AOA-AOG)/2
    IF(N.EQ.(NFEF+NFES+1)) CRE(N,1)=GC-(NFES-1)*(AOA-AOG)/2.D0
    CRE(N,2)=AO(N)+FO(N)/2.D0
    CRE(N,3)=BO(N)+EO(N)/2.D0
    CRE(N,4)=FO(N)/2.D0
  9 CONTINUE


    DO 11 N=1,M+1
    CRE(M+1+N,1)=CRE(N,1)
    CRE(M+1+N,2)=AO(N)/2.D0
    CRE(M+1+N,3)=BO(N)/2.D0
    CRE(M+1+N,4)=FO(N)/2.D0
 11 CONTINUE

C*   TH=THICKNESS OF THE BEAM (METER)
    TH=0.020D0

    DO 13 N=1,M+1
    CI(N)=CO(N)+TH*((DSIN(AL)-1)/DCOS(AL)+(DCOS(AL)-1)/DSIN(AL))
    AI(N)=CI(N)*DSIN(AL)
    BI(N)=CI(N)*DCOS(AL)
    EI(N)=EO(N)+2.D0*TH*(DCOS(AL)-1)/DSIN(AL)
    FI(N)=FO(N)+2.D0*TH*(DSIN(AL)-1)/DCOS(AL)
    CRE(2*(M+1)+N,1)=CRE(N,1)
    CRE(2*(M+1)+N,2)=AI(N)+FI(N)/2.D0
```

```
      CRE(2*(M+1)+N,3)=BI(N)+EI(N)/2.D0
      CRE(2*(M+1)+N,4)=FI(N)/2.D0
   13 CONTINUE

      DO 15 N=1,M+1
      CRE(3*(M+1)+N,1)=CRE(N,1)
      CRE(3*(M+1)+N,2)=AI(N)/2.D0
      CRE(3*(M+1)+N,3)=BI(N)/2.D0
      CRE(3*(M+1)+N,4)=FI(N)/2.D0
   15 CONTINUE

C     END OF THE FINITE ELEMENT NODE VALUES CONSTRUCTION

      L=0
      KK=1
   17 DO 18 J=KK,KK+M-1
      CRA(J-L,1)=CRE(J+1,1)
      CRA(J-L,2)=(CRE(J,2)+CRE(J+1,2))/2.D0
      CRA(J-L,3)=(CRE(J,3)+CRE(J+1,3))/2.D0
      CRA(J-L,4)=(CRE(J,4)+CRE(J+1,4))/2.D0
   18 CONTINUE
      KK=KK+M+1
      L=L+1
      IF(L.LE.3) GO TO 17
C      WRITE(11,*)'I J CRE(I,J)'
C      DO 23 I=1,NMN
C      DO 23 J=1,4
C      WRITE(11,*) I,J,CRE(I,J)
C  23 CONTINUE
C      WRITE(11,*)'I J CRA(I,J)'
C      DO 26 I=1,MM
C      DO 26 J=1,4
C      WRITE(11,*) I,J,CRA(I,J)
C  26 CONTINUE
C*     INPUTS FOR SUBROUTINE JACOBI
      NSMAX=15
      IFPR=0
      IOUT=6
      RTOL=0.00000001
C     N USING IN THE SUBROUTINE JACOBI IS THE SAME AS MN USING IN THE
C     MAIN AND OTHER SUBROUTINES

C     CALCULATION OF STRUCTURAL MASS MATRIX OF THE BODY 1 : SM(NM,NM)
      DO 28 I=1,NM-4
      DO 28 J=1,NM-4
      A(I,J)=0.D0
      B(I,J)=0.D0
   28 CONTINUE
      DO 35 I=1,NM
      DO 35 J=1,NM
      SM(I,J)=0.D0
      SS(I,J)=0.D0
   35 CONTINUE


      DO 80 K=1,M
      AR(K)=0.D0
      AQ1(K)=0.D0
      A2(K)=0.D0
      A3(K)=0.D0
      A4(K)=0.D0
      WI(K)=0.D0
      WI1(K)=0.D0
      WI2(K)=0.D0
      WI3(K)=0.D0
      WI4(K)=0.D0
      WI5(K)=0.D0
      WI6(K)=0.D0
      WI7(K)=0.D0
      WI8(K)=0.D0
      DO 80 I=1,NM
      DO 80 J=1,NM
      SM(I,J)=0.D0
      ES(K,I,J)=0.D0
   80 CONTINUE

      TWT=0.D0
      K=0
C*     RO=DENSITY OF BODY 1 MATERIAL
      RO=5750.D0
      DO 90 I=1,NM-5,3
```

```
      K=K+1
C     AR(K)=CROSS SECTIONAL AREA OF THE KTH FINITE ELEMENT
      AQ1(K)=(2.D0*CRA(K,2))*(2.D0*CRA(K,3))
      A2(K)=(2.D0*CRA(K+M,2))*(2.D0*CRA(K+M,3))
      A3(K)=(2.D0*CRA(K+2*M,2))*(2.D0*CRA(K+2*M,3))
      A4(K)=(2.D0*CRA(K+3*M,2))*(2.D0*CRA(K+3*M,3))
      AR(K)=AQ1(K)-2.D0*A2(K)-(A3(K)-2.D0*A4(K))
C     WRITE(*,*)'K AR(K)',K,AR(K)
C     WI(K)=AREA MOMENT OF INERTIA OF THE KTH FINITE ELEMENT
      WI1(K)=((2.D0*CRA(K,2))**3)*(2.D0*CRA(K,3))/12.D0
      WI2(K)=((2.D0*CRA(K+M,2))**3)*(2.D0*CRA(K+M,3))/36.D0
      WI3(K)=(2.D0*CRA(K+M,2))*(2.D0*CRA(K+M,3))/2.D0
      WI4(K)=(2.D0/3.D0*2.D0*CRA(K+M,2)+CRA(K+M,4))**2
      WI5(K)=((2.D0*CRA(K+2*M,2))**3)*(2.D0*CRA(K+2*M,3))/12.D0
      WI6(K)=((2.D0*CRA(K+3*M,2))**3)*(2.D0*CRA(K+3*M,3))/36.D0
      WI7(K)=(2.D0*CRA(K+3*M,2))*(2.D0*CRA(K+3*M,3))/2.D0
      WI8(K)=(2.D0/3.D0*2.D0*CRA(K+3*M,2)+CRA(K+3*M,4))**2
      WI(K)=WI1(K)-4.D0*(WI2(K)+WI3(K)*WI4(K))-(WI5(K)-4.D0
     + *(WI6(K)+WI7(K)*WI8(K)))

C     TW(K)=MASS OF THE KTH FINITE ELEMENT
      TW(K)=RO*CRA(K,1)*AR(K)
      RL=CRA(K,1)
      ES(K,I,I)=(1.D0/3.D0)*TW(K)
      ES(K,I,I+1)=0.D0
      ES(K,I,I+2)=0.D0
      ES(K,I,I+3)=(1.D0/6.D0)*TW(K)
      ES(K,I,I+4)=0.D0
      ES(K,I,I+5)=0.D0
      ES(K,I+1,I)=ES(K,I,I+1)
      ES(K,I+1,I+1)=(6.D0/5.D0)*(WI(K)/AR(K))/(RL*RL)*TW(K)+
     + (13.D0/35.D0)*TW(K)
      ES(K,I+1,I+2)=(1.D0/10.D0)*(WI(K)/AR(K))/RL*TW(K)+
     + (11.D0/210.D0)*RL*TW(K)
      ES(K,I+1,I+3)=0.D0
      ES(K,I+1,I+4)=-(6.D0/5.D0)*(WI(K)/AR(K))/(RL*RL)*TW(K)+
     + (9.D0/70.D0)*TW(K)
      ES(K,I+1,I+5)=(1.D0/10.D0)*(WI(K)/AR(K))/RL*TW(K)-
     + (13.D0/420.D0)*RL*TW(K)
      ES(K,I+2,I)=ES(K,I,I+2)
      ES(K,I+2,I+1)=ES(K,I+1,I+2)
      ES(K,I+2,I+2)=(2.D0/15.D0)*(WI(K)/AR(K))*TW(K)+
     + (1.D0/105.D0)*(RL*RL)*TW(K)
      ES(K,I+2,I+3)=0.D0
      ES(K,I+2,I+4)=-(1.D0/10.D0)*(WI(K)/AR(K))/RL*TW(K)+
     + (13.D0/420.D0)*RL*TW(K)
      ES(K,I+2,I+5)=-(1.D0/30.D0)*(WI(K)/AR(K))*TW(K)-
     + (1.D0/140.D0)*(RL*RL)*TW(K)
      ES(K,I+3,I)=ES(K,I,I+3)
      ES(K,I+3,I+1)=ES(K,I+1,I+3)
      ES(K,I+3,I+2)=ES(K,I+2,I+3)
      ES(K,I+3,I+3)=(1.0/3.0)*TW(K)
      ES(K,I+3,I+4)=0.D0
      ES(K,I+3,I+5)=0.D0
      ES(K,I+4,I)=ES(K,I,I+4)
      ES(K,I+4,I+1)=ES(K,I+1,I+4)
      ES(K,I+4,I+2)=ES(K,I+2,I+4)
      ES(K,I+4,I+3)=ES(K,I+3,I+4)
      ES(K,I+4,I+4)=(6.D0/5.D0)*(WI(K)/AR(K))/(RL*RL)*TW(K)+
     + (13.D0/35.D0)*TW(K)
      ES(K,I+4,I+5)=-(1.D0/10.D0)*(WI(K)/AR(K))/RL*TW(K)-
     + (11.D0/210.D0)*RL*TW(K)
      ES(K,I+5,I)=ES(K,I,I+5)
      ES(K,I+5,I+1)=ES(K,I+1,I+5)
      ES(K,I+5,I+2)=ES(K,I+2,I+5)
      ES(K,I+5,I+3)=ES(K,I+3,I+5)
      ES(K,I+5,I+4)=ES(K,I+4,I+5)
      ES(K,I+5,I+5)=(2.D0/15.D0)*(WI(K)/AR(K))*TW(K)+
     + (1.D0/105.D0)*(RL*RL)*TW(K)

C     TWT=TOTAL MASS OF THE BODY 1
      TWT=TWT+TW(K)
   90 CONTINUE

      WRITE(11,*)'IF'
      WRITE(11,*)'RO (kg/m3)  THICKNESS (m)',RO,TH
      WRITE(11,*)'TOTAL MASS OF THE BODY 1 TWT (kg)',TWT

      WRITE(*,*) 'ALL ES(I,J) MATRICES ARE CALCULATED'

C     ADDITION OF THE MATRICES
```

```
C     M: # OF ELEMENTS , N: # OF NODES , N=M+1
      IK=M*3+1
      IL=(M-1)*3+1
      DO 100 I=1,3
      DO 100 J=1,6
      SM(I,J)=ES(1,I,J)
  100 CONTINUE
      LT=-2
      K=0
      KT=1
      DO 150 K=1,M-1
      LT=LT+3
      KT=KT+3
      DO 150 I=KT,KT+2
      DO 150 J=LT,LT+8
      SM(I,J)=ES(K,I,J)+ES(K+1,I,J)
  150 CONTINUE
      DO 200 I=IK,IK+2
      DO 200 J=IL,IL+5
      SM(I,J)=ES(M,I,J)
  200 CONTINUE
      WRITE(*,*) 'SM(I,J) MATRIX IS OBTAINED'
      KT=0
      LSS=0
      DO  860 I=1,NM
      KT=KT+1
      DO 860 J=KT,NM
      IF(SM(I,J).EQ.SM(J,I)) GO TO 860
      WRITE(IO,*)'I J SM(I,J)',I,J,SM(I,J)
      WRITE(IO,*)'J I SM(J,I)',J,I,SM(J,I)
      LSS=LSS+1
  860 CONTINUE
      IF(LSS.NE.0) GO TO 865
      WRITE(*,*)'SM(I,J) IS A SYMMETRIC MATRIX'
      GO TO 870
  865 WRITE(*,*)'SM(I,J) IS NOT A SYMMETRIC MATRIX'


C     CALCULATION OF STRUCTURAL STIFFNESS MATRIX OF THE BODY 1:SS(NM,NM)
  870 DO 900 K=1,M
      DO 900 I=1,NM
      DO 900 J=1,NM
      AK(K,I,J)=0.D0
      SS(I,J)=0.D0
  900 CONTINUE
      K=0
      DO 920 I=1,NM-5,3
      K=K+1
C*    EE=MODULUS OF ELASTICITY OF BODY 1 MATERIAL
      EE=200000000000.D0
      RL=CRA(K,1)
      WW(K)=EE*WI(K)/RL
      AK(K,I,I)=AR(K)/WI(K)*WW(K)
      AK(K,I,I+1)=0.D0
      AK(K,I,I+2)=0.D0
      AK(K,I,I+3)=-AR(K)/WI(K)*WW(K)
      AK(K,I,I+4)=0.D0
      AK(K,I,I+5)=0.D0
      AK(K,I+1,I)=AK(K,I,I+1)
      AK(K,I+1,I+1)=12.D0/((RL)*(RL))*WW(K)
      AK(K,I+1,I+2)=6.D0/RL*WW(K)
      AK(K,I+1,I+3)=0.D0
      AK(K,I+1,I+4)=-12.D0/((RL)*(RL))*WW(K)
      AK(K,I+1,I+5)=6.D0/RL*WW(K)
      AK(K,I+2,I)=AK(K,I,I+2)
      AK(K,I+2,I+1)=AK(K,I+1,I+2)
      AK(K,I+2,I+2)=4.D0*WW(K)
      AK(K,I+2,I+3)=0.D0
      AK(K,I+2,I+4)=-6.D0/RL*WW(K)
      AK(K,I+2,I+5)=2.D0*WW(K)
      AK(K,I+3,I)=AK(K,I,I+3)
      AK(K,I+3,I+1)=AK(K,I+1,I+3)
      AK(K,I+3,I+2)=AK(K,I+2,I+3)
      AK(K,I+3,I+3)=AR(K)/WI(K)*WW(K)
      AK(K,I+3,I+4)=0.D0
      AK(K,I+3,I+5)=0.D0
      AK(K,I+4,I)=AK(K,I,I+4)
      AK(K,I+4,I+1)=AK(K,I+1,I+4)
      AK(K,I+4,I+2)=AK(K,I+2,I+4)
      AK(K,I+4,I+3)=AK(K,I+3,I+4)
      AK(K,I+4,I+4)=12.D0/((RL)*(RL))*WW(K)
```

```
      AK(K,I+4,I+5)=-6.D0/RL*WW(K)
      AK(K,I+5,I)=AK(K,I,I+5)
      AK(K,I+5,I+1)=AK(K,I+1,I+5)
      AK(K,I+5,I+2)=AK(K,I+2,I+5)
      AK(K,I+5,I+3)=AK(K,I+3,I+5)
      AK(K,I+5,I+4)=AK(K,I+4,I+5)
      AK(K,I+5,I+5)=4.*WW(K)
  920 CONTINUE
C     ADDITION OF THE MATRIX
C     M: # OF ELEMENTS , N: # OF NODES , N=M+1
      IK=M*3+1
      IL=(M-1)*3+1
      DO 1000 I=1,3
      DO 1000 J=1,6
      SS(I,J)=AK(1,I,J)
 1000 CONTINUE
      LT=-2
      K=0
      KT=1
      DO 1002 K=1,M-1
      LT=LT+3
      KT=KT+3
      DO 1002 I=KT,KT+2
      DO 1002 J=LT,LT+8
      SS(I,J)=AK(K,I,J)+AK(K+1,I,J)
 1002 CONTINUE
      DO 1004 I=IK,IK+2
      DO 1004 J=IL,IL+5
      SS(I,J)=AK(M,I,J)
 1004 CONTINUE
      KT=0
      LSS=0
      DO 1006 I=1,NM
      KT=KT+1
      DO 1006 J=KT,NM
      IF(SS(I,J).EQ.SS(J,I)) GO TO 1006
      LSS=LSS+1
      WRITE(*,*)'I J SS(I,J)',I,J,SS(I,J)
      WRITE(*,*)'J I SS(J,I)',J,I,SS(J,I)
 1006 CONTINUE
      IF(LSS.NE.0) GO TO 1008
      WRITE(*,*)'SS(I,J) IS A SYMMETRIC MATRIX'
      GO TO 1010
 1008 WRITE(*,*)'SS(I,J) IS NOT A SYMMETRIC MATRIX'

C     NOW, APPLY THE BOUNDARY CONDITIONS.
C     THERE ARE REVOLUTE JOINT AT Ao AND SLIDER JOINT AT D
C     1ST AND 2ND AND 14TH AND 15TH ROWS AND COLUMNS OF
C     MASS MATRIX SM(NM,NM) AND STRUCTURAL STIFFNESS
C     MATRIX SS(NM,NM) ARE DROPPED FOR 2-D ANALYSIS
 1010 DO 1015 I=3,(NFEF+2)*3+1
      DO 1015 J=3,(NFEF+2)*3+1
      B(I-2,J-2)=SM(I,J)
 1015 CONTINUE
      DO 1017 I=(NFEF+3)*3+1,NM
      DO 1017 J=3,(NFEF+2)*3+1
      B(I-4,J-2)=SM(I,J)
 1017 CONTINUE
      DO 1019 I=3,(NFEF+2)*3+1
      DO 1019 J=(NFEF+3)*3+1,NM
      B(I-2,J-4)=SM(I,J)
 1019 CONTINUE
      DO 1021 I=(NFEF+3)*3+1,NM
      DO 1021 J=(NFEF+3)*3+1,NM
      B(I-4,J-4)=SM(I,J)
 1021 CONTINUE

      DO 1023 I=3,(NFEF+2)*3+1
      DO 1023 J=3,(NFEF+2)*3+1
      A(I-2,J-2)=SS(I,J)
 1023 CONTINUE
      DO 1025 I=(NFEF+3)*3+1,NM
      DO 1025 J=3,(NFEF+2)*3+1
      A(I-4,J-2)=SS(I,J)
 1025 CONTINUE
      DO 1027 I=3,(NFEF+2)*3+1
      DO 1027 J=(NFEF+3)*3+1,NM
      A(I-2,J-4)=SS(I,J)
 1027 CONTINUE
      DO 1029 I=(NFEF+3)*3+1,NM
      DO 1029 J=(NFEF+3)*3+1,NM
```

```
      A(I-4,J-4)=SS(I,J)
 1029 CONTINUE

      CALL JACOBI(XX,EIGV,D,NM-4,RTOL,NSMAX,IFPR,IOUT)

C     XX(NM-4,NM-4)=EIGENVECTORS STORED COLUMNWISE
C     EIGV(NM-4)=EIGENVALUES
C      DO 1031 I=1,NM-4
C      WRITE(11,*)'I EIGV(I)',I,EIGV(I)
C 1031 CONTINUE
C      WRITE(11,*)'EIGENVECTORS STORED COLUMNWISE'
C      DO 1033 J=1,NM-4
C      DO 1033 I=1,NM-4
C      WRITE(11,*)'I J XX(I,J)',I,J,XX(I,J)
C 1033 CONTINUE

C     SEPARATION OF LONGITIDUNAL AND BENDING DEFORMATIONS
C     EIGENVALUES AND EIGENVECTORS

C     EIGVL(NM-4)=LONGITIDUNAL DEFORMATION EIGENVALUES
C     EIGVBL(NM-4)=BENDING DEFORMATION EIGENVALUES OF LEFT SIDE
C     EIGVBR(NM-4)=BENDING DEFORMATION EIGENVALUES OF RIGHT SIDE
C     XXL(NM-4,NM-4)=LONGITIDUNAL DEFORMATION EIGENVECTORS STORED
C     COLUMNWISE
C     XXBL(NM-4,NM-4)=BENDING DEFORMATION EIGENVECTORS OF LEFT SIDE
C     STORED COLUMNWISE
C     XXBR(NM-4,NM-4)=BENDING DEFORMATION EIGENVECTORS OF RIGHT SIDE
C     STORED COLUMNWISE

      IBL=0
      IBR=0
      IL=0
      DO 1050 J=1,NM-4
      IL1=0
      IL2=0
      IL3=0
      IL4=0
      IL5=0
      IL6=0
      DO 1052 I=2,NFEF*3+2,3
      IF((XX(I,J)).NE.0.D0) GO TO 1054
      IL1=IL1+1
 1054 IF((XX(I+1,J)).EQ.0.D0) GO TO 1056
      IL2=IL2+1
 1056 IF((XX(I+2,J)).EQ.0.D0) GO TO 1052
      IL3=IL3+1
 1052 CONTINUE
      DO 1058 I=(NFEF+2)*3,NM-6,3
      IF((XX(I,J)).NE.0.D0) GO TO 1060
      IL4=IL4+1
 1060 IF((XX(I+1,J)).EQ.0.D0) GO TO 1062
      IL5=IL5+1
 1062 IF((XX(I+2,J)).EQ.0.D0) GO TO 1058
      IL6=IL6+1
 1058 CONTINUE

C     FOR LONGITIDUNAL DEFORMATION
      IF(IL1.NE.0) GO TO 1100
      IF(IL2.NE.0) GO TO 1100
      IF(IL3.NE.0) GO TO 1100
      IF(IL4.NE.0) GO TO 1100
      IF(IL5.NE.0) GO TO 1100
      IF(IL6.NE.0) GO TO 1100
      IF(XX(1,J).NE.0.D0) GO TO 1100
      IF(XX(11,J).EQ.0.D0) GO TO 1100
      GO TO 1110

C     FOR BENDING DEFORMATION OF LEFT SIDE
 1100 IF(IL1.NE.3) GO TO 1102
      IF(IL2.NE.3) GO TO 1102
      IF(IL3.NE.3) GO TO 1102
      IF(IL4.NE.8) GO TO 1102
      IF(IL5.NE.0) GO TO 1102
      IF(IL6.NE.0) GO TO 1102
      IF(XX(1,J).EQ.0.D0) GO TO 1102
      IF(XX(11,J).NE.0.D0) GO TO 1102
      GO TO 1114

C     FOR BENDING DEFORMATION OF RIGHT SIDE
 1102 IF(IL1.NE.3) GO TO 1104
      IF(IL2.NE.0) GO TO 1104
```

```
      IF(IL3.NE.0) GO TO 1104
      IF(IL4.NE.8) GO TO 1104
      IF(IL5.NE.8) GO TO 1104
      IF(IL6.NE.8) GO TO 1104
      IF(XX(1,J).NE.0.D0) GO TO 1104
      IF(XX(11,J).NE.0.D0) GO TO 1104
      GO TO 1118

1104 WRITE(*,*)'EIGENVEC. NOT IN THE FORM OF LONGITIDUNAL OR BENDING'
      STOP
1110 IL=IL+1
      EIGVL(IL)=EIGV(J)
      DO 1112 I=1,NM-4
      XXL(I,IL)=XX(I,J)
1112 CONTINUE
      GO TO 1050
1114 IBL=IBL+1
      EIGVBL(IBL)=EIGV(J)
      DO 1116 I=1,NM-4
      XXBL(I,IBL)=XX(I,J)
1116 CONTINUE
      GO TO 1050
1118 IBR=IBR+1
      EIGVBR(IBR)=EIGV(J)
      DO 1120 I=1,NM-4
      XXBR(I,IBR)=XX(I,J)
1120 CONTINUE
1050 CONTINUE

C    ORDERING OF THE EIGENVALUES AND EIGENVECTORS FROM THE
C    SMALLEST ONE TO THE BIGGEST ONE
C
C    FOR BENDING DEFORMATION OF LEFT SIDE
C
      DO 1125 I=1,IBL-1
      DO 1125 J=I+1,IBL
      IF(EIGVBL(I).LE.EIGVBL(J)) GO TO 1125
      ZQ=EIGVBL(I)
      EIGVBL(I)=EIGVBL(J)
      EIGVBL(J)=ZQ
      DO 1127 JA=1,NM-4
      ZJ=XXBL(JA,I)
      XXBL(JA,I)=XXBL(JA,J)
      XXBL(JA,J)=ZJ
1127 CONTINUE
1125 CONTINUE
      DO 1129 I=1,IBL
      WRITE(*,*)'I EIGVBL(I)',I,EIGVBL(I)
      WRITE(11,*)'I EIGVBL(I)',I,EIGVBL(I)
1129 CONTINUE
C     DO 1132 J=1,IBL
C     DO 1132 I=1,NM-4
C     WRITE(*,*)'I J XXBL(I,J)',I,J,XXBL(I,J)
C     WRITE(11,*)'I J XXBL(I,J)',I,J,XXBL(I,J)
C 1132 CONTINUE
C
C    FOR BENDING DEFORMATION OF RIGHT SIDE
C
      DO 1251 I=1,IBR-1
      DO 1251 J=I+1,IBR
      IF(EIGVBR(I).LE.EIGVBR(J)) GO TO 1251
      ZQ=EIGVBR(I)
      EIGVBR(I)=EIGVBR(J)
      EIGVBR(J)=ZQ
      DO 1252 JA=1,NM-4
      ZJ=XXBR(JA,I)
      XXBR(JA,I)=XXBR(JA,J)
      XXBR(JA,J)=ZJ
1252 CONTINUE
1251 CONTINUE
      DO 1253 I=1,IBR
      WRITE(*,*)'I EIGVBR(I)',I,EIGVBR(I)
      WRITE(11,*)'I EIGVBR(I)',I,EIGVBR(I)
1253 CONTINUE
C     DO 1254 J=1,IBR
C     DO 1254 I=1,NM-4
C     WRITE(*,*)'I J XXBR(I,J)',I,J,XXBR(I,J)
C     WRITE(11,*)'I J XXBR(I,J)',I,J,XXBR(I,J)
C 1254 CONTINUE

C
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```
C    FOR LONGITUDUNAL DEFORMATION
C
     DO 1256 I=1,IL-1
     DO 1256 J=I+1,IL
     IF(EIGVL(I).LE.EIGVL(J)) GO TO 1256
     ZQ=EIGVL(I)
     EIGVL(I)=EIGVL(J)
     EIGVL(J)=ZQ
     DO 1257 JA=1,NM-4
     ZJ=XXL(JA,I)
     XXL(JA,I)=XXL(JA,J)
     XXL(JA,J)=ZJ
 1257 CONTINUE
 1256 CONTINUE
     DO 1258 I=1,IL
     WRITE(*,*)'I EIGVL(I)',I,EIGVL(I)
     WRITE(11,*)'I EIGVL(I)',I,EIGVL(I)
 1258 CONTINUE
C      DO 1260 J=1,IL
C      DO 1260 I=1,NM-4
C      WRITE(*,*)'I J XXL(I,J)',I,J,XXL(I,J)
C      WRITE(11,*)'I J XXL(I,J)',I,J,XXL(I,J)
C 1260 CONTINUE
C      WRITE(*,*)'IL IBL IBR',IL,IBL,IBR
C      WRITE(11,*)'IL IBL IBR',IL,IBL,IBR


C    CONSTRUCTION OF THE XXX(NM,NNM) MATRIX
C*    NNM=NUMBER OF VIBRATION MODES WILL BE USED IN THE CALCULATION
C    MAXIMUM VALUE OF NNM IS NM-4
     NNM=4
     DO 1261 I=1,2
     DO 1261 J=1,NNM
     XXX(I,J)=0.D0
 1261 CONTINUE
     DO 1262 I=3,(NFEF+2)*3+1
     XXX(I,1)=XXL(I-2,1)
     XXX(I,2)=XXBL(I-2,1)
     XXX(I,3)=XXBR(I-2,1)
     XXX(I,4)=XXBR(I-2,2)

C     XXX(I,1)=XX(I-2,5)
C     XXX(I,2)=XX(I-2,9)
C     XXX(I,3)=XX(I-2,16)
C     XXX(I,4)=XX(I-2,25)

 1262 CONTINUE
     DO 1263 I=(NFEF+2)*3+2,(NFEF+2)*3+3
     DO 1263 J=1,NNM
     XXX(I,J)=0.D0
 1263 CONTINUE
     DO 1264 I=(NFEF+3)*3+1,NM
     XXX(I,1)=XXL(I-4,1)
     XXX(I,2)=XXBL(I-4,1)
     XXX(I,3)=XXBR(I-4,1)
     XXX(I,4)=XXBR(I-4,2)

C     XXX(I,1)=XX(I-4,5)
C     XXX(I,2)=XX(I-4,9)
C     XXX(I,3)=XX(I-4,16)
C     XXX(I,4)=XX(I-4,25)

 1264 CONTINUE
C     DO 1265 J=1,NNM
C     DO 1265 I=1,NM
C     WRITE(*,*)'I J XXX(I,J)',I,J,XXX(I,J)
C     WRITE(11,*)'I J XXX(I,J)',I,J,XXX(I,J)
C 1265 CONTINUE

C    ORDERING OF THE TAKEN EIGENVALUES FROM THE BIGGEST ONE
C    TO THE SMALLEST ONE
     EVALUE(1)=EIGVL(1)
     EVALUE(2)=EIGVBL(1)
     EVALUE(3)=EIGVBR(1)
     EVALUE(4)=EIGVBR(2)
     DO 8250 I=1,4-1
     DO 8250 J=I+1,4
     IF(EVALUE(I).GE.EVALUE(J)) GO TO 8250
     ZQ=EVALUE(I)
     EVALUE(I)=EVALUE(J)
     EVALUE(J)=ZQ
```

```
8250 CONTINUE

      DO 8252 I=1,4
      WRITE(11,*)'I EVALUE(I)',I,EVALUE(I)
8252  CONTINUE

C     CALCULATION OF STRUCTURAL DAMPING MATRIX OF THE BODY 1:SD(NM,NM)

C     STRUCTURAL DAMPING COEFFICIENTS
      SDC1=0.02D0
      SDC2=0.02D0
C*    THE SMALLEST TWO FREQUENCIES OF TAKEN MODES
      OME1=DSQRT(EVALUE(4))
      OME2=DSQRT(EVALUE(3))
      ALFAK=2*OME1*OME2*(SDC1*OME2-SDC2*OME1)/((OME2)**2
     + -(OME1)**2)
      BETAK=2*(SDC1*OME1-SDC2*OME2)/((OME1)**2-(OME2)**2)
      DO 8263 I=1,NM
      DO 8263 J=1,NM
      SD(I,J)=ALFAK*SM(I,J)+BETAK*SS(I,J)
 8263 CONTINUE
      WRITE(*,*)'OME1 OME2',OME1,OME2
      WRITE(11,*)'OME1 OME2',OME1,OME2
      WRITE(*,*)'ALFAK BETAK',ALFAK,BETAK
      WRITE(11,*)'ALFAK BETAK',ALFAK,BETAK

C     CALCULATION OF TIME-INVARIANT MATRICES IN ADDITION TO THE
C     STRUCTURAL MASS, STRUCTURAL STIFNESS AND STRUCTURAL DAMPING
C     MATRICES
C
C     CALCULATION OF A MATRIX:A1(2,NM)
      DO 3000 K=1,M
      DO 3000 I=1,2
      DO 3000 J=1,NM
      AT(K,I,J)=0.D0
      A1(I,J)=0.D0
 3000 CONTINUE
      K=0
      DO 3010 I=1,NM-5,3
      K=K+1
      RL=CRA(K,1)
C     TW(K)=RO*AR(K)*RL
      AT(K,1,I)=(1.D0/2.D0)*TW(K)
      AT(K,1,I+1)=0.D0
      AT(K,1,I+2)=0.D0
      AT(K,1,I+3)=(1.D0/2.D0)*TW(K)
      AT(K,1,I+4)=0.D0
      AT(K,1,I+5)=0.D0
      AT(K,2,I)=0.D0
      AT(K,2,I+1)=(1.D0/2.D0)*TW(K)
      AT(K,2,I+2)=(1.D0/12.D0)*RL*TW(K)
      AT(K,2,I+3)=0.D0
      AT(K,2,I+4)=(1.D0/2.D0)*TW(K)
      AT(K,2,I+5)=-(1.D0/12.D0)*RL*TW(K)
 3010 CONTINUE
C     ADDITION OF THE MATRICES
      IK=M*3+1
      DO 3020 I=1,2
      DO 3020 J=1,3
      A1(I,J)=AT(1,I,J)
 3020 CONTINUE
      K=0
      LT=1
      DO 3030 K=1,M-1
      LT=LT+3
      DO 3030 I=1,2
      DO 3030 J=LT,LT+2
      A1(I,J)=AT(K,I,J)+AT(K+1,I,J)
 3030 CONTINUE
      DO 3040 I=1,2
      DO 3040 J=IK,IK+2
      A1(I,J)=AT(M,I,J)
 3040 CONTINUE
C
C     CALCULATION OF A*X MATRIX:YOO1(2,NNM)
C     A1(2,NM)*XXX(NM,NNM)=YOO1(2,NNM)
      DO 3080 I=1,2
      DO 3080 J=1,NNM
      YOO1(I,J)=0.D0
      DO 3080 K=1,NM
      YOO1(I,J)=YOO1(I,J)+A1(I,K)*XXX(K,J)
```

```
 3080 CONTINUE
C
C    CALCULATION OF  Mzz  ELEMENT:AMZZ(2,2)
      AMS=0.D0
      DO 3090 K=1,M
      AMS=AMS+TW(K)
 3090 CONTINUE
      AMZZ(1,1)=AMS
      AMZZ(1,2)=0.D0
      AMZZ(2,1)=0.D0
      AMZZ(2,2)=AMS
C
C    CALCULATION OF AA MATRIX:AA1(NM,NM)
      DO 3120 I=1,M
      DO 3120 J=1,NM
      DO 3120 K=1,NM
      AA1(J,K)=0.D0
      AAT(I,J,K)=0.D0
 3120 CONTINUE

      K=0
      DO 3130 I=1,NM-5,3
      K=K+1
      RL=CRA(K,1)

      AAT(K,I,I)=0.D0
      AAT(K,I,I+1)=21.D0*TW(K)/60.D0
      AAT(K,I,I+2)=3.D0*RL*TW(K)/60.D0
      AAT(K,I,I+3)=0.D0
      AAT(K,I,I+4)=9.D0*TW(K)/60.D0
      AAT(K,I,I+5)=-2.D0*RL*TW(K)/60.D0
      AAT(K,I+1,I)=-AAT(K,I,I+1)
      AAT(K,I+1,I+1)=0.D0
      AAT(K,I+1,I+2)=0.D0
      AAT(K,I+1,I+3)=-9.D0*TW(K)/60.D0
      AAT(K,I+1,I+4)=0.D0
      AAT(K,I+1,I+5)=0.D0
      AAT(K,I+2,I)=-AAT(K,I,I+2)
      AAT(K,I+2,I+1)=-AAT(K,I+1,I+2)
      AAT(K,I+2,I+2)=0.D0
      AAT(K,I+2,I+3)=-2.D0*RL*TW(K)/60.D0
      AAT(K,I+2,I+4)=0.D0
      AAT(K,I+2,I+5)=0.D0
      AAT(K,I+3,I)=-AAT(K,I,I+3)
      AAT(K,I+3,I+1)=-AAT(K,I+1,I+3)
      AAT(K,I+3,I+2)=-AAT(K,I+2,I+3)
      AAT(K,I+3,I+3)=0.D0
      AAT(K,I+3,I+4)=21.D0*TW(K)/60.D0
      AAT(K,I+3,I+5)=-3.D0*RL*TW(K)/60.D0
      AAT(K,I+4,I)=-AAT(K,I,I+4)
      AAT(K,I+4,I+1)=-AAT(K,I+1,I+4)
      AAT(K,I+4,I+2)=-AAT(K,I+2,I+4)
      AAT(K,I+4,I+3)=-AAT(K,I+3,I+4)
      AAT(K,I+4,I+4)=0.D0
      AAT(K,I+4,I+5)=0.D0
      AAT(K,I+5,I)=-AAT(K,I,I+5)
      AAT(K,I+5,I+1)=-AAT(K,I+1,I+5)
      AAT(K,I+5,I+2)=-AAT(K,I+2,I+5)
      AAT(K,I+5,I+3)=-AAT(K,I+3,I+5)
      AAT(K,I+5,I+4)=-AAT(K,I+4,I+5)
      AAT(K,I+5,I+5)=0.D0
 3130 CONTINUE

C    ADDITION OF THE MATRICES
C    M: # OF ELEMENTS , N: # OF NODES , N=M+1
      IK=M*3+1
      IL=(M-1)*3+1
      DO 3140 I=1,3
      DO 3140 J=1,6
      AA1(I,J)=AAT(1,I,J)
 3140 CONTINUE
      LT=-2
      K=0
      KT=1
      DO 3150 K=1,M-1
      LT=LT+3
      KT=KT+3
      DO 3150 I=KT,KT+2
      DO 3150 J=LT,LT+8
      AA1(I,J)=AAT(K,I,J)+AAT(K+1,I,J)
 3150 CONTINUE
```

```
      DO 3160 I=IK,IK+2
      DO 3160 J=IL,IL+5
      AA1(I,J)=AAT(M,I,J)
 3160 CONTINUE
C     WRITE(*,*) 'AA1(I,J) MATRIX IS OBTAINED'
      KT=0
      LSS=0
      DO 3170 I=1,NM
      KT=KT+1
      DO 3170 J=KT,NM
      IF(AA1(I,J).EQ.(-AA1(J,I))) GO TO 3170
      WRITE(*,*)'I J AA1(I,J)',I,J,AA1(I,J)
      WRITE(*,*)'J I AA1(J,I)',J,I,AA1(J,I)
      LSS=LSS+1
 3170 CONTINUE
      IF(LSS.NE.0) GO TO 3180
C     WRITE(*,*)'AA1(I,J) IS A SKEW SYMMETRIC MATRIX'
      GO TO 3190
 3180 WRITE(*,*)'AA1(I,J) IS NOT A SKEW SYMMETRIC MATRIX'
 3190 CONTINUE
C
C     CALCULATION OF AA*X MATRIX:YOO2(NM,NNM)
C     AA1(NM,NM)*XXX(NM,NNM)=YOO2(NM,NNM)
      DO 3200 I=1,NM
      DO 3200 J=1,NNM
      YOO2(I,J)=0.D0
      DO 3200 K=1,NM
      YOO2(I,J)=YOO2(I,J)+AA1(I,K)*XXX(K,J)
 3200 CONTINUE
C
C     CALCULATION OF Xtranpose*SM MATRIX:YOO3(NNM,NM)
      DO 3220 I=1,NM
      DO 3220 J=1,NNM
      XXXT(J,I)=XXX(I,J)
 3220 CONTINUE
C     XXXT(NNM,NM)*SM(NM,NM)=YOO3(NNM,NM)
      DO 3230 I=1,NNM
      DO 3230 J=1,NM
      YOO3(I,J)=0.D0
      DO 3230 K=1,NM
      YOO3(I,J)=YOO3(I,J)+XXXT(I,K)*SM(K,J)
 3230 CONTINUE
C
C     CALCULATOIN OF Mnn ELEMENT
C     SINCE EIGENVECTORS ARE NORMALIZED, Mnn MUST BE IDENTITY MATRIX
C     YOO3(NNM,NM)*XXX(NM,NNM)=AMNN(NNM,NNM)
      DO 3240 I=1,NNM
      DO 3240 J=1,NNM
      AMNN(I,J)=0.D0
      DO 3240 K=1,NM
      AMNN(I,J)=AMNN(I,J)+YOO3(I,K)*XXX(K,J)
 3240 CONTINUE
C     CHECKING THE AMNN(NNM,NNM) MATRIX IS AN IDENTITY MATRIX OR NOT
      K=0
      DO 3250 I=1,NNM
      DO 3250 J=1,NNM
      IF(I.NE.J) GO TO 3260
      IF(DABS(AMNN(I,J)-1.D0).LE.1.D-6) GO TO 3250
      K=K+1
C     WRITE(*,*)'I J AMNN(I,J)',I,J,AMNN(I,J)
      WRITE(13,*)'I J AMNN(I,J)',I,J,AMNN(I,J)
      GO TO 3250
 3260 IF(DABS(AMNN(I,J)).LE.1.D-7) GO TO 3250
      K=K+1
C     WRITE(*,*)'I J AMNN(I,J)',I,J,AMNN(I,J)
      WRITE(13,*)'I J AMNN(I,J)',I,J,AMNN(I,J)
 3250 CONTINUE
      IF(K.EQ.0) GO TO 3270
C     WRITE(*,*)'AMNN(I,J) IS NOT AN IDENTITY MATRIX.'
C     WRITE(*,*)'SO, THERE IS AN ERROR'
C     WRITE(13,*)'AMNN(I,J) IS NOT AN IDENTITY MATRIX.'
C     WRITE(13,*)'SO, THERE IS AN ERROR'
C     WRITE(*,*)'K',K
C     WRITE(13,*)'K',K

C     DO 3280 I=1,NNM
C     DO 3280 J=1,NNM
C     WRITE(*,*)'I J AMNN(I,J)',I,J,AMNN(I,J)
C     WRITE(13,*)'I J AMNN(I,J)',I,J,AMNN(I,J)
C 3280 CONTINUE
C 3290 FORMAT('I J AMNN(I,J)',I2,2X,I2,2X,D32.1)
```

```
 3270 CONTINUE
C
C   CALCULATION OF SM*X MATRIX:YOO4(NM,NNM)
C   SM(NM,NM)*XXX(NM,NNM)=YOO4(NM,NNM)
    DO 3300 I=1,NM
    DO 3300 J=1,NNM
    YOO4(I,J)=0.D0
    DO 3300 K=1,NM
    YOO4(I,J)=YOO4(I,J)+SM(I,K)*XXX(K,J)
 3300 CONTINUE
C
C   CALCULATION OF Xtranspose*AA*X MATRIX:YOO5(NNM,NNM)
C   XXXT(NNM,NM)*YOO2(NM,NNM)=YOO5(NNM,NNM)
    DO 3310 I=1,NNM
    DO 3310 J=1,NNM
    YOO5(I,J)=0.D0
    DO 3310 K=1,NM
    YOO5(I,J)=YOO5(I,J)+XXXT(I,K)*YOO2(K,J)
 3310 CONTINUE
C
C   TRANSPOZE OF A1 MATRIX:A1T(NM,2)
    DO 3320 I=1,2
    DO 3320 J=1,NM
    A1T(J,I)=A1(I,J)
 3320 CONTINUE
C   CALCULATION OF Xtranspose*Atranspose MATRIX:YOO6(NNM,2)
C   XXXT(NNM,NM)*A1T(NM,2)=YOO6(NNM,2)
    DO 3330 I=1,NNM
    DO 3330 J=1,2
    YOO6(I,J)=0.D0
    DO 3330 K=1,NM
    YOO6(I,J)=YOO6(I,J)+XXXT(I,K)*A1T(K,J)
 3330 CONTINUE
C
C   XXXT(NNM,NM)*SS(NM,NM)=YOO7(NNM,NM)
    DO 3340 I=1,NNM
    DO 3340 J=1,NM
    YOO7(I,J)=0.D0
    DO 3340 K=1,NM
    YOO7(I,J)=YOO7(I,J)+XXXT(I,K)*SS(K,J)
 3340 CONTINUE
C   YOO7(NNM,NM)*XXX(NM,NNM)=YOO8(NNM,NNM)
    DO 3350 I=1,NNM
    DO 3350 J=1,NNM
    YOO8(I,J)=0.D0
    DO 3350 K=1,NM
    YOO8(I,J)=YOO8(I,J)+YOO7(I,K)*XXX(K,J)
 3350 CONTINUE
C
C   XXXT(NNM,NM)*SD(NM,NM)=YOO9(NNM,NM)
    DO 3360 I=1,NNM
    DO 3360 J=1,NM
    YOO9(I,J)=0.D0
    DO 3360 K=1,NM
    YOO9(I,J)=YOO9(I,J)+XXXT(I,K)*SD(K,J)
 3360 CONTINUE
C   YOO9(NNM,NM)*XXX(NM,NNM)=YOO10(NNM,NNM)
    DO 3370 I=1,NNM
    DO 3370 J=1,NNM
    YOO10(I,J)=0.D0
    DO 3370 K=1,NM
    YOO10(I,J)=YOO10(I,J)+YOO9(I,K)*XXX(K,J)
 3370 CONTINUE
C
C   CALCULATION OF SOME TIME-INVARIANT MATRICES USING IN THE
C   CONSTRAINT EQUATIONS 6 AND 7
C   ICE4=NUMBER OF ELEMENTS BETWEEN Ao AND A
    ICE4=NFEF+2
    X=CRA(4,1)
    YE=0.D0
    RL=CRA(4,1)
C
C   CONSTRUCTION OF SHAPE FUNCTION MATRIX
C
    FAY(1,1)=1.D0-X/RL
    FAY(1,2)=6.D0*YE/RL*(X/RL-X*X/(RL*RL))
    FAY(1,3)=-YE*(1.D0-4.D0*X/RL+3.D0*X*X/(RL*RL))
    FAY(1,4)=X/RL
    FAY(1,5)=-6.D0*YE/RL*(X/RL-X*X/(RL*RL))
    FAY(1,6)=-YE*(-2.D0*X/RL+3.D0*X*X/(RL*RL))
    FAY(2,1)=0.D0
```

```
      FAY(2,2)=1.D0-3.D0*X*X/(RL*RL)+2.D0*X*X*X/(RL*RL*RL)
      FAY(2,3)=RL*(X/RL-2.D0*X*X/(RL*RL)+X*X*X/(RL*RL*RL))
      FAY(2,4)=0.D0
      FAY(2,5)= 3.D0*X*X/(RL*RL)-2.D0*X*X*X/(RL*RL*RL)
      FAY(2,6)=RL*(-X*X/(RL*RL)+X*X*X/(RL*RL*RL))
C
C   CONSTRUCTION OF SHAPE FUNCTION MATRIX FOR ROTATION
C
      CSI(1,1)=0.D0
      CSI(1,2)=-6.D0*X/(RL*RL)+6.D0*X*X/((RL)**3)
      CSI(1,3)=RL*(1.D0/RL-4.D0*X/(RL*RL)+3.D0*X*X/((RL)**3))
      CSI(1,4)=0.D0
      CSI(1,5)=6.D0*X/(RL*RL)-6.D0*X*X/((RL)**3)
      CSI(1,6)=RL*(-2.D0*X/(RL*RL)+3.D0*X*X/((RL)**3))
C
C   CONSTRUCTION OF BOOLEAN MATRIX
C
      DO 3500 I=1,6
      DO 3500 J=1,NM
      BOCE(I,J)=0.D0
 3500 CONTINUE
      ICE5=(ICE4-1)*3+1
      ICE6=(ICE4-1)*3
      DO 3510 I=1,6
      DO 3510 J=ICE5,ICE5+5
      IF(I.NE.(J-ICE6)) GO TO 3510
      BOCE(I,J)=1.D0
 3510 CONTINUE
C
C   FAY(2,6)*BOCE(6,NM)=YOO11(2,NM)
      DO 3520 I=1,2
      DO 3520 J=1,NM
      YOO11(I,J)=0.D0
      DO 3520 K=1,6
      YOO11(I,J)=YOO11(I,J)+FAY(I,K)*BOCE(K,J)
 3520 CONTINUE
C
C   YOO11(2,NM)*XXX(NM,NNM)=YOO12(2,NNM)
      DO 3530 I=1,2
      DO 3530 J=1,NNM
      YOO12(I,J)=0.D0
      DO 3530 K=1,NM
      YOO12(I,J)=YOO12(I,J)+YOO11(I,K)*XXX(K,J)
 3530 CONTINUE
C
C   CSI(1,6)*BOCE(6,NM)=YOO13(1,NM)
      DO 3540 J=1,NM
      YOO13(1,J)=0.D0
      DO 3540 K=1,6
      YOO13(1,J)=YOO13(1,J)+CSI(1,K)*BOCE(K,J)
 3540 CONTINUE
C   YOO13(1,NM)*XXX(NM,NNM)=YOO14(1,NNM)
      DO 3550 J=1,NNM
      YOO14(1,J)=0.D0
      DO 3550 K=1,NM
      YOO14(1,J)=YOO14(1,J)+YOO13(1,K)*XXX(K,J)
 3550 CONTINUE
C
C   CALCULATION OF SOME TIME-INVARIANT MATRICES USING IN THE
C   CONSTRAINT EQUATIONS 12 AND 13
      DO 3555 I=1,2
      DO 3555 J=1,6
      FAY(I,J)=0.D0
 3555 CONTINUE
      X=CRA(M,1)
      YE=0.D0
      RL=CRA(M,1)
C
C   CONSTRUCTION OF SHAPE FUNCTION MATRIX
C
      FAY(1,1)=1.D0-X/RL
      FAY(1,2)=6.D0*YE/RL*(X/RL-X*X/(RL*RL))
      FAY(1,3)=-YE*(1.D0-4.D0*X/RL+3.D0*X*X/(RL*RL))
      FAY(1,4)=X/RL
      FAY(1,5)=-6.D0*YE/RL*(X/RL-X*X/(RL*RL))
      FAY(1,6)=-YE*(-2.D0*X/RL+3.D0*X*X/(RL*RL))
      FAY(2,1)=0.D0
      FAY(2,2)=1.D0-3.D0*X*X/(RL*RL)+2.D0*X*X*X/(RL*RL*RL)
      FAY(2,3)=RL*(X/RL-2.D0*X*X/(RL*RL)+X*X*X/(RL*RL*RL))
      FAY(2,4)=0.D0
      FAY(2,5)= 3.D0*X*X/(RL*RL)-2.D0*X*X*X/(RL*RL*RL)
```

```
      FAY(2,6)=RL*(-X*X/(RL*RL)+X*X*X/(RL*RL*RL))
C
C    CONSTRUCTION OF BOOLEAN MATRIX
      DO 3560 I=1,6
      DO 3560 J=1,NM
      BOCE(I,J)=0.D0
 3560 CONTINUE
      DO 3570 I=1,6
      DO 3570 J=NM-5,NM
      IF(I.NE.(J-(NM-6))) GO TO 3570
      BOCE(I,J)=1.D0
 3570 CONTINUE
C
C    FAY(2,6)*BOCE(6,NM)=YOO15(2,NM)
      DO 3580 I=1,2
      DO 3580 J=1,NM
      YOO15(I,J)=0.D0
      DO 3580 K=1,6
      YOO15(I,J)=YOO15(I,J)+FAY(I,K)*BOCE(K,J)
 3580 CONTINUE
C
C    YOO15(2,NM)*XXX(NM,NNM)=YOO16(2,NNM)
      DO 3590 I=1,2
      DO 3590 J=1,NNM
      YOO16(I,J)=0.D0
      DO 3590 K=1,NM
      YOO16(I,J)=YOO16(I,J)+YOO15(I,K)*XXX(K,J)
 3590 CONTINUE

C    VELOCITY PROFILE INPUTS AND CALCULATIONS

C    ADLEN=LENGTH OF A-D
      ADLEN=0.565D0
C    AODLEN=LENGTH OF Ao-D (BODY 2)
      AODLEN=5.85D0
      TIME1=1.5D0
      TIME2=28.5D0
      TIME3=30.D0
C    THETA1I=INITIAL VALUE OF THE BOOM ANGULAR POSITION (radian)
      THETA1I=PI/180.D0*40.D0
C    THETA1F=FINAL VALUE OF THE BOOM ANGULAR POSITION (radian)
      THETA1F=PI/180.D0*80.D0
C     RGAMMA=DASIN(ADLEN,AODLEN)
      RGAMMA=0.09673198D0
      THETA2I=THETA1I-RGAMMA
      THETA2F=THETA1F-RGAMMA
      THETA3I=DATAN2((5.85D0*DSIN(THETA2I)+0.805D0),(5.85D0*
     + DCOS(THETA2I)-2.35D0))
      THETA3F=DATAN2((5.85D0*DSIN(THETA2F)+0.805D0),(5.85D0*
     + DCOS(THETA2F)-2.35D0))
      DISTANCEI=(5.85D0*DSIN(THETA2I)-3.455D0*DSIN(THETA3I)+
     + 0.805D0)/DSIN(THETA3I)
      DISTANCEF=(5.85D0*DSIN(THETA2F)-3.455D0*DSIN(THETA3F)+
     + 0.805D0)/DSIN(THETA3F)
      VZERON=DISTANCEF-DISTANCEI
      VZEROD=(0.5D0*TIME1+(TIME2-TIME1)+0.5D0*(TIME3-TIME2))
      VZERO=(DISTANCEF-DISTANCEI)/(0.5D0*TIME1+(TIME2-TIME1)+
     + 0.5D0*(TIME3-TIME2))
C     IF THE CRANE MOVES UPWARD DIRECTION (i.e.,FROM SMALLER THETA1
C     TO BIGGER THETA1), VZERO IS POSITIVE
C     IF THE CRANE MOVES DOWNWARD DIRECTION (i.e.,FROM BIGGER THETA1
C     TO SMALLER THETA1), VZERO IS NEGATIVE
      WRITE(*,*)'DISTANCEI DISTANCEF',DISTANCEI,DISTANCEF
      WRITE(11,*)'DISTANCEI DISTANCEF',DISTANCEI,DISTANCEF
      WRITE(*,*)'VZERON',VZERON
      WRITE(11,*)'VZERON',VZERON
      WRITE(*,*)'VZEROD',VZEROD
      WRITE(11,*)'VZEROD',VZEROD
      WRITE(*,*)'VZERO',VZERO
      WRITE(11,*)'VZERO',VZERO

C     CALCULATION OF INITIAL DEFLECTION OF BODY 1
C
C     SECOND METHOD USING F=KX EQUATION'S
C
C*
C     WF=MASS OF THE LIFTED LOAD (kg)
C      WF=4400.D0
C      WF=0.D0
C
      DO 1510 IJK=1,7
```

```
      IF(IJK.EQ.1) WF=3300.D0
      IF(IJK.EQ.2) WF=3300.D0
      IF(IJK.EQ.3) WF=4400.D0
      IF(IJK.EQ.4) WF=6225.D0
      IF(IJK.EQ.5) WF=10200.D0
      IF(IJK.EQ.6) WF=13650.D0
      IF(IJK.EQ.7) WF=15000.D0
C
      WRITE(14,*)'MASS OF THE LIFTED LOAD(kg) WF=',WF
      WRITE(14,*)
      K=0
      FORCE(1,1)=-(TW(1)*9.81D0/2.D0)*DSIN(THETA1I)
      FORCE(2,1)=-(TW(1)*9.81D0/2.D0)*DCOS(THETA1I)
      FORCE(3,1)=0.D0
      DO 8265 I=4,NM-5,3
      K=K+1
      FORCE(I,1)=-(TW(K)+TW(K+1))*9.81D0/2.D0*DSIN(THETA1I)
      FORCE(I+1,1)=-(TW(K)+TW(K+1))*9.81D0/2.D0*DCOS(THETA1I)
      FORCE(I+2,1)=0.D0
 8265 CONTINUE
      FORCE(NM-2,1)=-(TW(M)*9.81D0/2.D0+WF*9.81D0)*DSIN(THETA1I)
      FORCE(NM-1,1)=-(TW(M)*9.81D0/2.D0+WF*9.81D0)*DCOS(THETA1I)
      FORCE(NM,1)=0.D0


C     CALCULATION OF LEFT PART DEFLECTIONS:CMOT1(11,1)
C
C     APPLY THE B.C.'s
C     FROM 3TH ROW TO 14TH ROW OF FORCE MATRIX ARE TAKEN
      DO 8267 I=3,(NFEF+2)*3+1
      FORCEL(I-2,1)=FORCE(I,1)
 8267 CONTINUE

C     FROM 3TH TO 14TH, ROWS AND COLUMNS OF STRUCTURAL STIFFNESS
C     MATRIX ARE TAKEN WHICH WAS CONSTRUCTED AS AU1(11,11) MATRIX
C
C     CALCULATION OF INVERSE OF THE AU1(11,11) MATRIX
C
C     FIRST METHOD USING THE SUBROUTINE ELIM(AB,N,NP)

C     PREPARE THE REQUIRED PARAMETERS FOR THE SUBROUTINE ELIM(AB,N,NP)
C     ABXX(MN,NPSEL)=COEFFICENT MATRIX AUGMENTED WITH R.H.S. VECTORS
C     MN=NUMBER OF EQUATIONS
C     NPSEL=TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX=2*MN
C
      DO 8269 I=3,(NFEF+2)*3+1
      DO 8269 J=3,(NFEF+2)*3+1
      AU1(I-2,J-2)=SS(I,J)
 8269 CONTINUE

      DO 8270 I=1,(NFEF+2)*3-1
      DO 8270 J=1,(NFEF+2)*3-1
      ABXX1(I,J)=AU1(I,J)
 8270 CONTINUE
      DO 8272 I=1,(NFEF+2)*3-1
      DO 8272 J=(NFEF+2)*3,2*((NFEF+2)*3-1)
      IF(I.NE.(J-((NFEF+2)*3-1))) GO TO 8273
      ABXX1(I,J)=1.D0
      GO TO 8272
 8273 ABXX1(I,J)=0.D0
 8272 CONTINUE

      CALL ELIM(ABXX1,(NFEF+2)*3-1,2*((NFEF+2)*3-1))
      DO 8274 I=1,(NFEF+2)*3-1
      DO 8274 J=(NFEF+2)*3,2*((NFEF+2)*3-1)
      ANV1(I,J-((NFEF+2)*3-1))=ABXX1(I,J)
 8274 CONTINUE

C     SECOND METHOD USING THE SUBROUTINE INVERS(NSEL,SEL,SELI)

C     ANV1(11,11)=INVERSE OF THE AU1(11,11) MATRIX
C      CALL INVERS(11,AU1,ANV1)

      DO 8275 I=1,11
      CMOT1(I,1)=0.D0
      DO 8275 K=1,11
      CMOT1(I,1)=CMOT1(I,1)+ANV1(I,K)*FORCEL(K,1)
 8275 CONTINUE
```

```
C    CALCULATION OF RIGHT PART DEFLECTIONS:CMOT(NM-15,1)
C
C    APPLY THE B.C.'s
C    FIRST 15 ROWS OF FORCE MATRIX ARE DROPPED
     DO 1267 I=(NFEF+3)*3+1,NM
     FORCEN(I-((NFEF+3)*3),1)=FORCE(I,1)
1267 CONTINUE

C    FIRST 15 ROWS AND COLUMNS OF STRUCTURAL STIFFNESS
C    MATRIX ARE DROPPED WHICH WAS CONSTRUCTED AS AU(NM-15,NM-15) MATRIX
C
C    CALCULATION OF INVERSE OF THE AU(NM-15,NM-15) MATRIX
C
C    FIRST METHOD USING THE SUBROUTINE ELIM(AB,N,NP)

C    PREPARE THE REQUIRED PARAMETERS FOR THE SUBROUTINE ELIM(AB,N,NP)
C    ABXX(MN,NPSEL)=COEFFICENT MATRIX AUGMENTED WITH R.H.S. VECTORS
C    MN=NUMBER OF EQUATIONS
C    NPSEL=TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX=2*MN
C
     DO 1269 I=(NFEF+3)*3+1,NM
     DO 1269 J=(NFEF+3)*3+1,NM
     AU(I-((NFEF+3)*3),J-((NFEF+3)*3))=SS(I,J)
1269 CONTINUE

     DO 1270 I=1,NM-15
     DO 1270 J=1,NM-15
     ABXX(I,J)=AU(I,J)
1270 CONTINUE
     DO 1272 I=1,NM-15
     DO 1272 J=NM-14,2*(NM-15)
     IF(I.NE.(J-(NM-15))) GO TO 1273
     ABXX(I,J)=1.D0
     GO TO 1272
1273 ABXX(I,J)=0.D0
1272 CONTINUE

     CALL ELIM(ABXX,NM-15,2*(NM-15))
     DO 1274 I=1,NM-15
     DO 1274 J=NM-14,2*(NM-15)
     ANV(I,J-(NM-15))=ABXX(I,J)
1274 CONTINUE

C    SECOND METHOD USING THE SUBROUTINE INVERS(NSEL,SEL,SELI)

C    ANV(NM-15,NM-15)=INVERSE OF THE AU(NM-15,NM-15) MATRIX
C     CALL INVERS(NM-15,AU,ANV)

     DO 1275 I=1,NM-15
     CMOT(I,1)=0.D0
     DO 1275 K=1,NM-15
     CMOT(I,1)=CMOT(I,1)+ANV(I,K)*FORCEN(K,1)
1275 CONTINUE
     DO 1277 I=1,2
     CMOH(I,1)=0.D0
1277 CONTINUE
     DO 1279 I=3,(NFEF+2)*3+1
     CMOH(I,1)=CMOT1(I-2,1)
1279 CONTINUE
     DO 1281 I=(NFEF+2)*3+2,(NFEF+3)*3
     CMOH(I,1)=0.D0
1281 CONTINUE
     DO 1283 I=(NFEF+3)*3+1,NM
     CMOH(I,1)=CMOT(I-((NFEF+3)*3),1)
1283 CONTINUE

     DO 1280 I=1,M+1
     CRAT(I,1)=0.D0
1280 CONTINUE
     DO 1285 I=2,M+1
     CRAT(I,1)=CRAT(I-1,1)+CRA(I-1,1)
1285 CONTINUE

     DO 1284 I=1,NM
C     WRITE(*,*)'I FORCE(I,1)',I,FORCE(I,1)
     WRITE(14,*)'I FORCE(I,1)',I,FORCE(I,1)
1284 CONTINUE
C     WRITE(*,*)'CMOT(I,1)=INITIAL NODAL DEF. DISP. OF RIGHT SIDE'
     WRITE(14,*)'CMOT(I,1)=INITIAL NODAL DEF. DISP. OF RIGHT SIDE'
     DO 1286 I=1,NM-15
C     WRITE(*,*)'I CMOT(I,1)',I,CMOT(I,1)
```

```
      WRITE(14,*)'I CMOT(I,1)',I,CMOT(I,1)
 1286 CONTINUE
C     WRITE(*,*)'CMOT1(I,1)=INITIAL NODAL DEF. DISP. OF LEFT SIDE'
      WRITE(14,*)'CMOT1(I,1)=INITIAL NODAL DEF. DISP. OF LEFT SIDE'
      DO 1287 I=1,11
C     WRITE(*,*)'I CMOT1(I,1)',I,CMOT1(I,1)
      WRITE(14,*)'I CMOT1(I,1)',I,CMOT1(I,1)
 1287 CONTINUE
C     WRITE(*,*)'CMOH(I,1)=INITIAL NODAL DEF. DISP.'
      WRITE(14,*)'CMOH(I,1)=INITIAL NODAL DEF. DISP.'
      DO 1289 I=1,NM
C     WRITE(*,*)'I CMOH(I,1)',I,CMOH(I,1)
      WRITE(14,*)'I CMOH(I,1)',I,CMOH(I,1)
 1289 CONTINUE
C     WRITE(*,*)'CRAT(K,1) CMOH(I,1)'
      WRITE(14,*)'CRAT(K,1) CMOH(I,1)'
      K=0
      DO 1291 I=2,NM,3
      K=K+1
C     WRITE(*,*) CRAT(K,1),CMOH(I,1)
      WRITE(14,*) CRAT(K,1),CMOH(I,1)
 1291 CONTINUE


      DO 1288 I=1,M
C     WRITE(*,*)'I AR(I)',I,AR(I)
      WRITE(14,*)'I AR(I)',I,AR(I)
 1288 CONTINUE
      DO 1290 I=1,M
C     WRITE(*,*)'I WI(I)',I,WI(I)
      WRITE(14,*)'I WI(I)',I,WI(I)
 1290 CONTINUE
      DO 1292 I=1,M
C     WRITE(*,*)'I TW(I)',I,TW(I)
      WRITE(14,*)'I TW(I)',I,TW(I)
 1292 CONTINUE
C     TRANSPOZE OF XXX(NM,4)
      DO 1294 I=1,NM
      DO 1294 J=1,4
      XXXT(J,I)=XXX(I,J)
 1294 CONTINUE
C     XXXT(4,NM)*XXX(NM,4)=XXXZ(4,4)
      DO 1296 I=1,4
      DO 1296 J=1,4
      XXXZ(I,J)=0.D0
      DO 1296 K=1,NM
      XXXZ(I,J)=XXXZ(I,J)+XXXT(I,K)*XXX(K,J)
 1296 CONTINUE
C     DO 1297 I=1,4
C     DO 1297 J=1,4
C     WRITE(*,*)'I J XXXZ(I,J)',I,J,XXXZ(I,J)
C     WRITE(14,*)'I J XXXZ(I,J)',I,J,XXXZ(I,J)
C 1297 CONTINUE

C     INVERSE OF THE XXXZ(4,4)
C
C     FIRST METHOD USING THE SUBROUTINE ELIM(AB,N,NP)

C     PREPARE THE REQUIRED PARAMETERS FOR THE SUBROUTINE ELIM(AB,N,NP)
C     BXX(MN,NPSEL)=COEFFICENT MATRIX AUGMENTED WITH R.H.S. VECTORS
C     MN=NUMBER OF EQUATIONS
C     NPSEL=TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX=2*MN
C
      DO 1298 I=1,4
      DO 1298 J=1,4
      BXX(I,J)=XXXZ(I,J)
 1298 CONTINUE
      DO 1300 I=1,4
      DO 1300 J=5,2*4
      IF(I.NE.(J-4)) GO TO 1301
      BXX(I,J)=1.D0
      GO TO 1300
 1301 BXX(I,J)=0.D0
 1300 CONTINUE

      CALL ELIM(BXX,4,2*4)
      DO 1302 I=1,4
      DO 1302 J=5,2*4
      XXXI(I,J-4)=BXX(I,J)
 1302 CONTINUE
```

```
C    SECOND METHOD USING THE SUBROUTINE INVERS(NSEL,SEL,SELI)

C    XXXI(4,4)=INVERSE OF THE XXXZ(4,4) MATRIX
C
C     CALL INVERS(4,XXXZ,XXXI)
C
C     DO 1303 I=1,4
C     DO 1303 J=1,4
C     WRITE(*,*)'I J XXXI(I,J)',I,J,XXXI(I,J)
C     WRITE(14,*)'I J XXXI(I,J)',I,J,XXXI(I,J)
C 1303 CONTINUE

C    XXXI(4,4)*XXXT(4,NM)=XXXY(4,NM)
      DO 1304 I=1,4
      DO 1304 J=1,NM
      XXXY(I,J)=0.D0
      DO 1304 K=1,4
      XXXY(I,J)=XXXY(I,J)+XXXI(I,K)*XXXT(K,J)
 1304 CONTINUE
C    XXXY(4,NM)*CMOH(NM,1)=CETA(4,1)
      DO 1306 I=1,4
      CETA(I,1)=0.D0
      DO 1306 K=1,NM
      CETA(I,1)=CETA(I,1)+XXXY(I,K)*CMOH(K,1)
 1306 CONTINUE
      DO 1307 I=1,4
C     WRITE(*,*)'I CETA(I,1)',I,CETA(I,1)
      WRITE(14,*)'I CETA(I,1)',I,CETA(I,1)
 1307 CONTINUE
C*    CODIS=CONSTANT DISTANCE BETWEEN GROUND AND HOOK LOAD
      CODIS=1.D0

C    INITIALIZATION OF ETA VALUES
C     DO 1308 I=4,NNM+3
C     Y(I)=0.D0
C 1308 CONTINUE

      DO 1308 I=4,NNM+3
      Y(I)=CETA(I-3,1)
 1308 CONTINUE
C*
C    HORIZONTAL POSITION OF THE BODY 1 IS TAKEN AS INITIAL POSITION
C    OF THE SYSTEM
C    J=TIME STEP NUMBER
C    INITIALIZATION Y(2*(NNM+15)) MATRIX

      Y(1)=2.775D0
      Y(2)=2.57D0
      Y(3)=THETA1I

      Y(NNM+4)=2.775D0
      Y(NNM+5)=2.57D0
C     RGAMMA=DASIN(ADLEN,AODLEN)
      Y(NNM+6)=Y(3)-RGAMMA
      Y(NNM+10)=5.125D0
      Y(NNM+11)=1.765D0
      Y(NNM+12)=DATAN2((5.85D0*DSIN(Y(NNM+6))+0.805D0),
     + (5.85D0*DCOS(Y(NNM+6))-2.35D0))
      Y(NNM+9)=Y(NNM+12)
      Y(NNM+7)=Y(NNM+10)+((5.85D0*DSIN(Y(NNM+6))-3.455D0*
     + DSIN(Y(NNM+9))+0.805D0)/DSIN(Y(NNM+9)))*DCOS(Y(NNM+9))
      Y(NNM+8)=Y(NNM+11)+((5.85D0*DSIN(Y(NNM+6))-3.455D0*
     + DSIN(Y(NNM+9))+0.805D0)/DSIN(Y(NNM+9)))*DSIN(Y(NNM+9))
      Y(NNM+13)=Y(1)+19.5D0*DCOS(Y(3))+CMOH(37,1)*DCOS(Y(3))-
     + CMOH(38,1)*DSIN(Y(3))
      Y(NNM+14)=Y(2)+19.5D0*DSIN(Y(3))+CMOH(37,1)*DSIN(Y(3))+
     + CMOH(38,1)*DCOS(Y(3))
      Y(NNM+15)=3.D0/2.D0*PI
C    SINCE ANALYSIS FROM THE MECHANISM STATIC POSITION
C    DERIVATIVE OF ALL INITIAL VALUES ARE ZERO
      DO 1309 I=NNM+16,2*NNM+30
      Y(I)=0.D0
 1309 CONTINUE


C     WRITE(*,*)'INITIAL VALUES AT T='
      WRITE(IO,*)'INITIAL VALUES AT T='
C     WRITE(*,*)
      WRITE(IO,*)
      DO 1310 K=1,2*(NNM+15)
C     WRITE(*,9999) K,Y(K)
```

```
      WRITE(IO,9999) K,Y(K)
 1310 CONTINUE
 9999 FORMAT(6X,'Y(',I2,')=',D27.20)
C     DO 1310 K=1,2*(NNM+15)
C     WRITE(*,*)'K,Y(K)',K,Y(K)
C     WRITE(IO,*)'K,Y(K)',K,Y(K)
C 1310 CONTINUE


      IIIIZ=0
      NOO=0

CC    FORWARD EULER METHOD
C     NOO=0
C     I=0
C     T=0.D0
C     DT=9.877513959D-3/50.D0
C     WRITE(*,*)'PLEASE WAIT, PROGRAM IS RUNNING.'
C 1355 CALL TIND(T,Y,YP)
C     WRITE(16,*)'I T',I,T
C     DO 1360 K=1,2*(NNM+15)
CC     WRITE(*,*)'K YP(K)',K,YP(K)
C     IF(I.NE.(NOO*10000)) GO TO 1360
C     WRITE(16,*)'K YP(K)',K,YP(K)
C 1360 CONTINUE
C     WRITE(17,*)'I T',I,T
C     DO 1365 K=1,14
C     WRITE(*,*)'K LAMDA',K,SEQ(K+NNM+15,1)
C     IF(I.NE.(NOO*10000)) GO TO 1365
C     WRITE(17,*)'K LAMDA',K,SEQ(K+NNM+15,1)
C 1365 CONTINUE
C
C     IF(I.NE.(NOO*10000)) GO TO 1400
C     NOO=NOO+1
C 1400 I=I+1
C     IIIIZ=I
C     T=T+DT
C     IF(Y(3).GT.(PI/2.D0)) GO TO 1500
CC     WRITE(*,*)'I T',I,T
C     IF(I.NE.(NOO*10000)) GO TO 1367
C     WRITE(IO,*)'I T',I,T
C 1367 DO 1370 K=1,2*(NNM+15)
C     Y(K)=Y(K)+YP(K)*DT
CC     WRITE(*,*)'K Y(K)',K,Y(K)
C     IF(I.NE.(NOO*10000)) GO TO 1370
C     WRITE(IO,*)'K Y(K)',K,Y(K)
C 1370 CONTINUE
C
C     GO TO 1355
C

C     BACKWARD EULER METHOD

      T=0.D0
      CALL TIND(T,Y,YP)
      WRITE(16,*)'INITIAL VALUES AT T=',T
      DO 1311 K=1,2*(NNM+15)
C     WRITE(*,9998) K,YP(K)
      WRITE(16,9998) K,YP(K)
 1311 CONTINUE
 9998 FORMAT(6X,'YP(',I2,')=',D27.20)
C     DO 1311 K=1,2*(NNM+15)
C     WRITE(*,*)'K,YP(K)',K,YP(K)
C     WRITE(16,*)'K,YP(K)',K,YP(K)
C 1311 CONTINUE

      WRITE(17,*)'INITIAL VALUES AT T=',T
      DO 1312 K=1,14
C     WRITE(*,9997) K,SEQ(K+NNM+15,1)
      WRITE(17,9997) K,SEQ(K+NNM+15,1)
 1312 CONTINUE
 9997 FORMAT(6X,'LAMDA(',I2,')=',D27.20)
C     DO 1312 K=1,14
C     WRITE(*,*)'K LAMDA(K)',K,SEQ(K+NNM+15,1)
C     WRITE(17,*)'K LAMDA(K)',K,SEQ(K+NNM+15,1)
C 1312 CONTINUE
      IF(IJK.NE.1) GO TO 8000
      THE1=180.D0/PI*Y(3)
      WRITE(20,*) THE1,SEQ(NNM+16,1),SEQ(NNM+17,1)
      WRITE(21,*) THE1,SEQ(NNM+18,1),SEQ(NNM+21,1)
      SEQFA1=SEQ(NNM+16,1)+ SEQ(NNM+18,1)
```

```
      SEQFA2=SEQ(NNM+17,1)+ SEQ(NNM+21,1)
      WRITE(22,*) THE1,SEQFA1,SEQFA2
      WRITE(23,*) THE1,SEQ(NNM+19,1),SEQ(NNM+20,1)
      WRITE(24,*) THE1,SEQ(NNM+22,1),SEQ(NNM+26,1)
 8000 CONTINUE
C     THE1=180.D0/PI*Y(3)
C     SEQOI=-1.D0*SEQ(33,1)/1.D+3
C     WRITE(18,7000) Y(3),THE1,SEQOI,T

C     NOOM=5000 MEANS RESULTS ARE WRITTEN EACH 5000 STEPS
      NOOM=125000
C     NOOM2=13 MEANS RESULTS ARE WRITTEN EACH 13 STEPS
      NOOM2=50
      NZZZ=0
      NOO=1
      NOO2=1
      I=0
      T=0.D0

C     DT=9.877513959D-3/20.D0

      DT=(2.D0*PI/DSQRT(EVALUE(1)))/20.D0
      WRITE(11,*)'DT',DT
      DO 1313 K=1,2*(NNM+15)
      YI(K)=Y(K)
      YV(K)=Y(K)
 1313 CONTINUE
C     WRITE(*,*)'PLEASE WAIT, PROGRAM IS RUNNING'
 1314 I=I+1
      IIIIZ=I
      T=T+DT
      IF(T.GT.30.3D0) GO TO 1500
 1320 CALL TIND(T,YI,YP)
      DO 1323 K=1,2*(NNM+15)
      YI(K)=Y(K)+YP(K)*DT
 1323 CONTINUE
      DO 1326 K=1,2*(NNM+15)
      IF(DABS(YI(K)-YV(K)).GT.1.D-10) GO TO 1330
 1326 CONTINUE
 1328 DO 1332 K=1,2*(NNM+15)
      Y(K)=YI(K)
      YV(K)=YI(K)
 1332 CONTINUE
C     WRITE(*,*)'ITERATION COMPLETED AT THE STEP I',I
C     WRITE(*,*)'TIME T (sec)=',T
C     WRITE(*,*)'TOTAL NUMBER OF ITERATION NZZZ=',NZZZ
C
      FJK=(-(SEQ(NNM+16,1)+SEQ(NNM+18,1))*2.57D0-(SEQ(NNM+17,1)+
     + SEQ(NNM+21,1))*4.53D0-SEQ(NNM+19,1)*1.765D0-SEQ(NNM+20,1)*
     + 2.18D0+222792.772D0*4.125D0)
      Y3D=Y(3)*180.D0/PI
      WRITE(*,*)'Y3D FJK',Y3D,FJK
      IF(FJK.LE.0.D0) GO TO 1500
C     GO TO 1334
C
      IF(I.NE.(NOO2*NOOM2)) GO TO 7001
      IF(IJK.NE.1) GO TO 7001
      THE1=180.D0/PI*Y(3)
      WRITE(20,*) THE1,SEQ(NNM+16,1),SEQ(NNM+17,1)
      WRITE(21,*) THE1,SEQ(NNM+18,1),SEQ(NNM+21,1)
      SEQFA1=SEQ(NNM+16,1)+ SEQ(NNM+18,1)
      SEQFA2=SEQ(NNM+17,1)+ SEQ(NNM+21,1)
      WRITE(22,*) THE1,SEQFA1,SEQFA2
      WRITE(23,*) THE1,SEQ(NNM+19,1),SEQ(NNM+20,1)
C     WRITE(24,*) THE1,SEQ(NNM+22,1),SEQ(NNM+26,1)
      NOO2=NOO2+1
      GO TO 7001
      THE1=180.D0/PI*Y(3)
      SEQOI=-1.D0*SEQ(33,1)/1.D+3
      WRITE(18,7000) Y(3),THE1,SEQOI,T
 7000 FORMAT(1X,D20.13,8X,D20.13,8X,D20.13,8X,D12.5)
C     NOO2=NOO2+1
 7001 IF(I.NE.(NOO*NOOM)) GO TO 1334
 2100 WRITE(*,*)'I T',I,T
      WRITE(IO,*)'I T',I,T
      DO 1338 K=1,2*(NNM+15)
      WRITE(*,9999) K,Y(K)
      WRITE(IO,9999) K,Y(K)
 1338 CONTINUE
C     DO 1338 K=1,2*(NNM+15)
C     WRITE(*,*)'K,Y(K)',K,Y(K)
```

```
C     WRITE(IO,*)'K,Y(K)',K,Y(K)
C 1338 CONTINUE
      WRITE(16,*)'I T',I,T
      DO 1340 K=1,2*(NNM+15)
      WRITE(*,9998) K,YP(K)
      WRITE(16,9998) K,YP(K)
 1340 CONTINUE
C     DO 1340 K=1,2*(NNM+15)
C     WRITE(*,*)'K,YP(K)',K,YP(K)
C     WRITE(16,*)'K,YP(K)',K,YP(K)
C 1340 CONTINUE

      WRITE(17,*)'I T',I,T
      DO 1342 K=1,14
      WRITE(*,9997) K,SEQ(K+NNM+15,1)
      WRITE(17,9997) K,SEQ(K+NNM+15,1)
 1342 CONTINUE
C     DO 1342 K=1,14
C     WRITE(*,*)'K LAMDA(K)',K,SEQ(K+NNM+15,1)
C     WRITE(17,*)'K LAMDA(K)',K,SEQ(K+NNM+15,1)
C 1342 CONTINUE

      NOO=NOO+1
      WRITE(*,*)'I T',I,T
      WRITE(*,*)'NUMBER OF ITERATION  NZZZ=',NZZZ
      WRITE(IO,*)'NUMBER OF ITERATION  NZZZ=',NZZZ
 1334 NZZZ=0
      GO TO 1314
 1330 NZZZ=NZZZ+1
C     WRITE(*,*)'I T',I,T
C     WRITE(*,*)'I T IT NO',I,T,NZZZ
C     DO 1400 K=1,2*(NNM+15)
C     WRITE(*,*)'K YV(K) YI(K)',K,YV(K),YI(K)
C 1400 CONTINUE
      DO 1344 K=1,2*(NNM+15)
      YV(K)=YI(K)
 1344 CONTINUE
      GO TO 1320
C
 1500 WFS=(2.D0/3.D0)*WF/1.D+3
      Y3D=Y(3)*180.D0/PI
      RADIUS=Y(NNM+13)-Y(1)-1.83D0
      WRITE(*,*) WF,Y3D,RADIUS
      WRITE(*,*)'FJK',FJK
      WRITE(19,*) WF,WFS,Y3D,RADIUS
      WRITE(IO,*)'WFS Y3D RADIUS T',WFS,Y3D,RADIUS,T
      WRITE(IO,*)'WF FJK',WF,FJK
C
      WRITE(IO,*)
      WRITE(IO,*)'VALUES AT TIPPING'
C     WRITE(*,*)'I T',I,T
      WRITE(IO,*)'I T',I,T
C     DO 1444 K=1,2*(NNM+15)
C     WRITE(*,9999) K,Y(K)
C     WRITE(IO,9999) K,Y(K)
C 1444 CONTINUE
C     WRITE(*,*)'ITERATION NUMBER NZZZ=',NZZZ
      WRITE(IO,*)'ITERATION NUMBER NZZZ=',NZZZ
      DO 1444 K=1,2*(NNM+15)
C     WRITE(*,*)'K,Y(K)',K,Y(K)
      WRITE(IO,*)'K,Y(K)',K,Y(K)
 1444 CONTINUE
      WRITE(16,*)
      WRITE(16,*)'VALUES AT TIPPING'
      WRITE(16,*)'I T',I,T
C     DO 1446 K=1,2*(NNM+15)
C     WRITE(*,9998) K,YP(K)
C     WRITE(16,9998) K,YP(K)
C 1446 CONTINUE
      DO 1446 K=1,2*(NNM+15)
C     WRITE(*,*)'K,YP(K)',K,YP(K)
      WRITE(16,*)'K,YP(K)',K,YP(K)
 1446 CONTINUE
      WRITE(17,*)
      WRITE(17,*)'VALUES AT TIPPING'
      WRITE(17,*)'I T',I,T
C     DO 1448 K=1,14
C     WRITE(*,9997) K,SEQ(K+NNM+15,1)
C     WRITE(17,9997) K,SEQ(K+NNM+15,1)
C 1448 CONTINUE
      DO 1448 K=1,14
```

```
C     WRITE(*,*)'K LAMDA(K)',K,SEQ(K+NNM+15,1)
      WRITE(17,*)'K LAMDA(K)',K,SEQ(K+NNM+15,1)
 1448 CONTINUE

 1510 CONTINUE

      STOP
      END



      SUBROUTINE JACOBI(XX,EIGV,D,N,RTOL,NSMAX,IFPR,IOUT)
C..............................................................
      IMPLICIT REAL*8 (A-H,O-Z)
C.
C.SUBROUTINE
C.      TO SOLVE THE GENERALIZED EINGENPROBLEM USING THE  .
C       GENERALIZED JACOBI ITERATION.                   .
C.                                        .
C. INPUT VARIABLES
C.    A(N,N)  =STIFNESS MATRIX(ASSUMED POSITIVE DEFINITE  .
C.    B(N,N)  =MASS MATRIX(ASSUMED POSITIVE DEFINITE)    .
C.    XX(N,N)  =MATRIX STORING EINGENVECTORS ON EXXIT      .
C.    EIGV(N)  =VECTOR STORING EINGENVALUES ON EXXIT      .
C.     D(N)     =WORKING VECTOR                    .
C.    N        =ORDER OF MATRICES A AND B
C.    RTOL     =CONVERGENCE TOLERENCE
C.    NSMAX    =MAXIMUN NUMB'ER OF SWEEPS ALLOWED          .
C.              (USUALLY SET TO 15)                 .
C.    IFPR     =FLAG FOR PRINTING DURING ITERATION        .
C.       EQ.0  NO PRINTING
C.       EQ.1  INTERMEDIATE RESULTS ARE PRINTED        .
C.    IOUT     =OUTPUT DEVICE NUMBER                .
C.                                        .
C.---OUTPUT-
C.    A(N,N)  =DIAGONALIZED STIFFNESS MATRIX          .
C.    B(N,N)  =DIAGONALIZED MASS MATRIX            .
C.    XX(N,N)  =EINGENVECTORS STORED COLUMNWISE
C.    EIGV(N)  =EINGENVALUES                     .
C.                                        .
C..............................................................
C     REAL*8 XA,Y
C.
C      DIMENSION EIGV(NM-4),D(NM-4),XX(NM-4,NM-4)
C      DIMENSION A(NM-4,NM-4),B(NM-4,NM-4)
       DIMENSION EIGV(35),D(35),XX(35,35)
       DIMENSION A(35,35),B(35,35)

       COMMON/A2/ B
       COMMON/A3/ A
C
C   INITIALIZE EIGENVALUE AND EINGENVECTOR MATRICES
C
      DO 10 I=1,N
      IF(A(I,I).GT.0..AND.B(I,I).GT.0.)GO TO 4
      WRITE(*,*)'I A(I,I) B(I,I)',I,A(I,I),B(I,I)
      WRITE(IOUT,2020)
      STOP
    4 D(I)=A(I,I)/B(I,I)
   10 EIGV(I)=D(I)
      DO 30 I=1,N
      DO 20 J=1,N
   20 XX(I,J)=0.
   30 XX(I,I)=1.
      IF(N.EQ.1)RETURN
C
C   INITIALIZE SWEEO COUNTER AND EEGIN ITERATION
C
      NSWEEP=0
      NR=N-1
   40 NSWEEP=NSWEEP+1
      IF(IFPR.EQ.1)WRITE(IOUT,2000)NSWEEP
C
C   CHECK IF PRESENT OFF DIAGONAL REQUIRE ZEROING
C
      EPS=(.01**NSWEEP)**2
      DO 210 J=1,NR
      JJ=J+1
      DO 210 K=JJ,N
      EPTOLA=(A(J,K)*A(J,K))/(A(J,J)*A(K,K))
```

```
      EPTOLB=(B(J,K)*B(J,K))/(B(J,J)*B(K,K))
      IF((EPTOLA.LT.EPS).AND.(EPTOLB.LT.EPS))GO TO 210
C
C   IF ZEROING IS REQUIRED CALCULATE THE MATRIX ROTATION
C   ELEMENTS CA AND CG
C
      AKK=A(K,K)*B(J,K)-B(K,K)*A(J,K)
      AJJ=A(J,J)*B(J,K)-B(J,J)*A(J,K)
      AB=A(J,J)*B(K,K)-A(K,K)*B(J,J)
      CHECK=(AB*AB+4.*AKK*AJJ)/4.
      IF(CHECK)50,60,60
  50  WRITE(IOUT,2020)
      STOP
  60  SQCH=DSQRT(CHECK)
      D1=AB/2.+SQCH
      D2=AB/2.-SQCH
      DEN=D1
      IF(DABS(D2).GT.DABS(D1))DEN=D2
      IF(DEN)80,70,80
  70  CA=0.
      CG=-A(J,K)/A(K,K)
      GO TO 90
  80  CA=AKK/DEN
      CG=-AJJ/DEN
C
C    PERFORM THE GENERALIZED ROTATION TO ZERO THE
C    PRESENT OFF DIAGONAL ELEMENT
C
  90  IF(N-2)100,190,100
 100  JP1=J+1
      JM1=J-1
      KP1=K+1
      KM1=K-1
      IF(JM1-1)130,110,110
 110  DO 120 I=1,JM1
      AJ=A(I,J)
      BJ=B(I,J)
      AK=A(I,K)
      BK=B(I,K)
      A(I,J)=AJ+CG*AK
      B(I,J)=BJ+CG*BK
      A(I,K)=AK+CA*AJ
 120  B(I,K)=BK+CA*BJ
 130  IF(KP1-N)140,140,160
 140  DO 150 I=KP1,N
      AJ=A(J,I)
      BJ=B(J,I)
      AK=A(K,I)
      BK=B(K,I)
      A(J,I)=AJ+CG*AK
      B(J,I)=BJ+CG*BK
      A(K,I)=AK+CA*AJ
 150  B(K,I)=BK+CA*BJ
 160  IF(JP1-KM1)170,170,190
 170  DO 180 I=JP1,KM1
      AJ=A(J,I)
      BJ=B(J,I)
      AK=A(I,K)
      BK=B(I,K)
      A(J,I)=AJ+CG*AK
      B(J,I)=BJ+CG*BK
      A(I,K)=AK+CA*AJ
 180  B(I,K)=BK+CA*BJ
 190  AK=A(K,K)
      BK=B(K,K)
      A(K,K)=AK+2.*CA*A(J,K)+CA*CA*A(J,J)
      B(K,K)=BK+2.*CA*B(J,K)+CA*CA*B(J,J)
      A(J,J)=A(J,J)+2.*CG*A(J,K)+CG*CG*AK
      B(J,J)=B(J,J)+2.*CG*B(J,K)+CG*CG*BK
      A(J,K)=0.
      B(J,K)=0.
C.
C   UPDATE THE EINGENVECTOR MATRIX AFTER EACH ROTATION
C
      DO 200 I=1,N
      XXJ=XX(I,J)
      XXK=XX(I,K)
      XX(I,J)=XXJ+CG*XXK
 200  XX(I,K)=XXK+CA*XXJ
 210  CONTINUE
C
```

```
C   UPDATE THE EINGENVALUES AFTER EACH SWEEP
C
    DO 220 I=1,N
    IF(A(I,I).GT.0..AND.B(I,I).GT.0.)GO TO 220
    WRITE(IOUT,2020)
    STOP
220 EIGV(I)=A(I,I)/B(I,I)
    IF(IFPR.EQ.0) GO TO 230
C
C
C
    WRITE(IOUT,2030)
     DO 17 I=1,N
    VAEIG=DSQRT(EIGV(I))
 17 WRITE(IOUT,2010) VAEIG
    WRITE(IOUT,2040)
    DO 300 J=1,N
300 WRITE(IOUT,2050)(XX(I,J),I=1,N)
     IF(NSWEEP.LT.6)GO TO 230
C
       DO 32 J=1,N
       SUM=0.0
C
       DO 31 I=1,N
 31     SUM=SUM+XX(I,J)**2
C
        DO 33 I=1,N
 33      XX(I,J)=XX(I,J)/DSQRT(SUM)
C
 32     CONTINUE
C
C
C  CHECK FOR CONVERGENCE
C
230 DO 240 I=1,N
    TOL=RTOL*D(I)
    DIF=DABS(EIGV(I)-D(I))
    IF(DIF.GT.TOL)GO TO 280
240 CONTINUE
C
C   CHECK ALL OF DIAGONAL ELEMENTS TO SEE IF ANOTHER
C      SWEEP IS NECESSARY
C
    EPS=RTOL**2
    DO 250 J=1,NR
    JJ=J+1
    DO 250 K=JJ,N
    EPSA=(A(J,K)*A(J,K))/(A(J,J)*A(K,K))
    EPSB=(B(J,K)*B(J,K))/(B(J,J)*B(K,K))
    IF((EPSA.LT.EPS).AND.(EPSB.LT.EPS))GO TO 250
    GO TO 280
 250 CONTINUE
C
C  FILL OUT BOTTOM TRIANGLE OF RESULTANT MATRICES AND SCALE
C                          EINGENVECTORS
C
 255 DO 260 I=1,N
    DO 260 J=1,N
    A(J,I)=A(I,J)
260 B(J,I)=B(I,J)
    DO 270 J=1,N
    BB=DSQRT(B(J,J))
    DO 270 K=1,N
270 XX(K,J)=XX(K,J)/BB
    RETURN
C
C  UPDATE D MATRIX AND START NEW SWEEP,IF ALLOWED
C
 280 DO 290 I=1,N
 290 D(I)=EIGV(I)
    IF(NSWEEP.LT.NSMAX)GO TO 40
    GO TO 255
2000 FORMAT(' ',15X,'SWEEP NUMBER IN JACOBI =',I4)
2010 FORMAT(' ',E12.4,//)
2020 FORMAT(' ',10X,'ERROR SOLUTION STOP'/'MATRIX NOT POS DEF')
2030 FORMAT(' ','CURRENT EIGENVALUES IN JACOBI ARE'/)
2040 FORMAT(' ','CURRENT EINGENVECTORS ARE'/)
2050 FORMAT(' ',9E12.4,//)
    END
```

```
        SUBROUTINE ELIM(AB,N,NP)
      IMPLICIT REAL*8 (A-H,O-Z)
C         DIMENSION AB(30,60)
      DIMENSION AB(N,NP)
C****

C    DET= AB(1,1)*(AB(2,2)*AB(3,3)-AB(3,2)*AB(2,3))
C    1 -AB(1,2)*(AB(2,1)*AB(3,3)-AB(3,1)*AB(2,3))
C    2 +AB(1,3)*(AB(2,1)*AB(3,2)-AB(3,1)*AB(2,2))
C    WRITE(6,60) DET
C  60 FORMAT(' DET=',F14.4)
C****
C THIS SUBROUTINE SOLVES A SET OF LINEAR EQUATIONS.
C THE GAUSS ELIMINATION METHOD IS USED ,WITH PARTIAL PIVOTING.
C MULTIPLE RIGHT HAND SIDES ARE PERMITTED,THEY SHOULD BE SUPLIED
C AS COLUMNS THAT AUGMENT THE COEFFICIENT MATRIX.
C PARAMETERS ARE-
C    AB   COEFFICIENT MATRIX AUGMENTED WITH R.H.S VECTORS
C    N    NUMBER OF EQUATIONS
C    NP   TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX.
C    NDIM  FIRST DIMENSION OF MATRIX AB IN THE CALLING PROGRAM.
C  THE SOLUTION  VECTOR(S) ARE RETURNED IN THE AUGMENTED
C  COLUMNS OF AB.
C
C BEGIN  THE REDUCTION
       NDIM=N
       NM1=N-1
       DO 35 I = 1,NM1
C FIND THE ROW NUMBERS OF THE PIVOT ROW.WE WILL THEN
C INTERCHANGE ROWS TO PUT THE PIVOT ELEMENT ON THE DIAGONAL .
         IPVT = I
         IP1 = I + 1
         DO 10 J =IP1,N
           IF(DABS(AB(IPVT,I)).LT.DABS(AB(J,I))) IPVT = J
  10      CONTINUE
C CHECK TO BE SURE THE PIVOT ELEMENT IS NOT TOO SMALL,IF SO
C PRINT A MESSAGE AND RETURN.
           IF(DABS(AB(IPVT,I)).LT. 1.D-5) GO TO 99
C NOW INTERCHANGE ,EXECPT IF THE PIVOT ELEMENT IS ALREADY ON
C THE DIAGONAL ,DON'T NEED TO.
           IF(IPVT.EQ.I) GO TO 25
         DO 20 JCOL = I,NP
          SAVE = AB(I,JCOL)
          AB(I,JCOL)=AB(IPVT,JCOL)
          AB(IPVT,JCOL)=SAVE
  20      CONTINUE
C NOW REDUCE ALL ELEMENTS BELOW THE DIAGONAL IN THE I-TH ROW.CHECK
C CAN FIRST TO SEE IF A ZERO ALREADY PRESENT.IF SO,
C CAN SKIP REDUCTION FOR THAT ROW.
  25        DO 32 JROW = IP1,N
           IF(AB(JROW,I).EQ.0) GO TO 32
           RATIO = AB(JROW,I)/AB(I,I)
          DO 30 KCOL = IP1,NP
           AB(JROW,KCOL)=AB(JROW,KCOL)-RATIO*AB(I,KCOL)
  30       CONTINUE
  32      CONTINUE
  35     CONTINUE
C WE STILL NEED TO CHECK A(N,N) FOR SIZE.
       IF(DABS(AB(N,N)).LT.1.D-5) GO TO 99
C NOW WE BACK SUBSTITUTE
       NP1 = N + 1
       DO 50 KCOL = NP1,NP
        AB(N,KCOL)= AB(N,KCOL)/AB(N,N)
       DO 45 J=2,N
         NVBL= NP1-J
         L = NVBL + 1
         VALUE=AB(NVBL,KCOL)
        DO 40 K= L,N
          VALUE=VALUE-AB(NVBL,K)*AB(K,KCOL)
  40      CONTINUE
        AB(NVBL,KCOL)=VALUE/AB(NVBL,NVBL)
  45    CONTINUE
  50    CONTINUE
       RETURN
C MESSAGE FOR A NEAR SINGULAR MATRIX
  99   WRITE(6,100)
      WRITE(*,100)
 100  FORMAT('1',10X,'SOLUTION NOT FEASIBLE.ANEAR ZERO PIVOT WAS
    $ ENCOUTERED.')
      RETURN
```

```
      END


      SUBROUTINE INVERS(N,SM,SAVE)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     THIS PROGRAM CALCULATES THE INVERS OF THE MATRIX USING ROW
C     TRANSFORMATION
C
C     SAVE(I,J) IS THE INVERS OF THE SM(I,J)
C     N IS THE DIMENSION OF THE SM(I,J)
C
C      DIMENSION C(N,2*N),SE(N,2*N),SM(N,N),SAVE(N,N)
      DIMENSION C(36,72),SE(36,72),SM(N,N),SAVE(N,N)

      IO=6

      L=N
      NI=N
      N1=2*NI
      DO 1465 I=1,L
      DO 1466 II=1,N1
      C(I,II)=0
 1466 CONTINUE
 1465 CONTINUE
      DO 1468 I=1,NI
      DO 1469 II=1,NI
      SE(I,II)=0
 1469 CONTINUE
 1468 CONTINUE
      DO 1470 I=1,L
      C(I,NI+I)=1.0
      DO 1471 J=1,NI
      CZ=SM(I,J)
      C(I,J)=CZ
      SE(I,J)=CZ
 1471 CONTINUE
 1470 CONTINUE
      DO 1472 I=1,L-1
      P2=C(I,I)
      IF(P2.NE.0.) GO TO 1480
      IF(I.EQ.L) GO TO 1481
      DO 1483 J=I+1,L
      P1=C(J,I)
      IF(P1.EQ.0.) GO TO 1483
      DO 1485 K=1,N1
      CC=C(I,K)
      C(I,K)=C(J,K)
      C(J,K)=CC
 1485 CONTINUE
      GO TO 1480
 1483 CONTINUE
 1481 WRITE(*,1486)
      WRITE(IO,1486)
 1486 FORMAT(/,10X,'MATRIX IS SINGULAR')
      GO TO 1489
 1480 CC=C(I,I)
      DO 1490 J=I+1,N1
      C(I,J)=C(I,J)/CC
 1490 CONTINUE
      DO 1492 J=1,L
      IF(J.EQ.I) GO TO 1492
      CC=C(J,I)
      DO 1494 K=I+1,N1
      C(J,K)=C(J,K)-CC*C(I,K)
 1494 CONTINUE
 1492 CONTINUE
 1472 CONTINUE
      CC=C(L,L)
      DO 1475 J=I+1,N1
      C(L,J)=C(L,J)/CC
 1475 CONTINUE
      DO 1476 J=1,L
      IF(J.EQ.I) GO TO 1476
      CC=C(J,L)
      DO 1477 K=I+1,N1
      C(J,K)=C(J,K)-CC*C(L,K)
 1477 CONTINUE
 1476 CONTINUE
      DO 1497 I=1,L
```

```
      DO 1495 J=1,L
      TZ=C(I,NI+J)
      SAVE(I,J)=TZ
 1495 CONTINUE
 1497 CONTINUE
 1489 CONTINUE
C     DO 1500 I=1,N
C     DO 1500 J=1,N
C     WRITE(*,*)'I J SAVE(I,J)',I,J,SAVE(I,J)
C 1500 CONTINUE

      RETURN
      END



      SUBROUTINE TIND(T,Y,YP)

      IMPLICIT REAL*8(A-H,O-Z)

C     DIMENSION Y(2*(NNM+15)),YP(2*(NNM+15))
C
C    DIMENSIONS FOR FLEXIBLE BODY
C
C    DIMENSIONS OF MASS MATRIX ELEMENTS
C
C    DIMENSIONS OF Mzz ELEMENT
C     DIMENSION AMZZ(2,2),AR(M)
C    DIMENSIONS OF Mzo ELEMENT
C     DIMENSION CRA(MM,4),AT(M,2,NM),A1(2,NM),TW(M),EO(NM,1),
C    + ALFA(NM,1)
C     DIMENSION XXX(NM,NNM),ETA(NNM,1),VO(NM,1),TO1(2,2),C(2,NM)
C     DIMENSION AMZO(2,1)
C    DIMENSIONS OF Mzn ELEMENT
C     DIMENSION TT1(2,2),AMZN(2,NNM)
C    DIMENSIONS OF Moo ELEMENT
C     DIMENSION CA(NM,1),SM(NM,NM),VOT(1,NM),AMOO(1,1)
C    DIMENSIONS OF Mon ELEMENT
C     DIMENSION AAT(M,NM,NM),AA1(NM,NM),CB(NM,NNM)
C     DIMENSION AMON(1,NNM)
C    DIMENSIONS OF Mnn ELEMENT
C     DIMENSION XXXT(NNM,NM),CC(NNM,NM),AMNN(NNM,NNM)
C
C    DIMENSIONS OF COROLIS AND CENTRIFUGAL FORCE TERM MATRIX ELEMENTS
C
C    DIMENSIONS OF Qz ELEMENT
C     DIMENSION CD(2,NM),CE(2,1),CG(2,NNM),ETAD(NNM,1),CH(2,1)
C     DIMENSION QZ(2,1)
C    DIMENSIONS OF Qo ELEMENT
C     DIMENSION CI(1,NM),CJ(1,1),CL(1,NNM)
C     DIMENSION CM(1,1),QO(1,1)
C    DIMENSIONS OF Qn ELEMENT
C     DIMENSION CO(NNM,1),CS(NNM,NNM),CT(NNM,1)
C     DIMENSION QN(NNM,1)
C
C    DIMENSIONS OF EXTERNAL FORCE MATRICES ELEMENTS
C
C    DIMENSIONS FOR WEIGHT FORCE
C     DIMENSION A1T(NM,2),AFZW(2,1),DBA(1,2),TO1T(2,2)
C     DIMENSION DBB(1,2),ZS(2,1),AFOW(1,1),DBC(NNM,2),TT1T(2,2)
C     DIMENSION DBD(NNM,2),AFNW(NNM,1)
C
C    DIMENSIONS OF STIFFNESS AND DAMPING FORCE MATRIX ELEMENTS
C
C     DIMENSION SS(NM,NM),DD(NNM,NNM),SD(NM,NM),DD(NNM,NNM)
C     DIMENSION SD(NM,NM),SA(NNM,NM),SB(NNM,NNM)
C     DIMENSION FNSK(NNM,1),FSK(NNM+15,1),FNDK(NNM,1),FDK(NNM+15,1)

C
C    DIMENSIONS FOR RIGID BODIES
C
C    DIMENSIONS OF MASS MATRIX ELEMENTS
C
C     DIMENSION AMZZ2(2,2),AMZZ3(2,2),AMZZ4(2,2),AMZZ5(2,2),TO2(2,2)
C     DIMENSION FM2(2,1)
C     DIMENSION AMZO2(2,1),TO3(2,2),FM3(2,1),AMZO3(2,1),TO4(2,2)
C     DIMENSION FM4(2,1),AMZO4(2,1),AMOO2(1,1),AMOO3(1,1),AMOO4(1,1)
C     DIMENSION TO5(2,2),FM5(2,1),AMZO5(2,1),AMOO5(1,1)
C
```

```
C    DIMENSIONS OF COROLIS AND CENTRIFUGAL FORCE TERM MATRIX ELEMENTS
C
C     DIMENSION TT2(2,2),DM(2,1),QZ2(2,1),TT3(2,2),QZ3(2,1)
C     DIMENSION TT4(2,2),QZ4(2,1),QZ5(2,1),QO2(1,1),QO3(1,1)
C     DIMENSION QO4(1,1),TT5(2,2),DMQ(2,1),QO5(1,1)
C
C    DIMENSIONS OF EXTERNAL FORCE MATRICES ELEMENTS
C
C     DIMENSION AFZ2(2,1),AFZ3(2,1),AFZ4(2,1),AFZ5(2,1)
C     DIMENSION FM2T(1,2),FM3T(1,2),FM4T(1,2),RGR(2,1),TO2T(2,2)
C     DIMENSION DR(1,2),AFO2(1,1),TO3T(2,2),AFO3(1,1),TO4T(2,2)
C     DIMENSION AFO4(1,1),TO5T(2,2),FM5RT(1,2),FM5PT(1,2)
C     DIMENSION RGRP(2,1),AFO5R(1,1),AFO5P(1,1),AFO5(1,1)
C
C    DIMENSIONS OF CONSTRAINT EQUATIONS
C
C    FOR CONSTRAINT EQUATIONS 1,2,3,4,5 AND 6
C     DIMENSION ACB(14,NNM+15),ACBD(14,NNM+15),COE(1,2),FAY(2,6)
C     DIMENSION CSI(1,6)
C     DIMENSION BOCE(6,NM),GA(1,2),GB(1,6),GD(1,1),GH(1,NNM)
C    B DOT MATRIX DIMENSIONS
C     DIMENSION COED(1,2),GS(1,1),GZ(1,1),HH(1,NNM)
C
C    FOR CONSTRAINT EQUATION 7
C    THERE IS NO EXTRA DIMENSION
C
C    FOR CONSTRAINT EQUATIONS 8,9,10 AND 11
C    THERE IS NO EXTRA DIMENSION
C
C    FOR CONSTRAINT EQUATIONS 12 AND 13
C     DIMENSION HK(2,6)
C    B DOT DIMENSIONS
C     DIMENSION PA(2,1)

C    FOR CONSTRAINT EQUATION 14
C    THERE IS NO EXTRA DIMENSION
C
C
C    DIMENSIONS OF SYSTEM EQUATION
C
C    COMING FROM MASS MATRIX
C
C     DIMENSION AMM1(NNM+3,NNM+3),AMM2(3,3),AMM3(3,3),AMM4(3,3)
C     DIMENSION AMM5(3,3),AMM(NNM+15,NNM+15)
C
C    COMING FROM CONSTRAINT EQUATION MATRIX
C
C     DIMENSION SEL(NNM+29,NNM+29),ACBT(NNM+15,14)
C
C    COMING FROM CORIOLIS AND CENTRIFUGAL FORCE MATRIX
C
C     DIMENSION QQ(NNM+15,1)
C
C    COMING FROM EXTERNAL FORCE MATRIX
C
C     DIMENSION AFF(NNM+15,1)
C
C    COMING FROM B DOT Y MATRIX
C
C     DIMENSION PB(NNM+15,1),BDY(14,1)
C
C    COMING FROM GDOT MATRIX
C
C     DIMENSION GDOT(14,1)
C
C    DIMENSIONS FOR THE INVERSE OF THE SYSTEM EQUATION LEFT MATRIX
C
C     DIMENSION ABSEL(NNM+29,2*(NNM+29)),SELI(NNM+29,NNM+29)
C     DIMENSION AIDT(NNM+29,NNM+29)

C     DIMENSION OF THE SYSTEM EQUATION RIGHT MATRIX
C     DIMENSION SER(NNM+29,1)

C     DIMENSION OF THE SYSTEM EQUATION
C     DIMENSION SEQ(NNM+29,1)

C-----------------------------------------------------------------
      DIMENSION Y(2*(4+15)),YP(2*(4+15))
C
C     DIMENSIONS FOR FLEXIBLE BODY
C
```

```
C   DIMENSIONS OF MASS MATRIX ELEMENTS
C
C   DIMENSIONS OF Mzz ELEMENT
    DIMENSION AMZZ(2,2),AR(12)
C   DIMENSIONS OF Mzo ELEMENT
    DIMENSION CRA(48,4),AT(12,2,39),A1(2,39),TW(12),EO(39,1),
   + ALFA(39,1)
    DIMENSION XXX(39,4),ETA(4,1),VO(39,1),TO1(2,2),C(2,39)
    DIMENSION AMZO(2,1)
C   DIMENSIONS OF Mzn ELEMENT
    DIMENSION TT1(2,2),AMZN(2,4)
C   DIMENSIONS OF Moo ELEMENT
    DIMENSION CA(39,1),SM(39,39),VOT(1,39),AMOO(1,1)
C   DIMENSIONS OF Mon ELEMENT
    DIMENSION AAT(12,39,39),AA1(39,39),CB(39,4)
    DIMENSION AMON(1,4)
C   DIMENSIONS OF Mnn ELEMENT
    DIMENSION XXXT(4,39),CC(4,39),AMNN(4,4)
C
C   DIMENSIONS OF COROLIS AND CENTRIFUGAL FORCE TERM MATRIX ELEMENTS
C
C   DIMENSIONS OF Qz ELEMENT
    DIMENSION CD(2,39),CE(2,1),CG(2,4),ETAD(4,1),CH(2,1)
    DIMENSION QZ(2,1)
C   DIMENSIONS OF Qo ELEMENT
    DIMENSION CI(1,39),CJ(1,1),CL(1,4)
    DIMENSION CM(1,1),QO(1,1)
C   DIMENSIONS OF Qn ELEMENT
    DIMENSION CO(4,1),CS(4,4),CT(4,1)
    DIMENSION QN(4,1)
C
C   DIMENSIONS OF EXTERNAL FORCE MATRICES ELEMENTS
C
C   DIMENSIONS FOR WEIGHT FORCE
    DIMENSION A1T(39,2),AFZW(2,1),DBA(1,2),TO1T(2,2)
    DIMENSION DBB(1,2),ZS(2,1),AFOW(1,1),DBC(4,2),TT1T(2,2)
    DIMENSION DBD(4,2),AFNW(4,1)
C
C   DIMENSIONS OF STIFFNESS AND DAMPING FORCE MATRIX ELEMENTS
C
    DIMENSION SS(39,39),DD(4,4)
    DIMENSION SD(39,39),SA(4,39),SB(4,4)
    DIMENSION FNSK(4,1),FSK(19,1),FNDK(4,1),FDK(19,1)

C
C   DIMENSIONS FOR RIGID BODIES
C
C   DIMENSIONS OF MASS ELEMENTS
C
    DIMENSION AMZZ2(2,2),AMZZ3(2,2),AMZZ4(2,2),AMZZ5(2,2),TO2(2,2)
    DIMENSION FM2(2,1)
    DIMENSION AMZO2(2,1),TO3(2,2),FM3(2,1),AMZO3(2,1),TO4(2,2)
    DIMENSION FM4(2,1),AMZO4(2,1),AMOO2(1,1),AMOO3(1,1),AMOO4(1,1)
    DIMENSION TO5(2,2),FM5(2,1),AMZO5(2,1),AMOO5(1,1)
C
C   DIMENSIONS OF COROLIS AND CENTRIFUGAL FORCE TERM ELEMENTS
C
    DIMENSION TT2(2,2),DM(2,1),QZ2(2,1),TT3(2,2),QZ3(2,1)
    DIMENSION TT4(2,2),QZ4(2,1),QZ5(2,1),QO2(1,1),QO3(1,1)
    DIMENSION QO4(1,1),TT5(2,2),DMQ(2,1),QO5(1,1)
C
C   DIMENSIONS OF EXTERNAL FORCE MATRICES ELEMENTS
C
    DIMENSION AFZ2(2,1),AFZ3(2,1),AFZ4(2,1),AFZ5(2,1)
    DIMENSION FM2T(1,2),FM3T(1,2),FM4T(1,2),RGR(2,1),TO2T(2,2)
    DIMENSION DR(1,2),AFO2(1,1),TO3T(2,2),AFO3(1,1),TO4T(2,2)
    DIMENSION AFO4(1,1),TO5T(2,2),FM5RT(1,2),FM5PT(1,2)
    DIMENSION RGRP(2,1),AFO5R(1,1),AFO5P(1,1),AFO5(1,1)
C
C   DIMENSIONS OF CONSTRAINT EQUATIONS
C
C   FOR CONSTRAINT EQUATIONS 1,2,3,4,5 AND 6
    DIMENSION ACB(14,4+15),ACBD(14,4+15),COE(1,2),FAY(2,6)
    DIMENSION CSI(1,6)
    DIMENSION BOCE(6,39),GA(1,2),GB(1,6),GD(1,1),GH(1,4)

C   B DOT MATRIX DIMENSIONS
    DIMENSION COED(1,2),GS(1,1),GZ(1,1),HH(1,4)

C   FOR CONSTRAINT EQUATION 7
C   THERE IS NO EXTRA DIMENSION
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C   FOR CONSTRAINT EQUATIONS 8,9,10 AND 11
C   THERE IS NO EXTRA DIMENSION

C   FOR CONSTRAINT EQUATIONS 12 AND 13
    DIMENSION HK(2,6)

C   B DOT DIMENSIONS
    DIMENSION PA(2,1)

C   FOR CONSTRAINT EQUATION 14
C   THERE IS NO EXTRA DIMENSION
C
C
C   DIMENSIONS OF SYSTEM EQUATION
C
C   COMING FROM MASS MATRIX
C
    DIMENSION AMM1(4+3,4+3),AMM2(3,3),AMM3(3,3),AMM4(3,3)
    DIMENSION AMM5(3,3),AMM(4+15,4+15)
C
C   COMING FROM CONSTRAINT EQUATION MATRIX
C
    DIMENSION SEL(4+29,4+29),ACBT(4+15,14)
C
C   COMING FROM CORIOLIS AND CENTRIFUGAL FORCE MATRIX
C
    DIMENSION QQ(4+15,1)
C
C   COMING FROM EXTERNAL FORCE MATRIX
C
    DIMENSION AFF(4+15,1)
C
C   COMING FROM B DOT Y MATRIX
C
    DIMENSION PB(4+15,1),BDY(14,1)
C
C   COMING FROM GDOT MATRIX
C
    DIMENSION GDOT(14,1)
C
C   DIMENSIONS FOR THE INVERSE OF THE SYSTEM EQUATION LEFT MATRIX
C
    DIMENSION ABSEL(4+29,2*(4+29)),SELI(4+29,4+29)
    DIMENSION AIDT(4+29,4+29)

C   DIMENSION OF THE SYSTEM EQUATION RIGHT MATRIX
    DIMENSION SER(4+29,1)

C   DIMENSION OF THE SYSTEM EQUATION
    DIMENSION SEQ(4+29,1)


    COMMON /A4/XXX
    COMMON /A5/SM
    COMMON /A6/SS,SD
    COMMON /A7/AR,TW,AOA,AOG
    COMMON /A8/CRA,PI
    COMMON /A9/NFEF,NFES,M,NM,MM,MN,NMN,NNM
    COMMON /A10/ YOO1(2,4),YOO2(39,4),YOO3(4,39),YOO4(39,4),
   + YOO5(4,4),YOO6(4,2),YOO7(4,39),YOO8(4,4),YOO9(4,39),YOO10(4,4)
    COMMON /A12/ YOO11(2,39),YOO12(2,4),YOO13(1,39),YOO14(1,4),
   + YOO15(2,39),YOO16(2,4)
    COMMON /A13/ AMZZ,A1,AA1,AMNN,A1T,AMS
    COMMON /A14/ AT,AAT,XXXT,FAY,CSI,BOCE
    COMMON /A16/SEQ
    COMMON /A17/IIIIZ,NOO,NOOM,CODIS,RO,WF
    COMMON /A18/TIME1,TIME2,TIME3,VZERO

    IO=6
C
C   CONSTRUCTION OF MASS MATRIX OF BODY 1
C
C   CALCULATION OF Mzo ELEMENT
C
C   CONSTRUCTION OF Eo, ETA, ALFA, E MATRICES
    DO 50 I=1,NM
    EO(I,1)=0.D0
 50 CONTINUE
C
C   CALCULATION OF NODE VALUES
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```
      CR=0.D0
      L=0
      DO 55 I=4,NM-2,3
      L=L+1
      CR=CR+CRA(L,1)
C
C    CONSTRUCTION OF Eo MATRIX
      EO(I,1)=CR
   55 CONTINUE
C
C    CONSTRUCTION OF ETA MATRIX
      DO 57 I=1,NNM
      ETA(I,1)=Y(I+3)
   57 CONTINUE
C
C    CONSTRUCTION OF ALFA MATRIX
      DO 60 I=1,NM
      ALFA(I,1)=0.D0
      DO 60 K=1,NNM
      ALFA(I,1)=ALFA(I,1)+XXX(I,K)*ETA(K,1)
   60 CONTINUE
C
C    CONSTRUCTION OF E MATRIX
      DO 65 I=1,NM
      VO(I,1)=EO(I,1)+ALFA(I,1)
   65 CONTINUE
C
C    To MATRIX OF BODY 1
      TO1(1,1)=-DSIN(Y(3))
      TO1(1,2)=-DCOS(Y(3))
      TO1(2,1)=DCOS(Y(3))
      TO1(2,2)=-DSIN(Y(3))
C    TO1(2,2)*A1(2,NM)=C(2,NM)
      DO 90 I=1,2
      DO 90 J=1,NM
      C(I,J)=0.D0
      DO 90 K=1,2
      C(I,J)=C(I,J)+TO1(I,K)*A1(K,J)
   90 CONTINUE
C    C(2,NM)*VO(NM,1)=AMZO(2,1)
      DO 95 I=1,2
      AMZO(I,1)=0.D0
      DO 95 K=1,NM
      AMZO(I,1)=AMZO(I,1)+C(I,K)*VO(K,1)
   95 CONTINUE
C
C    CALCULATION OF  Mzn  ELEMENT
C
C
C    T MATRIX OF BODY 1
      TT1(1,1)=DCOS(Y(3))
      TT1(1,2)=-DSIN(Y(3))
      TT1(2,1)=DSIN(Y(3))
      TT1(2,2)=DCOS(Y(3))
C    TT1(2,2)*YOO1(2,NNM)=AMZN(2,NNM)
      DO 120 I=1,2
      DO 120 J=1,NNM
      AMZN(I,J)=0.D0
      DO 120 K=1,2
      AMZN(I,J)=AMZN(I,J)+TT1(I,K)*YOO1(K,J)
  120 CONTINUE
C
C    CALCULATION OF  Moo  ELEMENT
C
C    SM(NM,NM)*VO(NM,1)=CA(NM,1)
      DO 125 I=1,NM
      CA(I,1)=0.D0
      DO 125 K=1,NM
      CA(I,1)=CA(I,1)+SM(I,K)*VO(K,1)
  125 CONTINUE
      DO 130 I=1,NM
      VOT(1,I)=VO(I,1)
  130 CONTINUE
C    VOT(1,NM)*CA(NM,1)=AMOO(1,1)
      AMOO(1,1)=0.D0
      DO 135 K=1,NM
      AMOO(1,1)=AMOO(1,1)+VOT(1,K)*CA(K,1)
  135 CONTINUE
C
C    CALCULATION OF  Mon  ELEMENT
C
```

```
C    VOT(1,NM)*YOO2(NM,NNM)=AMON(1,NNM)
     DO 400 J=1,NNM
     AMON(1,J)=0.D0
     DO 400 K=1,NM
     AMON(1,J)=AMON(1,J)+VOT(1,K)*YOO2(K,J)
 400 CONTINUE
C
C    CONSTRUCTION OF THE CORIOLIS CENTRIFUGAL FORCE TERM MATRIX
C    OF BODY 1
C
C    CALCULATION OF Qz ELEMENT
C
C    TT1(2,2)*A1(2,NM)=CD(2,NM)
     DO 500 I=1,2
     DO 500 J=1,NM
     CD(I,J)=0.D0
     DO 500 K=1,2
     CD(I,J)=CD(I,J)+TT1(I,K)*A1(K,J)
 500 CONTINUE
C    CD(2,NM)*VO(NM,1)=CE(2,1)
     DO 520 I=1,2
     CE(I,1)=0.D0
     DO 520 K=1,NM
     CE(I,1)=CE(I,1)+CD(I,K)*VO(K,1)
 520 CONTINUE
     DO 550 I=1,2
     CE(I,1)=CE(I,1)*((Y(NNM+18))**2)
C     IF(IIIIZ.NE.(NOO*NOOM)) GO TO 550
C     WRITE(*,*)'I CE(I,J) Y(NNM+18)',I,CE(I,1),Y(NNM+18)
C     WRITE(15,*)'I CE(I,J) Y(NNM+18)',I,CE(I,1),Y(NNM+18)
 550 CONTINUE
C    TO1(2,2)*YOO1(2,NNM)=CG(2,NNM)
     DO 650 I=1,2
     DO 650 J=1,NNM
     CG(I,J)=0.D0
     DO 650 K=1,2
     CG(I,J)=CG(I,J)+TO1(I,K)*YOO1(K,J)
 650 CONTINUE
     DO 670 I=1,NNM
     ETAD(I,1)=Y(I+NNM+18)
 670 CONTINUE
C    CG(2,NNM)*ETAD(NNM,1)=CH(2,1)
     DO 700 I=1,2
     CH(I,1)=0.D0
     DO 700 K=1,NNM
     CH(I,1)=CH(I,1)+CG(I,K)*ETAD(K,1)
 700 CONTINUE
     DO 750 I=1,2
     CH(I,1)=2*CH(I,1)*Y(NNM+18)
 750 CONTINUE
C    CE(2,1)-CH(2,1)=QZ(2,1)
     DO 800 I=1,2
     QZ(I,1)=CE(I,1)-CH(I,1)
 800 CONTINUE
C
C    CALCULATION OF Qo ELEMENT
C
C    VOT(1,NM)*AA1(NM,NM)=CI(1,NM)
     DO 850 J=1,NM
     CI(1,J)=0.D0
     DO 850 K=1,NM
     CI(1,J)=CI(1,J)+VOT(1,K)*AA1(K,J)
 850 CONTINUE
C    CI(1,NM)*VO(NM,1)=CJ(1,1)
     CJ(1,1)=0.D0
     DO 860 K=1,NM
     CJ(1,1)=CJ(1,1)+CI(1,K)*VO(K,1)
 860 CONTINUE
     CJ(1,1)=CJ(1,1)*((Y(NNM+18))**2)
C    VOT(1,NM)*YOO4(NM,NNM)=CL(1,NNM)
     DO 890 J=1,NNM
     CL(1,J)=0.D0
     DO 890 K=1,NM
     CL(1,J)=CL(1,J)+VOT(1,K)*YOO4(K,J)
 890 CONTINUE
C    CL(1,NNM)*ETAD(NNM,1)=CM(1,1)
     CM(1,1)=0.D0
     DO 900 K=1,NNM
     CM(1,1)=CM(1,1)+CL(1,K)*ETAD(K,1)
 900 CONTINUE
     CM(1,1)=2*CM(1,1)*Y(NNM+18)
```

```
C   CJ(1,1)-CM(1,1)=QO(1,1)
    QO(1,1)=CJ(1,1)-CM(1,1)
C
C   CALCULATION OF Qn ELEMENT
C
C   YOO3(NNM,NM)*VO(NM,1)=CO(NNM,1)
    DO 920 I=1,NNM
    CO(I,1)=0.D0
    DO 920 K=1,NM
    CO(I,1)=CO(I,1)+YOO3(I,K)*VO(K,1)
 920 CONTINUE
    DO 930 I=1,NNM
    CO(I,1)=CO(I,1)*((Y(NNM+18))**2)
 930 CONTINUE
C   YOO5(NNM,NNM)*ETAD(NNM,1)=CT(NNM,1)
    DO 960 I=1,NNM
    CT(I,1)=0.D0
    DO 960 K=1,NNM
    CT(I,1)=CT(I,1)+YOO5(I,K)*ETAD(K,1)
 960 CONTINUE
    DO 970 I=1,NNM
    CT(I,1)=2*CT(I,1)*Y(NNM+18)
 970 CONTINUE
C   CO(NNM,1)+CT(NNM,1)=QN(NNM,1)
    DO 980 I=1,NNM
    QN(I,1)=CO(I,1)+CT(I,1)
 980 CONTINUE

C
C   CONSTRUCTION OF EXTERNAL FORCE OF BODY 1
C
C   THERE IS ONLY WEIGHT FORCE
C
C   CALCULATION OF  Fz  ELEMENT
C
    AFZW(1,1)=0.D0
    AFZW(2,1)=-AMS*9.81
C
C   CALCULATION OF  Fo  ELEMENT
C
C   VOT(1,NM)*A1T(NM,2)=DBA(1,2)
    DO 1212 J=1,2
    DBA(1,J)=0.D0
    DO 1212 K=1,NM
    DBA(1,J)=DBA(1,J)+VOT(1,K)*A1T(K,J)
 1212 CONTINUE
C
C   TRANSPOZE OF To MATRIX OF BODY 1
    DO 1213 I=1,2
    DO 1213 J=1,2
    TO1T(J,I)=TO1(I,J)
 1213 CONTINUE
C   DBA(1,2)*TO1T(2,2)=DBB(1,2)
    DO 1214 J=1,2
    DBB(1,J)=0.D0
    DO 1214 K=1,2
    DBB(1,J)=DBB(1,J)+DBA(1,K)*TO1T(K,J)
 1214 CONTINUE
    ZS(1,1)=0.D0
    ZS(2,1)=-9.81
C   DBB(1,2)*ZS(2,1)=AFOW(1,1)
    AFOW(1,1)=0.D0
    DO 1216 K=1,2
    AFOW(1,1)=AFOW(1,1)+DBB(1,K)*ZS(K,1)
 1216 CONTINUE
C
C   CALCULATION OF  Fn  ELEMENT
C
C   TRANSPOZE OF T MATRIX OF BODY 1
    DO 1219 I=1,2
    DO 1219 J=1,2
    TT1T(J,I)=TT1(I,J)
 1219 CONTINUE
C   YOO6(NNM,2)*TT1T(2,2)=DBD(NNM,2)
    DO 1220 I=1,NNM
    DO 1220 J=1,2
    DBD(I,J)=0.D0
    DO 1220 K=1,2
    DBD(I,J)=DBD(I,J)+YOO6(I,K)*TT1T(K,J)
 1220 CONTINUE
C   DBD(NNM,2)*ZS(2,1)=AFNW(NNM,1)
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```
      DO 1222 I=1,NNM
      AFNW(I,1)=0.D0
      DO 1222 K=1,2
      AFNW(I,1)=AFNW(I,1)+DBD(I,K)*ZS(K,1)
 1222 CONTINUE
C
C     CONSTRUCTION OF STIFFNESS FORCE MATRIX OF BODY 1
C
C     YOO8(NNM,NNM)*ETA(NNM,1)=FNSK(NNM,1)
      DO 1300 I=1,NNM
      FNSK(I,1)=0.D0
      DO 1300 K=1,NNM
      FNSK(I,1)=FNSK(I,1)+YOO8(I,K)*ETA(K,1)
 1300 CONTINUE
C     FSK(NNM+15,1)=STIFFNESS FORCE MATRIX
      FSK(1,1)=0.D0
      FSK(2,1)=0.D0
      FSK(3,1)=0.D0
      DO 1310 I=4,NNM+3
      FSK(I,1)=-FNSK(I-3,1)
 1310 CONTINUE
      DO 1320 I=NNM+4,NNM+15
      FSK(I,1)=0.D0
 1320 CONTINUE
C     DO 1330 I=1,NNM+15
C     WRITE(*,*)'I FSK(I,1)',I,FSK(I,1)
C     WRITE(13,*)'I FSK(I,1)',I,FSK(I,1)
C 1330 CONTINUE
C
C     CONSTRUCTION OF DAMPING FORCE MATRIX OF BODY 1
C
C     YOO10(NNM,NNM)*ETAD(NNM,1)=FNDK(NNM,1)
      DO 1340 I=1,NNM
      FNDK(I,1)=0.D0
      DO 1340 K=1,NNM
      FNDK(I,1)=FNDK(I,1)+YOO10(I,K)*ETAD(K,1)
 1340 CONTINUE
C     FDK(NNM+15,1)=DAMPING FORCE MATRIX
      FDK(1,1)=0.D0
      FDK(2,1)=0.D0
      FDK(3,1)=0.D0
      DO 1350 I=4,NNM+3
      FDK(I,1)=-FNDK(I-3,1)
 1350 CONTINUE
      DO 1360 I=NNM+4,NNM+15
      FDK(I,1)=0.D0
 1360 CONTINUE
C     DO 1370 I=1,NNM+15
C     WRITE(*,*)'I FDK(I,1)',I,FDK(I,1)
C     WRITE(13,*)'I FDK(I,1)',I,FDK(I,1)
C 1370 CONTINUE
C
C     CONSTRUCTION OF RIGID BODIES ELEMENTS
C
C     CONSTRUCTION OF MASS MATRICES
C
C     CALCULATION OF Mzz ELEMENTS
C
C     FOR BODY 2
C
C*    RO2=DENSITY OF BODY 2 MATERIAL
      RO2=7850.D0
C*    R2=RADIUS OF BODY 2 CROSS SECTION
      R2=0.025
C*    AL2=LENGTH OF BODY 2
      AL2=5.85D0
      AR2=PI*R2*R2
      AM2=RO2*AR2*AL2
      AMZZ2(1,1)=AM2
      AMZZ2(1,2)=0.D0
      AMZZ2(2,1)=0.D0
      AMZZ2(2,2)=AM2
C
C     FOR BODY 3
C
C*    RO3=DENSITY OF BODY 3 MATERIAL
      RO3=7850.D0
C*    R3=RADIUS OF BODY 3 CROSS SECTION
      R3=0.09D0
C*    AL3=LENGTH OF BODY 3
      AL3=3.455
```

```
      AR3=PI*R3*R3
      AM3=RO3*AR3*AL3
      AMZZ3(1,1)=AM3
      AMZZ3(1,2)=0.D0
      AMZZ3(2,1)=0.D0
      AMZZ3(2,2)=AM3
C
C     FOR BODY 4
C
C*    AOBO=LENGTH 0F OBo
      AOBO=2.350D0
C*    AOAO=LENGTH 0F OAo
      AOAO=0.805D0
C*    ROF=DENSITY OF THE FLUID
      ROF=891.D0
C*    AL4=LENGTH OF BODY 4
      AL4=3.440D0
C*    AAL4=LENGTH L
      AAL4=3.42881781D0
C*    AAL2=LENGTH L2
      AAL2=0.02D0
C*    RI4=INNER RADIUS OF BODY 4 CROSS SECTION
      RI4=0.115D0
C*    R4=OUTER RADIUS OF BODY 4 CROSS SECTION
      R4=0.123D0
C*    AAM3=MASS OF THE CYLINDER
      AAM3=161.5269215D0
C     WA=A VARIABLE LENGTH
C     WA=(-AOBO-AL3*DCOS(Y(NNM+9))+AL2*DCOS(Y(NNM+6)))/DCOS(Y(NNM+12))
      WA=(AOAO-AL3*DSIN(Y(NNM+9))+AL2*DSIN(Y(NNM+6)))/DSIN(Y(NNM+12))
C     AAL1=LENGTH L1 WHICH IS VARIABLE
      AAL1=WA
      AAM5=ROF*(AAL4-AAL2)*PI*(RI4*RI4-R3*R3)+ROF*AAL1*PI*R3*R3
      AM4=AAM3+AAM5
      AMZZ4(1,1)=AM4
      AMZZ4(1,2)=0.D0
      AMZZ4(2,1)=0.D0
      AMZZ4(2,2)=AM4
C
C     FOR BODY 5
C
C*    ROR=DENSITY OF THE ROPE
      ROR=2.D0*5124.789168D0
C     R5=RADIUS OF THE ROPE CROSS SECTION
      R5=0.01D0
C     AOC=LENGTH OF THE BODY 1
      AOC=19.5D0
C     AL5R=VARIABLE LENGTH OF THE ROPE
      AL5R=Y(NNM+14)-CODIS
C     AM5R=VARIABLE MASS OF THE ROPE
      AM5R=ROR*PI*((R5)**2)*AL5R
C*    AM5P=MASS OF THE LIFTED LOAD
      AM5P=WF
      AMZZ5(1,1)=AM5R+AM5P
      AMZZ5(1,2)=0.D0
      AMZZ5(2,1)=0.D0
      AMZZ5(2,2)=AM5R+AM5P
C
C     CALCULATION OF  Mzo  ELEMENTS
C
C     FOR BODY 2
C
      TO2(1,1)=-DSIN(Y(NNM+6))
      TO2(1,2)=-DCOS(Y(NNM+6))
      TO2(2,1)=DCOS(Y(NNM+6))
      TO2(2,2)=-DSIN(Y(NNM+6))
      FM2(1,1)=AM2*AL2/2.D0
      FM2(2,1)=0.D0
C
C     TO2(2,2)*FM2(2,1)=AMZO2(2,1)
C
      DO 1500 I=1,2
      AMZO2(I,1)=0.D0
      DO 1500 K=1,2
      AMZO2(I,1)=AMZO2(I,1)+TO2(I,K)*FM2(K,1)
 1500 CONTINUE
C
C     FOR BODY 3
C
      TO3(1,1)=-DSIN(Y(NNM+9))
      TO3(1,2)=-DCOS(Y(NNM+9))
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```
      TO3(2,1)=DCOS(Y(NNM+9))
      TO3(2,2)=-DSIN(Y(NNM+9))
      FM3(1,1)=AM3*AL3/2.D0
      FM3(2,1)=0.D0
C
C    TO3(2,2)*FM3(2,1)=AMZO3(2,1)
C
      DO 1510 I=1,2
      AMZO3(I,1)=0.D0
      DO 1510 K=1,2
      AMZO3(I,1)=AMZO3(I,1)+TO3(I,K)*FM3(K,1)
 1510 CONTINUE
C
C    FOR BODY 4
C
      TO4(1,1)=-DSIN(Y(NNM+12))
      TO4(1,2)=-DCOS(Y(NNM+12))
      TO4(2,1)=DCOS(Y(NNM+12))
      TO4(2,2)=-DSIN(Y(NNM+12))
      FM4(1,1)=AM4*AL4/2.D0
      FM4(2,1)=0.D0
C
C    TO4(2,2)*FM4(2,1)=AMZO4(2,1)
C
      DO 1520 I=1,2
      AMZO4(I,1)=0.D0
      DO 1520 K=1,2
      AMZO4(I,1)=AMZO4(I,1)+TO4(I,K)*FM4(K,1)
 1520 CONTINUE
C
C    FOR BODY 5
C
      TO5(1,1)=-DSIN(Y(NNM+15))
      TO5(1,2)=-DCOS(Y(NNM+15))
      TO5(2,1)=DCOS(Y(NNM+15))
      TO5(2,2)=-DSIN(Y(NNM+15))
      FM5(1,1)=(AM5R/2.D0+AM5P)*AL5R
      FM5(2,1)=0.D0
C
C    TO5(2,2)*FM5(2,1)=AMZO5(2,1)
C
      DO 1522 I=1,2
      AMZO5(I,1)=0.D0
      DO 1522 K=1,2
      AMZO5(I,1)=AMZO5(I,1)+TO5(I,K)*FM5(K,1)
 1522 CONTINUE
C
C    CALCULATION OF  Moo  ELEMENTS
C
C    FOR BODY 2
C
      AMOO2(1,1)=AM2*R2*R2/4.D0+AM2*AL2*AL2/3.D0
C
C    FOR BODY 3
C
      AMOO3(1,1)=AM3*R3*R3/4.D0+AM3*AL3*AL3/3.D0
C
C    FOR BODY 4
C
      AMOO4(1,1)=AM4*R4*R4/4.D0+AM4*AL4*AL4/3.D0
C
C    FOR BODY 5
C
      AMOO5(1,1)=AM5R*R5*R5/4.D0+AM5R*AL5R*AL5R/3.D0+AM5P*AL5R*AL5R
C
C    CONSTRUCTION OF CORIOLIS AND CENTRIFUGAL FORCE TERM MATRICES
C
C    CALCULATION OF  Qz  ELEMENTS
C
C    FOR BODY 2
C
      TT2(1,1)=DCOS(Y(NNM+6))
      TT2(1,2)=-DSIN(Y(NNM+6))
      TT2(2,1)=DSIN(Y(NNM+6))
      TT2(2,2)=DCOS(Y(NNM+6))
C    TT2(2,2)*FM2(2,1)=DM(2,1)
      DO 1530 I=1,2
      DM(I,1)=0.D0
      DO 1530 K=1,2
      DM(I,1)=DM(I,1)+TT2(I,K)*FM2(K,1)
 1530 CONTINUE
```

```
      QZ2(1,1)=DM(1,1)*((Y(2*NNM+21))**2)
      QZ2(2,1)=DM(2,1)*((Y(2*NNM+21))**2)
C
C     FOR BODY 3
C
      TT3(1,1)=DCOS(Y(NNM+9))
      TT3(1,2)=-DSIN(Y(NNM+9))
      TT3(2,1)=DSIN(Y(NNM+9))
      TT3(2,2)=DCOS(Y(NNM+9))
C     TT3(2,2)*FM3(2,1)=DM(2,1)
      DO 1540 I=1,2
      DM(I,1)=0.D0
      DO 1540 K=1,2
      DM(I,1)=DM(I,1)+TT3(I,K)*FM3(K,1)
 1540 CONTINUE
      QZ3(1,1)=DM(1,1)*((Y(2*NNM+24))**2)
      QZ3(2,1)=DM(2,1)*((Y(2*NNM+24))**2)
C
C     FOR BODY 4
C
      TT4(1,1)=DCOS(Y(NNM+12))
      TT4(1,2)=-DSIN(Y(NNM+12))
      TT4(2,1)=DSIN(Y(NNM+12))
      TT4(2,2)=DCOS(Y(NNM+12))
C     TT4(2,2)*FM4(2,1)=DM(2,1)
      DO 1550 I=1,2
      DM(I,1)=0.D0
      DO 1550 K=1,2
      DM(I,1)=DM(I,1)+TT4(I,K)*FM4(K,1)
 1550 CONTINUE
      QZ4(1,1)=DM(1,1)*((Y(2*NNM+27))**2)
      QZ4(2,1)=DM(2,1)*((Y(2*NNM+27))**2)
C
C     FOR BODY 5
C
      TT5(1,1)=DCOS(Y(NNM+15))
      TT5(1,2)=-DSIN(Y(NNM+15))
      TT5(2,1)=DSIN(Y(NNM+15))
      TT5(2,2)=DCOS(Y(NNM+15))
C     TT5(2,2)*FM5(2,1)=DMQ(2,1)
      DO 1554 I=1,2
      DMQ(I,1)=0.D0
      DO 1554 K=1,2
      DMQ(I,1)=DMQ(I,1)+TT5(I,K)*FM5(K,1)
 1554 CONTINUE
      QZ5(1,1)=DMQ(1,1)*((Y(2*NNM+30))**2)
      QZ5(2,1)=DMQ(2,1)*((Y(2*NNM+30))**2)
C
C     CALCULATION OF  Qo  ELEMENTS
C
C     FOR BODY 2
C
      QO2(1,1)=0.D0
C
C     FOR BODY 3
C
      QO3(1,1)=0.D0
C
C     FOR BODY 4
C
      QO4(1,1)=0.D0
C
C     FOR BODY 5
C
      QO5(1,1)=0.D0
C
C     CONSTRUCTION OF EXTERNAL FORCE MATRICES
C
C     WEIGHT OF THE BODIES ARE ONLY EXTERNAL FORCE AT BODIES 2,3 AND 4
C     THESE FORCES ARE DISTRIBUTED LOADS
C
C     CALCULATION OF  Fz  ELEMENTS
C
C     Fi IS WEIGHT OF THE BODY i
C
C     FOR BODY 2
C
      AFZ2(1,1)=0.D0
      AFZ2(2,1)=-AM2*9.81D0
C
C     FOR BODY 3
```

```
C
      AFZ3(1,1)=0.D0
      AFZ3(2,1)=-AM3*9.81D0
C
C    FOR BODY 4
C
      AFZ4(1,1)=0.D0
      AFZ4(2,1)=-AM4*9.81D0
C
C    FOR BODY 5
C
      AFZ5(1,1)=0.D0
      AFZ5(2,1)=-(AM5R+AM5P)*9.81D0
C
C    CALCULATION OF  Fo  ELEMENTS
C
      DO 1570 I=1,2
      FM2T(1,I)=FM2(I,1)
      FM3T(1,I)=FM3(I,1)
      FM4T(1,I)=FM4(I,1)
 1570 CONTINUE
      RGR(1,1)=0.D0
      RGR(2,1)=-9.81
C
C    FOR BODY 2
C
C    TRANSPOZE OF  To MATRIX
      DO 1600 I=1,2
      DO 1600 J=1,2
      TO2T(J,I)=TO2(I,J)
 1600 CONTINUE
C    FM2T(1,2)*TO2T(2,2)=DR(1,2)
      DO 1610 J=1,2
      DR(1,J)=0.D0
      DO 1610 K=1,2
      DR(1,J)=DR(1,J)+FM2T(1,K)*TO2T(K,J)
 1610 CONTINUE
C    DR(1,2)*RGR(2,1)=AFO2(1,1)
      AFO2(1,1)=0.D0
      DO 1620 K=1,2
      AFO2(1,1)=AFO2(1,1)+DR(1,K)*RGR(K,1)
 1620 CONTINUE
C
C    FOR BODY 3
C
C    TRANSPOZE OF  To MATRIX
      DO 1630 I=1,2
      DO 1630 J=1,2
      TO3T(J,I)=TO3(I,J)
 1630 CONTINUE
C    FM3T(1,2)*TO3T(2,2)=DR(1,2)
      DO 1640 J=1,2
      DR(1,J)=0.D0
      DO 1640 K=1,2
      DR(1,J)=DR(1,J)+FM3T(1.K)*TO3T(K,J)
 1640 CONTINUE
C    DR(1,2)*RGR(2,1)=AFO3(1,1)
      AFO3(1,1)=0.D0
      DO 1650 K=1,2
      AFO3(1,1)=AFO3(1,1)+DR(1,K)*RGR(K,1)
 1650 CONTINUE
C
C    FOR BODY 4
C
C    TRANSPOZE OF  To MATRIX
      DO 1660 I=1,2
      DO 1660 J=1,2
      TO4T(J,I)=TO4(I,J)
 1660 CONTINUE
C    FM4T(1,2)*TO4T(2,2)=DR(1,2)
      DO 1670 J=1,2
      DR(1,J)=0.D0
      DO 1670 K=1,2
      DR(1,J)=DR(1,J)+FM4T(1,K)*TO4T(K,J)
 1670 CONTINUE
C    DR(1,2)*RGR(2,1)=AFO4(1,1)
      AFO4(1,1)=0.D0
      DO 1680 K=1,2
      AFO4(1,1)=AFO4(1,1)+DR(1,K)*RGR(K,1)
 1680 CONTINUE
C
```

```
C    FOR BODY 5
C
C    TRANSPOZE OF  To MATRIX
     DO 1682 I=1,2
     DO 1682 J=1,2
     TO5T(J,I)=TO5(I,J)
 1682 CONTINUE
     FM5RT(1,1)=AM5R*AL5R/2.D0
     FM5RT(1,2)=0.D0
     FM5PT(1,1)=AL5R
     FM5PT(1,2)=0.D0
     RGRP(1,1)=0.D0
     RGRP(2,1)=-AM5P*9.81D0
C    FM5RT(1,2)*TO5T(2,2)=DR(1,2)
     DO 1684 J=1,2
     DR(1,J)=0.D0
     DO 1684 K=1,2
     DR(1,J)=DR(1,J)+FM5RT(1,K)*TO5T(K,J)
 1684 CONTINUE
C    DR(1,2)*RGR(2,1)=AFO5R(1,1)
     AFO5R(1,1)=0.D0
     DO 1686 K=1,2
     AFO5R(1,1)=AFO5R(1,1)+DR(1,K)*RGR(K,1)
 1686 CONTINUE
C    FM5PT(1,2)*TO5T(2,2)=DR(1,2)
     DO 1688 J=1,2
     DR(1,J)=0.D0
     DO 1688 K=1,2
     DR(1,J)=DR(1,J)+FM5PT(1,K)*TO5T(K,J)
 1688 CONTINUE
C    DR(1,2)*RGRP(2,1)=AFO5P(1,1)
     AFO5P(1,1)=0.D0
     DO 1690 K=1,2
     AFO5P(1,1)=AFO5P(1,1)+DR(1,K)*RGRP(K,1)
 1690 CONTINUE
     AFO5(1,1)=AFO5R(1,1)+AFO5P(1,1)
C
C    CONTRUCTION OF CONSTRAINT EQUATIONS
C
C    ACB:B MATRIX OF THE CONSTRAINT EQUATIONS
C    ACBD:B DOT MATRIX OF THE CONSTRAINT EQUATIONS

     DO 1700 I=1,14
     DO 1700 J=1,NNM+15
     ACB(I,J)=0.D0
     ACBD(I,J)=0.D0
 1700 CONTINUE
C
C    FOR CONSTRAINT EQUATIONS 1,2,3,4,5 AND 15
C
     ACB(1,1)=1.D0
     ACB(2,2)=1.D0
     ACB(3,NNM+4)=1.D0
     ACB(4,NNM+10)=1.D0
     ACB(5,NNM+11)=1.D0
C    B DOT MATRICES OF CONSTRAINT EQUATIONS 1,2,3,4,5 AND 15 ARE ZERO
C     DO 2080 J=1,NNM+15
C     ACB(3,J)=0.D0
C     ACBD(3,J)=0.D0
C 2080 CONTINUE
C     ACB(3,NNM+5)=1.D0
C
C    FOR CONSTRAINT EQUATION 6
C
C    CONSTRUCTION OF B MATRIX
C
C*    GAMMA=ANGLE BETWEEN RIGID BODIES 1 AND 2
C     GAMMA=DACOS(AOA,AL2)
     GAMMA=0.09673198077D0
     COE(1,1)=-DSIN(Y(NNM+6)+GAMMA)
     COE(1,2)=DCOS(Y(NNM+6)+GAMMA)
     ACB(6,1)=COE(1,1)
     ACB(6,2)=COE(1,2)
C    NFEF=NUMBER OF FINITE ELEMENTS IN THE FIRST PART
C    NFES=NUMBER OF FINITE ELEMENTS IN THE SECOND PART

C    COE(1,2)*TO1(2,2)=GA(1,2)
     DO 1750 J=1,2
     GA(1,J)=0.D0
     DO 1750 K=1,2
     GA(1,J)=GA(1,J)+COE(1,K)*TO1(K,J)
```

```
 1750 CONTINUE
C    GA(1,2)*YOO11(2,NM)=CI(1,NM)
     DO 1760 J=1,NM
     CI(1,J)=0.D0
     DO 1760 K=1,2
     CI(1,J)=CI(1,J)+GA(1,K)*YOO11(K,J)
 1760 CONTINUE
C    CI(1,NM)*VO(NM,1)=GD(1,1)
     GD(1,1)=0.D0
     DO 1780 K=1,NM
     GD(1,1)=GD(1,1)+CI(1,K)*VO(K,1)
 1780 CONTINUE
     ACB(6,3)=GD(1,1)
C    COE(1,2)*TT1(2,2)=GA(1,2)
     DO 1790 J=1,2
     GA(1,J)=0.D0
     DO 1790 K=1,2
     GA(1,J)=GA(1,J)+COE(1,K)*TT1(K,J)
 1790 CONTINUE
C    GA(1,2)*YOO12(2,NNM)=GH(1,NNM)
     DO 1800 J=1,NNM
     GH(1,J)=0.D0
     DO 1800 K=1,2
     GH(1,J)=GH(1,J)+GA(1,K)*YOO12(K,J)
 1800 CONTINUE
     DO 1830 J=4,NNM+3
     ACB(6,J)=GH(1,J-3)
 1830 CONTINUE
     ACB(6,NNM+4)=-COE(1,1)
     ACB(6,NNM+5)=-COE(1,2)
C*    AD=DISTANCE BETWEEN A AND D
     AD=0.565
     ACB(6,NNM+6)=-AL2*DCOS(GAMMA)
C
C    CONSTRUCTION OF B DOT MATRIX
C
     COED(1,1)=-DCOS(Y(NNM+6)+GAMMA)*Y(2*NNM+21)
     COED(1,2)=-DSIN(Y(NNM+6)+GAMMA)*Y(2*NNM+21)
     ACBD(6,1)=COED(1,1)
     ACBD(6,2)=COED(1,2)
C    COED(1,2)*TO1(2,2)=GA(1,2)
     DO 1850 J=1,2
     GA(1,J)=0.D0
     DO 1850 K=1,2
     GA(1,J)=GA(1,J)+COED(1,K)*TO1(K,J)
 1850 CONTINUE
C    GA(1,2)*YOO11(2,NM)=CI(1,NM)
     DO 1870 J=1,NM
     CI(1,J)=0.D0
     DO 1870 K=1,2
     CI(1,J)=CI(1,J)+GA(1,K)*YOO11(K,J)
 1870 CONTINUE
C    CI(1,NM)*VO(NM,1)=GD(1,1)
     GD(1,1)=0.D0
     DO 1880 K=1,NM
     GD(1,1)=GD(1,1)+CI(1,K)*VO(K,1)
 1880 CONTINUE
C    COE(1,2)*TT1(2,2)=GA(1,2)
     DO 1890 J=1,2
     GA(1,J)=0.D0
     DO 1890 K=1,2
     GA(1,J)=GA(1,J)+COE(1,K)*TT1(K,J)
 1890 CONTINUE
C    GA(1,2)*YOO11(2,NM)=CI(1,NM)
     DO 1910 J=1,NM
     CI(1,J)=0.D0
     DO 1910 K=1,2
     CI(1,J)=CI(1,J)+GA(1,K)*YOO11(K,J)
 1910 CONTINUE
C    CI(1,NM)*VO(NM,1)=GS(1,1)
     GS(1,1)=0.D0
     DO 1920 K=1,NM
     GS(1,1)=GS(1,1)+CI(1,K)*VO(K,1)
 1920 CONTINUE
     GS(1,1)=-GS(1,1)*Y(NNM+18)
C    COE(1,2)*TO1(2,2)=GA(1,2)
     DO 1930 J=1,2
     GA(1,J)=0.D0
     DO 1930 K=1,2
     GA(1,J)=GA(1,J)+COE(1,K)*TO1(K,J)
 1930 CONTINUE
```

```
C    GA(1,2)*YOO12(2,NNM)=GH(1,NNM)
     DO 1960 J=1,NNM
     GH(1,J)=0.D0
     DO 1960 K=1,2
     GH(1,J)=GH(1,J)+GA(1,K)*YOO12(K,J)
 1960 CONTINUE
C    GH(1,NNM)*ETAD(NNM,1)=GZ(1,1)
     GZ(1,1)=0.D0
     DO 1970 K=1,NNM
     GZ(1,1)=GZ(1,1)+GH(1,K)*ETAD(K,1)
 1970 CONTINUE
     ACBD(6,3)=GD(1,1)+GS(1,1)+GZ(1,1)
C
C    COED(1,2)*TT1(2,2)=GA(1,2)
     DO 1980 J=1,2
     GA(1,J)=0.D0
     DO 1980 K=1,2
     GA(1,J)=GA(1,J)+COED(1,K)*TT1(K,J)
 1980 CONTINUE
C    GA(1,2)*YOO12(2,NNM)=GH(1,NNM)
     DO 2010 J=1,NNM
     GH(1,J)=0.D0
     DO 2010 K=1,2
     GH(1,J)=GH(1,J)+GA(1,K)*YOO12(K,J)
 2010 CONTINUE
C    COE(1,2)*TO1(2,2)=GA(1,2)
     DO 2020 J=1,2
     GA(1,J)=0.D0
     DO 2020 K=1,2
     GA(1,J)=GA(1,J)+COE(1,K)*TO1(K,J)
 2020 CONTINUE
C    GA(1,2)*YOO12(2,NNM)=HH(1,NNM)
     DO 2050 J=1,NNM
     HH(1,J)=0.D0
     DO 2050 K=1,2
     HH(1,J)=HH(1,J)+GA(1,K)*YOO12(K,J)
 2050 CONTINUE
     DO 2060 J=1,NNM
     HH(1,J)=HH(1,J)*Y(NNM+18)
 2060 CONTINUE
     DO 2070 J=1,NNM
     ACBD(6,J+3)=GH(1,J)+HH(1,J)
 2070 CONTINUE
     ACBD(6,NNM+4)=-COED(1,1)
     ACBD(6,NNM+5)=-COED(1,2)
     DO 2080 J=1,NNM+15
     ACB(6,J)=0.D0
     ACBD(6,J)=0.D0
 2080 CONTINUE
     ACB(6,NNM+5)=1.D0
C
C    FOR CONSTRAINT EQUATION 7
C
     ACB(7,3)=1.D0
     DO 2120 J=4,NNM+3
     ACB(7,J)=YOO14(1,J-3)
 2120 CONTINUE
     ACB(7,NNM+6)=-1.D0
C    B DOT MATRICES OF CONSTRAINT EQUATION 7 IS ZERO
C     DO 2080 J=1,NNM+15
C     ACB(7,J)=0.D0
C     ACBD(7,J)=0.D0
C 2080 CONTINUE
C     ACB(7,NNM+5)=1.D0
C
C    FOR CONSTRAINT EQUATIONS 8 AND 9
C
C    AL2=LENGTH OF BODY 2
C    AL3=LENGTH OF BODY 3

     ACB(8,NNM+4)=1.D0
     ACB(8,NNM+6)=-AL2*DSIN(Y(NNM+6))
     ACB(8,NNM+7)=-1.D0
     ACB(8,NNM+9)=AL3*DSIN(Y(NNM+9))
     ACBD(8,NNM+6)=-AL2*DCOS(Y(NNM+6))*Y(2*NNM+21)
     ACBD(8,NNM+9)=AL3*DCOS(Y(NNM+9))*Y(2*NNM+24)
     ACB(9,NNM+5)=1.D0
     ACB(9,NNM+6)=AL2*DCOS(Y(NNM+6))
     ACB(9,NNM+8)=-1.D0
     ACB(9,NNM+9)=-AL3*DCOS(Y(NNM+9))
     ACBD(9,NNM+6)=-AL2*DSIN(Y(NNM+6))*Y(2*NNM+21)
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```
      ACBD(9,NNM+9)=AL3*DSIN(Y(NNM+9))*Y(2*NNM+24)
C
C     FOR CONSTRAINT EQUATIONS 10 AND 11
C
C     AOBO=DISTANCE BETWEEN O AND Bo

      ACB(10,NNM+7)=-DSIN(Y(NNM+9))
      ACB(10,NNM+8)=DCOS(Y(NNM+9))
      ACB(10,NNM+10)=DSIN(Y(NNM+12))
      ACB(10,NNM+11)=-DCOS(Y(NNM+12))
      ACBD(10,NNM+7)=-DCOS(Y(NNM+9))*Y(2*NNM+24)
      ACBD(10,NNM+8)=-DSIN(Y(NNM+9))*Y(2*NNM+24)
      ACBD(10,NNM+10)=DCOS(Y(NNM+12))*Y(2*NNM+27)
      ACBD(10,NNM+11)=DSIN(Y(NNM+12))*Y(2*NNM+27)
      ACB(11,NNM+9)=1.D0
      ACB(11,NNM+12)=-1.D0
      IF(Y(NNM+12).GT.(45.D0/180.D0*PI)) GO TO 2124
      ACB(10,NNM+12)=-(AL2*DCOS(Y(NNM+6))-AL3*DCOS(Y(NNM+9))-AOBO)/
     + DCOS(Y(NNM+12))
      ACBD(10,NNM+12)=-((-AL2*DSIN(Y(NNM+6))*Y(2*NNM+21)+AL3*
     + DSIN(Y(NNM+9))*Y(2*NNM+24))*DCOS(Y(NNM+12))+DSIN(Y(NNM+12))*
     + Y(2*NNM+27)*(AL2*DCOS(Y(NNM+6))-AL3*DCOS(Y(NNM+9))-AOBO))
     + /(DCOS(Y(NNM+12))*DCOS(Y(NNM+12)))
      GO TO 2126
 2124 ACB(10,NNM+12)=-(AL2*DSIN(Y(NNM+6))-AL3*DSIN(Y(NNM+9))+AOAO)/
     + DSIN(Y(NNM+12))
      ACBD(10,NNM+12)=-((AL2*DCOS(Y(NNM+6))*Y(2*NNM+21)-AL3*
     + DCOS(Y(NNM+9))*Y(2*NNM+24))*DSIN(Y(NNM+12))-DCOS(Y(NNM+12))*
     + Y(2*NNM+27)*(AL2*DSIN(Y(NNM+6))-AL3*DSIN(Y(NNM+9))+AOAO))
     + /(DSIN(Y(NNM+12))*DSIN(Y(NNM+12)))

C     B DOT MATRICES OF CONSTRAINT EQUATION 11 IS ZERO
C
C     CONSTRAINT EQUATIONS 12 AND 13
C
 2126 ACB(12,1)=1.D0
      ACB(13,2)=1.D0
C     TO1(2,2)*YOO15(2,NM)=CD(2,NM)
      DO 2152 I=1,2
      DO 2152 J=1,NM
      CD(I,J)=0.D0
      DO 2152 K=1,2
      CD(I,J)=CD(I,J)+TO1(I,K)*YOO15(K,J)
 2152 CONTINUE
C     CD(2,NM)*VO(NM,1)=DM(2,1)
      DO 2156 I=1,2
      DM(I,1)=0.D0
      DO 2156 K=1,NM
      DM(I,1)=DM(I,1)+CD(I,K)*VO(K,1)
 2156 CONTINUE
      ACB(12,3)=DM(1,1)
      ACB(13,3)=DM(2,1)
C     TT1(2,2)*YOO16(2,NNM)=CG(2,NNM)
      DO 2158 I=1,2
      DO 2158 J=1,NNM
      CG(I,J)=0.D0
      DO 2158 K=1,2
      CG(I,J)=CG(I,J)+TT1(I,K)*YOO16(K,J)
 2158 CONTINUE
      DO 2164 I=4,NNM+3
      ACB(12,I)=CG(1,I-3)
      ACB(13,I)=CG(2,I-3)
 2164 CONTINUE
      ACB(12,NNM+13)=-1.D0
      ACB(13,NNM+14)=-1.D0
C
C     B DOT MATRICES OF CONSTRAINT EQUATIONS 12 AND 13
C     TT1(2,2)*YOO15(2,NM)=CD(2,NM)
      DO 2166 I=1,2
      DO 2166 J=1,NM
      CD(I,J)=0.D0
      DO 2166 K=1,2
      CD(I,J)=CD(I,J)+TT1(I,K)*YOO15(K,J)
 2166 CONTINUE
C     CD(2,NM)*VO(NM,1)=DM(2,1)
      DO 2170 I=1,2
      DM(I,1)=0.D0
      DO 2170 K=1,NM
      DM(I,1)=DM(I,1)+CD(I,K)*VO(K,1)
 2170 CONTINUE
      DM(1,1)=-DM(1,1)*Y(NNM+18)
```

```
      DM(2,1)=-DM(2,1)*Y(NNM+18)
C     TO1(2,2)*YOO16(2,NNM)=CG(2,NNM)
      DO 2172 I=1,2
      DO 2172 J=1,NNM
      CG(I,J)=0.D0
      DO 2172 K=1,2
      CG(I,J)=CG(I,J)+TO1(I,K)*YOO16(K,J)
 2172 CONTINUE
C     CG(2,NNM)*ETAD(NNM,1)=PA(2,1)
      DO 2178 I=1,2
      PA(I,1)=0.D0
      DO 2178 K=1,NNM
      PA(I,1)=PA(I,1)+CG(I,K)*ETAD(K,1)
 2178 CONTINUE
      ACBD(12,3)=DM(1,1)+PA(1,1)
      ACBD(13,3)=DM(2,1)+PA(2,1)
      DO 2180 J=4,NNM+3
      ACBD(12,J)=CG(1,J-3)*Y(NNM+18)
      ACBD(13,J)=CG(2,J-3)*Y(NNM+18)
 2180 CONTINUE
C
C     CONSTRAINT EQUATION 14
C
      ACB(14,NNM+7)=DCOS(Y(NNM+9))
      ACB(14,NNM+8)=DSIN(Y(NNM+9))
      ACBD(14,NNM+7)=-DSIN(Y(NNM+9))*Y(2*NNM+24)
      ACBD(14,NNM+8)=DCOS(Y(NNM+9))*Y(2*NNM+24)

C     DO 2190 I=1,14
C     DO 2190 J=1,NNM+15
C     WRITE(*,*)'I J ACB(I,J)',I,J,ACB(I,J)
C     WRITE(IO,*)'I J ACB(I,J)',I,J,ACB(I,J)
C 2190 CONTINUE
C     DO 2195 I=1,14
C     DO 2195 J=1,NNM+15
C     WRITE(*,*)'I J ACBD(I,J)',I,J,ACBD(I,J)
C     WRITE(IO,*)'I J ACBD(I,J)',I,J,ACBD(I,J)
C 2195 CONTINUE
C
C     CONSTRUCTION OF SYSTEM EQUATION
C
C     CONSTRUCTION OF MASS MATRICES
C
C     FOR BODY 1
C
      DO 2200 I=1,2
      DO 2200 J=1,2
      AMM1(I,J)=AMZZ(I,J)
 2200 CONTINUE
      DO 2210 I=1,2
      AMM1(I,3)=AMZO(I,1)
      AMM1(3,I)=AMZO(I,1)
 2210 CONTINUE
      DO 2220 I=1,2
      DO 2220 J=1,NNM
      AMM1(I,J+3)=AMZN(I,J)
      AMM1(J+3,I)=AMZN(I,J)
 2220 CONTINUE
      AMM1(3,3)=AMOO(1,1)
      DO 2230 J=1,NNM
      AMM1(3,J+3)=AMON(1,J)
      AMM1(J+3,3)=AMON(1,J)
 2230 CONTINUE
      DO 2240 I=1,NNM
      DO 2240 J=1,NNM
      AMM1(I+3,J+3)=AMNN(I,J)
 2240 CONTINUE
C
C     FOR BODY 2
C
      DO 2250 I=1,2
      DO 2250 J=1,2
      AMM2(I,J)=AMZZ2(I,J)
 2250 CONTINUE
      DO 2260 I=1,2
      AMM2(I,3)=AMZO2(I,1)
      AMM2(3,I)=AMZO2(I,1)
 2260 CONTINUE
      AMM2(3,3)=AMOO2(1,1)
C
C     FOR BODY 3
```

```
C
   DO 2270 I=1,2
   DO 2270 J=1,2
   AMM3(I,J)=AMZZ3(I,J)
2270 CONTINUE
   DO 2280 I=1,2
   AMM3(I,3)=AMZO3(I,1)
   AMM3(3,I)=AMZO3(I,1)
2280 CONTINUE
   AMM3(3,3)=AMOO3(1,1)
C
C  FOR BODY 4
C
   DO 2290 I=1,2
   DO 2290 J=1,2
   AMM4(I,J)=AMZZ4(I,J)
2290 CONTINUE
   DO 2300 I=1,2
   AMM4(I,3)=AMZO4(I,1)
   AMM4(3,I)=AMZO4(I,1)
2300 CONTINUE
   AMM4(3,3)=AMOO4(1,1)
C
C  FOR BODY 5
C
   DO 2305 I=1,2
   DO 2305 J=1,2
   AMM5(I,J)=AMZZ5(I,J)
2305 CONTINUE
   DO 2307 I=1,2
   AMM5(I,3)=AMZO5(I,1)
   AMM5(3,I)=AMZO5(I,1)
2307 CONTINUE
   AMM5(3,3)=AMOO5(1,1)

C
C  CONSTRUCTION OF DIAGONAL MASS MATRIX
C
C  AMM(NNM+15,NNM+15)=TOTAL DIAGONAL MASS MATRIX
C
   DO 2310 I=1,NNM+15
   DO 2310 J=1,NNM+15
   AMM(I,J)=0.D0
2310 CONTINUE
C
C  CONTRIBUTION OF BODY 1
C
   DO 2320 I=1,NNM+3
   DO 2320 J=1,NNM+3
   AMM(I,J)=AMM1(I,J)
2320 CONTINUE
C
C  CONTRIBUTION OF BODY 2
C
   DO 2330 I=NNM+4,NNM+6
   DO 2330 J=NNM+4,NNM+6
   AMM(I,J)=AMM2(I-NNM-3,J-NNM-3)
2330 CONTINUE
C
C  CONTRIBUTION OF BODY 3
C
   DO 2340 I=NNM+7,NNM+9
   DO 2340 J=NNM+7,NNM+9
   AMM(I,J)=AMM3(I-NNM-6,J-NNM-6)
2340 CONTINUE
C
C  CONTRIBUTION OF BODY 4
C
   DO 2350 I=NNM+10,NNM+12
   DO 2350 J=NNM+10,NNM+12
   AMM(I,J)=AMM4(I-NNM-9,J-NNM-9)
2350 CONTINUE
C
C  CONTRIBUTION OF BODY 5
C
   DO 2355 I=NNM+13,NNM+15
   DO 2355 J=NNM+13,NNM+15
   AMM(I,J)=AMM5(I-NNM-12,J-NNM-12)
2355 CONTINUE
C    DO 2357 I=1,NNM+15
C    DO 2357 J=1,NNM+15
```

```
C     WRITE(*,*)'I J AMM(I,J)',I,J,AMM(I,J)
C     WRITE(13,*)'I J AMM(I,J)',I,J,AMM(I,J)
C 2357 CONTINUE
C
C    CONSTRUCTION OF SYSTEM EQUATION LEFT MATRIX=SEL(NNM+29,NNM+29)
C
    DO 2360 I=1,NNM+29
    DO 2360 J=1,NNM+29
    SEL(I,J)=0.D0
 2360 CONTINUE
C    TRANSPOZE OF CONSTRAINT COEFFICIENT MATRIX B
    DO 2370 I=1,14
    DO 2370 J=1,NNM+15
    ACBT(J,I)=ACB(I,J)
 2370 CONTINUE
C
C    CONTRIBUTION OF MASS MATRIX
C
    DO 2380 I=1,NNM+15
    DO 2380 J=1,NNM+15
    SEL(I,J)=AMM(I,J)
 2380 CONTINUE
C
C    CONTRIBUTION OF CONSTRAINT ,B TRANSPOZE MATRIX
C
    DO 2390 I=1,NNM+15
    DO 2390 J=1,14
    SEL(I,J+NNM+15)=ACBT(I,J)
 2390 CONTINUE
C
C    CONTRIBUTION OF CONSTRAINT ,B MATRIX
C
    DO 2392 I=1,14
    DO 2392 J=1,NNM+15
    SEL(I+NNM+15,J)=ACB(I,J)
C     SEL(J,I+NNM+15)=ACB(I,J)
 2392 CONTINUE
C     IF(IIIIZ.NE.(NOO*NOOM)) GO TO 9550
C     DO 2394 I=1,NNM+15
C     DO 2394 J=1,NNM+15
C     WRITE(*,*)'I J AMM(I,J)',I,J,AMM(I,J)
C     WRITE(13,*)'I J AMM(I,J)',I,J,AMM(I,J)
C 2394 CONTINUE
C     DO 2396 I=1,14
C     DO 2396 J=1,NNM+15
C     WRITE(*,*)'I J ACB(I,J)',I,J,ACB(I,J)
C     WRITE(13,*)'I J ACB(I,J)',I,J,ACB(I,J)
C 2396 CONTINUE
C     DO 2401 I=1,NNM+29
C     DO 2401 J=1,NNM+29
C     WRITE(*,*)'I J SEL(I,J)',I,J,SEL(I,J)
C     WRITE(IO,*)'I J SEL(I,J)',I,J,SEL(I,J)
C 2401 CONTINUE
C
C    CALCULATION OF INVERSE OF SYSTEM EQUATION LEFT MATRIX
C    SELI(NNM+29,NNM+29)
C
C    FIRST METHOD USING THE SUBROUTINE ELIM(AB,N,NP)

C    PREPARE THE REQUIRED PARAMETERS FOR THE SUBROUTINE ELIM(AB,N,NP)
C    ABSEL(NSEL,NPSEL)=COEFFICENT MATRIX AUGMENTED WITH R.H.S. VECTORS
C    NSEL=NUMBER OF EQUATIONS=NNM+29
C    NPSEL=TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX=2*(NNM+29)
C
 9550 DO 2402 I=1,NNM+29
    DO 2402 J=1,NNM+29
    ABSEL(I,J)=SEL(I,J)
 2402 CONTINUE
    DO 2404 I=1,NNM+29
    DO 2404 J=NNM+30,2*(NNM+29)
    IF(I.NE.(J-NNM-29)) GO TO 2403
    ABSEL(I,J)=1.D0
    GO TO 2404
 2403 ABSEL(I,J)=0.D0
 2404 CONTINUE
    CALL ELIM(ABSEL,NNM+29,2*(NNM+29))
    DO 2406 I=1,NNM+29
    DO 2406 J=NNM+30,2*(NNM+29)
    SELI(I,J-NNM-29)=ABSEL(I,J)
 2406 CONTINUE
```

```
C    SECOND METHOD USING THE SUBROUTINE INVERS(NSEL,SEL,SELI)
C     CALL INVERS(NNM+29,SEL,SELI)

C     DO 2407 I=1,NNM+29
C     DO 2407 J=1,NNM+29
C     WRITE(*,*)'I J SELI(I,J)',I,J,SELI(I,J)
C     WRITE(IO,*)'I J SELI(I,J)',I,J,SELI(I,J)
C 2407 CONTINUE

C    CHECKING THE INVERS, FEASIBLE OR NOT
C    SELI(NNM+29,NNM+29)*SEL(NNM+29,NNM+29)=AIDT(NNM+29,NNM+29)
     DO 2408 I=1,NNM+29
     DO 2408 J=1,NNM+29
     AIDT(I,J)=0.D0
     DO 2408 K=1,NNM+29
     AIDT(I,J)=AIDT(I,J)+SELI(I,K)*SEL(K,J)
 2408 CONTINUE
C     WRITE(*,*)'AIDT MUST BE IDENTITY MATRIX FOR THE FEASIBLE'
C     WRITE(*,*)'SOLUTION'
C     DO 2409 I=1,NNM+29
C     DO 2409 J=1,NNM+29
C     WRITE(*,*)'I J AIDT(I,J)',I,J,AIDT(I,J)
C     WRITE(IO,*)'I J AIDT(I,J)',I,J,AIDT(I,J)
C 2409 CONTINUE

     IF(IIIIZ.NE.(NOO*1000)) GO TO 2415
C    CHECKING THE AIDT(NNM+29,NNM+29) MATRIX IS AN IDENTITY
C    MATRIX OR NOT
     K=0
     DO 2411 I=1,NNM+29
     DO 2411 J=1,NNM+29
     IF(I.NE.J) GO TO 2413
     IF(DABS(AIDT(I,J)-1.D0).LE.1.D-6) GO TO 2411
     K=K+1
C     WRITE(*,*)'I J AIDT(I,J)',I,J,AIDT(I,J)
     WRITE(13,*)'I J AIDT(I,J)',I,J,AIDT(I,J)
     GO TO 2411
 2413 IF(DABS(AIDT(I,J)).LE.1.D-7) GO TO 2411
     K=K+1
C     WRITE(*,*)'I J AIDT(I,J)',I,J,AIDT(I,J)
     WRITE(13,*)'I J AIDT(I,J)',I,J,AIDT(I,J)
 2411 CONTINUE
     IF(K.EQ.0) GO TO 2415
C     WRITE(*,*)'AIDT(I,J) IS NOT AN IDENTITY MATRIX.'
C     WRITE(*,*)'SO, THERE IS AN ERROR'
C     WRITE(13,*)'AIDT(I,J) IS NOT AN IDENTITY MATRIX.'
C     WRITE(13,*)'SO, THERE IS AN ERROR'
C     WRITE(*,*)'K',K
C     WRITE(13,*)'K',K
C
C     CONSTRUCTION OF CORIOLIS AND CENTRIFUGAL FORCE TERM MATRIX
C     QQ(NNM+15,1)
C
C     CONTRIBUTION OF BODY 1
C
 2415 DO 2418 I=1,2
     QQ(I,1)=QZ(I,1)
 2418 CONTINUE
     QQ(3,1)=QO(1,1)
     DO 2420 I=1,NNM
     QQ(I+3,1)=QN(I,1)
 2420 CONTINUE
C
C     CONTRIBUTION OF BODY 2
C
     DO 2430 I=1,2
     QQ(I+NNM+3,1)=QZ2(I,1)
 2430 CONTINUE
     QQ(NNM+6,1)=QO2(1,1)
C
C     CONTRIBUTION OF BODY 3
C
     DO 2440 I=1,2
     QQ(I+NNM+6,1)=QZ3(I,1)
 2440 CONTINUE
     QQ(NNM+9,1)=QO3(1,1)
C
C     CONTRIBUTION OF BODY 4
C
     DO 2450 I=1,2
     QQ(I+NNM+9,1)=QZ4(I,1)
```

```
 2450 CONTINUE
    QQ(NNM+12,1)=QO4(1,1)
C
C    CONTRIBUTION OF BODY 5
C
    DO 2455 I=1,2
    QQ(I+NNM+12,1)=QZ5(I,1)
 2455 CONTINUE
    QQ(NNM+15,1)=QO5(1,1)
C     DO 2457 I=1,NNM+15
C     WRITE(*,*)'I QQ(I,1)',I,QQ(I,1)
C     WRITE(13,*)'I QQ(I,1)',I,QQ(I,1)
C 2457 CONTINUE
C
C    CONSTRUCTION OF EXTERNAL FORCE MATRIX=AFF(NNM+15,1)
C
C    CONTRIBUTION OF BODY 1
C
    DO 2460 I=1,2
    AFF(I,1)=AFZW(I,1)
 2460 CONTINUE
    AFF(3,1)=AFOW(1,1)
    DO 2470 I=1,NNM
    AFF(I+3,1)=AFNW(I,1)
 2470 CONTINUE
C
C    CONTRIBUTION OF BODY 2
C
    DO 2480 I=1,2
    AFF(I+NNM+3,1)=AFZ2(I,1)
 2480 CONTINUE
    AFF(NNM+6,1)=AFO2(1,1)
C
C    CONTRIBUTION OF BODY 3
C
    DO 2490 I=1,2
    AFF(I+NNM+6,1)=AFZ3(I,1)
 2490 CONTINUE
    AFF(NNM+9,1)=AFO3(1,1)
C
C    CONTRIBUTION OF BODY 4
C
    DO 2500 I=1,2
    AFF(I+NNM+9,1)=AFZ4(I,1)
 2500 CONTINUE
    AFF(NNM+12,1)=AFO4(1,1)
C
C    CONTRIBUTION OF BODY 5
C
    DO 2505 I=1,2
    AFF(I+NNM+12,1)=AFZ5(I,1)
 2505 CONTINUE
    AFF(NNM+15,1)=AFO5(1,1)
C     DO 2510 I=1,NNM+15
C     WRITE(*,*)'I AFF(I,1)',I,AFF(I,1)
C     WRITE(13,*)'I AFF(I,1)',I,AFF(I,1)
C 2510 CONTINUE
C
C    CONSTRUCTION OF B DOT Y MATRIX=BDY(14,1)
C
C    BDY(14,1) MATRIX INCLUDES '-' SIGN
    DO 2530 I=NNM+16,2*(NNM+15)
    PB(I-NNM-15,1)=Y(I)
 2530 CONTINUE
C    ACBD(14,NNM+15)*PB(NNM+15,1)=BDY(14,1)
    DO 2540 I=1,14
    BDY(I,1)=0.D0
    DO 2540 K=1,NNM+15
    BDY(I,1)=BDY(I,1)+ACBD(I,K)*PB(K,1)
 2540 CONTINUE
C    MULTIPLICATION OF THE BDY MATRIX WITH MINUS SIGN
    DO 2550 I=1,14
    BDY(I,1)=-BDY(I,1)
 2550 CONTINUE
C     DO 2555 I=1,14
C     WRITE(*,*)'I BDY(I,1)',I,BDY(I,1)
C     WRITE(IO,*)'I BDY(I,1)',I,BDY(I,1)
C 2555 CONTINUE
C
C    CONSTRUCTION OF GDOT MATRIX=GDOT(14,1)
C
```

```
      DO 2560 I=1,14
      GDOT(I,1)=0.D0
 2560 CONTINUE
C     IF THERE IS ANY GDOT VALUE, TRANSFER IT TO GDOT(14,1) MATRIX

C     FOR LINEAR MOTION
C      IF(T.LE.TIME1) GDOT(14,1)=VZERO/TIME1
C      IF((T.GT.TIME1).AND.(T.LE.TIME2)) GDOT(14,1)=0.D0
C      IF((T.GT.TIME2).AND.(T.LE.TIME3)) GDOT(14,1)=-VZERO/(TIME3-
C     + TIME2)
C      IF(T.GT.TIME3) GDOT(14,1)=0.D0

C     FOR CYCLOIDAL MOTION
      IF(T.LE.TIME1) GDOT(14,1)=VZERO/TIME1*(1-DCOS(2*PI/TIME1*T))
      IF((T.GT.TIME1).AND.(T.LT.TIME2)) GDOT(14,1)=0.D0
      IF((T.GE.TIME2).AND.(T.LE.TIME3)) GDOT(14,1)=-VZERO/(TIME3-
     + TIME2)*(1-DCOS(2*PI/(TIME3-TIME2)*(T-TIME2)))
      IF(T.GT.TIME3) GDOT(14,1)=0.D0

C      DO 2565 I=1,14
C      WRITE(*,*)'I GDOT(I,1)',I,GDOT(I,1)
C      WRITE(IO,*)'I GDOT(I,1)',I,GDOT(I,1)
C 2565 CONTINUE
C
C     CONSTRUCTION OF THE SYSTEM EQUATION RIGHT MATRIX =SER(NNM+29,1)
      DO 2570 I=1,NNM+29
      SER(I,1)=0.D0
 2570 CONTINUE
      DO 2580 I=1,NNM+15
      SER(I,1)=QQ(I,1)+AFF(I,1)+FSK(I,1)+FDK(I,1)
 2580 CONTINUE
      DO 2590 I=NNM+16,NNM+29
      SER(I,1)=BDY(I-NNM-15,1)+GDOT(I-NNM-15,1)
 2590 CONTINUE
C      DO 2600 I=1,NNM+29
C      WRITE(*,*)'I SER(I,1)',I,SER(I,1)
C      WRITE(13,*)'I SER(I,1)',I,SER(I,1)
C 2600 CONTINUE

C     MULTIPLICATION OF THE INVERSE OF THE SYSTEM EQUATION LEFT MATRIX
C     WITH SYSTEM EQUATION RIGHT MATRIX
C     SELI(NNM+29,NNM+29)*SER(NNM+29,1)=SEQ(NNM+29,1)
      DO 2610 I=1,NNM+29
      SEQ(I,1)=0.D0
      DO 2610 K=1,NNM+29
      SEQ(I,1)=SEQ(I,1)+SELI(I,K)*SER(K,1)
 2610 CONTINUE
C      DO 2620 I=1,NNM+29
C      WRITE(*,*)'I SEQ(I,1)',I,SEQ(I,1)
C      WRITE(13,*)'I SEQ(I,1)',I,SEQ(I,1)
C 2620 CONTINUE

      DO 2650 I=1,NNM+15
      YP(I)=Y(I+NNM+15)
 2650 CONTINUE
      DO 2660 I=NNM+16,2*(NNM+15)
      YP(I)=SEQ(I-NNM-15,1)
 2660 CONTINUE

      RETURN
      END
```