universe

Article

A Scale at 10 MeV, Gravitational Topological Vacuum, and Large Extra Dimensions

Ufuk Aydemir [1,2]

[1] School of Physics, Huazhong University of Science and Technology, Wuhan 430074, Hubei, China; uaydemir@hust.edu.cn
[2] Department of Physics and Astronomy, Uppsala University, Box 516, SE-751 20 Uppsala, Sweden

Received: 21 June 2018; Accepted: 16 July 2018; Published: 18 July 2018

Abstract: We discuss a possible scale of gravitational origin at around 10 MeV, or 10^{-12} cm, which arises in the MacDowell–Mansouri formalism of gravity due to the topological Gauss–Bonnet term in the action, as pointed out by Bjorken several years ago. A length scale of the same size emerges also in the Kodama solution in gravity, which is known to be closely related to the MacDowell–Mansouri formulation. We particularly draw attention to the intriguing incident that the existence of six compact extra dimensions originated from TeV-scale quantum gravity as well points to a length scale of 10^{-12} cm, as the compactification scale. The presence of six such extra dimensions is also in remarkable consistency with the MacDowell–Mansouri formalism; it provides a possible explanation for the factor of $\sim 10^{120}$ multiplying the Gauss–Bonnet term in the action. We also comment on the relevant implications of such a scale regarding the thermal history of the universe motivated by the fact that it is considerably close to 1–2 MeV below which the weak interactions freeze out, leading to Big Bang Nucleosynthesis.

Keywords: MacDowell–Mansouri formalism; Bjorken–Zeldovich scale; Gauss–Bonnet term; Einstein–Cartan formalism; Big Bang nucleosynthesis; cosmological constant; Kodama wavefunctions

1. Introduction

Bjorken points out in Ref. [1] that the MacDowell–Mansouri (MM) formulation of gravity [2] naturally reveals an induced scale of $\sim$10 MeV, or $\sim 10^{-12}$ cm, which he names after Zeldovich, inspired by Zeldovich's seminal papers [3,4]. The MM formulation unifies the tetrad and spin connection of the first order Einstein–Cartan formalism, which take values in $SO(3,1)$, into a grand connection that lives in $SO(4,1)$ (or $SO(3,2)$ for a negative cosmological constant). The resulting action, through breaking the $SO(4,1)$ symmetry down to the $SO(3,1)$, yields the usual Einstein–Hilbert term, a cosmological constant, and the Gauss–Bonnet (GB) term, which is topological in four dimensions [5–7].

Intriguingly, a length scale of 10^{-12} cm, as noted in Ref. [8], is also encountered in the context of so-called the Kodama wavefunction in gravity [9–13], analogous to the Chern–Simons solution in Yang–Mills theory in four dimensions, which is also an important element in Loop Quantum Gravity [14,15]. Actually, there is known to be a connection between the inner product of Kodama states and the MM formalism; Ref. [13] points out that the topological terms arising in the (extended) MM action and the inner product are the same.

Bjorken, additionally, suggests six extra spatial dimensions, assumed to be compactified on this induced scale of 10^{-12} cm, simply to account for the large factor multiplying the MM action [1]; $\sim 10^{120}$, which, quite remarkably, also happens to be the infamous number often encountered in the cosmological constant problem [16–19].

In this paper, we emphasize that the TeV-scale quantum gravity picture with large extra dimensions (LED) [20–26] (known as the ADD model) naturally reveals a scale of 10^{-12} cm as the compactification scale, provided that the number of extra spatial dimensions is set to six, with no need for an ad hoc assumption of the corresponding length scale. In order to be consistent with the known physics up to the TeV-scale, we adopt the well-known approach that only the graviton is allowed to propagate throughout the bulk experiencing the extra dimensions, while the Standard Model (SM) fields are localized to the usual four dimensions. This, in this scenario, would introduce a deviation in the gravitational interactions on scales smaller than 10^{-12} cm; the gravitational interaction has so far been tested down to the scale of 0.01 cm [27].

Moreover, we notice a combination of the "Bjorken–Zeldovich (BZ) scale" and the TeV scale, $M_{BZ}^3/M_{EW}^2 \sim 10^{-3}$ eV, which is in the order of the observed vacuum energy density in the present universe and in the ballpark of the anticipated neutrino masses [28,29]. Although it is most likely a coincidence, we present several toy models that illustrate its possible role as some sort of a see-saw-type suppression in obtaining the neutrino mass and the cosmological constant.

We also comment on possible other implications in cosmology. This scale is considerably close to 1–2 MeV below which the weak interactions freeze out, leading to Big Bang Nucleosynthesis (BBN). Premised on our current understanding of BBN, it is in general supposed that any deviation from the known radiation density around the decoupling temperature would change the time scale associated with BBN, and it is thus tightly constrained from the observations on the primordial abundances of light elements [30–32].[1]

2. The MacDowell–Mansouri Formalism and the Bjorken–Zeldovich Scale

Bjorken, in Ref. [1], discusses how a scale of ∼10 MeV is induced in the MM formalism through the GB topological term arising naturally in the formalism in addition to the usual Einstein–Hilbert action and a cosmological constant term.

The $SO(3,1)$ MM action, obtained through breaking the $SO(4,1)$ symmetry, is given as [1,2,5–7]

$$\mathcal{S}_{MM} = \frac{M_{Pl}^2}{64\pi H_0^2} \int d^4x \sqrt{-g}\, \frac{1}{4}\, F^{ab}_{\mu\nu} F^{cd}_{\lambda\sigma}\, \epsilon_{abcd}\, \epsilon^{\mu\nu\lambda\sigma}\,, \tag{1}$$

where the ϵ symbols denote Levi–Civita tensors, $F^{ab}_{\mu\nu} = R^{ab}_{\mu\nu} - H_0^2 \left(e^a_\mu e^b_\nu - e^a_\nu e^b_\mu \right)$, $R^{ab}_{\mu\nu} = R^{\rho\sigma}_{\mu\nu} e^a_\rho e^b_\sigma$ is the Riemann tensor, e^a_μ is the tetrad (vielbein), and $a, \mu = 0,1,2,3$ are the indices of the internal $SO(3,1)$ space and the four dimensional space-time, respectively. H_0 is the Hubble constant.

Note that $F^{ab}_{\mu\nu}$ is the $SO(3,1)$ projection of the curvature $F^{AB}_{\mu\nu}$, constructed from the generalized connection A^{AB}_μ $(A = 0,1,2,3,4)$ that lives in a local $SO(4,1)$. A^{AB}_μ takes the following form. $A^{4a}_\mu \equiv H_0 e^a_\mu$ and $A^{ab}_\mu \equiv w^{ab}_\mu$, where w is the spin connection that lives in the $SO(3,1)$ group.

The action in Equation (1) yields

$$\mathcal{S}_{MM} = \frac{M_{Pl}^2}{8\pi} \int d^4x \sqrt{-g} \left(\frac{1}{32 H_0^2} R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\lambda\sigma} \epsilon_{\alpha\beta\gamma\delta}\, \epsilon^{\mu\nu\lambda\sigma} + \frac{1}{2} R - \Lambda \right), \tag{2}$$

where the cosmological constant $\Lambda = 3H_0^2$ as it ought to be, and the first two terms are the GB and the Einstein–Hilbert terms, respectively. Notice the factor $\sim 10^{120}$ in front of the GB term in Equation (2), also multiplying the total MM action in Equation (1), which happens to be the infamous number in the cosmological constant problem ($\frac{M_{Pl}^4}{64\pi^2 \rho_\Lambda} \sim 10^{120}$). The possible role of this factor of the MM action

---

[1] Recently, the Atomki group in Hungary reported an anomaly in the ^{8}Be nuclear decay by internal e^+e^- formation at an invariant mass $m_{\bar{e}e} \cong 17$ MeV, with a statistical significance of 6.8σ [33]. See also Refs. [34–42] for the previous studies relevant to this observation. The observation has ignited interest in the high energy physics community to suggest explanations, some of which consider a hidden sector at around this energy scale whose effects have so far remained unnoticed [43–61].

in the resolution of the cosmological constant problem has not been demonstrated yet, to the best of our knowledge.

In the Friedmann–Robertson–Walker (FRW) background, where $ds^2 = -dt^2 + a^2(t)dx_i dx_i$, the GB term in Equation (2) becomes

$$\mathcal{S}_{GB} = -\frac{M_{Pl}^2 V(0)}{8\pi H_0^2}\int_0^t dt \frac{d}{dt}\dot{a}^3 \,, \tag{3}$$

where $V(0)$ is given through time dependent volume of region of interest dominated by dark energy, $V(t) = V(0)a^3 = V(0)e^{3H_0 t}$. Since in the semiclassical approximation the action is just the phase of the wavefunction, and for a topological term like the GB term the phase takes values in units of 2π, we can write the total amount of the action contributed by the GB term at time t, from Equation (3), as

$$|\mathcal{S}_{GB}| = \frac{M_{Pl}^2 V(0)\dot{a}^3}{8\pi H_0^2}\Bigg|_0^t \equiv 2\pi(N(t) - N(0)) \,. \tag{4}$$

Then, some sort of number density can be defined as

$$n \equiv \frac{N(t)}{V(t)} = \frac{M_{Pl}^2}{16\pi^2 H_0^2}\left(\frac{\dot{a}}{a}\right)^3 = \frac{H_0 M_{Pl}^2}{16\pi^2} \equiv \Lambda_{BZ}^3 \,, \tag{5}$$

which is time independent for the cosmological constant dominated space. Bjorken uses the term "darkness" for the quantity $N(t)$; we prefer to use the "Gauss–Bonnet number".

Once we put in the numerical factors, the Bjorken–Zeldovich scale yields

$$\Lambda_{BZ} \sim 10 \text{ MeV} \quad \text{or} \quad l_{BZ} = \frac{1}{\Lambda_{BZ}} \sim 2 \times 10^{-12} \text{ cm.} \tag{6}$$

Λ_{BZ} appears to be the scale up to which the MM formalism is valid. Next, we will see how a length scale of the same size comes about as the compactification scale of six extra dimensions originated from TeV-scale gauge-gravity unification. Considering this as the picture above Λ_{BZ}, $N(t)$ can be interpreted as an effective quantity, revealed below Λ_{BZ} upon integrated-over extra dimensions. This scenario, as we will see, accurately explains the factor 10^{120} in the MM action as well.

3. Bjorken–Zeldovich Scale from Large Extra Dimensions

In this section, we draw attention to an interesting incident regarding the onset of the scale of 10^{-12} cm from six compact extra (spatial) dimensions originated from TeV-scale gauge-gravity unification. If one imposes gauge-gravity unification at the TeV scale, the weakness of gravitational interactions can be explained via the existence of compact extra dimensions, large compared to the (inverse) TeV-scale [20–26].

For two test objects placed within a distance $r \gg R$, the gravitational potential becomes $V(r) \sim \frac{m_1 m_2}{M_U^{n+2} R^n}\frac{1}{r}$, where M_U is the unification scale of gauge and gravitational interactions, and R is the compactification scale of the extra dimensions. Imposing the requirement to get the right (reduced) Planck mass through the identification $M_U^{n+2}R^n = \overline{M}_{Pl}^2$, and assuming $M_U \sim 1$ TeV, we obtain

$$R = \frac{l_U^{1+2/n}}{l_{Pl}^{2/n}} \sim 2.0 \times (2.4)^{2/n} \times 10^{30/n-17} \text{ cm} \,, \tag{7}$$

where l_U and l_P are corresponding length scales for the TeV-scale and (reduced) Planck masses, respectively. As can be seen in Equation (7), for n = 6, we have

$$R \sim 2.7 \times 10^{-12} \text{ cm} \sim l_{BZ} \,, \tag{8}$$

a remarkable agreement with the Bjorken–Zeldovich length scale, given in Equation (6), revealed in the MM formalism, discussed previously.

Remarkably, existence of six extra spatial dimensions compactified on a scale of 10^{-12} cm in the MM framework, as also noted in Ref. [1], could also explain the factor $M_{Pl}^2/64\pi H_0^2 \sim 10^{120}$, multiplying both the MM action given in Equation (1) and the GB term in the action given in Equation (2). Extending the internal symmetry of the general MM action from $SO(4,1)$ to $SO(10,1)$, and breaking the symmetry down to $SO(9,1)$, in analogy with Equation (1), the action symbolically becomes

$$\mathcal{S}_{MM} \to \int d^4x \int \sqrt{-\tilde{g}}\, dy_1 ... dy_6 \; (F)^5_{(i\,\bar{\mu})} \cdot \epsilon^{(i)} \cdot \epsilon^{(\bar{\mu})} \;, \tag{9}$$

where we suppress the complete version of the tensors, and the indices run over ten values instead of original four. We expect $\langle F \rangle$ to take a value around the order of the (square of the) energy scale that sets the strength for the effective four-dimensional gravity, i.e., the (reduced) Planck mass square, $\langle F \rangle \sim \overline{M}_{Pl}^2$ (or $\langle F \rangle \sim \overline{M}_{Pl}^2 \sim M_U^8 R^6$ in the context of large extra dimensions picture discussed above). On the other hand, each integrated-over extra dimension contributes a factor in the order of the corresponding length scale, i.e., $\int dy \sim l_{BZ}$. Therefore,

$$\mathcal{S}_{MM} \quad \to \quad \underbrace{(\overline{M}_{Pl}^2 l_{BZ}^2)^3}_{\sim\, 10^{120}} \quad \int d^4x \, \sqrt{-g} \; (F)^2_{(a\,\mu)} \cdot \epsilon^{(a)} \cdot \epsilon^{(\mu)} \;, \tag{10}$$

which accurately accounts for the factor 10^{120} in the MM action given in Equation (1).

There are two main ways that the extra dimensions, in the context of the ADD model, would appear at the Large Hadron Collider (LHC). The first one is through the direct production of the graviton Kaluza–Klein (KK)-tower states, where the signal would appear as missing energy. The other signature would manifest itself via the exchange of virtual KK gravitons between the SM particles, which would give rise to enhancement in certain cross sections above the SM values [62,63]. Currently at the LHC, the lower limit for the gauge-gravity scale in the ADD model with six extra dimensions is set as $M_U > 2.6$ TeV with 95% confidence level (CL) by the CMS experiment [64–66], which translates into an upper bound on the corresponding length scale as $R < 0.8 \times 10^{-12}$ cm. This is still in the vicinity of the Bjorken–Zeldovich scale within an order of magnitude. Then, one wonders how robust the numerical agreement, given in Equation (8), is against the value of M_U. As can be seen in Equation (7), the outcome has some sensitivity against the value of M_U. Nevertheless, with a value of M_U up to around 10 TeV, we still get a required length scale up to an order of magnitude, which is generally acceptable when a scale is under discussion, as displayed in Figure 1. If, for instance, we take $M_U = 5$ TeV, the corresponding length scale becomes $R = 0.3 \times 10^{-12}$ cm; or, for $M_U = 10$ TeV, we have $R = 0.1 \times 10^{-12}$ cm $\cong d_{\text{proton}}$.

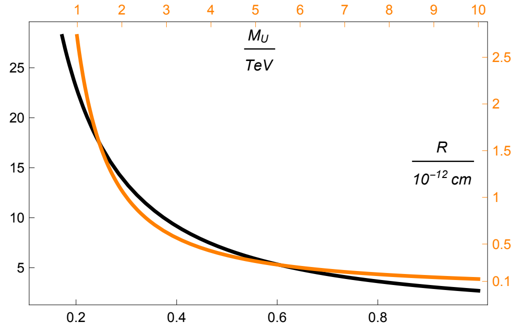

Figure 1. Compactification radius R vs. quantum gravity scale M_U in the case of six large extra dimensions. The black and orange plots denote the cases in which M_U takes values below and above 1 TeV, respectively.

4. A "See-Saw" Relation for the Small Cosmological Constant and the Neutrino Mass?

In the effort to understand the smallness of the cosmological constant, several numerical relations among the energy scales have been noticed (or proposed) in the literature that mimic a see-saw-type suppression mechanism [67–82].

In the case of the existence of an energy scale of ∼10 MeV, the relevant combination we notice is $\rho_\Lambda^{1/4} \overset{?}{=} M_{BZ}^3/M_{EW}^2 \sim 10^{-3}$ eV , where $M_{EW} \sim 1$ TeV. It is not straightforward to devise a realistic model yielding such a relation, since this requires a contribution in the amount of M_{BZ}^{12}/M_{EW}^8 in Lagrangian. Nevertheless, this type of term in the context of vacuum energy density contributions may be obtained in models where the cosmological constant problem is addressed by entertaining the possibility that the universe may be stuck in a false vacuum, split from the vanishing global vacuum in the amount of the cosmological constant [75,83].

When a new scale is under discussion, another question that comes to mind is the possibility of a (some sort of) see-saw mechanism that utilizes a relevant combination of the scales in the theory to explain the smallness of the neutrino mass. The relation $M_{BZ}^3/M_{EW}^2 \sim 10^{-3}$ eV is intriguing from this point of view as well, since it is in the vicinity of (at least one of the) the neutrino masses. Next, we will work on a hypothetical scenario just as an illustration of obtaining this combination in a model.

Consider a hidden sector with a quantum-chromodynamics (QCD)-like gauge interaction, where the symmetry group is $\mathcal{G}_h \equiv SU(N)$. Besides the corresponding gauge bosons, consider a real scalar field ϕ and a Dirac fermion F (for each family), both of which transform in some representation of $\mathcal{G}_h$, where the fermion does so vectorially (non-chirally). We assume that F has a confining scale of order 10 MeV and the SM fields are not charged under $\mathcal{G}_h$. The SM connects to the hidden sector through a portal coupling between the Higgs and the scalar ϕ. We also assume a discrete $\tilde{Z}_2$ symmetry that transforms F, ϕ, and the neutrinos in the following way. $F_{L(R)} \to \pm F_{L(R)}$, $\nu_{L(R)} \to \pm \nu_{L(R)}$, and $\phi \to -\phi$. Therefore, there are no mass or Yukawa-type terms (via the Standard Model Higgs) allowed at the tree level. The Yukawa-type terms involving the scalar ϕ do not contribute as mass terms at tree-level either, since we assume that $\langle \phi \rangle = 0$ so that the $SU(N)$ symmetry remains unbroken.

However, a mass term can be induced through the effective operator $\mathcal{L}^{eff} \supset \frac{c_\nu}{\Lambda^2} \overline{\nu} \nu \overline{F} F$, which is induced by integrating out the scalar field. We assume that a condensate forms, due to the possible nonperturbative characteristic of the $SU(N)$ vacuum, at $\sim M_{BZ}$, breaking the $\tilde{Z}_2$ symmetry; $\langle \overline{F} F \rangle \sim f^3 \sim (a \cdot M_{BZ})^3$, $\Lambda \sim m_\phi \sim \Lambda_{EW} \sim 1$ TeV. The scalar ϕ gets its mass via the portal coupling to the SM Higgs, i.e., $\lambda \phi^2 H^\dagger H$, which justifies its electroweak-scale mass. The extra fermion F acquires its mass via coupling to the condensate through the corresponding dimension-6 operator that yields a mass value on the order of the neutrino mass. The effective neutrino mass in this scenario becomes $m_\nu = (c_\nu a^3)\ M_{BZ}^3/M_{EW}^2 \lesssim 10^{-2}$ eV , provided that $c_\nu a^3 \lesssim 10$.

5. More on the Relevance in Cosmology

A scale around 10 MeV might be relevant also in terms of the thermal history of the universe. It is an energy scale considerably close to $T \sim 1-2$ MeV below which the weak interactions freeze out; the reaction rate $\Gamma \sim G_F^2 T^5$ drops below the expansion rate $H \sim \sqrt{g^*} T^2/M_{Pl}$, where g^* is given as $g^* = g_b + (7/8) g_f$ and g_b (g_f) denotes the total number of the effective bosonic (fermionic) degrees of freedom at around the background temperature T. Consequently, primordial neutrinos and possibly cold dark matter—if it exists—decouple from the rest of the matter, and the ratio of neutrons to protons freezes out. Any increase from the known radiation density would bring forward the Big Bang Nucleosynthesis (BBN) and hence would cause a larger Helium abundance in the present universe [30–32]. Therefore, if there is some unrevealed physics associated with such a scale of 10 MeV, they may have direct implications on our understanding of BBN, which is consistent with the current observations on the primordial abundances of light elements.

Since the effective MM action in 4D reveals the Einstein gravity with a cosmological constant and the GB term that does not have any effects in the equations of motion in 4D, the formalism at first sight only defines the graviton. This seemingly does not cause any problem in terms BBN since the gravitational interaction rate, as well known, is significantly suppressed compared to the expansion rate, i.e., $\Gamma \sim G_P^2 T^5 \ll H$, at $T \sim 1$ MeV. However, this is the case only if there is no any other relevant degrees of freedom obtained from the original action based on $SO(4,1)$, in addition to the terms given in Equation (2). Recall that the generalized connection A_μ^{AB} living in $SO(4,1)$ has 40 components. As also mentioned in Ref. [1], one may wonder whether some of these degrees of freedom can be identified with the (bosonic) degrees of freedom of the SM[2]. Then, several leftover terms may possibly define additional light degrees of freedom. One may expect at first that the relevant interactions are supposed to be suppressed, similar to the case with gravitons. However, one should not forget the enormous factor of 10^{120} multiplying the action in 4D, possibly arising due to being integrating over extra dimensions. If such identifications related to the SM are possible, then it is probably because of this large factor, and the same factor may amplify some interactions regarding these new light degrees of freedom, making them interact frequently enough to be in equilibrium at $T \sim 1$ MeV. Then, the model would be in tension with the constraints coming from BBN.

6. Discussion

In this paper, we aim to bring attention to the possibility of a gravitational scale at around $\sim$10 MeV, or 10^{-12} cm, induced in the (MM) extension to the Einstein–Cartan formalism in 4D, due to the topological GB term in the action, as suggested by Bjorken [1].

First, we point out that a scale of the same size, 10^{-12} cm naturally comes about in the context of large extra dimensions, originated from TeV-scale quantum gravity, as the compactification scale, if the number of extra spatial dimensions is set to six.[3] Apparently, these two approaches can be combined, where the four-dimensional MM formalism is the effective theory after the six extra dimensions are integrated over, which also explains the factor 10^{120} in the MM action. Second, we discuss that existence of such a scale may play a role in the smallness of the cosmological constant and the neutrino mass; to this end, we refer to some toy models as illustrations that generate a seesaw-type suppression mechanism. Note that we do not claim in this paper that this mechanism can directly be accommodated into the MM formalism. Instead, the main intention of this paper should be taken as an attempt to point out several coincidences regarding a scale at around 10 MeV, or 10^{-12} cm; the possibility that they are non-accidental deserves attention. Finally, we comment on possible implications in cosmology in the context of Big Bang Nucleosynthesis.

We note that if Nature contains six extra dimensions with a compactification scale of 10^{-12} cm, it raises the question why no Kaluza–Klein excitations with masses with a starting value of $\sim$10 MeV have been observed so far. However, their elusiveness would not be unanticipated since these modes are expected to be relatively suppressed [100].

Currently at the LHC, the ADD model with six extra dimensions is excluded with 95% CL for values $M_U \leqslant 2.6$ TeV by the CMS experiment [64–66], equivalent to an upper bound on the corresponding compactification length scale, $R < 0.8 \times 10^{-12}$ cm, which is in the ballpark of the Bjorken–Zeldovich scale within an order of magnitude. By the time the LHC searches are completed, we will have a compelling answer on the TeV-scale ADD model and hence on the MM-LED picture discussed in this paper. Note that a negative result along these lines does not necessarily invalidate Bjorken's original proposal in Ref. [1], which is not obliged to connect to the TeV-scale ADD model, and yet which can still include six compact extra dimensions. The most definitive answer regarding Bjorken's proposal will come from the gravitational inverse-square-law experiments. The current upper bound for the size of such extra dimensions, through the deviation in the effective gravitational interaction, is 0.01 cm [27].

---

[2] See, for instance, Refs. [84–96] for various geometric approaches to the Standard Model and beyond.

[3] Recall that existence of six compact extra dimensions is quite familiar from the string theory perspective [97–99].

Funding: This work is supported in parts by the National Natural Science Foundation of China (NSFC) under Grant No. 11505067 and the Swedish Research Council under contract 621-2011-5107.

Acknowledgments: We would like to thank James D. Bjorken and Djordje Minic for their comments and suggestions regarding the manuscript.

Conflicts of Interest: The author declares no conflict of interest. The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

References

1. Bjorken, J. Darkness: What comprises empty space? *Annalen der Physik* **2013**, *525*, A67–A79. [CrossRef]
2. MacDowell, S.W.; Mansouri, F. Unified Geometric Theory of Gravity and Supergravity. *Phys. Rev. Lett.* **1977**, *38*, 739. [CrossRef]
3. Zeldovich, Y.B. Cosmological Constant and Elementary Particles. *JETP Lett.* **1967**, *6*, 316–317. [CrossRef]
4. Zel'dovich, Y.B.; Krasinski, A. The Cosmological constant and the theory of elementary particles. *Sov. Phys. Uspekhi* **1968**, *11*, 381–393. [CrossRef]
5. Freidel, L.; Starodubtsev, A. Quantum gravity in terms of topological observables. *arXiv* **2005**, arXiv:hep-th/0501191.
6. Wise, D.K. MacDowell–Mansouri gravity and Cartan geometry. *Class. Quantum Gravity* **2010**, *27*, 155010. [CrossRef]
7. Wise, D.K. Symmetric space Cartan connections and gravity in three and four dimensions. *Symmetry Integr. Geom.* **2009**, *5*, 080. [CrossRef]
8. Randono, A. A Mesoscopic Quantum Gravity Effect. *Gen. Relativ. Gravit.* **2010**, *42*, 1909–1917. [CrossRef]
9. Kodama, H. Specialization of Ashtekar's Formalism to Bianchi Cosmology. *Prog. Theor. Phys.* **1988**, *80*, 1024. [CrossRef]
10. Smolin, L. Quantum gravity with a positive cosmological constant. *arXiv* **2002**, arXiv:hep-th/0209079.
11. Witten, E. A Note on the Chern–Simons and Kodama wave functions. *arXiv* **2003**, arXiv:gr-qc/0306083.
12. Randono, A. Generalizing the Kodama state. I. Construction. *arXiv* **2006**, arXiv:gr-qc/0611073.
13. Randono, A. Generalizing the Kodama state. II. Properties and physical interpretation. *arXiv* **2006**, arXiv:gr-qc/0611074.
14. Rovelli, C. Loop quantum gravity: The first twenty five years. *Class. Quantum Gravity* **2011**, *28*, 153002. [CrossRef]
15. Ashtekar, A.; Pullin, J. The Overview Chapter in Loop Quantum Gravity: The First 30 Years. *arXiv* **2017**, arXiv:gr-qc/1703.07396.
16. Weinberg, S. The Cosmological Constant Problem. *Rev. Mod. Phys.* **1989**, *61*, 1–23. [CrossRef]
17. Carroll, S.M. The Cosmological constant. *Living Rev. Relat.* **2001**, *4*, 1. [CrossRef] [PubMed]
18. Polchinski, J. The Cosmological Constant and the String Landscape. In *The Quantum Structure of Space and Time: Proceedings of the 23rd Solvay Conference on Physics, Brussels, Belgium, 1–3 December 2005*; Cornell University Library: Ithaca, NY, USA, 2006; pp. 216–236.
19. Padilla, A. Lectures on the Cosmological Constant Problem. *arXiv* **2015**, arXiv:hep-th/1502.05296.
20. Arkani-Hamed, N.; Dimopoulos, S.; Dvali, G.R. The Hierarchy problem and new dimensions at a millimeter. *Phys. Lett. B* **1998**, *429*, 263–272. [CrossRef]
21. Antoniadis, I.; Arkani-Hamed, N.; Dimopoulos, S.; Dvali, G.R. New dimensions at a millimeter to a Fermi and superstrings at a TeV. *Phys. Lett. B* **1998**, *436*, 257–263. [CrossRef]
22. Arkani-Hamed, N.; Dimopoulos, S.; Dvali, G.R. Phenomenology, astrophysics and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity. *Phys. Rev. D* **1999**, *59*, 086004. [CrossRef]
23. Arkani-Hamed, N.; Dimopoulos, S.; Kaloper, N.; Sundrum, R. A Small cosmological constant from a large extra dimension. *Phys. Lett. B* **2000**, *480*, 193–199. [CrossRef]
24. Antoniadis, I.; Dimopoulos, S.; Dvali, G.R. Millimeter range forces in superstring theories with weak scale compactification. *Nucl. Phys. B* **1998**, *516*, 70–82. [CrossRef]
25. Accomando, E.; Antoniadis, I.; Benakli, K. Looking for TeV scale strings and extra dimensions. *Nucl. Phys. B* **2000**, *579*, 3–16. [CrossRef]
26. Antoniadis, I.; Benakli, K.; Quiros, M. Direct collider signatures of large extra dimensions. *Phys. Lett. B* **1999**, *460*, 176–183. [CrossRef]

27. Kapner, D.J.; Cook, T.S.; Adelberger, E.G.; Gundlach, J.H.; Heckel, B.R.; Hoyle, C.D.; Swanson, H.E. Tests of the gravitational inverse-square law below the dark-energy length scale. *Phys. Rev. Lett.* **2007**, *98*, 021101. [CrossRef] [PubMed]
28. Ade, P.A.R.; Aghanim, N.; Arnaud, M.; Ashdown, M.; Aumont, J.; Baccigalupi, C.; Banday, A.J.; Barreiro, R.B.; Bartlett, J.G.; Bartolo, N.; et al. Planck 2015 results. XIII. Cosmological parameters. *Astron. Astrophys.* **2016**, *594*, A13.
29. Patrignani, C.; et al. [Particle Data Group]. Review of Particle Physics. *Chin. Phys. C* **2016**, *40*, 100001.
30. Carroll, S.M. *Spacetime and Geometry: An Introduction to General Relativity*; Pearson Education Inc.: London, UK, 2004.
31. Weinberg, S. *Cosmology*; Oxford University Press: Oxford, UK, 2008.
32. Langacker, P. *The Standard Model and Beyond*; CRC Press: Boca Raton, FL, USA, 2010.
33. Krasznahorkay, A.J.; Csatlós, M.; Csige, L.; Gácsi, Z.; Gulyás, J.; Hunyadi, M.; Kuti, I.; Nyakó, B.M.; Stuhl, L.; Timár, J.; et al. Observation of Anomalous Internal Pair Creation in ^{8}Be: A Possible Indication of a Light, Neutral Boson. *Phys. Rev. Lett.* **2016**, *116*, 042501. [CrossRef] [PubMed]
34. De Boer, F.W.N.; Fröhlich, O.; Stiebing, K.E.; Bethge, K.; Bokemeyer, H.; Balanda, A.; Buda, A.; Van Dantzig, R.; Elze, T.W.; Folger, H.; et al. A deviation in internal pair conversion. *Phys. Lett. B* **1996**, *388*, 235–240. [CrossRef]
35. De Boer, F.W.N.; van Dantzig, R.; van Klinken, J.; Bethge, K.; Bokemeyer, H.; Buda, A.; Muller, K.A.; Stiebing, K.E. Excess in e^+e^- pairs near 9 MeV invariant mass. *J. Phys.* **1997**, *G23*, L85–L96. [CrossRef]
36. De Boer, F.W.N.; Bethge, K.; Bokemeyer, H.; van Dantzig, R.; van Klinken, J.; Mironov, V.; Muller, K.A.; Stiebing, K.E. Further search for a neutral boson with a mass around 9-MeV/c2. *J. Phys.* **2001**, *G27*, L29. [CrossRef]
37. De Boer, F. Anomalous internal pair conversion signaling elusive light neutral particles. *AIP Conf. Proc.* **2006**, *802*, 146–152.
38. Krasznahorkay, A.; de Boer, F.W.N.; Csatlós, M.; Csige, L.; Gácsi, Z.; Gulyás, J.; Hunyadi, M.; Ketel, T.J.; van Klinken, J.; Krasznahorkay, A., Jr.; et al. e^+e^- pairs from a nuclear transition signaling an elusive light neutral boson. *AIP Conf. Proc.* **2005**, *802*, 236.
39. Krasznahorkay, A.; Gacsi, Z.; Ketel, T.J.; van Klinken, J.; de Boer, F.W.N.; Gulyás, J.; Csige, L.; Krasznahorkay, A. Jr.; Hunyadi, M.; Vitez, A.; et al. Lepton pairs from a forbidden M0 transition: Signaling an elusive light neutral boson? *Acta Phys. Polon. B* **2006**, *37*, 239–244.
40. De Boer, F.W.N.; Fields, C.A. A Re-evaluation of Evidence for Light Neutral Bosons in Nuclear Emulsions. *Int. J. Mod. Phys. E* **2011**, *20*, 1787–1803. [CrossRef]
41. Wojtsekhowski, B.; Nikolenko, D.; Rachek, I. Searching for a new force at VEPP-3. *arXiv* **2012**, arXiv:hep-ex/1207.5089.
42. Gulyas, J.; Ketel, T.J.; Krasznahorkay, A.J.; Csatlos, M.; Csige, L.; Gacsi, Z.; Hunyadi, M.; Krasznahorkay, A.; Vitez, A.; Tornyi, T.G. A pair spectrometer for measuring multipolarities of energetic nuclear transitions. *Nucl. Instrum. Method.* **2016**, *A808*, 21–28. [CrossRef]
43. Feng, J.L.; Fornal, B.; Galon, I.; Gardner, S.; Smolinsky, J.; Tait, T.M.P.; Tanedo, P. Protophobic Fifth-Force Interpretation of the Observed Anomaly in ^8Be Nuclear Transitions. *Phys. Rev. Lett.* **2016**, *117*, 071803. [CrossRef] [PubMed]
44. Feng, J.L.; Fornal, B.; Galon, I.; Gardner, S.; Smolinsky, J.; Tait, T.M.P.; Tanedo, P. Particle Physics Models for the 17 MeV Anomaly in Beryllium Nuclear Decays. *arXiv* **2016**, arXiv:hep-ph/1608.03591.
45. Gninenko, S.N.; Krasnikov, N.V.; Kirsanov, M.M.; Kirpichnikov, D.V. Missing energy signature from invisible decays of dark photons at the CERN SPS. *Phys. Rev. D* **2016**, *94*, 095025. [CrossRef]
46. Gu, P.H.; He, X.G. Realistic model for a fifth force explaining anomaly in $_8Be_* \to {}_8Be\ e^+e^-$ Decay. *Nucl. Phys. B* **2017**, *919*, 209–217. [CrossRef]
47. Jia, L.B.; Li, X.Q. The new interaction suggested by the anomalous ^8Be transition sets a rigorous constraint on the mass range of dark matter. *Eur. Phys. J. C* **2016**, *76*, 706. [CrossRef]
48. Kitahara, T.; Yamamoto, Y. Protophobic Light Vector Boson as a Mediator to the Dark Sector. *Phys. Rev. D* **2017**, *95*, 015008. [CrossRef]
49. Ellwanger, U.; Moretti, S. Possible Explanation of the Electron Positron Anomaly at 17 MeV in ^8Be Transitions Through a Light Pseudoscalar. *J. High Energy Phys.* **2016**, *2016*, 039, [CrossRef]

50. Chen, C.S.; Lin, G.L.; Lin, Y.H.; Xu, F. The 17 MeV Anomaly in Beryllium Decays and $U(1)$ Portal to Dark Matter. *arXiv* **2016**, arXiv:hep-ph/1609.07198.
51. Neves, M.J.; Helayel-Neto, J.A. TeV- and MeV-physics out of an $SU(2) \times U(1) \times U(1)$ model. *arXiv* **2016**, arXiv:hep-ph/1609.08471.
52. Kahn, Y.; Krnjaic, G.; Mishra-Sharma, S.; Tait, T.M.P. Light Weakly Coupled Axial Forces: Models, Constraints, and Projections. *arXiv* **2016**, arXiv:hep-ph/1609.09072.
53. Fayet, P. The light U boson as the mediator of a new force, coupled to a combination of Q, B, L and dark matter. *Eur. Phys. J. C* **2017**, *77*, 53. [CrossRef]
54. Neves, M.J.; Helayel-Neto, J.A. A Unified Hidden-Sector-Electroweak Model, Paraphotons and the X-Boson. *arXiv* **2016**, arXiv:hep-ph/1611.07974.
55. Kozaczuk, J.; Morrissey, D.E.; Stroberg, S.R. Light Axial Vectors, Nuclear Transitions, and the ^8Be Anomaly. *arXiv* **2016**, arXiv:hep-ph/1612.01525.
56. Chiang, C.W.; Tseng, P.Y. Probing a dark photon using rare leptonic kaon and pion decays. *Phys. Lett. B* **2017**, *767*, 289–294. [CrossRef]
57. Krasnikov, N.V. The muon $(g-2)$ anomaly and a new light vector boson. *arXiv* **2017**, arXiv:hep-ph/1702.04596.
58. Araki, T.; Hoshino, S.; Ota, T.; Sato, J.; Shimomura, T. Detecting the $L_\mu - L_\tau$ gauge boson at Belle II. *Phys. Rev. D* **2017**, *95*, 055006. [CrossRef]
59. Benavides, R.; Munoz, L.A.; Ponce, W.A.; Rodriguez, O.; Rojas, E. Minimal non-universal EW extensions of the Standard Model: A chiral multi-parameter solution. *arXiv* **2016**, arXiv:hep-ph/1612.07660.
60. Neves, M.J. The protophobic X-boson unified to the quantum electrodynamics. *arXiv*, **2017**, arXiv:hep-ph/1704.02491.
61. Delle Rose, L.; Khalil, S.; Moretti, S. Explanation of the 17 MeV Atomki Anomaly in a $U(1)'$-Extended 2-Higgs Doublet Model. *arXiv* **2017**, arXiv:hep-ph/1704.03436.
62. Csaki, C. TASI lectures on extra dimensions and branes. From fields to strings: Circumnavigating theoretical physics. Ian Kogan memorial collection (3 volume set). *arXiv* **2004**, arXiv:hep-ph/0404096.
63. Rizzo, T.G. Introduction to Extra Dimensions. *AIP Conf. Proc.* **2010**, *1256*, 27–50.
64. Roy, A. Search for Dark Matter and Large Extra Dimensions in the Photon + MET Final State in pp Collisions at $\sqrt{s} = 13$ TeV. *Springer Proc. Phys.* **2018**, *203*, 205–208.
65. Ghosh, S. Large Extra Dimensions Search in the Photon + MET Final State in pp Collisions at $\sqrt{s}$ = 13 TeV at CMS in LHC. *Springer Proc. Phys.* **2018**, *203*, 745–747.
66. Sirunyan, A.M.; et al. [The CMS Collaboration]. Search for new physics in the monophoton final state in proton-proton collisions at $\sqrt{s}$ = 13 TeV. *J. High Energy Phys.* **2017**, *2017*, 73. [CrossRef]
67. Banks, T. SUSY breaking, cosmology, vacuum selection and the cosmological constant in string theory. In Proceedings of the ITP Workshop on SUSY Phenomena and SUSY GUTS Santa Barbara, California, CA, USA, 7–9 December 1995.
68. Cohen, A.G.; Kaplan, D.B.; Nelson, A.E. Effective field theory, black holes, and the cosmological constant. *Phys. Rev. Lett.* **1999**, *82*, 4971–4974. [CrossRef]
69. Kim, J.E. Model dependent axion as quintessence with almost massless hidden sector quarks. *J. High Energy Phys.* **2000**, *2000*, 016. [CrossRef]
70. Kiritsis, E. Supergravity, D-brane probes and thermal superYang–Mills: A Comparison. *J. High Energy Phys.* **1999**, *1999*, 010. [CrossRef]
71. Arkani-Hamed, N.; Hall, L.J.; Kolda, C.F.; Murayama, H. A New perspective on cosmic coincidence problems. *Phys. Rev. Lett.* **2000**, *85*, 4434–4437. [CrossRef] [PubMed]
72. Banks, T. Cosmological breaking of supersymmetry? *Int. J. Mod. Phys. A* **2001**, *16*, 910–921. [CrossRef]
73. Chang, L.N.; Minic, D.; Okamura, N.; Takeuchi, T. The Effect of the minimal length uncertainty relation on the density of states and the cosmological constant problem. *Phys. Rev. D* **2002**, *65*, 125028. [CrossRef]
74. Chang, L.N.; Minic, D.; Okamura, N.; Takeuchi, T. Exact solution of the harmonic oscillator in arbitrary dimensions with minimal length uncertainty relations. *Phys. Rev. D* **2002**, *65*, 125027. [CrossRef]
75. Barr, S.M.; Seckel, D. The Cosmological constant, false vacua, and axions. *Phys. Rev. D* **2001**, *64*, 123513. [CrossRef]
76. Berglund, P.; Hubsch, T.; Minic, D. Relating the cosmological constant and supersymmetry breaking in warped compactifications of IIB string theory. *Phys. Rev. D* **2003**, *67*, 041901. [CrossRef]

77. Hsu, S.D.H.; Zee, A. A Speculative relation between the cosmological constant and the Planck mass. *Mod. Phys. Lett. A* **2005**, *20*, 2699–2704. [CrossRef]
78. Urban, F.R.; Zhitnitsky, A.R. The Cosmological constant from the ghost: A Toy model. *Phys. Rev. D* **2009**, *80*, 063001. [CrossRef]
79. Urban, F.R.; Zhitnitsky, A.R. The cosmological constant from the QCD Veneziano ghost. *Phys. Lett. B* **2010**, *688*, 9–12. [CrossRef]
80. Urban, F.R.; Zhitnitsky, A.R. The QCD nature of Dark Energy. *Nucl. Phys. B* **2010**, *835*, 135–173. [CrossRef]
81. Chang, L.N.; Minic, D.; Takeuchi, T. Quantum Gravity, Dynamical Energy-Momentum Space and Vacuum Energy. *Mod. Phys. Lett. A* **2010**, *25*, 2947–2954. [CrossRef]
82. Chang, L.N.; Lewis, Z.; Minic, D.; Takeuchi, T. On the Minimal Length Uncertainty Relation and the Foundations of String Theory. *Adv. High Energy Phys.* **2011**, *2011*, 493514. [CrossRef]
83. Garretson, W.D.; Carlson, E.D. Could there be something rather than nothing? *Phys. Lett. B* **1993**, *315*, 232–238. [CrossRef]
84. Lisi, A.G. An Exceptionally Simple Theory of Everything. *arXiv* **2007**, arXiv:hep-th/0711.0770.
85. Nesti, F.; Percacci, R. Chirality in unified theories of gravity. *Phys. Rev. D* **2010**, *81*, 025010. [CrossRef]
86. Lisi, A.G.; Smolin, L.; Speziale, S. Unification of gravity, gauge fields, and Higgs bosons. *J. Phys. A* **2010**, *43*, 445401–445408. [CrossRef]
87. Ne'eman, Y. Irreducible Gauge Theory of a Consolidated Weinberg-Salam Model. *Phys. Lett. B* **1979**, *81*, 190–194. [CrossRef]
88. Fairlie, D.B. Higgs' Fields and the Determination of the Weinberg Angle. *Phys. Lett. B* **1979**, *82*, 97–100. [CrossRef]
89. Fairlie, D.B. Two Consistent Calculations of the Weinberg Angle. *J. Phys. G* **1979**, *5*, L55. [CrossRef]
90. Ne'eman, Y.; Sternberg, S. Superconnections and internal supersymmetry dynamics. *Proc. Natl. Acad. Sci. USA* **1990**, *87*, 7875–7877. [CrossRef] [PubMed]
91. Ne'eman, Y.; Sternberg, S.; Fairlie, D. Superconnections for electroweak su(2/1) and extensions, and the mass of the Higgs. *Phys. Rep.* **2005**, *406*, 303–377. [CrossRef]
92. Chamseddine, A.H.; Connes, A. Noncommutative Geometry as a Framework for Unification of all Fundamental Interactions including Gravity. Part I. *Fortsch. Phys.* **2010**, *58*, 553–600. [CrossRef]
93. Chamseddine, A.H.; Connes, A. Resilience of the Spectral Standard Model. *J. High Energy Phys.* **2012**, *2012*, 104. [CrossRef]
94. Chamseddine, A.H.; Connes, A.; van Suijlekom, W.D. Beyond the Spectral Standard Model: Emergence of Pati-Salam Unification. *J. High Energy Phys.* **2013**, *2013*, 132. [CrossRef]
95. Aydemir, U.; Minic, D.; Takeuchi, T. The Higgs Mass and the Emergence of New Physics. *Phys. Lett. B* **2013**, *724*, 301–305. [CrossRef]
96. Aydemir, U.; Minic, D.; Sun, C.; Takeuchi, T. Higgs mass, superconnections, and the TeV-scale left-right symmetric model. *Phys. Rev. D* **2015**, *91*, 045020. [CrossRef]
97. Schwarz, J.H. Physical States and Pomeron Poles in the Dual Pion Model. *Nucl. Phys. B* **1972**, *46*, 61–74. [CrossRef]
98. Scherk, J.; Schwarz, J.H. Dual Field Theory of Quarks and Gluons. *Phys. Lett. B* **1975**, *57*, 463–466. [CrossRef]
99. Candelas, P.; Horowitz, G.T.; Strominger, A.; Witten, E. Vacuum Configurations for Superstrings. *Nucl. Phys. B* **1985**, *258*, 46–74. [CrossRef]
100. Han, T.; Lykken, J.D.; Zhang, R.J. On Kaluza–Klein states from large extra dimensions. *Phys. Rev. D* **1999**, *59*, 105006. [CrossRef]