Charmed baryon $\Omega_c^0 \to \Omega^- l^+ \nu_l$ and $\Omega_c^0 \to \Omega_c^- \pi^+ (\rho^+)$ decays in light cone sum rules

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The semileptonic $\Omega_c^0 \to \Omega^- l\nu$ and nonleptonic $\Omega_c^0 \to \Omega^- \pi^+$, $\Omega_c^0 \to \Omega^- \rho^+$ decays of the charmed Ω_c baryon are studied within the light cone sum rules. The form factors responsible for $\Omega_c \to \Omega$ transitions are calculated using the distribution amplitudes of the Ω_c baryon. With the obtained form factors, the branching ratios of $\Omega_c^0 \to \Omega^- l^+ \nu_l$, $\Omega_c^0 \to \Omega^- \pi^+$, and $\Omega_c^0 \to \Omega^- \rho^+$ decays are estimated. The results are compared with Belle data as well as the findings of other approaches.

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I. INTRODUCTION

The semileptonic weak decays of hadrons represent a very promising class of decays. The study of semileptonic decays can provide us with useful information about the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The investigation of these decays can play a crucial role in studying strong interactions, i.e., the form of the effective Hamiltonian. The decay amplitudes of semileptonic decays can be represented as a product of a well-understood leptonic current and a complicated hadronic current for describing the quark transitions. The hadronic part of the weak decays is usually parametrized in terms of form factors. The form factors belong to the nonperturbative region of QCD; hence, some nonperturbative methods are needed to calculate them. Among these methods, the QCD sum rules approach [1] occupies a particular place. The advantage of this method is that it is based on the fundamental QCD Lagrangian.

The lowest-lying Ω_c baryon predominantly decays weakly [2]. Up to now, several Ω_c^0 decays—such as $\Omega_c^0 \to \Xi^0 \bar{K}^{(*)0}$, $\Omega^- \rho^+$, and $\Omega^- l^+ \nu_e$—have been observed [3]. The first observation of semileptonic decays $\Omega_c \to \Omega^- e^+ \nu_e$ was achieved by the CLEO Collaboration [3] with $\mathcal{R} = \frac{\mathcal{B}(\Omega_c^0 \to \Omega^- e^+ \nu_e)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)} = 2.4 \pm 1.2$. This decay has been carefully investigated within different approaches such as the light-front quark model [4,5], the heavy quark expansion model [6], and the quark model [7]. However, the predictions of the branching ratios of $\Omega_c^0 \to \Omega^- l^+ \nu_e$ vary between 0.005 and 0.127, and this large variation deserves much more attention.

Recently, the Belle Collaboration announced the first observation of $\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu$ decay [8]. In this study, measurements of the branching ratios of $\Omega_c^0 \to \Omega^- l^+ \nu_l$ $(l = e \,\mathrm{or}\, \mu)$ decays are compared to the reference mode $\Omega_c^0 \to \Omega^- \pi^+$; namely, the branching ratios $\frac{\mathcal{B}(\Omega_c^0 \to \Omega^- e^+ \nu_e)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)}$ and $\frac{\mathcal{B}(\Omega_c^0 \to \Omega^- \mu^+ \nu_\mu)}{\mathcal{B}(\Omega_c^0 \to \Omega^- \pi^+)}$ are $1.98 \pm 0.13(\mathrm{stat}) \pm 0.08(\mathrm{syst})$ and $1.94 \pm 0.18(\mathrm{stat}) \pm 0, 10(\mathrm{syst})$, respectively.

The new measurement and the variation in the predictions of the branching fractions among different models need further attention. In the present work, we study the $\Omega_c^0 \to \Omega^- l^+ \nu_e$ decay within the light cone sum rules (LCSR) method (for more information about the LCSR method, see [9]).

The paper is organized as follows. In Sec. II, the LCSR for the relevant form factors responsible for $\Omega_c^0 \to \Omega^-$ transitions are derived. A numerical analysis, including the results for the form factors and decay widths, is presented in Sec. III. The last section contains our conclusion.

II. FORM FACTORS FOR $\Omega_c \to \Omega$ TRANSITION IN LCSR

To calculate the form factors for the $\Omega_c^0 \to \Omega^-$ transition, we consider the following correlation function:

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$$\Pi_{\mu\nu} = i \int d^4x e^{ip'x} \langle 0|T\{J_\mu^\Omega(x) J_\nu^{V-A}(0)\}|\Omega_c\rangle. \quad (1)$$

Here, the current $J_\mu^\Omega = \epsilon^{jkl} s^{j^T} C\gamma_\mu s^k s^l$ is the interpolating current of the Ω baryon, $J_\nu^{V-A}(0) = \bar{s}\gamma_\nu(1-\gamma_5)c$ is the current describing the $c \to s$ transition, and j, k, and l are the color indices.

In LCSR, the correlation function is calculated both at hadronic and QCD level at the deep Euclidean region, i.e., $p^2 \ll m_c^2$, $q^2 \ll m_c^2$. Then, the results of the calculations for the two representations of the correlation function are matched by using the quark-hadron duality ansatz. As a result, the sum rules for the relevant form factors are derived.

Let us first calculate the correlation function from the hadronic side. Inserting the complete set of baryon states carrying the quantum numbers of the Ω baryon and isolating the ground state for the correlation function, we obtain

$$\Pi_{\mu\nu} = \frac{\lambda\langle 0|J_\mu^\Omega|\Omega(p')\rangle\langle\Omega(p')|\bar{s}\gamma_\nu(1-\gamma_5)c|\Omega_c(p)\rangle}{m_\Omega^2 - p'^2}. \quad (2)$$

The first term of this matrix is defined as

$$\langle 0|J_\mu^\Omega|\Omega(p')\rangle = \lambda u_\mu(p'), \quad (3)$$

where λ is the decay constant and $u_\mu(p')$ is the Rarita-Schwinger spinor for the spin-$3/2$ Ω baryon.

The second matrix element is parametrized in terms of eight transition form factors,

$$\begin{aligned}
&\langle\Omega(p')|\bar{s}\gamma_\mu(1-\gamma_5)c|\Omega_c(p)\rangle \\
&= \bar{u}_\alpha(p')\left\{\left[\frac{p_\alpha}{m_{\Omega_c}}\left(\gamma_\nu F_1 + \frac{p_\nu}{m_{\Omega_c}} F_2 + \frac{p'_\nu}{m_\Omega} F_3\right) + F_4 g_{\nu\alpha}\right]\gamma_5 \right.\\
&\left. - \left[\frac{p_\alpha}{m_{\Omega_c}}\left(\gamma_\nu G_1 + \frac{p_\nu}{m_{\Omega_c}} G_2 + \frac{p'_\nu}{m_\Omega}\right) + G_4 g_{\mu\alpha}\right]\right\} u(p).
\end{aligned} \quad (4)$$

The summation over spins of the Ω baryon is performed via the formula

$$\sum_s u_\alpha(p')\bar{u}_\beta(p') = -(\not{p}' + m_{\Omega_c})\left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{p'_\alpha p'_\beta}{m_\Omega^2} + \frac{1}{3}\frac{p'_\alpha\gamma_\beta - p'_\beta\gamma_\alpha}{m_\Omega}\right]. \quad (5)$$

Before delving into the analysis, we make the following two remarks.

(i) The current J_μ^Ω also couples with spin-$1/2$ negative parity baryons, i.e.,

$$\langle 0|J_\mu^\Omega|B^-(p')\rangle \sim \left[\gamma_\mu - \frac{4}{m} p'_\mu\right] u(p', s), \quad (6)$$

where B denotes the negative parity baryon. Therefore, the structures with p'_μ and γ_μ terms also contain the contributions of the spin-1/2 baryon. Using this fact, from Eq. (5) we find that only the structure with the $g_{\alpha\beta}$ term is free of spin-1/2 baryon contributions. In other words, the structure with the $g_{\alpha\beta}$ term contains contributions coming only from spin-3/2 baryons.

(ii) Note that not all the Lorentz structures are independent, and to overcome this problem, a specific order of Dirac matrices is chosen. In this work, we choose the structure $\gamma_\mu \not{q} \gamma_\nu \not{p} (\gamma_\mu \not{q} \gamma_\nu \not{p} \gamma_5)$.

Taking these remarks into account, we obtain the correlation function from the hadronic part,

$$\begin{aligned}
\Pi_{\mu\nu} = \frac{1}{m_\Omega^2 - p'^2}&\left\{\frac{p_\mu}{m_{\Omega_c}}\left[F_1(2p_\nu\gamma_5 + (m_{\Omega_c} + m_\Omega)\gamma_\nu\gamma_5 - \not{q}\gamma_\nu\gamma_5) + F_2\frac{p_\nu}{m_{\Omega_c}}((m_\Omega - m_{\Omega_c})\gamma_5 - \not{q}\gamma_5)\right.\right. \\
&\left. + \frac{1}{m_\Omega} F_3 (p-q)_\nu((m_\Omega - m_{\Omega_c})\gamma_5 - \not{q}\gamma_5)\right] + F_4 g_{\mu\nu}((m_\Omega - m_{\Omega_c})\gamma_5 - \not{q}\gamma_5) \\
&- \frac{p_\mu}{m_{\Omega_c}}\left[G_1(2p_\nu + (m_\Omega - m_{\Omega_c})\gamma_\nu - \not{q}\gamma_\nu) + \frac{G_2 p_\nu}{m_{\Omega_c}}((m_\Omega + m_{\Omega_c}) - \not{q})\right. \\
&\left.\left. + G_3\frac{(p-q)_\nu}{m_\Omega}((m_\Omega + m_{\Omega_c}) - \not{q})\right] + G_4 g_{\mu\nu}(m_\Omega + m_{\Omega_c}) - \not{q}\right\} + \cdots,
\end{aligned} \quad (7)$$

where dots stand for the contributions of higher states and continuum, denoting the contributions arising from quarks starting from some threshold $s_{\rm th}$ value. In the following discussions, we denote the momentum of the Ω_c baryon as $p_\mu \to m_{\Omega_c} v_\mu$, where v_μ is its velocity.

At this point, a few words are in order. The number of independent form factors is considerably reduced if the heavy quark limit is applied. In this limit, the Ω_Q and Ω_Q^* are controlled with the same dynamics but described by different spinors. For this reason, the weak decays, $\Omega_Q \to \Omega$ and $\Omega_Q^* \to \Omega$, can be studied together.

Now let us turn our attention to the calculation of the correlation function from the QCD side with the help of the operator product expansion (OPE). Using the Wick theorem from Eq. (1) we get the correlation function

$$\Pi_{\mu\nu} = i\int d^4x e^{ip'x}\langle 0|\epsilon^{jkl}(C\gamma_\mu)_{\alpha\beta}(\mathcal{T}_\nu)_{\alpha_1\beta_1} \times\{S^{cj_1}_{\gamma\alpha_1}(x)s^j_\alpha(x)s^k_\beta(x)c^{j_1}_{\beta_1}(0) - S^{kj_1}_{\beta\alpha_1}(x)s^j_\alpha(x)s^l_\gamma(x)c^{j_1}_{\beta_1}(0) + S^{jj_1}_{\alpha\alpha_1}(x)s^k_\beta(x)s^l_\gamma(x)c^{j_1}_{\beta_1}(0)\}|\Omega_c\rangle, \quad (8)$$

where $S(x)$ is the s-quark propagator, and $\mathcal{T}_\nu = \gamma_\nu(1-\gamma_5)$. From Eq. (8), it follows that, to calculate the correlation function from the QCD side, we need the matrix element $\epsilon^{jkl}\langle 0|\bar{s}^j_\alpha(x)s^k_\beta(x)c^l_\gamma(0)|\Omega_c\rangle$. This matrix element can be parametrized in terms of the heavy baryon distribution amplitudes (DAs). The DAs of the sextet baryons with quantum numbers $J^P = \frac{1}{2}^+$ in the heavy quark mass limit are obtained in [10]. In this work, the DAs are classified by the total spin of two light quarks. If the polarization vector is parallel to the light cone plane, the matrix element $\epsilon_{jkl}\langle 0|q^j_{1\alpha}(t_1)q^k_{2\beta}(t_2)Q^l_\gamma(0)|\Omega_Q(p)\rangle$ can be expressed in terms of the four DAs in the following way:

$$\epsilon_{jkl}\langle 0|s^j_\alpha(t_1)s^k_\beta(t_2)c^l_\gamma(0)|\Omega_c(v)\rangle = \Sigma A_i(\Gamma_i C^{-1})_{\alpha\beta}(\gamma_5\bar{\not{v}}u_{\Omega_c})_\gamma \quad (9)$$

where

$$A_1 = \frac{1}{8}v_+ f^{(1)}\psi_2, \qquad \Gamma_1 = \bar{\not{n}},$$
$$A_2 = f^{(2)}\psi_3^{(\sigma)}, \qquad \Gamma_2 = \frac{1}{8}i\sigma_{\alpha\beta}\bar{n}_\alpha n_\beta,$$
$$A_3 = \frac{1}{4}\psi_3^{(s)}f^{(2)}, \qquad \Gamma_3 = 1,$$
$$A_4 = -\frac{1}{8v_+}\psi_4 f^{(1)}, \qquad \Gamma_4 = \not{n}. \quad (10)$$

Here, $n_\mu = \frac{x_\mu}{vx}$, $\bar{n}_\mu = 2v_\mu - \frac{1}{vx}x_\mu$, $\bar{v}_\mu = \frac{x_\mu}{vx} - v_\mu$, and $f^{(i)}$ are the decay constants of the Ω_c baryon, and $\psi^{(i)}$ are the distribution amplitudes. The Fourier transformations of the DAs are $\Psi(x_1,x_2) = \int_0^\infty d\omega_1 d\omega_2 e^{-i\omega_1 t_1}e^{-i\omega_2 t_2}\psi(\omega_1\omega_2)$, where ω_1 and ω_2 are the momentum of two light quarks along the light cone direction, and their total momentum is $\omega = \omega_1 + \omega_2$ with $t_1 = x_1 n$ and $t_2 = x_2 n$. The DAs can be written as

$$\psi(t_1,t_2) = \int_0^\infty d\omega\omega\int du e^{-i\omega v x_1}e^{-i\omega\bar{u}(x_2-x_1)}\psi(\omega,u). \quad (11)$$

In our case, since $x_1 = x_2$, we get

$$\psi(t,t_2) = \int_0^\infty d\omega\omega\int du e^{-i\omega vx}\psi(\omega,u). \quad (12)$$

Based on heavy-quark symmetry, we can use the same DAs for the baryon containing the charm quark and the b quark. In [10], the DAs for the Ω_b baryon were obtained, and we used the same DAs for Ω_c in this work:

$$\psi_2(\omega,u) = \omega^2 u(1-u)\sum_{n=0}^{2}\frac{a_n}{\epsilon_n{}^4}\frac{C_n^{3/2}(2u-1)}{|C_n^{3/2}|^2}\mathrm{e}^{-\omega/\epsilon_n},$$
$$\psi_4(\omega,u) = \sum_{n=0}^{2}\frac{a_n}{\epsilon_n{}^2}\frac{C_n^{1/2}(2u-1)}{|C_n^{1/2}|^2}\mathrm{e}^{-\omega/\epsilon_n},$$
$$\psi_3(\omega,u) = \frac{\omega}{2}\sum_{n=0}^{2}\frac{a_n}{\epsilon_n{}^3}\frac{C_n^{1/2}(2u-1)}{|C_n^{1/2}|^2}\mathrm{e}^{-\omega/\epsilon_n}, \quad (13)$$

where

$$|C_n^\lambda|^2 = \int_0^1 du[C_n^\lambda(2u-1)]^2, \quad (14)$$

with $|C_0^{1/2}|^2 = |C_0^{3/2}|^2 = 1$, $|C_1^{1/2}|^2 = 1/3$, $|C_1^{3/2}|^2 = 3$, $|C_2^{1/2}|^2 = 1/5$, and $|C_2^{3/2}|^2 = 6$. The parameters entering Eqs. (13) are obtained in [10], and we present their expressions in Table I for completeness. For the numerical calculations we take $A = 1/2$. Note that the variation of leading twist-2 distribution amplitudes with respect to the free parameter A is studied in [10]. When A varies in [0.3, 0.7], the DAs change by about 10%. Since we have taken $A = 0.5$, the results are expected to vary in the same order.

From Eq. (8), we obtain the following form using Eq. (9):

$$\Pi_{\mu\nu} = i\int d^4x\int_0^\infty d\omega\omega \times\int_0^1 du e^{i(p'-\omega v)x}\left\{\left[\sum A_i(Tr\Gamma_i\gamma_\mu)S\mathcal{T}_\nu\gamma_5\bar{\not{v}} - 2\sum A_i C\Gamma_i^T C^{-1}\gamma_\mu S\gamma_5\mathcal{T}_\nu\bar{\not{v}}\right]\right\}u_{\Omega_c}. \quad (15)$$

TABLE I. Values of the parameters appearing in DAs of the Ω_c baryon [see Eq. (13)].

Twist	a_0	a_1	a_2	ϵ_0	ϵ_1	ϵ_2
2	1	...	$\frac{8A+1}{A+1}$	$\frac{1.3A+1.3}{A+6.9}$	...	$\frac{0.41A+0.06}{A+0.11}$
3σ	1	...	$\frac{0.17A-0.16}{A-2}$	$\frac{0.56A-1.1}{A-3.22}$	...	$\frac{0.44A-0.43}{A+0.27}$
$3s$	...	1	...	...	$\frac{0.45A-0.63}{A-1.4}$	...
4	1	...	$\frac{-0.10A-0.01}{A+1}$	$\frac{0.62A+0.62}{A+1.62}$	...	$\frac{0.87A+0.07}{A+2.53}$

After integrating over x, one can obtain the explicit expressions of the correlation function at QCD level. Separating the coefficients of the Lorentz structures $[v_\mu \not{q} \gamma_\nu \gamma_5,\ \not{q}\gamma_5 v_\mu q_\nu,\ \not{q}\gamma_5 v_\mu v_\nu,\ \not{q}\gamma_5 g_{\mu\nu}\ (\not{q}\gamma_\nu v_\mu,\ \not{q} v_\mu q_\nu,\ \not{q} v_\mu v_\nu,\ \not{q} g_{\mu\nu})]$ from both representations of the correlation function, we get the desired sum rules for the transition form factors $F_1(q^2)$, $F_3(q^2)$, $F_2(q^2)+F_3(q^2)$, and $F_4(q^2)$ $[(G_1(q^2), G_3(q^2), G_2(q^2)+G_3(q^2))$ and $G_4(q^2)]$, respectively:

$$
\begin{aligned}
\frac{\lambda_\Omega}{(p-q)^2-m_\Omega^2} F_1(q^2) &= \Pi_1, & -\frac{\lambda_\Omega}{m_\Omega^2-(p-q)^2} G_1(q^2) &= \Pi_5, \\
-\frac{1}{m_\Omega}\frac{\lambda_\Omega}{m_\Omega^2-(p-q)^2} F_3(q^2) &= \Pi_2, & \frac{1}{m_\Omega}\frac{\lambda_\Omega}{m_\Omega^2-(p-q)^2} G_3(q^2) &= \Pi_6, \\
\frac{\lambda_\Omega}{m_\Omega^2-(p-q)^2}\left(F_2(q^2)+\frac{m_{\Omega_c}}{m_\Omega}F_3(q^2)\right) &= \Pi_3, & -\frac{\lambda_\Omega}{m_\Omega^2-(p-q)^2}\left(G_2(q^2)+\frac{m_{\Omega_c}}{m_\Omega}G_3(q^2)\right) &= \Pi_7, \\
\frac{\lambda_\Omega}{m_\Omega^2-(p-q)^2} F_4(q^2) &= \Pi_4, & -\frac{\lambda_\Omega}{m_\Omega^2-(p-q)^2} G_4(q^2) &= \Pi_8.
\end{aligned} \tag{16}
$$

Here, Π_i are the invariant functions for the Lorentz structures mentioned above. In general, the invariant functions can be written in the following form,

$$
\Pi_i = \int_0^1 du \int d\sigma\sigma \left\{ \frac{\rho_i^{(1)}}{\Delta} + \frac{\rho_i^{(2)}}{\Delta^2} + \frac{\rho_i^{(3)}}{\Delta^3} \right\}, \tag{17}
$$

where $\Delta = p'^2 - s(\sigma)$, $\sigma = \frac{\omega}{m_{\Omega_c}}$, and i runs from 1 to 8 (each value of i describes the corresponding structure). Since the explicit expressions of $\rho_i^{(1)}$, $\rho_i^{(2)}$, and $\rho_i^{(3)}$ are lengthy, we do not present them here. Applying Borel transformation with respect to the variable $-(p-q)^2$, we get the sum rules for the form factors:

$$
\begin{aligned}
F_1 &= \frac{e^{m_\Omega^2/M^2}}{\lambda_\Omega}\Pi_1^B, & G_1 &= -\frac{e^{m_\Omega^2/M^2}}{\lambda_\Omega}\Pi_5^B, \\
F_2 + \frac{m_{\Omega_c}}{m_\Omega}F_3 &= \frac{e^{m_\Omega^2/M^2}}{\lambda_\Omega}\Pi_3^B, & G_2 + \frac{m_{\Omega_c}}{m_\Omega}G_3 &= -\frac{e^{m_\Omega^2/M^2}}{\lambda_\Omega}\Pi_7^B, \\
F_3 &= -\frac{m_\Omega}{\lambda_\Omega}e^{m_\Omega^2/M^2}\Pi_2^B, & G_3 &= \frac{m_\Omega}{\lambda_\Omega}e^{-m_\Omega^2/M^2}\Pi_6^B, \\
F_4 &= \frac{e^{m_\Omega^2/M^2}}{\lambda_\Omega}\Pi_4^B, & G_4 &= -\frac{e^{m_\Omega^2/M^2}}{\lambda_\Omega}\Pi_8^B.
\end{aligned} \tag{18}
$$

Borel transformation and continuum subtraction are performed with the help of the formula

$$
\int_0^\infty d\sigma \frac{\rho(\sigma)}{[(p-q)^2-s(u)]^n} \Rightarrow \int_0^{\sigma_0} d\sigma \left\{ (-1)^n \frac{e^{-s(\sigma)} I_n(u,\sigma)}{(n-1)!(M^2)^{n-1}} \right\} - \frac{(-1)^n e^{-s(\sigma)}/M^2}{(n-1)!} \sum_{j=1}^{n-1} \frac{1}{(M^2)^{n-j-1}} \frac{1}{s'} \left(\frac{d}{d\sigma}\frac{1}{s'} \right)^{j-1} I_n \bigg|_{\sigma=\sigma_0}, \tag{19}
$$

where

$$
\sigma_0 = \frac{(s_{\rm th} + m_{\Omega_c}^2 - q^2) + \sqrt{(s_{\rm th} + m_{\Omega_c}^2 - q)^2 - 4m_{\Omega_c}^2(s_{th} - m_s^2)}}{2m_{\Omega_c}^2}, \tag{20}
$$

and

$$I_n = \frac{\rho_n(\sigma)}{\bar{\sigma}^n}. \qquad (21)$$

Numerical analysis of the obtained sum rules for the form factors is carried out in the next section.

Having determined the form factors for the $\Omega_c^0 \to \Omega^-$ transition, it is straightforward to calculate the width of the semileptonic $\Omega_c^0 \to \Omega^- l^+ \nu_l$ and nonleptonic $\Omega_c^0 \to \Omega^- \pi^+(\rho^+)$ decays.

First, we present the amplitudes of $\Omega_c^0 \to \Omega^- h$ $(h = \pi, \rho)$ and $\Omega_c^0 \to \Omega^- l^+ \nu_l$ in the helicity basis of $H_{\lambda_\Omega \lambda_{h(l)}}$ [4,11,12]. While $\lambda_\Omega = \pm 3/2, \pm 1/2$ corresponds to the helicity states of the Ω baryon, $\lambda_{h(l)}$ corresponds to the helicity states of the $\pi(\rho)$ and $l^+\nu_l$ pair, respectively. The helicity amplitudes are defined as

$$H^{V(A)}_{\lambda_\Omega, \lambda_W} = \langle \Omega(\lambda_\Omega) | \bar{s}\gamma_\mu(\gamma_5) c | \Omega_c(\lambda) \rangle \epsilon_W^{*\mu}(\lambda_W), \qquad (22)$$

where $\lambda = \lambda_W - \lambda_\Omega$ and $\epsilon_W^{*\mu}$ is the four-vector of the virtual W boson. Using these definitions of the helicity amplitude(s) and the matrix element $\langle \Omega | \bar{s}\gamma_\mu(\gamma_5) c | \Omega_c \rangle$ in terms of the form factors, we get the following relations [4]:

$$\begin{aligned}
H^{V(A)}_{3/2,1} &= \mp \sqrt{Q_\mp^2} F_4(G_4), \\
H^{V(A)}_{1/2,1} &= -\sqrt{\frac{Q_\mp^2}{3}} \left[F_1(G_1) \frac{Q_\pm^2}{m_1 m_2} - F_4(G_4) \right], \\
H^{V(A)}_{1/2,0} &= \sqrt{\frac{2Q_\mp^2}{3q^2}} \left[F_1(G_1) \frac{Q_\pm^2 m_\mp}{2m_1 m_2} \mp \left(F_2(G_2) + F_3(G_3) \frac{m_1}{m_2} \right) \frac{|\vec{p}_\Omega|^2}{m_2} \mp F_4(G_4) \tilde{m} \right], \\
\tilde{H}^{V(A)}_{1/2,t} &= \sqrt{\frac{2Q_\pm^2}{3q^2}} \frac{Q_\mp^2}{2m_1 m_2} F_1(G_1) m_\pm \mp (F_2(G_2)\tilde{m}_+ \mp F_3(G_3)\tilde{m}_- \mp F_4(G_4) m_1).
\end{aligned} \qquad (23)$$

In these expressions, $m_\pm = m_1 \pm m_2$, $Q_\pm^2 = m_\pm^2 - q^2$, $\tilde{m}_\pm = (m_+ m_- \pm q^2)/2m_1(m_2)$. Here, m_1 and m_2 are the mass of the Ω_c and Ω baryons, respectively. The remaining helicity amplitudes can be obtained by the symmetry relation

$$H^{V(A)}_{-\lambda,-\lambda_W} = \mp H^{V(A)}_{\lambda,\lambda_W}. \qquad (24)$$

From these helicity amplitudes, the decay widths of the semileptonic and nonleptonic decays are calculated as

$$\Gamma(\Omega_c \to \Omega l \nu) = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{\Omega_c}^2} \int_{m_l^2}^{(m_{\Omega_c} - m_\Omega)^2} dq^2 \frac{|\vec{p}_\Omega| (q^2 - m_l^2)^2}{q^2} H_l^2 \qquad (25)$$

and

$$\Gamma(\Omega_c \to \Omega h) = \frac{G_F^2 |\vec{p}_\Omega|}{32\pi m_{\Omega_c}^2} |V_{cs} V_{ud}^*|^2 a_1^2 m_h^2 f_h^2 H_h^2, \qquad (26)$$

$$H_l^2 = \left(1 + \frac{m_l^2}{2q^2}\right) H_\rho^2 + \frac{3m_l^2}{2q^2} H_\pi^2, \qquad (27)$$

where

$$H_\rho^2 = |H_{3/2,1}|^2 + |H_{1/2,1}|^2 + |H_{1/2,0}|^2 + |H_{-1/2,0}|^2 + |H_{-1/2,-1}|^2 + |H_{-3/2,-1}|^2, \qquad (28)$$

$$H_\pi^2 = |H_{1/2,t}|^2 + |H_{-1/2,t}|^2. \qquad (29)$$

Here, G_F is the Fermi coupling constant, and V_{ij} are the elements of the CKM. The factor $a_1 = C_1 + C_2/N_c$ comes from the factorization [13], where N_c is the color factor and $C_1 = -0.25$ and $C_2 = 1.1$ are the Wilson coefficients [11].

III. NUMERICAL ANALYSIS

In this section, we perform numerical analysis for the transition form factors obtained in the previous section. For this goal, first we present the input parameters in our numerical analysis:

$$\begin{aligned}
&m_c(1\,\text{GeV}) = 1.35 \pm 0.10\,\text{GeV}\,[2], \\
&m_s(1\,\text{GeV}) = 0.12 \pm 0.02\,\text{GeV}\,[2], \\
&m_\pi = 0.140\,\text{GeV}\,[2], \quad m_\rho = 0.77\,\text{GeV}\,[2], \\
&m_{\Omega_c} = 2.695\,\text{GeV}\,[2], \quad m_\Omega = 1.672\,\text{GeV}\,[2], \\
&f_\pi = 132\,\text{MeV}\,[2], \quad f_\rho = 216\,\text{MeV}\,[2], \\
&f^{(1)} = 0.093 \pm 0.01\,[14], \quad f^{(2)} = 0.093 \pm 0.01\,[14],
\end{aligned} \qquad (30)$$

$$V_{cs} = 0.97320 \pm 0.00011\,[2],$$
$$V_{ud} = 0.97401 \pm 0.00011\,[2]. \quad (31)$$

For the quark masses, $\overline{\text{MS}}$ scheme values are used. Besides these input parameters, LCSR involve two auxiliary parameters: the continuum threshold s_{th} and the Borel mass parameter M^2. The working region of M^2 is determined in such a way that the power corrections as well as the continuum contributions are suppressed. The working region of s_{th} is determined from the condition that the mass sum rules reproduce the mass with, say, 10% accuracy. Following these criteria, we obtain the working regions of s_{th} and M^2:

$$4.0\,\text{GeV}^2 \le s_{\text{th}} \le 4.5\ \text{GeV}^2,$$
$$3.0\ \text{GeV}^2 \le \text{M}^2 \le 4.0\ \text{GeV}^2. \quad (32)$$

In these working regions of s_{th} and M^2, both the conditions of the smallness of the subleading twist-3 and twist-4 contributions and the suppression of the higher states, as well as continuum contributions, are satisfied.

Having determined the working intervals of the threshold and Borel mass parameters, the next task is to find the best-fitting form factors. The LCSR predictions for the form factors are not applicable for the whole physical region, $m_l^2 \le q^2 \le (m_{\Omega_c}^2 - m_\Omega)^2$, but give reliable results for up to the $q^2 \le 0.5\ \text{GeV}^2$ region. Hence, we first obtain the form factors within QCD sum rules up to $q^2 \simeq 0$. Then, we extrapolate from the domain where LCSR predictions are reliable to the full physical region by applying the following z-expansion fit function [15,16]:

$$F_i(q^2) = \frac{1}{1 - \frac{q^2}{m_{R,i}^2}}(a_0^i + a_1^i(z(q^2) - z(0)) + a_2^i(z(q^2) - z(0))^2), \quad (33)$$

where

$$z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \quad (34)$$

and $t_\pm = (m_{\Omega_c} \pm m_\Omega)^2$, $t_0 = t_+(1 - \sqrt{1 - \frac{t_-}{t_+}})$.

Here, $m_{R,i}$ are the corresponding masses of the resonances for the $c \to s$ transition in the spectrum, i.e., $m_{D_s} = 1.97$ GeV.

The obtained parametrization that best reproduces the form factors predicted by the LCSR in the region $q^2 \le 1.1\ \text{GeV}^2$ is given in Table II. Note that a_0 corresponds to the form factor at $q^2 = 0$, i.e., $a_0 = F_i(q^2 = 0)$. To verify that the results of the form factors depend weakly on the chosen M^2 and s_{th} auxiliary parameters, we plot the variation of the form factors at $q^2 = 0$, on M^2 and $s_{\text{th}} = 4\ \text{GeV}^2$, in Fig. 1. The figure shows good stability of the form factors on M^2 and s_{th}.

TABLE II. Values of the form factors at $q^2 = 0$ and the fit parameters of a_i.

	a_0	a_1	a_2
F_1	-0.55 ± 0.05	6.391	-191.2
F_2	-0.68 ± 0.07	30.94	-1053
F_3	1.0 ± 0.2	-35.60	1117
F_4	0.16 ± 0.02	-7.24	239.9
G_1	-0.48 ± 0.02	3.513	-100.9
G_2	0.68 ± 0.07	-30.1	1053
G_3	-1.0 ± 0.2	35.60	-1117
G_4	-0.16 ± 0.02	7.24	-239.9

Using the obtained results for the form factors responsible for the $\Omega_c^0 \to \Omega^-$ transition, we calculate the branching ratio for $\Omega_c^0 \to \Omega^- l^+ \nu_l$ and $\Omega_c^0 \to \Omega^- h^+$ ($h = \pi$, ρ) decays by using Eqs. (34) and (33). The lifetime of the Ω_c baryon is taken as $\tau = (268 \pm 24 \pm 10) \times 10^{-15}$ s [2]. The Belle II Collaboration recently reported a lifetime of the Ω_c baryon, $\tau = 243 \pm 48 \pm 11$, which agrees with the previous measurements [17]. Using the values of the input parameters together with the decay width expressions, we obtain the branching ratios that are presented in Table III.

The $\Omega_c^0 \to \Omega^-$ transition has been studied in several models [4,7,11,18,19]. The obtained results are presented in Table III. Our results are close to the predictions of the light-front quark model [4] for the semileptonic part. However, there is a large discrepancy with other studies for these decays. In addition, our theoretical predictions do not match with the recent Belle II measurement [8], especially for the semileptonic $\Omega_c^0 \to \Omega^- l^+ \nu_l$ decay. A similar situation was also obtained in [4]. This discrepancy might be due to large $\frac{1}{m_c}$ corrections or the existence of new physics. This point needs further detailed analysis.

On the other hand, an upper bound for $\mathcal{R}_{\rho/\pi} > 1.3$ is set by the experiment. This bound does not contradict theoretical results; however, there is a considerable discrepancy among the predictions of different theoretical approaches. Hopefully, more experimental and theoretical efforts on this transition will lessen the discrepancy.

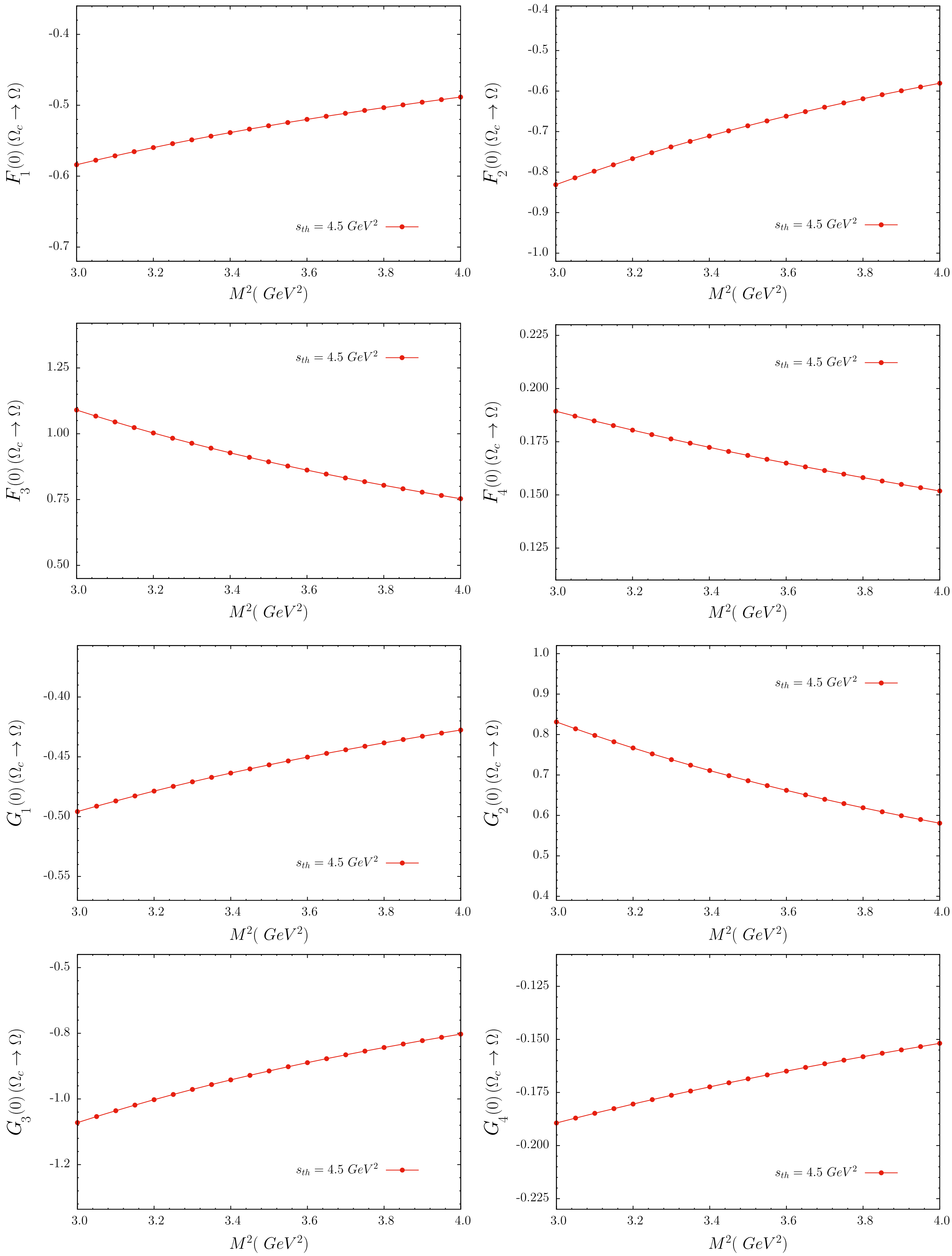

FIG. 1. Working regions of M^2 and s_0 as well as the value of residues for the considered states.

TABLE III. Branching fractions of the Ω_c^0 decays obtained via different models as well as experimental results. Note that $\mathcal{R}$ corresponds to the branching ratio of the considered decays.

States	Our result	Experiment [2,8]	Reference [4]	Reference [18]	Reference [19]	Reference [11]	Reference [7]
$\mathcal{B}_\pi$	29×10^{-3}	...	$(6 \pm 0.8) \times 10^{-3}$	66.5×10^{-3}	42.3×10^{-3}	...	
$\mathcal{B}_\rho$	63×10^{-3}	...	$(17 \pm 0.5) \times 10^{-3}$	361.1×10^{-3}	149×10^{-3}	...	
$\mathcal{B}_e$	20.6×10^{-3}	...	$(5.4 \pm 0.2) \times 10^{-3}$	...	...	...	127×10^{-3}
$\mathcal{B}_\mu$	19.6×10^{-3}	...	$(5.0 \pm 0.2) \times 10^{-3}$	...	...	...	
$\mathcal{R}_{\rho/\pi}$	2.18	>1.3	(2.8 ± 0.4)	5.4	3.5	9.5	
$\mathcal{R}_{e/\pi}$	0.71	$1.98 \pm 0.13 \pm 0.08$	(0.9 ± 0.1)	...	...	...	
$\mathcal{R}_{\mu/\pi}$	0.68	$1.94 \pm 0.18 \pm 0.10$	(0.9 ± 0.1)	...	...	...	

IV. CONCLUSION

In this work, the semileptonic and nonleptonic decays of Ω_c^0, namely, $\Omega_c^0 \to \Omega^- l^+ \nu_l$, $\Omega_c^0 \to \Omega^- \pi^+$, and $\Omega_c^0 \to \Omega^- \rho^+$, are studied within the framework of the light-cone sum rules by using the distribution amplitudes of the Ω_c baryon. In this study, DAs for Ω_b baryons are used for their c-quark counterpart depending on the heavy-quark symmetry. We first calculated the transition form factors for the $\Omega_c^0 \to \Omega^-$ decay in the LCSR method. Then, using the obtained results for the transition form factors, we predicted the branching ratios of the semileptonic $\Omega_c^0 \to \Omega^- l^+ \nu_l$ (where $l = e, \mu$) and nonleptonic $\Omega_c^0 \to \Omega^- \pi^+(\rho^+)$ decays. Finally, we compared our predictions with other approaches as well as recent Belle II results. We found that our results, especially on the branching ratio for the semileptonic $\Omega_c^0 \to \Omega^- l^+ \nu_l$ decay normalized to $\Omega_c^0 \to \Omega^- \pi^+$, are considerably smaller than existing experimental data. A similar discrepancy was also obtained in the light-front approach.

We found that there is a large deviation between the theoretical predictions and experimental results on the branching ratios of the semileptonic decays. Our results on the branching ratios for the semileptonic decay (normalized to $\Omega \to \pi$) are compatible with the light-front quark model. However, other approaches' results drastically differ from ours. Similar circumstances take place for the nonleptonic decays as well. Although there is only a lower bound for the ratio, $\mathcal{R}_{\rho/\pi}$, theoretical predictions differ among themselves. At this stage, it is hard to identify the reason for the discrepancies for both semileptonic and nonleptonic decays of the Ω_c baryon. These points need further studies from both experimental and theoretical sides, and new physics implications may even be implied.

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