MIDDLE SCHOOL STUDENTS' QUANTITATIVE REASONING IN PICTORIAL, SYMBOLIC AND ICONOGRAPHIC PROBLEMS

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ABSTRACT

MIDDLE SCHOOL STUDENTS' QUANTITATIVE REASONING IN PICTORIAL, SYMBOLIC AND ICONOGRAPHIC PROBLEMS

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The aim of this study was to examine middle school students' (5^{th}–8^{th} grade) quantitative reasoning in early algebra problems given in pictorial, symbolic, and iconographic forms and involved single-unit and multiple-unit substitutions. The study was carried out with a total of 60 students studying in two public middle schools in the town of Şarkışla, Sivas, in October of the 2021-2022 academic year. Participants were determined by the purposeful and convenience sampling method. In this study, a multiple case research design was used, and a total of seven problems were used in pictorial, symbolic, and iconographic forms. The collected data were analyzed with the content analysis method to explore middle grade students' quantitative reasoning. The results of the study showed that forms of the problems were not determinative in the students' quantitative reasoning. Instead, it was concluded that the students' quantitative reasoning differed according to the level of complexity of the relationship such as single unit substitution and multiple unit substitution within each form. Finally, the reasons behind students' explanations were generally similar at each grade level (5^{th}-8^{th} grade) so, there were no distinct differences between students' quantitative reasoning at different grade levels in the problems addressing pictographic, iconographic, and symbolic forms. Thus, this study concluded that middle school students, even 8th graders, did not demonstrate sufficient quantitative reasoning, especially in multiple-unit substitution problems in each of the three forms. In this sense, this study informs teachers and teacher educators about the forms of the problems used in early algebra teaching as well as the complexity of the relationship (i.e., single-unit vs multiple-unit substitutions) involved in the problem.

Keywords: Quantitative Reasoning, Middle Grade Students, Pictorial Problems, Symbolic Problems, Iconographic Problems

ÖZ

ORTAOKUL ÖĞRENCİLERİNİN RESİMSEL, SEMBOLİK VE İKONOGRAFİK PROBLEMLERDEKİ NİCELİKSEL MUHAKEMELERİ

Uyguç, Ayşenur
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Bu çalışmanın amacı, ortaokul öğrencilerinin (5-8. Sınıf) resimsel, sembolik ve ikonografik formda verilen ve tek birim ve çoklu birim yerine koyma sureci içeren erken cebir problemlerdeki niceliksel muhakemesini incelemektir. Çalışma, 2021-2022 eğitim-öğretim yılı ekim ayı içerisinde Sivas'ın Şarkışla ilçesinde iki devlet ortaokulunda öğrenim gören toplam 60 öğrenci ile gerçekleştirilmiştir. Katılımcılar amaçlı ve kolay ulaşılabilir örnekleme yöntemi ile belirlenmiştir. Çoklu durum araştırma deseni kullanılan bu çalışmada, resimsel, sembolik ve ikonografik formda olmak üzere toplam yedi problem kullanılmıştır. Toplanan veriler içerik analiz yöntemi ile analiz edilerek ortaokul öğrencilerin niceliksel muhakemeleri anlaşılmaya çalışılmıştır. Araştırmanın sonuçları problem formlarının öğrencilerin nicel muhakemelerinde belirleyici olmadığını göstermiştir. Bunun yerine, öğrencilerin nicel muhakemelerinin, her formda tek birim yerine koyma veya çoğulunu yerine koyma gibi ilişkinin karmaşıklık düzeyine göre farklılaştığı sonucuna varılmıştır. Son olarak, öğrencilerin açıklamalarının ardındaki nedenler genellikle her sınıf seviyesinde (5-8. Sınıf) benzerdir. Bu nedenle, farklı sınıf seviyelerindeki öğrencilerin piktografik, ikonografik ve sembolik biçimleri ele alan

problemlerdeki niceliksel muhakemelerinde belirgin bir fark yoktur. Dolayısıyla bu çalışma, orta okul öğrencilerinin, hatta 8. sınıf öğrencilerinin bile, özellikle üç formun her birindeki çoklu birim yerine koyma sureci içeren problemlerde yeterli nicel muhakeme gösteremedikleri sonucuna varmıştır. Bu anlamda, bu çalışma öğretmenlere ve öğretmen eğitimcilerine erken cebir öğretiminde kullanılan problemlerin biçimleri ve problemde yer alan ilişkinin karmaşıklığı (yani, tek ve çoklu birim yerine koyma) hakkında bilgi vermektedir.

Anahtar Kelimeler: Niceliksel Muhakeme, Orta Okul Öğrencileri, Resimsel Problemler, Sembolik Problemler, İkonografik Problemler

To mum and dad

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MNE: Ministry of National Education

NCTM: National Council of Teachers of Mathematics

CHAPTER 1

INTRODUCTION

Students start their education with arithmetic subjects such as numbers, counting, and arithmetic operations (addition, subtraction, multiplication, and division). In other words, students reason arithmetically from the beginning of their education. However, students are introduced to “algebra” in the second semester of the 6th grade, and after that, numbers are replaced by letters, symbols, and algebraic expressions (MNE, 2008). The transition from arithmetic to algebra is one of the most important stages of mathematics learning and coincides with the middle school years. Therefore, a meaningful transition from arithmetic reasoning to algebraic reasoning is required for students.

There were many parallel definitions related to algebra and algebraic reasoning in the literature. For instance, Kalman (2008) defined algebra as “a shorthand way to express quantitative reasoning.” and “elementary algebra actually is common sense written in symbols” (p.334). Moreover, Sfard (1995) defined it as general computational science. Similarly, Akkaya and Durmuş (2006) defined algebra as “a branch of mathematics that enables to transform the relations examined by using numbers and symbols into generalized equations” (p. 1). Algebraic reasoning included generalizing with numbers and operations, formalizing these thoughts using meaningful symbol systems, and examining the concepts of patterns and functions (Kalkan, 2014).

The algebra concept was considered one of the most critical subjects in mathematics, and it was so needed for students' mathematical development (Cai et al., 2011; Obioma, 2005). Actually, algebra is more abstract than arithmetic since it is related to symbols, letters, and algebraic expressions instead of numbers. For this reason, students had difficulty in understanding and comprehending algebra. Many researchers investigated students' difficulties, misconceptions, and misunderstandings in conceptualizing algebra (e.g., Carpenter & Levi, 2000; Carraher & Schliemann, 2007; Dede, 2004; Didiş & Erbaş, 2012; McNeil & Alibali, 2005; Samuel et al., 2016; Usta & Özdemir, 2018). For instance, Usta and Özdemir (2018) found that middle school students' algebraic thinking level was below the expected level, and they had difficulty in answering complex problems. One of the mistakes that students made was using arithmetic ways in solving algebra problems. They often assigned numerical values to the given quantities to find answers to the algebra problems. Besides, Didiş and Erbaş (2012) examined 10th grade students' achievements at algebraic word problems and the factors affecting their achievements. According to the findings, students who participated in the study had very low success in solving algebraic-verbal problems, and most of these students could not write appropriate equations for algebraic-verbal problems. In addition, the inability to understand and interpret the problem situation was found to be the reason why the students could not solve the algebraic problems successfully.

Some of these studies showed that students had difficulties in using and interpreting symbolic letters and had misconceptions about them. It was also shown that students failed to understand the meaning of operations while solving algebraic expressions, or equations, and that they did not know and could not interpret the different uses of variables (e.g., Dede, 2004; Herscovics & Linchevski, 1994; Linchevski & Herscovics, 1996). In order to eliminate all these difficulties that students may encounter in algebra and to have meaningful learning, it is necessary to ensure a smooth and meaningful transition from arithmetic to algebra.

As mentioned above, the transition from arithmetic to algebra was one of the middle school topics. Generally, the transition from arithmetic thinking to algebraic thinking did not happen automatically (Herscovics & Linchevski, 1994), and it was seen that the students had difficulties in this transition and in making sense of the subject of algebra. To eliminate them, quantitative reasoning was one of the necessary reasoning because quantities and the ability to reason between these quantities were included in every step of the subject of algebra (Kabael & Tanışlı, 2010). That is, it was needed for the smooth and easy transition from arithmetic to algebra and for the development of students' algebraic thinking. Therefore, it can be stated that quantitative reasoning acted as a bridge in the transition from arithmetic to algebra and formed the basis for algebraic thinking (Ellis, 2007).

Quantitative reasoning required understanding quantities, values of quantities, and quantitative relationships (Thompson, 1988). It was a central dimension of students' mathematical development (Smith & Thompson, 2007) and helped students to analyze the relationship between the quantities to reach the correct result when solving problems (Dwyer, 2003). Thus, in quantitative reasoning, numbers and the relationship between them took place after the quantities and relationship between them. In other words, when solving problems, students identified the quantities and analyzed the relationship between them at first. After that, they thought about and decided on the appropriate operations (Thompson, 1993). Quantitative reasoning contributed to the development of algebraic reasoning when quantitative reasoning was taught over the years together with appropriate teaching methods and techniques (Smith & Thompson, 2007). In this context, quantitative reasoning forms the basis for the development of algebraic reasoning (e.g., Britt & Irwin, 2011; Subramaniam & Banerjee, 2011).

Indeed, it was observed that the students who did not construct quantitative reasoning were not successful in solving arithmetic and algebraic problems according to Thompson (1988). Nonetheless, if the students cannot make sense of the quantities given in the problem and the relationships between the quantities, they saw algebra as a subject that contains meaningless symbols or letters. Thus, they had difficulty establishing equations and solving problems. However, as stated in a study by Smith and Thompson (2007), the focus on quantitative reasoning enabled students to solve algebraic problems by reasoning about quantities and the relationships between quantities. A number of studies have also shown that quantitative reasoning has a positive effect on students' problem solving processes (e.g., Moore et al., 2009; Moore, 2010; Moore & Carlson, 2012; Smith & Thompson, 2007). All these studies revealed that quantitative reasoning is necessary for student, and the inability to establish quantitative reasoning from an early age could cause many problems such as returning arithmetic methods in solving problems, affecting the development of algebraic reasoning negatively, and seeing algebra as a subject of meaningless symbols.

Understanding quantitative reasoning would make it easier for students to understand mathematics, more specifically algebra. Some of the studies related to the quantitative reasoning supported this thought (e.g., Britt & Irwin, 2011; Carraher & Schliemann, 2007; Smith & Thompson, 2007; Subramaniam & Banerjee, 2011; Thompson, 1988). On the other hand, as seen in many studies, the fact that students at different grade levels experienced various difficulties and had misconceptions in the field of algebra directed researchers to think about how students reason quantitatively. As the abovementioned studies in the literature were reviewed, any research particularly investigating and comparing the quantitative reasoning of middle school students in different forms of the problem (i.e., pictorial, symbolic, and iconographic) has not been encountered. Therefore, the current study investigated middle school students' quantitative reasoning in pictorial, symbolic, and iconographic problems.

1.1 Purpose and Research Questions of the Study

The aim of the current study was to investigate middle school students' quantitative reasoning in early algebra problems given in pictorial, symbolic, and iconographic forms. Specifically, the current study was conducted to address the following research question.

1. What are middle school (5^{th}-8^{th} grade) students' quantitative reasoning in early algebra problems given in pictorial, symbolic, and iconographic forms?

    1.1. Does middle school (5^{th}-8^{th} grade) students' quantitative reasoning differ by the forms of early algebra problems (i.e., pictorial, symbolic, and iconographic problems)? If so, how?

1.2. Does middle school students' quantitative reasoning in pictorial, symbolic, and iconographic problems differ by the grade level (5-8th grade)? If so, how?

1.3. Does middle school (5th-8th grade) students' quantitative reasoning differ in single-unit substitution and multiple-unit substitution problems given in three forms; pictorial, symbolic, and iconographic problems? If so, how?

In accordance with this purpose, a set of problems in different forms were prepared based on the literature and administered to the middle school students who participated in the study.

1.2 Significance of the Study

As seen in the literature, it is important for students to establish quantitative reasoning in many respects. First, a connection can be established between arithmetic and algebra. Particularly, it facilitates the learning of algebra for students. (e.g., Kindt et al., 2006; Meyer, 2001; Ramful & Ho, 2015; Smith & Thompson, 2007; Thompson, 1993). It also positively affects the problem-solving process of students and especially facilitated the solution of algebraic problems (e.g., Moore et al., 2009; Moore, 2010; Moore & Carlson, 2012; Smith & Thompson, 2007). In addition, numerous studies demonstrate the importance of quantitative thinking in the process of solving complicated and challenging problems. (e.g., Ellis, 2007; Kabael & Akın, 2016; Kabael & Kızıltoprak, 2014; Moore et al., 2009; Thompson, 1993; Thompson, 1988). As a result, it is important and necessary to investigate the students' quantitative reasoning and thinking related to it.

There were also studies related to quantitative reasoning in Turkiye (e.g., Akın, 2017; Danacı & Şahin,2021; Güvendiren, 2019; Kabael & Akın, 2016). Some studies investigated students' quantitative reasoning, and some others examined the relationship between quantitative reasoning and other reasoning or type of thinking. Some of them investigated the effect of particular training on the development of quantitative reasoning (e.g., Akın, 2017; Danacı & Şahin, 2021; Güvendiren, 2019).

However, students' quantitative reasoning was generally examined in one form of the problem such as a word problem or at a single grade level. For instance, while Kabael and Akın (2016) investigated 7th grade students' quantitative reasoning in word problems, Güvendiren (2019) and Dur (2014) examined 6th grade students' quantitative reasoning.

Hence, the relevant available literature is quite limited as no study examining the quantitative reasoning of all middle school students in problems of different forms and comparing the quantitative reasoning of students at different grade levels was encountered. As a result, it is expected that this study will contribute to the related literature by examining the quantitative reasoning of middle school students in pictographic, symbolic, and iconographic problems and comparing their quantitative reasoning.

This study is important for mathematics teachers in addition to its contribution to the associated literature because knowing students' approaches, understandings, and misunderstandings related to quantitative reasoning can help mathematics teachers plan their lessons. For instance, they can integrate various problems into their lessons, especially problems that students have difficulty with. In this way, students' quantitative reasoning can be improved, and misunderstandings or misconceptions can be eliminated. Therefore, better teaching can be provided by increasing the awareness of mathematics teachers on middle school students' quantitative reasoning.

The current study may also be helpful for mathematics teacher educators because they might design their courses based on these findings. In this way, pre-service teachers can learn more about quantitative reasoning which is the basis of algebra. In addition, the findings of this study may be noteworthy for textbook authors. The emphasis on quantitative reasoning in textbooks can be increased, and various problems (including problems in the current study) can be added to the textbooks. Especially, these problems can be included in textbooks before the 6th grade level because students are introduced to algebra for the first time in the 6th grade in my country. Therefore, quantitative reasoning can be developed before students learn algebra.

1.3 Definitions of Important Terms

Reasoning: It is the ability to think logically to understand an event, problem, or situation, to notice relationships, and to draw conclusions (Umay, 2003).

Quantitative reasoning: It was defined as understanding or interpreting the relationship between quantities (Smith & Thompson, 2007). In the current study, students' understanding of the relationships between the quantities given in the problems and reaching the correct answer indicated their quantitative reasoning.

Arithmetic reasoning: Arithmetic reasoning focused on numbers, operations, numerical methods, and the relationship between the numbers. In other words, arithmetic reasoning can be defined as mathematical operations (Bozkaya, 2020; Güvendiren, 2019; Smith & Thompson, 2007).

Algebraic reasoning: It can be defined as understanding the functions, representing and analyzing mathematical structures or situations in different ways using algebraic symbols, using mathematical models to represent and understand quantitative relationships, and analyzing the different situations in real life (NCTM, 2000).

Early algebra: The transition process between arithmetic and algebra is called early algebra (Turgut & Temur, 2017).

Pictographic Problems: The word pictograph is defined as *"one of the symbols belonging to a pictorial, graphic system"* (Merriam-Webster, n.d.). In the current study, the first three problems (see Table 3.2) are called pictographic problems since various fruits used in the given problems were symbolized by drawing.

Iconographic Problems: It was defined as *"representing something by pictures or diagrams"* (Merriam-Webster, n.d.). In the current study, the fourth and sixth problems (see Table 3.3) are called iconographic problems since triangles and squares were drawn as diagrams.

Symbolic Problems: Symbolic problems involve symbols such as letters. In the current study, the fifth and seventh problems (see Table 3.3) are called symbolic problems because letters were used in the problems.

Single-Unit Substitution Problems: Single-unit substitution involves directly substituting the given unit to another situation. In the current study, students are required to substitute the given unit directly to the addition operations in the pictographic substitution, symbolic substitution and iconographic substitution problems.

Multiple-Unit Substitution Problems: Multiple-unit substitution involves substituting the multiple of the given unit to another situation. In the current study, students first need to find the single-unit and then take the multiple of the given unit to substitute into the addition operations in the symbolic multiple-unit substitution and iconographic multiple-unit substitution problems.

1.4 My motivation to Conduct the Study

I took methods of teaching mathematics, and the nature of mathematical knowledge for teaching courses in my third and fourth years of the undergraduate teacher education program. I learned a lot about how mathematics topics should be taught, and students' misunderstandings or misconceptions about various mathematics topics. One of these was algebra. Both my observations during the internship period and my observations after I started in-service teaching were parallel to the education I received throughout my university life. I observed that students could not establish equations because they had difficulty making sense of algebra, one of the important topics of middle school mathematics. Especially, they cannot form equations using algebraic symbols since they cannot conceptualize the given quantities and relations.

In addition, I did research and read articles on the concept of equality, the equal sign, and its meanings in an elective course I took in my graduate education. Thus, I started to take an interest in these issues. Thereupon, my thesis supervisor suggested an article for me to read. I read both this article and various articles in parallel. I thought of focusing on students' quantitative reasoning. With the suggestions and support of my thesis supervisor, I decided to examine the middle school students’ quantitative reasoning in pictographic, symbolic, and iconographic problems. I think that this study will both contribute to the literature and be useful to me as a mathematics teacher and other mathematics teachers as well as mathematics teacher educators. The awareness of mathematics teachers about quantitative reasoning may increase by reading the findings of the study. Through the incorporation of various problems into their teachings, they can aid students in developing their quantitative reasoning.

CHAPTER 2

LITERATURE REVIEW

The purpose of the study was to explore middle school students (5-8th grade) quantitative reasoning in pictorial, symbolic, and iconographic problems. The review of the literature was explained throughout this chapter. The concept of quantitative reasoning and its definition were explained at the beginning of the literature. Subsequently, studies examining the relationship between quantitative, arithmetic, and algebraic reasoning were reviewed. Afterward, it was focused on students' quantitative reasoning experiences. These studies were examined under three main categories as studies on students' additive reasoning and studies on students' multiplicative reasoning, separately, and then studies examining students' additive and multiplicative reasoning together. Finally, studies conducted in Turkey were provided. This chapter concluded with a summary of the literature.

2.1 Quantitative Reasoning

Quantitative reasoning involved an examination of a problem situation within quantitative structures in the context of quantitative networks and quantitative relations. In other words, it was to discover the relationship between quantities in a problem (Thompson, 1993). According to Ramful and Ho (2015), quantitative reasoning was defined as "analyzing the quantities and relationships among quantities in a situation, creating new quantities, and making inferences with quantities" (p. 16). There were many parallel definitions in many different studies. Commonly, quantitative reasoning was considered to understand or interpret the relationship between quantities (e.g., Johnson, 2012; Nunes et al., 2015; Smith & Thompson, 2007; Thompson, 1993; Van den Heuvel-Panhuizen & Elia, 2020). To reason quantitatively, students could be able to compare quantities either additively or multiplicatively (e.g., Lobato & Siebert, 2002; Thompson, 1988). In this context, Van den Heuvel-Panhuizen and Elia (2020) analyzed quantitative reasoning under two main categories, which were additive and multiplicative reasoning (see Figure 2.1).

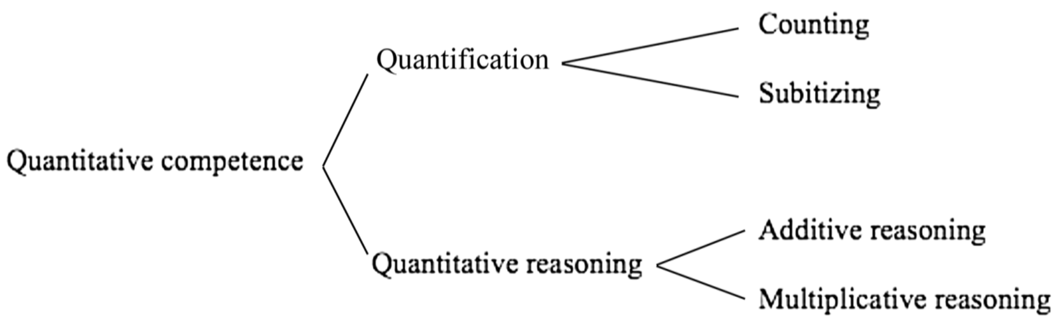


Figure 2. 1 Model of quantitative competence (Van den Heuvel-Panhuizen and Elia, 2020, p.807)

Their purpose was to analyze the relationship between different quantities where the relationship between quantities was either additive or multiplicative, or both. It was called additive reasoning if there was an additive relationship between the amounts. If there was a multiplicative relationship between the quantities, quantitative reasoning was called multiplicative reasoning (Van den Heuvel-Panhuizen & Elia, 2020).

In order to develop quantitative reasoning, students first needed to understand the concept of quantity. Thompson (1988) described quantity as the quantifiable property of an object. According to Charles (2011), the quantity was mathematically defined as the thing that could be counted or measured. The concept of quantity, such as weight, areas, and volume, was not easy for students to understand (Thompson, 2011). The important thing was to make sense of quantities so that the students did not have difficulty with other topics, especially algebra subjects (Smith & Thompson, 2007).

The other important point was that quantity, and quantitative reasoning was not the same as the number and numerical reasoning, respectively (Smith & Thompson, 2007). Actually, quantities expressed a numerical value when measured, but there was no need to measure quantities or to know their numerical value to reason about them. That is, whether quantities can be measured or not, what makes them a quantity was the capacity to be measured. For example, when comparing the height of people, it was not necessary to know the numerical value of how tall they are. Comparisons could be made without knowing the numerical value of the lengths (Thompson, 1993). Thus, students needed to make a meaningful transition from the number to the quantity.

Another concept needed to be learned to develop quantitative reasoning was a quantitative operation. A quantitative operation was defined as mental operations that enable the formation of a new quantity from two existing quantities in the problem. Also, the new quantity formed by quantitative operations indicated the relationship between the two quantities in mind (Smith & Thompson, 2007; Thompson, 1993; Troy, 1993). It was categorized the quantitative operations showing a relationship between quantities; "combine and compare quantities additively, and combine and compare quantities multiplicatively" (Troy, 1993, p.16-18).

Finally, the concepts of quantitative difference and quantitative ratio were important for developing quantitative reasoning (Smith & Thompson, 2007; Troy, 1993). The quantitative difference indicated a quantity resulted from the additive comparison of the two quantities. When making this comparison, it was taken into account how little or how much one quantity is over another (Smith & Thompson, 2007; Thompson, 1993). On the other hand, a quantitative ratio was the new quantity obtained from the multiplicative comparison of two quantities. For comparing multiplicatively, the critical point was how many times one quantity is compared to the other (Smith & Thompson, 2007).

To think quantitatively, students needed to compare the quantities such as prices, weights, widths, the number of objects, and so on. Many researchers stated that problems involving unknown quantities could be solved by using various methods. Of course, the algebraic solution has come to mind first among these methods, but these problems can also be solved without using equations. For example, Kindt et al. (2006) and Meyer (2001) gave different problems related to the prices or scales similar to the problems used in my study. They mentioned that these problems could be solved with or without equations. For example, charts, graphics, or tables can be used to compare quantities. Also, students can discover patterns among quantities by focusing on the relationship between them given in the problem. In this way, the unknown can be found by using the exchange method in shopping problems. In a problem illustrated with visuals, two umbrellas and a hat cost $80, one umbrella and two hats cost $76; and the price of a hat or an umbrella was asked. In this problem, the relationship between quantities was noticed when looking at the image to find the price of the hat or umbrella. When one umbrella was changed with a hat, the total cost was reduced by $4. If the remaining umbrella was replaced with the hat, the price of three hats was found by subtracting $4 from the price, so the cost of the three hats was found. Likewise, starting from the bottom of the line and going upwards, the price of the umbrellas was reached.

Similarly, Van Reeuwijk (2001) aimed to understand whether using "comparing quantities" was indeed beneficial in making sense of the subject of equations. In line with this purpose, a study included some problems involving the comparing the quantities. For example, these problems related to the scale, shopping, and word problems that need to be used equations for solving them. Some students in the study could reason about quantities well, while some could not. Solely, "comparing quantities" was a way of solving equations at the 6th grade level. It enabled students to reason about creating variables and solving equations. It was also seen that a flow created, and furthermore, starting with the comparison of the quantities on the scale and progressing to the problems that require using equations for solving helped the students to conceptualize the equations. This indicated that quantitative reasoning facilitated the understanding of algebra. Due to the construction of quantitative reasoning, students could construct the equations given in the problem more easily in the algebra problems.

At the same time, making a good sense of the relationships between quantities helped students transfer the learned information to another situation. For example, in the study by Lobato & Siebert (2002), nine students were given three tasks related to slope, and they were asked to answer these tasks before the teaching. Afterwards, these students were trained for ten days, and at the end of the training, the same tasks were given to the students again. However, the study focused on the work of one of these nine students. The results indicated that although the student knew the slope formula, he did not initially associate the slope with the steepness of the ramp. As a result of the training, it was seen that the student gradually established this connection. While establishing this connection, the importance of quantitative reasoning has been understood. At this duration, he first understood that the steepness consisted of two different quantities and then that there was a multiplicative relationship between these two quantities. He realized that for the steepness to remain the same, the ratio between these quantities must remain equal, and he realized that numerical operations were used here. In this way, he was able to establish the connection between them. Therefore, it was seen that quantitative reasoning was also effective in the transfer process, and quantitative, algebraic, and arithmetic reasoning were intertwined.

To solve the problem in a meaningful way by using algebra, the relationship between the quantities needed to be analyzed well, as seen in the studies mentioned above. From this point of view, quantitative reasoning was related to arithmetic and algebraic reasoning. It acted as a bridge between arithmetic and algebra. If students could understand and interpret the relationship between quantities correctly, they did not have difficulty in algebra, and algebra did no longer be a subject consisting of meaningless symbols for students. Therefore, there was a very strong relationship between quantitative, arithmetic, and algebraic reasoning (e.g., Kindt et al., 2006; Meyer, 2001; Ramful & Ho, 2015; Smith & Thompson, 2007; Thompson, 1993). In accordance with this purpose, studies related to the relationship between quantitative, arithmetic, and algebraic reasoning are presented below.

2.1.1 Relationship Between Quantitative and Arithmetic Reasoning

Arithmetic reasoning was the skill of finding the value of an unknown quantity with the help of known quantities. Arithmetic reasoning focused on numbers, operations, numerical methods such as trial and error, and the relationship between the numbers (e.g., Smith & Thompson, 2007; Güvendiren, 2019). Arithmetic and quantitative reasoning were not the same things, but there was a relationship between them. While quantitative reasoning focused on quantities and the relationship between quantities, arithmetic reasoning considered the values or numbers and the relationship between these. In fact, the focus point in both reasoning was relations (e.g., Thompson, 1993; Thompson, 2011). The important thing was to get students to focus on quantities and relations between them rather than the numbers in arithmetic operations in a given problem situation. If students did not change their focus from numbers to quantities, students used arithmetic solution methods such as the trial and error method, only focused on numbers and operations while solving problems (e.g., Johanning, 2004; Kabael & Akın, 2016; Smith & Thompson, 2007).

2.1.2 Relationship Between Quantitative and Algebraic Reasoning

One of the essential subjects of secondary school mathematics was teaching algebra (MNE, 2018). Because algebra was a continuation of arithmetic, one of the most critical features of secondary school mathematics was the transition from arithmetic to algebra (Zwanch, 2019). Many students had difficulties with this transition. They saw algebra as a subject made up of several symbols of which they had difficulty making sense and struggled in solving algebraic word problems (e.g., Kieran, 2007; Smith & Thompson, 2007).

Quantitative reasoning was critical for developing students' algebraic thinking and for the transition from arithmetic to algebra to be meaningful (e.g., Ellis, 2007; Kabael & Akın, 2016; Smith & Thompson, 2007). To solve problems by using algebraic symbols, it needed to identify variables, write algebraic equations and solve these equations. However, mathematical problems could also be solved using only quantities and relations between quantities without using variables and algebraic expressions. This approach was called quantitative reasoning (e.g., Ramful & Ho, 2015; Smith & Thompson, 2007). The problem given in Thompson's (2007) study is given below (p.8).

> **The problem:** I walk from home to school in 30 minutes, and my brother takes 40 minutes. My brother left 6 minutes before I did. In how many minutes will I overtake him? (Krutetski, 1976, p. 160)

The problem can be solved by using two ways which were algebraic and quantitative ways. If the problem was solved using the algebraic way, it was needed to establish equations by assigning letters to the quantities given in the problem. In the solution provided in the study, *t* was the duration of the path s/he walked, and $t + 6$ was the duration path his/her brother walked. d indicated the distance between school and home. $\frac{d}{30}$ and $\frac{d}{40}$ represented their speeds. The equation was established using the path formula, which was equal to multiplying the speed by the time. On the other hand, the problem was solved by focusing relationship between quantities. Since the duration was 30 and 40 minutes for him/her and his/her brother, respectively. S/he is 4/3 faster than his/her brother. The distance between them was reduced by 1/3. In order for the distance to disappear, it was needed three times as much as the minute his/her brother walked, which equaled the 18.

Using algebraic notations was the main difference between quantitative and algebraic reasoning. Quantitative reasoning did not include using variables and solving equations. The focal point was the quantities and the expression of the relationship between quantities. However, in algebraic reasoning, the critical point was transferring the relationship to the algebraic notations, which indicated a connection between them. When looking at the algebra definitions, this relationship can be understood. For example, Kalman (2008) defined algebra as "a shorthand way to express quantitative reasoning" (p.334). Basically, it was necessary to understand the relationship between the quantities given in the problem and be able to write this relationship by using algebraic symbols when necessary. However, students generally focused on arithmetical operations instead of focusing on quantitative reasoning (e.g., Johanning, 2004; Kabael & Akın, 2016; Smith & Thompson, 2007). For this reason, teachers' instructional practices played an important role in encouraging students to think quantitatively.

The transition process between arithmetic and algebra was called early algebra (Turgut & Temur, 2017). In other words, the period when students formed the basis of algebra with their arithmetic knowledge and began to think algebraically was defined as early algebra. It provided informal learning of algebraic concepts and rules by using arithmetic and geometric knowledge. At the same time, early algebra included informal symbolization, developing the arithmetic knowledge necessary for solving equations, and algebraic reasoning (Akkan et al., 2011). It was emphasized that the applying early algebraic activities was important so that students did not have difficulties in algebra (e.g., Carreher et al., 2017; Kieran, 2006; Mulligan & Vergnaud, 2006; Temur & Turgut, 2018; Turgut & Temur, 2017).

When examined early algebra activities or problems, some characteristics were defined. Firstly, "early algebra builds on background contexts of problems" (Carreher et al., 2017, p.236). Secondly, in early algebra, a formal language must be taught over time. Finally, early algebra was linked to primary school mathematics subjects (Carreher et al., 2017). Early algebra covered numbers, operations (addition, subtraction, multiplication, and division), relationship between numbers and operations, patterns, generalizations, ratio, proportion, rational numbers, mathematical reasoning, functional thinking and mathematical modeling. It was also related to the word problems including additive and multiplicative relationship, understanding relationship between quantities and conceptualizing of mathematical properties. (e.g., Carreher et al., 2017; Kieran, 2006; Mulligan & Vergnaud, 2006; Temur & Turgut, 2018). Some examples of early algebra problems were given below.

Example 1: Ali is 6 years old and Sevgi is 8 years old. When Ali is 8 years old, how old will Sevgi be? (Temur & Turgut, 2018, p.49).

Example 2: Mathematical properties such as commutative property (a+b=b+a)

Example 3: Mike has $8 in his hand and the rest of his money is in his wallet; Robin has exactly 3 times as much money as Mike has in his wallet. What can you say about the amounts of money Mike and Robin have? (Carreher et al., 2017, p.248).

Example 4: 8 + 4 = __+ 5

Example 5: Pattern starting from three and increasing by four

As a result, early algebra problems or activities were located between arithmetic and algebra, and formed the basis for algebra that is, it provided opportunities for students to understand and make sense of algebra. However, it was possible with robust quantitative reasoning that these early algebra problems can establish the connection between arithmetic and algebra. In this sense, when students could establish quantitative reasoning, they made a meaningful transition from arithmetic to algebra (Smith & Thompson, 2007; Dougherty, 2017).

Studies showed that quantitative reasoning acted as a bridge in the transition from arithmetic reasoning to algebraic reasoning. Quantitative reasoning played an important role in making this transition smooth and easy (e.g., Smith & Thomson, 2007; Ellis, 2007), and it enabled students to solve algebraic verbal problems (Smith & Thompson, 2007). Even, quantitative reasoning affected the problem-solving duration of undergraduate students positively. For example, Moore (2010) examined the effect of university students' quantitative reasoning on their problem-solving skills. Three university students participated in the study. The students were taught eight times in five weeks, and then one-on-one interviews were conducted. The result indicated that two of the students made sense of the relationship between different quantities, and so they were successful at solving the problems. However, the remaining student focused on the operations and the result instead of the quantitative relationship. This student could not be successful at solving the problems. This situation indicated that quantitative reasoning affected the problem-solving duration positively. Besides, student thoughts and solutions in this study indicated that while solving problems, students could remember constructs based on quantitative relations more easily, instead of memorized rules.

Similarly, Moore and Carlson (2012) aimed to examine the approaches of university students who take a general mathematics course in the process of solving daily life problems. The students, who were able to make sense of the relationships between the quantities given in the problems, were successful in the problem-solving process by reaching the correct results. Like the result of the Moore's study (2010), it was seen that quantitative reasoning positively affected the students' problem-solving process. Likewise, another study examined whether quantitative reasoning was effective in the problem-solving process of university students. The findings of the study found that students' understanding of quantitative relationships positively affected the problem-solving process (Moore et al., 2009).

Quantitative reasoning was very important for students as explained in many of the studies mentioned above. Therefore, many countries emphasized the development of students' quantitative reasoning. For instance, it was given great importance in China and Singapore's secondary school mathematics curricula, which were among the countries with superior performance in PISA, the Program for International Student Assessment (Cai et al., 2011). For this reason, different teaching approaches were considered to make quantitative relations meaningful in the transition from arithmetic to algebra in these countries (Cai et al., 2011). For instance, the bar model used in the Singapore education program, which has been known for its success in TIMSS exams, was quite effective in teaching algebra (Clarke, 2017). In the bar model, quantities given in the algebra problems were expressed with rectangular strips or bars instead of real objects, and the relationships between quantities were shown by the relationships between the lengths of the strips or bars (Kaur, 2019). That is, quantities and the relationship between quantities given in the algebra problems were concretized, which led to contributing to students' conceptualizing of the problems (Gedikli & Sevinç, 2020). Studies show that this method developed students' quantitative reasoning and, as a result, increased algebra performance (Cai et al., 2011).

Based on these studies, Gedikli and Sevinç (2020) investigated the solution methods, preferences, and justifications of 7th grade students who have learned to solve algebra problems with both equation and bar models. Results indicated that students preferred the bar model method first when solving problems. Most of the students reached the correct solution by using the bar model and then established the correct algebraic equation with the help of the bar model. Students also indicated that the bar model was more understandable, provided a more enjoyable process, and wanted to use the bar model in solving other problems. Thus, this method was both helpful in understanding the algebra problems and affected students' motivations positively. As a result, these studies showed that quantitative reasoning made it easier for students to understand algebra. In addition, several studies have shown that quantitative reasoning played a critical role in the solution process of complex and difficult problems (e.g., Ellis, 2007; Kabael & Akın, 2016; Kabael & Kızıltoprak, 2014; Moore et al., 2009; Thompson, 1993; Thompson, 1988). Therefore, there were many studies examining students' quantitative reasoning, and these studies are explained below.

2.2 Students' Quantitative Reasoning Experiences

As mentioned above, quantitative reasoning was very important in both arithmetic and algebraic reasoning. The fact that students can distinguish the quantities given in the problems and understand the relationship between quantities enabled them to solve the problem faster and easier, and use algebraic symbols meaningfully. Therefore, many researchers conducted studies investigating the students' quantitative reasoning experiences. While some of them investigated students' quantitative reasoning in additive problems, some of them investigated it in multiplicative problems, and some of them investigated quantitative reasoning in both additive and multiplicative problems.

2.2.1 Studies on Student's Additive Reasoning

Ramful and Ho (2014) investigated the 6th grade student's quantitative reasoning in solving problems involving the additive relation between quantities. According to the result, a student could easily solve the problem by focusing on the relationship between quantities in the easy tasks. However, in other tasks, including more complex problems, a student reasoned numerically (i.e., guess and checks strategy), focusing on numbers instead of quantitative reasoning. Similarly, Alsawaie (2008) found that 5th grade students who participated in the study could not determine the relationship between quantities given in the problem. Since they did not understand the relationships between quantities, they tried random numbers and hence could not reach the correct result. On the other hand, some of the 5th graders could find the correct result quantitatively. Even if these students used the number for quantities while solving the problems, the numbers that they used were not random numbers; that is, the numbers were given by considering the relations between the quantities. Thus, in both studies, researchers concluded that students tended to try numbers that were returned to numerical reasoning when solving problems. They suggested that the importance given to quantitative reasoning needed to be increased in education.

Furthermore, Thompson (1993) investigated six 5th grade students' quantitative reasoning in additive word problems. The teaching experiment was conducted for four days, and then interviews were conducted with these students. In this process, various complex problems were solved, and researchers analyzed students' thinking processes and the difficulties they experienced. Results indicated that students generally used appropriate calculations but they did not reach the correct answer because they did not know how to use numbers. They did not distinguish between "quantitative difference" (p.166) (a quantity resulted from the additive comparison of the two quantities) and "numerical difference" (p.166) (subtraction of numbers) because they thought that these were similar things. Therefore, it was concluded that the fact that students did not encounter such problems was one of the factors affecting students' quantitative reasoning negatively, and it was observed that as the difficulty levels of the problems increased, the thinking levels of the students could be improved.

These abovementioned studies brought to mind the question of how quantitative reasoning can be used in problem-solving. In this scope, Ramful and Ho (2015) investigated how the use of quantitative reasoning in problems that included additive relations between quantities and provided diagram models for representing the quantities. Their aim was to enable students to establish quantitative reasoning while solving problems. That is, their focus was on making it easier for students to learn quantitative reasoning. For this, they used many different additive problems. It was emphasized that using a diagram model while solving these problems would help students to improve their quantitative reasoning because the diagram made it easier to see the relationship between quantities. As a result, researchers explained that quantitative reasoning was necessary for understanding the algebraic reasoning in the problems. Teachers needed to use the model method to improve quantitative reasoning before introducing algebra. In order for students to develop their quantitative reasoning, questions about quantities and the relationships between them needed to be asked, and they needed to be allowed to talk about their reasoning.

2.2.2 Studies on Student's Multiplicative Reasoning

Multiplicative reasoning was important for understanding various mathematical topics such as algebra, rational numbers, and so on (e.g., Hackenberg & Tillema, 2009; Kosko, 2019; Norton et al., 2015). Hence**,** there were some studies focused on students' quantitative reasoning in problems, including the multiplicative relationship between quantities. For instance, Alexander et al. (2020) investigated 3^{th} to 5^{th} grade students' quantitative reasoning. Two tests, including visual objects, were given to the students. In the first test given, students needed to understand the relationship between different objects (quantity) and transfer this relationship to the second situation. For example, in the given situation, six cubes turn into three rectangular prisms. It was asked how many rectangular prisms equal two cubes in the second situation. The results showed that less than half of the students were able to answer all problems completely correctly, which indicated that students had difficulty interpreting multiplicative relationships and did not transfer the relationship to the second situation.

McMullen et al. (2013) also investigated the students at an early age quantitative reasoning but the relationship between the quantities was multiplicative. The study consisting of two sessions was conducted with 86 students aged four to eight years at the first grade level and kindergarten, and, in each session, students completed a task related to quantitative relations. The results showed that first-grade students focused more on quantitative relationships than kindergarten students. Hence, it was found that as the age level increased, the students reasoned about the relationship between quantities better.

As mentioned above, there was a relationship between multiplicative reasoning and other mathematical concepts. Thus, there were some studies focused on the relationship between them. For instance, Zwanch (2019) investigated the relationship between number sequences and multiplicative relationships. The study consisted of two stages, and middle grade students participated in this study. In the first stage, a survey was applied about the number sequences. In the second stage, written data were collected by interviewing the students who were selected according to the survey results. Results indicated that students who could clearly construct number sequences were able to establish the multiplicative relationship between quantities. Accordingly, they were able to express their relationship algebraically (e.g., $y = 5x$).

Besides, Ellis (2007) investigated the students' reasoning of the relationships between the quantities and the effect of this reasoning on generalization. The study consisted of two phases. In the first phase, various activities were carried out with the 34 students during 12 lesson hours, and seven of them were interviewed. In the second phase, training was carried out. Students who at both the interviewed and the training were more successful in reasoning about the multiplicative quantities. The results examined quantitative reasoning from two aspects which were “direct-measures reasoning” (p.466) and “emergent-ratio reasoning” (p.466). While the meaning of direct-measures reasoning was to be able to make sense of the relationship between two quantities, emergent-ratio reasoning was to create a new quantity by proportioning these two quantities. The results indicated that emergent-ratio reasoning was more important for generalizations. Also, in this study, it was seen that students focused not only on quantities and quantitative relations but also on numbers and operations. Thus, researchers suggested that teachers should give importance to learning of quantitative reasoning and organize lessons in which students would focus on quantitative reasoning.

Since multiplicative reasoning was important in the development of many mathematical subjects, it was focused on how multiplicative reasoning was developed. For this purpose, Bakker et al. (2014) investigated whether mini-games about multiplication were effective in the development of multiplicative reasoning of second and third grade level students. Researchers examined this study under three main categories which were playing games integrated into the lesson, only when played at home (without school intervention), and playing games at home with school intervention. The results indicated that playing games at home with school intervention affected the development of the multiplicative reasoning of students positively. At the same time, it was seen that the games integrated into the lessons at school had little effect, while the games played at home without school intervention had no effect on the development of students' multiplicative reasoning. In the next section, some studies related to additive and multiplicative reasoning were examined together are provided.

2.2.3 Studies on Relationship Between Additive and Multiplicative Reasoning

As mentioned above, quantitative reasoning was divided into two as an additive and multiplicative reasoning (see Figure 2.1), so there were some studies that examined additive or multiplicative reasoning separately. However, in some studies, quantitative reasoning has been examined in both two dimensions. Researchers asked about problems involving both additive reasoning and multiplicative reasoning. For example, Van den Heuvel-Panhuizen and Elia (2020) investigated kindergarten students' quantitative reasoning in both additive and multiplicative problems. A test containing problems related to both of them was administered to students in Netherlands and Cyprus. According to the result, students often had the most difficulty with multiplicative reasoning as in the study of Alexander et al. (2020). Also, supporting children in multiplicative reasoning would enable them to become better at additive reasoning, which indicated that multiplicative and additive reasoning were connected to each other.

Students who constructed the relationship between multiplicative and additive reasoning could easily establish these relationships in algebra (Britt & Irwin, 2011). However, students had difficulties distinguishing between additive and multiplicative or proportional reasoning. While they used multiplicative reasoning to solve problems including additive relations, they were able to solve problems with multiplicative relations in an additive reasoning, which led the students to the wrong results (De Bock, 2008; Tunç, 2020). For example, the problem given in De bock (2008)'s study *was "Given a picture to show Mr. Short's height is 6 paperclips. When we measure Mr. Short and Mr. Tall with matchsticks: Mr. Short's height is 4 matchsticks and Mr. Tall's height is 6 matchsticks. How many paperclips are needed for Mr. Tall's height?"* (p.125). The relationship between these quantities was actually multiplicative. However, since the difference between four matchsticks and six paper clips was two, the students added two to six matchsticks and reached the wrong result of six. On the contrary, the problem which was *"Sue and Julie were running equally fast around a track. Sue started first. When she had run 9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many laps had Sue run?"* (p.126) was given in the same study. In this problem, the relationship between quantities was additive but students thought that this relationship was multiplicative. They multiplied 15 by three since Julie ran three times as much as before, so reached the wrong answer of 45 laps.

It had been observed in some studies that students preferred additive reasoning instead of solving multiplicatively (Tourniaire & Pulos, 1985). On the other hand, some studies showed that students generally preferred to establish multiplicative reasoning, even if there was a problem that needed to be solved in additive reasoning (e.g., Fernandez et al., 2008; Modestou & Gagatsis, 2007; Van Dooren et al., 2005). From this point of view, it was thought that additive and multiplicative reasoning needed to be compared and investigated simultaneously not separately (Dooren et al., 2010).

In this context, Dooren et al. (2010) examined which type of reasoning, multiplicative or additive reasoning, students used in the given problems, and how this situation differed according to the grade level (i.e., 3rd, 4th, 5th, and 6th grade level). Verbal problems including additive and multiplicative relations were asked to the students. While the ratio between the different quantities was an integer in some problems, it was fractional in other problems. In this way, it was aimed to measure students' quantitative reasoning in all problem forms. Results demonstrated that as the grade level of the students increased, students generally solved problems multiplicatively; that is, they tended to think additively less frequently. In addition, if the ratio between the quantities was an integer, the students solved the problems with multiplicative reasoning, and if this ratio was a fraction, the solution with additive reasoning was preferred. In other words, the first point that the students paid attention to while solving the problems was the numerical values. Finally, there were students who used the wrong approach to the problem at all grade levels. In other words, there were students who used additive strategies in the problems that needed to be solved by establishing a multiplicative relationship and used multiplicative reasoning in the problems that needed to be solved by establishing an additive relationship. It was determined that this tendency was not related to age but rather related to the numbers given in the problem. In this case, it showed that the student's quantitative reasoning was not developed because they could not correctly decide whether the relationship between quantities was multiplicative or additive in the given problem.

Similarly, Fernández et al. (2012) explored students’ additive and multiplicative reasoning in middle and high school grade levels, and how integer or fractional relations between quantities affected students' additive and multiplicative reasoning. For this purpose, the test containing various additional and multiplicative word problems with integer and fractional ratios was prepared and applied to the students. According to the results, while middle school students generally constructed additive reasoning regardless of whether the relationship between quantities was additive or multiplicative, high school students established multiplicative reasoning regardless of whether the relationship between quantities was additive or multiplicative. Moreover, if the ratio between the number of quantities was integer, students used multiplicative ways. However, if the ratio between the number of quantities was a fraction, they used additive ways mostly. These results completely coincided with the conclusions of the study by Dooren et al. (2010).

In another study, Nunes et al. (2015) investigated young children’s quantitative reasoning in both forms. First grade level students’ quantitative reasoning was measured at first. A few months later, these children were given a mathematical reasoning test. Contrary to the result of the study of Dooren et al. (2010), researchers found that the youngest children were able to see the relationship between quantities successfully. Also, there was a strong relationship between young students’ quantitative reasoning and mathematical reasoning. Researchers stated that students' quantitative reasoning was very important for other mathematics subjects and suggested an advantage for their entire school life.

Similarly, Chen (2009) aimed to examine the quantitative reasoning of 2nd grade level students who were in two different countries, Taiwan and Hawaii. The study consisted of two stages. While concrete objects were used in the first stage, semi-tangible objects were used in the second stage. The result indicated that students from both countries were more successful in problems with concrete objects, and no difference was found between the quantitative reasoning of students in the two different countries. On the other hand, this success was significantly reduced in problems with semi-tangible objects, and a difference was seen in the quantitative reasoning of students in two different countries. Hence, the researcher stated that giving concrete objects to children and comparisons between them would contribute to the development of students' quantitative reasoning.

Besides, Degrande et al. (2017) investigated second, fourth, and sixth grade students' quantitative reasoning in both multiplicative and additive relations and how quantitative reasoning differed across grade levels. Visual objects were used in the given tasks, and students were asked to explore the relationship between these objects. Similar to Nunes et al.'s study (2015), the results of this study indicated that most of the students realized the quantitative relationship between the objects in the given tasks. Considering the grade levels, while students' quantitative reasoning in the multiplicative relations improved, no difference was observed in the additive relations as the grade level increased. When the grade levels were examined in more detail, it was found that the second-grade students were better in additive relations than multiplicative relations. In line with the results of Dooren et al.'s study (2010), students think more additively in their first year, and even if the relationship between the quantities given in the problem was multiplicative, students were more inclined to think additively.

While the above studies investigated the quantitative reasoning of primary or secondary school students, there were also some studies investigating the quantitative reasoning of high school students. For instance, Koedinger and Nathan (2004) explored how different forms of problems affected the performance of high school students who solve problems with quantitative reasoning. The study was conducted by two different groups who were high school students. The first group consists of students who had taken algebra courses before, while the second group was currently taking algebra courses. A test consisting of problems involving story, word, and symbolic equations was applied to these students. At the same time, problems of different difficulties have been prepared. Symbolic problems were found to be more difficult for students in both groups, which indicated that students had difficulties in quantitative reasoning in such problems. Therefore, the researchers concluded that forms of the problems changed the students' performance, that is, their quantitative reasoning differed according to forms of the problem. There were also some studies related to quantitative reasoning in the Turkish literature. Hence, studies related to quantitative reasoning in Turkiye are explained in the next section.

2.3 Research on Quantitative Reasoning in Turkiye

In many studies mentioned, developing quantitative reasoning was an important stage for Turkish students’ mathematics learning. For example, Kabael and Akın (2016) explored 7th grade students’ quantitative reasoning when solving algebraic verbal problems. In the given word problems, the nine students who participated in the study were expected to find the relationship between the number of coins and their value. The results showed that students generally used arithmetical ways and had difficulty with quantitative reasoning. Since they could not establish the relationship between the number of coins and their values to solve the problem, they used trial and error methods; that is, they gave random numbers parallel with the literature (Akkan et al., 2012). Thus, results indicated that quantitative reasoning was important for both arithmetic and algebraic strategies. Even if the students who could establish the relationship between quantities used arithmetic methods, they solved the problems by consciously assigning numbers. At the same time, if students reasoned quantitatively, they used algebraic strategies in a meaningful way. Therefore, these students both solved the problem faster and understood it better.

In addition, Güvendiren (2019) examined the quantitative reasoning of 6th grade students and whether the quantitative reasoning was related to covariational and functional thinking. In line with this purpose, a test with open-ended questions was administered to nine students, and then an interview was conducted with these students. According to the results of the study, it was seen that most of the participants did not have strong quantitative reasoning. In detail, students could establish relations between quantities whose numerical values were given directly. As the difficulty level of the problems increased (such as the existence of decimal numbers, and the number or value of coins), most of the students had difficulty establishing relations between quantities. Finally, it was found that quantitative reasoning affected functional, covariational, and algebraic thinking positively. Students with a high level of quantitative reasoning were able to reach high levels in other reasoning as well.

Thus, some researchers turned their attention to investigating how to develop quantitative reasoning since students have difficulties understanding the relationship between the quantities given in the problem, and quantitative reasoning had an important place in mathematics. For example, Akın (2017) investigated how instruction based on quantitative reasoning affected the quantitative reasoning and mathematical literacy performance of 8th grade students. A test consisting of open-ended questions was administered to the students before the instruction. According to the answers given, the students were divided into three categories as students with weak, moderate, and strong quantitative reasoning. Depending on these categories, participants were determined to teach in each school. The results of the study showed that the teaching approach based on quantitative reasoning improved the quantitative reasoning of all students in each category. This indirectly affected the students' mathematical literacy positively.

Similarly, Dur (2014) investigated 6th grade students' quantitative reasoning in the problem-solving process. In this study, an instruction was applied to the students and measured the students' quantitative reasoning in the problem-solving process before and after the instruction. According to the study results, before teaching the students, they generally tried to solve the problem without considering the quantitative relations. As a result, they had some difficulties when solving problems such as understanding the problems, using only one way, or using just arithmetical ways. After the instruction, students used tables, diagrams, or figures by focusing on the relations between the quantities given in the problem, so they were more successful, which indicated that students’ quantitative reasoning improved during teaching. In addition, researchers emphasize that the development of quantitative reasoning increased the improvement of algebra and their success in mathematics.

Moreover, another study examined whether mathematics history activities were effective in the development of quantitative reasoning in 7^{th} grade students. In the experimental study, one of the 7^{th} grade sections was applied to the history of mathematics activities, while the other was not applied. Before the instruction, students were given a pre-test that measured their quantitative reasoning. As a result of the pre-test, it was determined that the quantitative thinking levels of the students in both groups were close to each other. According to the results of the test applied after the teaching, it was seen that the scores of the class in which the mathematics history activities were applied increased compared to the first test. This showed that the use of mathematics history activities in lectures was effective in the development of students' quantitative reasoning (Danacı & Şahin, 2021).

2.4 Summary of the Literature Review

There were many parallel definitions of quantitative reasoning in the literature. In short, quantitative reasoning can be defined as understanding and interpreting the quantities and the relationships between quantities (e.g., Johnson, 2012; Nunes et al., 2015; Ramful & Ho, 2015; Smith & Thompson, 2007; Thompson, 1993; Van den Heuvel-Panhuizen & Elia, 2020). The relationship between the quantities given in the problems was either additive or multiplicative. Therefore, in order for students to think quantitatively, they needed to compare quantities additively or multiplicatively (e.g., Lobato & Siebert, 2002; Thompson, 1988). In this context, Van den Heuvel-Panhuizen and Elia (2020) examined quantitative reasoning under two categories as additive and multiplicative. If the relationship between quantities was additive, quantitative reasoning was called additive reasoning. If there was a multiplicative relationship between the quantities, quantitative reasoning was called multiplicative reasoning.

In order to develop quantitative reasoning, there were some things students need to understand. Students first needed to understand the concept of quantity. Second, students needed to realize that quantity and quantitative reasoning were not the same thing as numbers and numerical reasoning, respectively. Third, they needed to understand the concept of quantitative operation. Finally, they needed to make sense of the concepts of quantitative difference and quantitative ratio (e.g., Smith & Thompson, 2007; Thompson, 1993; Troy, 1993).

Many researchers stated that problems involving unknown quantities can be solved in many different ways. The first thing that comes to mind was algebraic ways, that is, solving problems by establishing equations. However, problems can be solved without using equations. For example, in the studies of Kindt et al. (2006) and Meyer (2001), many different problems were given, and it was shown that these problems could be solved by establishing the relationship between quantities. In another study, comparing quantities was found as a way to solve equations at the 6th grade level (Van Reeuwijk, 2001).

Quantitative reasoning acted as a bridge between arithmetic and algebra. In this respect, it related to both arithmetic reasoning and algebraic reasoning, and this relationship was seen in many studies (e.g., Kindt et al., 2006; Meyer, 2001; Ramful & Ho, 2015; Smith & Thompson, 2007; Thompson, 1993). Therefore, quantitative reasoning was of critical importance for a meaningful transition from arithmetic to algebra and for making sense of algebraic thinking (e.g., Ellis, 2007; Kabael & Akın, 2016; Smith & Thompson, 2007).

Actually, in order to make sense of algebra, it was necessary to understand the relationships between quantities. Otherwise, algebra became a subject including meaningless symbols for students. Quantitative reasoning also helped students to solve algebraic problems and more complex problems easily (e.g., Smith & Thomson, 2007; Ellis, 2007). It was even seen that constructing relationships between quantities well affected university students' problem-solving processes. That is, if students established the relationship between different quantities, they were successful in problem-solving (e.g., Moore et al., 2009; Moore, 2010; Moore & Carlson, 2012).

Since quantitative reasoning was critical and important for students, there were many studies examining students’ quantitative reasoning. While some studies examined students’ quantitative reasoning in additive relations, some examined it in multiplicative relations. In studies examining students' quantitative reasoning in problems involving additive relationships, it was found that some students could establish this reasoning while others did not. Students who could not construct quantitative reasoning focused on numbers and numerical operations Thus, they preferred arithmetic methods to solve problems (e.g., Alsawaie, 2008; Ramful & Ho, 2014; Thompson, 1993).

Some studies examined students' quantitative reasoning in multiplicative situations. For example, Alexander et al. (2020) found that students had difficulties understanding multiplicative relationships while McMullen et al. (2013) found that as the age of the students increased, they were better able to establish the relationships between the quantities. Some studies have explored whether there was a relationship between multiplicative reasoning and other topics. In this context, Zwanch (2019) found that it related to the number sequences and Ellis (2007) found that it related to the generalization.

Studies indicated that students had difficulties in distinguishing multiplicative and additive relations. It has been observed that while students used multiplicative reasoning in problems involving additive relations, they used additive reasoning in problems that require multiplicative relations which led students to reach the wrong answer (e.g., Fern´andez et al., 2008; De Bock, 2008; Modestou & Gagatsis, 2007; Tourniaire & Pulos, 1985; Tunç, 2020; Van Dooren et al., 2005). Hence, there were studies examining quantitative reasoning in both additive and multiplicative relations at the same time. Studies have shown that while students used additive reasoning more at the primary school level, they tended to use multiplicative reasoning as their age increased. At the same time, it was seen that they focused on multiplicative reasoning if the relationships between quantities were integers, and additive reasoning if the relationship between quantities were fractional (e.g., Dooren et al., 2010; Fernández et al., 2012).

Also, there were conducted studies related to quantitative reasoning in my country, Turkiye. For instance, Kabael and Akın (2016) found that students used arithmetical ways because they had difficulty constructing a quantitative relationship. Similarly, Güvendiren (2019) indicated that most of the students who participated in the study did not have strong quantitative reasoning, and as the difficulty level of the problems increased, it was seen that the students had more difficulty understanding the relationships. Moreover, instructions to develop quantitative reasoning were also tried since quantitative reasoning was important for students. It has been found that these trainings were also effective in improving quantitative reasoning (e.g., Akın, 2017; Danacı & Şahin, 2021; Dur, 2014).

Considering the importance suggested by the literature summarized above, in this current study, I focused on middle school students' quantitative reasoning in early algebra problems given in pictorial, symbolic, and iconographic forms. Also, I examined how middle school students' quantitative reasoning differed by forms of the problem and grade level (5-8th grade).

CHAPTER 3

METHODOLOGY

The aim of this study is to explore middle grade students' quantitative reasoning in pictorial, symbolic, and iconographic problems. In this chapter, the design of the study, participants, data collection tools, data collection procedure, data analysis, the researcher's role, procedure trustworthiness of the study, and limitations of the study are provided below.

3.1 Design of the Study

The study was designed as a case study which is one of the quantitative research methods. The case study provided an in-depth examination of one or more cases for researchers (Creswell, 2002; Fraenkel et al., 2012). A case can be many different things, such as a single person, a group of individuals, a class, an event, and so on (Fraenkel et al., 2012).

This study aimed to examine the middle school students' quantitative reasoning in pictorial, symbolic, and iconographic problems in depth. Hence, a case study was found to be suitable for the study. Case studies are divided into four types which are single-case holistic design, single-case embedded design, multiple-case holistic design, and multiple-case embedded design (Yin, 2003). If the design of the study is a single-case design, the study examines one case. On the other hand, if the study examines multiple cases, the design is a multiple-case design. In addition, the embedded design includes multiple units of analysis (see Figure 3.1).

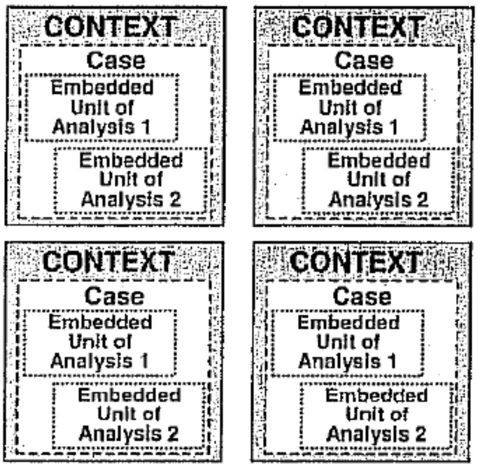


Figure 3. 1 Multiple-case embedded research design (Yin, 2003, p.40)

The present study was a multiple case study because there are four cases determined by the grade level, the quantitative reasoning of which were examined separately and then compared with one another. Moreover, the embedded design was suitable since the problems in the different forms constituted different units of analysis. Three different problem forms which are pictographic, symbolic, and iconographic were used in the study, and analyzed in two parts (Part 1: pictographic unit value problem, pictographic multiple-unit problem and pictographic substitution problem; Part 2: iconographic substitution problem, symbolic substitution problem, iconographic multiple-unit substitution problem and symbolic multiple-unit substitution problem). Therefore, this study involved two unit of analysis associated with these two parts of the problems. As a result, the design of this study was a multiple-case embedded design to examine students' explanations related to quantitative reasoning in depth.

3.2 Participants

This study was conducted with middle school students (i.e., 5th, 6th, 7th, and 8th grade level) in two public schools in Şarkışla, Sivas. The school 1 is a village school. There were four classes in total, one class for each grade level, and class sizes ranged from seven to fifteen students in the 2021-2022 academic year. The school 2 is located in the district center, but students come from the villages by bus. There were eight classes in total, two classes for each grade level, and class sizes ranged from 18 to 25 in the 2021-2022 academic year. In schools, there were some students at low, moderate, and high achievement levels. The researcher has been the mathematics teacher at the village school since the beginning of the 2020-2021 academic year.

The study was conducted with 60 middle school students. In each grade level, fifteen students participated the study. 26 of the students who participated in the study were boys, and 34 of them were girls. Also, 31 of the students were at the School 1 (i.e., village school), and 29 of them were at the School 2. Detailed demographic information in each grade level is given in Table 3.1.

Table 3. 1 Demographic Information of Students

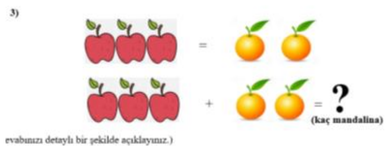
	The number of students at the first school		**The number of students at the second school**	
Grade Level	The number of boys	The number of girls	The number of boys	The number of girls
5th Grade	5	3	3	4
6th Grade	5	4	3	3
7th Grade	4	5	1	5
8th Grade	3	2	2	8

All students participated in the study on a voluntary basis, and participants in the study were selected according to the purposeful and convenience sampling method. In the convenience sampling method, due to some limitations, the participants are selected from among the people that the researcher can easily reach (Fraenkel et al., 2012). Hence, the convenience sampling method was used, and the students who participated in this study were selected from easily reached group of students.

3.3 Data Collection Tools

In order to understand the quantitative reasoning of the students, seven problems in three forms were prepared. While preparing the problems, it was benefited from the problems in Meyer's study (2001). However, the mathematical description of the problem was determined by the researcher regarding the purpose of the study, not citing the particular literature. The content and order of the problems were reviewed by the thesis supervisor. Then, the researchers arranged the content and order of the problems by focusing supervisor’s feedback. The data collection tool was divided into two parts involving seven problems presented in three different forms (i.e., pictographic, symbolic, and iconographic). The problems in Part 1 are below (see Table 3.2).

Table 3. 2 Data Collection Problems in the Part 1

<table>
<tr><th>Mathematical Description of the Problem</th><th>Problems</th><th>Purpose of asking</th></tr>
<tr><td>Pictographic Unit Value Problem</td><td></td><td>The purpose of this problem is to understand whether the student can see the relationship between the numbers of different objects given in the scale; in other words, see how many objects (lemon) are equal to one other object (pear).</td></tr>
<tr><td>Pictographic Multiple-Unit Problem</td><td>
(Adapted from Meyer (2001, p. 240))</td><td>The aim is to understand two things. Our first goal is to understand whether the student understands the relationship between the number of objects in the 1st scale model. Our second goal is to see whether students can transfer this relationship to the 2nd scale model or use it on the 2nd scale.</td></tr>
<tr><td>Pictographic Substitution Problem</td><td></td><td>The first aim of this problem is to understand whether the student can see the fractional relationship between the numbers of different objects (3 to 2). The second goal is to see if the students can transfer this relationship to an additive process.</td></tr>
</table>

As seen in Table 3.2, there were given three problems in the pictographic form in the first part of the data collection tool. The word pictograph is defined as *"one of the symbols belonging to a pictorial, graphic system"* (Merriam-Webster, n.d.). These three problems were called pictographic since various fruits used in the given problems were symbolized by drawing. The complexity of the relationship in the problems (i.e., direct substitution of the unit value vs multiple-unit substitution) is also increased from first to third problem. After the problems were asked in pictographic form, the students answered the problems in Part 2 of the data collection tool (see Table 3.3).

Table 3. 3 Data Collection Problems in the Part 2

Mathematical Description of the Problem	**Problems**	**Purpose of asking**
Iconographic Substitution Problem	4) □□ = △ □□+△△ = ? (kaç tane üçgen)	The aim of this problem is to understand whether the student can see the relationship between the numbers of different objects and can transfer this relationship to an additive process. However, in this problem, the objects are more formal objects, not the pictures of real objects.
Symbolic Substitution Problem	**5)** 2 U = 1 C 2 U + 2 C = ? (kaç tane C) (Adapted from Meyer (2001, p. 242))	The first goal is to understand whether the student can see the relationship between the numbers of different objects. The second goal is to see if the students transfer this relationship to an additive process. However, different from the previous problem, this problem involves symbols representing the quantities.

Table 3.3 (continued)

Iconographic Multiple-Unit Substitution Problem	6) □□ = △ □□+△△ = ? (kaç tane kare)	The aim of this problem is to understand two things. The first aim is to understand whether the student can see the relationship between the numbers of different objects. However, in this problem, the objects are more formal objects, and students are required to find the multiple of the given unit (triangle). The second aim is to see if the students substitute multiples of the units to an additive process.
Symbolic Multiple-Unit Substitution Problem	7) 2 U = 1 C 2 U + 2 C = ? (kaç tane U) (Adapted from Meyer (2001, p. 242))	The aim of this problem is to understand whether students can see the relationship between the numbers of different letters/algebraic symbols and can transfer this relationship to an additive process. However, different from the previous problem, this problem involves symbols representing the quantities, and the multiple of the given unit must be found, that is, how many U is equal to 2C

As seen in Table 3.3., there were given four problems in the iconographic and symbolic forms. The fourth and sixth problems are iconographic, which means *"representing something by pictures or diagrams"* (Merriam-Webster, n.d.). These problems were called iconographic because triangles and squares were drawn as diagrams. Also, the fifth and seventh problems are in the symbolic form because letters were used in the problems. The complexity of the relationship in the problems (i.e., direct substitution of the unit value vs multiple-unit substitution) is increased from fourth to seventh problem. Briefly, there were three different forms of problems which were pictographic, iconographic, and symbolic forms in the data collection tools. The same quantities were asked consecutively in symbolic and iconographic forms so seven problems in three different forms were divided into two parts:

- Part 1: Pictographic unit value problem, pictographic multiple-unit problem, and pictographic substitution problem
- Part 2: Iconographic substitution problem, symbolic substitution problem, iconographic multiple-unit substitution problem, and symbolic multiple- unit substitution problem

It is also important to note that the mathematical descriptions of the problems given in the first column of the Tables 3.2 and 3.3 were determined by the researcher in the light of the literature. In this way, it was aimed to examine the middle school students' quantitative reasoning in different forms of the problem.

3.4 Data Collection Procedure

Necessary permissions were obtained before conducting the study. Firstly, ethical committee permission was obtained from Middle East Technical University Human Subjects Ethics Committee (see Appendix A). Secondly, permission was obtained from the Ministry of National Education (see Appendix B) because the data were gathered from students at the middle schools. Permission was also obtained from the parents by sending a form (see Appendix C) since the students were under the age of 18. After the necessary permissions were obtained, first of all, the pilot study was carried out.

The pilot study was carried out before the main study in order to clearly decide on the correctness of the content and order of the problems, the time to be given, and the problems to be added or subtracted (Yin, 2011). Hence, in the present study, the pilot study was conducted in August 2021 with 11 students at the 5th, 6th, 7th, and 8th

grade levels. These students were selected by convenience sampling method, and all of them participated in the study voluntarily. Due to the pandemic conditions, one-on-one interviews were conducted using Zoom with one person from each grade level. Problems were sent to the remaining students via e-mail, and they were asked to explain the answers in detail and send them to the researcher. After conducting the pilot study, necessary modifications were made, and the data collection tool was finalized.

The data of the main study were collected in October at the beginning of the 2021-2022 academic year. Problems were given to the students during the lesson, and they were asked to answer the problems. During the answering problems, the researcher asked students to clearly write down how they found the result, what they thought, or their reasons. The study was completed face-to-face in approximately one class hour in each class, and written work was collected by the researcher. The same procedure was carried out at the School 2.

3.5 Data Analysis

In the present study, it was used content analysis method which is a qualitative data analysis method. Content analysis is the in-depth examination of the data obtained by the study. These obtained data are interpreted and explained by combining the data around certain concepts and themes via content analysis (Yıldırım & Şimşek, 2008). In addition, the obtained data is analyzed by coding in a qualitative research, and the way of coding was a data-driven coding frame. Coding is a method that makes data more meaningful (Saldana, 2011).

As mentioned above, seven problems asked the students were divided into two parts (see Table 3.2 and Table 3.3). Data analysis was made separately aligning with two parts. That is, first of all, pictographic problems were analyzed. Then, problems in symbolic and iconographic forms were analyzed together since the same problem were asked successively in these forms. In both parts, the correct and incorrect answers to the problems were determined, so two main categories were obtained as correct and incorrect answers. Based on these categories, a comparison across grade levels was made in both parts. After that, students' item base performances in each grade level were analyzed in detail for both parts. The codes and some sample statements were below (see Table 3.4).

Table 3. 4 The Description of Codes and Sample Student Statements

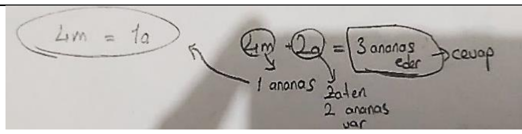
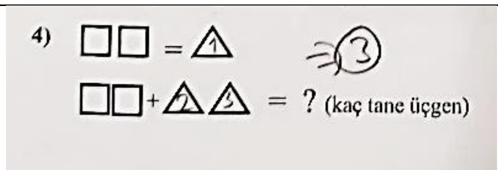
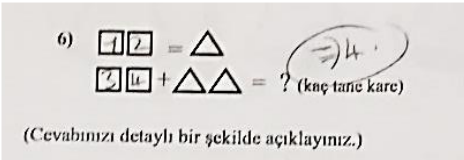
Category	**The description of the code**	**Sample Student Statement**
Correct Solutions	Identifying the relationship with addressing the weight	2 lemons are needed because, if the weight of the 2 lemons is equal to the weight of the 4 pears, the weight of the 1 pear is equal to the weight of the 2 lemons.
	Identifying the relationship without addressing the weight	If 2 pears are equal to 4 lemons, 1 pear is equal to 2 lemons. Answer 2.
	Referring the algebra	
	Using substitution method	Since 2 squares are equal to 1 triangle, the problem asked us to add 2 squares and 2 triangles. Thus, 2 squares are equal to 1 triangle. There are also 2 triangles, when you add them, you get 3 triangles.
	Focusing the number of the asked quantity on the image	

Table 3.4 (continued)

Incorrect Solutions	Misunderstanding the equality	Since there is 1 pear, I collect 3 lemons and 1 pear, and I find this result because the sum is 4.
	Understanding equality but not transferring the new situation	2 squares are equal to 1 triangle. 4 squares are equal to 2 triangles.
	Creating the visual sameness	We need to put one of the pineapples on the other pan. We should put 1 pineapple next to the bananas on the 1st scale to be equal**.**
	Focusing the numbers and operations	Because the sum of 3 apples and 2 oranges is equal to 5. If we count 2 more oranges, it will be 5.
	Focusing the number of the asked quantity on the image and operations	
	Answering the problem regardless of what kind of problem asked	Problem 4 is the same.

In addition to the coding process described above, cross-comparison was made between problems in both Part 1 (pictographic problems) and Part 2 (symbolic & iconographic problems). For this process, the scoring method was developed by using a rubric. In Part 1, students who answered all three pictographic problems correctly scored three. Students who answered pictographic unit value and pictographic multiple-unit problems correctly but not pictographic substitution problem scored two. Students who answered only pictographic unit value problem correctly but not pictographic multiple-unit and pictographic substitution problems scored 1 (see Table 3.5).

Table 3. 5 Scoring Method for Part 1 – Pictographic Problems

Pictographic Unit value Problem	**Pictographic Multiple-Unit Problem**	**Pictographic Substitution Problem**	**Overall Score**
+	-	-	**1**
+	+	-	**2**
+	+	+	**3**

There were cases where students answered the pictographic multiple-unit problem but not other two problems. Those students were not given the score 1 because the aim of this rubric is to test whether the particular order of the problems (i.e., the order that is given in this study) would play a role in students' responses.

Similar procedure was applied in Part 2 (symbolic & iconographic problems). Students who answered all four problems correctly scored four. Students who answered iconographic substitution, symbolic substitution, and iconographic multiple-unit substitution problems correctly but not symbolic multiple-unit substitution problem scored three. Students who answered iconographic substitution and symbolic substitution problems correctly but not iconographic multiple-unit substitution and multiple-unit substitution problems scored two. Students who answered only the iconographic substitution problem correctly but not others scored one (see Table 3.6).

Table 3. 6 Scoring Method for Part 2 – Symbolic and Iconographic Problems

Iconographic Substitution Problem	Symbolic Substitution Problem	Iconographic Multiple-Unit Substitution Problem	Symbolic Multiple-Unit Substitution Problem	Overall Score
+	-	-	-	**1**
+	+	-	-	**2**
+	+	+	-	**3**
+	+	+	+	**4**

Same as mentioned before, there were some other cases in this part, such as correctly answering only one problem but not the iconographic substitution problem or correctly answering the last two problems only. Those students were not given the score 1 or 2 because the aim of this rubric is to test whether the particular order of the problems (i.e., the order that is given in this study) would play a role in students' responses.

As a result, the students' explanations were analyzed by using content analysis method. Also, in the data analysis, the real names of the students were not used. Students were given nicknames, and if necessary these nicknames were used.

3.6 The Researchers Role

Johnson (1997) expressed that researcher bias is one of the threats to affect validity in qualitative research, and the results of the study can be affected by the researcher's opinions. Creswell (2009) emphasized that the researcher's role was very important in qualitative studies and stated that in qualitative studies, the researcher is the person who collects the data and is connected with the participants. The researcher's relationship and experiences with the participants should be explained (Creswell, 2009). Therefore, my role as a researcher in the study was explained in detail below in order to reduce bias.

I, as a researcher, have been working as the only mathematics teacher at the School 1 (i.e., village school) since the beginning of the 2020-2021 academic year. Therefore, I am the math teacher of 31 students who participated in the study. However, I did not know 29 students studying at the School 2. Before the students started answering the problems, I explained to the students in both schools what the study was, the purpose of the study, how long it would take, and how I would use the data. It was also stated to the students that their names will be anonymous, their written work will not be shared by anyone, and their performance would certainly not affect their grades at school. In order to reduce the bias, while students were answering the problems, the researcher was careful not to answer their questions related to the problems, such as what the problem is or whether my answer is correct, and not to interfere with their solutions. Also, I certainly did not tell the students anything about how to solve the problems. I correctly explained the findings of the study in the next section.

3.7 Trustworthiness of the Study

For the quality of the study, it is necessary to be sure of its validity and reliability. In the qualitative study, it was identified that there are four types of reliability and validity by the researchers (Lincoln & Guba, 1985; Miles & Huberman, 2016; Yıldırım & Şimşek, 2018). Since the study was a qualitative study, these four types were briefly explained below.

Validity is divided into internal validity and external validity. In this context, credibility is used to ensure internal validity. It is used to understand whether this study is actually measuring what it is intended to measure (Shenton, 2004). In this study, two methods were used to ensure credibility. The first method was triangulation. In this sense, the way of multiple researchers was used for ensuring triangulation in both developing data collection tool and data analysis. In the process of developing the data collection tool, the researcher formed the data collection tool by benefiting from the literature at first. Then, it was examined by the thesis supervisor, and necessary arrangements were made by her. In this way, the data collection tool was triangulated by the researcher and the thesis supervisor. In the data analysis, the collected data was examined by both the researcher and the mathematics teacher, and a consensus was reached. The second method was prolonged engagement. The researcher has been working as a mathematics teacher in School 1 (i.e., village school) since the beginning of the 2020-2021 academic year. At the same time, my teacher friend has been working as a mathematics teacher at the School 2 since the beginning of the 2020-2021 academic year. This indicates that both the researcher and the other mathematics teacher have been at the schools for a long time. Thus, students who participated in the study were quite comfortable while answering the problems. The credibility of the present study is ensured by using these two methods.

Transferability is used to ensure external validity. For ensuring transferability, the thick description was used. It means knowing the details of the study so that other researchers can use the results of the study. The researcher explained the design of the study, characteristics of the participants, data collection tool, data collection process, and data analysis process above in order to transfer the result of the study by the other researchers. Also, the expressions and drawings written by the students directly were also used to ensure transferability.

Furthermore, all necessary information related to the design, context, participants, and so on was explained in detail to ensure dependability. In addition, the findings were analyzed many times repeatedly and some students' ideas and answers were used directly in some parts of the findings. For ensuring confirmability, the researcher's role was explained, and detailed information related to the methodology of this study was given in the above parts. Students' data were also analyzed by both the researcher and the mathematics teacher.

3.8 Limitations of the Study

The participants of the study involved only but some students in two schools in Sivas province, Şarkışla district, using a convenient sampling method. That is, a total of 60 students, 15 at each grade level, participated in the study, which is a limitations of the study. In addition, only written data was collected in this study. This is second limitation of current study because the students who participated the study were not asked in detail why they thought so. Nevertheless, this study was considered as an exploratory study which may inform other researchers, mathematics teacher educators, or mathematics teachers who are teaching to middle school students.

CHAPTER 4

FINDINGS

In this chapter, the findings were explained in two main parts. In the first part, pictographic problems are examined, and in the second part, symbolic and iconographic problems are examined. Each part is explained under three sub-headings in itself. These subheadings are students' performances in problems across grades levels, students' item base performances on problems, and students' performances on problems across items.

4.1 Relationship Between Quantities Represented in Pictographic Form

There were three problems in the pictographic form in this set, and each problem had a different purpose of being asked (see Figure 4.1). The purpose of the first problem was to measure whether students could see the relationship between the numbers of different objects given on the scale; in other words, see how many objects (lemon) are equal to one object (pear). The goal of the second problem was to understand whether students understood the relationship between the number of objects in the first scale model and see whether they could transfer this relationship to the second scale model or use it on the second scale. Finally, the third problem aimed to understand whether students could see the relationship between the numbers of different objects and transfer this relationship to an additive process.

Figure 4.1a. Pictographic Unit Value Problem

Figure 4.1b. Pictographic Multiple-Unit Problem

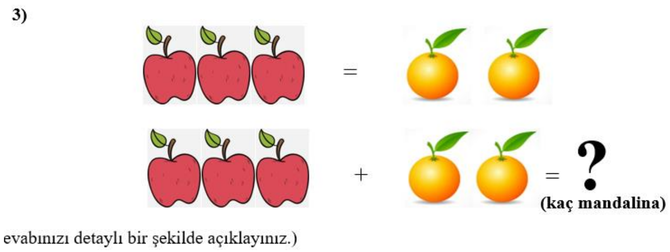


Figure 4.1c. Pictographic Substitution Problem

Figure 4. 1 The problems asked in pictographic form

I aimed to measure the middle-grade students' quantitative reasoning in three pictographic problems and presented students' performances in those pictographic problems, first overall performances across the grade levels and then item-based performances at each grade level.

4.1.1 Students' Performances in Pictographic Problems Across Grades Levels

While some students could reach the correct answers, some could get the wrong answers. Figure 4.2 demonstrates the number of students who reach the correct answer in each grade level (15 students at each grade level) for each problem.

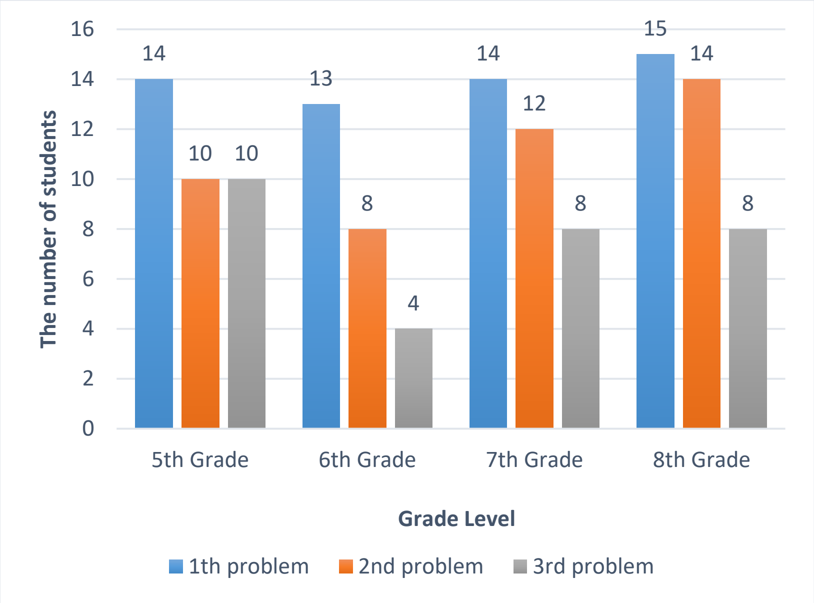


Figure 4. 2 The number of students who answered the problem correctly

The purpose of the first problem was to measure whether students could see the relationship between the numbers of different objects given on the scale; in other words, see how many objects (lemon) are equal to one object (pear). As seen in Figure 4.2, the first problem was answered correctly by 14 students in the 5th grade, 13 in the 6th grade, 14 in the 7th grade, and 15 in the 8th grade. Ten students from 5th grade, eight from 6th grade, 12 from 7th grade, and 14 from 8th grade answered the second problem correctly. In the third problem, ten students from 5th grade, four from 6th grade, eight from 7th grade, and eight from 8th grade answered correctly.

The number of students who correctly answered the problems decreased from the first problem to the third problem in each grade level. The first problem was asking how many lemons are equal to one pear and the one with the most correct answers at all levels, which indicated that students can easily establish the quantitative relationship between different objects given in this form. However, at all levels, there was a significant decrease in the number of students who answered the third problem correctly, especially in the 6th grade. The third problem was asking how many tangerines the sum of two apples and two tangerines are. This situation indicated that students had difficulty in both understanding the quantitative relationship between different objects and putting this relationship in another situation. Although the students generally understood the quantitative relationship between different objects in the first two problems, the number of students who put this relationship in the addition situation and reached the correct result decreased. Remarkably, the number of students who answered the third problem correctly in the 5th grade was higher than in the other levels. Another critical point was that while the majority of 7th and 8th graders answered the first two problems correctly, fewer ones answered the 3rd problem correctly.

4.1.2 Students' Item Base Performances on Pictographic Problems

4.1.2.1 Pictographic Unit Value Problem

Fifth Graders

Fourteen 5th grade students could reach the correct answer, and six of them did not give any explanation of solutions, so these students' reasoning or thinking was unclear to interpret. The remaining nine students gave an explanation of their solutions. For instance, Mert said, "If 2 pears are equal to 4 lemons, 1 pear is equal to 2 lemons. Answer 2." Like Mert, these nine students' expressions included "was equal to." In fact, these statements indicated that students did not or were unable to express their quantitative understanding completely because the quantity in the problem was the weight. In other words, the weight of the 1 pear was equal to the weight of the 2 lemons. Since they did not refer to the weight, it was not clear whether they really thought of the weight as a quantity. However, one student, Hale, reached the wrong result. Since she only wrote the answer, I cannot comment on her thoughts.

Sixth Graders

Thirteen 6th grade students found the correct answer, and all of them gave explanations for their answers. For example, Su stated that “2 lemons are needed because, if the weight of the 2 lemons is equal to the weight of the 4 pears, the weight of the 1 pear is equal to the weight of the 2 lemons.” This sentence indicated that she conceptualized the weight as quantity, and so it can be said that she clearly expressed her quantitative reasoning. The remaining eleven students who found the correct answer used the “equal to” but did not refer to the weight in their explanations like the 5th grade students, so it was not clear whether they reasoned thought the quantity as weight of the fruits but just shortcut in the explanations and did not mention the weight (and just stated the name of the fruit). Yet, one student’s, Burak, explanation was different. He said that “2 because there are 2 pears.” He found the correct result but focused on the number of objects in the image. Actually, he did not establish an equality relation in this problem.

On the other hand, two remaining students found the wrong answer to the first problem. For example, Yasemin explained that “Since there is 1 pear, I collect 3 lemons and 1 pear, and I find this result because the sum is 4.” She did not understand that the lemon and pear were on different sides of the scale and the relationship between the weight of these objects, which indicated that she could not conceptualize the concept of equality given on the scale model. Hence, she tended to add up the numbers given in the problem. The other student, Elif, indicated that “3 lemons are needed for the balance.” She emphasized that the scale should be in balance, but the reasons behind her thought were not clear.

Seventh Graders

Fourteen 7th grade students gave correct answers for the first problem. Twelve of them found that the unit value of the object (one pear was equal to the two lemons) but the quantity was the weight. Because their explanations did not include the weight, students did not or were unable to express their quantitative understanding completely like the other graders. The remaining two students, Selim and Esra, reached the correct answer, but their explanations were incorrect mathematically. Their explanations were below.

> Selim: It is needed an operation between +4 lemons and -2 pears. If we subtract -2 from +4, the scales are balanced.
>
> Esra: If we subtract 2 pears from 4 lemons, the answer is 2. If we do not remove 4 lemons from 2 pears, the answer will not be found.

Selim and Esra did not conceptualize the quantity. That is, they did not conceptualize the number of lemons on one pan and the number of pears on the other pan as quantitatively, and they did not know whether to operate the addition or take away for either lemon or pear. Thus, they got the result by subtracting the smaller number from the larger number without thinking the different quantity. Although they reached the correct answer, it was seen that the students could not make sense of the concept of quantity and quantitative reasoning.

On the other hand, only one student, Nihat, gave the wrong answer to the first problem. He stated that:

> There are 2 pears and 4 lemons at 1st scale. Accordingly, 1 pear should be placed on the other scale to carry an equal weight. Because there is 1 pear on the scale, there are two pears in total. There is no lemon on the other scale, so if we put 4 lemons, it will be equal.

He tried to form visual sameness in both scales. For this reason, he put one pear on the left side and four lemons on the right side of the scale. That way, he thought that since the visual of the two scales was the same, the scale would be in balance, which indicated that he did not establish the relationship between the different quantities.

Eighth Graders

All 8th grade students (i.e., 15 students) provided explanations parallel with the other grade levels. Only one student, Aynur, explained the equality in terms of weight (see Figure 4.3).

Figure 4. 3 The explanation given by Aynur

She conceptualized the relationship between the weights of the objects and found that the weight of the 1 pear was equal to the weight of the 2 lemons, which indicated that she completely reasoned quantitatively. However, the remaining students did not express the weight but those students might have operated on the quantity of weight although they did not express. For example, Eray and Aybuke's explanation showed how students approached to this problem.

> Eray: If 2 pears are equal to 4 lemons, 1 pear is equal to 2 lemons. Because half of 2 pears is equal to 1, half of 4 lemons is 2.
>
> Aybuke: If 2 pears are equal to 4 lemons, 1 pear is equal to 2 lemons.

As seen, they explained equality, but it was not clear whether they focus on weight or not. Therefore, it was seen that students could not fully express their quantitative understanding.

4.1.2.2 Pictographic Multiple-Unit Problem

In this problem, there were two scales, one of which had four bananas on the left side and one pineapple on the right side. The second had four bananas and two pineapples on the left side, and asked for the quantity on the right side (see Figure 4.1b).

Fifth Graders

Ten 5th grade students found the correct answer, and eight of them provided an explanation for their answer. For example, Aylin stated that:

> 4 bananas are equal to 1 pineapple. Then if there are 2 pineapples and 4 bananas on the other scale, and 4 bananas equal 1 pineapple, it will be 3 pineapples. They put 3 pineapples on the other scale and equalize it.

She thought that 4 bananas were equal to the 1 pineapple, so she inserted this relationship to the second scale. She was aware of 3 pineapples on the left side of the scale, so she thought three pineapples were needed for equality. Actually, the weight of the 4 bananas was equal to the weight of the 1 pineapple. In their words, 4 bananas are equal to the 1 pineapple, the thinking of which did not recognize that the scale in fact presents the comparison weights of pineapple and banana. Hence, students did not or were unable to express their quantitative understanding completely. However, the remaining student, Emre, said that "3 [*pineapples*] in order to be balanced." This indicated that he emphasized the concept of balance, but the sentence was not clear enough to analyze his perception.

On the other hand, five 5th grade students answered the problem incorrectly, and two of them provided explanations for those incorrect solutions.

> Derya: If we put 4 bananas and 1 pineapple as in the first one, we will obtain equality. Accordingly, with 4 bananas and 2 pineapples, we can obtain 6 fruit. If we put 4 bananas and 2 pineapples on the other scale, the equality is obtained.
>
> Deniz: It is needed four because the pineapple is heavier. When pineapples are combined with bananas, two and two are equal to four and it is equalized.

Derya put the 4 bananas and 2 pineapples on the right side of the balance scale. She thought equality occurred only if the same number of the same quantities was placed on both sides of the balance scale, indicating that she interpreted equality as sameness ($4a + 2b = 4a + 2b$). Although she reached the balance correctly in this way, she did not construct the relationship between the quantities (i.e., $a=?b$). On the other hand, Deniz was aware that the weight of the pineapple was heavier than the bananas, but she thought that 2 bananas were equal to 1 pineapple and substituted 2 pineapples for 4 bananas. She reached the result of 4 pineapples by adding 2 pineapples and 2 pineapples, which indicated that she did not understand equality and did not establish the relationship between the weights of the different quantities.

Sixth Graders

Eight 6th grade students answered the problem correctly, and six explained their solutions. Four of them who provided explanations used the unit and transferred it to the second scale equation. That is, they were aware of the relationship between the objects (bananas and pineapples) and put 1 pineapple instead of 4 bananas on the left side of the second scale, so they found that 3 pineapples should be placed on the right side of the second scale for equality. Nonetheless, like 5th graders, they did not express the equality of the weight of objects, which causes not to make very clear interpretations of their quantitative reasoning. However, two remaining students, Elif and Tarık, indicated that three pineapples should be needed for the scales to be in balance, like Emre at the 5th grade level, so their reasoning was unclear.

On the other hand, six 6th grade students reached the wrong answer, and one student did not find any answer. They provided different explanations for their solutions. For instance, Burcu said that "If 1 pineapple is put in the pan, it is equal to 4 bananas." She interpreted the relationship between the bananas and pineapples but did not transfer this relationship to the second scale, just only focused on the relationship.

Similarly, Emrah identified equality in the first scale. He stated that "In the second problem, 1 pineapple is equal to 4 bananas. If we put 2 pineapples, it is equal to 8 bananas." He could determine that 1 pineapple equaled 4 bananas, so 2 pineapples equaled 8 bananas. However, in his words, 1 pineapple equaled 4 bananas, the thinking of which did not recognize that the scale in fact presents the comparison weights of pineapple and bananas. He established the relationship, but again it was not clear whether she was constructing this by considering the weight. He also could not transfer this relationship to the second scale like Burcu; that is, he did not use the relationship in a different situation.

Moreover, Demir said that "There are 4 on the scale and 4 on the other scale. Thus, when we take one of the 2 pineapples and put it on the other pan, it will be equal." He tried to make the image on the second scale as in the first scale. Since there were 4 bananas and 1 pineapple on the first scale, he thought that the scales would be in balance if the pineapple next to the bananas was taken to the other pan. For this reason, he put one pineapple near the four bananas on the right side of the scale, but he did not think of the other pineapple on the left side of the scale. He tried to create the image on the first scale on the second scale but did not form the first scale on the new situation. This indicated that his quantitative reasoning was not developed enough.

Besides, the other student, Alperen, was affected by the first problem. He expressed that "On the scale, there are 4 bananas and 1 pineapple. When half of the desired fruits are placed on the other scale, there are 2 bananas and 1 pineapple." Since he bought half of the lemons in the first problem, he stated that half of the fruits should be placed on the other side of the scale in this problem. Another student, Nehir, like Deniz in 5th grade students, thought that 2 bananas equal 1 pineapple, so 4 bananas equal 2 pineapples. She reached the answer of 4 pineapples. Finally, Yasemin tended to add up the given numbers because she was not aware that pineapple and banana were in different quantities. She said, "Since there are 4 bananas and 2 pineapples on the scale, I collect with 6 pineapples, and the result is 12." Since there were 4 bananas and 2 pineapples on the left side of the scale, she put 4 bananas and 2 pineapples on the right side. Then, she added these numbers without thinking of different units. In short, she did not realize that there were two different quantities.

Seventh Graders

Twelve 7th grade students found the correct result. For instance, Ebru said that:

> 3 pineapples are required. If 4 bananas are equal to 1 pineapple, 2 pineapples are equal to 8 bananas, and 3 pineapples are equal to 12 bananas. 4 bananas and 2 pineapples are placed on one pan of the second scale, that is, 3 pineapples in total. So it takes 3 pineapples for the scales to be balanced.

Ebru analyzed the relationship between bananas and pineapples, and used the unit value in the second situation. Also, she found multiple of a given quantity (i.e., two pineapples equal to eight bananas). However, this situation showed that she did not or was unable to express the relationship between different quantities exactly because she did not explain the equality in terms of weights.

Three 7th grade students reached the wrong answer. Esra focused on the numbers (four and one). She added up the numbers and got the answer of 5 (i.e., $4 + 1 = 5$). She did not understand the relationship between the weights of different objects and the meaning of equality. Furthermore, explanations of Nihat and Selim were below.

> Nihat: We need to put one of the pineapples on the other pan. We should put 1 pineapple next to the bananas on the 1st scale to be equal.
>
> Selim: If we put 4 bananas on one side and two pineapples on the other side, it is equalized.

Both of them tried to create visual sameness in both scales. Nihat put one pineapple next to the bananas in the first scale and took one of the pineapples in the second scale to the right pan so that the number of quantities on the scales were the same. Selim, on the other hand, put the bananas on the one side and pineapples on the other side of the scale as it was on the first scale. These explanations indicated that both students focused on images to provide balance and interpreted equality as sameness.

Eighth Graders

Fourteen 8th grade students could reach the correct solution for the second problem (see Figure 4.4).

If 4 bananas are equal to 1 pineapple, how many pineapples are equal to the sum of 4 bananas and 2 pineapples.

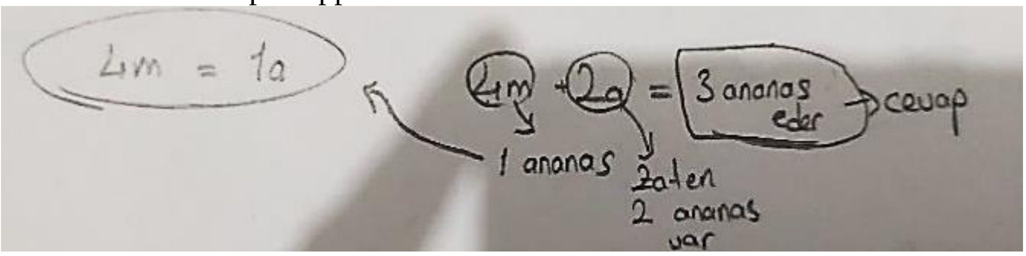


Figure 4. 4 The explanation given by Zeynep

Zeynep correctly interpreted the relationship between objects in the equation and was able to transfer this relationship to the second balance. She was the only one who used letters for objects and can express equality between quantities with letters ($4m = 1a$). When adding 4m and 2a, she can reach 3 pineapples by inserting 1a instead of 4m. This showed that she had algebra knowledge and could transfer the quantitative relationship to algebra. However, one student, Eda, did not find the correct result, as follows:

> If 1 pineapple is equal to 4 bananas and 4 bananas are equal to 2 pineapples, and so the result is 4 pineapples.

As seen with some students at other grade levels, the student equated 2 bananas to 1 pineapple and substituted 2 pineapples for 4 bananas. She reached a total of 4 pineapples. She did not understand equality on the first scale, so she established a wrong relationship. For this reason, she got the wrong answer.

4.1.2.3 Pictographic Substitution Problem

In this problem, there was one equality which had three apples on the left side and two tangerines on the right side. It was asking how many tangerines' weight is equal to the sum of the weights of 3 apples and 2 tangerines (see Figure 4.1c).

Fifth Graders

Ten 5^{th} grade students correctly responded to the third problem, and eight gave explanations of their solutions. Five students used substitution methods without finding the unit value. For instance, Murat stated:

> The result is four. Three apples are equal to two tangerines. If three apples are equal to two tangerines, adding three apples with two tangerines are equal to four.

He analyzed the relationship between quantities. That is, 3 apples were equal to 2 tangerines. He substituted two tangerines instead of the apples in the second equality and reached the four tangerines. However, it was not clear whether the focus was weight or the number of fruits as quantity. Therefore, it did not show complete evidence of constructing quantitative reasoning.

Solely, three students, Semih, Deniz, and Emre, focused on the number of tangerines in the image. They got the answer of four tangerines by counting the number of tangerines in the image. That is, they did not focus on the quantitative relationship as such they avoided to state the relationship in the problem properly, and they could not construct or show it.

On the other hand, one student did not answer the problem, and four students found the wrong result. Three of them who reached the wrong answer found the answer of five. For instance, Derya said that "Because the sum of 3 apples and 2 oranges is equal to 5. If we count 2 more oranges, it will be 5." This sentence indicated that she focused on the operation and number of the objects. She added the numbers three and two, so she reached the answer of five. She did not have an equality concept, which indicated that her quantitative reasoning was not developed enough.

Sixth Graders

Four 6^{th} grade students (Emrah, Su, Berkay, Burak) got the correct answer. They explained the relationship between the apples and tangerines, and inserted the two tangerines into the additive equality instead of the three apples. Nonetheless, the quantity in the third problem was weight as in the first two problems. Like the 5th graders, these students used the "equal to", but did not refer to the weight. Thus these explanations indicated that students did not or were unable to express their quantitative understanding completely.

However, two students did not find any answer, and nine students could not find the correct answer. Six of them focused on the number of apples and tangerines like some 5th graders. They added the number of apples (3) and tangerines (2), reaching the answer of 5. Furthermore, Osman tried to equalize two sides of the equal sign (see Figure 4.5).

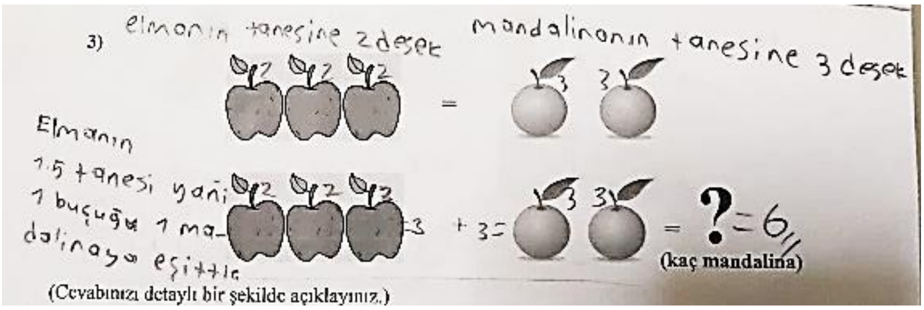


Figure 4. 5 The explanation given by Osman

He used 2 different quantities. He looked first at the numbers of quantities that are focused on the number of items. Based on this, he found that 1.5 of an apple was equal to 1 tangerine. However, he did not continue with this quantity to the solution. He also tried to equalize both sides of another quantity, weight. For this reason, he thought that apples corresponded to something like 2 kg/gr and 3 kg/gr for tangerines. That is, they focused on the weight as a quantity. Hence, he had a conception of equality and could establish equality both in terms of the number of objects and weight. However, he was confused about whether to use the number of objects or the weight. For this reason, he did not transfer the relationship to the total process. This showed that his quantitative reasoning was not stable.

Another student, Nuran, said, "3 apples are equal to 2 tangerines. 3 apples and 2 tangerines are equal to 3 tangerines." She thought that 3 apples were equal to 1 tangerine. She reached the result of 3 tangerines by collecting 1 tangerine and 2 tangerines. She could not construct equality, and as a result, she made the substitution incorrectly in the addition operation on the subsequent equality.

Seventh Graders

Eight 7th grade students found the correct result and provided explanations for solutions. Seven of them used the substitution method without using the unit value (see Figure 4.6).

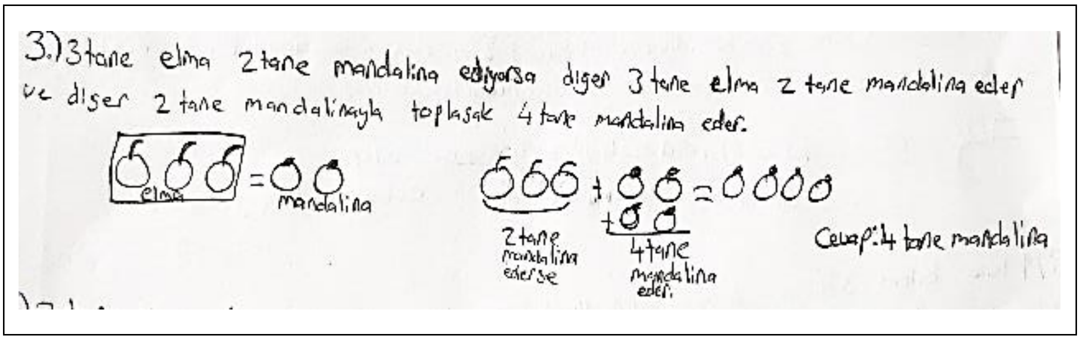


Figure 4. 6 The explanation given by Irmak

She comprehended the relationship between the objects ("three apples were equal to the two tangerines") and inserted the two tangerines instead of the three apples, so they reached the four tangerines. However, as with other grade levels, the focus of the students was not clear. In fact, it was the weights of the fruits that were equal, but the 8th grade students did not or could not express this clearly. On the other hand, only one student, Selen, focused on the number of tangerines in the image. She stated that "I added 2 tangerines and 2 tangerines, so the answer was 4". She reached the answer of 4 tangerines by counting the number of tangerines in the image.

However, seven 7th grade students found the wrong answer. Two of them (Nihat and Esra) added the number of the apple (3) and the number of the tangerine (2), so they reached the answer of 5. One student, Selin, understood the equality in the first situation, but she did not transfer the relationship to the addition process. She added 3 (the number of apples) and 2 (the number of tangerines) in the addition process. Since she made a mistake in the addition, she reached the result of 6. The other student, Simge, tried to equalize both sides in terms of kilograms, weight or numerical (see Figure 4.7).

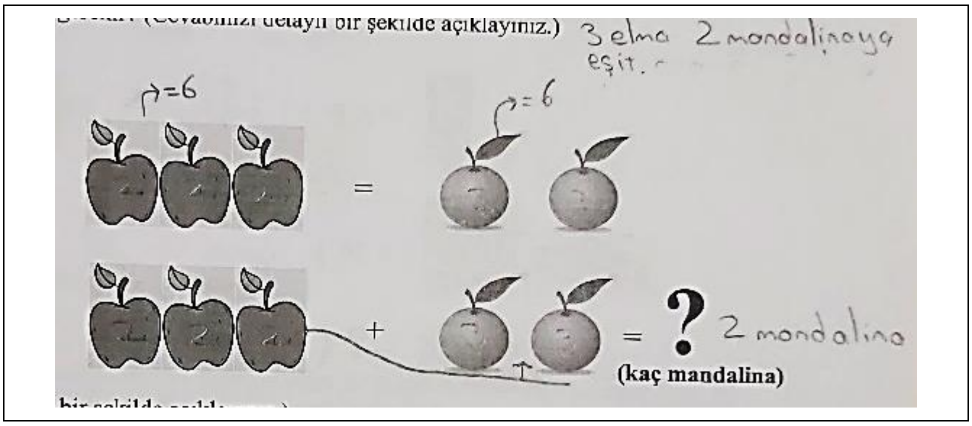


Figure 4. 7 The explanation given by Simge

She tried to make the two sides equal 6 by giving something like 2 to each apple and 3 to each tangerine. These numbers could be kilogram, weight, or just number. She also was aware that 3 apples were equal to 2 tangerines. Actually, she conceptualized the quantity and could establish equality but her thoughts were confused. Therefore, she changed her focus (equality in numerical or weight) and focused only on the number of tangerines in the addition operation, and found the result of 2 tangerines. In addition, the two remaining students (Selim and Cengiz) did not establish an equality relation in the first equality, so they did not reach the correct result.

Eight Graders

Eight 8^{th} grade students could reach the correct result, but seven focused on the relationship between objects. For example, Berk said that "Since 3 apples are equal to 2 tangerines, 1,5 apples are equal to 1 tangerine. The sum of 3 apples and 2 tangerines is 4 tangerines." He used the unit value of quantity given multiple. On the other hand, Ahmet used the substitution method (see Figure 4.8).

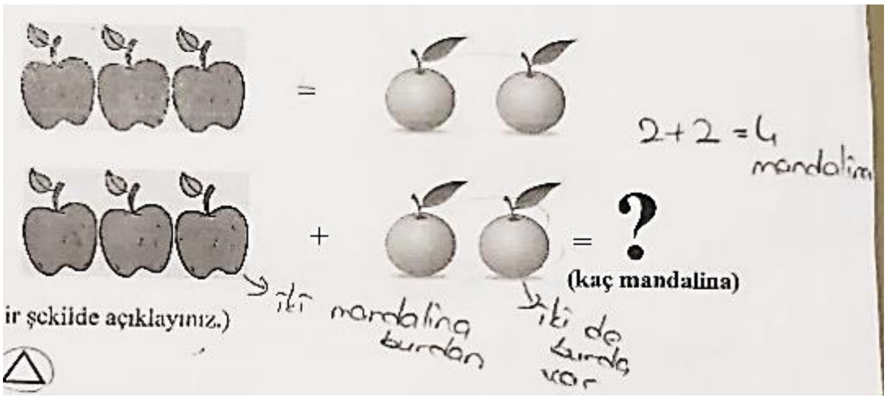


Figure 4. 8 The explanation given by Ahmet

Focusing on the relationship between the different objects, both of them reached the correct result. However, their focus was not clear. Students may have focused only on the numbers of quantities, or they may have thought about the weight. Nonetheless, the remaining student, Aynur, focused on how many tangerines were in the picture, so she counted the tangerines in the picture and reached the answer of 4. Actually, she focused on the image (see Figure 4.9).

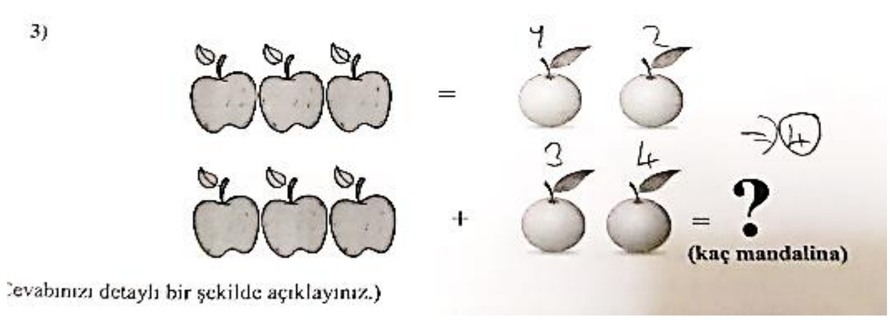


Figure 4. 9 The explanation given by Aynur

On the other hand, seven 8th grade students found the wrong answer. They provided different explanations. Two students (Simay and Aysel) from them focused the tangerines on the second equality. For example, Simay said that "In this problem, three apples are equal to two tangerines. 3 apples + 2 tangerines = 2 tangerines, nothing will change." She only focused on the number of tangerines in the additive process. Although she was aware that 3 apples were equal to 2 tangerines, she could not transfer the relationship to the additive situation.

In addition, the other student, Eda, added the number of apples (3) and tangerines (2), so they reached the answer of 5. She focused on the numbers of quantities and operation (i.e., addition). Emel and Nergis thought that 3 apples equaled the 1 tangerine, and so substituted 1 tangerine instead of 2 apples. Hence, they reached the answer of 3 by adding 2 tangerines and 1 tangerine. This indicated that students did not have an equality concept and did not understand the relationship between different quantities, so they reached the wrong answer. The remaining student, Meryem, said that "3 apples are equal to 2 tangerines. 2 tangerines are equal to 3 tangerines." She thought that 3 equals 2 means 2 equals 3. If this side of the equation was true, the other side was also true. That is, she did not have an equality concept.

4.1.3 Students' Performances on Pictographic Problems Across Items

In this section, it was examined the item-based results and compared which problems the students answered correctly. For this cross-comparison analysis, a scoring method was developed. In this scoring method, students who answered all three pictographic problems correctly scored 3. Students who answered unit value and multiple-unit pictographic problems (i.e., first and second problems) correctly but not pictographic substitution problem (i.e., the third problem) scored 2. Students who answered only unit value pictographic problem correctly but not multiple-unit and substitution problems scored 1 (see Table 4.1).

Table 4. 1 Scoring Method

Unit value Pictographic Problem	Multiple-Unit Pictographic Problem	Pictographic Substitution Problem	
			**Overall Score**
+	-	-	**1**
+	+	-	**2**
+	+	+	**3**

Figure 4.10 demonstrated that the number of students scoring 1, 2, and 3 at each grade level.

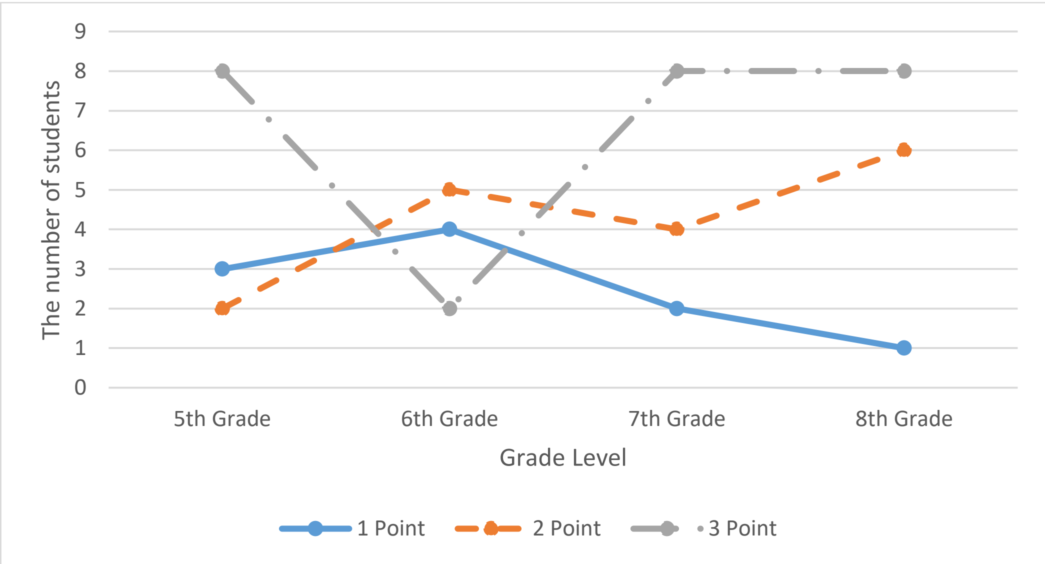


Figure 4. 10 The number of students who scored 1,2, and 3 at each grade level

The number of students who answered all three pictographic problems correctly (i.e., scored 3 points) at each grade level was higher than the number of students who answered the unit value and multiple problems (i.e., scored 2 points), and only unit value problem (i.e., scored 1 point) except at the 6th grade level. Since the number of students who answered the pictographic substitution problem correctly in 6^{th} grade students was quite low, the number of students who answered the first two problems (i.e., scored 2 points) at this grade level was higher. This showed that the 6^{th} grade students had difficulty establishing the quantitative relationship given in the third problem and transferring the relationship to the addition situation.

Especially, it was quite interesting that the number of students who answered all three problems correctly in the 5^{th} grade was higher than the number of students who reach the correct answer in the 6^{th} grade and was equal to the 7^{th} and 8^{th} grades. Therefore, it was shown that 5^{th} grade students' quantitative reasoning was not different from the other grade levels, and even more advanced than the 6^{th} grade students.

The majority of the students who answered the third problem correctly answered the first two problems correctly, which indicated that these students were able to construct quantitative reasoning in three problems. On the other hand, there were some students who answered the first two problems correctly and answered the third problem incorrectly. It was seen that these students established the relations between quantities. That is, the unit or unit of a given quantity was to find the multiple of a given quantity. However, it showed that they could not transfer the given relationship to the addition operation. In fact, all three problems asked were in pictographic form. This indicated that rather than forms of the problems, the complexity of the relationship (quantitative reasoning) that students need to establish in the problem was more important.

4.2 Relationship Between Quantities Represented in Iconographic and Symbolic Form

There were two problems in the iconographic and two problems in the symbolic form in this set, and each problem had a different purpose of being asked (see Figure 4.11).

This set of problems was asked to students right after the first set of problems in the pictographic form and therefore numbered as problems 4-7.

4) □□ = △ □□+△△ = ? (kaç tane üçgen) **Figure 4.11a.** Iconographic Substitution Problem
5) 2 U = 1 C 2 U + 2 C = ? (kaç tane C) **Figure 4.11b.** Symbolic Substitution Problem
6) □□ = △ □□+△△ = ? (kaç tane kare) **Figure 4.11c.** Iconographic Multiple-Unit Substitution Problem
7) 2 U = 1 C 2 U + 2 C = ? (kaç tane U) **Figure 4.11d.** Symbolic Multiple-Unit Substitution Problem

Figure 4. 11 The problems asked in the iconographic and symbolic forms.

In this regard, the purpose of the fourth problem was to understand whether students could see the relationship between the numbers of different objects and transfer this relationship to an additive process. However, in this problem, the objects were more formal objects. The goal of the fifth problem was to understand whether students could see the relationship between the numbers of different letters and could transfer this relationship to an additive process. In the sixth problem, actually, the purpose was similar to problem four, the only difference was that the problem asked how many squares the sum was. It was also measured whether the student could find the multiple of the given unit in the iconographic form and then substitute multiple to the additive process. Finally, the aim of the seventh problem was similar to the fifth problem. The difference was that the problem asked how many the letter “U” the sum was. That is, whether the student could find the multiple of the given unit in the symbolic form and then substitute multiple to the additive process was also measured. Therefore, I aimed to measure the middle-grade students' quantitative reasoning in two iconographic and two symbolic problems, and presented first students' overall performances across the grade levels and then item-based performances at each grade level.

4.2.1 Students' Performances on Iconographic and Symbolic Problems Across Grades Levels

While some students could reach the correct answers, some could get the wrong answers. Figure 4.12 demonstrates the number of students who reach the correct answer in each grade level (15 students at each grade level) for each problem.

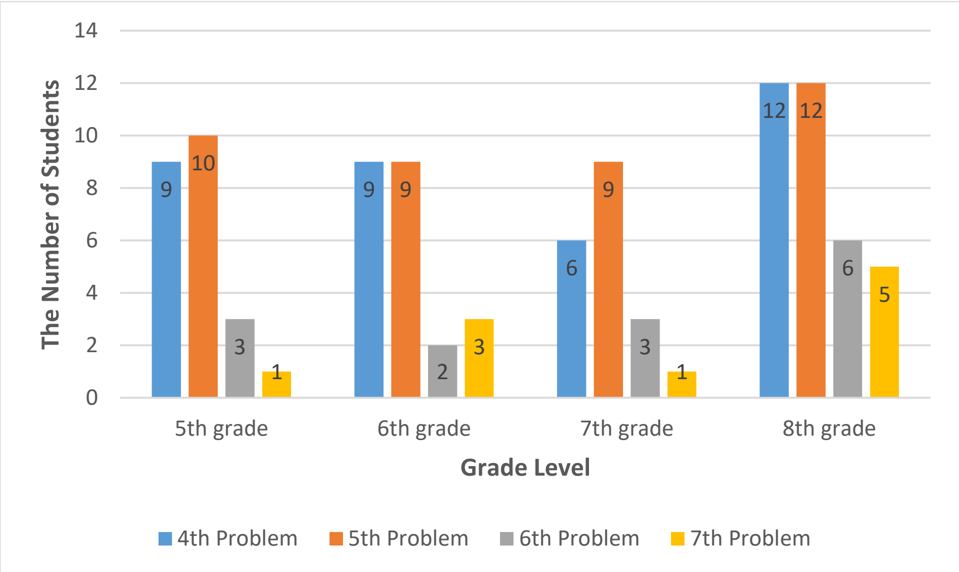


Figure 4. 12 The number of students who answered the problems correctly

As seen in Figure 4.12, the fourth problem was answered correctly by nine students in the 5th grade, nine in the 6th grade, six in the 7th grade, and twelve in the 8th grade. Ten students from 5th grade, nine from 6th grade, nine from 7th grade, and twelve from 8th grade answered the fifth problem correctly. In the sixth problem, three students from 5th grade, two from 6th grade, three from 7th grade, and six from 8th grade answered correctly. Finally, the seventh problem was answered correctly by one student in the 5th grade, three in the 6th grade, one in the 7th grade, and five in the 8th grade.

Although there were some exceptions, the number of students who correctly answered the problems generally decreased from the fourth problem to the seventh problem in each grade level. It was seen that the students especially had difficulties in the sixth and seventh problems, and hence the number of students who answered these problems correctly was much lower than the others. This indicated that students were better at making sense of the relationship between quantities and transferring this relationship to the addition situation in the fourth and fifth problems, asking how many triangles and the letter “C” are in the sum of quantities, respectively. However, most of the students had difficulty finding the multiple of the given quantity and transferring the multiple to the addition. In this case, it was seen that the number of students who answered the sixth and seventh problems correctly was quite low.

The number of students who answered all problems correctly was the highest at the 8th grade level and the number of students who answered each problem correctly at the 5, 6, and 7 grade levels were close to each other. One of the other remarkable points was that in the fourth problem, the number of students who answered correctly at the 7th grade level was lower than the 5th and 6th graders.

4.2.2 Students’ Item Base Performances on Iconographic and Symbolic Problems

4.2.2.1 Iconographic Substitution Problem

Nine 5th, nine 6th, six 7th, and twelve 8th grade students answered the problem correctly. Some of them gave explanations for answers, but some did not provide any explanations. Hence, these students’ reasoning or thinking was unclear to interpret. Five 5th, nine 6th, nine 7th, and eleven 8th grade level students who answered the problem correctly used the substation method. For instance, the explanations of Aylin (5th grade) and Ali (8th grade) were below.

> Aylin: If 2 squares are equal to 1 triangle, that is 1 triangle + 2 more triangle is equal to 3 triangles.
>
> Ali: Since 2 squares are equal to 1 triangle, the problem asked us to add 2 squares and 2 triangles. Thus, 2 squares are equal to 1 triangle. There are also 2 triangles, when you add them, you get 3 triangles.

They analyzed the relationship between different quantities; that is, two squares were equal to one triangle. Then, they substituted one triangle instead of the two squares in the addition and reached the answer of three triangles. However, two 5th grade students (Emre and Deniz) and one 8th grade student (Aynur) reached the correct answer, but their reasoning differed. They got the answer of three triangles by counting the number of triangles in the image. That is, they focused on the number of triangles on the image instead of the quantitative relationship (see Figure 4.13).

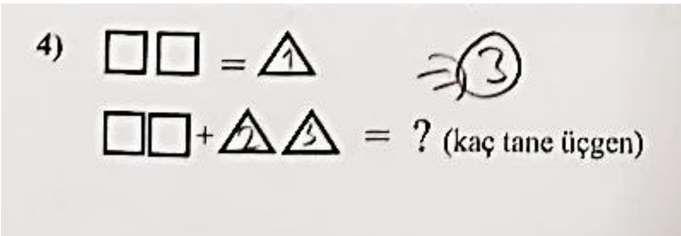


Figure 4. 13 The explanation given by Aynur

On the other hand, three 5th, three 6th, three 7th, and one 8th grade students reached the wrong answer of four. Some of these students conceptualized relationships between squares and triangles but did not insert the one triangle instead of the two squares in the addition. For instance, Murat (5th grade) conceptualized equality and was aware that two squares are equal to one triangle. However, he could not make the substitution in the addition operation on the subsequent equality. Thus, he reached the answer 4 by adding 2 and 2. Similarly, 8th grade student, Ahmet, saw the relationship between shapes and inserted one triangle instead of two squares. Even though he wrote "2+1", he might have been confused later, wrote 2+2, and reached the result of four triangles (see Figure 4.14).

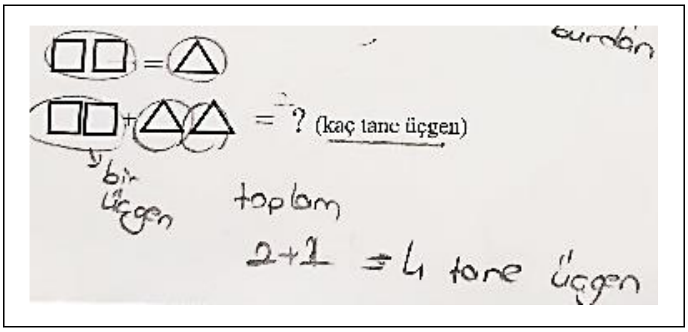


Figure 4. 14 The explanation given by Ahmet

The 5th grade student, Semih, stated, “2 squares are equal to 1 triangle. 4 squares are equal to 2 triangles.” He established the relationship between different objects. In fact, he found the multiple of the given unit (triangle). That is, he found four squares are equal to the two triangles. However, he could not make the substitution in the addition operation on the subsequent equality. The remaining students at all grade levels did not make sense of the relationship between quantities (i.e., squares and triangles), so they added the number of squares (two) and triangles (two), so they reached the answer of four.

Although three 7th grade and one 8th grade student established an equality relation, they reached the answer of two. For example, Asaf and Ecem (7th grade) understood an equality relation in this problem, but these students focused on all squares given in the problem. They thought that if two squares are equal to one triangle, four squares are equal to two triangles. Another 7th grade student, Simge, reached the result of two triangles, but her thinking was slightly different (see Figure 4.15).

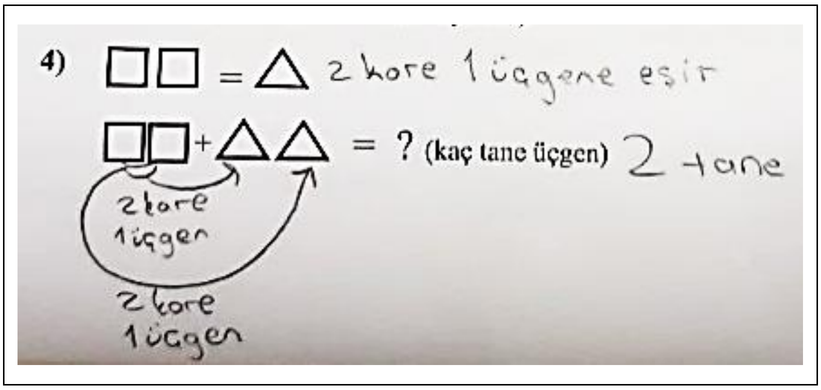


Figure 4. 15 The explanation given by Simge

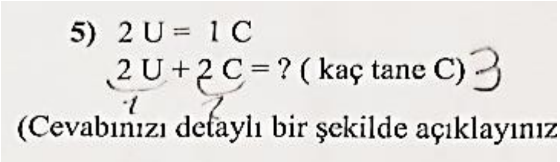
She analyzed the relationship, but she wrote that two squares equaled one triangle instead of each triangle. Thus, she found two triangles in total by adding one triangle and one triangle. Moreover, Simay (8^{th} grade) said that:

> 2 squares are equal to 1 triangle. Accordingly, 2 squares + 2 triangles = 1 triangle. For this to be two triangles, there must be four squares. In other words, the result does not change whether he gives one of these two triangles or not.

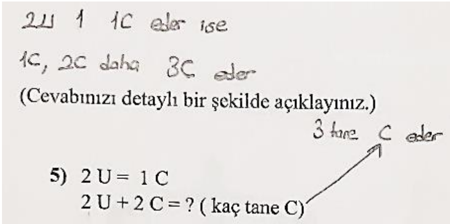
She established the relationship between the square and the triangle. He was aware that two squares are equal to one triangle, and she found that four squares are equal to two triangles. However, she focused only on the squares in the given addition operation. She substituted two triangles instead of the square but did not add up with a triangle in the addition operation.

4.2.2.2 Symbolic Substitution Problem

In this problem, the left side of the equality involves a quantity represented by the letter U where U can be any quantity such as weight of a banana, and the right side of the equality involves a quantity represented by the letter C where C can be any quantity such as weight of a pineapple. It was asked how many letters "C" are in the sum of 2U and 2C (see Figure 4.11b). Ten 5^{th}, nine 6^{th}, nine 7^{th}, and twelve 8^{th} grade level students correctly responded to the fifth problem and gave some explanations for their answers. Seven 5^{th}, nine 6^{th}, seven 7^{th}, and twelve 8^{th} grade level students who answered the problem correctly used substitution methods (see Figure 4.16). Below, I provided two distinctive responses to illustrate students' ways of thinking.



Since 2U is equal to C, 2U is equal to 1C. Adding 2C with 1C is equal to 3. The answer is 3.



If 2U is equal to 1C, adding 1C and 2C is equal to 3C. The answer is 3C.

Figure 4. 16 The explanation given by Demir (left) and Leyla (right)

Demir (6th grade) and Leyla (7th grade) established the relationship between the different quantities and transferred them to the addition situation. By writing 1C instead of 2U in the addition process, he reached the result of 3C.

On the other hand, two 5th, three 6th, two 7th, and two 8th grade students found the wrong answer of four. Students at the 5th, 6th, and 7th grade levels added two squares and two triangles without focusing on the different quantities in the problem. They just focused on the operation, result, and number of the letters. They did not focus on an equality concept, and their quantitative reasoning was not developed enough. However, Simay and Meryem (8th grade) could establish equality between letters. For example, Simay stated that “2U=1C and since the answer corresponds to 2U and 1C, 2C in the problem corresponds to 4U. Therefore, the answer is 4C.” She even found that 2C was equal to 4U in the addition process and found the multiple of the letter “C” in terms of the “U” type but the problem asked for the sum of quantities in terms of the letter “C”. She did not pay attention to it. Although she wrote the answer in U type, she expressed the result as C. She also did not focus on the addition operation, just focused on the relationship. This indicated that she did not transfer the relationship to the addition process.

Besides, Deniz (5th grade) and Simge (7th grade) reached the wrong answer of 2C. Simge’s explanations were below (see Figure 4.17).

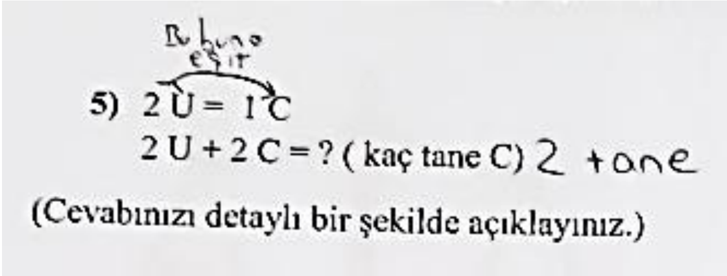


Figure 4. 17 The explanation given by Simge

She may have ignored the 2U in the addition or she may have thought of it as the sum of 1C and 1C. She reached the wrong answer of two because she could not establish the quantitative relationship.

Finally, Ecem (7th grade) said, “When adding all of them, the result is 4U and 3C.” The student was aware that the letters given in the problem were different letters but could not establish the relationship between the different quantities. Therefore, she tended to add the numbers of the letter U with each other and the letter C with each other, adding like terms, and reached the result of 4U and 3C.

4.2.2.3 Iconographic Multiple-Unit Substitution Problem

In this problem, there was one equality of two squares on the left side, and one triangle on the right side. It was asked how many squares are in the sum of two squares and two triangles (see Figure 4.11c). Three 5th, two 6th, three 7th, and six 8th grade students responded correctly to the sixth problem, and all of them used substitution methods. For instance, Emre (5th grade) stated that “2 squares + 4 squares = 6 because 1 triangle is equal to 2 squares.” He established the relationship between the different quantities that is two triangles were equal to the four squares. Then, he transferred four squares to the addition operation. By writing four squares instead of two triangles in the addition operation, he reached the result of six squares.

Three 5th, six 6th, six 7th, and four 8th grade students found the wrong answer of four squares. These students focused on the addition and numbers of the shapes as with the other problems. They added two squares and two triangles without focusing on the quantitative relationship. Among these students, Nihat (7th grade) and Aynur (8th grade) reached the answer of four but their thinking was different. They focused on the number of squares on the image, so counted the squares and found four squares (see Figure 4.18).

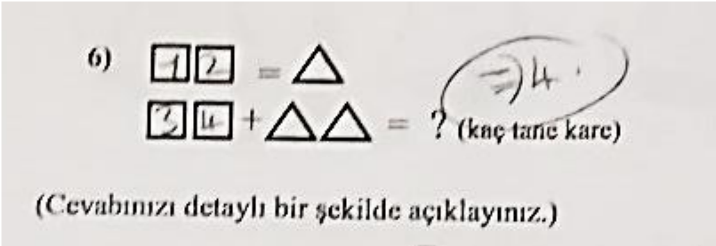
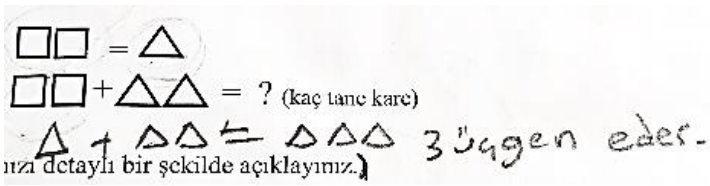


Figure 4. 18 The explanation given by Aynur

Three 5th, four 6th, one 7th, and two 8th grade students reached the wrong answer of three triangles. These students did not pay attention to whether they found the sum given in the problem in terms of a square or a triangle. For instance, Berkay said that "Problem 4 is the same.", so he reached the wrong result of three. Although the problem asked the sum in terms of a square, they reached the result of three triangles as in the fourth problem. The other student's explanation was below (see Figure 4.19).



1 triangle is equaled to 2 triangles, so equals 3 triangles.

Figure 4. 19 The explanation given by Emrah

Either, these students who reached the answer of three triangles, like Emrah, might have had difficulties in figuring out how many squares two triangles were equal to. That is, they may not be able to find the multiple of a triangle whose unit was given, so found the result in terms of triangles type. Or, they may not have paid attention to what the problem was asking, whether it was a square or a triangle.

Besides, Derya (5th grade), Nehir (6th grade), and Simay (8th grade) gave the wrong answer of two squares. For example, Simay said that "In this problem, two squares are equal to one triangle. 2 squares + 2 triangles = 2 squares. Even though it does not have a triangle there, the answer is still 2 squares." They focused on the just number of squares and did not think of the number of triangles and quantitative relationships, so she found the two squares.

Finally, Azra (5th grade) added the numbers of the shapes (square and triangle) given in the problem. Since she did not establish the relationship between the shapes, she tended to collect all the numbers seen in the problem. Hence, she could not recognize that there were two different quantities, operated with numbers only and reached the answer of seven by adding three and four.

4.2.2.4 Symbolic Multiple-Unit Substitution Problem

In this problem, the left side of the equality involves a quantity represented by the letter U where U can be any quantity such as weight of a banana, and the right side of the equality involves a quantity represented by the letter C where C can be any quantity such as weight of a pineapple. It was asking how many letters "U" are in the sum of 2U and 2C (see Figure 4.11d). One 5th, three 6th, one 7th, and five 8th grade students gave a correct response. While 5th grade student, Yaren, did not provide any explanation for her answer, the other students who gave a correct response established the relationship between the quantities and found that 2C was equal to the 4U. They inserted the 4U instead of the 2C and added 2U and 4U, so reached the 6U (see Figure 4.20).

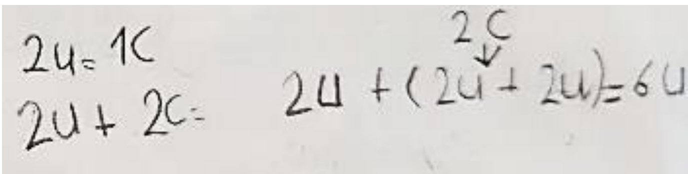


Figure 4. 20 The explanation given by Burak

On the other hand, the remaining students either answered the problem incorrectly or did not answer it. In this regard, six 5th, three 6th, six 7th, and ten 8th grade students provided the answer of four. Generally, students noticed that 2U was equal to 1C but they could not find the multiple of the given quantity. Hence, they did not substitute 4U instead the 2C. They added two and two, reaching the answer of four because they did not establish the quantitative relationship. However, Semih (5th grade) and Simay (8th grade) established the relationship between quantities. For example, Simay, stated that "2U=1C. 2U+2C are equal to 4U. She established equality and found that 2C was equal to 4U. She wrote 4U instead of 2C to reach the result of 4U without focusing on the letter U in the addition. That is, she established an equality relation, but she did not add two squares and four squares.

Besides, three 5th and one 6th grade students reached the answer of 2U. For instance, Deniz stated, "Two because there are two U. It is equal to 2U." Although she was aware that the letters in the addition process were different from each other and cannot be added, she did not pay attention to the relationship between the quantities. She focused on only the letter U in the addition process, so she reached the answer of 2U.

Two 6th grade students and one 7th grade student reached the answer of 3C. These students found the sum in terms of C, as in the fifth problem. They did not focus on what the problem asked. For example, Sema stated that "The result is 3U. Since 2U is equal to 1C, and 2U and 2C are equal to 3U." She did not focus on what the problem was asked. Actually, she found the sum in terms of C as she did in the fifth problem. However, she wrote as if she found the U type. The remaining 7th grade students', Zeynep, explanation was below (see Figure 4.21).

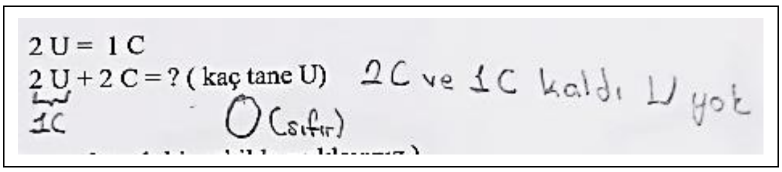


Figure 4. 21 The explanation given by Nur

Nur inserted the given equation into the addition operation and hence reached the result of zero U since there was only the letter C in the total; that is, there was no U left. This showed that she could not find the multiple of the given unit and transfer the multiple to the addition.

4.2.3 Students' Performances on Iconographic and Symbolic Problems Across Items

In this section, it was examined the item-based results and compared which problems the students answered correctly. For this cross-comparison analysis, a scoring method was developed. In this scoring method, students who answered all four problems correctly scored four. Students who answered iconographic substitution, symbolic substitution, and iconographic multiple-unit substitution correctly but not symbolic multiple-unit substitution problem scored three. Students who answered iconographic substitution and symbolic substitution problems correctly but not iconographic multiple-unit substitution and symbolic multiple-unit substitution problems scored two. Students who answered only the iconographic substitution problem correctly but not symbolic substitution, iconographic multiple-unit substitution, and symbolic multiple-unit substitution problems scored one (see Table 4.2).

Table 4. 2 Scoring Method

Iconographic Substitution Problem	**Symbolic Substitution Problem**	**Iconographic Multiple-Unit Substitution Problem**	**Symbolic Multiple-Unit Substitution Problem**	
4) □□ = △ □□+△△ = ? (kaç tane üçgen)	5) 2 U = 1 C 2 U + 2 C = ? (kaç tane C)	6) □□ = △ □□+△△ = ? (kaç tane kare)	7) 2 U = 1 C 2 U + 2 C = ? (kaç tane U)	**Overall Score**
+	-	-	-	**1**
+	+	-	-	**2**
+	+	+	-	**3**
+	+	+	+	**4**

Figure 4.22 demonstrated the number of students scoring 1, 2, 3, and 4 at each grade level.

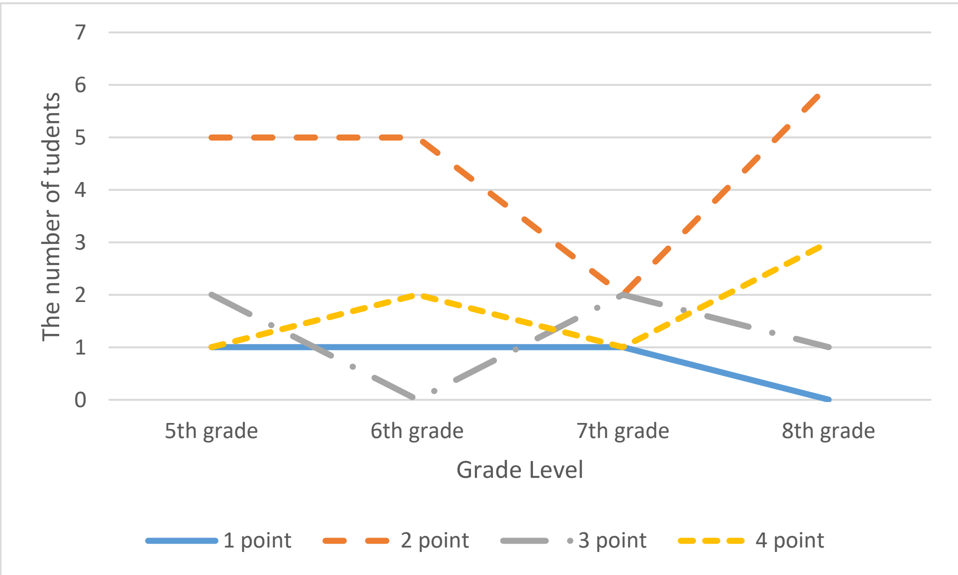


Figure 4. 22 The number of students who got 1, 2, 3, and 4 points in each grade level

As seen in the Figure 22, the rate of getting the correct answers was low, especially in the last two problems, which indicated that the students had more difficulty with the problems in this set. Generally, if students answered the iconographic substitution problem correctly, they also answered the symbolic substitution problem correctly. So, the number of students who got one point was almost non-existent, and the number of students who got two points was the most at all grade levels. Regardless of whether the given problem was iconographic or symbolic, most students were able to transfer a given relation to the addition operation and added these quantities in the fourth and fifth problems (i.e., iconographic and symbolic substitution problems).

On the other hand, the number of students who got three points and four points was quite low. That is, students did not establish the relations between different quantities in these problems which are the iconographic multiple-unit substitution problem and symbolic multiple-unit substitution problem. In this sense, although some students found the multiple of the given quantity, they could not transfer this relationship into the addition process. At the same time, the majority of students did not find the multiple of the given quantity, and they did not transfer it to the addition operation. Because they focused on different things, they reached the wrong answers. This situation indicated that students generally had difficulty finding the multiple of the given unit between shapes or letters and transferring the multiple to the addition process. In other words, they did not establish the relationship from the type of quantity given on the right side of the equation. It was seen that the quantitative reasoning of the students was not sufficiently developed. In fact, this indicated that the complexity of the relationship (quantitative reasoning) that students need to establish in the problem was more important rather than the forms of the problem.

Considering between grade levels, the number of students who got one, two, three, and four points from the 5^{th} grade to the 8^{th} grade level was close to each other in all four problems which are iconographic substitution, symbolic substitution, iconographic multiple-unit substitution, and symbolic multiple-unit substitution problems. Therefore, there was not distinct differences between students' quantitative reasoning at different grade levels in the problems addressing iconographic and symbolic substitution.

CHAPTER 5

CONCLUSION AND DISCUSSION

The aim of the present study was to investigate middle school students' quantitative reasoning in early algebra problems given in pictorial, symbolic, and iconographic forms. The results of this study were explained in the previous chapter in detail. In this chapter, these results are summarized and discussed. This chapter also included the implications of the study and recommendations for further research.

5.1 Discussion of the Findings

Based on the research questions, students' quantitative reasoning in different problems and in different grade levels were discussed respectively. While the findings of the current study were discussed, these findings were compared with the current literature.

5.1.1 Students' Quantitative Reasoning in Different Problems

A total of seven problems in three different forms which are pictographic, symbolic, and iconographic problems were asked to the students in the current study. When considering these three forms, the findings showed that the forms did not determine whether students have difficulty in quantitative reasoning. That is, there is no such thing as if students could not solve iconographic problems while solving pictographic problems. It was seen that while students could establish quantitative reasoning in some of the problems, and they could not establish quantitative reasoning in the other problems in the same form. Hence, this study concludes that while forms of the problems were not determinative in the students' quantitative reasoning, the level of complexity of the relationship such as single-unit substitution or multiple-unit substitution within each form seemed to play a role in students' quantitative reasoning. However, some studies in the literature have found the opposite findings. For instance, Koedinger and Nathan (2004) examined how different forms of the problems (story vs symbolic problems) affected the performance of high school students' quantitative reasoning. According to the results, students had more difficulties with symbolic problems. Contrary to the findings of the present study, students' quantitative reasoning differed according to the problem forms.

As stated above, the complexity of the relationship in the problems varies. For instance, the students needed to understand the relationship given in the pictographic substitution problem, iconographic substitution problem, and symbolic substitution problem and substitute given single-unit to the addition operation. However, in the iconographic multiple-unit substitution problem and symbolic multiple-unit substitution problem, which are more complex problems, they needed to find the multiple of the given quantity and substitute it into the addition operation. The findings revealed that the number of students who answered the problems correctly decreased as the level of the complexity of the relationship in the problem increased. Especially, in the multiple-unit substitution in two forms of the problems, the number of students who reached the correct result was quite low. This situation indicated that while most of the students could establish quantitative reasoning in simpler situations such as direct substitution of the unit value, they had difficulty in establishing quantitative reasoning in more complex situations such as multiple-unit substitution. As a result, complexity of the relationship in the problems played a more important role than the form of the given problem. As stated in the study of Thompson (1993), the fact that students are not sufficiently exposed to such problems may be one of the factors that affect negatively their quantitative reasoning in problems. This finding was also consistent with the findings of some other studies. Ramful and Ho (2014) found that 6^{th} grade students could easily solve the problems in the easy task, but in the more complex problems, they focused on numbers instead of the quantitative relationship. Similarly, Güvendiren (2019) found that most of the students had difficulty establishing relations between quantities as the complexity of the problems increased. Besides, Usta and Özdemir (2018) found that middle school students' performances were below expectations, and they had more difficulties especially in complex problems.

Another important finding in the current study was that the students who answered the iconographic substitution problem correctly answered the parallel symbolic substitution problem correctly. The only difference between the problems was that while in the iconographic substitution problem, shapes (i.e., square and triangle) were used, but letters (i.e., U and C) were used in the symbolic substitution problem. That is, the same thing was asked with different forms of the problems (iconographic and symbolic). The aim was to provide a kind of learning by giving these problems in parallel and successively. This was also the limitations because if the problems had not been presented in this way and in this order, it would be uncertain whether the result would be like this. In fact, the students may have applied the same procedure while solving the symbolic problem without thinking since the problems are the same except for their forms.

The students' quantitative reasoning in different forms of the early algebra problems would be discussed in more detail under two headings below: pictographic form, and symbolic and iconographic form.

5.1.1.1 Pictographic Form

Three problems in the pictographic form were asked to the students. Table 5.1 summarizes the characteristics of students' reasoning that reached the correct answer, which were in fact explained in detailed through sample student work in the previous chapter.

Table 5. 1 The characteristics of the students’ reasoning that resulted in correct answer

	5th grade	**6th grade**	**7th grade**	**8th grade**
Identifying the relationship with addressing the weight	–	+	–	+
Identifying the relationship without addressing the weight	+	+	+	+
Referring the algebra	–	–	–	+
Focusing on the number of the asked quantity on the image	+	–	+	+

+ : There are students with the stated reasoning

– : There is no student with the stated reasoning

As seen in table 5.1, 6th grade and 8th grade students focused on the weight as the quantity. Since the actual quantity was the weight, these students clearly expressed their quantitative reasoning. On the other hand, most of the students who found the correct result used expressions such as “1 pear is equal to 2 lemons” or “1 pineapple is equal to 4 bananas” when identifying the relationship, and did not refer to the weight. Although they established the relationship and reached the correct answer, they did not or could not express quantitative reasoning clearly or completely. However, the fact that students did not mentioned about the weight (i.e., quantity) does not show that they could not operate on quantities. If the students had been interviewed and asked, perhaps the students would have been able to said the weight as the quantity.

The difference between algebraic reasoning and quantitative reasoning was the use of algebraic notations. In algebraic reasoning, students were expected to be able to express their relationship in the problem algebraically (Kalman, 2008). As the present study was conducted at the beginning of the academic year, only 8th grade students learned the subject of algebra in the previous academic years (MNE, 2018). Therefore, it was expected that 8th grade students could use letters while answering the problems; that is, they could refer to algebra. However, only one student was able to express the relationship algebraically, which indicated that either 8th grade students cannot express quantitative reasoning algebraically, or they could not construct algebraic reasoning. Generally, in the literature, some studies showed that most students have difficulty in establishing algebraic reasoning and see algebra as a topic consisting of meaningless symbols (e.g., Carpenter & Levi, 2000; Carraher & Schliemann, 2007; Dede, 2004; Didiş & Erbaş, 2012; McNeil & Alibali, 2005; Samuel et al., 2016; Usta & Özdemir, 2018). Therefore, students' inability to express the relationship between the quantities in the problem algebraically may be due to their difficulties in understanding the algebra.

Quantity and quantitative reasoning were not the same as the number and numerical reasoning, respectively (Smith & Thompson, 2007). Students need to be able to make this distinction so that they can construct correct mathematical reasoning. Although there were students who reached the correct answer in the study, their mathematical reasoning was not correct because they cannot make this distinction. As seen in Table 5.1, some of the students focused on the number of quantities on the image, and counted only the number of quantity asked. Therefore, they did not conceptualize quantity, and so they could not construct the relationship between different quantities.

Table 5.2 presents a summary of the characteristics of students’ reasoning that reached the incorrect answer.

Table 5. 2 The characteristics of the students’ reasoning that resulted in incorrect answer

	5th grade	**6th grade**	**7th grade**	**8th grade**
Misunderstanding the equality	+	+	−	+
Understanding equality but not transferring the new situation	−	+	+	−
Creating the visual sameness	+	+	+	−
Focusing on the number of the asked quantity on the image and operations	+	+	+	+

+ : There are students with the stated reasoning

− : There is no student with the stated reasoning

As seen in Table 5.2, while some of the students could not establish the relationship between the given quantities, some of them constructed the relationship but could not transfer to the new situation. Some tried to create visual sameness because they thought that equality only occurs with the same quantities. Although these students established a balance or equality, they could not construct a quantitative relationship. On the other hand, common thought at all grade levels was focusing on numbers and operations. The students either tried to reach the result by counting the number of the asked quantity in the problem. Or, especially in the third problem, they collected three apples and two tangerines without paying attention to the different quantities and reached the result of five. Since students focused on numbers and operations, they did not identify the quantities and quantitative relations.

The problems asked in the pictographic form contain multiplicative relationships. In other words, students need to think multiplicatively and transfer the relationship to another situation. Especially, in the pictographic substitution problem, students generally had difficulty in establishing multiplicative relationship and transferring it to the addition operation. Similar results were found in the literature. For instance, Alexander et al. (2020) and Van den Heuvel-Panhuizen and Elia (2020) found the students had difficulty in multiplicative reasoning.

Considering the reasons why students have difficulty with problems in pictographic form, it may be because they are not sufficiently exposed to the problems in this form. When the mathematics books used in the public schools from the 1st to the 8th grade are examined (e.g., Çağlayan et al., 2021; Kayapınar et al., 2021; Oğan & Öztürk, 2021), it can be stated that problems in the pictographic form have not been covered in a sufficient amount. The concept of the weight and the concept of the equality are taught in the 4th grade. Although scale models are used, these concepts are explained through numerical values. Besides, in the 7th grade, the algebra is introduced by using scales. However, the emphasis is generally on finding the numerical value of the unknown. That is, the mathematics textbooks emphasize numbers, operations, and arithmetic in general. Since the students have not encountered such problems or activities, and the emphasis in the textbooks is on numbers and operations, the students participating in the current study may have focused on numbers and operations while solving the problems instead of the quantitative relationship. It is important to note that the researcher carried out an informal examination of the textbooks in relation to the findings of the study; however, a more systematic research on textbooks may provide more reliable information about the use of different forms of the problems in math textbooks.

5.1.1.2 Symbolic and Iconographic Form

Two symbolic and two iconographic problems were asked to the students in the present study. Most of the students who found the correct answer reached the correct result by using the substitution method. That is, they established the quantitative relations and transferred them to the new situation. However, two students in 5th grade and one student in 8th grade found the answer by counting the number of quantities (i.e., triangles) in the problem. Although these students found the correct result, they did not construct quantitative reasoning because they focused on the numbers and reasoned numerically. Since only two methods in the correct solutions were seen at all grade levels, a table like the one created in the other problem was not needed.

Table 5.3 summarizes the characteristics of students' the reasoning that reached the incorrect answer in the symbolic and iconographic form.

Table 5. 3 The characteristics of students' reasoning that resulted in the incorrect answer

	5th grade	**6th grade**	**7th grade**	**8th grade**
Focusing on the numbers and operations	+	+	+	+
Focusing on the number of the asked quantity on the image	+	+	+	+
Answering the problem regardless of what kind of problem asked	+	+	+	+
Understanding equality but not transferring the new situation	+	+	+	+

+ : There are students with the stated reasoning

– : There is no student with the stated reasoning

As seen in Table 5.3, the characteristics or reasoning behind the students' explanations are the same at all grade levels. In other words, the same characteristics or reasoning were seen at all grade levels. Most of the students usually focused on numbers and operations as in the pictographic problems. Since some of the students could not distinguish the quantities in the addition operations, they tended to add the given numbers in the addition operation or in both equality and addition operation without paying attention to the different quantities. Consistent with this result, Çelik and Güler (2013) found some students tended to add the given numbers in real-life problems. Similarly, in the study of Güvendiren (2019), it was observed that some students tended to add the numbers given in the problem.

On the other hand, some of the students focused on the quantity asked in the addition operation. For example, in the 6^{th} problem which is how many squares are in the sum of the 2 squares and 2 triangles, the students reached the answer of 2 squares because they only focused on the square in the addition operation. In fact, they identified that squares and triangles were different quantities, but they could not establish the relationship between triangles and squares. Hence, they reasoned incorrectly. As a result, they did not distinguish between number and numerical reasoning and between quantity and quantitative reasoning, respectively in symbolic and iconographic problem forms as well as in pictographic problems.

Another characteristic of students' reasoning, which was common to all other grade levels, was not to focus on what the problem was asking. The first possibility may be that they did not pay attention to what exactly was asked. However, the second possibility may be that the quantity asked in the problems contained multiple-unit substitution. In other words, the complexity of the relationship in the problem was higher. For this reason, the students may have found the answer to the problem of how many C or triangles with a lower complexity.

Considering the reasons why students have difficulty with problems in iconographic and symbolic forms, it may be because they are not sufficiently exposed to the problems in these forms. When all the mathematics books used in the public schools from the 1^{st} to the 8^{th} grade are (e.g., Çağlayan et al., 2021; Kayapınar et al., 2021; Oğan & Öztürk, 2021), problems in symbolic and iconographic forms are seen to be rarely used in the mathematics books, similar to pictographic problems. In the activities or problems in the mathematics book, numbers, operations or finding the value of the unknown are emphasized in general instead of the quantitative relationship. For example, problems such as $x + 3 = 8$, $x - 8 = -6$, and $4 - 4x = 9x - 20$ are included in the mathematics books. However, it seems there are not many activities or problems emphasizing the quantitative relationship between shapes and letters in the books, as used in the current study. Thus, the students have not encountered such problems and activities in their education life. The reason why the students participating in this study had difficulty in establishing quantitative reasoning may be that they may not encounter such problems in the mathematics books and that quantitative reasoning is not emphasized sufficiently in the books. However, again, it is important to note that the researcher carried out an informal examination of the textbooks in relation to the findings of the study and a more systematic research on textbooks may reveal a more detailed picture of the situation.

The characteristics students' answers to the seven problems of the three forms which are pictographic, symbolic, and iconographic were discussed separately above. When the answers asked in three different forms in the study were discussed together, one 5^{th} grade student, one 7^{th} grade student, and three 8^{th} grade students answered all problems correctly, which indicated that other students had difficulty constructing the quantitative reasoning. Similarly, in the study by Dooren et. al. (2010), it was found that very few students gave correct answers to all problems. On the other hand, it can be said that they had difficulties especially in establishing the multiplicative reasoning since the relationship between the quantities given in the problems was multiplicative. Similar conclusion was found in the literature. For example, Alexander et al. (2020) investigated the 3^{th} to 5^{th} grade students' quantitative reasoning in problems including multiplicative relationship between different quantities. Also, students needed to transfer this relationship to the second situation. According to the result, students had difficulty establishing multiplicative relationship and transferring this relationship to the second situation. Similar result was found in the study of Van den Heuvel-Panhuizen and Elia (2020) which was conducted with kindergarten students. Students were asked both additive and multiplicative problems in this study. The results showed that students had more difficulty in constructing multiplicative reasoning.

Besides, Kabael and Akın (2016) reached the conclusion that students generally had difficulty to find the relationship between the number of coins and their value. That is, they did not construct the quantitative reasoning. Dooren et. al. (2010) also found that students were unable to establish quantitative reasoning because they could not distinguish between multiplicative and additive reasoning, and that they were not particularly good at multiplicative reasoning. In another study, Dur (2014) found the 6th grade students solved the problems without considering the quantitative relations, so they had some difficulties when solving problems. As a result, these conclusions are consistent with the finding of current study.

On the other hand, some studies showed the opposite findings. Degrande et al. (2017) found that most of the students at second, fourth, and sixth grade levels identified the quantitative relationship between the objects in the tasks which include additive and multiplicative relationship. In another study, Nunes et al. (2015) investigated young children’s quantitative reasoning in both multiplicative and additive forms. They found that the youngest children were able to see the relationship between quantities successfully. These results are not consistent with the findings of present study.

The present study revealed that students' reasoning arithmetically was the most common inefficient preferences among all of the grade levels. The focus of arithmetic reasoning was numbers, operations, and numerical methods (e.g., Carraher & Schliemann, 2007; Smith & Thompson, 2007; Güvendiren, 2019), and arithmetic or numerical reasoning and quantitative reasoning were not the same things (e.g., Smith & Thompson, 2007; Thompson, 1993; Thompson, 2011). Studies indicated that students should focus on quantities and quantitative relations rather than numbers and operations. If they did not change this focus, they generally used arithmetic solution methods when solving problems (e.g., Johanning, 2004; Kabael & Akın, 2016; Smith & Thompson, 2007).

For instance, Alsawaie (2008) found that some of the 5th grade students who participated in the study did not understand the relationships between quantities and so they used the arithmetical methods. Similar results were found in many different studies (e.g., Akkan et al., 2012; Dur, 2014; Kabael & Akın, 2016; Ramful & Ho, 2014). Moreover, Thompson (1993) found that 5th grade students had difficulties in solving problems because they were unable to identify how to use quantities and values of them. It was also found that students did not distinguish between "quantitative difference" (p.166) and "numerical difference" (p.166) because they thought that these were similar things. Consequently, all these results were very similar to the results of the current study. When students did not understand the quantities and the relationships between the quantities in the problems, they turned to arithmetic methods. For example, in the current study, some students tended to add the numbers in the addition process without noticing that there were different quantities, look at the number of the asked quantity, or add all the numbers in the problem.

Alexander et al. (2020) observed that even in obvious situations, students did not reach the correct result, that is, they did not establish a relationship between quantities. It was concluded that this may be because it may be difficult for students in 3rd to 5th grade level to express themselves by writing. A similar conclusion was reached in the present study as well. Although this study was conducted with middle school students (i.e., 5-8th grade students) whose grade level was higher than the participants in the study by Alexander et al. (2020), it was observed that the students had difficulty expressing their thoughts in the current study. For instance, in Part 1 (pictographic problems), all students, except for two students, could not express the quantitative relationship completely because they did not refer the weight as the quantity. Therefore, the reason why the students could not fully express the quantitative relationship may be that they have difficulty in expressing their thoughts by writing. In addition to being unable to express thoughts in writing, students generally have difficulties in understanding the problem especially algebra problems. This has been observed in many studies (e.g., Bal & Karacaoğlu, 2017; Didiş & Erbaş, 2012; Güvendiren, 2019; Koedinger & Nathan, 2004). In the current study, students may have had difficulties because of lack of comprehension. In other words, the students' lack of understanding of the problem may have caused them to be unable to understand the quantitative relations given in the problems, so they could not construct quantitative reasoning.

Besides, it is seen that quantitative reasoning is not given much importance in the curriculum in Turkiye (MNE, 2018). The curriculum begins with arithmetic topics and continues with algebra in middle school. However, the emphasis generally is on numbers and operations. In algebra, the emphasis is on finding the numerical value of the unknown. Actually, as seen in many studies, quantitative reasoning establishes a connection between arithmetic and algebra. Nonetheless, it is not given sufficient importance in the curriculum applied in Turkiye. In addition, a similar situation appears in mathematic books. When mathematics books are also examined, it seems there are not many activities or problems emphasizing quantitative relations, so students may not have encounter problems or activities that emphasize quantitative relationship. Hence, the students participating in this study may have had difficulty in establishing relationships with the problems in the study. The findings in the literature support this idea. For example, Thompson (1993) found that concluded that one of the factors that negatively affect developing students' quantitative reasoning is that they do not encounter problems emphasizing the quantitative relationship.

5.1.2 Students' Quantitative Reasoning in Different Grade Levels

The second research question was how students' quantitative reasoning differs according to grade level. The number of students who answered the problems correctly was not very different and students' reasons seen at each grade level were almost the same when the examining explanations in both correct and incorrect answers. Therefore, there were no distinct differences between students' quantitative reasoning at different grade levels in the problems addressing pictographic, iconographic, and symbolic forms. However, the literature revealed the opposite results. For instance, McMullen et al. (2013) and Degrande et al. (2017) found that as the grade level increased, students' quantitative reasoning was developed. Therefore, these results contradicted the findings of the current study.

According to studies conducted by Degrande et al. (2017) and McMullen et al. (2013), 5th grade students were expected to have lower quantitative reasoning than other grade levels. However, the number of students who answered the third problem (there was one equality which had three apples on the left side and two tangerines on the right side. It was asking how many tangerines' weight is equal to the sum of the weights of 3 apples and 2 tangerines) correctly in the 5th grade was higher than in the other levels. It was also quite interesting that the number of students who answered all pictographic problems correctly in the 5th grade was higher than the number of students who reach the correct answer in the 6th grade, and was equal to the 7th and 8th grades. In addition, the number of 5th grade students who reached the correct answer in iconographic and symbolic problems was either the same or very close to other grade levels, in some cases, even a little more. Despite the fact that these differences are not very large, the performance of the 5th grade students as the smallest of the middle school students is quite remarkable contrary to the literature. In the Covid 19 pandemic, learning loss may have occurred due to the fact that schools continued in distance through video conferencing tools. The literature involves research supporting that during the Covid 19 period, students' lack of the participations, the limited use of the methods to teach mathematics by the teachers, the socio-economic status of the families and the lack of cooperation with the teachers are among the reasons for the loss of learning mathematics (Haser et al., 2022). Therefore, 6th and 7th graders in this study may have lower performance than 5th graders due to the learning loss that might have occurred during the pandemic period.

As stated above, the results of present study show that there is no distinct difference between grade levels and especially, 8th graders still have difficulties. As seen in some studies in the literature, quantitative reasoning is expected to evolve as the grade level increases. In other words, 8th grade students are not expected to have difficulties in establishing quantitative relationships. This may be due to the fact that students do not encounter different forms of early algebra problems enough. A similar conclusion was reached in the study of Thompson (1993). In the study, quantitative reasoning of 5th grade students in problems involving additive relations was examined. It was concluded that one of the factors that negatively affected the quantitative reasoning of the students was that the students did not encounter such problems. On the other hand, 8th graders showed higher performance than other grade levels in Part 2 (i.e., symbolic and iconographic problems). These students were preparing for high school entrance exam held at the end of the eight grade year. Therefore, it was expected that they would encounter more algebra problems than other students. For this reason, 8th grades might have had higher performance in Part 2.

5.2 Conclusion of the Study

The main research question of the present study is what are middle school students' quantitative reasoning in early algebra problems given in pictorial, symbolic, and iconographic forms? The answer to the sub-research questions used to answer main research question are briefly presented below.

1.1. Does middle school (5th - 8th grade) students' quantitative reasoning differ by the forms of early algebra problems (i.e., pictorial, symbolic, and iconographic problems)? If so, how?

It was observed that middle school students' quantitative reasoning did not differ by the forms of the early algebra problems which are pictorial, symbolic and iconographic. That is, there is no such thing as if students could not solve symbolic problems while solving iconographic problems. Actually, findings showed that in each form, students could establish quantitative reasoning in some of the problems, and they had difficulty in other problems in the same form. Hence, this indicated that while forms of the problems were not determinative in the students' quantitative reasoning, the level of complexity of the relationship seemed to play a role in students' quantitative reasoning.

1.2. Does middle school students' quantitative reasoning in pictorial, symbolic, and iconographic problems differ by the grade level (5^{th} – 8^{th} grade)? If so, how?

The number of students who got the right and wrong answers was close to each other and the characteristics or reasoning behind students' explanations were generally the same. Thus, there were no distinct differences between students' quantitative reasoning at different grade levels (5^{th} - 8^{th} grade) in the problems addressing pictographic, iconographic, and symbolic forms. When examined in detail, the number of students who answered the problems correctly was the highest at the 8^{th} grade level in these problems except for one problem. However, it was observed that 8^{th} graders still had difficulties in quantitative reasoning. Although the 5^{th} grades were at the lowest grade level, their performance was higher than other grade levels in some problems and equal in some other problems, which was a remarkable finding of the study.

1.3. Does middle school (5^{th} - 8^{th} grade) students' quantitative reasoning differ in single-unit substitution and multiple-unit substitution problems given in three forms; pictorial, symbolic, and iconographic problems? If so, how?

The findings of the current study showed that middle school (5^{th} – 8^{th} grade) students' quantitative reasoning differ in single-unit substitution and multiple-unit substitution problems given in three forms which are pictorial, symbolic and iconographic problems. For example, in Part 1 (pictographic problems), most of the students found the correct result pictographic unit-value problem and pictographic multiple-unit problem, but they had generally more difficulty answering the pictographic substitution problem. In Part 2 (symbolic & iconographic problems), while students generally did not have difficulty in the iconographic single-unit substitution problem and symbolic single-unit substitution problem, it was seen that they had difficulty in the iconographic multiple-unit substitution problem and symbolic multiple-unit substitution problem. This indicated that the complexity of the relationship in the problems such as single-unit substitution and multiple-unit substitution more influential than the forms of the problems.

5.3 Implications of the Study

In the previous sections, the findings of the current study are explained, summarized and discussed. In this section, some implications for mathematics education will be mentioned. In the current study, three problem forms which are pictographic, symbolic, and iconographic were used. The results showed that while most of the students did not have difficulty in constructing quantitative reasoning in pictographic unit value, pictographic multiple-unit, iconographic substitution, and symbolic substitution problem, they had difficulties in iconographic multiple-unit substitution and symbolic multiple-unit substitution problem. In light of these findings, mathematics teachers can plan their lessons by focusing on pictographic, symbolic and iconographic problems that used in the study. Solving and discussing especially problems that students had difficulty in a classroom environment can greatly contribute to the development of students' quantitative reasoning.

Besides, it was aimed to create learning while planning the order of the problems. If teachers can plan instruction in accordance with this order, it can be made easier for students to make sense of quantitative relationships. In addition, the problems used in current study contain visual. Thus, if these problems are included in the lessons by integrating the technology, more permanent learning may be provided for students. In these ways, the transition from arithmetic to algebra may become smooth and easy, and students may stop seeing algebra as a subject made up of meaningless symbols.

As stated above, the results of the study showed that there was no distinct difference between grade levels and 8^{th} graders still had difficulties in establishing relationship between quantities. The reason for this may be that students do not encounter such problems in textbooks or other sources, or they rarely encounter them. To eliminate this situation, teachers can prepare problems sets containing different forms of the problems for students and so give students the opportunity to experience this learning experience. In addition, textbook authors can add different forms of the problems to the books. In this way, students will encounter different forms of the problems in the books. Moreover, objectives related to quantitative reasoning can be added to the curriculum. Studies have revealed the importance of quantitative reasoning which is one of the types of mathematical reasoning, and the current study revealed that students had difficulties in constructing quantitative reasoning. Therefore, the development of quantitative reasoning of students can be ensured by adding objectives that specifically address and emphasize quantitative reasoning to the curriculum applied in Turkiye.

Finally, teachers or prospective teachers may not be aware of different forms of the problems or they may not know students answers or explanations because such problems are not very common in our curriculum or textbooks. If the teachers cannot understand the problems in different forms and the cognitive processes of the problems, they may not be able to transfer them to the students. Therefore, mathematics teacher educators can increase the importance given to quantitative reasoning in the education of pre-service teachers. For example, various problems such as iconographic, symbolic, and pictographic can be included in the teacher education courses. The answers of the students and the reasoning behind them can be emphasized and discussed by the pre-service teachers in those courses. For in-service teachers, a professional development may be designed and teachers' awareness can be increased by including such problems and by discussing the answers that students can give. In this way, it may contribute to the professional development of in-service teachers.

5.4 Recommendations for Further Research

There are some recommendations for further studies based on the conclusions of the current study. Firstly, the present study was conducted with 15 students in each grade level, a total of 60 students. This constitutes a limitation of the study. To expand the findings of the current study and remove its limitations, the study may be conducted with a larger number of middle school students in any location in Turkiye. In this way, more similarities and differences between the characteristics of students' quantitative reasoning in different problems may be found.

Secondly, the same study may be conducted with primary school students (i.e., 1^{st} - 4^{th} grade level). In this way, it is examined how the quantitative reasoning of primary school students differs according to the different problem forms and grade levels. Moreover, the development of quantitative reasoning from the 1^{st} grade to the 8^{th} grade can be examined by including both primary and middle school students in the study. Because quantitative reasoning act bridge between arithmetic and algebra, examining the quantitative reasoning of students at all grade levels (1^{st} - 8^{th} grade level) can be useful for mathematics teachers.

Moreover, it is used three different forms of the problems which are pictographic, iconographic, and symbolic forms in the current study. The further research may focus on the other forms of the problems. For example, word problems, problems that require comparing other things such as lengths, widths, or prices of quantities, or problems involving additive relationships can be asked to the students. Also, the relationships in the problem can be asked in a more complex structure. Thereby, information about the quantitative reasoning of the students in different forms of the problems may be obtained.

Besides, a similar study can be done with classroom teachers, where there are different forms of the problems and the numbers in the problems are chosen at their level of understanding (i.e., more complex problems). In this way, information about the teachers' quantitative reasoning in different forms of the problems may be obtained. Or a study can be done in which the reasoning characteristics of teachers and students are compared.

Lastly, written data was collected in this present study. This is also one of the limitations of the study because the students who participated the study were not asked why they thought so. Therefore, researchers who want to research this subject can use the clinical interview method, which gives the opportunity to probe students thought. In other words, more detailed information related to the students' quantitative reasoning can be obtained by clinical interviews.

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APPENDICES

A. METU Human Subjects Ethics Committee Approval

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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20 Mayıs 2021

Konu : Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi : İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr.Öğretim Üyesi Şerife SEVİNÇ

Danışmanlığını yaptığınız Ayşenur UYGUÇ'un "Ortaokul Öğrencilerinin Niceliksel Muhakemelerinin İncelenmesi" başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **158-ODTU-2021** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

Dr.Öğretim Üyesi Ali Emre TURGUT
İAEK Başkan Vekili

B. Permission Obtained from Ministry of National Education

T.C.
SİVAS VALİLİĞİ
İl Millî Eğitim Müdürlüğü

30

Sayı : E-92255297-605.01-26958177 — 23.06.2021
Konu : Araştırma İzni (Ayşenur UYGUÇ)

DAĞITIM YERLERİNE

İlgi : a) Ortadoğu Teknik Üniversitesi Rektörlüğünün 16/06/2021 tarihli ve 54850036-044-E.218 sayılı yazısı.
b) Valilik Makamının 21/06/2021 tarihli ve E-92255297-605.01-26761423 sayılı onayı.
c) Milli Eğitim Bakanlığı Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 21/01/2020 tarihli ve 81576613-10.06.02-E.1563890 sayılı 2020/2 no'lu genelgesi.

Ortadoğu Teknik Üniversitesi Matematik ve Fen Bilimleri Eğitimi Ana Bilim Dalı yüksek lisans öğrencisi Ayşenur UYGUÇ, "Ortaokul Öğrencilerinin Niceliksel Muhakemelerinin İncelenmesi" konulu tez çalışması kapsamında, onaylı bir örneği müdürlüğümüzde muhafaza edilen veri toplama araçlarının, gönüllülük esas olmak kaydıyla Şarkışla İlçesi Samankaya Şehit Süleyman Solak Ortaokulu ve Sefil Selimi Ortaokulundaki öğrencilere uygulanması valilik makamının ilgi (b) onayı ile uygun görülmüş olup onay örneği yazımız ekinde gönderilmiştir.

Söz konusu araştırma çalışmasının bitiminde, araştırma yapan kişi tarafından sonuç raporunun bir örneğinin CD ortamında müdürlüğümüze gönderilmesi hususunda;

Bilgilerinizi ve gereğini arz ve rica ederim.

Ebubekir Sıddık SAVAŞÇI
Milli Eğitim Müdürü

Ek:
1-İlgi (b) Onay Örneği (1 Sayfa)
2-Veri Toplama Araçları

Dağıtım:
Gereği:
-Ortadoğu Teknik Üniversitesi Rektörlüğü
(Öğrenci İşleri Daire Başkanlığı)
Dumlupınar Bulvarı 06800 Çankaya-ANKARA

Bilgi:
-Şarkışla İlçe Milli Eğitim Müd.

Güvenli Elektronik İmzalı
Aslı İle Aynıdır

Lütfi DENİZ
Şef

Adres : Muhsin Yazıcıoğlu Bulvarı No:23 SIVAS
Telefon No : 0 (346) 280 58 78
E-Posta: istatistik58@meb.gov.tr
Kep Adresi : meb@hs01.kep.tr

Bu belge güvenli elektronik imza ile imzalanmıştır.
Belge Doğrulama Adresi : https://www.turkiye.gov.tr/meb-ebys
Bilgi için: L.DENİZ- Şef
Unvan : Şef
İnternet Adresi: sivas.meb.gov.tr
Faks:3462805948

Bu evrak güvenli elektronik imza ile imzalanmıştır. https://evraksorgu.meb.gov.tr adresinden 247d-8dfd-34eb-8fcd-6775 kodu ile teyit edilebilir.

T.C.
SİVAS VALİLİĞİ
İl Millî Eğitim Müdürlüğü

Sayı : E-92255297-605.01-26761423 — 21.06.2021
Konu : Araştırma İzni (Ayşenur UYGUÇ)

VALİLİK MAKAMINA

İlgi : a) Ortadoğu Teknik Üniversitesi Rektörlüğünün 16/06/2021 tarihli ve 54850036-044-E.218 sayılı yazısı.
b) Milli Eğitim Bakanlığı Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 21/01/2020 tarihli ve 81576613-10.06.02-E.1563890 sayılı 2020/2 no'lu genelgesi.
c) Valilik Makamının 14/04/2021 tarihli ve E-92255297-605.01-24208414 sayılı onayı.

Ortadoğu Teknik Üniversitesi Matematik ve Fen Bilimleri Eğitimi Ana Bilim Dalı yüksek lisans öğrencisi Ayşenur UYGUÇ, "Ortaokul Öğrencilerinin Niceliksel Muhakemelerinin İncelenmesi" konulu tez çalışması kapsamında, Şarkışla İlçesi Samankaya Şehit Süleyman Solak Ortaokulu ve Sefil Selimi Ortaokulundaki öğrencilere yönelik araştırma çalışması yapmak istemektedir.

İlgi (a) yazı ekindeki çalışma; Valilik Makamının ilgi (c) onayı ile oluşturulan araştırma değerlendirme komisyonu tarafından incelenmiş olup çalışmanın, eğitim öğretimin aksatılmaması ve katılımcıların izni olmadan resim, video ve ses kayıtlarının alınmaması kaydıyla, Şarkışla İlçesi Samankaya Şehit Süleyman Solak Ortaokulu ve Sefil Selimi Ortaokulundaki öğrencilere uygulanmasında bir sakınca görülmemektedir.

Onaylarınıza arz ederim.

Fatih AYDIN
Müdür a.
Şube Müdürü

Ek: Araştırma Değerlendirme
Formu (1 sayfa)

OLUR

Ebubekir Sıddık SAVAŞÇI
Vali a.
Milli Eğitim Müdürü

Adres : Muhsin Yazıcıoğlu Bulvarı No:23 SIVAS
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Bu belge güvenli elektronik imza ile imzalanmıştır.
Belge Doğrulama Adresi : https://www.turkiye.gov.tr/meb-ebys
Bilgi için: L.DENİZ- Şef
Unvan : Şef
İnternet Adresi: sivas.meb.gov.tr
Faks:3462805948

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C. Parent Consent Form

VELİ ONAM FORMU

Sayın veli;

Çocuğunuzun katılacağı bu çalışma, "Ortaokul Öğrencilerinin Niceliksel Muhakemelerinin İncelenmesi " adıyla, 2021-2022 eğitim öğretim yılı birinci dönem içerisinde yapılacak bir araştırma uygulamasıdır.

Araştırmanın Hedefi: Ortaokul öğrencilerinin çeşitli matematik sorularındaki niceliksel muhakemelerini nasıl kullandıklarını incelemektir.

Araştırma Uygulaması: Anket (çeşitli matematik soruları) ve gerekirse görüşme şeklindedir.

Araştırma T.C. Milli Eğitim Bakanlığı'nın ve okul yönetiminin de izni ile gerçekleşmektedir. Araştırma uygulamasına katılım tamamıyla gönüllülük esasına dayalı olmaktadır. Çocuğunuz çalışmaya katılıp katılmamakta özgürdür. Araştırma çocuğunuz için herhangi bir istenmeyen etki ya da risk taşımamaktadır. Çocuğunuzun katılımı **tamamen sizin isteğinize bağlıdır**, reddedebilir ya da herhangi bir aşamasında ayrılabilirsiniz. Araştırmaya katılmama veya araştırmadan ayrılma durumunda öğrencilerin akademik başarıları, okul ve öğretmenleriyle olan ilişkileri etkilenmeyecektir.

Çalışmada öğrencilerden kimlik belirleyici hiçbir bilgi istenmemektedir. Cevaplar tamamıyla gizli tutulacak, sadece araştırmacılar tarafından değerlendirilecek ve bilimsel amaçla kullanılacaktır.

Uygulamalar, genel olarak kişisel rahatsızlık verecek sorular ve durumlar içermemektedir. Ancak, katılım sırasında sorulardan ya da herhangi başka bir nedenden çocuğunuz kendisini rahatsız hissederse cevaplama işini yarıda bırakıp çıkmakta özgürdür. Bu durumda rahatsızlığın giderilmesi için gereken yardım sağlanacaktır. Çocuğunuz çalışmaya katıldıktan sonra istediği an vazgeçebilir. Böyle bir durumda veri toplama aracını uygulayan kişiye, çalışmayı tamamlamayacağını söylemesi yeterli olacaktır. Yapılan çalışmaya katılmamak ya da katıldıktan sonra vazgeçmek çocuğunuza hiçbir sorumluluk getirmeyecektir.

Onay vermeden önce sormak istediğiniz herhangi bir konu varsa sormaktan çekinmeyiniz. Çalışma bittikten sonra bizlere telefon veya e-posta ile ulaşarak soru sorabilir, sonuçlar hakkında bilgi isteyebilirsiniz. Saygılarımızla,

Araştırmacı: Ayşenur UYGUÇ

İletişim bilgileri: 0536 522 7282 / aysenur.uyguc@gmail.com

Velisi bulunduğum .... sınıfı .... numaralı öğrencisi ....…………………………………’in yukarıda açıklanan araştırmaya katılmasına izin veriyorum. (Lütfen formu imzaladıktan sonra çocuğunuzla okula geri gönderiniz*).

…./…../……..

İsim-Soyisim İmza:

Veli Adı-Soyadı:

Telefon Numarası: