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*This study was conducted to investigate the characteristics of algebra tasks that middle school preservice mathematics teachers developed at the end of a three-week training related to examining and categorizing algebra items in the previous high school entrance examinations (2018-2020). Twenty-nine third-year middle school preservice mathematics teachers participated in the study. The data of the study included 12 algebra tasks generated by preservice teachers at the end of the training and their characteristics. Findings showed that preservice teachers were able to develop cognitively demanding algebra tasks. Most of the tasks aimed to assess students' knowledge in geometry and measurement learning areas besides algebra. Lastly, most of the tasks were related to manipulating symbols, while four tasks focused on modeling problems using equations or algebraic expressions.*

*Keywords: Task development, preservice teachers, cognitively demanding algebra tasks, high stakes assessment.*

## Introduction

Assessment can be categorized based on its purposes. Formative assessment or assessment for learning is used purposefully for learning (Laud & Patel, 2013). It provides students and the teacher with a rich stream of information that can be used to adjust instruction to meet students' needs and enhance their learning (Wiliam, 2007). On the other hand, other potential actors such as universities, policymakers, and administrators need summative data since they cannot deal with the vast quantity of evidence collected through formative assessment (Burkhardt & Schoenfeld, 2018). Hence, summative assessment or assessment of learning is crucial to measure students' level of accomplishment, especially in countries where the result of high-stake exams may cause a big change in students' lives.

The results of students' performance in high stake exams directly impact teachers and students in some countries like the UK and the USA (Burkhardt & Schoenfeld, 2018), including Turkey. The types of tasks in the exams and the valued competencies influence teachers' actions in the classroom (Barnes et al., 2000) and therefore students' learning. In some cases, most classrooms' learning activities were reformed and became parallel to the task structures covered in the exams (Burkhardt & Schoenfeld, 2018).

In the case of Turkey, the High School Entrance Exam (HSE) system was changed in 2018. One of the fundamental changes made in the HSE was the structure of the items covered in the exam. Before the change, HSE measured basic skills at the level of knowledge, comprehension, and application. Now, high-level skills such as making interpretation and inference and analytical thinking are measured (Biber et al., 2018). Reports informing the results of the HSE exams held in 2018 and 2019

showed that mathematics tests had the lowest rate of correct response of all the subjects (Ministry of National Education [MoNE], 2018a; 2019). One of the reasons for this situation might be that mathematics teachers were unprepared for the cognitively demanding assessment tasks and could not find sufficient resources to use in their classrooms (Biber et al., 2018). Since tasks in the mathematics textbooks are not compatible with the HSE items and the resources teachers use in their lessons are insufficient, preservice mathematics teachers' awareness about the cognitively demanding HSE items needs to be improved. In addition, whether preservice teachers can generate cognitively demanding tasks is worthy of investigation since these tasks can be used as teaching tools as well as assessment tools.

This study aimed to investigate the characteristics of algebra tasks that middle school preservice mathematics teachers (PMTs) developed after participating in a three-week training related to examining and categorizing algebra questions in the previous HSE examinations (2018-2020).

Algebra is a bridge between mathematics and other branches of science (Erbaş et al., 2009). Research points out the importance of developing students' algebraic thinking starting from kindergarten (Stephens et al., 2017), and one of the ways this could be achieved is through professional development (Kieran et al., 2016). Although there are studies focused on investigating the questions that PMTs pose during diagnostic algebraic thinking interviews (e.g., van den Kieboom, 2014), limited research exists on examining PMTs' generation of cognitively demanding algebra tasks.

Kaput (2008) focusing on arithmetic and algebra problems proposed a framework that included two core aspects for algebraic thinking. The two core aspects of algebra were "(A): algebra as the systematic symbolizing of generalizations of regularities and constraints" and "(B): algebra as syntactically guided reasoning and actions on generalizations expressed in conventional symbol systems" (Kaput, 2008, p. 11). Kaput stressed that while both aspects of algebra are significant, school algebra generally focuses on Core Aspect B, more specifically reasoning and actions on generalizations. While in Core Aspect A, students are encouraged to notice regularities, generalize and represent those generalizations. In this study, Kaput's (2008) framework was used to categorize PMTs' algebra tasks with respect to the core aspects of algebra.

Measuring students' ability requires the classification of levels of thinking. Bloom's revised taxonomy provides a measurement tool for thinking and classifies thinking into six cognitive levels of complexity: remembering, understanding, applying, analyzing, evaluating, and creating (Anderson et al., 2001). In a similar way, Smith and Stein (1998) proposed four categories of cognitive demand; (i) memorization; (ii) procedures without connection (PW/oC); (iii) procedures with connection (PWC); and (iv) doing mathematics in order to help teachers select and create cognitively demanding tasks to increase students' ability to think and reason. They identified the first two categories as low-level demands while the last two categories as high-level demands. We used Bloom's revised taxonomy and Smith and Stein's (1998) categorization to categorize the cognitive levels of PMTs' algebra tasks in the study.

## Methods

Basic qualitative research method (Merriam, 2009) was employed in this study in order to reveal the characteristics of algebra tasks generated by PMTs.

## Participants and Study Context

The study participants were 29 third-year middle school preservice mathematics teachers who were enrolled in a four-year middle grades (grades 5-8) mathematics teacher education program at a public university in Ankara, Turkey. The program mainly offers introductory education and mathematics courses in the first two years. The departmental courses mostly start in the third year of the program. They include courses that focus on developing pedagogical content knowledge, such as the Methods of Teaching Mathematics in Middle Schools I and II. In their fourth and last year in the program, PMTs take School Experience and Practice Teaching courses. Related to the context of this study, a semester before this study was conducted, PMTs were enrolled in the Assessment of Learning in Science and Mathematics course. Throughout this course, PMTs were introduced to the different types of assessment, including formative, summative, and diagnostic assessment types. They were also asked to develop different assessment instruments, including multiple-choice, short response, true-choice, and open-ended with their rubrics.

## Data Collection

The study's data were drawn from a study* that aimed to investigate middle school PMTs' conceptions of algebra, their awareness of the characteristics of algebra items in HSE, and their improvement in generating algebra tasks at the end of the training. The training was carried out in the Methods of Teaching Mathematics in Middle Schools II course, which focused on teaching "proportional and algebraic thinking", "statistics", and "probability", respectively. The Methods course was offered in the Spring Semester of 2020-2021 academic year by the first and second authors, and the researchers implemented the training of this study. The PMTs attended the course through online education via Zoom. The training took place as part of the algebraic thinking weeks.

The PMTs were asked to read the chapter on algebraic thinking by Van de Walle et al. (2013) and several other articles and a book chapter that were mainly intended for in-service and preservice teachers. These readings were discussed in class, and related activities were conducted in small groups. Usually, whole-class discussions took place after the small group discussions. At the beginning of the semester, two groups were assigned to design lesson plans focusing on algebraic thinking, choosing an objective from the curriculum. These plans were implemented through micro-teaching, and the groups received oral feedback from the instructors and their classmates. Then the training for this study started, which lasted for three weeks (12 class hours). PMTs were asked to analyze 2018, 2019, and 2020 HSE algebra items first individually and note the characteristics of the questions (objectives/contents addressed by the problems, and cognitive levels according to Bloom's revised taxonomy and Smith and Stein (1998), and their justifications). Then they discussed their analysis in the same groups of 4 or 5 each week. After small group discussions, whole-class discussions took place, and some sample items the researchers chose were discussed together. At the end of the training, groups were asked to generate two cognitively demanding open-ended algebra tasks in the same groups (6 groups and 12 tasks in total). They were also asked to write the related

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objectives/contents from the curriculum, expected student responses (both correct and incorrect), and cognitive levels according to Bloom's revised taxonomy and Smith and Stein's (1998) categorization.

**Data Analysis**

For the scope of this paper, we focused on 12 algebra tasks generated by PMTs at the end of the training and their characteristics. The data were analyzed through content analysis. Cognitive levels of the tasks were categorized based on both Smith and Stein's (1998) framework and Bloom's revised taxonomy (Anderson et al., 2001). We used Kaput's (2008) framework to determine which core aspect (A or B) each task focused on. Kaput's (2008) two core aspects and the cognitive levels of the tasks were determined separately by the researchers and later discussed to reach a consensus. Learning areas and contents were accessible from the national middle school mathematics curriculum (MoNE, 2018b).

## Findings

Findings showed that all the tasks developed by PMTs were cognitively demanding algebra tasks according to Smith and Stein's (1998) categorization and Bloom's revised taxonomy. We classified all tasks developed by PMTs as PWC. Furthermore, according to Bloom's revised taxonomy, we classified PMTs' tasks at two levels: nine tasks at the analyzing level, three tasks at the applying level. When we compared our and PMTs' decisions about the levels, we found medium to a high inter-rater agreement. Specifically, out of 12 tasks, we found the level of the eight tasks the same using Blooms' revised taxonomy (about 67% agreement); the disagreement was mostly between the levels of applying and analyzing. The agreement increased to about 92% for Smith and Stein's categorization.

Most of the algebra tasks generated by the PMTs aimed to assess students' knowledge in geometry and measurement learning areas. PMTs used geometric shapes to assess students' ability in doing operations with algebraic expressions in almost all these tasks. More specifically, in these tasks, students were expected to measure the area/perimeter/length of a single or complex shape by making addition, subtraction, or multiplication with algebraic expressions. On the other hand, the remaining tasks were prepared to assess students' understanding in only the algebra learning area. (See Table 1 for all 12 tasks)

To exemplify, in Task 11 (T11, see Table 1) generated by Group 6, students were required to use a proportional relationship to find the price of 1 L of Brand A and Brand B juice as algebraic expressions and solve first order inequalities with one unknown to reach the answer. This task assesses students' knowledge only in the algebra learning area. We classified the task's level of cognitive demand as PWC. Students need to engage with conceptual ideas, including using the information from the table provided, setting up an inequality to compare the prices of the juice brands for the same amount to complete the task and explain their reasoning. We also categorized it at the analyzing level considering Bloom's revised taxonomy since it requires relating parts to one another and an overall structure (Anderson et al., 2001) and making connections between different contents, including integers, ratios, and inequalities.

**Table 1: Characteristics of the tasks developed by PMTs**

| Groups | Ts | Smith & Stein | Bloom's Revised Taxonomy | A brief description of the contexts addressed by the tasks |
|---|---|---|---|---|
| G1 | T1* | PWC | Analyzing | This task asks students to calculate the basal metabolic rate to decide on the number of calories people need to take to not gain weight and to propose a diet to have less/more calories. |
| | T2 | PWC | Applying | This task asks students to find the shortest distance, which a goat can use to climb the top of the mountain using a right triangle. |
| G2 | T3 | PWC | Analyzing | This task asks students to divide a square field with one side $2x + 4$ cm into two equal parts to plant cotton to one part and corn to the other. Students are required to decide where two identical 360-degree rotatable fountains are set up to water the corn field's maximum area and find the least area in an algebraic expression where water cannot reach. |
| | T4 | PWC | Analyzing | In this task, students are expected to form a T shape by using all tangram pieces in the square-shaped tangram board with an area of $16x^2 - 8x + 4$ cm$^2$ and write the algebraic expression for the height of the T shape. |
| G3 | T5 | PWC | Analyzing | Given a scenario, students are asked to create an Atatürk corner using rectangular materials whose areas and short side lengths were given algebraically. They are also asked to express the area of Atatürk's picture algebraically. |
| | T6* | PWC | Applying | This task gives a situation where the discount is applied for the amount of the tickets purchased and asks students to write inequalities that express the given situations. It also asks students to find the profit if two groups of students buy the tickets together instead of separately. |
| G4 | T7 | PWC | Analyzing | The task gives information that there are two gardens whose perimeters are equal. The area of one is $9x^2 + 18x + 9$ cm$^2$, while the other's area is 35 cm$^2$ and asks the sum of the values *x* can take. (Each side is a natural number). |
| | T8 | PWC | Applying | In this task, a bus route is given on a map, and two different ticket types (students and adults) are defined. Ticket prices are given as algebraic expressions. Students are expected to find the cost of an adult ticket to go to a city in the route, examining two traveling situations and solving it. |
| G5 | T9 | PWC | Analyzing | This task asks students to create squares from cardboard with an area of $4x^2 - 8x + 4$ cm$^2$ and regular triangles and pentagons whose one side is half of the length of one side of the square. Students are expected to place these geometric shapes 3 cm apart on a rope with $15x + 45$ cm length and find how many geometric shapes are used. |
| | T10 | PWC | Analyzing | This task gives a situation where the water pipes are laid in a square garden with a side of $3x + 6$ m. The length of pipes to be laid adjacent to the garden walls is $2x + 4$ m. Pipes narrow by half after 3 m. The narrowed water pipes pass over each other. Students are required to find the area in the garden where the water pipe is not laid. |
| G6 | T11 | PWC | Analyzing | In this task, two types of juice brands (Brand A 200 ml and Brand B 500 ml) are defined, and the price of each is given as algebraic expressions, $4x + 6$ and $5x + 40$ for Brands A and B, respectively. Students are expected to find how many Turkish Liras (TL) Ayşe paid for 1 L of juice at most if she bought Brand A juice and made a profit and to explain their reasoning. (The money she pays is an integer.) |
| | T12 | PWC | Applying | In this task, students are given the information that the length of one side of a house with a square base is $2x + 2$ m, the area of one side of the house is two times the floor area, and the area of one window of the house is one-eighth of the side area. They are expected to find the area of the exterior of the house to be painted. |

**Two groups developed questions that had multiple parts. For those questions, the highest level of cognitive demand or taxonomy level was noted as the levels of the questions.*

Task 7, (T7, see Table 1), created by Group 4, required to use algebra, geometry and measurement knowledge. Students were expected to factorize the algebraic expression to find the perimeter of the square and solve the resulting first-order equation with one unknown to find the sum of the unknown ($x$) values. We classified the task's level of cognitive demand as PWC since the solution requires cognitive effort, including setting up an equation to find the values for $x$. Students cannot follow procedures mindlessly, and they need to make connections between different learning areas, algebra, geometry, and measurement. We also classified the task as analyzing according to Bloom's revised taxonomy since the task requires analytical skills. Specifically, students were asked to find the possible values for $x$, using the information that the perimeters of the shapes were equal given the areas and factoring 35 to find the length of one side of the rectangle.

When we examined the tasks developed by the groups according to the framework put forward by Kaput (2008), most of the tasks were closely related to Core Aspect B, which was explicitly about manipulating symbols. Specifically, while four out of 12 tasks were categorized closely related to Core Aspect A, the rest were found more closely related to Core Aspect B. This finding could be because, based on our analysis, PMTs aimed to assess students' abilities to make operations with algebraic expressions in most of the algebra tasks generated. To exemplify, T11 was a task categorized more closely with Core Aspect A. Students are expected to use algebraic expressions to set up an inequality to model the problem context in the task. In comparison, we found T7 primarily concerned about solving equations to find the possible values for $x$. The other tasks related to Core Aspect A also involved modeling the problem context using equations or algebraic expressions.

## Discussion

Mathematical tasks are at the center of students' learning since tasks give messages to the learners about what mathematics is and what doing mathematics includes (National Council of Teachers of Mathematics [NCTM], 1991). The tasks that mathematics teachers select, adapt, or develop and implement with their students have paramount importance since they also influence the level of students' learning. Hence mathematics teachers need to be aware of levels of demands to generate cognitively demanding tasks. On the other hand, some studies revealed that although higher-order thinking skills are crucial for education, what teachers understand from this and how they apply it in their instruction are unclear (Schulz & FitzPatrick, 2016). Turkish mathematics teachers can be an example of this situation since they had difficulty in finding resources for cognitively demanding tasks (Biber et al., 2018). This study showed that the algebra tasks PMTs developed were cognitively demanding. We classified all tasks generated as PWC. They were also at the analyzing (9 tasks) and applying (3 tasks) levels according to Bloom's revised taxonomy. This finding might indicate that PMTs were able to generate cognitively demanding algebra tasks. Even if PMTs created these tasks as possible HSE items, they might also be selective in the tasks they bring to the class and give place to the cognitively demanding tasks in the classrooms.

This study also showed that PMTs focused on geometry and measurement learning areas while generating cognitively demanding algebra tasks. They mainly preferred to assess students' learning in making operations with algebraic expressions by using geometric shapes. The same trend was also seen in HSE items. When we investigated the past HSE items, we observed that if the items were

prepared to assess students' algebra, geometry, and measurement knowledge, students were expected to do operations with algebraic expressions by using geometric shapes. Since PMTs investigated the HSE items of the last three years during the training, they might have been influenced by these items and tended to prepare similar items.

Regarding the analysis of core aspects of algebra, the findings showed that most of the generated tasks focused on calculation and solving instead of relating and representing. Kieran (2004) suggested several things to develop students' algebraic thinking. These include "a focus on both representing and solving a problem rather than on merely solving it" and "a focus on relations and not merely on the calculation of a numerical answer" (p. 141). Although these tasks were developed for HSE, what is being asked in the examinations influences what teachers and students value in their classrooms (Barnes et al., 2000). Therefore, it is essential that PMTs also try to generate tasks that focus on generalizing, relating, representing, and solving, and calculating. We suspect that this finding might also be due to the tendency of the past HSE items that PMTs examined in the training. Although some tasks were related to Core Aspect A, we identified the majority associated with Core Aspect B.

As part of this study, PMTs only examined 2018, 2019, and 2020 HSE algebra items. Examining only the past HSE items might be a limitation that influenced the characteristics of algebra tasks generated by PMTs. Hence using different task sources can be recommended for future studies. Investigating the HSE items holistically, without differentiating the learning area, can also be suggested to see the general characteristics of items that are not specific to the learning area. Furthermore, since demanding tasks used in the classrooms positively affect students' learning, encouraging PMTs to develop cognitively demanding tasks in algebra and different learning areas can also be suggested.

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