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Components of collective argumentation in geometric construction tasks

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ABSTRACT This study aims to examine the components of collective argumentation of pre-service middle school mathematics teachers during geometric construction activities. To scrutinize this issue, case study research was utilized. The participants were 14 pre-service middle school mathematics teachers who worked collectively by forming four groups. During the data collection process, the groups worked on four geometric construction tasks by using compass-straightedge or GeoGebra. The findings presented that the collective argumentation processes were depicted by means of eleven components. In more detail, the six components of Toulmin's argument model which are data, warrant, claim, backing, rebuttal, and qualifier were insufficient to represent collective argumentation. Instead of claim, the term conclusion was used in this study since the associated data and warrant were provided in the argumentation. The collective argumentation processes of the groups involved not only the mentioned six components but also the five additional components, which were named conclusion/data, target conclusion, guidance, challenger, and objection. The new components might be used while investigating the argumentation process in other disciplines.

Keywords: *Argumentation components, Collective argumentation, Geometric construction*

Geometrik inşa etkinliklerinde ortaklaşa argümantasyonun bileşenleri

ÖZ Bu çalışma, ortaokul matematik öğretmen adaylarının geometrik inşa etkinlikleri sürecindeki ortaklaşa argümantasyon bileşenlerini incelemeyi amaçlamaktadır. Bu konuyu araştırmak için durum çalışmasından yararlanılmıştır. Katılımcılar, dört grup oluşturarak ortaklaşa çalışan 14 ortaokul matematik öğretmen adayı olarak belirlenmiştir. Veri toplama sürecinde, gruplardan dört geometrik inşa etkinliği sırasında araç olarak pergel-çizgeç veya GeoGebra kullanarak çalışmaları istenmiştir. Çalışmanın bulguları, ortaklaşa argümantasyon sürecinin on bir bileşen aracılığıyla betimlenebildiğini ortaya koymuştur. Daha ayrıntılı ifade etmek gerekirse, Toulmin'in argüman modelinde veri, gerekçe, iddia, destek, çürütücü ve niteleyen olarak isimlendirilen altı bileşenin ortaklaşa argümantasyonu temsil etmekte yetersiz kaldığı görülmüştür. Ayrıca, bu çalışmada yer alan argümantasyon süreçlerinde katılımcılar birbiriyle bağlantılı veriler ve gerekçeler ortaya koymuştur. Bu nedenle, Toumin'in argümantasyon modelindeki iddia terimi yerine sonuç terimi kullanılmıştır. Grupların ortaklaşa argümantasyon süreçleri, sadece bahsedilen altı bileşeni değil, aynı zamanda sonuç/veri, hedef sonuç, rehber, meydan okuma ve itiraz olarak adlandırılan beş ek bileşeni de içermektedir. Yeni bileşenlerin, diğer disiplinlerdeki argümantasyon süreçleri araştırılırken kullanılabilmesi beklenmektedir.

Anahtar Sözcükler: *Argümantasyon bileşenleri, Geometrik inşa, Ortaklaşa argümantasyon*

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INTRODUCTION

Argumentation refers to a range of meanings depending on the discipline and context. In terms of mathematics, argumentation is expressed as a process in which a mathematical discourse is enhanced via pursuing logical connections (Smith, 2010). Argumentation is regarded as one of the themes at the heart of mathematics education research as well as mathematics research (Mariotti et al., 2018). In a similar vein, Conner et al. (2014a) emphasized the importance of comprehending, recognizing, and conducting arguments in mathematics. More precisely, Conner et al. (2014a) put emphasis on the term collective argumentation and explained it as "participating in discussions in a distinctively mathematical way can be framed as collective argumentation, where collective argumentation involves multiple people arriving at a conclusion, often by consensus" (p. 401). Collective argumentation usually involves a teacher and a small group of students who are participating in an investigative process collaboratively (Cervantes-Barraza et al., 2020). While analyzing collective argumentation, Toulmin's argument model (2003) is often used in mathematics education research (Carrascal, 2015; Conner et al., 2014b). Actually, in the investigation of any argumentation-related construct, Toulmin's model (2003) is one of the most frequently used frameworks (e.g., Boero et al., 2010; Krummheuer 1995; Pedemonte & Balacheff, 2016). Toulmin's model can be utilized to analyze arguments ranging from exploratory ones to more deductive ones (Boero et al., 2010). In this respect, the present study approaches the analysis of collective argumentation of pre-service middle school mathematics teachers by following Toulmin's argument model.

To prepare a setting which supports the groups of pre-service middle school mathematics teachers to engage in an argumentation collectively, geometric construction was decided to be the subject of the tasks. Geometric construction tasks were planned in a way that the steps of construction were not directly given to the participants. Hence, it was expected that they would need to think about the tasks thoroughly and justify the logic of steps in the process. The challenging environment provided by geometric constructions leads students to develop a deeper point of view towards geometry, improve their thinking and reasoning abilities (Stupel & Ben-Chaim, 2013), and apply not only the previous knowledge about geometry but also higher order thinking skills (Lim, 1997). Moreover, Barabash (2019) emphasized that geometric construction tasks can be modified based on the different levels of difficulty and also geometric construction presents a substantial source for exploration of geometric concepts by considering various creative approaches. Due to the mentioned benefits and applications of geometric construction, it was anticipated that it is an appropriate mathematical concept for this study.

In light of these points, the purpose of the study is structured as follows: to examine in detail the components of collective argumentation of pre-service middle school mathematics teachers during geometric construction tasks.

Components of Toulmin's Argument Model

According to Toulmin (2003), an argument may involve six components which are data, claim, warrant, backing, qualifier, and rebuttal. In the review of literature, it was seen that there are some differences in the definitions of the six components. Thus, to examine these components in detail and to determine the extent of them for this study, some studies in the literature (e.g., Boero et al., 2010; Brinkerhoff, 2007; Conner et al., 2014a, 2014b; Freeman, 2005; Knipping, 2008; Krummheuer, 1995; Metaxas et al., 2016; Nardi et al., 2012; Stephan & Rasmussen, 2002; Toulmin, 2003; Van Ness & Maher, 2018; Yu & Zenker, 2020) were compared by focusing on both forms and functions in the argument. By combining a variety of definitions of these components presented in the literature, the following table was prepared.

According to the definitions in Table 1, the common idea related to the definitions of data is the basis for the conclusion and warrant is any statement that justifies the connection between data and conclusion. While reviewing the related literature, it was noticed that the difficulty of distinguishing data from warrant in practice was stated in some studies (e.g., Knipping, 2008). Moreover, the terms claim and conclusion were used as having the same meaning in some studies (e.g., Stephan &

Rasmussen, 2002; Toulmin, 2003). For example, Van Ness and Maher (2018) stated that claim is a conclusion which can be declared before or after the data in the flow of an argument. In this perspective, a claim might be either a mathematical statement needed to be clarified or a solution to a problem. On the other hand, according to the study of Knipping (2008), the term claim is used when data and warrant are not provided, and the term conclusion is used in the case that data and warrant are provided. This perspective was also utilized in the present study.

Table 1.
Summary of Definitions of the Components of Argumentation

Component	Definition
Data	Some form of facts, evidence, statements, undoubted statements, specific piece of information or general information, and methods or mathematical relationships that function as the foundation, basis, ground, and inference of the claims/conclusions/argument, and also support, justify, and lead to the claims/conclusions/argument.
Warrant	A general statement, a rule, a principle or a definition that acts as bridge between data and claim/conclusion, functions as the rule of inference that authorizes the legitimacy of the step from data to claim/conclusion, justifies the relationship/connection between data and claim/conclusion, explains how data lead to the claim/conclusion, and provides more evidence to clarify an argument.
Claim/Conclusion	The statement being argued, established, justified, and inferred from data and also the assertion put forward for general acceptance or basic convictions.
Backing	The statement which supports warrants, describes the validity of warrants, and explains why warrants have the authority.
Rebuttal	Conditions/circumstances/exceptions under which conclusion/claim would not hold and the warrants would not be valid, and also all exceptions regarding the argument.
Qualifier	The statement which expresses the degree of confidence and the certainty of claim/conclusion and describes the strength of argument/claim/conclusion as determined by warrant.

Studies using Toulmin's model (2003) generally involve data, warrant, and claim as components. Many studies, however, do not cover backing, rebuttal, and qualifier or do not mention them in detail. Although instances, where backing is uttered, have minor differences such as explaining the authority of warrant and offering in the case that warrant is in doubt, the common ground of all definitions of backing is to support warrant. It can be summarized that rebuttal represents the statements which weaken the overall stance of the argument. For example, when a rebuttal is inserted as an exception regarding the statement in the warrant, the force of the warrant would be weakened (Boero et al., 2010). Lastly, qualifier expresses the certainty of the conclusion, and it may be represented implicitly or explicitly by stating a word such as certainly or probably in an argument (Metaxas et al., 2016).

It was seen that some studies (e.g., Boero et al., 2010; Conner et al., 2014a, 2014b; Verheij, 2005) did not directly use Toulmin's model and conducted some modifications on the display of the model in the light of the purposes and contexts of the studies. Although the components of argumentation are quite similar to Toulmin's model, some variations such as the different locations of the rebuttal and qualifier components are noticeable. For example, Verheij (2005) offered a layout starting with the data at the bottom and continuing to claim upwards, which does not match with what Toulmin developed as the layout of an argument. Variations of the application of the model are not limited to the layout of components. There are studies that identified the need for some additional components of argumentation. For example, Bench-Capon (1998) excluded the qualifier component and offered a new component, which was called presupposition. In more detail, the presupposition component was put forward as "supposed to represent propositions assumed to be true in the context, and so which do not need to be discussed but which can be made explicit if required" (p. 7). In addition, Bench-Capon (1998) stated that the claim of an argument might function as the data of another argument. In a similar vein, the idea that the conclusion of an argument may be the data of the following argument was taken into consideration in other studies (e.g., Conner et al., 2014a; Krummheuer, 1995). For example, Knipping (2008) offered to use a component called data/conclusion so as to represent the phrases that are both conclusion of an argument and the data of the next one and considered it as an indicator of the transition to a new argument. Similar to the data/conclusion component, Conner et al. (2014a) noticed that some statements function in favor of two components. Thus, they labeled some statements as data/claim and warrant/claim.

Collective Argumentation

Collective argumentation does not need to be developed in a direct manner. Throughout the interaction process, some revisions and corrections can be conducted, and the controversial points are aimed to be eliminated (Krummheuer, 1995). Yackel (2002) underlined the importance of collective argumentation as follows: “collective argumentation is a particularly useful construct for analyzing the nature of activity within mathematics classrooms that are characterized by collaborative problem solving and whole class discussions” (p. 424). In a similar vein, Brown (2017) remarked that collective argumentation “has the potential to create communicative spaces in the classroom where students have regular opportunities to ‘represent’, ‘compare’, ‘explain’, ‘justify’, ‘agree’ about and ‘validate’ their ideas” (p. 186).

Based on the social construction of knowledge, discussion conducted during a classroom activity has a critical role in learning (Mariotti et al., 1997). When students are involved in social interaction, it is seen that they start to get in charge of their own learning by being active and productive (Balacheff, 1999). Collaboration both with peers and with the experts is mentioned as a facilitator for promoting the conceptual understandings of students due to the numerous benefits to the overall structure of the instruction. Among these benefits, focusing on the content in a more thorough way, evoking the previous knowledge by means of argumentation, discussing alternative aspects of the concepts, offering more than one solution for the problem, developing problem-solving skills, increasing the quality of the discourse, and supporting higher level thinking of students can be listed (O’Donnell, 2006).

Rationale of the Study

As mentioned, it was observed that the use of the argument model of Toulmin for the examination of the argumentation process is a recurring theme among the studies. On the other hand, the argumentation model of Toulmin has been subject to some criticisms (Conner et al., 2014b; Mariotti et al., 2018; Pedemonte & Balacheff, 2016). For example, it was criticized since it is frequently used to examine arguments which are deductive in nature. The reason behind this situation was stated as the descriptions of the warrant component. In the case that warrant is explained by using the terms rule, principle, definition, algorithm or formula functioning as the bridge between data and claim, it seems that the argument takes a stand in a deductive way. However, all arguments in mathematics are not necessarily deductive (Conner et al., 2014b; Inglis et al., 2007). According to Pedemonte and Balacheff (2016), the knowledge base of the arguers is occasionally disregarded in the structure of argument and warrants are ambiguous in some cases when the rule used is not described explicitly. Due to such criticisms stated in the literature, this study aimed to have a critical look at the application of Toulmin’s model and carry out close scrutiny of the roles of the components of comprehensive argumentation processes and the possible new components of collective argumentation in class. In this manner, the components of collective argumentation were reconstructed since the structure of complex argumentation process was focused. Besides, there are some components which are difficult to discriminate from the flow of the argument due to some overlapping points and unclear edges. As stated, data and warrant can be presented among such components. Toulmin (1958) explained the distinction between the functions of data and warrant respectively as “in one situation to convey a piece of information, in another to authorise a step in an argument” (p. 99). Thus, to investigate the components of collective argumentation process in detail gains importance in terms of specifying the scope of each component. In this respect, this study might provide valuable feedback to other studies which plan to employ Toulmin’s model.

The narrower form of Toulmin’s model, which involves data, warrant, claim, and backing was used by some researchers (e.g., Krummheuer, 1995). On the other hand, the importance of counting all components of Toulmin's model while examining the whole range of the argumentation has been highlighted. Using the restricted form of Toulmin's model causes to downplay the functions of the other two components, which are qualifier and rebuttal (Inglis et al., 2007). For instance, Inglis et al. (2007) underlined the importance of qualifiers in the arguments in terms of presenting proper justification for the conclusion and stated that the use of the reduced version of Toulmin's model might lead to consider argumentation as covering the absolute conclusions only. In this manner, it can be stated that this study also called attention to the possibility of the presence of some other components in argumentation depending on the context that argumentation takes place.

Another point to note herein is that the social norms, which are arranged through the interactions in the classroom (Yackel, 2001), and the sociomathematical norms, which are identified as the norms particular to mathematics (Yackel & Cobb, 1996), have some overlapping points with the functions of the components of collective argumentation situated in the present study. For example, as a social norm, students are anticipated to justify their ideas and reasoning in the classroom, which could be considered as a feature underlying the warrant component. Students are also expected to probe questions in circumstances there were disagreements, which could be considered to be related to the functions of the components of rebuttal and objection in the argumentation depending on the presence of the reasoning proposed for the statement. To set up challenges to enrich the issue discussed in the classroom is also mentioned among the norms, which is quite relevant to the challenger component. In a similar vein, to promote the discussion in an inquiry-based mathematics classroom is one of the roles which are cast to teachers (Yackel & Cobb, 1996). The mentioned role of the teachers can be associated with the guidance component that emerged in the analysis of this study. As such social norms signify the characteristics of the interactions taking place in classrooms (Yackel, 2001), it can be analogized that the components of argumentation characterize how argumentation is enhanced by small groups or the whole classroom. In this respect, an in-depth investigation of the content of components might be explanatory in terms of the social norms.

As mentioned, the context of this study in terms of the mathematical domain is geometric construction. As Sanders (1998) stated, geometric construction "lends visual clarity to many geometric relationships" (p. 554). While working on geometric constructions, students should be encouraged to be active, evaluate, make presumptions, and discuss their ideas (Lim, 1997). In this manner, to lead the groups to have rich collective argumentation, geometric construction tasks were involved in this study. Moreover, the use of different tools, which are compass-straightedge and GeoGebra, while working on the tasks might give insight into the effect of using different tools on the collective argumentation.

All in all, the research question of this study is stated as follows: What are the components of collective argumentation processes of pre-service middle school mathematics teachers while working on geometric construction tasks?

METHOD

Research Design, Participants, and Context

Since it was critical to gain a clear understanding of the collective argumentation process, case study research was determined as the research design of this study. In more detail, Yin (2014) introduced four basic case study designs which are organized with respect to two issues, namely, the number of the cases and the number of the units of analysis. Based on the matrix of Yin (2014, p. 50), multiple-case holistic design was employed in this study. In this respect, there are four cases which are collective argumentation processes of each group throughout geometric construction tasks and there is one unit of analysis which is the component of collective argumentation.

Based on the results of the pilot study, this study was aimed to be conducted with juniors in Elementary

Mathematics Teacher Education program at a university in Ankara, Türkiye. The program provided a four-year education to train future mathematics teachers of grades 5-8. To graduate from the program, pre-service middle school mathematics teachers are required to complete elective courses and required courses, which cover mathematics, mathematics education, educational science, and common courses.

The participants were selected through purposeful sampling. The first criterion focused to determine the participants was the accessibility since it was expected that the researchers would spend the plenty of time with participants during the data collection process. Secondly, regarding the year level of the participants, it was anticipated that juniors and seniors would have the highest potential for gathering the detailed information in terms of the basis of the study. During the pilot study, it was seen that it was difficult to find voluntary seniors due to their occupation concerns and the high workload of the tasks of the study. Thus, for the main study, junior pre-service middle school mathematics teachers were determined as the participants. Of all the juniors in the program, the participants were also selected by following a criterion. Before the pilot study, it was planned to work with voluntary pre-service teachers. After the pilot study, it was seen that pre-service teachers who do not have the high GPAs and the relatively high grades in some courses regarding geometry and GeoGebra had difficulty in suggesting ideas, following the collective argumentation, and being an active participant. By aiming to avoid the isolation of any participant during the tasks and the need for the presence of the argumentation in the groups, it was decided to involve the juniors who have the highest success in the program. To be able to examine rich argumentation, the grades of all juniors in some related courses and the GPAs were listed and the upmost 14 juniors were determined as the participants. There were 12 females and 2 males, and all juniors had a GPA above 3.00 out of 4.00. More precisely, there were three pre-service teachers who have the GPAs in the range of 3.00-3.24, eight pre-service teachers with the GPAs ranged in 3.25-3.49, and lastly three pre-service teachers have the GPAs falling between 3.50 and 3.75.

By considering the description of collective argumentation of Conner et al. (2014a), which stated that “multiple people arriving at a conclusion, often by consensus” (p. 401), it was decided to form two groups of three participants and two groups of four participants. Moreover, the steps of geometric constructions are dependent on the tools used. While using different tools in a geometric construction, it is needed to consider different strategies and ideas (Pandiscio, 2002). The use of different tools during geometric constructions may present some new ways to develop students’ reasoning in geometry (Kuzle, 2013). By considering the possible effects of using different tools in geometric construction tasks on the components of argumentation, it was decided to include different settings in the present study. Thus, it was also arranged that one group of three juniors and one group of four juniors used compass-straightedge while the remaining two groups used GeoGebra during geometric construction tasks. Some characteristics of the groups are given in Table 2.

Table 2.
Participants of the Study

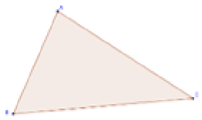
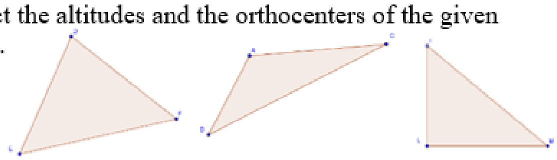
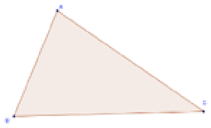
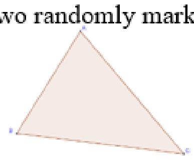
Group	Participants	Gender
Compass-straightedge Group 1 (CSG1)	P1	Female
	P2	Female
	P3	Female
Compass-straightedge Group 2 (CSG2)	P4	Female
	P5	Male
	P6	Female
	P7	Male
GeoGebra Group 1 (GG1)	P8	Female
	P9	Female
	P10	Female
GeoGebra Group 2 (GG2)	P11	Female
	P12	Female
	P13	Female
	P14	Female

Data Collection and Analysis

Before the administration of the study, the official permissions were taken from Applied Ethics Research Center in the subject university. At the beginning of data collection, how compass-straightedge and GeoGebra can be used in geometric constructions was explained to each group. The aim of this step is to ensure that all participants are familiar with the features of the mentioned tools and the entailments of geometric constructions. Compass-straightedge groups and GeoGebra groups worked on the tasks at different times and in different classrooms. In other words, during all tasks, CSG1 and CSG2 worked in separate tables located in a classroom. Similarly, GG1 and GG2 worked with separate computers located in another classroom. In each week, one of the tasks was applied to all groups. For CSG1 and CSG2, worksheets on which necessary information about the task was written and compass-straightedge packs were distributed to each member. For GG1 and GG2, worksheets were given to every group member, but each group worked with one computer in a face-to-face setting. GeoGebra files needed for the tasks were present on the desktops of the computers. All groups were asked to share their ideas with the group, discuss about them, and submit their documents as a group, not individually. By means of the documents and the video recordings of the groups in the four geometric construction tasks, data were collected.

To avoid leading the argumentation of groups while working on tasks, the steps of geometric constructions were not presented in the worksheets of the tasks. While CSG1 and CGS2 were asked to construct by using compass-straightedge, GG1 and GG2 were asked to construct by using the given GeoGebra files. All geometric construction tasks were related to triangle and circle. In more detail, Task 1 asked the groups to construct the circumcircle of a given acute triangle. Task 2 asked the construction of the altitudes and the orthocenters of the given acute, obtuse, and right triangles. Task 3 is basically related to the construction of the Euler line. More precisely, Task 3 involves the construction of the circumcenter, the orthocenter, and the centroid of a given acute triangle. The aim was to lead the groups search for the collinearity of the particular points on the Euler line via construction. Task 4 is about the construction of the Miquel point of a triangle. In more detail, the groups were asked to place random points X, Y, and Z on each side of a given acute triangle and then construct three circles, each of which passes through one vertex and two randomly marked points on the adjacent sides. Hereby, the groups were expected to work on the statements regarding the Miquel theorem. For example, they were expected to see the concurrency of three circles that they were intended to construct. What the worksheets of the mentioned tasks cover is summarized in Figure 1.

Figure 1.
Geometric Construction Tasks

Task 1	**Task 2**
Construct the circumcircle of the given triangle. 	Construct the altitudes and the orthocenters of the given triangles. 
Task 3	**Task 4**
Construct the Euler line of the given triangle by finding the orthocenter, the circumcenter, and the centroid. 	Place random points X, Y, and Z on each side of a given acute triangle. Construct three circles, each of which passes through one vertex and two randomly marked points on the adjacent sides. 

In data analysis, the six components in the model of Toulmin (2003) were utilized as the basis to portray the collective argumentation of the groups. Grounded on the studies which employ Toulmin's argument model, the operational definitions of the six components for the analysis of the present study were determined (See Table 1). Then, some other studies which cover the revisions of the model (e.g., Conner et al., 2014a; Knipping, 2008; Verheij, 2005) were also taken into consideration. During the analysis, it was seen that there are some statements which do not fit any component of Toulmin's model, but they also affect the flow of the argument. Thus, according to the transcripts of the collective argumentation of the groups, some additional components were proposed and used during the analysis as well.

FINDINGS

The collective argumentation processes of the groups contained not only the six main components of Toulmin's argument model which are data, warrant, conclusion, backing, rebuttal, and qualifier but also five additional components, which are conclusion/data, target conclusion, guidance, challenger, and objection. In this section, the new components are explained and then all components are instantiated by referencing to the dialogues from the collective argumentation processes.

Of the new components, conclusion/data and target conclusion were included with some adaptations from the study of Knipping (2008). Since the conclusion of an argument may be the data of the following argument (Conner et al., 2014a; Knipping, 2008), such statements in the argumentation stream were labeled as conclusion/data (C/D) in this study. The term conclusion/data was employed since it was considered that data should be the second term in the component since this component provides the data for the following argumentation process. In addition, Knipping (2008) employed the component entitled as target conclusion which was described as "the final conclusion of the argumentation" (p. 434). Hence, Knipping (2008) used the term target conclusion for all conclusions in the argumentation except the ones labeled as data/conclusion. However, target conclusion (TC) was used with a different meaning in this study due to the context of the tasks. It was used for the final answers and conjectures in the tasks which were reached by the consensus of all participants in the groups.

In addition, some new auxiliary components were also presented, namely, guidance (G), challenger (CH), and objection (O). When the expressions of the instructor would not directly fit into any one of the main six components, the need for a new component for such statements emerged. Thus, the expressions of the instructor which present some clues related to the task and affect the flow and direction of the argument were marked as guidance. Moreover, it was noticed that some statements of both the participants and the instructor could not properly be coded as rebuttal, but they somehow interfered with the flow of the discussion. Such statements were coded as challenger or objection. In more detail, the characteristics of the statements categorized as challenger were presented as follows; this kind of statements basically challenge ideas by leading others to think for a while, causing others to have question marks or to hesitate, leading the others to think out of the box, and putting a different case and point of view on surface regarding the concept of the task but without aiming to defeat the argument like the rebuttal component. For example, in Task 1, one participant stated that "what happens to the circumcenter when the given is an obtuse triangle" and then the rest of the group started to think about this extra case. As seen, this statement directly affected the flow of the argumentation and did not cover the purpose of refutation. When the arguers state an objection throughout the discussion without giving the reasoning behind their opposition, this kind of statements was coded as objection during the analysis. For example, in Task 1, a participant stated that "I think, it is not true, what you drew is incorrect" without explaining the reasoning and caused other participants to explain the method in order to convince.

Dialogues from different argumentation processes were selected to provide examples for each component since there is not an argumentation sequence involving all components. The selected argumentation processes are presented below.

Table 3.
Selected Argumentation Processes to Exemplify the Components

Argumentation	Components exemplified
Argumentation of CSG1 in Task 4	data (D), warrant (W), rebuttal (R), conclusion/data (C/D), challenger (CH), qualifier (Q), and target conclusion (TC)
Argumentation of GG1 in Task 1	backing (B), objection (O), and conclusion (C)
Argumentation of GG2 in Task 2	guidance (G)
Argumentation of CSG2 in Task 3	objection (O) and challenger (CH)
Argumentation of GG1 in Task 4	guidance (G)

The dialogues were presented in such a way that the explanation regarding participants' unclear expressions and also some extra descriptions to make the dialogues clearer were inserted in parentheses as italics. Moreover, triple dots placed between the lines mean that some other conversation took place at that moment, but they were not related to the focused components. These conversations were not included in the given excerpts, but they were indicated with the presence of triple dots. The capital letters which were given in the parentheses after the participants refer to the components of argumentation that the following lines were coded, namely, data (D), warrant (W), conclusion (C), backing (B), rebuttal (R), qualifier (Q), conclusion/data (C/D), target conclusion (TC), guidance (G), challenger (CH), and objection (O).

First of all, to depict the majority of components, which are data, warrant, rebuttal, conclusion/data, challenger, qualifier, and target conclusion, a comprehensive argumentation stream of CSG1 from Task 4 was explained below. As stated, Task 4 asked groups to place random points X, Y, and Z on each side of a given acute triangle and then construct three circles, each of which passes through one vertex and two randomly marked points on the adjacent sides.

P1 (D) These are three circles that we drew. I draw a triangle from the centers I obtained from them (*three circles*).
...
P2 (C/D) Does the one emerged at the outside look like a right triangle, doesn't it? (*She noticed that there is a similarity between the right triangle at the beginning and the triangle they drew by accepting centers as the vertices*)
P1 (W1) Yes, it is like something of the same triangle.
P2 (W1) Like miniature.
P1 (W1) Like its symmetric, the version of its direction was changed. We can also find its circumcircle.
...
P1 (R) Hmm, it does not happen like this (*She was stating that being symmetric is not a property of the triangles*). Its direction was changed only.
P2 (R) I think, it was getting smaller, and the direction changed.
...
P1 (C/D) The thing that we draw inside (*the new triangle*) is similar to this (*the given triangle,* ΔABC), isn't it?
...
I (CH) Well, what happens when the points X, Y, and Z change?
P2 (TC) They *always* looked similar.
P2 (Q) *Always*
P2 (W2) Here, it is like a right triangle, this is too (*She was showing different types of triangles and differently placed X, Y, and Z points*)
P2 (TC) Actually this one is *like* similar to this.
P2 (Q) *Like*
…
P2 (Q) I think
P2 (TC) Similar
...

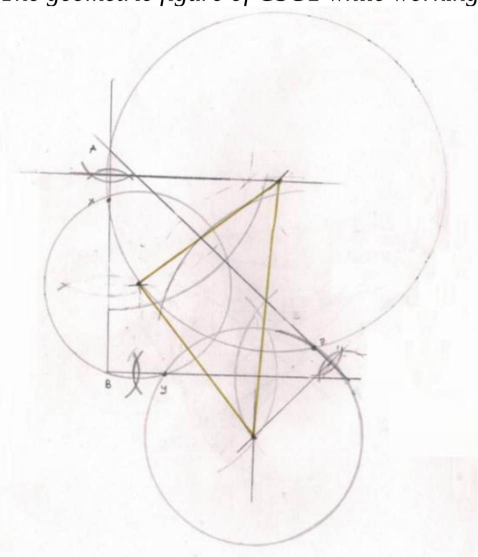
I How do you describe your connection now?

P2 (TC) Here we will say that the points X, Y, and Z on the triangle ABC, well. The centers of the circles passing through the points AXZ, BYX, and CZY are similar to the triangle ABC (*She was trying to sum up*)

P1 (TC) The triangle formed from the centers is similar to the triangle ABC.

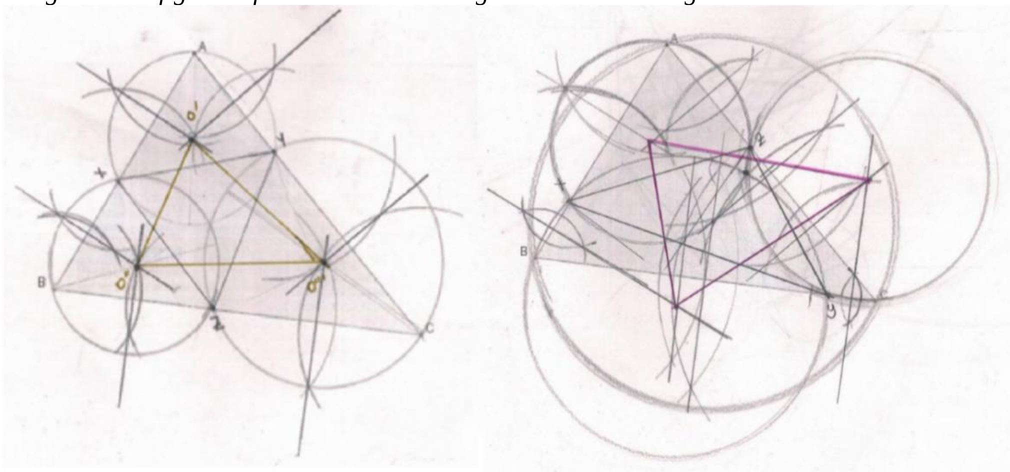
In the conversation given above, it can be concluded that participants of CSG1 were in a period where they were searching for a possible relationship among the three circles they constructed. The endeavor of P1 in terms of examining the characteristics of the geometric figures they formed resulted in new data in the argumentation. P1 declared that she could draw another triangle by using the centers of three circles. Since this statement constituted the basis of the following argumentation, it was coded as data (D). This data inspired P2 and she noticed that the new triangle also looks like a right triangle as well as the triangle they started to work on at first. Although they were given an acute triangle on the worksheet, they also decided to do the construction asked in Task 4 in different types of triangles. The right triangle that CSG1 mentioned at this point was one of the extra triangles that they focused. They continued to work on this issue and brought some justifications to the surface. They expressed some warrants which were stated as follows; the new triangle is a kind of miniature of the first one, the new one is like symmetric to the first one, and the new one is a version of the first one in which its status or direction was changed. These sentences were labeled as warrants (W1). In the meantime, they expressed the issues against some parts of their warrants. In more detail, they attempted to defeat the notion of being symmetric they stated earlier. Such expressions were coded as rebuttal (R) which was stated against warrant. Based on warrant and rebuttal, CSG1 concluded that the new triangle and the first triangle are similar. This statement was coded as a conclusion/data (C/D) since the related argumentation continued after this sentence and the idea of similarity performed as data of the following part of the argumentation. The figure that CSG1 drew at that period is presented below.

Figure 2.
The geometric figure of CSG1 while working on the right triangle in Task 4



Afterwards, a challenger (CH) was put forward by the instructor; "what happens when the points X, Y, and Z change?" Since this sentence caused the group to hesitate and search on this issue, it was coded as a challenger. Thus, CSG1 started to query about what they have found recently. P2 showed the cases for differently placed X, Y, and Z points through different types of triangles. These actions were also coded as the second warrant (W2) since she listed the occasions related to the conclusion they produced and the challenger. Some of the figures that CSG1 drew while working on the acute triangle for differently placed X, Y, and Z points were presented below.

Figure 3.
The geometric figures of CSG1 while working on the acute triangle in Task 4



After these justifications, CSG1 reached the target conclusion with the presence of a qualifier (Q). They declared the words "like" and "I think" which can be coded as qualifiers while expressing their final conjecture which also corresponds to target conclusion. As target conclusion (TC), CSG1 came up with the statement that there is a similarity between the initial triangle and the triangle they formed from the centers of three circles, which is a variation of Miquel's theorem (de Villiers, 2014).

As it can be seen in Table 3, to exemplify the components backing, objection, and conclusion, an argumentation stream of GG1 in Task 1 was selected. As mentioned, in Task 1, the groups were asked to construct the circumcircle of ΔABC which is an acute triangle. Although the main purpose is to give instances for the mentioned three components, sentences regarding some extra components were intentionally included in the following excerpt. More specifically, since backing (B) was provided to support the first warrant (W1), that warrant was also presented in the dialogue. In a similar vein, since objection (O) was stated against the second warrant (W2), it was also included. Besides, the related data (D) were presented so as to make the subsequent warrants interpretable.

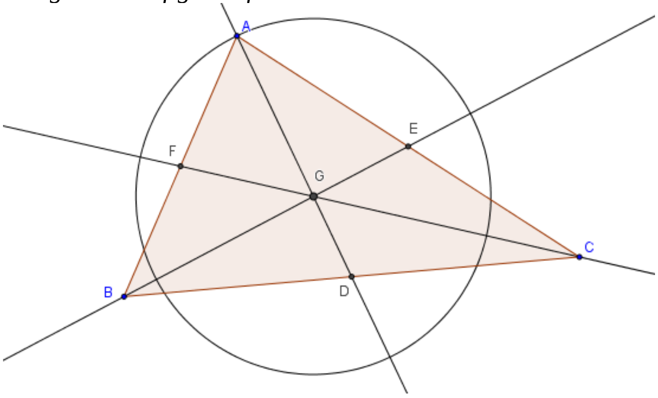
P8 (D) What if we use the centroid? (*The group thought that the centroid might give them the circumcenter of the given triangle*)
P10 (D) There is a feature about the ratio 2k, k.
P8 (D) Exactly, it was about the centroid.
P10 This equals to *a* too. Then, these three equal to *a* (*She mentioned the equality of the line segments she drew as radii through the vertices of* ΔABC)
P9 (W1) Now, if we draw the lines passing through the midpoints of these (*the sides of the triangle*), does the intersection of these three (*three medians*) give the centroid?
P8 Tell me again.
P9 (W1) We found a point by intersecting this and this (*She was moving her hand like drawing two medians. Therefore, she referred to the intersection of two medians by saying point*). We combined this with this (*She was pointing the vertex A and the midpoint of* $\overline{BC}$. *That is, she referred to one median*). Similarly, I think, the midpoint of this with this gives us the center (*She was pointing the vertex B and the midpoint of* $\overline{AC}$).
P10 I do not know it.
P9 (W1) Because this is the median, as we said about the ratio *a* and *2a*.
P8 (B) Hmm, we already know that the point of concurrency of the medians is the centroid.
P9 (W2) Now, to find the midpoint of this (*the side of the triangle*), here is the tool for drawing the line passing through the midpoint.
P8 Okay.
P9 (W2) We will find the midpoint from this.
P10(W2) That is, is it the perpendicular bisector, oops the midpoint?

P9 (W2) We need to find the midpoint. It could be either of them.
...
P9 (W2) Similarly, if we accept it as an example, then we combine the vertices. For example, we name them.
...
P10(W2) Now, you will combine these two points (*These points are the vertex A and the midpoint of* $\overline{BC}$)
P9 (W2) It will be the median. For example, draw a line (*a line passing from the vertex A and the midpoint of* $\overline{BC}$)
P10(O) I wonder if you draw a line segment (*instead of drawing line*).
...
P9 (C) But, it did not pass (*The circle did not pass through all vertices*)
P10(C) But, one minute. Why did not it pass from these? (*the vertices of the triangle*) But I selected the thing. Since these are not equal. We accepted them as equal and labeled as the radius (*She referred to the fact that the distances between the centroid and the vertices are not equal*)
P8 (C) Exactly, $|AG|$ and $|BG|$ are not equal.

According to the conversation given above, the participants of GG1 were trying to find an approach for the construction of circumcircle of ΔABC. They aimed to find the center of this circle as the first step. However, they considered that the center of that circle could be reached via finding the centroid. Thus, all their statements regarding finding the centroid and the ratio 2k:k which can be seen in the sentences at the beginning of the dialogue were coded as the data (D). Then, P9 asserted that the point of concurrency of the medians of ΔABC might give them the centroid. These statements of P9 were coded as the first warrant (W1). After the word median, P8 agreed with this idea and supported the warrant of P9. P8 declared that they already know that the point of concurrency of the medians is the centroid and this sentence was coded as a backing (B). Afterwards, P9 and P10 focused on the construction of the median by using the tools of GeoGebra.

As the first step, they found the midpoint of the sides and then drew lines between the vertices and the midpoints of the sides. In the meantime, P10 warned others about drawing line segments instead of the lines while constructing the medians. This statement was coded as an objection component (O) since she interfered in the construction process without explaining the reasoning of her expression. Moreover, others in GG1 hesitated for a while during the construction due to this objection but then continued to construct lines. After this objection, the discussion of the group continued with the expressions coded as a conclusion/data (C/D) and the third warrant (W3), but they were not presented in this excerpt. Finally, the issue in this argumentation stream ended up with a conclusion. As seen in the last part of the dialogue, GG1 observed that the circle they drew by accepting the centroid as the center and the distance between the centroid and the vertex A as the radius did not pass through other vertices B and C in ΔABC. The geometric figure that GG1 formed at the end of this idea is given in Figure 4.

Figure 4.
The geometric figure of GG1 in Task 1



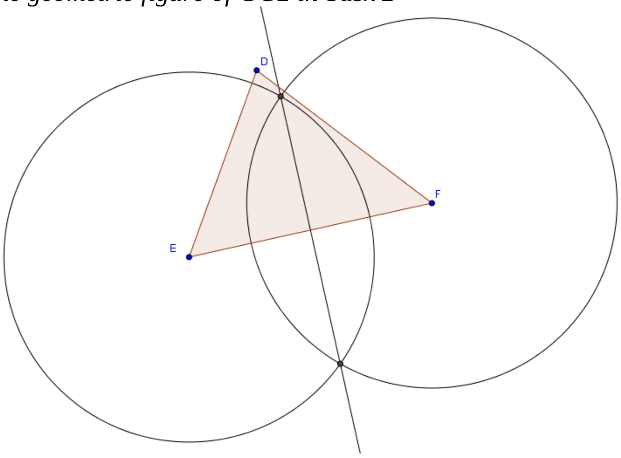
Finally, they noticed that they accepted the radius of the circle incorrectly. The sentences regarding this result were coded as conclusion (C).

To give an example for guidance, the dialogue from the argumentation of GG2 in Task 2 was presented below. As mentioned, Task 2 asked the construction of the altitudes and the orthocenters of the given acute, obtuse, and right triangles. There are two more components which are data (D) and warrant (W) related to guidance. Hence, the conversation of GG2 involving these extra components was presented below. However, the components after guidance were not taken into consideration at this point.

I What are you trying to do?
P13(D) We are trying to draw a perpendicular line passing from a point not on the line (*They were trying to remember how to draw a line that is perpendicular to a given line from a point not on the given line*). But, we could not do.
I (G) You can try it in an acute triangle. Maybe, it would be clearer for you. Since the orthocenter is outside of the triangle for an obtuse triangle, it might be confusing.
… (*They started to work on an acute triangle* ΔDEF)
P11(D) From the point D… A perpendicular line to EF (*She was stating to draw a line which is perpendicular to* $\overline{EF}$ *and passing from the vertex D*)
…
P12(W) We should take two circles and the radius should be more than the half of it (*She is pointing two circles by accepting the centers the vertices E and F. She mentioned that the radius is greater than the half of* $|EF|$). We tried it earlier, but we could not do it. Hmm, it is like the midpoint (*They noticed that their idea to construct the altitude of* $\overline{EF}$ *was not working. The line they constructed was not passing through the vertex D*)
I (G) Because the vertex D is a point which is not equidistant to other vertices E and F.
P13 Hmm, how can I find the length of $\overline{DE}$ on this part?

When the excerpt was read, it can be seen that GG2 was trying an approach for the construction of the orthocenter of triangle in Task 2. As P13 stated, GG2 was trying to remember how to draw a line that is perpendicular to a given line from a point not on the given line while working on an obtuse triangle. Since the instructor observed that they were having difficulty in adapting their idea to the obtuse triangle, she involved herself in their discussion. The instructor suggested trying the same approach in an acute triangle and these sentences were coded as guidance (G). Since the orthocenter of an obtuse triangle is outside of that triangle, the group might have difficulty in construction with this approach. After that, GG2 worked on an acute triangle ΔDEF by using the same construction idea for a while. P12 pointed that they tried to draw two circles by accepting the centers as the vertices E and F. She mentioned that the radius is greater than the half of $|EF|$. Then, they noticed that what they drew is not the altitude of $\overline{EF}$ since it did not pass through the vertex D. The geometric figure GG2 drew at that moment is presented in Figure 5.

Figure 5.
The geometric figure of GG2 in Task 2



The instructor noticed that GG2 was not completely clear why their idea did not work. Therefore, she wanted to give them a clue in the process. Thus, she stated that the vertex D is a point which is not equidistant to other vertices E and F to help them to see the further step. This statement was also coded as guidance (G). After this guidance, they started search for a way to find the points which are equidistant to the vertex D so that the perpendicular line would pass through the point D.

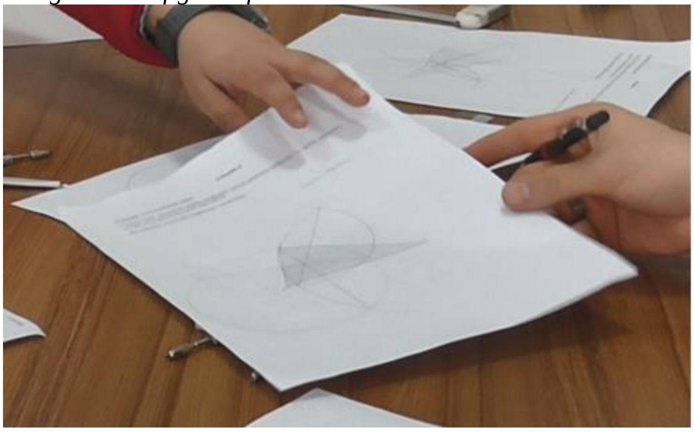
Conclusion/data and target conclusion are versions of previously stated components. Thus, more examples of objection, challenger, and guidance are also presented. In this respect, some parts from the argumentation of CSG2 in Task 3 and argumentation of GG1 in Task 4 are given below.

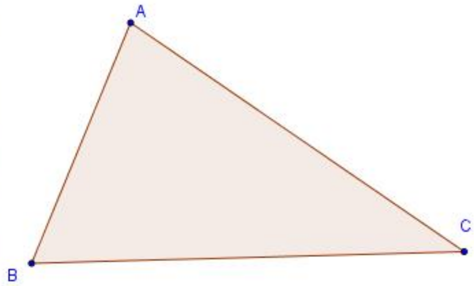
By means of the argumentation of CSG2 in Task 3, objection and challenger were exemplified. As stated, in Task 3, the groups were asked to construct Euler line by finding the circumcenter, the orthocenter, and the centroid of an acute triangle. At the time of the following dialogue, CSG2 could construct the circumcenter and the centroid, and they were working on the construction of the orthocenter.

P6 (O) Can I look what you drew there? I think, there is something wrong *(She was asking to look at the orthocenter construction)*
P7 (W) To construct it, I drew a perpendicular line from A to the side BC. Then, I drew a perpendicular line from B to the side AC *(He was showing two altitudes of ΔABC)*
P5 (W) We took the intersection *(of the altitudes)*. Thus, it is the orthocenter.
P6 (O) I think, we did it wrong, but I can't see why.
P7 We can try to construct it in a new page.
…
P6 (R) I got it, the wrong point. This is not the altitude of AC, you did not draw it from B. Look. There is problem here, that is what I am trying to say.
P7 Aa, okay, we did it wrong.

According to the conversation given above, P6 noticed something wrong with the construction of the altitudes of ΔABC. Since she did not present the reason for her idea and caused P5 and P7 to explain how they drew it, it was coded as objection (O). Moreover, the sentences of P5 and P7 were coded as warrant (W) since they tried to present the reasoning of their actions. Then, they decided to construct the orthocenter in a new page. After working on it for a while, P6 noticed why they could not construct the altitude of $\overline{AC}$. The screen capture from the video recordings of that discussion and the given ΔABC in the Task 3 are presented in Figure 6. As it can be seen, what CSG2 drew as the altitude of $\overline{AC}$ is not passing from the vertex B. Since she could present the reasoning of her idea for this time, this sentence was coded as rebuttal (R).

Figure 6.
The geometric figure of CSG2 in Task 3





To give an example for challenger, another excerpt from the argumentation of CSG2 in Task 3 was presented below.

P5 (CH) If it is an equilateral triangle.
P4 It is not, it is an acute triangle.
P5 (CH) I mean, if there is an equilateral triangle in this page. Then, all points will be the same point, maybe.
P6 Yes, maybe.
P7 I can try to find all points *(the circumcenter, the orthocenter, and the centroid)* of an equilateral triangle while you were writing what you have found.

At the end of the construction of Euler line in Task 3, CSG2 started to write what they have conducted. While writing, P5 asked about the circumcenter, the orthocenter, and the centroid in the case that an equilateral triangle was present at the worksheet. Then, he offered that they would find the same point via the construction of all points. Since his idea caused P7 to construct three points for an equilateral triangle, these sentences of P5 were coded as challenger (CH). As seen, the idea of P5 worked as an extension of the task and they focused on some extra cases regarding the task.

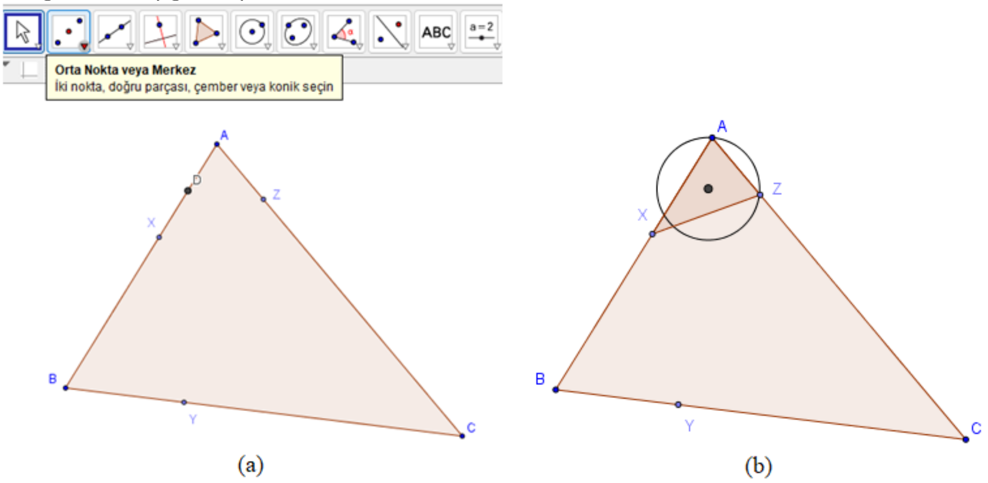
Another example for guidance can be given from the argumentation of GG1 in Task 4, which involves the construction of three circles, each of which passes through one vertex and two randomly marked points on the adjacent sides. Similarly, there are three more components which are data, warrant, and rebuttal until guidance. The conversation of GG1 related to the previous components of guidance was also presented below.

P8 (D) There is a triangle here *(She was pointing ΔAXZ).* If we draw the circumcircle of that triangle.
P9 Hmm
P10 Let's look at this *(the tools of GeoGebra).* Is there a tool for this?
P8 (D) There is ΔAXZ , let's try to construct the circumcircle of this triangle. *(She noticed that they could form a triangle by using the points A, X, and Z and the circumcircle of ΔAXZ is one of three circles aimed to construct in Task 4)*
P9 (W) I think, there is a method that we can find the center of the triangle.
P8 Where?
P9 In this side, this side *(She was pointing the left side of the toolbar of GeoGebra)*
P8 (W) Is this one? The midpoint or center

P9 Try with this.
P8 What will I do? Will I click here? *(She referred to clicking the tool)*
P9 (W) It *(the tool)* says two points or a line.
P10 (W) I suppose, we will select this and this *(two vertices of the triangle)*
P9 (W) If we select the triangle, it would find the center. I think, we will select the triangle from the vertices.
P10 (R) But it goes away after two times, the tool does not let me select the third vertex. *(She showed others that the tool the midpoint or center did not work)*
I (G) I think, it *(the tool)* is accepting the process differently. You may think about the construction of the circumcircle that we did in the previous weeks.

When the excerpt was read, it can be seen that GG1 was trying an approach for the construction of the circles asked in Task 4. As seen, P8 noticed that they could draw a triangle from the points A, X, and Z. She also stated that they could construct one of the intended circles by constructing the circumcircle of this triangle. These sentences constituted the data component (D). Based on this data, they started to examine the toolbar of GeoGebra to find a tool to construct the circumcenter. They attempted to use the tool “midpoint or center” but they could not manage how to use it. The entire process about finding and using the tool was coded as warrant (W). In the meantime, a rebuttal (R) against warrant was presented by P10 since she could not select all vertices of the triangle by using the mentioned tool. Therefore, they ended up with that the tool did not serve their purpose. The screenshot of the GeoGebra file of GG1 during this process is presented in Figure 7(a).

Figure 7.
The geometric figure of GG1 in Task 4



Since the instructor observed that they were having difficulty in both using the tool and finding the circumcenter, she was involved in their discussion. The instructor mentioned that the tool might be functioning in a different manner since the center of the triangle that the mentioned tool can supply is the centroid and not the circumcenter. In other words, as it can be seen in Figure (b), GG1 would find the centroid of ΔAXZ by using this tool and this was not the aimed construction in Task 4. Then, the instructor led them to think about the approach they used while constructing the circumcircle of a given triangle in one of the previous activities. This involvement of the instructor was coded as guidance (G). Based on this guidance, GG1 tried to remember the approach they used and presented some other warrants for finding the circumcenter of ΔAXZ.

DISCUSSION AND CONCLUSION

Different from the analysis in formal logic, which focuses solely on the dichotomy involving premises and conclusions throughout the examination of arguments, Toulmin's model offers six components for the analysis of arguments (Verheij, 2009). However, Toulmin's model might be limited with respect to analyzing the complex structure of arguments in practical discourse. Thus, researchers continued conducting studies to unfold new models and theories (Knipping, 2008). Although, prima facie, the six components proposed in Toulmin's model seemed to be sufficient in order to analyze the collective argumentation in the data analysis of this study, the findings illustrated that these components were not adequate and there was a need for some extra components to present a detailed analysis of the argumentation. Thus, five more components were added to the argumentation model. The need for the extra components might have originated from the fact that the discussion of the groups in this study was covering long periods and there were some statements which could not be categorized under the existing components. To consolidate the analysis with respect to the content of argumentation, all instances during the argumentation were taken into consideration by aligning them with the intonations in the video recordings. It may have been this detailed analysis that led to the inference of the extra components. Moreover, it could have stemmed from the fact that the functions of some existing components in Toulmin's model were simplified and divided into different components in this study. For example, the statement referred to as objection might be addressed under rebuttal in other studies. Another point to note is that all components were seen in the collective argumentation processes of both compass-straightedge groups and GeoGebra groups. Thus, it can be inferred that there is not a component which emerged as peculiar to the use of a specific tool during geometric construction tasks.

One of the newly used components in the argumentation is challenger. This component might partially originate from the nature of the concept of questioning proposed by Walton (2006). When an arguer questions a statement, the aim does not have to be to show that the statement at stake is false or true. That is, questioning can take a neutral stance or just refer to a phrase of doubt. Walton (2006) described it as "questioning a proposition represents a weaker kind of commitment than asserting it" (p.26). Originating from the instinct of questioning at a particular degree, the participants might have put forward some issues which were challenging for the rest of the group. The challenging issue was not asserted as true or false, but actually required an identification regarding the validity of the projected issue. To sum up, the unclear stance of the questioning act in terms of being valid or not might have turned into statements which created a challenging environment in the collective argumentation. Since it was observed that none of the existing components of Toulmin's model completely addressed the statements leading to a challenging issue, causing to have question marks, and directing the arguers to new attempts regarding the issue, the study implied the need for a component which was referred to as challenger. Moreover, it was seen that the statements coded as challenger were checked in a quicker manner in GeoGebra groups compared to compass-straightedge groups. Different settings, which emerge as a result of challenger, can be tried quickly by means of the movement of a free object belonging to the geometric figure (Schreck et al., 2012). In other words, the dragging feature of GeoGebra might facilitate the examination process of the applications of different cases and help the groups to come up with some generalizations and properties regarding the challenger (Stupel et al., 2018).

While students are dealing with a difficult problem, mathematics teachers might be coaching them by deflecting their attention to the needed issues in the problem and offering some ideas to use in the solution process. The behaviors of the teacher in the classroom have an impact on how the argumentation in the classroom is framed. It was stated that "arguments are produced by several students together, guided by the teacher" (Knipping, 2008, p. 432). However, Toulmin's model does not address a particular component to represent the stance of such actions of the teacher. To this end, the guidance component was employed during the analysis of the present study. Moreover, the instructor presented guidance depending on the difficulty levels of the tasks for the participants. For example, it was observed that compass-straightedge groups needed more guidance throughout the tasks. The reason underlying this result might be that these groups had to build their ideas by grounding on more solid bases and evoke their previous geometry knowledge; hence, they got stuck in more occasions. On the other hand, it can be stated that GeoGebra groups were more unconstrained in this issue since they had the chance to check the validity of their ideas quickly via dragging the movable points (Janičić, 2010) and using other tools of the program. The addition of guidance to the layout of argumentation was also observed in the study of Lin (2018), in which this component undertook three main functions, which are to complete conjecture, revise conjectures, and evoke argumentation.

In addition to the challenger and guidance components mentioned above, some extra components mentioned in the subsequent studies of Reid and Knipping were also employed with some modifications. The mentioned components were data/conclusion, which refers to the transition from one part to another in a discourse, and target conclusion, which stands for the final and main conclusions throughout the argumentation (Knipping, 2008; Knipping & Reid, 2013, 2015). As mentioned, the term target conclusion was kept, but its function was slightly changed. However, the term data/conclusion was reversed as conclusion/data because of the order of the functions of the combined components in the argument. Actually, the conclusion/data component has correspondence in the study of Walton (2006). It was asserted by Walton (2006) that the conclusion of an argument can function as a premise of the next argument.

Due to the differences between asserting that a statement as false or criticizing its validity (Walton, 2006), it was decided that all negative utterances in the argumentation of groups cannot be coded under the same component. When the components in the argumentation model of Toulmin were examined, it was observed that rebuttal undertakes the mentioned negative stance since it “provides conditions of exception for the argument” (Verheij, 2005, p. 348). In addition to rebuttal, another component referred to as objection came into play in the analysis of the data of the present study. In instances where the objection was uttered by a participant in a high interrogative and doubtful manner without stating even the reason, the rest of the group was led to have doubts and sometimes to give up the issue argued against. That is, the objection might have had the power to make others give up the issue argued against without presenting any solid counterargument. This finding is in accordance with what Walton (2006) proposed, which is two possible ways of attacking an argument. The first way is to present a counterargument to a statement by stating the underlying reasons and the second way is to utter a doubt regarding the statement without presenting the reason so that it cannot actually be rebutted due to the lack of a solid counterargument. Although there are not many statements coded as objection in all groups, the number of objections was higher in compass-straightedge groups. They might have difficulty in consolidating the underpinnings of their ideas and select not to offer the related explanation.

Since Toulmin’s model was used along with some modifications in a variety of studies (e.g., Conner et al., 2014a; Knipping, 2008; Verheij, 2005; Voss, 2005), the adapted version of the model, which was reconstructed with the inclusion of new components as well as keeping all six-component situated in the default version of model, might be used while investigating and analyzing the complex argumentation process. The new components might be used in other domains of mathematics, different from geometry. Since Toulmin’s model is independent of disciplines, the adapted version in this study could be applied to other disciplines. Moreover, the statements coded as guidance were the ones stated by the instructor only. However, the remaining components were not clear from this perspective. As an extension of this study, the characteristics of the arguers who declared the particular components might be investigated.

Acknowledgement

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TÜRKÇE GENİŞLETİLMİŞ ÖZET

Argümantasyon, disipline ve bağlama bağlı olarak farklı anlamlarda kullanılmaktadır. Matematik bağlamında argümantasyon, mantıksal bağlantıların takip edilerek matematiksel bir söylemin geliştirildiği bir süreç olarak ifade edilmektedir (Smith, 2010). Argümantasyon, matematik eğitimi araştırmalarının yanı sıra matematik araştırmalarının da temelinde yer alan temalardan biri olarak kabul edilmektedir (Mariotti vd., 2018). Benzer şekilde, Conner ve diğerleri (2014a) matematikte argümanları anlamanın, tanımanın ve yürütmenin önemini vurgulamış ve ortaklaşa argümantasyon kavramına odaklanmışlardır. Ortaklaşa argümantasyon genellikle bir araştırma sürecine işbirliği içinde katılan küçük bir öğrenci grubunu ve öğretmeni içermektedir (Cervantes-Barraza vd., 2020). Ortaklaşa argümantasyon süreçlerinin analizi için, matematik eğitimi araştırmalarında sıklıkla Toulmin'in argüman modeli (2003) kullanılmaktadır (Carrascal, 2015; Conner vd., 2014b). Aslında, argümantasyonla ilgili herhangi bir yapının araştırılmasında, Toulmin'in modeli (2003) en sık kullanılan çerçevelerden biridir (örn., Boero vd., 2010; Krummheuer 1995; Pedemonte & Balacheff, 2016). Bu çalışmada ortaokul matematik öğretmen adaylarının ortaklaşa argümantasyon süreçleri araştırılırken Toulmin'in argümantasyon modeline dayalı bir yaklaşım benimsenmiştir.

Ortaokul matematik öğretmen adaylarının ortaklaşa argümantasyon sürecine dahil olmalarını destekleyecek bir ortam hazırlamak adına, bu çalışmadaki etkinliklerin konusu geometrik inşa olarak belirlenmiştir. Geometrik inşa etkinlikleri, inşa aşamaları doğrudan katılımcılara verilmeyecek şekilde planlanmıştır. Bu nedenle, etkinliklerde iyice düşünmeleri ve süreçteki adımların mantığını gerekçelendirmeleri beklenmektedir. Geometrik inşa sürecinin sağladığı zorlu ortam, öğrencilerin geometriye karşı daha derin bir bakış açısı geliştirmelerine, düşünme ve akıl yürütme yeteneklerini geliştirmelerine (Stupel & Ben-Chaim, 2013) ve sadece geometri ile ilgili önceki bilgileri değil aynı zamanda üst düzey düşünme becerilerini de uygulamalarına alan sağlamaktadır (Lim, 1997). Ayrıca Barabash (2019), geometrik inşa etkinliklerinin farklı zorluk seviyelerine göre değiştirilebileceğini ve geometrik inşanın geometrik kavramların araştırılması için önemli bir kaynak sunduğunu vurgulamıştır. Geometrik inşanın bahsedilen faydaları ve uygulamaları nedeniyle bu çalışma için uygun bir matematiksel kavram olduğu öngörülmüştür.

Bu noktalar ışığında çalışmanın amacı, ortaokul matematik öğretmen adaylarının geometrik inşa etkinliklerindeki ortaklaşa argümantasyon sürecinin bileşenlerini incelemektir.

Toulmin'e (2003) göre bir argüman veri, iddia, gerekçe, destek, niteleyen ve çürütücü olmak üzere altı bileşen içerebilmektedir. Alan yazın taramasında altı bileşenin tanımlarında bazı farklılıklar olduğu görülmüştür. Bazı çalışmaların Toulmin'in modelini doğrudan kullanmadığı ve çalışmaların amaçları ve bağlamlarına göre modelin gösterimi üzerinde bazı düzenlemeler yaptığı görülmüştür (örn., Boero vd., 2010; Conner vd., 2014a, 2014b; Verheij, 2005). Toulmin'in modelinin kullanılmasındaki düzenlemelerin sadece bileşenlerin yerleşimi ve yapısı ile sınırlı olmadığı görülmüştür. Argümantasyon sürecini ortaya koyarken bazı ek bileşenlere ihtiyaç duyulduğunu belirten çalışmalar da bulunmaktadır. Ayrıca, Toulmin'in argümantasyon modeli bazı eleştirilere maruz kalmıştır (Conner vd., 2014b; Mariotti vd., 2018; Pedemonte & Balacheff, 2016). Alan yazında belirtilen eleştiriler nedeniyle bu çalışma, Toulmin'in modelinin uygulanmasına eleştirel bir bakış açısı getirmeyi ve kapsamlı argümantasyon süreçlerinin bileşenlerini ve ortaklaşa argümantasyonun olası yeni bileşenlerini yakından incelemeyi amaçlamıştır. Ek olarak, bazı örtüşen noktalar ve belirsiz kısımlar nedeniyle argümanın akışında ayırt edilmesi zor olan bileşenler de bulunmaktadır. Veri ve gerekçe bu bileşenler arasında düşünülebilir. Bu nedenle, ortaklaşa argümantasyon sürecinin bileşenlerini ayrıntılı olarak incelemek, her bileşenin kapsamını belirlemek açısından önem kazanmaktadır. Bu açıdan, bu çalışma Toulmin'in modelini kullanmayı planlayan diğer çalışmalara önemli sonuçlar sunma potansiyeline sahiptir. Bunun yanında, çalışman argümantasyonun gerçekleştiği bağlama göre farklı ve yeni bileşenlerin bulunma olasılığına da dikkat çekmektedir.

Ortaokul matematik öğretmen adaylarının geometrik inşa etkinliklerindeki ortaklaşa argümantasyon sürecini net ve derinlemesine anlamak önemli olduğundan, bu çalışmanın araştırma deseni olarak durum çalışması belirlenmiştir. Katılımcılar amaçlı örnekleme yoluyla, Ankara'daki bir devlet üniversitesinde öğrenim gören 14 üçüncü sınıf ortaokul matematik öğretmen adayı olarak belirlenmiştir. Conner ve diğerlerinin (2014a) ortaklaşa argümantasyon tanımını dikkate alarak, üçer kişilik iki grup ve dörder kişilik iki grup oluşturulmasına karar verilmiştir. Etkinlikler sırasında bir üç kişilik ve bir dört kişilik grup pergel-çizgeç kullanırken, diğer iki grup ise GeoGebra kullanmıştır. Grupların geometrik inşa etkinliğindeki çalışma kağıtları ve video kayıtları aracılığıyla veriler toplanmıştır. Veri analizinde, grupların ortaklaşa argümantasyon süreçlerini ortaya koymak için Toulmin'in modelindeki altı bileşen temel alınmıştır. Bu nedenle, Toulmin'in argüman modelini kullanan çalışmalar incelenerek, bu çalışmanın analizi için altı bileşenin kapsamlı tanımları belirlenmiştir. Sonrasında, Toulmin'in modelini revize ederek kullanan çalışmalar da (örn. Conner vd., 2014a; Knipping, 2008; Verheij, 2005) dikkate alınmıştır. Analiz sırasında, Toulmin'in modelinin herhangi bir bileşenine uymayan ancak argümanın akışını da etkileyen bazı ifadelerin olduğu görülmüştür. Böylece grupların ortaklaşa argümantasyonun içeriğine göre bazı ek bileşenler önerilmiş ve analiz sırasında da kullanılmıştır.

İlk bakışta, bu çalışmadaki ortaklaşa argümantasyon süreçlerini analiz etmek için Toulmin'in modelinde önerilen altı bileşen yeterli görünse de, bulgular bu bileşenlerin yeterli olmadığını ve argümantasyonun ayrıntılı analizini sunmak için bazı ek bileşenlere ihtiyaç duyulduğunu göstermiştir. Diğer bir deyişle, Toulmin'in argüman modelinin altı bileşeni olan veri, gerekçe, iddia, destek, çürütücü ve niteleyen ortaklaşa argümantasyonu temsil etmekte yetersiz kalmıştır. Grupların ortaklaşa argümantasyon süreçleri, sadece bahsedilen altı bileşeni değil, aynı zamanda sonuç/veri, hedef sonuç, rehber, meydan okuma ve itiraz olarak adlandırılan beş ek bileşeni de içermektedir. Yeni bileşenlerden sonuç/veri ve hedef sonuç, Knipping'in (2008) çalışmasından bazı uyarlamalar ile dahil edilmiştir. Bir argümanın sonucu bir sonraki argümanın verisi olabileceğinden (Conner vd., 2014a; Knipping, 2008), bu çalışmada argümantasyon akışındaki bu tür ifadeler sonuç/veri olarak sınıflandırılmıştır. Ayrıca, Knipping (2008) hedef sonuç bileşenini "argümanın nihai sonucu" (s. 434) olarak tanımlamıştır. Bu nedenle, Knipping (2008) veri/sonuç olarak etiketlenenler dışında argümantasyondaki tüm sonuçlar için hedef sonuç terimini kullanmıştır. Ancak bu çalışmada etkinliklerin bağlamı nedeniyle hedef sonuç farklı bir anlamda kullanılmıştır. Gruplardaki tüm katılımcıların fikir birliği ile ulaşılan son cevaplar ve varsayımlar için kullanılmıştır.

Bu çalışmada rehber, meydan okuma ve itiraz gibi bazı yeni yardımcı bileşenler de sunulmuştur. Öğretim elemanının ifadeleri, temel altı bileşenden herhangi birine doğrudan uymadığında, bu tür ifadeler için yeni bir bileşene ihtiyaç duyulmuştur. Böylece öğretim elemanının etkinlikle ilgili bazı ipuçları sunan, argümantasyonun akışını ve yönünü etkileyen ifadeleri rehber olarak kodlanmıştır. Ayrıca, hem katılımcıların hem de öğretim elemanının bazı ifadelerinin çürütücü olarak kodlanamadığı, ancak bir şekilde tartışmanın akışını engellediği fark edilmiştir. Bu tür ifadeler meydan okuma veya itiraz olarak kodlanmıştır. Daha detaylı olarak açıklamak gerekirse, grup üyelerini bir süre düşünmeye sevk eden, soru işaretlerine veya tereddütlere yol açan, kalıpların dışında düşünmeye sevk eden ve kavramla ilgili farklı bir durum ve bakış açısını ortaya koyan ancak çürütücü bileşeni gibi argümanı çürütmeyi amaçlamayan ifadeler meydan okuma olarak sınıflandırılmıştır. Örneğin, Etkinlik 1'de bir katılımcı "verilen geniş üçgen olduğunda çevrel çember merkezine ne olur" sorusunu yönelttiğinde, gruptakilerin doğrudan bu yeni durum hakkında düşünmeye başladığı görülmüştür. Bu ifade argümantasyonun akışını doğrudan etkilemiştir ve çürütme amacını kapsamamaktadır. Son olarak, katılımcılar argümantasyon süreçlerinde gerekçesini belirtmeden bir itirazda bulunduklarında, bu tür ifadeler analiz sırasında itiraz olarak kodlanmıştır. Örneğin, Etkinlik 1'de bir katılımcı gerekçesini açıklamadan "bence doğru değil, çizdiğiniz yanlış" demiş ve diğer katılımcıların ikna etmek için yöntemi açıklamalarına neden olmuştur.

Toulmin'in modelindeki altı bileşenin yanı sıra, karmaşık argümantasyon sürecini araştırırken ve analiz ederken olası yeni bileşenler dikkate alınmalıdır. Bu çalışmada ifade edilen yeni bileşenler, matematiğin diğer alanlarında kullanılabilir. Ayrıca Toulmin'in modelinin disiplinden bağımsız olduğu ifade edildiğinden, modelin bu çalışmadaki uyarlanmış versiyonunun diğer disiplinlerdeki ortaklaşa argümantasyon süreçleri araştırılırken kullanılabilmesi beklenmektedir.