Obtaining Linear FRFs for Model Updating in Structures with Multiple Nonlinearities Including Friction

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ABSTRACT

Most of the model updating methods used in structural dynamics are for linear systems. However, in real life applications structures may have nonlinearity. In order to apply model updating techniques to a nonlinear structure, linear FRFs of the structure have to be obtained. The linear dynamic behavior of a nonlinear structure can be obtained experimentally by using low forcing level excitations, if friction type of nonlinearity does not exist in the structure. However when the structure has multiple nonlinearities including friction type of nonlinearity, nonlinear forces due to friction will be more pronounced at low forcing level excitations. Then it will not be possible to measure linear FRFs by using low level forcing. In this study a method is proposed to obtain linear FRFs of a nonlinear structure having multiple nonlinearities including friction type of nonlinearity by using experimental measurements made at low and high forcing levels. The motivation is to obtain FRFs of the linear part of the system that can be used in model updating of a nonlinear system. The method suggested can also be used as a nonlinear identification method for nonlinear systems. The proposed method is validated with different case studies using SDOF and lumped MDOF systems and simulated experimental data. The effect of the excitation frequency, at which experiments are carried out, on the accuracy of the proposed method, is also demonstrated.

KEYWORDS

Nonlinear identification, nonlinearity, friction nonlinearity, model updating, nonlinear structures

1 INTRODUCTION

Structural modeling is one of the most important stages in the design of a structure. Since design stage is iterative, the need for structural modeling has come into prominence and accurate prediction of the dynamic response of a structure has become a vital step in the design stage. With the development of computation technology, finite element (FE) method has established itself as the most common numerical method used for obtaining the dynamic response of engineering structures. However, application of FE method may yield inaccuracies arising from numerical and modeling errors. Due to these errors, there are always discrepancies between the dynamic responses obtained by FE method and experiment; therefore FE models need to be updated by using the experimental results and changing some of the parameters used in the FE model.

Over the last three decades various model updating methods have been developed in order to have correct analytical models that reflect the real dynamic responses better. However, most of the model updating methods available in the literature are for linear systems. Berman [1] updated the analytical mass matrix by using incomplete set of measured modes in order to achieve the orthogonality of the mass matrix. In order to correlate the FE model and test results of an aerospace structure, Sidhu and Ewins [2] proposed a method in which the error matrix equation was used. Caesar [3] used Berman's direct system matrix update method, suggested improvements on this method and applied the extended method to a test model. In a later study, Caesar [4] described the methods for updating mass and stiffness matrices based on the eigendynamic behavior of linear structures. Visser and Imregun [5] investigated the use of a model updating technique by using FRFs. They discussed the requirement for minimum measured data for successful implementation of the technique and applied the updating technique to different systems in order to demonstrate the effectiveness of the method. Larsson and Sas [6] worked on model updating technique employing forced vibration testing. They proposed a set of updating equations based on force response data and investigated the limitation in the measurements and numerical aspects in the formulations. Bollinger [7] presented constrained optimization theory in order to improve FE model. Lammens et al. [8] optimized reduced analytical dynamic stiffness matrix by solving a linearized set of equations, and updated the FE model of an engine sub frame. Girard et al. [9] extended energy approach for the model updating of a rotating shaft mounted on hydrodynamic bearings. They applied the method to a simple shaft model including mass, stiffness and damping parameters. Billet et al. [10] used an updating method based on minimization of an error in constitutive relation in order to update a nuclear reactor building scale model. Mottershead et al. [11] compared the selection of different updating parameters in the model updating of an aluminum space frame. In a more recent work, Kozak et al. [12] presented a new error localization method and an updating routine and they applied the routine to different case studies.

Since most of the structures have nonlinear behavior, it is vital to have model updating techniques for nonlinear structures as well. In order to apply model updating techniques developed for linear systems to nonlinear structures and to correct linear system matrices, linear dynamic behaviors of the structure have to be experimentally obtained which may require identification of nonlinearity first. In early 1990's Benhafsi et al. [13] worked on the parametric identification of nonlinearities in structures by using describing function method. In a later study, Richards and Singh [14] studied on the identification of nonlinearities that are in the form of polynomial forms and they approximated the nonlinear elastic forces as polynomial functions. Chong and Imregun [15, 16] presented an identification procedure in terms of variable modal parameters for nonlinear systems. Adams and Allemang [17] derived a method for estimating the parameters of nonlinear models and demonstrate the implementation of this method on simulated data for SDOF and MDOF lumped parameter systems. One of most detailed nonlinear system identification literature survey was performed by Kerschen et al. [18] in which more than 400 papers are cited. Özer et al. [20] extended their previous study [19] and identified the nonlinearity in structures by using describing functions and Sherman-Morrison method. Jalali et al. [21] used the inversion of describing functions in order to identify the nonlinearities in the structure.

In a recent study by Arslan et al. [22], two different methods which are capable of identifying nonlinearities in structures are implemented on a test rig containing a nonlinear element. They used low forcing level excitations in the experiments in order to obtain the linear FRFs of the structure. However, when there are multiple nonlinearities including friction type of nonlinearity, it will not be possible to measure the linear FRFs at low level of force excitation. Since most of the model updating methods are applied to linear analytical models it is important to obtain linear FRFs of a nonlinear structure first. In the present study a method is proposed to obtain linear FRFs in order to update linear model parameters of a nonlinear structure having multiple nonlinearities including friction type of nonlinearity. The proposed method is validated with different case studies using SDOF and lumped MDOF systems. In these case studies simulated experimental data is used. The effect of the excitation frequency, at which experiments are carried out, on the accuracy of the proposed method, is also demonstrated with a case study. The method suggested can also be used as a nonlinear identification method.

2 THEORY

The proposed method in this study is based on the theory developed by Budak and Özgüven [23, 24] for expressing the nonlinear forcing vector in a nonlinear structure as a matrix multiplication form for harmonically excited nonlinear MDOF systems. They expressed the nonlinear internal force vector in a nonlinear MDOF system as

$$\{N(x,\dot{x})\} = [\Delta(x,\dot{x})]\{X\}e^{i\omega t} \tag{1}$$

where $\{N(x,\dot{x})\}$ stands for the nonlinear internal forcing vector, $\{X\}$ is a complex response amplitude vector and $[\Delta(x,\dot{x})]$ is the "nonlinearity matrix" which was first presented by Budak and Özgüven [23, 24] for certain types of nonlinearities, and later by Tanrıkulu et al. [25] for any type of nonlinearity by using describing functions. The elements of nonlinearity matrix are written in terms of describing functions, such that the describing function ν_{rj} represents the nonlinearity in the system by giving the best average restoring force between coordinates r and j.

The elements of $[\Delta(x,\dot{x})]$ are given as follows [23]:

$$\Delta_{rr} = \nu_{rr} + \sum_{\substack{j=1 \\ j\neq r}} \nu_{rj}, \quad r = 1, 2, ..., n \tag{2}$$

$$\Delta_{rj} = -\nu_{rj}, \ \ r \neq j, \ \ r = 1, 2, ..., n \tag{3}$$

The response level dependent nonlinear FRF matrix (in the form of receptances) for a nonlinear system can be written as follows:

$$\left[H^{NL}\right] = \left[-\omega^2 [M] + i\omega[C] + i[D] + [K] + \Delta\right]^{-1} \tag{4}$$

where $[M]$, $[C]$, $[D]$, $[K]$ represent the mass, viscous damping, structural damping and stiffness matrices, respectively. Considering the linear part of this nonlinear system, the linear FRF matrix (in the form of receptances) can be written as follows:

$$\left[H^{L}\right] = \left[-\omega^2 [M] + i\omega[C] + i[D] + [K]\right]^{-1} \tag{5}$$

Taking the inverses of $\left[H^{NL}\right]$ and $\left[H^{L}\right]$ matrices given in equations (4) and (5), and then subtracting the second from the first, the following equation is obtained:

$$[\Delta] = \left[H^{NL}\right]^{-1} - \left[H^{L}\right]^{-1} \tag{6}$$

After some matrix manipulations, the linear FRF matrix can be obtained as

$$\left[H^{L}\right] = \left[\left[H^{NL}\right]^{-1} - [\Delta]\right]^{-1} \tag{7}$$

For a nonlinear MDOF system with multiple nonlinearities including friction type of nonlinearity, nonlinearity matrix $[\Delta]$ can be partitioned as

$$[\Delta] = \left[\Delta_f\right] + [\Delta_{HF}] \tag{8}$$

where $\left[\Delta_f\right]$ is the nonlinearity matrix due to friction and $[\Delta_{HF}]$ is the nonlinearity matrix due to remaining nonlinearities that are dominant at high forcing levels of excitation. Substituting equation (8) into equation (6), the following equation can be obtained:

$$\left[\Delta_f\right] + [\Delta_{HF}] = \left[H^{NL}\right]^{-1} - \left[H^{L}\right]^{-1} \tag{9}$$

When the structure is excited at low forcing levels, $\left[\Delta_f\right]$ will be dominant and $[\Delta_{HF}]$ will have negligible terms. Then, at low forcing levels equation (9) can be approximated as

$$\left[\Delta_f\right] \cong \left[H^{NL}\right]^{-1} - \left[H^{L}\right]^{-1} \tag{10}$$

On the other hand, for high forcing levels, $[\Delta_{HF}]$ will be more pronounced compared to frictional nonlinear forces, therefore equation (9) can be approximated at high forcing levels as

$$[\Delta_{HF}] \cong \left[H^{NL}\right]^{-1} - \left[H^{L}\right]^{-1} \tag{11}$$

Then, by using equations (10) and (11), and measuring FRFs experimentally several times at the same frequency but at different forcing levels, the linear FRFs can be obtained and the nonlinearities can be identified as explained below.

Let us assume that a certain set of experiments are performed at a constant frequency ω, at different forcing levels. Firstly, let the system be excited at a low forcing level, and then (*n-1*) times at different high forcing levels. Using equations (10) and (11), the following equations can be written:

$$\left[\Delta_f\right]_1 = \left[H^{NL}\right]_1^{-1} - \left[H^L\right]_1^{-1} \tag{12}$$

$$\left[\Delta_{HF}\right]_2 = \left[H^{NL}\right]_2^{-1} - \left[H^L\right]_2^{-1} \tag{13}$$

$$\left[\Delta_{HF}\right]_3 = \left[H^{NL}\right]_3^{-1} - \left[H^L\right]_3^{-1} \tag{14}$$

$$\left[\Delta_{HF}\right]_4 = \left[H^{NL}\right]_4^{-1} - \left[H^L\right]_4^{-1} \tag{15}$$

…
..
.

$$\left[\Delta_{HF}\right]_n = \left[H^{NL}\right]_n^{-1} - \left[H^L\right]_n^{-1} \tag{16}$$

where subscript *1, 2, 3, … n* indicates forcing cases. If the equation at low forcing level (equation 12) is subtracted from each of the following ones, a new set of equations will be obtained:

$$\left[\Delta_{HF}\right]_2 - \left[\Delta_f\right]_1 = \left[H^{NL}\right]_2^{-1} - \left[H^{NL}\right]_1^{-1} \tag{17}$$

$$\left[\Delta_{HF}\right]_3 - \left[\Delta_f\right]_1 = \left[H^{NL}\right]_3^{-1} - \left[H^{NL}\right]_1^{-1} \tag{18}$$

$$\left[\Delta_{HF}\right]_4 - \left[\Delta_f\right]_1 = \left[H^{NL}\right]_4^{-1} - \left[H^{NL}\right]_1^{-1} \tag{19}$$

…
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$$\left[\Delta_{HF}\right]_n - \left[\Delta_f\right]_1 = \left[H^{NL}\right]_n^{-1} - \left[H^{NL}\right]_1^{-1} \tag{20}$$

Since the linear FRF matrix is not forcing level dependent, then these terms will be canceled out and they will not appear in the resulting equations as can be seen above. There are both zero and nonzero elements in the nonlinearity matrices at the left hand sides of the equations, and nonzero elements are related to nonlinear coordinates. These nonzero elements which can be written as polynomial functions of response amplitudes with unknown coefficients are the describing functions of the corresponding nonlinearities. Since it is always possible to take data points more than the unknown coefficients, least square fit can be used for obtaining the unknown coefficients. In order to find the unknown coefficients, polynomial fit for (*n-1*) data points is applied in a least square sense and the equation of the corresponding regression curve is obtained. For more complex nonlinearities where polynomial fit may be insufficient, nonlinear fit can also be used. Once the unknown coefficients are obtained, by comparing the terms of the regression equation with the corresponding describing functions, nonlinearities are identified and then linear FRFs can easily be calculated by using one of the equations from (12) to (16), one of which is given below.

$$\left[H^L\right]_1 = \left[\left[H^{NL}\right]_1^{-1} - \left[\Delta_f\right]_1\right]^{-1} \tag{21}$$

The equations (17) to (20) will be reduced to very simple algebraic equations for a SDOF system:

$$\Delta_{HF_2} - \Delta_{f_1} = \frac{1}{H^{NL}{}_2} - \frac{1}{H^{NL}{}_1} \tag{22}$$

$$\Delta_{HF_3} - \Delta_{f_1} = \frac{1}{H^{NL}{}_3} - \frac{1}{H^{NL}{}_1} \tag{23}$$

$$\Delta_{HF_4} - \Delta_{f_1} = \frac{1}{H^{NL}{}_4} - \frac{1}{H^{NL}{}_1} \tag{24}$$

$$\cdots$$

$$\cdot\cdot$$

$$\cdot$$

$$\Delta_{HF_n} - \Delta_{f_1} = \frac{1}{H^{NL}{}_n} - \frac{1}{H^{NL}{}_1} \tag{25}$$

For a SDOF system, since all the matrix inversions simplify to inversions of a scalar, it is much easier to obtain the right hand sides of the above equations by using experimental measurements. Then linear FRFs of the system can be calculated by using the following equation:

$$\frac{1}{H^{L}{}_1} = \frac{1}{H^{NL}{}_1} - \Delta_{f_1} \tag{26}$$

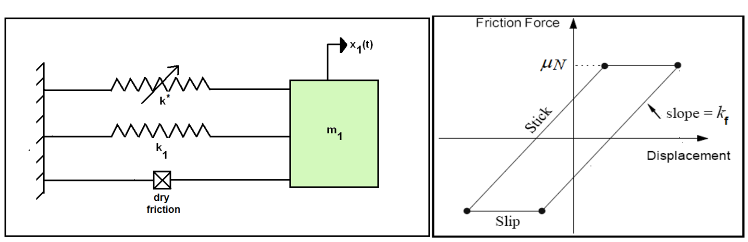
Equation (26) is valid for low forcing level at any frequency; therefore it is possible to obtain the linear FRFs over the desired frequency range. It should be noted that, although the above equations are valid for a harmonic excitation and therefore harmonic vibration of the system at any frequency, the difference between linear and nonlinear FRFs at an arbitrary frequency may be negligible, making it very difficult to identify the nonlinearity accurately. Hence, the identification can be made most accurately at frequencies where the nonlinearity has the maximum effect on the system response.

3 CASE STUDIES

In this section, applications of the proposed method to SDOF and MDOF nonlinear systems are presented. The first case study is purely a theoretical one and it illustrates the identification of nonlinearities and calculation of the linear frequency response of a nonlinear system from nonlinear FRF measurements. The second case study is an extension of the first one, in which polluted data is used in the analysis in order to simulate the experimental measurements more realistically. In the third case study, the proposed approach is applied to a MDOF system with multiple nonlinearities including friction. In this case study, again simulated experimental data is used. Finally, in order to study the effect of the excitation frequency at which measurements are made, on the accuracy of the proposed method, the same MDOF system is considered. The nonlinear parameters are identified by using simulated experimental FRF values obtained at different excitation frequency each time and the identified values are compared with each other. In obtaining all simulated experimental results, harmonic balance approach is used.

3.1 Application of the Approach to a SDOF System

In this case study, the proposed method is applied to a SDOF system with cubic stiffness and dry friction nonlinearities. The system and the dry friction model used in the case study are given in Fig. 1.



(a) (b)

Fig.1 (a) SDOF nonlinear system, (b) Dry friction model

The parameters of these nonlinear elements and the properties of the system are given as follows:

$$m_1 = 0.1\,kg,\;\; k_1 = 2x\,10^6\;\; N/m\;\; and\;\; \gamma(loss\ factor) = 0.01$$

$$k^* = 1x\,10^{10}\;\; N/m^3$$

$$\mu = 0.1,\;\; Normal\ Force = 10\ N\;\;\; and\;\;\; k_f\; = 3x\,10^5\;\; N/m$$

Firstly, the system is excited harmonically with a low forcing amplitude (F=0.01 N). The harmonic response of the system at this forcing level is compared with the linear frequency response of the system obtained disregarding both friction and cubic stiffness nonlinearity (Fig. 2).

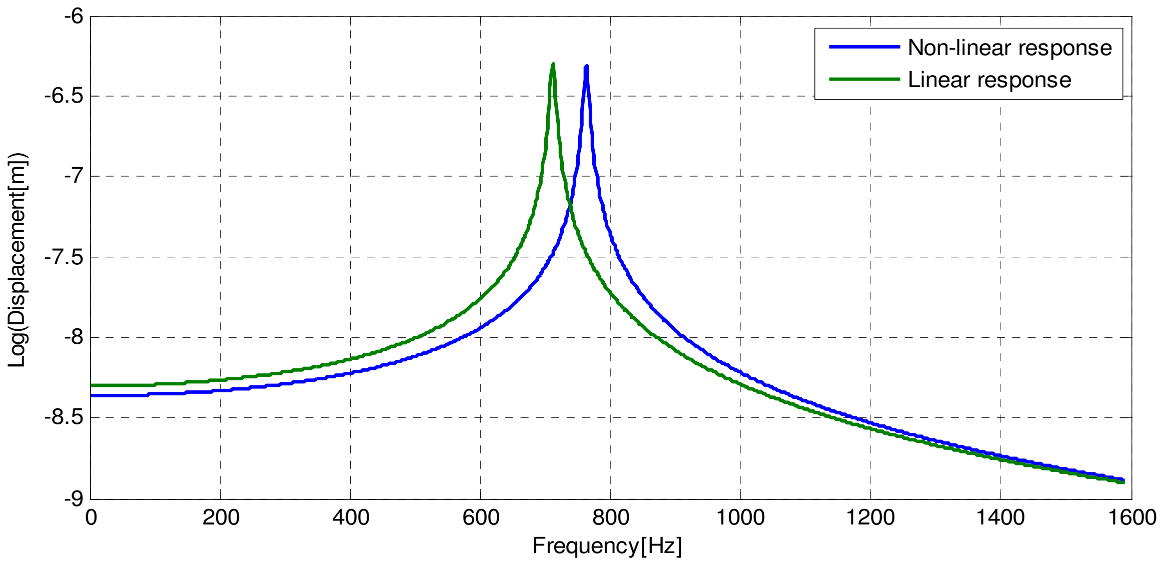
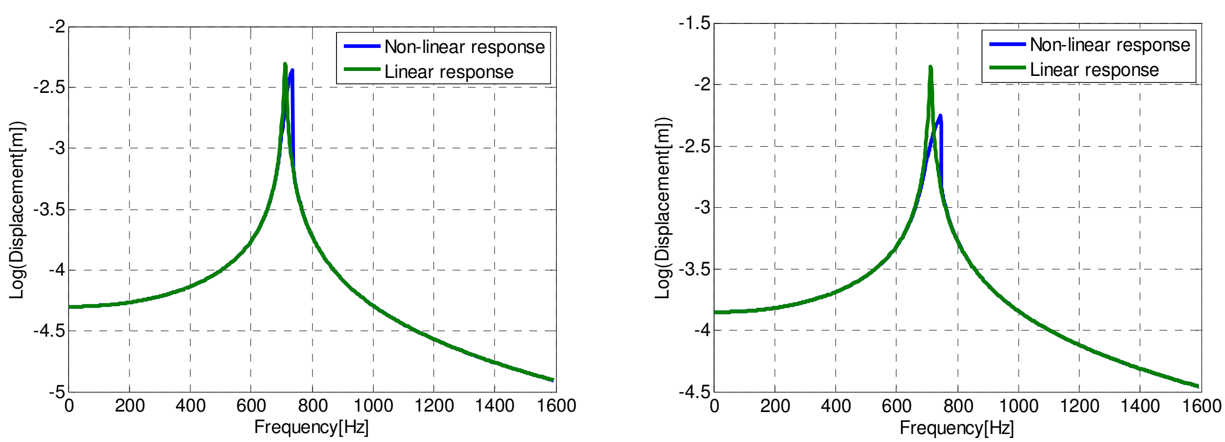


Fig.2 Harmonic response of the system for F=0.01 N

As can be seen in Fig. 2, since at low forcing level nonlinear internal forces due to cubic stiffness are negligible, the only nonlinear effect is due to friction. Frictional internal force causes a shift in the resonance frequency due to its stiffness component. Afterwards, 11 different higher excitation cases are considered by taking the amplitude of the harmonic excitation force between 100 N and 300 N and the responses of the system are obtained. In Fig. 3, nonlinear harmonic responses at only the forcing levels of F=130 N and F=280 N are shown.



(a) (b)

Fig.3 Harmonic response of the system for (a) F = 130 N, (b) F = 280 N

Since frictional nonlinearity is negligible at high forcing levels, as can be seen from the Fig. 3, only the cubic stiffness nonlinearity will be effective, and it will change the frequency response of the system around resonance causing a jump, which is a typical response behavior of cubic stiffness element. All above computations are based on the nonlinear identification made by using 11 different FRFs obtained at the excitation frequency of 710 Hz. By using "polyfit" function of MATLAB, equation of the regression curve is obtained and nonlinear parameters are identified by comparing the terms of the regression equation with the corresponding describing functions. The regression curve obtained can be seen in Fig. 4.

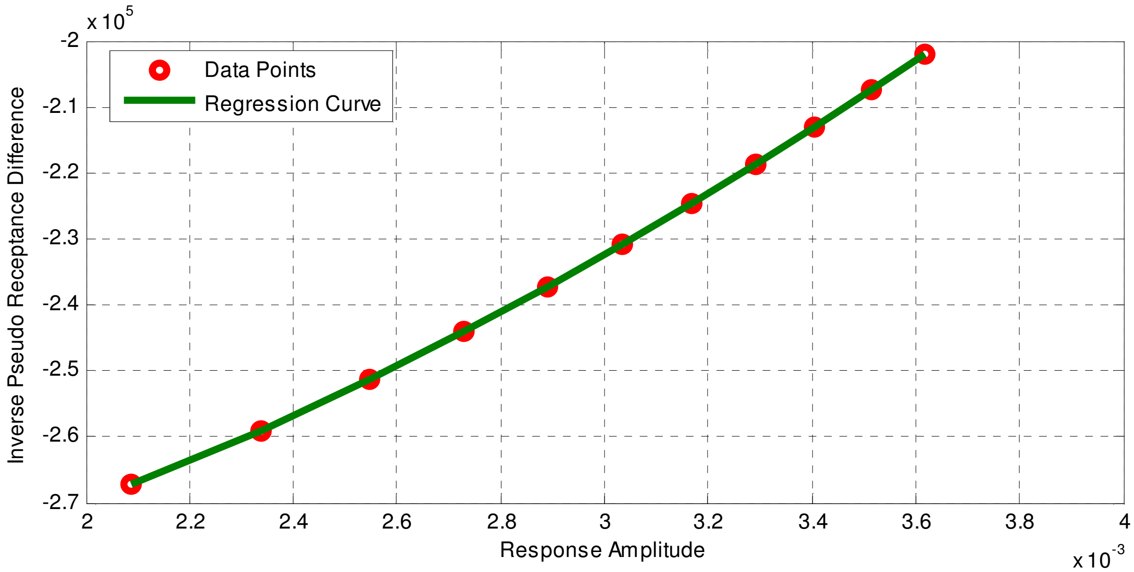


Fig.4 Polynomial regression curve for the available data points

The nonlinear parameters estimated are given below:

$$k^* = 9.9x\,10^9\ N/m^3 \quad and \quad k_f = 3x\,10^5\ N/m$$

It can be seen that the estimated parameters perfectly match with the actual values. The linear FRF of the system now can be calculated by using equation (26). The comparison of the estimated and actual linear frequency response is given in Fig. 5. As expected, there is a perfect match between estimated and actual linear frequency responses of the system.

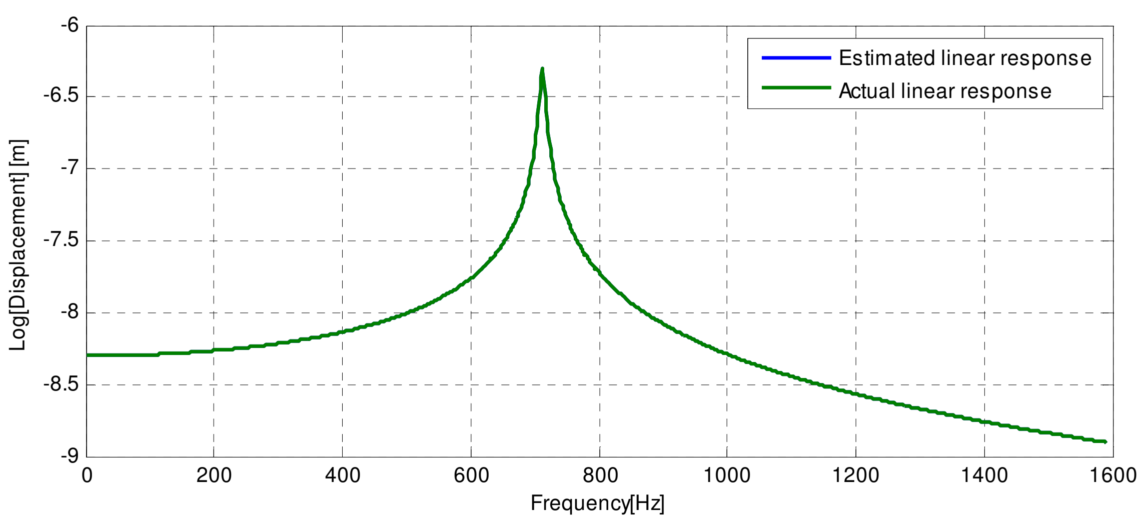


Fig.5 Comparison of the estimated and actual linear frequency responses of the system

3.2 Application of the Approach to a SDOF System with Polluted Data

In the second case study, in order to simulate the experimental data, theoretical data is polluted with 5% noise. The noise has normal distribution and standard deviation of 5% of the amplitude of the original response. In the analysis, SDOF nonlinear system with the same parameters as in the first case study is used. The system is excited with a low forcing amplitude and then with 11 different high forcing amplitudes. In Fig. 6, the FRF of the nonlinear system for F=0.01 N is compared with the linear frequency response of the system obtained disregarding both friction and cubic stiffness nonlinearity. The frequency responses of the system for F=130 N and F=280 N are given in Fig. 7.

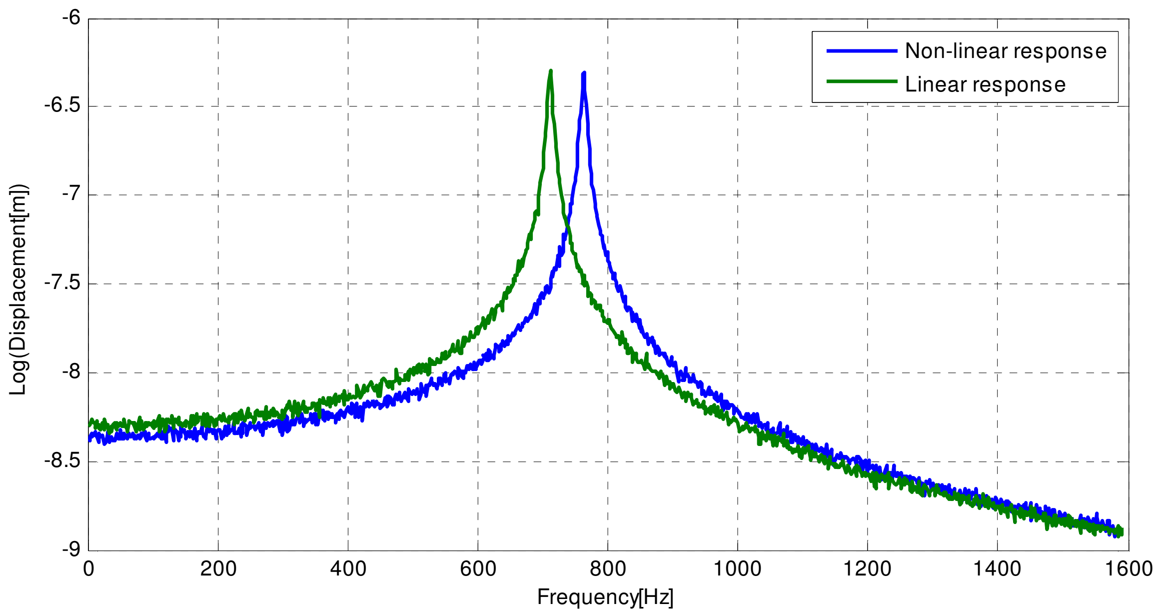
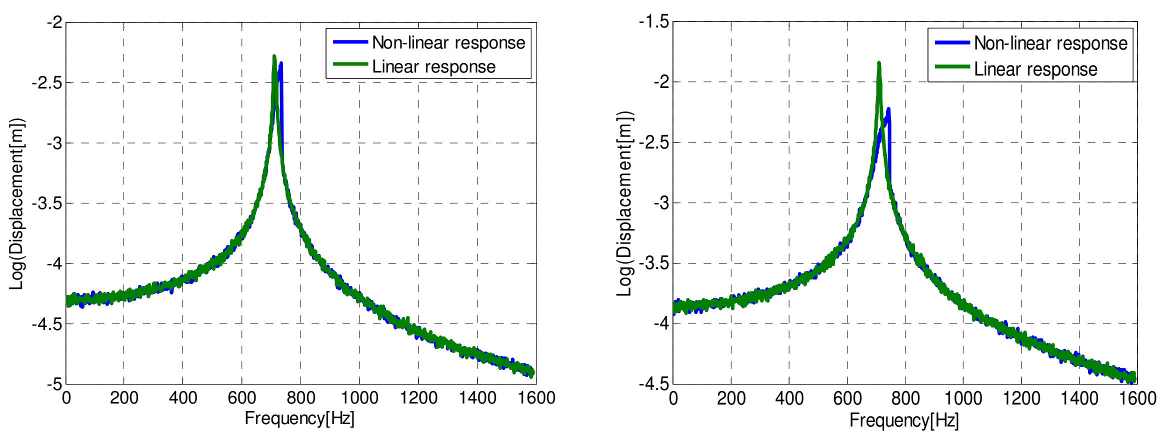


Fig.6 Frequency response of the system at F=0.01 N with 5% noise



(a) (b)

Fig.7 Frequency response of the system with 5% noise for (a) F = 130 N, (b) F = 280 N

At the excitation frequency of 710 Hz, 11 different data points are generated and the polynomial regression curve given in Fig. 8 is obtained.

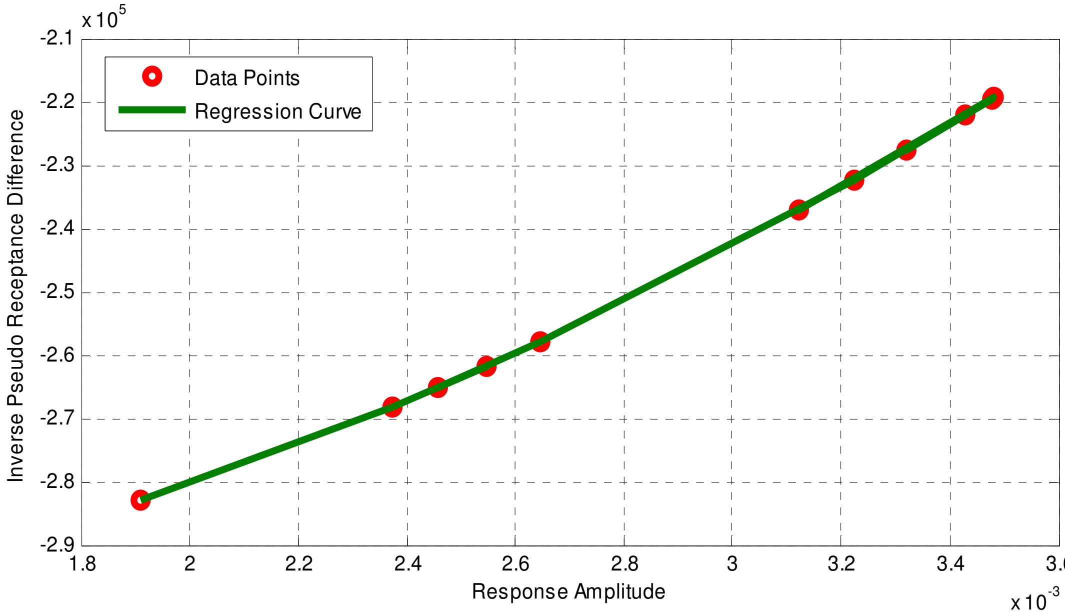


Fig.8 Polynomial regression curve for the available data points

Identified nonlinear parameters and comparison of these values with the actual ones are given in Table 1.

Table 1 Comparison of nonlinear parameters

Nonlinear Parameters	Estimated	Actual	% Error
k* (N/m^3)	9.9x 10^9	1x 10^{10}	1
k_f (N/m)	3.1x 10^5	3x 10^5	3.3

As can be seen in Table 1, there is a slight difference between the estimated and actual nonlinear parameters, mainly due to addition of noise to the theoretical data. Comparison of the estimated and actual linear frequency responses is given in Fig. 9. As expected, estimated linear frequency response matches perfectly with the actual one.

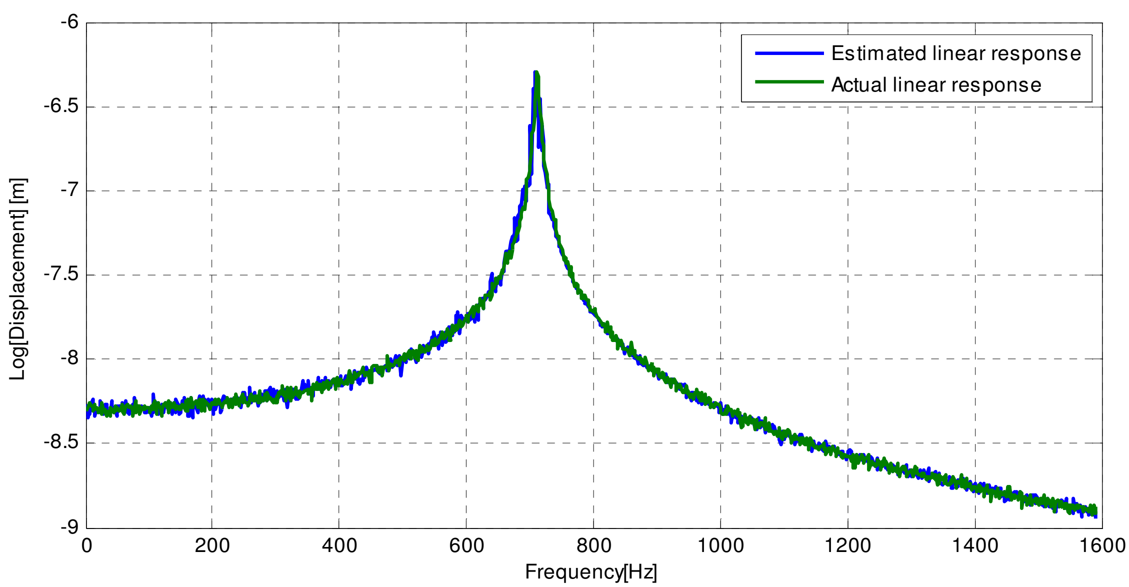


Fig.9 Comparison of the estimated and actual linear frequency response of the system with 5% noise

3.3 Application of the Approach to a MDOF System with Polluted Data

In the third case study, application of the proposed approach to a nonlinear MDOF system with polluted data is illustrated (Fig. 10). The same dry friction model given in previous case study is used here as well.

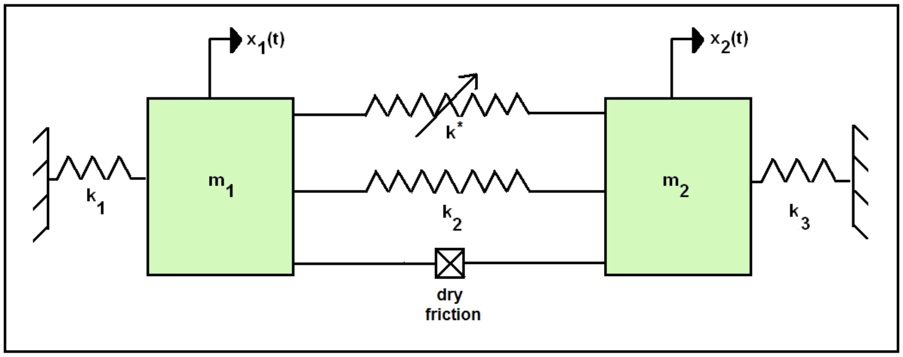


Fig.10 MDOF nonlinear system used in Case Study 3

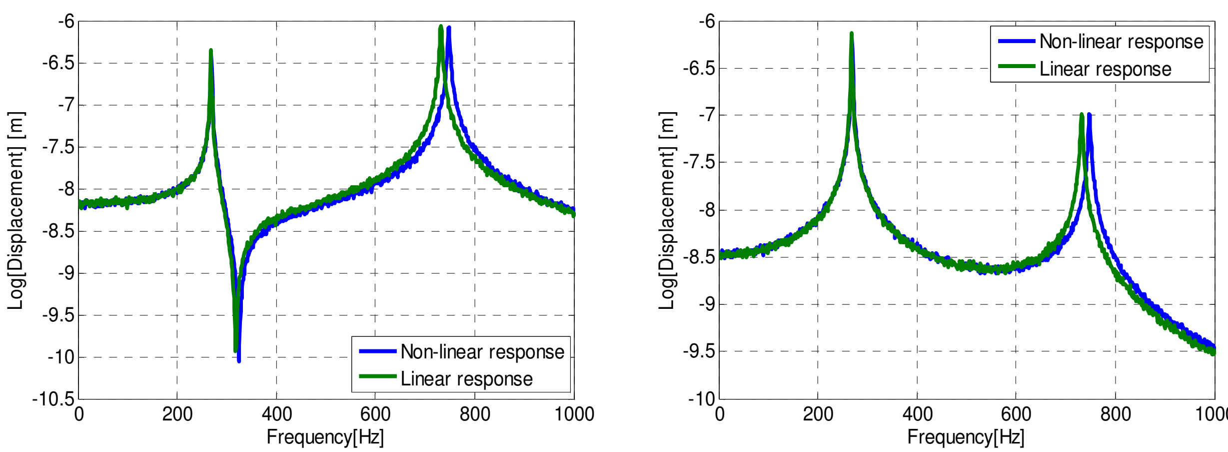
Parameters of the nonlinear element and the properties of the other system elements are given as follows:

$$m_1 = 0.1,\ m_2 = 0.5\,kg\ ,\ \ k_1 = k_2 = k_3 = 1x\,10^6\ N/m\ \ and\ \ \gamma(loss\ factor) = 0.005$$

$$\mu = 0.1,\ \ Normal\ Force = 10\ N\ \ \ and\ \ \ k_f\ = 8x\,10^4\ N/m$$

$$k^* = 1x\,10^{10}\ N/m^3$$

In order to simulate the experimental data, theoretical data is polluted with 5% noise. The noise has normal distribution and standard deviation of 5% of the amplitude of the original response. The system is excited with a low forcing level and then with 11 different high forcing levels. The frequency responses of the system at forcing levels of F=0.01 N and F=100 N are given in Fig. 11 and 12, respectively.



(a) (b)

Fig.11 Frequency responses of the system with 5% noise for F=0.01 N
(a) 1st coordinate, (b) 2nd coordinate

As can be seen in Fig. 11, for low forcing amplitudes, the only nonlinear effect is due to friction and frictional internal force causes a shift in the resonance frequency. The frictional nonlinearity is much more effective in the 2nd mode.

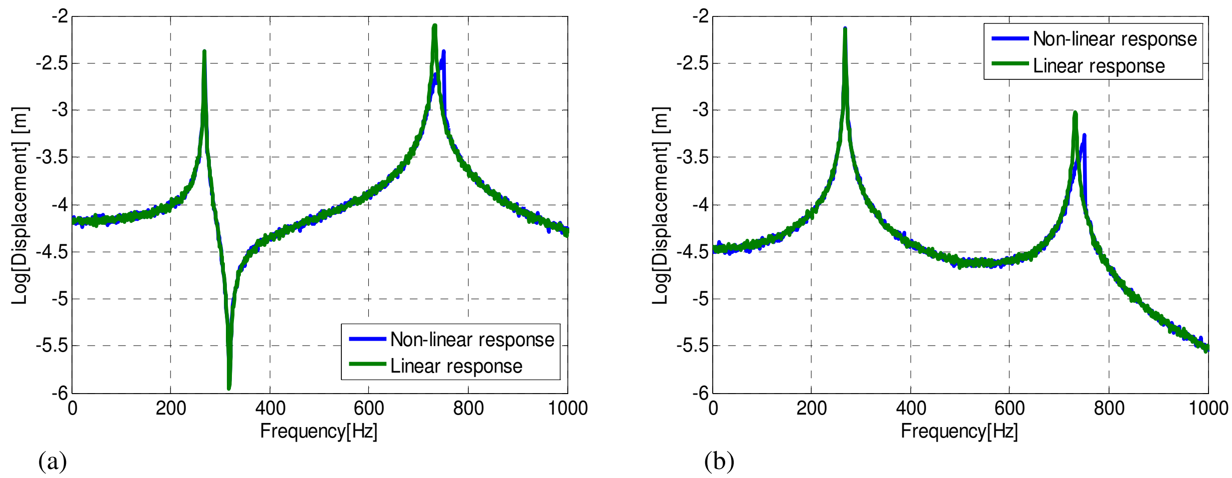


Fig.12 Frequency responses of the system with 5% noise for F=100 N
(a) 1st coordinate, (b) 2nd coordinate

In Fig. 12, it can be seen that, since frictional nonlinear internal forces are negligible for high forcing level, the nonlinearity changes the response of the system around 2nd resonance considerably by causing a jump, which is mainly due to cubic stiffness. As in the low forcing level case, 1st mode is not much affected from the existence of the stiffness nonlinearity in the system. The nonlinear parameters given in Table 2 are estimated as explained in the first case study (Fig. 13).

Table 2 Comparison of nonlinear parameters

Nonlinear Parameters	Estimated	Actual	% Error
k* (N/m^3)	9.9×10^9	1×10^{10}	1
k_f (N/m)	8.03×10^4	8×10^4	0.4

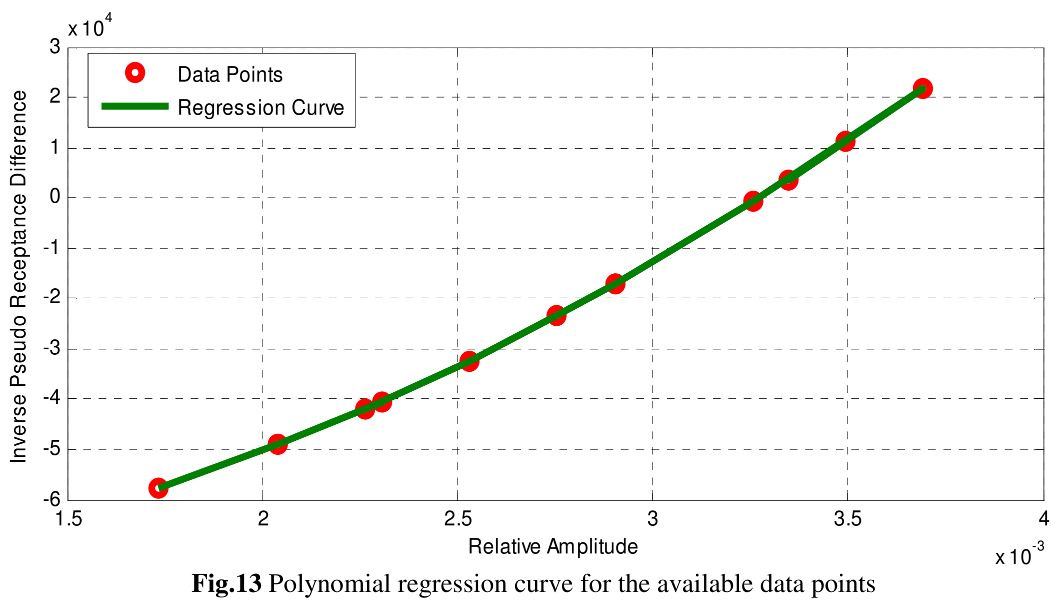


Fig.13 Polynomial regression curve for the available data points

As can be seen from the values in Table 2, estimated and actual nonlinear parameters are in perfect match. However, since in the proposed approach, excitation frequency is a free parameter (i.e., the equations are valid for any forcing level and therefore vibration at any frequency), the effect of the excitation frequency used in the experiments on the performance of the method should be analyzed.

3.4 Performance of the Method at Different Excitation Frequencies

As the last case study, in order to see the effect of excitation frequency on the proposed method, different simulated tests at various frequencies are performed for the MDOF nonlinear system given in the previous case study. For each test case, nonlinearities are identified and they are compared with the actual nonlinear parameters. Comparison of the results is given in Table 3.

Table 3 Comparison of nonlinear parameters identified by using different excitation frequencies

Excitation Frequency (Hz)	Estimated k* (N/m^3)	Actual k* (N/m^3)	% Error for k*	Estimated k_f (N/m)	Actual k_f (N/m)	% Error for k_f
725	9.9×10^9	1×10^{10}	3.3	8.03×10^4	8×10^4	0.4
730	9.9×10^9		3.3	7.67×10^4		4.1
745	9.9×10^9		3.3	16.3×10^4		103.8

Since the nonlinear forces affect the 2nd mode more, excitation frequencies are selected around the 2nd resonance of the system. As can be seen from Table 2, for all the excitation frequencies used, cubic stiffness values are successfully estimated. However, the frictional stiffness value is highly affected from the selection of excitation frequency, even though all frequencies are around the resonance region. It is observed that when the excitation frequency is closer to the 2nd resonance of the linear frequency response, estimated values become more accurate.

4 DISCUSSION AND CONCLUSIONS

When there are multiple nonlinearities including friction in a system, it is not possible to obtain the linear frequency response of the system by using low forcing levels in the experiments. In this paper, an approach for obtaining the linear FRFs of a nonlinear system with multiple nonlinearities including friction is proposed. The basic motivation is to have FRFs of the linear part of a nonlinear structure, so that they can be used in model updating.

In the method proposed, FRF values are measured at all coordinates that we are interested in, but at a constant frequency ω and at different forcing levels. First, the system is excited at a low forcing level, and then several times at different high forcing levels. By using the measured nonlinear FRFs, first the nonlinearities are identified and then linear FRFs are obtained in order to use them in model updating of nonlinear systems.

The method is validated with different case studies using SDOF and MDOF systems and simulated experimental data. In the first and second case studies, application of the approach is demonstrated on a SDOF nonlinear system. It is shown that the approach is very successful in identifying multiple nonlinearities in the system, as well as in determining the linear FRF. In simulated experimental case study it is observed that, the noise added to the data affects the values of the identified nonlinear parameters and linear frequency response. In the third case study, the method is applied to a MDOF nonlinear system by using simulated experimental data again. The results obtained show that nonlinearities can be identified very accurately by using nonlinear FRFs measured at low and high forcing levels. In the last case study, the effect of the excitation frequency used in the experiments on the performance of the method is investigated. From the results obtained it is concluded that excitation frequency should be close to the resonance frequency, otherwise the accuracy of the identified friction nonlinearity can be deteriorated considerably.

Consequently, it can be said that the method can successfully be used for the identification of nonlinearity and calculation of the linear FRFs of a nonlinear structure with multiple nonlinearities including friction. The applicability and accuracy of the approach proposed is demonstrated only on simple SDOF and MDOF systems. The method needs to be tested on real structures, in order to apply it to obtain FRFs of the linear part of a nonlinear structure that can be used in model updating of the nonlinear structure.

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