# Reanalysis of Dynamic Structures Using Successive Matrix Inversion Method

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## ABSTRACT

Increasing demand for high precision mechanical components has increased the importance of dynamic design of structures. Competitiveness dictates the necessity to predict the dynamic properties of structures when some modifications are to be made. This paper presents a new structural reanalysis approach which is an extension of Successive Matrix Inversion method presented for static analysis to dynamic analysis of structures. The method is based on exact calculation of Frequency Response Function (FRF) matrix of a modified structure using FRF matrix of the original structure and modifying mass, stiffness and damping matrices. Case studies are presented in order to demonstrate the application of the method. The results obtained are compared with exact values, as well as with those obtained by using Özgüven's Structural Modification method.

## NOMENCLATURE

| | |
|---|---|
| $i$ | Unit imaginary number |
| $N$ | Number of degrees of freedom |
| $\omega$ | Excitation frequency |
| $\eta$ | Structural damping proportionality constant |
| $r$ | Recursion factor |
| $m$ | Number of non-zero columns |
| $\left[M\right]$ | Mass matrix |
| $\left[K\right]$ | Stiffness matrix |

|  |  |
|---|---|
| $[H]$ | Structural damping matrix |
| $[C]$ | Viscous damping matrix |
| $[\Delta M]$ | Modification mass matrix |
| $[\Delta K]$ | Modification stiffness matrix |
| $[\Delta H]$ | Modification structural damping matrix |
| $[\Delta C]$ | Modification viscous damping matrix |
| $\{x\}$ | Displacement vector |
| $\left\{\ddot{x}\right\}$ | Acceleration vector |
| $\{F\}$ | Forcing vector |
| $[\alpha]$ | Receptance matrix of original system |
| $[\gamma]$ | Receptance matrix of modified system |
| $[I]$ | Identity matrix |

## 1. INTRODUCTION

Structural modifications are often required to enhance the dynamic behavior of an existing structure. It is almost impossible to design an optimum product that satisfies all the needs in the first trial, hence one of the basic practices of engineering is to build a work upon a previous one, thus saving time and effort. In such a design process, it may be necessary to study numerous alternative configurations in order to ensure a suitable dynamic response. In other instances, engineers often have difficulty when it is noticed that the dynamic properties of a structure do not meet the requirements of the design, and some modifications on the structures have to be made to obtain desired dynamic properties.

Structural modification is a general term used in structural dynamics that refers to either predicting the dynamic behavior of a modified structure (known as forward structural modification) or determining the modifications that should be made on an existing structure in order to obtain a required dynamic behavior (known as inverse structural modification). Forward structural modification, also known as reanalysis, is common in vibration optimization and in finite element model updating, whereas inverse methods are mostly used for eliminating vibration problems after a structure is constructed.

Several methods have been developed for both forward and inverse structural modification problems. The matrix inversion method proposed for finding receptances of locally damped structures from undamped counterparts [1] has later been combined with an efficient solution algorithm in order to avoid matrix inversion [2], and thus an efficient structural modification method has been developed for structural reanalysis problems [3]. The method is capable of finding frequency response functions (FRFs) of a modified structure from those of the original system. The method has been extended in the same work for structural modification cases with additional degrees of freedom (DOF). In a later work, the same approach has been followed and the matrix inversion formulation has been combined with Sherman-Morrison formula [4] and Woodbury formula [5] in order to avoid matrix inversion in structural modification problems when there are no additional degrees of freedom [6]. In both of the above methods [3, 6] the major part of the computations are limited to the coordinates where the modifications are made, which makes these methods favorable for systems with large DOFs and local modifications. The methods are very efficient as it is not required to invert a matrix or solve a new eigenvalue problem. Hager [7] also followed a similar approach, without the need for any matrix inversion or a solution of a new eigenvalue problem.

Many approaches have also been suggested for the solution of the inverse eigenvalue problem. Bucher and Braun [8] have developed a theory when a partial set of eigensolutions is available, where it is possible to find the necessary mass and stiffness modifications by using modal test results only. The work presents the general inverse problem with truncated information from measured FRFs. The solution is obtained by a linear combination of the original unmodified modal vectors. By this way, the effect of truncation is evaded. Park and Park [9] have developed a method that also uses test FRFs of the unmodified structure and enables one to analytically find necessary multiple mass, stiffness and damping modifications in order to obtain the required eigenvalues and eigenvectors. Akgün et al. [10] used the Sherman-Morrison-Woodbury formulas in the reanalysis problem for static systems. Huang and Verchery [11] presented a structural static reanalysis method that can be used in practical problems like progressive failure analysis. Kirsch et al. [12-14] presented a reanalysis approach based on Combined Approximations method for design, structural analysis and optimization. Chen, Yang and Lian [15] have compared several eigenvalue reanalysis methods for modified structures in terms of their computational efficiency and accuracy.

Bae, Grandhi and Canfield [16] have presented the so-called Successive Matrix Inversion (SMI) Method for Reanalysis. The method is given for static systems and it is an exact and a bound-free method (that is, there is no restriction in initial matrix size). In that work, comparisons of Successive Matrix Inversion Method with Cholesky decomposition, Gauss elimination and QR decomposition are also given. The method is used for local modifications such as replacement of aging aircraft parts and repair of battle damage. SMI is also used for reliability analysis for the uncertainties of material properties.

This paper presents a structural reanalysis approach which is an extension of the Successive Matrix Inversion method [16] presented for static analysis to dynamic analysis of structures. The method is based on exact calculation of FRFs of the modified structure by using those of the original structure, and the modifying mass, stiffness and damping matrices. The basic equation obtained in this method is the same as the one used in Özgüven's Structural Modification method [3]. However in the method presented here power series expansion is used in order to avoid matrix inversion, whereas a novel algorithm [2] is used in Özgüven's method to avoid matrix inversion, although matrix inversion is always an alternative for local modifications with small number of modified coordinates since the order of the matrix to be inverted is equal to the number of modified coordinates [3]. Case studies are presented in order to demonstrate the application of the method. Based on the numerical results, Successive Matrix Inversion method appears to be an efficient alternative method for reanalysis of dynamic structures subjected to local structural modifications.

## 2. SUCCESSIVE MATRIX INVERSION METHOD APPLIED TO DYNAMIC STRUCTURES

The equation of motion for an $N$ degrees of freedom system can be written as

$$\left[M\right]\left\{\ddot{x}\right\}+i\left[H\right]\left\{x\right\}+\left[K\right]\left\{x\right\}=\left\{F\right\} \tag{1}$$

where $\left[M\right],\left[H\right],\left[K\right]$ are mass, structural damping and stiffness matrices of the system, respectively, $\left\{x\right\}$ is the displacement vector, $\left\{F\right\}$ is the forcing vector and $i$ is the unit imaginary number. The dot shows differentiation with respect to time. The response of the system to a harmonic forcing at a frequency $\omega$ can be written as

$$\left\{x\right\}=\left[\left[K\right]-\omega^2\left[M\right]+i\left[H\right]\right]^{-1}\left\{F\right\} \tag{2}$$

from which the receptance matrix $\left[\alpha\right]$ can be obtained as

$$\left[\alpha\right]=\left[\left[K\right]-\omega^2\left[M\right]+i\left[H\right]\right]^{-1} \tag{3}$$

Now, if the structural modification matrices for mass, structural damping and stiffness are denoted by $\left[\Delta M\right]$, $\left[\Delta H\right]$, $\left[\Delta K\right]$, the equation of motion of the modified system will take the form

$$\left[M+\Delta M\right]\left\{\ddot{x}\right\}+i\left[H+\Delta H\right]\left\{x\right\}+\left[K+\Delta K\right]\left\{x\right\}=\left\{F\right\} \tag{4}$$

and the harmonic response of the system will be obtained as

$$\left[\left[K+\Delta K\right]-\omega^2\left[M+\Delta M\right]+i\left[H+\Delta H\right]\right]\left\{x\right\}=\left\{F\right\} \tag{5}$$

$$\left\{x\right\}=\left[\left[K+\Delta K\right]-\omega^2\left[M+\Delta M\right]+i\left[H+\Delta H\right]\right]^{-1}\left\{F\right\} \tag{6}$$

Then the receptance matrix of the modified system will be

$$\left[\gamma\right]=\left[\left[K+\Delta K\right]-\omega^2\left[M+\Delta M\right]+i\left[H+\Delta H\right]\right]^{-1} \tag{7}$$

$\left[\gamma\right]$ can also be obtained by premultiplying Eq. (5) by $\left[\alpha\right]$

$$\left(\left[I\right]-\left[P\right]\right)\left\{x\right\}=\left\{F'\right\} \tag{8}$$

where

$$\left\{F'\right\}=\left[\alpha\right]\left\{F\right\} \tag{9}$$

$$\left[P\right]=-\left[\alpha\right]\left[\left[\Delta K\right]-\omega^2\left[\Delta M\right]+i\left[\Delta H\right]\right] \tag{10}$$

Then from Eq. (8) and Eq.(9)

$$\left[\gamma\right]=\left(\left[I\right]-\left[P\right]\right)^{-1}\left[\alpha\right] \tag{11}$$

Thus we obtain the basic equation of the structural modification method of Özgüven [3]. While the size of the matrix inverted is reduced for locally modified systems by partitioning the system matrices, and furthermore a novel algorithm is suggested to avoid matrix inversion in Özgüven’s method, here in this work, power series expansion will be used for the inversion of matrix in Eq.(11) as successfully employed in Successive Matrix Inversion method [16]:

$$\left(\left[I\right]-\left[P\right]\right)^{-1}=\left[I\right]+\left[P\right]+\left[P\right]^2+\left[P\right]^3+\ldots \tag{12}$$

Let

$$\left[T\right]=\left[P\right]+\left[P\right]^2+\left[P\right]^3+\ldots \tag{13}$$

The elements of $\left[T\right]$ can be written as

$$T_{ij}=P_{ij}^{(1)}+P_{ij}^{(2)}+\ldots+P_{ij}^{(k)}+\ldots \tag{14}$$

where $P_{ij}^{(k)}$ is the $\left(i,j\right)$th element of $\left[P\right]^{(k)}$. If the k$^{th}$ recursion factor is defined as

$$r_{ij}^{(k)}=P_{ij}^{(k+1)}/P_{ij}^{(k)} \tag{15}$$

and if the recursion factor is constant through the expansion, Eq. (14) can be expressed as

$$T_{ij}=P_{ij}\left(1+r_{ij}+r_{ij}{}^2+r_{ij}{}^3+\ldots\right) \tag{16}$$

Using the series expansion for the recursive terms, $T_{ij}$ can be written as

$$T_{ij} = P_{ij} / \left(1 - r_{ij}\right) \tag{17}$$

Since the recursion factor is different through the series expansion, the modification matrix should be decomposed into separate matrices so that the variability of the recursion factor could be eliminated.

$$\left[\Delta K\right] - \omega^2 \left[\Delta M\right] + i\left[\Delta H\right] = \sum_{j=1}^{N} \left[\left[\Delta K^{(j)}\right] - \omega^2 \left[\Delta M^{(j)}\right] + i\left[\Delta H^{(j)}\right]\right] \tag{18}$$

where $\left[\Delta K^{(j)}\right]$, $\left[\Delta M^{(j)}\right]$, $\left[\Delta H^{(j)}\right]$ are matrices composed of the j$^{\text{th}}$ columns of stiffness, mass and structural damping matrices, respectively, and zero columns except the j$^{\text{th}}$ columns. For only one nonzero column of $\left[T\right]$, the recursion factor is a constant

$$r = P_{jj} \tag{19}$$

Then Eq. (17) becomes

$$T_{ij} = P_{ij} / \left(1 - r\right) \tag{20}$$

Now let

$$\left[Y\right] = \left[\left[\Delta K\right] - \omega^2 \left[\Delta M\right] + i\left[\Delta H\right]\right] \tag{21}$$

$$\left[Z\right] = \left[\left[K\right] - \omega^2 \left[M\right] + i\left[H\right]\right] \tag{22}$$

Then, the dynamic stiffness matrix, after the j$^{\text{th}}$ non-zero column of the structural modification is taken into consideration, can be written as

$$\left[Z^{(j)}\right] = \left[Z^{(j-1)}\right] + \left[Y^{(j)}\right] \tag{23}$$

where $\left[Z^{(j-1)}\right]$ and $\left[Z^{(j)}\right]$ are the modified dynamic stiffness matrices in (j-1)$^{\text{th}}$ and (j)$^{\text{th}}$ steps, respectively. $\left[Y^{(j)}\right]$ is a matrix which has the j$^{\text{th}}$ non-zero column of $\left[Y\right]$ matrix at its corresponding column, and zero columns elsewhere. Note that $\left[Z^{(0)}\right]$ denotes the initial $\left[Z\right]$ matrix, and for the modification matrix $\left[Y^{(j)}\right]$, $\left[Z^{(j-1)}\right]^{-1}$ refers to $\left[\alpha\right]$ and $\left[Z^{(j)}\right]^{-1}$ refers to $\left[\gamma\right]$ in Eq.(11). By using Eq. (12) and Eq.(13), the inverse of Eq. (23) can be obtained as:

$$\left[Z^{(j)}\right]^{-1} = \left(\left[I\right] + \left[T\right]\right)\left[Z^{(j-1)}\right]^{-1} \tag{24}$$

Thus, by updating these matrices for each nonzero column of the modification matrix, the modified FRF matrix can be obtained. Since each column of the modification matrix contributes to the dynamics of the system independently, the sequence of the columns used in the computation is not important. It should also be noted at this stage that for a local modification, $\left[Y\right]$ will be a highly sparse matrix with many zero columns and rows corresponding to the coordinates at which there is no structural modification, and that property of $\left[Y\right]$ will make the method presented here attractive.

The algorithm of the reanalysis method suggested for structural modifications using SMI can be summarized as follows:

**Step 1:**

Obtain the original FRF matrix and the modification matrix $\left[Y\right]$.

**Step 2:**

Partition the modification matrix to number of non-zero element columns (*m*) such that

$$\left[Y\right]=\left[Y^{(1)}\right]+\left[Y^{(2)}\right]+\ldots+\left[Y^{(m)}\right] \tag{25}$$

**Step 3:**

For each non-zero column of $\left[Y\right]$ ( j=1,2,…m ), the following computations are repeated:

$$\left[P^{(j)}\right]=-\left[Z^{(j-1)}\right]^{-1}\left[Y^{(j)}\right] \tag{26}$$

$$\left[Z^{(j)}\right]^{-1}=\left[Z^{(j-1)}\right]^{-1}+\left(\frac{1}{1-r^{(j)}}\right)\left[P^{(j)}\right]\left[Z^{(j-1)}\right]^{-1} \tag{27}$$

where $\left[P^{(j)}\right]$ will have all columns zero except the j-th column, since $\left[Y^{(j)}\right]$ is in the same form**.**

Then in the last step $\left[Z^{(m)}\right]^{-1}$ will give the receptance matrix of the modified structure. The algorithm of Successive Matrix Inversion method uses only matrix additions and matrix-vector multiplications to update the response matrix. In Eq. (26), $\left[Y^{(j)}\right]$ and therefore $\left[P^{(j)}\right]$ have many zero elements so that using sparse matrix properties in the storage and matrix operations in the algorithm summarized above will reduce the computational effort considerably.

If we have viscous damping $\left[C\right]$, instead of structural damping, then obviously $\left[H\right]$ and $\left[\Delta H\right]$ will be replaced by $\omega\left[C\right]$ and $\omega\left[\Delta C\right]$ respectively in all equations given above.

## 3. CASE STUDIES

In order to demonstrate the application of the structural modification method proposed in this study, two systems are analyzed and the results obtained are compared with exact values. The method is first applied to a discrete system for which the original system mass and stiffness matrices are given as follows:

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix} \text{[kg]} \qquad (28)$$

$$K = \begin{bmatrix} 5000 & -1000 & 0 & 0 & 0 & 0 \\ -1000 & 12500 & -5500 & 0 & -6000 & 0 \\ 0 & -5500 & 5500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 14000 & -6000 & 0 \\ 0 & -6000 & 0 & -6000 & 19000 & -4000 \\ 0 & 0 & 0 & 0 & -4000 & 4000 \end{bmatrix} \text{[N/m]} \qquad (29)$$

The damping is taken as structural (hysteretic) damping with a loss factor of $\eta = 0.05$.

The system is modified by placing additional masses ($M_4$ and $M_5$) at coordinates 4 and 5, respectively, and inserting additional stiffnesses ($k_2$ and $k_{3-4}$) between coordinate 2 and the ground, and between coordinates 3 and 4, respectively. The values of the modifying elements are given in Table 1.

**Table 1:** Modifying System Parameters

| | |
|---|---|
| **$M_4$ (kg)** | 7 |
| **$M_5$ (kg)** | 5 |
| **$k_2$ (kN/m)** | 4 |
| **$k_{3-4}$ (kN/m)** | 3.5 |

The receptance matrix of the modified system is calculated by using the Successive Matrix Inversion method generalized for dynamic systems in this work, as well as by using Özgüven's Structural Modification method. When the frequency response functions of the modified system are also calculated by analyzing the whole system itself it is observed that the three methods give exactly the same results. The point receptances of the modified system at coordinates 5 and 6 are compared with those of the original system in Figures 1 and 2, respectively.

As a second example, a beam with both ends fixed is considered. The beam is modeled using six finite elements and 10 degrees of freedom. The modulus of elasticity ($E$) of the beam is taken as 200 $GPa$ and the mass density ($\rho$) is taken as 8x$10^3$ $kg/m^3$. The length of the beam is 1200 $mm$. The original beam has a wall thickness of 3 $mm$. The beam is modified by reducing the wall thickness to 2 $mm$ and by adding a 50 $kg$ mass to the midpoint of the beam. The loss factor is again taken as 0.05, as in the first example.

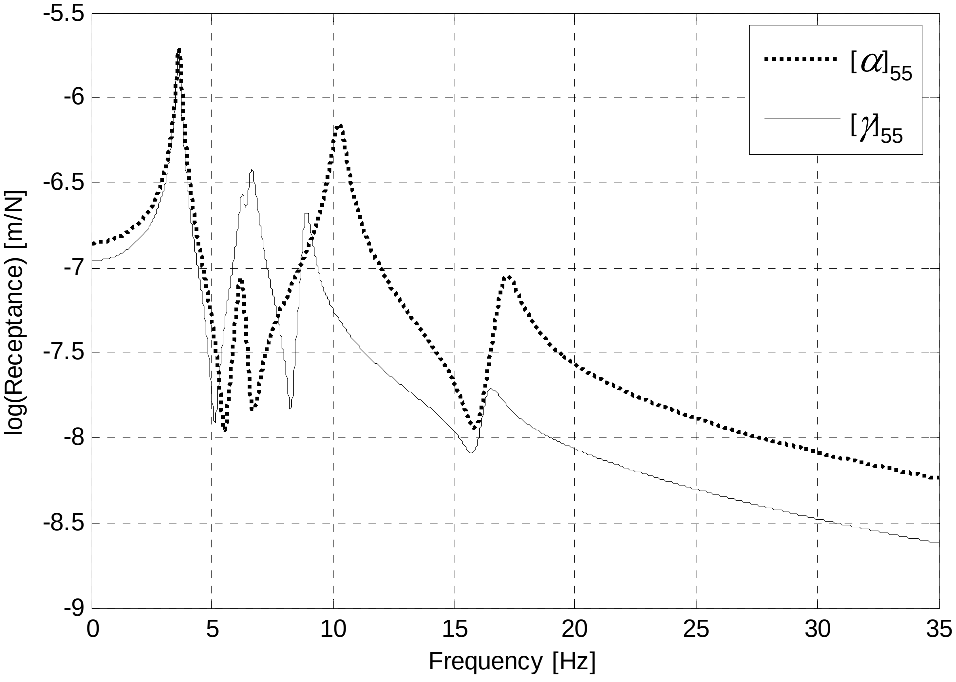

**Figure 1**: Point receptance at coordinate 5 for original and modified systems – Case Study 1

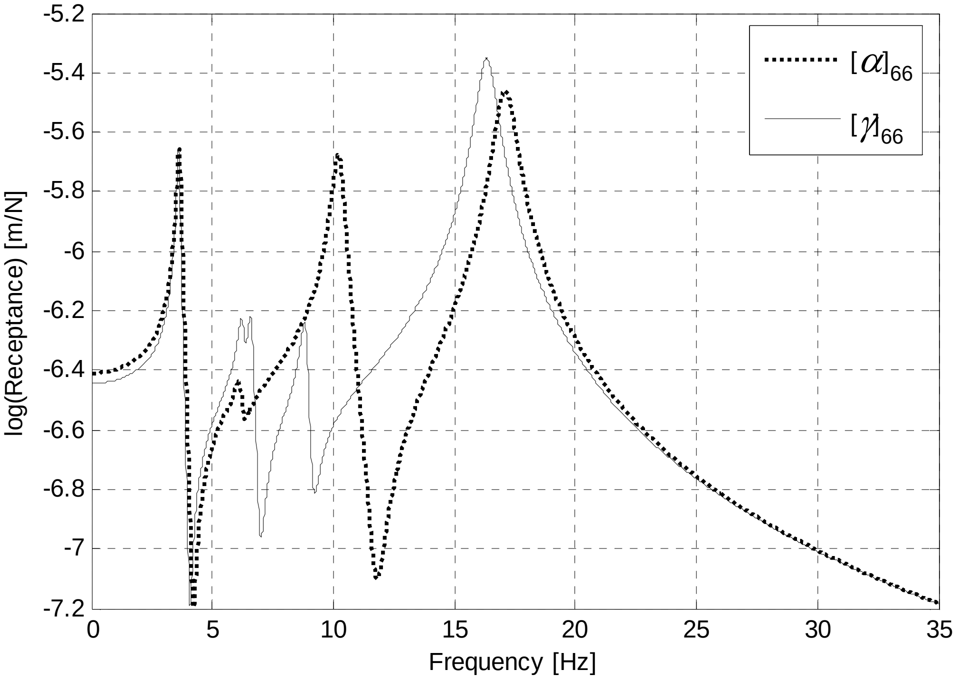

**Figure 2**: Point receptance at coordinate 6 for original and modified systems – Case Study 1

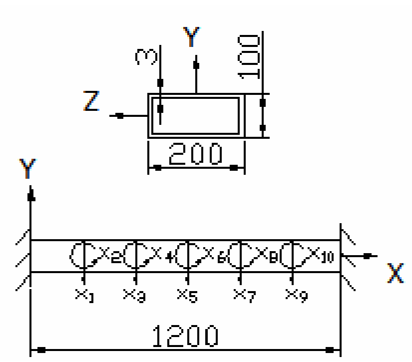

**Figure 3**: Case Study 2: Original beam

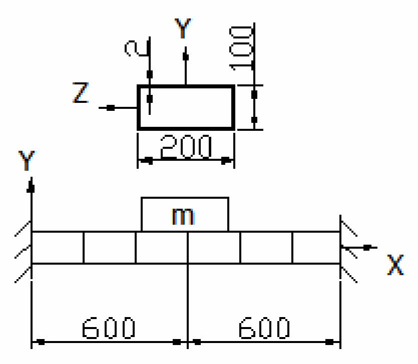

**Figure 4**: Case Study 2: Modified beam

The receptance matrix of the modified beam is calculated using the SMI method for dynamic systems, Özgüven's Structural Modification method and straight computation of the modified system, all of which again yield exactly the same results. Two modified point receptances are compared with those of the unmodified system in Figure 5 and Figure 6.

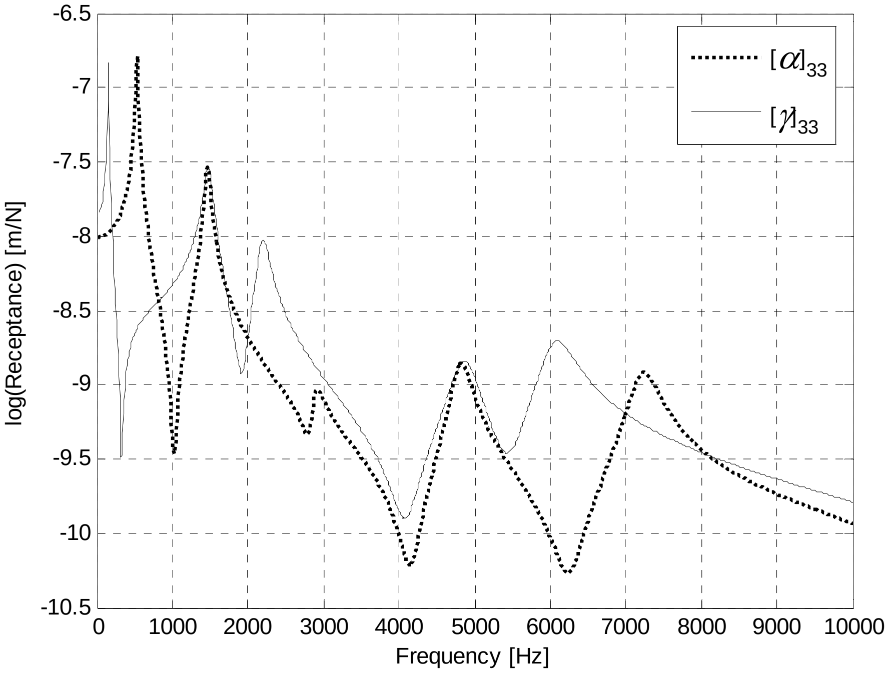

**Figure 5**: Point receptance at coordinate 3 for original and modified systems – Case Study 2

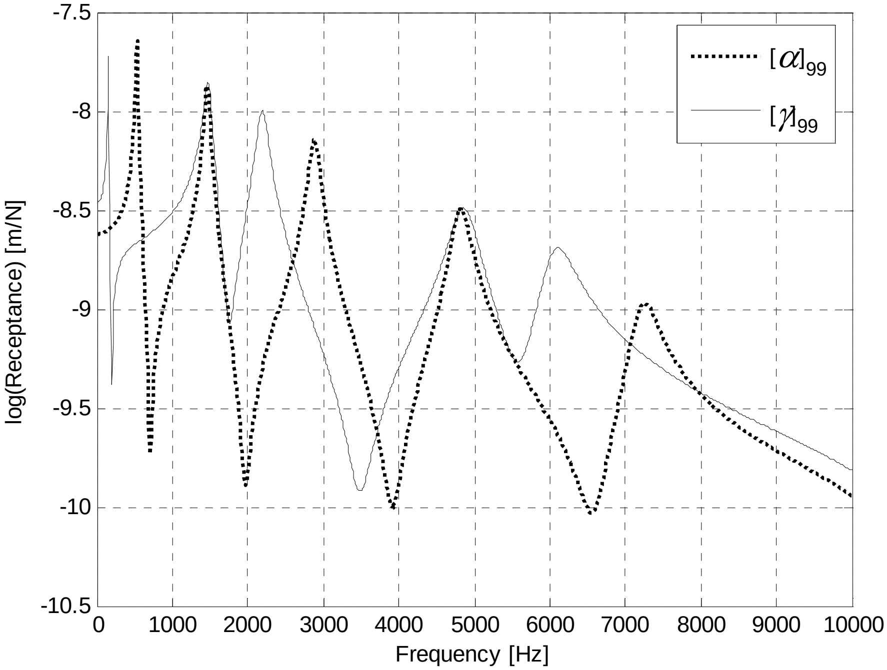

**Figure 6**: Point receptance at coordinate 9 for original and modified systems – Case Study 2

As demonstrated by the results of these case studies, Successive Matrix Inversion Method can successfully be applied for the analysis of dynamic structures subjected to structural modifications. The basic equation for the modified receptance matrix is the same both in Özgüven's Structural Modification Method and in the SMI Method presented here for dynamic systems. Owing to the exactness of both methods, the same results are obtained as can be expected. In SMI method, matrix inversion is replaced by a power series representation, thereby avoiding any possible numerical problems that may be encountered during matrix inversion. At this stage it is observed that using power series expansion to avoid matrix inversion in the method suggested here does not introduce any numerical inaccuracy. Furthermore it is observed that the method is very efficient for local modifications, where the modification matrix size is much smaller than the total size of the original system. The comparison of this method with other exact reanalysis methods from the computational effort point of view will be given in a following study.

## 4. CONCLUSION

The need for exact reanalysis techniques for dynamic structures, especially in the design stage has become crucial for today's complex structures. In spite of the tremendous increase in the availability and the sheer power of computational resources, methods with less computational burden are still desirable. In the present paper, an exact structural reanalysis approach called the Successive Matrix Inversion method originally presented for static analysis has been extended for application to dynamic structures. It has been shown that the Successive Matrix Inversion method can also successfully be used for dynamic analysis of structures. The method is based on exact calculation of Frequency Response Functions (FRF) of the modified structure using FRFs of the original structure along with the modifying mass, stiffness and damping matrices. The results of the method have been compared with the ones obtained by using Özgüven's Structural Modification method which is also an exact reanalysis method; as well as with those obtained by direct analysis of the modified system. It is observed that results of all three methods yield the same results indicating that the different algorithms used in the reanalysis methods do not introduce any numerical inaccuracy. Although the formulation using Successive Matrix Inversion method is presented only for structural damping, it is also readily applicable for structures with viscous damping. The comparisons of computational time requirements of these two methods as well as of other exact reanalysis methods for dynamic structures are planned to be presented in a following study.

# 5. REFERENCES

[1] **Özgüven, H. N.**, *Determination of Receptances of Locally Damped Structures,* Proceedings of the Second International Conference on Recent Advances in Structural Dynamics***,*** v.2, pp.887-892, Southampton, England, April 9-13, 1984.

[2] **Özgüven, H. N.**, *A New Method for Harmonic Response of Non-proportionally Damped Structures Using Undamped Modal Data,* Journal of Sound and Vibration***,*** v.117, n.2, pp.313-328, September 1987.

[3] **Özgüven, H. N.**, *Structural Modifications Using Frequency Response Functions,* Mechanical Systems and Signal Processing, v.4., n.1, pp.53-63, 1990.

[4] **Sherman, J.**, **Morrison W. J.,** *Adjustment of an Inverse Matrix Corresponding to Changes in the Elements of a Given Column or a Given Row of the Original Matrix,* Ann. Math. Statist., v.20, pp 621, 1949.

[5] **Woodbury, M,** *Inverting Modified Matrices,* Memorandum Report 42, Statistical Research Group, Princeton University, Princeton, NJ, 1950.

[6] **Sanliturk, K. Y.,** *An Efficient Method for Linear and Nonlinear Structural Modifications,* Proceedings of ESDA2002: 6th Biennial Conference on Engineering Systems Design and Analysis, ESDA2002/APM-028, Istanbul, Turkey, July, 2002.

[7] **Hager, W. G.,** *Updating the Inverse of a Matrix*, SIAM Rev., v.31, No.2, pp.221-239, 1989.

[8] **Bucher, I., Braun, S.,** *The Structural Modification Inverse Problem: An Exact Solution,* Mechanical Systems and Signal Processing, v.7, pp.217-283, 1993.

[9] **Park, Y. H., Park, Y. S.,** *Structural Modification Based on Measured Frequency Response Functions: An Exact Eigenproperties Reallocation,* Journal of Sound and Vibration, 237(3), pp.411-426, 2000.

[10] **Akgün, M. A., Garcelon, J. H., Haftka, R. T.,** *Fast Exact Linear and Non-linear Structural Reanalysis and the Sherman-Morrison-Woodbury Formulas,* International Journal for Numerical Methods in Engineering, v.50, pp.1587-1606, 2001.

[11] **Huang, C., Verchery, G.,** *An Exact Structural Static Reanalysis Method,* Communications in Numerical Methods in Engineering, v. 13, pp.103-112, 1997.

[12] **Kirsch, U.,** *A Unified Reanalysis Approach for Structural Analysis, Design, and Optimization,* Struct. Multidisc. Optim., v.25, pp.67-85, 2003.

[13] **Kirsch, U., Bogomolni, M.,** *Error Evaluation in Approximate Reanalysis of Structures,* Struct. Multidisc. Optim., v.28, pp.77-86, 2004.

[14] **Kirsch, U., Papalambros, P. Y.,** *Structural Reanalysis for Topological Modifications- A Unified Approach,* Struct. Multidisc. Optim., v.21, pp.333-344, 2001.

[15] **Chen, S. H., Yang, X. W., Lian, H. D.,** *Comparison of Several Eigenvalue Reanalysis Methods for Modified Structures,* Struct. Multidisc. Optim., v.20, pp.253-259, 2000.

[16] **Bae, H., Grandhi, R. V., Canfield, R. A.,** *Successive Matrix Inversion Method for Reanalysis of Engineering Structural Systems,* AIAA Journal, v.42, No.8, pp.1529-1535, 2004.