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Forecasting Gold Price Returns: A Time Series Analysis

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Abstract. Gold is a valuable metal that has historically been highly regarded by civilizations. It has been used in jewellery industries, decorative purposes and mainly in finance. Especially in uncertain financial environment investors invest in gold to protect themselves unwanted price movements. Forecasting the rate of return of gold is significant issue in financial environment. This paper aims that forecast and compare the return of gold using different time series model. ARIMA-GARCH, ARIMA-EGARCH, ARIMA-TGARCH, Simple exponential smoothing model, Holt's linear exponential smoothing model and Holt-Winters' exponential smoothing model has been used and evaluated using by measure of accuracy methods.

Keywords. Gold Price, Arima-Garch, Exponential Smoothing, Time Series Analysis, Prophet

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TERM PROJECT

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Contents

1 Introduction **2**

2 Literature Review **4**

3 Methodology **6**

3.1 Autoregressive Integrated Moving Average 6

3.1.1 Random Walk (RW) 7

3.2 Volatility Models 7

3.2.1 ARIMA-GARCH 8

3.2.2 Exponential GARCH Model(EGARCH) 8

3.2.3 Threshold GARCH Model (TGARCH) 9

3.3 Exponential Smoothing 9

3.4 Prophet 11

3.5 Measure of Accuracy 12

4 Data Analysis **13**

4.1 Data Description 13

4.2 Arima-Garch, Arima-Egarch and Arima-Tgarch Models 15

4.3 Smoothing Models 19

4.4 Forecast Results 20

5 Conclusion **21**

Chapter 1

Introduction

Gold is a precious metal that has been highly valued by civilizations throughout history. It holds a significant role in the financial environment due to some key factors. One of these key factors is gold is a safe investment asset for investors [6]. Gold's limited supply and its enduring value make it a reliable asset for long-term wealth preservation. It plays an exceptional role in diversifying the portfolios for investors. Price movements often have a low correlation with other financial assets such as stocks. Therefore, adding gold to the portfolio reduces the risk and provides stability. [5]

Gold is also an efficient store of value. [15] Unlike currencies that can be subject to inflation and devaluation, gold has preserved its worth over time. [11] The research has been done on five countries US, the UK, France, Germany, and Japan. The result indicates that gold is a long-term store of value. The other key factor, gold is hedging against inflation [15]. [2] Examine UK and US between 1971 to 2010 and find that gold is fully hedging ability in the long term long.

Gold is a crucial role in monetary systems. Central banks hold gold reserves as a means of maintaining financial stability and credibility. [20]Gold provides countries with a tangible and universally accepted store of value, allowing them to diversify their reserves and reduce reliance on any single currency. The presence of gold in the international monetary system ensures stability and confidence in the global financial landscape.

According to the World Gold Council, approximately 208,874 tonnes of gold have been extracted from the Earth so far. It is remarkable that about two-thirds of total gold has been mined since 1950. Given the extraordinary resilience possessed by gold, one can safely conclude that a substantial portion of this precious element remains intact today albeit being transformed into various shapes and components.

In the dynamic world of finance, accurate predictions and informed decision-making are outstanding to success. Forecasting plays a vital role in finance by providing valuable insights and helping individuals and organizations anticipate and prepare for future financial outcomes. By analyzing historical data, economic indicators, forecasting empowers financial professionals to make well-informed choices, mitigate risks, and optimize their financial strategies. Forecasting financial instruments is significant because it's a guide to investment decisions [7]. Since gold is a very precious material, price forecasting of gold is a highly popular research topic all over the World. Forecasting of gold price following for investors, producers, and all financial industries.

One of the fundamental techniques commonly used in statistics and econometrics is time series analysis. It involves utilizing temporal ordering data and focuses on analyzing these collections over a given period to forecast future outcomes with better accuracy. Apparently evident across fields from economics to environmental sciences due to its various applications, it helps enhance historical pattern insights besides aiding decision-making processes often associated with dynamic systems that get more complex over time. Time series modeling has been used during this analysis.

Volatility is a critical component of time series forecasting for many aspect. Under the volatile environment; to include volatility into the analysis allows better understanding of risk assessment, provides more accurate forecast result and helps decision making process for investors.

The main question of this paper is, what is the best forecast model for the return of gold? To find an answer to this question, $ARIMA-GARCH$, TGARCH, EGARCH, Exponential Smoothing, and Prophet models are built and compared. The data was taken at finance.yahoo.com. Data is weekly and run from January 2002 to June 2023.

Chapter 2

Literature Review

There are several research articles have been published about gold price forecasting. $ARIMA$ is one of the most common and traditional forecast methods. [4] In $ARIMA$ models, Box and Jenkins procedure has been followed.

[8] Published a volatility model that will lead many different studies in future. The Autoregressive Conditional Heteroskedasticity (ARCH) model describes the volatility in returns symmetrically. In this aspect, both positive and negative shocks effect volatility symmetrically. He is also introduce the Generalized Autoregressive Conditional Heteroskedasticity (GARCH). GARCH model is a symmetric volatility model but it also consider the volatility of previous periods.

In 1991, [17] introduced a Exponential GARCH Model and in 1994 [26] introduced Treshold Garch Model (TGARCH). Both models explain the asymmetry volatility which means that positive and negative shocks effect the return asymmetrically.

[24] Analyze gold price in India using the $ARIMA$ model. The data is monthly and covers years from 1990 to 2015. Forecasting results comparing by using the mean absolute error (MAE), maximum absolute error (Max AE), and mean absolute percentage error (MAPE). The measures of accuracy imply that $ARIMA(0,1,1)$ is the best fit in this specific time horizon. The author also suggests that $ARIMA$ is a useful tool for forecasting gold prices. It provides insights into future price movements and can aid decision-making for investors and market participants.

[10] Use the $ARIMA$ model to forecast futures gold prices between 2003 to 2014 in India and conclude that $ARIMA$ $(1,1,1)$ is the best fit for forecasting gold price. An Author also states that $ARIMA$ is a good tool to forecast gold prices but has some limitations. He mentioned that this model is useful for the short term and could not capture immediate changes in the price. The author also states that the purchase of gold was limited to weddings and other rituals in the past but now investors are also purchasing the gold and following the movements of gold price.

In addition to its many benefits, the $ARIMA$ model also has some limitations. [16] used the $ARIMA$ model to forecast gold price but the author pointed out that $ARIMA$ may not be enough to forecast the gold price because of other factors that have an influence on the gold price and make multivariate regression.

[1] Compare the different forecast methods for gold prices in different time zones. The Author compares the forecasts according to the RMSE values. The results show that $ARIMA$ gives the best forecast compared to Exponential Smoothing (ETS), Trend and Seasonal components (TBATS), and Multiple linear regression (MLR).

[14] Forecast the gold price by using the Multiple Linear Regression method. The aim of this article is to forecast the gold price based on economic factors such as inflation or price movements. The result of this paper is Multiple Linear Regression model may have a high accuracy in predicting gold price.

In literature, there many study exist that compare the symmetric and asymmetric models. [9] compere GARCH, EGARCH, TGARCH, IGARCH models for daily stocks index return of Romania(BET). Author compere the measure of accuricies and results indicates that TGARCH give the best result for BET index return. [13] Compere the APARCH, EGARCH and TGARCH models for gold's return. The Author conclude that among these assymetric models, EGARCH gives the best accuracy result.

[19] [25] Suggest ARIMA-GARCH hybrid model for gold price forecasting. They find that adding volatility to the ARIMA model improves the forecast results. Both of these studies suggest that if the residuals of ARIMA have a heteroscedasticity and hybrid model can solve the heteroscedasticity.

[18] Study focus used Exponential Smoothing (ETS) forecasting models which are Single Exponential Smoothing (SES), Double Exponential Smoothing (DES), and Holt-Winters Exponential Smoothing. monthly data from March 2016 to February 2021 Malaysia. Comparing the forecast result by using the measure of accuracy, Double Exponential Smoothing (DES) gives the best result.

[21] Author compares the forecast results of the market price of gold, silver, platinum, and crude oil prices by using Double Exponential Smoothing, Holt's Linear Trend, and Random Walk. The forecast result has been interpreted accordingly Sum Square Error, Mean Square Error, and Root Mean Square Error. Results indicate that Random Walk gives the best result for gold price.

[22] Analyse that is Gold price in London follows a random walk or not. The author selects the morning and afternoon fixings and the closing price, using the multiple variance ratio test. Data include 3061 observations from the years 1990 to 2001. The result indicates that only the closing price follows a random walk.

[12] Suggests a hybrid forecast model forecasting a precious metals' price. $ARIMA$, ELMAN neural network, NAR neural network, Long short-term memory neural network (LSTM), and Prophet model have been used during the analysis. Results comparing RMSE, MAE, and MAPE. For gold price forecasting, Prophet ICEEMDAN (improved complementary ensemble empirical mode), multi-model error correction gives the best result.

Chapter 3

Methodology

3.1 Autoregressive Integrated Moving Average

$ARIMA$ model was introduced by Box and Jenkins in [4]. The $ARIMA$ model is frequently used to forecast and predict future data on time series data.

$ARIMA$ defined with different parameters (p, d, q) that p is representing autoregressions' order, d is the level of non-seasonal differences and q is the number of lagged error values. [4]

$AR(p)$ part of $ARIMA$ represents the autoregressive process which is regressed on lagged values and p represents the order of autoregressions.

The equation for the $AR(p)$ component is;

$$Y_t = c + \phi_1 \cdot y_{t-1} + \phi_2 \cdot y_{t-2} + \ldots + \phi_p \cdot y_{t-p} + \varepsilon_t$$

The $MA(q)$ part of $ARIMA$ represents the moving average component that indicates the linear combination of past error terms from previous observations and q is the number of lagged error values.

The equation that calculates the $MA(q)$ component is as follows:

$$Y_t = c + \theta_1 \cdot \varepsilon_{t-1} + \theta_2 \cdot \varepsilon_{t-2} + \ldots + \theta_q \cdot \varepsilon_{t-q} + \varepsilon_t$$

The $ARMA(p, q)$ combines both $AR(p)$ and $MA(q)$ models.

$$Y_t = c + \phi_1 \cdot y_{t-1} + \phi_2 \cdot y_{t-2} + \ldots + \phi_p \cdot y_{t-p} + \theta_1 \cdot \varepsilon_{t-1} + \theta_2 \cdot \varepsilon_{t-2} + \ldots + \theta_q \cdot \varepsilon_{t-q}$$

Y_t is the value of the time series at t

c is the constant term

ϕ is autoregressive coefficients

θ is moving average coefficients

ε_t is error term at t

The integration component has been used to stabilize the time series data. It entails differencing the data in order to remove trends or seasonality, as indicated by the parameter d.

The equation for differencing is:

$$Y'_{(t)} = Y_{(t)} - Y_{(t-1)}$$

where:
$Y'_{(t)}$ shows the differenced value at t.
$Y_{(t)}$ shows the original value at t.
$ARIMA$ combines $AR(p)$, $MA(q)$ and integration components d. The complete equation of $ARIMA(p, d, q)$ is;

$$Y'_t = c + \phi_1 \cdot y'_{(t)} + \phi_2 \cdot y'_{t-2} + \ldots + \phi_p \cdot y'_{t-p} + \theta_1 \cdot \varepsilon_{t-1} + \theta_2 \cdot \varepsilon_{t-2} + \ldots + \theta_q \cdot \varepsilon_{t-q}$$

The methodology entails determining acceptable values for p, d, and q by using auto.arima function and by looking at the auto-correlation function (ACF) and partial auto-correlation function (PACF) plots. The selected model will be used for forecasting.

3.1.1 Random Walk (RW)

Random Walk is one of the approaches for forecasting stock market prices. The approach allowed researchers to presume that the most recent data are the greatest reference for forecasting future data. The equation of RW can be written as follows [21]

$$Y_{(t)} = Y_{(t-1)} + \varepsilon_t$$

To further analysis, intercept (constant term) adding to the equation.

$$Y_{(t)} = c + Y_{(t-1)} + \varepsilon_t$$

3.2 Volatility Models

Volatility in financial markets refers to the degree of change in a rate of return over a specific time period. It assesses the degree of risk or uncertainty of investment. Lower volatility implies more steady and predictable return movements, whereas higher volatility means larger return fluctuations.

In time series model, Generalized Auto-regressive Conditional Heteroskedasticity model's variations are commonly used for capturing the volatility especially in the context of assets' returns. The rate of return is the percentage change in the value of an investment over a given time period. In time series modelling, ARIMA model is using to represent the rate of return. Taking a logarithm and difference of ARMA model, represents the rate of return to an asset.

Volatility can be symmetric or asymmetric. Symmetric volatility implies that the negative and positive shocks effect equal of the volatility of return.

3.2.1 ARIMA-GARCH

While log-dif ARIMA models the rate of return, GARCH model capture the volatility. In this context, ARIMA-GARCH model is a hybrid model that combines the Autoregressive Integrated Moving Average (ARIMA) model and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The GARCH models the conditional variance of the residuals, capturing the volatility clustering and heteroscedasticity. [3] ARIMA-GARCH models illustrate the behavior of returns and their corresponding volatility, which provides a more comprehensive understanding of financial market dynamics.

$$
\begin{aligned}
\text{ARIMA Model (Conditional Mean):} \quad & y_t = c + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t \\
\text{GARCH Model (Conditional Variance):} \quad & \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \\
\text{Hybrid Model:} \quad & y_t = c + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t \\
& \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\end{aligned}
$$

Where;

σ_t^2 represents the conditional variance at t that represents the volatility of the time series.

ω is a constant that shows the baseline level of volatility.

$\alpha_1, \alpha_2, \alpha_i$ are the ARCH parameters that measure the impact of past squared residuals on the current variance.

β are the GARCH parameters that measures the impact of past conditional variances on the current variance.

ε_{t-i}^2 represents the squared residuals.

σ_{t-j}^2 represents the past conditional variances.

To identify the best fitting GARCH model, sum of coefficients and AIC output are commonly used. Sum of coefficient (β, ω, α) must not be exceed to 1. If it is exceed, it means that selected GARCH model is not appropriate for data. If the sum of coefficient is less than 1, selected GARCH model fits data well. To compare the different GARCH model, lowest AIC gives the best fit to data.

3.2.2 Exponential GARCH Model(EGARCH)

[17] used conditional variance for testing the possible asymmetry in volatility. The Exponential GARCH model (EGARCH) enables asymmetric responses of volatility to positive and negative shocks. The model examines the leverage effect, which occurs when negative shocks have a greater influence on volatility.The EGARCH model reflects this asymmetric

behavior by allowing the coefficients to be both positive and negative.

The equation for the EGARCH(p, q) model representing as follows:

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{j=1}^{q} \beta_j \log(\sigma_{t-j}^2) + \gamma\varepsilon_{t-1}$$

$\log(\sigma_t^2)$ indicates the natural logarithm of the conditional variance of the t.

ω indicates the constant term and α and β represent coefficients.

γ is the leverage term coefficient that captures volatility's asymmetrical response to shocks.

If $\gamma = 0$, it means that volatility respond symmetrically to both positive and negative shocks.

If $\gamma > 0$, it suggests a positive leverage effect which means positive shocks have a larger impact of volatility.

If $\gamma < 0$, it indicates a negative leverage effect which means negative shocks have a larger impact of volatility.

3.2.3 Threshold GARCH Model (TGARCH)

The Threshold GARCH model (TGARCH) includes a threshold mechanism to capture the impact of past volatility that exceeds a specified threshold level. [26] It enables for different volatility behaviors depending on whether the conditional variance is below or above the threshold.

The equation for the TGARCH model representing as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k\sigma_{t-k}^2 I_{t-k}^-$$

Where (I_{t-k}^-) represents threshold parameters that indicates the level at which the threshold mechanism becomes active. If γ is different than 0, the asymmetric volatility is presence in the model.

3.3 Exponential Smoothing

To predict time series accurately, Exponential Smoothing is a notable approach. It essentially distributes exponentially diminishing weights among past observations while increasing the weight of recent ones.

A simple exponential smoothing model (SES) may represent as below:

$$L_t = \alpha Y_t + (1 - \alpha) L_{t-1}$$
$$F_{t+1} = L_t$$

Utilizing cutting-edge state space models to enhance traditional exponential smoothing techniques, ETS (Error, Trend, Seasonality) enables thorough examination and projection of various dimensions present in any given time-series dataset. This includes capturing nuances such as errors within measurements, trend developments, as well as expected seasonal shifts over time.

The specific formulation of the ETS method varies depending on the combination of the error, trend, and seasonal components.

Holt's Linear Exponential Smoothing captures and forecasts a time series' linear trend. When a trend is evident in the data, including the trend component yields more accurate projections than simple exponential smoothing. However, Holt's method continues to assume a consistent trend throughout time and ignores seasonality.

The equation of Holt's Linear Exponential Smoothing;

$$L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + b_{t-1})$$
$$b_t = \beta(L_t - L_{t-1}) + (1 - \beta) b_{t-1}$$
$$F_{t+1} = L_t + b_t$$

Holt-Winters' Exponential Smoothing, often known as triple exponential smoothing, is a forecasting technique that contains both a trend and a seasonality component. It's especially beneficial for time series data with both trend and seasonal characteristics.

$$L_t = \alpha \left(\frac{Y_t}{S_t} \right) + (1 - \alpha)(L_{t-1} + b_{t-1})$$
$$b_t = \beta(L_t - L_{t-1}) + (1 - \beta) b_{t-1}$$
$$S_t = \gamma \left(\frac{Y_t}{L_t} \right) + (1 - \gamma) S_{t-m}$$
$$F_{t+1} = (L_t + b_t) \times S_{t-m+1}$$

Where;

Y_t is observed value at t

L_t is level component at t

α is the smoothing factor level that determines the weight that remains $0 < \alpha < 1$

L_{t-1} is previous level component

F_{t+1} is next period forecast

b_t is trend component at t

β Smoothing factor for the trend
γ is the smoothing factor for the seasonal index
m_t is the length of the seasonal cycle
S_t Seasonal index at time t

3.4 Prophet

The Prophet software, a tool for forecasting time series data, intends to forecast "at scale," which implies it is an automated forecasting tool that is easier to use when refining time series approaches. [23] The prophet function is developed by Facebook.

Prophet involves three model components: trend, seasonality, and holidays. They are merged in the following equation;

$$y_{(t)} = g_{(t)} + s_{(t)} + h_{(t)} + \varepsilon_t$$

where:
$g_{(t)}$ is the trend that reflects non-periodic variations in the time series value,
$s_{(t)}$ represents the seasonality.
$h_{(t)}$ indicates the effect of holidays may cause the irregulations in patterns.
ε_t is the error term

The linear trend equation with change-points that is a piece-wise constant rate of growth can be written as follows;

$$g(t) = \left(k + a(t)^T\delta\right)t + \left(m + a(t)^T\gamma\right),$$

where k is the growth rate,
δ is the rate adjustments,
m is the offset parameter,
and γ_j is set to $-s_j\delta_j$ for making the function continuous.
$a_j(t)$ defined as follows;

$$a_j(t) = \begin{cases} 1 & \text{if } t \geq s_j \\ 0 & \text{otherwise} \end{cases}.$$

Forecasting with Prophet function includes a certain time horizon that donated by H

$$\phi(T, h) = d(\hat{y}(T + h\|T), y(T + h)).$$

where;
$\hat{y}(T + h\|T)$ indicates the forecast for t
$d(y, y^{'})$ is a distance metric

3.5 Measure of Accuracy

There are several measures of accuracy used in time series analysis to evaluate the performance of forecast models.

Mean Squared Error (MSE)

MSE calculates the average squared difference between the actual and the forecasted values. Lower MSE represents better accuracy.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Root Mean Squared Error (RMSE)

RMSE is the square root of MSE. To obtain an accurate average measure of the mistakes made in the original data scale, this assessment tool is used. Like MSE, lower RMSE indicates better accuracy.

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

Mean Absolute Percentage Error (MAPE)

MAPE estimates the average percentage difference between the forecasted and the actual values. It is especially useful when comparing the accuracy across different time series.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100\%$$

Mean Absolute Error (MAE)

MAE calculates the average absolute difference between the forecasted values and the actual values in time series. Lower MAE indicates better accuracy.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

Chapter 4

Data Analysis

4.1 Data Description

The data analysis is conducted on the weekly gold price data between 2002 to 2023 that has been taken from Yahoo Finance. Data contains 1118 variables. R and Python software is used to code the models implemented for the dataset and obtain forecasts.

Table 4.1: Description of the variable

Variable	Description	Source
Gold Price	weekly gold closing price between 2002 to 2023	finance.yahoo.com/

To begin with the analysis, stabilizing the variance should be done by applying the logarithmic transformation. There are several ways to decide whether the logarithmic transformation is necessary or not. It can be observed in the plots and the lambda value may be calculated. In this dataset, the λ value is 0.0613 means that logarithmic transformation is necessary for stabilizing the data.

According to the time series plot of the gold price, gold has an increasing trend up to 2012. It decreased a limited amount until 2016 and it increased until 2022.

Table 4.2 displays the descriptive statistics of both differenced and transformed variables. Differenced variables' mean is around zero. Skewness and Kurtosis measure the shape and distribution of the data. Skewness measures the asymmetry of a distribution. Both of their Skewness values are negative, which means that data has a longer right tail. Kurtosis measures the degree of peakedness of a distribution. Differenced variables have a positive kurtosis value, which means that ıt has a more peaked distribution.

Jarque-Bera test measures the normality of a data set. p value of differenced value is 0.01 which smaller than 0.05, the data have not the normal distribution.

Figure 4.2 depicts the additive seasonal decomposition of Gold Price's seasonal, trend, and residual. As shown in the graph, the gold price has an increasing trend. The residuals are serially uncorrelated and have a mean near zero, corresponding to white noise.

Figure 4.1: TS Plot of Gold Price

Table 4.2: Descriptive statistics of the variables

	Gold Price Transformed Variables	Gold Price Differenced Variables
Count	1118	1117
Mean	6.910232	0.00719
Std. dev.	0.054994	0.023295
Min	5.632823	-0.090282
Max	7.618266	0.132951
Skewness	-1.074616	-0.2777
Kurtosis	-0.210916	1.96100
JB test	0.00	0.01

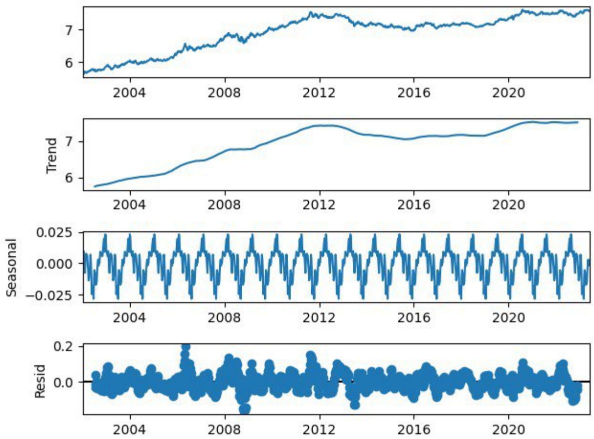

Figure 4.2: Seasonal Decomposition of Gold Price

$H0$: The time series data has a unit root
$H1$: The time series data does not have a unit root
If $p < 0.05$ Reject the $H0$ hypothesis and means that the time series is stationary.
If $p > 0.05$ Can not reject the $H0$ and means that the time series has a unit root.

4.2 Arima-Garch, Arima-Egarch and Arima-Tgarch Models

The Autocorrelation(ACF) Plot of the dataset shows a decreasing pattern which means that the data show nonstationary behaviors. Even though the data shows nonstationary behavior, there is no seasonal behavior detected in the plots.

The partial Autocorrelation(PACF) plot supports this view. To make sure of that perspective, the ADF test is applied.

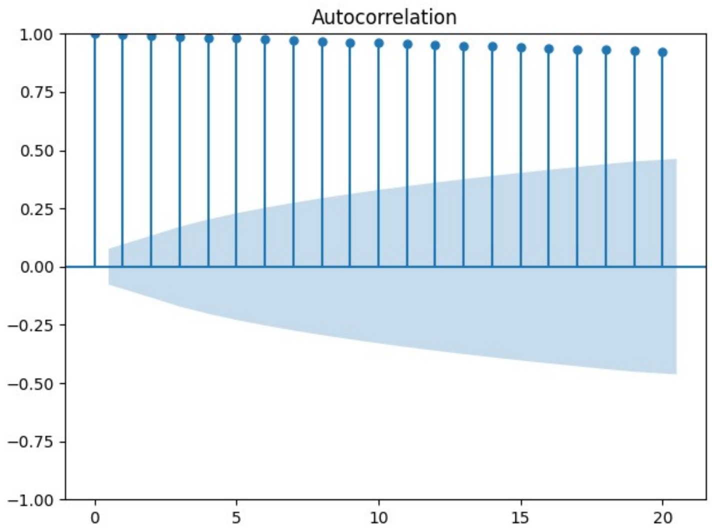

Figure 4.3: ACF Plot of Gold Price

The data is separated as test and train data. Among 1118 variables, 20% of data is separated as test data (223 variables), and 895 variables are separated as train data.

Table 4.3 represents the result of The Augmented Dickey-Fuller (ADF) test. ADF shows whether the time series has a unit root or not. ADF has the following hypothesis;

ADF test has applied the original data, transformed data, and the transformed differenced data. Both of the p values of original and transformed data are greater than 0.05 which means data has a regular unit root. The p-value of Transformed-differenced data is less than the p-value which means taking the difference solves the unit root problem.

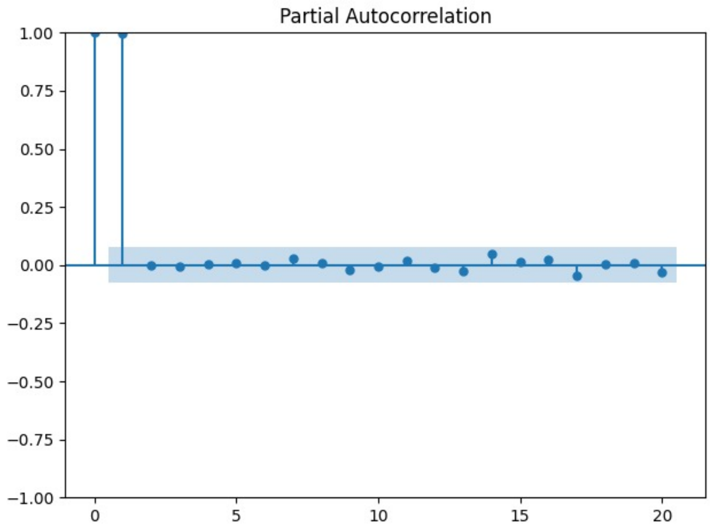

Figure 4.4: PACF Plot of Gold Price

Table 4.3: Stationarity test results

	ADF (c)			ADF (ct)			ADF (c)		
Variables	ADF-Stat	p-value	n-lags	ADF-Stat	p-value	n-lags	ADF-Stat	p-value	n-lags
Gold Price	-1,266064	0,947288	0	-0,098054	0,757236	0	-2,009071	0,596484	0
Transformed Gold Price	2,298034	0,996108	0	-2,008289	0,2895	0	-1,830429	0,596484	0
Transformed Differenced Gold Price	-33,392575	0	0	-33,56764	0	0	-33,600789	0	0

Table 4.4: Model output of $ARIMA(0,1,0)$ for Gold

	Estimate	Std. Error	z	p-value
sigma2	0.0006	0.0000198	28.874	0.000
intercept	0.0017	0.001	2.082	0.037

Log-likelihood	2070.180	AIC	-4136.360
BIC	-4126.769	HQIC	-4132.695

	z	p-value
Ljung-Box	0.06	0.80
Heteroskedasticity	0.69	0.00
Jarque-Bera (JB)	135.24	0.00

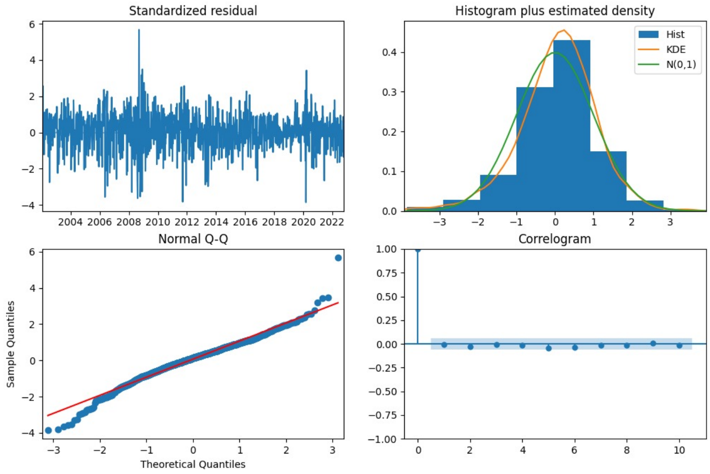

Figure 4.5: Residuals of ARIMA(0,1,0) model for Gold's Rate of Return

The Ljung-Box test is applied to check autocorrelation in the residuals of the model. Results represent that p-value is greater than the critical value 0.05 which means the null hypothesis that independently distributed residuals can not be rejected. The heteroskedasticity test also indicates a p-value equal to 0.00 which means the residual distribution has a heteroskedasticity. Figure 4.5 represents the residuals and they have a mean around zero.

Squared residuals are analyzed for volatility models. The Breusch-Pagan test has been applied to the squared residuals. It helps to detect heteroscedasticity in models. The p value of squared residuals' is 0.00032142269 which is below the significant level of 0.05. Therefore, it rejects the $H0$ and heteroscedasticity is present in the model.

The hypothesis of the Breusch-Pagan test is stated as follows:

$H0$: There is no heteroscedasticity present in the model.

$H1$: Heteroscedasticity is present in the model.

If $p < 0.05$ Reject the $H0$ hypothesis and means that heteroscedasticity is present in the model.

If $p > 0.05$ Can not reject the $H0$ There is no heteroscedasticity present in the model.

$ARIMA(0,1,0)$ has failed heteroscedasticity test. Therefore, $ARIMA - GARCH$ hybrid model was used to handle the existence of heteroscedasticity in the residuals.

To detect the best $GARCH$ model, information criteria have been analyzed. $GARCH(1,1)$ has the lowest AIC therefore, it is selected for further analysis.

If the sum coefficients of a GARCH model is more than 1, it indicates that GARCH model is not appropriate to fit the time series. In this GARCH model, the sum of the coefficients is less than 1 which indicates the model fits to data well. Table 4.5 represents the model outputs of GARCH(1,1) model. All p-values are smaller than 0.05 which means they all significant.

Table 4.5: Model output of $GARCH(1,1)$ for Gold

Log-likelihood	5027.02
BIC	-10026.9
AIC	-10046.0

	Coefficient	Std. Error	p-value
ω	0.00000002483	0.0000000000621	0.00000
α	0.100	0.03863	0.00964
β	0.8800	0.02500	0.00000

Table 4.6 indicates the model outputs of EGARCH(1,1) model. The model is significant since all p-values are smaller than significant level 0.05. The leverage coefficient (γ) are differ from zero, therefore, there is a leverage effect exists in model. In the presence of positive leverage effect, positive shocks have a larger impact on volatility than negative shocks.

Table 4.6: Model output of $EGARCH(1,1)$ for Gold

Log-likelihood	4918.81
BIC	-9799.64
AIC	-9823.62

	Coefficient	Std. Error	p-value
ω	-0.262375	0.060281	0.021393
α	0.045590	0.017229	0.008142
β	0.965218	0.007844	0.000000
γ	0.176763	0.029014	0.000000

Table 4.7: Model output of $TGARCH(1,1)$ for Gold

Log-likelihood	4971.02
BIC	-9908.07
AIC	-9932.05

	Coefficient	Std. Error	p-value
ω	0.000017	0.000018	0.033254
α	0.124832	0.036938	0.000726
β	0.884078	0.030731	0.000000
γ	-0.074760	0.028821	0.009489

Table 4.7 represents the model outputs of TGARCH(1,1) model. The sum of coefficients are smaller than 1 and p values are smaller than 0.05. On the contrary of EGARCH,

leverage effect is negative. Hence, negative shocks have a larger impact on volatility than positive shocks.

To validation of volatility models, necessary tests have been applied both residuals and squared residuals of models. Ljung-box used for checking the presence of auto-correlation in residuals and squared residuals. Arch-LM test also have been applied for making sure models capture all ARCH effects.

Table 4.8 represents the ljung-box test and Arch-LM test for garch model. For Ljung-box test p values are greater than significant value 0.05, we fail to reject null hypothesis which means there is no significant evidence of autocorrelation in the residuals of the models.

For Arch-LM test p value is greater than significant value 0.05, we fail to reject null hypothesis it suggests that there is no significant evidence of ARCH effects in the residuals of models.

Table 4.8: Arima-Garch(1, 1) Model Validation

		p-value
Ljung-Box	R	0.9473458
Ljung-Box	R^2	0.09069986
Arch-LM	R	0.2569138

4.3 Smoothing Models

Exponential smoothing methods are a type of forecasting approach in which past data in a time series are given exponentially decreasing weights.

Simple Exponential Smoothing (SES), data doesn't show any seasonal behavior or trend. In Simple Exponential Smoothing, previous data will be assigned exponentially declining weights, while more current observations will be given higher weights. This represents the idea that more recent data points may be more useful for anticipating future values. α identifies the point at which observation has exponential decay.

Holt's linear exponential smoothing is an extension of simple exponential smoothing that includes a trend component in the forecasting. β represents the weight given to the difference between the current level and the previous level when updating the trend. Table 4.9 represents the model coefficients of exponential smoothing models.

Table 4.9: Model coefficients of ES models

	SES		**Holt's Linear**				**Holt's Exponential**			
	α	**initial level**	α	β	**initial level**	**initial trend**	α	β	**initial level**	**initial trend**
Gold Price	1,000	5,661395	1,000	0,000	5,659738	0,0001680	1,000	0,000	5,659981	1,000

Table 4.10 represents the smoothing models Akaike information criterion (AIC), Bayesian information criterion (BIC), and Akaike's Information Corrected Criterion (AICC).

Table 4.10: Smooting Model output for Gold

	AIC	BIC	AICC
SES	-6677.464	-6677.870	-6677.419
Holt's Exponential	-6677.892	-6658.705	-6677.798
Holt's Linear	-6677.199	-6658.012	-6677.105

4.4 Forecast Results

Forecast Accuracy is assessed using the metrics RMSE, MAE, and MAPE, and the results are displayed in Table 4.11.

In ARIMA models, accuracy results are very close to each others. Despite of this similarity, ARIMA-EGARCH gives the smaller-better accuracy results.

Between the exponential smoothing models, Holt's linear gives the best accuracy result of root mean squared error (RMSE), mean absolute percentage error (MAPE), and mean absolute error (MAE).

Comparing the ARIMA-GARCH and SES model, ARIMA-GARCH gives a better result in three components.

Evaluating the SES and Prophet model, the Prophet model gives the better accuracy result for three of the component.

Holt's Linear Model gives a better result than Holt's Exponential Model MAE, MAPE and RMSE.

Table 4.11: Forecast Performance of Models for Gold's Rate of Return

	MAE	MAPE	RMSE
ARIMA-GARCH	0.10547759	0.014093591	0.12726806
ARIMA-EGARCH	0.10544935	0.014089825	0.12723649
ARIMA-TGARCH	0.10560867	0.014111097	0.12740856
SES	0.28670113	0.038242668	0.30876774
Holt's Exponential	0.10808989	0.014444217	0.13011528
Holt's Linear	0.10600436	0.014164132	0.12785518
Prophet	0.21787545	0.029072221	0.23448315

Comparing the mean absolute error (MAE) of all forecasts, ARIMA-EGARCH forecast model gives the lowest and better accuracy result.

Comparing the mean absolute percentage error (MAPE) of all forecasts, ARIMA-EGARCH forecast model gives the lowest and better accuracy result.

Comparing the root mean squared error (RMSE) of all forecasts, the ARIMA-EGARCH forecast model gives the lowest and better accuracy result.

Chapter 5

Conclusion

Gold is a very valuable metal with various uses. It uses not only in jewelry industries, and decorative purposes but also in finance. In finance, gold has been used as a safe investment tool. Especially in uncertain financial environments investors invest in gold to protect themselves from unwanted price movements. Gold is also used as a store of value.

Whereas gold is a very desirable material for humans for centuries, forecasting the price of gold is a very popular topic all over the world. The main question of this paper is what is the best forecast model for the gold price? Many different forecast techniques have been used for gold prices in literature. To answer the main question of this paper, time series analysis is used. ARIMA-GARCH, EGARCH, TGARCH, Simple exponential smoothing model (SES), Holt's Linear Exponential Smoothing model, and Holt-Winters' Exponential Smoothing model have been used during the data analysis.

To execute Arima models, the data has been prepared first. Data separated test and train, taking differences and transforming the data by taking logarithms. After the preparation, *auto.arima* function fits the $ARIMA(0,1,0)$ which it's fits the differenced ACF and PACF plots of differenced data. The output of $ARIMA(0,1,0)$ indicates that model has no autocorrelation but it has heteroscedasticity.

Gold is a volatile metal. ARIMA model captures only the linear components of data. Therefore, by using ARIMA-GARCH, error components are also added to the models. It allowed for capturing the volatility in the gold price. As a consequence, better forecast results are aimed at the ARIMA-GARCH forecast model. $ARIMA(0,1,0)-GARCH(1,1)$ hybrid model is selected for hybrid forecast. The lowest AIC GARCH model is selected for the hybrid forecast.

ARIMA-GARCH model represents the symmetric volatility. To examine the asymmetric volatility, EGARCH and TGARCH models have been added into analyse. The leverage of asymmetric models are different from zero which indicates the presence of asymmetric volatility.

Forecast accuracy result has been evaluated in the forecast section. Root mean squared error (RMSE), mean absolute percentage error (MAPE) and mean absolute error (MAE) has been used as a measure of accuracy.

ARIMA-EGARCH model's RMSE, MAE and MAPE result are 0.10544935 , 0.014089825 and 0.12723649 respectively. $ARIMA(0,1,0) - EGARCH(1,1)$ gives the best (lowest) forecast result in all forecast methods.

The aim of the analysis is to detect the best time series forecast method for return of weekly gold price data. Weekly data has been analyzed and it contains 1118 variables. The result represents that ARIMA-EGARCH forecast model gives the best forecast result and the volatility is a significant component in the time series forecast of gold return. The analysis also shows adding the volatility into analysis gives the best accuracy result among all forecast methods.

Bibliography

[1] A. Azzutti, Forecasting gold price: A comparative study, 02 2016.

[2] G. Bampinas and T. Panagiotidis, Are gold and silver a hedge against inflation? A two century perspective, International Review of Financial Analysis, 41, pp. 267–276, 10 2015.

[3] T. Bollerslev, Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics, 31(3), pp. 307–327, 4 1986.

[4] G. E. Box, G. M. Jenkins, G. C. Reinsel, and G. M. Ljung, *Time series analysis: forecasting and control*, John Wiley & Sons, 2015.

[5] W. Chkili, Dynamic correlations and hedging effectiveness between gold and stock markets: Evidence for BRICS countries, Research in International Business and Finance, 38, pp. 22–34, 9 2016.

[6] V. Coudert and H. Raymond-Feingold, Gold and financial assets: Are there any safe havens in bear markets?, Economics Bulletin, 31, pp. 1613–1622, 01 2011.

[7] F. Diebold and K. West, Forecasting and empirical methods in finance and macroeconomics, Journal of Econometrics, 105, pp. 1–3, 02 2001.

[8] R. F. Engle, Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, Econometrica, 50(4), p. 987, 7 1982.

[9] A. S. Gabriel, Evaluating the Forecasting Performance of GARCH Models. Evidence from Romania, Procedia - Social and Behavioral Sciences, 62, pp. 1006–1010, 10 2012.

[10] B. Guha, Gold price forecasting using arima model, Journal of Advanced Management Science, 4(2), pp. 117–121, 2016.

[11] S. Harmston, Gold as store of value, 1998.

[12] Y.-T. Huang, Y. , Q.-H. Yu, L. Ding, and Y.-J. Ma, Application of a hybrid model based on the prophet model, iceemdan and multi-model optimization error correction in metal price prediction, Resources Policy, 79, p. 102969, 12 2022.

[13] Y. Irene, M. Y. Wijaya, and A. Muhayani, World Gold Price Forecast using APARCH, EGARCH and TGARCH Model, InPrime Journal, 2(2), pp. 71–78, 5 2020.

[14] Z. Ismail, A. Yahya, and A. Shabri, Forecasting Gold Prices Using Multiple Linear Regression Method, American Journal of Applied Sciences, 6(8), pp. 1509–1514, 8 2009.

[15] J. F. Jaffe, Gold and Gold Stocks as Investments for Institutional Portfolios, Financial Analysts Journal, 45(2), pp. 53–59, 3 1989.

[16] D. G. Nambiar and V. M.Rajkumar, Forecasting price and analysing factors influencing the price of gold using arima model and multiple regression analysis, 2012.

[17] D. B. Nelson, Conditional heteroskedasticity in asset returns: a new approach, Econometrica, 59(2), p. 347, 3 1991.

[18] H. A. A. Rahman, M. F. R. Azizi, M. F. Saruand, and N. A. Shafie, Forecasting Of Malaysia Gold Price with Exponential Smoothing, Journal of science and technology, 14(1), 6 2022.

[19] Y. S. Roslindar, A. N. Azlinna, A. M. Hura, and Z. Roslinazairimah, Modelling gold price using ARIMA–TGARCH, Applied mathematical sciences, 10, pp. 1391–1402, 1 2016.

[20] C. R. Schenk, *The Global Gold Market and the International Monetary System*, 1 2013.

[21] S. A. Shukor, S. Sufahani, K. Khalid, M. H. A. Wahab, S. Z. S. Idrus, A. Ahmad, and T. S. Subramaniam, Forecasting Stock Market Price of Gold, Silver, Crude Oil and Platinum by Using Double Exponential Smoothing, Holt's Linear Trend and Random Walk, Journal of Physics: Conference Series, 1874(1), p. 012087, 5 2021.

[22] G. Smith, Tests of the random walk hypothesis for London gold prices, Applied Economics Letters, 9(10), pp. 671–674, 8 2002.

[23] S. J. Taylor and B. Letham, Forecasting at Scale, The American Statistician, 72, pp. 37–25, 5 2017.

[24] Tripathy, Forecasting gold price with auto regressive integrated moving average model, International Journal of Economics and Financial Issues, 7(4), pp. 324–329, 2017.

[25] S. R. Yaziz, R. Zakaria, and Suhartono, ARIMA and Symmetric GARCH-type Models in Forecasting Malaysia Gold Price, Journal of physics, 1366, p. 012126, 11 2019.

[26] J.-M. Zakoian, Threshold heteroskedastic models, Journal of Economic Dynamics and Control, 18(5), pp. 931–955, 9 1994.