Bypassing the Repeatability Issue in Nonlinear Experimental Modal Analysis of Jointed Structures by using the RCT-HFS Framework

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ABSTRACT

Mechanical joints, which are indispensable for almost all mechanical systems, are often an important source of nonlinearity due to frictional, backlash and/or preload effects. Recent studies have shown that the contact pressure distribution at bolted joint interfaces is the key parameter that governs joint friction and, therefore, the nonlinear damping mechanism in these systems. The problem is that this pressure distribution is susceptible to several different factors: bolt preload, bolt tightening order, surface roughness, surface flatness, and misalignments during the assembly process. These issues lead to considerable variability and repeatability problems in the nonlinear dynamics of jointed structures. Consequently, the accurate identification of nonlinear damping in jointed structures is still a challenging task. The combined use of the response-controlled stepped-sine testing (RCT) and the harmonic force surface concept (HFS) constitutes a framework that determines frequency response curves from the same measurement in two different ways; either by directly measuring them or by synthesizing them from the identified nonlinear modal parameters. Since any possible discrepancy of the frequency response curves obtained from the same measurement cannot be attributed to the repeatability issue, the RCT-HFS framework validates the accuracy of the identified nonlinear modal parameters in a sense bypassing the repeatability problem. In this study, this novel feature of the RCT-HFS framework is used in identifying and validating the accuracy of the modal model of a benchmark beam with a bolted lap joint.

Keywords: Joint nonlinearity, friction nonlinearity, response-controlled stepped-sine testing, harmonic force surface, repeatability

1. REPEATABILITY ISSUE

Vibration responses of jointed structures repeatedly measured under the same excitation condition can be very different from each other [1]. This poor repeatability in measurements makes it difficult to understand the nonlinear dynamics of mechanical joints and to develop a reliable and accurate mathematical model from experiments. Remarkable studies have been published recently that clarify the causes of the repeatability issue in jointed structures (e.g. [2-3]). In these studies, it is shown that the interfacial contact pressure distribution is the key metric that affects the nonlinear dynamics of mechanical joints as well as the measurement repeatability. It is also shown that this pressure distribution is very sensitive to various factors such as the topography of the contact interface (flatness and surface roughness), bolt tightening order, bolt preload, and alignment issues during the assembly process. Some of these factors change considerably not only in the case of disassembly/reassembly but even in the case of the repeated measurements of the same assembly, which highly affects the contact pressure and therefore the measurement repeatability. In [2], it is demonstrated that a well-controlled assembly procedure can highly improve measurement repeatability. Furthermore, it is revealed that lower surface roughness considerably reduces the sensitivity of the contact pressure to bolt preload and bolted tightening order, which also results in better repeatability.

Although significant progress has been made in understanding the nonlinear characteristics of mechanical joints and improving repeatability as discussed above, developing an accurate mathematical model for joints still remains a challenging problem. In a recent joint work [4], the nonlinear dynamics of a jointed beam have been studied by applying several different identification methods such as the Hilbert Transform method, Peak Finding and Fitting method to the free decay response data measured by impact testing and shaker ringdown testing. In the same work, a significant discrepancy was reported between the backbone curves and nonlinear modal damping curves identified from these methods with the ones obtained from classical force-controlled stepped sine testing. Whether this discrepancy is due to the theoretical/practical limitations of the studied identification methods or the repeatability issue remains ambiguous. On the other side, since the modal damping obtained from force-controlled testing is identified indirectly by assuming the input energy provided by the shaker is equal to the energy dissipated by bolted joints of the beam, it is also disputable whether this modal damping constitutes an accurate reference to validate other identification techniques.

An important advantage of the RCT-HFS identification framework [5] compared to the aforementioned identification methods is that it is capable of determining frequency response curves from the *same measurement* data set in two different ways; either by extracting them from the HFS or by synthesizing them from the identified nonlinear modal parameters based on the Single Nonlinear Mode (SNM) theory [6]. Since any possible discrepancy of the frequency response curves reproduced from the same measurement by two different approaches cannot be attributed to the repeatability issue, the RCT-HFS framework provides a reliable mean of validating its theoretical foundation, i.e. the SNM theory, in a sense bypassing the repeatability problem. Of course, if the structure suffers from poor repeatability, the problem is still there and can be studied separately by applying the RCT-HFS framework on repeated measurements and obtaining uncertainty bounds of identified nonlinear modal parameters as shown in the case of a real missile in [5].

In a short period of time, The RCT-HFS identification framework has been successfully applied to a wide range of nonlinear systems; a T-beam benchmark with local cubic stiffness nonlinearity and a real missile with moderate damping nonlinearity due to bolted joints [5], a metal strip that exhibits strong geometrical (distributed) nonlinearity [7, 8], a nonlinear micro-electromechanical device with stack-type piezo-actuator [9] and the control fin actuation mechanism of a real missile [10]. In this study, the framework is successfully applied to a recently proposed benchmark beam with bolted lap joint, namely the Orion beam [11].

2. APPLICATION OF THE RCT-HFS FRAMEWORK TO THE ORION BEAM

In this study, the RCT-HFS identification method is applied to a benchmark structure, the so-called Orion beam recently proposed in [11]. The structure consists of two thin beams connected by three bolted joints with contact patches on each connecting bolt. In [11], the Orion beam is subjected to a series of constant-force stepped-sine testing by using the experimental setup shown in Fig. 1. The beam is excited by a modal shaker at a point close to its clamped end. The response is measured by a laser vibrometer. All the details about the dimensions of the beam, data acquisition and control strategy can be found in [11]. Frequency response functions (FRFs) measured at different levels of excitation force amplitude and tightening torque are also provided in [12] as an open-source dataset to help different research groups to test the identification methods they have developed.

In the RCT-HFS framework, the usual practice is to conduct a series of response-controlled stepped-sine testing by keeping the displacement amplitude of a selected control point (usually the driving point) constant and to measure constant-response receptances which turn out to be quasi-linear even in the case of strongly nonlinear systems [5, 7-10]. Then, these receptances are processed by using standard linear modal analysis techniques to extract response-level dependent nonlinear modal parameters (natural frequency, modal damping ratio, and modal constant). On the other side, the HFS is constructed by using harmonic force spectra measured at different constant displacement amplitude levels. By cutting the HFS with constant force planes, one can extract constant-force frequency response curves including unstable branches if there are any. Finally, the accuracy of the identified nonlinear modal parameters is validated by comparing constant-force receptances synthesized from these parameters (in a Newton-Raphson solution scheme) with the ones extracted from the HFS. However, [11, 12] provide only the constant-force FRFs for the Orion beam but not the constant-response FRFs. In this study, this issue is solved by using the HFS concept in a novel way different than its usual implementation as explained below.

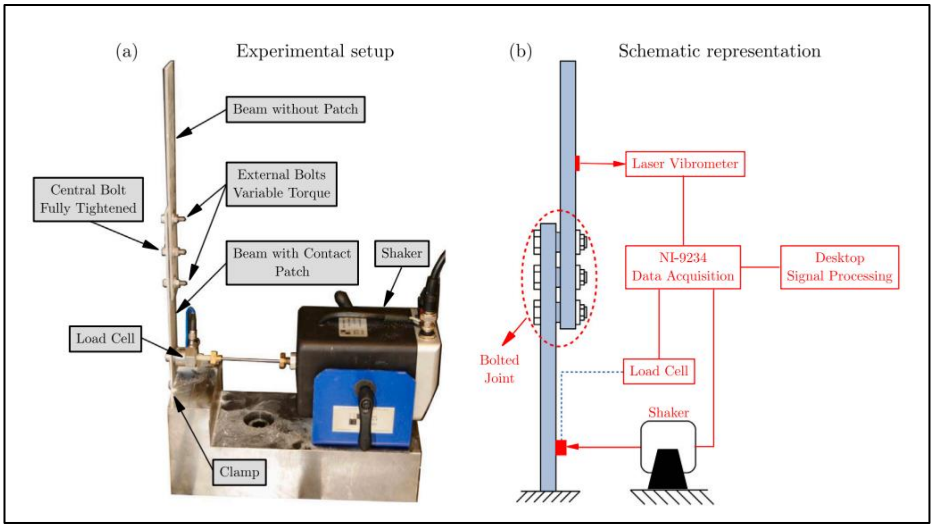
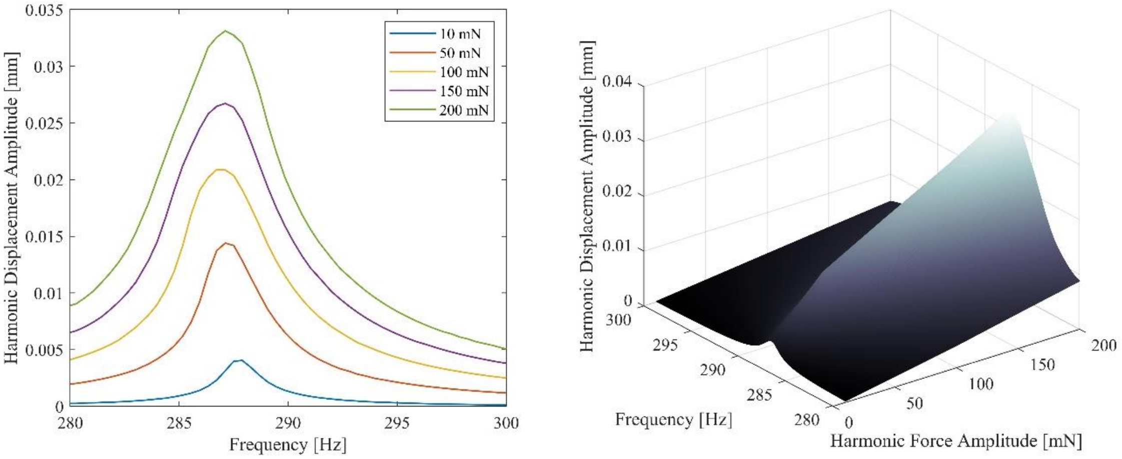


Fig. 1. Orion beam experimental setup [11]

In [12], constant-force mobility (velocity/force) data are measured around the 3rd mode of the Orion beam at 20 cNm tightening torque level. These data are converted into frequency response curves as shown in Fig. 2(a). Since the nonlinearity is relatively weak, the frequency response curves do not exhibit any jump and are very smooth. Consequently, these curves are merged to construct a smooth HFS as shown in Fig. 2(b). Cutting this HFS with constant displacement amplitude planes gives V-shaped harmonic force spectra. Finally, dividing selected constant displacement amplitudes with corresponding harmonic force spectra gives the quasi-linear constant-response FRFs as shown in Fig 3. It is important to note that the main motivation behind the invention of the HFS was accurately identifying unstable branches and turning points of constant-force FRFs that exhibit the jump phenomenon in the case of classical constant-force testing. Therefore, in the case of strongly nonlinear systems, the HFS is used to obtain constant-force FRFs from the measured constant-response measurements. However, as shown in this study, this procedure can be reversed in the case of weakly nonlinear systems. This novel implementation of the HFS concept can be very useful to identify the model parameters of nonlinear structures very accurately as shown below.



(a) (b)

Fig. 2. Construction of the HFS from the constant-force frequency response curves for the 3rd bending mode at 20 cNm tightening torque: (a) constant-force frequency response curves (b) Harmonic Force Surface (HFS)

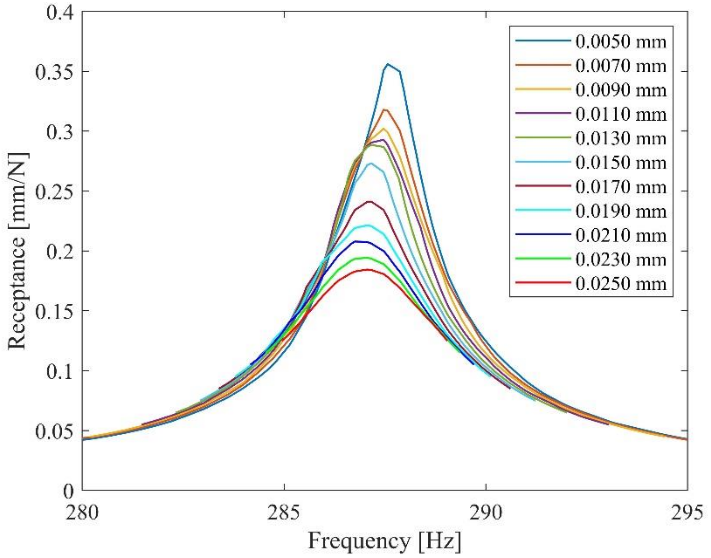
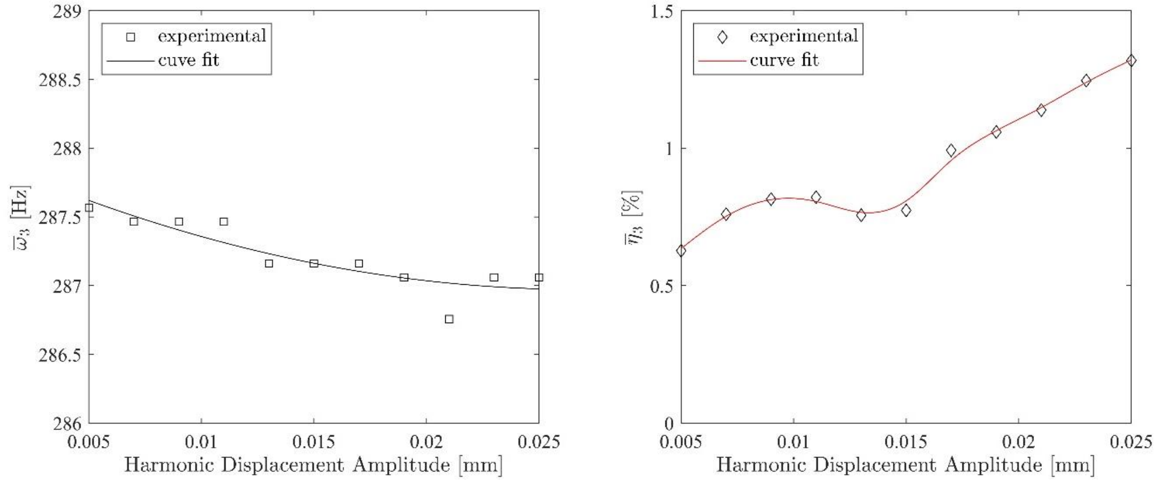


Fig. 3. Extraction of the quasi-linear constant-displacement FRFs from the HFS for the 3rd bending mode at 20 cNm tightening torque

Once the quasi-linear constant-response FRFs are obtained from the HFS as shown in Fig. 3, they can be processed by using standard linear modal analysis techniques to extract modal parameters corresponding to each displacement amplitude level. In this study, constant-response FRFs are processed by using the simple pick-picking algorithm, and response level depended nonlinear modal parameters are obtained as shown in Fig. 4. An important advantage of the RCT-HFS framework over most of the state-of-the-art identification techniques is that it identifies accurate modal models of nonlinear structures without necessitating the apriori knowledge of the location and/or the type of nonlinearity, and it can be used even for distributed nonlinearity. In [11], a Duffing-Van der Pol oscillator model is assumed for the Orion beam, and the parameters of that model are determined iteratively by using the measured constant-force FRFs. The issue with such parametric modeling is that it can be computationally expensive or not possible at all in the case of complex engineering systems.



(a) (b)

Fig. 4. Variation of modal parameters with response amplitude: (a) natural frequency (b) modal damping ratio

The final step of the RCT-HFS framework is the demonstration of the accuracy of the identified nonlinear modal parameters. To achieve this goal, constant-force FRFs are synthesized from the identified nonlinear modal model by using the SNM theory [6] and the Newton-Raphson solution scheme. These synthesized FRFs are compared with the ones directly measured by constant-force testing [11-12] in Fig. 5. As can be seen from the figure, the match between the synthesized and directly measured data is perfect, which proves the accuracy of the identified nonlinear modal model.

A very interesting and important observation that can be made from Fig. 4 and Fig. 5 is that although the nonlinear model parameters were experimentally obtained for the displacement levels between 0.005 mm and 0.025 mm as seen in Fig. 4, the constant-force FRFs corresponding to the 10mN and 200 mN covering response amplitudes below 0.005 mm and above 0.025 mm are also very accurately synthesized as shown in Fig. 5. This is achieved by a successful curve fitting and extrapolation process using the *fit* function of Matlab. A second-order polynomial is fitted to the natural frequency and smoothing spline curves are fitted to the modal damping ratio and modal constant, which satisfactorily extrapolates the modal parameters at amplitude levels below 0.005 mm and above 0.025 mm.

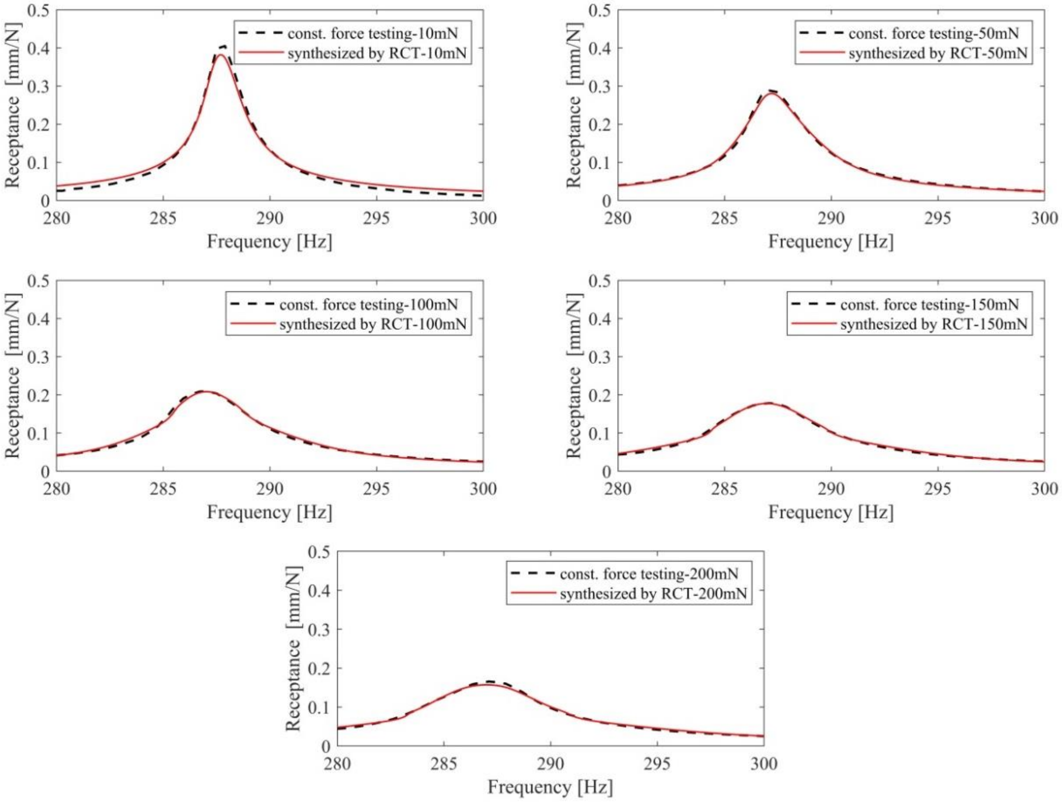


Fig. 5. Validation of the identified nonlinear modal parameters by comparing constant-force FRFs synthesized from these parameters with the ones directly measured by constant-force testing

3. DISCUSSIONS AND CONCLUSIONS

Poor repeatability in measurements of jointed structures makes it a challenging task to understand the nonlinear dynamics of mechanical joints and to develop reliable and accurate mathematical models from experiments. In this study, the RCT-HFS framework, which has been successfully applied to a wide range of structures in a short period of time, is applied for the nonlinear modal identification of a recently proposed benchmark beam with a lap joint, the so-called Orion beam. Since the experimental data available in the literature consists of constant-force FRFs, the HFS is constructed in a novel way by merging constant-force frequency response data, contrary to its usual implementation which uses constant-response measurements. Cutting the HFS with constant displacement amplitude planes gives quasi-linear constant-response FRFs. These FRFs are then processed with a simple peak-picking method to identify nonlinear modal parameters as functions of response amplitude. The perfect match between the constant-force FRFs obtained from direct measurement and the ones synthesized from the identified nonlinear modal parameters demonstrates the accuracy of the RCT-HFS method. Since the constant-force FRFs are obtained from the *same measurement* in two different ways, the accuracy of the identified nonlinear modal parameters is shown in a sense bypassing the repeatability problem. Of course, the repeatability problem is still there and can be studied separately by applying the RCT-HFS framework on repeated measurements and obtaining uncertainty bounds of identified nonlinear modal parameters. Also, since the nature of the RCT measurements requires keeping the vibration amplitude of the excitation point constant, unlike the constant amplitude forcing testing approach, the system does not move into an uncontrolled high amplitude oscillation regime at and around the resonance frequencies. In an RCT-HFS framework, the resonance is observed with decreased forcing amplitudes only; therefore, it avoids drastic changes in alignments and contact conditions as well as reduces the stress levels around the joint region which is expected to reduce the severity of the repeatability problem in comparison to constant amplitude forcing measurements. This point will be studied in future work.

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