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SUBSIDENCE PREDICTION MODEL BY A TWO DIMENSIONAL DISPLACEMENT DISCONTINUITY BOUNDARY ELEMENT METHOD

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Approval of the Graduate School of Natural and Applied Sciences.

Prof. Dr. Alpay ANKARA

Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science in Mining Engineering.

Prof.Dr. Gülhan ÖZBAYOĞLU

Chairman of the Department

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Msater of Science in Mining Engineering.

Prof.Dr. A. Günhan PAŞAMEHMETOĞLU

Supervisor

Examining Commitee in Charge:

Assoc.Prof.Dr. Seyfi Kulaksız (Chairman)

Prof.Dr. A. Günhan Paşamehmetoğlu

Assoc.Prof.Dr. Levent Tutluoğlu

Assoc.Prof.Dr. Abdürrahim özgenoğlu

Dr. Gürel Şenyur

ABSTRACT

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SARI, Dursun

Faculty of Engineering

Department of Mining Eng. M.S. Thesis

Supervisor: Prof. Dr. A. Günhan PASAMEHMETOGLU

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An attempt to model subsidence phenomena by a displacement discontinuity boundary element method was made in this thesis. The introduced method is modified from the solution previously given for a displacement discontinuity for the infinite-plane. Some changes made in the structure of the program to simulate the gravitational loaded semi-infinite plane in order to solve this problem.

The developed model is tested against available field data as far as the subsidence and strain values concerned for different longwall panel width to mine depth ratio using stress-free and displacement boundary conditions. In addition, this model is also compared with the already existing numerical and closed form analytic solution subsidence prediction methods.

Key Words: Subsidence, numerical modelling, boundary element, two dimensional displacement discontinuity method.

ÖZET

İKİ BOYUTLU SÜREKSİZ DEPLASMAN SINIR ELEMAN METODU İLE TASMAN BELİRLEME MODELİ

SARI, Dursun

Mühendislik Fakültesi

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Tez Yöneticisi: Prof. Dr. A. Günhan PAŞAMEHMETOĞLU

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Bu tezde, tasman olayı süreksiz deplasman sınır eleman nümerik metodu ile modellenmeye çalışıldı. Bu nümerik metod tasman problemini çözmek için, sonsuz düzlemdeki süreksiz deplasman çözümünden, yerçekimi yüklemeli yarı sonsuz düzleme uyarlandı.

Geliştirilen model, elde edilebilir arazi değerleri ile değişik uzunayak pano genişliğinin maden derinliğine oranı ve farklı sınır eleman durum tanımlamaları kullanılarak elde edilen sonuçları, tasman ve gerinim göz önüne alınarak karşılaştırıldı. Ayrıca bu model kaynaklarda elde edilebilir bazı nümerik tasman belirleme metodlarıyla da karşılaştırıldı.

Anahtar Kelimeler: Tasman, numerik modelleme, sınır eleman, iki boyutlu süreksiz deplasman metodu.

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CHAPTER 1

INTRODUCTION

Underground mining disturbs the existing equilibrium state of surrounding rock mass and induces large deformations and accompanying stress redistributions in the vicinity of the excavation. With the systems of total extraction allowing for full caving, the following ground movement extends over a wide region, including the ground surface. When the deformations involve the ground surface, damage to surface structures is a subject to concern. Engineering analysis is required in order to anticipate surface damage, and if possible to prevent it. Analysis involves precalculation of surface displacements (The term "subsidence" and "vertical displacement" can also be used) and strains caused by underground mining. A number of predictive methods have been developed to solve these parameters.

Predictive methods can be categorized into "empirical" and "phenomenological" approaches. Empirical approaches to the problem are based on statistical evaluation of data from the field and have proven useful in conditions of extraction under flat ground. Most recent work, however, has concentrated on phenomenological approaches in which the material properties and geometry of the mine are idealized in mathematical models; these predictive approaches contain the ability of intensify versatility. In early attemps closed-form solution methods were employed; applications were restricted in Scope. Recently, numerical methods used in conjunction with the digital computer have been employed.

Numerous computer programs for the analysis of subsidence are reported in the literature. However, a significant proportion of these have been written for a specific purpose and are generally either not well documented for use by outside groups or publicly unavailable. Additional factors are accessibility, availability and quality of documentation.

The aim of this study was to model the subsidence phenomena by a two dimensional displacement discontinuity boundary element method which is referred to in the following sections by the abbreviation TWODD. In order to solve this problem a method was developed from the solution previously given for a displacement discontinuity in a infinite-plane. In this method modifications for a semi-infinite plane with gravitational loading were done by changing the subroutine of influence coefficient and some other changes were introduced for the horizontal strain calculation. The model was idealized by homogeneous, isotropic, linear-elastic under plane strain conditions.

The subsidence and strain values obtained from this model for the different longwall panel width to mine depth ratios of stress-free and displacement boundary conditions were compared with the field British Coal (NCB) and the other numerical subsidence prediction methods available in the reviewed literature.

A brief review of the literature associated with subsidence prediction techniques is given in Chapter 2. In this chapter rather than giving the full description of these techniques, the comparison and the advantages or disadvantages of them are discussed. The analytical background of numerical analysis has to be undertaken especially for the ones which are added to the available TWODD program in order to get a brief idea about the structure of the new program. These studies are presented in Chapter 3. The computer modeling of this analysis and program input specifications in order to show how to execute the program are presented in Chapter 4. The experienced model analysis and discussion are given in Chapter 5. Finally, the conclusion and the recommendations for the future research are included in Chapter 6.

CHAPTER 2

REVIEW OF THE LITERATURE

2.1. A Review and Evaluation of Subsidence Prediction Techniques

A number of methods have been proposed or applied to predict surface ground movements (subsidence and horizontal strain) due to underground mining. These approaches can be broadly divided into three groups:

1. *Theoretical models* based on elastic, plastic, viscoelastic or other phenomenological models which are widely used in other engineering fields.

2. Numerical methods, mostly used as solutions to complex situations involving the phenomenological methods. Both these approaches assume that the strata in overburden behaves according to a specific and predictable manner. In using these models considerable information describing the behavior of the overburden is required, which has often limited applicability of these methods. Furthermore, in order to adopt their results to field data, a number of adjusting coefficients may have to be determined.

3. Empirical or semi-empirical methods are based largely on a combination of experience and detailed analysis of a large number of observed ground movements and have been successfully applied in the United Kingdom and European coalfields.

2.1.1. Empirical Technique

The British Coal (NCB) graphical method was derived from the analysis of an extensive field database collected over many years from a variety of mining conditions in the United Kingdom coalfields. A full description of the technique can be found in the NCB Subsidence Engineers Handbook (British Coal , 1975). Essentially the data were summarized in the form of a series of nondimensional graphs, where the subsidence is related to the ratio of the longwall face width to the depth of working and subsidence parameters (vertical displacement and horizontal strain) are calculated from this ratio. The accuracy of this method is claimed to be within ± 10 % although the conditions to which it is applied must be similar to those form which the handbook data are collected. It is used widely for comparing various other types of subsidence models.

2.1.2. Semi-Empirical Techniques

Two principal semi-empirical techniques were identified in the literature and it is purposed to mention each individually. These methods give no measure of intermediate strain and can not easily be adopted to do so. This is because they do not attempt to describe the mechanism of subsidence and rock failure, but produce a surface fit only.

(i) Profile Functions

The profile functions technique is based on a curve fitting procedure that employs a mathematical profile function to match the observed subsidence data. Once a fit is established, through the use of actual field data, the function is used to predict subsidence profiles over future areas of mining. This method has been successfully used in many parts of Eastern Europe (Hood, et al., 1981).

A number of different mathematical functions have been developed for critical mining areas by different researchers, but all the functions fit the general form as (Karmis, et al., 1986).

$$S(x) = S\ f(B,X,C) \qquad (1)$$

where S(x)= subsidence at point x, S= subsidence at the panel center, B= control parameter for the range of function, X= horizontal distance from panel center, and C= a constant.

The advantage of such a method is that it can be used easily with a computer, or pre-calculated tables. The main disadvantage of this technique is that, it can not negotiate excavations of complex shape or significant variation of mining or surface parameters, such as mining height, percent extraction and depth of excavation.

(ii) Influence Functions

This approach of subsidence prediction was initially developed by Dutch and German engineers (Bals, 1932; Karmis at al., 1981). It is based on the hypothesis that the affects of mining on subsidence of a point at surface can be superimposed. The extraction area can therefore be regarded as being composed of an infinite number of infinitesimal elements and the mathematical function which establishes the displacement of a surface point due to the infinite small extraction volume is known as the influence function. The subsidence of a surface point can therefore be obtained by integrating the influence of all the infinitesimal elements of the excavation. The method is extremely versatile and suffers none of the geometrical restrictions of the profile function and has been extensively used in the central and eastern European coalfields (Ren ,et al., 1987).

2.1.3. Additional Empirical Prediction Techniques

During the literature review, two additional empirical prediction techniques were identified; incremental design and stochastic methods.

Incremental design can be termed as the process of predicting future conditions from changes in current operations. In coal mining, the basic assumption is made that future subsidence can be predicted from previous experience. However, this process requires an existing database and the derivation of equations which assume that existing and future cases will behave in the same manner. A series of equations have been developed for flat lying ore bodies (Coates and Gyenge, 1973), but a review of the literature suggests that the process has never actually been used for subsidence prediction.

A number of stochastic models for subsidence prediction using probabilistic methods, were developed during the 1950's and 60's (Litwiniszyyen, 1957; Sweet and Bogdanoff, 1965). The stochastic model describes the behavior of a collection of discrete members or particles within a medium and may therefore applied to soils and specialized events such as block caving phenomena . However, its application to component layered strata must be questioned and probably accounts for the reason why no work has been undertaken in this area over the past ten years.

2.1.4. Theoretical Techniques

A review of the literature identified two principal theoretical approaches (Poon and Tammemagi, 1985): each of which will be summarized briefly.

(i) Mechanistic Models

The void-volume model proposes that the actions of discrete deformational and collapse mechanisms are the primary modes influencing subsidence development (Munson and Benzley, 1980). However, to facilitate this approach the actual collapse mechanisms must be known and physical models constructed to simulate the process (Whittaker, et al, 1986). The effect of scaling factors on this type of model must therefore raise questions concerning the validity of the results, especially since different types of material behavior are involved in the collapse mechanism.

(ii) Closed Form Elastic Solution

One method of studying subsidence development is to assume that the strata displacement behaves according to one of the constitutive equations of continuum mechanics over most of its range .The continuum theories were evolved from the analysis of a displacement discontinuity produced by a slit in an infinite half-space elastic media. Analytical procedures were subsequently developed for three types of underground excavation based on elastic ground conditions (Berry, 1960)

1. Non-closure
2. Partial closure
3. Complete closure.

Additional work (Berry and Sales, 1961 and 1962) extended the closed form solution to the transversely isotropic ground conditions in both two and three dimensions.This is progressed in the analysis of ground movement due to mining (Berry, 1978). The three dimensional solution was subsequently modified (Avasthi and Harloff, 1982) to bring the profiles into agreement with that data observed by the British coal in the United Kingdom and also extend its application to multiple and steeply dipping seams.Two-dimensional analysis of near surface single seam extraction is developed by superposition from solutions previously given for a displacement discontinuity, or dislocation, in an otherwise continuous, linearly elastic, infinite rock mass solution of a seam in a semi-infinite mass, the surface of which can be subjected to arbitrary prescribed tractions or traction free (Crouch, 1972).

2.1.5. Numerical Methods

Numerical models provide an excellent tool for the quantitative analysis of subsidence and strata mechanics problems and are not subject to the same restrictive assumptions required for the closed form analytical solutions especially for the complicated extraction patterns.

Finite element modeling is commonly applied to subsidence problem since it can readily accommodate non-homogenous media, non-linear material behavior and complicated mine geometries (Langland and Fletcher, 1976;

Ratigan and Goodman, 1981). Alternatively, finite difference models can be used for the large strain, non-linear phenomena associated with subsidence development (Trent, 1981).

Other elastic approaches employing numerical techniques include boundary element methods (Crouch and Starfield, 1981). A detailed discussion on the use of boundary element methods for subsidence modeling will be given in the following sections.

(i) Boundary Element Methods

In recent years, the boundary element techniques are gaining popularity among numerical methods. The main reasons for this growth are:

1. reduced set of equations
2. smaller amount of data
3. proper modeling of infinite domains

Many important practical problems in science and engineering can be modeled by Boundary Value Problems. These problems can be denoted by region of R included a boundary C. In the region, a partial differential equation models the physics of the problem, and this equation is solved according to the prescribed condition on the boundary C. If R is three-dimensional region then C is a boundary surface. In two dimensions, R is a plane region and C is a boundary contour (Figure 2.1).

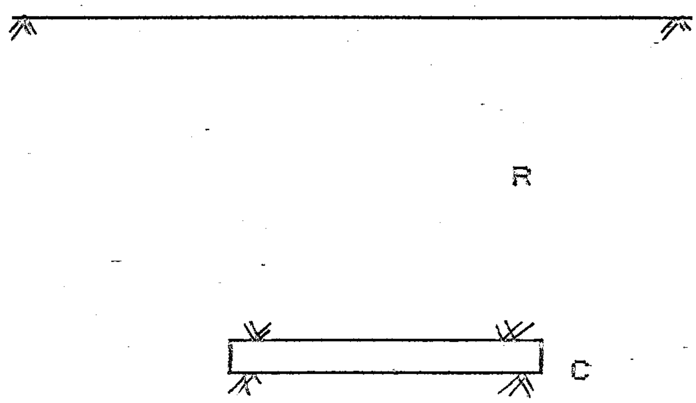


Figure 2.1 R and C in two dimensions.

In any boundary value problem only some of the related parameters on the boundary can be described; the others are found as a part of the solution.For example, when the displacements are specified on the boundary , then the stresses on the boundary or in the region of interest eventually can be found as part of the solution to the problem.

If the problem is solved for the singularities that satisfy the specified boundary conditions, and compute the rest of the boundary parameters in terms of these singular solutions, the unspecified boundary parameters are obtained indirectly. This procedure is called indirect boundary element method such as two dimensional displacement discontinuity and two dimensional fictitious stress methods. In the mathematical approach certain fundamental integral theorems are used to eliminate, the intermediate step, where the unknown boundary parameters are found directly from the related equations with specified parameters at each element of the contour. Accordingly, this procedure is called a direct boundary element method such as boundary integral method (Crouch and Starfield, 1981).

(ii) Boundary Elements and Finite Elements

To illustrate the difference between two types of numerical technique, the comparison will be done between these two methods. The finite element method requires that the whole region R be divided into a network elements as shown in Figure 2.2.

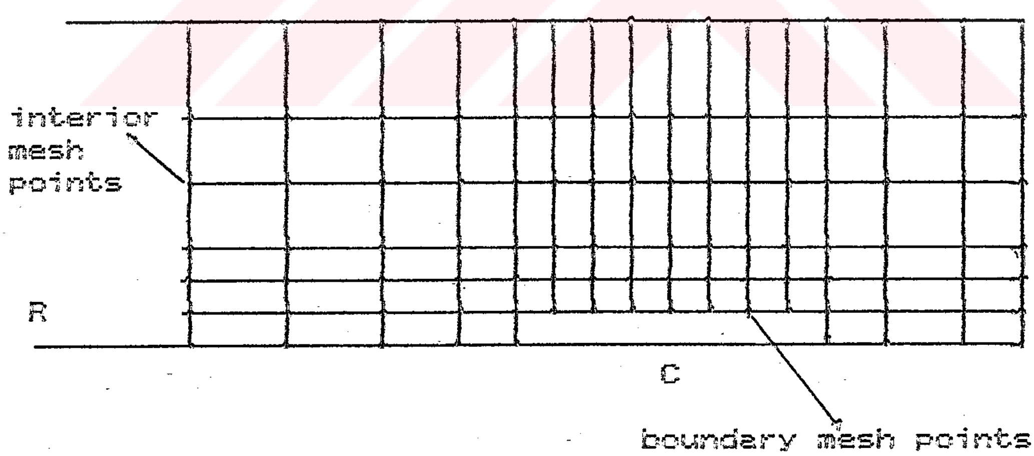


Figure 2.2 Finite element idealization.

The objective is to evaluate the solution to problem at the nodes or mesh points of the network; the solution between nodes is expressed in a simple, approximate form in terms of the values at the nodes. Relating this approximate solution to the original partial differential equations eventually leads to a system of linear algebraic equations in which the unknown parameters, the nodal values in R, are expressed in terms of the known nodal values at mesh points on the boundary C. There is a large number of unknown parameters, and hence a correspondingly large number of linear equations, each equation contains only a few of the unknown parameters explicitly.

In boundary element methods only the boundary C is divided into elements as in Figure 2.3. The numerical solution builds on the analytical solutions that have already been obtained for simple singular problems in such a way as to satisfy, approximately, the specified boundary conditions at each element on C. Because each of the singular solutions satisfies the governing partial differential equations in R, there is no need to divide R itself into a network of elements. The system of equations to be solved is much smaller than the system needed to solve the same boundary value problem by the finite element method.

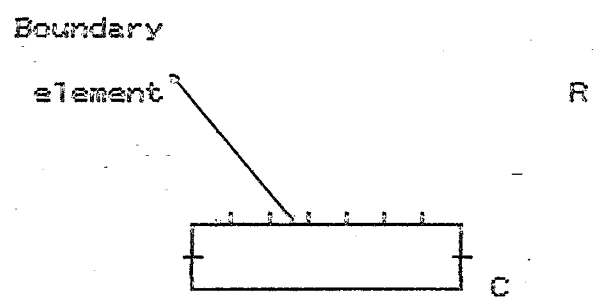


Figure 2.3 Boundary element idealization.

Boundary element methods are particularly attractive for exterior problems, where the contour C defines the boundary of a cavity in an infinite body. If the basic singular solution is chosen such that it satisfies the appropriate boundary conditions at infinity, then a linear combination of the solutions will also satisfy these boundary conditions. In solving a given problem, therefore, singularities need only be placed along the boundary contour C.

A finite element model of the problem, on the other hand requires that the network of elements be extended "sufficiently far" a way from the cavity so that the conditions imposed on the outer boundary of the network do not have appreciable effect on the solution in the vicinity of the cavity. The elements near the outer boundary can be made larger, as the solution does not vary a great deal from point to point at large distances from the cavity. It is necessary, however, to make the mesh transition between the inner and outer boundaries "sufficiently smooth" so that gradients in the solution can be accurately represented.

In boundary element only the elements number and position the inner boundary should be cared and this is rather easy to conduct "element refinement studies" to see how the computed results depend upon the element position along the boundary of the cavity. Similar studies for the finite element method can be very tedious to perform, because the entire finite element network must be re drawn in each case.

CHAPTER 3

THEORY OF TWODD PROGRAM AND ADDED PARTS

3.1. Introduction

In this chapter some important aspects of analytical solution for an elemental displacement discontinuity and computation of tangential stress along a boundary and displacement discontinuity in a half-plane will be explained. A full description of this approach can be found in the Boundary Element Methods in Solid Mechanics (Crouch and Starfield, 1981).

3.2. Displacement Discontinuity in an Infinite Solid

The displacement discontinuity method can be used for the practical problems in solid mechanics such as thin, slit-like openings or cracks. This method is based on the analytical solution to the problem which is specified by the condition that the displacements must be continuous everywhere except over a finite line segment in the x,y plane of an infinite elastic solid. The line segment can be a certain portion of the x-axis such as $|x| \leq a$, $y = 0$ (Figure 3.1). The segment's surfaces are on the negative and positive sides of y=0, and denoted by $y = 0_+$ for positive and $y = 0_-$ for negative sides. The displacement discontinuity $Di=(Dx,Dy)$ resulted in crossing from one surface to another can be defined as the difference in displacement between the two sides of the segment as follows:

$$D_i = U_i(X, 0_-) - U_i(X, 0_+)$$

or (1)

$$D_x = U_x(X, 0_-) - U_x(X, 0_+)$$

$$D_y = U_y(X, 0_-) - U_y(X, 0_+)$$

Because U_x and U_y are positive in the positive x and y co-ordinate directions, it follows that D_x and D_y are positive as illustrated in Figure 3.1 .

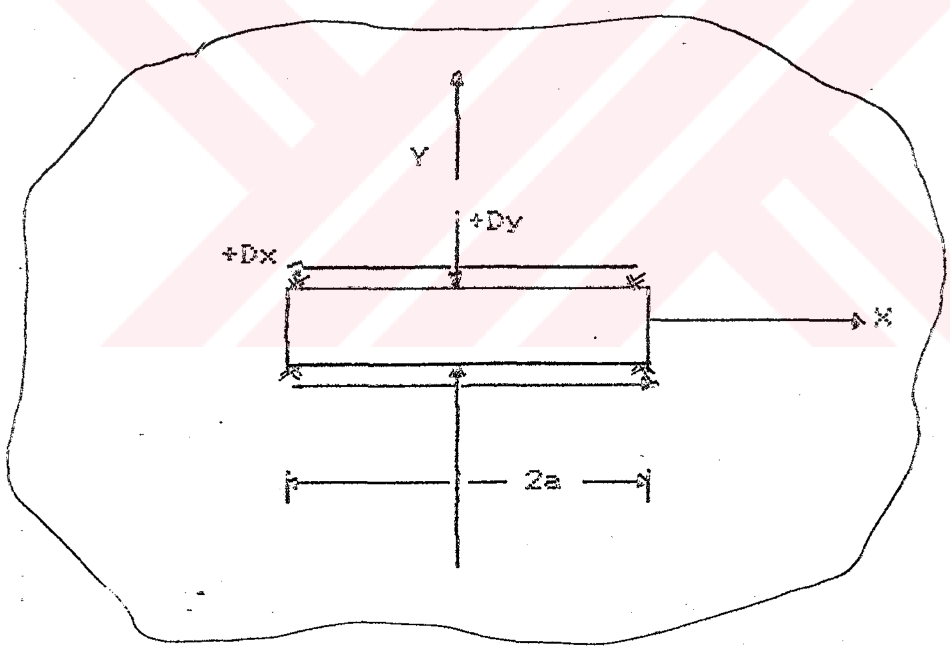


Figure 3.1 Constant displacement discontinuity components Dx and Dy.

The solution to the subject problem is given by Crouch (1976 a,b).

3.3. Program Module for a Half Plane

If the thickness of the excavated panel is uniform and sufficiently small in thickness compared with its sides and with its depth below the surface, its closure may be regarded as a constant displacement discontinuity with in an elastic half-plane.

The analytic solution for a constant displacement discontinuity over an arbitrarily oriented line segment is given by Crouch (1976 a,b). This solution is presented in Appendix A.

Half-plane problems can be solved by dividing the ground surface boundary into elements and giving the required surface stress or displacements conditions at these elements. This is difficult and a large number of boundary elements is needed.

The singular solutions given by Crouch that is mentioned above automatically meet the prescribed boundary conditions along the surface. When this program module is used a boundary elements need only be placed along internal contours.

3.4. Computation of Tangential Stress

The tangential stress along the boundary can also be computed by the displacement discontinuity method. Having found the boundary displacements, therefore the numerical differentiation formulas to compute their tangential stress can be used.

In this section only the numerical formulation of tangential stress which to be added to the TWODD main program rather than the other analytic steps that is involved will be given.

Standard finite difference approximations are used for the purpose of computing the derivatives of displacements at the ith element ($\delta Ux^-/\delta x$ or $\delta Ux^+/\delta x$) in order to find tangential stresses (σ_t^- and σ_t^+).

The forward and backward difference formulas are used for the first and last elements respectively, and central difference formulas is used for all others:

$$\left[\frac{\delta f}{\delta x}\right]_{x=x^i} = \frac{f(x^{i+1}) - f(x^i)}{x^{i+1} - x^i} \quad \text{(forward difference)}$$

(2)

$$\left[\frac{\delta f}{\delta x}\right]_{x=x^i} = \frac{f(x^i) - f(x^{i-1})}{x^i - x^{i-1}} \quad \text{(backward difference)}$$

$$\left[\frac{\delta f}{\delta x}\right]_{x=x^{i}} = \frac{f(x^{i+1}) - f(x^{i-1})}{x^{i+1} - x^{i-1}} \quad \text{(central difference)}$$

where;

$$f(x)=Ux^{+} \quad \text{or} \quad Ux^{-}$$

$$\frac{\delta f}{\delta x} = \frac{\delta Ux^{+}}{\delta x} \quad \text{or} \quad \frac{\delta Ux^{-}}{\delta x} \quad \text{(Displacement gradients)}$$

Hooke's law for the case of plane strain and finite difference formulas in terms of local co-ordinates are used for an element of arbitrary orientation as follows:

$$\sigma_{t}^{i-} - \sigma_{t}^{i+} = 2G/(1-v)[\delta Us^{i-}/\delta s - \delta Us^{i+}/\delta s] \qquad (3)$$

It is known that $\delta Us^{i-}/\delta s$ and $\delta Us^{i+}/\delta s$ have different values for the backward , forward and central finite difference. Subsituting these values to the equation (3) , the following formulas are produced:

1. Forward difference (First element)

$$\sigma_{t}^{i-} - \sigma_{t}^{i+} = 2G/(1-v)[Ds^{i+1}\cos(\beta^{i+1}-\beta^{i})/\Delta s^{i}$$
$$-Dn^{i+1}\sin(\beta^{i+1}-\beta^{i})-Ds^{i}]/\Delta s^{i} \qquad (4)$$

Where;

$$\Delta s^{i} = a^{i} + a^{i+1} \cos(\beta^{i+1} - \beta^{i})$$

2. Central difference (Central element)

$$\sigma_t^{i-} - \sigma_t^{i+} = 2G/(1-\nu)[\{Ds^{i+1} \cos(\beta^{i+1} - \beta^{i}) - Ds^{i-1} \cos(\beta^{i} - \beta^{i-1})\}/\Delta s^{i} \qquad (5)$$

$$-\{Dn^{i+1} \cos(\beta^{i+1} - \beta^{i}) + Dn^{i-1} \sin(\beta^{i} - \beta^{i-1})\}/\Delta s^{i}]$$

Where;

$$\Delta s^{i} = a^{i-1} \cos(\beta^{i} - \beta^{i+1}) + 2a^{i} + a^{i+1} \cos(\beta^{i+1} - \beta^{i})$$

3. Backward difference (Last element)

$$\sigma_t^{i-} - \sigma_t^{i+} = 2G/(1-\nu)[Ds^{i} - Ds^{i-1} \cos(\beta^{i} - \beta^{i-1})/\Delta s^{i-1} \qquad (6)$$

$$-Dn^{i-1} \sin(\beta^{i} - \beta^{i-1})/\Delta s^{i}]$$

Where;

$$\Delta s^{i} = a^{i} + a^{i-1} \cos(\beta^{i} - \beta^{i-1})$$

The tangential stresses on the two sides of the ith element (negative and positive) can be calculated as follows:

$$\sigma_t^{i+} = \tfrac{1}{2}[\sigma_t^{i-} + \sigma_t^{i+}] - \tfrac{1}{2}[\sigma_t^{i-} - \sigma_t^{i+}]$$

$$\sigma_t^{i-} = \tfrac{1}{2}[\sigma_t^{i-} - \sigma_t^{i+}] + \tfrac{1}{2}[\sigma_t^{i-} - \sigma_t^{i+}] \qquad (7)$$

The first part of the formula is continuous and given as:

$$\frac{1}{2}\left[\sigma_t^{i-} + \sigma_t^{i+}\right] = 2G \sum_{j=1}^{N} \left[2\cos^2\theta\bar{F}4 - \bar{F}5\sin 2\theta + y(\bar{F}6\cos 2\theta + \bar{F}7\sin 2\theta\right] Ds^j$$

$$+ 2G \sum_{j=1}^{N} \left[-\bar{F}5 - y - (\bar{F}6\sin 2\theta - \bar{F}7\cos 2\theta)\right] Dn^j$$

where;

$$\theta = \beta^i - \beta^j$$

Thus, (7) reduces to:

$$\sigma_t^{i+} = \sum_{j=1}^{N} A_{ts}^{ij} Ds^j + \sum_{j=1}^{N} A_{tn}^{ij} Dn^j - \frac{1}{2}(\sigma_t^{i-} - \sigma_t^{i+})$$

(8)

$$\sigma_t^{i-} = \sum_{j=1}^{N} A_{ts}^{ij} Ds^j + \sum_{j=1}^{N} A_{tn}^{ij} Dn^j + \frac{1}{2}(\sigma_t^{i-} - \sigma_t^{i+})$$

Where;

A_{ts}^{ij}, A_{tn}^{ij} = Influence coefficients which were calculated from the main part of the program.

$\frac{1}{2}\left[\sigma_t^{i-} - \sigma_t^{i+}\right]$ = Stress discontinuity calculated in the subroutine by using equations (4,5,6).

In this context, the calculated positive tangential stress is fictititous quantity. The negative one is meaningful, otherwise it will be misleading.

CHAPTER 4

MODIFICATIONS AND INPUT SPECIFICATIONS MADE ON THE AVAILABLE TWO DIMENSIONAL DISPLACEMENT DISCONTINUITY PROGRAM

4.1. General

In the previous chapters it was mentioned that by combining various program modules, new boundary element programs can easily be constructed on the main program. Due to this point of view the Two Dimensional Displacement Discontinuity program (TWODD) (Crouch and Starfield, 1981) is modified by making some changes on the structure of it. Modification to semi-infinite plane, tangential stress computation program, horizontal strains calculation, gravitational loading program and dividing the boundary into a number of elements are the changes done in the structure of the program. These changes in terms of computer language simulations will be briefly explained in this section, and the detailed parts of them will be given in appendices.

4.2. Modification to Semi-infinite Plane

All computer programs for Boundary Element Methods, have a very simple and similar structures. It will be useful to mention the computation steps that were carried out in the program briefly before explaining the modified part of the TWODD program.

1. Definition of the locations of all boundary elements and specify displacement or stress boundary conditions for each one.

2. Computation of the boundary influence coefficients and setting up this appropriate system of simultaneous linear equations by considering the boundary conditions at each element.

3. Solving the system of equations from step 2.

4. Computing the displacements and stresses at each boundary element.

5. Computing the influence coefficients for specified points within the region of interest, and hence compute the displacements and stresses at these points.

Step 1 is essentially an input operation involving geometric parameters and step 3 can be handled by standard methods of numerical analysis. Step 2,4 and 5 are similar to one another in that each involve computation of a set of influence coefficients.

Computation of the displacement and stress influence coefficients is the "heart" of a boundary element computer program. Different boundary element methods may employ different analytical expressions for this purpose, but the structure is essentially same as outlined above.

Because of this, a computer program for one boundary element method quite closely resembles a program for another method. Major differences occur only in the subroutine used to evaluate appropriate physical expressions for the method in question.

Consequently, boundary elements programs have modular character, and it is possible to go from one method to another merely by changing "modules" and, introducing a few new variables in the main program.

In the light of the above outlined points, by changing the subroutine COEFF that calculates the influence coefficients and introducing a few new variables in the main program, the TWODD is simulated to the new program for semi-infinite plane.

As outlined in section 3.3., the analytic solution of semi-infinite plane problems (so as influence coefficients) is based on superposition of actual, image and supplementary solution. These parts are calculated separately and added at the end of the subroutine to get the whole results of influence coefficient.

The changes in the subroutine COEFF are achieved by following the same procedure that has been done for the original program. Variable names in the program are chosen to correspond as closely as possible with the symbols used in the analytic solution in the previous chapter.

A small segment is given below from the subroutine

COEFF to give an idea about programming, but the whole program that is added to the original one is given in Appendix A1.

Example:

$$\bar{x} = (x-x^j)\text{Cos}\beta^j + (y-y^j)\text{Sin}\beta^j \quad \text{(Numerical idealization)}$$

XB = (X-CX)*CosB + (Y-CY)*SinB (Programming idealization)

$$F6(x,y)=f,_{\bar{x}\bar{y}\bar{y}}=1/(4\pi(1-v)[((\bar{x}-a)^2-\bar{y}^2)/(((\bar{x}-a)^2+\bar{y}^2)^2)$$

Appendix A (3)

$$-((\bar{x}+a)^2-\bar{y}^2)/(((\bar{x}+a)^2+\bar{y}^2)^2)]$$

FB6=CON*(((XB-A)**2-YB*YB/R1S**2-((XB+A)**2-YB*YB/R2S**2)

Where:

R1S = (XB-A) * (XB-A) + YB * YB

R2S = (XB+A) * (XB+A) + YB * YB

$$C_{ss}^{ij} = [-(1-2v)F2+2(1-v)\cos\beta F3+y(\sin\beta F4-\cos\beta F5) \quad \text{AppendixA (4)}$$

Where:

C_{ss}^{ij} = Influence coefficient for shear displacement at the midpoint of ith segment (U_s^i) due to a constant unit shear displacement discontinuity applied to the jth segment (D_s^j). Equation Appendix A (4) is expressed in the program by the following expression.

VXDS = -PR1*SINB*FB2+PR2*COSB*FB3+YB*(SINB*FB4-COSB*FB5)

4.3. Tangential Stress Computation Program

Following the numerical procedure explained in section 3. 4., the subroutine (SIGTAN) is added to the main program. Due to addition of this subroutine, some minor changes are necessary in the main program.

First of all, the displacement discontinuity components (DS,DN) are differentiated from the previously used array to calculate the other boundary stresses and displacements and stresses and displacements at specified points. Differentiation is done to make the sequence of the displacement discontinuity components in an order way.

The elements with different inclination in their local co-ordinates have different projection of components, and hence their angles are required(Chapter 3 (4,5,6)). To accomplish this the angle array ANG is added for this purpose.

The continuous part of tangential stress termed as SIGT1 is computed as usual by calculating the tangential stress along the boundary which is to the SIGXX in the main program.

The discontinuous part of tangential stress (Chapter 3 (8)) is introduced in the subroutine SIGTAN. This part is used in finding the solution of the real tangential stress. Rather than discontinuous positive tangential value (SIGTP), the negative one (SIGTN) has to be taken into account in finding the tangential stress.

In this subroutine, for each number of boundary element's tangential stresses on both sides (positive and negative) are calculated and the assigned names are inherently related the formulas given in section 3.4. Example below will give the readers a little idea about how the numerical model is formulated and incorporated to the computer program. The overall program related to this section to be is given in Appendix A2.

Example:

$$\beta^{i} = ANG(I) = F$$

$$\beta^{i+1} = ANG(I+1) = G$$

$$\beta^{i+1} - \beta^{i} = - ANG(I) + ANG(I+1) = G - F$$

$$\Delta S^{i} = DELS$$

$$a^{i} = A(I)$$

$$\Delta S^{i} = a^{i} + a^{i+1} \; Cos(\beta^{i+1} - \beta^{i}) \qquad \text{(Formulation)}$$

DELS = A(I) + A(I+1)*Cos(z) (Programming representation)

4.4. Calculation of Horizontal Strain

In subsidence engineering terminology, "strain" (e) is the change in length over a piece of ground or structure expressed either as dimension over the whole length or as a fraction of the unit length. Thus a strain may be expressed 0.01 meter in 10 meters as one part per thousand, or one millimeter per meter or simply as 0.001. The direction is always specified, extensions and compressions being indicated by + and - sign respectively.

First step in the strain calculation is to add a subroutine (STRAIN). This subroutine is called from the main program after computation of displacements at specified points at the surface in the global coordinates. In addition to this, XP STR, UX arrays are formed in the main program as well as in the subroutine and they are used in the iteration for calculation of surface strain values.

In the subroutine (STRAIN), the derivatives of displacements at the specified points are required, and hence standard finite difference approximations are used as previously used for calculation of tangential stress purpose in section (3.4). Using conditional control statements, positions of the specified point (first, middle, last) are checked. Following the same procedure and formulation as in the computation of tangential stress with the finite difference method introduced before strain at each specified point is evaluated. Programming portion related to this section can be found in Appendix A3.

4.5. Gravitational Loading Program

Like any other underground excavations, the extracted panel can also be subjected to uniform loading (in-situ stress) if present and/or gravitational loading due to the overlying strata weight.

From the empirical analysis which the data selected from the in-situ measurements made in

Scandinavia, Australia, South Africa, the U.S.A. and other parts of the European country, it can be deducted that gravitational loading varies with depth, and hence the vertical stress can be calculated by multiplying stress gradient (average unit weight of the overlying strata) with the depth of the point of intererest and the horizontal stress is a constant times vertical stress (Hoek and Brown, 1980).

In this program, the calculated gravitational loading is constant along the boundary (because the depth is not changing as it is horizontal) and the effect of it on the computational procedure is uniform throughout the structure of the program. It is introduced at the step where the uniform stress has been inserted previously and the other following computational steps will be just same as in the in-situ stress idealization. The procedure is simple and is explained below.

$$\sigma^{i}_{\bar{y}} = \gamma * (\bar{y})^{i}$$

PYY = GAMMA * YM(N) (YM(N) may be any array)

$$\sigma^{i}_{\bar{x}} = K * \sigma^{i}_{\bar{y}}$$

PXX = YK * PYY

$$\sigma^{i}_{\bar{x}\bar{y}} = 0$$

PXY = 0

Where:

γ = Unit weight of the rock

K = field stress ratio (PXX / PYY) constant

4.6. Dividing the Boundary into a Number of Elements

The original TWODD program has only a straight line segment type in order to divide the boundary into a number of elements. For the shapes other than straight line elements needed are difficult to symbolize, and hence circular arcs and elliptical arcs types are used.

In the following specifications, reference is made to the end points of the segment, described as the initial and final points. In deciding which is which, the rule is that when the boundary is traced from the initial to the final point and one faces the direction of travel, the solid material lies on the right hand side in other words following the counter clockwise direction.

1. Straight line segments

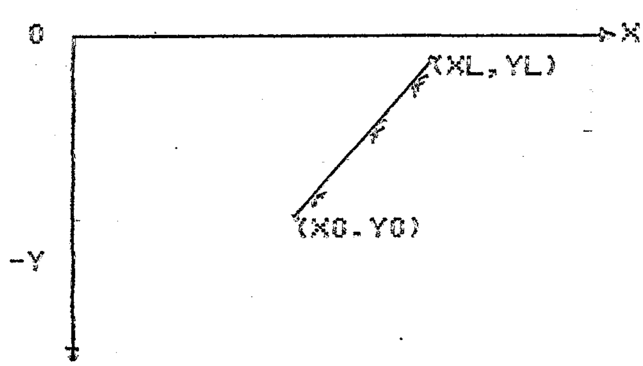


Figure 4.1 Straight line segment idealization.

X0,Y0 = co-ordinates of the initial point

XL,YL = co-ordinates of the final point

2. Circular segments

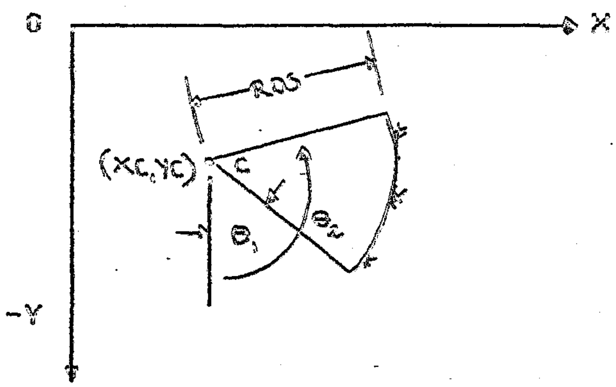


Figure 4.2 Circular segment idealization.

XC,YC = co-ordinates of center of circle

RDS = radius of circle

THET1 = polar angle of initial point

THET2 = polar angle of final point

Line CB is drawn from the center C in the direction of the +Y axis. The polar angles are measured in a counter-clockwise direction from CB.

3. Elliptical Segments

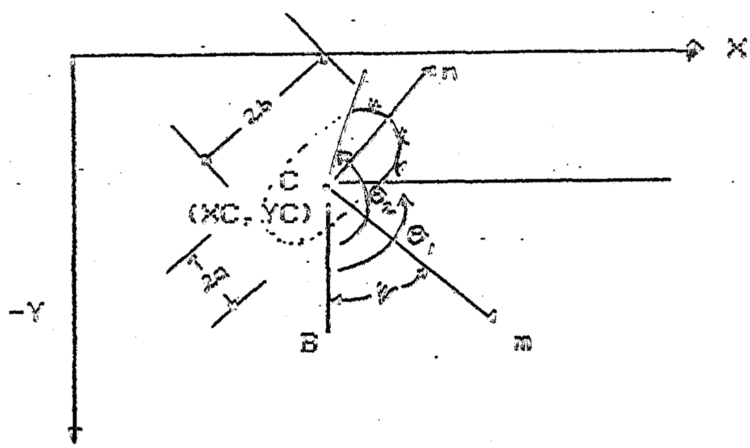


Figure 4.3 Elliptical segment idealization

XC,YC = co-ordinates of center C

semiax = length of one semi-axis (a)

RATIO = b/a, where b is the length of the other semi-axis

PSI = Polar angle of axis a

THET1 = Polar angle of initial point

THET2 = Polar angle of final point

Those polar angles are measured as described above

These segments may be used together or separately to form a boundary of virtually any arbitrary shape.

The listing for this section is available in Appendix A4.

4.7. Terminology for Program Input

In this section, the terminology for program input is given in order to show how to run program that consists of description of boundary contours, field points specification, units and input specification. The verification of the program is also pointed out in this part.

4.7.1. Description of Boundary Contours

All boundary contours are approximated by a straight line and/or a circular and/or an elliptical segments joined end to end. Each boundary segment is divided into NUM boundary elements. The element locations are specified by giving the x,y co-ordinates of the initial (XBEG, YBEG) and final (XEND, YEND) points of the segment, this corresponds to center and polar angle respectively for the circular and elliptical segments, and the value of NUM (NUM ≥ 1). In addition to those, the radius (RDS), axis ratio (RATIO),polar angle angle of axis a (PSI) have to be specified for the circular and elliptical segments. The computer program then numbers the boundary elements and computes their midpoint co-ordinates, lengths and orientations.

The number of boundary segments is called NUMBS. The number of element associated with the NUMBS depend on the related array size that is used for the calculation of boundary element specifications. For example, if the boundary elements are hundred, then the array used for this purpose (XM(100), YM(100), DN(100), DS(100) ...etc.) has to be 100 in size, and hence to solve system of algebraic equation array (C(200,200), B(200), D(200)) two times of the other array size in matrix form. This can be arranged depending on the memory capacity of the computer that is used. In general, the best results are obtained when the boundary elements all have approximately the same length. The more elements up to a point will give a better results as it approaches to the analytic solution due to decreasing the element size, hence getting a continuous line instead of discrete lines.

In giving the x,y co-ordinates of the boundary segment; a closed contour is traversed in the counter clockwise sense if the region of interest is outside the contour that is a cavity problem (Figure 4.4).

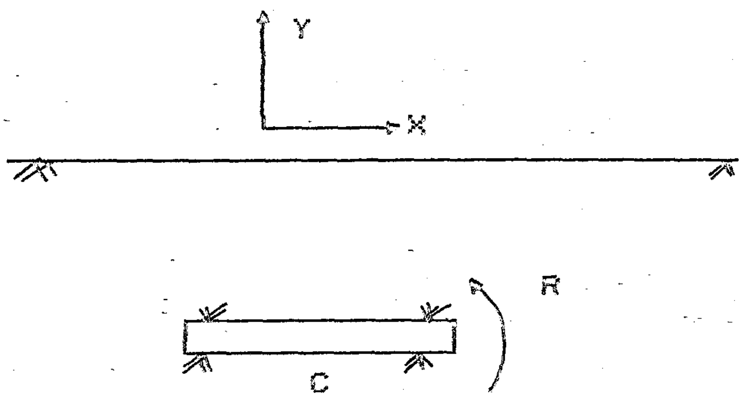


Figure 4.4 Traversing the closed contour of the problem.

4.7.2. Specification of Field Points

Field points are specific points within the region of interest (i.e. not on a boundary) where displacements and stresses and horizontal strains are to be computed. Equally spaced field points along a straight line are specified by giving the x,y co-ordinates of the beginning (XBEG, YBEG) and end (XEND, YEND) points of the line, and the number of desired intermediate points (NUMPB) along the line. The number straight line segments used to specify the field point locations in this way is called NUMOS. The value of NUMOS depend on the XP(N), UX(N) array size that means it is inherently related with the N. For example when the N is equal hundred the value of NUMOs will be hundred. The program is arranged to skip any field point lying within a distance of one element length from the midpoint of a boundary element, because the computed results are not accurate within this distance.

4.7.3. Symmetry Conditions

If symmetry conditions exist for a given problem, the amount of input data for the program can be reduced. KSYM is a parameter that specifies the kind of symmetry to be used, as illustrated in Figure 4.5.

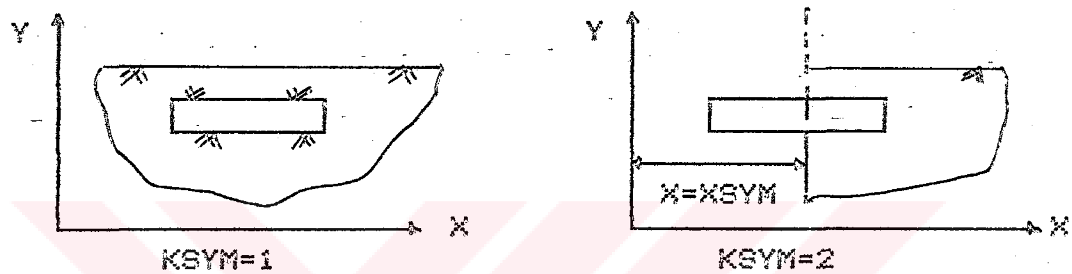


Figure 4.5 Specification of symmetry conditions using the parameter KSYM.

KSYM=1 means that no symmetry conditions are to be imposed, even if the problem has symmetry;

KSYM=2 means that a line parallel to the y axis (x=XSYM) is a line of symmetry;

If a problem has a line of symmetry (KSYM=2), boundary elements have to be defined for only one-half of the boundary contour as in Figure 4.5; the symmetrically placed image elements (Crouch and starfield, 1981) are supplied automatically by TWODD program.

There is two restrictions concerning the placement of boundary elements in this problem involving symmetry:

a boundary element can not lie along a line of symmetry, and it can not cross such a line. Thus, for example, if one end of an element touches a line of symmetry the other end can not. Field points, however, can be selected along a line of symmetry.

4.7.4. Units

Program TWODD is designed to work with any consistent set of units (e.g. SI units). For input, displacement boundary values must be given in the same units as are used to specify the co-ordinate locations, and stress boundary values and the initial stresses must be given in the same unit as Young's modulus. The computed values of the displacements and stresses will then have the corresponding units.

4.7.5. Input Specifications

Program is arranged to prepare a data file according to format specification given below. If the data file is written in a manner given below program will read this data file during execution.

Line 1.Free format; one like must be written.

Columns 1-72 of this line contain any desired information to identify the problem being solved.

Line 2.Format (3I4); one line must be written to specify the following control parameters.

NUMBS=number of straight line boundary segments (each containing at least one boundary element) used to define boundary contours.

NUMOS=number of other line segments (not on a boundary) along which displacements and stresses are to be computed.

KSYM =
- 1 no symmetry conditions imposed
- 2 X=XSYM is a line of symmetry
- 3 Y=YSYM is a line of symmetry
- 4 X=XSYM and Y=YSYM are lines of symmetry

Line 3.Format (F6.2, E11.4, 2F12.4): this line must be written to define the elastic constants and specify the locations of lines of symmetry (if any).

PR=Poisson's ratio (v)

E=Young's modulus (E)

XSYM=Location of line of symmetry parallel to y axis (XSYM is ignored if KSYM=1 or 3 in line 2)

YSYM=Location of line of symmetry parallel to x axis (YSYM is ignored if KSYM=1 or 2 in line 2)

Line 4.Format (3E11.4); One line must be given to define the control parameters of the stresses in the region interest.

GAMMA=Unit weight of the overburden

YK=Field stress ratio (PXX/PYY = YK)

PXY=(σ_{xy})

Line(s) 5.Format (I4, 4F12.4, 4I, 2E11.4): NUMBS lines (According to numbs these lines are arranged) must be written to define the locations and boundary conditions of boundary elements.

NUM=Number of equally spaced boundary elements along straight line segment, all elements having the same boundary conditions.

XBEG= X co-ordinate of beginning of line segment (center of ellipse or circle)

YBEG= Y co-ordinate of beginning of line segment (center of ellipse or circle

XEND= X co-ordinate of end of line segment (THETA1)

YEND= Y co-ordinate of end of line segment (THETA2)

RATIO= b/a ratio in ellipse

Radius= Radius of circle

PSI= Polar angle of axis a

KODE =
- 1 σ_s and σ_n prescribed
- 2 Us and Un prescribed
- 3 Us and σ_n prescribed
- 4 σ_s and Un prescribed

BVS=resultant shear stress (σ_s) or shear displacement (Us)

BVN=resultant normal stress (σ_n) or normal displacement (Un)

Line(s) 6.Format (4F12.4, I4); Numos lines must be written to define locations of points inside the region of interest where displacements and stresses, horizontal strains are to be computed.

XBEG= x co-ordinate of first point on line

YBEG= y co-ordinate of first point on line

XEND= x co-ordinate of last point on line

YEND= y co-ordinate of last point on line

NUMPB= number of equally spaced points between specified first and last points.

4.8. Verification of Modified TWODD

The modified TWODD program (Appendix B) was checked by using an analytic solution known as example problem. This problem was a circular problem in an semi-infinite plane.It was tested with uniform loading and gravitational loading case separately Appendix B1.

CHAPTER 5

ANALYSIS OF MODEL AND DISCUSSIONS

5.1. Analyzed Model Geometry

The two dimensional (plane-strain) elastic idealization is assumed to provide an adequate approximation to prototype conditions. A horizontal 2 m thick seam at 200 m depth is used as the excavation model in each analysis; panel width is kept as a variable for the analysis (Figure 5.1). Three kinds of panel width to depth ratio are used. The first one is 1.4 which is considered to be the critical ratio (at a given depth, the width of extraction for which the subsidence at the bottom of trough has a maximum possible value is termed "critical width". The ratio of critical width/depth is called "critical" ratio.) of extraction for European conditions (British Coal, 1975). The other width to depth ratios of the model geometry used are 0.5 and 0.75. Included in the model is a large region which extended beyond the presumed area of influence of mining transversely and longitudinally.

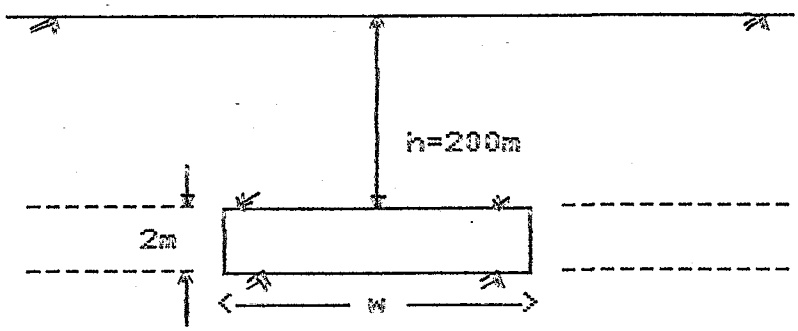


Figure 5.1 Model configuration of a longwall panel.

Hypothetical case history chosen is a representative longwall panel like the ones used by other researchers in analyzing subsidence problems by numerical modeling, rather than a specific field or work condition. Although the case history is very simple and avoided many of the real life complexities found in mining, it does allows valuable results to be obtained compared to the other numerical, theoretical, and empirical (NCB) subsidence prediction methods.

5.2. Material Properties Used in Example Problem

The typical laboratory values of Young's modulus determined from small samples of coal measure rocks are generally much greater than in-situ rock mass properties which in fact control ground motions. A reduction of laboratory values by approximately one order of magnitude was believed to be necessary. Therefore, after careful review, the average modulus of elasticity representative of field value of homogeneous overlying rock mass was chosen 8000 MN/m^2. The Poisson's ratio was taken as 0.17. The unit weight of the rock used for the calculation of overburden stress varying with depth was taken as 0.027 MN/m^3. Finally the value of average field ratio (it is generally named as " k " and it is the ratio of horizontal field sterss (σ_h) to vertical field stress (σ_v).) was taken from the empirical graph (data are chosen from the in-situ measurements of Scandnavia, Australia, South Africa , the U.S.A and other European countries.) that was arranged according to the depth of mining (Hoek and Brown, 1980).

The coal properties were taken equivalent to uniform overbürden properties as the whole medium was assumed to be homogeneous including the seam.

Material properties given here were used in each analysis, unless otherwise stated.

5.2.1. Influence of Material Properties on the Predicted Subsidence

Keeping the panel width to depth ratio 1.4 and applying free surface boundary conditions for all the extracted seam ($\sigma_s=0, \sigma_n=0$) in the modified TWODD program, elastic constants were varied systematically in order to asses their effects on subsidence.Results obtained are given below:

i. The effect of Poisson's ratio was examined by holding Young's modulus constant and varying the Poisson's ratio model within the range 0.0 to 0.30. Variations in the displacement or strain profiles as a function of Poisson's ratio were negligible order. Individual values of displacement and strains decreased very slightly as Poisson's ratio was increased. These results suggest that Poisson's ratio is not a significant variable for subsidence model analysis (Figure 5.2).

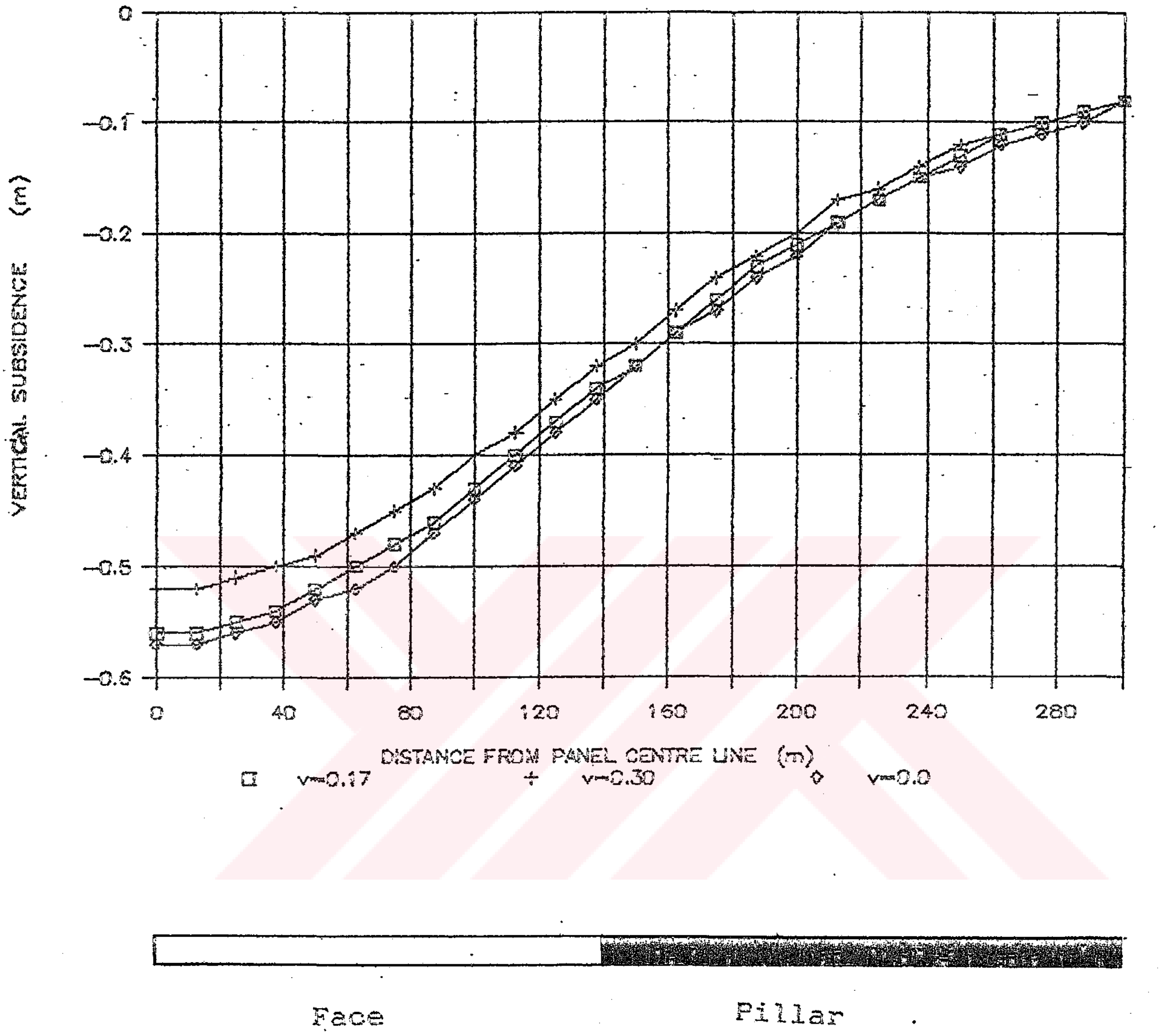


Figure 5.2 Effect of Poisson's ratio on subsidence

ii. Values of Young's modulus were varied within the range 2000 to 8000 MPa. Poisson's ratio was kept constant at 0.17 with the critical ratio of panel geometry.

Absolute values of vertical subsidence, however, significantly decreased with increased modulus of elasticity (Figure 5.3). This was only valid for the stress-free boundary conditions of the extracted seam surface (Figure 5.4); for the application of displacement boundary conditions of the extracted seam surface (Figure 5.6), the effects of changing Young's modulus on subsidence phenomena was in negligible order.

5.3. Extracted Seam and Ground Surface Interaction

With the experience obtained through operating the program values of width to depth ratio, the influence of the ground surface to the extracted seam was comparatively small for ratio greater than one (w/h > 1). Maximum subsidence values of .39m for w/h =0.75 and 0.55 m for w/h = 1.4 for stress-free boundary condition (will be discussed later) can be given as examples. This is in concurrence with Saint Venant's principle (Timoshenko, 1970).

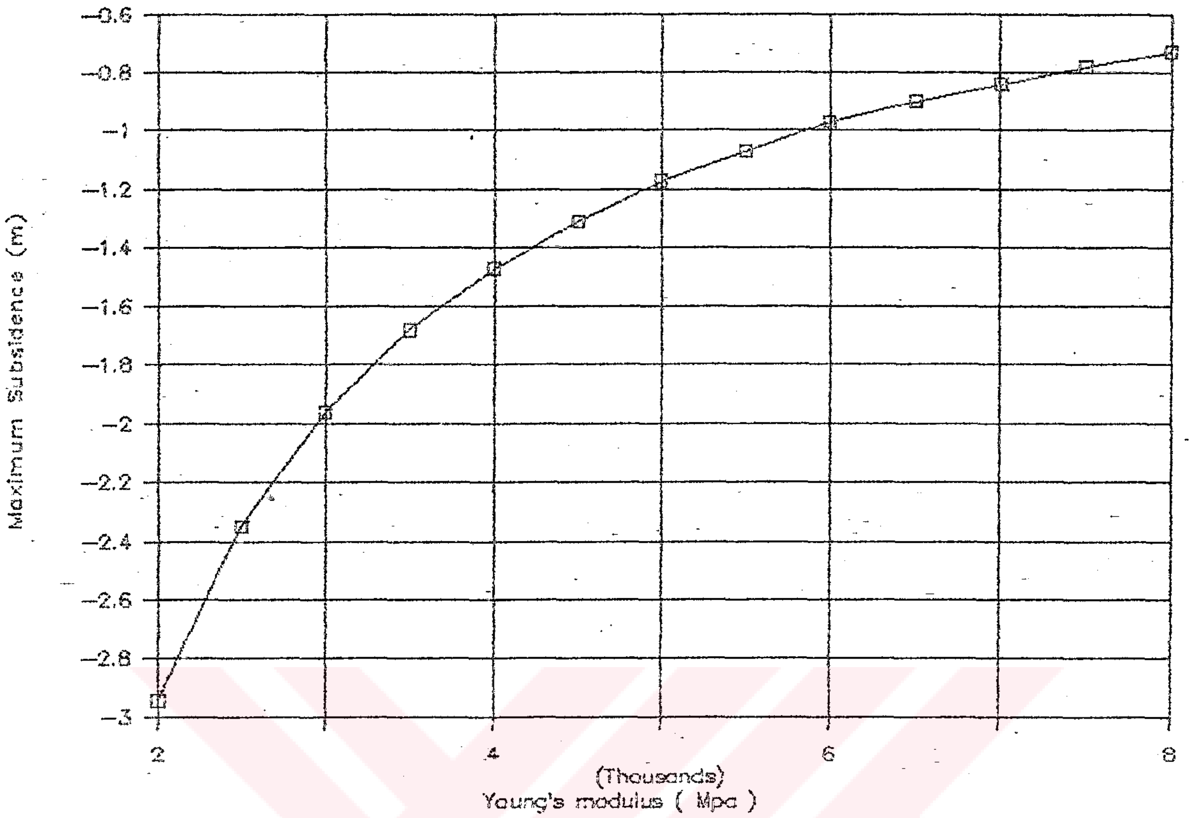


Figure 5.3 Effect of Young's modulus on maximum subsidence.

5.4. Basis for Comparative Analysis

Subsidence and strain profiles predicted by model analysis ought to be compared with actual profiles measured in the field. Therefore the extensive data collected by the British Coal (NCB) of the United Kingdom were employed as the basis for this comparison. NCB workers have utilized these data in order to develop empirical relationships for the prediction of ground movement due to mining. The NCB approach was based solely on geometric parameters, without specific reference to rock mass material properties. For any given width/depth ratio and the seam thickness, subsidence and strain profiles can be determined with the aid of graphs published in the NCB Subsidence Engineers Handbook (1966, 1975).

Besides the British Coal (NCB) empirical data, graphs of the results obtained from the numerical subsidence prediction methods those have used the finite element, boundary element and closed form solution (Aston, et al., 1986) were also used to make a comparison between the modified TWODD subsidence profiles and the others. As a remark, except one or two finite element models no strain profiles were available in the reviewed literature to make a comparison.

5.5. Analysis of Case-Study for Different Boundary Conditions

In the section, the program was experienced for the two major boundary conditions of the extracted panel width:

1. Stress-free (σ_s=0, σ_n=0) boundary condition
2. Displacement boundary conditions.

These boundary values were used for the different width to depth ratio one by one in more than one computer run. The boundary conditions for half of the used model were imposed due to symmetry about y-axis as mentioned in section 4.7.3 taking the advantage of reduced number of boundary elements , hence less running process time and economical output. The discussion of deducted results from the output of these running process as well as described boundary conditions were given in the following sections.

5.5.1. Analysis of Stress-free Boundary Condition at the Extracted Seam Surfaces

The stress-free boundary condition (at the instant of the extracted coal seam without any support application, the shear and normal stress of the seam walls were zero (Figure 5.4)) was applied to the program in two steps:

1. Application of stress-free boundary condition and using the displacements of the extracted seam surface

boundary of the first running for the input data of the second running.

2. The reduced Young's modulus (e.g. E=3450 for w/h=1.4) application for the stress-free boundary condition of the extracted seam surfaces.

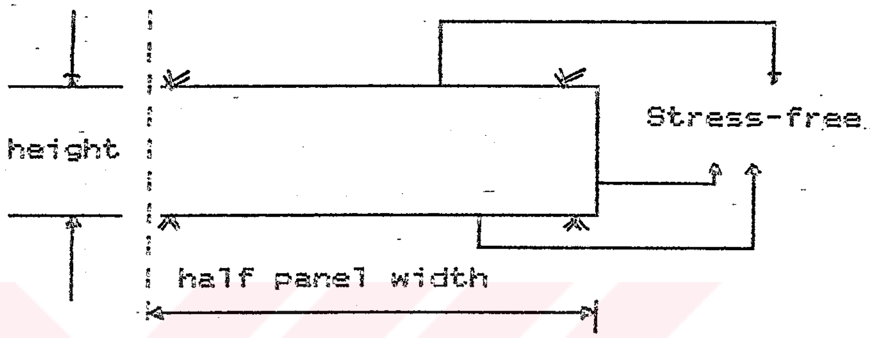


Figure 5.4 Stress-free boundary condition.

1. Iteration of Displacements of the Boundary of the Extracted Seam Surface

When the coal extracted the induced stresses and displacements at the surface of the immediate roof and floor of the face play an important role from the elastic analysis point of view. Thus an investigation about them might be useful to get an idea about the convergence potential of roof and floor which would be used for the input data for iterative running process.

The program was executed for the critical panel width and associated ground subsidence were drawn with the field (NCB) data (Figure 5.5). The displacements at the boundary deducted from this execution were used as a

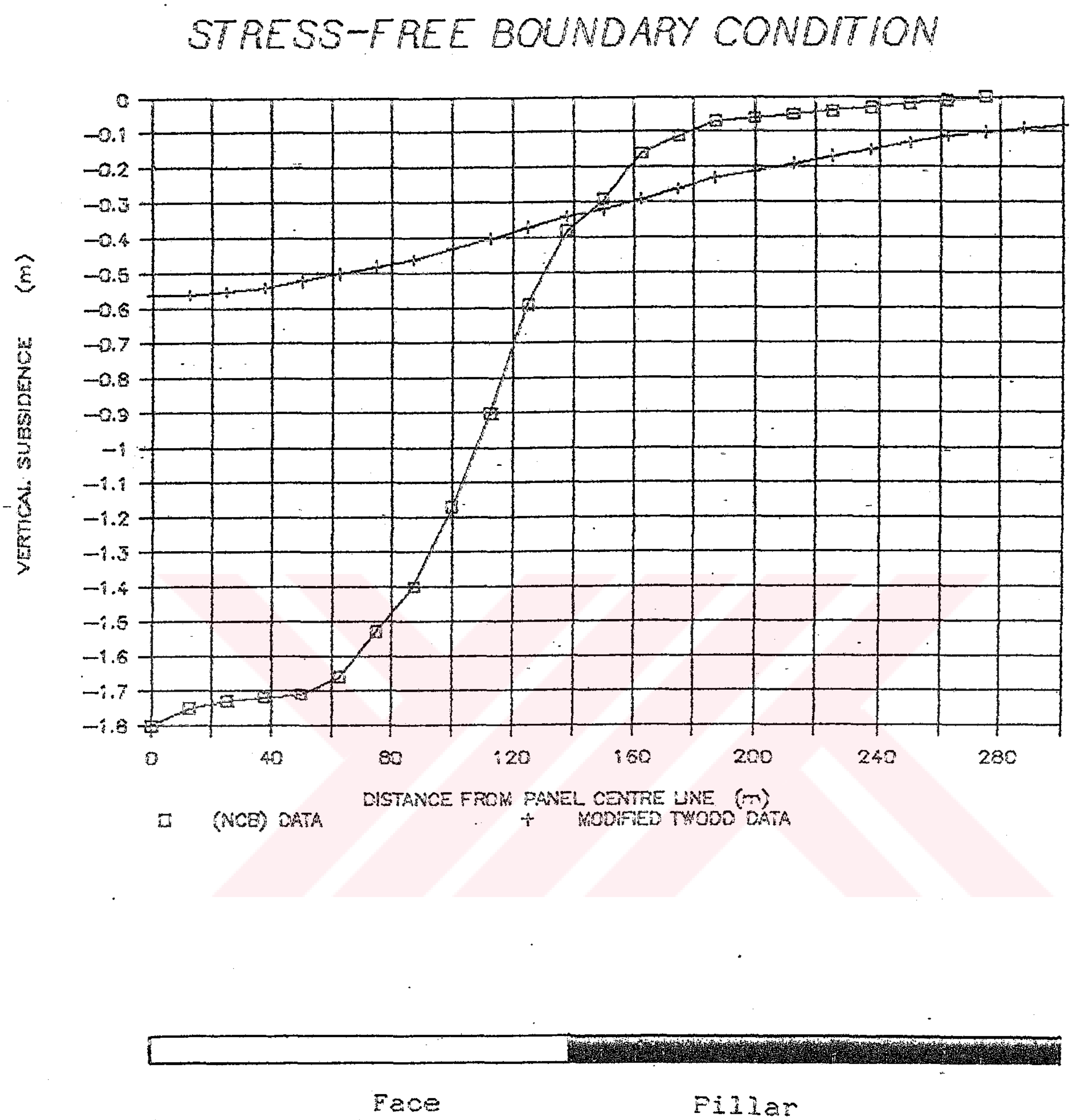


Figure 5.5 Subsidence profiles for w/h = 1.4 according to stress-free boundary condition.

displacement boundary condition of the input data for the iteration purpose. The resulted boundary condition was idealized in figure 5.6.

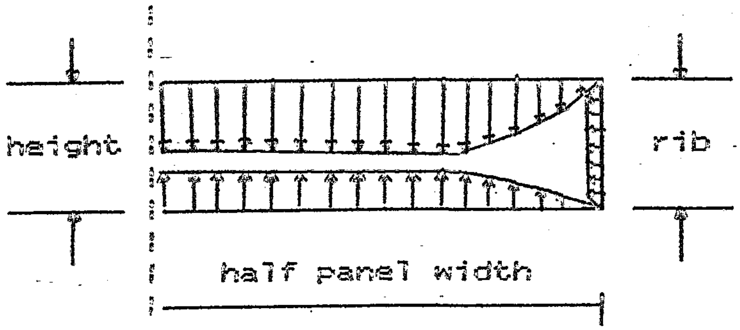


Figure 5.6 Displacement of the stress-free boundary condition idealization.

One might expected the ground subsidence would be similar to the empirical one, but it didn't even reach the maximum subsidence. This concluded results suggested that overlying strata didn't linearly affect by the displacement of the stress-free boundary condition solely (Appendix C).

2. Stress-free Boundary Condition Analysis by Reducing Young's Modulus

The resulted maximum subsidence from the previous section (output of second run) was almost one third of the empirical one. Due to this the reduction of Young's modulus within the range of one third to one fourth would have to give the empirical maximum subsidence value. This was experienced for the panel width ratios of 1.4 and 0.75. After several running processes, the desired maximum subsidence values were obtained. But the overlapping of the floor and roof at the boundary was noticed from the analysis of the resulted boundary displacement values. This is due to reduction of Young's modulus of elasticity that causes the boundary movement freely as there is no restriction to prevent it in anyway. Although this is physically impossible, but there is no objection mathematically (Crouch and Starfield, 1981). One of the ways to solve this problem would be to apply fixed displacement boundary condition that will be analyzed later.

The subsidence and strain profiles were drawn for the panel width to depth 1.4 and 0.75 in Figure 5.7, 5.8 and in Figure 5.9, 5.10 respectively. The correspondence between the model and the field profiles for both vertical subsidence and strain data was poor except the maximum values. Significant vertical subsidence for the model occurred over a wider area than has been demonstrated by field study. Furthermore, this subsidence curve was much flatter than the field curve. The location of the "half subsidence" or transition point occurred approximately over the rib for the model; this is at variance with field experience as it is over the face except the profiles of the panel width ratio less than one.

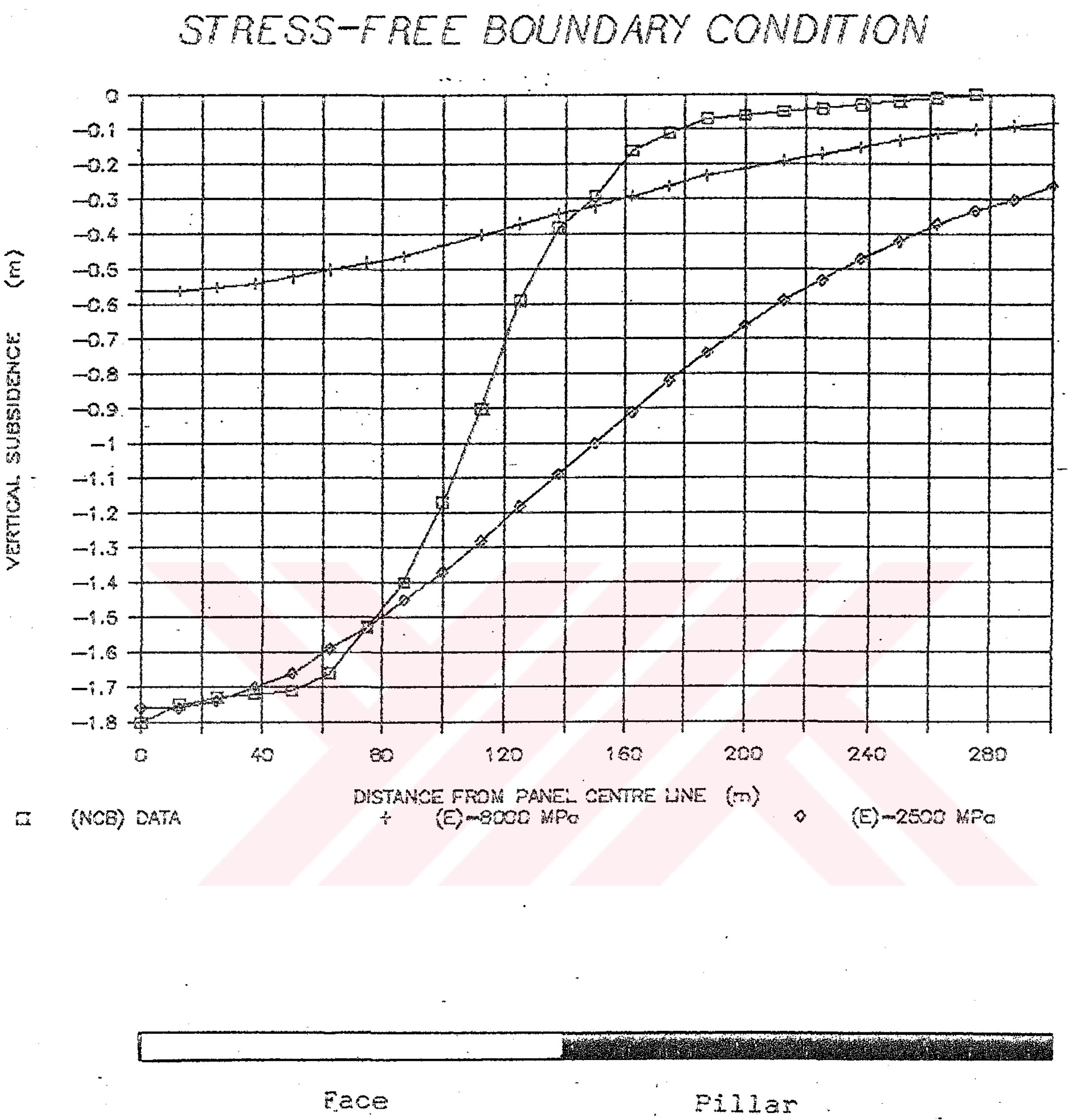


Figure 5.7 Subsidence profiles for w/h = 1.4 according to stress-free boundary condition with reduced Young's modulus.

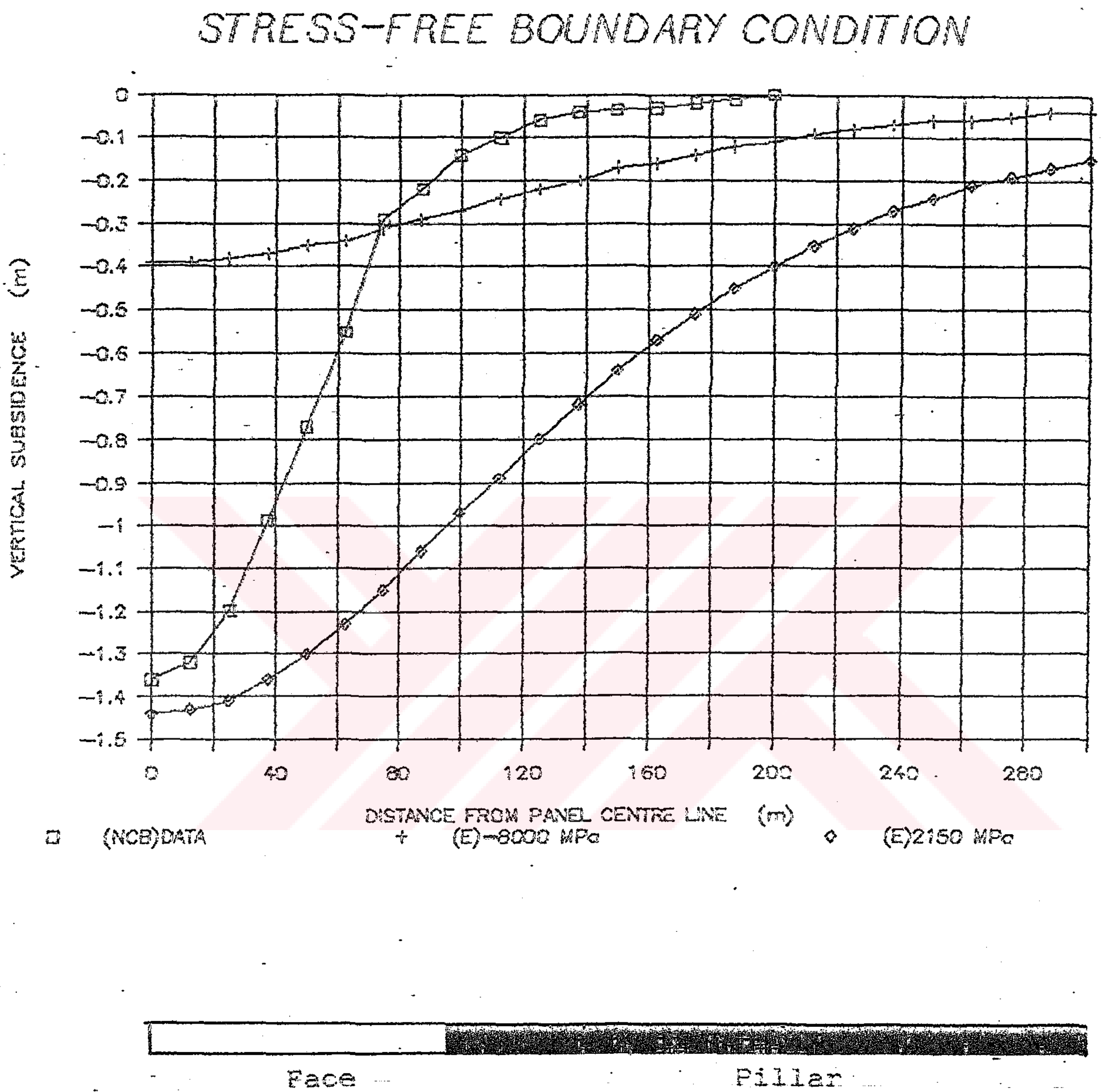


Figure 5.8 Subsidence profiles for w/h = 0.75 according to stress-free boundary condition with reduced Young's modulus.

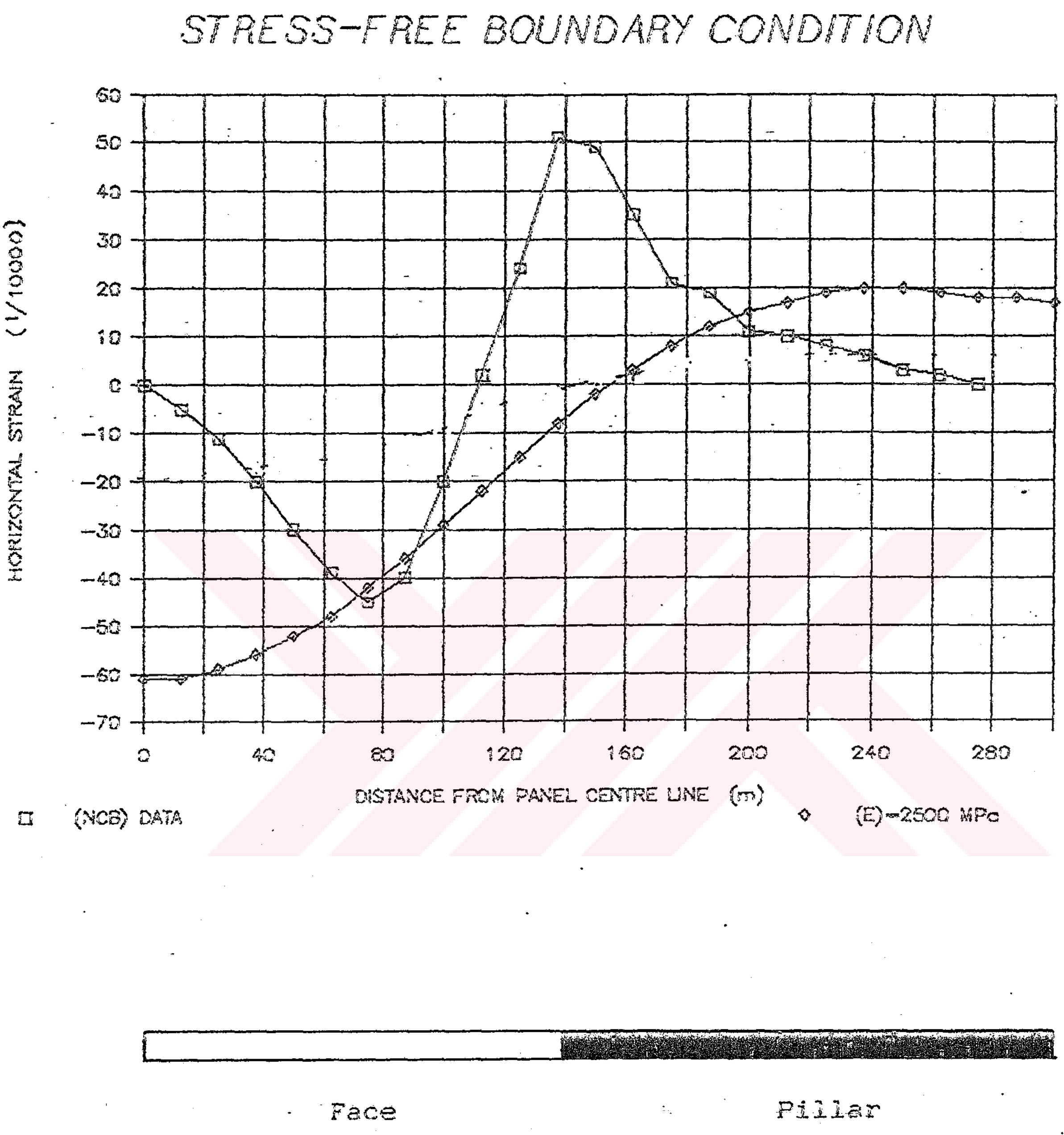


Figure 5.9 Strain profiles for w/h = 1.4 according to stress-free boundary condition with reduced Young's modulus.

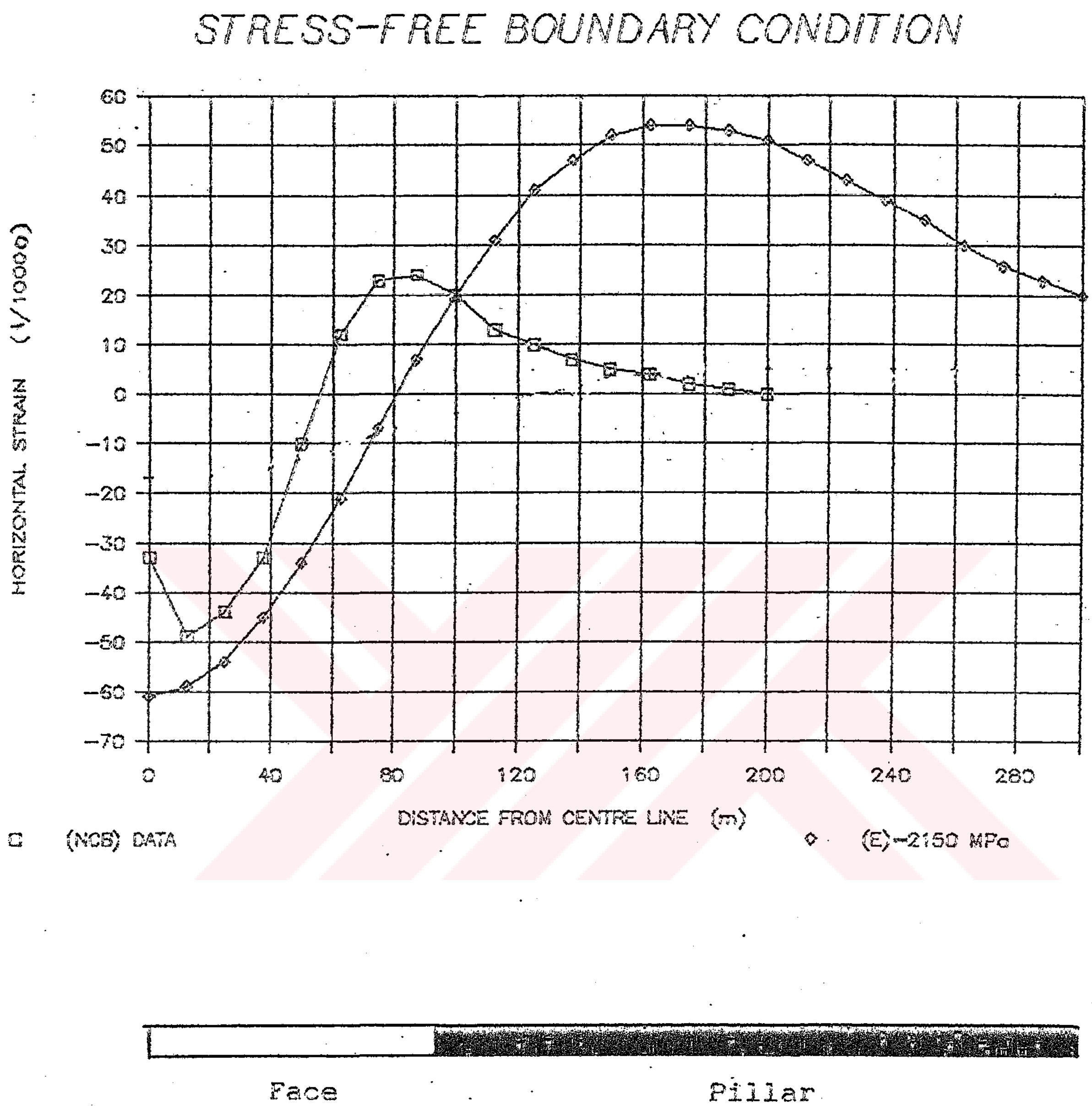


Figure 5.10 Strain profiles for w/h = 0.75 according to stress-free boundary condition with reduced Young's modulus.

5.5.2. Displacement Boundary Condition Analysis

It is difficult to get all the requisite information about many subsidence events especially at the seam boundary if they occurred in abandoned mines. Some qualitative conclusions rather than quantitative obtained from the analysis in section 5.4.1. These were used as a guide for the displacement boundary condition. The floor movement was ignored which was in concurrence with the other numerical methods using finite element model (Agloutantis and Karmis, 1988).

1. Displacement Boundary Condition Without Specifying Horizontal Displacement

As already mentioned, fixed vertical displacements were only assigned to the elements comprising roof of the face. Floor movement was assumed to be negligible in vertical direction for the case study under consideration. The initial horizontal displacements were not given to the boundary elements of the roof and floor. The stress-free boundary condition was applied to the ribside. The roof convergence was assumed to decrease piecewise linearly from element to element near the ribside of the excavation (Figure 5.11).

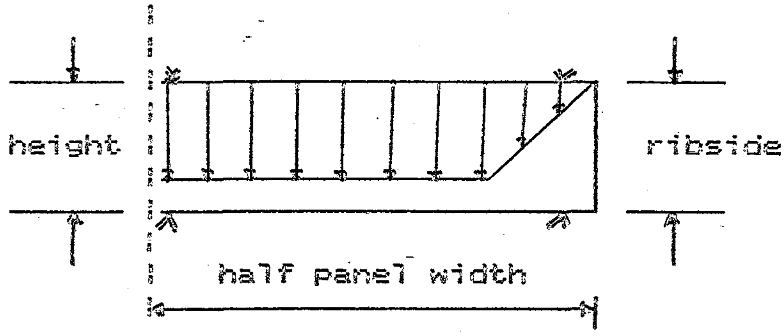


Figure 5.11 The roof convergence idealization.

The roof convergence was set to (1.40-1.95 m) for all elements except those close to the rib of the excavation. In such a case, roof was assumed to move less due to the supported provided by the intact rib, so convergence was reduced before the rib in order to achieve a smoother transition between simulated roof failure and roof bending.

For the three different width to depth ratio (w/h = 1.4 , w/h = 0.75 and w/h =0.5) the program was executed by applying several displacement values at the boundary to get the maximum subsidence for each of them (Appendix D).

The drawn subsidence with the field profiles were examined, and the below points were deducted (Figure 5.12, 5.13, 5.14):

(i) The significant vertical subsidence occurred over a wider area than field (NCB) curve. But when compared to the profile drawn from the reduced Young's modulus of free boundary condition it is better (Figure 5.7, 5.8).

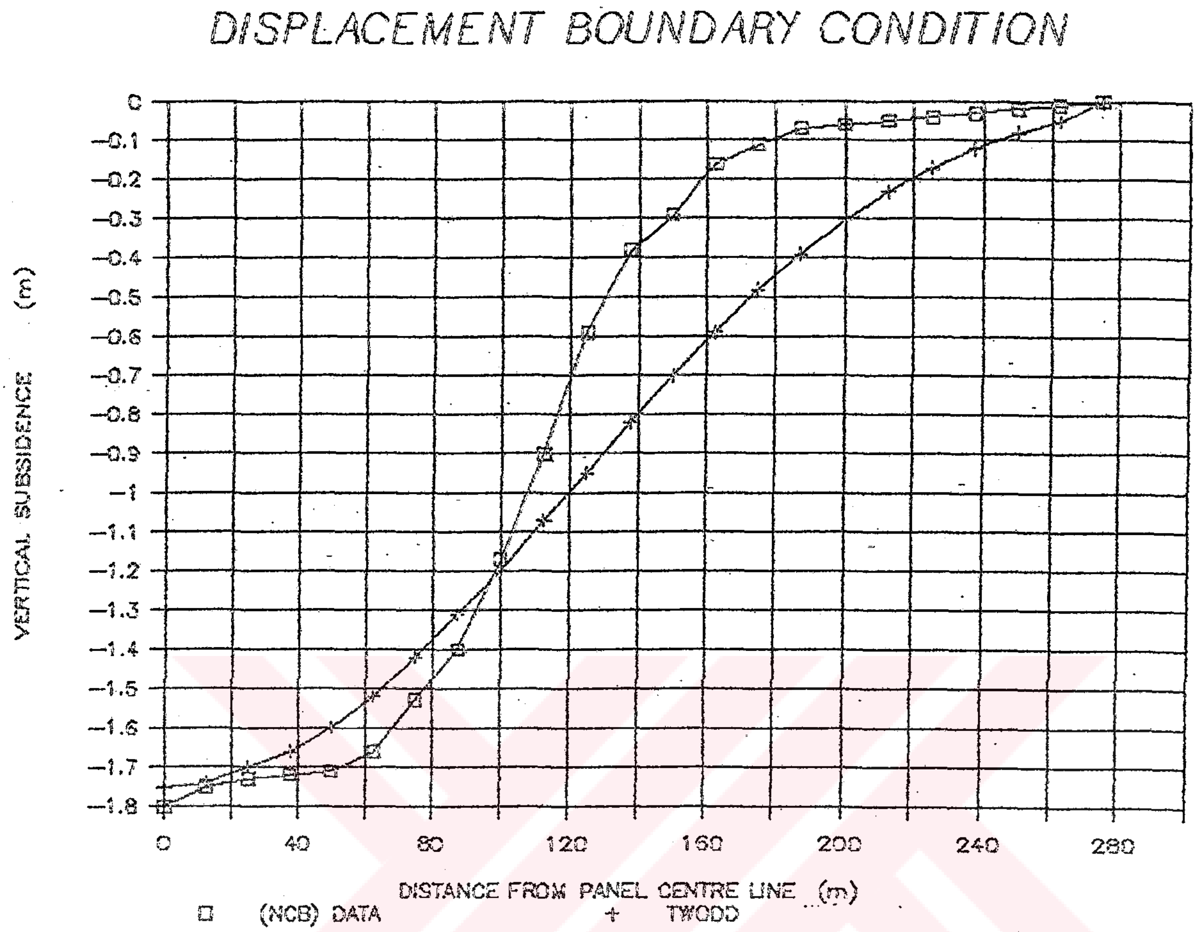
(ii) This curve is flatter than the field one.

(iii) The radius of influence area is greater than the empirical one except panel width to depth ratio of 1.4.

The drawn strain profiles (Figure 5.15, 5.16, 5.17) corresponding to those subsidence do not resemble to the field profile especially near the center-line of the panel width.

2. Displacement Boundary Condition Assigning Horizontal Displacement

In this section the same procedure of the previous section was applied to the simulation of the roof convergance. Floor's and rib's elements were fixed in order not to move in any directions (rigid boundary assumption). The modeled roof elements those near the ribside of the excavation were not assigned in any horizontal movement and the other roof elements up to the panel center-line were assigned to move only in vertical direction; zero in horizontal direction.



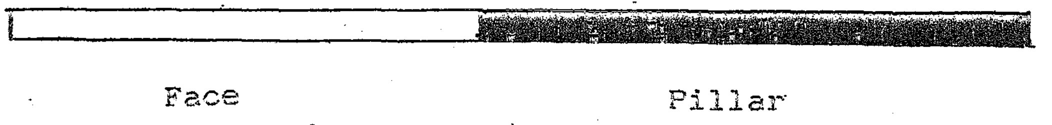


Figure 5.12 Subsidence profiles for w/h = 1.4 according to displacement boundary condition without assigning horizontal displacement.

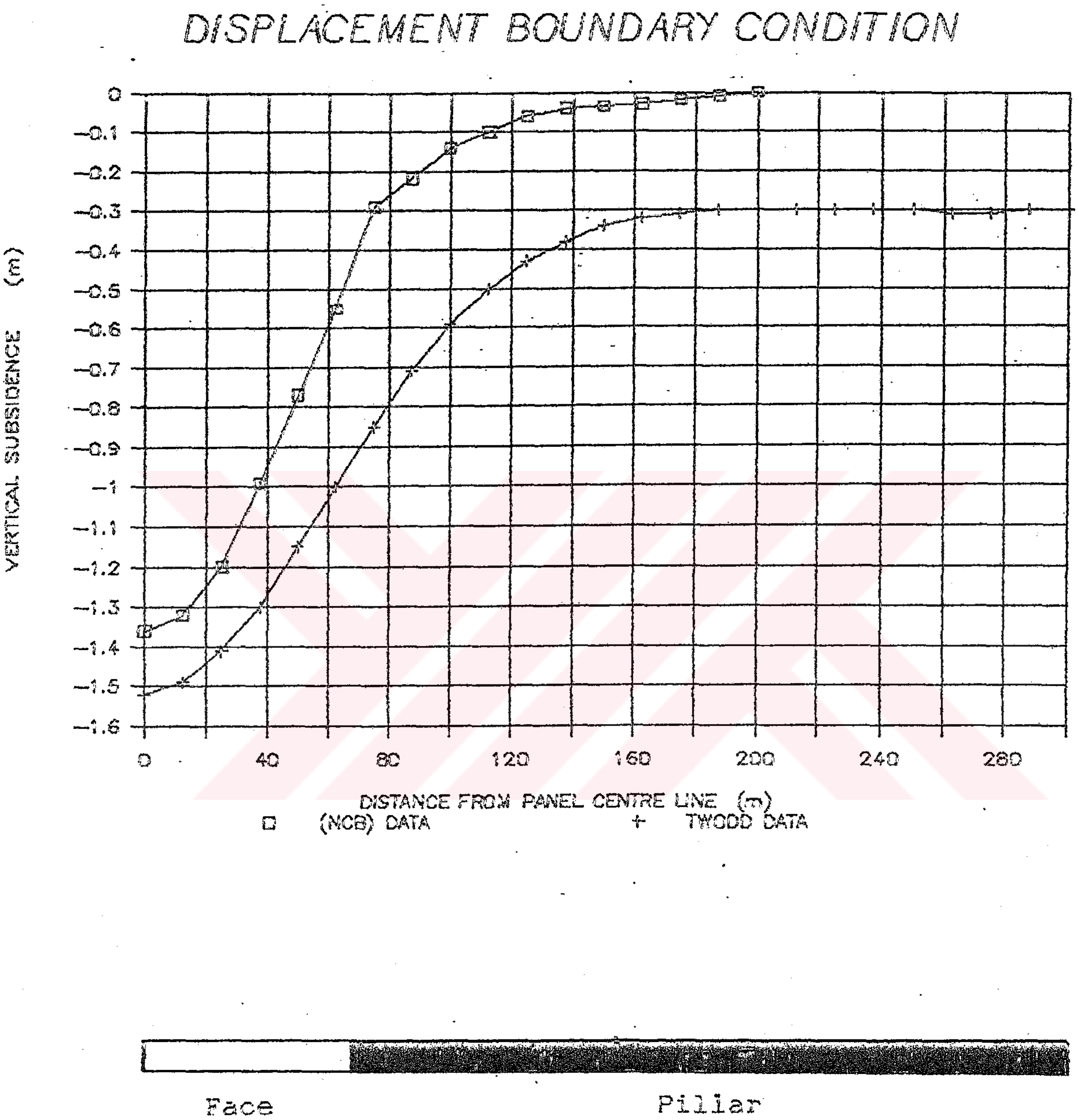


Figure 5.13 Subsidence profiles for w/h = 0.75 according to displacement boundary condition without assigning horizontal displacement.

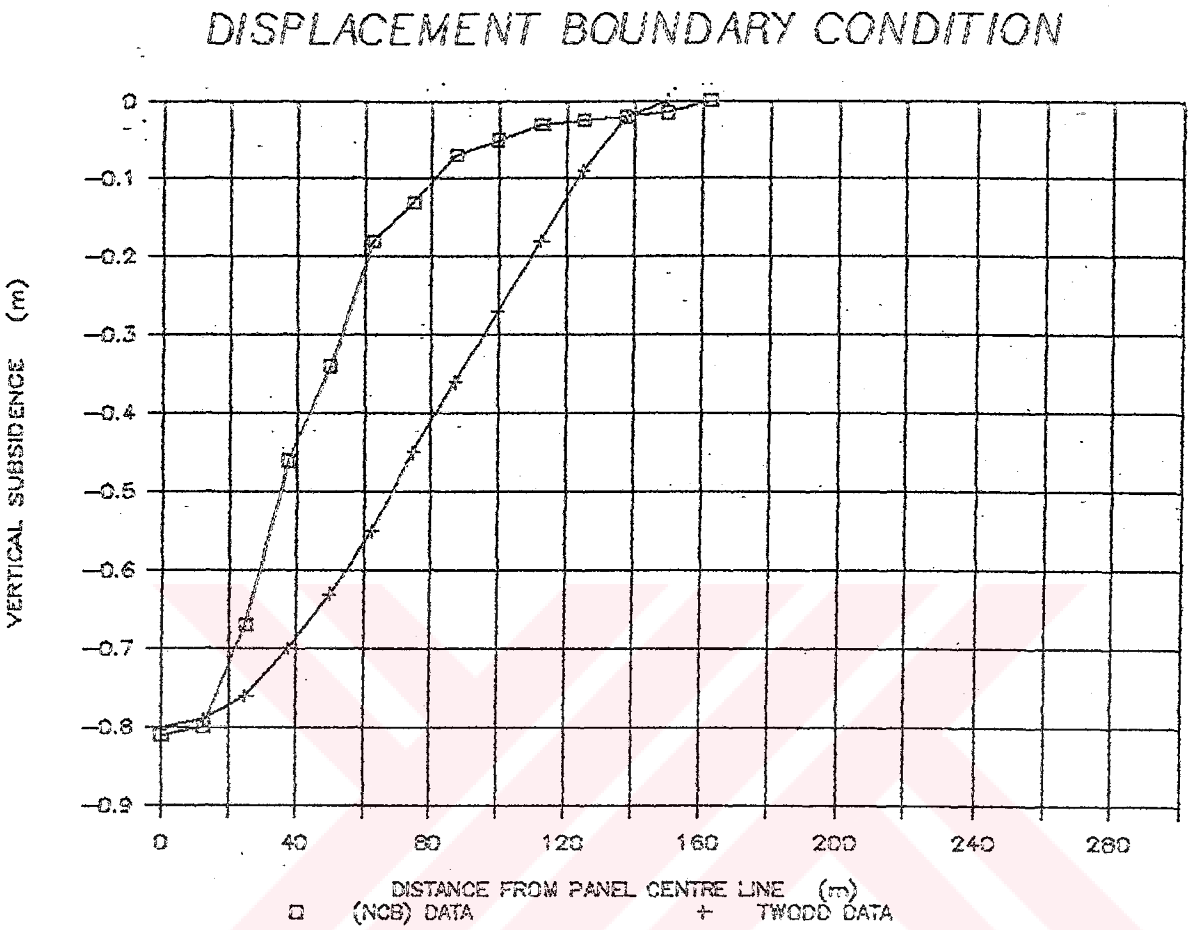


Figure 5.16 Subsidence profiles for w/h = 0.53 according to displacement boundary condition without assigning horizontal displacement.

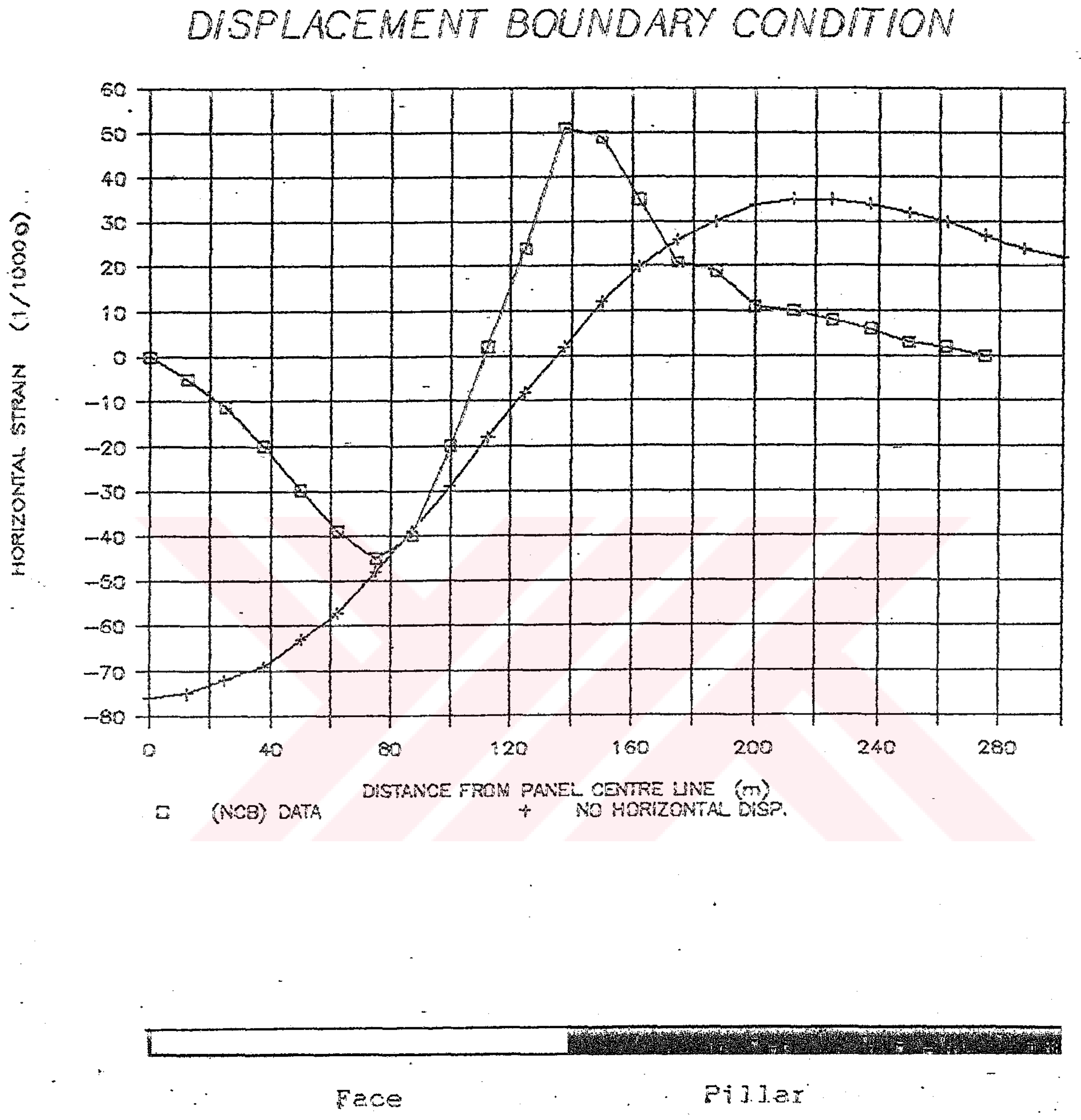


Figure 3.15 Strain profiles for w/h = 1.4 according to displacement boundary condition without assigning horizontal displacement.

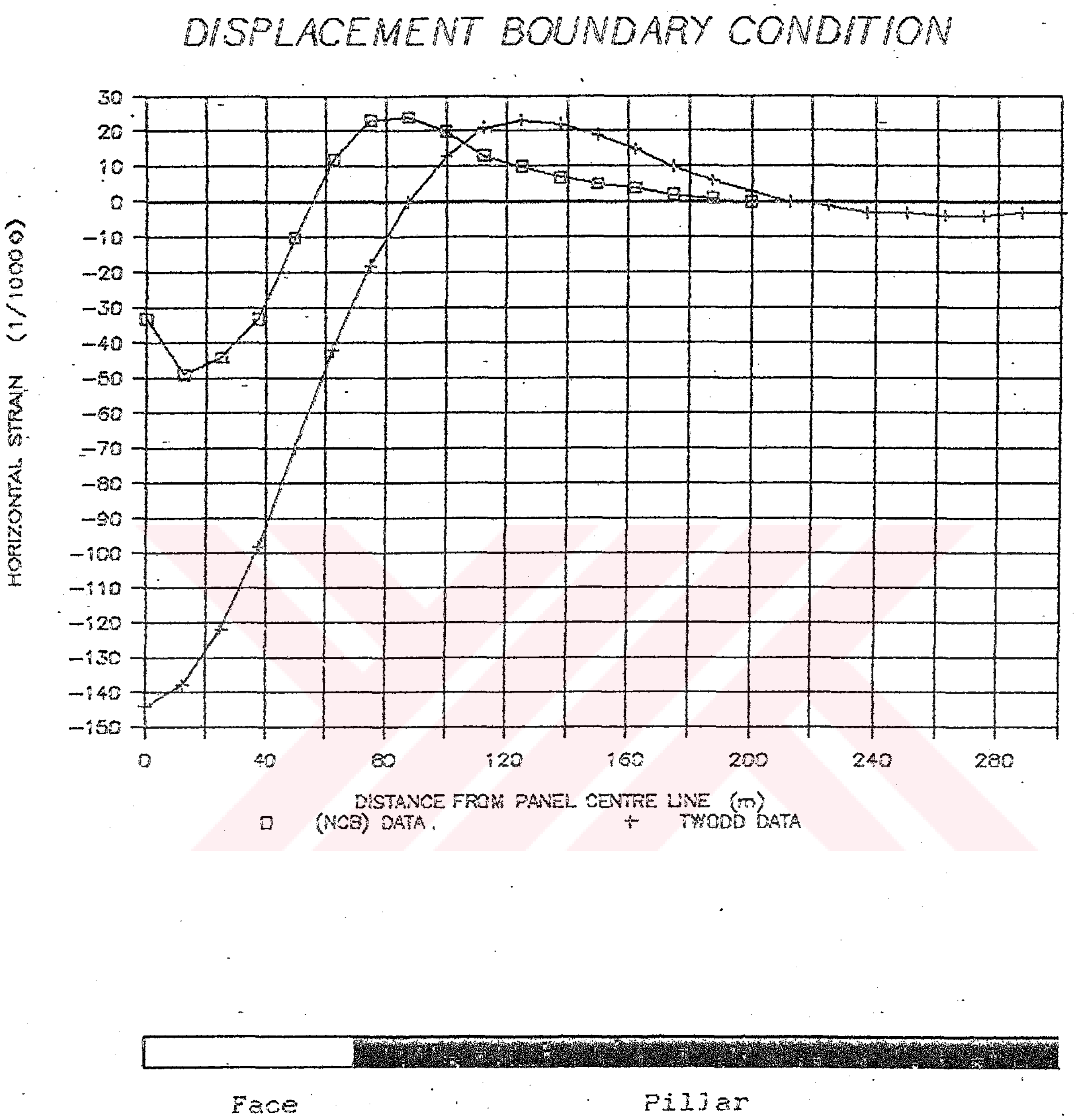


Figure 5.16 Strain profiles for w/h = 0.75 according to displacement boundary condition without assigning horizontal displacement.

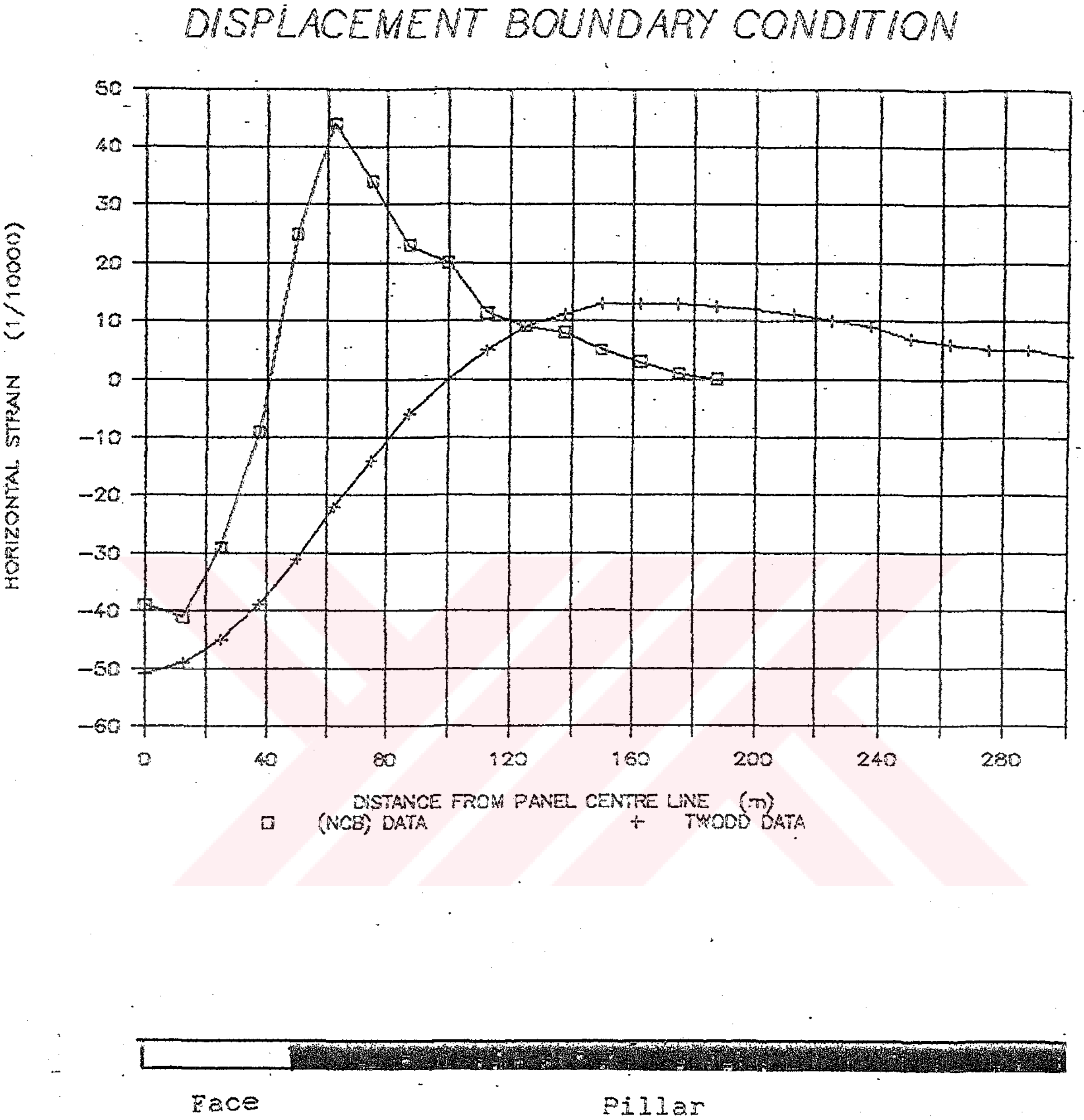


Figure 5.17 Strain profiles for w/h = 0.50 according to displacement boundary condition without assigning horizontal displacement.

From the three panel width to depth ratio running processes, the subsidence profiles with the empirical were drawn. The below points were revealed after analyzing these curves (Figure 5.18, 5.19, 5.20).

(i) The model curves' slope much closer to the field one.

(ii) The curve over the face part is parabolic and over the ribside matches the linear curve fitting.

(iii) Radius of the influence area almost same with the empirical one.

(iv) The strain values appeared to end abruptly close to the end of influence zone. This is attributable to the fact that the elements out of the influence zone are ignored to be taken into consideration.

The drawn strain profile (Figure 5.21, 5.22, 5.23) do not match the NCB profiles as like in the previous part. But for the panel width to depth ratio of 0.5 it fits the maximum compression strain almost the same point.

DISPLACEMENT BOUNDARY CONDITION

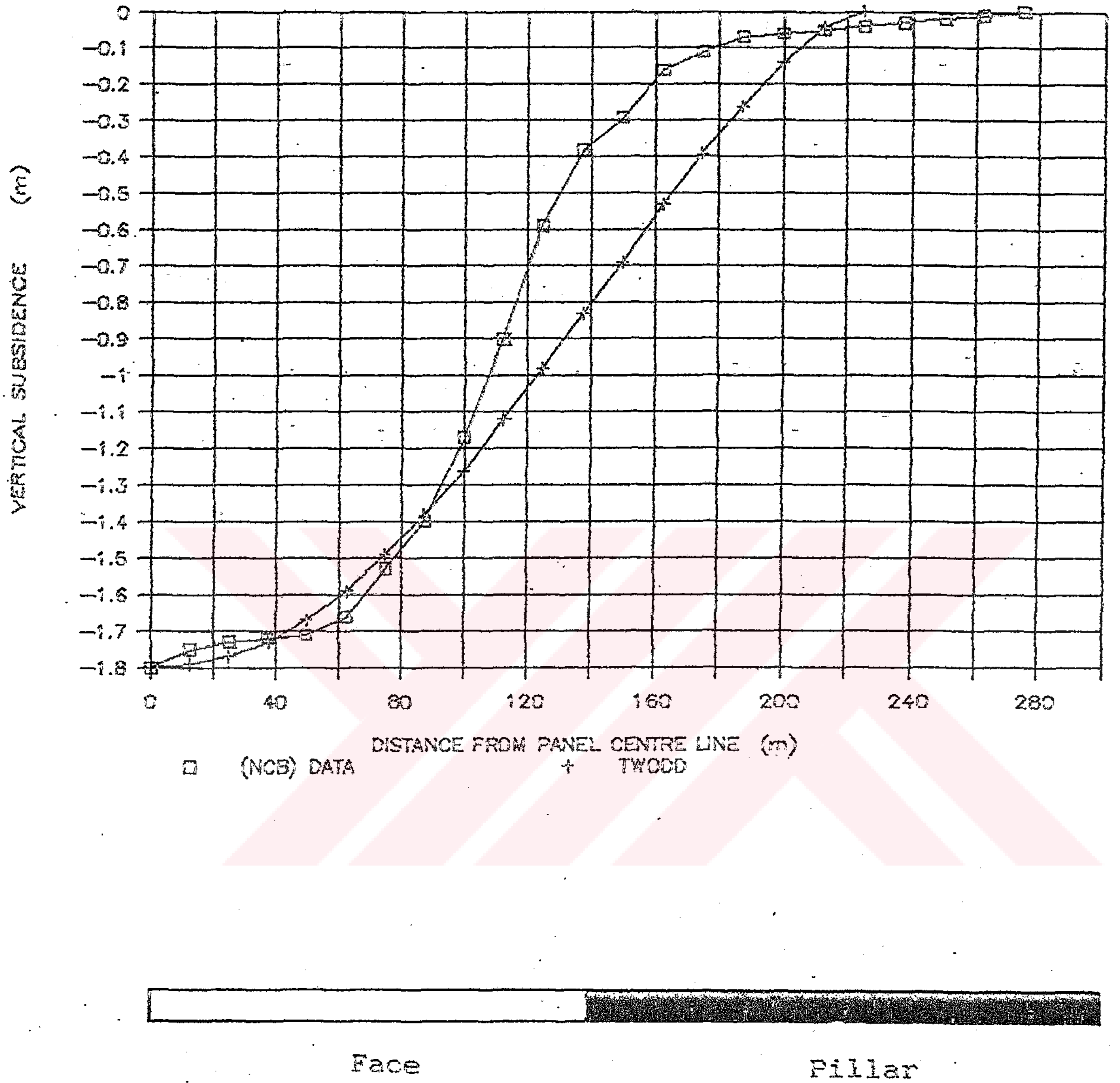
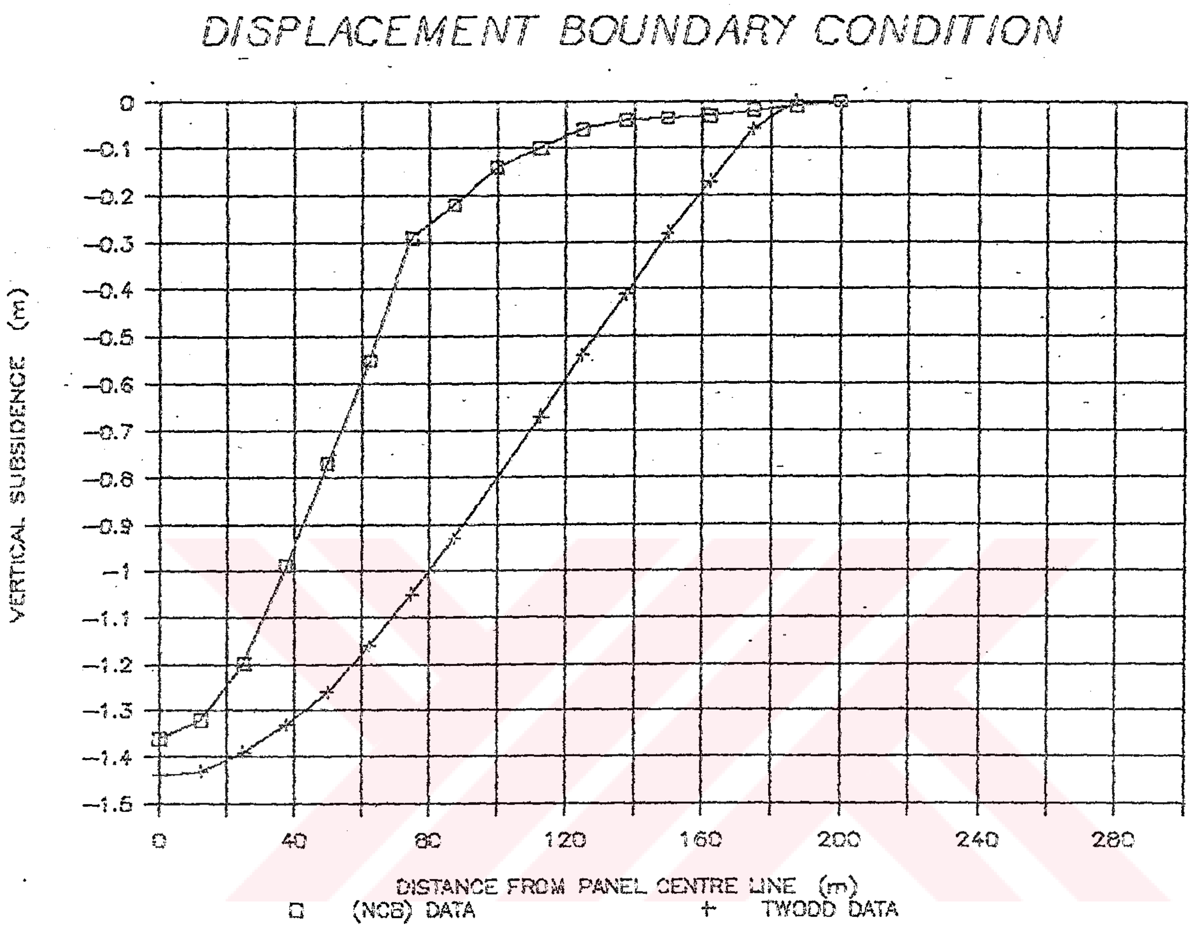


Figure 5.18 Subsidence profiles for w/h =1.4 according to displacement boundary condition with assigning zero horizontal displacement.



Face Pillar

Figure 5.19 Subsidence profiles for w/h = 0.75 according to displacement boundary condition with assigning zero horizontal displacement.

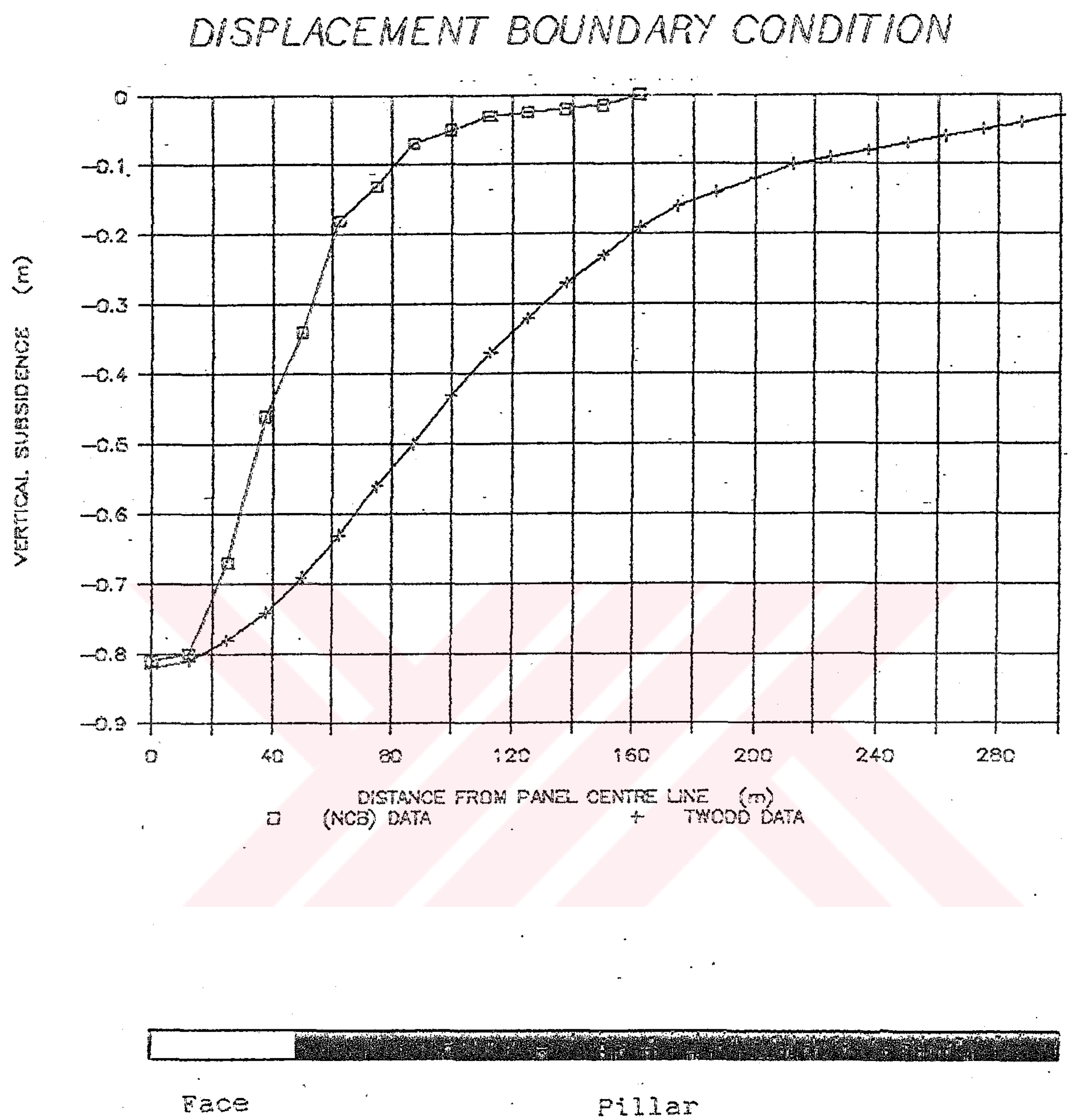


Figure 5.20 Subsidence profiles for w/h =0.50 according to displacement boundary condition with assigning zero horizontal displacement.

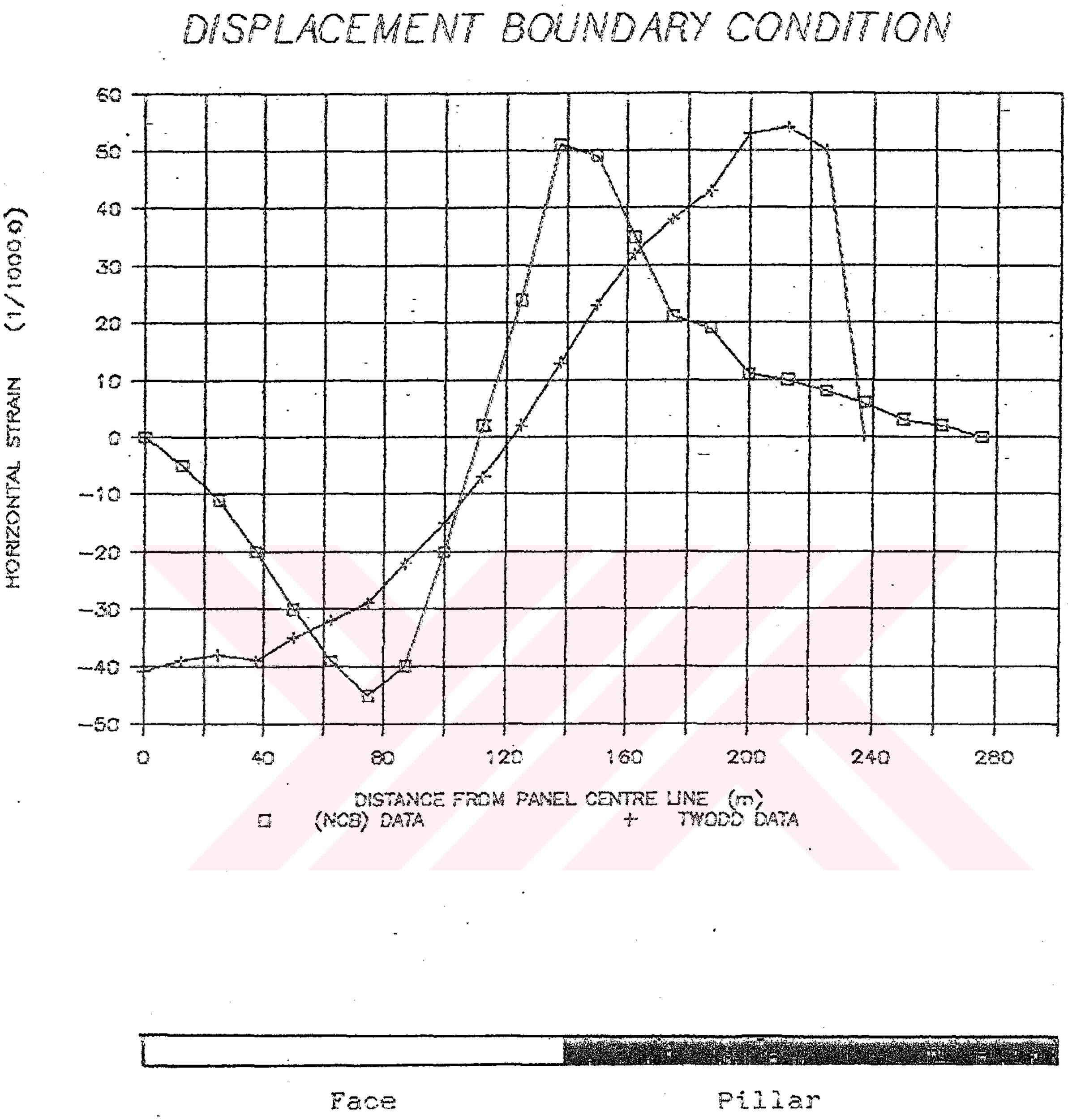
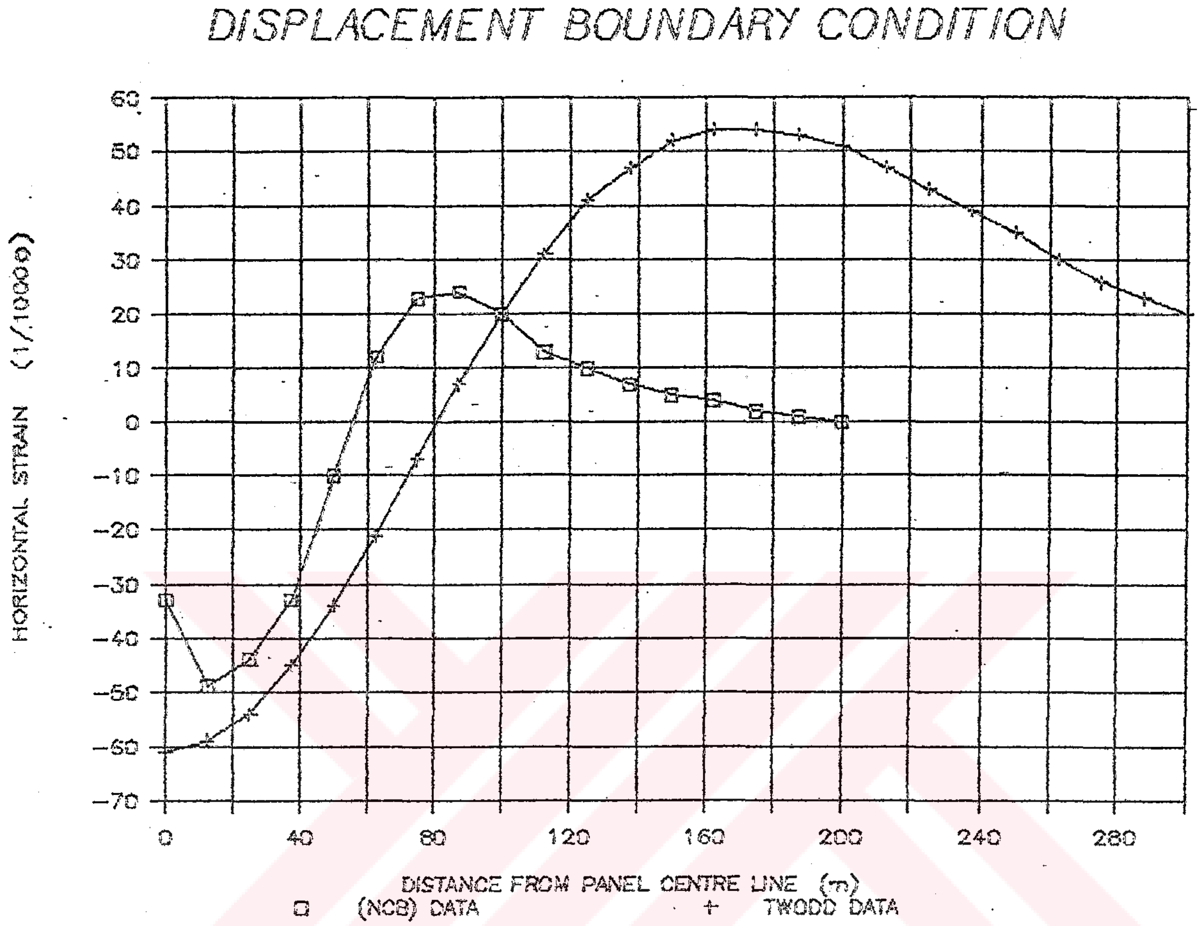


Figure 5.21 Strain profiles for w/h = 1.4 according to displacement boundary condition with assigning zero horizontal displacement.



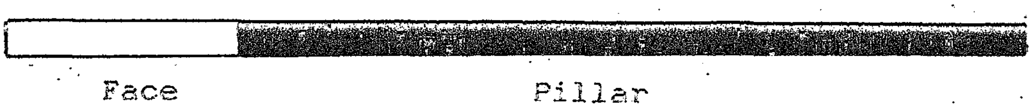


Figure 5.22 Strain profiles for w/h =0.75 according to displacement boundary condition with assigning zero horizontal displacement.

DISPLACEMENT BOUNDARY CONDITION

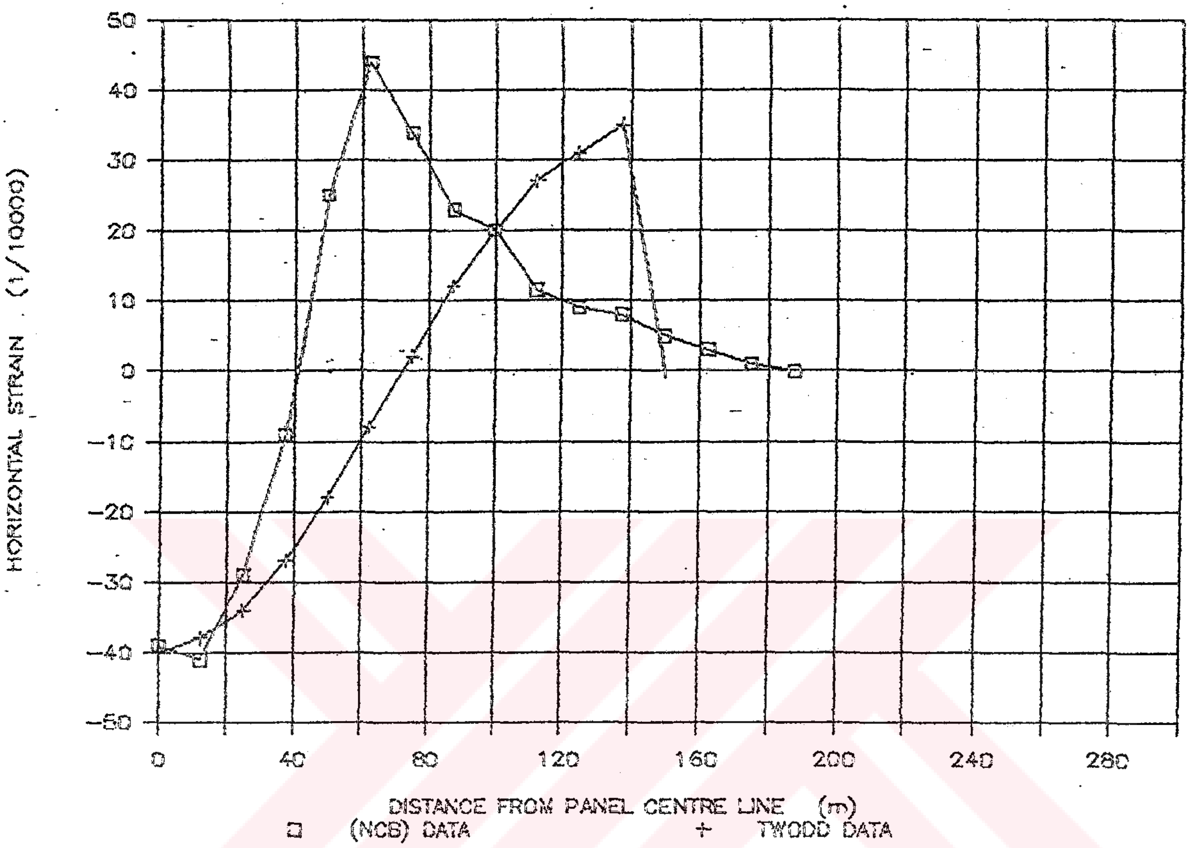


Figure 5.23 Strain profiles for w/h = 0.50 accordig to displacement boundary condition with assigning zero horizontal displacement.

5.6. Discussion

The introduced modified TWODD boundary element model capability was experienced on the three types of panel width to depth ratio of subsidence idealization. Due to structure of the program, the boundary values and material properties influenced the ground subsidence results very effectively. The results from the several computer running processes for the different boundary conditions and material properties were analysed and were discussed one by one:

1.First of all, the effects of elastic constants analysis were revealed that the changing of Poisson's ratio value was not a significant variable on mining subsidence (Figure 5.2). But for the same conditions of different Young's modulus introduced changing subsidence value (Figure 5.3). These results are in concurrence with the analytical approaches worked on subsidence phenomena (Poon and Tammemagi, 1985).

2.Stress-free boundary condition (at the instant of worked out panel width) displacement at the boundary contours were used as a displacement boundary condition for the purpose of obtaining information about the effects of free boundary on the vertical subsidence and the strain. The resulted maximum value was only thirty percent of the empirical one. Due to this, it was understood that, the subsidence mechanism was not solely bending of roof and heaving of floor, it was inherently related to the overburden modulus of elasticity. When the Young's modulus was reduced with in the range of one third to one fourth according to used panel geometry the correspondent maximum subsidence value was found, but with the overlapping of the roof and floor condition.

3. The partial closure of roof with neglecting of floor heaving was better than reduced Young's modulus analysis of stress-free boundary condition result as it prevented the overlapping of roof and floor (Figure 5.24).

4. Both of the analysis subsidence, especially strain profiles did not fit the empirical ones. The value and point of the maximum subsidence and strain generally match the NCB field values. These results indicated that only given the boundary conditions to the extracted boundary were not sufficient to simulate the strata behavior overall. Even the vertical displacement given at the roof without any horizontal displacement caused a maximum compression at the surface over the panel center that had to be zero according to the empirical data.

5. During the literature survey it was stated that the field modulus can be as little as 10 % (ten percent) of the laboratory determined value. Additional analysis using a finite element program revealed that even a ten-fold decrease in modulus was insufficient to explain the

difference between the analytical and empirical results. Young's modulus would have to be reduced between 30 and 50 times in order to make this conventional finite element computer program (SUBSID-B, GEOROC, ADINA) and NCB profiles comparable (Aston, et al. , 1987).In that discussion a boundary element (MSEAMS) was also used and the same result was found. For the comparison purpose the same case history was experienced by modified TWODD program and it was revealed that even one third of reduced Young's modulus was giving a comparable results with NCB profiles due to additional parts of gravitational loading semi-infinite plane idealization for numerical analysis approach .

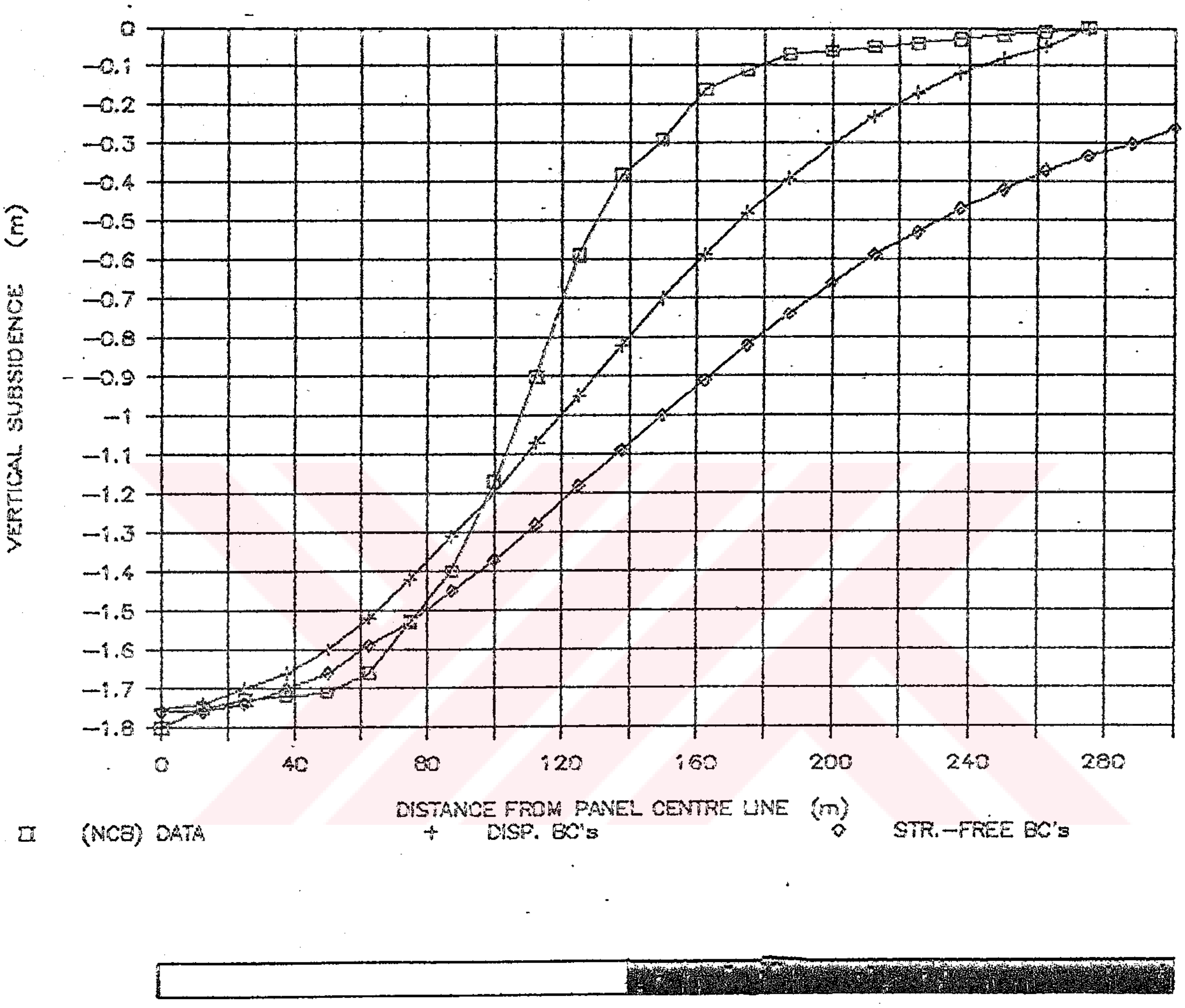


Figure 5.24 Comparison between stress-free boundary condition with reduced Young's modulus and displacement boundary condition.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

(i) The obtained maximum subsidence and strain values from the modified two dimensional displacement discontinuity boundary element numerical method were in close agreement with the field (NCB) results. The maximum subsidence values of the case history were better than the other conventional numerical subsidence prediction methods available in literature.

(ii) The subsidence profiles obtained from the model didn't exactly fit the field values as far as the area of influence and transition point were concerned. The discrepancies of the profiles with the field data may be due to isotropic, linear elastic idealization of the model. It was realized that subsidence phenomena were inherently associated with non-elastic and non-linear material behavior involving rock material disintegration, bed separation, block translation and rotation... etc.

(iii) The better results were observed from the critical (w/h = 1.4) ratio than the other sub-critical ratios especially less than one. Because the seam and ground surface interaction is in the range of effect limit.

(iv) The subsidence for the displacement boundary condition without specified horizontal displacement were more closer to the empirical profiles than the other experienced boundary conditions such as stress-free, displacement boundary conditions with specified horizontal displacement.

(v) The correspondence between the model and field (NCB) strain values were very poor. Although the discrepancy was high, to obtain strain values was an advantage over some numerical subsidence prediction technique reviewed in literature as they were lack in supplying these data .

The following points for the further research to supplement this study was recommended:

(1) It is supposed that the subsidence model can be improved by introducing non-linear and transversely isotropic non-homogeneous medium as the subsidence phenomena was realized to be inherently associated with these behaviors.

(2) Simulating the boundary conditions of the subsidence model more precisely and in a continuous manner by making the element size so small to be a point like as nodule points in finite element modeling can give better results for subsidence prediction.

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APPENDICES

APPENDIX A. ANALYTIC SOLUTION OF DISPLACEMENT DISCONTINUITY IN A HALF PLANE

This appendix includes the analytic solution of the problem for the displacement discontinuity in a half plane that is directly taken from the Boundary Element Methods in Solid Mechanics book (Crouch and Starfield, 1981).

Displacement Discontinuity in a Half Plane

The problem of a constant displacement discontinuity over the line segment $|x| \leq a$ $\bar{y} = 0$ in the semi-infinite region $y \leq 0$ is depicted in Figure A.1. The origin of the local $\bar{x}, \bar{y}$ co-ordinate system is at the point $x = Cx$, $y = Cy$ (where $Cy < 0$). The displacement discontinuity components are $\bar{Dx} > 0$ and $\bar{Dy} > 0$. The analytical solution to this problem can be obtained by using a procedure known as the method of images. The details of the derivation of this method, but a brief description of the method of images will help the reader to understand the basic form of the results.

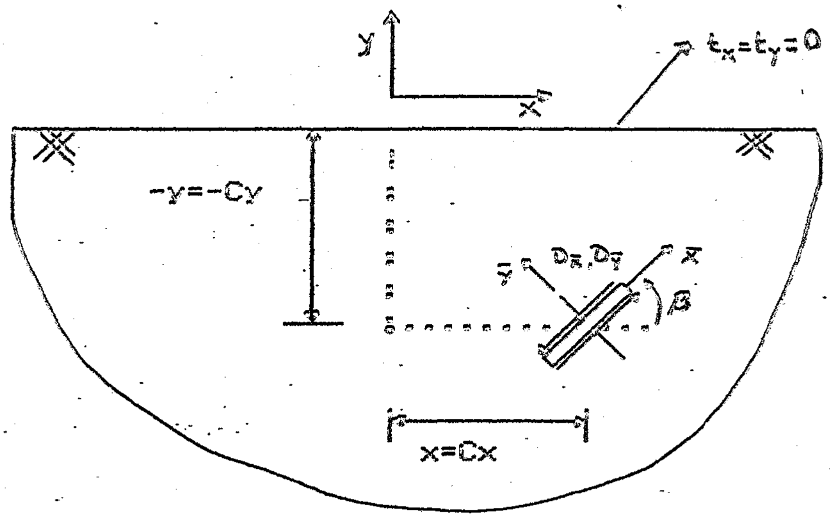

Figure A.1 Displacement discontinuity in half-plane $y \leq 0$.

The method of images is based on the principle of superposition. Using this method, the solution to the problem of Figure A.1 is found in two stages. In the first stage of the analysis, an infinite body containing two line segments with constant displacement discontinuities is considered, see Figure A.2. One of these in $y < 0$, represents the "actual" discontinuity while the other, in $y > 0$, represents its "image", reflected about the line $y = 0$. By symmetry, this arrangement makes shear traction $t_x = \sigma_{xy}$ vanish all along $y = 0$. The normal traction $t_y = \sigma_{yy}$ on $y = 0$, however, is not zero, and in the second stage of the analysis a supplementary solution is obtained to make it vanish. The supplementary solution, therefore relates to the elastic half-plane $y \leq 0$ with prescribed tractions $t_x = 0$ t_y not equal to zero along its surface.

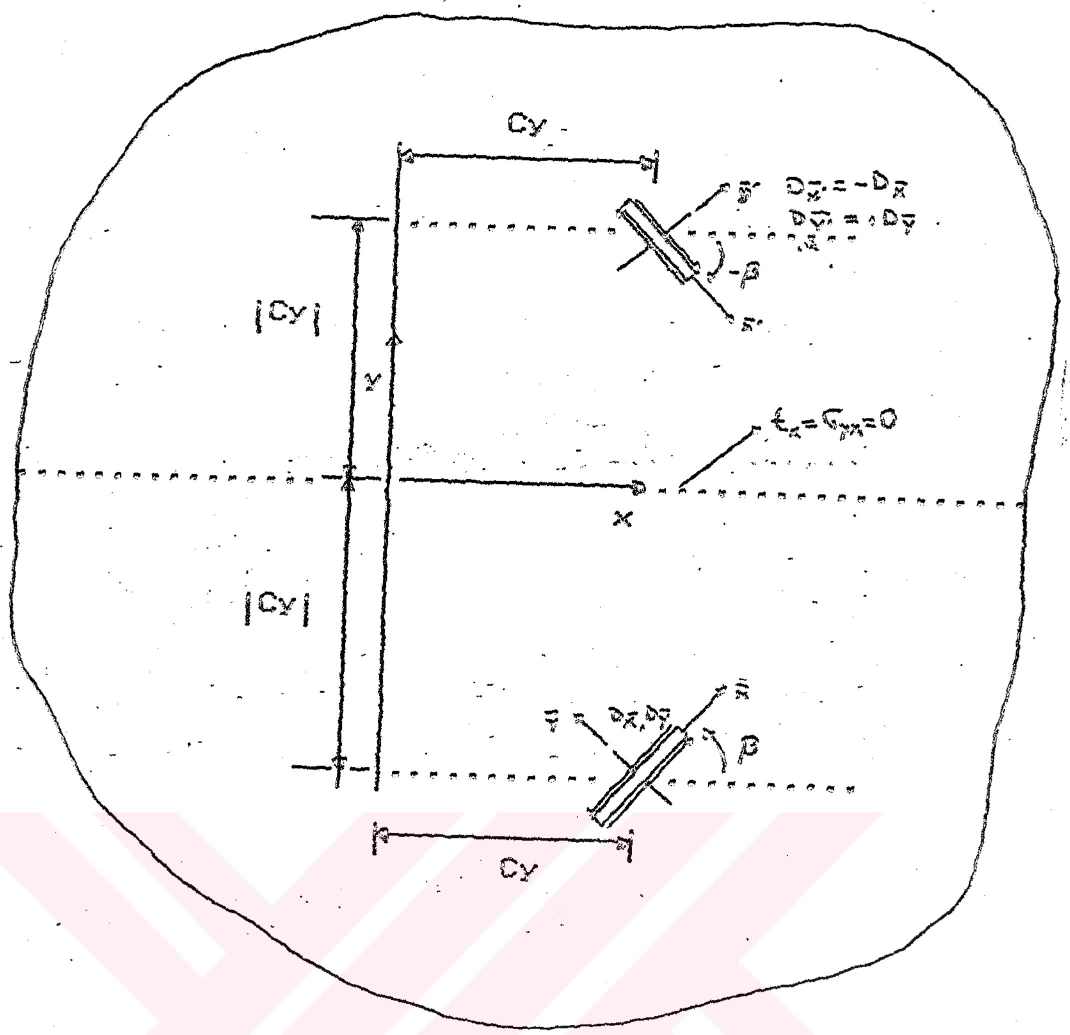

Figure A.2 Actual and image displacement discontinuties.

The displacements and stresses due to the actual displacement discontinuity will be denoted by U_i^A and σ_{ij}^A those due to its image by U_i^I and σ_{ij}^I, and those resulting from the supplementary solution by U_i^S and σ_{ij}^S. The complete solution for the half-plane $y \leq 0$ can then be written as:

$$U_i = U_i^A + U_i^I + U_i^S \qquad (1)$$

$$\sigma_{ij} = \sigma_{ij}^A + \sigma_{ij}^I + \sigma_{ij}^S$$

Expressions for the separate terms in (1) equation are given below.

(i) The displacement and stresses due to the actual displacement discontinuity can be written down as the local x,y co-ordinates are related to the global x,y co-ordinates by transformation formulas,

$$\bar{x} = (x-Cx)\cos\beta + (y-Cy)\sin\beta$$
$$\bar{y} = -(x-Cx)\sin\beta + (y-Cy)\cos\beta \qquad (2)$$

and the solution to the problem in question can be expressed interms of the function $f(\bar{x},\bar{y})$ defined in equation (3).This function will be denoted as $\hat{f}(\bar{x},\bar{y})$ to emphasize that it is associated with actual discontunity. Moreover, the derivatives of function $\hat{f}(\bar{x},\bar{y})$ will be denoted as $\delta\hat{f}(\bar{x},\bar{y})/\delta\bar{x}=\hat{F}2(\bar{x},\bar{y})$, etc., as in (3).

Function $f(\bar{x},\bar{y})$ in these equations is as follows:

$$f(\bar{x},\bar{y})=F1(\bar{x},\bar{y})=-1/4\pi(1-v)\ [\bar{y}(\arctan(\bar{y}/(\bar{x}-a))$$
$$-\arctan(\bar{y}/(\bar{x}+a)))-\ln\sqrt{((\bar{x}-a)^2+\bar{y}^2)}+(\bar{x}+a)\ln\sqrt{((\bar{x}+a)^2+\bar{y}^2)}]$$

$$f_{,\bar{x}}=F2(\bar{x},\bar{y})=1/4\pi(1-v)[\ln\sqrt{((\bar{x}-a)^2+\bar{y}^2)}-\ln\sqrt{((\bar{x}+a)^2+\bar{y}^2)}]$$

$$f_{,\bar{y}}=F3(\bar{x},\bar{y})=-1/4\pi(1-v)[\arctan(\bar{y}/(\bar{x}-a))-\arctan(\bar{y}/(\bar{x}+a))]$$

$$f_{,\bar{x}\bar{y}}=F4(\bar{x},\bar{y})=1/4\pi(1-v)[\bar{y}/((\bar{x}-a)^2+\bar{y}^2)-\bar{y}/((\bar{x}+a)^2+\bar{y}^2)]$$

$$f_{,\bar{x}\bar{x}} = F5(\bar{x},\bar{y}) = 1/4\pi(1-v)[(\bar{x}-a)/((\bar{x}-a)^2+\bar{y}^2)$$

$$-(\bar{x}+a)/((\bar{x}+a)^2+\bar{y}^2)]$$

$$f_{,\bar{x}\bar{y}\bar{y}} = F6(\bar{x},\bar{y}) = 1/4\pi(1-v)[((\bar{x}-a)^2-\bar{y}^2)/((\bar{x}-a)^2+\bar{y}^2)^2$$

$$-((\bar{x}+a)^2-\bar{y}^2)/((\bar{x}+a)^2+\bar{y}^2)^2]$$

$$f_{,\bar{y}\bar{y}\bar{y}} = F7(\bar{x},\bar{y}) = 2\bar{y}/4\pi(1-v)[(\bar{x}-a)/((\bar{x}-a)^2+\bar{y}^2)$$

$$-(\bar{x}+a)/((\bar{x}+a)^2+\bar{y}^2]$$

" $f_{,a}$ " denotes the partial derivative of the function with respect to given labels.

The required solution can be written as follows:

$$\hat{U}x = D\bar{x}[-(1-2v)\sin\beta\hat{F}2+2(1-v)\cos\beta\hat{F}3+\bar{y}(\sin\beta\hat{F}4-\cos\beta\hat{F}5)]$$

$$+D\bar{y}[-(1-2v)\cos\beta\hat{F}2-2(1-v)\sin\beta\hat{F}3-\bar{y}(\cos\beta\hat{F}4+\sin\beta\hat{F}5)]$$

(4)

$$\hat{U}y = D\bar{x}[(1-2v)\cos\beta\hat{F}2+2(1-v)\sin\beta\hat{F}3-\bar{y}(\cos\beta\hat{F}4+\sin\beta\hat{F}5)]$$

$$+D\bar{y}[-(1-2v)\sin\beta\hat{F}2+2(1-v)\cos\beta\hat{F}3-\bar{y}(\sin\beta\hat{F}4-\cos\beta\hat{F}5)]$$

and

$$\hat{\sigma}_{xx} = 2GD\bar{x}[2\cos^2\beta\hat{F}4+\sin2\beta\hat{F}5+\bar{y}(\cos2\beta\hat{F}6-\sin2\beta\hat{F}7)]$$

$$+2GD\bar{y}[-\hat{F}5+\bar{y}(\sin2\beta\hat{F}6+\cos2\beta\hat{F}7)]$$

$$\hat{\sigma}_{yy} = 2GD_{\bar{x}}[2\sin^2\beta \hat{F}4 - \sin 2\beta \hat{F}5 - \bar{y}(\cos 2\beta \hat{F}6 - \sin 2\beta \hat{F}7)]$$

$$+2GD_{\bar{y}}[-\hat{F}5 - \bar{y}(\sin 2\beta \hat{F}6 + \cos 2\beta \hat{F}7)] \qquad (5)$$

$$\hat{\sigma}_{xy} = 2GD_{\bar{x}}[\sin 2\beta \hat{F}4 - \cos 2\beta \hat{F}5 + \bar{y}(\sin 2\beta \hat{F}6 + \cos 2\beta \hat{F}7)]$$

$$+2GD_{\bar{y}}[-\bar{y}(\cos 2\beta \hat{F}6 - \sin 2\beta \hat{F}7)]$$

(ii) The displacements and stresses due to the image displacement discontinuity can similarly be found. The solution in this case can be expressed in terms of a function $\overset{I}{f}(\bar{x}',\bar{y}')$, defined for the image local co-ordinate system $\bar{x}',\bar{y}'$ in Figure A.1 as follows:

$$\overset{I}{f}(\bar{x}',\bar{y}') = -1/4\pi(1-\nu)[\bar{y}'(\arctan(\bar{y}'/(\bar{x}'-a)) - (\bar{y}'/(\bar{x}'+a)) \qquad (6)$$

$$-(\bar{x}'-a)\ln\sqrt{[(\bar{x}'-a)^2+\bar{y}'^2]} + (\bar{x}'+a)\ln\sqrt{[(\bar{x}'+a)^2+\bar{y}'^2]}]$$

The $\bar{x}',\bar{y}'$ and x,y co-ordinates are related by the transformation formula.

$$\bar{x}' = (x-Cx)\cos\beta - (y+Cy)\sin\beta$$

$$\bar{y}' = (x-Cx)\sin\beta + (y+Cy)\cos\beta \qquad (7)$$

which are obtained by replacing Cy and β in equation (2) by −Cy and −β.

Now, it can be shown (Crouch , 1976a,b) that the displacements and stresses for the supplementary solution are also expressible in terms of the function $\overset{I}{f}(\bar{x}',\bar{y}')$. It is possible, therefore, to quote the final results for the combined displacements $\overset{I}{U}i + \overset{S}{U}i$ and stresses $\overset{I}{\sigma}_{ij}$ + $\overset{S}{\sigma}_{ij}$. Denoting the derivatives of function $\overset{I}{f}(\bar{x}',\bar{y}')$ as:

$$\delta \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{x}' = \overset{I}{F2}(\bar{x}',\bar{y}') \quad , \quad \delta \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{y}' = \overset{I}{F3}(\bar{x}',\bar{y}')$$

$$\delta^2 \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{x}'\delta\bar{y}' = \overset{I}{F4}(\bar{x}',\bar{y}') \quad , \quad \delta^2 \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{y}'^2 = \overset{I}{F5}(\bar{x}'\bar{y}')$$

$$\delta^2 \overset{I}{f}(\bar{x}',\bar{y}')\delta \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{x}'\delta\bar{y}^{2\prime} = \overset{I}{F6}(\bar{x}',\bar{y}')$$

$$\delta^3 \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{y}'^3 = \overset{I}{F7}(\bar{x}',\bar{y}')$$

$$\delta^4 \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{x}'\delta\bar{y}'^3 = \overset{I}{F8}(\bar{x}',\bar{y}')$$

$$\delta^4 \overset{I}{f}(\bar{x}',\bar{y}')/\delta\bar{y}'^4 = \overset{I}{F9}(\bar{x}',\bar{y}')$$

it is found that the displacements $\overset{I}{U}i + \overset{S}{U}i$ in the global co-ordinate system

$$\overset{I}{U}x + \overset{S}{U}x = \bar{D}x[(1-2\nu)\sin\beta \overset{I}{F2} - 2(1-\nu)\cos\beta \overset{I}{F3}$$

$$+\{(3-4\nu)(y\sin 2\beta - \bar{y}\sin\beta) + 2y\sin 2\beta\}\overset{I}{F4}$$

$$+\{(3-4\nu)(y\cos 2\beta - \bar{y}\cos\beta) - y(1-2\cos 2\beta)\}\overset{I}{F5}$$

$$+2y(y\sin3\beta-\bar{y}\sin2\beta)F6^{I}-2y(y\cos3\beta-\bar{y}\cos2\beta)F7^{I}]$$

$$+D\bar{y}[(1-2v)\cos\beta F2^{I}+2(1-v)\sin\beta F3^{I}-\{(3-4v)(y\cos2\beta-\bar{y}\cos\beta)-y\}F4^{I}$$

$$+(3-4v)(y\sin2\beta-\bar{y}\sin\beta)F5^{I}-2y(y\cos3\beta-\bar{y}\cos2\beta)F6^{I}$$

$$-2y(y\sin3\beta-\bar{y}\sin2\beta)F7^{I}]$$

(8)

$$U_y^{I}+U_y^{S}=D\bar{x}[-(1-2v)\cos\beta F2^{I}-2(1-v)\sin\beta F3^{I}$$

$$-\{(3-4v)(y\cos2\beta-\bar{y}\cos\beta)+y(1-2\cos2\beta)\}F4^{I}$$

$$+\{(3-4v)(y\sin2\beta-\bar{y}\sin\beta)-2y\sin2\beta\}F5^{I}$$

$$+2y(y\cos3\beta-\bar{y}\cos2\beta)F6^{I}+2y(y\sin3\beta-\bar{y}\sin2\beta)F7^{I}]$$

$$+D\bar{y}[(1-2v)\sin\beta F2^{I}-2(1-v)\cos\beta F3^{I}-(3-4v)(y\sin2\beta-\bar{y}\sin\beta)F4^{I}$$

$$-\{(3-4v)(y\cos2\beta-\bar{y}\cos\beta)+y\}F5^{I}$$

$$+2y(y\sin3\beta-\bar{y}\sin2\beta)F6^{I}-2y(y\cos3\beta-\bar{y}\cos2\beta)F7^{I}]$$

Finally, the stresses $\sigma_{ij}^{I}+\sigma_{ij}^{S}$ associated with these displacements are

$$\sigma_{xx}^{I}+\sigma_{xx}^{S}=2GD\bar{x}[F4^{I}-3(\cos2\beta F4^{I}-\sin2\beta F5^{I})$$

$$+\{2y(\cos\beta-3\cos3\beta)3\bar{y}\cos2\beta\}F6^{I}$$

$$+\{2y(\sin\beta-3\sin3\beta)+3\bar{y}\sin2\beta\}F7^{I}$$

$$-2y(y\cos4\beta-\bar{y}\cos3\beta)F8^{I}-2y(y\sin4\beta-\bar{y}\sin3\beta)F9^{I}]$$

$$+2GD\bar{y}[F_5^I+(2y(\sin\beta-2\sin3\beta)+3\bar{y}\sin2\beta)F_6^I$$

$$-(2y(\cos\beta-2\cos3\beta)+3\bar{y}\cos2\beta)F_7^I \qquad (9)$$

$$-2y(y\sin4\beta-\bar{y}\sin3\beta)F_8^I+2y(y\cos4\beta-\bar{y}\cos3\beta)F_9^I]$$

$$\sigma_{yy}^I+\sigma_{yy}^S=2GD\bar{x}[F_4^I-(\cos2\beta F_4^I-\sin2\beta F_5^I)-(4y\sin\beta\sin2\beta-\bar{y}\cos2\beta)F_6^I$$

$$+(4y\sin\beta\cos2\beta+\bar{y}\sin2\beta)F_7^I$$

$$+2y(y\cos4\beta-\bar{y}\cos3\beta)F_8^I+2y(y\sin4\beta-\bar{y}\sin3\beta)F_9^I]$$

$$+2GD\bar{y}[F_5^I-(2y\sin\beta-\bar{y}\sin2\beta)F_6^I+(2y\cos\beta-\bar{y}\cos2\beta)F_7^I$$

$$+2y(y\sin4\beta-\bar{y}\sin3\beta)F_8^I-2y(y\cos4\beta-\bar{y}\cos3\beta)F_9^I]$$

$$\sigma_{xy}^I+\sigma_{xy}^S=2GD\bar{x}[\sin2\beta F_4^I+\cos2\beta F_5^I+(2y\sin\beta(1+4\cos2\beta)-\bar{y}\sin2\beta)F_6^I$$

$$+(2y\cos\beta(3-4\cos2\beta)+\bar{y}\cos2\beta)F_7^I$$

$$+2y(y\sin4\beta-\bar{y}\sin3\beta)F_8^I-2y(y\cos4\beta-\bar{y}\cos3\beta)F_9^I]$$

$$+2GD\bar{y}[(4y\sin\beta\sin2\beta+\bar{y}\cos2\beta)F_6^I-(4y\sin\beta\cos2\beta-\bar{y}\sin2\beta)F_7^I$$

$$-2y(y\cos4\beta-\bar{y}\cos3\beta)F_8^I-2y(y\sin4\beta-\bar{y}\sin3\beta)F_9^I]$$

The solution to the problem of figure A.1 is therefore (5) with (9) and (4) with (8) combination results. And the resultant tractions:

$$t_x = \sigma_{xy} (= \sigma^{A}_{xy} + \sigma^{I}_{xy} + \sigma^{S}_{xy})$$

$$t_y = \sigma_{yy} (= \sigma^{A}_{yy} + \sigma^{I}_{yy} + \sigma^{S}_{yy})$$

These tractions are equal to zero on the surface of the half plane. This has been checked when the verification is done.

APPENDIX A1. SUBROUTINE INFLUENCE COEFFICIENT

This appendix includes the listing of modified influence coefficient in fortran language.The line numbers show the place the modification is done in the program.

```
                                                                          Line Number
                                                                          -----------
      COMMON/S1/PI,PR,PR1,PR2,PR3,CON,CONS                                    10
      PR3=3.-4.*PR                                                            42
C*                                                                           459
C************************************************************************     .
C*                                                                     *     .
C*    THIS SUBROUTINE CALCULATES  INFLUENCE COEFFICIENTS OF SEMI       *     .
C*              INFINITE PLANE PROBLEM SOLUTIONS                       *
C*                                                                     *
C************************************************************************
      SUBROUTINE COEFF(X,Y,CX,CY,A,COSB,SINB,MSYM)
C*
      COMMON/S1/PI,PR,PR1,PR2,PR3,CON,CONS
      COMMON/S2/SXXS,SXXN,SYYS,SYYN,SXYS,SXYN,UXS,UXN,UYS,UYN
C*
      TXYDS=0.
      TXYDN=0.
      TYYDS=0.
      TYYDN=0.
      VXDS=0.
      UUXDS=0.
      COS2B=COSB**2-SINB**2
      SIN2B=2.*SINB*COSB
      COSB2=COSB**2
      SINB2=SINB**2
      SIN3B=SINB*COS2B+COSB*SIN2B
      COS3B=COSB*COS2B-SINB*SIN2B
      SIN4B=SIN2B*COS2B+COS2B*SIN2B
      COS4B=COS2B*COS2B-SIN2B*SIN2B
C************************************************************************
C*                                                                     *
C*     THIS PART OF THE SUBROUTINE IS FOR THE ACTUAL INFLUENCE         *
C*        COEFICIENT  CALCULATION OF SEMI-INFINITE PROBLEM             *
C************************************************************************
      XB=(X-CX)*COSB+(Y-CY)*SINB
      YB=-(X-CX)*SINB+(Y-CY)*COSB
C*
      R1S=(XB-A)**2+YB**2
      R2S=(XB+A)**2+YB**2
      FL1=0.5*ALOG(R1S)
      FL2=0.5*ALOG(R2S)
      FB2=CON*(FL1-FL2)
      IF(YB.NE.0.) GO TO 10
      FB3=0.
      IF(ABS(XB).LT.A)FB3=CON*PI
      GO TO 20
 10   FB3=-CON*(ATAN((XB+A)/YB)-ATAN((XB-A)/YB))
```

```
 20     FB4=CON*(YB/R1S-YB/R2S)
        FB5=CON*((XB-A)/R1S-(XB+A)/R2S)
        FB6=CON*(((XB-A)**2-YB**2)/R1S**2-((XB+A)**2-YB**2)/R2S**2)
        FB7=2.*CON*YB*((XB-A)/R1S**2-(XB+A)/R2S**2)
C*
        VXDS=-PR1*SINB*FB2+PR2*COSB*FB3+YB*(SINB*FB4-COSB*FB5)
        VXDN=-PR1*COSB*FB2-PR2*SINB*FB3-YB*(COSB*FB4+SINB*FB5)
        VYDS=PR1*COSB*FB2+PR2*SINB*FB3-YB*(COSB*FB4+SINB*FB5)
        VYDN=-PR1*SINB*FB2+PR2*COSB*FB3-YB*(SINB*FB4-COSB*FB5)
C*
        TXXDS=CONS*(2.*COSB2*FB4+SIN2B*FB5+YB*(COS2B*FB6-SIN2B*FB7))
        TXXDN=CONS*(-FB5+YB*(SIN2B*FB6+COS2B*FB7))
        TYYDS=CONS*(2.*SINB2*FB4-SIN2B*FB5-YB*(COS2B*FB6-SIN2B*FB7))
        TYYDN=CONS*(-FB5-YB*(SIN2B*FB6+COS2B*FB7))
        TXYDS=CONS*(SIN2B*FB4-COS2B*FB5+YB*(SIN2B*FB6+COS2B*FB7))
        TXYDN=CONS*(-YB*(COS2B*FB6-SIN2B*FB7))
C***********************************************************************
C*                                                                     *
C*      THIS PART OF THE SUBROUTINE IS FOR THE INFLUENCE COEFICIENT    *
C*       OF SUPLEMENTARY AND IMAGE SUPERPOSITION OF SEMI-INFINITE      *
C*                              PLANE                                  *
C***********************************************************************
        CF=(X-CX)*COSB-(Y+CY)*SINB
        HF=(X-CX)*SINB+(Y+CY)*COSB
C*
        R3S=(CF-A)**2+HF**2
        R4S=(CF+A)**2+HF**2
        FL3=.5*ALOG(R3S)
        FL4=.5*ALOG(R4S)
        FH2=CON*(FL3-FL4)
        IF(HF.NE.0.) GO TO 21
        FH3=0.
        IF(ABS(CF).LT.A)FH3=CON*PI
        GO TO 31
 21     FH3=-CON*(ATAN((CF+A)/HF)-ATAN((CF-A)/HF))
 31     FH4=CON*(HF/R3S-HF/R4S)
        FH5=CON*((CF-A)/R3S-(CF+A)/R4S)
        FH6=CON*(((CF-A)**2-HF**2)/R3S**2-((CF+A)**2-HF**2)/R4S**2)
        FH7=2.*CON*HF*((CF-A)/R3S**2-(CF+A)/R4S**2)
        FH8=-2.*CON*HF*((((CF-A)**2+HF**2)+2.*((CF-A)**2-HF**2))/R3S**3-
     *(((CF+A)**2+HF**2)+2.*((CF+A)**2-HF**2))/R4S**3)
        FH9=2.*CON*(((CF-A)**2+HF**2)*(CF-A)*((CF-A)**2-4.*HF**2)/R3S**4
     *-(CF+A)*((CF+A)**2+HF**2)*((CF+A)**2-4.*HF**2)/R4S**4)
C*
C*
       UUXDS=PR1*SINB*FH2-PR2*COSB*FH3+(PR3*(Y*SIN2B-YB*SINB)+2.*Y*SIN2B
     *)*FH4+(PR3*(Y*COS2B-YB*COSB)-Y*(1.-2.*COS2B))*FH5+2.*Y*(Y*SIN3B-YB
     **SIN2B)*FH6-2.*Y*(Y*COS3B-YB*COS2B)*FH7
C*
       UUXDN=PR1*COSB*FH2+PR2*SINB*FH3-(PR3*(Y*COS2B-YB*COSB)-Y)*FH4+PR3
     **(Y*SIN2B-YB*SINB)*FH5-2.*Y*(Y*COS3B-YB*COS2B)*FH6-2.*Y*(Y*SIN3B-Y
     *B*SIN2B)*FH7
       UUYDS=-PR1*COSB*FH2-PR2*SINB*FH3-(PR3*(Y*COS2B-YB*COSB)+Y*(1.-2.*
     *COS2B))*FH4+(PR3*(Y*SIN2B-YB*SINB)-2.*Y*SIN2B)*FH5+2.*Y*(Y*COS3B-Y
     *B*COS2B)*FH6+2.*Y*(Y*SIN3B-YB*SIN2B)*FH7
```

```
      UUYDN=PR1*SINB*FH2-PR2*COSB*FH3-PR3*(Y*SIN2B-YB*SINB)*FH4-(PR3*(Y
     **COS2B-YB*COSB)+Y)*FH5+2.*Y*(Y*SIN3B-YB*SIN2B)*FH6-2.*Y*(Y*COS3B-Y
     *B*COS2B)*FH7
C*
C*
      TTXXDS=CONS*(FH4-3.*(COS2B*FH4-SIN2B*FH5)+(2.*Y*(COSB-3.*COS3B)+3
     *.*YB*COS2B)*FH6+(2.*Y*(SINB-3.*SIN3B)+3.*YB*SIN2B)*FH7-2.*Y*(Y*COS
     *4B-YB*COS3B)*FH8-2.*Y*(Y*SIN4B-YB*SIN3B)*FH9)
      TTXXDN=CONS*(FH5+(2.*Y*(SINB-2.*SIN3B)+3.*YB*SIN2B)*FH6-(2.*Y*(CO
     *SB-2.*COS3B)+3.*YB*COS2B)*FH7-2.*Y*(Y*SIN4B-YB*SIN3B)*FH8+2.*Y*(Y*
     *COS4B-YB*COS3B)*FH9)
      TTYYDS=CONS*(FH4-(COS2B*FH4-SIN2B*FH5)-(4.*Y*SINB*SIN2B-YB*COS2B)
     **FH6+(4.*Y*SINB*COS2B+YB*SIN2B)*FH7+2.*Y*(Y*COS4B-YB*COS3B)*FH8+2.
     **Y*(Y*SIN4B-YB*SIN3B)*FH9)
      TTYYDN=CONS*(FH5-(2.*Y*SINB-YB*SIN2B)*FH6+(2.*Y*COSB-YB*COS2B)*FH
     *7+2.*Y*(Y*SIN4B-YB*SIN3B)*FH8-2.*Y*(Y*COS4B-YB*COS3B)*FH9)
      TTXYDS=CONS*(SIN2B*FH4+COS2B*FH5+(2.*Y*SINB*(1.+4.*COS2B)-YB*SIN2
     *B)*FH6+(2.*Y*COSB*(3.-4.*COS2B)+YB*COS2B)*FH7+2.*Y*(Y*SIN4B-YB*SIN
     *3B)*FH8-2.*Y*(Y*COS4B-YB*COS3B)*FH9)
      TTXYDN=CONS*((4.*Y*SINB*SIN2B+YB*COS2B)*FH6-(4.*Y*SINB*COS2B-YB*S
     *IN2B)*FH7-2.*Y*(Y*COS4B-YB*COS3B)*FH8-2.*Y*(Y*SIN4B-YB*SIN3B)*FH9)
C*
      UXDS=VXDS+UUXDS
      UXDN=VXDN+UUXDN
      UYDS=VYDS+UUYDS
      UYDN=VYDN+UUYDN
      UXS=UXS+MSYM*UXDS
      UXN=UXN+UXDN
      UYS=UYS+MSYM*UYDS
      UYN=UYN+UYDN
C*
      SXXDS=TXXDS+TTXXDS
      SXXDN=TXXDN+TTXXDN
      SYYDS=TYYDS+TTYYDS
      SYYDN=TYYDN+TTYYDN
      SXYDS=TXYDS+TTXYDS
      SXYDN=TXYDN+TTXYDN
      SXXS=SXXS+MSYM*SXXDS
      SXXN=SXXN+SXXDN
      SYYS=SYYS+MSYM*SYYDS
      SYYN=SYYN+SYYDN
      SXYS=SXYS+MSYM*SXYDS
      SXYN=SXYN+SXYDN
C*
C*
      RETURN
      END                                                          606
```

APPENDIX A2. COMPUTATION OF TANGENTIAL STRESS PROGRAM IN FORTRAN LANGUAGE

```
      DIMENSION TITLE(50),DS(100),DN(100),ANG(100),KOD(100)
      CALL SIGTAN (A,DS,DN,ANG,SIGT1,SIGTP,SIGTN,I,NUMBE)
      WRITE(6,17)I,D(IS),USNEG,USPOS,D(IN),UNNEG,UNPOS,UXNEG,UYNEG,
     *UXPOS,UYPOS,SIGS,SIGN,SIGT1,SIGTP,SIGTN
C************************************************************************
C*                                                                     *
C*    THIS SUBROUTINE CALCULATES TANGENTIAL STRESS ALONG THE           *
C*                         BOUNDARY                                    *
C************************************************************************
      SUBROUTINE SIGTAN (A,DS,DN,ANG,SIGT1,SIGTP,SIGTN,I,NUMBE)
C*
      COMMON/S1/PI,PR,PR1,PR2,PR3,CON,CONS

C*
C*
C*
      DIMENSION A(100),DN(100),DS(100),ANG(100)
C*
      F=ANG(I)
      G=ANG(I+1)
      Z=G-F
C*
      IF(I.EQ.1)GO TO 10
      H=ANG(I-1)
      V=F-H
      IF (I.EQ.NUMBE)GO TO 20
      DELS=A(I-1)*COS(V)+2*A(I)+A(I+1)*COS(Z)
      SIGTDI=CONS/PR2*(DS(I+1)*COS(Z)-DS(I-1)*COS(V)-DN(I+1)*SIN(Z)-DN
     *(I-1)*SIN(V))/DELS
C*
      GO TO 150
10    DELS=A(I)+A(I+1)*COS(Z)
      SIGTDI=CONS/PR2*(DS(I+1)*COS(Z)-DN(I+1)*SIN(Z)-DS(I))/DELS
C*    PRINT/,DELS,SIGT1,SIGTDI,DS(I),DN(I)
      GO TO 150
20    DELS=A(I)+A(I-1)*COS(V)
      SIGTDI=CONS/PR2*(DS(I)-DS(I-1)*COS(V)-DN(I-1)*SIN(V))/DELS
C*
150   SIGTP=SIGT1-SIGTDI
      SIGTN=SIGT1+SIGTDI
      RETURN
      END
```

APPENDIX A3. COMPUTATION OF HORIZONTAL STRAIN IN FORTRAN LANGUAGE

```
      DIMENSION UX(100),XP(100),STR(100)
C**************************************************************************
C*                                                                        *
C*   COMPUTATION OF HORIZONTAL STRAIN BY USING FINITE DIFFERENCE          *
C*                              METHOD                                    *
C*                                                                        *
C**************************************************************************
      WRITE (6,20)
      DO 892 NI=1,NUMP
      CALL STRAIN(UX,XP,NI,STR,NUMP)
      WRITE (6,22) NI,XP(NI),STR(NI)
C**************************************************************************
C*                                                                        *
C*      THIS SUBROUTINE CALCULATES THE HORIZANTAL STRAIN  AT              *
C*                     SPECIFIED POINTS                                   *
C**************************************************************************
      SUBROUTINE STRAIN(UX,XP,KI,STR,NUMP)
      DIMENSION UX(100),XP(100),STR(100)
      IF(KI.EQ.1) GOTO 13
      IF(KI.EQ.NUMP) GO TO 14
      FX=UX(KI+1)-UX(KI-1)
      XPS=XP(KI+1)-XP(KI-1)
      STR(KI)=FX/XPS
      GO TO 175
13    FX=UX(KI+1)-UX(KI)
      XPS=XP(KI+1)-XP(KI)
      STR(KI)=FX/XPS
      GOTO 175
14    FX=UX(KI)-UX(KI-1)
      XPS=XP(KI)-XP(KI-1)
      STR(KI)=FX/XPS
175   RETURN
      END
```

APPENDIX A4. DIVIDING THE SEGMENTS INTO ELEMENTS
PROGRAMMING IN FORTRAN LANGUAGE

```
      NUMBE=0
      NSEG=0
 701  IF(NSEG.EQ.NUMBS) GO TO 51
      NSEG=NSEG+1
      NELG=0.
      RELG=NELG
      READ(5,14) NUM,XBEG,YBEG,XEND,YEND,KODE,RDS,RATIO,PSI,BVS,BVN
C*************************************************************************
C*                                                                       *
C*  COMPUTATION OF BOUNDARY ELEMENTS BY USING CIRCULAR OR ELLIPTICAL     *
C*                         SEGMENTS                                      *
C*************************************************************************
      IF(RDS.EQ.0.) GO TO 601
      IF(RATIO.EQ.0.) RATIO=1.
      SINPSI=SIN(PSI*PI/180.)
      COSPSI=COS(PSI*PI/180.)
      GD=.1E-10
      GA=RATIO*COS((XEND-PSI)*PI/180.)
      IF(ABS(GA).LT.GD) GA=GD
      GB=RATIO*COS((YEND-PSI)*PI/180.)
      IF(ABS(GB).LT.GD) GB=GD
      CHI1=ATAN2(SIN((XEND-PSI)*PI/180.),GA)
      CHI2=ATAN2(SIN((YEND-PSI)*PI/180.),GB)
      DCHI=(CHI2-CHI1)/NUM
      IF(ABS(DCHI).LT.GD) GO TO 606
      GC=DCHI/ABS(DCHI)
      GO TO 605
 606  GC=-1.
 605  DCHI=DCHI+((YEND-XEND)/ABS(YEND-XEND)-GC)*PI/NUM
 601  NUMBE=NUMBE+1
      M=NUMBE
      CHI=CHI1+RELG*DCHI
      EX1=RDS*(COS(CHI)*SINPSI+SIN(CHI)*COSPSI*RATIO)+XBEG
      EY1=RDS*(COS(CHI)*COSPSI-SIN(CHI)*SINPSI*RATIO)-YBEG
      CHI=CHI+DCHI
      EX2=RDS*(COS(CHI)*SINPSI+SIN(CHI)*COSPSI*RATIO)+XBEG
      EY2=RDS*(COS(CHI)*COSPSI-SIN(CHI)*SINPSI*RATIO)-YBEG
      XM(M)=.5*(EX1+EX2)
      YM(M)=-.5*(EY1+EY2)
      DX=EX2-EX1
      DY=EY2-EY1
      IF(ABS(DX).LT.(.1E-13)*RDS) DX=0.
      IF(ABS(DY).LT.(.1E-13)*RDS) DY=0
      SW=SQRT(DX*DX+DY*DY)
      A(M)=.5*SW
      SINBET(M)=-DY/SW
      COSBET(M)=DX/SW
      ANG(M)=ATAN2(SINBET(M),COSBET(M))
      KOD(M)=KODE
      MN=2*M
```

```
      MS=MN-1
      B(MS)=BVS
      B(MN)=BVN
      NELG=NELG+1
      RELG=NELG
      IF(NELG.LT.NUM) GO TO 601
      GO TO 701
C***********************************************************************
C*                                                                     *
C*  COMPUTATION OF BOUNDARY ELEMENTS BY USING STRAIGT LINE SEGMENT     *
C***********************************************************************
 801  XD=(XEND-XBEG)/NUM
      YD=(YEND-YBEG)/NUM
      SW=SQRT(XD*XD+YD*YD)
      RELG=0.
      DO 110 NE=1,NUM
      NUMBE=NUMBE+1
      M=NUMBE
      RELG=RELG+1
      NELG=RELG
      XM(M)=XBEG+.5*(2.*NE-1.)*XD
      YM(M)=YBEG+.5*(2.*NE-1.)*YD
      A(M)=.5*SW
      SINBET(M)=YD/SW
      COSBET(M)=XD/SW
      ANG(M)=ATAN2(SINBET(M),COSBET(M))
      KOD(M)=KODE
      MN=2*M
      MS=MN-1
      B(MS)=BVS
      B(MN)=BVN
      IF(NELG.EQ.NUM) GO TO 701
C*
 110  CONTINUE
 51   WRITE(6,13)
      DO 115 M=1,NUMBE
      SIZE=2.*A(M)
      ANGLE=180.*ATAN2(SINBET(M),COSBET(M))/PI
      WRITE(6,15) M,KOD(M),XM(M),YM(M),SIZE,ANGLE,B(2*M-1),B(2*M)
 115  CONTINUE                                                          136
```

APPENDIX B COMPUTER PROGRAM FOR THE TWO DIMENSIONAL DISPLACEMENT DISCONTINUITY METHOD (TWODD) FORTRAN LISTING (*)

```
C*FILE    5(KIND=DISK,TITLE="TWD5",FILETYPE=7)
C*FILE    6(KIND=PRINTER,MAXRECSIZE=132)
C*************************************************************************
C*                                                                       *
C*      THIS PROGRAM IS A MODIFIED TWO DIMENSIONAL DISPLACEMENT          *
C*      DISCONTINUITY BOUNDARY ELEMENT METHOD FOR SEMI-INFINITE          *
C*                              PLANE                                    *
C*                                                                       *
C*************************************************************************
      COMMON/S1/PI,PR,PR1,PR2,PR3,CON,CONS
      COMMON/S2/SXXS,SXXN,SYYS,SYYN,SXYS,SXYN,UXS,UXN,UYS,UYN
      COMMON/S3/C(200,200),B(200),D(200)
C*
      DIMENSION XM(100),YM(100),A(100),COSBET(100),SINBET(100)
      DIMENSION TITLE(50),DS(100),DN(100),ANG(100),KOD(100)
      DIMENSION UX(100),XP(100),STR(100)
C*
      READ  (5,1)  (TITLE(I),I=1,20)
      WRITE (6,2)  (TITLE(I),I=1,20)
      READ  (5,3)  NUMBS,NUMOS,KSYM
      READ  (5,4)  PR,E,XSYM,YSYM
      READ  (5,5)  GAMMA,YK,PXY
      WRITE (6,6)  NUMBS,NUMOS
      GO TO (80,85,90,95), KSYM
 80   WRITE (6,7)
      GO TO 100
 85   WRITE (6,8) XSYM
      GO TO 100
 90   WRITE (6,9) YSYM
      GO TO 100
 95   WRITE (6,10) XSYM,YSYM
C*
 100  CONTINUE
      WRITE (6,11) PR,E
      WRITE (6,12) GAMMA,YK,PXY
C*
      PI=4*ATAN(1.)
      CON=1./(4*PI*(1.-PR))
      CONS=E/(1.+PR)
      PR1=1.-2.*PR
      PR2=2.*(1.-PR)
      PR3=3.-4.*PR
C*
C*    DEFINE LOCATION,SIZES,ORIENTATIONS AND BOUNDARY CONDITIONS OF
C*                BOUNDARY ELEMENTS
C*
      NUMBE=0
      NSEG=0
 701  IF(NSEG.EQ.NUMBS) GO TO 51
      NSEG=NSEG+1
      NELS=0.
```

```
      RELG=NELG
      READ(5,14) NUM,XBEG,YBEG,XEND,YEND,KODE,RDS,RATIO,PSI,BVS,BVN
C***************************************************************************
C*                                                                         *
C*  COMPUTATION OF BOUNDARY ELEMENTS BY USING CIRCULAR OR ELLIPTICAL       *
C*                          SEGMENTS                                       *
C***************************************************************************
      IF(RDS.EQ.0.) GO TO 601
      IF(RATIO.EQ.0.) RATIO=1.
      SINPSI=SIN(PSI*PI/180.)
      COSPSI=COS(PSI*PI/180.)
      GD=.1E-10
      GA=RATIO*COS((XEND-PSI)*PI/180.)
      IF(ABS(GA).LT.GD) GA=GD
      GB=RATIO*COS((YEND-PSI)*PI/180.)
      IF(ABS(GB).LT.GD) GB=GD
      CHI1=ATAN2(SIN((XEND-PSI)*PI/180.),GA)
      CHI2=ATAN2(SIN((YEND-PSI)*PI/180.),GB)
      DCHI=(CHI2-CHI1)/NUM
      IF(ABS(DCHI).LT.GD) GO TO 606
      GC=DCHI/ABS(DCHI)
      GO TO 605
 606  GC=-1.
 605  DCHI=DCHI+((YEND-XEND)/ABS(YEND-XEND)-GC)*PI/NUM
 601  NUMBE=NUMBE+1
      M=NUMBE
      CHI=CHI1+RELG*DCHI
      EX1=RDS*(COS(CHI)*SINPSI+SIN(CHI)*COSPSI*RATIO)+XBEG
      EY1=RDS*(COS(CHI)*COSPSI-SIN(CHI)*SINPSI*RATIO)-YBEG
      CHI=CHI+DCHI
      EX2=RDS*(COS(CHI)*SINPSI+SIN(CHI)*COSPSI*RATIO)+XBEG
      EY2=RDS*(COS(CHI)*COSPSI-SIN(CHI)*SINPSI*RATIO)-YBEG
      XM(M)=.5*(EX1+EX2)
      YM(M)=-.5*(EY1+EY2)
      DX=EX2-EX1
      DY=EY2-EY1
      IF(ABS(DX).LT.(.1E-13)*RDS) DX=0.
      IF(ABS(DY).LT.(.1E-13)*RDS) DY=0
      SW=SQRT(DX*DX+DY*DY)
      A(M)=.5*SW
      SINBET(M)=-DY/SW
      COSBET(M)=DX/SW
      ANG(M)=ATAN2(SINBET(M),COSBET(M))
      KOD(M)=KODE
      MN=2*M
      MS=MN-1
      B(MS)=BVS
      B(MN)=BVN
      NELG=NELG+1
      RELG=NELG
      IF(NELG.LT.NUM) GO TO 601
      GO TO 701
```

```
C***********************************************************************
C*                                                                     *
C*  COMPUTATION OF BOUNDARY ELEMENTS BY USING STRAIGT LINE SEGMENT     *
C***********************************************************************
 801    XD=(XEND-XBEG)/NUM
        YD=(YEND-YBEG)/NUM
        SW=SQRT(XD*XD+YD*YD)
        RELG=0.
        DO 110 NE=1,NUM
        NUMBE=NUMBE+1
        M=NUMBE
        RELG=RELG+1
        NELG=RELG
        XM(M)=XBEG+.5*(2.*NE-1.)*XD
        YM(M)=YBEG+.5*(2.*NE-1.)*YD
        A(M)=.5*SW
        SINBET(M)=YD/SW
        COSBET(M)=XD/SW
        ANG(M)=ATAN2(SINBET(M),COSBET(M))
        KOD(M)=KODE
        MN=2*M
        MS=MN-1
        B(MS)=BVS
        B(MN)=BVN
        IF(NELG.EQ.NUM) GO TO 701
C*      GO TO 602
 110    CONTINUE
 51     WRITE(6,13)
        DO 115 M=1,NUMBE
        SIZE=2.*A(M)
        ANGLE=180.*ATAN2(SINBET(M),COSBET(M))/PI
        WRITE(6,15) M,KOD(M),XM(M),YM(M),SIZE,ANGLE,B(2*M-1),B(2*M)
 115    CONTINUE
C***********************************************************************
C*      ADJUST STRESS BOUNDARY VALUES TO ACCOUNT FOR INITIAL STRESSES. *
C***********************************************************************
        DO 150 N=1,NUMBE
        NN=2*N
        NS=NN-1
        COSB=COSBET(N)
        SINB=SINBET(N)
        PYY=YM(N)*GAMMA
        PXX=YK*PYY
        PXY=0.
        SIGS=(PYY-PXX)*SINB*COSB+PXY*(COSB*COSB-SINB*SINB)
        SIGN=PXX*SINB*SINB-2.*PXY*SINB*COSB+PYY*COSB*COSB
        GO TO (120,150,130,140),KOD(N)
 120    B(NS)=B(NS)-SIGS
        B(NN)=B(NN)-SIGN
        GO TO 150
 130    B(NN)=B(NN)-SIGN
        GO TO 150
 140    B(NS)=B(NS)-SIGS
 150    CONTINUE
```

```
C*
C*    COMPUTE INFLUENCE COEFFICIENTS AND SET UP SYSTEM OF ALGEBRAIC
C*                        EQUETIONS
C*
      DO 300 I=1,NUMBE
      IN=2*I
      IS=IN-1
      XI=XM(I)
      YI=YM(I)
      COSBI=COSBET(I)
      SINBI=SINBET(I)
      KODE=KOD(I)
C*
      DO 300 J=1,NUMBE
      JN=2*J
      JS=JN-1
      CALL INITL
      XJ=XM(J)
      YJ=YM(J)
      COSBJ=COSBET(J)
      SINBJ=SINBET(J)
      AJ=A(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,SINBJ,+1)
      GO TO (240,210,220,230),KSYM
C*
 210  XJ=2.*XSYM-XM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      GO TO 240
C*
 220  YJ=2.*YSYM-YM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      GO TO 240
C*
 230  XJ=2.*XSYM-XM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      XJ=XM(J)
      YJ=2.*YSYM-YM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      XJ=2.*XSYM-XM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,-SINBJ,+1)
C*
 240  CONTINUE
      GO TO (250,260,270,280),KODE
C*
 250  C(IS,JS)=(SYYS-SXXS)*SINBI*COSBI+SXYS*(COSBI**2-SINBI**2)
      C(IS,JN)=(SYYN-SXXN)*SINBI*COSBI+SXYN*(COSBI**2-SINBI**2)
      C(IN,JS)=SXXS*SINBI**2-2.*SXYS*SINBI*COSBI+SYYS*COSBI**2
      C(IN,JN)=SXXN*SINBI**2-2.*SXYN*SINBI*COSBI+SYYN*COSBI**2
      GO TO 300
C*
 260  C(IS,JS)=UXS*COSBI+UYS*SINBI
      C(IS,JN)=UXN*COSBI+UYN*SINBI
      C(IN,JS)=-UXS*SINBI+UYS*COSBI
      C(IN,JN)=-UXN*SINBI+UYN*COSBI
      GO TO 300
```

```
C*
 270    C(IS,JS)=UXS*COSBI+UYS*SINBI
        C(IS,JN)=UXN*COSBI+UYN*SINBI
        C(IN,JS)=SXXS*SINBI**2-2.*SXYS*SINBI*COSBI+SYYS*COSBI**2
        C(IN,JN)=SXXN*SINBI**2-2.*SXYN*SINBI*COSBI+SYYN*COSBI**2
        GO TO 300
C*
 280    C(IS,JS)=(SYYS-SXXS)*SINBI*COSBI+SXYS*(COSBI**2-SINBI**2)
        C(IS,JN)=(SYYN-SXXN)*SINBI*COSBI+SXYN*(COSBI**2-SINBI**2)
        C(IN,JS)=-UXS*SINBI+UYS*COSBI
        C(IN,JN)=-UXN*SINBI+UYN*COSBI
C*
 300    CONTINUE
C*
C*      SOLVE SYSTEM OF ALGEBRAIC EQUATIONS
C*
        N=2*NUMBE
        CALL SOLVE(N)
C*      SEPARATION OF DS AND DN ARRAY FROM A COMBINED ARRAY
C*
        DO 311  K=1,NUMBE
        JN=2*K
        JS=JN-1
        DN(K)=D(JN)
        DS(K)=D(JS)
 311    CONTINUE
C*
C*
C*      COMPUTE BOUNDARY DISPLACEMENTS AND STRESSES.
C*
        WRITE(6,16)
        DO 600 I=1,NUMBE
        IN=2*I
        IS=IN-1
        DSI=DS(I)
        DNI=DN(I)
        ANGI=ANG(I)
        AI=A(I)
        XI=XM(I)
        YI=YM(I)
        SINBI=SINBET(I)
        COSBI=COSBET(I)
C*
        UXNEG=0.
        UYNEG=0.
        PYY=YI*GAMMA
        PXX=YK*PYY
        PXY=0.
        SIGYY=PYY
        SIGXX=PXX
        SIGXY=PXY
        DO 570 J=1,NUMBE
        JN=2*J
        JS=JN-1
        CALL INITL
```

```
      XJ=XM(J)
      YJ=YM(J)
      AJ=A(J)
      COSBJ=COSBET(J)
      SINBJ=SINBET(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,SINBJ,+1)
      GO TO (540,510,520,530),KSYM
C*
 510  XJ=2.*XSYM-XM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      GO TO 540
C*
 520  YJ=2.*YSYM-YM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      GO TO 540
C*
 530  XJ=2.*XSYM-XM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      XJ=XM(J)
      YJ=2.*YSYM-YM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      XJ=2.*XSYM-XM(J)
      CALL COEFF(XI,YI,XJ,YJ,AJ,-COSBJ,-SINBJ,+1)
C*
 540  CONTINUE
C*
      UXNEG=UXNEG+UXS*D(JS)+UXN*D(JN)
      UYNEG=UYNEG+UYS*D(JS)+UYN*D(JN)
      SIGXX=SIGXX+SXXS*D(JS)+SXXN*D(JN)
      SIGYY=SIGYY+SYYS*D(JS)+SYYN*D(JN)
      SIGXY=SIGXY+SXYS*D(JS)+SXYN*D(JN)
C*
 570  CONTINUE
      USNEG=UXNEG*COSBI+UYNEG*SINBI
      UNNEG=-UXNEG*SINBI+UYNEG*COSBI
      USPOS=USNEG-D(IS)
      UNPOS=UNNEG-D(IN)
      UXPOS=USPOS*COSBI-UNPOS*SINBI
      UYPOS=USPOS*SINBI+UNPOS*COSBI
      SIGS=(SIGYY-SIGXX)*SINBI*COSBI+SIGXY*(COSBI**2-SINBI**2)
      SIGN=SIGXX*SINBI**2-2.*SIGXY*SINBI*COSBI+SIGYY*COSBI**2
      SIGT1=SIGXX*COSBI*COSBI+2.*SIGXY*SINBI*COSBI+SIGYY*SINBI*SINBI
      CALL SIGTAN (A,DS,DN,ANG,SIGT1,SIGTP,SIGTN,I,NUMBE)
      WRITE(6,17)I,D(IS),USNEG,USPOS,D(IN),UNNEG,UNPOS,UXNEG,UYNEG,
     *UXPOS,UYPOS,SIGS,SIGN,SIGT1,SIGTP,SIGTN
C*
 600  CONTINUE
C*
C*    COMPUTE DISPLACEMENTS AND STRESSES AT SPECIFIED POINTS IN BODY.
C*
      IF(NUMOS.LE.0)GO TO 900
      WRITE(6,18)
      NPOINT=0
      DO 890 N=1,NUMOS
      READ(5,19) XBEG,YBEG,XEND,YEND,NUMPB
```

```
      NUMP=NUMPB+1
      DELX=(XEND-XBEG)/NUMP
      DELY=(YEND-YBEG)/NUMP
      IF(NUMPB.GT.0)NUMP=NUMP+1
      IF(DELX**2+DELY**2.EQ.0)NUMP=1
C*
      DO 890 NI=1,NUMP
      XP(NI)=XBEG+(NI-1)*DELX
      YP=YBEG+(NI-1)*DELY
      UX(NI)=0.
      UY=0.
      PYY=GAMMA*YP
      PXX=YK*PYY
      PXY=0.
      SIGXX=PXX
      SIGYY=PYY
      SIGXY=PXY
C*
      DO 880 J=1,NUMBE
      JN=2*J
      JS=JN-1
      CALL INITL
      XJ=XM(J)
      YJ=YM(J)
      AJ=A(J)
C*
      IF(SQRT((XP(NI)-XJ)**2+(YP-YJ)**2).LT.2*AJ)GO TO 890
C*
      COSBJ=COSBET(J)
      SINBJ=SINBET(J)
      CALL COEFF(XP(NI),YP,XJ,YJ,AJ,COSBJ,SINBJ,+1)
      GO TO (840,810,820,830),KSYM
C*
 810  XJ=2.*XSYM-XM(J)
      CALL COEFF(XP(NI),YP,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      GO TO 840
C*
 820  YJ=2.*YSYM-YM(J)
      CALL COEFF(XP(NI),YP,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      GO TO 840
C*
 830  XJ=2.*XSYM-XM(J)
      CALL COEFF(XP(NI),YP,XJ,YJ,AJ,COSBJ,-SINBJ,-1)
      XJ=XM(J)
      YJ=2.*YSYM-YM(J)
      CALL COEFF(XP(NI),YP,XJ,YJ,AJ,-COSBJ,SINBJ,-1)
      XJ=2.*XSYM-XM(J)
      CALL COEFF(XP(NI),YP,XJ,YJ,AJ,-COSBJ,-SINBJ,+1)
C*
 840  CONTINUE
C*
      UX(NI)=UX(NI)+UXS*D(JS)+UXN*D(JN)
      UY=UY+UYS*D(JS)+UYN*D(JN)
      SIGXX=SIGXX+SXXS*D(JS)+SXXN*D(JN)
      SIGYY=SIGYY+SYYS*D(JS)+SYYN*D(JN)
```

```
      SIGXY=SIGXY+SXYS*D(JS)+SXYN*D(JN)
C*
 880  CONTINUE
C*
      NPOINT=NPOINT+1
      WRITE(6,21) NPOINT,XP(NI),YP,UX(NI),UY,SIGXX,SIGYY,SIGXY
C*
 890  CONTINUE
C***********************************************************************
C*                                                                     *
C*  COMPUTATION OF HORIZONTAL STRAIN BY USING FINITE DIFFERENCE        *
C*                            METHOD                                   *
C*                                                                     *
C***********************************************************************
      WRITE (6,20)
      DO 892 NI=1,NUMP
      CALL STRAIN(UX,XP,NI,STR,NUMP)
      WRITE (6,22) NI,XP(NI),STR(NI)
 892  CONTINUE
 900  CONTINUE
C*    FORMAT STATEMENTS
C*
 1    FORMAT(20A4)
 2    FORMAT(1H1,/,25X,20A4,/)
 3    FORMAT(3I4)
 4    FORMAT(F6.2,E11.4,2F12.4)
 5    FORMAT(3E11.4)
 6    FORMAT(/, " NUMBER OF STRAIT-LINE SEGMENTS (EACH CONTAINING AT L
     *EAST ONE BOUNDARY ELEMENT) USED TO DEFINE BOUNDARIES =",I3,//,123H
     * NUMBER OF STRAIGHT-LINE SEGMENTS USED TO SPECIFY OTHER LOCATIONS
     *(I.E.,NOT ON A BOUNDARY) WHERE RESULTS ARE TO BE FOUND =, I3)
 7    FORMAT(/,32H NO SYMMETRY CONDITIONS IMPOSED.)
 8    FORMAT(/,18H THE LINE X = XS =,F12.4,23H IS A LINE OF SYMMETRY.)
 9    FORMAT(/,18H THE LINE Y = YS =,F12.4,23H IS A LINE OF SYMMETRY.)
 10   FORMAT(/,19H THE LINES X = XS =,F12.4,13H AND Y = YS =,F12.4,
     *23H ARE LINES OF SYMMETRY.)
 11   FORMAT(/," POISSION'S RATIO =",F6.2,//," YOUNG'S MODULUS =",
     *E11.4)
 12   FORMAT(/,31H UNIT WEIGHT  OF OVERBURDEN    =,E11.4,//,
     *          31H RATIO  OF FIELD STRESS        =,E11.4,//,
     *          31H XY-COMPONENT OF FIELD STRESS =,E11.4)
 13   FORMAT(1H1,/,27H     BOUNDARY ELEMENT DATA.,//,  "  ELEMENT
     *KODE  X (CENTER)  Y (CENTER)      LENGTH       ANGLE  US OR SIGMA-
     *S  UN OR SIGMA-N",/)
 14   FORMAT (I5,4F10.3,I1,I2,I1,I1,2E10.3)
 15   FORMAT(2I9,3F12.4,F12.2,2E15.4)
 16   FORMAT(1H1,/,  "     DISPLACEMENTS AND STRESSES AT MIDPOINTS OF
     *BOUNDARY ELEMENTS",//,40H ELEM     DS       US(-)     US(+)     DN ,
     * 61H   UN(-)    UN(+)    UX(-)     UY(-)    UX(+)    UY(+)  SIG-,
     * 29HS  SIG-N  SIGT1  SIGTP  SIGTN,/)
 17   FORMAT(2X,I3,4(2X,E7.1),(2X,F7.1),5(2X,E7.1),3(1X,E7.1),2(1X,E7.
     * 1))
 18   FORMAT(1H1,/,  "     DISPLACEMENTS AND STRESSES AT SPECIFIED POI
     *NTS IN THE BODY",//,  "    POINT    X CO-ORD    Y CO-ORD
     *UX          UY        SIGXX        SIGYY        SIGXY",/)
```

```
 19     FORMAT(4F12.4,I4)
 21     FORMAT(I9,2F12.4,2F12.6,3F12.1)
 20     FORMAT(1H1,/, "  HORIZONTAL  STRAIN AT SPECIFIED POINTS ",//,"
     * N-POINT   XP(NI)     STRAIN "/)
 22    FORMAT(I9,F12.4,F12.6)
       END
C*
C*
C*
       SUBROUTINE INITL
C*
       COMMON/S2/SXXS,SXXN,SYYS,SYYN,SXYS,SXYN,UXS,UXN,UYS,UYN
C*
       SXXS=0.
       SXXN=0.
       SYYS=0.
       SYYN=0.
       SXYS=0.
       SXYN=0.
C*
       UXS=0.
       UXN=0.
       UYS=0.
       UYN=0.
C*
       RETURN
       END
C***********************************************************************
C*                                                                     *
C*    THIS SUBROUTINE CALCULATES  INFLUENCE COEFFICIENTS OF SEMI      *
C*                 INFINITE PLANE PROBLEM SOLUTIONS                    *
C*                                                                     *
C***********************************************************************
       SUBROUTINE COEFF(X,Y,CX,CY,A,COSB,SINB,MSYM)
C*
       COMMON/S1/PI,PR,PR1,PR2,PR3,CON,CONS
       COMMON/S2/SXXS,SXXN,SYYS,SYYN,SXYS,SXYN,UXS,UXN,UYS,UYN
C*
       TXYDS=0.
       TXYDN=0.
       TYYDS=0.
       TYYDN=0.
       VXDS=0.
       UUXDS=0.
       COS2B=COSB**2-SINB**2
       SIN2B=2.*SINB*COSB
       COSB2=COSB**2
       SINB2=SINB**2
       SIN3B=SINB*COS2B+COSB*SIN2B
       COS3B=COSB*COS2B-SINB*SIN2B
       SIN4B=SIN2B*COS2B+COS2B*SIN2B
       COS4B=COS2B*COS2B-SIN2B*SIN2B
```

```
C************************************************************************
C*                                                                      *
C*      THIS PART OF THE SUBROUTINE IS FOR THE ACTUAL INFLUENCE         *
C*         COEFICIENT  CALCULATION OF SEMI-INFINITE PROBLEM             *
C************************************************************************
        XB=(X-CX)*COSB+(Y-CY)*SINB
        YB=-(X-CX)*SINB+(Y-CY)*COSB
C*
        R1S=(XB-A)**2+YB**2
        R2S=(XB+A)**2+YB**2
        FL1=0.5*ALOG(R1S)
        FL2=0.5*ALOG(R2S)
        FB2=CON*(FL1-FL2)
        IF(YB.NE.0.) GO TO 10
        FB3=0.
        IF(ABS(XB).LT.A)FB3=CON*PI
        GO TO 20
 10     FB3=-CON*(ATAN((XB+A)/YB)-ATAN((XB-A)/YB))
 20     FB4=CON*(YB/R1S-YB/R2S)
        FB5=CON*((XB-A)/R1S-(XB+A)/R2S)
        FB6=CON*(((XB-A)**2-YB**2)/R1S**2-((XB+A)**2-YB**2)/R2S**2)
        FB7=2.*CON*YB*((XB-A)/R1S**2-(XB+A)/R2S**2)
C*
        VXDS=-PR1*SINB*FB2+PR2*COSB*FB3+YB*(SINB*FB4-COSB*FB5)
        VXDN=-PR1*COSB*FB2-PR2*SINB*FB3-YB*(COSB*FB4+SINB*FB5)
        VYDS=PR1*COSB*FB2+PR2*SINB*FB3-YB*(COSB*FB4+SINB*FB5)
        VYDN=-PR1*SINB*FB2+PR2*COSB*FB3-YB*(SINB*FB4-COSB*FB5)
C*
        TXXDS=CONS*(2.*COSB2*FB4+SIN2B*FB5+YB*(COS2B*FB6-SIN2B*FB7))
        TXXDN=CONS*(-FB5+YB*(SIN2B*FB6+COS2B*FB7))
        TYYDS=CONS*(2.*SINB2*FB4-SIN2B*FB5-YB*(COS2B*FB6-SIN2B*FB7))
        TYYDN=CONS*(-FB5-YB*(SIN2B*FB6+COS2B*FB7))
        TXYDS=CONS*(SIN2B*FB4-COS2B*FB5+YB*(SIN2B*FB6+COS2B*FB7))
        TXYDN=CONS*(-YB*(COS2B*FB6-SIN2B*FB7))
C************************************************************************
C*                                                                      *
C*      THIS PART OF THE SUBROUTINE IS FOR THE INFLUENCE COEFICIENT     *
C*      OF SUPLEMENTARY AND IMAGE SUPERPOSITION OF SEMI-INFINITE        *
C*                            PLANE                                     *
C************************************************************************
        CF=(X-CX)*COSB-(Y+CY)*SINB
        HF=(X-CX)*SINB+(Y+CY)*COSB
C*
        R3S=(CF-A)**2+HF**2
        R4S=(CF+A)**2+HF**2
        FL3=.5*ALOG(R3S)
        FL4=.5*ALOG(R4S)
        FH2=CON*(FL3-FL4)
        IF(HF.NE.0.) GO TO 21
        FH3=0.
        IF(ABS(CF).LT.A)FH3=CON*PI
        GO TO 31
 21     FH3=-CON*(ATAN((CF+A)/HF)-ATAN((CF-A)/HF))
 31     FH4=CON*(HF/R3S-HF/R4S)
        FH5=CON*((CF-A)/R3S-(CF+A)/R4S)
```

```
      FH6=CON*(((CF-A)**2-HF**2)/R3S**2-((CF+A)**2-HF**2)/R4S**2)
      FH7=2.*CON*HF*((CF-A)/R3S**2-(CF+A)/R4S**2)
      FH8=-2.*CON*HF*((((CF-A)**2+HF**2)+2.*((CF-A)**2-HF**2))/R3S**3-
     *(((CF+A)**2+HF**2)+2.*((CF+A)**2-HF**2))/R4S**3)
      FH9=2.*CON*(((CF-A)**2+HF**2)*(CF-A)*((CF-A)**2-4.*HF**2)/R3S**4
     *-(CF+A)*((CF+A)**2+HF**2)*((CF+A)**2-4.*HF**2)/R4S**4)
C*
C*
      UUXDS=PR1*SINB*FH2-PR2*COSB*FH3+(PR3*(Y*SIN2B-YB*SINB)+2.*Y*SIN2B
     *)*FH4+(PR3*(Y*COS2B-YB*COSB)-Y*(1.-2.*COS2B))*FH5+2.*Y*(Y*SIN3B-YB
     **SIN2B)*FH6-2.*Y*(Y*COS3B-YB*COS2B)*FH7
C*
      UUXDN=PR1*COSB*FH2+PR2*SINB*FH3-(PR3*(Y*COS2B-YB*COSB)-Y)*FH4+PR3
     **(Y*SIN2B-YB*SINB)*FH5-2.*Y*(Y*COS3B-YB*COS2B)*FH6-2.*Y*(Y*SIN3B-Y
     *B*SIN2B)*FH7
      UUYDS=-PR1*COSB*FH2-PR2*SINB*FH3-(PR3*(Y*COS2B-YB*COSB)+Y*(1.-2.*
     *COS2B))*FH4+(PR3*(Y*SIN2B-YB*SINB)-2.*Y*SIN2B)*FH5+2.*Y*(Y*COS3B-Y
     *B*COS2B)*FH6+2.*Y*(Y*SIN3B-YB*SIN2B)*FH7
      UUYDN=PR1*SINB*FH2-PR2*COSB*FH3-PR3*(Y*SIN2B-YB*SINB)*FH4-(PR3*(Y
     **COS2B-YB*COSB)+Y)*FH5+2.*Y*(Y*SIN3B-YB*SIN2B)*FH6-2.*Y*(Y*COS3B-Y
     *B*COS2B)*FH7
C*
C*
      TTXXDS=CONS*(FH4-3.*(COS2B*FH4-SIN2B*FH5)+(2.*Y*(COSB-3.*COS3B)+3
     *.*YB*COS2B)*FH6+(2.*Y*(SINB-3.*SIN3B)+3.*YB*SIN2B)*FH7-2.*Y*(Y*COS
     *4B-YB*COS3B)*FH8-2.*Y*(Y*SIN4B-YB*SIN3B)*FH9)
      TTXXDN=CONS*(FH5+(2.*Y*(SINB-2.*SIN3B)+3.*YB*SIN2B)*FH6-(2.*Y*(CO
     *SB-2.*COS3B)+3.*YB*COS2B)*FH7-2.*Y*(Y*SIN4B-YB*SIN3B)*FH8+2.*Y*(Y*
     *COS4B-YB*COS3B)*FH9)
      TTYYDS=CONS*(FH4-(COS2B*FH4-SIN2B*FH5)-(4.*Y*SINB*SIN2B-YB*COS2B)
     **FH6+(4.*Y*SINB*COS2B+YB*SIN2B)*FH7+2.*Y*(Y*COS4B-YB*COS3B)*FH8+2.
     **Y*(Y*SIN4B-YB*SIN3B)*FH9)
      TTYYDN=CONS*(FH5-(2.*Y*SINB-YB*SIN2B)*FH6+(2.*Y*COSB-YB*COS2B)*FH
     *7+2.*Y*(Y*SIN4B-YB*SIN3B)*FH8-2.*Y*(Y*COS4B-YB*COS3B)*FH9)
      TTXYDS=CONS*(SIN2B*FH4+COS2B*FH5+(2.*Y*SINB*(1.+4.*COS2B)-YB*SIN2
     *B)*FH6+(2.*Y*COSB*(3.-4.*COS2B)+YB*COS2B)*FH7+2.*Y*(Y*SIN4B-YB*SIN
     *3B)*FH8-2.*Y*(Y*COS4B-YB*COS3B)*FH9)
      TTXYDN=CONS*((4.*Y*SINB*SIN2B+YB*COS2B)*FH6-(4.*Y*SINB*COS2B-YB*S
     *IN2B)*FH7-2.*Y*(Y*COS4B-YB*COS3B)*FH8-2.*Y*(Y*SIN4B-YB*SIN3B)*FH9)
C*
       UXDS=VXDS+UUXDS
       UXDN=VXDN+UUXDN
       UYDS=VYDS+UUYDS
       UYDN=VYDN+UUYDN
       UXS=UXS+MSYM*UXDS
       UXN=UXN+UXDN
       UYS=UYS+MSYM*UYDS
       UYN=UYN+UYDN
C*
       SXXDS=TXXDS+TTXXDS
       SXXDN=TXXDN+TTXXDN
       SYYDS=TYYDS+TTYYDS
       SYYDN=TYYDN+TTYYDN
       SXYDS=TXYDS+TTXYDS
       SXYDN=TXYDN+TTXYDN
```

```
      SXXS=SXXS+MSYM*SXXDS
      SXXN=SXXN+SXXDN
      SYYS=SYYS+MSYM*SYYDS
      SYYN=SYYN+SYYDN
      SXYS=SXYS+MSYM*SXYDS
      SXYN=SXYN+SXYDN
C*    PRINT/,SXXS,SXXN,SYYS,SYYN,SXYS,SXYN
C*
      RETURN
      END
C*
C*
C*
      SUBROUTINE SOLVE(N)
C*
      COMMON/S3/A(200,200),B(200),X(200)
C*
      NB=N-1
      DO 20 J=1,NB
      L=J+1
      DO 20 JJ=L,N
C*    IF(A(J,J).EQ.0.)A(J,J)=.00001
      XM=A(JJ,J)/A(J,J)
      DO 10 I=J,N
 10   A(JJ,I)=A(JJ,I)-A(J,I)*XM
 20   B(JJ)=B(JJ)-B(J)*XM
C*    IF(A(N,N).EQ.0.)A(N,N)=.00001
      X(N)=B(N)/A(N,N)
      DO 40 J=1,NB
      JJ=N-J
      L=JJ+1
      SUM=0.
      DO 30 I=L,N
 30   SUM=SUM+A(JJ,I)*X(I)
 40   X(JJ)=(B(JJ)-SUM)/A(JJ,JJ)
C*
      RETURN
      END
C**********************************************************************
C*                                                                    *
C*    THIS SUBROUTINE CALCULATES TANGENTIAL STRESS ALONG THE          *
C*                         BOUNDARY                                   *
C**********************************************************************
      SUBROUTINE SIGTAN (A,DS,DN,ANG,SIGT1,SIGTP,SIGTN,I,NUMBE)
C*
      COMMON/S1/PI,PR,PR1,PR2,PR3,CON,CONS

C*
C*
C*
      DIMENSION A(100),DN(100),DS(100),ANG(100)
C*
      F=ANG(I)
      G=ANG(I+1)
      Z=G-F
```

```
C*
      IF(I.EQ.1)GO TO 10
      H=ANG(I-1)
      V=F-H
      IF (I.EQ.NUMBE)GO TO 20
      DELS=A(I-1)*COS(V)+2*A(I)+A(I+1)*COS(Z)
      SIGTDI=CONS/PR2*(DS(I+1)*COS(Z)-DS(I-1)*COS(V)-DN(I+1)*SIN(Z)-DN
     *(I-1)*SIN(V))/DELS
C*    PRINT/,DELS,SIGT1,SIGTDI,DS(I),DN(I)
      GO TO 150
10    DELS=A(I)+A(I+1)*COS(Z)
      SIGTDI=CONS/PR2*(DS(I+1)*COS(Z)-DN(I+1)*SIN(Z)-DS(I))/DELS
C*    PRINT/,DELS,SIGT1,SIGTDI,DS(I),DN(I)
      GO TO 150
20    DELS=A(I)+A(I-1)*COS(V)
      SIGTDI=CONS/PR2*(DS(I)-DS(I-1)*COS(V)-DN(I-1)*SIN(V))/DELS
C*    PRINT/,DELS,SIGT1,SIGTDI,DS(I),DN(I)
150   SIGTP=SIGT1-SIGTDI
      SIGTN=SIGT1+SIGTDI
      RETURN
      END
C************************************************************************
C*                                                                      *
C*     THIS SUBROUTINE CALCULATES THE HORIZANTAL STRAIN  AT             *
C*                   SPECIFIED POINTS                                   *
C************************************************************************
      SUBROUTINE STRAIN(UX,XP,KI,STR,NUMP)
      DIMENSION UX(100),XP(100),STR(100)
      IF(KI.EQ.1) GOTO 13
      IF(KI.EQ.NUMP) GO TO 14
      FX=UX(KI+1)-UX(KI-1)
      XPS=XP(KI+1)-XP(KI-1)
      STR(KI)=FX/XPS
      GO TO 175
13    FX=UX(KI+1)-UX(KI)
      XPS=XP(KI+1)-XP(KI)
      STR(KI)=FX/XPS
      GOTO 175
14    FX=UX(KI)-UX(KI-1)
      XPS=XP(KI)-XP(KI-1)
      STR(KI)=FX/XPS
175   RETURN
      END
```

(*) Modified TWODD consists of a main program and five subroutines. The main program handles all input/output operations and contains the logic needed to define the boundary element locations, set up the system of algebraic equations and compute the unspecified boundary parameters (displacements or tractions) as well as the displacements and stresses at all field points, in terms of displacement discontinuity components at all the boundary elements. The influence coefficient are evaluated in subroutine COEFF, and the results are used in the main program to compute the displacement for both boundary points and field points. Subroutine INITL is used to initialize these computations for cases in which the symmetry conditions are imposed , so that image boundary elements are generated with in the program. The imaging process is done by making successive calls to COeff, after suitably defining the co-ordinates and inclinations of the image elements. The tangential stress value is computed using finite difference approximation in subroutine SIGTAN. The same approximation is also used in subroutine SITRAIN in order to calculate horizantel strain values.Finally, subroutine SOLVE solves the system of algebraic equations defined in the main program. The solution procedure uses Gaussian elimination. without pivoting.

As listed above, program twodd requires at least above 42K central memory locations. But it depends mainly on the used elements number.The first statement (the PROGRAM statement) is neded for Burroughs system;this statement must be changed to run program on other system or personel computers, systems(e.g to run in pc's this statement must be inserted after dimensions' statement).The process time to run the program is related to elements number.It takes twenty to fifty minutes according to used elements for the running of subsidence model.

APPENDIX B.1 EXAMPLE PROBLEM TO CHECK THE CORRECTNESS OF THE TWODD PROGRAM

A circular hole in an semi-infinite body under uniaxial tension at infinity problem shown in Figuer B 1.1 is considered for the checking the correctness of the modified TWODD firstly, then the same problem is experienced with gravitational loading case. The results are in concurrence with analytical solutions those can be found easily by using Kirsch equations.

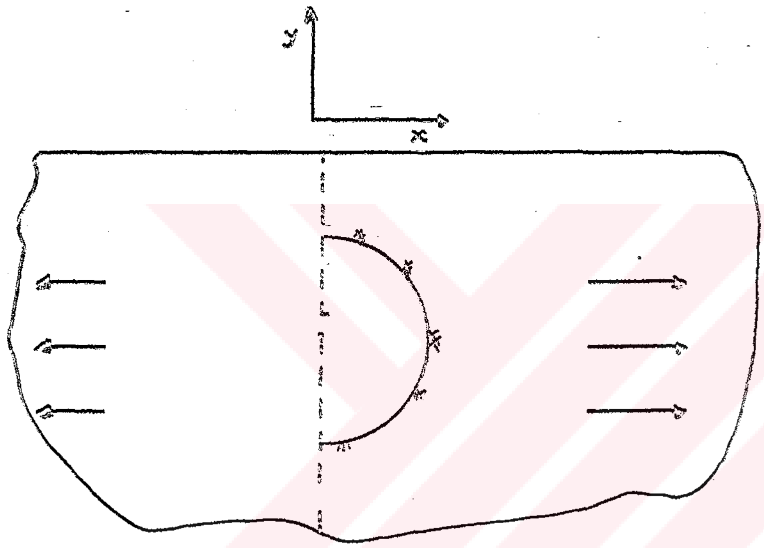

Figure B 1.1 Circular hole in an semi-infinite body under uniaxial tension at infinity.

In this example and the other experienced model for subsidence prediction, only the half of the problem is identified for specifying boundary condition due to symmetry.

Aditionaly, an extra boundary elemet must be specified with zero displacement (fixed point) in order to prevent rigid body motion in the body in any directions. The last element specified in the output is that one.

THE OUTPUT OF THE EXAMPLE PROBLEM

THE CIRCULAR HOLE IN A SEMI-INFINITE BODY UNDER UNIAXIAL TENSION AT INFINITY

STRAIGTH-LINE SEGMENTS (EACH CONTAINING AT L EAST ONE BOUNDARY ELEMENT) USED TO DEFINE BOUNDARIES = 1

NUMBER OF STRAIGHT-LINE SEGMENTS USED TO SPECIFY OTHER LOCATIONS (NOT ON A BOUNDARY) =1

THE LINE X = XS = 0.0000 IS A LINE OF SYMMETRY.

POISSION'S RATIO = 0.17

YOUNG'S MODULUS = .8000E+04

XX-COMPONENT OF FIELD STRESS = .1000E+03

YY-COMPONENT OF FIELD STRESS = 0.

XY-COMPONENT OF FIELD STRESS = 0.

1

BOUNDARY ELEMENT DATA.

ELEMENT	KODE	X (CENTER)	Y (CENTER)	LENGTH	ANGLE	US OR SIGMA-S	UN OR SIGMA-N
1	1	0.0160	-202.9997	0.0321	0.92	0.	0.
2	1	0.0481	-202.9987	0.0321	2.76	0.	0.
3	1	0.0800	-202.9967	0.0321	4.59	0.	0.
4	1	0.1120	-202.9936	0.0321	6.43	0.	0.
5	1	0.1437	-202.9895	0.0321	8.27	0.	0.
6	1	0.1754	-202.9844	0.0321	10.10	0.	0.
7	1	0.2068	-202.9782	0.0321	11.94	0.	0.
8	1	0.2381	-202.9711	0.0321	13.78	0.	0.
9	1	0.2691	-202.9630	0.0321	15.61	0.	0.
10	1	0.2998	-202.9539	0.0321	17.45	0.	0.
11	1	0.3302	-202.9438	0.0321	19.29	0.	0.
12	1	0.3603	-202.9327	0.0321	21.12	0.	0.
13	1	0.3900	-202.9207	0.0321	22.96	0.	0.
14	1	0.4193	-202.9077	0.0321	24.80	0.	0.
15	1	0.4482	-202.8938	0.0321	26.63	0.	0.
16	1	0.4766	-202.8790	0.0321	28.47	0.	0.
17	1	0.5046	-202.8632	0.0321	30.31	0.	0.
18	1	0.5320	-202.8466	0.0321	32.14	0.	0.
19	1	0.5588	-202.8291	0.0321	33.98	0.	0.
20	1	0.5851	-202.8108	0.0321	35.82	0.	0.
21	1	0.6108	-202.7916	0.0321	37.65	0.	0.
22	1	0.6359	-202.7716	0.0321	39.49	0.	0.
23	1	0.6603	-202.7509	0.0321	41.33	0.	0.
24	1	0.6840	-202.7293	0.0321	43.16	0.	0.
25	1	0.7070	-202.7070	0.0321	45.00	0.	0.
26	1	0.7293	-202.6840	0.0321	46.84	0.	0.
27	1	0.7509	-202.6603	0.0321	48.67	0.	0.
28	1	0.7716	-202.6359	0.0321	50.51	0.	0.
29	1	0.7916	-202.6108	0.0321	52.35	0.	0.
30	1	0.8108	-202.5851	0.0321	54.18	0.	0.

31	1	0.8291	-202.5588	0.0321	56.02	0.	0.
32	1	0.8466	-202.5320	0.0321	57.86	0.	0.
33	1	0.8632	-202.5046	0.0321	59.69	0.	0.
34	1	0.8790	-202.4766	0.0321	61.53	0.	0.
35	1	0.8938	-202.4482	0.0321	63.37	0.	0.
36	1	0.9077	-202.4193	0.0321	65.20	0.	0.
37	1	0.9207	-202.3900	0.0321	67.04	0.	0.
38	1	0.9327	-202.3603	0.0321	68.88	0.	0.
39	1	0.9438	-202.3302	0.0321	70.71	0.	0.
40	1	0.9539	-202.2998	0.0321	72.55	0.	0.
41	1	0.9630	-202.2691	0.0321	74.39	0.	0.
42	1	0.9711	-202.2381	0.0321	76.22	0.	0.
43	1	0.9782	-202.2068	0.0321	78.06	0.	0.
44	1	0.9844	-202.1754	0.0321	79.90	0.	0.
45	1	0.9895	-202.1437	0.0321	81.73	0.	0.
46	1	0.9936	-202.1120	0.0321	83.57	0.	0.
47	1	0.9967	-202.0800	0.0321	85.41	0.	0.
48	1	0.9987	-202.0481	0.0321	87.24	0.	0.
49	1	0.9997	-202.0160	0.0321	89.08	0.	0.
50	1	0.9997	-201.9840	0.0321	90.92	0.	0.
51	1	0.9987	-201.9519	0.0321	92.76	0.	0.
52	1	0.9967	-201.9200	0.0321	94.59	0.	0.
53	1	0.9936	-201.8880	0.0321	96.43	0.	0.
54	1	0.9895	-201.8563	0.0321	98.27	0.	0.
55	1	0.9844	-201.8246	0.0321	100.10	0.	0.
56	1	0.9782	-201.7932	0.0321	101.94	0.	0.
57	1	0.9711	-201.7619	0.0321	103.78	0.	0.
58	1	0.9630	-201.7309	0.0321	105.61	0.	0.
59	1	0.9539	-201.7002	0.0321	107.45	0.	0.
60	1	0.9438	-201.6698	0.0321	109.29	0.	0.
61	1	0.9327	-201.6397	0.0321	111.12	0.	0.
62	1	0.9207	-201.6100	0.0321	112.96	0.	0.
63	1	0.9077	-201.5807	0.0321	114.80	0.	0.
64	1	0.8938	-201.5518	0.0321	116.63	0.	0.
65	1	0.8790	-201.5234	0.0321	118.47	0.	0.
66	1	0.8632	-201.4954	0.0321	120.31	0.	0.
67	1	0.8466	-201.4680	0.0321	122.14	0.	0.
68	1	0.8291	-201.4412	0.0321	123.98	0.	0.
69	1	0.8108	-201.4149	0.0321	125.82	0.	0.
70	1	0.7916	-201.3892	0.0321	127.65	0.	0.
71	1	0.7716	-201.3641	0.0321	129.49	0.	0.
72	1	0.7509	-201.3397	0.0321	131.33	0.	0.
73	1	0.7293	-201.3160	0.0321	133.16	0.	0.
74	1	0.7070	-201.2930	0.0321	135.00	0.	0.
75	1	0.6840	-201.2707	0.0321	136.84	0.	0.
76	1	0.6603	-201.2491	0.0321	138.67	0.	0.
77	1	0.6359	-201.2284	0.0321	140.51	0.	0.
78	1	0.6108	-201.2084	0.0321	142.35	0.	0.
79	1	0.5851	-201.1892	0.0321	144.18	0.	0.
80	1	0.5588	-201.1709	0.0321	146.02	0.	0.
81	1	0.5320	-201.1534	0.0321	147.86	0.	0.
82	1	0.5046	-201.1368	0.0321	149.69	0.	0.
83	1	0.4766	-201.1210	0.0321	151.53	0.	0.
84	1	0.4482	-201.1062	0.0321	153.37	0.	0.
85	1	0.4193	-201.0923	0.0321	155.20	0.	0.

86	1	0.3900	-201.0793	0.0321	157.04	0.	0.
87	1	0.3603	-201.0673	0.0321	158.88	0.	0.
88	1	0.3302	-201.0562	0.0321	160.71	0.	0.
89	1	0.2998	-201.0461	0.0321	162.55	0.	0.
90	1	0.2691	-201.0370	0.0321	164.39	0.	0.
91	1	0.2381	-201.0289	0.0321	166.22	0.	0.
92	1	0.2068	-201.0218	0.0321	168.06	0.	0.
93	1	0.1754	-201.0156	0.0321	169.90	0.	0.
94	1	0.1437	-201.0105	0.0321	171.73	0.	0.
95	1	0.1120	-201.0064	0.0321	173.57	0.	0.
96	1	0.0800	-201.0033	0.0321	175.41	0.	0.
97	1	0.0481	-201.0013	0.0321	177.24	0.	0.
98	1	0.0160	-201.0003	0.0321	179.08	0.	0.

1

DISPLACEMENTS AND STRESSES AT MIDPOINTS OF BOUNDARY ELEMENTS

ELEM	DS	US(-)	US(+)	DN	UN(-)	UN(+)	UX(-)	UY(-)	SIG-S	SIG-N	SIGT1	SIGTP	SIGTN
1	.7E-03	.5E-03	-.2E-03	.7E-02	.1E-01	.3E-02	.4E-03	.1E-01	0.0	0.0	146.5	-4.9	297.9
2	.2E-02	.2E-02	-.5E-03	.7E-02	.1E-01	.3E-02	.1E-02	.1E-01	-0.0	0.0	146.1	-5.0	297.2
3	.4E-02	.3E-02	-.8E-03	.7E-02	.9E-02	.3E-02	.2E-02	.1E-01	0.0	0.0	145.3	-4.9	295.5
4	.5E-02	.4E-02	-.1E-02	.6E-02	.9E-02	.3E-02	.3E-02	.1E-01	0.0	0.0	144.1	-4.9	293.1
5	.6E-02	.5E-02	-.1E-02	.6E-02	.9E-02	.3E-02	.4E-02	.1E-01	-0.0	0.0	142.5	-4.8	289.9
6	.8E-02	.6E-02	-.2E-02	.5E-02	.9E-02	.3E-02	.4E-02	.9E-02	0.0	0.0	140.6	-4.7	285.9
7	.9E-02	.7E-02	-.2E-02	.5E-02	.8E-02	.3E-02	.5E-02	.9E-02	0.0	-0.0	138.2	-4.7	281.1
8	.1E-01	.8E-02	-.2E-02	.4E-02	.8E-02	.3E-02	.6E-02	.9E-02	0.0	0.0	135.5	-4.5	275.6
9	.1E-01	.9E-02	-.2E-02	.4E-02	.7E-02	.4E-02	.7E-02	.9E-02	-0.0	0.0	132.4	-4.4	269.3
10	.1E-01	.1E-01	-.3E-02	.3E-02	.7E-02	.4E-02	.7E-02	.9E-02	-0.0	0.0	129.0	-4.3	262.4
11	.1E-01	.1E-01	-.3E-02	.2E-02	.6E-02	.4E-02	.8E-02	.9E-02	0.0	-0.0	125.3	-4.2	254.7
12	.1E-01	.1E-01	-.3E-02	.9E-03	.5E-02	.4E-02	.9E-02	.9E-02	-0.0	0.0	121.2	-4.0	246.5
13	.2E-01	.1E-01	-.3E-02	-.1E-03	.4E-02	.5E-02	.1E-01	.9E-02	0.0	-0.0	116.9	-3.8	237.6
14	.2E-01	.1E-01	-.4E-02	-.1E-02	.4E-02	.5E-02	.1E-01	.9E-02	-0.0	-0.0	112.3	-3.7	228.2
15	.2E-01	.1E-01	-.4E-02	-.2E-02	.3E-02	.5E-02	.1E-01	.9E-02	0.0	0.0	107.4	-3.5	218.2
16	.2E-01	.1E-01	-.4E-02	-.4E-02	.2E-02	.5E-02	.1E-01	.9E-02	-0.0	0.0	102.2	-3.3	207.8
17	.2E-01	.1E-01	-.4E-02	-.5E-02	.9E-03	.6E-02	.1E-01	.8E-02	-0.0	-0.0	96.9	-3.1	196.9
18	.2E-01	.2E-01	-.4E-02	-.6E-02	-.2E-04	.6E-02	.1E-01	.8E-02	-0.0	0.0	91.4	-2.9	185.6
19	.2E-01	.2E-01	-.4E-02	-.7E-02	-.1E-02	.6E-02	.1E-01	.8E-02	-0.0	0.0	85.6	-2.7	173.9
20	.2E-01	.2E-01	-.4E-02	-.9E-02	-.2E-02	.7E-02	.1E-01	.8E-02	0.0	-0.0	79.8	-2.4	162.0
21	.2E-01	.2E-01	-.4E-02	-.1E-01	-.3E-02	.7E-02	.2E-01	.8E-02	0.0	-0.0	73.8	-2.2	149.8
22	.2E-01	.2E-01	-.4E-02	-.1E-01	-.4E-02	.8E-02	.2E-01	.7E-02	0.0	-0.0	67.7	-2.0	137.4
23	.2E-01	.2E-01	-.4E-02	-.1E-01	-.5E-02	.8E-02	.2E-01	.7E-02	-0.0	-0.0	61.5	-1.7	124.8
24	.2E-01	.2E-01	-.4E-02	-.1E-01	-.6E-02	.8E-02	.2E-01	.7E-02	0.0	-0.0	55.3	-1.5	112.1
25	.2E-01	.2E-01	-.4E-02	-.2E-01	-.7E-02	.9E-02	.2E-01	.7E-02	-0.0	-0.0	49.1	-1.3	99.4
26	.2E-01	.2E-01	-.4E-02	-.2E-01	-.9E-02	.9E-02	.2E-01	.7E-02	-0.0	-0.0	42.8	-1.0	86.6
27	.2E-01	.2E-01	-.4E-02	-.2E-01	-.1E-01	.9E-02	.2E-01	.6E-02	0.0	0.0	36.6	-0.8	74.0
28	.2E-01	.2E-01	-.3E-02	-.2E-01	-.1E-01	.1E-01	.2E-01	.6E-02	0.0	0.0	30.4	-0.6	61.4
29	.2E-01	.2E-01	-.3E-02	-.2E-01	-.1E-01	.1E-01	.2E-01	.6E-02	-0.0	-0.0	24.3	-0.3	49.0
30	.2E-01	.2E-01	-.3E-02	-.2E-01	-.1E-01	.1E-01	.2E-01	.6E-02	-0.0	-0.0	18.3	-0.1	36.8
31	.2E-01	.2E-01	-.3E-02	-.2E-01	-.1E-01	.1E-01	.2E-01	.5E-02	0.0	-0.0	12.5	0.1	24.8
32	.2E-01	.2E-01	-.2E-02	-.3E-01	-.1E-01	.1E-01	.2E-01	.5E-02	-0.0	-0.0	6.8	0.3	13.2
33	.2E-01	.1E-01	-.2E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	.5E-02	-0.0	0.0	1.2	0.6	1.9
34	.2E-01	.1E-01	-.2E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	.5E-02	0.0	-0.0	-4.1	0.8	-9.0
35	.2E-01	.1E-01	-.1E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	.4E-02	0.0	-0.0	-9.3	1.0	-19.5
36	.1E-01	.1E-01	-.1E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	.4E-02	-0.0	0.0	-14.1	1.1	-29.4
37	.1E-01	.1E-01	-.6E-03	-.3E-01	-.2E-01	.1E-01	.2E-01	.4E-02	-0.0	0.0	-18.8	1.3	-38.9

38	.1E-01	.1E-01	-.2E-03	-.3E-01	-.2E-01	.1E-01	.2E-01	.4E-02	-0.0	-0.0	-23.1	1.5	-47.7
39	.1E-01	.1E-01	.3E-03	-.3E-01	-.2E-01	.1E-01	.2E-01	.3E-02	-0.0	0.0	-27.2	1.6	-56.0
40	.9E-02	.1E-01	.7E-03	-.3E-01	-.2E-01	.1E-01	.2E-01	.3E-02	-0.0	0.0	-30.9	1.8	-63.6
41	.8E-02	.9E-02	.1E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	.3E-02	0.0	-0.0	-34.3	1.9	-70.6
42	.6E-02	.8E-02	.2E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.2E-02	0.0	-0.0	-37.4	2.0	-76.8
43	.5E-02	.7E-02	.2E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.2E-02	-0.0	-0.0	-40.1	2.1	-82.3
44	.3E-02	.6E-02	.3E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.2E-02	-0.0	0.0	-42.4	2.2	-87.1
45	.2E-02	.5E-02	.3E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.1E-02	-0.0	-0.0	-44.4	2.3	-91.1
46	.3E-03	.4E-02	.4E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.1E-02	-0.0	-0.0	-46.0	2.4	-94.4
47	-.1E-02	.3E-02	.4E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.8E-03	0.0	0.0	-47.2	2.4	-96.8
48	-.3E-02	.2E-02	.4E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.5E-03	0.0	0.0	-48.0	2.4	-98.4
49	-.4E-02	.6E-03	.5E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	.2E-03	0.0	0.0	-48.4	2.4	-99.2
50	-.6E-02	-.5E-03	.5E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	-.1E-03	-0.0	0.0	-48.4	2.4	-99.2
51	-.8E-02	-.2E-02	.6E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	-.4E-03	-0.0	0.0	-48.0	2.4	-98.4
52	-.9E-02	-.3E-02	.6E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	-.8E-03	-0.0	0.0	-47.2	2.4	-96.8
53	-.1E-01	-.4E-02	.7E-02	-.4E-01	-.2E-01	.1E-01	.2E-01	-.1E-02	0.0	-0.0	-46.0	2.4	-94.4
54	-.1E-01	-.5E-02	.7E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	-.1E-02	-0.0	-0.0	-44.4	2.3	-91.1
55	-.1E-01	-.6E-02	.8E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	-.2E-02	-0.0	-0.0	-42.4	2.2	-87.1
56	-.2E-01	-.7E-02	.8E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	-.2E-02	-0.0	-0.0	-40.1	2.1	-82.3
57	-.2E-01	-.8E-02	.9E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	-.2E-02	0.0	-0.0	-37.4	2.0	-76.8
58	-.2E-01	-.9E-02	.9E-02	-.3E-01	-.2E-01	.1E-01	.2E-01	-.3E-02	-0.0	0.0	-34.3	1.9	-70.6
59	-.2E-01	-.1E-01	.9E-02	-.3E-01	-.2E-01	.9E-02	.2E-01	-.3E-02	-0.0	-0.0	-30.9	1.8	-63.6
60	-.2E-01	-.1E-01	.1E-01	-.3E-01	-.2E-01	.9E-02	.2E-01	-.3E-02	0.0	0.0	-27.2	1.6	-56.0
61	-.2E-01	-.1E-01	.1E-01	-.3E-01	-.2E-01	.8E-02	.2E-01	-.3E-02	-0.0	-0.0	-23.1	1.5	-47.7
62	-.2E-01	-.1E-01	.1E-01	-.3E-01	-.2E-01	.8E-02	.2E-01	-.4E-02	0.0	-0.0	-18.8	1.3	-38.9
63	-.2E-01	-.1E-01	.1E-01	-.3E-01	-.2E-01	.7E-02	.2E-01	-.4E-02	-0.0	-0.0	-14.1	1.1	-29.4
64	-.2E-01	-.1E-01	.1E-01	-.2E-01	-.2E-01	.7E-02	.2E-01	-.4E-02	-0.0	0.0	-9.3	1.0	-19.5
65	-.3E-01	-.1E-01	.1E-01	-.2E-01	-.2E-01	.6E-02	.2E-01	-.5E-02	0.0	-0.0	-4.1	0.8	-9.0
66	-.3E-01	-.1E-01	.1E-01	-.2E-01	-.2E-01	.6E-02	.2E-01	-.5E-02	-0.0	-0.0	1.2	0.6	1.9
67	-.3E-01	-.2E-01	.1E-01	-.2E-01	-.1E-01	.5E-02	.2E-01	-.5E-02	0.0	0.0	6.8	0.3	13.2
68	-.3E-01	-.2E-01	.1E-01	-.2E-01	-.1E-01	.5E-02	.2E-01	-.5E-02	0.0	0.0	12.5	0.1	24.8
69	-.3E-01	-.2E-01	.1E-01	-.2E-01	-.1E-01	.4E-02	.2E-01	-.6E-02	0.0	-0.0	18.3	-0.1	36.8
70	-.3E-01	-.2E-01	.1E-01	-.2E-01	-.1E-01	.4E-02	.2E-01	-.6E-02	-0.0	-0.0	24.3	-0.3	49.0
71	-.3E-01	-.2E-01	.1E-01	-.1E-01	-.1E-01	.3E-02	.2E-01	-.6E-02	0.0	-0.0	30.4	-0.6	61.4
72	-.3E-01	-.2E-01	.1E-01	-.1E-01	-.1E-01	.2E-02	.2E-01	-.6E-02	-0.0	0.0	36.6	-0.8	74.0
73	-.3E-01	-.2E-01	.1E-01	-.1E-01	-.9E-02	.2E-02	.2E-01	-.7E-02	0.0	0.0	42.8	-1.0	86.6
74	-.3E-01	-.2E-01	.1E-01	-.9E-02	-.7E-02	.1E-02	.2E-01	-.7E-02	0.0	-0.0	49.1	-1.3	99.4
75	-.3E-01	-.2E-01	.1E-01	-.7E-02	-.6E-02	.6E-03	.2E-01	-.7E-02	-0.0	-0.0	55.3	-1.5	112.1
76	-.3E-01	-.2E-01	.1E-01	-.5E-02	-.5E-02	.3E-04	.2E-01	-.7E-02	0.0	0.0	61.5	-1.7	124.8
77	-.3E-01	-.2E-01	.1E-01	-.4E-02	-.4E-02	-.5E-03	.2E-01	-.7E-02	0.0	-0.0	67.7	-2.0	137.4
78	-.3E-01	-.2E-01	.1E-01	-.2E-02	-.3E-02	-.1E-02	.2E-01	-.8E-02	-0.0	-0.0	73.8	-2.2	149.8
79	-.3E-01	-.2E-01	.1E-01	-.4E-03	-.2E-02	-.2E-02	.1E-01	-.8E-02	0.0	0.0	79.8	-2.4	162.0
80	-.3E-01	-.2E-01	.1E-01	.1E-02	-.1E-02	-.2E-02	.1E-01	-.8E-02	-0.0	-0.0	85.6	-2.7	173.9
81	-.2E-01	-.2E-01	.1E-01	.3E-02	-.7E-04	-.3E-02	.1E-01	-.8E-02	-0.0	0.0	91.4	-2.9	185.6
82	-.2E-01	-.1E-01	.9E-02	.4E-02	.9E-03	-.3E-02	.1E-01	-.8E-02	-0.0	-0.0	96.9	-3.1	196.9
83	-.2E-01	-.1E-01	.9E-02	.6E-02	.2E-02	-.4E-02	.1E-01	-.8E-02	0.0	-0.0	102.2	-3.3	207.8
84	-.2E-01	-.1E-01	.8E-02	.7E-02	.3E-02	-.4E-02	.1E-01	-.9E-02	0.0	-0.0	107.4	-3.5	218.2
85	-.2E-01	-.1E-01	.8E-02	.8E-02	.4E-02	-.5E-02	.1E-01	-.9E-02	-0.0	0.0	112.3	-3.7	228.2
86	-.2E-01	-.1E-01	.7E-02	.9E-02	.4E-02	-.5E-02	.1E-01	-.9E-02	0.0	0.0	116.9	-3.8	237.6
87	-.2E-01	-.1E-01	.7E-02	.1E-01	.5E-02	-.5E-02	.9E-02	-.9E-02	0.0	-0.0	121.2	-4.0	246.5
88	-.2E-01	-.1E-01	.6E-02	.1E-01	.6E-02	-.6E-02	.8E-02	-.9E-02	0.0	0.0	125.3	-4.2	254.7
89	-.2E-01	-.1E-01	.6E-02	.1E-01	.7E-02	-.6E-02	.7E-02	-.9E-02	0.0	0.0	129.0	-4.3	262.4
90	-.1E-01	-.9E-02	.5E-02	.1E-01	.7E-02	-.6E-02	.7E-02	-.9E-02	-0.0	-0.0	132.4	-4.4	269.3
91	-.1E-01	-.8E-02	.5E-02	.1E-01	.8E-02	-.7E-02	.6E-02	-.9E-02	0.0	-0.0	135.5	-4.5	275.6
92	-.1E-01	-.7E-02	.4E-02	.2E-01	.8E-02	-.7E-02	.5E-02	-.9E-02	0.0	0.0	138.2	-4.7	281.1

93	-.9E-02	-.6E-02	.4E-02	.2E-01	.9E-02	-.7E-02	.4E-02	-.9E-02	0.0	0.0	140.6	-4.7	285.9
94	-.8E-02	-.5E-02	.3E-02	.2E-01	.9E-02	-.7E-02	.4E-02	-.9E-02	-0.0	-0.0	142.5	-4.8	289.9
95	-.6E-02	-.4E-02	.2E-02	.2E-01	.9E-02	-.7E-02	.3E-02	-.1E-01	0.0	-0.0	144.1	-4.9	293.1
96	-.4E-02	-.3E-02	.2E-02	.2E-01	.9E-02	-.8E-02	.2E-02	-.1E-01	0.0	0.0	145.3	-4.9	295.5
97	-.3E-02	-.2E-02	.1E-02	.2E-01	.1E-01	-.8E-02	.1E-02	-.1E-01	-0.0	-0.0	146.1	-5.0	297.2
98	-.9E-03	-.5E-03	.3E-03	.2E-01	.1E-01	-.8E-02	.4E-03	-.1E-01	-0.0	-0.0	146.5	-4.9	297.9

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DISPLACEMENTS AND STRESSES AT SPECIFIED POI NTS IN THE BODY

POINT	X CO-ORD	Y CO-ORD	UX	UY	SIGXX	SIGYY	SIGXY
1	0.0000	0.0000	0.000000	-0.000121	100.0	-0.0	0.0
2	12.5000	0.0000	-0.000007	-0.000118	100.0	-0.0	0.0
3	25.0000	0.0000	-0.000014	-0.000112	100.0	-0.0	0.0
4	37.5000	0.0000	-0.000019	-0.000101	100.0	-0.0	0.0
5	50.0000	0.0000	-0.000022	-0.000087	100.0	-0.0	-0.0
6	62.5000	0.0000	-0.000022	-0.000071	100.0	-0.0	-0.0
7	75.0000	0.0000	-0.000020	-0.000054	100.0	-0.0	0.0
8	87.5000	0.0000	-0.000016	-0.000037	100.0	-0.0	0.0
9	100.0000	0.0000	-0.000010	-0.000020	100.0	-0.0	-0.0
10	112.5000	0.0000	-0.000002	-0.000004	100.0	-0.0	0.0
11	125.0000	0.0000	0.000006	0.000010	100.0	-0.0	0.0
12	137.5000	0.0000	0.000015	0.000023	100.0	-0.0	0.0
13	150.0000	0.0000	0.000025	0.000033	100.0	-0.0	-0.0
14	162.5000	0.0000	0.000034	0.000043	100.0	-0.0	0.0
15	175.0000	0.0000	0.000043	0.000050	100.0	0.0	-0.0
16	187.5000	0.0000	0.000052	0.000056	100.0	0.0	-0.0
17	200.0000	0.0000	0.000060	0.000060	100.0	-0.0	0.0
18	212.5000	0.0000	0.000067	0.000064	100.0	-0.0	-0.0
19	225.0000	0.0000	0.000074	0.000066	100.0	0.0	0.0
20	237.5000	0.0000	0.000080	0.000068	100.0	0.0	0.0
21	250.0000	0.0000	0.000085	0.000068	100.0	0.0	-0.0
22	262.5000	0.0000	0.000089	0.000069	100.0	0.0	-0.0
23	275.0000	0.0000	0.000093	0.000068	100.0	0.0	-0.0
24	287.5000	0.0000	0.000096	0.000068	100.0	0.0	-0.0
25	300.0000	0.0000	0.000099	0.000067	100.0	0.0	0.0

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HORIZONTAL STRAIN AT SPECIFIED POINTS

N-POINT	XP(NI)	STRAIN
1	0.0000	-0.000001
2	12.5000	-0.000001
3	25.0000	-0.000000
4	37.5000	-0.000000
5	50.0000	-0.000000
6	62.5000	0.000000
7	75.0000	0.000000
8	87.5000	0.000000
9	100.0000	0.000001
10	112.5000	0.000001
11	125.0000	0.000001
12	137.5000	0.000001
13	150.0000	0.000001
14	162.5000	0.000001

15	175.0000	0.000001
16	187.5000	0.000001
17	200.0000	0.000001
18	212.5000	0.000001
19	225.0000	0.000000
20	237.5000	0.000000
21	250.0000	0.000000
22	262.5000	0.000000
23	275.0000	0.000000
24	287.5000	0.000000
25	300.0000	0.000000

OUTPUT FOR THE GRAVITATIONAL LOADED EXAMPLE PROBLEM

1

CIRCULAR HOLE IN A SEMI-INFINITE BODY UNDER GRAVITATIONAL LOADING

NUMBER OF STRAIGHT-LINE SEGMENTS (EACH CONTAINING AT L EAST ONE BOUNDARY ELEMENT) USED TO DEFINE BOUNDARIES = 1

NUMBER OF STRAIGHT-LINE SEGMENTS USED TO SPECIFY OTHER LOCATIONS (NOT ON A BOUNDARY) =1

THE LINE X = XS = 0.0000 IS A LINE OF SYMMETRY.

POISSION'S RATIO = 0.17

YOUNG'S MODULUS = .8000E+04

UNIT WEIGHT OF OVERBURDEN = .2700E-01

RATIO OF FIELD STRESS = .2000E-01

XY-COMPONENT OF FIELD STRESS = 0.

XY-COMPONENT OF FIELD STRESS = 0.

1

BOUNDARY ELEMENT DATA.

ELEMENT	KODE	X (CENTER)	Y (CENTER)	LENGTH	ANGLE	US OR SIGMA-S	UN OR SIGMA-N
1	1	0.0159	-202.9997	0.0317	0.91	0.	0.
2	1	0.0476	-202.9987	0.0317	2.73	0.	0.
3	1	0.0792	-202.9967	0.0317	4.55	0.	0.
4	1	0.1108	-202.9937	0.0317	6.36	0.	0.
5	1	0.1423	-202.9897	0.0317	8.18	0.	0.
6	1	0.1736	-202.9847	0.0317	10.00	0.	0.
7	1	0.2048	-202.9787	0.0317	11.82	0.	0.
8	1	0.2357	-202.9717	0.0317	13.64	0.	0.
9	1	0.2664	-202.9637	0.0317	15.45	0.	0.
10	1	0.2969	-202.9548	0.0317	17.27	0.	0.
11	1	0.3270	-202.9449	0.0317	19.09	0.	0.
12	1	0.3568	-202.9340	0.0317	20.91	0.	0.
13	1	0.3863	-202.9222	0.0317	22.73	0.	0.
14	1	0.4154	-202.9095	0.0317	24.55	0.	0.
15	1	0.4440	-202.8959	0.0317	26.36	0.	0.
16	1	0.4722	-202.8813	0.0317	28.18	0.	0.
17	1	0.4999	-202.8659	0.0317	30.00	0.	0.
18	1	0.5272	-202.8496	0.0317	31.82	0.	0.
19	1	0.5539	-202.8325	0.0317	33.64	0.	0.
20	1	0.5800	-202.8145	0.0317	35.45	0.	0.
21	1	0.6055	-202.7957	0.0317	37.27	0.	0.
22	1	0.6305	-202.7760	0.0317	39.09	0.	0.
23	1	0.6548	-202.7557	0.0317	40.91	0.	0.
24	1	0.6784	-202.7345	0.0317	42.73	0.	0.
25	1	0.7014	-202.7126	0.0317	44.55	0.	0.
26	1	0.7236	-202.6900	0.0317	46.36	0.	0.
27	1	0.7452	-202.6667	0.0317	48.18	0.	0.

28	1	0.7659	-202.6427	0.0317	50.00	0.	0.
29	1	0.7860	-202.6181	0.0317	51.82	0.	0.
30	1	0.8052	-202.5928	0.0317	53.64	0.	0.
31	1	0.8236	-202.5670	0.0317	55.45	0.	0.
32	1	0.8411	-202.5406	0.0317	57.27	0.	0.
33	1	0.8579	-202.5136	0.0317	59.09	0.	0.
34	1	0.8737	-202.4861	0.0317	60.91	0.	0.
35	1	0.8887	-202.4582	0.0317	62.73	0.	0.
36	1	0.9028	-202.4297	0.0317	64.55	0.	0.
37	1	0.9160	-202.4003	0.0317	66.36	0.	0.
38	1	0.9283	-202.3716	0.0317	68.18	0.	0.
39	1	0.9396	-202.3420	0.0317	70.00	0.	0.
40	1	0.9500	-202.3120	0.0317	71.82	0.	0.
41	1	0.9594	-202.2817	0.0317	73.64	0.	0.
42	1	0.9678	-202.2511	0.0317	75.45	0.	0.
43	1	0.9753	-202.2203	0.0317	77.27	0.	0.
44	1	0.9818	-202.1892	0.0317	79.09	0.	0.
45	1	0.9873	-202.1580	0.0317	80.91	0.	0.
46	1	0.9918	-202.1266	0.0317	82.73	0.	0.
47	1	0.9953	-202.0950	0.0317	84.55	0.	0.
48	1	0.9979	-202.0634	0.0317	86.36	0.	0.
49	1	0.9994	-202.0317	0.0317	88.18	0.	0.
50	1	0.9999	-202.0000	0.0317	90.00	0.	0.
51	1	0.9994	-201.9683	0.0317	91.82	0.	0.
52	1	0.9979	-201.9366	0.0317	93.64	0.	0.
53	1	0.9953	-201.9050	0.0317	95.45	0.	0.
54	1	0.9918	-201.8734	0.0317	97.27	0.	0.
55	1	0.9873	-201.8420	0.0317	99.09	0.	0.
56	1	0.9818	-201.8108	0.0317	100.91	0.	0.
57	1	0.9753	-201.7797	0.0317	102.73	0.	0.
58	1	0.9678	-201.7489	0.0317	104.55	0.	0.
59	1	0.9594	-201.7183	0.0317	106.36	0.	0.
60	1	0.9500	-201.6880	0.0317	108.18	0.	0.
61	1	0.9396	-201.6580	0.0317	110.00	0.	0.
62	1	0.9283	-201.6284	0.0317	111.82	0.	0.
63	1	0.9160	-201.5991	0.0317	113.64	0.	0.
64	1	0.9028	-201.5703	0.0317	115.45	0.	0.
65	1	0.8887	-201.5418	0.0317	117.27	0.	0.
66	1	0.8737	-201.5139	0.0317	119.09	0.	0.
67	1	0.8579	-201.4864	0.0317	120.91	0.	0.
68	1	0.8411	-201.4594	0.0317	122.73	0.	0.
69	1	0.8236	-201.4330	0.0317	124.55	0.	0.
70	1	0.8052	-201.4072	0.0317	126.36	0.	0.
71	1	0.7860	-201.3819	0.0317	128.18	0.	0.
72	1	0.7659	-201.3573	0.0317	130.00	0.	0.
73	1	0.7452	-201.3333	0.0317	131.82	0.	0.
74	1	0.7236	-201.3100	0.0317	133.64	0.	0.
75	1	0.7014	-201.2874	0.0317	135.45	0.	0.
76	1	0.6784	-201.2655	0.0317	137.27	0.	0.
77	1	0.6548	-201.2443	0.0317	139.09	0.	0.
78	1	0.6305	-201.2240	0.0317	140.91	0.	0.
79	1	0.6055	-201.2043	0.0317	142.73	0.	0.
80	1	0.5800	-201.1855	0.0317	144.55	0.	0.
81	1	0.5539	-201.1675	0.0317	146.36	0.	0.
82	1	0.5272	-201.1504	0.0317	148.18	0.	0.

83	1	0.4999	-201.1341	0.0317	150.00	0.	0.
84	1	0.4722	-201.1187	0.0317	151.82	0.	0.
85	1	0.4440	-201.1041	0.0317	153.64	0.	0.
86	1	0.4154	-201.0905	0.0317	155.45	0.	0.
87	1	0.3863	-201.0778	0.0317	157.27	0.	0.
88	1	0.3568	-201.0660	0.0317	159.09	0.	0.
89	1	0.3270	-201.0551	0.0317	160.91	0.	0.
90	1	0.2969	-201.0452	0.0317	162.73	0.	0.
91	1	0.2664	-201.0363	0.0317	164.55	0.	0.
92	1	0.2357	-201.0283	0.0317	166.36	0.	0.
93	1	0.2048	-201.0213	0.0317	168.18	0.	0.
94	1	0.1736	-201.0153	0.0317	170.00	0.	0.
95	1	0.1423	-201.0103	0.0317	171.82	0.	0.
96	1	0.1108	-201.0063	0.0317	173.64	0.	0.
97	1	0.0792	-201.0033	0.0317	175.45	0.	0.
98	1	0.0476	-201.0013	0.0317	177.27	0.	0.
99	1	0.0159	-201.0003	0.0317	179.09	0.	0.

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DISPLACEMENTS AND STRESSES AT MIDPOINTS OF BOUNDARY ELEMENTS

ELEM	DS	US(-)	US(+)	DN	UN(-)	UN(+)	UX(-)	UY(-)	SIG-S	SIG-N	SIGT1	SIGTP	SIGTN
1	.2E+02	-.3E-04	-.2E+02	.1E+04	.3E-03	-.1E+04	-.3E-04	.3E-03	-0.0	0.0	-13.5	-1.5	-25.4
2	.5E+02	-.8E-04	-.5E+02	.1E+04	.3E-03	-.1E+04	-.9E-04	.3E-03	-0.0	-0.0	-13.2	-0.1	-26.4
3	.9E+02	-.1E-03	-.9E+02	.1E+04	.3E-03	-.1E+04	-.2E-03	.3E-03	0.0	0.0	-13.0	2.8	-28.8
4	.1E+03	-.2E-03	-.1E+03	.1E+04	.3E-03	-.1E+04	-.2E-03	.3E-03	0.0	0.0	-13.8	1.9	-29.6
5	.2E+03	-.3E-03	-.2E+03	.1E+04	.3E-03	-.1E+04	-.3E-03	.3E-03	0.0	0.0	-14.0	-1.0	-26.9
6	.2E+03	-.3E-03	-.2E+03	.1E+04	.3E-03	-.1E+04	-.4E-03	.3E-03	0.0	0.0	-13.2	-1.2	-25.2
7	.2E+03	-.4E-03	-.2E+03	.1E+04	.4E-03	-.1E+04	-.4E-03	.3E-03	-0.0	0.0	-13.0	-1.0	-24.9
8	.3E+03	-.4E-03	-.3E+03	.1E+04	.4E-03	-.1E+04	-.5E-03	.3E-03	0.0	-0.0	-11.8	1.0	-24.7
9	.3E+03	-.5E-03	-.3E+03	.1E+04	.4E-03	-.1E+04	-.6E-03	.3E-03	-0.0	-0.0	-12.6	1.1	-26.2
10	.3E+03	-.5E-03	-.3E+03	.1E+04	.4E-03	-.1E+04	-.6E-03	.3E-03	-0.0	0.0	-12.5	0.5	-25.5
11	.4E+03	-.6E-03	-.4E+03	.1E+04	.5E-03	-.1E+04	-.7E-03	.3E-03	0.0	-0.0	-12.2	0.3	-24.8
12	.4E+03	-.6E-03	-.4E+03	.1E+04	.5E-03	-.1E+04	-.8E-03	.3E-03	-0.0	0.0	-12.0	-0.1	-23.9
13	.4E+03	-.7E-03	-.4E+03	.1E+04	.6E-03	-.1E+04	-.8E-03	.2E-03	0.0	-0.0	-10.2	2.2	-22.6
14	.4E+03	-.7E-03	-.4E+03	.1E+04	.6E-03	-.1E+04	-.9E-03	.2E-03	0.0	0.0	-11.5	1.4	-24.4
15	.5E+03	-.8E-03	-.5E+03	.1E+04	.6E-03	-.1E+04	-.1E-02	.2E-03	-0.0	0.0	-11.4	1.1	-23.9
16	.5E+03	-.8E-03	-.5E+03	.1E+04	.7E-03	-.1E+04	-.1E-02	.2E-03	0.0	0.0	-12.9	-1.5	-24.4
17	.5E+03	-.8E-03	-.5E+03	.9E+03	.7E-03	-.9E+03	-.1E-02	.2E-03	0.0	0.0	-10.8	-0.4	-21.1
18	.6E+03	-.9E-03	-.6E+03	.9E+03	.8E-03	-.9E+03	-.1E-02	.2E-03	-0.0	0.0	-8.4	2.4	-19.2
19	.6E+03	-.9E-03	-.6E+03	.9E+03	.8E-03	-.9E+03	-.1E-02	.2E-03	-0.0	-0.0	-10.1	1.0	-21.2
20	.6E+03	-.9E-03	-.6E+03	.9E+03	.9E-03	-.9E+03	-.1E-02	.2E-03	0.0	-0.0	-12.0	-1.8	-22.2
21	.7E+03	-.9E-03	-.7E+03	.9E+03	.1E-02	-.9E+03	-.1E-02	.2E-03	0.0	-0.0	-9.5	-0.1	-18.9
22	.7E+03	-.9E-03	-.7E+03	.8E+03	.1E-02	-.8E+03	-.1E-02	.2E-03	-0.0	0.0	-6.9	2.5	-16.3
23	.7E+03	-.9E-03	-.7E+03	.8E+03	.1E-02	-.8E+03	-.1E-02	.2E-03	0.0	0.0	-8.8	0.5	-18.1
24	.7E+03	-.1E-02	-.7E+03	.8E+03	.1E-02	-.8E+03	-.1E-02	.2E-03	0.0	-0.0	-10.7	-1.9	-19.5
25	.8E+03	-.9E-03	-.8E+03	.8E+03	.1E-02	-.8E+03	-.2E-02	.2E-03	0.0	-0.0	-8.1	0.2	-16.4
26	.8E+03	-.1E-02	-.8E+03	.7E+03	.1E-02	-.7E+03	-.2E-02	.2E-03	-0.0	-0.0	-5.5	2.4	-13.5
27	.8E+03	-.9E-03	-.8E+03	.7E+03	.1E-02	-.7E+03	-.2E-02	.2E-03	-0.0	0.0	-7.4	0.0	-14.9
28	.8E+03	-.9E-03	-.8E+03	.7E+03	.1E-02	-.7E+03	-.2E-02	.2E-03	-0.0	0.0	-9.4	-2.0	-16.7
29	.8E+03	-.9E-03	-.8E+03	.7E+03	.1E-02	-.7E+03	-.2E-02	.2E-03	0.0	-0.0	-6.8	0.4	-14.0
30	.9E+03	-.9E-03	-.9E+03	.6E+03	.1E-02	-.6E+03	-.2E-02	.1E-03	-0.0	0.0	-4.5	1.8	-10.8
31	.9E+03	-.9E-03	-.9E+03	.6E+03	.2E-02	-.6E+03	-.2E-02	.1E-03	-0.0	-0.0	-8.3	-2.5	-14.1
32	.9E+03	-.9E-03	-.9E+03	.6E+03	.2E-02	-.6E+03	-.2E-02	.1E-03	0.0	0.0	-8.0	-1.5	-14.5
33	.9E+03	-.8E-03	-.9E+03	.6E+03	.2E-02	-.6E+03	-.2E-02	.1E-03	0.0	0.0	-5.5	1.2	-12.2

34	.9E+03	-.8E-03	-.9E+03	.5E+03	.2E-02	-.5E+03	-.2E-02	.1E-03	-0.0	-0.0	-3.3	2.3	-8.9
35	.1E+04	-.8E-03	-.1E+04	.5E+03	.2E-02	-.5E+03	-.2E-02	.1E-03	0.0	0.0	-4.8	-0.1	-9.4
36	.1E+04	-.7E-03	-.1E+04	.5E+03	.2E-02	-.5E+03	-.2E-02	.1E-03	0.0	-0.0	-4.7	-0.1	-9.2
37	.1E+04	-.7E-03	-.1E+04	.4E+03	.2E-02	-.4E+03	-.2E-02	.9E-04	-0.0	0.0	-4.6	-0.5	-8.6
38	.1E+04	-.7E-03	-.1E+04	.4E+03	.2E-02	-.4E+03	-.2E-02	.9E-04	0.0	0.0	-5.7	-0.9	-10.4
39	.1E+04	-.6E-03	-.1E+04	.4E+03	.2E-02	-.4E+03	-.2E-02	.8E-04	-0.0	0.0	-3.9	1.9	-9.7
40	.1E+04	-.6E-03	-.1E+04	.3E+03	.2E-02	-.3E+03	-.2E-02	.7E-04	-0.0	-0.0	-2.2	2.5	-7.0
41	.1E+04	-.5E-03	-.1E+04	.3E+03	.2E-02	-.3E+03	-.2E-02	.5E-04	0.0	0.0	-2.1	0.6	-4.8
42	.1E+04	-.5E-03	-.1E+04	.3E+03	.2E-02	-.3E+03	-.2E-02	.5E-04	-0.0	-0.0	-3.3	-1.4	-5.1
43	.1E+04	-.4E-03	-.1E+04	.2E+03	.2E-02	-.2E+03	-.2E-02	.5E-04	0.0	-0.0	-3.2	-1.5	-4.9
44	.1E+04	-.4E-03	-.1E+04	.2E+03	.2E-02	-.2E+03	-.2E-02	.5E-04	0.0	-0.0	-3.9	-1.5	-6.4
45	.1E+04	-.3E-03	-.1E+04	.2E+03	.2E-02	-.2E+03	-.2E-02	.4E-04	-0.0	0.0	-3.6	1.4	-8.6
46	.1E+04	-.3E-03	-.1E+04	.1E+03	.2E-02	-.1E+03	-.2E-02	.2E-04	0.0	0.0	-2.3	2.7	-7.2
47	.1E+04	-.2E-03	-.1E+04	.1E+03	.2E-02	-.1E+03	-.2E-02	.3E-05	-0.0	-0.0	-2.2	0.0	-4.5
48	.1E+04	-.1E-03	-.1E+04	.7E+02	.2E-02	-.7E+02	-.2E-02	.2E-05	-0.0	-0.0	-2.6	-1.2	-4.1
49	.1E+04	-.7E-04	-.1E+04	.3E+02	.2E-02	-.3E+02	-.2E-02	-.3E-05	0.0	0.0	-2.6	-0.7	-4.6
50	.1E+04	-.9E-05	-.1E+04	.3E-02	.2E-02	-.1E-02	-.2E-02	-.9E-05	0.0	0.0	-2.6	-0.6	-4.6
51	.1E+04	.5E-04	-.1E+04	-.3E+02	.2E-02	.3E+02	-.2E-02	-.2E-04	-0.0	0.0	-2.7	-0.7	-4.6
52	.1E+04	.1E-03	-.1E+04	-.7E+02	.2E-02	.7E+02	-.2E-02	-.2E-04	0.0	0.0	-2.6	-1.1	-4.1
53	.1E+04	.2E-03	-.1E+04	-.1E+03	.2E-02	.1E+03	-.2E-02	-.2E-04	-0.0	0.0	-2.2	0.1	-4.5
54	.1E+04	.2E-03	-.1E+04	-.1E+03	.2E-02	.1E+03	-.2E-02	-.4E-04	-0.0	0.0	-2.2	2.8	-7.2
55	.1E+04	.3E-03	-.1E+04	-.2E+03	.2E-02	.2E+03	-.2E-02	-.6E-04	0.0	-0.0	-3.6	1.5	-8.7
56	.1E+04	.3E-03	-.1E+04	-.2E+03	.2E-02	.2E+03	-.2E-02	-.7E-04	0.0	0.0	-4.0	-1.3	-6.6
57	.1E+04	.4E-03	-.1E+04	-.2E+03	.2E-02	.2E+03	-.2E-02	-.7E-04	-0.0	-0.0	-3.2	-1.1	-5.3
58	.1E+04	.5E-03	-.1E+04	-.3E+03	.2E-02	.3E+03	-.2E-02	-.8E-04	0.0	-0.0	-3.3	-0.6	-5.9
59	.1E+04	.5E-03	-.1E+04	-.3E+03	.2E-02	.3E+03	-.2E-02	-.8E-04	-0.0	-0.0	-3.4	-0.6	-6.3
60	.1E+04	.6E-03	-.1E+04	-.3E+03	.2E-02	.3E+03	-.2E-02	-.9E-04	0.0	-0.0	-3.5	-0.7	-6.3
61	.1E+04	.6E-03	-.1E+04	-.4E+03	.2E-02	.4E+03	-.2E-02	-.9E-04	0.0	-0.0	-2.4	1.4	-6.2
62	.1E+04	.6E-03	-.1E+04	-.4E+03	.2E-02	.4E+03	-.2E-02	-.1E-03	0.0	0.0	-4.0	0.9	-8.8
63	.1E+04	.7E-03	-.1E+04	-.4E+03	.2E-02	.4E+03	-.2E-02	-.1E-03	0.0	0.0	-4.2	0.5	-9.0
64	.1E+04	.7E-03	-.1E+04	-.5E+03	.2E-02	.5E+03	-.2E-02	-.1E-03	0.0	-0.0	-4.5	0.3	-9.3
65	.1E+04	.8E-03	-.1E+04	-.5E+03	.2E-02	.5E+03	-.2E-02	-.1E-03	0.0	0.0	-4.7	0.1	-9.5
66	.9E+03	.8E-03	-.9E+03	-.5E+03	.2E-02	.5E+03	-.2E-02	-.1E-03	0.0	-0.0	-3.2	2.5	-8.9
67	.9E+03	.8E-03	-.9E+03	-.6E+03	.2E-02	.6E+03	-.2E-02	-.1E-03	0.0	-0.0	-5.5	1.3	-12.2
68	.9E+03	.8E-03	-.9E+03	-.6E+03	.2E-02	.6E+03	-.2E-02	-.2E-03	0.0	0.0	-8.0	-1.5	-14.5
69	.9E+03	.9E-03	-.9E+03	-.6E+03	.2E-02	.6E+03	-.2E-02	-.2E-03	0.0	0.0	-8.3	-2.4	-14.1
70	.9E+03	.9E-03	-.9E+03	-.6E+03	.2E-02	.6E+03	-.2E-02	-.2E-03	0.0	-0.0	-4.4	1.9	-10.7
71	.8E+03	.9E-03	-.8E+03	-.7E+03	.1E-02	.7E+03	-.2E-02	-.2E-03	-0.0	0.0	-6.8	0.4	-14.0
72	.8E+03	.9E-03	-.8E+03	-.7E+03	.1E-02	.7E+03	-.2E-02	-.2E-03	-0.0	-0.0	-9.3	-2.0	-16.6
73	.8E+03	.9E-03	-.8E+03	-.7E+03	.1E-02	.7E+03	-.2E-02	-.2E-03	0.0	0.0	-7.4	0.0	-14.8
74	.8E+03	.9E-03	-.8E+03	-.7E+03	.1E-02	.7E+03	-.2E-02	-.2E-03	-0.0	-0.0	-5.5	2.4	-13.5
75	.8E+03	.9E-03	-.8E+03	-.8E+03	.1E-02	.8E+03	-.2E-02	-.2E-03	-0.0	0.0	-8.1	0.3	-16.4
76	.7E+03	.9E-03	-.7E+03	-.8E+03	.1E-02	.8E+03	-.1E-02	-.2E-03	-0.0	-0.0	-10.7	-1.9	-19.4
77	.7E+03	.9E-03	-.7E+03	-.8E+03	.1E-02	.8E+03	-.1E-02	-.2E-03	-0.0	0.0	-8.8	0.5	-18.1
78	.7E+03	.9E-03	-.7E+03	-.8E+03	.1E-02	.8E+03	-.1E-02	-.2E-03	0.0	0.0	-6.9	2.5	-16.2
79	.7E+03	.9E-03	-.7E+03	-.9E+03	.1E-02	.9E+03	-.1E-02	-.2E-03	0.0	0.0	-9.4	-0.1	-18.7
80	.6E+03	.9E-03	-.6E+03	-.9E+03	.9E-03	.9E+03	-.1E-02	-.2E-03	-0.0	-0.0	-11.9	-1.8	-22.1
81	.6E+03	.9E-03	-.6E+03	-.9E+03	.9E-03	.9E+03	-.1E-02	-.2E-03	-0.0	0.0	-10.1	1.0	-21.1
82	.6E+03	.8E-03	-.6E+03	-.9E+03	.8E-03	.9E+03	-.1E-02	-.2E-03	0.0	-0.0	-8.4	2.3	-19.0
83	.5E+03	.8E-03	-.5E+03	-.9E+03	.8E-03	.9E+03	-.1E-02	-.3E-03	-0.0	0.0	-10.8	-0.5	-21.0
84	.5E+03	.8E-03	-.5E+03	-.1E+04	.7E-03	.1E+04	-.1E-02	-.2E-03	0.0	-0.0	-12.9	-1.5	-24.3
85	.5E+03	.7E-03	-.5E+03	-.1E+04	.7E-03	.1E+04	-.1E-02	-.3E-03	-0.0	-0.0	-11.4	1.0	-23.8
86	.4E+03	.7E-03	-.4E+03	-.1E+04	.6E-03	.1E+04	-.9E-03	-.3E-03	0.0	-0.0	-11.4	1.4	-24.2
87	.4E+03	.7E-03	-.4E+03	-.1E+04	.6E-03	.1E+04	-.8E-03	-.3E-03	0.0	-0.0	-10.1	2.2	-22.4
88	.4E+03	.6E-03	-.4E+03	-.1E+04	.5E-03	.1E+04	-.8E-03	-.3E-03	0.0	-0.0	-11.9	-0.2	-23.7

89	.4E+03	.6E-03	-.4E+03	-.1E+04	.5E-03	.1E+04	-.7E-03	-.3E-03	0.0	0.0	-12.2	0.2	-24.6
90	.3E+03	.5E-03	-.3E+03	-.1E+04	.5E-03	.1E+04	-.6E-03	-.3E-03	0.0	-0.0	-12.4	0.4	-25.3
91	.3E+03	.5E-03	-.3E+03	-.1E+04	.4E-03	.1E+04	-.6E-03	-.3E-03	-0.0	0.0	-12.5	0.9	-25.9
92	.3E+03	.4E-03	-.3E+03	-.1E+04	.4E-03	.1E+04	-.5E-03	-.3E-03	0.0	-0.0	-11.8	0.8	-24.4
93	.2E+03	.4E-03	-.2E+03	-.1E+04	.4E-03	.1E+04	-.4E-03	-.3E-03	0.0	-0.0	-12.9	-1.3	-24.6
94	.2E+03	.3E-03	-.2E+03	-.1E+04	.3E-03	.1E+04	-.4E-03	-.3E-03	-0.0	0.0	-13.2	-1.6	-24.7
95	.2E+03	.3E-03	-.2E+03	-.1E+04	.3E-03	.1E+04	-.3E-03	-.3E-03	-0.0	-0.0	-14.0	-1.6	-26.3
96	.1E+03	.2E-03	-.1E+03	-.1E+04	.3E-03	.1E+04	-.3E-03	-.3E-03	-0.0	0.0	-13.8	0.7	-28.3
97	.9E+02	.2E-03	-.9E+02	-.1E+04	.3E-03	.1E+04	-.2E-03	-.3E-03	0.0	-0.0	-13.4	1.7	-28.5
98	.5E+02	.9E-04	-.5E+02	-.1E+04	.3E-03	.1E+04	-.1E-03	-.3E-03	-0.0	-0.0	-13.4	1.2	-28.0
99	.2E+02	.3E-04	-.2E+02	-.1E+04	.3E-03	.1E+04	-.4E-04	-.3E-03	-0.0	-0.0	-13.4	1.1	-27.9

1 DISPLACEMENTS AND STRESSES AT SPECIFIED POINTS IN THE BODY

POINT	X CO-ORD	Y CO-ORD	UX	UY	SIGXX	SIGYY	SIGXY
1	0.0000	0.0000	0.000000	-0.000007	-0.0	0.0	0.0
2	12.5000	0.0000	-0.000000	-0.000007	-0.0	0.0	0.0
3	25.0000	0.0000	-0.000001	-0.000007	-0.0	0.0	0.0
4	37.5000	0.0000	-0.000001	-0.000007	-0.0	-0.0	0.0
5	50.0000	0.0000	-0.000002	-0.000008	-0.0	0.0	-0.0
6	62.5000	0.0000	-0.000003	-0.000008	-0.0	0.0	0.0
7	75.0000	0.0000	-0.000003	-0.000009	-0.0	0.0	-0.0
8	87.5000	0.0000	-0.000004	-0.000009	-0.0	0.0	-0.0
9	100.0000	0.0000	-0.000005	-0.000010	-0.0	0.0	0.0
10	112.5000	0.0000	-0.000006	-0.000010	-0.0	-0.0	0.0
11	125.0000	0.0000	-0.000006	-0.000010	-0.0	-0.0	-0.0
12	137.5000	0.0000	-0.000007	-0.000010	-0.0	0.0	0.0
13	150.0000	0.0000	-0.000008	-0.000010	-0.0	0.0	0.0
14	162.5000	0.0000	-0.000008	-0.000010	-0.0	0.0	0.0
15	175.0000	0.0000	-0.000009	-0.000010	-0.0	0.0	-0.0
16	187.5000	0.0000	-0.000009	-0.000010	-0.0	0.0	0.0
17	200.0000	0.0000	-0.000010	-0.000010	-0.0	-0.0	0.0
18	212.5000	0.0000	-0.000010	-0.000010	-0.0	0.0	0.0
19	225.0000	0.0000	-0.000011	-0.000010	-0.0	0.0	-0.0
20	237.5000	0.0000	-0.000011	-0.000009	-0.0	-0.0	-0.0
21	250.0000	0.0000	-0.000011	-0.000009	-0.0	-0.0	0.0
22	262.5000	0.0000	-0.000011	-0.000009	-0.0	0.0	-0.0
23	275.0000	0.0000	-0.000011	-0.000008	-0.0	-0.0	0.0

APPENDIX C THE OUTPUT OF STRESS-FREE BOUNDARY CONDITION'S

BOUNDARY DISPLACEMENT USED AS A DISPLACEMENT BOUNDARY CONDITION

OF THE SECOND RUNNING PROCESS (*).

1

THIS MODEL IS AN EXAMPLE OF BOUNDARY DISPLACEMENT OF THE FIRST RUNNING PROCESS TO BE THE SECOND'S INPUT DATA

NUMBER OF STRAIGHT-LINE SEGMENTS (EACH CONTAINING AT L EAST ONE BOUNDARY ELEMENT) USED TO DEFINE BOUNDARIES = 12

NUMBER OF STRAIGHT-LINE SEGMENTS USED TO SPECIFY OTHER LOCATIONS (NOT ON A BOUNDARY) = 1

THE LINE X = XS = 0.0000 IS A LINE OF SYMMETRY.

POISSION'S RATIO = 0.17

YOUNG'S MODULUS = .8000E+04

UNIT WEIGHT OF OVERBURDEN = .2700E-01

RATIO OF FIELD STRESS = .2000E+01

XY-COMPONENT OF FIELD STRESS = 0.

1

BOUNDARY ELEMENT DATA.

ELEMENT	KODE	X (CENTER)	Y (CENTER)	LENGTH	ANGLE	US OR SIGMA-S	UN OR SIGMA-N
1	4	1.5789	-202.0000	3.1579	0.00	0.	.3000E+00
2	4	4.7368	-202.0000	3.1579	0.00	0.	.3000E+00
3	4	7.8947	-202.0000	3.1579	0.00	0.	.3000E+00
4	4	11.0526	-202.0000	3.1579	0.00	0.	.3000E+00
5	4	14.2105	-202.0000	3.1579	0.00	0.	.3000E+00
6	4	17.3684	-202.0000	3.1579	0.00	0.	.3000E+00
7	4	20.5263	-202.0000	3.1579	0.00	0.	.3000E+00
8	4	23.6842	-202.0000	3.1579	0.00	0.	.3000E+00
9	4	26.8421	-202.0000	3.1579	0.00	0.	.3000E+00
10	4	30.0000	-202.0000	3.1579	0.00	0.	.3000E+00
11	4	33.1579	-202.0000	3.1579	0.00	0.	.3000E+00
12	4	36.3158	-202.0000	3.1579	0.00	0.	.3000E+00
13	4	39.4737	-202.0000	3.1579	0.00	0.	.3000E+00
14	4	42.6316	-202.0000	3.1579	0.00	0.	.3000E+00
15	4	45.7895	-202.0000	3.1579	0.00	0.	.3000E+00
16	4	48.9474	-202.0000	3.1579	0.00	0.	.3000E+00
17	4	52.1053	-202.0000	3.1579	0.00	0.	.3000E+00
18	4	55.2632	-202.0000	3.1579	0.00	0.	.3000E+00
19	4	58.4211	-202.0000	3.1579	0.00	0.	.3000E+00
20	4	61.3333	-202.0000	2.6667	0.00	0.	.2000E+00
21	4	64.0000	-202.0000	2.6667	0.00	0.	.2000E+00
22	4	66.6667	-202.0000	2.6667	0.00	0.	.2000E+00
23	4	69.3333	-202.0000	2.6667	0.00	0.	.2000E+00
24	4	72.0000	-202.0000	2.6667	0.00	0.	.2000E+00
25	4	74.6667	-202.0000	2.6667	0.00	0.	.2000E+00

26	4	77.3333	-202.0000	2.6667	0.00	0.	.2000E+00
27	4	80.0000	-202.0000	2.6667	0.00	0.	.2000E+00
28	4	82.6667	-202.0000	2.6667	0.00	0.	.2000E+00
29	4	85.3333	-202.0000	2.6667	0.00	0.	.2000E+00
30	4	88.0000	-202.0000	2.6667	0.00	0.	.2000E+00
31	4	90.6667	-202.0000	2.6667	0.00	0.	.2000E+00
32	4	93.3333	-202.0000	2.6667	0.00	0.	.2000E+00
33	4	96.0000	-202.0000	2.6667	0.00	0.	.2000E+00
34	4	98.6667	-202.0000	2.6667	0.00	0.	.2000E+00
35	4	101.2000	-202.0000	2.4000	0.00	0.	.1000E+00
36	4	103.6000	-202.0000	2.4000	0.00	0.	.1000E+00
37	4	106.0000	-202.0000	2.4000	0.00	0.	.1000E+00
38	4	108.4000	-202.0000	2.4000	0.00	0.	.1000E+00
39	4	110.8000	-202.0000	2.4000	0.00	0.	.1000E+00
40	4	113.1667	-202.0000	2.3333	0.00	0.	.5000E-01
41	4	115.5000	-202.0000	2.3333	0.00	0.	.5000E-01
42	4	117.8333	-202.0000	2.3333	0.00	0.	.5000E-01
43	4	120.1667	-202.0000	2.3333	0.00	0.	.5000E-01
44	4	122.5000	-202.0000	2.3333	0.00	0.	.5000E-01
45	4	124.8333	-202.0000	2.3333	0.00	0.	.5000E-01
46	4	128.0000	-202.0000	4.0000	0.00	0.	-.5000E-01
47	4	132.0000	-202.0000	4.0000	0.00	0.	-.5000E-01
48	4	136.0000	-202.0000	4.0000	0.00	0.	-.5000E-01
49	4	140.0000	-201.5000	1.0000	90.00	0.	-.1000E+00
50	4	140.0000	-200.5000	1.0000	90.00	0.	-.1000E+00
51	4	138.5000	-200.0000	3.0000	180.00	0.	.3000E+00
52	4	135.5000	-200.0000	3.0000	180.00	0.	.3000E+00
53	4	132.5000	-200.0000	3.0000	180.00	0.	.3000E+00
54	4	129.5000	-200.0000	3.0000	180.00	0.	.3000E+00
55	4	126.5000	-200.0000	3.0000	180.00	0.	.3000E+00
56	4	123.9167	-200.0000	2.1667	180.00	0.	.4000E+00
57	4	121.7500	-200.0000	2.1667	180.00	0.	.4000E+00
58	4	119.5833	-200.0000	2.1667	180.00	0.	.4000E+00
59	4	117.4167	-200.0000	2.1667	180.00	0.	.4000E+00
60	4	115.2500	-200.0000	2.1667	180.00	0.	.4000E+00
61	4	113.0833	-200.0000	2.1667	180.00	0.	.4000E+00
62	4	110.5000	-200.0000	3.0000	180.00	0.	.5000E+00
63	4	107.5000	-200.0000	3.0000	180.00	0.	.5000E+00
64	4	104.5000	-200.0000	3.0000	180.00	0.	.5000E+00
65	4	101.5000	-200.0000	3.0000	180.00	0.	.5000E+00
66	4	98.5000	-200.0000	3.0000	180.00	0.	.5000E+00
67	4	95.5000	-200.0000	3.0000	180.00	0.	.5000E+00
68	4	92.5000	-200.0000	3.0000	180.00	0.	.5000E+00
69	4	89.5417	-200.0000	2.9167	180.00	0.	.6000E+00
70	4	86.6250	-200.0000	2.9167	180.00	0.	.6000E+00
71	4	83.7083	-200.0000	2.9167	180.00	0.	.6000E+00
72	4	80.7917	-200.0000	2.9167	180.00	0.	.6000E+00
73	4	77.8750	-200.0000	2.9167	180.00	0.	.6000E+00
74	4	74.9583	-200.0000	2.9167	180.00	0.	.6000E+00
75	4	72.0417	-200.0000	2.9167	180.00	0.	.6000E+00
76	4	69.1250	-200.0000	2.9167	180.00	0.	.6000E+00
77	4	66.2083	-200.0000	2.9167	180.00	0.	.6000E+00
78	4	63.2917	-200.0000	2.9167	180.00	0.	.6000E+00
79	4	60.3750	-200.0000	2.9167	180.00	0.	.6000E+00
80	4	57.4583	-200.0000	2.9167	180.00	0.	.6000E+00

81	4	54.4444	-200.0000	3.1111	180.00	0.	.8000E+00
82	4	51.3333	-200.0000	3.1111	180.00	0.	.8000E+00
83	4	48.2222	-200.0000	3.1111	180.00	0.	.8000E+00
84	4	45.1111	-200.0000	3.1111	180.00	0.	.8000E+00
85	4	42.0000	-200.0000	3.1111	180.00	0.	.8000E+00
86	4	38.8889	-200.0000	3.1111	180.00	0.	.8000E+00
87	4	35.7778	-200.0000	3.1111	180.00	0.	.8000E+00
88	4	32.6667	-200.0000	3.1111	180.00	0.	.8000E+00
89	4	29.5556	-200.0000	3.1111	180.00	0.	.8000E+00
90	4	26.4444	-200.0000	3.1111	180.00	0.	.8000E+00
91	4	23.3333	-200.0000	3.1111	180.00	0.	.8000E+00
92	4	20.2222	-200.0000	3.1111	180.00	0.	.8000E+00
93	4	17.1111	-200.0000	3.1111	180.00	0.	.8000E+00
94	4	14.0000	-200.0000	3.1111	180.00	0.	.8000E+00
95	4	10.8889	-200.0000	3.1111	180.00	0.	.8000E+00
96	4	7.7778	-200.0000	3.1111	180.00	0.	.8000E+00
97	4	4.6667	-200.0000	3.1111	180.00	0.	.8000E+00
98	4	1.5556	-200.0000	3.1111	180.00	0.	.8000E+00
99	2	551.0005	-0.0010	0.0010	0.00	0.	0.

1

DISPLACEMENTS AND STRESSES AT MIDPOINTS OF BOUNDARY ELEMENTS

ELEM	DS	US(-)	US(+)	DN	UN(-)	UN(+)	UX(-)	UY(-)	SIG-S	SIG-N	SIGT1	SIGTP	SIGTN
1	.6E-01	.2E-02	-.6E-01	.2E+03	0.3	-.2E+03	.2E-02	.3E+00	.1E-12	.1E+02	-.2E+03	-.3E+03	-.3E+01
2	.2E+00	.7E-02	-.2E+00	.2E+03	0.3	-.2E+03	.7E-02	.3E+00	-.1E-06	.1E+02	-.2E+03	-.3E+03	-.3E+01
3	.3E+00	.1E-01	-.3E+00	.2E+03	0.3	-.2E+03	.1E-01	.3E+00	-.2E-06	.1E+02	-.2E+03	-.3E+03	-.2E+01
4	.4E+00	.2E-01	-.4E+00	.2E+03	0.3	-.2E+03	.2E-01	.3E+00	-.1E-07	.1E+02	-.2E+03	-.3E+03	-.2E+01
5	.5E+00	.2E-01	-.5E+00	.2E+03	0.3	-.2E+03	.2E-01	.3E+00	-.2E-07	.1E+02	-.2E+03	-.3E+03	-.1E+01
6	.7E+00	.3E-01	-.6E+00	.2E+03	0.3	-.2E+03	.3E-01	.3E+00	.2E-06	.1E+02	-.2E+03	-.3E+03	.9E-01
7	.8E+00	.3E-01	-.7E+00	.2E+03	0.3	-.2E+03	.3E-01	.3E+00	.2E-06	.1E+02	-.1E+03	-.3E+03	.2E+01
8	.9E+00	.4E-01	-.8E+00	.2E+03	0.3	-.2E+03	.4E-01	.3E+00	.9E-07	.7E+01	-.1E+03	-.3E+03	.6E+01
9	.1E+01	.4E-01	-.9E+00	.2E+03	0.3	-.2E+03	.4E-01	.3E+00	-.2E-06	.1E+00	-.1E+03	-.2E+03	.1E+02
10	.1E+01	.5E-01	-.1E+01	.2E+03	0.3	-.2E+03	.5E-01	.3E+00	.2E-06	-.2E+02	-.6E+02	-.1E+03	.2E+02
11	.1E+01	.5E-01	-.1E+01	.2E+03	0.3	-.2E+03	.5E-01	.3E+00	-.3E-06	-.5E+02	.4E+02	.3E+02	.4E+02
12	.1E+01	.3E-01	-.1E+01	.2E+03	0.3	-.2E+03	.3E-01	.3E+00	.3E-07	-.1E+03	.2E+03	.4E+03	.7E+02
13	.9E+00	-.1E-01	-.9E+00	.2E+03	0.3	-.2E+03	-.1E-01	.3E+00	.3E-06	-.3E+03	.5E+03	.9E+03	.1E+03
14	.5E+00	-.1E+00	-.6E+00	.2E+03	0.3	-.2E+03	-.1E+00	.3E+00	.4E-06	-.5E+03	.1E+04	.2E+04	.2E+03
15	-.4E+00	-.3E+00	.7E-01	.2E+03	0.3	-.2E+03	-.3E+00	.3E+00	-.5E-06	.9E+03	.2E+04	.3E+04	.3E+03
16	-.2E+01	-.6E+00	.1E+01	.2E+03	0.3	-.2E+03	-.6E+00	.3E+00	-.2E-07	-.1E+04	.3E+04	.5E+04	.5E+03
17	-.4E+01	-.1E+01	.2E+01	.1E+03	0.3	-.1E+03	-.1E+01	.3E+00	-.4E-06	-.2E+04	.3E+04	.6E+04	.8E+03
18	-.6E+01	-.2E+01	.4E+01	.1E+03	0.3	-.1E+03	-.2E+01	.3E+00	.6E-07	-.3E+04	.5E+04	.8E+04	.1E+04
19	-.9E+01	-.3E+01	.6E+01	.8E+02	0.3	-.8E+02	-.3E+01	.3E+00	.9E-07	-.4E+04	.3E+04	.5E+04	.9E+03
20	-.9E+01	-.2E+01	.6E+01	.5E+02	0.2	-.5E+02	-.2E+01	.2E+00	-.1E-06	-.4E+04	-.1E+04	-.6E+04	.3E+04
21	-.3E+01	-.9E+00	.3E+01	.2E+02	0.2	-.2E+02	-.9E+00	.2E+00	-.4E-07	-.3E+04	-.4E+04	-.1E+05	.4E+04
22	.1E+01	.2E+00	-.9E+00	-.1E+01	0.2	.2E+01	.2E+00	.2E+00	.9E-07	-.1E+04	-.4E+04	-.9E+04	.2E+04
23	.4E+01	.6E+00	-.3E+01	-.1E+02	0.2	.1E+02	.6E+00	.2E+00	.2E-06	-.3E+03	-.3E+04	-.5E+04	.8E+02
24	.5E+01	.6E+00	-.4E+01	-.2E+02	0.2	.2E+02	.6E+00	.2E+00	.7E-07	.8E+02	-.2E+04	-.3E+04	-.4E+03
25	.5E+01	.5E+00	-.5E+01	-.2E+02	0.2	.2E+02	.5E+00	.2E+00	-.3E-07	.2E+03	-.7E+03	-.1E+04	-.5E+03
26	.5E+01	.4E+00	-.3E+01	-.2E+02	0.2	.2E+02	.4E+00	.2E+00	.7E-08	.1E+03	-.3E+03	-.3E+03	-.4E+03
27	.5E+01	.3E+00	-.5E+01	-.2E+02	0.2	.2E+02	.3E+00	.2E+00	.9E-08	.1E+03	-.2E+03	-.9E+02	-.3E+03
28	.5E+01	.3E+00	-.5E+01	-.2E+02	0.2	.2E+02	.3E+00	.2E+00	.2E-06	.7E+02	-.2E+03	-.2E+03	-.1E+03
29	.5E+01	.3E+00	-.5E+01	-.2E+02	0.2	.3E+02	.3E+00	.2E+00	.3E-08	.1E+02	-.2E+03	-.4E+03	-.9E+00
30	.5E+01	.4E+00	-.5E+01	-.2E+02	0.2	.3E+02	.4E+00	.2E+00	-.1E-06	.2E+02	-.1E+03	-.3E+03	.1E+02
31	.5E+01	.4E+00	-.5E+01	-.2E+02	0.2	.2E+02	.4E+00	.2E+00	-.4E-07	.8E+02	.6E+02	.2E+03	-.1E+03

32	.5E+01	.3E+00	-.5E+01	-.2E+02	0.2	.2E+02	.3E+00	.2E+00	-.4E-07	.5E+02	.3E+03	.8E+03	-.1E+03
33	.5E+01	.3E+00	-.4E+01	-.2E+02	0.2	.2E+02	.3E+00	.2E+00	-.3E-07	.1E+02	.4E+03	.7E+03	.2E+02
34	.4E+01	.3E+00	-.4E+01	-.2E+02	0.2	.2E+02	.3E+00	.2E+00	-.4E-07	.6E+02	.4E+03	.6E+03	.1E+03
35	.4E+01	.3E+00	-.4E+01	-.2E+02	0.1	.2E+02	.3E+00	.1E+00	-.2E-08	-.8E+02	-.5E+02	.8E+02	-.2E+03
36	.4E+01	.3E+00	-.4E+01	-.2E+02	0.1	.2E+02	.3E+00	.1E+00	.4E-07	-.8E+01	-.6E+02	-.4E+02	-.7E+02
37	.4E+01	.3E+00	-.4E+01	-.2E+02	0.1	.2E+02	.3E+00	.1E+00	.1E-06	.3E+00	-.6E+02	-.1E+03	-.3E+02
38	.4E+01	.3E+00	-.4E+01	-.2E+02	0.1	.2E+02	.3E+00	.1E+00	-.2E-07	.7E+02	-.1E+03	-.1E+03	-.1E+03
39	.4E+01	.3E+00	-.4E+01	-.2E+02	0.1	.2E+02	.3E+00	.1E+00	-.4E-07	.1E+03	-.3E+03	-.4E+03	-.6E+02
40	.5E+01	.3E+00	-.4E+01	-.2E+02	0.0	.2E+02	.3E+00	.5E-01	.2E-06	-.6E+02	-.4E+03	-.8E+03	.1E+02
41	.5E+01	.4E+00	-.4E+01	-.2E+02	0.0	.2E+02	.4E+00	.5E-01	.7E-07	.5E+02	-.5E+03	-.1E+04	.1E+02
42	.5E+01	.4E+00	-.5E+01	-.2E+02	0.0	.2E+02	.4E+00	.5E-01	.5E-07	.1E+03	-.7E+03	-.1E+04	.5E+02
43	.6E+01	.6E+00	-.5E+01	-.1E+02	0.0	.1E+02	.6E+00	.5E-01	-.4E-07	.2E+03	-.7E+03	-.1E+04	.6E+02
44	.6E+01	.7E+00	-.5E+01	-.8E+01	0.0	.8E+01	.7E+00	.5E-01	.3E-06	.2E+03	-.3E+03	-.7E+03	-.8E+01
45	.6E+01	.8E+00	-.3E+01	-.2E+01	0.0	.2E+01	.8E+00	.5E-01	-.4E-06	-.3E+03	.9E+03	.2E+04	-.2E+02
46	.5E+01	.8E+00	-.4E+01	.4E+01	-0.1	-.4E+01	.8E+00	-.5E-01	.1E-06	-.2E+03	.8E+03	.2E+04	-.8E+03
47	.3E+01	.3E+00	-.3E+01	.7E+01	-0.1	-.7E+01	.3E+00	-.5E-01	-.1E-06	-.2E+03	.1E+04	.3E+04	-.2E+03
48	.2E+01	.2E-01	-.2E+01	.6E+01	-0.1	-.6E+01	.2E-01	-.5E-01	.1E-06	-.3E+03	.1E+04	.3E+04	.3E+03
49	.5E-01	-.1E+00	-.2E+00	-.2E+01	-0.1	.2E+01	.1E+00	-.1E+00	.3E-07	-.5E+03	-.4E+03	.1E+05	-.2E+05
50	.8E+00	.8E-01	-.7E+00	-.3E+01	-0.1	.3E+01	.1E+00	.8E-01	.1E-06	.4E+03	-.2E+04	-.8E+04	.4E+04
51	-.5E+01	-.9E+00	.4E+01	-.2E+01	0.3	.3E+01	.9E+00	-.3E+00	-.2E-06	-.3E+03	.9E+03	.9E+03	.9E+03
52	-.3E+01	-.2E+00	.3E+01	-.6E+01	0.3	.7E+01	.2E+00	-.3E+00	-.7E-07	-.3E+03	-.9E+03	-.3E+04	.7E+03
53	-.3E+01	-.7E-01	.3E+01	-.7E+01	0.3	.8E+01	.7E-01	-.3E+00	.1E-06	-.7E+02	-.1E+04	-.2E+04	.3E+03
54	-.1E+01	.3E+00	.2E+01	-.5E+01	0.3	.6E+01	-.3E+00	-.3E+00	.2E-06	-.5E+03	-.2E+03	-.1E+04	.9E+03
55	-.9E+00	.4E+00	.1E+01	-.2E+01	0.3	.2E+01	-.4E+00	-.3E+00	.6E-07	.9E+03	-.1E+04	-.2E+04	-.8E+03
56	-.4E+00	.3E+00	.7E+00	.5E+01	0.4	-.4E+01	-.3E+00	-.4E+00	-.1E-06	.5E+03	.2E+03	.1E+03	.2E+03
57	-.9E+00	.2E+00	.1E+01	.1E+02	0.4	-.1E+02	-.2E+00	-.4E+00	.4E-07	.3E+03	.6E+03	.2E+04	-.4E+03
58	-.2E+01	.4E-01	.2E+01	.1E+02	0.4	-.1E+02	-.4E-01	-.4E+00	-.1E-06	.2E+03	.9E+03	.2E+04	-.4E+03
59	-.2E+01	-.8E-01	.2E+01	.2E+02	0.4	-.2E+02	.8E-01	-.4E+00	-.2E-06	.1E+03	.9E+03	.2E+04	-.3E+03
60	-.3E+01	-.2E+00	.3E+01	.2E+02	0.4	-.2E+02	.2E+00	-.4E+00	.9E-07	.3E+02	.8E+03	.2E+04	-.2E+03
61	-.3E+01	-.2E+00	.3E+01	.2E+02	0.4	-.2E+02	.2E+00	-.4E+00	.4E-06	-.2E+03	.6E+03	.1E+04	-.1E+03
62	-.4E+01	-.2E+00	.3E+01	.2E+02	0.5	-.2E+02	.2E+00	-.5E+00	.3E-07	.5E+02	.5E+03	.1E+04	.4E+02
63	-.4E+01	-.2E+00	.4E+01	.2E+02	0.5	-.2E+02	.2E+00	-.5E+00	.3E-08	.3E+02	.4E+03	.7E+03	.3E+02
64	-.4E+01	-.2E+00	.4E+01	.2E+02	0.5	-.2E+02	.2E+00	-.5E+00	-.6E-07	-.6E+00	.3E+03	.7E+03	.1E+02
65	-.4E+01	-.2E+00	.4E+01	.2E+02	0.5	-.2E+02	.2E+00	-.5E+00	-.2E-06	-.3E+02	.3E+03	.5E+03	.4E+02
66	-.5E+01	-.2E+00	.4E+01	.2E+02	0.5	-.2E+02	.2E+00	-.5E+00	.5E-07	.3E+02	-.6E+01	.5E+02	-.6E+02
67	-.5E+01	-.2E+00	.4E+01	.2E+02	0.5	-.2E+02	.2E+00	-.5E+00	.2E-07	.1E+02	-.9E+02	-.1E+03	-.4E+02
68	-.4E+01	-.2E+00	.4E+01	.2E+02	0.5	-.2E+02	.2E+00	-.5E+00	-.1E-06	-.4E+01	-.1E+03	-.6E+02	-.1E+03
69	-.5E+01	-.2E+00	.4E+01	.3E+02	0.6	-.2E+02	.2E+00	-.6E+00	.6E-07	.1E+03	.4E+03	.7E+03	.5E+02
70	-.5E+01	-.2E+00	.5E+01	.3E+02	0.6	-.3E+02	.2E+00	-.6E+00	-.2E-06	.3E+02	.5E+03	.1E+04	-.3E+02
71	-.5E+01	-.3E+00	.5E+01	.3E+02	0.6	-.2E+02	.3E+00	-.6E+00	.3E-06	.5E+02	.4E+03	.9E+03	-.3E+02
72	-.6E+01	-.3E+00	.5E+01	.2E+02	0.6	-.2E+02	.3E+00	-.6E+00	.7E-07	.8E+02	.4E+03	.7E+03	-.1E+02
73	-.6E+01	-.3E+00	.6E+01	.2E+02	0.6	-.2E+02	.3E+00	-.6E+00	-.8E-07	.1E+03	.4E+03	.8E+03	.1E+02
74	-.6E+01	-.3E+00	.6E+01	.2E+02	0.6	-.2E+02	.3E+00	-.6E+00	.1E-05	.6E+02	.8E+03	.2E+04	.1E+01
75	-.7E+01	-.3E+00	.7E+01	.2E+02	0.6	-.2E+02	.3E+00	-.6E+00	.3E-06	-.6E+02	.2E+04	.3E+04	-.8E+02
76	-.9E+01	-.5E+00	.8E+01	.1E+02	0.6	-.1E+02	.5E+00	-.6E+00	-.4E-06	-.4E+03	.3E+04	.6E+04	-.3E+03
77	-.1E+02	-.1E+01	.1E+02	-.3E+00	0.6	.9E+00	.1E+01	-.6E+00	.4E-06	-.1E+04	.5E+04	.1E+05	-.5E+03
78	-.2E+02	-.2E+01	.1E+02	-.2E+02	0.6	.2E+02	.2E+01	-.6E+00	-.4E-06	-.2E+04	.6E+04	.1E+05	-.4E+03
79	-.2E+02	-.4E+01	.2E+02	-.6E+02	0.6	.6E+02	.4E+01	-.6E+00	-.4E-06	-.2E+04	.8E+03	.1E+04	.2E+03
80	-.2E+02	-.3E+01	.1E+02	-.9E+02	0.6	.9E+02	.3E+01	-.6E+00	-.5E-06	-.4E+04	-.8E+03	-.6E+04	.5E+04
81	-.1E+02	-.2E+01	.1E+02	-.1E+03	0.8	.1E+03	.2E+01	-.8E+00	.2E-05	-.3E+04	-.2E+04	-.8E+04	.4E+04
82	-.8E+01	-.1E+01	.7E+01	-.1E+03	0.8	.1E+03	.1E+01	-.8E+00	.1E-06	-.2E+04	-.2E+04	-.6E+04	.2E+04
83	-.6E+01	-.4E+00	.5E+01	-.2E+03	0.8	.2E+03	.4E+00	-.8E+00	-.6E-06	-.1E+04	-.2E+04	-.5E+04	.1E+04
84	-.4E+01	-.1E+00	.4E+01	-.2E+03	0.8	.2E+03	.1E+00	-.8E+00	.1E-05	-.7E+03	-.1E+04	-.3E+04	.6E+03
85	-.3E+01	.6E-02	.3E+01	-.2E+03	0.8	.2E+03	-.6E-02	-.8E+00	.5E-06	-.4E+03	-.1E+04	-.2E+04	.2E+03
86	-.2E+01	.6E-01	.2E+01	-.2E+03	0.8	.2E+03	-.6E-01	-.8E+00	.1E-05	-.2E+03	-.7E+03	-.1E+04	.3E+02

87	-.2E+01	.6E-01	.2E+01	-.2E+03	0.8	.2E+03	-.6E-01	-.8E+00	-.6E-07	-.8E+02	-.5E+03	-.9E+03	-.5E+02
88	-.1E+01	.5E-01	.1E+01	-.2E+03	0.8	.2E+03	-.5E-01	-.8E+00	-.3E-06	-.3E+02	-.3E+03	-.6E+03	-.6E+02
89	-.1E+01	.3E-01	.1E+01	-.2E+03	0.8	.2E+03	-.3E-01	-.8E+00	.1E-05	-.5E+01	-.3E+03	-.5E+03	-.6E+02
90	-.9E+00	.2E-01	.1E+01	-.2E+03	0.8	.2E+03	-.2E-01	-.8E+00	-.8E-07	.6E+01	-.2E+03	-.4E+03	-.5E+02
91	-.8E+00	.9E-02	.8E+00	-.2E+03	0.8	.2E+03	-.9E-02	-.8E+00	-.1E-05	.1E+02	-.2E+03	-.3E+03	-.4E+02
92	-.7E+00	.3E-02	.7E+00	-.2E+03	0.8	.2E+03	-.3E-02	-.8E+00	.6E-06	.1E+02	-.2E+03	-.3E+03	-.3E+02
93	-.6E+00	-.1E-02	.6E+00	-.2E+03	0.8	.2E+03	.1E-02	-.8E+00	.8E-06	.1E+02	-.2E+03	-.3E+03	-.2E+02
94	-.5E+00	-.3E-02	.5E+00	-.2E+03	0.8	.2E+03	.3E-02	-.8E+00	.1E-07	.1E+02	-.2E+03	-.3E+03	-.2E+02
95	-.4E+00	-.3E-02	.4E+00	-.2E+03	0.8	.2E+03	.3E-02	-.8E+00	-.2E-06	.1E+02	-.2E+03	-.3E+03	-.1E+02
96	-.3E+00	-.3E-02	.3E+00	-.2E+03	0.8	.2E+03	.3E-02	-.8E+00	-.3E-06	.1E+02	-.2E+03	-.3E+03	-.1E+02
97	-.2E+00	-.2E-02	.2E+00	-.2E+03	0.8	.2E+03	.2E-02	-.8E+00	-.4E-06	.1E+02	-.2E+03	-.3E+03	-.1E+02
98	-.6E-01	-.7E-03	.5E-01	-.2E+03	0.8	.2E+03	.7E-03	-.8E+00	-.6E-06	.1E+02	-.2E+03	-.3E+03	-.5E+02
99	.5E-01	-.9E-10	-.5E-01	.3E-01	-0.0	-.3E-01	-.9E-10	-.6E-11	.1E+06	.5E+05	.8E+05	.8E+05	.8E+05

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DISPLACEMENTS AND STRESSES AT SPECIFIED POINTS IN THE BODY

POINT	X CO-ORD	Y CO-ORD	UX	UY	SIGXX	SIGYY	SIGXY
1	0.0000	0.0000	0.000000	-0.668726	-20.4	-0.0	0.0
2	11.6667	0.0000	-0.029031	-0.665799	-20.1	-0.0	0.0
3	23.3333	0.0000	-0.057387	-0.657105	-19.4	-0.0	0.0
4	35.0000	0.0000	-0.084421	-0.642907	-18.3	0.0	-0.0
5	46.6667	0.0000	-0.109539	-0.623631	-16.8	-0.0	-0.0
6	58.3333	0.0000	-0.132230	-0.599846	-14.9	0.0	-0.0
7	70.0000	0.0000	-0.152088	-0.572238	-12.8	0.0	-0.0
8	81.6667	0.0000	-0.168824	-0.541574	-10.6	-0.0	-0.0
9	93.3333	0.0000	-0.182283	-0.508662	-8.2	-0.0	-0.0
10	105.0000	0.0000	-0.192433	-0.474310	-6.0	-0.0	-0.0
11	116.6667	0.0000	-0.199362	-0.439290	-3.8	-0.0	0.0
12	128.3333	0.0000	-0.203258	-0.404304	-1.7	-0.0	-0.0
13	140.0000	0.0000	-0.204388	-0.369966	0.1	0.0	-0.0
14	151.6667	0.0000	-0.203073	-0.336785	1.7	0.0	0.0
15	163.3333	0.0000	-0.199665	-0.305158	3.0	0.0	-0.0
16	175.0000	0.0000	-0.194527	-0.275379	4.1	0.0	0.0
17	186.6667	0.0000	-0.188015	-0.247638	5.0	0.0	-0.0
18	198.3333	0.0000	-0.180465	-0.222040	5.6	0.0	0.0
19	210.0000	0.0000	-0.172161	-0.198612	6.0	0.0	-0.0
20	221.6667	0.0000	-0.163429	-0.177321	6.2	0.0	-0.0
21	233.3333	0.0000	-0.154439	-0.158091	6.3	0.0	0.0
22	245.0000	0.0000	-0.145398	-0.140808	6.3	0.0	-0.0
23	256.6667	0.0000	-0.136457	-0.125342	6.2	0.0	-0.0
24	268.3333	0.0000	-0.127731	-0.111547	6.0	0.0	-0.0
25	280.0000	0.0000	-0.119306	-0.099276	5.8	-0.0	0.0

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HORIZONTAL STRAIN AT SPECIFIED POINTS

N-POINT	XP(NI)	STRAIN
1	0.0000	-0.002468
2	11.6667	-0.002459
3	23.3333	-0.002374
4	35.0000	-0.002235
5	46.6667	-0.002049
6	58.3333	-0.001824
7	70.0000	-0.001568

8	81.6667	-0.001294
9	93.3333	-0.001012
10	105.0000	-0.000732
11	116.6667	-0.000464
12	128.3333	-0.000215
13	140.0000	0.000008
14	151.6667	0.000202
15	163.3333	0.000366
16	175.0000	0.000499
17	186.6667	0.000603
18	198.3333	0.000679
19	210.0000	0.000730
20	221.6667	0.000760
21	233.3333	0.000773
22	245.0000	0.000771
23	256.6667	0.000757
24	268.3333	0.000735
25	280.0000	0.000722

(*) The output from program modified TWODD for the subsidence model in this appendix (or the other sections) gives the displacements and stresses at the midpoints of the boundary elements; this is arranged to print values of the displacements on the positive and negative sides of the elements, as well as the associated displacement discontinuity components (Ds=DS and Dn=DN). The displacement discontinuities are fictitious, and only the displacements and stresss on the negative side of boundary are physically meaningful.

APPENDIX D THE OUTPUT OF THE SUBSIDENCE MODEL ANALYSIS OF DISPLACEMENT BOUNDARY CONDITION SPECIFIED AT THE BOUNDARY WITHOUT GIVING HORIZONTAL DISPLACEMENT AT THE BOUNDARY

DISPLACEMENT BOUNDARY CONDITION FOR THE PANEL WIDTH/MINE DEPTH=1.4 WITHOUT HORIZONTAL DISPLACEMENT SPCIFIED AT BOUNDARY

NUMBER OF STRAIGHT-LINE SEGMENTS (EACH CONTAINING AT L EAST ONE BOUNDARY ELEMENT) USED TO DEFINE BOUNDARIES = 9

NUMBER OF STRAIGHT-LINE SEGMENTS USED TO SPECIFY OTHER LOCATIONS (NOT ON A BOUNDARY) = 1

THE LINE X = XS = 0.0000 IS A LINE OF SYMMETRY.

POISSION'S RATIO = 0.17

YOUNG'S MODULUS = .8000E+04

UNIT WEIGHT OF OVERBURDEN = .2700E-01

RATIO OF FIELD STRESS = .2000E+01

XY-COMPONENT OF FIELD STRESS = 0.

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BOUNDARY ELEMENT DATA.

ELEMENT	KODE	X (CENTER)	Y (CENTER)	LENGTH	ANGLE	US OR SIGMA-S	UN OR SIGMA-N
1	4	1.5217	-202.0000	3.0435	0.00	0.	0.
2	4	4.5652	-202.0000	3.0435	0.00	0.	0.
3	4	7.6087	-202.0000	3.0435	0.00	0.	0.
4	4	10.6522	-202.0000	3.0435	0.00	0.	0.
5	4	13.6957	-202.0000	3.0435	0.00	0.	0.
6	4	16.7391	-202.0000	3.0435	0.00	0.	0.
7	4	19.7826	-202.0000	3.0435	0.00	0.	0.
8	4	22.8261	-202.0000	3.0435	0.00	0.	0.
9	4	25.8696	-202.0000	3.0435	0.00	0.	0.
10	4	28.9130	-202.0000	3.0435	0.00	0.	0.
11	4	31.9565	-202.0000	3.0435	0.00	0.	0.
12	4	35.0000	-202.0000	3.0435	0.00	0.	0.
13	4	38.0435	-202.0000	3.0435	0.00	0.	0.
14	4	41.0870	-202.0000	3.0435	0.00	0.	0.
15	4	44.1304	-202.0000	3.0435	0.00	0.	0.
16	4	47.1739	-202.0000	3.0435	0.00	0.	0.
17	4	50.2174	-202.0000	3.0435	0.00	0.	0.
18	4	53.2609	-202.0000	3.0435	0.00	0.	0.
19	4	56.3043	-202.0000	3.0435	0.00	0.	0.
20	4	59.3478	-202.0000	3.0435	0.00	0.	0.
21	4	62.3913	-202.0000	3.0435	0.00	0.	0.
22	4	65.4348	-202.0000	3.0435	0.00	0.	0.
23	4	68.4783	-202.0000	3.0435	0.00	0.	0.
24	4	71.5217	-202.0000	3.0435	0.00	0.	0.
25	4	74.5652	-202.0000	3.0435	0.00	0.	0.

26	4	77.6087	-202.0000	3.0435	-0.00	0.	0.
27	4	80.6522	-202.0000	3.0435	0.00	0.	0.
28	4	83.6957	-202.0000	3.0435	0.00	0.	0.
29	4	86.7391	-202.0000	3.0435	0.00	0.	0.
30	4	89.7826	-202.0000	3.0435	0.00	0.	0.
31	4	92.8261	-202.0000	3.0435	0.00	0.	0.
32	4	95.8696	-202.0000	3.0435	0.00	0.	0.
33	4	98.9130	-202.0000	3.0435	0.00	0.	0.
34	4	101.9565	-202.0000	3.0435	0.00	0.	0.
35	4	105.0000	-202.0000	3.0435	0.00	0.	0.
36	4	108.0435	-202.0000	3.0435	0.00	0.	0.
37	4	111.0870	-202.0000	3.0435	0.00	0.	0.
38	4	114.1304	-202.0000	3.0435	0.00	0.	0.
39	4	117.1739	-202.0000	3.0435	0.00	0.	0.
40	4	120.2174	-202.0000	3.0435	0.00	0.	0.
41	4	123.2609	-202.0000	3.0435	0.00	0.	0.
42	4	126.3043	-202.0000	3.0435	0.00	0.	0.
43	4	129.3478	-202.0000	3.0435	0.00	0.	0.
44	4	132.3913	-202.0000	3.0435	0.00	0.	0.
45	4	135.4348	-202.0000	3.0435	0.00	0.	0.
46	4	138.4783	-202.0000	3.0435	0.00	0.	0.
47	1	140.0000	-201.5000	1.0000	90.00	0.	0.
48	1	140.0000	-200.5000	1.0000	90.00	0.	0.
49	4	139.3750	-200.0000	1.2500	180.00	0.	0.
50	4	138.6250	-200.0000	0.2500	180.00	0.	.5000E-01
51	4	138.3750	-200.0000	0.2500	180.00	0.	.1000E+00
52	4	138.1250	-200.0000	0.2500	180.00	0.	.4000E+00
53	4	137.8750	-200.0000	0.2500	180.00	0.	.8000E+00
54	4	136.2194	-200.0000	3.0611	180.00	0.	.1950E+01
55	4	133.1583	-200.0000	3.0611	180.00	0.	.1950E+01
56	4	130.0972	-200.0000	3.0611	180.00	0.	.1950E+01
57	4	127.0361	-200.0000	3.0611	180.00	0.	.1950E+01
58	4	123.9750	-200.0000	3.0611	180.00	0.	.1950E+01
59	4	120.9139	-200.0000	3.0611	180.00	0.	.1950E+01
60	4	117.8528	-200.0000	3.0611	180.00	0.	.1950E+01
61	4	114.7917	-200.0000	3.0611	180.00	0.	.1950E+01
62	4	111.7306	-200.0000	3.0611	180.00	0.	.1950E+01
63	4	108.6694	-200.0000	3.0611	180.00	0.	.1950E+01
64	4	105.6083	-200.0000	3.0611	180.00	0.	.1950E+01
65	4	102.5472	-200.0000	3.0611	180.00	0.	.1950E+01
66	4	99.4861	-200.0000	3.0611	180.00	0.	.1950E+01
67	4	96.4250	-200.0000	3.0611	180.00	0.	.1950E+01
68	4	93.3639	-200.0000	3.0611	180.00	0.	.1950E+01
69	4	90.3028	-200.0000	3.0611	180.00	0.	.1950E+01
70	4	87.2417	-200.0000	3.0611	180.00	0.	.1950E+01
71	4	84.1806	-200.0000	3.0611	180.00	0.	.1950E+01
72	4	81.1194	-200.0000	3.0611	180.00	0.	.1950E+01
73	4	78.0583	-200.0000	3.0611	180.00	0.	.1950E+01
74	4	74.9972	-200.0000	3.0611	180.00	0.	.1950E+01
75	4	71.9361	-200.0000	3.0611	180.00	0.	.1950E+01
76	4	68.8750	-200.0000	3.0611	180.00	0.	.1950E+01
77	4	65.8139	-200.0000	3.0611	180.00	0.	.1950E+01
78	4	62.7528	-200.0000	3.0611	180.00	0.	.1950E+01
79	4	59.6917	-200.0000	3.0611	180.00	0.	.1950E+01
80	4	56.6306	-200.0000	3.0611	180.00	0.	.1950E+01

79	4	59.6917	-200.0000	3.0611	180.00	0.	.1950E+01
80	4	56.6306	-200.0000	3.0611	180.00	0.	.1950E+01
81	4	53.5694	-200.0000	3.0611	180.00	0.	.1950E+01
82	4	50.5083	-200.0000	3.0611	180.00	0.	.1950E+01
83	4	47.4472	-200.0000	3.0611	180.00	0.	.1950E+01
84	4	44.3861	-200.0000	3.0611	180.00	0.	.1950E+01
85	4	41.3250	-200.0000	3.0611	180.00	0.	.1950E+01
86	4	38.2639	-200.0000	3.0611	180.00	0.	.1950E+01
87	4	35.2028	-200.0000	3.0611	180.00	0.	.1950E+01
88	4	32.1417	-200.0000	3.0611	180.00	0.	.1950E+01
89	4	29.0806	-200.0000	3.0611	180.00	0.	.1950E+01
90	4	26.0194	-200.0000	3.0611	180.00	0.	.1950E+01
91	4	22.9583	-200.0000	3.0611	180.00	0.	.1950E+01
92	4	19.8972	-200.0000	3.0611	180.00	0.	.1950E+01
93	4	16.8361	-200.0000	3.0611	180.00	0.	.1950E+01
94	4	13.7750	-200.0000	3.0611	180.00	0.	.1950E+01
95	4	10.7139	-200.0000	3.0611	180.00	0.	.1950E+01
96	4	7.6528	-200.0000	3.0611	180.00	0.	.1950E+01
97	4	4.5917	-200.0000	3.0611	180.00	0.	.1950E+01
98	4	1.5306	-200.0000	3.0611	180.00	0.	.1950E+01
99	2	551.0005	-0.0010	0.0010	0.00	0.	0.

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DISPLACEMENTS AND STRESSES AT MIDPOINTS OF BOUNDARY ELEMENTS

ELEM	DS	US(-)	US(+)	DN	UN(-)	UN(+)	UX(-)	UY(-)	SIG-S	SIG-N	SIGT1	SIGTF	SIGTN
1	-.9E-01	-.2E-01	.8E-01	-.5E+06	0.0	.5E+06	-.2E-01	.2E-05	.1E-12	-.4E+04	.5E+04	.5E+04	.4E+04
2	-.3E+00	-.5E-01	.2E+00	-.5E+06	0.0	.5E+06	-.5E-01	.3E-05	.1E-13	-.4E+04	.5E+04	.5E+04	.4E+04
3	-.5E+00	-.8E-01	.4E+00	-.5E+06	0.0	.5E+06	-.8E-01	.7E-06	.5E-03	-.4E+04	.5E+04	.5E+04	.4E+04
4	-.7E+00	-.1E+00	.6E+00	-.5E+06	-0.0	.5E+06	-.1E+00	-.1E-06	.4E-03	-.4E+04	.5E+04	.5E+04	.4E+04
5	-.9E+00	-.1E+00	.7E+00	-.5E+06	-0.0	.5E+06	-.1E+00	-.2E-05	.2E-03	-.4E+04	.5E+04	.5E+04	.4E+04
6	-.1E+01	-.2E+00	.9E+00	-.5E+06	-0.0	.5E+06	-.2E+00	-.5E-06	.4E-03	-.4E+04	.5E+04	.5E+04	.4E+04
7	-.1E+01	-.2E+00	.1E+01	-.5E+06	0.0	.5E+06	-.2E+00	.2E-05	.1E-04	-.4E+04	.5E+04	.5E+04	.4E+04
8	-.2E+01	-.3E+00	.1E+01	-.5E+06	0.0	.5E+06	-.3E+00	.5E-05	.8E-03	-.4E+04	.5E+04	.5E+04	.4E+04
9	-.2E+01	-.3E+00	.1E+01	-.5E+06	-0.0	.5E+06	-.3E+00	-.4E-06	.5E-03	-.4E+04	.5E+04	.5E+04	.4E+04
10	-.2E+01	-.3E+00	.2E+01	-.5E+06	0.0	.5E+06	-.3E+00	.3E-06	.6E-03	-.4E+04	.4E+04	.5E+04	.4E+04
11	-.2E+01	-.4E+00	.2E+01	-.5E+06	-0.0	.5E+06	-.4E+00	-.6E-05	.3E-03	-.4E+04	.4E+04	.5E+04	.4E+04
12	-.3E+01	-.4E+00	.2E+01	-.5E+06	-0.0	.5E+06	-.4E+00	-.7E-07	.1E-02	-.4E+04	.4E+04	.5E+04	.4E+04
13	-.3E+01	-.5E+00	.2E+01	-.5E+06	-0.0	.5E+06	-.5E+00	-.4E-05	.7E-03	-.4E+04	.4E+04	.5E+04	.4E+04
14	-.3E+01	-.5E+00	.3E+01	-.5E+06	-0.0	.5E+06	-.5E+00	-.3E-05	.7E-03	-.4E+04	.4E+04	.5E+04	.4E+04
15	-.4E+01	-.5E+00	.3E+01	-.5E+06	-0.0	.5E+06	-.5E+00	-.4E-05	.3E-03	-.4E+04	.4E+04	.5E+04	.4E+04
16	-.4E+01	-.6E+00	.3E+01	-.5E+06	0.0	.5E+06	-.6E+00	.4E-05	-.6E-03	-.3E+04	.4E+04	.5E+04	.4E+04
17	-.4E+01	-.6E+00	.4E+01	-.5E+06	-0.0	.5E+06	-.6E+00	-.6E-05	-.5E-03	-.3E+04	.4E+04	.5E+04	.4E+04
18	-.5E+01	-.7E+00	.4E+01	-.5E+06	0.0	.5E+06	-.7E+00	.9E-06	-.8E-03	-.3E+04	.4E+04	.5E+04	.3E+04
19	-.5E+01	-.8E+00	.5E+01	-.5E+06	-0.0	.5E+06	-.8E+00	-.1E-05	-.2E-02	-.3E+04	.4E+04	.5E+04	.3E+04
20	-.6E+01	-.8E+00	.5E+01	-.5E+06	0.0	.5E+06	-.8E+00	.3E-05	-.1E-02	-.3E+04	.4E+04	.5E+04	.3E+04
21	-.7E+01	-.9E+00	.6E+01	-.5E+06	0.0	.5E+06	-.9E+00	.8E-05	-.4E-03	-.3E+04	.4E+04	.5E+04	.3E+04
22	-.7E+01	-.1E+01	.6E+01	-.5E+06	0.0	.5E+06	-.1E+01	.8E-06	.1E-02	-.2E+04	.4E+04	.5E+04	.3E+04
23	-.8E+01	-.1E+01	.7E+01	-.5E+06	0.0	.5E+06	-.1E+01	.8E-05	-.1E-02	-.2E+04	.4E+04	.5E+04	.3E+04
24	-.9E+01	-.1E+01	.8E+01	-.5E+06	0.0	.5E+06	-.1E+01	.2E-05	-.1E-02	-.2E+04	.4E+04	.5E+04	.3E+04
25	-.1E+02	-.1E+01	.8E+01	-.5E+06	-0.0	.5E+06	-.1E+01	-.3E-05	.4E-03	-.2E+04	.4E+04	.5E+04	.2E+04
26	-.1E+02	-.1E+01	.9E+01	-.5E+06	-0.0	.5E+06	-.1E+01	-.2E-05	.2E-02	-.1E+04	.4E+04	.5E+04	.2E+04
27	-.1E+02	-.1E+01	.1E+02	-.5E+06	-0.0	.5E+06	-.1E+01	-.6E-05	-.2E-02	-.1E+04	.4E+04	.5E+04	.2E+04
28	-.1E+02	-.2E+01	.1E+02	-.5E+06	0.0	.5E+06	-.2E+01	.7E-05	-.1E-02	-.8E+03	.3E+04	.5E+04	.2E+04
29	-.1E+02	-.2E+01	.1E+02	-.5E+06	0.0	.5E+06	-.2E+01	.2E-05	.1E-02	-.5E+03	.3E+04	.5E+04	.2E+04

30	-.2E+02	-.2E+01	.1E+02	-.5E+06	0.0	.5E+06	-.2E+01	.3E-05	-.1E-02	-.9E+02	.3E+04	.6E+04	.1E+04
31	-.2E+02	-.2E+01	.2E+02	-.5E+06	-0.0	.5E+06	-.2E+01	-.5E-07	-.2E-02	.3E+03	.3E+04	.6E+04	.1E+04
32	-.2E+02	-.2E+01	.2E+02	-.5E+06	0.0	.5E+06	-.2E+01	.2E-05	.2E-03	.8E+03	.3E+04	.6E+04	.6E+03
33	-.2E+02	-.3E+01	.2E+02	-.5E+06	0.0	.5E+06	-.3E+01	.7E-06	-.1E-02	.1E+04	.3E+04	.6E+04	.2E+03
34	-.2E+02	-.3E+01	.2E+02	-.5E+06	0.0	.5E+06	-.3E+01	.5E-05	.5E-03	.2E+04	.3E+04	.7E+04	-.3E+03
35	-.3E+02	-.3E+01	.2E+02	-.5E+06	0.0	.5E+06	-.3E+01	.6E-05	.5E-03	.3E+04	.3E+04	.7E+04	-.8E+03
36	-.3E+02	-.4E+01	.3E+02	-.5E+06	0.0	.5E+06	-.4E+01	.2E-06	-.5E-03	.3E+04	.3E+04	.8E+04	-.2E+04
37	-.3E+02	-.4E+01	.3E+02	-.5E+06	0.0	.5E+06	-.4E+01	.3E-05	.3E-03	.4E+04	.3E+04	.8E+04	-.2E+04
38	-.4E+02	-.5E+01	.3E+02	-.5E+06	0.0	.5E+06	-.5E+01	.3E-05	.5E-03	.5E+04	.3E+04	.8E+04	-.3E+04
39	-.4E+02	-.6E+01	.4E+02	-.5E+06	0.0	.5E+06	-.6E+01	.1E-05	-.1E-02	.6E+04	.2E+04	.8E+04	-.4E+04
40	-.5E+02	-.7E+01	.4E+02	-.5E+06	0.0	.5E+06	-.7E+01	.5E-05	-.2E-03	.7E+04	.1E+04	.7E+04	-.5E+04
41	-.5E+02	-.8E+01	.4E+02	-.5E+06	-0.0	.5E+06	-.8E+01	-.4E-05	.2E-02	.8E+04	-.6E+03	.4E+04	-.6E+04
42	-.5E+02	-.9E+01	.5E+02	-.5E+06	-0.0	.5E+06	-.9E+01	-.2E-05	-.4E-03	.8E+04	-.3E+04	.2E+03	-.6E+04
43	-.6E+02	-.9E+01	.5E+02	-.5E+06	-0.0	.5E+06	-.9E+01	-.5E-05	-.1E-02	.8E+04	-.6E+04	-.6E+04	-.5E+04
44	-.5E+02	-.9E+01	.4E+02	-.5E+06	0.0	.5E+06	-.9E+01	.4E-05	.3E-02	.6E+04	-.9E+04	-.1E+05	-.4E+04
45	-.5E+02	-.8E+01	.4E+02	-.5E+06	-0.0	.5E+06	-.8E+01	-.1E-05	-.2E-02	.5E+04	-.1E+05	-.2E+05	-.3E+04
46	-.4E+02	-.7E+01	.3E+02	-.5E+06	0.0	.5E+06	-.7E+01	.2E-05	.9E-02	-.3E+04	-.9E+04	-.3E+05	.9E+04
47	-.5E+06	.4E+01	.5E+06	.3E+02	0.5	-.3E+02	-.5E+00	.4E+01	.9E-01	-.5E-02	.5E+04	.3E+05	-.2E+05
48	-.5E+06	.3E+01	.5E+06	.2E+02	-1.4	-.2E+02	.1E+01	.3E+01	-.2E-01	-.4E-02	-.1E+05	-.5E+04	-.2E+05
49	.1E+02	-.4E+01	-.2E+02	.5E+06	0.0	-.5E+06	.4E+01	-.6E-06	-.3E-03	.4E+04	.1E+05	.3E+05	-.1E+05
50	.1E+02	-.3E+01	-.1E+02	.5E+06	0.1	-.5E+06	.3E+01	-.5E-01	.2E-02	-.7E+04	.6E+04	.2E+05	-.7E+04
51	.9E+01	-.3E+01	-.1E+02	.5E+06	0.1	-.5E+06	.3E+01	-.1E+00	.1E-02	.6E+04	.2E+05	.5E+05	-.2E+04
52	.7E+01	-.3E+01	-.1E+02	.5E+06	0.4	-.5E+06	.3E+01	-.4E+00	-.4E-03	.2E+05	.4E+05	.7E+05	.1E+05
53	.5E+01	-.3E+01	-.8E+01	.5E+06	0.8	-.5E+06	.3E+01	-.8E+00	.2E-03	.1E+06	.1E+06	.1E+06	.1E+06
54	.2E+01	-.7E+01	-.9E+01	.5E+06	1.9	-.5E+06	.7E+01	-.2E+01	-.3E-02	.6E+04	.1E+05	.2E+05	.3E+04
55	-.6E+01	-.8E+01	-.2E+01	.5E+06	1.9	-.5E+06	.8E+01	-.2E+01	.2E-02	.6E+04	.8E+04	.2E+05	-.2E+04
56	-.1E+02	-.8E+01	.4E+01	.5E+06	2.0	-.5E+06	.8E+01	-.2E+01	-.4E-03	.7E+04	.4E+04	.1E+05	-.3E+04
57	-.2E+02	-.8E+01	.8E+01	.5E+06	2.0	-.5E+06	.8E+01	-.2E+01	.9E-03	.8E+04	.4E+03	.4E+04	-.3E+04
58	-.2E+02	-.8E+01	.9E+01	.5E+06	2.0	-.5E+06	.8E+01	-.2E+01	.1E-02	.8E+04	-.2E+04	-.4E+03	-.3E+04
59	-.2E+02	-.7E+01	.1E+02	.5E+06	2.0	-.5E+06	.7E+01	-.2E+01	.4E-02	.7E+04	-.3E+04	-.3E+04	-.2E+04
60	-.2E+02	-.7E+01	.1E+02	.5E+06	1.9	-.5E+06	.7E+01	-.2E+01	-.4E-02	.6E+04	-.3E+04	-.4E+04	-.2E+04
61	-.2E+02	-.6E+01	.1E+02	.5E+06	2.0	-.5E+06	.6E+01	-.2E+01	-.3E-02	.5E+04	-.3E+04	-.4E+04	-.1E+04
62	-.1E+02	-.6E+01	.9E+01	.5E+06	1.9	-.5E+06	.6E+01	-.2E+01	.6E-02	.4E+04	-.2E+04	-.4E+04	-.9E+03
63	-.1E+02	-.5E+01	.9E+01	.5E+06	2.0	-.5E+06	.5E+01	-.2E+01	-.7E-03	.4E+04	-.2E+04	-.3E+04	-.4E+03
64	-.1E+02	-.5E+01	.8E+01	.5E+06	1.9	-.5E+06	.5E+01	-.2E+01	.1E-02	.3E+04	-.1E+04	-.2E+04	.3E+02
65	-.1E+02	-.4E+01	.8E+01	.5E+06	1.9	-.5E+06	.4E+01	-.2E+01	.1E-02	.2E+04	-.7E+03	-.2E+04	.4E+03
66	-.1E+02	-.4E+01	.7E+01	.5E+06	1.9	-.5E+06	.4E+01	-.2E+01	.1E-02	.1E+04	-.1E+03	-.1E+04	.8E+03
67	-.1E+02	-.4E+01	.7E+01	.5E+06	1.9	-.5E+06	.4E+01	-.2E+01	-.1E-02	.9E+03	.3E+03	-.4E+03	.1E+04
68	-.1E+02	-.3E+01	.7E+01	.5E+06	2.0	-.5E+06	.3E+01	-.2E+01	.2E-02	.4E+03	.7E+03	.6E+02	.1E+04
69	-.9E+01	-.3E+01	.6E+01	.5E+06	2.0	-.5E+06	.3E+01	-.2E+01	.4E-03	-.3E+02	.1E+04	.5E+03	.2E+04
70	-.9E+01	-.3E+01	.6E+01	.5E+06	1.9	-.5E+06	.3E+01	-.2E+01	.1E-02	-.4E+03	.1E+04	.9E+03	.2E+04
71	-.9E+01	-.3E+01	.6E+01	.5E+06	2.0	-.5E+06	.3E+01	-.2E+01	.2E-02	-.8E+03	.2E+04	.1E+04	.2E+04
72	-.8E+01	-.3E+01	.6E+01	.5E+06	2.0	-.5E+06	.3E+01	-.2E+01	.2E-02	-.1E+04	.2E+04	.1E+04	.2E+04
73	-.8E+01	-.2E+01	.6E+01	.5E+06	2.0	-.5E+06	.2E+01	-.2E+01	-.2E-02	-.1E+04	.2E+04	.2E+04	.2E+04
74	-.8E+01	-.2E+01	.5E+01	.5E+06	1.9	-.5E+06	.2E+01	-.2E+01	-.1E-02	-.2E+04	.2E+04	.2E+04	.3E+04
75	-.8E+01	-.2E+01	.5E+01	.5E+06	2.0	-.5E+06	.2E+01	-.2E+01	.3E-02	-.2E+04	.2E+04	.2E+04	.3E+04
76	-.7E+01	-.2E+01	.5E+01	.5E+06	2.0	-.5E+06	.2E+01	-.2E+01	.4E-03	-.2E+04	.3E+04	.2E+04	.3E+04
77	-.7E+01	-.2E+01	.5E+01	.5E+06	1.9	-.5E+06	.2E+01	-.2E+01	.2E-02	-.2E+04	.3E+04	.2E+04	.3E+04
78	-.7E+01	-.2E+01	.5E+01	.5E+06	2.0	-.5E+06	.2E+01	-.2E+01	.3E-02	-.3E+04	.3E+04	.3E+04	.3E+04
79	-.6E+01	-.2E+01	.5E+01	.5E+06	1.9	-.5E+06	.2E+01	-.2E+01	-.2E-02	-.3E+04	.3E+04	.3E+04	.3E+04
80	-.6E+01	-.2E+01	.5E+01	.5E+06	2.0	-.5E+06	.2E+01	-.2E+01	.7E-03	-.3E+04	.3E+04	.3E+04	.4E+04
81	-.6E+01	-.1E+01	.4E+01	.5E+06	1.9	-.5E+06	.1E+01	-.2E+01	.7E-03	-.3E+04	.3E+04	.3E+04	.4E+04
82	-.6E+01	-.1E+01	.4E+01	.5E+06	2.0	-.5E+06	.1E+01	-.2E+01	-.1E-02	-.3E+04	.3E+04	.3E+04	.4E+04
83	-.5E+01	-.1E+01	.4E+01	.5E+06	1.9	-.5E+06	.1E+01	-.2E+01	.2E-02	-.3E+04	.3E+04	.3E+04	.4E+04
84	-.5E+01	-.1E+01	.4E+01	.5E+06	1.9	-.5E+06	.1E+01	-.2E+01	.2E-02	-.4E+04	.3E+04	.3E+04	.4E+04

85	-.5E+01	-.1E+01	.4E+01	.5E+06	2.0	-.5E+06	.1E+01	-.2E+01	.2E-02	-.4E+04	.4E+04	.3E+04	.4E+04
86	-.4E+01	-.1E+01	.3E+01	.5E+06	1.9	-.5E+06	.1E+01	-.2E+01	-.1E-02	-.4E+04	.4E+04	.3E+04	.4E+04
87	-.4E+01	-.9E+00	.3E+01	.5E+06	1.9	-.5E+06	.9E+00	-.2E+01	.5E-03	-.4E+04	.4E+04	.3E+04	.4E+04
88	-.4E+01	-.8E+00	.3E+01	.5E+06	1.9	-.5E+06	.8E+00	-.2E+01	-.1E-02	-.4E+04	.4E+04	.3E+04	.4E+04
89	-.3E+01-	-.8E+00	.3E+01	.5E+06	1.9	-.5E+06	.8E+00	-.2E+01	-.7E-03	-.4E+04	.4E+04	.3E+04	.4E+04
90	-.3E+01	-.7E+00	.2E+01	.5E+06	2.0	-.5E+06	.7E+00	-.2E+01	.8E-03	-.4E+04	.4E+04	.3E+04	.4E+04
91	-.3E+01	-.6E+00	.2E+01	.5E+06	1.9	-.5E+06	.6E+00	-.2E+01	-.1E-02	-.4E+04	.4E+04	.3E+04	.4E+04
92	-.2E+01	-.5E+00	.2E+01	.5E+06	1.9	-.5E+06	.5E+00	-.2E+01	.6E-03	-.4E+04	.4E+04	.3E+04	.4E+04
93	-.2E+01	-.4E+00	.2E+01	.5E+06	1.9	-.5E+06	.4E+00	-.2E+01	.1E-02	-.4E+04	.4E+04	.4E+04	.4E+04
94	-.2E+01	-.4E+00	.1E+01	.5E+06	1.9	-.5E+06	.4E+00	-.2E+01	.2E-03	-.4E+04	.4E+04	.4E+04	.4E+04
95	-.1E+01	-.3E+00	.1E+01	.5E+06	1.9	-.5E+06	.3E+00	-.2E+01	.4E-03	-.4E+04	.4E+04	.4E+04	.4E+04
96	-.9E+00	-.2E+00	.7E+00	.5E+06	2.0	-.5E+06	.2E+00	-.2E+01	-.1E-03	-.4E+04	.4E+04	.4E+04	.5E+04
97	-.5E+00	-.1E+00	.4E+00	.5E+06	2.0	-.5E+06	.1E+00	-.2E+01	-.2E-04	-.4E+04	.4E+04	.4E+04	.5E+04
98	-.2E+00	-.4E-01	.1E+00	.5E+06	1.9	-.5E+06	.4E-01	-.2E+01	-.3E-02	-.4E+04	.4E+04	.4E+04	.4E+04
99	.1E+00	-.7E-08	-.1E+00	.2E-02	0.0	-.2E-02	.7E-08	.2E-06	.3E+06	.4E+04	.6E+04	.6E+04	.6E+04

1 DISPLACEMENTS AND STRESSES AT SPECIFIED POINTS IN THE BODY

POINT	X CO-ORD	Y CO-ORD	UX	UY	SIGXX	SIGYY	SIGXY
1	0.0000	0.0000	0.000000	-1.749150	-62.1	0.0	0.0
2	11.6667	0.0000	-0.088535	-1.739481	-61.5	0.0	0.0
3	23.3333	0.0000	-0.175252	-1.710627	-59.6	0.0	0.0
4	35.0000	0.0000	-0.258349	-1.663164	-56.4	0.0	0.0
5	46.6667	0.0000	-0.335995	-1.598096	-52.0	0.0	0.0
6	58.3333	0.0000	-0.406460	-1.516817	-46.4	0.0	0.0
7	70.0000	0.0000	-0.468079	-1.421192	-39.6	0.0	0.0
8	81.6667	0.0000	-0.519388	-1.313594	-32.0	0.0	0.0
9	93.3333	0.0000	-0.559214	-1.196789	-23.6	0.0	0.0
10	105.0000	0.0000	-0.586804	-1.073888	-14.9	0.0	0.0
11	116.6667	0.0000	-0.601870	-0.948222	-6.2	0.0	0.0
12	128.3333	0.0000	-0.604691	-0.823134	2.1	0.0	0.0
13	140.0000	0.0000	-0.596102	-0.701803	9.7	0.0	0.0
14	151.6667	0.0000	-0.577423	-0.587040	16.2	-0.0	0.0
15	163.3333	0.0000	-0.550361	-0.481119	21.4	0.0	0.0
16	175.0000	0.0000	-0.516897	-0.385643	25.2	-0.0	0.0
17	186.6667	0.0000	-0.479040	-0.301552	27.5	0.0	0.0
18	198.3333	0.0000	-0.438723	-0.229137	28.6	-0.0	0.0
19	210.0000	0.0000	-0.397675	-0.168149	28.6	-0.0	0.0
20	221.6667	0.0000	-0.357358	-0.117883	27.7	-0.0	0.0
21	233.3333	0.0000	-0.318839	-0.077342	26.1	-0.0	0.0
22	245.0000	0.0000	-0.282952	-0.045390	24.1	-0.0	0.0
23	256.6667	0.0000	-0.250160	-0.020796	21.8	-0.0	0.0
24	268.3333	0.0000	-0.220696	-0.002362	19.4	-0.0	0.0
25	280.0000	0.0000	-0.194620	0.011001	17.1	-0.0	0.0

1

HORIZONTAL STRAIN AT SPECIFIED POINTS

N-POINT	XP(NI)	STRAIN
1	0.0000	-0.007599
2	11.6667	-0.007511
3	23.3333	-0.007276
4	35.0000	-0.006889
5	46.6667	-0.006348
6	58.3333	-0.005661

8	81.6667	-0.003906
9	93.3333	-0.002889
10	105.0000	-0.001828
11	116.6667	-0.000767
12	128.3333	0.000247
13	140.0000	0.001169
14	151.6667	0.001960
15	163.3333	0.002594
16	175.0000	0.003057
17	186.6667	0.003350
18	198.3333	0.003487
19	210.0000	0.003487
20	221.6667	0.003379
21	233.3333	0.003189
22	245.0000	0.002943
23	256.6667	0.002668
24	268.3333	0.002380
25	280.0000	0.002235