OPPORTUNITIES TO LEARN ALGEBRAIC THINKING AFFORDED BY THE TASKS IN MATHEMATICS TEXTBOOKS FOR GRADES 3 TO 5 IN TURKEY

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

OPPORTUNITIES TO LEARN ALGEBRAIC THINKING AFFORDED BY THE TASKS IN MATHEMATICS TEXTBOOKS FOR GRADES 3 TO 5 IN TURKEY

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The objective of this research is to investigate what the tasks in the primary school (3rd- 5th grades) mathematics textbooks in Turkey are for providing opportunities to learn algebraic thinking for students to enhance proficiency in algebra. The tasks were examined under the guidance of a specific emphasis on the five big ideas of algebraic thinking and their claims in the early grades. At each of these three school grades, the tasks were categorized into two groups: those that had been solved (worked examples) and those that still required a solution (to be solved). The study's findings showed the learning opportunities provided by primary school mathematics textbooks regarding algebraic thinking. A total of 6.3% of the tasks in textbooks offers algebraic thinking. From third to fifth grade, algebraic thinking ideas were distributed: 31.0%, 34.1%, and 34.9%, respectively. Equivalence, inequalities, and equations (38.9%) and functional thinking (34.1%) were the topics where the most learning opportunities related to algebraic thinking. In textbooks, only 50% of the expected claims were identified inside the textbooks. The findings indicate that textbooks provide only half the expected algebraic learning opportunities for elementary school students. The study's findings are significant for textbook authors, curriculum developers, and educators because they emphasize the need to create tasks that encourage the development of algebraic thinking and provide beneficial learning opportunities in this field. This investigation also informs the international community about elementary mathematics textbooks in Turkey.

Keywords: Algebraic Thinking, Early Algebra, Textbook Analysis, Task Analysis, Opportunity to Learn

ÖZ

TÜRKİYE'DE 3-5. SINIFLAR İÇİN MATEMATİK DERS KİTAPLARINDAKİ GÖREVLER İLE SAĞLANAN CEBİRSEL DÜŞÜNMEYİ ÖĞRENME FIRSATLARI

Aktürk, Gülümden
Yüksek Lisans, Matematik Eğitimi, Matematik ve Fen Bilimleri Eğitimi
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Bu araştırmanın amacı, Türkiye'de ilkokul (3.-5. sınıflar) matematik ders kitaplarındaki görevlerin cebirsel düşünmeyi öğrenme fırsatları açısından araştırmaktır. Görevlerin incelenmesi, erken sınıflarda cebirsel düşünmenin beş büyük fikri ve iddiaları kapsamında gerçekleştirilmiştir. Her bir iddia ilkokul öğrencilerinin cebirsel düşünme yeteneklerini geliştirmek için sunulması gereken öğrenme fırsatlarını temsil etmektedir. Görevler bu iddialar açısından incelenirken çözülmüş olanlar (çalışılmış örnekler) ve hala çözülmesi gerekenler (çözülecek olanlar) olarak iki ayrı gruba ayrıldı. Çalışmanın bulguları, ilkokul matematik ders kitaplarının cebirsel düşünme alanında sunduğu öğrenme fırsatlarına açıklık getirdi. Ders kitaplarındaki görevlerin toplam %6.3'ü cebirsel düşünmeyi sağlamaktadır. Cebirsel düşünme fikirleri üçüncü sınıftan beşinci sınıfa kadar şu şekilde dağılmaktadır: %31.0, %34.1, %34.9. Ders kitaplarının, en fazla %38.9 oranıyla eşdeğerlik, eşitsizlikler ve denklemler ile ilgili olan cebirsel düşünme fikri için ve %34.1 oranıyla ikinci olarak da fonksiyonel düşünme için öğrenme fırsatı sunduğu gözlemlenmiştir. Ders kitaplarında beklenen iddiaların sadece %50'si ders kitaplarında tespit edildi. Bulgular, ders kitaplarının ilkokul öğrencileri için beklenen cebirsel öğrenme fırsatlarının sadece yarısını sağladığını göstermektedir. Çalışmanın bulguları, cebirsel düşünmenin gelişimini teşvik eden ve bu alanda öğrenme fırsatları sağlayan görevler oluşturma ihtiyacını vurguladıkları için ders kitabı yazarları, müfredat geliştiriciler ve eğitimciler için önemli bir öneme sahiptir. Bu çalışmanın bulguları aynı zamanda uluslararası topluma Türkiye'deki ilköğretim matematik ders kitapları hakkında bilgi vermektedir.

Anahtar Kelimeler: Cebirsel Düşünme, Erken Cebir, Ders Kitabı Analizi, Soru Analizi, Öğrenme Fırsatları

To myself…

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As I write these words of appreciation, I am filled with an overwhelming sense of pride. I am extremely appreciative of my determination, confidence, and desire to overcome obstacles while composing this thesis. And I dedicate this thesis to my warrior spirit.

With profound gratitude,

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CHAPTER 1

INTRODUCTION

"Like love, algebra is where you find it." (Vennebush et al., 2005, p.86)

Throughout history, various civilizations, such as the Babylonians and Arabic mathematicians in the 9th century, have seen algebra as the discipline concerned with solving equations (Kieran, 2004). Algebra is derived from problems of "al-jabr, which means adding or multiplying both sides of an equation by the same thing to eliminate negative or fractional terms. These problems were similar to the issues of "al-muqabala," which means taking the same thing away from both sides or dividing the same thing into both sides (Mason, 1996, p.10). This perspective has persisted and evolved over time, leading to the development of abstract algebra. While the fundamental concepts of algebra remain unchanged, the methods and strategies employed in algebra education go through modifications.

The implementation of algebra education was initiated at the secondary school level since mistaken assumptions regarding the capacities of young kids, like psychological development, and the available empirical evidence pertaining to the challenges encountered by adolescents in the algebra domain (Carraher et al., 2001). Thus, pupils were introduced to algebra immediately after completing arithmetic instruction before attaining sufficient cognitive development necessary for abstract thinking. Consequently, at an early stage of their education, pupils began to be exposed to the use of symbols and the application of multiple operations to solve equations. This process, which continues today, is called traditional algebra education. It begins with introducing variable and symbolic-letter systems in secondary education, and the objective is for students to use symbols or letters to locate the unknown and apply numerous algebraic procedures to solve the question without mathematical meaning. Throughout ancient times, numerous students from various nations have been and continue to be introduced to conventional algebraic concepts within educational institutions.

Research undertaken during the 1970s and 1980s revealed a consistent trend of poor performance in mathematics, particularly in the field of algebra, in middle and high school students across different countries (Booth, 1984; Carpenter et al., 1982; Carraher & Schliemann, 2007; Pramesti & Retnawati, 2019; Schifter, 1999; X. Wang, 2015). For example, it was thought that there is a positive relation between students' attendance to these courses and the performance of high school mathematics courses, so Carpenter et al. (1981a) conducted a study on the attendance of high school students in mathematics courses in the United States. The findings showed that 46% of 17-year-old students complete a pre-algebra class, 72% Algebra 1 course, and 37% Algebra 2 course in at least one semester. Students' attendance in algebra courses needs to catch up to expectations. Over time, many studies have consistently demonstrated that these students show a notable lack of competence in algebra (Dede et al., 2002; Kieran, 1992; Lew, 2004).

According to Blanton (2010), the primary reason for the failure of algebra is the arithmetic-then-algebra approach. Since traditional algebra education begins immediately after arithmetic in the secondary grade, Herscovics and Linchevski (1994) linked this situation to a cognitive gap between arithmetic and algebra. They asserted that students in secondary schools need more preparedness for engaging with algebra and algebraic concepts. Because while arithmetic focuses on solving the problem and obtaining a numerical result, algebra focuses on recognizing relationships, expressing, deriving them, and manipulating them with procedures (Booth, 1988). Furthermore, the impact of students' lack of conceptual readiness for algebra on their cognitive development has led some scholars to link their difficulties in algebra to limitations in cognitive development. According to Mason (1996), the main issue with school mathematics is that generalizations, which are the heart of algebra and mathematics, cannot penetrate the classroom environment. In mathematics, students do not recognize this generalization action, which they are accustomed to performing from birth by observing their surroundings. The occurrence might be because school mathematics, influenced by the pragmatic approach, emphasized behavior rather than awareness and emotion. However, for behavior to be effective, it must be combined with knowledge and feeling.

Due to the cognitive cut between arithmetic and algebra and minimal emphasis on the act of generalizing, students encounter many difficulties in middle and high school. In addition, students cannot develop their algebraic thinking and algebraic skills because they need more time and space to think about algebra thoroughly at the secondary level (Blanton et al., 2015). Consequently, the challenges encountered by students in the field of algebra education have resulted in their failure to succeed in the subject (Schied, 1994). That means the conventional algebra-after-arithmetic way of teaching math in schools failed to sufficiently prepare students for formal algebra (Blanton, 2008).

Algebra is crucial in facilitating students' academic success and enhancing their chances for future financial opportunities and career options (Martinez et al., 2016; Stinson, 2002). For this reason, many educators emphasized the necessity of reconsidering the basic concepts of algebra and introducing them to children at an early age in primary education. They shared that this could yield beneficial results regarding mathematical ability in the 1990s. At that point, some educators came up with the concept of algebraic thinking and realized that encouraging students' early engagement with algebraic core concepts and ideas is crucial to their future algebraic comprehension and success in this field (Blanton et al., 2018; Freiman & Fellus, 2021; Kaput, 2008; Mason, 1996). Many kinds of research have been conducted, and Cai and Moyer (2008) claimed that numerous early algebra investigations or research have proven that primary school pupils are capable of algebraic thinking. To illustrate, Isler et al. (2014) indicated that students exposed to early algebra in elementary schooling show similar academic performance to those secondary education students who followed a traditional curriculum that was mainly arithmetic-based. In other words, at a young age, students have the cognitive ability to engage in algebraic thinking, and the introduction of early algebra has a beneficial impact on their achievement in algebra and other mathematics subjects (Mulligan et al., 2008). Thus, algebraic thinking became an essential expression for preparing students at a young age with the critical thinking skills required for algebraic success. The crucial query is: What exactly is algebraic thinking?

Algebraic thinking has been the subject of extensive research, resulting in many perspectives. The work of Cai and Moyer (2008) exemplified how studies have shown that this question lacks a straightforward solution. The relatively recent growth of the research discipline of algebraic thinking, characterized by a somewhat vague conceptual framework, could be one possible explanation. Possible factors contributing to the variation in the role of algebraic thinking in education might be policymakers' approaches, the perspectives of mathematics educators and mathematicians, and the objectives of a country's algebra curriculum, which is aimed at imparting algebraic thinking to students. This response pertains to the various dimensions of algebraic thinking and thus explains why algebraic thinking cannot be described definitively in mathematics education.

Kriegler (2008) recognized two elements of algebraic thinking: mathematical thinking abilities and fundamental algebraic ideas. Algebraic thinking is a method of thinking that emphasizes the fundamental concepts of algebra and requires mental and analytic development. In other words, it can be seen as the development of a mathematical system of thought in algebra's fundamental concepts. Therefore, algebra is not only a mathematical topic that uses a symbolic and numeric system of calculating and expressing something, but algebra also consists of a thinking system called algebraic thinking (Cai et al., 2005; Mason, 2018). In this way, algebraic thinking enhances students' understanding of algebraic and mathematical concepts, and students can establish connections between the concepts and analyze math problems without depending on rules or the letter-symbolic system (Carpenter et al., 2000). Similarly, from this perspective, the process of students' development of algebraic thinking encompasses not only facilitating their ability to learn mathematical concepts but also nurturing their perceptual ability regarding these concepts. Overall, algebraic thinking aims to help students overcome operational processes and achieve a high level of comprehension and cognitive capacity.

The value of algebraic thinking is growing, particularly within primary school mathematics, due to two crucial points. First of all, research indicates that young students exhibit proficiency in algebra when exposed to supportive educational environments, thus suggesting that individuals at a tender age possess the cognitive ability to comprehend algebraic thinking (Blanton & Kaput, 2004; Carpenter et al., 2003; Carraher et al., 2006). Secondly, the development of algebraic thinking from early childhood has been found to enhance cognitive processes and boost mathematical proficiency among pupils (Brizuela et al., 2013). Since imposing algebraic thinking on students, especially at an early age, provides a strong foundation and more profound comprehension of mathematical structure at a young age, developing algebraic thinking at an early age significantly impacts students. In other words, cultivating a solid foundation in early algebra enables students to acquire the essential cognitive abilities and logic to enhance their capacity to comprehend and apply algebraic principles in more sophisticated settings. Developing this capacity early will benefit students' later understanding of algebra and mathematics. Therefore, an early emphasis on algebraic thinking becomes crucial for students to grasp algebraic and mathematical concepts effectively in later grades and also for students to have a quality job and life (Kaya & Keşan, 2014).

Numerous countries have gained essential lessons from this particular circumstance and subsequently developed ways for kids to improve their abilities in algebraic thinking from a young age because providing kids with the opportunity to learn algebraic thinking abilities during their early education can significantly enhance their proficiency in algebra and mathematics, thereby leading to an improved career in the future. Törnroos (2005) highlighted the importance of providing learning opportunities for students to make them successful because there is a positive relationship between OTL and student achievement (Cueto et al., 2006). The view has been developed and supported by studies on incorporating algebraic thinking into the primary school curriculum (Blanton et al., 2015; Cai, 2004; Schliemann et al., 2003). At this juncture, the textbook, a potentially implementable curriculum, has a significant impact. Törnroos (2005) emphasized the significance of textbooks in providing educational opportunities and advocated analyzing textbooks to investigate the learning opportunities offered to students. The reason is that mathematics textbooks are essential, as they play a crucial role in supplying learning opportunities (Herman et al., 2005; Stylianides, 2009). To illustrate, while the Singapore primary mathematics curriculum does not explicitly address algebraic thinking, there exist activities in textbooks that facilitate the development of algebraic thinking through the analysis of textbooks (Fong, 2004). Furthermore, due to the potential variability of opportunities among various schools, courses, and classes, textbooks are crucial in affording equal learning opportunities for *all children* in mathematics (Cogan et al., 2001).

Additionally, analyzing the learning opportunities presented by tasks is highly beneficial to gain insights into the potential for fostering algebraic thinking as presented in textbooks. In this context, micro and macro research was conducted to evaluate and enhance textbooks. In recent years, for instance, the United States has been working to improve textbooks in order to enhance the opportunities for students to learn (Chavez et al., 2006). In particular, textbooks shape primary school-aged students' mathematical understanding and provide learning opportunities (Akkan et al., 2011; Kandemir & Yıldız, 2019; Toprak & Özmantar, 2019). For instance, Japan, one of the countries with the highest TIMSS scores, gives great importance to fundamental mathematics education in the primary school curriculum, allowing students to develop the skills necessary for success on such examinations (Mayer et al., 1991). This suggests that curriculum design and algebra textbooks should be re-conceptualized. More attention and effort are needed to foster algebraic thinking (Kilpatrick et al., 2001) since studies indicate that young children have the ability to think algebraically and that children at an early age can learn to think algebraically if the conditions that lead students to algebraic thinking are present (Blanton et al.,

2017. One of these studies is the Learning Through Early Algebra Progression (LEAP) project, which began in 2009 and is a multi-study longitudinal project. The project's primary objective is to develop an instructional sequence, interview assessments, and test the instructional sequence's impact on student learning to better understand the algebraic thinking capacity of primary school students and guide them toward success in this subject. With the Early Algebra Learning Progression (EALP) study within the LEAP project's scope, Fonger et al. (2015) and Blanton et al. (2018) established a conceptual framework for learning progressions that includes assessments that focus on curriculum progression, instructional sequence, and the five big ideas of algebraic thinking.

On the other hand, Blanton et al. (2015) designed an early algebra intervention for the third grade using the EALP results and these five big ideas. This intervention is a framework consisting of the algebraic thinking claims that third-grade students are expected to make within the scope of each big idea. This research on the best ways to teach early algebra focuses on creating opportunities for algebraic thinking.

These are big ideas:

(a) equivalence, expressions, equations, and inequalities;

(b) generalized arithmetic;

(c) functional thinking;

(d) variable; and

(e) proportional reasoning

A conceptual framework includes the big ideas of algebraic thinking and corresponding claims that primary school children will likely make. The framework encompasses fundamental principles essential for comprehending algebraic concepts. Additionally, it affords opportunities for fundamental algebraic thinking, including generalization, representation, and logical reasoning based on mathematical relationships (Kaput, 2008). This study adopts this analytic framework for investigating learning opportunities for algebra-related tasks in primary school textbooks.

Meanwhile, when examining Turkey's all success in TIMSS, TIMSS mathematics scores reveal that Turkish students consistently fail. On the other hand, algebra is one of the subjects in which students perform the worst in Turkey (MoNE, 2020). This poor performance caught the attention of why students have low scores in algebra, and the following question came to mind: Are students in Turkey given the opportunity to learn algebraic thinking in elementary math textbooks? Even though textbooks are a common source of education in Turkey, they are deemed inadequate by many (Baltacı & Biber, 2023; Gökçek & Hacısalihoğlu, 2013; Reçber & Sezer, 2018). This inadequate raises concern that this could be due to insufficient textbook tasks. With this concern, it is asked to examine the tasks in the primary school mathematics textbook and whether these tasks provide students with the opportunity to learn at the point of algebraic thinking and what kind of learning opportunities are offered to students in terms of algebraic thinking. The fact that algebra-related tasks in primary school textbooks are not clearly defined creates the risk of not fulfilling the opportunity of these tasks to engage students with early algebraic thinking (Demosthenous & Stylianides, 2014). This current research has shown that the key to success in the field of algebra is the algebraic thinking competencies that students acquire when they first start school. Thus, analyzing primary school mathematics textbooks regarding tasks focusing on developing students' algebraic thinking skills becomes necessary.

1.1 Purpose of the Study

Regarding the existing literature on algebraic thinking, mathematics textbooks, opportunities to learn, and the relationship between them, the present study aimed to investigate elementary mathematics textbooks and examine the tasks in the elementary school mathematics textbooks in Turkey for providing opportunities to learn algebraic thinking in Grades 3-5.

1.1.1 Research Question and Sub-questions

The following main research question and sub-questions guided the study.

What are the learning opportunities provided by tasks in mathematics textbooks with respect to algebraic thinking in grades 3 through 5 in Turkey?

- According to Blanton et al.'s (2015) five big ideas of algebra (equivalence, expressions, equations, and inequalities; generalized arithmetic; functional thinking; variable and proportional reasoning), what are the learning opportunities provided by the tasks in mathematics textbooks for grades 3, 4, and 5 in Turkey?

- How do the algebraic tasks in the textbooks evolve in grades from 3 to 5 concerning these big ideas?

1.2 Problem Statement and Rationale for Study

The TIMSS project evaluates the performance of students in the member countries and the factors that contribute to their success, and it also examines the performance in these member countries in an international comparison. Mayer et al. (1991) linked the differences in mathematics performance in this comparison to how much quality and more mathematics instruction a country provides for mathematics education for students to learn. According to the results of the TIMSS and PISA examinations, the students in Turkey have a mathematical accomplishment that is considerably lower than the average, and there is a significant gap in the pupils' mathematical achievement. Arıkan (2016), who thought that the explanation for this disparity in success was due to the variations in learning opportunities supplied to children in schools, conducted a study in this field by utilizing the data from PISA to examine several variables, including learning opportunities, the quality of the subject being taught, and so on. In other words, the reason why a country performs poorly in international comparative projects such as PISA and TIMSS may be the learning opportunity for math that is not given to students. OTL can determine whether poor performance is due to a lack of opportunity or other factors. If a country has a low score in a subject and these students are not given the opportunity to learn, they will not be successful in that subject. But if the opportunity to learn has been provided and performed unsuccessfully, then different factors can be examined. The outcomes of this study indicated that the opportunities for learning associated with the content sub-dimension were the most relevant factor to consider when attempting to predict mathematical performance. The literature also shows us that textbooks are the most important and influential resources that can be used to examine learning opportunities because they are accessible to every student. These reliable resources help students explore the learning opportunities available to them. Incikabi et al. (2023) mentioned that focusing on textbook tasks is crucial, particularly because they influence students' learning and critical thinking skills. Also, textbooks play a pivotal role in shaping the school environment and providing guidance for classroom instruction (Demosthenous & Stylianides, 2014). There is a prevailing belief among mathematics educators that the educational learning opportunities derived from textbooks primarily stem from the tasks they present (Brizuela et al., 2013).

The study by Blanton and Kaput (2011) underlined the significance of curricular and instructional material improvements to enhance algebraic thinking at the elementary school level. They pointed out the necessity of modifying these elementary school products to enhance algebraic thinking. However, the critical point is that, according to them, algebraic thinking should be incorporated throughout all areas of mathematics rather than being taught as a standard curriculum part. In this view, it is highlighted that rather than integrating algebraic thinking skills into only the early algebra curriculum, mathematics teaching materials should be constructed in a way that promotes this thinking system. The penetration of algebraic thinking into all mathematical subjects can allow students to apprehend mathematical principles more deeply because, considering that algebraic thinking includes capabilities inclusive of forming, expressing, and justifying mathematical generalizations, integrating this thinking system into all mathematical topics can enable students to broaden their mathematical thinking capabilities more comprehensively. In this way, students can find learning opportunities and are encouraged to expand their complete mathematical thinking, not only algebra, and make different mathematical connections. Due to this rationale, it is believed that the claims regarding five big algebraic thinking advocated by Blanton et al. (2015) for inclusion in primary school curricula ought to be incorporated into the tasks featured in textbooks since these claims related to big ideas of algebraic thinking present substantial learning opportunities for actively participating in the fundamental cognitive processes of algebraic thinking, including the abilities to generalize, represent, justify, and reason with mathematical relationships. It can be posited that the potential cause behind the lack of success in the education of algebra in Turkey could be attributed to the absence of the claims of big algebraic ideas within the tasks presented in primary school mathematics textbooks, and so students may have been deprived of the opportunity to learn knowledge about these essential ideas. Studies demonstrate that children who are given the opportunity to learn are more successful (Blanton et al., 2015; Guo and Liao, 2022; Hadar, 2017).

In the research of Blanton et al. (2015), the claims of algebraic big ideas represent the behaviors that primary school pupils expect them to take to improve their algebraic thinking. This current study views these claims as educational opportunities for elementary students and examines the learning opportunities provided by algebraic thinking tasks in Turkish primary school mathematics textbooks. Consequently, this study examines the mathematical tasks of primary school mathematics textbooks in Turkey about the five big ideas of algebraic thinking and their related claims. This study seeks to determine whether primary school textbooks provide insufficient learning opportunities as the cause of algebra's failure in Turkey. The study emphasizes the need for textbook authors and curriculum developers to include opportunities for algebraic thinking in primary school textbooks to improve algebra achievement in Turkey. The objective is for textbook authors and curriculum designers to contribute to developing algebraic thinking opportunities by illustrating the opportunities provided and not provided in elementary school textbooks. It also seeks to be an example analysis of algebraic thinking learning opportunities in primary school textbooks. In this manner, it can provide information about opportunities and limitations in Turkey and encourage research into educational opportunities in nations with high algebra scores. The most influential textbook can be identified as an example by comparing the learning opportunities and accomplishments supporting algebraic thinking in different countries' textbooks.

1.3 Algebraic Thinking

It is imperative to understand that algebraic thinking extends beyond the process of solving equations using symbols or letters, which encompasses the traditional algebraic concepts taught in middle and high school. It is acknowledged that algebraic thinking involves more than just doing calculations and arriving at solutions. Contrary to conventional methods of teaching algebra, algebraic thinking pertains to children's cognitive development. In other words, while manipulating symbols is generally associated with algebra, it is essential to accept that students' ability to notice relationships and generalize the process is associated with a prominent expression of algebraic thinking (Girit & Akyüz, 2016). Thanks to fostering students' cognitive development, algebraic thinking aims to enhance their comprehension of abstract concepts and promote the progression of their critical thinking abilities.

Algebraic thinking is a cognitive process characterized by applying algebraic principles in problem-solving and analytical reasoning. Algebraic thinking aims to cultivate students' capacity to generalize facts by promoting engagement in profound reasoning processes, thereby exceeding superficial observations. According to Kieran (2004), algebraic thinking aims to enhance students' cognitive capacities. Mason (2014) stated algebraic thinking holds significant value due to its ability to serve as a language that expresses generality and boundaries, and it empowers individuals to actively participate in critical analysis, effective problem-solving, and the comprehension of various relationships. Carraher and Schliemann (2018) provided additional support for this concept by characterizing algebraic thinking as a type of reasoning that employs diverse expressions or representations, such as tabular, graphical, and diagrammatic, to illustrate relationships among sets of elements, encompassing various forms of algebraic expressions. According to Blanton (2008), the primary objective of algebraic thinking is to facilitate students' ability to make informed assessments based on their observations of mathematical situations and to provide justifications for these assessments.

From this standpoint, students can both enhance their capacity to comprehend mathematical ideas better and foster their perceptual ability about these ideas by thinking algebraically. In other words, algebraic thinking promotes students' ability to recognize and justify mathematical principles. Consequently, students can engage in a deeper exploration of mathematics, so students' academic performance in both algebra and mathematics exhibits a positive trend.

Moreover, Mason (2014) mentioned that algebraic thinking should not be limited to secondary or higher school training specifically labeled as algebra. He associated the reason with the fact that everyone has inherent powers and competencies that can be developed to promote algebraic thinking. Mason asserted that everyone has inherent strengths and competencies that can be developed to promote algebraic thinking. According to him, infants start to display the ability to think algebraically as soon as they are born because algebra is a natural outcome of human cognitive abilities. As a result, that implies that students are capable of algebraic thought at an early age (Kaput, 2008; Lins et al., 2004; Brizuela et al., 2013; Tierney & Monk, 2008). At that point, early algebra becomes so important for learning formal algebra since algebraic thinking supports the implementation and growth of essential mental talents. Blanton (2008) also argued in favor of developing thinking algebraically at an early age by asserting that algebraic thinking is a concept that can be learnable and teachable. According to her, every student can learn how to think algebraically, and having this ability empowers students since early exposure to algebraic thinking enables children to strengthen and develop their sense of numbers, handle challenging computations, comprehend mathematical concepts, and better solve complex problems. At that point, starting algebraic thinking at an early age plays a key role in the development of algebraic thinking (Seeley, 2004).

On the other hand, Carraher and Schliemann (2018) have provided specific research demonstrating that cognitive-developmental constraints are to blame for children's difficulties with algebra. In other words, children's incapacity to think algebraically intellectually was blamed for their failure in algebra. It is explained as follows: the emphasis on arithmetic calculations in the first year of education and the subsequent practice with calculations prevent pupils from becoming mathematical thinkers. If children do not grow cognitively as algebraic thinkers, as in the previous sentence, this may indicate that they will struggle with algebra and mathematics. Hence, it is imperative for students' future success in algebra and math that they are introduced to algebraic thinking at an early stage, referred to as early algebra. Therefore, the significance of developing algebraic thinking early on was highlighted in the article.

1.3.1 Related Studies

Numerous research studies have been conducted to explore the fundamental reasons and processes underlying students' challenges in understanding algebra and develop approaches to enhancing algebraic teaching. Various of them have examined and emphasized the significance of algebraic thinking in the early grades. These studies support that every child possesses the cognitive ability to engage in algebraic thinking and can develop this skill if provided with early educational opportunities. The research findings also indicate that cultivating algebraic reasoning during the early years of education and augmenting children's algebraic thinking abilities can positively impact their mastery of algebra and mathematics in later stages of schooling. The growth of cognitive and mathematical abilities at the early stages of education is an important factor influencing students' school performance. This achievement, then, has the potential to positively impact the possibilities for getting high-quality employment and leading fulfilling lives.

Two of the important studies to enhance the instruction of algebraic thinking at an early age are the Learning through Early Algebra Progression (LEAP) project and the Early Algebra Learning Progression (EALP) (Blanton et al., 2015). Firstly, the Learning through Early Algebra Progression (LEAP) project was initiated in the 2000s to enhance, implement, and assess the development of algebraic thinking. Several long-term research projects have been successfully conducted under the LEAP initiative. The primary objectives of these projects are to conduct a comprehensive analysis of the developmental trajectory of students' comprehension of algebraic content and their acquisition of algebraic thinking skills. This research examines the conceptual progression and evolution of students' understanding and application of algebraic concepts during their early educational years. The comprehensive study has facilitated students in recognizing the variability of their learning capacity and algebraic thinking ability across different grade levels. The display of students' learning progression has provided insight into the development of their algebraic thinking, allowing for a comprehensive understanding of the subject matter. Implementing the Early Algebra Learning Progression (EALP) project has commenced through the utilization of the LEAP project. This project focused on a systematic examination of applications of algebraic thinking in the early grades. This project aimed to develop a framework for early algebra processes by providing comprehensive support through an integrated system of curriculum, instruction, and student learning in early algebra (Fonger et al., 2018). Therefore, a conceptual framework has been developed to account for learning progressions, which incorporates curriculum progression and focuses on five big ideas related to algebraic thinking and claims. This design instruction is based on Kaput's (2008) content strands and incorporates many early algebraic research studies to develop these significant concepts. The concepts encompassed in this framework consist of (a) equivalence, expressions, equations, and inequalities; (b) generalized arithmetic; (c) functional thinking; (d) variables; and (e) proportional reasoning. Numerous studies have investigated algebraic thinking from various ideas and perspectives within different frameworks. In this thesis, the concept of algebraic thinking is acknowledged as consisting of five big ideas proposed by Blanton et al. (2015).

At the same time, there are international studies related to mathematics education. The Trends in International Mathematics and Science Study (TIMSS), the Program for International Student Assessment (PISA), and the Progress in International Reading Literacy Study (PIRLS) are notable examples of extensive research endeavors that gather global data to enhance the standards of mathematics education (Uzun et al., 2010). The Programme for International Student Assessment (PISA), established by the Organisation for Economic Co-operation and Development (OECD) in 2000, aims to evaluate the educational progress and societal readiness of 15-year-old students. On the other hand, the International Association for the Evaluation of Educational Achievement (IEA) has devised the Trends in International Mathematics and Science Study (TIMSS), which centers its attention on examining educational policies, processes, and outcomes. The International Association for the Evaluation of Educational Achievement (IEA) was established in 1959. Since its establishment, the organization has extensively researched curriculum and school education to enhance understanding of educational processes in international and national contexts. Hence, the primary objective of the TIMSS initiative is to improve the quality of mathematics and science education. Every quadrennial period, an assessment is conducted to evaluate the state of mathematics and scientific education among students in the fourth and eighth grades. The grades in question were specifically chosen due to their association with the culmination of elementary school and the conclusion of lower secondary education, which hold significant importance in a student's educational trajectory (Mullis et al., 2005). Additionally, it gathers data about the academic milieu of students in each nation and the mathematics and science syllabi implemented in each respective country. Therefore, it provides an opportunity to thoroughly examine education policies and observe the consequences of alterations in educational practices.

1.4 Opportunity to Learn

International studies are valuable because they illustrate the similarities and differences between nations and allow each country to evaluate itself internally by providing scores and outputs for a country's educational accomplishments. Thus, the findings of these studies offer suggestions for enhancing education policies and their implementation, as well as the quality of education (Berberoğlu et al., 2003; Kartianom & Retnawati, 2018; Şen & Arcan, 2015).

For example, TIMSS's three-component curriculum supports the fact that TIMSS results enable the identification of the curriculum's prospective effects on student achievement. (Mullis et al., 2005). With this curriculum model, TIMSS aimed to examine the relationship between these three curricula and the impacts of these curricula on students by collecting information about the content of the curriculum, how it is implemented, how learning conditions are provided in the classrooms, and the learning opportunities provided to students. Nevertheless, it is possible to assert that numerous factors contribute to the diversity of mathematics education. For instance, it is well known that the mathematics education curriculum of each country varies in terms of the goals, approach of that country, and sociocultural contexts. It is extremely difficult to measure the effects of these factors on students' achievement. However, the results of the studies show that even if it is not mentioned in the intended curriculum, if students gain the ability to transfer their knowledge and skills to a problem or situation, they can be successful as a result of this problem or situation. That demonstrates the significance of providing students with learning opportunities to study and use what they have acquired in their coursework. In addition, Arıkan (2016) highlighted that the discrepancy between the mathematical performance of a nation and the mathematical accomplishment of children in that country might be attributable to the disparities in the learning opportunities supplied in schools. Likewise, he stated that if the gap between students' math performance is significant, no nation would want to be in this situation and that the disparity in the learning opportunities provided to pupils should be investigated. At this point, researching the opportunity to learn (OTL) provided to the students in the classroom environment has attracted more attention than measuring student achievement results. This concept, which emerged to ensure equality in education, emphasizes the importance of creating learning opportunities for every student. Husen made the frequently used definition of OTL as "whether or not… students have had the opportunity to study a particular topic or learn how to solve a particular type of problem presented by the test" (Husen, 1967, pp. 162-163, as cited in Kartianom & Retnawati, 2018).

The utilization of the Opportunity-to-Learn (OTL) framework has proven to be efficacious in elucidating variations observed among students in comparative studies at both the international and national levels, particularly in the context of educational attainment. In their seminal work, Stevens and Grymes (1993) comprehensively analyzed numerous studies on fulfilling students' needs and enhancing educational quality. As a result, they developed a conceptual framework aimed at comprehending the various factors contributing to the academic achievement gap. The framework comprises four variables, namely: (1) content coverage, (2) content exposure, (3) content emphasis, and (4) quality of instructional delivery. According to Floden (2002), OTL has two critical roles in international comparative studies. One of them is that it allows for a thorough examination of a country's curriculum. It assesses which subject is taught at which level in a country's curriculum; thus, a prosperous country's coverage of a certain subject at a certain level in its curriculum can serve as an example for other nations. Second, academics and policymakers may pay attention to OTL to interpret differences in achievement within or between nations. For instance, if a country's low achievement is associated with a lack of learning opportunities, this tells us not to look for the reason for low achievement in other criteria because if students do not have the opportunity to learn a subject, they cannot be expected to succeed in that subject (Törnroos, 2005).

Semerci (2004) stated that textbooks hold significant educational value, particularly in developed and developing nations. The author further supports this assertion by highlighting the removal of textbooks as the foremost priority in Japanese schools during an earthquake. Textbooks are a crucial link between the intended curriculum and the implemented curriculum, as exemplified in the triple program model of TIMSS discussed earlier, and are referred to as the potentially implemented curriculum. They assume distinct and significant roles within the various relationships in which they are involved. In simple terms, textbooks effectively disseminate policymakers' goals and objectives to students and teachers at the corresponding grade level. Textbooks are extensively accessible and readily obtainable written materials that have been authorized and endorsed, serving as a valuable resource for students by offering dependable information, illustrative instances, and suitable exercises tailored to students' proficiency levels. The textbook serves as an essential educational resource for students to learn and do mathematics. For teachers, textbooks serve as examples and instructional aids illustrating how the curriculum objectives should be elucidated systematically and according to specific criteria. As a result, textbooks benefit both students and teachers and assist both of them in achieving their objectives (Macintyre & Hamilton, 2010). Furthermore, researchers utilize textbooks to provide information regarding the mathematics curriculum and gather information about implementing school mathematics in a classroom setting (Jablonka et al., 2010). There are various arguments supporting the indispensability of textbooks as an educational resource. Notably, countries like Sweden, China, the USA, and Japan have emphasized the role of mathematics textbooks within their respective education systems (Fan et al., 2004; Mullis et al., 2008; Österholm & Bergqvist, 2013). Furthermore, textbooks are essential in facilitating educational opportunities by serving as instruments that strive to enhance the quality of education and foster equitable circumstances.

1.5 Textbooks Role in Turkey

To increase the quality of education in Turkey, to train successful students, and to benefit from these outputs, Turkey attended TIMSS-R, seen as a repetition of TIMSS 1995, for the first time in 1999 at the 8th grade level. Turkey later attended the fourth-grade level in 2011, 2015, and 2019 and participated at the eighth-grade level in 2007, 2011, 2015, and 2019 (Mullis et al., 2000, 2004, 2008, 2012, 2016, 2020). Table 1.1 shows Turkey's scores in TIMSS during the years it participated and the average scores of the TIMSS scale.

Table 1.1 reveals that Turkey has consistently scored below or near the average of the scale in TIMMS. In other terms, TIMSS demonstrates that the Turkish education system needs to be developed and qualified (Dogan & Tatsuoka, 2008). Comparing the first years of Turkey's participation in TIMSS to 2019, it is seen that although the scores progressed increasingly in both grade levels, with one exception, scores remained close to or below the average. In addition, as stated in the TIMSS 2019 Turkey Preliminary Report, while the most successful learning areas of students in Turkey are data and probability, it has been determined that they perform lower in algebra and geometry (Ministry of National Education Turkey, 2020). There may be numerous causes for Turkey's poor performance. Yayan and Berberoglu (2004) categorized these reasons under three main headings: family characteristics, teacher factors in the classroom environment, and affective measures taken by students. Güleç and Alkış (2003), on the other hand, investigated it under two headings: in-school and out-of-school factors. While the in-school factors were the education program and the school officials mentioned the educational environment, the out-of-school factors could include factors such as the family's socio-economic status, viewing television at home, and playing on a computer. According to the table, students in Turkey achieved an above-average score at the fourth-grade level only in 2019.

Table 1.1 TIMSS Results of Turkey and Average Score (Mullis et al., 2000, 2004, 2008, 2012, 2016, 2020)

Turkey	1995	1999	2003	2007	2011	2015	2019
4th grade	-	-	-	-	469 /500	483 /500	523 /500
8th grade	-	429 /487	-	432 /500	452 /500	458 /500	496 /500

That may be due to the reformist changes made about the elementary and secondary mathematics curriculum in Turkey (Arıkan et al., 2016) because with these reform movements, which especially affected the primary school mathematics curriculum, the student-centered classroom environment was centered on better student understanding. It was aimed at having teachers act only as facilitators while students were encouraged to establish interdisciplinary connections, use innovative teaching tools, and work as a group (Koc et al., 2007). It is anticipated that these reform movements in the curriculum will also have an impact on the preparation of textbooks because of the role of textbooks as one of the most important resources, providing students with the opportunity to learn both in school and outside of school, and their impact on students, teachers, and researchers. In Turkey, textbooks are the principal source of education and the most prevalent educational material (Aydemir, 2017; Bayrakçı, 2005; Gülersoy, 2013). With the "Free Textbook Distribution Project in Primary Education," it is aimed to disseminate textbooks free of charge so that every student can benefit from them, to ensure that every student benefits from the textbooks, and thus to provide equality of opportunity in education (Bayrakçı, 2005). Primary education, as the initial stage of education and training, is very important in developing students' motivation to learn and acquire fundamental knowledge and skills. Consequently, the importance of textbooks at the primary education level is increasing (Semerci, 2004; Tutak & Güder, 2012). However, the literature shows that textbooks are insufficient (Arslan & Özpınar, 2009; Gökçek & Hacısalihoğlu Karadeniz, 2013; Korkmaz et al., 2020; Taşdemir, 2011; Toluk & Orkun, 2002). In addition, the Ministry of National Education prepared the "Textbooks Evaluation Report" in 2021 by taking the teachers' opinions about the textbooks (Koçak & Karaca, 2011). As a result of this report, it was concluded that the tasks and activities in the textbooks were not sufficient and of high quality and that tasks and activities that could provide students with high-level critical thinking skills should be added to the textbooks.

Numerous studies have examined the learning opportunities in textbooks in Turkey and worldwide. While some of these studies focus on the physical characteristics of textbooks, such as the size and color of the text, others may examine the visual design, the texts and the structure of the texts, and the educational aspects, such as the purpose and content (Erbaş et al., 2012; Eroğlu, 2021; Şaban, 2019; Ubuz & Sarpkaya, 2014; Yıldırım, 2019; Yılmaz, 2018). According to Tertemiz et al. (2001), out of 36 criteria, the ones just discussed are the textbooks' four most important and prominent aspects. Textbooks have an essential role in any nation's educational system because they can influence many individuals due to the diverse functions they perform and the fact that they are written materials that are reliable and easily accessible. Thus, textbooks play an essential part in the educational process by allowing students to learn (Stylianides, 2009).

Meanwhile, it has been stated that textbooks hold a significant amount of relevance in education, particularly in basic school, compared to other levels (Yalçın, 2019). The rationale is that the primary education level is seen as the beginning of the students' educational and training careers, as well as the critical formative experience in such endeavors. According to Semerci and Semerci (2004) mentioned that especially primary school mathematics textbooks are very important among these textbooks. It was mentioned that the textbooks prepared with the appropriate task and content for the development of the students increase their success. Therefore, because of conducting an overall assessment of the mathematics textbooks used in primary schools in Turkey, they concluded that the textbooks ought to receive additional development to address the tasks of Turkish pupils more effectively, who have historically displayed poor academic performance. As a result, the creation of textbooks for elementary mathematics proposed that there should be more tasks, and they should be carefully selected (Dede & Arslan, 2019).

The fact that the learning opportunities in the textbooks are so significant and the poor performance of Turkish students in the field of algebra leads us to the issue of whether the textbooks allow the students the opportunity to learn algebra. Both factors contribute to the question. Examining the early algebra period is an extremely significant subject since this period is viewed as the beginning of algebraic thinking, particularly at the primary school level. This period is also seen as the time when textbooks had a more favorable influence on the students who were enrolled in elementary school. Because the learning opportunities that are presented to kids throughout this early algebra era on how to think algebraically will influence not just the students' achievement in algebra but also their attitude toward mathematics as a whole and their success in that subject. Students will be able to have a more successful academic and professional life due to this (Booker & Windsor, 2010).

1.6 Definitions of Important Terms

Algebraic thinking is the early cognitive development of students so they can solve problems that help them understand the relationships between quantities and look at the basic structure without using explicit algebraic rules (Kieran 1996).

A big idea of algebraic thinking is a fundamental concept or method for analyzing quantitative conditions and relationships (Cai et al., 2005).

Early algebra is the acquisition of algebraic knowledge, the ability to think algebraically at a younger age, and strategies and representations used to solve algebraic operations that are anticipated to be solved later (Carraher & Schliemann, 2018). According to Carraher and Schliemann (2007), this young age ranges roughly from 6 to 12.

A task is any exercise, problem, activity, or parts thereof that have a separate marker in the students' textbook (Stylianides, 2009, p. 270). Within the scope of this thesis, similarly, the term "tasks" encompasses any worked question and to-be-solved exercise and problem found in mathematics textbooks, as well as a range of questions designed for students to solve. However, it did not include activity.

CHAPTER 2

LITERATURE REVIEW

"Algebra is a gateway to opportunity, not a gate that blocks their way" (Carpenter et al., 2003, p. 7).

2.1 Algebra Instruction

It has been established via studies and research on algebra that dates back to the 1970s and 1980s that learners struggle with the subject and the specific types of struggles they experience (Kieran, 2004; Lee & Park., 2022). By the mid-1990s, algebra education in elementary grades and algebraic thinking became research topics researchers were interested in and focused on (Schifter, 1999). Numerous studies have been carried out to determine why students struggle with algebra, what they struggle with, and how they approach algebraic subjects because it is obvious that students have difficulty with algebra instruction worldwide (Blanton & Kaput, 2001).

Traditional algebra education is seen as memorizing facts and reaching the answer by making calculations and as a teacher-centered education. It is generally accepted that in traditional education, the introduction of letter symbols, which can represent an "unknown" that the student has to answer in an equation or represent a "variable" with a wide range of values in a functional relationship, is considered the first acquaintance of students with algebra (RAND Mathematics Study Panel & Ball, 2003; Usiskin, 1988). Another common point is that algebra is included in middle and high school after the arithmetic subject in formal algebra programs. The procedure used in almost every nation to instruct traditional algebra (Britt & Irwin, 2011; Subramaniam & Banerjee, 2011). Especially, students in middle and high school have difficulties with algebra. Arithmetic is taught independently of other mathematics subjects, making the transition between arithmetic and algebra particularly challenging because students are not encouraged to connect these two mathematics subjects (Carpenter et al., 2003). As such, students tried to memorize algebraic rules and operations rather than comprehend the relation between arithmetic and algebra and the purpose of algebra. However, many students fail to succeed because they cannot even memorize rules and procedures. Some can memorize the procedures correctly and perform the correct operations but cannot make sense of the procedures (Russell et al., 2011). In both cases, students cannot gain a sense of conceptual understanding of the expressions or strategies they utilize due to the focus of traditional curriculum and materials on the process.

By describing this operational process as a set of instructions, Subramaniam and Banerjee (2011) have made it clear that, beyond symbolic processes, quantitative relations and understanding of the operational combination of the quantity are of great importance. According to the National Council of Teachers of Mathematics [NCTM] (2000), the reason is that algebra is more than just representing letter symbols or doing math with letters. Algebra is a "succinct and manipulable language to express generality and constraints on that generality" (Mason, 2014, p. 77). They have asserted that algebra's main objective is to support students in expanding their understanding of quantities, how these quantities are integrated, and mathematical relationships, not just to do mechanical operations with symbols. Symbolic operations should be viewed in this context as a tool for comprehending the analysis, comparison, and connections of quantities. Otherwise, students cannot acquire a semantically algebraic perspective.

Moreover, students have lacked the motivation to study and understand algebra courses since they think they do not employ it in their daily lives and might not employ it in the future. They are unaware of algebra's importance in their daily lives. As a result, they believe their lack of experience with algebra won't impact how they live their lives since they can live productive lives without using algebra. They think that even if they are good at algebra, they can still lead successful lives without it (Cai & Moyer, 2008; Cai et al., 2011). This situation is because students are forced to learn algebra without having the ability to think algebraically in higher grades. Since secondary and high school students tend to focus on studying primarily for exams, they prefer to learn algebra at a basic level rather than a deeper understanding due to a lack of time and freedom. Students may suffer in math and algebra classes due to this case, exhibit poor cognitive growth, fail in their professional lives due to their academic failure, and experience troubled economic and career times in the future. Subramaniam and Banerjee (2011) have provided a pretty good summary of algebra's crucial role: "Algebra is a gateway to higher learning for some pupils and a barrier for others" (p. 89).

2.1.1 Algebra Reform in School

According to the studies and many others, questions regarding the effectiveness of algebra instruction have been raised, and reform movements have needed and started to alter how they approach this mathematical problem (Blanton & Kaput, 2001; Blanton & Kaput, 2005). According to what Schmittau (2011) mentioned, it has been noticed by teachers and educators that algebra problems, which are intended to be taught at the elementary school level, are challenging and abstract even for middle school students, especially in the Davydov curriculum adopted by the United States. In this elementary school curriculum, the disadvantages of an approach that progresses from arithmetic to algebra and becomes abstract have emerged. On this subject, it has been asserted in Russia that algebra is a field that provides the transition to higher levels of thinking with the work of Vygotsky (2012). In this context, algebra began to be seen not as a stage after arithmetic but as a generalization of the relations between quantities and the properties of these relations and reached a crucial milestone in the teaching of fundamental mathematics education. Algebra has been accepted as a teaching approach that aims to enable students to reach higher levels of development. This change aims to improve learners' ability to generalize and abstract mathematical concepts. Around the world, the role of algebra in elementary education started to change with the turn of the millennium (Kaput, 2008). Most recent research has focused on studies of algebraic thinking and developing it at the elementary school level (Kılıç, 2014; Moonpo et al., 2018).

The "Algebra for All" initiative, promoted by the National Council of Teachers of Mathematics [NCTM] in 2000, is a movement that seeks to require all students to study algebra during pre-K–12 in the United States, and then other countries also started this reform. Researchers justify why students should learn algebra before graduation from high school. The basis for the argument is the relationship described in the following sentences: If students become successful when they graduate from high school, the chance of going to university and graduating from there is high. Moreover, this initiative is based on the belief that algebraic thinking is an essential skill for all students and that the way of thinking algebraically is a critical component of mathematical literacy. Algebra became one of the five common strands in school mathematics from kindergarten to 12th grade. Therefore, the "Algebra for All" movement serves as a significant educational reform.

Algebra is for everyone, not just a certain group or level. This is how the RAND Mathematics Study Panel report expressed a similar sentiment (RAND Mathematics Study Panel & Ball). Since algebra has been stated as the "gatekeeper" of mathematics education from kindergarten through grade 12, it is absolutely crucial. Additionally, it is mentioned that teachers and decision-makers who place a high priority on research into the teaching, learning, and materials for algebra emphasize that funding studies into how algebra is portrayed in primary and secondary school curriculum materials can lead to positive changes for all students in the K–12 educational environment.

In 1999, the New Zealand Ministry of Education launched a professional development program called the Arithmetic Development in Mathematics Project (Britt & Irwin, 2011). This project was initiated to help students understand the operational processes of developing number sense. It aimed to measure algebraic thinking within the framework of arithmetic, with examples that encourage such thinking. Its foundation is to develop students' understanding of the underlying structure of algebraic operations and generalizability. The findings of this long-running project, which has had some impact on most schools in New Zealand, have important implications that the algebraic way of thinking can exist without algebraic symbols and expressions. It has been revealed that students became aware of the relationships between numbers and the basic structure of the strategy. Therefore, successfully implementing these mental processing strategies showed that students acquire algebraic thinking and gain generalization awareness. As a result, this project in New Zealand and the revision of the mathematics curriculum allowed teachers and students to understand algebra better. The New Zealand Numeracy Project encouraged seeing algebra in arithmetic and emphasized that mathematics curricula should be developed to reflect a generalizing thinking skill based on algebraic thinking.

The development of algebraic thinking in elementary school has been proven to be a successful strategy since there is no combating the lack of motivation like in middle and secondary school, according to Cai et al. (2011). Early exposure to algebraic thinking helps students develop their cognitive abilities, algebraic knowledge, and comprehension of other mathematical concepts, particularly arithmetic (Radford, 2011). Thus, students can gain opportunities for their future education and careers.

2.2 Early Algebra

Lins and Kaput (2004) have concentrated on two aspects of early algebra. One of these approaches means that 12–13-year-old students meet algebra at school for the first time, as in traditional algebra education, which has been common for many years, while the other approach includes the concept of early algebraic thinking, which became popular later and has an important place in mathematics education today. This approach introduces students to algebraic thought at a much earlier age.

Carraher et al. (2008) have agreed with the second approach and said that it is crucial to underline that early algebra does not mean beginning traditional algebra instruction at a young age. Carpenter and colleagues (2000) have also mentioned that early algebra has an entirely distinct objective from conventional algebra education, which involves manipulating algebraic symbols and using algebra procedures at a young age. In other words, it does not mean a pre-algebra course after arithmetic or teaching any aspects of the algebra course in earlier grades (Kilpatrick, 2011). Carraher and Schliemann (2018) have considered early algebra as an approach young children use to solve mathematical problems utilizing algebraic knowledge and thinking rather than algebraic rules and symbols. Instead of manipulating algebraic symbols in traditional algebra courses earlier, its goal is to enhance students' algebraic thinking, boost their abilities to prepare for algebra, and develop a rich understanding to improve mathematics.

It has explained the goal of early algebra as aiming to make them understand the concept of generalizability in-depth, allow students to engage with age-appropriate interpretations of algebraic concepts using their instincts for relationships, and explain the main purpose of mathematics to students rather than focusing on students' computation skills at the age of first contact with mathematics (Blanton & Kaput, 2011; Fonger et al., 2018; Lins & Kaput, 2004). This approach focuses on students' algebraic thinking skills to analyze mathematical problems, explore relationships, develop solution strategies, generalize, and justify (Kieran, 2004). Students' cognitive readiness will increase as a result. As students focus more on these conceptual processes, they will develop their ability to recognize, construct, explain, justify, and prove mathematical generalizations. It is described as "algebra readiness". This approach aims to enable students to understand their mathematical thinking skills and be more successful in advanced mathematics studies and mathematical operations. Due to the advancement of their cognitive development and capacity to make relevant observations, generate relations, and have a conceptually in-depth understanding of algebraic and mathematical topics, students who complete their secondary and high school education have a higher chance of being admitted to universities. A robust academic career boosts an individual's chances of achieving significant financial gains and a luxurious life, as one's chances of finding a qualified job are much higher.

On a global scale, it is inevitable that students face problems in algebra and mathematics from the very start of their training, and this is one of the elements that affect students' educational success. For this reason, algebra schooling has been a focal point of outstanding topics in studies. In line with the outcomes of this research, reform moves were initiated in the desire to change algebra education methods at an early age (Carraher & Schliemann, 2007). Many studies have been carried out, completed, and are now being conducted in this context. Numerous studies have been conducted to examine the root causes of the difficulties that algebra students have, their misconceptions, and their misunderstandings. From the instructor's perspective, numerous studies have also been carried out on teacher strategies, content knowledge of teachers, and trainer education at the university. In this context, the study mentioned above aims to understand how early algebra education is approached by the resources utilized in mathematics teaching and how the appropriate procedures should be used.

2.2.1 Algebraic Thinking

There is a very strong relationship between early algebra and algebraic thinking. This relationship demonstrates the value of instilling algebraic thinking skills in children at an early age. According to research by Blanton et al. (2015), children exposed to algebraic thinking processes in the first years of education develop their analytical and critical thinking skills. These skills will affect not only their success in algebra but also their success in all subjects of mathematics. Students can recognize generalizations, represent them verbally and symbolically, justify them, and reason them in all other areas of mathematics by developing their algebraic thinking skills (Kaput, 2008). Children who acquire these intellectual abilities at a young age will have a more comprehensive and meaningful understanding of algebra and mathematics in advanced grades. That helps students succeed in mathematics as they develop. The importance of this algebraic thinking skill is shown very clearly. For this reason, it is thought that early algebra and algebraic thinking should be emphasized in mathematics teaching (Smith & Thompson, 2014).

At this point, Watanabe (2011) has stated that one factor that most affects students' success in school algebra is that students gain algebraic experience at primary school age and learn how to think algebraically. The primary school period is an important period in which the foundations of mathematical thinking processes are laid, and algebraic concepts are discovered. These experiences form a foundation that influences students' mathematical abilities in later years. Blanton and Kaput (2011) have stated that starting algebraic thinking at an early age is important and critical as it will form the basis for students' conceptual understanding at an early age. They have underlined that the purpose of introducing algebra to young children goes beyond developing their understanding of the structural form and generality of mathematics and limiting them to computational activities.

Furthermore, they have claimed that with this preparation, young children might succeed mathematically in further education. They have also stated that researchers are coming to a consensus. For them, the experience of "building, expressing, and justifying mathematical generalizations," which they regard as the core of algebra and algebraic thought, should be the first step in formal education. Early exposure to these ways of thinking will provide students with the skills they need to solve any algebraic or mathematical problem they come across later on. Whatever the problems, the viewpoints and strategies they embrace will improve their prospects of success. In addition to the opportunity for more conceptual learning, elementary students are less concerned about exams and their futures and have more time to comprehend the underlying concepts of their topics. This enables students to achieve higher success in algebra courses and general mathematics education. This success increases students' potential to succeed in academic life and the business world. Thus, algebraic thinking is supposed to be taught in elementary school before studying formal algebra in school (Carpenter et al., 2000; Kilpatrick et al., 2001).

Along with the value of developing algebraic thinking early, it is an intriguing question to examine whether young children are capable of doing so. Students can have algebraic thinking abilities at an early age and have the ability to learn these abilities (Stephens et al., 2015). Blanton and Kaput (2004) noticed that students can think algebraically from an early age, and early algebra positively impacts students' ability to succeed in advanced-level grades and provides students with convenience for the future. A one-year study including third, sixth, and seventh graders was carried out in 2004. The conclusion that early algebraic thinking can assist pupils in the long term is supported by the finding that third-grade students can be as effective in standard schooling as middle-grade students. Using algebraic thinking capacity early is an important basis for students' understanding and learning of algebra (Brizuela et al., 2013). The abilities they gain in this early period make it easier to learn algebra and mathematics in the future.

Numerous studies have been conducted to understand algebraic thinking better and comprehend its impacts, and reform movements have emphasized the significance of these studies. These studies have demonstrated how significant algebraic thinking is to students' understanding of algebra in an educational setting. However, the studies have also shown that education approaches and methods in a country and the perspectives of math education researchers adopted to encourage algebraic thinking can vary. Cai and Knuth (2005) investigated the algebra curricula of five different nations, finding that although teaching algebraic concepts is primarily intended to help students comprehend quantitative relationships, every nation had a distinct objective, approach, and stage of algebra teaching. Depending on the researcher's perspective, the culture of a country, historical and societal factors, and teachers, there may be differences in how algebra is taught (Cai et al., 2005; Hemmi et al., 2019; Lins & Kaput, 2004). For example, when US teachers and Chinese teachers were compared, US teachers were more inclined to use concrete strategies and visual representations (Cai, 2004).

Kaput (2008) mentioned that the point of view of people who are primarily interested in mathematics education about mathematics should be changed in order to develop algebraic thinking skills in students. While algebra was once thought of as a course and manipulating symbols, it has since evolved into strands, a conceptual structure, and a representation system (Chazan & Edwards, 2010). According to Kaput (2008), the core aspect of algebra is expressing generalizations and a system of symbols. Three content strands have occurred in these two core aspects of algebraic reasoning.

Three algebraic strands represent these fundamental concepts:

> algebra as the study of generalizations of arithmetic, structures, and systems abstracted and quantitative reasoning;
>
> algebra as the study of the generalization of variability, functions, and relations;
>
> algebra as a tool for modeling both inside and outside mathematics (Blanton et al., 2018; Chazan & Edwards, 2010).

From another point of view, Kieran (1996) has stated that algebra education in schools consists of three important components: the generational, the transformational, and the global meta-level. The approach of Far Eastern countries to algebra education at the primary school level is different from the approach of America. While China and Japan, with a similar approach, use concrete materials as a tool and aim to teach algebraic subjects better, the approach in America uses concrete materials more and encourages progress with abstract strategies less. Cai (1998) has also recommended Chinese primary school textbooks because the textbooks contain examples with both arithmetic and algebraic solutions for a problem, and students are asked to solve other problems in this way. Students are thus given the chance to view several ways and compare them.

Lev S. Vygotsky, a renowned psychologist and developmental theorist, has concentrated on cognitive development and the development of cognition in a cultural and social setting. It is feasible to monitor students' cognitive development through a book or an instructor within the context of this theory, which discusses the impact of social contact and culture on cognitive development (Berger, 2005). Vygotsky has discussed the idea of higher mental function in relation to his research into the interaction between language development and thought, which attracted his interest. Later, he applied this term to mathematics (Rieber, 1997). Starting from the idea that children's language learning becomes abstract and generalized, he has continued concerning the fact that since algebra is adopted as a language, this language, algebra, raises the intellectual levels of children (Schifter, 1999). Suppose children gain the ability to generalize by focusing on concrete figures, i.e., through abstracting and generalizing arithmetic. In that case, it will advance their understanding and help them attain a new, higher plane of thought. This means that as students are exposed to the generalization of operations and thoughts and then acquire this skill, they reach the level of algebraic thinking. Thus, they can make freer operations in arithmetic and be aware of their thoughts at both the new and previous levels of thinking. Stating that this awareness provides freedom of movement and control in the system, Vygotsky's opinion was that reaching algebraic thinking opens up the opportunity for generalizations, synthesis, and new ways (Sierpinska, 1993).

Even though these studies all adopt various approaches to algebra, the following is consistent and obvious: elementary students can have the ability to recognize mathematical relationships, allowing them to develop an in-depth understanding of quantitative relationships and solve complex problems thanks to the algebraic way of thinking (Cai & Knuth, 2005; Carpenter et al., 2003). This way of thinking involves discovering mathematical relationships and generalizing these relationships to come up with strong, generally accepted ideas. It helps students gain a more comprehensive understanding of the mathematical world by encouraging their ability to understand, analyze, and solve mathematical connections. In this context, algebraic thinking plays an important role in mathematics education, emphasizing developing students' critical and creative thinking skills. At the same time, algebraic thinking strengthens students' logical reasoning, analytical thinking, and critical analysis skills. Blanton et al. (2011) defined the practices and abilities of algebraic thinking in general on four fundamentals: generalizing, representing, justifying, and reasoning with mathematical structure and relationships. These abilities enable students to deal with mathematical challenges more effectively and improve their problem-solving abilities. These skills deepen students' cognitive abilities and lead them to success in the academic and business worlds. Therefore, transferring algebraic thinking to students at an early age is very important in developing mathematical abilities in early periods. In primary education, visuals and concrete examples are used more than in other grades for students to understand more easily. Frequently using such examples creates an environment based on understanding, exploration, and questioning.

Exposing children to algebraic reasoning at a young age encourages them to think algebraically. The important point is giving students these opportunities to think algebraically. That is why, to attain a deeper understanding of algebra, it is also advised to adjust the curriculum, the textbooks, and the teacher training programs. The emphasis, however, should not be placed on beginning algebra instruction at a young age but instead on the need for curriculum revisions that will lead to a deeper comprehension of the subject matter.

2.3 Opportunity to Learn

Student opportunity to learn (also known as OTL) is one of the most crucial study subjects, particularly in the fields of mathematics and science education. First, let us investigate the thought process that led to this conclusion.

John B. Carroll, who established a name for himself in the literature with his first reference to learning opportunity, cited the learning opportunity for the first time in 1963 in his remarks on the value and breadth of content in education, and he built a model of school learning. This first mention of learning opportunity allowed Carroll to make a name for himself in the literature. According to this model, the degree to which a student can learn effectively while attending school is a function of the time necessary for learning and the time spent learning (Seel, 2012). This model means that a learner who puts in the necessary effort and time to learn anything will eventually achieve the objective (Carroll, 1963). Further, according to this model, the content of mathematical instruction should be carefully organized according to the levels and skills of the students. Similarly, Stephen and Bartlett (2016) mentioned that instructional time and content are OTL's two most significant aspects.

On the other hand, according to the research findings, this idea was developed to combat inequality in the United States. It has been noticed that children who come from households that are looked down upon for various reasons and whose standing is low (such as corruption) do not fare well academically. It has been a contentious problem that children from minority groups and low-income families are subjected to harsher educational settings, which results in lower academic performance (A.H. Wang, 2010; Meyers & Rogers, 2014; Schmidt et al., 2011). The concept that *every child should have an excellent educational environment and opportunities* was eventually agreed upon after numerous debates and investigations (Schliemann & Carraher, 1992).

Oakes (1986) devised a follow-up system to monitor homogeneous and heterogeneous groups responsible for creating inequity in the American school system. Additionally, he investigated how these groups were formed and how the classroom environment was modified according to the groupings. As a consequence of this, he has observed that various approaches were taken toward multiple groups. As a consequence of this, he has addressed the requirement of reform as well as the necessity of a single curriculum for all pupils to minimize inequality. He has also remarked that it would be easier to undertake studies on the education system and processes and to do research to improve the overall quality of education in a society that provides equal possibilities. (Gamoran, 1986).

Similarly, the IEA, which has been working since its inception to make it simpler for students to learn mathematics and science, has adopted the "If you want to measure change, do not change the measure" perspective. With the same viewpoint, the TIMSS project was started in 1990 to conduct regular measurements once every four years all over the world to assess the learning opportunities available to pupils as well as their level of achievement in mathematics. This project was started to conduct regular measurements once every four years. These measurements are taken to determine whether students are progressing in the abovementioned topics and evaluate the learning opportunities and student achievements now available. (Mullis et al., 2003).

At that point, the concept of *opportunity to learn* emerged because of a significant amount of work examining the methods and approaches aimed at providing educational equity and academic excellence to students who struggle academically as a result of being exposed to inequity (Abedi & Herman, 2010; Fuhrman, 2001; Stevens & Grymes, 1993). The significance of guaranteeing equitable access to opportunities is underscored by the imperative of providing inclusive educational environments that foster the growth and development of all students, enabling them to fully maximize their capabilities (Carroll, 1989; Liu, 2009). When interpreted in its most comprehensive sense, this concept pertains to the favorable circumstances students will encounter in the educational environment throughout their learning and instructional experiences. The concept of OTL can be examined with three teaching dimensions: time, content, and quality (Kurz, 2011). This may encompass a diverse array of activities, including pedagogical techniques educators employ during classroom instruction and inquiries into the educational institution's educational materials, instructional time, textbook coverage, and administrative framework (Schmidt & Maier, 2009). In his study, Floden (2002) provided instances of OTC from multiple viewpoints, highlighting that while OTC exhibits overall stability, it remains susceptible to diverse interpretations. For example, an individual may perceive the focal point of a textbook topic as an opportunity for learning.

In contrast, another researcher may view it as the time dedicated to learning within the school or curriculum. Additionally, the educational environment, including the school setting and the qualifications of the teachers, could also be regarded as potential opportunities (Winfield & Woodard, 1994). In contrast, the TIMSS International Curriculum Analysis (ICA) defines *opportunity* as the occurrence of potential learning experiences for students in a classroom rather than contingent upon teacher quality or instructional approaches (Houang & Schmidt, 2008).

The significance of OTL lies in its ability to positively impact student success by providing suitable educational environments and conditions within the classroom, coupled with opportunities for students. Guo and Liao (2022) demonstrated that, despite the utilization of diverse indicators of OTL, their investigation revealed a positive correlation between OTL and students' mathematical achievement. According to the findings of Carroll and Spearritt (1967), insufficient and ineffective learning outcomes may result when students are not afforded a high-quality learning environment. This finding was one of the conclusions derived from the study. Put simply, individuals raised in environments characterized by subpar educational standards face diminished potential for attaining intellectual acumen and achieving long-term success compared to their counterparts raised in more favorable educational settings. Nevertheless, Törnroos (2005) highlighted a crucial distinction between learning and learning opportunities. It is essential to recognize that for a student to acquire knowledge, they must first be allowed to engage in the learning process. One possible approach to demonstrating this concept is using an analogy. By likening the learning process to a seed, it becomes evident that the growth and development of this seed necessitate the presence of suitable conditions and surroundings conducive to its flourishing. Alternatively, adults must provide children with the requisite opportunities to facilitate their learning process. However, it is imperative to acknowledge the following aspect: despite the provision of an opportunity, there is no assurance that the seed will successfully undergo germination. The mere accessibility of an educational opportunity is a fundamental requirement. In other words, the absence of opportunities for acquiring knowledge inevitably leads to a lack of educational advancement.

In addition to the substantial influence of OTL on student achievement, international studies also hold a crucial position (Elliott, 2015). OTL refers to content coverage in large-scale international studies, and the role of OTL in international studies can be analyzed through two distinct categories. One argument suggests that if it is acknowledged that the primary cause of students' academic underachievement in a nation, particularly in mathematics, is a lack of educational opportunities, there is no necessity to expend effort in examining additional variables (Floden, 2002). Put simply, it is not worthwhile to try to figure out why children in a nation consistently perform poorly in math classes. As mentioned earlier, the opportunity to acquire knowledge is an essential prerequisite that initiates the process of learning. In the absence of educational opportunities, investigating surrounding circumstances becomes of limited utility. On the other hand, if OTC is not considered a singular factor influencing student achievement, Floden (2002) argues that it may become confounded with the effects of other variables, resulting in potentially misleading findings. The second reason is that it is beneficial to discover the appropriate extent of topic inclusion within a nation's curriculum and the optimal age group for studying it. For example, a country can potentially serve as a model for other nations by integrating early algebraic reasoning into its educational framework, including textbooks, and by achieving an impressive degree of student competence in algebra. Currently, the Trends in International Mathematics and Science Study (TIMSS) project is a prominent and significant illustration. TIMSS is an educational project initiated by the International Association for the Evaluation of Educational Achievement (IEA), established in 1959. The project's primary aim is to gather data on education policy practices and outcomes through comparative analyses of countries and in-depth investigations of specific nations. The TIMMS initiative was initiated by the IEA in 1990. This study, conducted by Mullis et al. (2003), holds significant global importance as it represents the inaugural effort in a series of comprehensive international studies to monitor shifts in student performance. This project aims to examine the educational programs and textbooks currently being implemented nationwide, as well as identify educational opportunities that foster comprehension of educational progress (Houang & Schmidt, 2008; Valverde et al., 2002). In summary, OTL in mathematics education is one of the key elements influencing student math success and plays a significant part in the study of the impacts of schooling (Guo & Liao, 2022).

As previously stated, the concept of OTL encompasses a wide range of applications and possesses considerable adaptability. Its primary objective, however, centers on enhancing the conditions and opportunities within the educational setting, with the ultimate aim of fostering the academic achievement of all students. The variables influencing these circumstances and prospects may encompass the variables that impact educational opportunity. According to Schmidt et al. (2009), the notion of learning opportunity pertains to the opportunity afforded to students to access and acquire knowledge on specific mathematical subjects within the confines of the classroom environment. Scholars also emphasize the significance of fundamental mathematical principles introduced in educational settings and their influence on achievement in their studies. The study's findings indicate that disparities in educational opportunities can result in unequal learning outcomes, ultimately contributing to the perpetuation of achievement gaps. At that point, textbooks play a significant role in influencing the learning opportunity. Numerous sub-factors exert an influence on the OTL in textbooks. The potential factors include the design of the textbooks, their layout, the content, and the tasks contained within them. This thesis will focus on analyzing mathematics tasks presented in mathematics textbooks, aiming to investigate their potential as effective learning opportunities.

2.3.1 The Role of Textbooks in Mathematics Education

Let us begin by understanding "curriculum" and figuring out how it connects to textbooks to comprehend the function of textbooks in the educational system. The curriculum is the basic structural framework governing all educational institutions' aspects. It serves as an underlying skeleton that gives instruction the identifiable form and direction it should follow in educational institutions all over the globe (Houang & Schmidt, 2008). This "skeleton" operates as an educational framework. A curriculum outlines the many learning opportunities that have been designed and scheduled. It is a structure developed by those in charge to bring order to the method of running education in a classroom to serve the students better. Because of this, following the curriculum is important and serves as a major source of education.

TIMSS ICA has produced a curriculum concept incorporating a threefold idea (Bazzini, 1990). This paradigm includes three distinct types of curricula: the intended curriculum, the implemented curriculum, and the accomplished curriculum (Mullis et al., 2003). The intended curriculum consists of official documents the country's education politicians created, such as an education program and a vision for the educational system. On the other hand, when someone talks about implementing a curriculum, they refer to the activities and events that take place in the classroom and affect the environment in line with the intended learning objectives and program. According to Houang and Schmidt (2008), the definition of "attained curriculum" is "the result of interaction in the classroom with students," also known as "student achievement." It is possible to refer to these components as the input, the procedure, and the output. In addition, they have argued that almost all students and instructors need an adequate comprehension of the subject matter taught in the curriculum and that most individuals need to be made aware of the issue. Because of this, they have no idea what the overarching aims and objectives of the educational program at the national level are supposed to be. Most students and instructors in practically every country use textbooks significantly in their daily work. Members of TIMSS nations who support the initiative have confirmed the findings (Haggarty & Pepin, 2002; Houang & Schmidt, 2008). At this point, textbooks serve as a resource that is comparable for the vast majority of students. It is the duty of the textbook to communicate to the students in the classroom the objectives to be learned during study. Students can accomplish the goals set for them by utilizing textbooks that contain exercises, assignments, and context tailored to correspond with their current level. They have access to textbooks inside and outside the classroom, reinforcing continuous learning, reinforcement, and self-assessment at any time. Because of this, it is referred to as a potentially implementable curriculum since it is between the intended and implemented curricula being utilized. This places it in the middle ground between the two (Figure 2.1).

According to Dede and Arslan (2019), textbooks, an integral element of the entire education process, are an important instructional tool that supports people in accomplishing the learning objectives defined in the curriculum. That is because textbooks contain information organized in a straightforward way to understand, and each person can have a textbook easily. Because they are tangible instruments that provide educational content in a logical and well-structured approach, textbooks are vital for both instructors and students to have in order to absorb the knowledge presented in a course entirely.

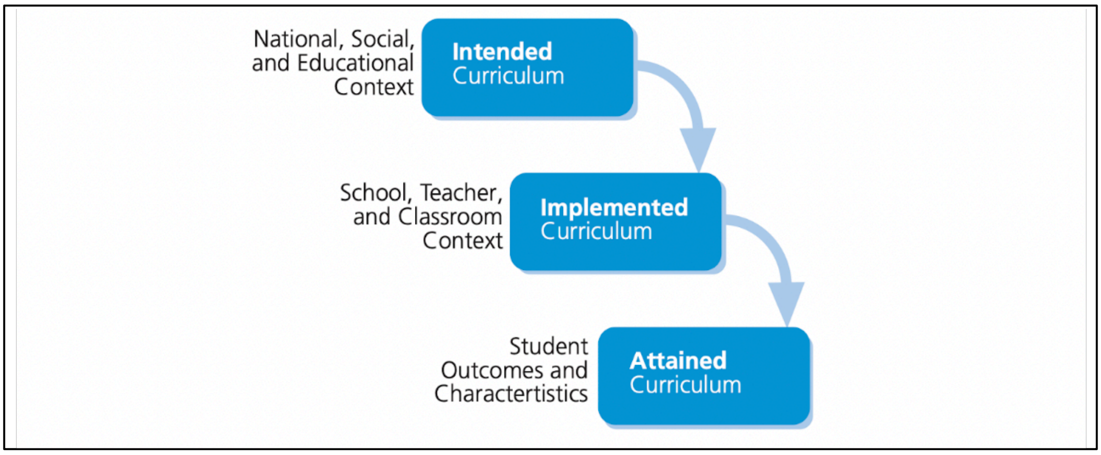


Figure 2. 1. TIMSS Curriculum Model (Mullis et al., 2005, p.5)

At that point, the model developed by Rezat (2006) provides a solution to whether textbooks should be considered tools or the objective of learning. The comprehensive representation of various functions performed by the tetrahedron model effectively addresses the classification of textbooks as either tools or the objective of education. The tetrahedron comprises four vertices, each representing a unique category: students, teachers, mathematics textbooks, and mathematics-related materials (Figure 2.2). One of the interactions that emerges from this tetrahedron reveals that teachers can operate as intermediaries between students and textbooks, in which case textbooks behave as objects. Textbooks serve as a reference for instructors since they include information regarding the topics taught in a course and their sequence, structure, time, and sample questions (Freeman & Porter, 1989; Güzel & Adbelli, 2011). This direction is helpful for teachers as they design their classes and activities. Students also have the option of focusing only on their textbooks for their mathematical education as opposed to the guidance of an instructor. When this occurs, the student's textbooks become an intermediary between students and their knowledge of mathematics.

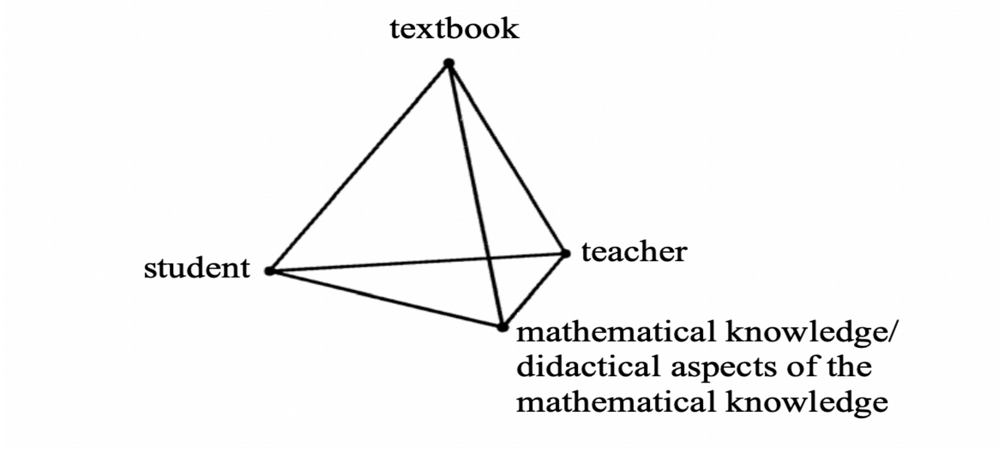


Figure 2. 2. The Model of Textbook Use (Rezat, 2006, p. 413)

First of all, they are textbooks, which are what make it possible for the country's curriculum policy and the objectives of the curriculum to reach the student in concrete terms in terms of operation since textbooks offer trustworthy information and are written and designed in a way that is understandable following the level of the students. These concrete materials help students better grasp and internalize the themes targeted in the curriculum and the educational aims of these topics. That allows students to not only follow the lesson being taught in the classroom but also to study a subject independently outside of the classroom. In addition, the textbooks provide exercises and questions for students to complete and questions for them to answer so that they can gauge how well they comprehend a topic as well as evaluate themselves. In the third relationship, the teacher conveys the mathematics information in the textbooks to the student without using the textbook. The fourth and last relationship states that the teacher displays the lesson over the textbook instead of explaining the information in the textbook.

In addition to the roles mentioned above and features of the textbooks presented by this model, it is worth noting that another notable attribute is their capacity to provide students with a valuable learning experience. According to Schmidt et al. (2001), providing learning opportunities is a crucial program component, contributing significantly to students' academic achievement. Hence, the influence of textbooks on educational prospects, particularly among children, holds considerable importance. Each student has the opportunity to learn as they access these resources because they are considered trustworthy and have received approval from educational policies. The pivotal issue pertains to the extent to which textbooks afford an avenue for acquiring knowledge. Hence, evaluating textbooks is significant in advancing educational materials, curricula, the education system, and the provision of enhanced educational opportunities, ultimately contributing to improved educational conditions and student achievement. Consequently, it serves as a valuable resource for educators, policymakers, and curriculum designers (Valverde et al., 2002).

2.3.2 The Relation Between OTL and Textbook

It has already been mentioned that OTL is versatile, and although it has a general purpose and meaning, it is, therefore, difficult to fully understand. Studies published in relevant academic journals have demonstrated that the amount of time spent training, the qualities of teachers, and the textbooks utilized significantly affect opportunities. However, Arıkan (2016) revealed that the content sub-dimension is the most important and effective variable in predicting the mathematical performance of the related learning opportunities. Elliott and Bartlett (2016) stated that training content is one of the two essential elements of OTL. Similarly, Herman et al. (2015) noted that the standard definition of OTL in the literature is preparing curriculum content, instructional strategies, available resources, and general assessment preparation. As it focuses schools' attention on content and processes, investigates issues of opportunity equity, and provides interim measures of change in teaching and learning, the OTL construct can significantly impact educational policy and practice.

At that point, the research has proven that textbooks have a unique position in providing opportunities for education and learning. Mathematics textbooks have a very important role in providing learning opportunities since they offer students a logically organized body of material as well as materials, examples, and activities that enhance learning and ensure that all students have an equal opportunity to learn. The fact that any student may access them and that they offer a diverse collection of content creates an environment in which learning can take place following the principle of equality. TIMSS is one of the most vital studies to show the importance of mathematics textbooks in this research field. It researches various textbooks available in multiple countries to examine how textbooks exploit educational opportunities. Its primary objective is to gain knowledge, within the context of a systematic methodology, of how textbooks affect learning opportunities and the constraints of such opportunities (Valverde et al., 2002).

Hadar (2017), who conducted a study in Israel to examine the relationship between the learning opportunities provided by mathematics textbooks and students' success in national exams, found a positive relationship between learning opportunities in textbooks and students' success. In other words, he concluded that the learning opportunities in the mathematics textbooks positively affected the students' success. It can be said clearly that textbooks can provide learning opportunities that can positively affect the success of their students. In addition, the findings also highlight a crucial point. If a textbook contains taxonomies that require high cognitive demands and students are exposed to them, these students have the opportunity to learn, and their success increases. The fact that students' academic performance improves as a direct result of the educational possibilities that may be found is evidence of the impact of learning opportunities in textbooks. According to a collection of research, the content, layout, and presentation of textbooks play an important role in assisting students in the development of skills such as more profound knowledge, critical thinking, and problem-solving abilities. However, the tasks presented to the students in the textbook are the main resources for providing learning opportunities because Stein et al. (1996) state that tasks are important tools for developing mathematical thinking and reasoning capacity. In addition, Hwang and Ham (2021) stated that students learn to do math by being busy with tasks and said that OTL affects the cognitive development process while doing math with different types of math tasks. In other words, tasks have an important effect on the development of students' mathematical success since the tasks afford individuals the opportunity to delve deeper into the field of mathematics (Henningsen & Stein, 1997). However, the learning opportunities offered to students may vary according to the tasks. Therefore, examining math tasks in math textbooks is important for recognizing and improving the learning opportunities offered because the tasks show what content students are exposed to (Elliott & Bartlett, 2016).

2.4 Theoretical Framework of the Study

The evolution of algebra teaching and learning over time has led to a longitudinal study of K–12 algebra and extensive research into early algebra. The Learning through an Early Algebra Progression project, which consists of a series of projects, was created with the question of how to prepare primary school students for algebra for middle school and beyond (Blanton et al., 2015). The purpose of the LEAP program is to deeply analyze the development of algebraic thinking in primary school students. It recognizes the capacity of elementary school students' abilities to acquire skills related to algebraic thinking. It seeks to promote the adoption of suitable instructional methods that can effectively facilitate the development of these skills. The ultimate objective is to enhance students' performance in algebra and mathematics as they progress to middle-class and higher grade levels. The outcomes of the LEAP project have provided significant knowledge regarding the adequate preparation of elementary school students to succeed academically in the domain of algebra at higher grade levels (Fonger, 2018).

Implementing the Early Algebra Learning Progression (EALP) has been supported through the utilization of the LEAP project. The goal is to create a theoretical framework as part of the project to support an integrated system for early algebra curriculum, instruction, and student learning (Fonger et al., 2015). This study systematically examines algebraic ideas and thinking strategies in the early grades, intending to develop a comprehensive framework to enhance students' abilities in algebraic thinking. The main goal is to provide students with suitable possibilities for learning to develop their skills in algebraic thinking, thereby promoting their overall growth in algebra and mathematics and eventually helping their success in the subject. In other words, the EALP is a valuable addition to the field of education as it develops assessment tools and instructional designs that both develop the impact of students' mathematical performance and also provide specific instructions on the effective instruction of algebraic concepts at an earlier stage.

These studies have been set up with Kaput's (2008) conceptual analysis of algebraic thinking practices and content strands. This initiative has established five primary standards by integrating Kaput's perspectives, content strands, and early algebraic studies. The five big ideas consist of "Equivalence, Expressions, Equations, and Inequalities; *Generalized Arithmetic*; *Functional Thinking*; *Variable* and *Proportional Reasoning*" (Blanton et al., 2015, p. 43).

The big idea behind *equivalence, expressions, equations, and inequalities* is to develop a relational understanding of the equal sign, write expressions and equations with symbols, and think about the relationships between quantities (Blanton et al., 2018). This *generalized arithmetic* entails generalizing arithmetic relations, analyzing the basic characteristics of numbers and operations, and considering the structure of arithmetic expressions rather than computational results. *Functional thinking* means noticing and generalizing relationships between covarying quantities and then using symbols, tables, and graphs to present and examine these relationships (Blanton et al., 2011). *Variable* is a linguistic tool that describes the use of symbolic notation to more simply express mathematical concepts. *Proportional reasoning* is a way of thinking about two generalized quantities linked to maintaining the ratio of one quantity to the other constant.

Based on the findings of EALP's study, an early algebra intervention framework for students in the third grade has been developed. This structure lays forth the EALP-organized big ideas and the claims about what third-graders are capable of doing (Blanton et al., 2015). Although there are numerous big ideas and approaches to algebra, this intervention framework has been adopted for this thesis because it is claimed that the big ideas, they created are the fundamental elements that either represent the core concept of algebra or a component that will feed this algebraic thinking skill. Furthermore, this framework incorporates the claim that students can perform at the required level due to a rigorous assessment of primary school students' algebraic thinking abilities. These statements can also be regarded as the claims that elementary school students are anticipated to make to enhance their algebraic thinking proficiency.

Despite the existence of numerous ideas and approaches in the field of algebra, this particular framework has been chosen for the purpose of the current study. At this juncture, it is possible to enumerate three significant arguments in the following manner:

1) This framework aims to support the cultivation of algebraic thinking and the advancement of it among primary school students because early algebra establishes a foundation that will affect students' math academic achievements and successful careers. Algebraic thinking is a significant part of cognitive development, substantially influencing students' lives.

2) The big ideas they create are representations of either the basic concept of algebra or will feed this algebraic thinking skill. Hence, it is posited that these ideas are inclusive within the realm of algebra.

3) These big ideas provide an opportunity to learn about applying algebraic thinking. The claims of the big ideas outlined in the early algebra instruction framework should be found in textbook tasks to strengthen algebra education.

2.5 Summary of Literature Review

In conclusion, the primary objective of the OTL initiative is to direct attention toward analyzing the circumstances and variables essential for establishing a fair and efficient educational setting that caters to the needs of all students. These analyses aim to ascertain optimal conditions for enhancing the quality of education and students' academic performance. In essence, the concept pertains to the various inputs and processes within an educational setting that are essential for facilitating students to achieve their intended learning objectives (Elliott & Bartlett, 2016). That is, the concept of learning opportunity holds substantial significance concerning its impact on learning outcomes, as noted by Törnroos (2005), because identifying and enhancing opportunities to learn are of paramount importance in establishing a fair and favorable educational setting that fosters the academic achievements of students. Moreover, providing sufficient and suitable opportunities for students to interact with educational content actively enhances their likelihood of attaining desired learning outcomes.

Textbooks represent a paramount educational resource that affords students a valuable avenue for acquiring knowledge. Textbooks are designed to provide students with the necessary tools to comprehend knowledge, develop skills, and achieve their maximum potential, laying the groundwork for their future success. Due to this rationale, textbooks hold significant value as an invaluable resource that presents abundant educational prospects. The crucial objective is for teachers and educators to enhance their students' learning processes and provide them with comprehensive learning encounters through meticulous investigation of the content and structure of textbooks. The existing body of literature highlights the importance of meticulous preparation for tasks featured in textbooks. That is primarily because students are mainly exposed to these tasks, and their engagement with these tasks facilitates their mathematical skills acquisition. The study conducted by Hadar (2017) also reveals that mathematics textbooks can encourage opportunities for mathematical learning. Furthermore, the study indicates that when textbooks incorporate tasks that demand a heightened level of comprehension, students tend to achieve higher scores due to their exposure to such tasks.

Algebra education garners global attention as a crucial matter impacting students' mathematical achievement, but learners see the topic of algebra as a significant challenge. Most students are introduced to algebra for the first time in secondary school and have no idea how to utilize the symbols or follow the rules. They are just attempting to memorize the policies and procedures. When they reach high school, youngsters encounter more difficult phrases, struggle, and fail. However, learning algebra is not simply about symbols and rules. If it were, pupils would not have as many problems as in conventional schooling.

The scope of algebra education needs to extend beyond just memorizing rules and symbols. In this particular scenario, it is anticipated that students exposed to algebra at a younger age, from an algebraic thinking perspective, will likely experience greater success in both their academic pursuits and professional endeavors. Numerous significant studies have been conducted on the initiation of algebra education in schools, specifically focusing on the cultivation of algebraic thinking during the early stages of development. The early introduction of algebraic thinking to students, coupled with the development of problem-solving strategies that emphasize semantic understanding rather than reliance on algebraic rules or symbols, has the potential to yield success across various mathematics subjects in subsequent grade levels, extending beyond the algebra domain. In other words, students will succeed in all mathematical disciplines in later grades, not just algebra, if they are exposed to algebraic thinking at a young age and build approaches through comprehending issues semantically. It is apparent that providing learning opportunities in mathematics textbooks that can impact students' cognitive abilities is crucial for their success in algebra. To promote future success in algebra among students, it is imperative to build on the approaches employed in primary mathematics textbooks to afford plenty of opportunities for mastery of algebraic concepts. This highlights the significance of algebraic thinking and its fundamental concepts regarding the educational opportunities offered within textbooks, facilitating cognitive and academic development.

Blanton (2008) emphasized the significance of utilizing tasks to foster algebraic thinking among students. By engaging students in tasks that prompt them to observe mathematical relationships, make comparisons, interpret findings, and conduct tests, algebraic thinking can be cultivated. She argued that algebraic thinking is not an additional component of the mathematics curriculum. Instead, algebraic thinking can be characterized as a cognitive framework, and instructional tasks play a crucial role in fostering students' engagement with this framework. By strategically posing questions that prompt students to reflect upon the five fundamental concepts and principles of algebraic thinking, tasks can effectively support the development of students' algebraic thinking abilities. Therefore, critical international studies such as TIMSS and PISA allow us to examine the educational conditions and learning potentials provided to students and measure the student achievement scores of countries with low performance in mathematics. Countries can strengthen education quality and improve student learning opportunities through this comprehensive work. Turkey, one of the countries with lower mathematics achievement, usually close to the middle score, shows the lowest performance in algebra. There has been curiosity surrounding the reasons behind the academic underperformance of students studying in Turkey in this crucial subject that significantly impacts their overall mathematical achievements throughout their educational journey. Even though textbooks are used as a common and essential resource in the education system in Turkey, the question of whether the success of algebra remains lower than in other fields since students are not provided with learning opportunities to support algebra education prompted us to conduct this research.

CHAPTER 3

METHODOLOGY

3.1 Research Design

This research aimed to provide a picture of the learning opportunities for algebraic thinking in Grades 3, 4, and 5 that were offered throughout the tasks in Turkish mathematics textbooks. Content analysis was used as the methodology in this study. When a researcher does not directly watch human behavior—when there are no circumstances involving direct observation, discussion, or testing—this is referred to as content analysis (Fraenkel et al., 2012). For instance, a researcher might examine written data in order to analyze human behavior and its relationships indirectly. The content analysis technique for analyzing textbook learning opportunities is the most appropriate method for data collection and offers a framework to guide data implementation. The framework developed by Blanton et al. (2015) was utilized in the research to assess the educational opportunities of the tasks featured in the textbooks. This framework focuses on examining algebraic content and incorporates the five fundamental concepts of algebraic thinking.

The subsequent section will be presented in the following sequence: The initial stage of the process involves explaining the selection of content areas, the alignment between textbook units and the content areas of frameworks, and the following selection of textbooks. Subsequently, the study's references regarding reliability and validity were addressed. The coding scheme employed in the study is elucidated. The coding procedure is subsequently clarified, accompanied by coding examples derived from the study.

3.2 Selection of the Content Area

The Turkish elementary school mathematics curriculum encompasses four distinct learning areas: Numbers and Operations, Geometry, Measurements, and Information Processing. In contrast, the secondary school mathematics curriculum is structured around five subject areas, namely Numbers and Operations, Algebra, Geometry and Measurement, Data Processing, and Probability (MEB, 2018).

Here, there are two important points. First, although algebra education is only included in the secondary school mathematics curriculum, this study aimed to examine primary school textbooks. Even if it is not included in the curriculum, textbooks may be designed to encourage algebraic thinking (Blanton & Kaput, 2011). They even mentioned that it is more critical to integrate algebraic thinking into mathematics teaching materials, such as textbooks, than curriculum. The second point is that the algebraic thinking concepts need more explicit visibility within the textbooks. It can be integrated into various mathematics subjects, particularly numbers and operations. Many studies have sought algebraic thinking on these issues alone since algebraic thinking is often found in natural numbers and operations (e.g., Palak, 2022). However, this thesis argues that algebraic thinking can and should be integrated into every mathematics subject (Vennebush et al., 2005). Therefore, this current study evaluated the presence of algebraic thinking concepts across each mathematics content area in textbooks.

3.3 Selection of the Textbooks

Numerous studies carried out to investigate and improve its instructional methods have shown that there is a widespread consensus among educators worldwide regarding the significance of early algebra. In their paper, Carraher and Schliemann (2007) mentioned two significant moments in American history that have substantially influenced the content and structure of algebra textbooks and curriculum at the primary grade level. One reason for the inclusion of algebra in early mathematics instruction is the endorsement by the National Council of Teachers of Mathematics (NCTM, 1989; 2000), which recognizes its potential to enhance students' mathematical comprehension and readiness for advanced academic levels. The other is that the RAND Mathematics Study Panel Report emphasizes algebra (RAND Mathematics Study Panel & Ball, 2003). The RAND project has examined algebra education in depth to determine how it is addressed, developed, and integrated into other topics in the K–12 mathematics curriculum due to the substantial impact of mathematics curriculum and instruction on students' learning opportunities. The RAND initiative highlights the significance of assessing and improving pre-K through high school algebra instruction.

Furthermore, according to Cuevas and Yeatts (2001), the period encompassing grades 3 to 5 is most important for cultivating algebraic ideas. NCTM (2000) asserts the significance of offering elementary school students' appropriate situations and contexts during this grade level range to foster the growth of their algebraic ideas. This preparation is seen as essential for their after-engagement with formal algebra and to ensure their success in the subject in subsequent years. As a result, grades 3, 4, and 5 were selected for this study on algebraic thinking in the early grades because they are crucial to developing algebraic abilities in thinking.

During the 2012–2013 academic year, the educational structure in Turkey experienced revision through the implementation of the "4+4+4 Law". The enactment of this legislation resulted in the expansion of the compulsory education system from eight years to a duration of twelve years. The structure of the elementary education system underwent a modification, wherein it now encompasses a four-year period for both elementary and middle school. The secondary education system also mandates a four-year attendance requirement at high school. When examining the mathematical curriculum in Turkey, it is observed that the introduction of algebraic content commences in the sixth grade within the middle-grade level. In many countries worldwide, algebra is taught for the first time at the secondary school level. The curricula for third-, fourth-, and fifth-grade mathematics do not include algebra as a required proficiency. Although algebra may not be explicitly included in the intended curriculum, the fundamental concepts of algebraic thinking have been integrated into other fields of math. Hence, it is crucial to comprehensively examine elementary mathematics textbooks and investigate possibilities for learning algebraic thinking within the tasks presented in these educational resources. By examining the tasks in elementary math textbooks, one can ascertain whether students are provided with the opportunity to engage in early algebra. Therefore, even though algebra is not included in Turkey's third-, fourth-, and fifth-grade curricula, this study focuses on these specific grade levels due to the recognition in the literature that they play a critical role in developing algebraic thinking skills in the early grades.

In Turkey, the Ministry of National Education ensures all students receive complimentary textbooks aligned with the approved national curriculum and standards. However, these textbooks exhibit variations based on the publishers responsible for their production. At every academic level, there exist textbooks produced by private publishers under the supervision of the Ministry of National Education, along with an authorized textbook developed by the Board of Education. These publishing houses may exhibit variations corresponding to each specific grade level. As an illustration, it can be observed that Tuna publications issued the 3rd-grade mathematics textbooks, whereas Sevgi publications published the 4th-grade mathematics textbooks. However, it is evident that MEB publications by the Morality of the Ministry of National Education offer a mathematics textbook customized to each educational level. The presence of mathematics textbooks catering to all educational levels within MEB publications and its status as a representative case among private publishing houses prompted a comprehensive analysis of the textbooks produced by MEB publications in the present study. The details of the mathematics textbooks for the 3rd, 4th, and 5th grades, developed by MEB Publishing for the academic year 2022-2023, are outlined below.

Table 3.1 Overall Information of Textbooks

Textbooks	*Grade Level*	*Pages*	*Number of Unit*
Primary School Mathematics 3 Textbook (Savaş et al., 2021).	The third-grade textbook	256 pages	six chapters
Primary School Mathematics 4 Textbook (Kayapınar et al., 2021).	The fourth-grade textbook	304 pages	six chapters
Middle School and Imam-Hatip Middle School Textbook Mathematics 5 (Cırıtcı et al., 2020).	The fifth-grade textbook	320 pages	six chapters

3.3.1 Brief Information Regarding the Textbooks

This study examined each task in the 3rd, 4th, and 5th grade mathematics textbooks within the scope of this thesis. Firstly, some brief information about these textbooks is given. Information on the current number of pages and units of the textbooks is given below:

The 3rd grade mathematics textbook consists of 256 pages and 6 units.

The 4th grade mathematics textbook consists of 304 pages and 6 units.

The 5th grade mathematics textbook consists of 320 pages and 6 units.

Upon looking at the content of these textbooks, it becomes apparent that a significant portion of them consist of questions. On the other hand, the lecture is shown very briefly at the beginning of the unit, and the information is usually given in an information box. It can be said that each book has more questions than information and text. That implies that the majority of the textbooks are dedicated to these tasks.

To clarify, the number of tasks in the textbooks is listed as follows:

3rd grade textbooks contain 160 questions with solutions and 369 questions to be solved. It consists of a total of 529 questions.

4th grade textbooks contain 205 questions with solutions and 434 questions to be solved. It consists of 639 questions in total.

5th grade textbooks contain 233 questions with solutions and 370 questions to be solved. It consists of 603 questions in total.

This means that there are an average of two tasks per page in each textbook.

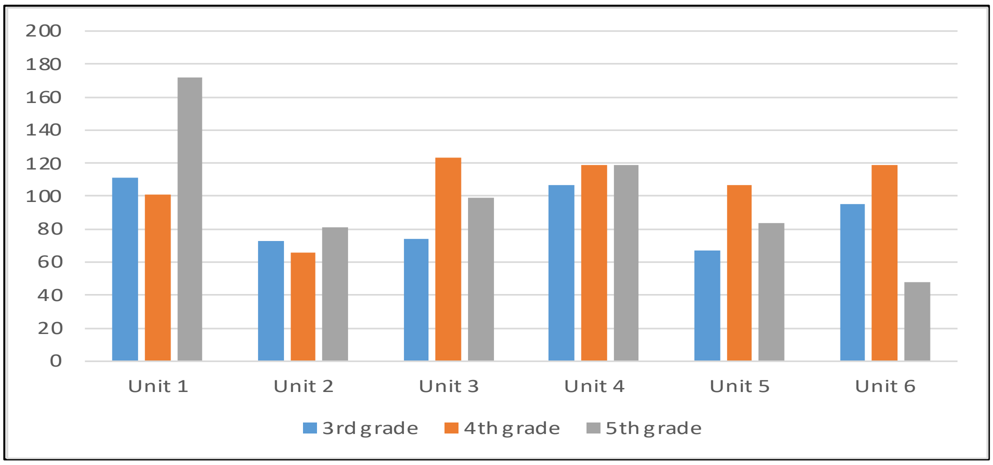


Figure 3. 1. Distribution of All Questions According to Units

Figure 3.1 shows that the distribution of all tasks according to the units of each textbook is shown in the figure in terms of grade level. The content of each unit differs according to the grade level. That is, the equivalent of units is different at every grade level. However, the order and contents of the units in the 3rd and 4th grades show an extreme similarity. Another finding is Unit 1 in the 5th-grade textbook is about natural numbers and operations of natural numbers, whereas they are covered in units 1, 2, and 3 of the 3rd and 4th-grade textbooks. In other words, the subject of natural numbers and operations of natural numbers is more dominant at these three levels than other subjects.

3.4 Data Analysis

The coding scheme presented in Figure 3.2 was utilized as a tool to analyze the claims made in the mathematics textbooks. Every task was organized in an Excel spreadsheet with columns for grade level, unit, page number, task title, task type, big idea, and claim. While examining the tasks, the primary objective has been to assess the potential overlap between the main purpose of the task and the claims being made. Hence, it is engaged in in-depth thought regarding the meaning of the claims and the fundamental goals of the tasks presented in the textbook. Although the main purpose of the tasks was focused on, it was observed that some questions contained more than one claim. For these tasks, the coding process was conducted under the assumption that there were two distinct claims. That is because some questions have more than one claim, and some have more than one question. Moreover, the textbook provides a set of examples that start with the solved questions, serving as a guiding tool for the reader. To-be-solved tasks are placed after worked example tasks. Hence, the analysis of the unresolved questions evaluates the to-be-solved tasks by considering the idea of the worked examples.

3.5 The Unit of Analysis

In the present study, tasks in textbooks were used as the unit of analysis. The tasks encompassed all the questions in the textbook except questions titled “activities” in textbooks. While examining textbooks for each grade, it was noticed that similar tasks were given different titles. In general, names like "example" and "let’s learn" were regularly used in primary school mathematics textbooks to introduce a topic, and tasks labeled "it’s your turn" were used to reinforce the topic. Towards the end of the topic, there are "topic evaluation" and "unit evaluation" tasks.

Tasks are categorized into two types within the scope of this research: worked examples and to-be-solved questions. Worked examples are tasks that are used as examples for students to practice right after a new lecture and topic. Typically, they appear at the very beginning of the unit or chapters. The unit's middle and final sections include tasks that still need to be completed, i.e., to-be-solved questions. They can be used to review the content from the chapter, get some extra practice, or assess the progress. The names of tasks at each grade level were analyzed for this thesis as the worked example and to-be-solved questions falling into two distinct types.

3.6 Coding Scheme

This framework, which was developed by Blanton et al. (2015) and includes the five big ideas of algebraic thinking and the claims students are expected to make within the scope of each claim, serves as the framework for this study. This framework is essentially a theoretical framework that belongs to the initial time of algebraic development. In the early phases of algebraic study, big ideas and claims are intended to foster the development of algebraic thought. Consequently, this research aims to analyze the tasks in each mathematics textbook using this conceptual framework and determine whether the tasks in this particular textbook do not support these claims. If it supports, it is to analyze which claims there are learning opportunities.

The conceptual framework comprises five big ideas: equivalence, expressions, equations, and inequalities (EEEI); generalized arithmetic (GA); functional thinking (FT); variable (Var); and proportional reasoning (PR). Every big idea is accompanied by its own set of claims. In essence, these claims represent what elementary school students are expected to do to improve their algebraic thinking. This study considers these claims as the learning opportunities for algebraic thinking in elementary mathematics textbooks. Therefore, each task in the textbook is analyzed within the framework of these claims, and the fact that the tasks reflect the claims means that students are given the opportunity to learn algebraic thinking about that claim.

A unique code is assigned to each claim in Figure 3.2 for this analysis. For instance, the EEEI-1 code is assigned to the initial big idea's initial claim. Then, based on this coding structure, each mathematical task provided in the textbooks is evaluated. The main strategy for coding the tasks was to use the solutions stated in the previous worked examples as a guide and code the to-be-solved questions accordingly. Furthermore, when reviewing the tasks, it is focused on the major learning opportunity, i.e., the main claim it presents; nonetheless, a small number of tasks are categorized into two different claims since there are cases where more than one question was asked for the same question. The left side of Figure 3.2 displays the five big ideas of algebraic thinking at a young age. The claims about the ideas are located on the right side of Figure 3.2, exactly opposite each significant notion. The concept of proportional reasoning is not present in Blanton et al. (2015)'s third-grade level study. Similarly to the framework, it is not expected that the concept of proportional reasoning will be covered in tasks prepared for students in grades K-3 to 5 mathematics textbooks in Turkey. As a result, the proportional reasoning idea and related claims are not included in this study.

Big ideas	Grade 3: Claims
Equivalence, Expressions, Equations, & Inequalities (EEEI)	• Interpret equations written in different formats (e.g., other than $a + b = c$) and evaluate as true or false • Solve open number sentences (e.g., 8 + 5 = ___ + 4), including by reasoning from the structural relationship in the equation • Use variable expressions to model linear problem situations • Identify the meaning of a variable used to represent an unknown quantity • Interpret an algebraic expression in the context of a problem • Model problem situations to produce linear equations of form $x + a = b$ • Analyze an equation to determine the value of a variable
Generalized Arithmetic (GA)	• Analyze information to conjecture an arithmetic relationship • Express the conjecture in words and/or variables • Identify values or domains of values for which a conjectured generalization is true • Describe the meaning of a repeated variable or different variables in the same equation • Identify a generalization in use (e.g., in computational work) • Justify an arithmetic generalization using either empirical arguments or representation-based arguments; examine limitations of empirical arguments
Functional Thinking (FT)	• Generate linear data and organize in a function table • Identify the meaning of a variable used to represent a varying quantity • Identify a recursive pattern and describe in words; use to predict near data • Identify a covariational relationship and describe in words • Identify a function rule and describe in words and variables • Use a function rule to predict far function values • Given a value of the dependent variable, determine the value of the independent variable (reversibility) • Construct a coordinate graph
Variable (Var)	• Use variables to represent arithmetic generalizations • Examine the meaning of a repeated variable or different variables in an equation or rule • Use variables to represent an unknown quantity (fixed or varying) • Understand that a variable represents the measure or amount of an object rather than the object itself • Interpret the meaning of a variable within a problem context • Use variables to represent linear problem situations • Describe a function rule using variables

Note. The big idea of Proportional Reasoning was not explicitly addressed at the third-grade level. Concepts associated with Variable were integrated in instruction throughout the other big ideas as appropriate.

Figure 3. 2. Big Ideas of Algebra and Claims (Blanton et al., 2015, p. 45)

3.7 Sample Coding from Study

In this section, we would like to give some examples to explain the coding process.

Figure 3.3 shows the sample coding of a task on page 121 of the 4th-grade textbook (Kayapınar et al., 2021). The question asks whether the given equations are true, and how they can be corrected if they are false. This query aims to enhance students' comprehension of the concept of equality, specifically pertaining to the equivalence of expressions on either side of an equation. In this context, it was coded as the interpretation of the equations written differently, which is the initial claim of the EEEI idea.

Question: Check the equivalences below. Write "T" for the true ones and "F" for the false ones. If false, write the correct ones in the opposite space.

Aşağıdaki eşitlikleri kontrol ediniz. Doğru olanların karşısına "D", yanlış olanların karşısına "Y" yazınız. Yanlış ise doğrusunu karşısındaki boşluğa yazınız.

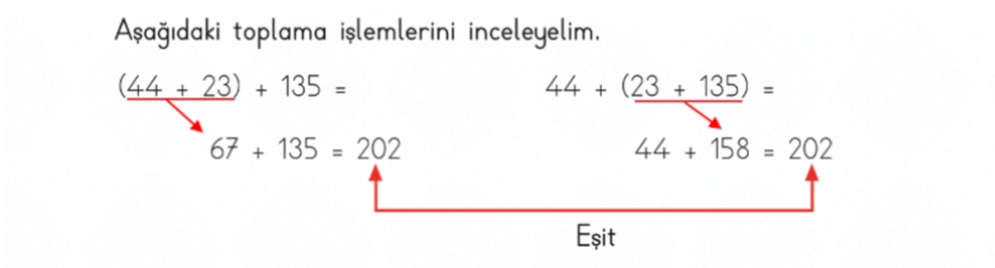
İşlemler	Doğru(D) Yanlış (Y)	İşlem Yanlışsa Doğrusu:
2400 ÷ 100 = 1 × 24		
1800 ÷ 100 = 18 × 10		
3000 ÷ 1000 = 300 × 100		
7800 ÷ 78 = 25 × 4		

Grade Level	Unit	Page No	Task Title	Task Type	Big Idea or Not	Big Idea	Claim
4th grade	Unit 3	121	Let's Learn	WE	yes	EEEI	1

Figure 3. 3. Sample Coding of 4th-grade Task from Kayapınar et al. (2021, p. 121)

The second example is in Figure 3.4 from the third-grade textbook (Savaş et al., 2021). Addition operations are presented along with a task designed to assist students in recognizing the relationship between these operations. This task is a component of the first unit, which focuses on natural numbers. This task serves as an illustration that highlights students' recognition and understanding of the commutative property of addition. The given example relates to the initial claim of the generalized arithmetic concept that students examine an arithmetic property. Hence, it is coded as GA-1 due to this reason.

Question: Let's examine the following collection processes.



Grade Level	Unit	Page No	Task Title	Task Type	Big Idea or Not	Big Idea	Claim
3rd grade	Unit 1	46	Example	WE	yes	GA	1

Figure 3. 4. Sample Coding of 3rd-grade Task from Savaş et al. (2021, p. 46)

Another sample task in Figure 3.5 has been classified as a solved question within the learning section titled "Let's Learn," specifically pertaining to the concept of natural numbers, as part of the curriculum for fourth-grade students in unit one (Kayapınar et al., 2021). This task presents a recursive pattern with the objective of identifying a relationship among a series of values. Additionally, the verbal explanation is written for the relationship's progression within this particular pattern. Hence, the third claim represents a valuable learning prospect marked as FT-3 within the functional thinking big idea.

Question: The number of pears given in the table below has increased to a certain extent. Let's examine the example.


Aşağıdaki tabloda verilen armut sayıları belirli bir oranda artmıştır. Örneği inceleyelim.

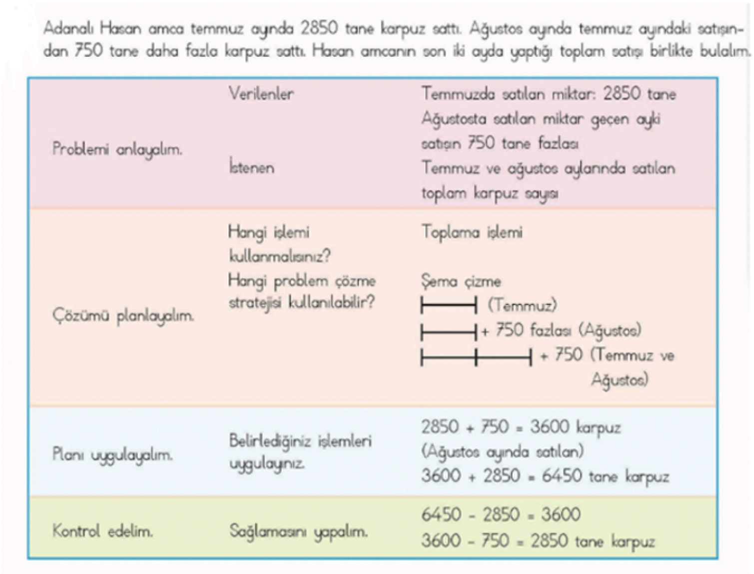
Adım Sayısı	1. Terim	2. Terim	3. Terim	4. Terim	5. Terim
Armut					
Örüntünün Kuralı	Örüntü 2 artarak devam etmiştir.				

Grade Level	Unit	Page No	Task Title	Task Type	Big Idea or Not	Big Idea	Claim
4th grade	Unit 1	36	Let's Learn	WE	yes	FT	3

Figure 3. 5. Sample Coding of 4th-grade Task from Kayapınar et al. (2021, p. 36)

Figure 3.6 shows a solved question on page 61 of Grade 4 (Kayapınar et al., (2021). Although this question is about the addition of natural numbers, the quantity of watermelons sold in the month of July is shown with the symbol while solving the question, and the number of watermelons sold in other months is also expressed with this symbol. The emphasis here overlaps with the 4th claim of the variable idea, as it is important to understand that this symbol represents the variable quantity of watermelons that were sold during the month of July. That is why it is coded as Var-4.

Question: Uncle Hasan from Adana sold 2850 watermelons in July. The sales in August exceeded those in July by a margin of 750 units. Let us determine the total sales generated by Uncle Hasan over the preceding two months.



Adanalı Hasan amca temmuz ayında 2850 tane karpuz sattı. Ağustos ayında temmuz ayındaki satışından 750 tane daha fazla karpuz sattı. Hasan amcanın son iki ayda yaptığı toplam satışı birlikte bulalım.

Problemi anlayalım.	Verilenler İstenen	Temmuzda satılan miktar: 2850 tane Ağustosta satılan miktar geçen ayki satışın 750 tane fazlası Temmuz ve ağustos aylarında satılan toplam karpuz sayısı
Çözümü planlayalım.	Hangi işlemi kullanmalısınız? Hangi problem çözme stratejisi kullanılabilir?	Toplama işlemi Şema çizme (Temmuz) + 750 fazlası (Ağustos) + 750 (Temmuz ve Ağustos)
Planı uygulayalım.	Belirlediğiniz işlemleri uygulayınız.	2850 + 750 = 3600 karpuz (Ağustos ayında satılan) 3600 + 2850 = 6450 tane karpuz
Kontrol edelim.	Sağlamasını yapalım.	6450 - 2850 = 3600 3600 - 750 = 2850 tane karpuz

Grade Level	Unit	Page No	Task Title	Task Type	Big Idea or Not	Big Idea	Claim
4th grade	Unit 2	61	Let’s Learn	WE	yes	Var	4

Figure 3. 6. Sample Coding of 4th-grade Task from Kayapınar et al. (2021, p. 61)

3.8 Validity

Fraenkel et al. (2012) defined *validity* as the "appropriateness, correctness, meaningfulness, and usefulness of the inferences" made. Alternatively, the concept of validity pertains to the extent to which data derived from a particular instrument or dataset substantiates the inferences made by a researcher based on said data. These inferences must possess relevance, significance, precision, and utility. Consequently, this section aims to examine the appropriateness of the study's framework, the significance of the study, and the practicality of the study's findings to establish the credibility of the present research.

Textbooks are frequently utilized as educational resources in primary school environments, in contrast to their comparatively lower usage at higher levels of education. In the present stage, textbooks, widely recognized as indispensable educational materials, are crucial in promoting learning opportunities. In their research, Blanton et al. (2015) developed and utilized a conceptual framework comprising five key principles and assertions of algebraic thinking to examine the content. The development of these five major concepts is primarily based on Kaput's (2008) theories concerning cognitive processes and the acquisition of concepts and claims, which are also reinforced by another relevant research in this domain. The study conducted by Blanton et al. (2015) involved a group of 106 third-grade students over a period of one year. This research has resulted in the identification of five big ideas related to algebraic thinking, as well as the formulation of corresponding claims for these concepts. In accordance with the ideas and claims, extensive research was conducted for this project, involving a multi-year curriculum progression. The study focused on the development of algebraic thinking among students in the third, fourth, and fifth grades, as examined by Blanton et al. (2018).

The assertions made by these ideas are well-suited and pertinent for the analysis of the instructional approach to algebraic thinking in elementary mathematics textbooks. Consequently, the tasks included in elementary mathematics textbooks must be analyzed to see whether there are ample opportunities for learning algebraic abilities at the first-grade levels, specifically K-3, K-4, and K-5, as examined in this research. Early algebra plays a crucial role in fostering students' cognitive development, enabling them to succeed in both algebraic and mathematical domains within their academic careers and enhancing their prospects in the professional world. Therefore, it is imperative to thoroughly analyze the tasks presented in textbooks to determine if they provide learning opportunities that build an understanding of algebraic thinking. This research yields a multitude of diverse recommendations that have a wide-ranging impact on a broad spectrum of individuals. Several strategies can be employed to promote the integration of early algebra in educational settings. One such approach involves engaging textbook authors in critically evaluating their materials to incorporate early algebraic concepts more effectively. Additionally, educators are encouraged to evaluate the existing curriculum to ensure its alignment with early algebra principles. Lastly, it may encourage teachers to incorporate classroom activities that foster the development of algebraic thinking skills among students.

3.9 Reliability

The concept of *reliability* is referred to as the extent to which a measurement proceure consistently yields identical outcomes when repeated trials are conducted (Carmines & Zeller, 1979). The assessment of coding reliability holds significance as it contributes to the establishment of the reliability of qualitative findings. The establishment of reliability in content analysis is achieved by assessing the level of agreement between two or more human coders when human coders are employed (Neuendorf, 2002). It can be referred to as inter-coder reliability. Inter-coder reliability (ICR) means the level of concurrence among multiple coders in assigning codes to qualitative text (MacPhail et al., 2016). In simpler terms, ICR measures the extent of agreement between two or more coders in their qualitative data coding. In this research, two coders worked independently and coded the tasks in the selected textbooks using the coding schema in Figure 3.1. The tasks are randomly selected to be 10 percent of the solved and 10 percent of the unanswered questions at each grade. The coding schema was utilized to examine each task, and subsequently, each task was assigned a code corresponding to its respective category. Two researchers came together to conduct an in-depth comparison of the coding outcomes, and the agreement level was found to be 85.71%. Since the percent agreement level is 85.71, 85.71% is the amount of data representing the research data. Upon thorough deliberation and analysis, two researchers discussed questions, and all disagreements on coding were eliminated. Complete compatibility has been successfully accomplished. Miles and Huberman (1994) stated that the initial code reliability between coders should be close to 80 percent, and eventually, the agreement between intercoder should be in the 90 percent range. Therefore, inter-rater reliability was provided.

CHAPTER 4

RESULTS

4.1 Results Regarding the Textbooks

4.1.1 General Results of Algebra-Related Task Distribution in Textbook

The framework was implemented in the textbook series employed within the educational system of Turkey, specifically targeting the three grades of state primary schools. The tasks provided within the mathematics textbooks are assessed based on the framework established by Blanton et al. (2015). The study showed that 6.3% of the overall number of tasks in the textbooks assigned to students in the fourth, fifth, and sixth grades (N = 1771) related to algebraic thinking.

Table 4.1 Distribution of Tasks According to Question Type and Total Tasks

	WE	*TBS*	*Total Algebra-Related Tasks*	*Total Tasks*
Third-grade Textbook	9	25	34	529
Fourth-grade Textbook	11	26	37	639
Fifth-grade Textbook	12	28	40	603
TOTAL	32	79	111	1771

The tasks that provide learning opportunities were investigated according to the question type: “worked examples” and “to-be-solved” questions. Table 4.1 shows the number of tasks in two sub-fields and each textbook's total number of tasks. The table shows that to-be-solved questions are more common than solved questions at all three grade levels. When examined according to sub-fields, an increase is seen in the number of tasks according to the class level. The fifth-grade textbook is the textbook with the most task-provided opportunities among both question types. In other terms, the 5th-grade textbook has the most algebra-related tasks. The 4th-grade textbook with a small margin and the 3rd-grade textbook with the least number of algebra-related tasks followed it.

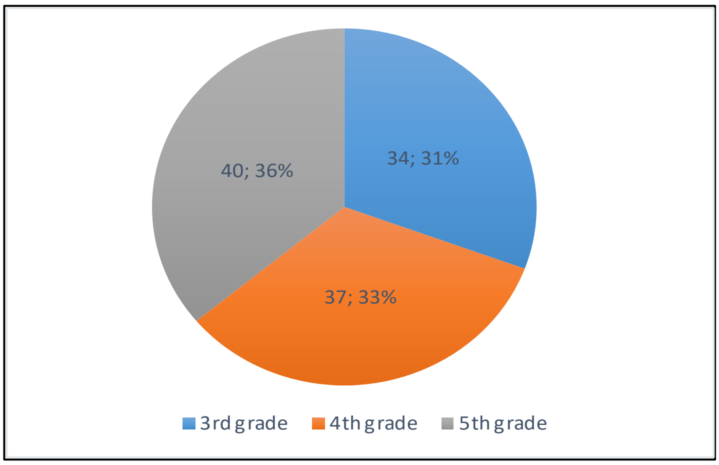


Figure 4. 1. Percentage Distribution of Algebra-related Tasks in the Textbook by Grade Level

That was a potential outcome because, as the grade level increases, more algebra-related tasks may be included in the textbooks to increase students' readiness for algebra. Likewise, the total number of algebra-related tasks increased from the third grade to the fifth grade. There is no noticeable positive correlation between the total amount of textbook tasks and grade level, with this trend particularly evident in the fourth grade. Moreover, according to Table 4.1, the proportion of algebra-related tasks about the total task number at each grade respectively is 6.4%, 5.8%, and 6.6%. In other words, 6.4% of the third-grade textbooks, 5.8% of the fourth-grade textbooks, and 6.6% of the fifth-grade textbooks consist of tasks that offer the opportunity to think algebraically. At this point, when the number of tasks is considered, the number of tasks increases according to the class level. However, when evaluating the proportion of algebra-related tasks in relation to the total task number at each class level, no significant association is found. The data shown in Figure 4.1 shows the percentage distribution of algebra-related tasks in the textbooks according to grade level.

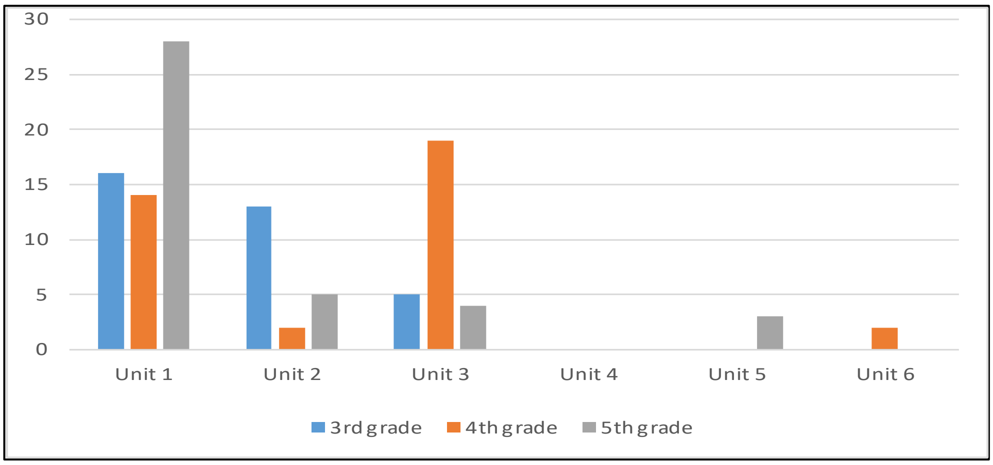


Figure 4. 2. Distribution of Tasks According to Units

Among the total number of algebra-related tasks identified, it is observed that 31% are included in third-grade textbooks, while 33% are contained inside fourth-grade textbooks. The remaining 36% of these tasks are located in fifth-grade textbooks. Currently, the majority of algebra-related tasks are found in fifth-grade textbooks. In addition, Figure 4.2 shows the distribution of the algebraic thinking-related tasks associated with the units at each grade level. According to this graph, the first units have more tasks that offer learning opportunities. When these units are examined in detail in Figure 4.3 it is seen that the first three units of the third grade are about natural numbers and operations with natural numbers.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6
3rd grade	Natural Numbers and Operations	Natural Numbers and Operations	Natural Numbers and Operations	Fraction Measurement	Geometry	Measurement
4th grade	Natural Numbers and Operations	Natural Numbers and Operations	Natural Numbers and Operations	Fraction Measurement Data Analysis	Geometry Data Analysis	Measurement
5th grade	Natural Numbers and Operations	Fractions	Decimals Percent	Geometry	Data Analysis	Measurement Data Analysis

Figure 4. 3. Grade Levels and Units (Cırıtcı et al., 2020; Kayapınar et al., 2021; Savaş et al., 2021).

Similarly, the first three units of fourth grade are about natural numbers and operations of natural numbers. In the fifth grade, only the first unit is about natural numbers and operations. That means that the tasks that offer the opportunity for algebraic thinking focus mostly on the topic of natural numbers and operations. In the fifth grade, algebra-related tasks were encountered in the second unit on fractions, in the third unit on decimals and percentages, and in the fifth unit on data analysis. In the fifth unit of the fourth grade, these tasks were encountered regarding measurements. According to the elementary mathematics curriculum in Turkey, while the time allocated in the planned curriculum to the first three units in the third grade is 52% of total instruction time, the time allocated to the first three units in the fourth grade is 45%. In the fifth grade, the time allocated to the first unit is 22% (MEB, 2018). At this point, while the time allocated to natural numbers and operations decreases as the class level progresses, the number of tasks that provide algebraic learning opportunities slightly increases.

4.2 Results Regarding the Ideas and Claims

4.2.1 Overall Results Regarding the Ideas

The analysis has focused on the distribution of the big ideas within the tasks that allow students an opportunity to acquire knowledge. Consequently, a frequency analysis was undertaken in order to ascertain the extent to which textbooks provide students the opportunity to engage in algebraic thinking.

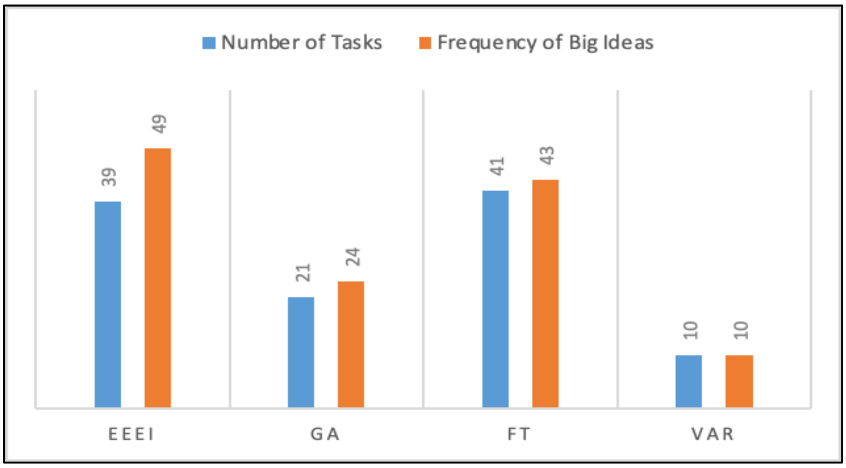


Figure 4. 4. The Number of Algebra Tasks and the Frequency of Big Algebra Ideas

When examining the tasks in consideration of the research question, the primary attention is on the central purpose of the task. In other words, it was focused on the main claim of algebraic thinking. In certain cases, however, tasks were encoded with two separate claims. The data presented in Figure 4.4 illustrates the distribution of algebra-related tasks and the corresponding frequency of big ideas of algebraic thinking across different grade levels. On average, 13.5 percent of tasks utilized double encoding. In other words, the number of claims exceeds the number of tasks.

The distribution of big ideas of algebraic thinking by grade level is depicted in Figure 4.5. The rates are 31.0%, 34.1%, and 34.9% respectively. As students' progress through school, textbooks emphasize algebraic thinking concepts more increases along with the grade levels.

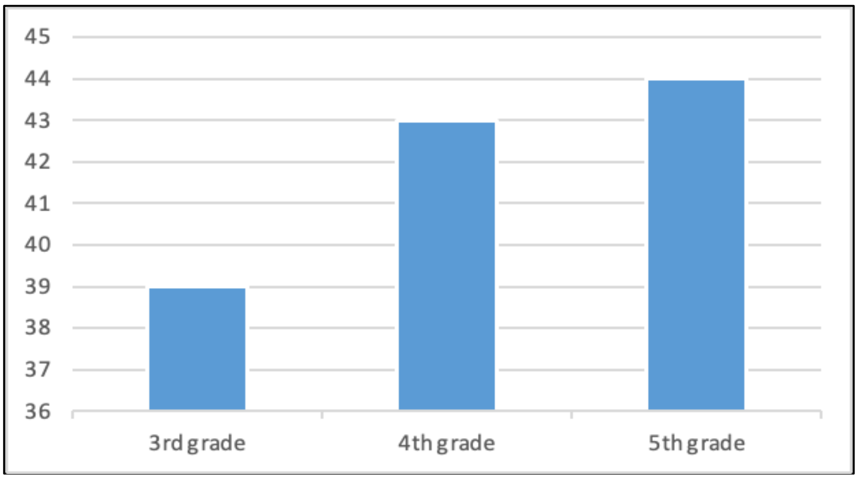


Figure 4. 5. Total Distribution of Big Algebra Ideas According to Grades

Figure 4.6 shows the percentage distribution of the big ideas of algebraic thinking. According to the figure, Equivalence, Expressions, Equations, and Inequalities (EEEI) is the most common idea across the three textbooks, with 39.0%. Functional Thinking (FT) was the second most common (34.0%), followed by Generalized Arithmetic (GA) (19.0%). Finally, there is the idea of variable exists (8.0%). The approaches employed by different countries in their curricula or textbooks to address the fundamental ideas of algebraic thinking may exhibit diversity (Cai et al., 2005).

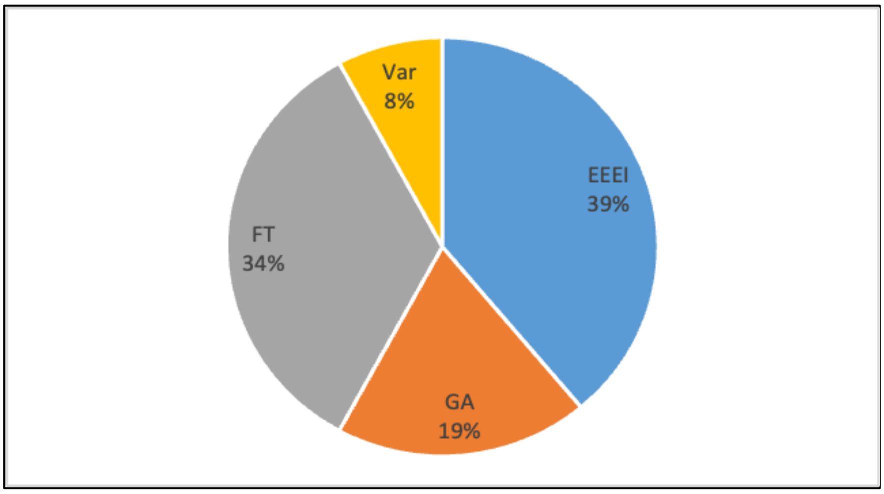


Figure 4. 6. Percentage Distribution of Big Ideas of Algebraic Thinking

For instance, while in China, there is a notable emphasis on equations and their solutions in the primary curriculum, the Russian elementary curriculum, specifically Davydov (1972/1990)'s approach, places greater emphasis on exploring the relationships between quantities through variables. Subsequently, the functional approach, which involves recognizing relationships and patterns followed by applying rules, is emphasized. Moreover, Figure 4.6 shows the general frequency distribution of big ideas of algebraic thinking. This figure explains that the EEEI idea, which has the highest percentage in big ideas of algebraic thinking with 39.0%, is seen 49 times in three textbooks.

Similarly, the number of tasks for the functional thinking idea is 43, for the generalized arithmetic idea is 24, and for the variable idea is 10. The variable idea is observed the least in the tasks. This idea also exists in the claims of other ideas. For example, in the case of a linear problem situation, the EEEI-3 claim means that the student should model the situation using a variable expression. In contrast, the Var-6 claim means that the student should represent the linear problem using a variable. In other words, since a clear distinction cannot be made between the ideas, some claims can be seen within each other. Another reason may be that the variable idea is sometimes seen as a side idea and is not included in the coding.

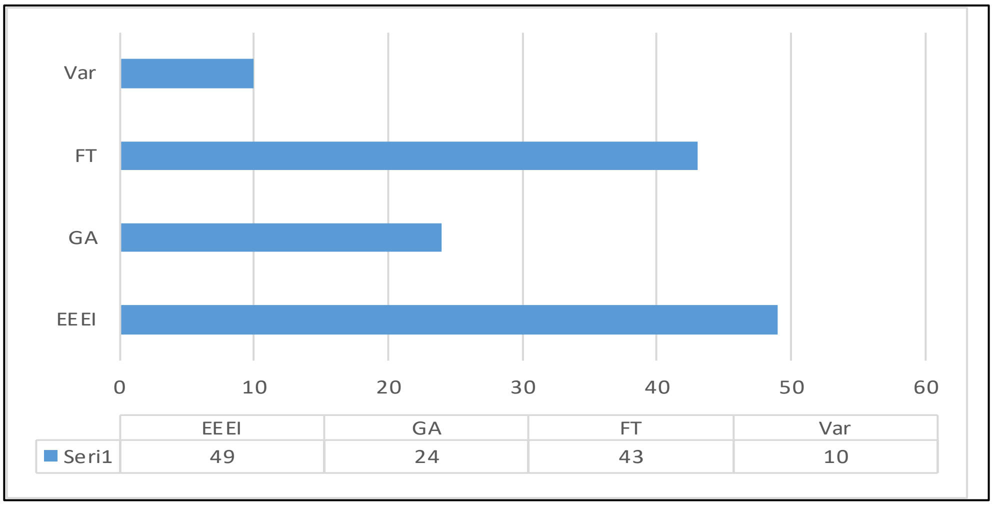


Figure 4. 7. Frequency Distribution of the Big Ideas of Algebraic Thinking

For example, in the Functional Thinking idea, the FT-5 claim refers to the student describing functional rules by word or variable. Since the student will use variables while defining this rule, Var-3 claims that using variables to define the unknown or variable is used simultaneously. In this case, if the main purpose of the question overlaps with FT-5, the Var-3 claim is not included during the coding process because it is seen as a secondary purpose. However, a thorough examination of the claims reveals that such claims were not present in the tasks. Within the scope of this study, the variable idea may be low for two reasons: it is not included in some tasks because it is not the primary idea, and the tasks offer fewer opportunities for the variable idea. Due to these reasons, the concept of variables may appear less prominent in textbooks.

The variation of ideas according to their classes is shown in Figure 4.8. When viewed from a broad perspective, Figure 4.8 reveals that FT is a recurring idea in fifth grade.

The least common idea is repeated three times. In the third and fourth grades, the idea is Var, while in the fifth, it is GA.

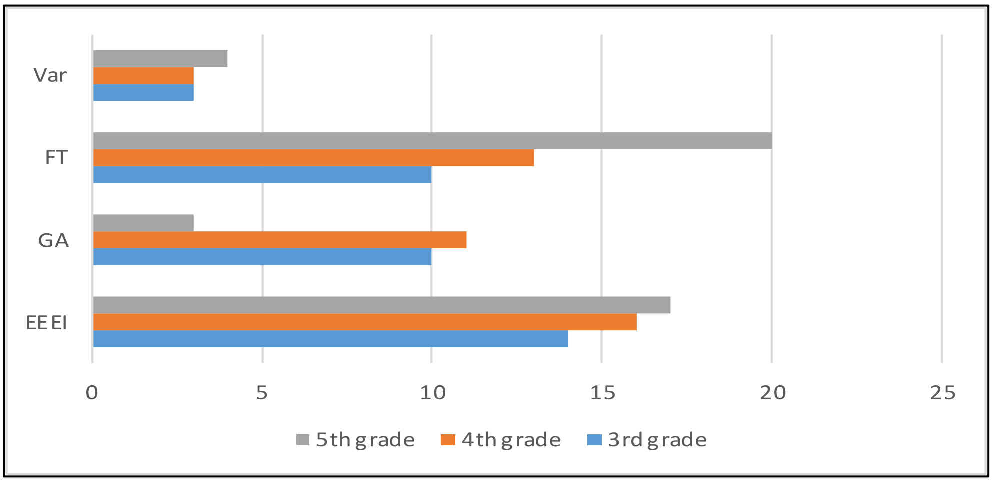


Figure 4. 8. Variation of Big Ideas of Algebraic Thinking According to Grades

According to the variation of ideas by grade level, the EEEI idea grows as the class level rises. Similarly, there is a positive relationship between the idea of FT and the class level. The GA idea is repeated at least in fifth grade, with a significant difference from other grade levels. The tasks about the idea of GA in the 5th-grade textbook are very few compared to other classes. In contrast, the idea of Var has similar or equivalent frequency across all grade levels. The possible reasons why the Var idea is the lowest at all grade levels for this have just been mentioned above.

4.2.2 Overall Results the Claims Regarding Algebraic Thinking

Figure 4.9 shows the most repetitive claims of each big idea of algebraic thinking at the grade level. Accordingly, the EEEI-2 claim was the most common, while the second most common claim was FT-3. EEEI-2 is a claim about students solving open-number sentences. FT-3, conversely, is about identifying the recursive pattern and finding close data using the relationship in the pattern. In third place, the GA-1 claim is repeated the most. GA-1, on the other hand, is associated with students' analysis of information to find an arithmetical relationship.

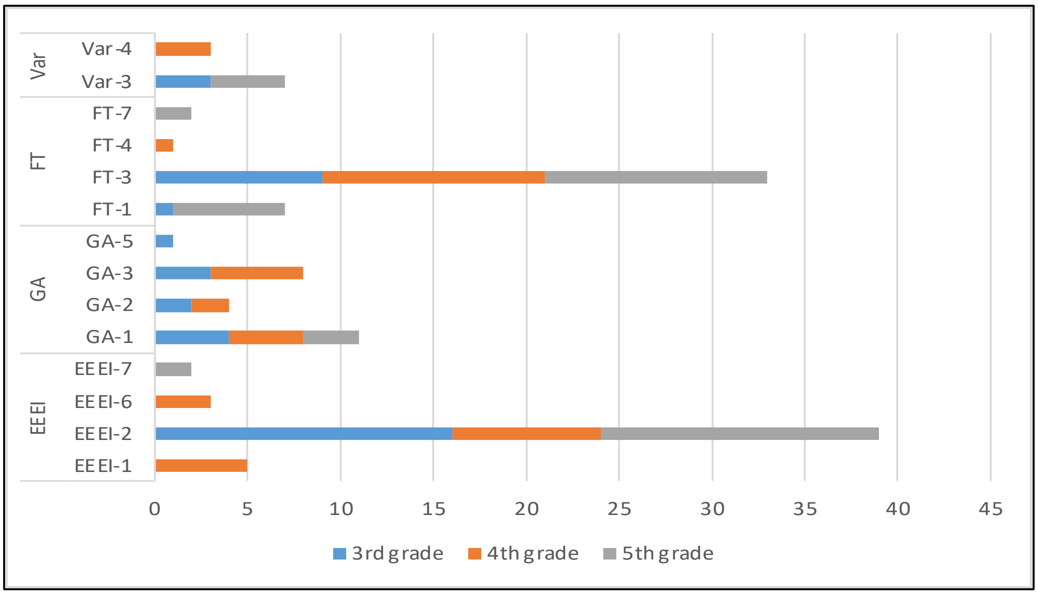


Figure 4. 9. Distribution of Claim Regarding Algebraic Thinking by Grades

In grades, EEEI-2 is mainly seen in the fifth grade and third grades. FT-3 is seen in the fourth grade and fifth grades. GA-1 is frequently found in the 3rd and 4th grades. The fourth most common claim is GA-3. Following this, FT-1 comes as the fifth claim. GA-3 determines the specific values or domains of values in which a proposed generalization holds true. FT-1, conversely, is a claim about generating linear data and displaying this data in the table.

Currently, it has been observed that 14 out of the total 28 claims have been encountered in the textbooks, while the remaining 14 claims have yet to be encountered. As illustrated in Figure 4.10, it can be observed that 50% of the claims are present, but the remaining 50% are absent. Claims not found in these three textbooks are listed in Figure 4.10.

Code	*Claim*
EEEI-3	Use variable expressions to model linear problem situations
EEEI-4	Identify the meaning of a variable used to represent an unknown quantity
EEEI-5	Interpret an algebraic expression in the context of a problem
GA-4	Describe the meaning of a repeated variable or different variables in the same equation
GA-6	Justify an arithmetic generalization using either empirical arguments or representation-based arguments; examine limitations of empirical arguments
FT-2	Identify the meaning of a variable used to represent a varying quantity
FT-5	Identify a function rule and describe in words and variables
FT-6	Use a function rule to predict far function values
FT-8	Construct a coordinate graph
Var-1	Use variables to represent arithmetic generalizations
Var-2	Examine the meaning of a repeated variable or different variables in an equation or rule
Var-5	Interpret the meaning of a variable within a problem context
Var-6	Use variables to represent linear problem situations
Var-7	Describe a function rule using variables

Figure 4. 10. Claims Regarding Algebraic Thinking Not Found in Textbooks

When these claims are examined, these claims are related to support the usage of mathematical variables as tools, whether the issues involved are linear and those requiring varying quantities and arithmetic generalizations. The purpose of these claims is for students to comprehend the meaning of the significance of variables in a context and to explain this significance. In addition, some claims concern the composition and interpretation of algebraic expressions. For example, they involve discovering function rules, predicting outcomes, deriving arithmetic generalizations, and achieving justification through empirical or representation-based arguments.

Claims not found in textbooks are primarily aimed at students expressing the meanings of variables in various situations. In addition, the textbooks did not find claims about students' use of variables, the justification and interpretation of an arithmetic generalization, and the definition of a function rule.

4.2.3 Details Regarding the Claims

1) Equivalent, Expressions, Equations, and Inequalities (EEEI)

The study's findings show that the textbook tasks offer the most opportunities for Equivalence, Expressions, Equations, and Inequalities (EEEI) ideas. It has seven claims, four of which have been observed in textbooks. No significant relationship was discovered between the distribution of these ideas and claims by grade level. In particular, the opportunities given to students to find a variable in open-numbered sentences draw attention. In addition, the tasks emphasize the meaning of the equal sign and make the students realize that the equivalence of the expressions in different formats is true or false. These are the opportunities that predominate at all three levels. In addition, as the level increased, there were opportunities for students to model a problem situation in the 4th-grade textbook and opportunities for students to analyze an equation and find the unknown in the 5th-grade. Linear equation cases were not found at all three levels. It was also observed that the result was checked at the end of most of the solved questions.

EEEI-1

It is a related claim to interpret the equations written in different formats and evaluate them as true or false. This claim has only been observed in the 4th-grade textbook. While establishing a relationship between addition and multiplication in the third grade, there are tasks to create two different equations for the same question and allow students to see that these two equations give the same result. However, in these tasks, various expressions are not written on both sides of the equal sign. That is why it is not coded. Figure 4.11 shows a question related to this claim in the fourth-grade textbook. The task presents an equation, and there are different expressions on both sides of the equals sign in that equation. It is desirable to evaluate the accuracy of this equation.

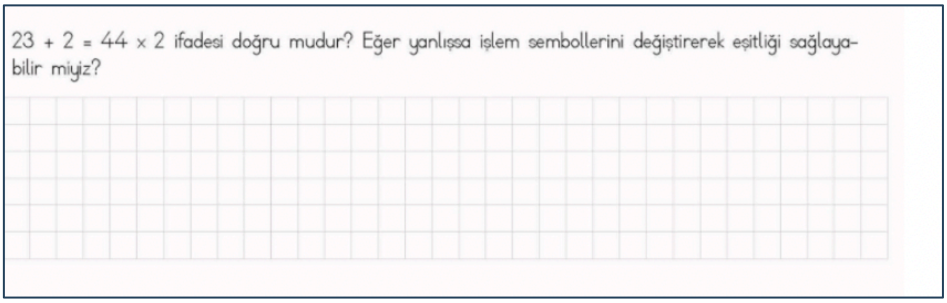

23 + 2 = 44 x 2 ifadesi doğru mudur? Eğer yanlışsa işlem sembollerini değiştirerek eşitliği sağlaya-bilir miyiz?

Figure 4. 11. Sample Coding of EEEI-1 Claim of Algebraic Thinking Ideas in Fourth-Grade Math Textbook (Kayapınar et al., 2021, p. 122).

EEEI-2

EEEI-2 is the most common of all claims found. At the same time, it is the most repeated claim in the three classes under the idea of the EEEI and is frequently repeated in every grade level. It was primarily seen in the third grade and the least in the fourth grade. This claim is about solving open-number sentences. It was expected that the values not given in the equation would be found in the tasks, and it is expected that equality would be ensured. The purpose of the study is for the student to solve the open-sentence question with its justifications. In the worked examples, it is observed that open-number sentences were solved with inverse operations or by the counting-up method. As an example of EEEI-2, the task in the third-grade textbook is shown in Figure 4.12.

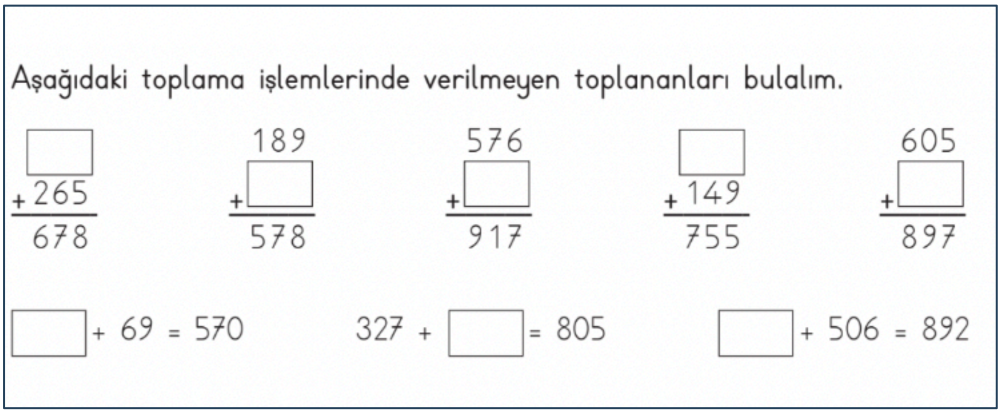


Figure 4. 12. Sample Coding of EEEI-2 Claim of Algebraic Thinking Ideas in Third-Grade Math Textbook (Savaş et al., 2021, p. 45)

EEEI-6

The EEEI-6 claim was first and only seen in the fourth grade. It is a very rare claim at this level. This claim aims to have students write the linear equation by correctly analyzing and modeling the question. This claim comes up with a worked question that aims for students to write and solve open sentences as understood from its solution. Right after the worked examples, a to-be-solved question is asked to the students for the same purpose.

EEEI-7

The EEEI-7 claim, which only appears in fifth grade, is about analyzing an equation to identify the value of a variable. It does not aim to find the answer to any question directly but aims to make students use their logic and analysis skills to conclude the variable value. Only in the fifth grade two questions were coded as this claim.

1) Generalized Arithmetic (GA)

In textbooks, the idea of generalized arithmetic is quite infrequent.

GA-1

The claim that is most frequently reiterated within the framework of the generalized arithmetic concept is GA-1. It is balanced at every grade level. The example from a third-grade textbook represents an example of GA-1 (Figure 4.13).

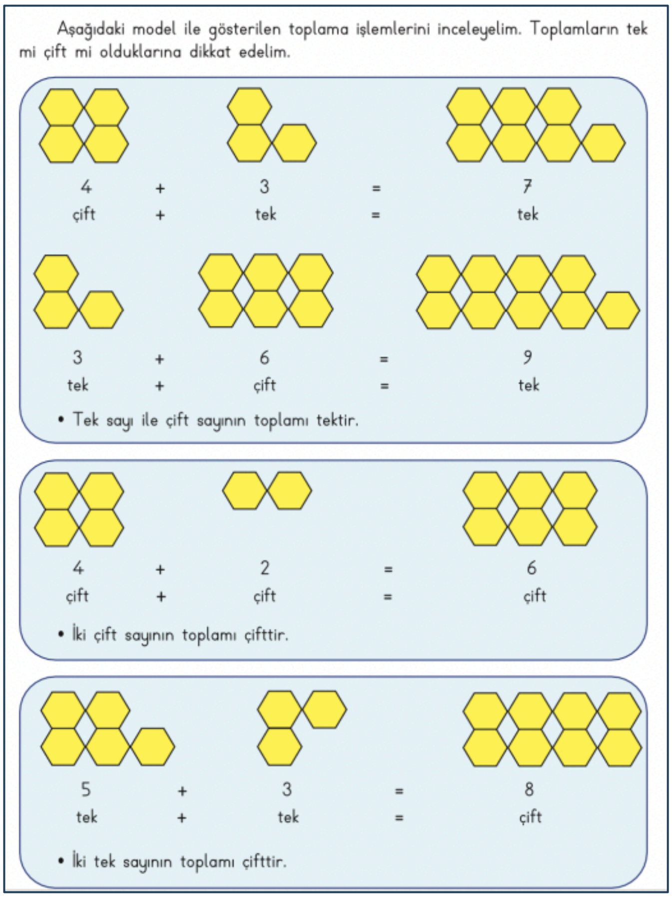


Figure 4. 13. Sample Binary Coding of Claims of Algebraic Thinking Ideas in Third-Grade Math Textbook (Savaş et al., 2021, p. 39)

The relevance of GA-1 lies in its objective of fostering students' ability to analyze relationships in arithmetic. Additionally, the objective of this task is to establish a specific description for this conjecture. The description is equally pertinent in relation to GA-2. Thus, the task involves binary coding.

GA-2

The GA-2 claim is about expressing the assumption in words or variables. For example, it is about noticing the commutative property in aggregation, generalizing it, and explaining it. That is, he expects students to write that the displacement of the numbers in the addition process will not affect the result. Tasks for the commutative property were observed in both addition and multiplication. This claim is only found in third and fourth-grade textbooks. However, the claim was observed only in worked questions. There are no to-be-solved questions expected from the student.

Aşağıdaki toplama işlemlerinde eksik olan yerlere uygun sayıları yazınız.

4 + (8 + 5) = (4 + ...) + 5

(13 + 5) + 22 = ... + (5 + 22)

563 + 281 = ... + 563

(45 + ...) + 28 = ... + (33 + 28)

Figure 4. 14. Sample Coding of GA-3 Claim of Algebraic Thinking Ideas in Third-Grade Math Textbook (Savaş et al., 2021, p.50)

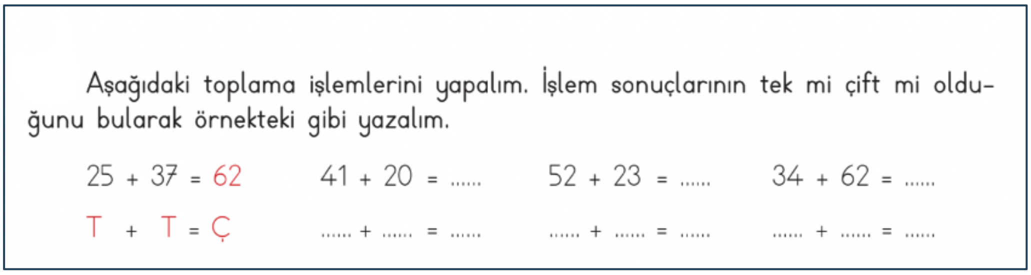
GA-3

After analyzing and generalizing a conjecture, the GA-3 claim expects students to identify values or domains for which a conjectured generalization is true. This claim is only observed in third and fourth-grade textbooks. An example from the third grade is given in Figure 4.14. In this example, it is aimed to find the unknown to ensure equality. But this equality is related to the commutative and associative property of addition. The primary purpose of the question is to recognize this property and find the correct values to ensure generalization.

GA-5

GA-5 claim expects to be able to express a generalization regardless of numbers. It has only been observed in a task in the third-grade textbook. Figure 4.14 shows the sample coding of GA-5 claim.

Figure 4.15 shows the example of GA-5. This task was coded as GA-5 because adding numbers was associated with the sum of odd and even numbers since a generalization was made about this relationship. That aims to encourage students to find generalizations in this task.


Aşağıdaki toplama işlemlerini yapalım. İşlem sonuçlarının tek mi çift mi olduğunu bularak örnekteki gibi yazalım.

25 + 37 = 62 41 + 20 = ... 52 + 23 = ... 34 + 62 = ...

T + T = Ç ...... + ... = ... + ... = ... + ... = ...

Figure 4. 15. Sample Coding of GA-5 Claim of Algebraic Thinking Ideas in Third-Grade Math Textbook (Savaş et al., 2021, p.40)

1) *Functional Thinking (FT)*

When examining the idea of functional thinking, there is an observable increase in the number of tasks representing this idea as students' progress from the 3rd to the 5th grade. The big idea of functional thinking is the second most common idea in textbooks. Students are asked to observe and express the regularities of a simple pattern in numbers and continue it. As the grade level increased, simple patterns in geometrical forms and table creation questions began to occur. They are also frequently given opportunities to define and maintain the recursive pattern.

However, opportunities were not provided to realize the covariational relationship, explore the functional rule, and create graphics.

FT-1

While the FT-1 claim is seen chiefly in the fifth-grade textbook, it does not appear in the fourth-grade textbook. The purpose of this claim is that students can generate linear data and then organize them in a function table. There is both a worked question and a to-be-solved question for this claim.

FT-3

Many tasks are associated with FT-3 claims. The objective of FT-3 is to identify a recursive pattern and employ it to make predictions regarding nearby data. The claims about seeing a covariational relationship, using a function rule, including variables, and making a coordinate graph are not in textbooks.

FT-7

In Figure 4.16, the task has the steps, and the corresponding quantities of bars are associated with each step. The main purpose of the task is to encourage students to determine the independent equation in a case where the dependent variable is provided. Since the purpose of the FT-7 clam is to make its students think of reversibility, this task has a strong connection to the FT-7.

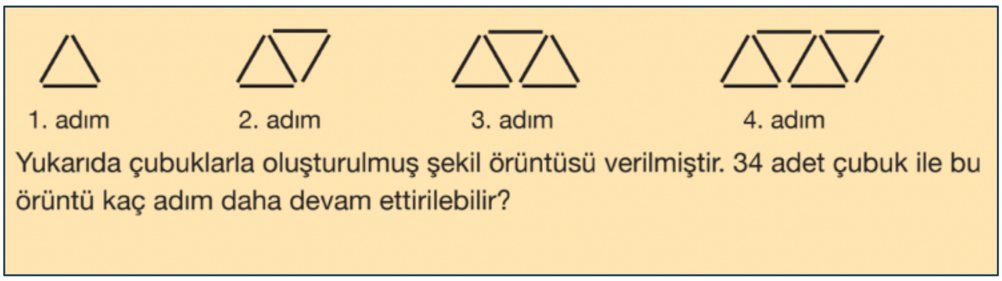


Figure 4. 16. Sample Coding of FT-7 Claim of Algebraic Thinking Ideas in Fifth-Grade Math Textbook (Cırıtcı et al., 2020, p. 29)

1) *Variable (Var)*

The idea of the variable is the least common idea in textbooks. Of the seven claims, only two have been observed in textbooks.

Var-3

The Var-3 claim is about using a variable to represent an unknown, fixed, or varying quantity. This claim is observed in third and fourth-grade textbooks. The places they are used in textbooks to express the fixed unknown in the questions that should be written in open sentences.

> Çıkanı 438, farkı 5098 olan çıkarma işleminin eksileni kaçtır?

Figure 4. 17. Sample Coding of Var-3 Claim of Algebraic Thinking Ideas in Fifth-Grade Math Textbook (Cırıtcı et al., 2020, p. 30)

For example, a task is given in Figure 4.17. A subtraction is given in this task. In the subtraction, the subtrahend is given as 438, and the difference is given as 5098. It is requested to find the minuend. This task comes after the worked example, and it is solved by representing the minuend as a variable in the worked example. Since the minuend is represented as a variable in the worked example, this task is coded as Var-3. In this task, it is also wanted to write and solve the open sentence, so the task is also coded as an EEEI-2 claim.

Var-7

There are tasks in the fourth-grade textbook that require using a variable to represent an object's quantity. These tasks coincide with the Var-7 claim because the claim expects students to focus on the represented quantity or measure, not the variable itself.

4.2.4 Results Regarding the Change of Ideas by Grade Level

While the third and fifth-grade tasks focus more on solving the EEEI-2, or open number syntax, in the fourth grade, claims are distributed more frequently. In addition to open-number syntax questions, claims from the 4th-grade textbook included evaluating the concept of equality, modeling the problem situation, and writing in a specific format. Only in the fifth grade were claims stated for students to analyze and comprehend an equation and determine the variable.

The claim to analyze information to find an arithmetic relationship is observed at every grade level and has a similar distribution. In the third and fourth-grade tasks, this relationship was explained with words and determined the values or value areas where the generalization was correct. Tasks linked to these claims were not observed in the fifth grade. Then, after noticing the generalization in a question, defining it with words, and determining the necessary values for it to be correct, the claim aiming to define the generalization outside of calculation was encountered in the third grade.

The claim about describing a recurring pattern with words and using it to infer nearby data is the claim that is repeated the most frequently across all grade levels. In the tasks for the third and fifth grades, the generation of linear data and the organization of a function table were observed. In addition to these, reversibility tasks are only introduced in the fifth grade. In general, covariational relationship and function rule claims were not encountered.

Only two of the seven claims of the Variable idea are presented in textbooks. These claims concern using a variable to represent an unknown quantity and noticing that the variable represents a quantity or measurement. However, the textbooks do not claim that a variable's meaning in a problem, rule, or generalization is understood. However, textbooks do not have claims about understanding the meaning of a variable in a problem, rule, or generalization.

CHAPTER 5

DISCUSSION

This study analyzed the learning opportunities provided through algebraic thinking-related tasks in Turkey's official third-grade, fourth-grade, and fifth-grade mathematics textbooks. During the investigation of the tasks, the algebraic thinking framework proposed by Blanton et al. (2015) is used, which consists of five ideas of algebraic thinking and their associated claims for elementary students to do. These ideas encompass significant aspects of algebraic thinking, and the claims outline what pupils in third grade should be able to do with the big ideas. In the context of this research, these claims are regarded as opportunities to learn algebraic thinking that should be provided to elementary school children. This study examines whether textbooks, commonly used educational materials in countries such as Turkey with low algebraic proficiency, provide learning opportunities for algebraic thinking among primary-level students. Additionally, the concept of algebra is often first introduced in the mathematics curriculum at the middle school level, typically in the sixth grade. However, algebra is inherently integrated into other mathematical topics. Therefore, despite the fact that it may not have a distinct presence in the elementary school curriculum, it is expected to be integrated into other learning objectives. For instance, concepts like the commutative property can be found within arithmetic, providing opportunities for students to engage in algebraic thinking. This integration allows for the inclusion of algebraic thinking in the elementary school curriculum without it being explicitly stated, providing opportunities for its incorporation in textbooks and instructional materials.

The results indicate that a few of the overall tasks contained within the three textbooks presented opportunities for using algebraic thinking (6.3%). Similarly, Demosthenous and Stylianides (2014) discovered a similar percentage (10.7%) while investigating algebra-related tasks in Cyprus's official grades 4-6 mathematics textbook. This implies that the limited amount of algebraic tasks in the primary school textbooks of a nation without objectives of early algebra in its curriculum is a probable circumstance.

As students progress from the 3rd to the 5th grade, there is a corresponding increase in the inclusion of algebra-related tasks within their textbooks. Although there is a slight rise in the prevalence of algebraic thinking concepts found in textbooks as grade level progresses, the observed change from third to fifth grade does not exhibit substantial growth. However, the fifth-grade level was expected to present the most significant opportunities for developing algebraic thinking. The fifth grade is a crucial level to equip kids with foundational knowledge and skills in preparation for their formal algebra study throughout their 6th-grade education. This situation emphasizes the need to place additional emphasis on fifth-grade textbooks that provide the opportunity to acquire algebraic thinking.

Upon examination of the distribution of algebra-related tasks among the units in the textbooks, it becomes evident that they primarily appear within the units related to natural numbers and operations of natural numbers. Although the focus of algebraic education taught in schools varies according to the countries' goals (Cai et al., 2005), it has been observed that algebraic thinking is mainly focused on numbers and operations (Palak, 2022). That is why some studies focus only on numbers and operations when analyzing algebraic thinking. However, since some researchers and studies think algebraic thinking can be ubiquitous, this thesis has examined every topic in the textbook. For example, Vennebush et al. (2005) mentioned number exploration, measurement tasks, and geometry investigations among the topics that will provide the most learning opportunities. They emphasized developing tasks and activities with rich content in these areas. In this study, algebra-related tasks were observed in the fourth grade on the measurement topic. Similarly, operations with fractions and decimals and data analysis topics provide learning opportunities for algebraic thinking in the fifth grade. The number of these tasks in these units is minimal. The result of this study also supports that the opportunity for algebraic thinking is primarily found in numbers and operations. Although algebraic thinking is mostly about numbers and operations, it should be more involved in every math topic. For example, Vennebush et al. (2005) research has demonstrated that tasks involving "Decreasing the volume" provide an essential opportunity for supporting algebraic thinking, as they include the intersection of three distinct mathematical domains: geometry, measurement, and algebra. The objective is to determine which dimension of a rectangular box should be increased by 1 unit to accomplish the most substantial decrease in the volume of such a box. Completing these tasks is crucial both for achieving each of the three standards stated in the mathematics curriculum and cultivating algebraic thinking.

Details of Algebraic Thinking Big Ideas Found in Textbooks

Blanton et al. (2015) posited that algebraic thinking encompasses five fundamental ideas. Upon close examination of the tasks presented in the textbooks, it becomes evident that elementary school textbooks offer very few opportunities for only four big ideas. Consistent with expectations, the idea of proportional reasoning is found to be absent in textbooks designed for students in grades 3-5, as noted by Blanton et al. (2015). In that order, the dominant categories of algebraic thinking big ideas are EEEI (Equivalence, Expressions, Equations, and Inequalities) idea, FT (Functional thinking) idea, GA (Generalized Arithmetic) idea, and Var (Variable) idea.

The most frequently recurring ideas are in the following order: EEEI (39%), FT (34%), GA (19%), and Var (8%). This means that the tasks in primary school textbooks are mainly designed to provide opportunities for the idea of Equivalence, Expressions, Equations, and Inequalities. Similarly, the Chinese primary school mathematics curriculum is focused on equations and solving equations (Cai, 2004). The second most frequent claim in textbooks derives from functional thinking, followed by generalized arithmetic.

This study's results underscore the presence of four discrete claims of each of EEEI, GA, and FT within the analyzed textbooks. This observation suggests that, barring the Idea of Var, most of the claims associated with the other big ideas are evident in these educational materials—at least fifty percent of the claims of these three big ideas are found in textbooks. Notably, it is discerned that the GA big idea exhibits the highest degree of inclusivity, with four out of the total six claims being represented within the textbooks under scrutiny.

Details of Algebraic Thinking Claims Found in Textbooks

The claims of five big algebraic thinking in this study's framework are actually expected of elementary school students. Still, they also represent opportunities that should be provided for them to think algebraically. According to the analysis, only fifty percent of the learning opportunities that should be provided for students to think algebraically in primary school mathematics textbooks are present. It is about the variety of opportunities rather than the number of tasks. That means that in Turkey, school math textbooks for grades 3-5 only provide half the opportunity for children to think about algebra through the tasks provided in the textbook. In other words, given the range of textbook tasks, students in Turkey have a 50% opportunity to benefit from algebraic thinking. Blanton and Kaput (2005) asserted that the existence of a diversity of types of algebraic thinking, along with their frequency, contributed to students' ability to develop algebraic thinking skills. The current study showed that textbooks have provided various algebraic thinking opportunities in mathematical tasks (50%). However, the diversity and amount of opportunities could be improved to create better student learning opportunities.

Floden (2002) argues that a learning opportunity is necessary for the starting point of the learning process. He also asserted that investigating additional factors contributing to kids' poor academic performance would result in minimal advantages, given the identification of insufficient or absent learning opportunities. Similarly, Henningsen and Stein (1997) emphasized that the textbook tasks have the potential to enhance students' thinking and limit their opinions on the subject. The TIMSS scores indicate that pupils in Turkey exhibit low academic performance, particularly in algebra (Mullis et al., 2000, 2004, 2008, 2012, 2016, 2020). In the present case, the potential cause of pupils' lack of success in algebra can be attributed to the limited exposure of learners to the processes inherent in algebraic thinking at an early age. In other words, the fact that tasks offer fewer opportunities to develop students' algebraic thinking contributes less to the growth of their algebraic proficiency.

The research findings show that the primary school mathematics textbooks in Turkey provide the opportunity to learn in the fields of equivalence, inequalities, and equations at most 39.0%. In particular, the textbooks provide students with the opportunity to solve open number sentences, that is, number sentence problems with the unknown, which is represented as an object or space. These tasks may lead students to evaluate their assumptions regarding the equal sign (Carpenter et al., 2003). There are also tasks for students to analyze the variety of equation types. Thus, students are given opportunities for a relational understanding of the equal sign. Thanks to these tasks, students have opportunities to understand the equivalence in an equation. However, the number of tasks related to this claim is a few. Yavuzköy Köse and Tanışlı (2011) examined mathematics textbooks from 1st to 5th grades from four different publishers and stated that textbooks were insufficient to provide a relational understanding of equal sign. Similarly, Powell (2012) examined eight different primary school mathematics textbooks common in the United States and found that no questions supported this meaning except one textbook. As a result of this study, it can be said that some tasks provide an opportunity for relational understanding, but they are insufficient numbers.

When looking at the curriculum, the most repeated claims in textbooks, as mentioned above, are the objectives of these three class levels. Since these claims were included in the curriculum, their presence in textbooks was not unexpected. On the other hand, the sixth grade is the first level in the curriculum where algebra and variables are mentioned, and the first formal equation-solving activity also appears in the seventh grade.

Secondly, 34.0% of the ideas are functional thinking, especially pattern recognition, functional relationships, and graphing. Within the framework of functional thinking, these textbooks provide students with opportunities to complete sequences of consecutive numbers, analyze geometric patterns, investigate recursive relationships, and determine values with near-term predictability. Uygur Kabael and Tanışlı (2010) emphasized the significance of this finding by stating that functional thinking should begin with a pattern in early algebra. Then, students should be permitted to comprehend the functional relationship through various representations. However, they reported that no exercises in Turkish primary school mathematics serve as a foundation for the concept of function, such as creating a function table. Findings are parallel with this statement because each of these textbooks in the study lacks an exploration of covariational relations, the establishment of functional relations, and the construction of graphs. It is essential to establish various functional thinking relationships with the tasks (Syawahid et al., 2020) because they observed that functions could be interpreted differently according to the viewpoints of primary school pupils. For instance, one student may observe a recursive pattern, whereas another may notice a covariation relationship. Moreover, each primary school student has the capacity to recognize these various connections of functions (Blanton et al., 2011). At this juncture, it is essential to provide opportunities with textbooks that encourage each student to recognize the various relationships of functions to improve their functional and algebraic thinking abilities in Turkey.

Moreover, Cai (2004) noted that although functions were introduced in the sixth grade in China, numerous opportunities for developing function sense were provided in elementary school mathematics textbooks. The textbooks examined within the scope of this current study need more task diversity. In fact, there are tasks founded on functional thinking, but, as stated previously, the tasks focus on students' understanding of the recursive pattern. That may be due to the absence of objectives for other functional thinking claims in the curriculum. As it is not included in the curriculum, it may not be included in the textbooks. That is why it is necessary to include more diverse tasks that emphasize the development of functional thinking skills in the curriculum and textbooks for younger students (Türkmen & Tanışlı, 2019).

In addition, only fifth-grade textbooks contained a limited amount of tasks about reversibility. These tasks are designed to allow students to identify the independent variable in a case given the dependent variable. In other words, while the number of steps is given in a pattern and the number of objects in that step is requested, the appropriate step is requested by giving the number of objects in fifth-grade tasks. While almost all of the tasks in textbooks focus on determining the dependent variable when the independent variable is given, it is notable that the fifth-grade textbook includes tasks for the inverse case. Thus, students are provided opportunities to develop their functional reversible thinking.

19.0% of algebra-related tasks are associated with the idea of generalized arithmetic, a field concerned with the relationships between arithmetic operations and their corresponding generalizations. Textbooks provide opportunities for students to discover the commutative and associative properties of addition and multiplication laws. While worked examples aim to discover these properties, to-be-solved questions allow students to find situations that will provide these properties. Also, some tasks aim to express these properties in words, but these tasks are all worked examples, and there is no question asking students to define properties in to-be-solved questions. In other words, it can be said that these tasks are aimed at giving information rather than giving students the opportunity to develop their ability to express property.

The emphasis on tasks related to addition's commutative and associative qualities is more frequent in the third grade than in other grades. In contrast, tasks pertaining to multiplication's commutative and associative properties are commonly seen in the fourth grade. In fifth grade, the learning opportunity provided for this idea could be more extensive. Also, in the fifth grade, we encountered tasks on the distribution property of multiplication over addition under the topic of parentheses in mathematics. However, these questions focus more on parenthetical order than on defining the distributive law. The first comprehension of the distributive law is introduced in the sixth grade in the math curriculum. In summary, students have opportunities to recognize, evaluate, and apply basic generalized arithmetic expressions, such as the commutative properties of addition and multiplication. Still, they were not given the opportunity to generalize these properties using variables, to define and evaluate this generalization.

8.0% of the algebra-related tasks relate to the idea of variables. The idea of the variable is included in other claims since it reflects a more natural way of teaching. Blanton et al. (2015) noted that they incorporated the variable idea throughout the other big ideas in the framework. The low percentage of the variable idea may be associated with the fact that this idea can be seen among the claims of other ideas. However, in the textbooks examined, the claims of other ideas containing the idea variable are absent from the examined textbooks. The reason for the low rate in this study is that coding is not included in the tasks that do not have a central claim. For example, a box is used instead of a variable in questions with open sentences, but in some cases, an asterisk or circle is also used. These tasks are not coded as one of the claims of the variable idea since their main purpose is the solution of equations. What is clear, however, is that the tasks were not designed to discover the meanings of the variables.

Only two of the variable idea's claims were identified as the primary claims. The claims are that students use variables to represent the unknown and comprehend that the variable represents a quantity as opposed to the object it represents. These claims are frequently observed in open-number sentence writing tasks. In other words, students were given the opportunity to use variable instead of unknown while creating open number sentences. In addition, opportunities are observed for students to understand that a variable represents some amount or quantity.

Details of Algebraic Thinking Claims Not Found in Textbooks

While 50% of the expected claims were discovered, the remaining 50% were not found. These claims are commonly summarized in the following manner. Designing tasks for these absent claims of algebraic thinking is most significant. The absence of opportunities to comprehend algebraic statements inside the framework of an issue and to utilize variable expressions for representing linear problem situations is apparent. A limited number of opportunities are available for pupils to analyze information to identify arithmetic relationships and ascertain values that would make generalizations accurate. Opportunities to explore arithmetic relationships, explain the properties of generalization, and express generalization are provided with worked examples. At this point, tasks should be designed with to-be-solved questions, in which students will make an arithmetic generalization in their own words and explain the properties of this generalization. Also, opportunities to interpret generalizations should be provided. Furthermore, textbooks do not provide learning opportunities for students to identify covariational relationships, formulate and employ function rules, and generate graphical representations. There are no opportunities for students to use variables in a generalization, linear equation, or function. In addition, there is no opportunity for pupils to investigate and comprehend the variable's meanings. Cai (2004) stated that although variable is not defined in Chinese primary school mathematics textbooks, there are messages for three distinct meanings of variable: variable as placeholder for unknowns, variable as pattern generalization, and variable as representation of relationships. Blanton et al. (2011) support this idea by stating that the role of a variable can vary according to different situations. However, only the first variable meaning is observed in elementary mathematics textbooks in Turkey. This means that textbooks offer very few learning opportunities for the variable idea. Variable term appears for the first time in Turkey's sixth-grade curriculum. Consequently, different meanings might appear in higher grades.

Additional Information

In general, firstly, the tasks presented in elementary school textbooks typically focus on the process of finding the unknown based on the known (Blanton, 2018). There needs to be more tasks that begin with an unknown variable in textbooks. Most tasks in this format do not aim to comprehend the unknown and operate on the unknown in conjunction with its justifications. Instead, the intent is to ensure that the rules are followed, as evidenced by the answered questions.

Second, there is an insufficient amount of instruction in the tasks in terms of asking questions. The absence of questions such as analyzing, observing, capturing, and expressing observations prevents students from analyzing a situation and recognizing and describing relationships. Blanton and Kaput (2011) defined the fundamental practices of algebraic thinking as generalizing, representing, justifying, and reasoning with mathematical structure and relationships. In these textbooks, students focus only on finding the answer to the question. Good textbooks should not just provide students and teachers with mathematical knowledge but also opportunities for them to interact with it and make sense of it (Erbaş et al., 2012). The tasks should, therefore, be supplemented with various practical questions for algebraic thinking. By posing multiple questions step by step within a single task, students will have more opportunities to discover and learn. Students can improve their mathematical literacy with the variety offered by the tasks (Hwang & Ham, 2021). Guo and Liao (2022) emphasized the importance of textbooks providing a balance between various tasks since the nature of the tasks affects mathematics success and related students' cognitive skills. Worked examples can be explicitly designed for algebraic thinking and can serve as an example for students by being solved with reasons. Moreover, worked examples in textbooks are frequently answered in more than one way, allowing students to recognize alternative approaches. In addition, the solutions to the problems in the textbooks are typically validated using inverse operations. For instance, division, which is the opposite of multiplication, determined the correct response to a question about multiplication but not with reasons.

Thirdly, a potential correlation was noticed between the claims presented in the textbooks and the curriculum. The two claims that appear most frequently in textbooks are solving open number syntheses problems, identifying a recursive relationship, and continuing the pattern. Examining the curriculum reveals that these claims are included in the primary education curriculum. In addition, the curriculum includes claims for examining the equality situation and analyzing the sum displacement properties.

5.1 The Implications and Limitations of the Study

The research findings significantly affect several stakeholders, including curriculum developers, textbook authors, educators, and researchers.

OTL has become an increasingly prominent research topic, and most studies have demonstrated a positive relationship between OTL and student achievement (Floden, 2002). That means the opportunity to learn any subject presented to a student positively affects that student's success. TIMSS, with significant international outputs, revealed that Turkey consistently performed poorly in mathematics, particularly algebra. Examining the mathematics textbooks aimed to determine whether the cause for this low performance was a lack of learning opportunities for algebra. According to the findings, the amount of algebra-related tasks is insufficient, and half of the expected claims could not be found in textbooks. This suggests that primary school mathematics textbooks do not provide sufficient opportunities for gaining algebraic thinking, which is the cause of the low algebra success score. Because according to Floden (2002), if students are not afforded the opportunity to learn, the subject has never been taught to them.

Furthermore, textbooks are crucial in elementary school because they are the foundational stage for acquiring mathematical concepts and abilities (Tutak & Güder, 2012). Consequently, the tasks in primary school mathematics textbooks are essential in developing students' algebraic thinking skills and making sense of big ideas of algebra by providing students with learning opportunities at an early age. This current study allows textbook authors to comprehend the importance of tasks that develop algebraic thinking in primary education since they prepare students for formal algebra. In this way, their awareness of algebraic thinking can be enhanced. Also, the current study recommended textbook authors should design algebra-related tasks for elementary textbooks even though the curriculum does not mention them directly. The present study's conclusions offer an examination of the tasks, focusing on the standpoint of algebraic thinking. This enables authors of textbooks to recognize algebra-related opportunities.

The framework employed in this study defines the anticipated learning opportunities aimed at building algebraic thinking within 3-5 grade textbooks. The study's findings clarify the existence or absence of specific learning opportunities throughout the textbooks. This approach aids textbook authors in determining the specific areas of focus and development required to enhance the presence of algebraic thinking opportunities in their textbooks. The concept of algebraic thinking has the potential to be incorporated into various mathematics topics. Hence, it is recommended that authors of educational textbooks prioritize considering diverse, frequent, and well-distributed learning opportunities for the development of algebraic thinking. This should be considered when constructing tasks to facilitate the acquisition of algebraic thinking skills.

The utilization of textbook research can potentially promote the relationship between the development of curricula and the field of educational research (Clements, 2007). The results of the current study show that learning opportunities most provided in textbooks are seen as an objective in the curriculum. This means that primary school mathematics textbooks in Turkey are prepared according to the mathematics curriculum in the country and are complied with. According to Johansson (2006), the concept of curriculum can be seen as a sequence of learning opportunities. To increase the provision of learning opportunities for the development of algebraic thinking in textbooks, it is necessary to make adjustments in the curriculum. The curriculum is seen as a skeleton outlining the many learning opportunities designed and scheduled (Houang & Schmidt, 2008). At this point, the results of the present study can inspire curriculum developers because they realize that curriculum must be guided if textbooks are designed to encourage early algebra education. Thus, it can encourage curriculum developers to re-examine the primary mathematics curriculum. This reexamination is not about establishing rigorous pre-algebra education but rather about defining algebraic thinking skills in the curriculum and providing all students with the opportunity to acquire them. The inherent structure of algebra enables its seamless integration into various other disciplines within the field of mathematics. Hence, as this research delineates, it is essential to include the learning opportunities sought in textbooks in the curriculum. It is especially important to prioritize algebraic thinking opportunities for learning that cannot be found in textbooks through the current study. They can refresh textbooks to guide them to provide a variety of opportunities for algebraic thinking.

Textbooks guide teachers on how mathematics should be explained regarding concepts, features, and examples related to the subject (Johansson, 2006). Drawing from the observations made in this study, it is evident that elementary school textbooks fall short of providing adequate opportunities for fostering algebraic thinking. Despite this deficiency, the potential for creating such learning opportunities within the classroom exists, provided there are well-trained and equipped elementary school teachers. The enhancement of elementary school teachers' knowledge and skills in algebraic thinking can ensure the future algebraic success of students (Welder and Simonsen, 2011). Beneficial advances in improving teachers' abilities encourage students to have positive attitudes toward mathematics and decrease anxiety (Alkaş Ulusoy et al., 2017). However, it is worth noting that many elementary school teachers lack the necessary algebraic thinking proficiency. This thesis underscores the significance of early-age algebraic thinking and its importance in supporting students through tasks. Furthermore, it aims to show teachers to recognize both present and absent learning opportunities within textbooks. This heightened teachers' awareness has the potential to redesign tasks.

With this newfound awareness, educators can develop textbook tasks to actively promote algebraic thinking opportunities in a classroom. For example, it is crucial for students in grades 3-5 to justify the generalizations they find with their reasons, as this fosters introducing mathematical proof (Carpenter et al., 2000). Consequently, it becomes imperative for elementary teachers to cultivate their own understanding of algebra and learn how to create enriching algebraic learning experiences for their students (Blanton & Kaput, 2003). Remarkably, the focus should be facilitating students' abilities to generalize mathematical concepts and articulate and substantiate these generalizations. This may involve introducing supplementary tasks that encourage students to extend their thought processes, such as mathematically proving the comprehensiveness of a set of solutions or exploring various avenues to generalize results (Sullivan et al., 2006). Thanks to awareness and suitable approaches, teachers can create learning opportunities in the classroom.

Within educational research, the pivotal concept of Opportunity to Learn (OTL) takes center stage, particularly in learning opportunities in mathematics. While schooling aims to convey society's deemed essential knowledge and skills, the exposure to specific content, skills, and analytical procedures encapsulated within OTL primarily dictates the efficacy of learning (Schmidt & Maier, 2009). In essence, providing learning opportunities to students is the primary objective of schooling. Consequently, the scrutiny of educational opportunity retains its paramount significance in academic research and public policy. This perspective contends this study has the potential to benefit academics by revealing learning opportunities related to algebraic thinking inside primary school mathematics textbooks. Thus, interest in OTL research within the country can increase, and the opportunity to learn algebraic thinking can be encouraged to be included in the policy discourse. Moreover, the results of this study show the international world whether primary school mathematics textbooks in Turkey offer learning opportunities to develop algebraic thinking and in which subjects. OTL allows it to compare the countries' curricula and textbooks and determine the most suitable conditions (Floden, 2002). Thus, the current study results help when comparing Turkey and other countries regarding learning opportunities. They can incorporate the findings of this study into their study or shape their study by the findings. For example, this study encourages a comparison of the algebraic thinking opportunities provided in their textbook between a country with high TIMSS scores and Turkey. Thanks to this comparison, the prosperous country's textbook can be taken as an example, and the learning opportunities offered in the textbooks can be improved.

This study examined algebraic thinking learning opportunities by examining textbooks, but additional factors create learning opportunities. Teachers are one of the most essential factors in the classroom environment for creating learning opportunities. At this point, future research can examine whether teachers provide learning opportunities not found in textbooks in the classroom setting. In addition, it may be a limitation of this study to examine only the textbooks of MEB publications, as there are other reputable publishing houses in Turkey. Future research may also investigate the textbooks of other publishing houses for educational purposes. In addition, this study is limited to examining 3-5 grade mathematics textbooks. At this point, future studies can expand the grade range and examine the first and second-grade textbooks because it is noticed that open sentence questions and geometric patterns tasks were included with the first grade. Especially in the 2nd grade, it is noticed that there are tasks for the conceptual understanding of open sentence questions and the meaning of the equal sign. That means some learning opportunities may have become available at a younger age. Future studies can examine these and analyze how learning opportunities develop through grade levels. Also, this study is limited to mathematics textbooks in Turkey, but this study can be used to examine other countries' textbooks because the methodology of this current study could be used to examine textbooks from other countries. Examining countries' textbooks for learning opportunities provided in the textbooks is essential to understanding opportunities for kids at early algebra to develop algebraic thinking. Thus, the tasks in the textbook can be designed to support algebraic thinking, and students can be provided with the opportunity to learn. Providing OTL, a prerequisite for learning, students can develop algebraic concepts and cognitive thinking abilities. Follow-up will increase their success in algebra and mathematics as students gain a deeper understanding of key concepts and skills.

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