MODELING EXCHANGE RATE VOLATILITY USING ARMA-GARCH APROACH WITH NON-GAUSSIAN DISTRIBUTION

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF APPLIED MATHEMATICS
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

YEŞİM GİRGİN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
FINANCIAL MATHEMATICS

SEPTEMBER 2023

Approval of the thesis:

MODELING EXCHANGE RATE VOLATILITY USING ARMA-GARCH APROACH WITH NON-GAUSSIAN DISTRIBUTION

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ABSTRACT

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September 2023, 84 pages

Modeling exchange rate volatility is a major concern for researchers, investors, and policymakers since it has a wide-ranging impact on the country's economy, including inflation, interest, investment, production, and foreign commerce [46]. Therefore, the primary goal of this research is to model the volatility of the exchange rate. For this purpose, the generalized autoregressive conditional heteroscedastic techniques comprising of symmetrical (GARCH) and asymmetrical (EGARCH, TGARCH, and APARCH) models are used in this study. Furthermore, aside from the studies conducted in the Turkish literature on that matter regarding models' distribution, various distributions which consist of skew normal, skew student t, and skew GED along with normal, student t, GED distributions are utilized for the error distribution in GARCH models.The data is taken from CBRT's closing prices in US dollars consisting of the period of June 2001 to June 2023, and it is divided into 4 sub-periods according to Chow Test results. The sub-periods as follows: from June 2001 to July 2013 (Period 1) , from July 2013 to October 2016 (Period 2), from October 2016 to February 2020 (Period 3), and from February 2020 to June 2023 (Period 4).

Convenient models for these periods are put forward based on model selection criteria such as Akaike (AIC), Schwarz (SC), and Log-Likelihood. In the end of the study, the results concluded as follows: ARMA(4,3)-EGARCH(1,1) with skew t for the entire data, ARMA(1,0)-TGARCH (1,1) with skew t distribution for Period 1, EGARCH (1,1) with skew t distribution for Period 2, ARMA(0,1)-GARCH (1,1) with t distribution and ARMA(0,1)-TGARCH (1,1) with skew GED distribution for Period 3, and ARMA(1,0)-EGARCH (1,1) with t distribution for Period 4 are the best-fitting model among the proposed models according to the selection criteria.

Keywords: Volatility, EGARCH, TGARCH, APARCH, Skew Distributions

ÖZ

GAUSSIAN OLMAYAN DAĞILIMLI ARMA-GARCH YAKLAŞIMI İLE DÖVİZ KURU OYNAKLIĞININ MODELLENMESİ

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Eylül 2023, 84 sayfa

Ülke ekonomisini enflasyon, faiz, yatırım, üretim, dış ticaret gibi birçok yolla etkilediği yadsınamaz olduğundan, döviz kuru oynaklığının modellenmesi araştırmacı, yatırımcı ve politika yapıcıların temel kaygısıdır [46]. Dolayısıyla, bu çalışmanın temel amacı döviz kuru oynaklığının modellenmesidir. Bunu sağlamak için bu çalışmada simetrik (GARCH) ve asimetrik (EGARCH, TGARCH ve APARCH) modellerden oluşan genelleştirilmiş otoregresif koşullu değişen varyans yaklaşımları kullanılmıştır. Ayrıca bu çalışmada, Türk literatüründe döviz kurlarının dağılımları konusunda yapılan çalışmalardan farklı olarak, GARCH modellerindeki hata dağılımları için çarpık normal, çarpık t ve çarpık GED, normal, t, GED dağılımları kullanılmıştır. Veriler, Haziran 2001-Haziran 2023 dönemini kapsayan TCMB ABD doları cinsinden kapanış fiyatlarından alınmakta olup, Chow Testi sonuçlarına göre 4 alt döneme ayrılmıştır. Alt dönemler şu şekildedir: Haziran 2001 - Temmuz 2013 (Dönem 1), Temmuz 2013 - Ekim 2016 (Dönem 2), Ekim 2016 - Şubat 2020 (Dönem 3) ve Şubat 2020 - Haziran 2023 (Dönem 4).

Bu çalışmada, Akaike (AIC), Schwarz (SC), Log-Likelihood gibi model seçim kriterleri esas alınarak bu dönemlere uygun modeller ortaya konulmaktadır. Çalışma sonunda şu sonuçlara ulaşılmıştır: Tüm veri seti için çarpık t dağılımlı ARMA(4,3)-EGARCH(1,1), Dönem 1 için çarpık t dağılımlı ARMA(1,0)-TGARCH (1,1), Dönem

2 için t dağılımlı ARMA(0,0)-GARCH (1,1) ve t dağılımlı ARMA(0,0)-TGARCH (1,1), Dönem 3 için t dağılımlı ARMA(0,1)-GARCH (1,1) ve çarpık GED dağılımlı ARMA(0,1)-TGARCH (1,1) ve Dönem 4 için t dağılımlı ARMA(1,0)-EGARCH (1,1) seçim kriterlerine göre önerilen modeller arasında en uygun modellerdir.

Anahtar Kelimeler: Oynaklık, EGARCH, TGARCH, APARCH, Çarpık Dağılımlar

To My Family,
Zahir, Emine, Dicle and Ardacan Girgin

ACKNOWLEDGMENTS

First and foremost, I would like to express my sincere gratitude to my supervisor, Prof. Dr. A. Sevtap Kestel, who provided the groundwork for my statistical knowledge. Attending her lectures was the starting point for the journey of this study, and her lectures were a great opportunity for me.

Many special thanks to my co-supervisor, Assoc. Prof. Özlem Türker Bayrak. She is the first person to introduce this joyful research field. I am thrilled to be able to work with her. Her patience, kindness, beneficial comments, excellent guidance, and vital support enabled my thesis to be completed.

I cannot express how grateful I am to my dear family, Zahir, Emine, Dicle, and Ardacan for being my biggest support and blessing in this life. I will be eternally thankful to them for their unconditional love and care.

During my research, my dearest companions, Fadime, Firdevs, Merve, and Tuğba, became my most powerful sources of motivation. They never left me alone when things got tough. They supported me wholeheartedly, waited for me always, and loved me sincerely. I am very lucky to have all of them.

I would also like to express my gratitude to the Banking Regulation and Supervision Agency (BDDK), of which I'm a part. My dear managers and coworkers have always supported me on this journey. I consider myself very thankful to have had the opportunity to work with them.

I am also thankful for the financial support provided by the Scientific and Technological Research Council of Türkiye (TÜBİTAK) during my research.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

ACF	Autocorrealtion Function
ADF	Augmented Dickey Fuller
AIC	Akaike Information Criteria
APARCH	Asymmetric Power ARCH
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criteria
CBRT	Central Bank Of The Republic Of Turkey
EGARCH	Exponential GARCH
EUR	Euro
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GBP	British pound Sterling
GED	Generalized Error Distribution
KPSS	Kwiatkowski-Phillips-Schmidt-Shin
LM	Lagrange Multiplier
MLE	Maximum Likelihood Estimation
OECD	Organization for Economic Co-operation and Development
PACF	Partial Autocorrealtion Function
PDF	Probability Density Function
PROP	Probability
SC	Schwarz
TGARCH	Treshold GARCH
TL	Turkish Lira
USD	ABD Dolar

CHAPTER 1

INTRODUCTION

It is well known to say that financial market behavior is almost uncertain. It has ups and downs, whose fluctuations can be from the market's state. Tranquil and volatile states are usually referred to as explaining the movements of a financial asset's return, and the existence of these states leads financial institutions and individual investors to deal with risk issues, which is another research area. Financial risk management has gained more importance in recent decades. Additionally, it is well known that the value of a financial asset is determined with respect to its level of risk.

Consequently, financial institutions, investors as individuals, academia, etc. are all interested in the analysis of financial time series that addresses the theory and practice of asset valuation throughout time. Tsay [51] mentions that although it is an extremely empirical discipline, theory serves as the groundwork for inference-making in other scientific disciplines as well. Thus, it is allowed to state that statistical theory and procedures constitute the analysis's core.

As previously stated, the rate of return on a financial asset cannot be predicted precisely. However, its volatility may be estimated, which is a significant factor in the financial field. It is known that basic assumptions are made in the classical analysis of the financial time series, namely, normality and linearity of the log-returns of the financial time series. However, Arlt [7] remarks that log return distributions are often skewed and more peaked than normal distributions, and linear models are unable to adequately capture these time series' distinct characteristics. As a result of this, so much research has been done on these characteristics of the financial series, more specifically, on the exchange rate series.

The bond market, stock market, commodity market, and exchange market are the four segments that constitute the financial market. In fact, among all financial markets, it can be said that the foreign exchange market has the largest volume of transactions.

Tokgoz [50] defines the exchange market as a financial market where all foreign currencies can be sold and bought except the local currency, i.e., it is a market where the foreign currency can be changed with the local currency. Hence, one can realize its importance in terms of future economic activities.

Exchange rates are regulated by supply and demand, and its impact on the country's economy is apparent in many ways, including inflation, interest, investment, production, and foreign trade [46]. Moreover, Ozdemir [27] points out the fact that keeping the currency rate stable can lead to economic stability for the country. Thus, one of the economy's most important indicators is the currency rate.

Different countries applied different exchange rate regimes over the years. The Bretton Woods Agreement, which was established in July 1944, fixed the currency rates among the major industrial nations [32]. This system is called fixed Peg (also known as "hard peg" occasionally). However, in the 1970s, it was collapsed, and the exchange rate system was replaced by a more flexible system. This system is called a flexible exchange rate. In today's world, the forces of the market, taking into account supply and demand, set the pricing. Moreover, other exchange rate regimes go between those two systems in accordance with their own independence levels. From the least flexibility to the most, the exchange rate regimes are briefly stated as follows:

(i) Monetary union which represents the use of a common currency in an area such as the Eurozone.

(ii) It has no separate legal tender, i.e., in this regime, another country's currency is used.

(iii) The currency board is a formal agreement between more than one currency on a fixed exchange rate.

(iv) The target zone arrangement which implies the exchange rate is permitted to vary within specific ranges.

(v) The crawling Peg signifying that the currency rate is updated on a regular basis.

(vi) Managed (dirty) float denotes a flexible exchange rate system with a certain amount of government interference.

(vii) Free (clean) float implying that the exchange rate is regulated by the market.

An important feature of the exchange rate is its volatility, which indicates a security's variation around its mean or average return over a time [53], i.e it demonstrates how widely values of the security vary from the average its value, and it is estimated from the standard deviations. Besides, It is know as a significant measure of the risk of investment in finance since the magnitude of volatility affects the possibility of a gain or loss's magnitude in the short term [36].Thus, many researchers seek to construct an appropriate, well-built, and plausible model for it.

The effects of fluctuations in exchange rates can be seen across a wide range of sectors, including international trade, global money flows, production, and investment [56]. For instance, due to the depreciation (appreciation) of the currency brought on by either domestic or international shocks, undeveloped and developing nations are more exposed to periods of significant exchange rate volatility [4].

Conversely, some factors have an impact on exchange rate volatility, such as capital flows, inflation, and interest rates across nations, according to Saglam and Basar [46]. Abdallah [3], however, states that there is no agreement in the literature as to what causes exchange rate volatility, but macroeconomic factors may generally account for a major portion of it.

1.1 Stylized Facts about Exchange Rate Volatility

Foreign exchange rates are shaped by nonlinear and non-stationary behavior. Some of the regularities for exchange rate volatility which can be regarded as stylized facts because of the significant amount of empirical evidence are as follows:

(i) It is important to note that the standardized fourth moment of the normal distribution is three, but the kurtosis of the distribution of exchange rate return is generally bigger than three, meaning that it has fatter tails. This feature also means excess kurtosis.

(ii) Taking into account a financial time series, heteroscedastic behavior often shows itself in volatility clustering. One can observe that there are some periods in the series whose variance is high, whereas some others have low volatility. Regarding this issue, Mandelbrot [42] mentioned that "large changes tend to be followed by large changes-of either sign-and small changes tend to be followed by small changes". Hsieh[38] also finds that the mean and variance in an empirical experiment on daily foreign exchange rates vary over time. Therefore, it is crucial to understand that the variance in financial data could vary rather than remains constant.

(iii) Observing high persistence, also known as long memory, which is the long-lasting effect of shocks on volatility, is a common phenomenon in the financial time series. Therefore, it is crucial for policymakers, in the event of shocks, to take precise action in accordance with their level of persistence [13].

(iv) According to many studies, such as Black's [15] study, volatility and price changes have a negative correlation. It means that, when compared to positive shocks of the same magnitude, volatility is greater after negative shocks. This was also concluded in the study of evidence of leverage effects and volatility spillover among exchange rates of selected emerging and growth-leading economies by Panda et al [40].

(v) Abdallah [3] remarks that large fluctuations in one currency are sometimes matched by large changes in another when comparing exchange rate returns for various currencies. Therefore, multivariate models in different markets are suggested when modeling cross-correlations for this regularity.

(vi) Weekends and holidays can affect volatility. The reason is that the information that is gathered on non-trading days accumulates and has an impact on volatility on trading days.

1.2 Objective of The Thesis

This thesis focuses on modeling the USD/TL exchange rate because of its importance emphasized in the previous sections. It is seen that different GARCH models are applied in various periods when similar studies are examined [45, 20, 48, 27, 46, 34, 56, 57]. In addition, it has been observed that the Normal and t distributions are utilized as distributions in the literature, and the application of skewed distributions is recent. Besides, there has not been a study that compares all models under different exchange rate distributions. Therefore, in this thesis, GARCH, EGARCH, TGARCH, and APARCH models are utilized under normal, skewed normal, t, skewed t, GED, and skewed GED, and it is aimed to find the best model according to AIC, BIC, and log-likelihood criteria. In line with the results obtained, this study is expected to contribute to understanding the volatility structure of the exchange rate by analyzing it accurately. Moreover, it may help forecast better the volatility.

The study is organized as follows: Chapter 2 summarizes the idea of the GARCH models as well as the literature regarding the studies and applications of GARCH models and their performance on the matter of modeling exchange rate volatility. Subsequently, Chapter 3 presents preliminary informations, proposed GARCH models, the distributions, and their features. Chapter 4 examines the USD/TRY exchange rate used in the study. Besides, it compares the models and provides final remarks and outcomes. Finally, Chapter 5 finalizes the thesis with concluding comments.

CHAPTER 2

LITERATURE REVIEW

For modeling the financial time series, firstly Engle [28] suggests the autoregressive conditional heteroscedasticity (ARCH) model, introducing it as a mean-zero, serially uncorrelated process with non-constant variance conditional on the past but constant unconditional variance. This model also displays persistence of the volatility shocks over time through its autoregressive part, so providing the variance as a function of past error makes it a desirable model. Although this idea has found many applications, ARCH has the drawback of having so many parameters and a long lag structure that leads to violations of the non-negative coefficients.

Afterwards, to get rid of these drawbacks, Bollerslev [16] introduces the generalized autoregressive conditional heteroscedasticity (GARCH) model offering a considerably more flexible lag structure with much fewer parameters than the ARCH model. Furthermore, these models have been developed, employed, and utilized in a wide range of fields, including the modeling and forecasting of exchange rate volatility [43, 41, 47, 22, 3].

Weiss [55] introduces a heteroscedastic model which can be reduced to the ARCH model and gives the asymptotic properties of the estimates as well as the sufficient conditions for them to hold.

Additionally, Bollerslev et al. [17, 18], Bera and Higgins [14], and others focus on a wide range of proposed ARCH processes in their surveys. For instance, Engle and Bollerslev proposed integrated variance, often known as IGARCH, in 1986 [11]. The Exponential GARCH (EGARCH) model, proposed by Nelson [44], aims to reflect the asymmetric effects. The Asymmetric Power ARCH (APARCH) model, which was developed by Ding et al. [25], is used for determining asymmetric adjustment, and lastly, Glosten et al. [33] introduce the GJR-ARCH model.

Regarding the parameter estimation of the models, Engle [28] suggests a two-step procedure, and it is generally seen as Maximum Likelihood Estimation (MLE) assuming a conditional normal distribution.

Furthermore, Fiorentini et al. [31] analyze the fact that maximum likelihood in GARCH estimation typically depends on a numerical approximation to the log-likelihood derivatives because a precise analytic differentiation is just time-consuming. Then they show that this is not the case in their study. On the other hand, when dealing with the analytical solution of maximization of the likelihood function, it is clear that the Berndt–Hall–Hall–Hausman (BHHH) algorithm has some advantages with regard to other numerical optimization techniques [8]. Besides this, some methods are proposed. For example, Hall and Yao [35] develop percentile-t, subsample bootstrap approximations to estimator distributions.

2.1 Studies on Exchange Rate Modeling

Normal (Gaussian) distribution is considered for the ARCH model's disturbance part in the beginning. However, various distributions are observed through empirical experiments since financial time series are often found to be very heavy-tailed and typically leptokurtic [16], [18]. In addition, there might be a case of having no information on error distribution as well. Thus, semiparametric ARCH-GARCH models are proposed for this problem [29].

Besides modifications to the models, Milhøj [43] remarks that when modeling variance in the daily exchange rate, using a Gaussian conditional distribution is not severely contradicted by evidence. Moreover, in the study carried out by Bollerslev and Wooldridge in 1992 [19], where a conditional Gaussian function was used on a quasi-likelihood estimator, even when the true distribution is fat-tailed, the estimation of the GARCH model with a normal distribution is viewed as consistent.

On the contrary, Bollerslev [16] notes that taking account of the true distribution in GARCH models could lead to results that are more efficient. Moreover, to cope with this issue, he utilizes the student's t-distribution for the foreign exchange rates. After that, Nelson [44] used the General Error Distribution (GED) with the EGARCH model. On top of that, Hsieh [39] modeling heteroscedasticity in daily foreign exchange rates gives estimates for ARCH and GARCH models in five foreign currencies, and conclude that non-normal densities such as GED, student-t, the normal-lognormal, and normal-Poisson with the EGARCH model fitted very well, as an example, for the Canadian dollar.

Nonetheless, Calzolari et al. [21] argue that the widely used Student's t-distribution and GED are problematic and propose the a-stable distribution, but the second moment of the a-stable distribution does not exist in most cases, so using this distribution in GARCH models will result in problematic interpretation. Moreover, Feng and Shi [30] employ the tempered stable distribution for the GARCH model and maintain that it surpasses both the Gaussian and the commonly used Student's t and GED.

On exchange rate modeling, the asymmetric volatility models (TGARCH, APARCH, GJR-GARCH, and EGARCH) are utilized in the literature such as the study of Longmore et al. [41]. As a result of the study, they remark that shocks to the exchange rate have a long memory, which means they last for a very long time, i.e., the exchange rate shows persistence. This persistence, asymmetry effect, and leverage effect can also be detected in the study of Yoon and Lee [47], who conduct the research on the daily won/dollar exchange rate regarding the asymmetry and volatility of exchange rates. Therefore, they use the TARCH model, which explains the asymmetry of the conditional variance about shocks, and the EGARCH model, which makes the conditional variance sign positive irrespective of the sign of the parameter in its equation, along with the GARCH model.

Jonathan Chipili's [22] work seeks to ascertain what causes the volatility. Thus, he uses the actual and nominal exchange rates of the key trading partners' currencies against the Zambian Kwacha. GARCH (1, 1), TGARCH (1,1), and EGARCH (1,1) are employed. The result concludes that EGARCH model gives the best fit to the data. Moreover, it is noted that different conditional volatility dynamics define the analyzed exchange rates. Therefore, he recommends that exploring the other GARCH specifications is also necessary instead of implementing a standard GARCH model if a large sample of currencies is taken into account.

Moreover, the daily returns of the currency rate series for 19 Arab countries are analyzed in Abdallah and Zakaria's article [3]. As a result of this study, it is found that GARCH models (symmetric and asymmetric) can sufficiently model volatility and capture the leverage effect and volatility clustering.

2.2 Studies on Turkish Exchange Rate Modeling

In Turkey, as in other countries, there have been several research on currency rates and their volatility using GARCH models. Aysoy et al. [9] conduct an empirical study on daily variations in the volatility- and day-of-week-effect on the Turkish foreign exchange market for the period from January 4, 1988, to December 29, 1995, employing the GARCH model. They conclude, aside from the periods of instability in 1988 and the financial crisis in 1994, there is little variation in Turkish daily foreign currency prices. Moreover, they demonstrate that the foreign exchange returns are subject to seasonal impacts, namely day-of-the-week effect, and have an ARCH effect as long as their model is not misspecified.

Agcaer [5] investigates as a whole and separately the impact of CBRT auctions and direct interventions in foreign exchange markets on the level and volatility of exchange rates. Then, he considers the daily data for the period February 2001 to November 2003 and employs the EGARCH model, which also provides information regarding the different effects of foreign exchange buying and selling transactions. As a result of this analysis, a positive effect is generally observed on the level of exchange rates through CBRT's auctions and direct intervention transactions.

Similar to Agcaer [5], Ozturk [45] expresses concern about the decisions and measures taken by CBRT affecting the level and volatility of the exchange rate. She also aims to show whether the Student-t distribution has greater explanatory power than the normal distribution or not. As a result of the empirical study, the leptokurtic property could not be captured by the Student-t distribution more successfully than the normal distribution. Another note from this study is that there is a considerably high leverage effect. In this regard, a huge difference between the GARCH and EGARCH variance equations is observed and when the volume of spot market trade is taken into account, the EGARCH model is found to produce significantly better results than the GARCH model does.

Like in the above articles, symmetric and asymmetric models were employed for modeling the volatility of the exchange rate return of the Organization for Economic Co-operation and Development (OECD) countries to find the best model for it in Caglayan and Dayoglu's article [20] for the period January 1993 to December 2006. On top of that, they aimed to find the best distribution for the data, which is characterized by excess kurtosis and fat tails. The results suggest that asymmetric conditional variance models with Student-t or GED distributions perform better for appropriate models.

Soytas et al. [48] deal with the daily TRY/USD, TRY/EUR, and TRY/GBP series for the period April 2002 to March 2009 in Turkish foreign exchange markets through moving average models, autoregression models, and ARCH models. It is observed that the GJR-GARCH (1, 1) model for the USD and GBP series and the EGARCH (1, 1) model for the EUR series performed better under the RMSE criteria. For the USD, GBP, and EUR series, AR models had the best performances based on the MAE criterion. Lastly, it is found that the volatility forecasting models' rankings were not much impacted by the financial crisis, but their performances tend to converge on the model that performed the worst during the crisis.

Another study concerning this issue belongs to Ozdemir [27], who aims to model the volatility of the exchange rate through the data set obtained by using the CBRT's daily closing prices in US dollars from January 2, 2009, to January 25, 2014. The EGARCH (1, 1), TGARCH (1, 1), and APARCH (1, 1) models are utilized in the study together with the symmetrical models ARCH (1) and GARCH (1, 1). In order to more accurately describe the thick-tailed aspect of the series, each model is also modeled under the Student-t and GED distributions, which are different from the normal distribution since the kurtosis coefficient was larger than 3 in the models under consideration. As the other studies resulted, asymmetric models provided the best results according to AIC, SC, and log-likelihood criteria. Lastly, as a result of empirical study, the TGARCH (1, 1) model under the student-t distribution is found to be the best model.

Saglam et al. [46] focus on the best volatility forecasting models for time series in the US dollars, British pounds, and euro using the ARCH, GARCH, EGARCH, and TARCH approaches. For this study, daily USD, GBP, and EUR observations between January 1, 2010 and November 31, 2015 are obtained from the Central Bank's Electronic Data Delivery System. They note that the exchange rates are shown to be impacted by the bad news in the market since market participants overreact to the bad news than the good news. Additionally, at the end of the empirical study, they concluded that EGARCH (1, 0), TGARCH (1), and ARCH (2) are the best models for the USD, EUR, and GBP, respectively.

Guler [34] carries out additional research on this subject. The indicative foreign currency-selling rate in Turkish Lira, which is equal to 1 US dollar, is collected for the study's exchange rate data. The data covers the dates January 1, 2006, through December 30, 2016, excluding weekends and federal holidays. To assess the exchange rate return volatility structure and to find out whether the volatility is influenced by return shocks and overnight interest rates, the GARCH (2, 1) and TARCH (2, 1) models were employed. Consequently, the TARCH (2, 1) model provided the best performance in terms of modeling the volatility of the exchange rate return according to the Akaike and Schwarz information criteria.

As a recent survey regarding the exchange rate volatility on GARCH Models under skew distribution, the study of Yıldırım et al. [57] can be given. The daily data is taken, and it covers the period of January 2016 to December 2018. ARMA-GARCH and ARMA-GARCH (M) models under t and skew t distributions are used. Results, for modeling the USD/TRY exchange rate, show that ARMA-GARCH models outperform ARMA-GARCH (M). Additionally, ARMA-NAGARCH model with skewed student t distribution is the best-fitted model, and ARMA-EGARCH model with student t distribution is the second best.

In summary, using different distributions in various volatility models for the exchange rate return is seen such as in the studies of Ozturk [45], Caglayan et al. [20], Ozdemir

[27], and lastly Yıldırım et al. [57] which are similar to the purpose of this thesis. However, in Turkish literature on the modeling volatility of the exchange rate return, it is not so common to utilize the models with skew distributions. Hence, in this study, skew normal, skew student t, and skew GED, along with normal, student t, and GED distributions, are used to model the volatility of the exchange rate return.

CHAPTER 3

METHODOLOGY

In this chapter, the modeling strategy and the corresponding methodology that are used in the analysis are discussed briefly. The first step of the modeling is investigating the data anomalies and investigating the stationarity of the series. In the literature, many stationarity tests are suggested. The most commonly used ones are the Augmented Dickey-Fuller (ADF) where the null hypothesis that a time series sample contains a unit root or the series is non-stationary, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) where the null hypothesis that a series is stationary. So, they are employed in Chapter 4. Based on the test results, appropriate transformation is applied to make the series stationary-like differencing. Furthermore, based on the autocorrelogram (ACF) and partial autocorrelogram (PACF) functions, and appropriate ARMA model is fitted for the mean of the data.

The model should be revised until the model residuals become serially uncorrelated which can be tested by the Ljung-Box Q test where the null hypothesis is that a series of residuals shows no autocorrelation for a certain number of lags, and the test statistic is:

$$Q(l) = k(k+2)\sum_{j=1}^{l}\frac{c_j^2}{k-j} \tag{3.1}$$

where l is the time lag, k is the lenght of the series, and c is the accumulated sample autocorrelations. Thus, if the test statistic which chi-square distributed has a significant p-value, the null hypothesis is rejected.

After fitting an appropriate mean model, the residuals are tested for the ARCH effect by the ARCH-LM test, where the null hypothesis is that a series of residuals show no ARCH effect and the test statistic is the standard F statistic used for regression on squared residual. If an ARCH effect is found, then it is modeled by one of the volatility models described through the Section 3.1. The model is revised until the residuals become uncorrelated.

Both in the mean and volatility modeling stages, the best model can be determined via AIC, BIC, and Log-Likelihood Criteria given in Section 3.2. The flow chart of the model fitting process can be found in Figure 3.1.

Besides, since financial time series hardly appear to be normal as demonstrated in many applications, normal, skew normal, t, skew t, GED, and skew GED distributions used in this study are discussed in Section 3.3.

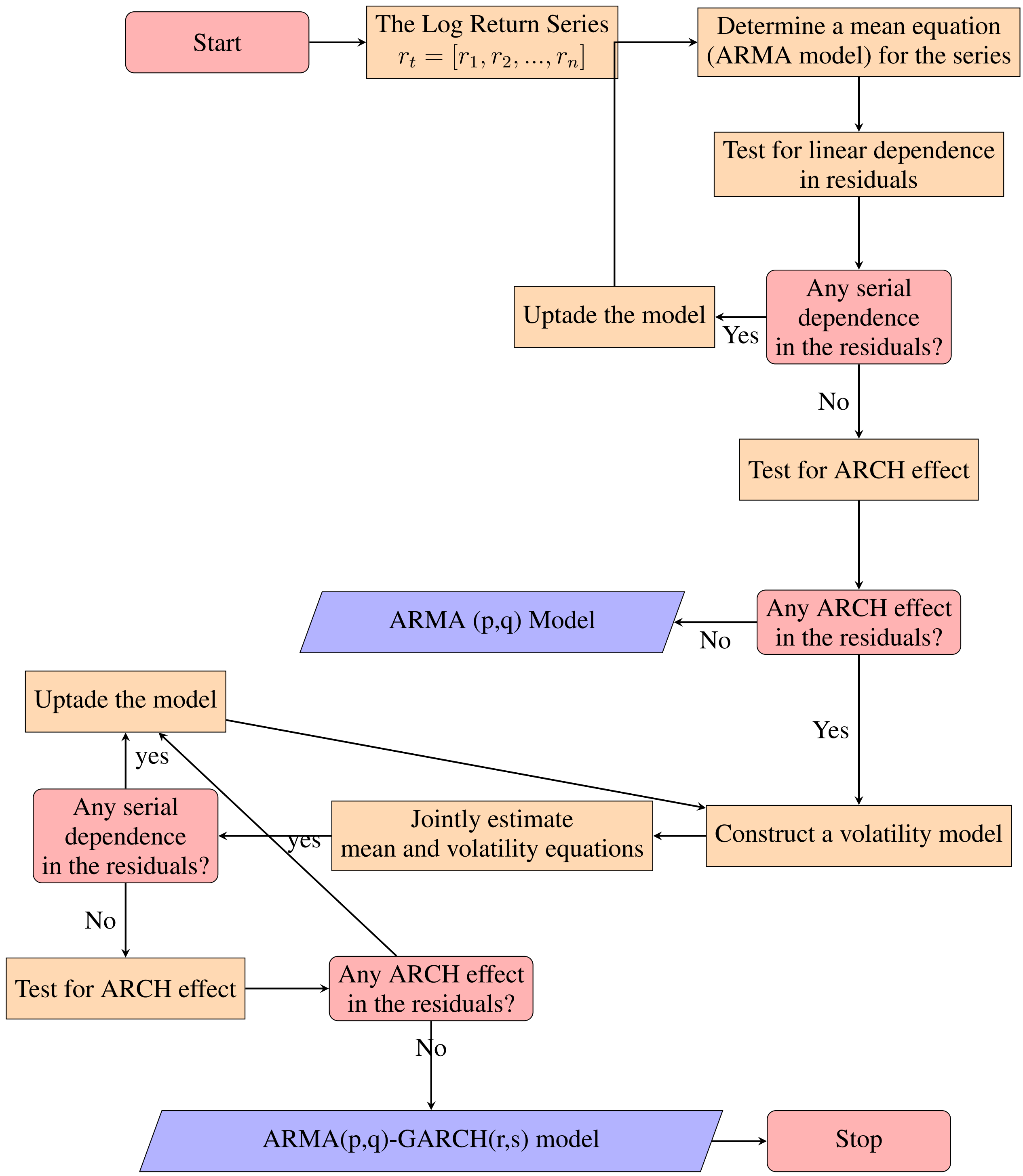


Figure 3.1: Flowchart of the Procedure to Construct ARMA-GARCH Models

3.1 Volatility Models

Engle [28] mentions that allowing for the conditional mean leads to a significant improvement in forecasts resulting from time series models. Consider the first-order autoregression, i.e., AR (1) process is expressed as

$$y_t = \gamma y_{t-1} + \epsilon_t \tag{3.2}$$

where y_t is a random variable at time t, ϵ_t is a white noise with $V(\epsilon_t) = \sigma^2$. According to Equation 3.2, the unconditional mean is zero, and the conditional mean, which is a forecast of today's value based on prior data is given by

$$E(y_t) = \gamma y_{t-1}. \tag{3.3}$$

Thus, this makes the conditional mean attractive to use as a forecasting tool. Furthermore, it is crucial to consider better forecasts and forecast intervals. The variance is useful to consider in this scenario. Although, the variance is treated as unconditional in traditional econometric methods, conditional variance, like conditional mean, can be a better tool when previous data is utilized to forecast variance.

The conditional variance is:

$$\begin{aligned} V_{t-1}(y_t) = E_{t-1}((y_t - E_{t-1}(y_t))^2) \\ = E_{t-1}((y_t - \gamma(y_t))^2) \\ = E_{t-1}(\epsilon_{t-1}^2) \\ = \sigma_t^2 \end{aligned} \tag{3.4}$$

where σ_t^2 is the conditional variance of ϵ_t. Besides, if white noise is homoscedastic then the result becomes

$$V_{t-1}(y_t) = E(\sigma_t^2) = \sigma^2. \tag{3.5}$$

The unconditional variance is:

$$
\begin{aligned}
V(y_t) &= V(\gamma y_{t-1} + \epsilon_t) \\
&= \gamma^2 v(Y_T) + V(\epsilon_t) \\
&= \frac{\sigma^2}{(1-\gamma^2)}.
\end{aligned}
\tag{3.6}
$$

As $\frac{1}{(1-\gamma^2)} > 1$, the unconditional variance is clearly bigger than the average conditional variance. On top of that, the realized variation in financial time series is frequently different from the average conditional variance. Thus, the results obtained in the research undertaken are likely to be incorrect, as indicated by the disparity described above.

Due to the importance of conditional volatility modeling, various models are constructed. For instance,

$$y_t = \epsilon_t x_{t-1} \tag{3.7}$$

where $V(\epsilon_t) = \sigma^2$. However, the problem with the above model is that the conditional variance of its error is dependent on only one exogenous variable, x, and it must be determined what causes the variance to change. Thus, the other model 3.8 is

$$y_t = \epsilon_t y_{t-1}. \tag{3.8}$$

It leads to the unconditional variance of its error being either zero or infinity. Thereupon, Engle (1982) introduced ARCH model to have a more attractive one.

3.1.1 ARCH(p) Model

This model was first used for modeling the variance of inflation observations in England. ARCH process permits the volatility to change over time depending on the past error terms, and it keeps the unconditional variance constant. The standard ARCH(p) model is expressed as follows:

Consider $y_t = \epsilon_t h_t^{\frac{1}{2}}$ where $V(\epsilon_t) = 1$ and assume $y_t|\psi_{t-1} \sim \mathcal{N}(0, h_t)$, ARCH model is given by

$$\begin{aligned} h_t &= \alpha_0 + \alpha_1 y_{t-1}^2 + ... + \alpha_p y_{p-1}^2 \\ &= \alpha_0 + \Sigma_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 \\ &= \alpha_0 + \alpha_1(L)\epsilon_t^2 \end{aligned} \tag{3.9}$$

where ψ_{t-1} is the information set available at time t, p is the order of the ARCH process, $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_p)$ is a vector of unknown parameters, h_t is σ_t^2, and L is the lag operator.

For example, ARCH (1) model is,

$$\sigma_t^2 = h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2. \tag{3.10}$$

For this process, the constraints $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ are imposed to avoid negative variance. If analyzing the error term process further, the orthogonality of it is observed, meaning that

$$E(\epsilon_t | \epsilon_{t-1}) = 0, t > 1. \tag{3.11}$$

Consequently, the errors that are serially uncorrelated. Moreover, they are not independent because they are linked by their second moment.

This ARCH(p) model has a number of flaws. Firstly, the given rule restricts coefficient " α", which shows that it is not so flexible when it comes to the parameter. Secondly, the model squares the past periods, resulting in the same volatility reaction for both negative and positive shocks, i.e., not showing the leverage effects. Thirdly, ARCH models react slowly to changes in shocks. Finally, the parameters of ARCH(p) are not easily estimated. The reason is that a large number of lagged squared error terms in the conditional variance have a tendency to be significant when determining the order of the ARCH process. These factors lead researcher to find or modify ARCH type models.

3.1.2 GARCH(p, q) Model

Due to the flaws of ARCH model mentioned in 3.1.1, of the ARCH model, generalized ARCH model (GARCH) is introduced by Bollerslev [16]. The GARCH (p, q) model is expressed as

$$\begin{aligned} h_t &= \alpha_0 + \Sigma_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \Sigma_{j=1}^{q} \beta_j h_{t-j}^2 \\ &= \alpha_0 + \alpha_1(L)\epsilon_t^2 + \beta_1(L) h_t \end{aligned} \tag{3.12}$$

where $p \geq 0, q \geq 0, \alpha_=(\alpha_0, \alpha_1, \ldots, \alpha_p)$ and $\beta = (\beta_1, \ldots, \beta_q)$ is a vector of unknown parameters, and $\alpha, \beta \geq 0$.

Therefore, this model shows two distributed lags. One is to capture high frequency impacts on preceding squared residuals, while the other is to capture longer term impact on lagged variance itself. For illustrative purposes, GARCH (1, 1) model is shown by

$$\sigma_t^2 = h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}. \tag{3.13}$$

The unconditional variance of the GARCH model is

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta_1}. \tag{3.14}$$

Through this adjustment, if it is defined, one can conclude that

$$\Sigma_{i=1}^{max(p,q)} (\alpha_i + \beta_i) < 1 \tag{3.15}$$

which means that the conditional variance changes over time, and unconditional variance becomes finite. That means covariance stationary, and high α would mean that volatility is reacting strongly to the market changes, and high β would mean volatility persistence.

3.1.3 EGARCH Model

Lack of consideration for the leverage effect, parameter constraints, and the burden of determining whether shocks on conditional variance persist or not are among GARCH models' three key drawbacks. Due to these challenges, Nelson [44] suggests that the exponential GARCH (EGARCH) which is described as

Let $\epsilon_t = \sigma_t z_t$ where $z_t \sim$ i.d.d with $E(z_t) = 0$ and $V(z_t) = 1$ given $g(z_t) = \theta z_t + \gamma[|z_t| - E|z_t|]$,

$$ln(h_t) = \alpha_0 + \Sigma_{i=1}^{p} \alpha_i g(z_t) + \Sigma_{j=1}^{q} \beta_j ln(h_{t-j}). \tag{3.16}$$

The parameter θ denoting the sign of error has an impact on the conditional variance as the multiply of z_t. Thus, if θ is zero, the positive and negative shocks would have the same effect on the variance. Besides, γ enables for a different size effect by multiplying $|z_t|$. So, the asymmetric effect on the series is considered on the contrary to GARCH (p,q) model. Moreover, there are no limitations on the parameters, and they can be either positive or negative because of the logarithm of the variance. Additionally, because $ln(h_t)$ is a linear process, its stationarity and ergodicity may be easily verified.

EGARCH (1, 1) model can be expressed as

$$ln(h_t) = \alpha_0 + \alpha_1[\theta z_t + \gamma[|z_t| - E|z_t|]] + \beta_1 ln(h_{t-1}). \tag{3.17}$$

Note that by inserting $\epsilon_t = \sigma_t z_t, z_t = \frac{\epsilon_t}{\sigma_t} = \frac{\epsilon_t}{\sqrt{h_t}}$ in Equation3.17, we obtain

$$ln(h_t) = \alpha_0 + \alpha_1[\theta \frac{\epsilon_t}{\sqrt{h_t}} + \gamma[|\frac{\epsilon_t}{\sqrt{h_t}}| - \sqrt{\frac{2}{\pi}}]] + \beta_1 ln(h_{t-1}). \tag{3.18}$$

3.1.4 GJR-GARCH and TGARCH Models

Despite the apparent benefits of EGARCH, empirical estimate of the model is technically difficult due to the use of extremely non-linear techniques [54].The distinction between positive shocks and negative shocks is concerned in this model, as well. The GJR-GARCH model is introduced by Glosten, Jagannathan, and Runkle [33]

$$ln(h_t) = \alpha_0 + \Sigma_{i=1}^{p}(\alpha_i \epsilon_{t-1}^2 + \gamma_i D_{t-i} \epsilon_{t-1}^2) + \Sigma_{j=1}^{q} \beta_j h_{t-j} \tag{3.19}$$

where

$$D_{t-i} = \begin{cases} 1 & \text{if } \epsilon_{t-i} < 0 \\ 0 & \text{if } \epsilon_{t-i} \geq 0. \end{cases} \tag{3.20}$$

According to this model, a dummy variable D_{t-i} has an important role for reflecting the leverage effects. When the shocks are positive D_{t-i} takes value 0, and the model becomes GARCH(p,q) model. Furthermore, when the shocks are negative, it would be 1. Additionally, γ parameter in the model denotes the leverage effect.

GJR-GARCH (1, 1) model is given as

$$ln(h_t) = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \gamma_1 D_{t-1} \epsilon_{t-1}^2 + \beta_1 h_{t-1}. \tag{3.21}$$

Furthermore, the threshold GARCH (TGARCH), introduced by Zokaian [59], demonstrates the asymmetry effect of the shocks on the variance, as well. Threshold ARCH (TARCH) or TGARCH model is similar to the GJR model. It differs mainly by using the standard deviation rather than the variance as given in Equation 3.22

$$\begin{aligned} &\text{Given } \epsilon_t = \sigma_t z_t \\ &\sigma_t^2 = \alpha_0 + \Sigma_{i=1}^{p} \alpha_i (|\epsilon_{t-i}| - \gamma_i \epsilon_{t-i}) + \Sigma_{j=1}^{q} \beta_j \sigma_{t-j} \\ &\text{where } Z_t \text{ is i.d.d, } EZ_t = 0, VZ_t = 1, Z_t \text{ independent of } \epsilon_{t-1} \text{ for all t.} \end{aligned} \tag{3.22}$$

Here, where $(\alpha_i)_{i=1,p}$ and $(\beta_j)_{j=1,q}$ are real scala squences.

3.1.5 APARCH Model

The Asymmetric Power ARCH (APARCH) model, proposed by Ding et al. [25] is utilized in determining the asymmetric effect. What this model offers is that leverage effect, thick tail, and excessive kurtosis can be detected. Additionally, this model covers some ARCH models as a special case. APARCH model is:

$$\sigma_t^\delta = \alpha_0 + \Sigma_{i=1}^p \alpha_i [|\epsilon_{t-i} - \gamma \epsilon_{t-i}]^\delta + \Sigma_{j=1}^q \beta_j \sigma_{t-1}^\delta \tag{3.23}$$

where δ is the power parameter, γ is the leverage parameter, and the other parameters $(\alpha_0, \alpha_1$ and $\beta_j)$ are the same as the GARCH parameters. Besides, it has some certain conditions to be fulfilled. Firstly, $\alpha_0 > 0, \alpha \geq 0, i = 1, ..., p$ and $\beta_j \geq 0, i = 1, ..., q$. Moreover, if $\alpha_i = \beta_j = 0$, then $\alpha_0 > 0$ to guarantee the conditional variance be positive. Lastly, for the stationary purposes, $0 \leq \Sigma_i^p \alpha_i + \Sigma_j^q \beta_j \leq 1$ should be satisfied.

3.2 Model Selection Criteria

In this study, the Akaike information criterion (AIC), Bayesian information criterion (BIC), and Log-Likelihood are considered when selecting the best statistical model among a set of candidate models. After defining the candidate models, the log-likelihood of each model is calculated using estimated parameters. Subsequently, a score is assigned to each model according to the creation that is used, and based on the score of the models, the best model whose score is the lowest determines the best fitting model [49].

For a particular model, the log-likelihood value may vary between negative infinity to positive infinity [58]. It, like the other criterion, can be used to compare candidate models. Finally, the higher the log-likelihood number, the better a model matches a dataset.

The maximum likelihood estimate and the number of parameters, independent variables, in the model are used to calculate a relative value according to the Akaike information criterion (AIC) which is

$$AIC = 2K - 2ln(L) \tag{3.24}$$

where K denotes the number of independent variables employed and L denotes the likelihood value estimated at parameter estimates. The score measures the model's goodness-of-fit alongside imposing penalties it for over-fitting the data, so a lower AIC score suggests better goodness-of-fit and less over-fitting [1].

Similar to AIC but unlikely, the Bayesian information criterion (BIC) considers the number of data observations and shown as

$$BIC = ln(n)K - 2ln(L) \tag{3.25}$$

where n is the number of data points.BIC values can be used to test the models. A model with a lower BIC value has smaller penalty terms and is, therefore, better [23]. After the adequacy of the models is discussed, model selection is used to determine which empirical model is the best fit for modeling the variance.

3.3 The Skewed Non-Gaussian Distributions

Literature suggests that it is need to use different distributions other than normal distribution for the exchange rate series. For this reason, in this thesis, various distributions are used, and they are presented in the following sections.

3.3.1 Normal Distribution

The normal distribution, also known as the Gaussian distribution, is the most frequently assumed form of distribution in technical stock surveys and other types of statistical investigations [2].

Moreover, the Central Limit Theorem (CLT), which is one of the prominent theorems, states that regardless of the distribution of the population, the sampling distribution of the mean will always be normally distributed as long as the sample size is large enough [52]. Thus, it can be said that its importance cannot be denied.

The mean and standard deviation are the only two parameters that can describe all normal distributions. The p.d.f. of the distribution is the following:

$$f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2} \tag{3.26}$$

where μ is the mean and σ is the standard deviation. Normal distribution is symmetrical around the mean so that the mean, median, and mode are all the same. Additionally, the kurtosis of the normal distribution 3.

Normal distribution has a skewness of zero, so that means it is symmetric. On the other hand, if the skewness is not zero, then the density function is according to Azzalini [10] as follows:

$$f(x;\lambda) = 2\phi(x)\Phi(\lambda x), \quad (x \in R), \tag{3.27}$$

where X $\sim$ N(0,1), $\lambda \in R$ is skewness parameter, ϕ is the normal PDF, and Φ is the normal cumulative density function (CDF).

3.3.2 The t Distribution

Known as the Student's t-distribution, the tails are heavier than the normal distribution's tail, so this distribution can be utilized for more extreme cases. This distribution is also used to estimate population parameters when the sample size is small or the population variance is unknown [37]. The p.d.f. is given as

$$f(x) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\Gamma(\frac{v}{2}}(1+\frac{t^2}{v})^{\frac{-(v+1)}{2}} \tag{3.28}$$

where v is the number of degrees of freedom, $v > 0$, and $x \in (-\infty, \infty)$.

Similar to the normal distribution, the t distribution is symmetric around the mean and is bell-shaped. If the parameter v is bigger than 1, then the mean of it equals 0, otherwise, it is undefined. Its median and mode are zero. Moreover, its skewness is zero if v>3, and its excess kurtosis is $\frac{6}{v-4}$ if v>4, ∞ for 2<$v \leq 4$.

If the skewness of it is not zero, then the p.d.f of the skew t distribution becomes

$$f(x|\mu,\sigma,\lambda,q) = \frac{\Gamma(\frac{1}{2}+q)}{v\sigma(\pi q)^{\frac{1}{2}}\Gamma(q)[1+\frac{|x-\mu+m|^2}{q(v\sigma)^2(1+\lambda sgn(x-\mu+m))^2}]^{(\frac{1}{2}+q)}} \tag{3.29}$$

$$\text{where } m = \sigma\frac{2q^{\frac{1}{2}}\Gamma(q-\frac{1}{2})}{\pi^{\frac{1}{2}}\Gamma(q)} \tag{3.30}$$

$$v = \frac{1}{q^{\frac{1}{2}}\sqrt{(1+3\lambda^2)\frac{1}{2q-2}-\frac{4\lambda^2}{\pi}(\frac{\Gamma(q-\frac{1}{2})}{\Gamma(q)})^2}}, \tag{3.31}$$

where sgn is the sign function.

3.3.3 The Generalized Error Distribution (GED)

This distribution is also referred to as generalized normal distribution or exponential power distribution since it is a symmetrical unimodal member of the exponential family. Nelson [44] developed and utilized for the EGARCH process. The p.d.f. of it is:

$$f(x|\mu,\sigma,p) = \frac{pe^{\frac{1}{2}|\frac{x-\mu}{\sigma}|^p}}{2p^{(1+\frac{1}{p})}\sigma\Gamma(\frac{1}{p})} \tag{3.32}$$

where domain of the $x \in (-\infty,\infty)$, μ is location parameter, σ is scale parameter, p is shape parameter, and $\Gamma(x)$ is the gamma function given by

$$\Gamma(x) = \int_0^t t^{x-1}e^{-t}dt. \tag{3.33}$$

If the p parameter is taken as 2, then Equation 3.32 becomes the p.d.f. of the Normal distribution. Moreover, if the p is taken as 1, then Equation 3.32 becomes the p.d.f. of the Double Exponential, or Laplace, distribution. Also, it should be noted that the distribution is leptokurtic if p is smaller than 2.

Furthermore, the non-centered skewed GED is given by

$$f(x|\mu, \sigma, \lambda, p) = \frac{pe^{\frac{-1}{p}|\frac{x-\mu+m}{v\sigma_p(1+_psgn(x-\mu_p+m))}|^p}}{2vp^{(1+\frac{1}{p})}\sigma\Gamma(\frac{1}{p})} \tag{3.34}$$

where sgn is the sign function that provides -1 for negative values of the argument and +1 for the positive values of it. Additionally, m and v are as follows:

$$m = \frac{2^{\frac{2}{p}}v\sigma_p\lambda_p\Gamma(\frac{1}{2}+\frac{1}{p})}{\sqrt{\pi}}, \tag{3.35}$$

$$v = \frac{\pi(1+3\lambda_p^2)\Gamma(\frac{3}{p}) - 16^{\frac{1}{p}}\lambda_p^2\Gamma(\frac{1}{2}+\frac{1}{p})\Gamma(\frac{1}{p})}{\pi\Gamma(\frac{1}{p})}. \tag{3.36}$$

CHAPTER 4

THE EMPIRICAL RESULTS

We implement the proposed approach on real life data set which known to have high volatility. So, daily closing prices in US dollars from June 2001 to June 2023 are obtained from the CBRT's database. The fundamental reason for selecting June 2001 as a starting point is that the floating exchange rate regime began at that time. It is important to consider the frequency of the data since, in 1988, Diebold [24] mentions that when implementing quarterly data instead of monthly data in an integrated process that is point-sampled, ARCH effects tend to decline as the sampling period grows. Moreover, Drost and Nijman [26] illustrate how the volatility structure changes with frequency and report that the GARCH (1, 1) model is not resistant to sampling interval specification. For this reason, the daily data of 5536 observations is used where holidays and weekends are excluded from this study. All analysis and applications of the proposed models for this study are done using R Studio 2023.06.1 and a computer with a 64-bit operating system.

Exploring the descriptive statistics of the data before observing the movement of the exchange rate may assist in comprehending its structure more thoroughly. Thus, they are reported in Table 4.1. As can be seen a Jarque-Bera value greater than 5, as well as p-value lower than 0.05, strongly suggest that this series does not have a normal distribution. Moreover, right-skewness (Skewness=2.47>0) and fat tails (Kurtosis=5.61>3) are observed, as mentioned in the literature regarding the exchange rate.

Table 4.1: The Descriptive Statistics

Mean	3.7
Median	1.79
Mode	1.35
Standard Deviation	4.16
Sample Variance	17.34
Kurtosis	5.61
Skewness	2.47
Range	19.52
Minimum	1.14
Maximum	20.66
Sum	20509.5
Count	5536
Jargue-Bera	12922.9
P-value	0

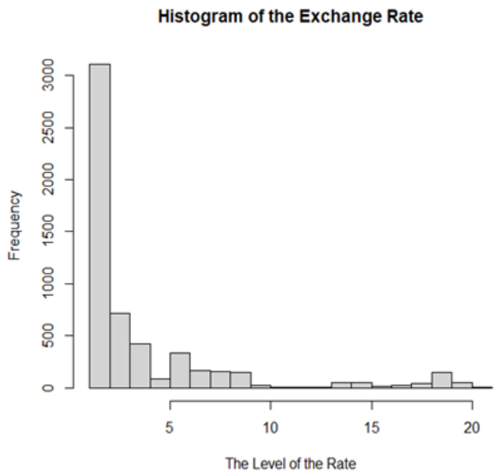


Figure 4.1: Histogram of the Exchange Rate

Further investigation for the distribution of the data can be done by the histogram given in Figure 4.1 which supports that the series violates a normal distribution, as well. Additionally, outliers and right-skewness are observed in the histogram, as in the descriptive statistics.

In Figure4.2, the time series of the data is given. The comments on the movement of the exchange rate and the story behind them are explained briefly in Section 4.3 period by period.

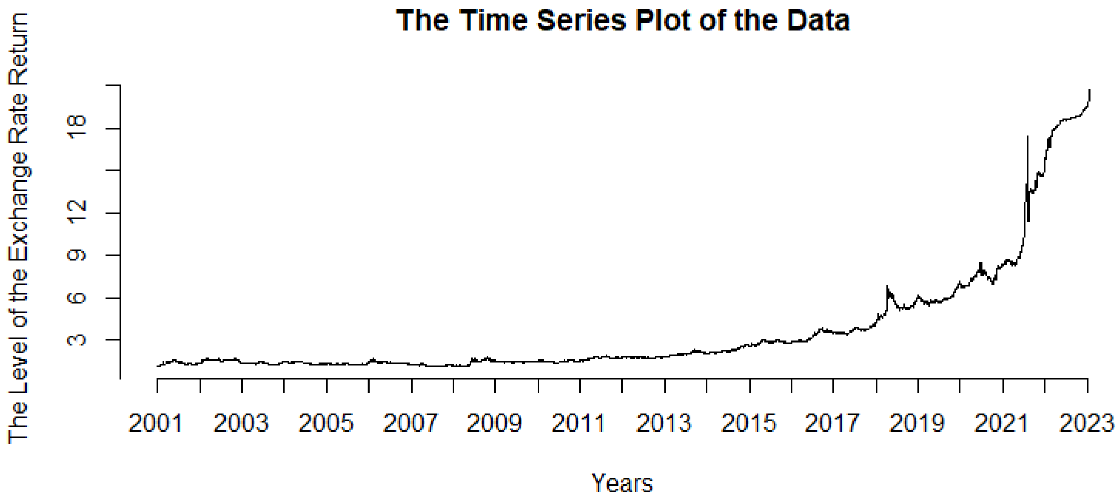


Figure 4.2: The Time Series Plot of the USD/TRY Exhange Rate

It is known that volatility exhibits itself on deviations from the mean caused by crises.So, outliers and shocks in the data are left the same without applying any procedures on them except for six sudden shocks which are shown in Figure 4.3, because they are troublesome and prevent the data from being modeled. The sudden shocks and the reasons for these shocks are as follows:

- (i) 30-31 October 2008 : The US Federal Reserve's decision to reduce interest rates to 1 percent and other countries following this had a positive impact on the stock markets. A sudden positive shock is observed for Turkish Lira.
- (ii) 14-15 August 2018 : Because of the Brunson case, the US placed economic sanctions on Turkey. After, a negative shock for the Lira is seen.
- (iii) 20-21 December 2021: Turkish government announced the exchange rate-protected deposit account (Kur Korumalı Mevduat Hesabı). Subsequently, a positive shock is noticed for the Turkish Lira.

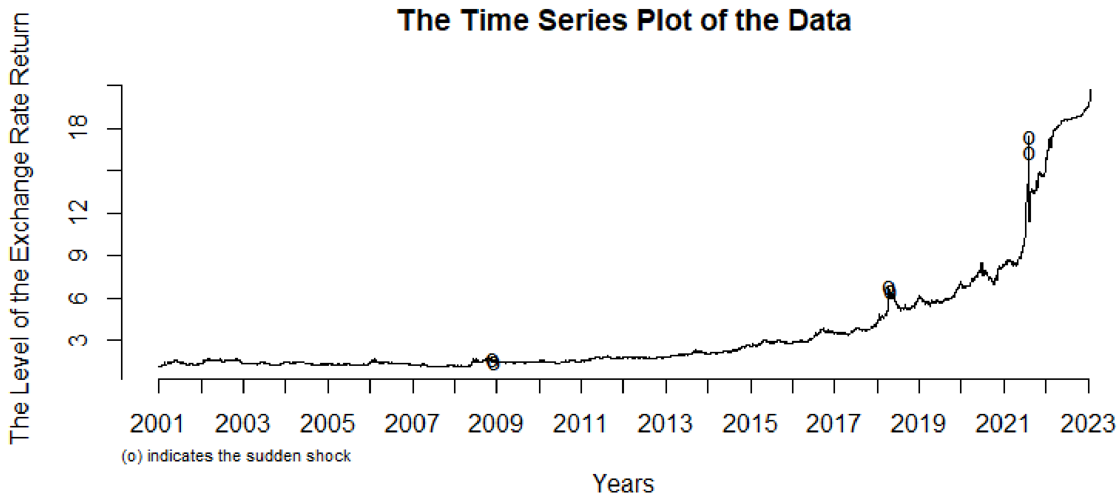


Figure 4.3: The Time Series Plot with the Shocks

In order to handle them, they are removed from the data set shown in Figure 4.4.

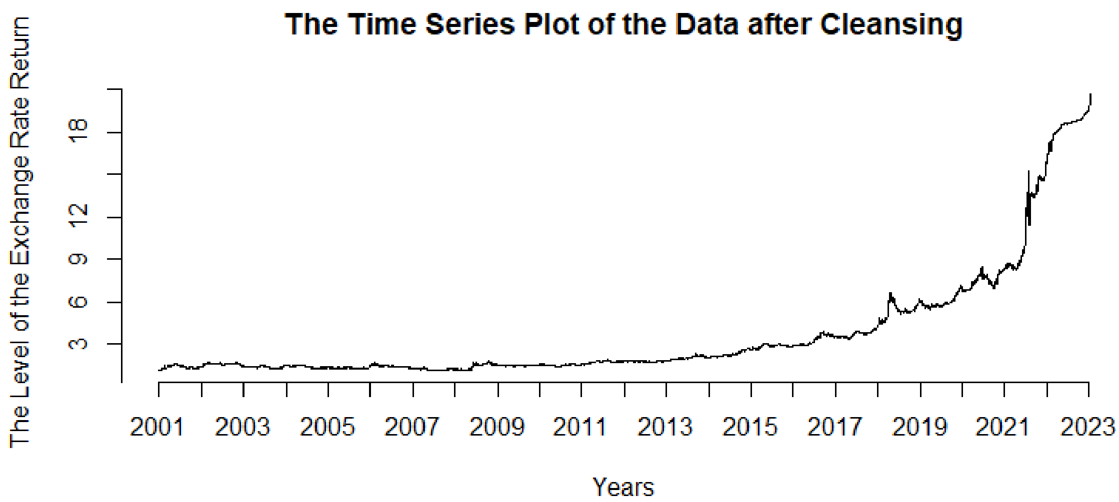


Figure 4.4: The Time Series Plot of the Data after Cleansing

4.1 Time Series Analysis for the Series

It is observed that the series is not stationary according to Figure 4.2. To specify the integration order ADF test for unit root and KPSS test for stationary are used. The results are given in Table 4.3.

Table 4.2: ADF and KPSS Test Results for the Entire Data

ADF		Test Statistic	P-value
	With constant ADF test statistic	13.8313	0.99
	Critical Values	-3.43	0.01
		-2.86	0.05
		-2.57	0.10
	With constant and trend ADF test statistic	8.7858	0.99
	Critical Values	-3.96	0.01
		-3.41	0.05
		-3.12	0.10
KPSS			
	With Constant	27.69	0.01*
	With Constant and Trend	7.85	0.01*

Note: (*) indicates significant at 5% significance level

The null hypothesis that the series has a unit root cannot be rejected with respect to the ADF result given in Table 4.3. Besides, the statistic value of the ADF test for the constant and trend model is also insignificant. So, it is seen that there is a stochastic trend in the data. This result is provided by the KPSS test, whose null hypothesis is that the series is stationary. The process is not stationary (p-value= 0.01<0.05) according to the KPSS test, and a stochastic trend is seen with respect to the model with constant and trend (p-value=0.01<0.05). For this reason, the difference in the logarithm of the series is considered for the empirical experiment in order to get rid of the unit root. The return series of the data set is constructed as follows:

$$r_t = log(\frac{P_t}{P_{t-1}}) \tag{4.1}$$

where r_t represents the return rate at time t.

In regards to that Figure 4.5, there is obviously volatility clustering in the series due to the fact that are some high volatility and low volatility clusters, and evidence indicates the time-varying volatility in daily USD/TRY returns.

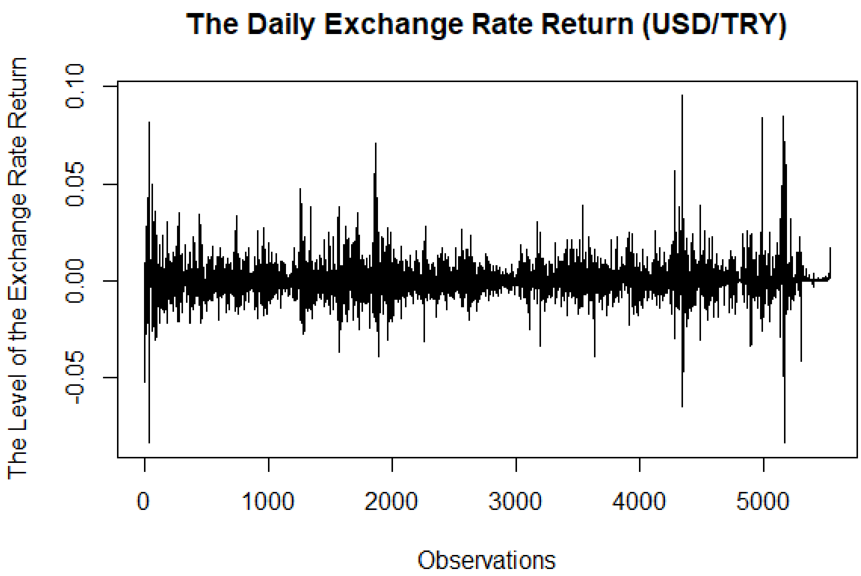


Figure 4.5: The Log-Return Series of the Exchange Rate Series

The ADF& KPSS tests are applied to the log-return data. The results are given in Table 4.10

Table 4.3: ADF and KPSS Test Results for the Log-Return Series

ADF		**Test Statistic**	**P-value**
	With constant ADF test statistic	-51.99	0.01*
	Critical Values	-3.43	0.01
		-2.86	0.05
		-2.57	0.10
	With constant and trend ADF test statistic	-52.16	0.01*
	Critical Values	-3.96	0.01
		-3.41	0.05
		-3.12	0.10
KPSS			
	With Constant	1.47	0.01*
	With Constant and Trend	0.08	0.10

Note: (*) indicates significant at 5% significance level

As can be seen from Table 4.10, the ADF test statistics are significant at 0.01 significance level for both models indicating the series does not have a unit root anymore which is shown by the KPSS test by an insignificant test statistic indicating stationarity of the series by a model including constant and trend. It must be noted that if only constant is considered, KPSS test result conflicts with the rest by indicating the series is not stationary (p-value=0.01).

Further, the acf and pacf plots of the data given in Figure 4.6, and it seems stationary.

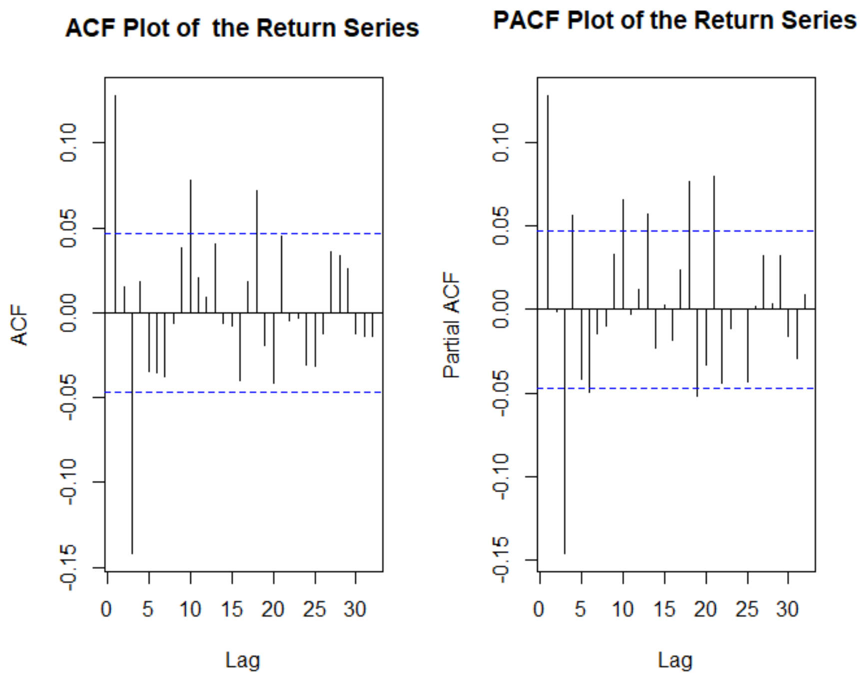


Figure 4.6: ACF-PACF Plots of the Return Series

The descriptive statistics and the histogram are given in Table 4.4 and Figure 4.7 in order to have an idea about the distribution of the log-return data. With respect to the JB test given in Table 4.4, the normality is rejected at 0.01 significance level for the series. The distribution is leptokurtic according to the kurtosis value, and the Histogram indicates fatter tails than the normal distribution. Moreover, although not severe, there might be a slight skewness. So, the models can be tried with symmetric non-normal distributions like t-distribution as well as skewed distributions to see the best-fitting one.

Table 4.4: The Descriptive Statistics of the Log-Return Data

Mean	0.0005
Median	0.00007
Sample Variance	0.00009
Kurtosis	26.24836
Skewness	-0.1747
Range	-0.2507
Minimum	-0.1554
Maximum	0.0954
Sum	2.8409
Count	5529
Jargue-Bera	158751
P-value	<2.2e-16

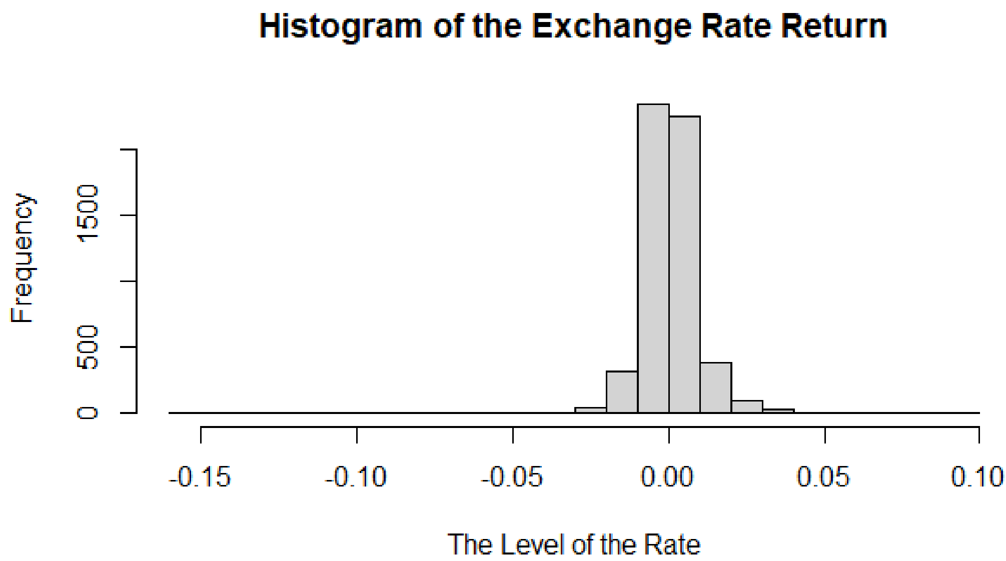


Figure 4.7: Histograms of the Log-Return Series

4.2 ARMA-GARCH Models for the Series

Autocorrelation in the ACF-PACF plots of the exchange rate return series is investigated, the mean equation was proposed as ARMA(4,3), and it is fitted to the data. According to the Box-Ljung test statistic ARMA(4,3) yields residuals having no serial correlation (p-value= 0.81). Additionally, ARCH effect is tested with the ARCH-LM test, whose null hypothesis is that there is no ARCH effect, and the test statistic for the model is found to be significant (p-value=0 for all lags) indicating there is a significant ARCH effect. This result is supported by the Ljung-Box-Pierce Q statistic of the squared residuals for both models, whose null hypothesis is that there is no ARCH effect (p- value=0 for all lags). Therefore, the model is obtained as a candidate mean model.

After finding ARCH effect in the model, we utilized the following models: GARCH model that ignores the asymmetry impact; EGARCH, TARCH, and APARCH models that consider the asymmetry impact. Furthermore, a practical distribution for the model estimate was attempted to be chosen among normal, skew-normal, t, skew-t, GED, and skew-GED, since it is observed that the data has fat tails and might have slight skewness. Generally, it is seen in the literature that GARCH (1,1), EGARCH (1,1), TGARCH (1,1), and APARCH (1,1) models are reported as adequate to remove ARCH effects.

2 out of 24 suggested models are found to be adequate according to the diagnostic checking for the log-return data. They are shown in bold in Figure4.5.

Table 4.5: The Empirical Models

The Empirical Models
ARMA(4,3)-GARCH(1,1) with normal distribution
ARMA(4,3)-GARCH(1,1) with skew normal distribution
ARMA(4,3)-GARCH(1,1) with student t distribution
ARMA(4,3)-GARCH(1,1) with skew student t distribution
ARMA(4,3)-GARCH(1,1) with GED distribution
ARMA(4,3)-GARCH(1,1) with skew GED distribution
ARMA(4,3)-EGARCH(1,1) with normal distribution
ARMA(4,3)-EGARCH(1,1) with skew normal distribution
ARMA(4,3)-EGARCH(1,1) with student t distribution
ARMA(4,3)-EGARCH(1,1) with skew student t distribution
ARMA(4,3))-EGARCH(1,1) with GED distribution
ARMA(4,3)-EGARCH(1,1) with skew GED distribution
ARMA(4,3)-TGARCH(1,1) with normal distribution
ARMA(4,3)-TGARCH(1,1) with skew normal distribution
ARMA(4,3)-TGARCH(1,1) with student t distribution
ARMA(4,3)-TGARCH(1,1) with skew student t distribution
ARMA(4,3)-TGARCH(1,1) with GED distribution
ARMA(4,3)-TGARCH(1,1) with skew GED distribution
ARMA(4,3)-APARCH(1,1) with normal distribution
ARMA(4,3)-APARCH(1,1) with skew normal distribution
ARMA(4,3)-APARCH(1,1) with student t distribution
ARMA(4,3)-APARCH(1,1) with skew student t distribution
ARMA(4,3)-APARCH(1,1) with GED distribution
ARMA(4,3)-APARCH(1,1) with skew GED distribution

For the adequate models the estimates and diagnostic checking are given in the following Tables 4.6 and 4.7, and for the inadequate models are shown in Tables A.1 and A.2.

From Table 4.6, the estimates of the all parameters of both models are found all significant at 0.05 significance levels, and both models hold the stationary condition since $\beta < 1$. Besides, an asymmetric effect in the series demonstrated by significant and positive γ in EGARCH models, and it indicates the fact that positive shocks have a bigger impact on increasing volatility than negative shocks. Furthermore, The shape and skewness parameters in skew t and GED are seen as a significant. They indicate the presence of the thicker tails since the shape value (degree of freedom) is around 4.5 for the skew t, and it is smaller than 2 in GED.

Table 4.6: The Estimate of the Models

Period : June 2001 -June 2023		
	ARMA(4,3)-EGARCH(1,1)	
	st	**GED**
c	0.000*	0.000*
ϕ_1	-0.597*	-0.827*
ϕ_2	0.983*	0.964*
ϕ_3	0.563*	0.814*
ϕ_4	-0.0385*	-0.074*
θ_1	0.695*	0.932*
θ_2	-0.899*	-0.853*
θ_3	-0.614*	-0.875*
ω	-0.141*	-0.177*
α	0.070*	0.063*
β	0.986*	0.982*
γ	0.345*	0.321*
δ	-	-
shape	4.400*	1.156*
skew	1.130*	-

(*) indicates significant at 5% significance level.

According to Table 4.7, it can be concluded that there is no ARCH effect or autocorrelation in the standardized residuals since all p-values are bigger than the 10% significance level. All in all, they are adequate models and seem to capture the volatility clustering.

Table 4.7: P-Values of the Diagnostic Tests for the Models

EGARCH						
	st			**GED**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.18	0.48	0.50	0.45	0.28	0.27
5	0.51	0.81	0.76	0.47	0.72	0.68
10	0.68	0.68	0.51	0.62	0.54	0.45
20	0.75	0.74	0.57	0.71	0.71	0.60

Finally, the selection criteria are reviewed in Table 4.8, and the lowest AIC and BIC and the maximum Log-Likelihood belong to the ARMA (4,3)-EGARCH (1,1) with skew t distribution model.

Table 4.8: Fit Measures of the Models

	Log-Likelihood	**AIC**	**BIC**
EGARCH-st	**19901.74**	**-7.1940**	**-7.1772**
EGARCH-GED	19832.94	7.1694	-7.1539

According to the results, only two models passed all tests successfully at the end of the experiment, which can be attributed to the structural changes in the series, since the abrupt changes are observed in the series. It can be said that they may effect the modeling the entire series. Therefore, it is crucial to investigate the presence of the breakpoints carried out in Section 4.3.

4.3 Breakpoint Detection

It is possible to see how certain changes appear smoothly and others emerge abruptly in a time series. External and internal effects can cause sudden and permanent change, leading to a structural breakpoint occurring in the series. The mean or in other parameters of the process that generates the series can change. So, the series may not be smooth throughout because of the breakpoints.

Modeling long-term data with structural breakpoints is possible, but prediction, estimation, and establishing relations can be problematic. Moreover, it can result in significant forecasting errors and model unreliability [6]. Thereupon, in this study before modeling, structural break tests are carried out on the data. For this reason, the "strucchange" package in R is utilized. In this package, the Chow test, whose null hypothesis says that there is no structural change point in the series, is applied, and it is found that its null hypothesis is rejected (p-value < 2.2e-16). Hence, there is a structural break in the series. After the test, "breakpoints" function in R is employed and optimal structural breakpoints are obtained given in Figure 4.8.

According to Figure 4.8, 3 breaks in the series occur. One of them is on the 1st of July 2013. This break is caused by an internal event called the Gezi Protests. The second one is on the 10th of October, caused by a coup attempt in July 2016. After the attempt, political measures for the safety of the country were taken and, of course, affected investors and financial markets in the country. The last break is observed on the 11th of February 2020 due to the COVID-19 Pandemic. Considering these breakpoints, the data set is divided into 4 sub-periods given in Figure 4.9 displaying the daily exchange rate (USD/TRY) for four different time periods.

(a) From June 2001 to July 2013 (Period 1): Ups and downs around the mean are

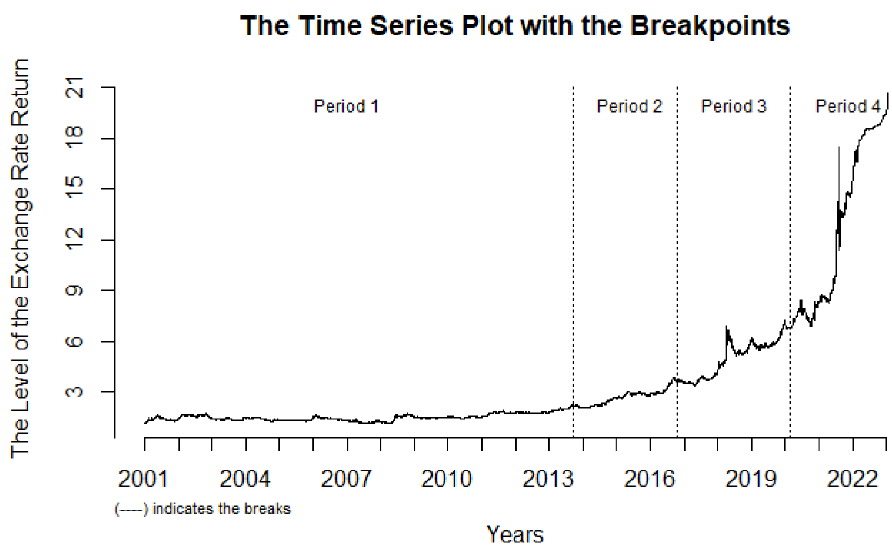


Figure 4.8: Breakpoints in the Data Set

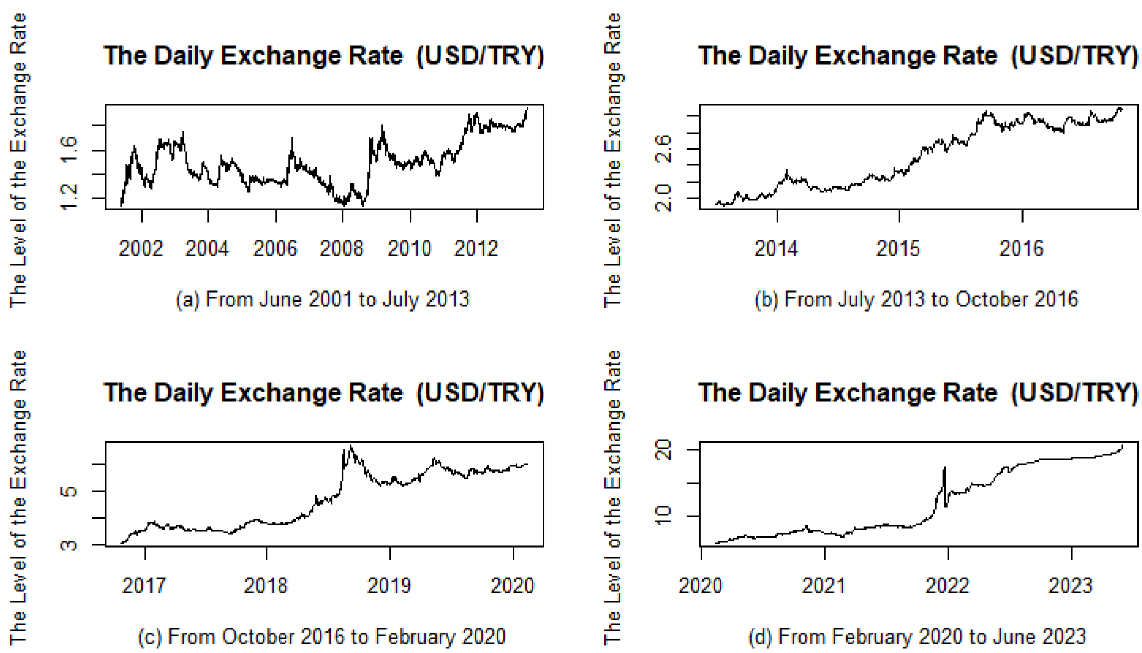


Figure 4.9: The Time Series of the Periods

observed in this interval. This shows that there may be a stochastic trend in the period. In 2001, a recession is observed in the international economy. Surely, it affected the whole world, but especially developing countries like Turkey. In February, a major economic downturn is provoked by a political event. The rapid capital withdrawals that accompanied the economic crisis, the increasing uncertainties in the economy, particularly in the exchange rate, and the loss of confidence in economic units all contributed to the decline of domestic demand and, as a result, the economy [12]. The currency rate fluctuation, which had declined significantly after April, increased with the negative events in the Argentine economy on July 16 and the potential of delaying the IMF loan tranche.

After the financial crisis in 2001, the Iraqi conflict in March 2003 happened, and again, economic uncertainty at that time caused another unexpected movement in the data. Following that, a period of stability was recorded until April of 2006. However, the volatility in global markets and the rise in risk aversion tendencies resulted in fluctuations in Turkey's financial markets and led to an increase in exchange rates in the second quarter of 2006, and the movement at the halfway point of the year can be defined as the sharpest depreciation beginning in the currency in the graph. Afterwards, the Turkish Lira appreciated to values similar to those seen in 2001 until June 2008.

Furthermore, there is apparent evidence of a trend, and the depreciation of the Turkish Lira that began with the financial crisis in 2008 continued until the end of 2014. In 2008–2009, the global financial crisis occurred, which is the most severe financial crisis since 1929.

(b) From July 2013 to October 2016 (Period 2): In December 2013- 2014, political events happened. Moreover, as a result of FED Chairman Yellen's remark concerning the rate hike in May 2015 and domestic events in September 2015, higher fluctuations are observed. So, the volatility in the rate was getting even higher. After a coup attempt in July 2016, political measures for the safety of the country were taken, which, of course, affected investors and financial markets, too. At the beginning of the following year, terrorist attacks, CBRT, and FED statements made the rate more volatile than ever.

(c) From October 2016 to February 2020 (Period 3): In March 2017, a political crisis occurred between Holland and Turkey. After that, a referendum was held in April. In the following year, the Brunson crisis was the biggest event since it affected relations between the USA and Turkey. So, this caused the depreciation of the Lira against the Dollar. In 2019, there was a local election, and its political environment caused another event. In the first half of the year, high variations are observed due to this uncertainty. In the second half of 2019, FED statements and a military operation in Syria in October 2019 played the biggest roles in the other variations of that period.

(d) From February 2020 to June 2023 (Period 4): This last period has the highest depreciation of the Lira and the fastest-growing trend among the given periods. The COVID-19 Pandemic lasted between the years 2020 and 2022, and during periods of political instability, the CBRT and FED decisions in terms of the rate appeared to reflect the exchange rate negatively. Especially during CBRT's monetary policy decisions, outliers can be observed. As a result, a graph with the most volatile currency of all time and the greatest depreciation of the Lira is observed in this period. In December 2021, the series is the most volatile period of the year since, at that time, the government announced the exchange rate-protected deposit account (Kur Korumalı Mevduat Hesabı). So, after a climbing trend, the sharpest decrease in the value of the exchange rate was observed. After that, due to the unstable economic and political situations, a trend can be seen in the graph until the end of April 2023.

4.4 Time Series Analysis for the Sub-Periods

From Figure 4.9, it is noticed that the series over the periods are clearly non-stationary. To determine the integration order (ADF) test for unit root and (KPSS) test for stationary are utilized for all periods, whose results are given in Table 4.9.

Table 4.9: ADF and KPSS Test Results

ADF		Period 1	Period 2	Period 3	Period 4
	With Constant	-1.9119 (0.346)	-0.737 (0.785)	-1.256 (0.591)	0.027 (0.958)
	Critical Values		-3.43 (0.01) -2.86 (0.05) -2.57 (0.10)		
	With constant and trend	-2.332 (0.438)	-2.303 (0.45)	-2.513 (0.361)	-2.238 (0.478)
	Critical Values		-3.96 (0.01) -3.41 (0.05) -3.12 (0.10)		
KPSS	**With Constant**	10.559 (0.01*)	11.416 (0.01*)	10.571 (0.01*)	11.435 (0.01*)
	With Constant and Trend	4.1562 (0.01*)	1.0022 (0.01*)	0.82654 (0.01*)	1.8859 (0.01*)

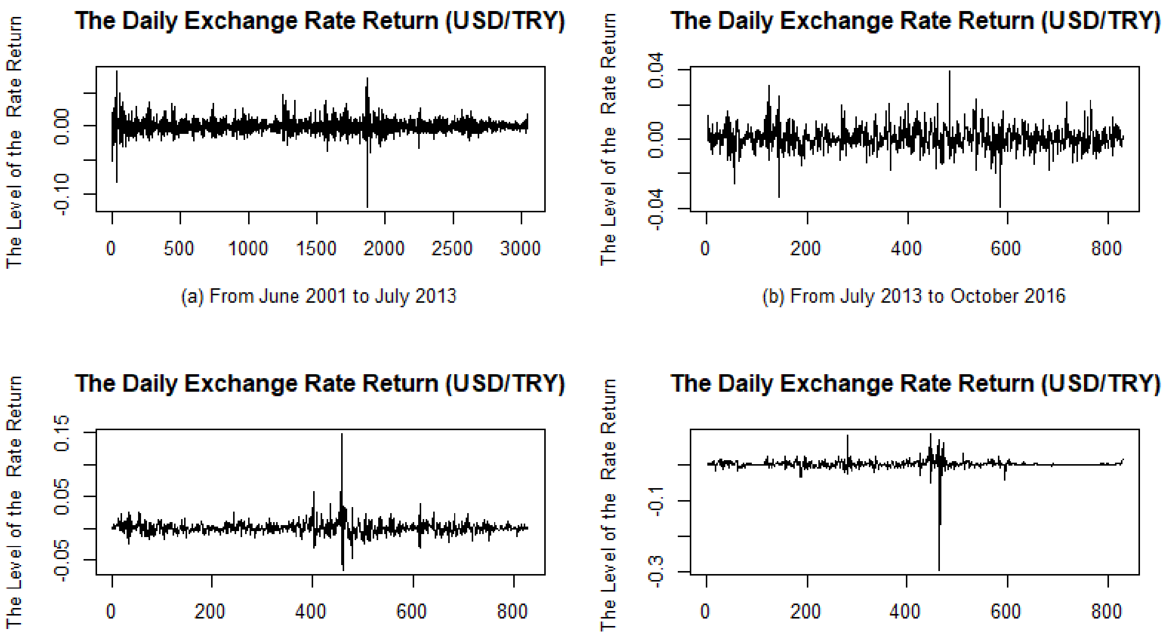
Note: P-values are given in the parenthesis, and (*) indicates significant at 5% significance level

According to the ADF results given in Table 4.9, the null hypothesis that the series has a unit root cannot be rejected, and the statistics value of the ADF tests for the constant and trend models are also insignificant for all periods. Thus, it is observed that there is a stochastic trend in all series. Besides, these results are supported by the KPSS test. The process is not stationary (p-value=0.01<0.05) for all periods according to the KPSS test results. Additionally, stochastic trends are seen with respect to the model with constant and trend (p-value=0.01<0.05) in all periods.

Thus, the series in the spans don't seem stationary. For this reason, the difference in the logarithm of the series in all periods is considered for the empirical experiment in order to get rid of the unit root. The return series of each period are formed as follows:

$$r_t = log(\frac{P_t}{P_{t-1}}) \tag{4.2}$$

where r_t represents the return rate at time t.



(a) From June 2001 to July 2013

(b) From July 2013 to October 2016

(c) From October 2016 to February 202

(d) From February 2020 to June 2023

Figure 4.10: The Log-Return Series of the Periods

In all periods, there is certainly volatility clustering according to Figure 4.10 since there are some high volatility and low volatility clusters, and time-varying volatility in daily USD/TRY returns is empirically demonstrated.

The ADF& KPSS tests are carried out for the log-return data. The results are given in Table 4.10

As can be seen from Table 4.10, in all periods, the ADF test statistics are significant at 0.01 significance level for both models indicating the series does not have a unit root anymore which is supported by the KPSS test by an insignificant test statistic indicating stationarity of the series by both models (p-value=0.1 for all models in all periods).

Further, by the acf and pacf plots of the periods given in Figure 4.11, it can be seen that the series in all periods are stationary.

Table 4.10: ADF and KPSS Test Results for the Log-Return Series

ADF		Period 1	Period 2	Period 3	Period 4
	With Constant	-13.184 (0.01*)	-20.031 (0.01*)	-20.695 (0.01*)	-18.131 (0.01*)
	Critical Values		-3.43 (0.01) -2.86 (0.05) -2.57 (0.10)		
	With Constant and Trend	-13.184 (0.01*)	-20.022 (0.01*)	-20.6965 (0.01*)	-18.1204 (0.01*)
	Critical Values		-3.96 (0.01) -3.41 (0.05) -3.12 (0.10)		
KPSS	**With Constant**	0.066 (0.10)	0.058 (0.10)	0.105 (0.10)	0.075 (0.10)
	With Constant and Trend	0.057 (0.10)	0.049 (0.10)	0.073 (0.10)	0.074 (0.10)

Note: P-values are given in the parenthesis, and (*) indicates significant at 5% significance level

To figure out the distribution of the log-return data, the descriptive statistics and the histogram are given in Table 4.11 and Figure 4.12, respectively. According to the JB test given in Table 4.11, the normality is rejected at 0.01 significance level for all periods. The distribution of them is leptokurtic according to the kurtosis value, and the Histogram indicates fatter tails than the normal distribution. Besides, although not severe, there might be a slight skewness in the first two periods. However, the last two periods are seem extremely skewed. Thus, the models can be tried with symmetric non-normal distributions like t-distribution as well as skewed distributions to see the best-fitting one.

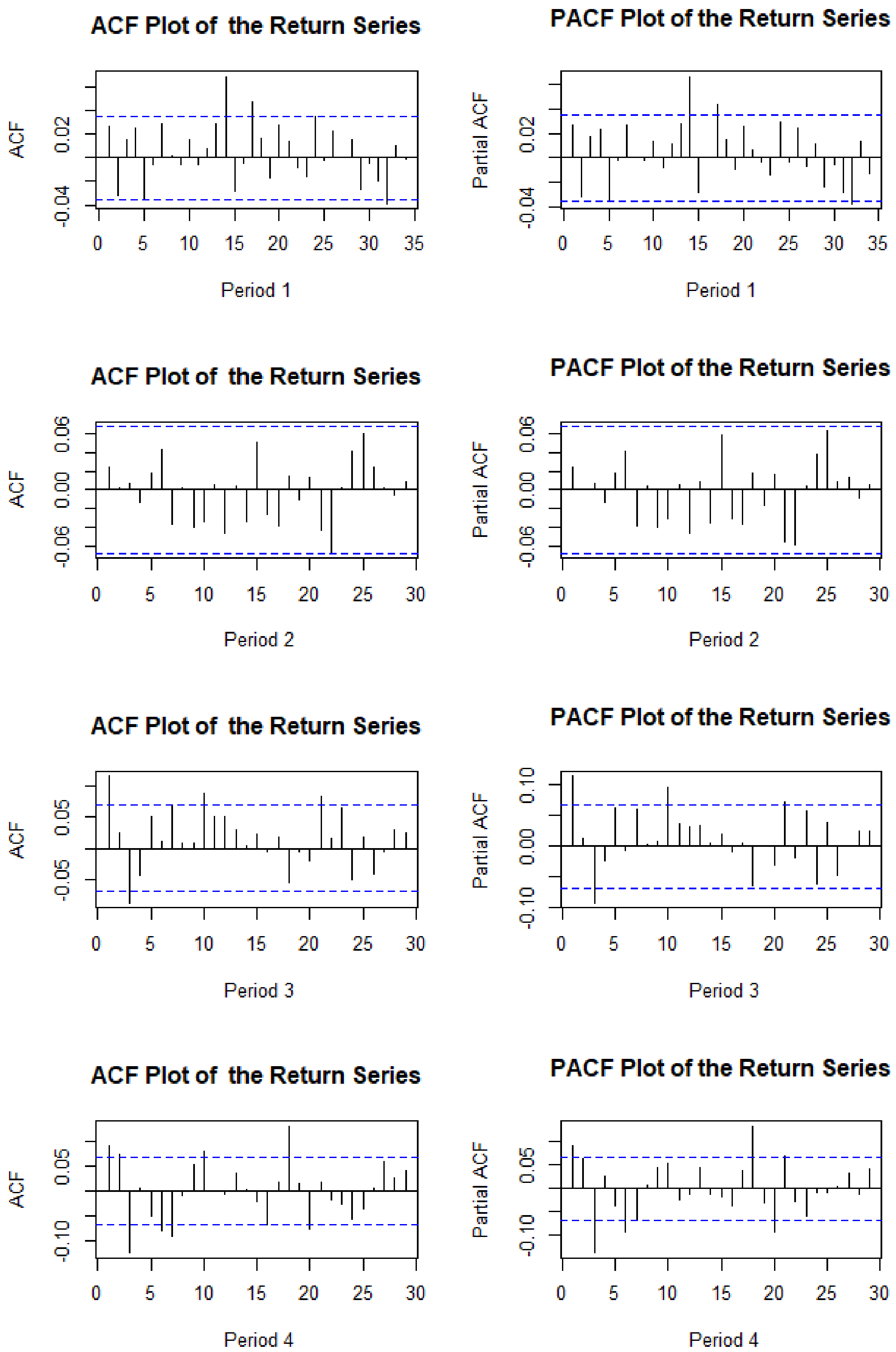


Figure 4.11: ACF-PACF Plots of the Return Data in the Periods

Table 4.11: The Descriptive Statistics of the Log-Return Data in all Periods

Periods	Period 1	Period 2	Period 3	Period 4
Mean	0.00015	0.00057	0.0008	0.00149
Median	-0.00045	0.00014	0.00026	0.00048
Sample Variance	0.00009	0.0005	0.00011	0.00022
Kurtosis	16.9353	3.49513	30.6626	200.4857
Skewness	0.01608	0.1312	2.0597	-9.2689
Range	-0.20077	-0.07792	-0.19329	-0.37843
Minimum	-0.11936	-0.03897	-0.06476	-0.29398
Maximum	0.08142	0.03895	0.12853	0.08445
Sum	0.46864	0.46802	0.66556	1.2374
Count	3044	829	829	830
Jargue-Bera	36376	424.34	33062	1401944
P-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16

4.5 ARMA-GARCH Models for the Periods

In this section, the procedure to construct ARMA-GARCH Models given in Figure 3.1 is applied for all periods. The empirical results are presented as the following.

4.5.1 The Results of Period 1

For the first step, the mean model needs to be constructed.According to ACF-PACF plots, ARMA(1,0) is fitted to the data, it is observed that ARMA(1,0) yields residuals having no serial correlation according to the Box-Ljung test statistic (p-value= 0.994). Additionally, after it is fitted to the data, ARCH effect is tested with the ARCH-LM test, whose null hypothesis is that there is no ARCH effect, and the test statistic for the model is found to be significant (p-value=0 for all lags) indicating there is a significant ARCH effect. This result is supported by the Ljung-Box-Pierce Q statistic of the squared residuals, whose null hypothesis is that there is no ARCH effect (p-value=0 for all lags). Therefore, the model is obtained as a candidate mean model.

After finding ARCH effect in the models, GARCH type models are proposed.

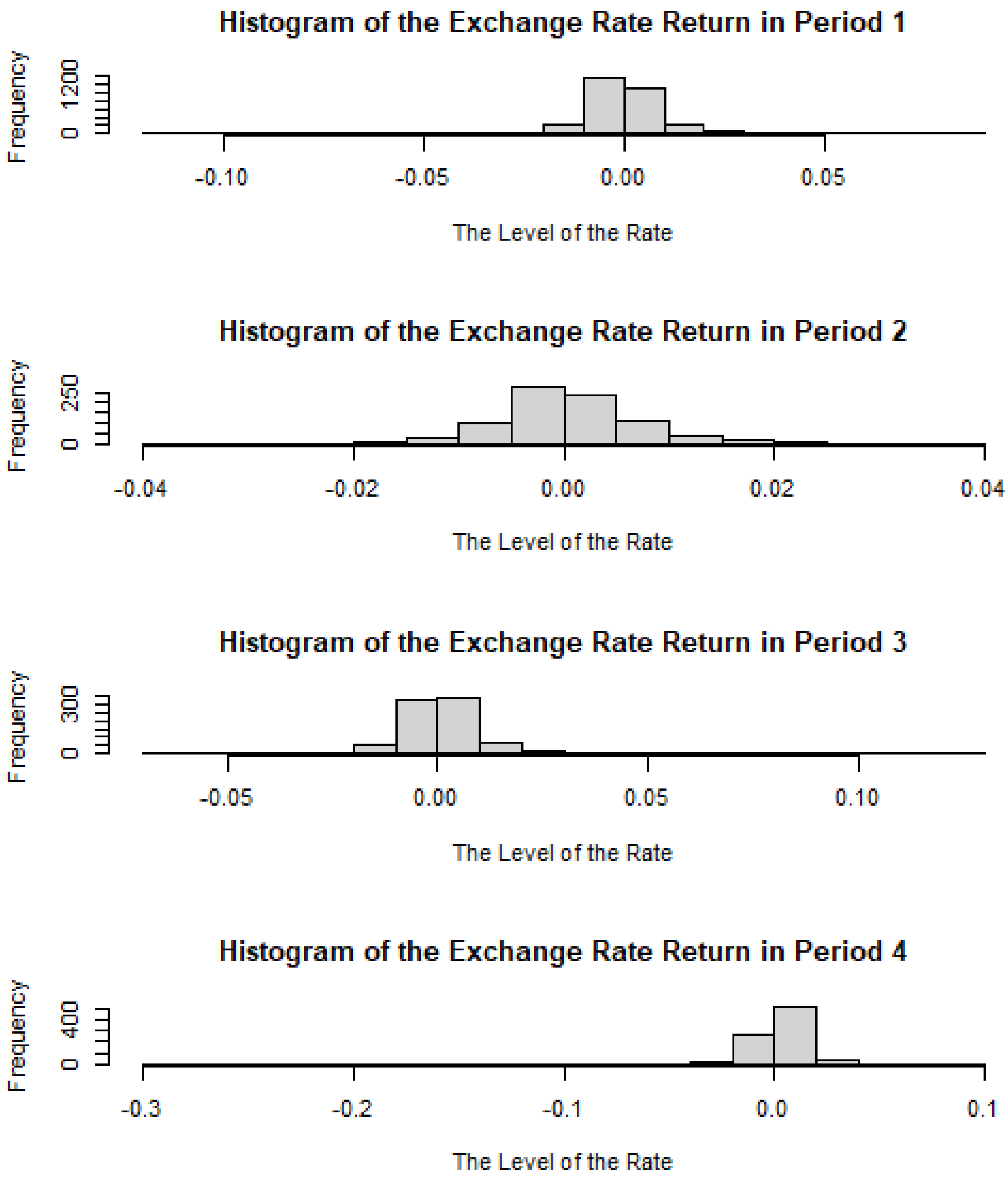


Figure 4.12: Histograms of the Log-Return Series in the Periods

19 out of 24 suggested models are found to be adequate according to the diagnostic checking for the log-return data. They are shown in bold in Figure4.12.

Table 4.12: The Empirical Models for Period 1

The Empirical Models
ARMA(1,0)-GARCH(1,1) with normal distribution
ARMA(1,0)-GARCH(1,1) with skew normal distribution
ARMA(1,0)-GARCH(1,1) with student t distribution
ARMA(1,0)-GARCH(1,1) with skew student t distribution
ARMA(1,0)-GARCH(1,1) with GED distribution
ARMA(1,0)-GARCH(1,1) with skew GED distribution
ARMA(1,0)-EGARCH(1,1) with normal distribution
ARMA(1,0)-EGARCH(1,1) with skew normal distribution
ARMA(1,0)-EGARCH(1,1) with student t distribution
ARMA(1,0)-EGARCH(1,1) with skew student t distribution
ARMA(1,0)-EGARCH(1,1) with GED distribution
ARMA(1,0)-EGARCH(1,1) with skew GED distribution
ARMA(1,0)-TGARCH(1,1) with normal distribution
ARMA(1,0)-TGARCH(1,1) with skew normal distribution
ARMA(1,0)-TGARCH(1,1) with student t distribution
ARMA(1,0)-TGARCH(1,1) with skew student t distribution
ARMA(1,0)-TGARCH(1,1) with GED distribution
ARMA(1,0)-TGARCH(1,1) with skew GED distribution
ARMA(1,0)-APARCH(1,1) with normal distribution
ARMA(1,0)-APARCH(1,1) with skew normal distribution
ARMA(1,0)-APARCH(1,1) with student t distribution
ARMA(1,0)-APARCH(1,1) with skew student t distribution
ARMA(1,0)-APARCH(1,1) with GED distribution
ARMA(1,0)-APARCH(1,1) with skew GED distribution

For the adequate models the estimates and diagnostic checking are given in the following 4.13 and 4.14, and for the inadequate models are shown in Tables A.3 and A.4.

Table 4.13: The Estimate of the Models for the Period 1

	Period 1 : June 2001-July 2013																			
	ARMA(1,0)-GARCH(1,1)				**ARMA(1,0)-EGARCH(1,1)**					**ARMA(1,0)-TGARCH(1,1)**						**ARMA(1,0)-APARCH(1,1)**			
	norm	**snorm**	**t**	**GED**	**norm**	**snorm**	**t**	**GED**	**sGED**	**norm**	**snorm**	**t**	**st**	**GED**	**sGED**	**norm**	**snorm**	**t**	**GED**
c	0.000	0.000	0.00*	0.00*	0.000	0.000	0.00*	0.00*	0.000	0.000	0.000	0.00*	0.000	0.00*	0.000	-0.000	0.000	0.00*	0.00*
ϕ	0.062*	0.04*	0.048*	0.052*	0.07*	0.05*	0.05*	0.06*	0.04*	0.07*	0.05*	0.06*	0.05*	0.06*	0.04*	0.06*	0.05*	0.06*	0.06*
ω	0.000	0.000	0.000	0.00	-0.25*	-0.26*	-0.35*	-0.31*	-0.31*	0.00*	0.000*	0.000*	0.00*	0.00*	0.00*	0.00	0.000	0.000	0.000
α	0.142*	0.138*	0.157*	0.15*	0.06*	0.05*	0.05*	0.061*	0.06*	0.13*	0.14*	0.16*	0.16*	0.15*	0.14*	0.11*	0.10*	0.10*	0.11*
β	0.856*	0.855*	0.842*	0.84*	0.97*	0.97*	0.96*	0.97*	0.97*	0.87*	0.87*	0.85*	0.85*	0.86*	0.86*	0.84*	0.79*	0.85*	0.84*
γ	-	-	-	-	0.25*	0.24*	0.29*	0.27*	0.26*	-0.25*	-0.24*	-0.26*	-0.26*	-0.24*	-0.25*	-0.09*	-0.09*	-0.14*	-0.11*
δ	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2.60*	3.01*	2.60*	2.52*
shape	-	-	6.472*	1.37*	-	-	6.28*	1.35*	1.37*	-	-	6.10*	6.30*	1.33*	1.34*	-	-	7.38*	1.38*
skew	-	1.186*	-	-	-	1.17*	-	-	1.17*	-	1.14*	-	1.18*	-	1.17*	-	1.17*	-	-

(*) indicates significant at5% significance level.

From Table 4.13, it is seen that the estimates of the variance equations and the ϕ parameter of all models are all significant at 0.05 significance levels except the constant term in the variance equations.

In GARCH models, all parameters that are in the variance equation are positive, and $\alpha + \beta$ is close but less than 1 indicating stationary and a high degree of persistence. Moreover, in EGARCH and APARCH models, the stationary condition is provided (β <1 for all EGARCH models and $-1 < \gamma < 1$ for all APARCH models), as well.

Furthermore, there is an asymmetric effect in the series, and it is provided by EGARCH, TGARCH and APARCH models. For instance, γ is significant and positive in EGARCH models, meaning that positive shocks have a greater impact on increasing volatility. In TGARCH models, the impact of negative news for the series is seen as $\alpha + \gamma$, otherwise, it could have seen impact as α. It is observed that γ is significant and negative in TGARCH models for the series. Thus, it also implies that positive shocks have a bigger impact on increasing volatility than negative shocks like APARCH models due to negative leverage parameters. Besides, it is seen that the power parameter, δ, in APARCH models is also significant (p-value is smaller than 5% significant level).

On top of that, the shape and skewness parameters in t, skew t, GED, and skew GED are found significant. Moreover, shape parameters in these distributions confirm that they have thicker tails than normal distribution as mentioned in Table 4.4 since the shape value (degree of freedom) is around 6-8 for the t and skew t. Also, it is smaller than 2 in GED and sGED. Therefore, considering the skewness and kurtosis may help to capture these behaviours of the log-return data.

According to Table 4.14, which shows the diagnostic results (p-values) on the standardized residuals of the fitted values, it can be concluded that there is no ARCH effect or auto-correlation in the standardized residuals since all p values are bigger than the 10% significance level except 2 p-values which are bigger than the 5% significance level. Hence, they are adequate models, and they seem to capture the volatility clustering.

Table 4.14: P-Values of the Diagnostic Tests for the Models in Period 1

GARCH

	norm			snorm			t			GED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.47	0.51	0.49	0.05	0.47	0.45	0.15	0.79	0.78	0.24	0.66	0.65
5	0.61	0.52	0.38	0.26	0.51	0.37	0.45	0.59	0.49	0.52	0.58	0.46
10	0.28	0.87	0.75	0.12	0.86	0.73	0.23	0.91	0.83	0.26	0.90	0.81
20	0.28	0.97	0.92	0.15	0.96	0.89	0.27	0.97	0.92	0.29	0.98	0.93

EGARCH

	norm			snorm			t			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.70	0.44	0.43	0.19	0.36	0.34	0.3	0.99	0.99	0.44	0.74	0.73	0.07	0.58	0.57
5	0.63	0.57	0.42	0.43	0.51	0.36	0.57	0.81	0.74	0.61	0.76	0.66	0.30	0.70	0.57
10	0.29	0.82	0.74	0.18	0.74	0.66	0.30	0.95	0.93	0.33	0.93	0.89	0.14	0.87	0.81
20	0.29	0.40	0.24	0.19	0.25	0.13	0.32	0.11	0.50	0.35	0.25	0.13	0.17	0.16	0.07

TGARCH

	norm			snorm			t			st			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.79	0.41	0.34	0.27	0.37	0.29	0.34	0.78	0.75	0.09	0.66	0.62	0.49	0.60	0.54	0.1	0.49	0.43
5	0.60	0.58	0.20	0.46	0.54	0.17	0.56	0.86	0.6	0.32	0.82	0.50	0.6	0.76	0.42	0.32	0.69	0.31
10	0.28	0.58	0.51	0.2	0.51	0.45	0.30	0.85	0.87	0.15	0.78	0.80	0.33	0.77	0.75	0.15	0.65	0.64
20	0.27	0.72	0.28	0.20	0.60	0.18	0.32	0.70	0.22	0.18	0.65	0.18	0.34	0.73	0.26	0.18	0.63	0.18

APARCH

	norm			snorm			t			GED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.56	0.96	0.96	0.07	0.72	0.73	0.28	0.88	0.88	0.32	0.9	0.9
5	0.65	0.68	0.66	0.34	0.64	0.65	0.58	0.76	0.73	0.59	0.72	0.69
10	0.28	0.93	0.92	0.16	0.89	0.89	0.29	0.96	0.94	0.3	0.95	0.93
20	0.32	0.99	0.99	0.19	0.92	0.9	0.35	0.99	0.99	0.35	0.99	0.99

Lastly, the selection criteria are considered given in Table 4.15 , and it is observed that the minimum AIC, BIC, and maximum log-likelihood belong to the ARMA (1, 0)-TGARCH (1, 1) with skew t distribution model among all models.

	Log-Likelihood	AIC	BIC
GARCH-norm	10576.77	-6.9460	-6.9361
GARCH-snorm	10606.16	-6.9646	-6.9528
GARCH-t	10659.97	-7.0000	-6.9881
GARCH-GED	10644.61	-6.9899	-6.9780
EGARCH-norm	10574.33	-6.9437	-6.9318
EGARCH-snorm	10600.82	-6.9605	-6.9466
EGARCH-t	10668.93	-7.0052	-6.9914
EGARCH-GED	10648.4	-6.9917	-6.9779
EGARCH-sGED	10671.47	-7.0062	-6.9904
TGARCH-norm	10547.41	-6.9260	-6.9142
TGARCH-snorm	10568.58	-6.9393	-6.9254
TGARCH-t	10659.16	-6.9988	-6.9849
TGARCH-st	**10678.72**	**-7.0110**	**-6.9952**
TGARCH-GED	10635.65	-6.9833	-6.9695
TGARCH-sGED	10657.08	-6.9968	-6.9809
APARCH-norm	10583.12	-6.9488	-6.9350
APARCH-snorm	10600.41	-6.9595	-6.9437
APARCH-t	10661.23	-6.9995	-6.9837
APARCH-GED	10647.56	-6.9905	-6.9747

Table 4.15: Fit Measures of the Models for Period 1

4.5.2 The Results of Period 2

In this period, according to ACF-PACF plots as a mean model ARMA(0,0) is suggested since the series is white noise. After, it is found that there is ARCH effect according to ARCH LM test (p-value=0 for all lags) and he Ljung-Box-Pierce Q statistic of the squared residuals (p-value=0 for all lags). After, heteroscedastic models are proposed.

14 out of 24 suggested models are found to be adequate according to the diagnostic checking for the log-return data. They are shown in bold in Figure 4.16.

Table 4.16: The Empirical Models for Period 2

The Empirical Models
GARCH(1,1) with normal distribution
GARCH(1,1) with skew normal distribution
GARCH(1,1) with student t distribution
GARCH(1,1) with skew student t distribution
GARCH(1,1) with GED distribution
GARCH(1,1) with skew GED distribution
EGARCH(1,1) with normal distribution
EGARCH(1,1) with skew normal distribution
EGARCH(1,1) with student t distribution
EGARCH(1,1) with skew student t distributio
EGARCH(1,1) with GED distribution
EGARCH(1,1) with skew GED distribution
GARCH(1,1) with normal distribution
TGARCH(1,1) with skew normal distribution
TGARCH(1,1) with student t distribution
TGARCH(1,1) with skew student t distribution
TGARCH(1,1) with GED distribution
TGARCH(1,1) with skew GED distribution
APARCH(1,1) with normal distribution
APARCH(1,1) with skew normal distribution
APARCH(1,1) with student t distribution
APARCH(1,1) with skew student t distribution
APARCH(1,1) with GED distribution
APARCH(1,1) with skew GED distribution

For the adequate models the estimates and diagnostic checking are demonstrated in the fol- lowing Tables 4.17 and 4.18, and for the inadequate models are shown in Tables A.5 and A.6, respectively.

The estimates and diagnostic checking of the models are given in the following Tables
.

Table 4.17: The Estimate of the Models for the Period 2

Period 2 : July 2013 -October 2016														
	ARMA(0,0)-GARCH(1,1)						**ARMA(0,0)-EGARCH(1,1)**					**ARMA(0,0)-TGARCH(1,1)**		
	norm	**snorm**	**t**	**st**	**GED**	**sGED**	**norm**	**snorm**	**t**	**st**	**GED**	**snorm**	**st**	**sGED**
c	0.000	0.000	0.000	0.000*	0.000	0.000*	0.000	0.000*	0.000	0.000*	0.000	0.000	0.000*	0.000*
ω	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	-2.001*	-1.78	-1.19*	-0.112*	-1.48	0.000*	0.00	0.000*
α	0.161*	0.153*	0.12*	0.105*	0.129*	0.109*	0.082*	0.08*	0.081*	0.072*	0.076*	0.029*	0.035*	0.032*
β	0.57*	0.591*	0.715*	0.758*	0.68*	0.748*	0.798*	0.82*	0.88*	0.989*	0.852*	0.956*	0.952*	0.953*
γ	-	-	-	-	-	-	0.221	0.205*	0.18*	-0.042*	0.198*	-1.00*	-1.00*	-1.00*
δ	-	-	-	-	-	-	-	-	-	-	-	-	-	-
shape	-	-	4.55*	4.572*	1.188*	1.174*	-	-	4.54*	4.010*	1.194*	-	4.524*	1.180*
skew	-	1.049*	-	1.096*	-	1.086*	-	1.059*	-	1.130*	-	1.095*	1.127*	1.117*

(*) indicates significant at 5% significance level.

From Table 4.17, the estimates of the variance equations and the ϕ parameter of all models are all significant at 0.05 except the some of the constant terms in the variance equations.

In GARCH models, all parameters that are in the variance equation are positive, and $\alpha+\beta$ is less than 1 indicating stationarity and it is not seen a high degree of persistence oppossed to Period 1. Moreover, in EGARCH, the stationarity condition is provided (β <1 for all EGARCH models), as well.

Additionally, there is an asymmetric effect in the series, and it is provided by EGARCH and TGARCH models. For instance, γ is significant and positive in EGARCH models, meaning that positive shocks have a greater impact on increasing volatility. It is observed that γ is significant and negative in TGARCH models for the series. So, it also implies that positive shocks have a bigger impact on increasing volatility than negative shocks.

Besides, the shape and skewness parameters in t, skew t, GED, and skew GED are found significant, and confirm that they have thicker tails than normal distribution as mentioned in Table 4.4 since the shape value (degree of freedom) is around 4.5 for the t and skew t. Also, it is smaller than 2 in GED and sGED.

According to Table 4.18, it can be concluded that there is no ARCH effect or autocorrelation in the standardized residuals since all p values are bigger than the 10% significance level except 4 p-values which are bigger than the 5% significance level. Hence, they are adequate models, and they seem to capture the volatility clustering well.

Table 4.18: P-Values of the Diagnostic Tests for the Models in Period 2

GARCH																		
	norm			snorm			t			st			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.22	0.85	0.86	0.22	0.91	0.91	0.20	0.75	0.75	0.19	0.64	0.64	0.21	0.87	0.87	0.19	0.70	0.70
5	0.71	0.93	0.94	0.71	0.94	0.95	0.73	0.98	0.98	0.73	0.98	0.98	0.72	0.98	0.98	0.72	0.98	0.98
10	0.55	0.97	0.97	0.56	0.97	0.97	0.62	0.96	0.96	0.65	0.94	0.94	0.6	0.96	0.96	0.63	0.94	0.94
20	0.83	0.98	0.99	0.83	0.99	0.99	0.86	0.99	0.99	0.88	0.99	0.99	0.85	0.99	0.99	0.87	0.99	0.99

EGARCH															
	snorm			snorm			t			st			GED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.28	0.92	0.92	0.26	0.85	0.85	0.25	0.74	0.74	0.18	0.02	0.02	0.27	0.82	0.82
5	0.81	0.97	0.97	0.81	0.97	0.97	0.82	0.97	0.97	0.76	0.05	0.09	0.82	0.97	0.97
10	0.59	0.97	0.97	0.60	0.97	0.97	0.61	0.93	0.93	0.59	0.21	0.31	0.61	0.95	0.95
20	0.82	0.99	0.98	0.83	0.98	0.98	0.84	0.98	0.98	0.72	0.71	0.83	0.84	0.99	0.98

TGARCH									
	snorm			t			GED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.24	0.09	0.10	0.24	0.12	0.13	0.26	0.12	0.12
5	0.88	0.45	0.43	0.87	0.52	0.50	0.88	0.52	0.50
10	0.67	0.52	0.54	0.63	0.54	0.55	0.64	0.54	0.56
20	0.86	0.92	0.92	0.85	0.92	0.91	0.85	0.92	0.92

Finally, the selection criteria are investigated given in Table 4.19, It is observed that the minimum AIC, BIC, and maximum log-likelihood belongs to the EGARCH (1, 1) with skew t distribution model.

Table 4.19: Fit Measures of the Models for Period 2

	Log-Likelihood	**AIC**	**BIC**
GARCH-norm	2953.224	-7.1151	-7.0924
GARCH-snorm	2954.022	-7.1146	-7.0862
GARCH-t	2989.995	-7.2014	-7.1730
GARCH-st	2991.919	-7.2037	-7.1695
GARCH-GED	2988.807	-7.1986	-7.1701
GARCH-sGED	2991.26	-7.2021	-7.1679
EGARCH-norm	2955.152	-7.1174	-7.0889
EGARCH-snorm	2956.306	-7.1177	-7.0836
EGARCH-t	2991.862	-7.2035	-7.1694
EGARCH-st	**3002.573**	**-7.2270**	**-7.1871**
EGARCH-GED	2990.212	-7.1995	-7.1654
TGARCH-snorm	2956.849	-7.1191	-7.0849
TGARCH-t	2992.696	-7.2055	-7.1714
TGARCH-GED	2990.204	-7.1995	-7.1654

4.5.3 The Results of Period 3

Outliers in all periods are left the same without applying any procedures to them, except for two outliers caused by the political crisis on August 14 and 15, 2018. Additionally, these two anomalies in the log-return data are apparent (the log-return value of these consecutive days is bigger than the value |0.2|), and they are replaced by the average of their previous and next observations.

Subsequently , as a mean model ARMA(0,1) is suggested based on the ACF-PACF plots, and fitted to the data. After specifying, it is obtained that ARMA(0,1) model provides residuals having no serial correlation according to the Box-Ljung test statistic (p-value= 0.92). Moreover, it is found that there is ARCH effect according to ARCH LM test (p-value=0 for all lags) and the Ljung-Box-Pierce Q statistic of the squared residuals (p-value=0 for all lags). Therefore, this model is obtained as a candidate mean model.

After, 11 out of 24 suggested models are found to be adequate according to the diagnostic checking for the log-return data. They are shown in bold in Figure 4.20

Table 4.20: The Empirical Models for Period 3

The Empirical Models
ARMA(0,1)-GARCH(1,1) with normal distribution
ARMA(0,1)-GARCH(1,1) with skew normal distribution
ARMA(0,1)-GARCH(1,1) with student t distribution
ARMA(0,1)-GARCH(1,1) with skew student t distribution
ARMA(0,1)-GARCH(1,1) with GED distribution
ARMA(0,1)-GARCH(1,1) with skew GED distribution
ARMA(0,1)-EGARCH(1,1) with normal distribution
ARMA(0,1)-EGARCH(1,1) with skew normal distribution
ARMA(0,1)-EGARCH(1,1) with student t distribution
ARMA(0,1)-EGARCH(1,1) with skew student t distribution
ARMA(0,1)-EGARCH(1,1) with GED distribution
ARMA(0,1)-EGARCH(1,1) with skew GED distribution
ARMA(1,0)-TGARCH(1,1) with normal distribution
ARMA(0,1)-TGARCH(1,1) with skew normal distribution
ARMA(0,1)-TGARCH(1,1) with student t distribution
ARMA(0,1)-TGARCH(1,1) with skew student t distribution
ARMA(0,1)-TGARCH(1,1) with GED distribution
ARMA(0,1)-TGARCH(1,1) with skew GED distribution
ARMA(0,1)-APARCH(1,1) with normal distribution
ARMA(0,1)-APARCH(1,1) with skew normal distribution
ARMA(0,1)-APARCH(1,1) with student t distribution
ARMA(0,1)-APARCH(1,1) with skew student t distribution
ARMA(0,1)-APARCH(1,1) with GED distribution
ARMA(0,1)-APARCH(1,1) with skew GED distribution

For the adequate models the estimates and diagnostic checking are shown in the following Tables 4.21 and 4.22, and for the inadequate models they are displayed in Tables A.7 and A.8.

Table 4.21: The Estimate of the Models for the Period 3

Period 3 : October 2016- February 2020													
	ARMA(0,1)-GARCH(1,1)					**ARMA(0,1)-EGARCH(1,1)**		**ARMA(0,1)-TGARCH(1,1)**			**ARMA(0,1)-APARCH(1,1)**	
	norm	**snorm**	**t**	**st**	**sGED**	**norm**	**snorm**	**norm**	**GED**	**sGED**	**norm**	**sGED**
c	0.000	0.00*	0.000	0.00*	0.000*	0.000*	0.000*	0.00*	0.000*	0.00*	0.000*	0.000*
θ	0.092*	0.089*	0.094*	0.093*	0.107*	0.06*	0.07*	0.059*	0.095*	0.105*	0.059*	0.11*
ω	0.000	0.000	0.000	0.000	0.000	-0.258*	-0.235*	0.000*	0.00	0.00*	0.000	0.00*
α	0.212*	0.177*	0.1635*	0.161**	0.16*	0.113*	0.103*	0.134*	0.135*	0.126*	0.134*	0.127*
β	0.787*	0.819*	0.8355*	0.837*	0.838*	0.972*	0.974*	0.876*	0.876*	0.886*	0.876*	0.883*
γ	-	-	-	-	-	0.237*	0.214*	-0.52*	-0.357*	-0.377*	-0.525*	-0.345*
δ	-	-	-	-	-	-	-	-	-	-	0.978*	1.188*
shape	-	-	4.287*	4.42*	1.124*	-	-	-	1.153*	1.168*	-	1.168*
skew	-	1.2*	-	1.105*	1.106*	-	1.161*	-	-	1.113*	-	1.112*

(*) indicates significant at 5% significance level.

From Table 4.21, the estimates of the variance equations and the θ parameter of all models are all significant at 0.05 significance levels except the some constant terms in the variance equations.

In GARCH models, all parameters that are in the variance equation are positive, and $\alpha + \beta$ is close but less than 1 indicating stationary and a high degree of persistence like in Period 1. Moreover, since $\beta < 1$ for all EGARCH models and $-1 < \gamma < 1$ for all APARCH models, EGARCH and APARCH models hold the stationary condition, as well.

Besides, an asymmetric effect in the series demonstrated by EGARCH, TGARCH and APARCH model'estimates. Significant and positive γ in EGARCH models, significant and negative γ in TGARCH models, and significant and negative leverage parameter γ in APARCH models indicate not just the leverage effect but also the fact that positive shocks have a bigger impact on increasing volatility than negative shocks.

Additionally, it is seen that the power parameter, δ, in APARCH models is also significant (p-value is smaller than 5% significant level).

Considering the shape and skewness parameters in t, skew t, GED, and skew GED, they all are found significant, and indicate that they have thicker tails than normal distribution since the shape value (degree of freedom) is around 4 for the t and skew t. Also, it is smaller than 2 in GED and sGED.

According to Table 4.22, which shows the diagnostic results (p-values) on the standardized residuals of the fitted values, it can be concluded that there is no ARCH effect or auto-correlation in the standardized residuals since all p values are bigger than the 10% significance level except two p-values which are bigger than the 5% significance level. Hence, they are adequate models, and they seem to capture the volatility clustering.

Table 4.22: P-Values of the Diagnostic Tests for the Models in Period 3

GARCH

	norm			snorm			t			st			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.13	0.89	0.88	0.13	0.75	0.75	0.18	0.66	0.66	0.17	0.67	0.67	0.29	0.67	0.67
5	0.33	0.98	0.98	0.32	0.97	0.97	0.38	0.96	0.96	0.36	0.96	0.96	0.45	0.96	0.96
10	0.21	0.93	0.94	0.2	0.95	0.95	0.25	0.95	0.95	0.22	0.95	0.95	0.28	0.95	0.95
20	0.57	0.91	0.9	0.57	0.98	0.98	0.64	0.99	0.99	0.6	0.99	0.99	0.66	0.99	0.99

EGARCH

	norm			snorm		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.08	0.44	0.44	0.11	0.33	0.33
5	0.16	0.86	0.87	0.2	0.71	0.74
10	0.17	0.89	0.91	0.18	0.82	0.87
20	0.56	0.76	0.79	0.56	0.8	0.84

TGARCH

	norm			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.08	0.36	0.36	0.36	0.37	0.38	0.51	0.29	0.29
5	0.17	0.7	0.72	0.37	0.67	0.69	0.39	0.49	0.53
10	0.17	0.76	0.82	0.32	0.78	0.83	0.3	0.66	0.74
20	0.54	0.72	0.76	0.73	0.79	0.83	0.71	0.77	0.83

APARCH

	norm			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.08	0.35	0.35	0.57	0.37	0.37
5	0.17	0.67	0.7	0.42	0.68	0.7
10	0.17	0.74	0.81	0.32	0.77	0.83
20	0.54	0.7	0.75	0.72	0.87	0.9

Finally, the selection criteria are considered given in Table 4.23 , and it is seen that the minimum AIC and the maximum Lo-Likelihood belong to the ARMA (0,1)-TGARCH (1, 1) with skew GED distribution model, and the minimum BIC belongs to the ARMA (0,1)-GARCH (1, 1) with t distribution model among all models. These two models are competing.

Table 4.23: Fit Measures of the Models for Period 3

	Log-Likelihood	**AIC**	**BIC**
GARCH-norm	2795.608	-6.7325	-6.7040
GARCH-snorm	2806.448	-6.7562	-6.7220
GARCH-t	2854.867	-6.8730	**-6.8389**
GARCH-st	2857.232	-6.8763	-6.8365
GARCH-sGED	2853.787	-6.8680	-6.8281
EGARCH-norm	2812.091	-6.7698	-6.7357
EGARCH-snorm	2819.396	-6.7850	-6.7452
TGARCH-norm	2816.435	-6.7803	-6.7461
TGARCH-GED	2856.149	-6.8737	-6.8338
TGARCH-sGED	**2859.606**	**-6.8796**	-6.8341
APARCH-norm	2816.441	-6.7779	-6.7380
APARCH-sGED	2859.832	-6.8778	-6.8265

4.5.4 The Results of Period 4

Firstly, according to ACF-PACF plots as a mean model ARMA(1,0) is suggested and fitted to the data, and the result is obtained that ARMA(1,0) provides residuals having no serial correlation according to the Box-Ljung test statistic (p-value=0.8017). Moreover, it is found that there is ARCH effect according to ARCH LM test (p-value=0 for all lags). Therefore, this model is obtained as a candidate mean model.

8 out of 24 suggested models are found to be adequate according to the diagnostic checking for the log-return data. They are shown in red color in Figure4.24.

Table 4.24: The Empirical Models for Period 1

The Empirical Models
ARMA(1,0)-GARCH(1,1) with normal distribution
ARMA(1,0)-GARCH(1,1) with skew normal distribution
ARMA(1,0)-GARCH(1,1) with student t distribution
ARMA(1,0)-GARCH(1,1) with skew student t distribution
ARMA(1,0)-GARCH(1,1) with GED distribution
ARMA(1,0)-GARCH(1,1) with skew GED distribution
ARMA(1,0)-EGARCH(1,1) with normal distribution
ARMA(1,0)-EGARCH(1,1) with skew normal distribution
ARMA(1,0)-EGARCH(1,1) with student t distribution
ARMA(1,0)-EGARCH(1,1) with skew student t distribution
ARMA(1,0)-EGARCH(1,1) with GED distribution
ARMA(1,0)-EGARCH(1,1) with skew GED distribution
ARMA(1,0)-TGARCH(1,1) with normal distribution
ARMA(1,0)-TGARCH(1,1) with skew normal distribution
ARMA(1,0)-TGARCH(1,1) with student t distribution
ARMA(1,0)-TGARCH(1,1) with skew student t distribution
ARMA(1,0)-TGARCH(1,1) with GED distribution
ARMA(1,0)-TGARCH(1,1) with skew GED distribution
ARMA(1,0)-APARCH(1,1) with normal distribution
ARMA(1,0)-APARCH(1,1) with skew normal distribution
ARMA(1,0)-APARCH(1,1) with student t distribution
ARMA(1,0)-APARCH(1,1) with skew student t distribution
ARMA(1,0)-APARCH(1,1) with GED distribution
ARMA(1,0)-APARCH(1,1) with skew GED distribution

For the adequate models the estimates and diagnostic checking of the models are given in the following Tables 4.25 and 4.26, and for the inadequate models they are shown in Tables A.9 and A.10, respectively.

From Table 4.25, except some of the constant components in the variance equations, the estimates of the variance equations and the ϕ parameters of all models are all significant at 0.05 significance levels.

All parameters that are in the variance equation are positive, and $\alpha + \beta$ is close but less than 1 indicating stationary and a high degree of persistence in GARCH models.

Table 4.25: The Estimate of the Models for the Period 4

Period 4 : February 2020-June 2023								
	ARMA(1,0)-GARCH(1,1)				**ARMA(1,0)-EGARCH(1,1)**			
	t	**st**	**GED**	**sGED**	**t**	**st**	**GED**	**sGED**
c	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
ϕ	0.329*	0.331*	0.294*	0.335*	0.331*	0.326*	0.298*	0.316*
ω	0.000	0.000	0.000	0.00	-0.156*	-0.153*	-0.221*	-0.239*
α	0.309*	0.307*	0.262*	0.263*	0.154*	0.163*	0.07*	0.081*
β	0.686*	0.692*	0.738*	0.734*	0.984*	0.984*	0.98*	0.978*
γ	-	-	-	-	0.999*	1.035*	0.51*	0.547*
δ	-	-	-	-	-	-	-	-
shape	3.496*	3.534*	0.802*	0.805*	2.132*	2.122*	0.716*	0.7*
skew	-	1.058*	-	1.039*	-	0.975*	-	0.937*

(*) indicates significant at 5% significance level.

Moreover, all EGARCH models hold the stationary condition since $\beta < 1$.

Besides, an asymmetric effect in the series demonstrated by significant and positive γ in EGARCH models, and it indicates the fact that positive shocks have a bigger impact on increasing volatility than negative shocks.

The shape and skewness parameters in t, skew t, GED, and skew GED are observed, and they all are seen significant. They indicate that they have thicker tails than normal distribution since the shape value (degree of freedom) is around 2-4 for the t and skew t. Also, it is smaller than 2 in GED and sGED.

According to Table 4.26, it can be concluded that there is no ARCH effect or autocorrelation in the standardized residuals since all p-values are bigger than the 10% significance level except one p-value which is bigger than the 5significance level. All in all, they are adequate models and seem to capture the volatility clustering.

Table 4.26: P-Values of the Diagnostic Tests for the Models in Period 4

GARCH

	t			st			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.99	0.87	0.87	0.99	0.87	0.87	0.78	0.96	0.96	0.78	0.95	0.95
5	0.12	0.99	0.99	0.10	0.99	0.99	0.05	0.99	0.99	0.08	0.99	0.99
10	0.45	1.00	1.00	0.35	0.99	0.99	0.22	0.99	0.99	0.31	0.99	0.99
20	0.73	1.00	1.00	0.67	1.00	1.00	0.52	1.00	1.00	0.62	1.00	1.00

EGARCH

	t			st			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.24	0.89	0.89	0.25	0.88	0.88	0.46	0.89	0.89	0.30	0.87	0.87
5	0.18	0.99	0.99	0.19	0.99	0.99	0.19	0.99	0.99	0.25	0.99	0.99
10	0.44	0.99	0.99	0.48	0.99	0.99	0.49	0.99	0.99	0.6	0.99	0.99
20	0.8	1.00	1.00	0.83	1.00	1.00	0.81	1.00	1.00	0.88	1.00	1.00

Lastly, the selection criteria are reviewed in Table 4.27, and the ARMA (1,0)-EGARCH (1,1) with t distribution model has been found the lowest AIC and BIC of all models, but it is seen the maximum Log-Likelihood value belongs to the ARMA (1,0)-EGARCH (1,1) with skew t distribution model. It is because of the the number of the parameters in the model.

Table 4.27: Fit Measures of the Models for Period 4

	Log-Likelihood	**AIC**	**BIC**
GARCH-t	3423.651	-8.2353	-8.2012
GARCH-st	3425.389	-8.2371	-8.1973
GARCH-GED	3408.929	-8.1998	-8.1657
GARCH-sGED	3408.052	-8.1953	-8.1555
EGARCH-t	3438.828	**-8.2695**	**-8.2296**
EGARCH-st	**3439.081**	-8.2677	-8.2222
EGARCH-GED	3421.216	-8.2270	-8.1872
EGARCH-sGED	3420.935	-8.2239	-8.1784

4.6 Summary

The empirical results for each time interval are presented the best models for their own period as the following.

<u>**The Entire Period (June 2001- June 2023)**</u>

ARMA(4,3)-EGARCH(1,1) with skew t

<u>**Period 1 (June 2001- July 2013)**</u>

ARMA(1,0)-TGARCH(1,1) with skew t

<u>**Period 2 (July 2013- October 2016)**</u>

EGARCH(1,1) with skew t

<u>**Period 3 (October 2016- February 2020)**</u>

ARMA(0,1)-GARCH(1,1) with t

ARMA(0,1)-TGARCH(1,1) with skew GED

<u>**Period 4 (February 2020- June 2023)**</u>

ARMA(1,0)-EGARCH(1,1) with t

CHAPTER 5

CONCLUSION

Modeling exchange rate volatility is an important concern for scholars, investors, and regulators. Therefore, modeling exchange rate volatility is necessary. So, in this study, volatility models are presented, and the best-fitting models are suggested. GARCH-type models are employed to accomplish the goal since the return rate has an ARCH effect, as Aysoy et al. [9] showed nearly four decades ago. The models used are GARCH as a symmetrical model, and EGARCH, TGARCH, and APARCH as asymmetrical models.

Besides, skew normal, skew student t, and skew GED distributions, as well as normal, student t and GED distributions for return errors are utilized to capture the leptokurtic behavior and the leverage effect of the CBRT's closing prices in US dollars from June 2001 to June 2023. However, after an investigation of breakpoints in the data, it is divided into four periods. Thus, four periods are studied separately, and the results are presented in Chapter 4.

It is observed that asymmetric models provide the best fitting, and there is a clear indication that leverage effects are present in the series for all sub-periods including the entire data set, since TGARCH and EGARCH models in the time intervals are found as the best-fitting model. This finding is in line with the findings of Oztürk [45], Caglayan et al. [20], Soytas et al. [48], Ozdemir[27], Saglam et al. [46], Guler [34], Yaman et al. [56], and Yıldırım et al. [57]. Added to that, as illustrated in Periods 3, a symmetric model like GARCH has the potential to offer a competing depending on the period characteristic. Because of that, analyzing of their forecasting performance can be beneficial for determining which model is the best model between two models.

Aside from selecting GARCH-type models, the distribution of residuals plays a key role when modeling volatility, as well. Non-normal and skew distributions are generally at the forefront of modeling regarding their performance since it is known that skewness and kurtosis can frequently be seen in the daily exchange rate of return. Table 4.4 which shows the descriptive statistics of all periods and the results of the non-normal and skew distributions given in Table 4.13 4.17 4.21 and 4.25, with respect to their significant shape and skewness parameters, revealed that using various distributions other than normal distribution to capture excess kurtosis, skewness and volatility clustering is beneficial. Moreover, we obtain that, as summarized in Section 4.6, the model with t, skew t, and skew GED specifications can provide the best model among the other models.

Another issue is in terms of the time intervals, since a whole and its sub-periods are separately taken in the research. Each time interval has its own scenarios mentioned in Section 4.3, and in this thesis, we are not only attempting to model the overall picture, but also period by period, i.e., scenario by scenario, so that it can be used in the future when dealing with a time span such as the sub-periods. In the beginning, the whole data set is taken into account, and only two models are found as a adequate model among 24 models that are proposed. Additionally, the high order in the mean model is seen. On the contrary, each sub-period has the mean model with the low order. Furthermore, distribution in the periods are different , since based on their features, for example, skew t distribution outperforms t distribution in the entire data set, Period 1 and Period 2. However, the reverse is true for Period 3 and 4.

In the study of Yıldırım [57], it is demonstrated that ARMA(1,1)-EGARCH (1,1) with t distribution is a better model than the skew ones after the ARMA(1,1)-NGARCH(1,1) with skew t distribution, and the data in this study covers January 2016 to December 2018. Furthermore, the empirical results in this thesis show that ARMA(0,1)-GARCH(1,1) with t and skew t specifications are found to be better than ARMA(0,1)-EGARCH (1,1) model with t distribution for the Period 3. The disparities between the two research could be attributed to the different time intervals. Yıldırıms' work covers from January 2016 to December 2018, whereas the data for Period 3 in the thesis covers from October 2016 to February 2020.

Additionally, as observed in the articles of Guler [34] and Ozdemir [27], TGARCH model plays an important role when modeling volatility since it is seen in Period 3 and 4 in this study with skew t and sGED, respectively.

Modeling exchange rate volatility is important since it has a broad impact on the country's economy. The volatility behavior may affect the behavior of the investors,especially in a country where they decide whether the investment should be made or not. In recent years in Turkey, it has been witnessed that unexpected high volatility can have severe consequences in all industries. Therefore, the studies on modeling and forecasting of it should be considered, and it is a need to take a measurement regarding the volatility of the exchange rate return. Furthermore, it is known that the positive shock means the depreciation of the Turkish Lira, and the leverage effect, in which it is concluded that positive shocks have a bigger effect than the negative shocks in all times, is observed in all time intervals. The reason behind the presence of this kind of leverage effect in the series is because of the past economic crisis and bad policies that make investors react suddenly and extremely. Therefore, policymakers need to take into account this stylized fact revealed in this research and take measures for the behavior of the market

Since this thesis has carried out the study of modeling the volatility of the USD/TL exchange rate return, it may pave the step for further research regarding volatility modeling and forecasting. Moreover, based on the findings, this study is expected to contribute to a better understanding of the volatility structure of the exchange rate.

APPENDIX A

INADEQUATE MODELS

The models that are found inadequate according to the procedure 3.1 are reported in the following sections.

A.1 The Entire Data Set

Period : June 2001-June 2023										
	ARMA(4,3)-GARCH(1,1)						ARMA(4,3)-EGARCH(1,1)			
	norm	**snorm**	**t**	**st**	**GED**	**sGED**	**norm**	**snorm**	**t**	**sGED**
c	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
ϕ_1	-0.368*	-1.326*	0.129*	-0.275*	-0.285*	-0.377*	-0.203*	-0.479*	0.041*	0.254*
ϕ_2	-0.426*	0.027*	0.912*	0.073*	-0.087*	0.924*	-0.510*	-0.472*	-0.491*	0.676*
ϕ_3	-0.946*	0.469*	-0.174*	-0.569*	-0.564*	0.388*	-0.707*	-0.925*	-0.655*	-0.006
ϕ_4	0.072*	-0.036*	0.043*	0.0581*	0.067*	-0.009	0.065*	0.061*	0.084*	0.021*
θ_1	0.439*	1.382*	-0.037*	0.362*	0.387*	0.451*	0.287*	0.556*	0.069*	-0.167*
θ_2	0.456*	0.032*	-0.929*	0.0432*	0.118*	-0.895*	0.537*	0.514*	0.503*	-0.691*
θ_3	0.975*	0.496*	0.108*	0.573*	0.579*	-0.433*	0.727*	0.953*	0.714*	-0.042*
ω	0.000	0.000	0.000	0.000	0.000	0.000	-0.175*	-0.160*	-0.152*	-0.162*
α	0.112*	0.106*	0.138*	0.134*	0.124*	0.122*	0.066*	0.063*	0.053*	0.066*
β	0.886*	0.893*	0.861*	0.864*	0.875*	0.877*	0.981*	0.982*	0.985*	0.984*
γ	-	-	-	-	-	-	0.274*	0.255*	0.361*	0.305*
δ	-	-	-	-	-	-	-	-	-	-
shape	-	-	5.631*	5.768*	1.190*	1.188*	-	-	4.410*	1.156*
skew	-	1.124*	-	1.109*	-	1.114*	-	1.117*	-	1.125*

Period : June 2001-June 2023												
	ARMA(4,3)-TGARCH(1,1)						ARMA(4,3)-APARCH(1,1)					
	norm	**snorm**	**t**	**st**	**GED**	**sGED**	**norm**	**snorm**	**t**	**st**	**GED**	**sGED**
c	0.000*	0.000*	0.000*	0.000*	0.000*	NA	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
ϕ_1	-0.183*	-0.156*	-0.845*	-0.502*	-0.846*	NA	0.565*	-0.334*	-0.849*	-0.869*	-0.854*	2.774*
ϕ_2	-0.489*	0.320*	0.961*	0.972*	0.959*	NA	0.075	-0.286*	0.959*	0.960*	0.954*	-2.763*
ϕ_3	-0.774*	0.793*	0.830*	0.469*	0.835*	NA	0.327*	-0.750*	0.834*	0.859*	0.841*	1.046*
ϕ_4	0.097*	-0.038*	-0.074*	0.028*	-0.079*	NA	-0.006	0.049*	-0.072*	-0.058*	-0.077*	-0.061*
θ_1	0.278*	0.240*	0.956*	0.602*	0.953*	NA	-0.492*	0.397*	0.957*	0.964*	0.959*	-2.696*
θ_2	0.529*	-0.306*	0.838*	-0.900*	-0.843*	NA	-0.118*	0.299*	-0.839*	-0.850*	-0.840*	2.562*
θ_3	0.808*	-0.834*	-0.885*	-0.522*	-0.896*	NA	-0.334*	0.758*	-0.886*	-0.898*	-0.901*	-0.858*
ω	0.000*	0.000*	0.000*	0.000*	0.000*	NA	0.000	0.000*	0.000	0.000	0.000*	0.000
α	0.143*	0.133*	0.187*	0.177*	0.169*	NA	0.086*	0.088*	0.171*	0.164*	0.166*	0.174*
β	0.892*	0.893*	0.865*	0.872*	0.876*	NA	0.901*	0.903*	0.875*	0.879*	0.876*	0.874*
γ	-0.254*	-0.272*	-0.201*	-0.217*	-0.205*	NA	-0.141*	-0.117*	-0.202*	-0.205*	-0.190*	-0.162*
δ	-	-	-	-	-	-	2.239*	2.164*	1.087*	1.100*	1.134*	1.421*
shape	-	-	4.270*	4.324*	1.122*	NA	-	-	1.087*	4.442*	1.129*	1.131*
skew	-	1.105*	-	1.124*	-	NA	-	1.104*	4.342*	1.124*	-	1.123*

(*) indicates significant at 5% significance level.

Table A.1: The Estimate of the Inadequate Models

GARCH

	norm			snorm			t			st			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.03	0.01	0.02	0.02	0.15	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.00	0.01	0.00
10	0.00	0.01	0.00	0.00	0.00	0.00	0.04	0.07	0.03	0.00	0.05	0.03	0.00	0.02	0.01	0.00	0.03	0.01
20	0.00	0.08	0.06	0.00	0.03	0.02	0.15	0.14	0.09	0.00	0.11	0.08	0.00	0.07	0.05	0.02	0.08	0.05

EGARCH

	norm			snorm			t			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.05	0.04	0.03	0.01	0.02	0.01	0.54	0.51	0.51	0.07	0.17	0.16
5	0.00	0.25	0.21	0.00	0.12	0.10	0.02	0.80	0.76	0.28	0.59	0.07
10	0.00	0.40	0.25	0.00	0.26	0.16	0.00	0.63	0.42	0.48	0.54	0.35
20	0.00	0.66	0.53	0.00	0.55	0.45	0.00	0.71	0.49	0.66	0.66	0.51

TGARCH

	norm			snorm			t			st			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.19	0.00	0.00	0.07	0.00	0.00	0.31	0.02	0.02	0.10	0.00	0.00	0.33	0.00	0.00	NA	NA	NA
5	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00	0.00	0.17	0.00	0.00	0.15	0.00	0.00	NA	NA	NA
10	0.00	0.00	0.01	0.00	0.00	0.02	0.45	0.03	0.00	0.38	0.00	0.00	0.32	0.01	0.00	NA	NA	NA
20	0.00	0.05	0.11	0.02	0.03	0.19	0.50	0.18	0.08	0.47	0.07	0.02	0.46	0.09	0.03	NA	NA	NA

APARCH

	norm			snorm			t			st			GED			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.00	0.05	0.00	0.00	0.28	0.01	0.00	0.01	0.02	0.02
5	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.7	0.00	0.00	0.16	0.00	0.00	0.02	0.02	0.01
10	0.00	0.02	0.01	0.00	0.00	0.00	0.24	0.00	0.00	0.19	0.00	0.00	0.28	0.03	0.0	0.09	0.12	0.02
20	0.00	0.10	0.09	0.00	0.06	0.05	0.30	0.02	0.00	0.24	0.00	0.00	0.45	0.11	0.04	0.25	0.21	0.06

Table A.2: P-Values of the Diagnostic Tests for the Inadequate Models

A.2 Period 1

Period 1 : June 2001-July 2013					
	ARMA(1,0)-GARCH(1,1)		ARMA(1,0)-EGARCH(1,1)	ARMA(1,0)-APARCH(1,1)	
	st	sGED	st	st	sGED
c	0.000	0.000	0.000	0.000	0.000
ϕ	0.027	0.025	0.045*	0.039*	0.035
ω	0.000	0.000	-0.357*	0.00*	0.000
α	0.148*	0.141*	0.069*	0.148*	0.095*
β	0.849*	0.853*	0.964*	0.847*	0.857*
γ	-	-	0.285*	-0.172*	-0.094*
δ	-	-	-	1.763*	2.602*
shape	6.834*	1.399*	6.492*	6.751*	1.437*
skew	1.181*	1.176*	1.181*	1.179*	1.171*

(*) indicates significant at 5% significance level.

Table A.3: The Estimate of the Inadequate Models for the Period 1

GARCH							EGARCH		
	st			sGED			st		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.02	0.63	0.62	0.02	0.53	0.51	0.08	0.86	0.85
5	0.14	0.56	0.43	0.13	0.53	0.4	0.32	0.79	0.7
10	0.08	0.89	0.79	0.07	0.88	0.76	0.15	0.92	0.89
20	0.12	0.96	0.9	0.11	0.96	0.9	0.18	0.08	0.03

APARCH						
	st			sGED		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.049	0.99	0.99	0.04	0.57	0.56
5	0.26	0.74	0.63	0.24	0.66	0.6
10	0.13	0.95	0.91	0.12	0.91	0.88
20	0.18	0.99	0.95	0.17	0.99	0.98

Table A.4: P-Values of the Diagnostic Tests for the Inadequate Models in Period 1

A.3 Period 2

Period 2 : July 2013 -October 2016										
	EGARCH(1,1)	**TGARCH(1,1)**			**APARCH(1,1)**					
	sGED	**norm**	**st**	**sGED**	**norm**	**snorm**	**t**	**st**	**GED**	**sGED**
ω	-0.11*	0.000	0.000	0.00	0.000	0.000	0.000	0.00*	0.000	0.000
α	0.06*	0.032	0.035*	0.03*	0.045	0.052	0.044	0.05*	0.04*	0.04*
β	0.99*	0.94*	0.95*	0.94*	0.88*	0.89*	0.88*	0.86*	0.89*	0.89*
γ	-0.05	-1.00	-1.00	-1.00	-0.125	-0.11*	-0.26*	-0.19	-0.21	-0.229
δ	-	-	-	-	2.89*	2.76*	2.85*	2.85*	2.79*	2.74*
shape	1.16*	-	4.51*	1.19*	-	-	4.19*	4.69*	1.14*	1.13*
skew	1.11*	-	-	-	-	1.09*	-	1.1*	-	1.11*

(*) indicates significant at 5% significance level.

Table A.5: The Estimate of the Inadequate Models for the Period 2

EGARCH				**TGARCH**								
	sGED			**norm**			**st**			**sGED**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.20	0.01	0.01	0.26	0.13	0.13	0.23	0.13	0.13	0.23	0.12	0.12
5	0.79	0.05	0.05	0.89	0.55	0.54	0.86	0.52	0.50	0.87	0.50	0.49
10	0.61	0.22	0.32	0.66	0.6	0.62	0.62	0.56	0.58	0.63	0.55	0.57
20	0.73	0.75	0.86	0.86	0.94	0.93	0.85	0.93	0.91	0.85	0.93	0.92

APARCH																		
	norm			**snorm**			**t**			**st**			**GED**			**sGED**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.15	0.42	0.42	0.15	0.42	0.42	0.20	0.48	0.48	0.19	0.53	0.53	0.20	0.37	0.37	0.19	0.43	0.43
5	0.74	0.94	0.94	0.74	0.94	0.94	0.81	0.95	0.95	0.79	0.97	0.97	0.82	0.93	0.92	0.80	0.94	0.93
10	0.62	0.74	0.77	0.62	0.74	0.77	0.61	0.80	0.81	0.63	0.84	0.85	0.63	0.78	0.80	0.62	0.79	0.81
20	0.87	0.98	0.97	0.87	0.97	0.96	0.85	0.98	0.97	0.86	0.98	0.98	0.86	0.98	0.97	0.86	0.97	0.96

Table A.6: P-Values of the Diagnostic Tests for the Inadequate Models in Period 2

A.4 Period 3

Period 3 : October 2016- February 2020												
	ARMA(0,1)-GARCH(1,1)	**ARMA(0,1)-EGARCH(1,1)**				**ARMA(0,1)-TGARCH(1,1)**			**ARMA(0,1)-APARCH(1,1)**			
	GED	**t**	**st**	**GED**	**sGED**	**snorm**	**t**	**st**	**snorm**	**t**	**st**	**GED**
c	0.000	0.000	0.000*	0.000*	0.000	0.000	0.000*	0.000	0.000*	0.000	0.000*	0.000
θ	0.102*	0.093*	0.094*	0.096*	0.108	0.066	0.089*	0.090*	0.077	0.091*	0.093*	0.096
ω	0.000	-0.361	-0.330*	-0.298	-0.282	0.000*	0.000	0.000	0.000	0.000	0.000	0.000
α	0.167	0.052	0.054	0.074*	0.075	0.118*	0.151*	0.146*	0.119*	0.152*	0.147*	0.136*
β	0.831*	0.963*	0.966*	0.970*	0.971*	0.890*	0.874*	0.887*	0.865*	0.87*	0.872*	0.874*
γ	-	0.313	0.307*	0.262	0.253	-0.530*	-0.246	-0.255	-0.492*	-0.227	-0.234	-0.332*
δ	-	-	-	-	-	-	-	-	1.197*	1.157*	1.158*	1.146
shape	1.106*	4.285*	4.367*	1.138*	1.157	-	4.410*	4.474*	-	4.410*	4.488*	1.153
skew	-	-	1.108*	-	1.109	1.155*	-	1.113*	1.165*	-	1.113*	-

(*) indicates significant at 5% significance level.

Table A.7: The Estimate of the Inadequate Models for the Period 3

	GARCH			**EGARCH**											
	GED			**t**			**st**			**GED**			**sGED**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.25	0.70	0.70	0.22	0.62	0.62	0.21	0.59	0.59	0.33	0.51	0.51	0.47	0.45	0.45
5	0.44	0.97	0.97	0.37	0.96	0.96	0.36	0.95	0.95	0.40	0.91	0.92	0.43	0.87	0.87
10	0.29	0.95	0.95	0.25	0.96	0.96	0.23	0.95	0.96	0.32	0.93	0.94	0.30	0.90	0.93
20	0.69	0.99	0.99	0.67	0.68	0.70	0.64	0.76	0.80	0.73	0.78	0.85	0.72	0.82	0.84

TGARCH									
	snorm			**t**			**st**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.12	0.25	0.25	0.23	0.42	0.42	0.25	0.37	0.37
5	0.19	0.46	0.50	0.33	0.74	0.75	0.32	0.65	0.67
10	0.18	0.62	0.71	0.26	0.83	0.87	0.23	0.77	0.83
20	0.54	0.71	0.78	0.68	0.83	0.86	0.66	0.84	0.88

APARCH												
	snorm			**t**			**st**			**GED**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.25	0.37	0.37	0.25	0.48	0.48	0.26	0.44	0.44	0.37	0.44	0.44
5	0.32	0.65	0.67	0.35	0.82	0.83	0.34	0.77	0.79	0.39	0.77	0.78
10	0.23	0.77	0.83	0.27	0.87	0.90	0.25	0.84	0.88	0.33	0.83	0.87
20	0.66	0.84	0.88	0.69	0.90	0.90	0.66	0.90	0.92	0.74	0.86	0.88

Table A.8: P-Values of the Diagnostic Tests for the Inadequate Models in Period 3

A.5 Period 4

Period 4 : July 2013 -June 2023														
	ARMA(1,0)-GARCH(1,1)		ARMA(1,0)-EGARCH(1,1)		ARMA(1,0)-TGARCH(1,1)						ARMA(1,0)-APARCH(1,1)			
	norm	**snorm**	**norm**	**snorm**	**norm**	**snorm**	**t**	**st**	**GED**	**sGED**	**snorm**	**t**	**st**	**GED**
c	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.00*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
ϕ	0.036	0.035	0.133*	0.133*	0.104*	0.127*	0.313*	0.311*	0.295*	0.281*	0.131*	0.06*	0.308*	0.294*
ω	0.000	0.000	-0.091*	-0.086*	0.000*	0.000*	0.000*	0.000*	0.000*	0.00*	0.000	0.000*	0.000	0.000
α	0.166*	0.163*	0.0919*	0.094*	0.148*	0.140*	0.563*	0.567*	0.391*	0.13*	0.137*	0.563*	0.456*	0.343*
β	0.833*	0.836*	0.985*	0.986*	0.881*	0.888*	0.701*	0.700*	0.748*	0.87*	0.893*	0.706*	0.748*	0.781*
γ	-	-	0.402*	0.391*	-0.446*	-0.485*	-0.079	-0.083	-0.081	-0.25*	-0.537*	-0.059*	-0.102	-0.042
δ	-	-	-	-	-	-	-	-	-	-	0.801*	0.701*	0.569*	0.757*
shape	-	-	-	-	-	-	2.470*	2.460*	0.731*	0.724*	-	2.314*	2.287*	0.699*
skew	-	1.076*	-	1.033*	-	1.077*	-	0.988*	-	0.985*	1.075*	-	0.975*	-

(*) indicates significant at 5% significance level.

Table A.9: The Estimate of the Inadequate Models for the Period 4

	GARCH						**EGARCH**					
	norm			**snorm**			**norm**			**snorm**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.00	0.21	0.21	0.00	0.519	0.19	0.00	0.57	0.57	0.00	0.51	0.51
5	0.00	0.57	0.56	0.00	0.54	0.53	0.00	0.96	0.96	0.00	0.95	0.95
10	0.00	0.89	0.89	0.00	0.87	0.88	0.00	0.99	0.99	0.00	0.99	0.99
20	0.00	0.98	0.98	0.00	0.98	0.97	0.00	0.99	0.99	0.00	0.99	0.99

	TGARCH																	
	norm			**snorm**			**t**			**st**			**GED**			**sGED**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.01	0.05	0.05	0.01	0.05	0.05	0.90	0.93	0.93	0.90	0.93	0.93	0.94	0.98	0.98	0.93	0.97	0.97
5	0.00	0.25	0.25	0.00	0.25	0.25	0.56	0.99	0.99	0.56	0.99	0.99	0.30	0.99	0.99	0.31	0.99	0.99
10	0.00	0.69	0.71	0.00	0.69	0.71	0.93	1.00	1.00	0.93	1.00	1.00	0.77	1.00	1.00	0.78	1.00	1.00
20	0.00	0.97	0.98	0.00	0.97	0.98	0.98	1.00	1.00	0.98	1.00	1.00	0.93	1.00	1.00	0.93	1.00	1.00

	APARCH											
	snorm			**t**			**st**			**GED**		
Lags	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**	Q	Q^2	**ARCH-LM**
1	0.02	0.03	0.03	0.89	0.96	0.96	0.89	0.96	0.96	0.85	0.86	0.86
5	0.00	0.12	0.13	0.63	1.00	1.00	0.63	1.00	1.00	0.15	0.99	0.99
10	0.00	0.49	0.51	0.95	1.00	1.00	0.95	1.00	1.00	0.49	1.00	1.00
20	0.00	0.92	0.94	0.99	1.00	1.00	0.98	1.00	1.00	0.82	1.00	1.00

Table A.10: P-Values of the Diagnostic Tests for the Inadequate Models in Period 4

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