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An Asset Market Approach to the Conditions on the Usage of Cryptocurrencies with Fiat Currencies in a Coexistence Economy: The Land under Zero

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Abstract

This study aims to construct a theoretical framework for the usage of cryptocurrencies together with fiat currencies in a coexistence economy to estimate the future effects of cryptocurrencies on the world economy. In this model, Bitcoin is used as a representative cryptocurrency, the uncertainties behind it are taken into consideration measuring by the variance of the expected returns on it. The model has three agents such as buyers, miners, and sellers. Buyers have to decide how much fraction of their initial wealth they spend either on Bitcoin or fiat money in the intention both to consume goods and services and to invest in the asset market. Buyers' problem of choice is solved by considering their risk attitudes. Miners select the optimal mining effort to validate the transactions to have maximum gain. They gain rewards as Bitcoin and transaction fees, but they incur the cost of electricity as long as they exert effort to mine Bitcoin. Sellers sell their goods and services in the goods market to buyers and have no important effect on the usage of Bitcoin in this model. The study reveals the coexistence

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conditions of cryptocurrencies and fiat money based on the agents' (buyers, miners, and sellers) optimization decisions. Overall, the "return-motive" creates the main intuition of using Bitcoin as a medium of exchange in the goods market. The uncertainties behind cryptocurrencies, the transaction and electricity fees, and people's risk attitudes construct the main determinants of the condition of using cryptocurrencies together with the fiat currencies.

Keywords: *Cryptocurrencies, Asset Market, Fiat Money, Uncertainty and Information.*
JEL Codes: *D82, E42, O30.*

Özet

Kripto Paraların Kağıt Paralar ile Ekonomide Birlikte Varoluş Koşullarına Bir Varlık Piyasası Yaklaşımı: Sıfırın Altındaki Bölge.

Bu çalışma kripto paraların dünya ekonomisi üzerindeki gelecek etkilerini tahmin edebilmek amacıyla kripto paraların kağıt para ile ekonomide birlikte kullanıma yönelik teorik bir çerçeve oluşturmayı amaçlamaktadır. Kurulan modelde, Bitcoin temsili kripto para olarak kullanılmaktadır, içerdiği belirsizlikler beklenen getirilerinin varyansıyla ölçülerek göz önünde bulundurulmaktadır. Modelde alıcı, madenci ve satıcı olmak üzere üç iktisadi aktör bulunmaktadır. Alıcılar hem mal ve hizmet tüketimi hem de varlık piyasasına yatırım yapma motifiyle birlikte ilk varlıklarının hangi orandaki kısmının Bitcoin'e ya da kağıt paraya harcayacağını belirlemek zorundadır. Alıcıların seçim problemi risk tutumlarını göz önünde bulundurarak çözülmektedir. Madenciler maksimum getiriyi sağlamak amacıyla işlemleri doğrulayarak optimal madencilik çabalarını belirlerler. Ödüllerini Bitcoin ve işlem ücreti şeklinde alırlar ancak Bitcoin madenciliği yaparken elektrik maliyetine de katlanmaktadırlar. Satıcılar alıcılara mal ve hizmetlerini mal piyasasında satarlar ve bu modelde Bitcoin'in kullanımı üzerinde bir etkileri bulunmamaktadır. Çalışma aktörlerin (alıcılar, madenciler ve satıcılar) optimizasyon kararlarına dayanan kripto paraların ve kağıt paraların birlikte var olma koşullarını ortaya koymaktadır. Sonuç olarak, "getiri motifi" Bitcoin'in mal piyasasındaki değişim aracı olarak kullanılmasının ana sezgisini oluşturur. Kripto paraların içerdiği belirsizlikler, işlem ve elektrik ücretleri, insanların risk tutumları kripto paraların kağıt paralar ile birlikte kullanılma koşullarını belirlemektedir.

Anahtar Kelimeler: *Kripto Paralar, Varlık Piyasası, Kağıt Para, Belirsizlik ve Bilgi.*
JEL Kodları: *D82, E42, O30.*

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1. Introduction

Cryptocurrencies have been gaining momentum since 2009, when the first cryptocurrency called Bitcoin was created by Satoshi Nakamoto, which is actually a

nickname. Actually, there exist now 9,103 cryptocurrencies (as of October 2019) according to "Coinranking" statistics with $249.26 billion total market capitalisation which shows that cryptocurrencies should be taken into consideration in a more comprehensive way. Obviously, it was not a coincidence that the first cryptocurrency was created after the third global crisis called "Great Recession". One can assert that in general, financial and technological innovations might lead to economic crises which seems partially contradictory with this paper's suggestion of the history of Bitcoin. However, actually it is not a contradiction since both of them could be happened one by one, that is, some technological innovations in the financial system could pave the way for the housing bubble in the U.S. and thereby the crisis; at the same time, new technological innovations such as Blockchain system, digital currencies and cryptocurrencies, digital payment systems, online and mobile banking could be aroused from the crisis as a response to the outcome of the crisis. Akgiray (2019) points out that the concomitants of the global crisis of 2007-2008 led to loss of reliance of the people on the firms, on the private banks and the central banks, on the markets, and on the governments. The people began to think that the financial system was not driven ethically and incorruptly, so there should exist some money that belongs to the public. From this point of view, these social facts underlay the creation of Bitcoin. In addition, according to Malherbe et al. (2019), creation of Bitcoin was an attempt to make a response to the failure of prevailing monetary system of the global order. Furthermore, Nakamoto (2008), the creator of Bitcoin, advocates that commerce on the Internet has been suffering from the increasing transaction costs and uncertainties of the third parties. This situation forms the reason why the first cryptocurrency was created. Cryptocurrencies have no central authority to control the transactions. They have a decentralized structure which is one of the most interesting features of the transaction process of cryptocurrencies. It could be easily seen that although cryptocurrencies were created with the intention of serving as a mediator of the commerce on the Internet, they have been used mostly as speculative assets. In 2009, 1000 Bitcoins were worth of $0.03 while today just 1 Bitcoin is worth of

around $9,288.99 and has $167.91 billion market cap size. No doubt, this increasing value and increasing market cap size of Bitcoin drew a lot of attention so there were several studies on some of the issues of cryptocurrencies such as their mining process, working principle, financial environment, illegal activities, taxation, environmental side (electricity usage), and monetary relations. In this paper, the issue of monetary relations will be investigated. Today, it is obvious that cryptocurrencies and fiat money are being used together in the economy. In the mainstream economics, fiat money has three functions such unit of account, medium of exchange, and store of value. Although cryptocurrencies have been being used as mostly store of value, they were actually created to serve as a medium of exchange. However, it could be said that cryptocurrencies could not fulfil undoubtedly the requirements of being a medium of exchange and a unit of account since their high and sometimes extreme volatility disable them to become a unit of account and although the number of websites that accept cryptocurrencies as a payment method have been increasing, they have still not been accepted by the majority of the traders (firms, individuals etc.) on the Internet (Carrick, 2016). Obviously, this problem begs the question of how cryptocurrencies and fiat (central bank issued) money could be used in an economy.

In a recent study held by Kang and Lee (2019) called “Money, Cryptocurrency, and Monetary Policy”, cryptocurrencies were taken into consideration by examining their performance on the economic welfare as a medium of exchange within a coexistence economy where cryptocurrencies and fiat money are used together. They used Bitcoin as a representative cryptocurrency since it is the top-rank cryptocurrency. They have found that Bitcoin, as a medium of exchange, can compete with fiat money as long as the inflation rate is sufficiently high, that is, the growth rate of the fiat money should be larger than the growth rate of Bitcoin considering the transaction fee and the government tax. However, at the end of the study, the result is surprising that as the inflation rate increases, the welfare gap between the coexistence economy and the money-only economy increases since when the inflation rate rises, fiat money is begun to be a substitute for Bitcoin which lead to increase Bitcoin transactions. Then,

increase in Bitcoin transactions causes to raise the time for validation and recording the transactions because of the limited number of transactions that each block can contain. As a result, effective consumption and economic welfare decreases in the coexistence economy when the inflation rate increases (Kang and Lee, 2019). Obviously, this situation produces a dilemma that Bitcoin can be used as a medium of exchange when the inflation rate is high but then it also generates a welfare loss in the coexistence economy. In both cases (the inflation rate is sufficiently high or low) Bitcoin could not be used effectively as a medium of exchange. This paper will solve the dilemma by inquiring the inefficiency problems of cryptocurrencies such as transaction costs, mining effort, and electricity costs. The main hypothesis of this study is that an upgraded cryptocurrency-fiat money framework which considers the optimization of the economic agents' problems under the risk and uncertainty conditions could explain the main condition of to use cryptocurrencies as a medium of exchange together with fiat currencies. The "return-motive" of cryptocurrencies could urge people to buy and use them as a medium of exchange. Therefore, depending upon fluctuations in the determinants of the expected returns on cryptocurrencies, people could choose to use both cryptocurrencies and fiat currencies as medium of exchanges which implies that there is a complementary relationship instead of perfect substitutability.

This paper consists of the two main parts and a conclusion part. Before the first part, there is a literature review of the previous studies related to cryptocurrencies and their effects on the economy. The first part consists of the theoretical framework that includes the economic agents' problem of choice. In the second part, an equilibrium is constructed to derive insights from the model. The new theoretical model was built under the risk and uncertainty problem and an asset market model is implemented to the coexistence economy of Bitcoin and fiat money. At the end, there is the conclusion part which derives insights from the new model in order to show the conditions on the usage of cryptocurrencies together with fiat currencies in the coexistence economy.

2. Literature Review

Although there are several studies on the different aspects of the cryptocurrencies, the number of extensive and persuasive studies is limited. In the first studies related to cryptocurrencies, most of the issues such as meeting the requirements of fiat money, their legal status, regulation and risks, criminal activities were not apparently clarified since they were very novel to grasp. Therefore, there are still lots of studies which advocates the different ideas about these issues of cryptocurrencies. It seems that just in the recent years, cryptocurrencies attracted more scholarly attention and thereby some in-depth analyses were held. Generally, the issues studied consist of cryptocurrencies' way of work, mining process, Blockchain technology, price volatility in the cryptocurrency market, monetary policy and central bank relations, regulation and potential risks, money laundering, predictive models to determine the future value of several cryptocurrencies, and relationship between fiat money and cryptocurrencies. The very first study related to cryptocurrencies was written by Nakamoto (2008), the creator of Bitcoin, to introduce what Bitcoin is about and how it is working. Actually, this article was the declaration of Bitcoin and thereby of the Blockchain technology-based cryptocurrencies. After Nakamoto, Surowiecki (2011) was one of the early authors about the issue of whether Bitcoin is being used as really a "currency" or just an investment to gain from it in the future. He emphasizes that Bitcoin were used mostly as an investment tool. He also points out that people could start to think about Bitcoin as a currency rather than think about Bitcoin as an investment when the bubble of Bitcoin price bursts. However, when the bubble bursts, then the enthusiasts of Bitcoin will lose their interest to Bitcoin which creates a kind of paradox. Then, Davis (2011) specifies the characteristic features of both the principles of Bitcoin and of its creator, Nakamoto in *The New Yorker.* He realized that Nakamoto was well-informed about cryptography and programming not only because of his invention but also because there were several attempts to hack Bitcoin system which were failed since Bitcoin was very well coded and secured. After relatively short reviews on the issue of cryptocurrency, in 2014, Bhatt, a retired professor of

Computer Science and Engineering from IIT Delhi, clarified the nature of cryptocurrencies in a detailed work with a historical starting point. He begins his article with the barter system which enables people to exchange goods and services. Then, he included a table of forms of money and several characteristics of those forms. He investigated whether cryptocurrencies fulfil the requirement of being a money or not. He concludes that cryptocurrencies could serve as a medium of exchange, but they have the problem with the other roles of money such as unit of account and store of value. Apart from the studies related to the working principle and to the characteristics of Bitcoin and other cryptocurrencies, there were also studies related to its political stance. Giotitsas and Kostakis (2014) emphasized that the problems of banking and financial system such as making new bank products that create value in the monetary terms but not in real production. Moreover, they implied that the integration of debt into the distribution of money destroys human relationships by creating unsustainable structures, so the digital currencies such as Bitcoin emerged to deal with these problems above. Ultimately, Bitcoin should be seen as a technological innovation itself. There are also the works related to the market of cryptocurrencies such as Gandal and Halaburda's (2016) detailed analysis of network effects on the competition of cryptocurrencies in the cryptocurrency market and Liu and Serletis' (2019) work about the volatility in the cryptocurrency market. Their findings are interesting that there exist statistically significant spillover effects from the cryptocurrency market to other financial markets in the U. S., Germany, the U.K., and Japan. Zhao (2015) analysed the competition among the cryptocurrencies and the competition of cryptocurrencies with fiat money. He concluded that cryptocurrencies have optimistic indications and they will achieve a higher development in the monetary transactions if cryptocurrencies make themselves more user-friendly and if they are supported by the several payments systems like Paypal. Furthermore; Corbet et al. (2017) carried out an empirical study on the effects of announcements of Fed's monetary policy on the volatility of cryptocurrencies.

They found that although Bitcoin seems to have an independent structure of its own, it has been affected by both the economic and the public policies like the fiat currencies.

This study falls under the issue of the relationship between fiat money and cryptocurrencies and derives insights from Kang and Lee (2019) framework. They construct a microeconomic model to analyse the coexistence economy of fiat money and Bitcoin as a representative cryptocurrency. The result of their analysis suggests that using Bitcoin as a medium of exchange and as a substitute to fiat money brings an economic welfare loss to the society. This research actually was held to deal with this problem. Benigno (2019) also analysed the coexistence economy by including credit market, interest rates, and government policies to his model. He concludes that competition between government-issued money and cryptocurrencies can restrict the ability of the central bank to use the interest rate as a policy instrument and can disable the inflation rate to reach to the attainable equilibrium level. Furthermore, Carrick (2016) looks at the relationship between fiat money and cryptocurrencies from a totally different perspective apart from the general idea of cryptocurrencies as a substitute to fiat money. He suggests that Bitcoin can be a complement to fiat currencies, especially emerging market currencies, not a substitute to fiat currencies. Another related and remarkable study, was held by Mehta et al. (2017), investigates Bitcoin as a world currency and the relationship between Bitcoin and the U.S Dollar. It can be said that Bitcoin harms the U.S. Dollar if the exchange rate versus the U.S. Dollar increases drastically.

Having taken the related literature into consideration, it can be said that most of the issues related to cryptocurrencies has been remain unclear and involves some uncertainties such that there has not been an explicitly applicable approach on the usage of them, there has not been a financial regulation, and whether they can be used instead of using fiat money. The first and the second issues will probably be determined by governments or financial regulation authorities such as central banks, audit institutions, etc., since they have the authority to let cryptocurrencies be used in

economy or not. However, the last issue is related to the main concern of this paper such that whether cryptocurrencies can meet the requirements of being a money like fiat money and thereby they can replace fiat money. Some of the authors of related literature argue that cryptocurrencies can meet the roles of fiat money totally whereas some of them point out that cryptocurrencies cannot meet the roles of money especially being a unit of account and a medium of exchange. It is explicit that cryptocurrencies have not meet the requirement of being a unit of account since their value generally is associated with their current dollar value even if a good or a service can be bought by them. This problem is probably because of lack of financial regulation to support them and the extreme volatility of the prices of cryptocurrencies. Furthermore, they cannot be seen as a store of value since they have again the same of problem of the extreme volatilities in their prices. Last but not least, the issue of their role of meeting the requirements of being a medium of exchange as a money is the main consideration of this paper and consists of the problem of how cryptocurrencies and fiat currencies can be used together in a most effective way with the cryptocurrencies' ambiguous issues mentioned above. In the next part, a theoretical framework of the relationship between cryptocurrencies and fiat money is given in the consideration of the related literature above by considering the problem of uncertainty and of choice with the risk preferences of individuals.

3. Theoretical Framework

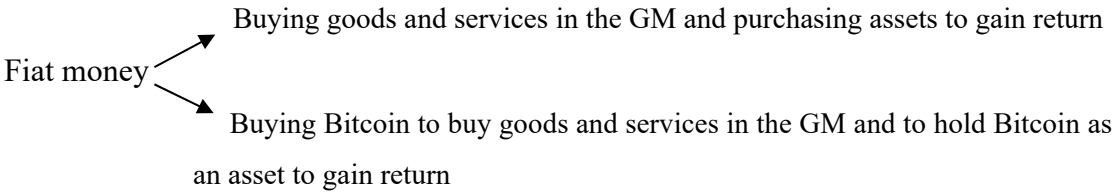
This study is inspired by the work of Kang and Lee (2019) to develop their ideas of the coexistence economy of fiat money and Bitcoin as a representative cryptocurrency. Their environment also is based on Lagos and Wright (2003) such that the both studies take the time as $t = 0, 1, ...,$ discrete and continues infinitely. Each period is divided into two subperiods when different economic activities occur. In Kang and Lee's model, there are two markets such as goods market (*GM*) and a currency market (*CM*) which operates in turn. There are three types of agents such as buyers, sellers, and miners. They assume that there is a continuous interaction

between buyers and sellers, and the number of miners is fixed at η, which is a positive integer. They suggest the quasi-linear functional forms for utility functions of the agents such as $U(q_t, X_t, H_t) = u(q_t) + X_t - H_t$ for buyers, $U(h_t, X_t, H_t) = -c(q_t) + X_t - H_t$ for sellers, and $U(e_t, X_t, H_t) = -e_t + X_t - H_t$ for miners where q_t is consumption in the GM, X_t and H_t are consumption and the labor supply, respectively in the CM, h_t is the labor supply used to produce goods in the GM, and e_t is the effort for mining in the GM. Then, they introduce the economic agents' problems by optimizing the costs and benefits of the behaviour of the economic agents. They take transaction fees, mining effort, growth rates and prices of fiat money and Bitcoin, and government taxation into consideration. After they introduce the algebraic forms, they construct stationary equilibria by clearing all markets and solving all problems of the economic agents in the model. The second part of their study relates to a quantitative analysis by evaluating the effects of monetary policy and Bitcoin transaction fees on economic activities and welfare replacing the hypothetical economy by the United States. According to their quantitative results, economic welfare in the money-only economy is higher than that in the coexistence economy. The lower welfare in the coexistence economy is firstly led by inefficient mining processes and increasing transaction time of Blockchain system since an increase in Bitcoin transactions leads to raising the time for validation of the transactions due the limited number of transactions that each block can contain at the same time. They also conclude that Bitcoin can be used only when the inflation rate is sufficiently high which means that the cost of holding fiat money is relatively high to the cost of holding Bitcoin.

Kang and Lee's (2019) work involves important insights to consider the relationship between fiat money and cryptocurrencies. Unfortunately, one of the problems of this study and some of the authors of the related literature is that they investigated and examined cryptocurrencies (mostly Bitcoin) to whether they can be pure substitutes for fiat money or not. They tested cryptocurrencies' performance and their situation in the economy by questioning the substitution ability of Bitcoin for fiat currencies. However, in the real situation, this is not the actual case since cryptocurrencies and

fiat currencies have been using together by not a substitutionary way but mostly a complementary way which forms the first consideration of this study. Today, people mostly use their fiat money to buy goods and services, but they also use Bitcoin for goods and services as well. Although the basket of available goods and services that can be bought by Bitcoin is much smaller than that of bought by fiat money, people have a tendency to buy Bitcoin not only to buy goods and services but mostly to hold it as a financial asset to gain by speculating in the cryptocurrency market. Therefore, people can have two different options if they possess Bitcoin which is most probably the main reason of why people have a tendency to buy or mine Bitcoin. The same illustration may seem to be made for fiat money but the return on Bitcoin in the cryptocurrency market is generally higher than the return on fiat money by investing it in a free-risk asset such as treasury bills, otherwise people will not have a tendency to invest in Bitcoin which is explicitly a risky asset. Secondly, Kang and Lee (2019) do not mention any kind of uncertainty and risk preferences in their framework. As it has been mentioned before in this paper, there exist some kinds of unclear issues such as lack of legal procedures, financial deregulation, decentralized ledger system, anonymous transactions, the problem of extreme volatility, money laundering and using cryptocurrencies in illegal activities in the background of Bitcoin and thereby of other cryptocurrencies. It should be that these unclear issues inevitably generate some uncertainties in the concept of cryptocurrencies. This study modifies and extends Kang and Lee's (2019) coexistence economy framework by adding risk preferences and uncertainty to the economic agents' problem of choice. Then, the new equilibrium is defined by solving the modified problems of the economic agents. The setting for the buyers can be shown as by Figure 1:

Figure 1: The Setting for Buyers



In Figure 1, it is assumed that a buyer has some fiat money sufficient to buy goods and services in the GM or to buy Bitcoin in the CM. Since buying Bitcoin could mean different purposes such as to buy goods and services in the GM to satisfy the buyer's own utility or to hold as an asset to make a profit in the next period. On the other hand, selecting to hold Bitcoin as an asset could also produce two different choices since the return on Bitcoin is determined in a stochastic process. Therefore, if the buyer has chosen to hold Bitcoin as an asset, could abandon to hold Bitcoin and could decide to buy goods and services in the GM depending upon the expected return on Bitcoin. In this decision tree, each node has of course some uncertainties. Each buyer can go through each of nodes, i.e. they can share their wealth both to buy directly goods and services and to buy Bitcoin either for hold as an asset or to buy goods and services. Therefore, the buyer assuming as a fully rational agent, has to make a decision of sharing its fiat money to buy goods and services directly or it may choose to spend its money to buy Bitcoin in the specific circumstances. These circumstances are defined by introducing the uncertainty and the risk functions which will specify the indifference conditions of the buyer over these different preferences.

3.1. Economic Agents' Optimization Problems

In this part, the optimal behaviour of each economic agent is introduced under the uncertainty conditions. In order to describe the optimization problems, mean-variance utility approach is used instead of the expected utility approach. The main reason of such a selection is that mean-variance approach is more efficient than maximization

of expected utility of final wealth with the utility functions exhibiting constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) in terms of mean-variance (Vigna, 2009, p. 10). For this reason, it is assumed that the agents choose mean-variance approach for their optimization problem. It is also assumed as in Kang and Lee's work that fiat money and Bitcoin are perfectly divisible, storable, and grow at the gross rates of γ and γ_b, respectively: $M_{t+1} = \gamma M_t$ and $B_{t+1} = \gamma_b B_t$. However, the growth of Bitcoin and fiat money is not included here since it is assumed that the mean-variance approach takes the opportunity costs of the future values of both Bitcoin and fiat money into account. Moreover, transaction fee rate, f, paid for buying goods and services by using Bitcoin to the miners is taken same as in Kang and Lee's framework. Delivery lags which are emerged when buyers make a transaction to buy goods and services are assumed to be zero in the model for the sake of simplicity.

3.1.1. Buyer's Problem

Before the optimization for a buyer's problem of choice under uncertainty is introduced, it is assumed that a buyer has initial wealth as W_0 which is held as fiat money. A buyer can share its income to buy directly goods and services and to buy Bitcoin for its speculative motive or for goods and services which can be purchased by Bitcoin. From the mean variance viewpoint, a buyer should seek to maximize its expected wealth and minimize the variance of its final wealth. Markowitz mean-variance portfolio theory is used for the purpose of to introduce the concept of riskiness. One of the interesting contributions of this paper is that both the asset motive of Bitcoin and the hypothetical asset motive of fiat money buying risk-free asset and the medium of exchange motive of Bitcoin and fiat money are brought together in a single expected return equation. In detail, the return function includes both the returns on the assets and the returns on the goods and services in the form of utility. The model is originally a *Capital Asset Pricing Model* (CAPM). Therefore, it is aimed that the most efficient frontier in the Markovitz mean-variance model should

be chosen. Let r_b and r_m denote the aggregate return on Bitcoin and the aggregate return on the market portfolio purchased by fiat money, respectively. In this context, the term "*aggregate return on Bitcoin*" means the total return on both speculating Bitcoin in the cryptocurrency market and using Bitcoin as a medium of exchange in the goods market. Let x denotes the fraction of the initial wealth in Bitcoin and $(1\text{-}x)$ is the fraction in risk-free asset. In addition, it is assumed that the asset return on Bitcoin in the time $t + 1$ is $m_{b,t+1}$ and the asset return on fiat money in the time $t + 1$ is $m_{m,t+1}$. Then, the portfolio of a buyer is such that

$$r_x = xr_b + (1\text{-}x)\, r_m, \text{ where} \quad (1)$$

$$r_b = x(m_{b,t+1}\,\alpha + (u(q_t^b)\,(1\text{-}\alpha))^{f}, \text{ and} \quad (2)$$

$$r_m = (1\text{-}x)\,(m_{m,t+1}\,\theta + u(q_t^b)\,(1\text{-}\theta)) \quad (3)$$

Then, the aggregate expected return can be written as

$$E(r_x) = \sum [x(m_{b,t+1}\,\alpha + (u(q_t^b)\,(1\text{-}\alpha))^{f}) + (1\text{-}x)\,(m_{m,t+1}\,\theta + u(q_t^b)\,(1\text{-}\theta))]\pi_s \quad (4)$$

which can be rearranged such that

$$= x \sum (m_{b,t+1}\,\alpha + (u(q_t^b)\,(1\text{-}\alpha))^{f})\,\pi_s + \sum (1\text{-}x)\,(m_{m,t+1}\,\theta + u(q_t^b)\,(1\text{-}\theta))\,\pi_s. \quad (4.1)$$

and

The expected return is

$$E(m_x) = \sum [x(m_{b,t+1} + (1\text{-}x)\,(m_{m,t+1})]\pi_s \quad (5)$$

Then we have,

$$E(m_x) = x\mu_b + (1\text{-}x)\mu_m \text{ where} \quad (5.1)$$

μ_b denotes the expected return of Bitcoin and μ_f denotes the rate of return on the market portfolio possessed by fiat money. α is the fraction of x to hold Bitcoin as a risky asset to gain return $m_{b,t+1}$ in the time $t+1$ and $(1\text{-}\alpha)$ is the fraction of x to use

Bitcoin as a medium of exchange to consume goods and services in the goods market. It is supposed that $u(q)$ is the gain from consumption as a return since all of the variables in the above equation are in percentage terms. f is the rate of transaction fee which enters to the equation as an exponential variable since all the parameters are in the range of between 0 and 1. θ stands for the fraction of (1-x) (wealth which is shared for fiat money) shared on the market portfolio and (1- θ) shows the fraction that spent on goods and services in the goods market. π_b is the probability that the state b which indicates the return on Bitcoin in the time $t + 1$ is positive and larger than the return on the risk-free asset in the time $t + 1$ will occur.

Then, the variance can be introduced to measure the riskiness of the choices. The variance of the probability distribution is such that

$$\sigma_x^2 = \sum[x(m_{b,t+1}\,\alpha + (u(q_t^b)\,(1-\alpha))^f) + (1-x)\,(m_{m,t+1}\,\theta + u(q_t^b)\,(1-\theta)) - E(r_x)]^2\,\pi_b \quad (6)$$

Substituting for $E(r_x)$, the equation becomes

$$\sigma_x^2 = \sum (xr_b - xr_m)^2\,\pi_b \quad (6.1)$$

$$\sigma_x^2 = \sum x^2(r_b - r_m)^2\,\pi_b \quad (6.2)$$

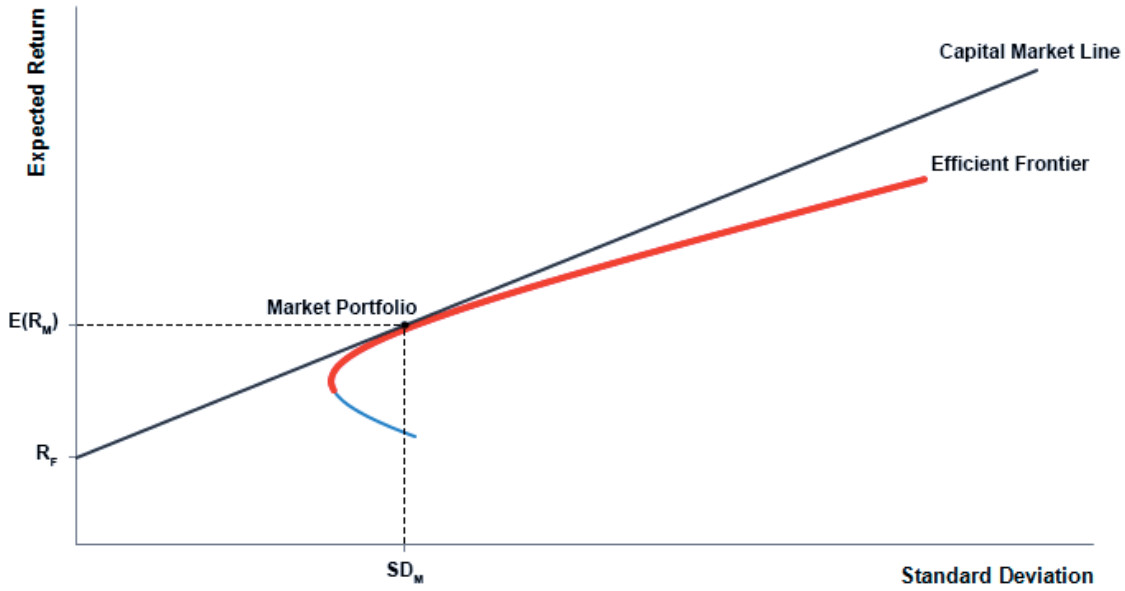
$$= x^2\,\sigma_b^2 + 2x(1-x)\,\sigma_{bm} + (1-x)^2\,\sigma_m^2 \text{ where} \quad (6.3)$$

σ_{bm} is the covariance of the return on Bitcoin and the market portfolio r_m.

After the mean and the variance of the assumed portfolio are introduced, a buyer's problem can be defined that she should both maximize the expected return on her portfolio and minimize the variance of the expected return. Then, it can be said that each buyer seeks to minimize the vector [-$E(r_x)$, σ_x^2] of their portfolio. The problem of choice could be solved by determining the most efficient allocation of the initial wealth W_0. Varian (2014) states that at the optimal choice of risk and return, the slope of the indifference curve must equal the slope of the budget line. In the Figure 2, the

capital market line shows the frontier with risk-free asset and the red curve indicates the frontier without risk-free asset. At $x = 0$ the red frontier becomes tangent to the capital market line and becomes the efficient frontier at this point.

Figure 2. Illustration of the Efficient Frontier



Source: http://financialmanagementpro.com/capital-market-line-cml/

In other words, *The Capital Asset Pricing Model* implies that if the efficient frontier is desired to be chosen then the expected return $E(r_x)$ on the portfolio x should satisfy the equation $E(m_b) = r_f + \frac{\mu_m - r_f}{\sigma_m} \sigma_b$. This means that the slope of this equation (or line) $\frac{\mu_m - r_f}{\sigma_m}$ which is called the price of risk should be equal to the marginal return on the expected return of the porfolio x. The proof can be given differentiating firstly the return function r_x and the standard deviation σ_x with respect to x such that

$$\frac{dr_x}{dx} = \mu_b - \mu_f \qquad (7)$$

and

$$\frac{d\sigma_x}{dx} = \frac{1}{\sigma_x}\left[x\,\sigma_b^2 + (1\text{-}2x)\,\sigma_{bm} + (x\text{-}1)\,\sigma_m^2\right] \tag{8}$$

$$\frac{dr_x}{d\sigma_x} = \frac{\frac{dr_x}{dx}}{\frac{d\sigma_x}{dx}} = \frac{\sigma_x(\mu_b - r_m)}{\left[x\sigma_x^2 + (1-2x)\sigma_{bm} + (x-1)\sigma_m^2\right]} \tag{9}$$

When the mean-standard deviation curve for r_b and r_m is tangent to the capital market line at $x = 0$, then the following equation can be derived such that

$$\frac{\mu_m - r_f}{\sigma_m} = \frac{dr_x}{d\sigma_x} \text{ when } x = 0 \text{ it equals } \frac{\sigma_m(\mu_b - \mu_m)}{\sigma_{bm} - \sigma_m^2}. \tag{10}$$

Solving for μ_m,

$$(\mu_m - r_f)(\sigma_{bm} - \sigma_m^2) = \sigma_m^2(\mu_b - \mu_m) \tag{11}$$

$$\frac{(\mu_m - r_f)(\sigma_{bm} - \sigma_m^2)}{\sigma_m^2} = (\mu_b - \mu_m) \tag{11.1}$$

$$\frac{(\mu_m - r_f)\sigma_{bm} - (\mu_m - r_f)\sigma_m^2}{\sigma_m^2} = \frac{(\mu_m - r_f)\sigma_{bm}}{\sigma_m^2} - (\mu_m - r_f) \tag{11.2}$$

$$\frac{(\mu_m - r_f)\sigma_{bm}}{\sigma_m^2} - (\mu_m - r_f) + \mu_m = \mu_b. \tag{11.3}$$

$$\mu_b = r_f + \frac{(\mu_m - r_f)\sigma_{bm}}{\sigma_m^2} \tag{11.4}$$

and if β is substituted for $\frac{\sigma_{bm}}{\sigma_m^2}$, then we have

$$\mu_b = r_f + \beta(\mu_m - r_f) \tag{11.5}$$

where β is referred to imply the excess rate of return of an asset as a percentage of the market excess rate of return. The main point of the problem of choice can be explained that if the beta of investing in Bitcoin is less than 1 then investing in Bitcoin can be considered as a conservative investment and its variance is less than that of the asset market. However, if the beta of investing in Bitcoin is higher than 1, it implies that investing in Bitcoin is riskier than the investing in the asset market since the

variance of the return on Bitcoin is higher than that of the asset market. On the other hand, a β value which is higher than 1 implies that investing in Bitcoin has higher expected return compared to the market portfolio's expected return. Therefore, there is trade-off between the expected return in investing Bitcoin and its certainty. It means higher the expected returns lower the certainty in higher returns. Now, the buyers' preferences can be determined by the value β and the buyers' risk attitudes. If β is higher than 1, only the risk-lover buyers will choose to invest in Bitcoin and thereby use Bitcoin as a medium of exchange. The risk-neutral and the risk-averse buyers will not choose to buy Bitcoin because of the higher variance of investing in Bitcoin than that of the market portfolio which signals that investing in Bitcoin is a risky preference. If β is equal to 1, then only the risk-neutral buyers will choose to buy Bitcoin since it means that the variances and the expected returns of both in investing Bitcoin or in the market portfolio are the same. The other possibility is that β is lower than 1. Then, only the risk-averse people could choose to buy Bitcoin since now investing in Bitcoin has a lower risk compared to investing in the market portfolio but at the cost of the lower expected returns. However, in the real-world conditions, since cryptocurrencies have more uncertainty than the assets which could be purchased by fiat-money, it should be that β is higher than 1 so that the buyers will have the propensity to buy and use Bitcoin as a medium of exchange.

3.1.2. Miner's Problem

Miners exert effort to validate Bitcoin transactions which are the results of the buyers' activity of purchasing Bitcoin as a financial asset and purchasing goods and services using Bitcoin. The main component of the miner's problem of choice is the mining effort, e_t, which is the same as in Kang and Lee's framework. The probability that a miner i will win the mining competition for each block is defined by the ratio of her/his own effort, $e_{i,t}$, to the aggregate effort of all miners to mine a block, $\sum_{j=1}^{\eta} e_j, t$. Then, the probability of miner i's winning the mining competition is

defined by $\frac{e_{i,t}}{\sum_{j=1}^{\eta} e_{j,t}}$ where η is the total number of miners. Each miner who win the mining competition receives the transaction fees, F_t, and the newly created Bitcoin S_t. However, because of the convenience conditions mentioned above, these variables should be expressed in the percentage terms. In addition to Kang and Lee's definition of miner's problem, the cost of electricity fee used for computer to validate transactions to mine Bitcoin is also introduced here. The cost of electricity fee is defined as a negative component of total endowment of mining effort of each miner since each miner will incur higher cost of electricity if they exert higher effort to validate more transactions. Therefore, the cost of electricity is expressed in the mining effort variable such that $C_e = e_{i,t} f^e n_t$ where $e_{i,t}$ is the mining effort of a miner, f^e is the cost of electricity fee rate, and n_t is the number of block mined in period t and miners take n_t as given. η shows the total number of miners in the mining competition. Then, the miner's problem can be written as

$$\pi_{i,t} = \underset{e_{i,t}}{\text{Max}} \left\{ \frac{(St + Ft)}{n_t} \frac{e_{i,t}}{\sum_{j=1}^{\eta} e_{j,t}} - e_{i,t}(1 + f^e) \right\} n_t. \tag{12}$$

$$\pi_{i,t} = \underset{e_{i,t}}{\text{Max}} \left\{ (S_t + F_t) \frac{e_{i,t}}{\sum_{j=1}^{\eta} e_{j,t}} - e_{i,t}(1 + f^e) \right\} n_t \tag{12.1}$$

It is assumed that a miner chooses her/his own effort level given other miners' effort. Then, if we take the first derivative of $\pi_{i,t}$ with respect to $e_{i,t}$ the equation becomes

$$(S_t + F_t) \frac{\sum_{j \neq i}^{\eta} e_j}{(\sum_{j \neq i}^{\eta} e_{j,t} + e_{i,t})(\sum_{j \neq i}^{\eta} e_{j,t} + e_{i,t})} - (1 + f^e) n_t = 0. \tag{12.2}$$

If it is assumed that all miners are homogeneous, then $e_{i,t}$ becomes e_t. Then, (14.2) becomes,

$$(S_t + F_t)\ \frac{(\eta-1)}{\eta^2 e_t} = (1 + f^e)\ n_t \tag{13}$$

where $\eta = \frac{\sum_{j\neq i}^{\eta} e_j}{e_t}$ which means that total mining effort over one miners' mining effort gives the total number of miners since we have assumption that all miners' efforts are the same.

Then, the optimal mining effort, $e_{i,t}^{*}$, can be shown as

$$e_{i,t}^{*} = \frac{(S_t+F_t)(\eta-1)}{\eta^2 n_t(1+ f^e)} \tag{14}$$

and if the optimum mining effort is put into the profit function the expected profit function becomes

$$\pi_t = \frac{(S_t+F_t)}{\eta^2} \tag{15}$$

which implies that when the number of miners increases, the expected profit will decrease.

3.1.3. Seller's Problem

Sellers sell their goods and services to the buyers in the form of both Bitcoin and fiat money in the goods market. They do not consume goods and services in the goods market, so they have no tendency to carry fiat money or Bitcoin. They incur the cost of production of goods and services $c(q)$ which is in the percentage term. It is assumed that there is no tax on sales. The seller's problem in the GM in period t is written as

$$\underset{q_t^s}{\text{Max}}\ \left\{-c(q_t^s) + x(v(q_t^s)(1-\alpha))^{f} + (1-x)\ v(q_t^s)(1-\theta)\right\} \tag{16}$$

where $x((1-\alpha))^{f}$ is the fraction of the wealth of a buyer that is shared to purchase goods and services by using Bitcoin and $(1-x)\ (1-\theta)$ is the fraction of the wealth of a

buyer that is shared to purchase goods and services by using fiat money. $v(q_t^s)$ is the return on the selling goods and services in the quantity q_t^s

If we take the first derivative of the profit function (18) with respect to q_t^s, then we have two equations such that

$$-c`(q_t^s) = x f(v(q_t^s))^{f-1}(1-\alpha)^{f} v`(q_t^s)). \quad (17)$$

$$-c`(q_t^s) = (1-x)\,(1-\theta)v`(q_t^s). \quad (18)$$

Then, the indifference condition between fiat money and Bitcoin for sellers is written as

$$x f(v(q_t^s))^{f-1}(1-\alpha)^{f} v`(q_t^s)) = (1-x)\,(1-\theta)v`(q_t^s). \quad (19)$$

4. Equilibrium

At the equilibrium, x (the fraction of the initial wealth shared in Bitcoin) solves the buyer's problem as opposed to the standard equilibrium models offer since in the standard equilibrium models the buyers choose her/his quantity of goods and services instead of choosing x. This is because of the implementation of the capital asset pricing model to cryptocurrency-fiat money relations in order to grasp idea of using Bitcoin as a medium of exchange instead or besides of fiat-money. The main aim of such a model is illustrating the actual motives of using cryptocurrencies as a medium of exchange. It can be said that the motive that leads people to use cryptocurrencies is actually their attractive asset value. This attractiveness serves cryptocurrencies both as a financial asset and as a medium of exchange.

i. In the goods market, the buyers' expenditures on goods and services should be equal to the sellers' income:

$x(v(q_t^s)(1\text{-}\alpha))^f = x(u(q_t^b)(1\text{-}\alpha))^f$ so that $v(q_t^s) = u(q_t^b)$ which shows the returns on the chosen quantity levels for the sellers and the buyers, respectively.

ii. The mining effort choice $\{e\}$ is a *Nash* equilibrium of the mining competition. The inference can be made from the relationship between the cost of electricity fee rate and the transaction fee rate. If the mining effort choice is in its equilibrium, then it is expected that if the cost of electricity rate increases, then the miners have to gain more Bitcoin as a mining competition reward to compensate the additional cost of electricity fee. Otherwise, the increase in the electricity fee rate will have to in- crease the transaction fee rate to protect the miners' expected profit which eventually will decrease the aggregate expected return on Bitcoin. To have a coexistence economy, the cost of electricity fee rate should be in accordance with the gains from the mining competition, otherwise Bitcoin could not be used as a medium of exchange since it will offer lower returns. In addition to buyers, sellers could also be affected negatively because of the increase in the cost of electricity fee rate and thereby the increase in the transaction fee rate unless the losses from the transactions with Bitcoin will be offset by the transactions with fiat money.

iii. The capital asset pricing model is at its own equilibrium point where the expected return on investing Bitcoin equals to the return on risk-free asset plus the return on portfolio with the excess rate of return of an asset as a percentage of the market excess rate of return β which is the main determinant of the buyers' preferences.

$$\mu_b = r_f + \beta(\mu_m - r_f).$$

Then the inference could be made such that the necessary condition for using Bitcoin besides with fiat-money is that the expected return on Bitcoin in the asset market

should be higher than the return on the market portfolio purchased by fiat-money. For this reason, the beta, β, should be higher than 1 to make Bitcoin more profitable over the market portfolio and to become an urge for people to purchase Bitcoin and thereby with the effect of this urge, people could use Bitcoin as a medium of exchange. This paper proposes that in these conditions, risk-lover people could choose to use Bitcoin as medium of exchange since as a first step to use Bitcoin as a medium of exchange, people seek to invest in Bitcoin to gain expected returns which involves high uncertainties at the same time. Risk-neutral people may also use Bitcoin as a medium of exchange depending on its variance of expected returns. However, even though the expected returns and the variances of investing in Bitcoin and in the market portfolio are the same (i.e. $\beta = 1$), the return as utility from the consumption on goods and services by using Bitcoin as a medium of exchange in the goods market could be lower than the return by using fiat money as a medium of exchange since the transaction fee of using Bitcoin will lower the expected returns on the consumption on goods and services purchased by Bitcoin. To compensate the transaction fee rate, the expected return in investing Bitcoin should be larger than the expected return in investing the market portfolio so that the aggregate return on Bitcoin will be larger than the aggregate return on fiat money. For risk-averse people, it can be said that they will probably choose to invest in the market portfolio if they have an investing motive, if they do not then they will share their whole wealth into the consumption on goods and services in the goods market (In this case, the fractions will be such that $x = 0$ and $\theta = 0$).

Overall, the electricity fee rate and the transaction fee rate are the important determinants of the general welfare level of the coexistence economy. If an increase in the cost of electricity fee rate (f^e) is not offset by an increase in the rewards as Bitcoin (S), then the increase will be offset by an increase in the transaction fee rate which will decrease the aggregate expected return on the Bitcoin. For this reason, the "return-motive" that lead people to buy Bitcoin and then use it as a medium of exchange will be damaged because of the lower returns on Bitcoin. In addition, if the

asset return on Bitcoin increases and the uncertainties could be lowered behind Bitcoin, then the increase in asset return on Bitcoin may improve the "return-motive" of people which eventually lead people to use Bitcoin as a medium of exchange to meet their needs of goods and services.

5. Conclusion and Discussion

This paper answers the question of how cryptocurrencies and fiat currencies could be used together in a coexistence economy. To be able to grasp the idea of an alternative currency like cryptocurrencies, a short history of Bitcoin was introduced in the introduction part. Most of the studies in the literature related to the relationship between cryptocurrencies and fiat currencies up to now have focused on cryptocurrencies by testing them whether they can be perfect substitutes to fiat currencies which seems unrealistic at least if we consider the today's conditions on the relationship. Therefore, this paper does not assume a perfect substitutability of cryptocurrencies, instead, it provides a coexistence economy where both cryptocurrencies and fiat currencies could exist together as in Kang and Lee's work (2019). However, the paper did not completely build on their work, it just gets inspired the idea of the coexistence economy so most of their framework were changed. The Capital Asset Pricing Model (CAPM) was used to show the "return-motive" of Bitcoin which is the main trigger leads people to buy Bitcoin as an asset and then with this motive, they can use Bitcoin as a medium of exchange as well. The preferences of buyers depend on the variance of the expected return on investing Bitcoin, the transaction fee that is paid to the miners, the cost of electricity that lowers the miners' total effort and expected profits. If the aggregate expected returns on Bitcoin is larger than the aggregate expected returns on fiat money, then the buyers which are risk-lovers or risk-neutrals will choose to use Bitcoin depending on the variance of the return. This paper proposes that if the uncertainties behind Bitcoin and thereby the variance of expected returns on Bitcoin could be decreased, then the probability of using it as a medium of exchange will increase. In this conjuncture, a

perfect substitutional relationship between cryptocurrencies and fiat currencies could not be expected because of the higher uncertainty behind cryptocurrencies.

The model in this paper has several assumptions that weaken the model's power of explanation. For instance, it is assumed that there is no government, so the model does not include the taxes paid to government. It is also assumed that the inflation is captured by the expected returns, therefore there is no explicit expression of growth rate of Bitcoin and fiat money. Since the model uses the Capital Asset Pricing Model, it defines the buyer's and the seller's problem in the percentage terms. However, the miner's problem is solved by using integers not using percentage terms. Delivery lags which are emerged when buyers make a transaction to buy goods and services are assumed to be zero. The model also assumes that all the goods and services in the goods market could be purchased by both Bitcoin and fiat money. Therefore, the further research could be made by relaxing these assumptions. The utility function of buyers could be modified to reflect the differences between the goods and services which could be purchased by Bitcoin and which could be purchased by fiat money. In reality, the amount of goods and services which could be purchased by Bitcoin is very limited compared to that of fiat currencies.

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