# Investigation of the Specialized Content Knowledge of the Pre-service Primary School Teachers about Multiplication

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This study aimed to investigate the specialized content knowledge of pre-service primary school teachers about multiplication through problem posing and the justification for the accuracy of the multiplication of a two-digit number and a single-digit number. The research questions were formulated based on the theoretical framework of Ball and her colleagues' teacher knowledge. The data were collected from second, third, and fourth-year pre-service teachers studying in the Faculty of Education of a state university using two open-ended questions. The data were analyzed with the content analysis method to provide a detailed perspective on the specialized content knowledge of pre-service teachers about multiplication. The study's findings showed that although the pre-service teachers had difficulties writing a problem related to multiplication operations in which one of the multipliers was zero, the number of correct problem statements for the given operation increased as the pre-service teachers proceeded in their education. Another finding of the study is that the majority of the pre-service teachers made correct interpretations about the correctness or incorrectness of the solutions to the multiplication of a two-digit number and a single-digit number. However, it was observed that the pre-service teachers based their justifications for the student solutions given to them on students' operational knowledge rather than their conceptual knowledge of the multiplication operation. The study's findings were discussed within the framework of the relevant literature, and some recommendations were made.

## Introduction

Teacher knowledge is undoubtedly crucial in the challenging and complex process of teaching mathematics. More specifically, specialized content knowledge is critical for teachers to teach mathematics effectively (Ball, 1990; Carpenter, Fennema, Franke, Levi, & Empson, 1999). Teachers with this knowledge can understand students' thinking and interpret the solutions they produce more easily with a teacher-specific perspective (Shulman, 1986; Ball, 1990; Ball, Thames, & Phelps, 2008). Having specialized content knowledge is a prerequisite for teachers to construct mathematical knowledge. For this reason, teacher training programs aim to train pre-service teachers in and through specialized content knowledge. One of the subjects which should be structured by primary school students and for which teachers' specialized content knowledge is important is multiplication (National Council of Teachers of Mathematics [NCTM], 2000; Ministry of National Education [MoNE], 2018). Pre-service teachers' knowledge about multiplication arouses curiosity (Apsari, Wulandari, & Novitasari, 2021; Harkness & Thomas, 2008; Whitacre & Nickelson, 2016; Ding, Xiaobao, & Capraro, 2013). There is a limited number of studies in the literature on the specialized content knowledge of pre-service primary school teachers and their exploration of the meaning and operation of multiplication. This study aimed to fill this gap by examining the specialized content knowledge of pre-service primary school teachers about multiplication. Specifically, this study focused on the pre-service primary school teachers' specialized content knowledge about multiplication in terms of writing a suitable problem for multiplication and the accuracy of the multiplication of a two-digit number and a single-digit number. The study's findings are expected to contribute to future research by providing new insights into helping teachers improve their specialized content knowledge.

### *Theoretical Framework*

Shulman (1986) classified teacher knowledge as content knowledge, pedagogical content knowledge, and curriculum knowledge. Following Shulman's (1986) studies on teacher knowledge, Ball et al. (2008) put forward a more specific theoretical framework for mathematics education. They included the knowledge that a mathematics teacher should have in this framework (Ball et al., 2008). In their model named "Mathematical Knowledge for Teaching," Ball et al. (2008) divided content knowledge and pedagogical content knowledge defined by Shulman (1986) into sub-components. Ball et al. (2008) categorized content knowledge into three components: common content knowledge, specialized content knowledge, and content knowledge at the horizon. Common content knowledge is generally defined as the mathematical knowledge possessed by any individual (Ball et al., 2008). The product of two two-digit numbers, the ordering of fractions, and the addition of rational numbers are examples of common content knowledge. However, specialized content knowledge can be considered special knowledge for mathematics educators (Ball et al., 2008). For example, being able to represent a mathematical expression in more than one way, justifying the steps of the operation, and deciding whether student thinking and solutions are generalizable are examples of specialized content knowledge. Therefore, this knowledge can be considered a special knowledge for mathematics educators. The third and final component of subject content knowledge is knowledge at the mathematical horizon, defined as "awareness of how mathematical subjects in the curriculum within the scope of mathematics are related" (Ball et al., 2008).

A teacher's knowledge would be incomplete without general content knowledge and specialized content knowledge. Content knowledge at the horizon provides the knowledge of how the objectives in the curriculum should be structured in practice. In addition, knowledge of content and student, knowledge of content and teaching, and knowledge of content and curriculum are used to transfer in-depth information about specific content to students. In this study, we focused on Ball et al.'s (2008) specialized content knowledge. More specifically, problem posing for multiplication was selected as one tool to evaluate their specialized content knowledge based on Ball et al.'s (2008) framework.

### *Problem Posing*

Problem posing, a significant part of the mathematics curriculum, is essential to learning and teaching mathematics (Brown & Walter, 1993; Moses, Bjork, & Goldenberg, 1990; Silver, 1994; English, 1998; Kilpatrick, Swafford, & Findell, 2001; NCTM, 2000). Problem posing is defined as the "generation of new problems and the re-formulation of a given problem" (Silver, 1994, p.19). English (1997b) states that constructing new problems from a given mathematical task is the main activity of problem posing. The NCTM (2000) reported that in addition to solving preformulated problems, it is necessary to engage in mathematics activities that include posing problems. Based on these considerations, it is assumed that there is a strong positive relationship between students' mathematical understanding and their ability to pose a problem (Leung, 1993). In more detail, students who can pose problems have many advantages, such as understanding mathematical concepts, enhancing problem-solving ability, developing diverse and flexible thinking, and feeling confident with mathematics (Silver, 1994).

Students can connect concepts, procedures, symbols, and semantic referents through conceptual understanding (Subramaniam & Banerjee, 2011). Students' difficulties in understanding operations and algorithms are grounded in teachers' approaches while teaching concepts and solving problems (Sharp, Garofalo & Adams, 2002). Problem-posing tasks affect students' understanding. In addition, teachers' ability to pose problems affects their teaching performance (Barlow & Cates, 2006). To provide relevant activities and different learning experiences for students, teachers must generate and reformulate problems (Crespo & Sinclair, 2008; Rowland, Huckstep, & Thwaites, 2003; Singer & Voica, 2012). Teachers' expertise and knowledge in problem posing are crucial as they can affect students' conceptual understanding (Crespo & Sinclair, 2008; Rowland et al., 2003; Singer & Voica, 2012). Problem posing is considered a means to investigate the knowledge of teachers who pose a problem (Ribeiro & Amaral, 2015). From this point of view, the study aimed to investigate pre-service primary school teachers' specialized content knowledge about multiplication through problem-posing tasks.

In the literature, there are many studies on problem posing (English, 1998; Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005; Chapman, 2012; Leung, 2013; Riberiro & Amaral, 2015; Lee, Capraro, & Capraro, 2018). Some of these studies on problem posing focused on the problem-posing processes of students and teachers (e.g., English, 1998; Chapman, 2012; Christou et al., 2005). More specifically, they focused on what kind of problems students and teachers pose and in which categories these problems can be grouped. In addition to the studies on students' and teachers' problem-posing processes, there are also studies investigating teachers' knowledge of problem posing although the number of these studies is limited (Ribeiro & Amaral, 2015; Lee et al., 2018). Research studies have shown that problem posing activities provide in-depth information about pre-service teachers' content knowledge (Barlow & Cates, 2006). Such activities create an environment for using field knowledge in a versatile manner as they allow pre-service teachers to express their existing knowledge on the subject as an appropriate mathematical sentence and to reuse their knowledge by thinking about the solution to the mathematical problem (Barlow & Cates, 2006). Therefore, including such activities in research studies ensures that the results of the studies examining the pre-service teachers' content knowledge are informative. For example, Lee et al. (2018) examined teachers' subject matter knowledge (SMK), knowledge of content and teaching (KCT), and knowledge of content and students (KCS) through the problems teachers pose. However, they did not investigate teachers' common content knowledge (CCK) and specialized content knowledge (SCK). Furthermore, Ribeiro and Amaral (2015) focused on prospective primary school mathematics teachers' specialized knowledge (MTSK) while posing a problem for a given expression in the context of division. Studies on teachers' or pre-service teachers' subject matter knowledge in problem posing are limited. For this reason, in this study, problem posing is one of the tools to evaluate pre-service primary school teachers' specialized content knowledge within the context of multiplication. Another tool to investigate pre-service teachers' specialized content knowledge is through their justification of students' solutions about the multiplication of a two-digit number and a single-digit number.

### *Justification of Students' Solutions*

Teachers need a deep understanding of mathematics to understand why students' methods are right or wrong (Ma, 1999). We can interpret students' understandings through student actions reflecting student thoughts (Leatham, Peterson, Stockero, & Van Zoest, 2015). For this reason, to conduct effective mathematics teaching, it is necessary to build mathematics instruction based on student thinking (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; National Council of Teachers of Mathematics, 2014). When teachers focus on student thinking, they can “interpret what the student does and says, and try to build a ‘model’ of the student’s conceptual structures” (von Glasersfeld, 1995, p.14). In other words, focusing on student thinking allows for a rational analysis of students’ mathematical understanding (van Es & Sherin, 2008; Jacobs, Lamb, & Philipp, 2010; Leatham et al., 2015).

A student’s solution is one way to give the teacher clues about the student’s thinking (English, 1997a; English, 2003; Leatham et al., 2015). In more detail, through the solutions students produce, teachers can analyze how students solve a problem, what strategies they use, and what misconceptions they have. Preservice teachers must gain experience in interpreting students’ thinking and methods mathematically. In the classroom, they must do this to promote conceptual understanding over practice (Harkness & Thomas, 2008). Since specialized content knowledge is a special type of knowledge for mathematics educators involving the ability to represent a mathematical expression in more than one way and justify the steps of the operation, teachers should have specialized content knowledge to decide whether students’ solutions are correct or not (Ball et al., 2008). From this point of view, in this study, pre-service primary school teachers were asked to justify the correctness of the two solutions about multiplication, including different mathematical details to evaluate their specialized content knowledge.

### *Definition of Multiplication and Multi-Digit Multiplication*

The most common meaning of multiplication is repeated addition. This meaning is important in dealing with and understanding multiplication in real-life situations (Whitacre & Rumsey, 2020). Some researchers name repeated addition as the primitive intuitive model for multiplication (Fischbein, Deri, Nello, & Marino, 1985, p. 4). This is the meaning of multiplication covered at the primary school level according to the Turkish primary school mathematics curriculum. Based on this meaning of multiplication, in a multiplication operation including two terms such as $a \cdot b$, one of the terms represents the number of groups, and the other represents the number of objects a group has. The number of groups gives information about how many times the objects in the group will be collected. Accordingly, the 9x5 operation means the total number of people in nine groups, each of which consists of five members, and the multiplication to count group members is based on addition (5+5+5+5+5+5+5+5+5) (Lampert, 1986). The numbers in the group must be matched precisely with the numbers in the other groups, emphasizing that the groups are equal. As a result, one of the multipliers indicates the number of identical collections, while the other indicates the size of the collection (Tirosh & Graeber, 1989). The meaning of repeated addition also feeds the belief that multiplying will always magnify the result (Tirosh & Graeber, 1989). However, numbers with decimal representations or negative integers can be problematic in performing multiplication operations by making sense of the group number. For example, 0.63 times 3 (Fischbein et al., 1985, p.6) is not a process that can be perceived intuitively. Therefore, the primitive intuitive model may not be sufficient when students work with different numbers. Multiplication has the meanings of rate, factor, array, and cartesian product besides repeated addition (Mulligan & Mitchelmore, 1997). In addition to these interpretations, multiplication also gains meaning in the examples of combination problems that are meaningful in the problems of area and other measurements that can be associated with the measurement subject (Van de Walle, Karp, & Bay-Williams, 2018). In problems related to the multiplication of area and other measurements, multiplication refers to a different unit than multipliers. For example, in area problems, the unit of multiplication of different unit lengths is the unit square, which in context refers to a different dimension of measurement.

Research studies have shown that students can interpret single-digit or multi-digit multiplication operations in different ways. Lampert (1986) states that while repeated counting may be sufficient when performing single-digit multiplications, this method is insufficient. It is technically more difficult for students when working with larger numbers. This is also valid in multi-digit multiplications (Whitacre & Nickerson, 2016). Multi-digit multiplications can also be performed by producing different methods and using mathematical facts and associations (Lampert, 1986). These methods can be meaningful in a real-life context students encounter or through concrete materials, as well as performing multiplication using mathematical facts and association and applying the multiplication algorithm correctly. Accordingly, multi-digit multiplications can be carried out with two main strategies: the number-based strategy and the digit-based algorithm (Hickendorff, Torbeyns, & Verschaffel, 2019). The number-based strategy can be implemented using four strategies: sequential, decomposition, varying, and column-based (Hickendorff et al., 2019).

Research studies showed that pre-service teachers can correctly apply the multi-digit multiplication algorithm but have difficulty making sense of it (Apsari et al., 2021; Harkness & Thomas, 2008; Whitacre & Nickelson, 2016). The rectangular model supports pre-service teachers' understanding of the multiplication algorithm when conceptualizing multipliers as 'partial' products (Whitacre & Rumsey, 2020). Simon and Blume (1994) investigated prospective teachers' interpretations of multiplication on the rectangular model. They found that the interpretations of the prospective teachers were limited in presenting justification for the relationship between the equal groups meaning of multiplication and length × width. Another study revealed that the commutative property, one of the three principles defined on the multiplication operation, can be confused with the associative property by pre-service teachers, and their performance in producing appropriate problems for the associative property is low (Ding et al., 2013). However, to teach multiplication, teachers need to understand why the multiplication algorithm works, connect the method to appropriate models, detect student errors, and select appropriate numbers to provide students with learning opportunities based on their specialized content knowledge on multiplication (Whitacre & Nickelson, 2016).

### *Multiplication in Turkish Primary School Curriculum*

In the Turkish primary school curriculum, the multiplication process is presented to the students as single-digit and multi-digit multiplication with the multiplication table and different problem types, including the meanings of multiplication suitable for the student's level (MoNE, 2018). Multiplication is not covered in the first grade. In the second grade, students learn the repeated addition meaning of multiplication, multiplication with natural numbers up to 10, and solving problems using multiplication as a single operation. In addition, in this process, the effect of 1 and 0 in multiplication is emphasized, and students are taught that changing the location of the multipliers will not change the product. The meaning of zero and the interpretation of zero in multiplication are covered in this grade. Explaining the role of 0 and 1 in multiplication in the second grade is an important step for the multiplication operations students will learn in the third grade. Third graders learn the multiplication of a two-digit number with another two-digit number. Moreover, third graders acquire knowledge of the traditional algorithm by giving meaning to the operations between the digits. Therefore, it can be said that the tens and units emphasized in the first grade find meaning in the multiplication process covered in the third grade. Furthermore, third graders learn multiplication by 10 and 100 and the rate meaning of multiplication. Therefore, by the end of this grade, students will learn the meanings of repeated addition and the rate of multiplication. Fourth graders, on the other hand, learn multiplication of three-digit and two-digit numbers, multiplication of two-digit natural numbers with 5, 25, and 50 using a shortcut, mental multiplication of three-digit numbers with 10, 100, and 1000, and commutative property of multiplication. Moreover, multiplication of natural numbers, including a maximum of three digits with natural numbers that are a maximum of nine times 10, 100, and 1000 are also taught at this grade (MoNE, 2018). Therefore, in fourth grade, it is significant for students to perform multiplication operations with three-digit numbers, predict the operation's result, and perform operations in their minds. Finally, it is recommended by the curriculum to present third- and fourth-graders some problem-posing activities about multiplication.

Considering the primary school curriculum, it is understood that pre-service teachers should have in-depth knowledge of the meanings of multiplication, the commutative property of multiplication, and calculation. While attention is drawn to the meaning of repeated addition and rate in the primary school curriculum, pre-service teachers are expected to know the commutative property of multiplication and to present problem-posing activities that the curriculum recommends. In addition, the sequential multi-digit multiplication strategy pointing to repeated addition, the column-based strategy pointing to adding the multiplication results obtained separately based on the place values, and the digit-based strategy emphasizing the use of algorithms are the basic strategies that are important for primary school students. Thus, primary school teachers are expected to make sense of these strategies related to their specialized content knowledge (Ball et al., 2008; Ding et al., 2013). Based on the theoretical framework of Ball et al. (2008), this study aimed to investigate pre-service primary school teachers' specialized content knowledge about multiplication in terms of writing a suitable problem for multiplication. Furthermore, the pre-service primary school teachers' specialized content knowledge on the accuracy of the multiplication of a two-digit number and a single-digit number and the reasons they presented for this were evaluated within the scope of their expertise.

In line with the aim of the study, the following research questions were addressed:

**1.** What is the nature of the pre-service primary school teachers' specialized content knowledge on multiplication?

> **a.** What kind of problems do pre-service primary school teachers write for multiplication operations (0x5 and 5x0)?
>
> > i. How are the problems written by the pre-service primary school teachers about the multiplication operations (0x5 and 5x0) distributed according to the year of study?
>
> **b.** How do pre-service primary school teachers justify the accuracy of student solutions (48x5) regarding the multiplication of a two-digit number and a single-digit number?
>
> > i. How are these justifications distributed according to the year of study?

# Method

## *Research Design*

Qualitative case study provides an in-depth investigation and understanding of the subject investigated in a certain context with one or more cases (Creswell, 2007; Yin, 2009). Yin (2009) categorized case studies into single-case holistic, multiple-case holistic designs and single-case embedded, multiple-case embedded designs. Whether a study is a single-case or multiple cases is related to the number of cases, and whether a study is holistic or embedded is related to the number of units of analysis. The case of the study is the pre-service primary school teachers, and the specialized content knowledge of the pre-service teacher is the unit of analysis. The study discussed the specialized content knowledge of the pre-service teachers about multiplication in the context of the problems written by the pre-service teachers and the justification of the pre-service teachers regarding the accuracy of the answers given by the students. In brief, the study employed the single-case holistic design since the aim was to gain insight into the specialized content knowledge of pre-service primary school teachers on multiplication in detail.

### *Context and Participants*

The study was carried out with pre-service primary school teachers (PTs) studying in the Classroom Teaching program of the Faculty of Education at a public university in Turkey. According to the package program created by the Council of Higher Education in 2007, PTs studying in this program complete the Basic Mathematics I and II courses at the end of the first year of their education and take the Instructional Principles and Methods and Instructional Technologies and Material Design courses at the end of the second year. In addition, they take Mathematics Teaching I and II courses in their third year and Teaching Practice I and II courses in their last year. When they graduate from this program, they work as teachers of first-, second-, third-, and fourth-grade students in primary schools.

The participants were selected using the purposive sampling method, and some criteria were considered during this selection process. First, the participants were selected from among PTs studying in the Classroom Teaching program. The participants were chosen from among the classroom teachers because multiplication has an important place in the primary school curriculum (MoNE, 2018), and classroom teachers teach this subject. Secondly, attention was paid to ensuring that the participants were easily accessible and close to the researcher so that the test could be administered to the participants in a short time. In addition, according to the ethical rules of scientific research, the participants were selected from among those who volunteered to participate. Finally, the participants were the PTs in their 2nd, 3rd, or 4th year at university. The aim was to compare the subject matter knowledge of pre-service teachers at different years of their education. In addition, the basic courses taken by pre-service teachers (e.g., Basic Mathematics I and II, Mathematics Teaching I and II, Instructional Principles and Methods, Instructional Technologies and Material Design, and Teaching Practice 1-2) help pre-service teachers improve their subject matter knowledge related to mathematics. In Basic Mathematics 1 and 2 courses, pre-service teachers learn to prove mathematical definitions and theorems and to perform operations on numbers and concepts. These courses also involve investigating the properties of multiplication, such as commutative and closure properties on different number sets, and enable pre-service teachers to make sense of algorithms.

Pre-service teachers learn to prepare materials and use technology to teach any mathematics subject through Instructional Principles and Methods and Instructional Technologies and Material Design courses. The aim of Mathematics Teaching I and II courses is to help pre-service teachers comprehend the basic strategies and methods that they can use in teaching mathematics, to introduce the primary school mathematics program, to equip pre-service teachers with knowledge and skills that are important in mathematics education, and to improve their skills to develop activities suitable for the objectives of the curriculum. In these courses, methods and sample activities on how to teach each mathematics subject in the primary school curriculum are examined. The methods of teaching multiplication are also covered in these courses. Finally, in Teaching Practices 1-2 courses, pre-service primary school teachers can prepare and implement appropriate lesson plans by gaining basic knowledge and skills about their fields and teaching formation. One of the subjects they teach or have the opportunity to prepare a lesson plan on is multiplication. In the study, one inclusion criterion was having completed at least two of these courses (e.g., Basic Mathematics I and II). Since the freshman pre-service teachers did not meet this requirement, the participants were selected from among the second-, third-, and fourth-year pre-service teachers. One hundred six volunteer PTs aged 19 to 21 participated in the research. Thirty of these participants were in the second year, 31 were in the third year, and 45 were in the last year of the program. The data was collected in the Spring Term of the 2017-2018 Academic Year.

### *Data Collection*

The study aimed to examine the specialized content knowledge of the PTs about multiplication. To this end, an instrument was developed and administered to the participants in a 40-minute session. Then, only two open-ended questions from this instrument were used in the current study. The participants were informed that their names and other personal information would be confidential. Each question and the related literature about the questions are explained below.

In the first question (see Figure 1), the comprehension dimension, one of the problem-posing situations put forward by Christou et al. (2005), was considered. In the case of problem posing based on comprehension, the mathematical equation or the operation/operations used in the solution are given, and they are asked to write a problem suitable for this equation or operation (English, 1998; Christou et al., 2005). Based on this, the PTs were asked to write a problem related to the 5x0 and 0x5 operations to examine their specialized content knowledge, a dimension under the conceptual framework Ball et al. (2008) created for teaching mathematics. Although 5x0 and 0x5 operations seem to be the same, they have different meanings (Van de Walle, Karp, & Bay-Williams, 2018). While the 5x0 operation refers to 5 clusters with 0 elements, the 0x5 operation means 0 clusters with 5 elements (Van de Walle, Karp, & Bay-Williams, 2018). Thus, considering the semantic difference that emerges when the place of zero is changed in multiplication, in the first question, the PTs were asked to write a mathematical problem suitable for the operations 5x0 and 0x5, and their specialized content knowledge in multiplication was examined.

In addition, specialized content knowledge of mathematics educators is about knowing the details of mathematics (Ball et al., 2008). Based on this definition, being able to interpret the meaning of 0 in 5x0 and 0x5 operations, knowing the different meanings of multiplication (e.g., repeated addition and rate), and knowing the differences in meanings of these two operations (5x0 and 0x5) are the information under the category of specialized content knowledge. For this reason, the question in Figure 1 was posed.

a. Write a mathematical problem that can correspond to the 5x0 operation. Solve the problem you wrote.
b. Write a mathematical problem that can correspond to the 0x5 operation. Solve the problem you wrote.

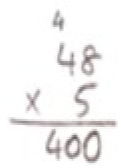
*Figure 1*. Question 1 inviting PTs to create a multiplication problem

In the second question (see Figure 2), solutions of two students (Ömer and Esma) to the 48x5 operation were given to the pre-service teachers, one correct with a non-traditional solution method and one incorrect. The solutions were adapted from the student solutions in

Ashlock's (2010) book. The researchers created the questions about student solutions to measure the specialized content knowledge under the conceptual framework on teaching mathematics created by Ball et al. (2008). Multi-digit multiplication requires splitting numbers into parts and performing calculations flexibly by applying the distributive property of multiplication over addition. The meaning of multi-digit multiplication becomes clear as this path makes the transitions between the digits in the algorithm meaningful. In multi-digit multiplication operations, which can be interpreted through different models, we aimed to focus on making sense of the multiplication algorithm, which is emphasized at the primary school level in Turkey. Therefore, the participants in this study were asked to perform a multiplication operation involving a single-digit multiplier, which is the most basic form of multi-digit multiplication.

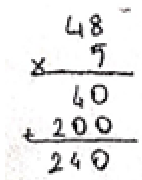
The solutions to the 48x5 operation and the second question posed about the solutions are given below:

Ömer's Solution




a. Is Ömer's solution correct or incorrect? Explain your answer with reasons.

Esma's Solution




b. Is Esma's solution correct or incorrect? Explain your answer with reasons.

*Figure 2*. Question 2, including sub-questions regarding Esma's and Ömer's solution

As can be seen in Ömer's solution, there is a mathematical difficulty related to "carrying." In Ömer's solution, we see a different use of carry in the multiplication algorithm. Here, the student combined 4 tens in the number 40, which he determined due to the 8x5 operation with the 4 tens in the number 48. From here, he got '8'. When he multiplied the number 8 by 5, he reached 40. Thus, it can be said that Ömer did not have a problem determining whether carrying would be performed, but carrying correctly in the calculation was the problem. In this solution, the amount represented by the numbers in the multiplication operation and its function in the calculation are disconnected. The PTs could attribute the difficulty of carrying over to Ömer's lack of knowledge and understanding about digit value, which is related to their specialized content knowledge.

On the other hand, Esma answered the question correctly but used an unconventional solution. In Esma's solution, the quantities represented by numbers resulting from multiplication and how these quantities function in the calculation complement each other correctly. Esma's solution is non-conventional in that she multiplied numbers by dividing them into digit values, which is also related to the specialized content knowledge of PTs. In summary, how the PTs commented on the correctness or incorrectness of Ömer's and Esma's solutions and how they justified their comments are related to their specialized content knowledge.

### *Data Analysis*

Content analysis (Strauss & Corbin, 1990) was performed to examine the specialized content knowledge of PTs about multiplication through the problems proposed by them and the justification of student solutions. For the analysis of the first question, two authors of this study read all the problems and coded them separately simultaneously. Based on the concept of multiplication (Van de Walle, Karp, & Bay-Williams, 2018), the type of problem posing (Christou et al., 2005), and the similarities and differences of participants' responses, Table 1 was created. Afterward, the coding was rediscussed and compared, and if needed, changes were made to ensure conformability.

Table 1
*Problem Types and Characteristics of the Problems*

| Problem types | | Problem characteristics |
|---|---|---|
| ***Appropriate Problem*** | | |
| | ***Appropriate Problem with Sufficient Representation of Multipliers*** | A problem in which the meanings of the number of groups and group size and the positions of multiplier are correctly interpreted. |
| | ***Appropriate Problem with Insufficient Representation of Multipliers*** | A problem in which the meanings of the number of groups and group size are correctly interpreted, while the positions of multiplier are incorrectly interpreted. |
| ***Inappropriate Problem*** | | A problem written about an operation other than multiplication (addition, subtraction, division) or about a problem not associated with multiplication |
| ***Not a Problem*** | | A statement that does not imply a question or a mathematical problem<br><br>Problem without content integrity |

For the given operations, clear and fluent problems in which the number of groups and the group size were expressed correctly, and the place of the multipliers were interpreted correctly were coded as "appropriate problem with sufficient representation of multipliers." In the problems evaluated in this category, the pre-service teacher considered the place of the multipliers and evaluated the operation as *the number of group x group size*. The problems in which the number of groups and the group size were determined correctly, but the place of the multipliers in the process was not considered were coded as "appropriate problem with insufficient representation of multipliers." In the problems in this category, the pre-service teacher correctly determined the number of groups and group size but misinterpreted the multipliers' place and thus did not evaluate the operation as *the number of group x group size*. In addition, the answers of the pre-service teachers who wrote the same problem for both processes were evaluated in this category, as they indicated that they did not consider the multiplication position. In addition, problems including another operation (addition, subtraction, division) or a problem not associated with multiplication were classified under the "Inappropriate Problem" category. Finally, the statements that did not imply a question or a problem were included in the "Not a Problem" category.

For the analysis of the second question, first, the answers of the PTs regarding the solution were grouped as correct and incorrect. Then, the justifications were named and grouped within themselves based on the participants' explanations. The inter-code method was used to analyze the data, and the reliability coefficient was found to be 91%, and no intervention was made to ensure inter-coder reliability. The coding completed by the two coders was terminated when the differences were discussed and agreed upon.

## Findings

The findings obtained from this study were examined under two headings. In the first part, in response to the research questions 1.a and 1.a.i, the problems written by the PTs for the multiplication operations where zero is the multiplier (0x5 and 5x0) and the differences that these problems show across the year of study of the PTs were examined. In the second part, in response to research questions 1.b and 1.b.i, the opinions of the PTs about the accuracy of the student solutions given to them for the multiplication of a two-digit number and a single-digit number, the justifications they presented and the differences in the justifications across the year of study of the PTs were examined.

### *Problems about Multiplication*

The problems created by the PTs regarding the problem where zero is the multiplier were classified under three categories: appropriate problem, inappropriate problem and not a problem. The characteristics of the answers in these categories and the frequency of the answers given by all the participants for the 5x0 and 0x5 problems are presented in Table 2.

Table 2
*Classification of the Problems Posed by the Participants Regarding the 5x0 and 0x5 Operations*

| | Problem types | 5x0 operation | 0x5 operation |
|---|---|---|---|
| ***Appropriate Problem*** | ***Appropriate Problem with Sufficient* Representation of Multipliers** | 16 (%15) | 8(%7,5) |
| | ***Appropriate Problem with Insufficient* Representation of Multipliers** | 7(%7) | 8(%7,5) |
| | ***Inappropriate Problem*** | 15(%14) | 10(%9) |
| | ***Not a Problem*** | 32(%30) | 29(%27) |
| | ***No answers*** | 36(%34) | 51(%48) |

As Table 2 shows, when the PTs were asked to write a problem regarding the 5x0 operation, approximately one-third (34%) of the participants left the question completely unanswered and did not write anything. The majority (54%) wrote statements with no question meaning or problems that did not have content integrity instead of mathematical problems. In addition, 15 PTs (14%) who participated in the research wrote a problem of addition, subtraction, or division operations instead of the multiplication operation given. Therefore, only 16 (15%) of the 106 participants could write a meaningful and correct problem related to the 5x0 operation by correctly interpreting the meanings and positions of 5 and 0 in the 5x0 operation. Finally, 7 PTs (%7) wrote the problem by correctly identifying the number of groups and the group size, although they ignored the place of multipliers.

When the problems written about the 0x5 operation are examined, it is seen that approximately half of the PTs (48%) did not answer the question. 29 PTs (27%) wrote statements that did not give a question meaning or problems that did not have content integrity. 10 PTs (9%) wrote a problem for other operations (addition, subtraction, or division operations) instead of multiplication. Among the participants, there were only 16 PTs (15%) who could correctly create an appropriate problem, including 8 PTs (7.5%) who could correctly interpret the meanings and the positions of 0 and 5 in the 0x5 operation and write a problem suitable for this operation and 8 PTs (7.5%) who ignored the place of 0 and 5 but correctly interpreted the number of group and group size for the 0x5 operation.

In multiplication operations, the first multiplier represents the group number, and the second multiplier represents the group size. Therefore, for 5x0 and 0x5 operations, it should be considered that the group size is zero and the group number is zero, respectively. As seen in Table 1, while the number of participants who left the question unanswered for the operation where zero is the second multiplier is lower compared to the operation where zero is the first multiplier, more participants could write a suitable problem for the multiplication operation. Therefore, it can be stated that the participants had more difficulty in finding examples where the number of groups was zero. Moreover, some answers include misinterpreting the place of zero, but the corresponding meaning of group number or group size was interpreted correctly. Finally, those who wrote a problem for another operation other than multiplication pointed out that the participants developed different meanings for the multiplication operation. Sample participant answers are given in Table 3:

Table 3
*Sample Participant Answers about Multiplication Problems*

| Problem regarding the 5x0 operation | Problem regarding the 0x5 operation |
|---|---|
| ***Appropriate Problem*** | |
| ***Appropriate Problem with Sufficient Representation of Multipliers*** | |
| *How many apples are there in total in 5 boxes containing 0 apple?* P84, 4th Year | *How many pencils are there in total in 0 pencil boxes containing 5 pencils?* P84, 4thYear |
| ***Appropriate Problem with Insufficient Representation Multipliers*** | |
| *Zübeyde has 0 lira. How many liras will Zübeyde have if her father gives her 0 lira 5 times?* P36, 3thYear | Zübeyde received 0 from the Scientific Research course. The teacher said, “Don't worry! I will multiply your grade by 5”. What is the latest exam grade of Zübeyde? P36, 3thYear |
| ***Inappropriate Problem*** | |
| *The crows ate 5 apples from a tree that had no apples. How many apples are left on the tree?* P50, 3th Year | |
| ***Not a Problem*** | |
| *When 0 is multiplied by any number, the result is 0.* P1, 2th Year | |

When the examples given in Table 3 are examined, it is seen that P84 correctly interpreted the position of the group number and size and their meanings in the problem situations he wrote for 5x0 and 0x5. For this reason, the problems were coded as “an appropriate problem with sufficient representation of multipliers.” In the example of P36, although the pre-service teacher interpreted the meanings of the group number and group size correctly, she misinterpreted their positions. This could be understood from the fact that the PT wrote the same problem situation for both 5x0 and 0x5. In both problems, the amount of money and the grade’s value are zero. In the first problem, there is 0 lira given 5 times, while in the second problem, there is a grade of 0 given 5 times. In both problems, 5 indicates the group number, and 0 indicates the group size. Thus, the problem written by P36 was coded as an appropriate problem with insufficient representation of multipliers. P50, on the other hand, was asked to write a problem situation related to multiplication. However, since he wrote a problem related to subtraction, the answer was coded as an inappropriate problem. Finally, since P1's answer did not indicate a problem, it was coded as “not a problem.”

The distribution of the problems written by the PTs for the 5x0 and 0x5 operations varies across the year of study. The distribution is given in Figure 3.

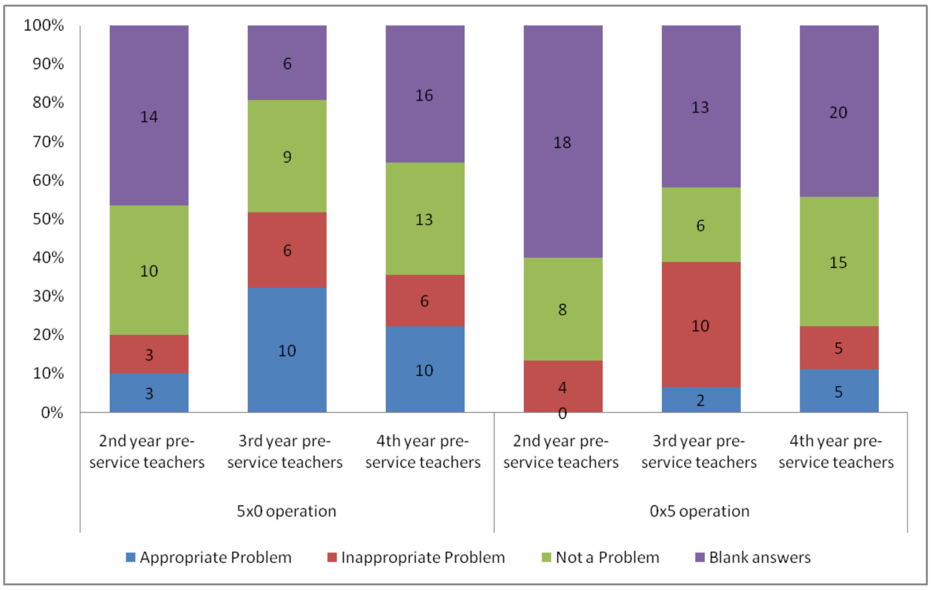



*Figure 3.* Investigation of the problems posed by the PTs regarding the 0x5 and 5x0 operations according to the year of study

When the types of problems written by the PTs for the 5x0 operation are examined according to their year of study, it is seen that almost half of the second-year PTs did not give any answers and left the question unanswered. Again, it was determined that the most common type of answer given by the same group of PTs was statements that did not indicate a problem. When we look at the distribution of the problems written about the 0x5 operation, it is seen that the distribution of the problem types that the second-year PTs posed for the 0x5 operation is like the distribution of the problem types they posed for the 5x0 operation. Most of the second-year PTs did not respond to the 0x5 operation either. Among the respondents, the most common type of answer is the statements that do not indicate a problem, while the type of answer that is not encountered is the problem suitable for the 0x5 operation.

The third-year PTs most frequently presented non-problem statements as answers for 5x0. While almost half of the third-year PTs did not write any answers, the most frequently recommended answer type is the incorrect problem, and the least recommended is the problem appropriate for the 0x5 operation.

One of the groups with the highest number of people who wrote a problem suitable for the given multiplication operation, 5x0, is the 4th year PTs. However, surprisingly, the group with the highest number of PTs who did not write any answers and left the question about 5x0 unanswered is again the fourth-year PTs. Similarly, the fourth-year PTs are the group who left most questions unanswered and wrote statements that do not indicate a problem for the 0x5 operation.

### *Interpretation of the Student Solutions by the Pre-service Teachers*

Two student solutions (solutions of Ömer and Esma) were given to the PTs for the 48x5 operation. They interpreted these solutions as incorrect or correct. In addition, some participants did not make any comments about the correctness of the solutions. In this section, the reasons why the participants considered these solutions to be correct or incorrect were examined.

***Justifications for Ömer's Solution.*** The participants classified Ömer’s solution (Figure 4) as correct or incorrect and provided justifications for their decisions. Five PTs (5%) left this question unanswered. One pre-service teacher (1%) thought that the solution was correct. One hundred participants (94%) stated that Ömer’s solution was incorrect; however, 29 (27%) of them did not provide any justification for their decision. Some PTs offered more than one justification. As can be seen in Table 3, 72 PTs provided a total of 75 justifications. Sample participant responses were represented in Table 4 as follows:

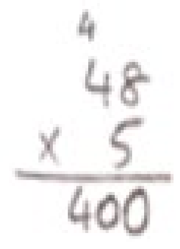



Figure 4. Ömer’s solution

Table 4
*Pre-service Teachers' Justifications Regarding Ömer's Solution as Correct or Incorrect**

| | Justification/Reason | Participant frequency | Sample participant responses |
|---|---|---|---|
| ncorrect | Incorrect implementation of the algorithm | 34 (%32) | It is wrong because while calculating 48x5, it should have been 4x5=20+4=24, but he calculated it as 40. P5, 2nd Year |
| | The answer is not the same as it should be | 24(%25) | The answer is incorrect. P68, 4th Year |
| | Different use of what 'carry' is | 8(%8) | It is incorrect. While doing the multiplication, the student wrote the 4 at hand as 40 as the result of the operation. P101, 4th Year |
| | Failure to grasp the multiplication operation | 5(%5) | It is incorrect. The student could not understand the multiplication operation. P9, 2nd Year |
| | The digits were not written in the correct place | 1(%1) | It is wrong because we have to multiply the units digit and write it in the units part and multiply the tens digit and write it in the tens part. P98, 4th Year |
| | Forgetting to multiply one of the digits | 1(%1) | It is incorrect because the student forgot to multiply one of the digits. P67, 4th Year |
| | Calculating 8x5 twice | 1(%1) | It is incorrect because the student wrote 8x5 twice. P40, 3rd Year |
| Correct | The answer is correct | 1(%1) | The answer is correct. P65, 4th Year |
| | Total number of justifications | 75 | |
| Total number of participants stating a justification | | 72 | |

*Some PTs stated more than one reason.
**Values are approximate.

As seen in Table 4, the justifications of the PTs for why Ömer's solution is incorrect are as follows: incorrect implementation of the algorithm (32%); the answer is not the same as it should be (25%); what is at hand was used differently (8%); the multiplication operation was not understood (5%); the digits were not written in the correct place (1%); the student forgot to multiply one of the digits (1%); 8x5 was written twice (1%); and the answer is correct (1%). Based on these answers, it can be stated that the PTs considered Ömer's answer or the multiplication algorithm when deciding whether the student's solution was correct or incorrect. This indicates that while the PTs were justifying Ömer's solution, they made a superficial decision by focusing on Ömer's knowledge of the algorithm of the multiplication operation or the result of the operation obtained by Ömer rather than explaining his conceptual understanding in detail. On the other hand, one participant stated that the student's solution was correct because he thought the answer was correct.

The distribution of the participants' comments on Ömer's solution by year of study is summarized in Table 5.

Table 5
*Distribution of Pre-service Teachers' Justifications for Ömer's Solution According to the Year of Study*

| Justification/Reason | 2nd Year | 3rd Year | 4th Year |
|---|---|---|---|
| | Participant frequency (Percentage) | Participant frequency (Percentage) | Participant frequency (Percentage) |
| Incorrect implementation of the algorithm | 8 (8%) | 9 (8%) | 17 (16%) |
| No justification | 10 (%9) | 10 (%9) | 9 (%8) |
| The answer is not the same as it should be | 10 (14%) | 4 (4%) | 10 (14%) |
| Different use of what carry is | - | 5 (5%) | 3 (3%) |
| Failure to grasp the multiplication operation | 1 (1%) | 2 (2%) | 2 (2%) |
| The digits were not written in the correct place | | | 1 (1%) |
| Forgetting to multiply one of the digits | | | 1 (1%) |
| Calculating 8x5 twice | | 1 (1%) | |
| The answer is correct | | | 1 (1%) |

When the reasons stated by the PTs regarding the correctness of the operation were analyzed according to the year of study of the PTs (Table 5), it was observed that the PTs had some common justifications for Ömer's solution regardless of their year of study, or there were some justifications specific to their year of study. As an unexpected finding, a senior student thought that Ömer's solution was correct, although his solution was incorrect. In addition to having trouble with multi-digit multiplication, this PT may also have misremembered the multiplication algorithm. On the other hand, among the reasons presented for the wrong solution, *the incorrect implementation of the algorithm* and the justification that *the answer is not the same as it should be* were presented as justifications by the PTs at each class year. Fourth-year PTs cited *the incorrect implementation of the algorithm* more often as justification. Some participants did not provide any justification by simply saying that the answer was wrong at each class year and their respective frequencies were close to each other. The participants who thought the solution was wrong due to the different use of what was at hand were the third-fourth-year PTs. Although the percentage is low, participants from all years of the study stated that the student's solution was wrong because the multiplication operation was not comprehended. The justifications of *the digits were not written in the correct place*, *forgetting to multiply one of the digits*, and *the answer is correct* were stated by only one fourth-year pre-service teacher. Performing the 8x5 operation twice is one of the reasons stated by a third-year pre-service teacher.

***Justifications for Esma's Solution***. Esma chose a non-traditional way to reach the solution and gave a correct answer (see Figure 4). Fifty-four participants (51%) thought this solution was correct, while 50 (47%) thought this solution was incorrect. Two PTs (2%) left this question unanswered. While 14 participants (13%) stated that the problem's solution was correct, 6 participants (6%) stated that the solution was wrong and did not provide any justification. Although most participants thought Esma's solution was correct, the number of PTs who thought this solution was incorrect is also quite high.

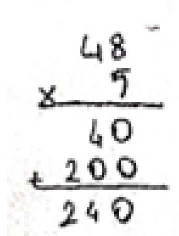



*Figure 4.* Esma's solution

As far as the justifications are concerned, the participants provided many justifications for the correctness or incorrectness of the solution. One pre-service teacher presented more than one justification. 40 PTs who stated that Esma's solution was correct produced 41 reasons. Table 6 presents the pre-service teachers' reasons for why Esma's solution is correct and sample participant responses.

Table 6
*Pre-service Teachers' Justifications Regarding Esma's Solution as Correct**

| Correct | Justification/Reason | Participant Frequency (Percentage) | Sample Participant Responses |
|---|---|---|---|
| | The answer is as it should be | 18 (17%) | Esma answered this question correctly, but what is expected is <br> 48 <br> x 5 <br> 240 (4) <br> 48 <br> x 5 <br> 240 (4) <br> P15, 2nd Year |
| | Multiplication which was operated based on digit value | 13(12%) | The answer is correct. <br> 48 → Birlik → 8x1=8 <br> → onluk → 4x10=40 <br> 5x8=40 5x40=200 <br> 200+40=240 <br> P33, 3rd Year |
| | The way to the solution is correct | 8(8%) | The answer is correct. She solved the problem using a different method. <br> P37, 3rd Year |
| | The digits were shifted. | 2(2%) | It's correct. She multiplied the units digit and then the tens digit, shifted one digit and added up. <br> P97, 4th Year |
| | Total number of justifications | 41 | |
| | Total number of participants | 40 (%38) | |

* One pre-service teacher stated more than one reason.

As seen in Table 6, the PTs considered Esma's solution to be correct for four reasons: *the answer is as it should be (17%), multiplication based on digit value (12%), correct way to the solution (7%), and digit shifting (2%).* Based on these reasons, we can say that the PTs mostly evaluated Esma's solution correctly, when they compared Esma's answer with the answer to the operation 48x5. Following this, the PTs drew attention to the conceptual dimension of Esma's solution and highlighted the function of the steps she used in the multiplication process. Accordingly, they pointed out that the number 5 was multiplied by the number 48, and the digits of 48 were considered. Some of the PTs emphasized that the digits were shifted. In this operation, they may have used the word 'shifting', referring to the fact that 200, obtained by multiplying 40 by 5, was written to the left of 40. Finally, there were also PTs who expressed the correctness of the solution in a very general way by presenting the reason as *the correct way to the solution.* Six of the 50 participants (5%) who found Esma's solution incorrect did not provide any justification for why the solution was incorrect. 44 PTs who explained that Esma's solution was incorrect by stating reasons produced 45 reasons, as seen in Table 7. In the table below, the reasons why the PTs found Esma's solution incorrect were exemplified:

Table 7
*Pre-service Teachers' Justifications as to why Esma's Solution is Incorrect**

| | Justification/Reason | Participant Frequency (Percentage) | Sample Participant Responses |
|---|---|---|---|
| Incorrect | Incorrect implementation of the algorithm | 16 (15%) | It is incorrect. She made a mistake in the operation in the ones digit. She was supposed to complete in the same place within the tenth digit. She could not fully comprehend how to multiply two-digit and one-digit numbers. P39, 3rd Year |
| | Incorrect way to solution | 10 (10%) | It is correct, but she reached the correct solution by chance, by following the wrong path. P101, 4th Year |
| | The necessity of performing a single operation for the solution | 7 (6%) | Esma's solution is wrong because the multiplication of a two-digit number and a one-digit number is simpler and requires only one operation. P8, 2nd Year |
| | Having no need to perform operations one after the other | 5 (4%) | Esma's solution is wrong because when multiplying a two-digit number and a single-digit number, there is no need to be written underneath. It is written next to it. P55, 3rd Year |
| | Thinking that multiplication was not understood | 4 (4%) | It is incorrect. Esma could not comprehend the multiplication operation. P44, 3rd Year |
| | Writing 200 instead of 20 | 2 (2%) | She unnecessarily wrote underneath. She added a zero next to 20. P96, 4th Year |
| | Confusion | 1 (1%) | It is incorrect. Esma knows some things. However, she got confused. In other words, she has some knowledge, but does not know what to use and where. P18, 2nd Year |
| | Total number of justifications | 45 | |
| | Total number of participants | 44 (%42) | |

* One pre-service teacher stated more than one reason.

The PTs attributed the incorrectness of Esma's solution to seven reasons: *incorrect implementation of the algorithm (15%), incorrect way to solution (10%), the necessity of performing a single operation for the solution (6%), having no need to perform operations one after the other (4%), thinking that multiplication was not understood (4%), writing 200 instead of 20 (2%), and confusion (1%).* Accordingly, most PTs (15%), who thought Esma's solution was incorrect even though it was correct, stipulated that the multiplication algorithm must be applied for 48x5 for the multiplication to be correct. Some PTs focused on the number of operations performed by Esma. They stated that while deciding on the correctness of Esma's solution, the solution should be performed with a single operation (6%) or that the operations performed one after the other was unnecessary and, therefore incorrect (4%). These views of the PTs imply that they expected to see a single number and a single operation line pointing to the result of the operation as applied in the multiplication algorithm. Some PTs (4%) stated that Esma did not understand the subject of multiplication or Esma knew something but did not know it exactly (1%), and they made such a decision when they saw Esma's solution. We understand that some of the PTs (2%) did not realize that Esma found 200 based on the result of 40x5 and expected her to write 20 instead of 200. The answers of the PTs show that they focused on applying multiplication algorithms without making sense of the algorithm. The justifications provided by the PTs for the accuracy of the operation are presented in Table 8 according to their year of study.

Table 8

*Distribution of Pre-service Teachers' Justifications for Esma's Solution According to Year of Study*

| | Justification/Reason | 2nd Year | 3rd Year | 4th Year |
|---|---|---|---|---|
| Correct | The answer is as it should be. | 4 (%4) | 6 (%6) | 8 (%7) |
| | Multiplication based on digit value | 2 (%2) | 5 (%5) | 6 (%6) |
| | The way to the solution is correct. | 3 (%3) | 4 (%4) | 1 (%1) |
| | The digits were shifted. | 1 (%1) | - | 1 (%1) |
| | Total | 40 PTs(%38) | | |
| Incorrect | Incorrect implementation of the algorithm | 1 (%1) | 7 (%7) | 8 (%8) |
| | Incorrect way to solution | 5 (%5) | 1 (%1) | 4 (%4) |
| | The necessity of performing a single operation for the solution | 5 (%5) | - | 2 (%2) |
| | Having no need to perform operations one after the other | - | 2 (%2) | 3 (%3) |
| | Thinking that multiplication was not understood | - | 1 (%1) | 3 (%3) |
| | Confusion | 1 (%1) | - | - |
| | Writing 200 instead of 20 | - | - | 2 (%2) |
| | Total | 44 PTs (42%) | | |

As seen in Table 8, when the justifications of the participants, who thought Esma's solution was correct, are examined, it is seen that the participants at different years of study also stated all the justifications other than digit shifting. When the justifications of the participants who thought that Esma's solution was incorrect were examined, it was seen that there were PTs from every year of study who thought that Esma's application of the multiplication algorithm was wrong and thus Esma's solution was wrong. Similarly, there were PTs from every year of study who focused on the necessary number of operations and pointed out that the student's understanding does not occur in the context of multiplication or that the student's knowledge is inaccurate due to the student's confusion in the use of information. The reason for "writing 200 instead of 20" was expressed by only a small number of fourth-grade students. The percentage of fourth-year students was found to be higher in all justifications.

## Discussion and Conclusion

The study's results revealed that 44% of the students could not write a problem for the 5x0 operation and 39% for the 0x5 operation. In addition, 34% and 48% of the PTs left the problem writing question unanswered for the 5x0 and 0x5 operations, respectively. In those operations, for the 5x0 operation, only 15% of the PTs and for the 0x5 operation, only about 9% of the PTs were able to pose the appropriate problem as a sufficient representation of multipliers. Accordingly, one of the reasons why the PTs were unable to propose and had difficulty in proposing the correct problem in this study may be because one of the multipliers in the problem they will write is zero. While it is possible to make zero visible in addition and subtraction, multiplication is a more difficult operation to make sense of zero and to make it visible as it makes the result of the operation zero (Semenza, Grana & Girelli, 2006). It is known that when zero is a multiplier, children have conceptual difficulties (Van de Walle et al., 2018), and the results of this study show that the PTs had similar difficulties. Studies conducted with teachers who teach at different grade levels on making sense of zero showed that teachers have the same tendency as pre-service teachers, and they adopt rote and rule-based teaching rather than highlighting students' conceptual learning (Ball, 1990; Cankoy, 2010; Karakuş, 2018). Another reason might be that PTs may experience a complexity about zero, as students have difficulty in making sense of zero in multiplication as stated in the related literature (Anthony & Walshaw, 2004; Reys, 1974; Whitney, Hirn, & Lingo, 2016; Levenson, Tirosh, & Tsamir, 2004). Also, Semenza et al.'s (2006) study showed that three neuropsychological patients' operational performances for Nx0 and 0xN differed.

In this study, some pre-service teachers did not reach an incorrect result; however, there are differences in their performance of posing a problem in the form of Nx0 and 0xN. For the 5x0, 7% of the PTs and for the 0x5, 8% of the PTs posed problems that were partially correct. In this respect, the findings of this study supported Semenza et al.'s (2006) study. The findings related to the PTs' problem-posing performances may indicate that they have insufficient specialized content knowledge in posing a problem where zero is the multiplier. Another reason for the low performance of the PTs in problem posing may be that they do not have sufficient knowledge about how to pose a mathematical problem. For the 5x0 operation, 34% of the PTs and for the 0x5 operation, about half of the PTs did not suggest any problems and left the question unanswered, which supports this possible reason. Other research studies also support PTs' poor performance related to problem posing. For example, PTs had difficulties writing problems related to multiplication operations involving natural numbers and writing problems about fractions (Luo, 2009; Toluk-Uçar, 2009) or negative integers (Işık, 2018). Considering the definition of specialized content knowledge (Ball et al., 2008) and viewing problem posing as a tool for transferring concepts between contexts (Mestre, 2002), problem posing serves as a tool for revealing teachers' specialized content knowledge. For this reason, the low problem-posing skills of pre-service teachers revealed that they had insufficient specialized content knowledge for the problems they posed.

When we examine how the responses of the PTs changed according to their year of study, it is seen that there are participants from each year who could not write a problem for both operations or left the problems unanswered. Again, for both operations, an incorrect problem was most frequently written by the third-year pre-service teachers. Most importantly, the percentage of the participants who wrote appropriate problems for both 5x0 and 0x5 operations increased as the year of study increased. These findings may be attributed to the fact that courses such as Mathematics Teaching I-II and Teaching Practice I-II contribute to pre-service teachers' specialized content knowledge, even though the PTs at different years of study lacked specialized content knowledge regarding posing problems and multidigit multiplication. More specifically, gaining knowledge about multiple representations about multiplication in the courses of Mathematics Teaching I-II and making observations of teaching multiplication in the courses of Practice I-II may have enhanced PTs' conceptual understanding of multiplication. Furthermore, taking these courses might have helped PTs gain an awareness of mathematical facts and student thinking. For these reasons, courses regarding methods to teach mathematics topics might be given in the early years of the Classroom Teaching program. Moreover, more teaching practice opportunities might be offered to PTs by increasing the number of teaching practice courses.

When the pre-service teachers' justifications were analyzed according to their year of study (Table 5 and Table 8), it was seen that the most frequently mentioned justifications were seen in almost every year of study, and most of the participants who gave justifications related to procedural knowledge were the senior PTs. This finding reveals the need for future teachers to improve their specialized content knowledge in multiplication, differentiating PTs from others who know how to apply algorithms. Teachers' knowledge should be more and deeper than the knowledge they will teach (Ball, 1990). At this point, this study agrees with the literature reporting that PTs have difficulty understanding the computational strategies of multiplication (Lo, Grant, & Flowers, 2008; Whitacre & Nickelson, 2016; Izsák, 2004; Fuson, 2003). In addition, future primary school teachers will expect their students to perform multiplication based on algorithms when multiplying two-digit and single-digit numbers. Moreover, PTs with high number sense skills could separate the numbers appropriately by thinking flexibly in the multiplication process and applying the multiplication algorithm correctly (Whitacre & Nickelson, 2016). For this reason, it can be said that few PTs at any level have sufficient number sense.

Another contribution of this study is that pre-service primary school teachers' reasons or justifications underlying their decisions about the accuracy of multi-step multiplication processes are added to the existing literature (Harkness & Thomas, 2008; Whitacre et al., 2020). The justifications put forward by the PTs for the accuracy of student solutions show that the PTs associated students' solutions with operational understanding of multiplication (Hiebert & Lefevre, 1985). For example, the participant examples in Table 3 and Table 6 indicate that although the student solutions given to them were correct, they decided that the solution was incorrect as they could not see the algorithm-based steps in the solutions or the operational steps they expected. It is necessary to pay attention to these justifications, which are based on procedural knowledge, because these justifications may indicate that PTs have a perception that superficial knowledge about the application of multiplication is sufficient. This perception may not always be valid in evaluating student solutions that require conceptual understanding for multiplication, as in the case of Esma's solution. While *the answer is/is not as it should be* is a reason often cited for an incorrect solution, *incorrect way to solution* was the reason for not accepting a correct student solution. Both justifications show that the PTs made decisions based on their procedural knowledge rather than their conceptual knowledge when evaluating a solution. Even if the answer to the questions in mathematics is correct, it may have been obtained by following a wrong process. In other words, more than the answer alone may be needed to make a solution correct or incorrect. The steps taken in the process are important. For example, a student who compares the numbers 0.4 and 0.675 can make the comparison correctly even though he bases the comparison on the idea that 0.675 is greater than 0.4 as the number with the longer decimal part is larger than the other (Nesher, 1987). In other words, the student may have obtained a correct result despite following the wrong path. In the question asked within the scope of this study, presenting the justification that *the answer should be different from what it should be* may indicate that the PTs had a result-oriented approach rather than the correct application of the process steps. The other justification, *incorrect way to solution*, shows that their conceptual knowledge about implementing the multiplication steps is insufficient.

## Recommendations

PTs need to know the meanings of multiplication when performed with different numbers (i.e., repeated addition, area, combination), practice these situations, and develop mind schemes for these meanings that emerge by learning different number sets. In this context, activities can be carried out to improve the problem-writing skills of PTs through activities that support their conceptual learning. Written problems should be questioned through class discussions, and the missing meanings of the concept of multiplication or meanings that do not suggest a problem should be discussed. A discussion environment can be created for PTs within the scope of classes, including the steps that make sense when performing multi-digit multiplication operations and students' misconceptions. Mathematical difficulties experienced by primary school teachers, secondary school teachers, or PTs can be discussed in these classes, emphasizing the mathematical aspects. Therefore, the Classroom Teaching program courses might focus more on the mathematical content knowledge and strategies to teach mathematical topics addressing multiplication. In addition to thinking about and solving multiplication problems, problem posing should be an indispensable part of method courses. Thus, the mathematical competencies of PTs will be developed along with their problem-posing skills (Leung, 1993). As a result, PTs will be able to use problem-posing skills in their professional lives to analyze students' mathematical understanding more accurately (Mestre, 2002) and teach them mathematics more effectively (Kilpatrick, et al., 2001; NCTM, 2000).

Moreover, more teaching practice opportunities might be offered to PTs by increasing the number of teaching practice courses. Since the time PTs and faculty members spend together increases with an increasing number of teaching practice courses, problem-posing activities can be performed as in-class activities. In line with this, there is more time to draw particular attention to problem-posing activities and gain experience. It can be ensured that pre-service teachers gain experience in problem-posing activities in multiplication and operations of addition, subtraction, and division. Increasing the number of teaching practice courses gives PTs more experience with the learning objectives related to problem-posing activities for multiplication presented in the Turkish primary school curriculum and more awareness of the importance of this learning objective.

Given that the present study was conducted only with the PTs studying in the Classroom Teaching program of a university in Turkey, the study's findings are limited to the participants. The same study can be replicated with PTs studying in the Classroom Teaching program of universities in different cities and regions. Thus, it may be possible to see if the results are the same for other primary school teaching programs in Turkey and to find common solutions for the deficiencies in the specialized content knowledge of the pre-service teachers. This study is limited to the questions aiming to reveal the specialized content knowledge of PTs studying in the Classroom Teaching program regarding the multiplication operation. Similar studies can be conducted with multi-digit multiplication, the multiplication algorithm in the decimal counting system, or other questions on making sense of the multiplication operation. As a further study, a longitudinal study can be conducted across three years involving some specific interventions designed to improve the PTs' specialized content knowledge. This way, any improvement within each cohort could be compared over time as well against other cohorts. Additionally, pre-service and in-service teachers' subject matter knowledge can be compared with the same instrument as a further study. Furthermore, given that the associative property of multiplication is more complex than the commutative property, the way pre-service teachers pose problems for operations that include the associative property of multiplication can be another subject of further research.

The confirmations of the PTs for Esma and Ömer's solutions are interesting. We observed that the PTs followed a highly procedural path in the given questions regarding the correct and incorrect multiplication operations. Also, they suggested justifications by not addressing the conceptual aspect of student solutions in detail, which revealed that previous mathematical knowledge of the PTs was not enough. We believe that the existence or probability of these reasons informs teacher educators about the planning of their lessons. Teacher educators can help PTs understand multiplication operations through different mathematical representations, including problem-posing activities. They can support PTs by emphasizing partial products when making multi-digit multiplication or provide lesson designs to make sense of the computational details underlying justification strategies.

As Harkness and Thomas (2008) stated, mathematics educators need to find ways to transform PTs, who have the approach of equating mathematics with memorizing definitions, formulas, or rules, into teachers who believe that children can also discover new ways and algorithms. Like Harkness and Thomas, the current study revealed that the PTs justified the sample student answers strictly depending on memorizing multiplication algorithms. Although memorizing multiplication algorithms is an aspect of performing multiplication, that is not the only one. To support PTs' understanding of multiplication algorithms and flexible thinking skills, lessons in teacher education programs should include examples of correct or incorrect student solutions, and classroom discussions should be held about justifications. It is important to provide students with environments where they can make sense of symbols and numbers rather than remembering and solving operations, and which enable them to transform from a mental mechanism that works as remembering and forgetting algorithms to people who are 'sense makers' (Lampert, 1986, p. 37). Thus, teachers can help students understand more than just performing operational steps (Lampert, 1986). Undoubtedly, for teachers to do this, they need to be equipped to approach the sample student answers considering the mathematical justifications. Therefore, considering the primary school levels, where the foundation of mathematics is laid, the importance of educating teacher candidates in this respect becomes evident. However, it is necessary to determine which algorithms fall into this scope, what kind of student solutions will help PTs, and how the samples are related to the content of the courses in teacher education programs.

This study examined PTs' specialized content knowledge considering the higher education courses taken by them. We shed light on the differences between classes in terms of special content knowledge on multiplication exemplifying their justification strategies. At this point, long-term studies are necessary to understand the scope of initial special content knowledge of PTs and how they develop it throughout the teacher education program. Future studies can explore PTs' multiplication knowledge with different methodologies, in different teacher education departments, or with more participants. Finally, further studies can examine the specialized content knowledge of PTs in different contexts and according to their year of study. These studies present detailed and important findings on the development of the content of teacher education courses.

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