# THE INCLUSIVE $B \to X_S\tau^+\tau^-$ DECAY IN THE TWO HIGGS DOUBLET MODEL

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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# ABSTRACT

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In this thesis, the inclusive $B \to X_s \tau^+ \tau^-$ decay is investigated. The main goal of this study is to calculate the contributions coming from the two Higgs doublet model to the $B \to X_s \tau^+ \tau^-$ decay, which was extensively studied in the Standard Model. We compute the differential branching ratio and the lepton polarization asymmetry of this decay and the results we obtain are compared with the current literature and the experimental ones where available.

Keywords: The Standard Model and Beyond, Two Higgs Doublet Model, Rare B-meson decay

# ÖZ

İKİ HIGGS DUBLETLİ MODELDE $B \to X_S\tau^+\tau^-$ BOZUNUMU

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Bu tezde, $B \to X_s\tau^+\tau^-$ bozunumu incelenmektedir. Bu çalışmanın asıl amacı, Standart Model'de geniş bir biçimde çalışılmış olan $B \to X_s\tau^+\tau^-$ bozunumuna iki Higgs dubletli model'den gelen katkıları hesaplamaktır. Bu bozunmanın bozunurluk oranı ve lepton polarizasyon asimetrisi hesaplanmakta ve elde edilen sonuçlar literatürdeki diğer sonuçlarla ve deneysel verilerle karşılaştırılmaktadır.

Anahtar Kelimeler: Standard Model ve Ötesi, İki Higgs Dubletli Model, Ender B-mezonu bozunumu

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# CHAPTER 1

# INTRODUCTION

In the past 30 years, an impressive progress has been achieved towards an understanding of particle physics. On the one hand, more and more powerful experimental tools became available, on the other a deeper insight in the theoretical understanding of elementary dynamics has been gained.

Today the basic theoretical framework particle physicists use to describe the strong and electroweak interactions is the Standard Model (SM)[1] . It is a gauge invariant quantum field theory with the symmetry group $SU(2) \times U(1)$ which is spontaneously broken by the Higgs mechanism. In the SM, the interactions of the quarks and leptons are mediated by vector bosons of the strong (gluons), weak ($W^{\pm}, Z$) and electromagnetic (photon) interactions. Quarks and leptons are fermions with spin-1/2 whereas the vector bosons have spin 1. Finally, there is the Higgs boson, a spin 0 particle, which is associated with the generation of all particle masses.

Although the SM has been very successful phenomenologically, and there is no experimental result till now which contradicts its predictions, it is not believed to be the ultimate theory of nature. One of the main reasons for this is that it has too many free parameters; e.g. the nine fermion masses, the three gauge coupling constants and the four Cabibbo-Kobayashi-Maskawa mixing parameters. These correspond to important physical quantities, but can not be computed in the context of the model. Another reason is the instability of the mass of the Higgs boson under radiative corrections in the presence of a high scale which is known as the Hierarchy problem. Additional unsatisfactory property is connected with the fact that the SM does not address basic questions such as the replication of families and the observed mass spectra. Finally, it does not unify all the fundamental interactions. These and many other unsatisfactory features and open questions of the SM lead the physicists to search beyond it and there are several classes of new approaches that address them such as supersymmetry [2], string theories [3], multi-Higgs doublet models [4], etc.

In general the rare decays of B-mesons have always been a good candidate for testing the SM at loop level and looking for new physics beyond it. The reason why it is just the B system that is so well suited for such studies can be traced back to the fact that these processes are induced by the flavor-changing neutral currents (FCNC) that change the flavor, but not the charge of the quark. In the SM, unitarity implies that there are no FCNC reactions at tree level. Quantum loops can, however, generate FCNC reactions, through box diagrams or penguin diagrams. Even at the loop level, as a consequence of the GIM mechanism [9], together with the unitarity of the CKM matrix, all the FCNC vertices vanish in the limit of degenerate quark masses, $m_u = m_c = m_t$. Of course, in reality, masses of the quarks are not equal and the FCNC vertices are not zero. However, the GIM mechanism may lead to a suppression of the amplitude, if the masses of the intermediate quarks are not very different. In case of $b \to s$ transitions, up-like quarks u,c and t run in the loop and t-quark is much heavier than the other quarks, which relaxes the GIM-suppression in box and penguin diagrams. This yields typical branching ratios of rare FCNC $b \to s$ transitions of the order of $10^{-6}$, which is by orders of magnitude larger than in rare K and D decays. Thus, rare B decays are more promising experimentally.

From the experimental point of view, the radiative $b \to s\gamma$ decay has been observed and measured by CLEO II Collaboration both in the inclusive $B \to X_s\gamma$ and exclusive $B \to K^*\gamma$ modes [5, 6]:

$$\begin{aligned} BR(b \to s\gamma) &= (3.15 \pm 0.35 \pm 0.32) \times 10^{-4} \\ BR(B^0 \to K^{*0}\gamma) &= (4.4 \pm 1.0 \pm 0.6) \times 10^{-5} \\ BR(B^- \to K^{*-}\gamma) &= (3.8 \pm 2.0 \pm 0.5) \times 10^{-5} \end{aligned} \qquad (1.1)$$

The $B \to X_s \tau^+ \tau^-$ decay has not been detected yet, but it is expected that important progress will come from operating B-factories CLEO, BaBar, Belle and also from planned future colliders, such as Tevatron Run II, Hera-B. Therefore, the main interest on the rare meson decays has been focused on the decays that can be potentially measurable in the near future. The inclusive process $B \to X_s \ell^+ \ell^-$ is a good candidate for this purpose and in this work we will study its differential branching ratio ($\mathcal{BR}$) and $\tau$ polarization asymmetry in the SM and the 2HDM.

The plan of the thesis is as follows: next two chapters, Chapter II and III, contain an overview of the main features of the models used in this work; the SM and the 2HDM. Chapter IV is devoted to the study of the inclusive $B \to X_s \tau^+ \tau^-$ decay. There we have calculated the differential $\mathcal{BR}$ and $\tau$ polarization asymmetry of this decay in the SM and the 2HDM. We present the conclusion of the thesis in Chapter V.

# CHAPTER 2

# THE STANDARD MODEL

The SM is the theory that explains the basic constituents of matter and the fundamental interactions between them. Today we know that matter is built out of quarks and leptons and we know four fundamental interactions; *the Electromagnetic* (QED), *the Strong* (QCD), *the Weak*, and *the Gravitational.* All these interactions (except the gravitational one) are collectively considered in the SM.

In this chapter, we give a brief summary of the main features of the SM. First, the concept of symmetry and the mechanism of symmetry breaking are considered. Later, the Glashow-Salam-Weinberg Theory is introduced and how the leptons, quarks and the Intermediate Vector Bosons gain mass through the Higgs Mechanism is explained. We also shortly give the reasons why the SM has to be modified.

## 2.1 Symmetry and Symmetry Breaking

According to Noether's Theorem, if the Lagrangian of a system is invariant under some group of transformations, there exist one or more conserved quantities (constants of motion) corresponding to these transformations.

Gauge Theories are constructed by using the invariance of the Lagrangian under local gauge transformations; Quantum Electrodynamics under local gauge transformations of the $U(1)$ group, Quantum Chromodynamics under $SU(3)$ and Weak interactions under $SU(2)$.

It happens sometimes that the system has an invariant Lagrangian under some specific transformations but a non-invariant vacuum. This type of symmetry is called *spontaneously broken symmetry.*

As an example, the Lagrangian of a self interacting real scalar field[1],

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4 \tag{2.1}$$

is invariant under the parity transformation

$$\phi \to -\phi. \tag{2.2}$$

We have two cases to investigate:

- for $\mu^2 > 0$, we have only one vacuum at $<\phi>_0 = 0$ [2] which is invariant under the parity transformation in eq.(2.2).

[1] We assume that $\lambda > 0$

[2] Here $<\phi>_0$ denotes vacuum expectation value of the field.

- For $\mu^2 < 0$, we have two vacuum states at $<\phi>_0 = \pm\sqrt{-\frac{\mu^2}{\lambda}}$ which correspond to two degenerate lowest-energy states.

One may define a shifted field,

$$\phi' \equiv \phi - <\phi>_0 \tag{2.3}$$

where the vacuum state becomes $<\phi'>_0 = 0$. Then the Lagrangian can be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi'\partial^\mu\phi' - \frac{1}{2}(\sqrt{-2\mu^2})^2\phi'^2 - \sqrt{-\mu^2\lambda}\phi'^3 - \frac{1}{4}\lambda\phi'^4. \tag{2.4}$$

This Lagrangian describes a scalar field $\phi'$ with real and positive mass. But it has lost the property of invariance under parity transformation due to the $\phi'^3$ term. Hence, one might choose either of the vacuum states, but in the charge of losing the invariance of the vacuum under parity transformation in eq.(2.2): The symmetry of the theory is *spontaneously broken.*

We can now investigate the spontaneous breaking of continuous symmetries by introducing a charged self interacting scalar field. The Lagrangian for such a field is given by

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - \mu^2(\phi^*\phi) - \lambda(\phi^*\phi)^2 \tag{2.5}$$

which is invariant under the global $U(1)$ symmetry,

$$\phi \to e^{-i\theta}\phi. \tag{2.6}$$

Proceeding as in the case of the Lagrangian in eq.(2.1), the two situations $\mu^2 < 0$ and $\mu^2 > 0$ produce different results. With a redefinition of the complex field in terms of two real scalar fields,

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \tag{2.7}$$

the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2) - \frac{\mu^2}{2}({\phi_1}^2 + {\phi_2}^2) - \frac{\lambda}{4}({\phi_1}^2 + {\phi_2}^2)^2 \tag{2.8}$$

which is invariant under the group $SO(2)$ of rotations in the plane

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{2.9}$$

For $\mu^2 > 0$, there is a unique vacuum at

$$< \phi_1 >_0 = < \phi_2 >_0 = 0 \tag{2.10}$$

where we have two scalar fields $\phi_1$ and $\phi_2$ with mass $m^2 = \mu^2 > 0$ and the Lagrangian preserves the $SO(2)$ symmetry.

As in the one scalar field case, $\mu^2 < 0$ leads to a spontaneous breakdown of the $SO(2)$ symmetry where we have a continuum of vacua at

$$< |\phi|^2 >_0 = \frac{< \phi_1 >_0{}^2 + < \phi_2 >_0{}^2}{2} = \frac{-\mu^2}{2\lambda} = \frac{v^2}{2} \tag{2.11}$$

which shows a $SO(2)$ symmetry *unless* a choice of vacuum is made. When a choice, say

$$\begin{pmatrix} < \phi_1 >_0 \\ < \phi_2 >_0 \end{pmatrix} = \begin{pmatrix} v \\ 0 \end{pmatrix} \tag{2.12}$$

is made, in terms of the shifted fields

$$\begin{pmatrix} {\phi_1}' \\ {\phi_2}' \end{pmatrix} = \begin{pmatrix} \phi_1 - v \\ \phi_2 \end{pmatrix} \tag{2.13}$$

the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu {\phi_1}' \partial^\mu {\phi_1}' - \frac{1}{2}(-2\mu^2){\phi_1}'^2 + \frac{1}{2}\partial_\mu \phi_2' \partial^\mu {\phi_2}' + other\ int.\ terms. \tag{2.14}$$

Hence after spontaneous breakdown of the $SO(2)$ symmetry, we have a scalar field ${\phi_1}'$ with real and positive mass $m^2 = -2\mu^2$ and a massless scalar boson ${\phi_2}'$.

The situation we have just explained is called the *Goldstone Theorem*[7] which states that 'massless spin-zero particle will occur for each broken generator of the original symmetry group 'and the massless particles are called *Goldstone bosons.*

## 2.2 Spontaneous Breaking of the Gauge Symmetries:The Higgs Mechanism

The spontaneous symmetry breaking mechanism we have outlined in the previous section can also be applied to the Gauge Symmetries. We require that the Lagrangian is invariant under *local* gauge transformation

$$\phi \rightarrow e^{iq\alpha(x)}\phi \tag{2.15}$$

and after the symmetry is broken down, the gauge bosons become massive.

In order to make the Lagrangian invariant under local gauge transformations, e replace the ordinary four derivative by the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu \tag{2.16}$$

nd introduce a gauge boson $A_\mu$. Then the Lagrangian in eq.(2.1) becomes

$$\mathcal{L} = (\partial_\mu + iqA_\mu)\phi(\partial^\mu - iqA^\mu)\phi^* - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.17}$$

vhere $F^{\mu\nu}$ is the electromagnetic tensor which is defined as

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{2.18}$$

This Lagrangian is invariant under the local gauge transformation in eq.(2.15) and under the transformation

$$A^\mu \to A^{\mu\prime} = A^\mu - \partial^\mu\alpha(x). \tag{2.19}$$

It is easy to see that a mass term like $\frac{1}{2}A_\mu A^\mu$ violates the local gauge invariance.

When $\mu^2 < 0$, there is a spontaneous symmetry breakdown, with the vacuum expectation value given by eq.(2.11) where we set the field $\phi$ as in eq.(2.7). By inserting the shifted fields in eq.(2.13), the Lagrangian becomes

$$\begin{aligned} \mathcal{L} = \quad & \frac{1}{2}\partial_\mu\phi_1{}'\partial^\mu\phi_1{}' + \frac{1}{2}\partial_\mu\phi_2{}'\partial^\mu\phi_2{}' - \frac{1}{2}(-2\mu^2)\phi_1'^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ + \quad & \frac{q^2v^2}{2}A_\mu A^\mu + qvA_\mu\partial^\mu\phi_2{}' + other\; int.\; terms. \end{aligned} \tag{2.20}$$

As we can conclude by the Lagrangian in eq.(2.20), after the spontaneous breakdown of the gauge symmetry, the vector boson $A^\mu$ gains a mass of $M_A = qv$.

It also contains a scalar field ${\phi_1}'$ with mass $M_{\phi_1'} = \sqrt{-2\mu^2}$ and a massless scalar boson ${\phi_2}'$ which is identified as the Goldstone boson.

The sixth term in the Lagrangian in eq.(2.20) looks a little bit confusing but this term can be eliminated by restricting the gauge parameter in eq.(2.15) to be in the form

$$\alpha(x) = -\frac{1}{qv}{\phi_2}'(x). \tag{2.21}$$

With this choice of the gauge (called *the unitary gauge*) the Lagrangian becomes

$$\begin{aligned} \mathcal{L} = \quad & \frac{1}{2}\partial_\mu {\phi_1}'\partial^\mu {\phi_1}' - \frac{1}{2}(-2\mu^2){\phi_1}' - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}{A_\mu}'A^{\mu\prime} \\ & + \frac{1}{2}q^2({\phi_1}' + 2v){\phi_1}'{A_\mu}'A^{\mu\prime} - \frac{\lambda}{4}{\phi_1}'^3({\phi_1}' + 4v). \end{aligned} \tag{2.22}$$

We say that the Goldstone boson $\phi_2$ has been *eaten* to give the photon mass.

## 2.3 Constructing the Standard Model

In this section we will explain how the SM is constructed and how the intermediate vector bosons gain mass through the Higgs mechanism [8]. The idea is to combine the $SU(2)$ group with the $U(1)$ electromagnetic group and construct a larger $SU_L(2) \otimes U(1)$ group. The gauge theory of the electroweak interactions based on this group is called *Glashow-Salam-Weinberg Theory* [1].

In $SU(2)$ group, where we introduce an isospin symmetry, the left handed components of the particles are described by an isospin doublet ($T = \frac{1}{2}$)

$$\mathrm{L} = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L = \begin{pmatrix} L\nu \\ L\ell \end{pmatrix} = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}. \tag{2.23}$$

Here $T_3 = \frac{1}{2}$ and $T_3 = -\frac{1}{2}$ components correspond to the left handed parts of the neutrino and the lepton respectively.

The right handed part of the lepton is described by a weak isospin singlet ($T = 0$)

$$\Re = R\ell = \ell_R \tag{2.24}$$

since there is no right handed component for the neutrino. In eq.(2.23) and eq.(2.24), $L$ and $R$ are the left handed and the right handed projection operators respectively, which are defined as,

$$\begin{aligned} L &= \frac{1}{2}(1-\gamma^5); \\ R &= \frac{1}{2}(1+\gamma^5). \end{aligned} \tag{2.25}$$

The three generators of the $SU(2)$ group are $\frac{\tau_i}{2}$ and the generator of the $U(1)$ group is $\frac{Y}{2}$, where the $\tau_i$ and $Y$ are the *Pauli spin matrices* and the *hypercharge* respectively. The hypercharge is related to the *weak isospin* ($T$) and the electric charge ($Q$) through the Gell-Mann-Nishijima formula

$$Q = T_3 + \frac{Y}{2}. \tag{2.26}$$

We associate gauge bosons to the $SU_L(2) \otimes U(1)$, where the subscript L stands for the fact that only the left handed components enter the interaction: a triplet $({W_\mu}^1, {W_\mu}^2, {W_\mu}^3)$ to the $SU_L(2)$ group and a singlet ($B_\mu$) to the $U(1)$ group respectively. The strength tensors are written as

$$ {W_{\mu\nu}}^i = \partial_\mu {W_\nu}^i - \partial_\nu {W_\mu}^i + g\epsilon^{ijk} {W_\mu}^j {W_\nu}^k $$

$$B_{\mu\nu} \;=\; \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{2.27}$$

We can write the free Lagrangian for the gauge fields using the strength tensors as follows;

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}{}^{i}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \tag{2.28}$$

For the leptons, the free Lagrangian is written as,

$$\begin{aligned} \mathcal{L}_{leptons} &= \bar{\mathrm{L}}i\,\partial\!\!\!/\mathrm{L} + \bar{\Re}i\,\partial\!\!\!/\Re \\ &= \bar{\ell}_L i\,\partial\!\!\!/\ell_L + \bar{\nu}_L i\,\partial\!\!\!/\nu_L + \bar{\ell}_R i\,\partial\!\!\!/\ell_R \\ \mathcal{L}_{leptons} &= \bar{\ell}i\,\partial\!\!\!/\ell + \bar{\nu}i\,\partial\!\!\!/\nu. \end{aligned} \tag{2.29}$$

Now, we can guarantee the gauge invariance of the theory if we write the two Lagrangians in terms of covariant derivatives. The convenient covariant derivatives for the left handed and the right handed components are

$$\begin{aligned} \mathrm{L} &: \partial_\mu + i\frac{g}{2}\tau^i W_\mu{}^i + i\frac{g'}{2}YB_\mu \\ \Re &: \partial_\mu + i\frac{g'}{2}YB_\mu \end{aligned} \tag{2.30}$$

where $g$ and $g'$ are introduced as the coupling constants of the $SU_L(2)$ and $U(1)$ respectively. Inserting the covariant derivatives in eq.(2.30), the Lagrangian in eq.(2.29) becomes

$$\begin{aligned} \mathcal{L}_{leptons} \rightarrow \quad & \mathcal{L}_{leptons} - g\bar{\mathrm{L}}\gamma^\mu(\frac{\tau^1}{2}W_\mu^1 + \frac{\tau^2}{2}W_\mu^2)\mathrm{L} - g\bar{\mathrm{L}}\gamma^\mu\frac{\tau^3}{2}\mathrm{L}W_\mu{}^3 \\ & -\frac{g'}{2}\bar{\mathrm{L}}\gamma^\mu Y\mathrm{L}B_\mu - \frac{g'}{2}\bar{\Re}\gamma^\mu Y\Re B_\mu. \end{aligned} \tag{2.31}$$

More explicitly, the Pauli spin matrices and isospin doublet and singlets can also be inserted to give

$$\begin{aligned}\mathcal{L}_{leptons} \rightarrow \quad & \mathcal{L}_{leptons} + \mathcal{L}_{charged} + \mathcal{L}_{neutral} \\ = \quad & \mathcal{L}_{leptons} - \frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma^5)\ell W_{\mu}{}^{+} + \bar{\ell}\gamma^{\mu}(1-\gamma^5)\nu W_{\mu}{}^{-}] \\ - \quad & \frac{g}{2}(\bar{\nu}\gamma^{\mu}\nu - \bar{\ell}_L\gamma^{\mu}\ell_L)W_{\mu}{}^{3} + \frac{g'}{2}(\bar{\nu}\gamma^{\mu}\nu + \bar{\ell}_L\gamma^{\mu}\ell_L + 2\bar{\ell}_R\gamma^{\mu}\ell_R)B_{\mu}\end{aligned} \tag{2.32}$$

where

$$W_{\mu}{}^{\pm} = \frac{1}{\sqrt{2}}(W_{\mu}{}^{1} \mp iW_{\mu}{}^{2}) \tag{2.33}$$

are recognized as the *charged gauge bosons.*

Let us define the rotated neutral fields

$$\begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix} = \begin{pmatrix} cos\theta_W & sin\theta_W \\ -sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} B_{\mu} \\ W_{\mu}{}^{3} \end{pmatrix} \tag{2.34}$$

or

$$\begin{pmatrix} W_{\mu}{}^{3} \\ B_{\mu} \end{pmatrix} = \begin{pmatrix} sin\theta_W & cos\theta_W \\ cos\theta_W & -sin\theta_W \end{pmatrix} \begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix}. \tag{2.35}$$

Here, $\theta_W$ is called the *Weinberg angle* which is related to the coupling constants of $SU(2)$ and $U(1)$ through the relations

$$sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}; \; cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}. \tag{2.36}$$

$A_{\mu}$ and $Z_{\mu}$ are recognized as the photon and the neutral gauge boson, respectively.

In terms of rotated fields the neutral part of the Lagrangian becomes,

$$\begin{aligned}
\mathcal{L}_{neutral} &= -\frac{g}{2}(\bar{\nu}_L\gamma^\mu\nu_L - \bar{\ell}_L\gamma^\mu\ell_L)(\sin\theta_W A_\mu + \cos\theta_W Z_\mu) \\
&+\frac{g'}{2}(\bar{\nu}_L\gamma^\mu\nu_L + \bar{\ell}_L\gamma^\mu\ell_L + 2\bar{\ell}_R\gamma^\mu\ell_R)(\cos\theta_W A_\mu - \sin\theta_W Z_\mu) \\
\mathcal{L}_{neutral} &= -\frac{gg'}{2\sqrt{g^2+g'^2}}(\bar{\nu}_L\gamma^\mu\nu_L - \bar{\ell}_L\gamma^\mu\ell_L)A_\mu \\
&-\frac{g^2}{2\sqrt{g^2+g'^2}}(\bar{\nu}_L\gamma^\mu\nu_L - \bar{\ell}_L\gamma^\mu\ell_L)Z_\mu \\
&+\frac{gg'}{2\sqrt{g^2+g'^2}}(\bar{\nu}_L\gamma^\mu\nu_L + \bar{\ell}_L\gamma^\mu\ell_L + 2\bar{\ell}_R\gamma^\mu\ell_R)A_\mu \\
&-\frac{g'^2}{2\sqrt{g^2+g'^2}}(\bar{\nu}_L\gamma^\mu\nu_L + \bar{\ell}_L\gamma^\mu\ell_L + 2\bar{\ell}_R\gamma^\mu\ell_R)Z_\mu \\
\mathcal{L}_{neutral} &= g\sin\theta_W(\bar{\ell}\gamma^\mu\ell)A_\mu - \frac{g}{2\cos\theta_W}(\bar{\nu}_L\gamma^\mu\nu_L)Z_\mu \\
&+\frac{g}{2\cos\theta_W}(\bar{\ell}_L\gamma^\mu\bar{\ell}_L)Z_\mu - g\frac{\sin^2\theta_W}{\cos\theta_W}(\bar{\ell}\gamma^\mu\ell)Z_\mu. \qquad (2.37)
\end{aligned}$$

or in another representation,

$$\mathcal{L}_{neutral} = g\sin\theta_W(\bar{\ell}\gamma^\mu\ell)A_\mu - \frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,\ell}\bar{\psi}_i\gamma^\mu(g_V^i - g_A^i\gamma_5)\psi_i Z_\mu. \qquad (2.38)$$

Here the vector and the axial vector couplings are defined as follows:

$$\begin{aligned}
g_V^i &= T_3^i - 2Q_i\sin^2\theta_W \\
g_A^i &= T_3^i. \qquad (2.39)
\end{aligned}$$

The electromagnetic current coupled to the photon and the electromagnetic charge

$$e = g\sin\theta_W \qquad (2.40)$$

are manifest in eq.(2.38), as a successful prediction of the SM.

Next step is to apply the Higgs mechanism to $SU_L(2) \otimes U(1)$ group in order to acquire mass for the gauge bosons $W^{\pm}$ and $Z^0$. We introduce the scalar doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.41}$$

and insert it into the Lagrangian

$$\mathcal{L}_{scalar} = \partial_\mu \phi^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \tag{2.42}$$

Gauge invariance under local transformations are guaranteed as soon as we introduce the covariant derivatives in eq.(2.30) as usual.

For $\mu^2 < 0$, the $SU_L(2) \otimes U(1)$ symmetry spontaneously breaks down after we make a choice of a vacuum state, say

$$< \phi >_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}. \tag{2.43}$$

Next, we associate a new field with each broken generator and redefine the scalar field. We define [8] the transformation

$$U(\vec{\xi}) = exp(-\frac{i}{2v} \vec{\xi} . \vec{\tau}) \tag{2.44}$$

and write

$$\begin{aligned} \phi &\rightarrow \phi' = U(\vec{\xi})\phi = < \phi >_0 = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}; \\ \mathrm{L} &\rightarrow \mathrm{L}' = U(\vec{\xi})\mathrm{L}; \\ \vec{W}_\mu &\rightarrow \vec{W}_\mu{}'. \end{aligned} \tag{2.45}$$

Here, $\eta$ describes the neutral Higgs boson.

Now, in terms of these new fields, our scalar Lagrangian in eq.(2.42) becomes;

$$\mathrm{L}_{scalar} = \left|(\partial_\mu + i\frac{g'}{2}YB_\mu + ig\frac{\vec{\tau}}{2}.\vec{W}_\mu)\begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}\right|^2 - \mu^2\frac{(v+\eta)^2}{2} - \lambda\frac{(v+\eta)^4}{4}. \tag{2.46}$$

Recognizing the fields $W^\pm$ in eq.(2.33) and $Z^0$ in eq.(2.34), the Lagrangian of the scalar sector that contains the vector bosons can be put in a form that we can easily identify the gauge boson masses;

$$\mathcal{L}_{scalar} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{g^2}{4}(v+\eta)^2(W_\mu{}^+W^{-\mu} + \frac{1}{2\cos^2\theta_W}Z_\mu Z^\mu) \tag{2.47}$$

$$\begin{aligned} m_W &= \frac{1}{2}gv; \\ m_Z &= \frac{gv}{2\cos\theta_W} = \frac{m_W}{\cos\theta_W} \end{aligned} \tag{2.48}$$

and the photon remains massless;

$$m_A = 0. \tag{2.49}$$

Hence we say that, by applying the Higgs mechanism to the electroweak gauge group $SU_L(2)\otimes U(1)$, we break the symmetry of this group to the symmetry of the electromagnetic group $U(1)$,

$$SU_L(2)\bigotimes U(1) \rightarrow U_{EM}(1). \tag{2.50}$$

*Charge* is the only remaining symmetry since the vacuum is invariant under the charge operator.

As we have explained above, the gauge bosons acquire mass through the spontaneous symmetry breaking (2.50). One can define a dimensionless parameter

$$\rho = \frac{{M_W}^2}{{M_Z}^2 \cos^2 \theta_W}. \tag{2.51}$$

At tree level, the predicted and experimentally verified value of $\rho \approx 1$, in the SM.

## 2.4 Lepton and Quark Masses

Besides, leptons acquire mass through a gauge invariant way called the *Yukawa coupling* of the leptons with the Higgs field.

$$\begin{aligned}
\mathcal{L}_{yuk} &= -G_y[\bar{\Re}(\phi^\dagger \mathrm{L}) + (\bar{\mathrm{L}}\phi)\Re] \\
&= -G_y \frac{v+\eta}{\sqrt{2}} \left[ \bar{\ell_R} (0 \quad 1) \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} + (\bar{\nu_L} \quad \bar{\ell_L}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ell_R \right] \\
\mathcal{L}_{yuk} &= -\frac{G_y v}{\sqrt{2}} \bar{\ell}\ell - \frac{G_y}{\sqrt{2}} \bar{\ell}\ell\eta
\end{aligned} \tag{2.52}$$

where $G_y$ is the *Yukawa coupling constant.* As one can observe, this recipe provides the leptons with the mass

$$M_L = \frac{G_y v}{\sqrt{2}}. \tag{2.53}$$

Quarks are introduced into the SM, in the same way as we did the leptons. We first introduce the left-handed quark doublets,

$$Q_L^u = \begin{pmatrix} u \\ d' \end{pmatrix}_L , \quad Q_L^c = \begin{pmatrix} c \\ s' \end{pmatrix}_L , \quad Q_L^t = \begin{pmatrix} t \\ b' \end{pmatrix}_L \tag{2.54}$$

and the right handed quark singlets

$$Q_R^u,\ Q_R^d,\ Q_R^s,\ Q_R^c,\ Q_R^t,\ Q_R^b. \tag{2.55}$$

A free Lagrangian for the quarks can be written as

$$\mathcal{L}_{quarks} = \bar{Q}_L^u i \not\partial Q_L^u + \bar{Q}_L^c i \not\partial Q_L^c + Q_R^u i \not\partial Q_R^u + \ldots + \bar{Q}_R^b i \not\partial Q_R^b. \tag{2.56}$$

In order to introduce the gauge boson-quark interactions through the covariant derivatives (2.30), first the quark hypercharges are determined from the Gell-Mann-Nishijima relation (2.26) as follows:

$$Y_{Q_L} = \frac{1}{3}; \quad Y_{Q_R^{u_i}} = \frac{4}{3}; \quad Y_{Q_R^{d_i}} = \frac{-2}{3}. \tag{2.57}$$

Then, the charged and the neutral weak interactions are determined as,

$$\begin{aligned} \mathcal{L}^Q_{charged} &= \frac{g}{2\sqrt{2}}[\bar{u}\gamma^\mu(1-\gamma_5)d' + \bar{c}\gamma^\mu(1-\gamma_5)s' + \bar{t}\gamma^\mu(1-\gamma_5)b']W^+_\mu + h.c. \\ \mathcal{L}^Q_{neutral} &= \frac{-g}{2\cos\theta_W} \sum_{Q=u,\ldots,b} \bar{Q}\gamma^\mu(g_V^Q - g_A^Q\gamma_5)\, Q\, Z_\mu \end{aligned} \tag{2.58}$$

with the vector and the axial vector couplings given by (2.39), for $i = Q$. Note that neutral current in eq.(2.58) is diagonal in quark flavours. This is known as the GIM Mechanism [9], which consists of adding a fourth quark flavor, the *charm* (c), to the quark sector and defining the doublets in eq.(2.54).

Quark masses are generated by using a $Y = -1$ Higgs doublet which is defined as the conjugate of the Higgs doublet in eq. (2.41),

$$\tilde{\phi} = i\tau_2\phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}. \tag{2.59}$$

The most general Yukawa Lagrangian for the interaction between the three generations of quarks and the Higgs scalars can be written as

$$\mathcal{L}_{yuk} = -\sum_{i,j=1}^{3}\left[g_{ij}{}^{(u)}\bar{Q}_R^{u_i}(\tilde{\phi}^\dagger Q_L^j) + g_{ij}{}^{(d)}\bar{Q}_R^{d_i}(\phi^\dagger Q_L^j)\right] + \ h.c. \tag{2.60}$$

Inserting the vacuum expectation values of $\phi$ and $\tilde{\phi}$, we obtain the mass term for the up quarks,

$$\frac{v}{\sqrt{2}}(\bar{u}' \quad \bar{c}' \quad \bar{t}')_R \, g_{ij}^{(u)} \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_L + h.c. \tag{2.61}$$

and for the down quarks

$$\frac{v}{\sqrt{2}}(\bar{d}' \quad \bar{s}' \quad \bar{b}')_R \, g_{ij}^{(d)} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L + h.c. \tag{2.62}$$

Here $g_{ij}^{(u)}$ and $g_{ij}^{(d)}$ are not diagonal matrices. The states with prime in Lagrangian (2.60) are not mass eigenstates but they are weak eigenstates. The weak eigenstates ($Q'$) are linear superposition of the mass eigenstates ($Q$) given by the unitary transformations,

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_{L,R} = U_{L,R}\begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \; ; \; \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{L,R} = D_{L,R}\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} \tag{2.63}$$

where $U_{L,R}$ and $D_{L,R}$ are the unitary matrices that diagonalize the mass matrices of quarks.

The charged weak current in eq.(2.58) will be proportional to

$$\begin{pmatrix} \bar{u}' & \bar{c}' & \bar{t}' \end{pmatrix}_L \gamma_\mu \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix}_L (U_L^\dagger D_L) \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{2.64}$$

so that the mixing between the mass eigenstates are described by

$$V = U_L^\dagger D_L. \tag{2.65}$$

For the neutral current, it will be proportional to

$$\begin{pmatrix} \bar{u}' & \bar{c}' & \bar{t}' \end{pmatrix}_L \gamma_\mu \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_L = \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix}_L (U_L^\dagger U_L) \gamma_\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \tag{2.66}$$

and since the matrix $U_L$ is unitary, there will be no mixing in the neutral sector. Hence FCNC will be suppressed by the unitarity of mixing matrix.

The quark mixing is restricted to the down quarks, so that $V \equiv D_L$, and the mixing matrix is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix [10]. It can be described in terms of four independent parameters and their values have to be extracted from the experimental data.

## 2.5 The Achievements and the Unsatisfactory Features of the Standard Model

The SM predictions about the elementary particle interactions have been extensively tested in accelerator experiments and agree well with the data measured up to the energies available at present. Three generations of quarks and leptons are experimentally established, together with the corresponding gauge bosons. The present experimental value for the W and Z masses are [47]

$$\begin{aligned} m_W &= 80.419 \pm 0.056\, GeV \\ m_Z &= 91.1882 \pm 0.0022\, GeV. \end{aligned}$$

The constraints on the top mass obtained from these results completely coincide with the direct $m_t$ measurement at the Tevatron of $m_t = 174.3 \pm 5.1\, GeV$ [47].

The only parameter in the SM lacking experimental detection is the Higgs boson. However there are some theoretical and experimental bounds on its value. The recent lower value established by LEP Collaborations [11] is given by $m_H > 95\, GeV$. It is also possible to obtain a theoretical lower bound on the Higgs boson mass by the requirement of the stability of the Higgs potential when quantum corrections are taken into account. By the assumption that the SM is valid up to an energy scale of $\Lambda \sim 1\, TeV$, it may be established that $m_H \geq 55\, GeV$. There are also some upper bounds on the Higgs boson mass. For $\Lambda \sim 1\, TeV$, lattice simulations and renormalization group analysis predict that, $m_H \leq 700\, GeV$.

In spite of the many experimental successes, the SM has many unsatisfactory features: First of all, it has too many free parameters, like the coupling constants, mixing angles, and the fermion masses etc. The ultimate goal is to explain everything with the least number of parameters. Secondly, the gravity is not included in the SM and other interactions considered in the SM are not unified, i.e. we still have three different coupling constants for the electromagnetic, the weak and the strong interactions. Moreover, the unobserved Higgs sector and the arbitrariness in Higgs mass can be considered among the unsatisfactory features of the SM. Hence, one is led to search for a new way *beyond the SM* as far as these features are considered. In the next chapter we will discuss the minimal extension of the SM, which is called the *two Higgs doublet model.*

# CHAPTER 3

# TWO HIGGS DOUBLET MODEL

In this chapter, we will consider the *two Higgs doublet model* (2HDM), which is the minimal extension of the SM. It consists of adding a second doublet to the Higgs sector [12]-[17]. The vacuum expectation values of these two doublets break the $SU(2) \otimes U(1)$ electroweak group down to $U_{EM}(1)$ group. We will investigate the general mechanism of this spontaneous symmetry breaking and later discuss some general properties of the 2HDM.

In the 2HDM, the scalar sector contains two electroweak doublets,

$$\Phi_1 = \begin{pmatrix} {\psi_1}^+ \\ {\psi_1}^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} ; \; \Phi_2 = \begin{pmatrix} {\psi_2}^+ \\ {\psi_2}^0 \end{pmatrix} = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix} . \tag{3.1}$$

We can guarantee the absence of FCNC at tree level, by requiring that all quarks of a given charge couple to a single scalar doublet [19]. In the SM, since there is only one Higgs doublet, the FCNC are suppressed automatically. When there are more than one scalar doublet, this can be achieved by the introduction of some *ad hoc* symmetries into the model. Hence for the 2HDM, the following discrete symmetries are introduced [1][4, 16, 17]

- Model I : $\Phi_1 \to -\Phi_1$
- Model II: $\Phi_1 \to -\Phi_1;\ Q_R^{d_i} \to -Q_R^{d_i};\ \Re_i \to -\Re_i$

where $Q_R^{d_i}$ $(i = 1, 2, 3)$ are the right handed negatively charged quarks and $\Re_i$ $(i = 1, 2, 3)$ are the right handed charged leptons. In Model I only $\Phi_2$ couples to fermions, while in Model II $\Phi_1$ couples to $Q = -\frac{1}{3}$ (down-type) quarks and leptons, and $\Phi_2$ couples to $Q = \frac{2}{3}$ (up-type) quarks. The part of the Lagrangian that contains the interaction of the fermions and the scalars, for the two models can be written as follows[17]:

$$\begin{aligned} Model\ I : \mathcal{L} &= g_{ij}{}^{(u)}\bar{Q}_L^i\tilde{\Phi}_2 Q_R^{u_j} + g_{ij}{}^{(d)}\bar{Q}_L^i\Phi_2 Q_R^{d_j} + g_{ij}{}^{(l)}\bar{\mathrm{L}}_i\Phi_2\Re_j + h.c. \\ Model\ II : \mathcal{L} &= g_{ij}{}^{(u)}\bar{Q}_L^i\tilde{\Phi}_2 Q_R^{u_j} + g_{ij}{}^{(d)}\bar{Q}_L^i\Phi_1 Q_R^{d_j} + g_{ij}{}^{(l)}\bar{\mathrm{L}}_i\Phi_1\Re_j + h.c \quad (3.2) \end{aligned}$$

where $\bar{Q}_L^i$ are the left handed quark doublets, $Q_R^{u_j}$ are the positively-charged right handed quark singlets and $\mathrm{L}_i$ are the left handed lepton doublets.

The vacuum expectation values of the doublets in eq.(3.1) that break the gauge group $SU_L(2) \otimes U(1)$ down to $U_{EM}(1)$ are given as

$$< \Phi_1 >_0 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix};\ < \Phi_2 >_0 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \cdot \quad (3.3)$$

[1] There is as well the case where no discrete symmetry is imposed and both up-type and down-type quarks have Flavor Changing couplings which is known as Model III [20], but we will discuss only Model I and II here.

where (comparing eq.(3.3) with eq.(3.1)), we take

$$< \phi_1 >_0 = < \phi_2 >_0 = < \phi_4 >_0 = < \phi_5 >_0 = < \phi_6 >_0 = < \phi_8 >_0 = 0$$

$$< \phi_3 >_0 = v_1; \; < \phi_7 >_0 = v_2. \tag{3.4}$$

The most general $SU(2) \otimes U(1)$ gauge invariant, renormalizable Higgs potential for the two doublets can be written as [18];

$$\begin{aligned} V &= \lambda_1(\Phi_1^\dagger\Phi_1 - v_1^2)^2 + \lambda_2(\Phi_2^\dagger\Phi_2 - v_2^2)^2 + \lambda_3[(\Phi_1^\dagger\Phi_1 - v_1^2) + (\Phi_2^\dagger\Phi_2 - v_2^2)]^2 \\ &+ \lambda_4[(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)] + \lambda_5[Re(\Phi_1^\dagger\Phi_2) - v_1v_2]^2 \\ &+ \lambda_6[Im(\Phi_1^\dagger\Phi_2)]^2. \end{aligned} \tag{3.5}$$

Next we calculate the matrix of second derivative of the potential,

$$M^2 = \frac{\partial^2 V}{\partial\phi_j\,\partial\phi_k}\bigg|_{minimum} \tag{3.6}$$

where "minimum "means setting the vacuum expectation values in eq.(3.4). This is nothing but the *scalar boson mass squared matrix.* A careful investigation of this mass squared matrix provides us with the mass eigenstates of the scalar bosons:

1. Diagonalization of the charged part of the mass squared matrix (indices 1,2,5,6) gives two massless Goldstone boson states:

$$G^\pm = \psi_1{}^\pm \cos\beta + \psi_2{}^\pm \sin\beta \tag{3.7}$$

and two massive charged Higgs boson states:

$$H^{\pm} = -\psi_1{}^{\pm} \sin\beta + \psi_2{}^{\pm} \cos\beta \qquad (3.8)$$

where the mixing angle $\beta$ is defined by

$$\tan\beta = \frac{v_2}{v_1}. \qquad (3.9)$$

The charged Higgs boson has a mass of

$$m_{H^{\pm}}{}^2 = \lambda_4(v_1{}^2 + v_2{}^2). \qquad (3.10)$$

2. Diagonalization of the $2 \times 2$ matrix including the indices 4 and 8 yields the *CP-odd states*:

$$\begin{aligned} G^0 &= \sqrt{2}[Im\psi_1{}^0 \cos\beta + Im\psi_2{}^0 \sin\beta] \\ A^0 &= \sqrt{2}[-Im\psi_1{}^0 \sin\beta + Im\psi_2{}^0 \cos\beta] \end{aligned} \qquad (3.11)$$

where the latter has the mass

$$m_{A^0} = \lambda_6(v_1{}^2 + v_2{}^2). \qquad (3.12)$$

The Goldstone bosons $G^{\pm}$ and $G^0$ are eaten by the $W^{\pm}$ and $Z^0$ bosons.

3. The mass squared matrix formed by the indices 3 and 7 gives the *CP-even states*:

$$\begin{aligned} H^0 &= \sqrt{2}[(Re\psi_1{}^0 - v_1)\cos\alpha + (Re\psi_2{}^0 - v_2)\sin\alpha] \\ h^0 &= \sqrt{2}[-(Re\psi_1{}^0 - v_1)\sin\alpha + (Re\psi_2{}^0 - v_2)\cos\alpha] \end{aligned} \qquad (3.13)$$

with masses

$$
\begin{aligned}
m_{H^0}{}^2 &= \frac{1}{2}\left[A + C + \sqrt{(A-C)^2 + 4B^2}\right] \\
m_{h^0}{}^2 &= \frac{1}{2}\left[A + C - \sqrt{(A-C)^2 + 4B^2}\right]
\end{aligned}
\tag{3.14}
$$

and mixing angle

$$
\tan\ 2\alpha = \frac{2B}{(A-C)} \tag{3.15}
$$

where

$$
\begin{aligned}
A &= 4v_1{}^2(\lambda_1 + \lambda_3) + v_2{}^2\lambda_5 \\
B &= (4\lambda_3 + \lambda_5)v_1 v_2 \\
C &= 4v_2{}^2(\lambda_2 + \lambda_3) + v_1{}^2\lambda_5.
\end{aligned}
\tag{3.16}
$$

As we have seen, by promoting the number of Higgs doublet from one to two, in addition to the Goldstone bosons that give mass to $W^\pm$ and $Z^0$, we get two charged scalars, $H^\pm$, one neutral CP-odd scalar, $A^0$, and two neutral CP-even scalars, $H^0$ and $h^0$. One can derive the Feynman rules of the 2HDM involving the couplings of Higgs scalars to fermions, their self-couplings etc. by writing down the corresponding interaction Lagrangian [15, 21].

As an extension of the SM, the 2HDM must also obey the constraints of this theory. As we have explained above, the absence of FCNC at tree level are guaranteed by introducing some *ad-hoc* discrete symmetry into the theory.

Another constraint that must be obeyed is the experimental value of the quantity

$$\rho = \frac{m_W{}^2}{m_Z{}^2 \cos^2 \theta_W} \approx 1. \tag{3.17}$$

One can show that, this constraint is satisfied for an arbitrary number of scalar multiplets.[22]

2HDM is the minimal extension of the SM which adds fewest new arbitrary parameters. The mixing angle $\alpha$, the ratio of the vacuum expectation values of the two doublets, $\tan\beta$, and the Higgs masses remain as free parameters in the 2HDM. However, there are lower limits for the masses of Higgs bosons [47]:

$$\begin{aligned} m_{H^\pm} &> 69.0\ GeV \\ m_{A^0} &> 84.1\ GeV \\ m_{h^0} &> 82.6\ GeV \quad (m_{h^0} < m_{H^0}). \end{aligned} \tag{3.18}$$

2HDM adds new phenomena to the SM, since five Higgs bosons instead of one, are introduced on SM. In the next chapter one of these new phenomena, the contributions coming from the 2HDM to the inclusive $B \to X_s \tau^+ \tau^-$ decay, will be investigated.

# CHAPTER 4

# THE INCLUSIVE $B \to X_S\tau^+\tau^-$ DECAY IN TWO HIGGS DOUBLET MODEL

The decays of B-mesons are induced by FCNC $b \to s$ transitions and occur only through electroweak loops in the SM. Therefore, they are rare and relatively more sensitive for the possible new physics beyond the SM. Further, they can be used to determine more precisely the fundamental parameters of the SM, such as CKM matrix elements, leptonic decay constants, etc.

Among the rare B-meson decays, the processes $B \to X_s\ell^+\ell^-(\ell = e, \mu, \tau)$ have received a considerable attention for a long time since they are measurable in the near future in the existing and the planned B-factories. Therefore, these decays have been studied extensively in many earlier work, in the framework of both the SM and the 2HDM [23]-[38]. Short-distance QCD effects have been calculated by several groups [39] and later these calculations were improved in refs. [24, 27] by including the next-to-leading QCD corrections. As for the long-distance contributions, mainly due to $\psi$ and $\psi'$ resonances, they were considered for the first time in refs.[25, 28], and later the discrepancy about the relative sign between the short and long distance terms was resolved via the analysis presented in ref.[29]. The process $B \to X_s \ell^+ \ell^-$ with $\ell = \tau$ was considered in SM in [42] and then in [43]. Later, the role played by Neutral Higgs Bosons was also included.

In this chapter, we study the differential $\mathcal{BR}$ and $\tau$ polarization asymmetry in the inclusive $B \to X_s \tau^+ \tau^-$ decay in the framework of the 2HDM. For this, the effective Hamiltonian describing the process $b \to s \tau^+ \tau^-$ at quark level is used since we model the inclusive $B \to X_s \tau^+ \tau^-$ decay by the process $b \to s \tau^+ \tau^-$, as implied by the Heavy Quark Effective Theory (HQET). First, we give the SM contributions to the Wilson coefficients of the effective Hamiltonian briefly. Then, we calculate the contributions coming from the charged Higgs bosons $H^{\pm}$ of the 2HDM to these Wilson coefficients. Later, we derive the differential $\mathcal{BR}$ and $\tau$ polarization asymmetry for the $B \to X_s \tau^+ \tau^-$ decay and give the numerical result and their discussions.

## 4.1 The Effective Hamiltonian

Weak decays of hadrons are mediated through weak interactions of quarks. Strong interactions among the quarks bind them into hadrons and they are characterized by typical hadronic energy scale of $O(1\,GeV)$, which is much lower than the scale of weak interactions, $O(m_W)$. These weak interactions are subject to some QCD corrections due to gluon exchanges between the quarks. Therefore, it is necessary to derive an effective low energy theory describing the weak interactions of quarks. The formal framework to do this is a method called the Operator Product Expansion (OPE)[44].

The effective Hamiltonian for the weak decays of the hadrons has the generic structure

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu)\, O_i(\mu)$$

where $G_F$ is the Fermi constant, $V^i_{CKM}$ is the Cabibbo-Kobayashi-Maskawa factors, $O_i$ are the relevant local operators which govern the decays in question and $C_i$ are the so-called Wilson coefficients which describe the strength of the corresponding operators entering the Hamiltonian. Here $\mu$ denotes a scale of the order of the mass of the decaying hadron.

The computation of such an effective Hamiltonian proceeds in two stages :

- Wilson coefficients $C_i(\mu)$ are calculated at a renormalization point comparable to the heavy mass, $\mu = O(m_W)$, in ordinary perturbation theory neglecting the light masses in comparison with the heavy ones. This step amounts to matching the full theory onto a five quark effective theory. In this process, $W^{\pm}$, $Z^0$ and the $t-$quark and generally all heavy particles with masses higher than $m_W$ are integrated out. The resulting coefficients

$C_i(m_W)$ depend generally on the masses of these heavy particles which have been integrated out.

- Then these functions must be scaled down to a mass of order $m_b$ using the so-called renormalization group method.

Following these steps above, one can obtain the effective Hamiltonian governing the $b \to s\tau^+\tau^-$ transitions in the SM, in terms of a set of operators [26]

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\, O_i(\mu) \tag{4.1}$$

where, neglecting the mass of the s-quark,

$$\begin{aligned}
O_1 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}), \\
O_2 &= (\bar{s}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}), \\
O_3 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\beta}), \\
O_4 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}), \\
O_5 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\beta}), \\
O_6 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}), \\
O_7 &= \frac{e}{16\pi^2} \bar{s}_\alpha \sigma_{\mu\nu} m_b R b_\alpha \mathcal{F}^{\mu\nu}, \\
O_8 &= \frac{g}{16\pi^2} \bar{s}_\alpha T^a_{\alpha\beta} \sigma_{\mu\nu} m_b R b_\beta \mathcal{G}^{a\mu\nu}, \\
O_9 &= \frac{e}{16\pi^2} (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{\tau}\gamma^\mu \tau), \\
O_{10} &= \frac{e}{16\pi^2} (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{\tau}\gamma^\mu \gamma_5 \tau).
\end{aligned} \tag{4.2}$$

Here, $\alpha$ and $\beta$ are $SU(3)$ colour indices, $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively, and $e$ and $g$ are the corresponding coupling constants. In eq.(4.2), $O_1$ and $O_2$ are *the current-current operators*, $O_3$,...,$O_6$ are usually named as *the QCD penguin operators*, $O_7$ and $O_8$ are *the magnetic penguin operators* and $O_9$ and $O_{10}$ are *the semileptonic electroweak penguin operators.*

Now our next task is to write the effective Hamiltonian for the inclusive decay $B \to X_s \tau^+ \tau^-$. It has been shown that the inclusive decay rates of heavy hadrons can be calculated in heavy quark effective theory (HQET)[45] and that the resulting branching ratio ($\mathcal{BR}$) can be calculated in the expansion in inverse powers of $m_b$:

$$\mathcal{BR}(B \to X_s) = \mathcal{BR}(b \to s) + \mathcal{O}(\frac{1}{m_b^2}).$$

The leading term in this expansion represents the decay of a free heavy quark and it can be calculated in perturbation theory. The remaining correction terms are of a non-perturbative origin, but they are suppressed by at least two powers of $m_b$. In this work we consider the leading term only. In this approximation the operator $O_8$ does not contribute. Also, the coefficients of $O_3 - O_6$ are small. Therefore, the analysis can be carried out by considering only the operators $O_1$, $O_2$, $O_7$, $O_9$ and $O_{10}$.

Then neglecting the s-quark mass, the effective Hamiltonian in eq.(4.1) leads to the following matrix element for $b \to s\tau^+\tau^-$:

$$\begin{aligned} \mathcal{M} &= \frac{G_F\alpha}{2\sqrt{2}\pi} V_{tb}V_{ts}^* \Big\{ C_9^{eff}(m_b)\ \bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{\tau}\gamma^\mu\tau + C_{10}(m_b)\ \bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{\tau}\gamma^\mu\gamma_5\tau \\ &- 2C_7^{eff}(m_b)\ \frac{m_b}{q^2}\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar{\tau}\gamma^\mu\tau \Big\}. \end{aligned} \tag{4.3}$$

## 4.2 $B \to X_s\tau^+\tau^-$ in the SM

Modeling the inclusive B-meson decay by b-quark decay, in the SM, the dominant contributions to the $B \to X_s\tau^+\tau^-$ decay come from the diagrams shown in fig.(4.1), when the dashed lines are exchanged with $W^+$-boson and the *unphysical Higgs*, $\phi^+$. There is also a box diagram with two W-bosons in the SM. The corresponding Wilson coefficients were calculated in ref.[26] and they are given as:

$$\begin{aligned} C_1^{SM}(m_W) &= 0\,, \\ C_2^{SM}(m_W) &= -1\,, \\ C_7^{SM}(m_W) &= \frac{1}{2}A(x)\,, \\ C_9^{SM}(m_W) &= \frac{1}{sin^2\theta_W}B(x) + \frac{-1+4\sin^2\theta_W}{\sin^2\theta_W}C(x) + D(x) - \frac{4}{9}\,, \\ C_{10}^{SM}(m_W) &= \frac{-1}{\sin^2\theta_W}(B(x) - C(x)) \end{aligned} \tag{4.4}$$

where

$$\begin{aligned} A(x) &= \frac{x}{12}\left[\frac{8x^2+5x-7}{(x-1)^3} - \frac{18x^2-12x}{(x-1)^4}\ln x\right] \\ B(x) &= -\frac{x}{4}\left[\frac{1}{(x-1)} - \frac{1}{(x-1)^2}\ln x\right], \end{aligned}$$

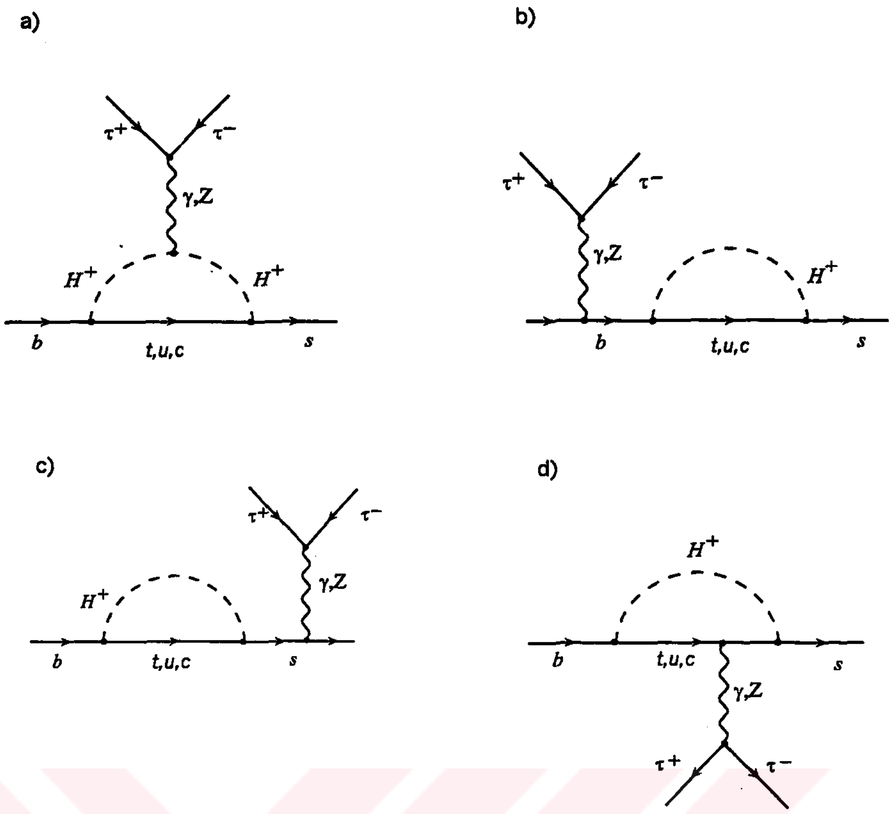



Figure 4.1: The charged Higgs boson exchange diagrams contributing to the $b \to s\tau^+\tau^-$ decay.

$$
\begin{aligned}
C(x) &= -\frac{x}{8}\left[\frac{x-6}{(x-1)}+\frac{3x+2}{(x-1)^2}\ln x\right] , \\
D(x) &= \frac{1}{18}\left[\frac{-19x^3+25x^2}{2(x-1)^3}+\frac{-3x^4+30x^3-54x^2+32x-8}{(x-1)^4}\ln x\right] , \\
x &= \frac{m_i^2}{m_W^2} , \qquad (4.5)
\end{aligned}
$$

and $\theta_W$ is the Weinberg angle with $\sin^2\theta_W = 0.23$.

## 4.3 $B \to X_s\tau^+\tau^-$ in the 2HDM

We will now investigate the charged Higgs boson contributions to the $B \to X_s\tau^+\tau^-$ process. The charged Higgs boson exchange diagrams in fig.(4.1) do not produce new operators and the operator basis remains the same as in the SM.

As the contributions coming from each diagram in fig.(4.1) are considered separately, they can be summarized as follows:

All the calculations are performed by using the Feynman-'t Hooft gauge and on-shell renormalization scheme. In all calculations, $p$ denotes the momentum of the *b-quark*, $k$ denotes the momentum of the *photon* or the *Z-boson*, and $m_j$ $(e_j)$ denotes the mass (charge) of the particle $j = b, s, ..., W$. All resulting amplitudes are given in terms of $y = m_i^2/m_H^2$, where $i = t, b$, or $c$ quark. We use the Feynman rules in Appendix-A during calculations. We give the detailed calculations of the vertex diagram in fig.(4.1-a), in Appendix-B.

We first consider the photon exchange diagrams. From fig.(4.1-b) and fig.(4.1-c), renormalized self-energy is given by,

$$\Sigma_\mu^{(b)} = i\Sigma_{ren}(p-k) \ \frac{i(\not p - \not k + m_b)}{[(p-k)^2 - m_b^2]} \ (-i\gamma_\mu e_b) \tag{4.6}$$

and

$$\Sigma_\mu^{(c)} = (-i\gamma_\mu e_b) \ \frac{i(\not p + m_s)}{[(p)^2 - m_s^2]} \ i\Sigma_{ren}(p) \tag{4.7}$$

where

$$\begin{aligned} \Sigma_{ren}(p) &= (\not p - m_s)\hat{\Sigma}_{ren}(p)(\not p - m_b) \qquad (4.8) \\ \hat{\Sigma}_{ren}(p) &= \frac{-ig^2}{32\pi^2}\left(\frac{m_i^2}{m_W^2}\right)\left[\xi^2(m_sR + m_bL + \not pR)\ I(2;1) - \xi\xi'(m_sR + m_bL)\ I(1;1)\right] \end{aligned}$$

with

$$I(n;m) \equiv \int_0^1 \frac{x^n(1-x)^m}{[m_H^2 x + m_i^2(1-x)]} dx. \tag{4.9}$$

There is no contribution for $\Sigma_\mu^{(b)}$ and $\Sigma_\mu^{(c)}$, since each term vanishes when one quark is on-shell. Then, the general form of the amplitude receiving contributions from the diagrams in fig.(4.1-a) and fig.(4.1-d) can be written in the form given by,

$$\mathcal{A}^{(\gamma)} = (\bar{s}\,\Gamma_\mu^{(\gamma)}\,b)\left(\frac{-i}{k^2}\right)(-ie)\,(\bar{\tau}\gamma^\mu\tau) \tag{4.10}$$

where

$$\Gamma_\mu^{(\gamma)} = F_1(y)(k_\mu \not{k} - k^2\gamma_\mu)L + F_2(y)\sigma_{\mu\nu}k^\nu R \tag{4.11}$$

with

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]. \tag{4.12}$$

Substituting eq.(4.11) into eq.(4.10) and using the Dirac Equation, we obtain

$$\mathcal{A}^{(\gamma)} = e\left[F_1(y)\;\bar{s}\gamma_\mu L\;b - F_2(y)\;\bar{s}\sigma_{\mu\nu}k^\nu R\;b\right]\left(\bar{\tau}\gamma^\mu\tau\right) \tag{4.13}$$

where the functions $F_1$ and $F_2$ receive the following contributions from the diagrams in fig.(4.1-a) and fig.(4.1-d):

$$\begin{aligned}
F_1^{(a)}(y) &= \frac{-\xi^2 y}{576\pi^2(y-1)^4}\left[(2-9y+18y^2-11y^3+6y^3\ln y)\right]; \\
F_1^{(d)}(y) &= \frac{\xi^2 y}{576\pi^2(y-1)^4}\left\{\left[-16+45y-36y^2+7y^3+6(3y-2)\ln y\right]\right\}; \\
F_2^{(a)}(y) &= \frac{-m_b y\xi}{192\pi^2(y-1)^4}\left\{\left[(y-1)[-6\xi'(y^2-1)+\xi(-1+5y+2y^2)]\right.\right. \\
&\quad \left.\left. +6y[2\xi'(y-1)-\xi y]\ln y\right]\right\};
\end{aligned} \tag{4.14}$$

$$
\begin{aligned}
F_2^{(d)}(y) &= \frac{-m_b y\xi}{192\pi^2(y-1)^4}\Bigg\{\Bigg[(y-1)[\xi(-2-5y+y^2)-6\xi'(3-4y+y^2)] \\
&\quad +6[-2\xi'(-1+y)+\xi y]\ln y\Bigg]\Bigg\}.
\end{aligned}
$$

By comparing the resulting amplitude in eq.(4.13) together with eq.(4.14), with the matrix element of the effective Hamiltonian $\mathcal{M}$, we find

$$
\begin{aligned}
C_7^{(\gamma)} &= \frac{1}{72(y-1)^4}\Big\{\xi y(y-1)[-6\xi'(3-8y+5y^2)+\xi(-7+5y+8y^2)] \\
&\quad +6(3y-2)[2\xi'(y-1)-\xi y]\ln y\Big\}; \\
C_9^{(\gamma)} &= \frac{-1}{108(y-1)^4}\Big\{\xi^2 y\Big[38-117y+126y^2-47y^3+6(4-6y+3y^3)\ln y\Big]\Big\}; \\
C_{10}^{(\gamma)} &= 0. \qquad (4.15)
\end{aligned}
$$

Next, we consider the Z-exchange diagrams. The corresponding amplitude is written as:

$$
\mathcal{A}^{(Z)} = (\bar{s}\Gamma_\mu^{(Z)}b)\left(\frac{1}{k^2-m_Z^2}\right)\left(\frac{ig}{4\cos\theta_W}\right)\left\{\bar{\tau}\gamma^\mu[(-1+4\sin^2\theta_W)-\gamma^5]\tau\right\} \qquad (4.16)
$$

where

$$
\Gamma_\mu^{(Z)} \equiv \Gamma_\mu^{(Z)}(b+c)+\Gamma_\mu^{(Z)}(a+d). \qquad (4.17)
$$

Here,

$$
\begin{aligned}
\Gamma_\mu^{(Z)}(b+c) &= \int_0^1 dx\; R_\mu^{fin}(b+c) \;+\; R_\mu^{div}(b+c) \\
\Gamma_\mu^{(Z)}(a+d) &= \int_0^1 dx\; R_\mu^{fin}(a+d) \;+\; R_\mu^{div}(a) \;+\; R_\mu^{div}(d) \qquad (4.18)
\end{aligned}
$$

where the divergent and the finite parts of the amplitude are given by,

$$
R_\mu^{div}(b+c) = \left(\frac{ig^3\sin^2\theta_W}{192\pi^2\cos\theta_W}\right)\mathcal{D};
$$

$$R_\mu^{div}(a) = \left(\frac{ig^3(1-2\sin^2\theta_W)}{128\pi^2\cos\theta_W}\right)\mathcal{D};$$
$$R_\mu^{div}(d) = \left(\frac{ig^3(-3+4\sin^2\theta_W)}{384\pi^2\cos\theta_W}\right)\mathcal{D} \tag{4.19}$$

with

$$\mathcal{D} = \left(\frac{m_i^2}{m_W^2}\right)\xi^2\left[\frac{2}{\epsilon} - \gamma_E + 4\pi - \log m_H^2\right]\gamma_\mu L \tag{4.20}$$

and

$$R_\mu^{fin}(b+c) = \frac{-ig^3\xi^2 m_H^2 R\sin^2\theta_W xy\log[x+y-xy]\gamma_\mu}{48\cos\theta_W m_W^2\pi^2};$$
$$R_\mu^{fin}(a+d) = \frac{1}{192\cos\theta_W m_W^2\pi^2}\Bigg\{ig^3\xi^2 m_H^2 R(1-x)y\Bigg[(3-4\sin^2\theta_W) + \frac{4\sin^2\theta_W y}{x+y-xy} + (6\sin^2\theta_W - 3)\log[1+x(y-1)] + (3-4\sin^2\theta_W)\log[x+y-xy]\Bigg]\Bigg\}\gamma_\mu. \tag{4.21}$$

where Euler Gamma, $\gamma_E = 0.5772$, and $\epsilon$ is a very small number, that is taken to 0, that comes from the method of dimensional regularization. We note that all the divergent terms given by eq.(4.19) vanish, when we add the resulting amplitudes of all the four $Z$-diagrams so that we are left with the final result:

$$\Gamma_\mu^{(Z)} = \frac{ig^3\xi^2 m_H^2 R y^2(y-1-\ln y)\gamma_\mu}{64\cos\theta_W m_W^2\pi^2(y-1)^2}. \tag{4.22}$$

Now, since $m_Z^2 >> k^2$, we can replace $\frac{1}{k^2-m_Z^2}$ with $\frac{-1}{m_Z^2}$ in eq.(4.16), and also write $m_Z = m_W/\cos\theta_W$, $e = g\sin\theta_W$ and finally $\frac{g^2}{2m_W^2} = 4\frac{G_F}{\sqrt{2}}$, and then compare the resulting expression with the matrix element of the effective Hamiltonian $\mathcal{M}$. This gives the following contributions:

$$C_7^{(Z)} = 0;$$

$$
\begin{aligned}
C_9^{(Z)} &= \frac{G_F \xi^2 (-1 + 4\sin^2\theta_W) m_H^2 y^2 (y - 1 - \ln y)}{8 \sin^2\theta_W m_W^2 (y-1)^2}; \\
C_{10}^{(Z)} &= \frac{G_F \xi^2 m_H^2 y^2 (y - 1 - \ln y)}{8 \sin^2\theta_W m_W^2 (y-1)^2}.
\end{aligned} \tag{4.23}
$$

The contributions to the Wilson coefficients of the operators $O_1$, $O_2$, $O_7$, $O_9$ and $O_{10}$ from the 2HDM are outlined as follows:

$$
\begin{aligned}
C_{1,2}^H(m_W) &= 0; \\
C_7^H(m_W) &= C_7^{(\gamma)} + C_7^{(Z)} = -\left[\xi\xi' G(y) - \frac{1}{6} |\xi|^2 \; A(y)\right]; \\
C_9^H(m_W) &= C_9^{(\gamma)} + C_9^{(Z)} = |\xi|^2 \left[\frac{1 - 4\sin^2\theta_W}{\sin^2\theta_W} \frac{x}{2} \; B(y) + y \; F(y)\right]; \\
C_{10}^H(m_W) &= C_{10}^{(\gamma)} + C_{10}^{(Z)} = |\xi|^2 \frac{-1}{\sin^2\theta_W} \frac{x}{2} \; B(y)
\end{aligned} \tag{4.24}
$$

where

$$
\begin{aligned}
F(y) &= \frac{1}{108}\left[\frac{47y^2 - 79y + 38}{(y-1)^3} + \frac{-18y^3 + 36y - 24}{(y-1)^4} \ln y\right]; \\
G(y) &= \frac{y}{12}\left[\frac{5y - 3}{(y-1)^2} - \frac{6y - 4}{(y-1)^3} \ln y\right]
\end{aligned}
$$

with $x$ given in eq.(4.5).

Note that the results for model I and model II can be obtained by the following substitutions:

$$
\begin{aligned}
&\xi = \xi' = 1/\tan\beta && \text{for Model I ,} \\
&\xi = -1/\xi' = 1/\tan\beta && \text{for Model II .}
\end{aligned} \tag{4.25}
$$

Our results for the charged Higgs contributions to the process $b \to s\tau^+\tau^-$ are in good agreement with ref.[26]. From now on the relevant Wilson coefficients can be written as

$$C_i^{2HDM}(m_W) = C_i^{SM}(m_W) + C_i^H(m_W) \tag{4.26}$$

taking the contributions of the charged Higgs into account.

As we have explained above, after the calculation of the Wilson coefficients at a renormalization point comparable to $m_W$, these coefficients must be scaled down to an order of $m_b$, so that large logarithms, $ln(\frac{m_W}{m_b})$, can be removed from the theory. The evolution of the Wilson coefficients from the higher scale $\mu = m_W$ down to the low energy scale $\mu = m_b$ is described by the renormalization group equation. The coefficients $C_7^{eff}(\mu)$ and $C_9^{eff}(\mu)$ at the scale $\mu = m_b$ are calculated in [26, 37]. As for the coefficient $C_{10}$, it is not modified as we move from $\mu = m_W$ to $\mu = m_b$ scale, i.e., $C_{10}(m_b) \equiv C_{10}(m_W)$.

The Wilson coefficients $C_9$ receives also long distance contributions, related to $c\bar{c}$ bound states, i.e., $J/\psi$, $\psi'$, $\cdots$ [26]-[29],[46]. These long distance effects, which proceed through the process $b \to s\psi \to s\tau^+\tau^-$, can be taken into account in an approximate manner by modifying the coefficient $C_9^{eff}$

$$C_9^{eff}(\mu) \;\to\; C_9^{eff}(\mu) + \left[g\left(\hat{m}_c, \hat{s}\right) + \kappa \sum_{V_i = J/\psi_i, \psi', \cdots} \frac{\hat{m}_{V_i}\hat{\Gamma}(V_i \to \ell^+\ell^-)}{(\hat{s} - \hat{m}_{V_i}^2) + i\hat{m}_{V_i}\hat{\Gamma}_{V_i}}\right](3C_1 + C_2) \;, \tag{4.27}$$

where we introduce the notation $\hat{m}_i = m_i/m_b$ , $\hat{s} = q^2/m_b^2$ , $q^2 = (p_{\tau^+} + p_{\tau^-})^2$ and

$$g\left(\hat{m}_i, \hat{s}\right) \;=\; -\frac{8}{9}\ln\left(\hat{m}_i\right) + \frac{8}{27} + \frac{4}{9}y_i$$

$$-\frac{2}{9}(2+y_i)\sqrt{|1-y_i|}\left\{\Theta(1-y_i)\left(\ln\frac{1+\sqrt{|1-y_i|}}{1-\sqrt{|1-y_i|}}-i\pi\right)\right.$$
$$\left.+\Theta(y_i-1)\,2\arctan\frac{1}{\sqrt{y_i-1}}\right\}, \qquad (4.28)$$

where $y_i = 4\hat{m}_i^2/\hat{s}$. $g(\hat{m}_i, \hat{s})$ arises from the one loop contributions of the four quark operators $O_1$ and $O_2$.

There are five known resonances in the $c\bar{c}$ system that can contribute to the decay channel $b \to s\tau^+\tau^-$. Their properties can be found in [47]. The phenomenological parameter in eq.(4.27), $\kappa = 2.3$, is chosen in ref.[48] in order to reproduce correctly the experimental result:

$$\mathcal{BR}(B \to K\ell^+\ell^-)|_{res} = \sum_{i=1}^{2} \mathcal{BR}(B \to \psi_i K)\,\mathcal{BR}(\psi_i \to \ell^+\ell^-) \simeq 7\times 10^{-5}.$$

Note that $\kappa = 0$ in eq.(4.27) corresponds to the short distance contribution.

Now our next task is to calculate the decay rate ($\Gamma$) of the $B \to X_s\tau^+\tau^-$ by using the amplitude in eq.(4.3). The general formula for obtaining the differential decay rate is given by;

$$d\Gamma = \frac{1}{2E_b}(2\pi)^4\delta^4(p_b - p_1 - p_2 - p_s)\frac{d^3p_1}{(2\pi)E_1}\frac{d^3p_2}{(2\pi)E_2}\frac{d^3p_s}{(2\pi)E_s}|\mathcal{M}|^2 \qquad (4.29)$$

where $p_b\,(E_b), p_1\,(E_1), p_2\,(E_2)$ and $p_s\,(E_s)$ are the momenta (energies) of the b-quark, $\tau^+, \tau^-$ and the s-quark, respectively.

When the incident b-quark beam is unpolarized and the final state polarizations are not measured, we must sum over final state polarizations and average over initial spins:

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2}\sum_{spins}|\mathcal{M}|^2. \tag{4.30}$$

More explicitly we can write:

$$\begin{aligned} |\overline{\mathcal{M}}|^2 = \left|\frac{G_F\,\alpha}{\sqrt{2}\pi}V_{tb}\,V_{ts}^*\right|^2 & \Big\{|C_9^{eff}|^2\,Q_{\mu\nu}^{(1)}\,L^{(1)\mu\nu} + |C_{10}|^2\,Q_{\mu\nu}^{(1)}\,L^{(2)\mu\nu} \\ &+ 4\frac{{m_b}^2}{q^4}|C_7^{eff}|^2\,Q_{\mu\nu}^{(2)}\,L^{(1)\mu\nu} + 2\,Re(C_9^{eff}\,C_{10}^*)\,Q_{\mu\nu}^{(1)}\,L^{(3)\mu\nu} \\ &+ i\frac{m_b}{q^2}\,Re(C_9^{eff}\,C_7^{eff*})\,Q_{\mu\nu}^{(3)}\,L^{(1)\mu\nu} \\ &+ i\frac{m_b}{q^2}\,Re(C_{10}\,C_7^{eff*})\,Q_{\mu\nu}^{(3)}\,L^{(3)\mu\nu}\Big\} \end{aligned} \tag{4.31}$$

where

$$\begin{aligned} Q_{\mu\nu}^{(1)} &= Tr[(\not{p}_s+m_s)\gamma_\mu\,L\,(\not{p}_b+m_b)\gamma_\nu\,L]\,; \\ Q_{\mu\nu}^{(2)} &= Tr[(\not{p}_s+m_s)\sigma_{\mu\alpha}\,q^\alpha\,R\,(\not{p}_b+m_b)\sigma_{\nu\beta}\,q^\beta\,L]\,; \\ Q_{\mu\nu}^{(3)} &= Tr[(\not{p}_s+m_s)\gamma_\mu\,L\,(\not{p}_b+m_b)\sigma_{\nu\beta}\,q^\beta\,L] \end{aligned} \tag{4.32}$$

give the hadronic tensors and

$$\begin{aligned} L_{\mu\nu}^{(1)} &= Tr[(\not{p}_1-m_\tau)\gamma_\mu\,(\not{p}_2+m_\tau)\gamma_\nu]\,; \\ L_{\mu\nu}^{(2)} &= Tr[(\not{p}_1-m_\tau)\gamma_\mu\,\gamma_5\,(\not{p}_2+m_\tau)\gamma_\nu\,\gamma_5]\,; \\ L_{\mu\nu}^{(3)} &= Tr[(\not{p}_1-m_\tau)\gamma_\mu\,(\not{p}_2+m_\tau)\gamma_\nu\,\gamma_5] \end{aligned} \tag{4.33}$$

give the leptonic tensors.

In deriving eqs.(4.32-4.33), the following projection operators summed over the spin states have been used:

$$\sum_{spin} q\overline{q} = \not{p}_q + m_q \,, \qquad for\ q = b, s\ ; \tag{4.34}$$

$$\sum_{spin} \tau(p_1)\overline{\tau}(p_1) = \not{p}_1 - m_\tau\,; \tag{4.35}$$

$$\sum_{spin} \tau(p_2)\overline{\tau}(p_2) = \not{p}_2 + m_\tau\,. \tag{4.36}$$

Calculating the traces in eqs.(4.32-4.33), substituting first into eq.(4.31), then into (4.29), we obtain the following double differential distribution:

$$\begin{aligned}
\frac{d^2\Gamma(B \to X_s\tau^+\tau^-)}{d\hat{s}\ dz} &= \frac{G_F^2 m_b^5}{192\pi^3}\frac{\alpha^2}{4\pi^2}(1-\hat{s})^2\left(1-\frac{4t^2}{\hat{s}}\right)^{\frac{1}{2}}|V_{tb}\ V_{ts}^*|^2 \\
&\times\left\{\frac{3}{2}|C_9^{eff}|^2\left[(1+\hat{s})-(1-\hat{s})\left(1-\frac{4t^2}{\hat{s}}\right)z^2+4t^2\right]\right. \\
&+6|C_7^{eff}|^2\left[\left(1+\frac{1}{\hat{s}}\right)-\left(1-\frac{4t^2}{\hat{s}}\right)\left(1-\frac{1}{\hat{s}}\right)z^2+\frac{4t^2}{\hat{s}}\right] \\
&+\frac{3}{2}|C_{10}|^2\left[(1+\hat{s})-(1-\hat{s})\left(1-\frac{4t^2}{\hat{s}}\right)z^2-4t^2\right] \\
&+12\ Re(C_9^{eff}C_7^{eff*})\left(1+\frac{2t^2}{\hat{s}}\right)+6\ Re(C_9^{eff}C_{10}^*)\left(1-\frac{4t^2}{\hat{s}}\right)^{\frac{1}{2}} \\
&\left.+12\ Re(C_7^{eff}C_{10}^*)\left(1-\frac{4t^2}{\hat{s}}\right)^{\frac{1}{2}}z\right\}
\end{aligned} \tag{4.37}$$

where $z = cos\theta$, $t = m_\tau/m_b$ and $\theta$ is the angle between the momentum of the B-meson and the momentum of $\tau^+$ in the center of mass frame of the dileptons $\tau^+\tau^-$.

Now integrating the double differential distribution in eq.(4.37), according to the angle variable $\theta$, one obtains the differential decay rate for $B \to X_s\tau^+\tau^-$:

$$\frac{d\Gamma(B \to X_s\tau^+\tau^-)}{d\hat{s}} = \frac{G_F^2 m_b^5}{192\pi^3}\frac{\alpha^2}{4\pi^2}(1-\hat{s})^2\left(1-\frac{4t^2}{\hat{s}}\right)^{\frac{1}{2}}|V_{tb}\ V_{ts}^*|^2\Delta \tag{4.38}$$

where

$$\Delta = [|C_9^{eff}|^2 - |C_{10}|^2]6t^2 + [|C_9^{eff}|^2 - |C_{10}|^2]\left[(\hat{s} - 4t^2) + (1 + \frac{2t^2}{\hat{s}})(1 + \hat{s})\right]$$
$$+ \; 12\, Re(C_7^{eff} C_9^{eff*})(1 + \frac{2t^2}{\hat{s}}) + \frac{4\,|C_7^{eff}|^2}{\hat{s}}(1 + \frac{2t^2}{\hat{s}})(2 + \hat{s}). \tag{4.39}$$

Sometimes the differential decay rate is normalized to $BR\,(B \to X_c \tau \overline{\nu})$ to reduce the dependence on the $V_{CKM}$ matrix elements and also on the b-quark mass. Then, we obtain the differential $\mathcal{BR}$:

$$\frac{d\,BR\,(B \to X_s \tau^+ \tau^-)}{d\hat{s}} = BR\,(B \to X_c \tau \overline{\nu}) \frac{\alpha^2}{4\pi^2 f(\frac{m_c}{mb})}(1 - \hat{s})^2 \left(1 - \frac{4t^2}{\hat{s}}\right)^{\frac{1}{2}} \frac{|V_{tb}\, V_{ts}^*|^2}{|V_{cb}|^2} \Delta \tag{4.40}$$

with $f(\alpha) = 1 - 8\alpha^2 + 8\alpha^8 - 24\alpha^4 \ln \alpha$.

## 4.4 Tau Polarization Asymmetry in $B \to X_s \tau^+ \tau^-$

We will now discuss the final state $\tau^-$ polarization. The polarized decay rates $\frac{d\Gamma(\hat{e}_i)}{d\hat{s}}$ are obtained by introducing the spin projection operator, which is

$$P = \frac{1}{2}(1 + \gamma_5 \not{N}_j) \tag{4.41}$$

for $\tau^-$. In eq.(4.41), $j$ denotes the *Longitudinal* ($L$), the *Normal* ($N$) and *Transverse* ($T$) components of the polarization where the four vectors $(N_\mu)_j$ satisfy;

$$N.p_2 = 0 \qquad and \qquad N^2 = -1\,. \tag{4.42}$$

We define the following unit vectors $N_j$ in the rest frame of $\tau^-$, for the L, N and the T components of the polarization:

$$N_L^\mu = (0, \hat{e}_L) = \left(0, \frac{\vec{p}_2}{|\vec{p}_2|}\right);$$

$$N_N^\mu = (0, \hat{e}_N) = \left(0, \frac{\vec{p}_s \times \vec{p}_2}{|\vec{p}_s \times \vec{p}_2|}\right); \tag{4.43}$$

$$N_T^\mu = (0, \hat{e}_T) = (0, \hat{e}_N \times \hat{e}_L)$$

where $\vec{p}_2$ and $\vec{p}_s$ are the three momenta of $\tau^-$ and the s-quark in the center-of-mass frame of the dileptons $\tau^+\tau^-$. The longitudinal unit vector $N_L$ is boosted by Lorentz transformation to the CM frame of $\tau^+\tau^-$:

$$(N_L^\mu)_{CM} = \left(\frac{|\vec{p}_2|}{m_\tau}, \frac{E_\tau}{m_\tau}\frac{\vec{p}_2}{|\vec{p}_2|}\right) \tag{4.44}$$

while the vectors of perpendicular directions are not affected by the boost.

The $\tau^-$ lepton polarization asymmetry is defined as

$$P_i = \frac{\frac{d\Gamma(N_j)}{d\hat{s}} - \frac{d\Gamma(-N_j)}{d\hat{s}}}{\frac{d\Gamma(N_j)}{d\hat{s}} + \frac{d\Gamma(-N_j)}{d\hat{s}}} \tag{4.45}$$

where the subindex $j$ is L, T or N. Here, $P_L$ denotes the longitudinal polarization, $P_T$ is the polarization asymmetry in the decay plane, and $P_N$ is the normal component to both of them.

To find $\frac{d\Gamma(\pm N_j)}{d\hat{s}}$ in eq.(4.45), we need to calculate the absolute square of the matrix element $\mathcal{M}$ given by eq.(4.3). But since the final $\tau^-$ lepton is polarized, the projection operator

$$\tau(p_2)\overline{\tau}(p_2) = \frac{1}{2}(\not{p}_2 + m_\tau)(1 + \gamma_5 \not{N}_i) \qquad (4.46)$$

should be used instead of the one given by eq.(4.36). If we sum over the spin states of the other final particles, s-quark and $\tau^+$ lepton, and average over the initial spins, then the hadronic tensors $Q^{(i)}_{\mu\nu}$, where $i = 1, 2, 3$, in eq.(4.32) remain unaltered. However, we should replace the terms $(\not{p}_2 + m_\tau)$ in the leptonic tensors $L^{(i)}_{\mu\nu}$, $i = 1, 2, 3$, in eq.(4.33) with

$$\frac{1}{2}(\not{p}_2 + m_\tau)(1 \pm \gamma_5 \not{N}_j). \qquad (4.47)$$

By evaluating the expression in eq.(4.45), the polarization components $P_L$, $P_N$ and $P_T$ are obtained as follows:

$$\begin{aligned} P_L(\hat{s}) &= \left(1 - \frac{4t^2}{\hat{s}}\right)^{\frac{1}{2}} [12\, C_7^{eff} C_{10}(1 - \hat{s}) \\ &\quad +2\, Re(C_9^{eff} C_{10})(1 + \hat{s} - 2\hat{s}^2)]/\Delta; \end{aligned} \qquad (4.48)$$

$$P_N(\hat{s}) = \frac{3\pi t}{2\Delta} Im(C_9^{eff*} C_{10}) \sqrt{\hat{s}}(1 - \hat{s}) \left(1 - \frac{4t^2}{\hat{s}}\right)^{\frac{1}{2}}; \qquad (4.49)$$

$$\begin{aligned} P_T(\hat{s}) &= \frac{3\pi t}{2\Delta\sqrt{\hat{s}}}(1 - \hat{s})[C_7^{eff} C_{10} - 4\, Re(C_7^{eff} C_9^{eff}) \\ &\quad -\frac{4}{\hat{s}}|C_7^{eff}|^2 + Re(C_9^{eff*} C_{10}) - |C_9^{eff}|^2 \hat{s}]. \end{aligned} \qquad (4.50)$$

where $\Delta$ is defined in eq.(4.39).

## 4.5 Numerical Results and Discussions

In the 2HDM masses of the charged and neutral Higgs bosons, $m_{H^\pm}$, $m_{h^0}$, $m_{A^0}$, $m_{H^0}$, the ratio of vacuum expectation values of Higgs bosons, $\tan\beta$, and the $H^0-h^0$ mixing angle $\alpha$ are all free parameters. In our analysis, we do not consider the contributions coming from neutral Higgs bosons $A^0$, $h^0$, $H^0$ so the parameter $\alpha$ is irrelevant. As for the charged Higgs boson masses and $\tan\beta$, there are some theoretical and experimental bounds on their values. A model independent lower bound of the mass of the charged Higgs boson, $m_{H^\pm} > 44$ GeV, can be extracted from the fact that charged $H^\pm$ pair is not observed in Z decays [49]. However there is no experimental upper bound for $m_{H^\pm}$ except $m_{H^\pm} \leq 1\ TeV$ coming from the unitarity condition [50]. It is possible to obtain a restriction for the ratio $\tan\beta/m_{H^\pm}$ and it has been estimated that $\tan\beta/m_{H^\pm} \leq 0.38\ GeV^{-1}$ in [51] and $\tan\beta/m_{H^\pm} \leq 0.46\ GeV^{-1}$ in [52] using the experimental BR's of $B \to \tau\bar{\nu}$ and $B \to X\tau\bar{\nu}$. Recently, the relation between $m_{H^\pm}$ and $\tan\beta$ has been estimated in [53] by using the CLEO measurement of the decay $B \to X_s\gamma$ [6, 54]. Otherwise, $\tan\beta$ is restricted from $Z \to b\bar{b}$ decay [55] as $\tan\beta > 0.7$.

In this work we study the invariant dilepton mass dependence of the differential branching ratio ($dBR/d\hat{s}$) and the final $\tau$-lepton polarization for the $B \to X_s\tau^+\tau^-$ decay in the 2HDM. We represent the results of the related analytical calculations in a series of graphs, in figs.(4.2-4.5). The input parameters used in the calculations are given in Table (4.1).

In figs.(4.2-4.3), the differential branching ratio $dBR(B \to X_s\tau^+\tau^-)/d\widehat{s}$ as a function of $\widehat{s}$ is plotted for three different values of $m_{H^\pm}$ and for the fixed value of $\tan\beta = 1$ in Model I and II, respectively. The three dimensional representations of these figures are also given. From these figures it follows that there is a considerable enhancement for the $dBR(B \to X_s\tau^+\tau^-)/d\widehat{s}$ in the 2HDM compared to the SM case, especially for the small values of $m_{H^\pm}$. For example, we find $dBR(B \to X_s\tau^+\tau^-)/d\widehat{s} = 1.39 \times 10^{-6}$ for $\widehat{s} = 0.58$ in the SM, which is in agreement with the ref.[42]. The corresponding results in Model I and II are $dBR(B \to X_s\tau^+\tau^-)/d\widehat{s} = 1.69 \times 10^{-6}$ and $dBR(B \to X_s\tau^+\tau^-)/d\widehat{s} = 1.49 \times 10^{-6}$, respectively, for $\tan\beta = 1$ and $m_{H^\pm} = 300$ GeV. We also observe that the enhancement of the rate is larger in Model I than in Model II. This occurs because in Model I, $\xi\xi' = \xi^2$ in $C_7^H$ so that overall sign of the contribution from magnetic moment type operators flips and causes a constructive interference between the coefficients $C_7$ and $C_9$.

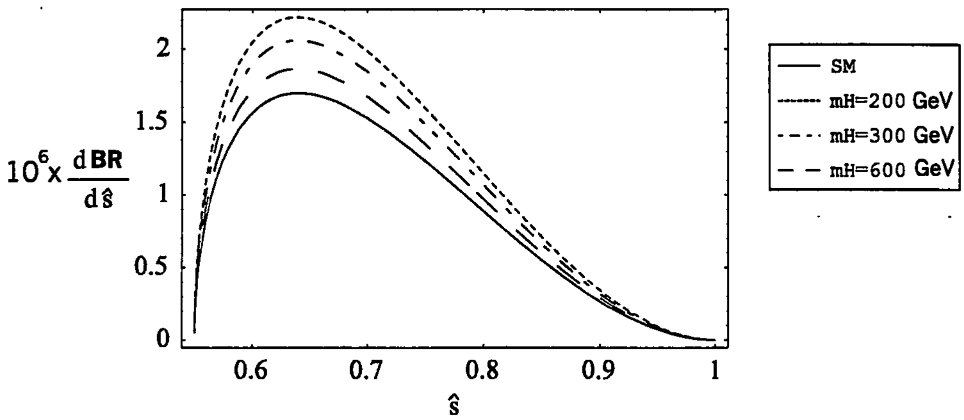
In fig.(4.4), we present the dependence of $dBR(B \to X_s\tau^+\tau^-)/d\widehat{s}$ on $\widehat{s}$, including the long distance effects, for three different values of $M_{H^\pm}$ and for $\tan\beta = 1$ in Model I and II, respectively. In these figures, curves with sharp peaks represent the LD contributions. Actually, there are five known resonances in the $c\bar{c}$ system [47] that can contribute to the $B \to X_s\tau^+\tau^-$ decay for which $\widehat{s} > 4m_\tau^2/m_b^2$, but we have considered only the lowest one $J/\Psi(3686\ MeV)$.

Fig.(4.5) represents the transverse, normal and longitudinal components of

Table 4.1: The input parameters used in numerical calculations.

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $m_b$ | $4.8 GeV$ | $m_c$ | $1.4 GeV$ |
| $m_\tau$ | $1.78 GeV$ | $\alpha$ | $1/129$ |
| $\|V_{cb}\|$ | 0.04 | $\|V_{tb}V_{ts}^*\|$ | 0.04 |
| $G_F$ | $1.17 \times 10^{-5}$ | $BR(B \to X_c \tau \bar{\nu})$ | 0.104 |
| $f(\frac{m_c}{m_b})$ | 0.54 | $sin^2\theta_W$ | 0.23 |

the final $\tau$ lepton polarization as a function of $\hat{s}$ for the fixed values of $m_{H^\pm} =$ 300 GeV and $\tan\beta = 1$, in the SM, Model I and II. Here the long distance effects from the five possible resonances are taken into account. We see that the effect of the charged Higgs boson exchange contribution is not very sizable for the polarization components of the $\tau$ lepton. As far as this contribution is concerned the only significant component may be $P_T$ , which has an average value of $< P_T >= -0.67, < P_T >= -0.66$ and $< P_T >= -0.63$, in the SM, Model I and Model II, respectively.



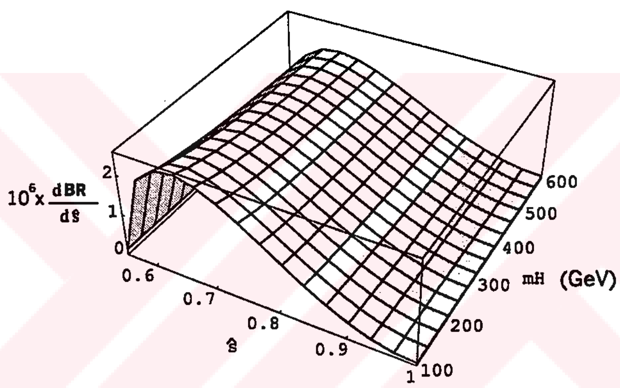




Figure 4.2: The differential branching ratio of $B \to X_s\tau^+\tau^-$ , as a function of $\hat{s}$ for different values of $m_H$ and for $\tan\beta = 1$ in Model I. In the second figure, this is shown by a three dimensional diagram.

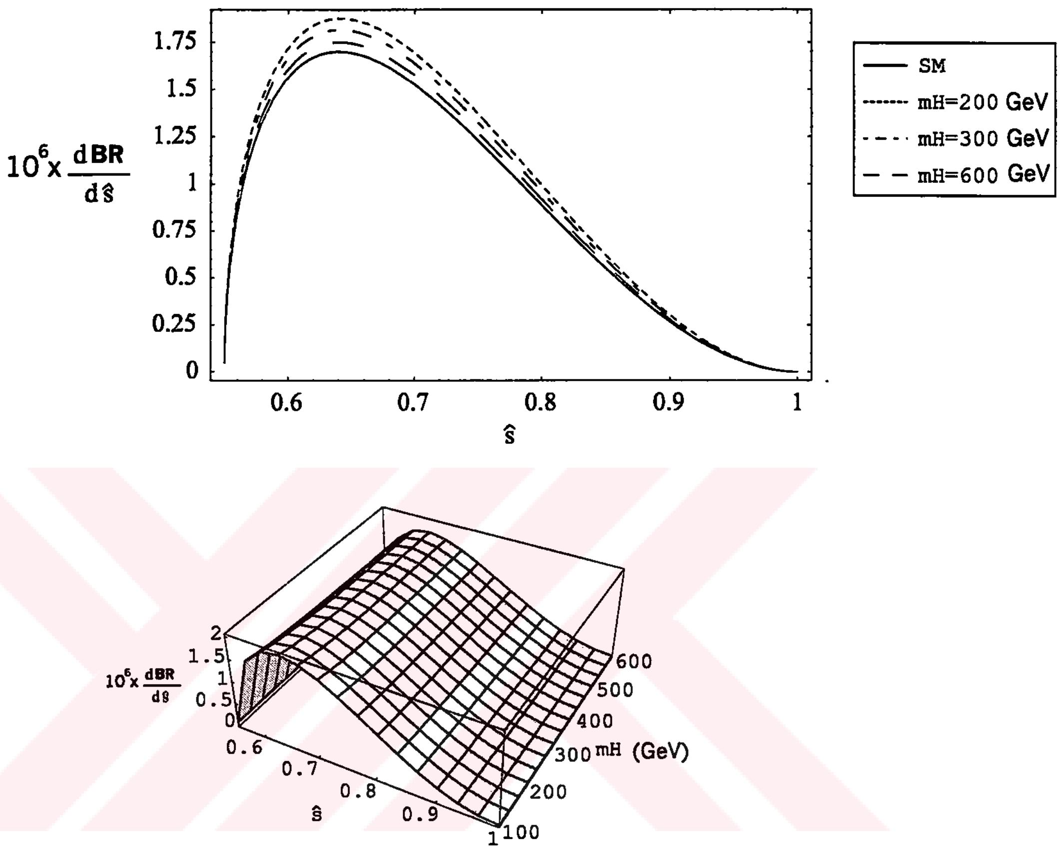



Figure 4.3: Same as fig.(4.2), but in Model II.

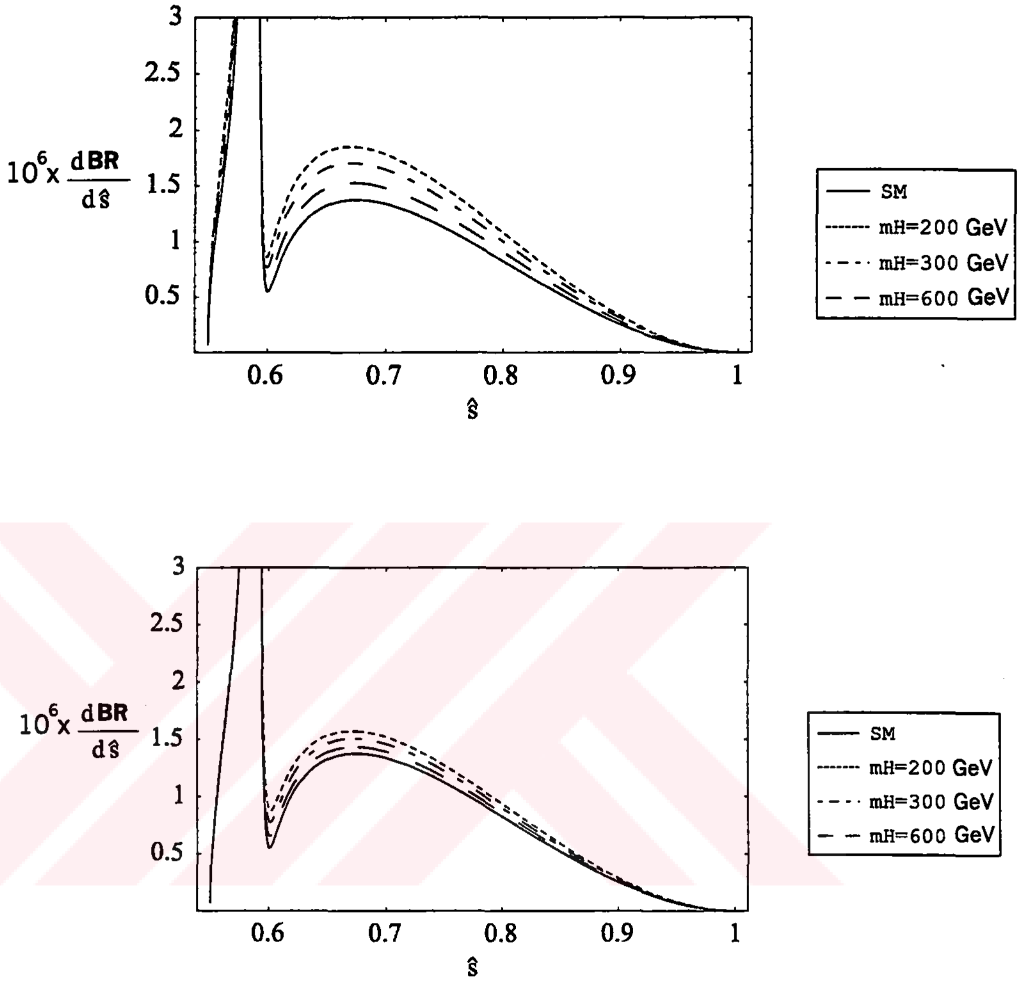



Figure 4.4: Same as fig.(4.2) and fig.(4.3), but with long distance effects included.

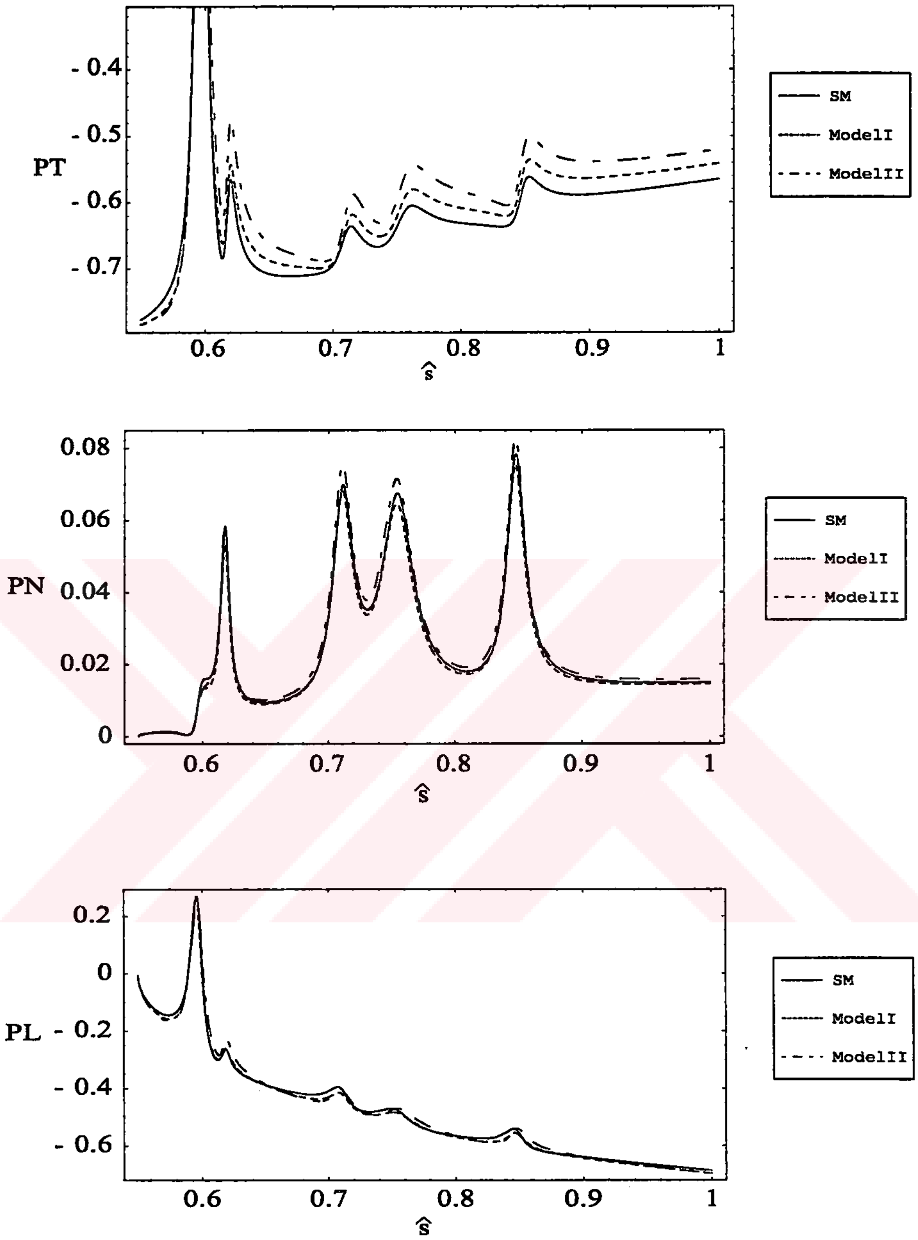



Figure 4.5: The transverse, normal and the longitudinal components of the polarization of $\tau^-$, as a function of $\hat{s}$, in the SM and Model I and II.

# CHAPTER 5

# CONCLUSION

Although the SM, describing the strong and electroweak interactions, is a phenomenologically successful and mathematically consistent and renormalizable theory, there are many open questions such as the origin of CP violation, the so-called hierarchy problem, etc. that lead physicists to search beyond it. In this direction, a series of alternative theories which address these questions have been proposed and 2HDM is one of them.

In this work, we have studied $B \to X_s \tau^+ \tau^-$ decay in the framework of the 2HDM. It is shown that the differential branching ratio $dBR(B \to X_s \tau^+ \tau^-)/d\hat{s}$ gets a considerable enhancement in comparison to the SM due to the extended Higgs sector. We have also observed that this enhancement is more sizable for the small values of $m_{H^\pm}$. We have compared our results with the available literature and shown that they are in good agreement.

As another experimentally observable parameter, we have investigated the $\tau$ lepton polarization in $B \to X_s \tau^+ \tau^-$ decay. We have studied the sensitivity of different polarization components of $\tau$ to the extended Higgs sector. It is seen that, as far as the charged Higgs contributions are concerned the only significant component may be $P_T$, which gets some enhancement compared to the SM case.

In conclusion, we can say that rare $B \to X_s \tau^+ \tau^-$ decay is an efficient and powerful probe for investigating the physics beyond the SM.

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# APPENDIX A

## Feynman Rules

Below we outline the useful Feynman rules needed for the calculation of the amplitudes regarding the diagrams in fig.(4.1). Note that the results for Model I and Model II can be obtained by the following substitutions:

$$\xi = \xi' = 1/\tan\beta \qquad \text{for Model I ,}$$
$$\xi = -1/\xi' = 1/\tan\beta \qquad \text{for Model II .} \tag{A.1}$$

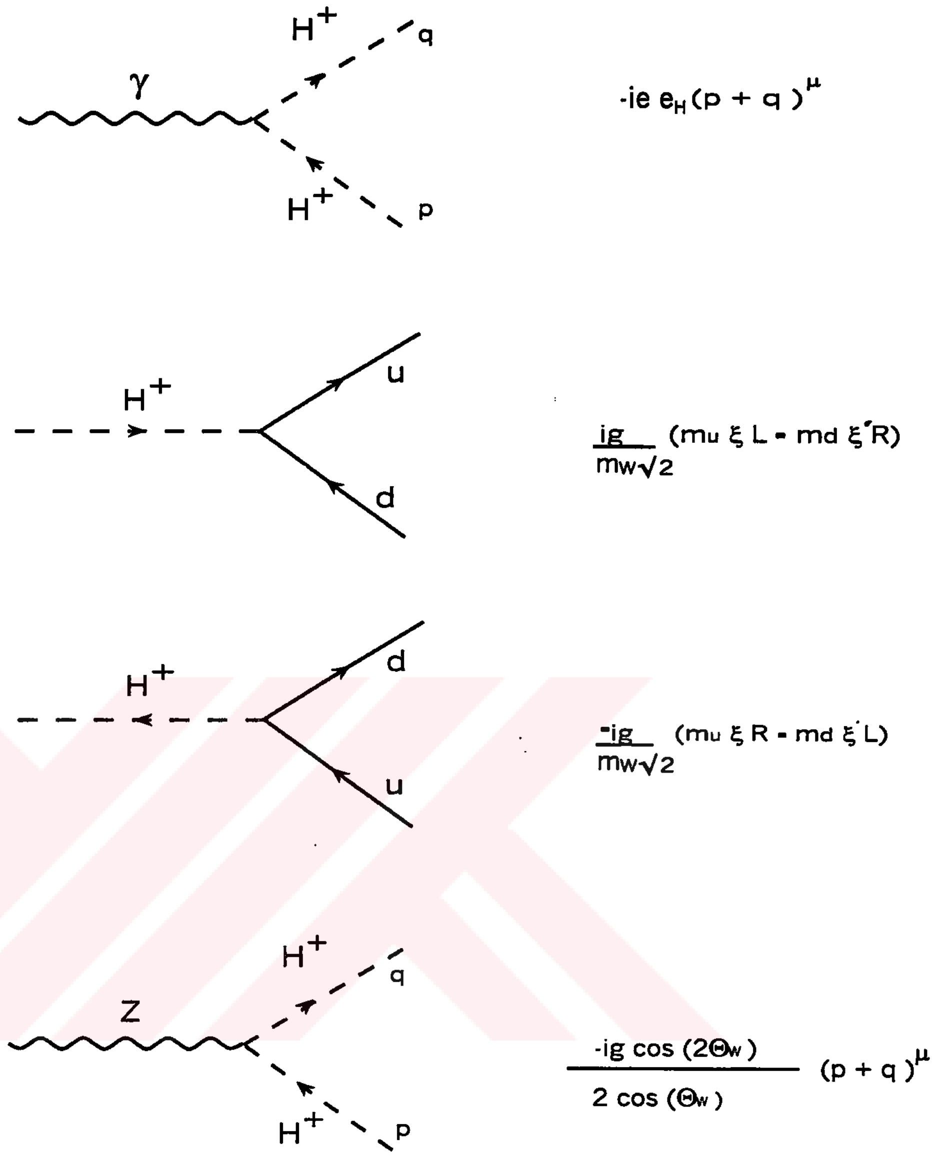


Figure A.1: Useful Feynman rules.

# APPENDIX B

## Renormalization of Vertex Diagrams

In this Appendix, we will make the calculations of one of the vertex diagrams contributing to $b \to s\tau^+\tau^-$ in 2HDM. We will consider the diagram in fig.(B.1). Using the Feynman rules in Appendix A, we write:

$$\begin{aligned}\Gamma^0_\mu &= \int \frac{d^4q}{(2\pi)^4}\left[\frac{-ig}{\sqrt{2}m_W}V^*_{is}(m_i\xi R - m_s\xi' L)\right]\left[\frac{i[(\not p + \not q) + m_i]}{(p+q)^2 - m_i^2}\right]\left(\frac{i}{(q+k)^2 - m_H^2}\right)\\ &\times[-iee_H(2q+k)_\mu]\left[\frac{ig}{\sqrt{2}m_W}V_{ib}(m_i\xi\ L - m_b\xi'\ R)\right]\times\left(\frac{i}{q^2 - m_H^2}\right) \qquad \text{(B.1)}\end{aligned}$$

where $q$ is the four momentum of $H^\pm$. With some arrangement, we can rewrite eq.(B.1) as:

$$\begin{aligned}\Gamma^0_\mu &= (i)^6\left(\frac{g}{\sqrt{2}m_W}\right)^2 ee_H(V_{ib}V^*_{is})\\ &\times \int \frac{d^4q}{(2\pi)^4}\frac{(m_i\xi R - m_s\xi' L)\ [(\not p + \not q) + m_i]\ (2q+k)_\mu\ (m_i\xi\ L - m_b\xi'\ R)}{[(p+q)^2 - m_i^2]\ [(q+k)^2 - m_H^2]\ [q^2 - m_H^2]}.\end{aligned}$$

(B.2)

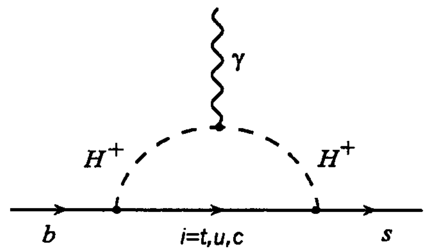


Figure B.1: One of the vertex diagrams contributing to $b \to s\tau^+\tau^-$ in 2HDM.

We can separately investigate the numerator and the denominator of eq.(B.2). The numerator

$$N = \left[(m_i \xi R - m_s \xi' L)\ [(\not p + \not q) + m_i]\ (m_i \xi\ L - m_b \xi'\ R)\ (2q+k)_\mu\right] \tag{B.3}$$

can be easily arranged to

$$N = \left[m_i^2 \xi^2 (\not p + \not q)\ L - m_i^2 m_b \xi \xi'\ R + m_s m_b \xi'^2 (\not p + \not q)\ R - m_i^2 m_s \xi \xi'\ L\right] (2q+k)_\mu . \tag{B.4}$$

As far as the denominator is concerned, using the method of Feynman parametrization

$$\frac{1}{ABC} = 2! \int_0^1 dx \int_0^{1-x} dy\ \frac{1}{[Ax + By + C(1-x-y)]^3} \tag{B.5}$$

an be rewritten as;

$$
\begin{aligned}
&= 2!\int_0^1 dx \int_0^{1-x} dy \frac{1}{\{[(p+q)^2-m_i^2]x+[(q+k)^2-m_H^2]y+[q^2-m_H^2](1-x-y)\}^3};\\
&= 2!\int_0^1 dx \int_0^{1-x} dy \frac{1}{(t^2-P(x,y))^3} \qquad \text{(B.6)}
\end{aligned}
$$

ıere we have used the abbreviations;

$$
\begin{aligned}
t &= q + y\ k + x\ p;\\
P(x,y) &= -y(1-y)k^2 - x(1-x)p^2 + 2xykp + (1-x)m_H^2 + xm_i^2. \qquad \text{(B.7)}
\end{aligned}
$$

lext, we must express the numerator N in eq.(B.4) in terms of $t$. This task is implified by noting that;

$$
\begin{aligned}
\int \frac{d^4t}{(2\pi)^4}\frac{t_\mu}{D^3} &= 0;\\
\int \frac{d^4t}{(2\pi)^4}\frac{t_\mu t_\nu}{D^3} &= \int \frac{d^4t}{(2\pi)^4}\frac{g_{\mu\nu}t^2}{4D^3}. \qquad \text{(B.8)}
\end{aligned}
$$

Using these identities, we rewrite $N$ and then insert it, together with the denominator in eq.(B.6), into eq.(B.2):

$$
\begin{aligned}
\Gamma_\mu^0 &= -i2!C\int_0^1 dx\int_0^{1-x} dy \int \frac{d^4t}{(2\pi)^4}\frac{1}{(t^2-P)^3}\\
&\quad \Big\{[\frac{1}{2}t^2\gamma_\mu + (2yk_\mu + 2xp_\mu - k_\mu)(y\not{k} - (1-x)\not{p})](m_b m_s \xi^2 q + m_i^2 \xi'^2 L)\\
&+ m_i^2 \xi\xi'(2yk_\mu + 2xp_\mu - k_\mu)(m_b R + m_s L)\Big\} \qquad \text{(B.9)}
\end{aligned}
$$

where

$$
C = \left(\frac{g}{\sqrt{2}m_W}\right)^2 ee_H (V_{ib}V_{is}^*). \qquad \text{(B.10)}
$$

This integral is divergent. We use the method of dimensional regularization in order to get rid of the divergent part of the integral. In this method, integrals are taken in $d = 4 - \epsilon$ dimension, where $\epsilon$ is a very small number, so that divergence in the resulting $\Gamma$ function can be removed. Later, $\epsilon$ is set to zero.

To perform the momentum integration in eq.(B.9), we use a trick called Wick rotation, i.e. $t_0 \to i\, t_0$. Then, using

$$\int \frac{d^dq}{(2\pi)^d} \frac{1}{(q^2 - s)^n} = \frac{i}{(4\pi)^{d/2}} (-1)^n \frac{\Gamma(n - d/2)}{\Gamma(n)} \frac{1}{s^{n-d/2}} \tag{B.11}$$

we obtain in $d = 4 - \epsilon$ dimension,

$$\begin{aligned} \Gamma^0_\mu &= \int_0^1 dx \int_0^{1-x} dy \frac{iC}{16\pi^2} m_i^2 \Big\{ \xi^2 (2/\epsilon - \gamma_E - \log[P/4\pi]) \gamma_\mu L \\ &- \frac{[(2y-1)k_\mu + 2xp_\mu][\xi\xi'(m_s L + m_b R) + \xi^2((x-1)\not p + y \not k)L]}{P} \Big\} \end{aligned} \tag{B.12}$$

where $\gamma = 0.5772$ is the Euler constant. The pole term $1/\epsilon$ in eq.(B.12) reflects the divergence of the integral in four dimension.

One can renormalize the $\Gamma$ function by including the counter terms. The general form of the renormalized vertex operator is,

$$\Gamma^{ren}_\mu = \Gamma^0_\mu + T_L \gamma_\mu L + T_R \gamma_\mu R. \tag{B.13}$$

To determine the coefficients of the counter terms, $T_L$ and $T_R$, we use the gauge invariance requirement,

$$k^\mu \Gamma^{ren}_\mu |_{on-shell} = 0 \tag{B.14}$$

which gives

$$(k^\mu \Gamma^0_\mu + T_L \not{k} L + T_R \not{k} R)|_{on-shell} = 0. \tag{B.15}$$

Now we find that,

$$\begin{aligned} k^\mu \Gamma^0_\mu|_{on-shell} &= i\frac{C}{16\pi^2} m_i^2 \int_0^1 dx \int_0^{1-x} dy \Bigg\{ \frac{m_s m_b}{P_0}[(1-x)\xi^2 + 2\xi\xi']x \not{k} R \\ &+ [\xi^2(2/\epsilon - \gamma_E - \log[\frac{P_0}{4\pi}]) + \frac{\xi^2}{P_0}(m_b^2(1-x-y) + m_s^2 y)x \\ &+ \frac{\xi\xi'}{P_0}(m_b^2 + m_s^2)x] \not{k} L \Bigg\} \end{aligned} \tag{B.16}$$

where $P_0 = P|_{on-shell}$.

Then from eq.(B.15) and eq.(B.16), we find

$$\begin{aligned} T_L &= \frac{-C}{16\pi^2} m_i^2 \int_0^1 dx \int_0^{1-x} dy \Bigg\{ \xi^2(2/\epsilon - \gamma_E - \log[\frac{P_0}{4\pi}]) \\ &+ \frac{\xi^2}{P_0}(m_b^2(1-x-y) + m_s^2 y)x + \frac{\xi\xi'}{P_0}(m_b^2 + m_s^2)x \Bigg\} \\ T_R &= \frac{-C}{16\pi^2} m_i^2 \int_0^1 dx \int_0^{1-x} dy \Bigg\{ \frac{m_s m_b}{P_0}[(1-x)\xi^2 + 2\xi\xi']x \Bigg\} \end{aligned} \tag{B.17}$$

We insert eq.(B.12) and eq.(B.17) into eq.(B.13), to find $\Gamma_\mu^{ren}$,

$$\begin{aligned} \Gamma_\mu^{ren} &= \frac{iC}{16\pi^2 P} m_i^2 \int_0^1 dx \int_0^{1-x} dy \Bigg\{ \xi^2[(1-2y)y k_\mu \not{k} L - \log[P/P_0]\gamma_\mu L] \\ &- i\Big[\xi\xi'(m_s L + m_b R)x + \xi^2[(-1+x+y)m_b R - m_s y L]\Big]\sigma_{\mu\nu}k^\nu \\ &+ \xi^2(m_b^2 - p^2)y\gamma_\mu L - 2\xi\xi'[2m_b m_s L + (m_b^2 + m_s^2)R]\gamma_\mu \Bigg\}. \end{aligned} \tag{B.18}$$

We evaluate y-integrations and obtain

$$\begin{aligned} \Gamma_\mu^{ren} &= \frac{iC}{96\pi^2} m_i^2 \int_0^1 \frac{dx}{[m_H^2(1-x) + m_i^2 x]} \Bigg\{ (-1+x)^3 \xi^2 (k_\mu \not{k} - k^2\gamma_\mu) L \\ &- 3m_b x(-1+x)(2\xi\xi' + (-1+x)\xi^2)\sigma_{\mu\nu}k^\nu R \Bigg\}. \end{aligned} \tag{B.19}$$

Next we evaluate the integrals over $x$, and by taking $m_s^2 = 0$, we determine the contribution from this diagram to $C_9$ Wilson coefficient of $B \to X_s \tau^+ \tau^-$ as;

$$C_9^{(a)} = \frac{-C\xi^2 y R[2 - 9y + 18y^2 - 11y^3 + 6y^3 \ln(y)]}{576\pi^2(-1+y)^4}(k^\mu \not{k} - k^2\gamma^\mu). \tag{B.20}$$

The contribution to $C_7$ is as follows,

$$\begin{aligned} C_7^{(a)} &= \frac{-C}{192\pi^2(-1+y)^4}\Big\{k^\nu \xi y(Lm_s + m_b R)\sigma^{\mu\nu}\Big[(-1+y)[-6\xi'(-1+y^2) \\ &+ \xi(-1+5y+2y^2)] + 6y[2\xi'(-1+y) - \xi y]\ln(y)\Big]\Big\}. \end{aligned} \tag{B.21}$$

In eqs.(B.20-B.21), we use the representation $y = \frac{m_t^2}{m_H^2}$.