DEVELOPING PRE-SERVICE MATHEMATICS TEACHERS’ UNDERSTANDING OF TEACHING MATHEMATICS FOR SOCIAL JUSTICE

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ABSTRACT

DEVELOPING PRE-SERVICE MATHEMATICS TEACHERS’ UNDERSTANDING OF TEACHING MATHEMATICS FOR SOCIAL JUSTICE

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The purpose of the study was to examine and develop pre-service elementary mathematics teachers’ understanding of teaching mathematics for social justice through employing design research. In line with this purpose, firstly, the needs of pre-service mathematics teachers were identified, and then an intervention in teaching mathematics for social justice was designed according to their needs and implemented. Needs analysis interviews were conducted with seven pre-service elementary mathematics teachers and four teacher educators. According to the needs analysis results, pre-service mathematics teachers did not have much knowledge about teaching mathematics for social justice, and teaching mathematics for social justice was not covered in any of the undergraduate courses they attended. The intervention's goals were determined by examining the results of the needs analysis study and reviewing the literature. Five pre-service mathematics teachers attended the 1st design experiment, and nine pre-service mathematics teachers attended the 2nd design experiment regarding teaching mathematics for social justice. Data were collected during both of the design experiments through initial and final semi-structured interviews, field notes of the researcher, classroom observations, reflective
papers, video recordings of classes, in-class activities, and individual lesson plans of pre-service mathematics teachers. The results of the study indicated that the intervention in teaching mathematics for social justice with a teaching and learning strategy grounded in social constructivism promotes pre-service mathematics teachers' understanding of teaching mathematics for social justice and their ability to prepare lesson plans based on teaching mathematics for social justice.

**Keywords:** teaching mathematics for social justice, pre-service teacher education, mathematics teacher education, social constructivism, design research
ÖZ

MATEMATİK ÖĞRETMEN ADAYLARININ SOSYAL ADALET ODAKLI MATEMATİK ÖĞRETİMİ ANLAYIŞLARININ GELİŞTİRİLMESİ

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toplantılar. Çalışmanın sonuçları, sosyal yapılandırılmışa dayanan bir öğretme ve öğrenme stratejisi ile sosyal adalet odaklı matematik öğretimi müdahalesinin matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışlarını ve sosyal adalet odaklı matematik öğretimi temeline dayanan ders planı hazırlama becerilerini geliştirdiğini göstermiştir.

Anahtar Kelimeler: sosyal adalet odaklı matematik öğretimi, hizmet öncesi öğretmen eğitimi, matematik öğretmen eğitimi, sosyal yapılandırılmış, tasarım araştırması
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LIST OF ABBREVIATIONS

TMSJ : Teaching Mathematics for Social Justice
PMT : Pre-service Mathematics Teacher
MoNE : Ministry of National Education
RWWM : Reading the World with Mathematics
WWWM : Writing the World with Mathematics
CHAPTER 1

INTRODUCTION

1.1. Background to the Study

In the modern era, the world faces many problems, such as poverty, racism, homelessness, terrorism, unemployment, global warming, and resource scarcity. Disruption of social justice and increased inequality can be stated as causes of these problems in local, national, and universal contexts (Turhan Türkkan, 2017). Especially with the Covid-19 pandemic which spread to worldwide in early 2020, societal inequalities and injustices become more visible such as access to healthcare and education, and the pandemic highlighted the disparities between different socioeconomic groups. According to Apple (2012), the role of education should be questioned while creating a society that helps individuals overcome such social issues. In that sense, education can have a transformative power on these social problems with some reorganizations. If we want permanent solutions to these problems, we need to integrate these issues into our education and make our students recognize them.

Social justice involves recognizing and addressing the broader systemic power structures that sustain the current societal conditions we experience, as mentioned by Bond and Chernoff (2015). Bell (1997) asserts that it encompasses a societal ideal where resources are distributed fairly, and all individuals feel both physically and psychologically protected. Gutstein (2006) also emphasizes providing for the material, social, psychological, spiritual, and emotional needs of human beings along with providing for economic needs. With increasing differences in society in recent years, it is essential to promote social justice. Inequitable educational opportunities and resources and limited access to highly qualified teachers raise the gap among different societal groups (Lalas, 2007). Within this concept, it has a significant value in how we will approach this rising gap in society.
To deal with the problems of society in a global context, United Nations (2022) stated 17 Sustainable Development Goals, which are an urgent call for action by all countries in a global partnership. These Sustainable Development Goals are mentioned in the 2030 Agenda for Sustainable Development, which provides a shared blueprint for peace and prosperity for people and the planet, now and into the future. United Nations Member States, including Türkiye, adopted this Agenda in 2015. In that sense, with the determined goals, it can be concluded that social justice is an essential component of sustainable development (Langhelle, 2000). Among 17 goals, especially Goal 16, it was aimed to promote peaceful and inclusive societies for sustainable development and provide access to justice for all. Moreover, some components of social justice can be encountered in other goals of the Agenda, such as no poverty, zero hunger, gender equality, quality education, and reduced inequalities (UN, 2022). In this manner, education can assist change in society to attain sustainable development goals.

As a proponent of the idea that education can help social transformation, Freire (2000) argues that education should aim for societal well-being, even within disciplines like mathematics which is considered an abstract field. Mathematics education equips individuals with the ability to analyze complex problems, evaluate evidence, and propose effective solutions. All of these are necessary for active participation in a democratic society. By integrating social justice themes into mathematics education, students can develop a deeper understanding of social inequalities and be empowered to address them through mathematical reasoning and action.

School mathematics is usually characterized as boring, irrelevant, passive, ignoring individual needs, and teaching rules without any rationale (Nardi & Steward, 2003). In other words, mathematics learned in the classroom is independent of the authentic life around students and mainly focuses on the cognitive level of learning. According to Hunt (1998), children require not just a solid academic foundation but also opportunities to apply their academic knowledge in fostering a democratic society where everyone can engage fully. In that sense, mathematics education can be a tool for creating solutions for social problems and making the world a better place.
Mathematics education should contribute to tackling the challenges prevalent in our society, such as increasing inequality, violations of human rights, and unsustainable economic growth (Cotton, 2013). Moreover, it can seek solutions to such problems by integrating social justice issues into the mathematics curriculum. At that point, teachers are the ones who are going to accomplish these goals in their classes. Therefore, the importance of teacher education is revealed. Mathematics teacher education programs are essential in terms of preparing pre-service teachers to teach mathematics for social justice. Dewey (1902) stated that the primary goal of education should be connecting education to the lives of students, not transmitting irrelevant facts that they cannot connect with their lives. Teaching mathematics for social justice can relate the abstract to the relevant by employing mathematics to uncover the effects of injustice (Bateiha & Reeder, 2014). Moreover, teaching mathematics for social justice can provide students with opportunities to learn mathematics in culturally specific and meaningful ways and improve marginalized groups economically and socially (Leonard & Evans, 2012).

According to Gutstein (2003), a pedagogy for social justice in mathematics education has three main goals: helping students develop sociopolitical consciousness (reading the world with mathematics), a sense of agency (writing the world with mathematics), and positive social and cultural identities. If students are asked questions that have meaning in their lives and begin to understand power relations in local and broader society, they will develop sociopolitical consciousness, which Freire called “conscientização” (Freire, 2000). Moreover, if they view themselves as capable of effecting meaningful change for social justice, they will develop a sense of agency (Ayers et al., 2009).

When mathematics education in Türkiye is considered, low-performance scores of students on both national and international exams are striking. Among the reasons for these low performances, students’ prejudice against mathematics and the use of rote-based teaching methods are prominent factors (Öztürk, 2018). The reform
movements in mathematics education globally and in Turkey emphasize the importance of employing real-life problems and contexts for students. This approach enables students to understand and give significance to the mathematics they are learning, rather than simply memorizing algorithms and repetitively practicing them (Doğan, 2012). Whereas, according to the results of a study conducted with middle school mathematics teachers, most of the teachers indicated that mathematics programs should include activities and include links to real-life and highlight the shortcomings in this regard (Bozkurt et al, 2021). In this respect, teacher education programs come into question while equipping pre-service teachers with related materials and a coherent understanding of current reform movements in education.

This study focuses on developing pre-service elementary mathematics teachers' understanding regarding teaching mathematics for social justice. The ages of potential students of are taken into consideration while selecting elementary pre-service mathematics teachers. According to Piaget, middle school students, who range in age from 11 to 14, are in the process of moving from the concrete operational stage to the formal operational stage. Students might start to comment on societal issues or their own lives and become informed citizens since the formal operational procedure helps them develop their abstract thinking and problem-solving skills.

There is a claim that university-based teacher education is overly academic and detached from the practical reality of "what works" (Down & Smyth, 2012), despite the fact that teacher education programs are viewed as places for social reform (McLaren & Baltodano, 2005). Teacher education for social justice must continually change and meet the needs of societies. Teacher education can be ‘deliberately designed and carried out to expose prospective teachers to a variety of ideological postures,’ thus enabling student teachers to ‘critically examine the damaging biases they may personally hold, and the inequalities and injustices present in schools and the society as a whole (Bartolome, 2007, p. 281).

In order to consider a more just world and more relevant mathematics education, curriculum and pedagogy need to be investigated from a social justice viewpoint. In
light of the needs of pre-service elementary mathematics teachers, the development and implementation of an intervention in teaching mathematics for social justice is the main goal of this study. The goal of the intervention was to enable pre-service teachers to understand the meaning of mathematics in light of the social principles woven throughout the rigorous standards of the traditional curriculum. Ultimately, pre-service teachers' development of concepts in teaching mathematics for social justice will help transform the way mathematics is taught, actively engage their future students in the learning process, and support the development of profoundly related knowledge (Darder, 2002).

1.2. Purpose of the Study

There has been a movement to teach mathematics in ways that involve social justice issues for quite some time (Frankenstein, 1983). Many scholars are working in this field, and producing research reports, and providing curriculum materials for teachers (Burton, 2003; Gutstein & Peterson, 2005). However, these works and curriculum materials will not appear in classrooms unless teachers practice them and believe that teaching mathematics for social justice makes a difference. Thus, activating social justice issues in mathematics lessons can be achieved by collaborating with pre-service mathematics teachers.

The purpose of the study is to examine and develop pre-service elementary mathematics teachers’ understanding of teaching mathematics for social justice. In line with this purpose, firstly, the needs of pre-service mathematics teachers were identified by collecting data from pre-service mathematics teachers and teacher educators, then an intervention in teaching mathematics for social justice was designed according to their needs and implemented, and reimplemented after making revisions. Besides, the study aims to examine how the TMSJ intervention contributes to their understanding of teaching mathematics for social justice and examine the development of the understanding of TMSJ of pre-service teachers.

1.3. Research Questions

In relation to the purposes stated above, the following research questions are attempted to be answered during the study.
Research Questions for Needs Analysis:

1. What are the current experiences of pre-service mathematics teachers regarding teaching mathematics for social justice?
2. What are the experiences of pre-service mathematics teachers regarding teaching mathematics for social justice from the perspectives of teacher educators?

Research Questions for Design Research:

“How can a teaching and learning strategy grounded in social constructivism promote pre-service mathematics teachers’ understanding of teaching mathematics for social justice?”

3. How does pre-service mathematics teachers’ understanding of teaching mathematics for social justice develop through the designed intervention grounded in social constructivism in terms of reading the world with mathematics and writing the world with mathematics?
4. What are the elements of a designed intervention to improve the understanding of pre-service mathematics teachers regarding teaching mathematics for social justice?

1.4. Significance of the Study

Making connections with students’ interests and real-life experiences while teaching mathematics can improve their learning. Rubel (2017) stated that embedding mathematics teaching within the context of social injustices they may encounter increases students’ interest in mathematics. Thanks to TMSJ, students gain a deeper understanding of the social injustices they observe in their lives, and they are also empowered to act upon these injustices (Berry III et al., 2020). In this sense, it is important to teach mathematics while integrating the social justice context. Mathematics teachers become crucial because they are the ones who will offer this kind of mathematics instruction. For this reason, it is necessary to examine the need for TMSJ in pre-service mathematics teacher education. This study is significant in terms of determining the needs of pre-service teachers related to teaching mathematics for social justice by collecting data from pre-service teachers and teacher educators in a state university in central Anatolia.
Teacher candidates need to develop teaching approaches that are social justice-oriented (Lee, 2011). These teaching approaches can be related to developing equitable pedagogy for all students, integrating students’ diverse cultures into the classroom, creating learning environments to reduce prejudice, and incorporating multiple knowledge construction processes (Banks, 2008). Pre-service teachers are informed about how they can utilize critical mathematics perspectives in their classes after attending the intervention in teaching mathematics for social justice. Also, this study provides teacher educators with an intervention related to TMSJ.

In terms of teaching for social justice, there is a need for an alternative approach to teaching mathematics that prevents the reproduction of inequities in society. MoNE (2018) states that mathematics is part of our lives, and connections between mathematics and real life and social sciences should be established. These connections can include issues such as bread waste, recycling, rights, and responsibilities for social security. However, there is no clear explanation in the elementary mathematics curriculum about how to make these connections while teaching mathematics. Despite the growing focus on incorporating real-life problems into mathematics education, there remained a lack of attention to critical real-world issues such as social justice, gender discrimination, poverty, democracy, equality, and peace within the new elementary mathematics curriculum (Doğan, 2012).

Therefore, in this study, issues related to social justice are included in elementary mathematics teacher education.

There are some case studies conducted with pre-service mathematics teachers about teaching mathematics for social justice (Alexandre, 2001; Bateiha, Reeder, 2014; Jong & Jackson, 2016). These studies are conducted in mathematics courses that integrate social justice issues. On the other hand, there are a limited number of studies that focus on learning to teach mathematics for social justice for pre-service and in-service teachers (Gau, 2005; Wright, 2015) and examine pre-service teachers’ beliefs in learning how to teach mathematics for social justice (Muller, 2008).

In Türkiye, social justice and mathematics education are not usually linked in studies. After examining the database of theses written in Türkiye that is provided by
the Higher Education Council on December 2023, there was only one dissertation related to developing students’ problem-posing skills and their awareness of social justice and equality values, which was conducted with seventh-grade students in a mathematics classroom (Turhan Turkkan, 2017). There exist studies that examine science teachers’ views on social justice (İdin & Aydoğdu, 2017) and social Specific to mathematics education; Turhan Turkkan and Karakuş (2018) examined the opinions of middle school mathematics teachers on the integration of mathematics courses and social issues studies teachers’ perceptions and experiences of social justice (Bursa & Ersoy, 2016).

First-time teachers find the opportunity to examine social justice issues and begin to discover their social justice beliefs in the period of pre-service education. For that reason, it is valuable to work with pre-service teachers in terms of teaching mathematics for social justice. The methodology for this research is design research. Designing an intervention related to teaching mathematics for social justice is significant because it contributes to the literature in terms of research and practice.

1.5. Definitions of Terms

Social Justice: Sharing power, knowledge, and resources equitably (Bogotch, 2000, p. 2).

Teaching for Social Justice: is enhancing learning opportunities for every student that involves implementing effective teaching methods, activities, and expectations tailored to individual needs. It also requires reforming educational structures and policies that restrict students' learning prospects. Moreover, it is addressing systemic injustices in education extends beyond the school setting, necessitating the transformation of societal structures that perpetuate inequality and injustice. (Chubbuck, 2010).

Teaching Mathematics for Social Justice: involves using mathematical concepts as a means to educate students about power dynamics, inequalities in resources, and disparities in opportunities among various social groups, aiming to inspire social and political activism. (Gutstein, 2006).
CHAPTER 2

LITERATURE REVIEW

2.1. Critical Pedagogy

Critical pedagogy is an educational philosophy that highlights the idea that there is social injustice in our world, in our communities, and in our institutions. The foundation of critical pedagogy is formed in the roots of critical theory which is developed by Max Horkheimer, Theodor Adorno, Herbert Marcuse, and Walter Benjamin belonging to the Frankfurt School (Kincheloe, 2008). As one of the critical pedagogues, Bourdieu (1998) states that one of the primary functions of schooling is to reproduce the current social order and to maintain unequal power relations in society. He states: “Often with a psychological brutality that nothing can attenuate, the school institution lays down its final judgements and its verdicts, from which there is no appeal, ranking all students in a unique hierarchy of all forms of excellence, nowadays dominated by a single discipline, mathematics.” (Bourdieu, 1998, p. 28). In other words, Bourdieu believes that school mathematics is crucial while reproducing the social order in society. Since mathematics is considered as one of the dominant disciplines, a curriculum that is more relevant and meaningful for students’ lives may be provided in schools in order to make students more successful in mathematics.

As another important critical pedagogue, Paulo Freire's (2000) work, "Pedagogy of the Oppressed" stands as a significant milestone in the advancement of critical pedagogy. In the book, Freire documented his research on literacy education conducted in rural Brazilian communities. His primary goal was to establish a theoretical framework that would enable students to engage critically with their own social contexts. Central to his approach was the concept of problem-posing education, wherein students' lived experiences served as the foundation for learning.
By encouraging the examination of real-world challenges, Freire aimed to empower students to become agents of change within their communities.

Freire (2000) believed that education is a political act because it includes social relations. All educational policies and practices have social implications, so it is impossible to be neutral in education. There are inequalities among people in capitalist societies, there are people who have the money and who do not have. Freire wasn’t comfortable with this situation and he believes that dominant ideas in education will continue to exist. Literacy was one of the important points he considered. He thinks that the idea of literacy does not only mean to read words. It is more than only reading words because every act of reading words implies a previous reading of the world and the following rereading of the world (Freire, 1985). Moreover, reading the word also leads to writing and rewriting the world which means resolving unjust situations. For Freire, taking a conscious practical action which is writing the world is essential to literacy.

2.2. Teaching for Social Justice

Teaching for social justice leads its participants to gain awareness, knowledge, and processes to analyze issues of justice and injustice in their own lives, communities, institutions, and the larger society (Bell, 1997). The focus of teaching for social justice should be on identifying oppression and addressing injustice (Garii & Appova, 2013). In this manner, students can be a part of and lead social change. All in all, teaching for social justice involves engaging students in problems relevant to their world, identifying barriers to their full humanity, and inspiring them to act to overcome those barriers (Gau, 2005).

Cochran-Smith (2004) identified six principles for teaching for social justice. These principles are:

1. Enable significant work within communities of learners: This principle means that teachers have high expectations from their students; in other words, they believe every student is capable of dealing with complex ideas.

2. Build on what students bring to school with them: This means that teachers build on the knowledge and interest of students.
3. Teach skills bridge gaps: Teachers make students use their prior knowledge and skills to learn new knowledge and skills.

4. Work with individuals, communities, and families: Teachers respect all students' family and cultural values and are connected with their students.

5. Diversify forms of assessment: Teachers do not use standardized tests to assess students' abilities and achievement; they use formative and cumulative assessment techniques to avoid inequalities created by standardized assessment.

6. Make inequity, power, and activism explicit parts of the curriculum: Teachers encourage students to think critically, make issues of inequities discussible in schools, and help students to explore how they can question the status quo.

Beyond addressing students' personal experiences, teaching for social justice focuses on developing curricula that foster an awareness of human oppression (Down & Smyth, 2012). It involves educating children about human rights, global poverty, and environmental issues. With teaching for social justice, students are not only equipped with academic knowledge at school but also become individuals who can make changes as active citizens in society. Reaching these outcomes might not be easy for teachers. To accomplish the aims of teaching for social justice, teachers need to implement an engaging curriculum and also a classroom environment where the values of trust and care are considered and students actively participate.

2.3. Critical Mathematics Education

There are various understandings and conceptualizations of social justice in mathematics education according to different scholars who work in the field. In some papers, it is interchangeably used with the word critical mathematics, whereas in some papers, it is interchangeably used with the word equity. In other words, the distinction is blurry, and these concepts are used alternatively. In this part, these different understandings will be examined.

In mathematics education, critical mathematics approach is pioneered by the works of Frankenstein (1983, 1990) and Skovsmose (1994, 2004). Frankenstein's works are
grounded in Freire's concept of critical pedagogy, particularly focusing on critical consciousness. Frankenstein argued that mathematics instruction should not remain neutral or divorced from societal issues but should instead serve to enhance students' critical awareness. According to Frankenstein (1983), Freire's theory urges mathematics educators to explore the non-positivist implications of mathematical knowledge, the role of quantitative reasoning in fostering critical consciousness, the perpetuation of hegemonic ideologies through mathematics anxiety, and the alignment between the curriculum and the cultivation of critical consciousness.

Critical mathematical literacy is an important concept that Frankenstein (1990) developed. It includes the ability to ask basic statistical questions in order to deepen one's appreciation of particular issues and the ability to present data to change people's perceptions of those issues. In this manner, individuals question how society is structured and they can act to change it. In her study with adult learners in which she integrated some issues like race, gender, and class inequality into mathematics teaching, Frankenstein (1990) found that adult students usually see these problems as their personal problems rather than problems woven into the institutional fabric of the society. It was found that adult students were resistant to this kind of mathematics curriculum. In that point, providing self-confidence and motivation seems crucial to change their perspective on mathematics learning.

Skovsmose (1994) explained that critical mathematics education has concerns related to equity and social justice, and it can contribute to the creation of critical citizenship and support democratic ideals. Similar to Frankenstein’s mathematical literacy, Skovsmose mentioned about mathemacy which has roots in the spirit of critique and it enables people to participate in the understanding and transformation of their society. Like Freire’s expansion of literacy to more than reading and writing, by mathemacy, Skovsmose mentioned the expansion of mathematical knowledge. In this way, after training students to think critically in mathematics, mathematics can become a tool for democracy, and mathemacy can be used for the purpose of empowerment.

Gutierrez is one of the figures who used equity as social justice. According to Gutierrez (2002), equity is realized when individuals in positions of authority are
unable to accurately forecast students' success and involvement in mathematics solely based on factors such as race, socioeconomic status, ethnicity, gender, cultural beliefs, language proficiency, and religious affiliations. According to Gutierrez (2009), in order to frame equity, there are two important concepts as “learning to play the game” and “learning to change the game”. “Learning to play the game” overlaps with the classical definition of equity which supports students to do well on standardized tests and have opportunities to engage in quality mathematics. On the other hand, “learning to change the game” overlaps with social justice in mathematics education, and it is supporting students to use mathematics to acknowledge hegemony in society and can address social and political issues of importance to their communities. In this study, the focus will be mainly on the second concept of Gutierrez which is “learning to change the game”.

The discussion on critical mathematics education is extended by Aslan Tutak et al. (2011), focusing on three domains that address social inequalities such as ethnomathematics, equity in mathematics education, and culturally responsive teaching. In their understanding of critical mathematics education, they stated that the concepts of multiculturalism and equality should be included in mathematics education as well as emphasizing democratic values and critical consciousness. They concluded that, in the field of mathematics education, there is a need for more research on critical mathematics education. Also, we need to provide mathematics teachers with curriculum suggestions on critical mathematics education.

2.3.1. Teaching Mathematics for Social Justice

Gutstein is an important figure in teaching mathematics for social justice and the one whose classification is utilized in this study. Gutstein adapted Freire's ideas regarding “reading the word (learning to read text)” to the concept of teaching and learning mathematics for social justice and called it as “reading the mathematical word” and defined it as the act of "developing mathematical literacy and mathematical power" (Gutstein, 2003, p. 45). In defining mathematical power, he used the following definition of NCTM (2000):

Students confidently engage in complex mathematical tasks...draw on knowledge from a wide variety of mathematical topics, sometimes
approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress...are flexible and resourceful problem solvers...work productively and reflectively...communicate their ideas and results effectively...value mathematics and engage actively in learning it (p. 3).

Figure 2.1. Dialectical Relationships between Concepts (Source: Gutstein, 2016)

Upon Freire’s ideas, Gutstein also defined “reading the world with mathematics” and “writing the world with mathematics”. According to Gutstein (2003, p.45) reading the world with mathematics is to use mathematics to understand the relations of power, resource inequities, and disparate opportunities, between different social groups and to understand explicit discrimination based on race, class, gender, language and other differences. Using real-world topics that include mathematics is the main source for students to be able to read the world with mathematics. On the other hand, understanding social reality does not liberate people; although it is both a precondition and effect of changing the world intentionally. In that sense, writing the world with mathematics is defined as using mathematics to change the world (Gutstein, 2006, p.27). The concepts of reading and writing the world with mathematics are linked to each other and do not follow a linear path. Moreover, in order to achieve reading the world with mathematics, reading the world and reading the mathematical word are two interdependent and required processes (Gutstein, 2016). The dialectical relationships between the concepts are demonstrated in Figure 2.1.
2.4. Applications of Teaching Mathematics for Social Justice in the Literature

Gutstein (2003) conducted practitioner research in an urban, Latino school for 2 years about teaching and learning mathematics for social justice. He collected through many different data sources as students’ standardized test scores, reflections, observations of the researcher, student work (student journals, unit tests, real-world mathematics projects, etc.). He has some larger goals for teaching for social justice and some mathematics-specific objectives (Table 2.1.). The curriculum he used has two main components: Mathematics in Context and a series of real-world projects. Mathematics in Context is a comprehensive curriculum which is developed by National Science Foundation, and it emphasizes real-world contexts are necessary for students to learn mathematics. The results of the study indicated that the writings and efforts on real-life projects show that almost all of the students started reading the world with mathematics and they become aware of the social justice issues in their lives. Also, results indicated that almost all of the students developed mathematical power in which they developed their own solutions to the problems, solved problems in different ways, and they developed their mathematical and personal confidence by communicating their conclusions both orally and written.

<table>
<thead>
<tr>
<th>Goals of Teaching for Social Justice</th>
<th>Specific Mathematics-Related Objectives</th>
</tr>
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<tbody>
<tr>
<td>Develop Sociopolitical Consciousness</td>
<td>Read the World Using Mathematics</td>
</tr>
<tr>
<td>Develop Sense of Agency</td>
<td>Develop Mathematical Power</td>
</tr>
<tr>
<td>Develop Positive Social/Cultural Identities</td>
<td>Change Dispositions Toward Mathematics</td>
</tr>
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</table>

(Source: Gutstein, 2003)

Wright (2016) conducted a critical research model of participatory action research in which there were three action research cycles. The aim of the research was to understand the experiences of teachers who committed to a program about teaching mathematics. Participants of the study are five newly qualified teachers. Data is collected through audio recordings of meetings with teachers, and a series of semi-structured empathetic interviews conducted with each teacher, and student surveys
designed by teachers. During the research, teachers are informed about the theoretical framework of the study, and then the researcher provided some resources about teaching mathematics for social justice, teachers planned and reflected on their lessons according to the main focus of the study. The findings of the study indicated that teachers question their beliefs about the nature of mathematics, and they concluded that students’ mathematical understanding and awareness of social justice issues are linked to each other. Moreover, in terms of students’, it is seen that making mathematics more meaningful and relevant increased students’ involvement in the classes. Also, such a collaborative group that supported each other increased teachers’ self-efficacy in terms of integrating social justice issues in their mathematics classroom.

Gau (2005) investigated teachers’ conversations around teaching mathematics for social justice as they participate in a graduate course that is designed like a lesson study. Eight secondary mathematics teachers attended the study and they develop, implemented, observe, revise, and re-teach mathematics lessons connecting social justice issues to the mathematics classes during a semester. Data collection tools are pre and post interviews, teachers’ interactions during the course, written lesson plans, and written reflections.

The study has three phases which are named teaching for social justice, teaching mathematics for social justice, and lesson study. Results of the study indicated that teachers conceptualize teaching mathematics for social justice, gaining awareness of social issues and empowering students to take action and change their world. In their conceptualization, they did not focus on the mathematical component of teaching mathematics for social justice. They emphasized, students first know the mathematical knowledge, then they can use this knowledge to understand social issues around them. In that sense, they had difficulty in terms of balancing between teaching mathematics and teaching for social justice. Teachers also have some concerns about how to integrate such fragile issues like racism into their classes.

In national literature, Turhan Turkkan (2017) conducted action research for 10 weeks to examine the effect of mathematics education integrated with socio-mathematical
subjects at secondary school seventh grade level on developing students’ problem-posing skills and their awareness of social justice and equality values. Participants of the study are 17 seventh-grade students who are attending to mathematics practice course in a public school. Social topics which are included in the mathematics practice course are equality for disabled people, gender equality, race and ethnicity equality, income distribution, unemployment and child labor, poverty, equality in terms of age and faith, and sharing of natural resources. On the other hand, mathematical topics which are integrated with social issues are integers, rational numbers, percentages, ratio and proportion, equations, problem posing, tables and graphs, central tendency and dispersion measures, and probability. Both quantitative and qualitative data are collected in the study. Quantitative data is collected through the Determination Form of the Awareness Levels of Social Justice and Equality Values and the Problem Posing Skill Test, these data collection tools are applied before and after the practice. Qualitative data is collected through semi-structured interviews, course video recordings, student self-evaluations of each course. Turhan Turkkan (2017) concluded that before the practice students’ awareness of social justice and equality values and problem-posing skills were low, and there was a significant difference between pretest and post-test scores of students in favor of post-test scores.

Qualitative findings supported quantitative findings and also evaluation of the instruction by the students indicated that teaching of mathematics combined with socio-mathematical issues was generally regarded favorably by the students.

When it is searched for what is done with students while teaching mathematics for social justice, in the literature, there are qualitative and mixed studies that focus on teaching mathematics for social justice in practice, which is similar to one of the aims of this study. The studies conducted by Brantlinger (2013), Gutstein (2016), Hagerman and Porath (2018), Kokka (2019), Ross and Shelton (2018), Rubel et al. (2016), Unfried and Canner (2018) and Turner et al., (2009) are at all levels including K-12 and higher education.

Turner et al. (2009) conducted a critical ethnographic study and examined 23 Latino sixth-grade students' participation in a mathematics club that focuses on teaching
mathematics in a critical way. One of the projects completed by the students in this study was about redesigning community parking spaces; the students work on this issue, and at the end, they shared their work with the city council. Then, they were encouraged to see that their work was taken into consideration by the city council. In the end, the students stated that they could benefit the community with mathematics.

Brantlinger (2013) conducted practitioner research to examine reformist critical mathematics education in a remedial high school classroom. Although the researcher spent more than 120 hours developing the activities to be used in the lessons, he had difficulty in finding activities for some geometry topics, and some of the activities he developed were not directly included in the high school mathematics curriculum. This is one of the major drawbacks he encountered during the study, although the researcher still thought that these examples with quantitative content would attract students' interest. He experienced active and passive resistance to some of the activities, especially in the first ones, and later, he saw that this resistance was expressed even by successful students who stated that what they did was not preparation for university mathematics. In this sense, the researcher encountered many difficulties during the implementation.

Gutstein (2016), employed a research that has characteristics of practitioner research, ethnography and critical, collaborative action research which was one of the good examples of integrating social justice into mathematics education. The participants were the 12th grade students, and the aim of the study was to examine complexities in actualizing RWWM and examine the student learning through the process. The results indicated that students’ mathematical knowledge and also sociopolitical knowledge developed after integrating social justice into mathematics classes.

Rubel et al. (2016) employed field testing in a low-income neighborhood to explore the integration of social justice into mathematics teaching. They found an improvement in student engagement and success. Ross and Shelton (2018) have similar results in their qualitative study, in which they integrate social justice into an introduction to statistics course. Also, Hagerman and Porath (2018) conducted a case study with 8th-grade students to examine how teaching for, with, and about social
justice developed in a learner-centered environment. The results indicated that teaching for social justice is attained by giving choices to students and developing positive relations with the teachers, and teaching for social justice is attained while students take active roles in creating their environment. Kokka (2019) employed a case study to examine how social justice mathematics provides opportunities to engage in healing practices with middle school students. Also, this study had positive results in three dimensions as sociopolitical consciousness, mathematics learning, and well-being. Another study that employs social justice in mathematics education was a qualitative study conducted by Unfried and Canner (2018). They identified an increase in the empowerment of their undergraduate students in terms of realizing their talents and the role of mathematics in society.

While these studies are examined in terms of their similarities and differences, some of the studies focused on higher-order thinking skills (Brantlinger, 2013; Gutstein, 2016; Ross & Shelton, 2018; Unfried & Canner, 2018; Turner et al., 2009), whereas some of them focused on lower thinking skills (Hagerman & Porath, 2018; Kokka, 2019; Rubel et al., 2016); In terms of mathematical topics, the researches include topics such as statistics, data analysis, measurement, proportional reasoning, probability, algebraic reasoning, number sense, and discrete mathematics. In terms of social justice topics, the studies include topics such as access (food, roads, health), racism and racial profiling, inequality, immigration, elections, criminalization, sexism, and local issues. It is seen that there are good practices in the literature that focus on both higher-order thinking skills and lower-order thinking skills, and it can be concluded that if mathematics is linked to students’ local lives and problems, it would be more meaningful for students. While advocates of TMSJ consider it as being for all students, most of the interventions in the literature take place with Latino or black students (Brantlinger, 2013; Gutstein, 2016; Kokka, 2019; Rubel et al., 2016; Turner et al., 2009), TMSJ studies can be done in different contexts with privileged students, and it would be more meaningful if it is linked to the students’ local lives and problems. Furthermore, after analyzing studies, it can be concluded that student resistance should be taken into account. For these reasons, to have good practices in real classrooms, pre-service teachers should have knowledge and practice about integrating social justice issues into their classes.
2.5. Social Constructivism

Constructivism is a theory about knowledge and learning; it explains what “knowing” is and how one “knows.” According to constructivism, knowledge is described not as truths to be discovered or transmitted but as emergent, developmental, nonobjective, viable constructed explanations by humans engaged in meaning-making in cultural and social communities of discourse (Fosnot, 2013). On the other hand, learning is not the passive transformation of information; it is constructed so that learners build new knowledge upon the foundations of previous learning (Bada & Olusegun, 2015). Constructivism as a learning theory is based on the theories of Vygotsky and Piaget. Although all constructivists embrace the learner's active engagement in knowledge construction, cognitive constructivism, radical constructivism, and social constructivism are three separate theoretical viewpoints of constructivism on a continuum.

Social constructivism is based on the ideas of Lev Vygotsky, who claims social interaction is an integral part of learning (Kalina & Powell, 2009). Vygotsky (1978) emphasized the impact of the environment on learning by considering knowledge as a social and cultural entity. Social context is crucial in the learning process because individuals are active agents of society and cannot be separated from it. Zone of Proximal Development is one of the important concepts that Vygotsky defines as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86).

2.6. Summary of Literature Review

The concept of teaching mathematics for social justice has its roots in the work of Freire who is a prominent figure in critical pedagogy. Critical pedagogy is an educational philosophy that underscores the presence of social injustice in the world. Freire (2000) emphasized that education is a political act and it is not possible to be neutral in education. He established two crucial concepts for literacy as reading the
Gutstein's ideas for teaching mathematics for social justice based on Freire’s ideas were utilized in this study. Gutstein (2006) refers two important concepts as reading the world with mathematics and writing the world with mathematics. Gutstein argued that through mathematics, students can increase their awareness of social justice issues, mobilize for social change, and develop positive social and cultural identities.

Gutstein (2003) conducted practitioner research with middle school students in teaching mathematics for social justice, and results indicated that students became more aware of social justice issues, and they also developed mathematical power by developing their own solutions. Turhan Turkkan (2017) integrated socio-mathematical subjects into mathematics education for middle school students, and it is seen that students were more interested in mathematics classes. These studies indicated the positive effect of teaching mathematics for social justice and encouraged to conduct more studies in terms of teaching mathematics for social justice. Since teachers are the ones who will implement teaching mathematics for social justice lessons, it is very essential that teachers are well informed in order to be able to see these positive effects in mathematics education. In studies conducted with teachers, Wright (2016) concluded that by making teachers familiar with TMSJ, mathematics was more relevant and meaningful for their students, and this increased student involvement. Moreover, Gau (2005) concluded that teachers developed their ideas on TMSJ, gain awareness on social justice issues and empower their students to change the world, however, teachers had concerns about including fragile issues into their classes, and had difficulty in terms of balancing between teaching mathematics and teaching for social justice. In light of these background information and related studies, the development of an intervention for pre-service teachers, which is designed towards their needs and with their active participation based on social constructivism, can both improve pre-service mathematics teachers' understanding of TMSJ and have positive effects for their future students.
In this chapter, detailed information about the method of the study is provided. The study’s design, context, participants, data collection procedures, and tools are presented. Then, data analysis, trustworthiness, researcher role, and ethical considerations are described.

3.1. Research Design

Design research was conducted in this study to examine and develop pre-service elementary mathematics teachers' understanding of teaching mathematics for social justice. Design research in education serves various purposes, such as designing and examining interventions, learning activities, or programs for professional development (Bakker, 2019). Educational design research can be defined as a genre of research in which the iterative development of solutions to practical and complex educational problems also provides the context for empirical investigation, which yields theoretical understanding that can inform the work of others (McKenney & Reeves, 2012, p. 83). The impulse behind the emergence of design research is to bridge the gap between educational research and its application in policy and practice (van den Akker et al., 2006). This approach seeks to formulate research-driven solutions for intricate issues within educational practice. Drawing on previous studies and a thorough examination of existing literature, design researchers design and refine workable and efficient interventions by iteratively studying various iterations (prototypes) of these interventions in their intended contexts. Through this process, they engage in reflective analysis of their research methodologies, ultimately aiming to derive designing principles (Plomp, 2010). Another motivation for design research is about developing empirically grounded theories (i.e., local instruction theories) through a study of the forms of learning and means of
supporting and organizing the process of learning (Gravemeijer & Cobb, 2006). With this motivation, it aims to develop educational products and a theoretical understanding of how these products can be used in education.

Educational design research is defined as the systematic study of designing, developing, and evaluating educational interventions, such as programs, teaching-learning strategies, and materials, products, and systems as solutions to such problems, which also aims at advancing our knowledge about the characteristics of these interventions and the processes to design and develop them (Plomp, 2010, p. 9). Another study defines it as the study of learning in context through the systematic design and study of instructional strategies and tools (Design-Based Research Collective, 2003, p. 5). It is aimed to overcome some problems of educational research by utilizing design research (Bakker, 2019). Firstly, in design research, the development of new educational approaches is based on the knowledge base available in the research. Secondly, practitioners could benefit from design research products because they are not carried out in isolated situations.

Design research is considered an appropriate approach for this study with its suitable characteristics with the purpose and the nature of the study. As Plomp (2013) emphasized, design research aims to design, develop, and evaluate educational interventions such as programs, learning processes and environments, teaching, and learning materials. Since the focus of this study is to design an intervention for developing pre-service mathematics teachers’ understanding of teaching mathematics for social justice, the aims of the design research and this study coincide. In addition, this study also examines the learning environment grounded in social constructivism on pre-service mathematics teachers' understanding of teaching mathematics for social justice.

Design research is called by a variety of concepts due to its diverse history in different countries including educational design research (to distinguish it from other disciplines like architecture) (McKenney & Reeves, 2012; Plomp & Nieveen, 2013); design experiment (Brown, 1992); design-based research (Design-Based Research Collective, 2003); developmental research (to develop and improve curriculum in Netherlands) (Gravemeijer, 1994); and formative experiment (Reinking & Watkins,
Although different names are used in the literature, design research authors have some common ideas regarding design research characteristics: interventionist, iterative, process-oriented, utility-oriented, and theory-oriented (Van den Akker et al., 2006, p.5). These characteristics can be explained as follows:

- **Interventionist**: the research aims to design an intervention in a real-world setting.
- **Iterative**: the research incorporates cycles of analysis, design and development, evaluation, and revision.
- **Process-oriented**: the focus is on understanding and improving interventions (a black-box model of input-output measurement is avoided).
- **Utility-oriented**: the merit of a design is measured, in part, by its practicality for users in real contexts.
- **Theory-oriented**: the design is (at least partly) based on a conceptual framework and upon theoretical propositions, whilst the systematic evaluation of consecutive prototypes of the intervention contributes to theory building.

Design research with the aforementioned characteristics is considered an appropriate research method to accomplish the aim of this study. Initially, this study is interventionist due to the fact that it aims to develop pre-service mathematics teachers’ understanding of teaching mathematics for social justice with the designed intervention. Secondly, in design research, researchers typically start with a prototype of a design, conduct experiments to test the design, analyze the outcomes, and refine the design based on the findings for subsequent iterations. This study is iterative because there are two macrocycles. Thirdly, it is process-oriented since it aims to examine the teaching and learning process designed and implemented based on needs analysis and related literature in teaching mathematics for social justice. Moreover, the practicality of the designed intervention and utilizing needs analysis and theoretical knowledge while developing the intervention are indicators consistent with the design research's characteristics. Lastly, the theory generated from this dissertation is developing design principles for developing pre-service teachers’ understanding of teaching mathematics for social justice by utilizing social constructivism.
Several researchers have proposed alternative classifications for the phases of design research projects. McKenney and Reeves (2012) name the three core phases of a design research study: analysis and exploration, design and construction, evaluation and reflection. Then, Gravemeijer and Cobb (2013) list the three phases of a design research study: preparing for the experiment, experimenting in the classroom, and conducting retrospective analyses. This study will utilize the classification of Gravemeijer and Cobb (2013) since it aligns with designing an intervention regarding developing pre-service mathematics teachers’ understanding of teaching mathematics for social justice.

3.1.1. Phases of Design Research

Preparing for the experiment: In this phase, what the researcher wants to design (task, sequences, learning environments) and what type of knowledge the researcher wants to generate is decided (Bakker, 2019). In design research, the researcher starts by clarifying instructional endpoints and starting points. In other words, firstly, the researcher identifies learning goals and then analyses the consequences of earlier instructions. Broadly, before implementing planned tasks in the classroom, the researcher predicts improvements in students’ thinking and understanding. In addition, the researcher makes conjectures about how to develop students’ thinking and understanding.

Experimenting in the classroom: In this phase, a design experiment is carried out in the classroom. The goal of the design experiment is to test and refine the conjectured local instructional theory, as well as to understand how it works rather than to see whether it works (Gravemeijer & Cobb, 2013). This is an iterative process that consists of testing, improving, and understanding what is happening.

Conducting retrospective analysis: The last phase consists of retrospective analyses of the entire data set collected during the experiment. In this phase, a complete set of data is analyzed in an iterative process (Gravemeijer & Cobb, 2013).

Design-based research is utilized in this study due to its appropriateness for the aim of the study. In that sense, the general aim of this research is to add to the knowledge
base about how pre-service mathematics teachers’ understanding can be supported in teaching mathematics for social justice. With this general aim, the main purpose of the study is to examine and develop pre-service mathematics teachers’ understanding of teaching mathematics for social justice. Based on this main purpose, it was aimed to explore the experiences of pre-service mathematics teachers in teaching mathematics for social justice by collecting data from pre-service mathematics teachers and teacher educators. Furthermore, it was aimed to examine the development of concepts related to teaching mathematics for social justice. The design of the study is presented in Figure 3.1.

**Figure 3.1. Phases of the Study**

Concerning the purposes stated above, the following research questions are tried to be answered during the study.

Research Questions for Needs Analysis:

1. What are the current experiences of pre-service mathematics teachers regarding teaching mathematics for social justice?
2. What are the experiences of pre-service mathematics teachers regarding teaching mathematics for social justice from the perspectives of teacher educators?

Research Questions for Design Research:

“How can a teaching and learning strategy grounded in social constructivism promote pre-service mathematics teachers’ understanding of teaching mathematics for social justice?”
1. How does pre-service mathematics teachers’ understanding of teaching mathematics for social justice develop through the designed intervention grounded in social constructivism in terms of reading the world with mathematics & writing the world with mathematics?

2. What are the elements of a designed intervention to develop the understanding of pre-service mathematics teachers regarding teaching mathematics for social justice?

Figure 3.2. demonstrates the steps of the study according to the design research phases, which are classified by Gravemeijer and Cobb (2013).

3.1.2. Procedures of the Study

As aforementioned, there are three phases of design research: preparing for the experiment, experimenting in the classroom, and conducting retrospective analysis (Gravemeijer & Cobb, 2006, p. 19). In this section, information about the processes carried out at these stages will be given.

3.1.2.1. Phase 1- Preparing for the Experiment

In this study, the initial attempt was to identify the goals of the intervention (Gravemeijer & Cobb, 2013), which would be designed to teach mathematics for social justice for pre-service elementary mathematics teachers. I decided on the core goals of the domain by reviewing the related literature on teaching mathematics for social justice and the middle school mathematics curriculum.

In order to understand the current situation and reveal the need for teaching mathematics for social justice in the mathematics teacher education program, I conducted interviews with pre-service mathematics teachers and teacher educators. These interviews included questions about the purpose of mathematics, learning and teaching mathematics for social justice, and the current situation in mathematics teacher education programs.
The middle school mathematics curriculum was examined in order to have a coherent approach to the intervention since it is thought that the outcomes of this intervention would be reflected in middle school students through pre-service elementary mathematics teachers. Eight key competences are stated in the curriculum. These competences are students' skills at the national and international levels ranging from personal, social, academic, and business lives which are determined according to Türkiye Qualifications Framework. The competences are: communication in the mother tongue, communication in foreign languages,
mathematical competence and core competences in science and technology, digital competence, learning to learn, social and civic competences, initiative and entrepreneurship, and cultural awareness and expression (MoNE, 2018). Teaching mathematics for social justice mainly covers 2 of these competences: mathematical competence and core competences in science and technology, and social and civic competences. Mathematical competence is identified as the development and application of mathematical thinking to solve a range of problems encountered in daily life. Social and civic competences are related with individuals to participating fully in civil life based on knowledge of social and political concepts and structures and a commitment to democratic and active participation. It is seen that these competences are in line with TMSJ in terms of its goals for teaching mathematics and social justice.

In the framework of these competences, some special goals of the mathematics curriculum are also considered in the development of the program related to teaching mathematics for social justice (MoNE, 2018). These are:

Students,
- will be able to develop and use mathematical literacy skills effectively.
- will be able to understand and use mathematical concepts in daily life.
- will be able to easily express their own thoughts and reasoning in the problem-solving process, and will be able to see the deficiencies or gaps in the mathematical reasoning of others.
- will be able to use mathematical terminology and language correctly to explain and share mathematical thoughts in a logical way.
- will be able to make sense of the relations between people and objects and the relations of objects with each other by using the meaning and language of mathematics.
- will be able to develop the skills of doing research, producing and using information.

In the middle school mathematics curriculum, more specifically, some principles are mentioned, which are considered in the implementation of the program (MoNE,
2018). It is stated that mathematics should be considered a part of life, and every
opportunity should be used for the development of mathematical thinking. For
instance, issues such as bread waste, recycling, healthy and planned life, tax
awareness, social security rights, and obligations encountered in daily life should be
emphasized, and examples should be given on these subjects. Also, it emphasizes
associating values such as equality, justice, and sharing with appropriate objectives.

Some root values are determined as justice, friendship, honesty, self-control,
patience, respect, love, responsibility, patriotism, and helpfulness in the curriculum.
Therefore, by mentioning daily life problems, problem-solving, and issues related to
justice, the middle school mathematics curriculum emphasizes the concept of
teaching mathematics for social justice at some points. In that sense, it can be
concluded that TMSJ is in line with competences and principles in the middle school
mathematics curriculum, so an intervention in this topic will present examples for
pre-service elementary mathematics teachers and raise their awareness to implement
the middle school mathematics curriculum in a more effective way.

Considering the middle school mathematics curriculum’s aims, principles, and
values, reviewing related literature on teaching mathematics for social justice, and
results of the needs analysis study with the pre-service teachers and teacher
educators, the core ideas of teaching mathematics for social justice intervention are
determined as teaching for social justice, teaching mathematics for social justice,
reading the world with mathematics, and writing the world with mathematics.
Initially, I had not planned to include the topic of teaching for social justice in this
intervention. I had planned to focus only on mathematics teaching throughout the
intervention. According to the results of the needs analysis study, I identified that
some pre-service teachers did not have much knowledge about teaching for social
justice, and those who had heard about that concept did not have a deep
understanding. In order to implement social justice-oriented mathematics teaching in
the classroom, a teacher needs to be able to create an environment in the classroom
that is generally committed to the principles of social justice. Therefore, I decided to
include this topic at the beginning of this intervention. In terms of teaching
mathematics for social justice, I utilized Gutstein's (2006) concepts as reading the
world with mathematics and writing the world with mathematics. According to these core ideas, the goals of the program are determined as follows:

- improve pre-service teachers’ understanding regarding teaching for social justice
- develop pre-service teachers’ understanding of teaching mathematics for social justice
- develop pre-service teachers’ understanding regarding reading the world with mathematics
- develop pre-service teachers’ understanding regarding writing the world with mathematics
- include social justice perspective in pre-service teachers’ mathematics teaching.

As a theory of teaching and learning in the teaching mathematics for social justice intervention, social constructivism is determined because of its suitable characteristics for teaching for social justice (Adams et al., 2016) and the results of the needs analysis study with pre-service mathematics teachers. This study will utilize social constructivism while preparing lesson plans for teaching mathematics for social justice intervention.

The design and implementation of mathematics teaching for social justice intervention are influenced explicitly by social constructivism through its emphasis on collaborative learning and the construction of knowledge through social interactions. In the intervention, pre-service mathematics teachers engage in group discussions, class discussions, problem-solving activities, and reflective exercises to actively construct their understanding of how to teach mathematics for social justice. For example, pre-service mathematics teachers work together during the intervention to analyze real-world mathematical problems related to social justice issues such as unemployment. Then, they collaboratively design instructional strategies that promote critical thinking and empower students to use mathematics as a tool for addressing social justice issues. Integrating social constructivism into the instructional strategies ensures that pre-service mathematics teachers not only gain theoretical knowledge but also develop practical skills for creating inclusive and socially conscious mathematics classrooms.
Along with identifying core concepts and utilizing social constructivism as a learning approach in this intervention, as stated, it is considered essential for pre-service mathematics teachers to develop their practical skills in terms of developing TMSJ lesson plans. Jeram and Davids (2020) stated that teachers should be provided with the skills necessary for the conversion of theoretical ideas into practice. With this idea, it was aimed to make pre-service teachers connect their theoretical knowledge with practice in this study. Therefore, individual lesson plan preparation and presenting those lesson plans are also included as a part of the TMSJ intervention.

Content and Instructional Sequence: The content and instructional sequence of the TMSJ intervention, which was decided according to the needs analysis study and literature review, is presented in this section (Table 3.1).

The Content of the TMSJ Intervention:

1. Teaching for Social Justice
2. Teaching Mathematics for Social Justice
   a. Reading the World with Mathematics
   b. Writing the World with Mathematics

<table>
<thead>
<tr>
<th>Table 3.1. Instructional Sequence</th>
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<tbody>
<tr>
<td>Lesson (for 2 class hours)</td>
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<tr>
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<td>2</td>
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<tr>
<td>6</td>
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<tr>
<td>0</td>
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</tbody>
</table>

After deciding on the instructional sequence of the TMSJ intervention, the researcher decided on the materials that would be utilized and developed lesson plans for the intervention. Moreover, how the related resources on the subject could be utilized
consistently for social constructivism in the course content is determined. The advisor of the researcher gave feedback on these materials and gave feedback on developed lesson plans. In addition, another person with a Ph.D. from the education faculty also gave feedback on the materials and lesson plans. Based on the determined instructional sequence, and the received feedbacks the default outline of the planned intervention is determined as follows (Table 3.2.).

The needs analysis study conducted with pre-service mathematics teachers and teacher educators determined this intervention's content and teaching and learning approach. Moreover, the initial interviews with pre-service teachers were also significant in providing their understanding of the subject and making necessary revisions on the content. Therefore, in the initial interviews with pre-service mathematics teachers, their understanding of teaching mathematics and teaching mathematics for social justice was asked.

| Lesson 0 | Initial interviews are conducted with PMTs to identify their understanding of teaching mathematics and TMSJ. |
| Lesson 1 | A warm-up activity is implemented to meet with pre-service teachers. |
|          | Pre-service teachers share their expectations about the TMSJ intervention. |
|          | The schedule of the TMSJ intervention is given. |
|          | PMTs share how they define social justice by using an educational technology tool. |
|          | PMTs criticize the definition of social justice in different resources. |
|          | PMTs work in groups on the activity about teaching for social justice. The activity is about the stories of 3 beginning teachers and how they enact social justice teaching in their classes. The cases are from Agarwal et al. (2010). |
|          | PMTs discuss what can be done to solve social justice challenges in their groups in the sample stories and what might be essential points of teaching for social justice. |
|          | PMTs discuss how they could define teaching for social justice in class. |
|          | After introducing the definition of teaching for social justice, PMTs discuss what teachers could do to maintain social justice in class. |
Lesson 2  PMTs answer a quiz on an educational technology tool that included questions on social justice and social injustices worldwide.

Two mathematics problems with social justice context that can be solved with the same mathematical operation are shared with PMTs, and they criticize the importance of context.

PMTs discuss: “Is teaching mathematics neutral?” in the class, and then the idea of critical pedagogy is shared with them.

PMTs are given another sample mathematics question with a social justice focus to solve and they discuss the opportunities it provided for the students.

PMTs criticized the definition of TMSJ through different resources.

A poster describing the purpose of TMSJ is shared with PMTs and they interpret the poster.

PMTs read a text “Why Social Justice in Mathematics Education?” from Berry III et al. (2020) as part of the Think-Pair-Share Activity. Firstly, they individually identify important aspect, they share their ideas with their peers, and they share their ideas in the class.

PMTs write a reflective paper on what is the role of a teacher beyond teaching the subject matter.

Lesson 3  PMTs solve a mathematics puzzle as a warm-up activity.

PMTs share what they remember from the previous class and changes in their thoughts.

PMTs discuss benefits of TMSJ.

A sample TMSJ lesson plan which is developed by researcher on unemployment was implemented in the class.

A sample TMSJ lesson plan from Berry et al. (2020) about “Sampling Disaster” is shared with PMTs and they identify social justice goals and mathematical objectives of the lesson. PMTs categorize two dimensions of social justice goals: reading the world with mathematics and writing the world with mathematics.

A class discussion is conducted on RWWM and WWWM.

PMTs draw a diagram of the relationship between RWWM and WWWM, and compare their diagram with Gutstein (2003)’s diagram.

Lesson 4  PMTs solve a mathematical puzzle as a warm up activity.

PMTs discuss the components of TMSJ lesson plans, and what is crucial for them while developing TMSJ lesson plans.

PMTs write a social justice topic that they can integrate to their classes.

A class discussion is conducted on one of the social justice topics they
Lesson 4  write regarding how they can develop a TMSJ lesson plan. A lesson plan draft is developed by all the PMTs. PMTs write a reflective paper on challenging and most powerful learning moments of the lesson and their ideas on RWWM and WWWM. At the end of the class, information about what is expected from PMTs while developing their individual TMSJ lesson plan is given

Lesson 5  PMTs work in groups to find a slogan for RWWM or WWWM. Groups share their slogans and they evaluate other groups slogans. PMTs share their TMSJ lesson plan ideas, and receive feedback from their friends and the researcher.

Lesson 6  PMTs presented their individual lesson plans on TMSJ, give feedback to their friends and receive feedback on lesson plans.

Lesson 0  Final interviews are conducted with the PMTs to identify their understanding of TMSJ and views on the intervention.

Lesson 1  started with a warm-up activity because the intervention was grounded in social constructivism and, therefore, required high interaction among pre-service mathematics teachers. Warm-up activity named as the favorite things is planned to be done, which would allow pre-service teachers to share with their friends and to keep in mind their friends' favorite things. In this activity, they form a circle, and one of them is chosen as the starter and shares her/his name and her/his favorite color first, and the next one on the circle repeats what the previous person said and adds his/her name and favorite color. The activity continues with their favorite movies, food, and books. Then, after pre-service mathematics teachers share their expectations of the intervention, they are given a schedule. PMTs are asked what words come to their minds when social justice is mentioned, and a word cloud is created from their answers using an educational technology tool. After getting answers, the meaning of social justice is discussed using different definitions. Since the researcher conjectured that pre-service mathematics teachers would develop a deeper understanding through the activities and tasks by working in groups rather than individually, they are given a group activity on teaching for social justice. Moreover, the researcher also conjectured that pre-service mathematics teachers
were required to examine sample cases of teachers from the literature to develop their understanding of teaching for social justice. In this activity, 3 cases focus on three beginning teachers’ experiences of teaching for social justice (Agarwal et al., 2020). PMTs work in groups, and in their groups, they discuss some questions related to the experiences of those beginning teachers, and they try to figure out the markers of teaching for social justice. At the end of the activity, the class forms a definition of teaching for social justice and compares it with its definitions from the literature.

Lesson 2 started with an interactive quiz through an educational technology tool, and this quiz is conducted to make a fun lesson start and reinforce the pre-service mathematics teachers' understanding of social justice. This quiz includes questions that will develop their thoughts on social justice and examples of social injustices worldwide. In addition, the fact that this quiz is done by using a technology tool causes a fun competition among the students and increases interaction. The researcher conjectured that examining mathematics problems in different social justice contexts will make PMTs question the importance of the social justice context in mathematics problems. Therefore, TMSJ is introduced with two mathematics problems to the PMTs. Through these sample problems, a class discussion on whether teaching mathematics is objective or subjective is planned. Then, PMTs discuss the opportunities of TMSJ after another sample mathematics question was introduced to them. After PMTs develop their understanding by the examples, they criticize different definitions of TMSJ from the literature. A think-pair-share activity to make pre-service teachers discuss and generate ideas together is planned to be implemented during the intervention. After the lessons in which they question teaching for social justice and TMSJ, PMTs are asked to write a reflective paper on the role of a teacher beyond teaching subject matter.

Lesson 3 began with a warm-up activity consisting of a mathematics puzzle. After solving the puzzle, the pre-service mathematics teachers share what they remember from previous lessons and what is interesting to them, and they also share and discuss as a class what they think about opportunities for TMSJ. The researcher conjectured that there is a need for a TMSJ lesson plan implementation for the pre-
service mathematics teachers as they are students in order to develop their understanding of how to teach mathematics with a social justice focus. Therefore, a TMSJ lesson plan on unemployment was implemented to the pre-service teachers as an example of lesson flow and as an example of how to conduct the social justice discussions in a class. Then, it was planned to share a sample TMSJ lesson plan with PMTs. The initial plan was to share another lesson plan with the PMTs as an example, but based on the circumstances of the earthquake disaster in Türkiye in February 2023, a lesson plan on Sampling Disaster (Berry et al., 2020) is planned to be shared to guide them how they can integrate such a disaster issue in their mathematics classes. In this example, pre-service teachers are asked whether they could make a categorization of social justice goals as RWWM and WWWM. Finally, the pre-service mathematics teachers are asked to show how they defined the relationship between these concepts on a diagram, because it was conjectured that they would be able to identify the relationship and develop conceptual understanding through these concepts by drawing a diagram that shows their relationship.

*Lesson 4*, as a starting activity, involves PMTs solving a mathematics puzzle. Afterward, pre-service mathematics teachers discuss what components should be included in a TMSJ lesson plan. Then, through an interactive presentation, the PMTs discuss what situations should be given importance and what problems could be encountered while preparing and implementing TMSJ lesson plans. After these discussions, an agreement on the components of the TMSJ lesson plans was expected. The researcher conjectured that a TMSJ lesson plan developed jointly with the pre-service mathematics teachers through classroom discussion would contribute to developing their skills in developing individual TMSJ lesson plans. Therefore, each pre-service mathematics teacher writes a social justice topic, and a TMSJ lesson plan draft is developed with the pre-service mathematics teachers on a topic selected from among the written social justice topics with a joint decision. At the end of the lesson, the pre-service mathematics teachers wrote a reflective paper to reflect on their learning process and ideas on RWWM and WWWM. Lastly, information about what is expected from their individual TMSJ lesson plans and a rubric to evaluate their lesson plans are shared with PMTs (Appendix A).
Lesson 5 is planned that pre-service mathematics teachers will prepare a slogan for RWWM or WWWM in group work, and this would deepen their understanding of the concepts. Also, group members are expected to acquire new perspectives on RWWM and WWWM by evaluating the slogans of the other groups. Then, PMTs share their TMSJ lesson plan ideas, give feedback to their friends, and receive feedback from their friends and the researcher. During Lesson 6, PMTs present their TMSJ lesson plans, provide feedback to their peers using the lesson plan presentation rubric (Appendix B), and receive feedback on their lesson plans from their friends and the researcher. Final interviews are carried out with the PMTs to reveal their understanding of TMSJ and their views on the intervention.

3.1.2.2. Phase 2 - Experimenting in the Classroom

This is the phase of conducting the design experiment to test, refine, and observe how the implementation works in the classroom (Gravemeijer & Cobb, 2013). This study is conducted in two macrocycles (1st experiment and 2nd experiment). The 1st design experiment took place face-to-face at the Faculty of Education Building of the university during the semester break in February 2023. The 2nd design experiment occurred face-to-face at the Faculty of Education in March and April 2023.

The 1st Experiment was conducted in February 2023 with five pre-service mathematics teachers. The researcher was the instructor during this intervention. She was responsible for conducting initial and final interviews, taking video recordings in the class, and conducting lessons according to paying attention to the critical points in TMSJ. After each lesson, the researcher wrote field notes on events and tasks that she thought had a role in developing concepts related to TMSJ and the events and tasks that do not support learning about TMSJ. During the 1st experiment, lessons were conducted based on the planned outline. Since the medium of instruction at the university where the intervention was implemented was English, all materials used in the classroom were prepared in English. The lessons in the intervention were grounded in social constructivism, so the pre-service mathematics teachers had to express their opinions and discuss them extensively during the experiment. The PMTs stated that they preferred these discussions to be held in Turkish. Interaction is very important in this study, and PMTs pointed out that language could be a barrier
to their discussions; therefore, the discussions were held in Turkish. Some revisions were made in the planned outline before the 2nd experiment, according to data gathered by initial and final interviews, video recordings of class discussions, pre-service mathematics teachers’ reflective papers, and in-class activities.

In the first lesson, after the pre-service mathematics teachers filled in the voluntary participation form, a warm up activity was implemented, and information was received about their expectations from the intervention. Then, information was given about the content. Afterward, discussions were held on the concepts of justice and social justice, and an activity was done to find the definition of the concept of teaching for social justice. In the second lesson, two mathematics problems with the same answer that was asked in different contexts was shared with the students, and they discussed whether mathematics teaching is neutral or not. Later, information about TMSJ was given on another example mathematics problem. After starting with a warm-up activity and asking what they remembered from the previous lessons, in the third lesson, pre-service mathematics teachers did a think-pair-share activity on why social justice is important in mathematics education. Then, a sample lesson on TMSJ was applied to the pre-service mathematics teachers. Moreover, they were further asked to define the concepts of RWWM and WWWM on a different sample lesson plan. In the fourth lesson, they were informed about what to pay attention to in a TMSJ lesson plan, and they prepared a TMSJ lesson plan draft as a whole group. Then, they wrote a reflective paper on the subject. In the fifth lesson, they wrote a slogan about RWWM and WWWM as group work. PMTs started to prepare their draft TMSJ lesson plans and received feedback from the researcher and other pre-service mathematics teachers. In the last lesson, they practiced their lesson plans in the classroom and received feedback from their friends and the researcher.

Revisions After the 1st Experiment: After watching videos of the lessons, reviewing pre-service mathematics teachers’ in-class activities, reflection papers, and initial and final interviews, some revisions were made to the intervention. Time constraint for preparing individual TMSJ lesson plans was the biggest problem observed during the 1st experiment. The whole intervention was intensively completed in one week since the students did not have classes during the semester break in February 2023. After
the pre-service mathematics teachers engaged in the topic, they needed more time to prepare lesson plans and think about them deeply. Therefore, the 2nd experiment was planned for a longer interval, with one week between classes. Apart from that, the first activity was about TSJ. It was planned to discuss three cases with three groups, but the time was enough to discuss only one case as a group. Therefore, in the 2nd experiment, the planning included only one case. Pre-service mathematics teachers shared their desire to see more sample lesson plans to be more engaged in the subject; therefore, in the 2nd experiment, more sample lesson plans, including those of pre-service mathematics teachers of the 1st experiment, were given to the students as resources. In the TMSJ lesson plan preparation part as a group, since there were few people in the 1st experiment, a lesson plan was prepared as a whole group, but this process was not very productive. This lesson plan preparation activity was planned as a group activity in the 2nd experiment since it was observed that pre-service teachers were active in other group activities. Moreover, a TMSJ lesson plan template is developed by the researcher and it is given to PMTs as it provided a structural order to the lessons. As a small change in the experiment, there were mathematics puzzle questions at the beginning of classes to make students warm up before the lesson. Since it took longer time to solve these puzzles, more simple questions were chosen, and they were planned as peer activities during the 2nd experiment.

The 2nd experiment was conducted with nine pre-service mathematics teachers during March and April 2023 after making revisions in the 1st experiment. Due to the diversity and intensity of the courses pre-service mathematics teachers took, it was planned to be on Saturdays for six lessons (i.e. 12 class hours). After conducting initial interviews with pre-service mathematics teachers, 2nd experiment started. As in the 1st experiment, in this experiment, the pre-service mathematics teachers thought that language could be a barrier to their discussions, so although all materials were in English, the discussions were conducted in Turkish. A revised outline of the intervention is provided in Table 3.3. Revisions are written in italics.

Through the overview provided in Table 3.3., the 2nd experiment was conducted. Since the TMSJ intervention was planned to be delivered through social
constructivism, it was crucial to create an environment where pre-service mathematics teachers could freely share their ideas and engage with each other. Therefore, the beginning of lessons included warm-up activities that would lead PMTs to share their ideas. For instance, at the first lesson, we started with a warm-up activity in which PMTs shared their favorite color, movies, food, and books, and everyone in the class tried to learn these characteristics of each other and memorize them. This activity was essential to ensure PMTs got to know each other and shared their ideas freely in class. Moreover, activities carried out in the classroom were usually based on examples. PMTs tried to identify the common features and group these features in the examples. The researcher was the instructor at the same time, so she supported discussions with her questions. Thus, PMTs developed their own understanding through group and class discussions.

Table 3. Revised Outline of the TMSJ Intervention

<table>
<thead>
<tr>
<th>Lesson 0</th>
<th>Initial interviews are conducted with PMTs to identify their understanding of teaching mathematics and TMSJ.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 1</td>
<td>A warm-up activity is implemented to meet with pre-service teachers. In this activity, they share their favorite color, movies, food, books, etc., and remember what their friends shared. Pre-service teachers share their expectations about the TMSJ intervention. The schedule of the TMSJ intervention is given. PMTs share how they define social justice by using an educational technology tool. PMTs criticize the definition of social justice in different resources. <em>PMTs work in groups on the activity about Teaching for Social Justice. The activity is about the story of one beginning teacher and how she enacted social justice teaching in her classes. The case is from Agarwal et al. (2010).</em> PMTs discuss what can be done to solve social justice challenges in their groups in the sample stories and what might be essential points of Teaching for Social Justice. PMTs discuss how they could define Teaching for Social Justice in class. After introducing the definition of Teaching for Social Justice, PMTs discuss what teachers could do to maintain social justice in class.</td>
</tr>
<tr>
<td>Lesson 2</td>
<td>PMTs answer a quiz on an educational technology tool that included questions on social justice and social injustices worldwide. Two mathematics problems with social justice context that can be solved with the same mathematical operation are shared with PMTs, and they criticize the importance of context. PMTs discuss: “Is teaching mathematics neutral?” in the class, and then the idea of critical pedagogy is shared with them. PMTs are given another sample mathematics question with a social justice focus to solve and discuss the opportunities it provided for the students. PMTs criticized the definition of TMSJ through different resources. A poster describing the purpose of TMSJ is shared with PMTs and they interpret the poster. PMTs read a text “Why Social Justice in Mathematics Education?” from Berry III et al. (2020) as part of the Think-Pair-Share Activity. Firstly, they individually identify important aspect, they share their ideas with their peers, and they share their ideas in the class. PMTs write a reflective paper on what is the role of a teacher beyond teaching the subject matter.</td>
</tr>
<tr>
<td>Lesson 3</td>
<td><em>PMTs work in pairs to solve a mathematics puzzle as a warm-up activity.</em> PMTs share what they remember from the previous class and changes in their thoughts. PMTs discuss benefits of TMSJ. A sample TMSJ lesson plan which is developed by researcher on unemployment was implemented in the class. A sample TMSJ lesson plan from Berry et al. (2020) about “Sampling Disaster” is shared with PMTs and they identify social justice goals and mathematical objectives of the lesson. PMTs categorize two dimensions of social justice goals: reading the world with mathematics and writing the world with mathematics. A class discussion is conducted on RWWM and WWWM. PMTs draw a diagram of the relationship between RWWM and WWWM, and compare their diagram with Gutstein (2003)’s diagram. <em>Sample lesson plans were shared with PMTs.</em></td>
</tr>
<tr>
<td>Lesson 4</td>
<td><em>PMTs work in pairs to solve a mathematical puzzle as a warm up activity.</em> PMTs discuss the components of TMSJ lesson plans, and what is crucial for them while developing TMSJ lesson plans.</td>
</tr>
</tbody>
</table>
Lesson 4  PMTs write a social justice topic that they can integrate to their classes.

*TMSJ lesson plan template is shared with PMTs.*

*PMTs work on groups on TMSJ lesson plans.*

PMTs write a reflective paper on challenging and most powerful learning moments of the lesson and their ideas on RWWM and WWWM.

At the end of the class, information about what is expected from PMTs while developing their individual TMSJ lesson plan is given.

Lesson 5  PMTs work in groups to find a slogan for RWWM or WWWM.

Groups share their slogans and they evaluate other groups slogans.

PMTs share their TMSJ lesson plan ideas, and receive feedback from their friends and the researcher.

Lesson 6  PMTs presented their individual lesson plans on TMSJ, give feedback to their friends and receive feedback on lesson plans.

Lesson 0  Final interviews are conducted with the PMTs to identify their understanding of TMSJ and views on the intervention.

### 3.1.2.3. Phase 3 - Conducting Retrospective Analysis

The data obtained from different data collection tools such as initial and final interviews with pre-service mathematics teachers, reflective papers, field notes, video recordings of the class, observations, in-class activities, individual lesson plans were analyzed by thematic analysis to examine the development of pre-service teachers’ understanding of TMSJ. More detailed information about this phase is explained in detail in the data analysis section.

### 3.2. Context of the Study

The context of the study is a four-year elementary mathematics teacher education program in a Faculty of Education in an English-medium state university in Central Anatolia, Türkiye. Graduates of this elementary mathematics teacher education program can teach middle school mathematics to grades 5 to 8. In the program's first year, teacher candidates are responsible for the courses of mathematics, introduction
to education and some education courses, and also an introduction course from the computer education and instructional technology department, English and Turkish language, and Principles of Atatürk and History of Turkish Revolution courses. In the program's second year, education, mathematics, and courses on teaching mathematics are compulsory, and there are some elective courses. In the third and fourth years, mathematics, teaching mathematics, practice in teaching, education, and elective courses are offered to the teacher candidates. There is no emphasis on teaching mathematics for social justice in teaching mathematics courses in the undergraduate teacher education program. A detailed table with the course names and credits for each semester appears in Appendix C.

The designed intervention in teaching mathematics for social justice is composed of 6 lessons, each consisting of 2 class periods. Participants were given a participation certificate after completing the intervention voluntarily. The main focus of the intervention was the critical approach to mathematics education. Teaching and learning mathematics were discussed in terms of social, political, and cultural perspectives. Students were expected to participate in discussions actively. Students were encouraged to speak their ideas freely, and they were supposed to respect the opinions of others. Since there were discussions in the class, pre-service teachers were expected to value everyone’s ideas and not consider discussions personally.

The intervention included lessons to train pre-service mathematics teachers about teaching mathematics for social justice. At the end of the intervention, they were expected to prepare a mathematics lesson plan with a social justice focus. During the intervention, they discussed concepts such as teaching for social justice, and teaching mathematics for social justice with sub-concepts such as reading the world with mathematics and writing the world with mathematics in groups or in class. They encountered sample lesson plans that included a social justice focus on teaching mathematics. Pre-service mathematics teachers first prepared lesson plans as a group, then prepared lesson plans individually, and they presented the individual lesson plans during the intervention. This intervention was implemented with pre-service mathematics teachers in two macrocycles: 1st experiment and 2nd experiment.
3.3. Participants

Data were collected during Phase 1 and Phase 2 of the study. In Phase 1, data were collected from pre-service elementary mathematics teachers and teacher educators as needs analysis in teaching mathematics for social justice. In phase 2, data were collected during 1st experiment and 2nd experiment in the classroom. An overview of the participants of the study is given in Table 3.4.

<table>
<thead>
<tr>
<th></th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td></td>
</tr>
<tr>
<td>Participants of Needs Analysis Interviews with Pre-Service Teachers</td>
<td>7</td>
</tr>
<tr>
<td>Participants of Needs Analysis Interviews with Teacher Educators</td>
<td>4</td>
</tr>
<tr>
<td>Phase 2</td>
<td></td>
</tr>
<tr>
<td>Participants of 1st Experiment</td>
<td>5</td>
</tr>
<tr>
<td>Participants of 2nd Experiment</td>
<td>9</td>
</tr>
</tbody>
</table>

3.3.1. Participants of Needs Analysis

Needs Analysis study was conducted with the pre-service mathematics teachers and teacher educators to reveal pre-service teachers’ familiarity or unfamiliarity with the concepts teaching for social justice and teaching mathematics for social justice.

3.3.1.1. Participants of Needs Analysis Interviews with Pre-Service Teachers

This study utilizes purposeful sampling to reveal pre-service mathematics teachers’ experiences regarding teaching mathematics for social justice. Purposeful sampling is intentionally selecting individuals and sites to learn or understand the central phenomenon (Creswell, 2015). The logic of purposeful sampling is selecting cases that are information-rich (Patton, 2015). By choosing these information-rich cases, researchers can learn a great deal to answer their research questions.

Participants are selected among junior and senior pre-service teachers who are enrolled in the elementary mathematics education department to understand the
phenomenon deeply. Therefore, among the types of purposeful sampling, criterion sampling, in which all cases that meet some predetermined criteria are studied, is utilized in this study (Patton, 2015).

The criteria were being a junior or senior student at the elementary mathematics education department in the state university in Central Anatolia, Türkiye. The reason for determining these criteria was that these pre-service elementary mathematics teachers were familiar with the departmental subjects, such as knowledge of basic concepts and principles of teaching and learning, planning, implementing, and evaluating instruction.

In line with this purpose, seven senior pre-service elementary mathematics teachers are interviewed to identify their experiences and needs in terms of teaching mathematics for social justice. The descriptive information of the participants is presented in Table 3.5, including the year in the program, gender, and type of high school they graduated. In terms of gender, there was only one male participant in the needs analysis study. Actually, this gender distribution is consistent with the gender distribution of pre-service mathematics teachers enrolled in 3rd and 4th grade in the university. There are a total of 89 junior and senior students enrolled in the department of elementary mathematics education, and only 20 of them were male.

<table>
<thead>
<tr>
<th>Participant Number</th>
<th>Year in the Program</th>
<th>Gender</th>
<th>Type of High School Graduated</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST1</td>
<td>4</td>
<td>Female</td>
<td>Medical Vocational High School</td>
</tr>
<tr>
<td>ST2</td>
<td>3</td>
<td>Female</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>ST3</td>
<td>4</td>
<td>Female</td>
<td>General High School</td>
</tr>
<tr>
<td>ST4</td>
<td>4</td>
<td>Female</td>
<td>General High School</td>
</tr>
<tr>
<td>ST5</td>
<td>4</td>
<td>Female</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>ST6</td>
<td>4</td>
<td>Male</td>
<td>General High School</td>
</tr>
<tr>
<td>ST7</td>
<td>4</td>
<td>Female</td>
<td>Science High School</td>
</tr>
</tbody>
</table>

3.3.1.2. Participants of Needs Analysis Interviews with Teacher Educators

There are five teacher educators at the Department of Elementary Mathematics Education of the state university where the study is conducted. Interviews were
conducted with four of them to understand pre-service teachers’ experiences with teaching mathematics for social justice. The descriptive information of the participants is presented in Table 3.6, including their gender, years of experience as a teacher educator, and academic title.

Table 3.6. Descriptive Information of Participants of Needs Analysis with Teacher Educators

<table>
<thead>
<tr>
<th>Participant Number</th>
<th>Years of Experience as a Teacher Educator</th>
<th>Gender</th>
<th>Academic Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE1</td>
<td>6</td>
<td>Female</td>
<td>Assist Prof.</td>
</tr>
<tr>
<td>TE2</td>
<td>10</td>
<td>Female</td>
<td>Prof. Dr.</td>
</tr>
<tr>
<td>TE3</td>
<td>6</td>
<td>Female</td>
<td>Assist. Prof.</td>
</tr>
<tr>
<td>TE4</td>
<td>22</td>
<td>Male</td>
<td>Prof. Dr.</td>
</tr>
</tbody>
</table>

3.3.2. Participants of 1st Experiment

Participants of the 1st experiment were teacher candidates who willingly attended the intervention in teaching mathematics for social justice. 1st experiment was completed with five pre-service mathematics teachers whose descriptive information are given in Table 3.7.

Table 3.7. Descriptive Information of Participants of 1st Experiment Experiment

<table>
<thead>
<tr>
<th>Participant Number</th>
<th>Year in the Program</th>
<th>Gender</th>
<th>Type of High School Graduated</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>3</td>
<td>Female</td>
<td>Science High School</td>
</tr>
<tr>
<td>P2</td>
<td>3</td>
<td>Male</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>P3</td>
<td>3</td>
<td>Female</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>P4</td>
<td>3</td>
<td>Male</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>P5</td>
<td>3</td>
<td>Female</td>
<td>Anatolian High School</td>
</tr>
</tbody>
</table>

3.3.3. Participants of 2nd Experiment (Main Study)

The 2nd experiment was conducted with a total of nine pre-service mathematics teachers who willingly agreed to participate during March and April 2023. Information about these nine participants is given in Table 3.8.
### Table 3. 8. Descriptive Information of Participants of 2nd Experiment

<table>
<thead>
<tr>
<th>Participant Number</th>
<th>Year in the Program</th>
<th>Gender</th>
<th>Type of High School Graduated</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE1_Can</td>
<td>3</td>
<td>Male</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>DE2_Ceren</td>
<td>3</td>
<td>Female</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>DE3_Defne</td>
<td>3</td>
<td>Female</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>DE4_Kerem</td>
<td>4</td>
<td>Male</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>DE5_Leyla</td>
<td>3</td>
<td>Female</td>
<td>Imam Hatip High School</td>
</tr>
<tr>
<td>DE6_Mine</td>
<td>3</td>
<td>Female</td>
<td>Anatolian High School</td>
</tr>
<tr>
<td>DE7_Mira</td>
<td>4</td>
<td>Female</td>
<td>Science High School</td>
</tr>
<tr>
<td>DE8_Selin</td>
<td>4</td>
<td>Female</td>
<td>General High School</td>
</tr>
<tr>
<td>DE9_Umut</td>
<td>3</td>
<td>Male</td>
<td>Anatolian High School</td>
</tr>
</tbody>
</table>

### 3.4. Data Collection Instruments

Design research utilizes different data collection tools in order to evaluate the outcomes of the design and refine the design process (Design-Based Research Collective, 2003). In this study, data collection tools through the 1st experiment and the 2nd experiment are needs analysis interviews for pre-service teachers and teacher educators, initial and final interviews with pre-service teachers, field notes of the researcher, external classroom observations, reflective papers, in-class activities and individual lesson plans of pre-service teachers, and video recordings of classes. In this section, these tools are stated in detail.

#### 3.4.1. Semi-Structured Interview Protocols

In order to conduct interviews with pre-service teachers and teacher educators, I developed 4 semi-structured interview forms (2 interview forms for the needs analysis and 2 interview forms for the design experiment). Specific information is required from all participants in the semi-structured interviews in which exact wording or the order of the questions is determined during the interview (Merriam & Tisdell, 2016). After forming the interview schedules, I shared them with three experts to ensure the content and face validity of the interview protocol, the clarity of the wording, and their suitability for the pre-service elementary mathematics teachers and teacher educators. After making necessary changes according to the experts' reviews, the final version of the interview protocols is obtained.
3.4.1.1. Needs Analysis Interview Protocol for Pre-Service Teachers

Semi-structured interviews were conducted with pre-service elementary mathematics teachers to reveal their experiences in teaching mathematics for social justice in depth. For the needs analysis with pre-service teachers, I wanted to explore their PMTs' views on teaching and learning mathematics and their experiences of the term social justice during their undergraduate studies. Also, I wanted to understand their views on teaching for social justice and teaching mathematics for social justice to explore their basic conceptions of the concept and its use in the teacher education field. In addition, I wanted to see their views on taking part in an intervention about teaching mathematics for social justice are also inquired. After obtaining the final version of the interview protocol, a pilot interview was conducted with a pre-service elementary mathematics teacher, and there was no need to change the interview questions. The final version of the interview protocol involved two main sections: demographic information and needs analysis questions on teaching mathematics for social justice for pre-service teachers (Appendix D).

3.4.1.2. Needs Analysis Interview Protocol for Teacher Educators

In developing needs analysis interview questions with teacher educators, I followed the same procedure as the needs analysis interview with pre-service teachers. I developed the needs analysis interview questions for teacher educators, and the final version involved two sections: demographic information and needs analysis questions on teaching mathematics for social justice for teacher educators (Appendix E).

3.4.1.3. Initial and Final Interviews Protocol for Pre-Service Teachers

Individual interviews were conducted with pre-service mathematics teachers before and after attending 1st experiment and 2nd experiment. After developing initial and final interview questions, they were revised based on the opinions of experts. The initial and final interview questions focused on how pre-service mathematics teachers’ understanding evolved while attending the teaching mathematics for social
justice intervention. Initial interview questions mainly focus on expectations from the intervention and their previous knowledge regarding TMSJ (Appendix F). Final interview questions mainly focus on their understanding regarding TMSJ and their reflections on the intervention (Appendix G).

### 3.4.2. Field Notes

Field notes are records of what the researcher hears, sees, experiences, and thinks while engaged in the process of gathering and reflecting data in a qualitative study (Bogdan & Biklen, 2007). The researcher wrote reflective field notes after each class with a focus on identifying pre-service teachers’ perceptions and experiences regarding TMSJ. The researcher wanted to ensure that the data gathered from the interviews was connected to actual classroom events and tones. The researcher also wanted to better understand the participants' experiences by knowing the environment in which they formed.

### 3.4.3. Classroom Observations Protocol

Observations play a pivotal role in qualitative research, encompassing a range of activities. These activities involve hanging around in the setting, getting to know people and understanding their routines, and documenting actions and interactions, and having a checklist to identify actions (Marshall & Rossman, 2016). In this study, I designed the observation protocol based on the following dimensions: a) verbal descriptions of the setting, the people, and the activities; b) direct quotations of what participants said; and c) observer’s comments (Merriam, 1998).

One of the main purposes of conducting classroom observations was to contextualize the data. To accomplish this aim, in the 2nd experiment, two external observers attended the lesson at different times. One was a Ph.D. student, and the other was a Ph.D. candidate in Curriculum and Instruction Department. They took notes according to the identified dimensions, reviewed their notes after the lessons, and shared them with the researcher.
3.4.4. Reflective Papers

Pre-service mathematics teachers wrote two reflective papers throughout the study that included their ideas parallel to what was covered in both experiments' lessons. These reflective papers had a significant value in understanding how their construction process of concepts related to TMSJ developed. Pre-service mathematics teachers wrote their first writing after discussing TSJ and TMSJ. They wrote their second reflection paper after having detailed knowledge about TMSJ and being familiar with the sample lesson plans on TMSJ to explain what the concepts of RWWM and WWWM meant to them and the challenges and most powerful learning moments in that lesson.

3.4.5. Video and Audio Recordings of Classes

Both of the experiments in the classroom were recorded, and group discussions in the class were audio-recorded to identify each pre-service teacher’s ideas. All of the video and audio recordings of the lessons were transcribed by the researcher, and during transcription, the researcher took notes about the events, tasks, and environment that developed pre-service mathematics teachers’ knowledge and practices on TMSJ.

3.4.6. In-Class Activities

In-class activity sheets of the pre-service teachers during both experiments were collected. Pre-service teachers encountered the TMSJ concept in an environment that supported their participation throughout both experiments. In this process, they had to complete various activity sheets individually and as groups during the lessons; some sample activity worksheets can be found in Appendix H. These activity sheets also were used as data in this study.

3.4.7. Individual Lesson Plans

Participants of the experiments prepared an individual lesson plan at the end of the intervention as a final project, and they presented their lesson plans to the class.
Therefore, it was aimed at how they can integrate the concepts we covered in the intervention into their teaching. I developed a lesson plan evaluation rubric to evaluate their TMSJ lesson plans in terms of appropriateness of the mathematics content, social justice content, balance between them, etc (Appendix B).

3.5. Data Collection Procedures

In this study, I first collected data from pre-service teachers and teacher educators for the needs analysis study. Then, I collected data through the implementation of the intervention. Two permissions from the Middle East Technical University Human Subjects Ethics Committee were taken throughout the study. Firstly, the necessary permission for conducting needs analysis interviews with pre-service teachers and teacher educators was taken from the Middle East Technical University Human Subjects Ethics Committee (0418-ODTUİAEK-2022, see appendix I). Afterwards, permission to implement the intervention was taken (0040-ODTUİAEK-2023, see appendix J).

After receiving approval from the ethics committee at the beginning of August 2022, I sent invitation e-mails to the junior and senior pre-service elementary mathematics teachers for the needs analysis interviews. Interviews were conducted with the pre-service teacher who responded this e-mail. The interviews with pre-service mathematics teachers were scheduled with the participants face-to-face or online in August and fall of 2022. The needs analysis interviews lasted approximately 30 minutes with pre-service teachers, and I took memos during the interviews. Participants signed a consent form before the interview, and with their permission, the interviews were recorded. Simultaneously, the data for the needs analysis with teacher educators were collected. I sent invitation e-mails to the teacher educators to schedule interviews, and they were conducted according to their availability. The interviews with teacher educators were conducted during the fall term of 2022 and lasted approximately 30-40 minutes, and I took memos during the interviews. Three interviews with teacher educators were conducted face-to-face, and only one was conducted online. After teacher educators signed a consent form, the interviews were recorded with permission. Participants in the needs analysis interviews were informed about the confidentiality of their records and their voluntary participation.
I collected data through different instruments such as initial and final interviews, field notes, classroom observations, reflective papers, video and audio recordings of the classes, in-class activities, and individual lesson plans during the 1\textsuperscript{st} experiment and 2\textsuperscript{nd} experiment. Teaching mathematics for social justice intervention was announced to the university's junior and senior pre-service elementary mathematics teachers through e-mail. This e-mail gives brief information about the intervention. Students were informed that the 1\textsuperscript{st} experiment would be during the semester break (February 2023). Some students answered the e-mail by clarifying their desire to attend the intervention, but they will be in their hometowns during the semester break. Those students' names were noted so they would be called again to participate in the 2\textsuperscript{nd} experiment. Therefore, pre-service elementary mathematics teacher participated in the experiments of their own will. For the 2\textsuperscript{nd} experiment, a group of pre-service mathematics teachers who expressed interest in participating in the 1\textsuperscript{st} experiment but were unable to do so due to scheduling conflicts were contacted through email. The pre-service teachers who still wanted to participate and whose schedule fitted participated in the 2\textsuperscript{nd} experiment. Five pre-service mathematics teachers participated in the 1\textsuperscript{st} experiment for one week during February 2023, and nine pre-service mathematics teachers participated in the 2\textsuperscript{nd} experiment for four weeks during March and April 2023.

Teaching mathematics for social justice intervention was face-to-face, but all initial and final interviews with pre-service teachers were conducted online for both experiments. One week before each experiment, pre-service teachers were invited to initial interviews via e-mail, and one week after each experiment, pre-service teachers were invited to final interviews via e-mail. All initial interviews were transcribed before the experiment, and the final interviews were transcribed after each experiment ended. Initial interviews lasted approximately 30 minutes, and final interviews lasted approximately 60 minutes. I took memos during the initial and final interviews. Throughout both interventions, all lessons were video-recorded, group works were audio-recorded, and all students' work was collected. In addition, as the researcher, I wrote field notes after each lesson, and a Ph.D. student and a Ph.D. candidate observed different lessons and shared their observation notes with the researcher.
The participants of the TMSJ intervention were informed that they would attend the study voluntarily and could leave whenever they wanted. Informed consent forms containing detailed information about the process were read and signed by participants of TMSJ intervention before it started (Appendix K). Voluntary participants participated in the study; they were informed that they could leave the study at any time. Confidentiality was provided by using pseudonyms. Besides, no one except the researcher had permission to access the data that could reveal participants’ identities.

3.6. Data Analysis

The researcher analyzed the various types of data from needs analysis interviews with pre-service teachers and teacher educators; and the design experiments (initial and final interviews, in-class activities, reflection papers, field notes, class observations, and video recordings of classes. In this part, firstly, I explained how I analyzed the needs analysis interviews with pre-service teachers and teacher educators. Then, I explained how the retrospective analysis is conducted with the entire data set in the design experiment.

3.6.1. Needs Analysis of Pre-Service Teachers Regarding Teaching Mathematics for Social Justice

Thematic analysis was conducted through MAXQDA to analyze the needs and experiences of pre-service elementary mathematics teachers in terms of teaching mathematics for social justice. Data were gathered from pre-service elementary mathematics teachers and teacher educators at the Department of Elementary Mathematics Education in the state university in Central Anatolia, Türkiye. While doing thematic analysis, the researcher utilized steps that Braun and Clarke (2006) identified as familiarizing with data, generating initial codes, searching for themes, reviewing themes, defining and naming themes, and producing the report. Firstly, interviews were transcribed verbatim, and then all of them were read to get an overall view of the information provided by the participants. After forming initial codes, sub-themes, and themes, the researcher shared the coding process and sub-
themes and themes with the supervisor. Some themes, sub-themes, and codes were restructured after a meeting with the supervisor. A second meeting was conducted with the supervisor on the restructured codes, sub-themes, and themes. Some revisions were also made at the second meeting, and themes of the needs analysis were finalized with a joint decision with the supervisor. The themes that emerged from needs analysis interviews with pre-service teachers were reasons for choosing the department, purpose of learning mathematics, social justice as a concept in education, the conception of teaching for social justice, and teaching mathematics for social justice. The themes of the needs analysis interviews with teacher educators were teaching, social justice as a concept in education, and teaching mathematics for social justice.

3.6.2. Analysis of Design Experiment

The analysis of the design experiment consisted of processes similar to those used in needs analysis interviews with pre-service mathematics teachers and teacher educators. Furthermore, a comparison of data was made in the design experiment analysis. First of all, initial and final interviews, the videos of the lectures, and the audio of group discussions in the class were transcribed, and pre-service teachers’ TMSJ lesson plans were reviewed. It was aimed to examine the development of pre-service teachers' understanding related to TMSJ. Later, the researcher coded the data, created sub-themes from these codes, and then themes from the sub-themes. In addition, a constant comparative analysis was conducted to compare the pre-service teachers' understanding before and after the experiment. Comparison of different or the same data sets is a constant comparative method (Merriam, 2015). When comparing the data sets, the researcher mainly focused on pre-service mathematics teachers’ understanding of TMSJ. A comparison was made with the data from initial and final interviews and the reflection papers of PMT’s. The researcher collaborated with the supervisor to review and discuss the coding process and identified sub-themes and themes. Subsequent to this discussion, certain themes were reorganized and finalized through a collaborative decision-making process.

While examining how PMT’s understanding of TMSJ developed through the experiment, Mira was selected as a case that showed a considerable change in her
understanding of TMSJ. In selecting Mira’s case, all transcripts of the classroom and group works, in-class works, reflection papers, and field notes were examined. The change in Mira's understanding of TMSJ and perspective on teaching, as well as her active participation in class discussions and group work, were the reasons for the selection. For example, in the initial interview, Mira identified that she had not encountered the concept of TMSJ. Therefore, her ideas were based on predictions. In the initial interview, her ideas ranged from equal opportunity in mathematics education to caring for individual differences or achieving social justice through teachers’ attitudes. However, in the final interview, her ideas were more focused, and she defined TMSJ as hitting two birds with one stone, teaching mathematics learning outcomes and social justice consciousness at the same time. All the data about Mira were analyzed, and the incidents that caused the change in her thoughts were identified. In this way, how her understanding of TMSJ developed was examined.

3.7. Trustworthiness

The procedures employed to assure validity and reliability are related to the trustworthiness of a research project (Patton, 2002). Lincoln and Guba (1985) mention four criteria qualitative researchers should consider in pursuit of trustworthiness: credibility, transferability, dependability, and confirmability. The researcher followed the following procedures based on these criteria to ensure the trustworthiness of the study.

Firstly, credibility is related to the internal validity of the study and, according to Merriam (1998), deals with answering the question: “How congruent are findings with the reality?”. The researcher was actively involved in all the processes of the design experiment: designing, experimenting, analyzing, and making revisions. She was both the instructor of the TMSJ intervention and a researcher at the same time, so she had a prolonged engagement with all the processes of the research and took field notes after the lessons. Hence, the researcher deeply tried to understand the context. Also, data triangulation was used to accomplish the internal validity of the research. Data were obtained from many different data sources (i.e., interviews, field notes of the researcher, classroom observations, reflection papers, lesson plans,
records of lessons, and audios of group works) and analyzed by cross-verifying with each other. For instance, reflection papers were used to triangulate the findings of final interviews with the pre-service mathematics teachers. Besides, observers’ notes were used to triangulate the data gathered from video recordings of the classroom and the researcher's field notes.

Secondly, transferability is concerned with the extent to which the findings of the study can be applied to other situations (Merriam, 1998). In this study, thick descriptions were an essential guide for researchers or implementers. The researcher provided information about the participants and participation procedures of the study, how and when the experiment took place, and in what ways data were collected.

Thirdly, dependability relates to reliability and deals with the consistency and replicability of the findings (Lincoln & Guba, 1985). This criterion was achieved primarily through data triangulation. Moreover, an audit trail of the processes of the design experiment that includes records of the entire research processes ensured that the research steps could be followed. In addition, data analysis was carried out as a process with the supervisor and she provided effective feedback, and the final analysis was carried out in line with the joint decisions taken after doing several meetings with the supervisor until agreement was reached.

Lastly, confirmability is about how “how much are the findings shaped by the participants and conditions and not due to researcher bias?” (Lincoln & Guba, 1985). In this study, confirmability is assured through using data collection triangulation. The researcher kept reflective field notes after the lessons in which she analyzed the lessons and PMT’s understanding of TMSJ. In addition, the researcher and the supervisor co-worked during the research process and analysis of the data.

3.8. The Role of the Researcher

In the study, I was active both in the designing process and in the experimenting of the designed intervention because, in design research, the teacher can also be the
researcher (Bakker, 2019). After deciding the content of the intervention, I designed tasks and activities and revised them. I supported the creation of a discussion environment and followed the discussions. In addition, after each class, I took field notes in which I monitored pre-service mathematics teachers’ understanding and the effect of the tasks and activities on their learning regarding TMSJ. In that sense, I was also a data source for this study and took an active role throughout the process.

After graduating from the Department of Secondary Mathematics Education, I worked as a mathematics teacher of the 5th graders in a private school in İstanbul. This experience provided me with many insights into teaching and learning mathematics. As a mathematics teacher, what I observed was that, unfortunately, most of my students had negative attitudes toward mathematics and that these attitudes were developed at an early age. In fact, one of the most significant reasons for these negative attitudes was that students did not connect the mathematics they were learning to their lives. We were teaching mathematics by doing many activities and giving importance to the active participation of the students in the school where I worked. Still, it seemed that this was not very effective. I thought that maybe this was related to our method of teaching mathematics, that the mathematics lessons in which the students actively participated and were taught through activities might not have made enough connections with their lives.

After a year of study, I completed my master's degree in Curriculum and Instruction, where I conducted a study on pre-service teachers' thinking skills and reflective thinking tendencies. Since pre-service teacher education is a field that I particularly care about and love to study, I thought that I wanted to work with pre-service teachers during the process of deciding of the topic of my dissertation because I think that pre-service teachers can adapt to innovative ideas and teaching methods very quickly and are prone to self-improvement. As a result of these thoughts, I wanted to work with pre-service mathematics teachers because teaching mathematics and using different techniques in mathematics teaching is a field that I liked very much. Based on my own teaching experience, social justice-oriented mathematics teaching, which aims to make students both love mathematics and develop their awareness of social justice in addition to their mathematics learning outcomes, was very compatible with
my goals of teaching and teaching mathematics. I think we can both make students love mathematics lessons and contribute to their growth as individuals who question social justice issues in society at the same time. After deciding on the topic, I thought that it would be useful to design an intervention for pre-service teachers in this topic because I think that if I reach pre-service mathematics teachers, I can somehow reach their students in the future and reach a large number of students as an effect of this intervention.

Before planning the intervention, I wanted to see pre-service mathematics teachers' and teacher educators' ideas and experiences about this topic, and I did a needs analysis study. Then, I designed an intervention. Since this intervention was not part of a course and my thesis monitoring committee members stated that students might participate but may not continue, I had some concerns about finding participants and making them attend until the end of the intervention. When I invited pre-service mathematics teachers for this intervention, I indicated that I would give them incentives. I realized these concerns were unfounded after contacting pre-service mathematics teachers. The topic attracted the interest of the pre-service mathematics teachers. I was a little relieved when they shared that they liked coming to the lessons because it was an environment where they could share their ideas comfortably, and I guided them with questions to construct their understanding. During the interventions, I did not give them knowledge but aimed to develop their understanding, sometimes just through discussions and sometimes through discussions on examples related to the topic.

Conducting this research as design research was the part of this study that improved me the most and challenged me the most. I had to carefully decide on each step while designing the intervention and collecting the data in the right way while implementing it. I collected data from many different sources during the interventions and had to make data triangulation while analyzing them. This process developed me both as a researcher and as a teacher educator. As a researcher, I designed an intervention in which I played an active role in all processes and implemented this intervention; after doing revisions to the 1st intervention, I developed the 2nd intervention and analyzed the results. As an emerging teacher
educator, I implemented this intervention, which consisted of 12 lesson hours with pre-service mathematics teachers twice, and I had the chance to observe the process and the results.

3.9. Limitations of the Study

The most critical limitation of this study was the February 2023 earthquakes that took place just before the 1st experiment. The earthquake caused both the date to change and the lessons in the university conducted online afterward. During this process, there were problems with planning of the experiments. The 1st experiment was planned to start on 13 February 2023 during the semester break and last for one week. Unfortunately, before we started the 1st experiment, the February 2023 earthquakes happened, and since she was affected by the earthquakes, one of the pre-service teachers could not participate. Since other participants were not directly affected by the earthquakes, the experiment was postponed by one week, started on February 20, and was conducted face-to-face. It was postponed because the whole country was actually affected psychologically after the earthquakes. In the following spring semester, due to the earthquakes, the lessons started to be held online, but all of the participants of the 2nd experiment were in Ankara during this period. Since the intervention was designed with a learning and teaching approach grounded in social constructivism, in which interaction is crucial in the classroom, it was essential to conduct the lessons face-to-face. Since the participants were in the city, the 2nd experiment was conducted face-to-face on Saturdays while pre-service mathematics teachers' other classes continued online. It was on Saturdays because no two hours were available on weekdays that fit into each participant’s course schedule.
CHAPTER 4

RESULTS

There are two main sections in this chapter. The results of the needs analysis regarding teaching mathematics for social justice from the perspective of pre-service teachers and teacher educators and the results of the design experiment are presented in separate sections.

4.1. Results of Needs Analysis

This section contains the findings of the needs analysis study with pre-service teachers and teacher educators. Findings are separately presented in relation to the research questions of the study.

4.1.1. Needs Analysis Regarding Teaching Mathematics for Social Justice from the Perspectives of Pre-Service Teachers

![Figure 4.1. Needs Analysis with PMTs](image-url)
The needs analysis interviews with the pre-service teachers constituted the findings related to the 1st research question of the study. Drawing on the interviews with PMTs, they had different reasons for choosing the mathematics teacher education department. Moreover, their views about the purpose of learning mathematics, social justice as a concept in education, teaching for social justice, and teaching mathematics for social justice are explained in Figure 4.1.

4.1.1.1. Reasons for Choosing Mathematics Teacher Education Department

The results revealed that pre-service mathematics teachers have various sources of motivation for choosing the elementary mathematics education department. The findings scrutinized that the reasons for choosing the department vary from self-directed, family-directed, university exam ranking, and having role model teachers.

Self-directed: Directed from the needs analysis, I could capture “the self” as a defining concept of PMTs’ decision to become a mathematics teacher. The love of mathematics was strongly emphasized by most of the participants. ST4, for instance, mentioned the love of mathematics in relation to the love of children and nurturing the mathematics learning of children. Additionally, being able to teach something to their friends made them feel happy. They indicated the pleasures of satisfying the learning needs of their friends and how this impacted their future aspirations. Moreover, some of them highlighted that they see the teaching profession as a sacred profession, so they choose their careers based on this idea.

Family-directed: Some pre-service mathematics teachers had chosen teaching as a profession because of the guidance of their families. Having someone with a teaching career in their family made them more predisposed to this profession. ST3 stated that her sister graduated as a classroom teacher, and her mother worked as a teacher for a while. Therefore, she pointed out that these might cause her predisposition to the teaching profession.

University Exam Ranking: The results revealed that some pre-service elementary mathematics teachers chose this department because of their university ranking. ST7
stated that her university exam ranking was low for the department she wanted to study. She was tired and did not want to prepare for the university exam again. She chose this university and the department because she wanted to study at the best university.

Having Role Model Teachers: A good teacher can improve us in many areas of our lives and guide us in choosing teaching as a profession. ST4 commented on this issue as her mathematics teachers were outstanding in middle and high school; she was very good with them. They helped her grow in many ways, which was why she liked teaching mathematics, and wanted to be a teacher like them.

4.1.1.2. Purpose of Learning Mathematics

The views of pre-service mathematics teachers about the purposes of learning mathematics actually provide information about how they position mathematics learning in their lives and their teaching. Based on the study's findings, PMTs stated the purpose of learning mathematics is that mathematics is in every aspect of our lives, builds thinking skills, and mathematics is a gatekeeper.

Mathematics in every aspect of our lives: Some pre-service elementary mathematics teachers stated that they appreciate mathematics in every aspect of their lives. One reason is that mathematics makes their lives easier as they encounter it in many ways in their daily lives. ST4 highlighted this by saying: “For example, you're going to buy something for your house and have a budget. You have to use mathematics about this budget in terms of what you have, what expenses you have, and what income you have. It is very, very necessary.” ST4 emphasized that the budget arrangement was one of the prominent factors she felt as a student. Pre-service teachers also pointed out that mathematics can be regarded as the building block of our lives, and it needs to be applied in many areas, such as doing daily grocery shopping, and in many professions, such as architecture and tailoring. In particular, one of the PMTs emphasized its relationship with arts; she clarified that the mathematics of compositions are indicators that it is in every aspect of our lives. She stated, "I'm also very interested in the relationship of mathematics with arts and painters. For
example, there are compositions by famous artists like Beethoven. I think mathematics was used a lot in those compositions. That's why it's crucial.” (ST2)

Building Thinking Skills: Another purpose of learning mathematics mentioned by the pre-service teachers is that mathematics develops thinking skills. ST6 emphasized the mind-developing role of mathematics, as people can start to think more analytically and create solutions more easily through mathematics, and it also improves thinking. In addition to this view, some of them mentioned that it provides us with different perspectives and has an impact on success in different fields.

Gatekeeper: A pre-service mathematics teacher shared how mathematics changed her life and how mathematics opened new doors in her life by stating:

Mathematics was the only thing that improved my life. I went to university, went to new cities, and met many people thanks to mathematics. So, I can say that I paved the way for my life thanks to mathematics. I have always loved mathematics since childhood. Mathematics helped me get into a good high school. I came to a good university because I did mathematics well. For example, I am in demand from people as a teacher because I do mathematics well. It actually has a huge role in my life. (ST7)

4.1.1.3. Social Justice as a Concept in Education

The findings on social justice as a concept in education focused on the need for social justice in education and their familiarity with social justice as a concept in the teacher education curriculum.

The Need for Social Justice in Education: Some pre-service mathematics teachers exemplify injustices in the education system through the differences between rich and poor or differences between east and west regions of the country in terms of quality of education and access to education. This situation emphasizes the need for social justice in the education system. One of them stated that:

Well ..., for example, students in good financial situations can take private lessons and go to a tutoring center. Students with poor financial status are actually left with only the education they receive at school. And if the family
is uneducated, that is an even worse factor. The family cannot take care of them. The teacher can only pay a little attention within the limits of the possibilities because they don't have enough resources and can't use technology such as computers. We need social justice to overcome the gap between those students. (ST7)

Lack of Familiarity with Social Justice in Teacher Education Curriculum: The findings revealed that pre-service mathematics teachers had a lack of familiarity with social justice concepts in their teacher education curriculum. Some of them emphasized that they had never encountered this concept in any course during their undergraduate education. However, some stated that they were slightly familiar with the concept of social justice in some courses of their undergraduate education. For example, ST2 stated in an education course that they talked about students coming from a certain environment, family, and culture when they start school. She pointed out that all of these are related to social justice. ST2 mentioned that she was introduced to social justice concepts in that course. According to her statements, it can be concluded that social justice is not directly emphasized but indirectly mentioned in that course.

4.1.1.4. The Conception of Teaching for Social Justice

According to the findings, the conceptions of pre-service teachers who had a lack of familiarity with the concept of social justice throughout their teacher education curriculum on teaching for social justice are centered on a single dimension. Pre-service teachers’ definitions of teaching for social justice are mainly focused on the fact that it is equal opportunity for all.

Equal Opportunity for All: Most pre-service teachers defined teaching for social justice as providing equal opportunities for all, while one mentioned it as providing equal access to education, especially for students with diverse socioeconomic status. ST2 stated that when she heard teaching for social justice, what came to her mind was that students from diverse backgrounds and environments. She mentioned that teaching for social justice is providing opportunities in a way that can make everyone more equal. She added, "Providing the same opportunities for all students is a little
difficult. But, as I understood, it is at least making an effort for this.”. Along with these ideas, ST3 emphasized that teaching for social justice is related to not discriminating between students in class; teachers should behave equally to the students to create a more socially just environment in their classrooms.

4.1.1.5. Teaching Mathematics for Social Justice

The findings related to PMTs’ conceptions regarding teaching mathematics for social justice are categorized under their understanding, knowledge of sources of TMSJ, recommended social justice topics for TMSJ, opportunities and concerns regarding TMSJ, and their thoughts and preferences regarding TMSJ.

Understanding of Teaching Mathematics for Social Justice: As with the concept of teaching for social justice, pre-service teachers also lacked knowledge about teaching mathematics for social justice. Some pre-service teachers were not able to relate the concept of social justice with mathematics teaching and, therefore, could not give an example of this concept, as one of them stated: “At the moment, I cannot associate any social justice issue with teaching mathematics, so I guess I cannot give such a specific example” (ST1). Some PMTs shared that they had no idea how teaching mathematics for social justice would contribute to students' mathematics learning because they could not connect it with teaching mathematics, as one of them stated: “I couldn't associate social justice with mathematics teaching, so I don’t know what kind of an effect it would have on teaching mathematics; I don’t know it right now; I couldn't think of it” (ST3). Another pre-service teacher, ST7, also stated that she did not know about teaching mathematics for social justice; therefore, she was unsure about its effect on teaching mathematics. Regarding reading the world with mathematics, one pre-service teacher said that it is not possible to raise students' awareness of social justice issues by teaching mathematics; it is only possible by doing different types of activities. In terms of writing the world with mathematics, ST3 stated that she could not associate teaching mathematics with acting for change on social justice issues.

Knowledge of TMSJ Sources: All the PMTs except one indicated they had no idea of sources related to TMSJ. ST1 stated that she did not know any TMSJ sources, and
ST3 noted that she had no information about TMSJ sources. Moreover, ST6 also emphasized that he had no information about TMSJ sources. On the other hand, only one of the PMTs indicated that she could benefit from the work of Non-Governmental Organizations while preparing her mathematics lessons in a social justice context.

Recommended Social Justice Topics for TMSJ: Since some pre-service teachers could not make a connection between teaching mathematics and social justice, they could not suggest any social justice issue that could be integrated into mathematics teaching. For instance, ST7 stated: "Hmm. I can't think of anything right now. Nothing comes to my mind."

One PMT stated that it would be good to integrate religious, sectarian, and racial differences into mathematics teaching, mainly based on her own experiences involving social injustice in her hometown, but added that she had no idea how to do this. ST3 stated that:

\[I \text{ mean, here again, in my hometown, there is a lot of sectarian mixing in Hatay. There are differences in religion and race. Maybe these can be good if they are included in teaching, but I don't know how they can be included in mathematics teaching.}\]

In addition, some PMTs emphasized integrating poverty, rights-based education, and equality topics into mathematics teaching without elaboration on how to do it.

Opportunities of Teaching Mathematics for Social Justice: The findings revealed that pre-service teachers identified opportunities in different aspects by integrating social justice issues into teaching mathematics. Some pre-service teachers stated that through this integration, students' awareness of social justice issues may increase, and they may want to take action in some situations thanks to their increased awareness. For example, ST4 explained that TMSJ would raise students' awareness. She added that when we included social justice topics in mathematics teaching, students would love mathematics and do mathematics by being aware of some social justice topics. ST4 emphasized that students would learn mathematics, and their
awareness of social justice topics would increase; with this increase in their awareness, students would demand a change, and they would be willing to make changes.

One pre-service teacher stated that students could take this action only when they were adults. She stated that:

*If they realize this when they are young, they can defend their rights better when they grow up because they have realized it when they were young. At least they learn that they have rights. They can say that I don't deserve to get less because I am a woman.* (ST7)

Some PMTs emphasized that it would contribute to students' positive attitudes toward mathematics because mathematics classes would integrate social issues that attract students' attention. Besides, one PMT indicated that integrating social justice issues into teaching mathematics would be an example for other courses; other teachers may also prefer to use this if they see benefits.

Concerns Regarding Teaching Mathematics for Social Justice: According to the findings, the pre-service teachers had concerns about integrating social justice issues in teaching mathematics. Some PMTs indicated that students would only focus on solving the mathematical problem, and they would not focus on the context of the problem. Therefore, students would miss the social justice outcome, and social justice outcomes would not be observed. Some pointed out that students would not be able to transfer what they learned in the classroom regarding a social justice issue to the outside of the classroom. For example, one PMT stated: “Maybe in class, they might say, why did you give less to girls? Or you gave more to boys. They can say that, but I don't know if they can take it to the next level. Maybe that concept of social justice stays in the classroom” (ST7).

Another important point that emerged from the findings was the importance of the appropriateness of social justice topics for students' age groups. According to some pre-service teachers, it would be challenging for teachers to discuss social justice issues that are not appropriate for the age group of the students, and students will not
be able to achieve social justice outcomes in the end. Lastly, if there is disharmony between what students learn at school and the situation at their homes, this will reduce the effectiveness of integrating social justice issues into teaching mathematics. A PMT shared her ideas on this issue: “In a family or in a society that is not conscious about this issue, the child learns at school that inequality is wrong, but when they go home, for example, it is very normal. In this case, what we try to do as teachers will not work” (ST1).

Curriculum Transmission Preference for TMSJ: PMTs' curriculum transmission preferences on integrating social justice issues into mathematics teaching are that they believe they could integrate social justice issues into mathematics teaching through in-class and extracurricular activities. Some pre-service teachers thought social justice issues could be taught by paying attention to situations such as equal distribution and justice by the teachers during activities in class. Some of them stated that they could integrate daily life social justice issues into mathematics problems. Regarding extracurricular activities, some PMTs pointed out that project work as an extracurricular activity could be beneficial for students to understand social justice issues.

Thoughts on TMSJ Intervention: While examining the needs of pre-service elementary mathematics teachers, a question was asked in the interview about their ideas on participating in an intervention in teaching mathematics for social justice. The findings indicated that some pre-service mathematics teachers stated that they found such intervention beneficial, and some stated they would like to participate in such an intervention. ST6 emphasized: “I mean, I would want to gain experience. Maybe I am looking at it from the wrong point of view right now. I would want to attend it to change my point of view.” Some of the participants indicated their perceptions of teaching mathematics for social justice, and others indicated the following:

*I would very much like to take part in such a training. I honestly don't know how to integrate social justice into my lessons, but I would like to integrate it. Because the world is changing, and we need to eliminate these inequalities.*
Everyone needs to access their rights fairly. If there is such a training, I would love to participate. (ST1)

For example, I am saddened that I can't think of any examples right now. So, maybe when we are pre-service teachers or after we become teachers, we should have a training on teaching mathematics for social justice regarding what can be done in middle school education. I think this training could be good. (ST3)

Teaching Preference for TMSJ Intervention: The results of the PMTs Teaching Preferences for the intervention in TMSJ are listed in Table 4.1.

### Table 4.1. Pre-Service Teachers’ Teaching Preferences for TMSJ Intervention

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning Activities Preference</td>
<td></td>
</tr>
<tr>
<td>Group Work</td>
<td>6</td>
</tr>
<tr>
<td>Both Individual and Group Work</td>
<td>1</td>
</tr>
<tr>
<td>How to Receive TMSJ</td>
<td></td>
</tr>
<tr>
<td>Face to Face</td>
<td>6</td>
</tr>
<tr>
<td>Face to Face or Online</td>
<td>1</td>
</tr>
</tbody>
</table>

Most pre-service teachers said they would prefer the intervention to be planned by including group activities for reasons such as learning from peers in the group, gaining different perspectives, and creating new ideas with brainstorming. Regarding their preferences for face-to-face or online classroom environments, most pre-service teachers emphasized their preference for face-to-face classrooms, stating that face-to-face is better in terms of focusing on work, communication, and effectiveness.

Based on these findings obtained from pre-service mathematics teachers, the intervention in teaching mathematics for social justice is designed face-to-face. It utilizes social constructivism as a teaching and learning theory.

### 4.1.2. Needs Analysis Regarding Teaching Mathematics for Social Justice from the Perspectives of Teacher Educators

The findings of the interviews conducted with teacher educators on the experiences of PMTs regarding TMSJ encompass the answer to the 2nd research question of the
study. The findings revealed three themes shown in Figure 4.2, which are teaching, social justice as a concept in education, and teaching mathematics for social justice.

![Diagram showing three themes: Teaching, Teaching Mathematics for Social Justice, and Social Justice as a Concept in Education.]

**Figure 4.2. Needs Analysis with Teacher Educators**

### 4.1.2.1. Teaching

Under the teaching theme, two categories flourished: teaching mathematics orientation and mathematics teacher education orientation.

Teaching Mathematics Orientation: Most teacher educators emphasized the significance of constructing mathematical knowledge by students themselves. They highlighted that students should learn mathematics. They stated that mathematics is not a subject to be memorized but rather a subject to be learned by understanding. To make the students understand mathematics, one female teacher educator explained that she emphasizes the classroom environment where mathematics is co-constructed. She also mentioned that mistakes are learning opportunities for students by stating:

*We decide together with students on how to proceed with mathematical knowledge construction in the class based on the mathematical knowledge*
Some teacher educators highlighted the importance of teaching mathematics concepts correctly to the students, focusing on different dimensions. One teacher educator said, “I think that knowing the concepts correctly gives me information about the conceptual structure of that person, whether that person's conceptual structure is strong mathematically” (P3). Another teacher educator said, "Students are likely to encounter mathematical concepts and symbols in daily life, these symbols or representations related to mathematics. Therefore, I attach great importance to it" (P2).

Another point that some teacher educators place importance on teaching mathematics is emphasizing mathematics in daily life. One teacher educator mentioned that mathematics is necessary to meet our needs in our daily lives. Another teacher educator emphasized that mathematics is a useful tool by stating:

Mathematics is a very useful tool that we can adapt to specific life situations. We can understand some aspects better and analyze and examine some aspects by using mathematics, from very technical to social issues. Therefore, mathematics has such a powerful aspect. (P4)

Drawing on the interviews with teacher educators, according to some of teacher educators developing reasoning skills is another critical point in teaching mathematics. P3 indicated that from mathematics lessons, students could learn to analyze a problem situation they encounter, and could reach the results by following problem solving processes, and she indicated this as:

I think one of the most important things a child can learn from mathematics lessons is to be able to analyze a problem situation they encounter. To be able to determine the data necessary for the solution of that problem, either by doing research, asking if they need to ask around, or drawing from their mathematical knowledge. To be able to design a solution and when that solution does not work, to see and evaluate the shortcomings of that solution, to think about how to revise it, and try it again, and to do this patiently without giving up until the result is reached. (P3)
Lastly, one teacher educator pointed out that making students recognize the method for learning mathematics should also be considered in teaching mathematics. P3 expressed that it is crucial to give the opportunity to the students to understand how they learn mathematics because students can learn in different ways, such as by writing or visual representations. She pointed out that this is about realizing themselves.

Mathematics Teacher Education Orientation: Teacher educators indicated some points they consider essential in mathematics teacher education. Some teacher educators emphasized the significance of raising awareness of pre-service teachers on equity in mathematics education. While one of the teacher educators emphasized the importance of a fair and qualified educational environment in the classroom, the other one defined it in terms of being a conscientious teacher in the classroom and said:

*Being a conscientious teacher is not just an emotional conscience; I am not just talking about an affective conscience. It could be about the students’ socioeconomic status, disability, or anything that bothers them. Recognizing that there is a right to learn mathematics independent of all these and not putting a filter to transfer the mathematical knowledge that you have as a teacher. (P3)*

Another critical aspect in teacher education identified by one teacher educator (P3) is that pre-service teachers need advanced mathematical knowledge when they become teachers because even if they are not going to teach this advanced mathematical knowledge to their students, they will utilize this knowledge while making sense of the subject and developing activities on that subject. Another teacher educator (P4) explained that since pre-service teachers need to do measurement and evaluation when they become teachers, it is important to equip them in this regard.

4.1.2.2. Social Justice as a Concept in Education

From the drawings of interviews with teacher educators, social justice in education includes both ways to achieve social justice in their classes and the need for social justice in education. Information about them is given in detail below.
Ways to Achieve Social Justice: Ensuring social justice in the classroom is vital because the learning environment pre-service teachers encounter during their teacher education will form the basis of the learning environment they will create in the future. Teacher educators have used different ways to achieve social justice in their classrooms. One teacher educator tried to achieve social justice in her class by co-creating a learning environment and shared decision-making. In addition to this, another teacher educator tried to achieve this by setting classroom norms. To illustrate, she pointed out that:

*One should not interrupt or offend someone because they said a wrong thing. Listening to each other within a framework of mutual respect and being able to discuss in that framework. You are having a discussion. You have a discussion there, but you set a social norm when you discuss. You discuss within the framework of those norms. So, they become your rules.* (P2)

The Need for Social Justice in Education: Some teacher educators emphasize the need for social justice, especially in public schools, regarding the gap between students from low socioeconomic status and students from high socioeconomic status. P1 said that she encountered this situation while observing pre-service teachers in their practice teaching in public schools as students from low socioeconomic status do not have the same opportunity in the classes as students from the high socioeconomic status, and she stated that:

*I saw this often when I went to the observation in practice teaching, and I felt very sad. I observed this when we used to go to Çiğdem. There are children who are collecting paper in that region, but now they have moved to a different place. These children go to the same public school with the children of well-educated families live in the same neighbourhood, and I saw that those children collecting paper were never involved in the classroom discussions when I went to observe pre-service teachers. Pre-service teachers tried to give everyone the floor with what they see from us, they chose randomly from the students. And the teacher told them a few times in class that they would not talk. I was also upset when I heard this, but we could see that they participated when the floor was given. Because they were not given the opportunity. Do you know what I mean? I mean, they were not given the opportunity.* (P1)

4.1.2.3. Teaching Mathematics for Social Justice

The findings related to teacher educators’ conceptions of teaching mathematics for social justice are categorized under TMSJ in the undergraduate curriculum, the need
TMSJ in Undergraduate Curriculum: To understand whether there is a need for teaching mathematics for social justice, it is guiding to look at whether there is a place for it in the mathematics teacher education undergraduate curriculum. The findings of the interviews with teacher educators show that there is no direct emphasis on teaching mathematics for social justice in undergraduate mathematics education curriculum. P3 commented that they did not establish a relationship between social justice and mathematics teaching. Moreover, P4 explained that they did not have such an emphasis on social justice, especially in mathematics problems. A teacher educator stated that in one of the lessons in the undergraduate curriculum, there was a topic of equity which has a slightly relevant emphasis on teaching mathematics for social justice and said:

There is an Equity chapter in a course. In that course, we were talking about equity in detail as content. It was a more specialized chapter but also dealt with general issues. It included gifted children, children who are not attentive, children who speak different languages, and what methods can be applied to them. (P1)

As can be dedicated to these findings, teaching mathematics for social justice is not directly emphasized in undergraduate mathematics teacher education curriculum.

The Need for TMSJ Curriculum in Schools: Although there is no specific focus on teaching mathematics for social justice in the middle school curriculum, one teacher educator explained that teaching mathematics for social justice is related to the values section of the curriculum in which concepts such as justice, friendship, honesty, self-control, patience, respect, love, responsibility, patriotism, and benevolence are specified. She stated, "These values are defined in our curriculum, but teachers have great difficulty associating these values with the learning outcomes" (P1). Therefore, she pointed out that TMSJ can guide teachers in this respect by showing ways to deliver values to the students. Furthermore, she also mentioned that to accomplish TMSJ in schools, there is a need for effective in-
service training for teachers and emphasized the necessity of practicing the sample activities developed in this regard with the following explanation:

*It would be beneficial to try out these kinds of sample activities for the learning areas we just talked about, wouldn't it? Students will give some reactions to these activities, and then the teachers will try to understand these reactions. We can give the teacher training first and teachers implement these activities directly and then we can share these good examples with the MoNE experts. (P1)*

Similar to P1’s comments, another teacher educator emphasized the need to raise awareness on TMSJ for both pre-service teachers and teachers. P4 pointed out that materials and good examples have significant contributions to this awareness raising, but while doing so, "*It would be more successful to do this by emphasizing the examples where mathematics is a tool that we can benefit in a very natural way.*"

Another teacher educator explained that thanks to teaching mathematics for social justice, students will have different experiences. Therefore, she explained the need to include different experiences in mathematics teaching as follows:

*We need to offer children different experiences and give them the opportunity to discover their interests. Maybe someone will be interested in this field. That's why I think we could include this subject but not very intensively, but rather integrate different situations into mathematics education so that children can have different experiences. Therefore, it would be nice. (P3)*

Opportunities of Teaching Mathematics for Social Justice: One of the opportunities of TMSJ identified by teacher educators is that it raises awareness about social justice issues. A teacher educator stated the following about this: “*Even if nothing happens, it raises awareness. Among forty people, maybe someone will be interested and continue reading and researching that issue.*” (P3). The same teacher educator also stated that it would serve as an example for the use of mathematics in our lives:

*For example, we have a topic like graphs. I think the examples of graphs in the textbooks are ridiculous because they are very simple. Why do we need graphs? To read more complex data. When we already have bigger data, reading it and making sense of it can actually be the answer to where we use mathematics in daily life. (P3)*
Some teacher educators mentioned that thanks to TMSJ, mathematics can be used as a tool to convey social justice outcomes as well as mathematics outcomes. One stated, “Mathematics outcomes can be used as a tool for achieving social justice outcomes” (P1). Additionally, a teacher educator said, "I think it contributes to learning mathematics" (P4) and focused that mathematics taught through examples students can encounter in their daily lives by utilizing TMSJ will enable them to understand mathematics better.

Concerns Regarding Teaching Mathematics for Social Justice: Two teacher educators mentioned the concerns that could be encountered while implementing TMSJ lesson plans in the classroom. According to one teacher educator, one of the concerns regarding TMSJ is using inappropriate examples for students’ lives. In order to overcome this problem, P4 stated that "topics that touch children's lives" should be integrated into mathematics lessons. Another teacher educator stated that social justice issues should be integrated into the lesson so that "It will not suppress the mathematics content" (P3). Thus, she emphasized that it is appropriate for students to receive social justice and mathematics outcomes.

TMSJ Curriculum Transmission Preference: Regarding the TMSJ curriculum transmission preference, two teacher educators highlighted that it would be better if it is applied in mathematics classes. One of them stated: “I think definitely over the tasks, in class...because it is not possible for every student to spare time in out of class, that's why” (P2).

Two teacher educators stated that TMSJ can be applied in class and through extracurricular activities. P1 stated with a focus on out-of-school learning: “I think it can be applied both in class and during extracurricular activities. For example, I think out-of-school learning can also be very effective. I don't know, trips, but to where? You need to think about these things well.”

4.2. Results of the Design Experiment

This section consists of two sub-sections regarding the findings of the design experiment. In the first sub-section, the understanding of TMSJ of pre-service
mathematics teachers who participated in the 2nd experiment is presented by comparing their initial and final understanding of the sub-concepts of TMSJ and their TMSJ lesson plans. In the second sub-section, the case of a pre-service mathematics teacher from the 2nd experiment, Mira, is presented to show how her understanding changed and how the materials, activities, and discussions influenced her understanding of TMSJ.

4.2.1. Understanding of TMSJ

The following sub-section presents PMTs’ initial and final understanding related to TMSJ. Their initial and final understanding regarding TMSJ is derived from the data from initial and final interviews and reflective papers they completed during the lessons. The initial interviews included questions about their views on teaching, their understanding of TMSJ and RWWM, and WWWM dimensions of TMSJ. Final interviews included questions on their views on teaching the final understanding of TMSJ with RWWM and WWWM dimensions. Reflection papers were also analyzed to understand their final understanding.

4.2.1.1. Understanding of the Concept of TMSJ

In this section, PTMs’ initial and final understandings regarding the concept of TMSJ are explained. They were asked to explain how they could describe TMSJ in the initial and final interviews.

4.2.1.1.1. PMTs’ Initial Understanding of the Concept of TMSJ

The findings of the needs analysis interviews revealed that pre-service mathematics teachers were unfamiliar with the concept of TMSJ in their undergraduate courses. However, the participants of the design experiment were asked whether they had any prior knowledge about this topic, as they might have knowledge about it from different sources. Also, they were asked how they would define this concept in the initial semi-structured interviews. Their preliminary views about the concept of TMSJ were gathered through this question.
I found that all participants stated they were unfamiliar with the concept of TMSJ. Some of them stated as follows:

*I haven't come across an article or a written text about it. I haven't come across a conversation about it before, to be honest.* (Umut)

*No, I haven't heard of it. I mean, I haven’t heard it at school or in classes.”* (Leyla)

*No, I have never encountered it!* (Defne)

I clearly saw that the definitions regarding TMSJ by pre-service mathematics teachers who had no prior knowledge of this subject were based on their predictions. Many PMTs emphasized that TMSJ could be related to providing equal opportunity in mathematics education. For instance, Define commented on this issue:

*When I talk about social justice, I define it as educating all children on the same level, taking into account who has difficulties and where they have difficulties, and getting them to the same level at the end of the day or end of the semester. I defined it as being able to give them all the same opportunities.*

Similarly, Kerem pointed out, *"When social justice is reflected in education and especially in mathematics, what I think is that all students should have equal mathematics education.”* In addition to providing equal opportunities for all students, Mira also stated that social justice in mathematics teaching could be provided through the “teacher's attitudes” or behaviors towards students in the class, as well as through emphasizing *"liberal education"* based on the concept of social justice.

Another prediction of some pre-service mathematics teachers about TMSJ was that it could be using social justice in the context of the problems. The pre-service teachers were able to make such a comment because they were used to integrating contexts from daily life into mathematics lesson plans in their undergraduate courses according to the results of the needs analysis with teacher educators. Ceren highlighted her ideas:

*It can be about graphs, surveys. How is this problem constituted in this region before applying the solution method for this problem? For example, there were so many girls who could not go to school. But we tried to solve it*
with this method, or we applied that method in this region. How many girls are there now? How much have we achieved? Or how much of an impact this solution has had by having students comment on it in the form of surveys, graphs, etc.

Lastly, some pre-service teachers commented that to accomplish TMSJ, ‘caring for individual differences’ could have a significant effect. In this regard, Leyla talked about cultural, racial, and religious differences by stating: “For example, we are culturally very diverse. We will have students from different races and different religions. I wonder if the subject of this education is to be able to maintain and manage that fair environment despite those differences?” On the other hand, Selin explained her thoughts that it could be about teaching mathematics to students through different teaching techniques as follows:

*We have different teaching techniques according to the interests of children; some children are more visual, and some are more verbal. It can be about approaching the visual student with more visual materials and approaching the verbal one with more verbal materials. It creates this meaning for me.*

As a result, in terms of caring for individual differences, pre-service teachers focused on differences in culture and learning culture.

In the initial interviews, the pre-service teachers' views on TMSJ were based on their predictions. Most of them thought of it as providing equal opportunity in mathematics education, and a few stated that social justice issues could be used in the context of the problems. According to the findings of the first interviews, the pre-service teachers' understanding of TMSJ is not very comprehensive at the beginning; they do not have detailed knowledge about how it can be implemented in the classroom.

**4.2.1.1.2. PMTs’ Final Understanding of the Concept of TMSJ**

In my analysis, I compared the participants’ views taken from their initial interviews with the final interviews and I found a clear picture of how the intervention had an influence on their conceptual understanding. To explicate, the findings of the final
Interviews with pre-service teachers revealed that most of the pre-service teachers explained that TMSJ is integrating social justice issues into mathematics teaching. In this respect, Can, who emphasized “I could not make a connection with mathematics teaching” in the initial interviews, explained: "If we want to make our students achieve some outcomes related to social events or social justice, we can do it in the mathematics class since our major is mathematics. We do not need to do it out of the mathematics class” in the final interview.

Similarly, Leyla whose initial ideas focused on “caring for individual differences,” pointed out: “We try to eliminate inequalities among students by integrating social justice issues into mathematics classes; we try to reduce those injustices as much as we can.” Similar to the idea of integrating social justice issues into mathematics teaching, some pre-service teachers emphasized that TMSJ is also integrating daily events into mathematics teaching. Selin, who pointed out in the initial interview that TMSJ could be “teaching mathematics through different teaching techniques,” commented: “After this training, I realized that TMSJ is integrating daily life into mathematics.” Kerem, similar to his initial ideas: “using social justice in the context of the problems to raise awareness,” stated in the final interview: “For example, taking statistics about cyberbullying and making a lesson plan including them can be very effective. You can also teach mathematical outcomes to students with real statistics and examples from daily life.”

In addition to the previous comments on the meaning of TMSJ, pre-service teachers also explained TMSJ based on two dimensions (RWWM and WWWM) which are emphasized during the TMSJ intervention. 7 of the pre-service teachers highlighted that TMSJ is about raising awareness about social justice issues through mathematics; this definition focuses on RWWM. To illustrate, Umut, whose initial idea about TMSJ was “providing equal opportunity in mathematics education,” stated: “We use mathematics to show students social injustices in life and make them aware of them. We raise their awareness.” Mira's initial comment on TMSJ was: “It can be provided by teachers’ attitudes.”. She emphasized in the final interview that: “This is the case of hitting two birds with one stone; we both teach mathematics outcomes and social justice consciousness. I can actually teach mathematics easily.”
Moreover, Ceren, who commented on the initial interviews as “…I think TMSJ can effect on raising awareness on social justice issues…” stated in the final interview that:

Before the TMSJ intervention, I was saying that we can raise students' awareness, but I didn't know how to implement it. I didn't know what the classroom environment would be like. Now, I know how to make students raise awareness on social justice issues while teaching mathematics.

Some of the pre-service teachers focused on WWWM while explaining their understanding of TMSJ in the final interviews. They explained that TMSJ is taking action regarding social justice issues. Kerem commented: “It is integrating social justice issues into mathematics in order to take action on these issues.” Similarly, Selin pointed out: “It is like being able to contribute to the flow of life, to make a change.”

Additionally, Defne’s initial ideas about TMSJ focused on: “providing equal opportunity in mathematics teaching.”. In the final interviews, besides her ideas on integrating social justice issues into mathematics teaching, she also focused on promoting students to be sensitive to social justice issues. She stated: "TMSJ is about doing something to make students more sensitive, more empathetic by an education integrated with mathematics so that all individuals can access equal rights.”

No prior Knowledge
- Conceptions based on Predictions

TMSJ Intervention

Conceptual Understanding of TMSJ
- Final Conception

Figure 4.3. PMTs Understanding of Concept of TMSJ
In the Figure 4.3, how pre-service mathematics teachers’ understanding regarding the concept of TMSJ is explained. Since they had no prior knowledge at the beginning, their understanding was based on predictions. After participating in TMSJ intervention, they reached their final conception that they had a conceptual understanding of TMSJ. In that sense, it can be concluded that TMSJ intervention influenced their understanding of the concept of TMSJ.

4.2.1.2. PMTs’ Understanding of RWWM

In the initial interviews, since the pre-service teachers had no prior knowledge about teaching mathematics for social justice as a concept, it was thought that they would not be able to elaborate specifically on RWWM, and therefore, a question directly involving the concept of RWWM was not asked. However, a question related to the definition of RWWM was asked. In the final interviews, the pre-service teachers were asked different questions about RWWM and WWWM, the two TMSJ sub-concepts covered in the TMSJ intervention. In addition to the final interviews, their writings in the 2nd reflection were also analyzed as a source of information on their understanding of RWWM and WWWM. In 2nd reflection paper, they shared their views on RWWM and WWWM and the challenging and powerful learning moments they encountered.

4.2.1.2.1. PMTs’ Initial Understanding of RWWM

PMTs were asked how mathematics teaching would have an impact on raising awareness of the students about social justice issues. When this question was asked, some pre-service teachers answered that since they could not connect social justice to mathematics education, they had no idea about the connection between raising awareness on social justice issues and mathematics teaching. Leyla emphasized: “I have no idea about raising awareness through mathematics teaching. Social justice and mathematics are so far apart. I can't make the connection between them. I think I'm thinking from one side right now.”

Another pre-service teacher, Selin, shared her similar ideas: “I don't know. It just feels like we cannot raise awareness by teaching mathematics. When you ask it like
that, it doesn't sound like there is a connection between social justice and mathematics."

Two PMTs pointed out that teachers’ attitudes towards students can raise awareness of social justice issues in a mathematics class. Kerem explained his thoughts: “I teach mathematics to students, and they develop social justice awareness. So, I think this depends more on the teacher than on the mathematics teaching, and it depends on the teacher's attitude, behavior, or narratives in the lesson.”

Besides, Kerem and some other pre-service teachers mentioned new generation questions; they said such an awareness could be created by including social justice in the context of the problems. Kerem further explains this:

\[
\text{We have new generation questions that are content-based, concept-based, and story-based. Maybe in those questions or those activities, we can integrate this with messages that include social justice awareness. Other than that, as I said, I think it depends on the teacher. It depends on the teacher's attitude and behavior in the lesson.}
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4.2.1.2.2. PMTs’ Final Understanding of RWWM

PMTs’ final interviews and the 2\textsuperscript{nd} reflection papers were examined in order to reveal their final understanding of RWWM. In the final interview, pre-service teachers were asked how they defined RWWM after participating in the TMSJ intervention. In the 2\textsuperscript{nd} reflection paper, pre-service teachers were asked what their thoughts were in terms of RWWM and WWWM. Pre-service teachers who had difficulties making connections between mathematics and social justice in the initial interviews had more developed ideas in the final interviews.

The most common view among pre-service teachers regarding RWWM was raising awareness of social justice issues through mathematics. All of the pre-service teachers indicated the issue of raising awareness. To illustrate, Leyla who commented on the initial interview as: “I have no idea about raising awareness through mathematics teaching. Social justice and mathematics are so far apart.”,
commented on the final interviews as: “RWWM is raising awareness about a situation of injustice by using mathematics. Having knowledge about social justice issues with the resources offered to us by mathematics.” Similar to Leyla’s final ideas, Mira also commented on the 2nd reflection that: “RWWM is raising awareness on the social justice issues.”

Another pre-service teacher, Can, who said in the initial interview that: “I have no idea about how mathematics teaching could raise awareness on social justice issues,” focused on empathizing through mathematics in terms of RWWM in final interviews. Can conveyed his message as: “I think RWWM is making students empathize about a social justice issue.” Defne also indicated that RWWM is “using mathematics in the affective domain, building empathy in social justice issues.”

Two pre-service teachers, Mine and Selin also emphasized that students will recognize differences through mathematics thanks to RWWM. Selin stated in the initial interview: “I don’t know. It just feels like we cannot raise awareness by teaching mathematics.”. In addition to her ideas of raising awareness on social justice issues through mathematics, she also commented on recognizing differences through mathematics in the final interview: “recognizing that there are different groups in society, such as minorities.”

Lastly, according to the findings of the final interviews and 2nd reflection papers, 5 of the pre-service teachers commented that RWWM is intertwined with WWWM. Leyla pointed out, "I think it is incomplete when RWWM and WWWM are not together. They complement each other.” Similarly, Kerem stated in his reflection: “RWWM and WWWM are feeding each other. If your awareness regarding something increases, then your reactions also change.”

4.2.1.3. PMTs’ Understanding of WWWM

Similar to the reasoning behind not asking a question directly, including the RWWM concept in the initial interviews, a question involving the WWWM concept was not asked in initial interviews. Nevertheless, there was a question related to the meaning
of WWWM in the initial interview. The pre-service teachers were asked different questions about WWWM during the final interviews. In addition to the final interviews, similar to the situation with RWWM, their writings in the 2nd reflection paper were also analyzed as a source of information to interpret their understanding of WWWM.

4.2.1.3.1. PMTs’ Initial Understanding of WWWM

Regarding WWWM, pre-service teachers were asked a question about how mathematics teaching could make students take action on social justice issues. Some pre-service teachers explained that they have no idea how to make students take action on social justice issues in teaching mathematics. To illustrate, Defne stated: “I can't think of anything about it right now.”. Similarly, Kerem stated: “I can’t think of it”. Similarly, Kerem stated: “I can’t think of it”.

Some pre-service teachers reasoned about this issue and came to some conclusions regarding making students take action on social justice issues in a mathematics lesson. First of all, some of them emphasized that if a problem involving the social justice concept was used in a mathematics lesson, some students might internalize the sub-message to take action regarding that social justice issue. Mine communicated in this issue as:

Children may feel that they can do something to solve the social justice issues and problems they encounter in the mathematical tasks. Actually, the student doesn't have that idea about taking action in his/her mind, but after solving a question, the idea that I can do something may arise.

One pre-service teacher, Ceren, pointed out that learning mathematics increases the analytical thinking skills of the students, and thanks to this, students may develop awareness regarding social justice issues and take action on these issues. In other words, Ceren remarked that learning mathematics, which increases students’ analytical thinking, could increase students' motivation to take action.

Three of the PMTs’ mentioned that middle school students cannot take action on social justice issues by focusing on the idea of transferring actions to adult life. Can
commented, "I think they can take action in their adult lives." Similarly, Umut stated: "After all, when the students grow up and get a profession, they can do something and take action. I think more future-oriented."

Only one pre-service teacher's, Mira's, thoughts about taking action were a little more developed. She commented that students may influence other people's ideas on a social justice issue by conveying the message, "After gaining awareness, students can change other people's opinions on the social justice topic." This quote can serve as an example for taking action on social justice topics.

4.2.1.3.2. PMTs' Final Understanding of WWWM

PMTs’ final interviews and the 2nd reflection papers were examined in order to reveal their final understanding of WWWM. In the final interviews, pre-service teachers were asked how they defined WWWM after participating in the TMSJ intervention. In the 2nd reflection paper, pre-service teachers were asked what their thoughts were in terms of RWWM and WWWM.

In terms of WWWM, all pre-service teachers’ ideas are based on taking action through mathematics, which is the core idea of WWWM. For example, in the initial interview, Defne stated: "I can't think of anything about it right now." After participating in the TMSJ intervention, she commented on WWWM in a final interview: "WWWM is taking action. It is a list of things we can do about social justice issues, from what I can do individually to what we can do as a society.” Similar to the case of Defne, Leyla commented in the initial interview that: “I could not find a connection between taking action in social justice issues and mathematics education.” Her ideas changed in the final interview: “It is about taking action and trying to change things in society. I can do it by using mathematics.”

Umut’s idea regarding WWWM in the initial interviews was that students could take action when they grow up, in other words, in their adult lives. He stated WWWM is “taking action” and commented in the final interview: “Students can make some changes in their lives according to social justice issues covered in the class.” This
sentence indicates that he is no longer future-oriented in terms of taking action. Moreover, in the reflection, he commented about WWWM: “It is precautions that can be taken before the social justice issues occur or solutions that can be produced to solve those social justice issues.”

In the initial interview, Mine’s idea on WWWM was that students might internalize the sub-message in the context of the problem and then take action by themselves. However, in the final interview, she commented on her initial ideas to explain how she understands WWWM after participating TMSJ intervention:

I think with the teacher's guidance while taking action on social justice issues, students can be more courageous to do something. Doing something together, even making a poster at school. I had never thought about these things before participating in this intervention; I thought the students themselves would think through the question. But doing something together can be more effective.

In the initial interviews, Mira’s idea about WWWM was the most congruent idea with the concept of WWWM. She stated that students might influence other people and their ideas on social justice issues. In the final interviews, she developed her ideas and explained different ways to provide WWWM. She stated that WWWM is taking action through mathematics, and it can be preparing a poster or using the power of social media, especially in this recent period. She also added that even if one student tell one person about that social justice issue and that person tells a few, they can reach many people and reaching people like that is actually part of taking action.

In terms of the ways to accomplish WWWM, some pre-service teachers emphasized the same points as Mira mentioned: “preparing a poster, using power of social media, telling someone about that social justice issue.” Moreover, Can stated in his reflection that “being a member of a civil society organization, doing a project about a social justice issue might be ways for taking action in a social justice issue.” Furthermore, in the final interview, Can also mentioned: “students can share their ideas with their family to increase their awareness or warn people if they misbehave.” Defne also mentioned in her reflection paper “students changing their own misbehavior is also a way for taking action.”
4.2.1.4. PMTs’ Views about Opportunities of TMSJ

In initial and final interviews, pre-service mathematics teachers were asked about the effects we might encounter when TMSJ lesson plans are implemented in the classrooms. Since the concepts of RWWM and WWWM were included in the TMSJ intervention, the opportunities related to these concepts were asked and analyzed separately in the final interviews. In this section, the opportunities pre-service teachers mentioned are presented.

4.2.1.4.1. PMTs’ Initial Views about Opportunities of TMSJ

Since the pre-service teachers' ideas about TMSJ were not very detailed before they attended the TMSJ intervention, their answers about the opportunities of TMSJ were not detailed in the initial interviews.

Some pre-service teachers stated that positive effects could be observed in terms of mathematics learning. For example, Mira believes that TMSJ can facilitate mathematics learning; she commented that TMSJ would make mathematics learning easier because she thinks associating mathematics lessons with real-life issues would facilitate learning.

Mine believes that TMSJ might increase students’ interest in mathematics. She stated that covering social justice topics and giving examples from real life can attract students to mathematics lessons and increase students’ interest in mathematics.

Regarding personal development, two of the pre-service teachers pointed out that TMSJ would positively affect students' personal development. One of them, Mine explained her opinion on this issue as follows: “If students incorporate the concept of social justice in a given mathematics problem into their life as a guide, a path, a way of behavior, a habit, or a way of thinking, they can improve in a positive way.”

Additionally, some pre-service teachers emphasized that TMSJ would create some opportunities for social development in terms of creating a more just society. Ceren
stated, "As individuals become more aware of the social justice issues, these inequalities in the society will decrease." Another pre-service teacher, Umut, commented on this issue: “Injustices can be prevented by students who are aware of those injustices.”

4.2.1.4.2. PMT’s Final Views about Opportunities of TMSJ

RWWM is defined as using mathematics to understand social justice issues in the world whereas WWWM is defined as using mathematics to change the world (Gutstein, 2006). In that sense, they would create different opportunities for the students. For that reason, the opportunities of RWWM and WWWM were asked and analyzed separately in the final interviews.

PMT’s Final Views about Opportunities of RWWM: The opportunities identified by pre-service teachers about RWWM were examined under three categories: opportunities for mathematical learning, personal development, and social development.

First of all, in terms of opportunities for mathematics learning, all of the pre-service teachers emphasized that it can increase interest in mathematics. Kerem stated that students were more interested in statistics and examples from daily life, and by making lesson plans like these, teachers could increase their interest and raise awareness. Similarly, Mine commented: “Integrating a social justice topic into the mathematics lesson can attract their attention and increase interest in the lesson.” Another pre-service teacher, Umut pointed out that since social justice issues are related to daily life and if teachers use examples from daily life in mathematics class, they can attract students more or increase their interest in mathematics.

Another opportunity of RWWM regarding mathematics learning that most of the pre-service teachers emphasized was that it would facilitate mathematics learning. Mine commented that students will learn mathematics without realizing it while they are talking about a social justice issue in the class. She pointed out that it will make it easier for them to understand mathematics topics since the topic is from daily life.
Defne also pointed out: “Our aim is to teach students mathematics, and when we make connections with social justice issues, using such examples makes mathematics easier for them to understand.”

In terms of personal development, the most emphasized opportunity of RWWM was developing empathy. In the initial interview, none of the pre-service teachers commented on developing empathy. In the final interview, Can commented: “Students can empathize with issues they have an awareness of at school.” Kerem pointed out: “RWWM develops students’ empathy skills.” Kerem also discussed that RWWM can make students adopt different perspectives by adding his ideas: “We emphasize topics they have never encountered or situations they have never experienced before so that we can change their perspectives.” Umut mentioned another opportunity of RWWM, such as increasing awareness on social justice issues; he commented that “including the social justice topic in the mathematics lesson will increase awareness in the students.” Defne explained that RWWM can give rise to promoting sensitive individuals and make students assess themselves. Defne commented that RWWM could develop sensitivity, and students could start to question their own behaviors. For example, if bullying was discussed in the class, they could question themselves as: 'Am I doing such things?' She also added that thinking this way could be beneficial for them in becoming better individuals.

Improving general culture was also an opportunity highlighted by a pre-service teacher regarding RWWM. Leyla, commented that students would learn about the world and they would have general cultural knowledge if we give them such information about social justice issues by integrating it into our activities.

Regarding social development, pre-service teachers emphasized two main opportunities of RWWM: raising public awareness and creating a more tolerant society. All of the pre-service teachers highlighted that RWWM would raise public awareness. Kerem stated that since individuals have awareness, society would also have awareness about social justice issues. Leyla commented: “They will become more conscious, so they will definitely try to change something; we will become a more conscious society.” Ceren, along with her ideas about “RWWM would raise
public awareness,” added, “the society would be more tolerant.” Selin also pointed out that if everyone was raised as conscious individuals about these social justice issues, everyone's behavior and understanding towards each other would actually change in a positive way.

PMT’s Final Views about Opportunities of WWWM: As in RWWM, the opportunities identified by pre-service teachers about WWWM were examined under three categories: opportunities for mathematical learning, personal development, and social development.

In terms of opportunities of WWWM in mathematics learning, as in RWWM, the same issues are mentioned by pre-service teachers. Some of the pre-service teachers emphasized that WWWM would give rise to increasing interest in mathematics and facilitate mathematics learning. Mira emphasized that WWWM will increase interest in mathematics by stating:

I said in the lesson I taught in the class that in our next lesson, we will prepare a poster on this subject. Research the data. Think about what we can do. Preparing a poster by combining social justice and mathematics. Mathematics seems difficult and unbearable for students, and since we don't give it directly as a mathematics subject, it will be more interesting to them.

Leyla pointed out that if she presented social justice topics to the students by putting it into activity, by putting it into action, the students’ perspective towards mathematics would change, and their interest in mathematics would increase. She also focused on that WWWM facilitates mathematics learning by adding: “it will facilitate their understanding of mathematics.” Mine also commented that WWWM would facilitate students understanding of mathematics.

In terms of opportunities of WWWM in personal development, some pre-service teachers emphasized that it would empower students’ personal development. Mine commented that WWWM contributes to students’ personal development by promoting to learn and think that something can be done in this world and to feel that they are at a level where they can stop something wrong and that they have that
power. Another opportunity some pre-service teachers mentioned was that WWWM would improve the confidence of students. Leyla commented that WWWM would give students self-confidence. They would say that I can change something in this world, and this will increase their self-confidence. Furthermore, Leyla also emphasized that WWWM can raise awareness of students by stating: “I may not be able to create that awareness in some students without moving on to the take action part. So, it is much more effective to use both RWWM and WWWM together.” One of the pre-service teachers, Kerem focused on WWWM can improve communication skills. He commented that in WWWM there were some suggestions like, contacting an aid organization about a social justice issue, or preparing a petition or a poster presentation informing people about the issue or asking for help. In that sense, he emphasized that it can increase students' communication skills. Another opportunity Mira identified regarding WWWM was that WWWM can give a purpose to life. Mira pointed out:

If I am alive, I must have a purpose. Therefore, in order to give them a purpose, it has a positive effect. In other words, being useful to someone, being able to cause changes in someone's life, rather than I've lived and gone, actually gives them a purpose to live.

In addition to this, Mira also explained WWWM can give rise to improvement in thinking skills by commenting: “Students will say that: [What can I do in life? What more can I do? What problems does humanity have?] It develops the ability to think. I think it improves people in many ways.” One opportunity identified by Umut was that WWWM enables opportunities for self-assessment. He pointed out:

For example, in my lesson plan, students will ask themselves: [Do I finish my food? Or how efficiently do I use the food I have? What am I doing if half of the food I cook goes to waste? I am wasting this food, so I should cook less or put less food to my plate.] It provides personal development and improvement for students to direct their own lives. It enables students to evaluate themselves.

In terms of opportunities of WWWM for social development, some of the pre-service teachers stated that WWWM could raise others’ awareness of social justice issues. Can conveyed the message that if a student were interested in the social
justice topic discussed in class, then he/she would tell it to his/her parents. For instance, this message could be about obeying the traffic rules, and students could raise awareness of their families by warning them. Besides, Umut mentioned that WWWM could lead to a more just and equitable life by stating that if we apply the awareness we have gained to our lives, our lives actually become easier, more livable, more sustainable. Then, we start to live a fairer and more equal life. Moreover, Leyla commented that society will benefit from WWWM. She illustrated her ideas: “In my lesson plan, students will plant trees. It will be beneficial for the environment and the society. I think it’s a very good thing. It will prevent desertification. It will lead to such positive results.” In addition to these opportunities, Ceren emphasized that students can recognize societal problems thanks to WWWM. She stated: “While taking action, students will realize how important the problem is and how it affects people if it is not solved.”

4.2.1.5. PMTs’ Concerns for TMSJ

In the initial and final interviews, the pre-service teachers were asked questions about what difficulties they might encounter while implementing the TMSJ lesson plans. In the initial interviews, this question was asked only regarding TMSJ, while in the final interviews, it was asked on RWWM and WWWM dimensions since different factors might emerge while implementing TMSJ lesson plans in terms of RWWM and WWWM. Additionally, in the final interviews, the pre-service teachers were asked what solutions they could find for the concerns they stated might be encountered while implementing TMSJ (including RWWM and WWWM dimensions) lesson plans.

4.2.1.5.1. PMTs’ Initial Concerns for TMSJ

In the initial interviews, the pre-service teachers' definitions of TMSJ were mostly based on their predictions, and their concerns that might be encountered were based on their predictions. Since they did not have a clear understanding of TMSJ as a concept, they could not elaborate on the concerns they mentioned. Their concerns for TMSJ are student-related, parent-related, school management-related, school-related, curriculum-related, and educational source-related.
First, some pre-service teachers stated that social justice topics may contradict students’ truth, which can be an obstacle while implementing TMSJ lesson plans. Since students come from diverse backgrounds, the social justice topics covered in the class might not align with their views. Mine conveyed this idea: “Every child grows up in a different environment and culture. There may be certain stereotypes. It can be difficult to break them.” Another student-related concern mentioned by some pre-service teachers is the unintended violation of sensitive topics. They pointed out that students may be upset by the situations in some activities that include social justice issues. To illustrate, Ceren stated:

For example, in the question, we say Ela will buy three apples, but since she has no money, she borrows from her family. I think this is a bad question. After all, we don't know the economic situation of the children. In the example, we say she has insufficient money and borrows from her family. For example, that child may not have a family. So, I think it is not appropriate to use such examples.

Regarding parent-related concerns, some pre-service teachers highlighted that social justice topics in mathematics teaching might contradict parents’ truth. In that sense, Leyla stated that: “Parents will create challenges. People are not one type; what is right for me will not be right for them.” Moreover, according to Selin, parents' high expectations of national exams can be another parent-related concern for teachers. She stated: “Parents will say that: Do we need to do these things, teacher? Let's solve more questions in the class. What will the children do in national exams?”

Some pre-service teachers also identified one school management-related concern. They mentioned that conventional school management can be an obstacle while implementing TMSJ lesson plans. Mira commented that I might have some challenges with an administration that is rigid and does not offer me the opportunities that I should be offered.

Some pre-service teachers' school-related concerns were based on the location of the school. They stated that location is crucial because it is also related to the culture of the school stakeholders. Can stated that where the school is located is important because not all places in the country have the same culture. He added that he could
have difficulties in other parts of the country except his hometown because he is unfamiliar with their culture.

Some pre-service teachers identified their curriculum-related concern as pacing with the curriculum. Mira stated that it is difficult to keep up with the curriculum even in normal conditions. Integrating social justice context into the problems while trying to teach mathematics would make it more difficult. She highlighted that it would not work in terms of pacing with the curriculum.

Finally, some pre-service teachers stated they would have difficulty in finding educational resources on TMSJ because they had never encountered such a resource before. For example, Ceren pointed out: “I think we won't find many resources because I think the focus of teaching mathematics is more on the theoretical parts. So, I think there will be no resources.”

4.2.1.5.2. PMTs’ Final Concerns for TMSJ

After participating in the TMSJ intervention, pre-service teachers developed their understanding of TMSJ; therefore, as in the case of opportunities, they indicated more concerns for TMSJ in the final interviews than in the initial interviews. In addition, since there will be differences in the implementation of RWWM and WWWM dimensions, the concerns they are supposed to encounter are examined under these headings.

PMTs’ Final Concerns for RWWM: PMTs’ concerns for RWWM are categorized as student-related, parent-related, school management-related, school-related, curriculum-related, and educational source-related, as in the initial concerns for TMSJ. Besides pre-service teachers’ concerns, their solutions for those concerns are explained in the following.

Student-Related Concerns: Regarding student-related concerns, some pre-service teachers emphasized that social justice topics discussed in the class might contradict students’ truth. Based on her experience, Mira commented that there might be
students who have been brought up in such a way that they are closed to new ideas. She also explained her experience while teaching that she had prepared a mathematics problem focusing on a social topic, but students said that they did not have to learn it. To overcome this issue, she proposed to cooperate with allies like school management, parents, counselors, and class teachers by stating:

*I will try to cooperate with the administration and especially with the parents as trying to overcome this issue. I cannot solve this problem on my own. What can be done? There may be parent meetings and additional seminars. I can get help from the administration while organizing these meetings and seminars. I will ask parents for their participation. I will work in cooperation with them. When I work in cooperation with them, the problems with the students will be reduced...I can talk with their class teachers and counselors. I have some ideas in my mind to solve this...*

As another solution to this problem, Umut focused on caring for the sensitivity to the culture of the community. He pointed out, "So, *I think the teacher's role here is not to live in the extremes but to draw a path close to the culture.*"

Another student-related concern, Can and Kerem mentioned was social justice topics may be regarded as overweighing mathematics for the students. Can conveyed the message: "*If we plan a mathematics lesson with too much focus on social justice, students may think that we are in mathematics class but not doing anything related to mathematics.*" As a solution to this problem, Can stated: "*The teacher should balance between social justice and mathematics outcomes.*" Kerem commented on a solution regarding building teacher authority and stated that the authority of the teacher is important. If the teacher is a little more serious in that lesson, it can be more effective if he/she gives the students a message about the seriousness of this social justice subject. In that sense, students would not try to disrupt the lesson or behave in a more informal manner in response to teachers’ authority.

A student-related concern emphasized by a pre-service teacher, Umut, was that RWWM might cause emerging cliques in the classroom. He communicated: "*After the lesson, for example, children may start judging their classmates based on their thoughts. [You don't think like this; we exclude you.] There may be groupings in the*
classroom.” He offered two solutions to overcome this concern: creating a positive climate for discussion and getting anonymous opinions on sensitive issues by stating:

> It is very crucial to manage the discussion in the classroom. Discussion on the topic within the framework of respect and love for people. Of course, as classroom management, it is necessary to set rules before starting the discussion. There may be rules like not interrupting people, starting by asking for the floor, and respecting others’ ideas. Maybe in sensitive issues, we can anonymously collect their ideas. For example, how can I do this? I'll hand out a piece of paper, little papers like. I can say write your opinions there, don’t write your names. So, it won’t be clear whose idea it is, but different ideas can be shared in class.

Some pre-service teachers emphasized that social justice topics may not be interesting for the students as a student-related concern for RWWM. To illustrate, Mine stated: “Maybe students won't like the social justice topic we discuss in the class. They won't even want to listen to it.” To overcome such a concern, she emphasized enabling students to empathize with the social justice topic and stated: “I try to make them feel like the topic is related to them. I think of my own lesson plan. It's about disabilities. I can say that you could be disabled. We are all candidates for disability. Therefore, I can talk to motivate them, such as I think you should pay more attention. It is a problem when they are stubborn. I need to say something to motivate them without offending them.

Leyla and Mira also commented that students’ low motivation due to standardized exams may be another student-related concern for RWWM. Leyla pointed out: “There will be many students who will not study that subject if it will not be on the exam. Because their actions are according to the standardized exams.” Actually, Leyla did not have a solution for this concern. However, Mira repeated her previous ideas on cooperating with allies (school management, parents, counselors, and class teachers) to overcome this issue because she stated that she could not solve this issue alone.

Lastly, a student-related concern that some pre-service teachers emphasize was an unintended violation of sensitive topics that also occurred in initial interviews. Defne commented on this issue:
Considering that we are in a disadvantaged class, we may have unintentionally touched on a subject that may hurt some students. For example, we are talking about something related to being disabled. In fact, it is a normal situation that should be in society. But maybe there is a disabled person in their own family, or they have a disability themselves. I think those students may experience bad feelings.”

Her ideas to solve this issue are based on having background information about students. Defne stated that if she would integrate a social justice issue into a lesson, she needed to know the background information of her students. In that sense, she could choose a topic that won’t really hurt them. For example, if there is a disabled or a non-native student or a refugee student in her class, she would not focus on these issues and would choose slightly different topics. Another solution to this concern explained by Umut was focused on getting support from a guidance counselor; he commented: “I think with the help of the guidance counselor, we can overcome this issue; this can be a solution.”

Parent-Related Concerns: In terms of parent-related concerns for RWWM, most of the pre-service teachers highlighted the idea that social justice topics may contradict the truth of the parents. This concern was also highlighted in the initial interviews. Leyla pointed out:

*Social justice issues may be contrary to the parents' views. For example, let's say I say something about immigration. There may be a parent who is against it and never wants immigrants to come to our country. According to him/her, what I teach the students may be wrong. It may cause problems in this regard. If it is different from his/her own mindset, he/she may reject it directly instead of raising awareness about it. I just want to say that if their thoughts are different, teachers and parents may have disagreements about that subject and that parents may create problems.*

As a solution, Leyla focused on avoiding controversial issues and stated that she would try not to mention those kinds of controversial subjects or sensitive issues. For example, there are tons of different opinions on immigration issues. Instead, she would try to discuss social justice issues that are relevant to everyone in general. As another solution, Mira highlighted organizing seminars for parents and stated: “In fact, seminars should be organized, and parents should be informed on social justice issues. This is not something that only teachers can overcome.”
Another parent-related concern that was also emphasized in the initial interviews was parents’ having high expectations on exams. Selin pointed out that parents might say that students are not learning mathematics in the class; they should learn mathematics and get good grades in the exam, and then they can learn these things later. Similarly, Can commented: “Parents may say that you talk about good things, but isn't this about social studies lessons? Why are you doing this in mathematics class? My child is not successful at exams”. To overcome this issue, Can has two suggestions, which are emphasizing to parents that he is preparing students for life, and he has program fidelity. He stated:

*I can actually say these to parents. Life is not just about school success. You also face some injustices in your life, as do your children. They can learn mathematics and also have a perspective on life... However, we are never behind in our mathematics class curriculum. We are doing this as an extra, and I can convince you that this is preparing the child for life faster and easier.*

School Administration-Related Concerns: In terms of school administration-related concerns for RWWM, some pre-service teachers emphasized that school administration may pressure curriculum fidelity. Mira commented: “There is a strict curriculum, and the management is constantly monitoring it. They give directives to the teachers as: “Do this, do that, don't bother with these issues,” especially in public schools.” As a suggestion for this concern, Umut commented on justifying himself through the sample lesson plans and stated: “I can present sample lesson plans to the management. I can say that I am teaching mathematics. I believe students should learn mathematics first, and my examples are related to social justice issues.” with such a dialog with school administration Umut thinks that he could solve the problems with the school administration; because school administration would realize that mathematics is on the focus. Leyla thought that when she is implementing TMSJ lesson plans in her classroom in the future, she may encounter some barriers from the school management; because of the fact that the social justice issues discussed in the class might contradict the truth of the school management in the similar vein to the situation with the parents. Again, as with the parents, Leyla said she would try to solve this situation by avoiding choosing controversial social justice issues in her classes.
School-Related Concerns: PMTs were also concerned about the difficulties they might encounter in the schools. They highlighted that other teachers might display negative reactions with the belief that it is unnecessary to integrate social justice issues into mathematics or other classes. Ceren emphasized the discussion in the lesson of TMSJ intervention and stated that as a result of this discussion, it was accepted that implementing social justice issues in mathematics class was not easy, so other teachers who did not know about the subject would have negative reactions. Besides, Leyla commented that other teachers would find her as a teacher too idealistic with the idea of believing to create a change by saying: “Other teachers may be prejudiced against me. They would think I would try to do this for a while. After that, I would get bored and tired, I was too idealistic.” To solve such a problem with the other teachers of the school, Ceren commented that she could share her sample lesson plans with the other teachers, and in that sense, she can share what she is doing in the classroom. Thus, Ceren believes she could change their negative reactions and take a step towards integrating social justice issues into their lesson plans.

In accordance with the idea of receiving negative reactions from other teachers, Ceren also emphasized that her students might receive negative reactions from other students of the school. Moreover, she communicated that her students might be bullied by other students and told by other students that what they have learned regarding social justice issues is wrong. She stated that, especially for male students, it might be reacted as that they are too emotional, and that might be considered as bullying for male students. To overcome this issue, Ceren, similar to her ideas in dealing with the negative reactions of other teachers, highlighted she could share her lesson plans with other teachers and try to collaborate with them regarding integrating social justice issues in their classes. In this way, if more teachers integrate social justice issues into their teaching, negative reactions from other students to their own students will decrease.

Lastly, some PMTs mentioned that a school context that is closed to change might be regarded as another concern. In that sense, Defne particularly emphasized the importance of the context of the school and highlighted that in a school in a
disadvantaged environment, what is said may remain in the classroom, and no change may be observed in students' ideas or behaviors. Mira commented on this issue:

*I teach them mathematics with integrating social justice issues, but in their groups of friends, they exclude someone, they bully; because this is considered normal in that school environment. If the school context is contrary to what I do, things won't change.*

To solve this concern, Mira pointed out that she could collaborate with other teachers and the school administration to make all of the students of the school more aware of social justice issues. She stated firstly, teachers and school administration should be aware of these issues through necessary training. In this way, in an environment where everyone speaks a common language, awareness raising can be observed in students.

Curriculum-Related Concerns: The only curriculum-related concern emphasized by pre-service teachers is pacing with the curriculum. Umut communicated that since social justice issues are also discussed during mathematics classes, time is of the essence. It might create problems in terms of catching up with the curriculum. To overcome this, he states that he could succeed in raising students' awareness of social justice issues by including a social justice issue in the context of the examples in the classroom or by including a social justice context to all the problems in the assignment. Similar to Umut’s ideas, Ceren also highlighted assigning homework questions to the students based on a social justice context. Moreover, she argues that students' awareness of the social justice issue can be increased by a discussion about this homework in the next lesson.

Educational Resource-Related Concerns: PMTs stated that they would have difficulty finding educational resources that include social justice context in mathematics lessons. Can stated that he researched before preparing a TMSJ lesson plan, but he could not find any resources, whereas when he was searching for lesson plans related to mathematics topics, he could find many examples of lesson plans. Other pre-service teachers stated that they had not encountered any resources
integrating social justice issues into the mathematics course before, that these resources were not very common, especially in Turkish, and that they thought they would have difficulties accessing the resources in this sense.

PMTs’ Final Concerns for WWWM: PMTs’ concerns for WWWM are categorized as student-related, parent-related, school management-related, school-related, curriculum-related, and as in the initial concerns for TMSJ. Besides pre-service teachers’ concerns, their solutions for those concerns are explained in the following.

Student-Related Concerns: Having uninterested students take action on social justice issues is the most prominent concern that PMTs mentioned. They also communicated that being uninterested could be due to various reasons. For example, Ceren explained that the reason why students do not want to take action might be because of procrastination, while Can stated that students who become aware of the social justice issue will not want to take action because they think that they will not be graded from WWWM outcomes in the mathematics lesson. To overcome such problems while implementing TMSJ lessons, Defne commented that choosing topics that attract students might be a solution. Thanks to this, students may participate more in WWWM activities because they are interested in the social justice topic and want to reach a conclusion about it. Moreover, if the topic does not directly have an effect on students' lives, Can commented that he could make students empathize with the social justice topic. In this way, when students empathize with social justice issues, they will be more willing to take action to contribute solutions to them or at least raise others’ awareness of that issue. Ceren highlighted the idea that she would not force students to participate in WWWM activities; she would work with willing students. She would share what they do during those activities and this would probably draw uninterested students.

Another student-related concern PMTs emphasized was that students might consider taking action as an extra burden. Students may be more likely to react, especially if other teachers in the school do not provide such activities. Kerem commented on this as students might communicate with him: “Other teachers do not make such activities. Why are we doing it?” Selin stated that to overcome this concern, actions
can be determined based on the students' ideas, and these actions can be taken together because students will enjoy taking part in processes in which the teacher plays an active role. Moreover, Kerem also stated that collaborating with other mathematics teachers in the school and doing the same activities throughout the school year would make students feel more comfortable in WWWM activities.

Parent-Related Concerns: The most prominent concern regarding the parents was contradicting their truth for RWWM. Actually, when we focus WWWM, PMTs also declared the similar concerns in terms of WWWM that actions taken might contradict parents’ truth. Ceren commented on this issue as I might get negative feedback from the parents, because what we try to do contradicts their truth. While taking action, students might volunteer in some NGOs, and some parents may not want their children to volunteer in those organizations. To overcome this issue with parents, Mine suggested informing parents at parent-teacher conferences about her views on mathematics education and then asking for their support. By informing parents in the parent-teacher conferences, they would not be surprised about what is happening in the mathematics class, and they will learn what the teacher aims by doing different activities. Umut communicated that he could take action in small paces in terms of implementing TMSJ lesson plans. In that sense, he aims that what he does in the class should be suitable for that community where students come from.

Another parent-related concern Kerem stated was parents might think that actions taken during WWWM are political. He stated: “For example, if we want to work in partnership with a charity organization, most of them have a connection with political parties. Parents might assume that this teacher supports political party A or political party B. So, I could also get into conflict with the parents.” Kerem was concerned about this, but he did not have a solution to this concern.

School Administration-Related Concerns: The concerns emphasized by pre-service teachers about school administration were that the administration might not allow in-school or out-of-school actions. In terms of in-school actions, some pre-service teachers mentioned that the school administration might not allow the exhibition of
products in the school. The pre-service teachers stated that displaying posters to raise awareness about a social justice issue among other students might not be desirable on the board of the classroom or the school, and they might also have difficulties in obtaining permission for some activities that could be done within the school. Mine stated that she could collaborate with the school administration and parents. Defne commented that she could share good practices with the school administration and that she would try to convince the school administration by showing the benefits of these actions to students through good practices.

In terms of out-of-school concerns of the school management, pre-service teachers mentioned that the school administration can bring up things like the difficulty of getting permission and resource issues when going on a school field. Leyla commented that if she could build confidence in her teacher's capability, the school administration would be more open to out-of-school activities.

School-Related Concerns: In terms of WWWM, pre-service teachers commented that they could get negative reactions from other teachers because their students might also want to take part in such activities and demand them from their teachers. Can commented that other teachers would see this as an extra burden and complain about it. To overcome such problems in terms of WWWM, Mine emphasized that she could share her instructional resources with the other teachers and encourage them by sharing what she was doing in class with them.

Curriculum-Related Concerns: Like in RWWM, in terms of WWWM, the only concern that pre-service teachers mentioned is pacing with the curriculum. Mine commented that, especially for WWWM activities, there is a need for extra time, and she might have some problems while implementing WWWM activities in class. In a similar vein, Defne commented that there is a need for extra time in terms of WWWM. As a solution, Umut mentioned that he could give homework on some parts of the WWWM activities to the students so it would not take too much time in the lesson. So that students can do something regarding WWWM themselves and obtain social justice outcomes.
4.2.1.6. PMTs’ Initial and Final Views on Roles of a Teacher

Even if PMTs did not know what TMSJ is as a definition and were unfamiliar with the concept, they were asked questions about their roles as teachers to find out whether there were some similarities between their thoughts about teaching and TMSJ in the initial interviews. Their final views on this topic were taken from 1st Reflective Paper (which included a question related role of a teacher beyond teaching subject matter by giving examples) and from the questions in the final interview with them.

4.2.1.6.1. PMTs’ Initial Views on Roles of a Teacher

In the first interview, pre-service teachers were asked a question about the areas in which they would like to contribute to the development of their students other than teaching mathematics. Pre-service teachers’ answers to this question are categorized under two sections: supporting individual development and supporting social development.

To begin with, in terms of supporting individual development, some pre-service teachers aimed to promote their students’ sensitive thinking. For instance, Mine commented:

I want my students to be sensitive to the environment, to animals, to people. So, it's not just going into the classroom and explaining math. I would like to teach them how to be a good person and be sensitive. Knowing math doesn't mean they are good students or people.

In addition, two of the pre-service teachers aimed to promote their students’ critical thinking skills. One of them commented: “Of course, first of all, I would like to raise individuals who question and do not accept what they see directly.”.

Two of the pre-service teachers, Kerem and Mira, also aimed to develop their students in terms of self-awareness. They wanted their students to orient their interests, and Mira explained this: “I want them to pursue their own interests. I want
them to recognize their interests and talents and pursue them.” Similarly, Kerem commented that he would like to “raise individuals who can pursue their own interests.”

As a last point in supporting individual development, one of the pre-service teachers, Can aimed to develop his students’ interpersonal skills by stating: “I think it is important to develop interpersonal skills. I would like to help students in this regard. I think I would like to improve them to make it easier for them to communicate with individuals when they go to university.”

In terms of supporting social development, there were not many aims that pre-service teachers wanted to develop in their students. Umut had a static view of citizenship. Therefore, he wanted to “raise good citizens who are beneficial for the country and the nation.” On the other hand, without emphasizing citizenship Mira aimed to “raise students who are beneficial to the society.”

4.2.1.6.2. PMTs’ Final Views on Roles of a Teacher

During the TMSJ intervention, pre-service teachers wrote a reflective journal after I introduced the topics of Teaching for Social Justice and Teaching Mathematics for Social Justice. There was a question in that reflective journal and it was: In your opinion, what’s the role of a teacher beyond teaching the subject matter? Please explain in detail with some examples. This question was aimed to examine PMT ideas on the non-academic roles of a teacher after discussing some important issues in terms of TSJ and TMSJ. The findings in this section are from pre-service teachers’ answers to that reflective paper and final interviews.

After the discussions and tasks pre-service teachers completed during TMSJ intervention, their views on the roles of a teacher changed in some respects. To begin with, the most prominent view on the role of a teacher was that he/she should raise awareness of social justice issues in the final interviews, although no one mentioned raising awareness of social justice issues in the initial interviews. In order to accomplish this, pre-service teachers stated teachers could integrate social justice
issues into their lessons. Mine commented that: *If I include real-life examples of social justice issues in my teaching, I can raise my students’ awareness of these issues.*

Promoting a positive classroom environment was another point that pre-service teachers mentioned regarding the roles of teachers. While describing this positive classroom environment, Ceren and Leyla focused on the idea that “everyone should have a voice in the classroom and share their ideas freely.” Umut, Can, and Defne emphasized, *"Everyone should respect each other’s ideas."* Kerem stated that a teacher should care about individual differences; he/she might have students with disabilities or immigrant students, and all of these students should feel cared in the classroom. Lastly, Mine pointed out that students might have differences but would be accepted into this positive classroom environment with their differences.

In the initial interviews, only one pre-service teacher, Can, mentioned that he aims to develop his students’ interpersonal skills. In the final interviews, more pre-service teachers commented about developing students’ interpersonal skills. Kerem states that a teacher should create opportunities for students to communicate with each other and encourage them to express themselves. In a similar vein, Selin commented that a teacher should guide his/her students on how to behave in social relationships or communicate with their friends.

Mira and Kerem emphasized that teachers could empower students to make changes in society. Mira commented that students could play a major role in creating a better society. Similarly, Kerem stated that: *We usually underestimate students' power to change society. If a teacher raises awareness of this, it is an important step towards a better society.*

Another important point regarding views on the roles of a teacher is that some pre-service teachers mentioned that a teacher should be a role model for the students. Mira commented on this: *Teachers can be role models for students by showing positive attitudes in terms of communication skills.* Moreover, Selin conveyed her message: *If a teacher says, "Don't belittle your friends who make a mistake," and*
then belittles the students who cannot solve the question correctly, this will negatively affect the student's behavior.

4.2.1.7. PMTs’ TMSJ Lesson Plans

Pre-service teachers developed a mathematics lesson plan focused on a social justice context as a learning outcome of TMSJ intervention. TMSJ Lesson Plan Evaluation Rubric was developed to evaluate their lesson plans. In this section, pre-service teachers’ lesson plans were examined based on this rubric. While developing their TMSJ lesson plans, they first chose a social justice topic and a mathematics topic that they wanted to connect. Then, before the last lesson, they shared their ideas with their classmates and gave each other feedback on the appropriateness and feasibility of their draft lesson plans. They also received feedback from me.

Table 4.2. Pre-Service Teachers’ Lesson Plan Topics and Grade Levels

<table>
<thead>
<tr>
<th>PMT</th>
<th>Social Justice Topic</th>
<th>Mathematics Topic</th>
<th>Grade Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can</td>
<td>Vegan Meal Prices</td>
<td>Ratio &amp; Proportion</td>
<td>7th Grade</td>
</tr>
<tr>
<td>Ceren</td>
<td>Immigrants</td>
<td>Percentages</td>
<td>7th Grade</td>
</tr>
<tr>
<td>Define</td>
<td>Food Waste</td>
<td>Scientific Notation of Numbers</td>
<td>8th Grade</td>
</tr>
<tr>
<td>Kerem</td>
<td>Cyber Bullying</td>
<td>Probability of Simple Events</td>
<td>8th Grade</td>
</tr>
<tr>
<td>Leyla</td>
<td>Erosion</td>
<td>Least Common Multiple &amp; Greatest Common Divisor</td>
<td>8th Grade</td>
</tr>
<tr>
<td>Mine</td>
<td>Employment of People with Disabilities</td>
<td>Mean, Median, Peak Values</td>
<td>7th Grade</td>
</tr>
<tr>
<td>Mira</td>
<td>Visually Impaired People</td>
<td>Least Common Multiple &amp; Greatest Common Divisor</td>
<td>8th Grade</td>
</tr>
<tr>
<td>Selin</td>
<td>Bullying</td>
<td>Decimal Notation &amp; Fractions</td>
<td>5th Grade</td>
</tr>
<tr>
<td>Umut</td>
<td>Food Insecurity</td>
<td>First Order Equations</td>
<td>7th Grade</td>
</tr>
</tbody>
</table>

Table 4.2. contains information about which social justice and mathematics topics the pre-service teachers chose and at which grade level they prepared TMSJ lesson plans. As seen in the table, pre-service teachers chose very diverse topics regarding social justice issues. Only two of them focused on the same mathematics topics as Least Common Multiple and Greatest Common Divisor; all the others had chosen
different mathematics topics. These findings revealed that pre-service teachers could make relations with diverse social justice and mathematics topics as aimed with the TMSJ intervention.

Table 4.3. Lesson Plan Evaluation Rubric Scores

<table>
<thead>
<tr>
<th>PMT</th>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can</td>
<td>2.7</td>
</tr>
<tr>
<td>Ceren</td>
<td>2.7</td>
</tr>
<tr>
<td>Defne</td>
<td>2.8</td>
</tr>
<tr>
<td>Kerem</td>
<td>2.7</td>
</tr>
<tr>
<td>Leyla</td>
<td>2.5</td>
</tr>
<tr>
<td>Mine</td>
<td>2.8</td>
</tr>
<tr>
<td>Mira</td>
<td>2.9</td>
</tr>
<tr>
<td>Selin</td>
<td>2.8</td>
</tr>
<tr>
<td>Umut</td>
<td>2.8</td>
</tr>
</tbody>
</table>

The lesson plan evaluation rubric I developed consists of 10 items. These items include teachers’ background knowledge related to social justice issues to see what the teachers’ point of view regarding that social justice issue. Also, there are items related to the relevancy of mathematical learning outcomes with social justice goals according to the information of target students and the balance between mathematical learning outcomes and social justice goals. Target students are important because they provide insight regarding the relevance of that social issue to their lives. Assessment was another significant criterion for evaluating TMSJ lesson plans; moreover, it should have two dimensions: social justice and mathematics. Lastly, clarity of the teaching strategies and including RWWM and WWWM dimensions are other criteria for evaluating TMSJ lesson plans. Pre-service teachers could get scores from 0 to 3 (0= Does not meet expectations, 1= Needs Improvement, 2= Meets Expectations, 3= Exceeds Expectations). In Table 4.3, their lesson plan evaluation rubric scores are given. As can be seen from the table, the lowest score is 2.5 and the highest score is 2.9 for pre-service mathematics teachers according to TMSJ lesson plan evaluation rubric. When these scores out of 3 are evaluated, it can be said that in general, all lesson plans are in line with the exceeds expectations category.
After examining pre-service teachers’ lesson plans, they all provided background information, information about target students and the benefits of the TMSJ lesson plan. This provides an opportunity to understand the context in which they are going to implement TMSJ lesson plans. Can, Ceren, and Defne, Kerem, Leyla did not provide a detailed explanation of the background of social justice issues, but it was adequate. The second item was related to the relevancy of mathematics learning outcomes for the grade level. All the pre-service teachers obtained the learning outcomes from the Mathematics Curriculum (MoNE, 2018). However, in Mine’s and Leyla’s TMSJ lesson plans, the activity's prerequisite knowledge sometimes became the dominant learning outcome. Therefore, which mathematical topics will be connected to a social justice issue is critical while writing TMSJ lesson plans.

The TMSJ lesson plans included RWWM and WWWM dimensions i.e., raising awareness and taking action on social justice issues. In terms of alignment of social justice content with the mathematical content, pre-service teachers’ lesson plans were satisfactory except for Leyla; sometimes, in her lesson plan, social justice goals and mathematical learning outcomes do not progress smoothly. In the lesson plans of the pre-service teachers, social justice content and mathematics content were balanced, but in Ceren's lesson plan, social justice was slightly more dominant. Pre-service teachers explained their instructional strategies clearly, and they mentioned how they were going to assess social justice goals. Can’s and Kerem’s TMSJ lesson plans were missing in assessing mathematical learning objectives because both of them ended the lessons with taking action part, and they did not focus on how they were going to assess mathematical learning objectives.

4.2.2. Mira’s Initial and Final Understanding and Changes in her Understanding and Views regarding TMSJ

Mira was one of the pre-service teachers who attended the TMSJ intervention. Mira is a pre-service mathematics teacher who states she is very happy when she teaches someone something and enjoys teaching very much. Therefore, she chose this department of her own will and with the guidance of her family, who know their daughter's potential. She was chosen to reflect on pre-service teachers' understanding
and lesson plan development process regarding TMSJ, because she actively participated in class discussions and group work and explicitly shared her opinions. Differences were observed in Mira’s understanding between the answers she gave in the initial interview, and the reflective papers and the final interview after attending the TMSJ intervention. This section presents how classroom materials, activities, and interactions influenced Mira’s understanding of TMSJ.

4.2.2.1. Mira’s Initial and Final Understanding of the Concept of TMSJ

Mira’s Initial and Final Understanding of the Concept of TMSJ was revealed through the initial and final interviews correspondingly. In the initial interviews, she was asked how she would define the concept of TMSJ. Her initial definition was based on her predictions. In the final interviews, she was again asked a question on how she would define the concept of TMSJ.

4.2.2.1.1. Mira’s Initial Understanding of the Concept of TMSJ

In the initial interview, Mira stated that she had not encountered the concept of TMSJ before. Her initial thoughts focused more on equal opportunity in mathematics education. She also emphasized that caring for individual differences. Moreover, she stated that social justice might be achieved through mathematical tasks, the teacher's attitudes, and the opportunities offered to students. As can be seen, her initial ideas varied. She made some predictions, but these predictions had very different characteristics. According to the findings of the initial interview, it is seen that she was not very sure about where to position the concept in her mind.

4.2.2.1.2. Mira’s Final Understanding of the Concept of TMSJ

In her final interview, Mira commented that social justice is a concept she looks for in the lesson plans and the questions she examines. For example, in the lesson plan that she taught at the internship that day, she talked about air pollution; she also mentioned that the use of electric scooters is beneficial to the environment. She highlighted that electric scooters are good for us, but without harming other people.
Mira explained that she was constantly thinking about how she could prepare a lesson plan integrating a social justice issue.

She defines TMSJ as a case of hitting two birds with one stone, both teaching mathematics learning outcomes and social justice consciousness. With this statement, she emphasized the dimension of RWWM, which is related to raising awareness of social justice issues. In addition to this, she emphasized that integrating social justice context could make the lesson more interesting as it will include current events. She highlighted that, in that way, she can teach mathematics easily. She also focused on the WWWM dimension, stating: “I think that change starts from childhood; thanks to TMSJ, as mathematics teachers, we can cause important changes in society with our students.”

4.2.2.1.3. Change in Mira’s Understanding of the Concept of TMSJ

When PMTs’ initial and final understanding regarding the Concept of TMSJ is examined, it is seen that their understanding is developed. Their initial understanding is based on their predictions because they had no prior knowledge regarding TMSJ. How the TMSJ intervention materials, activities, and discussions shaped PMTs' understanding of the Concept of TMSJ is analyzed by the case of Mira.

In the second lesson, which consisted of two class hours, after introducing teaching for social justice in the first lesson, PMTs are introduced to the concept of TMSJ and critique the power of mathematics and mathematics teaching.

Two mathematics problems from Gutstein & Peterson (2006) are used as introduction questions to the TMSJ. They had similar results in terms of mathematical operations, but the context of them was crucial in terms of the message.

A math problem is shared with the PMTs at the beginning of the lesson:

- A group of youth aged 14, 15, and 16 go to the store. Candy bars are on sale for 13 TLs. They buy a total of 12 candy bars. How much do they spend?
A discussion about this question was as follows:

Umut: Why did he give their ages?
Mira: It is a confusing bit of information.
Can: Is it directly proportional to their age?
Researcher: How much money did they spend?
Umut: 12*13

Pre-service teachers started to question the context of the problem with this example.

After this question, another mathematics problem is shared with PMTs:

- Factory workers aged 14, 15, and 16 make children's clothing. Each worker earns 13 TLs an hour and works 12 hours daily. How much does each worker make in one day?

Researcher: What did the question make you think?
Mine: Why are fourteen, fifteen, and sixteen-year-old children working?
Researcher: They also make children's clothes.
Selin: And 13 liras for an hour.
Researcher: Right?
Selin: They are working; at least they shouldn't work for 13 liras for an hour.
Researcher: What was emphasized in the first question?
Mira: Eating too much chocolate?
Researcher: You are close, excessive consumption and unhealthy eating. What is the message of the second question?
Umut: Students’ ages are probably similar to the ages of the children in the problem.
Researcher: What can be the reason for asking this question?
Umut: Empathizing.
Researcher: What else?
Mira: Raising awareness about child labor.
Kerem: I thought the opposite; I thought such a problem should not be asked in the classroom; it’s like normalization [child labor].
Mira: That's what I thought.
Researcher: It becomes normalized if they directly ask it like this and just do it thirteen times twelve times and pass on to the next question. But you can ask the students, "Do you think there is something different about this question? Did something catch your attention?" and maybe you can provide that awareness.

………………[discussion continues on child labor]

It was a guided think-aloud time with the pre-service teachers. The contexts they encountered in these sample mathematics problems served as examples of how they could integrate the social justice context into problems. Kerem's question about
whether such questions normalize injustices was very crucial in the flow of the discussion. It is seen that Mira was also confused about integrating such context into problems. After Kerem’s question, the researcher explained to the pre-service teachers how not to normalize such questions.

Researcher: After looking at these problems. Do you think mathematics teaching is neutral? Are we neutral as mathematics teachers?
Mira: We are not.
Selin: Exactly, we can't be neutral. It is obvious from the problems.
Researcher: Is it obvious from the previous problems? What do you think?
Selin: We can actually give the idea within the problem. For example, after solving the problem, if we ask: do you know anyone like this? or do you think there is something strange here? Asking these children will raise their awareness, and in this way, they learn mathematics. And he also solves the problems. So, it makes sense.
Mira: We took a book analysis course. We analyzed the book in this way. Even in the question, even in the photo next to the question, you know that it is not neutral. They are trying to give a message. In fact, many messages are tried to be given through mathematics education. That’s why I don't think it's neutral, it's even in the books.

With these two questions congruent with the philosophy behind TMSJ, critical pedagogy, it is observed that students criticized teaching mathematics. This dialog showed the idea of teaching mathematics is not neutral for PMTs. Mira connected her prior knowledge with the discussion and rationalized her thoughts according to her previous learning. Sample mathematics questions and class discussions might have contributed to PMTs’ understanding of TMSJ.

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<thead>
<tr>
<th>Film</th>
<th>Miray’s Preference</th>
<th>Sena’s Preferences</th>
<th>Hakan’s Preferences</th>
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<td>Kurak Günler</td>
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Figure 4.4. Mathematics Question with Social Justice Focus
After this discussion of two mathematics problems, PMTs are given another mathematics question (Figure 4.4.) regarding TMSJ to solve.

Class discussion related to the question in Figure 4.4. was as follows:

**Researcher:** What do you think about this question?
**Umut:** We are doing addition and subtraction, and we are also using the average. I'm actually doing this addition and subtraction or finding the average for people's choices to find the most appropriate option so that all people can meet on a common agreement without realizing it.
**Mina:** Can we call them real-life problems?
**Researcher:** Yes, and what else?
**Umut:** It is done for the democratic decision-making process.
**Researcher:** So, this is actually just a mathematics question when you look at it, but when you look at the context, it makes you think of different things, right?
**Ss:** Yes.
**Researcher:** Do you have anything else to add? What can you say?
**Mina:** Also, when children encounter this question, they wonder which movie Miray, Sena, or Hakan should go to and try to solve it. Maybe they don't even realize that they are doing mathematics. It becomes interesting for children.
**Researcher:** Ok. If you give examples from current movies, these are also the ones in the movies right now. There is a joint decision-making process in the question. The decision is made according to everyone's choice. We use mathematics in this too. So, after looking at the examples, what did TMSJ make you feel?
**Umut:** We could have given social justice outcomes without making students realize it.
**Mira:** As we said at first, we have never seen anything like TMSJ; we were unfamiliar with it. We didn't emphasize social justice issues much because we were unaware of them. From now on, instead of just solving the question like that. Did you notice anything? What is something else you noticed in these questions? We can touch on the issue of social justice.
**Kerem:** With the help of mathematics, we can address everyday problems and problems that we can experience everywhere. We can overcome it with social justice. Ultimately, we are neither far from Sena's nor Hakan's preference. We did this with mathematics. We can use social justice issues in the context of the problems.
**Defne:** It's not what one person wants. It's all democracy. That's good, a joint decision.

In the needs analysis, it is seen that PMTs were familiar with using different contexts in mathematics problems, but there was no social justice focus. This dialog with the
previous dialog shows that they find using social justice in the context of the problems useful. Problems, including social justice context, made them think they could raise awareness and make mathematics learning interesting for the students. Also, Mira stated that she became aware of using social justice context in the problems and would use such context to raise her students’ awareness.

After these sample problems, I shared with students many definitions of teaching mathematics for social justice by different scholars. After having a small discussion on which definition was closer to them, I shared with them a poster about TMSJ (Figure 4.5) and asked them what it meant to them.

Class discussion on the poster:

Kerem: There is a person who has been discriminated against. He is a minority.
Mira: If a strong person is against you, his/her followers will also be against you without any reasoning about whether he is wrong or right.
Researcher: Let's also focus on the writing.
Selin: Ninety-nine percent is more than one percent. But one percent is actually not that different from ninety-nine percent.
Researcher: Focus a little more on the “use your numbers” part.
Can: What we can do with words, we can do with numbers.
Researcher: What do we use numbers for?
Can: Equality.
Researcher: Equality for whom?
Can: The equality between one percent and ninety-nine percent. What one person thinks is also important. Just because ninety-nine percent think the same thing doesn’t mean that the other doesn't have a say.
Researcher: You are closed.
Kerem: He wrote "them" there.
Researcher: Are you talking about marginalization? Did I understand correctly?
Kerem: Exactly.
Selin: Ninety-nine percent of society can be the same, but we cannot be one hundred percent without that remaining one percent.
Researcher: There are arrows in the picture. Also, consider them.
Can: We are 99 percent and can provide equality with numbers.
Researcher: Much closer. It is generally thought that things like seeking rights related to social justice can always be expressed in words, right?
Ss: Yes.
Researcher: So, we can show inequality by using numbers. Here it says that you can be ninety-nine percent. You can be the majority. But you still need to use these numbers for one percent.
Until this poster, pre-service teachers encountered mathematics problems that aimed to raise awareness in the social justice context, but in this poster, it was more about taking action part, that is, WWWM. This visual aid and classroom discussion with guided questions of the researcher contributed their understanding of TMSJ developed.

4.2.2.2. Mira’s Initial and Final Understanding of RWWM

Mira’s initial and final understanding of RWWM was revealed through initial and final interviews and 2nd reflective paper. In the initial interviews, since she was not familiar with the concept of TMSJ, she was asked a question related to the meaning of it. In the final and 2nd reflective paper, she was directly asked about her understanding of RWWM.

4.2.2.2.1. Mira’s Initial Understanding of RWWM

In the initial interview, Mira was asked how mathematics teaching could have an impact on raising students' awareness of social justice issues. Her first answer to this
question was that she had no idea because she had no previous knowledge of the subject. However, later in the interview, she makes some comments regarding RWWM. For instance, she emphasized that she can use social justice in the context of the problems. She tried to give an example to illustrate the way of her thinking. Her example was based on a context where a school administration provides students with free books and transportation, but the other school does not provide these opportunities. Her question for this context was based on the differences between students. However, she could not form a clear question for the context. She mentioned that she can utilize such real-life context in the problems. Even if her ideas were not in a framework, she had some reasoning and ideas regarding RWWM during the initial interview. Actually, the fact that she first said that she had no idea about RWWM and then began to comment on issues related to RWWM during the interview indicates that she started to think about how she could connect social justice and mathematics teaching with the questions asked in the interview.

4.2.2.2.2. Mira’s Final Understanding of RWWM

Mira’s final understanding of RWWM was obtained from the final interview and 2nd reflection paper. In the initial interview, Mira said that she had no idea when she was asked about her opinions regarding RWWM, and then as the interview progressed, she proposed some ideas about how to raise awareness of social justice in students. In the final interview, her views regarding RWWM were clearer. She defined RWWM as raising awareness of social justice issues through mathematics. She pointed out that students should have knowledge about a social justice issue first, and then they can realize how to take action to solve that issue accordingly. She emphasized that with RWWM, students could learn about a social justice issue, the shortcomings of that issue, problems that occur, and how society behaves according to that issue; this can be done by observing the data and doing research; thereafter, students will gain awareness.

4.2.2.2.3. Change in Mira’s Understanding of RWWM

Mira’s understanding of RWWM was not clear in the initial interview; however, she had a more developed understanding in the final interview.
In lesson 3, students were exposed to a TMSJ lesson plan I had developed on unemployment rates. The data for this lesson plan was taken from TUIK. This lesson plan aimed to raise pre-service teachers' awareness about how they would feel in a TMSJ lesson if they were students. After completing this lesson about unemployment rates, and discussing how they are calculated, a detailed lesson plan developed by Powell (2020) was given to the students as an example. This lesson plan was chosen as an example because its subject was about the precautions to be taken before and after the disaster. I wanted to show this lesson plan to the pre-service teachers as an example because I thought that this awareness was especially important after the February 2023 earthquakes. In fact, this lesson plan was planned for a high school course and focused on disaster as social justice and statistics and probability as mathematics. After the lesson plans were distributed to the pre-service teachers, they worked in groups and were given an activity sheet to answer about this lesson plan. In this activity sheet, they were asked to indicate the social justice goals and mathematics outcomes of the lesson plan and explain if there was anything that was interesting to them in this lesson plan.

I didn't want to directly give them the definitions of RWWM and WWWM. I aimed for them to create the categories themselves, thinking that there could be different categories related to social justice outcomes through an example, and then I aimed to name these categories. First, the students found the social justice goals in the example, I wrote the goals that the students said on the board, and then the discussion was as follows:

Researcher: When you look at these goals? Can you form different categories? Can you group some of them?
Leyla: Like before and after.
Researcher: What do you mean?
Leyla: We said that they should be prepared before the disaster happens.
Researcher: You mean before and after the disaster?
Leyla: What can we do before? We will reach a lot of people. We will create awareness. After the disaster, we can participate in NGOs.
Researcher: Yes, what else?
Umut: Singular and plural?
Researcher: What do you mean?
Umut: Individual aid, the aid provided by NGOs and the state.
Researcher: I think we can say individual and social.
Umut: Exactly
Researcher: What else?
Ceren: Directly and indirectly help?
Researcher: Don't think in terms of aid. Here, you are trying to support students in two ways, you know, in terms of social justice; think about categorizing it.
Ceren: Physical and, well.....
Define: Emotional?
Ceren: Exactly, whatever the opposite of physical is, spiritual.
Researcher: Okay, let's make it a little clearer. What are we trying to do emotionally? We can discuss this.
Ceren: Actually, I think emotionally, there is awareness or something like that. Physically, it is to volunteer for an NGO.
Researcher: Exactly. What was written in the lesson plan?
Ceren: Take action.
Researcher: Yes, that's actually what I wanted to ask. We can actually categorize them into two groups. The first one is about raising awareness. The second one is to make students take action, like becoming a member of an NGO or making them talk to their school administration in order to find out what kind of plans they have. It takes an approach in which students take an active role. This comes from Gutstein. He divided the TMSJ into two: RWWM and WWWM. He defines the part where we first become aware of as RWWM and he defines WWWM as the part where we take action. Are we clear on this?
Ss: Yes.

Mira did not take an active role in this conversation, but she listened to it as actively as the other participants of the class. At the end of this discussion, the pre-service teachers defined the categories through a sample lesson plan that included both RWWM and WWWM dimensions. Then the discussion followed:

Researcher: If we want to raise awareness of our students, what can we do?
Mira: We can use movies.
Researcher: Yes, it can be.
Kerem: We can do it like in this lesson plan.
Researcher: Yes, what was this lesson plan based on?
Mira: Being fair with data.
Kerem: Article.
Researcher: Yes, was the data true or not?
Umut: It was true.
Researcher: We are talking about real data, right? By using real data, we can create awareness in our students about different social events. This is RWWM.

It is seen that Mira, who did not actively participate in the initial conversation, is active in giving examples about RWWM in the rest of the conversation. Sample lesson plan and the class discussion developed her understanding of RWWM.
4.2.2.3. Mira’s Initial and Final Understanding of WWWM

Mira’s initial and final understanding of WWWM was revealed through initial and final interviews and 2nd reflective paper. Given her unfamiliarity with the TMSJ concept during the initial interviews, she was questioned about its meaning. In the final interviews and the second reflective paper, she was directly questioned about her understanding of WWWM.

4.2.2.3.1. Mira’s Initial Understanding of WWWM

Mira was asked about the role of mathematics teaching in making changes related to social justice events regarding WWWM in the initial interview. She stated that students who gain awareness can make changes related to social justice events. Mira emphasized that her future students, as members of society, can reach people from diverse backgrounds, share their awareness with those people, and influence their opinions. This can be an example regarding WWWM, but it does not encompass various aspects of WWWM. In Mira’s thoughts, there is not a focus on students taking action to make change in societal issues, in this sense, her thoughts about WWWM are limited in the first interview.

4.2.2.3.2. Mira’s Final Understanding of WWWM

In the final interview and in the 2nd reflective paper, her ideas regarding WWWM were asked. In the initial interview, Mira commented that her students could make a change regarding a social justice issue by talking to someone and influencing their thoughts. In the final interview, her views were more detailed. Mira directly stated that WWWM is taking action on social justice issues through mathematics. She commented:

*After RWWM, I'm moving on to action by WWWM. It is using various ways to bring others to an awareness of a social justice issue. These ways can be preparing a poster for others or using the power of social media to recognize the issues, especially in this last period. If we tell one person about a social justice issue and that one person tells a few people, reaching people like that is actually part of taking action.*
In her initial interview, Mira only focused on interacting with other people and influencing their opinions on social justice issues. In the final interviews, she focused on a variety of ways to take action which indicated the development in her understanding of WWWM.

### 4.2.2.3.3. Change in Mira’s Understanding of WWWM

Pre-service teachers’ understanding of WWWM was realized together through the class discussion I mentioned in the title of 5.2.2 Mira’s Initial and Final Understanding of RWWM. The following discussion is from their group discussion, where they tried to find social justice goals and mathematics outcomes of the sample lesson plan.

*Leyla:* Students can gain awareness, but how do they incorporate it into their daily lives?
*Mira:* Taking action part is interesting. Different from normal lesson plans. Can we write it here?
*Leyla:* Sure. We can write about taking action...
*Mira:* Taking action part at the end of the lesson plan is important...
*Ceren:* The lesson plan makes students take action...
*Leyla:* There are two here?
*Mira:* Exactly. provide students taking action...
*Leyla:* The progression of the questions, he doesn't want them to come to a conclusion at once, for example, but by making them realize and question. The preparation of the questions is important. Did you understand?
*Mira:* I understand.

[Researcher comes]

*Mira:* Are we going to prepare a lesson plan like this?
*Researcher:* Yes.
*Ceren:* This is different from the lesson plans we prepared in class.
*Mira:* It is really different
*Ceren:* Why didn't they teach us like this? It's not strict, it's flexible.
*Leyla:* What we do is easier; at least you know what to do.

In this group work, pre-service teachers tried to find the social justice goals in the lesson plan, especially the taking action part that attracted their attention. Mira stated that taking action is different from what they do in the lesson plans they prepared. As in the case of RWWM, sample lesson plan and the class discussion and group discussion developed her understanding of WWWM.
4.2.2.4. Mira’s Initial and Final Views of Opportunities of TMSJ

Mira’s initial and final views of opportunities at TMSJ were revealed through initial and final interviews. In the initial interviews, since she was not familiar with the concept of TMSJ, her views of the opportunities of TMSJ were based on her predictions. In the final interviews, she was asked to specify the opportunities for RWWM and WWWM.

4.2.2.4.1. Mira’s Initial Views of Opportunities of TMSJ

In the initial interview, Mira was asked a question about the effects of applying TMSJ in her class. Mira stated that a positive effect could be observed in terms of mathematics learning. She highlighted that TMSJ can facilitate mathematics learning because associating mathematics with social justice issues would make it easier for students to understand mathematics topics. She stated that real-life connections are crucial for students to understand mathematics. In terms of personal development, she stated that with TMSJ, students will be more aware of the differences. Therefore, their personal development will be positively affected. Lastly, in terms of social development, she highlighted that TMSJ would lead to a more just society by raising awareness of social justice issues in society. She emphasized that if her students, whose awareness about social justice increases, communicate with other people on these social justice issues, it will increase other people’s awareness as well, and this will create a more just society.

4.2.2.4.2. Mira’s Final Views of Opportunities of TMSJ

Mira identified her views of opportunities of TMSJ in terms of RWWM and WWWM in the final interviews. In terms of mathematics learning, she expressed the same opportunities for RWWM and WWWM, which are increasing interest in mathematics and facilitating mathematics learning. In her initial interviews, she did not emphasize increasing interest in mathematics through TMSJ. However, in the final interviews, for RWWM, she pointed out that students' interest in mathematics would increase towards the mathematics lessons because social justice topics will be of interest to students, and there will be discussion in the classroom regarding those social justice topics or questions would be asked, or activities would be carried out.
In a similar vein, for WWWM, she stated that since students are expected to take action by combining mathematics and social justice, the mathematics topic will not seem difficult to them and will be more interesting. She also stated that WWWM will facilitate their understanding. In that sense, her views were similar to the views in the initial interview; she highlighted that while solving real-life problems, students will understand mathematics easily.

In terms of opportunities for personal development, she remarked on distinctive opportunities for RWWM and WWWM. She commented that RWWM can give rise to promoting sensitive individuals. She stated: “They will develop in terms of being sensitive and not hurting people. This is more than that I can't harm people; I should also do good to people.” She highlighted that WWWM can give students a purpose to live. Mira pointed out:

\begin{quote}
If I am alive, I must have a purpose. Therefore, in order to give them a purpose, it has a positive effect. In other words, being useful to someone, being able to cause changes in someone's life, rather than I've lived and gone, actually gives them a purpose to live.
\end{quote}

Moreover, she also emphasized that WWWM can give rise to improvement in thinking skills. She stated that with WWWM, students will think about what more they can do after causing a change in a social justice issue; they will do research about it, and therefore, their thinking skills will improve.

Lastly, in terms of opportunities for social development, she commented that with RWWM and WWWM, public awareness will rise, and a more tolerant society will be built. According to Mira, with RWWM and WWWM, raising students and people who embrace the concept of social justice will increase social awareness. Moreover, such a society will be more tolerant towards each other. For example, while a disabled person crossing the street, cars will stop and wait for that person to cross.

4.2.2.4.3. Change in Mira’s Views of Opportunities of TMSJ

Opportunities and concerns were discussed together in class discussions based on the sample lesson plans and components of the TMSJ lesson plans. In this section, the focus will be more on the dialogs that focus on opportunities.
Another class activity regarding understanding the Concept of TMSJ was reading a chapter from the book (Berry III et al) about Why Social Justice in Mathematics Education? In the chosen part, the contributions of TMSJ are explained. The think-pair-share activity was done in class. After reading the article, PMTs wrote the three important ideas for them from the reading, then they shared their ideas with pairs and wrote new ideas if they appeared after discussion with their peers. Lastly, a class discussion was held, and they wrote the ideas that they considered important after the class discussion to their activity sheets again.

The class discussion in this activity was as follows:

**Researcher:** What conclusions did you reach? Why should social justice be in mathematics education?

**Selin:** If children learn about the lives of others, if they realize those differences, then discrimination is actually more likely will be at a lower level.

**Kerem:** Their ability to empathize actually increases.

**Selin:** Exactly.

**Kerem:** When we have a lot of information about other people's lives, the ability to analyze and think logically increases. This leads to making decisions accordingly. Increased ability to make decisions, increased empathy. Learning about other people's lives and cultures.

**Researcher:** Learning about other cultures, okay... What else?

**Mira:** Thanks to the TMSJ lesson, students can discover their power in numbers and realize that they can change their own lives and the lives of others thanks to this power.

**Umut:** It creates a more conscious society.

**Mira:** A more conscious society is being formed. We can use power in numbers to make society more aware.

**Researcher:** What else?

**Kerem:** I want my students to feel the power to change the lives of others and their own lives...

**Researcher:** Feeling the power to change.

**Leyla:** There is a typical question: Why am I learning mathematics? To avoid such a question, we can integrate problems we encounter daily into the activities in mathematics lessons.

**Researcher:** Can we say mathematics teaching is integrated with daily life problems?

**Leyla:** Yes

**Researcher:** What else?

**Umut:** Since we associate it with daily life, mathematical knowledge becomes more useful and memorable.
Can: What I would like to say is that as students become more aware of what social change and inequality are, they realize that they have a role in this change.

In this discussion, the pre-service teachers concluded that TMSJ would facilitate mathematics teaching and create a more conscious society, improve students' empathy and thinking skills, and make them realize their power to change their lives and others’ lives. These ideas are similar to Mira’s final views on the opportunities of TMSJ. Pre-service teachers' learning processes related to teaching mathematics for social justice start with sample questions, then they share their thoughts with their peers through the text they read in a pair activity and finally express their thoughts on a class discussion. All in all, these have an impact on their views about the opportunities of TMSJ.

4.2.2.5. Mira’s Initial and Final Concerns for TMSJ

Mira’s initial and final concerns for TMSJ were revealed through initial and final interviews. In the initial interviews, since she was not familiar with the concept of TMSJ, her views of the concerns of TMSJ were based on her predictions. In the final interviews, she was asked to specify the concerns for RWWM and WWWM. Also, she was asked what she could do as a teacher to address these concerns.

4.2.2.5.1. Mira’s Initial Concerns for TMSJ

Mira was asked what was her concerns regarding implementing TMSJ lesson plans. She mostly highlighted the concerns regarding school management and curriculum. She claimed that rigid school management would not support a focus on social justice issues in classes. Moreover, she added school management might not provide her with opportunities to implement the TMSJ lesson plans. Additionally, she pointed out that her curriculum-related concern is pacing with the curriculum. She mentioned that, especially after the school experience course, her views about keeping up with the curriculum changed and expressed her concerns about keeping up with the curriculum.
4.2.2.5.2. Mira’s Final Concerns for TMSJ

Mira’s final concerns for TMSJ were examined for RWWM and WWWM. She did not have many concerns in the initial interview, after attending the TMSJ intervention she had more concerns on her mind, but also, she had solutions to solve them. In other words, she actually thought about the scenarios she might experience as a teacher and generated solutions to them.

Regarding student-related concerns, she had two concerns for RWWM and she had one concern for WWWM. Firstly, she emphasized that social justice issues discussed in the class might contradict with students’ truth. Mira stated that when social justice issues contradict students' truths, it can be very challenging for the teacher because students will not want to listen to the teacher, and this will disrupt the flow of the lesson in general. Another student-related concern for RWWM was the low motivation of students due to standardized exams. Mira highlighted that since the success criterion is the central exams, students only focus on the exam, so they do not want to focus on things other than solving questions in mathematics. She also emphasized that students do not want to improve themselves in different fields since they are standardized exam-oriented. Her student-related concern for the WWWM was students’ being closed to change themselves. Mira stated that the first action she expects from students in the WWWM dimension is to change their own misbehaviors. She pointed out that some students may be attached to their misbehaviors in this regard, they will definitely not want to change them, so she thinks that she may have some problems. For all these concerns, she has one solution cooperating with allies (school management, parents, other teachers). Mira pointed out that trying to deal with the problems alone could lead to more negative outcomes. Therefore, she plans to move forward faster by cooperating with allies. If she can get the administration and parents on her side, she thinks that students will be more positive in TMSJ lessons, as they will hear similar things from everyone.

Regarding parent-related concerns, for RWWM, she emphasized that social justice issues discussed in the class may contradict with parents’ truth. In a similar vein, for WWWM, she emphasized that actions might contradict the parents’ truth. Especially
Regarding WWWM, she stated that when a student's parent does the opposite of a behavior or attitude that the student has learned in the classroom, the student will be in a dilemma and will take the parent's behavior as correct. With emphasizing that parents may be closer to change than students because they are adults, Mira pointed out that if seminars on social justice are organized for parents, their awareness of social justice issues may increase.

Regarding school-management-related concerns, for RWWM, she pointed out school management might apply pressure for curriculum fidelity; for WWWM, she pointed out that school administration might not allow in-school actions; for example, they do not allow exhibitions of posters on the school boards. To overcome this issue, she again considers cooperating with school management.

Regarding school-related concerns, she emphasized that school context might be closed to change. She also stated that she could solve this issue with cooperating with her allies. Lastly, she pointed out that she might have difficulty in finding educational resources for TMSJ lesson plans.

In the initial interview, she commented on concerns regarding school management and curriculum. In the final interview, she stated her concerns regarding students, parents, school management, school, and educational resources. Her concerns increased in number, her ideas developed, and she also developed ideas on how she could deal with her concerns. One of the most striking findings related to the change in her concerns was that in the final interview, she highlighted that she no longer has concerns about implementing TMSJ lesson plans. She explained her ideas:

*I think I can prove to my students that there are actually points where solving ten questions is less productive than solving two questions. Students think that they should solve many questions in a lesson. I think I can convince them in order to understand a mathematics subject, they do not need to solve many questions, but they also need to understand the parts I have explained. In this sense, integrating social justice issues into the mathematics course will not create a curriculum-related problem for me. I think I can do TMSJ in my classes. I do not think I will have problems in terms of falling behind the curriculum.*
4.2.2.5.3. Change in Mira’s Concerns for TMSJ

As in the opportunities of TMSJ, opportunities and concerns were discussed together in class discussions based on the sample lesson plans and components of the TMSJ lesson plans. In this section, the focus will be more on the dialogs that focus on concerns.

After the pre-service teachers encountered the first example of a TMSJ lesson plan, I first asked them what they thought about this lesson plan. The pre-service teachers reflected their concerns in the following dialog:

Mira: I'm so confused.
Researcher: Confused? Why?
Selin: Actually, the lesson plan is very informative, but it made me question how much it can be applied in our country.
Umut: I think this can be considered as a waste of time. We can solve questions instead.
Mira: I think so too.
Umut: This is due to the education system.
Researcher: You say there may be problems.
Mira: Yes, a very good context was chosen to raise awareness. It was supported very well, especially with questions. But the focus has shifted a little more to social justice. This is how it should be, but as we said, it seems a bit impossible to do this in public schools.
Leyla: There can be only one lesson with such an activity in a semester. Mostly, we can solve questions in our classes during the semester.
Researcher: Of course, this is not something you can do in every lesson.
Kerem: What I liked about this lesson plan is that it uses a variety of sources: watching a video and reading an article. Then, there is a group work. It was very informative to have all of these.
Ceren: If my teacher did this, my interest in mathematics would increase.
Umut: But there doesn't seem to be much mathematics, so I didn't know.
Researcher: Social justice is a little dominant in this lesson plan. It was about disaster. I chose this because we have also experienced earthquakes recently, unfortunately. What can be done about it? We normally think that nothing can be done in mathematics class. It shows us that a little bit.

In the class discussion, it can be observed that they have some concerns regarding implementing TMSJ lesson plans in their classes due to the education system. Pre-service teachers think that what will be demanded from them as mathematics teachers will be solving more problems, not including social justice context in their
classes. As I said at the beginning of this section, the concerns and opportunities are discussed together, therefore we can also identify some opportunities like that Ceren stated in this discussion, TMSJ can increase interest in mathematics.

After this discussion, the sample TMSJ lesson plan was discussed in detail in the class, and after a break, a small reminder was given to the pre-service teachers about crucial things about TMSJ lesson plans, i.e., having strong foundations in mathematics, identifying specific social justice goals, addressing appropriate grade level and content, etc. Then, I asked a class discussion question: “What do you consider when making a TMSJ lesson plan?” This question caused Mira to clarify her concerns, and the following dialog took place between her and Umut in the class:

**Mira:** As we did in the previous activity, we look at the social justice goals. What are their goals, and what are they doing to accomplish these goals? We look at the taking action part at the end. That part is very important. We can also look at its relationship with mathematics to some extent.

**Umut:** To some extent?

**Mira:** Yes, there will be mathematics, but it doesn't have to be predominantly mathematics.

**Umut:** I think the emphasis should be on mathematics.

**Kerem:** A mathematics lesson plan generally has one outcome. There are two outcomes in TMSJ lesson plans. One is the social justice outcome, and the other one is the mathematics outcome from the curriculum. How did we combine those two? This is actually important. I think that we can give both of them fifty-fifty percent weight.

**Umut:** I think mathematics’ weight should be sixty or seventy percent.

**Mira:** But I don't think this is a lesson plan that we apply all the time. As I said, mathematics should not be so dominant.

**Umut:** Are you saying that you will implement a lesson like this once a month or once a year?

**Mira:** Do you believe that you can make every lesson plan with a social justice context?

**Umut:** It depends on the mathematics subject I will teach. For example, like in the sample problem, the employment of fifteen-year-old children. I can ask students: What caught your attention in the problem? What should be in the question, what should not be in it, and how else could it be asked? I think the goal is to be able to touch on a social justice topic, at every point, at every moment I can touch.

**Mira:** No, I'm okay with that anyway. But, there is a strict curriculum given to us. There is a strict system, and there are outcomes. You can't always focus on social justice while teaching your students mathematics outcomes. So, let's talk about some realities. If I am
going to prepare a TMSJ lesson plan, I think my weight should not be too much on mathematics. I'm okay with fifty percent, fifty percent.

**Umut:** So, what you mean is that you will prepare 2-3 lesson plans in a semester.

**Mira:** Exactly, I can make a few TMSJ lesson plans, but of course, I will touch on social justice throughout the course.

**Sadık:** So then, we're in agreement with implementing one or two such comprehensive lesson plans and touching on social justice issues throughout the course.

**Mira:** Agreed.

Mira was concerned that implementing the TMSJ lesson plans might be difficult due to the strict curriculum, as she stated in the first interview, so she thought she could implement it 2 or 3 times in a semester. With the participation of other pre-service teachers, this discussion progressed to the point that parents and other factors in the school can also cause difficulties in the implementation of TMSJ lesson plans.

Afterward, questions to guide pre-service teachers in preparing TMSJ lesson plans were shared (in terms of content, context, when, and how). Then, pre-service teachers were divided into groups of 3 and developed a draft lesson plan. At the end of the TMSJ lesson plan preparation process in the groups, the following dialog occurred between Mira, Umut, and researcher:

**Mira:** We shouldn't think utopian about some issues; what I observed in the internship makes me think in this way. Maybe it would be different if I were the teacher.

**Researcher:** It will be different when it's your class.

**Umut:** We can make a change.

**Mira:** Okay, as I said, it might change when you are the teacher.

**Researcher:** Exactly. You will know your students.

**Mira:** I hope so, teacher.

In the initial interview and before preparing a TMSJ lesson plan, Mira's ideas about the curriculum were very clear, and she thought that the curriculum could not be caught up. After the first draft lesson plan, she prepared with her groupmates, it was seen that her ideas started to change gradually. In the last conversation, she stated that when she became a teacher, maybe it would be different in her class. What she said in the last interview is also consistent with this idea. The process of lesson plan preparation changed her concerns regarding TMSJ.
4.2.2.6. Mira’s Initial and Final Views on the Roles of a Teacher

Mira’s Initial and Final Views on the Roles of a Teacher changed during the TMSJ intervention. In the initial interview, she was asked a question to reveal her understanding of the roles of a teacher. Her final views are gathered through a reflective paper and final interviews.

4.2.2.6.1. Mira’s Initial Views on the Roles of a Teacher

In the initial interview, Mira was asked, rather than teaching mathematics which areas she would like to contribute to her students’ development. Mira commented that she aims to develop her students’ self-awareness related to their interests. She emphasized that she would guide students to recognize their own interests and pursue a career according to their talents. She stated that she could recognize this by observing the students and communicating with them out of class. Mira aims to guide her students to their talents by supporting and appreciating them. For example, when she sees a student drawing during a mathematics lesson, she first tells the student that he/she should not draw during the mathematics lesson, and then she can have a private conversation with him/her outside of the class. In this conversation, she can first make the student realize his/her talent by appreciating his/her drawing, and then she can ask the student if he/she would like to progress in this field and try to guide him/her if necessary.

Another important point Mira mentioned in the initial interview was she aims to raise students who are beneficial to society. In that sense, she commented that she is a role model for her students, and with her behaviors, she will be a model for her students. Especially in terms of benefits to society, she mentioned the value of “sharing.” She believes that she can utilize interdisciplinary tasks or examples to accomplish her aim.

In the initial interview, Mira focused on two main roles of a teacher, which she aims to accomplish in her teaching career: developing awareness of students’ self-interests and raising students who are beneficial to society. Neither of them focused on social justice, or integrating social justice in mathematics teaching.
4.2.2.6.2. Mira’s Final Views on the Roles of a Teacher

In the reflective paper, she was asked what a teacher's role was beyond teaching the subject matter, and her answer and focus points differed from the initial interview. Mira, who never mentioned social justice in the initial interview, firstly highlighted raising social justice awareness as a role of a teacher in the reflective paper. In addition, she also emphasized raising cultural awareness in her students, which is related to social justice. She stated that a teacher should use social justice context in her/his lessons. Beyond raising students’ awareness on social issues, she also mentioned that as a teacher, she aims to empower students to make changes in society.

Moreover, Mira also focused on how she would promote students’ interpersonal skills. In this context, she emphasized the importance of communication skills and respect for each other and other ideas. To accomplish this aim, she emphasized that the teacher should be a role model for students and be a model with his/her behaviors. For instance, she states that her behaviors toward students in the class might be model behaviors for her students.

All in all, changes were observed in Mira’s views regarding the roles of a teacher. It can be inferred that what she focused on corresponds with the goals of Teaching Mathematics for Social Justice after participating TMSJ intervention.

4.2.2.6.3. Change in Mira’s Views on the Roles of a Teacher

There were some changes in Mira’s views on the role of a teacher. How materials, activities, and discussions supported pre-service mathematics teachers’ views on a teacher's role is exemplified by Mira's case.

In the first part of the lesson, the definition of social justice is discussed so that all PMTs have a common concept in mind when they discuss other related concepts. In the second part of the first lesson, the pre-service teachers read the story of Allison, a beginning teacher. This story is taken from the article by Agarwal et al. (2010); it is
about the situations that a beginning teacher, Allison, a fifth-grade teacher, experienced concerning social justice in her class. Allison was working in a racially diverse classroom, and she wants her students to feel safe and to integrate social justice issues into the classes; she sees the mandated curriculum as a barrier because it has no focus on such issues. Pre-service teachers first read Allison's story by themselves, then some questions related to Allison’s story were given to them, they discussed the answers to these questions within their groups of 3, and finally, the groups shared what they thought in the class, and a class discussion about social justice issues that might occur in the class is held.

Mira, Can, and Ceren were group mates for this activity. The points they emphasized in the group discussion were giving voice to students, caring for differences (especially racial), group work, and fostering classroom discussion in terms of teaching for social justice. Additionally, they proposed some ideas regarding maintaining teaching for social justice, such as conveying to the students that there is no absolute truth, everyone can learn from each other, and reflecting social justice awareness to the behaviors of a teacher.

In the class discussion, Allison's social justice challenges were discussed first. Later, what she did to solve these challenges and what pre-service teachers could do differently to solve these challenges were discussed. Then, it was concluded that what Allison learned in the teacher education curriculum and what she experienced were inconsistent. Afterward, the essential points of teaching for social justice were discussed. A class discussion about this last point was as follows:

**Researcher:** What are the important points of teaching for social justice after reading this example?

**Leyla:** I like the sentence in the reading that I want everyone to express their thoughts freely, but I think it is very important to be able to make that distinction in terms of what should be said or should not be said for everyone to be in a healthy environment.

**Researcher:** So, is this something like moderating a discussion?

**Leyla:** It may be a discussion or, even when I am going to ask a question, what should be a student’s limit in order not to hurt the other students? Of course, I want him/her to say what he thinks.

**Mira:** Are we restricting the students?
Leyla: Maybe.
Mira: I think when they learn how to communicate, it will not be a problem.
Researcher: Do you mean being aware of what and how much they can express their ideas?
Mira: Yes.
Leyla: I think so.
Can: We also said we should ask non-subjective questions for this [students’ not hurting others by their ideas].
Researcher: How?
Mira: Considering differences is also important while asking questions.
Researcher: Yes. After all, we will probably have students from diverse backgrounds.
Mira: Giving a voice is also important.
Researcher: Yes, giving students the right to speak.

As the dialog between pre-service teachers indicated, class discussion of what should be said or should not be said in the class led Mira to emphasize the importance of developing students' interpersonal skills. She thinks that students with developed communication skills can create a classroom environment in which students will not hurt each other with what they say. Also, as can be seen in the dialog, Leyla, who was undecided about her ideas on the issue, clarified her ideas after discussing with Mira.

Another class discussion on about Allison's case:

Researcher: So, let's move on to the last question. What might be other essential points of teaching for social justice?
Mira: The teacher should be able to reflect the awareness of social justice in his/her lesson, attitudes, and classroom management. Because the teacher is a role model.
Can: How can she/he reflect this?
Mira: Allison added different topics to the curriculum. She wanted her students to evaluate situations from different perspectives.
Can: Hmm.
Mira: You can add something related to social justice to the curriculum to raise awareness among the students. Also, the teacher can be a role model and bring social justice awareness to the class.

As can be seen in the dialog above, Allison's example provided an example for Mira that social justice issues could be integrated into the lesson, and she embraced this idea. In her dialog with Can, Mira expressed that a teacher could integrate social
justice issues, make students analyze events from different perspectives, and demonstrate an understanding of social justice in her attitudes and classroom management.

From these sample dialogues and the opinions shared by Mira, it is seen that the example of Allison, who tries to ensure social justice in her class, and the classroom discussions had an influence on her thoughts about the roles of a teacher.

4.2.2.7. Mira’s TMSJ Lesson Plan

Mira developed a mathematics lesson plan focused on a social justice context as a learning outcome of TMSJ intervention. In the 5th lesson of the intervention, she stated that she was thinking of preparing a lesson plan about visually impaired people since pre-service teachers were expected to come to that class thinking about their TMSJ lesson plans. Pre-service teachers shared their ideas with their friends and received feedback from their friends and Mira. Accordingly, in the 6th lesson, they presented their lesson plans to the class as teachers as a micro-teaching activity in the class. Mira’s lesson plan was evaluated according to the Lesson Evaluation Rubric and scored 2.9. The lesson plan prepared by Mira was detailed, and the necessary sections were indicated, but at some points, there was a little more talk about social justice, which was the reason for her score.

In her lesson plan, Mira emphasized the problems visually impaired people might encounter due to the occupation of yellow stripes in the pavement and integrated this social justice issue into Greatest Common Divisor and Least Common Multiple topics of 8th grade mathematics. The activity and the lesson flow she prepared for the TMSJ lesson appear in Appendix L. The lesson plan was about yellow lines, which students always see, that will attract their attention. It was discussed in class how parked cars and street electricity poles on these lines can be a big problem for visually impaired people, so Mira’s lesson plan was interesting as a social justice issue. In that sense, the mathematics learning outcomes were also intertwined with the topic. In addition, the mathematics learning outcomes were also intertwined with the topic. In that sense, students raised awareness of social justice issues such as RWWM, and at the end of the class, they discussed what can be done to overcome such problems as WWWM.
4.3. Summary of the Results

In this section, firstly, the needs of pre-service mathematics teachers regarding teaching mathematics for social justice were summarized, and then the results of the design experiment were summarized.

4.3.1. Summary of Needs Analysis Results

In order to determine the needs of pre-service teachers regarding teaching mathematics for social justice, semi-structured interviews were conducted with pre-service teachers and teacher educators. The needs analysis findings revealed that pre-service teachers were not familiar with the concept of social justice in their undergraduate courses. Moreover, pre-service elementary mathematics teachers had not encountered TMSJ in any course in their undergraduate curriculum, also they could not associate social justice and mathematics teaching when asked. They explained they wanted to participate in an intervention in teaching mathematics for social justice and preferred the intervention to include group work because they stated during group work, they can learn from their peers, gain different perspectives, and develop new ideas with brainstorming. Also, according to the results of the needs analysis conducted with pre-service teachers and teacher educators, it was seen that pre-service teachers had lack of familiarity with the concept of social justice in their teacher education curriculum. Therefore, it was decided to include teaching for social justice in the content of the TMSJ intervention and to plan learning activities based on social constructivism.

4.3.2. Summary of Design Experiment Results

While designing TMSJ intervention, I conjectured that utilizing teaching and learning approach grounded in social constructivism would develop pre-service mathematics teachers’ understanding of TMSJ and their ability to develop TMSJ lesson plans. To accomplish this conjecture, I developed my lesson plans to include activities in which pre-service teachers would interact with each other to a high degree. In particular, I aimed to develop their thinking about TMSJ by sharing
concrete examples with them before giving information about the topic and by having them conduct class discussions or group discussions about these examples and features of TMSJ. Since I was using social constructivism, I used scaffolding to give pre-service teachers the opportunity to develop their thinking and designed a learning environment where students were active. The embodiment and processes are shown in Figure 4.6.

In the initial interviews before the TMSJ intervention, all of the pre-service teachers expressed that they were unfamiliar with the concept of TMSJ. Therefore, their opinions on TMSJ are primarily based on their predictions. The majority believed it could provide equal opportunities in mathematics education, and a few mentioned the potential integration of social justice issues into problem contexts. According to findings, initially, pre-service teachers possess a limited and not very comprehensive understanding of TMSJ. They lack detailed knowledge regarding its practical implementation in the classroom. In the final interviews, however, it was observed that they defined TMSJ as integrating social justice issues into the mathematics lessons with the knowledge of how to do. They also defined TMSJ regarding the concepts of RWWM and WWWM covered in the intervention.

![Figure 4.6. Elements of the Design Research](image)

Pre-service teachers were asked how teaching mathematics could raise students' awareness (RWWM) in the initial interviews; they expressed that they could not
connect social justice with teaching mathematics, except for one student who said that it could only happen with the teachers’ attitudes. However, in the final interviews and in their reflection papers, all of them explained that RWWM was raising awareness through mathematics; some of them also emphasized it as empathizing with mathematics and recognizing differences through mathematics.

In response to the question about whether students can take action on social justice issues through mathematics teaching about WWWM in the initial interviews, some pre-service teachers said that they had no idea how this could happen; maybe students could take action as internalizing the sub-message in the problems, while others emphasized that students could only take action when they become adults. In the final interviews, the pre-service teachers first defined WWWM as taking action through mathematics and then were able to identify different ways to provide WWWM. The pre-service teachers who emphasized that students could make change only in adult life changed their minds in the final interviews and stated that students could also cause changes in their own lives or in the lives of others.

The pre-service teachers also expressed some opportunities and concerns about TMSJ. In the final interviews, it is seen that there was also an improvement in their views regarding opportunities and concerns. Finally, while there was no emphasis on social justice in the first interview regarding the roles of teachers. After participating the intervention, the pre-service teachers expressed their views on both raising awareness about social justice and empowering students to take action on social justice issues as roles of a teacher in their reflection papers.

As an important outcome of TMSJ intervention, pre-service teachers developed TMSJ lesson plans. The lesson plans they developed have both RWWM and WWWM dimensions. The lesson plans can be shown as an example of putting the knowledge they received into practice. First of all, they prepared lesson plans as a group in the classroom according to the given template. In their group, TMSJ lesson plans, mathematics topics of the lessons were data analysis, ratio, proportion, and percentage. They did not associate different topics from mathematics with the subjects of social justice. However, this was their first trial. In their individual TMSJ
lesson plans, it was seen that they associated many different mathematics topics, such as the probability of simple events, first-order equations, least common multiple, and greatest common divisor with social justice issues. In conclusion, it can be seen that the lesson plans developed in accordance with social constructivism improved the pre-service teachers' understanding of TMSJ and their lesson plan preparation skills. In this sense, the activities and discussions used in the TMSJ intervention influenced the pre-service teachers' understanding of TMSJ.
CHAPTER 5

DISCUSSION

This study aimed to examine and improve pre-service teachers’ understanding of teaching mathematics for social justice. TMSJ intervention was developed and implemented, and after revisions were made, it was reimplemented. Before developing the intervention, a needs analysis study was conducted with pre-service teachers and teacher educators to identify pre-service teachers’ needs on TMSJ. In the first part of this chapter, the results of the needs of pre-service teachers regarding TMSJ and design experiment are discussed. Also, the implications of the study and recommendations for further research are explained in this chapter.

5.1. Discussion of the Findings

The results of the research are discussed in accordance with the research questions; first, the discussion of findings of the needs analysis on TMSJ, then the discussion of the findings of the design research are presented.

5.1.1. Discussion of Findings of the Needs on Teaching Mathematics for Social Justice

Jong and Jackson (2016) examined the elementary pre-service teachers’ conceptions of teaching mathematics for social justice; the most common themes that emerged in the study were being unsure about TMSJ and defining it as an opportunity or access to education. Actually, the results of the needs analysis of this study were also consistent with those findings. Pre-service teachers identified teaching for social justice as equal opportunity for all, and they had lack of knowledge on TMSJ. In that sense, it can be concluded that there is a need for pre-service teachers to more profound understanding in terms of teaching mathematics for social justice.
According to the results of the needs analysis interviews with teacher educators, they do not address social justice-oriented mathematics teaching in their lessons. As McLeman and Piert (2013) stated, there is also a need for teacher educators to engage in critical self-reflection on their teaching practices to adequately equip pre-service teachers for this particular approach. Because they are the ones who will initiate change in teaching mathematics. Moreover, Harrison (2015) also emphasized the necessity of teacher education programs to prepare for teaching for social justice.

5.1.2. Discussion of the Findings of the Design Experiment

Register et al. (2020), in their study, make pre-service teachers prepare TMSJ lesson plans. PMTs could not integrate mathematics and social justice, and they either focused on mathematics or social justice issues separately in their lesson plans. In this study, Leyla and Selin’s lesson plans had shortcomings in this regard. Contrary to their findings, all of the pre-service teachers focused on WWWM dimension in their lesson plans. However, consistent with the findings of Bartell (2013), some pre-service teachers struggled to balance social justice with mathematics content; sometimes, social justice was more dominant in some lesson plans.

Vargas and Gutierrez (2018) developed a professional development program for pre-service mathematics teachers, and the findings of the study indicated that pre-service teachers redefined mathematics after participating the professional development program, also pre-service teachers emphasized that thanks to TMSJ their students could be critical participants of the society which is congruent with WWWM in this study.

Another professional development program on teaching mathematics for social justice developed by Cahill and Bostic (2021). Attendees who participated in professional development sessions exhibited more favorable beliefs regarding teaching mathematics for social justice, aligning with the intended goal of the professional development. Similarly, in this study, pre-service teachers stated that they would use TMSJ lesson plans in the future and expressed positive attitudes towards implementing of TMSJ lesson plans, in this sense. To some degree, this
research expands the existing body of literature by providing preservice teachers with the chance to participate in an activity grounded in an equity perspective, as suggested by Gutiérrez (2009).

Incorporating social justice issues into mathematics can increase students' awareness on social justice issues and interest and success in mathematics, however it is crucial to ensure that these topics are integrated effectively into the curriculum and mathematics lessons. Including real-life examples may not be sufficient if they are not presented in a meaningful and engaging way for the students. For instance, similar to the results of study conducted by Brantlinger (2013), students may resist the inclusion of social justice topics in mathematics instruction, because they may feel that it does not support them preparing for university entrance exams. Therefore, it is necessary to provide pre-service teachers with comprehensive training on how to effectively incorporate TMSJ into their mathematics lessons. In this way, they can share the various opportunities of implementing TMSJ lessons to their students.

5.2. Implications for Theory and Practice

The findings of this study provide researchers and teacher educators with information about pre-service mathematics teachers' understandings of teaching mathematics for social justice and how they develop these understandings. Some implications are presented for researchers and teacher educators who will benefit from this study.

According to the needs analysis study findings and the initial interviews with the pre-service teachers who participated in the TMSJ intervention, the pre-service teachers had no prior knowledge of TMSJ. After participating in the study, it was observed that the activities, discussions, and tasks developed based on social constructivism improved pre-service teachers' understanding of TMSJ. Therefore, teacher educators can integrate the activities developed in this intervention into their courses and improve pre-service teachers' understanding of TMSJ. In this way, they can pave the way for future teachers to implement TMSJ in their classrooms. Furthermore, teacher educators can utilize the learning theory of the study, social constructivism, to train pre-service teachers in different areas.
The people who will be most affected by the development of pre-service teachers' understanding of TMSJ are their future students. With this development in the understanding of pre-service teachers, their future students will become more aware of social justice issues and be able to take action regarding those issues, and their interest in mathematics will increase as these interesting and real-life topics are included in mathematics. Therefore, it is important to develop pre-service teachers' understanding of TMSJ. In addition, the intervention developed can be applied to teachers as in-service training, and their understanding of TMSJ can be developed. However, it is essential to recognize that in-service training may need different requirements compared to pre-service teacher education. Teachers who have been in the profession for longer may have different levels of familiarity and comfort with TMSJ concepts. Therefore, any intervention or training aimed at enhancing their understanding of TMSJ should be tailored to address their specific needs and challenges. In that sense, before implementing an intervention with the teachers, conducting a needs analysis study could be guide the most appropriate way for the researchers or teacher educators.

5.3. Recommendations for Further Research

In this study, firstly, the needs of pre-service mathematics teachers who are studying at a state university in the Central Anatolian region regarding TMSJ were identified. Then, a TMSJ intervention was developed in accordance with the emerging needs. The needs analysis interviews were conducted with pre-service teachers and teacher educators, and revealed the need of pre-service teachers’ in TMSJ. In this sense, further needs analysis studies could be conducted in different universities. In addition, it is also crucial to determine teachers’ needs regarding TMSJ. In this sense, needs analysis studies on teaching mathematics for social justice can also be conducted with teachers.

According to the results of this study, it was seen that the social constructivism-based teaching mathematics for social justice intervention improved pre-service teachers' understanding of TMSJ and their ability to prepare TMSJ lesson plans. These results are for the context in which the intervention was implemented. Further studies could
be conducted to implement TMSJ intervention and analyse its influence on pre-
service mathematics teachers’ understanding. Since the pre-service teachers were 3rd
and 4th year students, this intervention could be coordinated with the practice in
teaching courses and pre-service teachers could implement the lesson plans in their
schools. As another suggestion, follow-up studies could be conducted with
participants of the TMSJ intervention to find the possible influence of the
intervention in their teaching.

In addition, the TMSJ intervention could be implemented with teachers.
Furthermore, further studies could be conducted with teachers in order to determine
the influence of the intervention on their teaching. This could be done by conducting
follow-up studies after the intervention to assess any sustained changes in how they
teach mathematics. Additionally, it would be beneficial to compare the outcomes of
teachers who have participated in the TMSJ intervention and those who have not to
determine the true impact of the intervention on their teaching effectiveness.

The lessons in this study were planned on the basis of social constructivism, and the
positive impact on pre-service teachers' understanding of TMSJ and their ability to
develop TMSJ lesson plans was observed. According to the needs and characteristics
of the pre-service teachers or teachers to be implemented, the TMSJ intervention
could also be designed with different learning and teaching strategies and techniques
which could lead to further learning opportunities for pre-service teachers or teacher
educators.

The effects of social justice-oriented mathematics teaching on students are crucial to
determine its potential benefits and drawbacks. For this reason, conducting studies
on the outcomes of implementing pedagogy that focuses on teaching mathematics for
social justice to students is also recommended. These studies could examine how
students' achievement or attitudes toward mathematics changed by implementing
TMSJ lessons and also the extent to how social justice outcomes are developed. In
addition, studies on how to integrate social justice issues into mathematics teaching
can also be conducted. These studies will provide guidance for teachers and pre-
service teachers in developing and implementing TMSJ lessons.
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APPENDICES

A. TMSJ LESSON PLAN EVALUATION RUBRIC

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Exceeds Expectations (Level 3)</th>
<th>Meets Expectations (Level 2)</th>
<th>Needs Improvement (Level 1)</th>
<th>Does not meet expectations (Level 0)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>The background information about the social justice issue is acquired.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics learning outcomes are relevant for the grade level.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The social justice goals chosen are clear and relevant to students.</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Lesson plan includes raising awareness on SJ issues</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Lesson Plan includes taking action about social justice issues.</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Social justice content aligns with mathematical content.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social justice and mathematics content are balanced.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Instructional strategies to engage students are clearly explained.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The assessment of mathematics learning outcomes is clear.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The assessment of social justice goals is clear.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target students and the benefits of the lesson for them are clearly explained.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## B. LESSON PLAN PRESENTATION EVALUATION RUBRIC

Your Name (evaluator):

Lesson Plan Topic:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Excellent (5 points)</th>
<th>Good (4 points)</th>
<th>Average (3 points)</th>
<th>Poor (2 points)</th>
<th>Very Poor (1 point)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>The lesson was well-designed.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics content is relevant for the grade level.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social justice content aligns with mathematical content.</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>The task is inspirational.</td>
<td></td>
<td></td>
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<tr>
<td>Instructions for the activity are clearly presented.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>The assessment strategies are relevant with objectives.</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
C. ELEMENTARY MATHEMATICS EDUCATION PROGRAM

D. NEEDS ANALYSIS INTERVIEW FOR PRE-SERVICE MATHEMATICS TEACHERS

Öğretmen Adayları için Sosyal Adalet Odaklı Matematik Öğretimine Yönetilir
İhtiyaç Analizi Görüşme Soruları

Görüülülen Kiş (takma isim):

Tarih:

Değerli öğretmen adayı,

Bu araştırma, ODTÜ Eğitim Bilimleri Bölümü Doktora öğrencisi Gonca Çınar Bent tarafından Prof. Dr. Hanife Akar danışmanlığında doktora tezi kapsamında yürütülmektedir. Bu çalışma ile matematik öğretmen adayları için sosyal adalet odaklı matematik öğretimi ile ilgili bir uygulama geliştirilmesi ve bu programın etkilerinin incelenmesi amaçlanmaktadır. İlköğretim matematik öğretmenliği alanında bir aday öğretmen olarak sizin görüşleriniz bu çalışma için büyük bir önem taşımaktadır.


Görüşmeye başlamadan önce sormak istediğiniz soru veya sorular varsa cevaplamaktan memnun olun. Araştırma yapışınız katkıdan dolayı şimdiye çok teşekkür ederim.

Arş. Gör. Gonca ÇINAR BENT
Orta Doğu Teknik Üniversitesi
Eğitim Bilimleri Bölümü
Eğitim Programları ve Öğretim
İletişim: gcinar@metu.edu.tr
Kişisel Bilgiler

1. Kendinizi tanır mısınız?
   - Mezun olunan lise:
   - Liseden mezuniyet yılı:
   - Sınıf:
   - Odtü’deki kaçınıncı yılınız:
   - Ortalama

Sosyal Adalet Odaklı Matematik Öğretimine Dair İhtiyaç Analizi Görüşme Soruları

1. Matematik öğretmenliği bölümünü seçmenizin nedenleri nedenleri nelerdir?
   a. Özellikle hangi unsurlar bu bölümü seçmenizde etken oldu?

2. Matematik öğrenmenin öğrencilerin hayatlarına ne gibi faydalar sağlayacağını düşünüyorsunuz?
   a. Size göre insanlar neden matematik öğrenmelidir?
      - Meslek kazandırma
      - Günlük hayatta kullanacakları matematik ile tanıştırma vb.

3. a. Matematik öğretim programının ulaşmaya çalıştığı genel amaçlara baktığınızda size en önemli görünen madde ya da maddeler hangisidir/hangileridir? Nedenini açıklar mısınız?
   b. Bu listedeki amaçlar dışında eksik gördüğünüz veya dahil olması gerektirdiğini düşündüğünüz diğer amaçlar nelerdir? Açıklar mısınız?

4. Lisans eğitiminizde düşündüğünüzde sosyal adalet kavramı ile nasıl karşılaştınız? Açıklar mısınız?
   - Ders
   - Konu
   - Etkinlik

5. Sosyal adalet odaklı öğretim (teaching for social justice) size ne ifade ediyor? Aklıma ilk gelenleri paylaşabilir misiniz?

6. Sosyal adalet ile ilgili özellikle hangi konular matematik öğretimi ile ilişkilendirilebilir? Nedenleri ile açıklayabilir misiniz?
7. Matematik öğretimine sosyal adalet konularının (eşitsizlik, yoksulluk, insan hakları vb.) dahil edilmesini nasıl değerlendiriyorsunuz?
   - Duyuşsal açıdan
   - Bilişsel açıdan
   - Toplumsal açıdan

8. Geleceğin öğretmeni olarak, sosyal adalet konularını matematik dersinize nasıl dahil edebilirsiniz?
   - Ders içi etkinlik
   - Ders dışı (proje vb.)

9. Sosyal adalet konularını dersinize dahil etmek istediğinizde hangi kaynaklardan yararlanacağını düşünüyorsunuz?
    Alternatif. Hangi kaynakların bu alanda size yol gösterici olacağını düşünüyorsunuz?

10. Size göre matematik öğretiminin sosyal adalet konuları ile ilgili öğrencilerin farkındalığını oluşturmadan nasıl bir etkisi olabilir? Örnek vererek açıklayabilir misiniz?

11. Size göre matematik öğretiminin sosyal adalet konuları ile ilgili öğrencilerin harekete geçmelerinde nasıl bir etkisi olabilir? Örnek vererek açıklayabilir misiniz?

12. a. Sosyal adalet odaklı matematik öğretimi (teaching mathematics for social justice) ile ilgili bir eğitim verilmesine dair düşünceniz nedir?
   a. Böyle bir eğitimin nasıl planlanmasını tercih edersiniz?
      - Bireysel etkinlikler
      - Grup çalışması etkinlikleri
      - Çevrimiçi
      - Yüz yüze

**Matematik Dersi Öğretim Programı'nın Özel Amaçları**

1739 sayılı Millî Eğitim Temel Kanunu’nda belirlenmiş olan Genel Amaçlar ve Temel İlkeler doğrultusunda

Matematik Dersi Öğretim Programı'nın ulaştıça çalıştığı genel amaçlar şu şekilde sıralanabilir:
Öğrenci;

1. Matematiksel okuryazarlık becerilerini geliştirebilecek ve etkin bir şekilde kullanabilecektir.

2. Matematiksel kavramları anlayabilecek, bu kavramları günlük hayatta kullanabilecektir.

3. Problem çözme sürecinde kendi düşünce ve akıl yürütmelerini rahatlığıyla ifade edebilecek, başkalarının matematiksel akıl yürütmelerindeki eksiklikleri veya boşlukları görebilecektir.

4. Matematiksel düşüncelerini mantıklı bir şekilde açıklamak ve paylaşmak için matematiksel terminolojisi ve dili doğru kullanabilecektir.

5. Matematiğin anlam ve dilini kullanarak insan ile nesneler arasındaki ilişkileri ve nesnelerin birbirleriyle ilişkilerini anlamlayabilir ve anlamları anlayabilecektir.

6. Üst bilişsel bilgi ve becerilerini geliştirebilecek, kendi öğrenme süreçlerini bilinçli bir biçimde yönetebilecektir.

7. Tahmin etme ve zihinden işlem yapma becerilerini etkin bir şekilde kullanabilecektir.

8. Kavramları farklı temsil biçimleri ile ifade edebilecektir.


10. Sistemli, dikkatli, sabırlı ve sorumlu olma özelliklerini geliştirebilecektir.

11. Araştırma yapma, bilgi üretme ve kullanma becerilerini geliştirebilecektir.

12. Matematiğin sanat ve estetikle ilişkisini fark edebilecektir.
13. Matematiğin insanlığın ortak bir değeri olduğunu bilincinde olarak matematiğe değer verecektir.

Kaynak
http://mufredat.meb.gov.tr/Dosyalar/201813017165445-
MATEMAT%C4%B0K%20%C3%96%C4%9ERET%C4%B0M%20PROGRAMI% 202018v.pdf
E. NEEDS ANALYSIS INTERVIEW FOR TEACHER EDUCATORS

İlköğretim Matematik Öğretmenliği Öğretim Üyeleri için Sosyal Adalet Odaklı Matematik Öğretimine Yönelik İhtiyaç Analizi Görüşme Soruları

Görüşülen Kişi (takma isim): Tarih:

Değerli Hocam,

Bu araştırma, ODTÜ Eğitim Bilimleri Bölümü Doktora öğrencisi Gonca Çınar Bent tarafindan Prof. Dr. Hanife Akar danışmanlığında doktora tezi kapsamında yürütülmektedir. Bu çalışma ile matematik öğretmen adayları için sosyal adalet odaklı matematik öğretimi ile ilgili bir müdahale geliştirilmesi ve etkilerinin incelenmesi amaçlanmaktadır. İlköğretim matematik öğretmenliği alanından mezun olacak öğretmen adaylarını yetiştirir bir öğretmen eğiticisi olarak sizin görüşleriniz bu çalışma için büyük bir önem taşımaktadır.


Görüşmeye başlamadan önce sormak istediğiniz soru veya sorular varsa cevaplamakta memnun olurum. Araştırmaya yaptığı katkıdan dolayı şimdiden çok teşekkür ederim.

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İletişim: gcinar@metu.edu.tr
Kişisel Bilgiler

1. Kendinizden bahseder misiniz?
   - Eğitim
     o Lisans
     o Yüksek lisans
     o Doktora
   - Mesleki deneyim
   - Çalışılan kurumdaki deneyim
   - Verdiğiniz dersler

Sosyal Adalet Odaklı Matematik Öğretimine Dair İhtiyaç Analizi Görüşme Soruları

1. a. Matematik öğretim programının ulaşımda çalıştığı genel amaçlara baktığımızda hangi madde ya da maddeler sizin için önem taşımaktadır? Nedenini açıklar mısınız?
   b. Bu amaçlar dışında sizin için önemli olan noktalar nelerdir?

2. Yukarıda bahsettiğiniz lisans derslerinde özellikle matematik öğretimi boyutunda hangi noktalara önem veriyorsunuz?
   a. Matematik öğretmen adaylarına matematik öğretme ile ilgili kazandırmayı hedeflediğiniz yeterlikler nelerdir?

3. a. Verdiğiniz lisans derslerinde sosyal adalet kavramına nasıl yer veriyorsunuz?
   - Ayrılan süre
   - Kullanılan kaynak
   b. Sosyal adalet kavramını öğrencilere tanıtırken matematik öğretimi ile nasıl bir ilişki kuruyorsunuz?

4. Matematik öğretimine sosyal adalet ile ilgili konuların dahil edilmesi ile ilgili düşünceleriniz nelerdir?
   - Matematik kazanımları
   - Sosyal adalet kazanımları
   - Toplumsal fayda
   - Zaman problemi (müfredatı yetiştirme açısından)
5. Okullardaki matematik öğretimine sosyal adalet ile ilgili konuların nasıl dahil edilebileceğini düşünüyorsunuz?
   - Ders içi etkinlik
   - Ders dışı (proje vb.)

Matematik Dersi Öğretim Programı'nın Özel Amaçları

1739 sayılı Milli Eğitim Temel Kanunu'nda belirlenmiş olan Genel Amaçlar ve Temel İlkeler doğrultusunda

Matematik Dersi Öğretim Programı'nın ulaşmaya çalıştığı genel amaçlar şu şekilde sıralanabilir:

Öğrenci;
1. Matematiksel okuryazarlık becerilerini geliştirebilecek ve etkin bir şekilde kullanabilecektir.
2. Matematiksel kavramları anlayabilecek, bu kavramları günlük hayatta kullanabilecektir.
3. Problem çözme sürecinde kendi düşünce ve akıl yürütümlerini rahatlıkla ifade edebilecek, başkalarının matematiksel akıl yürütümlerindeki eksiklikleri veya boşlukları görebilecektir.
4. Matematiksel düşüncelerini mantıksal bir şekilde açıklamak ve paylaşmak için matematiksel terminolojiyi ve dili doğru kullanabilecektir.
5. Matematiğin anlamlı ve dilini kullanarak insan ile nesneler arasındaki ilişkileri ve nesnelerin birbirleriyle ilişkilerini anlamlandırabilecektir.
6. Üst bilişsel bilgi ve becerilerini geliştirebilecek, kendi öğrenme süreçlerini bilinçli biçimde yönetebilecektir.

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7. Tahmin etme ve zihinden işlem yapma becerilerini etkin bir şekilde kullanabilecektir.

8. Kavramları farklı temsil biçimleri ile ifade edebilecektir.


Kaynak
F. INITIAL INTERVIEW QUESTIONS

İlköğretim Matematik Öğretmen Adayları için Ön Görüşme Soruları

Görüşülen Kişi (takma isim): Tarih:

Değerli Öğretmen Adayı,

Bu araştırma, ODTÜ Eğitim Bilimleri Bölümü Doktora öğrencisi Gonca Çınar Bent tarafından Prof. Dr. Hanife Akar danışmanlığında doktora tezi kapsamında yürütülmektedir. Bu çalışma ile matematik öğretmen adayları için sosyal adalet odaklı matematik öğretimi ile ilgili bir müdahalenin geliştirilmesi ve etkilerinin incelenmesi amaçlanmaktadır. İlköğretim matematik öğretmenliği alanında bir aday öğretmen olarak sizin görüşlerinizi ve katılımınızı bu çalışma için büyük bir önem taşımaktadır.


Görüşmeye başlamadan önce sormak istediğiniz soru veya sorular varsa cevaplamaktan memnun olurum. Araştırmaya yaptığınız katkıdan dolayı şimdişen çok teşekkür ederim.

Arş. Gör. Gonca Çınar BENT
Orta Doğu Teknik Üniversitesi
Eğitim Bilimleri Bölümü
Eğitim Programları ve Öğretim İletişim: gcinar@metu.edu.tr
Kişisel Bilgiler
1. Kendinizden bahseder misiniz?
   - Mezun olunan lise:
   - Liseden mezuniyet yılı:
   - Sınıf:
   - OdTÜ’deki kaçinci yılınız:
   - Ortalama:

Ön Görüşme Soruları
1. Matematik öğretmenliği bölümünü seçmenizin nedenleri nelerdir?
2. Matematiğin hayatımızda nasıl bir yeri olduğunu düşünüyorsunuz?
   - Gündelik yaşam
   - Meslek edinme
   - Bilim
   - Toplumsal
3. Gelecekte öğretmenliğin yaptığınızı düşündüğümüzde sizin sınıfınızdan mezun olan bir öğrencinin matematik kazanımları dışında hangi alanlarda gelişimine katkıda bulunmayı düşünüyorsunuz?
   - Duyuşssal kazanımlar
   - Toplumsal kazanımlar
4. Matematik öğretimine sosyal adalet konularını dahil eden bu programa neden katılmak istediğinizi açıklayabilir misiniz?
5.
   a. Sosyal adalet odaklı matematik öğretimi (teaching mathematics for social justice) kavramı ile daha önce nasıl karşılaştınız?
   b. Bu kavramı nasıl tanımlarsınız?
6. Size göre matematik öğretiminin öğrencilerin sosyal adalet konuları ile ilgili farkındalığını oluşturmada nasıl bir rolü vardır? Örnek vererek açıklayabilir misiniz?
7. Size göre matematik öğretiminin öğrencilerin sosyal adalet konuları ile ilgili değişim yapabilme lerede nasıl bir rolü vardır? Örnek vererek açıklayabilir misiniz?
8. Bir matematik öğretmen adayı olarak sosyal adalet odaklı matematik öğretimini sınıflarınızda uygularken ne gibi etkileri olacağını düşünüyorsunuz?
   - Matematik dersine yönelik
   - Kişisel gelişimlerine yönelik
   - Toplumsal fayda

9. Bir matematik öğretmen adayı olarak sosyal adalet odaklı matematik öğretimini sınıflarınızda uygularken ne gibi zorluklarla karşılaşacağınızı düşünüyorsunuz?
   - Öğrenci kaynaklı
   - Program kaynaklı
   - Yönetim kaynaklı
   - Veli kaynaklı
   - Eğitsel kaynak
   - Okul iklimi
İlköğretim Matematik Öğretmen Adayları için Son Görüşme Soruları

Görüşülen Kişi (takma isim): Tarih:

Değerli Öğretmen Adayı,

Bu araştırma, ODTÜ Eğitim Bilimleri Bölümü Doktora öğrencisi Gonca Çınar Bent tarafından Prof. Dr. Hanife Akar danışmanlığındaki doktora tezi kapsamında yürütülmektedir. Bu çalışma ile matematik öğretmen adayları için sosyal adalet odaklı matematik öğretimi ile ilgili bir müdahale geliştirilmesi ve etkilerinin incelenmesi amaçlanmaktadır. İlköğretim matematik öğretmenliği alanında bir aday öğretmen olarak sizin görüşleriniz ve katılımınız bu çalışma için büyük bir önem taşımaktadır.


Görüşmeye başlamadan önce sormak istediğiniz soru veya sorular varsa cevaplamanıza memnun olurum. Araştırmaya yaptığınız katkıdan dolayı şimdiden çok teşekkür ederim.

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Eğitim Bilimleri Bölümü
Eğitim Programları ve Öğretim
İletişim: gcinar@metu.edu.tr
Kişisel Bilgiler

1. Kendinizden bahseder misiniz?
   - Mezun olunan lise:
   - Liseden mezuniyet yılı:
   - Sınıf:
   - Odtü’deki kaçınıcı yılınız:
   - Ortalama:

Son Görüşme Soruları

1. Sosyal adalet odaklı matematik öğretimi (teaching mathematics for social justice) kavramı sizin için ne ifade ediyor?
2. Bugünkü ifadeniz sosyal adalet odaklı matematik öğretimi kavramı ile ilgili ilk düşüncelerinize ne gibi benzerlik veya farklılıklar içeriyor?
3. Sizin öğretmenliğe bakış açınız ile sosyal adalet odaklı matematik öğretimi arasında nasıl bir ilişki görüyorsunuz?
   - Benzerlik
   - Farklılık
4. Sosyal adalet odaklı matematik öğretimi içeren bu programa katılmmanın öğretmenliğe dair düşüncelerinizi nasıl etkilediğini düşünüyorsunuz?
   - Matematik kazanımları
   - Sosyal adalet kazanımları
   - Etkinlik temelli öğrenme
5. Sosyal adalet odaklı matematik öğretimi içeren bu programa katılmının gündelik hayata dair bakış açınızı nasıl etkilediğini düşünüyorsunuz?
6. Bu programa katılmakla edindiğiniz bilgi ve teknikleri öğretmenlik meslekte hayatınızda nasıl kullanmayı düşünüyorsunuz?
7. Bir matematik öğretmen adayı olarak sosyal adalet odaklı matematik öğretimini sınıflarında uygularken dünyayı matematikle okuma (reading the world with mathematics) ve dünyayı matematikle yazma (writing the world with mathematics) içeriklerinde ne gibi faydalar göreceğinizi düşünüyorsunuz?
   - Matematik dersine yönelik
- Kişisel gelişimlere yönelik
- Toplumsal fayda

7. a. Bir matematik öğretmen adayı olarak sosyal adalet odaklı matematik öğretimini sınıflarımızda uygularken dünyayı matematikle okuma (reading the world with mathematics) ve dünyayı matematikle yazma (writing the world with mathematics) içeriklerinde ne gibi zorluklarla karşılaşıcağınızzı düşünüyorsunuz?
   - Öğrenci kaynaklı
   - Program kaynaklı
   - Yönetim kaynaklı
   - Veli kaynaklı
   - Eğitsel kaynak
   - Okul iklimi

b. Bu zorluklarla baş etmek için ne tür önlemler alabileceğinizi düşünüyorsunuz?

8. Bu programın içeriğinde aklınızda kalan en önemli noktalar nelerdir?

9. Bu programın daha etkili olması için nelerin değiştirilmesini önerirsiniz?

10. Bunların dışında belirtmek istediğiniz noktalar var mı?
H. SAMPLE CLASS ACTIVITIES

Week 1-Teaching for Social Justice Activity Sheet

Questions
1. What Social Justice challenges does Allison face in their teaching life?
2. What does Allison do to solve these challenges?
3. Are the situations they encounter consistent with what they learned in the teacher education program?
4. As you read in this example, what essential points does teaching for social justice contain?

Week 2- Think-Pair-Share Activity Sheet

Name:

Read the given text and answer the parts.

Think
On your own, write three ideas you have about this issue:
1.____________________________________________________________________
   ______________
   2.____________________________________________________________________
   ______________
   3.____________________________________________________________________
   __________
Pair
Discuss your ideas with a partner. Put a check by any ideas above, that your partner also wrote down. Then, write down ideas your partner had that you did not have:
1._________________________________________________________________
____________________________________________________________________
_____________
2._________________________________________________________________
____________________________________________________________________
_____________
3._________________________________________________________________
____________________________________________________________________
__________

Share
Review all of your ideas and circle the one you think is most important. One of you will share this idea with the whole group.
As you listen to the ideas of the whole group, write down three more ideas you liked:
1._________________________________________________________________
____________________________________________________________________
_____________
2._________________________________________________________________
____________________________________________________________________
_____________
3._________________________________________________________________
____________________________________________________________________
__________
I. ETHICAL PERMISSION FOR NEEDS ANALYSIS INTERVIEWS
J. ETHICAL PERMISSION FOR TEACHING MATHEMATICS FOR
SOCIAL JUSTICE INTERVENTION

27 OCAK 2023

Koşulu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (IAEK)

İlge: İnsan Araştırmaları Etik Kurulu Başkanı

Sayın Prof. Dr. Hamide AKAR

Dannanılığınızı gerektiririniz Gencə Çınar Bent’in “Matematik Öğretmen Adaylarının
Sosyal Adak Odaklı Matematik Öğretimine Buğün Bilgi ve Uygulamalarına Geliştirilmesi”
bağlı olduğu araştırmasına İnsan Araştırmaları Etik Kurulu tarafından uygun görülen
0040-ODTU/AEKE-2023 protokollamayla ile onaylanmıştır.

Bilgilerinize uygun duruma atanım.

Başkan

Oye Oye

Oye Oye

Oye Oye

178
K. INFORMED CONSENT FORM

ARAŞTIRMAYA GÖNÜLLÜ KATILIM FORMU
Bu araştırma, ODTÜ Eğitim Bilimleri Bölümü Doktora öğrencisi Gonca Çınar Bent tarafından Prof. Dr. Hanife Akar danışmanlığında doktora tezi kapsamında yürütülmektedir. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir?
Bu tasarım araştırmasında, “Sosyal Adalet Odaklı Matematik Öğretmeyi Öğrenme” konusunda matematik öğretmen adaylarına yönelik 6 haftalık bir müdahale geliştirmek, uygulamak ve revize edip tekrar uygulamak amaçlanmaktadır.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz?
Araştırmaya katılmayı kabul ederseniz, sizden 6 hafta sürecek bir uygulamaya katılmanız beklenmektedir. Bu uygulama sırasında video kaydı alınacak, sınıf içi yapılan etkinlikler ve konuyla ilgili yazdığınız günlüklar gelişimizi gözlemlemek için toplanacaktır.

Sizden Topladığız Bilgileri Nasıl Kullanacağız?
Araştırmaya katılmınız tamamen gönüllülük temelinde olmalıdır. Sizden elde edilen verilerde kimliğiniz gizli tutulacak ve elde edilen veriler sadece araştırmacılar tarafından değerlendirilecektir. Elde edilecek bilgiler bilimsel yayımlarda kullanılmaktır.

Katıldığınızla ilgili bilmeniz gerekenler:
Araştırmayla ilgili daha fazla bilgi almak isterseniz:
Uygulama sonunda, bu çalışmaya ilgili sorularınız cevaplanacaktır. Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Çalışma hakkında daha fazla bilgi almak için Eğitim Bilimleri Bölümü öğretim üyelerinden Prof. Dr. Hanife Akar (E-posta: hanif@metu.edu.tr) ya da doktora öğrencisi Gonca Çınar Bent (E-posta: gcinar@metu.edu.tr) ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katıldığım.

(Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyisim       Tarih       İmza
L. MİRA’S TMSJ LESSON

PROBLEM 1)

Municipal officials placed yellow colored tactile paving in the form of square prisms with reliefs on the upper surface, which would facilitate the visually impaired individuals in determining the direction, between the tactile paving stones in the form of rectangular prisms on the sidewalks.

These tactile paving stones were placed on a flat pavement with a length of less than 30 meters, without dividing, overlapping and without gaps as in the image.

What is the maximum number of yellow colored tactile paving stones in the form of square prisms with embossed upper surfaces placed on this pavement?

PROBLEM 2)

The tactile paving on a sidewalk consists of rectangular tiles that measure 30 cm by 60 cm. Each tile has a series of raised bumps that are arranged in a pattern to indicate to visually impaired individuals that they are approaching a hazard, such as a street crossing or a staircase. The bumps are spaced 50 cm apart.
If a section of sidewalk is 15 meters long, what is the least number of tiles that can be used to cover the entire section, assuming there are no obstructions or interruptions in the paving?

**SOLUTION:**
To solve this problem using LCM and GCD, we need to find the least common multiple (LCM) of the length of the section and the width of the tile, and then divide by the greatest common divisor (GCD) of the length of the section and the spacing between the bumps.

The width of each tile is 60 cm, or 0.6 meters.
The length of the section is 15 meters.
The spacing between the bumps is 50 cm, or 0.5 meters.
The LCM of 15 and 0.6 is 30, so we can fit 30 tiles across the length of the section.

The GCD of 15 and 0.5 is 0.5, so we need to use 1 tile for every 0.5 meters of length.
Since there are 15 meters in total, we need to use 30 tiles for every 15 meters.

Therefore, the least number of tiles that can be used to cover the entire section is:

\[(30 \text{ tiles across}) \times (30 \text{ tiles for every } 15 \text{ meters}) = 60 \text{ tiles}\]

So, 60 tiles are needed to cover the 15-meter section of sidewalk with tactile paving, assuming no obstructions or interruptions.

**Closing** (assessing students’ thinking, summarizing)

- What will you say and/or do at the end in order to assess students’ learning?
- What will you write on the board and/or use as students share their strategies/thinking?
- What do you expect to see or hear that lets you know about students’ understandings of mathematical big ideas that you intended for them to learn?
• How would you close the lesson to connect different student strategies and connect them to the mathematical goal of the lesson?

With the participation of all students, after the answers of the activity are discussed on the board and the correct solutions are reached, the teacher raised awareness among the students about the difficulties faced by visually impaired people as s/he wanted. The teacher makes the students feel that it is time to take action and asks the students to share their ideas about what they can do to make the already difficult life of the visually impaired a little easier with the exit card below. Exit card is distributed to students and students write their ideas on these cards.

Then the teacher collects all the exit cards and a taking action plan is created on the board in the light of the ideas on the exit cards.
M. CURRICULUM VITAE

GONCA ÇINAR BENT
Middle East Technical University
Faculty of Education
Department of Educational Sciences

EDUCATION


MS, Department of Educational Sciences, Institute of Educational Sciences, Ondokuz Mayıs University, Samsun, Türkiye. Major: Curriculum and Instruction. Thesis: The Relationship between Thinking Styles and Reflective Thinking Tendencies of Pre-Service Teachers. 2016.


ACADEMIC POSITIONS

Research Assistant. February 2023- March 2023. Middle East Technical University, Ankara, Türkiye, Department of Educational Sciences.


PUBLICATIONS


Reports

Presentations at International Conferences


Dönmez Yapucuoğlu, M., Çınar, G. The Evaluation of the 4th Grade Mathematics Program in the Framework of Tylerian Evaluation Model. 5th International Congress on Curriculum and Instruction. Muğla, Türkiye, 26-28 October 2017


Özer, B., Gelen, İ., Hızlı Alkan, S., Çınar, G., Duran, V. The Views of Teacher Trainees regarding to the Most Common Mistakes Teachers Make. 3rd International Congress on Curriculum and Instruction. Adana, Türkiye. 22-24 Ekim 2015.

Giriş


uçuruma nasıl yaklaşacağız, bu durum için neler yapacağız önemli bir değere sahiptir.


**Sosyal Adalet Odaklı Matematik Öğretimi**

Gutstein, sosyal adalet için matematik öğretiminde önemli bir figürdür ve bu çalışmadada onun yaptığı sınıflandırma kullanılmıştır. Gutstein, Freire'nin "kelimeyi okuma (metni okumayı öğrenme)" ile ilgili fikirlerini sosyal adalet odaklı matematik öğretme ve öğrenme kavramına uyarlanmış ve bunu "matematiksel kelimeyi okuma" olarak adlandırılmış ve "matematik okuryazarlığı ve matematiksel güç geliştirme" eylemi olarak tanımlanmıştır (Gutstein, 2003, s. 45).

Gutstein, Freire'nin fikirlerinden yola çıkarak "dünya matematikle okumak" ve "dünya matematikle yazmak" kavramlarının tanımlarını da yapmıştır. Gutstein'a
göre (2003, s.45) dünyayı matematikle okumak, farklı sosyal gruplar arasındaki güç ilişkilerini, kaynak eşitsizliklerini ve eşitsiz fırsatları anlamak ve ırk, sınıf, cinsiyet, dil ve diğer farklılıklarla dayalı açık ayrımcılığı anlamak için matematik kullanmaktadır. Matematik içeren gerçek hayat konularını kullanmak, öğrencilerin dünyayı matematikle okuyabilmeleri için ana kaynağıdır. Öte yandan, sosyal gerçekliği anlamak insanları özgürleştirmez; ancak dünyayı bilinçli bir şekilde değiştirmenin hem ön koşulu hem de sonuçudur. Bu anlamda dünyayı matematikle yazmak, dünyayı değiştirmek için matematik kullanmak olarak tanımlanır (Gutstein, 2006, s.27). Dünyayı matematikle okuma ve dünyayı matematikle yazma kavramları birbirleriyle bağlantılıdır. Dahası, dünyayı matematikle okumayı başarmak için dünyayı okumak ve matematiksel kelimeleri okumak birbirine bağlı ve gerekli iki süreçtir (Gutstein, 2016).

Gutstein'a (2003) göre, matematik eğitiminde sosyal adalet pedagojisinin üç ana hedefi vardır, bunlar; öğrencilerin sosyopolitik bilinc (dünyayı matematikle okumak) geliştirmesi, eylemlilik duygusu (dünyayı matematikle yazmak) geliştirmelerine ve olumlu sosyal ve kültürel kimlikler geliştirmelerine yardımcı olmaktır. Öğrencilere hayatlarında anlam ifade eden sorular sorulup ve yerel ve daha geniş toplumdaki güç ilişkilerini anlamaya başlarlarsa, öğrenciler Freire'nin "conscientização" (Freire, 2000) olarak adlandırdığı sosyopolitik bilinci geliştireceklerdir. Dahasi, kendilerini sosyal adalet için anlamlı bir değişim yaratabilecek kapasitede görürlere, bir eylemlilik duygusu geliştireceklerdir (Ayers vd., 2009).

Bu çalışmanın amacı, ilköğretim matematik öğretmeni adaylarının sosyal adalet için matematik öğretimi anlayışlarını incelemek ve geliştirmektir. Bu amaç doğrultusunda, öncelikle matematik öğretmen adayları ve öğretmen eğitimcilerinden veri toplanarak öğretmen adaylarının ihtiyaçları belirlenmiş, ardından ihtiyaçlar doğrultusunda sosyal adalet odaklı matematik öğretimine yönelik bir müdahale tasarlanarak uygulanmış ve revizyonlar yapılarak yeniden uygulanmıştır. Ayrıca çalışma, bu müdahalenin öğretmen adaylarının sosyal adalet odaklı matematik öğretmeni anlayışlarına nasıl katkı sağladığı ve öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışlarının gelişimini incelemeyi amaçlamaktadır.
Yukarıda belirtilen amaçlarla ilişkili olarak, çalışma sırasında aşağıdaki araştırma soruları yanıtlanmaya çalışılmıştır:

İhtiyaç Analizi için Araştırma Soruları:
1. Matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimine ilişkin mevcut deneyimleri nelerdir?
2. Öğretmen eğiticilerinin perspektifinden matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimine ilişkin deneyimleri nelerdir?

Tasarım Araştırması için Araştırma Soruları:
Temel Soru: "Sosyal yapılandırma ile dayanan bir öğretme ve öğrenme stratejisi matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışını nasıl geliştirebilir?"

1. Matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimine anlayışı, dünyayı matematikle okuma ve dünyayı matematikle yazma boyutlarında, sosyal yapılandırılmasına dayanarak tasarlanmış bir müdahale yoluyla nasıl gelişir?
2. Matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimine ilişkin anlayışlarını geliştirmek için tasarlanmış bir müdahalenin unsurları nelerdir?

Bu araştırmanın bir kaç önemi vardır, bunlar; hizmet öncesi öğretmen eğitiminde sosyal adalet odaklı matematik öğretimine karşı ihtiyaçın saptaması, bunun yanında ihtiyaç analizi bulgularına ve alanyazına göre geliştirilen ve uygulanan müdahale ile öğretmen adaylarının, sosyal adalet odaklı matematik öğretimini anlayışlarının geliştirilmesi ve öğretmen eğiticilerince sosyal adalet odaklı matematik öğretimi ile ilgili bir müdahalenin içeriğinin sunulmasıdır. Öğretmen adaylarının sosyal adalet odaklı öğretim yaklaşımları geliştirmeleri gerekmektedir (Lee, 2011). Bu öğretim yaklaşımları, tüm öğrenciler için eşitlikçi pedagoji geliştirme, öğrencilerin farklı kültürlernini sınıfa entegre etme, önyargıyi azaltacak öğrenme ortamları yaratma ve çoklu bilgi yapılandırma süreçlerini dahil etme ile ilgili olabilir (Banks, 2008).

Yöntem

Bu çalışmada, ilköğretim matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimini anlayışlarını incelemek ve geliştirmek amacıyla tasarım


yapma. Bu çalışmada, matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışlarını geliştirmeye yönelik bir müdahale tasarlamak olduğundan bu duruma uygunduğunu için Gravemeijer ve Cobb'un (2013) sınıflandırmasını kullanacaktır.

**Tasarım Araştırmasının Aşamaları**

Deney için hazırlanık: Bu aşamada, araştırmacının ne tasarlamak istediğini (görev, sıralama, öğrenme ortamları) ve ne tür bir bilgi üretmek istediğini karar verilir (Bakker, 2019). Tasarım araştırmasında araştırmacı, öğretimin bitiş ve başlangıç noktalarını netleştirek işe başlar. Genel olarak, planlanan görevleri sınıfta uygulamadan önce, araştırmacı öğrencilerin düşünme ve anlamalarındaki gelişmeleri tahmin eder. Buna ek olarak, araştırmacı öğrencilerin düşünme ve anlamalarının nasıl geliştirileceği konusunda varsayımlandırılmaktadır.

Sınıfta deney yapma: Bu aşamada, sınıf bir tasarım deneyi gerçekleştirir. Tasarım deneyinin amacı, varsayılan yerel öğretim teorisini test etmek ve iyileştirmek ve işe yarayıp yaramadığını görmekten ziyade nasıl çalıştığını anlamaktır (Gravemeijer & Cobb, 2013). Bu aşama, test etme, geliştirme ve neler olduğunu anlamayı içeren yinelemeli bir süreçtir.

Geriye dönük analiz yapılması: Son aşama, deney sırasında toplanan tüm veri setinin geriye dönük analizlerinden oluşur. Bu aşama, eksiksiz bir veri seti yinelemeli bir süreçte analiz edilir (Gravemeijer & Cobb, 2013)

*Bu Çalışmanın Aşamaları*

**Deney için Hazırlık:** Bu çalışmada ilk olarak yapılan şey, ilköğretim matematik öğretmeni adaylarına sosyal adalet odaklı matematik öğretmek üzere tasarlanacak müdahalenin (Gravemeijer & Cobb, 2013) hedeflerini belirlemek olmuştur. Müdahalenin temel hedeflerine, sosyal adalet odaklı matematik öğretimi ile ilgili alanyazını ve ortaokul matematik müfredatı ile ilgili belgeleri gözden geçirdik karar verdim. Ayrıca, mevcut durumu anlamak ve matematik öğretmeni yetiştirme programında sosyal adalet odaklı matematik öğretimine duyarız ortaya çıkarmak için matematik öğretmeni adayları ve öğretmen eğitimcileri ile görüşmeler
yaptım. Bu görüşmeler, matematiğin amacı, sosyal adaylar odaklı matematik öğrenme ve öğretme ve matematik öğretmeni yetiştirme programlarındaki mevcut durum hakkında sorular içerenyordu.


Geriye Dönük Analizin Yapılması: Matematik öğretmen adaylarıyla yapılan ön ve son görüşmeler, yansıtıcı kağıtlar, alan notları, sınıfın video kayıtları, gözlemler, sınıf içi etkinlikler, bireysel ders planları gibi farklı veri toplama araçlarından elde edilen veriler, öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışlarının gelişimini incelemek için tematik analiz ile analiz edilmiştir.
Katılımcılar

Veriler çalışmanın 1. ve 2. aşamalarında toplanmıştır. 1. aşamada veriler ilköğretim matematik öğretmen adayları ve öğretmen eğitimcilerinden sosyal adalet odaklı matematik öğretimi ile ilgili ihtiyaç analizi çalışması ile toplanmıştır. 2. Aşamada ise veriler 1. tasarım deneyi ve 2. tasarım deneyi sırasında sınıf ortamında toplanmıştır. Çalışmanın katılımcılarına genel bir bakış Tablo 1'de verilmiştir.

<table>
<thead>
<tr>
<th>Tablo 1 Çalışmanın Katılımcıları</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. aşama</strong></td>
</tr>
<tr>
<td>Öğretmen Adayları ile Yapılan İhtiyaç Analizi Görüşmelerinin Katılımcıları</td>
</tr>
<tr>
<td>Öğretmen Eğitimcilerle Yapılan İhtiyaç Analizi Görüşmelerinin Katılımcıları</td>
</tr>
<tr>
<td><strong>2. aşama</strong></td>
</tr>
<tr>
<td>1. Tasarım Deneyinin Katılımcıları</td>
</tr>
<tr>
<td>2. Tasarım Deneyinin Katılımcıları</td>
</tr>
</tbody>
</table>

Veri Toplama Araçları

Tasarım araştırması, tasarımın sonuçlarını değerlendirmek ve tasarım sürecini iyileştirmek için farklı veri toplama araçlarından yararlanır (Design-Based Research Collective, 2003). Bu çalışmada, veri toplama araçları, öğretmen adayları ve öğretmen eğitimcileri için ihtiyaç analizi görüşmeleri, 1. tasarım deneyi ve 2. tasarım deneyi boyunca öğretmen adaylarıyla yapılan ilk ve son görüşmeler, araştırmacının saha notları, sınıf dışı gözlemler, yansıtıcı kağıtlar, öğretmen adaylarının sınıf içi etkinlikleri ve öğretmen adaylarının bireysel ders planları ve derslerin video kayıtları ve grup çalışmalarındaki ses kayıtlarıdır.

Yarı Yapılandırılmış Görüşme Protokolleri: Öğretmen adayları ve öğretmen eğitimcileriyle görüşmeler yapmak için dört adet yarı yapılandırılmış görüşme formu geliştirilmiştir (İhtiyaç analizi için 2 görüşme formu ve tasarım deneyi için 2 görüşme formu). Soruların tam ifadesinin ya da sırasının görüşme sırasında belirlendiği yarı yapılandırılmış görüşmelerde tüm katılımcılardan spesifik bilgiler istenmektedir (Merriam ve Tisdell, 2016). Görüşme çizelgeleri oluşturulduktan sonra,
görüşme protokolünün kapsam ve görüş geçmişliğini, ifadelerin netliğini ve ilköğretim matematik öğretmen adayları ve öğretmen eğitmenleri için uygunluğunu sağlamak için bunları üç uzmana paylaşildı. Uzmanların görüşlerine göre gerekli değişiklikler yapıldıktan sonra görüşme protokollerinin son halı elde edilmiştir.

Öğretmen adayları için ihtiyaç analizi görüşme protokolü ilköğretim matematik öğretmen adaylarının sosyal adlet odaklı matematik öğretimi deneyimlerini derinlemesine ortaya çıkarmak amacıyla yarı yapılandırılmış görüşme soruları içermektedir. Öğretmen eğitmenleri için ihtiyaç analizi görüşme protokolü öğretmen eğitmenlerin gözünden öğretmen adaylarının sosyal adlet odaklı matematik öğretimi ile ilgili deneyimlerini yansıtmayı amaçlayana sorular içermektedir. Öğretmen adayları için ile yapılan ilk ve son görüşme protokolleri ise matematik öğretmen adaylarının sosyal adlet için matematik öğretimi müdahaleine katılırken anlayışlarının nasıl geliştiğine odaklanan sorular içermektedir.

**Saha Notları:** Araştırmacının nitel bir çalışmada veri toplama ve yansıma sürecinde duyduklarının, gördüklerinin, deneyimlediklerinin ve düşündüklerinin kayıtlarıdır (Bogdan & Biklen, 2007). Araştıracı, öğretmen adaylarının sosyal adlet odaklı matematik öğretimine ilişkin algı ve deneyimlerini belirlemeye odaklanarak her dersten sonra yansıtıcı saha notları yazmıştır.

Yansıtıcı Yazılar: Matematik öğretmen adayları, çalışma boyunca her iki deney dersinde işlenen konulara paralel olarak kendi fikirlerini içeren iki yansıtıcı makale yazmışlardır. Bu yansıtıcı yazılardan sosyal adalet odaklı matematik öğretimi ile ilgili kavramları oluşturma süreçlerinin nasıl geliştiğini anlamak açısından önemli bir değer sahiptir.

Derslerin Video ve Ses Kayıtları: İki tasarım deneyi de kaydedilmiş ve her bir öğretmen adayının fikirlerini belirlemek için sınıflı grup tartışmaları ses kayına alınmıştır. Derslerin tüm video ve ses kayıtları araştırmacı tarafından deşifre edilmiş ve deşifre sırasında araştırmacı matematik öğretmen adaylarının sosyal adalet odaklı bilgi ve uygulamalarını geliştiren olaylar, görevler ve ortam hakkında notlar almıştır.

Sınıf İçi Etkinlikler: Her iki tasarım deneyi sırasında öğretmen adaylarının sınıf içi etkinlik kağıtları toplanmıştır. Katılımcıların müdahale boyunca, dersler sırasında bireysel ve grup olarak çeşitli etkinlik kağıtlarını doldurması gerekmiştir.


Veri Toplama Süreçleri

Bu çalışmada ilk olarak ihtiyaç analizi çalışması için öğretmen adayları ve öğretmen eğitimcilerinden veri toplanmıştır. Ardından, müdahalenin uygulanmasını yoluya veri toplanmıştır.

Ağustos 2022'nin başında etik kuruldan onay alınarak onay alındıktan sonra, ihtiyaç analizi görüşmeleri için üçüncü ve son sınıf ilköğretim matematik öğretmen adaylarına


Sosyal adalet odaklı matematik öğretimi müdahalesi yüz yüze yapılmıştır, ancak öğretmen adaylarıyla yapılan tüm ilk ve son görüşmeler her iki deney için de
çevrimiçi olarak gerçekleştirilmişdir. Her deneyden bir hafta önce öğretmen adayları e-posta yoluyla ilk görüşmelere davet edilmiş ve her deneyden bir hafta sonra öğretmen adayları e-posta yoluyla son görüşmelere davet edilmiştir. Tüm ilk görüşmeler deneyden önce, son görüşmeler ise her deney sona erdikten sonra yazıya dökülmüştür. İlk görüşmeler yaklaşık 30 dakika, son görüşmeler ise yaklaşık 60 dakika sürmüştür. İlk ve son görüşmeler sırasında notlar alınmıştır. Her iki uygulama boyunca tüm dersler videoya kaydedilmiş, grup çalışmalarını ses kayına alınmış ve tüm öğrencilerin çalışmaları toplanmıştır. Ayrıca araştırmacı her dersin ardından saha notları yazmış ve bir doktora öğrencisi ile bir doktora adayı farklı dersleri gözelemleyerek gözlem notlarını araştırmacı ile paylaşılmıştır.

Sosyal adalet odaklı matematik öğretimi müdahalenin katılımcıları çalışmaya gönüllü olarak katılacakları ve istediğini zaman ayrılabilecekleri konusunda bilgilendirilmişdir. Çalışmaya gönüllü olarak katılan katılımcılar; istediğini zaman çalışmadan ayrılabilcekleri konusunda bilgilendirilmişlerdir. Takma isimler kullanılarak gizlilik sağlanmıştır. Ayrıca, araştırmacı dışında hiç kimse katılımcıların kimliklerini ortaya çkarabilecek verilere erişim iznine sahip olmamıştır.

Veri Analizi

Araştırmacı, öğretmen adayları ve öğretmen eğitimcileriyle yapılan ihtiyaç analizi görüşmelerinden ve tasarım deneylerinden (ilk ve son görüşmeler, sınıf içi etkinlikler, yansıtma kağıtları, saha notları, sınıf gözlemleri ve derslerin video ve ses kayıtları) elde edilen çeşitli veri türlerini analiz etmiştir. İlk olarak, öğretmen adayları ve öğretmen eğitimcileriyle yapılan ihtiyaç analizi görüşmelerini nasıl analiz ettiklerini analiz etmiştir. Ardından, tasarım deneyindeki tüm veri setiyle geriye dönük analizin nasıl yapıldığını açıkladım. Ardından, tasarım deneyindeki tüm veri setiyle geriye dönük analizin nasıl yapıldığını açıkladım.

Öğretmen Adaylarının Sosyal Adalet Odaklı Matematik Öğretimine İlişkin İhtiyaç Analizi: İlköğretim matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimi konusundaki ihtiyaç ve deneyimlerini analiz etmek için MAXQDA kullanılarak tematik analiz yapılmıştır. Veriler, İç Anadolu Bölgesi'ndeki bir devlet üniversitesinin İlköğretim Matematik Eğitimi Bölümü'nde öğrenim gören ilköğretim

Öğretmen adayları ile yapılan ihtiyaç analizi görüşmelerinde ortaya çıkan temalar; bölüm seçme nedenleri, matematik öğrenme amacı, eğitimde bir kavram olarak sosyal adalet, sosyal adalet odaklıöğretim anlayışı ve sosyal adalet odaklı matematik öğretmeni olmuştur. Öğretmen eğitimcileri ile yapılan ihtiyaç analizi görüşmelerinin temaları ise eğitim, eğitimde bir kavram olarak sosyal adalet ve sosyal adalet odaklı matematik öğretmeni olmuştur.

**Tasarım Deneyinin Analizi:** Tasarım deneyinin analiz süreci, matematik öğretmen adayları ve öğretmen eğitimcileriyle gerçekleştirilen ihtiyaç analizi görüşmelerinin veri analiz süreci ile benzer süreçlerden oluşmuştur. Ek olarak, tasarım deneyi analizinde verilerin karşılaştırılması da yapılmıştır. Öncelikle ilk ve son görüşmeler, derslerin videoları ve sınıftaki grup tartışmalarının ses kayıtları yazıya dökülmuş ve öğretmen adaylarının sosyal adalet odaklı matematik dersi planları incelenmiştir. Daha sonra araştırmacı verileri kodlanmış, bu kodlardan alt temalar ve daha sonra alt temalardan, temalar oluşturulmuştur. Buna ek olarak, öğretmen adaylarının tasarım deneyi öncesi ve sonrası anlayışlarını karşılaştırmak için sürekli karşılaştırmalı analiz yapılmıştır. Farklı ya da aynı veri setlerinin karşılaştırılması sürekli karşılaştırma yöntemidir (Merriam, 2015). Araştırmacı veri setlerini karşılaştırmırken temel olarak matematik öğretmen adaylarının sosyal aday etik matematik öğretimi anlayışlarına odaklanmıştır. İlk ve son görüşmelerden ve matematik öğretmen adaylarının yansıtma kağıtlarından elde edilen verilerle bir karşılaştırma yapılmıştır.
Araştıracı, kodlama sürecini gözden geçirmek ve tartışmak için süpervizör ile iş birliği yapmış ve alt temaları ve temaları belirlemiştir. Bu tartışmanın ardından, belirli temalar yeniden düzenlenmiştir ve ortak bir karar verme süreciyle son haline getirilmiştir.


**Bulgular**

Bu çalışmada, bulgular iki ana kısımda anlatılmaktadır. Öğretmen adayları ve öğretmen eğitimcilerinin bakış açısından sosyal adalet odaklı matematik öğretimine ilişkin ihtiyaç analizi sonuçları ve tasarım deneyinin sonuçları ayrı bölümlerde sunulmuştur.

**İhtiyaç Analizi Bulguları**

Öğretmen adaylarının sosyal adalet odaklı matematik öğretimine ilişkin ihtiyaçlarını belirlemek amacıyla öğretmen adayları ve öğretmen eğitimcileri ile yarı
yapılandırılmış görüşmeler yapılmıştır. İhtiyaç analizi bulguları, öğretmen adaylarının lisans derslerinde sosyal adayla kavrama aşına olmadıklarını ortaya koymmuştur. Ayrıca, ilköğretim matematik öğretmen adayları lisans müfredatlarında herhangi bir derste sosyal adayda odaklı matematik öğretimi ile karşılaşmamışlardır ve soruluğunda sosyal aday ile matematik öğretimini ilişkilendirememişlerdir. Öğretmen adayları, sosyal adayda odaklı matematik öğretiminin yönelik bir uygulamaya katılmak istediklerini ve uygulamanın grup çalışması içermesi tercih ettiklerini çünkü grup çalışması sırasında arkadaşlarından öğrenebileceklerini, farklı bakış açıları kazanabileceklerini ve beynin fırtınası ile yeni fikirler geliştirileceklerini belirtmişlerdir. Ayrıca, öğretmen adayları ve öğretmen eğitimcileri ile yapılan ihtiyaç analizi sonuçlarına göre, öğretmen adaylarının öğretmen eğitimi müfredatında sosyal adayla kavrama aşına olmadıkları görülmüştür. Bu nedenle, sosyal adayda odaklı matematik öğretimi müdahaleinin içeriğine sosyal adayda odaklı öğretimin dahil edilmesine ve öğrenme etkinliklerinin sosyal yapılandırmacılığa dayalı olarak planlanmasına karar verilmiştir.

**Tasarım Deneyinin Sonuçları**

İlk görüşmelerde öğretmen adaylarına matematik öğretiminin öğrencilerin sosyal adalet konular ile ilgili farkındalığını (dünyayı matematikle okuma) nasıl artırabileceğini sorulmuş; öğretmen adayları sosyal adalet ile matematik öğretimi arasında bir bağlantı kuramadıklarını ifade etmiş, bir öğrenci ise bunun ancak öğretmenlerin tutumlarıyla olabileceğini söylemiştir. Ancak, son görüşmelerde ve yansıtma kağıtlarında, tüm öğretmen adayları dünyayı matematikle okuma kavramını matematik yoluyla farkındalık yaratmak olarak açıklamış; bazıları da bu kavramı matematikle empati kurmak ve matematik yoluyla farklılıkları tanımak olarak dile getirmişler.

İlk görüşmelerde, öğretmen adaylarına dünyayı matematikle okuma kavramı ile ilgili sorulan matematik öğretimi yoluyla öğrencilerin sosyal adalet konularında harekete geçip geçemeceğini ilişkin soruya bazı öğretmen adayları bunun nasıl olabileceği konusunda bir fikirleri olmadığını, belki öğrencilerin problemlerdeki alt mesaji içselleştirerek harekete geçebileceklerini söylerken, bazıları da öğrencilerin ancak yetişkin olduklarında harekete geçebileceklerini vurgulamışlardır. Son görüşmelerde, öğretmen adayları öncelikle dünyayı matematikle okuma kavramını matematik yoluyla eyleme geçme olarak tanımlamışlar ve ardından dünyayı matematikle okuma anlamında neler yapılabileceği farklı yollarla belirlenebilmiştir. İlk görüşmelerde öğrencilerin ancak yetişkin olduklarında değişşim yaratabileceklerini vurgulayan öğretmen adayları son görüşmelerde fikir değiştirek öğrencilerin kendi hayatlarında ya da başkalarının hayatlarında da değişşim yaratabileceklerini belirtmişlerdir.

Öğretmen adayları ayrıca sosyal adalet odaklı matematik dersleri uygulama ile ilgili bazı faydaları ve endişelerini dile getirmişlerdir. Son görüşmelerde, belirttiğleri faydalar ve endişelere ilişkin görüşlerinde de bir gelişme olduğu görülmektedir. Son olarak, öğretmenlerin rollerine ilişkin ilk görüşmede sosyal adalete vurgu yapılmazken, müdahaleye katılduktan sonra, öğretmen adayları yansıtma kağıtlarında öğretmenin rolleri olarak hem sosyal adalet konusunda farkındalık yaratma hem de öğrencileri sosyal adalet konularında harekete geçmeleri için güçlendirme konusundaki görüşlerini dile getirmişlerdir.

Tartışma

Bu kısımda, öğretmen adaylarının sosyal adalet odaklı matematik öğretimine ilişkin ihtiyaçlarının ve tasarım deneyinin sonuçları tartışılmıştır. Ayrıca, çalışmadan yapılabilecek çıkarımlar ve daha sonraki araştırmalar için öneriler de bu bölümde açıklanmıştır.

Sosyal Adalet Odaklı Matematik Öğretimi İhtiyaçlarına İlişkin Bulguların Tartışılması

Jong ve Jackson (2016) ilköğretim öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışlarını incelemiş; çalışmada ortaya çıkan en yaygın temalar sosyal adalet odaklı matematik öğretimi hakkında bilgi sahibi olmama ve bunu bir fırsat ya da edeetime erişim olarak tanımlama olmuştur. Aslında, bu çalışmanın ihtiyaç analizi sonuçları da bu bulgularla tutarlıdır. Yapılan ihtiyaç analizi çalışmasında, öğretmen adayları sosyal adalet odaklı öğretimi herkes için fırsat
eşitliği olarak tanımlamışlardır ve sosyal adalet odaklı matematik öğretimi konusunda bilgi eksiklikleri bulunduğu görülmektedir. Bu bağlamda, öğretmen adaylarının sosyal adalet odaklı matematik öğretimi konusunda daha derin bir anlayışa sahip olmalarına ihtiyaç olduğu sonucuna varılabilir.


**Tasarım Deneyi Bulgularının Tartışılması**


**Teori ve Uygulama için Çıkarımlar**

Bu çalışmanın bulguları araştırmacıları ve öğretmen eğitimcilerine matematik öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışlarını ve bu anlayışların nasıl geliştirildikleri hakkında bilgi sağlamaktadır. Bu çalışmadan faydalanacak araştırmacılar ve öğretmen eğitimcileri için bazı çıkarımlar sunulmuştur.


Öğretmen adaylarının sosyal adalet odaklı matematik öğretimi anlayışlarının gelişmesinden en çok etkilenecek olan kişiler, onların gelecekteki öğrencileridir.
Öğretmen adaylarının anlayışlarında bu gelişmeyle, gelecekteki öğrencileri sosyal aday konularının daha fazla farkına varacak ve bu konularla ilgili harekete geçebilecek, bu ilgi çekici ve gerçek hayattan konular matematiğe dahil edildiğçe matematiğe olan ilgileri de artacaktır. Bu nedenle, öğretmen adaylarının sosyal adayta matematik öğretimi anlayışını geliştirmek önemlidir. Ayrıca, geliştirilen müdahale öğretmenlere hizmet içi eğitim olarak uygulanabilir ve sosyal adayta matematik öğretimi anlayışları uygulanabilir.

Sonraki Araştırmalar için Öneriler


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