A CASE STUDY ON MIDDLE SCHOOL TEACHERS' MATHEMATICS QUALITY OF INSTRUCTION

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

AZİME ATAY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
THE DEPARTMENT OF ELEMENTARY EDUCATION

MARCH 2024

Approval of the thesis:

A CASE STUDY ON MIDDLE SCHOOL TEACHERS' MATHEMATICS QUALITY OF INSTRUCTION

submitted by **AZIME ATAY** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Elementary Education, the Graduate School of Social Sciences of Middle East Technical University** by,

Prof. Dr. Sadettin KİRAZCI
Dean
Graduate School of Social Sciences

Prof. Dr. Semra SUNGUR
Head of Department
Department of Elementary Education

Prof. Dr. Erdinç ÇAKIROĞLU
Supervisor
Department of Mathematics and Science Education

Examining Committee Members:

Prof. Dr. Mine IŞIKSAL-BOSTAN (Head of the Examining Committee)
Middle East Technical University
Department of Mathematics and Science Education

Prof. Dr. Erdinç ÇAKIROĞLU (Supervisor)
Middle East Technical University
Department of Mathematics and Science Education

Assoc. Prof. Dr. İffet Elif YETKİN-ÖZDEMİR
Hacettepe University
Department of Mathematics and Science Education

Assist. Prof. Dr. Işıl İŞLER-BAYKAL
Middle East Technical University
Department of Mathematics and Science Education

Assist. Prof. Dr. Gönül ERHAN
Başkent University
Department of Mathematics and Science Education

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: Azime ATAY

Signature:

ABSTRACT

A CASE STUDY ON MIDDLE SCHOOL TEACHERS' MATHEMATICS QUALITY OF INSTRUCTION

ATAY, AZİME
Ph.D., The Department of Elementary Education
Supervisor: Prof. Dr. Erdinç ÇAKIROĞLU

March 2024, 285 pages

This study aimed to investigate aspects of the quality that middle school mathematics teachers highlighted and the quality of instruction while teaching the area of circles and sectors. Data were collected from middle school teachers who worked in different public schools throughout the teaching of the area of circle and the area of the sector in 7th grade. The schools were placed in different districts of the city. Participant teachers graduated from Elementary Mathematics Education programs at the universities and experienced teachers who have been working more than 5 years. In the data collection process, the teachers came together and talked about how they teach the area of the circle and the area of the sector. After group discussions, the teachers instructed their lessons and the researcher took video-record of the instruction. The data were analyzed within the Mathematical Quality for Instruction framework. Findings indicated that the teachers knew the content they teach and only three instances were observed for the Errors and Imprecision dimension. They received mid-or-high scores for the Richness of Mathematics dimension in many segments. Explanations, Mathematical Sense-Making and Mathematical Language were the most frequently used dimensions. The Linking Between Representations is

the least used sub-dimension by the teachers. Sub-dimensions of Working with Students and Mathematics dimensions were observed many times during instructions. In the group discussion, both Ali and Efe stated possible students' difficulties and errors and students' thinking about the content They corrected students' errors but generally, correction was procedural. However, they rarely used Common Core Aligned Student Practices. They preferred teacher-centered teaching methods, and so students' contributions were limited.

Keywords: Mathematical Quality of Instruction, area of circle, area of sector.

ÖZ

ORTAOKUL MATEMATİK ÖĞRETİMİNİN KALİTESİ ÜZERİNE BİR ÖRNEK OLAY ÇALIŞMASI

ATAY , Azime
Doktora, İlköğretim Bölümü
Tez Yöneticisi: Prof. Dr. Erdinç ÇAKIROĞLU

Mart 2024, 285 sayfa

Bu çalışma ortaokul matematik öğretmenlerinin dairenin alanı ve daire dilimin alanını öğretirken, kaliteli bir öğretim için nelere odaklandıklarını ve öğretimlerinin kalitesini araştırmayı amaçlamıştır. Veriler 7. sınıfta dairenin alanı ve daire diliminin alanının öğretilmesi sürecinde farklı devlet okullarında görev yapan ortaokul öğretmenlerinden toplanmıştır. Okullar şehrin farklı semtlerinden seçilmiştir. Çalışmaya katılan öğretmenler üniversitelerin İlköğretim Matematik Öğretmenliği programından mezun olmuş, beş yıldan daha uzun süredir öğretmenlik yapan tecrübeli öğretmenlerdir. Veri toplama sürecinde öğretmenler bir araya gelerek daire ve daire diliminin alanını nasıl öğrettiklerine dair öğretimsel deneyimlerini paylaşmıştır.. Grup tartışmalarının ardından öğretmenler derslerini anlatmış ve bu dersler video ile kayıt altına alınmıştır. Veriler Öğretimin Matematiksel Kalitesi (MQI) çerçevesinde analiz edilmiştir. Bulgular, öğretmenlerin öğrettikleri konuyu hakim olduklarını göstermiştir. Dolaysıyla Hatalar ve Belirsizlik boyutu için dil kullanımında muğlaklık oluşturan yalnızca üç örnek gözlemlenmiştir. Buna paralel olarak öğretmenler Matematiksel Zenginlik boyutunda da birçok segmentte orta ve yüksek puanlar almıştır. Açıklamalar, Matematiksel Anlamlandırma ve Matematiksel Dil en sık kullanılan alt boyutlar olmuştur. Temsiller Arası Bağlantı Kurmak alt boyutu ise en az kullanılan alt boyut olmuştur. Öğrencilerle ve Matematikle Çalışma boyutunun alt bileşenlerine derslerde yer verilmiştir. Öğretmenler öğrencilerin hatalarını fark edip düzeltmişlerdir. Fakat bu düzeltmeler genelde işlemsel düzeyde gerçekleşmiştir. Öğretmenler grup tartışmasında olası öğrenci zorluklarını ve hatalarını belirtmelerine rağmen, ders anlatırken bir önlem planlamadıkları görüldü. Ortak Temelde Oluşturulmuş Öğrenci Uygulamaları en az kullanılan boyut olmuştur. Öğretmenler öğretmen temelli bir öğretim tercih ettikleri için öğrenci katlısının sınırlı kaldığı gözlemlenmiştir.

Anahtar Kelimeler:Öğretimin matematiksel kalitesi, dairenin ve daire diliminin alanı

To my Family

ACKNOWLEDGMENTS

Firstly, I would like to thank my supervisor Prof. Dr. Erdinç ÇAKIROĞLU for his patience, support, and encouragement in the writing process of this thesis. I have learned a lot in this process. I also want to thank my committee members Prof. Dr. Mine IŞIKSAL-BOSTAN, Assoc. Prof. Dr. İffet Elif YETKİN-ÖZDEMİR, Assist. Prof. Dr. Işıl İŞLER BAYKAL and, Assist. Prof. Dr Gönül ERHAN for their time, comments, and suggestions to improve my thesis.

I want to thank my family for their support throughout my life. I want to thank my husband Erol MUTLU and my son Ali Efe MUTLU who were my biggest support and encouragement reasons.

I would like to thank my friends Merve KIRNAP DÖNMEZ AND Pınar YILDIZ who encouraged and supported me all this process. I also want to thank my patient friends Berna TEKİN and Tuğba AYDOĞAN for being my friends for more than fifteen years.

Lastly, I want to thank my participant teachers and students who voluntarily took part in my study. I express my sincere thanks to you for spending time on my research.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

MQI : Mathematical Quality of Instruction

SMQR : Student Mathematical Questioning and Reasoning

MKT : Mathematical Knowledge for Teaching

MSMT : Middle School Mathematics Teacher

SMQR : Students Mathematical Questioning and Reasoning

CHAPTER 1

INTRODUCTION

Mathematics is one of the most important subjects taught at school because mathematics is related to real life directly and it also provides a basis to other areas such as physics, chemistry, and computer science. The improvement of student learning in mathematics is a central focus of policymakers, and researchers in many countries (Harniss et. al., 2002; Jaworski, 2006). To improve student learning outcomes, many different methods have been tried. One of these methods is changing the curriculum. “Competency-based curriculum’ which was developed to cover the “irrelevance of much knowledge-based education to occupational performance and the failure of educational qualifications to predict occupational success” (Raven, 2001, p. 253) was widely accepted educational reform of the last decades. The educational reform movement also affected the Turkish educational system and Middle School Mathematics curricula were updated in 2005 because of socio-economic (globalization), political (European Union), philosophical (Constructivism), and educational (Student-centered teaching) reasons (İnal, 2005). The new curricula are based on the idea that “Every child can learn mathematics” and focus on process skills, and mathematical thinking skills and want students to participate in his/her own learning actively. However, teachers who implement curricula in the classrooms, are the decisive factors of what students receive during instruction. The research showed that teachers are important for students learning (Bobis et al., 2012; Hill et al., 2005; Rockoff, 2004; Teddlie & Reynolds, 2000).

The researches on pre-service teachers education and mathematics teaching indicated that work of teaching and teacher knowledge to teach effectively are very complex process (Ball et. al., 2008) Which variables in teacher education affect the student’s learning and achievement? The researchers investigated the effect of teachers’ years of experience, teachers’ beliefs, teachers’ content knowledge and pedagogical content knowledge, and lesson artifacts (assignments, books, lesson plans). Identifying the types of classroom practice that affect student outcome is critical for the education researchers (Blazar, 2015). The required teacher knowledge to teach mathematics effective defined Ball et al in Mathematical Knowledge for Teaching framework. Mathematical Knowledge for teaching is a content specific framework that describes content knowledge and pedagogical content knowledge required for effectively teach the mathematics.

To understand the aspects that affect learning and achievement generally standardized test are used. However, standardized test results report the result, and give no information about the how to improve the quality of instruction (Boston et. al, 2015). To improve the students learning more attention should be paid the instruction that students received in the classroom (Charalambous et. al., 2012). However, understanding effect of teaching process on student achievement faced problems such as developing an appropriate tool to measure the quality. Researcher decided to work on this problem and some observation protocols were developed to analyze instructions. Instructional Quality Assessment (IQA), the Classroom Observation Instrument, Inside the Classroom Observation and Analytic Protocol (ICOAP), and Reformed Teaching Observation (RTOP) are some of the classroom observation frameworks that are used to measure the quality of instruction. However, result of studies also highlights the importance of subject-specific framework (Charalambous & Praetorious, 2018). QUASAR The Mathematical Quality of Instruction (MQI) framework is a content-specific classroom observation framework. It focus on mathematical quality of instruction and eliminates problems related to other factors. Therefore, it meets the aspects of mathematics education the best. In this research, the MQI framework was used to analyze quality of mathematics instruction.

1.1. Aim of the Study and Research Questions

Mathematics is an important subject of the school. Mathematics is not only a school subject but also it forms a base for many profession. Increasing the quality of mathematics education is one of the important goal of Turkish education system. This study is structured as a qualitative study that researches mathematical quality of middle school mathematic teachers' instruction, and the quality aspects that highlighted. It is centered on multiple case studies. This study tries to answer the following two main research questions:

1. What aspects of the instruction do middle school mathematics teachers highlight and what is the quality of these aspects?

2. How is the quality of instruction of middle school mathematics teachers in implementing the instruction as they observed through the Mathematical Quality of Instruction (MQI) instrument?

1.2. Significance of the Study

Quality of education is affected by many factors. Some of the factors are, teachers, students, the district that school located, socio-economic level of families and so on. Teacher knowledge is one of the important factor that affect student achievement. How teaching occur in the classroom is highly related to students achievement. Looking into instruction process may give information about how student learn, which kind of activities support permanent learning, what teacher do to support student learning, and what factors affect student high order thinking skill. However, each observation protocols has some strong and week aspect. To minimize the lose of data because of the observation protocol, using an content speci fic observation rubric will be helpful.

MQI separates the instruction into segments and examines each segment in detail. Examining the quality of mathematics instruction gives information about the needs of the teachers, and teaching. It shows strong and week aspects of the instruction. It has the potential to inform educational policy by addressing the need for mathematics education in Turkey. It suggests ways to modify or adapt the content of mathematics teacher education, professional development programs, and in-service training workshops. Also, school profiles are different, so it also gives information about effect of students' profiles on instructional quality of mathematics.

1.3. Definition of Important Terms

Area

Amount of space occupied by a two dimensional figure

Circle

A circle is a shape in the plane that is formed by combination of all points that are situated in an equal distance from a center.

Sector

A sector is a section of a circle bounded by two radii and corresponding circular arc.

Mathematical knowledge for teaching

"Mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students (Ball et al., 2008, p.399)"

Middle school mathematics teacher

Teachers who teach mathematics to from fifth to eighth-grade students are named middle school mathematics teachers. They generally graduated from Elementary Mathematics Education programs at universities. The teachers working in public middle schools were selected as participants in this study.

Mathematical Quality of Instruction (MQI)

A framework that was developed to investigate quality of mathematics instruction. It is a content-specific framework and used to analyze video records of instruction.

CHAPTER 2

LITERATURE REVIEW

The goal of this study is to investigate the instructional quality of mathematics lessons, by observing their mathematics instruction. In this chapter, an overview of the literature about the teaching the area concept, mathematical knowledge for teaching and mathematical quality of instruction is presented.

2.1. Teaching Area Concept

Measurement is one of the central components of the primary and secondary mathematics curriculum and includes measurement of length, measurement of area, and measurement of volume. Measurement is a part of our everyday life. Therefore, learning the measurement concept is important for students not only for achievement in mathematics but also for their everyday life skills.

Area measurement is one of the significant topics in the mathematics curriculum and it is covered in the measurement strand or geometry strand in different countries (NCTM, 2000). In Tukey, it is mandated in the geometry and measurement strand. Area measurements are included in the middle school mathematics curriculum of different grades. The measurement of the area of the rectangle (square is presented as a special rectangle) is covered in the fifth-grade curriculum, and the measurement of the area of triangles, and parallelograms is covered in the sixth-grade curriculum. In the seventh-grade curriculum, the measurement of the area of the rhombus, trapezoids, circles, and sectors is included. Lastly, in the eighth-grade curriculum, the surface area of the right circular cylinder is included. So, the teaching of the area of measurement is an important topic of school mathematics in Turkey, too.

The area is the measure of two-dimensional bounded and closed surfaces and a surface can be divided into equal parts by using an area measurement unit; the measure of the area is expressed by the number of these measurement units (Smith et al., 2016). The area measurement constructs a connection between the concrete world of real measurement units and the abstract world of mathematics (Hiebert, 1981). The calculation of the measure of the area requires a transition from the use of physical objects to doing mathematical operations (formulas) (Kordaki & Potari, 2002; Lehrer, 2003; Zacharos, 2006). Therefore, area measurement is a difficult topic for students to understand. The literature lists five difficulties related to learning the area concept. These are; a) conservation of the area, b) understanding and using measurement units, c) spatial structuring of the rectangle, d) multiplicative composition, and e) confusing the area and perimeter (Smith et al., 2016, p. 241).

The conservation of the area is defined as the understanding that the amount of 2D closed shape does not change when the shape is moved or divided into parts (Smith et al., 2016). That is, the quantity of an object remains the same when it is rearranged and length, area, and volume can be divided into smaller equal parts (units) (Outhred & McPhail, 2000). Understanding the conservation of the area is the first step in understanding area measurement.

The second difficulty that students face is related to understanding and using the area measurement units. Covering a surface with square units and counting the square unit to find the measurement of the area does not mean that students understand the unit square is the measurement unit of the area (Kamii & Kysh, 2006; Kordaki & Potari, 1998). To understand that the measurement unit of the area is the unit square, it is necessary to understand that the square can be divided into small units, and although the square unit is discontinuous on its own, it becomes continuous with repeated use in area measurement processes. To understand the continuity of unit squares, students should know the process of covering a surface with unit squares to measure the area and why these rules are necessary. While measuring with unit square "*The units must "cover" the quantity exactly - there can be no overlap between units and no part can be left uncovered* (Hiebert, 1981, p.40)". The use of unit squares without leaving uncovered space or overlapping units and understanding the repeated use of units is the key to the process of abstraction of area measurement. Students face difficulty in understanding why no uncovered space should be left (O'Keefe &

Bobis, 2008). Olkun et al. (2014) conducted research in 4 different cities with the participation of 248 students studying in 4th, 6th, 8th and 9th grades. Most of the participants do not consider the unit squares as the measurement unit of the area.

Moving from physically covering a space with square units to abstraction of measurement includes representing unit squares with two parallel lines to measure the areas. However, representing units using two parallel lines is more difficult than expected, and covering an area with square units is not clear enough for students (Outherd & Mitchelmore, 2004). To calculate the area of a rectangular region without covering and counting square units, students must visualize the rectangular array model, and this arrangement both speeds up counting and forms the basis of the area formula of rectangles (Smith et al.,2016). Understanding the multiplicative relation between the numbers of rows and columns and the total area of the rectangle is the idea behind the area formula. Huang and Witz (2013) conducted research with 23-grade students and none of the participants talked about the array model to find the area. When teaching the area concept, using formulas is given more importance than the conceptual understanding of the area measurement. In the literature, many researchers are investigating the construction of the area formula. Although dividing the rectangular area into small squares is an important step in reasoning about the area formula, it is not given as an area measurement strategy, it is just part of the teaching process (Zacharos, 2006). Dividing the area into unit squares is crucial when calculating the area of irregular polygons. When teaching the area of polygons such as squares, rectangles, triangles, or parallelograms, the focus of instruction is the correct application of the area formula. So, students try to apply a formula while calculating the area of irregular polygons. Kamii and Kysh (2006) revealed that one-third of eighth-grade students had difficulty finding the area of an irregular polygonal region, and some of these students used the area formula to calculate the area of the polygonal region, while some of them calculated its perimeter. Huang and Witz (2011), conducted an experimental study with 120 fourth-grade students. They applied three different curricula in 4 different classrooms to improve students' conceptual understanding of the area formula and students' performance in solving area measurement problems. The control group received the traditional curriculum that focus on numerical calculations of area measurement. Group 1 was treated with a geometry motions curriculum that focuses on 2-D geometry motions to explain the rationale of the area formula rather than operations conducted to calculate the measure of area. In the lessons of Group 2, a mix of the 2-D geometry motions curriculum and the traditional curriculum was applied. The research results showed that the group that was treated with both the conceptual knowledge of the area formula and calculation of the area with numerical data, was more successful in explaining their solutions and solving questions that require high-level thinking skills than the other groups. Students who received only applications of area formulas were successful in solving simple problems but showed low success in solving questions that required high-level thinking skills.

Students are good at memorizing the area formula and doing operations that the area formula includes (Huang & Witz, 2013). However, correctly performing the operation of the area formula does not mean that students understand the meaning of the area measurement. Two-fifths of the fourth-grade students who participated in Huang and Witz's (2013) study could not distinguish the area concept and the area measurement. Reducing the area formula to arithmetic operations causes overgeneralization of the formula and results in the low success of the area measurement content (Erdem & Gürbüz, 2018; Zacharos, 2006). The result of a study conducted with seventh-grade students showed that students generalized the area formula of the rectangle (multiply the length of two sides to find the area) and they multiplied the length of three sizes of the triangle to calculate the area of triangles (Erdem & Gürbüz, 2018). Also, students' success in solving area measurement problems is very low. Kaya (2019) conducted research with 16 sixth-grade students and asked them to solve two real-life problems including the measurement. More than half of the participant’s answers were incorrect. When the students’ answers were analyzed in detail, it was determined that the majority of the students did not know what the area refers to and tried to reach a numerical result by performing operations with the given numbers.

Students who can explain the area concept and have a good understanding of the multiplicative relationship that the area formula developed on are competent in applying the area formula, recognizing the geometric object, realizing and correcting their errors, and justifying their answer when solving area measurement and perimeter measurement questions (Huang & Witz, 2011; Huang, 2014). Explaining the connections between the square units and the area formula, and using models while teaching area content helps students to learn the area concept better (Erdem & Gürbüz, 2018).

Students face difficulty in distinguishing the perimeter and the area concepts. They used the area formula and the perimeter formula interchangeably (Smith et al., 2013; Smith et al., 2016). Some 4th, 6th, 8th, and 9th-grade students calculated the perimeter when they were asked to find the area of polygons (Olkun et al., 2014). Another research was conducted by Kamii and Kysh (2006). They asked to eight-grade students to find the area of irregular polygons. However, some students calculated the perimeter of the polygons instead of the area of them.

The studies that were conducted with the pre-service teachers and in-service teachers show that the teachers also face difficulty in understanding the area concept and distinguishing the area and perimeter concepts. Yeo (2008) investigated the effects of teachers' MTK on teaching the area and perimeter to the fourth grade. Although students can verbally express the area of a rectangle and square, they cannot define what the area is. In the first lesson, the teacher asked students to define the area, the students answered "The base times the length". This shows that students understand the area as a formula. The teacher gave examples about the area instead of guiding the students to define the area concept. Also, when the teachers were asked to define the area and perimeter, they tried to remember the formula of the area and the perimeter. The complexity of the area and perimeter concepts demonstrated the importance of the teacher's subject matter knowledge and pedagogical content knowledge. The key knowledge about the concepts of area and perimeter must be well understood by the teacher himself, and the teacher must be able to determine which activities will contribute to the student's understanding and carry out these activities appropriately in the classroom environment.

Simon and Blume (1994) conducted a study by informally observing 26 pre-service and video-typed them through 12 hours of instruction. Pre-service teachers were expected to solve problems related to the area of rectangles. Researchers asked them to find the area of the rectangular region of the desk by covering it with the unit rectangles. They easily covered the surface and counted the unit rectangles. However, pre-service teachers tried to find the area of the rectangular surface by multiplying the number of rectangles in a column and number of the rectangles in a row. It shows that memorizing the area formula as “base times height” resulted in overgeneralization of the area formula to the situation when the unit rectangles are used as a unit of measure. Also, they did not question if the unit square was appropriate to measure an area. the results indicated that pre-service teachers memorized the area formula and used the unit squares to cover the surface without understanding the multiplicative relation under the area formula and its relation with the unit squares.

Another study that shows that pre-service teachers’ knowledge of the area was formula-oriented was conducted by Runnalls and Hong (2020). Pre-service teachers applied the area formula to solve the questions, but they had difficulty in explaining the solution steps. Pre-service teachers were given questions that were solved incorrectly by the students. they were asked to find out the mistakes of the students. pre-service teachers answered that the students' solutions were incorrect because their application of the area formula was incorrect.

The teacher thinks that measuring an area is covering the surface with unit squares and counting the numbers of the unit squares not dividing the surface into equal parts (Outhred & McPhail, 2000). Teachers’’ emphasis on the area measurement as covering the surface causes students to face difficulty when measuring the area of irregular shapes (Kordaki and Potari, 1998; Outherd and Mitchelmore, 2004; Zacharos, 2006) and in cases where students cannot cover the surface with concrete materials (Outhred & McPhail, 2000).

Reinke (1997) conducted a study to investigate pre-service teachers' understanding of the relation between the area and the perimeter. The participants of the study were 76 pre-service teachers. 26 of the participants gave wrong answers to the are question and 67 participants gave wrong answers to the perimeter question. Approximately

22% of the candidates approached the perimeter question as an area question. 8 participants indicated that without knowing the value of the sides, the perimeter cannot be calculated. The results showed that the pre-service teachers faced difficulty in distinguishing area and perimeter.

Ma (2010) investigated American and Chinese teachers' content knowledge and pedagogical content knowledge around the area and perimeter relation. She conducted interviews with American and Chinese teachers using scenarios. Findings of the research showed that Chinese teachers' content knowledge was better than American teachers' content knowledge and content knowledge of the teachers affected the teaching practice of the teachers. Teachers who know what area and perimeter are and the relationship between them direct students to explore and prove their ideas. However, teachers who did not have enough content knowledge about the area and perimeter faced difficulty in understanding students' ideas and guiding students to prove their ideas. That is teachers content knowledge of the mathematical contents affects their teaching practices

Students have difficulty understanding the area, area measurement, and perimeter. Students' difficulties can be listed under the topics; difficulty in understanding the conservation of the area, difficulty in understanding measurement units, difficulty in understanding area formulas, and difficulty in distinguishing the area and perimeter. However, the studies conducted with pre-service and end-in-service teachers indicated that teachers face difficulty in understanding the area content, the area formula, and distinguishing the area and perimeter, too. The research also showed that teachers' content knowledge and pedagogical content knowledge affect their teaching practices.

2.2. Teacher Knowledge

Increasing the students' achievement is the key focus of the education. One of the important factors that affect students' achievement is the teachers. Teachers' knowledge about the content and how to teach the content are highly related to students' achievement (Bobis et al., 2012; Hill et al., 2005). The teacher should have a deep understanding of mathematical concepts and pedagogical strategies to ensure effective teaching of mathematics (Baumert et al., 2010). What the teacher should know for effective mathematics teaching? This question tried to be answered by Ball et al (2008). They defined two main categories which are subject matter knowledge and pedagogical content knowledge using Shulman's (1986) pedagogical content knowledge notion. They defined the "Mathematical Knowledge for Teaching" framework to distinguish what a mathematics teacher should know differently than any adult who received a mathematics education. (Ball et al., 2005; Hill et al., 2008; Ball et al., 2008). That is, MTK was built on the idea that "*teachers need to understand and use mathematics in ways that are specific to the work of teaching and that often differ from the ways in which mathematics is attuned to needs of other workplaces*" (Stylianides & Ball, 2008, p.398). MTK is a widely accepted and used framework to evaluate the knowledge of mathematics teachers. Mathematical knowledge for teaching includes two main categories (*i*) subject matter knowledge and (*ii*)pedagogical content knowledge (see Figure 1).

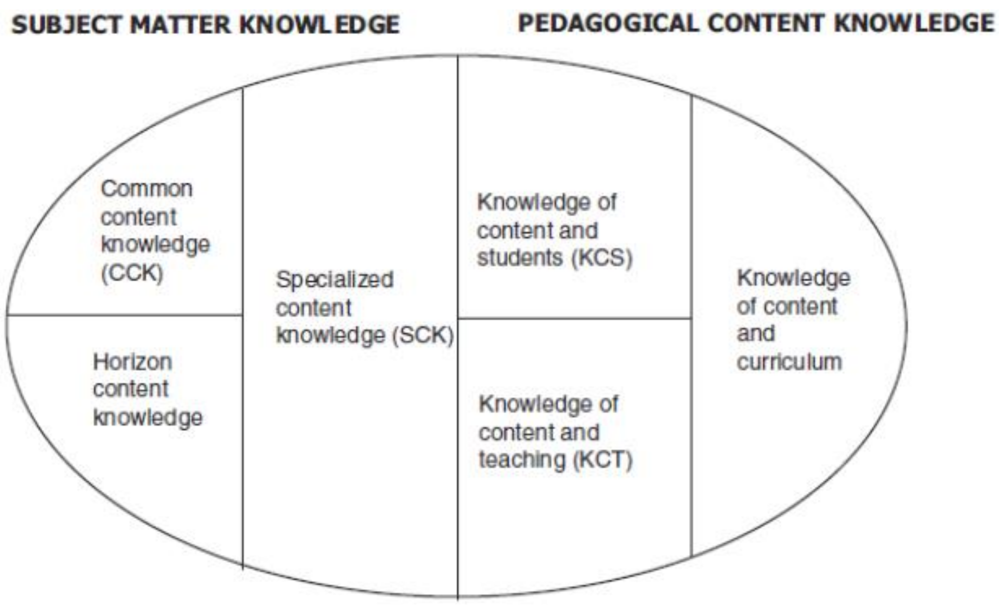


Figure 1. Mathematical Knowledge for Teaching (Ball et al., 2008, p.403)

Subject matter knowledge means knowledge of mathematics and it consists of three sub-categories which are (*i*) common content knowledge, (*ii*) horizon content knowledge, and (*iii*) specialized content knowledge. Although common content knowledge and specialized content knowledge are defined as separate knowledge in theory, Ball and her colleagues also state that there are difficulties in measuring these two knowledge dimensions separately in classroom practices (Ball & Rowan, 2004; Ball et al., 2008). Therefore, for this study common content knowledge, specialized content knowledge and horizon content knowledge will be analyzed together under the topic of content knowledge.

Common content knowledge is used to express the mathematical knowledge of the facts, procedures, terms, or operations that any educated adult to know (Hill & Ball, 2004). Common content knowledge includes being able to identify students' correct or incorrect answers, being able to recognize incorrect definitions and questions in the textbook, and being able to use mathematical terms and mathematical language correctly (Ball et al., 2008). Correctly performing the area formula to find the area of a rectangle is an example of common content knowledge. Teachers should know the area formula and be able to perform multiplication correctly, but knowing the formula and correctly performing the operations is not specific to teaching (Hill & Ball, 2004). Teachers should know the rules, definitions of terms, and facts of the topic that they teach. Lack of common content knowledge of teachers causes insufficient teaching process (Ball et al., 2005; Ball et al., 2008). Common content knowledge is crucial for teachers but not sufficient for effectively teaching mathematics (Ball et al., 2008).

Specialized content knowledge is mathematical knowledge and skills that are important and used only during teaching mathematics (Hill & Ball, 2004; Ball et al., 2008). Specialized content knowledge is needed only for teaching, other professions that use mathematics do not need specialized content knowledge (Ball et al., 2008). Specialized content knowledge includes making explanations to answer the question "why", using different representations to solve a question and linking these representations to make sense of mathematics, making connections between the topics being taught and previous learning of students, modifying tasks, asking mathematical questions to develop the mathematics, or adapting mathematical content of textbook to the teaching process. For example, knowing that a data set can be represented with a graph or with a table, making connections between the graph and the table, and explaining the difference between interpreting the data from these two representations is related to the specialized content knowledge. Another example of specialized content knowledge is explaining why squares are the special form of rectangles. Bair and Rich (2011) identified four components of development of the specialized content knowledge as the "*ability to explain and justify their work, use multiple representations, recognize and generalize relationships among conceptually similar problems, and pose problems* (p. 299)", and defined five levels of specialized content knowledge for each component. Level 0 specified the common content knowledge that a beginner college student expected to know and Level 4 represented the deep mathematical content knowledge and knowing how to use mathematical content knowledge for teaching. Teacher education program expected to increase pre-service teachers MTK. Kleickmann et. al. (2012) found the largest difference in content knowledge and pedagogical content knowledge between first-grade pre-service teachers and last-grade pre-service teachers. Content knowledge is a combination of common content knowledge and specialized content knowledge, and teachers need both common content knowledge and specialized content knowledge to teach mathematics (Hill & Ball, 2004). In order for a teacher to be able to question whether mathematical concepts are understood, the teacher must first have a deep and relational understanding of mathematics and mathematical ideas (Bair & Rich, 2011).

Pedagogical content knowledge is defined as teacher knowledge that is necessary to make specific content understandable to students, it is a combination of the content knowledge and pedagogical knowledge of teachers and it constructs a teacher's professional expertise (Depaepe et al., 2013). Pedagogical content knowledge of a teacher is personal and changes with the experience (Van Dijk & Kattmann, 2007). Pedagogical content knowledge is the knowledge that teachers need in the classroom. In other words, pedagogical content knowledge is about how to teach mathematics. Pedagogical content knowledge is divided into three subcategories: (*i)* knowledge of the content and students, (*ii*)knowledge of content and teaching, and (*iii)* knowledge of content and curriculum.

Knowledge of content and students refers to combining the knowledge of mathematics and the knowledge of students (Ball et al., 2008). A teacher must know how to motivate students, how to draw their attention, what is easy and difficult for them, which kinds of explanations confuse them, or what they will think when content is presented (Ball et al., 2008). Knowledge of content and students requires deep mathematical content knowledge, and being familiar with students and students' thinking (Hill et al., 2008). For example, a teacher needs to know that when students may confuse area and perimeter and calculate perimeter instead of area, or some students may divide the area of a rhombus by 2 since they think the area formula of rhombus $\frac{e.f}{2}$ means dividing the total area by 2.

Knowledge of content and teaching is the knowledge that combines knowing about mathematics and knowing about teaching (Ball et al., 2008). Knowledge of content and teaching includes sequencing tasks for instruction, choosing which examples to use, and knowing the advantages and disadvantages of using a representation to teach a topic and all these tasks require the interaction of pedagogical issues that determine students' learning and knowledge of mathematical content (Hill & Ball, 2004; Ball et al., 2008). During the instruction, a teacher should know how many hours to spend on a topic, if further explanation is needed when to ask questions to students, in which mathematical content to make connections, or in which order to present tasks. For example, a teacher should know that s/he should explain the relation between the square, the rectangle, and the parallelogram. Otherwise, students will think that they are separate geometric figures, and never be able to generalize that a rectangle is a specific form of the parallelogram.

An example illustrating the relationship between common content knowledge, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching is as follows: Ordering a list of decimals (common content knowledge), generating a list of important mathematical issues to order decimals (specialized content knowledge), determining which decimals will cause difficulty for students (knowledge of content and students), and deciding what to do to remediate students difficulties (knowledge of content and teaching) (Ball et al., 2008, p.404).

Knowledge of content and curriculum is teachers' knowledge of the curriculum they will teach. Teachers should know what students learned previously and what they will learn next to make connections between mathematical content. For example, teachers' knowledge of the subjects in the mathematics curriculum they will teach, being aware of the curriculum outcomes, and being aware of the relationship between different mathematics subjects while preparing their lessons can be considered within the scope of content and curriculum knowledge (Aslan-Tutak & Köklü, 2016).

2.3. Mathematical Quality of Instruction (MQI)

Viewing the same instruction video, different audiences often have different perspectives about the instruction. One audience may think the teacher does not use manipulatives, so the instruction is not well-enough. Another audience may think the students are participating actively. Therefore, the instruction is good enough. It is useful to use a common framework to prevent judging instructional quality from an individual point of view.

Mathematical Quality of Instruction (MQI) was piloted and developed between 2003 and 2012. It is a standardized framework that is developed to assess the quality of mathematics instruction. That is, the MQI framework is a content-focused observation protocol specific to mathematics (Charalambous & Litke, 2018). That is, the focus is mathematics and teaching of mathematics rather than pedagogical knowledge, school distinct, or classroom climate. To develop the MQI framework, an iterative process involving cycles of fine-grained observation of video-recorded lessons and theoretical discussion of relevant literature on teaching mathematics and the knowledge needed to reach this subject matter (Charalambous & Litke, 2018). MQI dimensions focus on interactions between teacher and content, teacher-student interaction around the content, and interaction between students and content.

MQI breaks out instruction into different dimensions, so it allows us to see the weak and strong features of an instruction. Since its first development, the dimensions of the MQI framework have developed and changed. For this research, the 4-Point version of MQI which was developed in 2014 is used. It includes five dimensions and 16 sub-dimensions. MQI is designed to be used to analyze videotaped lessons, not live lesson observations. MQI rubric advice dividing instruction into 3.5. to 7.5-minute segments. The Classroom Work is Connected to Mathematics (CWCM) dimension is scored as "Yes" or "No". Each of the other four dimensions has a rubric describing not present, low, mid, and high. Each segment also gets an overall code related to the main dimensions. Segments are viewed and coded according to the MQI 4-Point rubric (See Appendices).

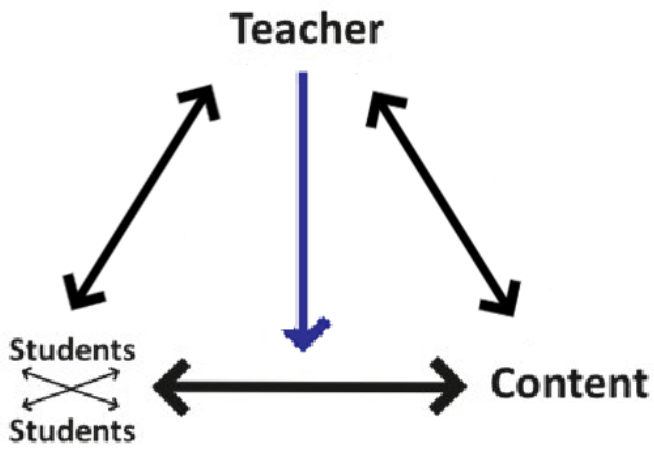


Figure 2. The Instructional triangle

2.3.1. Classroom Work is Connected to Mathematics

This dimension is about the focus of the segments. This dimension tries to capture whether the instruction time is spent on the mathematical content or not. If the focus of half or more of the segment is mathematical content, it is scored "yes". The teacher may introduce a new topic, review previous learning, or solve a problem or the students may work on the content as a group or individually. If half or more of the segment is spent on cutting, pasting activities without any connection to mathematical content, distributing or gathering materials, or talking on non-mathematical issues, it is coded as "no".

2.3.2. Richness of Mathematics (RM)

The richness of the mathematics dimension is about the depth of mathematics introduced in the classroom. This dimension tries to capture evidence of an explanation of what a definition means, an explanation of the logic behind facts and procedures, precise language use, different solution methods, and a comparison of this method. RM dimension tries to answer the following questions (MQI Training Modules):

- Does the segment include explanations of why facts are true, a procedure works or some solution methods are appropriate for some type of problems?
- Does the segment convey making sense of the solutions, the relation between numbers, definition of the term?
- Does the segment involve an examination of solution methods or a comparison of different solution methods, making generalizations or using clear and understandable language?

Students' comments or explanations, textbooks, and curriculum materials that contribute to rich mathematical instructions are also scored in the RM dimension. Richness elements focus on the instructional triangle's interaction between the teacher and the content.

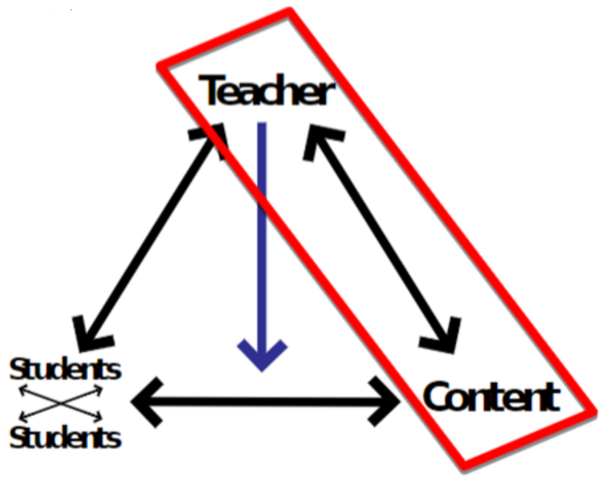


Figure 3.Instructional Triangle- Richness of Mathematics

This dimension includes seven items. Six items, except the overall richness of mathematics, are grouped into two broad categories; meaning of fact and procedures and key mathematical practices. Items listed under these two broad categories are given in the table below.

Table 1. Sub-dimensions of Richness of Mathematics

Meaning of Fact and Procedures (Meaning-oriented codes)	Key Mathematical Practices (Practice-oriented codes)
- Linking Between Representation - Explanations - Mathematical Sense Making	- Multiple Procedures or Solution Methods - Patterns and Generalizations Mathematical Language

2.3.2.1. Linking Between Representation

This code refers to links across different representational families. Representational families mean tables, graphs, stories, written symbols, or manipulatives. The link within a representational family is not coded. For example, a manipulative and a real-life situation describing the same area is coded as a link between representations. However, two different symbolic representations of numbers ½ and 0.5 are not coded as links between representations. The link can be drawn by students, teachers, or both. This dimension is not about the teacher's use of representation. If the teacher uses many different representations without any link between them, it is scored as "not present".

2.3.2.2. Explanations

This code refers to answers to the question "why". It tries to capture an explanation of why a procedure works, why a solution method is useful, and why the answer is true. It also codes justification of the definition but not the definition itself. It also does not code how a procedure is done or, a statement of facts. For example, all rectangles are a parallelogram is not an explanation, it is a fact. So, it is not coded under the explanation dimension. However, classifying rectangles as a parallelogram because rectangles satisfy the properties of a parallelogram- their opposite sides are equal, their opposite angles are equal, and their diagonals bisect each other- is an explanation. That is, definitions are considered explanations if they are used to explain statements.

2.3.2.3. Mathematical Sense-Making

This code focuses on meaning. That is, it captures the extent to which teachers or students talk about the meaning of numbers, explore relationships between numbers, the relationships between different content, the connection between different representations and ideas, and discuss the reasonableness of a solution. For example, making sense of formulas, definitions (what counts as a rectangle, what does not count as a rectangle), using estimations, using number sense, discussing the reasonableness of an answer (why the length of a human cannot be 3 meters), and so on. In many cases, explanation and mathematical sensemaking overlap, and the instants are coded in both dimensions. However, in some cases, an explanation does not qualify as Sense-Making. For example, a teacher may explain why an object is the transformation of another object without meaning to mathematical ideas. Some instances of Sense-Making are not scored under Explanation. For example, using estimation meets the criteria of Sense-Making but without an explanation of why an estimation is true, it does not count as an Explanation.

2.3.2.4. Multiple Procedure or Solution Methods

This code is discussed if multiple solution methods occur or are discussed in the segment. Multiple solution methods can be used for a single problem. For example, Student A can calculate the area of a rectangle using the multiplication of rows and columns while Student B calculates the area by counting unit squares. Multiple solution methods can be used for a significant problem type. For example, Student A compares two fractions using a common numerator, and Student B compares them using the area model.

2.3.2.5. Pattern and Generalizations

This code is used if the students first examine instances or examples and use the knowledge that they interfere with to develop patterns or generalizations. This dimension is scored if the segment includes inference of a mathematical pattern, derivation of a mathematical property, construction, or testing definition of a term. For example, finding the area of different rectangles by counting unit squares and making generalizations of the relationship between the number of squares in rows and columns and the area of the rectangle. To discover a pattern, build a definition, or make a generalization, the class should work on more than one example, at least two examples. Patterns, generalizations, and definitions should be developed during instruction, not just stated by the teacher. For example, if the teacher stated that all rectangles are a parallelogram without first discovering it by comparing the properties of two polygons, it does not count as Patterns and Generalizations

2.3.2.6. Mathematical Language

This code scores the mathematical language use of the teacher. It includes teachers' fluent use of mathematical language, supporting and encouraging students' accurate use of mathematical language, and explaining the meaning of mathematical terms. Students' mathematical language use is not coded except if it is high.

2.3.2.7. Overall Richness of the Mathematics

Each segment gets an overall richness code which shows the depth of the mathematics offered to students. This code is not the average of the first six richness codes. It is an overall estimate of the richness of the mathematics

2.3.3. Working with Students and Mathematics

Working with students and the mathematics dimension captures whether the teacher can understand what students mean, respond to students' contributions, realize students' mistakes, and correct these errors. Students' contribution includes, but is not limited to, ideas, claims, solutions, explanations, and comments. Students' mistakes or mathematical errors mean students' wrong solutions, misconceptions, and incorrect deductions which allow the teacher to see students' difficulties. Also, it tried to capture to what extent the teachers use students' contributions to construct the instruction. This dimension undertakes to answer the following questions:

- Do students face difficulty with the content and make mistakes?
- Does the teacher correct students' mistakes? If so, how?
- Do students share their ideas and contribute to the course?
- Does the teacher use these contributions to build up the instruction?

This code focuses on student-teacher interactions around the content.

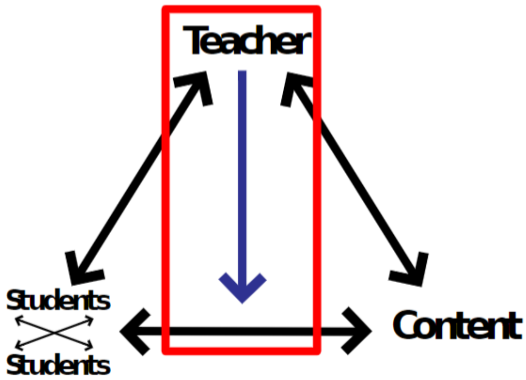


Figure 4. Instructional Triangle- Working with Students and Mathematics

This dimension includes 3 sub-categories which are listed below.

2.3.3.1. Remediation of Students Errors and Difficulties

This code focuses on the instance in which students' misconceptions and difficulties are remediated. There are two types of remediation; conceptual remediation and procedural remediation. In conceptual remediation, the teacher seeks the root of the misunderstanding and then repairs it. For example, the teacher said some students use the perimeter formula although the question asks to calculate the area of a circle, and the teacher explains the area and the perimeter concepts and shows the difference between the two. In procedural remediation, the teacher corrects students' mistakes by demonstrating procedures. For example, the teacher warns a student that he skips a step. Saying the student's answer is false or giving the correct answer is not a remediation.

2.3.3.2. Teacher Uses Student Mathematical Contribution

This item captures if the teacher uses student mathematical contribution while constructing instruction. Students' contributions can be student comments, answers, discussion, solutions to a problem, student work, generalization, ideas, and so on. For example, when a student offers a solution method, the teacher asks other students to comment on her ideas. Students' contributions can be verbal or written.

2.3.3.3. Overall Working with Students and Mathematics

This code shows the overall interaction of teacher and students. It is not the average of the first two working with students and mathematics dimension, it is an overall estimate of teacher-student interaction around the content

2.3.4. Errors and Imprecision

This dimension is about the teachers' language errors or imprecision, errors in mathematical notation, or lack of clarity in the presentation of the content, evaluating an incorrect solution as correct, solving a question incorrectly, defining a term incorrectly or missing some key condition of the definition. Students' errors and imprecision are not coded if it is not endorsed by the teacher. Also, if the teacher realizes her mistake and corrects it, it is not scored. The Errors and Imprecision dimension looks at the interaction between the teacher and the content.

This dimension consists of four sub-categories that aim to capture problematic aspects of the course.

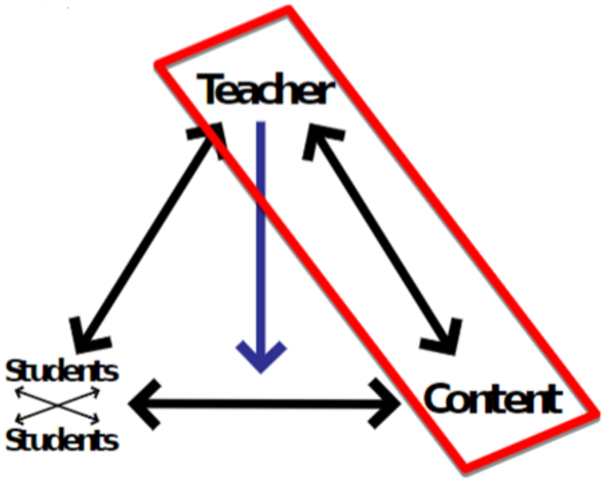


Figure 5. Instructional Triangle-Errors and Imprecision

2.3.4.1. Mathematical Content Errors

These dimensions focus on the events that are mathematically incorrect. Content errors include solving a problem incorrectly, making incorrect definitions, missing key conditions of a definition, endorsing an incorrect answer or comment, and saying a solution is incorrect when it is correct—for example, defining 3+(7+2) = (3+7) + 2 as the commutative property of addition although it was the associative property of the addition. The errors that are corrected within the segment are not counted.

2.3.4.2. Imprecision in Language and Notations

This code tries to capture the problematic use of mathematical language or notations. It can be incorrect use of a notation or misuse use of mathematical language. For example, errors in the use of equal signs such as 4x3=12+7=19 which results in an incorrect number sentence, saying the result of multiplication should be greater than factors, or spelling “angle” as “angel”.

2.3.4.3. Lack of Clarity in Presentation of Mathematical Content

This code tries to capture muddled, confusing, or distorted mathematical points, language errors, or inexplicit explanations of the teacher. The question "What did the teacher mathematically say?" is the guiding question of the dimension. That is, the dimension is about the instances that confuse students and prevent learning. For example, when talking about the division of a number by 4, the teacher says the division makes "4 groups" and then for the same operation he says the division makes "groups of 4". Although, the division both gives the number of groups and the number of a group's members, referring to both for the same division is confusing.

2.3.4.4. Overall Errors and Imprecision

This code tries to capture the presence of the teacher's mathematical error. Its point is not the average of the first three dimensions. It is an overall estimate of the teacher's errors.

2.3.5. Common Core Aligned Student Practice (CCASP)

This dimension is about students' involvement in the mathematical tasks, contribution, and participation in meaning-making and reasoning. Students' meaningful engagement with mathematics includes; constructing reasonable arguments, commenting on other's ideas, developing models, and using appropriate tools. This dimension tries to answer the following question:

- Do students actively engage with mathematics?

The CCASP dimension focuses on the interaction between students and the content (see Figure). It is impossible to know what is going on in a student's mind. Therefore, CCASP looks for observable behaviors of the students to catch evidence of students' mathematical thinking.

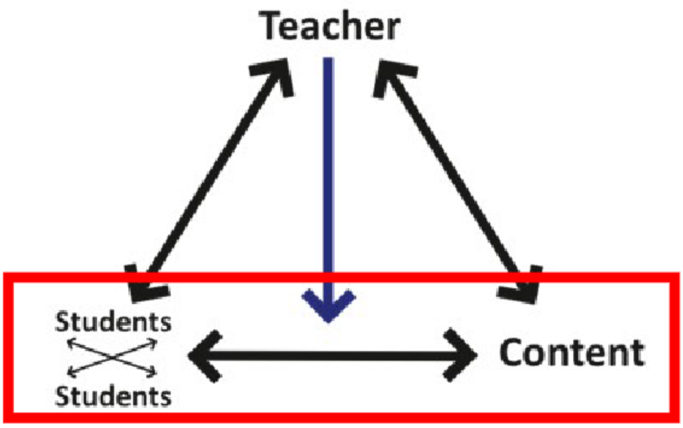


Figure 6. Instructional Triangle- CCASP

CCASP dimension involves six sub-categories which are given below. The first three codes focus on how students engage with mathematics. Two codes are about types of mathematical tasks that students work on and the last one is the overall code of CCASP.

2.3.5.1. Students Provide Explanations

This item focuses on the student's mathematical explanation for a fact, procedure, solution method, and explanation of why something is true. Students' explanations can be self-constructed or co-constructed with the teacher. Students can make an explanation by initiating themselves or they can give an explanation to a question from the teacher. If the students' explanations are suitable for the Explanation code in RM, they should be coded in both of the dimensions. Unlike the Explanation code in RM, the student's explanation must not be correct. Students' incomplete or incorrect explanations also indicate that students are trying to make sense of the content. It shows the student is thinking mathematically.

2.3.5.2. Student Mathematical Questioning and Reasoning

This dimension focuses on students' mathematical questions and mathematical thought. Students' contributions do not have to be complete or correct. Some examples of students' mathematical questioning and reasoning include providing counter-claims, asking mathematically motivating questions that request explanations, making conjectures, and using ideas from different mathematical topics to reason about the content of the lesson. For example, since the sum of the interior angles of a triangle is 180 degrees, a triangle cannot have two right angles or a student asked what would happen if a number was divided by zero.

2.3.5.3. Students Communicate about Mathematics of the Segment

This item tries to capture the extent to which students communicate their mathematical ideas during the course of the segment. Some examples of student contribution are sharing solution methods with the class (this sharing can occur with words or without words), asking questions, making definitions, engaging in discussion, and commenting on others' ideas.

2.3.5.4. Task Cognitive Demand

This code focuses on the cognitive demand of the enacted task and ignores the initial demand of the task that is present in the textbook or the cognitive demand of the task the teacher set up. It captures student engagement in tasks in which they think deeply. Some examples of cognitively demanding tasks are making connections between different representations, determining the meaning of concepts, making conjectures, and looking for patterns and justification.

2.3.5.5. Students Work with Contextualized Problems

This dimension focuses on students' engagement with contextualized problems which are story problems, real-world applications, or experiments. This dimension includes solving such problems, making sense of relationships, discussing different solution methods, using different representations to make sense of relationships, or creating contextualized problems. The segment is given a point in this dimension according to the teachers' scaffolding. If the teacher scaffolds students heavily, the segment reaches a lower point although students engage with a contextualized problem.

2.3.5.6. Overall Common Core Aligned Students Practice

Each segment gets an overall common core aligned student practice point. It captures evidence of student involvement in doing mathematics. However, its point is not the average of the four Common Core Aligned Students Practice sub-dimensions. The RM and CCASP dimensions look very similar at first. However, the focuses of them are different. RM focuses on the depth of mathematics offered to students. Students do not have to be an active participant of the instruction. In the RM dimension, mainly the teacher’s work is scored. Students’ contributions are scored even if they are true. However, the CCASP focuses on the student's contributions and is scored even if they are wrong or incomplete.

2.4. Studies on the Mathematical Quality of Instruction

Teachers’ MTK is an important factor that affects student learning (Bobis et al., 2012; Hill et al., 2005). However, it is not the only one. The instruction that students receive in the classroom needs to be investigated in more detail to improve the students’ learning (Chalambous et al., 2012). To investigate the instruction in detail, the MQI observation protocol was develop. A strong aspect of MQI is it is specific to mathematics lessons, and content-specific dimensions of the MQI allow administrators to prioritize those aspects of the meth instruction to improve students learning (Charalambous & Litke, 2018). MQI can be used for all content in the mathematic curriculum of K12. Validity studies of the MQI showed that MQI can be applicable to different mathematical content. Hill et al., (2012). Indicated that subject matter content does not affect the rater agreement with master scores and MQI can be applied to different content.

The MQI framework is used in many research for different purposes. Multiple studies investigate the connection between teachers’ MTK and the quality of their instruction (Santagata & Lee, 2019; Hill et al. 2008, 2012, 2015). Some studies explore the contribution of teachers' MKT and curriculum materials to the quality of instruction (Charalambous & Hill, 2012; Hill & Charalambous, 2012).

Students learning is always accepted as an indicator of the quality of instruction. Therefore, some researchers focus on the association between teachers' MQI scores and students' test scores (Blazer et al, 2016; Blazer & Kraft, 2017; Kane & Straiger, 2012; Hill et al., 2011). The MQI framework has also been used as a tool for professional development (Kraft & Hill, 2017; Hill et al. 2016; Mitchell & Marin, 2015).

Santagata and Lee (2019) investigate the association between the quality of instruction and MKT in a sample of novice teachers. Ten first-year elementary school teachers were selected as participants. To score MTK of participants, an MKT survey completed by participants was used. While scoring instructional quality instead of segment-level codes, whole lesson codes of MQI were preferred. Three mathematics lessons of each teacher were videotaped and analyzed by using MQI. Research results reported a linear and positive relation between teachers' MTK and MQI scores. However, in some cases, teachers' high levels of MKT scores did not correspond to a high level of MQI scores while in no cases low level of knowledge ever result in a high level of instructional quality. A strong, positive, and significant relationship was found between teachers' MKT and *Mathematics is Clear and not Distorted* dimensions of MQI. Also, a positive but not statically significant association between *Tasks and Activities Develop Mathematics* and *Lesson is Mathematically Dense* dimensions. They found a positive association between Efficient Use of Lesson Time and teacher MTK and Students are Engaged and teacher knowledge. Other relationships were weak.

Adkins (2017) tried to establish how the teacher of successful students delivered the mathematics. She tried to establish the MQI methods (dimension) that were used by these teachers. She found that the teachers knew the mathematics that they teach. That is, they have at least an average MTK score, and only %3 of the 80 segments were scored for teacher error. The teachers got the highest score in the Richness of Mathematics dimension. The teachers corrected students' errors and used their contributions in Working with Students and Mathematics dimension. The least used dimension was CCASP. Students procedurally communicated with mathematics and they rarely made conjectures or conclusions using mathematical reasoning. The relative weakness of teachers was guiding students and making generalization of mathematical reasoning.

Hill et al.,(2015) worked with 272 fourth and fifth-grade teachers from four schools to explore the relationship between instructional quality and teachers' background characteristics, teachers' mathematical knowledge, instructional resources, teachers' habits, and school environment. They collected data from three different sources which are video-recorded lessons, teachers' surveys, and students' demographic and test scores. They recorded the mathematics lessons of the teachers over three years. Results of the study showed that teachers' mathematical knowledge and the school environment explained the variating in mathematics-specific teaching dimension; other factors explained very little variation in any dimension.

The study of Hill et al. (2011) video-typed six lessons for each of the 24 teachers. They also conducted interviews with teachers. The results indicated that higher quality instruction and teachers were observed in the more affluent schools. They also found a modest relationship between teacher quality and student characteristics. The research findings state that it is difficult to separate teacher characteristics and teacher quality. Also, they found a high correlation between teachers' MTK and the quality of the instruction.

Based on the idea that any observation instrument highlights some features of the instruction while ignoring some others, Charalambos and Litke (2018) aimed to examine the affordances and limitations of MQI. They analyzed three fourth-grade teachers' lessons which are part of the National Center of Teachers Effectiveness at Harvard University, using the MQI framework. Each lesson was scored in 7.5-minute segments and each segment was also assigned an overall score for all MQI dimensions. Data analysis shows that two instructions can have the same holistic MQI score for different reasons. For example, one instruction gets a mid-richness score because of a rich connection between representations while the other instruction gets mid Richness score because of a mathematical explanation. examining each aspect of instruction shows the strong and weak aspects of the

instruction separately and allows a chance to improve the weak part of each instruction privately. This research also indicates that the MQI framework, as it promised, focuses on the content-related feature of the instruction to describe its quality of it. However, in the course of data analysis, researchers detected some aspects of instruction that are not captured by the MQI framework. MQI dimension does not focus on generic instructional aspects such as; classroom management and organization, how the lesson is structured and implemented by teachers to back up student learning, and student engagement. Although MQI tries to surface content-specific aspects of instruction, there is no dimension focusing on the appropriate use of tools, and teaching mathematics equitably (equitable participation and explicit presentation of mathematics. Since MOI uses video recordings of lessons, raters sometimes cannot see what students are talking about and doing. Also, the researcher noted that MQI gives no information about student learning. However, Kane and Staiger (2012) and Blazar and Kraft (2017) founded a positive relation between the students learning and MQI scores. Kane and Staiger (2012) evaluated videos of instructions from The Measures of Effective Teaching (MET) Project (A large-scale project conducted with the participation of 3000 volunteer teachers) and found a significant relation between the MQI score of instructions and students' test scores (ranging from r=0.12 to r=0.16). Blazar and Kraft (2017) aim to investigate teachers' effect on student achievement. They worked with 310 teachers from 52 different schools. The results indicated that there is a relationship between teachers' MQI scores and students' learning (in both cognitive and non-cognitive aspects).
In 2014 Hill et al. (2016) started to deliver virtual mathematics coaching to the teacher. They worked with 142 teachers, 72 teachers in the MQI coaching group, and 70 teachers in the control group. Research results state that teachers in the MQI coaching group asked more substantive questions to the students, used rich mathematical vocabulary, and allowed more student talk in the classroom.

Hill and Charalambos (2012) used cross-case analysis to investigate the unique contribution of both MKT and curriculum materials and their joint contribution to the quality of instruction. They defined four cases and compared the instruction of two or three teachers for each case. Findings from the research showed that both MKT and curriculum material affect instruction. MKT of teachers has an influence on the richness of mathematical language used during instruction, teachers' explanation related to facts and procedure, avoiding error, and key mathematical points that teachers attract attention. Teachers with stronger MKT quickly understood students' ideas and used them while constructing the instruction. Also, well-defined and closely followed curriculum materials can lead to high-quality mathematics instruction although the teacher had a low MTK score.

Chalambous et al. (2012) investigated the effect of curriculum materials and teachers' MTK on the quality of instruction while teaching integer substruction. The participants of the study were three mathematics teachers with different levels of MTK and they used different curriculum materials. They worked on the joint and distinct contribution of MTK and curriculum materials to the instructional quality. The results stated that the MTK has a positive effect on teachers' use of different representations, providing explanations, use of mathematical language, ability to use students' contribution to develop mathematics, and moving mathematics to reach a goal. Curriculum materials were used to construct the meaning of integer subtraction, support teachers' mathematical language use, and provide explanations, and multiple representations use. Also, well-constructed curriculum materials supported teachers' instructions. Teachers who have higher MTK levels were able to meet deficiencies in curriculum materials.

Another study that investigates the relationship between MTK and instructional quality was conducted by Hill et al. (2008). They scored the instructions of the teachers using the MQI rubric and implemented a paper-pencil assessment to score teachers' MKT, and then they correlated these two scores. They conducted an in-depth qualitative analysis to confirm the relationship between instructional quality and MTK. Participants of the study were 10 teachers who taught various grades from second to sixth. The analysis of the data showed that *"there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction"* (p. 496). Other factors; teachers' belief about mathematics should be learned; how to make mathematics fun for students; teachers' belief about curriculum materials, and their use: and attainability of curriculum materials for teachers have very little effect on the instructional quality. However, these factors are all shaped by the teachers’ knowledge. The high level of MTK suggested avoidance of error and the teachers with a high level of MTK (4 out of 10) offered denser and more rigorous mathematics and chose the examples wisely to ensure equitable opportunities to learn. The lower-knowledge teachers’ instructional quality was not constant across lessons. When the lessons went well, it was generally thanks to curriculum materials.

CHAPTER 3

METHODOLOGY

This study aims to investigate the quality of mathematics instruction using the MQI lens. This chapter presents an overview of the methodology of this research. The chapter begins with the re-expression of the purpose statement and the research questions. The research design, the justification for the use of qualitative research design and case study, and the context of the study are presented. Then, participants of the study, data collection tools, and data analysis techniques are given respectively. Lastly, trustworthiness, the researcher's role, and ethical issues are addressed.

3.1. Research Questions

The purpose of this study is to investigate the quality of the instruction. Therefore, this study tries to answer the following research questions;

1. What aspects of the instruction do middle school mathematics teachers highlight and what is the quality of these aspects?
2. How is the quality of instruction of middle school mathematics teachers in implementing the instruction as they observed through the Mathematical Quality of Instruction (MQI) instrument?

3.2. Research Design

A qualitative research design was applied to investigate the quality of mathematics lessons in middle school. Qualitative research can be defined as research in which qualitative data collection methods such as observation, interview, and document analysis are used, and a qualitative process is followed to reveal events realistically and holistically in their natural environment (Yıldırım & Şimşek, 2013). Interviewing, observing, and analyzing are central activities of qualitative research because they all help to understand the underlying meaning (Merriam & Tisdell, 2015). Therefore, qualitative inquiry is best suited for research on the daily life of human beings. In education, many questions related to the teaching and learning process are asked by researchers, educators, or policymakers. What are the factors that affect student learning? How do teachers teach mathematics in middle school? What kind of activities do they do? What kinds of problems do they solve? What sorts of things do students do in the classroom? Do teachers and students use manipulatives or technology? To answer these questions, the daily routine of the instruction needs to be observed, and some interviews with teachers and students need to be conducted. That is, to answer many questions related to the education process, qualitative inquiry is the best way of research.

Qualitative research starts with emerging questions, data collection generally takes place in the participants' setting, particular data is used to develop general themes, and the researcher interprets the meaning of the data (Creswell, 2007). Qualitative research continues in the natural setting with a detailed data analysis. In qualitative research, the researcher wants to see a more complete picture of what is going on in a natural setting, not just seek to understand "to what extent" or "how well" something is done (Fraenkel et al., 2012). Therefore, the researcher focuses on the context of the research and tries to get rich information about the context. This research is qualitative, and it aims to investigate the quality of mathematics instruction in the classroom (natural setting) using qualitative data collection methods observation and group discussion.

3.2.1. Case Study

To answer the research question, a case study as a qualitative approach was applied in this study. A case study is defined as "an in-depth description and analysis of a bounded system (Merriam & Tisdell, 2015, p. 37)". The case study aims to understand the case in its natural environment, in depth, by considering its complexity and context (Punch, 2005). The case studies aim to find an answer to the questions “how” and “why” in a real-life event (Yin, 2003) and the case study method reveals links with their causes which are not possible to discern with correlational research (Yin, 1994). The researcher understands, describes in detail, and defines the current issues, their causes, and consequences without controlling the variables (Leymun et. al., 2017). Although there is a specific purpose and research questions, the general aim is to understand the case in all aspects as possible (Punch, 2005). Therefore, the researcher deeply explores a case by collecting data from multiple sources (Creswell, 2007; Leymun et. al. 2017).

The most characteristic feature of a case study is that the object of the study, the case, is limited (Merriam & Tisdell, 2015). A case can be a group of people, a school, a classroom, or a program. Three different types of case studies are defined by Fraenkel et.al. (2012). In an intrinsic case study, the case is a specific individual or situation. The researcher defines the case in detail. For example, the researcher works with a student to find out why that student having trouble with learning mathematics content. In an instrumental case study, the researcher works with a specific case to understand the larger picture of the situation. The third one is the multiple case study. In multiple case studies, the researcher works on more than one case as a part of an overall study.

A multiple-case study approach was applied for this study. The case study is characterized by the unit of analysis, not by the topic of the study (Merriam & Tisdell, 2015). This study investigates the quality of mathematics instruction by using qualitative data collection methods (observations and group discussion). Participants of the study is three middle school mathematics teachers. This study is a case study since it works on a bounded system (instruction of three mathematics teachers) in its natural environment (classroom) without controlling variables.

3.2.2. Context of the Study

The focus of this study is the mathematical quality of instruction. Since the focus is instructional quality, providing some information about the school where the observations took place will be helpful in understanding the setting of the study. The study was conducted in two different public middle schools in Central Anatolia. One of the schools was an imam-hatip middle school. Imam-hatip middle schools are a type of middle school with religious courses in addition to common middle school courses. The boys and girls received education in separate classrooms. The classrooms included 25 or 26 students. There were non-Turkish speaker students in the classroom. The socio-economic levels of some parents were under the average. Some parents were working as seasonal workers. Therefore, the students of the school have attendance problems. The school's success level is a little under the average level of achievement. The second school was a middle school, and in Turkey middle schools are co-educational. The classrooms include 35 or 36 students. all the students were Turkish speakers and the socio-economic levels of the parents were above the average. Since the school was close to the campus area of a public university, most of the parents were university staff. The school was known as one of the most successful schools in the city.

3.2.3. Selection of the Participants

Two types of sampling, probability, and nonprobability are defined in the literature (Merriam & Tisdell, 2015). Probability sampling includes simple random sampling, stratified sampling, systematic sampling, and cluster sampling methods. Probability sampling allows the researcher to generalize the result of the research. In qualitative research, the general aim is not to generalize the result of the study, but to get a rich and detailed picture of the case at hand. Therefore, in a qualitative study, participants can be selected using nonprobability sampling methods. Purposeful sampling is one of the nonprobability sampling methods. Purposeful sampling is used when the researcher wants to understand, define, and describe the case in detail, and therefore must select participants from which the most detailed data can be collected (Merriam & Tisdell, 2015). This research aims to collect rich data about the instruction of mathematics in middle school. Therefore, a purposeful sampling method was employed to select the participants of the study.

The aim and the content of the data to be collected in a study are the main factors that affect the selection process of the participants. This study aims to evaluate the instructional quality of mathematics teaching. To analyze the process deeply, a few participants were selected using purposeful sampling. The participants of this study are three middle mathematics teachers. While determining the study group, 12 middle school mathematics teachers were interviewed. The primary criteria were that the teachers volunteer to participate in the study and allow their instruction to be videotaped and to be examined and evaluated.

There are many factors that affect the success of students. While choosing the participants, care was taken to select teachers who work in schools with different levels of success, from different social environments, and with diverse student profiles to ensure diversity. In addition to the aspects of schools they work in, the academic characteristics and personal characteristics of the participants were also effective in the selection of the participants. Teachers' understanding of the importance of academic study, their willingness to take part in a long-term study, being open to communication, and being open to criticism were also considered.

Table 2. Knowledge about MSMTs teaching experience and graduate programs

	Teacher Ali	**Teacher Efe**	**Teacher Yusuf**
Level of Education	Master Degree	Bachelor’s degree	Bachelor’s degree
Graduated Program	Elementary mathematics education	Elementary mathematics education	Elementary mathematics education
Teaching experience	8	17	12

3.2.3.1. Teacher Ali

Teacher Ali has been teaching in middle school for eight years. He graduated from an elementary mathematics education program and had a master's degree in mathematics education. He has been teaching in fifth, sixth, seventh, and eighth grades in the semester that the data were collected. He was willing to develop his professional skills. He took part in different professional development programs provided by the Ministry of National Education. In a semi-structured interview, he noted that he considered student knowledge, difficulties, and thinking while planning the instruction. He also said that he awarded students who correctly solved problems with a “well done” seal, and it highly motivated students. He said he generally used direct instruction while teaching. He sometimes employed pair teaching and group work techniques. He indicated that he sequenced questions from easiest to harder. While solving a new type of problem of the hardest problem, he noted he gave clues to students to solve the problem. That is, he scaffolded students to make them solve the problem. He selected problems that were like problems in national exams.

He was working in an imam-hatip middle school. In imam-hatip middle school, boys and girls receive education in different classrooms. The region where the school was located was classified as a disadvantaged region. The socio-economic level of the parents was low. Many families came from different cities. The success level of the school was low.

3.2.3.2. Teacher Efe

Teacher Ali has been teaching in middle school for seventeen years. He graduated from an elementary mathematics education program, and he was a master's student in mathematics education program. He was willing to develop his professional skills. He took part in different professional development programs provided by the Ministry of National Education. He has been teaching in seventh grade in the semester that the data were collected. In a semi-structured interview, he said he talked about the history of mathematics during instructions. He focused on why mathematics was important for daily life. He said he introduced the new content in a daily life context. He talked about how mathematics was important for hunter-gatherer people. He indicated that students' understanding of mathematics was important. Therefore, he noted that he focused on the meaning of mathematical content rather than number of the questions solved in the classroom. For students learning, he noted students must take notes and forced students to take notes during instruction. He also talked about the importance of the curriculum knowledge. He indicated that he sequenced questions from easiest to harder. He said he used non-routine problems, and he awarded students who correctly solved problems with a “plus sign”. When a student collects three “plus signs”, they call the parents of the student to congratulate them. He said this method motivated students to solve the problems.

He was working in a public middle school as a mathematics teacher. Public middle schools are co-educated in Turkey. The school was in the campus area of a public university and most of the parents were working at the university. The education level of the parents was high. The socio-economic level of the parents was mid or high. The teacher indicated that students’ readiness for the lesson was high. The success level of the school was above the average in national exams.

3.2.3.3. Teacher Yusuf

Teacher Ali has been teaching in middle school for twelve years. He graduated from an elementary mathematics education program and he was a master's student in mathematics education program. He was willing to develop his professional skills. He took part in different professional development programs provided by the Ministry of National Education. He also participated in a dynamic geometry workshop conducted by a public university. He has been teaching in fifth, sixth, seventh, and eighth grades in the semester that the data were collected. In a semi-structured interview, he said he focused on the mathematical thinking process in the classroom. He said he used different solution methods in the classroom and forced students to share their own solution methods.

He was working in a public middle school as a mathematics teacher. Public middle schools are co-educated in Turkey. The school was in a disadvantaged region of the city. There were non-Turkish speaker students in the classroom. The socio-economic level of the parents was low and some students dropped out the school to work. The success level of the school was below the average.

3.3. Data Collection Process

The data collection process includes two phases: pre-instruction and implementation. The first data source is pre-instruction group discussion videos. Almost one month before the instructions, the teachers came together and discussed “how they teach” the content. They shared their experiences about the teaching area of the circle and the teaching area of the sectors. Each teacher shared activities, problems, and teaching methods that they used during instruction. They also talked about student changes and misconceptions related to the area of circle and sector.

The second source of data is videotaped instructions. Direct observation is one of the effective ways of investigating teacher effectiveness (Mangiante, 2011). To conduct a valid observation two components are required, a valid observation form and a trained observer (Goe et.al., 2011) To fulfill these requirements, the researcher completed the MQI training Modula provided by Harward University and used the MQI 4-Point scale to analyze observations. To capture the instructional quality of a teacher, observation of at least two lessons is suggested by MQI research (Ho & Kane, 2013; Santagata & Lee, 2019). Each teacher observed two times which were inconsecutive instructions. To answer research questions, the teachers videotaped while both teachers teaching the same content. The objective of the instructions was to “Calculate the area of the circle and sector.”. The teachers were first observed while teaching the area of the circle. Their second observation took place while teaching the area of the sector of a circle. In addition to the main data source, the middle school mathematics syllabus and middle school mathematics textbook were used to better understand the data and provide explanations. The researcher was in the classroom as an observer and took notes related to instruction. Teachers’ and students’ work that they did on the board was not visible in the video recording. For these works, the researcher’s field notes were used.

Table 3. The data collection process

Events	Date
Selection of participant	May, June, July, and August 2018
The participants were introduced to each other	9 January 2019
The discussion about teaching the area of the circle	13 Mach 2019

Table 3. (continues)

The discussion about teaching the area of sector	20 March 2019
The first observation of Efe (Teaching the area of the circle)	29 April 2019
The second observation of Efe (Teaching the area of the sector)	6 May 2019
The first observation of Ali (Teaching the area of the circle)	14 May 2019
The second observation of Efe (Teaching the area of the sector)	21 May 2019

3.4. Data Analysis

The qualitative data analysis includes steps; organizing and preparing raw data for analysis, reading through all data, coding the data, forming themes and descriptions, interrelating the themes/descriptions, and interpreting the meaning of themes/descriptions (Creswell, 2007). In the literature, many different analysis methods were used in the analysis of qualitative data. In this study, discourse analysis and content analysis were used together. Content analysis technique enables researchers to work on human behavior indirectly (Fraenkel et. al, 2012). Discourse analysis is a linguistic approach and it focuses on the language of the speech (Merriam and Tisdell, 2015). To analyze the pre-instruction group discussion video records, an adapted version of the MQI framework was applied as an analysis framework. However, the MQI 4-point version was developed to score instructions. Therefore MQI 4-point version was adapted to score group discussions. Instructions video records were evaluated, arranged, and interpreted according to the MQI framework components and sub-components, constituting the study's theoretical framework. Findings were supported by narratives and direct quotations.

3.4.1. MQI as an Analysis Framework

As the Mathematical Knowledge for Teaching framework (Ball et.al., 2008) started to be used widely to measure teacher knowledge in mathematics, the researchers wanted to know how and to what extent the teachers translate their knowledge into classroom instruction (Hill et al., 2008). To meet this need, many classroom observation protocols were developed. MQI is one of these classroom observation protocols. MQI is used to evaluate instruction processes around the three interactions as illustrated in the instructional triangle; teacher-student relation around the content, teacher-content relation, and student-student relation around the content (Santagata & Lee, 2021). MQI is a mathematics-specific framework and aims to evaluate content-focused aspects of mathematics instruction (Charalambos & Litke, 2018). The development process of MQI is iterative, and since its first development, many versions of the MQI have been released (Charalambos & Litke, 2018; Santagata & Lee, 2021). The current version of it contains four main dimensions and twenty sub-dimensions.

Table 4. MQI Dimensions

Richness of Mathematics	Working with Students and Mathematics	Error and Imprecision	Common Core Aligned Student Practice
-Linking Between Representations -Explanations -Mathematical Sense-Making -Multiple Procedure and Solution Ways Pattern and Generalizations -Mathematical Language Overall Richness of Mathematics	-Remediation of Student Errors and Difficulties -Teacher Uses Student Mathematical Contributions -Overall Working with Students and Mathematics	-Mathematical Content Errors -Imprecision in Language or Notation -Lack of Clarity in Presentation of Mathematical Content -Overall Error and Imprecision	-Students Provide Explanations -Students Mathematical Questioning and Reasoning -Students Communicate about the Mathematics of the Segment -Task Cognitive Demand -Student Work with Contextualized Problems Overall Common Core Aligned Student Practice

The score each item, a 4-point scale (Not Present (NP), Low, Mid, High) is used. For an overview of the MQI domain and coding protocol see Appendix A.

The MQI was developed to measure the mathematical content-related work that teachers do with students during instruction (Center for Education Policy Research, 2023). This study aims to analyze, the quality of mathematics instruction in middle schools using the MQI framework. Two lessons of two middle school mathematics teachers were analyzed using the MQI framework.

3.4.2. Adaptation of MQI to Analyze Group Discussions

Teachers' Mathematical Knowledge for Teaching (MKT) is an important characteristic of teacher quality, and it is expected that high Mathematical Knowledge for Teaching results in high-quality instruction. However, only looking at teachers' MKT is not enough to understand the quality of the instruction. The MQI framework, which helps to understand what the teacher and the students do around the content, is a tool to measure instructional quality. The MQI framework is developed to use analyzing video-record of the instruction. However, this research also aims to evaluate the quality elements of the instruction that teachers focus on while talking about instruction. For this reason, by referencing the MQI 4-point scale and the works of Hangül (2018) and Strand (2016) the following scoring protocol was developed to analyze group discussion. Hangül investigated the knowledge source of the teacher educators about the quality of mathematics instruction. Teacher educators watched a 27-minute mathematics instruction and took notes. She interviewed teacher educators about the mathematical quality of instruction. She analyzed data using the MQI framework. The MQI Framework was originally developed to analyze video records of the instruction. To analyze teacher educators' notes and interviews through the lens of the MQI framework, she developed three criteria; depth, consistency, and non-direct use. Strand (2016) conducted a ten-week professional development program and investigated intermediate-grade teachers' MQI-related noticing. To analyze teachers' speech with the MQI framework, she developed a five-level noticing framework. Level 0 is the non-noticing level. Level 1 is defined as noticing mathematics. Level 2 is defined as noticing other features of instruction without evidence and Level 3 is defined as noticing other features of instruction with evidence. Level 3 is defined as noticing mathematical features of instruction without evidence and lastly, Level 4 is defined as noticing mathematical features of instruction with evidence. With the help of related literature, the MQI 4-Point scale was adopted to analyze the speech of teachers.

Table 5. Adapted version of MQI 4-Point scale

Richness of Mathematics					
Linking Between Representations	**Explanations**	**Mathematical Sense-Making**	**Multiple Procedure and Solution Ways**	**Pattern and Generalizations**	**Mathematical Language**
NP: Different representations may be mentioned, but no connection is actively made. Low: The link is mentioned in a pro forma way. Mid: The link between representation are explained in detail. High: The link between representations are explained in detail, use of link and representation during instruction is discussed or explained.	NP: No explanation occurred, or explanations were incorrect. Low: Some brief explanations occurred. Mid: The explanations were detailed. High: The explanations are detailed and how to use explanations during instruction is discussed or explained.	NP: No mathematical sense-making present or they were incorrect. Low: A brief focus on the meaning. Mid: More than a brief focus on the meaning. High: More than a brief focus on the meaning and how to use sense-making during instruction are discussed or explained	NP: No multiple solution methods are discussed. Low: The teachers briefly mentioned a second method or procedure, Mid: Multiple procedures and solution methods are mentioned and discussed. High: Multiple procedures and solution methods are mentioned and the connection between them is discussed. The reason for selecting a solution procedure is explained and discussed. Different methods are compared. How to use multiple procedures during instruction is discussed.	NP: No patterns and generalizations are mentioned. Low: A brief mention of developing a generalization or building a definition but the work is undeveloped. Mid: There is work on developing a generalization or building a definition, but the work is not finalized. High: Patterns and generalizations are finalized and, how to use patterns and generalizations during instruction is discussed.	NP: No mathematical terms are used. Low: Low density of mathematical language. Mid: Middling density of mathematical language. High: The density of mathematical language is high. Teachers talk about student language errors or how to increase students' mathematical language use density.

Table 5. (continued)

Working with Students and Mathematics	
Remediation of Student Errors and Difficulties	**Teacher Uses Student Mathematical Contributions**
NP: No possible student errors and difficulties are mentioned or discussed. Low: Teachers talk about possible student errors and difficulties, and a brief conceptual remediation or brief or moderate procedural remediations are discussed. Mid: Teachers talk about moderate conceptual remediation or extended procedural remediation. High: Teachers engage in conceptual remediation by addressing the possible source of student errors or misconceptions, discussing how these errors may cause broader misunderstanding, and discussing pe-remediation ways.	NP: Teachers mention no possible student contributions. Low: Teachers talk about student contribution in a pro forma way. Mid: Teachers talk about possible student contributions and how to use these contributions to develop mathematics. High: Teachers talk about possible student contributions and the high density of the use of this contribution to develop mathematics is discussed.

Error and Imprecision		
Mathematical Content Errors	**Imprecision in Language or Notation**	**Lack of Clarity in Presentation of Mathematical Content**
NP: No content error occurs. Low: A brief content error occurred. Mid: Content errors occur in a part(s) of the discussion. High: Content error occurs in most of the discussion.	NP: No imprecision in Language or Notation occurs. Low: A brief instance of imprecision occurred. Mid: Imprecision occurs in part(s) of the discussion or obscures the mathematics. High: Imprecision occurs in most of the discussion.	NP: None. Low: Brief lack of clarity and it does not obscure the mathematics. Mid: Lack of clarity occurs in part(s) of the discussion. High: Lack of clarity occurs in most of the discussion and it obscures the mathematics.

Table 5. (continued)

Common Core Aligned Student Practice				
Students Provide Explanations	**Students Mathematical Questioning and Reasoning**	**Students Communicate about the Mathematics of the Segment**	**Task Cognitive Demand**	**Student Work with Contextualized Problems**
NP: Teachers do not mention any student explanations. Low: Teachers talk about one or two brief student explanations. Mid: Teachers talk about sustained student explanations. High: The teachers talk about sustained student explanations and how to use these explanations to develop mathematics.	NP: No student mathematical questioning and reasoning was discussed. Low: Teachers mentioned one or two brief students' mathematical questioning and reasoning. Mid:	NP: No student contribution was discussed. Low: Teachers talk about brief student contributions. Mid: Teachers talk about substantive student contributions, but they do not explain how to use students' contributions to develop math. High: Teachers talk about substantive student contributions but do not explain how to use them to develop math.	NP: Teachers talk about the cognitively undemanding tasks. Low: A brief example of a cognitively demanding task or low cognitively demanding tasks are presented. The teachers talk about a cognitively demanding task but they mention heavily scaffolding the students. Mid: Teachers talk about middling cognitively demanding tasks or high cognitively demanding tasks by scaffolding the students. High: The teachers talk about high levels of cognitively demanding tasks.	NP: Teachers do not mention any contextualized problems. Low: The contextualized problems that they mention are executed as a routine exercise. Mid: Teachers talk about non-routine contextualized problems and some students' reasoning about the problem. High: Teachers talk about non-routine contextualized problems, possible solutions ways of the problem, different representations of the solution, and students' engagement with the problem during instruction.

3.4.3. Data Analysis of Pre-Instruction Group Discussion

The researcher asked participants to come together and discuss the teaching area of the circle and the area of the sector. Teachers shared their experiences how they teach these contents and why they used some specific methods. They also gave information about their students' understanding, difficulties, achievement levels, and classroom profiles. The group discussion was video recorded and transcribed by the researcher. Group discussion videos were analyzed using the adopted version of the MQI instrument. The teachers' speech was coded according to MQI categories. Then, their sharing was categorized according to the MQI 4-Point scale.

3.4.4. Data Analysis of Instruction Videos

The instructions were video recorded and transcribed by the researcher. Each lesson was divided into 7-minute segments. Each segment was scored independently using the MQI 4-Point scale. After watching each segment, a score was assigned for each item listed in Table 2. To score the lessons, the MQI 4-point scale is used (See Appendix A)

3.4.5. Trustworthiness

Yıldırım and Şimşek (2013) stated that validity and reliability are important for qualitative studies and there are some measures to keep validity and reliability high. To avoid the effect of came presence on the instruction data, the researcher started to visit the classrooms three weeks before the data collection. The environment in which the study is conducted is important to interpret the data. The environments of the school are described in the study to increase the validity and reliability of the research. During the data collection, the data collection was not limited to one method. The group discussion and instruction videos were used to collect data about the mathematical quality of instruction.

Hill et al. (2012) indicated that the MQI is sensitive to the rater quality. Therefore, I completed the training provided by Harvard University (2023). To calculate the inter-coder reliability in qualitative research, at least %10 of the data should be scored by different coders (MacNealy, 1999). The % 10 (2 segments for each instruction) of the instruction segments will be scored by a MQI certificated rater. Then two raters came together and reconciled their scores. They discussed the divergent ratings to generate a consensus about the quality of instruction.

3.4.6. The Researcher's Role

Qualitative research is interpretive research and researchers may reflect their bias, values, and culture in the interpretation of the data (Creswell, 2007). Therefore, defining the researcher's role is important for qualitative research (Merriam, 2009). In qualitative research, trust between the researcher and the participants is important. After I selected my participants, I stayed in contact with them until the start of the data collection process. I conducted interviews data at the campus area of a university. All participants were familiar with the campus area. I designed the room where the interview took place for privacy. I introduced teachers to each other almost one month before the data collection, and they talked informally. I wanted them to know each other, trust each other, and be willing to share information about their instruction. During the interview I only asked questions to start the discussion "How do you introduce the area of circle? What are the key points of this topic? What is the most difficult part of this content for students?".

The second data source was the classroom videos. I was in the classroom as a nonparticipant observer. To avoid the effect of the video on the research data, I started my video record three weeks before the data collection. I observed two or four lesson hours of in each week. When the actual data collection processes the teacher and students got used to my presence.

3.4.7. Ethical Issues

The official permission from the Applied Ethics Research Center at Middle East Technical University (METU) was obtained before the data collection. The approval form of the Human Subjects Ethics Committee is given in Appendix B. I also applied to the Ministry of National Education to collect data from middle school. The approval of the Ministry of National Education is given in Appendix C. Before conducting the group discussion, I informed participants personally about the aim of the study and data collection process. I gave an informed consent form, and they reported their voluntariness. To hide the identity of teachers and students, I used pseudonyms instead of their real names. In the next chapter, the findings of the study will be shared.

CHAPTER 4

FINDINGS

This study aimed to examine the quality of mathematics instruction in middle school classrooms by observing how the teachers approach mathematics education through the lens of the Mathematical Quality of Instruction (MQI) framework. Data were gathered from group discussions and video recordings of instructions. Three middle school mathematics teachers participated in group discussions. Lessons of two mathematics teachers were video recorded. Instructions of one participant were not available for video recording.

This chapter summarized the current study's findings in six main sections and related sub-sections. In the first and second sections, a summary of group discussions was presented, and then the group discussions were analyzed using the adopted MQI framework. The MQI scores were given with evidence from teachers' speeches. In the remaining sections, the lessons of teachers were analyzed. Firstly, the mathematics instruction of teachers was described. These narratives of lessons are used to peek inside the world of instruction of two teachers during four lessons. Each lesson was viewed through the lens of the MQI framework. Using MQI lenses to analyze lessons allows researchers a broad point of view of effective teaching. The MQI framework describes the essential dimensions of qualified mathematics instruction under four main dimensions: Linking Between Representation, Woking with Students and Mathematics, Errors and Imprecision, and Common Core Aligned Student Practice.

4.1. Pre-Instruction Focus Group Discussion: Teaching Area of Circle

The researcher asked the teachers to come together and discuss how they teach the area of the circle. The teacher explained their own instruction and commented on each other’s applications. Group discussion aimed to collect more data about the teachers’ classroom application and the quality of mathematics instruction. Therefore, a narrative of the group discussion was given before the MQI scores.

The discussion lasted one hundred minutes. They first talked about how they introduced the area of the circle. All participants said he used an interesting task or real-life problem to gather the attention of the students. Then, they discussed the questions they solved. Of course, the teacher sometimes discussed mathematical content other than the area of the circle. They gave examples from their teaching practice and commented on each other’s ideas. The teachers shared their knowledge and experiences.

The teacher talked about how they introduced the area of the circle. Efe said he used signals from a base station or played a marble game to show a circular region. He explained the aim of the game. Ali said he used the sheep problem. However, Ali also indicated that using the current trend of students would be more attractive for them. He said he sometimes used computer game examples to introduce mathematical content. The game PUBG could be used to introduce the area of the circle. Efe and Yusuf said they did not know much about the game. Ali explained how the game was constructed, and they discussed how it could be integrated into the instruction.

Yusuf said he used dynamic geometry software GeoGebra or manipulatives to show the relation between the area of the circular region and the rectangle. Yusuf explained how to use manipulatives to make sense of the area of the circle. He said the circular region was divided into sectors and rearranged to form a parallelogram or rectangle. Efe had difficulty understanding the relation between the area of a rectangle and a circle. Yusuf explained to Efe that the rectangle's height equals the radius, and the rectangle's base equals half of the perimeter. Therefore, the area of the rectangle would be equal to the multiplication of the radius by half of the perimeter, that is, r. πr. He said to make the learning permanent; the students could cut cardboard circles into sectors and rearrange them. However, Efe objected to cutting the cardboard circles and offered to watch the video of the process from EBA TV.

Efe said he used regular polygons while introducing the perimeter and the area of the circle. He said, “I draw squares both inside and the outside of the circle and calculate the perimeters and areas of the squares. The circle's perimeter is smaller than the perimeter of the outside square and bigger than the perimeter of the inside square. Then, I draw a hexagon both inside and outside of the circle. The circle's perimeter is bigger than the perimeter of the inside hexagon and smaller than the perimeter of the outside hexagon. The area of the hexagons is closer to the area of circles than the area of the squares.”. He said if he drew a regular polygon with one hundred sides, then the area and perimeter of the polygons would get closer to the area and perimeter of the circle. He indicated that he used this method to show the value of the pi.

Ali showed the picture of a square, a parallelogram, and a circle drawn on a grid paper. He said he asked the students which area was the biggest. Efe commented on Ali’s idea. Efe said the answer to the question of which area was bigger was self-evident. Asking to calculate the area of the polygons was more challenging because the students could not find the area of the circle by counting the squares on grid paper.

Yusuf said he asked the students to draw a rectangle. Students drew a standard rectangle, and Yusuf drew a square and said it was a rectangle. He continued the instruction and asked the students to draw a parallelogram, and the students drew a common parallelogram. He drew a rectangle and claimed it was a parallelogram, too. The students objected to his ideas by claiming that he was wrong. He said he drew a table, wrote down aspects of each polygon, and asked students if a square met the elements of a rectangle. The teachers discussed definitions of trapezoids and polygons. They focused on the deficiencies of definitions given in the textbooks. They drew some figures and debated if they were polygons or not. Efe claimed that some definitions were problematic because of the Turkish language. He said definitions in mathematics textbooks should be updated after careful work.

Efe said students mostly asked what the pi was when introducing the circle. He said when the teacher wrote down the perimeter formula” $2\pi r$”, the students memorized the formula. By solving more exercise questions, the learning became permanent. But the question “What is the pi?” continued to occupy students’ minds. Therefore, they decided that making sense of pi was an essential part of the teaching circle. Yusuf spoke of an activity that he used. The students draw a square, fill it with many dots, and count the number of dots. They draw the inner tangent circle of the square and count the dots inside of the circle. Lastly, they divide the number of dots inside the circle by the number of total dots inside the square and multiply it by 4. The result is close to the number pi.

They discussed the pi. Efe said pi could not be represented as a simple fraction if pi was an irrational number. However, we define the pi as the ratio of the circumference of any circle to its diameter. He explained that the circumference and diameter value did not have to be an integer, and the ratio was not a simple fraction. Efe explained how he make sense of pi. He asked students to find a number whose square equals 5. The students offered to calculate squares of 2.5,2.4,2.3, and 2.2. He said they performed the multiplications and saw the square of none of these numbers was equal to 5. In this way, students realize there are numbers whose exact value cannot be written as a rational number. Yusuf said the students in his seventh grade cannot perform multiplication with decimals. Also, some students could not perform four basic operations. Yusuf said making sense of the pi this way was impossible in his classroom. Ali agreed with Yusuf and said seventh-grade students in his school also had difficulty performing multiplication with decimals.

Teachers said they started to solve questions by application of the area formula of the circle. In these questions, the radius of a circle was given, and the measurement of the area of the circle was asked. Then, they moved to the questions where the perimeter measurement was known, and the measurement of the area was asked. They talked about the questions that the measurement of the circle’s area was given, and the radius of the circle was asked to find. Lastly, they said they used questions requesting the difference between the area of a square or the area of a rectangle and the area of the circle inside them. Ali and Efe said they used half and quarter circles while solving questions related to the area of the circle. Yusuf said he did not use half or quarter because the students did not know how to calculate the area of sectors.

Ali said some students in seventh-grade classrooms cannot perform multiplication of r^2. Efe said he generally used contextualized problems in his instruction and mentioned two contextualized problems that he used. The first was a painting problem, and the second was a dart game problem.

They talked about how students perform operations without reading and understanding the context of the problem. Efe said some students do not understand the meaning behind the mathematical procedures and only do calculations. Ali said some of his students performed division or multiplication, which was not asked in the question. He mentioned some students missed the first step of problem-solving, which is understanding the problem. They discussed the importance of examining the meaning of operations to avoid misuse of the operations. However, it was not detailed.

Efe mentioned that students had difficulties solving probability questions that required finding areas. He said since students did not understand the meaning of area, they faced trouble in the following years. This difficulty is not evaluated in this research.

The video recordings of group discussions were analyzed using the adopted MQI framework. The teachers' speeches were analyzed in detail, and their explanations were scored under the related MQI dimension. The findings of the MQI are presented with evidence.

4.1.1. Findings Related to MQI

The classroom applications were highly related to teachers' knowledge (teacher knowledge about the content, teacher knowledge about the student, teacher knowledge about the teaching, and teacher knowledge about the curriculum) (Ma, 2010), and the MQI framework includes teacher knowledge. The teacher's knowledge of the content is scored in Richness of Mathematics and Errors and Imprecisions dimensions. If the teacher knows the content he teaches, his instruction generally gets high scores in the Richness of Mathematics dimension and low scores in the Errors and Imprecisions dimension. Therefore, the teachers' explanations about the content were scored under the Richness of Mathematics dimension. The Working with Students and Mathematics dimension is highly related to the teacher’s knowledge about the content and student. Since the Working with Students and the Mathematics dimension focus on the teacher-student interaction around the content. The teachers’ explanations about student difficulties, student errors, and how they use student contribution during instruction were scored under the Working with Students and the Mathematics dimension. The teachers’ explanations about students’ possible explanations, students’ possible questions and reasoning (it is related to teacher knowledge about the content and students), and the problems and tasks used during instructions (teacher knowledge about the content and teaching) were scored under the Common Core Aligned Student Practice dimension.

Teachers’ group discussion was analyzed according to the adopted MQI rubric. For the Richness of Mathematics dimension, Linking Between Representations, Explanations, Mathematical Sense-Making, and Mathematical Language were scored. No evidence related to the dimensions of Multiple Procedures and Solution Methods and Patterns and Generalization were observed.

Only two instances achieved a score for Linking Between Representations. The Linking Between the Representations dimension focuses on the link between different representational families. The link between the representations of the same families is not counted as a link. Yusuf mentioned using manipulatives to introduce the area of the circle. Also, in geometry courses, the shape does not count as a representation. Therefore, the link between the formula (symbolic representation) and the shape (pictural representation) was not scored as ink between representations. He explained how to use manipulatives to make sense of the area formula of the circle. Yusuf explained how to relate a rectangle's area with the circle's area. Students cut the cardboard circle into identical sectors and rearranged rectors to form a rectangle. When the number of sectors increased, the rearranged shape would look like a rectangle. Students would realize that the area of the rearranged shape could be calculated by multiplication of base and height, which were equal to half perimeter and radius, respectively. They discussed it further, but the application of manipulatives in the classroom was not discussed in detail. Therefore, it scored as “Mid.” The other idea came from Ali. He said forming a circle by arranging identical triangles helped make sense of the area formula. His idea was not discussed and scored as “Low.”

The teacher knew the area formula of the circle and the definition of the pi. For the Explanation dimension, only one instance was scored. Efe talked about how he explained why pi was an irrational number and how he made sense of the irrational number. They discussed the pi for a long time. Efe said “Pi is an irrational number. However, they define pi as the ratio of the circle’s perimeter to the circle’s diameter. Is it a contradiction? Since when we write the ratio $\frac{circle'sperimeter}{circle'diameter}$, we write it as $\frac{a}{b}$.” Yusuf and Ali confused for a few minutes. Then, they said the circle’s perimeter of the circle’s diameter did not have to be an integer. Pi is a ratio, but it is not a simple fraction. The teachers’ discussion about Mathematical Sense-Making can be grouped under two main headings: making sense-of-area formula of the circle and making sense of the pi. All three teachers told the methods they used to make sense of the area formula. Yusuf said he distributed circular cardboard to students. Students cut the cardboard into sectors and rearranged sectors to form a parallelogram. Students realized that when the number of sectors increases, the rearrangement of the sectors more looks like a rectangle. Then, students tried to calculate the area of the rectangle and discovered the relation between the rectangle and the circle. The area of the rectangle was equal to the multiplication of the radius and half the perimeter of the circle. Efe said, he did not know this method and Yusuf explained to Efe why the area of the rectangle was equal to the multiplication radius by half perimeter. Yusuf explanations were detailed and scored as “High”. Ali said, he also used Yusuf's method to make sense of the circle’s area using dynamic software, GeoGebra instead of manipulatives. He did not explain how he used it. So, it was scored as “Low”. Efe’s method approximates the area of a circle by using regular polygons. Efe explained how he used the regular polygons. However, he did not talk about the process that students went through. So Efe’s explanation scored as “Mid” for mathematical sense-making. Ali talked about how he used polygons drawn on a grid paper and asked students to compare the area of these polygons with the area of the circle. Students faced difficulty comparing the polygons' area with the circle's area. Ali's explanation was not detailed, so scored "Low". Efe developed Ali's activity and claimed that asking about the value of the areas instead of comparing them would be more challenging for students. Students could not find the value of the circle's area by counting the unit square. As a result, they would realize counting unit squares was not an appropriate method for calculating the circle's area and needed another solution way to find the circle's area. Efe's explanation was detailed and scored "High" for Mathematical Sense-Making.

Yusuf explained the activities that they used to make sense of the pi. Yusuf talked about an activity in which students drew a square and filled the square with ordinary dots. Then, they drew the inner tangent circle of the square. Students counted the dots inside the circle and the total dots inside the square. Yusuf said the ratio of the number of total dots to the number of dots inside the circle was close to the value of pi. However, this activity was not meaningful to making sense of the pi, and scored as "Not Present". Efe said " I generally ask students to find a number whose square is equal to 5. Students tried to find a number and approximate the 5. For a while they realized that finding such a number was not possible." Students understood that some numbers square root could not be found in the set of rational numbers. They would realize that irrational numbers existed. His explanation was detailed and scored "High" for Mathematical Sense-Making. Ali did not talk about how he presented the pi. The teachers used mathematical language correctly and effectively. No floppy use or error was observed.

For Working with Students and Mathematics dimension, Remediation of Student Errors and Difficulties, and Teacher uses Student Mathematical Contribution were scored. Yusuf and Ali mentioned possible student difficulties and errors. Ali said his students would confuse the area and perimeter formula, and face difficulty operating with r^2. Yusuf agreed with Ali and said his students would face the same difficulties. Efe also said some students perform operations with a given number without understanding what the problem asks for. The other teachers agreed with Efe and said they had such students. The teacher knew their students' possible errors and difficulties related to the content but they offered no remediation. Therefore, for the

Remediation of student Error, they scored “Not Present. Efe talked about how students contribute to the lesson and how he uses student contribution to develop mathematics and it was coded as “Mid”. When students were asked to find the square root of 5, they would try to approximate the 5. The teacher helped students to understand that some numbers were not rational.

All sub-dimensions of CCASP were scored at least once. For Student Provide Explanation only Efe mentioned how students provided explanations about which circle was bigger. Efe claimed students would answer “Arae of this circle is bigger because it covers more space or this circle is bigger because its circumference is bigger.” It was scored as “Low” because the explanations were brief. Only Efe mentioned Student Mathematical Questioning and Reasoning and Student Communicate about the Mathematics of the Segment while he was explaining how he made sense of pi and his explanation scored “Mid”. He mentioned some substantive contributions of the students. Students tried to find the square root of 5 and tried to challenge the teacher. For Students Work with Contextualized Problems all three teachers offered examples of problem context. Ali said he generally used a sheep problem. A sheep was tied to a fence and the problem asked to find the area of the field where the sheep could eat the grass. However, Efe said its context was not attractive for students living in the city center. They did not discuss it further and scored “Low”. Ali’s second context suggestion was using a computer game that was popular among students. He said it was very remarkable for students. Ali said he sometimes used computer game problems and students showed great interest in these problems. They discussed how the computer game PUBG could be used effectively to attract the attention of students and to present the topic of the circle’s area. They talked about this context for a long and scored “High”. Yusuf mentioned that creating a problem using a traffic radar working system was related to students’ daily life and he sometimes used traffic radar signals to show the circles in real life. It was not detailed and scored “Low”. Efe talked about more than one context. He said he generally brought marbles or darts to the classroom and planned a game. For the marble game, Efe asked students to draw a circle on the ground and asked students in which circle the possibility of stopping the marble is the biggest. Students discussed the size of the circle and provided evidence about which circle was bigger. For the

Table 6. Pre-instruction Group Discussion: Evidence and score of Richness of Mathematics Dimension

Richness of Mathematics	**Teacher**	**Evidence**	**Score**
Linking Between Representation	Yusuf	He offered to use a circular cardboard and cut it into sectors. Then, he said they could rearrange the sector to form a parallelogram or rectangle. In this way, the students can discover the area of the circle meaningful.	Mid
	Ali	He offered to use triangles to form a circle. When the number of circles increases, it will look like a circle. He did not detail his method and it was not discussed.	Low
Explanations	Efe	He explained why pi is an irrational number and he talked about how he explained it in the classroom.	High
	Yusuf	He said the circular region was divided into sectors and rearranged to form a parallelogram or rectangle. The height of the rectangle equals the radius and the base equals half of the perimeter. He explained that by increasing the number of sectors, the rearranged shape would be more like a rectangle. He made a connection between the perimeter of the circle and the area of the circle. He said to make the learning permanent, they could let the students perform this process using manipulatives themselves.	High
	Ali	He said he used the same method with Yusuf (Dividing circles into sectors) using the dynamic geometry software, GeoGebra.	Low

Table 6. (continues)

Mathematical Sense-Making	Efe	He said he used regular polygons to introduce the perimeter of the circle and the area of the circle. First, he drew the largest square that could be drawn inside the circle and the smallest square that could be drawn outside the circle. They calculated the perimeters and areas of the squares. Then, he drew the largest regular hexagon that can be drawn inside the circle and the smallest regular hexagon that can be drawn outside the circle. He increased the side of the polygons and expected the students to form the conclusion that when the number of sides increased, the perimeter and the area of the inner and exterior polygons got closer. He said he also used this method to make sense of the pi.	Mid
	Ali	Ali offered to use polygons on a grid paper. He said they could ask the students which area was bigger. The students would find the area of a square, a parallelogram, or a rectangle by counting the units.	Low
	Efe	Efe developed Ali's offer about using grid paper and counting squares to find the area of polygons. He said they could ask to calculate the area of the polygons and the circle instead of comparing them. Because, when the students tried to calculate the area of the circle, they realized that counting unit squares was not appropriate for finding the area of the circle	High
	Efe	He said he was drawing inner polygons and exterior polygons to show the number pi. His explanations were not detailed.	Low
	Yusuf	Yusuf explained an activity that he used to make sense of the pi. He said they draw a square and fill it with dots. Then they draw the inner tangent circle of the square. Lastly, they divide the number of dots inside the circle by the total number of dots inside the square and multiply the result by 4. The result is close to the value of the pi. But this activity is not appropriate to make sense of the pi.	NP

Table 6. (continued)

	Efe	Efe said they asked the students to find the number whose square was equal to 5. He said the students would say the square of the 2.5 could be equal to 5. He said he calculated the square of 2.5 and showed it was bigger than 5. Then they calculated the squares of 2.4, 2.3, and 2.2 and showed it was between 2.2 and 2.3. They continued to try and showed there was a number whose exact value was not known.	High
Multiple Procedures and Solution Methods	Not Present		
Patterns and Generalizations	Not Present		
Mathematical Language	Ali, Efe, and Yusuf	All teachers used mathematical language correctly, and effectively. However, no student language error was mentioned.	Mid

Table 7. Pre-instruction Group Discussion: Evidence and score of Working with Students and Mathematics

Working with Students and Mathematics	**Teacher**	**Evidence**	**Score**
Remediation of Student Errors and Difficulties	Yusuf and Ali	They said students in his classrooms confused the perimeter and area concepts. No remediation was mentioned.	NP
	Yusuf and Ali	He said some students in his seventh grade could not perform finding the square of the r. No remediation was mentioned.	NP
Teacher uses Student Mathematical Contribution	Efe	He said he explained to the students that the value of the pi was not exactly known. The students would ask the question about how an unknown number could be used in calculations. He said the students would try to find the square root of the 5.	Mid

Table 8. Pre-instruction Group Discussion: Evidence and score of Common Core Aligned Student Practice (CCASP)

CCASP	Teacher	Evidence	Score
Students Provide Explanations	Efe	Efe offered to draw two circular regions on the PUBG map. Then he said he would ask students which area they would choose, and why. He said the students would answer as its area is bigger or its region is bigger. The student explanations were not discussed in detail.	Low
Students Mathematical Questioning and Reasoning	Efe	Efe said when he told students that the value of the pi was not known exactly, students asked "How can we use a number whose exact value is unknown?"	Low
	Efe	Efe said when he asked students to find a number whose square equals 5, students tried to find an example.	Mid
Students Communicate about the Mathematics of the Segment.	Efe	Efe said the students would contribute to finding the square root of 5 and explained possible student contributions.	Mid
	Efe	Efe offered to draw two circular regions on the PUBG map. Then he said he would ask students which area they would choose, and why. He said students would contribute with brief explanations.	Low
	Ali	Ali said the sheep problem to introduce the area of the circle was commonly used. Efe said that the context	Low

Table 8. (continues)

		of the problem was not appropriate for students in the city center. They did not discuss it further in the introduction of the lesson.	
	Yusuf	Yusuf offered that the traffic radar system could be used to write a contextualized problem about the area of the circle. He said he used explaining its working principle and asked how many kilometers away the system can detect you. They did not discuss it further.	Low
Students Work with Contextualized Problems			
	Ali	Ali offered to use the PUBG map as the context of the problem and said it was very remarkable for students. he explained the game to the other teachers. They discussed how the game can be used in the classroom in detail. They offered more than one context to use during instruction. They discussed these contexts.	High
	Efe	Efe said he used the wave of a base station and playing with marble as an example of circular region problem context. Efe said he used real-world applications of the marble game and explained it in detail.	High
	Efe	He said he generally used contextualized problems in his instruction to relate mathematics to real life. He mentioned a painting problem.	Mid
	Efe	Efe said he brought a dart to the classroom and asked the probability of hitting which area is the smallest. He said it requires students to calculate the area of circles and segments.	Mid
	Ali	He showed pictures of a square, a parallelogram, and a circle that was drawn on a grid paper and asked which area was bigger.	Low
	Efe	Efe commented on Ali’s idea and said that asking to calculate the area of the square, parallelogram, and	High

		circle would be more challenging. Since the students could not find the area of the circle by counting unit squares.	
		Table 8. (continues)	
Task Cognitive Demand	Ali	Ali said there was only one safe area in the game and it got smaller in time and the color of the area changed when it became smaller. He offered to ask the difference between the two areas.	Low
	Efe	He said they could give three different circular regions and ask which region would be safer for them.	Low
	Ali	Ali showed a question including three circular regions and asked which area was the biggest. The circles were drawn on grid paper.	Low
	Ali, Efe, and Yusuf	They said they used questions that required the application of the area formula of the circle.	Low
	Ali, Efe, and Yusuf	They said they used questions that the perimeter was given and the area was asked.	Low
	Ali, Efe, and Yusuf	They said they used questions that asked the difference between the area of the square or rectangle and the area of the circles inside them.	Mid

dart game, he brought a dart and asked students about the possibility of shooting which area was the biggest. Students discussed the area of the segment and circle. they explained which area was bigger and why. His real-world applications were detailed and scored “High”.

The only dimension that was not scored is Errors and Imprecision. The teachers made no content errors. They used mathematical language precisely. They discussed mathematical content and commented on each other’s ideas. The scores of dimensions and the evidence are given in Table 6, Table 7, and Table 8.

4.2. Pre-Instruction Group Discussion: Teaching Area of Sector

The researcher asked the teachers to come together and discuss how they teach the area of the sector. The teachers gave examples from their teaching practice and commented on each other’s applications. The discussion lasted seventy minutes. Group discussion aimed to collect more data about the teachers’ classroom application and the quality of mathematics instruction. Therefore, a narrative of the group discussion was given before the MQI scores.

The teacher talked about how they introduced the area of the sector. Yusuf and Ali said they used half and quarter relations. That is, they first asked about the area of a half circle and a quarter circle. Then they related half and quarter with the central angle of sectors. They said they used one-sixth of the circle and related it with the central angle. Efe said he generally used a problem in a real-life context to attract the attention of the students. He indicated that a pizza problem or a cake problem best fit the content. Ali said he used a wheel of fortune problem. Efe developed the idea of Ali and said painting a wheel of fortune context could be used. They talked about how to write a painting problem that could help them to introduce and develop mathematics. Teachers discussed that if they should use one or two wheels of fortune with different sectors of different sizes. They discussed that the central angle of the sectors on the Wheel of Fortune should be different than 60, 90, or 180 since students should understand the need for a central angle to find the area. If they could find the area with direct proportion, they did not realize the need for the central angle. Efe said the student should understand the relation between the central angle and the area. They talked about painting a wheel of fortune problem in detail.

Efe said he first asked about the area of the sector to make students think about how the central angle was related to the area. He first gave the total area of the circle and asked for the area of a sector. He expected students to realize the ratio between the central angle and the area of the sector. However, Yusuf remarked that students in his classrooms would face difficulty in relating 360 degrees with the total area of the circle. So, Yusuf said he used half and quarter circles to show the relation between the central angle of a sector and the area of the sector. He claimed that if he used a sector with an unknown angle, students could not interfere and that they needed an angle to calculate the area of a sector. Ali agreed with Yusuf and claimed his students would also face difficulty relating the central angle and the area. Efe objected to Yusuf's idea and using half, quarter, or one-third of the circle may cause overgeneralization. Students would try to find a ratio between the circle's area and the sector's area and miss the main point of area calculation. Ali said he divided the circle into identical sectors, gave the measurement of the circle's area, and expected students to realize the ratio between the total area and the sectors' area. Efe said Ali's method was similar to Yusuf's method. They continued to discuss the relation between half and quarter with the central angle. Efe said students started to use area models or real-life examples to express half or quarter in grade two. However, in the classroom, this area model never related to the central angle. So, students faced difficulty while making connections. Yusuf insisted that his students would better understand if he used half and quarter sectors at first. Ali agreed with Yusuf.

Efe said they would cover the length of the arc before the area of a sector. While calculating an arc length, they would use the central angle. When they were working on the area of the sectors, students would remember their previous learning. Therefore, students would make the connection between the area of the sector and its central angle. Ali and Yusuf said their students would not relate their previous learning and the area of the sector. Ali and Yusuf claimed their students would realize the relation between the central angle and the sectors' area after introducing the half, quarter, and the sectors with a central angle that was proportional to 360 degrees. Teachers agreed that the students would realize the direct proportion between the central angle and the area of the sector.

They agreed on the sequence of the questions. They said they first solved the question with direct proportion, then they introduced the area formula of the sectors. They said they solved some questions that required the application of the formula, and it was important to memorize and remember the formula. Efe said he used a third method to solve the question which was dividing the total area by 360 and multiplying the result with the central angle. Yusuf agreed with Efe saying he was using this method frequently. Yusuf indicated that telling students the relationship between the total area and a unit of angle helped students to understand the meaning of the area of sectors. Efe mentioned a difficulty. If the total angle of the circle was not divisible by 360, students faced difficulty in performing the operations. Efe offered to reduce the division and multiply the central angle with a fraction.

Efe talked about how he introduced the pi. He said he first asked students to find a number whose square equals 36. Then he asked to find the number that’s square equals 7. In this way, students were convinced of the existence of the irrational numbers. Efe mentioned the computer in Japan that was calculating the value of pi. He said they watched a video about the men who memorized and recited digits of the pi. In this way, the teacher claimed students’ curiosity about mathematics increased. He also added that if he realized a student’s lack of knowledge related to the previous learning, he spent class time to compensate for their previous learning.

4.2.1. Findings Related to MQI

The classroom applications were highly related to teachers’ knowledge and teachers’ knowledge affects the quality score of their instruction. If the teachers know the content that they teach, the instruction segment generally receives a “High” score for the Richness of Mathematics dimension and a “Not Present “or “Low” score for the Errors and Imprecision dimensions. The selection of tasks and order of presenting tasks is affected by teachers' knowledge of content and teaching. Also, teachers try to select attractive tasks for students. Using appropriate tasks or problems is related to teachers' knowledge of the content and students. Therefore, teachers' explanations that showed teachers' knowledge were scored under the related MQI dimension. For example, the teacher explained why the area formula of the sector includes the square of the radius. This explanation showed the teacher's knowledge of the content, and it was scored under the Explanation and Mathematical Sense-Making dimensions.

Teacher group discussion was analyzed according to the adopted MQI rubric. For the Richness of Mathematics dimension, Linking Between Representations, Explanations, Mathematical Sense-Making, Multiple Procedures and Solution Methods, and Mathematical Language were scored. All participants knew the area formula of sectors. Therefore, they discussed how to introduce the topic. They commented on each other's application and tried to find the best way of presenting the topic. No evidence related to the Patterns and Generalization were observed. There was only one instance scored for Linking Between Representations and it received a "Mid" score. Efe mentioned the area model of fractions and the symbolic representation of one-fourth. He said that the area model was never connected to the central area of a circle. Also, for Explanations, only one instance scored. Yusuf explained why a given sector was half of a circle by using central angles.

Teachers were very determined to make sense of why a central angle was needed to find the area of the sectors. They talked about more than one way to make sense of the area of a sector. Yusuf and Ali said they used half and quarter circles to construct student knowledge on their previous knowledge. Students would realize a half circle had a 180-degree central angle and its central angle was half of the whole angle. Ali and Efe's activities scored as "Mid". Efe said he did not write the central angle of sectors to make sense that the central angle was necessary to find the area of a sector. He insisted that if students used the relation between the central angle to find the sector's area, they would overgeneralize it and try to find the area of all sectors using the relation between the central angles. While finding the area of the sector with a central angle like 70 degrees, using the relation between the numbers would be challenging. Efe's explanation about his instruction was scored as "High". Efe also said he wanted students to compare the area of sectors t without calculating the exact value of areas to show the relation between the central angles and the area of sectors, and it received a “Mid” score. Teachers were careful about the use of mathematical terms and Mathematical Language scored “Mid”.

For Working with Students and Mathematics dimension, Remediation of Student Errors and Difficulties, and Teacher uses Student Mathematical Contribution were scored. The teachers knew the content and their students. Yusuf and Efe mentioned possible student difficulties and errors. However, their ideas about students’ possible errors and difficulties were different. Yusuf claimed that his students would face difficulty in connecting the total area of a circle and the 360-degree central angle. Efe said it was obvious for his students. Yusuf also said that his students would not realize the direct proportion between the central angle of a sector and its area. Efe indicated that the students would remember how they found the length of an arc and they would offer to use direct proportion as they did when finding the length of an arc. To remediate the students’ errors and difficulties, Yusuf said they started with half and quarter circles. Efe pointed out a procedural difficulty that the students face difficulty finding the area if the total area was not divisible by 360. For multiplication like this, he offered to reduce the fraction and then multiply the central angle with the fraction. Teacher uses Student Mathematical Contribution scored once. Efe said students would connect the finding length of the arc to the finding of the area of the sector. He said he constructed the direct proportion method on this student's explanation and scored “High”. Yusuf and Ali said students would realize the direct proportion between the central angle of the sector and the area of the sector. Both instances scored as “High”.

All sub-dimensions of Common Core Aligned Student Practice (CCASP) were scored at least once. For Students Provide Explanation all three teachers talked about some student explanations. Efe said students would provide an explanation about why the area of a sector was bigger than the area of another sector. His explanations were detailed, but he did not mention how he used student contribution to develop math, and it was scored as “Mid”. Ali and Yusuf talked about student justification of the relationship between half or quarter and the central angle. Ali claimed that to compare the area of sectors with an unknown central angle, students would reference the area of half and the area of the quarter. The teacher used this student's contribution to explain why a central angle was needed to find the area of a sector. Ali’s explanations scored “High". Yusuf said students would explain why a quarter is one-fourth of a circle using the central angle. He said he used this student explanation to introduce the area formula of the sector and scored as “High”

Yusuf and Efe mentioned Student Mathematical Questioning and Reasoning (SMQR). They shared their experience related to students’ explanations about the proportion between the central angle and the area of a sector. According to Yusuf, only a few students could realize the relation between the central angle of a sector and the area of it. His explanation was not detailed and developed and scored as “Low”. Efe claimed students would refer to their knowledge about the length of the arc. Students would use their previous knowledge to reason the area of sector. Efe said students could provide some explanation about the area of sectors. Ali also said students would compare sectors with a half or a quarter to decide which one was bigger. Efe’s and Ali’s explanations scored “Mid”.

All three teachers’ sharing about student contributions scored for Students Communicate about the Mathematics of the Segment dimension. They talked about some brief student contributions and some substantial student contributions. Ali and Efe talked about how students could compare the area of the sector without calculating the value of areas. Ali said they would use half and quarter circles as references. Efe said they could provide explanations using sectors’ perimeter or radius of circles. Ali and Efe’s explanations received a “Mid” score. Efe also said students could remember their learning about the length of the arc and explain that the central angle could be used to find the sector’s area. His explanation was scored as “High”. All three teachers agreed that students could realize and provide an explanation about the ratio between central angles of sectors and the area of sectors after some examples with central angles like 90 degrees, 180 degrees, and 60 degrees.

For Students Work with Contextualized Problems all three teachers offered examples of problem context. Ali said he used a pizza or a cake problem to attract the attention

Table 9. Pre-Instruction group discussion: Score and evidence of Richness of Mathematics

Richness of Mathematics	**Teacher**	**Evidence**	**Score**
Linking Between Representation	Efe	He mentioned that the students were familiar with the area model of one-fourth, but they did not relate it to the central angle.	Mid
Explanations	Yusuf	He explained why a sector was half. It was a brief explanation.	Low
Mathematical Sense-Making	Ali	He said when students saw a sector, they could compare it with half or quarter.	Mid
	Yusuf	He indicated that he used the half and quarter sectors, and asked students how they knew it was a half. He expected students to explain the central angle of the sector was 180-degre which was half of 360-degree. Then he used a sector with a central angle-like 70-degree to connect the area measurement of the sector and the central angle.	Mid
	Efe	He used sectors with an unknown central angle and asked students what they needed to know to find the area of the sector. He expected students to discover that a central angle was necessary to find the area of a sector. He also said that students would use previous learnings to find the area of the sector.	Mid
	Efe and Ali	They discussed why they needed the radius of the circle and the central angle of the sector.	Low

Table 9. (continues)

	Efe	Efe indicated that he asked students why the area of a sector was bigger than the area of another sector.	Mid
Multiple Procedures and Solution Methods	Ali, Yusuf, and Efe	The teachers said direct proportion or ratio between angles and areas could be used as a solution method in addition to the area formula of the sector.	Mid
	Efe	He said he used a third method to find the area of the sector which was dividing the total area of the circle by 360 and multiplying the result with the central angle of the sector.	Mid
Patterns and Generalizations	Not present.		
Mathematical Language	Ali, Efe, and Yusuf	All teachers used mathematical language correctly and effectively. However, no student language error was mentioned.	Mid

Table 10. Pre-Instruction group discussion: Score and evidence of Working with Students and Mathematics

Working with Students and Mathematics	Teacher	Evidence	Score
Remediation of Student Errors and Difficulties	Yusuf	He said the students in his school would face difficulty connecting the total area of a circle and the 360-degree central angle. But he offered no remediation.	NP
	Yusuf	He said only a few students would realize the proportion between the central angle and the area of a sector. He said he used half and quarter circles and related the central angle of half and quarter circles to the area of the sector.	Mid
	Efe	He said if he used the half, quarter, and one-sixth of the circle while introducing the area of the sectors, his students would overgeneralize this idea. They would try to find a factional relation for every central angle. So, he said he used sectors with a central angle which was not divisible by 360.	Mid
	Yusuf	Yusuf said he used half and quarter to explain the relationship between the area of the sector and the central angle. According to his claim, his students could not interfere that they need the central angle to calculate the area of the sector if he used a sector with an unknown central angle.	High
	Efe	He said while finding the area of a sector using the method, dividing the total area of the circle by 360 and multiplying the result with a central angle, students faced difficulty in performing division if the total area was not divisible by 360. He offered to reduce the division and multiply the central angle with a fraction.	High

Table 10. (continues)

Teacher uses Student Mathematical Contribution	Ali, Efe, and Yusuf	They said the students would realize the direct proportion between the central angle of the sector and the area of the sector. They said they could use students' contributions to develop mathematics.	High
	Efe	Efe said that students would connect the finding length of the arc to the finding of the area of the sector. They would interfere that they could use the sector’s central angle to find the area of it. He used this student's idea to show the direct proportion between the central angle and the area of a sector.	High

Table 11. Pre-instruction Group Discussion: Evidence and score of Common Core Aligned Student Practice (CCASP)

CCASP	Teacher	Evidence	Score
Students Provide Explanations	Efe	He said he asked students why the area of a sector was bigger than the area of another sector.	Mid
	Ali	He said when students saw a sector, they could justify why it is bigger by comparing half or quarter. He used this student contribution to construct the need for the central angle to find the area of a sector.	High
	Yusuf	He said students would explain why the sector was one-fourth of the circle by using a central angle and he used this explanation to introduce the area of sectors.	High
SMQR	Yusuf	He said only a few students would realize the proportion between the central angle and the area of a sector.	Low
	Efe	He said before the area of the circle, they would cover the central angle of sectors and the perimeter of a sector. So, students can use this previous learning to reason how to calculate the area of a sector.	High
	Ali	Ali said students would compare the size of sectors with unknown central angles with a half or a quarter to decide which one is bigger.	Mid
Students Communicate about the Mathematics of the	Ali	He said when students saw a sector, they could compare it with half or quarter.	Mid
	Efe	Efe said students could provide explanations about the size of sectors.	Mid
	Efe	He said students could use their knowledge about the length of the arc and the central angle of sectors	High

Segment.		to reason the area of sectors. He said students would realize the similarity between finding the length of an arc and finding the area of a sector.	
	Table 11. (continues)		
	Ali, Efe, and Yusuf	They said the students would realize the ratio between the central angles and the area of the sector after they introduced half, quarter, and some sectors with central angles like 60-degree, and 120-degree. They would use direct proportion to calculate the area of the sector.	High
Students Work with Contextualized Problems	Efe	He said he used a pizza problem or a cake problem to introduce the area of the sector.	Low
	Yusuf	He developed the pizza problem. He said they could divide the pizza with different central angles and asked who would eat the most. They did not discuss it further.	Low
	Ali	He said he used a problem context about the Wheel of Fortune, but it was not developed.	Low
	Efe	Efe developed Ali's Wheel of Fortune problem and asked it as a painting problem. He said if we asked about the cost of painting, students would notice that they needed the central angles of the sectors. They discussed this problem context for a long time.	High
Task Cognitive Demand	Ali	Ali said he divided a circle into identical sectors and asked for the area of one of them.	Low
	Efe	He said he used a pizza problem or a cake problem to introduce the area of the sector.	Low
	Yusuf	He developed the pizza problem. He said they could divide the pizza with different central angles and asked who would eat the most.	Mid

	Efe	Efe developed Ali's Wheel of Fortune problem and asked it as a painting problem. He said if we asked about the cost of painting, students would notice that they needed the central angles of the sectors. They discussed this problem context for a long time.	High
	Table 11. (continues)		
	Ali and Yusuf	Ali and Yusuf commented on Efe's problem text. They said using two Wheel of Fortune would be more supportive. They insisted on using halves and quarters to introduce the sectors' areas.	High
	Ali, Efe, and Yusuf	They exemplified the routine exercise they did. They were related to the application of direct proportion and the application of the area formula.	Low

of students. Yusuf said he was using pizza problems, too. Ali said he used Wheel of Fortune context. They discussed on Wheel of Fortune problem for a long time. All these three problems scored as “Low” because they were not discussed in detail. Efe developed Ali’s Wheel of Fortune problem and offered to use it as a panting problem. The teachers discuss how to ask a painting problem to support student learning. Therefore, this discussion was scored “High” for the Students Work with Contextualized Problems dimension. While introducing the content, they used contextualized problems which were highly cognitively demanding problems. After introducing the content, they started to solve questions. The questions they claimed that they solved were low-level cognitive demanding tasks.
For the Task Cognitive Demand dimension, the contextualized problems and exercises that teachers offered were scored. The cake problem of Yusuf received a “Mid” score. Ali said he used identical sectors to introduce the area of sector. Efe and Yusuf objected to him and claimed using identical sectors would not help students understand the area of sectors. Ali’s task was received a “Low” score. The painting problem that the teacher discussed received a “High” score. The painting problem required students to understand that a central angle was needed to calculate the sector’s area. It also required students to provide explanations about their solutions. Yusuf and Ali insisted on using two Wheel of Fortune in the problem context. They claimed using half and quarter would help students understand the area formula of the sector more easily.

The only dimension that was not scored is Errors and Imprecision. The teachers made no content errors. They used mathematical language correctly. The scores of dimensions and the evidence are given in Table 9, Table 10, and Table 11.

4.3. First Observation of Ali: Area of Circle

The first observation of Ali’s lesson took place while he was teaching the area of circle and it took place in the seventh-grade classroom. The lesson was videotaped and the researcher was in the classroom as a passive observer. The observation process lasted for two consecutive lesson hours because covering the objective of the area of the circle lasted two hours. His 7th-grade class was composed of boys because the school was imam-hatip middle school. Therefore, girls and boys receive education in separate classrooms. 26 students registered in this classroom. 4 students were absent on the day of observation.

4.3.1. Summary of the Lesson

The instruction started on time. Only one and a half minutes were spent before starting instruction. This time was spent greeting the students and opening the smart board. Ali started the lesson by informing students of the lesson's objective that they would learn how to calculate the area of a circle. He showed a map on the smart board. The students said it was a map of a computer game. Almost all of the students have known the game. Ali said there are different circular regions in this game and these regions are called “safe areas”. Ali showed two circular regions on the map asking students which area would they prefer to hide. Some students answered all together as “the bigger one”.

> *Ali: By saying bigger one, what do you mean?*
> *Ufuk: The one with a big diameter.*
> *Serdar: The one with a big circumference.*
> *Ali: What else is bigger when its diameter is bigger?*

Students said, its area, circumference, and length are bigger. Then, Ali asked, “How do we find the area of a circle?”. Some of the students chorused “2.π.r”. Hakan said “Pi times diameter”. The teacher reminded students that a diameter includes two radii. Ali turned back to the first question and asked again which area they would prefer. Ufuk said he would choose the one with a bigger area. Ali drew a circle on the board and showed the map again.

> *Ali:* Why area of this one is bigger? How do you know?
> *Ufuk:* It looks so.
> *Erdem:* It covers more space.
> *Ali*: In where it covers more space?

Some students answered Ali's question and said it covered more space on the map. Serdar said its circumference was bigger. Ali again asked why it had a bigger circumference. Serdar replied, "It coves more space and it has a bigger radius". Ali came up with a discussion "If a circle has a bigger radius will it have a bigger area?". The students hesitated and did not answer. Ali directed students that this statement was true. Then, he asked Cem which one of the circles was bigger. Cem said the one on the right side is bigger. When the teacher asked the reason, Cem said it had a bigger circumference. Ali asked, "How do you know it has a bigger circumference?". Cem did not answer. No students explained the reason. However, some students just said "$2.\pi.r$" without any explanation. Ali also did not explain the question related to circumference. He prompted a new question "How can we calculate the area of circles?". Serdar suggested adding radii. Ali allowed him to make additions to the board. Ali just said without doing addition it is possible to see which one is bigger. Mert suggested finding the circumference first, then multiplying the circumference by 2. Ali asked what they expected to find at the end of this operation. Some of the students claimed the result of this operation is equal to circumference, some others claimed it is equal to radius. Ali explained the result of this operation is twice the circumference. Hakan claimed that they could find the area by multiplying circumference by circumference. Another student suggested multiplying circumference with pi.

Ali asked again how they calculated the area of the circle. Some students said they could use a formula. However, Ali reminded them that they did not know the formula yet. Also, he asked, "How does the formula construct?". Students did not answer. Then, he showed a picture of a square and a parallelogram which are divided into unit squares. He wanted the students to find the area of the square and the parallelogram. The students said they could calculate area by counting unit squares. Hakan remained the area formula of the rectangle is "base times height". Students suggested finding the area of the circle by counting unit squares. Ali wanted them to compare the area of the circle and the area of the square. The students did not arrive at a consensus. Some of them claimed the area of the circle is bigger, some of them claimed it is smaller and one of the students claimed that they are equal. Kerem said the area of the circle is smaller and he drew a square whose sides are tangent to the circle. He said the circle did not cover the corner of the square. Ali asked how they found the exact measure of the difference between these two areas. Erdem said, “Do we divide it by 2?”. Ali answered immediately and drew the diagonal of the square “Division by 2 is used while finding the area of triangles because when the square is divided by diagonal, two triangles occur.”. The students said they could find the area of the square tangent to the circle and the corners of the square not covered by the circle are four triangles. Ali drew border lines of the left area and showed that one edge of the side shape is not linear. He said it looks like a triangle, but it is not a triangle.

Ali split the circle, that he drew on the board, into four equal sectors from the center by drawing two diameters. He said that the students can think of this circle as cake or bread. The length of the radius was determined as 5 cm. Some students summed up the length of 4 radii and said the answer was 20. Some of them said it is not an exact triangle but they still need to use the area formula of the triangle. Ali suggested rearranging the sectors of the circle and drawing a new shape that resembles a parallelogram. Then, he asked for a measurement of the central angles of the sectors. Some students answered as 5. Ali just warned them not to confuse length and angle. He did no more explanation about students’ confusion. Then, he asked if the area of the circle and the area of the new shape of rearranged sectors were equal. Some students claimed they were the same, however; some students hesitated. Some students talked at the same time and it showed they are facing difficulty with area and circumference content. Ali wanted them to define what “*area*” is. Ufuk said” place occupied by an object in space” then he corrected himself and said it is the definition of volume. Ali then explained area is 2 dimensional and defined the area as “The region occupied by the shape in the plane”. Then, he asked again if the areas of the two shapes were the same. Some students said the circle has a bigger area because it seems so. Ali suggested dividing the circle into 8 sectors from the center and rearranging them. Some students still claimed they have different areas. Students’ explanations showed that their confusion about area and circumference is the reason for their claim. Ali split the circle into 16 sectors from the center and rearranged them. He asked students “What do these three new rearranged shapes look like?" Hakan said they look like a rhombus. Erdem said they resemble rectangles. Ali repeated they resembled a rectangle and asked what would happen if he split them into infinite parts. He opened a prepared material on dynamic geometry software, GeoGebra, and showed students what would happen if they split the circle into more parts and rearranged them. Students chorused that it looks like a rectangle. Ufuk and Yusuf suggested using the formula "base times height". However, they had difficulty explaining what base and high mean for rearranged shapes. Ufuk said they should use "base times height divided by 2". He explained that the new shape includes triangles so they should use the area formula of the triangle.

Ali said the height of the new shape was equal to the radius and asked measurement of the base. Students said it is equal to the diameter. Mert said it is equal to a part of circumference but he said he did not know the relation. Ali explained that he split a circle into 16 pieces and a long side is equal to additions of 8 arc lengths. Then, students said it was equal to half of the circumference. He asked for the formula of the circumference and students said "2 times pi times radius". Ali tried to generalize the area formula of the new shape but the students' attention was on numbers. The students tried to calculate the area of the circle. Ali came up with the conclusion that the base of the rectangle is equal to "π times r". To find the area of the rectangle, he said they need to multiply height "r" with the base "π times r". Then students concluded that it is equal to "π times r^2". Ali explained that the area of the circle can be calculated by multiplying half of the circumference by the radius. He calculated the area of the circle with radius 5 incorporation with students and asked what is unit of the area measurement was. Some students said it is a cubic centimeter. Ali said it was a square centimeter and asked students to copywrite the things written on the board and two and a half minutes passed while the students were writing. Then the first hour of the lesson ended.

The first five minutes of the lesson were spent while students were copywriting the things on the board. After the students finished writing Ali asked, "How is the area of a circle found?". The students said it is "r square times pi". Ali asked students how they got this conclusion, and students reminded them that "r times pi is equal to half of the perimeter". Ali concluded that multiplying half of the perimeter with radius will give the measurement of the area. He opened two examples on the smart board to solve. The first one was “ What is the area of a circle with a radius of 6 cm?”. Ali asked again what the measurement unit of the area was.

Ali: What is the unit of measure? Centimeter, meter, cubic centimeter, or square meter.

Students all together: Square centimeter.

Ali: Why the area of this is square centimeter?

Murat: When two same units are multiplied the result becomes square.

Yusuf: Because it is two-dimensional.

The students started to talk about the result of the problem and the discussion did not come to an end. One of the students found a false answer. The teacher asked how he got the answer. The student explained his solution process and they realized he used pi two times in the multiplication. Ali highlighted the area formula written on the board again. After another student explained his solution, the teachers solved the question using the area formula. Ali asked the question that he asked before “Why the unit of measure is square centimeter?”. Some students expressed that they did not understand why the unit of measure is a square centimeter. Ali explained the reason why the measure of the area is the square unit as it comes from the multiplication of two radii. However, some students had different confusion about the computing area of a circle.

Hakan: Why are we multiplying with 3?

Ali: We have already covered the topic pi last week. We discussed what pi is.

Ali also reminded the students what they covered in the first lesson hours and how they got the area formula using the perimeter of the circle. They solved another question. In the next question, the diameter of the circle was given. One of the students used diameter instead of radius. His friend warned him that the diameter was given not the radius. Ali drew a circle and showed diameter is given as 8 and its radius is 4. The other question was about finding the radius when the total area was given. However, some students explained their solution, the teacher noticed that these students divided the total area by 2.

Ali: Why did you divide the area by 2?

Hakan: Because it is a square of something.

Some students have different reasons to divide the area by 2. Their explanation revealed that they confused the area and the perimeter formula of the circle. After checking the students' solutions one by one, Ali solved the question interactively with the students. The next question was about finding the circle's diameter when the circle's total area was given. Similar to the previous solution process, some students had problems related to the square of a number. The students asked how they found the 7 when r^2 equaled 49.

Ali: Whichever number we multiply by itself makes 49? The square of a number means multiplying the number with itself.

To solve the new question, Ali called a student. The teacher wanted the student to write the area formula which is $\pi.r^2$. The equation that students needed to solve was $3.r^2= 27$, but students could not do the procedure. Ali asked a new simple world question which described the equation that the student should solve. The new question was "If the price of 3 apples is 27, what is the price of one apple?", and the student gave the correct answer as 3. Ali, again, emphasized that r^2 indicates multiplying r by itself. They solved another question about finding the area of the circle when the perimeter of the circle was given. For the next question, the shape of a square and the tangent circle of this square were given. The students were asked to find the difference between the area of the square and the area of the tangent circle. The students got confused for a few minutes because this question was different than previous questions. Ali asked the students how they could find the answer.

Ali: How can we find the grey area? I do not want you to do the operations. I ask you to describe the steps of the solving process.

Selim: First, we should find the area of the circle. Then, we should find the area of the square. If we subtract the circle's area from the square's area, we can find the answer.

Ali solved the question on the board by explaining the solution steps. In the last 4 minutes, the teacher announced the students' exam grades.

4.4. Finding Related to MQI

In this section, Ali’s instruction while teaching the area of a circle is evaluated using the MQI 4-Point framework. Each 7-minute segment was evaluated separately. Segment scores and quality evidence of the segment are given in the tables in detail.

4.4.1. Richness of Mathematics

Richness evidence of the instruction is presented in Table 12 and Table 13. In the first table segment codes and evidence of meaning-oriented codes are given. Meaning-oriented codes are Linking Between Representations, Explanations, and Mathematical Sense-Making. In the second table segment score, evidence of practice-oriented codes, and the overall richness scores of the segments are presented. Practice-oriented codes are Multiple Procedures or Solution Methods, Patterns and Generalizations, and Mathematical Language. The distribution of segment scores and subdimension of Richness of Mathematics is given in Figure 7.

Eight of the segments received “NP” for Linking Between Representations because in geometry shapes do not count as a representation. The shape is considered the “thing itself”. Only two segments were coded as “Low” for the Linking Between Representations dimensions. In the third segment, the teacher linked the representation of the circle to cake and bread, but it was not detailed. In the ninth segment, the teacher used a real-life context to make the equation meaningful to the students. The teachers asked a simple word problem “Price of 3 apples is 27 TL. What is the price of one apple?” to a student who faced difficulty in solving equation $3r^2=27$. Only the fifth segment, the teacher linked two geometric shapes and the symbolic representation of the area formula of the circle. However, the link within the same representational family do not count as a link and are not scored in MQI rubric.

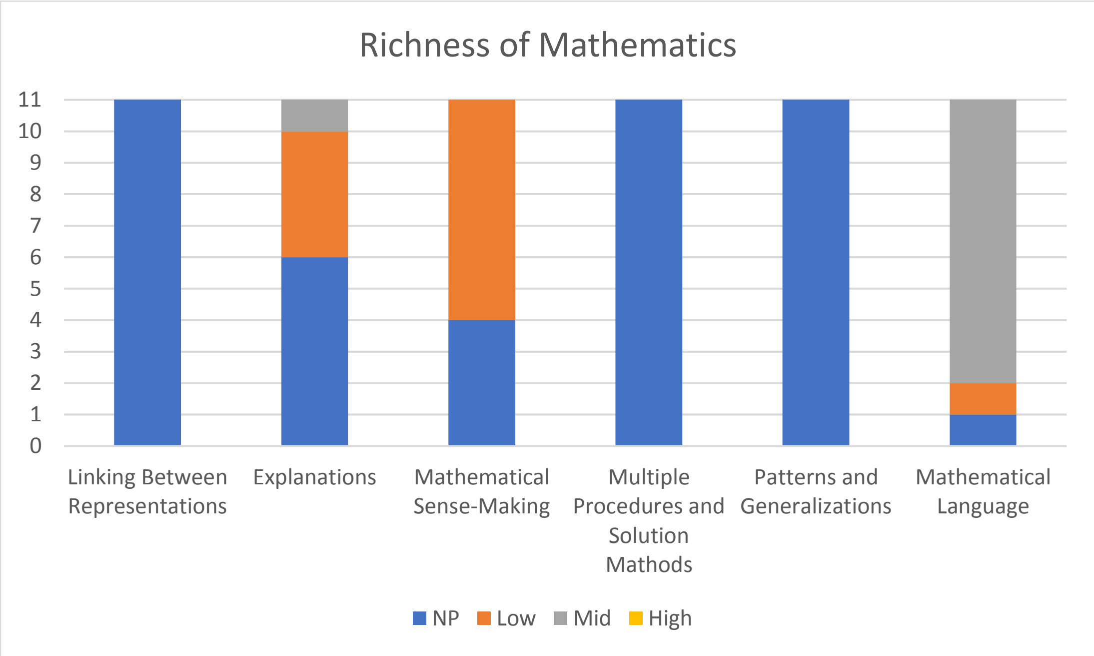


Figure 7. Segment Scores of Richness of Mathematics

Segments received different Explanation codes. In the first segment, the teacher asked why the area of the one circle was bigger than the other. The students made some explanations, but these explanations were not developed. Ali explained why the circle's perimeter formula included 2r. his explanation was not detailed enough. So, the segment received a "Mid" score instead of "High". Segment 2 received a code of "Mid" because students explained why the area of the square was bigger than the area of the circle by translating the square onto the circle, pictures of both shapes were shown on the smart board. The discussion about the area of the circle and the area of rearranged sectors started in Segment 3, however, the discussion came to a conclusion in Segment 5. In segment 3, the given explanation was just stated and not developed or discussed in detail, so segment 3 scored as "Low". These explanations developed a little bit in segment 4 and coded as "Mid". Segment 5 received a "High" code of Explanation because the focus of the segment was why the area formula of the circle includes r^2. The teacher explained the area formula of the circle by using the area formula of the rectangle. Ali explained why the base of the rectangle was equal to half of the circle's perimeter and the height of the rectangle was equal to the circle’s radius. In segment 7, the teacher asked “Why measurement of the area expressed as cm^2 or m^2?”. The teacher explained that cm times cm (cmxcm) resulted in cm^2. His explanation was not wrong but not the mathematical explanation of the unit of area measurement. Therefore, the segment is coded as “Low”. The rest of the segment got “NP” for Explanation.

The aim of the first lesson was the construction of the circle’s area formula and making sense of the area formula. Segments 2,4 and 5 received a “High” Mathematical Sense-Making score. In segment 2, the teacher tried to make sense of why counting unit squares was not an appropriate method to find the area of the circle. Also, a student compared the area of a square and the area of a circle by translating the square into a circle. In segment 3, the teacher wanted to develop the area formula of the circle but, some students had problems related to the conservation of the area. As a result, the teacher had to explain the procedure that he had done, and the equality of the length and the segment coded as “Low” Mathematical Sense-Making. In segments 4 and 5, the teacher cooperated with students to make sense of the area formula of the circle. They worked on the relationship between the circle and the rearranged shape. Students faced difficulty in understanding the relation between the sides of the rectangle and the circle’s radius and perimeter. The teacher showed the relation between the shapes. However, some students faced difficulty in understanding why the area of the two shapes was equal. These students had problems related to the conservation of the area. By writing the area of the arranged sectors, they discovered why the area formula includes the multiplication of the length of the radius with itself. Segment 6 was spent on coping. In segment 7 the teacher focused on the measurement unit of the area. However, the student's answer was isolated, and the teacher only explained the procedure. It was not enough to get “High”. In Segments 8 and 9, the teacher focused on the meaning of numbers because some students faced difficulty in calculating the square roots of numbers. The tenth segment received a “High” score because the teacher explained to a student why his answer was not reasonable.

For Multiple Procedures or Solution Methods, only two segments received “Mid”, and the other segments got “NP”. In segment 2, to find the area of a square and a parallelogram, they used two different methods. Students first calculated the areas by counting unit squares, then they calculated the same areas by using formulas. Since the methods were not compared or the effectiveness or appropriateness of methods was not discussed. Therefore, the segment was scored “Mid”. A student offered another solution method in segment 7. Still, his method was not compared to the teacher’s solution, and the segment got a score of “Mid.”

The teacher started the discussion to develop the area formula of the circle in segment 3, which went on in segments 4 and 5. Throughout these three segments, the teacher and students tried to construct a generalization to find the area of the circle. The teacher divided the circle into sectors and rearranged them to form a new shape. The teacher showed that when the number of segments increased the rearrangement of the sectors more look like a rectangle. For Pattern or Generalization, segments 3, 4, and 5 got “Low,” “Mid,” and “High,” respectively. In segment 4, the teacher helped students to generalize that the base of the rearranged rectangle was half of the circle’s perimeter, and the height of the rectangle was equal to the radius of the circle. The students realized that the area of the circle and the area of rearranged shape were the same. In segment 5, the symbolic representation of the circle’s area formula was constructed. The other segments were coded as “NP.”

For the Mathematical Language dimension, segments got a “Mid” or “High” score because the teacher was careful about using mathematical language and pressed students to use mathematical terms accurately. In segment 1, the teacher warned a student to use the term circle accurately, and the segment got a score of “High”. The other segments that were scored as “High” were segment 3 and segment 4. In segment 3, the teacher defined the area. In segment 4, they use mathematical terms in dense.

The overall Richness of Mathematics scores, which show the depth of mathematics offered to students, are given in the table below. Although segments got different scores for richness codes, the mathematics offered to students got “Mid” or “High” scores. The teacher tried to make sense of the circle's area and construct students’ knowledge on what they already knew.

Table 12. First observation of Ali: Score and evidence of Richness of Mathematics-I

	Linking Between Representations		**Explanation**		**Mathematical Sense-Making**	
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**
S1	NP	The shapes do not count as a representation in geometry.	Mid	The teacher asked, "Why is the area of one of the circles bigger?" they tried to explain it. The teacher explained why the formula of the circle's perimeter includes 2r. However, his explanation was not explicit and detailed enough.	NP	Nothing related to sense-making happened.
S2	NP	The shapes do not count as a representation in geometry.	Mid	A student added radii to prove which circle was bigger, and the teacher talked about why this procedure was inappropriate. A student explained why the area of the shown circle is smaller than the given square.	High	The teacher tried to make sense of why counting unit squares was not a reasonable procedure for calculating the area of a circle. They also compare the area of a square and the area of a circle whose diameter equals the length of a side of the square.

Table 12. (continues)

S3	Low	The link between the circle and cake and bread is made in a pro forma way. But this link is not developed.	Low	Why do the circle and rearrangement of its sector have equal areas? The explanations are isolated and not developed.	Low	The main idea of the procedure was developed on the conservation of the area. However, some students had problems related to the conservation of the area. The teacher focused on the equality of the radii, and his explanation was not enough to make sense.
S4	NP	Different representation of the same family is not coded.	High	The teacher explained why the height of the rearranged sectors was equal to the circle's radius, but it was not detailed. They also explained why the base of the rearranged sectors was equal to half the circle's perimeter. The teacher briefly explained why the circle's area and the rearranged sectors' total area were equal.	High	The segment focuses on the relationship between rearranged sectors and the rectangle and making sense of the area formula that "radius times half of the circumference."

Table 12. (continues)

S5	NP	The teacher linked the circle's radius and perimeter, the sides of rearranged sectors, and the symbolic representation of the rectangle's area formula and the circle's area formula.	High	The teacher justified the area formula of the circle. He explained why the area formula includes r^2 and was the segment's focus.	High	The focus of the segment was making sense of the circle's area formula by using the rectangle's area formula.
S6	NP	Students spent over 5 minutes copying the things on the board.	NP	Students spent over 5 minutes copying the things on the board.	NP	Students spent over 5 minutes copying the things on the board.
S7	NP	The shapes do not count as a representation in geometry.	Low	Explanation of why the measurement unit of the area is written as cm^2, m^2, etc. The students' and the teacher's explanations were not detailed and connected.	Mid	They focused on the meaning of measurement units of area, but this work is substantial.
S8	NP	The shapes do not count as a representation in geometry.	NP	A student made a mistake while solving the problem. The teacher just reminded the area formula of the circle.	Low	The teacher tried to explain the relationship between numbers, such as 49 equals the square of 7.

Table 12. (continues)

S9	Low	The teacher linked the equation $3.r^2=27cm^2$ to a problem in a real-life context to make it easier to solve the question, but this link was not detailed.	NP	The teacher or students offered no mathematical explanation.	Low	The teacher tried to explain the relationship between numbers, such as 9 equals the square of 3.
S10	NP	The shapes do not count as a representation in geometry.	NP	The teacher or students offered no mathematical explanation.	High	A student found the area of the inner tangent circle of a square bigger than the area of the square. The teacher made the students examine the reasonableness of his answer.

Table 13. First observation of Ali: Score and evidence of Richness of Mathematics-II

	Multiple Procedure or Solution Methods		Patterns and Generalization		Mathematical Language		Overall
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**	**Score**
S1	NP	There was no evidence of multiple procedures and solution methods.	Low	They tried to make a generalization about which circle was bigger, but the generalization was not developed during the segment.	High	The terms area, perimeter, radius, diameter, and length were used, and the teacher pressed the students for accurate use of the terms.	Mid
S2	Mid	They calculated the area of a square and a parallelogram, counting unit squares and using the area formula. However, no comparison was made.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	Middling use of the terms area, perimeter, radius, diameter, and length.	Mid
S3	NP	There is no evidence of multiple procedures and solution methods.	Low	The teacher tried to develop a generalization related to finding the area of a circle, but the generalization was not	High	Middling use of mathematical terms such as perimeter, area, triangle, base, height. The teacher reminded the meaning of area and defined the	Low

				developed in this segment.		area.	
Table 13. (continues)							
S4	NP	Only one procedure was presented.	Mid	The generalization of long sides of the shapes, which were constructed by rearrangement of sectors, was equal to half of the circle's perimeter, and its height was equal to the circle's radius but not detailed enough.	High	Dense use of mathematical terms such as perimeter, area, triangle, base, height, arc, angle, rectangle, and rhombus.	High
S5	NP	There was no evidence of multiple procedures and solution methods.	High	The area formula of the circle was developed using the generalization of the area formula of the rectangle.	Mid	Middling use of mathematical terms such as perimeter, area, triangle, base, height.	High
S6	NP	Students spent over 5 minutes copying the things on the board.	NP	Students spent over 5 minutes copying the things on the board.	NP	Students spent over 5 minutes copying the things on the board.	NP
S7	Mid	A student offered a second way to solve one question, but his	NP	No generalization was developed. No pattern was	Mid	Middling uses mathematical terms such as perimeter, area, radius, pi,	Mid

		solution was not compared to the teacher's.		discovered, and no definition was built.		and unit of measure.	

Table 13. (continues)

S8	NP	There was no evidence of multiple procedures and solution methods.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	Middling uses mathematical terms such as perimeter, area, radius, pi, and unit of measure.	Low
S9	NP	There was no evidence of multiple procedures and solution methods.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	Middling uses mathematical terms such as perimeter, area, radius, pi, and unit of measure.	Low
S10	NP	There was no evidence of multiple procedures and solution methods.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	Middling uses mathematical terms such as perimeter, area, radius, pi, and unit of measure.	Mid

4.4.2. Working with Students and Mathematics

In this section, the score and evidence of the Working with Students and Mathematics are given. The Working with Students and Mathematics dimension is highly related to teachers' knowledge of content and students. If the teacher knows possible students' errors and difficulties related to the content that is taught, he can plan pre-remediation activities. The teacher also can hear, understand, and use students' contributions to develop mathematics. The segments coded "NP" or "Low" for both Remediation of Students Errors and Difficulties and Teacher Uses Students Mathematical Contribution. The distribution of segment scores and subdimension of Working with Students and Mathematics is given in Figure 8.

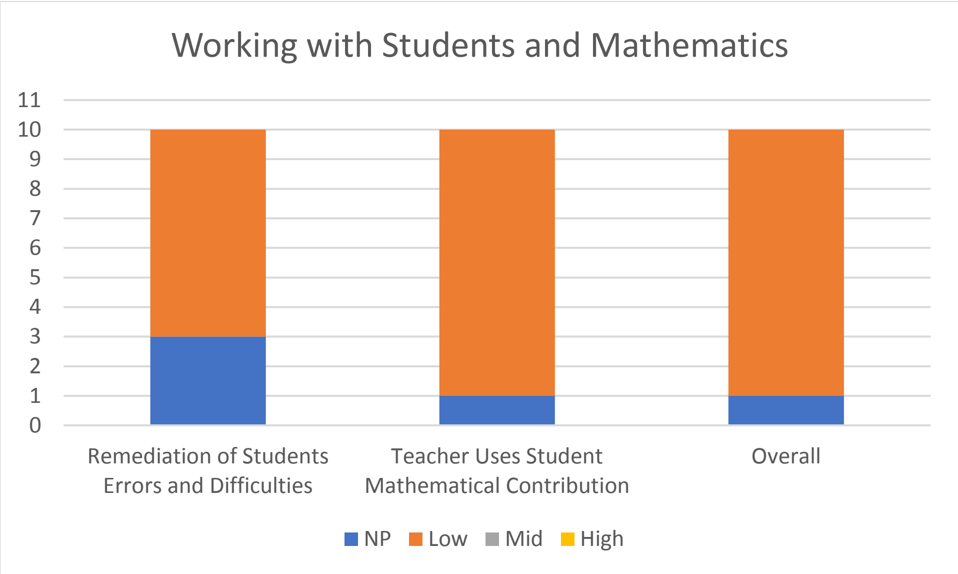


Figure 8. Segment Scores of Working with Students and Mathematics

The teacher's remediations were procedural in all segments, and no conceptual remediation occurred. Some students encountered difficulty with the mathematics of the lesson, and some students had problems with procedures. More than one student faced difficulty with the conservation of the area. However, the teacher said the areas of the shapes were equal and gave an example to clarify the conservation of the area. He said "You have a garden, and you translate this garden to another place. Does the space covered by the garden change?" Translating a garden without changing its shape was not possible. Also finding the sands that form the surface of the garden was not applicable. Since the example was not appropriate to explain the conservation of the area, it did not help to understand the conservation of the area. Some students had problems understanding what the perimeter means and what the area means. They used the perimeter formula to find the area of the circle. The teacher defined the area at the beginning of the class, but he did not focus on the definition of the area and the perimeter when he witnessed students' confusion. The teacher just reminded the area formula of the circle and no conceptual remediation occurred. Students' confusion went on throughout the instruction. Another student difficulty occurred with the value of the π. The students asked why π was 3. The teacher responded that they discussed it last week. Some students faced difficulty with the square of the numbers. Students multiplied the number by 2 to find its square or they divided the value of r^2 by 2 to find the r. The teacher explained that r^2 means r times r, and the square means multiplying a number by itself. The teacher performed the operations to show how to multiply r by r. Other procedural remediations were related to students' operational errors. The teachers warned students "You made a mistake while multiplying or you made mistakes in substruction".

Ali's instruction proceeded with interaction with students. Ali always asked questions to students, and they answered these questions. However, students' answers were generally short, one or two-word answers. The teacher asked how to solve the questions and students described the solution steps or just said the results. In segment 2, a student showed why the area of the square was bigger than the area of the circle. He drew the tangent square of the circle which was identical to the given square. However, the student's explanation was not developed. With the teacher's questions, students got involved in the instruction. Most of the time, their contributions were not mathematically substantial, and it was the teacher who developed the mathematics. Therefore, for the Teacher Uses Students Mathematical Contribution dimension, nine segments were scored as "Low", and one segment was scored as "NP".

Table 14. First observation of Ali: Score and evidence of Working with Students and Mathematics

Segment	Remediation of Student Errors and Difficulties		Teacher Uses Students Mathematical Contribution		Overall
	Scores	**Evidence**	**Scores**	**Evidence**	**Scores**
S1	NP	No misunderstanding or difficulties with the content occurred.	Low	The teacher developed the math, not the students' answers and the students' answers and the teacher's responses were in a pro forma way. T: In which circle, the chance of staying alive is bigger? S. In the big one. T: What's big? S: Diameter. T. If the diameter is bigger, what else becomes bigger?	Low
S2	Low	Brief procedural remediations: A student offered to find the area of the circle by adding radii. The teacher let him add on the board and asked what the result meant. Another student offered to multiply the perimeter by 2 to find the area. The teacher asked what multiplication result indicated.	Low	The teacher developed the math, not the students' answers and the students' answers and the teacher's responses were in a pro forma way. T: How can you calculate the area of the circle S: We can find the perimeter and then multiply it by 2. T: What will you find in this way? S: Perimeter. S: Radius. T: You will get the measurement of 2 perimeters.	Low

Table 14. (continues)

S3	Low	Some students had problems related to the conservation of the area but the teacher only said that the two areas were equal and gave a real-life example about moving a field to another space and dividing a garden bed into parts. However, the example was not clear enough and did not help students understand the conservation of the area. A student defined the volume instead of the area. The teacher just said it was the definition of the volume and defined the area.	Low	Students contributed to the lesson with short answers and short explanations. T: How can we find the area? … S: We drew a square around the circle. We can find the area of the square. There are triangles in the corners. T: Are they triangles? They are not triangles.	Low
S4	Low	The teacher answered a student who objected to the equality of the areas by explaining the procedure he did.	Low	The teacher used only a few students' answers to develop mathematics. The students' answers were shot.	Low

Table 14. (continues)

S5	Low	When the teacher asked what x times x equals, a student said it was equal to 2x. The teacher explained it by reminding the procedure. Some students faced difficulty deciding the unit of the measurement of the area. However, the teacher was not remediated.	Low	The teacher developed the math using some ideas from the students. These ideas were short and not detailed. S: How these are equal? T: They are equal. I divided it into 16 pieces and I got these shapes. I divided it into 8, I got these. I divided it into 4 pieces and I got this one. … T: Like which geometric shape does this look like? S: Rectangle. S: Faruk said, it looks like a rhombus.	Low
S6	NP	Students copied the notes for more than 5 minutes of the segment. They	NP	Students copied the notes for more than 5 minutes of the segment.	NP
S7	Low	Procedural remediation: Ali warned a student who multiplied π two times to find the area. The teacher said to find half of the perimeter, π had already been used as a multiplicator.	Low	Students contributed and the teacher responded in a pro forma way. For example: T: Who will answer? S: Square of 5, 5 times 5 is 25. We multiply π with 25 and it is equal to 75.	Low
		A student asked why 3 was used as the value of π. The teacher said they covered and discussed π before.		T: 75 cm^2. If the unit was a meter, we would say meter square.	

Table 14. (continues)

S8	Low	Procedural remediations: A student divided the circle's area by 2 and the teacher asked students if the area formula included 2 as the coefficient and showed the area formula. Some students used diameter to find the circle's area. The teacher drew a circle and showed the diameter and asked what the radius was. Some students offered to divide 49 by to when r^2=49. The teacher explained that r^2 shows r times r.	Low	Students contributed and the teacher responded in a pro forma way. For example: S: Why 49 is equal to 7? T: Whichever number we multiply by itself gives 49? It is r square. The square of the r is equal to 49. It means multiple 2 r and it is equal to 49. It means multiple two same numbers.	Low
S9	NP	No misunderstanding or difficulties with the content occurred.	Low	Students contributed and the teacher responded in a pro forma way. For example: T. What was the formula of the perimeter* S: 2 times π times r.	Low
				T: The formula of the perimeter is 2 times pi times r. 2 times pi times r are equal to 24.	

Table 14. (continues)

S10	Low	Moderate procedural remediation occurred. T: What is the area of the square? S: 36. T. You got 66. How does this happen? The area of the square is 36 but the area of the shapes in the corners is 66. Does it make sense?	Low	Students contributed and the teacher responded in a pro forma way. For example: T: Tell it again. What we will do Selim? You said we will first calculate the area of the circle. S: 27. T: Then we will find the area of the square. S: 36 T: Selim said if we subtract the circle's area from the square's area, we will get the answer.	Low

Nine segments scored “Low” for overall Working with Students and Mathematics and one segment received a “NP”. Several students’ misconceptions and errors occurred during the instruction. However, the teacher’s remediations were procedural. Some of the misconceptions were because of the lack of knowledge of students. For example, a student asked why they used 3 as a value of π. The teacher said they had discussed the value of π before. The teacher and students were in dialogue during the instruction. However, Teacher Uses Students Mathematical Contribution scored “Low” because the students’ contribution was insufficient to construct or develop mathematics. The teacher’s questions were directive in many cases, and it resulted in short sentences answers. Therefore, the segments scored “Low” for overall code.

Working with Students and Mathematics code shows how the teacher responds to students and how the teacher uses students’ contributions. Note that the Working with Students and Mathematics code does not show the depth of the mathematics that students face. The richness of Mathematics codes has criteria to measure the depth of the mathematics. Ali’s instruction scored “Mid” or “High” for some segments in Richness of Mathematics codes, although it was scored “Low” for Working with Students and Mathematics codes. Since the teacher developed the mathematics and the instruction was mathematically rich. The segment scores and evidence from instruction are given in Table 14.

4.4.3. Common Core Aligned Student Practices (CCASP)

Common Core Aligned Student Practice dimension includes five subdimensions which are Students Provide Explanations, Students Mathematical Questioning and Reasoning (SMQR), Students Communicate about the Mathematics of the Segment, Task Cognitive Demand, and Students Work with Contextualized Problems. CCASP focuses on evidence of student involvement in the tasks. It tries to capture the extent to which students engage in and work with the mathematics of the segment. Student explanations, student mathematical questions, students' reasoning, tasks, and problems that students work with are scored for the CCASP dimension. The tasks and the problems that the teacher selects to use during instruction are affected by teachers' knowledge of content and teaching and teacher knowledge of content and students. If the teacher knows what is easy and what is difficult for his students, he will select tasks that support students' learning.

All the dimensions of CCASP were scored. The Distribution of segment scores is given in the Figure 9.

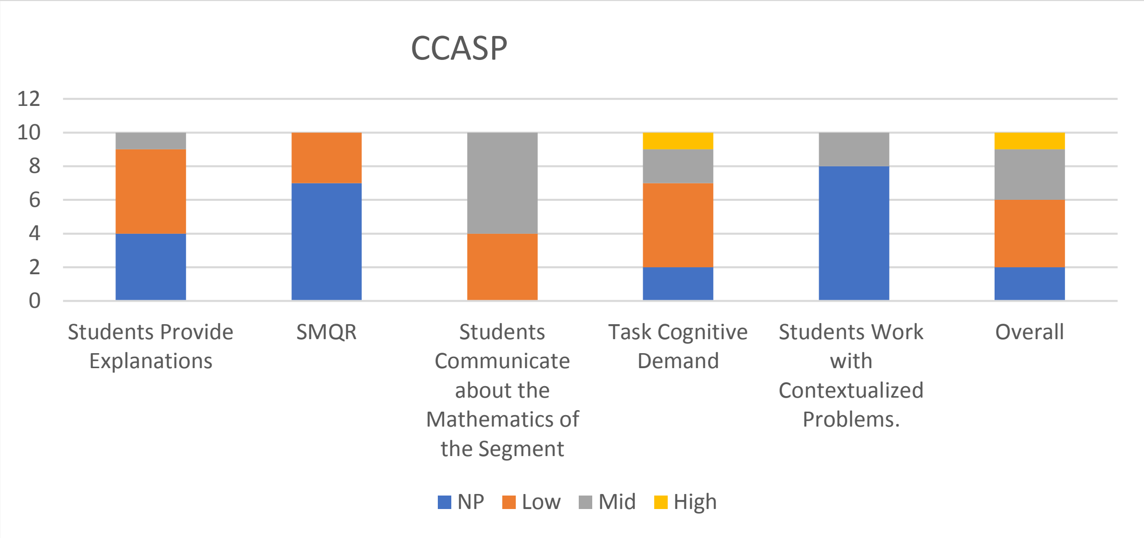


Figure 9. Distribution of Score of CCASP

Evidence and segment scores of the CCASP are presented in Table… and Table... For the Students Provide Explanations dimension four segments scored "NP". Only segment 1 scored "Mid" because the students' explanations were frequent. They explained why one circle was bigger than the other by justifying their answer. The other five segments scored "Low" because students provided brief explanations. For Example, in segment 7, the teacher asked what was the measurement unit of the area and why the measurement unit was a meter square. Some students said it was a fact. Some students said it was measured by a meter square. In segment 4 a student explained that the area of the rearranged shape could be found by multiplying base with height and dividing the result by 2. This student's explanation was wrong but it was scored for Student Provide Explanation dimension. In segment 8 a student explained his solution steps. He said he divided the total area first by pi, then by 2 because the area formula includes r^2. The student's explanation was wrong but it was scored. Since in this dimension, students' explanations were scored even if they were wrong.

The instruction continued with mutual dialogue between teachers and students. However, students' contributions were not substantial. Seven segments received an "NP" score for SMQR. In segment 7, a student asked a question about the value of pi. He said "Why are dividing the total area by 3? Why are we using 3? The teacher said they already learned the pi, they discussed it for a while a few weeks before. In segment 8, a student asked a question related to r^2. He asked, "Why 49 is equal to 7?" He faced difficulty in understanding the operation done with the r^2. The teacher explained the square of a number means to multiply that number by itself. In segment 9, a student explained that the tetragon had to be a square since the circle best fit in the tetragon and tangent to the tetragon on all four sides. All these three segments were scored "Low".

The dialogue between students and the teacher continued throughout the lesson. Therefore, at least some brief student contributions occurred in all segments. For Students Communicate about the Mathematics of the Segment dimension no segment was scored "NP". Four of the segments got a "Low" score since students contributed with one-or-two words or defined the solution steps partially. For example, in segment 4 the teacher asked, "What shape does this shape look like?". The students answered "Rectangle" "Rhombus" and "Trapezoid". In segment 5, a student repeated the circle's area formula to solve the question. The other five segments, segments 1, 2, 3, 7, 9, and 10, were scored "Mid". For example, in segment 3, a student defined the area. Another student commented on this definition and said it is the definition of volume, not the area. Some substantial student contributions occurred and some students solved the question publicly.

At the beginning of the lesson, the teacher presented a story problem and asked students how they decided which circle was bigger. So, segment 1 received a "Low" score for Task Cognitive Demand. Then the teacher wanted students to explain how they could find the area of a circle. It was a highly cognitively demanding task for seventh-grade students. So, segment 2 scored "High" for Task Cognitive Demand. However, the teacher scaffolded heavily and offered a solution method, and so segment 3 scored "Mid" instead of "High". The other segment that received a "Mid" score was segment 9 because students worked on a middling cognitive demanding task that asked the difference between the area of the square and the area of its inner circle. Its solution way was not obvious to the students. Segment 6 scored "NP" because in segment 6 students took notes and the teacher summarized what they covered so far. The teacher solved the question, and students engaged in no observable cognitively demanding process so segment 10 got an "NP" score. Students worked on simple questions that required the application of the area formula and asked students low cognitive demanding work. Therefore, segments 4, 5, 7, and 8 scored "Low".

The teacher started the instruction by introducing a story problem, and they worked on it through the first two segments. So first two segments scored "Mid" for the Student Work with Contextualized Problems dimension. In segment 3, the teacher drew a circle, rearranged segments, and talked about pure geometry. In segment 4, and segment 5 they developed the area formula of the circle, and they scored "NP". After they developed the area formula, they solved pure geometry tasks that required application of the area formula, and segments scored "NP" for Student Work with Contextualized Problems dimension.

Overall CCASP that scored students' involvement in doing mathematics was scored "NP" for segments 6 and 10. Segment 6 was dominated by the teacher talk who summarized what they did so far without any important student contributions. In segment 10, the teacher solved the question himself, and no evidence of CCASP occurred. Only segment 2 was scored "High", and segments 1, 3, and 9 were scored "Mid". The rest of the segments scored "Low" for overall CCAPS.

The scores of segments and evidence of CCASP dimensions were given in the Table 15 and Table 16.

Table 15. First observation of Ali: Score and evidence of CCASP-I

	Students Provide Explanations		Student Mathematical Questioning and Reasoning (SMQR)		Students Communicate about the Mathematics of the Segment	
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**
S1	Mid	Student explanations were frequent. S: In the big one, there is more space to hide. S: The space it occupies is bigger, and also, its radius is bigger.	NP	No student mathematical questioning and reasoning occurred.	Mid	Some brief and some substantial contributions occurred. T: Why its perimeter is bigger? S: The area it covers is larger. Also, its radius is bigger than the other circle.
S2	Low	Two brief student explanations were present. S: It is bigger since its perimeter is bigger. A student explained why the area of the square is bigger by drawing the tangent square of the circle.	NP	No student mathematical questioning and reasoning occurred	Mid	Students offered one one-or-two-word answers to the teacher's questions. T: How do we find the area? S: By counting the unit square. S: Base times height. Some substantial contributions occurred. For example, a student compared the area of a square and a circle.

Table 15. (continues)

S3	NP	No student explanation occurred.	NP	No student mathematical questioning and reasoning occurred	Mid	Students offered one-or-two-word answers to the teacher's questions. However, a student commented on another student's definition. T: Who will define the area? S: The place an object occupies in space. S: It's volume.
S4	Low	A student explanation was given below: S: The area of the rearranged shape can be calculated by multiplying height with the base and dividing it by 2. Since it includes triangles. The student's explanation was not true but it counts as the student's explanation.	NP	No student mathematical questioning and reasoning occurred	Low	Students' brief contributions were frequent. T: What shape does this shape look like? S: Rectangle, rhombus. S: It is height times the side.

Table 15. (continues)

S5	NP	No student explanation occurred.	NP	No student mathematical questioning and reasoning occurred	Low	Students’ brief contributions were frequent. S: 2 times pi times radius over 2. … S: r times pi.
S6	NP	No student explanation occurred.	NP	No student mathematical questioning and reasoning occurred	Low	A few brief student contributions occurred.
S7	Low	A few brief student explanations occurred about the measurement unit of the area.	Low	A student asked, “Why does the value of π equal to 3?”. The students learned how to calculate the perimeter of a circle and a sector, and they already knew π.	Mid	Students’ brief contributions were frequent. Students presented their solutions to the board. Two students corrected another student’s solution. S: It is diameter. You should multiply by the radius. S: If you use the radius, the result will be 48.

Table 15. (continues)

S8	Low	A student explained his solution method. He divided the total area by 2 because the area formula includes r^2. This explanation was wrong but it was scored as a student explanation.	Low	A student asked, “Why does 49 equal 7?”	Low	Students’ brief contributions were frequent. S: It is not an integer. 49 over 2.. T: Why do you divide it by 2? S: It's square…
S9	Low	A student explained that the tetragon had to be a square if it was tangent to the circle.	Low	A student explained that the tetragon should be square because the circle exactly fits in it.	Mid	Students’ brief contributions were frequent. A student explained that the tetragon should be square because the circle exactly fits in it.
S10	NP	No student explanation occurred.	NP	No student mathematical questioning and reasoning occurred	Mid	Students’ brief contributions were frequent. A student shared the solution method publicly.

Table 16. First observation of Ali: Score and evidence of CCASP-II

	Task Cognitive Demand		**Student Work with Contextualized Problems**		**Overall**
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**
S1	Low	Direct instruction was conducted with some student explanations. T: If its diameter is bigger, what else is bigger? S: The surface it covers. S: Its perimeter.	Mid	The teacher presented a story problem and they discussed a solution to the problem.	Mid
S2	High	The teacher asked how to calculate the area of the circle which was a cognitively demanding task.	Mid	They continued to work on the story problem presented in the previous segment	High
S3	Mid	The students continued to work on the task of calculating the area of the circle. However, the teacher scaffolded the students by dividing the circle into sectors.	NP	The students worked on pure geometry tasks.	Mid

Table 16. (continues)

S4	Low	Direct instruction was conducted with some SMQRs.	NP	The students worked on pure geometry tasks.	Low
S5	Low	Direct instruction was conducted. The students recalled the perimeter formula of the circle. The students were heavily scaffolded by the teacher. T: To what does this length equal? S: Half of the perimeter. T: What is the length? S: 5.	NP	The students worked on pure geometry tasks.	Low
S6	NP	The students took notes. Then the teacher summarized what they learned by asking questions to the students.	NP	The students took notes. Then the teacher summarized what they learned by asking questions to the students.	NP
S7	Low	The students worked on low cognitive demanding tasks which required the application of the area formula of the circle.	NP	The students worked on pure geometry tasks.	Low

Table 16. (continues)

S8	Low	The students worked on low cognitive demanding tasks which required the application of the area formula of the circle.	NP	The students worked on pure geometry tasks.	Low
S9	Mid	The students worked on a task that required a middling cognitive demand. It was asking about the difference between the area of the square and the area of its inner circle. Its solution was not obvious.	NP	The students worked on pure geometry tasks.	Mid
S10	NP	The teacher solved the question given in the previous segment.	NP	The students worked on pure geometry tasks.	NP

4.5. Second Observation of Ali: Area of Sector

The second observation of Ali's lesson took place while he was teaching the area of sectors, and it took place in the seventh-grade classroom. The lesson was videotaped and audiotaped, and the researcher was in the classroom as a passive observer. The observation process lasted for two consecutive lesson hours because covering the objective of the area of the sector lasted two hours. His 7th-grade class was composed of boys since it was an imam-hatip middle school. 26 students registered in this classroom. There were only 11 students on the day of observation. The observation took place in June, 2 weeks before the summer holiday began. The teacher said the students at the school had attendance problems and in June only a few students came to school although the classes were going on.

4.5.1. Summary of the Lesson

The first minutes were spent greeting the students. Then Ali said they were going to solve some questions about the area of the circle before moving on to a new topic. He drew a rectangle and a half circle in it. He asked the students to find the difference between the area of the rectangle and half circle. The students solved the question individually and Ali checked their answers. He gave feedback on the students' solutions true or false and said where they made an error. Then he started to solve the question on the board interactively with the students. He opened a new question, the square and a quarter circle inside the square, asking the difference between the area of the circle and the square. Firstly, some students said there was an error and that the result was meaningless.

> *Hasan:* There is an error.
> *Yusuf:* Yes. There is an error.
> *Ali:* What is the error?
> *Hasan:* We multiply 36 with 3, then we divide it by 2.
> *Yusuf:* Oh no. we should divide it by 4.

While discussing the solution, Yusuf realized their mistake that the given shape was a quarter circle, not a half circle. They needed to divide the total area of the circle by

4 to find the area of the quarter circle. Ali waited for the students to solve the question individually and checked their solutions. If the students found the wrong answer, Ali warned them about the operation steps they did error. Then he started to solve questions asking the students to describe what they should do to find the difference between the area of the two shapes. However, Cem asked, “How do we **k**now it is a quarter circle?”. The teacher said it was given in the text of the question. Then he drew a square and a circle tangent to the sides of the square. He divided the square into four identical squares and said the shape given in the question was one of these four parts. Ali solved the question on the board. Another student asked, “How do we understand quarter means divide it by 4?”. The teacher explained “The quarter means one-fourth. You divide it by 2 to find the half. You divide it by 4 to find a quarter.” One of the students asked where he made a mistake but the teacher did not hear him.

They moved to a new question. It was a rectangle including 8 identical circles. The difference between the area of the rectangle and the total area of the circle was asked. The teacher let the students solve the question individually. He checked their answers and gave feedback on their solutions. Then, Ali asked which steps they should take to solve the question. Three of the students said they needed to find the circumference. One of the students suggested multiplying the circumference and the area. The students’ explanations showed that they were still confused about the circumference and area content and their formulas. Ali allowed Yiğit to speak and Yiğit suggested a correct solution method. The teacher solved the question and directed a real-life problem. Ali wanted the students to use pi equal to 3,14. Some students said they did not know how to do multiplication with decimals. Ali reminded them they learned multiplication with decimals in fifth grade. A student claimed that they needed to find the diameter to find the area. The teacher asked the students the area formula of the circle and recalled that r^2 represents the square of r, not the diameter.

Ali drew two identical circles with a 48 square centimeter area. He divided the first one into one-half and two-quarter sectors. He divided the second one into three different sectors. He said they were the wheels of the fortune and they were going to paint each sector a different color. He asked the students painting which sectors would cost the most. The students had different ideas about the teacher’s question.

Ali: Which part cost the most?

Han: The green one.

Ali: Why?

Han: Its area is bigger.

Mert: Its perimeter is longer.

Ali: Han, how do you know its area is bigger?

Han: It seems so.

Ali: Why is the orange one bigger than the blue one?

Yusuf: The orange one looks like bigger than a half.

Yusuf showed that the orange one was bigger than half by drawing diameter. Ali started a discussion using Yusuf's idea and they checked all sectors against half and quarter. Lastly, they arranged the area of the sectors from the biggest to the smallest. The first hour of the lesson finished.

At the beginning of the second hour, Ali asked the students to calculate the area of sectors. Some students said they did not know. Yusuf wanted to learn the measurement of sectors’ angles. Cem said the area of the blue sector was equal to 24 square centimeters. The blue sector was the half of the circle, to find the area he said he divided the total area by 2. Then, they calculate the area of the quarter sectors. However, when they tried to calculate the area of the sectors that were not equal to half or a quarter, they faced difficulty. Some students said the orange sector’s area was equal to 30 square centimeters. They claimed that the orange sector was equal to a half and a half of a quarter. The teacher asked more questions about how they knew, the students faced the problem that they did not know the exact measurement of sectors. Some students said they needed angles to calculate the exact value of the area.

Ali: If we know the measurement of the angles, can we calculate the area?

Students: Yes.

Ali: How?

Students: Proportion.

Mert: If 360 degrees is equal to something, 200 is equal to something?

Ali confirmed the students' idea and wrote down the proportion "The area of a circle with 360 degrees angles is equal to 48 square centimeters. What is the measurement of the area of sector with 100 degrees?". The teacher asked a new question about finding the area of a sector and permitted the students to solve the question personally. He checked students' solutions and gave personal feedback. A student calculated perimeter instead of area. Confusing area and perimeter was a problem while teaching the area of the circle. Ali just reminded the student of the question asking for the area of the sector. Ali called Yusuf to solve the question on the board. A student asked if it was an inverse proportion. Ali only asked, "Why it should be an inverse proportion?". Other students said it was a direct proportion. The teacher made no more explanation. Ali resolved the question using the area formula of the sector, $A = \pi . r^2 . \frac{\alpha}{360°}$. They solved another question asking for the area of a sector.

In the next question, there were two circles concentric circles. The difference between the area of the sector of the small circle and the big circle, the central angle of both sectors was 75°, was asked. Students had difficulty solving this question. Only two students, Hasan and Yusuf, were able to solve the question. Ali asked Hasan and Yusuf which steps should be followed for a solution, respectively, and explained the solution methods of both two students to the rest of the students. Two students said they did not understand multiplication operations and simplification. The teacher explained the multiplication again. Ali posed a new question, and the students solved it individually. Two students solved the question correctly. The second hour had ended, and Ali only verbally explained the solution to the question.

4.5.2. Finding Related to MQI

In this section, Ali's instruction while teaching the area of sectors was evaluated using the MQI 4-Point framework. Each 7-minute segment was evaluated separately. Segment scores and quality evidence of the segment are given in the tables in detail.

4.5.2.1. Richness of Mathematics

Richness evidence of the instruction is presented in two separate tables. In the first table segment codes and evidence of meaning-oriented codes are given. Meaning-oriented codes are Linking Between Representations, Explanations, and Mathematical Sense-Making. In the second table segment score, evidence of practice-oriented codes, and the overall richness scores of the segments are presented. Practice-oriented codes are Multiple Procedures or Solution Methods, Patterns and Generalizations, and Mathematical Language. The distribution of segment scores and subdimension of Richness of Mathematics is given in Figure 10.

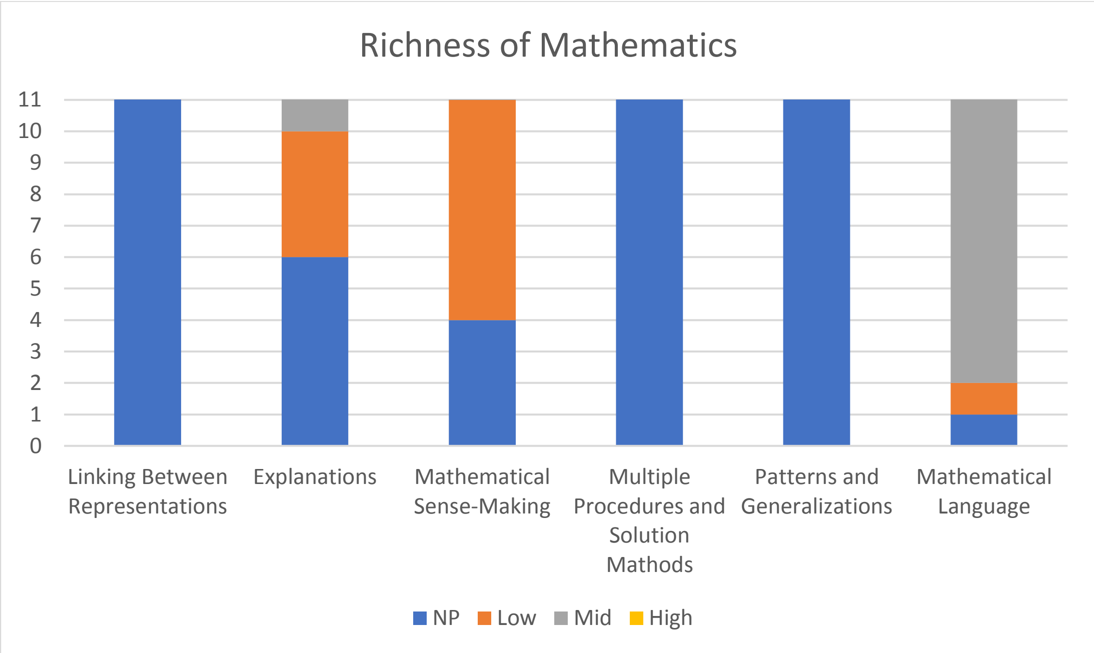


Figure 10. Distribution of Richness of Mathematics score

The instruction aimed to teach how to calculate the area of the sector. Since it was a geometry lesson, the teacher used pictures of geometric shapes and symbolic representations. While coding MQI, for geometry the shapes are not counted as representation. Therefore, eight segments scored as “NP” for the Linking Between Representation codes. In segment 3, which scored “Mid”, the teacher used the circle as an area model of fractions and linked this model with oral and symbolic representations of fractions. The teacher explained the quarter meant one-fourth of a whole. One-fourth of a circle had a 90-degree central angle, and the central angle was one-fourth of a 360-degree. Another segment that got a code different than "NP" was segment 6. Segment 6 scored a "High" score. The teacher showed the link between the area of the sectors and the symbolic representation of equality and inequality. They sequenced the area of the sector from the smallest to the biggest by comparing each sector with half and quarter.

The instruction of Ali generally scored different than "NP" for Explanation. Students asked "Why" and the teacher had to explain. Only segment 4 and segment 10, where the teacher and the students solved questions just by describing solution steps, received "NP". Segments 1, 2, 3, and 5 scored "Mid", while segments 6 and 7 scored "High". In segment 1, a student's explanation occurred. A student explained to another student why his solution was true. This explanation took place between a few students and was not discussed as a whole group. Some students faced difficulty in understanding why a sector with a 90-degree central angle one-fourth of a circle was. In segments 2 and 3, the teacher explained why the given sector was one-fourth of the whole, which means the sector area was a quarter of a whole circle. In segment 5, the teacher explained why the solution method of a student was incorrect although he found the answer correct. The other explanation of the students occurred in segment 6. The students explained his way of comprising the area of sectors. He explained why the orange sector had the biggest area by comparing the orange sector with half. His explanation was clear and detailed and scored as "High." In the next segment, they decided that the central angle of the sector needed to calculate the area of the sector. Segments 6 and 7 received "High" scores because the focus of the instruction was the comparison of the area of the sectors and the way of the comparison of the area of sectors. In segments 8 and 9, they solved questions and brief explanations related to solutions to the questions that occurred and got a "Low" Explanation score. In segment 10, the teacher described the solution steps of the question. Explanation about "how" is not scored as the Explanation.

In many cases, Explanation and Mathematical Sense-Making overlap. Therefore, the instruction scored differently than "NP" for many segments of the Mathematical Sense-Making dimension. Only segments 4, and 10 received "NP" for Mathematical

Sense-Making. In segment 1, the teachers explained the solution to a question. To explain the solution of the question, the teacher showed the equality of the radius of the circle and the sides of the rectangle. In segments 2 and 3, the teacher tried to make sense of the quarter. He explained why a given circle is a quarter and one-fourth of a whole circle. The segments scored “Low” while they were scored “Mid” for Explanations because not all the explanations qualified as Mathematical Sense-Making. S5 scored “Mid” for Explanations while it scored “High” for Mathematical Sense-Making. The teacher and students discussed why a solution method of a student was incorrect although he found the correct answer. The teacher's explanation about the student’s solution was detailed but it was not the focus of the instruction. Therefore, it was scored “Mid” for the Explanation. Segments 6 and 7 focused on the quantities and compared the area of sectors using half and quarter relations, and they were scored as “High”. In segments 8 and 9, the teacher made explanations related to the understanding of relationships between numbers. They used the relationship of the numbers to solve the questions. The focus on the meaning of numbers was brief and scored “Low”. The student's and the teacher’s explanation of “how” his solution was proceeded was not an example of the Mathematical Sense-Making.

In the first hour of the instruction, they solved questions about finding the area of the circle and they only applied the circle’s area formula for solutions. Therefore, the first five segments received “NP” for Multiple Procedures or Solution Methods codes. Segments 6, 8, 9, and 10 received “Mid” because they used more than one method to solve the questions. In segment 6, they compared the area of sectors using more than one method. In segments 8 and 9, they used both area formula and direct proportion to solve the questions. In the tenth segment, two students offered two different solution methods. The solution methods were not compared, and no explicit connection between the methods was made. That is why these segments could not reach “High” scores.

The class developed generalization only in segment 7 and this segment received a “Mid” score. Students developed that they needed the central angles of sectors to

Table 17. Second observation of Ali: Scores and evidence of Richness of Mathematics-I

	Linking Between Representations		Explanation		Mathematical Sense-Making	
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**
S1	NP	The shapes do not count as a representation in geometry.	Mid	A student explained to another student why his solution method was true. The teacher explained the height of the rectangle was equal to the radius of the circle and that the base of the rectangle was equal to the diameter of the circle.	Low	The teacher showed the connection between the radius of the circle and the sides of the rectangle.
S2	NP	The shapes do not count as a representation in geometry.	Mid	The teacher explained his solution. After finding the area of the circle, he divided it by 2 because the shape included a half circle. The teacher briefly explained why the sector inside the square is the quarter of a circle.	Low	A short discussion occurred about why the given sector was the quarter of a circle.

Table 17. (continues)

S3	Mid	The area model of the one-fourth was visually presented, and the teacher linked the area model and symbolic representation of one-fourth.	Mid	The teacher explained why the given sector was one-fourth of a circle.	Mid	The teacher explained why the quarter means one-fourth.
S4	NP	The shapes do not count as a representation in geometry.	NP	The teacher described the solution steps, and it does not count as an explanation.	NP	Nothing related to sense-making happened.
S5	NP	A real-life problem was solved but no link occurred.	Mid	The teacher explained why the solution of the student was incorrect, although the achieved answer was correct.	High	The teacher explained why the solution of the student was incorrect by explaining what 4 means. The student said it was the diameter. The teacher showed using diameter to find the area of the circle was inappropriate for other circles whose diameter and square of the radius had different numbers.

Table 17. (continues)

S6	High	An explicit link between the area model of the sectors and symbolic representation of equality and inequalities occurred.	High	The focus of the segments was comparison of sectors' area and a student explained why the orange sector had the biggest area by comparing it with half.	High	The focus of the segment was the relationship between the area of the sectors.
S7	NP	The shapes do not count as a representation in geometry.	High	Explanations about why the central angles of sectors were needed to find the area of sectors.	High	They discussed why they need angles to calculate the sectors' area and the reasonableness of using angles to find the area.
S8	NP	The shapes do not count as a representation in geometry.	Low	A student briefly explained why they used direct proportion to calculate the area of sectors	Low	The teacher explained the relationship between numbers, 108 is 3 times 36.
S9	NP	The shapes do not count as a representation in geometry.	Low	The teacher warned a student because his solution was not true, but it occurred in an isolated instance.	Low	The teacher explained the relationship between numbers (45 and 360, and sector's area and circle's area, 48).
S10	NP	The shapes do not count as a representation in geometry.	NP	The teacher described the solution steps, and it does not count as an explanation.	NP	Nothing related to sense-making happened.

Table 18. Second observation of Ali: Scores and evidence of Richness of Mathematics-II

	Multiple Procedure or Solution Methods		Patterns and Generalization		Mathematical Language		Overall
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**	
S1	NP	There was no evidence of multiple procedures and solution methods.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	High	The teacher pressed the students to use a circle instead of a round shape and to use a rectangle while students were calling it a square.	Mid
S2	NP	The questions were solved using only one method.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	High	The mathematical language was used in the moderate density and the teacher pressed a student to use the word circle instead of round shape.	Low

Table 18. (continues)

S3	NP	The questions were solved using only one method.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	High	The mathematical language was used in the moderate density. The teacher explained the quarter means.	Mid
S4	NP	The question was solved using only one method.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The mathematical language was used in the moderate density.	Low
S5	NP	A student solved the question using a different way, but it was incorrect.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The mathematical language was used in the moderate density.	Mid
S6	Mid	They compared the areas of the sectors by referencing half and quarter. They calculated the areas of the sectors using half and quarter relations and then compared them.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	Middling uses mathematical terms such as area, perimeter, radius, half, quarter, and arc.	High

Table 18. (continues)

S7	NP	They calculated the area of sectors using proportion.	Mid	They developed that they need central angles of the sectors to find the area of the sectors. They also generalized that they could use proportion to find the areas of sectors.	Low	Middling use of mathematical terms such as area, angle, radius, and proportion but sloppy use of central angle. The teacher used the word angle instead of the word central angle.	High
S8	Mid	They calculated the area of a sector by using direct proportion and the area formula of the sector. The teacher made a comparison between two methods but it was not detailed and substantial.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	Middling uses mathematical terms such as area, angle, radius, and proportion.	Mid
S9	Mid	A student found the sector’s area using direct proportion. The teacher offered to use relationship between numbers (45 is one-eight of the 360, and the area must be equal to one-eight of 48).	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The mathematical language was used in the moderate density.	Mid

Table 18. (continues)

S10	Mid	Two different students offered two different methods.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The mathematical language was used in the moderate density.	Low

find the area of the sectors. However, the generalization was not detailed. The other segments scored “NP”.

The teacher used mathematical language carefully and generally corrected students’ misusing. Therefore, Mathematical Language codes of the segments “Mid” or “High”. For example, some students used the word round shape instead of the circle. The teacher corrected these students and pressed them to use the word circle.

The depth of the mathematics offered to students was “Low” or “Mid” when they were solving questions, and “Mid” or “High” when the teacher introduced the content. For example, the first five segments received “Low” or “Mid” scores when the class worked on solving questions about the area of the circle. They solved exercise questions that required one or two steps to solve. Segments 6 and 7 scored as “High” when the class discovered how to find the area of a sector. The teacher made more explanations and tried to make sense of why the central angle was needed to find the area of sectors.

The evidence and segment scores were given in the Table 17 and Table 18.

4.5.2.2. Working with Students and Mathematics

In this section, the score and evidence of the Working with Students and Mathematics are given. The Working with Students and Mathematics dimension is highly related to teachers' knowledge of content and students. If the teacher knows possible students’ errors and difficulties related to the content that is taught, he can plan pre-remediation activities. The teacher also can hear, understand, and use students’ contributions to develop mathematics. The segments coded “NP”, “Low” od “Mid” both Remediation of Students Errors and Difficulties. Segments received “Low”, “Mid” and “High” for Teacher Uses Students Mathematical Contribution. The distribution of segment scores and subdimension of Working with Students and Mathematics is given in Figure 11.

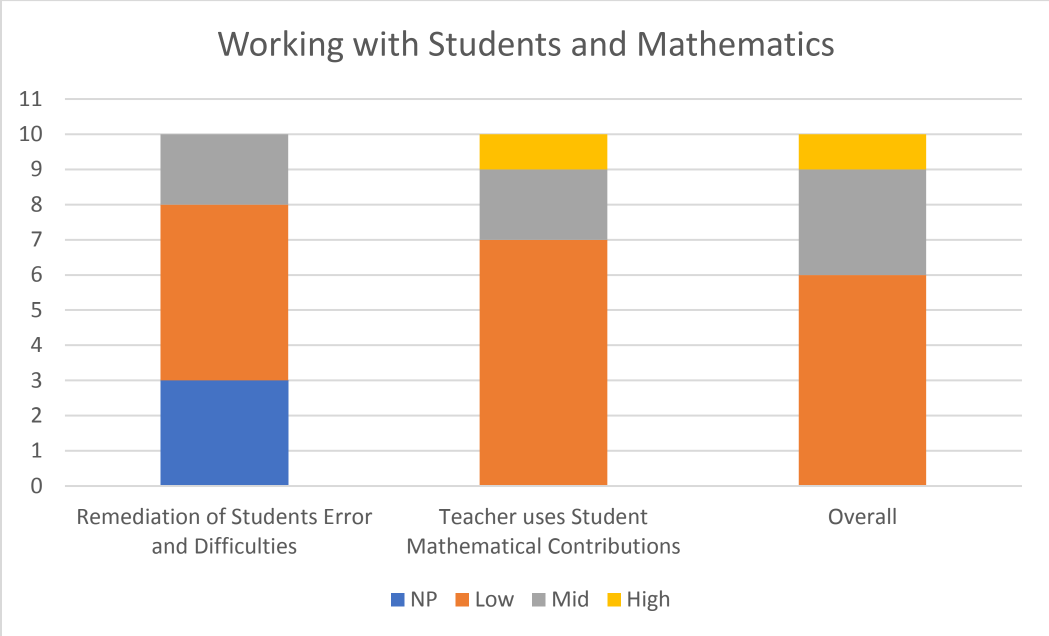


Figure 11. Distribution of Score of Woking with Students and Mathematics

Evidence of Working with Students and Mathematics and the score of the segments is given in the Table…. The segments coded “NP”, “Low” or “Mid” for Remediation of Student Errors and Difficulties. In segment 4, more than one student's difficulties were observed. In segment 4, a student explanation about the solution was wrong. The students claimed they need to multiply the rectangle’s area with the rectangle’s perimeter to find the difference between areas of rectangle and inner circles. The teacher did not ask further question to understand source of student error. Ali only said it was not a correct way to solve the question. Two students used the perimeter formula and calculated the perimeter while it was asking for the area. The teacher said nothing related to this situation. Another student said he did not learn how to multiply with decimals. However, the teacher replied they covered multiplication with decimals in fifth grade and no remediation occurred. Although a student error occurred, no teacher remediation observed, and the segment got a “NP” score. Only in two segments, segments 6 and 10, no student errors, misunderstandings, or difficulties were observed. In the segments which were scored “Low”, only procedural remediations were observed. Only in segment 2 and 3 extended procedural remediation observed and they received a “Mid” score. The teacher explained the relation between the quarter and the 90-degree sector using the area model in segment 2. However, in segment 3 another student asked how the teacher knew the quarter was one-fourth. The teacher talked about how to find half and a quarter of bread.

During the instruction, the teacher always interacted with students. However, most of the students' contributions were not substantial or in a pro forma way. For the Teacher uses Student Mathematical Contributions code, the segments received "Low", "Mid" or "High" scores. In the segments that were scored "Low," the teacher asked the students solution steps of the questions. In segment 6, the teacher used and developed a student's idea to compare areas of sectors, and it received a "Mid" score. A student compared the area of a sector with a half circle and said "This sector is the biggest one because it is greater than half. When I draw the diameter of the circle, I can find the half. This sector has more region than a half". The teacher used this idea to compare areas of other sectors. In segment 7, the teacher led a whole group discussion, and it was also scored "Mid". The only segment scored as "High" was segment 10. The teacher identified two students with their solution methods, explained them to the class, and compared them.

The instruction proceeded with dialogue between the teacher and the students. However, in most parts of the instruction, the teacher developed the mathematics and the students' contributions were in a pro forma way. No conceptual remediation was observed. The teacher ignored some student difficulties or procedural remediation was conducted. The teacher and the students' interactions were generally on the solution steps, and it was not devoted to the development of mathematics. The segments got "Low", "Mid" or "High" scores for overall Working with Students and Mathematics.

The segments scored mostly "Mid" for the Richness of Mathematics code. The mathematics that the students engaged in was rich. However, it was the teacher who pushed the mathematics. The student's contributions were generally insubstantial and the teacher's remediations were procedural. So, the segments scored "Low" for overall Working with Students and Mathematics.

The evidence and segments scores were presented in Table 19.

Table 19. Second observation of Ali: Scores and evidence of Working with Students and Mathematics

Segment	**Remediation of Student Error and Difficulties**		**Teacher Uses Students Mathematical Contribution**		**Overall**
	Scores	**Evidence**	**Scores**	**Evidence**	**Scores**
S1	Low	The teacher gave personal feedback to the students' solutions "I ask for half of the area". A student remediated another student's solution procedure.	Low	The teacher asked students to describe the solution steps. T: How can we find the area of this? S: First, we find the area of the circle. … S: We find the area of the rectangle. then, we find the area of the other shape. We subtract.	Low
S2	Mid	Some procedural remediations occurred. T: Your subtraction is wrong. A student had difficulty understanding the quarter and the sector with 90-degree central angles. The teacher explained it by drawing a whole circle.	Low	The teacher asked students to describe the solution steps. T: Which shapes do we have here? S: There is a square. T: What else? S: It looks like a triangle.	Low

Table 19. (continues)

S3	Mid	One of the students asked how the teacher knew the quarter was one-fourth. The teacher explained the quarter by using half and one-fourth of the bread as an example. The teacher gave students personal feedback about their solution “You need to find the area.”	Low	The teacher asked students to describe the solution steps. T: What we are going to find the first? S: The area of the rectangle. T: We will find the area of the rectangle first. What are we going to do next? ..	Mid
S4	NP	A student claimed they needed to multiply the area of the rectangle and the perimeter of the rectangle to find the difference between the area of the rectangle and the total area of the 8 circles. The teacher did not ask any further questions to understand the source of the student’s error. Another student confused the area and the perimeter concepts and calculated the perimeter instead of the area. The teacher did not remediate. A student had difficulty multiplying with decimals. The teacher said they covered multiplying with decimals in fifth grade.	Low	The teacher asked students to describe the solution steps. T: Yiğit, tell the solution steps. S: We find the area of the rectangle. we find the area of one circle and multiply it by 8. We subtract the Rectangle’s area from the total area of the circles. ….. T: … How can we find the area of the rectangle? S: Base time height.	Low

Table 19. (continues)

S5	Low	A student used the diameter instead of the square of the radius to find the area. The teacher reminded the area formula and said the area formula did not include the diameter. He said they did not need a diameter and had to find the square of the radius. A student divided the area by 5 instead of multiplying it by 5. The teacher pointed out he should multiply it.	Low	The teacher asked students to describe the solution steps. T: I am asking about the procedure that you conduct. How did you find the 4? S. It is the diameter. T: If it was the 3, would you multiply it by 6? What is the area formula?	Low
S6	NP	No misunderstanding or difficulties occurred.	Mid	A student explained why the orange sector had the biggest area by comparing it with half. The teacher used the student's explanations to develop the math and compare the area of the other sectors.	Mid
S7	Low	A student calculated the area of the purple sector by assuming it was a quarter. The teacher showed that to be a quarter, it should have a right central angle.	Mid	The teacher orchestrated a whole group discussion about what was necessary to calculate the area of a sector. They concluded that they needed a central angle and a radius. The discussion was not sustained systematically. That is why the segment gets Mid instead of High.	Mid

Table 19. (continues)

S8	Low	A student asked if they needed to use inverse proportion. The teacher asked why. Another student answered that it was a direct proportion because if the central angle decreased, the area of the sector decreased too.	Low	The teacher communicated with students while solving the question.	Low
S9	Low	The teacher helped a student identify his error. T: If the 360-degree circle's area is 144, the 75-degree sector's area should be smaller, isn't it?	Low	The teacher talked with students about the solution steps of the question.	Low
S10	NP	No misunderstanding or difficulties occurred.	High	The teacher identified two students with their solution methods (Yusuf and Hasan). Then the teacher solved the question using both methods and compared them.	High

4.4.2.3.Common Core Aligned Student Practices (CCASP)

Common Core Aligned Student Practice dimension includes five subdimensions which are Students Provide Explanations, Students Mathematical Questioning and Reasoning (SMQR), Students Communicate about the Mathematics of the Segment, Task Cognitive Demand, and Students Work with Contextualized Problems. CCASP focuses on evidence of student involvement in the tasks. It tries to capture the extent to which students engage in and work with the mathematics of the segment. Student explanations, student mathematical questions, students' reasoning, tasks, and problems that students work with are scored for the CCASP dimension. The tasks and the problems that the teacher selects to use during instruction are affected by teachers' knowledge of content and teaching and teacher knowledge of content and students. If the teacher knows what is easy and what is difficult for his students, he would select tasks that support students' learning.

All the dimensions of CCASP were scored. The Distribution of segment scores is given in Figure 12.

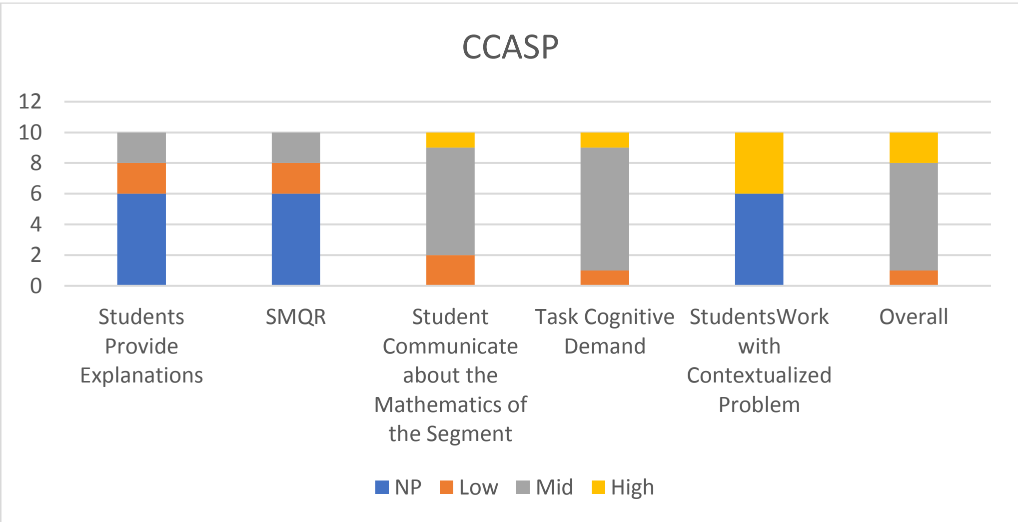


Figure 12. Distribution of score of CCASP

Evidence and segment scores of the CCASP are presented in the Table and Table. For the Students Provide Explanations dimension six segments scored "NP". A student claimed the question was wrong because its answer was meaningless. Therefore, segment 1 scored “Mid” although the student’s claim was wrong. The other segment that received a “Mid” score was segment 6 since frequent brief student explanations occurred. Segments 3 and 8 scored “Low”. During segment 3, a student explained his solution method. However, his solution was wrong. For Students Provide Explanations, wrong student explanations also counted and scored “Low”. In segment 8 a student explained the direct proportion between the sector’s central angle and its area.

The instruction continued with mutual dialogue between teachers and students. However, students’ contributions were not generally substantial, that is the students did not engage in mathematical thinking. Six segments received an “NP” score for SMQR. Segment 1 and segment 6 were scored “Mid”. In the first segment, a student explained his solution method and commented on his friend’s solution pointing out his mistake. Segment 6 scored “Mid” because a student explained why the orange area was bigger than the blue area by comparing the region with half. Segments 2 and 3 were scored “Low” since students asked questions about the quarter and one-fourth.

The teacher engaged students in the lesson by asking questions. The instruction went on the teacher’s questions and students’ answers. So, For Students Communicate about the Mathematics of the Segment dimension no segment was scored “NP”. Only segments 2 and 3 received a “Low” square because students' contributions were brief. Two students presented their solution on the board and segment 10 scored “High”. The other seven segments scored “Mid” because some brief student contributions, students’ explanations, or students’ share of solution methods were observed. For example, in segment 4, in addition to some brief student contributions, a student summarized the complete steps of the solution. In segment 5, a student solved the question on the board and some students commented on his solution.

The teacher stated the instruction by presenting the area of the circle problems. Students worked on these questions in three segments. These questions' solutions were not obvious and scored “Mid” for the Task Cognitive Demand dimension. The teacher asked a story problem that required a middling cognitive demand and students worked on this problem in segment 4 and segment 5. Then the students moved to the new content and asked students to justify the area of which sector was bigger. The students justified their answers by referring to the previous learning. After working on a middling cognitively demanding task, the teacher asked students to establish what was necessary to calculate the area of the sector. It was a highly cognitively demanding task, and was cored “High”. In segment 8, students worked on a low cognitive demanding pure geometry task. Segments 9 and 10 were scored “Mid” because the students worked on a middling cognitively demanding task.

At the beginning of the lesson, they solved questions related to the area of the circle and these questions were pure geometry questions. So, the Student Work with Contextualized Problems dimension was scored “NP” for the first three segments. then, the teachers presented a story problem about the area of the circle. Students continued to work on this problem in segment 5 and both segments were coded “Mid”. The teacher moved on to new content that was the area of the sectors and presented a story problem. They worked on it for two segments, and segment 6 and segment 7 received a “High “score. After he introduced the area formula, the teacher solved pure geometry problems and the segments scored “NP”.

Overall CCASP that scored students’ involvement in doing mathematics was scored “Low” in segment 2. In segment six, they started to discuss how to calculate the area of the sectors. Students contributed to the lesson and provided explanations and justifications. So, segment 6 and segment 7 scored “High”. The other segments scored “Mid” because students’ contributions were not substantial or sustained in these segments.

The evidence and segments scores were presented in Table 20 and Table 21.

Table 20. Second observation of Ali: Scores and evidence of CCASP-I

	Students Provide Explanations		Student Mathematical Questioning and Reasoning (SMQR)		Students Communicate about the Mathematics of the Segment	
Segment	**Scores**	**Evidence**	**Scores**	**Evidence**	**Scores**	**Evidence**
S1	Mid	A student claimed there was a logical mistake in the question because the area of the circle was bigger than the area of the square. His claim was wrong. A student explained the length was 4 cm because it was a radius.	Mid	A student explained his solution method to his friend and helped him to understand his mistake. A student explained the length was 4 cm because it was a radius.	Mid	Some brief contributions were present. For example: T:…. The area of a rectangle S: a times b. the area of a square is a times a. A student explained his solution method to a classmate and told him what his mistake was (Divide it by 2 since it is asking half area.).
S2	NP	No student explanations were observed.	Low	A student asked "How do you know a quarter of the round shape is there?	Low	Some brief contributions were present. For example: T: Which shapes are there? S: Square T: What else? S: It seems like a triangle.

Table 20. (continues)

S3	Low	A student explained his solution method. His solution method was wrong but it still counted as student explanation. S: We should find the area of the rectangle first. Then divide the area by 2. T: Why do you divide it by 2? S: To find the perimeter.	Low	A student asked “How do you understand a quarter is 4?	Low	Some brief contributions were present. For example: T: …What we are going to find first? S: The area of the rectangle. T: Good. What we will do next? S: Divide it by 2.
S4	NP	No student explanations were observed.	NP	No student mathematical questioning or reasoning was presented.	Mid	A student narrated the complete steps of the solution. S: We calculate the area of the rectangle. We calculate the area of a circle and then multiply it by 8. We subtract the total area of 8 circles from the area of the rectangle. Some brief contributions were present.

Table 20. (continues)

S5	NP	No student explanations were observed.	NP	No student mathematical questioning or reasoning was presented.	Mid	A student presented his solution to the board. Some students comment on his solution. S: He multiplied by 5, by money, instead of multiplying 4 by pi.
S6	Mid	Student explanations were frequent. Some examples of the explanations are given below. S: It covers more space. S: Its perimeter is bigger. S: It is bigger than half (He showed why his claim was true on the board). S: I divided the total area by 2 since the blue region equals half.	Mid	A student explained why the orange area was bigger than the blue area by comparing the region with half. The student made a connection between the area model of the half and the sectors of a circle.	Mid	A student explained why the orange area was bigger than the blue area by comparing the region with half. Some brief contributions were present. T: If the total area of the circle is 48, what is the area of the blue sector? S: 24. I divided 48 by 2.

Table 20. (continues)

S7	NP	No student explanations were observed.	NP	No student mathematical questioning or reasoning was presented.	Mid	Some brief contributions were present. T: What do we need to find the area? S: The angle. ,,, T: I we know the measure of the angle, can we find? S: Yes T: How? S: Proportion. A student described the steps of proportion to solve the question.
S8	Low	A student explained "The area and the central angle of the sector are directly proportional because if the central angle decreases, the area decreases too. "	NP	No student mathematical questioning or reasoning was presented.	Mid	A student solved the question publicly and narrated his solution steps. Some brief contributions were present. S: 3 simplified with 3. S: Dividing 108 by 36 is easier.

Table 20. (continues)

S9	NP	No student explanations were observed.	NP	No student mathematical questioning or reasoning was presented.	Mid	A student solved the question publicly and narrated his solution steps. S: If the 360-degree circle's area equals 48, what is the area of the 45-degree sector? I used the proportion. 360 x equals 45 times 48. Another student talked about his solution but he did not describe his solution in detail. Some brief contributions were present.
S10	NP	No student explanations were observed.	NP	No student mathematical questioning or reasoning was presented.	High	Two students solved a question in different ways and they presented their solutions publicly and narrated their solution steps. They compare their solutions briefly. . Some brief contributions were present.

Table 21. Second observation of Ali: Scores and evidence of CCASP-II

	Task Cognitive Demand		Student Work with Contextualized Problems		Overall
Segment	**Scores**	**Evidence**	**Scores**	**Evidence**	**Scores**
S1	Mid	The students worked on a question asking the difference between the area of a rectangle and the area of a half circle that is inside the rectangle. It was a middling cognitively demanding task. Its solution was not obvious.	NP	The students worked on pure geometry questions.	Mid
S2	Mid	The students worked on a question asking the difference between the area of a square and the area of a quarter circle that is inside the square. It was a middling cognitively demanding task. Its solution was not obvious.	NP	The students worked on pure geometry questions.	Low

Table 21. (continues)

S3	Mid	The students worked on a question asking the difference between the area of a rectangle and the area of 8 circles inside the rectangle. It was a middling cognitively demanding task. Its solution was not obvious.	NP	The students worked on pure geometry questions.	Mid
S4	Mid	The students worked on the word problem. Its solution was not obvious, and it was a middling cognitively demanding task.	High	A story problem was presented, and the students worked on how to solve the problem individually. The teacher checked the students' solutions and gave personal feedback.	Mid
S5	Mid	Students presented their solutions to the word problem given in the previous segment.	High	The student went on working on the story problem and presented their solution methods on the board.	Mid
S6	Mid	Students tried to find out the area of which sector was the biggest. They justified their claims. It was a middling cognitively demanding task.	High	The teacher presented a story problem. The students expressed ideas about which area is bigger and how to decide on it.	High

Table 21. (continues)

S7	High	The students tried to establish what was necessary to calculate the area of a sector. It was a highly cognitively demanding task.	High	They continued to work on the story problem presented in the previous segment. The students identified what was needed to solve the question.	High
S8	Low	The students worked on a low cognitive demanding task; Direct proportion was used.	NP	The students worked on pure geometry questions.	Mid
S9	Mid	They worked on a low cognitive demanding task (A question asking to find the area of a sector. That can be solved by direct proportion or application of the formula) and a middling cognitive demanding task (A question asking to find the area of the minor circular segment between two circles with the same center).	NP	The students worked on pure geometry questions.	Mid
S10	Mid	They continued to work on a middling cognitively demanding task presented in the previous segment.	NP	The students worked on pure geometry questions.	Mid

4.6. First Observation of Efe: Area of Circle

The first observation of Efe's lesson took place while he was teaching the area of circle, and it took place in the seventh-grade classroom. The lesson was videotaped and audiotaped and the researcher was in the classroom as a passive observer. The observation process lasted for two consecutive lesson hours because covering the objective of the area of the circle lasted two hours. His 7th-grade class was composed of both boys and girls since it was a coeducational school. 36 students registered in this classroom. 5 students were absent on the day of observation. The observation took place in the last week of April.

4.6.1. Summary of the Lesson

The lesson started on time. Efe began the lesson by reminding the students that they had already learned how to find the perimeter of the circle and the length of an arc. Then he said the length is one-dimensional and the area is two-dimensional which are length and height. Efe informed students that they were going to learn something related to the area of the circle. He then showed a map from a game and almost all of the students knew the game. Efe pointed out some circular regions on the map and the students said these areas were called "safe areas". The teacher drew two circles and a point the same distance from the center of the circles on the board. Efe said, "The one who first enters the circles wins the game. Which area would they prefer to go to?". Some students answered they would choose the blue area and Ozan explained the reason for their choice as "Since the blue one has a bigger area, we can reach there faster." Efe wanted the students to explain how they decided which area is bigger. Elif expressed the reason as "it covers more surface". The teacher confirmed Elif's idea and explained covering more surfaces means occupying more places. Another student mentioned the radius of the circle.

> *Mehmet:* It has a longer radius.
> *Efe:* Is the area of a circle with a longer radius larger?
> *Students:* Yes.
> *Efe:* Yeah. It seems there is a proportion between the area and the radius. Since we do not know how to calculate the area, we do not know the proportion between area and radius.

Another student said they intuitively knew that the blue area was bigger since the blue area seemed bigger. Efe confirmed the student's idea and reminded them that “the surface occupied by a shape is called an area in mathematics. The teacher said the students already knew how to calculate the area of regular shapes such as squares, and rectangles. He showed a picture of a square, a parallelogram, and a circle divided into unit squares. He said, “ You can find the area of the square and the parallelogram by counting unit square.” He counted the unit squares inside the square and the parallelogram and also reminded the area of the square and the parallelogram. He asked how to find the area of the circle.

> *Efe*:….. If we want to count unit squares inside the circle, a problem occurs. It is not a regular shape. We need to do something. What can we do?
>
> *Semih*: We can find the radius of the circle and then calculate the area of it.
>
> *Efe*: I mean we can count the unit square inside of a square. To count the unit square inside of a parallelogram, we draw the altitude of the parallelogram and we get a triangle portion. If we cut the triangle portion from one side and paste the triangle side to the opposite side, we get a rectangle. We can easily count the unit square inside of a rectangle. You have already known these. To find the area of the circle by counting unit squares, how can we break the circle into parts?

A student tried to suggest using the area formula. However, her knowledge was deficient. Efe warned the students that he did not want to talk about the knowledge picked up here and there by listening. He asked why the area formula includes the square of r, not the square of π. Efe said that they, as a class, always construct the formula by themselves. He directed the students to break the circle into parts and combine the parts to get a unit square. A student said its area was approximately 7-unit squares and some others said the parts did not fit perfectly to form a unit square. Some other students expressed their ideas on finding the area of the circle. Then, the teacher explained it was a problem for mathematicians for many years and it was not possible to form unit squares with parts of circles. He drew a circle on the board and slid it into 4 equal sectors. Then he split it into 8 and 16 identical sectors respectively. He dyed half of the sectors red and the rest blue. He rejoined the blue and red sectors to get a new shape that looked like a parallelogram. The students chorused “It is a rectangle”. Efe said it was not a rectangle and opened a picture of a circle split into 16 sectors and the new shape obtained by the reunion of these sectors. The students understood the reunion of the sectors formed a parallelogram. Efe asked students to split the circle into more parts like 1000, 1000000, or infinite. Then, they concluded that the more sectors the circle was divided into, the more rectangle-like the new shape became. They started to talk about the length and width of the rectangle. The students easily realized that the width of the rectangle was equal to the radius of the circle. Although some of the students hesitated about the length of the rectangle, they found out the length was equal to half of the circumference of the circle.

Efe: How do we calculate the circumference of the circle?
Students: 2 times r times π.

Then, they realized the long side of the rectangle was “r.π”. To find the area of the rectangle length “r.π” was multiplied by width “r”. Efe reminded the students that they had already learned algebraic expression and that r times r was represented as r^2. Efe emphasized the commutative property of the multiplication. He informed the students that they constructed the area formula of the circle and said “It is more meaningful now, why the area formula of the circle includes π and r^2”. He interactively calculated the area of the initial circles with the students. The teacher waited for the students to copywrite the things on the board. Lastly, Efe summarized how they constructed the area formula of the circle and what each term in the formula means by asking questions. The first hour of the lessons finished.

In the second hour, they were started by reminding and clarifying what they learned in the first hour. Efe clarified that the rectangle was constructed by translating parts of the circle. So that the circle and the rectangle had equal areas, then they found the area of a circle with a radius of 7 cm. The students said the area was equal to 147. Efe asked for the unit of the measure and underlined the importance of the unit of the measure. for the next question, they used pi as 3.14 and r as 10 cm. Some students faced problems while multiplying 100 with 3.14 and said the area was 3.14 square meters.

Efe: The area formula is $\pi.r^2$. What is the result of 3,14 times 100? 314 square centimeters.

Arda: 3.14 square meters.

Efe: No. You are wrong. It cannot be a square meter. What was the common conversion factor for the area?

Students: 100.

Efe: Yes. So if you want to say it as 3.14, its measurement unit should be square decimeters. We would say 3 square decimeters and 14 square centimeters. To convert a square centimeter to a square meter, we should divide it by 10000.

The lesson continued with the solution of a new question. For this question Efe wanted students to use $\frac{22}{7}$ as value of π. While solving the question, the teacher reminded the students to simplify the equation before multiplying. He also offered to use easy multiplication tricks that they learned before. To calculate 22x28, he found the tens of 22 and rewrote the operation as 20x28+2x28.

After solving the question, Efe went back to the area formula and explained again where r^2 came from. He summed up the relation between the circle and the constructed rectangle. Then, he said the formula "$\pi.r^2$" is used to find the area of any circle. They solved a new question that asked to find the radius when the measurement of the area was given. Then the students copywrite the question and the answer to their notebooks.

Efe talked about the circle. He said the circle was the first geometric shape that human beings recognized. The first man saw the sun and the moon and realized that they were a circle and painted circles on the walls of calves. He reminded they had watched a video about pi while they were learning the circumference of the circle. Efe opened 3 questions on the smart board. Two of them were about finding the area of the circle when the radius was given. One of them was about finding the area of the circle when the diameter was given. Lastly, the teacher drew a square and 4 identical circles in it. He said they used this shape while covering the circumference of the circle and asked the students to calculate the difference between the total areas of the circles and the area of the square. Efe checked the students' solutions individually and gave feedback if the solution was true or false. He also warned the students about the steps in that they made mistakes. The second hour had ended and they did not solve this question on the board.

4.6.2. Finding Related to MQI

In this section, Efe's instruction while teaching the area of a circle is evaluated using the MQI 4-Point framework. Each 7-minute segment is evaluated separately, and quality evidence of the segment is given in the tables in detail.

4.6.2.1. Richness of Mathematics

Richness evidence of the instruction is presented in two separate tables. In the first table segment codes and evidence of meaning-oriented codes are given. Meaning-oriented codes are Linking Between Representations, Explanations, and Mathematical Sense-Making. In the second table segment score, evidence of practice-oriented codes, and the overall richness scores of the segments are presented. Practice-oriented codes are Multiple Procedures or Solution Methods, Patterns and Generalizations, and Mathematical Language. The distribution of segment scores and subdimension of Richness of Mathematics is given in Figure 13.

Nine segments received "NP" for linking between representations since in geometry the shapes are not counted as representation. The link between the geometric shapes and the symbolic representation of these shapes is not coded. In segment 3, the teacher used a real-life example to make the students imagine how to break the circle into pieces and was scored "Low". The teacher asked what would happen if bread was divided into one thousand pieces. Then he wanted students to imagine dividing the bread into more pieces. Only the circle was visually present and the second representation was not visually present. In segment 4, the teacher linked the circle, rearranged sectors, rectangles, and the area formula of the circle. However, the link within the same representational family do not count as a link between representation in MQI rubric.

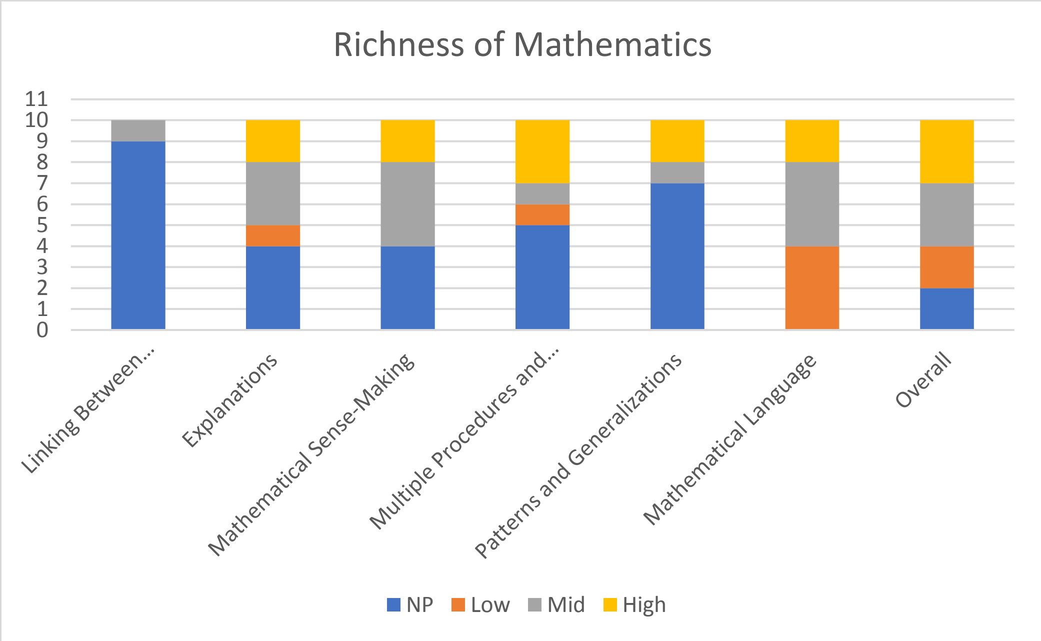


Figure 13. Distribution of Richness of Mathematics scores

In the first hour of the lesson, they developed the area formula of the circle, and the teacher and students made many mathematical explanations. During the development process, the teachers asked “Why?” for many times. Therefore, the first four segments received “Mid” or “High” explanation codes. In the first segment, the teacher asked which circle was bigger and wanted students to reason their answer. Students explained their thinking and correct student explanation was coded. So, the first segment scored “Mid” for the Explanation. The second, third, and fourth segments were scored “High”. In segment 2, the teacher explained why counting unit squares was not an appropriate method for calculating the circle’s area. Before the teacher's explanation, students also provide some brief explanations. In segments 4 and 5, the teacher and students worked on the construction of the circle’s area formula. The focus was on why the circle and rearranged sectors had equal areas and the relation between these two shapes. In the second hour, they solved questions using the area formula of the circle. The questions were like exercise and little explanation was made. Only segment 7 and segment 8 received a “Low” score since the teacher explained why minus 6 could not be the answer and the existence of the irrational numbers. The rest of the segments received “NP”.

Efe focused on making sense of the circle's area formula throughout the instruction. The first hour of the instruction was built on making sense of the area formula of the circle and the development of the area formula. Therefore, segments 2, 3, and 4 were coded as "High". In segment 2, the teacher tried to make sense that counting unit squares to measure the area of the circle was not an appropriate method. In segment 3, he used real-life examples to explain infinity and he focused on the relationship between the numerator and the denominator. The fourth segment was devoted to making sense of the circle's area formula and explanation of "why area formula includes r^2 and pi". In the second hour of the instruction, they usually solved exercise questions. The teacher focused on the relationship between the number and the meaning of the numbers. So, the sixth segment scored "High" for Mathematical Sense-Making. In segment 7, was scored "Mid", the teacher said 36 had two roots, and explained that (-6) cannot be the measure of the radius. Therefore, the answer should be (+6). Efe used the square root to make sense of the existence of the irrational numbers and so the pi. His explanations were detailed and segment 8 scored "High". In segment 9, the teacher used the partial product to show the relation between the numbers. Segments 1, 5, and 10 were coded as "NP".

Through ten segments, only the first segment scored "Mid" for multiple procedures and solution methods. They calculated the square's area and the parallelogram's area, they used area formulas and counting unit squares. Nine segments scored as "NP" because only the area formula of the circle was applied to solve the questions.

For Patterns and Generalization, only segment 3 received a "Mid" score and segment 4 received a "High" while other segments received "NP". During the first four segments, the focus of instruction was the development of the area formula of the circle. The teacher spent the effort to develop generalizations and pressed students to make generalizations. In segment 3, they made generalizations about the sides of the rectangle constructed by rearrangement of sectors. In segment 4, they developed the area formula by connecting the circle and the rectangle. The teacher also explained why the area formula included r^2 referring to the rectangle's area formula. After the

Table 22. First observation of Efe: Score and evidence of Richness of Mathematics-I

	Linking Between Representations		Explanation		Mathematical Sense-Making	
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**
S1	NP	The shapes do not count as a representation in geometry.	Mid	A student explained why the area of the blue circle was bigger than the area of the red circle by mentioning the surface occupied by the circles. Another student explained by comparing radii.	NP	Nothing related to sense-making happened.
S2	NP	The shapes do not count as a representation in geometry.	High	The focus of the segment was why counting unit squares was an inappropriate method to find the area of a circle.	High	The teacher tried to make sense of why counting unit squares was not a reasonable procedure for calculating the area of a circle.

Table 22. (continues)

S3	Low	The teacher connected the circle with bread to explain infinity.	High	The teacher explained why the height of the rearranged sectors was equal to the radius of the circle, but it was not detailed. The teacher explained why the new shape (rearrangement of sectors) would be close to a rectangle in detail.	High	The teacher tried to make sense of the infinity by using real-life examples. He also mentioned the meaning of numbers such as $\frac{1}{1000}$, $\frac{1}{1000000}$, and the relationship between the numerator, denominator, and the result of division numerator to denominator.
S4	NP	The teacher linked the circle, the rearrangement of sectors, and the symbolic representation of the area formula of the circle. However, the link within the same representational family not scored in MQI in rubric.	High	The student said the addition of two long sides of the rectangle equals the perimeter of the circle. The teacher explained why the base of the rearranged sectors was equal to the half perimeter of the circle in detail.	High	The focus of the segment is the relationship between the area of rearranged sectors and the area formula of the rectangle and making sense of the area formula of the circle that “radius times half of the circumference” and why the area formula includes r^2.

Table 22. (continues)

S5	NP	The shapes do not count as a representation in geometry.	NP	The teacher re-explained what they did in S3 and S4. The teacher talked and the students took notes.	NP	The teacher re-explained what they did in S3 and S4. The teacher talked and the students took notes.
S6	NP	No linking occurred.	NP	No mathematical explanations were offered by the teacher and students.	High	While multiplying 49 by 3, the teacher multiplied 50 by 3 and subtracted 3 from 150. While multiplying 28 with 22, the teacher first multiplied 28 with 20 and then multiplied 28 with 2. He lastly added 560 with 56.
S7	NP	No linking occurred.	Low	When finding the root of the equation $r^2=36$, the teacher explained why r cannot be equal to -6.	Mid	When finding the root of the equation $r^2=36$, the teacher explained why r cannot be equal to -6.

Table 22. (continues)

S8	NP	No linking occurred.	Low	The teacher made an explanation about the existence of irrational numbers, but his explanation occurred as an isolated instance.	High	The teacher used the square root to make sense of irrational numbers and the existence of the pi.
S9	NP	No linking occurred.	NP	No mathematical explanations were offered by the teacher and students.	Low	The students performed the multiplication 225x3, and the teacher used the partial product.
S10	NP	No linking occurred.	NP	No mathematical explanations were offered by the teacher and students.	NP	Not present.

Table 23. First observation of Efe: Score and evidence of Richness of Mathematics-II

	Multiple Procedure or Solution Methods		**Patterns and Generalization**		**Mathematical Language**		**Overall**
Segment	**Scores**	**Evidence**	**Scores**	**Evidence**	**Scores**	**Evidence**	
S1	Mid	They calculated the area of a square and a parallelogram both counting unit squares and using the area formula. No comparison of the methods was made.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	High	The teacher used mathematical language in dense and reminded the meaning of the area, perimeter, and circle.	Mid
S2	NP	There was no evidence of multiple procedures and solution methods.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	Middling density of using mathematical language.	High

Table 23. (continues)

S3	NP	There was no evidence of multiple procedures and solution methods.	Mid	The generalization of long sides of the shapes which were constructed by rearrangement of sectors was equal to half of the perimeter of the circle and its height was equal to the radius, but not detailed enough.	High	Middling density of using mathematical language. The teacher introduced the term infinity and explained it with examples.	High
S4	NP	There was no evidence of multiple procedures and solution methods.	High	The area formula of the circle was developed using the generalization of the area formula of the rectangle.	Mid	Middling density of using mathematical language. Mathematical terms, perimeter, circle, the area of the rectangle, and algebraic expression were used.	High
S5	NP	The teacher re-explained what they did in S3 and S4. The teacher talked and the students took notes.	NP	The teacher re-explained what they did in S3 and S4. The teacher talked and the students took notes.	Mid	Middling density of using mathematical language. The teacher talked and the students took notes.	NP

Table 23. (continues)

S6	NP	Only the area formula was used to solve the questions.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The middling density of using mathematical language while solving questions.	Mid
S7	NP	Only the area formula of the circle was used while solving the questions.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Low	The low density of mathematical language.	Low
S8	NP	No procedures or solution methods were used.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Low	The low density of mathematical language. The teacher talked about the history of circles, the existence of irrational numbers, and pi.	Low
S9	NP	Only the area formula of the circle was used while solving the questions.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The middling density of using mathematical language while solving questions.	Low

Table 23. (continues)

S10	NP	Only the area formula of the circle was used while solving the questions.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The middling density of using mathematical language while solving questions.	NP

development of the area formula, the class worked on exercise problems, and no pattern was discovered, and no generalization was developed.

Efe used mathematical language fluently all through the instruction and pressed students for accurate use of mathematical language. Therefore, segments generally received a “Mid” or “High” score. Only segment 7 and segment 8 coded “Low” where students were copying, and the teacher summarized what they did and learnt.
All segments received an overall richness score. While assigning an overall score, middling use of mathematical language is not considered as an element of richness. Therefore, segment 5 and segment 10 received “NP” overall scores, because they do not include any richness elements except Mathematical Language. The first four segments received “Mid” and “High” overall richness scores. The mathematics offered to students in the first lesson hour segments were in-depth and constructed to support the conceptual understanding of the students. Also, segment 6 received a “High” score, where the teacher focused on the relationship between numbers. When the class started to solve exercise questions, the depth of the mathematics offered to students was reduced and the following segment got “Low” scores. Since they just applied the area formula of the circle to solve the questions, few Richness of Mathematics elements occurred.

The evidence and segments scores were presented in Table 22 and Table 23.

4.6.2.2. Working with Students and Mathematics

In this section, the score and evidence of the Working with Students and Mathematics are given in Table 24. The Working with Students and Mathematics dimension is highly related to teachers' knowledge of content and students. If the teacher knows possible students’ errors and difficulties related to the content that is taught, he can plan pre-remediation activities. The teacher also can hear, understand, and use students’ contributions to develop mathematics. The segments coded “NP”, “Low” or “Mid” for Remediation of Students Errors and Difficulties and Teacher Uses Students Mathematical Contribution. No segment scored “High” for Working with Students and Mathematics dimension. The distribution of segment scores and subdimension of Working with Students and Mathematics is given in Figure 14.

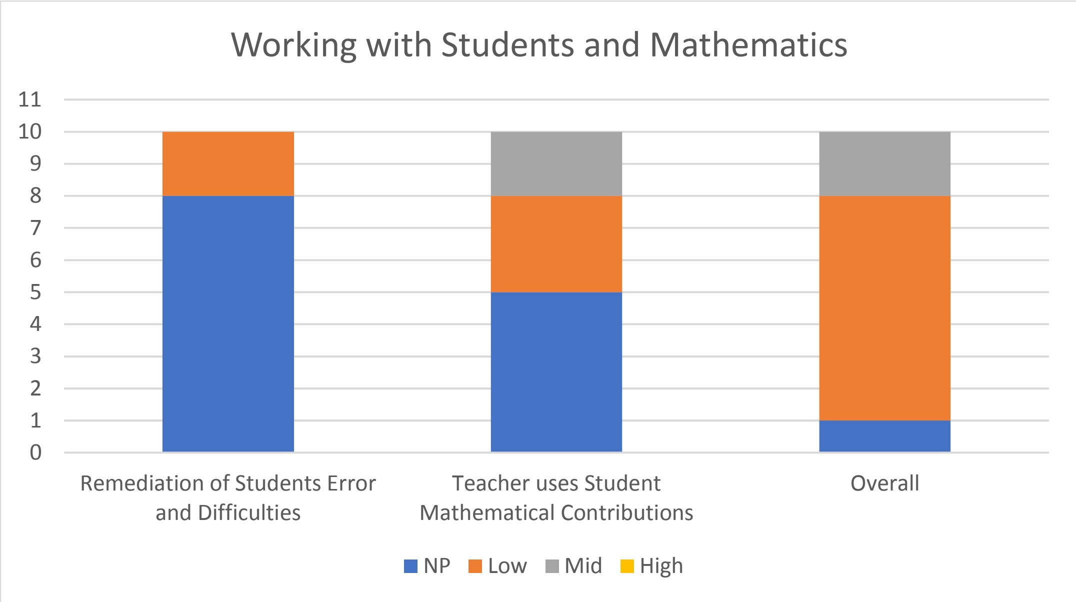


Figure 14. Distribution of Working with Students and Mathematics scores

Evidence of Working with Students and Mathematics is given in the Table. The segments coded “NP” or “Low” for Remediation of Student Errors and Difficulties. The instruction was dominated by the teacher's talk. Therefore, a few student errors or difficulties, in segment 6 and segment 10, were observed. In segment 6, a student had difficulty deciding the unit of measure when the value of pi was 3,14. The teacher reminded students that a one-meter square is 10000 times bigger than a centimeter square, not 100 times. Efe performed the multiplication and explained the unit of measure. In the tenth segment, the teacher checked students’ solutions and told them in which steps they made wrong calculation. These segments were scored “Low” because the teacher's remediations were procedural.

The segments scored “NP”, “Low” and “Mid” for Teacher Uses Students Mathematical Contribution. As mentioned before, the instruction was dominated by the teacher’s talk, and the teacher solved the questions himself. These segments were coded “NP” because the students’ contributions were not substantial. In the segments coded “Low” the students’ contributions and the teacher’s response were in a pro forma way. Mostly the teacher developed the math. In segment 2, although the segment was dominated by the teacher's talk, the teacher highlighted a student's idea and used it to develop mathematics. So, the segment was scored "Mid." The other segment that scored "Mid" was segment 4. Students explained why the side of the rectangle was equal to the radius and half of the circle's perimeter. Efe used these students' explanations to develop the area formula of the circle. Most of the time was spent on the teacher's talk. However, the teacher used some student answers to develop math.

For overall Working with Students and Mathematics, segment 2 and segment 4, scored "Mid" and segment 7 scored "NP". The other segments received a "Low" score. The teacher interacted with students, but the interaction was not substantial in most of the segments. It was the teacher who developed the mathematics. He heavily scaffolded the students while developing the area formula of the circle. Segments 2 and 4, were scored "Mid" since the teacher used students' ideas to go further. Also, the teacher solved the questions himself at first. Therefore, student contributions were limited.

Working with Students and Mathematics code shows the teacher and students' interaction around the content. Efe's instruction scored "Mid" or "High" for some segments in Richness of Mathematics dimension, although it was scored "Low" for Working with Students and Mathematics codes. The instruction was generally dominated by the teacher and the mathematics developed by the teacher.

Table 24. First observation of Efe: Score and evidence of Working with students and Mathematics

Segments	**Remediation of Student Error and Difficulties**		**Teacher Uses Students Mathematical Contribution**		**Overall WSM**
	Scores	Evidence	Scores	Evidence	Scores
S1	NP	No misunderstanding or difficulties with the content occurred.	Low	The teacher developed the math, not the students' answers and the students' answers were in a pro forma way. S: Since the blue one has a bigger area, it is easy to reach there. T: What's bigger? S: Its area.	Low
S2	NP	No misunderstanding or difficulties with the content occurred.	Mid	The segment was dominated by the teachers' talk and a few short students' answers occurred such as; we got the area as 5 when we added the parts together or it was a rectangle. However, the teacher highlighted a student's idea "Elif said that the blue circle covers more surface and I agree with her. The blue circle occupies more space on the board and so on the map." Therefore, the segments scored mid.	Mid

Table 24. (continues)

S3	NP	No misunderstanding or difficulties with the content occurred.	Low	The segment was dominated by the teachers' talk. A few short students' answers occurred and the teacher developed the math, not the students' ideas. T: What does this shape look like? S: A parallelogram. T: It looks like a parallelogram now. You are right about that it looks like a rectangle… If I divide it into more parts, does it more look like a rectangle? S: Yes.	Low
S4	NP	No misunderstanding or difficulties with the content occurred.	Mid	The segment was dominated by the teachers' talk. A few students' answers occurred and they contributed to the development of mathematics. S: It is not the whole perimeter, because there are white parts in the whole perimeter. S: The addition of two long sides becomes equal to the perimeter of the circle. The teacher used students' ideas to develop mathematics. That is, he developed the area formula using students' explanations.	Mid

Table 24. (continues)

S5	NP	No misunderstanding or difficulties with the content occurred.	NP	The segment was dominated by the teachers' talk. A few short student' answers occurred and they were in a pro forma way. The students repeated the area formula they developed in the previous segment and took notes.	Low
S6	Low	A student had difficulty deciding the unit of the area while the value of the π was 3,14. The teacher reminded the procedural relation between the standard units of the area.	NP	The segment was dominated by the teachers' talk. A few short student' answers occurred and they were in a pro forma way. The interaction, that does not gain favor to the development of math, is not counted. T: …the square of 14 means 14 times 14, doesn't it? S: 196 T: Let me write it as 14 times 14 times 22 over 7. S: Yes.	Low
S7	NP	The teacher solved the questions himself. No misunderstanding or difficulties with the content occurred.	NP	The segment was dominated by the teachers' talk	NP

Table 24. (continues)

S8	NP	The segment was dominated by the teachers' talk. The teacher talked about irrational numbers and the history of math.	NP	The segment was dominated by the teachers' talk	Low
S9	NP	No misunderstanding or difficulties with the content occurred.	Low	Students solved questions on the board and the teacher explained their solution to the class.	Low
S10	Low	Procedural remediations occurred. The teacher checked students' solutions individually and said "You did subtraction wrong" or "Your first multiplication is true but the second multiplication is wrong".	NP	The students work on the question individually. The teacher gave personal feedback.	Low

4.6.2.3. Common Core Aligned Student Practice (CCASP)

Common Core Aligned Student Practice dimension includes five subdimensions which are Students Provide Explanations, Students Mathematical Questioning and Reasoning (SMQR), Students Communicate about the Mathematics of the Segment, Task Cognitive Demand, and Students Work with Contextualized Problems. CCASP focuses on evidence of student involvement in the tasks. It tries to capture the extent to which students engage in and work with the mathematics of the segment. Student explanations, student mathematical questions, students' reasoning, tasks, and problems that students work with are scored for the CCASP dimension. The tasks and the problems that the teacher selects to use during instruction are affected by teachers' knowledge of content and teaching and teacher knowledge of content and students. If the teacher knows what is easy and what is difficult for his students, he will select tasks that support students' learning.

All the dimensions of CCASP were scored. The Distribution of segment scores is given in Figure 15.

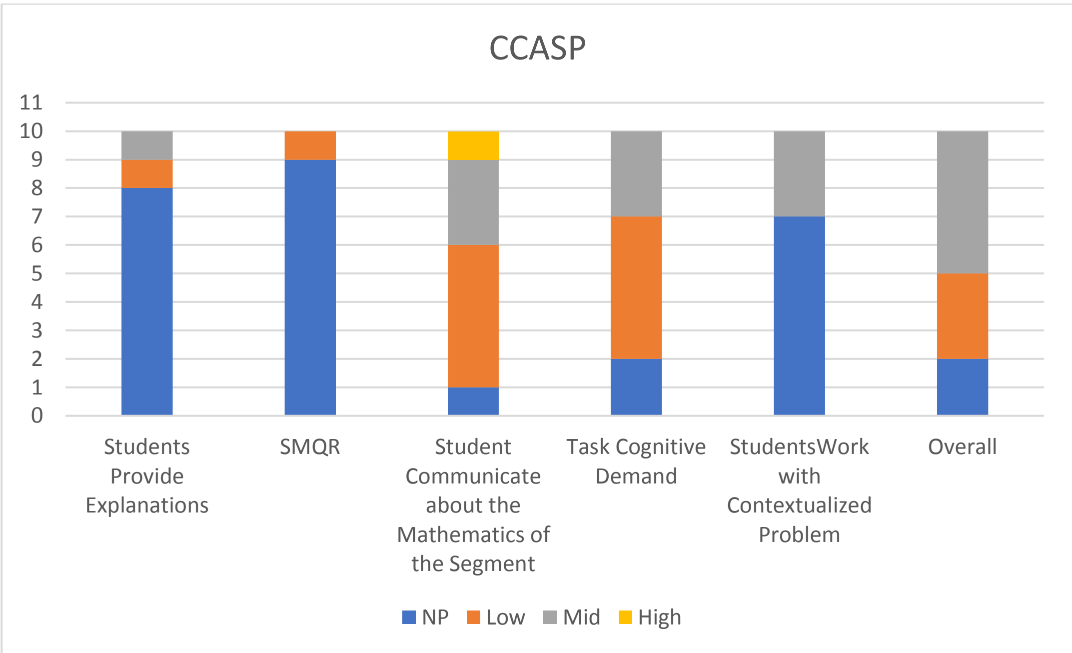


Figure 15. Distribution of CCASP scores

Evidence and segment scores of the CCASP are presented in the Table and Table. For Students Provide Explanations dimension eight segments scored “NP”. There were only two instances in which students provided explanations. In segment 1, students explained which circular region they would choose, and why. Student explanations were frequent, so the segment scored as “Mid”. In segment 4, a student explained the relationship between the base of the rectangle and the perimeter of the circle. The student’s explanation was substantial but not sustained. The teacher maintained the explanation, and the segment scored “Low”.

The instruction was dominated by the teacher’s talk and most of the time the teacher developed the mathematics. Therefore, only two instances, in segment 3 and segment 4, scored for SMQR. In segment 3, the teacher explained the infinity and a student asked how it was possible to perform operation with an unknown. The teacher divided the circle into sectors and rearranged the sectors to get a rectangle. A student asked if all geometric shapes were obtained by dividing a circle into parts and then rearranging parts. The teacher answered “No” and the students did not sustain the questioning. So, the segment received a “Low” score.

One segment, segment 8, scored “NP” for Students Communicate about the Mathematics of the Segment dimension. Segment 8 was dominated by the teacher’s talk and no mathematical student contribution occurred. Segment 1, and segment 4, received a “Mid” score because some brief and some substantial student contributions occurred. In the first segment, students voiced their thinking about which circle was bigger. They explained their reasoning. In segment 4, students contributed to development of circle’s area formula. A student solved the question on the board, and segment 10 was scored “Mid”. In segment 9, more than one student presented their solutions on the board and scored “High”. The rest of the segments scored “Low” since students' contributions were brief, limited to one-or-two-word answer to the teacher’ questions.

At the beginning of the lesson, the teacher presented a story problem and asked students how they decided which circle was bigger. So, segment 1 received a “Low” score. Then the teacher wanted students to explain how they could find the area of a

Table 25. First observation of Efe: Score and evidence of CCASP-I

	Students Provide Explanations		Student Mathematical Questioning and Reasoning (SMQR)		Students Communicate about the Mathematics of the Segment	
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**
S1	Mid	Students' explanations were frequent. S. Its area is bigger, so we can reach there more easily. S: The surface it occupied is bigger. S: Its radius is bigger.	NP	No student questioning and reasoning occurred.	Mid	There were some brief contributions and some substantial contributions from students. The students produced ideas to decide which circle had a bigger area.
S2	NP	No student explanations were observed.	NP	No student questioning and reasoning occurred.	Low	Student contributions were very brief. In the second part of the segment, the teacher offered a solution method, and the students only contributed with one-or-two-word answers.

Table 25. (continues)

S3	NP	No student explanations were observed.	Low	A student asked if the infinity was unknown, how can a person perform operation with something unknown.	Low	The segment was dominated by the teacher's talk, and the students only contributed with one-or-two-word answers.
S4	Low	A student explanation occurred. S: The addition of two long sides of the rectangle equal to the perimeter of the circle.	Low	A student asked “Can we say that shapes other than circles, such as squares and rectangles, are formed by dividing a circle a million times?	Mid	There were some brief contributions and some substantial contributions from students. The student explained that the base of the rectangle equals half the perimeter of the circle.
S5	NP	No student explanations were observed.	NP	No student questioning and reasoning occurred.	Low	Student contributions were very brief. An example is given below: S: Pi times r square? T: Somebody can ask what r means. S: Radius.

Table 25. (continues)

S6	NP	No student explanations were observed.	NP	No student questioning and reasoning occurred.	Low	Student contributions were very brief. An example is given below: T: … the square of 14 means 14 times 14, doesn't it? S:196 … S: 560
S7	NP	No student explanations were observed.	NP	No student questioning and reasoning occurred.	Low	Student contributions were very brief. An example is given below: T: r^2 equals 36. Which number… S: 6 T: There is one more number. S: -6
S8	NP	No student explanations were observed.	NP	No student questioning and reasoning occurred.	NP	The segment was dominated by the teacher's talk.
S9	NP	No student explanations were observed.	NP	No student questioning and reasoning occurred.	High	The students presented their solutions to the board.

Table 25. (continues)

S10	NP	No student explanations were observed.	NP	No student questioning and reasoning occurred.	Mid	A student solved the question on the board. Then the students worked on the next problem individually. The teacher described the solution method.

Table 26. First observation of Efe: Score and evidence of CCASP-I

	Task Cognitive Demand		**Student Work with Contextualized Problems**		**Overall**
Segment	Score	Evidence	Score	Evidence	Score
S1	Low	Direct instruction with some student explanations occurred. S: It covers more space. …. S: Its radius is longer.	Mid	The teacher presented a story problem and the students worked on it. They tried to produce a solution to decide which circle was bigger.	Mid
S2	Mid	The teacher asked the students how to calculate the area of the circle. It was a cognitively demanding task for the students. However, in the second part of the segment, the teacher offered a solution method.	Mid	The students continued to work on the problem presented previous segment. They expressed their ideas about how to calculate the area of the circle. However, the teacher offered a solution way.	Mid
S3	Mid	They continued to work on the area of the circle. However, the teacher scaffolded the students by dividing the circle into sectors.	NP	They worked on a pure geometry task. The teacher divided the circle into sectors and rearranged the sectors to form a rectangle.	Low

Table 26. (continues)

S4	Low	They continued to work on the area of the circle. However, the teacher developed the mathematics and he developed some student explanations	Mid	The students continued to work on the problem presented previous segment. They worked on developing the area formula of the circle.	Mid
S5	NP	The teacher summarized how they developed the area formula of the circle. Then the students took notes.	NP	The teacher summarized how they developed the area formula of the circle. Then the students took notes.	NP
S6	Low	The students worked on low cognitive demanding tasks which required the application of the area formula of the circle	NP	They worked on a pure geometry task.	Low
S7	Low	The students worked on low cognitive demanding tasks which required the application of the area formula of the circle	NP	They worked on a pure geometry task.	Low
S8	NP	The segment was dominated by the teacher's talk and the students took notes.	NP	The segment was dominated by the teacher's talk and the students took notes.	NP

Table 26. (continues)

S9	Low	The students worked on low cognitive demanding tasks which required the application of the area formula of the circle	NP	They worked on a pure geometry task.	Mid
S10	Mid	The students worked on a task that required a middling cognitive demand. It was asking about the difference between the area of the square and the area of 4 inner circles. Its solution was not obvious.	NP	They worked on a pure geometry task.	Mid

circle. It was a highly cognitively demanding task for seventh-grade students. However, the teacher scaffolded heavily and offered a solution method. Therefore, segments 2 and 3 scored “Mid” instead of “High”. Also, segment 10 scored “Mid”, since the student worked on a cognitively demanding task. The question required more than the application of the area formula and the solution was not obvious. Segments 5 and 8 scored “NP” since the teacher’s talk dominated the segment. He summarized what they learned and talked about the history of math. The other segments score “Low” because students work on simple pure geometry questions.

The teacher started the instruction by introducing a story problem, and they worked on it through the first two segments. So first two segments scored “Mid” for the Student Work with Contextualized Problems dimension. In segment 3, the teacher drew a circle, rearranged segments, and talked about pure geometry. In segment 4, the teacher turned back to the story problem and solved it. Therefore, S4 scored as “Mid” for Students Work with Contextualized Problems dimension. After they developed the area formula of the circle, they worked on pure geometry and segments received an “NP” score.

Overall CCASP that scored students’ involvement in doing mathematics was scored “NP” for segment 5 and segment 8. These segments were dominated by the teacher talk, and no important student contribution occurred. Segments 3, 6, and 7 received “Low” scores because the teacher’s talk was dominated the segments, and only a few students' contributions observed. The other segments scored “Mid”. No segment scored “High” for Overall CCASP.

The evidence and segments scores were presented in Table 25 and Table 26.

4.7. Second Observation of Efe: Area of Sector

The second observation of Efe’s lesson took place while he was teaching the area of sectors, and it took place in the seventh-grade classroom. The lesson was videotaped and audiotaped, and the researcher was in the classroom as a passive observer. The observation process lasted for two consecutive lesson hours because covering the objective of the area of the circle lasted two hours. His 7th-grade class was composed of both boys and girls since it was a coeducational school. 36 students registered in this classroom. 4 students were absent on the day of observation. The observation took place in the first week of May.

4.7.1. Summary of the Lesson

The first minutes were spent greeting the students. Then Efe drew a circle on the board and he said he wanted to make a wooden wheel of fortune. Therefore, he divided the circle into 3 sectors with different central angles. He said he wanted to paint all three sectors with different colors and called a painter cost of painting. He asked the students how the painter explained the cost of the painting. A student said the painter informed the cost of 200 or 180 degrees of the sectors, and the teacher could calculate the cost of each sector using this information. Another one said he needed to know the radius of the circle. Efe reminded the students that the painter did not know the shape of the object. Ezgi said he could inform the teacher about the cost of a square centimeter or a square meter. Efe confirmed Ezgi’s idea and asked the students how to calculate the cost of each sector separately. Efe wanted the students to decide on a reward for each sector of the Wheel of Fortune. Kağan claimed they should choose a more valuable reward for the blue sector because the chance of stopping the Wheel of Fortune in the blue sector was less likely.

> *Efe:* Why?
> *Kağan:* It has the smallest area.
> *Efe:* You said its area is smaller than others. How do you know?
> *Kağan:* It seems so.

Efe said it could seem smaller, but it was not meaningful for mathematicians. A mathematician should do an operation and be sure that it is smaller than others. Then, he asked if the given information was enough to calculate the cost of the painting. Ferit said they needed the central angle of the sector. Ahmet said they also needed the radius. Mehmet said the cost could be calculated by using perimeters of sectors.

Efe: How will you find it? Can you explain Mehmet?

Mehmet: Using perimeter, we can find the radius of the circle then we can calculate the area.

Efe: You can find the whole area of the circle, can't you? How will you find the area of the blue sector?

Mehmet: Oh, no. We cannot find it.

Mehmet realized it was not possible to find the area of the sector just knowing the radius. Bekir recommended using proportion using angles of the sectors. Some other students also confirmed Ferit's idea that the central angle was necessary. The teacher declared that they agreed that both the radius of circles and central angles of the sector were necessary. Then he repeated Bekir's words and approved Bekir's idea. Efe asked about the type of proportion between the area of a sector and the central angle of the sector. The students said it was a direct proportion. The students explained if the central angle of a sector gets larger, its area also gets larger. Efe and the students interactively ordered sectors from the smallest to the biggest according to central angles. Then, they decided to find the area of the whole circle first. Bekir had already reminded the area formula of the circle and the teacher repeated it and asked what should be the value of π.

Efe: In another class, one of the students wrote π equals 3 cm. What do you think? Is it true?

Elif: It cannot be cm. We do not know the exact value of the π.

Eda: Since π is infinite, it cannot be length.

Efe: You are confusing the value of the π and the unit of the π. We have discussed the value of the π and, decided it is not possible to know the exact value of the π. The students wrote π equals 3 cm. Is it true?

Elif: No, it is not true. π is not a length.

Efe: π is a ratio and constant number. It does not have a unit.

They turned back to the question and calculated the area of the circle and the cost of painting the whole circle which was 150 TL. The teacher asked how to find the cost of the blue sector. Sina explained his solution way and did the operation on the board. He said he used the direct proportion. If the area of the circle with a 360-degree central angle was 300 cm^2, the area of the sector with a 60-degree central angle was 50 cm^2. Since painting 1 cm^2 cost 0.5 TL, painting 50 cm^2 area cost 25 TL. Efe explained Sina's solution to the students. Efe asked the students if there was anybody who used a different solution method. Deniz said he said he divided the total area of the circle by 360 and then multiplied the quotient by 60 to find the area of the blue sector.

Efe asked the students how to write the proportion they used in symbolic sentences. Tuna stated the area formula of the sector. However, the teacher asked how did he get the formula. Tuna had difficulty explaining. Elif reminded while learning the perimeter of the sector, they first calculated the whole perimeter of the circle and then divided 60 by 360 to find the perimeter of a small part. She offered to use this way to calculate the area of the sector. Efe confirmed her and said he would explain it in detail. He offered to divide the whole area of the circle by 360. The students said the result of the division was the area of the sector with a 1-degree central angle. Then the teacher multiplied the $\frac{300}{360}$ by 60. Then he said the area of the circle could be different than 300 and the central angle could be different than 60- degrees. The only number that would stay constant in operation $\frac{300}{360}$. 60 was 360. Therefore, to use this operation to solve other questions, he said they needed to generalize this operation to get a formula. The first hour of the lesson ended.

At the beginning of the second hour, Efe wrote $r^2.\pi$ for the area of a circle and named the central area of the sector as α. Then, he noted the area formula of a sector as $\frac{\pi r^2}{360}$. 60. Throughout this process, the students actively incorporated with the teacher. The students copywrite what the teacher noted on the board. While the students were taking notes, Efe reminded them what they talked about π and the history of mathematics in previous courses. Efe posed a question related to finding the area of the sector and wanted the students to explain how to solve the question. İnci offered to find the area of the circle, divide it by 360, and then multiply the result by 90.

Tuana offered to find the area of the circle and then divide it by 4 since the sector was one-fourth of the circle. Efe solved the question in both ways and reminded the students of the area formula of the sector. He said the logic of all three ways of the solution was the same and they could use all three ways to solve the questions. He wanted the students to note all solutions in their notebooks. He drew a new circle and sector and asked for the area of the sector with an 80-degree central angle. He solved the question using the area formula of the sector and proportion. The students noted the question and solution ways and the second hour of the lesson ended.

4.7.2. Finding Related to MQI

In this section, Efe's instruction while teaching the area of sectors is evaluated using the MQI 4-Point framework. Each 7-minute segment is evaluated separately and quality evidence of the segment is given in the tables in detail.

4.7.2.1. Richness of Mathematics

Richness evidence of the instruction is presented in two separate tables. In the first table segment codes and evidence of meaning-oriented codes are given. Meaning-oriented codes are Linking Between Representations, Explanations, and Mathematical Sense-Making. In the second table segment score, evidence of practice-oriented codes, and the overall richness scores of the segments are presented. Practice-oriented codes are Multiple Procedures or Solution Methods, Patterns and Generalizations, and Mathematical Language. The distribution of segment scores and subdimension of Richness of Mathematics is given in Figure 16.
Richness of Mathematics shows deepness of mathematics that is presented to students.

Richness dimensions score both the teacher' work and students' work. If the students' contributions are not completed or not correct, it is not scored.

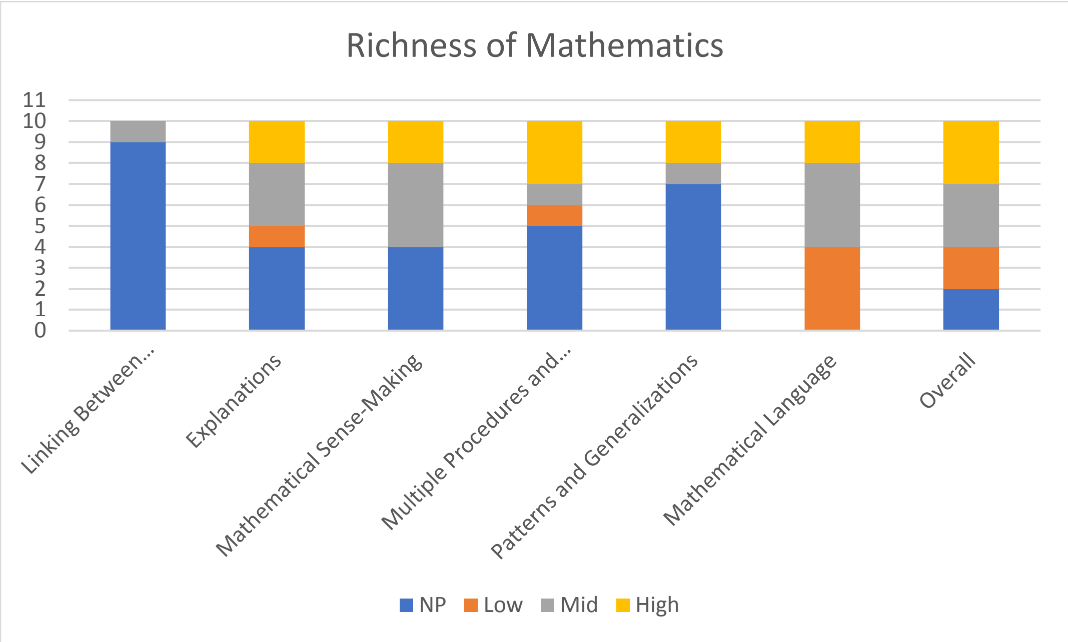


Figure 16. Distribution of Richness of Mathematics scores

The objective of the instruction was learning to calculate the area of the sectors. It was a geometry lesson; the shapes do not count as representation. Therefore, only segment 1 received a “Mid” score for the Linking Between Representations code. The teacher connected the circle and the real-life situation. The connections were made between the circles or the sectors and the symbolic representations. The other segments were coded as “NP”.

Through two-hour instruction, both the teacher and students offered mathematical explanations. Segments 2, 8, and 9 received a “Mid” score. In the second segment, a student explained that the chance of stopping the wheel of fortune in the blue area was the smallest because the blue area was the smallest. The explanations in segments 8 and 9 were related to why a solution methods was true or why a solution was true. In segment 3, which was coded “High”, the teacher explained the direct proportion between the central angles of the sectors and the areas of the sectors. The teacher made this explanation after the students mentioned this relation. The other segment that received a “High” score was segment 5. The teacher explained the reasonableness of the area formula of sectors by using students' previous knowledge.

He explained why the formula includes the central angle of the sectors and the round angle.
The explanations are also used to make sense of the mathematical content. Therefore, in many situations, the instances that are counted for the Explanations are also counted for the Mathematical Sense-Making. The segments that were coded as "NP" for Explanation were also coded as "NP" for Mathematical Sense-Making. Only segment 4 was scored "Mid" for Mathematical Sense-Making although it was scored "Low "for Explanation. The teacher's explanation about pi was scored in both dimensions. However, the student's statement about the relations of the numbers scored only for Mathematical Sense-Making.

The teacher stated the area formula after working on it. The teacher endeavored to make students explore the area formula of the sectors by themselves. For this reason, students first discovered the direct proportion between the central angle of sectors and the areas of sectors. So, the first three segments were spent constructing the context and no procedure or solution methods were introduced. Secondly, Efe introduced the area formula of the sectors and encouraged students to use relationships between numbers while solving the questions. Students used these methods while solving the question and compared them. For each question they used at least two different solution methods. Therefore, the segments where the class solved questions were -segment 5 and segment 8- scored "Mid" while segments 4, 5 and 9" scored "High" for Multiple Procedures and Solution Methods.

The segments where the teacher and the student work on how to find the area of the sectors were also rich concerning Patterns and Generalizations. In segment 1, the teacher introduced the context of the problem. Nothing related to Patterns and Generalization occurred and the segment was scored "NP". Segments 2, 3, and 5 scored "High" because the students and the teacher developed generalizations regarding the area of sectors. They developed that the central angle and the radius of the sector were necessary to find the area. They generalized that when the central angle of a sector of the circle gets wider, the area of this sector also gets wider. No pattern was discovered, and no generalization was developed while the class working on problem-solving. Therefore, segments 4, 6, 7, 8, 9 and 10 were scored "NP".

Table 27. Second observation of Efe: Scores and evidence of Richness of Mathematics-I

	Linking Between Representations		Explanation		Mathematical Sense-Making	
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**
S1	Mid	The teacher linked a real-life situation (painting a wheel of fortune) to mathematical content	NP	No mathematical explanation was offered by the teacher or students.	NP	Nothing related to sense-making happened.
S2	NP	The shapes do not count as a representation in geometry.	Mid	A student explained that the chance of stopping the wheel of fortune in the blue area was the smallest because the blue area was the smallest.	Mid	A student explained that the chance of stopping the wheel of fortune in the blue area was the smallest because the blue area was the smallest.
S3	NP	The shapes do not count as a representation in geometry.	High	The teacher explained why the solution method of a student was inappropriate. The teacher explained there was a direct proportion between a sector's	High	The focus of the segment was making sense of what they needed to calculate the area of sectors. The teacher explained the relationship between the central angles and the areas of sectors of the same circle. The teacher started a discussion about what is

				central angle and area.		the unit of pi but it was not finalized.

Table 27. (continues)

S4	NP	No linking occurred.	Low	The teacher briefly explained that pi does not have a unit like cm or m because it is a ratio and constant number.	Mid	The teacher explained the meaning of the pi. A student used the relationship between numbers to solve a question and the teacher re-explained his solution.
S5	NP	The shapes do not count as a representation in geometry.	High	The teacher explained the reasonableness of the area formula of the sectors. He developed knowledge of why the area formula of sectors includes 360 in the denominator and the central angle of sectors as multiplicative with the students.	High	A student used her knowledge about the perimeter of sectors to explain how to find the area of sectors. The focus of the segment was making sense of the area formula of sectors.
S6	NP	The teacher reminded the area formula that they constructed and with it as an algebraic expression	NP	The teacher reminded the area formula that they constructed and with it as an algebraic expression and students copied writings on	NP	The teacher reminded the area formula that they constructed and with it as an algebraic expression and students copied writings on the board.
		and students copied.		the board.		

Table 27. (continues)

S7	NP	The teacher talked about the history of mathematics while students were copying.	NP	The teacher talked about the history of mathematics while students were copying.	NP	The teacher talked about the history of mathematics while students were copying.
S8	NP	The shapes do not count as a representation in geometry.	Mid	A student explained why his solution method was true in detail.	Mid	A student explained the whole-quarter relation by using the central angles of the circle and the sector.
S9	NP	The shapes do not count as a representation in geometry.	Mid	The teacher explained which solution methods were appropriate.	Mid	The teacher explained which solution methods were appropriate.
S10	NP	The shapes do not count as a representation in geometry.	NP	No mathematical explanation was offered by the teacher or students.	NP	Nothing related to sense-making happened.

Table 28.Second observation of Efe: Scores and evidence of Richness of Mathematics-II

	Multiple Procedure or Solution Methods		Patterns and Generalization		Mathematical Language		Over all
Segment	Score	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**	**Score**
S1	NP	There was no evidence of multiple procedures and solution methods.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Low	Low-density mathematical language used.	Low
S2	NP	There was no evidence of multiple procedures and solution methods.	Mid	They generalized that the central angle was necessary to find areas of sectors but it was not finalized.	High	The teacher used mathematical language with moderate density. The teacher pressed students to use central angle instead of the word angle.	Mid
S3	NP	They used the area formula of the circle. They used direct proportion to find the area of sectors.	High	They generalize to find the area of a sector, (1) central angles of sectors and the radius are needed(2) direct proportion can be applied (3) area of a sector and central angle are directly proportional.	Mid	The teacher used mathematical language with moderate density.	High

Table 28. (continues)

S4	High	A student solved the question by using relationships between numbers. The teacher solved the same question by using direct proportion. The teacher explained how to choose which method to use.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The teacher used mathematical language with moderate density.	High
S5	High	The teacher repeated student solution conducted in the previous segment and compared it with his solution method. A short discussion occurred.	High	They worked on developing generalizations to find the area of sectors.	High	The teacher warned a student for accurate use of the words area and perimeter. The teacher used mathematical language with moderate density.	High
S6	NP	The teacher reminded the area formula and with it as an algebraic expression and students copied writings on the board.	NP	The teacher reminded the area formula that they constructed and with it as an algebraic expression and students copied writings on the board.	Low	The teacher reminded the area formula that they constructed and with it as an algebraic expression and students copied writings on the board.	NP

Table 28. (continues)

S7	NP	The teacher talked about the history of mathematics while students were copying.	NP	The teacher talked about the history of mathematics while students were copying.	Low	The teacher talked about the history of mathematics while students were copying.	NP
S8	Mid	The teachers solved the question by using direct proportion and the whole-quarter relationship.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The teacher used mathematical language with moderate density.	Mid
S9	High	The teacher introduced three different solutions for a single question and discussed which method was easier. The teacher solved another question using the area formula of the sectors and direct proportion.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Mid	The teacher used mathematical language with moderate density.	Mid
S10	Low	A student mentioned which solution method was easier but it was not detailed.	NP	No generalization was developed. No pattern was discovered, and no definition was built.	Low	The low density of mathematical language use.	Low

The teacher generally used mathematical language in moderate density. He was careful about the accurate use of mathematical terms and pressed the students for accurate use. Therefore, six segments received “Mid” or “High” the mathematical language score. The first segment scored “Low” for the Mathematical Language because the teacher introduced the context and used a few mathematical terms. Segments 6, 7, and 10 also scored “Low” because the teacher repeated what they learned and talked about the history of mathematics while students were taking notes.

The depth of the mathematics offered to students in each segment scored using an overall score. Segments 6 and 7 received “NP” for the overall score because the students were copying the writings on the board, and it was not possible to observe. Segment 1 and segment 10 scored Low”. During the development part of the instruction, both the teacher and students offered explanations about solution methods, developed generalizations, and made an effort to explain the relationship between numbers. Therefore, segments 3,4,5 scored “High” for the overall richness code. Segments 2.8, and 9 received a “mid” overall richness score.

The evidence of dimensions and score of segments were given in the Table 27 and Table 28.

4.7.2.2. Working with Students and Mathematics

In this section, the score and evidence of the Working with Students and Mathematics are given. The Working with Students and Mathematics dimension is highly related to teachers' knowledge of content and students. If the teacher knows possible students’ errors and difficulties related to the content that is taught, he can plan pre-remediation activities. The teacher also can hear, understand, and use students’ contributions to develop mathematics. The segments coded “NP” or “Low” for both Remediation of Students Errors and Difficulties and Teacher Uses Students Mathematical Contribution. The distribution of segment scores and subdimension of Working with Students and Mathematics is given in Figure 17.

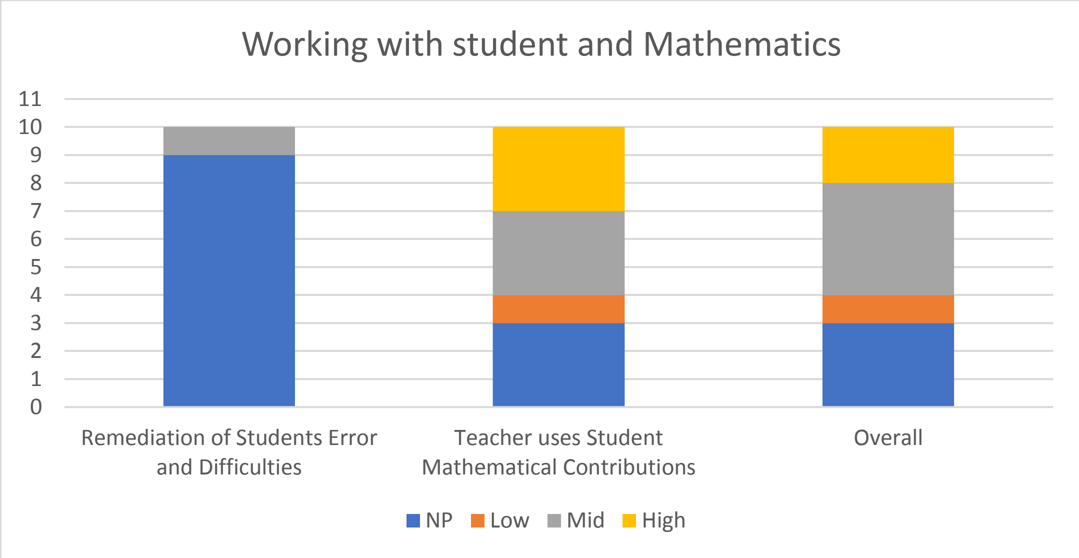


Figure 17. Distribution of Working with Student and Mathematics scores

Evidence of Working with Students and Mathematics and scores of the segments are given in the table below. Eight segments were coded "NP", one segment was coded "Low", and one segment was coded "Mid" for Remediation of Student Errors and Difficulties. In eight of segments no student errors or difficulties were observed. In segment 3, a student conducted an unnecessary operation. The teacher warned the students and explained the meaninglessness of his operation. Also, in segment 3 a pre-remediation occurred. The teacher said a student in a different class wrote π= 3 cm and asked the students if it was true. The teacher explained number what pi was and said it did not have a unit of measure. Therefore, the third segment received a "Mid" score. In segment 9, a student asked what α means.

During the instruction, the teacher interacted with students. Although the teacher's talk was dominant, Efe was willing to use students' substantial contributions to develop mathematics. In the first five segments, they work on how to find the area of a sector. the students actively participated and the teacher highlighted correct student ideas and used them to construct the content. So, For the Teacher uses Student Mathematical Contributions code, the segments receive "Mid" or "High" scores. In the second hour, the students took notes, then they solved questions. Segments 6, 7, and 10 scored "NP". Since the teacher identified a student with his solution and used

Table 29. Second observation of Efe: Scores and evidence of Working with Students and Mathematics

	Remediation of Student Error and Difficulties		**Teacher Uses Students Mathematical Contribution**		**Overall**
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**
S1	NP	No misunderstanding or difficulties with the content occurred.	Mid	The teacher identified the key idea of a student and explained its reasonableness. S: He put a price per square centimeter or square meter. T: The sentence "He put a price per square centimeter or square meter." Is very good. … For a painter, the important thing is the surface that he would paint. The teacher developed the math and connected the students' explanation to the area content.	Mid
S2	NP	No misunderstanding or difficulties with the content occurred.	Mid	The teacher used students' ideas to develop math. T: What do we need to find the area of a sector? … S: We can measure the angle of the black, blue, and red regions. S: We need the radius.	Mid

Table 29. (continues)

S3	Mid	A student calculated the perimeter of the circle and sectors. Then he calculated the radius again. Lastly, he calculated the area of the circle and sectors. The teacher said finding the perimeter was unnecessary find the area when the radius was given. A pre-remediation occurred. The teacher said a student in a different class wrote π=3 cm and asked if it was true.	High	The teacher identified a student with the idea “$r^2.\pi$ shows the area of 360 degrees” and used the student's idea to forward the instruction.	High
S4	NP	No misunderstanding or difficulties with the content occurred.	High	The teacher identified a student’s method “Sina said that “Sina said that...” and re-explained the students the solution method again. They used the student's idea in the following segment “I think Sina’s ideas were very good. Mert said the central angle of a sector and its area is directly related.	High
S5	NP	No misunderstanding or difficulties with the content occurred.	Mid	A student's explanation was very close to the area formula of the sector. The teacher said, “I understand what you mean. You are close. You can express it more understandable.”. He did not develop the student’s explanation. He asked for new ideas. The teacher referred to Sina’s idea and re-explained it to solve the question.	Mid

Table 29. (continues)

S6	NP	No misunderstanding or difficulties with the content occurred.	NP	The segment was dominated by the teacher talk.	NP
S7	NP	No misunderstanding or difficulties with the content occurred.	NP	The segment was dominated by the teacher talk.	NP
S8	NP	No misunderstanding or difficulties with the content occurred.	High	The teacher identified a student method which was using the relation between 90 degrees sector and the quarter to find the area of a sector.	Mid
S9	Low	A student asked what α was. The teacher explained that it was a symbol used to express the angle.	Low	The teacher explained why it was inappropriate to use a student's method (using multiplicative relations) that they used before.	Low
S10	NP	No misunderstanding or difficulties with the content occurred.	NP	The segment was dominated by the teacher talk.	NP

it to solve the questions as a second method, the segment received a “High” score. During segment 9, the teacher referred to a student’s solution method and explained why it was inappropriate to use that solution method. So, the segment received a “Low” score, because the explanation was about the procedures of the question.

For overall Working with Students and Mathematics, the segments got “NP”, “Low”, “Mid” and “High scores. In the first hour of the instruction, the teacher and the students interacted to find out how to calculate the area of the sector and the segments of the first hour scored “Mid” or “High”. However, in the second hour, the segment dominated by teacher’s talk and the teacher solved the question himself. Three segments of the second hour scored “NP” and one segment scored “Low”. Only one segment received a “Mid” score.

This instruction received a very close score for Richness of Mathematics and Working with Students and Mathematics dimensions. The student engaged in rich mathematics in the first hour of the instruction, and the teacher developed mathematics with the ideas of the students. As a result, both dimensions scored similar.

The score of segments and evidence were given in the Table 29.

4.7.2.3. Common Core Aligned Student Practices (CCASP)

Common Core Aligned Student Practice dimension includes four subdimensions which are Students Provide Explanations, Students Mathematical Questioning and Reasoning (SMQR), Students Communicate about the Mathematics of the Segment, Task Cognitive Demand, and Students Work with Contextualized Problems. CCASP focuses on evidence of student involvement in the tasks. It tries to capture the extent to which students engage in and work with the mathematics of the segment. Student explanations, student mathematical questions, students' reasoning, tasks, and problems that students work with are scored for the CCASP dimension. The tasks and the problems that the teacher selects to use during instruction are affected by teachers’ knowledge of content and teaching and teacher knowledge of content and students. If the teacher knows what is easy and what is difficult for his students, he would select tasks that support students' learning.

All the dimensions of CCASP were scored. The distribution of segment scores is given in the Figure 18.

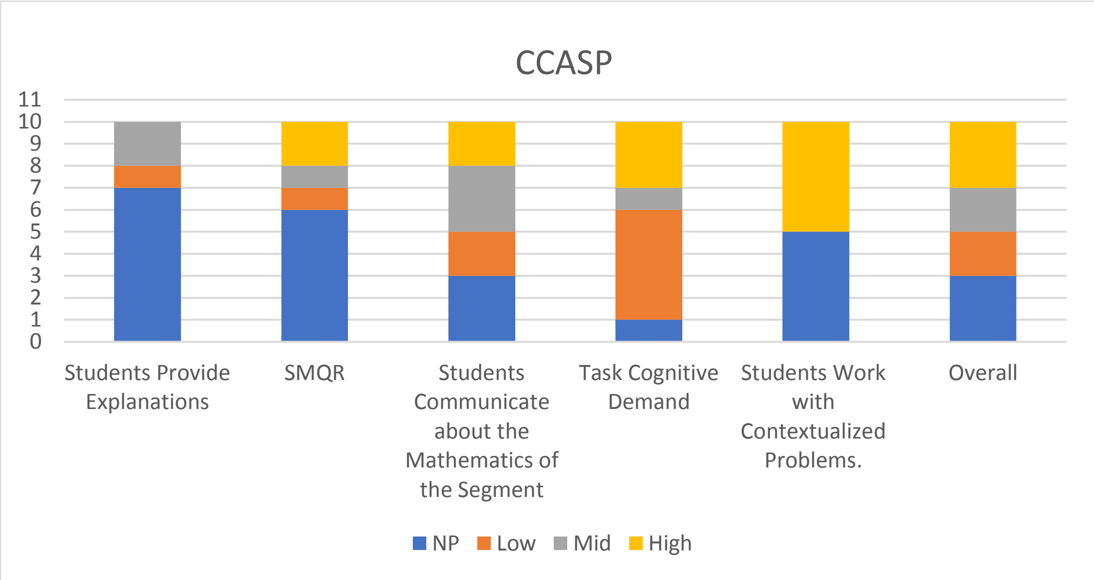


Figure 18**.** Distribution of CCASP scores

Evidence and segment scores of the CCASP are presented in the Table and Table. For the Students Provide Explanations dimension seven segments scored "NP". Students' explanations were observed in segments 2, 5, and 8. In segment 2 a student explained the chance of stopping the Wheel of Fortune in the blue sector. His explanation was more than brief and scored "Mid". In the fifth segment, a student indicated the area formula of the sector. The teacher asked students to clarify what the formula meant. The student's explanation was not complete and received a "Low" score. Another student explanation occurred in segment 8. The student explained why his solution worked.

Efe's instruction was generally dominated by the teacher's talk. Students' contributions were not frequent. Therefore, six segments scored "NP", one segment scored “Low”, one segment scored “Mid” and two segments scored “High” for the SMQR dimension. In segments 2, 3, and 5 students’ explanations occurred. The second segment included only one instance of student explanation and scored “Mid”. In the third segment, student contributions were frequent and substantial. So, the segment scored “High”. In segment 5, a student made a connection between the previous learning and the new content. She said the length of the arc and the central angle of the sector were directly proportional. The area of the sector and its central angle were directly proportional, too, and they could write a direct proportion as they did to find the length of the arc. Some other student explained their idea about how to find the area of the sector and the segment received a “High” score. In segment 8 a student said they could divide the total by 4 to find the area of the sector with 90 degrees because the sector was one-fourth of the circle. the student’s justification was the only reasoning that occurred in the segment and scored “Low”.

The teacher engaged students in the lesson by asking questions. The instruction went on the teacher’s questions and students’ answers at some points. So, For Students Communicate about the Mathematics of the Segment dimension three segments were scored “NP”. Segments 6 and 7 scored “NP since they were dominated by the teacher’s talk while students taking notes. In segment 10, the teacher solved the question by explaining the solution steps, so, it scored also scored “NP”. Segments 1 and 9 scored “Low”. In the first segment, the teacher presented the problem situation and some students contributed to the instruction briefly. In segment 9, students described the solution methods partially. The teacher clarified and developed the students’ methods. Segments 2, 4, and 8 scored “Mid”. In segment 2, some brief student contributions accompanied a student explanation. Segment 3 and segment 5 scored “High” because substantive student contribution occurred.

The teacher started the instruction by presenting a real-world problem. Students worked on this problem through the five segments. Therefore, the first five segments scored “High” for the Student Work with Contextualized Problems dimension. In the fifth segment, they developed the area formula of the sector and talked about different solution methods. In segments 6 and 7, the teacher summarized what they did and talked about the history of math while students took notes. So, they scored

"NP". Through the last three segments, students worked on pure geometry tasks and solved no contextualized problem. As a result, they scored "NP".

The first segment started with the presentation of a real-world problem. The segment was dominated by the teacher's talk and a few students' answers occurred. So, received a "Low" score for Task Cognitive Demand. In the second segment, the teacher developed the problem and asked students how to calculate the area of sectors and what was necessary to know to find the area. It was a highly cognitively demanding task and segment 2, and segment 3 scored "High". Students explained their ideas about how to calculate the area of the sector and segment 4 received a "Mid" score. In the fifth segment, a student drew a connection between previous learning and the new concept. Therefore, segment 5 scored "High". Segments 6, 8, 9, and 10 scored "Low" because students work on simply pure geometry tasks" that required low cognitive work. The only segment that scored "NP" was segment 7.

Overall CCASP that scored students' involvement in doing mathematics was scored "NP" in three segments. Segments 6 and 7 scored "NP" because these segments were dominated by the teacher's talk and only some insignificant student contributions occurred. The teacher solved the questions o, and the tenth segment scored "NP", too. Segments 1 and 9 received a "Low" score since few brief student contributions were observed. While segment 4 and 8 received a "Mid" and segments 2, 3, and 5 scored "High". The score of segments were presented in Table 30 with evidence.

Table 30. Second observation of Efe: Scores and evidence of CCASP-I

	Students Provide Explanations		**Student Mathematical Questioning and Reasoning (SMQR)**		**Students Communicate about the Mathematics of the Segment**	
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**	**Evidence**
S1	NP	No student explanations were observed.	NP	No student explanation, questioning, or reasoning was presented.	Low	Some brief student contributions were presented. S: For example, a 180-degree sector costs 200 TL, how much cost that sector?
S2	Mid	A student explained that the probability of stopping the wheel fortune on the blue area was the smallest because it had the smallest area.	Mid	A student explained that the probability of stopping the wheel of fortune on the blue area was the smallest because it had the smallest area.	Mid	Some brief student contributions were presented. S: We can measure the angle of the black, blue, and red sectors. S: If you find one of them you can find the other. A student explained that the probability of stopping the wheel of fortune on the blue area was the smallest because it had the smallest area.

Table 30. (continues)

S3	NP	No student explanations were observed.	High	A student formed a conclusion that if the total area of the circle covers 360 degrees, the central angle of the sector could be used to find the area of a sector. Two students commented on measuring unit of π: "It cannot be cm because we do not know the exact value of the π." and "Since the digits of the π are infinite, it cannot be a length".	High	A student formed a conclusion that if the total area of the circle covers 360 degrees, the central angle of the sector could be used to find the area of a sector. Some brief student contributions were presented. T: …. How can I find? S: Direct proportion. : …how do we know if two quantities are directly proportional or inversely proportional? S: If they increase or decrease with a constant ratio.
S4	NP	No student explanations were observed.	NP	No student explanation, questioning, or reasoning was presented.	Mid	A student presented his solution to the board. Some brief student contributions were presented. T: How did you find it? S: I divided 300 by 60. T: What is the name of your method? S: Proportion.

Table 30. (continues)

S5	Low	A student stated the area formula of the sector. The teacher wanted her to explain what the formula meant. Her explanation was not complete.	High	A student connected the area of the sector to the perimeter of the sector. She said, that to find the area of the sector, they could use the method that they used to find the perimeter of the sector. Students explained how to find the area of the sector.	High	A student connected the area of the sector to the perimeter of the sector. Some students offered solutions to the question. Some brief student contributions were presented.
S6	NP	No student explanations were observed.	NP	No student explanation, questioning, or reasoning was presented.	NP	The segment was dominated by the teacher's talk. Some infrequent one-or-two-word student contributions occurred.
S7	NP	No student explanations were observed.	NP	No student explanation, questioning, or reasoning was presented.	NP	The segment was dominated by the teacher's talk. Some infrequent one-or-two-word student contributions occurred.

Table 30. (continues)

S8	Mid	A student explained that to find the area of the 90-degree sector, they could divide the total area by 4. Since the 90-degree sector was one-fourth of a circle.	Low	A student engaged in reasoning “Since the central angle of the sector is 90 degrees, the area of the sector is one-fourth of the area of the circle.	Mid	A student engaged in reasoning “Since the central angle of the sector is 90 degrees, the area of the sector is one-fourth of the area of the circle. Some students expressed their solution methods, but the teacher conducted the solution on the board. Some brief student contributions were presented.
S9	NP	No student explanations were observed.	NP	No student explanation, questioning, or reasoning was presented.	Low	Students described the solution steps partially.
S10	NP	No student explanations were observed.	NP	No student explanation, questioning, or reasoning was presented.	NP	No student contributions occurred.

Table 31. Second observation of Efe: Scores and evidence of CCASP-II

	Task Cognitive Demand		Student Work with Contextualized Problems		Overall
Segment	**Score**	**Evidence**	**Score**	**Evidence**	**Score**
S1	Low	The segment was dominated by the teacher's talk and one or two student explanations occurred.	High	The teacher presented a word problem. The students expressed ideas about how to find the cost of painting sectors.	Low
S2	High	The students tried to establish what was necessary to calculate the area of a sector. It was a highly cognitively demanding task.	High	They continued to work on the problem given in the first segment. The students present and justify their ideas about how to find the area of a sector. They also discussed what was necessary to find the area.	High
S3	High	The students tried to establish what was necessary to calculate the area of a sector. The teacher and students also discussed how to calculate the area. They were highly cognitively demanding tasks.	High	They continued to work on the problem given in the first segment. The students present and justify their ideas about how to find the area of a sector. They also discussed what was necessary to find the area.	High

Table 31. (continues)

S4	Mid	The students presented their solution strategy to find the area of the sectors and the cost of painting	High	They continued to work on the problem given in the first segment. They calculated the area of the sectors and the costs of the painting.	Mid	
S5	High	A student drew a connection between the solution method of the sectors' perimeter and the solution method of the sectors' area. They continued to discuss how to calculate the area of the sector.	High	They continued to work on the problem given in the first segment. They calculated the area of the sectors and the costs of the painting.	High	
S6	Low	Direct instruction with SMQR input at some point.	NP	They talked about the formula of the sectors. Then, the students took notes.	NP	
S7	NP	The segment was dominated by the teacher's talk and the students took notes.	NP	The segment was dominated by the teacher's talk and the students took notes.	NP	

Table 31. (continues)

S8	Low	They worked on low cognitive demanding tasks. They solved the question by using more than one method (by application of the area formula of the sectors, by using the relation between 90-degree and whole angle, and by using the direct proportion)	NP	They worked on pure geometry tasks.	Mid
S9	Low	Direct instruction with a few student contributions. The teacher solved the questions himself.	NP	They worked on pure geometry tasks.	Low
S10	Low	Direct instruction with a few student contributions. The teacher solved the questions himself.	NP	They worked on pure geometry tasks.	NP

CHAPTER 5

CONCLUSION and DISCUSSION

This study provides a detailed look inside the mathematics instruction of two middle school mathematics teachers and aspects of instruction that teachers highlighted in group discussion. The group discussions were conducted almost one month before the instruction. Teachers shared their own teaching experiences. That is, they talked about their teaching methods, activities they implemented, problems they solved, and difficulties their students faced related to the area of circle and sector. Two inconsecutive lessons of each teacher were videotaped and then analyzed using the MQI framework. The MQI enables the observer to investigate the instruction and analyze the mathematical quality of the instruction. (Kane & Staiger, 2012). Instructions took place in the seventh-grade classrooms. Lessons were divided into 7-minute segments for analysis.

The lesson was introduced and developed by the teacher and the teachers got the highest in the Richness of Mathematics sub-dimensions while teaching the area of the circle. The distribution of richness scores of Ali's instructions' segments was presented in Figure 19 and Efe's instructions' segment scores were presented in Figure 20. The finding of this research about the MQI dimension is similar to the finding of Adkins (2017). As seen in the Figures, teachers used the Linking Between Representations dimension only in 6 segments out of 40 segments and only one segment received a "High" score. The teachers were good at explaining the content and constructing a lesson to make sense of the mathematics. The segments received an Explanation score for 26 segments and 18 of these segments scored "Mid" or "High". The teacher put effort into making students make sense of the area formula of the circle and they received a Mathematical Sense-Making score for 29 segments and 20 of these segments scored "Mid" or "High".

Multiple Procedures and Solution Methods were observed only 3 times in total while teaching of circle's area. However, in the teaching sector's area, multiple solution methods were observed more frequently. The reason for this result can be the mathematics curriculum of middle school. In the mathematics curriculum (MEB, 2018), the use of direct proportion as a second solution method while teaching sectors' area was suggested. No second way was offered to find the area of the circle.

Another richness dimension that was used the least is Pattern and Generalizations. Teachers developed generalizations in 10 segments out of 40 segments. The teacher generalized the area formula of circles and sectors. While solving questions, they introduced no problem that required to development of a pattern or a generalization.

They also used mathematical language carefully and precisely. Only in a few instances of floppy use of language was observed. Efe used the word "regular shape" while he was talking about convex polygons and Ali used "angle" instead of "central angle". The study by Adkins (2017) presented similar results that teachers were careful about mathematical language.

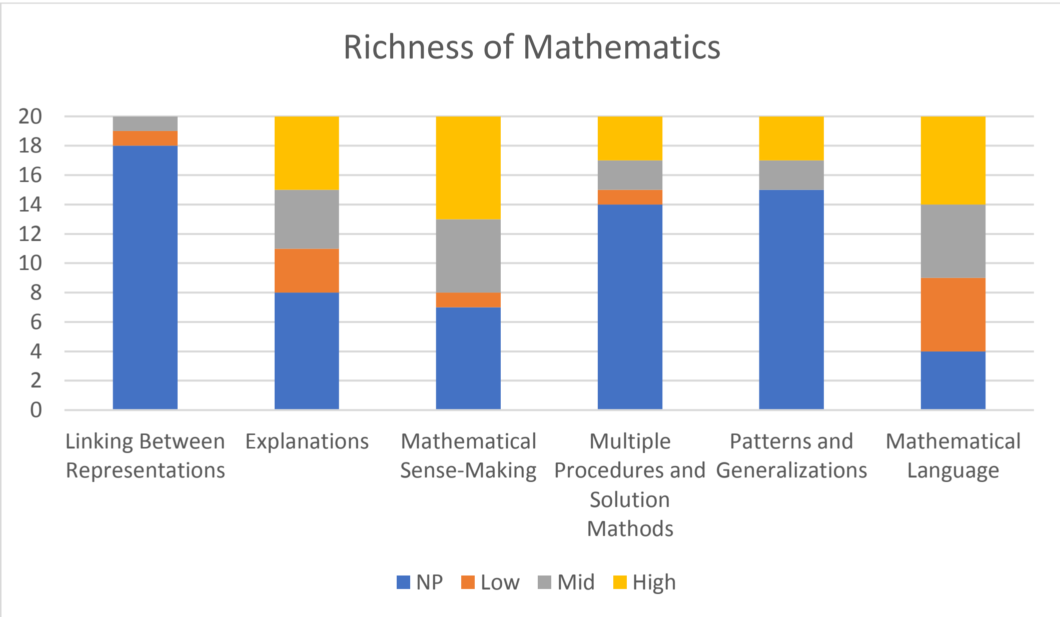


Figure 19. Distribution of Richness score in Ali's instructions

The use of Explanations and Mathematical Sense-Making shows that the teacher knew the content they teach. The findings of the group discussions support this result. The teachers explained the content, offered more than one teaching method, and talked about the important terms of the content such as pi, central angle, area, perimeter, and measurement unit.

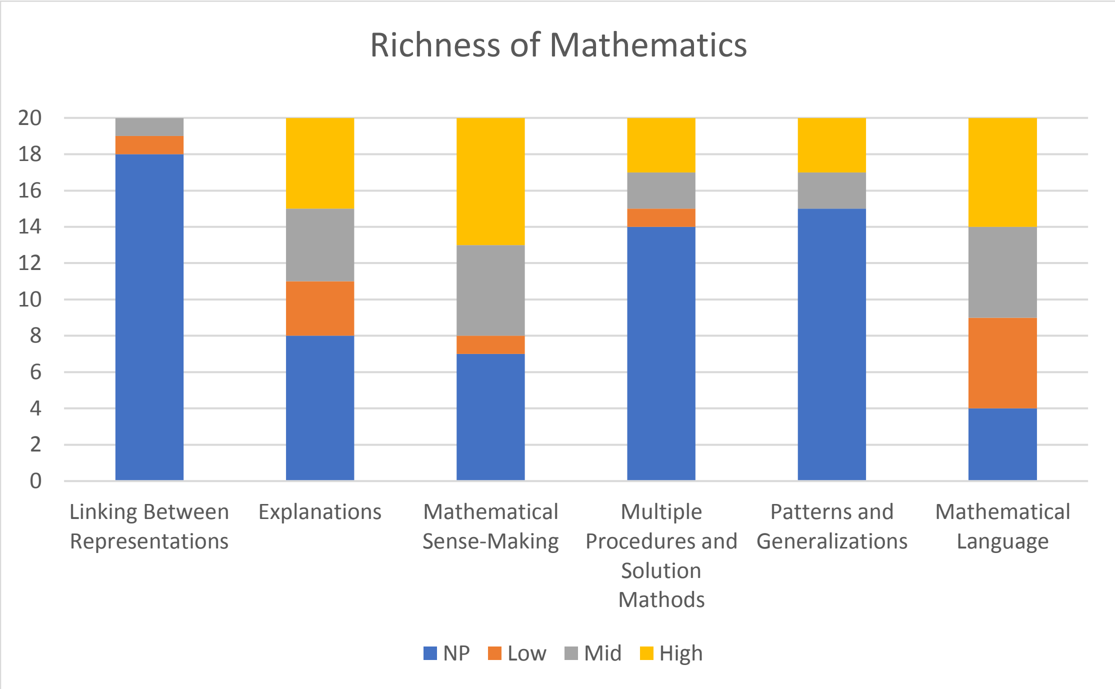


Figure 20.Distribution of Richness score in Efe's instructions

There was only one subject that teachers had insufficient knowledge. Both teachers had insufficient knowledge about the unit of measure of the area. Both Ali and Efe explained that the measure of the area is a meter square because of the multiplication of two lengths. The research in the literature indicated that teachers also faced difficulty in understanding unit of area measurement (Kordaki & Potari, 1998; Ma, 2010; Outhred & McPhail, 2000; Outherd & Mitchelmore, 2004; Zacharos, 2006; Reinke, 1997). As a result, students face difficulty in understanding the measurement unit of the area (Kamii & Kysh, 2006; Kordaki & Potari, 1998; O'Keefe & Bobis, 2008).

Teachers corrected students' errors and used students' contributions in Working with Students and Mathematics dimension (See Figure 21 and Figure 22). In the pre-instruction group discussion, Ali mentioned possible student errors and difficulties. He knew the content that he was going to teach and his students. He said his students faced difficulty in distinguishing the area and perimeter concepts and their formulas and the studies in the literature stated similar results (Smith et al., 2013; Smith et al., 2016; (Olkun et al., 2014). At the beginning of the lesson, Ali defined the area and briefly reminded students what the perimeter was. However, during the instruction, students used the perimeter formula to find the area, or they divided the area by 2 to find the radius. In this situation, the teacher wrote down the area formula and performed the operations, but the students' confusion continued throughout the lesson. The instants sored "Low" for remediation of student errors and difficulties dimension. Another student difficulty that Ali mentioned was difficulty in performing operations with r^2. Students multiplied the radius by 2 to find the r^2, they divided the total area by 2 to find the radius. The teacher showed the procedure and explained that r^2 means r times r. However, students' confusion continued. The literature indicated some similar result that students face difficulty in justifying the area formula of the circle and they memorize and use it without understanding (Demir et al., 2022; Lehmann, 2024; Rejeki & Putri, 2018).

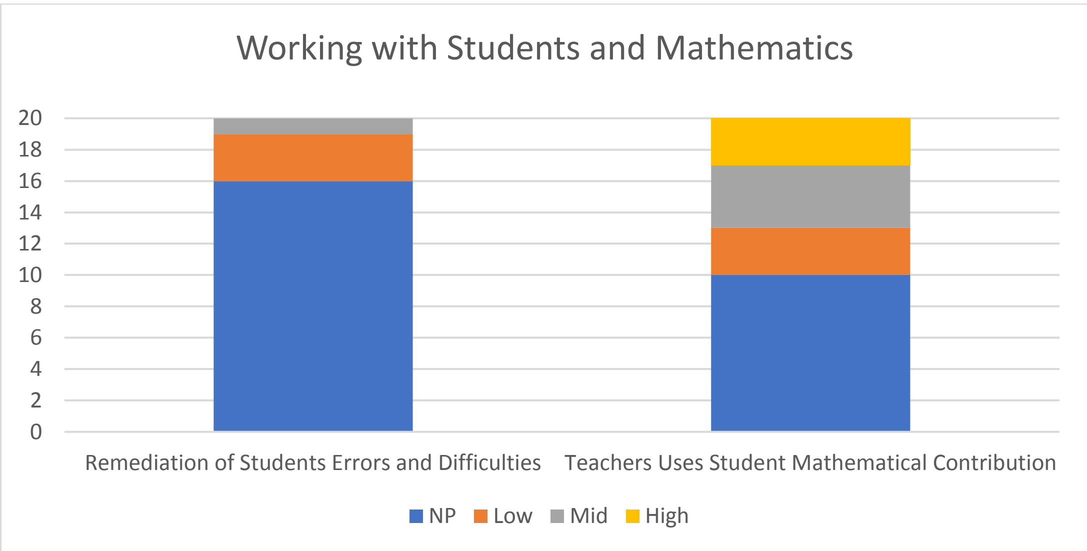


Figure 21.Distribution of Working with Students and Mathematics in Ali's instructions

The other difficulty that Ali said was difficulty in multiplication with decimals. When Ali asked students to use the value of pi as 3.14, students said they could not perform the multiplication because they did not know how to multiply with decimals. Ali said they had covered the topic in fifth grade. The teacher knew the curriculum and planned a lesson using previous learning. However, the students did not remember their previous learning. In the group discussion, Ali said these difficulties were related to a lack of students' previous knowledge and he did not have time to cover previous topics. Also, he said he did not want to lose the students' attention who already knew previous content. The reasons for the difficulties in the multiplication of decimals are not having adequate knowledge about the numerical value of the decimals and misplacing the decimal point (Brueckner, 1928). In Ali's classroom, some students face difficulty in the numerical value of the decimal and in Efe's classroom, some students face difficulty in putting decimal points after multiplication.

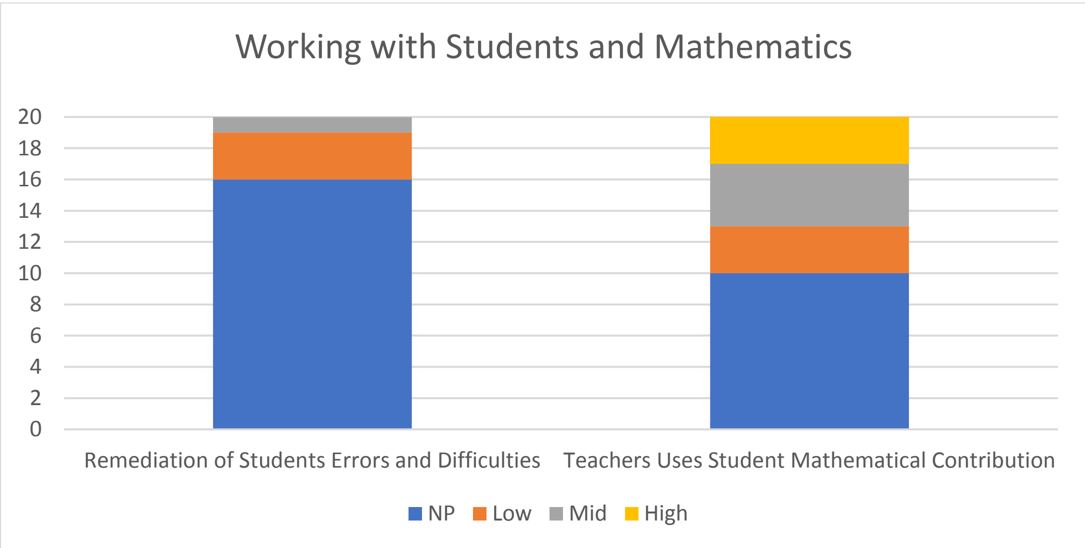


Figure 22.Distribution of Working with Students and Mathematics in Efe's instructions

Students in Ali's classroom faced difficulty in understanding the meaning of pi. Ali did not explain what the pi was. He said they discussed the pi a week before.

Although Ali's behavior may be scored as a lack of content knowledge, the group discussion data showed he knew the pi. During group discussion, they talked about pi for a while, and Ali also contributed to this discussion and said while teaching the perimeter of the circle, he mentioned that pi is the ratio of a circle's perimeter to its diameter. However, Efe explained it was a ratio and gave more information about pi. Although both teachers knew the pi, only Efe explained what pi was.

Effective teachers know and respond to the needs of their students (Anthony & Walshaw, 2009). The participant teachers heard their students and used students' contributions to move the instruction forward. However, substantial student contributions were very rare. The teachers let students to share their solutions, to explain their ideas, and to discuss about the content. In Ali's classroom, the students faced a lack of previous knowledge, and their contributions were very limited. However, Efe's talk dominated the instructions, and students had little opportunity to share.

Teachers used direct instruction and question-answer techniques for all instructions. Teachers used direct instruction at the beginning segment of the instruction and techniques such as question-answer and discussion (Yeo, 2008). Ali used the question-answer technique throughout instructions, but students' contributions were generally procedural, and these contributions were one-or-two-word contributions. Efe gave very little opportunity for students to share their ideas. Student explanations and mathematical reasoning were rarely scored. Therefore, the least used dimension was CCASP (Adkins, 2017). The distribution of CCASP scores in Ali's instruction were presented in Figure 23 and İn Efe's instructions were presented in Figure 24. The teachers used contextualized problems to introduce the topic. However, they heavily scaffold students when the tasks force students cognitively.

Both instructions of Efe were dominated by the teacher's talk. Ali used the question-answer technique more frequently. However, students' contributions to the development of mathematics were high in Efe's classroom. The students' explanations were rare but substantial. In Ali's classroom, there were many instances in which students talked about non-mathematical topics, and students' explanations were brief or wrong. Also, in Ali's classroom students conducted operations with the given numbers without thinking about the context of the problem.

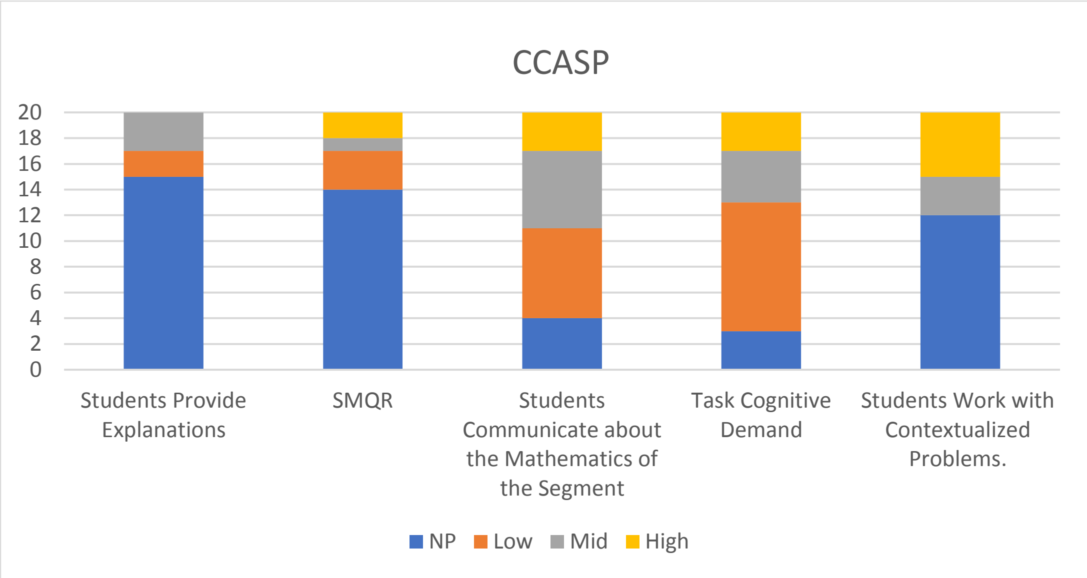


Figure 23.Distribution of CCASP scores in Ali's instructions

The cognitive levels of the tasks were high when the teacher introduced mathematical content. However, teachers highly scaffolded students, they offered a way of solving and reducing the task's cognitive level. The studies in the literature indicated similar results (Alcazar, 2017; Henningsen & Stein; 1997; Sarpkaya, 2011). The cognitive level of the tasks was lowered by the teachers by scaffolding. Also, the cognitive levels of the questions that they solved were low. Henningsen and Stein (1997) stated reasons for the decrease in cognitive demands of tasks during instruction as (1) converting the problem into a routine problem, (2) removing the challenging aspect of the problem (3) not giving the student enough time to develop cognitive demand (4) not focusing on the completeness or accuracy of the answer, and (5) classroom management problems. High-level cognitive demand problems can be perceived as risky and unclear by students, and the teacher reduces the complexity of the task to decrease anxiety (Doyle, 1998). In the observed lesson, the teachers did not give enough time for students to think or produce a solution method for high-level cognitive demanding tasks. Also, the teachers reduce the cognitive level by scaffolding. They gave hints and offered solution methods. In Ali's classroom, the classroom management problems are also a reason for the decrease in the cognitive level of the tasks.

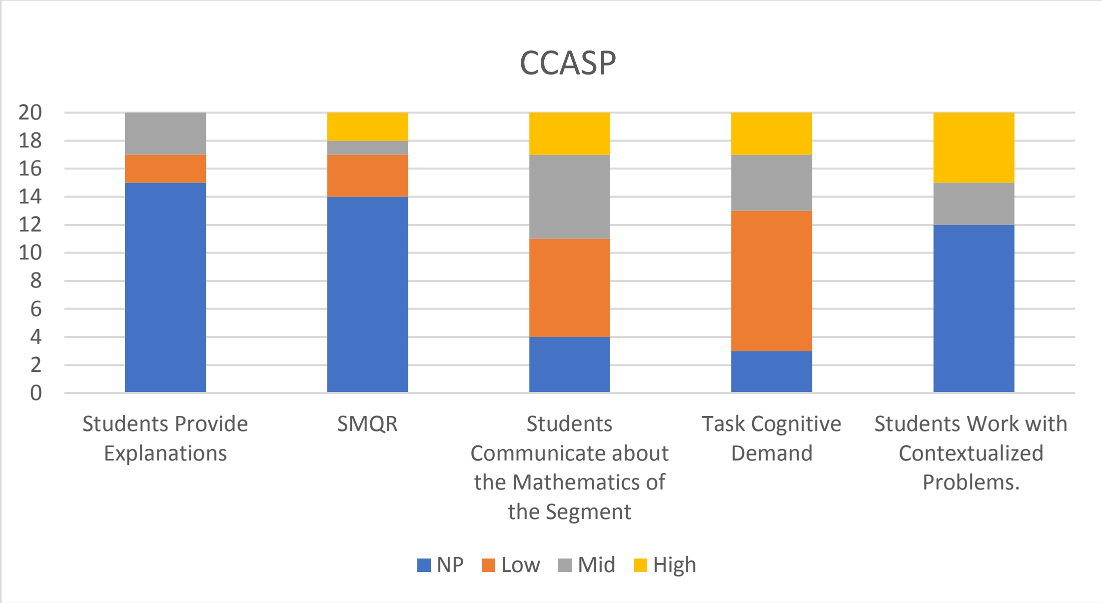


Figure 24.Distribution of CCASP scores in Efe's instructions

During group discussion, Ali said he introduced the area of sectors by using the half and quarter relation. In his instruction, he asked students to find the area of a half circle and two-quarter circles and helped students to realize the direct proportion between the sectors' central angle and the area. He forced students to use a central angle by working with sectors with unknown central angles. Students concluded that they needed to use a central angle. Ali said his students had difficulty with making connections between their previous knowledge and new learning. So, they could not connect their knowledge about the arc's length to the area of a sector.

During group discussion, Efe said using sectors with central angles 60, 90, 120, or 180 to introduce the area of sectors would cause overgeneralization. He claimed students would try to find multiple relations between any central angle and 360 degrees. First, he worked with sectors with unknown central angles and the students interfered they need the central angle of the sector. a student made the connection between the length of an arc and the area of the sector. She said that they could use the same method to find the area of the sector. Efe said his students could refer to the length of the arc and distinguish the direct proportion between the central angle and the area of a sector.

In the group discussion, Efe said he used a real-life context when introducing a new topic. He said using contextualized problems was important to make sense of the mathematics. Both teachers started with a contextualized problem. However, they solved routine questions in the following.

The teacher MKT was not affected by the school distinct but the MQI score of the instruction was affected by the school distinct. The students in Ali’s school came from low-socio economic level families. Many students did not attend the school regularly. So, they missed many mathematical topics and faced a lack of knowledge. In Ali’s classroom the students’ difficulties were related to lack of knowledge and students’ explanations generally were not substantial. The students of Efe’s school come from socioeconomic families but their parents are well-educated. Any instances of students' lack of previous knowledge were observed. The students were easily connected to the new topic and their previous learnings. Hill et al., (2015) stated a similar result indicating that the school environment affects instructional quality.

Ali’s MTK helped him to explain to students the operation $3r^2$=27 with a simple contextualized problem since algebraic notation was new for him and he faced difficulty in conducting operations with algebraic notations He asked a student “If the cost of 3 apples is 27, what is the cost of one apple?”. The student correctly answered the new contextualized question because he was familiar with simple contextualized problems. The context may be helpful or unhelpful for students depending on the context (Leinonen, 2021). In this case the context help student to solve the question which was presented in algebraic notation. Ali new the content and how to express it in a context. The study of Hill et al., (2008) showed that the teachers with high MTK choose examples wisely to provide an equitable opportunity to students.

5.1. Implications

This study aims to investigate the quality of mathematics instructions. The findings of this study have implications for researchers, teacher educators, and teacher education programs. Findings showed that teachers had a rich content knowledge of the content. Therefore, they did explanations and constructed tasks to make sense of the content. However, they rarely used multiple representations and rarely made generalizations. Also, the teachers were aware of student difficulties, but they did not plan any pre-remediation. If the teachers were aware of how to plan pre-remediation tasks, the students may face less difficulty in understanding the content. They also face difficulty with CCASP dimensions. Improving teachers' ability to more effectively apply CCASP dimensions may help students to participate more actively during instruction and allow them to be more skillful in mathematical content. The teacher educators should be aware of quality aspects of mathematics instruction and help pre-service teachers to understand and use this aspect during instruction. also, in-service teachers can inform about MQI dimensions with professional development programs.

5.2. Limitation of the Study

This study has some limitations. I investigated the quality of mathematics instruction using the MQI rubric and some may argue to assess instructional quality with some other observation protocols. I also did not use classroom artifacts but for some researchers, classroom artifacts can be important factors that affect instructional quality. Another limitation is, I observed the lesson of two teachers in a public school. Both of teachers were graduated from Elementary Mathematics Education programs. I also select may participants with purposeful sampling and they might not represent the MSMT family. Therefore, the findings of the study might be different with different teachers.

REFERENCES

Adkins, A. (2017). *Instructional strategies used by effective mathematics teachers in adventist elementary schools in Florida: An MQI analysis.* (Publication No. 10689066.) [Doctoral dissertation, Andrews University]. ProQuest Dissertations & Theses Global.

Alcazar, M. T. M. (2007). *Alignment of cognitive demand: Peruvian National assessment mandated curriculum teaching and textbook in second grade math.* (Publication No. 3267160.) [Master dissertation, University of Delaware]. ProQuest Dissertations & Theses Global.

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the west. Journal of Mathematics Education, 2(2), 147-164

Aslan-Tutak, F. & Köklü, O. (2016). Öğretmek İçin Matematik Bilgisi, In İ. Ö. Zembat, E. Bingölbalı, S. Arslan (Eds.*) Matematik Eğitiminde Teoriler* (701-717). Pegem.

Bair, S. L., & Rich, B. S. (2011). Characterizing the development of specialized mathematical content knowledge for teaching in algebraic reasoning and number theory. *Mathematical Thinking and Learning,* 13(4), 292-321. https://doi.org/10.1080/10986065.2011.608345

Ball, D. L., Hill, H.C, & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator,* 29(1), 14-17, 20-22, 43-46.

Ball, D. L., & Rowan, B. (2004). Introduction: measuring instruction. *The Elementary School Journal*, 105(1), 3-10. https://doi.org/10.1086/428762

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education,* 59(5), 389-407. https://doi.org/10.1177/0022487108324554

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A.,& Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal,* 47(1), 133-180. https://doi.org/10.3102/0002831209345157

Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review*, 48, 16-29. https://doi.org/10.1016/j.econedurev.2015.05.005.

Blazar, D., Litke, E., & Barmore, J. (2016). What does it mean to be ranked a “high” or "low" value-added teacher? Observing differences in instructional quality across districts. *American Educational Research Journal*, 53(2), 324-359 https://doi.org/10.3102/0002831216630407

Blazar, D., & Kraft, M. A. (2017). Teacher and teaching effects on students' attitudes and behaviors. *Educational Evaluation and Policy Analysis*, 39(1), 146-170.https://doi.org/10.3102/016237371667

Blazar, D., Braslow, D., Charalambous, C. Y., & Hill, H. C. (2017). Attending to general and mathematics-specific dimensions of teaching: Exploring factors across two observation instruments. *Educational Assessment*, 22(2), 71-94. https://doi.org/10.1080/10627197.2017.1309274

Boston, M., Bostic, J., & Lesseig, K. (2015). The mathematical quality of instruction protocol for classroom observations. Retrieved from http://k12education.gatesfoundation.org/resource/the-mqi-protocol-for-classroomobservations/

Bobis, J., Higgins, J., Cavanagh, M., & Roche, A. (2012). Professional knowledge of practising teachers of mathematics. *Research in Mathematics Education in Australasia 2008-2011*, 313-341. https://doi.org/10.1007/978-94-6091-970-1_15

Brueckner, L. J. (1928). Analysis of difficulties in decimals. The Elementary School Journal, 29(1), 32-41.

Charalambous, C. Y., Hill, H. C., & Mitchell, R. N. (2012). Two negatives don't always make a positive: Exploring how limitations in teacher knowledge and the curriculum contribute to instructional quality. *Journal of Curriculum Studies*, 44(4), 489-513. doi:10.1080/00220272.2012.716974 https://doi.org/10.1080/00220272.2012.716974

Charalambous, C. Y., & Litke, E. (2018). Studying instructional quality by using a content-specific lens: the case of the Mathematical Quality of Instruction framework. *ZDM*, 50, 445-460. https://doi.org/10.1007/s11858-018-0913-9

Charalambous, C.Y., Praetorius, A.K.(2018). Studying mathematics instruction through different lenses: setting the ground for understanding instructional quality more comprehensively. *ZDM Mathematics Education* 50, 355–366. https://doi.org/10.1007/s11858-018-0914-8

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five traditions*. Sage Publications.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education*, 34, 12-25. https://doi.org/10.1016/j.tate.2013.03.001

Demir, M., Zengin, Y., Özcan, Ş, Urhan, S., & Aksu, N. (2022). Students' mathematical reasoning on the area of the circle: 5E-based flipped classroom approach. *International Journal of Mathematical Education in Science and Technology,* 54(1), 99–123.https://doi.org/10.1080/ 0020739X.2022.2101955

Doyle, W.(1988).Work in Mathematics Classes: the context of students' thinking during instruction. Educational Psychologist, 23,167-180. https://doi.org/10.1207/s15326985ep2302_6

Erdem, Z. Ç., & Gürbüz, R. (2018). Matematik modelleme etkinliklerine dayalı öğrenme ortamında yedinci sınıf öğrencilerinin alan ölçme bilgi ve becerilerinin incelenmesi. *Adıyaman University Journal of Educational Sciences*, 8(2), 86-115. https://doi.org/10.17984/adyuebd.468376

Fraenkel, J. R., Wallen, N. E. & Hyun, H., H., (2012). *How to design and evaluate research in education* (8 ed.). Order Department, McGraw Hill Publishing Co.

Goe, L., Holdheide, L., & Miller, T. (2011). A Practical Guide to Designing Comprehensive Teacher Evaluation Systems: A Tool to Assist in the Development of Teacher Evaluation Systems. *National Comprehensive Center for Teacher Quality*. https://doi.org/10.1007/s11092-010-9107-x

Hangül, T. (2018). *Öğretmen eğitimcilerinin öğretimin matematiksel kalitesine yönelik değerlendirmeleri üzerine bir inceleme* (Publication No:537772) [Doctoral dissertation, Marmara University]. YÖK Ulusal Tez Merkezi.

Harniss, M. K., Stein, M., & Carnine, D. (2002). Promoting mathematics achievement. Interventions for academic and behavior problems II: Preventive and remedial approaches, 571-587.

Henningsen, M., and Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28, 524–549.

Hiebert, E. H. (1981). Developmental patterns and interrelationships of preschool children's print awareness. *Reading Research Quarterly*, 236-260. https://doi.org/10.2307/747558

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for research in Mathematics Education,* 35(5), 330-351. https://doi.org/10.2307/30034819

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. https://doi.org/10.3102/00028312042002371

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. Journal for Research in Mathematics Education, 39(4), 372-400 https://doi.org/10.5951/jresematheduc.39.4.0372

Hill, H. C., Blunk, M., Charalambous, C., Lewis, J., Phelps, G. C., Sleep, L., et al. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, 26(4), 430-511. https://doi.org/10.1080/07370000802177235

Hill, H. C., Charalambous, C. Y., Blazar, D., McGinn, D., Kraft, M. A., Beisiegel, M., ... & Lynch, K. (2012). Validating arguments for observational instruments: Attending to multiple sources of variation. *Educational Assessment*, *17*(2-3), 88-106. https://doi.org/10.1080/10627197.2012.715019

Hill, H. C., & Charalambous, C. Y. (2012). Teacher knowledge, curriculum materials, and quality of instruction: Lessons learned and open issues. *Journal of Curriculum Studies*, 44(4), 559-576. https://doi.org/10.1080/00220272.2012.716978

Hill, H. C., Umland, K. L., & Kapitula, L. R. (2011). A validity argument approach to evaluating valueadded scores. *American Educational Research Journal*, 48(3), 794-83 Assessment, 17(2-3), 88-106. https://doi.org/10.3102/0002831210387916

Hill, H. C., Blazar, D., & Lynch, K. (2015). Resources for Teaching: Examining Personal and Institutional Predictors of High-Quality Instruction. *AERA Open*, *1*(4). https://doi.org/10.1177/2332858415617703

Hill, H. C., & Charalambous, C. Y. (2012). Teacher knowledge, curriculum materials, and quality of instruction: Lessons learned and open issues. *Journal of Curriculum Studies*, 44(4), 559-576. https://doi.org/10.1080/00220272.2012.716978

Hill, H. C., Kraft, M., & Herlihy, C. (2016). Developing common core classrooms through rubric-based coaching: Early findings report. http://cepr.harvard.edu/files/cepr/files/mqi-coaching-researchfindings.pdf. Accessed 10 Sep 2023.

Ho, A. D., & Kane, T. J. (2013). The Reliability of Classroom Observations by School Personnel. Research Paper. MET Project. *Bill & Melinda Gates Foundation*.

Huang, H. M. E., & Witz, K. G. (2011). Developing children's conceptual understanding of area measurement: A curriculum and teaching experiment. *Learning and Instruction*, 21(1), 1-13. https://doi.org/10.1016/j.learninstruc.2009.09.002

Huang, H. M. E., & Witz, K. G. (2013). Children conception of area measurement and their strategies for solving area measurent problems. *Journal of Curriculum and Teaching*, 2(1), 10-26. https://doi.org/10.5430/jct.v2n1p10

Huang, H. M. E. (2014). Third to fourth-grade students' conceptions of multiplication and area measurement. *ZDM*, 46(3), 449-463. https://doi.org/10.1007/s11858-014-0603-1

İnal, K. (2005). Yeni ilköğretim müfredatının felsefesi. Muhafazakar Düşünce Dergisi, 2(6), 75-92.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. Journal of mathematics teacher education, 9(2), 187-211. https://doi.org/10.1007/s10857-005-1223-z

Kamii, C., & Kysh, J. (2006). The difficulty of "lenght x width": Is a square the unit of measurent?.Journal of Mathematical Behaviour, 25, 105-115. https://doi.org/10.1016/j.jmathb.2006.02.001

Kane, T. J., & Staiger, D. O. (2012). Gathering feedback for teaching: combining high-quality observations with student surveys and achievement gains. Research Paper. MET Project. Bill & Melinda Gates Foundation.

Kaya, D., (2019). 6. sınıf öğrencilerinin alan ölçme ile ilgili problem çözme becerileri. *International Journal of Educational Studies in Mathematics*, 6(4), 144-171.

Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., & Baumert, J. (2013). Teachers' content knowledge and pedagogical content knowledge: The role of structural differences in teacher education. *Journal of Teacher Education*, 64(1), 90-106. https://doi.org/10.1177/0022487112460398

Kordaki, M., & Potari, D. (1998). Children's approaches to area measurement through different contexts. *Journal of Mathematical Behavior*, 17(3), 303-316. https://doi.org/10.1016/S0364-0213(99)80065-2

Kraft, M. A., & Hill, H. C. (2017). Developing ambitious mathematics instruction through web-based coaching: An experimental trial. Harvard University Working Paper.

Lehmann, T. H. (2024). Learning to measure the area of circles, Research in Mathematics Education, https://doi.org/10.1080/14794802.2024.2304329

Lehrer, R. (2003). Developing understanding of measurement. *A research companion to principles and standards for school mathematics*, 179-191.

Leinonen, J., Denny, P., & Whalley, J. (2021, February). Exploring the effects of contextualized problem descriptions on problem solving. In Proceedings of the 23rd Australasian Computing Education Conference (pp. 30-39).

Leymun, Ş. O., Odabaşı, F., & Yurdakul, İ. K. (2017). Eğitim ortamlarında durum çalışmasının önemi. Eğitimde Nitel Araştırmalar Dergisi, 5(3), 367-385.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Routledge.

MacNealy, M. S. (1999). *Strategies for empirical research in writing*. New York: Addison Wesley Longman https://doi.org/10.2307/358974

Mangiante, E. (2011). Teachers matter: Measures of teacher effectiveness in low-income minority schools. Educational Assessment, Evaluation and Accountability, 23(1), 41-63. https://doi.org/10.1007/s11092-010-9107-x

Milli Eğitim Bakanlığı [MEB]. (2018). *Matematik dersi öğretim pogramı* (Primay and Middle School 1, 2, 3, 4,5,6,7 and 8 grade). Ankara.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. John Wiley & Sons.

Merriam, S.B. and Tisdell, E. J. 2015. *Qualitative research: A guide to design and implementation* (4th ed.). Jossey Bass.

Mitchell, R. N., & Marin, K. A. (2015). Examining the use of a structured analysis framework to support prospective teacher noticing. *Journal of Mathematics Teacher Education*, 18, 551-575. https://doi.org/10.1007/s10857-014-9294-3

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: The National Council of Teachers of Mathematics.

O'Keefe, M., & Bobis, J. (2008, June 28-July 1). Primary teachers' perceptions of their knowledge and understanding of measurement. *Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, Australia,* 391-398.

Olkun, S., Çelebi, Ö., Fidan, E., Engin, Ö., & Gökgün, C. (2014). Birim kare ve alan formülünün Türk öğrenciler için anlamı. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 29(29-1), 180-195

Outhred, L. N., & Mitchelmore, M. C. (2000). Young children's intuitive understanding of rectangular area measurement. Journal for Research in Mathematics Education, 31(2), 144-167. https://doi.org/10.2307/749749

Outhred, L., & McPhail, D. (2000). A framework for teaching early measurement. In J. Bana, & A. Chap man (Eds.), *Mathematics education beyond 2000* (pp. 487-494). Perth: Mathematics Education Research Group of Australasia Incorporated.

Punch, K. F. (2005). Introduction to social research: Quantitative and qualitative approaches (2nd ed.) Sage.

Raven, J., & Stephenson, J. (Eds.). (2001). Competence in the learning society. New York: P. Lang.

Reinke, K. S. (1997). Area and perimeter: Preservice teachers' confusion. *School Science and Mathematics*, 97(2), 75-77. https://doi.org/10.1111/j.1949-8594.1997.tb17346.x

Rejeki, S., & Putri, R. I. I. (2018). Models to support students' understanding of measuring area of circles. Journal of Physics: Conference Series, 948, 1–8. https://doi.org/10.1088/1742-6596/948/1/ 012058

Rockoff, J. (2004). The impact of individual teacher on student achievement evidence form panel data. The American Economic Review, 94(2), 247-252.

Runnalls, C., & Hong, D. S. (2020). "Well, they understand the concept of area": pre-service teachers' responses to student area misconceptions. *Mathematics Education Research Journal*, 1-23. https://doi.org/10.1007/s13394-019-00274-1

Santagata, R., & Lee, J. (2021). Mathematical knowledge for teaching and the mathematical quality of instruction: A study of novice elementary school teachers. *Journal of Mathematics Teacher Education*, *24*(1), 33-60. https://doi.org/10.1007/s10857-019-09447-y

Sarpkaya, G. (2011). *İlköğretim ikinci kademe cebir öğrenme alani ile ilgili matematiksel görevlerin bilişsel istemler açisindan incelenmesi: Matematik ders kitaplari ve sınıf uygulamalar* (Publication No: 298508) [Doctoral dissertation, Gazi University]. YÖK Ulusal Tez Merkezi.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14. https://doi.org/10.2307/1175860

Smith III, J. P.,Males, L. M., & Gonulates, F. (2016). Conceptual limitations in curricular presentations of area measurement: Onenation's challenges. *Mathematical Thinking and Learning*, *18*(4), 239-270. https://doi.org/10.1080/10986065.2016.1219930

Smith III, J. P.,Males, L. M., Dietiker, L. C., Lee, K., &Mosier, A. (2013). Curricular treatments of length measurement in the United States: Do they address known learning challenges?. *Cognition and Instruction*, 31(4), 388-433.

Simon, M. A., & Blume, G. W. (1994). Building and understanding multiplicative relationships: A study of prospective elementary teachers. *Journal for Research in Mathematics Education*, 472-494.https://doi.org/10.2307/749486

Strand, K. L.(2016). *An investigation into intermediate grades teachers' noticing of the mathematical quality of instruction.* (Publication No. 10065522) [Doctoral disseration, Portland State University]. ProQuest Dissertations & Theses Global.

Stylianides, A. J., & Ball, D. L. (2008). Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education*,11,307-332. https://doi.org/10.1007/s10857-008-9077-9

Teddlie C., Reynolds D. (2000). The international handbook of school effectiveness research. Falmer Press.

Van Dijk, E. M., & Kattmann, U. (2007). A research model for the study of science teachers' PCK and improving teacher education. *Teaching and Teacher Education,* 23(6), 885-897. https://doi.org/10.1016/j.tate.2006.05.002

Yeo, J. K. K. (2008). Teaching area and perimeter: Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, Australia* Vol. 621-627.

Yıldırım, A., & Şimşek, H. (2013). *Sosyal bilimlerde nitel araştırma yöntemleri*. Seçkin.

Yin, R. K. (1994). *Case study research: Designs and methods* (2nd ed.). Thousand Oaks, CA: Sage

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Sage.

Zacharos, K. (2006). Prevailing educational practices for area measurement and students' failure in measuring areas. The Journal of Mathematical Behavior, 25(3), 224. https://doi.org/10.1016/j.jmathb.2006.09.003

APPENDICES

A. MQI 4-POINT VERSION

Classroom Work is Connected to Mathematics	
Score here for whether the focus is on *mathematical content* during half or more of the segment (3.75 minutes or more total for a 7.5-minute segment).	
No	**Yes**
Focus for majority of the segment (at least 3.75 minutes for a 7.5-minute segment) is on non-mathematical topics, or student activities that have no clear connections to developing mathematical content. Examples: • Gathering or distributing materials, other administrative issues • Disciplinary issues that severely impinge upon instructional time • Students doing an activity (cutting, pasting, coloring) that is not clearly connected to mathematics ("bad reform")	Focus is on mathematical content for majority of the segment (at least 3.75 minutes for a 7.5-minute segment). Examples: • Teacher reviewing content from a prior lesson • Teacher introducing content • Students practicing content • Students working on a warm-up problem while teacher takes attendance

Back to Table of Contents

Richness of the Mathematics

This dimension attempts to capture the depth of the mathematics offered to students. The codes within this dimension are grouped into two broad categories: codes that capture the extent to which instruction focuses on the *meaning of facts and procedures* (Linking Between Representations, Explanations, and Mathematical Sense-Making), and codes that capture the degree to which instruction focuses on *key mathematical practices* (Multiple Procedures or Solution Methods, Patterns and Generalizations, and Mathematical Language).

For all codes within this dimension, the aspect of instruction must be substantially correct to count as Low, Mid or High. Richness elements that are not correct should be ignored (though the segment can be still credited for other correct elements within the same code).

Back to Table of Contents

Linking Between Representations

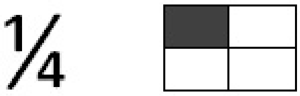
This code refers to teachers' and students' explicit linking and connections between different representations of a mathematical idea or procedure. To count, these links must occur across different representational "families" e.g., a linear graph and a table both capturing a linear relationship. So, two different representations that are both in the symbolic family (e.g., 1/4 and 0.25) are not candidates for being linked.

For Linking Between Representations to be scored above a Not Present:

- At least one representation must be visually present
- The explicit linking between the two representations must be communicated out loud

For Linking Between Representations to be scored Mid or High, two conditions must be satisfied:

- Both representations must be visually present
- The correspondence between the representations must be explicitly pointed out in a way that focuses on meaning (e.g., pointing to the numerator in 1/4, then commenting that you can see that one in the figure, pointing to the four in the denominator, pointing to the four partitions in the whole. "You can see the 1 in the 1/4 corresponds to the upper left-hand box, which is shaded, showing one piece out of four total pieces...")



For geometry, we do not count shapes as a representation that can be linked—we consider those to be the "thing itself." However, links can be scored in geometry if the manipulation of geometric objects is linked to a computation, e.g., showing that two 45-degree angles can be combined to get a 90 degree angle and linking that to the symbolic representation 45 + 45 =90.

Note: If links are made but underlying representation/idea is incorrect, do NOT count as linking between representations.

Not Present	Low	Mid	High
No linking occurs. Representations may be present, but no connections are actively made.	Links are present in a pro forma way; For example, the teacher may show the above figure and state that one quarter is one part out of four. These links will not be very explicit or detailed; both representations need not be present.	Links and connections have the features noted under High, but they occur as an isolated instance in the segment.	Links and connections are present with extended, careful work characterized by one of the following features: • Explicitness about how two or more representations are *related* (e.g., pointing to specific areas of correspondence) OR • Detail and elaboration about the relationship between two mathematical representations (e.g., noting meta-features; providing information about under what conditions the relationship occurs; discussing implications of relationship) These links will be a characterizing feature of the segment, in that they may in fact be the focus of instruction. They need not take up the majority or even a significant portion of the segment; however, they will offer significant insight into the mathematical material.

Back to Table of Contents

Explanations			
Mathematical explanations focus on why, e.g.: • Why a procedure works (or doesn't work) • Why a solution method is appropriate (or inappropriate) • Why an answer is true (or not true) • In geometry: justification using a definition, why an object is symmetrical, why a second figure is a transformation of the first • In data analysis: why you would choose a specific graph to represent a set of data, why median is different than mode or mean of a dataset, etc. Do NOT count *"how"* e.g., simply providing descriptions of steps (first I did x, then I did y) or definitions unless meaning is also attached. Note: Do NOT count incorrect or incomplete explanations as explanations.			
Not Present	**Low**	**Mid**	**High**
No mathematical explanations are offered by the teacher or students or the "explanations" provided are simply descriptions of steps of a procedure.	A mathematical explanation occurs as an isolated instance in the segment.	Two or more brief mathematical explanations occur in the segment OR an explanation is more than briefly present but not the focus of instruction.	One or more mathematical explanation(s) is a focus of instruction in the segment. The explanation(s) need not be most or even a majority of the segment; what distinguishes a High is the fact that the explanation(s) are a major feature of the teacher- student work (e.g., working for 2-3 minutes to elucidate the simplifying exampleabove).
Scoring Help - Explanations			
Examples of explanations: • Explaining the reason for steps in simplifying fractions (dividing by 2/2 is same as dividing by 1; anything divided by 1 is still itself) • Explaining why particular steps in a complex problem are justified or work to achieve the solution • Classifying triangles as polygons because they are closed and made up of line segments that do not cross • Explaining why a formula can be used to find an outcome (why ℓ x *w* works to find area) Note that when scoring, you can count the build-up to an explanation as part of the explanation. Ask yourself: Was the point of the instruction to provide the explanation, even if it only emerged at the end? If so, you may score that clip as a High. To help understand the difference between the Explanations code and the Mathematical Sense-Making code, see the Scoring Help for Sense Making.			

Back to Table of Contents

Mathematical Sense-Making			
This code captures the extent to which the teacher or students attend to one or more of the following: • The meaning of numbers • Understanding relationships between numbers • The relationships between contexts and the numbers or operations that represent them • Connections between mathematical ideas or between ideas and representations • Giving meaning to mathematical ideas • Whether the modeling of and answers to problems make sense Examples include: • Focusing on value of quantities (e.g., "7/8 is close to 1") • The meaning of quantities (e.g., "the six represents the number of groups") • Discussing reasonableness of an expression, solution method, or answer • Using estimation or number sense • Giving meaning to procedures (e.g., “1/4 x 2/3 means taking 1/4 *of* 2/3 *of* a whole”) • Giving meaning to expressions or equations For word problems, score for activities like explaining why an operation is called for by a problem, why certain numbers are used in the operation, reasonableness of answer, reasonableness of solution method, etc. In geometry, include making sense of definitions (what counts as a polygon, what does not count as a polygon), formulas, by elaborating them, applying them, finding counter-examples, etc. rather than just stating/executing them. Do not count “Give me examples of a circle” – instead, count cases where the definition or formula has meaning made around it. If sense-making is partially correct and partially incorrect, only score the portion that is correct (e.g., would be a High, but vague for parts, thus receives a Mid).			
Not Present	**Low**	**Mid**	**High**
Not present or incorrect.	Teacher and/or students focus briefly on meaning. For instance, a student may remark that 7/8 is "almost 1" or attends to reasonableness of the solution method.	Teacher and/or students focus on meaning more than briefly (e.g., several instances within the segment or one somewhat long instance), but this work is not sustained or substantial.	Teacher and/or students focus on meaning in sustained way during segment. Need not be the entire segment, but must be substantial.

Back to Table of Contents

Scoring Help - Mathematical Sense-Making
In many cases, Sense-Making overlaps with events already scored in Explanations. For example, a teacher may provide the explanation that dividing both the numerator and denominator by 4 is in essence dividing by 4/4. And because dividing by 4/4 is the same as dividing by 1, dividing by 4/4 actually does not change the value of the original fraction. This explanation would also count as sense-making, as the teacher is giving meaning to the fraction 4/4 and the procedure of making equivalent fractions. While many explanations will also qualify as Sense-Making, some will not. For example, a teacher who walks through an algebraic/geometric proof may get credit for explanations for explaining why a solution is true without meaning to the mathematical ideas. There also are instances of Sense-Making that do not count under Explanations. For example, attention to any of the following may be scored as Sense-Making without meeting the criteria for Explanations: • The value or meaning of quantities • The reasonableness of an expression or answer • Using estimation or number sense • Making sense of word problems Finally, it is important to note that instances that count under both Sense-Making and Explanations won't necessarily earn the same score point. For both codes, we ask raters to assess the *quantity* of the code, i.e., whether it occurs at all, is brief, or is more extended. However, a High for Explanations means that the instance is *the main feature of the segment*, whereas under Sense-Making it just needs to be *sustained*. Additionally, when scoring Explanations, you can count the build-up as a part of the explanation, even if that build-up is procedural. When scoring Sense-Making, only count time in which sense is actually being made; we do not count related procedural work or build-up as sense-making.

Back to Table of Contents

Multiple Procedures or Solution Methods			
Multiple procedures or solution methods occur or are discussed in the segment: • Multiple solution methods for a single problem (including shortcuts) • Multiple procedures for a given problem type Defined as, e.g.: • Taking different mathematical approaches to solving a problem (e.g., comparing fractions by finding a common denominator AND comparing fractions by finding a common numerator) • Solving or discussing how to solve a word problem using two different strategies. If the initial strategy or strategies occurred in a prior segment, score Multiple Procedures in the subsequent segment (i.e., no need to go back and adjust your score in the initial segment). Note: Do NOT count incorrect procedures or solution methods.			
Not Present	**Low**	**Mid**	**High**
No evidence of multiple procedures or solution methods for single problem or a given problem type.	Teacher or student briefly mentions a second procedure or method, but the method is not discussed at length or enacted ("we also showed yesterday that you can do it XYZ").	Multiple procedures or solution methods occur or are discussed in the segment (e.g., solving division problems in two ways), but does not include the special features listed in High, or feature these only momentarily (e.g., "this method is easier than the other" without explicit discussion of why).	Multiple procedures or solution methods occur or are discussed in the segment, and include special features: • Explicit comparison of multiple procedures or solution methods for efficiency, appropriateness, ease of use, or other advantages and disadvantages • Explicit discussion of features of a problem that cues the selection of a particular procedure • Explicit connections between multiple procedures or solution methods (e.g., how one is like or unlike the other)
Scoring Help - Multiple Procedures and Solution Methods			
You will need to use some judgment when deciding whether to count two methods as distinct from one another. We consider methods distinct when they feature two different mathematical paths to the solution. For instance, in the case of comparing fractions, we would NOT consider it distinct if student A compares 3/5 and 7/10 by finding a common denominator of 10, and student B finds a common denominator of 50. However, we would consider finding common numerators and finding common denominators to be distinct methods.			

Back to Table of Contents

Patterns and Generalizations

This code is meant to capture instruction during which the class *first* examines instances or examples, *then* uses this information to develop or work on a mathematical generalization; to notice, extend or generalize a mathematical pattern; to derive a mathematical property; or to build and test definitions.

Examples of this activity include:

- Examining particular cases and then noticing and extending a pattern (e.g., looking at the sum of the angles in 3, 4, 5, and 6-sided regular polygons and extending the pattern or generalizing to an n-sided regular polygon)
- Saying whether mathematical procedures work in all cases
- "Building up" a mathematical definition or deriving a mathematical property (e.g., defining "polygons" after considering different examples and non-examples of polygons)

Notes:

- Patterns, generalizations and definitions must be based on *at least two examples* (either explicitly worked on or referred to)
- Do NOT count incorrect generalizations, incorrect pattern noticing, or incorrect definition building
- Do NOT count when teachers and/or students *state* generalizations, patterns, or definitions without first developing them from examples

Not Present	Low	Mid	High
No generalizations are developed or worked on; no patterns are noticed or extended; no definitions are built or tested.	There is brief work on developing a generalization or building a definition, but this work is undeveloped and/or is not the primary focus of the segment. OR Teachers and/or students engage in pattern-noticing and/or extending. This is done in a pro forma way (e.g. red, blue, blue, red, blue, blue, ??, blue blue)	There is work on developing a generalization, extending a pattern or building a definition, but the work is not finalized. For instance, a pattern may be noticed, extended, or reasoned about but not codified ("it looks like when we increase the coefficient, the line might get steeper"). OR Teachers and/or students develop a generalization, extend a pattern, or build a definition, but the work is not complete, clear or detailed.	The pattern or generalization is codified, AND the work is complete, clear and detailed. For instance, the teacher and/or students may carefully develop a generalization from examples in detail; or summarize and codify a pattern by describing how the pattern is generated.

Back to Table of Contents

Mathematical Language			
This code captures how fluently the teacher (and students) use mathematical language and whether the teacher supports students' use of mathematical language. Examples: • Fluent use of technical language • Explicitness about mathematical terminology • Encouraging students to use mathematical terms			
Not Present	**Low**	**Mid**	**High**
Score here when NO mathematical terms are used. Teacher uses non-mathematical terms to describe mathematical ideas and procedures AND/OR teacher talk is characterized by sloppy/incorrect use of mathematical terms.	Low density of mathematical language. Not necessarily an indication that teacher is not "fluent" in mathematics, but simply a segment where few mathematical terms are used, or the same term is used over and over without features of High. Also score as Low when segment has middling density, but sloppy use.	Teacher uses mathematical language as a vehicle for conveying content, with middling density. However, the segment has few or none of the special features listed under High. Also score as Mid when segment has both features of High but includes some linguistic sloppiness or low density.	Teacher uses mathematical language correctly and *fluently*. Can be achieved in two ways: 1. Density of mathematical language is high during periods of teacher talk. 2. Moderate density, but also explicitness about terminology, reminding students of meaning, pressing students for accurate use of terms, encouraging student use of mathematical language. Instances of students using sophisticated mathematical vocabulary can also count toward a High.

Back to Table of Contents

Overall Richness of the Mathematics			
This code captures the depth of the mathematics offered to students. Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of richness.			
Not Present	**Low**	**Mid**	**High**
Elements of richness are present but are all incorrect OR Elements of rich mathematics are not present.	Elements of rich mathematics are minimally present. Note that there may be isolated Mid scores in the codes of this dimension.	Elements of rich mathematics are more than minimally present but the overall richness of the segment does not rise to the level of a High. For example, a segment may be characterized by some Mid scores in the codes of this dimension or by an isolated High along with substantial procedural focus, etc.	Elements of rich mathematics are present, and either: a) There is a combination of elements that together saturate the segment with rich mathematics either through meaning or mathematical practices. OR b) There is truly outstanding performance in one or more of the elements.
Scoring Help - Overall Richness of the Mathematics			
In scoring Overall Richness, we assign a score of Not Present when there are no elements of richness present in the segment, or the components of richness that are present are all incorrect. For this code, we do not consider middling density of Mathematical Language to be an element of richness. That is, a segment could get a score of Low or Mid for Mathematical Language and still get a score of Not Present for Overall Richness.			

Back to Table of Contents

Working with Students and Mathematics

This dimension captures whether teachers can understand and respond to students' mathematical contributions (utterances or written work) or mathematical errors. Student contributions include, but are not limited to, questions, claims, explanations, solution methods, ideas, etc. By students' mathematical errors, we mean those incorrect student contributions that offer opportunities for addressing student difficulty.

Back to Table of Contents

Remediation of Student Errors and Difficulties
With this code, we mean to record instances of remediation in which student misconceptions and difficulties with the content are addressed. *Conceptual remediation* gets at the root of student misunderstandings, rather than repairing just the procedure or fact. Conceptual remediation includes: •*Identifying/addressing the source of student errors or misconceptions*: "I noticed that some of you seem to think that 1.024 is a larger number than 1.1. I think you were noticing the number of digits to the right of the decimal point rather than thinking about the place value." •*Pointing to underlying meaning when responding to errors*: "I noticed that some of you seem to think that 1.024 is a larger number than 1.1. Both numbers start with 1. But what value do they have in the tenths place? Zero tenths, one tenth." *Procedural remediation* corrects student problems with procedures (e.g., re-demonstrating the procedure for addition of fractions with unlike denominators without reference to why the procedure works or sense-making around the quantities). To score an instance of procedural remediation, there must be *more than a simple correction* of a student mistake. Examples such as "no, that is not correct" or "you should have gotten 9" should be considered simple *corrections* rather than remediation because they do not address student difficulty. Examples of corrections could include correcting a misunderstanding about a definition ("This is an expression." "No, it's an equation.") or correcting the result of the calculation without talking about the calculation. If some portion of the remediation muddles the mathematics, the score may be adjusted downward. Notes: • Remediation can occur during active instruction or small group/partner/individual work time. • Remediation must have mathematical content. • If teacher prompts a student to remediate another student, it can be scored as present as long as the remediation is correct. • Pre-remediation (calling students' attention to a common error) counts as a Mid or High, depending upon the amount of detail and clarity. It demonstrates teacher familiarity with student thinking.

Not Present	Low	Mid	High
No remediation occurs for any of the following reasons: • There are no student misunderstandings or difficulties with the content •Remediation does not go beyond correcting students' answers •The teacher chooses not to remediate • The teacher remediation is confusing or off-track	Brief conceptual remediation occurs. OR Brief or moderate procedural remediation occurs.	Moderate (neither brief nor at length) conceptual remediation or extensive procedural remediation occurs. OR Brief pre-remediation occurs.	Teacher engages in conceptual remediation *systematically* and *at length*. Examples include: • Identifying the source of student errors or misconceptions • Discussing how student errors illustrate broader misunderstanding and then addressing those errors • Extended pre-remediation

Scoring Help - Remediation of Student Errors and Difficulties			
In scoring this code, it is helpful to first identify whether any student difficulty exists in the segment. If there is any student difficulty, then the teacher's response can be categorized according to whether or not it was remediation and if so, what type.			
Examples - Remediation of Student Errors and Difficulties			
Not Present	**Low**	**Mid**	**High**
[correction, not remediation] Teacher notices student has gotten the wrong answer and says, "No, that is not correct. You should have gotten 9."	[brief conceptual remediation] "Remember, you need to keep both sides of your equation equivalent to each other, so you can't perform an operation on only one side."	[moderate conceptual remediation] "I noticed that some of you forgot to multiply both sides of the equation by x. What happens if you multiply one side by x and not the other?" A few students offer reasons, and the teacher summarizes their ideas by saying, "The sides wouldn't be equivalent anymore."	[systematic conceptual remediation] "I noticed that some of you forgot to multiply both sides of the equation by x. What happens if you multiply one side by x and not the other?" The class continues to discuss at length why you need to multiply on both sides.

Back to Table of Contents

Teacher uses Student Mathematical Contributions			
This item captures the extent to which teacher uses student mathematical contributions to move instruction forward. Contributions can include, but are not limited to, student answers to questions (including one-word answers), comments, mathematical ideas, explanations, representations, generalizations, questions to the teacher, and student work. If some portion of the response to students muddles the mathematics, the score may be adjusted downward. This code can be used in whole-group or small-group/individual time segments.			
Not Present	**Low**	**Mid**	**High**
No or very few student responses and only pro forma use of student ideas to develop the mathematics. For example, class may be dominated by teacher talk with very few student comments. OR Teacher uses student contributions but in a way that muddles or confuses the mathematics of the lesson. OR Student contributions occur but the teacher ignores them	Students contribute and the teacher responds in a pro forma way.	The teacher uses student contributions to some degree in the development of the mathematics. Teacher may engage in features listed under High briefly, but instruction generally proceeds *without* strong use of student mathematical ideas.	Students' mathematical ideas are woven *at length* into the *development* of mathematical ideas during the segment. Teacher "hears" what students are saying, mathematically, and responds appropriately during instruction. In particular, teacher may comment on students' mathematical ideas, elicit further student clarification of ideas, ask other students to comment on ideas, expand on and reinforce student utterances, etc. Other markers include: • Identifying key ideas in student statement ("Mark had an interesting idea…") • Highlighting key features of student questions ("Mark was asking about whether this would work in all cases…") • Identifying a student with an idea ("Mark's method")

Back to Table of Contents

Scoring Help - Teacher uses Student Mathematical Contributions
This code is intended to measure how the teacher responds to and uses the mathematics that students contribute, regardless of the quality of the student contribution. That is, student contributions can be brief and procedural in nature; what we are looking for in this code is the length and quality of teacher uptake and use. Note that this is not a quantity code. If a teacher responds to multiple student contributions throughout the segment, but always does so in a pro forma way, score as a Low. Several types of teacher responses may qualify as pro forma: • The students regularly contribute basic calculations or answer-bounded questions during instruction, and the teacher acknowledges correct responses and uses them in the course of instruction, perhaps to move a calculation forward on the board. • Different students contribute many solutions or explanations, but the teacher doesn't make use of them beyond acknowledging students who are correct. • A student provides a contribution, and the teacher recognizes that it is interesting but decides not to take it up at that moment. Do not count use of "student" ideas that are not truly coming from the students as a feature of High (e.g., calling Claire's repeating of the addition procedure "Claire's idea" when the teacher just really wants to talk about the addition algorithm.)

Back to Table of Contents

Overall Working with Students and Mathematics			
This code provides an overall evaluation of the teacher-student interactions around the content. Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the teachers' interactions with the students around the content. If some portion of the response to students or remediation muddles the mathematics, the score may be adjusted downward.			
Not Present	**Low**	**Mid**	**High**
No or few interactions between teacher and students. There is no remediation and little use of student ideas OR Student mathematical contributions or difficulties occur, but teacher does not respond to or use those contributions. OR Teacher responses to student contributions are unclear or lead the segment off-track.	Teacher and students interact over content, but teacher responses are pro forma – moving instruction along with limited input from students. AND/OR There may be brief remediation.	Teacher and student interaction goes beyond pro forma exchanges to feature some use of student ideas, moderate conceptual remediation or extended procedural remediation. Portions of the clip may also feature a mix of strong and weak elements, or less-than-skillful use of student ideas.	Teacher weaves student ideas into the development of the mathematics and/or conceptually addresses misconceptions for clip. This must be done with some level of teacher skill at "hearing," understanding, and appropriately responding to student contributions or difficulties.

Back to Table of Contents

Errors and Imprecision

This dimension is intended to capture teacher errors or imprecision in language and notation, or the lack of clarity/precision in the teacher's presentation of the content.

Do NOT count errors that are noticed and corrected within the segment.

Back to Table of Contents

Mathematical Content Errors			
The code is intended to capture events in the segment that are mathematically incorrect. For example: • Solving problems incorrectly • Defining terms incorrectly • Forgetting a key condition in a definition • Equating two non-identical mathematical terms Mathematical errors that are made by students and endorsed by the teacher (e.g., leaving it on the board, saying it is correct, adopting an incorrect definition of fractions) should be counted here. Also score here if the teacher evaluates a correct solution method as incorrect. Do not count • Intentional errors (teacher following a wrong student idea or doing a procedure incorrectly to make a point) • Errors that are corrected within the segment			
Not Present	**Low**	**Mid**	**High**
None.	A brief content error. Does not obscure the mathematics of the segment.	Content errors occur in part(s) of the segment. OR Error(s) obscure the mathematics, but for only part of the segment.	Content errors occur in most or all of the segment. OR The errors obscure the mathematics of the segment.

Examples - Mathematical Content Errors			
Not Present	**Low**	**Mid**	**High**
	When solving a multi-step problem, the teacher makes a calculation error in the last step, which results in an incorrect answer. Other similar problems are solved correctly.	The teacher's discussion of the solution to a problem is incorrect. This discussion is more than brief, but correct mathematics also occurs more than briefly during the segment.	The teacher uses an inappropriate metaphor for most of the segment (e.g., in a graph comparing distance and time, the teacher refers to the upward slope as runner going up the hill, flat slope as runner running straight).

Back to Table of Contents

Imprecision in Language or Notation			
This code is intended to capture problematic uses of mathematical language or notation. For example: • Errors in notation (mathematical symbols) • Errors in mathematical language • Errors in general language ***Definitions*** • ***Notation*** includes conventional mathematical symbols (such as +, -, =) or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents, etc. Errors in notation might include inaccurate use of the equals sign, parentheses, or division symbol. By "conventional notation," we do not mean use of numerals or mathematical terms. • ***Mathematical language*** includes technical mathematical terms, such as "angle," "equation," "perimeter," and "capacity." If the teacher uses these terms incorrectly, record as an error. When the focus is on a particular term or definition, also score errors in spelling or grammar. • Teachers often use "***general language***" to convey mathematical concepts (i.e., explaining mathematical ideas or procedures in non-technical terms). General language also includes analogies, metaphors, and stories. Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. If the teacher is unclear in his/her general talk about mathematical ideas, terms, concepts, or procedures, record as an error.			
Not Present	**Low**	**Mid**	**High**
None.	Brief instance of imprecision. Does not obscure the mathematics of the segment.	Imprecision occurs in part(s) of the segment. OR Imprecision obscures the mathematics, but for only part of the segment.	Imprecision occurs in most or all of the segment. OR Imprecision obscures the mathematics of the segment.

Back to Table of Contents

Scoring Help - Imprecision in Language or Notation
We have specifically identified some commonly used imprecise terms and phrases that raters should be on the lookout for. These fall into two categories: phrases whose usage *automatically* results in a score of (at least) a Low, and "gray area" phrases whose isolated usage should not necessarily result in a score above Not Present, but can be considered in conjunction with other language use. If gray area phrases occur once or twice, we typically ignore them. However, if they occur repeatedly, or if they occur in combination with other linguistic or notational imprecision, we do consider them when scoring Imprecision. Note that these are not exhaustive lists of all phrases that might count towards score above Not Present. *Automatically score as an imprecision*: • referring to "bigger" or "smaller" equivalent fractions • different variations on "you can't subtract a bigger number from a smaller" • different variations on "multiplication makes a number bigger" • misuse of "expression" and "equation" • misuse of equals sign • "reducing" fractions (instead of simplifying) *"Gray area" phrases*: • "timesing", "minusing" • "top" and "bottom" for numerator and denominator • "alligator mouth" for greater-than and less-than symbols • "carrying" • "canceling" • "borrowing" (instead of regrouping) • "line" instead of line segment • brief reference that pi is 3.14 without mentioning this is an approximation.

Examples - Imprecision in Language or Notation			
Not Present	**Low**	**Mid**	**High**
	1) The teacher misuses "expression" or "equation" once or twice in a lesson on representing patterns. 2) Teacher uses term like "reduce" instead of "simplify", and this does not obscure the mathematics being taught.	1) The teacher uses the word "expression" instead of "equation" one or two times during a segment specifically about equations or the nature of equality. 2) The teacher uses terms like "reduce" and tells students that reducing makes fractions smaller.	The teacher's language and notation is sloppy throughout the segment.

Back to Table of Contents

Lack of Clarity in Presentation of Mathematical Content
This code is intended to capture when a teacher's utterances cannot be understood. For example: • Mathematical point is muddled, confusing, or distorted • Language or major errors make it difficult to discern the point • Teacher neglects to clearly solve the problem or explain content Teacher's launch of a task/activity lacks clarity (the "launch" is the teacher's effort to get the mathematical tasks/activities into play). If the launch is problematic, score for the launch plus amount of time students are confused/off-task/engaging in non-productive explorations

Not Present	Low	Mid	High
None.	Brief lack of clarity. Does not obscure the mathematics of the segment.	Lack of clarity occurs in part(s) of the segment. OR Lack of clarity obscures the mathematics, but for only part of the segment.	Lack of clarity occurs in most or all of the segment. OR Lack of clarity obscures the mathematics of the segment.

Scoring Help - Lack of Clarity

Definition: You have to ask: "What, mathematically, was the teacher trying to say?"
Examples:

- Discussion of why 7 + -3 = 4 heads toward "-4 is too small to be the answer"
  - This is not wrong, but the mathematical point is not clear.
- Teacher endorses conflicting definitions for same concept
  - "The area is a number of square units needed to cover the figure, and we've talked before about the box like a gift that somebody gives you. The box itself and everything inside the box is the area, but the wrapping paper around it would be like surface area and we talked about that and we talked about the perimeter is walking around the fence around an area."
- Talking through a division problem and alternating back and forth between "making 3 groups" and "making groups of 3."
- Garbling a task launch, e.g., by asking initially "How much TV is watched in the US?" when students really must draw a graph to show "How many TVs in US vs. Europe vs. rest of the world?"

Examples -

Not Present	Low	Mid	High
	The launch of task is unclear, but the teacher clarifies quickly. A sentence or phrase is unclear, but the main mathematical point is not affected.	To introduce inverse operations, teacher explains that multiplication and division are "best friends" and "if you know something about one, you know something about the other." Examples later in the segment make the point clearer.	Teacher states that the lesson is going to be on surface area and volume. When students are asked to describe a cardboard box using math terms, the teacher endorses correct and incorrect student suggestions. The teacher then tries to define volume by asking whether a twelve foot TV would fit into the box. Surface area is mentioned numerous times but never defined. It is unclear if the teacher is using surface area as a synonym for volume or whether the term is simply never defined.

Back to Table of Contents

Overall Errors and Imprecision			
This code intends to capture the overall presence of teacher errors in doing and talking about mathematics. Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the errors and imprecision in instruction.			
Not Present	**Low**	**Mid**	**High**
No errors occur. Do not score as Not present if Low, Mid or High is marked in any category above.	Small, momentary error(s) occur. For example, small slips in language, a brief lack of clarity, or a minor error in solving an exercise. These typically do not obscure the mathematics of the segment.	One or more errors, for example, persistent misuse of language, a lack of clarity in a portion of the segment and/or mathematical errors, but these typically obscure the math for part of the segment.	Either there are many small errors, a consistent lack of clarity, or one large error that obscures the mathematics of the segment.

Back to Table of Contents

Common Core Aligned Student Practices (CCASP)

This dimension attempts to capture evidence of students' involvement in tasks that ask them to "do" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. During active instructional segments, this mainly occurs through student mathematical statements, including reasoning, explanations, and asking questions. During small group/partner/individual work times, this mainly occurs through work on a non-routine task.

The CCASP dimension captures the same kind of student meaningful engagement with mathematics envisioned in the eight Standards of Mathematical Practices listed in the Common Core State Standards for Mathematics[1], which say that students should:

1. Make sense of problems and persevere in solving problems
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Although there is not a one-to-one correspondence between the CCASP codes and the 8 Common Core Standards, the CCASP dimension includes many of the observable student behaviors contained in the Common Core. For instance, the Common Core practice "Model with mathematics" is addressed in the MQI code Students Work with Contextualized Problems.

1 National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers

Back to Table of Contents

Students Provide Explanations

Students provide a mathematical explanation for an idea, procedure, or solution.

Examples:

- Students explain why a procedure works
- Students explain the procedure they used to solve a particular problem by attending to the meaning of the steps involved in this procedure rather than simply listing those steps
- Students explain what an answer means
- Students explain why a solution method is suitable or better than another method
- Students explain an answer based on an estimate or other number-sense reasoning

Notes:

- Explanations can be initiated by the teacher or self-initiated
- Explanations can be co-constructed with the teacher or constructed individually
- Explanations do not have to be complete or correct
- If a student's explanation meets the criteria for the Explanations code in Richness, it should be counted in both places
- Only give credit for things you actually hear students say

Not Present	Low	Mid	High
No instances of student explanation are present.	One or two brief student explanations are present.	Student explanations are more sustained or more frequent, but they are not characteristic of the segment.	Student explanations characterize much of the segment.

Scoring Help – Students Provide Explanations

When students are working independently or in groups and you cannot hear anything they say, assign the segment a score of Not Present. If you can hear student explanations or reasoning during these times, score them as usual.

Back to Table of Contents

Student Mathematical Questioning and Reasoning (SMQR)			
Students engage in mathematical thinking that has features of important mathematical practices. There must be clear evidence of students engaging in such practices. Examples include but are not limited to: • Students provide counter-claims in response to a proposed mathematical statement or idea (whether from another student, the teacher, or a text) • Students ask mathematically motivated questions requesting explanations (e.g., "Why does this rule work?" "What happens if all the numbers are negative?") • Students make conjectures about the mathematics discussed in the lesson (e.g., "I've been trying to make a triangle with two obtuse angles, and I don't think you can.") • Students form conclusions based on patterns they identify or on other forms of evidence (e.g., "It looks like, for polygons, every time we add a side we add another 180 degrees.") • Students engage in reasoning about a hypothetical or general case (e.g., "Because the sum of the angles of any triangle is 180 degrees, a triangle should have at least two acute angles.") • Students use ideas from a different mathematical topic to reason about the content of the lesson (e.g., student uses ideas from symmetry to reason about equivalent fractions in a pie chart) • Students make a connection between the topic of the lesson and another mathematical area (e.g., a student notes the connection between area models for multiplication and area in measurement) • Students comment on the *mathematics* of one another's contributions (this must go beyond stating "I did it another way" or simply agreeing or disagreeing) An explanation captured under the Student Explanations code should *also* be coded as SMQR only if the statement includes an additional SMQR element. For example, a conjecture and an explanation of the conjecture should be counted under both codes. (e.g., "I don't think that the output in that table will ever be 0 because all of the other outputs are odd numbers.") Notes: • Students' contributions do not have to be complete or correct • Only give credit for things you actually hear students say			
Not Present	**Low**	**Mid**	**High**
No instances of student mathematical questioning or reasoning are present.	One or two instances of brief student mathematical questioning or reasoning are present.	Student mathematical questioning or reasoning is more sustained or more frequent, but it is not characteristic of the segment.	Student mathematical questioning or reasoning characterizes much of the segment.
Scoring Help - SMQR			
When students are working independently or in groups and you cannot hear anything they say, assign the segment a score of Not Present. If you can hear student explanations or reasoning during these times, score them as usual.			

Back to Table of Contents

Students Communicate about the Mathematics of the Segment			
This item captures the extent to which students communicate their mathematical ideas during the course of the segment, either in whole-group or small group settings. Examples of *substantive* student contributions include, but are not limited to, students presenting solution methods publicly (with or without words), asking mathematical questions, describing the meaning of a term, offering an explanation, discussing solution methods, commenting on the reasoning of others, etc. In cases in which students are working in pairs or small groups, code substantive student contributions when you can a) hear them (e.g., a student and teacher are talking as teacher circulates, or you can overhear pairs of students) or b) the teacher's directions are very clear, and we can reasonably expect students to be having a substantive exchange for the duration of the small group work (e.g., a turn and talk). However, if it is not clear what students are talking about in small groups/pair work, score as Not Present.			
Not Present	**Low**	**Mid**	**High**
Not present or minimally present. Students may contribute a word or phrase infrequently during whole- group instruction, but the segment primarily features teacher talk.	Student contributions are very brief. For example, students offer one- or two-word answers to questions or a partial description of steps, and they occur regularly during the segment.	There are some substantive student contributions, but these do not characterize the segment.	Substantive student contributions characterize the segment.
Scoring Help - Students Communicate About The Mathematics Of The Segment			
Note that the difference between Not Present and Low is the prevalence of brief, one- or two-word answers, and the difference between Mid and High is the prevalence of *substantive* student contributions. The difference between Not Present/Low and Mid/High is whether there exist *any* substantive student contributions (i.e. a segment with a single substantive student contribution must be scored at least a Mid, and a segment with no substantive student contributions may not score above a Low). For instance, a student may provide one step of a procedure, followed by the teacher giving the next step. This would count as a Low. If the student narrates a complete set of steps for a problem, it would be counted as a Mid. Student explanations and SMQR-type responses count here. In addition, this code encompasses additional types of substantive student contributions under Mid and High, including descriptions of choices students made while solving word problems, definitions, and so forth.			

Back to Table of Contents

Task Cognitive Demand
This code captures student engagement in tasks in which they think deeply and reason about mathematics. This code refers to the *enactment* of the task, regardless of the initial demand of the curriculum/textbook task or how the teacher sets up the task for students. Notes: • Student confusion does not necessarily suggest that students are engaging with the content at a high cognitive level. • Working on review tasks or on ideas discussed in previous lessons does not necessarily mean that students use lower order thinking skills. • This code should not be confounded with the difficulty of the task or whether it is appropriate for a certain grade- level. • Code a student presentation of a solution method at the same level of cognitive demand as the task itself was coded.

Not Present	Low	Mid	High
Students are engaged in cognitively undemanding activities. Examples of cognitively *undemanding* activities include: • Recalling and applying well-established procedures • Recalling or reproducing known facts, rules, or formulas • Listening to a teacher presentation with limited student input • Going over homework with little additional student work (e.g., reporting numerical answers) • Unsystematic exploration (i.e., students do not make *systematic and sustained progress in developing mathematical strategies or understanding*)	There is a brief example of a cognitively demanding activity, e.g. • A momentary think- pair-share where students define a term • Direct instruction with one or two examples of student explanations or SMQR • Tasks with a momentary high cognitive demand element • Tasks that are not completely routine, but are heavily scaffolded for students with hints or directions	Segment features mix of demanding and undemanding tasks and activities, e.g. • Tasks with variable enactment (e.g., demanding tasks followed by a transition to undemanding tasks; or, when working in small groups, some groups work on a high-demand task while some groups work on an undemanding task) • Direct instruction with student explanations and/or SMQR input at certain points • Tasks with middling cognitive demand	Students engage with content at a *high* level of cognitive demand. Examples of cognitively *demanding* activities include when students: • Determine the meaning of mathematical concepts, processes, or relationships • Draw connections among different representations or concepts • Make and test conjectures • Look for patterns • Examine constraints • Explain and justify

Back to Table of Contents

Students Work with Contextualized Problems			
Students work with contextualized problems (e.g., story problems, real-world applications, experiments that generate data). This includes solving such problems; discussing solutions to such problems; writing expressions or equations to represent contextualized situations; making sense of contextualized relationships through tables, graphs or other representations; or creating contextualized problems/situations to match expressions/equations. Note: Do not count when the teacher or student mentions a contextualized example for illustrative purposes (e.g., "you can think of 1/4 as a quarter and 1 whole as a dollar when you are converting fractions to decimals" or "remember yesterday when we solved the hat problem?"), but the students do not work on it. Note: This is not a duration code; the difference between a Low, Mid, and High is amount of teacher scaffolding, not length. In the case of two or more different tasks with different levels, score to the highest level.			
Not Present	**Low**	**Mid**	**High**
Students do not work with contextualized problems or a contextualized problem is mentioned but not worked on.	The contextualized problems are executed as mostly rote/routine exercises. Teacher heavily scaffolds the presentation, for example, by telling students which procedure is to be applied, helping them write out the expression or equation, and so forth. Also include here times where there is data collection *without* reference to the underlying relationships or shape of the data. For instance, students may be collecting and marking down ice cream preferences in preparation for later plotting the graph and discussing.	Some student reasoning about contextualized problems is required for at least a portion of the problem execution; however, solution paths may be co-constructed or scaffolded by teacher to some extent. For instance: • Students play some role in deciding how to solve the problem • The problem starts off as non-routine but teacher hints at a solution method	Students are allowed significant opportunities to think and reason mathematically about contextualized problems. Students might need to choose which operation to apply, decide which kind of graph is appropriate for their data, or figure out how to write an expression that represents a pattern. The characterizing feature of this segment is that the teacher will not be doing much of the cognitive work of solving the problem.

Back to Table of Contents

Scoring Help - Students Work with Contextualized Problems
When scoring, first determine if a contextualized problem is present during the segment. When determining whether or not a problem is contextualized, it may be necessary to refer to previous segments to determine what problem or task was assigned. Special cases: • Probability experiments (such as rolling dice, spinning a spinner, or pulling colored chips out of a bag) are NOT contextualized problems, unless there is an additional context (such as pulling colored chips out of a bag that represent socks in a drawer). • Working with data is generally contextualized. If the data are completely void of context (for example, if students are asked to find the median of a set of numbers), it is not contextualized; but, if there is any meaning to the data involved (for example, students take a list of names, count the letters in each name, and then find the median), then it should be counted as contextualized. If there is a contextualized problem, determine whether students are working on it (they do not have to finish or 'solve' the problem, just work on it in some way). If students are working on a contextualized problem, determine how much scaffolding or support the students are given. It may be necessary to refer to previous tasks and segments in order to infer whether the task is routine or not. Use your best judgment.

Back to Table of Contents

<table>
<tr><th colspan="4">Overall Common Core Aligned Student Practices</th></tr>
<tr><td colspan="4">This code attempts to capture evidence of students' involvement in "doing" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning.
• During active instruction segments, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking.
• During small group/partner/individual work time, this mainly occurs through work on a non-routine task.

Note: This is an overall code for each segment. It is not an average of the codes in this dimension, but an overall estimate of the student participation in meaning-making and reasoning.</td></tr>
<tr><th>Not Present</th><th>Low</th><th>Mid</th><th>High</th></tr>
<tr><td>There are no examples of student involvement in cognitively demanding classroom work. Tasks are largely procedural in nature or heavily scaffolded by the teacher.

For example, there may be inquiry-response-evaluation-type teacher lectures with no examples of student explanation, questioning, or reasoning.

Also score as Not Present if there are unproductive explorations in which the majority of the students are off-track mathematically.</td><td>There are few examples of student engagement in mathematical practices such as explanation, questioning, and reasoning. Tasks may be largely procedural in nature, but occasional student participation or a brief cognitively demanding task occurs.</td><td>Students engage with content at mixed level. Students may provide substantive explanations or ask mathematically motivated questions. This may also include tasks with variable enactment (high and then low during segment). This can also include instances in which some students/groups are engaged in tasks at a high level and others are not.
Students may also engage in a task with middling cognitive demand.</td><td>Students contribute substantially to the building of mathematical ideas through posing questions, offering explanations, looking for patterns, making conjectures, and engaging in other types of reasoning. Such contributions are a major feature of the segment, with many student contributions or extended work on a cognitively demanding task.</td></tr>
</table>

Back to Table of Contents

B. APPROVAL OF THE METU HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 /057

20 Şubat 2020

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof.Dr. Erdinç ÇAKIROĞLU

Danışmanlığını yaptığınız **Azime ATAY'ın "Ortaokul Matematik Öğretmenlerinin Görev Tasarım Süreçleri: Bir Ders Araştırması"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **057-ODTU-2020** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

Prof.Dr. Mine MISIRLISOY
Başkan

Prof. Dr. Tolga CAN
Üye

Doç.Dr. Pınar KAYGAN
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

Dr. Öğr. Üyesi Şerife SEVİNÇ
Üye

Dr. Öğr. Üyesi Müge GÜNDÜZ
Üye

Dr. Öğr. Üyesi Süreyya Özcan KABASAKAL
Üye

C. CURRICULUM VITAE

Name/Surname: Azime ATAY
Nationality: Turkey
E-Mail

EDUCATION

M.A. (2013-2015)	**Erciyes University, Turkey**
B. A (2006-2011)	Middle East Technical University, Turkey.

WORK EXPERIENCE

September 2012- November 2012	Şanlıurfa	Ministry of Education	Elementary Mathematics Teachers
November 2012-	Kayseri	Erciyes University	Research Assistant

PUBLICATION

Journal Papers

Yıldız P., Atay A., "Ortaokul Beşinci Sınıf Öğrencilerinin Eşit İşaretine İlişkin Anlamalarının İncelenmesi", Ahi Evran Üniversitesi Sosyal Bilimler Enstitüsü Dergisi, vol.5, pp.426-438, 2019

Yildiz, P., Atay, A., & Çiftçi, S. K. (2020). Preservice Middle School Mathematics Teachers' Definitions of Algebraic Expression and Equation. *International Journal of Contemporary Educational Research*, *7*(2), 156-164.

Proceedings

Çiftçi Karadağ Ş. K., Atay Mutlu A., Yıldız P. "Elementary School Teachers' Understanding of Area Measurement Units", 18th International Primary Teacher Education Symposium , Antalya, Turkey, 16 - 20 October 2019, pp.413-414

Yıldız P., Atay Mutlu A., Çiftçi Karadağ Ş. K. "Prospective Mathematics Teachers' Understanding of Algebraic Expression and Equation", 18th International Primary Teacher Education Symposium , Antalya, Turkey, 16 - 20 October 2019, pp.417-418

Yıldız P., Çiftçi S.K., Atay A., Şengil Akar Ş., "Prospective Middle School Mathematics Teachers' Definitions of Algebraic Expression", 28. International Conference on Educational Science, Ankara, Turkey, 26-28 April 2019, pp.694-697

Yıldız P., Atay A., "Ortaokul Beşinci Sınıf Öğrencilerinin Eşit İşaretine İlişkin Anlamaları", VI. International Eurasian Educational Research Congress- EJER 2019, Ankara, Turkey, 19-22 June 2019, pp.1343-1344

Bayazıt İ. , Atay A., Yıldız P., "Exploring Level Of Cognitive Demand Of Geometry Questions In Transition Exam From Elementary Education To Secondary Education", 4th International Symposium of Turkish Computer and Mathematics Education , İzmir, Turkey, 26-28 September 2019, pp.---

Koyuncu, İ., Atay, A, Macun, Y., "Investigation Of Prospective Middle School Mathematics Teachers' Mathematical
Modeling Self-Efficacy", 4th International Symposium of Turkish Computer and Mathematics Education, İzmir, Turkey, 26-28 September 2019, pp. 719-720.

Şengil Akar Ş., Atay A., Çiftçi S.K., Yıldız P., "Investigation of Primary School Teacers' Area Measurement Process ", 28. International Conference on Educational Science, Ankara, Turkey, 26-28 April 2019, pp.768-771

Ciftçi S.K., Yetkin Özdemir İ.E., Atay A., Yıldız P., "Examining The Mathematical Content Knowledge Of In-Service Primary School Teachers Through A Professional Development Process", 4th International Symposium of Turkish Computer and Mathematics Education, İzmir, Turkey, 26-28 September 2019, pp. 565-566.

Yıldız P., Çiftçi S.K., Atay A., Şengil Akar Ş., "İlköğretim Matematik Öğretmen Adaylarının Denklem Kavramına İlişkin Tanımları", VI.. International Eurasian Educational Research Congress - EJER 2019, Ankara, Turkey, 19-22 June 2019, pp.1290-1291.

Çiftçi S.K., Atay A., Yıldız P., Şengil Akar Ş., Çankaya A., "Sınıf Öğretmenlerinin Cetvele İlişkin Anlayışlarının İncelenmesi", VI. International Eurasian Educational Research Congress- EJER 2019, ANKARA, TURKEY, 19-22 June 2019, pp.1292-1294.

Yıldız P., Atay A., "Views of Prospective Elementary Mathematics Teachers' About Use of Concrete Materials In Mathematics Teaching", International Conference on Mathematics and Mathematics Education- ICMME 2019, KONYA, TURKEY, 11-13 July 2019, pp.349-351.

Atay A., Yıldız P., "Classroom Design of Prospective Middle School Mathematics Teachers", International Conference on Mathematics and Mathematics Education-ICMME 2019, KONYA, TURKEY, 11-13 July 2019, pp 359-361

Atay, A. "Analysing Mathematical Tasks In Lesson Plans Done By Prospective Mathematics Teachers- International Conference on Mathematics and Mathematics Education -ICMME 2018, ORDU, TURKEY, 27-29 June, pp.540-541.

Bayazit, İ., Atay, A. "9. Sınıf Öğrencilerinin Gerçek Yaşam Problemlerinin Çözümünde Geometri Bilgilerini ve Orantısal Akıl Yürütme Becerilerini Kullanma Yeterliklerinin İncelenmesi", 2. Türk Bilgisayar ve Matematik Sempozyumu , Adıyaman, Turkey, 16 - 18 May 2015, pp.71.

Bayazıt, İ., Kırnap Dönmez, S. M., Atay, A "İlköğretim Matematik Öğretmen Adaylarının Problem Çözme Konusundaki Pedagojik Alan Bilgilerinin İncelenmesi", 11. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, Adana, Turkey, 11 - 14 September 2014, pp.1350.

Bayazit, İ., Kırnap Dönmez, S. M., Atay, A "Öğretmen Adaylarının Gerçek Yaşam Problemlerini Çözmedeki Yeterliliklerinin Kullandıkları Yaklaşımlar" 1. Türk Bilgisayar ve Matematik Sempozyumu , Trabzon, Turkey, 16 - 18 May 2015, pp.71.

D. TURKISH SUMMARY / TÜRKÇE ÖZET

ORTAOKUL ÖĞRETMENLERİNİN ÖĞRETİMİNİNİN MATEMATİKSEL KALİTESİ ÜZERİNE BİR DURUM ÇALIŞMASI

1. Giriş

Matematik, gerçek hayatla doğrudan ilgili olması ve aynı zamanda fizik, kimya, bilgisayar bilimleri gibi diğer alanlara da temel oluşturması nedeniyle okulda öğretilen en önemli derslerden biridir. Matematikte öğrenci başarısının geliştirilmesi, birçok ülkede politikacılar ve araştırmacıların ilgisini çekmektedir (Harniss & diğerleri, 2002; Jaworski, 2006). Öğrencilerin başarılarını artırmak için birçok farklı yöntem denenmiştir. Bu yöntemlerden biri de müfredatta değişiklik yapılmasıdır. Eğitim reformu hareketi Türk eğitim sistemini de etkilemiş ve sosyo-ekonomik (küreselleşme), politik (Avrupa Birliği), felsefi (Yapılandırmacılık) ve eğitimsel (Öğrenci merkezli öğretim) nedenlerle 2005 yılında Ortaokul Matematik müfredatı güncellenmiştir (İnal, 2005). Yeni öğretim programları "Her çocuk matematiği öğrenebilir" düşüncesinden yola çıkarak süreç becerileri ve matematiksel düşünme becerilerinin gelişimine odaklanmakta ve öğrenci merkezli eğitim anlayışını benimsemektedir. Fakat müfredatın uygulayıcısı olan öğretmenler öğrencilerin neyi nasıl öğrendiğini belirleyen en önemli faktörleridir. Alanyazında yer alan çalışmalar, öğretmenlerin öğrencilerin başarısı üzerinde etkili olduğunu göstermiştir (Bobis vd., 2012; Hill vd., 2005; Rockoff, 2004; Teddlie & Reynolds, 2000).

Öğretmen eğitiminde hangi değişkenler öğrencinin öğrenmesini ve başarısını etkiler? Öğretmenlerin mesleki tecrübe sürelerinin, öğretmenlerin inançlarının, öğretmenlerin alan ve pedagojik alan bilgisinin ve ders materyallerinin (ödevler, kitaplar, ders planları) etkisi üzerine araştırmalar gerçekleştirilmiştir. Öğrenmeyi ve başarıyı etkileyen faktörleri anlamak için bazı araştırmacılar sınıf içinde gerçekleşen öğretim sürecine bakmanın gerekliliği üzerinde durmuştur. Öğretimin kalitesini araştımak için çeşitli sınıf içi gözlem protokolleri geliştirilmiştir. Öğretim Kalitesi Değerlendirmesi (IQA), Sınıf İçi Gözlem Aracı, Sınıf İçi Gözlem ve Analitik Protokolü (ICOAP) ve Yenilenmiş Öğretim Gözlem Protokolü (RTOP), öğretimin kalitesini ölçmek için kullanılan sınıf gözlem çerçevelerinden bazılarıdır. Öğretimin Matematiksel Kalitesi (MQI) çerçevesi, matematik öğetimine özgü bir sınıf gözlem çerçevesidir. Bu nedenle matematik eğitiminin beklentilerini en iyi şekilde karşılamaktadır. Bu araştırmada matematik öğretiminin kalitesini analiz etmek için MQI çerçevesi kullanılmıştır.

1.1. Çalışmanın Amacı ve Araştırma Soruları

Bu çalışma ortaokul matematik öğretmenlerinin öğretiminin matematiksel kalitesini ve öğretmenlerin vurguladıkları öğretim yönlerini ve bunların matematiksel kalitesini araştıran nitel bir çalışmadır. Bu çalışma farklı öğretmenlerin öğretimi değerlendirin bir çoklu durum çalışmasıdır. Bu çalışma aşağıdaki iki temel araştırma sorusunu yanıtlamaya çalışmaktadır:

1. Ortaokul matematik öğretmenleri öğretimin hangi yönlerini vurgulamaktadır ve bu yönlerin niteliği nedir?

2. Ortaokul matematik öğretmenlerinin öğretiminin kalitesi Öğretimin Matematiksel Kalitesi (MQI) çerçevesi ile değerlendirildiğinde nasıldır?

1.2. Bu çalışmanın önemi

Öğretimin Matematiksel Kalitesi (MQI) çerçevesi, ders sürecini bölümlere ayırır ve her bölümü ayrıntılı olarak inceler. Matematik öğretiminin kalitesinin incelenmesi öğretmenlerin ihtiyaçları hakkında bilgi verir. Bu çalışma Türkiye'de matematik eğitiminin ihtiyaçlarını güçlü ve zayıf yönlerini ortaya çıkararak eğitim politikasına yön verme potansiyeline sahiptir. Matematik öğretmen eğitiminin, mesleki gelişim programlarının ve hizmet içi eğitim çalıştaylarının içeriğini değiştirme veya uyarlamaya yönelik ipuçları verebilir. Ayrıca çalışmanın gerçekleştirildiği okul profilleri birbirinden oldukça farklıdır, dolayısıyla öğrenci profillerinin matematiğin öğretim kalitesi üzerindeki etkisi hakkında da bilgi verir.

2. Literatür Taraması

Bu çalışma ortaokul matematik öğretmenlerinin derslerinin matematiksel kalitesini araştırmayı amaçlamaktadır. Bu bağlamda, iki öğretmenin dairenin alanı ve daire diliminin alanını anlattıkları dersleri gözlemlenmiştir. Bu nedenle, alan ölçmenin öğretimine ilişkin alan yazın, öğretmenlerin bilgisine ilişkin alanyazın ve Öğretimin Matematiksel Kalitesi (MQI) ilişkin alan yazın sırasıyla sunulmuştur.

2.1. Alan Ölçmenin Öğretimi

Ölçme, ilk ve orta matematik müfredatının merkezi bileşenlerinden biridir ve uzunluk ölçümünü, alan ölçümünü ve hacim ölçümünü içerir. Ölçme günlük hayatımızın bir parçasıdır. Bu nedenle ölçme kavramının öğrenilmesi öğrenciler için sadece matematik başarısı açısından değil aynı zamanda günlük yaşam becerileri açısından da önemlidir.

Alan ölçümü matematik müfredatındaki önemli konulardan biridir ve farklı ülkelerde ölçme veya geometri başlığı altında ele alınmaktadır (NCTM, 2000). Türkiye'de geometri ve ölçme başlığı altında yer almaktadır. Alan ölçme farklı sınıflardaki ortaokul matematik müfredatında yer almaktadır. Dikdörtgenin alanının ölçümü (kare özel bir dikdörtgen olarak gösterilmektedir) beşinci sınıf müfredatında, üçgen ve paralelkenarın alanının ölçümü ise altıncı sınıf müfredatında yer almaktadır. Yedinci sınıf müfredatında eşkenar dörtgen, yamuk, daire ve daire diliminin alan ölçümüne yer verilmektedir. Son olarak sekizinci sınıf müfredatında dik dairesel silindirin yüzey alanına yer verilmektedir. Dolayısıyla alan öğretimi Türkiye'de de okul matematiğinin önemli bir konusudur.

Alan, iki boyutlu sınırlı ve kapalı yüzeylerin ölçüsüdür ve bir yüzey, alan ölçüm birimi kullanılarak eşit parçalara bölünebilir; alanın ölçüsü bu ölçü birimlerinin sayısıyla ifade edilir (Smith vd., 2016). Alan ölçümü, gerçek ölçü birimlerinin somut dünyası ile matematiğin soyut dünyası arasında bir bağlantı kurar (Hiebert, 1981). Alanın ölçüsünün hesaplanması, fiziksel nesnelerin kullanımından matematiksel işlemlerin (formüllerin) yapılmasına geçişi gerektirmektedir (Kordaki & Potari, 2002; Lehrer, 2003; Zacharos, 2006). Bu nedenle alan ölçümü öğrenciler için anlaşılması zor bir konudur. Literatürde alan kavramının öğrenilmesiyle ilgili beş zorluk listelenmektedir. Bunlar; a) alanın korunumu, b) ölçü birimlerinin anlaşılması ve kullanılması, c) dikdörtgeninin yapılanması, d) çarpımsal ilişkiler ve e) alan ve çevrenin karıştırılması (Smith vd., 2016, s. 241).

Öğrencilerin karşılaştığı ilk zorluk alanın korunumunun anlaşılmasıdır. Öğrencilerin karşılaştığı ikinci zorluk ise alan ölçü birimlerini anlama ve kullanmayla ilgilidir. Bir yüzeyin birim karelerle kaplanması ve alanın ölçüsünü bulmak için birim karenin sayılması, öğrencilerin birim karenin alanın ölçü birimi olduğunu anlamaları anlamına gelmez (Kamii & Kysh, 2006; Kordaki & Potari, 1998). Alanın ölçü biriminin birim kare olduğunu anlamak için karenin küçük birimlere bölünebileceğini, birim karenin kendi başına süreksiz olmasına rağmen alan ölçüm süreçlerinde tekrar tekrar kullanılmasıyla sürekli hale geldiğini anlamak gerekir.

Bir alanı fiziksel olarak birimler karelerle kaplamaktan ölçümün soyutlanmasına geçiş, alanları ölçmek için birim karelerin iki paralel çizgiyle temsil edilmesini içerir. Ancak birimleri iki paralel doğru kullanarak temsil etmek beklenenden daha zor olup, bir alanı birim karelerle kaplamak öğrenciler için yeterince açık değildir (Outherd & Mitchelmore, 2004). Kare birimler ile kaplamadan ve birimleri saymadan dikdörtgen bir bölgenin alanını hesaplamak için öğrencilerin dikdörtgensel dizi modelini görselleştirmeleri gerekir ve bu süreç öğrencilerin birim kare sayısını hesaplamalarını kolaylaştırır hem de dikdörtgenlerin alan formülünün temelini oluşturur (Smith vd., 2016). Alan formülünün arkasındaki fikir, satır ve sütun sayıları ile dikdörtgenin toplam alanı arasındaki çarpımsal ilişkiyi anlamaktır.

Öğrenciler alan formülünü ezberleme ve alan formülünün içerdiği işlemleri yapma konusunda iyidirler (Huang & Witz, 2013). Ancak alan formülü işleminin doğru yapılması öğrencilerin alan ölçümünün anlamını anladığı anlamına gelmemektedir. Huang ve Witz'in (2013) araştırmasına katılan dördüncü sınıf öğrencilerinin beşte ikisi alan kavramı ile alan ölçümünü ayırt edememiştir. Alan formülünün aritmetik işlemlere indirgenmesi formülün aşırı genellenmesine neden olmakta ve alan ölçüm içeriğinin başarısının düşük olmasına neden olmaktadır (Erdem & Gürbüz, 2018; Zacharos, 2006).

Öğrenciler çevre ve alan kavramlarını ayırt etmekte zorluk çekmektedir. Alan formülünü ve çevre formülünü birbirinin yerine kullanmaktadırlar (Smith ve diğerleri, 2013; Smith ve diğerleri, 2016). 4, 6, 8 ve 9. sınıf öğrencilerinden bazıları çokgenlerin alanını bulmaları istendiğinde çevreyi hesaplamışlardır (Olkun vd., 2014). Bir başka araştırma ise Kamii ve Kysh (2006) tarafından yapılmıştır. Sekizinci sınıf öğrencilerinden düzensiz çokgenlerin alanını bulmaları istenmiştir. Ancak bazı öğrenciler çokgenlerin alanı yerine çevresini hesaplamışlardır.

Öğretmen adayları ve öğretmen adayları ile yapılan araştırmalar öğretmenlerin de alan kavramını anlamada, alan ve çevre kavramlarını ayırt etmede zorluk yaşadıklarını göstermektedir. Öğretmenler bir alanı ölçmenin, yüzeyi birim karelerle kaplamak ve yüzeyi eşit parçalara bölmeyen birim karelerin sayısını saymak olduğunu düşünmektedir (Outhred ve McPhail, 2000). Öğretmenlerin alan ölçümünü yüzeyi kaplamak olarak vurgulaması, düzensiz şekillerin alanını ölçerken (Kordaki ve Potari, 1998; Outherd ve Mitchelmore, 2004; Zacharos, 2006) ve öğrencilerin yüzeyi kaplayamadığı durumlarda zorluk yaşamasına neden olmaktadır (Outhred & McPhail, 2000).

2.2. Öğretmen Bilgisi

Öğrencilerin başarısının arttırılması eğitimin temel odak noktasıdır. Öğrencilerin başarısını etkileyen önemli faktörlerden biri de öğretmenlerdir. Öğretmenlerin içerikle ilgili bilgileri ve içeriğin nasıl öğretileceği, öğrencilerin başarısıyla büyük ölçüde ilişkilidir (Bobis vd., 2012; Hill vd., 2005). Öğretmenin etkili matematik öğretimini sağlamak için matematiksel kavramlar ve pedagojik stratejiler konusunda derin bir anlayışa sahip olması gerekir (Baumert vd., 2010). Etkili matematik öğretimi için öğretmenin bilmesi gerekenler nelerdir? Bu soru Ball ve diğerleri (2008) tarafından yanıtlanmaya çalışılmıştır. Shulman'ın (1986) pedagojik alan bilgisi kavramını kullanarak alan bilgisi ve pedagojik alan bilgisi olmak üzere iki ana kategoriyi tanımlamışlardır. Bir matematik öğretmeninin, matematik eğitimi almış herhangi bir yetişkinden farklı olarak neyi bilmesi gerektiğini ayırt etmek için "Matematiği Öğretme Bilgisi" çerçevesini tanımladılar. (Ball ve diğerleri, 2005; Hill ve diğerleri, 2008; Ball ve diğerleri, 2008). Yani Matematiği Öğretme Bilgisi, öğretmenlerin matematiği, öğretim işine özgü ve matematiğin diğer mesleklerin ihtiyaçlarına uyum sağlama yöntemlerinden sıklıkla farklı şekillerde anlaması ve kullanması gerektiği fikri üzerine inşa edilmiştir (Stylianides & Ball, 2008, 398). Matematiği Öğretme Bilgisi matematik öğretmenlerinin bilgilerini değerlendirmek için yaygın olarak kabul edilen ve kullanılan bir çerçevedir. Öğretime yönelik matematik bilgisi iki ana kategoriyi içerir: (i) konu alanı bilgisi ve (ii)pedagojik içerik bilgisi.

2.3. Öğretimin Matematiksel Kalitesi (MQI)

Öğretmenlerin Matematiği Öğretme Bilgisi öğrenci öğrenmesini etkileyen önemli bir faktördür (Bobis vd., 2012; Hill vd., 2005). Ancak öğretimi etkileyen tek durum bu değildir. Öğrencilerin öğrenmesini geliştirmek için öğrencilerin sınıfta aldıkları öğretimin daha ayrıntılı olarak araştırılması gerekmektedir (Chalambous vd., 2012). Öğretimi ayrıntılı olarak incelemek için MQI gözlem protokolü geliştirilmiştir. MQI'nin güçlü bir yönü, matematik derslerine özel olmasıdır ve MQI'nin içeriğe özgü boyutları, öğrencilerin öğrenmesini geliştirmek için yöneticilerin öğretiminin bu yönlerine öncelik vermesine olanak tanır (Charalambous & Litke, 2018). MQI, birinci sınıf ile on ikinci sınıf arasında yer alan matematik müfredatındaki tüm içerik için kullanılabilir. MQI'nin geçerlilik çalışmaları, MQI'nin farklı matematiksel içeriklere uygulanabileceğini göstermiştir.

MQI çerçevesi birçok araştırmada farklı amaçlarla kullanılmaktadır. Öğretmenlerin Matematiği Öğretme Bilgisi ile öğretim kalitesi arasındaki bağlantıyı araştıran çok sayıda çalışma vardır (Santagata ve Lee, 2019; Hill ve diğerleri 2008, 2012, 2015). Bazı çalışmalar öğretmenlerin Matematiği Öğretme Bilgisinin ve müfredat materyallerinin öğretimin kalitesine katkısını araştırmaktadır (Charalambous ve Hill, 2012; Hill ve Charalambous, 2012).

Öğrencilerin öğrenmesi her zaman öğretimin kalitesinin bir göstergesi olarak kabul edilir. Bu nedenle bazı araştırmacılar öğretmenlerin MQI puanları ile öğrencilerin test puanları arasındaki ilişkiye odaklanmaktadır (Blazer vd, 2016; Blazer ve Kraft, 2017; Kane ve Straiger, 2012; Hill vd., 2011). MQI çerçevesi aynı zamanda mesleki gelişim için bir araç olarak da kullanılmıştır (Kraft & Hill, 2017; Hill vd. 2016; Mitchell & Marin, 2015).

Adkins (2017) başarılı öğrencilerin öğretmenlerinin matematiği nasıl aktardığını belirlemeye çalışmıştır. Bu öğretmenlerin kullandığı MQI yöntemlerini (boyutunu) belirlemeye çalışmasında başarılı öğrenciler yetiştiren öğretmenlerin öğrettikleri konuya ilişkin matematiksel alan bilgilerinin yeteli olduğunu ortaya koymuştur. Bu öğretmenler ortalama bir Matematiği Öğretme Bilgisi puanına sahipler ve çalışmada gözlemlenen derslere ait 80 bölümün yalnızca %3'ünde öğretmenlerin içerik hatası tespit edilmiştir. Öğretmenler Matematiksel Zenginlik boyutunda yüksek puanı almışlardır. Öğretmenler öğrencilerin hatalarını düzelterek onların katkılarını Öğrencilerle ve Matematikle Çalışma boyutuna ilişkin puan almışlardır. En az kullanılan boyut Ortak Temelde Oluşturulmuş Öğrenci Uygulamalarıdır. Öğrenciler matematikle işlemsel olarak iletişim kurmuşlar ve nadiren matematiksel akıl yürütmeyi kullanarak tahminler veya sonuçlar çıkarmışlardır. Öğretmenlerin göreli zayıflığı öğrencilere rehberlik etmek ve matematiksel akıl yürütmede genelleme yapmak olarak saptanmıştır.

3. Yöntem

Bu bölümde nitel araştırma tasarımının ve durum çalışmasının kullanımının gerekçesi ve çalışmanın bağlamı sunulmaktadır. Daha sonra sırasıyla araştırmanın katılımcılarına, veri toplama araçlarına ve veri analiz tekniklerine yer verilmiştir.

3.1 Araştırma Deseni

Ortaokuldaki matematik derslerinin kalitesini araştırmak amacıyla nitel bir araştırma tasarımı uygulanmıştır. Nitel araştırma, gözlem, görüşme, doküman analizi gibi nitel veri toplama yöntemlerinin kullanıldığı, olayların doğal ortamında gerçekçi ve bütüncül bir şekilde ortaya çıkarılması için nitel bir sürecin takip edildiği araştırmalar olarak tanımlanabilir (Yıldırım ve Şimşek, 2013). Görüşme, gözlem ve analiz nitel araştırmanın merkezi faaliyetleridir çünkü bunların hepsi altta yatan anlamın anlaşılmasına yardımcı olur (Merriam ve Tisdell, 2015). Bu nedenle nitel araştırma, insanların günlük yaşamına ilişkin araştırmalar için en uygun yöntemdir. Eğitimde, öğretme ve öğrenme süreciyle ilgili birçok soru araştırmacılar, eğitimciler veya politika yapıcılar tarafından sorulmaktadır. Öğrencinin öğrenmesini etkileyen faktörler nelerdir? Öğretmenler ortaokulda matematiği nasıl öğretir? Ne tür faaliyetler yapıyorlar? Ne tür soruları çözüyorlar? Öğrenciler sınıfta neler yaparlar? Öğretmenler ve öğrenciler somut materyalleri veya teknolojiyi kullanıyor mu? Bu sorulara cevap verebilmek için öğretimin günlük rutinine uyulması, öğretmen ve öğrencilerle bazı görüşmelerin yapılması gerekmektedir. Yani eğitim süreciyle ilgili birçok soruyu cevaplamak için nitel araştırma en iyi araştırma yoludur.

Nitel araştırma ortaya atılan sorularla başlar, veri toplama genellikle katılımcıların ortamında gerçekleşir, belirli veriler genel temaları geliştirmek için kullanılır ve araştırmacı verilerin anlamını yorumlar (Creswell, 2007). Niteliksel araştırma, doğal ortamda ayrıntılı bir veri analizini gerçekleştirir. Nitel araştırmalarda araştırmacı, yalnızca bir şeyin "ne ölçüde" veya "ne kadar iyi" yapıldığını anlamaya çalışmak yerine, doğal ortamda olup bitenlerin daha bütünsel bir resmini görmek ister (Fraenkel vd., 2012). Bu nedenle araştırmacı araştırmanın bağlamına odaklanır ve bağlama ilişkin zengin bilgiler elde etmeye çalışır. Bu araştırma nitel bir çalışmadır ve sınıftaki (doğal ortamda) matematik öğretiminin kalitesini nitel veri toplama yöntemleri gözlem ve grup tartışması kullanarak araştırmayı amaçlamaktadır.

3.2. Çalışmanın Bağlamı ve Katılımcılar

Bu çalışmanın odak noktası öğretimin matematiksel kalitesidir. Odak noktası öğretim kalitesi olduğundan, gözlemlerin yapıldığı okul hakkında biraz bilgi vermek, çalışmanın ortamını anlamada yardımcı olacaktır. Araştırma Orta Anadolu'daki iki farklı devlet ortaokulunda gerçekleştirilmiştir. Okullardan biri imam hatip ortaokuludur. İmam-hatip ortaokulları, ortak ortaokul derslerinin yanı sıra dini derslerin de verildiği bir ortaokul türüdür. Bu okullarda kız ve erkek öğrenciler ayrı sınıflarda eğitim alırlar. Sınıflarda ortalama 25 veya 26 öğrenci kayıtllıydı. Sınıfta Türkçe bilmeyen öğrenciler vardı. Bazı ebeveynlerin sosyo-ekonomik düzeyleri ortalamanın altındaydı. Bazı ebeveynler mevsimlik işçi olarak çalışıyordu. Bu nedenle okuldaki öğrenciler devam sorunu yaşamaktadır. Okulun başarı düzeyi ortalama başarı düzeyinin biraz altındadır. İkinci okul bir devlet ortaokuluydu ve Türkiye'de ortaokullar karma eğitim vermektedir. Sınıflarda 35 veya 36 öğrenci bulunmaktadır. Öğrencilerin tamamı Türkçe konuşuyordu ve ebeveynlerin sosyo-ekonomik düzeyleri ortalamanın üzerindeydi. Okul bir devlet üniversitesinin kampüs alanına yakın olduğundan velilerin çoğu üniversite personeliydi. Okul şehrin en başarılı okullarından biri olarak biliniyordu.

Bir araştırmada toplanacak verilerin amacı ve içeriği katılımcıların seçim sürecini etkileyen temel faktörlerdir. Bu çalışma matematik öğretiminin öğretim kalitesini değerlendirmeyi amaçlamaktadır. Süreci derinlemesine analiz etmek için amaçlı örnekleme kullanılarak üç katılımcı seçilmiştir. Bu çalışmanın katılımcıları üç ortaokul matematik öğretmenidir. Çalışma grubunu belirlerken 12 ortaokul matematik öğretmeniyle görüşme yapılmıştır. Katılımcıların seçiminde birincil kriter, öğretmenlerin çalışmaya katılmaya gönüllü olmaları ve öğretimlerinin videoya kaydedilmesine, incelenip değerlendirilmesine izin vermeleridir. Ayrıca öğretmenelrin paylaşmaya açık olmaları da katılımcı seçimi etkileyen bir diğer faktördür. Her üç katılımcı da grup tartışması sürecinde yer almıştır. Sadece iki katılımcının dersleri gözlemlenebilmiştir.

3.3. Veri Toplama Süreci

Veri toplama süreci öğretim öncesi ve uygulama olmak üzere iki aşamadan oluşmaktadır. İlk veri kaynağı öğretim öncesi gerçekleştirilen grup tartışma videolarıdır. Öğretimin gerçekleştirilmesinden yaklaşık bir ay önce öğretmenler bir araya gelerek içeriği “nasıl öğreteceklerini” tartıştılar. Dairenin alanının öğretimi ve daire diliminin alanının öğretimi ile ilgili deneyimlerini paylaştılar. Her öğretmen öğretim sırasında kullandıkları etkinlikleri, problemleri ve öğretim yöntemlerinden bahsetti. Ayrıca öğrencilerin daire ve sektör alanına ilişkin değişimlerinden ve kavram yanılgılarından da bahsettiler.

İkinci veri kaynağı videoya kaydedilmiş ders anlatım süreçleridir. Doğrudan gözlem, öğretmen etkinliğini araştırmanın etkili yollarından biridir (Mangiante, 2011). Geçerli bir gözlem gerçekleştirmek için iki bileşen gereklidir; geçerli bir gözlem formu ve eğitimli bir gözlemci (Goe ve diğerleri, 2011). Bir öğretmenin öğretim kalitesini yakalamak için MQI araştırması en az iki dersin gözlemlenmesini önermektedir (Ho & Kane, 2013; Santagata & Lee, 2019). Her öğretmen birbirini takip etmeyen talimatları iki kez gözlemledi. Araştırma sorularını yanıtlamak için, öğretmenler her iki öğretmen de aynı içeriği öğretirken videoya kaydettiler. Derslerin kazanımı “Dairenin ve daire diliminin alanını hesaplar” idi. Öğretmenler ilk olarak çemberin alanını öğretirken gözlemlendi. İkinci gözlemleri daire diliminin alanını öğretirken gerçekleşti. Verilerin daha iyi anlaşılması ve açıklamalar yapılabilmesi için ana veri kaynağının yanı sıra ortaokul matematik müfredatı ve ortaokul matematik ders kitabı da kullanılmıştır. Araştırmacı sınıfta gözlemci olarak bulunarak öğretimle ilgili notlar almıştır. Öğretmen ve öğrencilerin tahtada yaptıkları çalışmalar video kaydında görünmüyordu. Bu çalışmalar için araştırmacının alan notlarından yararlanılmıştır.

3.4. Veri Analizi

Nitel veri analizi aşağıdaki adımları içerir; Ham verileri analiz için organize etme ve hazırlama, tüm verileri okuma, verileri kodlama, temalar ve açıklamalar oluşturma, temalar/açıklamalar arasında ilişki kurma ve temaların/açıklamaların anlamlarını yorumlama adımları kullanaılmaktadır (Creswell, 2007). Literatürde nitel verilerin analizinde birçok farklı analiz yöntemi kullanılmıştır. Bu çalışmada söylem analizi ve içerik analizi birlikte kullanılmıştır. İçerik analizi tekniği, araştırmacıların insan davranışları üzerinde dolaylı olarak çalışabilmesine olanak sağlar (Fraenkel vd., 2012). Söylem analizi dilbilimsel bir yaklaşımdır ve konuşmanın diline odaklanır (Merriam & Tisdell, 2015). Eğitim öncesi grup tartışma video kayıtlarını analiz etmek için, MQI çerçevesinin uyarlanmış bir versiyonu analiz çerçevesi olarak kullanılmıştır. Ancak MQI'nin 4 seviyeli puanlama anahtarı ders videolarının analizi için geliştirilmiştir. Bu nedenle MQI'nin seviyeli puanlama anahtarı grup tartışmalarını puanlamak için uyarlanmıştır. Ders video kayıtları, çalışmanın teorik çerçevesini oluşturan MQI çerçeve bileşenleri ve alt bileşenlerine göre değerlendirilmiş, düzenlenmiş ve yorumlanmıştır. Bulgular anlatılar ve doğrudan alıntılarla desteklenmiştir.

4. Bulgular ve Tartışma

Bu bölümde mevcut çalışmanın bulguları ana bölümler ve ilgili alt bölümlerde özetlenmiştir. Bu bölümde öğretmenlerin matematik derslerinin özetleri yer almaktadır. Bu ders anlatımları, öğretmenlerin öğretim sürecini incelemek ve öğretimin kalitesini araştırmak için kullanılmıştır.

4.1. Grup Tartışması: Dairenin Alanının Öğretimi

Öğretmenler ders anlatımını gerçekleştirmeden önce bir araya gelerek çemberin alanının nasıl öğretileceğini tartıştılar. Tartışma yaklaşık yüz dakika sürdü. Öğretmenlik uygulamalarından örnekler verdiler ve birbirlerinin fikirleri hakkında yorum yaptılar. Öğretmenler bilgi ve deneyimlerini paylaştılar.

Öğretmenlerin grup tartışması uyarlanan MQI değerlendirme tablosuna göre analiz edildi. Öğretmenlerin tartışmasında Matematiksel Zenginlik boyutu için Temsiller Arası Bağlantı Kurmak, Açıklama, Matematiksel Anlamlandırma ve Matematiksel Dil alt boyutları puanlanmıştır. Çoklu İşlemler ve Çözüm Yöntemleri ve Örüntüler ve Genelleme alt boyutlarına ilişkin herhangi bir bulguya rastlanmamıştır. Temsiller

Arası Bağlantı Kurmak alt boyutuna ilişkin yalnızca iki örneğe yer verilmiştir. Yusuf dairenin alanını tanıtmak için somut materyallerin kullanımından bahsetti. Dairenin alan formülünü anlamlandırmak için manipülatiflerin nasıl kullanılacağını detaylı olarak anlattı. Konuyu daha ayrıntılı olarak tartıştılar ancak manipülatiflerin sınıfta kullanımı ayrıntılı olarak tartışılmadı. Bu nedenle "Orta" olarak puanlanmıştır. Açıklama boyutunda ise yalnızca bir örnek puanlanmıştır. Efe, pi sayısının neden irrasyonel bir sayı olduğunu nasıl açıkladığını ve irrasyonel sayıyı nasıl anlamlandırdığını anlattı. Öğretmenlerin Matematiksel Anlamlandırma boyutuna ilişkin tartışmalarını iki ana başlıkta toplamak mümkündür; dairenin alan formülünün anlamını anlamak ve pi sayısını anlamlandırmaktır. Her üç öğretmen de alan formülünü anlamlandırmak için kullandıkları yöntemleri anlattılar. Efe ve Yusuf pi'yi anlamlandırmak için yaptıkları etkinlikleri anlattılar. Öğretmenler matematik dilini doğru ve etkili kullanmışlardır. Herhangi bir yanlış kullanım veya dil hatası gözlemlenmemiştir.

Öğrencilerle ve Matematikle Çalışmak ve boyutu için Öğrenci Hatalarının ve Zorluklarının Düzeltilmesi ve Öğretmenin Öğrencilerin Matematiksel Katkısını kullanması boyutlarına ilişkin veriler elde edilmiştir. Yusuf ve Ali olası öğrenci zorluklarından ve hatalarından bahsettiler ancak herhangi bir çözüm önermediler. Sadece Efe öğrencilerin derse nasıl katkı sağladığını ve öğrenci katkısını matematiğin geliştirilmesinde nasıl kullandığını anlattı ve "Orta" olarak kodlandı.

Ortak Temelde Oluşturulmuş Öğrenci Uygulamaları boyunun tüm alt boyutları en az bir kez puanlanmıştır. Öğrenci Açıklama Üretmeleri alt boyutunda öğrencilerin hangi dairenin daha büyük olduğuna dair nasıl açıklama yaptıklarından sadece Efe bahsetti ve bu "Düşük" olarak puanlandı. Öğrencilerin Bağlamsal Problemler Üzeninde Çalışması için her üç öğretmen de bağlamsal problemlere ilişkin örnekler sundu. Ali'nin bağlamsal önerisi öğrenciler arasında popüler olan bir bilgisayar oyununun kullanılmasıydı. Öğrenciler için çok dikkat çekici olduğunu söyledi. Bu bağlamdan uzun süre bahsettiler ve "Yüksek" olarak puanlanmıdı. Efe birden fazla bağlamdan bahsetti.

Puanlanmayan tek boyut Hatalar ve Belirsizliklerdir. Öğretmenler içerik hatası yapmamıştır. Matematiksel dili titizlikle kullanmıştır.

4.2. Grup Tartışması: Dairenin Diliminin Alanının Öğretimi

Araştırmacı öğretmenlerden bir araya gelerek daire alanını nasıl öğrettiklerini tartışmalarını istemiştir. Öğretmenler kendi öğretim uygulamalarından örnekler vererek birbirlerinin uygulamaları hakkında yorum yaptılar. Tartışma yetmiş dakika sürdü. Grup tartışması ile öğretmenlerin sınıf uygulamaları ve matematik öğretiminin kalitesi hakkında daha fazla veri toplamayı amaçlanmıştır.

Öğretmenlerin grubu tartışması, uyarlanan MQI değerlendirme tablosuna göre analiz edildi. Matematiksel Zenginlik boyutu için Temsiller Arası Bağlantı Kurmak, Açıklama, Matematiksel Anlamlandırma ve Matematiksel Dil alt boyutları puanlanmıştır. Katılımcıların tamamı daire diliminin alan formülünü biliyordu. Bu nedenle konunun nasıl tanıtılması gerektiğini tartıştılar. Birbirlerinin başvuruları hakkında yorum yaptılar ve konuyu en iyi şekilde sunmanın yolunu bulmaya çalıştılar. Örüntü ve Genelleme ile ilgili herhangi bir kanıt gözlemlenmemiştir.

Öğrencilerle ve Matematikle Çalışmak ve boyutu için Öğrenci Hatalarının ve Zorluklarının Düzeltilmesi ve Öğretmenin Öğrencilerin Matematiksel Katkısını kullanması boyutlarına ilişkin veriler elde edilmiştir. Öğretmenler öğretecekleri konuya dair alan bilgisine sahipti ve kendi öğrencilerini tanıyordu. Yusuf ve Efe olası öğrenci zorluklarından ve hatalarından bahsettiler. Ancak öğrencilerin olası hata ve zorluklara ilişkin düşünceleri farklıydı. Yusuf, öğrencilerinin dairenin toplam alanı ile 360 derecelik merkez açısını ilişkilendirmede zorluk yaşayacaklarını iddia etti. Efe, öğrencileri için bunun bariz olduğunu söyledi.

Ortak Temelde Oluşturulmuş Öğrenci Uygulamaları boyunun tüm alt boyutları en az bir kez puanlanmıştır. Öğrencilerin Açıklama Üretmeleri alt boyutunda her üç öğretmen de bazı öğrenci açıklamalarından bahsetti. Efe, öğrencilerin bir sektörün alanının diğer bir sektörün alanından neden daha büyük olduğuna dair açıklama yapacaklarını söyledi. Açıklamaları ayrıntılıydı ancak matematiği geliştirmek için öğrenci katkısını nasıl kullandığından bahsetmedi ve “Orta” olarak puanlandı. Ali ve Yusuf, öğrencilerin yarım veya çeyrek ile merkez açı arasındaki ilişkiyi kurabileceklerinden bahsetti. Ali, bilinmeyen bir merkez açıya sahip daire dilimlerinin alanlarını karşılaştırmak için öğrencilerin yarımın alanını ve çeyreğin alanını referans alacaklarını söyledi. Ali bir daire diliminin alanını bulmak için neden merkezi açıya ihtiyaç duyulduğunu açıklamak için bu öğrencinin katkısını kullandı. Ali'nin açıklamaları "Yüksek" puan aldı

4.3. Ali’nin İlk Dersinin Gözlemi: Dairenin Alanı

Ali'nin dersinin ilk gözlemi dairenin alanını öğretirken yedinci sınıflarla gerçekleşti. Ders videoya kaydedilmiş ve araştırmacı pasif gözlemci olarak sınıfta yer almıştır.

Ali derse zamanında başladı. Öğretime başlamadan önce yalnızca bir buçuk dakika harcandı. Bu süre öğrencileri selamlamak ve akıllı tahtayı açmakla geçti. Ali, öğrencilere dersin amacını, dairenin alanının nasıl hesaplanacağını öğreneceklerini açıklayarak derse başladı. Akıllı tahtada bir harita gösterdi. Öğrenciler bunun bir bilgisayar oyununun haritası olduğunu söylediler. Öğrencilerin neredeyse tamamı oyunu biliyordu. Bu bölümde Ali'nin dairenin alanı öğretim videosu MQI 4-seviyeli analiz çerçevesini kullanılarak değerlendirilmektedir. Ders 7 dakikalık bölümlere ayrılmış ve her 7 dakikalık bölüm ayrı ayrı değerlendirilmiştir.

Öğrencilerle ve Matematikle Çalışmak kodu, öğretmenin öğrencilere nasıl yanıt verdiğini ve öğretmenin öğrencilerin katkılarından nasıl yararlandığını gösterir. Öğrencilerle ve Matematikle Çalışmak kodunun öğrencilerin karşılaştığı matematiğin derinliğini göstermektedir. Ali'nin dersi Matematiksel Zenginliği kodlarında bazı bölümler için "Orta" veya "Yüksek" puan alırken, Öğrencilerle ve Matematik ile Çalışmak kodlarında "Düşük" puan aldı.

Öğrencilerin Açıklamalar Üretmeleri boyutu dört bölümde puanlanamamıştır. Öğrencilerin açıklamaları sık olduğu için sadece 1. bölüm “Orta” olarak puanlandı. Cevaplarını gerekçelendirerek neden bir dairenin diğerinden daha büyük olduğunu açıkladılar. Diğer beş bölüm ise öğrencilerin kısa açıklamalar yapması nedeniyle

“Düşük” puan aldı. Örneğin 7. bölümde öğretmen alanın ölçü biriminin ne olduğunu ve ölçü biriminin neden metrekare olduğunu sordu. Bazı öğrenciler bunun bir gerçek olduğunu söyledi. Bazı öğrenciler ise metre kare ile ölçüldüğünü söyledi. 4.bölümde bir öğrenci, yeniden düzenlenen şeklin alanının taban ile yükseklikle çarpılıp sonucun 2'ye bölünmesiyle bulunabileceğini açıklamıştır. Bu öğrencinin açıklaması yanlıştır.

Eğitim, öğretmen ve öğrenciler arasındaki karşılıklı diyalogla devam etti. Ancak öğrencilerin katkıları çok fazla değildi. Yedi segment Öğrencilerin Matematiksel Sorgulama ve Muhakeme Yapması için “NP” puanı aldı. 7. bölümde bir öğrenci pi'nin değeri hakkında bir soru sordu. “Toplam alanı neden 3’e bölüyoruz?” dedi. Neden 3 kullanıyoruz? Öğretmen pi'yi zaten öğrendiklerini, birkaç hafta önce biraz tartıştıklarını söyledi. 8. bölümde bir öğrenci r2 ile ilgili bir soru sordu. "49 neden 7'ye eşit?" diye sordu. Yarıçağın karesi ile yapılan işlemi anlamakta zorluk yaşadı. Öğretmen bir sayının karesinin o sayıyı kendisiyle çarpmak anlamına geldiğini açıkladı. 9. Bölümde bir öğrenci, dairenin tetragona en iyi şekilde uyacağı ve tetragona dört taraftan da teğet olacağı için tetragonun kare olması gerektiğini açıkladı. Bu üç bölümün tümü “Düşük” olarak puanlandı.

Geometride şekiller bir temsil olarak sayılmadığından, yedi segment Temsiller Arası Bağlantı Kurmak alt boyunda puanlanmamıştır. Geometri derslerinde geometrik şekil “şeyin kendisi” olarak kabul edilir. Temsiller Arası Bağlantı Kurmak boyutu için yalnızca iki segment “Düşük” olarak kodlanmıştır.

4.4. Ali’nin İkinci Dersinin Gözlemi: Daire diliminin Alanı

Bu derste bütün Matematiksel Zenginlik alt boyutları en bir kez gözlemlenmiştir. Ez az gözlemlenen alt boyut Temsiller Arası Bağlantı Kurmak, Çoklu İşlemler ve Çözüm Yöntemleri ve Örüntüler ve Genelleme boyutlarıdır. Çoğu durumda Açıklama ve Matematiksel Anlamlandırma boyutları birbiri ile çakışmaktadır. Bu nedenle, Matematiksel Anlam Oluşturma boyutunun birçok bölümü için öğretim “NP”den farklı puan aldı. Matematiksel Anlamlandırma alanında yalnızca 4. ve 10. bölümler “NP” aldı. 1. bölümde öğretmenler bir sorunun çözümünü açıkladılar.

Sorunun çözümünü açıklamak için öğretmen dairenin yarıçapı ile dikdörtgenin kenarlarının eşitliğini gösterdi. 2. ve 3. bölümlerde öğretmen çeyreği anlamlandırmaya çalıştı. Belirli bir dairenin neden tam dairenin çeyreği ve dörtte biri olduğunu açıkladı. Açıklamaların tamamı Matematiksel Anlamlandırma olarak nitelendirilmediğinden bölümler “Düşük” puan alırken, Açıklamalar için “Orta” puan aldı. S5, Açıklamalar alanında “Orta” puan alırken, Matematiksel Anlamlandırma alanında “Yüksek” puan aldı. Öğretmen ve öğrenciler, bir öğrencinin doğru cevabı bulmasına rağmen çözüm yönteminin neden yanlış olduğunu tartıştılar. Öğretmenin öğrencinin çözümüne ilişkin açıklaması ayrıntılıydı ancak öğretimin odak noktası değildi

Ders, öğretmen ve öğrenciler arasındaki karşılıklı diyalogla üzerine kurulmuştu. Ancak öğrenci katkıları genel olarak matematiksel olarak önemli değildi, yani öğrenciler matematiksel düşünmeyle meşgul değildi. Altı segment Öğrencilerin Matematiksel Sorgulama ve Muhakeme Yapması boyutu için “NP” puanı aldı. Segment 1 ve segment 6 “Orta” olarak puanlandı. Birinci bölümde bir öğrenci kendi çözüm yöntemini anlatmış ve arkadaşının çözümünü kendi hatasına işaret ederek yorumlamıştır. 6. Bölüm “Orta” olarak puanlandı çünkü bir öğrenci turuncu alanın mavi alandan neden daha büyük olduğunu bölgeyi yarımla karşılaştırarak açıkladı. Öğrencilerin çeyrek ve dörtte bir ile ilgili sorular sorması nedeniyle 2. ve 3. bölümler “Düşük” olarak puanlandı.

Öğretmen sorular sorarak öğrencilerin derse katılımını sağladı. Öğretim, öğretmenin soruları ve öğrencilerin cevaplarıyla devam etti. Dolayısıyla, Öğrencilerin Matematik ile İlgili İletişim Kurması için hiçbir segment "NP" olarak puanlanmadı. Öğrencilerin katkıları kısa olduğu için yalnızca 2. ve 3. bölümler “Düşük” kareyi aldı. İki öğrenci çözümlerini tahtada sundu ve 10. bölüm “Yüksek” puan aldı. Diğer yedi bölüm ise “Orta” olarak puanlandı çünkü bazı kısa öğrenci katkıları, öğrenci açıklamaları veya öğrencilerin çözüm yöntemlerine ilişkin paylaşımları gözlemlendi. Örneğin 4. bölümde bazı kısa öğrenci katkılarına ek olarak bir öğrenci çözümün tüm adımlarını özetledi. 5. bölümde bir öğrenci tahtadaki soruyu çözdü ve bazı öğrenciler onun çözümü hakkında yorum yaptı.

4.5. Efe'nin İlk Dersinin Gözlemi: Dairenin Alanı

Efe, öğretim boyunca matematiksel dili akıcı bir şekilde kullandı ve öğrencilere matematiksel terimleri doğru kullanmaları konusunda uyarılarda bulundu yaptı. Bu nedenle segmentler genel olarak "Orta" veya "Yüksek" puan aldı. Yalnızca öğrencilerin kopyaladığı bölüm 7 ve bölüm 8 "Düşük" olarak kodlanmıştır ve öğretmen onların yaptıklarını ve öğrendiklerini özetlemiştir.

Tüm segmentler genel bir Matematiksel Zenginlik puanı verilmektedir.. Genel puan verilirken matematik dilinin orta düzeyde kullanılması zenginlik unsuru olarak değerlendirilmemektedir. Dolayısıyla 5. bölüm ve 10. bölüm Matematiksel Dil dışında herhangi bir zenginlik unsuru içermediğinden "NP" genel puanlarını almıştır. İlk dört segment "Orta" ve "Yüksek" genel zenginlik puanları aldı. İlk ders saati dilimlerinde öğrencilere sunulan matematik derinlemesine hazırlanmış ve öğrencilerin kavramsal anlamalarını destekleyecek şekilde yapılandırılmıştır. Ayrıca öğretmenin sayılar arasındaki ilişkiye odaklandığı 6. bölüm "Yüksek" puan aldı. Sınıf alıştırma sorularını çözmeye başladığında öğrencilere sunulan matematiğin derinliği azaldı ve bir sonraki bölüm "Düşük" puanlar aldı. Soruların çözümünde sadece dairenin alan formülünü uyguladıkları için az sayıda Matematiksel Zenginlik unsuru oluştu.

Efe'nin ikinci ders anlatımı Öğrenci Hatalarının ve Zorluklarının Düzeltilmesi için "NP" veya "Düşük" olarak kodlanmıştır. Derse öğretmenin konuşması hakimdi. Bu nedenle bölüm 6 ve bölüm 10'da birkaç öğrenci hatası veya zorluğu gözlemlendi. 6. bölümde pi değeri 3,14 iken bir öğrenci ölçü birimine karar vermekte zorlanmıştır. Bölümler Öğretmenin Öğrencilerin Matematiksel Katkısını Kullanması boyutunda "NP" ya da "Orta" olarak puanlanmıştır.

Öğretmen derse bir hikaye problemini tanıtarak başladı ve ilk iki bölüm boyunca bunun üzerinde çalıştılar. Yani ilk iki bölüm, Öğrencilerin Bağlamsal Problemler Üzerinde Çalışması boyutu için "Orta" puan aldı. 3. bölümde öğretmen bir daire çizdi, parçaları yeniden düzenledi ve saf geometriden bahsetti. 4.bölümde öğretmen hikaye problemine geri döndü ve çözdü. Dolayısıyla , Öğrencilerin Bağlamsal

Problemler Üzerinde Çalışması boyutu için S4 “Orta” olarak puanlanmıştır. Çemberin alan formülünü geliştirdikten sonra saf geometri üzerinde çalıştılar ve segmentlere “NP” puanı verildi.

Segment 8, Öğrencilerin Matematik ile İlgili İletişim Kurması boyutunda "NP" puanı aldı. 8. bölüme öğretmenin konuşması hâkim oldu ve matematik öğrencisinin katkısı olmadı. Bölüm 1 ve bölüm 4, bazı kısa ve bazı önemli öğrenci katkıları oluştuğundan "Orta" puan aldı. Birinci bölümde öğrenciler hangi dairenin daha büyük olduğuna dair düşüncelerini dile getirdiler. Gerekçelerini açıkladılar. 4. bölümde öğrenciler dairenin alan formülünün geliştirilmesine katkıda bulundular. Bir öğrenci soruyu tahtada çözdü ve 10. bölüm “Orta” olarak puanlandı. 9.bölümde birden fazla öğrenci çözümlerini tahtada sunarak “Yüksek” puan aldı. Öğrencilerin katkıları kısa olduğundan ve öğretmenin sorularına verilen bir veya iki kelimelik yanıtlarla sınırlı olduğundan geri kalan bölümler "Düşük" puan aldı.

4.6. Efe’nin İkinci Dersinin Gözlemi: Daire diliminin Alanı

Dersin amacı daire diliminin alanını hesaplamayı öğrenmekti. Bu bir geometri olduğu için şekiller temsil olarak kabul edilmemektedir. Bu nedenle Temsiller Arası Bağlantı Kurmak kodu için yalnızca 1. bölümde “Orta” puan almıştır. Öğretmen daire ile gerçek yaşam durumu arasında bağlantı kurdu. Daireler veya daire dilimi ile sembolik temsiller arasında bağlantılar kuruldu. Diğer segmentler “NP” olarak kodlandı.

Öğretmen alan formülünü üzerinde çalıştıktan sonra açıkladı. Öğretmen öğrencilerin sektörlerin alan formüllerini kendi kendilerine keşfetmelerini sağlamaya çalıştı. Bu nedenle öğrenciler öncelikle sektörlerin merkez açısı ile sektörlerin alanları arasındaki doğru orantıyı keşfettiler. Dolayısıyla ilk üç bölüm bağlamı oluşturmaya harcandı ve hiçbir prosedür veya çözüm yöntemi tanıtılmadı. Efe ikinci olarak sektörlerin alan formülünü tanıtarak öğrencileri soruları çözerken sayılar arasındaki ilişkileri kullanmaya teşvik etti. Öğrenciler soruyu çözerken bu yöntemleri kullanmışlar ve karşılaştırmışlardır. Her soru için en az iki farklı çözüm yöntemi kullanmışlardır. Dolayısıyla sınıfın soru çözdüğü bölümler (bölüm 5 ve bölüm 8)

“Orta” puan alırken, 4, 5 ve 9” bölümleri Çoklu İşlemler ve Çözüm Yöntemleri için “Yüksek” puan aldı.

Öğrenci Hatalarının ve Zorluklarının Düzeltilmesi için sekiz bölüm “NP”, bir bölüm “Düşük” ve bir bölüm de “Orta” olarak kodlandı. Segmentlerin sekizinde herhangi bir öğrenci hatası veya zorluğu gözlemlenmedi. 3.bölümde bir öğrenci gereksiz bir işlem gerçekleştirdi. Öğretmen öğrencileri uyardı ve yaptığı işlemlerin anlamsız olduğunu söyledi. Ayrıca 3. segmentte bir ön iyileştirme meydana geldi. Ders sırasında öğretmen öğrencilerle etkileşimde bulundu. Öğretmenin konuşması baskın olsa da Efe, öğrencilerin önemli katkılarını matematiğin geliştirilmesi için kullanmaya istekliydi. İlk beş bölümde bir sektörün alanının nasıl bulunacağı üzerinde çalışılıyor. Öğrenciler aktif olarak katıldılar ve öğretmen doğru öğrenci fikirlerini vurguladı ve bunları içeriği oluşturmak için kullandı. Yani, Öğretmenin Öğrencilerin Matematiksel Katkıları Kullanması kodunu kullandığı için segmentler “Orta” veya “Yüksek” puanlar alır.

Ortak Temelde Oluşturulmuş Öğrenci Uygulamaları boyutunun tüm alt boyutları en az bir kez puanlanmıştır. Öğrenci Açıklama Üretmeleri boyutu için yedi bölüm “NP” olarak puanlanmıştır. 2, 5 ve 8. bölümlerde öğrencilerin açıklamaları yer almaktadır. 2. bölümde bir öğrenci, mavi sektörde Çarkıfelek'i durdurma şansını açıkladı. Açıklaması kısa olmaktan da öteydi ve "Orta" olarak puanlandı. Beşinci bölümde bir öğrenci sektörün alan formülünü belirtmiştir. Öğretmen öğrencilerden formülün ne anlama geldiğini açıklamalarını istedi. Öğrencinin açıklaması tamamlanmadı ve “Düşük” puan aldı. 8. bölümde başka bir öğrenci açıklaması daha gerçekleşti. Öğrenci, çözümünün neden işe yaradığını açıkladı.

Efe'nin anlatımına genellikle öğretmenin konuşması hakimdi. Öğrencilerin katkıları sık değildi. Dolayısıyla Öğrencilerin Matematiksel Sorgulama ve Muhakeme Yapması boyutunda altı segment “NP”, bir segment “Düşük”, bir segment “Orta” ve iki segment “Yüksek” puan aldı. 2, 3 ve 5. bölümlerde öğrencilerin açıklamaları gerçekleşti. İkinci bölüm yalnızca bir öğrenci açıklaması örneğini içeriyordu ve “Orta” olarak puanlandı.

5. Conclusion and Discussion

Bu çalışma, iki ortaokul matematik öğretmeninin matematik öğretimine ve öğretmenlerin grup tartışmasında vurguladığı öğretim yönlerine ayrıntılı bir bakış sunmaktadır. Grup tartışmaları öğretimden neredeyse bir ay önce gerçekleştirilmiştir. Öğretmenler kendi öğretim deneyimlerini paylaştılar. Yani öğretim yöntemlerinden, uyguladıkları etkinliklerden, çözdükleri problemlerden, öğrencilerinin daire ve daire diliminin alanına ilişkin karşılaştıkları zorluklardan bahsetmişlerdir. Her öğretmenin ardışık olmayan iki dersi videoya kaydedildi ve ardından MQI çerçevesi kullanılarak analiz edildi. Dersler yedinci sınıflarda gerçekleştirildi. Dersler analiz için 7 dakikalık bölümlere ayrıldı.

Öğretmenler tüm derslerde doğrudan anlatım ve soru-cevap tekniklerini kullanmışlardır. Alan alanyazında da benzer çalışmalar bulunmaktadır. Öğretmenler öğretimin başlangıcında doğrudan öğretimi kullanmış, daha sonra soru-cevap ve tartışma gibi teknikleri kullanmışlardır (Yeo, 2008).

Grup tartışmalarının ve ders videolarının bulguları, öğretmenler öğrettikleri matematiksel içeriğin alan bilgisine sahip olduğu göstermektedir. Yalnızca iki örnekte matematiksel dilin muğlak kullanımı gözlemlenmiştir. Efe dışbükey çokgenlerden bahsederken “düzgün şekil” kelimesini, Ali ise “merkez açı” yerine “açı” kelimesini kullanmıştır. Her iki öğretmenin de alanın ölçü birimine ilişkin bilgileri yetersizdir. Ali ve Efe, iki uzunluğun çarpımı nedeniyle alanın ölçüsünün metrekare olduğunu açıkladılar. Literatürdeki araştırmalar öğretmenlerin de alan ölçümü birimlerini anlamada zorluk yaşadıklarını göstermiştir (Kordaki ve Potari, 1998; Ma, 2010; Outhred ve McPhail, 2000; Outherd ve Mitchelmore, 2004; Zacharos, 2006; Reinke, 1997).

Öğretmenlerin matematiksel alan bilgileri, öğretmenlerin görev yaptığı okullardan etkilenmezken öğretimin MQI puanı okul farklılığından etkilenmiştir. Ali’nin okulundaki öğrenciler sosyo-ekonomik düzeyi düşük ailelerden geliyordu. Birçok öğrenci düzenli olarak okula gidemedi. Bu nedenle pek çok matematik konusunu gözden kaçırdılar ve bilgi eksikliğiyle karşı karşıya kaldılar. Ali'nin sınıfında öğrencilerin yaşadığı zorluklar bilgi eksikliğinden kaynaklanıyordu ve öğrencilerin açıklamaları genellikle matematiksel olarak önemli değildi. Efe okulunun öğrencileri yüksek sosyoekonomik düseye sahip ailelerden gelmektedir ve anne ve babaları iyi eğitimlidir. Öğrenciler yeni konu ile önceki öğrendikleri arasında kolayca bağlantı kurdular. Hill ve diğerleri (2015) okul ortamının öğretim kalitesini etkilediğini belirterek benzer bir sonuç belirtmiştir.

Ali'nin sınıfındaki öğrenciler pi'nin anlamını anlamakta zorluk çekiyorlardı. Ali pi'nin ne olduğunu açıklamadı. Pi konusunu bir hafta önce tartıştıklarını söyledi. Ali'nin davranışı içerik bilgisi eksikliği olarak değerlendirilse de grup tartışması verileri onun pi'yi bildiğini gösterdi. Grup tartışmasında bir süre pi hakkında konuştular ve Ali de bu tartışmaya katkıda bulunarak çemberin çevresini öğretirken pi'nin çemberin çevresinin çapına oranı olduğundan bahsettiğini söyledi. Ancak Efe bunun bir oran olduğunu açıkladı ve pi hakkında daha fazla bilgi verdi. Her iki öğretmenin de pi hakkında bilgisi olmasına rağmen sadece Efe pi'nin ne olduğunu açıklamıştır. Bulgular öğretmenlerin alan bilgisinin sinin her zaman yüksek MQI puanıyla sonuçlanmadığını gösterdi.

Efe'nin her iki dersinde de öğretenin konuşması dersi domine etmiştir. Ali öğrencilerle iletişim kurmak için soru-cevap tekniğini sık sık kullandı. Ancak Efe'nin sınıfında öğrencilerin derse katılımı az olmasına rağmen, öğrencilerden gelen fikirler dersin matematiksel gelişimine daha fazla katkı sağlamıştır. Öğrenciler önemli açıklamalarda bulundular. Ali'nin sınıfında öğrencilerin matematik dışı konular hakkında konuştuğu, öğrencilerin açıklamalarının kısa veya yanlış olduğu birçok durum vardı. Ayrıca Ali’nin sınıfındaki öğrenciler problemin içeriğini düşünmeden verilen sayılarla işlemler yapmışlardır.

Öğretmen matematiksel içeriği tanıttığında görevlerin bilişsel düzeyleri yüksekti. Ancak öğretmenler öğrencilere yüksek düzey bilişsel çaba gerektiren sorularda açıklamalar yapmış, ve sorunun bilişsel düzeyinin düşmesine sebep olmuşlardır. Ayrıca öğretmenler bilişsel düzeyi düşük sorular çözmüştür. Grup tartışmasında Efe, yeni bir konuyu tanıtırken gerçek hayattan bir bağlam kullandığını söyledi. Bağlamsal problemler kullanmanın matematiği anlamlandırmak için önemli olduğunu söyledi. Her iki öğretmen de bağlamsal bir problemle başladı. Ancak aşağıda rutin soruları çözdüler.

Ali'nin alan bilgisi, öğrencilere $3r^2=27$ işlemini basit bir bağlamsal problemle açıklamasına yardımcı oldu. Cerilen eşitlikte yarıçapın değerini bulmakta zorlanan bir öğrenciye “3 elmanın fiyatı 27 olduğuna göre bir elmanın fiyatı nedir?” diye sormuştur. Öğrenci yeni bağlamsal soruyu doğru yanıtladılar çünkü basit bağlamsal problemlere önceki sınıflardan aşinaydı. Ancak cebirsel ifade onun için yeniydi ve cebirsel notasyonlarla işlem yapmakta zorluk çekiyordu. Hill ve arkadaşlarının (2008) çalışması, Matematiği Öğretme Bilgisi değeri yüksek öğretmenlerin öğrencilere fırsat eşitliği sağlamak için örnekleri akıllıca seçtiklerini göstermiştir.

E. THESIS PERMISSION FORM / TEZ İZİN FORMU

(Please fill out this form on computer. Double click on the boxes to fill them)

ENSTİTÜ / INSTITUTE

Fen Bilimleri Enstitüsü / Graduate School of Natural and Applied Sciences	☐
Sosyal Bilimler Enstitüsü / Graduate School of Social Sciences	☒
Uygulamalı Matematik Enstitüsü / Graduate School of Applied Mathematics	☐
Enformatik Enstitüsü / Graduate School of Informatics	☐
Deniz Bilimleri Enstitüsü / Graduate School of Marine Sciences	☐

YAZARIN / AUTHOR

Soyadı / Surname :
Adı / Name :
Bölümü / Department : Choose an item. / Choose an item.

TEZİN ADI / TITLE OF THE THESIS (**İngilizce** / English):

TEZİN TÜRÜ / DEGREE: **Yüksek Lisans** / Master ☐ **Doktora** / PhD ☒

1. **Tezin tamamı dünya çapında erişime açılacaktır. /** Release the entire work immediately for access worldwide. ☐

2. **Tez iki yıl süreyle erişime kapalı olacaktır.** / Secure the entire work for patent and/or proprietary purposes for a period of **two years. *** ☐

3. **Tez altı ay süreyle erişime kapalı olacaktır.** / Secure the entire work for period of **six months**. ***** ☒

***** *Enstitü Yönetim Kurulu kararının basılı kopyası tezle birlikte kütüphaneye teslim edilecektir. / A copy of the decision of the Institute Administrative Committee will be delivered to the library together with the printed thesis.*

Yazarın imzası / Signature .... **Tarih** / Date ....
(Kütüphaneye teslim ettiğiniz tarih. Elle doldurulacaktır.)
(Library submission date. Please fill out by hand.)

Tezin son sayfasıdır. / This is the last page of the thesis/dissertation.