Prices of Production In a Monetary Economy: A Note

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Abstract

Prices of production in a monetary economy are defined with reference to the concept of monetary equilibrium as defined by Keynes. The distinction between the supply price and the demand price of capital assets is recognized as a fundamental insight of Keynes. The function of the prices of production is to generate just enough quasi-rents accruing to a capital asset to equate the two prices of a newly purchased capital asset. Accordingly, equal amounts of money will earn the same expected rate of return in all industries, if equal amounts of money were invested in newly produced capital assets in respective industries.

1. Introduction

The purpose of this note is to introduce a model of prices of production in a monetary economy, assuming a linear technology. It is introductory in the sense that all details and implications of the model are not developed, either because this has not been possible or because doing so would have blurred the main points we wish to emphasize. In particular, some results are only meant to be suggestive, being obtained within simplified frameworks. The model attempts to integrate monetary and value theories using the concept of monetary equilibrium as defined by Keynes, i.e. equalization of the marginal efficiencies of all assets (real or financial) to that of money. We understand from the term "monetary economy" a "capitalist economy". Capitalist production is generalized commodity production and the latter is
not possible without a generalized value equivalent, which is what money is. One essential feature of a "monetary economy" is implicit in the definition of monetary equilibrium, namely that "money" has a marginal efficiency. In other words, "money" is an asset that commands an interest or "money" earns "money". Thus, any theory of price formation must include the interest on capital employed, i.e. the opportunity cost of the monetary value of the resources employed in the production of commodities, as part of costs. The next section discusses how this should be done in an economy that employs only variable (or circulating) capital. Section 3 introduces constant capital and the ingredients of a model of prices of production in a monetary production economy that employs constant capital are developed. In Section 4 a simple example is developed to illustrate the main arguments of the paper. A final section is devoted to further observations about the nature of the model developed.

2. Production without constant capital

The general process of capitalist production is best described by the following Marxian circuit:

\[ M \rightarrow C \rightarrow [P] \rightarrow C' \rightarrow M' \]

where \( M \) is a given sum of money used in purchasing or hiring inputs shown by the vector \( C \). As a result of technical transformation of inputs during production process \([P]\), a new vector \( C' \) is obtained—which if sold, will result in a new sum of money \( M' \). We assume here that \([P]\) does not use any constant capital, all capital employed (\( M \)) is of circulating nature.

The circuit makes clear the fundamental idea that capitalist production starts and ends (or rather should end) with money. Obviously, the object of capitalist production is to end up with a higher sum of money than what is advanced at the beginning of the process, the difference \( M' - M \) being the realized profit as a sum of money. Capitalists seek to maximize profits. We can restate the problem in the usual way, by defining profit as the difference between revenue and cost. Revenue is simply \( pC' \), \( p \) being a vector of prices of outputs. Definition of costs on the other hand, must involve opportunity cost of the money advanced at the beginning of the process; i.e. we must determine the highest-valued forsaken opportunity of the monetary outlay \( M = wC \), \( w \) being the input price vector.

This is accomplished by considering the alternative circuit for a given sum of money over the same period as that of the process \([P]\) that is available in a capitalist economy, which is:
M...M' = (1 + i)M,

where i is the interest rate per period. This means that the opportunity cost of using M in production is (1 + i)M, so that the appropriate profit maximization problem is:

Max M' - (1 + i)M = pC' - (1 + i)wC.

Assuming a simple textbook case in which C consists only of labour we have M = wL; w being the money wage rate and L is labour. If only one output is produced, we have M' = pL; where p is the price of output and aL describes the technical possibilities of transforming L into output, assumed to be possible at a constant rate a. In defining M' as pL we are implicitly assuming that any output produced will be sold. In other words the capitalist does not perceive any realization problems. Thus the problem becomes:

Max pL - (1 + i)wL.

and the first order condition is:

p = (1 + i)w/a = (1 + i)w/

where l = w/a is the per unit labour requirement. Thus, the competitive price of output appears as a mark-up over unit cost (w/l), the rate of mark-up being the interest rate.

It follows trivially that the realized rate of profit (r) is:

r = (M' - M)/M = (pL - wL)/wL = pL/w - 1 = i,

so that under conditions of perfect competition all capitalists will earn profit at a rate i, the interest rate relevant for the period of production, provided period of production is the same in all cases. This means that in the n-commodity general case we can write p; = (1 + r);, r; being the industry specific profit rate. If r; > i, the jth industry will attract more capital and output will expand. The new entrants (or incumbents that employ more capital) will be able to charge a lower price to capture market share and still remain profitable. As a result price will fall towards the

Note that total cost is (1 + i)L, so that the cost function is C(w, i, x) = (1 + i)x, x being output. Hence, marginal cost, MC, is MC = (1 + i)L, so that the standard result that p = MC (also = AC, average cost) holds in the present formulation.
competitive level\(^2\). Thus, competition entails profit rate equalization at the limit, while the process of competition is a state of flux through which the allocative function of the price mechanism is realized.

3. Production with constant capital

Consider a firm producing a commodity \(X\) using a specific type (type A) machine, labour and certain raw materials. Suppose further that a unit of \(X\) requires \(B\) hours of processing of a certain amount of raw materials by the machine. The appropriate definition of the amount of labour required per unit of \(X\) must be the amount of labour hours necessary to keep the machine operating for \(B\) hours. This may be different from \(B\) depending on the nature of the technology in use. For example, type A machines may require continuous manning by one worker so that per unit labour requirement must at least be \(B\) hours. If the machine requires two workers to operate it, per unit labour requirement must at least be \(2B\) hours. The technology may permit one worker to set more than one machines to work for \(B\) hours. In this case per unit labour requirement will be less than \(B\). On the other hand, more labour may be required in addition to what is required for the physical operation of the machine--such as additional workers to maintain a constant flow of raw materials throughout \(B\) hours.

We may suppose that per unit labour requirements of \(X\) production is \(l\) hours in the sense that \(l\) hours of labour is required to operate a type A machine for \(B\) hours and that \(l\) hours represents the best practice organization of the production process for \(X\) using type A machines. The term "best practice" here means "technically efficient" so that if less than \(l\) hours of labour is employed, a unit of \(X\) cannot be produced; and if more than \(l\) hours of labour is employed, output remains to be one unit per \(B\) hours of machine operation\(^3\). We may then define \(k = B/l\), and differentiate different techniques

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\(^2\) In general, an adjustment should be made for the "normal" remuneration for the risk and trouble of productively employing capital in the \(j\)th industry, \(n_j\). If we allow for this industry specific remuneration, profit rate of equalization can be stated as \(r_j = i + n_j\). See Pivetti (1991) on this issue, where a monetary theory of distribution "along classical lines" is developed within a circulating capital framework.

\(^3\) Suppose \(l > B\), then a unit of \(X\) can be produced in \(B\) hours if more than one workers are present during the \(B\) hours of machine operation. If not, then a unit of \(X\) can only be produced in \(l\) hours. Optimal organization of the workplace may involve an adjustment of the number of machines and workers in such a way as to reduce idleness of workers and machines. Such adjustments are obviously not independent of the level of output that can
by their k-ratios. It follows from our definitions that capital-labour substitution does not make sense within the same technique, since by definition $l$ hours of labour is required to operate the machine for $b$ hours. This means that it is not possible to increase $b/l$, while decreasing the ratio (employing more labour than is necessary) can only increase costs without increasing output. A different technique is possible only with a different type machine, say with type B machines. It is well known that orthodox theory infers from a comparison of $k_A$ and $k_B$, the relative magnitudes of wage-rental ratios that would prevail in the two cases, and conversely from ex-ante knowledge of wage-rental ratios it seeks to infer the choice of technique. The controversy surrounding the issue is well known and should not retain us here.

We note two points to motivate the approach that will be developed below. First, if more than one type of machines are involved in a production process, description of technology in terms of the machine hours necessary per unit of output becomes increasingly arbitrary. This is because when more than one machines are employed, it becomes increasingly arbitrary to determine the necessary hours of each type of machine per unit of output. Moreover, even if this is possible, the necessary hours of different type machines cannot be added to yield an "aggregate machine hours" necessary per unit of output, since the rental prices of different machines are not necessarily the same. Imagine a situation in which a firm rents type A machines from Firm A and type B machines from Firm B. The question, and this is the second point we wish to raise, is how will equilibrium rental rates charged by Firm A and Firm B be determined, and if equilibrium means profit rate equalization, in what sense the profits of the two firms are to be defined and equalized? The business of renting machines involves a monetary outlay in purchasing the machine, and then renting it out over the life time of the machine. Different machines, or more generally different capital assets, involve different monetary outlays and different economic lifetimes, so that their profitability can only be compared after appropriate discounting of prospective incomes that will accrue to the assets. Moreover, different techniques mean a specific configuration of different capital assets, which will be operated in combination to generate an (prospective) income stream to justify the investment in the "technique", so that it is neither possible nor necessary to determine the "contribution" of any single asset to the outcome.

The description of the technology to be adopted is as follows. We

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be profitably produced. But this is not an issue that need be addressed here in describing the technical possibilities of production.
suppose that there are \(m = 1, \ldots, K\) types of capital assets and \(j = 1, \ldots, N, \ldots, N+K\) industries in the economy. The first \(N\) industries produce consumption goods both for the consumers and producers (intermediate inputs). The last \(K\) industries produce the capital assets. The technology of a typical firm in the \(j\)th industry consists of \([B_j, A_j, l]\), where \(B_j\) is a \(K \times 1\) vector of stock of capital assets of the firm, \(A_j\) is an \(N \times 1\) vector of intermediate inputs expressed as per unit requirements, so that \(a_{ij}\) (component of \(A_j\)) is the amount of commodity \(i\) required as intermediate input per unit of commodity \(j\), while \(l\) is the per unit requirement of homogenous labour. We assume that the technique \([B_j, A_j, l]\) represents the "best" technique in the sense that it would be adopted by a new entrant, who has the option of choosing among alternatives. The per unit labour requirement, \(l\), should be understood to mean the total labour required to operate the technique in use at a unit level\(^4\).

This approach completely bypasses capital measurement problems, since capital is specified in physical terms as a vector \(B\). The vector \(B\) specifies a technique as a particular combination of equipment which must be operated in combination to produce a given output. Different techniques typically involve a different set of equipment, that is a different \(B\). Labour requirement of different techniques will be different, but there is no basis for defining "capital intensity" once by a technique one understands a specific set of equipment and associated workplace organization. If a technique uses less of the homogenous labour we may define it to be more "automated", but this has nothing to do with "capital intensity". Finally we note that this formulation implicitly assumes constant returns with respect to \(A_j\) and \(l\). That is, if \(A_j\) and \(l\) are increased proportionately, output of the \(j\)th industry will increase proportionally. The rate of capital utilization, that is the services that flow from the stock of capital, must also increase\(^5\). If the variation in output is large enough so that the required flow of services cannot be met by the existing stock of capital within a period, the firm in question must expand capacity by purchasing additional machines.

The technology of the whole economy is given by \([B, A, l]\), where \(B\) is

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\(^4\) One objection here would be the fact that labour is not homogenous, so that it is not possible to determine per unit labour requirement of a technique as a single number \(l\). This is no doubt true. However, the assumption is not necessary in what follows. The crucial variable is the total variable capital including total wage payments. The assumption that \(l\) is known could be interpreted to mean that total wage bill changes linearly with output.

\(^5\) It is immaterial from the point of view of the theory of price formation that will be developed below whether or not capital utilization increases proportionately under the described circumstances, as will become clear later when price formation is considered. Thus, it is unnecessary to specify anything about returns to scale properties of "capital" in describing the technology.
Kx(N+K) matrix obtained by writing B as the jth column, A is an Nx(N+K) matrix of interindustry flows obtained by writing A_j as the jth column, and I is a 1x(N+K) vector of labour requirements with elements I_j. For future reference we note that A and I can be partitioned as A = [A_1, A_2] and I = [I_1, I_2], where A_1 is an NxN matrix of interindustry flows in the consumption goods industries, while A_2 is an N xK matrix of the same flows for machine producing industries. I_1 and I_2 are lxN and lxK vectors of per unit labour requirements for the same set of industries as in the partition of A.

Note here the fundamental asymmetry between the matrices A and B. A is a flow matrix. A unit of output of jth good requires p_j A_j of monetary outlay, p_j being the lxN price vector of consumption goods, in addition to w_j per unit wages advanced. B is a stock matrix and the equipment as specified by the jth column provides a flow of combined services to the production of the jth good. We assume that all components of B_j are owned by the relevant firm so that no monetary outlays are advanced for the services of capital assets that make up B_j. Obviously the opportunity cost of owning the machines must be accounted for in the price of output. The problem is how this will be done. We follow the logic of the circuit defined in the previous section reproduced here at a unit level for the jth industry:

\[ V_j - C - \{P\} - L - P_j \]

\[ M = P_j A_j + w_j I \]

Here \( V_j = p_j A_j + w_j \) is per unit variable capital in the jth industry and \( p_j \) is the price of output. The opportunity cost of \( V_j \) is \( (1 + i) V_j \), \( i \) being the rate of interest relevant over the period of production which is assumed to be the same for all industries. We shall refer to \( i \) as the short-term interest rate. Thus, if

\[ p_j = (1 + i)(p_j A_j + w_j) = (1 + i)V_j \]

as would be in the absence of constant capital, no rental income can be imputed to the services of the stock of capital owned by the firm. This means that we must have \( p_j > (1 + i)V_j \), the difference being imputed to the capital stock employed in the jth industry. In other words, we are treating the capital stock of the industry as a rent earning asset, \( p_j - (1 + i)V_j \) being the amount of rent, or rather quasi-rent, imputed to the capital assets of the industry for each unit of output produced and sold. The total quasi-rent imputed will obviously depend on the level of output. The margin by which price must exceed unit variable cost in different industries, i.e. the quasi-rent imputed to the capital assets of different industries per unit of output, cannot be the same, simply because different industries differ in terms of both structure and

\[ \text{If some machines are hired, then rental charges should be treated as part of variable capital.} \]
economic life time of the capital stock. We model these differences as industry specific mark-up pricing:

\[ p_j = (1 + m_j) v_j \]  

(1)

where \( m_j \) is the mark-up rate in the \( j \)th industry. The term "mark-up" immediately connotes "imperfect competition" in the minds of those who understand from perfect competition price-taking behaviour and marginal cost pricing. It must be stressed that perfect competition is assumed throughout this paper in the classical sense of profit rate equalization. Firms employ two types of capital in the present case: variable and constant. The rate of profit on variable capital is the short-term interest rate as we have seen, and is the same for each industry. Mark-up pricing is necessary for the definition and equalization of profits over constant capital. The equilibrium mark-up rates and prices will be determined as those rates and prices which result in profit rate equalization—in a specific sense to be defined—for all industries which employ different constant capitals as specified by the matrix \( B \). In other words, the mark-up rates in the present setting are competitive mark-ups as will become clear.

The price equations for the economy are:

\[ \mathbf{p} = (\mathbf{p} \mathbf{A} + \mathbf{w} \mathbf{l}) \mathbf{M} \]  

(2)

where \( \mathbf{M} \) is an \((N+K) \times (N+K)\) diagonal matrix with \((1 + m_j)\) as the \( j \)th diagonal element, \( \mathbf{p} \) is the price vector which can be partitioned as \([\mathbf{p}_1, \mathbf{p}_2]\) and in the same way as \( \mathbf{A} \) is partitioned. Using the partitioned format we can write:

\[ [\mathbf{p}_1, \mathbf{p}_2] = (\mathbf{p}_1 [\mathbf{A}_1, \mathbf{A}_2] + \mathbf{w}[\mathbf{l}_1, \mathbf{l}_2]) \mathbf{M} \]  

(3)

or

\[ p_j = (p_j \mathbf{A}_1 + \mathbf{w}_j) \mathbf{M}_j \]  

(3a)

\[ p_j = (p_j \mathbf{A}_2 + \mathbf{w}_j) \mathbf{M}_2 \]  

(3b)

where \( \mathbf{M}_1 \) is the \( N \times N \) matrix of competitive mark-up rates in the consumption good industries, and \( \mathbf{M}_2 \) is the \( K \times K \) matrix of competitive mark-up rates in the machine producing industries. If the mark-up matrix \( \mathbf{M} = [\mathbf{M}_1, \mathbf{M}_2] \) were to be known, it would follow from (3a) that:

\[ p_j = \mathbf{w}_j \mathbf{M}_j (\mathbf{I} - \mathbf{A}_1 \mathbf{M}_1)^{-1} \]  

(4)

where \( \mathbf{I} \) is the appropriate unit matrix. We can then solve for \( \mathbf{p}_2 \) by substituting for \( \mathbf{p}_1 \) in (3b) from (4) to get:

\[ p_j = \mathbf{w}_j (\mathbf{I}_2 \mathbf{M}_1 (\mathbf{I} - \mathbf{A}_1 \mathbf{M}_1)^{-1} \mathbf{A}_2 + \mathbf{I}_2) \mathbf{M}_2 \]  

(4a)

These solutions will obviously make sense only if the matrix \((\mathbf{I} - \mathbf{A}_1 \mathbf{M}_1)^{-1}\) is non-negatively invertible, which is possible if and only if the dominant
root of the non-negative matrix $A, M_i$ is less than unity. This requirement places restrictions on the mark-up matrix $M_i$. Since the dominant root of $A, M_i$ is increasing in the elements of the matrix, the higher the mark-ups the higher will be the dominant root. Thus, there is an upper limit to the possible mark-up rates, but there is no unique way of determining a maximum mark-up rate for each industry.

To complete the solution we have yet to determine the $M$ matrix. We start by considering the relevant opportunity cost of money invested in constant capital of an industry. Let $Q_j = p_j B_j$ be the current supply price of constant capital of the $j$th industry, which is also the amount of money that has to be invested in the capital stock of the $j$th industry by a new entrant. The opportunity cost of investing $Q_j$ amount of money in industry $j$ for $T_j$ periods is the present value of what would be obtained by holding a long-term interest-bearing asset (long-term government bond) that costs $Q_j$ and pays $i Q_j$ in each of the $T_j - 1$ periods and $(1 + i)Q_j$ in the last period, which is given by:

$$S = \sum_{t=1}^{T_j} \frac{Q_j}{(1 + i)^{t-1}} + \frac{(1 + i)Q_j}{(1 + i)^T_j}$$

(5)

where $T_j$ is the industry-specific time horizon relevant for discounting and $i$ is the long-term rate of interest. Simple algebraic manipulation will reveal that $S = Q_j$. Thus, investing $Q_j$ amount of money in a plant in the $j$th industry must generate just enough quasi-rents over the $T_j$ periods, which when discounted at a certain rate $r_j$ must yield the same present value as that of the interest-bearing asset that can be purchased with $Q_j$. In other words we must have:

$$Q_j = \sum_{t=1}^{T_j} \left( \pi_j \gamma_j \right)$$

(6)

where $\gamma_j = v_j Y_j$ is the total variable capital expected to be advanced at time $t$, $Y_j$ being the industry output expected at time $t$; and $\pi_j = (m_j - i)$, so that $\pi_j \gamma_j$ is the total quasi-rent that is expected to accrue to the constant capital of the $j$th industry at time $t$. The rate of discount $r_j$ that appears in (6) is nothing other than the marginal efficiency of constant capital of the $j$th industry, as defined by Keynes. We claim that $r_j$ is the appropriate rate of profit to be considered in the case of production with constant capital. To see why we first consider the equalization of marginal efficiencies of different constant capitals.

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7 The result $S = Q_j$ holds for any other scheme of repayment of the capital, provided interest is computed only for the outstanding balance in each period.
Now, Keynes proposed, in Chapter 17 of his *General Theory*, the equalization of marginal efficiencies of all assets to that of money as the condition for monetary equilibrium. The essence of Keynes' argument is that monetary equilibrium will obtain only if there is an asset such that the marginal efficiencies of all assets converge to that of this asset. This asset is money and Keynes, to put briefly, attributed this "peculiar" role played by money to uncertainty and to the fact that money is the only asset that can command a liquidity premium (Keynes, 1973: Chp. 17). If we denote by \( i \) the liquidity premium of money -or the long-run interest rate- then \( i = r_j \), \( j = 1, \ldots, N+K \), is the condition for monetary equilibrium. We can now consider the sense in which \( r \) is the appropriate rate of profit in the case of production with constant capital. Let \( r \) be any other rate of discount such that \( r < r_j = i^* \). Define \( Q_r \) to be the corresponding discounted sum obtained by replacing \( r \) by \( r \) in (6). This means \( Q_r > Q \), implying a capital gain in the case of investing in the plant as specified by \( B \). Thus, the industry will attract new investment and output will expand. The new entrants can sell at a lower price to gain market share and remain profitable. This means a fall in \( m \), until such time that the plant generates just enough expected income stream to satisfy \( Q_r = Q_r \), or what is the same thing \( r = r_j = i^* \). Thus, unless all industries have the same marginal efficiency of constant capital, the system cannot be in competitive equilibrium. This is the sense in which \( r \) is the appropriate profit rate: if \( r_s \) are equalized across industries, there will be no incentive for shifting capital between industries; and this is precisely the definition of competitive equilibrium as specified by profit rate equalization.

In other words, the Keynesian notion of monetary equilibrium is interpreted here as a state in which profit rates are equalized across industries. Note that the marginal efficiency of a capital asset is not a "rental" rate, it reflects the opportunity cost of the money invested in the asset. In the circulating capital case the money invested \( (M) \) returns at the end of the production period together with a mark-up, namely \( (1 + i)M \). The same must be true for constant capital, i.e., the money invested in a capital asset must earn the necessary interest in each period, and also it must be recovered over the period during which the asset is held. This is precisely what the

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8 It must be admitted that in this note we are simply assuming that there is a short-term and a long-term interest rate in the economy, without offering any explanation as to how and why these rates come to be established.

9 This is true if scrap values of capital assets are zero or negligible at the end of their lifetimes. If not, the difference between the supply price and the scrap value must be recovered. We here note a subtle point. In defining the marginal efficiency of a capital asset we have used \( T_j \), the lifetime of the asset. Consider a capitalist investing in constant capital
definition of marginal efficiency accomplishes in equilibrium: not only the money invested in an asset earns on average \( r \) over the period of the lifetime of the asset, but also the capital invested in the asset is recovered over the lifetime of the asset.

We wish to determine the industry-specific mark-up rates from (6) by imposing the condition for monetary equilibrium, so that \( r_j = i \). It is readily seen that even if we replace \( r_j \) with \( i \) in (6), additional assumptions are needed to proceed. One way could be to assume a steady-state situation in which \( \pi_j \) and \( V \) is the same in each period, as we will do in a specific example below. While the assumption of steady-state is analytically convenient, it conceals some important aspects of the solution. The other alternative is to take expectations as given, as did Keynes, and solve for the current period mark-up rates. Thus, we can rewrite (6) in monetary equilibrium as:

\[
Q_j = \frac{\pi_j V_j}{1 + \theta} \sum_{t=1}^{T} \frac{\pi_i V_i}{(1 + \gamma)^t} \left( (1 + \theta)^t \right) \Lambda_j
\]

where \( \Lambda_j \) is the discounted sum of total quasi-rents expected to accrue to the

in an industry with borrowed funds. The lender is unlikely to wait over the entire lifetime of the asset in order to be repaid, and in general financial institutions will not wait that long. There must, therefore, be a period over which debts must be settled. We call that period the amortization period and assume that it lasts \( \tau \) years. The parameter \( \tau \) may be industry- or even customer-specific and also it may depend on the type of asset financed. Taking \( \tau \) as given and assuming \( \tau < T_j \), we can decompose (6) as:

\[
Q_j = \sum_{t=1}^{\tau} \pi_i V_i (1 + \theta)^t \sum_{t'=1}^{T} \pi_j V_j (1 + \theta)^t
\]

From this, it follows that at the end of \( \tau \) periods the capital asset becomes a purely rent-earning asset, the discounted sum of these rents being the second term in the above equation, which is also the resale value of the asset at time \( \tau \). This is the gist of capitalist production: a successful investment becomes a purely rent-generating asset after some time. The question is how to judge \textit{ex ante} whether or not an investment will be successful. The deriving force of capitalist production is nothing other than the continuous search for rent-generating investments. Note also that the mark-up in the amortization period (Period 1) will be higher, because capital is repaid during this period. A firm can afford a lower mark-up and price in the pure rent-earning period (Period 2). This can be one explanation why prices tend to fall in mature industries. Finally, entry into an industry will always involve a period of capital loss, if there are existing firms that are in the second period of their lives; because entrants can only charge the price of the existing firms which will in general be lower than what is required in Period 1. But if successful entry is accomplished, pure rents to be earned may more than compensate for the initial capital loss. These are the sort of considerations that flow from the model.
constant capital over the period $t=2,\ldots,T$, and since $V = vY$, it depends on the expectations of future unit costs and demand conditions. The term $\pi_j V_j$ is the current period quasi-rent. Noting that $\pi = m - i$, it follows from (7) that:

$$m_j = i + (1 + i)(Q/V_j - A_j/V_j)$$

(8)

so that subject to expectations of future demand conditions, the current period mark-up rate in the $j$th industry is proportional to constant capital to variable capital ratio in the current period. To be able to solve for $m_j$s from (8), however, we need to know $Q_j = p_jB_j$ and $V_j = (p_jA_j + w_l)Y_j$. In particular, current period outputs must be known. It follows that, it is not possible to solve for prices of production from the knowledge of technology alone, even if the interest rate structure is known. In other words, a general equilibrium framework is needed for the complete solution.

4. An example

The solution is conceptually simple. The mark-up rates in every industry are determined in such a way as to yield the same marginal efficiency of constant capitals in all possible alternative uses. Consider a simple case where two goods ($x$ and $y$) and a machine ($m$) are produced. Machines are assumed to last forever. Suppose the machine can be produced in one period by labour alone so that $p_m = (1 + i)w_m$. Suppose further that constant capital requirements of $x$ is $B$ machines, while that of $y$ is only one machine. Also suppose $l = l_x = l_y$ and that no intermediate inputs are required. Then,

$$v = v_x = w_l = v_y, \quad \pi_x = m_x - i \quad \text{and} \quad \pi_y = m_y - i$$

Finally, we assume the economy to be in a steady-state situation with perfect foresight and a particular net output vector $(X, Y, 0)$, so that the (net) output of machines is zero. Note that because machines last forever, there is no difference between the marginal efficiencies of existing machines and newly produced machines. Thus:

$$Q_x = \beta p_m = \pi_x v_X i \quad \text{and} \quad Q_y = p_m = \pi_y v_Y i$$

so that $\pi_x = \beta p_m i / v_X$ or $(m_x - i)v_X = i Q_x$, implying:

$$m_x = i + (Q_x/v_X)i$$

where $V_x = vX = $ total variable capital employed in the $x$ industry. Likewise we can solve for $m_y$,

$$m_y = i + (Q_y/V_y)i$$

The commodity prices are given by:

$$p_x = (1 + m_x)v, \quad p_y = (1 + m_y)v$$

Some important aspects of the solution should be noted. First, note the
crucial role played by $p_m$, the asset price in solving the model. Contrary to
the impression one gets from the way Equations (3)-(4a) are obtained, it
should now be clear that solving for commodity prices is not possible
independently of asset prices. Second, the solution requires the knowledge of
three nominal variables ($w$, $i$, $i$) in addition to the knowledge of output
levels. In other words, we need a "general theory" of employment (output),
interest and prices (including asset prices); so that determination of (relative)
commodity prices is not in general possible without determining output,
interest rates and asset prices. Moreover, because of the special assumption
that machines are produced by labour alone, it has been possible to determine
the price of machines without the knowledge of machine output (investment
demand). In the general case, investment demand would assume a critical
position, because of its critical role in determining capital asset prices.

Now, let us consider the relation of prices to costs. The total cost of
producing $X$ has two components. First component is the variable capital
costs, which are given by $(1 + i)vX$ as we have seen in section one. The
second is the opportunity cost of $0$ (omitting the subscript $x$), the value of
constant capital employed by the firm, which is simply $iQ$ in each period.
This follows from the assumption that machines last forever, so that the value
of the shares of an $x$-industry firm must be equivalent (in equilibrium) to that
of a perpetual bond that pays $iQ$ in each period. Thus,

$$C(w, i, i, X) = (1 + i)vX + iQ$$
$$= (1 + i)vX + \pi vX$$
$$= (1 + m)vX,$$

where, we have used the fact that $Q = \pi vX/i$ and $\pi = m - i$ in obtaining the
last two equations. From the last equation it follows that:

$$MC = (1 + m)v = AC = p,$$

where $MC$ and $AC$ denote marginal and average cost, respectively. In other
words, the pricing rule that comes out of the present formulation is that of
marginal (= average) cost pricing, cost being interpreted in the proper sense
of opportunity cost to include interest charges of both variable and constant
capitals. From the second expression in the definition of the cost function, we
can obtain a useful decomposition of $MC (= AC)$ as follows:

$$MC = AC = (1 + i)v + \pi v = (1 + i)v + iQ/X = AVC + ACC,$$

where, $AVC = (1 + i)v$ and $ACC = \pi v = iQ/X$. AVC is the average (and
also marginal, denoted MVC) variable capital cost, which is simply the
opportunity cost of per unit variable capital. The ACC curve may be
interpreted as the "average constant capital cost" curve, as it shows the per
unit quasi-rent ($\pi v$) imputed to the constant capital stock of the firm. The
relations can be depicted in the following figure:

**Figure 1**

The ACC curve is a rectangular hyperbola, the area under the curve and the horizontal axis (such as the rectangle A) being equal to $iQ$ total amount of quasi-rent that must be imputed to constant capital in equilibrium for any level of output. It follows that $AC (=MC)$ being the vertical sum of the MVC and ACC curves, is a monotonically decreasing function of output\(^{10}\). If, say, the constant capital of the firm increases to $Q$, the ACC and the AC curves will shift vertically upwards in such a way that the area under the ACC curve (A) becomes equal to $iQ$ for any level of output.

Now, suppose we have $B > 1$ and $\gamma = Y/X = 1$ in the present example. Because the two industries have the same MVC, but the $x$ industry employs more constant capital, the AC curve of the $y$ industry will lie below that of the $x$ industry. Given that the outputs of the two industries are the same, measured in respective units, it follows that we must have $p_x > p_y$, despite the fact that the two industries have the same per unit variable capital cost. If the prices of the two industries were to be the same, no one would invest in the $x$ industry which requires $B$ times as much monetary outlay for constant capital compared to the $y$ industry. On the other hand, suppose $B = 1$ but $\gamma > 1$. That is, both industries employ the same constant capital and the same per unit variable capital, but $y$ industry produces more output than $x$ industry in equilibrium. Then, the AC curves of the two industries coincide and since

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\(^{10}\) This result would ordinarily be interpreted as increasing returns to scale, whereas in fact no such assumption has been made. In fact this and any other result of the present example would hold under the explicit assumption of constant returns to scale.
Y > X we have \( p_x > p_y \). Most economists when confronted with this result, will attribute the higher price of x to some sort of imperfect competition\(^\text{11}\). However, there is nothing to be gained by shifting resources from y to x, since the competitive prices in the sense of equating the marginal efficiencies in the two industries is defined precisely to rule out any such gain. Hence, the result that \( p_x > p_y \) is the perfectly competitive solution. The two industries of our example will have the same price if and only if \( \beta = 1 \) and \( \gamma = 1 \). The latter requirement means that they must be employing the same total variable capital in equilibrium, but obviously there is no basis for supposing that having the same constant capital and unit variable capital costs should mean employing the same total variable capital in equilibrium.

Consider, finally, the income distribution possibilities in our example. Given our assumption about the composition of net output, the value of net output \((Z = p_X X + p_Y Y)\) is given by:

\[
Z = (1 + m_x)w_x X + (1 + m_y)w_y Y
\]

so that:

\[
w/Z = \frac{1}{(1 + m_x)l_x X + (1 + m_y)l_y Y}
\]

or

\[
o = \frac{1}{(1 + l)L + i Q/w}
\]

where, we have used \( m = (i Q/V + i) \) for the respective industry. Here, \( Q = Q_x + Q_y \), \( L = (l_x X + l_y Y) \) total employment, and \( o \) is the real wage rate in terms of the value of the net output. This is the wage-profit curve of the economy that is in a steady-state with a particular net output. The denominator of the equation is the value of net output expressed in terms of labour (or the total labour content of the net output), which is the sum of the necessary labour for producing the net output \((L)\) and the surplus labour \((iL + i Q/w)\). The real wage appears to be a monotonically decreasing function of \((i, o)\), as expected. But, strictly speaking, the monotonic relation is valid if and only if the level and composition of net output is independent of income distribution. Because, if the level and/or the composition of output changes with income distribution, so will \( L \). Hence, in a steady-state situation with a particular net output vector, there can only be one point on the wage-profit curve: the point that corresponds to an income distribution pattern which generates a composition of demand that is consistent with the

\(^{11}\) If \( \beta = 1 \), a unit of X and Y requires \( l \) hours of labour, and from the way we have defined \( l \) we may suppose that, also \( l \) hours of machine operation. Let \( r \) and \( w \) denote hourly rental rates of machines and labour, respectively. Then, the unit cost of X and Y is simply \( c = (w + r)l \). In equilibrium we must have \( p_x = p_y = c \), which would be the conclusion of the orthodox theory under the assumed conditions.
particular net output. For example, the wage-profit frontier implies that a lower real wage rate will be associated with a higher employment, given the interest rate structure; although the share of wages in net output (oL) will be higher. Clearly, this result is meaningful, only if a) higher employment is consistent with the interest rate structure in the first place, and b) the new pattern of income distribution is consistent with the new composition of net output resulting from higher employment. In other words, the model must be solved within the framework of a "general theory" of employment, interest and asset prices, as we have already noted above. The "grand wage-profit curve" that would be obtained from such a framework is unlikely to result in any simple monotonic relationship.

5. Some further observations on the nature of prices of production in a monetary economy

First, consider the Ricardian corn model. We may imagine a capitalist as renting a plot of land for a fixed amount of rent in terms of corn, R. He then advances a wage fund, wL, where w is the money wage rate and L is total labour employed. Suppose that the output obtained from that plot of land by the application of L is X. We must have that \( p(X - R) = (1 + r)wL \), where \( r \) is the rate of profit and \( p \) is the price of corn; so that \( p = (1 + r)w \lambda \), where \( \lambda = L/(X - R) \) = labour content of a unit of net-of-rent output. Consider another capitalist that employs a variable capital of \( wL' \) on another plot of land rented for \( R' \) units of corn and obtains \( X' \) of corn output. We must still have \( p(X' - R') = (1 + r)wL \) or \( p = (1 + r)w \lambda' \), which is possible if and only if \( \lambda = \lambda' \), i.e. if and only if the labour content of a unit of net-of-rent output is the same on any plot of land. The Ricardian theory of rent precisely accomplishes this, and we can have \( p = (1 + r)w \lambda \). Note that, \( \lambda = \lambda' \) implies, upon rearranging, that \( X/L - X'/L' = (R/L - R'/L') \), so that the difference in the (average) productivity of labour on the two plots of land is absorbed by the per worker rents imputed to the two plots of land. There is one crucial point to note, however. Consider the prices of the two plots of land, denoted \( Q \) and \( Q' \), of our example. Clearly, we must have \( Q = pR/d \) and \( Q' = pR'/d \), \( d \) being the relevant rate of discount employed in discounting perpetual income streams. Hence, \( p(R/L - R'/L') = d(Q/L - Q'/L') \), so that the corn economy is always in "monetary equilibrium" in the sense that marginal efficiencies of different plots of land (\( d \)) are the same, and the burden of adjustment falls entirely on asset (land) prices. While this assumption may be justified in an economy where the only "capital asset" is a non-produced asset, it cannot be extended to an economy where there are produced capital
assets that have a supply price. A produced capital asset in effect has two prices: a supply price and a "asset" (or demand) price which is the discounted sum of current and expected incomes that will accrue to the asset\textsuperscript{13}. The logic of the marginal efficiency calculus is to equate the two prices of a capital asset, and since the supply price cannot (fully) adjust, the income that accrues to the asset must adjust to make monetary equilibrium possible. In general, let X and Y be any two industries with perpetual constant capitals, then using our previous notation:

\[ i \left( \frac{Q_x}{L_x} - \frac{Q_y}{L_y} \right) = \left( \frac{p_x X}{L_x} - \frac{p_y Y}{L_y} \right) \]

obtained from the fact that \((1 + r)w = \left( \frac{p_x X}{L_x} - i \frac{Q_x}{L_x} \right) = \left( \frac{p_y Y}{L_y} - i \frac{Q_y}{L_y} \right)\).

Thus, the difference in quasi-rents per unit of labour employed that must be imputed to constant capitals in equilibrium, is to be generated by the difference in revenue per unit of labour employed in the two industries. Prices of production of this note are so defined as to make this possible. This contrasts with the corn model, where the differences in rents are fully accommodated by changes in asset prices. It follows that employment (and so outputs) in the two industries must stand in a specific relation to total constant capital \((Q = p_B)\) employed. If demand conditions change so as to make the attainment of this specific relation impossible for a given distribution of constant capitals between industries \((Q_x, Q_y)\), then \(B_s\) (and perhaps prices of capital assets) would adjust, i.e. investment in constant capital is an integral part of the process of establishing the prices of production.

Next, a further clarification of the nature of the prices of production obtained in this note is in order. This follows from the nature of the marginal efficiency of capital, which is ".. here defined in terms of the expectations of yield and of the current supply price of the capital asset. It depends on the rate of return expected to be obtainable on money if it were invested in a newly produced asset; not on the historical result of what an investment has yielded on its original cost." (Keynes, 1973: 136, emphasis added). We may refer to the marginal efficiency of capital as defined by Keynes as the \textit{ex ante} marginal efficiency of capital as opposed to \textit{ex post} marginal efficiency of capital which refers to the marginal efficiency of constant capital that is already being employed in a given industry. Thus, the \textit{ex ante} marginal efficiencies must be equalized in monetary equilibrium, i.e. mark-up rates

\textsuperscript{13} This aspect of a capitalist economy has been stressed forcefully by Minsky; he writes: "A fundamental insight of Keynes is that an economic theory that is relevant to a capitalist economy must explicitly deal with these two sets of prices...that there are two sets of prices to be determined, and they are determined in different markets and react to quite different phenomena" (Minsky, 1982: 79).
and prices must be so determined as to enable equal amounts of money to earn the same expected rate of return in all industries, if equal amounts of money were invested in newly produced capital assets in respective industries. In other words, the prices of production defined so far are not the prices that would be obtained by considering the cost structure of existing firms in a given industry. They are the prices which would obtain at the limit of the competitive process when \textit{ex ante} marginal efficiencies of constant capital in different industries are equated to that of money. Such a state comes about through commodity price adjustment (as opposed to asset price adjustment), which is possible if and only if there is a sufficient level of investment in all lines of industry in pursuit of quasi-rent differentials.

If investment is low for whatever reason, then the price mechanism cannot function, and monetary equilibrium in the sense of equating \textit{ex ante} marginal efficiencies of constant capitals in all industries to that of money cannot be realised. In that case, the \textit{ex post} marginal efficiency of constant capitals that is already being employed in some industries will be equated to \(i\) through a fall in the demand price of constant capital in those industries. Hence, as in the corn model, changes in asset prices will absorb the differences in quasi-rents imputed to constant capitals. In other words, the economy will settle in a position of quasi-equilibrium in which \textit{ex post} marginal efficiencies of constant capital in some industries are equalized through falling asset prices. Such a state will be characterised by low investment and income and a fall in commodity prices under the assumed conditions will only aggravate the situation, since \textit{ex ante} marginal efficiencies of constant capitals will be even lower. This means that asset prices will have to be even lower, resulting in an even lower inducement to invest. What is in fact required is an increase in effective demand and also prices, so that \textit{ex ante} marginal efficiencies of constant capitals can increase and a higher level of investment may become possible. This is what, we believe, Keynes was saying.

References


Özet
Parasal bir ekonomide üretim fiyatları üzerine bir not

Parasal bir ekonomide üretim fiyatları Keynes'in parasal denge kavramına dayanarak tanımlanmıştır. Sermaye mallarının arz ve talep fiyatları arasındaki ayrım, Keynes'in temel bir öneresi olarak kabul edilmiştir. Buna göre üretim fiyatlarının fonksiyonu, sermaye mallarına aitfedilen rant gelirlerinin bu malların iki fiyatını eşitleyecek şekilde oluşmasını sağlamaktır. Böylece eşit miktarda para, yeni üretilen sermaye mallarına yatırılma durumunda, her sektörde aynı beklenen getiriyi sağlayacaktır.