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RATIONAL EXPECTATIONS, CURRENCY SUBSTITUTION, AND THE DEMAND FOR MONEY DURING THE GERMAN HYPERINFLATION

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The literature estimating the demand for money during the German hyperinflation is based on three different data sets, numerous expectation formations, and provides several different conclusions. This paper attempts to determine the relative explanatory power of the three data sets based on a common model which incorporates the essential characteristics of the earlier models. A two-step instrumental variable estimation technique is used; the first step being the estimation of the "rational" expectations, and the second step the estimation of the parameters of the demand equation. The findings of this study show that the empirical results are dependent on the data employed, and that the seemingly different results obtained in the literature come from expectation formations and underlying money demand specifications.

1) INRODUCTION

The persistent debate concerning the appropriate procedure for estimating the demand function for real cash balances during the German hyperinflation is centered around the specific formulation of expectations. Unfortunately this discussion is further hampered by the availibility of three independent data sets collected by Cagan (1959), Barro (1970), and Abel, Dornbush, Huizinga and Marcus (1979). These data sets differ from one another mainly in the selection of the price variable and the source from which the variables were

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recorded[1]. Empirical studies estimating the money demand function during the German hyperinflation do not completely account for the differences among the data sets before comparing their results. As a consequence, these comparisons may be unfitting and inconclusive. Thus, before comparisons are conducted to determine the relative explanatory power of each model, a comparison must first be conducted to determine the relative explanatory power of each data set.

The purpose of this paper is to determine the relative explanatory power of the three data sets for explaining the economic events that occurred during the German hyperinflation[2]. To insure that the explanatory power of each data set is a direct result of the information contained in the data set and not from the model used to conduct the experiment, it is necessary to test each data set with a common model and estimation technique.

In order to build a general model that incorporates the essential characteristics of the models found in the literature, a brief literature review is necessary. Cagan's (1956) original work was based on adaptive expectations, followed by Barro's (1970) work which imposed a second order gama distribution on the lagged coefficients in formulating the expectations. Sargent and Wallace (1973, 1976, 1977) provided a striking alternative to the formulation of expectations which was the catalyst for the rational expectation hypothesis. The results produced by these works are consistent with the 'quantity theory of money' in that only money matters in determining the price level.

Frenkel (1977) presented yet another alternative in formulating inflationary expectations. In his work, he discussed the problems of

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1. The description of the price variable and its source are shown below for each data set:

Cagan: P is an index of wholesale prices, collected from Zahlen zun Geldentwertung in Deutschland 1914 bis 1923, Sonderheft zu Wirtschaft und Statistik, 1, 1925, p. 33.

Barro: P is an index of the cost of living, collected from Statistisches Jahrbuck für das Deutsche Reich, 1924/23, international table 1.

Abel et al.: P is the consumer price index, collected from Zahlen zun Geldentwertung in Deutschland 1914 bis 1923, Sonderheft zu Wirtschaft und Statistik, 1, 1925, p. 33.

2. Only the time span all three data sets had in common was used. The exchange rate between the Mark and the Pound was collected from Graham (1930) and was used to supplement the data sets lacking the exchange rate data.

unobservable expected inflation, and thus proposed the forward premium on foreign exchange as a direct measure of inflationary expectations. Abel, et al. (1979) and Salemi (1980) question the theoretical reason for the forward premium on foreign exchange as da determining factor of money demand. Abel et al. (1979) argue that in order for the forward premium on foreign exchange to be a proxy for inflationary expectations, one needs to assume the restrictive purchasing power parity hypothesis. Hence, they suggest that one interpretation of the forward premium on foreign exchange as a determining factor of money demand is the possible existence of currency substitution between domestic and foreign monies.

Based on their interpretation of the forward premium on foreign exchange, Abel et al. (1979), estimated the German money demand equation with both depreciation and inflationary expectations. The expectations for depreciation were proxied by the forward premium on foreign exchange, while the actual rate of inflation was used as a proxy for inflationary expectations. Their study concluded that both depreciation and inflationary expectations were significant in estimating the German money demand equation.

Until the work by Abel et al. (1979), no one had applied the concept of foreign currency as an alternative form of wealth when empirically estimating the demand for real cash balances during the German hyperinflation. While their paper is important for this reason, their conclusions are quite different from those of Sargent and Wallace (1973, 1976, 1977). A direct comparison of these works highlight that Abel et al. (1979) relied on an external source for the expected rate of depreciation and used 'perfect foresight' for the expected rate of inflation, rather than letting the model internally form the expected values from the past values of all the available monetary variables. In doing so, they assumed that the actual rate of inflation is equal to the expected rate of inflation plus some white noise. It can be shown that this assumption caused "errors in variables" in their model, and led to inconsistent estimates of the parameters[3].

3. In addition, rather than using the percentage changes of the variables in their estimation of the money demand equation, Abel et al. used the levels of the varibles. As Frenkel (1979, p. 2) states, "a regression using the levels of the exchange rate is completely dominated by the last few observations due to the rapid acceleration of the depreciation of the mark".

2) THE MODEL

The model used here is designed to be as general as possible and to be a superset of the models used in earlier studies. It incorporates the essential characteristics of those models, specifically rational expectations and currency substitution. As with other models based on rational expectations, this model forms expectations with the available information contained in the past values of all variables in the model. The information sets used for the formation of the expectations consist of the past values of all the statistically significant variables.

The assumption of currency substitution allows the model to be more comprehensive and less restrictive than some of the models stated above. The existence of currency substitution allows residents of the country in question to diversify their monetary wealth between alternative currencies. The portfolio choice depends on the relative expected rate of returns, which in this case is the expected rate of deprectiation.

The demand equation presented here is similar in nature to the equation developed by Cagan.

$$\frac{M_t}{P_t} = e^{-\alpha_0 - \alpha_1 \pi_t - \alpha_2 \varnothing_{1t} - \varepsilon_t} \qquad (1)$$

where: M_t = demand for nominal balances
P_t = price level
π_t = expected rate of inflation
$\varnothing_t$ = expected rate of devaluation
ε_t = random variable, normally distributed with mean zero and constant variance
e = natural log base
α_0, α_1, α_2 are parameters

Equation (1) states that the demand for real cash balances is negatively related to the expected rate of inflation and the expected rate of devaluation. The expected inflation rate (π_t) is defined to be the expected value of the log ratio of P_{t+1} and P_t, and the expected rate of devaluation ($\varnothing_t$) is defined to be the expected value of the log ratio of r_{t+1} and r_t.

$$\pi_t = E_t (\ln P_{t+1} - \ln P_t) = E_t (X_{t+1}) \quad (2)$$

$$\varnothing_t = E_t (\ln r_{t+1} - \ln r_t) = E_t (\rho_{t+1}) \quad (3)$$

Where E_t is the expectations operator at time (t), r represents the exchange rate, and P represents the price level. For convenience, X_{t+1} represents the log ratio of prices at time t+1, and ρ_{t+1} represents the log ratio of exchange rates at time t+1. Equation (1) can be rewritten in its log linear form.

$$\ln M_t - \ln P_t = -\alpha_o - \alpha_1 E_t (X_{t+1}) - \alpha_2 E_t (\rho_{t+1}) - \varepsilon_t) \quad (4)$$

The information contained in equation (4) at time (t) must also be contained at time (t–1).

$$\ln M_{t-1} - \ln P_{t-1} = -\alpha_o - \alpha_1 E_{t-1} (X_t) - \alpha_2 E_{t-1} (\rho_t) - \varepsilon_t \quad (5)$$

Taking the difference between equation (4) and equation (5) yields:

$$\mu_t - X_t = -\alpha_1 [E_t X_{t+1} - E_{t-1} X_t] - \alpha_2 [E_t \rho_{t+1} - E_{t-1} \rho_t] - n_t \quad (6)$$

where: $\mu_t = (\ln M_t - \ln M_{t-1})$

$X_t = (\ln P_t - \ln P_{t-1})$

$n_t = \varepsilon_t - \varepsilon_{t-1}$

Equation (6) states that the percentage change of real cash balances is negatively related to the change of the expected rate of inflation and the change of the expected rate of devaluation.

It was assumed above that ε_t was normally distributed with zero mean and constant variance and orthogonal to past values to M, P, and r[4].

$$E_{t-1} \varepsilon_t = E (\varepsilon_t / M_{t-1}, M_{t-2}, \ldots, P_{t-1}, P_{t-2}, \ldots, r_{t-1}, r_{t-2}, \ldots) = 0 \quad (7)$$

Taking expectations of equation (6) conditional on information known in the previous period (Ω_{t-1}) yields:

$$E_{t-1} (\mu_t - X_t) = -\alpha_1 [E_{t-1} (X_{t+1} - X_t)] - \alpha_2 [E_{t-1} (\rho_{t+1} - \rho_t)] \quad (8)$$

Since

$$\mu_t - X_t = E_{t-1} (\mu_t - X_t) - n_t \quad (9)$$

Equation (6) can then be written as:

$$\mu_t - X_t = -\alpha_1 [E_{t-1} (X_{t+1} - X_t)] - \alpha_2 [E_{t-1} (\rho_{t+1} - \rho_t)] - n_t \quad (10)$$

Rewritten in terms of information sets, equation (10) becomes

$$\mu_t - X_t = -\alpha_1 E (X_{t+1} - X_t) / \Omega_{t-1} - \alpha_2 E (\rho_{t+1} - \rho_t) / \Omega_{t-1} - n_t \quad (11)$$

4. However, n_t can be correlated with current and future values of X, μ and ρ.

3) TWO-STEP INSTRUMENTAL VARIABLE ESTIMATION BY OLS

In its simplest form, equation (9) can be solved for

$$(\mu_t - X_t) - E_{t-1}(\mu_t - X_t) = -n_t \qquad (12)$$

where n_t is orthogonal to the information set which includes variables at time (t–1). This will permit the usage of OLS estimation to find inefficient but consistent estimates for $-\alpha_1$ and $-\alpha_2$[5]. In order to estimate equation (10), a two-step instrumental variable estimation technique is utilized. The first step of this technique is the construction of the instrumental variables. This involves the identification of the relevant variables to be contained in the information sets. Once the information sets have been determined, they are then used to estimate values for the expectations of inflation and currency depreciation. The second step involves the estimation of the parameters of equation (10) using the previously estimated values of the expectations as the instrumental variables. Specifically, the estimated values for the expectations of inflation and currency depreciation are used as the independent variables in the estimation of equation (10).

However, before the first step of the procedure can be conducted, the notion of 'rational' expectations and the information sets must first be addressed. The literature suggests that the information set for inflation consists of past values of inflation along with past values of devaluation, and that the information set for the rate of devaluation should include the past values of the rate of devaluation. On the other hand, 'rational' expectations assumes that information sets could conceicably consist of the past values of all the variables in the model However, the relevant information sets consist of only those variables that are statistically significant in predicting the expectations.

4) EMPIRICAL RESULTS

To determine the 'rational' information sets used in forming the expectations of inflation and currency depreciation, I regressed $(\rho_{t+1} - \rho_t)$ and $(X_{t-1} - X_t)$ on Ω_{t-1} using every possible combination of

5. The estimation technique depends on the non-trivial solution to $E_{t-1}(\mu_t - x_t) \neq 0$, and $E_{t-1}\, n_t = 0$

TABLE 1 Rational Information Sets
coefficients (t statistic)

	const.	X_{-1}	X_{-2}	X_{-3}	ρ_{-1}
Cagan					
$X_{t+1} - X_t$	.083	.79	.288	−.088	−.76
	(.866)	(1.5)	(.43)	(.16)	(1.7)
$\rho_{t+1} - \rho_t$	.058				−.147
	(.48)				(.74)
Barro					
$X_{t+1} - X_t$	−.02	−.24	−.04	−.84	.508
	(.87)	(.69)	(.10)	(3.8)	(8.5)
$\rho_{t+1} - \rho_t$	.021	−.27	.52	−.50	.07
	(.21)	(1.5)	(2.1)	(1.3)	(.06)
Abel et al.					
$X_{t+1} - X_t$	.12	−.67	−.94	−.44	−.19
	(1.6)	(1.3)	(1.4)	(.76)	(.77)
$\rho_{t+1} - \rho_t$	.11				−.22
	(1.02)				(.89)

ρ_{-2}	ρ_{-3}	μ_{-1}	μ_{-2}	μ_{-3}	R^2	D-W
.07	−.23				.23	1.59
(.17)	(.43)					
.663	−.334				.13	2.16
(2.1)	(1.06)					
.15	−.27	.81	−.52	.93	.89	1.95
(1.7)	(2.1)	(.99)	(.62)	(1.3)		
−.70	.74	−.62	−2.0	3.83	.62	2.92
(.56)	(1.16)	(.26)	(.83)	(1.8)		
−.36	−.25	2.08	.015	.206	.37	1.43
(1.29)	(.91)	(2.12)	(.10)	(1.64)		
.002	−.045				.04	2.64
(.026)	(.60)					

variables in the information sets[1]. Due to the small sample size of the data sets, the information sets were constructed by using only a three period lag for each variable in order to allow an adequate number of degrees of freedom for conducting statistical tests. In order to determine the statistically significant variables for the information sets, a likelihood ratio test was used to compare the expectations of each information set with the expectations of the other information sets. A summary of the results is found below in Table 1.

The information above demonstrates the difference between the three data sets. The data sets of Barro and Abel et al. suggest similar information sets for the expected rate of inflation, while the data sets of Cagan and Abel et al suggest similar information sets for the expected rate of currency depreciation. However, Cagan's data suggests quite different information sets for the expected rate of inflation, and Barro's data suggests different information sets for the expected rate of depreciation.

TABLE 2 Estimation of the Money Demand Equation

$$\mu_t - x_t = -\alpha_1 [E_{t-1} (x_{t+1} - x_t)] - \alpha_2 [E_{t-1} (P_{t+1} - P_t)] - n_t$$

coefficients (t statistic)

Data Sets	const.	$-\alpha_1$	$-\alpha_2$	R^2	F
Caqan	−.12	−.019	.16	.012	.19
	(2.23)a	(.05)	(.42)		
Barro	−.046	−.13	−.016	.08	1.48
	(1.73)	(1.68)	(.274)		
Abel et al.	−.083	.388	.217	.211	3.35
	(2.64)a	(2.05)	(.559)		

a) significant at the 95 % level (t statistic).

6. The object is to maximize the likelihood of the α's and not the likelihood of the parameters of the expectation equations. Since the values for the maximum likelihood of the α's are not directly related to the values of the maximum likelihood of the expectation equation coefficients, one can therefore relax the strict theoretical form of the equation for the expectations.

7. The likelihood ratio test is formed by taking twice the difference between the value of the log likelihood functions of the two regressions and comparing it with the Chi squared distribution.

8. One should note that in Table 1 above, the R^2's are meaningless since the objective is not to maximize the likelihood of the expectation equations, but rather to maximize the likelihood of equation (10). The Durbin-Watson statistic is also useless since the serial correlation rho represents the 'extra' information between time (t+1) and time (t—1). The addition of this new information would be inconsistent with the formulation of the expectations.

The second step of the two step procedure was conducted by using the estimated values for the 'rational' expectations of inflation and currency depreciation as the instrumental variables (e.g. independent variables) in estimating the parameters in equation (10). Specifically, I regressed $(\mu_t - X_t)$ on the estimated values of the 'rational' expectations of inflation and currency depreciation.

The results of all three data sets with 'rational' expectations, suggest that neither the expected rate of inflation nor the expected rate of depreciation have any significant effect on real cash balances.

5) CONCLUDING REMARKS

The results of this study highlight two important points. First, each data set appears to be capturing different aspects of the German hyperinflation. This is evident by the description of the price variable contained in each data set. Consequently, the information sets include different relevant variables and produce different estimated values for the expectations of inflation and currency depreciation. This explains the lack of consistency among the signs, magnitudes and significances of the coefficients in the regression model. Therefore, it is not appropriate to draw conclusions concerning the specification of the model based on empirical results utilizing inconsistent data sets. The fact that all three data sets failed to support or refute the underlying theoretical model presented here highlights the second point. Namely that the seemingly different results obtained in the literature seem to be generated by the formation of the expectations and the underlying money demand specifications.

In order to draw any conclusions on the appropriate specification of the money demand equation, empirical tests of alternative specificiations must be conducted utilizing a common data set. Unfortunately, the results obtained in the literature are not based on a common data set, and therefore one must reserve judgment on the correct model specification until these models have been re-estimated with a common data set.

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ÖZET

Bu yazıda Alman hiperenflasyonu döneminde para talebi yaygın olarak kullanılan modeller çerçevesinde ve iki aşamalı araç değişken tahmin yöntemi yoluyla ele alınmaktadır. İlk aşamada rasyonel bekleyişler ikinci aşamada ise talep denkleminin parametreleri tahmin edilmektedir. Çalışma ampirik bulguların kullanılan verilere göre değiştiğini ve yazında farklı gibi görünen sonuçların bekleyişlerin oluşma sürecinden ve para talebi belirlemelerinden kaynaklandığını göstermektedir.