EXAMINING A SEVENTH-GRADE TEACHER'S MATHEMATICAL DISCOURSE RELATED TO ALGEBRAIC EXPRESSIONS

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ABSTRACT

EXAMINING A SEVENTH-GRADE TEACHER'S MATHEMATICAL DISCOURSE RELATED TO ALGEBRAIC EXPRESSIONS

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The aim of the study is to examine seventh-grade algebraic expression lessons in the context of a Turkish middle school through the discursive perspective. Specifically, the study is concerned with Sfard's (2008) mathematical discourse categories and analyzes how they occur in the mathematical discourse in instruction (MDI) (Adler & Ronda, 2015), which are explanatory talks and exemplifications, during the seventh-grade algebraic expressions lessons. The case study method was employed in this study. The participants of the study are Teacher Esra and her seventh-grade algebra class in a public middle school located in the central town of Afyonkarahisar. Participants were determined with a purposive sampling method. Seven algebraic expressions lessons were video-typed and observed throughout the instruction. The observed lessons were transcribed and analyzed through the commognitive theory and MDI. The findings of the study showed that analyzing the mathematical discourse revealed the implicit nature of algebra learning. Another significant finding indicated that the teacher's discourse was dominant, and students' discourses were generally repetitioning of the teacher's discourse, yes/no answers, and one or two-

word answers. Word-use, visual mediators, routines, and narratives are the mathematical discourse characteristics that generate the discourse.

Keywords: Algebra, Algebraic expressions, Mathematical Discourse, Commognitive Theory, Mathematical Discourse in Instruction

ÖZ

BİR YEDİNCİ SINIF MATEMATİK ÖĞRETMENİNİN CEBİRSEL İFADELERLE İLGİLİ MATEMATİKSEL SÖYLEMLERİNİN İNCELENMESİ

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Çalışmanın amacı, söylemsel perspektif yoluyla bir devlet okulu bağlamında yedinci sınıf cebirsel ifadeler derslerini incelemektir. Çalışma özellikle Sfard'ın (2008) matematiksel söylem kategorileriyle ilgilenmekte ve bunların Matematik Öğretimindeki Söylemler (MDI) (Adler ve Ronda, 2015) çerçevesi kapsamında nasıl ortaya çıktığını analiz etmektedir. Bu çerçevedeki söylemler yedinci sınıf cebirsel ifadeler dersleri sırasında açıklayıcı konuşmalar ve örneklendirmeler bağlamında analiz edilmiştir. Bu çalışmada durum çalışması yöntemi kullanılmıştır. Çalışmanın katılımcıları, Afyonkarahisar merkez ilçesinde bulunan bir devlet ortaokulundaki Öğretmen Esra ve öğretmenin yedinci sınıf öğrencileridir. Katılımcılar amaçlı örnekleme yöntemi ile belirlenmiştir. Cebirsel ifadeler derslerinin öğretildiği yedi matematik dersi videoya kaydedilmiş ve öğretim boyunca gözlemlenmiştir. Gözlemlenen dersler yazıya dökülmüş ve commognitive teori ve MDI yoluyla analiz edilmiştir. Çalışmanın bulguları, matematiksel söylemin analiz edilmesinin cebir öğreniminin örtük doğasını ortaya çıkardığını göstermiştir. Bir diğer önemli bulgu, öğretmen söyleminin baskın olduğunu ve öğrencilerin söylemlerinin genellikle öğretmen söyleminin tekrarı, evet/hayır cevapları ve bir veya iki kelimelik cevaplar

olduğunu göstermiştir. Ayrıca kelime kullanımı, görsel aracılar, rutinler ve anlatılar, matematiksel söylemin inşa edilmesini sağlayan söylem unsurlarıdır.

Anahtar Kelimeler: Cebir, Cebirsel ifadeler, Matematiksel Söylem, Bilişsel Teori, Öğretimde Matematiksel Söylem

To My Lovely Daughter Ece

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LIST OF ABBREVIATIONS

CCSSI	Common Core State Standards Initiative
CDA	Critical Discourse Analysis
FD	The Field of Discourse
MD	Mode of Discourse
MDI	Mathematical Discourse in Instruction
MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics
SFL	Systemic Functional Linguistics
TD	Tenor of Discourse

CHAPTER 1

INTRODUCTION

Algebra is regarded as one of the earliest disciplines within the field of mathematics. The history of its content spans the entirety of human civilization (Tabak, 2004). According to Usiskin (1995), algebra is "the language of generalization" (p.31) and provides a simple expression to describe what we do repeatedly. In more detail, algebra involves the manipulation of variables, modeling concrete situations using expressions, and performing operations on these expressions with the aim of simplifying, solving, and interpreting them (Carraher & Schliemann, 2007). Furthermore, algebra enables "forming generalizations from experiences with numbers and computations, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts of pattern and functions" (Van de Walle, Karp, & Bay-Williams, 2009, p.254).

In addition, NCTM (2000) describes the ability of algebraic thinking as "one must be able to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze the change in various contexts" (p. 2). Similarly, Kaput (1999) defined five different modes of algebraic reasoning. These are algebra as the generalization and formalization of patterns, semantically directed manipulation of formalisms, the study of structures derived from computations and relationships, analysis of functions and relations, and collection of language that are used for modeling and controlling phenomena. Generalization and formalization serve as fundamental abilities, symbolic manipulation provides the necessary tools, abstract structures enhance comprehension, functional thinking applies these principles to dynamic relationships, and lastly, modeling combines all these elements to describe and manage real-world phenomena (Kaput, 1999). Consequently, the ideas that are

commonly agreed upon are that algebra emphasizes the process of identifying and promoting patterns and relationships, utilizes symbolic notation, serves as the application of quantitative reasoning and mathematical models, and helps enhance problem-solving skills.

The widespread role of algebraic thinking highlights its fundamental importance in mathematics education, positioning algebra at the heart of school mathematics as a pattern-and-order science. For instance, Kaput (1998) emphasized that it is difficult to encounter a branch of mathematics that does not include algebraic thinking in a central manner. This is particularly evident in numbers and computations, where algebraic thinking is predominantly observed (Van de Walle et al., 2009). For instance, the method of calculating 5 + 7 by completing 10 (5 + 7 = 2 + 3 + 7 = 2 + 10) exemplifies a generalization of taking away 3 from one addend and moving it to the other one. This process can be generalized algebraically as (a + b = (a – 3) + (b + 3)). Another topic that is closely related to algebra is proportional reasoning. Every proportional scenario yields a linear function with a graph that passes through the origin, and the slope of the graph is the constant ratio in the proportion (Van de Walle et al., 2009). Additionally, recurring patterns in operations, such as the commutative property (a + b = b + a) and the associative property ((a + b) + c = a + (b + c)), further demonstrate the application of algebraic principles. Furthermore, algebra is utilized to clarify measurement formulas and determine the area formulas of various geometric shapes in geometry (Van de Walle, et al., 2009). Thus, with the meaningful use of symbolic systems, study of patterns, and generalizations from arithmetic, algebra serves as an extensive, unifying, and connective topic among other mathematical concepts.

The importance of algebraic thinking and its conceptual development is emphasized in numerous national and international standards and curriculum documents (Common Core State Standards Initiative [CCSSI], 2010; Ministry of National Education [MoNE], 2013, 2018, 2024; National Council of Teachers of Mathematics [NCTM], 2000). For example, NCTM (2000) regards algebra as one of the essential concepts in mathematics curriculum that enables learners to comprehend other domains of mathematics and establish connections between them. Likewise, CCSSI

(2010) places a great emphasis on algebra and algebraic thinking from as early as primary school by using letters for unknown quantities, identifying arithmetic patterns, or solving word problems via drawings and using symbols to represent the unknown. In the same manner, the MoNE (2013, 2018) officially reserves a considerable amount of classroom time for algebra from middle to high school, as well as in preschool and primary school. Although the algebra learning area was not officially included in earlier grades in the previous curricula (MoNE, 2013, 2018), the new curriculum published in 2024 includes objectives for the development of algebraic thinking under the title of 'Algebraic Thinking and Changes,' starting from the first grade (MoNE, 2024). It is evident that the curriculum emphasizes and promotes the teaching of algebra and the development of algebraic thinking by promoting early grade-level learning. Furthermore, algebra holds a significant position in the realm of mathematics education, serving as a crucial prerequisite for advanced mathematical studies and prospective career opportunities (NCTM, 2000; Usiskin, 1995). Considering its crucial value, algebra teaching and the development of algebraic thinking "constitute a major component of the school mathematics curriculum" (NCTM, 2000, p. 37).

Research studies in mathematics education also emphasize the significance of algebra instruction in teaching and learning mathematics (Blanton & Kaput, 2005; Cai & Knuth, 2011; Caspi & Sfard, 2012; Chazan, 2008; Lannin et al., 2006; MacGregor & Stacey, 1999; Martinez & Brizuela, 2006; Usiskin, 1995; Uygur-Kabael & Tanışlı, 2010; Warren & Cooper, 2005; Warren et al., 2006). For instance, Usiskin (1995) claims that algebra is a kind of language that relates quantities and enables students to answer identical questions at once. Students who study algebra should also be able to understand the "transition from arithmetic to algebra" (Cai & Knuth, 2011, p. viii) and generalizations of patterns. They should also be able to understand the relationships between quantities (Caspi & Sfard, 2012; Chazan, 2008; MacGregor & Stacey, 1999; Warren & Cooper, 2005), recursive rules, and trans how to transition from one variable to another (Lannin et al., 2006). It is evident that the teaching of algebra is further highlighted by the fact that algebra has its own language, relates quantities through its language, and enables students to develop

higher-level skills through the transition from arithmetic operations to more complex ones.

Moreover, teachers should be aware of the fact that despite the great emphasis on algebra in curricula and literature, numerous research studies have documented that students have difficulties in terms of learning the algebra content and responding to related tasks (Bednarz, 2001; Carraher & Schliemann; Girit & Akyüz, 2016; Kieran, 1992; Kieran, 2007; Linchevski & Livneh, 1999). To understand why algebra is challenging for students, it is necessary to identify the types of errors students frequently make and then investigate the reasons behind them. For this reason, many researchers have explored the reasons that could be traced to the errors and difficulties experienced in the process of algebra learning (Bamberger et al., 2010; Bednarz, 2001; Blanton et al., 2015; Cai & Knuth, 2011; Chazan, 1996; Kieran, 2007; Robert & Roux, 2019; Stacey & MacGregor, 1997; Stephens et al., 2013; Usiskin, 1988). One of them is the separation of arithmetic and algebra in school curriculums, where arithmetic is the focus in elementary school and algebra in middle and high schools, which poses challenges for later algebra instruction (Cai & Knuth, 2011; Kieran, 2007). Introducing algebraic activities in the early grades aims to mitigate these challenges by fostering algebraic thinking from a young age (Chazan, 1996; Jupri et al., 2014; Kieran, 2007). Other studies on arithmetic and algebra have noted that students who face difficulties in arithmetic operations also struggle with addition and subtraction operations in algebraic expressions (Booth, 1988; Erbaş et al., 2010; Herscovics & Linchevski, 1994; Jupri et al., 2014; Pillay et al., 1998; Seng, 2010).

Another difficulty is that students often struggle with understanding the concept of variables, which can represent changing quantities, unknowns, or generalizations of patterns (Blanton et al., 2015; NCTM, 2000). Research indicates that students frequently misinterpret variables as letters that stand only for numbers, a misconception that hinders their grasp of algebraic relationships (Bamberger et al., 2010; Bednarz, 2001; Usiskin, 1988). Furthermore, students have difficulties understanding the equal sign since they cannot grasp their relational meanings, which hinders their skills in solving equations. (Bamberger et al., 2010; Blanton et al.,

2015; Stephens et al., 2013). Another significant challenge that students experience is difficulty in integrating verbal structures with symbolics to explain quantitative relations due to the difficulties between the grammatical rules of algebra and natural language. (Bednarz, 2001; Blanton et al., 2019; Carraher et al., 2008; Stacey & MacGregor, 1997). Therefore, despite the importance of learning algebra, these challenges highlight the critical need for meticulous observations and analyses to elicit the teaching of algebra.

Establishing connections between everyday language and algebraic language is crucial for addressing these difficulties and misconceptions (Robert & Roux, 2019). Sfard (2008) explains these two kinds of languages as colloquial discourse and literate or mathematical discourse. While the former refers to discourses that arise every day or spontaneously and arise naturally as a result of repetitive daily activities, the latter refers to discourses that are characterized by their extensive use of written symbols and their wide range of procedures for utilizing specific notations. Colloquial discourse is the language that students engage in their everyday lives and bring into the classroom, whereas mathematical discourse is commonly employed in schools or professional contexts.

More specifically, Sfard (2008) defined the *mathematical discourse* used in the classroom as a form of communication that involves the use of expressions referring to mathematical objects and a process by which individuals can express their mathematical thoughts at different cognitive levels. In this study, any instance of articulating the word discourse is intended to refer to mathematical discourse. Researchers asserted that linking colloquial discourse with mathematical discourse enhances students' comprehension and utilization of mathematical signifiers (Barwell, 2016; Kim, Ferrini-Mundy & Sfard, 2012; Mahlaba, 2021; Sfard, 2008). In educational settings, it is considered essential to promote students' transition from the discourse they employ in their daily lives to mathematical discourse. For this reason, in the present study, the examination of mathematical discourse in algebra courses could be enlightening in terms of recognizing the transition from everyday language to algebraic language and the difficulties arising from miscommunication issues during this transition.

Current research on teaching and learning algebra primarily relies on a cognitivist perspective, whereas a limited number of studies focus on discourse through a communicational perspective of cognition (Caspi & Sfard, 2012; Ioannou, 2017; Johansson & Kilhamn, 2022; Nisa et al., 2012; Roberts & Roux, 2019; Wallach et al., 2022). The "communicational approach to cognition" is a sociocultural perspective that conceptualizes thinking as a form of communication, emphasizing the inseparability of cognition and communication (Sfard, 2001, p.22). This approach deals with discursive processes, which refer to communicational instances related to discourse and are more easily accessible compared to traditional cognitive processes that occur within the mind (Sfard, 2001; 2008). Moreover, this approach states that learning occurs in a context, unlike cognitive approaches. In addition, it claims that learning takes place in a communicational environment based on cultural and historical factors (Nardi, Reyve, Stedler, Viirman, 2014; Sfard, 2001). By adopting the discursive approach to obtain a deeper understanding of teaching and learning algebra, the present study focuses on observable communication processes and features of teaching and learning within a socio-cultural context rather than the processes of the mind.

Due to the need to examine the teaching environment through communicational components, researchers have developed various frameworks and theories based on the discursive approach (Adler & Ronda, 2015; Halliday & Hasan, 1989; Sfard, 2008; Van Dijk, 1995; 2001). One of them is the *commognitive theory*, developed by Sfard (2008), which is widely utilized and particularly designed for analyzing mathematical discourse. Combining communication and cognition, the commognitive theory considers learning as a process of transforming an individual's discourse by having them engage with a community of discourse (Sfard, 2008). In theory, Sfard (2008) defined mathematics as a discourse and expressed that *word use*, *visual mediators, routines*, and *endorsed narratives* are characteristics of mathematical discourse. She stated that *word use* is explained by the specific words uttered in the discourse. These are usually relevant to quantities, shapes, and their abstract relationships. In addition, *visual mediators* are visible objects generated specifically for mathematical discourse, such as symbols, tables, and graphs. Sfard (2008) defined *routines* as repetitive characters in a given discourse. In mathematical

discourse, these include categorizing shapes and finding similar or different situations. Lastly, *endorsed narratives* are known as truths, endorsed by other discourse characteristics. These are theories, definitions, and proofs endorsed by other discourse characteristics. Analyzing these discursive characters and their relationships with each other reveals the hidden features of teaching mathematics. Thus, the roles of words, visual mediators, routines, and endorsed narratives are explicitly examined in a discourse from a communicational perspective. In the present study, analyzing mathematical discourse in middle school algebra courses through the commognitive theory might help map the communication environment where algebra teaching and learning occurs, give information about the communicational elements, shed light on the implicit nature of algebra, and reveal the communicational breakdowns in the course of teaching algebra.

Moreover, Adler and Ronda (2015) developed a framework known as Mathematical Discourse in Instruction (MDI) based on the discursive approach, which analyzes the discourse in a class, provides a detailed depiction of mathematics education, and interprets the mathematical materials that are available to learn. The framework focuses on three components of teaching mathematics under the object of learning. These are "exemplification," "explanatory talk," and "learner participation" (Adler & Ronda, 2015, p.237). Firstly, "explanatory talk" is the language the teacher uses to name and legitimize mathematical ideas and concepts (Adler & Ronda, 2015, p.241). Then, "exemplification" includes using examples and tasks to provide learners with opportunities to experience variation in mathematical concepts (Adler & Ronda, 2015, p.239). Lastly, "learner participation" concentrates on the interaction between students and the teacher, as well as on the opportunities for students to participate in mathematical discourse and reasoning (Adler & Ronda, 2015, p.245). Thus, MDI offers a comprehensive and organized approach to examine the discourse used in mathematics instruction. By analyzing the instruction in terms of distinct discursive elements, such as explanatory talk, exemplification, and learner participation, MDI offers a more detailed understanding of how mathematics is both taught and learned. In addition to commognitive theory, this study intends to analyze algebra lessons through the MDI framework. Thus, the framework might reveal the explanations explicitly stating the rationale behind the mathematical procedure, determine the

examples used to elaborate on the algebra topic and identify how these two elements are employed to engage students in classroom discussions within a rich communication environment.

Both discursive approaches mentioned above (commognitive theory and MDI) are based on the socio-cultural theory, emphasizing that learning occurs within a social and cultural context and that cultural tools and social interaction are significant factors in the learning process. The present study focuses on the social environment in a classroom where algebra instruction takes place and the interaction between the teacher and the students, which aims to teach students algebraic expressions. The commognitive theory enables the analysis of the characteristics of mathematical discourse used in the classroom based on distinct rules and meta-rules (Sfard, 2008), whereas MDI provides a comprehensive depiction of mathematics instruction through a learning objective (Adler & Ronda, 2015). MDI mainly emphasizes examining the teacher's discourse, the way they define and clarify mathematical concepts, the structure of the examples they use to elaborate on the topic, and the interaction with students (Adler & Venkat, 2014). One of the main constituting elements of MDI, explanatory talk, is described as an examination of the naming of mathematical objects and legitimations (Adler & Ronda, 2015). However, the framework is insufficient to provide a distinct definition of what constitutes legitimated mathematical language, which complicates the process of distinguishing between subcategories (Mosvold & Fauskanger, 2022). Commognitive theory would fill this gap by analyzing the classified mathematical characteristics in the discourse, including word-use, visual mediators, routines, and endorsed narratives, and by offering a comprehensive definition for each category. Thus, this study aims to uncover the implicit nature of algebra teaching by focusing not only on cognitive processes of learning but also mostly on communicative elements, which are relatively easy to observe in the classrooms mentioned in both theories. Moreover, since this study focuses on teaching and learning algebra from a communicative perspective, it might reveal the difficulties that may arise from communicational breakdowns throughout the teaching of algebra. In conclusion, the current study examines the explanatory talks and exemplifications in MDI through the analysis of the characteristics of the mathematical discourse in the commognitive framework to

reveal the hidden structure of teaching algebra in seventh grade, where the transition from arithmetic to algebra is more obvious.

1.1. The Aim of the Study and Research Questions

The aim of this study is to examine the discursive environment of a seventh-grade classroom in a Turkish middle school to gain insight into the teaching of algebraic expressions. Considering the purpose of the study, research questions and sub-questions are presented as follows:

1. What is the nature of the characteristics of mathematical discourse (word-use, visual mediators, routines, endorsed narratives) during explanatory talks and exemplifications in the context of algebraic expressions lessons in a seventh-grade classroom of a public middle school?
    - What is the nature of the characteristics of mathematical discourse during explanatory talks and exemplifications in the topic of concepts in algebraic expressions?
    - What is the nature of the characteristics of mathematical discourse during explanatory talks and exemplifications in the topic of operations in algebraic expressions?

1.2. Significance of the Study

This study aims to examine mathematical discourse from the discursive perspective in order to provide further insights into the teaching of algebra, especially algebraic expressions. The study holds great importance due to its contribution to both academic literature and the educational environment. The major contribution of the dissertation is to provide the literature with a case study that focuses on examining the mathematical discourse using the discursive approach in Turkish middle schools. By eliciting unique features of mathematical discourse, such as the use of symbolic artifacts and meta-rules, the discursive approach highlights and suggests to researchers a better way to understand the learning of mathematics (Sfard, 2000). Rather than simply investigating the acquisition of new mathematical concepts and teaching a set of rules, this approach refers to the awareness of the combination of

the meaning of learned terms in the social context, related to visual expressions and meta-rules to change in mathematical discourse (Sfard, 2000; 2001). Thus, this study provides a valuable contribution by offering a discourse analysis of the teaching of algebraic expressions from a sociocultural perspective in the context of Turkish middle schools. Thus, researchers and mathematics educators might develop an understanding of how concepts, definitions, theorems, and visual aids play an essential role in teaching algebra, considering both cognitive and communicational elements.

In recent years, a couple of research studies have been conducted in Turkey focusing on analyzing mathematical discourse in a variety of mathematics concepts as a result of the awareness of the role and significance of discourse analysis mentioned above (Akkuş, 2015; Baki & Çelik, 2018; Emre-Akdoğan et al., 2018; Emre-Akdoğan et al., 2020; Genç & Erdem, 2016; Şahin-Doğruer et al., 2015; Tangül-Kabael & Ata-Baran, 2017; Uğurel & Moralı, 2010). For instance, Emre-Akdoğan et al. (2018) examined the development of the discourse on geometric translation in high school students and their teachers. Baki and Çelik (2018) examined the discourse on statistics that takes place in mathematics instruction within the context of language. In addition, Tangül-Kabael and Ata-Baran (2017) investigated mathematical discourses and the ways of analyzing the mathematical discourses of a pre-service secondary school mathematics teacher who had been trained in the context of mathematical communication. However, no research has been conducted in our country to examine mathematical discourse in the context of algebra teaching in middle or high school education. Therefore, this study might provide a deeper awareness of the complex process of teaching algebra by revealing the mathematical communicational components that are critical for understanding the process of learning through the discursive approach and filling the gap in this research area.

This study also contributes to the literature by providing a deeper analysis of algebraic expressions lessons using the commognitive theory (Sfard, 2008) and the MDI framework (Adler & Ronda, 2015). MDI provides a comprehensive and structured approach for analyzing the discourse used in teaching practices while also helping make mathematics available to be learned clearly and visibly during

instruction (Adler & Ronda, 2015). The framework examines discourse under the object of learning, focusing on three key components, which are exemplification, explanatory talk, and learner participation. Commognitive theory examines the mathematical characteristics of discourse, which are word-use, visual mediators, routines, and endorsed narratives (Sfard, 2008). In this study, the characteristics of the mathematical discourse (word-use, visual mediators, routines, and endorsed narratives) used during algebraic expressions instruction are examined under the categories of exemplification and explanatory talk in accordance with the object of learning in MDI. Thus, the study aims to shed light on the teaching of algebraic expressions by examining the characteristics of the mathematical discourse used in the relevant phases of the lessons (exemplification or explanatory talk). In this manner, mathematics educators can develop an understanding of the words, visual aids, routines, and narratives used in explanatory talks and exemplifications in algebraic expressions lessons, as well as the role of the characteristics of mathematical discourse in the teaching process. Moreover, teachers can gain insights into how and when to use these characteristics in classroom discussions by examining them in a communicative context.

Another contribution of this study is to ensure that teachers are aware of the role of mathematical communication in the classroom. The significance of communication in teaching and learning was emphasized in the curriculum and research studies (MoNE, 2013; 2018; 2024; Nardi et al., 2014; NCTM, 2000; Planas et al., 2018; Toptaş, 2015). The mathematics curriculum stresses the importance of developing students' communicational skills and their use of mathematical language (MoNE, 2013; 2018; 2024), and the skills that students are expected to acquire are listed as follows:

> "• Realizing that mathematics is a language with its own symbols and terminology,
> • Using mathematical symbols and terms effectively and correctly,
> • Using mathematical language appropriately and effectively within mathematics itself, in different disciplines and in daily life,
> • Concrete models, figures, pictures, graphs, tables, symbols, etc. express mathematical ideas using different forms of representation,
> • Expressing mathematical ideas verbally and in writing,

- Using everyday language, mathematical language, and symbols; relating mathematical language to everyday language and symbols,
- Interpreting the accuracy and meaning of mathematical ideas" (MoNE, 2013, p. V)

In addition, NCTM (2000) also suggested that educational programs enable students to "organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely" (p. 60). As indicated above, the curriculum prioritizes the enhancement of students' mathematical communication abilities by focusing on various aspects of mathematical discourse. Besides, research studies have also emphasized the importance of using mathematical language and communication skills. For instance, Toptaş (2015) stated that since the correct, effective, and conscious use of mathematical language is crucial in learning mathematical concepts, teachers should be aware of the language they use and enhance students' mathematical terminology.

Another study conducted by Nardi et al. (2014) emphasized the significance of communication in mathematics education and considered it as a distinct element of thinking. They concluded that mathematical communication relies heavily on how the participants of discourse use symbols and words to represent their ideas. Another important study carried out on the use of mathematical language and communication skills emphasized the significance of the language used in mathematics education and stated that it is an essential component in students' comprehension, application, and expression of mathematical concepts (Planas et al., 2018). They also asserted that the ability of students to use mathematical terminology empowers them to engage in classroom discussions, thereby enhancing their mathematical thinking processes. Developing a healthy communication environment is essential for enhancing students' mathematical communication abilities. Therefore, examining the discourse in the classroom is essential for teachers to realize the specific components of mathematical discourse emphasized by the curriculum to develop mathematical communicational skills.

The study might also contribute to the literature by revealing the difficulties and miscommunication issues associated with learning algebra. As aforementioned, algebra is usually considered a difficult subject for students to grasp. Students encounter challenges when transitioning from arithmetic to algebra, misunderstand the abstract nature of variables, have an erroneous sense of the equal sign, and have difficulty combining verbal descriptions with symbolic systems (Bamberger et. al., 2010; Bednarz, 2001; Blanton et. al., 2015; Booth, 1988; Cai & Knuth, 2011; Carraher & Schliemann, 2007; Carraher et. al., 2008; Girit & Akyüz, 2016; Kieran, 1994; Kieran, 2007; Linchevski & Livneh, 1999; Stacey & Chick, 2006; Stacey & MacGregor, 1997; Usiskin, 1988; Warren et al., 2016). Sfard (2008) explains the reasons behind miscommunication issues with the notion of commognitive conflict, which refers to a situation in a communicational environment when individuals utilize the same mathematical elements in distinctive ways. Prawiti et al. (2020) examined the factors contributing to the commognitive conflict experienced by students. The results of the study indicated that the conflict occurred due to the improper use of visual mediators and narratives employed in transitioning written word problems to mathematical notations. If the teacher successfully encourages students to solve this conflict, they can acquire new knowledge. However, if the teacher proceeds without helping them, students are likely to experience commognitive conflict again. Güçler (2013) states that examining the discourse enables teachers to notice the communicational breakdowns related to the inconsistencies between the discourses of teachers and students. Moreover, she points out the effects of teachers' discourse on students' understanding of the topic and the potential factors influencing students' learning. Thus, teachers should be aware of the effects of their discourse on students' understanding, commognitive conflicts, and minimize students' difficulties and miscommunication issues. As a result, this study might provide a significant contribution to the current literature by demonstrating the impact of teacher's mathematical discourse on the development of students' mathematical understanding.

The discursive approach provides the opportunity to make a connection through a communication environment since it is related to written, spoken, or any other communicational components that exist in a particular discourse (Sfard, 2000).

Moreover, the discursive approach provides insights into the teaching and learning environment by determining the discourse patterns and relationships between communication elements and failures (Güçler, 2013). Therefore, the general aim of the study is to examine the mathematical discourse used in algebra lessons. Examining the discourse in mathematics classrooms could show us how mathematical language is being used, as well as the use of specific words, symbolic systems, and routines.

1.3. Definition of Important Terms

In this part, the operational definitions of key concepts used in the study are presented to enhance the clarity of the following chapters.

1.3.1. Algebra

Algebra is referred to as "the language of generalization" (Usiskin, 1995, p. 31). It is a mathematical discipline that encompasses the manipulation of variables and modeling concrete situations using expressions with the aim of simplifying, solving, and interpreting them (Carraher & Schliemann, 2007). In this study, algebra refers to algebra in the middle school level o that is related to the generalization of arithmetic, manipulations of symbols and variables, modeling the situations using expressions, and simplifying them.

1.3.2. Discourse

In general terms, discourse is “ways of using language” (Gee, 1999, p. 17). Particularly, discourse is “any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system” (Sfard & Kieran, 2001, p. 47). In this study, discourse refers to ways of using language in the classroom. That is to say, it refers to the use of communicational ingredients in a specific context with the help of a symbolic system.

1.3.3. Mathematical Discourse

Mathematical discourse is a communication type that includes expressions that refer to mathematical objects and a process by which individuals can express their mathematical thoughts at different cognitive levels (Sfard, 2008). In this study, mathematical discourse refers to being familiar with mathematical terminology and using language correctly to explain and share mathematical ideas in a logical way to realize meaningful learning. Sfard explained mathematical discourse in four categories as given below.

- *Word-Use*

Word use refers to specific words that are usually relevant to quantities, shapes, and their abstract relationships (Sfard, 2008). In this study, word-use refers to specific terms used in teaching and learning algebraic expressions.

- *Visual Mediators*

Visual mediators are visible objects generated specifically for mathematical discourse as symbols, tables, and graphs (Sfard, 2008). In this study, visual mediators refer to symbols, shapes, models and tables used in teaching and learning algebraic expressions.

- *Endorsed Narratives*

Endorsed narratives are known as truths. These are theories, definitions, and proofs endorsed by other characteristics of discourse (Sfard, 2008). In this study, endorsed narratives refer to theories and definitions used in teaching and learning algebraic expressions.

- *Routines*

Routines are considered to be repetitive characters in the given discourse. In mathematical discourse, these include categorizing shapes and finding similar or different situations (Sfard, 2008). In this study, routines refer to specific patterns used in teaching and learning algebraic expressions.

1.3.4. Explanatory Talk

Explanatory talk is the language used by the teacher to name and legitimize mathematical ideas and concepts (Adler & Ronda, 2015, p.241). It refers to teachers' explanations and clarifications about concepts and procedures.

1.3.5. Exemplification

Exemplification includes the use of examples and tasks to provide learners with opportunities to experience variations in mathematical concepts (Adler & Ronda, 2015). It refers to the use of tasks and examples within algebraic expressions lessons.

CHAPTER 2

LITERATURE REVIEW

The purpose of the study is to examine the mathematical discourse in order to provide further insights into the teaching of algebra, especially algebraic expressions, in the context of a Turkish middle school through the discursive perspective. The study is specifically concerned with the mathematical discourse categories, and it analyzes how these categories are displayed in the mathematical discourse in instruction (MDI). In accordance with this aim, this chapter discusses the relevant literature under five sub-headings, each of which clarifies the fundamental concepts by referencing related literature the importance of algebra and algebra learning, defining discourse and discourse analysis, mathematical discourse frameworks, studies related to mathematical discourse, and studies related to algebra and mathematical discourse. Following these sections, the chapter concludes with a summary of the related literature.

2.1. Importance of Algebra and Algebra Learning

Algebra is considered one of the most significant topics in the field of mathematics as well as in professional life in various branches of business. Schonfeld (1995) stated that algebra is an academic passport to almost every career field and educational path. Too often, children who struggle with algebra face significant challenges in their future careers, such as being pushed into low-paying jobs and having difficulty in accessing training programs for their desired career paths (Schonfeld, 1995; Usiskin, 1995). Likewise, Usiskin (1995) indicated that without adequate algebra knowledge, “students are kept from many jobs or even entering programs that will get them a job, lose control over parts of their lives, and must rely on others to do things for them, making them more likely to make unwise decisions”

(p. 31). Individuals fail to get a job in fields like finance, engineering, astronomy, statistics, business, economics, physics, chemistry, etc., with a lack of algebra knowledge (Schonfeld, 1995; Usiskin, 1995). Therefore, proficiency in algebra plays a crucial role in entering fulfilling careers and succeeding in academic fields.

Algebra is defined as "the language of generalization" and offers concise expressions for describing recurrent activities (Usiskin, 1995, p.31). It is the generalization of mathematical ideas from specific instances, supporting these generalizations through argumentation and formulating them with expressions (Blanton & Kaput, 2005). Moreover, algebra has a distinct symbolic system that expresses the unknowns with letters and generalizes the experiences through this special system (Carreher & Schliemann, 2007). In other words, algebra is the generalization of arithmetic through the application of letters and symbols depending on the fundamental principles of arithmetic (Tabak, 2004).

Algebraic thinking, on the other hand, encompasses several important aspects. It involves using arithmetic to formulate expressions and generalizing patterns to define functional relationships. Additionally, it employs modeling as a means of generalization and derives generalizations of mathematical systems extracted from operations and relations (Blanton & Kaput, 2005). Similarly, NCTM (2000) highlights that through algebraic thinking, "students understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze the change in various contexts" (p. 2). In short, algebraic thinking is the ability to recognize patterns and relationships, utilize symbolic systems to express these relationships, use models to demonstrate quantitative relationships and enhance problem-solving skills.

The branch of algebra lies at the heart of school mathematics as the significance of the ability to think algebraically is vital for effective mathematics education. It has been underlined that encountering any topic of mathematics that is not related to algebraic thinking is challenging (Kaput, 1998). Algebraic thinking is predominately applied in a variety of domains, including number and computation, operation

concepts, proportional reasoning, measurement, geometry, and data analysis (Van de Walle et al., 2009). For example, the Make Ten strategy exemplifies the generalization of taking away from the first addend and then adding the same number to another addend to complete the second on ten and multiples of ten (e.g., 7 + 18 = (7 - 2) + (18 + 2) = 5 + 20). This procedure can be generalized as a + b = (a – 2) + (b + 2). Other operations where algebraic thinking is explicitly applied are the commutative property (e.g., a + b = b + a) and the associative property (e.g., (a + b) + c = a + (b + c)). Furthermore, algebraic symbols or formulas are explicitly used in calculating the perimeter, area, or volume of geometric shapes and figures. In conclusion, with the wide range of uses of algebra, it plays a crucial role in combining and integrating with other branches of mathematics.

In addition, a pile of research studies has emphasized that extensive knowledge of algebra and enhanced algebraic reasoning help students improve their problem-solving abilities (Bednarz, 2001; Blanton & Kaput, 2005; Blanton et. al., 2019; Booth et al., 2017; Egodawatte, 2009; Herscovics & Linchevski, 1994; Kieran, 1981; MacGregor & Stacey, 2007; Mulungye et al., 2016; Tirosh et al., 1998). Algebraic reasoning provides a systematic way to represent problems with a specific symbolic system (Blanton et al., 2019; Egodawatte et al., 2009; Herscovics & Linchevski, 1994; Mulungye et al., 2016). With a better understanding of variables, symbols, and equal signs, algebraic reasoning facilitates students' understanding and enhances their problem-solving abilities (Booth et al., 2015). Besides, Egodawatte (2009) stated that algebra helps students transform verbal problems into algebraic expressions, so that they gain a broader understanding of the relation between verbal and symbolic systems. On the other hand, algebraic reasoning is closely related to proportional reasoning as the use of symbols, numbers, and relations between them (Blanton & Kaput, 2004).

Numerous national and international mathematics education standards and curriculum documents emphasize the significance of enhancing students' algebraic thinking (CCSSI, 2010; MoNE, 2013, 2018, 2024; NCTM, 2000). NCTM (2000), for instance, underscores that students should be able to comprehend (1) patterns, relations, and functions, (2) explain and examine mathematical situations using

algebraic symbolic systems, (3) use modeling to express relationships, and (4) examine the change in different contexts from kindergarten to high school. CCSSI (2010) even incorporates algebra standards from prekindergarten to high school. In Turkey, MoNE (2013, 2018, 2024) allocates a significant amount of class time from middle school to high school to algebraic thinking. Moreover, the newest curriculum offered by MoNE contains course objectives related to algebra and indicates the durations of algebra lessons from the first grade to high school (MoNE, 2024). Thus, algebra holds substantial importance and prominence in school mathematics. Despite the extensive emphasis on the significance of algebra in numerous studies and curricula, students frequently experience difficulties in learning algebra. These difficulties are outlined in detail below.

2.1.1. Difficulties in Algebra Learning

Despite its fundamental importance in mathematics education and professional contexts, teaching and learning algebra often present several challenges (Al-Rababaha et al., 2020; Booth & Koedinger, 2008; Booth et al., 2015; Egodawatte, 2009; Mulungye et al., 2016; Pramesti & Retnawati, 2019; Uçar Sarımanoğlu, 2019; Wang, 2015). Stacey and Chick (2006) stated that these difficulties may stem from students' abstraction levels, previous knowledge of arithmetic, ability in symbolic manipulation, and the ability to move from procedural to structural thinking. Booth stated (1988) that in order to determine the challenges students face in learning algebra, it is essential to identify common errors and investigate the underlying reasons behind these mistakes. The difficulties encountered by students are summarized below, as identified in the literature on algebra learning.

2.1.2. Difficulties with arithmetical operations

Research studies have shown that students often struggle with addition and subtraction when dealing with algebraic expressions involving arithmetic operations (Booth, 1988; Erbaş et al., 2010; Herscovics & Linchevski, 1994; Jupri et al., 2014; Pillay et al., 1998; Seng, 2010). For instance, Herscovics and Linchevski (1994) found that some students have trouble with operations where an unknown is

represented by a letter while combining algebraic expressions. The authors defined this difficulty as incompetence of operation with unknowns. In addition, Pillays et al. (1998) stated that students' insufficient understanding of distributive and commutative laws causes not only a misunderstanding of the full structure of equations but also their failure to comprehend the relationship between expressions. In line with these, another study conducted by Jupri et al. (2014) indicated that due to the lack of competence in arithmetical calculations in early grades, students have difficulty in implementing the commutative and distributive laws in solving algebra examples in middle schools. To sum up, errors related to arithmetic operations in algebra learning include failing to apply the order of operations correctly, incorrect use of the distributive property, overlooking the effects of coefficients 1 and -1, and neglecting the negative integer before the parentheses (Jupri et al., 2014; Seng, 2010).

2.1.3. Difficulties with the notion of variable

Another challenge that students face in learning algebra is related to their understanding of the concept of variable. Usiskin (1988) posited that the concept of the variable is versatile. He identifies five different forms of variables that students typically encounter when they begin learning algebra.

1. A = LW
2. 40 = 5x
3. sin x = cos x. tan x
4. 1 = n. (1/n)
5. y = kx

Figure 2. 1. Different forms of variable (Usiskin, 1988, p.9)

As seen in Figure 2.1, all of these equations are operations with two numbers, and the result equals the third. The first exemplifies a *formula* in which the letters represent length, area, and width. The second is an *equation* where x is an unknown. In addition, the third is an *identity* where x represents an argument of a function. The equation four is a *property* where n refers to the generalization of an arithmetic

pattern. Lastly, the fifth one is the equation of *a function* where the variability of the x is seen as obvious.

Moreover, according to Blanton et al. (2015), the term variable refers to symbolic notation that represents mathematical ideas and contains different roles in different contexts. In school mathematics, variables are seen in three different contexts, i.e., changing quantity, unknown, and generalizations of patterns (NCTM, 2000). Usiskin (1988) emphasized that all the different uses of variables could be confusing for students to learn.

Relevant literature reveals that students experience difficulties in using variable notation to represent quantities and the corresponding connections between these quantities (Bamberger et al., 2010; Bednarz, 2001; Blanton et al., 2015; Uçar-Sarımanoğlu, 2019; Usiskin, 1988; Warren et al., 2016). For instance, "many students think all variables are letters that stand for numbers" (Usiskin, 1988, p.10). However, in geometry, a variable usually represents a point (e.g., A, B, C for the vertexes of the triangle), or in logic, a variable stands for propositions (p, q). The idea that a letter merely substitutes for a numeral, rather than representing a variable with a range of possible values, reflects a fundamental misunderstanding of the concept (Bamberger et. al., 2010). Similarly, Booth (1988) stated that students regard letters as representing specific values and accept that different letters stand for distinct numerical values. Thus, some students accept that expressions like x + y + z and x + p + z cannot be equal, because there exist different letters. However, this is a misunderstanding.

In the same way, Uçar-Sarımanoğlu (2019) determined that middle school students' misconceptions of variables are related to deficiency in their understanding of arithmetic operations that produce difficulty in operations between variables and misconceptions regarding letters being used for integers. She also asserted that disregarding letters in algebraic expressions leads to errors. Failures caused by disregarding the letters are a common misconception in algebra learning (Booth et al., 2017; Küchemann, 1978; MacGregor & Stacey, 1997; Uçar-Sarımanoğlu, 2019). Uçar-Sarımanoğlu (2019) illustrated this finding with an example: One of the

students in the study solved the equation 2a + 8 = 10 and then recorded 10-8=2/2a=1, demonstrating a failure to properly consider the variable a.

2.1.4. Difficulties with the use of equal sign

It has been reported by the research that many students have a flawed understanding of the equal sign while engaging with algebra tasks (Bamberger et al., 2010; Blanton, et al., 2015; Herscovics & Linchevski, 1994; Jupri et al., 2014; Knuth et al., 2006; Stacey & Chick, 2004; Stephens et al., 2013; Warren et al., 2016). Knuth et al. (2006), for example, indicated that most students have a lack of understanding of the equal sign, which results in poor performance in solving equations. Likewise, Stephens et al. (2013) found that some students do not comprehend the meaning of the equal sign and have obstacles to interpreting, transforming, and solving questions.

Bamberger et al. (2010) indicated that students incorrectly view the equal sign as representing a balanced relationship on each side. Many students have an understanding of the equal sign demonstrates the conclusion of the arithmetic operation instead of the balancing relation (Kieran, 1981). In early grades, where arithmetic operations are frequently performed, students may develop misconceptions about the equal sign. Often, they perceive it as a concluding signal, rather than a symbol indicating equality. For instance, they may mistakenly interpret the equal sign as denoting the result, as exemplified by expressions such as a + 3 (Kieran, 1981). Many students perceive that the equal sign represents “do something” instead of “the same as” and overlook the relational characteristics (Knuth et al., 2005, p.69). This understanding usually causes further problems when students encounter an algebraic equation consisting of symbols on both sides (e.g., 5x-4=2x+1).

2.1.5. Difficulties with transitioning verbal descriptions to symbolic systems

According to related literature, one of the most common reasons for the difficulty is the challenges in combining verbal descriptions with symbolic systems to express

relationships between quantities (Bednarz, 2001; Blanton et al., 2019; Carraher et al., 2008; Kieran, 1994; Linchevski & Livneh, 1999; Stacey & MacGregor, 1997). This is because "the grammatical rules of algebra not being the same as ordinary language rules" (Stacey & MacGregor, 1997, p. 110). Robert and Roux (2019) stated that most students participating in their study used colloquial language rather than objectified mathematical words. Jupri and Drijvers (2016) investigated students' difficulties in solving word problems on linear equations with one variable. They found that students have difficulty in transforming phrases, words, and sentences into algebraic symbols and formulating equations. For this reason, it is essential to establish the connections between daily language and algebra language to eliminate students' difficulties and misconceptions.

To sum up, the review of the literature on teaching and learning algebra has revealed there are four primary categories of student difficulties in algebra: difficulties in arithmetic operations, the notion of variables, the use of equal signs, and the transition of verbal descriptions to symbolic systems (Al-Rababaha et al., 2020; Bamberger et al., 2010; Bednarz, 2001; Blanton, et al., 2015; Blanton et al., 2019; Booth, 1988; Booth et al., 2015; Booth & Koedinger, 2008; Carraher et al., 2008; Egodawatte, 2009; Herscovics & Linchevski, 1994; Jupri et al., 2014; Kieran, 1994; Knuth et al., 2006; Linchevski & Livneh, 1999; Mulungye et al., 2016; Pillay et al., 1998; Pramesti & Retnawati, 2019; Seng, 2010; Stacey & Chick, 2004; Stacey & MacGregor, 1997; Stephens et al., 2013; Uçar Sarımanoğlu, 2019; Wang, 2015; Warren et al., 2016). Kieran (2007) stated that focusing on relationships rather than just a numerical answer of a calculation, concentrating on operations besides their inverses, and attending to both numerals and letters instead of just numbers enable the smooth transition from arithmetic to algebra. Furthermore, the separation of arithmetic and algebra in school mathematics, that is, focusing on arithmetic primarily in elementary school and focusing on algebra primarily in middle and high school, makes algebra instruction challenging for later grades (Cai & Knuth, 2011; Kieran, 2007). Discussions about the idea of introducing algebraic activities in early grades bring strong expectations to mitigate certain challenges that later graders may encounter while learning algebra (Kieran, 2007). This notion is rooted in the movement of "Algebra for All" (Chazan, 1996). In addition to making algebra

accessible to all students, the Algebra for All movement introduces younger students to algebra earlier. Thus, the development of algebraic thinking in early grades requires exploring appropriate ways of teaching arithmetic and understanding the factors that make the transition from arithmetic to algebra challenging for students (Cai & Knuth, 2011).

This section elaborated and explained the importance of algebra and the difficulties in teaching and learning algebra. As mentioned earlier, the present study examines algebra teaching from a discursive perspective to gain insight. For this reason, it is aimed to analyze mathematical discourse in algebra lessons. The following section provides an overview of the literature related to discourse and discourse analysis.

2.2. Defining the Discourse and Discourse Analysis

Recent studies in mathematics education have paid special attention to language, which is the primary teaching and learning medium (e.g. Halliday, 1978; Halliday & Hasan, 1989; Kieran, 2012; Lavie et al., 2019; Morgan, 2006; Moschkovich, 2003; Sfard; 2000; Sfard, 2007, Pimm; 2004; Viirman, 2015). According to the European Society for Research in Mathematics Education (ERME), examining the role of language in mathematics teaching and learning "has led us to identify traditional themes concerning the language of the learner (the language and linguistic skills brought to the mathematics classroom by learners), the language of the teacher and the classroom (the language and linguistic skills brought to the mathematics classroom by teachers), and the language of mathematics (the language and linguistic features of the texts that arise within the practice of mathematics)" (Planas, Morgan & Schütte, 2018, p.196). This has led to a greater focus on language and various semiotic structures used in mathematical activities, the roles that these structures play in teaching and learning mathematics and benefiting from semiotic theories and adapting them for the needs of researchers (Fairclough, 1995; Halliday & Hasan, 1989; Sfard, 2000). The attention to language and other communication components has made researchers increasingly interested in "ways of using language" (Gee, 1999, p. 17), which is called *discourse*. In this study, the studies conducted on using

language in mathematics education are referred to as discourse studies. According to Gee (1999), *discourse* is:

> "a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or *social network* or to signal (that one is playing) a socially meaningful *role.*" (p. 131).

Discourse is closely associated with the language. In the most general sense, the term discourse is defined as "language-in-use" (Rymes, 2009, p. 12). It has been regarded as language usage in a specifically communicative context (Schwandt, 2001). Researchers made their own discourse definitions, such as a specific way of speaking about and comprehending the world (Phillips & Jorgensen, 2004), different ways of talking that make sense of experiences from a specific perspective (Fairclough, 1995), the point of view that is not questioned, and communicating with different verbal interactions (Atkins & Wallens, 2012). The shared aspect in these definitions appears to be their focus on a mode of communication within a certain environment.

Moreover, Sfard and Kieran (2001) defined discourse as "any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system" (p. 47). Their research intended to explore effective ways of learning and highlighted that efficient communication substantially affects students' learning processes. They analyzed students' discourse and characterized the discourse as encompassing both communicating with others and with him/herself. They accepted that thinking activity is a kind of communication with themselves. Furthermore, their definition does not restrict discourse only to a language activity but also includes other symbolic systems similar to mathematical activities, which involve verbal, symbolic, and visual tools. In this regard, their conceptualization of discourse distinguishes itself from other definitions by highlighting discourse as the act of communicating internally and externally.

Gee (1999) described five aspects of discourse in a communicative context: semiotic, activity, material, political, and sociocultural. *The semiotic aspect* is the sign

systems, such as different languages, gestures used in a communicational environment, or images for more effective expressions. In addition, the mathematical symbolic system is also a specific sign system. *The activity aspect* is the activities in which interlocutors are engaged in a specific social environment. Activities consist of a series of actions that exist in an order. Gee (1999) addressed *the material aspect* as the physical properties of the discourse. These properties are composed of time, space, people, and objects in an interaction. *The political aspect* is the social norms of an interaction. The distribution of power, the position of people, and anything else acknowledged as a social norm in a culture are considered political aspects. *The sociocultural aspect* combines personal and sociocultural knowledge, feelings, values, identity, and relationships related to interaction (Gee, 1999).

Potter (2003) also addressed the theoretical principles of discourse from his point of view in four aspects. The first aspect states that discourse is an action-oriented issue, and actions play a fundamental role in generating discourse. This view describes how actions are done. The second aspect accepts that discourse is constructed in sequential, institutional, and rhetorical ways. This means actions do not take place independently; rather, they are answers to other actions. The third aspect asserts that actions are constructed institutionally. For instance, medical discourse (settings and terms that medical staff use), mathematical discourse (such as symbolic systems, graphs, and tables used in mathematics), and political discourse (statements, definitions, and issues relevant to politics) are generated in different ways within different institutional contexts. Potter (2003) lastly addressed that actions are constructed rhetorically. This is relevant to rhetorical psychology and emphasizes that descriptions were constituted against actual or potential options. In addition, Philips and Jorgensen (2004) stated that discourse is both constructed and constructive. It is a construct because discourse consists of various sources such as words, rhetorical means, and grammatical items, and it is constructive in the psychological environment, which consists of beliefs, emotional feelings, and intentions (Philips & Jorgensen, 2004). As seen from these points of view, discourse has been handled in a variety of dimensions within a communicative environment.

More specifically, *mathematical discourse* is a communication type including expressions that refer to mathematical objects and a process by which individuals can

express their mathematical thoughts at different cognitive levels (Sfard, 2008). Besides, mathematical discourse consists of conversations, raising questions, writing to articulate mathematical expressions, and procedures in classroom (Higgins, Cermak-Rudolf, & Blanke, 2009). The Turkish curriculum also mentions the discursive elements, which are being familiar with mathematical terminology and using language correctly to explain and share mathematical ideas in a logical way (MoNE, 2013, 2018, 2024). Weaver, Dick, and Rigelman (2005) concentrated on analyzing students' mathematical discourse as it occurs during the execution of mathematical activities. They defined five types of mathematical discourses that students utilized: *questioning, explaining, justifying, challenging,* and *generalizing.* Students ask *questions* to clarify their comprehension of mathematical ideas or procedures. They try to *explain* mathematical ideas or procedures by describing how they do, but they do not tell why. They provide *justifications* for the validity of mathematical ideas or procedures. Students make statements or ask questions to *challenge* the validity of the idea. Finally, they make *general* statements that represent the transition from a specific sample to a general truth. These are the discursive issues that teachers and students utilize throughout the instruction.

Discourse analysis is defined as the study of language to explore how people use it in their daily lives (Jones, 2012). Jones (2012) made four assumptions about language. First, he suggests that language is ambiguous. Second, language should always be considered in a context. Third, the way we use language is not independent of our identity and the communities we belong to. Finally, fourth, language is always accompanied by tone, facial expressions, and gestures. Based on these assumptions of language, discourse analysis is the study of language in a way that explains how sentences get together to compose texts and interactions and how those appear in social life (Jones, 2012). Moreover, Johnstone (2008) defines discourse analysis as the examination of various characteristics and functions of language in use. Thus, Johnstone (2008) and Jones (2012) investigate language as multi-dimensional, emphasizing its functions beyond merely articulating words.

Moreover, Philips and Jorgensen (2004) asserted that discourse analysis should focus on the linguistic characteristics of the text (speech, writing, visual image), processes

produced in the text (discursive practice), and broad social practices. According to Alwright (1988), discourse analysis involves *a turn-taking analysis* that is relevant to several aspects of turn-giving and turn-taking practices; *a topic analysis* that is relevant to the language usages as students' imitation of linguistic samples and communicative expressions of target language; *a task analysis* that is relevant to classroom tasks from the cognitive aspect. As a result, while analyzing discourse, it is important to consider various linguistic features, the context in which the discourse takes place, and the roles of individuals participating in the discourse. In addition, discourse analysis is conducted through a process. Jones (2012) identified these processes as comprising six distinct procedures: (i) "making word frequency lists"; (ii) "calculating type-token ratio" (how many different kinds of words occur in text in relation to the total number of words); (iii) "analyzing concordances" (words in the contexts of the sentences; spot patterns in word usage); (iv) "analyzing collocation" (words appearing together); (v) "analyzing keywords"; and finally, vi) "creating dispersion plots" (where words occur in texts) (p. 97). Through these processes, the words used in a discourse, the frequency of use of these words, the relationship between them, and their meanings in the context could be analyzed.

In conclusion, discourse analysis allows us to consider language as multidimensional and interpret it in a context. For instance, language components include gestures, facial expressions, tones, symbols, and images. Colloquial and professional life discourses are contextualized in different conditions. Schools are the first step that children encounter in professional life. The teacher's primary purpose should be bridging the colloquial and professional life discourses. Through discourse analysis in the classrooms, we can infer how discourse helps teachers and students to make this connection. More specifically, in mathematics education, analyzing the discourse provides the opportunity to how written, verbal, and semiotic ingredients help teachers and students to construct knowledge. The present study aims to analyze the mathematical discourse throughout the algebraic expression lessons to gain a deeper insight.

2.3. Mathematical Discourse Frameworks

Researchers have developed several mathematical discourse frameworks in order to map the discourse used in the learning environment. There are five types of

frameworks in the literature that are mostly used in analyzing mathematical discourse: Systemic Functional Linguistics (SFL) (Halliday, 1985), Critical Discourse Analysis (Fairclough & Wodak, 1997), Sociomathematical Norms (Yackel & Cobb, 1996), Commognitive Theory (Sfard, 2001), and Mathematical Discourse in Instruction (MDI) (Adler & Ronda, 2015). All these frameworks explained in detail below are based on the sociocultural theory.

Sociocultural theory adopts the idea that children learn through their interactions with the society they belong to (Vygotsky, 1981). According to this theory, learning occurs through takes place by participating in activities and interacting with other participants in a social context (Vygotsky, 1981). With the advent of Vygotsky's sociocultural theory, mathematics educators have been interested in social context in mathematics classrooms. Thus, the definition of this term has become important during the learning process. With the prominence of social context, Atweh, Bleicher, and Cooper (1998) explained that there are three components of social context: variables, settings, and construct. Variables could consist of fixed factors such as ability, knowledge, and gender or manipulable factors such as teaching styles and curriculum. Settings are relevant for analyzing values, power relationships, and the structure of the context. Atweh et al. (1998) described construct as an interactive environment that the teacher and the students socially construct in order to promote learning. Researchers interested in mathematical discourse are often associated with the construct of a social context. Therefore, defining and understanding the construct may help to analyze the discourse in the classroom.

2.3.1. Systemic Functional Linguistics

Systemic Functional Linguistics (SFL) is conducted by Halliday (1978) and is closely associated with the sociocultural theory. In his point of view, language consists of "the exchange of meanings in interpersonal contexts of one kind or another" (Halliday, 1978, p. 2). Halliday's (1985) theory regards language as social semiotics, and according to this theory, meaning is constructed in a social context (Morgan, 2002), where language is actively used and learning takes place (Morgan,

2006). The theory of social semiotics concentrates on social interactions and how systems of meaning are constructed by interlocutors (Chapman, 2003; Morgan, 2006). Social semiotics is important with regards to acknowledging functions performed by different semiotic sources and various uses of language (Atweh et al., 1998; Morgan, 2006). As a social semiotic linguist, Halliday presented the Systemic Functional Linguistics (SFL) model, which regards language as "a very useful descriptive and interpretive framework and meaning-making source" (Eggins, 2004, p. 2). Firstly, Halliday and Martin (1993, as cited in Ugurel & Boz-Yaman, 2017) defined language, in their model of SFL, as a source of meaning, not a system or rule. Secondly, SFL regards the whole text as a source of meaning, rather than just a single sentence. Thirdly, SFL investigates the relationships between text and context. Regarding language as social semiotics leads researchers to develop four theoretical claims:

> "1. Language use is functional 2. Its function is to make meanings 3. These meanings are influenced by the social and cultural context in which they are exchanged 4. The process of using language is a semiotic process, a process of making meanings by choosing" (Eggins, 2004, p. 3).

On the other hand, according to Morgan (2006), social semiotics conceptualize the context as not only the place where particular activities occur and learning takes place (*the context of the situation*), but also considering the culture outside the classroom (*the context of culture*). The context of the situation includes the activity, the participants, the tools, and other features of the specific environment. On the other hand, the context of the culture involves the values, goals, and history of the society that participants share in common.

Halliday and Hasan (1989) scrutinized social context in three categories as *field*, *tenor*, and *mode* of discourse, a framework they referred to as Systemic Functional Analysis (SFL). *The field of discourse (FD)* includes subject matter and the settings of the social action in which participants are engaged (Atweh et al., 1998). Besides, FD is the language and its properties that participants use (Morgan, 2002). *The tenor of discourse (TD)* is related to the role of the participants and their statuses (Atweh et al., 1998). TD investigates the relationship between participants in a specific context

(Morgan, 2002). *The mode of discourse* (MD) is relevant to the way in which the communication takes place and which channel (writing or speaking) the participants use (Morgan, 2002). In other words, MD is defined as "interpreting how the participants accomplish these activities through the nuances of the language used" (Atweh et al., 1998, p.66). Uğurel and Boz-Yaman (2017) conducted a study in a high school that included classroom discussions about proof and proving. The participants were 2 teachers and 13 high school students. The data was collected from verbal discourses between two teachers and students. They used the SFL model to analyze classroom discussions. The findings were based on FD in their study. Because both teachers adopted a teacher-centered manner, they did not provide students with a discussion environment. The results showed that teachers' discourse affected students' cognition of proof and proving. As SFL interprets the discourse in a social context, discourse is not only described in a communicational environment but also described in where, how, and with whom the activities are taking place and what are the effects of culture.

SFL, primarily utilized in language studies, analyzes the discourse within a communicative context. It examines the role of participants, the topic of discourse, and the means (written or spoken) used by the participants to communicate. However, SFL is inadequate for examining the content of mathematical discourse. Therefore, while SFL can be used to map the communicative elements in the classroom, it is incapable of examining the mathematical content that students need to learn, as well as the challenges they may encounter or the instructional strategies employed to address these challenges.

2.3.2. Critical Discourse Analysis

Critical Discourse Analysis (CDA) is a methodological approach that mainly examines the social power, dominance, and inequality presented in a text, talk, and other communicative tools (Van Dijk, 1995, 2001). CDA primarily analyzes the discourse, language, and communication from a socio-political point of view (Van Dijk, 1995). There are certain research requirements to be met to effectively achieve the objectives of discourse analysis. First, in CDA, social problems with political

issues are issues of concern rather than contemporary fashions and paradigms. Second, CDA is usually *multidisciplinary* and particularly concentrates on relations between society and discourse. Third, CDA attempts to *explain* discourse with regard to properties of social structure and social interaction. Fourth, *power*, *inequality*, and *dominance* are the focus of interest that society reproduces or resists via text and talk. (Van Dijk, 1995, 2001). Fairclough and Wodak (1997) outlined the core principles of CDA as follows:

> "1) CDA addresses social problems 2) Power relations are discursive 3) Discourse constitutes society and culture 4) Discourse does ideological work 5) Discourse is historical 6) The link between text and society is mediated 7) Discourse analysis is interpretive and explanatory 8) Discourse is a form of social action" (p. 271-280)

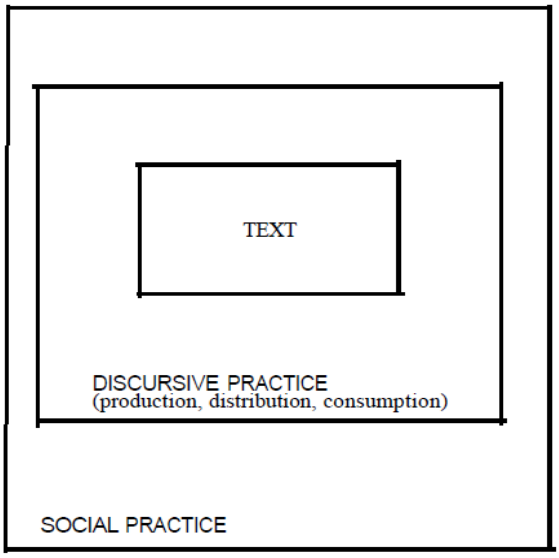


Figure 2. 2. Three-dimensional conception of discourse

CDA contains three dimensions: "(i) a language text, spoken or written, (ii) discourse practice (text production and text interpretation), (iii) sociocultural practice" (Fairclough, 1995, p. 97). Figure 2.1 describes Fairclough's three-dimensional conception of discourse. In his framework, Fairclough (1992; 1995) analyzes the discourse from different points of view. The first dimension includes linguistic features of discourse. The analysis of text must be done by taking vocabulary, grammar, and text structure into consideration. The second dimension expresses that discourse is constituted, distributed, and used in society. Fairclough indicated that

‘force’ is to explain for what purpose the text was used socially, ‘coherence’ to establish meaningful relationships and understanding of the text completely, and ‘intertextuality’ to explain how texts are related historically with each other. The third dimension includes two concepts, namely ideology and hegemony as a social practice. Ideology is presented both in the structure of discourse and in discursive events. For example, in the classroom, ideological assumptions exist in social identities and roles of teachers and students in a relationship. Hegemony, on the other hand, explains the power that is obtained through integrating groups and constituting alliances. CDA helps to reveal the relationship between language and society and defines this relationship through different dimensions.

CDA also examines the discourse primarily in terms of communicative elements like SFL. Contrary to SFL, CDA is more interested in the socio-political aspects of discourse. In other words, it centers on identifying the dominant participant in a discourse and examining how this dominance influences the communication environment. This framework is also inadequate for analyzing mathematical teaching as it generally examines the interaction between teacher-student or student-student, and the impact of this interaction on discourse.

2.3.3. Socio-Mathematical Norms

The concept of *socio-mathematical norms* was formed by Yackel and Cobb (1996). These norms are relevant to classroom discussions and discourse of the classroom (Wood, Cobb & Yackel, 1993). Socio-mathematical norms differ from classroom social norms in the way that they are only specific to mathematical norms in the classroom. Yackel and Cobb grounded their theoretical perspective on constructivism, symbolic interactionism, and ethnomethodology. Firstly, they focused on cognitive perspectives with constructivism; then, they developed a sociologic perspective through mathematical activities according to symbolic interactionism and ethnomethodology perspectives. From this point of view, students’ reasoning and sense-making abilities develop through their participation in the interactive construction of shared and identical mathematical meanings. The purpose of their framework is to investigate how students enhance mathematical

beliefs and values and how they achieve mathematical autonomy. They adopt the view that learning mathematics is both an active process of individual construction and a process of enculturation of mathematical practices with wider society. The researchers primarily focused on the initiated and guided development of social norms that indicated classroom microculture characters by teachers. Afterward, they extended their previous work to concentrate on the normative aspects of mathematics discussions that appeared in students' mathematical activities. They called these norms specifically *socio-mathematical norms*, which are the normative understandings considered to be mathematically different, elegant, and efficient in a classroom. It is important to note that there is a difference between socio-mathematical norms and social norms. In social norms, students explain their ways of thinking and finding solutions, whereas in socio-mathematical norms, students acquire acceptable mathematical statements.

Yackel and Cobb (1996) stated that socio-mathematical norms enable us to analyze and elaborate on students' and teachers' mathematical practices in the classroom. These norms facilitate the investigation as to how students and teachers conceptualize mathematical explanations based on criteria such as similarities, differences, efficiency, and sophistication. To illustrate, students distinguish varying solutions and compose different solutions to one example (they know that the solutions of "16+14+8=…." such as "20+10+8=38" and "15+15+8=38" are different, but both are considered acceptable). This reflects that they have some opinions about the acceptable mathematical explanations. Yackel and Cobb (1996) concluded that the constitution of socio-mathematical norms may differ significantly across different socially and structurally constructed classroom environments. Moreover, socio-mathematical norms help us to clarify the role of teachers in the represented mathematical community.

Socio-mathematical norms refer especially to the norms observed in mathematics classrooms, distinguishes them from SFL and CDA. Socio-mathematical norms are informed by the diverse interpretations of mathematical expressions among mathematics classrooms with distinct cultural backgrounds. However, similar to previously discussed frameworks, this approach also lacks a defined unit of analysis for examining the teaching of mathematics.

2.3.4. Commognitive Theory

Contrary to the frameworks mentioned earlier, Sfard has constructed a detailed theory for analyzing mathematical discourse. Sfard (2001) developed *the commognitive theory* based on sociocultural theory, paying attention to the relationships between communication and cognition to analyze mathematical discourse. According to this theory, thinking is considered as a way of communication, so it is accepted that speech and thought are both essential. In addition, since human behavior and thought are shaped by the society to which they belong, communicative problems between individuals result in misunderstanding and communication failures according to this theory. Therefore, Sfard (2001) prioritized communication in mathematics education and developed the commognitive theory based on the communicational approach. This approach relies on self-communication, which means thinking individually, asking questions to oneself, waiting for answers, and arguing the answer with one's own self and others. Therefore, this theory emphasizes the importance of the communication that individuals establish with themselves and their community and discusses the effects of communicative problems on individuals' learning.

Sfard (2001) asserted that meaningful learning can only occur when students are actively engaged as participants in the learning process and called this as a "*participationist (communicational) approach to cognition"* (p. 22). This approach is closely linked to sociocultural theory, which posits that children learn through their interactions with the society to which they belong (Vygotsky, 1981). More specifically, sociocultural theorists such as Vygotsky (1981) define learning as active participation in a particular activity, rather than merely the acquisition of conceptual schemes. Sfard (2008) stated that students learn mathematics through active participation in an activity and engagements in interactions, where communication and the use of mathematical discourse play essential roles. In other words, in the "*communicational approach to cognition,* thinking is considered as one way of communication, where communication takes place with own self (Sfard, 2001, p. 22). This approach has two distinct features. First, human behavior and thinking are interdependent with the social environment, emphasizing the interplay between

individual and social dimensions. Second, this approach includes both speech and thought without hierarchical distinction, demonstrating that thinking is inherently embedded in social context and that both thinking and speaking equally influence an individual's learning processes.

Based on this approach, as mentioned earlier, Sfard (2001) defined discourse as "any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system" (p. 28). Therefore, while analyzing the discourse, acts of communication and factors that influence its effectiveness should be included (situational clues and participants' histories). She stated that in a specific discourse, there are two critical aspects: *mediating tools* (or simply *mediators*) that people use to communicate and *meta-discursive rules* that organize the communication (Sfard, 2001). Elaborating on this, Sfard (2008) explains further that *mediating tools* signify that people use symbolic tools, such as language, during communication. These tools contain different meanings as the social environment changes. In other words, the meaning of the tool has different interpretations for different people and in different contexts. While *meta-discursive rules* provide guidance for the performance of communication, they are interpretive in nature (Sfard, 2001). Therefore, the perspective of the observer and a participant who is implementing the rule may differ due to their respective roles. It is therefore essential to identify the mediating tools employed by individuals within a particular context, as well as meta-discursive rules that guide the flow and order of communication, in order to address the issues of miscommunication.

From a different perspective, the communicational approach argues that communicative problems among students and teachers engaged in a discourse cause student failure (Sfard, 2001; 2008). According to this approach, failure occurs when students are unable to conduct an effective dialogue with their teacher, partner, and themselves. In other words, failure arises when the discourse is interpreted differently by the participants in the communication (Sfard, 2008). For instance, when the teacher elucidates that the equal sign signifies a state of equilibrium, the pupils' comprehension that the equal sign just represents the outcome of a

mathematical operation leads to a breakdown in communication and failure. In order to figure out the reason for the failure, Sfard (2001) conducted a detailed study on the communicational way. She stated that we may change the way of our understanding of learning and teaching by paying attention to communication. Therefore, Sfard (2008) analyzed the students' dialogues to determine their mathematical discourse and to contribute to the theory and practice of education. For this purpose, she conducted analyses focusing on two types of rules: object-level rules and meta-level rules. Both of these rule types can be transmitted through communicative actions. While object-level (cognitive, content-related) rules are relevant to the explicit aims of the given activity, meta-discursive or meta-level rules are less visible but not less influential. The latter rules include participants' interests in the way the interaction is being managed and the weight of the relationship among participants. While object-level rules represent visible aspects of discourse, meta-level rules engage with guiding the way of communicational activities. This analysis is made by determining the ways in which interactions are managed (communication with the partner or themselves) and issues of the relationships between interlocutors. While object-level rules are implied in explicit ways, meta-level rules are located in the forms of utterances and in procedures of interaction.

In conclusion, Sfard (2008) has developed a comprehensive framework for examining mathematical discourse within classroom dialogues, focusing on identifying both object-level and meta-level rules. This framework depicts the defining characteristics of mathematical discourse and clarifies the relationships between these characteristics. The characteristics of mathematical discourse offered by Sfard (2008) comprise word-use, visual mediators, routines, and endorsed narratives. Word-use and visual mediators are related to object-level rules that correspond to the visible aspects of a discourse, whereas the routines are closely associated with the meta-level rules that guide the way of communicational actions. These mathematical discourse characteristics are described in detail below.

2.3.4.1. Word-use

Sfard (2008) emphasized that words and their uses are central characteristics of discourse, asserting that learning depends heavily on the use of words (Kim & Lim,

2018). Word uses change gradually in the process of learning, progressing through four hierarchical levels. In the initial phase, known as *passive use*, students cannot use the word in their speech, but they may react to dialogues involving the word. For instance, students who are not acquainted with the meaning of 'like term' but try to find them when asked is an example of passive use. In the second phase, called *routine-driven*, students are able to use the word in specific routines. In the *phrase-driven,* students are able to use the word with phrases. The last phase of using the word is *object-driven* which involves the use of stable meaning independent from the context. The development process of word use is illustrated in Figure 3.9.

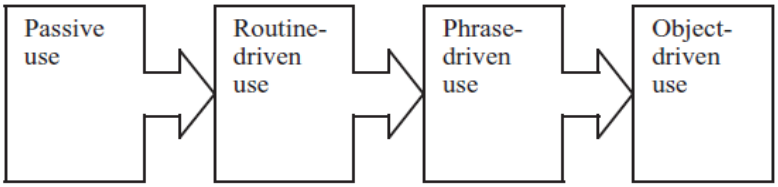


Figure 2. 3. The development process of word use (Sfard, 2008, p. 182)

Word-use analysis can be conducted according to the four hierarchical stages outlined above, and can also be examined through the lenses of objectification levels, i.e., processual and structural (Güçler, 2016; Kim & Lim, 2018). Objectification involves two key discursive moves: reification and alienation. The reification process contains replacement statements about process and action with about state and object. Following reification, alienation involves presenting the phenomena as occurring by themselves without involving a human being (Sfard, 2008, p. 50). Kim and Lim (2018) categorized these objectification levels as *processual,* which conceptualizes the concept as a process, and *structural*, which conceptualizes the concept as an object in the structure (Figure 3.10).

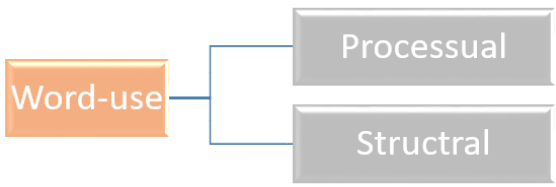


Figure 2. 4. Processual and structural process of word-use

In the literature, some researchers employ a four-category analysis of word-use (Emre-Akdoğan, 2015), while others choose two (Sarı-Arıkan, 2019; Kim et al., 2017). These preferences were decided by considering the nature of the subjects the researchers studied. In her study, Sarı-Arıkan (2019) handled the word-use analysis as if the teacher's statements about the quadrilateral include process, algorithm, and action; the word usage is *process-based*; if it uses the quadrilateral as an explicit mathematical object, word use is considered as *object-based (structural)*. On the other hand, Emre-Akdoğan (2015) analyzed a high school teacher and his students' word usage according to four hierarchical stages. In the first stage, passive use, the participants were unable to employ the words in sentences and instead used pronouns such as this and that. In the routine-driven stage, the participants used the words as an action. Then, in the phrase-driven stage, the participants used the words in the sentences with their general phrases. Lastly, the participants used words as their objectified forms in the object-driven stage. In this study, it was decided to use the four-stage analysis of word use since all stages can be easily followed. Therefore, it is possible to analyze in detail the hierarchical structure of the words used in algebra teaching.

2.3.4.2. Visual Mediators

Visual mediators are visible elements of mathematical discourse. They are described as images that individuals use to identify the object of the discourse and organize the communication (Sfard, 2008). Mathematical discourse is described as primarily visually mediated by symbolic artifacts. These artifacts are icons, including "algebraic symbols, designed diagrams, graphs, and drawings" (Sfard, 2008, p.148). In mathematical discourse analysis, it is not enough to list which visual mediators are used by the individuals participating in the discourse. In order to complete the analysis, it is important to investigate how the participants use these visual aids (routines), their lexical preferences in the process of using these aids, and which mathematical narratives they use these visual aids to affirm (Güçler, 2016). In this study, algebraic expressions, symbols, and geometric figures are analyzed as visual mediators.

2.3.4.3. Routines

Routines are a collection of metarules that establish recurring patterns in discourse and they could be idiosyncratic (Sfard, 2008). For instance, the repeating patterns exhibited by a student when engaging in mathematical tasks might be different from those observed in another student or teacher. In order for a recurring pattern to be acknowledged as a routine, it needs to be extensively practiced within the community of discourse and approved by the majority of the community, particularly by the experts. Examples of routines observed in algebra lessons could be finding like terms or simplifying a given algebraic expression. According to commognitive theory, the first stage of routine analysis is to identify the *prompts* that lead to the emergence of the routine. This theory considers it necessary to analyze *how* and *when* a routine is implemented. *How* the routine is implemented includes the method and action used when implementing the routine. *When* a routine is applied includes the mathematical conditions under which participants consider the use of a routine appropriate. When routines are used, they are discussed in terms of two separate terms: *applicability* and *closure*. While applicability underlines the circumstances in which a participant would apply a particular routine, closure investigates the circumstances in which a participant considers the routine to be complete.

2.3.4.4. Endorsed narratives

Narratives are "any sequence of utterances framed as a description of objects, of relations between objects, or of processes with or by objects, that is subject to endorsement or rejection with the help of discourse-specific substantiation procedures" (Sfard, 2008, p. 134). Narratives within a discourse that are endorsed by the discourse community and the other characteristics of the discourse are regarded as true. These are generally definitions, theorems, or axioms in mathematics. In other words, in order for a narrative to be endorsed, it should be confirmed by the words, visual mediators, and as well as routines existing in the discourse.

2.3.5. Mathematical Discourse in Instruction

Another framework was developed by Adler and Ronda (2015) to analyze Mathematical Discourse in Instruction (MDI). The framework provides a detailed

depiction of mathematics education and interprets the mathematical materials that are available for learning. It focuses on four components of teaching mathematics: "the object of learning," "exemplification," "explanatory talk," and "learner participation" (Adler & Ronda, 2015, p.237). Firstly, "the object of learning" expresses the specific mathematical content or concept that learners are expected to know and be able to do (Adler & Ronda, 2015, p.238). Secondly, "explanatory talk" is the language used by the teacher to name and legitimize mathematical ideas and concepts (Adler & Ronda, 2015, p.241). Thirdly, "exemplification" includes the use of examples and tasks to provide learners with opportunities to experience variation in mathematical concepts (Adler & Ronda, 2015, p.239). Lastly, "learner participation" concentrates on the interaction between students and the teacher, as well as the opportunities for students to participate in mathematical discourse and reasoning (Adler & Ronda, 2015, p.245). These elements interact with each other to develop mathematical discourse in instruction and provide an extensive framework for an elaborated description of mathematics teaching practices (Adler, 2017; Adler & Ronda, 2015; Adler & Venkat, 2014). Thus, MDI's purpose is to make a contribution to the field of mathematics education research and enhance the understanding of effective mathematics learning.

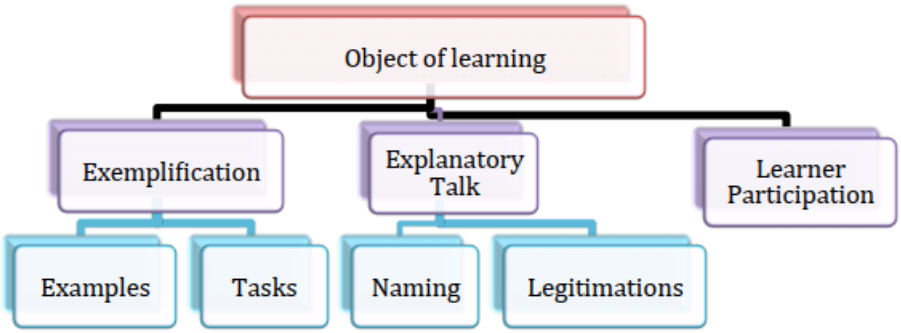


Figure 2. 5. Constitutive elements of MDI and their interrelation (Adler & Ronda, 2015, p.239)

Figure 1.1 shows the constituents of MDI framework. *The object of learning* is the main component of the instruction that emphasizes the aim of the lesson (Adler & Ronda, 2015, p.238). In other words, the focus is on what students are expected to grasp and be able to do regarding the subject matter being taught. For instance, Adler

and Ronda (2015) analyzed a 10th-grade lesson whose object of learning is dividing algebraic fractions. The *explanatory talk* in the dividing algebraic expressions lesson includes the teacher's explanations and clarifications about the concept and procedures that the concept includes. *Exemplification* is the use of tasks and examples within the dividing algebraic expressions lesson. For example, $\frac{2(x+y)}{2}$, $[\frac{2(x+y)}{2}]^2$, $[\frac{2(x+y)}{2}]^2 x\ ab$ are the set of examples that the teacher writes on the board. Lastly, interactions between students and the teacher, as well as among the students, express the *learners' participation* in MDI. These are "yes/no answers," "phrase answers to the how/what questions," or "answers to why questions" (Adler & Ronda, 2015, p.242). Thus, the MDI framework provides an elaborate description of mathematics teaching and interpretes variations in the mathematical resources accessible for learning. Furthermore, it helps to identify patterns and variations in teaching practices.

Discourse analysis has been applied in the literature to examine the process of teaching many subjects within the mathematics curriculum. For example, there is a considerable amount of study that examines discourse in mathematics education in the context of limit and infinity (Güçler, 2013; Kim & Lim, 2018; Kim, Sfrad & Ferrini-Mundy, 2005), polygons (Barwell, 2015), functions (Güçler, 2016; Viirman, 2011), pre-calculus and calculus (Berger, 2013; Nardi, Ryve, Stadler & Viirman, 2014), fractions (Ngin, 2018), algebra and abstract algebra (Ioannou, 2016; van Jaarsveld, 2018), and geometric translation (Emre-Akdoğan, 2015; Emre-Akdoğan et. al., 2018). Most of the research studies included university students (Ioannou, 2016; Kim & Lim, 2018; Nardi et. al, 2014; Viirman, 2011), pre-service teachers (Berger, 2013; van Jaarsveld, 2018), high school students (Emre-Akdoğan, 2015; Emre-Akdoğan et. al., 2018), elementary, middle, and high school students and their teachers (Barwell, 2015; Kim et. al., 2005; Ngin, 2018) as participants. It was noticed that the majority of the research studies were conducted with high school and undergraduate level, while those at middle and elementary levels were less.

In conclusion, it is apparent that algebra learning holds great value in terms of both individual career development and greater societal engagement. Besides, the

importance of algebra education is emphasized by its historical origins, its integrative role within many mathematical disciplines, and the difficulties encountered by learners. On the other hand, the difficulties related to learning algebra and the emphasis on the importance of early algebra education necessitate a deeper insight into algebra instruction for educators and policymakers. The analysis of mathematical discourse through frameworks like Sfard's (2008) commognitive theory and Adler and Ronda's (2015) MDI shed light on the complexities of teaching and learning algebra. Therefore, analyzing the mathematical discourse provides insight into teaching and learning environment by determining the discourse patterns and relationship between communication elements and failures (Güçler, 2013).

2.3.6. The Summary of Mathematical Discourse Frameworks

In recent years, mathematical discourse has become popular with the interest of communication and means of communication in mathematics education. The role of discourse has been investigated extensively in mathematics classrooms. Moreover, researchers develop theories and frameworks in order to study the effects and role of discourse in a communicational environment based on sociocultural theory.

The first framework is Systemic Functional Linguistics (SFL), presented by Micheal Halliday, which is based on sociocultural theory (Eggins, 2004). This framework analyzes the discourse within the social context. According to SFL, discourse is constituted in a social context and differs as the context changes. There are three components of this framework, namely the field of discourse (FD), tenor of discourse (TD), and mode of discourse (MD). SFL interprets the discourse in a social context, and it is important to know where, how, and with whom the activities are taking place. The role of participants and the communicational methods used by the participants are emphasized. Hence, SFL offers a concise evaluation of the communication environment and role of the participants. Nevertheless, SFL does not give the opportunity to address the mathematical content and analyze teaching this content. Critical Discourse Analysis (CDA) is another framework which was developed by Fairclough (1995). This framework scrutinizes the language from a

sociopolitical point of view. Social problems and power relations are issues of concern. The main focus of this framework is to investigate the dominant participant who directs the discourse and the role of this participant in the formation of the discourse. Similar to SFL, CDA is not appropriate for analyzing the mathematical discourse characteristics throughout the algebra lessons. SFL is concerned with the environment in which the discourse occurs, while CDA focuses on the sociopolitical factors in the discourse. Therefore, these two frameworks were not used in this study due to their inadequacy in analyzing algebra lessons.

Another framework named Socio-mathematical norms was originated by Yackel and Cobb (1996). The role of norms, especially mathematical norms, is being investigated in mathematics classrooms. Contrary to SFL and CDA, socio-mathematical norms are specifically designed for the analysis of mathematical norms. According to this framework, socio-mathematical norms arise from the different mathematics classrooms with varied cultural backgrounds. This framework is once again insufficient for the analysis of algebra lessons and teaching algebra, as it primarily focuses on mathematical norms in classrooms with varying socio-cultural backgrounds.

Commognitive theory was developed by Sfard (2001) to associate communication and thinking. Sfard generated this theory to analyze mathematical discourse specifically. This theory is also based on sociocultural theory like others. According to the theory, student's behavior and thinking are not independent from social environment. Speech and thought are equally important and neither is considered superior to the other. Communicational mediators as auditory, visual, or symbolic tools have an extensive role in the theory and mathematical discourse consisting of word use, visual mediators, endorsed narratives, and routines, which can be observed easily in a mathematics classroom.

The present study is primarily based on Sfard's commmognitive theory, because it is appropriate for analyzing the algebraic discourse as algebra has its own words, symbols, narratives, and routines. Mathematics, and specifically algebra, have a common symbolic system. People can understand mathematical expressions and

communicate with each other via this system. In the classroom, teachers and students use this system and mediators to communicate and learn mathematics. Sfard contends that meaningful learning occurs and misconception could be lower, if only participants truly communicate. In the communication process, participants' use of mathematical expressions, specific words, visual mediators, narratives, ways, and focus of conversations influence the effectiveness of communication. In order to map mathematical discourse employed during algebra teaching in the present study, these items were analyzed through Sfard's theory. Thus, the algebraic content taught in lessons could be analyzed in a communication environment. While Sfard's framework is useful for analyzing the mathematical discourse characteristics in algebra lessons, an additional framework, such as MDI, is required for a comprehensive analysis of the overall instructional process in general terms. MDI places particular emphasis on key aspects like "the object of learning," "exemplification," "explanatory talk," and "learner participation" in the context of mathematics instruction (Adler & Ronda, 2015, p.237). Therefore, the interaction of these aspects contributes to the development of mathematical discourse in instruction and offers a more comprehensive foundation for a detailed description of mathematics teaching practices. MDI lacks the ability to define the characteristics of mathematical discourse clearly, whereas commognitive theory aims to address this gap by examining the categorized mathematical characteristics in a discourse.

2.4. Studies Related to Mathematical Discourse

In the relevant literature, many research studies address the importance of examining the discourse on mathematics education (Atweh et al., 1998; Baki & Çelik, 2018; Emre-Akdoğan, 2018 et al.; Güçler, 2013; Güçler, Wang & Kim, 2015; Tabach & Nachieli, 2015). For instance, Güçler (2013) stated that analyzing the mathematical discourse throughout the instruction reveals that despite students and the instructor using the same features of discourse (words, visual mediators, endorsed narratives), they confirm different phenomena ('limit is a number' instead of 'limit is a process'), and this is the result of miscommunication. Thus, examining the discourse enlightens the communicational breakdowns in individuals' discourses by elaborating how, when, and in what ways they occur (Güçler, 2013; Sfard, 2008). Tabach and Nachieli

(2015) examined classroom events in which the prospective teachers' definitions of functions and they observed some changes in their discourse towards defining functions based on an accepted mathematical definition. By analyzing the discourse, the researchers were able to trace the development of students' understanding and applications of mathematical definitions, particularly in the context of functions. This analysis provided important insights into the learning processes and the efficacy of instructional strategies in fostering students' comprehension and use of mathematical definitions. Moreover, Atweh, Bleicher, and Cooper (1998) stated that by analyzing the discourse, it is possible to determine patterns, distinctions, and underlying beliefs that influence classroom interactions, so they gained a deeper understanding of the influence of the social environment on the processes of teaching and learning. Consequently, the aforementioned studies highlight the critical role of discourse analysis in confronting the difficulties in mathematics education and fostering a deeper understanding of the mechanisms that influence the teaching and learning of mathematics.

Mathematical discourse is a process in which students engage in discussions about mathematical reasoning at different cognitive levels and focus on expressing mathematical procedures through talking, questioning, and writing (Knott, 2010). Discussing the discourse only with a limited perspective is not preferred. As mentioned above, discourse should be handled in different aspects, such as semiotic, activity, material, political, and socio-cultural (Gee, 1999). These aspects could be observed and enhanced in the classroom environment in different contexts. Besides these different aspects, discourse can be considered in two domains: formal and informal discourse. Students start to build their formal discourse at school, while informal discourse is built in their everyday lives (Caspi & Sfard, 2012).

Students attend the classroom with the knowledge and language that they learn and practice in their daily lives. This informal discourse is essential for developing formal discourse (Caspi & Sfard, 2012; Moschkovich, 2003). Moschkovich (2003) examined mathematical discourse and students' talk in the classroom. She explored how every day and mathematical discourses help us to determine mathematical content students use in classroom discussions. She observed a mathematics

classroom working on two-dimensional figures in which students talked about geometrical figures' properties and tried to identify similarities and differences between them. The classroom discussions enabled the researcher infer that students use both everyday and mathematical discourse to discuss and construct the mathematical content. Therefore, these two discourses were inseparable in the mathematics classroom.

Soucy-McCrone (2005) investigated how mathematical discourse is enhanced by exploring the development of classroom discussions in a 5th-grade mathematics classroom and the contribution of the teacher to this development for 6 months. She used the term discussion as an aspect of mathematical discourse and described it as a small-group or whole-class discussion in which participants made sense of mathematical problems. She analyzed the observed data according to Cobb's (1999, as cited in Soucy McCrone, 2005) classification of discourse, which included (a) discourse patterns in small groups or whole class discussions, (b) levels of response that contained short recall or complex reasoning, (c) the role of teacher and students, (d) socio-mathematical norms in the classroom, and (e) mathematical practices in the classroom. Discourse patterns and level of response categories were used to describe the development of classroom discussions over time, along with students increased communicational competence. Findings revealed that students participate in discussions and mathematical discourse when both the teacher and students take responsibility for working together and communicating mathematical concepts. Therefore, Soucy-McCrone concluded that the teacher has an important role in guiding classroom discussions and maintaining students' curiosity about investigating mathematical discourse and understanding the content.

Providing guidance and teacher education programs may help teachers realize how they can promote students' mathematical discourse. Nathan and Knuth (2003) conducted a two-year study that presents a middle school mathematics teacher's changing her classroom practices after a professional development program. This program was based on the practical argument method, which involved reflecting on our actions and connecting the objectives of these actions with ways and means of the objectives (Fenstermacher, 1994). The researchers focused on whole-class

discussions with students and the teacher about mathematical content. The teacher had the opportunity to notice why she did those actions in the classroom. Then, she changed her teacher-centered instruction, and students participated more in classroom discussions than earlier. The researchers concluded that students' contributions to the discussions were central to productive discourse, and the teacher had an essential role in initiating and facilitating discourse. Therefore, they recommended that discourse-based practices should be promoted also in teacher education programs.

Novice teachers might find it difficult to engage their students in discursive practices. In the study on providing guidance for promoting classroom discourse, Benneth (2010) examined two new mathematics teachers' improvement of mathematical discourse with a mentor's assistant and engaging their students in whole-class discussions. Contrary to teachers' perceptions about themselves, observational data showed that they were not including their students in activities. These teachers worked with the mentor, examined the observational data, and had recognizable improvements in their ability to enhance mathematical discourse. With the help of the communicational environment and facilitating mathematical discourse, their students' mathematical understanding was also improved. Benneth's (2010) study showed that the role of teachers is to increase students' engagement in classroom discussions and promote mathematical discourse to facilitate understanding.

Previous studies on mathematical discourse emphasize that there is an important effect of classroom discussions on students' mathematical development (Back, 2000; Moschkovich, 2003; Nathan & Knuth, 2003; Soucy-McCrone, 2005; Walshaw & Anthony, 2008). Teachers have an essential role in generating and carrying out a discussion environment in the classroom. Back (2000) observed primary school mathematics instruction to analyze teacher's and students' talks engaged in the classroom. The purpose of the study was to explore the possible relationship between classroom discourse, mathematical discourse, and school mathematics discourse. She attended two lessons as a helping out parent and took field notes and audio records. In the analysis process, she focused on the words because she wondered about how

the use of talk encouraged students to learn mathematics and develop their mathematical thinking and expressions. Back (2000) noticed that the teacher asked questions to assess students' knowledge and understanding and she used students' responses as a base to create a discussion environment. Including students in the conversations gave the teacher the opportunity to refine and develop students' mathematical discourse. The teacher was involved in conversations both with individual students and with the whole class during the lesson. She regularly clarified the task, explicated on the students' responses, and offered guidelines to contribute to classroom talk. Students' contributions guided the teacher to move the mathematical discourse forward. The teacher restated what students said and offered guidelines to explain their mathematical thinking and understanding with their own adopted guidelines.

Walshaw and Anthony (2008) provided a critical review of the role of mathematics teachers in promoting mathematical discourse. The purpose of the study was to provide suggestions to teachers and mathematics educators about how to establish mathematical communication in the classroom. They used a classification to review the literature on discourse, including "participating rights and obligations," "articulating thinking," "fine-tuning mathematical thinking through language," and "shaping mathematical argumentation" (p. 522). After reviewing the related literature Walshaw and Anthony (2008) presented that effective use of discourse in the classroom provided students to improve mathematical reasoning. By explaining their ideas, stimulating, challenging, and extending other students' ideas, students had the responsibility for their own learning. Walshaw and Anthony (2008) suggested some implications for teachers aimed at improving students' discourse to foster the development of mathematical understanding. In order to support classroom discussions, teachers should create an environment that help students communicate, reason, and critique the ideas. Teachers should help their students enhance mathematical discourse by listening to the students cautiously and supporting important mathematical thinking. They should provide students with a rich discussion environment to enhance social engagement. This interaction would help students to use mathematical language properly and support understanding. In order to reach productive discourse, teachers should let students communicate with

mathematical concepts and their relationships to study mathematical structures. Students' informal discourse learned from everyday activities and socio-cultural background influences the formal discourse they learned at school. An effective teacher should regard this distinction and try to bridge colloquial and mathematical discourse for meaningful understanding. They should engage every student in classroom discussions even if they are unwilling to. Therefore, students become eager to learn mathematics. A review of the literature led Walshaw and Anthony (2008) to notice that there needs to be longitudinal studies for analyzing the relationship between the discourse and the learning process.

To summarize, the previous studies have showed that discourse should be handled in different ways and in different contexts. Teachers have a crucial role in enhancing mathematical discourse and developing understanding in the classroom. They should consider both students' informal and formal discourses to promote their mathematical understanding. They should create discussion environments in order to help students discuss their ideas with each other and the teacher. Apart from these, teachers who do not have sufficient experience in promoting discourse should be provided with training programs and guidance.

2.5. Studies Related to Algebra and Mathematical Discourse

In the Turkish Middle School Mathematics Curriculum, students are expected to represent and indicate relationships among variables and solve some problems involving patterns, algebraic concepts, and processes (MoNE, 2018, 2024). While the more procedural aspects of algebra are handled across the high school grades, the broader aspects of algebraic thinking and its conceptual underpinnings are reflected in the elementary and middle-grade levels. As mentioned above, algebra can be defined as a "sub-category of mathematical discourse" (Caspi & Sfard, 2012, p. 470), and elementary algebra can be defined as a meta-discourse of arithmetic (Sfard, 2008). Therefore, it is prominently noted that middle school algebra, in which students meet formal algebra for the first time, is the basis for the future study of algebra and mathematics. For this reason, a bulk of studies have been conducted on algebra teaching and the role of mathematical discourse in this process (Caspi &

Sfard, 2012; Johansson & Kilhamn, 2022; Nisa et al., 2021; Robert & Roux, 2019; Wallach et al., 2022).

Along with understanding the importance of algebra in the mathematics curriculum, researchers have focused on how to improve students' algebraic thinking. Researchers have indicated that students in middle school classes should demonstrate algebraic thinking skills in accordance with their grade levels (Allen, 2000; Fennell et al., 1997; Moore-Harris, 1998; Moses, 2000). According to Braswell et al. (2001), students should know the quantity and relationships between quantities and transmit underlying reasoning skills beyond arithmetic. They should be able to compare and contrast mathematical understandings and generate their own examples. Middle school students should be able to establish relationships between data and graphs and use multiple representations of mathematics. Students who are in middle school should know the procedures of data gathering, organizing the data, and interpreting the findings.

It is very important that students acquire those skills mentioned above, but they first should make the connection between informal and formal algebra. In order to examine this connection, Caspi and Sfard (2012) observed 5th and 7th graders as they learned algebraic discourse in a 24-month period. The study aimed to map the development of algebraic thinking, the relationship between informal and formal discourse, and the transition from informal to formal discourse. They named the informal algebra meta-arithmetic. Findings showed that structures of algebraic formulas are similar to arithmetic expressions, and thus, students might build arithmetic competence first and strengthen their knowledge of these concepts. Moreover, informal algebra ideas seem to start before the introduction of formal algebra in the 7^{th} grade. Algebraic expressions may break into colloquial discourse through media. Therefore, they suggest to conduct a more detailed study for the early development of algebra and meta-arithmetic. According to Caspi and Sfard (2012), the teachers' aim should be eliciting the spontaneously developed discourse and helping students construct their own formal algebraic discourse in order to develop algebraic thinking. This process helps learners to understand what they know and how they can employ that knowledge. Their research findings indicated that in the

education of pre-service elementary and middle school teachers, there is a need for more focus on algebra and algebraic reasoning, the applications of arithmetic and algebra, and the treatment of student transitions from arithmetical to algebraic thinking (Day & Jones, 1997; Lodholz, 1990). For this reason, the analysis of mathematical discourse throughout the development of algebraic thinking is essential at primary and middle school levels.

For instance, Wallach, Heyd-Metzuyanim, and Band (2022) investigated how middle school mathematics teachers addressed the shift from process to object in algebra instruction and explored potential obstacles and opportunities for helping students develop algebraic thinking skills. By analyzing the visual mediators, word use, endorsed narratives, and routines used by the teachers, it was aimed to determine whether their discourses were object-oriented or process-oriented. According to the findings, teachers who used physical and iconic symbols adopted a process-oriented discourse, while teachers who used formal symbols adopted an object-oriented discourse. Since objectification plays a crucial role in students' learning, this study highlighted the importance of using formal algebraic symbols to develop an object-oriented discourse. Another study by Johansson & Kilhamn (2022) examined how teachers used and introduced algebraic concepts in their lessons and how they handled objectification and symbolization in this process. In this study, sixth-grade teachers' algebra lessons were analyzed to examine how teachers performed objectification through vocabulary use, symbols, routines, and narratives. The findings revealed that objectification is present in teacher discourse, but it is often implicit, which can make it difficult for students to recognize. Therefore, it was emphasized that teachers should make the objectification process more explicit. It was also noted that objectification is closely related to the use of formal algebraic symbols, so it is important to move from concrete and iconic representations to formal and arbitrary representations.

Another study conducted by Robert and Roux (2019) investigated 8th and 9th-grade students' approaches to linear equations. The study indicated that students employ a ritualized discourse rather than an explorative one when solving linear equations, which means that they work with disobjectified entities and rigidly apply certain

rules. The findings revealed that students, with the help of the teacher's prompting, started to produce endorsed narratives about mathematical objects and tried to explore the meaning of routine. This finding provides a potential clue of how educators can offer guidance to shift student thinking toward exploratory discourse. On the other hand, Nisa, Lukito, and Masriyah (2021) examined the mathematical discourse of 10th-grade students while analyzing absolute value equations using the Commognitive Framework. The results revealed that students primarily use exploratory discourse and are able to use mathematical theorems more than in ritualized discourse, which indicates that they are able to use mathematical theorems to construct endorsed narratives.

In conclusion, previous studies have shown that there is a close connection between students' algebraic thinking and mathematical discourse. Analyzing their mathematical discourse may help researchers and educators to realize how students enhance algebraic thinking. Teachers' choices of activity, directions of classroom discussions, and students' colloquial discourse all play a crucial role in the development of algebraic thinking. Based on these findings, understanding how students and teachers use mathematical discourse to develop algebraic thinking is the central focus of the present study.

2.6. Summary of Literature

Algebra knowledge is of great importance for both academic achievement and career development (Schonfeld, 1995; Usiskin, 1995). It is considered an essential component of mathematical thinking, aiding students in developing a deeper understanding of mathematical concepts, including numbers, operations, ratios, measurements, geometry, and data analysis (Kaput, 2005). Numerous research has consistently shown that a comprehensive knowledge of algebra and an improved ability in algebraic reasoning help students enhance their problem-solving skills (Bednarz, 2001; Blanton & Kaput, 2005; Blanton et. al., 2019; Booth et al., 2017; Egodawatte, 2009; Herscovics & Linchevski, 1994; Kieran, 1981; MacGregor & Stacey, 2007; Mulungye et al., 2016; Tirosh et al., 1998). In addition, algebra provides a systematic way of representing problems with a specific symbolic system,

which helps students have a better understanding of key concepts such as variables, symbols, and equality signs (Blanton et al., 2019; Booth et al., 2015).

Despite the significance of algebra, students often face some difficulties in learning it. Difficulties in learning algebra stem from a variety of factors that make it difficult for students to understand and apply mathematical concepts. One of the primary difficulties is associated with arithmetic operation skills. Some students encounter challenges when performing arithmetic operations such as addition and subtraction in algebraic expressions (Booth, 1988; Erbaş et al., 2010; Herscovics & Linchevski, 1994; Jupri et al., 2014; Pillay et al., 1998; Seng, 2010). In particular, they have difficulty in performing operations when an unknown variable is represented by a letter (Herscovics & Linchevski, 1994). Moreover, their lack of understanding of the distributive and commutative laws prevents them from grasping the full structure of equations and the relationship between expressions (Jupri et al., 2014). Another difficulty is related to the utilization of variable notation. Some students struggle with representing quantities and establishing connections between these quantities using variables (Bamberger et al., 2010; Bednarz, 2001; Blanton et al., 2015; Uçar-Sarımanoğlu, 2019; Usiskin, 1988; Warren et al., 2016). For instance, a common misunderstanding among students is that all variables are simply letters representing numbers (Usiskin, 1988). This leads them to ignore the fact that variables represent a broader concept, not just a number. Another difficulty that students encounter is related to the use of the equality sign. There is a misunderstanding among students regarding the interpretation of the equal sign (Bamberger et al., 2010; Blanton, et al., 2015; Herscovics & Linchevski, 1994; Jupri et al., 2014; Knuth et al., 2006; Stacey & Chick, 2004; Stephens et al., 2013; Warren et al., 2016). Thinking that the equality sign symbolizes the result of an arithmetic operation, they have difficulty in comprehending that it actually represents a balance between two sides (Bamberger et al., 2010). This misunderstanding becomes particularly problematic when they encounter algebraic equations full of symbols. Finally, an important difficulty indicated in the literature is the transformation of verbal descriptions into symbolic systems. Students struggle with the process of transforming verbal explanations into symbolic systems (Bednarz, 2001; Blanton et al., 2019; Carraher et al., 2008; Kieran, 1994; Linchevski & Livneh, 1999; Stacey & MacGregor, 1997). This makes it

challenging for them to make connections between everyday language and algebraic language, and prevents students from expressing relationships between quantities (Robert & Roux, 2019). Such difficulties and misunderstandings represent significant obstacles in algebra learning. Therefore, it is necessary to examine the algebra instruction to determine and overcome these problems effectively.

Examining mathematical discourse in algebra teaching enables us to comprehend the communicative dynamics in teaching and learning processes. It reveals how students and teachers comprehend, articulate, and conceptualize the mathematical concepts they use. Moreover, discourse analysis helps to identify the disconnections and misunderstandings that arise in the communication environment between students and teachers (Güçler, 2013). It provides a means to observe how students utilize mathematical definitions and concepts by allowing an examination of their learning processes. Thus, discourse analysis is crucial for assessing the effectiveness of teaching strategies and deepening students' conceptual understanding (Tabach & Nachieli, 2015). Furthermore, discourse analysis enables us to understand the social and cultural context of classroom interactions. In other words, the language and knowledge that students acquire in their daily lives play a critical role in developing formal mathematical discourse in the classroom (Caspi & Sfard, 2012; Moschkovich, 2003). Consequently, analyzing discourse in algebra teaching can uncover the miscommunication problems that may arise during teaching processes.

A range of frameworks has been developed to facilitate the analysis of discourse, each offering unique perspectives for understanding how language affects the learning process. These frameworks include SFL (Halliday & Hasan, 1989), CDA (Van Dijk, 1995), socio-mathematical norms (Yackel & Cobb, 1996), Commognitive Theory (Sfard, 2008), and MDI (Adler & Ronda, 2015). Notably, the latter three frameworks were specifically designed to analyze mathematical discourse. Socio-mathematics norms are primarily intended to investigate mathematical norms through the socio-cultural aspects. Commognitive theory, widely used in mathematics education, posits that behavior and thought are interlinked with the social environment, with communicational mediators such as word-use, visual mediators, routines, and endorsed narratives. In the present study, the commognitive

theory is adopted to analyze the teaching of algebra, exploring how teachers and students utilize these mathematical discourse characteristics to communicate mathematically in a classroom setting. In addition to this theory, MDI is employed to analyze the mathematical discourse in instruction, focusing on aspects such as “the object of learning,” “exemplification,” “explanatory talk,” and “learner participation” in mathematics instruction (Adler & Ronda, 2015, p.237). Since MDI alone cannot clearly characterize mathematical discourse, commognitive theory examines categorized mathematical characteristics in discourse to fulfill this gap. Hence, these factors collectively contribute to make a thorough analysis of mathematical discourse in instruction and provide a detailed framework for describing mathematics teaching.

CHAPTER 3

METHODOLOGY

The general purpose of the study is to examine the teaching of algebraic expressions in the context of a Turkish middle school classroom through the discursive approach. More specifically, this study aims to examine the discursive environment of a seventh-grade Turkish middle school classroom in order to gain insights into the teaching of algebraic expressions. In accordance with this purpose, this chapter presents the research design and procedures used in this study. The following sections discuss the context of the study, data collection, data analysis, trustworthiness, and the role of the researcher.

3.1. Research Design

The qualitative research design has attracted a great deal of attention in response to the need to gain an in-depth understanding of particular activities (Fraenkel & Wallen, 2012). This is mainly because it is essential to have a clear understanding of interrelated and explanatory procedures in order to fully evaluate and understand the issues at stake in such designs (Denzin & Lincoln, 2017). Furthermore, the motivation for using a qualitative research design is to enhance a multifaceted and deep understanding related to the fundamental phenomenon of the study and explore the context related to the issue (Bogdan & Biklen, 2007).

According to Creswell (2007), "*qualitative research* begins with assumptions, a worldview, the possible use of a theoretical lens, and the study of research problems inquiring into the meaning individuals or groups ascribe to a social or human problem" (p. 37). Furthermore, a detailed description of qualitative research has been provided by Denzin and Lincoln (2017, p. 43) as follows;

> "*Qualitative research* is a situated activity that locates the observer in the world. Qualitative research consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of or interpret phenomena in terms of the meanings people bring to them."

Two fundamental issues stand out regarding the aforementioned definitions of qualitative research: adopting an interpretive approach and reflecting natural settings. Researchers need to spend a considerable amount of time in the field in order to picture the natural environment under inquiry (Creswell, 2007; Fraenkel et al., 2012). In addition, they use thick and rich descriptions to bring readers to the scene (Creswell, 2009). However, the collected data is descriptive in nature and presented as words or pictures instead of quantities (Bogdan & Biklen, 2007). For this reason, it is inevitable to come across the researcher's interpretation (Creswell, 2007).

This study examines the nature of mathematical discourse during the teaching of algebraic expressions lessons in a seventh-grade classroom. In this respect, the qualitative research design is convenient for this study due to the use of a wide range of empirical materials - interviews, observations, field notes, etc. - and illustrative practices to make accurate interpretations and get a solid understanding of the issues in this design (Denzin & Lincoln, 2017). Furthermore, the goal of qualitative research is to deploy a great variety of interrelated practices to gain a clear understanding of the central phenomenon through an in-depth investigation (Denzin & Lincoln, 2017; Fraenkel et al., 2012), which fits in with the purpose of the study. In other words, the study required an in-depth investigation and a detailed data collection process to identify the characteristics of mathematical discourse - word use, visual mediators, endorsed narratives, and routines - in the context of explanatory talk and exemplification in the teaching of algebraic expressions. All in all, the qualitative research design was adopted to ascertain the characteristics of mathematical discourse. In line with this purpose, it was necessary to obtain an in-depth comprehension of the characteristics of mathematical discourse while teaching algebraic expressions in seventh grade. In other words, to achieve this purpose and

respond to research questions, it was essential to obtain an explicit understanding of the characteristics of mathematical discourse, which are word use, visual mediators, routines, and endorsed narratives in the context of explanatory talk and exemplification that reflect the process of teaching mathematics.

In the related literature, there are various classifications for qualitative research methods (Creswell, 2007; Meriam, 2009). Creswell's (2007) categorization consists of five approaches: phenomenology, ethnography, narrative research, grounded theory, and case study. On the other hand, Meriam (2009) classifies qualitative research into seven categories: phenomenology, grounded theory, basic qualitative research, narrative analysis, ethnography, case study, and critical research. This study is generated as a qualitative case study. A description of the case study is given below.

3.1.1. Case Study Research

This study employs the case study research design – a qualitative research design - (Creswell, 2007; Meriam, 2009; Yin, 2003) to get detailed answers to the research questions. It is defined by Creswell (2007) as “an issue explored through one or more cases within a bounded system” (p.73). In a case study, the researcher investigates one case or multiple cases over time by collecting extensive and in-depth data from many information sources, reporting a case definition, and describing case-based themes (Creswell, 2007). Yin (2003) differentiates the case study from other approaches and defines it in a more detailed way. Firstly, he presented the objective of the case study, which is investigating a contemporary fact with its real-life context. Then, he stated that there could be more variables than the existing data points, that the data need to be collected in a triangulation fashion, that results should benefit from theoretical perspectives developed to guide the collection and analysis of data, and that obtained results should rely on more than one source. Since the case study approach enables researchers to obtain the holistic and significant aspects of actual events, it is used when an in-depth exploration of a complex phenomenon is the goal of the study (Yin, 2003). Briefly, researchers concentrate on a bounded system and use observations, interviews, and detailed documentation for extensive

data collection. Thus, the researcher obtains a comprehensive understanding of the complicated phenomena and their significant elements. In this regard, since the aim of the present study is to examine the mathematical discourse in a seventh-grade middle school classroom to gain insights during the teaching of algebraic expressions, it was determined that the case study method is appropriate for the objectives of the study. An in-depth analysis is needed in a natural environment to reveal the mathematical discourse in the classroom.

In the case study, it is critical to state what would be described as a case. In general terms, a case is "a phenomenon of some sort occurring in a bounded context" and is the unit of analysis of the study (Miles & Huberman, 1994, p.25). Comprehensively, Denzin and Lincoln (2017) defined the case as "an instance, incident, or unit of something and can be anything—a person, an organization, an event, a decision, an action, a location like a neighborhood, or a nation-state" (p. 600). In the present study, the case is the mathematical discourse in a seventh-grade algebraic expressions instruction within the context of algebra instruction. This study is restricted to the mathematical discourse used in the seventh-grade algebraic expressions lessons in a middle school located in the countryside near the city center of Afyonkarahisar.

There are various categorizations related to the case study method based on different criteria in the related literature (Merriam, 2009; Stake, 1995; Yin, 2003). To illustrate, Merriam (2009) categorized case studies as "historical, organizational, observational, instrumental, and collective or multisite case studies" (p. 54). On the other hand, Stake (1995) identified three types of case studies which are intrinsic study, instrumental study, and collective study. Comprehensively, "intrinsic case study" focuses on a particular case, such as a student having a learning difficulty or investigation of an agency or a program, rather than focusing on other cases or a general problem (Stake, 1995, p.3). The instrumental case study aims to gain a general grasp of the issue and learn more about it (Stake, 1995). It focuses on a problem or concern and choosing a bounded case in order to represent this problem (Creswell, 2007). Lastly, in the collective case study, the researcher may need to concentrate on more than one case to learn more about the specific issue (Stake,

1995). Moreover, according to Yin (2003), there are three classifications of case studies: exploratory, explanatory, and descriptive (p.3). Firstly, the exploratory case study aims to develop proper hypotheses for further inquiries. Secondly, the explanatory case study examines the causes of the general phenomenon. Lastly, the descriptive case study aims to portray the phenomenon instead of generalizing it. As the purpose of the study is to map the teaching process of algebraic expressions through the discursive approach, the descriptive case study design was adopted to provide a representation of the phenomenon rather than to generalize it. Below are the types of case study designs developed by Yin (2003).

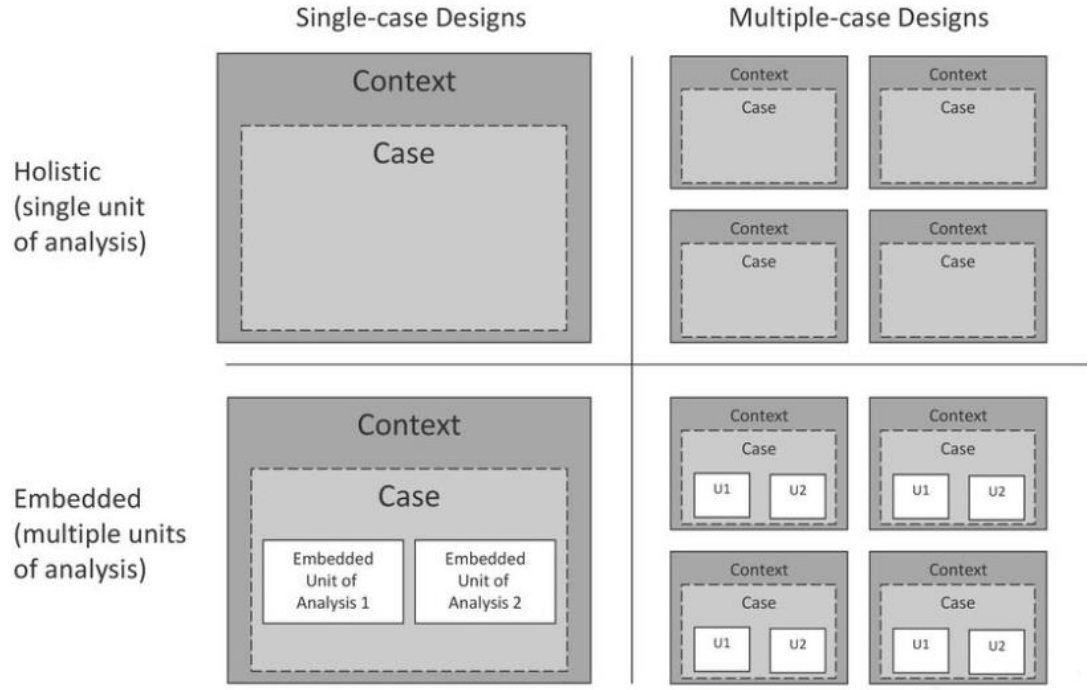


Figure 3. 1. Basic types of designs for case studies (Yin, 1995, p. 40)

Yin (2003) regulated a 2x2 matrix to demonstrate the types of case study designs given in Figure 3.1. There are four fundamental case study designs presented in the matrix that are grouped according to the quantity of cases and the quantity of units of analysis. In the vertical column, there are two types of case studies, which are single-case and multiple-case designs. Similarly, in the horizontal column, there are also two types of case studies, which are holistic and embedded case designs. In Figure 3.1, the matrix represents the single-case holistic design, single-case embedded design, multiple-case holistic design, and multiple-case embedded design.

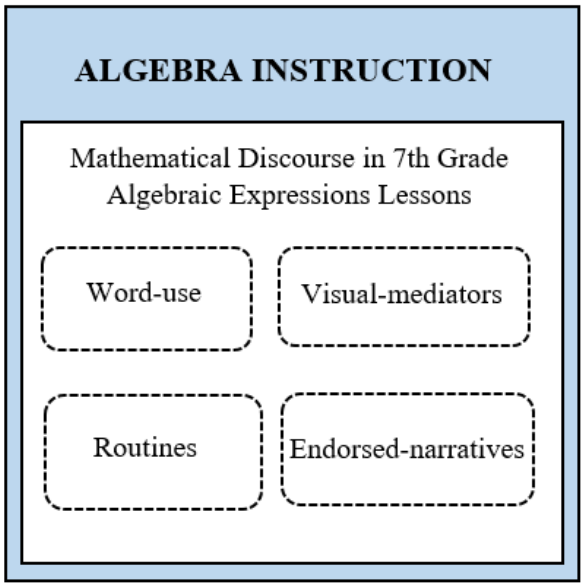


Figure 3. 2. Single-case embedded design (Adapted from Yin, 2003)

In the present study, in accordance with its purpose, the context is algebra instruction in seventh grade. The mathematical discourse used in seventh-grade algebraic expression lessons represents the case. Word-use, visual mediators, routines, and endorsed narratives are units of analysis (Figure 3.2). In this manner, the present study adopted the embedded single-case design according to Yin's (2003) classification (Figure 3.1). It was thought that examining the mathematical discourse in a single classroom would be informative enough for the teaching of algebraic expressions. In addition, working with a single case provided the researcher with the necessary time and energy to examine the mathematical discourse categories used in the algebraic expressions course. The visual of the study design is presented in Figure 3.2.

The next section provides the procedures that were used to determine the participants, followed by an elaboration of the context of the study.

3.2. Participants

To determine the participants, purposeful sampling was used, which involves selecting participants who might provide useful information and rich data for understanding the phenomenon (Cresswell, 2007). In purposeful sampling,

researchers select participants that they believe will provide the data they need (Fraenkel et al., 2012). In other words, according to Meriam (2009), purposeful sampling is "based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (p.77). In the present study, it is believed that teacher Esra (pseudonym) and her 7th-grade students will provide rich data in order to analyze the discourse used throughout the teaching of algebraic expressions. The primary participant is the teacher Esra in the present study.

The criteria for determining the participants are described in three items. First, the algebraic expressions lessons, as well as the observations, needed to be videotaped to avoid overlooking any mathematical dialogues taking place in the classroom. Second, the classroom size needed to be reasonable for the researcher to record the dialogues but still large enough to provide a rich communicational environment. Third, the teacher had to be willing to participate in the study, have a rich knowledge of mathematics, and give permission for video recording. Considering these criteria, it was decided to study with Teacher Esra, who graduated from a public university in Ankara, and observe her lessons in her seventh-grade classroom. The school was eager to study and the class size was suitable. The researcher chose teacher Esra and her seventh-grade students because she considered that Esra would make a significant contribution to the study due to her extensive teaching experience in seventh grade, her master's thesis and professional career, and her high level of education.

3.2.1. Teacher Esra and her 7th Grade Students

Esra is the pseudonym of the observed teacher in this study. She graduated from the Elementary Mathematics Education program at a state university in Ankara in 2007. She took the courses of Fundamentals of Mathematics, Discrete Mathematics, Calculus, Analytic Geometry, Elementary Geometry, Discrete Mathematics, Basic Algebraic Structures, Differential Equations, and Linear Algebra. She also took some mathematics education courses such as Instructional Planning and Evaluation, Methods of Mathematics Teaching, Probability and Statistics, Instructional

Developmental Media in Mathematics Education, Understanding in Mathematics Education, Textbook Analysis in Mathematics Education, and Computer Application in Education. She completed general education and technology courses. Additionally, she completed teacher training courses in the first and fourth years of her bachelor's degree. All of the courses the teacher took during her bachelor's degree are listed in Appendix C. She graduated with a GPA of 3.12, which is considered successful on a 4.00 scale. Moreover, she completed a master's degree in Elementary Science and Mathematics Education program at the same university in 2011. She also took some inquiry and mathematics education courses in the master's program (see Appendix D), graduating with a GPA of 3.64 (over 4.00). Her master's thesis was on the typical errors made by 7th-grade students and possible misconceptions in the concept of graphs.

Esra worked as a mathematics teacher at a private school for two years after graduating from university. Afterward, she took the Public Personnel Selection Examination (PPSE) and obtained the necessary score required to be appointed by the MoNE for positions in public middle schools in 2011. Esra has been teaching in public schools for six and a half years. She had been working for four years in the present school at the time of data collection. She was teaching 6th and 7th-grade students at the time of the study. There were 25 (13-year-old) students, 13 of whom (52%) were boys and 12 (48%) were girls, in the seventh grade. Most of the students came from low socioeconomic backgrounds. According to the teacher, they were low achievers, and their parents placed little value on academic pursuits. The researcher chose teacher Esra and her 7th grade students because she considered that Esra would make a significant contribution to the study due to her extensive teaching experience in seventh grade, her master's thesis and professional career, and her high level of education. Another criterion for choosing the school, the teacher, and her class was the willingness of the teacher and the school management to study with the researcher. After expanding on the participants, the context of the study is given below.

3.2.2. Learning Environment

The present study was conducted in a seventh-grade classroom in order to investigate the mathematical discourse during the teaching of algebraic expressions in a public

middle school in Turkey. The school is located in a central town in Afyonkarahisar and follows followed the national educational system. The distance between the school and the city center of Afyonkarahisar is 20 kilometers.

The school building had 16 classrooms, but only 12 of them were actually in use. There was a library, conference room, science lab, and gymnasium in the school, but there were not any mathematics classrooms or computer labs. There were 13 different kinds of social, scientific, athletic, and artistic clubs that students could enroll in. The school had a principal, two vice-principals, a guidance counselor, and sixteen branch teachers, only three of whom were mathematicians. The school had 291 students, 148 females and 143 males, ranging from fifth grade to eighth grade. Each grade level had 2, 3, or 4 sections during the academic year when the study was conducted. In particular, there were four sections of seventh graders. The researcher decided to conduct the study in only one section of the mathematics class to use her time and energy wisely and analyze the mathematical discourse thoroughly.

7-B, one of the classes where Teacher Esra taught, was the observed classroom with 25 students 13 boys and 12 girls. According to the teacher, the students were typically low achievers and came from low socioeconomic backgrounds. Their parents placed little value on academic pursuits. The mathematics teacher used a mathematics textbook titled 7th Grade Mathematics Textbook (7. Sınıf Matematik Ders Kitabı) prepared by Erenkuş and Eren-Savaş (2018). It was designed by a private publisher, reviewed and confirmed by the Turkish Board of Education (Talim Terbiye Kurulu Başkanlığı-TTKB) to be used in public schools for five school years. The textbook served as a guide for the teacher to instruct according to the curriculum content. In accordance with the curriculum, the algebra learning area in the textbook consists of two main sub-learning areas, which are ‘Algebraic Expressions’ and ‘Equality and Equation’. The Algebraic Expressions include *performing addition and subtraction operations with algebraic expressions, multiplying a natural number by an algebraic expression,* and *labeling a letter with a number pattern rule, and finding the desired term.* The sub-learning area of Equality and Equations includes *understanding the principle of equality conservation, recognizing first-degree equations with one unknown and setting up first-degree equations with one unknown*

according to real-life situations, solving first-degree equations with one unknown, and *solving problems that require posing first-degree equations with one unknown*. The teacher followed this sequence to develop her students' algebraic thinking. The present study is restricted to two topics: *Performing addition and subtraction operations with algebraic expressions* and *multiplying a natural number by an algebraic expression*.

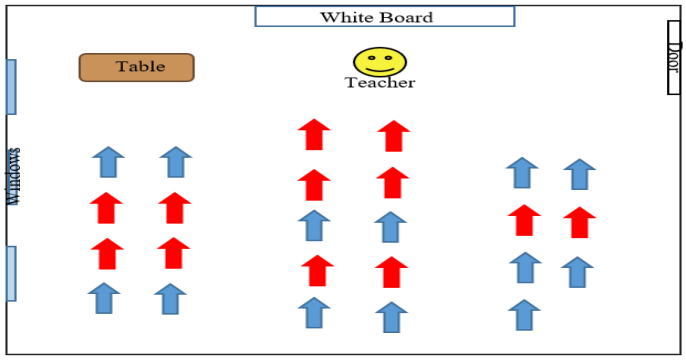


Figure 3. 3. The classroom layout

The physical conditions of the classroom were similar to those of a typical one in the country. There was a whiteboard, the teacher's table next to the board, and three rows of student desks in front of the whiteboard (See Figure 3.3). In the classroom, there were no educational technology instruments. The teacher usually stood in front of the whiteboard, taught the students face-to-face, and visualized the mathematical examples via the board (Figure 3.4). She sometimes walked around the classroom and checked on the students' notebooks and solutions. The students generally sat at their desks and went to the board only when they got the permission to solve a problem.

Figure 3. 4. Capture from an algebra lesson

Although the students were generally seated at their desks, the classroom had a fluid communication atmosphere. A pleasant communication environment in the classroom makes students willing to participate in lessons. The next section is related to the data collection tools used in this study.

3.3. Context of the Study

The case study offers an elaborate description of the situation and the setting (Yin, 2003). In order to obtain a comprehensive and clear picture of the research setting, the context was delineated in detail. Information concerning the context of the study is presented in this section under two subheadings: Algebra education in Turkish middle schools and the learning environment.

3.3.1. Algebra Education in Turkish Middle Schools

Turkish education has a centralized system; all students and teachers follow the same curricula throughout the country. The curriculum is designed by the Board of Education in the Ministry of National Education (MoNE, 2018, 2024). The curriculum consisted of three different levels, which are elementary level (1-4 grades), middle level (5-8 grades), and secondary level (9-12 grades) at the time of study. This study was carried out based on the algebraic expressions lessons conducted in a seventh-grade classroom, following the mathematics curriculum of 2018.

There were two guidebooks for the mathematics curriculum; one of them covered grades 1-8, and the other one covered grades 9-12 (MoNE, 2018). These official guidebooks include the perspective and purpose of the teaching program, the approach used in the assessment procedure, mathematics learning areas and the structure of the program, the distribution of subject matters and course hours according to grade levels, and objectives and some explanations regarding these objectives. On the other hand, in 2024, a new skill-based curriculum was developed and presented to the public. Starting from the 2024-2025 academic year, the new

curriculum will be gradually implemented for grades 1, 5, and 9. Other grade levels will continue mathematics education based on the 2018 curriculum. Since the 2018 curriculum was in force at the time the present study was conducted, the main emphasis was on the specific aspects of this curriculum. Furthermore, several fundamental changes taking place in the new curriculum are also presented, particularly in the realm of algebra.

The former middle school mathematics curriculum contained five learning areas, which were Numbers and Operations, Algebra, Geometry and Measurement, Data Analysis, and Probability (MoNE, 2018). The new curriculum contains themes such as Numbers and Quantities, Algebraic Thinking and Changes with Operations, Geometric Shapes, Geometric Quantities, Transformation, Statistical Research Process, and From Data to Possibility (MoNE, 2024). Algebra classes are introduced in 1st, 2nd, and 3rd-grade levels within the context of the subject of "Geometric Patterns" under the learning area of Geometry. Official algebra lessons begin in 6th grade in the national middle school mathematics curricula (MoNE, 2018) with the subject of Algebraic Expressions. Although algebra lessons start from the 6th grade in the 2018 curriculum, algebra-related gains can be found starting from primary school levels. In addition, the new curriculum includes arithmetic operations, the relationships between these operations, and the relationships between operations in word problems, which serve as the initial stage of algebraic thinking under the theme From Operations to Algebraic Thinking starting in 1st grade (MoNE, 2024). Thus, the new curriculum aims to develop algebraic thinking in students at a younger age.

In the learning area of Algebra subjects, the number of objectives and course hours are given in Table 3.1 according to grade levels in the middle school mathematics curriculum (MoNE, 2018). Concerning the table, there are 10 course hours with three objectives in the 6th grade, 30 course hours with seven objectives in the 7th grade, and 55 course hours with thirteen objectives in 8^{th} grade. The subjects of the algebra learning area are also presented in Table 3.1 according to grade levels in the middle school mathematics curricula. In line with this, there are two subjects in seventh grade: Algebraic Expressions and Equality and Equations.

Table 3. 1. Subjects, number of objectives, and course hours of the algebra learning area according to grade levels in the middle school mathematics curricula (MoNE, 2018)

Grade Levels	Subjects	Number of Objectives	Course Hours
6th Grade	Algebraic Expressions	3	10
7th Grade	Algebraic Expressions	3	10
	Equality and Equation	4	20
8th Grade	Algebraic Expressions and Identities	4	15
	Linear Equations	6	30
	Inequalities	3	10
Total		**23**	**95**

In the present study, the reason for choosing the 7th-grade level is that these students have already met algebraic expressions in 6th grade and are now learning more advanced topics. The learning objectives of 7th-grade algebra courses are presented in Table 3.2. The Turkish and English versions of the objectives, explanations, and some samples are given in Appendix A and Appendix B. In the curriculum, there are some codes before each objective. Figure 3.5 explains the meaning of the numbers and letters in the codes.

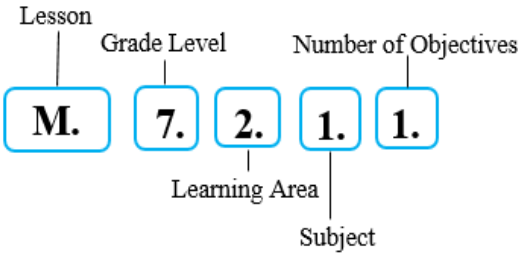


Figure 3. 5. The explanation of codes

In accordance with Figure 3.5, the code of M.7.2.1.1. refers Mathematics, 7th grade, Algebra, Algebraic Expressions, and the first objective, respectively.

Table 3. 2. The subjects and learning objectives in the 7th-grade mathematics curriculum in the learning area of Algebra (MoNE, 2018, p. 67-68) (Translated by the researcher)

M.7.2.1	**Algebraic Expressions**
M.7.2.1.1	Perform addition and subtraction operations with algebraic expressions.

Table 3.2. (continued)

M.7.2.1.2.	Multiply a natural number with an algebraic expression.
M.7.2.1.3.	Denote the rule of number patterns with a letter and find the desired term.
M.7.2.2.	**Equality and Equations**
M.7.2.2.1.	Understand the principle of equality conservation.
M.7.2.2.2.	Recognize first-degree equations with one unknown and pose first-degree equations with one unknown according to real-life situations.
M.7.2.2.3.	Solve first-degree equations with one unknown.
M.7.2.2.4.	Solve problems that require posing first-degree equations with one unknown.

In the present study, the researcher decided on the subject of algebraic expressions to investigate mathematical discourse during instruction. Particularly, the focus was on the first and second objectives of Algebraic Expressions (MoNE, 2018), which are:

- Perform addition and subtraction operations with algebraic expressions.
- Multiply a natural number with an algebraic expression.

3.4. Data Collection Tools

Qualitative research contains various data sources such as photographs, observation notes, texts, interviews, video recordings, and audio recordings (Stake, 2010). To address the construct validity, researchers need to "use multiple, not just single, sources of evidence" (Yin, 2003, p.54). It is important to realize that no single data source is inherently superior to others. In fact, the use of multiple sources is enormously complementary, and high-quality case studies involve more than one source (Yin, 2003). Therefore, researchers might use convenient data-collecting sources based on the aim of the study for triangulation. The data collection instruments used in this study are video recordings of algebra instruction, informal interviews with the teacher, and field notes. The following sections provide more information on these data collection sources.

3.4.1. Video Recordings of Algebra Instructions

In this study, video recordings were the primary data collection tools. Throughout the algebraic expressions lessons, the camera was positioned in a way to capture the

widest view in the classroom, and the lessons were videotaped. During the recordings, particular attention was paid to the blackboard, the teacher's movements, and the most accurate angle covering most of the students. Afterward, the researcher transcribed the recorded videos. Due to the significance of emphasizing each word and phrase in discourse analysis, the classroom dialogues, mathematical operations, and figures written on the board were noted in detail.

3.4.2. Informal Interviews with the Teacher

Patton (2002) states that interviewing enables the researcher to understand the viewpoint of another individual. This section includes interviews with the teacher during break times or after classes through unrecorded informal conversations. In these interviews, the teacher was asked questions to clarify the researcher's observations, such as the researcher's explanations of some analogies the teacher used in the lessons, how the teacher chose the situations that provided motivation to the students, and what the teacher considered when associating daily life with mathematical language. Some interview questions asked during the informal interviews are listed below, and additional interview questions are presented in Appendix F.

- The language students use at school and in daily life varies. Considering the algebra course, do you think that the language used in daily life affects students' success in mathematics?
- You utilize some analogies in the lessons (such as Casper and smiling face). Could you please explain the purpose of using them and how you created them?
- You follow a certain routine when solving exercises in algebra class. What is the reason for this?

3.4.3. Field Notes

Bogdan and Biklen (2007) state that fields notes are "the written account of what the researcher hears, sees, experiences and thinks in the course of collecting and

reflecting on the data in a qualitative study" (pp. 118–119). In addition to the video recordings, taking field notes documenting the mathematical expressions that do not appear in the videos and the researcher's notes about each event that occurred in the classroom are essential validate the data. As a nonparticipant observer, the researcher attended all the algebra lessons and paid attention to taking careful field notes, which were edited and compared with the video recordings immediately after the lessons.

3.5. Data Collection Procedure

Before the main study observations, a pilot study was conducted during the fall semester of 2017-2018. After determining the criteria and selecting the teacher and her classroom, the researcher got Ethics and Ministry approvals for the study in November and December 2018. Afterward, the actual observations took place throughout the fall semester of 2018-2019 in algebra lessons. The timeline for the data collection procedure is given in Table 3.3.

Table 3. 3. Timeline for data collection

Date	**Event**
December 2017-January 2018	Pilot Study
November 2018	Permission from the *Research Center for Applied Ethics*
November-December 2018	Permission from *Afyonkarahisar Provincial Directorate for National Education*
December 2018- January 2019	Actual Classroom Observations

The major data sources were video recordings of the lessons, observations, field notes, and informal interviews with the teacher. In the fall semester of 2018-2019, the teacher Esra and her seventh-grade classroom were observed during the algebraic expressions lessons. As a nonparticipant-observer in this study, the researcher observed the algebraic expressions lessons and took field notes about the procedure. At the same time, all lessons were videotaped to avoid missing any details of the teaching of algebraic expressions. The camera was positioned to get the widest view of the classroom, and the researcher sat in the back row to record all the

communication flow and took notes of everything written on the board. Figure 3.6 below depicts the layout of the classroom during the observations.

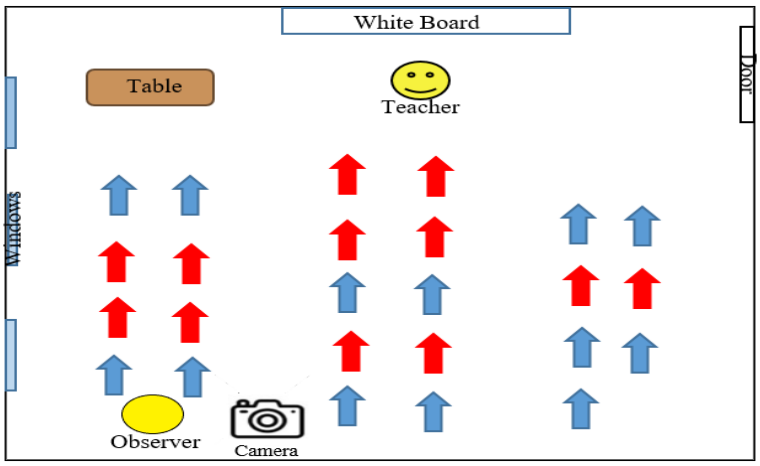


Figure 3. 6. The classroom layout during the observations

The video recordings were transcribed after the sessions. The transcriptions included the teacher's and students' statements as well as the photos of the visual mediators. The language used in the classroom and in the transcriptions was Turkish. Only the necessary parts and utterances were translated into English. The researcher watched the videos several times during the transcription process to ensure that she understood and transcribed them correctly. Field notes were utilized to confirm the accuracy of numbers, symbols, graphics, and expressions during the transcription process. Comparing the transcriptions and the field notes was critical for the accuracy of the data, as various numerical and syntactical aspects were not visible in the videos. Below is the analysis process following the collection of the data.

3.6. Data Analysis

After collecting data through the sources mentioned above, the video recordings of the algebra lessons were transcribed verbatim. The video recordings were watched repeatedly during the transcription process, attempting to capture every conversation uttered in the classroom. The print screen feature was used to photograph the symbols, calculations, and solutions for algebraic equations that were written on the board during the lessons. Then, these transcripts and photographs were compared

with the field notes, and the inconsistencies between them were addressed. In this way, the raw versions of the transcripts were compiled and prepared for analysis. The following abbreviations were used to generate the transcripts in a certain order:

T: Teacher

G= Girl

B= Boy

RC= Responses from the class

[…]= Waiting for a while

"… " = Written on the board

{…} = What the teacher articulated and made the students write on their notebooks

(…) = Notes taken by the researcher regarding classroom activities

… = Irrelevant utterance omitted from the transkript

These abbreviations helped compose the transcripts in a specific order. In addition, the photographs containing symbols, formulas, and figures taken from the videos were also placed in the sections with the relevant dialogues. After the raw transcription process, the instructional flow was divided into the following topics in the order of the objectives:

Table 3. 4. Topics of the Algebraic Expressions instructional sequence

Lessons	Topics	Objectives
Lesson 1 *Lesson2*	Concepts in algebraic expressions	• Write an algebraic expression appropriate to a verbally given situation, and a verbal situation appropriate to a given algebraic expression (6^{th} Grade). a) It is stated that the letters used in algebraic expressions represent numbers and are called variables. b) It is emphasized that expressions containing at least one variable and operation are algebraic expressions. c) Term, constant term, like term, and coefficient concepts are discussed

Table 3.4. (continued)

Lesson 2 *Lesson 3* *Lesson 4* *Lesson 5* *Lesson 6* *Lesson 7*	Operations in algebraic expressions	• Perform addition and subtraction operations with algebraic expressions. • Multiply an algebraic expression by a natural number

As seen in Table 3.4, the algebraic expressions lessons were divided into topics determined to be consistent with the curriculum objectives. The researcher identified two topics, which are concepts in algebraic expressions and operations in algebraic expressions. Table 3.4 gives a comprehensive list of all topics, instruction sequences, and corresponding curriculum objectives.

The analysis process began after dividing the lesson flow into topics in accordance with the frameworks chosen for the analysis. Firstly, the object of learning was determined as stated in MDI (Mathematics Discourse in Instruction) (Adler & Ronda, 2015) in Figure 3.7. In the present study, the object of learning is teaching and learning algebraic expressions. Due to the low participation of learners, the focus was on exemplification and explanatory talks in the present study. The first step was to identify the explanatory talk and exemplification expressions used in the algebraic expressions lessons. Then, these expressions were analyzed according to Sfard's (2008) mathematical discourse categories (word-use, visual mediators, endorsed narratives, and routines).

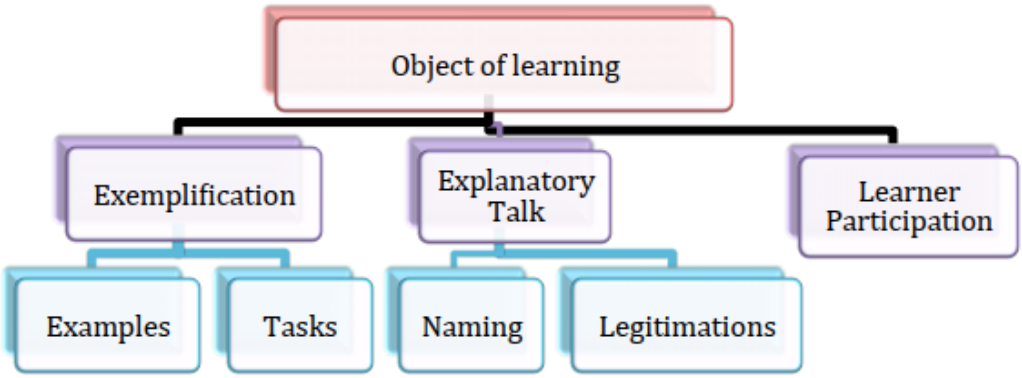


Figure 3. 7. Constitutive elements of MDI and their interrelation (Adler & Ronda, 2015, p.239)

Table 3. 5. An illustration of explanatory talk and exemplification expressions

Line	Explanatory Talk	Line	Exemplification
2.140	T: So, I asked Süleyman a question. I said, “2 apples plus 3 apples” make? He said “5 apples”. Then, I said “2 apples plus 3 oranges”. Süleyman hesitated. He said, “They are mixed”. As you see, I can add the same kind of fruit. Here, Süleyman added the apples and said 5 apples. But he couldn't add apples and oranges, could he?	2.146	T: Now, here comes "2x-3y+x-2y" (writes on the board). Which are like terms? Tell me, Büşra.
		2.147	Büşra: -3y and -2y.
		2.148	T: Can I combine these two?
		2.149	SC: We can combine them, yes.
		2.150	T: Are there any other like terms?
		2.151	
	RC: No	2.152	RC: Yes, there are.
2.141	T: So, we can make a generalization. I can combine like terms, but I cannot combine unlike terms.	2.153	T: Tell me.
		2.154	B: x and 2x.
2.142			T: x and 2x. I can combine them as well, right?
		2.155	RC: Yes

In Table 3.5, there is an example for each explanatory talk and exemplification expression. The line column describes the lesson and the dialogue from which the excerpt is taken. For instance, line 2.140 indicates that the dialogue is extracted from the second lesson and placed in the 140th sequence in the transcripts. In the explanatory talk, the teacher explains the rule of combining like terms. In the exemplification, the teacher and the students are solving a problem about combining like terms. All transcripts of the algebraic expressions lessons were coded using the Maxqda program, primarily in two categories as explanatory talk and exemplification, as indicated in Table 3.5 above. Thereafter, these two main categories were analyzed according to mathematical discourse categories, as specified in Sfard's commognitive framework.

In the framework, she addressed four categories of mathematical discourse: word use, visual mediators, endorsed narratives, and routines. There are four main categories and four sub-categories under word-use. These categories are given in Figure 3.8.

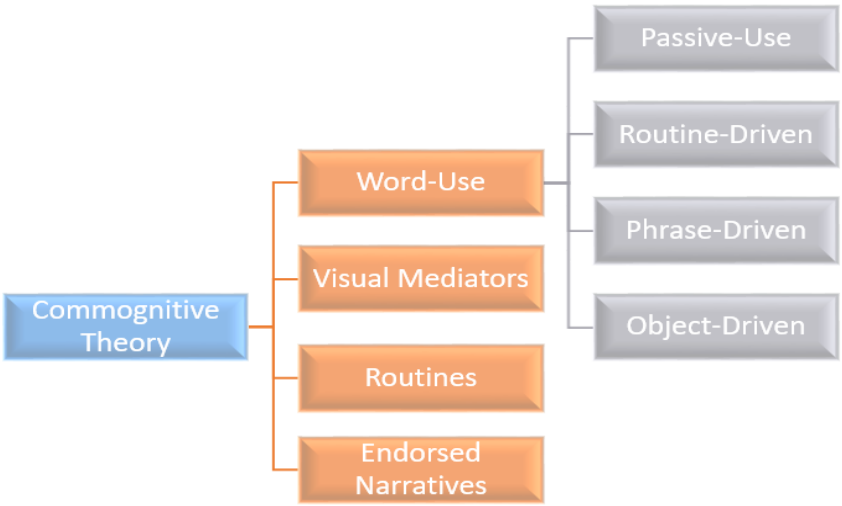


Figure 3. 8. Sfard's Commognitive Theory

Before analyzing each of them, the transcriptions and field notes were reviewed and analyzed concurrently, border notes were taken, and related videos were watched repeatedly when the researcher needed clarification. Thus, the researcher became acquainted with the data to conduct an accurate analysis. Each mathematical category is discussed in detail below.

3.6.1. Data Analysis of the Word-Use

Word use in algebraic expressions lessons involves the utilization of specific words and terms that are consistently employed throughout the instruction. This section elaborates on how to analyze word use in algebraic expressions through the commognitive framework. Firstly, the specific words which were clarified after the pilot study and have a crucial role in the teaching of algebraic expressions (given in Table 3.6) were analyzed using Sfard's commognitive framework. Before the word-use analysis, each word was identified in the transcripts and marked with different colors. The use of words is analyzed qualitatively based on their context. Therefore, the words are considered not only within the sentence but also within the context in which they are used. Table 3.6 below demonstrates the assigned colors for each word in the transcription. Later, a four-column table was constructed, including words, lines, utterances, and word-use categories. Appendix G contains an example of a table from the first lesson's analysis of word use.

Table 3. 6. Colors assigned for each word

Words	Color
Term	
Unknown	
Coefficient	
Sum of Coefficients	
Number of Terms	
Constant Term	
Like Term	
Algebraic Expression	

It is necessary to specify certain circumstances that were considered when the word-use analysis table was created. For instance, it was included in the analysis that some of the words were not uttered clearly although they were mentioned in the dialogues. The teacher sometimes discussed a word even if she did not pronounce it. To illustrate, in the excerpt below, the teacher and the students referred to *like terms* with x^2y without pronouncing like terms. This kind of use is included in the analysis.

> *2.64 Teacher:* …Yes, what else? x^2y?
> *2.65 RC:* $7x^2y$
> *2.66 Teacher:* Can it be like this? I am just asking. Look at here $\frac{2x^2y}{3}$.
> *2.67 RC:* Yes, it can.
> *2.68 Teacher:* Yes, because where is 2? Where are both 2's?
> *2.69 RC:* Above
> *2.70 Teacher:* Above. Wright…

As another example, in the following excerpt, the teacher explains how to find an *unknown*, but does not pronounce the word.

> *1.162 Teacher*: Let me show you. (Circles the unknowns) (Circles x's in 2x and -4x) They both have x, they are like terms. (She circles y and xy in -3y and 2xy, respectively) one has y and the other one has xy, not like terms. Do you understand?

She explains to students how to find the unknowns in algebraic expressions in the given dialogue. Even if she does not pronounce the word 'unknown,' she performs

the action that finds the unknown. These kinds of statements are also included in the analysis.

Moreover, words that are repeatedly pronounced in the same sentence or dialogue are counted as a single value in the analysis table. In line 2.197 below, like term was pronounced three times, but it was counted only once for the analysis because they were in the same dialogue and context.

> *2.197 Teacher:* Did you understand? Then, they were both *like terms*? (indicating 2x+3x) I was able to sum up. They were not *like terms* (indicating 2x-3y). So, I could not do anything. There were *like terms* in the expression, and I brought them together. I was able to add them. Is there any question?
> *2.198 RC:* No.

On the other hand, the words that the teacher used in a heading were excluded from the analysis by the researcher. For example, in the excerpt below, the teacher introduces the headings of the lesson to the students. Utterances like this are excluded from the analysis as *like term* is pronounced as one of the headlines.

> *1.1 Teacher*: There are four headings in algebraic expressions. 1) Like terms, 2) Addition and subtracting in algebraic expressions, 3) Multiplying in algebraic expressions, and 4) Patterns and Relations. So, I have 4 headings, and the first one is Like Terms…

Another example of the words used in the title is given in line 1.44. The teacher reminds the students which terms they have learned before. The terms mentioned in this sentence are not included in the analysis because they are pronounced in the title.

> *1.44 Teacher:* We have learned before (Writing on the board) a) Number of terms b) Terms c) Coefficient d) Constant term e) Sum of coefficients. Do you remember?

After marking the terms to be examined in the transcripts and determining which phrases to include in the evaluation, word use was analyzed using four hierarchical stages; *passive-use, routine-driven, phrase-driven,* and *object-driven*. In the first stage, the students were unable to use the words in sentences and instead used

pronouns such as this and that. In other words, if the word is pronounced wrongly or used incorrectly without knowing its meaning, it is considered *passive-use*. This type of word use often occurs in the early stages of learning. For instance, students who are not acquainted with the meaning of like terms may try to find them when they are asked. The present study does not include any passive-use of words. In the second stage, called *routine-driven*, students are able to use the word in specific routines. When pronouncing a word, if it contains an action, it becomes routine-driven. For example, in the following excerpt, the teacher performs an operation for finding the terms by circling them (Figure 3.9).

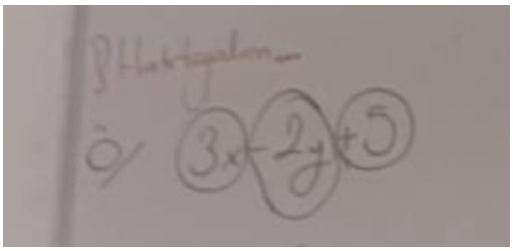


Figure 3. 9. Finding the terms

> *1.125 Teacher*: …(Shows 3x-2y+5 on the board) Does anyone have difficulty circling? Does anyone remember? Huh? I am going until the sign, and I am circling with the sign, is it true? This is how our friend did it (She is talking about the student in front of the board). We grouped the negative sign and 2y together because it belongs to 2y. Since the positive sign belongs to 5, we grouped them together. We are always going until the sign and get the circle. Here you go (Repeating how to find the terms here).

Above, the teacher performs an operation for finding the 'terms' in a given algebraic expression, even if she is not pronouncing it. The circling is an action; thus, the term is used as routine-driven.

In the *phrase-driven* stage, students are able to use words with phrases. If a word is in a context that defines and describes its components, it is a phrase-driven use. Below, a dialogue given as an example of phrase-driven use.

> *1.9 Teacher:* We are finding a *rule*. While expressing the rule this year, we will use letters such as a, b, c… with unknown expressions. We can express the rules with them. Thus, we will be using algebraic expressions.

In the excerpt above, the teacher defines the *rule*. Although she used the continuous tense as if it is an action, when we look at the whole context, the excerpt indicates a definition of the rule. The researcher considered such usages of the words as phrase-driven.

The last phase of the word-use is *object-driven,* which involves the use of stable means independent from the context. If the word is used as a mathematical object independent from the context, it is an object-driven use. Students use words in their objectified forms independent from the context in the object-driven stage. The purpose of the word-use is to make students proceed from passive use to object-driven use. In other words, students are able to use words as mathematical objects, learn their meanings, recognize the actions they require, and use them in different contexts. All words are coded according to the four level categories of commognitive theory. Then, all the words coded in the word use table are also coded in the Maxqda program. Therefore, it was determined in which phase the words were employed as explanatory talks and exemplifications in the previously examined lessons.

3.6.2. Data Analysis of the Visual Mediators

As mentioned before, visual mediators are visible objects that serve as complementary elements of the discursive process (Sfard, 2008). In the study, visual mediators include expressions, symbols, and figures written on the board. The researcher re-examined the transcripts to identify the visual mediators and noted them along with their context.

3.6.3. Data Analysis of the Routines

Routines are repetitive characters in the discourse (Sfard, 2008). Recurring patterns and procedures were investigated during the teaching to determine the routines. The researcher examined the questions What are the *prompts* to initiate the routine, *how* and *when* do the routines occur, and what are applicability and closure conditions. The procedure for analyzing a routine is demonstrated in the table below.

Table 3. 7. Analyzing the process for 'finding the like terms' routine

		Description	Discourse *What is said*	What is Done	Analysis
Routine Name		Naming the repetitive actions that correspond to a topic in the curriculum	*Teacher*: Guys, what is the first title? What is written above in the first title? *Class*: Like terms	(Showing the board)	Finding the like terms
Prompt		Situations that enhance the likelihood of starting the routine	*Teacher*: Are there any like terms?	(showing the task 2x-3y+x-2y)	Asking students to find the like terms initiates the routine
How		Defining how the routine is constituted	*Beril:* 3y and 2y *Teacher*: Are there any other like terms? *RC:* Yes Furkan: x and 2x	(highlighting y's in the task) (highlighting x's in the task)	Students mark the like terms with the teacher's instructions and find the like terms
When	*Applicability*	Possible circumstances of implementing the routines	*Teacher*: Find the like terms and simplify them		Find the like terms in a given algebraic expression before addition or subtraction
	Closure	The signal for successful completion of the routine	*Beril:* … *Furkan:* …	(writes 3y and 2y together) (writes x and 2x together)	Write the like terms together

As seen in Table 3.7, the analysis process for the 'finding the like terms' routine was clarified. After identifying a pattern of repeated actions or procedures, the researcher referred to it as a routine and labeled it according to the subtopics of algebraic

expressions (MoNE, 2018). Then, the conditions that increased the likelihood of initiating the routine, which are called the prompts. As shown in the table, the teacher asks the students to identify like terms, and then ‘finding the like terms’ routine begins. Defining the routine involves systematic movements of the action. In the table, students determine the like terms one by one. The teacher and the students find the like terms in order to make calculations with algebraic expressions. Finally, the closure is the signal for the successful completion of the routine. The students write the like terms together to indicate that the routine has successfully been completed. The researcher used the same procedure for every routine identified during the instruction.

3.6.4. Data Analysis of the Endorsed Narratives

Endorsed narratives are known as truths in the discourse (Sfard, 2008). To determine the endorsed narratives, the researcher analyzed the transcripts in detail. The definitions, properties, or rules used in the algebraic expressions lessons were coded as endorsed narratives. The identified narratives were analyzed by examining whether they were used in the participants' word use, routines, and visual mediators and whether these components also validated the narratives. For instance, in this study, ‘like term’ was defined, and it was analyzed whether the definition was used in the participants' word-use, visual mediators, or routines.

3.7. Pilot Study

The pilot study is a crucial stage in qualitative research because it is a smaller investigation designed to assess the feasibility of processes and collect data before a larger investigation is conducted (Van Teijlingen & Hundley, 2002). The purpose of the pilot study is to identify any potential problems so that the researcher can address them before the actual study (Fraenkel et al., 2012). Since the pilot study predicts and helps minimize potential problems encountered in the main study during the data collection and analysis process, it is not a waste of time or energy. Hence, the researcher needed to conduct a pilot study to determine the best way of clarifying the implicit nature of mathematical discourse, decide how to use the theoretical

framework to map the algebraic expressions instruction, and determine the most suitable classroom and teacher to collect ample data.

Therefore, the pilot study was carried out in two different middle schools with two different mathematics teachers and their seventh graders students during algebra classes in the fall semester of the 2017–2018 academic year. These schools are located in the city center of Afyonkarahisar. The teachers and their classes were selected for the pilot study due to their willingness to collaborate with the researcher and allowing her to observe their lessons. More specifically, the participants were teacher Ece and her 37 seventh-grade students, - 21 girls and 16 boys - in School A, and teacher Zeynep and her 36 seventh-grade students 15 girls and 21 boys -in School B (Table 3.8). To prevent revealing teachers' names, pseudonyms were assigned to each of them. Before spending nearly two weeks observing their algebra lessons, the researcher first introduced herself to the students to help them become accustomed to her presence. Consequently, the algebra lessons progressed in a natural manner.

Table 3. 8. Pilot study participants

School	Teacher	7^{th} Grade Students
A	Ece	21 Girls 16 Boys
B	Zeynep	15 Girls 20 Boys

During the observations, the teaching strategies, methods and lecturing styles of the teachers were investigated to determine their appropriateness for analyzing mathematical discourse in a class similar to the one observed in the pilot study. Although it is not the primary focus of the examination of mathematical discourse in the classroom, it has been discovered that the teacher's strategies and methods are effective in creating a fluid communication environment in the classroom. Therefore, the initial objective was to evaluate whether was a communication environment that could reveal the implicit discourse in the lessons.

Teacher Zeynep utilized pre-made documents in the algebra lessons. These files include video recordings of lectures on algebra topics as well as examples of

problem solutions. She used an interactive whiteboard to play these videos, from which the students tried to learn algebra. During the algebra sessions, the dialogues in the classroom were generally as follows:

Teacher Zeynep: …(Starts the video)
Students: …(Watching and trying to learn algebra topics from the video)
Teacher Zeynep: Did you understand?
Students: Yes, we did.

According to the observations of Teacher Zeynep's lessons, the students stated that they comprehended the algebra topics after watching the videos. However, in the problem-solving process, it was noticed that they did not comprehend the topics because they were unable to apply the information provided in the video to solve algebra problems. The teacher was not lecturing her students; instead, video recordings were fulfilling her role. Therefore, they communicated at a minimum level due to the lack of live interaction with an interlocutor in the lesson. The lack of an effective communicational environment brought along the failure.

On the other hand, Teacher Ece established a more communicative classroom environment. After introducing herself, the researcher started classroom observations in Teacher Ece's classroom. In the observed lesson she explained the algebra topics herself in addition to using the interactive whiteboard. In the meantime, she encouraged her students to be active listeners by asking some questions, providing examples from daily life, and repeating specific terminology and definitions. The students were eager to ask questions regarding the topic. As a result, a healthy communicational environment was created in the classroom. However, the class size was too large for each student to voice their ideas openly. Some students had difficulty asking questions about the concepts they did not comprehend.

Via the pilot observations, the researcher was able to become acquainted with the students, the teachers, and the classroom atmosphere in general. Throughout the algebra lessons, she determined how to take effective field notes, where to place the

camera, and where to sit in the classroom for optimal observation. Concurrently, she tested the usability of the observation form (Appendix E). She took notes of specific terms pronounced during the algebraic expressions lessons. Afterward, by comparing these terms with the textbook, the actual terms (words) (Table 3.9) to be observed in the main study were determined. Some words that were not used in either of the classes were removed from the observation form. As in the observation tool, the visual expressions that the teacher drew on the board were also noted. After this, it was decided that the visual mediators should be noted alongside their context rather than separately since the figures alone would be meaningless. Like visual mediators, the routines should also be noted in their context. All dialogs containing the routines should be included in the field notes to be analyzed within their context.

Table 3. 9. The words included in the observation tool, but decided to be removed from the main study following the pilot study

Words	Observation Form	Main Study
Term	✓	✓
Variable	✓	x
Unknown	✓	✓
Coefficient	✓	✓
Sum of Coefficients	✓	✓
Number of Terms	✓	✓
Constant Term	✓	✓
Like Term	✓	✓
Algebraic Expression	✓	✓
Addition in Algebraic Expressions	✓	x
Subtraction in Algebraic Expressions	✓	x
Multiplication in Algebraic Expressions	✓	x
Modeling	✓	x
Patterns and Relations	✓	x
Sign	✓	x
Symbol	✓	x
Notation	✓	x
Representative Number	✓	x
General Number	✓	x
Step	✓	x
Rule	✓	x
Letter Expression	✓	x

In summary, the analysis of the pilot study demonstrated that observing a classroom with a smaller size and choosing a teacher who establishes a fluid communication environment would provide richer data in the main study. These circumstances were considered in the selection of the participants for the main study.

3.8. Trustworthiness of the Study

Qualitative research studies have a subjective perspective in nature. As a result, researchers ensure the trustworthiness of the study. According to Lincoln & Guba, research findings should be "worth paying attention to" (Lincoln & Guba, 1985, p. 290). Moreover, the trustworthiness of a research study is relevant to the procedures that are implemented to ensure validity and reliability (Bakker & van Eerde, 2015). In addition, qualitative research is "based on different assumptions about reality and a different worldview and should consider validity and reliability from a perspective congruent with the philosophical assumptions underlying the paradigm" (Meriam, 2009, p. 211). In this respect, Lincoln and Guba (1985) suggested terms like "credibility, transferability, dependability, and conformability equivalent for internal validity, external validity, reliability, and objectivity" (p. 300). These issues are addressed respectively below.

Firstly, credibility is relevant to internal validity and concerns with the "quality of data and the soundness of the reasoning that has led to the conclusions" (Bakker & van Eerde, 2015, p. 444). To ensure the credibility of a qualitative study, there are some issues that the researcher should pay attention to, which are persistent observation in the field and prolonged engagement, triangulation, debriefing or peer review, member checking, and thick description (Cresswell, 2007; Meriam, 2009). In the present study, trustworthiness was ensured through prolonged engagement in the field, triangulation, and thick description. Triangulation refers to the researcher's utilization of many sources, methodologies, investigators, and theories for providing corroborative evidence (Cresswell, 2007). Data was collected in the classroom through observations, video recordings, and informal interviews to compare and cross-check the results of the study. The researcher began the observations before the

actual study in order to build trust with the teacher and the students and become familiar with the research environment.

Secondly, transferability is related to external validity (Lincoln & Guba, 1985) and concerns with "how do the findings inform other contexts" (Bakker & van Eerde, 2015). In order to understand the study's background for future analysis, it is important to produce a detailed and comprehensive description (Lincoln & Guba, 1985). A thick description of the data collection and analysis process, data collection instruments, the role of the researcher, and the characteristics of the participants were made in an attempt to ensure transferability to other settings.

Thirdly, dependability is related to the degree of consistency within the data and the interpretation of the data, and the replicability of the results (Lincoln & Guba, 1985; Merriam, 1998). In order to ensure dependability, it was suggested that the procedures employed in the study be reported in detail, thereby allowing future researchers to replicate the study, even if the results may not be identical (Shenton, 2009). In case studies, the aim of dependability is to ensure that repeating the same procedures in an identical context will result in consistent conclusions and findings. (Yin, 2003). In the present study, the researcher elaborately documented the design of the study, the data collection procedure, and the data analysis process. Furthermore, dependability might be demonstrated by means of a peer review to obtain additional insights into the study. In this study, peer review was done by a second coder to analyze the data and discuss the coding procedure with the researcher. The second coder is a mathematics teacher educator and works as an associate professor at a public university. The intercoder reliability is calculated as %82. After that, consensus was achieved by discussing the inconsistent elements in the analyses of the two coders.

3.9. The Researcher’s Role

I adopted a nonparticipant-observer role during the whole data collection process. The nonparticipant observer role lets me be in a passive role. I am not involved in the activities (Fraenkell & Wallen, 2009) and am placed in a position to observe the

whole process in a way that does not interrupt the instruction. As a nonparticipant-observer, I let students know that I was a researcher but would not have an active role during the classroom activities, but only take field notes. (Creswell, 2007).

I met the teacher Esra, whose lessons I observed, approximately one month prior to the start of the algebra lessons. During this process, she provided personal details, such as her graduation and the duration of her teaching experience. I provided a brief description of the purpose of this investigation and expressed my desire to simply observe the natural progression of her algebra lessons. To avoid interrupting the natural flow of the lessons, I started classroom visits two weeks before the beginning of the algebra courses. This allowed the students to become acquainted with my presence as a researcher. Upon entering the classroom, I introduced myself and provided a clear explanation of my purpose for observing the lessons. Consequently, both the teacher and the pupils became accustomed to my presence in the classroom prior to the beginning of the algebra lessons. Throughout the lessons, I observed the lessons from the back corner of the classroom without interrupting the lesson's progression and took field notes. After the lessons, I had discussions with the teacher regarding the lessons. Thus, I gained insight into the teaching of algebra. The nonparticipant-observer role enabled me to investigate a natural classroom environment.

CHAPTER 4

FINDINGS

The general purpose of the study is to examine seventh-grade algebraic expression lessons in the context of a Turkish middle school through the discursive perspective. Specifically, the study is concerned with Sfard's (2008) mathematical discourse categories and analyzes how they occur in the mathematical discourse in instruction (MDI) (Adler & Ronda, 2015), which are explanatory talks and exemplifications, during the seventh-grade algebraic expressions lessons. In addition, learner participation is also considered, but it is concluded that students' contribution is minimal.

In accordance with this aim, this chapter presents the findings of the study in two main topics, each of which is determined according to the curriculum objectives of algebraic expressions. These topics are concepts in algebraic expressions and operations in algebraic expressions, which are analyzed during the explanatory talks and exemplification parts of the lessons. Later, these topics were examined under subheadings based on the mathematical discourse characteristics in Commognitive Theory.

Each topic is determined as related to algebraic expression objectives, as seen in Table 4.1. In this regard, the findings of two algebraic expressions topics are presented in sub-sections based on the mathematical discourse categories (word-use, visual mediators, routines, endorsed narratives) determined by Sfard (2008), as stated in the methodology chapter. The analysis of algebraic expressions instruction concluded that the teacher's discourse is dominant, and students' discourses are generally formed by the repetition of her discourse. For this reason, the focus is generally on the teacher's mathematical discourse, and the teacher's discourse was analyzed in detail in accordance with both frameworks.

Table 4. 1. Topics of algebraic expression instruction

Lessons	Topics	Objectives
Lesson 1 *Lesson2*	Concepts in algebraic expressions	• Write an algebraic expression appropriate to a verbally given situation and a verbal situation appropriate to a given algebraic expression (6^{th} Grade) a) It is stated that the letters used in algebraic expressions represent numbers and are called variables. b) It is emphasized that expressions containing at least one variable and operation are algebraic expressions. c) Term, constant term, like term, and coefficient concepts are discussed
Lesson 2 *Lesson 3* *Lesson 4* *Lesson 5* *Lesson 6* *Lesson 7*	Operations in algebraic expressions	• Perform addition and subtraction operations with algebraic expressions. • Multiply an algebraic expression by a natural number

4.1. Mathematical Discourse Characteristics during the Explanatory Talks and Exemplifications in the Topic of Concepts in Algebraic Expressions

This section provides a detailed analysis of word-use, visual mediators, routines, and endorsed narratives employed in explanatory talks and exemplifications within the topic of concepts in algebraic expressions. At the beginning of the algebra instruction, the teacher started with a short review of the sixth-grade topics to make students remember. These reminders are related to a 6th-grade objective and sub-objectives given below:

> M.6.2.1.1. Write an algebraic expression appropriate to a verbally given situation and a verbal situation appropriate to a given algebraic expression
> a) it is stated that the letters used in algebraic expressions represent numbers and are called variables,
> b) it is emphasized that expressions containing at least one variable and operation are algebraic expressions,
> c) term, constant term, like term, and coefficient concepts are discussed (MoNE, 2018, p.61).

Before recalling the prior topics, she drew a concept map that included like terms, addition and subtraction with algebraic expressions, multiplying a natural number

with an algebraic expression, and patterns and relations (Figure 4.1). The focus is on the first three items in this study.

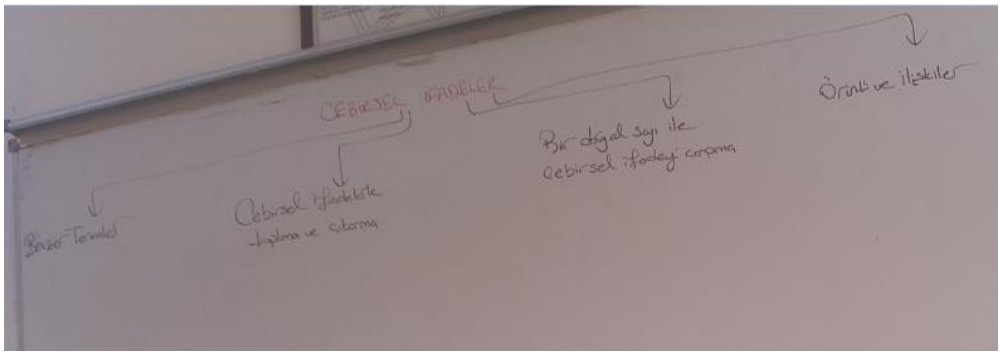


Figure 4. 1. The concept map about the topics of algebraic expressions

After that, the teacher started with the subject of like terms and continued with other subjects respectively. At the same time, she defined keywords and their applications in algebraic expressions. The study represents these concepts and their usage in the next section.

4.1.1. Word-use in the topic of concepts in algebraic expressions

Word-use in algebraic expressions lessons involves the utilization of specific words and terms that are consistently employed throughout the instruction. This section provides a comprehensive analysis of the word-use in the topic of concepts in algebraic expressions through commognitive framework. In addition, the use of words is analyzed qualitatively regarding their context. Therefore, in some situations, even if the word is not pronounced, it was included in the analysis because it is mentioned in the context. Since the specific words in algebraic expressions are not usually verbs, generally nouns, the researcher had to consider the context to determine whether the use of words contains an action. For this reason, words are analyzed in a dialogue rather than in a sentence.

Results of the analysis show that the teacher generally uses words in an object-driven manner. In other words, she uses the mathematically objectified forms of the words. The teacher also uses the routine-driven and phrase-driven usage of the words

throughout the instructions. In phrase-driven usage, she defines some specific terms, and in routine-driven usage, she explains how to perform the operations step by step. Throughout the instructional sequence, the researcher did not determine passive-use forms because they are used in the early stages of instruction. These students were introduced to the subject of algebraic expressions in the sixth grade.

Below, the specific words used in concepts in algebraic expressions and their analysis based on the commognitive framework are given in detail. These words are *term, unknown, coefficient, the sum of coefficients, number of terms, constant, like term,* and *algebraic expressions*. As shown in Table 4.2, the findings of the word use on the topic of concepts in algebraic expressions indicate that the use of routine-driven words (%12) is the least, while the use of object-driven words (%52) is the most used.

Table 4. 2. Word-use in the topic of concepts in algebraic expressions

	Routine-driven	Phrase-driven	Object-driven	Total
Term	2	1	8	11 (%14)
Unknown		1	10	11 (%14)
Coefficient		2	3	5 (%6)
The sum of coefficients	3	2	2	7 (%9)
Number of terms	1			1 (%1)
Constant		1		1 (%1)
Like term	6	16	17	39 (%48)
Algebraic expressions		4	2	6 (%7)
Total	12 (%12)	27 (%36)	42 (%52)	

In addition, each word-use is also considered whether it is used in *explanatory talks* or *exemplifications* (Table 4.3). In the exemplification part of the instruction, words used in routine-driven twice, words used in phrase-driven 10 times, and words used in object-driven eight times. On the other hand, in the explanatory talk part, there are 10 times routine-driven use, 17 times phrase-driven use, and 34 times object-driven use.

Table 4. 3. Word-use in explanatory talks and exemplifications

	Explanatory Talk	Exemplification
Routine-Driven	10	2
Phrase-Driven	17	10
Object-Driven	34	8
Total	61 (%75)	20 (%25)

Table 4.3 reveals that word-use categories in concepts in algebraic expressions vary with a higher frequency determined in explanatory talks (%75) and a lower frequency determined in exemplifications (%25). This finding indicates that the teacher used a greater number of explanatory expressions. In other words, this indicates that she utilized definitions, explanations, and properties of mathematical structures more. Below is given a comprehensive analysis of each specific word through the commognitive framework.

4.1.1.1. Term

The *term* is one of the words uttered in the teaching of concepts in algebraic expressions lessons. The instruction started with the teacher's briefing, in which she emphasized the significance of comprehending the *terms* in algebraic expressions. Despite its inclusion in the 6th-grade curriculum, she elaborated on its meaning and usage once again. Then, she reminded students how to find the terms as below.

> *1.131 Teacher:* ...Does anyone have difficulty in circling? Is there anyone who does not remember? Huh? I am going up to the sign, and from the sign, I am circling it, right? That is what your friend did (referring to the student on the board). The minus sign since it belonged to 2y, we grouped them together (-2y). The plus sign, since it belongs to 5, we grouped them together. We always go up to the sign and circle them together. Here you go.

As seen above, although *the term* is not explicitly mentioned in the dialogue, it was included in the analysis because the teacher describes how terms are identified. She makes an explanatory talk regarding the concept of *the term* and explains how to determine terms in a given algebraic expression. The emphasis is on explanations that include both a process and an action. Therefore, it is an example of routine-driven use.

1.57 Teacher: Hah -2y I accept as a *term* what is in the circle.
1.106 Teacher: Now you told me that there are *terms*. What were they? 3x, -2y, +5. Are 3x and -2y like term?

Above, in dialogue 1.57, the teacher explains what they can acknowledge as a term. This type of usage exemplifies phrase-driven use because she says that it is a term inside the circle. On the other hand, line 1.106 exemplifies an object-driven use. Here the teacher uses the term as a mathematical object. She utters the terms respectively in a given algebraic expression as they are known mathematical objects. These two excerpts are both uttered in explanatory talk.

Table 4. 4. Categorization of the use of *term* in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.47	*Teacher*: 3, I'm writing it down right away. So, who tells the **terms**, finger? Yes Sudenaz.		Object-Driven	Explanatory talk
1.48	*Sudenaz:* 3x			
1.49	*Teacher:* Let's put a comma to separate them. What is the other one? Tell			
1.50	me, yes.			
1.51	*Girl:* 2x			
1.52	*RC:* -2..			
	Teacher: Wait a minute, wait a minute, don't interfere, don't interfere, whomever I give			
1.53	permission to talk, she			
1.54	should answer			
1.55	*Girl:* -2x			
	Teacher: Minus..			
	Girl: -2y			
1.57	*Teacher*: Hah, -2y I accept as a **term** what is in the circle…	She describes the condition under which they accept the term.	Phrase-Driven	Explanatory talk
1.57	*Teacher:* ... Are there any other **terms**?		Object-Driven	Explanatory talk
1.58	*RC:* Yes, there is.			
1.59	*Teacher:* Sultan			
1.60	*Sultan:* +5			

Table 4.4. (continued)

1.63	*Teacher:* I already know what it is, This is a plus. We determined the **terms**, don't forget the signs! Minus belongs to 2y, 3x is already clear.		Object-Driven	Explanatory talk
1.106	*Teacher:* Now you told me that there are **terms**. What were they? 3x, -2y, +5. Are 3x and -2y like term?		Object-Driven	Explanatory talk
1.131	*Teacher:* ...Does anyone have difficulty in circling? Is there anyone who does not remember? Huh? I am going up to the sign, and from the sign, I am circling it, right? … The minus sign, since it belonged to 2y, we grouped them together (-2y). The plus sign, since it belongs to 5, we grouped them together….	Even if she does not pronounce the term, she explains and circles the term -2y.	Routine-Driven	Explanatory talk
1.141	*Teacher*: If there are 2 elements, it means an expression with two **terms**, we can talk about subtraction. But, expression with three elements, we can not talk about subtraction. Is that right?		Object-Driven	Explanatory Talk
1.164	*Teacher:* Ok, let's look, both of them are exactly the same, that is, the **terms** are alike. I mean the unknowns should be the same. When there is a different unknown, there are not like terms....		Object-Driven	Explanatory Talk

Table 4.4 represents the analysis of the *term* through Sfard's commognitive theory and explanatory talks and exemplifications of the lessons. All word usage occurred in the lessons' explanatory talk parts. In other words, the teacher used the term while making explanations about concepts in algebraic expressions. Given the table, it is clear that the term is pronounced mainly by the teacher. However, considering the

dialogues in the context, students also use this word implicitly, although they do not pronounce it explicitly. For instance, they determine the terms in a given algebraic expression between lines 1.47 and 1.55. The teacher asks what the terms are, and the students determine the terms in the expression even if they do not pronounce it. The term is mostly used as object-driven. However, phrase-driven and routine-driven usage are also observed.

4.1.1.2. Unknown

Unknown is another frequently pronounced word in teaching and learning concepts in algebraic expressions. To begin with, in excerpt 1.8, the teacher provides a clear explanation of the word unknown in the given excerpt. She states that they are going to utilize letters like a, b, c, and more as unknowns. Due to the teacher's utilization of interpretive language in this quote, it is accepted as explanatory talk. Additionally, it is also acknowledged as phrase-driven use since it expresses the way the unknown is used in the lesson.

> *1.8 Teacher:* While expressing the rules this year, we will use *unknowns* such as a, b, c, etc. we will express with them.

In another dialogue below, where the teacher employs explanatory expressions to ascertain whether the unknowns are identical in order to find like terms in a given algebraic expression. Both the teacher and students grasp the meaning of the unknown and use it as a mathematical entity. For instance, in line 1.109, the boy is aware of what is unknown, and he realizes that both expressions have an unknown. For this reason, it is accepted that the unknown is used as object-driven.

> *1.106 Teacher:* ...Now you told me that there are terms. Which terms were they? 3x, -2y, +5. Are 3x and -2y like terms?
> *1.107 RC:* They are like terms.
> *1.108 Teacher:* In what way?
> *1.109 Boy:* They both have an *unknown.*
> *1.110 Teacher:* Else
> *1.111 RC:* […]
> *1.112 Techer:* Or let's put it this way. Are the *unknowns* alike?
> *1.113 RC:* Yess. Teacher, there [...]

1.114 Teacher: Are the *unknowns* alike?
1.115 RC: Nooooo

Above, dialogues exemplifying both the phrase-driven and object-driven use of the *unknown* are scrutinized. It is noteworthy that when the teacher first utters unknown, she uses explanatory and descriptive expressions as phrase-driven use and in subsequent use, it is employed as a mathematical object or entity as object-driven.

Table 4. 5. Categorization of the use of unknown in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.9	*Teacher:* We are finding a rule. While expressing the rule this year, we will use letters such as a, b, c… with **unknown** expressions. We can express the rules with them. Thus, we will be using algebraic expressions.		Phrase-Driven	Explanatory talk
1.28	*Teacher:* Guys, when we are defining algebraic expressions, we use something like this. Algebraic expressions refer to expressions that consist of **unknowns** and entail mathematical operations.		Object-Driven	Explanatory talk
1.28 1.29 1.30 1.31 1.32	*Teacher*: ...Are there any **unknowns**? *RC*: Yes *Teacher:* What are these? *RC*: 2y, 3x SC: -2y, 3x *Teacher:* So x and y...		Object-Driven	Explanatory talk
1.106	*Teacher*:…Expressions containing both operations and **unknowns** are referred to as algebraic expressions.		Object-Driven	Explanatory talk
1.106 1.107 1.108 1.109	*Teacher*: ...Now you told me that there are terms. Which terms were they? 3x, -2y, +5. Are 3x and -2y like terms? *RC*: They are like terms. *Teacher*: In what way? *Boy*: They both have an **unknown**.		Object-Driven	Explanatory talk

Table 4.5. (continued)

1.112 1.113 1.114 1.115 1.116 1.117 1.118 1.119	*Teacher*: Or let's put it this way. Are the unknowns alike? *RC*: Yess. Teacher, there are. *Teacher*: Are the **unknowns** alike? *RC*: Nooooo *Teacher:* There is an … *RC*: x *Teacher*: There is a… *RC*: y	Object-Driven	Explanatory talk
1.131	Teacher:…{Expressions that contain **unknowns** and involve operations are called algebraic expressions}.	Object-Driven	Explanatory Talk
1.143	*Teacher*: So, they both have **unknown** x and they are identical. Ah, then okay, we can say they are like terms. Let's do it this way. Are -3y and 2xy like terms?	Object-Driven	Explanatory Talk
1.154 1.155	Teacher: Now, do we consider minus and plus signs or **unknowns**? RC: (Most of the students answer all together) **Unknowns**.	Object-Driven	Explanatory Talk
1.164	*Teacher:* Ok, let's look, both of them are exactly the same, that is, the terms are alike. I mean the **unknowns** should be the same. When there is a different unknown, there are not like terms....	Object-Driven	Explanatory Talk
1.192	*Teacher:* Wait a minute, we don't call it x2, we call them as x^2y. What is next? $-5x^2y$. They are like terms as the **unknowns** are the same.	Object-Driven	Exemplification

The analysis of the *unknown* is given in Table 4.5. All uses except one occurred in the explanatory talks during the instruction. For instance, in line 1.192, the unknown is uttered in the exemplification part, while the rest of them are used in explanatory talks. In line 1.9, the teacher used the word as phrase-driven. She describes letters such as a, b, c,.. as unknowns. All the remaining uses are object-driven, treating the word as a mathematical object. Moreover, students also used this word

individualistically, as in line 1.109, and in unison, as in line 1.155, as a response from the class (RC).

4.1.1.3. Coefficient

Coefficient is the other word that the teacher mentioned in the concepts in algebraic expressions courses. The following excerpt features an attempt by the instructor to define the coefficient. She emphasizes by spelling that the explanation is in the name of the co-efficient. Which is the multiple of the term. This is a phrase-driven use of words as it defines the coefficient. Moreover, this word is uttered during the teacher's explanatory talks.

> *1.64 Teacher:* Coefficient, that is, the co-efficient in terms (spelling); remember, what are the numbers that are coefficient? For example, what is the coefficient of the first term? Look, it is in the name, a number, who will tell the coefficient? Samet
> *1.65 Samet:* 3
> *1.66 Teacher:* What's the coefficient in this term? 3. Next, the other one, the second term, Zeki?

In the next conversation below, the teacher asks students to the coefficients of the given expression (3x-2y+5), and they respond by stating the coefficients in order (3, -2, 5). They pronounce the coefficient as if it were a mathematical entity. Therefore, this represents a utilization of object-driven use in the context of explanatory discussions.

> *1.79 Teacher:* What are the coefficients? (3x -2y +5)
> *1.80 RC:* (All together) 3, 2, no -2, +5

During the next part of the conversation, they determine the coefficients of the provided algebraic expressions one by one. The teacher emphasizes the sign of the numbers when deciding the coefficients in lines 1.87 and 1.91. Here again, the process of determining the coefficients is carried out through explanatory talks. This is obviously a demonstration of the routine-driven use because finding the coefficients includes an action.

1.85 Teacher: It would be a mistake not to consider them. The *coefficients*. What's the first *coefficient*?
1.86 RC: 3
1.87 Teacher: Let's say +3, group it (in parentheses), then?
1.88 RS: -2
1.89 Teacher: Next?
1.90 RS: +5
1.91 Teacher: What was I doing? Now, I'm adding the plus sign.

Although word use is generally expected to progress in a hierarchical fashion, from passive-use to object-driven, the teacher may occasionally make exceptions to follow the hierarchy. For instance, the teacher initially used the word coefficient as phrase-driven and object-driven. In the next use, she reverted to the routine-driven use. In other words, she can return to the routine-driven or phrase-driven use if she determines that it is necessary for the students to review it.

Table 4. 6. Categorization of the use of coefficients in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.64	*Teacher:* **Coefficient**, that is, the co-efficient in terms (spelling); remember, what are the numbers that are coefficient? For example, what is the coefficient of the first term? Look, it is in the name, a number, who will tell the coefficient? Samet		Phrase-Driven	Explanatory talk
1.66	*Teacher:* What's the co-efficient in this term? 3. Next, the other one, the second term, Zeki?		Phrase-driven	Explanatory Talk
1.79	*Teacher:* What are the **coefficients**? (3x -2y +5)		Object-Driven	Explanatory talk
1.80	*RC:* (All together) 3, 2, no -2, +5			
1.85	*Teacher*: It would be a mistake not to consider them. The **coefficients**. What's the first coefficient?		Object-Driven	Explanatory talk
1.86	*RC:* 3			
1.87	Teacher: Let's say +3, group it (in parentheses), then?			
1.88	*RS:* -2			
1.89	*Teacher*: Next?			

Table 4.6. (continued)

1.90	*RS:* +5		
1.91	*Teacher:* What was I doing? Now, I'm adding the plus sign		
1.133	*Teacher*: Usually, addition is asked, not subtraction. Then can I subtract 3 **coefficients**? How many numbers do we need to subtract?	Object-Driven	Explanatory talk
1.134	*RC:* 2		
1.135	*Teacher*: There are 3 coefficients here.		

Table 4.6 shows that all uses of coefficient occurred in the explanatory talks. The teacher used the word as a phrase-driven in lines 1.64 and 1.66, and the rest of the usages are object-driven. Although the teacher usually pronounced the word explicitly, students referred to it implicitly when determining the coefficients.

4.1.1.4. Sum of coefficients

The *sum of coefficients* refers to a group of words that the teacher addresses following the learning coefficients. It is commonly used in exercises involving addition and subtraction in algebraic expressions. The following excerpt provides guidance on the aspects that warrant consideration when calculating the sum of the coefficients for 3x-2y+5. In the excerpt, although the sum of coefficients is not explicitly uttered, the teacher aims to draw attention to the factors that should be considered when summing up the coefficients. Since the descriptive expressions are utilized the phrase-driven use is employed.

> *1.81 Teacher:* This example asks me to add these. Where should I pay attention to while adding (3x-2y+5)?
> *1.82 RC:* The negatives and positives.
> *1.83 Teacher:* So, where should I focus on? I should focus on the group. Right?
> *1.106 Teacher:* I have 6 positives in my bag. We also found the *sum of the coefficients*.

In line 1.106, the sum of coefficients is used as a mathematical object following the usage of definitions, interpretations, and actions of finding it. It is used as a final statement as an object-driven use. The whole usage of the sum of coefficients is given in Table 4.7 below.

Table 4. 7. Categorization of the use of the sum of coefficients in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.77	*Teacher*: Yes, item e is related to the sum of coefficients. It says what are we going to do with the coefficients?		Routine-Driven	Explanatory talk
	RC: (all together) Sum.			
1.78				
1.81	*Teacher:* This example asks me to add these. Where should I pay attention to while adding (3x-2y+5)?		Phrase-Driven	Explanatory talk
	RC: The negatives and positives.			
1.82	*Teacher:* So, where should I focus on? I should focus on the group.			
1.83	Right?			
1.85	*Teacher:* It would be a mistake not to consider them. The coefficients. What's the first coefficient?		Routine-Driven	Explanatory talk
	RC: 3			
1.86	*Teacher:* Let's say +3, group it (in			
1.87	parentheses), then?			
	RS: -2			
1.88	*Teacher*: Next?			
1.89	*RS*: +5			
1.90	*Teacher:* What was I doing? Now,			
1.91	I'm adding the plus sign.			
1.106	*Teacher:* I have 6 positives in my bag. We also found the **sum of the coefficients**.		Object-Driven	Explanatory talk
1.129	*Teacher:* What happens if we do not parenthesize when collecting the coefficients? Would we make a mistake?		Phrase-Driven	Explanatory talk
1.130	*RC:* Yes			
1.131	*Teacher:* When we don't group things, we make mistakes somewhere.			
1.132	*Girl*: Teacher, you said that the **sum of the coefficients.** Can we subtract the coefficients?		Routine-Driven	Explanatory talk
1.135	*Teacher*: Usually, addition is asked, but subtraction is not. Can I subtract 3 coefficients? (Voices in unison from the class) How many numbers does it have to be? How many numbers do I need for subtraction?		Phrase-Driven	Explanatory talk
	RC: 2			
	Teacher: There are 3 coefficients			
1.136	(The class continues to discuss			
1.137	among themselves)			

Table 4.7 represents the analysis of *the sum of coefficients* in the topic of concepts in algebraic expressions. As seen in the table, all usages are employed in the explanatory talks. In addition, the sum of coefficients is mostly used as routine-driven and phrase-driven. As in lines 1.85 to 1.91, in routine-driven, the teacher and students perform the calculation of the sum of coefficients. In phrase-driven use, as in lines 1.129 to 1.131, the teacher explains how to find the sum of coefficients. Moreover, the teacher mostly used the sum of the coefficient, and students generally used one-word answers in the discourse (e.g., in lines between 1.85 to 1.91).

4.1.1.5. Number of terms

The number of terms pronounced only once in the topic of concepts in algebraic expressions. In the excerpt below, the teacher explains how to calculate the number of terms. She claims that if each term is circled with its sign, they may count the circles to determine the number of terms. Since there is an action of calculating the number of terms, it is acknowledged as routine-driven use.

> *1.46 Teacher:* What are we doing here? We are counting the circles, there are 1, 2, and 3 circles, so how many terms? What is the number of terms?

Table 4. 8. Categorization of the number of terms in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.46	*Teacher*: What are we doing here? We are counting the rounds, there are 1, 2, 3 rounds. So what is the number of terms?		Routine-Driven	Explanatory talk

Table 4.8 represents the only use of *the number of terms* in the topic of concepts in algebraic expressions. This usage is routine-driven because the teacher is performing the act of finding the number of terms and taking part in the explanatory talks.

4.1.1.6. Constant

The constant is also uttered once in the related topic, and it is related to the definition of the constant. In line 1.73 the teacher emphasizes that the constant is an unchanging term in an algebraic expression. This is an example of phrase-driven use.

1.73 Teacher: Yes, the constant term, the constant, the unchanging. Only a few people know?

Table 4. 9. Categorization of the number of terms in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.73	*Teacher*: Yes, the *constant* term, the constant, the unchanging, stable. How many people say?	Writes on the board	Phrase-Driven	Explanatory talk

Table 4.9 demonstrates the utilization of the constant term in the topic of concepts in algebraic expressions. The only use of the constant term is phrase-driven, as the usage involves defining the term. In addition, this usage takes part in the explanatory talks.

4.1.1.7. Like term

The most frequently pronounced word is *like term* in the topic of concepts in algebraic expressions. The teacher dedicated the majority of the initial two lessons to explaining the concept of like terms and conducting corresponding activities. Considering the expressions related to the like term, the findings revealed that the utilization of phrase-driven use, which is employed primarily to delineate like terms, and object-driven use, applied in the context of accepting it as a mathematical object or as a concluding statement, are utilized with nearly equivalent frequency. On the other hand, routine-driven use of utterances, including process and action-based activities, is comparatively less prevalent in the discourse. The dialogues regarding the utilization of like terms are elaborated on below.

1.117: Teacher: Like terms. Everyone! If there are such different expressions, they are not like terms. Do you get the point? Okay? If there is an expression like 3x -2x + 5, I would say that 3x and -2x are like terms. We will discuss like terms in more detail soon. Let's write the board on your notebooks.

In the preceding dialogue, the teacher makes an explanation related to the like terms. She describes which expressions are similar and which are not. Therefore, in an

explanatory talk, the teacher uses like term in a phrase-driven use. Moreover, in the following dialogue, the teacher proceeds to make explanations about like terms. She indicates that both expressions (2x and -4x) have x and they could accept that they are like terms. This usage is also an example of phrase-driven use in explanatory talks.

> *1.141 Teacher: …*Are 2x and - 4x like terms?
> *1.142 RC:* Yes. They both have x.
> *1.143 Teacher:* So, they both have unknown x and they are identical. Ah, then okay, we can say they are like terms...

After that, the teacher proceeds with the instruction by providing an example of two expressions that are not like terms in line 1.143 given below. She asks whether they are like terms or not. The responses from the class are both like terms and not like terms. Then, a student answers that they are like terms due to the presence of both x and y. In this conversation, the teacher uses the like term as a mathematical object without providing any further explanation. She employs the term as an object-driven usage. However, students have not progressed to phrase-driven or object-driven use. Instead, they use the word as routine-driven as they focus on how to find like terms.

> *1.143 Teacher:* …Let's do it this way. Are -3y and 2xy like terms?
> *1.144 RC:* Yes
> *1.145 RC:* No
> *1.146 Teacher:* Who says they are like terms? Who says not? (Fingers up for both situations). Why are they like terms, Şevket?
> *1.147 Şevket:* There is x and y.

In the next dialogue, they continue to discuss the exercise -3y and 2xy. The teacher explains to the students why these terms are not like terms. Therefore, she returns to the phrase-driven use. She clarifies that the terms cannot be considered like terms since one of them contains the unknown y, while the other one has xy.

> *1.160 Teacher:* Look at this; one has y, and the other one has xy. Are they like terms?
> *1.161 RC:* Nooo
> *1.162 Teacher:* They are not like terms because they are not the same (indicating the unknown).

Afterward, they attempt to figure out whether 3x and y are like terms (line 1.175) The teacher lets a girl speak. She says that they are not like terms because one of them has x as an unknown, and the other one has y. Consequently, she has decided that they are not like terms. She comes to this conclusion by considering the definition of like term. Phrase-driven use of the like term used by the girl.

> *1.175 Teacher:* What else? 3x and y. Fingers up. Yes.
> *1.176 Girl:* Not (Like term)
> *1.177 Teacher:* It is not, why?
> *1.178 Girl:* One has x and one has y.
> *1.179 Teacher:* Yes, that's true. They are not. I marked it as false.

After the teacher's informative and explanatory talks, she moved to the exemplifications, which is a series of exercises. The first exercise is to determine if x^2 and x are like terms. Selim asserts that they are not like terms due to the presence of a square on one of x. Besides, Şevket states that x^2 is the result of multiplying two x's, whereas the other term has only one x. In this dialogue, students employ phrase-driven use by utilizing descriptive expressions.

> *1.193 Teacher:* Well, keep going. Are x^2 and x like terms? Selim?
> *1.194 Selim:* They are not.
> *1.195 Teacher:* Not? Why?
> *1.196 Selim:* There is a square on top of x.
> *1.197 Teacher:* Wait a minute, wait a minute. Şevket said something. Tell us, Şevket.
> *1.198 Şevket:* The square of x is multiplication of 2 x's. But there is only one x here.
> *1.199 Teacher:* Oh, your friend says that x has 2 on it and that it is substituting for the multiplication of two x's, but there is one x there, and they are not like terms.

In the next exercise, students articulate terms that are similar to -2x. They have acquired the meaning and usage of the like terms and are capable of generating an expression that is similar to the given one. Consequently, students have transitioned to object-driven use.

> *2. 42 Teacher:* ...Yes, the other example is -2x, let's see if you can tell me a like term. Finger. Yakup
> *2.43 Yakup:* x

2.44 Teacher: can it be x?
2.45 RC: It can.
2.46 Teacher: Let's write a few more.
2.47 Samet: -3x
2.48 Teacher: -3x This can also be, shall we write one more? Zeki?
2.49 Zeki: 5x
2.50 Teacher: 5x well done. Yes, we found like terms. We could write more different ones too...

Table 4. 10. Categorization of the like terms in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.11	*Teacher*: There are three operations we will learn, addition, subtraction, and multiplication, but not division. Then, we will talk about patterns and relations. At first, there are **like terms**.	Students draw the concept map in their notebook while the teacher explains it	Object-Driven	Explanatory talk
1.106	*Teacher:* ...Now you told me that there are terms. Which terms were they? 3x, -2y, +5. Are 3x and -2y **like terms**?		Routine-Driven	Explanatory talk
1.107				
1.108	*RC:* They are like terms.			
1.109	Teacher: In what way?			
1.110	*Boy:* They both have an unknown.			
1.111	*Teacher:* Else			
1.112	*RC:* […]			
1.113	*Teacher:* Or let's put it this way. Are the unknowns alike?			
1.114	*RC:* Yess. Teacher, there [...]			
1.115	*Teacher:* Are the			
1.116	unknowns alike?			
1.117	*RC:* Nooooo			
1.118	*Teacher:* There is an?	The teacher shows the concept map on the bord		
1.119	*RC:* x			
1.120	*Teacher:*There is a? *RC:* y			
1.121	*Teacher:* What is the first heading? *RC:* Like terms			

Table 4.10. (continued)

1.23	*Teacher:* **Like terms**. Guys, if there are such different unknowns, they are not **like terms.** Did you understand? If there is an expression like 3x-2x+5, I could say that 3x and -2x are like terms. We will continue with **like terms** in detail. First, let's write the board in your notebook...		Phrase-Driven	Explanatory talk
1.141 1.142 1.143	*Teacher:* … Are 2x and -4x **like terms**? *RC:* Yes. They both have x. *Teacher:* So, they both have unknown x and they are identical. Ah, then okay, we can say they are **like terms**...		Phrase-Driven	Explanatory talk
1.143 1.144 1.145 1.146 1.147	*Teacher:* …Let's do it this way. Are -3y and 2xy like terms? *RC:* Yes *RC:* No *Teacher:* Who says they are **like terms**? Who says not? Why are they **like terms**, Şevket? *Şevket:* There is x and y.	Fingers up for both situations	Object-Driven	Explanatory talk
1.157 1.158 1.159	*Boy:* I think they are not same *Teacher:* So, **like**..? *Boy:* Not same.		Routine-Driven	Explanatory talk
1.160 1.161 1.162	*Teacher:* Look at this; one has y, and the other one has xy. Are they **like terms**? *RC:* Nooo *Teacher:* They are not **like terms** because they are not the same	Indicating the unknowns	Phrase-Driven	Explanatory talk
1.62	*Teacher*: … Let me show you. They both have x, they are **like terms**. one has y and the other one has xy, not **like terms**. Do you understand?	Circles unknowns. Circles x's in 2x and -4x. She circles y and xy in -3y and 2xy respectively	Routine-Driven	Explanatory talk

Table 4.10. (continued)

1.164	*Teacher*: Yes, then I see that all of them are exactly the same; that is, two terms are the same. It means the unknowns should be the same. When we see a different unknown, it is not the same. Let's do some examples and discuss together whether they are **like terms**! Okay? Let's write this first, then continue. There is a note about **like term**. Let's write! Note: In an algebraic expression, terms with the same letters and the same powers of the letters are called **like terms**.		Phrase-Driven	Explanatory talk
1.166	*Teacher:* Let's put a tick next to the **like** ones and a cross next to the **not-like** ones. Let's write first, then we will do it together. Write and lean back […] Speed up, speed up […] Who have finished writing lean back and let me see. How many people haven't finished yet...		Object-Driven	Exemplification
1.66	*Teacher:* ...Yes, the first one. Are 4x and x **like terms**? Finger. Finger. This many people? Finger. Buse.		Phrase-Driven	Exemplification
1.67	*Buse:* They are **like terms**..			
1.68	*Teacher:* Are they? Why?			
1.69	*Buse:* Multiplication by 4...x exists			
1.70	*Teacher:* Wait a minute, wait a minute, stay out of this. Tell me.			
1.71	*Buse:* x exists.			
1.72	*Teacher:* What exists both of them?			
1.73	*Buse:* There is x			
1.174	*Teacher:* Then they are **like terms** (a)	She puts a tick next to 4x and x		

Table 4.10. (continued)

1.175	*Teacher:* What else? b) 3x and y. Fingers up. Yes.		Phrase-Driven	Exemplification
1.176	*Girl:* Not (Like term)	The girl means they are not like terms		
1.177	*Teacher:* It is not, why?			
1.178	*Girl:* One has x and one has y.			
1.179	*Teacher:* Yes, that's true. They are not. I marked it as false.	The teacher puts a cross next to example b		
1.180	*Teacher:* c) xy and 2xy. Do you want to explain Yakup? Are they **like**		Phrase-Driven	Exemplification
1.181	**terms** or not?			
1.182	*Yakup:* [...] which question?			
1.183	*Teacher:* We are doing c,			
1.184	Yakup.			
1.185	*Yakup:* They are like terms.			
1.186	*Teacher:* Are they like terms, why?	She puts a tick next to c) xy and 2xy		
1.187	*Yakup:* xy... they both have xy			
	RC: Both have xy			
	Teacher: Both have xy, yes they are **like terms**…			
1.187	*Teacher:* ...The other one? $3x^2y$ and $-5x^2y$. Tell me, boy.		Routine-Driven	Exemplification
1.188	*Boy:* **Like term**			
1.189	*Teacher:* **Like term**, yes, what are the same then?			
1.190	*Boy:* x2y...			
1.191	*RC:* Squares are the same.			
1.192	*Teacher:* Wait a minute, we do not call it x2; we are calling it x^2y, and what is the other one, $-5x^2y$. These are **like terms** because the unknowns are the same.	She puts a tick next to d) $3x^2y$ and $-5x^2y$		
1.193	*Teacher:* Well, keep going. Are x^2 and x **like**		Phrase-Driven	Exemplification
	terms? Selim?			
1.194	*Selim:* They are not.			
1.195	*Teacher:* Not? Why?			
1.196	*Selim:* There is a square on top of x.			

Table 4.10. (continued)

1.197	*Teacher:* Wait a minute, wait a minute. Şevket said something. Tell us,		Phrase-Driven	Exemplification
1.198	Şevket. *Şevket:* The square of x is multiplication of 2 x's.			
1.199	But there is only one x here. *Teacher:* Oh, your friend says that x has 2 on it and that it is substituting for the multiplication of two x's, but there is one x there, and they are not **like terms**.			
1.200	*RC:* I think like terms	Various ideas coming from the class at the same time	Object-driven	Exemplification
1.201	*RC:* Not			
1.202	*Teacher:* How many people agree with Şevket that they are not like terms? Those who agree with Süleyman and say they are **like terms**? The others, those who don't raise their hands, are they abstaining? I think they are **like terms**.	Fingers are raised		
1.203	*Şevket:* We have written a note on the top.		Phrase-Driven	Exemplification
1.204	*Teacher:* Wait a minute, wait a minute, Şevket says that we have writen a note above, read it,			
1.205	Şevket. *Şevket:* In an algebraic expression, terms with the same letters and	Reading the note		
1.206	powers of letters are called **like terms**. *Teacher:* Suleyman, you			
1.207	read it too. Read aloud to your friends *Süleyman:* Note: In an algebraic expression, terms with the same letter powers are called **like terms**.			

Table 4.10. (continued)

1.208	*Teacher*: What did you notice there?		Phrase-Driven	Exemplification
1.209	*Süleyman*: The powers will be the same	The bell rings		
1.210	*Teacher:* It says the powers will be the same. Right? Then are they like			
1.211	terms?	They answer		
1.212	*RC:* Nooo	all at once		
	Teacher: They are not, is			
1.213	it our final decision?			
1.214	*RC:* Yes			
	Teacher: Yes, their powers should be the same…			
1.214	*Teacher:* …Let's do the others right away.	Pointing to option f, f) x and 1/x	Routine-Driven	Exemplification
1.215	*RC:* **Like terms**, not like terms, **like terms**, not like terms.	Answers come from the class		
1.216	*Teacher:* Wait a minute.			
1.217	*Boy:* Teacher, one of them has 1			
1.218	*RC:* No no ...			
1.219	*Teacher:* Those who say **like terms**? This is a controversial issue. Let's continue later.	The class could not agree on a solution		
1.220	*RC:* Not **like terms**,			
1.221	teacher			
	Teacher: Let's continue when we come back.			
2.7	*Teacher:* Ah. Here there is casper one. One has 2 and the other has one. The definition says that terms with the same powers are **like terms**. If one has one and the other has two, are they like	She points to the powers	Phrase-Driven	Exemplification
2.8	terms?			
	RC: Noo, their powers are not same.			

Table 4.10. (continued)

2.9 2.10 2.11 2.12 2.13 2.14 2.15 2.16 2.17 2.18 2.19	*Teacher:* Exactly the other Those who say they are **like terms**? Those who say they are not? Yes, why are they **like terms,** Merve? *Merve:* Because there is Casper and there is one *RC:* What casper *Teacher:* Is it on the top or on the bottom? *RC:* At the bottom *Teacher:* Where is Casper, the one on top or the one on the bottom? Merve: At the bottom, casper one *Teacher:* What else? *RC:* They are **like terms**. *Teacher:* Are they? *RC:* We simplify and they become same, one of them is x and one of them is not clear	Showing f) 1/x with x The teacher draws a denominator line on x and writes 1 in the denominator	Phrase-Driven	Exemplification
2.20	*Teacher:* Guys, we can say... They both should be in the same position... Here, one is below and one is above. They are same, okay, but their positions are not same. that is why we can not say they are **like terms**. Because their positions are different, one is up, and the other one is down. If they were side by side, I would accept they are **like terms**…		Phrase-Driven	Explanatory talk
2.20 2.21	*Teacher:* …Then, what do we call them? They are not **like terms**. *RC:* Not	Teacher puts a cross next to option f	Object-Driven	Exemplification

Table 4.10. (continued)

2.22	*Teacher:* Exercise g) x^2y and $4xy^2$, fingers! Like terms or not?. Tell me.		Phrase-Driven	Exemplification
2.23	*Girl:* It is not.			
2.24	*Teacher:* Your friend says it is not.			
2.25	*RC:* Like terms, no, no.!			
2.26	*Yakup:* Teacher, there is 2 over x in one of them and there is 2 over y in the other.			
2.27	*Teacher:* One has x^2 and one has y^2. Then they are not *like terms*.			
2.28	*RC:* Noottt (In unison)			
2.29	*Teacher*: So, like here, are you looking at this? (Shows the exercise d) If the powers and unknowns were the same as here, this was ok, we would say they are like terms. But when we look at the powers here, one of the x has square and other not, one of the y has square and other not We can say that they are not like terms	Points the two different terms in exercise g. Puts a cross next to exercise g	Object-Driven	Exemplification

Table 4.10 represents the flow of the usage of *like terms* in the topic of concepts in algebraic expressions. The two lessons are devoted to mostly like terms. Although the like term is mainly uttered by the teacher, students also use the word with longer sentences, different from other words. The teacher and the students used the like terms both in the explanatory talks and exemplification parts of the lessons. At the beginning of the instruction, routine-driven and phrase-driven usages are predominant, while object-driven usages are also observed towards the end.

4.1.1.8. Algebraic expression

Algebraic expression is the last used word in the topic of concepts in algebraic expressions. In the excerpt below, the teacher describes the phrase of algebraic expression. She says that they find rules and use unknowns a, b, or c for those rules. Due to the given description, this usage can be classified as phrase-driven. Besides, this usage exists in the explanatory part of the lesson.

1.9 Teacher: We are finding rules. When we were expressing the rules this year, we were using expressions like a, b, and c with unknown expressions. We will express them with them. So, in a way, we will be using algebraic expressions.

In the dialogue below they are using algebraic expression as a mathematical object. In lines 1.205 and 1.207, students are reading the definition of the like terms. The word algebraic expression is employed in this definition to refer to a mathematical entity. This is an object-driven use in the explanatory part of the lesson.

1.205 Şevket: (Reading the note) In an *algebraic expression*, terms with the same letters and powers of letters are called like terms.
1.206 Teacher: Suleyman, you read it too. Read aloud to your friends
1.207 Süleyman: Note: In an *algebraic expression,* terms with the same letter powers are called like terms.

The teacher writes a series of examples on the board (-2x, $3x^2$, x^2y, -2/x, -3x/4, 4xy) and asks students to write expressions that are similar to the given algebraic expressions. Here, the teacher uses the algebraic expressions as a mathematical object. This phrase is included in the exemplifications and it is used in an object-driven manner.

2.34 Teacher: We say example. Example, example. Coming up
2.35 Teacher: Write *algebraic expressions* that are similar to the expressions given below.
2.36 Teacher: So, what do you understand from this? I will give you *algebraic expressions* and you will write the ones similar to mine. I leave it to you, okay, it is an open-ended activity. We said example, then let's write.

Table 4. 11. Categorization of the algebraic expressions in the concepts in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
1.9	*Teacher*: We are finding rules. When we were expressing the rules this year, we were using expressions like a, b, c with unknown expressions. We will express them with them. So in a way we will be using **algebraic expressions.**		Phrase-Driven	Explanatory Talk

Table 4.11. (continued)

1.28	*Teacher:* Guys, when we are defining algebraic expressions, we use something like this. **Algebraic expressions** refer to expressions that consist of unknowns and entail mathematical operations.	Phrase-Driven	Explanatory Talk
1.106	*Teacher*: …Expressions containing both operations and unknowns are referred to as **algebraic expressions**.	Phrase-Driven	Explanatory Talk
1.131	*Teacher*: …Expressions that contain unknowns and involve operations are called **algebraic expressions**.	Phrase-Driven	Explanatory Talk
1.164	*Teacher*: …Let's write! Note: In an **algebraic expression**, terms with the same letters and the same powers of the letters are called like terms.	Object-Driven	Explanatory Talk
1.205	*Şevket:* In an **algebraic expression**, terms with the same letters and powers of letters are called like	Object-Driven	Exemplification
1.206	terms. *Teacher:* Suleyman, you read it too. Read aloud to		
1.207	your friends *Süleyman:* Note: In an **algebraic expression**, terms with the same letter powers are called like terms		
2.34	*Teacher:* Example, here comes. Write algebraic expressions that are similar to the expressions given below.	Object-Driven	Exemplification
2.35	*Teacher*: So what do you understand from this? I'll give you algebraic expressions and you'll write the ones similar to mine…		

Table 4.11. (continued)

2.37	*Teacher*: a) -2x b) 3x2 c) x2y d) -2/x e) -3x/4 f) 4xy		Object-driven	Exemplification
2.38	*Teacher*: Süleyman, is that your voice, or is it just me? Let's see. First, you write and then we will do it together. This time, he gave me an algebraic expression, and we will write similar expressions.			
2.39	*Teacher:* Small a small b bullet point. *Teacher:* Speed up. Süleyman was able to write the bullet points first so far. Come on, son. You can go on and write your own **algebraic expression**. We will ask you some questions and then we will write them down and add them.	Whilst the students are writing, she walks around in between		

In Table 4.11, the usage flow of the algebraic expression in the topic of concepts in algebraic expressions is provided. The phrase is generally used in explanatory talks of the lesson. For instance, as in lines 1.28 or 1.106, the teacher used the phrase in a phrase-driven manner to describe. Then she moved to the examples and used the phrase as object-driven. Furthermore, as in lines 1.205 and 1.207, the students were able to use the algebraic expression in an object-driven manner.

4.1.2. Visual mediators in the topic of concepts in algebraic expressions

The findings from the lessons related to the concepts in algebraic expressions showed that the primary used visual mediator is algebraic notations. Additionally, the teacher presented a concept map to the students before initiating the algebraic expressions lessons. As indicated on this map (Figure 4.2), the teacher informed the students that they would learn algebraic expressions under the four subheadings. These topics are like terms, addition and subtraction in algebraic expressions, multiplication of a natural number with an algebraic expression, and patterns and relationships.

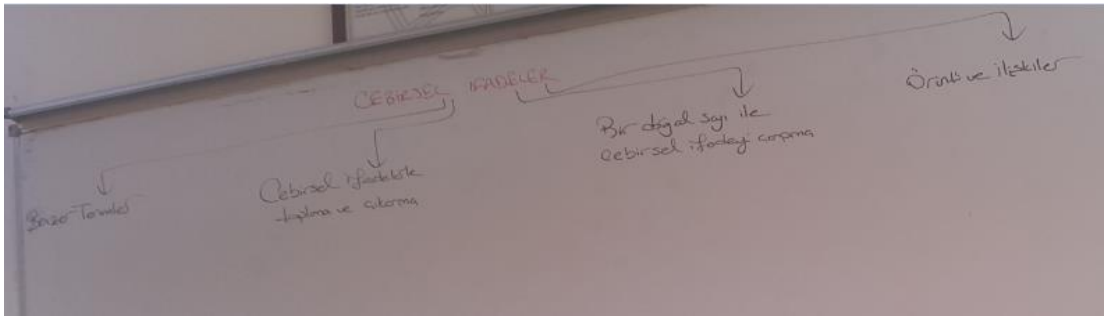


Figure 4. 2. Concept map related to algebraic expressions topics

Following that, the teacher writes the concepts they are going to learn on the board and proceeds to write the corresponding algebraic notations. As seen in Figure 4.3, letters such as x and y are utilized to represent unknowns. Symbolic notations are the predominantly used visual mediators in algebraic expressions lessons.

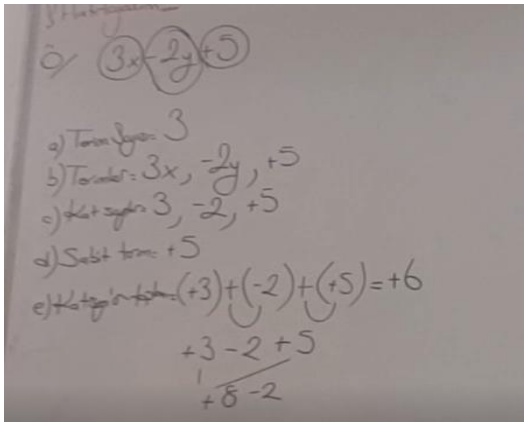


Figure 4. 3. Concepts and corresponding algebraic notations

In addition, the teacher circles the terms in a given algebraic expression in order to determine the number of terms. For instance, in Figure 4.4, she demonstrates that the expression 3x-2y+5 consists of three terms, as shown by three circles.

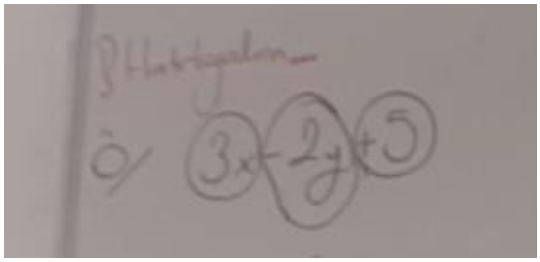


Figure 4. 4. Circles representing the terms

As seen in Figure 4.5 and Figure 4.6, algebraic notations are mostly used as visual mediators. In Figure 4.5 the teachers wrote expressions and then let students write like terms to those expressions. Moreover, she put ticks or false to indicate whether terms are like terms or not.

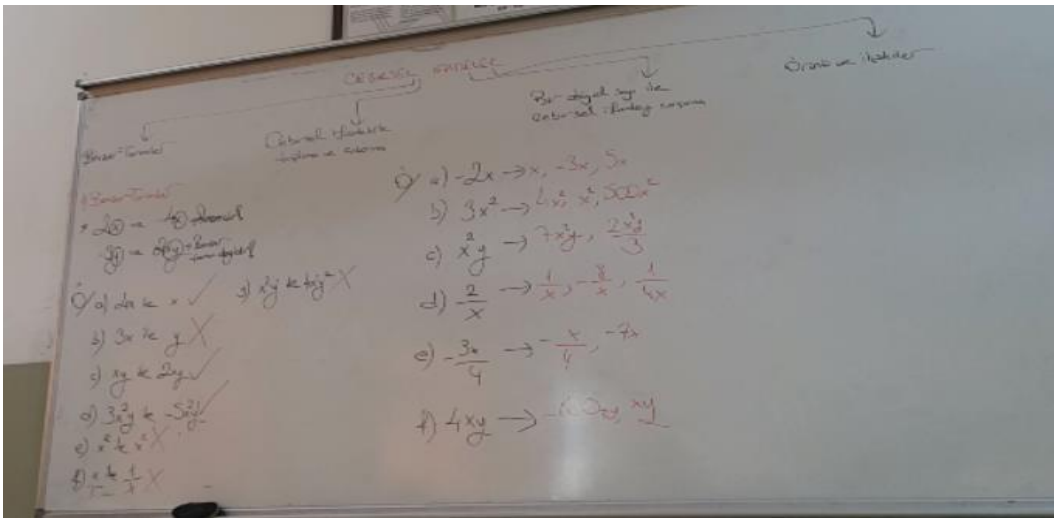


Figure 4. 5. Algebraic notations

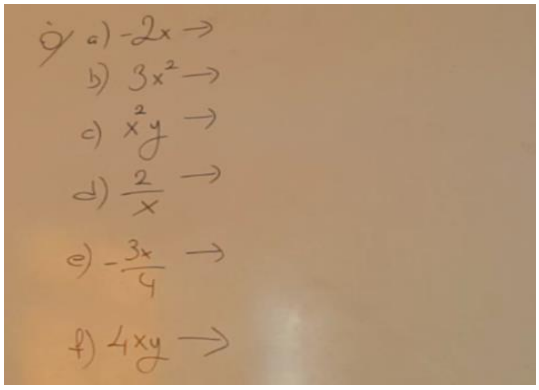


Figure 4. 6. Algebraic notations and arcs

To sum up, algebraic notations are mostly used as visual mediators in concepts in algebraic expressions. Besides, the teacher used a concept map to provide students with an overview of the topics they would be learning. Additional visual mediators used in teaching algebraic expressions contain circles, arrows, and arcs to demonstrate like terms or a number of terms. The findings of routines, which are another mathematical discourse characteristic identified in commognitive theory, are elaborated upon in the next section.

4.1.3. Routines in the topic of concepts in algebraic expressions

In this section, it was focused on routines related to concepts in algebraic expressions that were analyzed in algebra lessons. Discursive patterns, which are recurring activities in the communication environment, were analyzed to identify routines during the concepts in the algebraic expressions topic. There are three routines determined in this section. These are *finding the like terms, writing a like term,* and *Casper routine.* The frequencies of performing routines are given in Table 4.12.

Table 4. 12. Frequency of routines in concepts in algebraic expressions

Routines	Explanatory Talk	Exemplification	Total
Finding the like terms	2	3	5 (%36)
Writing a like term	0	6	6 (%43)
Casper routine	0	3	3 (%21)
Total	2 (%14)	12 (%86)	

The most frequently used routine was writing a like term (%43), and following that, finding the like terms (%36) was the second most frequently used routine. The least frequently used routine was the Casper routine (%21). The majority of the routines are observed in the exemplification part of the instruction (%86). Below, a more detailed explanation of the routines is given.

4.1.3.1. Finding the like terms

At the beginning of the instruction, the teacher applied *finding the like terms* routine to identify similar terms within the given expressions. By this means, she ensured her students recognized the terms containing the same unknowns. Firstly, she started with simple exercises. For instance, she said that 2x and -4x are like terms, but -3y and 2xy are not like terms. Later she continued with more complex ones. During the instruction on the topic of concepts in algebraic expressions, the noticeable issue is finding the like terms routine is first introduced by the teacher, and after a couple of exercises, the students could use the routine alone.

Below, a classroom dialogue is given related to the example above (2x and -4x) to finding the like terms routine. Firstly, the teacher introduces the like terms, which

sets the stage for the lesson's focus. By asking the students if 2x and -4x are like terms, the teacher prompts the students to engage with finding the like terms. Response from the classroom, confirming that both terms contain the variable x, leads the teacher to clarify and affirm that since both terms share the same unknown, they are indeed like terms.

> *1.141 Teacher: ...*Ok, I'm cleaning the board. The first heading is like terms. Write the heading. Are 2x and - 4x like terms?
> *1.142 RC:* Yes. They both have x.
> *1.143 Teacher:* So, they both have unknown x and they are identical. Ah, then okay, we can say they are like terms...
> …
> *1.162 Teacher:* Let me show you. (Circles unknowns) (Circles x's in 2x and -4x) They both have x, they are like terms. (She circles y and xy in -3y and 2xy respectively) one has y and the other one has xy, not like terms. Do you understand?

This dialogue serves as a practical application of the routine, showing students how to identify like terms. The teacher's method of circling and explaining directly on the board provides a clear visual reinforcement of the concept, which is crucial for student understanding. The teacher ensures students grasp the difference between like and unlike terms, marked by her explanation of why y and xy are not like terms. This confirms the successful completion of the routine, leaving students with a firm understanding of how to identify and differentiate terms based on their variables, a foundational skill in algebraic simplification. She closes the routine by marking the like terms (2x and -4x) and putting a cross next to not like terms (xy and -3y).

In another example below, the teacher wants students to determine whether the given algebraic expressions are like terms. She asks Yakup if he could explain whether xy and 2xy are like terms or not. Yakup confirms that xy and 2xy are like terms, justifying his answer by noting that both expressions include xy. Then, the classroom supports Yakup's answer and emphasizes that both terms contain xy. After that, the teacher also confirms their responses and agrees that these are like terms because they share the same variables. She puts a tick next to the example.

> *1.180 Teacher:* xy and 2xy. Do you want to explain Yakup? Are they like terms or not?

1.181 Yakup*:* [...] which question?
1.182 Teacher: We are doing c, Yakup.
1.183 Yakup*:* They are like terms.
1.184 Teacher: Are they like terms, why?
1.185 Yakup xy... they both have xy
1.185 RC: Both have xy
1.186 Teacher: Both have xy; yes, they are like terms (c. puts a tick next to xy and 2xy).

These two dialogues illustrate how finding the like term routine is applied within the topic of concepts in algebraic expressions. The routine is comprised of a few meta-rules, including prompt, how, and when. These meta-rules are given below.

Table 4. 13. Finding the like terms routine in the topic of concepts in algebraic expressions

Prompt	Routine	How?	When?	
			Applicability	Closure
Asking for the like terms in the given algebraic expressions	*Finding the like terms*	Marking the terms with the same unknown in the given algebraic expression	Finding the like terms in the given algebraic expressions	Marking the like terms in algebraic expressions

Table 4.13 indicates the meta-rules for finding the like terms routine. The prompt is the teacher's asking about the like terms. After the teacher's request, they are trying to notice if the expressions include like terms. Then, students or the teacher mark the like terms in accordance with the teacher's instructions defining how the routine occurs. It is known that applicability is the condition when the routine actually occurs. Here, the routine occurs in order to find the like terms in the given algebraic expressions. For a successful ending, the closure of the routine is marking the like terms.

4.1.3.2. Writing a Like Term

After finding the like terms in the given algebraic expressions, the teacher asked students to form a like term. She wrote some expressions on the board and wanted students to try to generate new terms (Figure 4.7). Thus, *writing a like term* routine begins. The following excerpt explains how they have formed the terms.

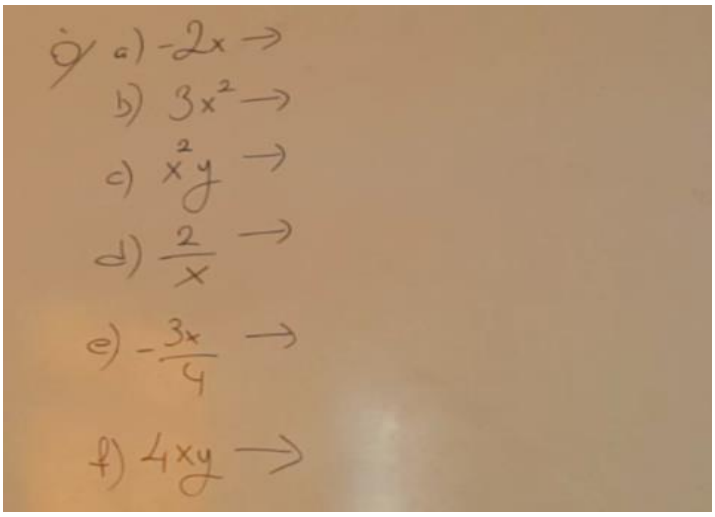


Figure 4. 7. Examples for generating like terms

2.42 Teacher: …Yes, the first example is -2x. Tell me a like term. Fingers up. Yakup?
2.43 Yakup: x
2.44 Teacher: Can it be x?
2.45 Class: Yeessss, it can.
2.46 Teacher: Let's write a few more.
2.47 Can: -3x
2.48 Teacher: -3x. It can be, too. Let's write one more. Zeki?
2.49 Zeki: 5x
2.50 Teacher: 5x, good for you. Yes, we have to find like terms and we can write differently.

The teacher asked for the first exercise (-2x), and the whole class attempted to generate like terms. Then, she encourages further participation by asking for more examples, thereby deepening the students' engagement with the concept. Can and Zeki contribute -3x and 5x, respectively, both of which the teacher affirms as correct like terms. This dialogue effectively reinforces the understanding that like terms are expressions that share the same variable and algebraic form but can differ in their coefficients.

Afterward, the teacher asked about the other exercises respectively, and the whole class attempted to create like terms one by one. For instance, below, she wanted to draw attention to form expressions with exponential power x^2y. This time the exercise involves more complex expressions with both variables and exponents. Writing a like term routine starts with the teacher asking to generate another example

of a like term for x^2y. The class responds with $7x^2y$, proposing a term that shares the same variables and exponent structure. The teacher then challenges the students further by referencing a more complex expression $\frac{2x^2y}{3}$, and asking if this could also be considered a like term. The class confirms that they are like terms. The teacher specifically points to the exponential power, prompting the students to acknowledge that the position of the 2 as an exponent is crucial for maintaining the like term relationship. The students' correct recognition that the exponent 2 remains constant above the variable x confirms their understanding.

2.50 Teacher: …Yes another? x^2y, say it?
2.51 Class: $7x^2y$
2.52 Teacher: Would it be like this? I'm just asking, look here $\frac{2x^2y}{3}$
2.53 Class: It can be, it will.
2.54 Teacher: Because where is 2 (talking about exponential power of 2)?
2.55 Class: Above (they indicated that 2 is an exponential power as in the given exercise)
2.56 Teacher: Above. Isn't it? That's true. No problem.

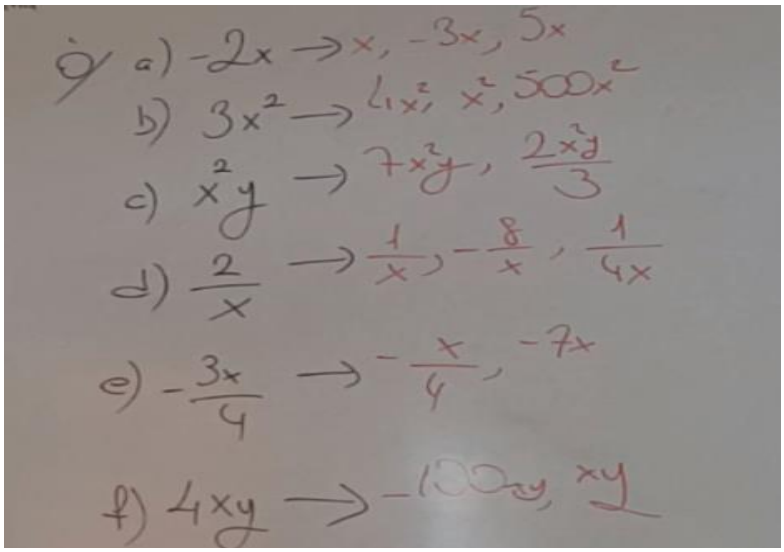


Figure 4. 8. Examples for generating like terms

In the excerpt above, the teacher does not need further explanations as the students can easily generate the like terms. They performed all exercises and found similar terms for each given expression. They completed all exercises and identified like terms for each given expression.

Table 4. 14. Writing a like term routine to a given expression

Prompt	Routine	How?	When?	
			Applicability	Closure
Asking to generate like terms for a given algebraic expression	Writing a like term	Generating expressions with the same unknown and different coefficients as a given algebraic expression	When generating like terms for a given algebraic expression	Writing the like term

When Sfard's analysis was implemented on these dialogues (Table 4.14), the routine of generating starts with the teacher asking for like terms similar to a given algebraic expression (x^2y) (*prompt*). Afterward, the students form expressions with the same unknown, exponential, and different coefficients to the given algebraic expression ($7x^2y$) (*how*). The only use of this routine in the algebra lessons was during the process of generating like terms to a given algebraic expression (*applicability*). The *closure* of the routine was the teacher's writing the like terms that students generated on the board (Figure 4.8). The whole process is given in Table 4.14.

4.1.3.3. The Casper Routine

The Casper routine was named after the cartoon called 'Casper the Friendly Ghost'. During an informal conversation, the teacher said that the appearance and disappearance of the friendly ghost Casper inspired her. In mathematics and particularly in algebraic expressions, there are symbols representing operations, like multiplying and adding, and coefficient 1 that may be hidden. For instance, there is a hidden coefficient 1 in front of x and a hidden multiplication sign between 2 and x in 2x, but they are not always written explicitly. The hidden expressions can be made visible or invisible, depending on the nature of the operations. Therefore, the teacher employs a routine called the Casper routine in algebraic expression lessons in such cases.

Below, the teacher continues to algebraic expressions focusing on the implicit elements in algebraic expressions. The discussion centers on the expression x^2 and

implicitly accompanying elements. The teacher asks the class, specifically targeting Meryem, what precedes x^2. When Meryem does not respond, the classroom mentions about the Casper. Then, the teacher clarifies that there is an implicit 1 as the coefficient of x^2, explaining that it is usually not written to simplify the expression visually, referring to it as making the expression more handsome.

> *2.53 Teacher:* Just x^2. What is in front of x^2? Rabia, what is in front of x^2? Sorry, Meryem.
> *2.54 Meryem*: […]
> 2.*55 RC*: Casper
> *2.56 Teacher:* There's a hidden 1 here with Casper, isn't there? But we don't make it visible right now. To make it more handsome, right? ...

Thus, the teacher ensures students recognize and remember that every algebraic term has a coefficient, even if it is 1 and not explicitly shown. In another example, below, students are asked to construct an expression that can be a like term to $-\frac{3x}{4}$. Sevgi responds $\frac{-1x}{4}$, which prompts a discussion about simplifying expressions by removing visually unnecessary elements (such as the coefficient 1), referred to here as the Casper mode. Again, the teacher uses Casper to engage the students in thinking about the implicit aspects of algebraic expressions, similar to the invisible character Casper the Friendly Ghost, to illustrate how the coefficient 1 is typically invisible in algebraic expressions. When Sevgi writes the expression as $\frac{-1x}{4}$, the teacher takes advantage of demonstrating that although her answer is mathematically accurate, it is more convenient to write it simply as $\frac{-x}{4}$ in algebra. Then, the teacher removed the Casper, meaning she removed 1. Thus, she focuses on simplifying algebraic expressions by addressing how coefficient 1 is typically handled in algebraic expressions.

> *2.78 Teacher*: … Yes, another, $-\frac{3x}{4}$? Sevgi?
> *2.79 Sevgi*: $\frac{-1x}{4}$
> *2.80 Teacher*: -1x, what is the Casper mode?
> *2.81 Class*: -x
> *2.82 Teacher*: In other words, I mean this, your friend said this $\frac{-1x}{4}$ (writing on the board). We do not write it like this; we write its Casper mode, what are

we are going to write? (Answers from the classroom suddenly start to come in) Wait a minute, Zeki says.
2.83 Zeki: $-\frac{x}{4}$
2.84 Teacher: Yeah, let's see, we're removing the Casper. (Deletes the 1 in front of the x)

Table 4. 15. The Casper routine in concepts in algebraic expression

Prompt	Routine	How?	When?	
			Applicability	Closure
Asking for comments or to remove symbols and numbers that actually exist but do not appear physically in algebraic expressions	Casper Routine	Removing coefficient 1 in a given algebraic expression	Generating like terms for a given algebraic expression	Removing the coefficient 1 in an expression Ex: Removing 1 in $\frac{-1x}{4}$ and writing $\frac{x}{4}$

As stated in Table 4.15, the Casper routine is used when generating like terms in a given algebraic expression (applicability). The routine starts by asking students to comment on or remove symbols and numbers that actually exist but do not appear physically in algebraic expressions (prompt). Afterward, students and the teacher remove coefficient 1 or make coefficient 1 visible, which does not normally appear according to the situation (how). The closure of the routine is removing coefficient 1.

4.1.4. Endorsed narratives in the topic of concepts in algebraic expressions

The final discursive element in this study is endorsed narratives. Narratives refer to any series of spoken or written statements that are presented as descriptions of objects, relationships between objects, or actions involving objects. These narratives can be endorsed or rejected based on specific procedures used to support or validate the discourse (Sfard, 2008). Once they are endorsed, narratives are regarded as mathematical facts like definitions, theorems, and axioms. This section presents narratives that have been endorsed by the use of words, visual mediators, and routines in the context of algebraic expressions. In algebraic expressions lessons,

many narratives both related to properties and definitions of algebraic ingredients endorsed. This section aims to focus on narratives commonly used in concepts in algebraic expressions courses, supported by the use of words, visual mediators, and routines. These narratives include the expressions that the teacher shared with the students as notes and recorded in their notebooks, as well as the explanations pronounced during the process of doing exercises.

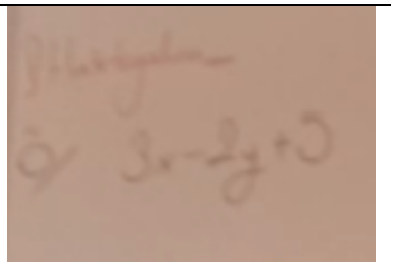
4.1.4.1. Algebraic expressions refer to expressions that consist of unknowns and entail mathematical operations

One of the narratives endorsed in the discourse was related to the definition of algebraic expressions. It was pronounced three times during the concepts in algebraic expressions topic. Moreover, defining algebraic expressions was mostly uttered in the explanatory talk parts of the lesson. The teacher first uses this definition at the beginning of the lesson to introduce algebraic expressions. She explains that the given expression (3x+2y+5) is actually an algebraic expression since it contains both unknowns and operations. The definition of algebraic expression was endorsed by the word-use in the discourse. Throughout the instruction, unknowns, terms, number of terms, and constant terms were used to support the definition and use of algebraic expressions. For instance, in the excerpt below, the teacher defines algebraic expression. She supports this definition with the use of words such as unknown and term.

> *1.106 Teacher:* Expressions containing both operations and *unknowns* are referred to as algebraic expressions… Now you told me that there are *terms*. What were they? 3x, -2y, +5…

Besides, it was endorsed visually many times with the symbolic notations of algebraic expressions on the board (3x-2y+5). Finally, the related routines that endorse the algebraic expressions are ‘finding the like terms routine,’ ‘writing a like term,’ and ‘Casper routine.’ The narrative related to the definition of algebraic expression is given in Table 4.16. As seen in table the narrative was endorsed by words, visual mediators, and routines.

Table 4. 16. Definition of algebraic expressions in the topic of concepts in algebraic expressions

Narrative	Endorsed Word-Use	Endorsed Routine	Endorsed Visual Mediator
Algebraic expressions refer to expressions that consist of unknowns and entail mathematical operations.	*1.28 T:* Guys, when we are defining algebraic expressions, we use something like this. *Algebraic expressions* refer to expressions that consist of *unknowns* and entail mathematical operations.	Finding the Terms Writing a like term Casper Routine	

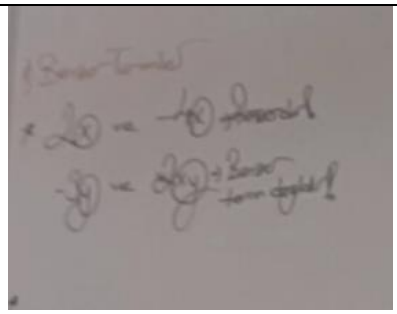
4.1.4.2. Terms with the same letters and the same powers of the letters are called Like Terms

Another narrative uttered in the concepts in algebraic expressions topic is the definition of like terms. This narrative was utilized by the teacher and students eight times throughout the instruction. The initial narrative was employed in the explanatory talk part, whereas subsequent narratives were uttered in the process of exemplification. In the following excerpt, the teacher makes an explanation about the like terms for the first time. Then, she asks the students to write this definition as a note in their notebooks. In the following process, they referred to the note they wrote while doing exercises.

> *1.164 Teacher:* Yes, then I see that all of them are exactly the same; that is, two terms are the same. It means the unknowns should be the same. When we see a different unknown, it is not the same. Let's do some examples and discuss together whether they are like terms! Okay? Let's write this first, then continue. There is a note about like term. Let's write! Note*: '*In an algebraic expression, terms with the same letters and the same powers of the letters are called like terms'.

Throughout the instruction, the most commonly uttered word-use is like terms. More than half of the word-use is actually related to like terms. Moreover, they use some visual mediators to endorse the narrative. These are algebraic symbols, and they circle or mark the unknowns to show like terms (Table 4.17). The related routines that endorsed the narrative are 'finding the like term' and 'writing a like term'.

Table 4. 17. Definition of like terms in the topic of concepts in algebraic expressions

Narrative	Endorsed Word-Use	Endorsed Routine	Endorsed Visual Mediator
Like terms in an algebraic expression refer to terms that have identical variables and corresponding exponents.	*1.143 T:* So, they both have unknown x and they are identical. Ah, then okay, we can say they are like terms...	Finding the like terms Writing a like term	

4.2. Mathematical Discourse Characteristics during the Explanatory Talks and Exemplifications in the Topic of Operations in Algebraic Expressions

This section provides a detailed analysis of word-use, visual mediators, routines, and endorsed narratives employed in explanatory talks and exemplifications within the topic of operations in algebraic expressions. After explaining the definition of algebraic expressions and associated terminology, as well as a series of activities pertaining to these concepts, the teacher proceeded to present the operations of addition and subtraction within algebraic expressions. Afterward, the teacher moved to multiplication in algebraic expressions.

4.2.1. Word-use in the topic of operations in algebraic expressions

As mentioned before, word-use in algebraic expressions lessons involves the usage of specific words and terms that are consistently employed throughout the instruction. This section offers an in-depth examination of the word-use in the topic of operations in algebraic expressions using commognitive framework. As discussed in section 4.1.1, the use of words is analyzed qualitatively regarding their context. Hence, in certain circumstances, even if the word is not uttered, it was included in the analysis because it is mentioned in the context. Similar to the previous section, the researcher again excluded the words from the analysis that the teacher used in a heading. The analysis focused on word usage across four subcategories: *routine-driven, object-driven,* and *phrase-driven*. Passive-use was not encountered in the

topic of operations in algebraic expressions. A hierarchical progression was anticipated among these categories as students progressed through the learning process.

A comprehensive analysis of the specific terms used in operations in algebraic expressions, based on the commognitive framework, is provided below. The following terms are included: *term, unknown, coefficient, the sum of coefficients, like terms, adding and subtracting, and multiplying*. As given in Table 4.18, the findings of the word-use on the topic of operations in algebraic expressions indicate that the use of phrase-driven words (%22) is the least, while the use of object-driven words (%46) is the most.

Table 4. 18. Word-use in the topic of operations in algebraic expressions

	Routine-driven	Phrase-driven	Object-driven	Total
Term			3	3 (%5)
Unknown	1		5	6 (%10)
Coefficient			1	1 (%2)
The sum of coefficients	5		2	7 (%11)
Like terms	13	10	19	42 (%67)
Algebraic expressions		2	1	3 (%5)
Total	19 (%31)	12 (%20)	31 (%49)	

Furthermore, each instance of word use is also evaluated based on whether it is employed in explanatory talks or as exemplifications. In the explanatory talk part of the instruction, words used were routine-driven three times, phrase-driven six times, and object-driven 21 times. Moreover, in the exemplification part, there are 58 times routine-driven use, 28 times phrase-driven use, and 47 times object-driven use.

Table 4. 19. Word-use in explanatory talks and exemplifications

	Explanatory Talk	Exemplification
Routine-Driven	3	58
Phrase-Driven	6	28
Object-Driven	21	47
Total	30 (%18)	133 (%82)

Table 4.19 indicates that the frequency of word-use categories in operations in algebraic expressions differs, with explanatory discussions containing the lowest frequency (18%) and exemplifications having the highest frequency (82%).

Findings indicate that a greater number of exemplifications were used throughout the instruction of operations in algebraic expressions. In other words, this points out that mostly examples and tasks are used to provide learners with opportunities to experience in operations in algebraic expressions. Below is given a comprehensive analysis of each specific word through the commognitive framework.

4.2.1.1. Term

In section 4.1.1, the teacher emphasized the importance of terms in algebraic expressions and their crucial role in the formation of such expressions. Additionally, she explained how to determine the terms in algebraic expressions. However, in this section, the term has been used as a mathematical entity or objectified form independent of the context. Below are two excerpts from the operations in algebraic expressions lessons. In this dialogue, the teacher is preparing to proceed to the operations in algebraic expressions. The teacher is getting ready to discuss how to perform operations on algebraic expressions.

> *2.142 Teacher:* Then, can we make a generalization like this? I can add like terms. But I can't add *terms* that are not like terms.
> …
> *2.144 Teacher*: ... … if they are different *terms*, it stays like this, I cannot operate on them. So, what is the result of this? It is here (Shows 2x-3y on the board)

She emphasizes the importance of recognizing the terms in order to correctly perform addition in algebraic expressions. In this context, as the term is used to explain addition in algebraic expressions and is treated as a mathematical object, demonstrating an object-driven usage. Besides, the term is used in explanatory talks. Table 4.19 provides the analysis of the term utilized in the teaching of operations in algebraic expressions according to the mathematical discourse categories and its uses in explanatory talk or exemplification.

Table 4. 20. Categorization of the use of *term* in the operations in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
2.108	*Teacher:* I give two examples, okay? 2x+3x. Yes, what kind of **terms** are these?	Writes 2x+3x on the board	Object-driven	Explanatory talk
2.142	*Teacher:* Then can we make a generalization like this? I can add like terms. But I can't add **terms** that are not like terms.		Object-driven	Explanatory talk
2.144	*Teacher:* … if they are different **terms**, it stays like this, I cannot operate on them. So, what is the result of this? It is here.	Shows 2x-3y on the board	Object-driven	Explanatory talk

Table 4.20 represents the analysis of the term through Sfard's commognitive theory and MDI framework. As in section 4.1.1, all word usage occurred in the lessons' explanatory talk parts. In other words, the teacher used the term while making explanations about operations in algebraic expressions. The table clearly indicates that the term is predominantly uttered by the teacher. All the usages of the term are object-driven. This implies that the term is used as a mathematical object regardless of the context.

4.2.1.2. Unknown

Unknown is also a commonly pronounced word in teaching and learning operations in algebraic expressions. Below is a dialogue giving an explanation of the role of the unknown during operations in algebraic expressions. Since the students were familiar with the meaning of the unknown from prior lessons, the teacher used the word as a mathematical object. This excerpt illustrates object-driven use of the word.

> *3.15 Teacher:* Then let's remember again, in algebraic expressions, I operate on the numbers but not the x's and unknowns. Because I attach them to the end (to the end of the parentheses). Is that right?

Furthermore, as the teacher employed unknown when instructing on the addition of algebraic expressions, this usage is categorized as explanatory talk.

The following discourse is an excerpt from the dialogue concerning simplifying the example x+2x+5x. The teacher clarifies how unknowns are handled in addition in algebraic expressions. Once again, the word unknown is used as a mathematical object. Thus, this is an object-driven use. Besides, this use was realized in the exemplification part of the lesson.

> *3.53 Teacher:* Wait. Normally, what do we do when we calculate? We add the numbers, attach the *unknown* at the end, and find the result. Now, I took this (pointing to 2x), I took this also (pointing to 5x). This one (points to x), remains. I need to perform an operation on this as well…

In line 4.132, the teacher clarifies the distinction between summing up terms in algebraic expressions that contain unknowns and those that do not. After that, she explains the process of combining constant terms, which are terms without unknowns, in algebraic expressions. This again illustrates the unknown being used as object-driven and exemplification part. The unknown is utilized as a mathematical entity or as a concluding expression. It can also be considered as an exemplification since it is used during the example solution.

> *4.132 Teacher:* It does not necessarily have to be unknown. They don't necessarily have to include an *unknown*. As we can see here, +4 and -2 are just numbers, and I operate with them. Just as I sum up them when they are accumulated in the bag, here I can sum up them in the same way. Even though +4 and -2 do not contain *unknowns* like x or y next to them, they are alike. So I can sum up them…

Consequently, students are acquainted with the unknown as it is included in the 6th-grade curriculum. Furthermore, the preceding topic concepts in algebraic expressions involve both phrase-driven and object-driven usages. In this section, as seen in Table 4.20, the word unknown is used just as object-driven in the topic of operations in algebraic expressions. The teacher did not need to provide further explanations or illustrate how to find the unknown. The word unknown is employed as a mathematical entity in this section. In addition, while just two uses are in explanatory talks, the remaining ones are predominantly used in the exemplification part of the instruction.

Table 4. 21. Categorization of the use of unknown in the operations in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
3.15	*Teacher:* Then let's remember again, in algebraic expressions, I operate on the numbers but not the x's and **unknowns**. Because I attach them to the end (to the end of the parentheses). Is that right?		Object-driven	Explanatory talk
3.53	*Teacher:* Normally, what do we do when we operate? We add the numbers, attach the **unknown** to the end, and find the result. Now, I look here. I took this, and I took this. This remains. I need to perform an operation on this as well.	Shows 2x Shows 5x Shows x	Object-driven	Exemplification
4.132	*Teacher*: … They don't necessarily have to include an **unknown**. As we can see here, +4 and -2 are just numbers, and I operate with them. Just as I sum up them when they are accumulated in the bag, here I can sum up them in the same way. Even though +4 and -2 do not contain **unknowns** like x or y next to them, they are alike. So I can sum up them…		Object-driven	Exemplification
4.144 4.145 4.146	*Teacher:* Our friend has pasted Casper here. Now, Mustafa, which ones look similar? Which ones look the same when you look at them? For example, if it's an apple, it's an apple; if it's an orange, it's an orange. Which ones? Just a second, don't interrupt (warns the class). Look at the **unknowns**. *Mustafa:* Teacher, is it -1 and -7? *Teacher:* Look at the **unknowns**. What is the unknown? +4x, -1x, what do they all have at the end? X, so these are like terms. Now I look here, -7y. Look, one has x and the other has y, so they are not similar, Mustafa. You were here when I explained this topic.	Points to the front of -x	Object-driven	Exemplification

Table 4.21. (continued)

5.87	*Teacher:* I want to say something. Now look here. Whether there is an unknown next to it or not, I will always do the same thing. There is an unknown expression next to them, but I can combine them, which makes -11. These both don't have any unknown, so I can combine them, and I get -7. I make the same operation in both situations. The only difference is that this one has an unknown. Do you understand?	Points to -5y - 6y Points to +4 - 11	Object-driven	Exemplification
6.101	*Teacher:* ... Until now, I used numbers and **unknowns** when I asked you an exercise. That is, I directly gave you the algebraic expressions, right?		Object-driven	Explanatory talk
7.78	*Teacher:* Büşra, tell me, +x plus +x equals +2x. Yes, wait, listen... In algebraic expressions, we generally put the **unknowns** front and numbers back...		Object-driven	Exemplification

The analysis of the unknown is given in Table 4.21. Two of the uses occurred in explanatory talks, and all other uses are in exemplifications. The teacher's discourse predominantly prevailed, and she employed the term. As in line 4.15, the students generally join lessons with short phrases.

4.2.1.3. Sum of Coefficient

The sum of coefficients is one of the most frequently used phrases in operations in algebraic expressions. In the previous topic, concepts in algebraic expressions, much attention is given to understanding the meaning and how to calculate the sum of coefficients. On the other hand, in the current topic, the class is well acquainted with the sum of coefficients and they commonly refer to it as a mathematical entity. Object-driven usage is observed more frequently. Additionally, there are also usages determined that are routine-driven and phrase-driven.

For instance, in the excerpt below, while the teacher is making explanations about operations in algebraic expressions, she clarifies how coefficients are added or

subtracted while the variables remain unchanged. This is an example of phrase-driven usage in explanatory talks.

> *2.219 Teacher:* Exactly. The coefficients are added or subtracted, so I don't touch the type, do you understand? x remains x, y remains y. I only perform operations with the numbers.

In the following dialogue, although the sum of coefficients is not directly mentioned, it is included in the analysis due to its usage in the context. Below, the teacher guides students to identify and combine like terms. They perform the action of summing up coefficients without explicitly mentioning them. This is a routine-driven use during the exemplification part of the lesson.

> *4.47 Teacher:* Then we will use x^2. Now, which are the other like terms?
> *4.48 Boy:* … (Underlines +3xy and +4xy)
> *4.49 Teacher:* Write it down and combine them.
> *4.50 Boy:* … (Writes 3xy+4xy= (+3+4) and thinks)
> *4.51 Teacher:* What remains there, that you don't use?
> *4.52 Boy:* xy (Completing the remaining calculation)

In line 5.87, the teacher demonstrates the process of combining terms in algebraic expressions, emphasizing that the procedure is the same whether or not the terms contain unknowns. The excerpt is extracted from the exemplification part of the lesson and exemplifies the routine-driven use of the sum of coefficients in order to combine like terms.

> *5.87 Teacher:* I want to say something. Now, look here. I will always do the same thing, whether there is an unknown or not (pointing to -5y-6y). There is an unknown here, I'm combining them, and it equals -11 (pointing to +4-11). There is no unknown here, and if I combine them, it equals -7. I am doing the same operation in both cases. The only difference is that there is an unknown in this one. Did you understand? I'll write up here -11y -7 (Writes -11y-7 on the board).

After a few routine-driven and phrase-driven usages of the sum of coefficients, they used predominantly object-driven manner in operations in algebraic expressions. For instance, in the next dialogue, they are calculating the perimeter of a square with one side length of x+3.

6.55 Girl: … (Writes (x+3)+ (x+3)+ (x+3)+ (x+3)+ (x+3) on the board)
6.56 Teacher: Let's remove the parenthesis. Since there is no sign in front of the parenthesis, it comes out exactly the same (Helps the student to take +(x+3) out of the parenthesis). Let's continue. How many x's are there?
6.57 Girl: 4
6.58 Teacher: There are 4 x's +4x…

Although the sum of coefficients is not explicitly mentioned, in order to calculate the perimeter of a square with a given side length expressed as an algebraic expression, they need to sum the coefficients. The student, who is already acquainted with this phrase and knows the routine, expresses that the result is directly equal to 4x with the help of the teacher. This is an example of object-driven usage because it is employed as a terminating statement regarding the solution in the exemplification section of the lesson.

6.91 Teacher: These are like terms. There is 1 in front of x. Now I ask, +3x -1x, which one remains (Points to 3x and -x)? Sivri.
6.92 Sivri: The positive.
6.93 Teacher: The positive, how many?
6.94 Sivri: +2x

In another dialogue above, they also use the sum of coefficients in object-driven manner. The operation of combining the terms +3x and -1x results in a sum of coefficients and equals 2x.

According to Table 4.22, the instruction flow starts with routine-driven and phrase-driven uses and continues with mostly object-driven uses. However, as indicated in lines 5.87 and 6.180, the teacher returned to routine-driven uses to meet students' requirements.

Table 4. 22. Categorization of the use of unknown in the operations in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
2.215	*RC*: coefficient {Coefficients are added or subtracted when adding or subtracting like terms}		Phrase-driven	Explanatory talk

Table 4.22. (continued)

2.219	*Teacher:* Exactly. The coefficients are added or subtracted, so I don't touch the type, do you understand? x remains x, y remains y. I only perform operations with the numbers.		Phrase-driven	Explanatory talk
4.22	*Boy:* …	Writes $2x^2$-x^2=(2 - 1) and hesitates	Routine-driven	Exemplification
4.23	*Teacher:* You used 2, you used -1, what did you not use?			
4.24	*Boy:* …			
4.25	*Teacher:* Look, you summed up the numbers. There are x^2s left. You pasted it there. +2 -1 What	Points to x^2		
4.26	is in the bag?	Hears +1 sounds from the class and he writes +1 on the board		
4.27	*Boy:* …			
4.28	*Teacher:* He is doing. Go on, equals...			
	Boy: …	Writes +1x^2		
4.47	*Teacher*: Then we will use x^2. Now, which are the other like terms?	Underlines +3xy and +4xy	Routine-driven	Exemplification
4.48	*Boy:* …			
4.49	*Teacher:* Write it down and combine them.	Writes 3xy+4xy= (+3+4) and thinks		
4.50	*Boy:* ...			
4.51	*Teacher:* What's left there that you don't use?			
4.52	*Boy:* xy			
4.149	*Teacher*: Go on, Şevket.	Underlines +4x and -1x and combines	Routine-driven	Exemplification
4.150	*Şevket*: …			
5.26	*Sudenaz:* …	Marks the terms that are like. Writes -3x-4x on the bottom line	Routine-driven	Exemplification
5.27	*Teacher*: Put the equation. Can we combine it now even if we don't put the parenthesis?			
5.28	*RC:* Yes			
5.29	*Teacher*: I'm asking, look at the x's, watch well. I'm doing it without parenthesis, look at me right now. +3 -4 which one is left?			
5.30	*RC:* The minus, one more.			
5.33	*Teacher*: Let's give it a try, okay? Now, listen, I'm asking, -8y -5y what will remain in my bag?	Sudenaz writes -8y -5y on the board	Object-driven	Exemplification
5.34	*RC:* Minus, -13y			

Table 4.22. (continued)

5.44	*Teacher*: +8y+4y What remains in the bag? Plus? How many?		Object-driven	Exemplification
5.45	*Balı*: 12y			
5.46	*Teacher*: 12y, ok…			
5.46	*Teacher*:…The other one? -5x -2x, now what's in the bag? What's in the bag?		Object-driven	Exemplification
5.47	*Balı*: Minus.			
5.48	*Teacher*: Yes. How many?			
5.49	*Balı*: 7			
5.87	*Teacher*: I want to say something. Now, look here. I will always do the same thing, whether there is an unknown or not. There is an unknown here, I'm combining them, and it equals -11. There is no unknown here, and if I combine them, it equals -7. I do the same operation in both cases. The only difference is that there is an unknown in this one. Did you understand? I'll write up here -11y -7.	Points to -5y -6y Points to +4 -11 Writes -11y-7 on the board	Routine-driven	Exemplification
6.22	*Teacher*: What's in front of the x's?		Routine-driven	Exemplification
6.23	*RC:* Casper			
6.24	*Teacher*: Let's put Casper 1. +1x, +1x, +1x makes how many x, finger?	Shows 3 x's.		
6.25	Mehmet.			
6.26	*Mehmet:* 3x, 4, ... 3 *Teacher*: 3, what? 3x…			
6.26	*Teacher*:…+1 +2 more?	Writes 3x+3	Object-driven	Exemplification
6.27	*Girl:* +3			
6.28	*Teacher*: +3 yes guys, the circumference equals to this			
6.41	*Teacher*: Look here 1x plus 1x?		Object-driven	Exemplification
6.42	*RC:* +2x			
6.43	*Teacher*: +2x +2x more, +6x.			

Table 4.22. (continued)

6.44	*Teacher*: What is +2 plus +2?		Object-driven	Exemplification
6.45	*RC:* +4			
6.46	*Teacher*: What is -5 plus -5?			
6.47				
6.48	*RC:* -10			
6.49	*Teacher*: Which one remains?			
6.50	*RC:* The negative one. *Teacher*: How much remains from the negative? 6 remains.			
6.55	*Girl:* …	Writes (x+3)+ (x+3)+ (x+3)+ (x+3)+ (x+3) on the board Helps the student to take +(x+3) out of the parenthesis	Object-driven	Exemplification
6.56	*Teacher*: Let's remove the parenthesis. Since there is no sign in front of the parenthesis, it comes out exactly the same. Let's continue. How many x's are there?			
6.57	*Girl:* 4			
6.58	*Teacher*: There are 4 x's +4x…			
6.58	*Teacher*: …What is in the bag +3 +3 +3 +3 +3?		Object-driven	Exemplification
6.59	*Girl:* +12	Writes +4x+12		
6.91	*Teacher*: These are like terms. There is 1 in front of x. Now I ask, +3x -1x, which one remains? Sivri.	Points to 3x and -x	Object-driven	Exemplification
6.92	*Sivri:* The positive.			
6.93	*Teacher*: The positive, how many?			
6.94	*Sivri:* +2x			
6.95	*Teacher*: Tell me +4 +2, Merve?		Object-driven	Exemplification
6.96	*Merve:* +6			
6.162	*Teacher*: … Let's do the other side. x +1. Can we do it like this? It's a bit messy, let's combine? How many x's are there?	Writes x+x+1+1+1 under the model Points to x+x+1+1	Object-driven	Exemplification
6.163	*RC:* 2			
6.164	*Teacher*: Positive or negative?			
6.165	*RC:* Positive			
6.166	*Teacher*: +2x...			
6.174	*Teacher*: First, combine x's. 2x plus 1x?		Object-driven	Exemplification
6.175	*Sinem:* 3			
6.176	*Teacher*: Well done.			

Table 4.22. (continued)

6.179	*Zeki:* There are 3 x's, right, and 3 1's ? combine them and... The answer is the same.		Routine-driven	Exemplification
6.180	*Teacher*: Yes. So your friend tells us. He says 1x plus 1x plus 1x plus 1x makes 3x, +1 plus +1 plus +1 makes +3, the same result. He is right.			
7.26	*Samet:* +x	writes -3x-2+x+3	Object-driven	Exemplification
7.27	*Teacher*: Okay, good, there are -3x and +x. Which one remains?			
7.28	*Samet:* Negative			
7.29	*Teacher*: It's a negative. We can write it next to the equal sign, there is an equal above. Negative... How many x's are there,?			
7.30	*Samet:* 2 x's.			
7.31	*Teacher*: 2 good for you, 2 what?			
7.32	*Samet:* -2x			
7.35	*Teacher*: Then? -2, +3 Which one is more? more than the plus, how much more?	Samet points to the positive	Object-driven	Exemplification
7.36	*Samet:* 1			
7.61	*Teacher*: …Let's combine them. There are like terms +2x -x		Object-driven	Exemplification
7.62	*SC:*+x			
7.78	*Teacher*: Büşra tell me,...+x plus +x, +2x yes, wait, now listen…		Object-driven	Exemplification

Table 4.22 represents the analysis of the sum of coefficients in the topic of operations in algebraic expressions. As seen in the table, the first two usages are employed in the explanatory talks, and the remaining usages occurred in exemplifications. All of the uses in explanatory talks are in phrase-driven manner, while those in exemplifications are routine-driven and object-driven. Analysis of the table reveals that the initial two usages are phrase-driven. Afterward, routine-driven usage took place, and at the end of the instruction, object-driven usage was observed. As in 6.180, it was assumed that the hierarchy is maintained even when she

occasionally transitions to routine-driven. Subsequently, at the end of the instruction, they reached the object-driven usage as intended.

4.2.1.4. Like term

Like term is the most frequently uttered phrase in operations in algebraic expressions, similar to the concepts in algebraic expressions. There are 30 instances of object-driven usage. Seven of them are employed in explanatory talks, while the remaining ones are uttered in the exemplification part. On the other hand, the phrase-driven level has been used ten times. All of them are uttered in the exemplification part of the lesson. While the routine-driven level is utilized 14 times, completely in the exemplification of the instruction.

In the dialogue below, the class identifies and combines like terms in the algebraic expression 2x-3y+x-2y. Firstly, they determine that 2x and +x are like terms. Afterward, they decide that -3y and -2y are like terms too. The teacher initially crosses out 2x and x, and then underlines 3y and -2y. In this case, routine-driven use is employed, as they performed the action of finding the like term in the exemplification part of the lesson.

> *2.150 Teacher:* Are there any other like terms?
> *2.151 RC:* Yes, there are.
> *2.152 Teacher:* Tell me.
> *2.153 Boy:* x and 2x.
> *2.154 Teacher:* x and 2x. I can combine them as well, right? Then I can do this: watch carefully, look. these are like terms, and these are like terms. Is that right?
> *2.155 RC:* Yes

In the following dialogue, they identify the like terms when combining algebraic expressions. In line 2.197, the teacher uses the like term in an object-driven manner. After combining the algebraic expressions 2x-3y+x-2y, the teacher clarified the process. Therefore, this usage takes place in the explanatory part of the lesson.

> *2. 197 Teacher:* Do you understand? Then, these two are like terms (points 2x+3x) , I could combine them. These are not like terms (Points 2x-3y), I

could not do any operations, I couldn't even move the pencil. Then, there are like terms in the expression, and I've combined them. Any question?
2.198 RC: No

In the excerpt below, they are working on the algebraic expression -7x+3y+2x-y. The teacher asks Sudenaz for like terms. With the guidance of the teacher, Sudenaz first verbally says the like terms and then marks them (+3y and -y). This excerpt employs the routine-driven use of like terms during the exemplification part of the lesson.

3.132 Teacher: Which are they like terms? First, can you tell verbally?
3.133 Sudenaz: +3y and -y
3.134 Teacher: Okay, put a mark on them, group them.
3.135 Sudenaz: … (Marks +3y and -y)

Below, there is an excerpt from exercise 7a-2b+4c-c+3d-2a+10b. They are finding the like terms respectively to combine the expression. The teacher is marking -10b and 2b in line 3.184, while Samet is underlining +4c and -c in line 3.185 to indicate they are like terms. Since they perform the action of finding like terms, routine-driven use is again observed here.

3.182 Teacher: What is in the bag? Yes, there are 12 negatives, I'm pasting b nest to it. Are there any other like terms?
3.183 RC: Yes
3.184 Teacher: Yes, who is coming? (Marks -10b and -2b) Samet come here.
3.185 Samet: ... (Underlines +4c and -c)

Below is an excerpt from combining the expression -(-4+2x+2). In line 4.124 the teacher clarifies which terms can be considered as like terms. She explains that -2 and -2x are not like terms since -2x contains an unknown, whereas -2 does not. Thus, the teacher employed the phrase-driven use in this instance by providing explanations of like terms while a solution of example.

4.124 Teacher: One has x, and the other doesn't. How can they be like terms? Finger. Think carefully and pay attention. Balı tell me which are they like terms?
4.125 Balı: -2 and -2x
4.126 Teacher: One of them has an x next to it, and the other doesn't….

Above, the teacher explained how to find and determine the like terms using numerous examples related to it. Subsequently, in lines 6.91 and 7.61, she employed the word as a mathematical object in an object-driven use.

> *6.91 Teacher:* Let's combine +2. These are like terms. x has 1 in front of it. I'm asking now, +3x and -1x which ones left? Sivri?
> *6.92 Sivri:* From the positive
> *7.61 Teacher:* Let's combine them, there are like terms. +2x and -x

In the prior topic, concepts in algebraic expressions, like terms were mainly used in routine-driven and phrase-driven manner. However, as seen in Table 4.23, object-driven use is mainly observed in the topic of operations in algebraic expressions. At the beginning of the instruction, in line 2.104, the teacher states that they have previously learned like terms and she uses it as object-driven. For a while, they continue to use the word as object-driven. As they solve exercises related to operations in algebraic expressions, the teacher returns to routine-driven and phrase-driven usages since she considers that it is necessary for students' understanding. For instance, in line 3.103 she returned the phrase-driven use. In addition, in line 3.140, they perform the action of finding the like term to combine algebraic expressions as a routine-driven use. Therefore, even though they have progressed in a hierarchical progressive use, from passive-use to object-driven use, they are more adaptable and can return to other forms of use as necessary. After considering that students are capable of using and understanding at object-driven level, the word is used at that level.

Table 4. 23. Categorization of the use of like terms in the operations in algebraic expressions

Line	What is said	What is done	Type of word use	Explanatory talk/ Exemplification
2.104	*Teacher:* We have learned the **like terms**. Any problems? What is next? Addition and subtraction in algebraic expressions.		Object-driven	Explanatory talk

Table 4.23. (continued)

2.108 2.109	*Teacher:* ...I am writing two examples, okay? 2x+3x. Yes, which terms are they? *RC:* 2x+..., **Like terms**, **like terms**.	Writes 2x+3x	Object-driven	Explanatory talk
2.120	*Şevket:* Teacher, do we subtract **like terms**?		Object-driven	Explanatory talk
2.142	*Teacher*: Then, can we make a generalization? I can combine **like terms**, but I can not combine unlike terms.		Object-driven	Explanatory talk
2.146 2.147	*Teacher:* Here comes 2x-3y+x-2y. Which are **like terms**? Tell me. Tell me, Büşra. *Büşra:* -3y and -2y		Object-driven	Exemplification
2.150 2.151 2.152 2.153 2.154 2.155	*Teacher:* Are there any other **like terms**? *RC:* Yes, there are. *Teacher:* Tell me. *Boy:* x and 2x. *Teacher:* x and 2x. I can combine them as well, right? Then I can do this: watch carefully, look. these are **like terms,** and these are **like terms**. Is that right? *RC:* Yes	First crosses out 2x and x, then underlines 3y and -2y	Routine-driven	Exemplification
2.197 2.198	*Teacher:* Do you understand? Then, these two are **like terms**, I could combine them. These are not like terms, I could not do any operations, I couldn't even move the pencil. Then, there are like terms in the expression, and I've combined them. Any question? *RC:* No	Points 2x + 3x Points 2x – 3y	Object-driven	Explanatory talk

Table 4.23. (continued)

2.203 2.204 2.205 2.206	*Teacher:* ...Let's go on. Then I have to look carefully, open my eyes, whether they are **like terms** or not. *Süleyman:* Teacher, we solved one more question. *Teacher:* Süleyman, we mixed them together for you, isn't that right? ... Isn't it a mixture of the third question and the second question? There are **like terms** and unlike terms. Is that right? *RC:* Mixed	Object-driven	Explanatory talk
2.209 2.211 2.212 2.213 2.214 2.215 	*Teacher:* …Note {While adding or subtracting in algebraic expressions, **like terms** are added or subtracted} … *Teacher:* So what do we add or subtract when combining **like terms**? *RC:* Numbers Teacher: Number, what are they? *Boy:* Coefficient *RC:* {Coefficients are added or subtracted when adding or subtracting **like terms**}	Object-driven	Explanatory talk
3.103 	*Teacher*: Lean back, we'll do it soon. What will we pay attention to in this question? We will be careful about whether they are the same kind, it means **like terms** or not.	Phrase-driven	Exemplification

Table 4.23. (continued)

3.104	*Teacher:* If they are **like terms**, I can combine them, right? Come on, are there any like terms? No, they are not. Did you study at home?	Examines the solutions	Object-driven	Exemplification
3.108	*Teacher:* ...Which are they **like terms**? Finger. Ladylike and gentlemanly finger-raisers, Merve?		Object-driven	Exemplification
3.109	*Merve:* +8 and -3			
3.110	*Teacher:* So this and that are like terms, this is different... Then what am I going to do with **like terms**? Let's write them down. What is that?	Points 8x and -3x Points 5y Writes down 8x and -3x	Routine-driven	Exemplification
3.118	*Teacher:* We generally don't say +5x, what do we say? 5x. Look, the result of these two is 5x. But I can't touch these, why? Because one is x and the other is y.		Phrase-driven	Exemplification
3.119	*Şevket:* They are not **like terms.**			
3.120	*Teacher:* I'm writing this as it is. Here the result 5x+5y.			
3.121	*Teacher:* Some of them parenthesize, take 8, take 5, and take -3, but they did not pay attention to the fact that these two are **like terms,** while this one is not. We can not take this, friends. As I did, I can combine like terms; if I can not touch them, I will write it as it is.		Object-driven	Exemplification

Table 4.23. (continued)

3.123	*Teacher:* ... Then I will combine **like terms** in one place and paste them above, got it?		Object-driven	Exemplification
3.128	*Teacher:* Somebody come. Sudenaz, come forward. Let's group the **like terms**. Which ones are **like terms**?		Object-Driven	Exemplification
3.129	*Sudenaz*:	Marks -7x and -y		
3.130	*Teacher:* Are they **like terms**? Decide carefully.			
3.132	*Teacher:* Which are they **like terms**? First can you tell verbally?		Routine-driven	Exemplification
3.133	*Sudenaz:* +3y and -y			
3.134	*Teacher:* Okay, put a mark on them, group them.			
3.135	*Sudenaz:* …	Marks +3y and -y		
3.138	*Teacher:* Others, which are they **like**		Routine-driven	Exemplification
3.139	**terms**?			
3.140	*Sudenaz*: -7x and +2x *Teacher:* Then put a sign under them so that we can understand. Now, the ones above are **like terms;** the ones below are **like terms**. What am I doing? Some of them continue the operation here, but no. Let's take these two **like terms** and paste them down...	Points to -7x +3y+2x-y=		
3.146	*Teacher:* What else? Yakup Kenan, let's do the others. Take the pen. Which ones are **like terms**?		Object-driven	Exemplification
3.147	*Yakup Kenan:* +3y and -y			

Table 4.23. (continued)

3.165	*Teacher:* Let's do an example and make it a little difficult by working together to find **like terms.** Come on, write down. Yes, write it. We are doing together.	Writes on the board 7a -2b +4c-c+3d-2a+10b=	Object-driven	Exemplification
3.166	*Teacher:* Lean back if you wrote. Don't do anything. Zeki, take the pen and, find a **like term**, do the operation below. Which one do you choose?	Marks 7a and -2a	Routine-driven	Exemplification
3.167	*Zeki*: ...			
3.180	*Teacher:* ...Let's see, are there any other **like terms**?	Underlines -2b and -10b	Routine-driven	Exemplification
3.181	*Merve:* …			
3.182	*Teacher*: What is in the bag? Yes, there are 12 negatives, I'm pasting b nest to it. Are there any other **like terms**?	Marks -10b and -2b	Routine-driven	Exemplification
3.183	*RC:* Yes			
3.184	*Teacher:* Yes, who is coming? Samet come here.			
3.185	*Samet:* ...	Underlines +4c and -c		
3.188	*Teacher:* Are there any other like terms?		Object-driven	Exemplification
3.189	*RC:* No			
3.190	*Teacher:* There is 8d			
3.191	*RC*: But there is not any **like term** to it.			
3.202	*Teacher:* Because they are different kinds, there are a, b, c, d. There are no other **like terms.** You will write now.		Object-driven	Exemplification
3.201	*Girl:* Teacher, are we going to do the operation if they were **like terms**?			
3.202	*Teacher:* Of course we are. Did you understand?			

Table 4.23. (continued)

4.2	*Teacher:* 2xy+4x^{2}y-6xy which ones are **like terms**? Write them first.	Writes 2xy+4x2y-6xy	Object-driven	Exemplification
4.3	*Zeki:* 2xy and -6xy	Marks 2xy and -6xy, parenthesize and writes the coefficients.	Routine-driven	Exemplification
4.8	*Teacher:* Zeki said that two of them are **like terms.** He've combined, equals -4xy, but he couldn't touch 4x^{2}y, because this is a different kind, not like term.		Phrase-driven	Exemplification
4.13	*Teacher:* Here comes. Yes, bonus question. You have 3 minutes. Put down the **like terms** and paste the result of the operation above. Come on, who did it? Lean back.	Writes 2x^2+3xy-x^2+4xy=	Object-driven	Exemplification
4.15 4.16 4.17	*Teacher*: Wait a minute, tick the **like terms** first. *Boy:* ... *Teacher:* one, two, what else? Okay, go on. Let's write first. *Boy:* ...	Marks 2x^2 and -x^2 Writes down 2x^2-x^2	Routine-driven	Exemplification
4.47 4.48	*Teacher:* Then we will use x2. What are the other **like terms**? *Boy:* ...	Marks +3xy and +4xy	Routine-driven	Exemplification
4.79 4.80 4.90 4.92	*Teacher:* +1 I'm writing. 3x+1 did you understand? I continue. *Girl:* Does it stay like that? *Teacher:* Are these **like terms**? *Girl:* No, they are not.	Points 3x and +1	Object-driven	Exemplification

Table 4.23. (continued)

4.114	*Teacher:* Ah, are there **like terms** here this time? Let's look at the prior one. Are they **like terms**?	Shows 3x+1	Object-driven	Exemplification
4.115	*RC:* No			
4.116	*Teacher:* Then it stayed as it is. **Like terms** or not?	Shows -3x-1		
4.117	*RC:* Not			
4.118	*Teacher:* It stays **like this**. Any **like terms**?	Points to +4-2x-2		
4.119	*RC:* Yes			
4.120	*Teacher:* Then I will combine them. I need to find the simplest situation. Which ones are **like terms**? Sultan?		Routine-driven	Exemplification
	Sultan: -2x-2			
4.121	*Teacher:* Are they **like terms**?			
4.122	*Sultan:* Yes.			
4.123				
4.124	*Teacher:* One has x, and the other doesn't. How can they be like terms? Finger. Think carefully, and pay attention. Balı tell me which are they like terms?		Phrase-driven	Exemplification
	Balı: -2 and -2x			
4.125	*Teacher:* One of them has an x			
4.126	next to it, and the other doesn't….			
4.126	*Teacher:* ... Samet?		Phrase-driven	Exemplification
4.127	*Samet:* +4 and -2			
4.128	*Teacher:* +4 and -2. Look they are alone. Let's combine them.			
4.131	*Girl:* Can I ask a question? In order to be **like terms,** is it necessary to be x or y in front of them?		Phrase-driven	Exemplification
4.132	*Teacher:* It does not necessarily have to be unknown. As we can see here, +4 and -2 are just numbers and I can combine them. Just as I combine them which are in the bag, in the same way, I combine +4 and -2 even if they don't have unknowns as x and y. Did you understand?			

Table 4.23. (continued)

4.144 4.145	*Teacher:* Our friend pasted the casper here. Now Mustafa, which ones are **like terms**? Which ones look similar? For example, if it is an apple, it is an apple or if it is an orange, it is an orange. Which ones? Don't get involved. Attention to unknowns. *Mustafa:* Are -1 and -7?	Phrase-driven	Exemplification
4.146	*Teacher:* Look at the unknowns. What are the unknowns? +4x and -1x What exists at the end of them? There is x, these are **like terms.** I came here -7y. Look, one has x and the other y, then they are not like terms...	Phrase-driven	Exemplification
4.148 4.149 4.150	*Teacher:* Go on Şevket. Explain. *Şevket:* 4x and -1x *Teacher:* Pay attention to the sign, you didn't do it before. Negative and positive, positive and positive. Put the equal sign Şevket. Put it in front of the parenthesis. What else? Let's write this. Since +3x and -7y are not **like terms**, I can not combine them. We paste them as they are.	Object-driven	Exemplification
5.12 5.13	*Teacher:* The answer is -2x-7. Mustafa, Are they **like terms**? *Mustafa:* No	Object-driven	Exemplification

Table 4.23. (continued)

5.14	*Teacher:* Why? One has x, and one has nothing; these are not **like terms**. So as to be like terms, both of them should have x's or not. Do you understand?		Phrase-driven	Exemplification
5.23	*Teacher*: ... Are there any **like terms**?		Routine-driven	Exemplification
5.24	*Sudenaz*: Yes			
5.25	*Teacher*: Show them and combine.	Marks the like terms and writes		
	Sudenaz: ...			
5.44	*Teacher:* Which ones are **like terms**? Negative times negative positive 4y; negative times positive -2y; the sign of 8y is already clear +8y. Which ones are **like terms**?...	He finds the like terms and combine	Object-driven	Exemplification
5.45	*Balı:* …			
5.61	*Teacher:* Which ones are **like terms**? +2x and +6x		Object-driven	Exemplification
5.77	*Teacher:* Which ones are **like terms**? Mustafa?		Object-driven	Exemplification
5.78	*Mustafa:* 5y and 6y			
5.79	*Teacher:* What else?			
5.80	*RC:* Yes			
5.81	*Teacher:* Let Mustafa say. Are there any other **like terms**?			
5.82	*Mustafa:* +4 and -11			
5.83	*Teacher:* Yes, there is nothing nest to them, so they are **like terms**.		Phrase-driven	Exemplification
5.100	*Teacher:* ... Firstly we write the sides. x+x+1+x+2 combine. Are there any **like terms**?		Object-driven	Exemplification
5.101	*RC:* Yes			
5.102	*Teacher:* Show me. Şevket +1x +1x+1x equals?			
5.103	*Şevket:* 3x			

Table 4.23. (continued)

6.20	*Teacher:* +1 we will continue. +x+2 are there any like terms? Which ones?		Routine-driven	Exemplification
6.21	*Süleyman:* x+1, x+2			
6.22	*Teacher:* Are they **like terms**? Which ones are **like terms**? What is in front of the x's?		Object-driven	Exemplification
6.41	*Teacher*: Let's combine. Remove the parenthesis. Guys, let's mark the **like terms**. 1x plus 1x?	Marks the like terms	Routine-driven	Exemplification
6.91	*Teacher:* Let's combine +2. These are **like terms**. x has 1 in front of it. I'm asking now, +3x and -1x which ones left? Sivri?		Object-driven	Exemplification
6.92	*Sivri:* From the positive			
7.61	*Teacher:* Let's combine them, there are **like terms**. +2x and -x		Object-driven	Exemplification

Table 4.23 represents the flow of the usage of like terms within the context of operations in algebraic expressions. In the topic of operations in algebraic expressions, the most pronounced word is like term. Although the teacher's use was dominant, as in section 4.1.1, the students participated actively in the activities related to the like term. Although the use of the like term was also noticed in exemplifications, it was mainly used in exemplifications. At the beginning of the instruction, routine-driven usages are predominant, while phrase-driven and object-driven usages are observed towards the end.

4.2.1.5. Algebraic expression

Algebraic expression is the last analyzed word in the topic of operations in algebraic expressions. In the excerpt below, the teacher only uttered expression. But she intended to use algebraic expression. She uses the word while performing operations

on algebraic expressions and making explanations. In other words, she uses it as a mathematical object. This use is considered as object-driven in explanatory talks.

> *2.114 Teacher*: ... parenthesis and say what is the sign of this here
> *2.115 RC:* Plus
> *2.116 Teacher:* +2x +.. Excuse me (+2+3) I'll paste the x at the end. When you look, what was collected in the +2+3 bag?
> *2.117 RC:* Plus 5
> *2.118 Ö:* Then my expression is 5x. Let's not say +5x, it's fine even if I don't put plus. Let it be properly, let it look properly. I can directly call it 5x

The next excerpt is again uttered in explanatory talks. She used the algebraic expression with a definition. Thus, this use is considered as phrase driven.

> *3.15 Teacher:* Then let's remember again, in algebraic expressions, I process the numbers, but I don't take the x's, the unknowns, because I paste them at the end (at the end of the parenthesis). Is that right?

4.2.2. Visual mediators in the topic of operations in algebraic expressions

The findings reveal that the primary visual mediator is algebraic notation in the topic of operations in algebraic expressions. Besides that, signs, arrows, circles, geometric figures, and modeling tiles were also frequently employed for several purposes. Moreover, a limited quantity of geometric shapes and modeling tiles was also utilized. For instance, in Figure 4.9, the teacher starts with the example of 2x+3x. She draws an arc to signify that the coefficients within brackets should be summed while demonstrating addition operations in algebraic expressions.

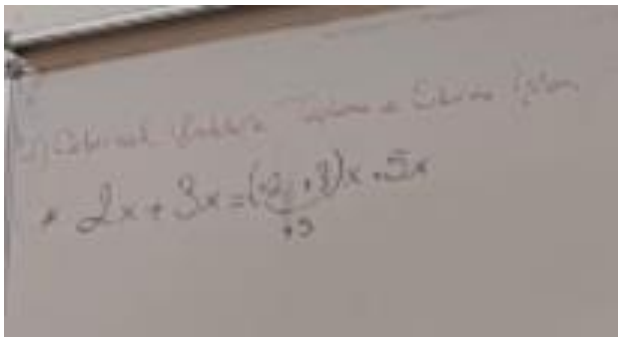


Figure 4. 9. Algebraic notations and arcs in addition and subtracting in algebraic expression example

Following that, the teacher writes a couple of examples on the board as in Figure 4.10, which are related to simplifying algebraic expressions. In these examples algebraic notations are again predominantly used visual mediator. In addition, arrows and arcs are used to indicate like terms. Arcs are also used to indicate that the coefficients in brackets should be added.

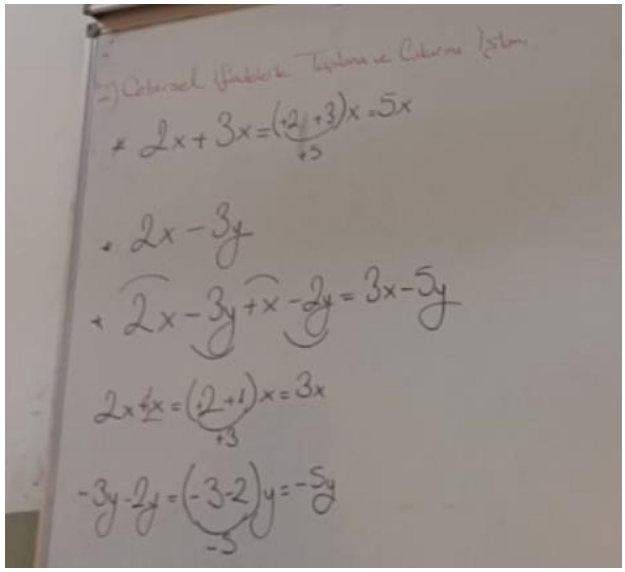


Figure 4. 10. Algebraic notations, arcs, and arrows in addition and subtraction in algebraic expression examples

Figure 4.11 represents another simplifying example of algebraic expressions. The teacher again employs arrows and arcs to illustrate like terms and draws an arc to indicate that the coefficients within the paratheses should be summed. Finally, she marks the simplest version of the expression by covering it in a rectangle.

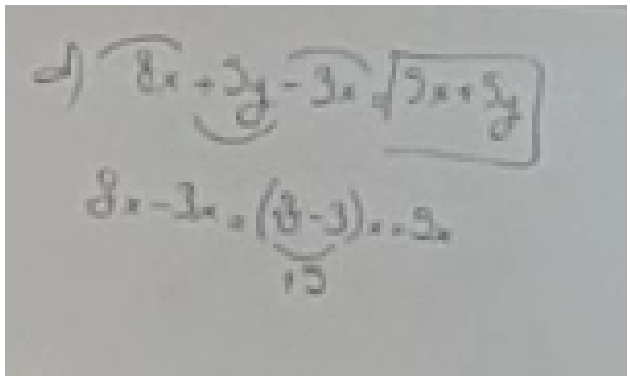


Figure 4. 11. Algebraic notations and arcs in addition and subtracting in algebraic expression example

Figure 4.12 illustrates the visual mediator of multiplying the algebraic expression within the parenthesis by a negative value. Besides algebraic notations, in order to multiply the algebraic expressions within parentheses by a negative value, arcs are used to indicate that each sign in the parenthesis should be multiplied. Then each term that is removed from the brackets is again indicated by arcs accompanied by its sign.

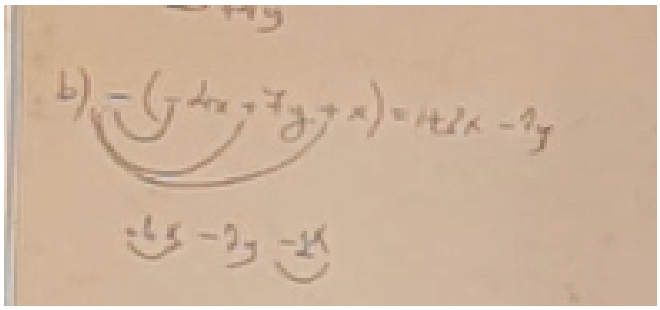


Figure 4. 12. Algebraic notations and arcs in addition and subtracting in algebraic expression example

On the other hand, Figure 4.13 illustrates the visual mediator of multiplying the algebraic expression within the parenthesis by a positive value. Again, algebraic notations and arcs are used to visualize the removing the parenthesis process. Arcs indicate that each sign in the parenthesis should be multiplied by the positive value.

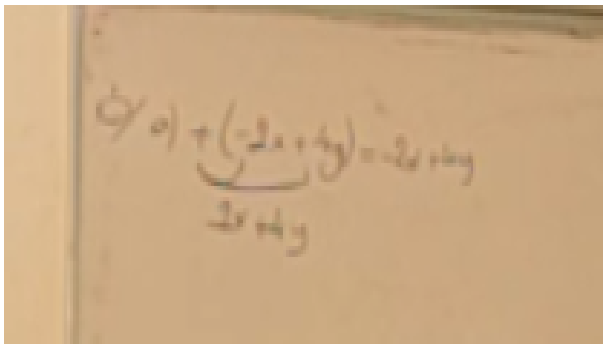


Figure 4. 13. Algebraic notations and arcs in addition and subtracting in algebraic expression example

Furthermore, Figure 4.14 depicts the simplification of an algebraic expression with two parentheses. In this context, arcs are employed to remove both parentheses. After removing the parentheses, arcs are again used to indicate like terms.

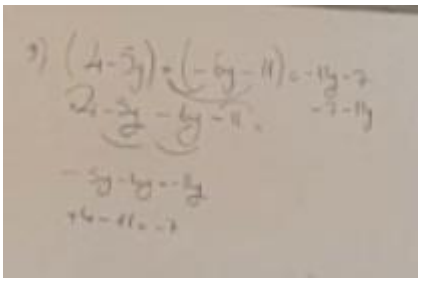


Figure 4. 14. Algebraic notations and arcs in addition and subtracting in algebraic expression example

Figure 4.15 illustrates another example of employing modeling tiles in operations on algebraic expressions. The example involves the construction of an algebraic expression in accordance with a given model, with yellow tiles representing x and pink tiles representing the number 1.

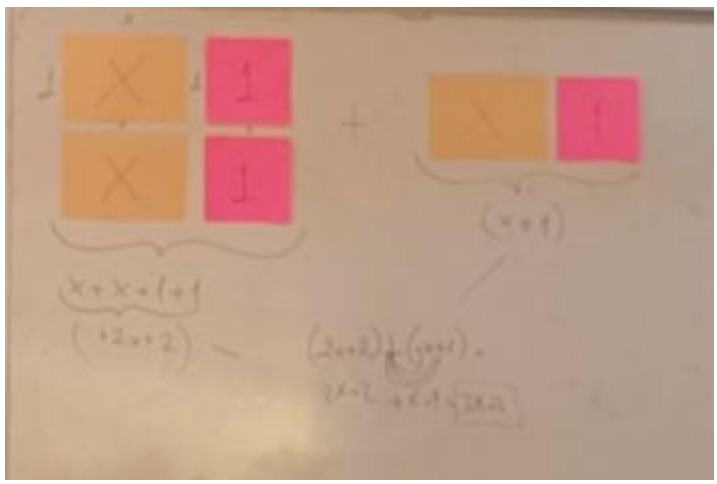


Figure 4. 15. Algebraic notations and modeling tiles in addition and subtracting in algebraic expression example

Figure 4.16 is a visual representation of the perimeter of a rectangle whose side lengths represent an algebraic expression. Here again, algebraic notations, arrows, and arcs are used along with a rectangle.

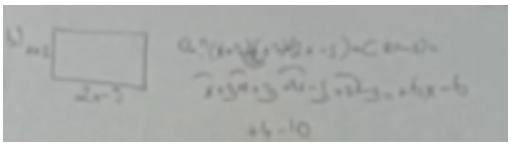


Figure 4. 16. Algebraic notations and geometric figures in addition and subtracting in algebraic expression example

As a conclusion, algebraic notations are mostly used as visual mediators in operations in algebraic expressions. Additional visual mediators used in teaching algebraic expressions contain circles, arrows, and arcs to demonstrate like terms. In addition, geometric figures and modeling tiles were utilized in operations in algebraic expressions, contrary to concepts in algebraic expressions. The following section comprises routines, which represent other mathematical discourse characteristics.

4.2.3. Routines in the topic of operations in algebraic expressions

This section specifically focuses on the routines related to operations in algebraic expressions. An analysis is conducted on discursive patterns that are repetitive actions in the discourse, in order to determine routines while the instruction of operations in algebraic expressions. there are four routines specified in this section. These are *finding the like terms, casper routine, addition and subtraction in algebraic expressions*, and *removing the parenthesis*. Table 4.24 presents the frequencies of routines.

Table 4. 24. Frequency of routines in operations in algebraic expressions

Routines	Explanatory Talk	Exemplification	Total
Finding the like terms	0	14	14 (%28)
Casper routine	0	6	6 (%12)
Addition and subtraction in algebraic expressions	2	24	26 (%52)
Removing the parentheses	2	3	4 (%8)
Total	4 (%8)	46 (%92)	

In the topic of operations in algebraic expressions, the most commonly used routine is addition and subtraction in algebraic expressions (%52). Following that finding the like terms (%28) is the second most frequently used routine. Removing the parenthesis routine (%8) is the least frequently used one. According to Table 4.24 the great majority of the routines are in the exemplification part of the lesson (%92). An extended description of the routines is provided below.

4.2.3.1. Finding the like terms

Finding the like terms routine is also applied in operations in algebraic expressions. Students have learned this routine in the prior topic of concepts in algebraic expressions. In the present topic, they can apply the routine in more complex examples related to combining algebraic expressions.

For instance, in the following dialogue, they apply the routine of finding the like terms to simplify the expression 2x-3y+x-2y. The teacher asks students to determine the like terms. Büşra states that -3y and -2y are like terms. Then the teacher asks whether they can combine these terms, and the class confirms the answer. Afterward, she asks whether there are any other like terms. The class answered that there are other like terms in the algebraic expression. Then, a boy says that x and 2x are also like terms.

> *2.146 Teacher:* Now, here comes 2x-3y+x-2y (writes on the board). Which are these like terms? Tell me, Büşra.
> *2.147 Büşra:* -3y and -2y.
> *2.148 Teacher:* Can I combine these two?
> *2.149 RC:* We can combine them, yes.
> *2.150 Teacher:* Are there any other like terms?
> *2.151 RC:* Yes, there are.
> *2.152 Teacher:* Tell me.
> *2.153 Boy:* x and 2x.
> *2.154 Teacher:* x and 2x. I can combine them as well, right?

Below, they are combining another expression 7a -2b +4c-c+3d-2a+10b. The teacher asks Zeki to find the like terms. He highlights 7a and -2a. After that the teacher asks for any other like terms. This time, Merve underlines -2b and -10b, and Samet highlights +4c and -c. Thus, they have found all of the like terms in the given expression.

> *3.166 Teacher:* ...Zeki, take the pen and find a like term, do the operation below. Which one do you choose? (She shows the operation 7a -2b +4c-c+3d-2a+10b= ? on the board)
> *3.167 Zeki:* ...(indicates 7a and -2a by drawing with an arc.)
> ...
> *3.180 Teacher:* Let's see, are there any other like terms?

3.181 Merve: ...(Underlines -2b and -10b with an arc to indicate that they are like terms)
3.182 Teacher: ...Are there any other like terms?
3.183 RC: Yes
3.184 Teacher: Yes, who's coming? (underlines -2b and -10b) Samet, come.
3.185 Samet: (He draws an arc under +4c and -c and writes -c as -1c.)
3.186 Teacher: Well done, write it below

Another representative illustration of the routine is observed when the teacher asks for the like terms in the algebraic expression given on the board (-(-4x+7y+x)). This routine occurred before the addition and subtraction of algebraic expressions.

4.144 Teacher: …Now Mustafa, which ones seem similar? (Points to "-(-4x+7y+x) = +4x-7y-1x" on the board) When we look at them, they seem the same? If it is an apple than the other one is an apple or if it is an orange than the other one is also an orange. Wait a minute, don't interrupt (Warning the class) Look at the unknowns!
4.145 Mustafa: Teacher, -1 and -7?
4.146 Teacher: Look for the unknown. What is the unknown? +4x, -1x, what exists at the end of both. Here is x, these are like terms (showing +4x and -1x). Come here, -7y. There is x and there is y, so these are unlike terms Selim.
4.147 Teacher: Go on Berk. Tell us.
4.148 Şevket: (He marks +4x and -1x, than calculates)

The teacher asks students to find the like terms in order to perform the calculation. Mustafa gives the wrong answer and the teacher warns him to look at the unknowns. She emphasizes their similarities, and later another student called Şevket completes the solution (Figure 4.17). He marks 4x and -1x to indicate the like terms.

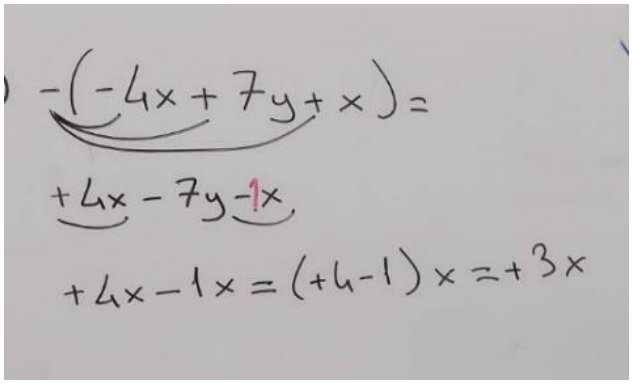


Figure 4. 17. Şevket's solution for finding the like terms before addition and subtraction

Table 4. 25. Finding the like terms routine in algebraic expressions lessons

Prompt	Routine	How?	When?	
			Applicability	Closure
Asking for the like terms in the given algebraic expressions	***Finding the like terms***	Marking the terms with the same unknown in the given algebraic expression	Finding the like terms in the given algebraic expressions	Marking the like terms in algebraic expressions
			Finding the like terms in given algebraic expressions before addition and subtraction	Highlighting the like terms in algebraic expressions

As inferred from the dialogues above and through Sfard's framework, Table 4.25 indicates the meta-rules for finding the like terms routine. The *prompt* is the teacher's asking about the like terms. After the teacher's request, students mark the like terms in accordance with the teacher's instructions defining *how* the routine occurs. We know that a*pplicability* is the condition when the routine actually occurs. This routine includes finding the like terms in algebraic expressions and finding the like terms before addition and subtraction in algebraic expressions. For a successful ending, the *closure* of the routine is marking, highlighting, or underlining the like terms.

4.2.3.2. The Casper routine

It was mentioned before, there are symbols or coefficient 1 that are sometimes hidden in algebraic expressions. These can be either implicit or visible. The teacher and students call these situations Casper. For instance, in the dialogue below, the teacher and the students are calculating 2x+x. In order to combine 2x and x, they decide to write the coefficient of x. One student called Serkan cannot make sense of where 1 came from. The teacher tells him that they need to add coefficients in algebraic expressions for addition; however, the coefficient of x is invisible. They should make it visible and write 1 in front of x.

2.172 Teacher: I wonder if any number exists in front of x? (Showing 2x+x)
2.173 RC: Yeesss, it does.
2.174 Teacher: What is it?
2.175 RC: Casper
2.176 Teacher: Here is a Casper that does not appear. (Puts 1 in front of x) Is it true?…
2.177 Teacher: … Listen, listen, let's do it like this at first. Now I have 2x and +x. What did we say in front of +x?
2.178 RC: Casper
2.179 Teacher: Watch carefully, let me put 1 here.
2.180 Serkan: Teacher, where did 1 come from?
2.181 Class: Casper 1
2.182 Teacher: Serkan, in addition, when we are adding the numbers, there is no number in front of x. I made the Casper 1 visible in front of x.
2.183 Serkan: Okay, there is Casper 1.

In another dialogue below, after combining the terms, the teacher asks what is in the bag, which means what the result is. The class answers +10x. Then the teacher wonders do they need to write the positive sign. They determine that it is unnecessary to write the positive sign.

3.8 RC: 10, x, x, 10x
3.9 Teacher: I am pasting x to there, right? (After closing the parenthesis, she writes x) Tell me, what is in the bag?
3.10 RC: +10x
3.11 Teacher: Do I need to put the positive sign?
3.12 RC: No, no.
3.13 Teacher: No, I can put it in or not. Is there a problem?
3.14 RC: No

The following excerpt refers to an operation about combining algebraic expressions (x+2x+5x). They make visible the hidden coefficient 1 in order to combine the terms. They write 1 in front of x and continue to the operation.

3.44 Mehmet: Teacher, there is Casper 1 (Shows x in algebraic expression x+2x+5x)
3.45 Teacher: Where?
3.46 RC: In front of x
3.47 Teacher: Ah, Sinem, go to your seat. What is here (pointing to x)
3.48 RC: x
3.49 Teacher: What's in front of this?
3.50 RC: Casper 1
3.51 Teacher: (writes 1 in front of x)

There is another dialogue related to Casper routine below. In line 3.87, a student asks the teacher to explain the rationale behind the existence of one in front of x. She says that 1 is the multiplicative identity, and there is a multiplication sign between 1 and x. Therefore, she explains that there is no need to write 1, but it also exists invisibly (Figure 4.18). She asserts that they can make Casper visible if they need to.

3.87 Boy: Why there is 1?
3.88 RC: There is Casper
3.89 Teacher: Let's tell it like this. There is 4 in front of x, there is -3 in front of x, there is -x, so it means -1 in front of x. I'm asking you, does it sound better to say 1x or x
3.90 RC: x, 1x
3.91 Teacher: Okay, let's ask, does it sound better to say 1x or x? (writes 1x and x on the board)
3.92 RC: 1x, x
3.93 Teacher: We get stuck here, let me give a few examples, in exponential numbers, we call x instead of x to the power of 1 (x^1). For example, we do not say x over 1 (x/1), we say x. Sometimes we use more handsome modes. These are Casper 1s. Sometimes we use them visible, sometimes invisible. For example, here, it is invisible (shows 4x-3x-x); we made it visible. Do you understand?
3.94 RC: Yeesss.
3.95 Teacher: Is there a problem? Did I make myself clear?
3.96 RC: Yes, we understand
3.97 Teacher: First of all, 1 is the multiplicative identity. Multiplying 1 and x equals x. We don't say 1x, we call it x.

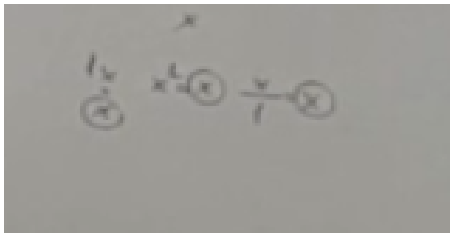


Figure 4. 18. The visual mediator of the Casper routine

Table 4. 26. The Casper routine in operations in the algebraic expression

Prompt	Routine	How?	When	
			Applicability	Closure
The teacher asks whether there is a number in front of x When there is a term with coefficient 1, combining like terms	Casper Routine	Making coefficient 1 visible that does not appear in a given algebraic expression	Adding and subtracting algebraic expressions containing terms with coefficient 1	Writing 1 in front of the term, that has an invisible coefficient 1. Ex: In 2x+x, writing 1 in front of x to make 2x+1x

Table 4.26. (continued)

The teacher asks whether it is necessary to write the plus sign	If the sign of the resulting term is positive, delete the plus sign in front of the term	When the positive term is obtained as a result of the operation in algebraic expressions	Delete the positive sign in front of the term with a positive coefficient Ex: Writing the result as 10x not +10x

As stated in Table 4.26, the Casper routine is used when adding and subtracting algebraic expressions and when the positive term is obtained as a result of the operation in algebraic expressions (*applicability*). The routine starts by asking students whether there is a number in front of x, whether, when combining algebraic expressions, there is a term with coefficient 1, and whether it is necessary to write the plus sign (*prompt*). Afterward, students and the teacher remove coefficient 1 or make coefficient 1 visible, which does not normally appear according to the situation (*how*). The *closures* of the routine are removing coefficient 1 and writing the hidden coefficient 1.

4.2.3.3. Addition and subtraction in algebraic expressions

In the addition and subtraction in algebraic expression routine, the teacher explains that operations can only be performed with like terms. To add or subtract the algebraic expressions, they first group the like terms, then they calculate the coefficients. Unlike terms are left alone. In the dialogue below, there is an example of simplifying 2x-3y+x-2y (Figure 4.19). The teacher marks 2x and x with an upper arch and -3y and -2y with an arch below. Then she writes coefficients of 2x and x in parenthesis below and -3y and -2y underneath. Results are 3x and -5y. The teacher implements the routine and helps her students understand the procedures.

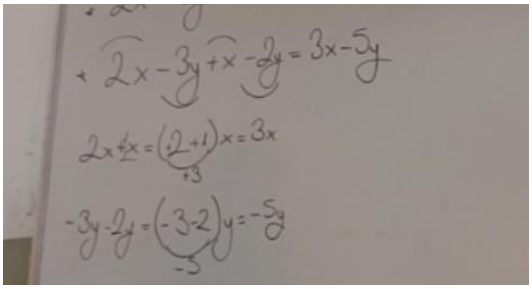


Figure 4. 19. Performing the Addition and Subtraction Routine

2.178 Teacher: … (2x-3y+x-2y on the board.) I will calculate these in a different place, then I will stick the x next to the result. Listen, listen, let's do this first. Now I have 2x and x. What is in front of x?
2.179 Class: Casper
2.180 Teacher: Watch carefully. I will place 1 here (places 1 in front of x).
Teacher: Listen, now, the other one. Let's get y's together here. I've opened a parenthesis. What am I going to write?
2.181 Class: -3
2.182 Teacher: Then?
2.183 Class: -2
2.184 Teacher: Hoopp (Closing the parenthesis and writing y next to it) What is in the basket? (She means, what is the result for the sum in the parenthesis*)*
2.185 Class: -5
2.186 Teacher: -5y. I may not put the positive sign here (indicating in front of 3x). But I must put the sign here (shows in front of -5y). Now I am bringing them together (writes {-3y-2y=(-3-2)y=-5y} on the board) and writing the results here (Writes =3x-5y next to 2x-3y+x-2y). I cannot calculate these two (shows 3x-5y). These are just apples and oranges.

In the dialogue below, addition and subtraction in algebraic expressions routine is being implemented in the example 4x-3x-x (Figure 4.19). The teacher describes raw expressions as messy, they need to be tidied up. First of all, she asks students if there is any Casper. Then they determine the Casper 1 before -x. They make the Casper visible and write -1x. After all the coefficients are made visible, they open a parenthesis, write the coefficients, and paste the unknown x next to the parenthesis. The teacher calls the parenthesis a bag. She asks them what is in the bag, and they combine the coefficients. Later they write the unknown next to the result. Thus, the teacher and the students implemented the addition and subtraction routine in algebraic expressions.

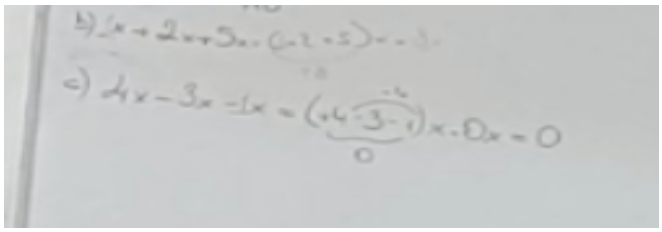


Figure 4. 20. Performing the Addition and Subtraction Routine

3.66 Teacher: (She writes 4x-3x-x=?) Write and lean back. If they are messy, we tidy up. When? After writing in parentheses. For those who have finished, lean back; I will check it. Time is not over, come on, tidy up. Those who have finished lean back, so I can understand who has finished. (She checks all

students' solutions. 14 students got it right.) Lean back. Let's have a look. Does any Casper exist?
3.67 RC: Yes..
3.68 Teacher: Which one?
3.69 RC: x, -x
3.70 Teacher: (writes -x as -1x) Are all of the numbers visible now?
3.71 RC: Yes
3.72 Teacher: I think so. I opened the group; which number will I write first?
3.73 RC: +4
3.74 Teacher: Then?
3.75 RC: -3
3.76 Teacher: Then?
3.77 RC: -1
3.78 Teacher: Let's stick x at the end. Guys, let me ask you something about this. (pointing to 10x) What is between 10 and x? I mean, what does 10x mean? 10 and x....?
3.79 RC: We multiply.
3.80 Teacher: There is a Casper multiplication sign there again. Do you get the point? What does it mean, actually? 10x means multiply x by 10. Let's keep it in your mind. Now let's tidy the messy minuses together (she marks the negative numbers in parenthesis)
3.81 RC: -4
3.82 Teacher: Ah, we add +4 -4?
3.83 RC: Zero...
3.84 Teacher: If there are four positive and four negative numbers in the bag, there remains zero. Tell me, what is the handsome version of 0x?
3.85 RC: Zero
3.86 Teacher: Yes. Why? Because there is already a Casper multiplication sign between zero and x, what do we call it? We all it zero. Do you understand? Who has understood? (Fingers are raised, a student says he/she didn't understand) You didn't understand, what didn't you understand?

In line 4.2, this time, they combine the algebraic expression $2xy+4x^2y$ -6xy. The teacher asks again what are like terms. She lets the student named Zeki do the operation on the board. He marks 2xy and -6xy to indicate they are like terms. With the guidance of the teacher, they open parenthesis, as they have done previously, and write the coefficients of like terms within the parenthesis. Thus, he obtains the result (+2-6) xy = -4xy. The term $4x^2y$ remains unchanged as there are no other like terms.

4.2 Teacher: (She writes the operation $2xy+4x^2y$ -6xy= on the board and tells Zeki to come to the board) Which ones are like terms? Write them first.
4.3 Zeki: (He marks 2xy and -6xy. He opens a parenthesis on the below line and starts to write 2x..... Then the teacher erases what Zeki writes)

4.4 Teacher: Süleyman is being unfair and disrespectful to you right now. (He warns Süleyman by saying).
4.5 Zeki: (With some guidance from the teacher, he makes the operation 2xy-6xy=(+2-6)xy=-4xy on the board).
4.6 Teacher: Can you write it?
4.7 Zeki: (He writes the algebraic expression $-4xy+4x^2y$)
4.8 Teacher: Zeki says that teacher, two of them are like terms, and he writes the result -4xy; he can not touch this ($4x^2y$) because this is different, so these are not like terms; he pasted it next to it as it was. Did everyone do like this ($2xy+4x^2y-6xy=-4xy+4x^2y=$)?
4.9 RC: Yes
4.10 Teacher: Does anyone do it differently?
4.11 RC: None

Below, they are doing an exercise with two unknowns and parenthesis (-5x-(-4y+2x)+8y). The teacher says that it is a bonus question. Firstly, students solve it in their notebooks, and then the teacher checks their answers. She gives feedback on all solutions. After checking the notebooks, the teacher let Balı to calculate the exercise on the board. With the help of the teacher, Balı removes the parenthesis and combines the algebraic expression. Thus, they perform addition and subtraction in algebraic expressions routine.

5.44 Teacher: (Writes -5x-(-4y+2x)+8y= on the board) Yes, this is a bonus question. Lean back. No fingers, I will look at all of your solutions. If the finishers lean back, I can see who has finished. Not talking. No finishers? No, come on. (She starts to check the solutions and puts a tick for the right answers. She explains why some of them are wrong) Speakers, lean back. Who's coming? Balı, will you come? Come. (Balı started to do the operation. While Balı is doing it, the teacher is explaining the solution.) Now your friend writes the sign of 5x. What is that? Negative with negative, positive 4y; positive with negative, -2x; the sign of 8y is already clear +8y. Which ones are like terms? Put the sign. Whoops, the others? While your friend is doing it... Now he puts signs, but let's write them properly, you are making mistakes because of that. (Balı writes 4y) What is written next to it, Balı? +8y+4y, what is in the bag? Positive? How many?
5.45 Balı: 12y
5.46 Teacher: 12y it is over. The other? Below. -5x -2x, now what is in the bag? What's in the bag?
5.47 Balı: Negative
5.48 Teacher: Yes. How many do you have?
5.49 Balı: 7
5.50 Teacher: There are 7 x's. Let's write them above.
5.51 Balı: +12y-7x

In the dialogue below, they are doing a modeling exercise in algebraic expressions. In Figure 4.21, the teacher posts tiles on the board. The yellow ones represent x, and the pink ones represent number 1. They are trying to combine representing algebraic expressions. The first one stands for 2x+2, while the second one stands for x+1. After generating the expression, they are performing the same routine for operation in algebraic expression. They combine like terms as they did before.

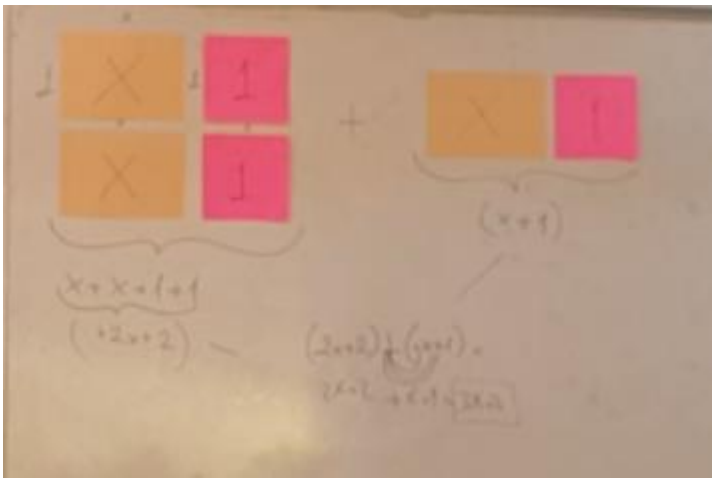


Figure 4. 21. Performing the addition and subtraction routine with modeling tiles

6.162 Teacher: Let's find the algebraic expression. There is x, again x, 1, and another 1. (Writes x+x+1+1 under the model) Is it true? Let's do the other side. x +1. Can we do it like this? It's a bit messy, let's combine them. How many x's are there?
6.163 RC: 2
6.164 Teacher: Positive or negative?
6.165 RC: Positive
6.166 Teacher: +2x...
6.167 RC: +2
6.168 Tecaher: Look, there is a group here, is it true? (Puts +2x+2 in parentheses) Do I have a group here too? (She also puts x+1 in other parentheses) What does it say to do with these at the end?
6.169 RC: Combine
6.170 Teacher: I am combining, look, I am getting them together. Watch. It says combine (2x+2) +(x+1). Oh, I know this. I give as a figure; this time, it says to write the expressions and then calculate. Do you understand? I did not write the positive sign here. Is there a problem? (She is talking about the positive sign in front of 2x) Is there a sign in front of it? (showing (2x+2))
6.171 RC: Noo!
6.172 Teacher: Is there anything in front of this? (Showing (x+1). She puts a positive sign in front of x) Then let someone come to the board. Sinem, do it right, come, I don't want any mistakes. Is it ok? Let's combine.

6.173 Sinem: (She makes the solution on the board)
6.174 Teacher: First, combine the x's. What is 2x more 1x?
6.175 Sinem: 3 (Sinem completes the operation)

Table 4. 27. Addition and subtraction routine in operations in algebraic expressions

Prompt	Routine	How?	When?	
			Applicability	Closure
Asking students to perform addition and subtraction operations in algebraic expressions	Addition and subtraction in algebraic expressions	Finding like terms in the addition or subtraction operation in algebraic expressions and calculating with their coefficients	Addition and subtraction in algebraic expressions with one unknown	Calculating the coefficients and writing next to unknown
Writing addition and subtraction exercises in algebraic expressions on the board			Addition and subtraction in algebraic expressions with two unknowns	
Asking for the simplest version of the algebraic expressions			Addition and subtraction in algebraic expressions with four unknowns	Writing the simplest version of the algebraic expressions
Asking students about the perimeter of geometric figures with algebraic expressions		Writing all lengths of the sides of the figures with addition sign and combining like terms and numbers	Finding the perimeter of figures with algebraic expressions	Writing the perimeter
Asking addition and subtraction operations in algebraic expressions with modeling tiles		Writing the modeled algebraic expressions and combining like terms	Addition and subtraction in algebraic expressions with modeling	Writing the simplest version of the expressions

As can be inferred from Table 4.27, the prompts in the routine are asking students to perform addition and subtraction in algebraic expressions, writing these exercises on the board, asking for the simplest version of the expressions, asking to calculate the perimeter of geometric figures with algebraic expressions, and asking addition and subtraction operations in algebraic expressions with modeling tiles. The addition and subtraction routine occurred by combining the like terms (How). The applicability conditions are addition and subtraction in algebraic expressions with one unknown, with two unknowns, with four unknowns, finding perimeter of figures with algebraic expressions, and addition and subtraction in algebraic expressions with modelling. There are some closures indicating the end of the operations like writing the result on the board, writing the simplest version of the algebraic expressions, and writing the perimeter with algebraic expressions.

4.2.3.4. Removing the parentheses

Removing the parenthesis routine is applied to algebraic expressions, including parenthesis. For instance, in line 4.84, the teacher writes the expression -(3x+1) on the board and draws attention to the negative sign before the parenthesis. The teacher explains students how to get rid of the parenthesis. She distributes the negative sign to each terms in the parenthesis. Then writes -3x-1 on the board.

> *4.84 Teacher:* ... Now look here. Watch. (She writes -(3x+1) on the board) What is in front of the same expression this time?
> *4.85 RC:* Negative
> *4.86 Teacher:* Who does the negative sign belong to?
> *4.87 RC:* Parentheses
> *4.88 Teacher:* To parentheses. What is in front of 3x?
> *4.89 RC:* Positive
> *4.90 Teacher:* Then, when there are two signs, we put the smiling face. Positive and negative?
> *4.91 RC:* -3, -3x
> *4.92 Teacher:* What's next?
> *4.93 RC:* +1
> *4.94 Teacher:* Whoops, positive and negative?
> *4.95 RC:* Positive, negative, positive.
> *4.96 Teacher:* Do negative and positive get along or not?
> *4.97 RC:* Can't get along.

4.98 Teacher: Because one of them says a, the other says b. The two of them speak different languages. Then they cannot get along, it is -1. This time it becomes -3x-1. Do you understand? We get it. Continue.

Below, the teacher solves the exercise -(-4+2x+2) with the removing the parentheses routine. The purpose is to distribute the negative sign to the whole group to get rid of the parenthesis. In order to distribute the negative sign, she draws arrows to each term. Since there are three terms in this exercise, she draws three arrows. When she applies the negative sign to the group, the sign of each term is changed. The excerpt below gives the dialogue of solving the expression of -(-4+2x+2) between the teacher and the class with the removing the parentheses routine. Figure 4.22 indicates the visual mediators of this routine.

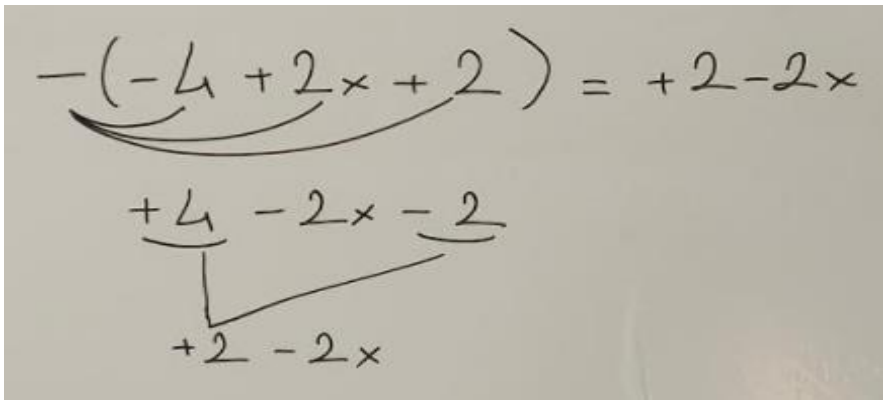


Figure 4. 22. Performing removing the parentheses routine

4.100 Teacher: Now look at this (She writes "-(-4+2x+2)" on the board). Who does the minus sign outside belong to?
4.101 Class: Belongs to parentheses.
4.102 Teacher: What is inside the parentheses?
4.103 Class: Smiling face, negative sign, positive sign...
4.104 Teacher: What are the terms?
4.105 Class: -4, +2x, +2
4.106 Teacher: Then the minus sign outside belongs to these three terms. Let's continue. Are negative times negative?
4.107 Class: Positive, +4
4.108 Teacher: What's next?
4.109 Class: +2x
4.110 Teacher: Hoop, Negative times positive?
4.111 Class: Negative, Negative 2x
4.112 Teacher: Here, Negative times positive?
4.113 Class: Negative 2.

Table 4. 28. Removing the parentheses in algebraic expressions

Prompt	Routine	How?	When?	
			Applicability	Closure
Algebraic expression exercises with parentheses	**Removing the parenthesis**	Using distributive property in order to remove parentheses	Algebraic exercises with parentheses	Terms removed from parentheses are written with their signs.

Table 4.28 shows the analysis of the removing the parenthesis routine according to Sfard's framework. The *prompt* is asking students an algebraic expression with a parenthesis. In order to remove the parenthesis, the teacher applied the distributive property (*how*). She draws arrows that she called 'smiling faces' for each term in the group. Then she changes the sign of each term if the sign outside is negative. Removing the parenthesis routine can be *applied* in algebraic exercises with parenthesis. The *closure* of the routine is writing the terms removed from the parenthesis with their signs.

4.2.4. Endorsed narratives in the topic of operations in algebraic expressions

In the topic of operations in algebraic expressions, endorsed narratives are the last discursive element in analyzed discourse. As previously mentioned, narratives should be endorsed by word usage, visual mediators, and routines to become definitions, theorems, and axioms. This section focuses on narratives frequently used in the topic of operations in algebraic expressions endorsed by other discursive elements. As in the topic of concepts in algebraic expressions, these narratives represent the teacher's expressions recorded as notes and explanations uttered during the exercise sessions. These are related to operations with like terms, operations in algebraic expressions, and definitions of like terms.

4.2.4.1. Terms with the same letters and the same powers of the letters are called Like Terms

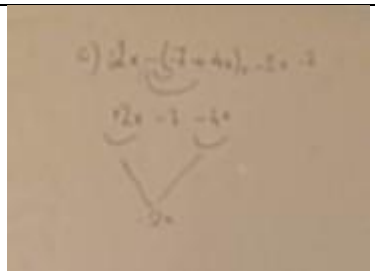
The definition of the like terms is uttered three times in the topic of operations in algebraic expressions. All three narratives are expressed in the exemplification part

of the lessons. In the following excerpt, throughout combining the expression 2x-(7+4x), the teacher reminds the definition of the like terms.

> *5.14 Teacher:* Why? One has x, and one has nothing; these are not like terms. So as to be like terms, both of them should have x's or not. Do you understand?

Like terms are the most uttered words in the topic of operations in algebraic expressions as well as in the topic of concepts in algebraic expressions. The teacher used like terms many times and used visual mediators to endorse the definition (Table 4.29). They used algebraic symbols and arcs to indicate the situation of like terms. Furthermore, they perform finding the like term routine to endorse the narrative.

Table 4. 29. Definition of like terms in the topic of operations in algebraic expressions

Narrative	Endorsed Word-Use	Endorsed Routine	Endorsed Visual Mediator
Like terms in an algebraic expression refer to terms that have identical variables and corresponding exponents.	*5.14 Teacher:* Why? One has x, and one has nothing; these are not like terms. So as to be like terms, both of them should have x's or not. Do you understand?	Finding the like terms	

4.2.4.2. Definition of addition and subtraction in algebraic expression

There are thirteen narratives related to defining addition and subtraction in algebraic expressions. Six of them are uttered in the explanatory part of the instruction, while seven of them are uttered in the exemplification part. The following excerpts are derived from the teacher's definition of addition and subtraction in algebraic expressions during the explanatory part of the lessons. In line 2.209, she wants students to write the note regarding addition and subtraction in algebraic expressions on their notebooks. In addition, in line 2.215, the class utters the note they have already written.

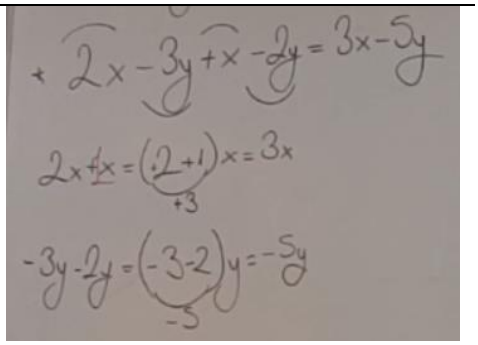
2.209 Teacher: A little later we are going to learn with the examples. Note {While adding or subtracting in algebraic expressions, like terms are added or subtracted}
2.215 RC: Coefficient {Coefficients are added or subtracted when adding or subtracting like terms}

In the following excerpt, the teacher clarified the circumstances in which they can perform addition and subtraction operations on algebraic expressions. She explains that they can perform operations with like terms but not with unlike ones.

3.110 Teacher: So this and that are like terms (Points 8x and -3x), this is different (Points 5y) Now, I am able to perform operations on like terms, but I cannot do anything unlike ones. Then what am I going to do with like terms? Let's write them down (Writes down 8x and -3x). What is that?

Table 4.30 presents an example of this narrative that is endorsed by other discursive elements of the discourse.

Table 4. 30. Definition of addition and subtraction in algebraic expressions

Narrative	Endorsed Word-Use	Endorsed Routine	Endorsed Visual Mediator
While adding or subtracting in algebraic expressions, like terms are added or subtracted	*2.209 Teacher:* A little later we are going to learn with the examples. Note {While adding or subtracting in algebraic expressions, *like terms* are added or subtracted}	Finding the like terms Addition and subtraction in algebraic expressions Removing the parenthesis	

4.3. Summary of the Findings

In this chapter, the discourse on teaching algebraic expressions is investigated with a focus on word-use, visual mediators, routines, and endorsed narratives. Additionally, it is determined whether these discursive elements are used during the explanatory talk part of the teaching or the exemplification part. The teaching of algebraic expressions is examined through the two main topics. These are concepts in

algebraic expressions and operations in algebraic expressions. The teacher reminds students of the specific concepts related to algebraic expressions that exist in the 6th grade's objectives in the first section. In the second section, she continues with the 7th-grade algebra objective, which is 'performing addition and subtraction operations with algebraic expressions'. This study is expected to contribute to the existing body of literature by concentrating on the discourse of the teaching and learning of algebraic expressions at the 7th-grade level.

Table 4.31 displays the presence of shared and distinct words used in both themes. Common words are term, unknown, coefficient, the sum of coefficients, like terms, and algebraic expressions. Besides, the different words used in the topic of concepts in algebraic expressions are coefficient, number of terms, and constant. Whereas there are not any different words in the topic of operations in algebraic expression. The most frequently used words in both topics are like terms.

The words in the topic of concepts in algebraic expressions are primarily used in an object-driven manner (%52), followed by a phrase-driven manner (%36), and the least commonly used in routine-driven (%12) manner. In the topic of operations in algebraic expressions, object-driven (%46) is the most commonly determined usage, as seen in the previous topic. While routine-driven (%41) is the second detected and phrase-driven (%27) is the last frequently detected usage. In the first topic, words are predominantly employed in the explanatory talks. In the second topic, words are mostly utilized in the exemplification part of the lessons.

Word use levels develop hierarchically. Learners are expected to transition from passive-use to object-driven use. Although word use is expected to progress in a hierarchical manner, the teacher may sometimes make exceptions to follow the hierarchy. For example, in Table 4.6 the teacher initially used the word coefficient as phrase-driven and object-driven. In the next one, she returned to the routine-driven use. In other words, she can return to the routine-driven or phrase-driven after the use of object-driven.

Algebraic notation is predominantly used visual mediator in both topics. The teacher initially starts the lesson by presenting a concept map that includes the topics to be

learned. In this way, students acquire an understanding of the general framework of the topic they will be studying. Other visual mediators employed in the instruction of concepts in algebraic expressions are circles, arrows, and arcs. For example, the teacher circles each term to indicate the number of terms in an algebraic expression. The number of circles represents the number of terms in the given expression. Moreover, the teacher and students use arcs to indicate like terms in the given expression and use arrows to generate a like term for a given expression.

Algebraic notations, circles, arrows, and arcs are also employed as visual mediators in the topic of operations in algebraic expressions. Furthermore, geometric figures and modeling tiles are utilized for visualization. For example, the teacher draws a rectangle to calculate the perimeter of a rectangle whose sides are represented as algebraic expressions. Likewise, they use algebra tiles to construct a model to represent the given algebraic expression or to generate an algebraic expression based on a given model.

There have been a greater number of routines performed on the topic of operations in algebraic expressions compared to the topic of concepts in algebraic expressions. In concepts in algebraic expressions finding the like terms, writing a like term, and Casper routines are performed, while in operations in algebraic expressions finding the like terms, casper routine, addition and subtraction in algebraic expressions, and removing the parenthesis routines are performed (Table 4.30). In the initial topic, the routines are performed with the same frequency in both the explanatory talk and exemplification parts of the lessons. Regarding the second topic, the routines are predominantly performed in the exemplification parts of the lesson.

Narratives are the last discursive characteristics endorsed by the other characteristics of the discourse. In the topic of concepts in algebraic expressions there are two endorsed narratives. These are ‘algebraic expressions refer to expressions that consist of unknowns and entail mathematical operations’ and ‘terms with the same letters and the same powers of the letters are called like terms’. Similarly, there are also two endorsed narratives in the topic of operations in algebraic expressions. These are ‘terms with the same letters and the same powers of the letters are called like terms’

and ‘definition of addition and subtraction in algebraic expression’. These narratives are endorsed by the use of words, visual mediators, and routines.

Table 4. 31. Discursive elements in concepts in algebraic expressions and operations in algebraic expressions

	Concepts in Algebraic Expressions	Operations in Algebraic Expressions
Word-Use	term, unknown, coefficient, the sum of coefficients, number of terms, constant, like term, and algebraic expressions	term, unknown, the sum of coefficients, like terms, algebraic expressions
Visual Mediators	Concept map Algebraic notations Circles Arcs Arrows	Algebraic notations Circles Arcs Arrows Geometric figures Modelling tiles
Routines	Finding the like terms Writing a like term Casper routine.	Finding the like terms Casper routine Addition and subtraction in algebraic expressions Removing the parenthesis
Endorsed Narratives	Algebraic expressions refer to expressions that consist of unknowns and entail mathematical operations Terms with the same letters and the same powers of the letters are called Like Terms	Terms with the same letters and the same powers of the letters are called Like Terms Definition of addition and subtraction in algebraic expression

CHAPTER 5

CONCLUSIONS AND DISCUSSION

The purpose of the present study is to examine seventh-grade algebraic expression lessons in the context of a Turkish middle school through the discursive perspective. In more detail, the study is concerned with the mathematical discourse categories (Sfard, 2008) and analyzes how they occur in the MDI (Adler & Ronda, 2015), which are explanatory talks and exemplifications, during the seventh-grade algebraic expressions lessons. Moreover, learner participation, one of the elements of MDI, is also determined and discussed in the concepts of algebraic expressions and operations in algebraic expressions. In accordance with this aim, the prior chapter presented the findings of the study in two main topics, each of which was determined according to the curriculum objectives of algebraic expressions.

This chapter discusses the discourse on teaching and learning algebraic expressions, focusing on the mathematical discourse characteristics, which are word use, visual mediators, routines, and endorsed narratives, and compares the results with the related literature. Each aspect of the mathematical discourse characteristic is analyzed in separate sections and evaluated in terms of MDI, including explanatory talks and exemplifications. Beyond that, the chapter features implications for teaching and learning algebra and suggestions for further research in the following sections.

5.1. Word-Use in Algebraic Expression Instruction

Word-use in algebraic expressions is investigated under two topics, which are concepts in algebraic expressions and operations in algebraic expressions. During the observations the teacher made students remember specific concepts regarding algebraic expressions that exist in the 6^{th} grade's objectives in the former topic. Then,

she utilized these concepts in performing operations in algebraic expressions related to the 7th-grade objectives in the other topic. In this section, the words utilized in teaching and learning algebraic expressions were identified and discussed in detail.

The words used in concepts in algebraic expressions are term, unknown, coefficient, the sum of coefficients, number of terms, constant, like term, and algebraic expressions. On the other hand, words used in operations in algebraic expressions are term, unknown, the sum of coefficients, like terms, and algebraic expressions. As a result, the common words utilized in both topics are term, unknown, the sum of coefficients, like term, and algebraic expressions. The least were analyzed according to passive-use, routine-driven, phrase-driven, and object-driven categories in the word-use category of Sfard's (2008) commognitive theory. Since passive-use generally corresponds to the usage in preschool periods, which are the first years of learning life, this category was not encountered (Emre-Akdoğan, 2015). Therefore, all word usages were analyzed in the remaining three categories.

The findings reveal that these words were mostly used as object-driven (52%) in concepts in algebraic expressions. The following was phrase-driven (36%), while the least frequent usage was routine-driven (12%) (see Table 4.2). In the same way, object-driven (%49) usage is the most utilized way in operations in algebraic expressions, and contrarily, routine-driven is the second most used category, and phrase-driven (%20) is the least used one (see Table 4.18). Generally, object-driven is the most commonly uttered word-use category in algebraic expression instruction. Sfard (2008) suggests that word use should progress hierarchically from passive-use to object-driven. In accordance with this consideration, in the present study the fact that object-driven is the most utilized usage may suggest that words used in algebraic expressions instruction have reached an objectified level of mathematical, which was expected for effective learning. The objectification mentioned here can be generally accepted as understanding a concept. To be more specific, it is stated in the literature that objectification is a distinct manner of discussing mathematical objects as existing independently of human discourse (Kontorovich, 2018). In this study, the objectified use of the like terms could be described as demonstrating like terms in algebraic expressions without any definition or action. In this way, the teacher and

students might use the word as a mathematical object independent of their own thinking (Sfard, 2008). However, it was determined in the present study that the teacher used words hierarchically as routine-driven, phrase-driven, and object-driven most of the time, but occasionally, she returned to routine-driven or phrase-driven usage (e.g., see Table 4.10 and Table 4.22). The teacher's backward usage of the word category might be because she considers that the students have not yet progressed to an object-driven usage and cannot objectify the words mathematically.

More specifically, the teacher primarily performs the action of finding like terms by highlighting 8x and -3x (in line 3.110) (routine-driven) in accordance with the hierarchy. For example, as in Table 4.22, she solves the example of combining the expression 8x+5y-3x. After that, she gives an explanation as to why they are similar (in line 3.118) (phrase-driven). Finally, she demonstrates which terms are similar and which are not without providing any explanation or action (in line 3.121) (object-driven). Yet, when the teacher transitions to a new example (-7x+3y+2x-y), despite having advanced to the objectified level, she could return to routine-driven or phrase-driven usage. In Table 4.22, lines 3.135 (Marking +3y and -y) and 3.140 (Pointing -7x and +2x) are examples of returning the routine-driven use. As a matter of fact, she expressed during the informal interviews that she was reluctant to move on to further concepts before the students completely comprehended them, and she reiterated these concepts when necessary. Sfard (2008) states that using the same mathematical word or signifier in a different manner causes commognitive conflict, an occasion in which different people act regarding different metarules of a discourse. In this situation, the teacher's use of a word in an objectified way, while students' use of the word in a description or an action is the reason for the conflict. Correspondingly, Nachlieli and Heyd-Metzuyanim (2022) stated that commognitive conflict occurs when interlocutors use the same words in different ways. That is, although individuals seem to discuss the same topic, the two parties fail to understand each other as the meta-rules of discourse are not the same (e.g., the teacher uses the like terms as object-driven, but the students remain at the level of routine-driven). For this reason, in this study, the teacher's flexible usage of the word-use levels according to students' needs demonstrates that she might be acquainted with students' word-use levels. Therefore, in parallel with the existing literature, it is

considered as an essential step for effective learning by preventing commognitive conflicts that may stem from miscommunication issues (Nachlieli & Heyd-Metzuyanim, 2022; Sfard, 2008). Consequently, it might be concluded that in order to avoid any commognitive conflicts, the teacher pays attention to determining students' word use levels and adapts her own level of usage accordingly.

In this study, word usage was investigated, both in explanatory talks and exemplifications parts of the instruction as well. The findings of the study reveal that in concepts in algebraic expressions, words are mostly uttered in explanatory talks (%66), while in operations in algebraic expressions, words are mostly used in exemplification parts (%82) (see Table 4.3 and Table 4.19). In other words, she utilized definitions, explanations, and properties of mathematical structures more in concepts in algebraic expressions, while she used examples and tasks to present students' words in operations in algebraic expressions more. Learner participation was not presented in a distinct section because the teacher's discourse was dominant, and students' discourses were generally repetitioning of the teacher's discourse, yes/no answers, and one or two-word answers. Sfard (2008) explains this condition as one of the interlocutor's discourses would be authoritative over the others who set up the rules. Mahlaba and Mudaly (2022) explain this dominance of the teacher through students' ritualistic participation, where learners engage in discourse mainly to conform to societal expectations or to receive positive evaluations from teachers rather than to explore and understand the subject matter. This type of participation is characterized by imitation and repetition of routines without a deep understanding, which limits learners' ability to engage independently and meaningfully in discourse (Mahlaba, 2021; Mahlaba & Mudaly, 2022). In this study, the teacher's discourse is obviously dominant, and she is the rule maker. Students generally repeat her discourse. As Mahlaba (2021) stated, it might be concluded that students adopt ritualistic discourse that confirms and repeats the teacher's discourse instead of employing exploratory discourse and constructing their own mathematical meanings.

According to the findings, like term was the most frequently pronounced word in both topics (see Tables 4.2 and 4.18). Tirosh et al (1998) found that the strategy of 'collecting like terms' is frequently used in simplifying algebraic expressions, and

teachers commonly use this strategy before operations in algebraic expressions (p.51). The study showed that the teacher devoted most of the two lessons to clarifying and practicing the concept of like terms before operations in algebraic expressions. The findings revealed that object-driven use, which refers to considering words as a mathematical object or as a final statement (Sfard, 2008), was commonly used in both topics that were concepts in algebraic expressions and operations in algebraic expressions. Besides, in the first topic, phrase-driven and object-driven usage were used almost equally, while routine-driven usage was noticed less. The teacher employed relatively more phrase-driven usage in the first topic than in the second topic. In other words, she associated more with fixed phrases rather than complete routines (Sfard, 2008), which means that she made more descriptive statements. By incorporating phrase-driven usage more frequently in the first topic, the teacher may have intended to enhance the students' mathematical understanding of the notion of like term. In the literature, Tirosh et al. (1998) found that one of the teachers who attended their study spent a significant amount of time exercising "collecting like term," and her students appeared to have acquired this competence compared to others (p. 51). Also in another study, Ignacio and Sison (2022) stated that the students' word-use level progresses to phrase-driven after describing the mathematical terms, and the level of discourse is enhanced from colloquial to literate discourse with phrase-driven usage. According to the analysis of the like term given in Table 4.10 in lines 1.23, 1.143, and 1.162, the teacher uses more colloquial language to describe the like term, such as different unknowns for unlike terms, identical unknowns, or same unknowns. But then, as in line 1.64, she describes like term more officially as 'terms with the same letters and the same powers of the letters are called like terms'. The table obviously demonstrates the transition from colloquial to official discourse of like term. The development of phrase-driven use thus plays a pivotal role in enabling learners to move from a colloquial understanding of mathematics to a more formal and structured mathematical discourse, ultimately contributing to their overall mathematical proficiency and understanding (Ignacio & Sison, 2022; Tyskerud, 2018). Thus, it might be concluded that the teacher's transition from colloquial to mathematical discourse observed in the phrase-driven use facilitates students' progression to a more formal and structured mathematical discourse.

On the other hand, the findings indicate that, in the latter topic, object-driven use of the like term was the most commonly utilized usage, routine-driven was the second most commonly used, and the least commonly utilized usage was phrase-driven. The teacher employed a greater degree of routine-driven use in the second topic compared to the first topic. In the first topic, the teacher primarily focused on teaching the concept of like terms, while in the second, she utilized like terms in the context of addition and subtraction in algebraic expressions. For instance, in lines 2.150 to 2.155, they search for like terms to simplify 2x-3y+x-2y. They first highlight like terms and then combine coefficients. That is, they used like term as an action. In routine-driven usage, the like term is formed by the process, procedure, and the realization of certain actions. Sarı-Arıkan (2019) named routine-driven usage as process-based and object-driven usage as object-based. She emphasized that in the teaching of a mathematical concept, it is crucial to prioritize process-based use, and objectification should be followed by object-based usage. In Table 4.23, it appears that there is no clear hierarchy in the utilization of the like term. However, the solution of the exercises typically begins with a routine-driven use and progresses towards an object-driven use as a concluding statement. In addition, Sarı-Arıkan (2019) indicated that the process-based use was mainly in the explanation of the subject rather than solving examples. In the present study, routine-driven usage of like terms was observed in the explanatory talks of the lesson on concepts in algebraic expression and exemplification in operations in algebraic expressions. This finding differs from the study of Sarı-Arıkan (2019) in some aspects. In other words, the findings of this study indicate that the hierarchy in word-use levels was not always consistently followed, contrary to Sarı-Arıkan (2019). The teacher successfully restarted the routine-driven, phrase-driven, and object-driven hierarchical cycle in each new example in line with the student's requirements. Furthermore, unlike Sarı-Arıkan's (2019) findings, this study employed routine-driven use not only in explanatory talks but also in exemplifications. The teacher may have returned to routine-driven use at each stage of the lesson to ensure that students comprehensively understand the mathematical contents.

5.2. Visual-Mediators in Algebraic Expression Instruction

The teacher used concept maps, algebraic notations, circles, arcs, and arrows as visual mediators in concepts in algebraic expressions. She predominantly utilized algebraic notations. At the beginning of the instruction, she initially started the lesson by presenting a concept map that included the topics to be learned. Concept maps serve as a powerful tool for organizing information, fostering meaningful learning, and strengthening students' comprehension (Evans & Jeong, 2023). In this way, students recognize the topics they will learn in an organized manner and might gain an insight into the general framework of the topic. She used other visual mediators like circles, arrows, and arcs to indicate terms, unknowns, and like terms. For instance, the teacher circled each term to indicate the number of terms in an algebraic expression. The number of circles represents the number of terms in the given expression. Moreover, she used arcs to indicate like terms in the given expression and used arrows to generate a like term for a given expression. These kinds of visual mediators play a crucial role in mathematical communication and learning (Nachlieli & Heyd-Metzuyanim, 2022; Tabach & Nachlieli, 2015). These mediators embody abstract expressions and facilitate the learning process by making complex concepts easier to understand ((Nachlieli & Heyd-Metzuyanim, 2022). Thus, they help learners acquire and remember knowledge more easily by bridging symbolic and verbal expressions (Nardi et al., 2014). In this study, the teacher usually calls the arcs employed in removing the parenthesis as smiling faces. Thus, this usage could make students more easily remember the subjects. When the teacher mentions smiling faces, students can easily recall the process of removing the parenthesis. Consequently, the utilization of a variety of visual mediators might enhance students' mathematical understanding by integrating verbal expression with symbolic expressions and concretizing complex mathematical concepts.

Moreover, the teacher typically employed the aforementioned mediators in the classroom as follows. She initially illustrated the appropriate visual mediator to employ for each situation. Afterward, she encouraged students to utilize these mediators in a similar manner. However, as Mahlaba (2021) stated that even if students' utilizing visual mediators similar to their teacher may result in a more

cohesive learning experience, students could develop a ritualistic discourse, which is based on reaching answers without understanding the procedures. As in this study, the teacher's request for the students to use visual mediators according to her preferences sometimes may cause confusion among the students. The students were unable to construct their own discourses because the teacher directed them to use the same mediators when identifying like terms or indicating the terms in a given algebraic expression. In some cases, the students were unable to identify like terms on which they should focus, as they were preoccupied with remembering the mediator used by the teacher.

Furthermore, algebraic notations, circles, arrows, and arcs are also employed as visual mediators in the topic of operations in algebraic expressions. In addition, geometric figures and modeling tiles are utilized for visualization as well. Based on the literature review, it was highlighted that using diverse visual mediators in mathematics classes enhances the comprehension of mathematical expressions by making them more apparent, fosters the classroom communication environment, and improves mathematical understanding by establishing links between various visual representations (Mahlaba, 2021; Nachlieli & Heyd-Metzuyanim, 2022; Nardi et al., 2014; Tabach & Nachieli, 2015; Zayyadi et al., 2019). In the present study, the teacher's use of various visual aids on operations in algebraic expressions may have supported students' mathematical understanding. For example, the teacher drew a rectangle to calculate the perimeter of a rectangle whose sides are represented as algebraic expressions. Likewise, she used algebra tiles to construct a model to represent the given algebraic expression or to generate an algebraic expression based on a given model. With the use of algebraic expressions to calculate the perimeters of geometric figures, students realized that algebraic expressions could be used in different representations. Utilizing diverse visual mediators to present algebraic expressions might enhance students' comprehension of complex mathematical concepts and increase communication within the classroom. The next section discusses the findings of routines, which is another mathematical discourse characteristic.

5.3. Routines in Algebraic Expression Instruction

Routines are another characteristic of mathematical discourse in commognitive theory (Sfard, 2008). The findings indicated that one of the elements that ensures the instruction of algebraic expressions occurs through some recursively performed activities called routines. The teaching activities that explicitly refer to the routines in the discourse on algebraic expressions have the potential to promote student learning. Each routine was concluded to have meta-rules that initiate the patterns as prompts, defining how the routine occurs and when the routine arises. Thus, the analysis of routines provided a general idea about the direction of communication and who leads discursive activities in the classroom.

The most important finding from the analysis of seven algebra lessons is that, as in previous discourse characteristics, the teacher's routines are dominant, and students' routines are generally formed by the repetition of her discourse. Therefore, it was focused on the teacher's routines to document the teaching process of algebraic expressions. The dominance of the teacher's discourse is consistent with the existing literature about students' adoption of particular elements of the teacher's discourse (Emre-Akdoğan et al., 2018). Furthermore, the literature emphasized that the teacher's discourse affects the students' discourse (Roberts & Le Roux, 2019), and they organize the discourse in order for the students to participate (Tabach & Nachlieli, 2011). In this study, the reasons for the teacher's dominance might be due to the high number of low-achieving students in the classroom, as mentioned by the teacher during an informal meeting, and the nature of direct instruction. This assumption might be true because it was noticed that the students could successfully end the routine when they applied the teacher's routine along the same line. On the other hand, when they tried to implement a routine independent of the teacher, they made conceptual mistakes and could not complete the routine. This result shows that most students in the study had problems developing algebraic thinking skills and limited knowledge about algebra to perform their own routines. In fact, in the present study teacher-centered and direct teaching might help students who are low-achievers to remember and repeat the learned discourse and make new connections to expand their further discourses. All in all, when students are low achievers, it

might be the right approach for the teacher's routines to be dominant during the teaching and learning process since the conversations and discursive activities are generally directed by the teacher.

The predomination of the teacher's discourse led this study to analyze the teacher's repetitive activities throughout the lessons to document the instruction of algebraic expressions. Moreover, the commognitive theory (Sfard, 2008) provided a framework to determine the iterative actions that the teacher performed in the lessons. There are five routines relevant to the curriculum were ascertained (MoNE, 2018). These are finding the like terms, writing a like term, Casper routine, addition and subtraction in algebraic expressions, and removing the parentheses. In the course of the lessons, initially, the teacher presented the routine, and then she asked about examples that enabled students to use the same routine. Three routines were identified in concepts in algebraic expressions. These are finding the like terms, writing a like term, and the Casper routine. The most frequently employed routine was writing a like term (43%), nearly the same frequency as finding the like terms (%36), while the least utilized routine was the Casper routine (%21). Routines were primarily executed in the exemplification part of the lessons (%86), and the rest of them were performed in explanatory talks (%12). Moreover, in algebraic expression operations, there are four kinds of routines. These are finding the like terms, Casper routine, addition and subtraction in algebraic expressions, and removing the parentheses. Similarly, routines were commonly performed in exemplification parts (%92). The most frequently used routine was addition and subtraction in algebraic expressions (%52), and the least performed routine was removing the parenthesis (%8). In concepts in algebraic expressions, algebraic routines such as finding the like term and writing a like term were primarily observed. While, in operations in algebraic expressions, routines such as addition and subtraction, algebraic expression and removing the parenthesis are dominated by routines showing the steps of algebraic operations. Recognizing and finding like terms before performing algebraic operations is a crucial step in combining algebraic expressions. In the literature, it was noted that there is a common challenge in performing algebraic operations sequentially, as in arithmetic (Booth, 1988; Erbaş et al., 2010; Herscovics & Linchevski, 1994; Jupri et al., 2014; Pillay et al., 1998; Seng, 2010). For instance,

students often mistakenly perform the correct order of algebraic operations (e.g., brackets, multiplication and division, addition and subtraction) (Herscovics & Linchevski, 1994). The routines identified in this study are believed to prevent students from making mistakes by sequentially demonstrating the operations, specifically illustrating when the routine is executed, how it occurs, and how it ends. The teacher's elaboration of specific algebraic operations like finding the like terms or removing the parenthesis as routines, might enhance students' algebraic operation skills.

More specifically, n algebraic expressions lessons, the teacher applied finding the like terms and writing a like term routine before activities about simplifying the expressions. Spending enough time and effort on these routines, which were applied to discover the unknown and to notice the variable, might provide a basis for further operations with algebraic expressions, like addition and subtraction. In the literature, it is stated that if sufficient time is not spent on these routines, students have difficulties mastering the skills for algebraic expression operations (Tirosh, Even, & Robinson, 1998). The spiral nature of the Turkish mathematics curriculum (MoNE, 2018) demands that the basic steps be indicated in a systematic way as well. Therefore, no matter how complicated the topic is, the students might develop advanced skills with ease and enhance their discourse with carefully designed routines.

One of the interesting findings of the study is the teacher’s use of the Casper routine within the other routines, like addition and subtraction in algebraic expressions. The routine was named through the influence of the cartoon, whose character was a lovely ghost. The Casper routine is relevant to the rule of terms with no coefficient (e.g., x) actually having the coefficient 1 in algebra. Furthermore, the 1 can be made visible or invisible, like Casper the ghost, depending on the nature of the operations. In this context, the students could easily notice this rule during activities about simplifying algebraic expressions that have terms with no coefficient (e.g., 2x-3y+x-2y), and most of the students stated that Casper was in front of x. This means x has a coefficient of 1. Such concrete situations from their daily lives may encourage the students to attend the lessons even if they are uninterested in mathematics. Kim,

Sfard, and Ferrini-Mundy (2005) also claimed that colloquial discourse has a positive effect on mathematical discourse. Here, Casper is a well-known character in a child's world, and the teacher takes advantage of this fame to teach mathematical rules. Thus, the implicit nature of algebra is embodied to some extent to make it easier for students to comprehend. Referring to daily life in classroom routines might enable students to develop a positive attitude toward algebra, enhance classroom communication, increase students' understanding, and enable the development of further discourse.

Another exciting contribution of this study to the literature is regarding addition and subtraction in algebraic expressions and removing the parentheses routines. These routines were generally used in exercises about simplifying algebraic expressions. Students might recognize and use the previous three routines (finding the like terms, writing a like term, and Casper routine) to solve the simplifying activities successfully. Tirosh, Even, and Robinson (1998) named these routines as "collecting like terms" (p. 59). They concluded that presenting this method as a ritual helped students master exercises about simplifying algebraic expressions. On the contrary, Mahlaba (2021) stated that these kinds of routines are included in a ritualistic discourse, characterized by following certain procedures and applying these procedures without understanding. These ritualistic discourses involve the search for social acceptance (e.g. the teacher's acceptance). Instead, they argue that meaningful learning will occur with explorative discourse, which involves a more flexible and meaning-focused approach where students can be more flexible. In this study, the teacher regarded the students as low achievers, so she implemented more ritualistic routines and expected them to repeat these routines until they learned them. This may be an appropriate application for low-level classes. As a result of their inadequate mathematical competence, students may be incapable of creating their own routines. Although teaching with routines seems to hinder students' creativity, decisions such as how and when to apply previously learned patterns are judgments that individuals will make on their own. Here, in algebraic expressions lessons, the teacher decided to use finding like terms and writing a like term routine to implement addition and subtraction in the algebraic expressions routine. In the next exercises, students also used routines they had learned before to perform a new one. We may conclude that

as the number of patterns and rules the students learned increased, the routines they can use in further mathematics topics might increase. Thus, lecturing with a focus on routines has great potential to increase students' understanding. Consequently, routines are not patterns that constrain students but rather discursive activities that greatly contribute to their learning. In the next section, the results related to the final mathematical discourse characteristic endorsed narratives are discussed.

5.4. Endorsed Narratives in Algebraic Expression Instruction

Narratives are the last discursive characteristics endorsed by the other discourse characteristics. In the topic of concepts in algebraic expressions there are two endorsed narratives. These are ‘algebraic expressions refer to expressions that consist of unknowns and entail mathematical operations’ and ‘terms with the same letters, and the same powers of the letters are called like terms.’ Similarly, there are also two endorsed narratives on the topic of operations in algebraic expressions. These are ‘terms with the same letters and the same powers of the letters are called like terms’ and ‘definition of addition and subtraction in the algebraic expression.’ These narratives are endorsed by the use of words, visual mediators, and routines.

In the first topic, there were two narratives. The first one is *algebraic expressions, which refer to expressions that consist of unknowns and entail mathematical operations.* This narrative was endorsed by words like unknowns, terms, number of terms, and constant terms, which were used to support the definition and use of algebraic expressions. For instance, the teacher defined that ‘expressions containing both operations and unknowns are referred to as algebraic expressions.’ She supported the definition of algebraic expression with words like unknown and algebraic expression. In addition, she endorsed the narrative with a visual mediator, which represents algebraic notations. She explains that the expression 3x+2y+5 is actually an algebraic expression since it contains both unknowns and operations. Moreover, finding the like terms routine, writing a like term, and Casper routine are discursive patterns that endorsed the narrative. The role of narratives, supported by other discourse characteristics, is creating and substantiating mathematical discourse to serve as foundational elements that guide the understanding and communication of

mathematical ideas (Tasara, 2018). Thus, approved narratives have become definitions, theorems, and proofs accepted by the mathematical community (Nardi et al., 2014). The other narrative that exists in concepts in algebraic expressions is *terms with the same letters, and the same powers of the letters are called like terms*. The narrative was endorsed by the like term many times. Like term was the most frequently uttered word in algebra lessons. Furthermore, the narrative was endorsed by some visual mediators. These are algebraic symbols, circles, and arcs to indicate like terms. The related routines that endorsed the narrative are finding the like term and writing a like term.

In operations in algebraic expressions, there were also two narratives. The first one was again *terms with the same letters, and the same powers of the letters are called like terms* as in the first topic. The second narrative was related to *the definition of addition and subtraction in the algebraic expression*. The narrative was endorsed by like term many times. Moreover, finding the like terms, addition and subtraction in algebraic expressions, and removing the parenthesis are routines that endorsed the narrative. Visual mediators were, as in other narratives, algebraic notations, circles, and arcs, and different from them, there were geometric figures and modeling tiles.

5.5. Implications of the Study

The results of the study reveal critical insights for enhancing algebraic expressions instruction that could be useful to numerous individuals in mathematics education. These are mathematics teachers, preservice teachers, curriculum developers, teacher educators, mathematics education researchers, and mathematics textbook writers.

This study employs the commognitive framework (Sfard, 2008), for examining the mathematical discourse during the teaching of algebraic expressions. Furthermore, this study fills a significant gap in the literature by examining the discourse during the teaching of algebra at the seventh-grade level. Thus, mathematics education researchers and mathematics teachers could use the explanatory nature of the discursive analysis to gain insight into algebra teaching. They could comprehend the power of commognitive theory, where used words, visual mediators, routines, and

endorsed narratives were carefully analyzed in mathematical discourse in instruction (Sfard, 2008). Moreover, the framework demonstrates the relationship between these mathematical discourse characteristics and how this relationship occurs in accordance with the mathematical context. Thus, teachers and researchers can realize the role of these mathematical discourse characteristics in instruction.

More specifically, teachers can notice the importance of word-use levels on students learning. If the teachers and students use the word levels synchronically, the misconceptions that stem from miscommunication issues can be reduced. In other words, teachers should pay attention to determining students' level of word use and adapt their own levels according to them in order to avoid commognitive conflicts. Moreover, the colloquial discourse also has an important role in developing mathematical discourse (Kim et al., 2005). The teachers should be aware of the role of using daily life instances to develop a meaningful and comprehensive understanding. Therefore, mathematics teachers should utilize real-life slices to enhance students' learning as well. In addition, textbook writers and curriculum developers should also consider the role of using words and colloquial language. They should pay attention to the role of using words from students' daily lives to facilitate students' transition from colloquial discourse to mathematical discourse.

Another mathematical discourse characteristic that is important in discourse analysis is routine. Teaching algebra rules with a certain routine was an impressive way to help students learn. Documenting the discursive routines in algebra lessons presents the opportunity to improve classroom communications by determining how, when, and in what circumstances disconnections occur. This situation may help mathematics teachers to plan lessons by considering these communicative components. They could realize which routine should be used and how and when they were demonstrated in the lessons. Therefore, students' learning process of algebraic expressions should be promoted by routines that are expected to encourage them to establish further routines and explorations. In other words, teachers should have comprehensive knowledge of the curriculum and be aware of the students' prior knowledge in order to generate new routines to promote further learning.

Moreover, through the examination of the MDI framework, educators, researchers, teacher educators, textbook writers, and curriculum developers could determine the characteristics of mathematical discourse within commognitive theory. In addition, they should be aware of the use of diverse visual mediators to objectify the implicit nature of algebra. Thus, they could enhance communication in the classroom. The next section presents the limitations and recommendations of the present study.

5.6. Limitations and Recommendations of the Study

The study is conducted as a case study that attempts to investigate the teaching of algebraic expressions at the seventh-grade level by focusing on only one classroom. The findings of this study may be influenced by certain limitations. It provided a comprehensive overview of each of these restrictions, suggestions to resolve them, and recommendations for future research.

First of all, the present study is limited to seventh-grade algebraic expressions instruction. Further research can be conducted on another level of middle school to see how the mathematical discourse characteristics occur. Moreover, this study is restricted to analyzing mathematical discourse in the context of algebra. New studies could be conducted by examining the discourse of different concepts in the curriculum. Thus, investigating the difference in word-use, visual mediators, routines, and endorsed narratives within different contexts.

Another limitation of the study is that the findings are obtained from only one classroom that exists near the city center in a public middle school where the students are low achievers. New studies could be conducted in schools in the city center or private schools to compare the discourse they used.

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APPENDICES

A. THE CONCEPT OF ALGEBRA LEARNING OBJECTIVES OF 7th GRADE/ SINIF CEBİR ÖĞRENME ALANI KAZANIMLARI

M.7.2. CEBİR

M.7.2.1. Cebirsel İfadeler

M.7.2.1.1. Cebirsel ifadelerle toplama ve çıkarma işlemleri yapar.

Cebirsel ifadelerle toplama ve çıkarma işleminde uygun modeller kullanılır.

M.7.2.1.2. Bir doğal sayı ile bir cebirsel ifadeyi çarpar.

Örneğin 5 (x + 3) = 5x + 15

M.7.2.1.3. Sayı örüntülerinin kuralını harfle ifade eder, kuralı harfle ifade edilen örüntünün istenilen terimini bulur.

a) *Adımlar arasındaki farkı sabit olan örüntülerle sınırlı kalınır. Değişken kullanımının önemi ve gerekliliği vurgulanır.*

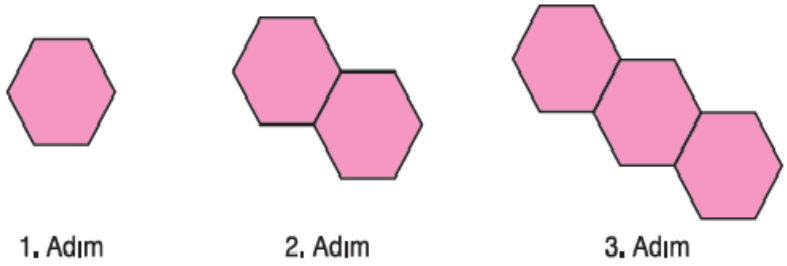
b) *Sayı örüntüleri incelenerek örüntünün kuralını bir değişken ile (örneğin n cinsinden) yazmaya yönelik çalışmalar yapılır. Örneğin ilk dört terimi 3, 9, 15 ve 21 olan bir aritmetik örüntünün kuralı 6n–3 olarak ifade edilir.*

c) *Günlük hayat durumlarında veya şekil örüntülerindeki ilişkileri örüntüye dönüştürerek kuralı bulmaya yönelik çalışmalara da yer verilir.*

ç) *Günlük hayat durumu örneği: Birinci hafta 7 kelebekle koleksiyona başlayan Emine, sonraki her hafta koleksiyonuna 5 kelebek eklemektedir. Kelebek sayısının hafta sayısıyla ilişkisini cebirsel ifade olarak belirtiniz.*

Hafta	1	2	3	...	n
Toplam kelebek sayısı	7	12	17	...	...
İlişki	5 . 1 + 2	5 . 2 + 2	5 . 3 + 2	...	5 . n + 2

d) *Şekil örüntüsü örneği: Her adımda mevcut altıgenlerden yalnız biriyle ortak kenara sahip olacak şekilde altıgen eklenerek oluşturulan şekil örüntüsünde, altıgen sayısı ile toplam kenar sayısı arasındaki ilişkinin cebirsel kuralı nedir?*



Altıgen sayısı	*1*	*2*	*3*	...	*n*
Toplam kenar sayısı	*6*	*11*	*16*	...	...
İlişki	*5 . 1 + 1*	*5 . 2 + 1*	*5 . 3 + 1*	...	*5 . n + 1*

M.7.2.2. Eşitlik ve Denklem

Terimler veya kavramlar: eşitlik, derece, bilinmeyen, denklem

M.7.2.2.1. Eşitliğin korunumu ilkesini anlar.

a) *7 + 2 = +3 gibi eşitliklerin bozulmaması için yerine gelecek sayıyı bulmaya yönelik çalışmalar yapılır.*

b) *Ekleme ve çıkarma durumlarında eşitliğin korunduğunu göstermek için terazi veya benzeri denge modellerine yer verilir.*

c) *Eşitliğin her iki tarafına aynı sayının eklenmesi veya çıkarılması veya iki tarafın aynı sayıyla çarpılması veya bölünmesi durumunda eşitliğin korunması ele alınır.*

M.7.2.2.2. Birinci dereceden bir bilinmeyenli denklemi tanır ve verilen gerçek hayat durumlarına uygun birinci dereceden bir bilinmeyenli denklem kurar.

M.7.2.2.3. Birinci dereceden bir bilinmeyenli denklemleri çözer.

Denklemlerdeki katsayılar tam sayılardan seçilir.

M.7.2.2.4. Birinci dereceden bir bilinmeyenli denklem kurmayı gerektiren problemleri çözer.

B. THE CONCEPT OF ALGEBRA LEARNING OBJECTIVES OF 7th GRADE/7. SINIF CEBİR ÖĞRENME ALANI KAZANIMLARI (MoNE, 2018)

M.7.2. ALGEBRA

M.7.2.1. Algebraic Expressions

M.7.2.1.1. Students perform addition and subtraction operations with algebraic expressions.

Students use appropriate models for addition and subtraction with algebraic expressions.

M.7.2.1.2. Students multiply a natural number with an algebraic expression.

For example 5 (x + 3) = 5x + 15

M.7.2.1.3. Students denote the rule of the number patterns with a letter and find the desired term.

a) *The difference between the steps is limited to patterns that are constant. The importance and necessity of variable use is highlighted.*

b) *By examining number patterns, studies are carried out to write the rule of the pattern with a variable (such as n). For example, an arithmetic pattern with the first four terms 3, 9, 15 and 21 is referred to as rule 6n 9 3.*

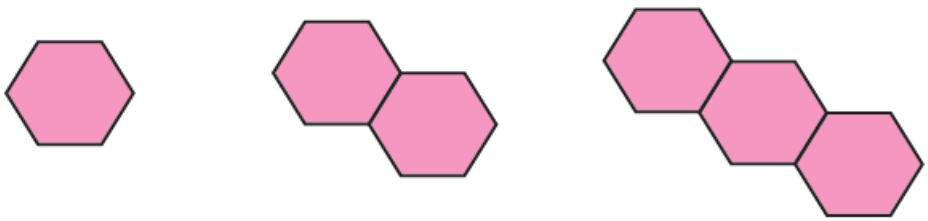
c) *Studies in order to find the rule by transforming the relationships in real life situations or shape patterns are also included.*

d) *An example of real life situation: Emine, who started collecting with 7 butterflies in the first week, adds 5 butterflies to her collection every week. Compose the algebraic expression which includes the relationship between the number of butterflies and the number of weeks.*

Week	*1*		*2*	*3*	...	*n*
Number of Butterflies	*7*		*12*	*17*	...	...
The Relationship	*5.1 + 2*		*5.2 + 2*	*5.3 + 2*	...	*5.n + 2*

d) *What is the algebraic rule of the relationship between the number of hexagons and the total number of edges?*

Example of a shape pattern: In the figure, pattern formed by adding hexagons to have common edge with only one of the available hexagons in each step. What is the algebraic rule of the relationship between the number of hexagons and the total number of edges?



1. Step 2. Step 3. Step

Number of Hexagons	*1*	*2*	*3*	...	*n*
Total number of Edges	*6*	*11*	*16*	...	...
The Relationship	*5.1 + 1*	*5.2 +1*	*5.3 + 1*	...	*5.n + 1*

M.7.2.2. Equality and Equations

Terms or Conceptions: equality, degree, unknown, equation

M.7.2.2.1. Students understand the principle of equality conservation.

a) *In order to conserve equalities like* $7 + 2 = \Delta + 3$ *, doing exercises to find value of* Δ*.*

b) *Scales or similar equilibrium models are used to indicate that conserving the equality in addition and subtraction situations.*

c) *Considering the conserve of equality with addition or subtraction of the same number and multiplied or divided by the same number on both sides of equation.*

M.7.2.2.2. Students recognize first degree equations with one unknown and posing a first degree equation with one unknown according to real life situations.

M.7.2.2.3. Students solve first degree equations with one unknown.

The coefficient in the equations are selected from the whole numbers.

M.7.2.2.4. Students solve problems with requiring posing first degree equations with one unknown.

C. COURSES IN ELEMENTARY MATHEMATICS TEACHER EDUCATION

The list of mathematics, mathematics teaching, and general education courses in the Elementary Mathematics Education program of the selected teacher's graduated university are given at table below. The codes of the courses are given on the left, and the class hours and credits of the courses exist on the right of the table.

Elementary Mathematics Education Program Courses List

First Semester		
MATH 111	Fundamentals of Mathematics	(3-0)3
MATH 117	Calculus I	(4-2)5
MATH 115	Analytic Geometry	(3-0)3
ENG 101	English for Academic Purposes I	(4-0)4
EDS 200	Introduction to Education	(3-0)3
Second Semester		
MATH 112	Discrete Mathematics	(3-0)3
MATH 118	Calculus II	(4-2)5
MATH 116	Basic Algebraic Structures	(3-0)3
ENG 102	English for Academic Purposes II	(4-0)4
CEIT 100	Computer Applications in Education	(2-2)3
Third Semester		
PHYS 181	Basic Physics I	(4-2)5
STAT 201	Introduction to Probability and Statistics I	(3-0)3
MATH 219	Introduction to Differential Equations	(4-0)4
ELE 221	Instructional Principles and Methods	(3-0)3
EDS 220	Educational Psychology	(3-0)3
HIST 2201	Principles of Kemal Atatürk I	NC
Fourth Semester		
MATH 201	Elementary Geometry	(3-0)3
PHYS 182	Basic Physics II	(4-2)5

STAT 202	Introduction to Probability and Statistics II	(3-0)3
ELE 225	Measurement and Assessment	(3-0)3
ENG 211	Academic Oral Presentation	(3-0)3
HIST 2202	Principles of Kemal Atatürk II	NC
Fifth Semester		
MATH 260	Linear Algebra	(3-0)3
ELE 341	Methods of Teaching Mathematics I	(2-2)3
TURK 305	Oral Communication	(2-0)2
Elective I		(3-0)3
Elective II		(3-0)3
Sixth Semester		
ELE 310	Community Service	(1-2)2
ELE 329	Instructional Technology and Material Development	(2-2)3
ELE 342	Methods of Teaching Mathematics II	(2-2)3
EDS 304	Classroom Management	(3-0)3
TURK 306	Written Communication	(2-0)2
Elective III		(3-0)3
Seventh Semester		
ELE 301	Research Methods	(3-0)3
ELE 419	School Experience	(1-4)3
ELE 465	Nature of Mathematical Knowledge for Teaching	(3-0)3
Elective IV		(3-0)3
Elective V		(3-0)3
Eight Semester		
ELE 420	Practice Teaching in Elementary Education	(2-6)5
EDS 416	Turkish Educational System and School Management	(3-0)3
EDS 424	Guidance	(3-0)3
Elective VI		(3-0)3

D. COURSES IN SCIENCE AND MATHEMATICS TEACHING M. S. PROGRAM/FEN VE MATEMATİK ÖĞRETİMİNDE DERSLER YÜKSEK LISANS PROGRAMI

The list of courses in the Science and Mathematics Teaching M. S Degree program that the teacher took is given in the table below. The codes of the courses are given on the left, and the class hours and credits of the courses exist on the right of the table.

Science and Mathematics Teaching Master's Degree Program Courses List		
8220506	Quantitative Data Analysis in Education	(3-2)4
4200540	Test Construction in Science and Mathematics Education	(3-0)3
4300521	Distance Education: Theory, Research, and Application	(2-2)3
4100611	Self-Regulation in Elementary School Set	(3-0)3
8220509	Educational Inquiry	(3-0)3
4200456	Laboratory Applications in Mathematics Teaching	(2-2)3
8220572	Independent Study in Elementary Mathematics and Science Education	(3-0)3
8220500	M.S Thesis	(0-0)0
8220550	Seminar in Elementary Mathematics and Science Education	(0-0)0
8220810	Special Studies	(0-0)0

E. OBSERVATION PROTOCOL/DERS GÖZLEM PROTOKOLÜ

Okul: **Sınıf:** **Saat:** **Tarih:**

Öğretmen: **Gözlemci:**

Öğrenme Alanı: **Alt Öğrenme Alanı:**

1. Sayın gözlemci gözlediğiniz ders esnasında aşağıda verilen kelimelerin nasıl ve ne sıklıkta kullanıldığını not ediniz. Bu kelimelerden farklı kelimeler kullanılmışsa eğer listenin altına ekleyip nasıl ve ne sıklıkta kullanıldığını belirtiniz.

Kullanılan Kelimeler	**Ne Sıklıkta**	**Nasıl Kullanıldı**
Terim		
Değişken		
Bilinmeyen		
Katsayı		
Sabit Terim		
Benzer Terim		
Cebirsel İfade		
Cİ Çıkarma		
Cİ Toplama		
Cİ Çarpma		
Modelleme		
Örüntü		
İşaret		
Sembol		
Notasyon		
Temsilci Sayı		
Genel Sayı		
Sıra		
Adım		

2. Sayın gözlemci gözlediğiniz ders esnasında aşağıda verilen görsel araçların nasıl ve ne sıklıkta kullanıldığını not ediniz. Bunlardan farklı araçlar kullanılmışsa eğer listenin altına ekleyip nasıl ve ne sıklıkta kullanıldığını belirtiniz.

Resim	
Sembol	

Tablo	
Grafik	
Materyal (Cebir Karoları, Eşit Kollu Terazi, vs.)	
Teknoloji (Akıllı Tahta, Eba, Morpa, Yazılım, Video, vs.)	

3. Sayın gözlemci gözlediğiniz ders esnasında tanımlar ve teoremler nasıl ve ne zaman kullanılmıştır, not ediniz.

Tanım/ Teorem	**Ne Zaman**	**Nasıl**

Sayın gözlemci gözlediğiniz ders esnasında yer alan rutinleri not ediniz.

- Cebir dersinde cebirsel ifadeler nasıl oluşturulur?
- Cebir dersinde cebirsel ifadelerle toplama işlemi nasıl yapılır?
- Cebir dersinde cebirsel ifadelerle çıkarma işlemi nasıl yapılır?
- Cebir dersinde bir doğal sayı ile cebirsel ifade nasıl çarpılır?

F. INTERVIEW QUESTIONS EXAMPLE/MÜLAKAT SORULARI ÖRNEĞİ

- Considering the language you use in the classroom, do you think that mathematics has a different language?
- The language students use at school and in daily life varies. Considering the algebra course, do you think that the language used in daily life affects students' success in mathematics?
- You utilize some analogies in your lessons (such as grandfather-grandson, balloon, Casper, and smiling face). Could you please explain the purpose of using these and how you created them?
- Do these analogies affect the development of students' mathematical discourse and their level of understanding of mathematics? If so, how do you establish the connection between this everyday discourse and mathematical discourse?
- In lessons, some students do not participate in discussions and solutions to exercises. Sometimes you intervene in such situations and encourage them to participate, why? Why do you think these students do not want to participate in discussions?
- You follow a certain routine when solving exercises in algebra class. What is the reason for this?
- What is your purpose of asking for bonus examples in lessons and what type of examples do you choose? There are certain rules that the class must follow after the examples you are asked. How do you determine these?
- Throughout the algebraic expressions lessons, after writing some exercises and problems on the board, sometimes you need the change the numbers, why?
- You ask students to write the definitions on their notebooks. Sometimes you give these definitions at the beginning of the topic and sometimes at a different time in the lessons. What is the best time to give these definitions (beginning, during, or end of the topic)? Why?

- It is generally observed that you have two types of routines in algebra classes. These are behavioral routines and mathematical routines related to solving algebraic exercises and problems. How and why did you create these routines?

G. WORD USE ANALYSIS EXAMPLE/KELİME KULLANIMI ANALİZİ ÖRNEĞİ

Words	Line	Utterances	Word-Use Categories
Terim	40	Ö: Bekle bekle. (Tahtaya yazarken) a)terim sayısı, b) terimler (başlıkta geçiyor analize dahil değil) başka c) Katsayı doğru mu d) sabit terim bide şey vardı e) katsayılar toplamı topluyorduk biz katsayıları Yazılıda bunun aynısını sormuştum. Var mı hatırlayan? Yok. Aa çok ayıp. Kaç tane terim var? (Soru cümlesinde kullanılmış, dahil değil)	Object-Driven (Nesne-Bazlı) (matematiksel bir nesne olarak kullanmış)
	44 45 46 47 48 49 50 51 52	Ö: 3 hemen yazıyorum. Peki terimleri kim söyleyecek bana, parmak. Evet Sudenaz. Sude: 3x Ö: virgül koyalım ayırmak için. Diğeri ne? Söyle kızım. K: 2x SC: -2.. Ö: Bi dakka bi dakka, Karışma karışma, ben kimi kaldırıyorsam kime söz hakkı veriyorsam o cevaplasın K: -2x Ö: eksiii K: -2y	Object-Driven (Nesne-Bazlı) (Öğretmen öğrencilerle birlikte terimleri buluyor. Matematiksel nesne olarak kullanmışlar)
	53	Ö: Hah -2y yuvarlağın içindekini bir terim olarak alıyorum. -2y (tahtaya yazıyor)...	Phrase-Driven (Tabir-Bazlı) (Neyin terim olduğunu söylüyor)
	53 54 55 56	Ö: … Başka var mı terim? SC: var var Ö: Sultan Sultan: +5	Object-Driven
	59	Ö:Yok ben onun zaten ne olduğunu biliyorum, artı olduğunu biliyorum. Arkadaşlar ne yaptık terimleri ayırdık, işaretleri unutmuyoruz. Eksi 2y ye ait, 3x zaten belli.	Object-Driven (sonlandırma ifadesi olabilir?) (Nesne-Bazlı) Routine-Driven da olabilir
	60	Ö: Katsayı, yani terimlerdeki kat-sayı (heceliyor), hatırla, kat olan sayılar neler, mesela ilk terimin katsayısı kaç?	Object-Driven (Nesne-Bazlı)
	62	3 evet. İlk terim 3x ti. Bu terimdeki kat olan sayı kaç 3. Başka, diğeri ikinci terim Zeki?	Object-Driven, (Nesne-Bazlı)
	101	Ö:...Şimdi bana dediniz ki orda terimler var. Hangi terimlerdi onlar 3x, -2y, +5. 3x ile -2y benziyor mu?	Object-Driven,
	125	Ö: …Yuvarlağa almada zorlanan var mı? Hatırlamayan var mı? Hıı? İşleme kadar gidiyordum işlemden itibaren alıyordum doğru mu? Arkadaşımız böyle yaptı (Tahtaya çıkan öğrenciden bahsediyor) – 2y ye ait olduğu için onunla beraber grupladık. + 5'e ait olduğu için onunla beraber grupladık. Her zaman işleme kadar gidip yuvarlağımızı alıyorduk. Hadi bakalım. *(Burada terimleri nasıl bulacaklarını tekrar ediyor)*	Routine-Driven (Terim ifadesi diyalogda geçmese da, terimlerin nasıl bulunduğu anlatıldığı için analize dahil edildi)
	134	Ö: 2 eleman, yani iki terimli bi ifade olursa tabiki çıkarma işlemini bana sorabilirler. Ama 3'lü bi ifadede bana çıkarma işlemini sormazlar. Doğru mu? Mantığım böyle diyor sanki bana.	Object-Driven (Nesne-Bazlı)
	157	Ö: evet o zaman bakıyorum tamamı birebir yani iki terimi her şeyi aynı oluyor ya bazı terimlerin aynı olması gerekiyor. Yani bilinmeyenlerin aynı olması gerekiyor. Farklı bi ifade gördüğümüz zaman aynı olmuyor. Hatta ben altına şöyle yapalım bi kaç tane vereyim, benzer olup olmadıklarını beraber tartışalım. Tamam mı? Önce şunu bi yazalım, önce şunu bi yaz. Ondan sonra devam edelim. Hemen bi notumuz var benzer terimlerle ilgili. Not. Şöyle diyelim.	Object-Driven (Nesne-Bazlı) (benzer terim ifadesini açıklarken burada terim sözcüğünü matematiksel bir nesne olarak kullanıyor)

	157	Ö: {Bir cebirsel ifadede harfli ve harflerin kuvvetleri aynı olan terimlere benzer terimler denir.}	**Object-Driven**
	197 198 199	Şevket: (Notu okuyor) Bir cebirsel ifadede harflerin ve harflerin kuvvetleri aynı olan terimlere benzer terim denir. Ö: Süleyman bir de sen oku Süleyman. Oku yüksek sesle arkadaşlarına Süleyman: Not: Bir cebirsel ifadede harflerin kuvvetleri aynı olan terimlere benzer terim denir.	**Object-Driven** (Nesne-Bazlı) *(Benzer terim tanımı içerisinde matematiksel isim olarak kullanılıyor Bi üstte öğretmen burada da öğrenciler kullanıyor tanımı)*
Bilinmeyen	8	Ö: Biz kuralları bu sene ifade ederken bilinmeyen ifadelerle a, b, c gibi ifadeler kullanıyorduk. Onlarla ifade edeçez.	**Phrase-Driven** (Bilinmeyenin ne olduğunu açıklıyor)
	25	Ö: Arkadaşlar cebirsel ifadenin tanımını yaparken şöyle bişey kullanmıştık. İçerisinde bilinmeyenler bulunan ve işlem içeren ifadelere biz cebirsel ifadeler diyoruz.	**Object-Driven** (Nesne-Bazlı) (tanım içerisinde yer almış)
	25 26 27 28 29	Ö: …Var mı bilinmeyen? SC: Var Ö: Ne bunlar? SC: 2y, 3x SC: -2y, 3x Ö: Yani x ve y…	**Object-Driven**
	101	Ö: …İçinde bilinmeyen bulunan ve işlemler bulunan ifadelere biz ne diyoruz, cebirsel ifadeler diyoruz…	**Object-Driven** (Nesne-Bazlı)
	101 102 103 104	Ö: …Şimdi bana dediniz ki orda terimler var. Hangi terimlerdi onlar 3x, -2y, +5. 3x ile -2y benziyor mu? SC: Benziyor Ö: Ne yönden benziyor? E: İkisinde de bilinmeyen var.	**Object-Driven** (Nesne-Bazlı)
	107 108 109 110 111 112 113 114	Ö: Ya da şöyle söyleyelim. İçindeki bilinmeyenler aynı mı? SC: Aynı. Hocam orda […] Ö: Bilinmeyenler aynı mı? SC: Hayıırrr Ö: Bi tane ne var SC: x Ö: Bi tane ne var? SC: y	**Object-Driven** (Nesne-Bazlı) Object-Driven (Nesne-Bazlı)
	125	Ö: …İçerisinde bilinmeyen bulunan ve işlem içeren ifadelere cebirsel ifadeler denir.}	Object-Driven (Tanımı tekrar ediyor matematiksel bir isim olarak kullanıyor)
	135	Ö: Yani ikisinde de bilinmeyen x var ve bunlar da birbirinin aynısı. Aaa o zaman tamam buna benzer dedik. Bi de şöyle yapalım "-3y ve 2xy" benzer midir?	Object-Driven (Nesne-Bazlı)
	146 147	Ö: Şimdi eksi ile artıya takılıyor muyum? Bilinmeyenler mi beni ilgilendiriyor? SC: (Çoğunluk bağırıyor) Bilinmeyenleeerrr	Object-Driven (Nesne-Bazlı)
	155 156	Ö: Hatta şöyle yapayım ben size düzgün bir şekilde göstereyim. (Bilinmeyenleri yuvarlak içine alıyor) (2x ve -4x ifadelerinde x'leri yuvarlak içine alıyor) İkisinde de x var "benzerdir". (-3y ve 2xy ifadelerinde sırayla y ve xy'yi yuvarlak içine alıyor) birinde y birinde xy var "benzer terim değildir". Aradaki farkı anladık mı? SC: Evet	Routine-Driven (Öğretmen burada bilinmeyen kelimesini kullanmasa bile bilinmeyen ifadeleri bulma eylemi gerçekleştiriyor)
	157	Ö: Evet o zaman bakıyorum tamamı birebir yani iki terimi her şeyi aynı oluyor ya bazı terimlerin aynı olması gerekiyor. Yani bilinmeyenlerin aynı olması gerekiyor. Farklı bi ifade gördüğümüz zaman aynı olmuyor. Hatta ben altına şöyle yapalım bi kaç tane vereyim, benzer olup olmadıklarını beraber tartışalım. Tamam mı? Önce şunu bi yazalım, önce	Object-Driven (Nesne-Bazlı)

		şunu bi yaz. Ondan sonra devam edelim. Hemen bi notumuz var benzer terimlerle ilgili. Not. Şöyle diyelim.	
	184	Ö: Bi dakka biz ona x2 demiyorduk ne diyorduk x^2y diğeri ne $-5x^2y$ bilinmeyenler aynısı olduğu için bunlar benzerdir. (d) $3x^2y$ ile $-5x^2y$ yanına tik koyuyor)	Object-Driven (Nesne-Bazlı)
Katsayı	40	Bekle bekle. (Tahtaya yazarken) a)terim sayısı, b) terimler başka c) [redacted] doğru mu d) sabit terim bide şey vardı e) katsayılar toplamı topluyorduk biz katsayıları Yazılıda bunun aynısını sormuştum. Var mı hatırlayan? Yok. Aa çok avıp. Kaç tane terim var?	Object-Driven (Nesne-Bazlı) (Başlık olarak kullanılmış analize dahil değil)
	60	[redacted], yani terimlerdeki kat-sayı (heceliyor), hatırla, kat olan sayılar neler, mesela ilk terimin [redacted] kaç? Adı üstünde bak sayı, [redacted] bana kim söyler? Sanatçım	Phrase-Driven (Tabir-Bazlı)
	62	Bu terimdeki [redacted] kaç 3. Başka, diğeri ikinci terim Zeki	Phrase-Driven (Tabir-Bazlı)
	75 76	Ö: [redacted] hangileriydi SC: (hep birden) 3, 2, -2, +5	Object-Driven (Öğrenciler katsayıları söylüyor)
	81	Ö: Almazsam hata olurdu. [redacted]. İlk [redacted] ne? SC: 3 Ö: Hadi +3 diyelim grup (paranteze alıyor), sonra? S: -2 Ö: Sonra S: +5 Ö: Ne yapıyordum? Şimdi artılarımı koyuyorum.	Object-Driven
	127 128 129	Ö: Genelde toplama sorulur, çıkarma sorulmaz. O zaman 3 tane [redacted] ben çıkarabilir miyim? (sınıftan hep bir ağızdan sesler geliyor) Kaç tane sayı olması gerekiyor? Benim çıkarma işlemi yapabilmem için kaç tane sayı olması gerekiyor? SC: 2 Ö: Orda 3 tane [redacted] var.	Object-Driven
Katsayılar Toplamı	40	Bekle bekle. (Tahtaya yazarken) a)terim sayısı, b) terimler başka c) Katsayı doğru mu d) sabit terim bide şey vardı e) [redacted] topluyorduk biz katsayıları Yazılıda bunun aynısını sormuştum. Var mı hatırlayan? Yok. Aa çok avıp. Kaç tane terim var?	Object-Driven (Nesne-Bazlı) (Başlık olarak kullanılmış analize dahil değil)
	73	Ö:Evet e şıkkı [redacted]. Kat olan sayıları ne yap diyor S: (hep birden) Topla	Routin-Driven (Rutin-Bazlı)
	77 78	Ö: Bana [redacted] diyor. Toplarken ben neye dikkat edeceğim. SC: eksiyle artıya Ö: Yani neye dikkat edeceğim. Gruba dikkat edeceğim. Doğru mu?	Phrase-Driven (Tabir-Bazlı)
	81 82 83 84 85 86 87 88 89 90 91 92 93 94	Ö: Almazsam hata olurdu. [redacted]. İlk [redacted] ne? SC: 3 Ö: Hadi +3 diyelim grup (paranteze alıyor), sonra? S: -2 Ö: Sonra S: +5 Ö: Ne yapıyordum? Şimdi artılarımı koyuyorum. Gruplarımı belirledim yapıyorum. Ne var burada SC: Gülen yüz Ö: iki tane gülen yüz. 3'ün işareti ne? SC: Artı Ö: Artı ile eksi SC: eksi 2 Ö: Artı ile artı SC: Artı Ö: Çünkü anlaşıyorlar. Artı 5. Aaa bu nasıl halde SC: Toplayalım SC: Çok dağınık Ö: Dağınık, biz onu ne yapacağız biraz toparlayalım. Artlı olanları bir yere alayım, eksili olanları aynı şekilde toparlayalım. Kim söylüyor artılı olanları toparlarsam, Sinem? Sinem: +8	Routine-Driven (Bu diyalogda öğretmen "katsayılar toplamı" ifadesini telaffuz etmese de katsayılar toplamı bulma eylemini yapıyor adım adım)

	95 96 97 98 99 100 101	Ö: 3 tane artım var 5 tane daha geldi +8 bi de -2'm var. Torbada ne kalır? SC: +6 Ö: Hangisi fazla? SC: Artı Ö: Kaç fazla S: 6 Ö: Torbamda 6 tane artım var. Katsayılar toplamını da bulduk.	
	122 123 124	Ö: Katsayıları toplarken paranteze almasaydık ne olurdu? Hata yapar mıydık? SC: Evet Ö: Gruplamadığımız zaman mutlaka bi yerlerde hata yaparız.	Phrase-Driven, (Tabir-Bazlı) (Routine-Driven) ???
	126	K: Öğretmenim hani siz katsayılar toplamı dediniz ya, kat sayıların çıkarması… çıkarçaz mı?	Passive-use (edilgen kullanım)
	127 128 129	Ö: Genelde toplama sorulur, çıkarma sorulmaz. O zaman 3 tane katsayıyı ben çıkarabilir miyim? (sınıftan hep bir ağızdan sesler geliyor) Kaç tane sayı olması gerekiyor? Benim çıkarma işlemi yapabilmem için kaç tane sayı olması gerekiyor? SC: 2 Ö: Orda 3 tane katsayı var. (Sınıf kendi arasında tartışmaya devam ediyor)	Object-Driven, (Nesne-Bazlı)
Terim Sayısı	40	Bekle bekle. (Tahtaya yazarken) a) terim sayısı, b) terimler başka c) Katsayı doğru mu d) sabit terim bide şey vardı e) katsayılar toplamı topluyorduk biz katsayıları Yazılıda bunun aynısını sormuştum. Var mı hatırlayan? Yok. Aa çok avıp. Kaç tane terim var?	Object-Driven, (Nesne-Bazlı) (Başlıkta geçiyor analize dahil değil)
	42	Yapıyorduk bunlarda, yuvarlağa aldığımızı sayıyorduk 1, 2, 3 yuvarlak var dolayısıyla benim terim sayım kaç tane oluyor?	Routin-Driven, (Terim sayısını nasıl bulduğunu gösteriyor)
Sabit Terim	69	Evet sabit terim, sabit olan değişmeyen, aa-bit. Bu kadar kişi mi biliyor? Sudenaz	Phrase-Driven,
Benzer Terim	1 2	Ö: …Cebirsel ifadelerde 4 tane başlığım var. 1) Benzer Terimler… Ö: 4 tane başlığım var ilki benzer terimler…	Object-Driven, (Nesne-Bazlı) (Başlıkta geçiyor analize dahil değil)
	10	Ö: …Yine aynı şekilde toplama çıkarma çarpmamız var ama bölmemiz yok yani üç işlem göreceğiz. Uzantısı olarak örüntü ve ilişkilerden bahsedeceğiz. Çünkü bu kuralları bulurken cebirsel ifadelerden bahsetmiş olacağız. En başta zaten temel ne var benzer terimler var. (Öğrenciler tahtadaki kavram haritasını deftere yazarken öğretmen tekrar kavram haritasını anlatıyor).	Object-Driven,
	101 102 103 104 105 106 107 108 109 110 111 112 113	Ö: …Şimdi bana dediniz ki orda terimler var. Hangi terimlerdi onlar 3x, -2y, +5. 3x ile -2y benziyor mu? SC: Benziyor Ö: Ne yönden benziyor? E: İkisinde de bilinmeyen var Ö: Başka S: […] Ö: Ya da şöyle söyleyelim. İçindeki bilinmeyenler aynı mı? SC: Aynı. Hocam orda […] Ö: Bilinmeyenler aynı mı? SC: Hayırrr, Ö: Bi tane ne var SC: x Ö: Bi tane ne var?	Passive-use (Burada öğrenciler "benzer terim" ifadesinin ne demek olduğunu aslında bilmiyor. Fakat öğretmen sorular sorduğunda doğru ya da yanlış cevaplar vermeye çalışıyorlar.

	114 115 116	SC: y Ö: (tahtayı göstererek) Arkadaşlar ilk başlığımız neydi? Yukarda ne yazıyordu ilk başlıkta? SC: Benzer terimler	
	117	Ö: Benzer terimler. Arkadaşım içinde böyle farklı ifadeler varsa bunlar benzer olmuyor. Anladık mı olayı? Tamam mı? Eğer ben orda mesela şöyle bi ifade olsaydı 3x -2x + 5 gibi bir ifade olsaydı ben şunu söyleyecektim. 3x ile -2x benzer terimlerdir diyecektim. Biraz sonra benzer terimlere geçeceğiz detaylı bir şekilde. Önce tahtayı bi yazalım defterimize. […]	Phrase-Driven (Tabir-Bazlı) (Öğretmen benzer terimlerin ne olduğu ve olmadığına dair bilgiler veriyor)
	134	Şimdi siliyorum tahtayı. Birinci başlığımız benzer terimler. Başlığını at. "1.Benzer terimler" Yaslan…	Object-Driven (Nesne-bazlı) (Başlıkta geçiyor analize dahil değil)
	134 135 136	Ö: …"2x ve – 4x" benzer midir? SC: Eveeettt, ikisinde de x var Ö: Yani ikisinde de bilinmeyen x var ve bunlar da birbirinin aynısı. Aaa o zaman tamam buna benzer dedik…	Phrase-Driven (Tabir-Bazlı)
	136 137 138 139 140	Ö:…Bi de şöyle yapalım "-3y ve 2xy" benzer midir? SC: benzer SC: Değil Ö: Benzerdir diyenler? Benzer değildir diyenler? (Her iki seçenek için de parmaklar kalkıyor) Neden benzerdir Şevket'cim? Şevket: Hocam x ve y var	Object-Driven (Passive-use) (Öğrenciler)
	149 150 151	E: Aynı değil bence Ö: Yani benzer? E: Değil *(öğrenci tam olarak benzer kavramını kullanamıyor, bunun yerine aynı diyor)*	Passive-Use
	153	Ö: Arkadaşlar birinde x var birinde xy var. Bunlar aynı mı? SC: Değil Ö: Olmadıkları için bunlar benzer değil.	Phrase-Driven (Tabir-Bazlı)
	155	Ö: Hatta şöyle yapayım ben size düzgün bir şekilde göstereyim. (Bilinmeyenleri yuvarlak içine alıyor) (2x ve -4x ifadelerinde x'leri yuvarlak içine alıyor) İkisinde de x var "benzerdir". (-3y ve 2xy ifadelerinde sırayla y ve xy'yi yuvarlak içine alıyor) birinde y birinde xy var "benzer terim değildir". Aradaki farkı anladık mı?	Routine-Driven (Benzer terimi nasıl bulacaklarını gösteriyor sınıfa)
	157	Ö: evet o zaman bakıyorum tamamı birebir yani iki terimi her şeyi aynı oluyor ya bazı terimlerin aynı olması gerekiyor. Yani bilinmeyenlerin aynı olması gerekiyor. Farklı bi ifade gördüğümüz zaman aynı olmuyor. Hatta ben altına şöyle yapalım bi kaç tane vereyim, benzer olup olmadıklarını beraber tartışalım. Tamam mı? Önce şunu bi yazalım, önce şunu bi yaz. Ondan sonra devam edelim. Hemen bi notumuz var benzer terimlerle ilgili. Not. Şöyle diyelim. NOT: Bir cebirsel ifadede harfli ve harflerin kuvvetleri aynı olan terimlere benzer terimler denir	Phrase-Driven (Tabir-Bazlı)
	159	Ö: Benzer olanların yanına tik benzer olmayanların yanına çarpı koyalım. Önce bi yazalım hep beraber yapıcaz. Yaz arkana yaslan. […] Hızlan, hızlan […] Yazma işlemini bitirenler arkasına yaslansın bende göreyim. Kaç kişi var bitirmeyen…	Object-Driven (Nesne-bazlı)
	159 160 161 162 163 164 165 166 167	Ö:…Evet birincisi. 4x ile x benzer midir? Parmak. Parmak. Bu kadar kişi mi? Parmak. Buse Buse: Benzerdir. Ö: Benzerdir. Sebep? Buse: 4 ile çarpma…x var Ö: Bi dakka bi dakka karışma. Söyle Buse: x var Ö: İkisinde de ne var? Buse: x var Ö: O zaman bunlar benzerdir. (-a) 4x ile x yanına tik koyuyor)	Phrase-Driven (Tabir-Bazlı) *(Phrase-driven çünkü öğrenci burda ikisinde de aynı bilinmeyen olduğunu söylüyor)* (Öğretmenin 159 da soru sorması ve 161 de busenin söylediğini tekrar etmesi analize katılmamıştır)

	168 169 170 171 172	Ö: Başka 3x ile y. Parmak. Söyle. K: Değildir. (Benzer) Ö: Değildir, Sebep? K: Birisinde x var birisinde de y var. Ö: Evet değildir hemen çarpımı koydum. (-b) 3x ile y yanına çarpı işareti koyuyor.)	Phrase-Driven (Tabir-Bazlı) *(Burada da öğrenci tanımı kullanarak benzer olma durumu bakıyor)*
	173 174 175 176 177 178 179 180	Ö: xy ile 2xy. Yakup söylemek ister misin? Benzer midir değil midir? Yakup: […] hangi soru? Ö: c'yi yapıyoruz Yakup'cum. Yakup: Benzerdir. Ö: Benzerdir, sebep? Yakup: xy… ikisinde de xy var SC: İkisinde de xy var Ö: İkisinde de xy var, evet benzerdir (c) xy ile 2xy yanına tik koyuyor.)…	Phrase-Driven (Tabir-Bazlı) *(Burada da öğrenci tanımı kullanarak benzer olma durumu bakıyor)*
	180 181 182 183 184 185	Ö:…Diğeri? $3x^2y$ ile $-5x^2y$. Söyle oğlum. E: Benzer Ö: Benzerdir, evet, neler aynı o zaman E: Hocam x2y… SC: Kareler aynı Ö: Bi dakka biz ona x2 demiyorduk ne diyorduk x^2y diğeri ne $-5x^2y$ bilinmeyenler aynısı olduğu için bunlar benzerdir. (d) $3x^2y$ ile $-5x^2y$ yanına tik koyuyor)	Passive-Use *(Burada öğrenci benzer olma koşulu olan bilinmeyen kavramını anlamamış))*
	186 187 188 189	Ö: Evet devam ediyorum. x^2 ile x. Sivri? Sivri: Değildir. (Benzer) Ö: Değildir. Sebep? Sivri: Hocam onun üstünde kare var	Phrase-Driven (Tabir-Bazlı)
	190 191 192	Ö: Bi dakka bi daka Şevket bi şey söyledi. Söyle Şevketcim. Şevket: Hocam x'in karesi 2 tane x'in çarpımıdır. Ama orda bi tane x var. Ö: Haa, diyor ki arkadaşınız x'in üzerinde 2 vardır o iki tane x'in çarpımının yerine geçer, ama orda bi tane x vardır benzer değildir diyor.	Phrase-Driven (Tabir-Bazlı) (Öğrenci benzer terim olma koşulundan bahsediyor)
	193 194	SC: Bence benzer SC: Değil (Sınıftan çeşitli fikirler geliyor aynı anda) Ö: Şevket'e katılanlar benzer değildir diyen kaç kişi var? (parmaklar kalkar) Süleyman'a katılıp benzerdir diyenler? Diğerleri, kaldırmayanlar çekimser mi oluyor? Bence benzer.	Object-Driven (Nesne-bazlı) *(Benzer terim kavramı bir matematiksel kavram, isim olarak kullanılıyor)*
	195 196 197 198 199 200 201 202 203 204 205	Şevket: Hocam yukarıya bi not yazdık. Ö: Bi dakka bi dakka Şevket diyor ki yukarıya bi not yazdık oku bakalım Şevket'cim onu? Şevket: (Notu okuyor) Bir cebirsel ifadede harflerin ve harflerin kuvvetleri aynı olan terimlere benzer terim denir. Ö: Süleyman bir de sen oku Süleyman. Oku yüksek sesle arkadaşlarına Süleyman: Not: Bir cebirsel ifadede harflerin kuvvetleri aynı olan terimlere benzer terim denir. Ö: Ne dikkatini çekti orda? Süleyman: Kuvvetleri aynı olacak (Zil çalıyor) Ö: Kuvvetleri aynı olacak diyor. Di mi? O zaman bunlar benzer mi? SC: Değiiiiiiiiil (Hep birden cevaplıyorlar) Ö: Değildir, Son kararımız mı? SC: Eveeeeet Ö: Evet kuvvetlerinin de aynı olması gerekir. Hemen diğerlerini de yapalım. (f şıkkını gösteriyor, f) x ile 1/x) SC: Benzer, değil, benzer, değil (Sınıftan cevaplar geliyor)	Phrase-Driven (Tabir-Bazlı) *(Burada benzer olma durumunun tanımından faydalanarak bir tartışma yapılıyor)*
	200 201 202 203 204 205	Ö: Ne dikkatini çekti orda? Süleyman: Kuvvetleri aynı olacak (Zil çalıyor) Ö: Kuvvetleri aynı olacak diyor. Di mi? O zaman bunlar benzer mi? SC: Değiiiiiiiiil (Hep birden cevaplıyorlar) Ö: Değildir, Son kararımız mı? SC: Eveeeeet	Phrase-Driven (Tabir-Bazlı)

	206 207 208 209 210 211 212 213	Ö: Evet kuvvetlerinin de aynı olması gerekir. Hemen diğerlerini de yapalım. (f şıkkını gösteriyor, f) x ile 1/x) SC: Benzer, değil, benzer, değil (Sınıftan cevaplar geliyor) Ö: Bi dakka söyle E: Hocam birinde 1 var SC: Hayır hayır … Ö: Benzer diyenler (Sınıf çözümde karara varamadı) Tartışmalı bi konu gelince devam edelim. SC: Benzer değil hocam Ö: Gelince devam edelim.	Object-Driven
Cebirsel İfade	1	Ö: …Cebirsel ifadelerde 4 tane başlığım var…	Object-Driven (Başlık analize dahil değil)
	8	Ö: Kural buluyoruz. Biz kuralları bu sene ifade ederken bilinmeyen ifadelerle a, b, c gibi ifadeler kullanıyorduk. Onlarla ifade edecez. Yani bir bakıma Cebirsel ifadeleri kullanmış olucaz.	Phrase-Driven (cebirsel ifadelerin hangi ifadeleri içerdiğini söylüyor)
	25	Ö: Arkadaşlar cebirsel ifadenin tanımını yaparken şöyle bişey kullanmıştık. İçerisinde bilinmeyenler bulunan ve işlem içeren ifadelere biz cebirsel ifadeler diyoruz…	Phrase-Driven (Tabir-Bazlı)
	101	Ö: …İçinde bilinmeyen bulunan ve işlemler bulunan ifadelere biz ne diyoruz. Cebirsel ifadeler diyoruz…	Phrase-Driven (Tabir-Bazlı)
	124	Ö: …Biz bi Cebirsel ifadelerin tanımını da yazalım tekrardan defterimize hatırlayalım kalıcı olsun. Tamam mı? […] Başlıkları yazın…. {İçerisinde bilinmeyen bulunan ve işlem içeren ifadelere cebirsel ifadeler denir}	Phrase-Driven (Tabir-Bazlı)
	157	Ö: Evet o zaman bakıyorum tamamı birebir yani iki terimi her şeyi aynı oluyor ya bazı terimlerin aynı olması gerekiyor. Yani bilinmeyenlerin aynı olması gerekiyor. Farklı bi ifade gördüğümüz zaman aynı olmuyor. Hatta ben altına şöyle yapalım bi kaç tane vereyim, benzer olup olmadıklarını beraber tartışalım. Tamam mı? Önce şunu bi yazalım, önce şunu bi yaz. Ondan sonra devam edelim. Hemen bi notumuz var benzer terimlerle ilgili. Not. Şöyle diyelim. {Bir cebirsel ifadede harfli ve harflerin kuvvetleri aynı olan terimlere benzer terimler denir.}	Object-Driven
	197 198 199	Şevket: (Notu okuyor) Bir cebirsel ifadede harflerin ve harflerin kuvvetleri aynı olan terimlere benzer terim denir. Ö: Süleyman bir de sen oku Süleyman. Oku yüksek sesle arkadaşlarına Süleyman: Not: Bir cebirsel ifadede harflerin kuvvetleri aynı olan terimlere benzer terim denir	Object-Driven (Nesne-Bazlı) *(Benzer terim tanımı yapılırken cebirsel ifade matematiksel bir isim olarak kullanılıyor)*
Cİ Çıkarma Cİ Toplama (Rutin olarak ele alınabilir)	1	Ö: …Cebirsel ifadelerde 4 tane başlığım var. 2) Cebirsel ifadelerle toplama ve çıkarma…	Object-Driven (Nesne-Bazlı) (Başlık)
	2	Ö: 4 tane başlığım var ilki benzer terimler 2) Bütün her şeyde ne yapıyoruz? Bi toplama çıkarma işlemini yapıyoruz.	Rutin-Bazlı (Başlık)
	10	Ö: Yine aynı şekilde toplama çıkarma çarpmamız var ama bölmemiz yok yani üç işlem göreceğiz.	Object-Driven (Nesne-Bazlı) (Başlık)
	15 16 17 18	Ö: İçinde ne var bu ifadenin (Tahtaya 3x+2y+5 yazıyor) K1: çarpı artı y E1: toplama çıkarma Ö: Toplama çıkarma var başka ne var?	Object-Driven (Nesne-Bazlı)
	29	Ö: Yani x ve y. İşlem olarak ne var? Toplamayla çıkarma. İlk biz ne yapıyorduk? Yuvaya alıyorduk. Şöyle yuvarlağa alıyorduk. Hatırladık mı?	Object-Driven
	101	Üstüne biraz daha farklı şeyler olacak. Toplaması gelecek, çıkarması gelecek, çarpması gelecek.	Object-Driven (Nesne-Bazlı)

Cİ Çarpma (Rutin olarak ele alınabilir)	1	Ö: …Cebirsel ifadelerde 4 tane başlığım var. Bir doğal sayı ile cebirsel ifadeyi çarpma …	Object-Driven (Başlık) (Nesne-Bazlı)
	4	Ö. Çarpma işlemimiz var Bölme işlemimiz yok	Object-Driven (Nesne-Bazlı)
	10	Ö: …Yine aynı şekilde toplama çıkarma çarpmamız var ama bölmemiz yok yani üç işlem göreceğiz…	Object-Driven (Nesne-Bazlı)
	101	Ö: …Üstüne biraz daha farklı şeyler olacak. Toplaması gelecek, çıkarması gelecek, çarpması gelecek.	Object-Driven (Nesne-Bazlı)
Modelleme			
Örüntü ve İlişkiler	1	Ö: …Cebirsel ifadelerde 4 tane başlığım var. 4) Örüntü ve ilişkiler	Object-Driven (Başlık) (Nesne-Bazlı)
	4	Ö: Niye örüntü ve ilişkiler oraya girmiş olabilir sanki biraz alakasız gibi mi duruyor? Neden girmiş olabilir şevket?	Object Driven
	6 7 8	Ö: Örüntülerde biz ne buluyoruz genel anlamda? Ş: Kural, üçgen, kare… Kural buluyoruz. Ö: Kural buluyoruz. Biz kuralları bu sene ifade ederken bilinmeyen ifadelerle a, b, c gibi ifadeler kullanıyorduk. Onlarla ifade edecez. Yani bir bakıma cebirsel ifadeleri kullanmış olucaz. O yüzden örüntü ve ilişkileri bu başlık altına koyduk.	Phrase-Driven (Tabir-Bazlı)
	10	Ö: Hayır hayır sığar. Okları biraz aşağı doğru yaparsın, tamam? Hadi bakalım. Yine aynı şekilde toplama çıkarma çarpmamız var ama bölmemiz yok yani üç işlem göreceğiz. Uzantısı olarak örüntü ve ilişkilerden bahsedeceğiz. Çünkü bu kuralları bulurken cebirsel ifadelerden bahsetmiş olacağız. En başta zaten temel ne var benzer terimler var. (Öğrenciler tahtadaki kavram haritasını deftere yazarken öğretmen tekrar kavram haritasını anlatıyor).	Object-Driven (Başlık) (Nesne-Bazlı)
	101	Ö:.. Toplaması gelecek, çıkarması gelecek, çarpması gelecek. Aynı şekilde örüntülerle ilişkilendireceğiz, konuyu bilmiş olacağız.	Object-Driven?? (Çünkü örüntüyü bir matematiksel kavram olarak biliyor ve kabul ediyor)
İşaret			
Sembol			
Notasyon			
Temsilci Sayı			
Genel Sayı			
Sıra			
Adım			
Kural	6 7 8	Ö: Örüntülerde biz ne buluyoruz genel anlamda? Ş: Kural, üçgen, kare… Kural buluyoruz. Ö: Kural buluyoruz. Biz kuralları bu sene ifade ederken bilinmeyen ifadelerle a, b, c gibi ifadeler kullanıyorduk. Onlarla ifade edecez.	Object-Driven (Nesne-Bazlı)
	10	Ö: Hayır hayır sığar. Okları biraz aşağı doğru yaparsın, tamam? Hadi bakalım. Yine aynı şekilde toplama çıkarma çarpmamız var ama bölmemiz yok yani üç işlem göreceğiz. Uzantısı olarak örüntü ve ilişkilerden bahsedeceğiz. Çünkü bu kuralları bulurken cebirsel ifadelerden bahsetmiş olacağız. En başta zaten temel ne var benzer	Routine-Driven

H. APPROVAL OF THE METU HUMAN SUBJECTS ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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08 KASIM 2018

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof.Dr. Mine IŞIKSAL BOSTAN

Danışmanlığını yaptığınız doktora öğrencisi Tuba Ceylan ÇELİKER'in **"Ortaokul Matematik Öğretmenlerinin Öğrencilerinin Cebirsel Gelişim Sürecinde Matematiksel Söylem Kullanımlarının İncelenmesi"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2018-EGT-150** protokol numarası ile **08.11.2018 - 30.12.2019** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan V

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

Prof.Dr. Yaşar KONDAKÇI
Üye

Doç. Dr. Zana ÇITAK
Üye

Doç. Dr. Emre SELÇUK
Üye

Doç.Dr. Üyesi Pınar KAYGAN
Üye

I. OFFICIAL PERMISSIONS TAKEN FROM THE MINISTRY OF NATIONAL EDUCATION/MEB ARAŞTIRMA İZNİ ONAYI

T.C.
MİLLÎ EĞİTİM BAKANLIĞI
Yenilik ve Eğitim Teknolojileri Genel Müdürlüğü

Sayı : 81576613-605.01-E.623041 09.01.2019
Konu : Araştırma Uygulama İzin Talebi

ORTA DOĞU TEKNİK ÜNİVERSİTESİ REKTÖRLÜĞÜNE
(Sosyal Bilimler Enstitüsü)

İlgi: a) Temel Eğitim Genel Müdürlüğünün 04/01/2019 tarih ve 70297673-605.01-E.296276 sayılı yazısı
b) Millî Eğitim Bakanlığının 22/08/2017 tarihli ve 35558626-10.06.01-E.12607291 (2017/25) sayılı genelgesi

İlgi (a) yazı ile Genel Müdürlüğümüze intikal eden Orta Doğu Teknik Üniversitesi Sosyal Bilimler Enstitüsü İlköğretim Bölümü Doktora Programı öğrencisi Tuba Ceylan ÇELİKER'in "Ortaokul Matematik Öğretmenlerinin ve Öğrencilerinin Cebirsel Gelişim Sürecinde Matematiksel Söylem Kullanımlarının İncelenmesi" konulu doktora tezi kapsamında geliştirdiği veri toplama araçlarının Afyonkarahisar ili Işıklar İmam Hatip Ortaokulu, Şemsettin Karahisari Ortaokulu, Özlem Özyurt Ortaokulu, Kadınana İmam Hatip Ortaokulu, Yunus Emre Ortaokulu, Kadınana Ortaokulu ve Ankara İli Kalecik Ahiler Ortaokulunda görev yapmakta olan öğretmenlere ve buralarda öğretim gören öğrencilere uygulanmasına yönelik izin talebi Genel Müdürlüğümüzce incelenmiştir.

Denetimi il, ilçe millî eğitim müdürlükleri ve okul/kurum idaresinde olmak üzere, eğitim öğretim faaliyetlerini aksatmadan, gönüllülük esasına göre; onaylı bir örneği Bakanlığımızda muhafaza edilen, uygulama sırasında da mühürlü ve imzalı örnekten çoğaltılmış veri toplama aracının ilgi (b) genelge doğrultusunda uygulanmasına izin verilmiştir.

Gereğini bilgilerinize rica ederim.

Güvenli Elektronik İmzalı
Aslı İle Aynıdır
09 Ocak 2019

Anıl YILMAZ
Bakan a.
Genel Müdür V.

[redacted]
Erdoğan GÜRLER
Bilgisayar İşletmeni

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J. CURRICULUM VITAE

TUBA CEYLAN ÇELİKER

ARAŞTIRMA GÖREVLİSİ

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Akademik Görevler

ARAŞTIRMA GÖREVLİSİ 21.01.2009	AHİ EVRAN ÜNİVERSİTESİ/EĞİTİM FAKÜLTESİ/MATEMATİK VE FEN BİLİMLERİ EĞİTİMİ BÖLÜMÜ/MATEMATİK EĞİTİMİ ANABİLİM DALI

Projelerde Yaptığı Görevler:

1. İlköğretim İkinci Kademede Görev Alan Matematik Öğretmenlerine Dinamik Matematik Yazılımı Tanıtımı Geogebra Örneği, Yükseköğretim Kurumları tarafından destekli bilimsel araştırma projesi, Yürütücü:MUHARREM AKTÜMEN, Araştırmacı:AVNİ YILDIZ, Araştırmacı:TUĞBA HORZUM, Araştırmacı:TUBA CEYLAN ÇELİKER, , 01/08/2009 - 01/08/2011 (ULUSAL)
2. Ortaokul Öğrencilerinin Matematik Derslerinde Yaşadıkları Can Sıkıntısının Öz Yeterlik Kaynakları Ve Matematik Kaygısı İle İlişkisi, Yükseköğretim Kurumları tarafından destekli bilimsel araştırma projesi, Yürütücü:HÜSEYİN ŞİMŞEK, Araştırmacı:SULTAN SELEN KULA, Araştırmacı:BİLAL ÖZÇAKIR, Araştırmacı:TUBA CEYLAN ÇELİKER, , 04/07/2018 - 04/01/2020 (ULUSAL)

Eserler

Uluslararası hakemli dergilerde yayımlanan makaleler:

1. ŞİMŞEK HÜSEYİN,KULA SULTAN SELEN,ÖZÇAKIR BİLAL,CEYLAN ÇELİKER TUBA (2020). The Relation Between Academic Boredom of Students with Mathematics Self-Efficacy and Mathematics Anxiety. Acta Didactica Napocensia, 13(2), 30-42., Doi: 10.24193/adn.13.2.2 (Yayın No: 8992733)

Uluslararası hakemli dergilerde yayımlanan makaleler:

2. AKTÜMEN MUHARREM,HORZUM TUĞBA,CEYLAN ÇELİKER TUBA (2013). Modeling and Visualization Process of the Curve of Pen Point by GeoGebra. European Journal of Contemporary Education, 4(2), 88-99., Doi: 10.13187/ejced.2013.4.88 (Yayın No: 8992725)

B. Uluslararası bilimsel toplantılarda sunulan ve bildiri kitaplarında (proceedings) basılan bildiriler :

1. ŞİMŞEK HÜSEYİN,KULA SULTAN SELEN,ÖZÇAKIR BİLAL,CEYLAN ÇELİKER TUBA (2019). The Relation Between Academic Boredom Of Secondary School Students in Mathematics Lessons And Sources of Methematics Self-efficacy and Mathematics Anxiety. International Conference on New Horizons in Education (INTE 2019) (Özet Bildiri/Sözlü Sunum) (Yayın No: 9022467)
2. OLKUN SİNAN,ŞALDIRDAK Banu,ÇAKMAKTEPE Mehmet,YILMAZ ÇİĞDEM,CEYLAN ÇELİKER TUBA,KASAR Neslihan,Azimi Hadi,Peynircioğlu Aslı (2009). A study on elementary school students' Speed of Multiplication. The International Conference on Primary Education (Özet Bildiri/Sözlü Sunum) (Yayın No: 9022460)

C. Yazılan ulusal/uluslararası kitaplar veya kitaplardaki bölümler: C2. Yazılan ulusal/uluslararası kitaplardaki bölümler:

1. Bir dinamik matematik yazılımı: Geogebra ve ilköğretim 6-8. sınıf matematik dersleri için örnek etkinlikler (2010)., AKTÜMEN MUHARREM,HORZUM TUĞBA,YILDIZ AVNİ,CEYLAN ÇELİKER TUBA, Yok, Basım sayısı:1, Sayfa Sayısı 95, ISBN:978-605-125-189-9, Türkçe(Bilimsel Kitap) (Yayın No: 29326)

D. Ulusal hakemli dergilerde yayımlanan makaleler :

1. AKTÜMEN MUHARREM,YILDIZ AVNİ,HORZUM TUĞBA,CEYLAN ÇELİKER TUBA (2011). İlköğretim matematik öğretmenlerinin GeoGebra yazılımının derslerde uygulanabilirliği hakkındaki görüşleri. Türk Bilgisayar ve Matematik Eğitimi Dergisi, 2(2), 103-120., Doi: 10.16949/turcomat.81758 (Kontrol No: 9022431)

E. Ulusal bilimsel toplantılarda sunulan ve bildiri kitaplarında

1. CEYLAN ÇELİKER TUBA,ISIKSAL BOSTAN MINE (2021). 7. Sınıf örüntü ve ilişkiler konusu öğrenme sürecinin söylemsel açıdan incelenmesi. 14. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (Özet Bildiri/Sözlü Sunum) (Yayın No: 9022486)
2. AYTEKİN CAHİT,CEYLAN ÇELİKER TUBA (2013). Sınıf Öğretmeni Adaylarının Farklı Toplama Çıkarma Algoritmaları Üzerine Yorumları. 12. MATEMATİK SEMPOZYUMU [TOPLUMDA MATEMATİK] (Özet Bildiri/Sözlü Sunum) (Yayın No: 9022470)
3. AKTÜMEN MUHARREM,HORZUM TUĞBA,CEYLAN ÇELİKER TUBA (2010). Önünde engel bulunan bir kalemin ucunun izinin parametrik denkleminin hesaplanması ve Geogebra ile görselleştirme. 9. Matematik Sempozyumu Sergi ve Şenlikleri (Özet Bildiri/Sözlü Sunum) (Yayın No: 9022483)
4. ŞALDIRDAK Banu, ÇAKMAKTEPE Mehmet, YILMAZ ÇİĞDEM, CEYLAN ÇELİKER TUBA, KASAR Neslihan,Peynircioğlu Aslı,OLKUN SİNAN (2009). İlköğretim Öğrencilerinin Çarpma İşlemini Yapma Hızları Üzerine Bir Çalışma. 8. Ulusal Sınıf Öğretmenliği Eğitimi Sempozyumu (Özet Bildiri/Sözlü Sunum) (Yayın No: 9022476)
5. CEYLAN ÇELİKER TUBA,YILMAZ ÇİĞDEM,ÇAKMAKTEPE Mehmet (2009). 2002-2008 Yılları Arasında Türkiye'de Yayınlanan Dergilerde Bulunan Matematik Eğitimi ile İlgili Makalelerin Teknik Analizi. I. Ulusal Matematik Eğitimi Öğrenci Kongresi (Özet Bildiri/Sözlü Sunum) (Yayın No: 9022479)

F. İdari Görevler

1. AYDEP Ölçme Değerlendirme Komisyon Üyesi
2. Matematik Eğitimi A.B.D Akreditasyon Komisyon Üyesi
3. Dekanlık Birim Kalite Koordinatörlüğü Üyesi
4. Matematik ve Fen Bilimleri Eğitimi Bölümü Kalite Komisyon Üyesi
5. Yön Çalıştayı Stratejik Plan Komisyonu
6. Sosyal Transkript Hazırlama Komisyonu
7. Çocuk Eğitimi Uygulama ve Araştırma Merkezi Müdürlüğü Yönetim Kurulu Üyesi

K. TURKISH SUMMARY / TÜRKÇE ÖZET

BİR YEDİNCİ SINIF MATEMATİK ÖĞRETMENİNİN CEBİRSEL İFADELERLE İLGİLİ MATEMATİKSEL SÖYLEMLERİNİN İNCELENMESİ

1. Giriş

Cebir “genelleme dilidir” (Usiskin, 1995, s.31) ve tekrar tekrar yaptığımız şeyleri tanımlamak için basit bir ifade tanımlar. Daha ayrıntılı olarak cebir, değişkenlerin manipüle edilmesini, ifadeler kullanarak somut durumların modellenmesini ve bu ifadeler üzerinde basitleştirme, çözme ve yorumlama amacıyla işlemler yapılmasını içerir (Carraher ve Schliemann, 2007). Dahası, cebir "sayılar ve hesaplamalarla ilgili deneyimlerden genellemeler oluşturmayı, bu fikirleri anlamlı bir sembol sistemi kullanarak resmileştirmeyi, desen ve fonksiyon kavramlarını keşfetmeyi" sağlar (Van de Walle, Karp ve Bay-Williams, 2009, s.254).

Cebirsel düşünme ise “kişinin örüntüleri, ilişkileri ve fonksiyonları anlayabilmesi, cebirsel semboller kullanarak matematiksel durumları ve yapıları temsil edebilmesi ve analiz edebilmesi, nicel ilişkileri temsil etmek ve anlamak için matematiksel modelleri kullanabilmesi ve çeşitli bağlamlardaki değişimi analiz edebilmesi” olarak tanımlamaktadır (NCTM, 2000, s. 2). Kaput (1999) ise benzer şekilde, cebirsel akıl yürütmenin beş farklı modunu olduğu ileri sürmüştür. Bunlar, örüntülerin genelleştirilmesi ve oluşturulması olarak cebir, biçimselliklerin anlamsal olarak yönlendirilmiş manipülasyonu, hesaplamalardan ve ilişkilerden türetilen yapıların incelenmesi, fonksiyon ve ilişkilerin analizi ve fenomenleri modellemek ve kontrol etmek için kullanılan dil koleksiyonudur. Genelleme ve oluşturma temel yetenekler olarak hizmet eder, sembolik manipülasyon gerekli araçları sağlar, soyut yapılar kavrayışı geliştirir, işlevsel düşünme bu ilkeleri dinamik ilişkilere uygular ve son olarak modelleme tüm bu unsurları gerçek dünya fenomenlerini tanımlamak ve yönetmek için birleştirir (Kaput, 1999). Sonuç olarak, cebirin kalıpları ve ilişkileri

belirleme ve geliştirme sürecine vurgu yaptığı, sembolik gösterimden yararlandığı, nicel akıl yürütme ve matematiksel modellerin uygulanmasına hizmet ettiği ve problem çözme becerilerinin geliştirilmesine yardımcı olduğu konusunda ortak bir görüş birliği vardır.

Cebirsel düşüncenin önemi ve kavramsal gelişimi, çok sayıda ulusal ve uluslararası standart ve öğretim programında vurgulanmıştır (Common Core State Standards Initiative [CCSSI], 2010; Milli Eğitim Bakanlığı [MEB], 2013, 2018, 2024; National Council of Teachers of Mathematics [NCTM], 2000). Örneğin, NCTM (2000), cebiri, öğrencilerin matematiğin diğer alanlarını anlamalarını ve bunlar arasında bağlantılar kurmalarını sağlayan matematik müfredatındaki temel kavramlardan biri olarak görmektedir. Benzer şekilde, CCSSI (2010), bilinmeyen miktarlar için harfleri kullanarak, aritmetik desenleri belirleyerek veya çizimlerle kelime problemlerini çözerek ve bilinmeyeni temsil etmek için semboller kullanarak ilkokuldan itibaren cebire ve cebirsel düşüncenin gelişimine büyük önem vermiştir. Benzer şekilde, MEB (2013, 2018) resmi olarak ortaokuldan liseye, okul öncesi ve ilkokulda cebire hatırı sayılır miktarda ders saati ayırmıştır. Cebir öğrenme alanı önceki müfredatlarda (MEB, 2013, 2018) resmi olarak daha önceki sınıflara dahil edilmemiş olsa da 2024 yılında yayınlanan yeni müfredat, birinci sınıftan itibaren ‘Cebirsel Düşünme ve Değişimler’ başlığı altında cebirsel düşüncenin geliştirilmesine yönelik kazanımlar içermektedir (MEB, 2024). Öğretim programının erken sınıf düzeyinde öğrenmeyi teşvik ederek cebir öğretimini ve cebirsel düşüncenin gelişimini vurguladığı ve teşvik ettiği açıktır. Dahası, cebir, ileri matematik çalışmaları ve olası kariyer fırsatları için önemli bir ön koşul olarak hizmet ederek matematik eğitimi alanında önemli bir konuma sahiptir (NCTM, 2000; Usiskin, 1995). Cebirin önemi göz önüne alındığında, cebir öğretimi ve cebirsel düşüncenin geliştirilmesi "okul matematik müfredatının önemli bir bileşenini oluşturmaktadır" (NCTM, 2000, s. 37).

Matematik eğitimindeki araştırmalar da cebir öğretiminin matematik öğretimi ve öğrenimindeki önemini vurgulamaktadır (Blanton ve Kaput, 2005; Cai ve Knuth, 2011; Caspi ve Sfrad, 2012; Chazan, 2008; Lannin ve diğ., 2006; MacGregor ve Stacey, 1999; Martinez ve Brizuela, 2006; Usiskin, 1995; Uygur-Kabael ve Tanışlı, 2010; Warren ve Cooper, 2005; Warren ve diğ., 2006). Örneğin, Usiskin (1995)

cebirin nicelikleri ilişkilendiren ve öğrencilerin aynı soruları aynı anda cevaplayabilmelerini sağlayan bir tür dil olduğunu ileri sürmektedir. Cebir öğrenen öğrenciler ayrıca "aritmetikten cebire geçiş"i (Cai & Knuth, 2011, s. viii) ve desenlerin genellemelerini anlayabilmektedirler. Ayrıca nicelikler arasındaki ilişkileri (Caspi & Sfard, 2012; Chazan, 2008; MacGregor & Stacey, 1999; Warren & Cooper, 2005), yinelemeli kuralları ve bir değişkenden diğerine nasıl geçiş yapılacağını (Lannin vd., 2006) anlayabilmektedirler. Cebir öğretiminin önemi, cebirin kendine has diline sahip olması, nicelikleri dili aracılığıyla ilişkilendirmesi ve öğrencilerin aritmetik işlemlerden daha karmaşık işlemlere geçiş yoluyla daha üst düzey beceriler geliştirmelerini sağlamaları nedeniyle açıkça vurgulanmıştır.

Literatür ve öğretim programlarında cebir öğreniminin önemine dair vurgular yapılmasına rağmen, çok sayıda araştırmanın öğrencilerin cebir içeriğini öğrenme ve ilgili görevlere cevap verme konusunda zorluklar yaşadığını belgelemiştir (Bednarz, 2001; Carraher ve Schliemann; Girit ve Akyüz, 2016; Kieran, 1994; Kieran, 2007; Linchevski ve Livneh, 1999). Cebirin öğrenciler için neden zor olduğunu anlamak için, öğrencilerin sıklıkla yaptığı hata türlerini belirlemek ve ardından bunların nedenlerini araştırmak gerekmektedir. Bu nedenle, birçok araştırmacı cebir öğrenme sürecinde yaşanan hatalara ve zorlukların nedenlerini araştırmıştır (Bamberger vd., 2010; Bednarz, 2001; Blanton vd., 2015; Cai ve Knuth, 2011; Chazan, 1996; Kieran, 2007; Robert ve Roux, 2019; Stacey ve MacGregor, 1997; Stephens vd., 2013; Usiskin, 1988). Bu zorluklardan biri, ilkokulda aritmetiğin, ortaokul ve lisede ise cebirin ön planda olduğu okul müfredatlarında aritmetik ve cebirin ayrılması ve bu durumun daha sonraki cebir öğretimi için zorlukları ortaya çıkarmasıdır (Cai ve Knuth, 2011; Kieran, 2007). Ayrıca aritmetik ve cebir üzerine yapılan diğer çalışmalar, aritmetik işlemlerde zorluk çeken öğrencilerin cebirsel ifadelerdeki toplama ve çıkarma işlemlerinde de zorluklar yaşadığını belirtmiştir (Booth, 1988; Erbaş vd., 2010; Herscovics & Linchevski, 1994; Jupri vd., 2014; Pillay vd., 1998; Seng, 2010). Bir diğer zorluk ise öğrencilerin değişken kavramını anlamakta zorluk çekmeleridir. Araştırmalar, öğrencilerin değişkenleri yalnızca sayıları temsil eden harfler olarak yorumladıklarını ve bu yanlış anlamanın cebirsel ilişkileri kavramalarını engellediğini göstermektedir (Bamberger vd., 2010; Bednarz, 2001; Usiskin, 1988). Bir başka yaşanan zorlukta, öğrenciler eşittir işaretinin ilişkisel

anlamlarını kavrayamadıkları için onu anlamakta zorluk çekerler ve bu da denklemleri çözme becerilerini engellemektedir. (Bamberger vd., 2010; Blanton vd., 2015; Stephens vd., 2013). Öğrencilerin yaşadığı bir diğer önemli zorluk ise cebir ve doğal dilin dilbilgisi kuralları arasındaki farklılıklar nedeniyle nicel ilişkileri açıklamak için sözel yapıları sembollerle bütünleştirmede zorluk çekmeleridir. (Bednarz, 2001; Blanton ve diğerleri, 2019; Carraher ve diğerleri, 2008; Stacey ve MacGregor, 1997). Bu nedenle, cebir öğrenmenin önemine rağmen, bu zorluklar cebir öğretimini ortaya çıkarmak için titiz gözlemler ve analizlere yönelik kritik ihtiyacı vurgulamaktadır.

Günlük dil ile cebirsel dil arasında bağlantılar kurmak, bu zorlukları ve yanlış anlamaları azaltma açısından çok önemlidir (Robert ve Roux, 2019). Sfard (2008) bu iki tür dili konuşma dili söylemi ve okuryazar veya matematiksel söylem olarak açıklar. İlki her gün veya kendiliğinden ortaya çıkan ve tekrarlayan günlük faaliyetlerin bir doğal sonucu olarak ortaya çıkan söylemleri ifade ederken, ikincisi yazılı sembollerin kapsamlı kullanımı ve belirli notasyonları kullanma prosedürlerinin geniş yelpazesi ile karakterize edilen söylemleri ifade etmektedir. Konuşma dili söylemi, öğrencilerin günlük yaşamlarında kullandıkları ve sınıfa getirdikleri dildir; matematiksel söylem ise genellikle okullarda veya profesyonel bağlamlarda kullanılmaktadır. Daha spesifik olarak, Sfard (2008) sınıfta kullanılan matematiksel söylemi, matematiksel nesnelere atıfta bulunan ifadelerin kullanımını ve bireylerin matematiksel düşüncelerini farklı bilişsel düzeylerde ifade edebilecekleri bir süreci içeren iletişim biçimi olarak tanımlamıştır. Bu çalışmada, söylem kelimesinin dile getirildiği herhangi bir ifade, matematiksel söyleme atıfta bulunmak üzere tasarlanmıştır. Araştırmacılar, günlük konuşma dilini matematiksel söylemle ilişkilendirmenin öğrencilerin matematiksel göstergeleri anlama ve kullanma becerilerini geliştirdiğini ileri sürmüşlerdir (Barwell, 2016; Kim, Ferrini-Mundy ve Sfard, 2012; Mahlaba, 2021; Sfard, 2008). Eğitim ortamlarında, öğrencilerin günlük yaşamlarında kullandıkları söylemden matematiksel söyleme geçişlerini teşvik etmenin önemli olduğu düşünülmektedir. Bu nedenle, bu çalışmada cebir derslerinde matematiksel söylemin incelenmesi, günlük dilden cebirsel dile geçişi ve bu geçiş sırasında yanlış iletişim sorunlarından kaynaklanan zorlukları fark etmek açısından aydınlatıcı olduğu düşünülmektedir.

Günlük dil ve matematiksel söylemin öneminin anlaşılması üzerine araştırmacılar söylemsel yaklaşıma dayalı çeşitli çerçeveler ve teoriler geliştirmişlerdir (Adler ve Ronda, 2015; Halliday ve Hasan, 1989; Sfard, 2008; Van Dijk, 1995; 2001). Bunlardan biri, özellikle matematiksel söylemi analiz etmek için tasarlanmış ve yaygın olarak kullanılan Sfard (2008) tarafından geliştirilen commognitive teoridir. İletişim ve bilişi birleştiren commognitive teori, öğrenmeyi bir bireyin söyleminin bir söylem topluluğuyla etkileşime girerek dönüştürme süreci olarak ele almaktadır (Sfard, 2008). Teoride Sfard (2008), matematiği bir söylem olarak tanımlamış ve *kelime kullanımı, görsel aracılar, rutinler* ve *onaylanmış anlatıların* matematiksel söylemin karakteristikleri olduğunu ifade etmiştir. Kelime kullanımının söylemde dile getirilen belirli kelimelerle açıklandığını belirtmiştir. Bunlar genellikle nicelikler, şekiller ve bunların soyut ilişkileriyle ilgilidir. Ek olarak, görsel aracılar, semboller, tablolar ve grafikler gibi özellikle matematiksel söylem için üretilen görünür nesnelerdir. Sfard (2008) rutinleri, bir söylemdeki tekrar eden karakterler olarak tanımlamıştır. Matematiksel söylemde, bunlar şekilleri kategorize etmeyi ve benzer veya farklı durumları bulmayı içermektedir. Son olarak, onaylanmış anlatılar, diğer söylem özellikleri tarafından onaylanan gerçekler olarak bilinmektedir. Bunlar, diğer söylem karakteristikleri tarafından onaylanan teoriler, tanımlar ve kanıtlar olmuştur. Bu söylemsel karakterleri ve birbirleriyle olan ilişkilerini analiz etmek, matematiğin öğretiminin gizli özelliklerini ortaya çıkarmaktadır. Böylece, kelimelerin, görsel aracıların, rutinlerin ve onaylanmış anlatıların rolleri, bir söylemde iletişimsel bir bakış açısından açıkça incelenmektedir. Mevcut çalışmada, ortaokul cebir derslerinde matematiksel söylemi commognitive teori aracılığıyla analiz etmek, cebir öğretiminin ve öğreniminin gerçekleştiği iletişim ortamını haritalamaya, iletişimsel unsurlar hakkında bilgi vermeye, cebirin örtük doğasına ışık tutmaya ve cebir öğretimi sırasında iletişimsel bozuklukları ortaya çıkarmaya yardımcı olduğu düşünülmektedir.

Ayrıca, Adler ve Ronda (2015), bir sınıftaki söylemi analiz eden, matematik eğitiminin ayrıntılı bir tasvirini sağlayan ve öğrenmek için mevcut matematiksel materyalleri yorumlayan söylemsel yaklaşıma dayalı Öğretimde Matematiksel Söylem (MDI) olarak bilinen bir çerçeve geliştirmişlerdir. Çerçeve, öğrenme nesnesi altında matematik öğretiminin üç bileşenine odaklanmaktadır. Bunlar "örnekleme",

“açıklayıcı konuşma” ve “öğrenci katılımıdır” (Adler ve Ronda, 2015, s.237). İlk olarak, “açıklayıcı konuşma”, öğretmenin matematiksel fikirleri ve kavramları adlandırmak ve meşrulaştırmak için kullandığı dildir (Adler ve Ronda, 2015, s.241). Ardından, “örnekleme”, öğrencilere matematiksel kavramlarda çeşitlilik deneyimleme fırsatları sağlamak için örnekler ve görevler kullanmayı içermektedir (Adler ve Ronda, 2015, s.239). Son olarak, “öğrenen katılımı” öğrenciler ve öğretmen arasındaki etkileşime ve öğrencilerin matematiksel söyleme ve akıl yürütmeye katılma fırsatlarına odaklanmaktadır (Adler ve Ronda, 2015, s.245). Bu nedenle, MDI matematik eğitiminde kullanılan söylemi incelemek için kapsamlı ve organize bir yaklaşım sunmaktadır. Açıklayıcı konuşma, örneklendirme ve öğrenci katılımı gibi farklı söylemsel unsurlar açısından eğitimi analiz ederek, MDI matematiğin hem nasıl öğretildiği hem de öğrenildiği konusunda daha ayrıntılı bir anlayış sunmaktadır. Commognitive teoriye ek olarak, bu çalışma cebir derslerini MDI çerçevesi aracılığıyla analiz etmeyi amaçlamaktadır. Bu nedenle, çerçeve matematiksel prosedürün arkasındaki mantığı açıkça belirten açıklamaları ortaya çıkarabilir, cebir konusunu ayrıntılı olarak açıklamak için kullanılan örnekleri belirleyebilir ve bu iki unsurun öğrencileri zengin bir iletişim ortamında sınıf tartışmalarına dahil etmek için nasıl kullanıldığını belirleyeceği düşünülmektedir.

Yukarıda belirtilen her iki söylemsel yaklaşım da (commognitive teori ve MDI) öğrenmenin sosyal ve kültürel bir bağlamda gerçekleştiğini, kültürel araçların ve sosyal etkileşimin öğrenme sürecinde önemli faktörler olduğunu vurgulayan sosyo-kültürel teoriye dayanmaktadır. Mevcut çalışma, cebir eğitiminin gerçekleştiği bir sınıftaki sosyal ortama ve öğrencilere cebirsel ifadeleri öğretmeyi amaçlayan öğretmen ile öğrenciler arasındaki etkileşime odaklanmaktadır. Commognitive teori, sınıfta kullanılan matematiksel söylemin özelliklerinin ayrı kurallara ve meta-kurallara dayalı olarak analizini sağlamaktadır (Sfard, 2008), oysa MDI bir öğrenme hedefi aracılığıyla matematik eğitiminin kapsamlı bir tasvirini yapmaktadır (Adler ve Ronda, 2015). MDI esas olarak öğretmenin söylemini, matematiksel kavramları tanımlama ve açıklama biçimlerini, konuyu ayrıntılandırmak için kullandıkları örneklerin yapısını ve öğrencilerle etkileşimi incelemeye vurgu yapmaktadır (Adler ve Venkat, 2014). MDI'nin temel oluşturucu unsurlarından biri olan açıklayıcı konuşma, matematiksel nesnelerin isimlendirilmesinin ve meşrulaştırmaların

incelenmesi olarak tanımlanmaktadır (Adler ve Ronda, 2015). Ancak, çerçeve meşrulaştırılmış matematiksel dili oluşturan şeyin net bir tanımını sağlamak için yetersizdir ve bu da alt kategoriler arasında ayrım yapma sürecini karmaşıklaştırır (Mosvold ve Fauskanger, 2022). Öte yandan commognitive teori, kelime kullanımı, görsel aracılar, rutinler ve onaylanmış anlatılar dahil olmak üzere söylemdeki sınıflandırılmış matematiksel özellikleri analiz ederek ve her kategori için kapsamlı bir tanım sunarak bu boşluğu doldurmuş olacaktır. Bu nedenle, bu çalışma yalnızca öğrenmenin bilişsel süreçlerine değil, aynı zamanda her iki teoride belirtilen sınıflarda gözlemlenmesi nispeten kolay olan iletişimsel unsurlara da odaklanarak cebir öğretiminin örtük doğasını ortaya çıkarmayı amaçlamaktadır. Dahası, bu çalışma cebiri iletişimsel bir perspektiften öğretmeye ve öğrenmeye odaklandığından, cebir öğretimi boyunca iletişimsel sorunlardan kaynaklanabilecek zorlukları ortaya çıkarabilmektedir. Sonuç olarak, bu çalışma, yedinci sınıfta aritmetikten cebire geçişin daha belirgin olduğu cebir öğretiminin gizli yapısını ortaya çıkarmak için bilişsel çerçevedeki matematiksel söylemin özelliklerinin analizi yoluyla MDI'daki açıklayıcı konuşmaları ve örneklemeleri incelemektedir.

1.1 Çalışmanın Amacı ve Araştırma Soruları

Bu çalışman yedinci sınıf cebirsel ifadeler derslerini bir Türk ortaokulu bağlamında söylemsel perspektifle incelemeyi amaçlamaktadır. Çalışma özel olarak Sfard'ın (2008) matematiksel söylem kategorileriyle ilgilenmekte ve bunların öğretimde matematiksel söylemde (MDI) (Adler ve Ronda, 2015) nasıl ortaya çıktığını analiz etmektedir, yani yedinci sınıf cebirsel ifadeler dersleri sırasında açıklayıcı konuşmalar ve örneklemelerdir. Ayrıca, öğrenci katılımı da dikkate alınmış, ancak öğrencilerin katkısının asgari düzeyde olduğu sonucuna varılmıştır. Bu amaç doğrultusunda, bu bölüm çalışmanın bulgularını cebirsel ifadelerin müfredat hedeflerine göre belirlenen iki ana başlık altında sunmaktadır. Bu başlıklar cebirsel ifadelerdeki kavramlar ve cebirsel ifadelerdeki işlemlerdir ve derslerin açıklayıcı konuşmalar ve örnekleme kısımlarında analiz edilmektedir. Her bir başlık, Tablo 4.1'de görüldüğü gibi cebirsel ifade hedefleriyle ilişkili olarak belirlenmiştir. Bu bağlamda, iki cebirsel ifade konusunun bulguları, metodoloji bölümünde belirtildiği gibi Sfard (2008) tarafından belirlenen matematiksel söylem kategorilerine (kelime

kullanımı, görsel aracılar, rutinler, onaylanmış anlatılar) dayalı alt bölümlerde sunulmuştur. Cebirsel ifadeler öğretiminin analizi, öğretmenin söyleminin baskın olduğu ve öğrencilerin söylemlerinin genellikle onun söyleminin tekrarlanmasıyla oluştuğu sonucuna varmıştır. Bu nedenle, odak genellikle öğretmenin matematiksel söylemi üzerindedir. Çalışmanın amacı göz önünde bulundurulduğunda, araştırma soruları ve alt sorular aşağıdaki gibi sunulmuştur:

1. Bir devlet ortaokulunun yedinci sınıfında cebirsel ifadeler dersleri bağlamında açıklayıcı konuşmalar ve örneklemeler sırasında matematiksel söylemin özelliklerinin (kelime kullanımı, görsel aracılar, rutinler, onaylanmış anlatılar) doğası nedir?
   - Cebirsel ifadelerdeki kavramlar konusundaki açıklayıcı konuşmalar ve örneklemeler sırasında matematiksel söylemin özelliklerinin doğası nedir?
   - Cebirsel ifadelerdeki işlemler konusundaki açıklayıcı konuşmalar ve örneklemeler sırasında matematiksel söylemin özelliklerinin doğası nedir?

1.2 Çalışmanın Önemi

Bu çalışma, cebir öğretimine özellikle de cebirsel ifadelerin öğretimine yönelik daha derinlemesine bir anlayış sunmak amacıyla, matematiksel söylemi söylemsel perspektiften incelemeyi hedeflemektedir. Çalışma, akademik literatüre ve eğitim ortamına katkıları nedeniyle büyük önem taşımaktadır. Çalışma, Türkiye'deki ortaokullarda matematiksel söylemi söylemsel yaklaşımla inceleyen bir vaka çalışması sunarak literatüre önemli bir katkı sağlamaktadır. Söylemsel yaklaşım, matematiksel söylemin sembolik nesneler ve meta kurallar gibi benzersiz özelliklerini ortaya çıkartarak, matematik öğrenimini daha iyi anlamak için araştırmacılara önerilerde bulunmaktadır. Bu çalışma, Türk ortaokullarında cebirsel ifadelerin öğretimine yönelik söylem analizi sunarak, araştırmacılar ve matematik eğitmenlerinin cebir öğretiminde kavramlar, tanımlar, teoremler ve görsel araçların oynadığı önemli rolü anlamalarına yardımcı olmayı amaçlamaktadır.

Çalışma, Sfard'ın commognitive teorisi ve MDI çerçevesi kullanılarak cebirsel ifadeler derslerinin derinlemesine bir analizini sunarak literatüre katkı sağlamaktadır. MDI çerçevesi, öğretim uygulamalarında kullanılan söylemi analiz etmek için

kapsamlı ve yapılandırılmış bir yaklaşım sunarken, matematiğin öğrenilmesini daha açık ve görünür hale getirmeye yardımcı olmaktadır. Çalışma, cebirsel ifadeler öğretiminde kullanılan matematiksel söylemin özelliklerini incelemeyi amaçlamakta ve bu sayede matematik eğitmenlerinin, cebirsel ifadeler derslerinde kullanılan sözcükler, görsel araçlar, rutinler ve anlatılar hakkında bir anlayış geliştirmelerine olanak tanımaktadır. Öğretmenler, bu özellikleri sınıf tartışmalarında nasıl ve ne zaman kullanacaklarına dair içgörüler kazanabileceği düşünülmektedir.

Bu araştırma ayrıca, cebir öğreniminde karşılaşılan zorluklar ve yanlış iletişim sorunlarını ortaya koyarak literatüre katkıda bulunacağı düşünülmektedir. Cebir genellikle öğrenciler için zor bir konu olarak kabul edilir ve öğrenciler aritmetikten cebire geçişte, değişkenlerin soyut doğasında, eşitlik işaretinin yanlış anlamında ve sözel açıklamaları sembolik sistemlerle birleştirmekte zorluk yaşamaktadırlar. Sfard, iletişimsel çevrede bireylerin aynı matematiksel unsurları farklı şekillerde kullanmaları durumunu ifade eden commognitive çatışma kavramı ile yanlış iletişim sorunlarının nedenlerini açıklamaktadır. Öğretmenlerin söylemlerinin öğrencilerin matematiksel anlayışlarını nasıl etkilediğini fark etmeleri ve söylemsel çatışmaları en aza indirgemelerinin önemli olacağı düşünülmektedir. Bu çalışma, öğretmen söyleminin öğrencilerin matematiksel anlayışlarının gelişimi üzerindeki etkisini göstererek literatüre önemli bir katkı sağlayabileceği düşünülmektedir.

2. Alanyazın Taraması

Cebir öğretimi matematik eğitiminde ve profesyonel hayatta son derece önemli olduğu düşünüşmektedir. Literatürde cebirin, matematiksel kavramları genelleştirme ve sembolik sistemler aracılığıyla ifade etme yeteneği sağladığı ifade edilmiştir (Blanton & Kaput, 2005; Carreher & Schliemann, 2007). Araştırmacılar, cebirin bir akademik pasaport olduğunu, cebir bilgisi yetersiz olan bireylerin çeşitli mesleklerde ve eğitim yollarında zorluklarla karşılaşabileceğini belirtmektedir (Schonfeld, 1995; Usiskin,1995). Bunun yanında cebirsel düşünme, örüntüleri tanıma, sembolik sistemleri kullanma ve modelleme yoluyla nicel ilişkileri anlamaya katkıda bulunmaktadır (Blanton & Kaput 2005; NCTM 2000). Bu yüzden literatürde de

belirtildiği üzere cebir ve cebirsel düşünme becerileri bireylerin hayatlarında son derece önemli olduğu düşünülmektedir.

Bununla birlikte cebir, okul matematiğinin kalbinde yer almakta ve sayı işlemleri, orantı, ölçme, geometri ve veri analizi gibi birçok alanla doğrudan ilişkili olduğu düşünülmektedir. (Van de Walle et al. 2009). Cebirsel düşünmenin becerisinin aynı zamanda öğrencilerin problem çözme becerilerini geliştirdiğini ve semboller, değişkenler ve eşitlik işareti ile ilgili anlayışlarını güçlendirdiği vurgulanmaktadır (Booth et al. 2015; Egodawatte 2009).

Literatürde cebirin önemi vurgulansa da öğrenciler cebir öğrenirken birtakım zorluklarla karşılaşmaktadırlar (Bamberger et al., 2010; Bednarz, 2001; Blanton et al., 2015; Booth, 1988; Erbaş Çetinkaya & Ersoy, 2010; Herscovics & Linchevski, 1994; Jupri et al., 2014; Knuth et al., 2006; Pillay et al., 1998; Seng, 2010; Stacey & Chick, 2004; Stephens et al., 2013; Uçar-Sarımanoğlu, 2019; Usiskin, 1988; Warren et al., 2016).
Bunlardan ilki aritmetik işlemlerle ilgili zorluklardır (Booth, 1988; Erbaş et al., 2010; Herscovics & Linchevski, 1994; Jupri et al., 2014; Pillay et al., 1998; Seng, 2010). Öğrenciler, cebirsel ifadelerde toplama ve çıkarma gibi işlemleri uygulamakta zorlanmakta, özellikle dağıtma ve yer değiştirme kurallarını anlamakta sıkıntı yaşamaktadır (Booth 1988; Jupri et al. 2014). Bir diğer zorluk değişken kavramının anlamı ile ilgili yaşana zorluklardır (Bamberger et al., 2010; Bednarz, 2001; Blanton et al., 2015; Uçar-Sarımanoğlu, 2019; Usiskin, 1988; Warren et al., 2016). Öğrenciler, değişkenlerin yalnızca bir sayı olduğunu düşünerek, değişkenlerin soyut doğasını anlamakta güçlük çekmektedir (Usiskin 1988; Blanton et al. 2015). Bir başka zorluk ise eşitlik işaretinin yanlış kullanımı ile ilgilidir (Bamberger et al., 2010; Blanton, et al., 2015; Herscovics & Linchevski, 1994; Jupri et al., 2014; Knuth et al., 2006; Stacey & Chick, 2004; Stephens et al., 2013; Warren et al., 2016). Birçok öğrenci, eşitlik işaretini aritmetik işlemin sonucu olarak görmektedir ve iki taraf arasındaki dengeyi kavramada zorluk yaşamaktadır (Knuth et al. 2006; Stephens et al. 2013). Cebirsel öğreniminde yaşanan en önemli zorluklardan bir tanesi ise sözlü açıklamaların sembolik sistemlere dönüştürülmesi ile ilgilidir (Bednarz, 2001; Blanton et al., 2019; Carraher et al., 2008; Kieran, 1994; Linchevski

& Livneh, 1999; Stacey & MacGregor, 1997) Öğrenciler, günlük dili cebirsel sembollere dönüştürmekte zorlanmakta, dilin kuralları ile cebir dilinin kuralları arasındaki farkı anlamada zorluk yaşamaktadır (Stacey & MacGregor 1997; Jupri & Drijvers 2016).

Cebir öğrenimi ve öğretiminde yaşanan zorluklar araştırmacıları matematik derslerini derinlemesine inceleme imkanı sunan söylem analizine yönlendirmiştir. Kısaca dilin kullanımı olarak tanımlanan söylem (Rymes, 2009), dilin belirli bir sosyal bağlamda kullanılması anlamına gelmektedir (Fairclough, 1995) ve matematik eğitimi araştırmalarında son dönemde söylem analizine artan bir ilgi olduğu düşünülmektedir (Atweh et al., 1998; Baki & Çelik, 2018; Emre-Akdoğan, 2018; Güçler, 2013; Güçler, Wang & Kim, 2015; Tabach & Nachieli, 2015). Söylem, öğrencilere matematiksel kavramları öğrenme süreçlerinde yardımcı olan dilsel ve görsel araçları içermektedir (Sfard, 2001; Sfard, 2008). Matematik eğitiminde söylem analizi öncüsü olan Sfard (2001), düşünmenin bir iletişim şekli olduğunu ve anlamlı öğrenmenin ancak öğrencilerin bu sürece aktif katılımıyla gerçekleşebileceğini savunmaktadır. Bu bakış açısına göre, matematiksel söylemin analizinde kullanmak üzere commognitive teoriyi geliştirmiştir (Sfard, 2008). Bu teoriye göre matematiksel söylem kelime kullanımı, görsel arabulucular (semboller ve diyagramlar), rutinler (tekrar eden kalıplar) ve onaylanmış anlatılar (matematiksel teoriler ve tanımlar) olarak olmak üzere belirli matematiksel söylem karakterlerinin analizini içermektedir (Sfard, 2008). Diğer yandan, Adler ve Ronda (2017) matematik öğretiminde söylem analizi üzerine Matematik Öğretiminde Söylem (MDI) çerçevesi adlı bir başka teorik çerçeve geliştirmiştir. Bu çerçeve ise matematik öğretimi süreçlerini “öğrenme nesnesi," "örneklendirme," "açıklayıcı konuşma" ve "öğrenci katılımı" gibi bileşenler üzerinden analiz etmektedir (Adler ve Ronda, 2015, sy. 237). Bu çerçeveler, matematiksel söylemin özelliklerini ve öğretim süreçlerini kapsamlı bir şekilde inceleyerek, matematik eğitimi araştırmalarına katkıda bulunmaktadır. Commognitive teori daha çok matematiksel söylemin bilişsel ve iletişimsel yönlerine odaklanırken, MDI çerçevesi matematik öğretim süreçlerinin yapısal ve etkileşimsel yönlerini analiz etmektedir (Adler ve Ronda, 2015; Sfard, 2008). Cebir derslerinin analizinde her iki yaklaşımı da kullanmak cebir öğretiminin örtük doğasını ortaya çıkararak ayrıntılı bir analizi

sunmaya yardımcı olacağı düşünülmektedir. Ayrıca commognitive teori öğretimdeki matematiksel söylem karakterlerini inceleyerek cebir öğretimini içerik anlamında analiz etmeye imkan tanırken, MDI bu karakterlerin derslerin hangi aşamalarında yer aldığına dair bir analiz yapmaya imkan sağlayacağı düşünülmektedir. Bu yüzden bu çalışmada 7. Sınıf cebirsel ifadeler dersleri commognitive teori ve MDI teorik çerçeveleri kullanılarak analiz edilecektir. Böylece cebir öğretiminin örtük doğasının açığa çıkarılması amaçlanmaktadır.

3. Yöntem

Çalışmanın genel amacı, bir devlet okulunda yedinci sınıf cebirsel ifadelerin öğretimini söylemsel yaklaşım ile incelemektir. Özellikle, bu araştırma, cebir derslerindeki söylemsel unsurları inceleyerek cebirsel ifadelerin öğretimine dair içgörüler elde etmeyi hedeflemektedir. Bu amaç doğrultusunda nitel bir araştırma deseni olan durum çalışması kullanılmıştır (Merriam, 2009; Stake, 1995; Yin, 2003). Durum çalışmalarında araştırmacı, bir durum tanımını bildirerek çok sayıda bilgi kaynağından kapsamlı ve derinlemesine veriler toplar ve böylece zaman içinde bir veya birden fazla vakayı araştırmaktadır (Creswell, 2007). Yin (2003) çalışmaları durum sayısı ve analiz birimlerine göre gruplandırarak durum çalışmalarını dört farklı kategoride incelemiştir. Bunlar durum sayılarına göre tekli veya çoklu durum çalışmaları ve bütüncül veya iç içe geçmiş durum çalışmaları olarak adlandırılmıştır.
Bu çalışma amacına uygun olarak yedinci sınıf cebir öğretimi bağlamında oluşturulmuştur. Bu bağlam kapsamında edinci sınıf cebirsel ifade derslerinde kullanılan matematiksel söylem, durumu temsil eder. Kelime kullanımı, görsel aracılar, rutinler ve onaylanmış anlatılar ise çalışmada yer alan analiz birimleridir. Bu çalışmada Yin'in (2003) durum çalışması sınıflamalarından tek durumlu gömülü desen kullanılmıştır. Mevcut çalışmada matematiksel söylemi tek bir sınıfta incelemenin cebirsel ifadelerin öğretimi için yeterince bilgilendirici olacağı düşünülmüştür. Ayrıca, tek bir vaka ile çalışmak cebirsel ifadeler dersinde kullanılan matematiksel söylem kategorilerini incelemek için araştırmacıya gerekli zaman ve enerjiyi sağlamıştır.

3.1 Çalışmanın Bağlamı ve Katılımcılar

Bu çalışma Afyonkarahisar'ın merkezi bir ilçesinde bulunan ve ulusal eğitim sistemini takip eden, şehir merkezine uzaklığı 20 kilometre olan bir devlet okulunda görev yapan bir matematik öğretmeni Esra ve öğretmenin yedinci sınıf öğrencileri ile yürütülmüştür. Öğretmen Esra Ankara'daki bir devlet üniversitesinin İlköğretim Matematik Öğretmenliği programından mezun olmuştur. Ayrıca, aynı üniversitede İlköğretim Fen ve Matematik Eğitimi programında 2011 yılında yüksek lisans derecesini tamamlamıştır. Esra, üniversiteden mezun olduktan sonra iki yıl özel bir okulda matematik öğretmeni olarak çalışmıştır. Daha sonra, Kamu Personeli Seçme Sınavı'na (KPSS) girmiş ve 2011 yılında MEB tarafından kamu ortaokullarındaki pozisyonlara atanmak için gereken puanı almıştır. Altı buçuk yıldır kamu okullarında öğretmenlik yapmaktadır. Veri toplama sırasında mevcut okulda dört yıldır çalışmakta ve yedinci sınıf öğrencilerine ders vermektedir. Bu çalışmanın da yürütüldüğü yedinci sınıfta 25 (13 yaşında) öğrenci bulunmaktadır; bunlardan 13'ü (%52) erkek ve 12'si (%48) kız öğrenciden oluşmaktadır. Öğrencilerin çoğu düşük sosyoekonomik geçmişe sahiptir. Bu yüzden öğretmene göre, düşük başarı gösteriyorlardı ve ebeveynleri akademik çalışmalara çok az değer vermektedir. Öğretmen Esra ve yedinci sınıf öğrencileri, Esra'nın yedinci sınıftaki öğretmenlik deneyimi ve yüksek eğitim düzeyi nedeniyle araştırmaya önemli bir katkı sağlayacağını düşündüğü için seçilmiştir. Okulu, öğretmeni ve sınıfını seçerken bir diğer kriter de öğretmenin ve okul yönetiminin araştırmacı ile çalışmaya istekli olması olmuştur.

3.2 Veri Toplama Araçları ve Süreci

Nitel araştırmalar gözlem notları, görüşmeler, fotoğraflar, metinler, görüşmeler, video kayıtları ve ses kayıtları gibi çeşitli veri kaynaklarını kullanmaktadır (Stake, 2010). Durum çalışmalarında birden fazla veri kaynağı kullanmak çalışmanın geçerlik ve güvenirliği sağlamak için önerilmektedir (Stake, 2010; Yin, 2003). Bu nedenle bu çalışmada kullanılan veri toplama araçları cebir eğitiminden elde edilen video kayıtları, öğretmenle yapılan gayri resmi görüşmeler ve gözlem notları olmuştur. Daha sonra veri toplama süreci başlamıştır.

Öncelikle bu çalışmada, araştırmacı, matematiksel söylemin örtük doğasını açıklamanın en iyi yolunu belirlemek, cebirsel ifadeler öğretimini haritalamak için teorik çerçeveyi nasıl kullanacağını kararlaştırmak ve yeterli veri toplamak için en uygun sınıf ve öğretmeni belirlemek üzere bir pilot çalışması yapılmıştır. Pilot çalışma, 2017-2018 akademik yılının güz döneminde Afyonkarahisar şehir merkezinde bulunan iki farklı ortaokulda, iki farklı matematik öğretmeni ve yedinci sınıf öğrencileri ile gerçekleştirilmiştir. Öğretmenlerin öğretim stratejileri, yöntemleri ve ders anlatım tarzları incelenmiş ve öğretmenlerin stratejilerinin ve yöntemlerinin sınıfta akıcı bir iletişim ortamı yaratmada etkili olduğu anlaşılmıştır. Ayrıca pilot çalışmada yapılan gözlemler aracılığıyla, araştırmacı öğrenciler, öğretmenler ve sınıf atmosferi hakkında genel bir bilgi edinmiş ve veri toplama sürecini optimize etmek için en iyi yöntemleri belirlemiştir. Araştırmacı, derslerin gözlemi sırasında etkili saha notları almayı, kamerayı nereye yerleştireceğini ve sınıfta nerede oturacağını kararlaştırmıştır. Ayrıca, gözlem formunun kullanılabilirliğini test etmiş ve gözlem aracıyla kaydedilen ve esas çalışmada yer alacak kellimeler tespit edilmiştir.

Asıl çalışmanın veri toplama süreci 2018-2019 güz döneminde yapılmıştır. Öğretmen Esra ve yedinci sınıf öğrencileri cebirsel ifadeler derslerinin öğretimi süresince gözlemlenmiştir. Araştırmacı katılımcı olmayan bir gözlemci rolüyle, cebirsel ifadeler derslerini gözlemlemiş ve dersler hakkında saha notları almıştır. Aynı zamanda, cebirsel ifadelerin öğretiminin hiçbir ayrıntısını kaçırmamak için tüm dersler videoya alınmıştır. Kamera, sınıfın en geniş görüş açısını elde edecek şekilde konumlandırılmıştır. Araştırmacı tüm iletişim akışını kaydetmek için arka sırada oturmuş ve tahtaya yazılan tüm matematiksel ifadeleri not almıştır.

3.3 Veri analizi

Veri analizi süreci cebir dersleri sona erdiğinde elde edilen video kayıtlarının araştırmacı tarafından transkript edilmesi ile başlamıştır. Transkriptlerin en doğru şekilde oluşturulması için araştırmacı video kayıtlarını birden fazla kez izlemiş ve gözlem notları ile karşılaştırmıştır. Böylece derslerde kullanılan sayılar, semboller, grafikler ve ifadelerin doğruluğu teyit edilmiştir. Bu esnada ders sırasında tahtaya yazılan semboller, hesaplamalar ve çözümler, ekran görüntüleriyle desteklenerek

görsel aracılar oluşturulmuştur. Video kayıtlarının transkript edilmesinin ardından cebir dersleri öğretim programında yer alan cebir kazanımları göz önünde bulundurularak iki ana başlık altında ele alınmıştır. Bunlar cebirsel ifadelerde kavramlar ve cebirsel ifadelerde işlemlerdir. Her başlık MDI çerçevesinde yer alan açıklayıcı konuşmalar ve örneklendirmeler kategorilerinde analiz edilmiştir. MDI'da yer alan öğrenen katılımı ise ayrı bir başlıkta incelenmemiştir çünkü öğretmenin söylemi daha baskınken öğrencilerin söylemi öğretmeninkinin tekrarı ya da kısa cevaplı katılımlar şeklinde olmuştur. Ayrıca bu iki ana başlık Sfard'ın (2008) teorisinde bulunan kelime kullanımı, görsel arabulucular, onaylanmış anlatılar ve rutinler olarak adlandırılan matematiksel söylem kategorilerine göre analiz edilmiştir.

Bu kategorilerden ilki olan kelime kullanımı cebirsel ifadelerde yer alan özel terimleri içermektedir. Tespit edilen her bir kelime hiyerarşik olarak pasif kullanım, rutin odaklı kullanım, ifade odaklı kullanım ve nesne odaklı kullanım olarak ayrıntılı bir şekilde analiz edilmiştir. Kelime kullanımı nicelikten ziyade bağlama göre analiz edilmiş olup kelime kullanım tablosuna göre analiz edilmiştir. Görsel aracılar sınıfta tahtaya yazılan cebirsel ifadeler, semboller, şekiller, oklar, yaylar ve geometrik şekillerden oluşmaktadır. Bu araçların hangi bağlamda ve hangi amaçla kullanıldığı analiz edilmiştir. Bir diğer matematiksel söylem karakteri olan rutinler tekrar eden kalıplar olarak tanımlanmıştır (Sfard, 2008). Belirli işlemlerin tekrar tekrar yapıldığı durumlar incelenmiş ve bu süreçlerin nasıl ortaya çıktığı, nasıl uygulandığı ve ne zaman tamamlandığı analiz edilmiştir. Son matematiksel söylem karakteri olan onaylanmış anlatılar dersler esnasında kabul gören matematiksel doğrular olarak ele alınmıştır. Cebirsel ifadelerde kullanılan tanımlar, özellikler veya kurallar, diğer karakterler olan kelime kullanımı, görsel arabulucular ve rutinler aracılığıyla desteklenip desteklenmediğine göre analiz edilmiştir.

4. Bulgular ve Tartışma

Bu bölümde çalışmanın amacı doğrultusunda elde edilen bulgular cebirsel ifadelerde kavramlar ve cebirsel ifadelerde işlemler olmak üzere iki ana başlık halinde sunulmuştur. Her iki ana başlık MDI (Adler ve Ronda, 2015) çerçevesi kapsamında

açıklayıcı konuşmalar ve örneklendirmeler olarak ve commognitive teori (Sfard, 2008) kapsamında matematiksel söylem karakteristikleri alt başlıkları olarak sunulmuştur. Bu karakteristikler kelime kullanımı, görsel aracılar, rutinler ve onaylanmış anlatılardır.

4.1 Cebirsel İfadelerde Kelime Kullanımı

Cebirsel ifadelerde kullanılan kelimeler, cebirsel ifadelerdeki kavramlar ve cebirsel ifadelerdeki işlemler olmak üzere iki başlık altında incelenmiştir. Bu bölümde, cebirsel ifadeleri öğretme ve öğrenmede kullanılan kelimeler belirlenmiş ve ayrıntılı olarak tartışılmıştır.

Cebirsel ifadelerdeki kavramlarda kullanılan kelimeler terim, bilinmeyen, katsayı, katsayıların toplamı, terim sayısı, sabit terim, benzer terim ve cebirsel ifler olmuştur. Cebirsel ifadelerdeki işlemlerde kullanılan kelimeler ise terim, bilinmeyen, katsayıların toplamı, benzer terimler ve cebirsel ifadelerdir. Her iki konuda da kullanılan ortak kelimeler terim, bilinmeyen, katsayıların toplamı, benzer terim ve cebirsel ifadeler olmuştur. Her bir kelime Sfard'ın (2008) commognitive teoreminde bulunan kelime kullanımı kategorisinde pasif-kullanım, rutin-bazlı, tabir-bazlı kullanım ve nesne-bazlı kullanım kategorilerine göre analiz edilmiştir. Pasif-kullanım genellikle öğrenme yaşamının ilk yılları olan okul öncesi dönemdeki kullanıma karşılık geldiğinden bu çalışmada bu kategoriye rastlanmamıştır (Emre-Akdoğan, 2015). Bu nedenle kalan üç kategoride tüm sözcük kullanımları analiz edilmiştir.

Bulgular, bu kelimelerin cebirsel ifadelerdeki kavramlarda çoğunlukla nesne-bazlı (%52) kullanıldığını ortaya koymaktadır. Bir sonraki kullanım tabir-bazlı (%36) iken en az kullanılan kullanım rutin-bazlı kullanım (%12) olmuştur. Aynı şekilde, cebirsel ifadelerdeki işlemler konusunda nesne-bazlı kullanım (%49) en çok kullanılmıştır. Ayrıca, rutin-bazlı ikinci en çok kullanılan ve tabir-bazlı kullanım (%20) ise en az kullanılmıştır. Genel olarak, cebirsel ifadelerin öğretiminde nesne-bazlı kullanım en sık dile getirilen kelime kullanım şekli olmuştur. Sfard (2008), kelime kullanımının pasif kullanımdan nesne odaklıya doğru hiyerarşik olarak ilerlemesi gerektiğini öne sürmektedir. Bu düşünceye uygun olarak, mevcut çalışmada nesne-bazlı kullanımın

en çok kullanılan kullanım şekli olması, cebirsel ifadeler öğretiminde kullanılan kelimelerin etkili öğrenme için beklenen matematiksel olarak nesnelleştirilmiş bir seviyeye ulaştığı düşünülmektedir. Burada sözü edilen nesnelleştirme, genel olarak bir kavramı anlamak olarak kabul edileblmektedir. Daha spesifik olmak gerekirse, literatürde nesnelleştirmenin, matematiksel nesneleri insan söyleminden bağımsız olarak var olarak tartışmanın belirgin bir yolu olduğu belirtilmektedir (Kontorovich, 2018). Bu çalışmada, benzer terimlerin nesnelleştirilmiş kullanımı, herhangi bir tanımlama veya eylem olmaksızın cebirsel ifadelerde benzer terimleri göstermek olarak tanımlanabilmektedir. Böylece, öğretmen ve öğrenciler kelimeyi kendi düşüncelerinden bağımsız olarak matematiksel bir nesne olarak kullanabilmekteler (Sfard, 2008). Mevcut çalışmada öğretmenin kelime kullanımları çoğunlukla rutin odaklı, ifade odaklı ve nesne odaklı olarak hiyerarşik olarak kullandığı, ancak öğretmenin ihtiyaç duydukça rutin odaklı veya ifade odaklı kullanıma geri döndüğü belirlenmiştir. Öğretmenin kelime kullanım kategorilerinde hiyerarşik olarak geriye gitmesi, öğrencilerin henüz nesne odaklı kullanıma geçmediklerini ve kelimeleri matematiksel olarak nesnelleştiremediklerini düşünmesinden kaynaklanmış olabilir.

Bununla birlikte bu çalışmada, cebir öğretiminde kelime kullanımı dersin hem açıklayıcı konuşmalarında hem de örneklendirme kısımlarında incelenmiştir. Çalışmanın bulguları, cebirsel ifadelerdeki kavramlarda kelimelerin çoğunlukla açıklayıcı konuşmalarda (%66), cebirsel ifadelerdeki işlemlerde ise kelimelerin çoğunlukla örneklendirme kısımlarında (%82) kullanıldığını ortaya koymaktadır. Başka bir deyişle, cebirsel ifadelerdeki kavramlarda daha çok tanım, açıklama ve matematiksel yapıların özelliklerinden yararlanırken, cebirsel ifadelerdeki işlemlerde öğrencilerin kelimelerini sunmak için daha çok örnek ve görevlerden yararlanmıştır.

Öğrenci katılımı, öğretmenin söylemi baskın olduğundan ve öğrencilerin söylemleri genellikle öğretmenin söyleminin tekrarı, evet/hayır cevapları ve bir veya iki kelimelik cevaplar olduğundan belirgin bir bölümde sunulmamıştır. Sfard (2008) bu durumu, söyleme katılan muhataplardan birinin söyleminin kurallarını oluşturarak diğerleri üzerinde otorite kurması şeklinde açıklamaktadır. Mahlaba ve Mudaly (2022) ise, bu durumu öğrencilerin konuyu keşfetmek ve anlamak yerine, esas olarak toplumsal beklentilere uymak veya öğretmenlerden olumlu değerlendirmeler almak

için söyleme ritüelistik katılım göstermeleri ile açıklanmaktadır. Bu tür katılım, bir anlayış olmaksızın rutinlerin taklidi ve tekrarı ile karakterize edilmekte ve bu da öğrencilerin söyleme bağımsız ve anlamlı bir şekilde katılma yeteneklerini sınırlamaktadır (Mahlaba, 2021; Mahlaba ve Mudaly, 2022). Bu çalışmada, öğretmenin söylemi açıkça baskındır ve o kural koyucu rolünde yer almaktadır. Öğrenciler genellikle onun söylemini tekrarlamışlardır. Mahlaba'nın (2021) da belirttiği gibi, öğrencilerin keşfedici söylemi kullanmak ve kendi matematiksel anlamlarını oluşturmak yerine öğretmenin söylemini doğrulayan ve tekrarlayan ritüelistik söylemi benimsedikleri sonucuna varılabileceği düşünülmüştür.

4.2 Cebirsel İfadelerde Görsel Aracı Kullanımı

Cebirsel ifadelerde görsel aracı olarak kavram haritaları, cebirsel gösterimler, daireler, yaylar ve oklar kullanılmıştır. En yaygın kullanılan görsel aracı cebirsel gösterimler olmuştur. Öğretmen ilk olarak dersin başında, öğrenilecek konuları içeren bir kavram haritası sunarak derse başlamıştır. Kavram haritaları, bilgileri düzenlemek, anlamlı öğrenmeyi teşvik etmek ve öğrencilerin kavrayışını güçlendirmek için güçlü bir araç görevi görmektedir (Evans ve Jeong, 2023). Bu sayede, öğrenciler öğrenecekleri konuları organize bir şekilde tanıyıp ve konunun genel çerçevesi hakkında fikir edinebilecekleri düşünülmüştür. Öğretmen derslerde terimleri, bilinmeyenleri ve benzer terimleri belirtmek için daireler, oklar ve yaylar gibi diğer görsel aracıları kullanmıştır. Örneğin öğretmen cebirsel bir ifadedeki terim sayısını belirtmek için her terimi daire içine almıştır. Daire sayısı, verilen ifadedeki terim sayısını temsil etmektedir.

Dahası, verilen ifadedeki benzer terimleri belirtmek için yaylar kullanılmıştır ve verilen bir ifade için benzer bir terim oluşturmak için oklar kullanılmıştır. Bu tür görsel aracılar matematiksel iletişim ve öğrenmede ifadelerin somutlaştırılmasında önemli bir rol oynamıştır (Nachlieli ve Heyd-Metzuyanim, 2022; Tabach ve Nachlieli, 2015). Bu aracılar soyut ifadeleri somutlaştırmış ve karmaşık kavramların anlaşılmasını kolaylaştırarak öğrenme sürecini kolaylaştırmıştır (Nachlieli ve Heyd-Metzuyanim, 2022). Böylece, sembolik ve sözel ifadeleri birbirine bağlayarak

öğrencilerin bilgiyi daha kolay öğrenmelerine ve hatırlamalarına yardımcı olmuşlardır (Nardi vd., 2014). Bu çalışmada, öğretmen genellikle parantezi kaldırmada kullanılan yayları gülen yüzler olarak adlandırmıştır. Dolayısıyla, bu kullanım öğrencilerin konuları daha kolay hatırlamalarını sağlamıştır. Öğretmen gülen yüzlerden bahsettiğinde, öğrenciler parantez kaldırma sürecini kolayca hatırlayabilmişlerdir. Sonuç olarak, çeşitli görsel aracıların kullanımı, sözel ifadeyi sembolik ifadelerle bütünleştirerek ve karmaşık matematiksel kavramları somutlaştırarak öğrencilerin matematiksel anlayışını artırabileceği düşünülmüştür.

Öğretmen genellikle yukarıda belirtilen görsel aracıları sınıfta şu şekilde kullanmıştır. Başlangıçta her durum için kullanılacak uygun görsel aracıyı göstermiştir. Daha sonra, öğrencileri bu aracıları benzer şekilde kullanmaya teşvik etmiştir. Ancak, Mahlaba'nın (2021) belirttiği gibi, öğrencilerin öğretmenlerine benzer görsel aracıları kullanmaları daha tutarlı bir öğrenme deneyimiyle sonuçlansa bile, öğrenciler prosedürleri anlamadan cevaplara ulaşmaya dayanan ritüel bir söylem geliştirebilecekleri düşünülmüştür. Bu çalışmada olduğu gibi, öğretmenin öğrencilerden tercihlerine göre görsel aracıları kullanmalarını istemesi bazen öğrenciler arasında karışıklığa neden olabilmektedir. Öğrenciler, öğretmenin benzer terimleri belirlerken veya verilen bir cebirsel ifadedeki terimleri belirtirken aynı aracıları kullanmalarını yönlendirmesi nedeniyle kendi söylemlerini oluşturamadığı durumlar olmuştur. Bu durumda, öğrenciler öğretmenin kullandığı aracıyı hatırlamakla meşgul oldukları için odaklanmaları gereken benzer terimleri belirleyememişlerdir.

Öğretmen cebirsel ifadelerde işlemler konusunda cebirsel gösterimler, daireler, oklar ve yayları görsel aracılar olarak kullanılmıştır. Ayrıca, görselleştirme için geometrik şekiller ve modelleme karolarından da faydalanmıştır. Literatür taramasına dayanarak, matematik derslerinde çeşitli görsel aracıların kullanılmasının, matematiksel ifadelerin daha belirgin hale getirilerek anlaşılmasını artırdığı, sınıf içi iletişim ortamını geliştirdiği ve çeşitli görsel temsiller arasında bağlantılar kurarak matematiksel anlayışı geliştirdiği vurgulanmıştır (Mahlaba, 2021; Nachlieli ve Heyd-Metzuyanim, 2022; Nardi vd., 2014; Tabach ve Nachieli, 2015; Zayyadi vd., 2019). Mevcut çalışmada, öğretmenin cebirsel ifadelerdeki işlemlerde çeşitli görsel

yardımcıları kullanması öğrencilerin matematiksel anlayışını destekleyeceği düşünülmüştür. Örneğin, öğretmen kenarları cebirsel ifadeler olarak gösterilen bir dikdörtgenin çevresini hesaplamak için bir dikdörtgen çizmiştir. Benzer şekilde, verilen cebirsel ifadeyi temsil etmek veya verilen bir modele dayalı bir cebirsel ifade üretmek için bir model oluşturmak amacıyla cebir karolarını kullanmıştır. Geometrik şekillerin çevrelerini hesaplamak için cebirsel ifadelerin kullanılmasıyla, öğrenciler cebirsel ifadelerin farklı temsillerde kullanılabileceğini fark etmişlerdir. Cebirsel ifadeleri sunmak için çeşitli görsel aracıların kullanılması, öğrencilerin karmaşık matematiksel kavramları anlamalarını sağlayabilir ve sınıf içindeki iletişimi artırabileceği düşünülmüştür.

4.3 Cebirsel İfadelerde Rutinler

Rutinler, commognitive teorideki matematiksel söylemi inceleyen bir diğer karakterdir (Sfard, 2008). Bulgular, cebirsel ifadelerin öğretimini sağlayan unsurlardan birinin rutinler adı verilen bazı yinelemeli olarak gerçekleştirilen etkinlikler aracılığıyla gerçekleştiğini göstermiştir. Cebirsel ifadelerin öğretiminde rutinlere açıkça atıfta bulunan öğretim etkinliklerinin, öğrenci öğrenmesini teşvik etme potansiyeline sahip olduğu sonucuna ulaşılmıştır. Her rutinin, rutinin nasıl gerçekleştiğini ve ne zaman ortaya çıktığını tanımlayan, örüntüleri tetikleyiciler olarak başlatan meta kurallar vardır. Bu nedenle, rutinlerin analizi, iletişimin yönü ve sınıfta söylemsel etkinlikleri kimin yönettiği hakkında genel bir fikir sağlamıştır.

Cebir derslerinin analizinden elde edilen en önemli bulgu, önceki söylem karakteristiklerinde olduğu gibi, öğretmenin rutinlerinin baskın olduğu ve öğrencilerin rutinlerinin genellikle onun söyleminin tekrarıyla oluşmuş olduğudur. Bu nedenle, cebirsel ifadelerin öğretim sürecini belgelemek için öğretmenin rutinlerine odaklanılmıştır. Öğretmenin söyleminin baskınlığı, öğrencilerin öğretmenin söyleminin belirli unsurlarını benimsemesiyle ilgili mevcut literatürle tutarlı olmuştur (Emre-Akdoğan vd., 2018). Dahası, literatür öğretmenin söyleminin öğrencilerin söylemini etkilediğini (Roberts ve Le Roux, 2019) ve öğrencilerin katılımını sağlamak için söylemi organize ettiğini vurgulamıştır (Tabach ve Nachlieli, 2011). Bu çalışmada, öğretmenin baskınlığının nedenleri, öğretmenin

gayri resmi bir toplantıda belirttiği gibi, sınıftaki düşük başarı gösteren öğrenci sayısının çok olması ve doğrudan öğretimin doğası olduğu düşünülmüştür. Bu varsayım doğru olabilir çünkü öğrencilerin öğretmenin rutinini aynı doğrultuda uyguladıklarında rutini başarılı bir şekilde sonlandırabildikleri fark edilmiştir. Öte yandan, öğretmenden bağımsız bir rutin uygulamaya çalıştıklarında kavramsal hatalar yapmışlar ve rutini tamamlayamamışlardır. Bu sonuç, çalışmadaki öğrencilerin çoğunun cebirsel düşünme becerilerini geliştirmede sorun yaşadıklarını ve kendi rutinlerini gerçekleştirmek için cebir hakkında sınırlı bilgiye sahip olduklarını göstermektedir. Aslında, mevcut çalışmada öğretmen merkezli ve doğrudan öğretim, düşük başarı gösteren öğrencilerin söylemi hatırlamalarına ve tekrarlamalarına ve daha fazla söylemlerini genişletmek için yeni bağlantılar kurmalarına yardımcı olmuş olabilir. Sonuç olarak, öğrenciler düşük başarı gösterdiğinde, konuşmalar ve söylemsel etkinlikler genellikle öğretmen tarafından yönetildiği için öğretme ve öğrenme sürecinde öğretmenin rutinlerinin baskın olması doğru bir yaklaşım olacağı düşünülmüştür.

Öğretmenin söyleminin baskın olması, bu çalışmayı cebirsel ifadelerin öğretimini incelemek için dersler boyunca öğretmenin tekrarlayan etkinliklerini analiz etmeye yöneltmiştir. Dahası, commognitive teori (Sfard, 2008), öğretmenin derslerde gerçekleştirdiği yinelemeli eylemleri belirlemek için bir çerçeve sağlamıştır. Analizler sonucunda öğretim programı ile bağlantılı olarak beş rutin belirlenmiştir (MEB, 2018). Bunlar benzer terimleri bulma, benzer terim yazma, Casper rutini, cebirsel ifadelerde toplama ve çıkarma ve parantezleri kaldırma rutinleridir. Dersler sırasında, öğretmen önce rutini sunmuş ve ardından öğrencilerin aynı rutini kullanmalarını sağlayan örnekler hakkında sorular sormuştur. Cebirsel ifadelerdeki kavramlar konusu kapsamında üç tane rutin belirlenmiştir. Bunlar benzer terimleri bulma, benzer terim yazma ve Casper rutinidir. En sık kullanılan rutin benzer terim yazma (%43) idi ve benzer terimleri bulma rutini ile neredeyse aynı sıklıktaydı (%36), en az kullanılan rutin ise Casper rutini (%21) olmuştur. Rutinle genellikle dersleri örneklendirme bölümünde (%86), geri kalan az sayıda rutin ise açıklayıcı konuşmalarda (%12) gerçekleştirilmiştir. Diğer yandan cebirsel ifadelerde işlemler konusunun öğretiminde dört çeşit rutine rastlanmıştır. Bunlar benzer terimleri bulma, Casper rutini, cebirsel ifadelerde toplama ve çıkarma, parantezden çıkarmadır. İlk

konu ile benzer şekilde, burada da rutinler çoğunlukla derslerin örneklendirme aşamasında gerçekleşmiştir (%92). Cebirsel ifadelerde işlemler konusunda en sık kullanılan rutin toplama ve çıkarma (%52), en az kullanılan rutin ise parantezden çıkarma olmuştur (%8). Cebirsel ifadelerdeki kavramlarda öncelikli olarak benzer terimi bulma ve benzer terimi yazma gibi cebirsel rutinler gözlenmiştir. Cebirsel ifadelerdeki işlemlerde ise toplama ve çıkarma, cebirsel ifade ve parantez kaldırma gibi rutinlerde cebirsel işlemlerin adımlarını gösteren rutinler baskındır. Cebirsel işlemleri yapmadan önce benzer terimleri tanımak ve bulmak cebirsel ifadeleri birleştirmede önemli bir adım olacağı düşünülmüştür. Literatürde, aritmetikte olduğu gibi cebirsel işlemleri ardışık olarak gerçekleştirmede yaygın bir zorluk olduğu belirtilmiştir (Booth, 1988; Erbaş vd., 2010; Herscovics ve Linchevski, 1994; Jupri vd., 2014; Pillay vd., 1998; Seng, 2010). Örneğin, öğrenciler cebirsel işlemlerin (örn. parantez, çarpma ve bölme, toplama ve çıkarma) doğru sırasını sıklıkla yanlışlıkla gerçekleştirirler (Herscovics ve Linchevski, 1994). Bu çalışmada belirlenen rutinlerin, özellikle rutinin ne zaman gerçekleştirildiğini, nasıl gerçekleştiğini ve nasıl sona erdiğini göstererek, işlemleri ardışık olarak göstererek öğrencilerin hata yapmasını önlediği düşünülmektedir. Öğretmenin benzer terimleri bulma veya parantezleri kaldırma gibi belirli cebirsel işlemleri rutin olarak ayrıntılı olarak açıklaması, öğrencilerin cebirsel işlem becerilerini geliştirebileceği düşünülmüştür.

4.4 Cebirsel İfadelerde Onaylanmış Anlatılar

Anlatılar, diğer matematiksel söylem karakteristikleri tarafından onaylanan bir başka söylem karakteristiğidir. Cebirsel ifadelerdeki kavramlar konusunda iki onaylanmış anlatıya rastlanmıştır. Bunlar, 'cebirsel ifadeler, bilinmeyenlerden oluşan ve matematiksel işlemler içeren ifadeleri ifade eder' ve 'aynı harflere sahip terimler ve harflerin aynı kuvvetleri benzer terimler olarak adlandırılmıştır.' Benzer şekilde, cebirsel ifadelerdeki işlemler konusunda da iki onaylanmış anlatı vardır. Bunlar, 'aynı harflere sahip terimler ve harflerin aynı kuvvetleri benzer terimler olarak adlandırılır' ve 'cebirsel ifadede toplama ve çıkarma tanımıdır'. Bu anlatılar, kelimelerin, görsel aracıların ve rutinlerin desteğiyle onaylanmış olurlar.

İlk konuda iki anlatı bulunmuştur. Birincisi, bilinmeyenlerden oluşan ve matematiksel işlemler içeren ifadeleri ifade eden cebirsel ifadelerdir. Bu anlatı, cebirsel ifadelerin tanımını ve kullanımını desteklemek için kullanılan bilinmeyenler, terimler, terim sayısı ve sabit terimler gibi kelimelerle desteklenmiştir. Örneğin, öğretmen 'hem işlemleri hem de bilinmeyenleri içeren ifadelere cebirsel ifadeler denir' diye tanımlamıştır. Cebirsel ifadenin tanımını bilinmeyen ve cebirsel ifade gibi kelimelerle desteklemiştir. Ayrıca, cebirsel notasyonları temsil eden görsel bir aracıyla da anlatıyı desteklemiştir. 3x+2y+5 ifadesinin hem bilinmeyenleri hem de işlemleri içerdiği için aslında bir cebirsel ifade olduğunu açıklamaktadır. Dahası, benzer terimleri bulma rutini, benzer bir terim yazma ve Casper rutini, anlatıyı destekleyen söylemsel kalıplardır. Diğer söylem özellikleriyle desteklenen anlatıların rolü, matematiksel fikirlerin anlaşılmasını ve iletilmesini yönlendiren temel unsurlar olarak hizmet etmek üzere matematiksel söylemi oluşturmak ve doğrulamaktır (Tasara, 2018). Böylece, onaylanmış anlatılar matematik topluluğu tarafından kabul edilen tanımlar, teoremler ve kanıtlar haline gelmiştir (Nardi vd., 2014). Cebirsel ifadelerdeki kavramlarda var olan diğer anlatı, aynı harflere sahip terimler ve terimlerin aynı kuvvetleri benzer terimler olarak adlandırılmıştır. Anlatı, benzer terim tarafından birçok kez onaylanmıştır. Benzer terim, cebir derslerinde en sık kullanılan kelime kullanımıydı. Dahası, anlatı bazı görsel aracılar tarafından onaylanmıştır. Bunlar, benzer terimleri belirtmek için cebirsel semboller, daireler ve yaylardır. Anlatıyı onaylayan ilgili rutinler, benzer terimi bulmak ve benzer bir terim yazmaktır.

Cebirsel ifadelerdeki işlemlerde de iki anlatı vardı. Birincisi, yine aynı harflere sahip terimlerdi ve harflerin aynı kuvvetleri ilk konudaki gibi benzer terimler olarak adladırılır. İkinci anlatı, cebirsel ifadedeki toplama ve çıkarma tanımıyla ilgiliydi. Anlatı, benzer terim tarafından birçok kez onaylanmıştır. Ayrıca, benzer terimleri bulma, cebirsel ifadelerde toplama ve çıkarma ve parantezi kaldırma, anlatıyı destekleyen rutinlerdir. Görsel aracılar, diğer anlatılarda olduğu gibi, cebirsel notasyonlar, daireler ve yaylardı ve bunlardan farklı olarak geometrik şekiller ve modelleme karoları vardır.

4.5 Öneriler

Bu çalışma, yalnızca bir sınıfa odaklanarak yedinci sınıf düzeyinde cebirsel ifadelerin öğretimini araştırmayı amaçlayan bir durum çalışması olarak yürütülmüştür. Bu çalışmanın bulguları belirli sınırlamalardan etkilenmiş olacağı düşünülmüştür. Her şeyden önce, mevcut çalışma yedinci sınıf cebirsel ifadeler öğretimiyle sınırlıdır. Matematiksel söylem özelliklerinin nasıl oluştuğunu görmek için ortaokulun başka bir düzeyinde daha fazla araştırma yapılabileceği düşünülmüştür. Dahası, bu çalışma matematiksel söylemi cebir bağlamında analiz etmekle sınırlıdır. Müfredattaki farklı kavramların söylemini inceleyerek yeni çalışmalar yürütebileceği düşünülmüştür. Böylece, farklı bağlamlarda kelime kullanımındaki, görsel aracılardaki, rutinlerdeki ve onaylanan anlatılardaki fark araştırabileceği düşünülmüştür. Çalışmanın bir diğer sınırlaması, bulguların yalnızca öğrencilerin düşük başarı gösterdiği bir kamu ortaokulunda şehir merkezine yakın bulunan bir sınıftan elde edilmiş olmasıdır. Kullandıkları söylemi karşılaştırmak için şehir merkezindeki okullarda veya özel okullarda da farklı çalışmalar yürütülebileceği düşünülmüştür.

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