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Research Article

Facilitating online learning environment in math classes: Teachers' views and suggestions

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This study aims to reveal the views and suggestions of in-service mathematics teachers about supporting online learning. Many times, studies indicate the teachers' views related to the advantages and disadvantages of online learning. However, limited studies investigate teachers' views for improving the online learning environment. Thus, this study reveals teachers' recommendations for supporting e-learning. To achieve this, a case study was conducted to describe teachers' views. Data were collected from four middle and high school mathematics teachers through interviews, reflections, and lesson plans for online mathematics education courses. Teachers' views and suggestions related to supporting online learning were coded under five categories: norms, selection of manipulatives, building a community of learners, participation in group work, and assessment in the online learning environment.

Keywords: Online learning; Mathematics teachers; Preservice teachers

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1. Introduction

In the past few decades, using online modes supported different opportunities for teaching and learning mathematics. Internet development extended the classroom concept flexibly with access to learning materials anytime. Borba et al. (2017) emphasize, "Flipped classrooms change the notion of what is in and outside of the classroom and also change the roles of students and teachers" (p.230). One reason for this change is that computers, the internet, new technological devices, and online social media are essential to the new generation's life (Engelbrecht et al., 2020).

Especially after the COVID-19 pandemic, online education has become an essential part of our lives. While online education can be defined as a way of bringing teachers and students together independent of time and place in the medium of technology (Moore & Kearsley, 2011), online learning can be defined as access to learning experiences with technology (Carliner, 2003). Students can use computers, iPads, tablets, and smartphones in these environments. Many online learning platforms, such as Zoom meetings, Microsoft Teams meetings, WhatsApp, and Google Classroom, can also be used.

According to Sun and Chen (2016), there are three types of learning in online education: synchronous, asynchronous, and a combination of both. While synchronous learning provides

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real-time interaction between student and teacher, the students can use the internet to access the learning materials and have a chance to meet experts in the fields they are studying; asynchronous online learning provides access to online materials at any time. Researchers reveal that online learning has advantages and disadvantages. Finch and Jacobs (2012) report that e-education saves time and travel expenses, increases the opportunity to access and work together with expert professional educators, provides students with flexibility in accessing courses, and allows the adjustment of topics and content. In addition, Westberry (2009) emphasizes that online learning supports students' cognition levels, reflections, and information processing. Online learning systems can be used to identify learners' needs and give appropriate materials for learners to achieve the instructional goals.

Online learning also has disadvantages, such as a lack of interaction and motivation (Almosa, 2002). The most crucial disadvantage is that teachers need more online teaching experience as many teachers adapt their classroom-based teaching approaches to the online platform rather than finding new approaches (Livy et al., 2022). Planning and implementing practices and innovative strategies in online education is essential to successfully promote and expand online education (Palvia et al., 2018). Teachers need to support interactive activities, innovate in designing and gathering materials and learning methods, and find creative ways to motivate students to keep their enthusiasm on online platforms.

The practices that help students stay engaged in learning are known as instructional methods. In order to help students, stay active in online learning, teachers need different instructional strategies related to norms, selection of manipulatives, building a community of learners, participation in group work, and assessment in the online learning environment (Wills, 2020). Researchers emphasize the importance of establishing and sustaining norms that can be defined as classroom regularities to support effective mathematical learning (Stephan, 2020), as it is important to create an environment where students discuss their ideas without any shyness. Additionally, students must be supported with manipulatives to make sense of the mathematical ideas before they become abstract. In a practical environment, students must also take responsibility where everyone can learn, share, and work in groups to discuss their ideas. Finally, the teacher needs to use practical assessment tools to give valuable feedback to the students. Thus, this study extracts teachers' views under these categories to support students' mathematical understanding in the online learning environment.

1.1. Integration of Technology into Mathematics in Online Settings

Various technologies influenced teaching and learning mathematics before online learning became popular. Effective technology integration enriches mathematics learning quality (Aydogan & Gokce, 2021). Some of the most common ways to use technology in online settings are dynamic geometry software [DGS], MATLAB, homework submission, evaluation resources (i.e., Webwork), learning management systems such as Canvas, and online platforms for assessment like STACK and Classkick. While the assessment tools provide opportunities for students to take immediate teacher feedback and save time for the class (Rasila et al., 2015), DGS support students in visualizing mathematical ideas with different representation (e.g., symbolic, graphical) in interactive content. Additionally, with these tools, students can explore the connections, make conjectures, and prove mathematical ideas. This allows students to become active participants by developing high-level thinking skills and conceptual learning (Lee & Bednarz, 2012).

In addition to these tools, many Massive Open Online Courses [MOOCs] were designed and applied in undergraduate mathematics courses. Some of these courses used flipped classrooms (Naccarato & Karakok, 2015) approaches to support students' mathematical ideas. By using tablets with online whiteboards (i.e., selecting freehand writing tools) and permission to screen share mathematical concepts, graphs, symbols, and solution processes, students have more opportunities to interact and participate in online mathematics courses (Karal, 2015).

One of the most critical challenges to creating a productive online learning environment is supporting student-centered activities where students can interact with the content, teacher, and peers (Engelbrecht & Harding, 2005). The significance of active learning, where students share their ideas, collaborate, and discourse mathematical ideas, has been emphasized by many researchers for a long time (Steffe & Thompson, 2000). Thus, researchers emphasize using interactive learning materials where students actively engage, feel part of learning (Engelbrect et al., 2020; Petty & Farinde, 2013), and collaborate (Taranto et al., 2020) in online learning. It is essential to substitute physical tools and resources with virtual manipulatives and integrate new pedagogical approaches to support students' mathematical understanding. Unless teachers use the tools effectively to facilitate the development of higher-order thinking and provide scaffolding, studies show that face-to-face students outperform students who learn fully online mathematical courses (Trenholm et al., 2019). For example, teachers can use interactive learning activities such as virtual and augmented reality applications, simulations, and videos to support a multi-dimensional and interactive learning environment (Collins & Halverson, 2018). The literature also highlights that students' success, self-confidence, and attitudes change positively when interactive activities are used appropriately (Hamari et al., 2016; Hung et al., 2014).

Supporting different representations and motivating students to use them in online learning also provokes students' interest and promotes more profound thinking. Virtual materials are essential tools to support students' logical thinking and problem-solving skills (Moyer-Packenham & Westenskow, 2016; Reys et al., 2020). These tools can be used instead of authentic materials in online learning. As these tools also have dynamic features, it is found that they are helpful not only for students but also for preservice teachers for visualizing solutions and experiencing spatial reasoning skills for different geometric objects (Livy et al., 2022). Another essential tool is discussion platforms where students can engage with interactive cloud-based collaborative web platforms such as Padlet. Using these tools, students can upload and share files visually to virtual bulletin boards in real time and think about their strategies (Fisher, 2017). The studies revealed that this type of discussion environment facilitated recording multiple responses and seeing different responses on time (Lowe & Humphrey, 2018). Finally, using a shared link for Google Jamboards that includes multiple pages might help access virtual representations during synchronous teaching and collaboration for posting multiple solutions to a problem (Sweeney et al., 2021).

1.2. Teacher's Role in Online Learning

Even though various technological tools can be used in online learning to support students' conceptual understanding, the teacher factor remains as the critical point of using them effectively (Marbán & Mulenga, 2019). Teachers' beliefs, attitudes, and self-efficacy are essential elements in their use of technology. Since teachers' pedagogical decisions and practices are influenced by their beliefs (Beswick, 2012), it is essential to change their beliefs to positively support the effective use of technology. As individuals' beliefs originate, change, or develop over their lifetime (Francis et al., 2014), it is critical to help teachers when they have difficulties using ICT, designing materials, and choosing applications in line with the materials. One of the crucial points is that teacher creativity is vital to their success in motivating students to online learning (Rahayu & Wirza, 2020). Teachers need to create models and appropriate learning strategies for their students. By utilizing online media packages effectively, they can make a big difference in online learning. However, even though the schools have the highest level of technology, teachers' attitudes and beliefs limit their use of technology in the classrooms (Kellogg & Kersaint, 2004). For example, there are various tools for teaching and learning mathematics, such as Geogebra, Sketchpad, Cabri, and Maple. However, teachers often restrict their use of technology with Word, PowerPoint, and Excel (Baki et al., 2009). One of the reasons is teachers' past habits and fears of using practical tools. Thus, it is crucial to support teachers in creating effective online learning environments where they can use tools willingly. This study investigates in-service teachers' views and suggestions related to supporting online learning. In order to achieve this, interviews were conducted with four in-

service mathematics teachers who took classes related to online mathematics education during their graduate study. Additionally, the teachers' reflections and lesson plans were used to extract views and suggestions supporting online education.

2. Method

2.1. Research Design

The current study uses a qualitative research design to investigate mathematics teachers' views and experiences of supporting online learning. Case studies have been commonly used in education research where the aim is to examine a specific phenomenon in a bounded system (Merriam, 2009). If the researcher selects the case because it is intrinsically interesting and wants to understand the phenomena fully, these studies can also be characterized as particularistic or intrinsic case studies (Merriam, 2009). This study can be characterized as a particularistic case study since the aim is to investigate four teachers' views and suggestions rather than investigating them for general understanding.

2.2. Participants

A purposeful sampling method was used as experienced participants related to online learning were selected (Bernard, 2002). All participant teachers took a course related to teaching online mathematics education. Besides being teachers, they were also graduate students. While the two teachers in the study worked in the public school at the middle school level, one of them worked as a high school teacher in a private school, and one worked as a high school teacher in a public school. Demographic information of the participants is shown in Table 1.

Table 1
Teachers' information

Information	*Shirley*	*Giselle*	*Susan*	*Tina*
Gender	Female	Female	Female	Female
Experience	6 Years	3 Years	3 Years	6 Years
School Type	Public	Public	Public	Private
Location	City Center	Village	City Center	City Center
Level	High School	Middle School	Middle School	High School
Education	PhD student	PhD student	PhD student	PhD student

2.3. Data Collection and Analysis

This study used the semi-structured interview data collection tool to understand the mathematics teachers' views and suggestions on online learning in depth. For this purpose, two semi-structured interviews were carried out on the online platform together with the recorder. While the first interview was conducted to determine the backgrounds and perspectives of teachers related to mathematics education, the second interview was conducted to reveal their own experiences with online education. Each interview approximately lasted one hour and included around 25 questions. Experts' views were taken to determine the quality of interview questions. Based on the feedback of experts, the questions were revised. Additionally, the triangulation technique was used to support reliability. Besides interviews, teachers' reflections and lesson plans that were prepared for online learning were collected. After collecting the data, the interviews were transcribed, and participants were given pseudonyms.

In this study, data analyses were performed using the content analysis. Content analysis can detect relationships, divide data into sub-categories, and draw conclusions directly from the data. Thus, the analysis in this study included categories and the conceptual links among these categories. The analysis started with a particular incidence from the interviews, reflections, and lesson plans that could be compared with other incidents from the data. These comparisons helped us create tentative categories, and later on, they were compared within themselves.

The data analysis of this study was categorized under five parts: norms, selection of manipulatives, building a community of learners, participation in group work, and assessment in the online learning environment. Even though these categories comprise the teacher's activities to support face-to-face and online learning, how it is applied changes based on the environment. This study aims to extract teachers' views and suggestions of teachers related to supporting online learning environments.

3. Results

3.1. Norms

One of the teacher's role is to establish the norms by leading the negotiation and renegotiation of them with his/her students. Students often do not explain their ideas or try to make justifications independently, which may cause difficulty supporting the ongoing discourse in the classroom. Even though the basic ideas behind the norms are the same, the practices that support the norms change based on the environment. Thus, teachers need to know the norms supporting effective online learning. When the participants in this study were asked about the norms that would support online learning, Tina stated that "..participate and prepared for the class, raising a hand to speak, on timing, safety, kindness and respect, student agency through discussion, math talk, and reducing distractions like background noise, siblings".

Tina emphasized the importance of attending the online session on time. She revealed she would ask them to use the microphone in small groups but turn it off during the classroom discussion or while she was talking. She also emphasized using a chat box or private chat box, not a microphone, if they have problems at home (i.e., young sisters/brothers distract them). Like Tina, Susan highlighted, "Raising virtual hands and eliminating background noise while someone is speaking are important." She also stated that it might take time for students to adjust to this environment, but the teacher needs to support the norms until they are established.

Giselle said creating and facilitating norms online is more complex than in a face-to-face classroom. She stated, "Through the class discussions, group work, and activities, we need norms like tolerance of mistakes and sharing ideas freely. In the online classes, sustaining those norms is harder than face-to-face". She explained the reasons for having more difficulty in supporting norms in the online environment as having technical problems such as internet connection quality or the lack of knowledge related to using technological tools. Thus, she said she supported how to use Blackboard, Canvas, and Google Classroom at the beginning of online lessons. Students might need to learn how to use interactive sites to upload images of their drawings or screenshots. In addition to this, she also stated she needed to support the use of tools such as Desmos and virtual manipulatives as resources. Another essential tool in online learning is videoconferencing tools. Shirley emphasized that she tried to support using Zoom at the beginning of online lessons. She helped students to use audio, chat box, video streaming, and share application tools in this environment. All teachers stated that once students learned about the tools, supporting norms was the most essential part of the online environment. They highlighted the use of kind and respectful language, reducing or eliminating background noise, raising a virtual hand when talking, being helpful to other people when they need technological help, supporting the idea that "making mistakes is okay," and also stated they asked their students to leave a message when they had to leave from the computer.

All teachers expressed that new norms needed to be established for online learning. For example, in face-to-face classes, students might ask questions outside the class or in your office. In the online environment, answering everyone's questions during class is difficult, so they suggested using Google Slides, where students can write their questions. Another important point they focused was that students need to participate in small group instruction by taking responsibility and collaboratively. In an online environment, they stated they used breakout rooms with different numbers of participants. While visiting students in different small groups, they asked students to get responsibility, share the screen, and take notes to solve the problems. Thus, they tried to

establish and sustain new norms in online learning. Based on the analysis of lesson plans and reflections about those plans, it was found that all teachers incorporated norms into their plans. For example, Giselle wrote in her lesson plan:

> "Remind students the rules of group work in breakout rooms. Microphones and cameras must be open if students and their environment are available. Support talking and sharing ideas. If they need help, they can use the chat box to ask questions."

All the teachers emphasized asking questions, explaining the ideas, and listening to each other in their lesson plans. In their reflections, teachers emphasized the importance of discussing the problems with the whole class. However, they also mentioned time limitations. Tina stated that she could not prevent the lack of time; thus, she kept the group discussions short. So, she said she could not wait for each student to explain their opinion or ask questions about whether they agreed with their classmates. Even though they might have had some problems, all the teachers were knowledgeable about norms and supported them during their teaching.

3.2. Selection of Manipulatives

Teachers may have misconceptions regarding the proper use of manipulatives and collaborative groups. For example, many times, manipulatives are given to students, and the teacher tells them how to use the tools. However, students may need help connecting or seeing the relationships between the manipulatives and the mathematics they are meant to embody since students have a different knowledge than the teacher. The teacher needs to develop students' mathematical understanding using manipulatives rather than tell the students the relationship between the tool and the mathematics. Even though the manipulatives change from face-to-face to online learning, the ways used to support students' understanding of mathematics are the same. Tina emphasized that the teacher needs to know how to choose the manipulative by stating, "...the tools should be chosen and integrated to the lesson plans regarding the mathematical goals and the needs of students as well as classroom dynamics". Moreover, she said:

> "I would be purposeful in selecting the manipulative by choosing them correctly regarding the classroom culture or norms, user-friendliness, and accessibility from everywhere and every student. Besides, I consider the time and examine whether it supports students' procedural and conceptual understanding, mathematical goals of the lesson, students' collaboration, or distracts them more. "

She also added that she used hands-on manipulatives such as spaghetti, beans, and paper clips. She added, "I ask students to show how they use tools rather than showing how to use them myself. Next, we discuss the manipulatives' effectiveness with the class". She also emphasized using manipulatives by integrating them into solution of a problem and real life. She highlighted the importance of using them by the students when constructing a formula or modelling a mathematical problem rather than the teacher's presentation of the tool. She also said that she chooses the manipulatives based on the lesson's goal and added that she would select the manipulatives to support students' collaboration and their awareness of digital literacy. She emphasized that using tools for visualization purposes might not be helpful for students, and teachers need to support students' understanding by connecting it with different representations. Giselle stated that manipulatives' aim is crucial in effectively using them in online learning. She said:

> "If you want to use them in the exploration process, the materials should serve as a way to conceptual understanding like fraction bars, base-ten blocks, and algebra tiles. If you want to improve students' procedural fluency, you can use games drag-drop activities."

She also added that using the breakout rooms feature, she tries to use virtual manipulatives during the small group work. This way, students have a chance to discuss the answers to the questions with group members, experience using it themselves and participate more actively. She also said that while students work on the problem by using manipulatives as a group, she visits each breakout room and asks conceptual questions while they are using them. Like Giselle, Susan also emphasized selecting the manipulatives based on lesson objectives. She emphasized that she

let students select their manipulatives. She added, "for example, for the fraction topic, each student can demonstrate fraction models by uploading photos to the online environment. They can use materials from home such as lego blocks". Finally, she stated that the most important thing is that asking conceptual questions during manipulatives is essential, and students need to practice them by using them in small groups. Shirley emphasized either students' or teacher's selection when using the tools. She said:

> "...depending on the lesson's purpose, I could use both ways to select a manipulative. For instance, if the manipulative I intend to use has a few forms that all serve the mathematical goal, I can let my students choose the manipulative. Moreover, I also need to try the manipulatives to teach students about the unsuitable ones, or I can use them when they have a question about using them…"

She also mentioned about the evaluation of the manipulatives. She said that she considers how her students use it, how it relates to the mathematical goal, and how it is used to build conceptual or procedural understanding. She added that in an online environment, once students work on the virtual manipulative and model the problem with tools by discussing the solution in breakout rooms, they can get screenshots of their solutions, post them to Google Slides where everyone can see and discuss those solutions during the whole group discussion. The teacher or the student's colleagues might ask questions regarding their use of manipulatives in these discussions.

In the lesson plans and reflections, all the teachers supported using different manipulatives. For example, in her lesson plan, Susan emphasized modeling the first-degree equations with virtual algebra tiles. She asked students to share pictures of the models they had created using Google Slides. Similarly, Tina also asked students to use Google Slides after taking pictures of the papers, where they constructed the medians of the triangle and labeled the vertices and centroid. Their reflections emphasized that students were comfortable sharing their work with slides. Shirley wrote in her reflection regarding the effectiveness of the tools students used:

> "The one thing that I did not expect was that the learning tool was not very user-friendly. It produced some results that were not within my expectations. If I checked the tool more thoroughly or used software such as Geogebra to create it, I would not have encountered such a situation".

So, she emphasized the use of online manipulatives in detail (i.e., whether it caused any confusion or misconception) before using it in the lesson.

3.3. Building a Learner of Communities

The teacher's role is to see the classroom as a mathematics learning community and support the community to create and sustain the norms and mathematical practices taken-as-shared by all community members and orchestrate discussions. The teacher needs to consider the classroom as a community where everyone has an opportunity to learn and share. In the learning community, the teachers share the difficulties faced by their students and reflect on what they can do to address them. Tina stated that:

> "I can use multiple modalities (images, animated gifs, text, voice, and emojis) to build a community of learners by choosing quick and productive routines and strategies that get students talking and enjoying. I also use a diary to announce the news in school or to share the celebrations of those born on that day. To establish a student-to-student collaboration, interactive slides might be one of the best ways to learn each other's hobbies or interests, or whatever they want to share about their lives, experiences, or family routines."

She also highlighted representing a picture and pose a "Would you rather?" question to understand whether students are present or not and hear their ideas mathematically. She also said that even if time is limited, using at least one of them might support building strong math communities. Giselle also emphasized that morning meeting is a good starting point to create social interaction space in the class as students might know more about each other. She emphasized sharing their life, stories, and interests by interactive slides. On the other hand, she stated talking is the best way to socialize the class and interactive slides including emojis might be important to observe students' emotional situations. She said:

> "Icebreakers can also build a community while launching a math lesson. Icebreakers could reveal students' prior knowledge, attract students' attention, and prepare students for the lesson. Students discuss, share their ideas, and criticize each other's responses. Thus, a classroom community can be built. Integrating small-group works using breakout rooms can also help build small communities in the class".

Susan emphasized that in her lessons, she prefer to use breakout rooms and check-in strategies to build a community of learners. In the breakout rooms, students prefer to use images and their voices to share ideas about math questions. Each student leverages his/her voice to tell others about mathematical ideas. In breakout rooms, they upload images about their work to tell mathematical strategies and thinking way. She also emphasized using open-ended problems with context from real life. This way, multiple strategies of students might emerge due to the structure of the problem. On the other hand, Shirley stated that a teacher's most essential role is building a strong classroom community. She said:

> "Although sparing time to talk to my students about their daily lives might sound like I am wasting time, I think spending just a few minutes every day like this, brings my students and me closer to each other, and after some time, I can know what my students are thinking without asking much."

Therefore, she believes that allocating time to build a community of learners is crucial. She also added that she used multiple modalities to overcome students' difficulties, such as images, animated gifs, text, voice, and emoji. Using them informs her about her students' feelings in general or about a specific math subject. She uses them to check her students' well-being, creating a bond with them and building a community of learners in the classroom. Moreover, she uses them to get feedback from her students about the mathematical subjects they learn. She also emphasized the importance of asking her students whether the topic they learned was complex or easy and if they needed additional practice to understand it at the end of the lesson.

Teachers also highlighted the importance of building learner communities in lesson plans and reflections. For example, Giselle stated she would ask students how they feel and open a shared slide where they can paste pictures of different modes (i.e., smiley face, upset face) at the beginning of the lesson. On the other hand, Tina wrote in her lesson plan that she would apply the jigsaw method in the groups to support everyone in taking responsibility for the group work. She wrote:

> "I will give a problem to students in the group and ask them to solve it by using different representations. Next, I will ask them to discuss different solutions in their group. I will ask different students to defend the solution during the classroom discussion. This way, everyone in the group will work."

In the reflections, Shirley emphasized the importance of including real-life examples and asking students how the examples relate to the mathematical topic she would teach. She said asking students to write their ideas on shared slides was very useful. This way, even shy students had a chance to share their ideas. Thus, the discussion was more powerful.

3.4. Participation in Group Work

In small groups, students may need to learn what questions to ask their friends or how to work collaboratively. Teachers need to support students in small groups to create effective conversations and questions rather than physically create the groups and leave students to work. In an online learning environment, it might include more challenges for teachers. So, teachers need to know the practices that will help students develop productive small-group collaborative relationships. Lampert and Blunk (1998) emphasize teachers' role in small groups as follows: first, the teacher needs to explain why working in small groups can be helpful for the students; second, the teacher should emphasize to the students the importance of practicing their skills to work with other people. Finally, the teacher should convince the students that she is not the sole knowledge resource in the classroom.

When Tina was asked about the structures that would encourage participation during small-group work, she stated that she would create structures that are an individual space or "own-a space" for each student to write, drag or drop, upload photos or images to encourage participation during small-group work. This way, she added, she would allow each student to explain their thinking. She said she would monitor breakout rooms by visiting them simultaneously and asking students to record their thinking on Google Slides. She emphasized the importance of clarifying the directions before students start to work and giving each group different links with different slides. In her lesson plans, she focused on asking students to record their group work on slides or in a class notebook such as Padlet. She highlighted the importance of using virtual manipulatives to support multiple representations (i.e., numerical, graphical, algebraic). She gave an example as follows:

> "I use the breakout rooms and want groups to solve the equation systems by using graphs, algebra tiles and tables. Then, I ask them the reasoning and definition of their exploration in their own words and write their ideas to Google slides. Next, I choose one of the students in each group and want him/her to explain briefly what they get, which increases students' self-confidence."

Giselle also emphasized using breakout rooms and forming groups randomly. She said she visit each breakout room and ask questions to support students' mathematical thinking and take notes on students' misconceptions and strategies. Like Tina, she also emphasized using Google Slides to follow each group's work. She emphasized the importance of social and socio-mathematical norms such as being tolerant of mistakes, listening and respecting each other, and finding different solutions for practical small group outputs. She highlighted encouraging shy students as spokespersons to help them gain self-confidence. She added that she also provide anonymous solutions to support those shy students if they do not want to. Giselle also focused on the use of Padlet, where students can upload photos of their paper-pencil solutions or screenshots of their solutions. She also stated she support multiple representations:

> "I ask students to defend their thinking in multiple ways and create space for multiple representations in the working area of the group with the strategies such as "multiple choice corners", "defend your choice", and "mine, yours, ours strategies".

Susan also stated the importance of controlling group work and asking extended questions during breakout rooms. She said she tried to ensure each student's participation in small group work. While traveling across breakout rooms, she asked why and how questions to observe whole groups' thinking way. In the Zoom platform, she said students had the opportunity to share their ideas on screen so they could easily display their strategies by uploading photos or writing on white-board.

Shirley stated that she creates heterogeneous groups and encourages students to participate in group work. She added that sometimes students might hesitate to speak in groups of 4 or more. If there were such students in her classroom, Shirley highlighted employing 'pair-share' routines so that her students could share their opinions more comfortably. Lastly, she said she would encourage participation during small-group work and launch a rich task using a hook or story related to the student's prior knowledge and interest. This way, her students become more motivated to solve the problem and support connecting to mathematics in real life. To monitor her students in online classrooms, she said she views students' groups' slides and enters their breakout rooms. She also emphasized the use of interactive slides as well as multiple representations with virtual manipulatives.

In the lesson plans, all the teachers emphasized creating breakout rooms and giving students time to discuss the problem. They said they would visit each group while working and ask some questions. Teachers also focused on using virtual manipulatives while solving the problem given to the group. Susan said that she would divide students into breakout groups of four in each group, then ask them to explain the relationship between the area of a rectangle and the distributive rule of algebraic expressions using virtual algebra tiles. She also wrote that she would ask each group to demonstrate constructed rectangles using algebra tiles and explain their

reasoning. All teachers also mentioned the importance of using Google Slides, where each group included their solution on a different slide. In her reflection, Giselle said, "Group works on breakout rooms were good; all groups discussed and shared their solutions on Google Slides in a given time." Teachers stated that using shared slides is essential as all the students can work on a different slide, and when the time finishes, the teacher can have a chance to show all the solutions visually in whole class discussion.

3.5. Assessment in the Online Learning Environment

The practice of assessment includes actions of creating summative as well as formative assessment tasks. Using formative assessments, the teachers can get student feedback and revise the lesson's content based on their interaction. The teacher's knowledge of content and curriculum is crucial in preparing both procedural and conceptual assessment tasks to help her understand and evaluate the students' mathematical thinking. Her pedagogical knowledge, particularly about students' misconceptions, is essential to creating formative assessment tasks to remediate students' difficulties. Creating assessment tasks can help a teacher understand students' problem-solving skills. As with the other practices, assessment also changes when we move to the online environment.

When Tina was asked about supporting students to gain prior knowledge about a concept before they investigate it during the synchronous class time, she stated that she asks students to watch videos from Khan Academy before the lessons and then create short videos related to what they understand from the topics they watch. She added that during class time, she asks questions about the videos students watch before the lesson. She gave an example:

> "I use a 3-act video for the 10th grades to teach the types of functions. Students can take photos of real-life functions, such as linear, constant motion problems, bamboo growth, quadratic functions, roller coasters, bridges, exponential functions, bacterial growth, or compound interest. Then, they can model these functions by Desmos and represent them with Google Slides or take videos by Flipgrid. I suggest grouping them according to the function types. The students can find the similarities and differences of the functions."

She emphasized using flipped classrooms as it supports differentiated instruction and assessment. This way, she said her students could collect data concerning their interests rather than having the data provided to them. Making students collect data is a significant way to make students active participants from passive learners. She emphasized that independent investigation raise students' experience of math in real life and help them to improve several problem-solving strategies and reasoning skills.

Giselle also emphasized the use of flipped classrooms by stating that it allows students to independently learn concepts from videos and extra materials, and then they can practice with teachers. Assigning videos from national assessment platforms, instructional slides, YouTube, and Khan Academy also help students prepare for practice with the teacher in synchronous class. She highlighted the importance of learning concepts through videos and virtual manipulatives on these platforms. On the other side, Susan emphasized the importance of using open-ended tasks for formative assessments. She also suggested the use of Khan Academy to support prior knowledge. She stated she gives online math games for students to practice. In order to remediate students' misconceptions, she emphasized the use of online manipulatives.

Shirley said that to ensure that our students are accountable for their answers and honest while answering them, she asks them open-ended questions requiring them to apply general rules and theories found on the internet. She establishes rules explaining specific situations where students can use the internet. Additionally, she focused on selecting questions that give her students more opportunity to take ownership and allow them to find the relationship to their world. She also stated she asks them to use images and videos as evidence of independent work and creativity to support a unique answer. She emphasized giving her students multimedia homework since using it engage even the most reluctant students. She emphasized the use of demonstrations. She said:

> "For instance, if we want our students to follow some instructions to construct something like a tower or a bridge, then we can use a video that involves the instructions. Alternatively, if we have some concepts that we have difficulty demonstrating ourselves, we can use videos or simulations. Using videos or demonstrations in synchronous class is more beneficial for students as we can intervene and explain the concepts by stopping the video and discussing essential points."

She added that for introducing a subject such as geometric objects, the teacher can ask students to collect data by taking photos of the geometric objects in their homes. This way, students can be in charge of their own learning and can transform from passive attendees to active participants. In addition, collecting data increases their motivation and deepens their investment in the studied concepts. Thus, she emphasized collecting data rather than providing the data to students. Finally, she also emphasized using tools such as Classkick, where the teacher can observe students' answers simultaneously and give feedback.

In their lesson plans and reflections, the teachers emphasized the importance of formative assessments. For example, in her lesson plan, Shirley stated that she would ask students to show their results on the whiteboard after they had solved the problem. She also added that she would discuss the answers, the solution processes they had followed, and alternative ways of solving the question. Tina said she would assess students' knowledge by using the class notebook in Google Slides and asking them to write a keyword or a simple sentence about what they learned in that lesson. She also wrote that she would ask them to use an emoji to define their feelings about the lesson. In her reflections, Shirley emphasized preparing Google Slides with separate worksheets for each group. She also said she would focus on students' unexpected questions, misconceptions, and solutions while planning her instruction. In other words, all the teachers emphasized the importance of assessment, especially the formative one, during instruction.

4. Discussion and Conclusion

Norms, a crucial component of student-centered classrooms, can be defined as patterns and routines established by the classroom community where the patterns become hidden regularities that guide the classroom members' actions and are taken for granted during the interaction in the classroom culture (Wood, 1998). They create a risk-free environment where students and teachers focus on reasoning rather than just answering (Stephan & Whitenack, 2003). For example, social norms for whole classroom discussion may consist of explaining and justifying solutions, understanding other students' explanations, stating agreement and disagreement, and questioning different solutions where conflicts occur (Cobb et al., 1991). In an online environment, the teacher needs to support safety, kindness and community norms, student agency, productivity, lesson workflow, small group instruction, breakout meetings, productive struggle, materials, and manipulatives management (Wills, 2020). In order to achieve this, at the beginning of the lesson, an introductory section is essential as the teacher needs to support students in terms of the goals of the lesson use of the platform (Bardelle & Di Martino, 2012; Voigt et al., 2020) and classroom norms. This way, all students can have a chance to participate in the class.

During online learning, as the teacher might have poor communication with students due to limited eye contact (e.g., they may turn off their cameras), it might be challenging to establish classroom norms. Additionally, cultural norms can change based on country context in online learning (Grothaus, 2023). For example, while students might have problems in some countries because of a lack of affective engagement due to ineffective communication with their friends, in others, students might not verbally join videoconferences as they are mostly with their families at home. Thus, teachers should know the needs of their students' before supporting the norms. By establishing classroom norms at the beginning of the semester together with the community of the class, teachers can prevent students from misbehavior. They can create a democratic atmosphere where everyone can discuss their ideas without offending each other. However, teachers often have barriers to integrating online learning, such as a lack of confidence, an unwillingness to change their practices, a failure to understand the advantages of online learning, and beliefs related to ICT, knowledge, and experiences (Mailazar et al., 2021). Thus, in order to support these

norms, teachers first need to overcome these barriers. Teachers might start using breakout rooms to conduct group discussions and motivate students. This way, they can recognize their students more closely and learn more about their knowledge. Studies showed that teachers in online environments often make assumptions about their students' pre-existing knowledge and fail to provide detailed explanations to questions of their students (Ramasamy, 2009). Thus, by supporting a student-centered environment, teachers can have a chance to recognize their students' features and help them if needed.

Using manipulatives is also critical to support students' mathematical understanding. Teachers might plan their lessons by leading students to use physical manipulatives (such as beans, spaghetti, etc.), and then they might require students to take a photo of their activities. Next, they can support digital manipulatives such as applets and software. The essential point for including manipulatives in the learning environment is whether they lead students to express their ideas and allow them to use different representations. Teachers might give students rich tasks that require high-level cognitive domain during manipulatives. This way, students can explore different ways of solving problems using manipulatives effectively. If they have difficulties, teachers might monitor students and select strategies that are helpful for students. Here, it is essential to state that teachers must have technological pedagogical content knowledge to support digital tool use effectively. Studies showed that teachers struggled to integrate digital pedagogical resources, relate with students, and find instructional activities (Cao et al., 2021).

The COVID-19 pandemic has made quick changes in the classrooms. Many researchers suggested using this crisis as an opportunity for educational shifts (Zhao & Watterston, 2021). One of the changes related to teachers' practices in online learning is supporting communities of learners. In this environment, teachers might start the lesson with a conversation where students can tell each other about their daily lives. Using breakout rooms also might help to know students closely. Additionally, supporting students in working collaboratively using online communication tools was found as the most beneficial engagement strategy among learners (Martin & Bollinger, 2018). During the lesson students might be asked to write their ideas about the question in the space provided in Google Slides. Observing many answers at the same time might support students' thinking differently. The teacher might discuss these ideas and ask students to defend them. Moreover, the teacher might ask them to support in different ways based on their choice. They might defend by using a microphone, chat box or note their ideas on interactive Google Slides. This way, they can take responsibility for their learning. Teachers can also use different strategies to appeal to students' attention. For example, the teacher might ask real word questions and have discussion with guiding questions (Martin & Bollinger, 2018); find different solutions of the problems, estimate the answer to the problem, and use different representations. These activities support students to collaborate and reflect on their learning.

Another essential point in online learning is assessment. Both formative and summative assessments are critical. However, teachers might have difficulties in summative assessment as there might be different problems (i.e., internet connection, camera problem). Teachers need to talk about being honest digital citizens. They can give rich tasks where the answers vary based on the conditions given in the problem. During the formative assessment, they can check students' learning by asking questions. Students can answer using different tools such as a chatbox or microphone or writing it on the interactive slides. This way, the teacher might facilitate the whole class discussion.

In summary, the teachers in this study stated that they had different experiences that allowed them to improve their teaching practices in online learning. They improved their practices, such as using technology, time management, and formative assessment. They tried interactive teaching by promoting argumentation, using interactive tools, and differentiated teaching. However, the most challenging part for them was interaction with students as they could not see their students. Future studies might investigate how these experiences might be used in school curricula and teacher education programs. One limitation of this study is about generalizability, as purposeful

sampling was used in the selection of participants. This means that there were a limited number of in-service mathematics teachers that were investigated through in-depth interviews. These teachers were selected as they took the online mathematics education course during their graduate degree and they were knowledgeable about online education. Future studies might be conducted with a broader sample size or diversity in teaching contexts to enhance the generalizability of the findings. Researchers might include a more diverse range of teaching environments and student demographics and investigate the long-term impact of these strategies on students' learning outcomes and engagement. Moreover, they can explore the integration of recently emerging technologies such as artificial intelligence to personalize learning and support teachers.

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