INVESTIGATION OF STUDENTS' ALGEBRAIC THINKING: A TEACHING EXPERIMENT WITH 7TH GRADERS

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

INVESTIGATION OF STUDENTS' ALGEBRAIC THINKING: A TEACHING EXPERIMENT WITH 7TH GRADERS

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The aim of the study is to examine the development of 7th-grade algebraic thinking of students during algebra instruction, employing a teaching experiment model that spanned two months. Data were collected from six 7th-grade students during the spring semester of 2023-2024. The Chelsea Diagnostic Algebra Test was administered at the beginning and end of the teaching experiment to assess algebraic thinking levels of students. The teaching experiment, consisting of 8 sessions, was designed with a focus on various components of algebraic thinking. During the sessions, the students worked in groups on worksheets covering topics such as properties of the number system, the meaning of the equal sign, relational thinking, variables as unknowns and varying quantities, quantitative reasoning, repetitive patterns, growing patterns, and multiple representations. Data were collected through worksheets, guiding questions, and video recordings to examine algebraic thinking of students. Qualitative findings indicated that students were able to generalize properties of the number system, analyze and find general rules of repetitive and growing patterns, perform quantitative reasoning, grasp different meanings of variables, comprehend the meaning of the equal sign in algebra, and utilize multiple representations. Quantitative findings showed a significant increase in algebraic

thinking levels of students after the teaching experiment. Thus, it was concluded that the algebra teaching experiment had a positive impact on the development of algebraic thinking of students.

Keywords: Algebra, Algebraic Thinking, Teaching Experiment, Middle School Students

ÖZ

ÖĞRENCİLERİN CEBİRSEL DÜŞÜNMELERİNİN İNCELENMESİ: 7. SINIF ÖĞRENCİLERİ İLE BİR ÖĞRETİM DENEYİ

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Bu çalışmanın amacı, cebir öğretimi sırasında 7. sınıf öğrencilerinin cebirsel düşünmelerinin gelişimini iki ay süren bir öğretim deneyi modeli kullanılarak incelemektir. Veriler, 2023-2024 bahar döneminde altı 7. sınıf öğrencisinden toplanmıştır. Öğrencilerin cebirsel düşünme düzeylerini belirlemek için öğretim deneyinin başında ve sonunda Chelsea Diagnostic Algebra Testi uygulanmıştır. Sekiz oturumdan oluşan öğretim deneyi, cebirsel düşünmenin çeşitli bileşenlerine odaklanarak tasarlanmıştır. Oturumlar sırasında öğrenciler gruplar halinde sayı sisteminin özellikleri, eşittir işaretinin anlamı, ilişkisel düşünme, bilinmeyen ve değişen çokluklar olarak değişkenler, niceliksel muhakeme, tekrarlayan örüntüler, büyüyen örüntüler ve çoklu temsiller gibi konuları kapsayan çalışma kâğıtları üzerinde çalışmışlardır. Öğrencilerin cebirsel düşünmelerini incelemek için çalışma kâğıtları, yönlendirici sorular ve video kayıtları aracılığıyla veriler toplanmıştır. Nitel bulgular, öğrencilerin sayı sisteminin özelliklerini genelleyebildiklerini, tekrarlayan ve büyüyen örüntüleri analiz edebildiklerini ve genel kurallarını bulabildiklerini, niceliksel muhakeme yapabildiklerini, değişkenlerin farklı anlamlarını kavrayabildiklerini, cebirde eşittir işaretinin anlamını kavrayabildiklerini ve çoklu temsilleri kullanabildiklerini göstermiştir. Nicel

bulgular, öğretim deneyinin ardından öğrencilerin cebirsel düşünme düzeylerinde anlamlı bir artış olduğunu göstermiştir. Böylece, cebir öğretimi deneyinin öğrencilerin cebirsel düşünmelerinin gelişimi üzerinde olumlu bir etkisi olduğu sonucuna varılmıştır.

Anahtar Kelimeler: Cebir, Cebirsel Düşünme, Öğretim Deneyi, Ortaokul Öğrencileri

To myself…

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

ABBREVIATIONS	
Ministry of National Education	MoNE
National Council of Teachers of Mathematics	NCTM
Chelsea Diagnostic Algebra Test	CDAT
Concepts in Secondary Mathematics and Science Team	CSMST

CHAPTER 1

INTRODUCTION

Thinking is an innate human trait that distinguishes us from other living and non-living things (Fisher, 2005). We have to think in order to solve a problem or an issue that we come across in our daily lives. Burton (1984) defined thinking as a tool used to be aware of and control what is happening around us. Every conscious person thinks when making decisions, explaining situations or events, solving problems, or making predictions. In this way, thinking becomes indispensable in every moment of life when done properly and effectively. It enables constructions to develop life in a good way. Mathematics is also known to improve thinking. Mathematics is a science that aims to create and develop thinking skills (Samo & Kartasasmita, 2017). At the same time, mathematics is a language of thinking (Umay, 1992). In other words, mathematics is a way of thinking that we use consciously or unconsciously in our daily lives. This way of thinking can be called mathematical thinking.

There is no single definition of mathematical thinking. According to Henderson et al. (2002), mathematical thinking is the use of mathematical techniques, concepts, and processes to solve problems or questions. Burton (1984) defined mathematical thinking as the function of known mathematical dynamics, processes, and certain operations to understand and control events in the environment. According to Alkan and Güzel (2005), mathematical thinking is a form of thinking with a quality whose usefulness and efficiency can be measured in solving problems and satisfying needs. In other words, since mathematical thinking is used consciously or unconsciously to solve problems at every stage of life, it is a way of thinking that can be used not only in places where mathematical concepts and expressions are present but also in everyday life. This is because people try to solve problems by using thinking in all areas of life (Biltze, 2003).

The ability to use mathematical thinking to solve real-life problems is a goal of mathematics education and is also important for learning and teaching mathematics (Stacey, 2006). In the Principles and Standards published by the National Council of Teachers of Mathematics (NCTM) (2000), the importance of mathematical thinking is strongly emphasized. In fact, the Turkish mathematics curriculum states that mathematics is part of life and that mathematical thinking should be developed and applied to find solutions to problems encountered in everyday life (Ministry of National Education, [MoNE], 2018). With the increasing need for mathematics in daily life, there is a greater need for mathematical thinking in professional fields (NCTM, 2000). In other words, mathematical thinking is not only a way of thinking used by mathematicians but also a way of thinking that people in all professional groups should use (Alkan & Güzel, 2005). In other words, an individual should have the ability to think mathematically, regardless of his or her field of study and profession.

Mathematical thinking takes different forms according to different branches of mathematics, such as arithmetic, algebra, and geometry (Carroll, 1994). Algebraic thinking, one of these forms, can be seen as a subset of mathematical thinking. Algebraic thinking involves making connections between algebraic relationships by using symbols to describe concrete, semi-concrete, or abstract concepts in algebraic relationships, uncovering ideas by using different and multiple representations, and drawing conclusions through the process of reasoning (Kaya & Keşan, 2014). Students use algebraic thinking when they can transfer an idea from concrete to abstract situations and go beyond small numbers (Vance, 1998). Algebraic thinking is used as a tool for learning algebra, but it is also used to develop students' mathematical understanding (Kamol & Har, 2010). However, algebraic thinking is not limited to learning algebra because it also includes skills such as problem-solving, using multiple representations, and reasoning (Çelik, 2007). Therefore, we can say that algebraic thinking encompasses the fundamental skills required for mathematical proficiency.

It is crucial for teachers to understand how students think and reason algebraically as they generate solutions to problems. Understanding how students' thinking and reasoning develop can improve students' learning of mathematics (Kamol & Har, 2010). An educational activity that fails to develop students' thinking misses its main purpose. As a matter of fact, the algebra learning area in the mathematics curriculum is designed to foster algebraic thinking (MoNE, 2018). In other words, it is intended to support the development of students' algebraic thinking. Given the importance of algebraic thinking, this study aimed to investigate the development of students' algebraic thinking and to document how the teaching experiment supported this development.

1.1 Problem Situation

Mathematics has been adopted as a purpose or a tool to make sense of life and to find solutions to problems that arise since the beginning of life, so it is not just a fundamental subject in school. Although the teaching of mathematics, which has an abstract structure, is done first with concrete structures and operations, it involves abstract thinking because it is a mental system (Umay, 1996). In addition to this, mathematics, which is an abstract science, reaches its meaning through algebra, which is one of the sub-learning areas of mathematics and requires abstraction (Altun, 2013). In other words, as algebra requires abstract thinking, it acts as a language between mathematics and other disciplines (Erbaş et al., 2009).

Different definitions have been given to the concept of algebra, which is a sub-learning area of mathematics. MacGregor and Stacey (1999) stated that algebra is an element of mathematical language that aims to specify the relationships between numbers. Dede and Argün (2003) stated that algebra is a language, a school subject, and a tool for problem-solving and thinking. O'Bannon et al. (2002) defined algebra as the language of patterns, rules, and symbols. Cai et al. (2011) described algebra as the ‘gatekeeper’ of mathematics education. Given the importance of algebra for mathematics, the NCTM (2000) emphasized that it is imperative for every student to learn algebra and outlined principles and standards for improving students’ learning of algebra from preschool through high school. In this way, the teaching of algebra can be said to be at the forefront of mathematics education. Indeed, the NCTM (2000) emphasizes the importance of algebraic competence in the adult world and higher education. That is, algebra should not be seen only as a course but as a tool for finding solutions to problems encountered in everyday life (Akkaya, 2006). Therefore, the knowledge and skills that students acquire in algebra also play a vital role in their daily lives.

However, difficulties are also experienced in learning algebra, where abstract structures are used (NCTM, 2000). Numerous studies have reported that students struggle with understanding algebra and algebraic concepts (Burton, 1988; Dede & Argün, 2003; Yenilmez & Avcu, 2009). The reason for these difficulties is the inadequacy of the content, learning, and teaching of algebra (Dede & Peker, 2007). In order to eliminate the difficulties that students may encounter in learning algebra, it is necessary to prepare a foundation for algebra from an early age and to develop their understanding of algebra. The development of algebraic understanding takes time, and algebraic thinking should be developed in the early years of primary education (Carpenter & Levi, 2000). Also, to cultivate an appreciation for algebraic thinking, it is essential to develop an understanding of algebra that highlights its importance (Chazan, 1996). That is, algebraic thinking is considered a crucial component of algebra (Trybulski, 2007). Improving students’ knowledge and skills in learning algebra positively impacts the development of algebraic thinking (Kaya & Keşan, 2014). In this study, a teaching experiment is designed to improve students’ knowledge and skills in learning algebra for seventh-grade students and to explore how effectively it affects students’ algebraic thinking.

1.2 Statement of the Purpose and Research Questions

The purpose of this study was to investigate the development of 7th-grade students' algebraic thinking during the teaching experiment.

The research questions of the study were as follows:

1. What is the change in algebraic thinking levels of 7th-grade students before and after participating in the teaching experiment?
2. How does the algebraic thinking of 7th-grade students evolve during the teaching experiment?

1.3 Significance of the Study

Algebra is one of the content standards in mathematics. Algebra acts as a gateway for students to think abstractly (Witzel et al., 2003), and according to Piaget (1965) by the end of the 11th year, around age 12, students typically reach the stage of abstract operations. Since children can think abstractly during this period, it is likely that their algebraic thinking will also develop. Since the period of abstract operations coincides with the period when students are in 7th grade, this study focused on working 7th graders. At the same time, algebra is the focus of mathematics learning at every grade level, as it is a way of expressing mathematics (Lacampagne, 1995). Indeed, there is a need to improve the teaching and learning of algebra as it is the foundation for mathematics at the high school and university levels (Rakes et al., 2010). However, algebra is not limited to mathematics; it also has a place in everyday life. The existence of algebra in everyday life requires the development of an understanding of it (Williams & Molina, 1997). Similarly, people unconsciously use algebraic thinking to analyze the situations they encounter in everyday life (Davidenlo, 1997).

Algebraic thinking involves using mathematical symbols and tools to analyze and uncover information in different situations. It includes expressing findings through writing, tables, graphs, equations, and diagrams, and interpreting this information through mathematical insights such as solving for unknowns, checking assumptions, and discovering functional relationships (Herbert & Brown, 1997). The development of algebraic thinking skills is related to students' education in the area of algebra sub-proficiency and their knowledge, skills, and experiences (Yenilmez & Teke, 2008). However, according to the research, it has been found that students have difficulties and misconceptions in understanding the concepts of algebra such as patterns, equality, algebraic expressions, variables, unknown, equations and they have misconceptions (Dede, 2004; Ersoy & Erbaş, 2005; Knuth et al., 2006; Kocamaz & İkikardeş, 2021; MacGregor & Stacey, 1993). Since the development of algebraic thinking affects the development of mathematical thinking, it is necessary to create a classroom environment that supports students' algebraic understanding and algebraic thinking.

It is thought that the worksheets will play an effective role in this process, as it is necessary to focus on the teaching that will enable the development of students' algebraic thinking in the abstract operations stage. The literature review shows that the worksheets help teach concepts to the students, assess the level at which these concepts are understood, promote active learning, ensure that educational objectives are met, and connect mathematics to real-life situations (Ev, 2003). In addition, the worksheets help students to define and analyze important mathematical issues while guiding teachers to identify students' misconceptions and misunderstandings (Toumasis, 1995). As the worksheets make the lesson more interesting, fun, and enjoyable, they help to motivate students towards the lesson (Aktepe, 2012). Since algebra is a subject that students find difficult, it is believed that the worksheets can develop student's understanding of algebra and algebraic thinking. In this study, worksheets will be prepared using open-ended questions. Indeed, Fadhillah and Toyib (2024) found that worksheets based on an open-ended approach for the algebra subject effectively improved students' reasoning skills, and they argued that the open-ended approach improves mathematical reasoning skills. Therefore, this study supports the idea that preparing worksheets using the open-ended approach will improve algebraic thinking. Additionally, worksheets can positively influence students' communication with one another, as they are suitable for group work (Tan, 2008). In group work, students can collaborate to understand the activity or problem, use the given information to develop a solution-oriented strategy, and validate their solutions. Şengül and Tekcan (2023) suggested using group work to overcome students' difficulties in learning algebra and to improve their algebraic thinking skills. Gelici and Bilgin (2007) found that cooperative learning based on group work is effective in students' algebra learning. Therefore, in this study, students will work in groups while solving the worksheets prepared using an open-ended approach.

In this study, worksheets will be used to design teaching content aimed at investigating and developing the algebraic thinking of 7th-grade students. The teaching experiment is the research method by which we can most easily observe this development. While the teaching experiment is accepted as learning and change, it seeks answers to the questions of how change can occur in students' knowledge, skills, and experiences, i.e., in their existing mathematical schemata, and how these changes can be expressed (Steffe & Thompson, 2000). For this reason, the aim of this study is to investigate the development of 7th-grade students' algebraic thinking during the teaching experiment.

1.4 Limitations

This study is limited to 7th-grade students attending a public secondary school in Niğde during the 2023-2024 academic year. It focuses on algebraic activities aligned with specific grade-level outcomes. Since the study is confined to the algebra learning area, its findings may not be generalizable to other areas of mathematics. Since the researcher intervenes in the process in the role of both teacher and researcher, there is no outside eye for observing the process.

1.5 Assumptions

It was assumed that the students participated in this study honestly and impartially, and that the researcher maintained objectivity throughout the execution and interpretation of the study.

1.6 Definition of Key Terms

Algebra: It is the language of patterns, rules, and symbols that aim to determine the relationships between numbers, that is, it is an element of mathematical language and a tool for problem-solving and thinking (Dede & Argün, 2003; MacGregor & Stacey, 1999; O'Bannon et al., 2002).

Algebraic Thinking: It focuses on understanding relationships rather than numerical results. It involves making generalizations about numbers and operations, working with numbers and symbols (such as unknowns, equations, and parameters), recognizing the significance of the equal sign, and exploring the concepts of patterns and functions (Kieran, 2004; Van de Walle et al., 2013).

Mathematical Thinking: It is the application of mathematical techniques, concepts, and processes in solving problems or issues (Henderson et al., (2002).

Teaching Experiment: It seeks answers to the questions of how a change can occur in students' knowledge, skills, and experiences, that is, in their existing mathematical schemas, and how these changes can be expressed (Steffe & Thompson, 2000).

CHAPTER 2

LITERATURE REVIEW

This chapter includes a literature review of algebraic thinking, algebraic thinking components, teaching experiments, and related studies on algebraic thinking, teaching experiments, and both.

2.1 Algebraic Thinking

Looking at the literature, it becomes evident that there is no universally accepted definition of algebraic thinking. This is because various researchers have defined it by focusing on different aspects, reflecting the complexity and multifaceted nature of algebraic thinking. Moreover, since limiting algebraic thinking to a single definition would vulgarize its actual value in mathematics, different definitions have been proposed by researchers. According to Blanton and Kaput (2004), algebraic thinking is a routine of mind that interpenetrates mathematics, in which students form, justify, and state assumptions for mathematical constructions and relationships. Kamol (2005) stated that since algebraic thinking is a section of mathematical thinking, it is a way of thinking that makes use of the mathematical thinking skills of reasoning, representation, and problem-solving to understand algebra. According to Ranford (2011), algebraic thinking is not about whether or not to use symbols because algebraic thinking is concerned with indefinite quantities that are designed by reasoning in specific ways. Lawrence and Hennessy (2002) claimed that, in general, algebraic thinking is all understanding that enables us to have an opinion about the world by converting information into mathematical language to guess and clarify events.

According to Kieran (2004), in order to think algebraically, it is necessary to base relations instead of numerical results, to deal with basic operations and their inverses, to be able to represent a question while solving it, to be able to work together with numbers and letters, for example, to be familiar with unknowns, equations and parameters, and to pay attention to the meanings of the equal sign. Van de Walle et al. (2013) stated that algebraic thinking includes making generalizations about numbers and their basic computation, shaping them using symbols, and discovering the concepts of patterns and functions.

Lins (1992) characterized algebraic thinking in terms of three types of thinking: arithmetical, intrinsic, and analytical. According to Lins, algebraic thinking is the method of putting in order the world by modeling events and orienting these models according to these types of thinking. Blanton (2008) emphasized that improving and stating generalizations using arithmetic and defining numerical and geometric patterns to determine functional relationships are two main points of algebraic thinking. Carpenter and Levi (2002) focus on two topics as the basis of algebraic thinking: generalization and the use of symbols to represent mathematical ideas and solve problems. According to Carpenter and Levi, generalization and formalization are the basis of algebraic thinking, where students uncover important and powerful mathematical ideas and relationships and build unifying ideas and representations.

Kriegler (2008) divided algebraic thinking into two main components: mathematical thinking tools and basic algebraic ideas. Mathematical thinking tools are also divided into three: problem-solving skills, representation skills, and quantitative reasoning skills. Basic algebraic ideas are also divided into three: algebra as generalized arithmetic, algebra as a language, and algebra as a tool for functions and mathematical modeling. Kaput (2008) divided algebraic thinking into two basic aspects. The first is algebra as the systematic expression of generalizations with traditional symbolic systems, and the second is algebra as reasoning and actions of generalizations expressed in traditional symbolic systems. Also, Kaput (2008) identified three strands containing these two core aspects. These are generalizing to arithmetic and quantitative reasoning, functional thinking, and modeling.

National Council of Mathematics Teachers (NCTM) (2000) explained that algebra education from kindergarten to 12th grade should develop students' understanding of patterns, relations, and functions, and their ability to represent and analyze using symbols, while at the same time, this education needs to provide for students to utilize mathematical models to make sense of quantitative relationships, and to analyze the alteration in contexts. What NCTM emphasizes for algebra training positively affects the development of algebraic thinking. As a matter of fact, Ceyhan (2012) found that increasing algebra achievement positively affected algebraic thinking and increased students' algebraic thinking levels.

Kriegler (2008) stated that algebraic thinking has turned into an inclusive expression for mathematics teaching and learning, which uses critical thinking skills to help students achieve success in the field of algebra learning. In order to develop and maximize students' algebraic thinking, there is no need to change the curriculum because this can be done by changing the teaching methods (Lawrence & Hennessy, 2002). The methods chosen in teaching algebra affect students' mental activities and ensure the effective and lifelong development of their algebraic thinking skills (Kaya & Keşan, 2014).

2.1.1 Algebraic Thinking Components

In this study, the components of algebraic thinking were formed by synthesizing various studies (Blanton, 2008; Kaput, 2008; Van de Walle, 2013). Soycan (2023) and Ayber (2017) created algebraic thinking components by drawing from a variety of similar studies. In this study, the components of algebraic thinking were shaped by these studies. Table 2.1 provides a detailed description of these components.

Table 2.1 Algebraic Thinking Components

Generalizing Arithmetic and Quantitative Reasoning		
Properties of the number system	Generalizing basic operation properties Generalizing conjectures derived from basic properties Generalizing the properties of odd and even numbers	
Meaningful use of symbols	The meaning of the equal sign and relational thinking	
	Meaning of variables	Using of variables as unknowns Using of variables as varying quantities
Quantitative reasoning	Establishing the relationship between quantities and interpreting and analyzing it	
Functional Thinking		
Repetitive patterns	Define and expand the repeat unit Finding the general rule of the pattern	
Growing patterns	Number patterns	Analyze and expand the pattern Recursive relationship Finding the general rule of the pattern
	Shape patterns	Analyze and expand the pattern Recursive relationship Finding the general rule of the pattern
Modeling		
Multiple representations	Context Table Verbal description Symbols Graphs	

2.1.1.1 Generalizing to Arithmetic and Quantitative Reasoning

Algebra and arithmetic are closely related because algebra builds upon the foundations of arithmetic, and arithmetic provides opportunities for symbolization, generalization, and the development of algebraic thinking, facilitating the transition from basic numerical operations to more complex algebraic concepts (Van Amerom, 2002). Thus, students should make sense of the arithmetic ideas they developed in primary school together with their algebraic ideas (Herscovic & Linchevski, 1994). The development of children's thinking processes through generalization, that is, the development of their algebraic thinking is emphasized by many researchers (Blanton, 2008; Mason et al., 2005; Van de Walle et al., 2013). Mason (1996) emphasized that generalization is at the center of mathematics education. On the other hand, Kaput (2008) stated that central to algebraic thinking is the complex symbolization process that allows for generalization and reasoning about it. The generalization skill is important for algebraic thinking as it plays a role in the development of algebraic knowledge and skills by revealing the mental processes of the students. Generalizing to arithmetic and quantitative reasoning involves generalizing arithmetic operations and their properties, generalizing the properties and relationships of the number system, and reasoning about them (Kaput, 2008). Also, generalized arithmetic, which is a base of algebra, focuses on the general, process that transforms objects into what they are (Mason, 1996). In this study, generalizing arithmetic and quantitative reasoning is categorized under three subheadings: properties of the number system, the meaning of symbols, and quantitative reasoning.

Through generalized arithmetic, students recognize, make sense of, and justify patterns in the properties and operations of the number system (Blanton, 2008). In addition, Blanton emphasizes that there is more than one way to generalize arithmetic. For example, when the results of operations 27+12 and 12+27 are the same, and students calculate a series of operations like this, they can generalize and express the commutative property of addition as a+b=b+a symbolically. Students can generalize the distributive property in multiplication, additive identity property, additive inverse property, and so on arithmetic properties. Also, generalizing operations with odd and even numbers is pretty much a goldmine for algebraic thinking. Students make the transition from arithmetic to algebraic thinking by analyzing, conjecturing, justifying, and symbolizing situations related to numbers and operations (Blanton, 2008).

The correct use of the meaning of the equal sign in the given situations allows students to generalize and justify mathematics concepts (Carpenter et al., 2003). The equal sign is used with different meanings in arithmetic and algebra. Kieran (1981) states that the equal sign means "do something signal" in arithmetic, that is, the result comes after the equal sign, while in algebra it means a symbol for equivalence, that is, it is perceived as a "relational symbol". The equal sign is the key point of relational thinking. Since the equal sign has an important effect on relational thinking, students who can develop a relational perspective towards the equal sign can conceptualize the meaning of this concept and thus ensure correct use (Stephens, 2006). Students have misconceptions about understanding the equal sign, and these misconceptions restrict their understanding, representation, and use of basic arithmetic ideas, while also creating difficulties in learning algebra (Carpenter et al., 2003). Thus, understanding the meaning of the equal sign conceptually and using it appropriately in the given situations affects the development of algebraic thinking.

The use of variables in generalizing arithmetic is another important issue of algebraic thinking. For students to effectively use variables in their generalizations, they need to understand the various meanings of variables and be proficient in their different applications. According to Van de Walle et al. (2013), the variable has two meanings. These are variable as unknowns and variable as quantities that vary. For example, in the equation $3y-16=20$, since y has a single numerical value, it has an unknown meaning. On the other hand, since x and y represent a set of values in the $2y=4x-10$ function, it means variables as quantities that vary. While the concept of variable is important for the transition from arithmetic to algebra, it is the focal point for middle and high school mathematics learning and teaching (Schonfeld & Arcavi, 1999). In addition, the concept of variable has a central importance in the teaching of algebra subjects.

Belue (2015) highlighted that studies on quantitative reasoning in the literature, show that it positively impacts students' ability to generalize correctly. Quantitative reasoning involves the process in which a student conceptualizes a situation, identifies the relevant quantities, and analyzes the relationships between them (Moore & Carlson, 2012). While quantitative reasoning emphasizes the relationship between quantities, algebraic thinking transfers this relationship to algebraic representations, underscoring the connection between the two (Uyguç, 2023). Therefore, quantitative reasoning plays a crucial role in bridging arithmetic and algebra, providing a foundation for the development of algebraic thinking. (Ellis, 2007).

2.1.1.2 Functional Thinking

One of the forms of algebraic thinking involves functional thinking (Blanton & Kaput, 2004), which focuses on the relationship between quantities and allows studies in the field of algebra (Tanışlı, 2011). For this reason, functional thinking plays an important role in the development of algebraic thinking. Functions are at the center of functional thinking and there is a strong connection between them (Blanton, 2008; Vollrath, 1986). The acquisition of functional thinking depends on learning the concept of function, which is important in the development of algebraic thinking. Functions are touched upon with patterns at the primary and secondary school levels. The concept of pattern is important to support the development of functional thinking at an early age. (Kabael & Tanışlı, 2010; Warren & Cooper, 2008). Students' ability to identify and extend the patterns, as well as express the general rule of those patterns using symbols, fosters their understanding of functional relationships (Çayır & Akyüz, 2015). This engagement with patterns enhances students' functional thinking.

Patterns can differ based on their structure and presentation leading researchers to categorize them in various ways. In this study, patterns are classified into two main groups: repetitive and growing. Repetitive patterns consist of a sequence in which a group of items is repeated as the pattern progresses, characterized by a repeating cycle of items known as the repeat unit (Warren & Cooper, 2006; Zaskis & Liljedahl, 2002). Liljedahl (2004) defines the repeat unit in repetitive patterns as a small part of the pattern. For example, in the pattern 1 2 1 2 1 2 ..., the repeat unit is 1 2, and it represents the smallest part of the pattern. Warren and Cooper (2006) mentioned that there are different and various ways to create repetitive patterns. For example, 4 5 6 4 5 6 4 5 6, re mi do re mi do re mi do, and so on. On the other hand, growing patterns are patterns in which the relationship between terms increases or decreases systematically according to a certain rule (Warren, 2005a). The relationship between consecutive terms in growing patterns can be formed by adding or subtracting a fixed number or shape, changing it in a ratio, and increasing the differences between the terms (Karaz, 2021). In this study, growing patterns will be represented using numbers and shapes. For example, 8, 14, 20, 26, 32, …, 4, 7, 11, 16, 22, … and 6, 36, 216, 1296, … are examples of the number pattern. The following are examples of shape patterns (Karaz, 2021).

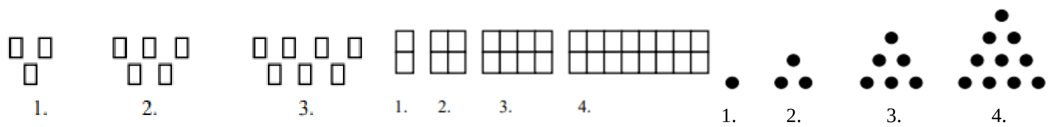


Figure 2.1 Examples of Shape Patterns

Since identifying the pattern, maintaining this pattern, and finding the general rule allow students to think functionally and relationally, repeating patterns and growing patterns provide an early development of functional thinking (Warren & Cooper, 2006).

2.1.1.3 Modeling

The concept of representation is the form in which a mathematical concept and a relationship come together at one point (NCTM, 2000). Students' ability to learn a subject meaningfully is related to their ability to express knowledge with different forms of representation (Akkan et al., 2016). It can be said that students who use multiple representations are more efficient and successful in solving problems by forming and using different methods than those who do not (McGrowan & Tall, 2001). The use of multiple representations in problem-solving should be highlighted in all areas of mathematics as it provides students with different ways of thinking and facilitates their understanding of the problem and mathematical concepts (Erbaş, 2005). At this point, the importance of using multiple representations in algebra teaching should be emphasized.

Representations allow students to develop their ideas about algebra (NCTM, 2000). The use of multiple representations in algebra teaching has the opportunity to make this teaching process more useful and valuable (Friedlander & Tabach, 2001). At this point, it will be beneficial for students to use multiple representations in algebra learning. That is because multiple representations contribute to students' understanding by increasing their efficiency in learning algebra topics (Özgün Koca, 2001). The use of multiple representations in algebra teaching can show students that algebra does not only consist of operations and help them see different aspects of algebra (Kaya and Keşan, 2014). Teaching algebra using multiple representations would positively affect students' development in algebra and support the development of algebraic thinking (Çıkla Akkuş, 2004). At the same time, multiple representations contribute to the development of algebraic thinking by transforming abstract concepts into tangible, visible data. Thus, the ability to utilize multiple representations and transform between representations is critical to the development of algebraic thinking (NCTM, 2000). According to Van de Walle et al. (2013), multiple representations include context, tables, verbal descriptions, symbols, and graphs. In this study, the development of algebraic thinking will be supported by multiple representations, which serve as a key aspect of the modeling component highlighted by Van de Walle.

2.2 The Teaching Experiment

Experimental research was not sufficient to examine how students learn mathematical concepts and how these concepts develop (Steffe & Thompson, 2000). When examining how students construct mathematical concepts and knowledge, mathematics teaching should also be included (Cobb & Steffe, 2011). At this point, the teaching experiment model, which has been widely used in mathematics education in recent years, will be convenient. The teaching experiment is not only a tool but also a methodology to investigate the nature of students' mathematics learning and the development of mathematical thinking and to seek answers to research questions (Czarnocha & Maj, 2008). The teaching experiment aim to find out what students can think, that is, what passes through their minds and what their mathematical knowledge is - the kind of mathematical knowledge they have (Steffe, 1991). The teaching experiment allows researchers to test the suitability and usability of new teaching methods and techniques (Engelhardt et al., 2004).

The teaching experiment is a conceptual tool that is based on Piaget's clinical interview and fills a gap in the field for studying students' mathematical learning and development (Steffe & Thompson, 2000). The aim of clinical interviews is to try to understand students' existing knowledge and thinking without altering them (Engelhardt et al., 2004). The teaching experiment aims to understand how students' mathematical knowledge changes and develops by influencing their mathematical knowledge (Steffe & Thompson, 2000). While clinical interviews seek to understand students' present knowledge, the teaching experiment seeks to understand how students' knowledge progresses over a long period of time (Steffe & Thompson, 2000). Thus, the teaching experiment goes beyond a clinical interview (Steffe & Thompson, 2000). At this point, what differentiates the teaching experiment from clinical interviews could be that it offers the opportunity to change students' thinking.

In teaching experiments, the models that reveal students' mathematical activities and behaviors represent how we make sense of their mathematical realities (Cobb & Steffe, 2011). At this point, Steffe and Thompson (2000) mention two related phrases: students' mathematics and mathematics of students. "Students' mathematics" means the mathematical realities that students have differently from the teacher-researcher, while "mathematics of students" means the teacher-researcher interpretations of the students' mathematics, that is, their mathematical realities (Steffe & Thompson, 2000). "Students' mathematics" includes what students do and think while working on mathematical activities (Steffe & Thompson, 2000). As a matter of fact, the essential role of the researcher is to model "students' mathematics" (Steffe & Thompson, 2000). "Mathematics of students" is these models created and the change in students' thinking (Steffe, 1988).

The teaching experiment includes a set of teaching episodes and one-on-one interviews that take place over a period ranging from six weeks to two years and put students' explanations of their mathematical behavior into a phraseological pattern (Cobb & Steffe, 2011). A teaching episode involves a teacher, a student or students, an observer, and a method of recording the process (Czarnocha & Maj, 2008; Engelhardt et al., 2004; Steffe & Thompson, 2000). The teaching episodes in the teaching experiment provide opportunities to explore and examine students' mathematical constructions (Cobb & Steffe, 2011). In a teaching experiment, students are often the focus group and students can be worked with in different ways: individually, in groups, or as a whole class (Arslan & Sağlam Arslan, 2016). The teaching experiment allows new and different teaching techniques and methods to be tried out and is more reminiscent of the classroom environment as the number of students increases (Engelhardt et al., 2004).

One of the distinctive features of a teaching experiment is the role of the researcher as a teacher, and in this role, the researcher learns about students' mathematical knowledge and how they construct it (Steffe, 1991). The teacher-researcher can conduct the teaching experiment in accordance with its purpose (Aşık & Yılmaz, 2017). The role of the teacher-researcher in the teaching experiment is to create appropriate situations and types of learning to elicit students' mathematical knowledge and to ask critical questions to reveal how this knowledge is formed (Steffe, 1991). Also, another role of the teacher-researcher is to interpret and analyze the mathematical knowledge that each student reveals after each teaching episode (Cobb, 2000). Since the teaching experiment is a method that covers a long period of time, the interaction between the teacher-researcher and the students is high (Cobb & Steffe, 2011). Steffe and Thompson (2000) point out that this interaction can take two different forms; "responsive-and-intuitive" and "analytical". Teacher-researchers elicit students' reasoning through responsive and intuitive interaction (Steffe & Thompson, 2000). Teacher-researchers move from responsive and intuitive interaction to analytic interaction when they are able to make inferences about students' reasoning and test these inferences (Steffe & Thompson, 2000).

According to Cobb (2000), there are three stages of a teaching experiment. These are the instructional design and planning, teaching episodes, and retrospective analysis. In the instructional design and planning stage, hypotheses are generated in light of the research purpose, and appropriate instructional activities are planned and designed to address these hypotheses. The teaching episodes' stage involves the implementation of the prepared instructional activities in a sequence and the continuous revision of the activities between the teaching episodes (Cobb, 2000). At the stage of teaching episodes, the process is recorded (Cobb & Steffe, 2011). These records are used to analyze students' mathematical development (Cobb & Steffe, 2011) and to revise subsequent teaching episodes, that is prospective analysis and to conduct retrospective analysis (Steffe & Thompson, 2000). Prospective analysis is an analysis that should be carried out quickly and after each teaching experiment, allowing for the development and revision of hypotheses and appropriate adjustments for other teaching episodes (Molina et al., 2007). In other words, prospective analysis is conducted during and between teaching episodes and the data collected is analyzed to identify the shortcomings of the next teaching episodes and to identify the interventions that need to be made so that the necessary adjustments and improvements can be made accordingly (Arslan & Sağlam Arslan, 2016). Retrospective analysis is one of the most significant stages of a teaching experiment and it is even more effort than the teaching episodes (Steffe & Thompson, 2000). Retrospective analysis is the process of analyzing all the data after it has been collected and placing it in a broader theoretical context to determine how students' reasoning emerged and developed (Cobb et al., 2014). In a sense, retrospective analysis is a form of general evaluation at the end of a teaching experiment and is done in order to determine whether the aim has been achieved or not, i.e. to determine the effect of the practices on the students (Arslan & Sağlam Arslan, 2016). It is emphasized that there are no standardized criteria for how these processes involved in the teaching experiment will take place (Steffe & Thompson, 2000). Since the purpose of each teaching experiment is different and this process is shaped according to the perspective of the researcher who designs it, there are differences in the design of teaching experiments.

2.3 Related Studies

2.3.1 Studies on Algebraic Thinking

In the literature, the development of algebraic thinking has been investigated using different learning approaches. Tekcan (2022) investigated the effect of a mastery-centered learning environment on algebraic thinking skills. The study was conducted to investigate how the enriched learning environment within the framework of mastery learning principles affects students' algebraic thinking skills. The participants of the study were twelve 5th-grade students. In the study, six lesson plans were prepared based on Kaput's (1999) five algebraic thinking themes. Since one of the themes covered more than the other, two lesson plans were prepared. These lesson plans were implemented over a period of five weeks. Each of Kaput's algebraic thinking themes was designated as a learning unit. A post-test was given after each unit. The Algebraic Thinking Skills Test was administered before and after the implementation part of the study. According to the results of the study, it was found that the learning environment prepared according to the principles of mastery learning positively affected the students' algebraic thinking skills. On the other hand, Çağdaşer (2008) investigated the effect of the constructivist approach on algebraic thinking. The aim of the study was to document how algebra teaching with a constructivist approach affected students' algebraic thinking levels. The study was conducted using an experimental design, and fifty-five 6th-grade students participated. In this study, algebra teaching with a constructivist approach consisted of 10 activities and covered 10 class periods. The Algebraic Thinking Levels Test was administered at the beginning and the end of the constructivist instruction. As a result of the study, it was found that algebra teaching with a constructivist approach increased the algebraic thinking levels of 6th-grade students.

In addition, Kaş (2010) investigated whether instruction designed with worksheets affects students' problem-solving and algebraic thinking skills. Sixty-three 8th-grade students were the participants of the study. The model of the study was a quasi-experimental model with a pre-test and post-test control group. Algebraic Problem Solving Skills Test, Algebraic Thinking Level Determination Test, and Mathematics Problem Solving Attitude Scale constitute the data collection instruments of the study. While traditional instruction was used for the control group, worksheet instruction was used for the experimental group. Five worksheets were used in the worksheet instruction. As a result of the applied instruction, it was found that the experimental group improved their problem-solving and algebraic thinking skills compared to the control group. Also, Girit and Akyüz (2015) investigated the reasoning and solution strategies of students across different grade levels when generalizing patterns. The purpose of the study was to identify students' conceptions of generalizing patterns during the stage when algebraic thinking begins to develop. A total of 154 students from grades 6 to 8 participated in the study. All students were given a pattern test, and two students from each grade level were selected for an activity-based interview. The findings revealed that to enhance their algebraic thinking, students need to improve their understanding of the concept of variables and work with patterns presented with different representations.

There are studies in which different learning approaches were used to investigate the development in 7th graders' algebraic thinking. Palabıyık (2010) investigated the effect of pattern-based algebra instruction on 7th-grade students' attitudes toward mathematics and algebraic thinking skills. The participants of the study consisted of forty 7th-grade students. The study used a quasi-experimental model with a pre-test and post-test control group. The whole study lasted for six weeks. The control group was taught algebra using the activities in the mathematics curriculum. The experimental group was taught algebra with ten activities designed based on patterns. According to the results of the study, pattern-based algebra instruction positively affected students' algebraic thinking skills. On the other hand, Arabacı (2016) investigated the effect of activity-based algebra instruction on the algebraic learning and thinking of 7th-grade students. The experimentally designed study was conducted with twenty-six 7th-grade students. While the control group received no instruction, the experimental group received activity-based algebra instruction consisting of 8 activities for seven weeks. According to the results of the study, students in the experimental group were more successful than those in the control group. It can be said that activity-based algebra instruction supports the algebraic learning and thinking of 7th-grade students.

Several studies have examined the development of algebraic thinking using multiple representation-based instruction. Kaya (2015) investigated the effect of computer software-supported multiple representation-based algebra instruction on students' algebraic reasoning skills, algebraic thinking levels, and attitudes toward mathematics. The study was experimental research, and pre-tests and post-tests were administered to experimental and control groups. The participants of the study consisted of sixty 7th-grade students. The tests used to collect data in the study were the Chelsea Diagnostic Algebra Test, Algebraic Reasoning Assessment Tool, and

Mathematics Attitude Scale. Students in the control group were taught algebra using the traditional method. Students in the experimental group were taught computer-assisted multiple-representation algebra with activities. According to the results of the study, a significant difference was found between the mean scores of the students in the experimental group on the Chelsea Diagnostic Algebra Test and the scores of the students in the control group. In other words, it can be said that computer software-assisted multiple representation-based algebra instruction improved the algebraic thinking levels of 7th-grade students more than traditional instruction.

Similarly, Moseley and Brenner (1997) investigated how multiple representation-based curriculum instruction affects students' conceptual change toward algebraic thinking. The study used an experimental methodology and conducted clinical interviews before and after the implementation of the multiple representation curriculum. Twenty-seven junior high school students in pre-algebra classes participated in the study. Algebraic variables were taught to the experimental group using the multiple representation-based curriculum and to the control group using the traditional method. The results of the study showed that the students in the experimental group were more likely to show signs of algebraic thinking than the students in the control group. As a result, it can be said that teaching with a multiple representation-based curriculum has a positive effect on students' algebraic thinking. Also, Kusumaningsih et al. (2018) aimed to improve students' algebraic thinking skills with multiple representation strategies using the realist approach. The participants of the study were seventy-two 5th-grade students, and it was a quasi-experimental study with a pre-test and post-test control group. The experimental group was taught multiple representation strategies using a realist approach, while the control group was taught using scientific methods. In light of the results of the study, it was found that the algebraic thinking skills of 5th-grade students who learned with multiple representation strategies improved more than those who learned with scientific methods.

2.3.2 Studies Employing a Teaching Experiment

In the literature, the development of students' mathematical knowledge and skills using the teaching experiment model has been investigated from different perspectives. Walkington (2017) investigated how an teaching experiment affected 8th-grade students' learning of algebra and their interest in algebra. This study was conducted with one hundred and seventy-one 8th-grade students. The teaching experiment examined how students solved algebra problems related to subjects outside of school and how they created and solved their own algebra problems. According to the results of the study, it was concluded that the teaching experiment improved students' understanding of algebra learning and increased their interest in algebra.

While Walkington (2017) examines the development of students' understanding of algebra using the teaching experiment model, there are also studies that specified algebra learning, that is, examine the development of students' understanding of growing patterns. Warren and Cooper (2008) conducted a teaching experiment involving the generalization of growing shape patterns. They worked with 45 students between the ages of 8 and 9. The teaching experiment consisted of two lessons. The first lesson focused on identifying and maintaining how simple growing shape patterns expand, while the second lesson focused on describing and continuing the growing shape patterns for each position. According to the results of the study, it was found that the student's ability to understand the growing shape patterns and to form the general term by understanding the relationship between the pattern and its position increased. Likewise, Warren (2005a) conducted a teaching experiment that examined children's ability to generalize general rules in growing patterns. This study was conducted with 45 children with an average age of 9 years. The teaching experiment was conducted in 2 lessons, and each lesson lasted 1 hour. The lessons focused on continuing growing patterns, finding the desired step in the patterns, and determining the number of the given steps. According to the results of the study, it was found that the children improved in making sense of growing patterns and in forming the general organization of patterns.

Molina et al. (2008) conducted a teaching experiment in which elementary school students were encouraged to use relational thinking to solve true/false number sentences. The aim of the study was to determine the extent to which students used relational thinking and to identify the methods they used to solve true/false sentences. The six-session teaching experiment was conducted over the course of a year with 26 eight-year-old Spanish students. The first two sessions focused on students' understanding and exploration of the equal sign, and the last four sessions focused on students' use of relational thinking. They analyzed the strategies the students used to solve true/false number sentences, focusing on the degree of use of relational thinking. As a result of the study, students' use of relational thinking was classified according to 6 distinct and sequential behaviors. These behaviors are no relational thinking behavior, simple relational thinking behavior, equal relational thinking behavior, one-shot relational thinking behavior, frequent relational thinking behavior, and all relational thinking behavior.

On the other hand, there are studies that examine the development of students' relational thinking with the teaching experiment model. Kızıltoprak (2014) conducted a study examining the development of relational thinking in 5th-grade middle school students. When the study aimed to reveal how this skill develops, a teaching experiment was conducted. A teaching process consisting of eight episodes supporting relational thinking was carried out with six 5th-grade students. The episodes included activities aimed at developing students' relational thinking skills. As a result of the study, it was determined that all students' relational thinking skills improved. The study also concluded that relational thinking enables students to recognize and use the basic properties of numbers and operations. Similarly, Demir (2022) used hypothetical learning trajectories to uncover 7th-grade students' mathematical thinking about algebraic expressions, equality, and equations. In this study, which was designed as a teaching experiment, instructional content was created with twenty-four 7th graders, which lasted six weeks, took place in two stages, and progressed in the form of class discussions. Among the participants, three students were selected, and their progress was under observation. As a result of the study, it was found that these three students were able to perform addition, subtraction, and multiplication operations with algebraic expressions, pattern generalization, conservation of equality, and setting up and solving equations. It was found that the activities implemented throughout the teaching experiment had a positive impact on the development of relational thinking in these students. It was also found that the functional and algebraic thinking of these students developed positively.

As in Demir's (2022) study, there are different studies in which the teaching experiment model affects the functional development of students. However, in these studies, the development of functional thinking was investigated from a different perspective using growing patterns. Miller (2016) investigated students' transition from recursive thinking to functional thinking using growing patterns. The study was conducted with a total of 18 students in the 2nd and 3rd grade levels. The study used the teaching experiment and clinical interviews to examine the role of growing patterns in generalizing functional relations. The teaching experiment consisted of a pre-test, three 45-minute lessons, and clinical interviews. As a result of the study, it was found that students were able to define and express the functional relationship of growing patterns. This shows that students can think functionally, which contradicts the idea that students at this grade level can only think recursively. Likewise, Kulaç (2023) investigated the development of functional thinking of 7th-grade students with a hypothetical learning route prepared with growing shape patterns. This study was conducted with twenty-one 7th-grade students. A teaching experiment model was used to examine the development of 7th-grade students' understanding of functional relations. A functional thinking test was administered at the beginning and end of the teaching experiment. For the teaching experiment, activities appropriate for the purpose of the study were developed and implemented in a total of 8 class periods over four weeks. According to the results of the study, as a result of the teaching experiment, it was found that the extension of shape patterns contributed to the development of 7th-grade students' functional thinking and enabled them to generalize functional rules.

Kızıltoprak (2014) determined that the students perceived the equal sign as a relational symbol in addition to finding the result in his study. Similarly, Deniz (2024) investigated the cognitive processes of students regarding the relational interpretation of equality and variable concepts in the transition from arithmetic to algebra. The participants of the study were four 5th-grade students. The teaching experiment model was used to investigate the students' cognitive processes. According to the results of the study, students' awareness of different interpretations of the equal sign and different representations of variables increased. It was observed that they improved their ability to use different interpretations of the equal sign and different representations of variables effectively.

Kızıltoprak (2014) found that students' perception of the equals sign as a relational symbol facilitated finding the unknown in his study. On the other hand, Marum et al. (2011) examined the development of the use of unknowns with early algebra instruction from a different perspective. They investigated how the effect of the early algebra learning process improved students' ability to use the concept of variable in the sense of unknown. The participants of the study consisted of 301 students with grade levels ranging from 3rd to 5th grade. The classroom teaching experiment model was used in the study. When the results of the study were examined, it was found that students improved their skills in using the unknown meaning of variables and used letters to represent unknown quantities.

2.3.3 Studies on Algebraic Thinking and Teaching Experiment

Limited studies were found in which students' algebraic thinking development was examined using the teaching experiment model. One of these studies was conducted by Store et al. (2010), who found that using multiple representations had a greater impact on the development of algebraic thinking. They sought to identify instructional practices that provide context for the development of algebraic thinking. A 3-day teaching experiment was conducted with twenty-five 5th-grade students. The instructional practices were the use of multiple representations and strategies, the organization of student responses, the promotion of sociomathematical norms for justifying the solution, and the promotion of the norm of sociomathematically meaningful language. According to the results of the study, it can be said that the use of multiple representations and strategies among these practices supports algebraic thinking more. Similarly, Yüce (2022) also examined the development of algebraic thinking using the teaching experiment model. However, she approached the development of algebraic thinking from a different perspective with a teaching process involving growing shape and number patterns. She investigated the algebraic thinking of 4th-grade students. The study was conducted with six 4th-grade students and lasted for four months. Since the aim of the study was to develop students' algebraic thinking with growing shape and number patterns, appropriate early algebra instructional content was created. The instructional content consisted of 14 activities, including 6 number patterns and eight shape patterns. After each activity, individual and focus group interviews were conducted with students according to the context of the activity. This was done in order to better understand the students' algebraic thinking. The data obtained were analyzed according to functional levels of thinking, generalization, and justification strategies, which are components of algebraic thinking in early algebra. According to the results of the study, it was found that the teaching experiment aimed at teaching early algebra was effective in terms of various components of 4th-grade students' algebraic thinking.

2.4 Summary of the Literature Review

Algebraic thinking is a way of thinking that involves basic operations and making generalizations about them, working with symbols, such as unknowns, equations, and parameters, knowing the meaning of the equal sign, exploring the topics of patterns and functions and is a sub form of mathematical thinking (Kamol, 2005; Kieran, 2004; Van de Walle et al., 2013). Kaput (2008) identified three strands for algebraic thinking, which are generalizing to arithmetic and quantitative reasoning, functional thinking, and modeling. Blanton (2008) stated that two main points of algebraic thinking are emphasized: generalizing arithmetic to find arithmetic in algebra and using numerical and geometric patterns to determine functional relationships.

Algebraic thinking components were formed by synthesizing studies of Kaput (2008), Blanton (2008), and Van de Walle (2013). In this study, algebraic thinking components are divided into three subheadings, which are generalization to arithmetic and quantitative reasoning, functional thinking, and modeling. Generalization to arithmetic and quantitative reasoning is divided into three subheadings, which are the properties of the number system, the meaning of symbols, and quantitative reasoning. The properties of the number system include generalizing basic operation properties, conjectures derived from basic properties, and the properties of odd and even numbers. The meaning of symbols consists of the meaning of variables, which includes using variables as unknowns and as varying quantities, and the meaning of the equal sign and relational thinking. Quantitative reasoning includes establishing the relationship between quantities and interpreting and analyzing it. Functional thinking is divided into two subheadings, which are repetitive patterns and growing patterns. Repetitive patterns include defining and expanding the repeat unit and finding the general rule of the pattern. Growing patterns include shape and number patterns, which involve analyzing and expanding the pattern, recursive relationships, and finding the general rule of the pattern. Modeling has one subheading, which is a multiple representation. Multiple representations include context, table, verbal description, symbols, and graphs.

Numerous studies have been conducted to improve the development of students' algebraic thinking. These studies have found that the use of worksheets (Kaş, 2010), developing understanding of the concept of variables and generalizing patterns (Girit & Akyüz, 2015), multiple representation-based teaching (Kaya, 2015; Kusumaningsih et al., 2018; Moseley & Brenner, 1997), pattern-based teaching (Palabıyık, 2010), activity-based teaching (Arabacı, 2016), a mastery-centered learning environment (Tekcan, 2022), and the constructivist approach (Çağdaşer, 2008) positively affect the development of algebraic thinking.

The chosen teaching method impacts students' learning of algebra and the development of their algebraic thinking (Lawrence & Hennessy, 2002; Kaya & Keşan, 2014). At this point, it has been seen that the teaching experiment is effective for developing algebraic thinking (Store et al., 2010; Yüce, 2022). The teaching experiment, which is a new and different version of the clinical interview, is a method that aims to find out what kind of mathematical knowledge students have; that is, it is a method that examines students' mathematical learning and development (Engelhardt et al., 2004; Steffe, 1991; Steffe & Thompson, 2000). While clinical interviews concern students' existing knowledge and thoughts (Engelhardt et al., 2004), the teaching experiment concerns how this knowledge and thoughts change and develop (Steffe & Thompson, 2000). The teaching experiment differs from clinical interviews in a distinctive way. A teaching experiment is a method that involves a series of teaching episodes, a teacher, a student or students, an observer, and a method for recording the process over a period ranging from six weeks to two years (Cobb & Steffe, 2011; Steffe & Thompson, 2000). The primary role of the researcher in a teaching experiment is to model students' mathematics, that is, students' mathematical ideas and thoughts (Steffe & Thompson, 2000). Since there are no standardized criteria for planning a teaching experiment (Steffe & Thompson, 2000), differences can be seen in the design of the teaching experiment for its purpose.

In the literature, several previous research studies have been conducted using the teaching experiment method in the field of algebra learning, both in Turkey and other countries. In these studies, it was found that thanks to the teaching experiment method, students' understanding of algebra learning improved (Walkington, 2017), and their understanding of growing patterns improved (Kulaç, 2023; Miller, 2016; Warren, 2005a; Warren & Cooper, 2008), the use of growing patterns improved their functional thinking (Kulaç, 2023; Miller, 2016), their relational thinking skills improved (Demir, 2022; Kızıltoprak, 2014), their use of variables as unknowns improved (Marum, 2011), and they were able to perceive the equal sign as a relational symbol (Deniz, 2024; Kızıltoprak, 2014). In addition, it was found that the teaching experiment involving growing shape and number patterns (Yüce, 2022) and the use of multiple representations (Store et al., 2010) improved students' algebraic thinking. In the literature review conducted, a teaching experiment study aimed at the development of students' algebraic thinking by using all the algebraic thinking components could not be reached. Therefore, the aim of this study is to investigate the development of 7th-grade students' algebraic thinking during the teaching experiment.

CHAPTER 3

METHOD

This chapter discusses the study's methodology, covering ten subtopics: research design, the design and piloting of the worksheets, participants of the study, data collection tools and procedures, data analysis, the role of the researcher, and the trustworthiness of the study.

3.1 Research Design

In the study, a teaching experiment model was used to examine the development of the algebraic thinking of the students and to allow new plans to develop these. The purpose of the teaching experiment is to directly observe students' learning and reasoning (Steffe & Thompson, 2000). The teaching experiment includes a sequence of teaching episodes and one-to-one interviews conducted over a long period of time (Cobb & Steffe, 2011). The teaching experiment allows us to analyze students' progress throughout the process (Cobb & Steffe, 2011). Thus, this research used the teaching experiment to uncover the algebraic thinking, levels, and developments of 7th-grade secondary school students.

3.2 Designing and Piloting the Worksheets

In this study, the worksheets used during the teaching experiment were prepared, adapted, and aligned with the components of algebraic thinking and the objectives of 7th-grade mathematics. In selecting and adapting the activities in the worksheets a pool of activities was compiled and presented to two experts in mathematics education (Professors in this field). For each component of algebraic thinking, the experts were provided with two different activities, and they selected the most appropriate ones for the corresponding component. Additionally, they reviewed the activities and provided suggestions for potential improvements. The algebraic thinking component and the experts' activity choices are listed in Table 3.1.

Table 3.2 Algebraic Thinking Component and the Experts' Activity Choices

Related Algebraic Thinking Components	Activities	Expert 1	Expert 2	Selected Activity
Properties of the number system	1a	x	x	x
	1b			
The meaning of the equal sign and relational thinking	2a	x	x	x
	2b			
Use of variables as unknowns	3a	x	x	x
	3b			
Use of variables as varying multiplicities	4a	x		
	4b		x	x
Quantitative Reasoning	5a			
	5b	x	x	x
Repetitive Patterns	6a	x	x	x
	6b			
Growing Patterns	7a	x	x	x
	7b			
Multiple Representations	8a	x		
	8b		x	x

The experts agreed on most of the activities but expressed differing opinions regarding two components: "use of variables as varying multiplicities" and "multiple representations". To finalize the activity selection for these components, the researcher sought additional input from the experts. After further discussion, the experts reached a consensus on the activities to be used. Necessary revisions were then made to the selected activities based on the expert feedback received. The content of the worksheets is presented in Table 3.2.

Table 3.3 Content of the Worksheets

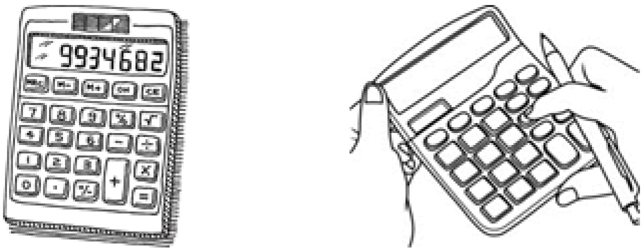
	The Name of the Activity	The Related Algebraic Thinking Components
Teaching Episode 1	The Calculator	Properties of the number system
Teaching Episode 2	The Ali Captain's Ship	The meaning of the equal sign and relational thinking
Teaching Episode 3	Let's Go to the Bazaar	Use of variables as unknowns
Teaching Episode 4	Ali's Shopping	Use of variables as varying quantities
Teaching Episode 5	The Gasoline Tank	Quantitative reasoning
Teaching Episode 6	Box-Penny	Repetitive patterns
Teaching Episode 7	Urban Transformation	Growing patterns
Teaching Episode 8	The Table Organization	Multiple representations

The Calculator activity and Ali's Shopping activity were adapted from "Activity 14.10" and "Figure 14.9", respectively, in the book "Teaching Primary and Secondary School Mathematics with a Developmental Approach" by Van de Walle et al. (2013), with permission from Özmantar, who translated Chapter 14 into Turkish. The Ali Captain's Ship activity was adapted from "Ali Kaptanın Gemisi Etkinliği" in the Master's thesis titled "The Effect of Realistic Mathematics Education on Students' Success on 7th-Grade Topics of Equality and Equations and Student Opinions" written by İşcan (2022). The Let's Go to the Bazaar activity and the Urban Transformation activity were developed and used in the TUBITAK-supported project entitled “A University-School Collaboration Model for Promoting Pre-service Teachers’ Pedagogical Content Knowledge about Students” (Grant No. 215K049) conducted by Hülya Kılıç (2016-2018). In this study, the Urban Transformation activity has been adopted, and the Let’s Go to The Bazaar activity was used as a ready-made resource. The Gasoline Tank activity is adapted from “Problem 3:Tır” as presented in Akın’s (2016) doctoral thesis titled “An Analysis of the Supporting Middle School Students’ Mathematical Literacy Through Strengthening Their Quantitative Reasoning” by Akın (2016). The Box-Penny activity was adapted and developed from the “Kutu-Kuruş Problem” in the book “Matematiksel Akıl Yürütme” by Aydoğan Yenmez and Gökçe (2022). The Table Organization activity was adapted and developed from “Etkinlik 7” in the Post-Instruction Algebraic Thinking Process Evaluation Form in the Master’s thesis entitled “The Effect of Inquiry-Based Instruction Supported by Socio-Mathematical Norms on Students’ Algebraic Thinking Processes” by Köken (2022). The necessary permissions for the activities were provided (Appendix B).

The pilot study was conducted in the same public secondary school in Niğde where the main study was conducted. Adhering to the curriculum plan of the mathematics course, the pilot study started in the week when the topic of algebraic expressions was covered. The implementation period of the pilot study in schools was two months: one week in November, four weeks in December, and three weeks in January. The pilot study involved two groups of students with achievement similar to the participants in the main study, comprising six students categorized as low, medium, and high achievers, based on input from mathematics teachers and the students' course scores. Data collection included videotaped observations and interviews, allowing for an examination of how the teaching experiment influenced the algebraic thinking of students across different achievement levels. As a result of the pilot study findings, various revisions and adjustments were made to the worksheets used in the main study. Below are some of the revisions made based on the pilot study findings:

The Calculator activity (see Figure 3.1) was revised by removing the picture at the beginning to prevent students from using the numbers in the image as hints for answering the questions. The primary goal of this activity is to encourage students to make generalizations. Therefore, the question in option b was restructured to better align with this objective, ensuring a focus on algebraic reasoning rather than relying on visual cues (see Figure 3.2).

CALCULATOR ACTIVITY



Answer the following questions.

a) When you do addition, subtraction, multiplication and division operations separately with any number on a calculator, can you generate the same number on the display of the calculator as you got at the beginning? If so, how?

Is it valid for all numbers? If so, write the algebraic expression.

b) If you cannot use the "0" and "1" keys, can you generate the numbers "0" and "1" on the calculator display? If so, how?

Once you have generated 0 and 1, are there other situations in which you can generate 0 and 1 besides the numbers/methods you used? How many are there? Explain why. Write the algebraic expression for these situations.

Figure 3.1 Options a and b Parts of the Calculator Activity Before the Pilot Study

CALCULATOR ACTIVITY

Answer the following questions.

a) When you do addition, subtraction, multiplication and division operations separately with any number on a calculator, can you generate the same number on the display of the calculator as you got at the beginning? If so, how?

Is it valid for all numbers? If so, write the algebraic expression.

b) If you cannot use the "0" and "1" keys, can you generate the numbers "0" and "1" on the calculator display? If so, how?

Once you have generated 0 and 1, are there other situations in which you can generate 0 and 1 besides the numbers/methods you used? **Can these situations be generalized as an expression? If so, write the algebraic expression.**

Figure 3.2 Options a and b Parts of the Calculator Activity After the Pilot Study

In the Ali Captain's Ship activity, several adjustments were made for clarity and to improve students' understanding. In option b, the phrase "with using an equation" was changed to "with using algebraic expressions" because students were unclear about the term "equation." The revised phrase aligns with their familiarity with algebraic expressions, making the task more accessible. In option c, the phrase "in this port" was added to clarify that the boxes should be moved when the ship is in the port of Cyprus. This change was necessary because one student misunderstood the context. Option e was added because the activity took less time than anticipated, and the additional question ensured that the class time was fully utilized (see Figure 3.3).

e) Ali Captain will leave all his weights when he arrives at the port of Mersin. Ali Captain wants to make another voyage with his ship again. For this voyage he will load 12 blue boxes on the right side of the ship. How and what color boxes should Ali Captain load on the left side of the ship in order to be able to start the voyage?

Figure 3.3 Option e Part of the Ali Captain's Ship Activity After Pilot Study

In the Let's Go to the Bazaar activity, the expression "by setting up an equation" in options b and e was changed to "by using algebraic expressions" because the students did not realize what they should do when they heard the expression "equation." Therefore, it was deemed appropriate to use the familiar word algebraic expression.

In Ali's Shopping activity, two main revisions were made for clarity and better understanding.

- In option b, since the task involved writing algebraic expressions, it was emphasized that option a should be solved without using algebraic expressions.
- During the pilot study, one student struggled with option a, interpreting the question as allowing multiple answers such as "pencil and eraser," "eraser only," or "pencil only." To address this, the question in option a "How many different ways can Ali do this shopping?", was revised for clarity. The new statement is: "Since Ali spent all 35 TL in his pocket to buy pencils and erasers, how many erasers and pencils can he buy and do this shopping in different ways?" This rewording makes the task clearer by specifying that the total amount must be spent, and it focuses on finding the different combinations of items purchased.

In the Gasoline Tank activity, several changes were made after the pilot study to improve clarity and understanding.

- Option d was originally one question, but after the pilot study, it was split into two parts (e and f): Option e asked students to find the algebraic expression, and option f asked them to draw the graph. This separation aimed to break down the task and focus on different skills.

- Since option e required students to write algebraic expressions, a new instruction was added for earlier options, stating that the tasks should be solved without using algebraic expressions. This was meant to help students realize that they could find solutions without necessarily relying on algebraic expressions.
- In the activity, there was confusion regarding the table, as some students misunderstood the data as representing the amount of gasoline filled in a given period of time. To clarify this, the sentence "shows the amount of gasoline (liters) in the gas tank of the truck in this period of time" was underlined, and the table was made more explicit to ensure students interpreted it correctly.
- Additionally, the phrase "per minute" was changed to "in 1 minute", as students found the original terminology unclear. This minor change was intended to make the time-related expressions easier to understand.
- Finally, an extra option was added to the activity because it took less time than initially planned, allowing students to engage further with the material (see Figure 3.4).

b) How many minutes will pass when 125 liters of gasoline are in the tank of this truck? (Solve without using algebraic expressions)

Figure 3.4 The Added Option to The Gasoline Tank Activity After Pilot Study

In option c of the Box-Penny activity, a warning was added stating: "Make sure that the amounts of money in the boxes are arranged in a sequence starting from the first box". This warning was provided to ensure that students recognize the repetitive pattern necessary to solve the question.

During the pilot study, one of the students proposed dividing the box numbers into multiples of two, three, and four while attempting to solve the problem, which led to confusion and made it difficult to identify the correct relationship. To avoid this kind of misunderstanding and ensure that students focus on recognizing the intended repetitive pattern, this warning was included.

In question 1 of the Urban Transformation Activity, the expression “by using the table” was added to make it more understandable. For question 2, option b, the original prompt “What about n years later?” was revised to “Then find how many flats will be in the apartment after n years by using the table” to make the task more specific and easier for students to follow.

In question 3, option a, the expression “What about after “s days?” was replaced with “Find from the table how many triangles will be used to form the flower after s days (Find the pattern rule)” to emphasize the need for students to find the pattern and use the table effectively. Additionally, Additionally, since the activity required less time than anticipated, option b was added to extend question 3 and ensure that the class time was fully utilized (see Figure 3.5).

b) The number of streets and the number of flowers planted according to the flowering work for street arrangement are given in the table below. What is the number of flowers planted on the 5th street? Find the number of flowers planted in "xth Street" from the table. (Find the pattern rule.)

Number Of Streets	Number of Flowers Planted
1	2
2	5
3	8
4	
5	
⋮	⋮
x	?

Figure 3.5 A New Option Added to the Urban Transformation Activity After Pilot Study

In the Table Organization activity, it was noted that option d required students to write an algebraic expression. To prevent students from jumping to algebraic expressions too early, a condition was added that required the questions in the previous options to be solved without using algebraic expressions. This adjustment was made after observing that one student immediately provided the algebraic expression in the first option. Additionally, another option was added to extend the class engagement, as illustrated in Figure 3.6.

c) Are there any chairs that remain unchanged or fixed in this arrangement example?
Are there any chairs that have been changed or added in this arrangement example?
Show them.

Figure 3.6 The Option Added to the Table Organization Activity After Pilot Study

Throughout the teaching experiment, guiding questions were prepared for each worksheet to guide students in completing the activities and to assess their algebraic thinking. After the pilot study, outdated questions were removed and new questions were added based on the adjustments made to the activities in the worksheets. For example, in the Ali Captain's Ship activity, three new guiding questions were formulated for the option added after the pilot study. These questions were:

- Is there a possibility that the color and number of the boxes to be loaded on the left side of the ship will be different from what you found?
- Can yellow boxes be used on the left side of the ship to balance the weight of the blue boxes on the right side of the ship as you did in the previous options?
- Can both yellow and blue boxes be used on the left side of the ship to balance the weight of the blue boxes on the right side of the ship as you did in the previous options?

As another example, in the Let's Go to the Market activity, since the term "equation" was replaced with "algebraic expressions" after the pilot study, a guiding question was modified accordingly. The original question, "Can you explain how you wrote the equation?” was replaced with the question, “Can you explain how you wrote the algebraic expression?”.

In another example, in the Calculator activity, since the students had difficulty in expressing odd and even numbers with symbols during the pilot study, additional guiding questions were added to help them. These questions were:

- What can we do to indicate that the unknowns in an algebraic expression are odd or even numbers?
- Can we use the letter “T” for an odd number and the letter “Ç” for an even number to indicate that the unknowns in an algebraic expression are odd or even?

The final versions of the worksheets used in this study were provided in Appendix C, while the final versions of the guiding questions were included in Appendix D.

3.3 The Context and Participants of the Study

The study was conducted in a public secondary school in Niğde in the spring semester of the 2023-2024 academic year. The school building was old**,** with three floors and 18 classrooms. There were 43 teachers and 528 students at the school. The socio-economic status of the students was relatively uniform, with most families belonging to the middle-class civil servant category. These families were financially stable, not struggling to make ends meet, but not wealthy either. This common socio-economic background likely provided a consistent context for the study.

Six 7th-grade students were included in the study. The study utilized criterion sampling, a type of purposive sampling, to select participants. The criteria focused on 7th-grade students with varying levels of achievement, specifically low, medium, and high. To determine students' achievement levels, input was gathered from mathematics teachers, alongside the students’ scores in mathematics.

- Students categorized as high achievers had mathematics course averages above 80.
- Those classified as medium achievers had averages between 60 and 80.
- Students identified as low achievers had averages ranging from 40 to 60.

The rationale for selecting students with different achievement levels was to examine how the teaching experiment influences and develops algebraic thinking among students at these various levels. Participation in the study was voluntary, ensuring the continuity of the research and the active involvement of all students. The individual characteristics of the participants in the study are presented in Table 3.3.

Table 3.4 Individual Characteristics of the Participants

Participant	Gender	Age	Mathematics Achievement Level
Melek	Female	13	High
Zülal	Female	13	Medium
Selin	Female	13	Low
Cem	Male	13	High
Güney	Male	13	Medium
Emre	Male	13	Low

Participants worked in groups of 3 in the teaching experiment. The groups were formed by considering the participants' math achievement, i.e., there were participants with low, medium, and high math achievement in one group. The group is heterogeneous in terms of mathematics achievement. The students participating in the study were given pseudonyms. The participants in the first group were Cem, Güney, and Selin, while the participants in the second group were Melek, Zülal, and Emre.

In the mathematics course, students were taught all algebra topics according to the 7th-grade mathematics curriculum (MoNE, 2018). The algebra topics covered included Algebraic Expressions and Equality and Equations, and these topics were completed in the fall semester in line with the curriculum plan. Based on this planning, the implementation period of this study was set for two months, spanning three weeks in February, four weeks in March, and one week in April.

3.4 Data Collection Tools and Procedures

In the study, a teaching experiment with algebraic thinking components was planned and implemented, and the effectiveness of algebraic thinking of 7th-grade students was investigated. The study involved planning and implementing a 2-month teaching experiment. The data collection tools used in the teaching experiment conducted in this study were the Chelsea Diagnostic Algebra Test (CDAT), worksheets, guiding questions and video recordings.

3.4.1 Chelsea Diagnostic Algebra Test (CDAT)

The CDAT was used at the beginning and end of this 2-month teaching experiment. The CDAT was developed by the Concepts in Secondary Mathematics and Science Team (CSMST) (Hart et al., 1985; 1998) to determine the algebraic thinking levels of British children aged 13-15. CSMST has set criteria for constructing and analyzing the test and according to these criteria; students use 6 different ways to interpret the letters. Küchemann (1998) describes these ways as follows.

1. Letter evaluated: The letter is given a numerical value. For example, if u=v+1 and v=1, what can be said about u?
2. Letter not used: The letter is ignored or recognized without attaching any meaning to it. For example, if n-246=762, what is n-247=?
3. Letter as an object: A letter can be seen as an object in itself or as an abbreviation for an object. For example, what is 2a+5b+a=?

4. Letter as a specific unknown: When operating directly with letters, they are used as a particular but unknown number. For example, if you add 4 to 3n, what will it be?
5. Letter as a generalized number: It is recognized that there are cases where the letter can take on more than one value. For example, if L+M+N=L+P+N, is this always true?
6. Letter as a variable: The letter is understood to represent more than one unstated value, and the existence of a systematic relationship between these value sets is accepted. For example, is 2n or n+2 larger? Explain.

The whole test was adapted into Turkish by Çıkla Akkuş (2004). While the test consists of 22 items, taking sub-items into consideration, there are 55 items in total. When scoring the test, correct answers are scored as 1, and incorrect answers are scored as 0. In this way, the lowest score to be obtained from the test is 0 and the highest score is 55. The initial Cronbach's reliability of the test was 0.70. According to Çıkla Akkuş (2004), three research assistants from the Department of Mathematics Education and a mathematics teacher were involved in translating the test into Turkish. In the translation process, the conformity of this translation to the English version was checked by mathematicians, and its form and content validity was assessed by a mathematician and two mathematics teachers. 120 seventh-grade students participated in a pilot study and the KR-20 reliability coefficient of the test was calculated as 0.93. The discrimination power of the items is between 0.20 and 0.60 and the item difficulties vary between 0 and 0.94. This test was included in Appendix A. The necessary permissions for the use of this test were obtained (Appendix B).

The six criteria determined for constructing CDAT according to the CSMST were named the dimensions of the CDAT. The algebraic thinking components to which the activities are related (see Table 3.2) are called activity dimensions. The relationship between the test and the activity dimensions is given in Table 3.4.

Table 3.5 The Relationship Between CDAT and Activities

		Dimensions of Activities							
		Properties of the number system	The meaning of the equal sign and relational thinking	Use of variables as unknowns	Use of variables as varying quantities	Quantitative reasoning	Repetitive patterns	Growing patterns	Multiple representations
Dimensions of CDAT	Letter evaluated		✓	✓					
	Letter not used								
	Letter as an object	✓		✓					
	Letter as a specific unknown	✓	✓	✓		✓		✓	✓
	Letter as a generalized number	✓				✓		✓	✓
	Letter as a variable				✓				

3.4.2 Worksheets

During the teaching experiment, eight worksheets were used, with students working in groups. The worksheets were created using an open-ended approach, and specific instructions were deliberately omitted due to the nature of the teaching experiment. his approach allowed for a detailed examination of students' algebraic thinking development from different aspects and perspectives. The researcher The researcher provided guidance through targeted questions, offering direction as needed. In addition, since the questions in the worksheet were sequentially connected, they served as a guiding framework for the students.

The teaching experiment took place over eight sessions, each lasting 40 minutes. As the teaching experiment was conducted as an extracurricular activity, the study was carried out with two separate groups, one class hour per week for 8 weeks, during times agreed upon by the students' teachers. At the end of each teaching episode, the worksheets were collected and analyzed during both prospective and retrospective analysis phases.

3.4.3 The Guiding Questions

The guiding questions were prepared to support and direct the students during their group work on their worksheets, particularly at points where they encountered difficulties. In addition, these questions were written in a way to encourage students to provide detailed explanations in areas deemed important by the researcher. By tailoring the questions to the specific content of each teaching episode, they were intended to elicit students' algebraic thinking. The guidance and explanations central to the worksheets were conveyed through these questions (see Appendix D).

3.4.4 Video Recordings

In this study, video recordings were used to document the students' work on the worksheets. The worksheets were applied to the two separate groups over an 8-week period, with each session lasting one class hour. This entire implementation process was recorded with a camera. In a small classroom, the students were seated in a U-shape, with the researcher positioned in the center. This allowed the researcher to closely observe and intervene as needed throughout the process. The camera was positioned to focus on the students' written responses on the worksheets. These recordings were used to monitor and track the students' discussions and to analyze the development of their algebraic thinking during the prospective and retrospective analysis phases.

3.5 Data Analysis

Before the teaching experiment, the hypothesis was first determined in accordance with the research purpose. The extent to which this hypothesis was realized during the teaching experiment was evaluated. Then, the teaching experiment was designed and planned to test this hypothesis. The data were analyzed in two stages: prospective analysis during the study and retrospective analysis after the completion of the study. The prospective analysis included a short and quick analysis of the students' worksheets, interview questions, and video recordings at the end of each teaching episode. As a result of these analyses, adjustments, and improvements were made for the following teaching episode. At this point, adjustments, additions, and improvements were made in the questions to guide students and in the next activity to be implemented. The researcher and the students did not know each other, and the researcher knew very little about their mathematical knowledge. Therefore, in the prospective analysis stage, the researcher made adjustments and improvements in the management of the teaching episodes. These refinements allowed for more student-led discussions among themselves, enabling the researcher to better gauge when to intervene and when to pose guiding questions. The retrospective analysis, on the other hand, is an analysis in which all the data obtained at the last stage, after the completion of the teaching experiment, are evaluated together. At this point, it includes a detailed analysis of the CDAT administered at the beginning and end of the teaching sections, the student worksheets administered throughout the eight teaching episodes, and the video recordings of the entire teaching experiment process.

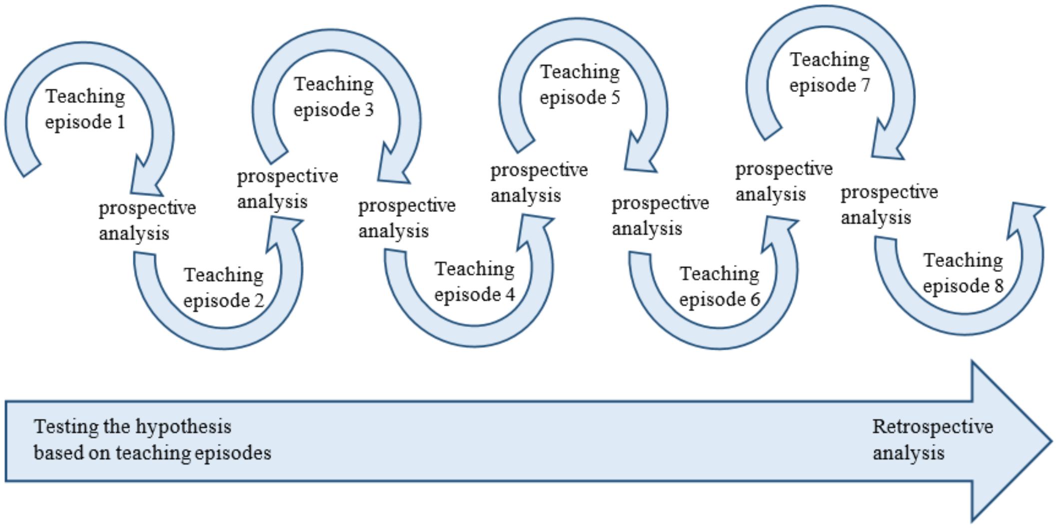


Figure 3.7 Data Analysis Scheme

Quantitative data obtained from the Chelsea Diagnostic Algebra Test were analyzed using the SPSS program. Descriptive statistics, namely mean, median, and standard deviation values, were used to evaluate algebraic thinking levels to analyze the sample's basic characteristics. These descriptive statistics were examined separately for both before and after the teaching experiment. The frequencies of correct and incorrect answers given to the test items were determined, and the correct answer was scored as 1 point and the incorrect answer as 0 points. Since the obtained data set is small, it is suitable for nonparametric test use (Siegel, 1957). The Wilcoxon Signed Rank Test, a nonparametric test designed to be used in repeated measurements, was used in this study. It is used to test the significance of the difference between the scores when the participants are measured under two related conditions (Büyüköztürk, 2023). In this study, the Wilcoxon Signed Rank Test was used to determine whether there was a significant difference in the algebraic thinking levels of the students before and after the teaching experiment. The algebraic thinking level was taken as the dependent variable, and the teaching experiment was the independent variable.

Qualitative data were analyzed within the framework of students' emerging and effective algebraic thinking. For this purpose, descriptive analysis was conducted using algebraic thinking components. Descriptive analysis is defined as the analysis of the data obtained by making interpretations and inferences based on predetermined categories or themes (Yıldırım & Şimşek, 2006). In the study, a literature review on algebraic thinking was conducted first (Blanton, 2008; Kaput, 2008; Van de Walle, 2013), and algebraic thinking components were created. These components are properties of the number system, the meaning of the equal sign and relational thinking, using variables as unknowns and as varying quantities, quantitative reasoning, repetitive patterns, growing patterns, and multiple representations. Then, the data set related to students' algebraic thinking in the algebra teaching experiment conducted in this study was analyzed. Direct quotations were used throughout the process to represent the participants' ideas or thinking.

The qualitative raw data of the study were coded independently by two researchers. The number of "consensus" and "disagreement" in their coding was recorded, and Miles and Huberman's (1994) formula "[Percent of Agreement = Consensus ÷ (Agreement + Disagreement) × 100]" was applied to determine the inter-coder reliability. With an agreement rate of 89%, which exceeds the acceptable threshold of 70%, the coding reliability was deemed sufficient. Table 3.5 provides examples of the analysis process used in the study.

Table 3.6 An Example of Qualitative Data Analysis

Team	Code by the first researcher	Code by the second researcher	Example of dialog	Explanations
The Meaning of the Equal Sign and Relational Thinking	The equal sign as a relational symbol	The equal sign as a relational symbol	*Güney: 6s=8m* *Cem: 6 yellow boxes weigh the same as 8 blue boxes.*	Both researchers coded this dialog in the same way.
	Relational thinking	The equal sign as a relational symbol and Relational thinking	*Güney: The 3 yellow boxes on the left side fall into the sea, so 3 yellow boxes remain here (he is talking about the left side of the ship). How much will be transferred from right to left and what color?* *Selin: It will be transferred in blue.* *Cem: I think 2 blues from the right side should go to the left side.* *Selin: Yes.* *Cem: Because if 2 blues go from here, there will be 2 blues left here (talking about the right side). The yellows had already neutralized each other.*	The second researcher stated that there were also findings regarding the use of the equal sign as a relational symbol in this dialog. The researcher explained that the students' attempts to equalize the weight of boxes on the left and right sides of the ship indicated this.

3.6 The Role of Researcher

In this study, designed within the framework of a teaching experiment, the researcher took an active role in the primary data collection and analysis process, which aligns with the nature of teaching experiment. Therefore, the researcher took part in the teaching experiment with two different roles: as a researcher and as a teacher. The fact that the researcher stayed in the study environment for two months, i.e., this long-term study, supported the researcher's active communication and interaction with the students in a positive way. In this way, the researcher supported the teaching. The researcher reviewed the academic studies in algebra and algebraic thinking, created worksheets, and planned the pilot teaching experiment by identifying teacher actions and strategies to promote productive mathematical discussions. As a result of the pilot teaching experiment, the researcher gained experience in conducting a teaching experiment, and the researcher revised the teaching style and teaching plan and developed the final version of the teaching experiment. In the process of the teaching experiment conducted within the scope of the research, the researcher prepared plans and activities for the realization of the application, asked creative questions to focus students on the purpose of teaching, prepared the appropriate environment to create interaction among students, and provided opportunities for students to express their thoughts by supporting group discussions. In this way, students' algebraic thinking was revealed under the researcher's control in each teaching episode.

3.7 Validity and Reliability

In this study, the teaching experiment method was used. Validity and reliability are two significant issues for qualitative and quantitative research to be trustworthy (Fraenkel et al., 2011). However, the criteria applied to ensure the trustworthiness of qualitative research differ from those used in quantitative research because the bases of qualitative research and quantitative research are different. Since the term validity and reliability are primarily associated with quantitative research, it is possible to enhance the trustworthiness of qualitative research by providing some different criteria. According to Lincoln & Guba (1985), these criteria are credibility, transferability, dependability, and confirmability, which provide the trustworthiness of the research. These criteria, coined by Lincoln and Guba (1985), are used instead of internal validity, external validity, reliability, and objectivity.

Credibility is the researcher’s accurate explanation of the processes that impact what participants think, feel, and do (Lodici et al., 2006). According to Lincoln and Guba (1985), the role of credibility is to enhance the likelihood of the credibility of the findings and to ensure that their credibility is demonstrated by confirming the results. In this study, within the scope of credibility, the researcher interacted with the participants for an extended period and thus presented the facts and situations from the participants’ perspectives. In the study, data triangulation was made using data from many different sources, such as camera recordings taken during the teaching episodes, students’ worksheets, clinical interviews, etc., which supported each other. Expert opinion was obtained about the worksheets and clinical interviews used in the study, and a pilot study was conducted to measure whether they served the purpose.

Transferability is the criteria used to indicate that the study’s findings could be used in different but related research areas (Lincoln & Guba, 1985). In order to fulfill this criterion, the researcher should provide the necessary and detailed information about the research, such as place, time, process, and participants. Within the scope of transferability, the conceptual framework of this study, the process of preparing the teaching experiment, the sample of the research, the design of the process, the implementation process, the analysis process, the role of the researcher, and the result of the study are explained and described in detail.

Dependability is the criteria for ensuring the trustworthiness of the research with the findings of the research to show that the research can be repeated and consistent (Merriam & Tisdell, 2015). Within the scope of dependability, the researcher collected written and oral data from the students throughout the teaching experiment.

To increase the degree of dependability, the researcher recorded the whole data collection process in detail. Since the researcher was also an implementer, she was able to follow the data collection and storage process. During the data analysis process, experts were consulted, and their feedback and comments were taken into account to achieve consistent results.

The conclusions and findings of the research should be based on the data source rather than on the researcher's bias, personal views, or opinions (Lincoln & Guba, 1985). In this way, the confirmability of the research is ensured. The researcher attempted to act objectively in the process of collecting, describing, analyzing, and interpreting the data and the accuracy of the results was confirmed by an expert. The role of the researcher was described in detail to show that the researcher acted objectively. In addition, transcribed data and video recordings were preserved for confirmation.

The Kuder-Richardson Reliability Coefficient (KR20) value was calculated for the CDAT pretest and posttest. KR20 reliability coefficient value is 0.87 for the pretest and 0.81 for the posttest. When $0.8 < r < 0.89$, the test is considered good reliability. In this respect, the pretest and posttest of CDAT is highly reliable.

CHAPTER 4

FINDINGS

The purpose of the study is to investigate the development of 7th-grade students' algebraic thinking during the teaching experiment. This chapter presents the results of the analyses related to the problems identified in the study. In addition to the descriptive statistics of the Chelsea Diagnostic Algebraic Test, the qualitative analysis of the worksheets used during the teaching experiment is presented.

4.1 Findings Related to the Students' Algebraic Thinking Levels

4.1.1 Descriptive Statistics

The Chelsea diagnostic algebra test was administered to six 7th-grade students before and after the teaching experiment. This section presents descriptive statistics about the level of algebraic thinking of the 7th-grade students. The mean and standard deviation for the CDAT scores of the 7th-grade students before and after the teaching experiment are presented in Table 4.1.

Table 4.1 Descriptive Statistics Results for Students' Algebraic Thinking

	N	Mean	Std. Deviation	Percentiles		
				25th	50th (Median)	75th
PRECDAT	6	31,50	9,586	22,00	34,50	40,00
POSTCDAT	6	39,33	6,861	32,50	42,00	45,00

Table 4.1.1 shows that 7th grade students obtained the highest score on CDAT after the teaching experiment (M=39.33, SD=6.861) and the lowest score on CDAT before the teaching experiment (M=31.50 SD=9.586).

4.1.2 Inferential Statistics

A nonparametric test was used to analyze the data in terms of inferential statistics in this study. Siegel (1957) stated that when having small sample size data, say 6, there is no option but to use nonparametric test. Since the size of this data set is 6 and does not meet the assumptions of the parametric test, the nonparametric test is used.

A null hypothesis was written to determine whether there is a significant increase in the level of algebraic thinking of 7th grade students after implementing the teaching experiment. This null hypothesis indicated, "There is no significant increase in the level of algebraic thinking of 7th grade students after the implementation of the teaching experiment".

Table 4.7 Wilcoxon Signed Rank Test Results for CDAT Pre-Test and Post-Test Scores

	POSTCDAT - PRECDAT
Z	-2,207
Asymp. Sig. (2-tailed)	0,027

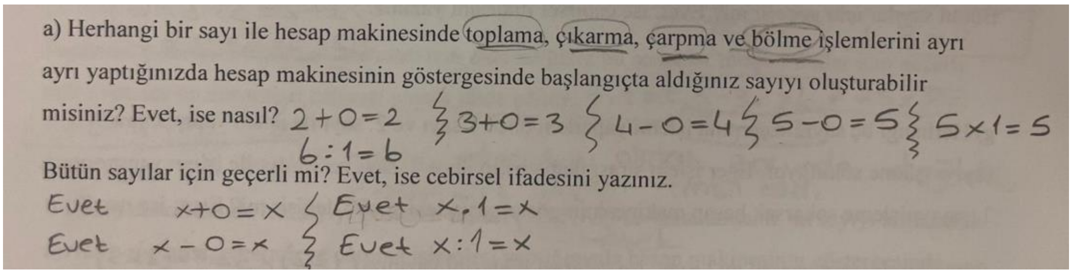
The Wilcoxon Signed Rank Test was used to compare the level of algebraic thinking of the 7th-grade students before and after the implementation of the teaching experiment. The Wilcoxon Signed Rank Test revealed a statistically significant increase in 7th-grade students' level of algebraic thinking after implementing the teaching experiment, $z = -2.207$, $p < .05$, with a large effect size ($r = .90$). The median score on the Chelsea Diagnostic Algebra test increased from before the teaching experiment (Md = 34.50) to after the teaching experiment (Md = 42). These results could mean that the algebraic thinking of the 7th-grade students was developed after the teaching experiment.

4.2 Findings Related to the Students' Algebraic Thinking

This section includes the qualitative analysis of the 8 worksheets prepared for each of the algebraic thinking components during the teaching experiment in terms of these components.

4.2.1 Properties of the Number System (Calculator Activity)

The Calculator activity deals with the properties of the number system, which is a subcategory of generalizing arithmetic and quantitative reasoning, one of the components of algebraic thinking. This activity involves generalizing basic operation properties, conjectures derived from basic properties and the properties of odd and even numbers. The activity consists of thirteen independent options.


a) Herhangi bir sayı ile hesap makinesinde toplama, çıkarma, çarpma ve bölme işlemlerini ayrı ayrı yaptığınızda hesap makinesinin göstergesinde başlangıçta aldığınız sayıyı oluşturabilir misiniz? Evet, ise nasıl? 2+0=2 3+0=3 4−0=4 5−0=5 5×1=5 6:1=6

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

Evet x+0=x Evet x.1=x

Evet x−0=x Evet x:1=x

Figure 4.1 The Answer of the First Group for Option a for the Calculator Activity

How the first group generalized the identity element property from the basic operation properties in the question in option a is given below.

… Güney: I don't understand the question.

Researcher: It says with any number, so we choose a random number. When you add it up, the calculator display asks you to recreate the number you have chosen. How is this possible?
Güney: Neutralization is possible.
Researcher: When you add any number, for example the number 2, on a calculator, can you create the number 2 on the calculator's display?
Güney: We have to add up to zero.
Cem: 2 plus 0 equals 2.
Researcher: If you tried this with another number, would you still get the same number?
Güney: Is the second number going to change?
Researcher: Would you reach the same number if the second number changed?
Cem and Güney: No.
Researcher: Then what about the second number?
Güney: Zero.

Looking at the dialog above, it is seen that the students had difficulty in understanding the question. The researcher helped them by providing the necessary explanations and an example to help them understand the question better.

How the first group was able to generalize the identity element property in four operations is given below.

… Researcher: Can you give different examples?
Güney: 3 plus 0 equals 3.
Researcher: Does it apply to all numbers?
Cem: Yes, it is valid.
Researcher: It wants you to write its algebraic expression.
Cem: We write x plus 0.
Güney: x plus zero equals x.
Researcher: Now you have to do it for subtraction, multiplication and division.
Güney: I think the second number is 0 again for subtraction.

Cem: I agree. Since zero is ineffective, it should be zero.

Güney: 4 minus 0 equals 4. 5 minus 0 equals 5. x minus 0 equals x.

Researcher: Why?

Güney: Since 4 minus 0 equals 4 and 5 minus 0 equals 5, it means that it happens with all numbers. Since x can also change, that is, it is not definite, we used x. x minus zero equals x.

Researcher: Can you do it for multiplication?

Güney: The first number should be zero. No, I said it wrong.

Cem: I think the first number should be 1 because the identity element in multiplication is 1.

Selin: Yes, I think so.

Cem: Whatever number we multiply by 1 is still the same number.

Researcher: Can you give an example?

Selin: If we multiply 5 by 1, we get 5.

Researcher: Does it apply to all numbers?

Selin and Cem: Yes.

Cem: x times 1 equals x. Since x is unknown, it can be any number. Since 1 is an identity element, the result is still x no matter what x is.

Researcher: How about for division?

Güney: The second number should be 1 again.

Selin and Cem: I agree.

Güney: Because if we divide 5 by 1, we get 5. If we divide 6 by 1, we get 6.

Researcher: What is the algebraic expression?

Güney: x divided by 1 equals x because x can be any number, so dividing by 1 percent gives x again.

The dialog shows that students first gave examples to show that they reached the same number that they wrote at the beginning by doing addition, subtraction, multiplication and division operations separately. In other words, when zero was doing addition and subtraction operations with any number, and when one was doing multiplication and division operations with any number, they found that they ended up with the same number as they started with. The students were able to form their algebraic expressions by stating that the four situations they created were valid in four operations with all numbers. In their algebraic expressions, they represented any number with the symbol x to emphasize that when the four operations were performed separately, the initial number was reached. They were able to write algebraic expressions as $x+0=x$, $x-0=x$, $x\cdot 1=x$ and $x\div 1=x$.

How the second group generalized the identity element property from the basic operation properties in the question in option a is given below.

... Melek: It is asking if we can get the same result when we do these operations separately.
Researcher: Yes.
Zülal: I think it will come out. 2 times 1, 2 for example.
Melek: Yes. For example, 2 plus 0 makes 2. 2 minus 0 makes 2. 2 times 1 is 2. 2 divided by 1 is 2. Yes, it does.
Researcher: Does it apply to all numbers? I mean, is it valid for all numbers separately for addition, subtraction, multiplication and division?
Melek: When we do it with 3, it is the same. Valid.
Zülal: It's the same.

The students were able to give examples showing that they reached the same number as they initially wrote by performing addition, subtraction, multiplication and division operations separately. When any number with zero was added and subtracted, and any number with one was multiplied and divided, they found that they reached the same number as they had originally written.

The students were confused while writing algebraic expressions. By asking questions, the researcher made the students realize the numbers that changed and stayed the same in the operations in the examples they gave.

Researcher: Then it wants you to write the algebraic expression of these operations separately.

Melek: x becomes zero and y becomes one. When we write it as an algebraic expression, it is like 2 plus x, 2 minus x, 2 times y and 2 divided by y.

Researcher: Well, you said it is valid for all numbers. But it only applies to 2?

Zülal and Melek: It can also be with 3.

Melek: Why did we write 2 here?

Researcher: If you say that it applies to all numbers, your algebraic expression is valid for all numbers.

Melek: It should cover all of them. Then what are we going to do?

Researcher: Now if we look at addition. You added 2 and 0 and found 2. You made another number.

Emre, Melek and Zülal: 3 and 0 add up to 3.

Researcher: So what has changed here?

Melek: The result changes according to the number we give.

Researcher: What hasn't changed?

Melek: Zero. Then we will write x instead of 2.

Researcher: Why?

Melek: Because 2 changes. It can be 3,4,5. Then it will be x plus zero.

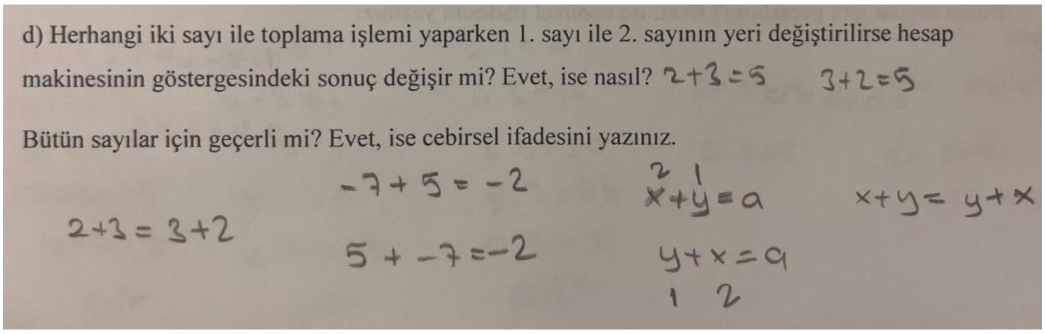
Researcher: What is the result of the process?

Melek: x

Researcher: How did you write your algebraic expressions?

Melek: We gave x to the changing numbers, so x plus 0, x minus 0, x times 1 and x divided by 1, all equal to x.

The dialog shows that the students represented 0 and 1 with symbols and wrote a value for the number they were doing operations, for example, $2+x=2$. After the researcher's guidance, they realized that the number they were doing operations could take different values, that it should be represented by the symbol x, and that they could not get the same result if there were no 0 and 1. Thus, the students were able to form their algebraic expressions by stating that the four situations they created were valid in four operations with all numbers. They were able to write algebraic expressions as $x+0=x$, $x-0=x$, $x\cdot 1=x$ and $x\div 1=x$.


d) Herhangi iki sayı ile toplama işlemi yaparken 1. sayı ile 2. sayının yeri değiştirilirse hesap makinesinin göstergesindeki sonuç değişir mi? Evet, ise nasıl? 2+3=5 3+2=5

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

2+3 = 3+2

-7 + 5 = -2

5 + -7 = -2

x+y = a

y+x = a

x+y = y+x

Figure 4.2 The Answer of the Second Group for Option d for the Calculator Activity

The dialog below shows how the first group generalized the commutative property of the basic operation properties in the question in option d.

... Selin: 1 plus 2 makes 3. If we change their places, the result will be the same.
Cem: Addition has the commutative property.
Selin and Güney: Yes.
Researcher: Can you tell me through your example?
Selin: 3 plus 2 equals 5.
Güney: Let's do it the other way.
Selin: 2 plus 3 equals 5. So no matter what, if we change their places, the result is the same.
Researcher: For example, in the operations "2 plus 3" and "3 plus 2", can an expression other than the result of the operation be written opposite the equal sign?
Selin: What do you mean?
Researcher: For example, you said that 3 plus 2 is 5 and 2 plus 3 is 5. Can you write something other than 5 opposite the equal sign in "2 plus 3" and "3 plus 2"?
Selin: No.
Cem: We can write 4 plus 1.
Researcher: Okay, let's do it like this. You said that 3 plus 2 is equal to 5. You said that 5 is equal to something else.

Cem: 2 plus 3.

Researcher: What can you write opposite the equal sign in the "3 plus 2" operation?

Cem and Güney: We can write 2 plus 3. (They wrote the equation 3+2=2+3)

Researcher: So what did you express by writing like this?

Cem: When number 1 and number 2 are switched, the result does not change.

Researcher: Does it apply to all numbers?

Güney: It applies to all numbers.

Selin and Cem: Yes.

The dialog below shows how the second group generalized the commutative property of the basic operation properties in the question in option d.

... Melek: It doesn't change.

Zülal: Yes.

Researcher: Can you show it?

Melek: 2 plus 3.

Zülal: 2 plus 3 is 5. 3 plus 2.

Melek: Five again.

Researcher: For example, you said that 3 plus 2 is 5 and 2 plus 3 is 5. Can you write something other than 5 opposite the equal sign in "2 plus 3" and "3 plus 2"?

Melek: Do we equate 2 plus 3 to a different number?

Researcher: Well, let's do it like this. You said that the result of these two operations is equal to 5. Then what can we say about these two operations.

Melek, Zülal and Emre: Equal to each other.

Researcher: Can you show this equality?

Emre: 2 plus 3 equals 3 plus 2.

Researcher: Does it apply to all numbers?

Melek and Zülal: Yes, it is valid whether it is negative or positive.

Researcher: Can you give me another example?

Melek: -7 plus 5.

Zülal: 5 plus -7.

Melek: These are equal. It's -2.

The dialogues above show that the students first gave examples to show that the result does not change when fist number and second number are switched. Since the researcher wanted the students to write by establishing an equality between the operations in which two numbers are substituted and the operations in which they are not substituted, she guided them with questions to write in this way. She made them realize that these two addition operations are equal to each other.

How the first group constructed the algebraic expression of the situation they created is given below.

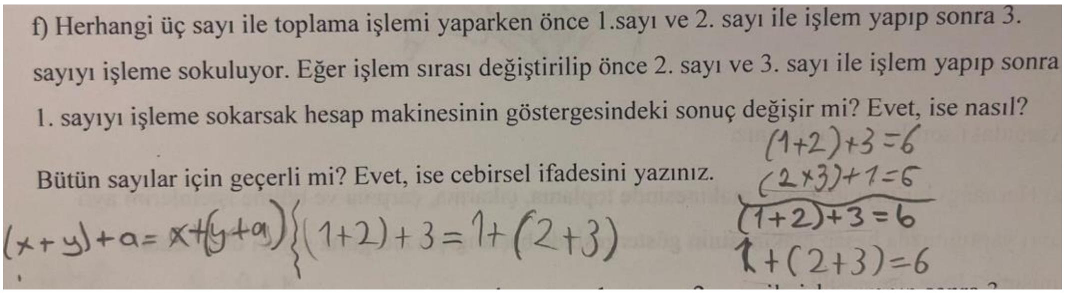
… Researcher: Then write the algebraic expression.
Cem: If x plus y is 6, then y plus x is 6.
Researcher: You said it is valid for all numbers. But you gave a value to the result.
Cem: Then we can give another algebraic expression to the result.
Researcher: Can you pay attention to the last statement you wrote in the example you gave? Can we write based on it?
Güney and Cem: Yes.
Güney: x plus y equals x plus y. To neutralize.
Cem: x plus y equals y plus x.
Güney: I forgot to change their places.
Cem: Such results do not change.
Researcher: What do x and y represent?
Cem: Any number.

How the second group constructed the algebraic expression of the situation they created is given below.

… Researcher: Then write the algebraic expression.
Melek: x plus y equals a. Again y plus x equals a.

Researcher: When you gave the last example, you said that these two addition operations are equal to each other. Can we show this in algebraic expression?
Melek: x plus y equals y plus x.
Researcher: What do x and y represent?
Melek and Zülal: Any number.

As seen in the dialogues, since both groups did not write an algebraic expression considering the last equality they created in the situation they gave as an example, the researcher directed them to write it in that way. In the algebraic expression they created, they showed that they could add any two numbers with the symbols *x* and *y* and wrote $x+y = y+x$. Thus both groups were able to generalize the commutative property in addition.


f) Herhangi üç sayı ile toplama işlemi yaparken önce 1.sayı ve 2. sayı ile işlem yapıp sonra 3. sayıyı işleme sokuluyor. Eğer işlem sırası değiştirilip önce 2. sayı ve 3. sayı ile işlem yapıp sonra 1. sayıyı işleme sokarsak hesap makinesinin göstergesindeki sonuç değişir mi? Evet, ise nasıl?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

$(1+2)+3=6$
$(2+3)+1=6$
$(1+2)+3=6$
$1+(2+3)=6$

$(x+y)+a = x+(y+a)$ $(1+2)+3 = 1+(2+3)$

Figure 4.3 The Answer of the First Group for Option f for the Calculator Activity

In the dialog below, it is shown how the first group generalized the associative property from the basic operation properties in the question in option f.

... Cem: Let's give an example. We can call fist number as 1, second number as 2 and third number as 3.
Selin: How do we do it?
Cem: First we add 1 and 2, which equals 3. Then we add 3 with 3, equals 6. If we change the order of operations...
Güney: Since it has the commutative property, it will still be the same.
Researcher: Can you show it?

Cem: First we will add 2 and 3 and then we will add 1.
Güney: What did you say?
Cem: 2 plus 3 equals 5. If we add 5 and 1, we get 6 again. The result does not change.
Güney: We have to put parentheses.
Researcher: Why do you need to put parentheses?
Cem and Güney: We will provide transaction priority.
Selin: Yes.
Researcher: Then where will you use the parenthesis?
Güney: We will put the first and second numbers in parentheses because the 1st number and the 2nd number are added and then the 3rd number is added. So we do not include the 3rd number in the parenthesis. In the other one, we will put second and third number in parentheses. We do not put the 1st number in parentheses.

The dialog above shows that the students first can give examples to show that when the order of addition of numbers changes, the result does not change. The students used parentheses to emphasize which numbers would be added first.

The following dialog shows how the researcher made the students realize that these two addition operations are equal to each other and that they do not need to change the order of the numbers because the parenthesis emphasizes the transaction priority.

Researcher: You said that you wanted to emphasize the transaction priority with parentheses. You wrote 1 plus 2 in parentheses, then closed the parenthesis, wrote 3 and wrote equals 6. In the other one, you wrote 2 plus 3 in parentheses, then closed the parenthesis, wrote 1 and wrote equals 6. So, is it absolutely necessary to change the order of the numbers to emphasize with parentheses that the numbers 2 and 3 will be added first?
Cem: We can do it like this. We can exclude 1 in the parenthesis and make 1 plus and 2 plus 3 in the parenthesis.
Selin: How do we write? I don't understand.
Cem: Since 2 plus 3 is in parentheses, we will take it as a single number and not separate it. 1 plus 2 plus 3 in parentheses equals 6.

Güney: We did the transaction priority without moving the numbers.

Researcher: Well, you found the results of the expressions in the previous two questions equal. How did we write in those questions?

Cem: We wrote it as equality.

Researcher: Can we write it that way in this question?

Cem and Güney: We can write.

Cem: 1 plus 2 in parentheses then plus 3 equals 1 plus and 2 plus 3 in parentheses.

Researcher: Does it apply to all numbers?

Cem: Yes, because the parenthesis has transaction priority.

Researcher: What is the algebraic expression?

Güney: x plus y and plus a but x plus y in parentheses.

Cem: Equals x plus and y plus a in parentheses.

As seen in the dialog above, the researcher first made the students realize that they could write (1+2) +3=6 and 1+(2+3) =6 by without changing order of the numbers. Then, she made them realize that these two expressions are equal to each other, that is, they can write (1+2) +3 =1+(2+3). The students were able to form the algebraic expression by stating that the situation they formed is valid for all numbers. In the algebraic expression they formed, they showed that they could add any three numbers with three different symbols and they were able to write $(x+y) +z=x+(y+z)$.

In the dialog below, it is shown how the second group generalized the associative property from the basic operation properties in the question in option f.

... Zülal: Let's give numbers one by one. Let's make it 2, 3 and 4.

Melek: Let's just give 1, 2 and 3.

Zülal: Okay.

Emre: I think it won't change.

Melek: 1 and 2 add up to 3. Then we add 3. It becomes 6. First we added 2 and 3 which is 5. Then we added 1.

Zülal: Six again.

Melek: Yes. It doesn't change.

Researcher: Does it apply to all numbers?

Zülal: Yes.

Melek: Let's give another example. Let's give 4,5 and 6. 4 plus 5 is 9. Plus 6.

Zülal: It'll be 15.

Melek: 5 plus 6 is 11. Plus 4, 15. Again, they are equal to 15. Anyway, even if we add all 3 numbers at the same time, nothing will change.

Zülal: Yes.

The dialog above shows that students can give examples to show that the result does not change when the order of addition of numbers changes. At first, the students didn't used parentheses to emphasize which numbers would be added first.

The following dialog shows that when the students had difficulty in how to make this emphasis, the researcher reminded them with questions that parentheses provide the transaction priority in an operation.

Researcher: It asks you to write algebraic expression. You need to show that you change the order of operations when writing the algebraic expression.

[...]

Researcher: We use parentheses in some operations. What property do parentheses give to a process?

Zülal, Melek and Emre: Transaction priority.

Researcher: Can you use the property of the parenthesis to do the operation that is asked of you in this question?

Melek: Then we do it like this. a plus b in parentheses, plus c equals…

Zülal: x.

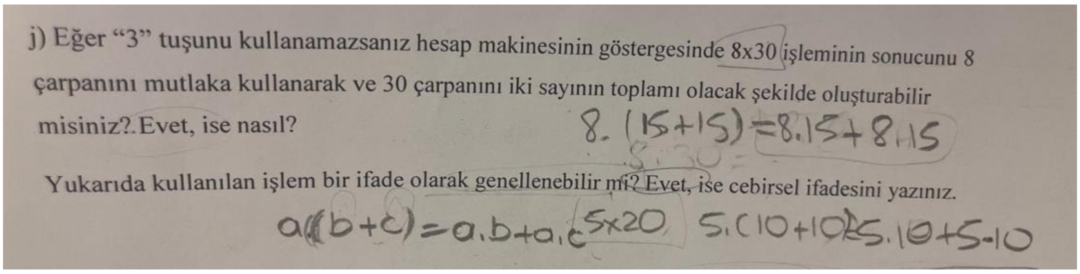
Emre: b plus c in parentheses then plus a.

Looking at dialog, after researcher guidance, the students used parentheses to emphasize which numbers would be added first, so they write like this $(a+b)+c=x$ and $(b+c)+a=x$.

The following dialog shows how the researcher made the students realize that these two addition operations are equal to each other and that they do not need to change the order of the numbers because the parenthesis emphasizes the transaction priority.

… Researcher: Last time you said that these two operations are equal to each other.
Melek: a plus b would be in parentheses, plus c equals b plus c in parentheses then plus a.
Researcher: Is it absolutely necessary to change the order of the numbers to emphasize that the second and third numbers will be added first?
Melek: Do you need to emphasize...
Researcher: Why did you use parenthesis?
Zülal, Melek and Emre: For transaction priority.
Researcher: Is it absolutely necessary to change the order of the numbers to emphasize that the second and third numbers will be added first because the parenthesis emphasizes the transaction priority?
Melek: It is not necessary.
Melek: We write it like this. a plus b in parentheses, plus c equals plus a then b plus c in parentheses.
Zülal: I don't understand.
Melek: We will put b and c directly in parentheses without changing the numbers. We don't need to change the numbers.

The dialog above shows that the researcher first made the students realize that these two expressions are equal to each other, that is, they can write $(a+b)+c=(b+c)+a$. Then, the researcher made them realize that they did not need to change the order of the numbers because the parenthesis emphasized the transaction priority that is, they can write $(a+b)+c=a+(b+c)$. Thus, the students were able to form the algebraic expression by stating that the situation they formed is valid for all numbers.


j) Eğer "3" tuşunu kullanamazsanız hesap makinesinin göstergesinde 8x30 işleminin sonucunu 8 çarpanını mutlaka kullanarak ve 30 çarpanını iki sayının toplamı olacak şekilde oluşturabilir misiniz?. Evet, ise nasıl?

8.(15+15)=8.15+8.15

Yukarıda kullanılan işlem bir ifade olarak genellenebilir mi? Evet, ise cebirsel ifadesini yazınız.

a(b+c)=a.b+a.c 5x20 5.(10+10)=5.10+5.10

Figure 4.4 The Answer of the Second Group for Option j for the Calculator Activity

How the first group generalized the assumption *a(b+c) =ab+ac* derived from the properties of the basic operations in the question in option j is given below.

... Cem: Here we can find it by doing the distributive property. We can do it like this. We can do 8 times and 15 plus 15 in parentheses.
Researcher: Why did you think so?
Cem: We can't use the key 3. We will find 30 by adding two numbers. I said 15 plus 15, but it could be 12 plus 18. It could be two different numbers that add up to 30. Then we multiply by distributing 8. So 8 times 15 plus 8 times 15.
Researcher: Why did you put parentheses?
Güney and Selin: For transaction priority.
Cem: If we hadn't, it would have been 8 times 15 plus 15.
Selin: Yes.
Güney: Because multiplication has priority. Also, since it goes from left to right, multiplication makes first.
Researcher: Can this process used to be generalized as an expression?
Cem and Selin: Yes.
Researcher: Why?
Cem: Because no matter what number we put, it will work.
Researcher: Show me please.
Cem: Let's try 4 times 20.
Güney: 4 times 10 plus 10 in parentheses.
Selin: We write equals 4 times 10 plus 4 times 10.

The dialog above shows that the students represented 30 as the sum of two numbers as asked in the question and were able to enclose this addition in parentheses to ensure the precedence of operations. In other words, they wrote it as $8\cdot$ (15+15) and used the distributive property to write $8\cdot 15+8\cdot 15$ opposite the equal sign. In order to indicate that the situation they created was generalizable, the students created the same situation by experimenting with different numbers.

How the first group constructed the algebraic expression of this situation is given below.

Researcher: So how do you write the algebraic expression of this situation?
Güney: We can do it like this. We can say 15 plus 15, 30. b plus y, 30. We write a times b plus y in parentheses.
Selin: So a times parenthesis b plus y equals...
Güney: But there is such a problem. b and y can be any number, but their sum must be 30.
Cem: What if it's a different number? There could be different numbers.
Güney: What do you mean?
Cem: b plus y equals x.
Güney: But the question says 30.
Cem: But it can be a different number. It can be in different numbers.
Researcher: You said that the statement is generalizable.
Güney: We will not put a limit then, as it can be any number.
Researcher: Then what is b plus y?
Selin and Güney: x
Researcher: Then what is your algebraic expression?
Selin: We wrote a times a plus b plus y in parentheses, so a times b plus a times y.

As seen in the dialog, one student in the first group ignored that the expression was generalizable and claimed that b plus c would be 30 by focusing on the values in the question. The researcher clarified this situation by emphasizing that this expression is generalizable. Thus, the students were able to form the algebraic expression of this situation and write $a(b+y) = ab+ay$.

How the second group generalized the assumption $a(b+c) = ab+ac$ derived from the properties of basic operations in the question in option j is given below.

... Melek and Zülal: Yes, we can create it.
Zülal: There will be transaction priority again.
Melek: Yes.
Researcher: Why will there be transaction priority?
Melek: Multiplication comes before addition, so we have to put parentheses, like 8 times...
Zülal: 15 plus 15 in parentheses.
Researcher: What is the result of this operation?
Melek: 8 times 30.
Researcher: What can you write opposite the equal sign in the operation you did other than 8 times 30?
Melek: We can write the result of the operation 240.
Researcher: What else can we write other than the result of the process?
Zülal: 8 times
Melek: 5 times 6.
Researcher: Let's do it like this. I want you to solve this operation you wrote, but how can you do it if you don't do the operation inside the parenthesis first?
Melek: We can distribute. 8 times 15 plus 15.
Zülal: No, it will still be 8 times 15.
Melek: Yes, 8 times 15 plus 8 times 15.

The dialog above shows that the students represented 30 as the sum of two numbers as asked in the question and were able to enclose this sum in parentheses to ensure the precedence of operations. In other words, they wrote it as $8\cdot(15+15)$. However, they had difficulty in writing $8\cdot 15+8\cdot 15$ opposite the equal sign at the beginning.

With the guidance of the researcher, they remembered and were able to use the distributive property.

How the second group constructed the algebraic expression of this situation is given below.

Researcher: Can this operation be generalized as an expression? Can I actually write such an operation for all numbers?
Zülal and Melek: Yes, it can be generalized.
Researcher: How?
Melek: For example, let's try 5 times 20. Here, if we make 5 times 10 plus 10 in parentheses, then it equals 5 times 10 plus 5 times 10.
Researcher: So how do we write the algebraic expression of this situation?
Zülal: a times b plus b in parentheses.
Melek: Those numbers do not have to be the same. Let's write it like this. a times b in parentheses plus c equals a times b plus a times c.
Researcher: Why did you write a, b and c like that?
Melek: a is the first factor. b and c are the sum of the second factor.
Zülal: They can be different numbers.

As seen in the dialog, to indicate that the situation they created was generalizable, the students created the same situation by experimenting with different numbers. They were able to form the algebraic expression of this situation and write *a(b+c) =ab+ac*.

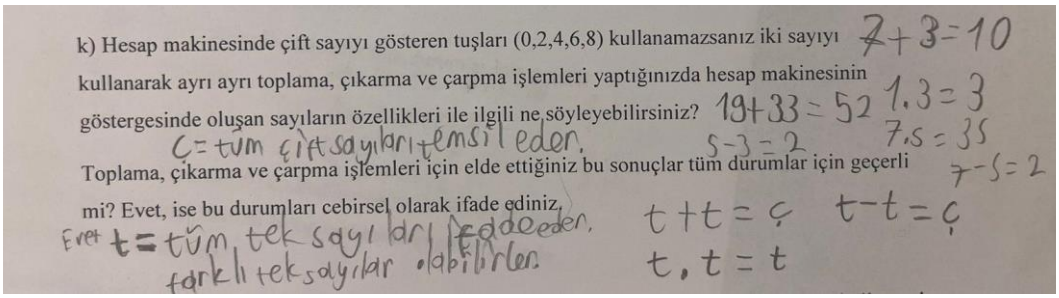


Figure 4.5 The Answer of the First Group for Option k for the Calculator Activity

How the first group generalized the properties of odd numbers in the question in option k is given below.

... Cem and Selin: I don't understand.

Güney: Actually it says. We will do addition, subtraction and multiplication using two numbers. I didn't understand the rest either.

Researcher: The question mentions that you cannot use some digits on the calculator. How many digits are there?

Güney: 10.

Researcher: 0,2,4,6, and 8 of 10 digits cannot be used. What is special about these numbers?

Güney and Selin: Even numbers.

Researcher: You will add, subtract and multiply two numbers without using these. Which numbers do you use?

Cem: Odd numbers.

Researcher: Let's give an example. For example, when you add the numbers 19 and 33, what can you say about the properties of the number that appears on the calculator's display?

Selin: It is 52.

Researcher: What can you say about the characteristic of this number?

Cem: An even number.

Researcher: Try other examples please.

Güney: 7 plus 3, 10.

Researcher: What can you say about the characteristic of this number?

Cem, Selin and Güney: Even number again.

Researcher: Do these results apply to all situations?

Selin, Güney and Cem: Yes, it is.

Researcher: Why?

Cem: Because the sum of odd numbers is equal to even numbers.

Looking at the dialog above, it is seen that the first group had difficulty in understanding the question. The researcher helped them by providing the necessary explanations and an example to help them understand the question better. Then, the students performed addition operations with odd numbers by giving examples. They were able to determine the properties of the numbers they found. In other words, they were able to find that if adding any two odd numbers, the result is always an even number.

The following dialogue shows how the first group decided what the symbols *t* and *ç* covered.

Researcher: It wants you to write the algebraic expression of this situation.
Güney: t plus t equals ç.
Cem: But we don't know if the t's are equal. If we write t in both, they have to be equal.
Researcher: Does it have to be the same?
Cem: It doesn't have to be the same.
Güney: It has to be the same because we write t in both.
Cem: But if it is the same, it is not a generalization. It has to include all odd numbers.
[...]
Researcher: What is your decision?
Cem: t is all odd numbers.
Researcher: What happens then?
Selin and Cem: ç represents all even numbers.

After discussing among themselves and reaching a consensus, the students correctly stated what *t* and *ç* stand for. They state that *t* is all odd numbers and *ç* is all even numbers. The students were able to form algebraic expression for addition of odd numbers. They write the algebraic expression as $t+t=ç$.

How the first group generalized the properties of odd numbers and formed the algebraic expression for subtraction and multiplication operations is given below.

Researcher: Now you need to do multiplication and subtraction.
Cem: Let's do multiplication. 1 times 3 becomes 3.
Selin: 8 times 1.
Cem: But 8 is even number.
Selin: Uh, yeah.
Cem: 7 times 5 is 35.
Researcher: What can you say about the characteristic of this number?
Güney and Cem: Odd number.
Cem: Then the product of odd numbers is odd.
Researcher: Are these results valid for all situations?
Cem and Güney: Yes.
Cem: The product of odd numbers is always odd.
Researcher: It wants you to write its algebraic expression.
Cem: t times t equals t.
Güney: There is also subtraction. 5 minus 3 equals 2. It is even number. 7 minus 5 equals 2. Again even numbers.
Researcher: Are these results valid for all situations?
Cem and Güney: Yes.
Researcher: Why?
Güney: Subtracting an odd number from an odd number is an even number.
Researcher: It wants you to write its algebraic expression.
Güney: t minus t equals ç.

The dialog shows that the students performed subtraction and multiplication operations with odd numbers separately by giving examples. They were able to determine the properties of the numbers they found as a result of these operations. In other words, they were able to determine that when any two odd numbers are subtracted, the result is always an even number, and when any two odd numbers are multiplied, the result is always an odd number. The students were able to form their algebraic expressions for subtraction and multiplication of odd numbers. They write the algebraic expressions as *t-t=ç* and *t·t=t*. Thus. they were able to generalize the properties of odd numbers.

How the second group generalized the properties of odd numbers in the question in option k is given below.

... Zülal: We won't use them. Then we use an odd number.
Melek and Zülal: 1, 3, 5, 7 and 9
Zülal: 1 plus 3...
Melek: I'll tell you something, it's always even number.
Zülal: But there is also subtraction.
Melek: Addition is always even number. Multiplication is always odd number.
Researcher: Please show it on the example.
Melek: 1 plus 3 becomes 4. 3 plus 5 is 8. 5 plus 7 is 12. Yes, it's always even.
Zülal: Subtraction?
Melek: It's also even numbers.
Zülal: Yes. 3 minus 1 becomes 2.
Melek: 5 minus 3 is 2. 7 minus 5 is 2.
Zülal: Multiplication?
Melek: 1 times 3 is 3. 5 times 3 is 15. 7 times 5 is 35. The result is even number in addition and subtraction, but odd number in multiplication.
Zülal: Yes.
Researcher: Does it apply in all cases?
Melek: It applies in all cases where we don't use even numbers.
Zülal: It is valid in odd numbers.

In the dialogues above, students performed addition, subtraction, and multiplication operations with odd numbers separately by giving examples. They were able to determine the properties of the numbers they found as a result of these operations. In other words, they were able to determine that when you add or subtract any two odd numbers, the result is always an even number, and when you multiply any two numbers, the result is always an odd number. The students were able to state that the situations they created were valid for all odd numbers.

How the second group formed the algebraic expression for addition, subtraction and multiplication of odd numbers is given below.

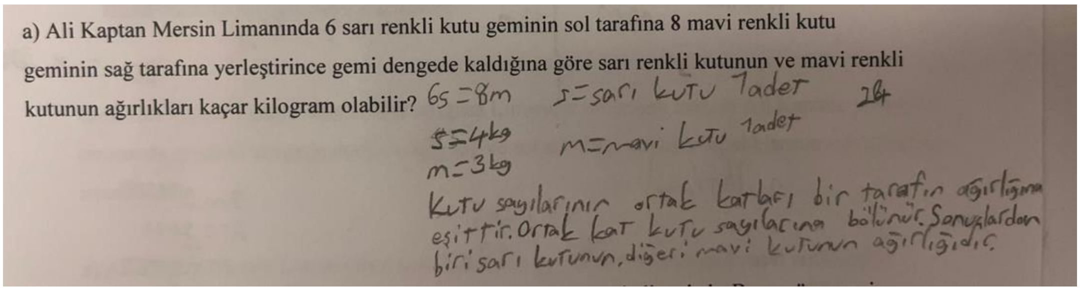
Researcher: It wants you to write algebraic expressions separately.
Melek: Let's write with addition first. a plus b equals c.
Researcher: How do we know that a and b are odd and c is even in an algebraic expression?
Melek: We cannot understand that way.
[...]
Researcher: In an algebraic expression, can we show an odd number with the letter "t" and an even number with the letter "ç" to indicate whether the unknowns are odd or even numbers?
Melek and Zülal: Yes.
Zülal: t plus t equals ç.
Melek: t minus t equals ç.
Zülal: t times t equals t.

According to the dialogues, the students struggled with how to express odd and even numbers in algebraic expressions. The researcher guided them by suggesting them to represent odd numbers with the symbol *t* and even numbers with the symbol *ç*. They used *t* to represent all odd numbers and *ç* to represent all even numbers in their algebraic expressions. The students were able to form their algebraic expressions as $t+t=ç$, $t-t=ç$, and $t \cdot t=t$. Thus, they were able to generalize the properties of odd numbers.

A general evaluation of this activity shows that students were able to generalize the properties of basic operations, assumptions derived from basic properties, and properties of odd and even numbers.

4.2.2 The Meaning of the Equal Sign and Relational Thinking (Ali Captain’s Ship Activity)

Ali Captain’s Ship activity is related to the meaning of the equal sign and relational thinking, which are subcategories of the meaningful use of symbols category. This activity consists of five interconnected options.


a) Ali Kaptan Mersin Limanında 6 sarı renkli kutu geminin sol tarafına 8 mavi renkli kutu geminin sağ tarafına yerleştirince gemi dengede kaldığına göre sarı renkli kutunun ve mavi renkli kutunun ağırlıkları kaçar kilogram olabilir?

6s = 8m   s = sarı kutu 1 adet   24
s = 4 kg   m = mavi kutu 1 adet
m = 3 kg
Kutu sayılarının ortak katları bir tarafın ağırlığına eşittir. Ortak kat kutu sayılarına bölünür. Sonuçlardan biri sarı kutunun, diğeri mavi kutunun ağırlığıdır.

Figure 4.6 The Answer of the First Group for Option a for the Ali’s Captain Ship Activity

The following dialog shows how the first group perceived the equal sign as a relational symbol for the question in option a.

… Cem: I think let’s make a first equation. Let’s make equality like 6 yellow equals 8 blue.

Güney: 6 yellow equals 8 blue. Well, not just blue. Wouldn’t it be better to put x or something?

Cem: Let’s call blue an m. Let’s call yellow an s.

Researcher: And what do the letters m and s stand for?

Cem and Selin: Blue and yellow boxes. s is the yellow box; m is the blue box.

Researcher: So the yellow and blue boxes were equal?

Cem: Weight.

Güney: 6s=8m

Cem: 6 yellow boxes weigh the same as 8 blue boxes.

[...]

Researcher: What does m mean?

Cem: The weight of a blue box.

Researcher: What does s stand for?

Selin: The weight of a yellow box.

Researcher: What does it want from you in the first option?

Cem: It is asking how many kilograms the boxes can weigh. To do this, we first need to find the common multiples of 6 and 8.

Güney: Yes.

Researcher: How do you find the common multiples of 6 and 8?

Cem: We can multiply 6 by 8, or we can write multiples of both one by one.

Selin: Yes.

Researcher: Then try to do it.

Cem: The least multiple is 24. We should choose the least multiple to make it easier.

[...]

Researcher: So how much do a yellow box and a blue box weigh?

Cem: We have to multiply 6 by 4 to get 24. The weight of yellow should be 4.

Güney: Both of those common multiples are...

Cem: Since 6 find its common multiple in the fourth number, s becomes 4 kilograms. Since 8 also find the third number, m becomes 3 kilograms.

In the dialog above, the students showed that they perceived the equal sign as a relational symbol by forming equality between the weights of 6 yellow boxes on the left side of the ship and 8 blue boxes on the right side, i.e. $6s=8m$. They were able to state that the symbols s and m in the equality they formed are the weight of a yellow box and the weight of a blue box, respectively. In order to find the values of s and m that would satisfy the equality they formed, they found the least common multiple of the number of boxes. The values they determined were 4 kilograms for s and 3 kilograms for m.

In this dialog, the second group's thoughts on whether there are other values that the weights of one yellow and one blue box can take are given.

... Researcher: Are there any other values that the weights of the yellow and blue colored boxes can take?

Zülal and Emre: Yes.

Zülal and Melek: It continues to be their common multiple.

Researcher: What do you get when you take the common multiple of 6 and 8?

Melek and Emre: Equality

Researcher: Yes, we get equality, but in what area?

Melek and Zülal: The weight on the right and left side.

Researcher: And?

Melek: The common multiple of 6 and 8 is equal to the total kilograms on the left and right.

Researcher: So how do we get the weight of the yellow and blue box?

Melek: Dividing the total weight by the number of boxes separately gives the weight of the boxes.

Researcher: Okay, is this how we can find other values?

Melek and Zülal: Yes.

In the dialog above, students were able to explain how to reach these values by stating that there are other values that the weights of the boxes can take. In order to find other values, students mentioned that they could reach the weights of the yellow and blue boxes by finding other common multiples of 6 and 8 and dividing these values by the number of boxes.

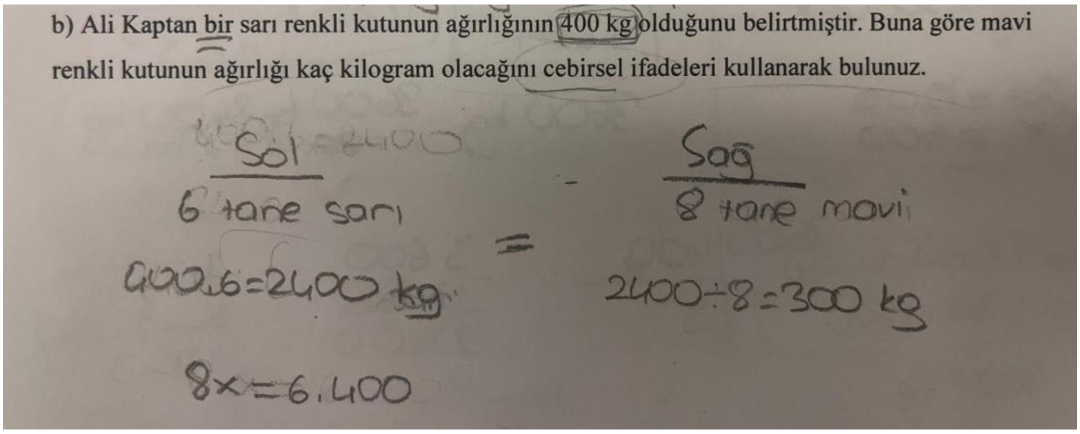


Figure 4.7 The Answer of the Second Group for Option b for the Ali's Captain Ship Activity

The first group stated that they could write 400 kilograms given in the question instead of *s* by using the equality 6*s*=8*m* they created in the previous option. In other words, here students wrote 400 instead of *s* and expressed it as 6·400=8*m*. Students were able to form equality by using the relational symbol meaning of the equal sign and expressing the relationship on the right and left sides of the equal sign with algebraic expressions. They solved the equality and found the weight of a blue box to be 300 kilograms. However, the second group had difficulty in forming equality by using the relational symbol meaning of the equal sign and algebraic expressions in the question in option b. Their thoughts on this issue are given in the dialog below.

... Melek and Zülal: Then we will divide 400 by 6...
Researcher: It wants you to solve using algebraic expressions.
Zülal: Look now, since 400 kilograms is equal in both, let's say x. x divided by 6, x divided by 8
Melek: Equals 400.
Zülal: No, not 400.
Melek: No, x times 6 equals 400, y times 8 equals 400.
Zülal: Cross?
Melek: We're going to multiply.

Zülal: But 400x.
Melek: No, the total weight was 400 kilograms.
Researcher: What has a total weight of 400 kilograms?
Melek and Zülal: The one in yellow (they mean the left side).
Researcher: No, one yellow box weighs 400 kilograms.
Melek: The weight of one yellow box.
Zülal: Hmmmm...
[...]

In the dialogue above, the students initially had difficulty grasping that 400 kilograms is the weight of a yellow box and the researcher pointed this out to them.

At first, the students tried to solve the question without using algebraic expressions because they had difficulty in solving it using algebraic expressions.

Researcher: You will do it with the algebraic expression. What do we need to form the algebraic expression?
Melek and Zülal: The unknown.
[...]
Researcher: Do you know the number of boxes?
Melek, Zülal and Emre: Yes.
Researcher: Do you know the weight of one yellow box?
Melek, Zülal and Emre: Yes.
Researcher: What does it want from you?
Melek and Zülal: Weight of the blue box.
Researcher: What is your unknown?
Melek and Zülal: Weight of the blue box.
Researcher: Based on this, you should create an algebraic expression.
Zülal: If 6 yellow boxes are 400, what are 8 blue boxes?
Melek: 6 of them are not 400. One of them is 400.
[…]

Melek: Let’s write something like this and try to turn it into an algebraic expression. 400 times 6, 2400. Total weight is 2400.
Zülal: 2400 divided by 8.
Melek: No, 3 divided by 3. We found 3 kilograms.
Researcher: 3 kilograms?
Melek: We will call the number of blue boxes x.
Zülal: We don’t know the number of boxes.
Researcher: You don’t know the number of boxes?
Melek: We know the number of boxes but not the weight of the blue box.

The dialog above shows that the students still had difficulty in solving the problem without using algebraic expressions.

The researcher guided them to write the number and weight of the boxes on the left and right sides of the ship.

Researcher: Let’s do it this way, you are very confused. Write yourself a right and left side. State what is on the right and left side.
Melek and Zülal: Six yellow boxes on the left. 8 blue boxes on the right.
Researcher: What information did this question give you?
Melek and Zülal: One yellow weighs 400 kilograms.
Researcher: Then what happened on the left?
Melek and Zülal: 4 times 6, 2400.
Researcher: What does it want from you?
Melek, Zülal and Emre: The weight of a blue box.
Researcher: We are trying to equalize the weight of the loads on the left and right side of the ship, right?
Melek: Yes.
Zülal: Then 2400 divided by 8 is what I said. You divided by 3.
Melek: Let’s write it normally and then write it algebraically.
Melek and Zülal: One of them weighs 300 kilograms.

After researcher's guidance, they multiplied the number of yellow boxes by the weight of the yellow box and divided the result by the number of blue boxes to arrive at the weight of one blue box, 300 kilograms.

While solving the question using algebraic expressions, they had difficulty in deciding what the unknown was. The researcher guided the students with questions and made them realize that the unknown was the weight of the blue box. The researcher also guided the students with questions to write the equation.

Researcher: It asks you to find it using the algebraic expression.
Melek: We know 2400 kilograms.
Zülal: Let's say x for the same ones. Let's say x for 2400.
Researcher: Back to the beginning again. What is your unknown here?
Melek, Zülal and Emre: Blue box weight.
Researcher: What are you going to call it?
Melek, Zülal and Emre: x
Researcher: From the very beginning of the problem, we are trying to make the weight of the loads on the right and left side of the ship equal, right?
Melek: Yes. We will equalize them. 400 times 6 equals 2400. We will equalize. We will call the weight of a blue box x. Equals 2400.
[...]
Researcher: (showing 400x6 on the written text) You know that this is the left side. What is the left side equal to?
Melek: Right.
Researcher: Then write down the information on the right side.
Melek and Zülal: 8 times x.
Melek: Why have we been struggling for so many hours? It's easy. But it's done now.
Researcher: Finally, explain how you wrote the algebraic expression.
Emre: A yellow box weighs 400. 400 times 6, since there are 6 of them. There are 8 of the blue box, but we don't know its weight, so we said x. These are equal to each other.

The dialog above shows that the students find the equality $400 \cdot 6 = 8x$ by making them equalize the right and left boxes weights to each other. As can be understood from this, the students have difficulty perceiving the equal sign as a relational symbol and thinking relationally.

Both groups found that they would load 3 yellow boxes by determining the number of boxes to add to the right side to ensure balance on the right and left sides of the ship by thinking relationally in the question in option c. However, both groups had difficulty establishing equality using algebraic expressions in this question. The first group's thinking on this issue is shown in the dialog below.

... Researcher: Can we solve using algebraic expressions?

Güney: Yes

Researcher: How can you solve it?

[...]

Researcher: What do we focus on when we write the algebraic expression? What do we need?

Cem: Unknown.

Researcher: What is your unknown here?

Cem: The number of yellow boxes to add on the right side.

Researcher: You don't know the number of yellow boxes. You always talk about establishing equality, so can you establish equality and write what is on the right side and what is on the left side?

Cem: There are 6 yellow boxes on the left side.

Researcher: What are we trying to make equal from the very beginning of the question?

Cem: Weights.

Researcher: How much does a box of 6 yellows weigh?

Cem:6s

Researcher: Don't you know the weight of the yellow box?

Cem: We know, 6 times 400.

Researcher: What should it be equal to?

Güney: 8 blue boxes.

Researcher: But there are not 8 boxes now. What is left?

Güney: 4 boxes.

Researcher: How do you find the weight of 4 blue boxes?

Cem: 4 times 300

Researcher: You wrote the total weights of the boxes on the right and left sides. The question tells us to add a yellow box on the right side. What is the weight of the yellow box?

Selin, Cem and Güney: 400 kilograms

Researcher: Do you know how many we should add?

Selin, Cem and Güney: No.

Researcher: How can we express it?

Selin, Cem and Güney: Let's say x.

Cem: 6·400= 4·300+400x

Güney: 400x?

Cem: Yes, because we don't know how many x will be.

Güney: Okay, but 400x what?

Cem: 400 is kilograms, x is the number of boxes

Güney: Okay.

Researcher: What are we going to do now?

Cem: 2400=1200+400x

Researcher: How do you find x?

Güney: We have to leave x alone, so 1200 will be move to on the other side. It'll be 2400-1200. 1200 equals 400x.

Cem: We divide both sides by 400. So x is left alone.

Güney: x becomes 3.

When you look at the dialog above, it is seen that the students had difficulty in writing an equality using algebraic expressions. The researcher guided them with questions to write the equality. The researcher started by asking them to determine the number of boxes on the left and right sides of the ship. She also directed them to create an equality expression between these two sides. Then, she focused on how to express the unknown, that is, the number of yellow boxes to be loaded on the right side, and had them express the load to be added on the right side. Finally, the students solved the equality they created using algebraic expressions and found the unknown, *x*, to be 3.

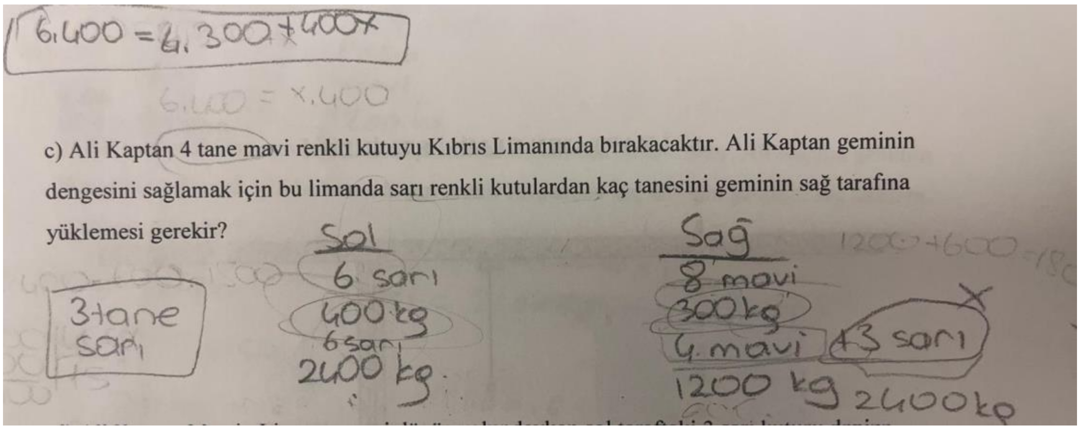


Figure 4.8 The Answer of the Second Group for Option c for the Ali's Captain Ship Activity

The dialog below shows how the second group established equality by using algebraic expressions in the question in option c.

... Researcher: Can we solve it using algebraic expressions?
Melek: 4 plus x equals 6.
Researcher: But is it saying that the number of boxes is equal?
Zülal: The number of boxes is not equal; the weight is equal.
Melek: 6 times 400 equals 4 times 300 plus x
Researcher: x what?
Melek and Zülal: x is yellow box
Researcher: x is the weight or number of yellow boxes?
Melek: x is the weight of the yellow box. We'll say 3 times x.

Researcher: Do you know the weight of the yellow box?

Melek and Zülal: Yes.

Emre: One of them is 400.

Researcher: What do you not know in the question?

Melek and Zülal: Number of yellow boxes.

Researcher: What are you going to call it?

Melek: x. Then we will say x times 400.

Researcher: Can you explain how you wrote the algebraic expression?

Melek: Now there are six yellow boxes. One of them weight is 400 kilograms. To equalize here, the weight of a box on the right side is 300, there are 4 left. This is 1200 here. We said how many yellow boxes we get from the port will equalize. We called x the number of boxes. We multiplied x by 400. This will come to 1200 kilograms.

Emre: I didn't understand. (Here the group members explain how they solved the question)

[...]

Emre: Ok. We multiplied 400 by 6. The left side weighs 2400. We multiplied 4 blue 300 for the right side. We multiplied 400 by x, that is, the number of yellow boxes, and added it.

The dialog above shows that the students needed guidance from the researcher while writing an equality using algebraic expressions. The researcher highlighted that the amount of boxes on the right and left sides of the ship were equal to each other. In addition, the researcher guided the students with questions to find the unknown. In other words, she guided them on how to express the number of yellow boxes to be loaded on the right side and the load to be added to the right side. The equality they created using algebraic expressions is $6 \cdot 400 = 4 \cdot 300 + 400x$. Then the student was able to express how they formed the equality.

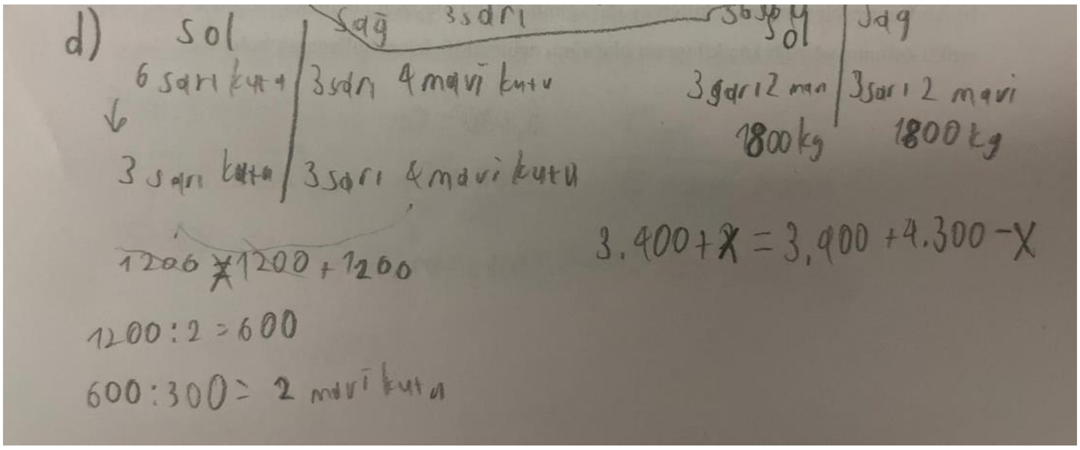


Figure 4.9 The Answer of the First Group for Option d for the Ali's Captain Ship Activity

The following dialog shows the relational thinking of the first group towards the question in option d.

... Researcher: Before moving on to the next option, please write down the last condition of the left and right sides of the ship.

Selin: There are six yellow boxes on the left side.

Güney: 3 yellow and 4 blue on the right side.

Researcher: You can read the other option.

Güney: The 3 yellow boxes on the left side fall into the sea, so 3 yellow boxes remain here (he is talking about the left side of the ship). How much will be transferred from right to left and what color?

Selin: It will be transferred in blue.

Cem: I think 2 blues from the right side should go to the left side.

Selin: Yes.

Cem: Because if 2 blues go from here, there will be 2 blues left here (talking about the right side). The yellows had already neutralized each other.

Güney: Yes.

Researcher: How will you express it? How will you show it?

Güney: Let's show it in operation. 1200 equals 900 plus...

Researcher: 900 what?

Güney: Weight of 3 yellow boxes.

Cem: 3 yellow boxes weigh 1200 because the yellow box weighs 400.

Güney: Yes.

Cem: 1200=1200+1200

Researcher: Can there be such equality?

Cem: So it's not equal.

Researcher: What are you going to do then?

Güney: Neutralization.

Cem: Since the 1200 on the left side takes away the 1200 on the right side, that is, since their weights are balanced, there is one 1200 left. We divide it in half and distribute it to the two sides.

Güney: 600 divided by 300 is 2 blue boxes because the blue box weighs 300 kilograms.

Researcher: What happened in the final situation?

Güney: 3 yellow, 2 blue on the left side.

Cem: 3 yellow, 2 blue on the right side.

As can be understood from the dialog above, the students stated that after 3 yellow boxes fell from the left side of the ship, 2 blue boxes should be transferred from right to left by using relational thinking in order to maintain the balance on the right and left sides of the ship. In order to explain this idea, they tried to establish equality about the weights of the remaining boxes on the right and left sides of the ship and found out how much boxes to transfer to the left side in order to achieve this equality. They stated that they had to transfer 2 blue boxes by dividing the weight they found by the weight of a blue box.

The relational thinking of the second group for the question in option d are given below.

... Melek: 6 yellow on the left, 4 blue and 3 yellow on the right.

Zülal: 3 yellow fall into the sea and 3 yellow remain.

Melek: This time it asks us which color to give from the one on the right.

Zülal: 3 yellow... 1200 kilograms left here (talking about the left side.)

Melek: This time we'll give and take on the right side.

Zülal: Yes.

Melek: To transfer it to the left side of the ship.

Zülal: If we give 3 yellows...

Melek: 1 minute. Let's take 3 yellow boxes. We have 3 yellow boxes left.

Zülal: Let's give 2 blues then.

Melek: Our left side is 1200. Our right side is 2400 kilograms. We will transfer from here to here (talking about the right side to the left side).

Zülal: 2 blue boxes. Each weighs 300 kilograms and when you multiply by 2, it is 600. When you subtract 600 from 2400, it is 1800. This is 1800 here (she means the left side).

Melek: We could just do it like this. If we give one of the blue ones across, it will be equal again.

Zülal: 1?

Melek: Yes.

Zülal: One weighs 300 kilograms. That's 1500. There will be 2.

Melek: How can there be two?

Zülal: You will achieve balance. You'll lose 600 kilograms, 2 blue. 2400 minus 600 is 800.

Melek: Wait, wait, and let's do it. There are 2 blue and 3 yellow left here, 3 yellow plus 2 blue on the left side.

Researcher: What about the weights then?

Melek: Then the left side is 3 yellow plus 2 blue, which is 1800 kilograms. The right side is 2 blue plus 3 yellow equals 1800 kilograms. Then we need to give 2 blue to the opposite side, the left side.

As can be understood from the dialogue above, the students discussed among themselves to find out which color and number of boxes to move to the left side of the ship. At this point, they used relational thinking and determined that 2 blue boxes should be transferred from right to left to ensure the equality of the boxes weights on the left and right sides of the ship. In order to achieve this equality, they found out how much boxes they had to transfer from the right side to the left side and stated that this weight corresponded to the weight of 2 blue boxes.

The students were asked how they could solve this question differently. The first group's thought is given below.

... Researcher: Could you solve this question in a different way? For example, can you solve it using algebraic expressions? ...You can start by writing the weights on the right and left sides.
Cem: Three yellow boxes on the left.
Researcher: How do we write?
Cem: 3 times 400.
Researcher: What does the question ask you to do on the left side?
Cem: We will add to it.
Researcher: What will you add?
Cem: Plus x.
Researcher: x what?
Güney: We don't know how much to add.
Researcher: What was on the right side?
Cem: 3 times 400 plus 4 times 300 minus x.
Researcher: Why did you make minus x?
Cem: Because we take x from the right and add it to the left. We do this for equalization.
Güney: We wrote $3 \cdot 400+x=3 \cdot 400+4 \cdot 300-x$.

According to the dialog above, the researcher asked the students if they could solve the question using algebraic expressions and when they did not answer, the researcher encouraged them to create equality by having them determine the weights of boxes on the right and left sides of the ship. In order to preserve the equality of weights of boxes on the right and left sides of the ship and by thinking relationally, the students stated that the weights of boxes to be added to the left side, *x*, should be subtracted from the right side. Students were able to form the equality for this question by using algebraic expressions.

The second group's thoughts on how they could solve this question in a different way are given below.

... Researcher: Could you solve this question in a different way?
Melek: We would solve it using algebraic expressions. Now what we don't know here is how many boxes to transfer to the left side.
Zülal: 2 blue is x.
Melek: No, we will say x to the number of boxes we will give.
Zülal: Okay.
Melek: You know; we don't know if we're going to give the blue one. Now on our left side 3 yellow plus x equals...
Researcher: Were the numbers of boxes equal or...
Melek, Zülal and Emre: Kilograms
Melek: 1200 plus x equals 2400 minus x.
Researcher: What is x?
Zülal: How did you get this number of boxes from 2400?
Melek: Not the number of boxes but kilograms.
Emre: Kilogram.
Melek: x, that's 1200 kilograms.
Zülal: It's not happening right now, that's wrong.
Melek: How is it wrong? We move to 1200 on the other side and it became 2x. We got it right again. Divide 1200 by 2. When we move to x on the other side, we don't get 1x, we get 2x. That's a minus. 2x equals 1200. x equals 600.
Researcher: What does it want from you?
Melek: How many boxes does it want us to transfer to the left side? Actually we can continue.
Researcher: Yes. How?

Melek: We found 600 kilograms.

Researcher: Then you can think how can we get 600 kilograms from the right side?

Melek: Then we need to take two blue ones.

Researcher: Can you explain how you wrote the algebraic expression?

Emre: The total weight of the left side is 1200, since 3 fell into the sea. We will transfer the boxes from the right side to the left side, both will be equal. We called the box we will transfer x.

Melek: No, we said x per kilogram.

Emre: We will transfer kilograms from the right side to the left side. 1200+x=2400-x.

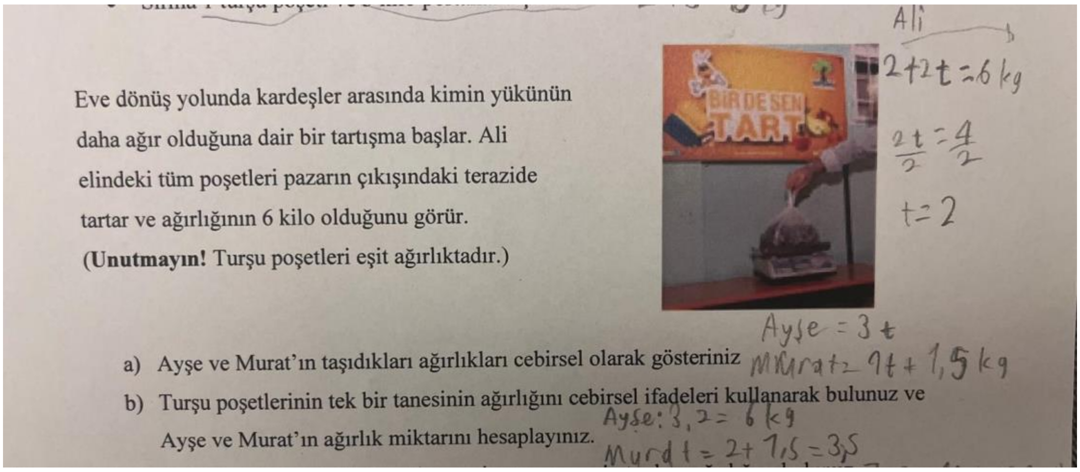
Melek: x would be 600. This corresponds to 2 blue boxes.

According to the dialog above, the students discussed among themselves and decided what the unknown was. In other words, the unknown was called *x*, the weights of boxes to be transferred to the left side. In order to maintain equality and by thinking relationally, the students stated that the weights of boxes to be added to the left side, *x*, should be subtracted from the right side. Students were able to form the equality for this question using algebraic expressions, i.e. the equality is $1200+x=2400-x$. They stated that the unknown is 600 and this corresponds to the weight of 2 blue boxes.

Students' work examined throughout the activity shows that both groups perceived the equal sign as a relational symbol, were able to think relationally, and improved in writing equations using algebraic expressions.

4.2.3 The Use of Variables as Unknowns (Let's Go to the Bazaar Activity)

Let's Go to The Bazaar activity is related to the use of variables as unknowns, which is a subcategory of the meaning of variables. This activity consists of 5 options.



Eve dönüş yolunda kardeşler arasında kimin yükünün daha ağır olduğuna dair bir tartışma başlar. Ali elindeki tüm poşetleri pazarın çıkışındaki terazide tartar ve ağırlığının 6 kilo olduğunu görür. (**Unutmayın!** Turşu poşetleri eşit ağırlıktadır.)

Ali
$2+2t=6$ kg
$\frac{2t}{2}=\frac{4}{2}$
$t=2$

a) Ayşe ve Murat'ın taşıdıkları ağırlıkları cebirsel olarak gösteriniz

Ayşe = 3t
Murat = 1t + 1,5 kg

b) Turşu poşetlerinin tek bir tanesinin ağırlığını cebirsel ifadeleri kullanarak bulunuz ve Ayşe ve Murat'ın ağırlık miktarını hesaplayınız.

Ayşe: 3.2 = 6 kg
Murat = 2+1,5 = 3,5

Figure 4.10 The Answer of the First Group for Option a and b for the Let's Go to the Bazaar Activity

The algebraic thinking related to the use of variables as unknowns in the question in option a of the first group are presented in the following dialog.

... Researcher: What does option a want from you?

Selin: We are asked to show the weights carried by Ayşe and Murat as algebraic expressions.

Cem: Let's find Ayşe's weight first. If there are 3 pickle bags, since we don't know its weight, we need to give an unknown to the weight.

Güney: x.

Cem: We can say t for its weight because it is pickle to avoid confusion.

Researcher: What was Ayşe carrying?

Selin: There are 3 pickle bags.

Cem and Selin: It's 3t.

Researcher: What is Murat carrying?

Selin: 1 pickle bag.

Güney and Selin: I mean 1 t.

Researcher: Does he only carry pickle bags, Murat?

Selin: A kilo of bananas and half a kilo of apples.

Cem: Let's call it 1m for bananas.

Güney: Let's say 0.5e.

In the dialog above, the students expressed that the weight of the pickle bag was unknown and indicated it with the symbol *t*. Thus, the students expressed the weights carried by Ayşe as 3*t*.

However, they acted as if the weights of the fruits were unknown, although they knew the weight of the fruit carried by Murat. The researcher made them realize their mistake by asking them questions.

Researcher: Do we know the weight of bananas and apples? Do we know the weight of fruits?
Selin and Cem: We know.
Researcher: If we know, can we act as if it is unknown?
Güney: No.
Researcher: Then what is the algebraic expression of the weight that Murat carries?
Güney: t+1.5 kilograms

After the researcher's guidance, the students expressed the weights carried Murat as *t*+1.5. It can be seen that Ayşe and Murat were able to express the weights they were carrying algebraically.

The algebraic thinking of the second group regarding the use of variables as unknowns in the question in option a are presented in the following dialog.

... Researcher: What does option a want from you?
Melek and Zülal: It wants us to write the weights carried by Ayşe and Murat as algebraic expressions.
Melek: Are we going to call the pickles x now? But we know the weight of the pickles, they are equal.
[...]
Melek: Let's call the pickle 5x, we don't know the kilogram of the pickle. 5x plus 2 kilograms of tangerines.

Researcher: Why did you say 5x? You have to express algebraically the weights carried by Ayşe and Murat separately.
Melek: Oh separately... Okay.
[...]
Melek: Let's call Ayşe 3x. We do not know the kilogram of the pickle bag.
Zülal: Ayşe 3x.
Researcher: Why did you say 3x?
Melek: x is the kilogram of the pickle bag.
Researcher: What is Murat's algebraic expression?
Melek: x plus 1 plus 0.5 kilograms.
Zülal: Can't we just write 1.5?
Melek: Let's just write 1.5.

Looking at the dialog above, the students had difficulties in understanding the question and they thought that they would write the sum of the weights carried by Ayşe and Murat. The researcher clearly explained to them what was asked in the question. Then, they stated that the weight of the pickle bag was unknown and represented it with a symbol x. They expressed the weights carried by Ayşe and Murat as $3x$ and $x+1.5$.

In the following dialog, the algebraic thinking of the first group on how they found the unknown by using variables as unknowns in the question in option b are revealed.

... Researcher: What does option b want from you?
Selin: Find the weight of a single pickle bag by making an equation and calculate the amount of weight carried by Ayşe and Murat.
Cem: We can use Ali's bags to find the weight of the pickle bag because the weight of Ali's bags is known.
Güney: 2 kilograms of tangerines
Cem: 2 kilograms of tangerines + 2t equals to…
Güney: 6 kilograms.
Researcher: Where did you get 2 kilograms of tangerines + 2t?

Selin: Ali bought 2 kilograms of tangerines and 2 bags of pickles.
Researcher: So you wrote the weight that Ali carried as an algebraic expression. What is the weight that Ali is carrying?
Cem, Selin and Güney: 6 kilograms.
Güney: To leave the unknown alone, we move to 2 on the other side of the equality as the negative 2. 2t is equal to 6 minus 2. Thus, 2t is 4. We divide by 2 on both sides of the equality.
Cem: Leave t alone. t is equal to 2.
Researcher: You know the value of t. It wants you to find the amount of weight carried by Ayşe and Murat.
Güney: 3 times 2…
Cem: If it's 3t, we substitute 2 for t. 3 times 2...
Cem and Güney: Ayşe carries a weight of 6 kilograms.
Güney: For Murat, 2 plus 1.5 equals 3.5 kilograms.

Looking at the dialog above, the students were able to form equality by algebraically expressing the weight that Ali was carrying. They showed that it was $2t+2=6$, and they used this to find that the weight of a pickle bag was 2 kilograms. After finding the weight of the pickle bag, they used the algebraic expressions they created in option a to find that the weights carried by Ayşe and Murat were 6 and 3.5 kilograms, respectively.

In the following dialog, the algebraic thinking of the second group on how they found the unknown by using variables as unknowns in the question in option b are revealed.

... Researcher: What does option b want from you?
Melek: Find the weight of the pickle bag algebraically and find the weights carried by Ayşe and Murat.
Zülal: It said 6 kilograms.
Researcher: 6 kilograms of what?
Melek: The total weight of Ali's bag. Now we cannot find 6 kilograms minus 2x minus 2 kilograms.

Researcher: You know the weight of the bags carried by Ali. In the previous option, you expressed the weights carried by Ayşe and Murat algebraically. Can you also express Ali's weight algebraically?
Melek and Zülal: Yes.
Zülal: 2x+2
Researcher: What was the total weight that Ali was carrying?
Melek: 6 kilograms.
Zülal: It is equal to 6.
Melek and Zülal: We can move to 2 on the other side of the equality.
Melek: 2x equals 4 kilograms. x equals 2 kilograms.

According to dialog, since the students had difficulty establishing an equality using the weight Ali was carrying, the researcher guided them. She asked them to express the weight Ali was carrying algebraically, and they formed it as $2x+2=6$. They then found that the weight of a pickle bag was 2 kilograms.

The following dialog shows how Ayşe and Murat find out how many kilograms the weights they are carrying are.

Researcher: What is the rest of the question asking you?
Melek: We will calculate the weights carried by Ayşe and Murat.
Zülal: Ayşe 3 pickle bags, 3x.
Melek: We found the pickle bag.
Zülal: 2 kilograms. 3 times 2.
Melek: Ayşe carries 6 kilograms. Murat would weigh 3.5 kilograms.
Zülal: We replaced x with 2. 2 plus 1.5.

Looking at the dialog above, after finding the weight of the pickle bag, they used the algebraic expressions they created in option a to find that the weights carried by Ayşe and Murat were 6 and 3.5 kilograms, respectively.

The dialog below shows how the first group constructed algebraic expressions using the unknown meaning of the variable in the question in option e and how they found the unknown.

... Güney: Since we don't know the amount of carrot, we can say x. The white cabbage is three times as much.

Cem: Then 3x.

Güney: Yeah, 3x. That's 500g more cucumber than twice the amount of carrots.

Güney and Cem: 2x+500 grams

Cem: A quarter of the carrot is the green pepper, so x times $\frac{1}{4}$.

Güney: Yes.

Researcher: What does the rest of the question say?

Cem: So, the sum of all the ingredients here is equal to 5.5 kilograms of pickles.

Güney: Then let's add them all together and establish equality.

Cem: x plus...

Selin and Cem: 3x.

Selin: Plus, 2x plus 500 grams.

Cem and Selin: Plus x times $\frac{1}{4}$.

Researcher: What does this equal to?

Selin: 5.5 kilograms.

Researcher: What are you going to do now?

Güney: We need to simplify it.

Cem: There are grams here and kilograms here. We can write this (he is talking about 500 grams) as 0.5 kilograms.

Güney: We can convert kilograms to grams.

Cem: Yes.

Güney: 5500 grams.

Researcher: How can you organize the algebraic expression you wrote?

Güney: 6x plus 500 grams plus x times $\frac{1}{4}$ *equals 5500 grams.*

Cem: We can move to 500 grams on the other side and leave the unknowns alone.

Güney: Yes. It will act as negative.

Cem and Güney: 6x plus x times $\frac{1}{4}$ equals 5500 minus 500 which equals 5000.

Researcher: How do we add 6x plus x times $\frac{1}{4}$?

[...]

According to above dialog, the students determined the amount of carrot as unknown and expressed it with a symbol *x*. They determined the other ingredients by using algebraic expressions using the amount of carrot and formed equality. However, they had difficulty in organizing the equality when adding the expressions 6*x* and $\frac{x}{4}$.

The following dialog shows how the researcher guided the students by asking them to remember fraction addition.

Researcher: How do we add in fractions?

Selin, Cem and Güney: Make the denominator equal.

Researcher: Then make their denominators equal.

Cem: Their hidden denominator is one (he is talking about 6x and x times $\frac{1}{4}$).

Researcher: Does x times $\frac{1}{4}$ have no denominator?

Cem: Yes, we do. Then we write it as $\frac{x}{4}$.

Researcher: What happens then?

Cem: $\frac{6x}{1}$ plus $\frac{x}{4}$ equals 5000 grams. To make the denominators equal, we multiply 1 by 4.

Güney: So $\frac{24x}{4}$

Cem: Plus $\frac{x}{4}$ equals 5000 grams.

Researcher: Can we collect it now?

Selin: $\frac{25x}{4}$ equals 5000 grams.

Researcher: How do you find x?

Güney: 25 divided by 4. It is not an exact division.

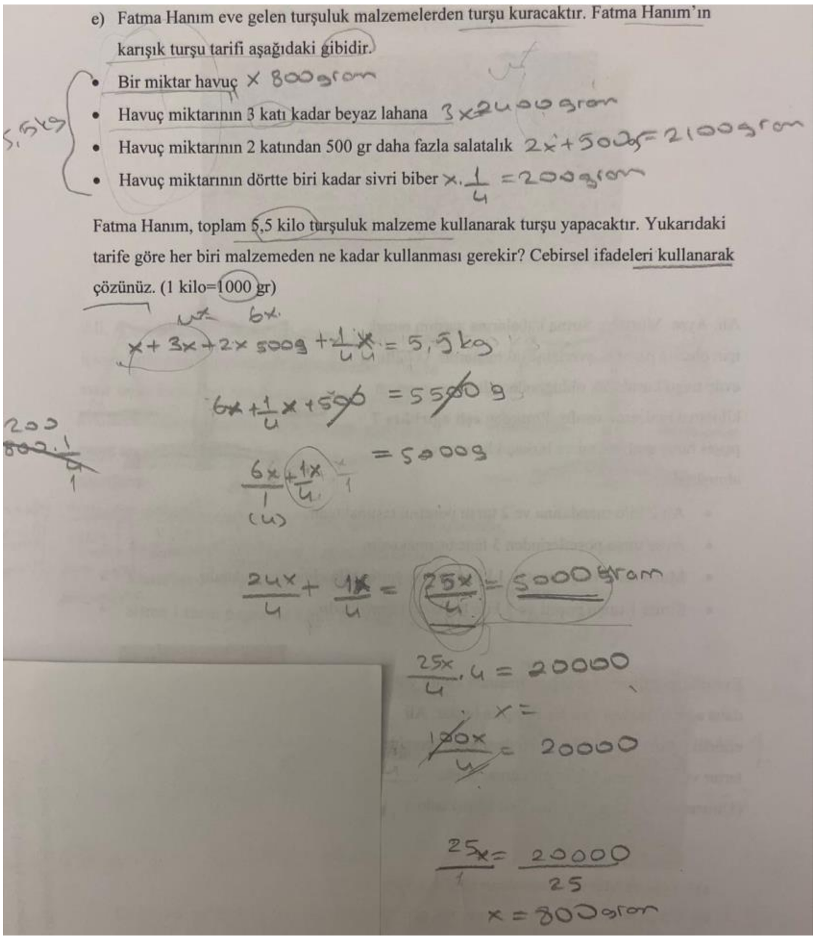
[...]

After researcher's questions, the students were able to add the two expressions and find $\frac{25x}{4}$ =5000 grams. However, they had difficulty trying to find the value of *x*.

The researcher explained the meaning of this equation to the students and guided them as follows.

Researcher: Let's read the expression. 25x divided by 4 equals 5000 grams. So when you divide 25x by 4, you get 5000 grams.
Güney: What if we multiply 5000 by 4?
Cem: Let's multiply. We will cross this division as a cross.
Cem and Güney: 25x equals 5000 times 4.
Güney: Equals 20,000.
Cem: We divide each side by 25. x is 800 grams.
Researcher: Then determine the quantities of the materials.
Cem: Carrots 800 grams.
Researcher: What is the amount of white cabbage?
Güney: 3 times so...
Researcher: What was the value of x?
Selin: 800 grams.
Researcher: What is the amount of white cabbage?
Selin: 3 times 800.
Cem: 2400 grams.
Researcher: What is the amount of cucumber?
Selin: 2 times 800.
Güney: Plus 500.
Cem: 2100 grams.
Researcher: What is the amount of green pepper?
Cem: Since 800 times 14*, we divide 800 by 4.*
Güney: 200 grams.

As seen in the dialog, the students multiplied both sides of the equation by 4 to get rid of the number 4 in the denominator of the fraction and reached the weight of 800 grams of carrot. Using the weight of the amount of carrots, they found out how many kilograms the weights of the other ingredients were. Thus, they were able to use algebraic expressions to determine how much pickle ingredient to use.


e) Fatma Hanım eve gelen turşuluk malzemelerden turşu kuracaktır. Fatma Hanım'ın karışık turşu tarifi aşağıdaki gibidir.

- Bir miktar havuç × 800gram
- Havuç miktarının 3 katı kadar beyaz lahana 3×2400gram
- Havuç miktarının 2 katından 500 gr daha fazla salatalık 2x+500=2100gram
- Havuç miktarının dörtte biri kadar sivri biber $x.\frac{1}{4}$ =200gram

Fatma Hanım, toplam 5,5 kilo turşuluk malzeme kullanarak turşu yapacaktır. Yukarıdaki tarife göre her biri malzemeden ne kadar kullanması gerekir? Cebirsel ifadeleri kullanarak çözünüz. (1 kilo=1000 gr)

$x+3x+2x\ 500g+\frac{1}{4}x = 5,5kg$

$6x+\frac{1}{4}x+500 = 5500g$

$=5000g$

$\frac{6x}{1}+\frac{1x}{4}$

$\frac{24x}{4}+\frac{1x}{4}=\frac{25x}{4}=5000gram$

$\frac{25x}{4}.4=20000$

$\frac{25x}{1}=\frac{20000}{25}$

$x=800gram$

Figure 4.11 The Answer of the Second Group for Option e for the Let's Go to the Bazaar Activity

The dialog below shows how the second group constructed algebraic expressions using the unknown meaning of the variable in the question in option e and how they found the unknown.

... Melek: It said some carrots, let's call it x.

Zülal: x unknown.

Melek: 3 times, then 3x.

Zülal: White cabbage.

Melek: 500 grams more cucumber than 2 times the amount of carrots.

Zülal: 3x plus 500 grams.

Melek: 2x+500. It says the amount of carrots.

Zülal: Oh yes.

Melek: The amount of carrots is $\frac{1}{4}$ *green pepper, x times* $\frac{1}{4}$*. Now we need to find x.*

Researcher: You have expressed the amount of each material algebraically. How much material is there in total?

Melek and Zülal: 5.5 kilograms.

Researcher: Then can you show this?

(They wrote the equation x+3x+2x+500 grams+ x. $\frac{1}{4}$*= 5.5 kilograms)*

Zülal: Then let's convert it to grams first, 5500 grams.

Researcher: How can you organize the algebraic expression you wrote?

Melek: Let's add up the x's. It makes 7x.

Researcher: How did you find the 7x?

Emre: 7 not 6.

Melek: That's 4x, 6x. There's another x.

Researcher: The expression is x times $\frac{1}{4}$*.*

Zülal: 6x...

Melek: Then let's not take it.

Researcher: How is that going to happen?

Zülal: Let's write 6x plus x times $\frac{1}{4}$ *first.*

Melek: Plus 500 grams.

Zülal: Let's subtract 500 grams directly.

Melek: Let's just take it off. That leaves 5000 grams.

Zülal: 6x+ x times $\frac{1}{4}$ *equals 5000 grams.*

As seen in the dialog, the students expressed that the amount of carrots was unknown and indicated it with the symbol *x*. Based on the amount of carrots, they determined the amount of other ingredients using algebraic expressions and formed an equation. However, they could not add the expressions 6*x* and $\frac{x}{4}$ while forming this equation.

The following dialog shows how the researcher guided the students by asking them to remember fraction addition.

... Researcher: How do we add 6x plus x times 14*? How do we add in fractions?*
Melek: Make the denominator equal.
Zülal: $\frac{6x}{1}$*, we will expand it with 4.*
Melek: $\frac{24x}{4}$ *plus* $\frac{4x}{4}$
Zülal: $\frac{25x}{4}$
Melek: $\frac{28x}{4}$ equals *5000 grams.*
Zülal: What? What did you do?
Melek: I got it, $\frac{28x}{4}$*.*
Zülal: Melek?
Melek: Why did I equalize the denominator; it is already equal. Yes, $\frac{25x}{4}$ *equals 5000 grams. We will divide these.*
Zülal: Didn't we turn it upside down and multiply it?
Melek: Why are we going to multiply?
Zülal: Fractions.
Melek: We'll divide this by that and find x.
Researcher: Let's read the expression. 25x divided by 4 equals 5000 grams. So when you divide 25x by 4, you get 5000 grams.
[...]

With the necessary guidance, the students were able to add these two expressions and reached the expression $\frac{25x}{4}$=5000. However, they had difficulties on how to find the value of *x*.

The researcher explained the meaning of this equation to the students and guided them as follows.

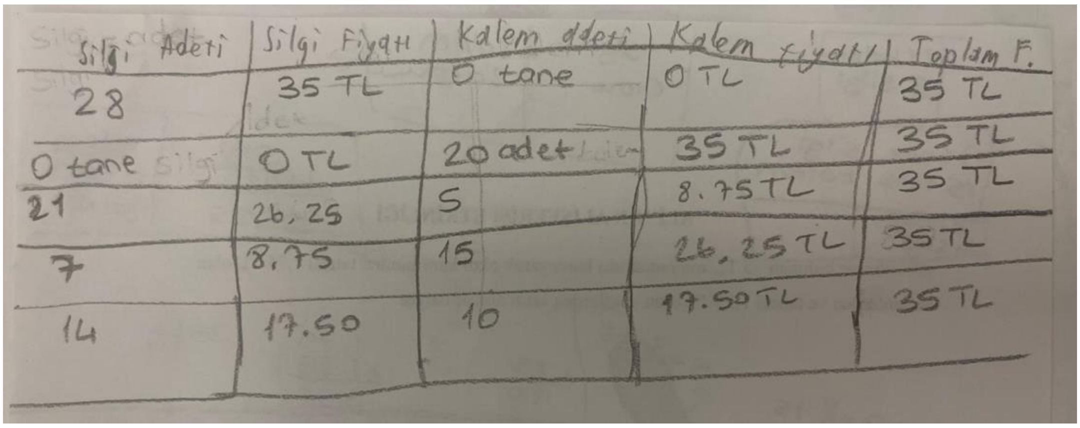
… Researcher: What would happen if 25x was not divided by 4?
Zülal: Then we will multiply.
Melek: Then we multiply $\frac{25x}{4}$ *by 4. We will multiply 5000 by 4.*
Zülal: $\frac{25x}{4}$ *times 4 is* $\frac{100x}{4}$.
Melek: Equals 20000 grams.
Zülal: $\frac{100x}{4}$ *equals 25x.*
Melek: 25x equals 20000.
Emre: x equals 800 grams.
Researcher: After finding the unknown, how can we find the weight of all the ingredients used for pickles?
Melek: Yes. Then the carrot is 800 grams. White cabbage 2400 grams. Cucumber 2100 grams. If it's 14*, let's find* 14 *times 800. That's 200 grams.*

Looking at the dialog above, in order to get rid of the number 4 in the denominator of the fraction $\frac{25x}{4}$, students multiplied both sides of the equation by 4 to find the weight of the carrot. They found the weight of the other ingredients from the weight of the carrot. Thus, they were able to find how much of the pickle ingredients they should use by using algebraic expressions.

If an evaluation is made from the beginning to the end of the activity, the students were able to use the variable in the sense of the unknown and improved themselves in finding the value of the unknown.

4.2.4 The Use of Variables as Varying Quantities (Ali’s Shopping Activity)

Ali’s Shopping activity is related to the use of variables as varying quantities, which is subcategories of the meaning of variables. This activity consists of 5 interconnected options.



Silgi Adeti	Silgi Fiyatı	Kalem adeti	Kalem fiyatı	Toplam F.
28	35 TL	0 tane	0 TL	35 TL
0 tane silgi	0 TL	20 adet	35 TL	35 TL
21	26,25	5	8.75 TL	35 TL
7	8,75	15	26,25 TL	35 TL
14	17.50	10	17.50 TL	35 TL

Figure 4.12 The Answer of the First Group for Option a for Ali’s Shopping Activity

The algebraic thinking related to the use of variables as varying quantities in the question in option a of the first group are presented in the following dialog.

... Researcher: So, what does this question asking you?
Cem: It asks how many erasers and pencils we can buy and make this exchange in different ways.
Researcher: And on what condition does it want you to do this?
Güney: It want to spend all 35 liras.
Selin: Let’s determine their numbers first.
Cem: I think we should first add up the price of the eraser and the pencil. Then we divide it by 35. We can find out how many of each we should buy.

Güney: When we divide it, we have 2 liras left. We can buy a pencil or an eraser, but there is still money left over.
Researcher: Were you able to spend all 35 liras?
Cem: No.
Researcher: How will you proceed? In order to find out how many different ways this shopping can be done, can you give different values to the number of pencils and erasers and then check whether the sum of their prices is 35 TL according to the given numbers?
Cem: The price of an eraser is 1.25. I think we should start with pairs because it will be complete if we make pairs.
Güney: We can also do something like that. We can look at the price of the eraser and divide it by 35. We can only buy the eraser.
Cem: The question says that pencils and erasers were bought, which means that these two were bought.
Researcher: The question says that it bought pencils and erasers, but it does not specify how it bought them.
Selin: Yes.
Cem: Then let's divide it.
Güney: 4 erasers of 1.25 is 5 liras. 35 divided by 5 is 7. 4 times 7 is 28. That's 28 erasers. Let's multiply 8 by 1.25 and make sure it works.
Selin: Let's give it a try. It's 35.
Güney: We bought 28 erasers. That's 35 TL.
Selin: It was 0 pens.

In the dialog above, the students gave values to the number of pencils and erasers and found their first possibilities about how many different ways they could do this shopping with the guidance of the researcher. In their first possibility, they stated that they spent the entire 35 TL by buying 28 erasers without buying any pencils.

In the dialog below, the first group's thoughts on whether this exchange was done in a different way than the possibility they found are given.

... Researcher: Are there other values that the number of pencils and erasers can take?
Güney: We can only try to buy it from the pen.
Cem: Yes.
Güney: Let's make it.
Cem: Let's multiply by 4. When we multiply by 4, the price of the pen becomes a whole number. That's 7 liras, unless I miscalculated.
Güney: You can have 2. No, don't.
Cem: 4 pieces is 7 liras. Then let's divide 35 by 7. It is 5.
Güney: We multiply 5 by 4.
Selin: That'll be 20.
Güney: Let's verify it and multiply it (talking about multiplying the price of a pen by 20)
Selin: 35 liras will do.

The dialog above shows how the students expressed their second possibility for this purchase. In the first possibility, they only bought erasers, so in this possibility they thought they could only buy pencils. In the second possibility, they stated that they spent the entire 35 liras by buying 20 pencils without buying any erasers.

While the students were thinking about finding other possibilities, they bought the least number of erasers and pencils and how they found the situation where their prices were equal is given below.

.... Researcher: Are there other values that the number of pencils and erasers can take? In one of the possibilities you have found so far, you took an eraser without buying a pencil. In the other one, you took a pencil without buying an eraser. Can we get it from both?
Cem: Yes. We can find out by trial and error.
Güney: I just thought of something. When you add erasers and pencils together, it costs 3 liras. Instead, we can find the least common multiple. We can go from there.
Researcher: How?

Güney: Until we reach the same price.

Cem: I don't understand.

Selin: Say it again.

Güney: We will make the price of the eraser and the pencil equal by finding the least common multiple. We can subtract it from the unit price.

Researcher: I understand that you are talking about equalizing the price of erasers and pencils. How are you going to do that?

Güney: We will buy erasers and pencils. The prices of what we buy will be equal. We will find the least common multiple.

Cem: We will find the least common multiple.

Güney: The same number that the two have in common. We'll equalize the price and we'll take the smallest one.

Researcher: Will you try to equalize the prices so that the number of erasers and pencils purchased is the minimum?

Selin: So we will multiply a number with 1.25. We multiply 1.75 by another number. Will these be equal to each other?

Güney: Yes.

Selin: For example, can we multiply 1.25 by 5?

Güney: If we multiply by 5...

Cem: 6.25.

Güney: Yes, that's 6.25. If we multiply 1.75 by 6...

Cem: But we will multiply 1.75 by a smaller number than we multiplied the eraser by. The number of the pencil is more.

Güney: I got it. They equalize at 8.75.

Selin: How many did you multiply?

Güney: I used 5 for the pencil and 7 for the eraser.

Cem: Yes, they both equalize at 8.75.

In the dialog above, the students buy the least number of erasers and pencils and equalize their prices. The students gave different values to the number of erasers and pencils and tried them and found that the prices of 7 erasers and 5 pencils were equal to each other and equal to 8.75 liras.

How the first group found other possibilities by decreasing and increasing the number of erasers and pencils using this equality is given below.

Researcher: What are you going to do next?
Güney: We'll subtract 7 from the eraser and add 5 to the pencil. To keep it balanced.
Cem: So we add 5 to 20?
Selin: No, we will subtract.
Güney: No, we add 5 to the pencil and subtract 7 from the eraser.
[...]
Güney: Now when we buy 28 erasers, it costs 35 liras. We'll take out 7 erasers. When we subtract 7 erasers...
Cem: 21 erasers.
Güney: We will add 5 to the pen.
Cem: So that's 21 erasers and 5 pencils.
Researcher: Can you decrease or increase the number of erasers and pencils according to the number of erasers and pencils in this situation?
Cem: We will subtract 5 from 20 pencils. We will add 7 erasers to 0 erasers.
Güney: I don't understand.
Cem: We subtract 5 pencils from 20 pencils. This way we lose 8.75. We add 7 erasers to 0 erasers to get 8.75 back.
Researcher: Are there other values that the number of pencils and erasers can take?
Güney and Selin: I think we found it.
Cem: In the probability of 21 erasers and 5 pencils, we subtract 7 of the erasers. We lose 8.75. We earn 8.75 by adding 5 pencils.
Researcher: Are there other values that the number of pencils and erasers can take?
Cem, Selin and Güney: No, we found them all.

As seen in the dialog above, they proceeded with the first possibility they found (28 erasers cost 35 liras and 0 pencils cost 0 lira) and found another possibility by subtracting 7 erasers to 28 erasers and adding 5 pencils to 0 pencil. The students were aware that by subtracting 7 pencils, 35 liras would decrease by 8.75 and by adding 5 pencils, 35 liras would increase by 8.75, and they maintained the balance in this way. Then, they proceeded with the second possibility they found (0 erasers cost 0 lira and 20 pencils cost 35 liras) and found another possibility by adding 7 erasers to 0 eraser and subtracting 5 pencils to 20 pencils. Lastly, they proceeded with the third possibility they found (21 erasers cost 26.25 lira and 5 pencils cost 8.75 liras) and found another possibility by subtracting 7 erasers to 21 erasers and adding 5 pencils to 5 pencils. As a result, the students expressed all the possibilities by finding three other possibilities that Ali could make this purchase.

In the following dialog, the algebraic thinking related to the use of variables as varying quantities in the question in option a of the second group are given.

... Melek: We write the number of pencils, the price and the number of erasers, the price. When we give 5 erasers... Let's give 4 so that it comes out whole number, not decimal.

Zülal: How much for the pen?

Melek: Let's give 4 for the eraser. When it's 4, it becomes 5 liras.

Zülal: It is less.

Melek: It'll be 5 liras.

Zülal: Okay, but we have 35 liras, we have to spend it all.

Melek: Yes, we need to give a lot to the pencil. Let's give 8 for the eraser. This time the eraser will be 10 liras.

Zülal: Let's give 16 or something.

Melek: If we give 12, the price of an eraser will be 15. How many liras do we have left? We have 20 liras left. 20 liras are for 1.75 liras.

Zülal: Is it possible?

Melek: It could be divided. That makes 12. The price is 20 liras.

Researcher: Are you sure?

Zülal: Let's multiply 12 by 1.75.

Melek: The price is too high.

Zülal: I think we divided it wrong.

Melek: I think we divided it wrong too. It's 25 liras. If the number of pens is 12, that's 25 liras.

[…]

Melek: Just now, 12 of them cost 15 liras. Anyway, let's give 7 for the eraser and multiply it. That's 8 liras 75 pennies. Let's subtract 8 liras 75 pennies from 35 liras. That's 26 liras 25 pennies. Let's divide. (They divided 26.25 by 1.75)

Zülal: 15.

Melek: Then if the number of pens is 15, it will be 26 liras 25 pennies.

In the dialog above, the students' ideas about how many different ways Ali did this exchange are given. By giving different values to the number of pencils and erasers, the students found the first possibility of how many different ways they could do this shopping. In their first possibility, they stated that when they bought 7 erasers, they bought 15 pencils and spent the entire 35 liras.

In the following dialog, the second group's thoughts on whether this exchange was done in a different way than the possibility they found are given below.

... Researcher: Are there other values that the number of pencils and erasers can take?

Melek: Yes.

Researcher: How can you find it? Then can you find other possibilities by decreasing one and increasing the other number of pencils and erasers?

Melek: Let's decrease the number of pencils by one and increase the number of erasers by one.

Researcher: How much difference does this decreasing and increasing make?

Zülal: 1 lira makes a difference.

Melek: No, 50 penny makes a difference.

Researcher: Then is all 35 liras spent?

Melek: No. Then if we increase the eraser by 2, it will be 2.5 liras, if we decrease the pencil by 2, it will be 3.5 liras. It didn't work again.

As seen in the dialog above, since the students had difficulty finding other possibilities for this exchange.

The researcher guided them and asked them to buy the minimum number of erasers and pencils and find the situation where their prices are equal. This is shown in the following dialog.

Researcher: Then let's do it like this. In which case will the prices of the erasers and pencils purchased be equal to each other with the minimum number of pieces?
Melek: Then let's try it. Taking 2 doesn't work, let's try it differently.
Zülal: Will 10 liras 50 pennies equalize?
Melek: But it says the least. Let's think the least.
Zülal: 2 doesn't work, 3 doesn't work either.
Melek: It cannot be 3.5 liras because 3.5 liras cannot be divided into 1.25 liras. Let's take 3 from this.
Zülal: Which one, the pen?
Melek: Yes, if we take 3 pencils, we get 5.25 liras. If we divide 5.25 by 1.25…
Zülal: There are 4 times. It doesn't divide exactly.
[...]
Melek: Let's give the pen 5.
Zülal: 8.75 liras.
Melek: Maybe this will equalize it. Let's divide it.
Zülal: Okay, we'll give 5 for this and 7 for that.
Researcher: So when are the prices equal?
Emre: 5 pencils are equal to the price of 7 erasers, which are 8.75 liras.

The students gave different values to the number of erasers and pencils and tried them and found that the prices of 7 erasers and 5 pencils were equal to each other and equal to 8.75 liras.

The researcher then guided them to decrease or increase the number of erasers and pencils using this equality which is the price of 7 erasers and 5 pencils is the same and 8.75 liras. This is shown in below.

Researcher: In this case, can we decrease or increase the number of erasers and pencils according to the number of erasers and pencils in the equation?
Melek: Yes.
Zülal and Emre: I don't understand.
Melek: Now, for example, if we received 3 liras from the eraser, we need to add 3 liras to the pencil. We will ensure equality. If we take 7 erasers from here, 0 erasers remain. We need to add 5 pencils here.
Zülal and Emre: I understand.
Melek: When I bought 15 pencils, the price was 26.25 liras. If you buy 7 erasers, the price is 8.75 liras. 35 liras are in total. We had found this. Now let's do as I just said. Let's buy 20 pencils and make 0 erasers.
Zülal: Okay.

They proceeded with the first possibility they found (7 erasers cost 8.75 liras and 15 pencils cost 26.25 liras) and found another possibility by increasing 15 pencils by 5 and decreasing 7 erasers by 7 erasers. In other words, they increased the price of pencils by 8.75 liras and decreased the price of erasers by the same amount and they maintained the balance in this way.

The dialog below shows how the students expressed their third possibility for this purchase.

Melek: The price of the pencil is already 35 TL. Now, when you take back 10 pencils (she is talking about reducing by 10), the price of the eraser should be added 17.50 liras. How many should we divide it by?
Zülal: 1.25.
Melek: That's 14 erasers. We'll add 14 erasers here. 21 erasers and 5 pencils (they added and subtracted from the first possibility)

They proceeded with the first possibility they found (7 erasers cost 8.75 liras and 15 pencils cost 26.25 liras) and found another possibility by decreasing 15 pencils by 10 and increasing 7 erasers by 14 erasers. In other words, they decreased the price of pencils by 17.5 liras and increased the price of erasers by the same amount and they maintained the balance in this way.

The dialog below shows how the students expressed their fourth possibility for this purchase.

Researcher: Are there other values that the number of pencils and erasers can take?
Melek: Let's make the pen 15.
Zülal: We have done that. Let's be pen 10.
Melek: That's 17.5. Subtract 17.5 from 35...
Zülal: Left 17.5 to buy erasers.
Melek: How many erasers do we buy?
Zülal: 17.5 divided by 1.25. That's 14.
Researcher: Are there other values that the number of pencils and erasers can take?
Emre, Melek and Zülal: None.

In their fourth possibility, the students tried to buy 10 pencils and calculated that they had 17.55 liras left from 35 liras. They found that they could buy 14 erasers with the remaining money. However, they could not find a final possibility.

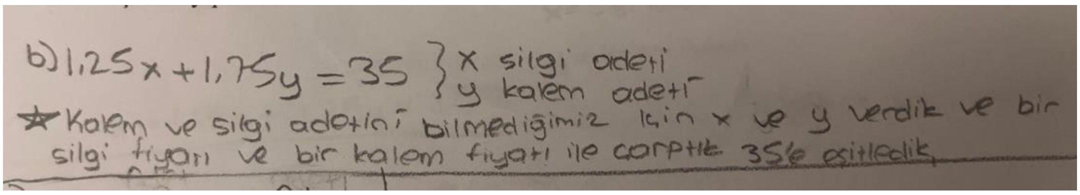


Figure 4.13 The Answer of the Second Group for Option b for Ali's Shopping Activity

The following dialog shows how the first group constructed the algebraic expression for the probabilities they found for the question in option b.

... Güney: There is no unknown...
Researcher: You are asked to write an algebraic expression involving the situations or possibilities you find in option a.
Güney: Okay.
Cem: Then we will do something, 1.25 times, for example, we can call the number of erasers s. Plus 1.75 times the number of pencils we can say k. 1.25 times s plus 1.75 times k equals 35 TL.
Selin: Did you make an addition sign there?
Cem: Yes. The sum of the two makes 35.
Researcher: Can you explain why you wrote it like that?
Cem: Since we don’t know the number of pencils or erasers, we multiply the number of erasers "s" by 1.25. We multiply 1.75 by the number of pencils "k". Thus, we can find 35.
Güney: Multiply the eraser price by the unknown number of erasers "s" and multiply the pencil price by the unknown number of pencils "k". The two are added together. This equals 35.

The dialog below shows how the second group constructed the algebraic expression for the probabilities they found for the question in option b.

... Emre: We will write its algebraic expression and explain it verbally.
Melek: Here the unknown is number. Now 1.25 times x equals 1.75 times y equals 35.
Emre: I don’t understand.
Melek: Now we don’t know the number. We call the number of an eraser x and the number of a pencil y.
Emre: Okay, I don’t understand how we wrote it.
Melek: What didn’t you understand?
Emre: You just said x times what you just said.

Melek: 1.25 times x plus 1.75 y is 35. Because 1.25 is the price of 1 eraser and 1.75 is the price of 1 pencil.
Emre: Okay.
Researcher: Can you explain how you wrote the algebraic expression?
Zülal: Since we don't know the number of pencils and erasers, we gave x and y. We multiplied by the prices and got 35.

As can be seen in the dialogues above, both groups were able to write algebraic expressions showing the probabilities they found for how many different ways Ali could exchange pencils and erasers. In addition, the students were able to explain how they wrote the algebraic expression. In other words, since they did not know the number of pencils and erasers, they stated that their unknowns were the number of pencils and erasers and expressed them with two different symbols. They were able to multiply the price of the eraser by the symbol for the number of erasers and the price of the pencil by the symbol for the number of pencils and show that their sum equals 35.

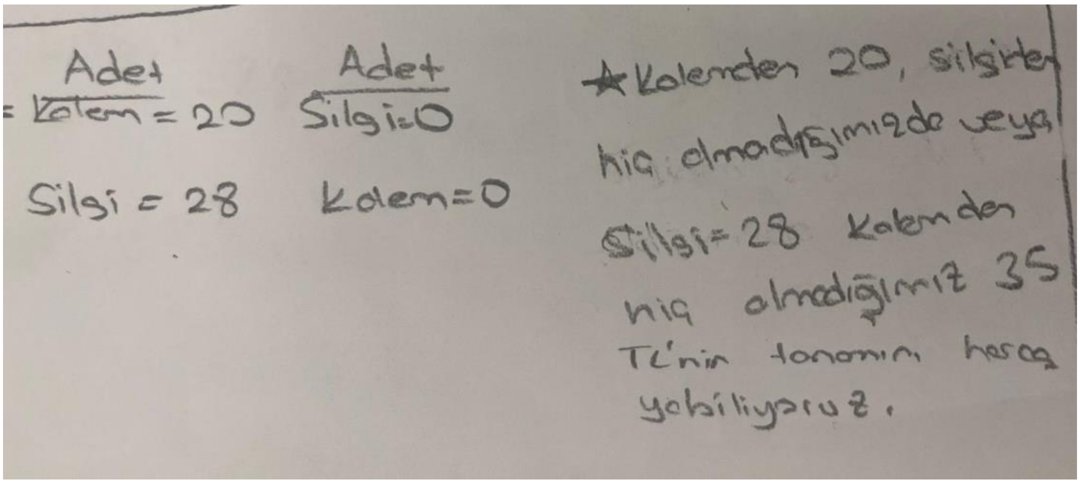


Figure 4.14 The Answer of the Second Group for Option c for Ali's Shopping Activity

The following dialogue reveals how the second group realized that the possibilities they found in the question in option c were missing and how they created the missing possibility.

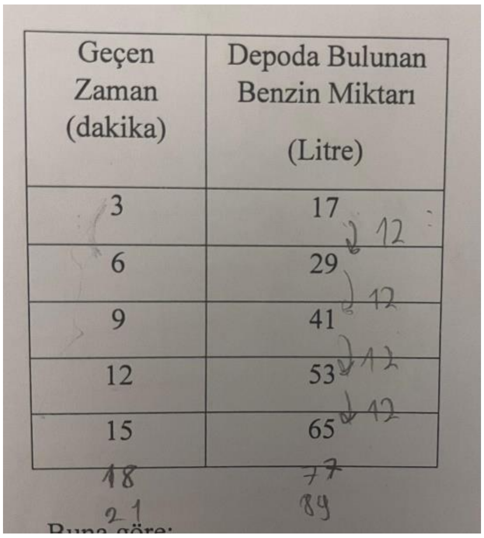
... Zülal: We made for the pencil. Let's try the eraser too. 35 divided by 1.25. There are 28 erasers.
Melek: Eraser 28, pencil 0.
Zülal: What's the number of pencils?
Melek: With 20 pencils, the eraser becomes zero.
Researcher: It wants you to explain.
Melek: When we buy 20 pencils or 28 erasers... But we didn't specify that we said 0.
Zülal: When we take 20 from the pencil, the eraser is 0. When we take 28 from the eraser, the pencil is 0.
Melek: We are not explaining it. When we buy 20 pencils and no erasers or 28 erasers and no pencils, we can spend the whole 35 TL.
Researcher: As you noticed, you found a different possibility than the one you found in option a.
Melek: Yes, eraser 28, pencil 0. We will add this to the table.

As seen in the dialogue above, the second group realized that they had forgotten a possibility about how many different ways Ali did the shopping. They stated that this possibility was when they did not buy any pencils. In other words, they found that they spent all 35 TL when they bought 28 erasers without buying any pencils. In this way, students in the second group were able to find all possibilities.

Students' work examined throughout the activity shows that both groups were able to use variables as varying quantities and were able to find all the values that the variables could take.

4.2.5 Quantitative Reasoning (The Gasoline Tank Activity)

The Gasoline Tank activity deals with establishing the relationship between quantities and interpreting and analyzing it, which is a subcategory of quantitative reasoning, one of the components of algebraic thinking. The activity consists of six interconnected options.



Geçen Zaman (dakika)	Depoda Bulunan Benzin Miktarı (Litre)
3	17
6	29
9	41
12	53
15	65

Figure 4.15 The Answer of the First Group for Option a for The Gasoline Tank Activity

In the following dialog, the question in option a shows how the first group establishes the relationship between quantities and interprets this relationship to find the amount of gasoline in the tank in a certain minute.

... Cem: There is a pattern here.
Selin and Güney: Yes.
Selin: What is the amount of increase in the pattern?
Güney: It increases twelve by twelve.
Researcher: What is increasing in this way?
Güney: The amount of gasoline in the tank. We can subtract 12 from 17, leaving 5.

Researcher: Option a asks for the amount of gasoline in the tank at 21 minutes.
Cem: We can do 12 times 21 plus 5.
Güney: Yes.
Selin: Let's try this.
Researcher: Why do you multiply 12 by 21?
Cem: Because here 12 is multiplied by 1 because it is the first step. Here is 12 multiplied by 1 and 5 added.

In the dialog above, students were able to find the amount of gasoline in the gas tank at the 21st minute by establishing the quantitative relationship. However, students had difficulty in finding the quantitative relationship. They thought of adding 5 by multiplying 21 by 12 and stated that 21 was the number of steps and 12 was the amount of increase in the amount of gasoline in the tank. They reached a wrong solution by perceiving minutes as the number of steps.

At this point, the researcher guided them and warned them to examine the information in the table carefully, emphasizing that the elapsed time and the amount of gasoline in the tank were given.

… Researcher: Could you examine the information in the table carefully? The table shows the elapsed time and the amount of gasoline in the tank.
Güney: Uh-huh. Here (the elapsed time) increases by thirds, here (the amount of gasoline in the tank) increases by twelve.
Researcher: Then can you find the amount of gasoline in the tank at minute 21 by paying attention to the increase in minutes and liters? How can we find this?
Güney: We can continue the pattern.
Selin: The elapsed time will be 18. Gasoline will be 77.
Güney: 89 in the 21st minute.

As seen in the dialog, they found that the amount of increase in elapsed time was 3 and the amount of increase in the amount of gasoline in the tank was 12. Taking into account the increase amounts, they increased the elapsed time by three and the amount of gasoline in the tank by twelve and found that the amount of gasoline in the tank at the 21st minute was 89 liters.

How the second group establishes the relationship between the quantities in the question in option a and how they find the amount of gasoline in the tank in a certain minute by interpreting this relationship is given in the dialog below.

... Melek: We go rhythmically.
Zülal: Yes.
Melek: Now let's calculate how much gasoline goes there (referring to the amount of gasoline in the tank).
Emre and Zülal: It increases twelve by twelve.
Melek: Here it goes in threes. (Talking about the past time) Let's write it here like in the table. 3 6 9...
Zülal: It's already written.
Melek: Let's write it down already there is space to go downwards. 3 6 9 12 15 18 21 (elapsed time). 17 29 41 53 65 77 89 (amount of gasoline in the tank). That's 89.
Researcher: How did you do it?
Melek and Zülal: We went rhythmically.
Melek: First we found the rule of the pattern and then we continued it.
Researcher: What is the rule of the pattern?
Emre: The liter increases twelve by twelve. The elapsed time increases by three.
Zülal and Melek: 89 liters at 21 minutes.

In the dialog above, students were able to find the amount of gasoline in the gas tank in the 21st minute by establishing the quantitative relationship. They found that the increase in the elapsed time was 3 and the increase in the amount of gasoline in the tank was 12. Taking into account the amount of increase, they increased the elapsed time by three and found the amount of gasoline in the tank twelve by twelve and the amount of gasoline in the tank at the 21st minute was 89 liters.

The dialog below shows how the first group tried to find the amount of gasoline filled per minute using algebraic expressions by establishing a ratio-proportion in option c.

... Güney: We can do proportionality.
Cem: If 17 in 3 minutes, what is 1 minute?
Researcher: Does the information in the table give you the amount of gasoline filled in the elapsed time or does it give you the amount of gasoline in the gas tank in the elapsed time?
Selin, Cem and Güney: It gives what is found.
Researcher: What is required in option c?
Selin: 1 minute says how many liters of gasoline is filled.
Researcher: Did you understand the difference?
Cem: Then we do it like this. We put 12 instead of 17. Because 12 liters are filled.
Selin: Can you say it again?
Cem: If it fills 12 liters in 3 minutes, it fills x liters in 1 minute.
Researcher: It wants you to solve without using algebraic expressions.

According to the dialog above, they first thought that the amount of gasoline filled in 3 minutes was 17, but this thought was wrong. The researcher asked the students a question and emphasized that the amount of gasoline given in the table was the amount found. The researcher also warned them not to use algebraic expressions.

The dialog below shows how the first group found the amount of gasoline that fills the tank in one minute by interpreting and analyzing the relationship between quantities.

... Cem: If it is 12 in 3 minutes, we can find the amount of filling in 1 minute by dividing 12 by 3.
Güney: Yes.
Selin: Then it's 4 liters.
Güney: 4 liters per minute.

As a result, the students determined that the amount of gasoline filled in 3 minutes was 12 and found 4 liters by dividing 12 by 3 to find the amount filled per minute.

How the second group thinks about this question is given in the following dialog.

... Melek: If it's 17 in 3 minutes, we divide 17 by 3.
Researcher: Can you look at the information in the table? What information does it give you?
Melek and Zülal: Time elapsed and the amount of gasoline in the tank.
Researcher: So the information in the table gives you the amount in the tank or the amount filled in the tank?
Melek and Emre: It gives what is found. It does not give what is full.
Researcher: Then what is required in option c?
Emre: 1 minute.

According to the dialog above, the second group, like the first group, thought that the amount of gasoline given in the table was the amount of gasoline filled, so they thought that the amount of gasoline filled in 3 minutes was 17. The researcher guided the students to realize that the amount of gasoline given in the table was the amount found.

How the second group found the amount of gasoline that fills the tank in one minute by interpreting and analyzing the relationship between quantities is shown in the dialog below.

… Emre: It's 4.
Melek and Zülal: How?
Emre: If 3 minutes fills 12 gallons. 1 minute fills 4 liters of gasoline.
Melek: It makes sense.
Zülal: I don't understand.

Melek: Now the gasoline found here. It was here in 3 minutes (17 liters). Then they added 12 liters of gasoline here after 3 minutes. That's why we divided 12 by 3 minutes. 1 minute is 4 liters.
Zülal: I see.

After distinguishing the difference between the amount of gasoline filled and the amount of gasoline found, the students found that the amount of gasoline filled in 3 minutes was 12. Thus, they divided 12 by 3 to find the amount of gasoline filled per minute and reached 4 liters.

The dialog below shows how the first group found the amount of gasoline in the tank without filling the tank by interpreting and analyzing the relationship between quantities in the question in option d.

... Güney: Subtract 12 from 17 to get 5. That's 5 liters.
Cem: I mean the gasoline at the very beginning.
Selin: Yes.
Researcher: Why did you subtract 12 from 17?
Cem: Because 17 is the amount of gasoline after you start filling up.
Güney: 3 minutes later.
Cem: If 12 is the filling amount, if we subtract the full amount from the found amount, we find the amount found at the beginning.
Güney: Filling in 3 minutes.

Looking at the dialog above, they were able to find the amount of gasoline in the tank before filling the tank with gasoline. Since there was 17 liters of gasoline in the tank in 3 minutes, the students found that there was 5 liters of gasoline in the tank at the beginning by subtracting 12 liters from the amount of gasoline filled in 3 minutes.

How the second group thinks about this question is given in the following dialog.

… Zülal: So now if 3 minutes is 17, you subtract 12 and you get 5.

Melek: But here it says... Before they start filling the gasoline. Not filled; there was petrol in the tank. There are 17 liters in the tank. Not filled yet. It tells you how many liters of petrol are in the tank before it starts filling. There are 17 liters in the tank.

Zülal: I don't think it's 17.

Melek: How is it not 17?

Zülal: 17 in 3 minutes.

Melek: It has not been filled.

Zülal: Time has passed. We need to make time zero.

Melek: Time has passed, yes. Then let's go back twelve by twelve.

Zülal: We'll go backwards. That's 5 liters. Minute...

Melek: That's 5 in 2 minutes then.

Zülal: Not 2 minutes. It's three by three.

Melek: Three by three...

Zülal: Then 0 minutes and 5 minutes.

Emre: 5 liters in the tank.

Zülal: There were 5 liters in 0 minutes. Twelve by twelve and then it increases.

Researcher: Why did you do it this way?

Zülal: 17 liters of gasoline in 3 minutes. We made time zero. So we went back in time three by three, twelve by twelve in liters because 12 liters are filled in 3 minutes.

Looking at the dialog above, they count backwards to find the amount of gasoline in the tank before filling the tank with gasoline. Using the information that 12 liters of gasoline were filled in 3 minutes to reset the minute to zero, they counted backwards three by three in minute and twelve by twelve in liters from the statement given in the first row of the table (17 liters of gasoline in the tank in 3 minutes). Thus, they reached 5 liters of gasoline in zero minutes.

The dialog below shows how the first group shows the relationship between quantities in the question in option e with algebraic expressions.

... Selin: In one, it increases by thirds. In the other it increases twelve by twelve. That's how the relationship is.

Güney: First of all, we can use the pattern thing. The elapsed time is 3n. The amount of gasoline in the tank is 12n plus 5.
Researcher: What does the "n" stand for?
Güney: "n" is the number of things...
Researcher: What will you replace n with?
Güney: 1 2 3...
Researcher: So, how does this give the relationship between the elapsed time and the amount of gasoline in the tank?
Cem: It doesn't.
Researcher: It wants the algebraic expression showing the relationship between elapsed time and the amount of gasoline.
Cem: It could be like this. If it's 17 liters in 3 minutes...
Güney: 12
Selin: No 17.
Cem: The amount of gasoline found. x minutes, we can do 12n+5. So x is the minutes that are multiples of 3. n here...
Güney: It is how many minutes have passed.

In the dialog above, students needed the researcher's guidance as they had difficulty in writing this expression because they wrote two different algebraic expressions without establishing a relationship between the elapsed time and the amount of gasoline in the tank and taking into account their increased amounts.

At this point, the researcher asked questions to make them realize that minutes are unknown. Thus, the students were able to write the algebraic expression that shows the relationship between the elapsed time and the amount of gasoline in the tank. This is shown in the following dialog.

... Researcher: You called x a minute and n a minute. Let's do it like this. The liter changes according to the minute, right?
Cem: Yes.

Researcher: Since the liter value changes according to the minute, can the minute be our unknown?

Güney: I see. It was 4 liters per minute. We can go from that. How much was there in the beginning?

Cem: There were 5 liters.

Güney: 9 per minute.

Researcher: How did it become 9?

Güney: I added the initial 5 liters of gasoline.

Cem: So you added 4 to 5.

Güney: 9 liters in 1 minute. In two minutes…

Cem: We'll add 4 to 9. This time it will be 13 liters.

Güney: Yes, 13 liters.

Researcher: Well, what about the algebraic expression?

Güney: 4n+5. Because 4 fills in a minute. "n" represents minutes. 5 represents the initial gasoline.

Selin and Cem: Yes.

Researcher: Can you explain again how you wrote the algebraic rule?

Güney: 4 represents the amount of gasoline filled per minute.

Cem: n represents minutes. So we multiplied n by 4 to find how many minutes it takes to fill up.

Güney: 5 liters, we added the amount of gasoline that was originally in the tank.

Researcher: How did you decide that this algebraic rule you wrote is valid for every situation in this activity?

Güney: Let's try.

Cem: Let's give a number "n".

Güney: Let's try 3 for example. Multiplying 4 by 3 gives 12, and adding 5 gives 17.

Cem: Let's try one more. Let's try the sixth minute this time.

Güney: 29 liters.

Cem: That happens too.

According to dialog, the researcher’s questions led them to use the information that 4 liters of gasoline filled the tank in 1 minute and that there were 5 liters of gasoline in the tank at the beginning. In other words, to find the amount of gasoline in the tank at the end of 1 minute, they added 4 and 5 to find 9 liters. To find the amount of gasoline at the end of 2 minutes, they added 4 to 9 to get 13 liters. In this way, they said that the algebraic expression showing the relationship between the elapsed time and the amount of gasoline in the tank was 4*n*+5. They were able to explain how they wrote this algebraic expression and proved its validity for every question in the scope of this question. They explained that n represents minutes, 4 represents the amount of gasoline filled per minute, they multiplied them to find out how much gasoline was filled as the minutes passed, and finally they added 5 liters in the initial gasoline. To verify the validity of the algebraic expression, they substituted different minutes for *n*.

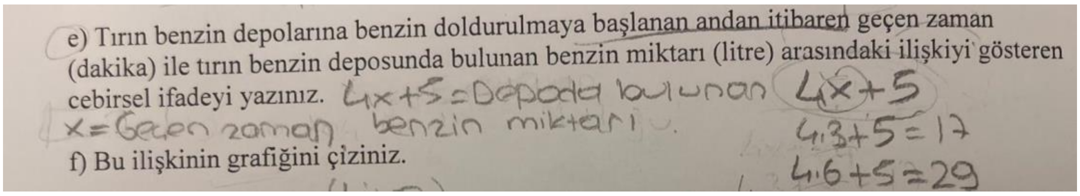

e) Tırın benzin depolarına benzin doldurulmaya başlanan andan itibaren geçen zaman (dakika) ile tırın benzin deposunda bulunan benzin miktarı (litre) arasındaki ilişkiyi gösteren cebirsel ifadeyi yazınız. 4x+5 = Depoda bulunan benzin miktarı. 4x+5
x = Geçen zaman 4.3+5=17
f) Bu ilişkinin grafiğini çiziniz. 4.6+5=29

Figure 4.16 The Answer of the Second Group for Option e for The Gasoline Tank Activity

The following dialog shows how the second group shows the relationship between quantities in the question in option e with algebraic expressions.

... Zülal: We will write the algebraic expression between this (the amount of gasoline in the tank) over time.
Melek: It goes up in threes. I’ll write it as an exponential expression, but it won’t work.
Zülal: No. It’s a multiple of 3. If we say 3x, if we say 12x.
[...]

Zülal: Not 12. At first there were five liters.

Melek: So that's 12x plus 5 then? Yes, 12x+5.

Zülal: We will find 1 minute as a pattern.

Melek: Here we will find it as a pattern. For example, 12 times 1 plus 5 is 17. 12x+5 is correct then.

Emre: Right.

Researcher: What does the "x" stand for?

Emre: What we add to the gasoline in the tank.

Melek: It increases twelve by twelve so it shows how many times it increases.

[…]

Researcher: And how does this relate to the elapsed time?

Melek: We called it 3x. 12x plus 5 is the amount of gasoline in the tank.

Researcher: Let's do it like this. I will ask you a few questions, let's proceed in that way. According to the table, how do liter values change according to minutes?

Melek, Zülal and Emre: Increasing.

Researcher: Since the liter value changes according to the minute, can the minute be our unknown?

Melek: Yes.

[…]

Researcher: How do we express the unknown?

Melek: x.

Zülal: 3x and 12x plus 5. Because it is the same. If we substitute 2 for x. 6 gives 29.

[…]

Melek: 2 minutes don't pass. It is asking about the relationship with minutes. We did not call x a minute here. How many steps x has increased here (he is talking about the expression 12x+5)? Our unknown is the minute.

[…]

Researcher: You have decided that the minute is unknown. Well, when writing the algebraic expression, do you take into account the amount of gasoline filled in 1 minute that you found in option c? When writing the algebraic expression, do you

take into account the amount of gasoline in the tank of the truck before it starts to fill up?
[…]
Melek and Zülal: 3x plus 5.
Zülal: If we substitute 3 for x, we get 14.
Melek: We found 4 liters there, that's how many liters are filled in 1 minute, so we will put 4 here.
Zülal: Yes.
Melek: If we substitute 4 for x, make 17.

In the dialog above, students had difficulty in writing this expression and the researcher tried hard to guide them. At first, the students wrote two different algebraic expressions without establishing a relationship between the elapsed time and the amount of gasoline in the tank and taking into account their increase amounts. Since they focused on using the increments, it took a long time for them to consider using the amount of gasoline filled per minute.

The researcher used questions to attract and direct their attention as shown in the following dialog.

… Researcher: Let's try again. Since the liter changes according to the minute, can the minute be our unknown?
Zülal: Yes, it can be, we don't know how much it is. For example, if it says 89 L was filled in how many minutes, it is unknown.
[…]
Melek: … Time becomes unknown. But I will say 3x, but we need to put something next to 3x.
Zülal: Yes.
Researcher: Why do you say 3x?
Melek: Because it increases by threes and threes.
Zülal: Yes.

Researcher: Well, when writing the algebraic expression, do you take into account the amount of gasoline filled in 1 minute that you found in option c?
Zülal: If it's 4 liters in 1 minute.
Melek: Should we say 4x then? Yes, if we say 4x, because 4 liters are filled in 1 minute. We can't find it because we go in 3 minutes.
Zülal: 4x plus 5? 4 times 3 plus 5 is 17. 4 times 6 plus 5 is 29. It is what is in the table.
Melek: Okay, then we got it right.
Emre: I don't understand.
Melek: We did not take into account how many liters of gasoline are filled in one minute. x is the unknown, minutes. We said 4x because we wanted to find the amount of gasoline that fills up over time. Then we added 5 liters of gasoline that was in the tank at the beginning.

In the dialog above, students were able to write the algebraic expression showing the relationship between the elapsed time and the amount of gasoline in the tank and they were able to determine that the algebraic expression was $4n+5$. They were able to explain how they wrote this algebraic expression and proved its validity for each question in the scope of this question. They explained that n represents minutes, 4 represents the amount of gasoline filled per minute, they multiplied them to find out how much gasoline was filled as the minutes passed, and finally they added 5 liters from the initial tank. To verify the validity of the algebraic expression, they substituted different minutes for n.

As a general evaluation, it was observed that students were able to establish a relationship between quantities and interpret this relationship. However, it can be said that they have difficulty in showing this relationship using algebraic expressions and need improvement in this regard.

4.2.6 Repetitive Patterns (Box-Penny Activity)

The Box-Penny activity deals with the repetitive patterns, which is a subcategory of functional thinking, one of the components of algebraic thinking. This activity involves defining and expanding the repeat unit and finding the general rule of repetitive patterns. The activity consists of four options.

The dialog below shows how the first group thought about the relationship between the amounts of money in the boxes in option c.

... Cem: I think we should write up to a certain point and then write the rule verbally.
Selin: It could be.
Researcher: Then start from the beginning and take turns to find the amounts of money in the boxes.
Selin: How far will we write?
Cem: Let's write as far as we can.
Güney: There's one penny in box 1. 5 pennies in the 2nd. 10 pennies in the 3rd. 25 pennies in the 4th.
[...]
Cem: It's 5 pennies in the 9th.
Researcher: Why?
Cem: Sorry, it's 10 pennies.
Güney: It can be divided into three, its Rüzgar.
[...]
Güney: The 14th box is 10 pennies.
Researcher: How?
Güney: Aaaa, 5 pennies.
Selin: The 15th box is 5 pennies.
Cem: No, 10 pennies
Selin: Yes.
[...]

Cem: I think this is enough (they wrote until box 26).

In the dialog above, students determined the amount of money in a certain amount of boxes in order to determine the relationship between the amounts of money in the boxes. They made mistakes while determining the amounts of money in some boxes. However, when the researcher questioned them about how and why they did so, the students realized their mistakes and found the correct answers.

The first group's thinking about how they found the repetition unit and the general rule in a repetitive pattern is given in the dialog below.

… Güney: It doesn't seem to follow a pattern.
Selin: It changes after a while. The pattern breaks down. Should we keep it going?
Güney: No need. Let's say... 4 divided by 2 pennies, 3 divided by 10 pennies...
Researcher: It already tells us that in the question stem.
Cem: Actually, one step of the pattern is very long. For example, it is like a step of the pattern until here (talking about Box 16). Because after this, the pattern... Did I do it wrong?
Güney: Well, let's look diagonally.
Selin: It depends on the way we write.
Cem: There is a step until there (pointing to the 12th box). After that (he means after the 12th box) it continues as before.
Researcher: How does it work? If you want, let one of you read from the beginning and the other one read after Box 12 and let's see together.
Cem and Güney: 1 penny, 5 pennies, 10 pennies, 25 pennies, 1 penny, 10 pennies, 1 penny, 25 pennies, 10 pennies, 5 pennies, 1 penny, 25 pennies.
Researcher: How far did you read?
Güney: Box 1 to Box 12.
Cem: Box 13 to Box 24.
Researcher: What happens after that?
Cem: It continues in the same way.
Researcher: Are you sure?

Cem: Let's write some more.

[...]

Güney: It will be if we write box 36 (they wrote until box 35).

Researcher: Was it the same again?

Güney: Let's read it again then.

Cem and Güney: 1 penny, 5 pennies, 10 pennies, 25 pennies, 1 penny, 10 pennies, 1 penny, 25 pennies, 10 pennies, 5 pennies, 1 penny, 25 pennies. (Cem reads from Box 1 to Box 12, Güney reads from Box 25 to Box 36)

Researcher: What did you notice here?

Güney: It goes back to the beginning every 12 times.

Selin: Yes.

Cem: The amount of money in the boxes repeats every 12 times.

According to the dialog above, Cem, one of the students in this group, suggested that the parts from box 1 to box 12 and from box 13 to box 24 repeat each other. In order to support this idea, the researcher asked the students to compare the amounts of money in the boxes. In order to be sure of their idea, the students continued the pattern until the 36th box and proved that their idea was correct. Students were able to express that the pattern returned to the beginning every 12 boxes. As a result, students were able to write and explain the general rule of the pattern by determining the repetition unit in a repetitive pattern.

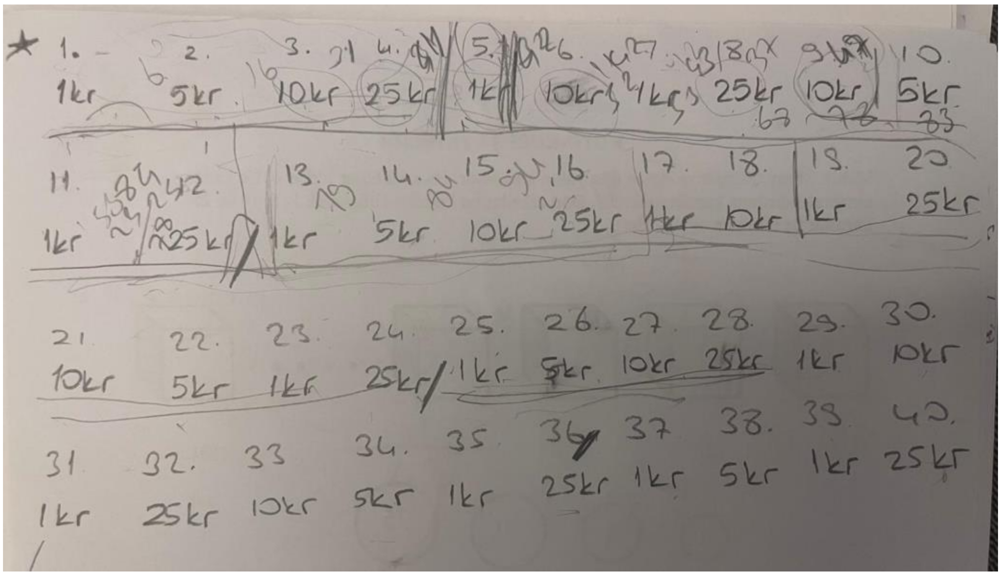


Figure 4.17 The Answer of the Second Group for Option c for Box-Penny Activity

The following dialog shows the second group's thinking about how they found the repetition unit and the general rule in a repetitive pattern for the question in option c.

... Melek and Zülal: We will look at the relationship between them.

Zülal: Let's list and write again.

Melek: Are we going to write up to 101?

Researcher: If you are looking for a relationship, start writing from the beginning as asked in option c.

Melek: 1 penny for the 1st box, 5 pennies for the 2nd box

Emre: 10 pennies for the 3rd box, 25 pennies for the 4th box

Melek: 5 pennies in box 5. 5 pennies in the 6th box

Zülal: No, 10 pennies. Last time it was multiples of 3.

[...]

(they determine the amount of money until the 36th box)

Melek: 1 penny is always thrown at odd numbers.

Zülal: No. 3 is an odd number.

Melek: One point on 33...

Melek: If there were prime numbers...

[...]

Melek: We threw either 5 pennies or 25 pennies in multiples of two.

Researcher: What you said is about how children put pennies in the boxes.

In the dialog above, students found the amount of money in a certain amount of boxes to determine the relationship between the amounts of money in the boxes. At first, students did not realize that there was a repetitive situation in finding the relationship between the amounts of money in the boxes.

Since the students had difficulty in determining the relationship, the researcher guided them with questions as follows.

... Researcher: Do you notice repetitive situations in the amounts of money in the boxes?

Zülal: 1 5 10 25 1 10 1 25 15 10 1...

Melek: 1 5 10 25 (says the first 4 boxes) 1 5 10 25 (says Box 13 to Box 16). 1 10 1 25 (says from box 5 to box 8) 1 10 1 25 (says from box 17 to box 20). Repeating.

Researcher: Are the money in the boxes in between not repeated? (referring to the section from Box 9 to Box 12)

Melek: 10 5 1 25 is repeated here (referring to Box 21 to Box 24). It goes in this order like this. 1 5 10 25 1 10 1 25 10 5 1 25 10 5 1 25 goes like this. What do we get from this?

Researcher: You noticed repetitive situations. How often do the amounts of money in the boxes repeat?

Melek: One in 4 boxes.

Researcher: Every 4 boxes the same thing is written?

Melek: No it is different.

Researcher: We are aware that the same thing has to be written after a repetitive situation, right?

Melek and Zülal: Yes.

[...]

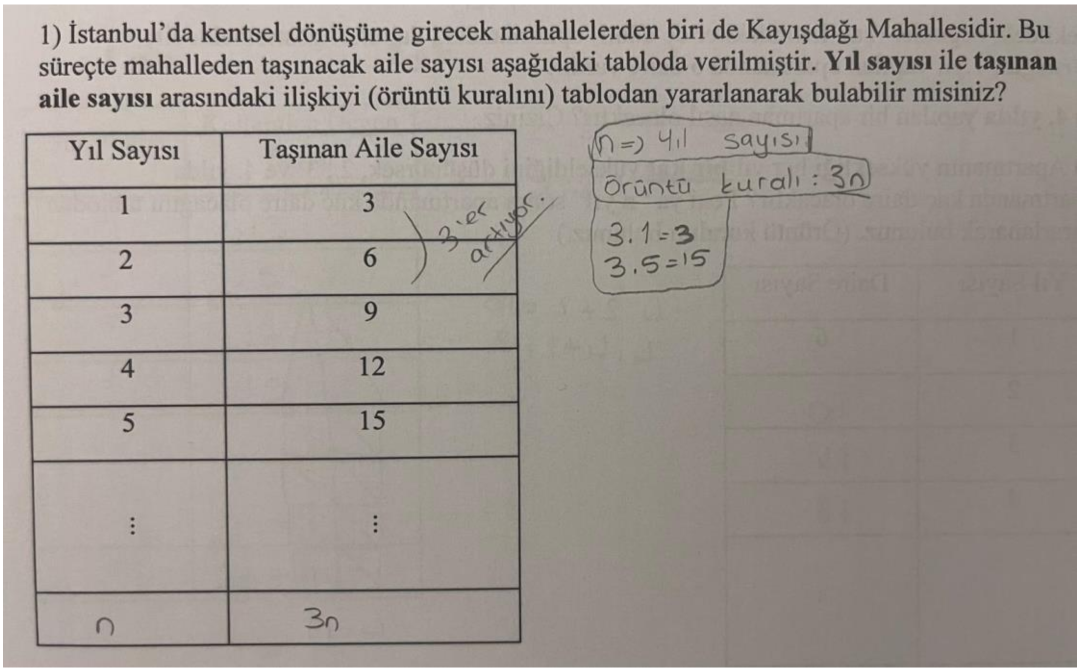
Researcher: In repetitive situations, nothing different should come in between. The same thing should follow.
Melek: These 4 numbers (the first four boxes) and these 4 numbers (Box 5 to Box 8) are not the same.
Zülal: I don't understand.
Melek: We took them separately, the same should be repeated.
Researcher: Start at the beginning and identify the repetitive situations. Where does the repetition of Box 1 begin?
Melek and Zülal: This is where it happens (Box 13).
Researcher: Try to compare.
Melek: Zülal, you read from box 1. Emre, you read from box 13.
Zülal and Emre: 1 5 10 25 1 10 1 25 10 5 1 25.
Researcher: Did you say the same thing?
Zülal and Emre: Yes.
Researcher: Then how many boxes does it repeat every time?
Melek, Zülal and Emre: It repeats every 12 boxes.

The dialog above shows that the researcher tried to make them realize whether there were repeating situations in the boxes and directed them to identify the repeating situations. The students noticed repetitive situations in the amounts of money in the boxes. However, they had difficulty in expressing how often there was repetition. The researcher emphasized that the repeating situations should come one after the other and made the students realize that the parts from the 1st box to the 12th box and the parts from the 13th box to the 24th box repeat each other. In line with the instructions, students were able to express that the pattern returns to the beginning every 12 boxes. As a result, it was revealed that students had difficulty in determining the repetition unit in a repetitive pattern and writing the general rule of the pattern.

It can be said that students made positive progress in determining the unit of repetition in a repetitive pattern and in writing the general rule of that pattern.

4.2.7 Growing Patterns (Urban Transformation Activity)

The Urban Transformation activity deals with the growing patterns category, which is a subcategory of functional thinking, one of the components of algebraic thinking. This activity involves analyzing and expanding number and shape patterns, finding the recursive relationship in these patterns and the general rule of these patterns. The activity consists of three independent questions.



1) İstanbul'da kentsel dönüşüme girecek mahallelerden biri de Kayışdağı Mahallesidir. Bu süreçte mahalleden taşınacak aile sayısı aşağıdaki tabloda verilmiştir. **Yıl sayısı** ile **taşınan aile sayısı** arasındaki ilişkiyi (örüntü kuralını) tablodan yararlanarak bulabilir misiniz?

Yıl Sayısı	Taşınan Aile Sayısı
1	3
2	6
3	9
4	12
5	15
⋮	⋮
n	3n

3'er artıyor.

n => yıl sayısı

Örüntü kuralı : 3n

3.1=3

3.5=15

Figure 4.18 The Answer of the First Group for the First Question for Urban Transformation Activity

In the dialog below, the algebraic thinking of the first group about writing the pattern rule for the first question are presented.

... Cem: Since 3 families move every year, there is an increasing pattern of 3 families each year.

Researcher: Okay, what does the question ask you?

Cem: It wants the pattern rule, then it becomes 3n.

Selin and Güney: Yes, 3n.
Cem: For example, on the fifth day it becomes a multiple of three, it becomes 15.
Researcher: What is the n?
Cem and Güney: The number of years.

In the dialog above, Cem, one of the students in the first group, correctly analyzed the relationship between the number of years and the number of families moved and revealed the recursive relationship in the pattern. Cem found the general rule of the pattern and his groupmates joined him. In other words, the students stated that the unknown in the pattern rule they wrote was the number of years and they expressed the pattern rule as $3n$ by calling the unknown n. They are also aware that the reason for multiplying n by 3 is that three families move every year.

In the following dialog, the algebraic thinking of the second group about writing the pattern rule for the first question are revealed. The researcher guided the students about writing the pattern rule.

... Melek: 1 year, 2 years. Okay, then 3 families moved in one year. We'll make a pattern for it.
Emre: It's increasing three by three.
Zülal: 3, 6, 9, 12, 15...
[...]
Melek: What will be unknown here, will it be a year?
Zülal and Emre: There is also the number of families that have moved.
Melek: But if we know the number of families moved, we can find the number of years. If we know the number of years, we can find the number of moved families.
Researcher: According to the information in the table, how does the number of moving families increase with the number of years?
Emre, Melek, Zülal: It increases by three.
[...]

In the dialog above, the students could not decide whether the unknown was the number of years or the number of families that moved. They knew that if they knew the number of families moved, they could find the number of years or if they knew the number of years, they could find the number of families moved, but they did not aware that families would move over years.

At the dialog below, the researcher guided them by asking them questions. By asking questions, the researcher made the students realize that the number of years is unknown.

… Researcher: I mean, since the number of families moved according to the number of years change, wouldn't the number of years be unknown?
Melek and Zülal: Yes, 3n.
Researcher: Do you understand what I mean?
Emre: I understand that we say the number of years, n.
[...]
Researcher: So, what happened to the number of families moving according to the number of years?
Emre, Melek and Zülal: Increasing.
Researcher: So what is the number of years here now?
Melek and Zülal: Unknown.
Researcher: Then the rule of our pattern?
Melek and Zülal: 3n
Researcher: What is n?
Emre, Melek and Zülal: The number of years.
[...]
Researcher: Can you tell us how you wrote the pattern rule?
Emre: We looked at the table. The number of years increases one by one and the number of families moving increases three by three. We called the number of years n. Since the number of families moving in increases by three by three, it becomes 3n.

After these questions, the students were able to write the general rule of the pattern. In the pattern rule they wrote, the students stated that the unknown was the number of years and they expressed the pattern rule as $3n$ by calling the unknown n. They were also aware that the reason why they multiplied n by 3 was because three families moved every year.

The dialogues below respectively present the thoughts of the two groups on the validity of the general rule of the pattern they found for each situation within the scope of the question.

... Researcher: So, is this pattern rule, or rather algebraic rule, that you wrote valid for every situation within the scope of this question?
Cem: Yes.
Researcher: How did you decide this?
Cem: Because they are always multiples of 3. So we can find it by multiplying the number of years by 3.
Güney: Let's try.
[...]
Güney: 3 times 1 equals 3. Or 3 times 5 equals 15.
Researcher: Does it match the information in the table?
Cem and Güney: Yes.

…

... Researcher: Well, how did you decide that this pattern rule, that is, the algebraic expression you wrote is valid for every situation within the scope of this question?
Zülal: It comes out when we type the number we want instead of n.
Melek: Yes.
Researcher: Have you tried it?
Emre: No.
Melek: Let's try.

Zülal and Melek: For example, 3 times 1 equals 3. 3 times 2 equals 6. 3 times 3 equals 9. 3 times 4 equals 12. 3 times 5 equals 15. And so on.

In the dialogues above, students in both groups decided that the general rule of the pattern is valid for every situation within the scope of the question by trying the pattern rule they created. In other words, they wrote different numbers of years instead of *n*, instead of the unknown term, checked whether they matched the information in the table, and stated that the general rule of the pattern was valid for every situation within the scope of the question.

In the dialogue below, the first group's thoughts on whether they were aware of the fact that the general rule of the pattern they found after creating their solutions to this question was the number of families moved are given.

... Researcher: How do you think the pattern rule you wrote in the bottom blank on the table can be placed?
Cem and Güney: n for the number of years. 3n for the number of families moved.
Researcher: Why did we write n and 3n?
Selin: Because the rule of the pattern is 3n.
Researcher: What does this 3n give us
Selin: Increased by three.
Researcher: So, what does it give us when we multiply 3 by n?
Cem: The number of families moving in any given year.

In the dialog below, the second group's thoughts about their awareness that the general rule of the pattern they found after creating their solutions for this question is the number of families moved are given.

... Researcher: How do you think the pattern rule you wrote in the bottom blank on the table can be placed?
Emre: We increase the number of years one by one.
Melek: n for the year.

Emre: Algebraically?

Researcher: So, how is your pattern rule placed? What was your number of years?

Melek and Zülal: n.

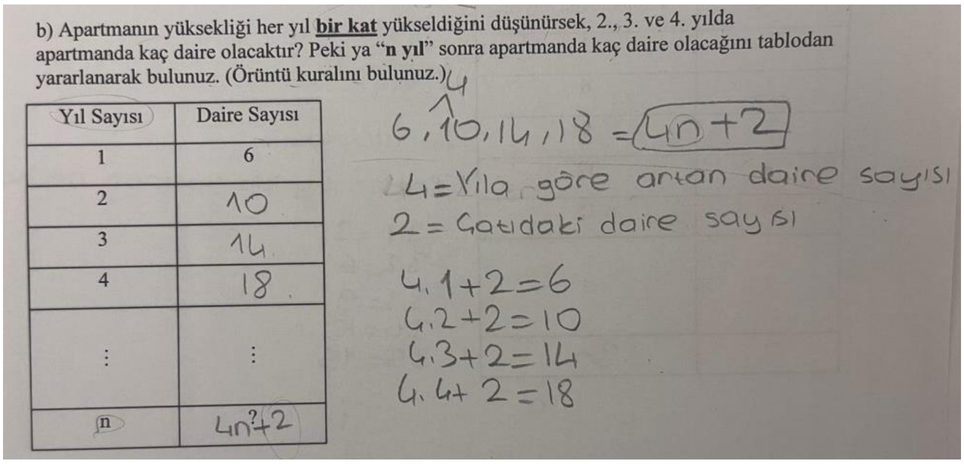
Researcher: What is the number of families that moved?

Melek and Zülal: 3 but the number of families moving every year.

Researcher: Okay, what do you do to find out the number of families that moved?

Melek: We multiply n by 3. Then we will write 3n.

In the dialogues above, some of the students in both groups had difficulty in grasping that the general rule of the pattern they found gives the number of families moved. In other words, the students thought that they found the pattern rule for this question when they multiplied 3 by *n*, but they did not realize that this rule actually gives the number of families moved in the nth year, that is, in any year. The researcher guided them by asking questions. The researcher reminded the students of the purpose of why they multiplied *n* by 3 and increased their awareness of this situation.



b) Apartmanın yüksekliği her yıl **bir kat** yükseldiğini düşünürsek, 2., 3. ve 4. yılda apartmanda kaç daire olacaktır? Peki ya "**n yıl**" sonra apartmanda kaç daire olacağını tablodan yararlanarak bulunuz. (Örüntü kuralını bulunuz.)

Yıl Sayısı	Daire Sayısı
1	6
2	10
3	14
4	18
⋮	⋮
n	4n+2

$6, 10, 14, 18 = \boxed{4n+2}$

4 = Yıla göre artan daire sayısı

2 = Çatıdaki daire sayısı

$4.1+2=6$

$4.2+2=10$

$4.3+2=14$

$4.4+2=18$

Figure 4.19 The Answer of the Second Group for Option b of the Second Question for Urban Transformation Activity

In the following dialog, the algebraic thinking of the second group about writing the pattern rule for option b of the second question are revealed.

... Emre: Four by four.

Melek: Four by four. 6,10, 14 then.

[...]

Zülal: The number of flats goes 6 10 14 18.

Melek: Emre, do you understand?

Emre: Yes, I understand, I wrote it down.

Zülal: Increased by four

Melek: This will be 4n+2 (referring to the last space in the column of the number of flats in the table)

Researcher: I don't understand how you wrote 4n+2, can someone tell me? Did you talk among yourselves about how you wrote it?

Melek: Emre, do you understand how we write?

Emre: That's why it has been increasing every year...

Melek: Where does the 2 come from?

Emre: From the roof.

Melek: Are you sure?

Emre: Yes.

Melek: Emre, now it increases four by four. This is two more than four, so we add it to two. Not from the roof, I mean.

Researcher: I think Emre is right.

Melek: Through the roof. There 4 there 2 (shows on the figure the apartments on the floor and the flats on the roof)

Researcher: Does the roof ever change?

Zülal and Melek: No.

[...]

Researcher: So can you tell me one more time how you wrote this pattern rule?

Zülal: What we call n is the number of years. Then we said 4n, it increases four by four, that is, it increases by four according to the year 4n. Two is the number of flats on the roof.

In the dialog above, the students in the second group analyzed the relationship between the number of years and the number of flats, revealed the recursive relationship in the pattern and were able to write the general rule of the pattern. In other words, in the pattern rule they wrote, they stated that the unknown is the number of years and by calling the unknown *n*, they expressed the pattern rule as $4n+2$. They were also aware that the reason why they multiplied *n* by 4 was because four flats were made every year. They are also aware that the general rule of the pattern they found, namely $4n+2$, gives the number of flats in any year. However, while explaining how they wrote the general rule of the pattern, the students had difficulty in coming a common opinion on why they added 2. While explaining why they added 2 in the general rule of the pattern, Melek, one of the students in this group, explained that she added 2 because the number of flats in the pattern progressing in the form of 6 10 14... increases by four every year, and the number of flats is always 2 more than a multiple of four. However, her groupmate Emre thought that the reason why they added 2 was because of the 2 flats on the roof, which never change, that is, remain constant. Both students were not wrong because one of them expressed numerically why they added 2, while the other expressed what +2 represented in the visual in the given question. The researcher intervened at this point and stated that Emre was also right. In this way, the students realized that +2 represents the 2 flats on the roof that remain constant.

In the following dialog, the algebraic thinking of the first group about writing the pattern rule for this question are presented.

... Güney: 6 in the first year, 10 in the second year.
Researcher: How did it become 10?
Güney: 4 more flats came. Here again 4 flats came, 14 in the third year and 18 in the fourth year.
[...]
Researcher: You have seen how it has progressed until the fourth year. What does it want from you now?

Selin, Cem and Güney: Pattern rule.

Selin: It increases by four. Then we can write 4n here (in the last space in the number of flats column in the table).

Cem: 4n+2 because it increases by two each time.

Researcher: Why did you add 2?

Cem: Because it starts with six and increases by four. And since it increases by four...

Güney: When we subtract 4 from 6, we get 2. So we add two to get six.

Cem: We find the number of flats by adding two to multiples of four.

Researcher: So what was n?

Selin, Cem and Güney: The number of years.

[...]

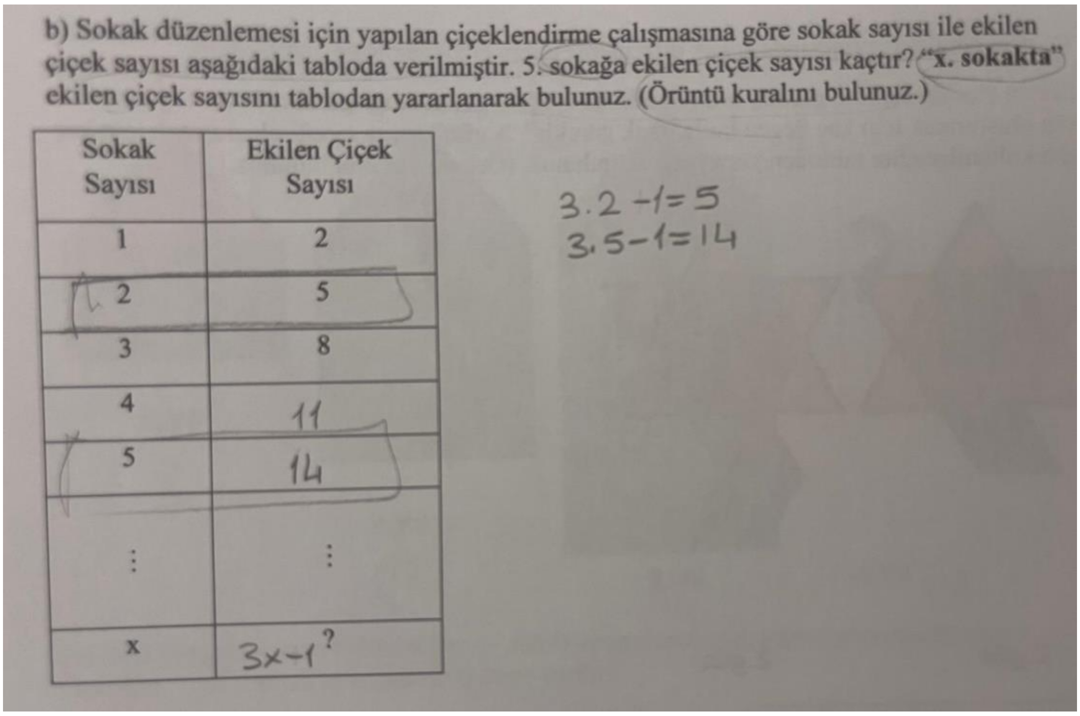
Researcher: ... But I still don't understand why you added +2.

Cem: For example, in the third year, it increased by 3 times as much as 4 times, plus 2 more flats.

Researcher: Where do those 2 flats come from?

Cem and Güney: It comes from 2 flats on the roof and they all have roofs.

In the dialog above, the students in the first group revealed the recursive relationship in the pattern by analyzing the relationship between the number of years and the number of flats and were able to write the general rule of the pattern. In other words, in the pattern rule they wrote, they stated that the unknown is the number of years and by calling the unknown n, they expressed the pattern rule as $4n+2$. They were aware that the reason why they multiplied n by 4 was because four flats were made each year. They are also aware that the general rule of the pattern they found, namely $4n+2$, gives the number of flats in each year. However, students had difficulty in explaining what +2 represents when explaining how they wrote the general rule of the pattern. When explaining why they added 2 in the general rule of the pattern, the students explained that the number of flats in the pattern increased by four, and they added 2 because the number of flats was 2 more than a multiple of four. The students did not think wrong, but at first they could not say what +2 represented in the visual in the given question. When the researcher intervened at this point and asked them where the 2 circles came from, the students realized that +2 represented the 2 flats on the roof that remained constant.



b) Sokak düzenlemesi için yapılan çiçeklendirme çalışmasına göre sokak sayısı ile ekilen çiçek sayısı aşağıdaki tabloda verilmiştir. 5. sokağa ekilen çiçek sayısı kaçtır? "x. sokakta" ekilen çiçek sayısını tablodan yararlanarak bulunuz. (Örüntü kuralını bulunuz.)

Sokak Sayısı	Ekilen Çiçek Sayısı
1	2
2	5
3	8
4	11
5	14
⋮	⋮
x	3x-1 ?

3.2 -1=5
3.5-1=14

Figure 4.20 The Answer of the Second Group for Option b of the Third Question for Urban Transformation Activity

The following dialog shows how the first group thought while writing the pattern rule for option b of the third question.

... Cem: It increases by three each time. Then...

[...]

Güney: It increases by three, 11 and 14.

Cem: Exactly. In the fifth street the number of flowers becomes 14.

Selin: I said 3x+ but I couldn't do the rest (talking about the gap in the number of flowers used in the table)

Güney: 3x-1.

Researcher: Why -1? Why not +1?

Cem: Because there is missing a flower from the amount of increase.

Güney: Yes

Cem: So here, when we multiply 2 by 3, we get 6 flowers, but because one flower is missing, we get 5 flowers.

Güney and Selin: Yes.

The following dialog shows how the second group thought while writing the pattern rule for option b of the third question.

... Zülal and Melek: It increases by three.

Zülal: 5 8 11...

[...]

Melek: It asked the fifth street about the number of flowers planted.

Emre and Melek: Three by three.

Melek: 14.

Researcher: Now you find the pattern rule.

Melek and Zülal: 3x-1

Melek: Do you understand, Emre?

Emre: -1?

Melek: It increases by three but one less.

Researcher: Can you explain how you wrote the pattern rule?

Melek: 3x is the number of flowers planted per day. Then what is -1? -1 is something that does not exist.

Zülal: That means, if it is zero, it decreases three by three, or if we subtract 3 from 2, it becomes -1.

Melek: -1 is something that does not exist, how can the number of flowers planted be -1?

Researcher: It would be better if you explain why you removed one.

[...]

In the dialogues above, the students in both groups expressed the pattern rule they wrote as $3x$-1. Students were able to explain how they wrote the general rule of the pattern. In other words, they explained that the reason why they multiplied *x* by 3 was because the number of flowers planted on each street increased by three and the reason for subtracting 1 is that the number of flowers is always 1 less than the amount of increase, i.e. 3. However, students in the second group had difficulty in explaining why they subtracted -1.

At this point, the researcher guided them by asking questions as follows.

... Researcher: How can the number of flowers vary?
Emre, Zülal and Melek: It depends on the number of streets.
Zülal: Then the number of flowers increased according to the number of streets, i.e. 3 is one more than the number of flowers planted on the first road.
Researcher: So you have to subtract 1 every time?
Melek: Yes, because each time, for example, 2 times 3 is one less, 3 times 3 is one less.

After researcher's questions, the students in the second group realized that the number of flowers planted varied according to the number of streets. In general, compared to all the questions in this activity, in this question, both groups were better able to express what the numbers they multiplied and then subtracted the unknown in the pattern rule represented and how they wrote the pattern rule.

A thorough evaluation of all the questions in this activity reveals that both groups improved in analyzing and extending number and shape patterns, finding the recursive relationship in these patterns, and finding the general rule of these patterns.

4.2.8 Multiple Representations (Table Organization Activity)

The Table Organization activity deals with the multiple representations category, which is a subcategory of modeling, one of the components of algebraic thinking. This activity deals with 5 multiple representations which are context, table, verbal description, symbols, and graphs. The activity consists of five interconnected options.

In the question in option b of the first group, how the context of multiple representations affects students' algebraic thinking is given below.

... Cem: We can do it like this and we won't use algebraic expressions. We can divide 76 by 2...

Güney: We will make four.

Cem: We subtract 2. 2, right?

Güney: Yes 2.

Selin: Let's try it then.

Güney: 38.

Researcher: What does 38 mean what you found.

Güney: There are 38 tables.

Selin: I think the number of tables.

Cem: Yes, the number of tables.

Researcher: Why did you divide 76 into 2?

Cem, Selin and Güney: For increasing by two.

Researcher: I wonder if this method would be valid for the situation you found in option a (they found that 22 people could sit at 10 tables).

Cem: 22 divided by 2 is 11. No. First we were going to subtract 2 and then we were going to divide by 2.

Researcher: Why do you subtract 2?

Cem: Because they increase two by two and they are all two more than two times two.

Güney: Yes. Two more than all of them.

Cem: So all the seats are two more than twice the number of two.

Güney: If we subtract 2 from 4, 2 is the initial 2.

Researcher: What do you mean?

Güney: In the beginning there are 4 chairs. There are 6 chairs at two tables. Now, since they are increasing two by two, if we subtract 2 from 4, it is 2.
Cem: We multiply 2 by 2.
Researcher: I don't understand.
Selin: We take them out because they increase two by two.
Researcher: But you said that the reason for dividing it by 2 is because it increases two by two.
Selin: Oh yes.
Researcher: Can you explain to me why you subtract 2?
Cem: Those two chairs are over there. Since the two tables were joined, the two chairs there were taken. So there were two missing.
Researcher: Is it like that every time?
Güney: The other one is missing 4 chairs (he is talking about a combination of 3 tables), but this one is missing 2 and the other one is missing 4, so we subtract 2 from all of them.

Looking at the dialog above, the students tried a different way before continuing the pattern while finding how many tables 76 people could sit at. First, they stated that 76 would be divided by two, and then two would be subtracted from the result. However, they changed what they did because it did not fit the result of 22 people sitting at ten tables in option a. In other words, they said they would first subtract two from 76 and then divide by 2. Their operation was correct and led them to the result, but the students could not explain why they did it this way. In other words, they could not fully express that the reason for subtracting 2 was to subtract a constant term or two chairs, and the reason for dividing by two was due to the amount of increase.

The researcher suggested they try a different solution.

… Researcher: You can try another solution.
Cem: We can continue the pattern.
Güney: We can start at 22.

Selin: 27.

Cem: Not 27.

Selin: After 22, I continued until 76. (She counted the tables after 22 by numbering them from 1)

Güney and Cem: 22 people at 10 tables.

Güney: There are already 10 tables, so that makes 37.

Selin: Oh yes. I counted wrong.

Researcher: Can you explain how you found it?

Selin: From 22, we increased by two by two until the 76th person. At 37, 76 people sit at the table.

As can be seen dialog above, the students continued the pattern by increasing by two until 76 people were reached. Because 22 people could sit at ten tables, they counted two by two, starting from the 22nd person. This way, they found that 76 people could sit at 37 tables.

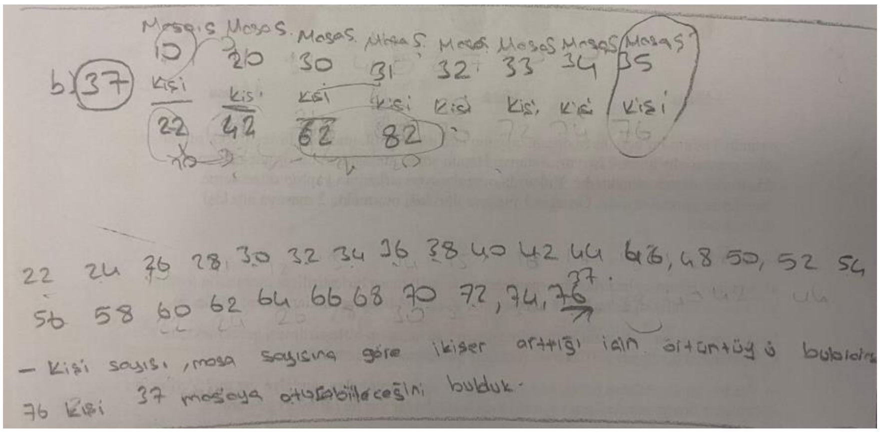


Figure 4.21 The Answer of the Second Group for Option b for the Table Organization Activity

In the following dialog, the question in the second group’s option b shows how the context of multiple representations affects students’ algebraic thinking.

... Emre: We divide 76 by 2.

Melek: Why did you divide it into two?

Emre: They are increasing two by two.

Melek: Yes, but there are 12 differences between 22 people sitting at 10 tables.

Researcher: How will you do it?

[…]

Melek: 22 people at table 10. 24 at table 11, 26 at table 12...

Zülal: Why did you do that? Let's go directly as a multiple of 10. If there are 22 people in 10, 44 in 20, 66 in 30, 76 in 40.

Melek: Let's write it down.

Zülal: No, it's not 76, it's 88.

Melek: You added it up wrong. It's 54, not 44. It goes twenty by twenty, here it is. 32 54 76

Zülal: Are you sure? 22 44... Aren't we collecting?

Melek: No. If there are 22 people at every 10 tables. Another 10 tables need to add another 22 people.

Zülal: Okay, you added 22, that's 44.

Melek: I added 10 here. 44 66... It doesn't work. Let's just write normally.

Zülal: We're going to do something later. 72 in 34, 74 in 36... Oh why did I increase it by two?

Melek: Zülal, what are you calculating?

Zülal: It increased ten by ten. After 66, it becomes 88. We continue it one by one.

Melek: Hmmm okay.

Zülal: 70 72 74 ... at 35 it becomes 76.

Researcher: Are you sure about your solution?

Melek: I am not sure.

Zülal: But that's what happens when you do it.

Researcher: If you want to, you can do it up to 20 tables and see if the number of people increases the way you do it.

Zülal: It was 42. It didn't happen.

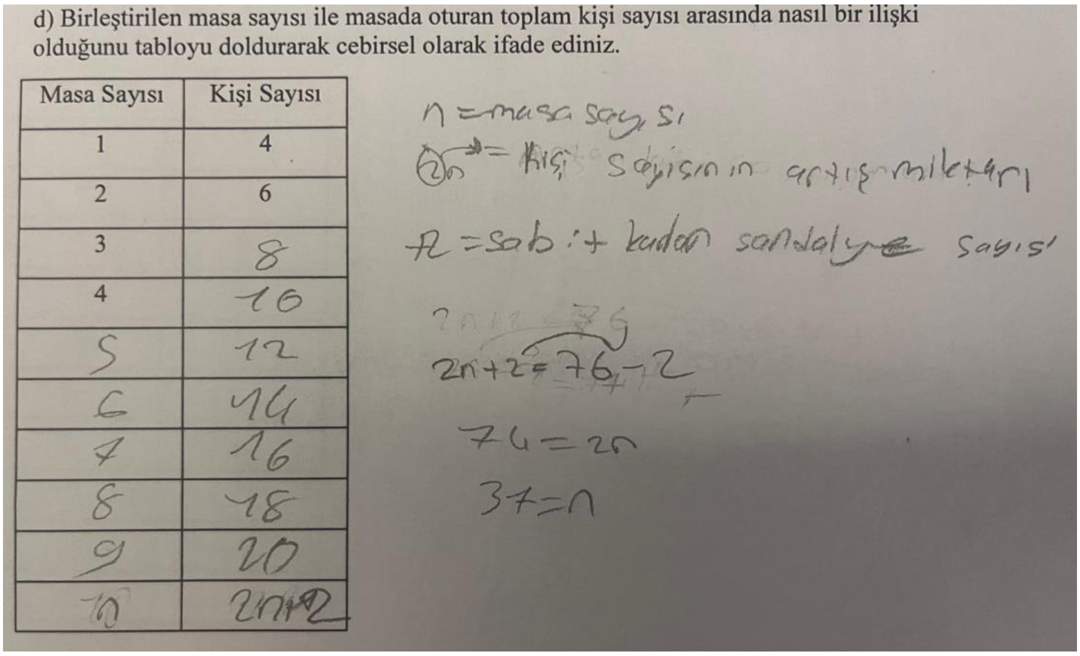
Melek: Yes. It goes twenty by twenty when you go ten by ten.

Looking at the dialog above, the students continued the pattern while finding how many tables 76 people could sit at. However, they first talked about dividing 76 by 2, but they decided that this idea was not correct and did not dwell on it. In order to make it easier to continue the pattern, the students tried to find the number of people for the number of tables in multiples of 10 and tried to find how many tables 76 people could sit at. Based on the statement that 22 people sit at ten tables, which they found in the previous question, they added 22 to the number of people every time they added ten tables, but the pattern did not progress as they did. For them to realize that they had made a mistake, the researcher suggested they determine the number of people by increasing the number of people by two by two up to 20 tables and having them check the correctness of their solution. In other words, the students realized that 22 people were not added for every 10 table increases, but they still could not reach a solution.

The researcher suggested they try to continue the pattern by counting by twos.

… Researcher: Can't you count the number of people in twos then?
Zülal: It would be very difficult.
Melek: Let's count.
[...]
Emre, Melek and Zülal: 76 people at 37 tables.

In the dialog above, they continued the pattern by increasing by two until they reached 76 people who could sit at 37 tables.


d) Birleştirilen masa sayısı ile masada oturan toplam kişi sayısı arasında nasıl bir ilişki olduğunu tabloyu doldurarak cebirsel olarak ifade ediniz.

Masa Sayısı	Kişi Sayısı
1	4
2	6
3	8
4	10
5	12
6	14
7	16
8	18
9	20
n	2n+2

n = masa sayısı

2n = kişi sayısının artış miktarı

2 = sabit kalan sandalye sayısı

2n+2 = 76 − 2

74 = 2n

37 = n

Figure 4.22 The Answer of the Second Group for Option d for the Table Organization Activity

In the following dialog, in the question in option d, the first group's thoughts on how they found the pattern rule by using tables and symbols from multiple representations are given.

... Güney: Let's write the algebraic expression at the bottom of the table. There are 20 people at 9 tables.

Selin: Yes.

Güney: 2n can be plus 2.

Selin: Yes.

Cem: Yes, plus two fixed chairs. There are increasing chairs in 2n. Then the number of tables becomes n.

[…]

Researcher: Well, can you explain the algebraic expression you wrote?

Güney and Cem: n is the number of tables.

Selin: We multiplied it by 2 because it increases by two.

Cem: We added 2. Those are fixed chairs.

The thoughts of the second group on how they found the pattern rule by using tables and symbols from multiple representations in the question in option d are given below.

... Melek: The question asks for a table.
Zülal: Also the algebraic expression.
Melek: Let's write the algebraic expression at the bottom of the table.
Zülal: 4 6 8 10 12 14 16...
Melek and Zülal: 2n+2
Melek: Let's explain. n is the number of tables. Plus 2 added chairs.
Zülal: Isn't the number of people increasing?
Melek: They add chairs and then the number of people increases. Increasing number of people?
Zülal: The number of people. Increasing number of people at each table.
Researcher: Then what does 2 multiplied by n mean?
Melek: This is the amount of increase.
Zülal: No.
Melek: Yes, the amount of increase, right?
Emre: Increased number of chair.
Melek: Okay, here is the amount of increase.
Researcher: If you want, first talk about why you multiplied n by 2.
Melek: Isn't it the amount of increase?
[...]

The dialogues above show that both groups filled the missing spaces in the table by applying the pattern rule that is, by increasing the number of people two by two, taking into account the amount of increase. It is seen that the students were able to write the relationship between the number of tables and the number of people by using algebraic expressions. Both groups determined the general rule of the pattern as $2n+2$ and expressed it with the symbol n by stating that the unknown is the number of tables. Students in the first group were able to explain how they wrote the algebraic expression.

However, the students in the second group had difficulty in finding a common sight while explaining why they multiplied *n* by 2 and why they added 2 to this product. At this point, the researcher guided them by reminding them of what they did in the previous question.

… Researcher: What did we do in option c?
Melek: We found the chairs that were fixed and the chairs that were added. So the 2 that we added are the fixed ones?
Zülal: The number of chairs is fixed then.
Melek: What is 2n then? Then it is the amount of increase.
Researcher: Well, can you explain the algebraic expression you wrote?
Emre: n is the number of tables. We made 2n because of the increasing number of people. We added 2, which is the number of chairs that remains constant. That's 2n plus 2.

As seen above, the students were able to explain that the reason why they multiplied *n* by 2 was because the number of chairs increased by two and the reason why they added 2 to this product was because of the chairs that were fixed.

The first group thought as follows while confirming the validity of the general rule of the pattern they found for each situation within the scope of the question.

… Researcher: So, is this pattern rule, or rather algebraic rule that you wrote valid for every situation within the scope of this question?
Güney: Let's try. 2 times 4 plus 2 equals 10. Let's try again. 2 times 6 plus 2 equals 14.

While the second group verified the validity of the general rule of the pattern they found for each situation within the scope of the question, they thought as follows.

… Researcher: So, is this pattern rule, or rather algebraic rule that you wrote valid for every situation within the scope of this question?

Melek: Yes, but let’s try.

Zülal: Let’s try with option b. 2n plus 2 equals 76. 2n equals 76 minus 2, which is 74. n becomes 37.

In the dialogues above, students in both groups decided that the general rule of the pattern is valid for every situation within the scope of the question by trying the pattern rule they created. In other words, they wrote number of tables instead of *n* instead of the unknown term, checked whether they matched the information in the table, and found that the general rule of the pattern was valid for every situation within the scope of the question.

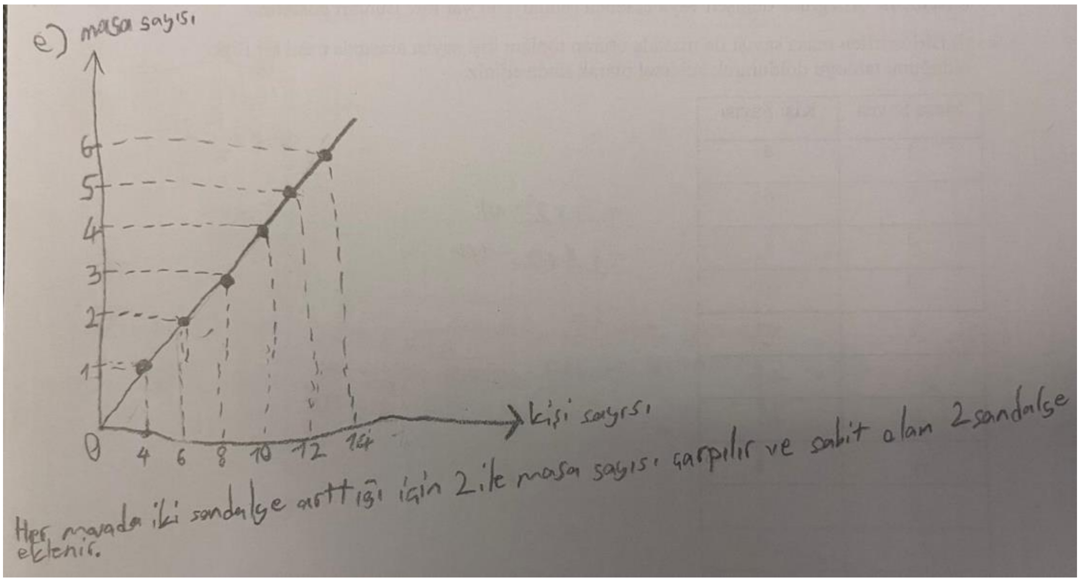


Figure 4.23 The Answer of the First Group for Option e for the Table Organization Activity

In the dialog below, the question in option e reveals how the first group expresses the pattern rule using different representations by using graphical and verbal description from multiple representations.

... Güney: What kind of graphic?

Selin: We drew something like this for a while. (She drew the horizontal and vertical axis)

Cem: Let's make a line graph.

Güney: Yes, like a line graph. Number of tables... (vertical axis)

Selin: Number of people here (horizontal axis)

Güney: How many people were at 1 table?

Selin: Four. Let's continue. 6 8 10 12 14.

(they started to draw the graph as 4 people sitting at 1 table and started at the zero point by showing the number of people at each table)

Researcher: Why did you start your graph from point 0?

Güney: Because 0 people sit at 0 tables.

Researcher: You are asked to verbally explain the relationship between the number of people and the number of tables.

Cem: Since there are two more chairs at each table, multiply 2 by the number of tables and add the 2 fixed chairs.

The following dialog reveals how the second group expressed the pattern rule using different representations by using graphical and verbal description from multiple representations in the question in option e.

... Zülal: Let's call this the number of tables (vertical axis). And let's call this the number of people (horizontal axis)

Melek: Should we make a line graph or a column graph?

Zülal: Let's write the number of tables as 1 2 3 4 5. Let's write 4 6 8 10 12 for the number of people.

[...]

Zülal: 1 in 4. 2 in 6. 3 in 8. 4 in 10. 5 in 12.

(they draw the graph as if there are 4 people sitting at 1 table, showing the number of people at each table, starting from the zero point)

Researcher: Why did you start your graph from point 0?

Zülal: There are no tables at zero...

Emre: Zero people.

Melek: Zero people at zero tables.

Researcher: You are asked to verbally explain the relationship between the number of people and the number of tables.

Melek: We explain 2n+2. n is the number of tables. We multiplied 2 by n because the number of people increases according to the number of tables. We added 2 for the number of chairs that remain constant.

As can be seen in the dialogues above, students drew a graph of the relationship between the number of tables and the number of people and explained it verbally. While drawing the graph, both groups paid attention to the fact that the starting point of the graph should be zero and increased the number of people by two as the number of tables increased. They explained this relationship by emphasizing that they added 2 to the number of tables because the number of chairs at each table increased by two, or they added 2 to the number of tables because the number of people increased, emphasizing that the number of chairs was constant.

A general evaluation of this activity shows that both groups were able to use multiple representations effectively. They were able to transition between different forms of representation with ease. Table 4.3 summarizes the findings of the qualitative analysis, demonstrating that students showed improvement in each component of algebraic thinking.

Table 4.8 Development of Algebraic Thinking

The Name of the Activity	Development of Algebraic Thinking
The Calculator	The students were able to generalize the properties of basic operations, conjectures derived from basic properties, and properties of odd and even numbers.
The Ali Captain's Ship	The students perceived the equal sign as a relational symbol, were able to think relationally and improved in writing equations using algebraic expressions.
Let's Go to The Bazaar	The students were able to use the variable in the sense of the unknown and improved themselves in finding the value of the unknown.
Ali's Shopping	The students were able to use variables as varying quantities and were able to find all the values that the variables could take.
The Gasoline Tank	The students were able to establish a relationship between quantities and interpret this relationship. However, it can be said that they have difficulty in showing this relationship using algebraic expressions and need improvement in this regard.
Box-Penny	It can be said that students made positive progress in determining the unit of repetition in a repetitive pattern and in writing the general rule of that pattern.
Urban Transformation	The students improved in analyzing and extending number and shape patterns, finding the recursive relationship in these patterns, and finding the general rule of these patterns.
The Table Organization	The students were able to use multiple representations effectively. They were able to effectively transform multiple representations between each other.

The following table presents the analysis of qualitative data from an alternative perspective.

Table 4.9 Development of Algebraic Thinking Components

	Algebraic Thinking Components							
The Name of The Activity	Properties of the number system	The meaning of the equal sign and relational thinking	Use of variables as unknowns	Use of variables as varying quantities	Quantitative reasoning	Repetitive patterns	Growing patterns	Multiple representations
The Calculator	✱							
The Ali Captain's Ship		●						
Let's Go to The Bazaar			▲					
Ali's Shopping				✚				
The Gasoline Tank					◆			
Box-Penny						■		
Urban Transformation							✦	
The Table Organization								✶

✱: Improvements were observed in generalizing basic operation properties, basic property assumptions, and odd and even number properties.
●: Progress was achieved in perceiving the equal sign as relational symbol and thinking relationally.
▲: Development was seen in using the variable in the sense of the unknown and finding the value of the unknown.
✚: The use of variables as varying quantities had improved.
◆: The ability to establish a relationship between quantities and to interpret this relationship has improved, but the ability to express this relationship using algebraic expressions needs to be developed further.
■: Positive progress was detected in determining the unit of repetition in a repetitive pattern and in writing the general rule of that pattern.
✦: The improvement was reached in analyzing and extending number and shape patterns, finding the recursive relationship and general rule in these patterns.
✶: The improvement of the effective use of multiple representations and their transformation between each other was observed.

CHAPTER 5

DISCUSSION, CONCLUSION and RECOMMENDATION

In this chapter, conclusions, discussions and recommendations regarding the findings of the study are given.

5.1 Discussion

5.1.1 Generalizing Arithmetic and Quantitative Reasoning

Properties of the Number System

Properties of the number systems is a subcategory of the generalizing arithmetic and quantitative reasoning category and is a component of algebraic thinking according to the conceptual framework of this study. According to this component, students are expected to generalize the basic operation properties, the conjectures derived from these properties, and the properties of odd and even numbers. According to the study's findings, students could generalize the basic operation properties, the assumptions derived from these properties, and the properties of odd and even numbers, and they improved themselves in this regard. It can be stated that this result improves students' algebraic thinking. As a matter of fact, according to the quantitative findings supporting the qualitative findings, the students' algebraic thinking levels increased after the teaching experiment.

The importance of generalization is emphasized in order to ensure a smooth transition from arithmetic thinking to algebraic thinking (Carpenter & Levi, 2000).

Indeed, generalizing important mathematical ideas, such as the properties of numbers and four operations with numbers, affects the development of arithmetic understanding (Blanton, 2008), and in this way, the transition from arithmetic thinking to algebraic thinking is achieved. However, Pillay et al. (1998) reported in their study that most of the 7th and 8th-grade students lacked knowledge about commutative and distributive laws, which are fundamental for algebra, or even had no knowledge at all. In other words, the students could not generalize the situations related to both commutative and distributive properties, the transition from arithmetic to algebra was not achieved, and algebraic thinking did not develop. At this point, it was indirectly revealed how important the generalization of the properties of the number system is for the development of students' algebraic thinking.

Looking at the literature, Blanton et al. (2015) investigated the effects of comprehensive algebra instruction in their experimental study. As a result of the study, it was found that students could generalize the commutative property in addition and that subtracting the same number from itself is equal to zero. It was also found that the experimental group performed better than the control group and could use algebraic strategies more. According to the study's findings, students have algebraic thinking capacities and can think algebraically. In their teaching experiment, Isler et al. (2013) investigated the development of students' understanding of generalized arithmetic in the context of even and odd numbers. They found that students could develop, explain, and generalize conjectures about the sum of two odd numbers, the sum of two even numbers, and the sum of an odd and an even number. Carpenter and Levi (2000) found that students were able to generalize the properties of basic operations with zero, i.e., when zero is added to and subtracted from any number, it is equal to the same number, and if we subtract itself from any number, the result is zero. They also generalized some of the properties of odd and even numbers, namely that the sum of two odd integers is an even number, and the sum of an even and an odd integer is an odd integer. Although their explanations were not always sufficient to justify these conjectures, they could express them using algebraic expressions, which influenced the development of their algebraic thinking. The findings of previous studies support the results of this study.

The Meaning of the Equal Sign and Relational Thinking

In the conceptual framework of the study, the meaning of the equal sign, a subcategory of the meaningful use of symbols category, and relational thinking are components of algebraic thinking. According to this component, it is hoped that students will know the meaning of the equal sign, use it by its meaning, and think relationally. According to the data analysis, after the instructional section aimed at developing the correct use of the meaning of the equal sign and relational thinking, students were able to perceive the equal sign as a "relational symbol" in algebraic terms rather than "writing the result of the operation after the equal sign" in arithmetic terms, they were able to think relationally, and they improved in writing equality using algebraic expressions. This affects the development of algebraic thinking. The test analysis results also support these findings, that is, students' algebraic thinking levels increased.

The correct and effective use of the meanings of symbols is essential for the beginning and development of algebraic thinking (Adıyaman, 2019). However, when the studies in the literature were examined, it was found that students could not perceive the equal sign as a relational symbol (Behr et al., 1975; Demir, 2022; McNeil et al., 2019; Yaman et al., 2003). The meaning that the equal sign symbolizes a relationship is vital for algebra learning (Carpenter et al., 2005). It is also argued that the backbone of algebraic thinking is the relational symbol meaning of the equal sign, which is the idea that two sides are equal and interchangeable (Fyfe et al., 2018). We can conclude that perceiving the equal sign as a relational symbol is critical for algebraic thinking.

Molina and Ambrose (2006) conducted a study to develop students' understanding of equal sign and relational thinking, and according to the findings of this study, they observed that students' relational thinking improved. Similarly, Kızıltoprak (2014) applied the teaching experiment model and found that students perceived the equal sign as a relational symbol, improving their relational thinking skills. Demir (2022) found that the activities implemented during the teaching experiment positively affected the development of students' relational thinking, which in turn led to the development of algebraic thinking, proving the consistency of this study's findings. In the teaching experiment conducted in this study, the meaning of the equal sign, and the relational thinking component were found to affect the development of students' algebraic thinking.

The Use of Variables as Unknowns

The use of variables in the sense of the unknown is a subcategory of the meaning of variables and is a component of algebraic thinking according to the conceptual framework of this study. According to the quantitative data findings of the study, it was found that students improved themselves in using variables in the sense of the unknown and finding the value of the unknown after the teaching section aimed at improving the use of variables in the sense of the unknown. It can be stated that this result improved students' algebraic thinking. As a matter of fact, according to the quantitative findings supporting the qualitative findings, students' algebraic thinking levels increased after the teaching experiment. Similarly, Marum et al. (2011) examined the development of student's skills in the use of the unknown meaning of the concept of variable using a teaching experiment model and found that students were able to use the unknown meaning of variables, represent them using unknown letters, and improve in this regard. Kieran (2004) argued that focusing on the unknown meaning of variables and working on them can improve algebraic thinking, and according to the result of Marum et al. (2011), it can be said that the development of the use of the unknown meaning of variables also improves algebraic thinking. These inferences also prove the findings of the study.

When we look at the literature, there are negative studies on the use of the meaning of the unknown of the variable (Sünkür et al., 2012; Usta & Gökkurt Özdemir, 2018; MacGregor & Stacey, 1997). MacGregor and Stacey (1997) conducted a study to learn about algebraic symbols in the prior knowledge students between the ages of 11 and 15 have while learning algebra. As a result of this study, it was found that most of the students could not interpret the letter symbols in algebra as generalized numbers or even as unknowns. In addition, Akkan (2009) stated in his study that 5th-grade students could not comprehend the difference between the meanings of unknown and varying quantities of the variable and that they confused the ways of using them; that is, they used them interchangeably. Şimşek and Soylu (2018) also found a similar result in their study with 7th-grade students. Chevallard (1989) stated that this may be due to traditional algebra teaching (as cited in Arzarello et al., 1993). Another study that indirectly supports this idea is conducted by Carraher et al. (2001) to examine whether elementary school students can work on unknowns. They found that children aged between 8 and 9 years could use letters to represent unknowns; that is, they comprehended the meaning of the unknown and could perform operations with letters and symbols representing unknowns without valuing them.

Similarly, Ergöz (2000) stated in her experimental study that the education that enables the transition from arithmetic to algebra provides a better understanding of the unknown concept. Berg (2012) also sought evidence on how elementary school students develop algebraic thinking by giving them a series of tasks and observing them. According to the study's results, she found that students think algebraically when they can find unknowns, work with variables, and generalize. The conclusion drawn from this is that the meaning of the unknown of the variable is essential for algebraic thinking.

The Use of Variables as Varying Quantities

Using variables in the sense of varying quantities is another subcategory of the meaning of variables. According to the study's conceptual framework, this subheading is a component of algebraic thinking. According to the quantitative findings, students were able to use variables in the sense of varying quantities and find all the values that variables can take after the instructional part aimed at developing the use of variables in the sense of varying quantities. It can be stated that this component improved students' algebraic thinking. According to the test analysis in the quantitative findings, the increase in students' algebraic thinking levels after the teaching experiment supports the qualitative findings. Similarly, Stephens et al. (2017) applied early algebra instruction to third to fifth-grade students for three years. As a result, students were increasingly able to reason about relationships between varying quantities and improved their use of variables in terms of varying quantities.

There are many negative studies in the literature on the use of variables in the sense of varying quantities (Rosnick, 1981; Clement et al., 1981; Akkan, 2009; Knuth et al., 2005). Soylu (2006), in his study to reveal students' errors and learning difficulties regarding the concept of variable, found that students used variable as a label of an object instead of using it in the sense of varying quantities. Similarly, Akgün (2007) conducted a study to reveal the competencies of 8th-grade students towards the concept of variables and their understanding of different uses of this concept and concluded that they had difficulties in using variables in terms of different quantities and that their level of knowledge on this subject was insufficient. Akkan (2009) also found that 5th-grade students could not comprehend the difference between the unknown and varying quantities of the variable and used them interchangeably. Soycan (2023) examined how mathematics textbooks from Grade 1 to Grade 4 contribute to students' algebraic thinking and concluded that the use of variables in the sense of varying quantities is not included in the textbooks. This result proves why students have such difficulties and fail in this subject. At this point, this study has gained significant meaning by developing students' algebraic thinking by developing them in this subject.

Some mathematics educators argue that algebra should be taught using the meaning of varying quantities before the unknown meaning of variables (Fey & Good, 1985; Usiskin, 1988). In fact, Gürbüz and Özdemir (2020), in their study in which they investigated the abstraction processes of 6th-grade students toward the concept of variables by using a teaching experiment model, concluded that the development of the use of variables progresses depending on the use of variables in terms of different quantities, supporting the arguments of Fey and Good (1985) and Usiskin (1988). In other words, it can be said that the meaning of variables in terms of varying quantities is indispensable for algebra teaching. This will affect the development of their algebraic thinking.

Quantitative Reasoning

According to the conceptual framework adopted in this study, quantitative reasoning, a subcategory of generalizing arithmetic and quantitative reasoning, constitutes one of the components of algebraic thinking. According to this component, students are expected to establish, interpret, and analyze the relationship between quantities. According to the study's qualitative findings, it was revealed that students were able to establish the relationship between quantities and interpret the relationship after the teaching section aimed at developing quantitative reasoning. Quantitative reasoning is another factor affecting algebraic thinking. The quantitative findings found that students' algebraic thinking levels increased according to the CDAT analysis. Quantitative and qualitative findings support each other. Similarly, Dur (2014) examined the development of 6th-grade students' quantitative reasoning skills using a teaching experiment model and found that students' quantitative reasoning skills improved, positively affecting their algebraic development. Since algebraic development will improve algebraic thinking (Yenilmez & Teke, 2008), the statement that quantitative reasoning improves algebraic thinking can be said according to the results of Dur's (2014) study. In addition, this shows that the findings of Dur's (2014) study and this study are consistent.

Thompson (1988) argued that quantitative reasoning is the building block for algebraic thinking and found that students with inadequate quantitative reasoning skills performed poorly and inadequately in solving arithmetic and algebraic problems. The findings of this study indirectly proves what Thompson (1988) argued. Similarly, Smith and Thompson (2007), who argued that algebraic knowledge is more meaningful with quantitative reasoning, found that K-8 students have arithmetic, quantitative reasoning, and algebraic thinking skills and that the algebraic thinking levels of students with low quantitative reasoning skills are negatively affected in parallel. In addition, Güvendiren (2019) examined the algebraic thinking of 6th-grade students in quantitative reasoning, covariational, and functional thinking and found that quantitative reasoning is the focal point for developing other forms of thinking. The conclusion to be drawn from this study is that quantitative reasoning is one of the main factors affecting the development of algebraic thinking. Indeed, in the teaching experiment conducted in this study, the quantitative reasoning component was found to affect the development of students' algebraic thinking.

5.1.2 Functional Thinking

Repetitive Patterns

According to the study's conceptual framework, repetitive patterns in the subcategory of functional thinking are another component of algebraic thinking. According to this component, students are asked to determine the repetition unit in repetitive patterns and find the general rule of these patterns. According to the data analysis, students made positive progress in determining the unit of repetition and expressing the general rule after the teaching section aimed at developing a conceptual understanding of repetitive patterns, contributing to their functional development. Their algebraic thinking development was also positively affected by this situation. The test analysis results support these findings; that is, there was an increase in students' algebraic thinking levels. Similarly, Warren (2005b) conducted a study using the teaching experiment model and found that the majority of the students were able to create a repetitive shape pattern, continue it, and find a repetition unit and that they obtained these by using a functional relationship. Since algebraic and functional thinking are interrelated (Blanton & Kaput, 2004), we can conclude that Warren's (2005b) study influenced students' algebraic thinking.

Kabael and Tanışlı (2010) view patterns as the foundation for the development of functional relationships. This perspective is supported by Warren and Cooper's (2006) examination of repetitive patterns, which revealed that students' understanding of these patterns led them to discover functional thinking. Surprisingly, even young students were able to articulate their functional thinking clearly. The study's findings underscore the role of repetitive patterns in fostering functional thinking.

Tanışlı (2008) found that in repetitive patterns, students could find the repetition unit, determine how the shapes in this repetition unit have a functional relationship with each other, continue the pattern to a specific step, and form a repetitive pattern. Students who determine the repetition unit in the pattern will have laid the foundation of algebra by analyzing the relationships between the shapes and numbers in the repetition unit, making a transition from a concrete situation to an abstract situation (Orton, 1992; as cited in Threlfall, 1999, p. 26). In addition, Palabıyık (2010) found that teaching algebra with patterns positively affected algebraic thinking. It can be concluded that teaching with repetitive patterns affects algebra positively, and this affects algebraic thinking.

Growing Patterns

According to the conceptual framework adopted in this study, the growing pattern, a subcategory of functional thinking, is a component of algebraic thinking. According to this component, students are expected to be able to analyze number and shape patterns, expand the patterns by using the recursive relationship, and express the general rule. The qualitative findings of this study revealed that after the instructional section aimed at developing a conceptual understanding of growing patterns, students showed improvement in analyzing and extending these patterns by using the recursive relationship and finding the general rule in these patterns. In this way, it contributed to the development of both functional and algebraic thinking. This is in line with the results of other studies (Blanton et al., 2015; Stephens et al., 2017; Kulaç, 2023), which found that growing patterns have an effect on the development of functional thinking. According to the quantitative findings in this study, the increase in students' algebraic thinking levels after the teaching experiment supports the quantitative findings.

Looking at the literature, Warren and Cooper (2008) examined the generalization of growing shape patterns with a teaching experiment and found that students' ability to understand these patterns and to form general terms for patterns increased. Warren and Cooper's (2008) study is consistent with the findings of this study. In addition, Markworth (2010) examined how expanding shape patterns, a practical context, affect students' development of functional thinking with the teaching experiment model. It was found that these patterns are indeed a practical context in the development of functional thinking. Similarly, Miller (2016) investigated 2nd and 3rd-grade students' transition from recursive thinking to functional thinking with a teaching experiment model using growing patterns. According to the study's findings, it was observed that they could define and express the functional relationship and think functionally. In light of the results of these studies, while proving the effect of growing patterns on functional thinking, it is revealed that the teaching experiment also contributes to students' functional thinking. In addition, Yüce (2022) used growing patterns to improve students' algebraic thinking in her teaching experiment, and it was found that students' algebraic thinking improved. Accordingly, the conclusion that growing patterns affect the development of algebraic thinking with the teaching experiment conducted in this study is supported by other studies.

5.1.3 Modeling

Multiple Representation

According to the conceptual framework of this study, multiple representations, a subcategory of modeling, are one of the components of algebraic thinking. According to this component, students are expected to use context, table, verbal explanation, symbol, and graph representations effectively and to be able to make the transition between these representations. According to the qualitative data analysis of this study, it was found that students could use multiple representations effectively and make transitions between representations after the instructional part aimed at improving students' use of multiple representations. Accordingly, it was found that students' ability to use multiple representations affected their algebraic thinking. According to the quantitative findings that support the qualitative findings, students' algebraic thinking levels increased after the teaching experiment. Kaya and Keşan (2014) also supported this study's findings by arguing that for students to develop their understanding of algebra and algebraic thinking, they should be able to switch between algebraic concepts and show them with multiple representations.

In many studies conducted at different grade levels, it has been observed that student's ability to transform between representations of algebraic concepts is low, and they have difficulty in this regard (Gök & Cansız Aktaş, 2019; Gürbüz & Şahin, 2015; Mercan, 2020; Sert, 2007; Baloğlu Demir, 2022). This situation may cause students to have a negative attitude towards algebra (Yılmaz, 2011) and can be said to affect students' understanding of algebra. Since algebra is an integral part of algebraic thinking and the development of students' understanding of algebra also improves their algebraic thinking (Trybulski, 2007; Kaya & Keşan, 2014), it can be stated that the effect of multiple representations on algebra and algebraic thinking is undeniable.

There are many studies proving the positive effects of using multiple representations on algebra learning (Brenner et al., 1995; Swafford & Langrall, 2000; Özgün Koca, 2001; Akkuş & Çakıroğlu, 2010). Janvier (1978) stated that by integrating multiple representations into algebra instruction, students are able to use their representations of the meaning of algebraic concepts more effectively, which leads to conceptual understanding. In this way, meaningful algebra teaching can be realized. In addition, there are studies proving that multiple representations improve algebraic thinking (Faujiah, 2024; Kaya, 2015; Kusumaningsih et al., 2018; Liadiani, 2020; Moseley &

Brenner, 1997). It is argued that the effective use of multiple representations provides students with better algebraic thinking capability (Liadiani, 2020). In addition, Ataş (2019) concluded in her study that the use of multiple representations improved and developed in parallel with the increase in students' algebraic thinking levels. As a result, the importance of the appropriate use of multiple representations for the development of algebraic thinking was also evident in this study.

Nilkland (2004) examined changes in algebraic thinking and reasoning by providing learning environments in which students could share their thoughts and demonstrate their understanding and found that such a learning environment improved students' algebraic thinking. The results of Nikald (2004) support this study, which showed that students' algebraic thinking levels increased after the instruction during the teaching experiment. Another study that supports the consistency of this study is the study by Store et al. (2010), who found that using multiple representations and strategies in a teaching experiment is an instructional practice that supports algebraic thinking more. In this study, it was concluded that the multiple representation component of the teaching experiment was influential in the development of students' algebraic thinking.

5.2 Conclusion

The conceptual framework of the study consists of three main components: generalizing arithmetic and quantitative reasoning, functional thinking, and modeling, which are argued to develop algebraic thinking based on the results of research on the development of algebraic thinking in the literature. Under the conceptual framework of this study, we sought to answer how students' algebraic thinking developed with the teaching experiment model. In addition, the development of the components of properties of the number system, the meaning of the equal sign and relational thinking, the use of variables as unknowns, the use of variables as varying quantities, quantitative reasoning, repetitive patterns, growing patterns, and multiple representations were examined separately in the teaching experiment implemented for 8-weeks. The qualitative and quantitative findings of the study indicate that teaching algebra with a teaching experiment affects students' algebraic thinking.

In addition, the data analysis of the 8 worksheets, each representing an algebraic thinking component, used during the teaching experiment showed that the development of each algebraic thinking component was observed. The fact that there was a significant difference between the students' scores on the Chelsea Diagnostic Algebra Test before and after the teaching experiment shows that the instruction increased the students' level of algebraic thinking. These results show how algebraic thinking can improve with the development of algebraic thinking components.

It can be emphasized that the application of the teaching experiment model rather than traditional teaching has a positive effect on the development of algebraic thinking. These results may shed light on how students can succeed in algebra and develop their algebraic understanding.

5.3 Limitations of Teaching Experiment

A teaching experiment aims to experience the participants' learning and reasoning directly. For this reason, the researcher assumes the role of both teacher and researcher, allowing for first-hand data collection while actively managing the process. However, this dual role may introduce a potential limitation, as the researcher's bias could affect the study. In this study, the researcher's bias was minimized to reach accurate and unbiased results by balancing the roles of the researcher and the teacher. Various data collection tools, such as video recordings of teaching episodes, guiding questions, CDAT, and student worksheets, were used to reduce the researcher's bias. Additionally, a retrospective analysis of data, —including detailed examination of the CDAT, student worksheets, and video recordings—was conducted to further minimize research bias. Finally, participants were selected voluntarily and the purpose of the study was clearly explained to them, serving as another measure to reduce potential researcher bias.

Another limitation of the study could be the unfamiliarity between the participants and the researcher, as they had no prior interaction. This lack of familiarity may have impacted the teaching experiment, as the students were engaging with someone they had never met before. This unfamiliarity could potentially influence how students responded to questions, expressed their thoughts, and approached the learning process. To mitigate this, the researcher extended the study period to allow for more interaction with the students, fostering a level of comfort. Additionally, guiding questions were employed throughout the process to encourage students to articulate their algebraic thinking more effectively.

5.4 Recommendations

As a result of the effectiveness of the teaching experiment conducted in this study, studies can be conducted to improve mathematics teachers' awareness of algebraic thinking through in-service training. In this direction, teachers can contribute to students' algebraic thinking by providing curriculum integration with the knowledge they have acquired and developed about algebra.

The study was conducted with six students in the seventh-grade level of middle school to investigate their algebraic thinking with a teaching experiment. However, the teaching experiment may be effective at other ages and grade levels. For this reason, it may be useful to test the method at different levels of education. This will allow the results of this study to be compared with the results of future studies, which may lead to more general conclusions on this topic. Also, this study can be conducted with more students. This can effectively reduce the margin of error in the results of the study.

This study was conducted over eight weeks. In future research, the teaching experiment can be applied for longer periods and cover different learning areas. If the implementation lasts longer and includes a different topic, there may be a chance to obtain more evidence about students' algebraic thinking development.

Within the framework of this research, instructional content for the development of algebraic thinking was created using the teaching experiment model. This instructional content was applied to students by organizing group work and focusing on the development of groups. This method can be applied to students individually to observe their individual development.

As a result of this study, it was found that the teaching experiment improved students' algebraic thinking. The content of this teaching experiment can serve as an example and guide mathematics teachers in designing course content to enhance their students' algebraic thinking. Therefore, studies can be conducted on how to adapt instruction similar to the content of this teaching experiment to the mathematics curriculum and textbooks. This will make it easier for mathematics teachers to put it into practice. In addition, a guidebook on sample applications can be published.

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APPENDICES

A. Chelsea Diagnostic Algebra Test (CDAT)

1) Belirtilenlere göre aşağıdaki boşlukları doldurunuz.

a) $x \longrightarrow (x+2)$ b) $x \longrightarrow (4x)$

$6 \longrightarrow$ ….. $3 \longrightarrow$ ......

$r \longrightarrow$ …..

2) Aşağıdakilerden en küçük ve en büyük olanı yazınız

n+1, n+*4*, n-*3*, n, n-*7* en küçük en büyük

....

3) Hangisi daha büyüktür, *2n* ya da *n+2?*

Yanıtınızı açıklayınız:

....

....

4) a) n'ye 4 eklendiğinde *"n+4"* olarak yazılır. Aşağıdaki ifadelerin her birine 4 ekleyiniz.

8	*n+5*	*3n*
....	....	....

b) n, 4 ile çarpıldığında *"4n"* olarak yazılır. Aşağıdaki ifadelerin her birini 4 ile çarpınız.

8	*n+5*	*3n*
....	....	....

5) a + b = 43   ise   a + b + 2 = ...

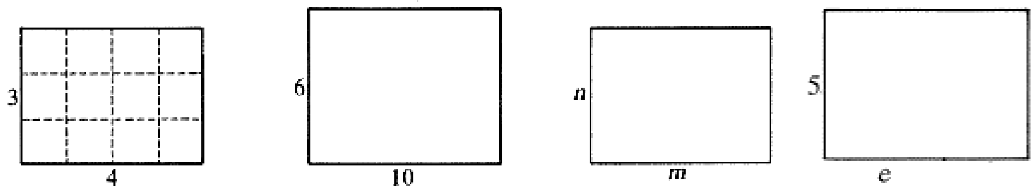
n - 246 = 762   ise   n - 247 = ...

e + f = g   ise   e + f + g = ...

6) a +5 = 8   ise   a nedir? ....

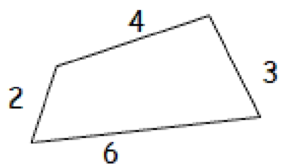
b + 2, 2b’ye eşit ise b nedir?.........................

7) Aşağıdaki şekillerin alanı nedir?

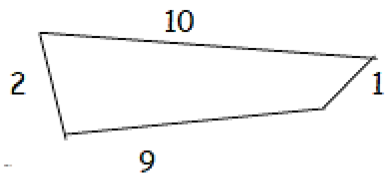


Alan = …….   Alan = …….   Alan = …….   Alan = …….

8) Yandaki şeklin çevresi, 6+3+4+2 = 15’tir.

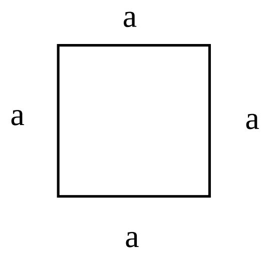


Buna göre, aşağıdaki şeklin çevresi nedir?



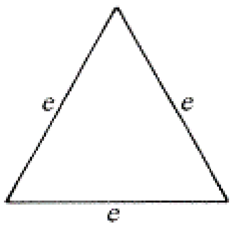
Çevre =

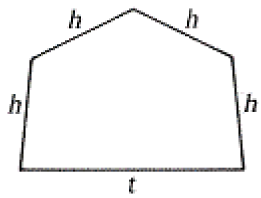
9) Yandaki karenin kenar uzunluğu a birimdir.

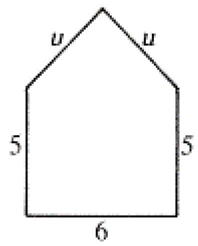


Bu karenin çevresi, Ç = 4a olarak gösterilir.

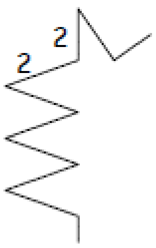
Buna göre, aşağıdaki şekillerin çevrelerini nasıl yazarız?







Çevre =   Çevre =   Çevre =



Bir kısmı çizilmeyen yandaki şeklin toplam n kenarı vardır ve her bir kenar uzunluğu 2cm'dir.

Çevre =

10) Kırtasiyede satılan bilgisayar dergilerinin tanesi 8, müzik dergilerinin tanesi 6 liradır. b harfi satın alınan bilgisayar dergilerinin sayısını, m harfi de müzik dergilerinin sayısını gösteriyorsa;

8b+6m neyi göstermektedir? ....

Toplam kaç tane dergi alınmıştır?..

11) Eğer u = v+3 ve v = 1 ise, u =? ....

Eğer m = 3n+1 ve n = 4 ise, m =? ....

12) Eğer Özlem'in Ö, Atakan'ın da A kadar misketi varsa, ikisinin sahip olduğu toplam misket miktarını nasıl yazarsınız?

....

13) a+3a ifadesi sade haliyle 4a olarak yazılır.

Buna göre; aşağıdaki ifadeleri yazılabiliyor ise sade halleriyle yazınız.

2a + 5b = ...	3a - (b + a) = ...
(a + b) + a = ...	a + 4 + a - 4 = ...
2a + 5b + a = ...	3a – b + a = ...
(a - b) + b = ...	(a + b) + (a - b) = ...

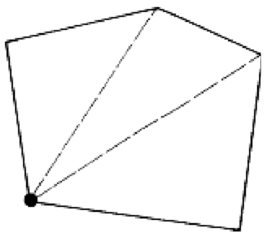
14) Eğer r = s + t ve r + s + t = 30 ise r = ...

15) Yandaki gibi bir şekilde köşegen sayısı kenar sayısından 3 çıkarılarak bulunabilir.

Buna göre; 5 kenarlı bir şeklin 2 köşegeni vardır.

57 kenarlı bir şeklin ....köşegeni vardır.

k kenarlı bir şeklin ....köşegeni vardır.



16) Eğer c + d = 10 ve c, d'den küçük ise c = ...

17) Ahmet'in haftalık kazancı 20 liradır ve fazla mesai yaptığı her saat başına 2 lira daha almaktadır.

Eğer s harfi yapılan fazla mesai saatini ve k harfi de Ahmet'in toplam kazancını gösteriyorsa; s ile k arasındaki ilişkiyi gösteren bir denklem yazınız: ....

Eğer Ahmet 4 saat fazla mesai yaparsa, toplam kazancı ne olur?...

18) Aşağıdaki ifadeler ne zaman doğrudur? Her zaman, Asla, Bazen?

Doğru yanıtın altını çiziniz. Yanıtınız "Bazen" ise ne zaman olduğunu açıklayınız.

A+B+C = C+A+B Her zaman Asla Bazen, ...

L+M+N = L+P+N Her zaman Asla Bazen, ...

19) a = b + 3 iken b 2 artırıldığında a ne olur?

....

f= 3g + 1iken g 2 artırıldığında f ne olur?

....

20) Isırgan büfede kekler k liraya, börekler b liraya satılmaktadır.

Eğer 4 kek ve 3 börek alırsam, 4k + 3b ifadesi ne anlama gelir? ....
....

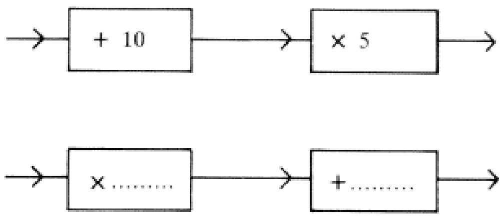
21) Kırtasiyede satılan mavi kalemlerin her biri 5, kırmızı kalemlerin her biri 6 liradır. Biraz mavi ve kırmızı kalem alırsam, toplam 90 lira ödüyorum. Eğer m alınan mavi kalem sayısını, k alınan kırmızı kalem sayısını gösteriyorsa, m ve k hakkında ne yazabilirsiniz?

....

....

....

22) Yandaki makineyi herhangi bir sayı ile besleyebilirsiniz.



Aynı etkiye sahip başka bir makine bulabilir misiniz?

B. The Permission for Activities and CDAT

Permission for the Calculator and Ali’s Shopping activities was obtained by sending an e-mail to Mehmet Fatih Özmantar on 25.10.2023.

Feyza Gün 25 Eki 2023 Çar 13:05
Alıcı:

Merhaba Mehmet Fatih hocam,
Ben ODTÜ Matematik Eğitimi yüksek lisans öğrencisi ve OHÜ Matematik Eğitimi araştırma görevlisi Feyzanur Gün. "İlkokul ve Ortaokul Matematiği Gelişimsel Yaklaşımla Öğretim" adlı kitabta türkçeye çevirdiğiniz 14. bölümünde yer alan "Şekil 14.1", "Şekil 14.9" ve "Etkinlik 14.10" yer alan soruları tezimde izniniz dahilinde kullanmak isterim. Sizin için herhangi bir sorun teşkil eder mi?
İyi günler...
Ar. Gör. Feyzanur Gün

Mehmet Fatih Özmantar 25 Eki 2023 Çar 18:54
Alıcı: ben

Merhaba Feyza,
İfade ettiğin bu kısımları kullanmanda bir mahsur yok. Elbette kullanabilirsin.
Çalışmalarında başarılar ve bol şans dilerim.
Fatih Özmantar

The permission for the activity of Ali Captain’s Ship was obtained by sending an e-mail to Pınar İşcan on 10.10.2023.

Feyza Gün 1 Eki 2023 Paz 19:16
Alıcı:

Merhaba Pınar hocam,
Ben ODTÜ Matematik Eğitimi yüksek lisans öğrencisi ve OHÜ Matematik Eğitimi araştırma görevlisi Feyzanur Gün. "Gerçekçi Matematik Eğitiminin 7. Sınıf Eşitlik Ve Denklemler Konusunda Öğrencilerin Başarılarına Etkisi Ve Öğrenci Görüşleri" tezinizde yer alan "Ali Kaptanın Gemisi" ve "Borcum Ne Kadar" etkinliklerinizi tezimde izniniz dahilinde kullanmak isterim. Sizin için herhangi bir sorun teşkil eder mi?
İyi günler...
Ar. Gör. Feyzanur Gün

Feyza Gün 10 Eki 2023 Sal 15:38
Pınar hocam tekrardan merhaba, Daha önce yazdığım maile yönelik olumlu ya da olumsuz cevabınızı beklemekteyim. Hatırlatmak istedim. iyi günler Ar. Gör. Feyzanur

PINAR İŞCAN 17 Eki 2023 Sal 19:29
Merhaba iyi günler Tezinizde etkinliklerimi kullanabilirsiniz .:) 10 Eki 2023 Sal 15:38 tarihinde Feyza Gün <gunfeyzanur@gmail.com> şunu yazdı:

Permission was obtained by sending an email to Hülya Kılıç on 03.10.2023 for the Let's Go to the Bazaar and Urban Transformation activities.

These activities were developed and used under the TUBITAK-supported project entitled " A University-School Collaboration Model for Promoting Pre-service Teachers' Pedagogical Content Knowledge about Students" with the number 215K049.

Feyza Gün ‹
Alıcı:
3 Eki 2023 Sal 16:02

Merhaba Hülya hocam,
Ben ODTÜ Matematik Eğitimi yüksek lisans öğrencisi ve OHÜ Matematik Eğitimi araştırma görevlisi Feyzanur Gün. TÜBİTAK destekli, 215K049 numaralı "Öğretmen Adaylarının Öğrenciye İlişkin Pedagojik Alan Bilgilerinin Gelişimi İçin Fakülte-Okul İşbirliği Modeli" başlıklı yürüttüğünüz projeniz kapsamında geliştirilen "Kentsel Dönüşüm", "Pazara Gidelim" ve "Karışık Meyce Suyu" adlı etkinlikleri tezimde izniniz dahilinde kullanmak isterim. Sizin için herhangi bir sorun teşkil eder mi? Ayrıca Nil Arabacı'ya ulaştığımda tezinin bu TÜBİTAK projesi kapsamında yürütüldüğünü ve telif hakları projeye ait olduğunu belirtti. Bu yüzden sizden izin almam gerektiğini söyledi. Size onun aracılığı ile ulaştım.
İyi günler...
Ar. Gör. Feyzanur Gün

Hulya Kilic ‹ ›
Alıcı: oguzhan, ben
3 Eki 2023 Sal 16:36

Merhaba Feyza,
TUBITAK projesi kapsaminda bu etkinliklerin gelistirilip kullanildigini tezinde belirtmek sartiyla elbette kullanabilirsin. Simdiden kolay gelsin & Iyi calismalar.

•••

Permission for the Gasoline Tank activity was obtained by sending an e-mail to Ayça Akın on 25.09.2023.

Feyza Gün ‹
Alıcı:
25 Eyl 2023 Pzt 18:13

Merhaba Ayça hocam,
Ben ODTÜ Matematik Eğitimi yüksek lisans öğrencisi ve OHÜ Matematik Eğitimi araştırma görevlisi Feyzanur Gün. "Ortaokul Öğrencilerinin Matematik Okuryazarliklarinin Niceliksel Muhakemelerinin Güçlendirilerek Desteklenmesinin İncelenmesi" adlı doktora tezinizde Öğretim IV: Niceliklerin Değişiminin Analizi ve Temsili bölümünde yer alan "Bisikletler Problem 1" ve "Problem 3:Tır" sorularınızı tezimde izniniz dahilinde kullanmak isterim. Sizin için herhangi bir sorun teşkil eder mi?
İyi günler...
Ar. Gör. Feyzanur Gün

---

ayça akın ‹
Alıcı: ben
27 Eyl 2023 Çar 09:13

Merhaba Feyza Hocam,

"Ortaokul Öğrencilerinin Matematik Okuryazarlıklarının Niceliksel Muhakemelerinin Güçlendirilerek Desteklenmesinin İncelenmesi" adlı doktora tezimde Öğretim IV: Niceliklerin Değişiminin Analizi ve Temsili bölümünde yer alan "Bisikletler Problem 1" ve "Problem 3:Tır" sorularınızı tezinizde ve tezinizden üreteceğiniz makalede atıfta bulunarak ve referans vererek kullanmanızda sakınca yoktur. Bu konuda belirttiğim şartlarda izin veriyorum. Çalışmanızda başarılar dilerim.

Sevgiler
Doç. Dr. Ayça AKIN
Antalya Belek Üniversitesi

The permission for the Box-Penny activity was obtained by sending an e-mail to Arzu Aydoğan Yenmez and Semirhan Gökçe on 04.01.2024.

Feyza Gün 4 Oca Per 23:22 (14 saat önce)
Alıcı:

Merhaba Arzu ve Semirhan hocalarım,
Ben ODTÜ Matematik Eğitimi yüksek lisans öğrencisi ve OHÜ Matematik Eğitimi araştırma görevlisi Feyzanur Gün. Matematiksel Akıl Yürütme kitabınızda yer alan Kutu-kuruş problemini tezimde izniniz dahilinde kullanmak isterim. Sizin için herhangi bir sorun teşkil eder mi?

İyi günler...
Ar. Gör. Feyzanur Gün

Arzu Aydogan 07:01 (7 saat önce)
Merhaba Feyzanur Gün hocam, belirttiğiniz problemi tezinizde kullanabilirsiniz. Çalışmalarınızda kolaylıklar dilerim.Doç. Dr. Arzu Aydoğan Yenmez 4 Oca 2024 Per

---

Semirhan Gökçe 10:19 (3 saat önce)
Alıcı: ben

Merhaba Feyzanur Hocam,
Matematiksel Akıl Yürütme kitabımızda yer alan Kutu-Kuruş problemini çalışmalarınızda kullanabilirsiniz.

The permission for the activity Table Organization was obtained by sending an e-mail to Sümeyra Köken on 11.10.2023.

Feyza Gün 11 Eki 2023 Çar 21:48
Alıcı:

Merhaba Sümeyra hocam,
Ben ODTÜ Matematik Eğitimi yüksek lisans öğrencisi ve OHÜ Matematik Eğitimi araştırma görevlisi Feyzanur Gün. "Sosyo-Matematiksel Normlarla Desteklenmiş Sorgulama Temelli Öğretimin Öğrencilerin Cebirsel Düşünme Süreçlerine Etkisi" tezinizde yer alan Öğretim Sonrası Cebirsel Düşünme Süreçleri Değerlendirme Formunda yer alan "Etkinlik 4" ve "Etkinlik 7" yi tezimde izniniz dahilinde kullanmak isterim. Sizin için herhangi bir sorun teşkil eder mi?
İyi günler...
Ar. Gör. Feyzanur Gün

Sümeyra Köken 28 Eki 2023 Cmt 09:28
Alıcı: ben

Merhaba Feyza Hocam, tabi ki iznim vardır, kullanabilirsiniz. Sonrasında araştırmanızın sonuçlarını bizimle de paylaşırsanız çok memnun oluruz. Şimdiden kolay gelsin.
İyi çalışmalar.

11 Eki 2023 Çar, saat 21:48 tarihinde Feyza Gün şunu yazdı:

Permission was obtained by sending an email to Oylum Akkuş Çıkla on 25.09.2023 for the Chelsea diagnostic algebra test.

Feyza Gün 25 Eyl 2023 Pzt 21:22

Alıcı:

Merhaba Oylum hocam,
Ben ODTÜ Matematik Eğitimi yüksek lisans öğrencisi ve OHÜ Matematik Eğitimi araştırma görevlisi Feyzanur Gün. "The Effects of Multiple Representations-Based Instruction on Seventh Grade Students' Algebra Performance, Attitude Toward Mathematics, and Representation Preference" adlı doktora tezinizde Türkçeye uyarlamış olduğunuz "Chelsea Tanılayıcı Cebir Testini" ve aynı tez için geliştirmiş olduğunuz "5. Etkinliği" tezimde izniniz dahilinde kullanmak isterim. Sizin için herhangi bir sorun teşkil eder mi?
İyi günler...
Ar. Gör. Feyzanur Gün

oylum akkus 25 Eyl 2023 Pzt 21:31

Alıcı: ben

Merhaba Feyzanur,

Izin icin mail atmana cok sevindim. Tabi ki kullanabilirsin.

Iyi gunler,
Oylum

C. The Worksheets

The Worksheet 1

Hesap Makinesi Etkinliği

Aşağıdaki soruları cevaplayınız.

a) Herhangi bir sayı ile hesap makinesinde toplama, çıkarma, çarpma ve bölme işlemlerini ayrı ayrı yaptığınızda hesap makinesinin göstergesinde başlangıçta aldığınız sayıyı oluşturabilir misiniz? Evet, ise nasıl?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

b) Eğer “0” ile “1” tuşlarını kullanamazsanız hesap makinesinin göstergesinde “0” ve “1” sayılarını oluşturabilir misiniz? Evet, ise nasıl?

Eğer 0 ve 1’i oluşturduysanız, bu kullandığınız sayılar/yöntemler dışında 0 ve 1’i oluşturmak için başka durumlar var mı? Bu durumların bir ifade olarak genellenebilir mi? Evet, ise cebirsel ifadesini yazınız.

c) Herhangi iki sayı ile bir çarpma işlemi yaptığınızda çarpanlardan biri “0” olduğunda hesap makinesinin göstergesinde nasıl bir sonuç oluşur?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

d) Herhangi iki sayı ile toplama işlemi yaparken 1. sayı ile 2. sayının yeri değiştirilirse hesap makinesinin göstergesindeki sonuç değişir mi? Evet, ise nasıl?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

e) Herhangi iki sayı ile çarpma işlemi yaparken 1. sayı ile 2. sayının yeri değiştirilirse hesap makinesinin göstergesindeki sonuç değişir mi? Evet, ise nasıl?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

f) Herhangi üç sayı ile toplama işlemi yaparken önce 1.sayı ve 2. sayı ile işlem yapıp sonra 3. sayıyı işleme sokuluyor. Eğer işlem sırası değiştirilip önce 2. sayı ve 3. sayı ile işlem yapıp sonra 1. sayıyı işleme sokarsak hesap makinesinin göstergesindeki sonuç değişir mi? Evet, ise nasıl?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

g) Herhangi üç sayı ile çarpma işlemi yaparken önce 1.sayı ve 2. sayı ile işlem yapıp sonra 3. sayıyı işleme sokuluyor. Eğer işlem sırası değiştirilip önce 2. sayı ve 3. sayı ile işlem yapıp sonra 1. sayıyı işleme sokarsak hesap makinesinin göstergesindeki sonuç değişir mi? Evet, ise nasıl?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

h) Herhangi iki farklı sayının toplama işlemi yazıldıktan sonra hesap makinesi bir hata vererek toplama işleminin ardına "-" tuşunu ve 2. toplananı ekrana yazıyor. "=" tuşuna bastıktan sonra hesap makinesinin göstergesinde nasıl bir sonuç oluşur?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

ı) Herhangi iki farklı sayının çarpma işlemi yazıldıktan sonra hesap makinesi bir hata vererek çarpma işleminin ardına "÷" tuşunu ve 2. çarpanı ekrana yazıyor. "=" tuşuna bastıktan sonra hesap makinesinin göstergesinde nasıl bir sonuç oluşur?

Bütün sayılar için geçerli mi? Evet, ise cebirsel ifadesini yazınız.

j) Eğer “3” tuşunu kullanamazsanız hesap makinesinin göstergesinde 8x30 işleminin sonucunu 8 çarpanını mutlaka kullanarak ve 30 çarpanını iki sayının toplamı olacak şekilde oluşturabilir misiniz? Evet, ise nasıl?

Yukarıda kullanılan işlem bir ifade olarak genellenebilir mi? Evet, ise cebirsel ifadesini yazınız

k) Hesap makinesinde çift sayıyı gösteren tuşları (0,2,4,6,8) kullanamazsanız iki sayıyı kullanarak ayrı ayrı toplama, çıkarma ve çarpma işlemleri yaptığınızda hesap makinesinin göstergesinde oluşan sayıların özellikleri ile ilgili ne söyleyebilirsiniz?

Toplama, çıkarma ve çarpma işlemleri için elde ettiğiniz bu sonuçlar tüm durumlar için geçerli mi? Evet, ise bu durumları cebirsel olarak ifade ediniz.

l) Hesap makinesinde tek sayıyı gösteren tuşları (1,3,5,7,9) kullanamazsanız iki sayıyı kullanarak ayrı ayrı toplama, çıkarma ve çarpma işlemleri yaptığınızda hesap makinesinin göstergesinde oluşan sayıların özellikleri ile ilgili ne söyleyebilirsiniz?

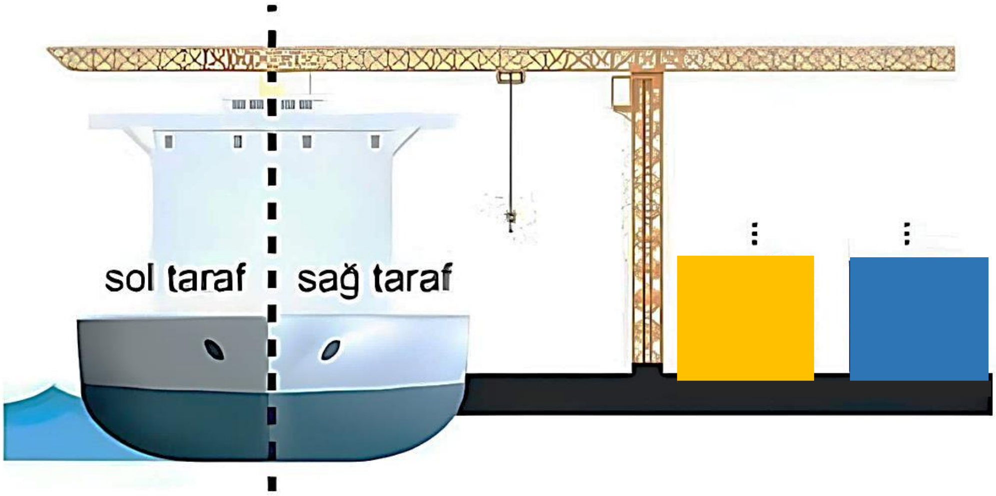
Toplama, çıkarma ve çarpma işlemleri için elde ettiğiniz bu sonuçlar tüm durumlar için geçerli mi? Evet, ise bu durumları cebirsel olarak ifade ediniz.

m) Herhangi bir sayı tek sayı ve bir çift sayı hesap makinesinde toplama, çıkarma ve çarpma işlemlerini ayrı ayrı yaptığınızda hesap makinesinin göstergesinde oluşan sayıların özellikleri ile ilgili ne söyleyebilirsiniz?

Toplama, çıkarma ve çarpma işlemleri için elde ettiğiniz bu sonuçlar tüm durumlar için geçerli mi? Evet, ise bu durumları cebirsel olarak ifade ediniz.

The Worksheet 2

Ali Kaptan’ın Gemisi Etkinliği



Gemisini Mersin Limanından Kıbrıs Mağusa Limanına götürmek isteyen Ali Kaptan, yolculuk esnasında geminin dengede durması için sağ ve sol taraftaki yüklerin ağırlıklarının eşit olmasını istemektedir.

a) Ali Kaptan Mersin Limanında 6 sarı renkli kutu geminin sol tarafına 8 mavi renkli kutu geminin sağ tarafına yerleştirince gemi dengede kaldığına göre sarı renkli kutunun ve mavi renkli kutunun ağırlıkları kaçar kilogram olabilir?

b) Ali Kaptan bir sarı renkli kutunun ağırlığının 400 kg olduğunu belirtmiştir. Buna göre mavi renkli kutunun ağırlığı kaç kilogram olacağını cebirsel ifadeleri kullanarak bulunuz.

c) Ali Kaptan 4 tane mavi renkli kutuyu Kıbrıs Limanında bırakacaktır. Ali Kaptan geminin dengesini sağlamak için bu limanda sarı renkli kutulardan kaç tanesini geminin sağ tarafına yüklemesi gerekir?

d) Ali Kaptan Mersin Limanına geri dönüş yolundayken sol taraftaki 3 sarı kutusu denize düşüyor. Ali Kaptan geminin dengesini sağlamak için sağ taraftaki hangi renk kutudan kaçar tane geminin sol tarafına aktarması gerekir?

f) Ali Kaptan Mersin Limanına vardığında bütün ağırlıklarını bırakacaktır. Ali Kaptan gemisi ile tekrar yolculuğa çıkmak istiyor. Bu yolculuk için geminin sağ tarafına 12 mavi renkli kutu yükleyecektir. Ali kaptan yolculuğa çıkabilmek için geminin sol tarafına nasıl ve ne renkte kutular yüklemesi gerekir?

The Worksheet 3

Pazara Gidelim Etkinliği

Ali, Ayşe, Murat ve Sırma babalarına yardım etmek için onunla pazar alışverişine çıkmışlardır. O hafta evde turşu kurulacak olduğundan alınması gereken kilolarca malzeme vardır. Pazardan **eşit ağırlıkta 7 poşet** turşu malzemesi ve birkaç kilo meyve

- Ali 2 kilo mandalina ve 2 turşu poşetini taşımaktadır.
- Ayşe turşu poşetlerinden 3 tane taşımaktadır.
- Murat ise 1 turşu poşeti, 1 kilo muz ve yarım kilo elma taşımaktadır.
- Sırma 1 turşu poşeti ve 3 kilo portakal taşımaktadır.

Eve dönüş yolunda kardeşler arasında kimin yükünün daha ağır olduğuna dair bir tartışma başlar. Ali elindeki tüm poşetleri pazarın çıkışındaki terazide tartar ve ağırlığının 6 kilo olduğunu görür. (**Unutmayın!** Turşu poşetleri eşit ağırlıktadır.)

a) Ayşe ve Murat'ın taşıdıkları ağırlıkları cebirsel olarak gösteriniz

b) Turşu poşetlerinin tek bir tanesinin ağırlığını cebirsel ifadeleri kullanarak bulunuz ve Ayşe ve Murat'ın taşıdıkları ağırlık miktarını hesaplayınız.

c) Pazardan alınan turşuluk malzemenin ve meyvenin toplam ağırlığını bulunuz.

d) Ali, Ayşe, Murat ve Sırma'nın taşıdıkları ağırlık miktarlarını karşılaştırınız.

e) Fatma Hanım eve gelen turşuluk malzemelerden turşu kuracaktır. Fatma Hanım'ın karışık turşu tarifi aşağıdaki gibidir.

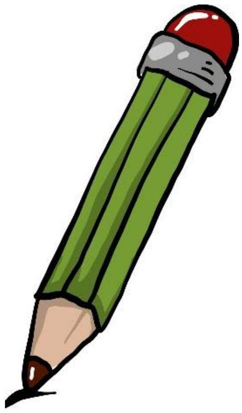
- Bir miktar havuç
- Havuç miktarının 3 katı kadar beyaz lahana
- Havuç miktarının 2 katından 500 gr daha fazla salatalık
- Havuç miktarının dörtte biri kadar sivri biber

Fatma Hanım, toplam 5,5 kilo turşuluk malzeme kullanarak turşu yapacaktır. Yukarıdaki tarife göre her biri malzemeden ne kadar kullanması gerekir? Cebirsel ifadeleri kullanarak çözünüz. (1 kilo=1000 gr)

The Worksheet 4

Ali'nin Alışverişi Etkinliği

Ali cebinde bulunan 35 TL'nin tamamını harcayarak okul kantininden tanesi 1,75 TL olan kalemlerden ve tanesi 1,25 TL olan silgilerden satın almak istiyor.



1.25 TL 1,75 TL

Buna göre;

a) Ali cebinde bulunan 35 TL'nin tamamını kalem ve silgi almak için harcadığı için kaç tane silgi ve kalem alarak bu alışverişi farklı şekillerde yapabilir? Tablo çizerek gösteriniz. (Cebirsel ifadeleri kullanmadan çözünüz.)

b) Bu durumu gösteren cebirsel ifadeyi yazınız ve sözel olarak açıklayınız.

c) Ali alışverişinde hiç kalem almadığı durumda veya hiç silgi almadığı durumda parasının tamamını harcayabilir mi? Açıklayınız.

d) Oluşturduğunuz cebirsel ifade "c" şıkkında verdiğiniz yanıta uygun mudur? Açıklayınız.

e) Oluşturduğunuz cebirsel ifadeyi de dikkate alarak son durumda Ali bu alışverişi kaç farklı şekilde yapabilir?

The Worksheet 5

Benzin Deposu Etkinliği

Bir tır petrol istasyonuna benzin doldurmaya gelmiştir. Benzin tırın deposuna sabit hızla dolmaktadır. Aşağıdaki tablo tırın benzin deposuna benzin doldurulmaya başlanan andan itibaren geçen zamanı (dakika) ve <u>bu sürede tırın benzin deposunda bulunan benzin miktarını</u> (litre) göstermektedir.

Geçen Zaman (dakika)	Depoda Bulunan Benzin Miktarı (Litre)
3	17
6	29
9	41
12	53
15	65

Buna göre;

a) 21. dakikada bu tırın benzin deposunda kaç litre benzin bulunur? (Cebirsel ifadeleri kullanmadan çözünüz)

b) Tırın benzin deposunda 125 litre benzin bulunduğunda kaç dakika geçmiş olur? (Cebirsel ifadeleri kullanmadan çözünüz)

c) Bu tıra 1 dakikada kaç litre benzin doldurulmaktadır? (Cebirsel ifadeleri kullanmadan çözünüz)

d) Bu tır benzin istasyonuna geldiğinde (benzin doldurulmaya başlamadan önce) deposunda kaç litre benzin vardır? (Cebirsel ifadeleri kullanmadan çözünüz)

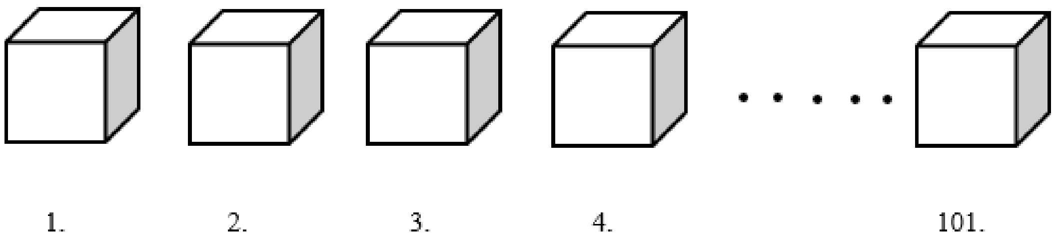
e) Tırın benzin depolarına benzin doldurulmaya başlanan andan itibaren geçen zaman (dakika) ile tırın benzin deposunda bulunan benzin miktarı (litre) arasındaki ilişkiyi gösteren cebirsel ifadeyi yazınız.

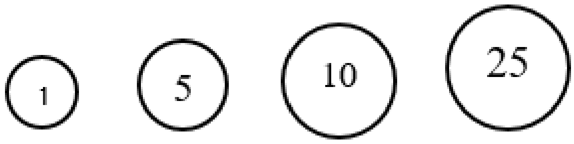
f) Bu ilişkinin grafiğini çiziniz.

The Worksheet 6

Kutu-Kuruş Etkinliği

Mahir, Miray, Rüzgâr ve Artun dört yakın arkadaştır. Bu arkadaşlar 1’den 101’e kadar numaralandırılmış kutulara sahiptir. Dört arkadaş bu kutuları kullanarak 1, 5, 10 ve 25 kuruşlar ile bir oyun oynuyorlar.





Oyun sırasında,

- İlk olarak Mahir, 1 numaralı kutudan başlayarak sırayla her birine 1’er kuruş atıyor.
- Sonra Miray, 1 numaralı kutudan başlayarak her ikinci kutudaki parayı 5 kuruş ile değiştiriyor.
- Daha sonra Rüzgâr, 1 numaralı kutudan başlayarak her üçüncü kutudaki parayı 10 kuruş ile değiştiriyor.
- Son olarak Artun, 1 numaralı kutudan başlayarak her dördüncü kutudaki parayı 25 kuruş ile değiştiriyor.

a) 6. Kutudaki para miktarı nedir? Açıklar mısınız?

b) 16. Kutudaki para miktarı nedir? Açıklar mısınız?

c) Son durumda kutulardaki para miktarlarını belirleyerek kutulardaki para miktarları arasında nasıl bir ilişki olduğunu sözel olarak açıklayarak yazınız.

(Kutulardaki para miktarlarının 1. Kutudan başlanarak bir sıra haline getirmeye özen verin)

d) Son durumda kutulardaki toplam para miktarlarını doğru hesaplayan kişi oyunun kazananı olacaktır. Sizce oyunun kazananı toplam para miktarının ne kadar olduğunu belirtmiştir?

The Worksheet 7

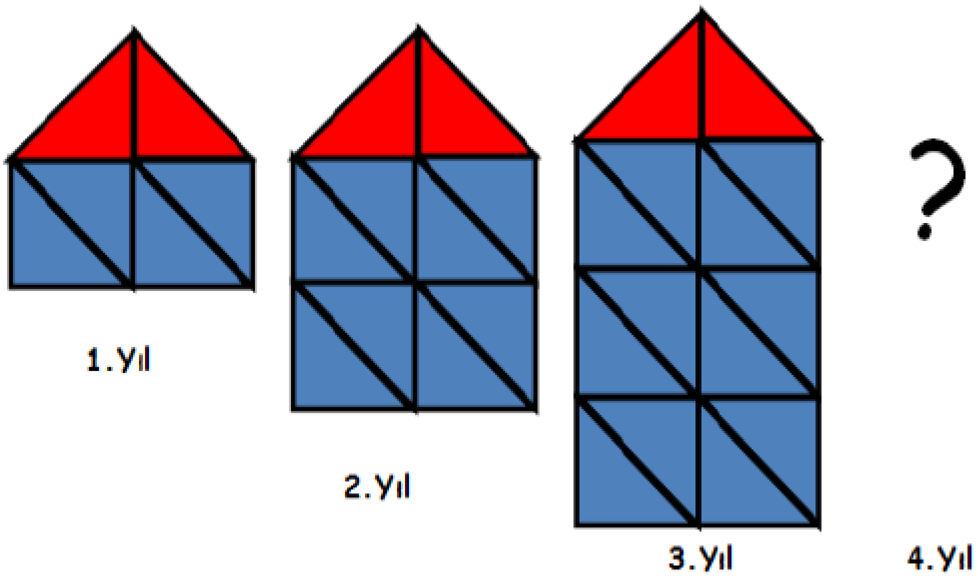
Kentsel Dönüşüm Etkinliği

Mevcut depreme dayanıksız, ekonomik ömrünü tamamlamış binaların yaşanabilir, depreme dayanıklı, sosyal donatıları, otoparkı, yeşil alanları olan kaliteli yaşam alanlarına dönüştürme sürecine (projesine) kentsel dönüşüm denir. Kentsel dönüşümün bu tür yararları olmasına rağmen bu süreç içinde bazı ailelerin mahallerinden başka yerlere taşınması da gerekir.

1) İstanbul'da kentsel dönüşüme girecek mahallelerden biri de Kayışdağı Mahallesidir. Bu süreçte mahalleden taşınacak aile sayısı aşağıdaki tabloda verilmiştir. **Yıl sayısı** ile **taşınan aile sayısı** arasındaki ilişkiyi (örüntü kuralını) tablodan yararlanarak bulabilir misiniz?

Yıl Sayısı	Taşınan Aile Sayısı
1	3
2	6
3	9
4	12
5	15
⋮	⋮

2) Kentsel dönüşüm projesi kapsamında inşa edilen apartmanlar şekildeki gibidir. **Yeni yapılan apartmanların yüksekliği her yıl biraz daha artmaktadır.**



Şekildeki üçgenler (çatı katındakiler de dâhil) apartmandaki daireleri göstermektedir. **Örneğin**, 1.yıl yapılan apartmanda 6 daire vardır.

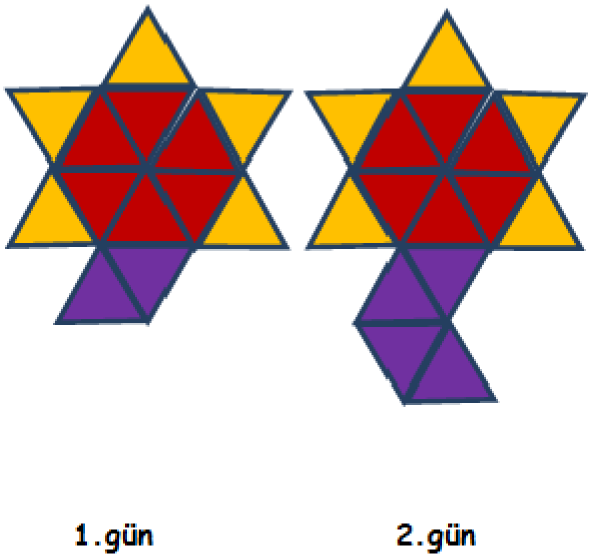
a) **4. yılda** yapılan bir apartman nasıl olacaktır? Çiziniz.

b) Apartmanın yüksekliği her yıl **bir kat** yükseldiğini düşünürsek, 2., 3. ve 4. yılda apartmanda kaç daire olacaktır? Peki ya "**n yıl**" sonra apartmanda kaç daire olacağını tablodan yararlanarak bulunuz. (Örüntü kuralını bulunuz.)

Yıl Sayısı	Daire Sayısı
1	6
2	
3	
4	
⋮	⋮
n	?

3) Kentsel dönüşüm projesinde sokak düzenlemesi için çiçeklendirme yapılacaktır.

a) Ekilen çiçeğin sapının günlük uzama miktarı aşağıda gösterildiği gibidir. 3. ve 4. günde çiçeği oluşturmak için kaç üçgen kullanmak gerekir? “**s gün**” sonra çiçeği oluşturmak için kaç üçgen kullanılacağını tablodan yararlanarak bulunuz. (Örüntü kuralını bulunuz.)



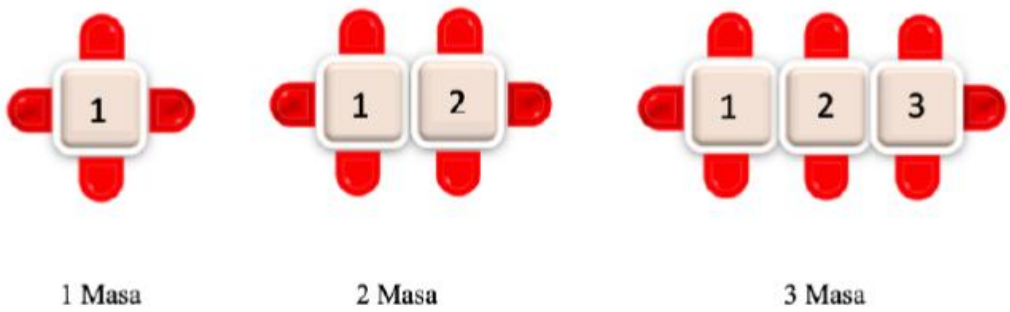
Gün Sayısı	Kullanılan Üçgen Sayısı
1	
2	
3	
4	
⋮	⋮
s	?

b) Sokak düzenlemesi için yapılan çiçeklendirme çalışmasına göre sokak sayısı ile ekilen çiçek sayısı aşağıdaki tabloda verilmiştir. 5. sokağa ekilen çiçek sayısı kaçtır? "**x. sokakta**" ekilen çiçek sayısını tablodan yararlanarak bulunuz. (Örüntü kuralını bulunuz.)

Sokak Sayısı	Ekilen Çiçek Sayısı
1	2
2	5
3	8
4	
5	
⋮	⋮
x	?

The Worksheet 8

Masa Organizasyonu Etkinliği



Asuman Hanım bir açık hava organizasyonu için dört kişilik masaları birleştirerek büyük bir şölen masası oluşturmak istiyor. Asuman Hanım şölen masası için bir organizasyon şirketinden destek almaktadır. Yukarıda organizasyon şirketinin yaptığı düzenleme örneklerini görmektesiniz. Örneğin 1 masaya dört kişi oturmakta 2 masaya altı kişi oturmaktadır.

Buna göre;

a) Asuman Hanımın hazırladığı organizasyon için 10 masa birleştirilirse masalarda toplam kaç kişi oturabilir? Çözümünüzü açıklayınız. (Cebirsel ifadeleri kullanmadan çözünüz)

b) Organizasyona 76 kişinin katılabilmesi için kaç masanın birleştirilmesi gerekmektedir? Çözümünüzü açıklayınız. (Cebirsel ifadeleri kullanmadan çözünüz)

c) Verilen bu düzenleme örneğinde değişmeyen veya sabit kalan sandalye var mı? Verilen bu düzenleme örneğinde değişen veya eklenen sandalyeler var mı? Bunları gösteriniz.

d) Birleştirilen masa sayısı ile masada oturan toplam kişi sayısı arasında nasıl bir ilişki olduğunu tabloyu doldurarak cebirsel olarak ifade ediniz.

Masa Sayısı	Kişi Sayısı
1	4
2	6
3	
4	

e) Cebirsel olarak ifade ettğiniz bu ilişkinin grafiğini çizerek bu ilişkiyi sözel olarak açıklayınız.

D. The Guiding Questions About Activities

Hesap Makinesi Etkinliği

Etkinlikte yer alan soruları anladınız mı?

Farklı sayılar kullanarak aynı işlemi/işlemleri yaptığınızda soruda sizden istenenlere ulaşabiliyor musunuz?

Cebirsel ifadeyi nasıl yazdığınızı açıklayabilir misiniz?

Yazdığınız bu cebirsel ifadenin bu soru kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz?

Herhangi bir sayı mesela 15 sayısı ile hesap makinesinde toplama işlemi yaptığınızda hesap makinesinin göstergesinde 15 sayısını oluşturabilir misiniz? Aynı durumu çıkarma, çarpma ve bölme işleminde yapabilir miyiz? (a şıkkı için)

İki sayı kullanarak yaptığınız bir işlemde işlemin sonucunu "0" veya "1" bulabilir misiniz? (b şıkkı için)

Aynı iki sayısı ile işlem yaptığınızda nasıl sonuçlara ulaşabilirsiniz? (b şıkkı için)

Hesap makinesinin göstergesinde "1" sayısını oluşturmak için "0" oluşturduğumuz gibi iki aynı sayıyı kullanabilir miyiz? (b şıkkı için)

Herhangi iki sayı mesela 25+17 işlemi yapıp sonucunu bulduktan sonra aynı işlemi 1. sayı ile 2. sayının yeri değiştirip yaptığınız hesap makinesinin göstergesindeki sonuç değişir mi? (d şıkkı için)

Mesela "25+17=" ve "17+25=" işlemlerinde eşittir işaretinin karşısına işlemin sonucundan başka bir ifade yazılabilir mi? (d şıkkı için)

Herhangi iki sayı mesela 25x17 işlemi yapıp sonucunu bulduktan sonra aynı işlemi 1. sayı ile 2. sayının yeri değiştirip yaptığınız hesap makinesinin göstergesindeki sonuç değişir mi? (e şıkkı için)

Herhangi üç sayı mesela 25, 17, 26 sayıları ile toplama işlemi yaparken önce 1. ve 2. sayıyı toplayıp sonra 3. sayıyı ekleyerek sonuca ulaşıldı. Eğer işlem sırası değiştirilip önce 2. ve 3. sayıyı toplayıp sonra 1. sayıyı eklersek hesap makinesinin göstergesindeki sonuç değişir mi? (f şıkkı için)

Parantez olmasaydı bu işleme bakan bir kişi önce 2. ve 3. Sayıyı toplayacağını sonra 1. Sayısı toplaması gerektiğini anlar mıydı? (f şıkkı için)

Önce 2. ve 3. Sayının toplanacağının vurgusunu yapmak için sayıların sıralamasını mı değiştirmemiz kesinlikle gerekli mi? (f şıkkı için)

Parantez bir işleme ne özelliği katar? Parantezin özelliğini bu sorudaki bizden istenen işlemi yapabilmek için kullanabilir miyiz? (f, g ve j şıkkı için)

Herhangi üç sayı mesela 25, 17, 26 sayıları ile çarpma işlemi yaparken önce 1. ve 2. sayıyı çarpıp sonra 3. sayıyı çarparsak sonuca ulaşıldı. Eğer işlem sırası değiştirilip önce 2. ve 3. sayıyı çarpıp sonra 1. sayıyı çarparsak hesap makinesinin göstergesindeki sonuç değişir mi? (g şıkkı için)

Parantez olmasaydı bu işleme bakan bir kişi önce 2. ve 3. Sayıyı çarpılacağını sonra 1. sayının çarpılması gerektiğini anlar mıydı? (g şıkkı için)

Önce 2. ve 3. Sayının çarpılacağının vurgusunu yapmak için sayıların sıralamasını mı değiştirmemiz kesinlikle gerekli mi? (g şıkkı için)

Herhangi iki farklı sayının mesela 15 ve 16 sayılarının toplama işlemi yazıldıktan sonra hesap makinesi bir hata vererek toplama işleminin ardına "-" tuşunu ve 2. toplananı yani 16 sayısını ekrana yazıyor. "=" tuşuna bastıktan sonra hesap makinesinin göstergesinde nasıl bir sonuç oluşur? (h şıkkı için)

Herhangi iki farklı sayının mesela 15 ve 16 sayılarının çarpma işlemi yazıldıktan sonra hesap makinesi bir hata vererek çarpma işleminin ardına "÷" tuşunu ve 2. çarpanı yanı 16 sayısını ekrana yazıyor. "=" tuşuna bastıktan sonra hesap makinesinin göstergesinde nasıl bir sonuç oluşur? (ı şıkkı için)

8 çarpanını 30 çarpanını oluşturan iki sayının toplamı ile nasıl çarpabilirim? (j şıkkı için)

"8.(20+10)=" işleminde eşittir işaretinin karşısına neler yazabilirsiniz? (j şıkkı için)

Hesap makinesinde çift sayıyı gösteren tuşları (0,2,4,6,8) kullanamazsanız iki sayıyı kullanarak mesela 19 ve 33 sayıları ile toplama işlemi yaptığınızda hesap makinesinin göstergesinde oluşan sayının özellikleri ile ilgili ne söyleyebilirsiniz? Aynı durumu çıkarma ve çarpma işleminde yaptığınızda hesap makinesinin göstergesinde oluşan sayının özellikleri ile ilgili ne söyleyebilirsiniz? (k şıkkı için)

Hesap makinesinde tek sayıyı gösteren tuşları (1,3,5,7,9) kullanamazsanız iki sayıyı kullanarak mesela 28 ve 42 sayıları ile toplama işlemi yaptığınızda hesap makinesinin göstergesinde oluşan sayının özellikleri ile ilgili ne söyleyebilirsiniz? Aynı durumu çıkarma ve çarpma işleminde yaptığınızda hesap makinesinin göstergesinde oluşan sayının özellikleri ile ilgili ne söyleyebilirsiniz? (l şıkkı için)

Herhangi bir tek sayı ve çift sayı mesela 12 ve 13 ile hesap makinesinde toplama işlemi yaptığınızda hesap makinesinin göstergesinde oluşan sayıların özellikleri ile ilgili ne söyleyebilirsiniz? Aynı durumu çıkarma ve çarpma işleminde yaptığınızda hesap makinesinin göstergesinde oluşan sayının özellikleri ile ilgili ne söyleyebilirsiniz? (m şıkkı için)

Cebirsel ifadede bilinmeyenlerin tek veya çift sayı olduğunu belirtmek için ne yapabiliriz? (k, l ve m şıkkı için)

Cebirsel ifadede bilinmeyenlerin tek veya çift sayı olduğunu belirtmek için tek sayıyı "T" harfi ve çift sayıyı "Ç" harfi ile gösterebilir miyiz? (k, l ve m şıkkı için)

Çıkarma işleminde her zaman birinci sayı ikinci sayıdan büyük olmak zorunda mı? Olmadığı bir durumu gösterir misiniz? (k, l ve m şıkkı için)

Ali Kaptanın Gemisi Etkinliği

Etkinlikte yer alan soruları anladınız mı?

Geminin sağ ve sol tarafındaki yüklerin ağırlığını eşit yapmaya çalışıyoruz değil mi?

Sarı ve mavi renkli kutu sayılarının en küçük ortak katını bulursak onların en yakın hangi durumda eşitleyebileceğimizi görebilir miyiz? (a şıkkı için)

Mavi ve sarı renkli kutuların sayılarından doğru bulduğunuz en küçük ortak kattan yola çıkarak diğer eşit oldukları durumları bulabilir miyiz? (a şıkkı için)

Sarı ve mavi renkli kutunun ağırlıklarının alabileceği başka değerler var mıdır? (a şıkkı için)

Mavi renkli kutunun ağırlığını bilmediğimiz için bilinmeyenimiz olmaz mı? Bilinmeyeni nasıl ifade edebiliriz? (b şıkkı için)

Cebirsel ifadeyi nasıl yazdığınızı açıklayabilir misiniz? (b şıkkı için)

Cebirsel ifadeyi yazarken neye odaklanıyoruz? (b şıkkı için)

Bu soruyu farklı bir şekilde çözebilir miydiniz? (b şıkkı için)

4 tane mavi renkli kutunun ağırlığı kaç tane sarı renkli kutunun ağırlığına eşit olabilir? (c şıkkı için)

Bu soruyu farklı bir şekilde çözebilir miydiniz? Mesela cebirsel ifadeleri kullanarak çözebilir miyiz? (c, d ve e şıkkı için)

Sol taraftan 3 sarı renkli kutu düştükten sonra sol tarafta kalan yük miktarı ne kadardır? (d şıkkı için)

Geminin sol tarafına yüklenecek kutunun renk ve sayısının bulduğunuzdan başka bir şekilde olma ihtimali var mıdır?(e şıkkı için)

Bir önceki şıklarda yaptığınız gibi geminin sağ tarafında bulunan mavi kutuların ağırlığını dengelemek için geminin sol tarafında sarı kutular kullanılabilir mi? (e şıkkı için)

Bir önceki şıklarda yaptığınız gibi geminin sağ tarafında bulunan mavi kutuların ağırlığını dengelemek için geminin sol tarafında hem sarı hem de mavi kutular kullanılabilir mi? (e şıkkı için)

Pazara Gidelim Etkinliği

Etkinlikte yer alan soruları anladınız mı?

Bu sorudaki cebirsel ifadeleri oluşturmak için neye ihtiyacımız var? (a şıkkı için)

Turşu poşetinin ağırlığını bilmediğimiz için bilinmeyenimiz olmaz mı? Bilinmeyeni nasıl ifade edebiliriz? (a şıkkı için)

Meyvelerin ağırlıklarını biliyor muyuz? Ağırlığını bildiğimiz bir nesneye veya meyveye bilinmeyen olarak davranabilir miyiz? (a şıkkı için)

Ayşe ve Murat'ın taşıdıkları ağırlık miktarlarını cebirsel ifade olarak nasıl yazdığınızı açıklayabilir misiniz? (a şıkkı için)

Ali'nin taşıdığı ağırlık miktarının cebirsel ifadesini yazarak turşu poşetinin ağırlığını bulabilir misiniz? (b şıkkı için)

Cebirsel ifadeyi nasıl yazdığınızı açıklayabilir misiniz? (b şıkkı için)

Bilinmeyen için bulduğunuz değeri Ayşe ve Murat'ın için oluşturduğunuz cebirsel ifadelerde yerine koyarak taşıdıkları ağırlık miktarını bulabilir misiniz? (b şıkkı için)

Pazardan toplam kaç poşet turşuluk malzeme alınmıştı? Bir turşu poşetinin ağırlığını bulduğumuz için kaç kilo turşuluk malzeme aldığımızı bulabilir miyiz? (c şıkkı için)

Ali, Ayşe ve Murat'ın taşıdıkları ağırlık miktarını bildiğimiz için Sırma'nın taşıdığı ağırlık miktarını bulabilir misiniz? (d şıkkı için)

Sırmanın taşıdığı poşetlerin ağırlık miktarını nasıl buldunuz? (d şıkkı için)

Dört çocuğun taşıdıkları ağırlık miktarını nasıl karşılaştırdınız açıklayabilir misiniz? (d şıkkı için)

Beyaz lahana, salatalık ve sivri biber miktarları havuç miktarı üzerinden ifade edilmektedir değil mi? Bu yüzden havuç miktarı bilinmeyenimiz olabilir mi? Bilinmeyeni nasıl ifade edebiliriz? (e şıkkı için)

Havuç, beyaz lahana, salatalık ve sivri biber miktarları cebirsel ifade olarak yazabilir misiniz? (e şıkkı için)

Toplam 5,5 kilo turşuluk malzeme kullanılacağı için havuç, beyaz lahana, salatalık ve sivri biber miktarlarını bulabilir misiniz? (e şıkkı için)

Yazdığınız cebirsel ifadeyi nasıl düzenleyebilirsiniz? (e şıkkı için)

Aynı birim veya aynı cins olan terimleri toplayıp çıkarabiliriz değil mi? (e şıkkı için)

Kg ve gram ile nasıl işlem yapabiliriz? Kg ve gram ile işlem yapabilmek için birbirlerine dönüştürmemiz gerekir mi? (e şıkkı için)

Bilinmeyeni bulduktan sonra turşu için kullanılan tüm malzemelerin ağırlığını nasıl bulabilir miyiz? (e şıkkı için)

Cebirsel ifadeyi nasıl yazdığınızı açıklayabilir misiniz? (e şıkkı için)

Ali'nin Alışverişi Etkinliği

Etkinlikte yer alan soruları anladınız mı?

Satın alınabilecek kalem ve silgi adetlerini belirlerken 35 TL'nin tamamının harcandığına dikkat ediyor musunuz? (a şıkkı için)

Oluşturacağınız tabloda satın alınan kalem ve silgi sayısını, bu sayıya göre onların fiyatının ve bunların toplamının ne olduğunu gösterebilir misiniz? (a şıkkı için)

Bu alışverişin kaç farklı şekilde yapılabileceğini bulmak için kalem ve silgi sayısına farklı değerler verip daha sonra verilen sayılara göre onların fiyatının toplamın 35 TL yapıp yapmadığını kontrol eder misiniz? (a şıkkı için)

(ilk buldukları ihtimale yönelik) Kalem ve silgi sayılarının birini azaltıp diğeri artırarak diğer ihtimalleri bulabilir misiniz? (a şıkkı için)

Hangi durumda satın alınan silgi ve kalemlerin adet sayısı en az olacak şekilde fiyatları birbirine eşit olur? Bulduğunuz bu durumda eşitlikteki silgi ve kalem sayılarına göre silgi ve kalem sayılarını azaltıp arttırabilir miyiz? (a şıkkı için)

Kalem ve silgi sayısının alabileceği başka değerler var mıdır? (a şıkkı için)

Tabloyu oluştururken nelere dikkat ettiniz? (a şıkkı için)

Kalem sayısı ve silgi sayısı bilmediğimiz için bilinmeyenlerimiz olabilir mi? Bilinmeyenleri nasıl ifade edebiliriz (b şıkkı için)

Oluşturduğunuz tablodaki bilgiler size cebirsel ifadeyi yazma konusunda yardımcı olabilir mi? (b şıkkı için)

Cebirsel ifadeyi nasıl yazdığınızı açıklayabilir misiniz? (b şıkkı için)

Oluşturduğunuz tablo size cebirsel ifadeyi yazma konusunda nasıl yardımcı oldu? (b şıkkı için)

Oluşturduğunuz cebirsel ifadeyi açıklamak için yazdıklarınızın yeterli ve doğru olduğunu düşüyor musunuz? (b şıkkı için)

Cebirsel ifadeyi oluştururken tüm durumları dikkate aldınız mı? (c şıkkı için)

Yazdığınız bu cebirsel ifadenin bu etkinlik kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz? (d şıkkı için)

Benzin Deposu Etkinliği

Etkinlikte yer alan soruları anladınız mı?

Tablodaki bilgiler size geçen süredeki benzin dolum miktarını mı veriyor yoksa geçen süredeki benzin deposunda bulunan benzin miktarını mı veriyor?

Tabloda verilen dakika ve litre değerleri nasıl değişmektedir?

Dakika ve litre değerlerinin artış miktarlarına dikkat ederek 21. Dakikada depoda bulunan benzin miktarını bulur musunuz? (a şıkkı için)

Dakika ve litre değerlerinin artış miktarlarına dikkat ederek 125 litre benzinin depoda bulunması için kaç dakikanın geçtiğini bulabilir misiniz? (b şıkkı için)

Tabloda verilen dakika değerleri ve litre değerlerinin artışı miktarına dikkat ederek tıra 1 dakikada dolan benzin miktarını bulabilir misiniz? (c şıkkı için)

Tıra deposuna 1 dakikada dolan benzin miktarından yola çıkarak benzin doldurulmaya başlamadan önce tırın deposunda bulunan benzin miktarını bulabilir misiniz? (d şıkkı için)

Dakikaya göre litre değeri değiştiği için dakika bilinmeyenimiz olabilir mi? Bilinmeyeni nasıl ifade edebiliriz? (e şıkkı için)

Geçen zaman ile benzin miktarı arasındaki ilişkiyi gösteren kuralı yazarken benzin doldurulmaya başlamadan önce tırın deposunda bulunan benzin miktarını dikkate alıyor musunuz? (e şıkkı için)

Cebirsel ifadeyi yazarken b şıkkında bulduğunuz 1 dakikada dolan benzin miktarını dikkate alıyor musunuz? (e şıkkı için)

Cebirsel kuralı nasıl yazdığınızı açıklayabilir misiniz? (e şıkkı için)

Yazdığınız bu cebirsel kuralın bu etkinlik kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz? (e şıkkı için)

Grafik nasıl çizilir? Eksenlere ne ad verebiliriz? (f şıkkı için)

Alınan yol-zaman grafiklerini hatırlıyor musunuz? O grafiklerle bu çizmeniz gereken grafiği nasıl bağdaştırabilirsiniz? (f şıkkı için)

Grafik 0 litreden mi başlıyor? Grafikte başlangıçta bulunan depodaki 5 litredeki benzin miktarını nasıl gösterebilirsiniz? Aslında 0. Dakikada depoda 5 litre benzin yok mudur? (f şıkkı için)

Grafiği çizerken neler dikkat ettiniz? (f şıkkı için)

<u>Kutu-Kuruş Etkinliği</u>

Etkinlikte yer alan soruları anladınız mı?

6'nın hangi sayılarının katı olduğunu bulursanız 6. Kutudaki son durumdaki para miktarını belirlemenize yardımcı olur mu? (a şıkkı için)

16'nın hangi sayıların katı olduğunu bulursanız 16. Kutudaki son durumdaki para miktarını belirlemenize yardımcı olur mu? (b şıkkı için)

Yaptığınız açıklamaların yeterli ve doğru olduğunu düşüyor musunuz? (a ve b şıkkı için)

Birinci kutudan başlayarak sırasıyla kutulardaki son durumdaki para miktarını bulabilir misiniz? (c şıkkı için)

Eğer Mahir, Miray, Rüzgar ve Artun'un para attıkları kutuları katlarına göre ayırırsanız kutular arasındaki ilişkiyi bütüncül olarak görebilir misiniz? (c şıkkı için)

Kutulardaki para miktarlarında tekrar eden durumlar fark ediyor musunuz? Kaç kutuda bir tekrar ediyor kutulardaki para miktarları? (c şıkkı için)

Kutulardaki para miktarları arasındaki ilişkiyi nasıl bulduğunuzu anlatabilir misiniz? (c şıkkı için)

Kutulardaki para miktarları arasındaki ilişkiyi açıklamak için yazdıklarınızın yeterli ve doğru olduğunu düşüyor musunuz? (c şıkkı için)

<u>Kentsel Dönüşüm Etkinliği</u>

Etkinlikte yer alan soruları anladınız mı?

Bir yılda taşınan aile sayısı üç olurken iki yılda taşınan aile sayısı altı kişi olduğunda yıl sayası bir artarken taşınan aile sayısı üç artmış değil mi? (1. soru için)

Tablodaki bilgilere göre yıl sayısı artıkça taşınan aile sayısı nasıl değişmektedir? (1. soru için)

Yıl sayısına göre taşınan aile sayısı değiştiği için yıl sayısı bilinmeyenimiz olabilir mi? Bilinmeyeni nasıl ifade edebiliriz? (1. soru için)

Tabladaki boşluğa yazdığınız örüntü kuralını nasıl yerleştirirsiniz? (1. soru için)

Yıl sayısı ile taşınan aile sayısı arasındaki ilişkinin örüntü kuralını nasıl yazdığınızı anlatabilir misiniz? (1. soru için)

Tablodaki bilgiler size cebirsel kuralı yazma konusunda nasıl yardımcı oldu? (1. soru için)

Yazdığınız bu cebirsel kuralın bu soru kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz? (1. soru için)

4. yılda yapılan bir apartmanı çizebilmek için önceki yıllarda yapılan apartmanlardaki yıl geçtikçe değişimin ne olduğuna dikkat edebilir misiniz? (2. soru a şıkkı için)

Apartmanın yüksekliği her yıl bir kat yükseldiği için 1. yılda altı daire varken 2. yılda 1 kat daha eklersek kaç daire eklemiş oluruz? (2. soru b şıkkı için)

Tabloyu doldururken nelere dikkat ettiniz? (2. soru b şıkkı için)

"n yıl" sonraki daire sayısı için oluşturduğunuz örüntü kuralını nasıl yazdığınızı anlatabilir misiniz? (2. soru b şıkkı için)

Tablodaki boşlukları doldurmak size örüntü kuralını yazma konusunda nasıl yardımcı oldu? (2. soru b şıkkı için)

Yazdığınız bu örüntü kuralının bu soru kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz? (2. Soru b şıkkı için)

Bu örüntü kuralını yazarken sabit terim ve artış miktarı nedir? (2. ve 3. soru için)

Çiçeğin sapının günlük uzama miktarı üçgen sayısı ile belirtildiği için 1. günde iki üçgen varken 2. günde dört üçgen olduğu için gün sayısı bir artarken üçgen sayısı 2 artmış değil mi? (3. soru için)

Çiçeğin sapının günlük uzama miktarı üçgen sayısı ile belirtildiği için 1. günde iki üçgen varken 2. günde dört üçgen olduğu için 3. ve 4. günde kaç üçgen olur? (3. soru için)

Tabloyu doldururken nelere dikkat ettiniz? (3. soru için)

"s gün" sonraki çiçeğin oluşturabilmesi için kullanılması gereken üçgen sayısını nasıl yazdığınızı anlatabilir misiniz? (3. soru için)

Tablodaki boşlukları doldurmak size örüntü kuralını yazma konusunda nasıl yardımcı oldu? (3. soru için)

Yazdığınız bu örüntü kuralının bu soru kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz? (3. soru için)

1. sokakta iki çiçek, 2. sokakta 5 çiçek ve 3. Sokakta 8. çiçek olduğu için sokak sayısı bir artarken çiçek sayısı 3 artmış değil mi? (3. soru b şıkkı için)

1. sokakta iki çiçek, 2. sokakta 5 çiçek ve 3. Sokakta 8. çiçek olduğu için 5. sokakta kaç çiçek ekilmiş olur? (3. soru b şıkkı için)

Tabloyu doldururken nelere dikkat ettiniz? (3. soru b şıkkı için)

"x. sokaktaki" ekilen çiçek sayısını nasıl yazdığınızı anlatabilir misiniz? (3. soru b şıkkı için)

Tablodaki boşlukları doldurmak size örüntü kuralını yazma konusunda nasıl yardımcı oldu? (3. soru b şıkkı için)

Yazdığınız bu örüntü kuralının bu soru kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz? (3. Soru b şıkkı için)

Masa Organizasyonu Etkinliği

Etkinlikte yer alan soruları anladınız mı?

Bir masaya dört kişi otururken iki masaya altı kişi oturduğunda masa sayası bir artarken kişi sayısı iki artmış değil mi?

Masa sayısı artıkça masalarda oturabilecek toplam kişi sayısı nasıl değişmektedir?

Masa sayısı arttıkça oturan kişi sayısını ikişer ikişer arttırdığınızda birleştirilen 10 masada toplam kaç kişi oturur? (a şıkkı için)

76 kişinin oturabileceği masa sayısını bulabilmek için artış miktarı kadar yani ikişer ikişer sayabilir miyiz? (b şıkkı için)

Masa sayısına göre kişi sayısı değiştiği için masa sayısı bilinmeyenimiz olabilir mi? Bilinmeyeni nasıl ifade edebiliriz? (d şıkkı için)

Tablodaki boşlukları doldurmak size cebirsel ifadeyi yazma konusunda yardımcı olabilir mi? (d şıkkı için)

Tabloyu doldururken nelere dikkat ettiniz? (d şıkkı için)

Birleştirilen masa sayısı ile masada oturan toplam kişi sayısı arasındaki ilişkinin cebirsel ifadesini nasıl yazdığınızı anlatabilir misiniz? (d şıkkı için)

Yazdığınız bu cebirsel ifadenin bu etkinlik kapsamındaki olan her durum için geçerli olduğuna nasıl karar verdiniz? (d şıkkı için)

Grafiği çizerken cebirsel ifadedeki “2n+2” artı 2’yi belirtmemize gerek var mı? 0 masada kaç kişi oturur? (e şıkkı için)

Grafiğiniz nereden başmalı? Başlangıç noktası neresidir? Eksenlere ne ad verebiliriz? (e şıkkı için)

Grafiği çizerken nelere dikkat ettiniz? (e şıkkı için)

Alınan yol-zaman grafiklerini hatırlıyor musunuz? O grafiklerle bu çizmeniz gereken grafiği nasıl bağdaştırabilirsiniz? (e şıkkı için)

Birleştirilen masa sayısı ile masada oturan toplam kişi sayısı arasındaki ilişkiyi açıklamak için yazdıklarınızın yeterli ve doğru olduğunu düşüyor musunuz? (e şıkkı için)

E. Parent Approval Form

Sevgili Anne/Baba,

Bu çalışma Orta Doğu Teknik Üniversitesi Matematik Eğitimi Bölümü yüksek lisans öğrencisi Feyzanur Gün'ün ve Doç. Dr. Bülent Çetinkaya ve Doç. Dr. Arzu Aydoğan Yenmez danışmanlığında yürütülen tez çalışması kapsamında yapılmaktadır. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Bu çalışmanın amacı nedir?

Çalışmanın amacı, 7. sınıf öğrencilerinin cebirsel düşünme yapılarının ve düzeylerinin öğretim deneyi sırasındaki gelişimini incelemektir.

Çocuğunuzun katılımcı olarak ne yapmasını istiyoruz?

Bu amaç doğrultusunda, çocuğunuzdan 8 hafta boyunca sürecek olan bu çalışma için çalışma başlamadan ve bittikten sonra bir test çözmesini, çalışma boyunca her hafta 40 dakikalık derslere katılmasını, bu dersler boyunca 8 farklı çalışma yaprağını 3'er kişilik gruplar halinde yapmasını ve her ders sonrasında yapılacak olan bire bir görüşmelerde ders sırasında yaptığı çalışma yaprakları hakkında sorular cevaplaması isteyeceğiz. Çocuğunuzun cevaplarını/davranışlarını ses kaydı, görüntü kaydı ve yazılı biçiminde toplayacağız. Sizden çocuğunuzun katılımcı olmasıyla ilgili izin istediğimiz gibi, çalışmaya başlamadan çocuğunuzdan da sözlü olarak katılımıyla ilgili rızası mutlaka alınacak.

Çocuğunuzdan alınan bilgiler ne amaçla ve nasıl kullanılacak?

Çocuğunuzdan alacağımız cevaplar tamamen gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Elde edilecek bilgiler sadece bilimsel amaçla (yayın, konferans sunumu, vb.) kullanılacak, çocuğunuzun ya da sizin ismi ve kimlik bilgileriniz, hiçbir şekilde kimseyle paylaşılmayacaktır.

Çocuğunuz ya da siz çalışmayı yarıda kesmek isterseniz ne yapmalısınız?

Katılım sırasında sorulan sorulardan ya da herhangi bir uygulama ile ilgili başka bir nedenden ötürü çocuğunuz kendisini rahatsız hissettiğini belirtirse, ya da kendi belirtmese de araştırmacı çocuğun rahatsız olduğunu öngörürse, çalışmaya sorular tamamlanmadan ve derhal son verilecektir.

Bu çalışmayla ilgili daha fazla bilgi almak isterseniz;

Çalışmaya katılımınızın sonrasında, bu çalışmayla ilgili sorularınız yazılı biçimde cevaplandırılacaktır. Çalışma hakkında daha fazla bilgi almak için ODTÜ Matematik Eğitimi Bölümü yüksek lisans öğrencisi Feyzanur Gün (E-posta: [redacted]) ya da Doç. Dr. Bülent Çetinkaya (E-posta: [redacted]) ya da Doç. Dr. Arzu Aydoğan Yenmez (E-posta: [redacted]) ile iletişim kurabilirsiniz. Bu çalışmaya katılımınız için şimdiden teşekkür ederiz.

Yukarıdaki bilgileri okudum ve çocuğumun bu çalışmada yer almasını onaylıyorum (Lütfen alttaki iki seçenekten birini işaretleyiniz.)

Evet, onaylıyorum___ Hayır, onaylamıyorum___

Annenin adı-soyadı: _______________
Bugünün Tarihi:________________

Çocuğun adı soyadı ve doğum tarihi:_________________

(Formu doldurup imzaladıktan sonra araştırmacıya ulaştırınız)

F. Ethical Approval

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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29 KASIM 2023

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Bülent Çetinkaya

Danışmanlığını yürüttüğünüz ***Feyzanur GÜN'ün "Öğretim Deneyi Sürecinde Öğrencilerin Cebirsel Düşünme Yapılarının İncelenmesi"*** başlıklı araştırmanız İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek 0485-ODTUİAEK-2023 protokol numarası ile onaylanmıştır.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan

Prof.Dr. İ. Semih AKÇOMAK
Üye

Doç. Dr. Ali Emre Turgut
Üye

Doç. Dr. Şerife SEVİNÇ
Üye

Doç.Dr. Murat Perit ÇAKIR
Üye

Dr. Öğretim Üyesi Süreyya ÖZCAN KABASAKAL
Üye

Dr. Öğretim Üyesi Müge GÜNDÜZ
Üye

G. Permission Obtained from Ministry of Education

T.C.
NİĞDE VALİLİĞİ
İl Millî Eğitim Müdürlüğü

Sayı : E-61900286-20-95680164 01.02.2024
Konu : Araştırma İzni
(Feyzanur Gün)

VALİLİK MAKAMINA

İlgi: a) Milli Eğitim Bakanlığına Bağlı Okul Ve Kurumlarda Yapılacak Araştırma Ve Araştırma Desteğine Yönelik İzin Ve Uygulama Yönergesi.
b) Orta Doğu Teknik Üniversitesi Rektörlüğü Öğrenci İşleri Daire Başkanlığının 19/01/2023 tarihli ve 552 sayılı yazısı.

Orta Doğu Teknik Üniversitesi Matematik ve Fen Bilimleri Eğitimi Anabilim Dalı, Matematik Eğitimi yüksek lisans programı öğrencisi Feyzanur Gün, Doç. Dr. Bülent Çetinkaya'nın danışmanlığında yürütmekte olduğu "Öğretim Deneyi Sürecinde Öğrencilerin Cebirsel Düşünme Yapılarının İncelenmesi" başlıklı tez çalışmasını, değerlendirme formunda belirtilen okullarda uygulamak istemektedir.

Feyzanur Gün' ün, gerekli özen ve hassasiyeti göstererek çalışmasını uygulaması Müdürlüğümüz Araştırma Değerlendirme Komisyonu tarafından yapılan değerlendirme sonucunda, Müdürlüğümüzce uygun görülmektedir.

Makamlarınızca da uygun görülmesi halinde olurlarınıza arz ederim.

Ahmet ŞENEL
Milli Eğitim Müdür Yardımcısı

OLUR

Halil İbrahim YAŞAR
Vali a.
Milli Eğitim Müdürü