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Archimedes's *Measurement of the Circle* in Arabic: Texts and Translations

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Abstract

Measurement of the Circle is a short treatise by Archimedes on the area and the perimeter of a circle. It was translated into Arabic in the 9th century, along with other works attributed to Archimedes. Various versions of the Arabic translation of *Measurement of the Circle* were also produced. In this article, the critical editions of three Arabic versions of *Measurement of the Circle* are presented together with their English translations.

I Introduction

Measurement of the Circle (henceforth *MC*), in its extant form, is a short treatise by Archimedes (ca. 287–212 BCE) that contains three propositions pertaining to the perimeter-diameter ratio and the area of a circle. Due to its interest both for Greek geometers working in the Euclidean tradition and other mathematicians interested in its applications to measurement and astronomy, *MC* attracted the attention of many Greek mathematicians until the end of Late Antiquity.

In the 9th century many scientific and philosophical texts, both Greek and non-Greek, were translated into Arabic during what has been called the "translation movement."[1] Among the translated texts are some of the treatises of Archimedes such as *On the Sphere and the Cylinder* and *MC*, along with a number of shorter works attributed to Archimedes but not extant in Greek. These works later served as inspiration and starting point for a great amount of original research by scholars in various Islamicate societies, in addition to derivative versions.

The impact of the Arabic *MC* was not limited to the Islamicate world. Two Hebrew and two Latin translations of the Arabic *MC* were made in Western Europe in the Middle Ages. The Hebrew translations, which seem to have been made in the 12th and 13th centuries, are anonymous. The Latin translations, one probably by Plato of Tivoli (fl. first half of the 12th century) and the other by Gerard of Cremona (ca. 1114–1187), gave rise to a wave of mathematical activity in which several new versions of the *MC* as well as other treatises on the subject were written.

Three Arabic versions of the *MC*, called the *Fatih*, *Columbia*, and the *Riżā* versions in this article, are extant, as well as the well-known *taḥrīr* of Naṣīr al-Dīn

[1] Dates are CE unless otherwise specified.

al-Ṭūsī (1201–1274).[2] Despite the importance of their study for the history of mathematics, to date no critical edition of these three versions of the Arabic *MC* have been published. To be sure, two of them, namely the *Fatih* and *Columbia* versions, have been studied by Knorr (1989) in detail; however, Knorr's work only contains translations of these two texts and not critical editions.[3] The purpose of this article is to attempt to fill this gap by presenting the critical editions of the *Fatih*, *Columbia*, and the *Riżā* versions of *MC* together with their English translations.[4]

I.1 *Measurement of the Circle* in Greek Mathematics

I.1.1 The Greek text of Measurement of the Circle

As stated above, the subject of *MC* is the perimeter-diameter ratio and the area of a circle.[5] The extant Greek text has three propositions:[6] *MC* 1 states that the area of a circle is equal to the area of a right-angled triangle one of whose legs is equal to the radius of the circle and whose other leg is equal to the perimeter of the circle;[7]

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[2] See Sections I.3.1, I.3.2, and I.3.3, respectively, for brief descriptions of these versions and Section II for information on the manuscripts. The reader should be advised that Knorr (1989) denotes the *Fatih* and the *Columbia* versions and Naṣīr al-Dīn al-Ṭūsī's *taḥrīr* by **AF**, **AR**, and **AT**, respectively.

[3] The Arabic *On the Sphere and the Cylinder* has received even less attention than the Arabic *MC*. The only study dedicated to the Arabic *On the Sphere and the Cylinder* of whose existence I am aware is a brief survey of its transmission by Lorch (1989).

[4] I have not edited Naṣīr al-Dīn al-Ṭūsī's *taḥrīr* for two reasons. First, the Arabic text has already appeared in a (noncritical) edition (al-Ṭūsī 1939, 377–389), and an English translation has been published by Knorr (1989, 577–583). Second, no survey of the manuscripts of this *taḥrīr*, of which there are dozens, has been undertaken; accordingly, any attempt at a critical edition would have been premature.

[5] For detailed studies of the works of Archimedes, see Dijksterhuis (1987) and Heath (1897); there is also a brief overview in Netz (2004–2017, I.10–13). An English translation of *MC* can be found in Heath (1897, 91–98).

[6] The division of *MC* into three propositions using proposition numbers is due to Johan Ludvig Heiberg (1972, I.232–243), the editor of the Greek text; as Netz (2012, 194) points out, the extant Greek manuscripts are not so divided. Heiberg's numbering can be defended by the observation that the three propositions have enunciations at their beginnings and there is none in the middle of the third proposition, where the second half of the proof begins. However, Heiberg (1972, I.240) records in the critical apparatus that manuscript A, the lost archetype for several other manuscripts, marks the middle of the third proposition as the beginning of the fourth proposition (δʹ). This is presumably because some of the manuscripts deriving from A have δʹ at this point; unfortunately Heiberg does not give more details.

[7] Henceforth, the convention of referring to propositions in mathematical texts by the name of the text in question (in italics) followed by the proposition number is adopted.

MC 2 states that the ratio of a circle to the square of its diameter is 11 : 14; *MC* 3 states that the perimeter of a circle exceeds three times its diameter by an amount smaller than 1/7 of the diameter and greater than 10/71 of the diameter.[8]

Since Archimedes himself refers to the result of *MC* 3 in his *Sand Reckoner*, and to *MC* 1 in his *Method*, his authorship of a text on the measurement of the circle that contains at least *MC* 1 and *MC* 3 is certain (Heiberg 1972, II.230.3–6, 440.10–12). As to the title *Measurement of the Circle* (κύκλου μέτρησις), which is reported in one manuscript (Heiberg 1972, I.232), while there is no reason to suppose that it was coined by Archimedes himself, it is attested, for instance, by Hero (fl. ca. 62) in *Metrica* (Acerbi and Vitrac 2014, 212.1–2, 16–17, 240.7–9). Unlike some other works of Archimedes, such as the two books of *On the Sphere and the Cylinder* or *Quadrature of the Parabola*, *MC* contains no introductory letter that might have given us further information on the circumstances of its composition and circulation.

Knorr's (1989, 375–400) study contains excerpts from Hero, Pappus (fl. 300–350), and Theon (fl. 350–400),[9] as well as versions of *MC* 1 preserved in Pappus's *Collection* and Theon's *Commentary on Ptolemy's Book I*. It is evident from the testimony of these authors that *MC* as originally written by Archimedes contained other propositions besides those preserved in the extant Greek text.[10]

MC is closely related to several of the works of Archimedes by its subject matter and approach. One very conspicuous strain in the works of Archimedes is the metrical study of areas and volumes of various geometric figures. This strain is represented by works such as *On the Sphere and the Cylinder*, *MC*, *Conoids and Spheroids*, *Spiral Lines*, and *Quadrature of the Parabola*, where the "indirect method" that is usually attributed to Eudoxus of Cnidus (ca. 400–ca. 347 BCE) is used for studying areas and volumes of various geometrical figures.[11]

The Greek text of *MC*, together with other treatises of Archimedes then extant in Greek, was edited by Johan Ludvig Heiberg in 1880–1881 in three volumes (Heiberg 1880–1881). After the discovery of two manuscripts, one of which is the famous

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8 In modern notation, $3\frac{10}{71} < \pi < 3\frac{1}{7}$.

9 Unless otherwise stated, "Theon" refers to Theon of Alexandria.

10 One such example is the so-called "Sector Theorem" on the area of sectors of circles, cited by Hero (Acerbi and Vitrac 2014, 240.7–9). The statement of this theorem is given below in Section I.1.2; Pappus's proof is translated by Knorr (1989, 394–395).

Using textual comparisons of *MC* 1 with Theon's *Commentary on Ptolemy's Book I*, Knorr (1989, 404–405) has argued that the extant Greek text of *MC* is descended from Theon's *Commentary* and is not a direct copy of Archimedes's own text; this view has come under criticism from Vitrac (1997, 20).

11 As Dijksterhuis (1987, 130), among many others, has pointed out, the commonly used term "method of exhaustion" for this procedure is misleading, since nothing is exhausted. It is for this reason that I have adopted his phrase "indirect method."

Archimedes Palimpsest dating from the 10th century,[12] Heiberg published a second edition in 1910–1915, again in three volumes, the first two of which are available today as a reprint (Heiberg 1972).

I.1.2 Measurement of the Circle *after Archimedes*

The writings of Greek mathematicians after Archimedes abound with references to *MC* and uses of the results established in it. Few of the works of Archimedes were the object of such interest, and Knorr (1978, 217) is probably right in supposing that one reason for this is that the reader does not need to know much mathematics to understand *MC*; for this, familiarity with Books I–VI and XII of Euclid's *Elements* as well as some basic arithmetic would have been sufficient. As the references by various Greek astronomers to *MC* or the results contained therein suggest, the central role of the circle in Greek astronomy is likely to have been another reason for the relative popularity of *MC* among Greek mathematicians.

Without undertaking a detailed survey, I present some references to *MC* in post-Archimedes Greek mathematics below. I have chosen five texts, namely Hero's *Metrica*, Ptolemy's *Syntaxis*, Theon of Smyrna's *Exposition of the Mathematical Things Useful in the Reading of Plato*, Proclus's *Commentary on the First Book of Euclid's Elements*, and an anonymous *Commentary on Isoperimetric Plane Figures.* These texts are chosen to illustrate the variety of the mathematical contexts in which the *MC* was cited and used: metrical (in the *Metrica*), astronomical (in the *Syntaxis* and the *Exposition of the Mathematical Things Useful in the Reading of Plato*), and purely geometrical (in the *Commentary on the First Book of Euclid's Elements* and *Commentary on Isoperimetric Plane Figures*). I have differentiated between explicit references, where the title of *MC* or the name of Archimedes is mentioned, and implicit references, where it is not. I have left out Theon's *Commentary on Ptolemy's Book I* and Pappus's (fl. 300–350) *Collectio* since they have already been examined in detail by Knorr (1989, 375–400). Eutocius's commentary on *MC* has also not been included here since it is treated in more detail below in Section I.1.3.

1. In *Metrica* I.26 Hero cites *MC* 1 and *MC* 2 explicitly (Acerbi and Vitrac 2014, 212.1–2, 16–17, 240.7–9). However, *MC* 1 is cited in the "product format,"[13] which differs from the extant Greek text: the area of a rectangle one of whose sides is equal to the perimeter of the circle and whose other side is equal to the radius of the circle is equal to twice the area of the circle. He also cites explicitly a now lost work of Archimedes titled *On Plynths and Cylinders* which states that the perimeter-diameter ratio of a circle is greater than 211875 : 67441

[12] See Netz, Noel, Tchernetska, and Wilson (2011) for images and transcriptions of, and commentary on, the Archimedes Palimpsest.

[13] The expression is due to Knorr (1989, 377).

and smaller than 197888 : 62351.[14] But he rejects the use of these numbers since they are ill-suited to calculations, and he opts for the value 22 : 7 for the perimeter-diameter ratio of a circle without, however, citing *MC* 3 explicitly. Hero uses the results of *MC* 1 and *MC* 2 along with the perimeter-diameter ratio 22 : 7 later in the book numerous times: to calculate areas of sectors of the circle (I.30–31, 33) and ellipses (I.34), the lateral surface areas of cylinders and cones (I.36, 37), the surface areas of spheres (I.38) and segments of spheres (I.39), the volumes of cylinders and cones (II.1), spheres (II.11), segments of spheres (II.12), and tori (II.13).

One more explicit reference to *MC* in the *Metrica* has already been mentioned: the so-called "Sector Theorem."[15] This result states that the area of a sector of the circle is equal to half of the area of the rectangle one of whose sides is equal to the perimeter of the sector and whose other side is equal to the radius of the circle where the sector is located (Acerbi and Vitrac 2014, 240.7–9).

2. In *Syntaxis* VI.7, in a discussion of solar eclipses, Ptolemy (ca. 100–ca. 170) states that the value he uses for the perimeter-diameter ratio of a circle, 3;8,30 : 1, is about halfway between 3 1/7 and 3 10/71, the simple values used by Archimedes (Heiberg 1898, 513.1–5). He then uses this ratio to calculate the overlapping parts of the disks of the sun and the moon during a solar eclipse; here he cites *MC* 1 implicitly but in the product format (Heiberg 1898, 514.5–6).
3. In a discussion on the sphericity of the universe and Earth, and their respective sizes, Theon of Smyrna (fl. early 2nd century) mentions in *Exposition of the Mathematical Things Useful in the Reading of Plato* that Eratosthenes shows that the size of Earth is approximately 252000 stadia and Archimedes shows that the perimeter of a circle, when straightened out, is 3 1/7 times its diameter; the diameter of Earth would therefore be approximately 80182 stadia (Hiller 1878, 124.10–19). Later on, he also cites *MC* 2 implicitly (Hiller 1878, 126.12–14).[16]

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[14] These numbers are corrupt. See Acerbi and Vitrac (2014, 213, n. 255).

[15] See note 10.

[16] A comparison of *Exposition* (Hiller 1878, 126.8–127.6), and Theon's *Commentary on Ptolemy's Book I* (Rome 1931–1943, II.395.2–396.12), reveals that they are nearly identical. Based on this similarity, Knorr (1989, 493, n. 14) suggests that the passage of the *Exposition* is a later, Byzantine, interpolation and the original version is the one found in Theon's *Commentary.* Against this, Vitrac (1997, 62) proposes that such citations originate from a work combining the results of *MC* and *On the Sphere and the Cylinder* to produce procedures to calculate surfaces with circular elements. Of the two suggestions, that of Vitrac seems to me the more convincing. Based on the similarity of the contexts where Theon of Smyrna and Theon cite these results of *MC*, I might only add that if, indeed, they draw from a common source as Vitrac suggests, it was probably an *astronomical*

4. At the end of his commentary on *Elements* I.45 (on the construction of a parallelogram equal to a given rectilinear figure in a given rectilinear angle) Proclus offers his conjecture that this problem was at the origin of the problem of the squaring of the circle, “for if it is worthy of inquiry to find a parallelogram equal to a given rectilinear figure, it is also worthy of inquiry whether it is possible to find rectilinear figures equal to curvilinear figures.” He then cites *MC* 1 explicitly (Friedlein 1873, 422.24–423.5).[17]
5. In the anonymous *Commentary on Isoperimetric Plane Figures* (around the middle of the 5th century),[18] the author wants to prove that the circle has the greatest area among all figures of the same perimeter. Since he has already proved that, among all polygons of the same perimeter and number of sides, the regular one has the greatest area, it will suffice to prove that the circle has greater area than any regular polygon having the same perimeter as the circle. At the end of the proof of this, the author cites *MC* 1 explicitly (Hultsch 1876–1878, III.1158.22–1160.4; Acerbi, Vinel, and Vitrac 2010, 130.17–130.23).[19]

I.1.3 Eutocius's Commentary to Measurement of the Circle

Among all Greek mathematicians of antiquity, it is Eutocius of Ascalon (b. ca. 480) who engaged with the treatises of Archimedes the most by writing detailed commentaries on them. His commentaries on *MC*, on both books of *On the Sphere and the Cylinder* as well as on both books of *Planes in Equilibrium* are extant (Heiberg 1880–1881, III.1–371).[20] In addition, he wrote commentaries on the first four books of Apollonius's (b. ca. 240 BCE) *Conics*; these also survive.[21]

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source containing an exposition on the size and shape of Earth (perhaps the treatise of Adrastes that Theon of Smyrna says he follows (Hiller 1878, 120.6–9)).

[17] According to Knorr (1989, 433, 525–526), this citation may be interpolated. Even if this is true, it does not affect the point made here about references to *MC* being widespread in Greek mathematics after Archimedes.

[18] I have taken the date from Knorr (1989, 168).

[19] See Hultsch (1876–1878, III.1156.25–1160.4) or Acerbi, Vinel, and Vitrac (2010, 129.9–130.23) for the whole argument.

[20] For a detailed study of Eutocius's commentary on *MC* with a view to determining the properties of the Greek text of *MC* that was used by him, see Knorr (1989, 513–534). In particular, according to Knorr (1989, 521), the extant version of Eutocius's commentary on *MC* is due to Isidore of Miletus (6th century) “and copied out by some disciple of his.” See Decorps-Foulquier (2009) for another study of Eutocius's commentary on *MC* with a view to using it to elucidate the textual history of *MC*.

[21] I have taken Apollonius's date from Toomer (1990, xi).

Eutocius's practice as a commentator on the works of Greek mathematicians of antiquity, and Archimedes in particular, has been investigated in detail by Decorps-Foulquier (1998, 89–97). It was part of an established tradition of commentary "whose rules were codified by grammarians, rhetors, and philosophers," and its main goal was to "explain clearly that which is difficult to understand" (Decorps-Foulquier 1998, 89–90).[22]

In the brief introduction to his commentary on *MC*, Eutocius makes some remarks on the goals of Archimedes as well as the history of the problem of the quadrature of the circle. Thus Archimedes's goal in *MC* 1 is, according to Eutocius, "to exhibit to which rectilinear figure the circle would be equal, a matter investigated long ago by famous philosophers before him" (Heiberg 1880–1881, III.264.12–14). Both Hippocrates of Chios and Antiphon (both 5th century BCE) had "investigated this problem carefully and came up with fallacies any reader of Eudemus's history of geometry and Aristotelian *Ceria* would know well" (Heiberg 1880–1881, III.264.15–20).[23] Eutocius continues by attributing to Heraclides's *Life of Archimedes* the notion that *MC* is "necessary for everyday purposes" (Heiberg 1880–1881, III.266.1–2);[24] he then cites the upper and lower bounds on the perimeter-diameter ratio given in *MC* 3. Even though these bounds are approximate, Archimedes actually "found a straight line equal to the perimeter of a circle by using some spirals" (Heiberg 1880–1881, III.266.6–7).[25]

Eutocius's comments on *MC* 1 are also quite brief and they address a possible objection concerning the lack of an important element in Archimedes's proof (Heiberg 1880–1881, III.266.8–268.17). Since *MC* 1 asserts the equality of the area of a circle and the area of a right-angled triangle whose sides are equal to the radius and perimeter of the circle, it might be thought that Archimedes left out the step of supplying a line equal to the perimeter of the circle. Eutocius retorts to this imagined

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[22] See also Netz (1998) for a list of textual practices of "deuteronomic texts," which include commentaries such as that of Eutocius.

[23] It is possible that Eutocius uses the word "famous" (κλεινῶν) in the quotation in the previous sentence sarcastically. The fallacy of Hippocrates presumably refers to taking the quadratures of lunes as implying the quadrature of the circle; that of Antiphon was to inscribe polygons in a given circle until the circle was exhausted. See Knorr (1986, 25–39) for a discussion.

[24] The translation is due to Knorr (1989, 494, n. 38).

[25] Archimedes proves in *Spiral Lines* 18 that "if a straight line should touch the spiral drawn in the first rotation at the end of the spiral, and a certain <line> is drawn from the point, which is <the> start of the spiral, at right <angles> to the start of the rotation … the line between the tangent and the start of the spiral shall be equal to the circumference of the first circle" (Netz 2004–2017, II.114). This result gives the perimeter of a circle as equal to a straight line constructed using spirals. See Netz (2004–2017, II.114–124) for an English translation of *Spiral Lines* 18 and a discussion.

objection that "it is clear to all that the perimeter of the circle is a one-dimensional quantity and a straight line is of the same kind" (Heiberg 1880–1881, III.266.23–25). Hence, "even if it has not yet been possible to produce a straight line equal to the perimeter of a circle, that there is, by nature, some straight line equal [to the perimeter] is itself not doubted by anyone" (Heiberg 1880–1881, III.266.26–268.2).[26]

Eutocius does not comment on *MC* 2; in contrast, his comments on *MC* 3 are quite detailed and take up the bulk of his commentary. As already mentioned in Section I.1.1, *MC* 3 states that the perimeter of a circle exceeds three times its diameter by an amount smaller than 1/7 of the diameter and greater than 10/71 of the diameter. Archimedes proves this by using regular hexagons circumscribed and inscribed around and in the given circle, followed by four angle bisections in both cases to construct circumscribed and inscribed regular 96-gons to establish upper and lower bounds for the perimeter-diameter ratio, respectively. The angle bisector theorem (*Elements* VI.3) and the Pythagorean theorem (*Elements* I.47) are used to calculate the side lengths of the constructed polygons at each step, which necessitate the calculation of square roots.[27] Eutocius starts his commentary on *MC* 3 by pointing out that the proof makes constant use of square roots, but it is impossible to calculate these exactly unless one starts with a square number. Since the way to do this approximately has already been described by Hero in his *Metrica*, by Pappus, Theon, and by the many commentators of Ptolemy's *Syntaxis*, he dispenses with such explanation (Heiberg 1880–1881, III.268.19–270.6). The rest of his comments on *MC* 3 are dedicated to the justification of the calculations at each angle bisection. Eutocius's providing details for these calculations is consistent with the requirement, stated above, that commentaries "explain clearly that which is difficult to understand" since *MC* gives only the results of these calculations.

After the calculations, Eutocius's closing remarks offer a defense of Archimedes against accusations that his approximations to the perimeter-diameter ratio of a circle are not as accurate as they could have been. He starts by mentioning Apollonius's approximations in his *Rapid Delivery* (Ὠκυτόκιον), which were indeed more accurate than those of Archimedes but not useful toward Archimedes's goal, which was to find numbers that would be useful in real life. Therefore, the criticisms of Sporus of Nicaea against Archimedes that the latter did not find a straight line equal to the perimeter of the circle are wide of the mark. According to Eutocius, Sporus says in his *Ceria* that his own teacher, Philo of Gadara, found more accurate approximations than those of Archimedes (that is, 3 1/7 and 3 10/71). However, all of these writers have ignored Archimedes's goal, which was to find approximations

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[26] I have followed Heiberg's *dubium* to translate ζητούμενον as "doubted" (Heiberg 1880–1881, III.269).

[27] For a detailed explanation of the proof of the propositions in *MC*, see Dijksterhuis (1987, 222–240).

that would be practical. Instead, they used multiplications and divisions involving myriads, which are difficult to understand for one not versed in advanced logistics, such as that of Magnus. In fact, anyone wishing to obtain more accurate results could simply have used Ptolemy's approach in the *Syntaxis*. Eutocius adds that he could have done so, but did not, since he knew that to find a straight line equal to the perimeter of a circle is impossible and, in any case, Archimedes's approach suffices to find more accurate results.[28]

The brief summary presented here and in Section I.1.2 makes it clear that *MC* had a significance that was out of proportion to its short length in the Greek mathematical sciences of Late Antiquity. Hero of Alexandria and the writers of the metrical works transmitted under his name were evidently interested in the *MC* due to its subject. However, *MC* was also important in Greek astronomy, where the shape and the size of Earth had been a central concern from very early on; one need only remember, for example, the various arguments for the sphericity of Earth given by Aristotle and the measurements made by Eratosthenes of its circumference.[29] Finally, *MC* was a treatise whose content was crucial for writers on isoperimetric figures.

I.2 Reception of *Measurement of the Circle* in Arabic among Abbasid Scholarly Circles in the Ninth Century

I.2.1 Ibn al-Nadīm on Measurement of the Circle

In contrast to his reports on Euclid's *Elements* and Ptolemy's *Almagest*,[30] Ibn al-Nadīm gives no details on the question of who translated the works of Archimedes into Arabic. *On the Sphere and the Cylinder* and a work titled *The Quadrature of the Circle*, presumably *MC* itself, are mentioned at the beginning of the list of the works of Archimedes given by Ibn al-Nadīm.[31]

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[28] See Heiberg (1880–1881, III.300.15–302.17) for the whole passage. An English translation of part of this passage can be found in Knorr (1989, 504–505).

[29] For a summary of the arguments of Aristotle as well as various calculations of the size of Earth, both those of Eratosthenes and others, see Heath (1913, 235–236, 337–350).

[30] See Flügel (1871–1872, I.265.20-23, 267.29–268.4) for Ibn al-Nadīm's reports on the translation of the *Elements* and the *Almagest* into Arabic.

[31] The full list given by Ibn al-Nadīm is as follows: (i) *The Sphere and the Cylinder* (*Kitāb al-kura wa-l-usṭuwāna*), two books; (ii) *The Quadrature of the Circle* (*Kitāb tarbīʿ al-dāʾira*), one book; (iii) *The Subdivision the Circle into Seven Equal Parts* (*Kitāb tasbīʿ al-dāʾira*), one book; (iv) *Mutually Tangent Circles* (*Kitāb al-dawāʾir al-mutamāssa*), one book; (v) *Triangles* (*Kitāb al-muthallathāt*), one book; (vi) *Parallel Lines* (*Kitāb al-khuṭūṭ al-mutawāziya*); (vii) *Lemmas on the Elements of Geometry* (*Kitāb al-maʾkhūdhāt fī uṣūl al-handasa*); (viii) *Assumptions* (*Kitāb al-mafrūḍāt*), one book; (ix) *Properties of Right-Angled Triangles* (*Kitāb khawāṣṣ al-muthallathāt al-qāʾima al-zawāyā*),

I.2.2 Al-Kindī's Epistle to Yūḥannā ibn Māsawayh on the Third Proposition of Measurement of the Circle

Abū Yūsuf Yaʿqūb ibn Isḥāq al-Kindī (ca. 801–ca. 866) was an Abbasid scholar of the 9th century who wrote hundreds of works that cover a wide range of topics such as philosophy, logic, arithmetic, music, geometry, astronomy and astrology, psychology, politics, and medicine.

Most of the mathematical works of al-Kindī are lost. However, even a cursory examination of the titles of these works reveals that al-Kindī had an abiding interest in the geometrical properties of circular and spherical figures and their applications to astronomy. As examples, we may note the following mathematical and astronomical works, whose subjects display striking overlap with those of the Greek texts mentioned in Section I.1.2. This overlap goes a long way in explaining al-Kindī's interest in *MC*, which shall be treated in more detail below.[32]

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one book; (x) *Construction of Water Clocks that Throw Little Balls* (*Kitāb ālat sāʿāt al-māʾ allatī tarmī bi-l-banādiq*), one book. In the Arabic title of (iv), I have followed Sezgin (1974, 134) in correcting the title to *al-mutamāssa* from Flügel's *al-mumāssa*. This correction is suggested not only by the fact that mutually interacting objects are referred to with Form VI verbs in Arabic mathematical texts, but also by the reading in Ibn al-Qifṭī's list of the works of Archimedes.

As to other biobibliographers, Ṣāʿid al-Andalusī (1029–1070) in his *Ṭabaqāt al-Umam* begins his list of the works of Archimedes with (i) *Heptagon in the Circle* (*Kitāb al-musabbaʿ fī al-dāʾira*) and (ii) *Measurement of the Circle* (*Kitāb misāḥat al-dāʾira*), which are presumably identical to (iii) and (ii) in Ibn al-Nadīm's list, respectively. Ṣāʿid al-Andalusī's list also includes (iii) *The Sphere and the Cylinder* (*Kitāb al-kura wa-l-usṭuwāna al-makhrūṭa*) at the end (Cheikho 1912, 29.2–3).

Finally, Ibn al-Qifṭī's (1172–1248) entry on Archimedes in his *Taʾrīkh al-Ḥukamāʾ* contains a list of the works of Archimedes which is a simple amalgamation of Ṣāʿid al-Andalusī's list followed by Ibn al-Nadīm's list, except that Ibn al-Qifṭī skips (iii) in Ṣāʿid al-Andalusī's list and (iii) in Ibn al-Nadīm's list, probably to avoid repetition (Lippert 1903, 67.10–15). We therefore get no additional information from Ibn al-Qifṭī on the works of Archimedes in Arabic.

For lists of extant manuscripts containing the works of Archimedes in Arabic, see Sezgin (1974, 128–136). The reader should be warned that the correspondence between the works of Archimedes listed by the biobibliographers on one hand, and the titles given in the manuscripts listed by Sezgin on the other, is not perfect, due first to the inevitable variations of the titles in medieval manuscripts, and second, to the fact that some works attributed to Archimedes are extant in manuscripts but are not listed by the biobibliographers.

[32] Most of the known titles of works of al-Kindī come from Ibn al-Nadīm's list (Flügel 1871–1872, I.255–261). This has been supplemented from titles in other biobibliographical sources and translated into English by Adamson and Pormann (2012, l–lxii). For the entries given below, the Arabic titles are from Ibn al-Nadīm's (Flügel 1871–1872, I.256–257) list, and the numbers at the beginning of each entry, as well as the English translations of the titles, are from Adamson and

- 43: That the world and everything in it is spherical in shape (*Fī anna al-ʿālam wa-kullamā fīhi kuriyy al-shakl*),
- 45: That the largest solid shape is the sphere, and the largest plane figure is the circle (*Fī anna al-kura aʿẓam al-ashkāl al-jirmiyya wa-l-dāʾira aʿẓam min jamīʿ al-ashkāl al-basīṭa*),
- 47: On the flattening (projection) of a sphere (*Fī tasṭīḥ al-kura*),[33]
- 81: The proposition of Archimedes on the approximation of the ratio between the diameter of a circle and its circumference (*Qawl Arshimīdis fī taqrīb qadr quṭr al-dāʾira min muḥīṭihā*),[34]
- 83: On the approximation of the chord of the circle (*Fī taqrīb watar al-dāʾira*),
- 84: On the approximation of the chord of the ninth (*Fī taqrīb watar al-tusʿ*),
- 85: On the measurement of an *īwān* (*Fī misāḥat īwān*),[35]
- 87: On the manner of constructing a circle whose area is equal to the surface of a given cylinder (*Fī kayfiyyat ʿamal dāʾira musāwiya li-saṭḥ usṭuwāna mafrūḍa*).

A discussion of the shape and size of the earth in an astronomical work of al-Kindī titled *The Great Art* (*Fī al-ṣināʿa al-ʿuẓmā*) provides evidence that all three propositions of *MC* were known to him (Rashed 1993, 12–13; Ahmad 1987, 174–176).[36]

We find al-Kindī's deepest engagement with *MC* in an epistle, whose title has already been given under number 81 in the above list, to Yūḥannā ibn Māsawayh (d. 857).[37] From the beginning of the epistle, we learn that Yūḥannā ibn Māsawayh had

Pormann's (2012, lii–liv) list, with minor changes to the translations. More important changes are noted where appropriate.

[33] Adamson and Pormann's (2012, lii) translation "On calculating the surface of a sphere" of this title is misleading.

[34] For this entry, which is an epistle of al-Kindī that I shall treat in more detail below, I have taken both the Arabic title and the English translation from Rashed (1993, 13–14) since the Arabic title as reported by Ibn al-Nadīm is corrupt.

[35] An *īwān* is a vaulted hall that is closed on three sides and open to the outside on the remaining side.

[36] *The Great Art* is not mentioned in the biobibliographical sources. The contents of this treatise, which is extant in a single manuscript, have been examined by Rosenthal (1956). Much of the treatise consists of either literal translations or paraphrases of Ptolemy's *Almagest*, interspersed with additional material, most of which derives from Theon's *Commentary on Ptolemy's Book I* (Rosenthal 1956, 439–440, 446). Ibn al-Nadīm reports a work of Theon titled *Introduction to the Almagest* (*Al-mudkhal ilā al-Majisṭī*) in an "old translation" (*bi-naql qadīm*) (Flügel 1871–1872, I.268.29); Rosenthal (1956, 446) suggests that this text may have been the one used by al-Kindī in composing the *The Great Art*.

[37] For an edition and an English translation of the epistle, see Rashed (1993).

asked al-Kindī to explain the proof of *MC* 3 in detail. Al-Kindī agrees to provide an explanation, saying "it is possible in this case to extend the statement and to expand it in a way which would not be necessary in this art for those people who are well-versed in it" (Rashed 1993, 32), which suggests that al-Kindī did not consider Yūḥannā ibn Māsawayh to be "well-versed" in geometry, even though it is clear that Yūḥannā must have been familiar with at least some of Euclid's *Elements*, as the references to the *Elements* in the epistle indicate.[38]

The details of the proof and calculations of *MC* 3 take up the rest of the epistle. For the first part of the proof of *MC* 3, where Archimedes uses a regular 96-gon circumscribed around a given circle in order to obtain the upper bound of 3 1/7 for the perimeter-diameter ratio, al-Kindī starts with a justification of the inequality

$$\frac{265}{153} < \sqrt{3}$$

used by Archimedes; he does not, however, attempt to explain the choices of the numbers 265 and 153. He proceeds to the calculations associated with the bisections. For the second part of the proof, al-Kindī starts by constructing a side of the regular 96-gon inscribed in the given circle using four angle bisections. After that, his way of proceeding is similar to the first part: first, a justification of the inequality

$$\sqrt{3} < \frac{1351}{780}$$

without, again, attempting to explain the choices of the numbers, followed by the calculations associated with the bisections.

The repeated references to the *Elements* in the epistle,[39] the detailed calculations of the various side lengths, and the repeated statements of the number of sides of the regular polygons that can be constructed with the various sides are all consistent with al-Kindī's stated desire to help Yūḥannā ibn Māsawayh in understanding *MC* 3.

I.2.3 Banū Mūsā's Book for Knowing the Measurement of Plane and Spherical Figures

The Banū Mūsā were three brothers—Muḥammad (d. 873), Aḥmad, and al-Ḥasan—who worked as courtiers, ministers, and scholars in the 9th century, with the focus of their scholarship on mathematical sciences. They were the authors of a work on

[38] The correspondence between al-Kindī and Yūḥannā ibn Māsawayh was not limited to *MC* as al-Kindī is known to have written another epistle, on the soul, to Yūḥannā (Adamson and Pormann 2012, lxii).

[39] For an explanation of al-Kindī's peculiar term for the *Elements*, namely "Principal Books" (*al-aqāwīl al-ūlā*), see Rashed (1993, 52–53).

the measurement of geometric figures, extant in a Latin translation made by Gerard of Cremona (ca. 1114–1187) under the title *Verba filiorum Moysi filii Sekir, i.e. Maumeti, Hameti, Hasen* as well as an edition made by Naṣīr al-Dīn al-Ṭūsī (1201–1274) under the title *Book for Knowing the Measurement of Plane and Spherical Figures* (*Kitāb maʿrifat misāḥat al-ashkāl al-basīṭa wa-l-kuriyya*).[40]

The treatise contains an introduction by the Banū Mūsā followed by 18 (Naṣīr al-Dīn al-Ṭūsī) or 19 (Gerard of Cremona) propositions. The introduction starts with a justification for the composition of the work.[41] The Banū Mūsā claim that there is a need for the science of measurement of geometric figures, but that none of their contemporaries properly understands this science (Clagett 1964, 238–239). Even though "there are some things which some of the early savants understood and wrote about in their books," the knowledge of such things is available but not common in the Banū Mūsā's time (Clagett 1964, 239). The authors also make it clear that they assume a working knowledge in the "books of geometry in common usage" in their time (Clagett 1964, 241). These remarks are followed by a discussion of the concepts of length, width, and breadth of geometric figures (Clagett 1964, 240–244).

Propositions 2–6, which are the same for both versions, concern the area and the perimeter of the circle, with Propositions 2 and 3 used as preliminary results in the proofs of the later propositions. Proposition 4 states that the product of half of the diameter of any circle with half of its perimeter is equal to the area of the circle. This is of course equivalent to *MC* 1 but the proof is different. Proposition 5 states that the ratio of the diameter of any circle to its perimeter is unique, which is not proved in the extant Greek text of *MC*. Proposition 6 takes up the calculation of the perimeter-diameter ratio according to "the method used by Archimedes" (Rashed 1996, 74–75), but it is supplemented with the intermediate calculations, in the same way as in Eutocius's commentary or al-Kindī's epistle. Thus, the evidence of Propositions 4–6 indicates that the Banū Mūsā were already familiar with the contents of *MC* by the time of the composition of their treatise.

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[40] This treatise is not listed in Ibn al-Nadīm's list of the works of the Banū Mūsā (Flügel 1871–1872, I.255–261). Both versions have been critically edited and translated, Gerard's Latin translation by Clagett (1964, 223–367) with an English translation and Naṣīr al-Dīn al-Ṭūsī's version by Rashed (1996, 1–137) with a French translation. The attribution of the Latin translation to Gerard of Cremona is made secure by the appearance of the title in a list of Gerard's translations written some time after his death by some of his associates (Burnett 2001, 277). It is important to note that the two versions differ to some extent, seemingly due to Naṣīr al-Dīn al-Ṭūsī's editorial choices (Rashed 1996, 7–11).

[41] As Rashed (1996, 58, n. 1) points out, the introduction in Naṣīr al-Dīn al-Ṭūsī's version is truncated. Hence I refer to Gerard's version for the introduction.

I.2.4 Thābit ibn Qurra's Measurement of Plane and Solid Figures

Finally, brief mention must be made of the appearance of the contents of *MC* in the treatise *Measurement of Plane and Solid Figures* (*Fī misāḥat al-ashkāl al-musaṭṭaḥa wa-l-mujassama*) by Thābit ibn Qurra (d. 901), another outstanding mathematical scholar of the 9th century.[42] After the areas of rectilinear plane figures, Thābit considers "figures with curvature" (*al-ashkāl dhawāt al-taqwīs*) and the first of these is the circle (Rashed 2009b, 191.11–12). First, the area of the circle is equal to the product of half of its diameter with half of its perimeter, which is equivalent to *MC* 1. And if the diameter of the circle is known, the perimeter can be known approximately by multiplying the diameter by 3 1/7, which is equivalent to using the upper bound for the perimeter-diameter ratio given in *MC* 3. The area of the circle can also be found approximately by multiplying the diameter by itself and then removing 1/7 of the result and then half of 1/7 of the result, which is equivalent to *MC* 2 (Rashed 2009b, 191.12–19). Thābit also discusses the area of the sector of the circle, which is equal to the product of half of the diameter with half of the length of the arc of the sector (Rashed 2009b, 191.22–193.2). Nowhere in discussions of the measurement of the area and perimeter of a circle does Thābit explicitly mention the name of Archimedes. However, he does so later on three times in discussions concerning the area and volume of a sphere and the area of a segment of a sphere, also mentioning *On the Sphere and the Cylinder* by name once (Rashed 2009b, 195.28, 199.1–6, 209.1–3). Since Thābit was not only an associate of the Banū Mūsā but a competent mathematical scholar in his own right, and since *MC* and Archimedes's authorship of it was known among Abbasid scholars since the mid-9th century at the latest,[43] it is very likely that Thābit was in fact familiar with *MC* and its mathematical details.

I.3 The Arabic, Latin, and Hebrew Texts of *Measurement of the Circle*

I.3.1 The Fatih *Version*

Of the three Arabic versions of *MC* edited in this article, the *Fatih* version, extant in two manuscripts,[44] is the one that has received the most attention from historians of mathematical sciences. Presumably, the reason for this interest is that one manuscript containing it, namely İstanbul, Süleymaniye Manuscript Library,

[42] An edition and French translation of this treatise has been published by Rashed (2009b).

[43] See Section I.2.2.

[44] These are İstanbul, Süleymaniye Manuscript Library, Fatih 3414 and Bursa, İnebey Manuscript Library, Haraççıoğlu 1174. See Section II.1 for more information on the manuscripts.

Fatih 3414, has been known to historians of mathematics since 1936 at the latest.[45] Despite this interest, no critical edition of the *Fatih* version has appeared so far. However, the *Fatih* version has been studied in detail by Knorr (1989, 421–494) as part of a more wide-ranging study on the medieval tradition of *MC*. Knorr (1989, 455–463) also includes a facsimile of the folios of Fatih 3414 containing the *Fatih* version. Unfortunately, these reproductions are not of good quality: not only does the Arabic text look thicker than it is in reality, the lines in the diagrams appear faded. It is therefore difficult to use these images for critical study. The English translation of the *Fatih* version that Knorr provides is accurate, despite a style that is sometimes excessively literal (Knorr 1989, 436–438, 484–489). He also provides a convenient collection of variant readings between the *Fatih* version and the Hebrew and Latin versions (Knorr 1989, 438–441, 489–491).

One feature of the *Fatih* version deserves brief comment, and that is that the first two of the three numbers in *MC* 3 that are greater than 10000 are either transmitted or translated incorrectly in the *Fatih* version. These numbers are 349450 and 23409, which appear in the *Fatih* version as 9450 and 3409, respectively. Knorr (1989, 482–483) explains these corrupted numbers as the result of a scribal error, based on the presence of the correct forms of these numbers in the Latin translation of Gerard of Cremona and the Greek text of *MC* itself.[46] I argue below that the correct numbers in the Latin translation of Gerard of Cremona are due to deliberate correction.[47] As to the corrupt numbers in the *Fatih* version, since both of them lost their multiples of 10000, it is an *ad hoc* explanation to consider them as scribal errors. Indeed, using other numbers in *Fatih* 3 as templates, we see that the number 23409 would have been rendered as *al-thalātha wa-l-ʿishrīn alfan wa-l-arbaʿimiʾa wa-l-tisʿa*, and 349450 would have been rendered as *al-thalāthimiʾa wa-l-tisʿa wa-l-arbaʿīn alfan wa-l-arbaʿimiʾa wa-l-khamsīn* (both genitive). In order for 23409 to be corrupted into 3409, the second word (*wa-l-ʿishrīn*) would have to be dropped and the third word (*alfan*) would have to be changed into *alf*. For 349450 to be corrupted into 9450, the first and third words (*al-thalāthimiʾa* and *wa-l-arbaʿīn*) would have be dropped and the fifth word (*alfan*) would again have to be changed into *alf*. A more economical explanation of these corruptions would take into account the numerical representation of multiples of 10000, which are written in Greek with M with the number of 10000s on top,

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45 This is the year of publication of Max Krause's *Stambuler Handschriften islamischer Mathematiker* (Krause 1936, 457), the earliest mention of Fatih 3414 known to me, though, to be sure, it only mentions the *Kitāb al-maʾkhūdhāt* contained in Fatih 3414 and not the *Fatih* version of *MC*.

46 Earlier, Knorr (1989, 422) claims that these corrupt numbers are not found in the Hebrew and Latin versions. This statement is misleading since the Hebrew version does not contain *MC* 3 at all while the Latin translation attributed to Plato of Tivoli also has 9450 and 3409.

47 See Section I.3.5.

since one letter with one or two letters on top of it representing numbers would more easily be corrupted in transmission or be translated erroneously.

I.3.2 Columbia Preliminaries *and the* Columbia *Version*

The *Columbia* version is preceded by another text, henceforth called *Columbia Preliminaries*, in the unique manuscript containing it, namely New York, Columbia University Rare Book and Manuscript Library, Or. 45. *Columbia Preliminaries* consists of four propositions, and it is edited and translated in this article in addition to *Columbia*.

Columbia Preliminaries carries the name of an author in the manuscript whereas *Columbia* does not. I follow Knorr (1989, 543, 552) in reading that name as Abū al-Rashīd ʿAbd al-Hādī, even though the last word might equally be al-Bāriʾ. He also suggests that the author of both *Columbia Preliminaries* and *Columbia* was one Abū al-Rashīd Mubashshir ibn Aḥmad ibn ʿAlī ibn ʿUmar al-Rāzī, a brief notice about whom can be found in Suter (1900, 126). Suter in turn bases his information on Ibn al-Qifṭī (Lippert 1903, 269–270). According to Ibn al-Qifṭī, this Abū al-Rashīd Mubashshir ibn Aḥmad was "very skilled in calculation, properties of numbers, and algebra" (*kathīr al-maʿrifa bi-l-ḥisāb wa-khawāṣṣ al-aʿdād wa-l-jabr wa-l-muqābala*), as well as other subjects; he died in 1193 (AH 589) (Lippert 1903, 269.11–12, 270.3). Knorr's identification of Abū al-Rashīd ʿAbd al-Hādī with Abū al-Rashīd Mubashshir ibn Aḥmad is based on the occurrence of a supposed "from the calculator" (*min al-ḥāsib*) in the manuscript. However, this reading is wrong and it should read "from the margin" (*min al-ḥāshiya*).[48] It follows that there are no grounds for identifying Abū al-Rashīd ʿAbd al-Hādī with Abū al-Rashīd Mubashshir ibn Aḥmad.

In view of some terminological differences between *Columbia Preliminaries* 1–3 and *Columbia*, it is certain that they were authored by different individuals.[49] In *Columbia Preliminary* 1, a square is a *murabbaʿ mutasāwī al-aḍlāʿ*. The area bounded by the line *alif jīm* and the arc *alif jīm* is referred to as a *qaws*. In *Columbia Preliminary* 2, *qaws* is again used to refer to areas bounded by a line and and arc having the same endpoints. Finally, in *Columbia Preliminary* 3, a square is again referred to as a *murabbaʿ mutasāwī al-aḍlāʿ*.[50]

We find similar uses of the term *murabbaʿ mutasāwī al-aḍlāʿ* in two early 9th-century algebra texts. These are al-Khwarizmī's *Kitāb al-jabr wa-l-muqābala* and

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[48] See note 188.

[49] Against Knorr (1989, 543, 552), who attributes both *Columbia Preliminaries* and *Columbia* to Abū al-Rashīd ʿAbd al-Hādī.

[50] The mathematical terminology of the second paragraph of *Columbia Preliminary* 3, which I suspect is an interpolation (see note 154), has no noticeable difference with respect to the *Fatih*, *Columbia*, and *Riżā* versions.

Ibn Turk's *Al-ḍarūrāt fī al-muqtarināt min kitāb al-jabr wa-l-muqābala.*[51] The two authors are most likely roughly contemporary (Sayılı 1985, 91). Sayılı (1985, 84) has already drawn attention to the ways Ibn Turk and al-Khwarizmī use the word *murabbaʿ*. Ibn Turk typically refers to squares as "equilateral right-angled quadrilateral surface" (*saṭḥ murabbaʿ mutasāwī al-aḍlāʿ qāʾim al-zawāyā*).[52] In contrast, al-Khwarizmī mostly refers to squares and rectangles indiscriminately as "surface" (*saṭḥ*), but he also uses "square surface" (*saṭḥ murabbaʿ*) and he sometimes specifies that with "equilateral and equiangular" (*mutasāwī al-aḍlāʿ wa-l-zawāyā*).[53] Based on these varying uses of the word *murabbaʿ*, Høyrup (1986, 474, n. 28) suggests that "the value of *murabbaʿ* was changing first in the circle of court mathematicians around Al-Maʾmūn." If this is correct, this may indicate that *Columbia Preliminaries* 1–3 were composed in the first half of the 9th century and hence that *MC* was known to mathematical scholars in Abbasid society at that time.

We also find in al-Khwarizmī a usage of *qaws* similar to that in *Columbia Preliminaries* 1 and 2, a fact that may strengthen the suggestion made above on the date of composition of *Columbia Preliminaries*. In the chapter on measurement in the *Kitāb al-jabr wa-l-muqābala*, al-Khwarizmī describes how to calculate the "area of the arc" (*taksīr al-qaws*) (Rashed 2009a, 207.4, 10). The procedure involves taking the difference between a sector of a circle and a triangle. The fact that the phrase *taksīr al-qaws* is repeated twice makes it unlikely that a scribal error is involved and that *qaws* refers to the region bounded by an arc and its chord, just as in *Columbia Preliminaries.*[54]

Columbia 1 and 2, which correspond to *Fatih* 1, are not different from it in the essential ideas of the proofs, but the diagrams are drawn differently and have different letterings.[55] *Fatih* does not label the points around the circle and the circumscribed and inscribed polygons completely, whereas *Columbia* does. Moreover, in the labeling of the letters around the circle in *Columbia* 1, there is a peculiarity which may have implications for the circumstances of the composition of *Columbia.* After labeling the corners of the square in the circle with *alif* through *dāl* and the corners of the triangle with *hāʾ* through *ḥāʾ*, *Columbia* 1 labels the cardinal points on the circle with *ṭāʾ* through *mīm* counterclockwise, skipping *yāʾ*. Next, it starts to la-

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[51] These treatises are edited and translated by Rashed (2009a) and Sayılı (1985), respectively.

[52] For only four examples among many, see Sayılı (1985, 145.8, 17–18, 146.13–14, 149.17). The reader of Sayılı's edition should be warned that no line numbers are included in the Arabic text and I count the lines from the top, starting with the first Arabic line.

[53] For examples of *saṭḥ* used for both squares and rectangles, see Rashed (2009a, 117.1–3). For an example of *saṭḥ murabbaʿ mutasāwī al-aḍlāʿ wa-l-zawāyā*, see Rashed (2009a, 119.7–8).

[54] Hence, Rashed's (2009a, 206) editorial addition of "portion limited by" to "the arc" to translate *taksīr al-qaws* is unwarranted.

[55] See Knorr (1989, 543–546) for a brief review of the differences between *Columbia* and *Fatih.*

bel the midpoints of the eighths of the circle, again counterclockwise, starting with *nūn*. It then uses *ṣād* for the next one, which shows that the author is using the "Western" system of *abjad* notation. The same usage is also found in the diagram of *Columbia* 2, where *ṣād* is used after *nūn* for a corner of the octagon, again in the counterclockwise direction. It has recently been suggested by Thomann (2018, 167) that the "Eastern" system of *abjad* notation has developed in conjunction with the Arabic translations of Syriac and Greek astronomical texts in the first half of the 9th century and that the "Western" system is older than the "Eastern" one. Together with the suggestion I have made above that the terminology of *Columbia Preliminaries* indicates a date of composition in the first half of the 9th century, one is tempted to see a similar date of composition for *Columbia* as well, though obviously not by the same person. However, I see no reason why a composition by an individual in the Maghrib can be ruled out.

A textual comparison makes it clear that the enunciation of *Columbia* 4 is closer than that of *Fatih* 3 to a literal translation of the Greek text of *MC* 3: *Fatih* 3 has

كلّ خطّ محيط بدائرة فإنّه زائد على ثلاثة أضعاف قطرها بأقلّ من سبع القطر وبأكثر من عشرة أجزاء من أحد وسبعين جزءًا من القطر،

while *Columbia* 4 has

محيط كلّ دائرة ثلاثة أمثال قطرها، ويزيد أيضًا بأقلّ من سبع القطر وبأكثر من عشرة أجزاء من أحد وسبعين منه،

and *MC* 3 has

> παντὸς κύκλου ἡ περίμετρος τῆς διαμέτρου τριπλασίων ἐστὶ καὶ ἔτι ὑπερέχει ἐλάσσονι μὲν ἢ ἑβδόμῳ μέρει τῆς διαμέτρου, μείζονι δὲ ἢ δέκα ἑβδομηκοστομόνοις. (Heiberg 1972, I.236.8–11)

It is also obvious that the enunciation of *Fatih* 3 is mathematically sounder than the one in *Columbia* 4 and the Greek text of *MC* 3. In the latter, the perimeter of the circle is said to be, first, three times the diameter, then is said to yet also exceed it by an amount between 1/7 and 10/71 of the diameter, which makes for a clumsy wording since it implies that the same thing is equal to another thing and yet exceeds it. By contrast, in *Fatih* 3, the perimeter exceeds three times the diameter by an amount between 1/7 and 10/71 of the diameter, which is a mathematically correct wording. Except for these initial divergences between *Columbia* 4 and *Fatih* 3, the rest of the two enunciations are nearly identical, except at the very end, where a simple *minhu* in *Columbia* 4 corresponds to *min al-quṭr* in *Fatih* 3.

An examination of Table 1 suggests that neither the epistle of al-Kindī nor the *Book for Knowing the Measurement of Plane and Spherical Figures* of the Banū Mūsā is likely to be among the source(s) of *Columbia* 4 and 5. The fractions in Eutocius's commentary on *MC* that are missing in the epistle of al-Kindī and the treatise of the Banū Mūsā are for the most part present in *Columbia*; the two exceptions to this are 5448723 and 5472132, which have no fractions.

Table 1: Some numbers in Eutocius's commentary on *MC* and various other texts

Eutocius	al-Kindī	Banū Mūsā (al-Ṭūsī)	Banū Mūsā (Gerard)	*Columbia*
$1350534\frac{1}{2}\frac{1}{64}$	1350534	1350534	$1350534\frac{1}{4}$	$1350534\frac{1}{2}\frac{1}{64}$
$1373943\frac{1}{2}\frac{1}{64}$	1373943	1373943	$1373943\frac{1}{4}$	$1373943\frac{1}{2}\frac{1}{64}$
$5448723\frac{1}{16}$	5448723	5448723	5448723	5448723
$5472132\frac{1}{16}$	5472132	5472132	5472132	5472132
$4064928\frac{1}{36}$	4064928	4064928	4064928	$4064928\frac{1}{36}$
$4069284\frac{1}{36}$	4069284	4069284	4069284	$4069284\frac{1}{36}$

In *Columbia* 3, numbers are expressed in lexical numerals as in *Fatih* 2.[56] The same is generally true in *Columbia* 4; one obvious change is the repeated use of the word *ribwa* ("ten thousand, myriad") to express multiples of 10000. This word appears for the first time to express 1350534 1/2 1/64. Even though the numbers 23409, 326041, and 349450, all of which involve multiples of 10000, had appeared before, none of them is expressed with *ribwa*. Toward the end of *Columbia* 4, Hindu-Arabic numerals are first used to write 5448723 after the 500 myriads (where 500 is written in lexical numerals). Hindu-Arabic numerals are used consistently until the end of *Columbia* 4, where the last two numbers in the proposition, namely 4673 1/2 and 96, are again written in lexical numerals. There seems to be no discernible pattern to this usage of Hindu-Arabic numerals.

I.3.3 The Riżā *Version*

The principal difference of the *Riżā* version compared to *Fatih* or *Columbia* is its organization: *Riżā* 2 corresponds to *Fatih* 3, whereas *Riżā* 3 corresponds to *Fatih* 2. Since the proof of *Fatih* 2 uses the result of *Fatih* 3, this arrangement is mathematically sounder. The proof of *Riżā* 3 is different from that of *Fatih* 2, but with no

[56] The remarks on the use of numeration systems for the Arabic versions can only be tentative due to the small number of manuscripts for each version.

noticeable shortening or clarification. There is also what seems to be an interpolation in *Riżā* 3.[57] The appearance of the words "their counterparts" (*naẓāʾiruhumā*) and "contradiction" (*khulf*) in *Riżā* 1, which do not appear in *Columbia* 1 and 2, suggests that a text that was closely related to the *Fatih* version was used a source for the *Riżā* version. This suggestion is strengthened by the fact that *Riżā* 1 never mentions all objects around the diagram by letters as *Columbia* does, but rather mentions the first occurrence of such objects with letters and then simply states that the same argument holds for the remaining objects, as *Fatih* does.

Just like *Columbia* 4 and 5, *Riżā* 2 is closely related to *Fatih* 3 but it is expanded with the intermediate calculations. A phrase found in *Riżā* 2 allows us to state with certainty that an Arabic version of Eutocius's commentary on *MC* was used as a source for *Riżā*. After taking the difference 93636 − 23409 = 70227, *Riżā* 2 takes the square root of this number as 265, and states that the line *BG* is greater than 265 "by an insignificant amount imperceptible to the senses" (*bi-shayʾ yasīr lā yudrik al-ḥiss*). This is a fairly exact translation of the Greek μόριον ἐλάχιστον καὶ ἀνεπαίσθητον that is used to describe the excess of the square root of 70227 over 265 in Eutocius's commentary on *MC* (Heiberg 1880–1881, III.272.7).[58] Since such close correspondence in two verbal expressions for the same mathematical object is unlikely to be the result of mere coincidence, we have to conclude that an Arabic translation of Eutocius's commentary or a closely related text was used as a source for the *Riżā* version.

Riżā 2 differs from *Fatih* 3 and *Columbia* 4 and 5 in that the numbers are often expressed in the sexagesimal *abjad* system. This system is first used to write the square root of 349450 as 591;8,34 where the 591 is written in Hindu-Arabic numerals and the fractional parts are written in sexagesimal *abjad*. From then on, sexagesimal *abjad* is used to write the fractional parts of the numbers appearing in intermediate calculations as well as the integer parts of some large numbers (with at least four sexagesimal places).

In *Riżā*, the numbers found in *MC* 3 and Eutocius's commentary were not simply converted to sexagesimal, but the calculations seem to have been redone from scratch. One example of this, among many others, is the number 591;8,34 mentioned above, corresponding to *MC*'s 591 1/8, which would have been expressed in sexagesimal as 591;7,30. Another clear sign of a recalculation of the numbers is given by the small numerical errors in the text. These, however, are not so large as to invalidate the conclusions of *Riżā* 2.

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[57] See note 267.

[58] Indeed, $\sqrt{70227} = 265.00377$.

I.3.4 The Hebrew Translations of Measurement of the Circle

There are two known Hebrew translations of *MC* made in the Middle Ages, which have recently been edited and translated into French by Lévy (2011).[59] Neither translation carries the name of a translator. One of them (henceforth denoted **HA**, following Lévy), closely related to the *Fatih* version, is extant in a single manuscript,[60] and its existence has been known since the end of the 19th century. **HA** contains only the translations of *MC* 1 and 2, and part of the enunciation of *MC* 3. The second translation (henceforth denoted **HB**, again following Lévy), which was identified by Lévy, is extant in two manuscripts,[61] and it contains only the translation of *MC* 1. A comparison of the texts of **HA** and **HB** reveals that they have different sources.

A number of observations on similarities between **HA** and **HB** and the Arabic versions edited in this article may be made which, while quite weak if taken in isolation, together might indicate that Arabic versions related to the *Columbia* and *Riżā* had been in circulation in Western Europe when the Hebrew translations were made.

First, the diagram for *MC* 1 in **HA** is, as Lévy (2011, 113) points out, different from the diagram in the *Fatih* version and the two Latin translations in that it presents two circles/squares for the two parts of the proof, but it should be noted that the diagram in **HA** is similar to the diagram in the *Columbia* version. The circle/square on the left in **HA** resembles the diagram of *Columbia* 1 in that both have a circle and an inscribed square with horizontal and vertical sides. In addition, **HA** has the sides *BF* and *FA* of the inscribed octagon obtained by subdividing the arc *BA* in two halves at *F*, which is similar to the diagram of *Columbia* 1 (although of course *Columbia* draws the octagon in its entirety). The line segments *NS* in **HA** and *PF* in *Columbia* 1 are similarly positioned, from the center to the lower left (**HA**) or the lower right (*Columbia* 1). The similarities for the diagrams for the first part of *MC* 1 in **HA** and *Columbia* 1 are unlikely to be independent inventions. However, the lettering in both diagrams are completely different. Likewise, the circle/square on the right in **HA** resembles the diagram of *Columbia* 2 but not only are some lines in *Columbia* 2 are absent in **HA**, the lettering in the two diagrams are completely different.

Secondly, the enunciation of *MC* 1 in **HB** agrees particularly closely with the enunciation of *Riżā* 1:

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[59] The information presented in this paragraph summarizes Lévy (2011, 103, 104), including the footnotes.

[60] Vatican, Biblioteca Apostolica, MS Ebr. 384, ff. 412r–412v.

[61] Berlin, Staatsbibliothek, MS Heb. 204, ff. 156r–157r; Hamburg, Staats- und Universitätsbibliothek, MS Levy 113, ff. 104r–105r.

כל עגולה הנה שטחה שוה לשטח הנצב הזויות אשר אחת מצלעותיו המקיפים בזוית הנצבת שוה
לחצי קוטרה והצלע האחר שוה לקו המקיף בה (Lévy 2011, 124.4–6)

and

كلّ دائرة فإنّ بسيطها كالمثلّث القائم الزاوية الذي أحد ضلعيه المحيطين بالقائمة كنصف قطرها
والآخر كمحيطها.

These two enunciations have the following similarities to the enunciation in the *Fatih* version, in distinction to the *Columbia* version: First, they follow the radius-perimeter order in stating the equalities for the legs of the triangle, and second, the words and expressions used for the right-angled triangle are definite. They differ from the *Fatih* and *Columbia* versions in that they both have words to denote areas, even though these words do not correspond to each other phonetically (Arabic *basīṭ*, Hebrew שטח) and only one instance is used in *Riżā* (for the circle) where **HB** uses two (one for the circle and one for the triangle). However, at the end of the proof of *MC* 1 in the *Riżā* version, we see the Arabic *saṭḥ* used (*fa-saṭḥ al-dā'ira ka-saṭḥ al-muthallath*), which corresponds phonetically to the Hebrew שטח.

The instantiation of *MC* 1 in **HB** has the following similarities to the instantiations in both the *Columbia* and *Riżā* versions: **HB** and *Riżā* agree in introducing the circle *ABGD* followed by an identification of its center *E*; in contrast, *Fatih* and *Columbia* both introduce the center further into the proof. Second, both **HB** and *Columbia* describe the right-angled triangle by its three vertices and they specify at which one the right angle is located, whereas *Fatih* refers to the right-angled triangle by one letter *E* and *Riżā* does not refer to it by any letter at all.

I.3.5 The Latin Translations of Measurement of the Circle

Approximately one century before William of Moerbeke (b. ca. 1220–1335; d. before 1286) translated many of the works of Archimedes from Greek into Latin, *MC* had already been translated twice into Latin from Arabic. These translations, both of which are closely related to the *Fatih* version, have been edited by Clagett (1964, 15–58).[62] The first translation, which is anonymous, and extant in three manuscripts,[63] has been conjectured by Clagett (1964, 17) to have been made by Plato of Tivoli (fl. first half of the 12th century). Clagett's reason for suggesting that Plato of Tivoli was the translator of this translation, which shall henceforth be denoted **LP**, is the

[62] Much of what is presented below summarizes Clagett's arguments.

[63] Paris, Bibliothèque Nationale, Lat. 11246, ff. 37v–39r; Paris, Bibliothèque Nationale, Lat. 7224, ff. 63r–65r; Dublin, Trinity College, D.2.9, ff. 54r–55r. The second and third manuscripts are copies of the first.

fact that the text of **LP** follows Plato's translation of the *Liber Embadorum* of Abraham bar Ḥiyya from the Hebrew in the main manuscript (Bibliothèque Nationale, Lat. 11246).

LP contains Latin translations of an Arabic text closely related to *Fatih* 1, *Fatih* 2, and the first half of *Fatih* 3. Clagett (1964, 17) points out that there are numerous errors in the rendering of numbers in Proposition 3; he is quick to add that these could be due to a scribe rather than to the translator.[64] While an examination of Clagett's critical apparatus shows many such instances to be simple scribal errors indeed, a few are due rather to the deficiencies of the numbers in *Fatih* 3 itself. The most obvious of these are the numbers 349450 and 23409 in the first half of Proposition 3. Bibliothèque Nationale, Lat. 11246, and following it, the other two manuscripts, do not have the multiples of 10000 in these two numbers. Clagett (1964, 26.87) corrects these numbers, but, as explained above,[65] the absence of the multiples of 10000 in these two numbers is due to errors in the transmission of the *Fatih* version and not to a scribal error. The third number greater than 10000 in the first half of Proposition 3, namely 14688, is correctly rendered, as it is in the *Fatih* version. Another such example is the absence of *et unius octave* in 591 1/8 (Clagett 1964, 26.89). The fraction 1/8 is also absent in the two Arabic manuscripts of the *Fatih* version. The Arabic copy with which Plato worked, then, possibly had a common ancestor with these two Arabic manuscripts.

The argument by Clagett (1964, 30–31) that the second translation, which is also anonymous on all extant manuscripts, is in fact by Gerard of Cremona (ca. 1114–1187), is convincing. This translation will henceforth be denoted **LG**. His argument is based on the presence of **LG** in a manuscript dedicated to Gerard's works, terminological similarities between **LG** and other works of Gerard, and the mention of an item *Archimenidis tractatus I* in a document, written after Gerard's death by some of his associates, containing the list of his translations.[66] Since Gerard is not known to have translated any other work of Archimedes, this reference is likely to *MC*. Judging by the relatively high number of extant manuscripts (twelve), **LG** seems to have been much more popular than **LP**; possibly this was due to Gerard's

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[64] On the same page, Clagett claims that the Arabic version of *MC* exists only in the version of Naṣīr al-Dīn al-Ṭūsī, based on his examination of a number of manuscripts of the Arabic text. Since manuscripts of al-Ṭūsī's *taḥrīr* vastly outnumber the manuscripts of the *Fatih*, *Columbia*, and *Riżā* versions, it is not surprising that he should have come to this erroneous conclusion. In any case, since al-Ṭūsī himself was a practicing mathematical scholar, the numbers in his *taḥrīr* are correct. Perhaps it was this that led Clagett to ascribe the errors in the numbers in **LP** to a scribe rather than the translator.

[65] See Section I.3.1.

[66] For a recent edition of this text, see Burnett (2001). The item in question is the sixth, whose title is read by Burnett as *Liber Archimedis tractatus .i.*

prestige as a translator. This popularity is also indicated by the fact that several other Latin texts about the circle quadrature problem composed during the Middle Ages took **LG** as a source. **LG** contains translations of all three propositions of the *Fatih* version.

A comparison of numbers in the first half of Proposition 3 in *Fatih* and **LG** leads us to temper Clagett's (1964, 31) judgement about the "accuracy regarding numbers" of **LG**. While Clagett never makes this explicit, it may be surmised that a major factor in his assessment is the correct rendering of the numbers 349450 and 23409. As I have argued above, these numbers must have appeared as 9450 and 3409 in *Fatih* 3.[67] The fact that they appear correctly in **LG**, then, must be due to a correction, either by Gerard himself or by someone else in either the Latin or Arabic tradition. Indeed, anyone who had studied the proof of Proposition 3 and was competent in arithmetic would have been able to compute the correct forms of the numbers, since $349450 = 571^2 + 153^2$ and $23409 = 153^2$.

II Description of the Manuscripts

I have obtained the list of manuscripts to use from Sezgin (1974, 131). Of the seven manuscripts listed by him, two (Esat 2034 and Sipahsālār 690) contain the text of Naṣīr al-Dīn al-Ṭūsī's *taḥrīr* of *MC* and they have not been taken into consideration. I have also been unable to obtain a copy of a third manuscript (Leningrad GPB 144). The remaining four manuscripts are described below according to the versions they were used to establish.

II.1 The *Fatih* Version

The *Fatih* version was established using the following manuscripts:

*F: İstanbul, Süleymaniye Manuscript Library, Fatih 3414, 1286 (*AH *684)*

Since I have described this manuscript in some detail before (Coşkun 2018, 61–63), I shall give only a summary here. This carefully written and drawn manuscript of 75 folios was copied by Muḥammad ibn ʿUmar ibn Abī Jarāda (henceforth Ibn Abī Jarāda), who lived in the 13th century (AH 7th century).[68] It contains the *Fatih* version of *MC* (ff. 2v–6v), an Arabic translation of *On the Sphere and the Cylinder*, part of an Arabic translation of Eutocius's commentary on *On the Sphere and the Cylinder*, and finally, an Arabic translation of a work titled *Ma'khūdhāt Mansūba ilā Arshimīdis*. The colophons give the years of copying as 1277 (AH 676) for *On the Sphere and the Cylinder* and as 1286 (AH 684) for Eutocius's commentary on *On the*

[67] See Section I.3.1.

[68] See the entry about him in Suter (1900, 158, no. 385).

Sphere and the Cylinder and *Ma'khūdhāt Mansūba ilā Arshimīdis.* The colophon for the *Fatih* version does not give a date.

The title for the *Fatih* version is written on top of f. 2v with large letters in red ink. The propositions are numbered using the Arabic *abjad* system, again with large letters in red ink. Proposition 3 is mistakenly divided into two propositions, according to the two halves of the proof.[69] There is one scholium on f. 6r, written in the same hand as the text but with red ink. There is water damage affecting mostly, but not exclusively, the bottom parts of the pages.

H: *Bursa, İnebey Manuscript Library, Haraççıoğlu 1174, possibly 14th century*

This manuscript that contains 47 folios with 23 lines per page probably dates from the 14th century.[70] The folio numbers are written at the upper left corners of the rectos, once with Arabic positional numerals and once with modern Western numerals. However, there is a difference between the two numerations, with the Arabic positional numbers running from 98 to 144 and the modern Western numerals running from 1 to 47.[71] In addition to these, there is another folio at the beginning of the manuscript that is marked as "8" in Arabic positional numerals. This indicates that a chunk of the manuscript with 89 folios dropped just after this folio and another chunk of 7 folios dropped from just before it.

The text and the diagram letters are written in one hand in a readable *naskhī* with brown ink, with pointing often provided. The diagrams are carefully drawn and there are no empty spaces in which a diagram should have been drawn but was not. Individual propositions are not numbered. Rather, the subdivisions of the text, including the beginnings of propositions, are marked with a purple bar over the first few words. Occasionally, some letters have been retraced, and some corrections

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[69] See note 119.

[70] In his list of manuscripts of *MC*, Sezgin (1974, 131) reports that this manuscript dates from the 6th century AH, referring to Ritter (1950, 102). Ritter (1950, 102–103) in turn reports that a manuscript named "Haraççızade, Heyet ve Hikmet 22" and containing *MC* dates from the 8th century AH. According to Ritter (1950, 103), this "Heyet ve Hikmet 22" contains 144 leaves, which suggests very strongly that it is none other than Haraççıoğlu 1174, since this latter is also numerated up to the number 144 by Arabic positional numerals (see below). It might be conjectured that sometime between 1950 and 1974 large chunks of "Heyet ve Hikmet 22" were lost and then the remainder was simply called "Haraççıoğlu 1174" and renumerated with Western numerals. However, I was unable to obtain positive confirmation of this in my communication with the İnebey Manuscript Library staff. Even though Ritter (1950, 103) does not make clear why he dates "Heyet ve Hikmet 22" to the 8th century AH, possibly he obtains this information from a colophon in the now lost parts of the full collection of 144 leaves. Therefore, I shall provisionally use the date of 8th century AH, or the 14th century, as the date of **H**.

[71] The "1" in Western numerals is not written but is inferred from the "2" on the next recto.

made, with a pen with a thicker nib and with black ink; as far as I can tell, these are in the same hand as the main text.

The works in the manuscript are as follows:[72]

1. (*Fatih*) *Kitāb Arshimīdis fī misāḥat al-dāʾira*: ff. 1v–4r. There is neither a title nor a colophon for this text. The identification of the text as a copy of the *Fatih* version is on the basis of a comparison with the corresponding text in **F**.
2. *Maqālatā*[73] *Arshimīdis fī al-kura wa-l-usṭuwāna*: ff. 4v–47r. The title is written in the same way as the surrounding text. Just below the title is the expression *iṣlāḥ Thābit ibn Qurra* ("correction of Thābit ibn Qurra"). The two colophons for this text (at the end of the two books) carry neither dates nor names of copyists.

II.2 *Columbia Preliminaries* and the *Columbia* Version

The *Columbia* version is preceded by another text, which I call *Columbia Preliminaries* in this article and which consists of four preliminary propositions. *Columbia Preliminaries* and the *Columbia* version were established using the following manuscript:

C: *New York, Columbia University Rare Book and Manuscript Library, Or. 45, possibly 13th or 14th centuries*

Since a detailed description of this manuscript, which possibly dates from the 13th or 14th century, can be found online, I shall give only the details relevant to *Columbia Preliminaries* and the *Columbia* version.[74] Most of the manuscript, including the two texts edited in this article, is written in the same hand in a readable *naskhī*

[72] In the folio numbers in what follows, I use the Western numerals at the upper left corners. The reader should be aware that other authors, such as Sezgin (1974, 129), use the Arabic positional folio numbers.

[73] Written *maqālatay* in the manuscript.

[74] A detailed description and images of the manuscript are made available online, by the University of Pennsylvania Libraries, at https://openn.library.upenn.edu/Data/0032/html/ms_or_045.html (accessed on 24 July 2023). Since the folios themselves are unnumbered, I have used the folio numbers assigned to the images of the individual folios on that web page. The date of the manuscript is estimated to be in the 13th or 14th century based on the paper and writing; in any case, no author whose works are in this manuscript lived later than the early 13th century.

with brown ink.[75] Pointing is often provided. The diagrams are carefully drawn and there are no empty spaces left for diagrams.

Following Knorr (1989, 543–546, 552–576), I have edited *Columbia Preliminaries* and the *Columbia* Version from the following two texts:

1. (*Columbia Preliminaries*) *Ashkāl nāfiʿa fī kitāb Arshimīdis*: ff. 24r–25r. The title is written at the first line of the text and in the same way as it. The colophon carries neither a date nor the name of the copyist. Only two propositions in this text are numbered with the Arabic *abjad* system.[76]
2. (*Columbia*) *Qawl mansūb ilā Arshimīdis fī misāḥat al-dāʾira*: ff. 25r–30v. The title is written at the first line of the text and in the same way as it. The colophon carries neither a date nor the name of the copyist. The proposition numbers, which are not consistently given, are written as Arabic *abjad* numbers, either in the text or next to the diagrams.[77] For *Columbia* 4 (corresponding to the first half of *Fatih* 3) there are eight scholia; all but the first are written in a different hand ($\mathbf{C^2}$) and in darker ink. For *Columbia* 4 and *Columbia* 5, a third hand ($\mathbf{C^3}$) marks certain parts of the text as interpolations. It therefore seems that one scribe carelessly copied some marginal notes in the exemplar into the main text and another scribe then tried to correct this by crossing these parts out. Numbers in this text are written variously as lexical numerals and Hindu-Arabic numerals.[78]

II.3 The *Riżā* Version

The *Riżā* version was established using the following manuscript:

R: *Mashhad, Central Library of Āstān-i Quds-i Riżavī, 5634, date unknown*

This manuscript contains six folios, with 21–23 lines per page. The folio numbers are written at the upper left corners of the rectos with Arabic positional numerals, except for the first folio, which contains no folio number. In addition, the pages are

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[75] The online description of the manuscript (see note 74 for the link), claims that the manuscript is "copied in the same hand." However, as Rashed and Papadopoulos (2017, 400) point out, the first treatise in the collection, a fragment of a translation of Menelaus's *Spherics*, is written in a different hand. In addition, the notes starting from f. 129r are written in different hands.

[76] They are the second and the fourth, marked with *bāʾ* and *dāl*. The reader should be aware that my numeration of the propositions of *Ashkāl nāfiʿa fī kitāb Arshimīdis* differs from Knorr's (1989, 552–554). See notes 148 and 155.

[77] There is no numbering for *Columbia* 1. For *Columbia* 2, only the diagram is marked with a number.

[78] For more information on the ways in which numbers are written in *Columbia*, see Section I.3.2.

numbered at the middle of the bottom margins with Arabic positional numerals, starting from f. 1v.

The text and the diagram letters are written in one hand in *nastaʿlīq* with black ink, with pointing often provided. However, there is a tendency for the pointing to become sparse toward the end of the text of the *Riżā* version. The diagrams are carefully drawn and there are no empty spaces left for diagrams. Individual propositions are not numbered with the Arabic *abjad* system and there is no other mechanism to indicate where one proposition ends and the next one begins. There are no scholia.

The works in the manuscript are as follows:

1. (*Riżā*) *Risālat Arshimīdis fī misāḥat al-dāʾira wa-nisbat muḥīṭihā ilā quṭrihā wa-nisbat basīṭihā ilā murabbaʿ quṭrihā*: ff. 1v–3v. The title is written at the first line of the text and in the same way as it. The colophon carries neither a date nor the name of the copyist. There is no proposition numbering. Numbers in this text are written in various forms: as lexical numerals, Hindu-Arabic numerals, and sexagesimal numerals with the *abjad* system. In the sexagesimal system, zeroes in sexagesimal places are written in a variety of forms, some of which are reproduced as color images below.
2. *Risālat Arshimīdis fī al-khiffa wa-l-thiql*:[79] ff. 4v–5r. The title is written at the first line of the text and in the same way as it. The colophon carries neither a date nor the name of the copyist.
3. *A fragment of an untitled treatise*: ff. 5v–6v. Since the treatise starts with "He said: Weight is the comparison of lightness and heaviness with each other using the balance" (*qāla al-wazn huwa qiyās al-khiffa wa-l-thiql baʿḍihā ilā baʿḍ bi-l-mīzān*), the subject is mechanics. The abrupt ending of the text shows that this is a fragment. There is no colophon.

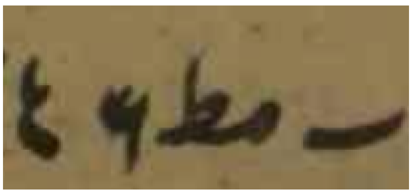

Figure 1: 2,49,0,0. Taken from **R** 2v.

[79] For an uncritical edition of this text made from a Parisian manuscript, see Zotenberg (1879); this was translated into English by Clagett (1959, 52–55). Another translation, this time into German, was made from a manuscript in Gotha by Wiedemann (1906).

Figure 2: Two examples of 16,0,0. Taken from **R** 2v and 3r, respectively.

III Editorial Principles

III.1 Text

Since the manuscripts used for establishing the Arabic texts are inconsistent in their use of diacritical pointing, I have corrected missing or erroneous diacritical pointing in the manuscripts silently whenever the readings of the words involved are clear from the context (both mathematical and grammatical), which is most often the case.

The lack of diacritical pointing frequently leaves one in doubt about the person, number, and gender of imperfect verbs. As to gender, I have harmonized the gender of third person singular imperfect verbs with the gender of their subject. For imperfect verbs that take an object, which are typically used for geometrical constructions, my choice has been to put the verb in the first person plural since perfect verbs with objects tend to be in the first person plural in the texts, and there is no reason to suppose that imperfect verbs would follow a different pattern.[80]

In general, in cases where a word cannot be read unambiguously, the context does not remove the ambiguity, and the principles stated above do not apply, the correct reading must either be determined from other manuscripts or be conjectured. I have indicated conjectures concerning diacritical pointing (“read.”), vocalization (“voc.”), or the consonantal skeleton (“corr.”) in the critical apparatus. In these cases, I have recorded what I see in the manuscripts exactly (that is, with no implicit correction of diacritics, as opposed to the greater number of entries in the critical apparatus),[81] together with a superscript asterisk with the siglum of the manuscript (for example, $\mathbf{F}^*$).

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[80] Another clue is given by the frequent occurrence of the imperfect forms of the verb *waṣala*, used for joining two points by a line segment, where the consonantal skeleton does not include a *wāw*, thus ruling out the third person singular passive and leaving *naṣilu* as the only plausible reading.

[81] The reader should be aware that in addition to the usual diacritical pointing and *ḥarakāt*, **F** often uses a sign resembling a check mark, to distinguish *sīn* from *shīn* and *rāʾ* from *zāʾ*. I have used the Unicode sign U+065A (ٚ ,“Arabic Vowel Sign Small V Above”) to render this sign in the critical apparatus.

In cases where the text cannot be read in the manuscript due to physical damage,[82] or illegible consonantal skeleton, this is indicated in the critical apparatus ("illeg.").[83] If an illegible word or words in one manuscript can be read in other manuscripts, no editorial intervention is necessary; these are simply noted in the critical apparatus. Otherwise, the text must be restored; this is noted in the text by curly brackets.

In preparing the editions and translations of the *Fatih* and *Columbia* versions, I have used Knorr's (1989) work extensively. In many cases, when it is clear that the readings of the Arabic manuscripts are faulty, he translated the text using what he thought must be the correct reading, and he explained some of these in his footnotes.[84] I have sometimes followed him when emending the Arabic text, and I have pointed this out in the critical apparatus with his name in parentheses.[85] I have also discussed some of the major points of agreement or disagreement with him in footnotes to the translations, noting the footnote number in the critical apparatus.

I have not reported a number of minor faults in the manuscripts such as variations in spelling (when the intended word is clear), minor damages to the manuscript where the word is still legible, and overflows of a line into the left margin. I have similarly not reported words at the bottom of pages that replicate the first word on the following page. I have also standardized the spelling of number words, where, for example, the omission of long vowel *alif* in the number words is especially common in the manuscripts.

I have reported the manuscript readings of sexagesimal numbers in the critical apparatus only in cases of significant errors involving the shapes of the letters. I have reproduced the various signs to denote empty sexagesimal places with the zero numeral (•) without reporting the signs in the critical apparatus.

Some entries in the critical apparatus are discussed in footnotes in the translation; these entries contain the relevant footnote numbers.

Folio numbers have been indicated in the margins to the Arabic text. Of the pious invocations, only the *basmala* has been included in the Arabic texts.

I have divided the *Fatih* version into three propositions, following the extant Greek text of *MC*.[86] I have similarly divided the *Riżā* version into three propositions.

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[82] This is especially common in Fatih 3414 (**F**). See Section II.1.

[83] In cases of illegibility, I have not indicated which characters in a word are illegible.

[84] However, the reader should be aware that he was not entirely consistent in pointing out when his translations supposed a reading different from that in the manuscripts.

[85] One exception to this rule is imperfect verbs, which he tended to read in the same way as I do. I have not pointed these out to avoid encumbering the critical apparatus.

[86] See Section I.1 for the division of the Greek text into propositions.

For the *Columbia* version, I have followed the letters used in the manuscript for the proposition numbering.[87]

Two more editorial interventions have been made for the sake of readability. First, I have split the text into paragraphs. In doing this, I have followed Heiberg's (Heiberg 1972, I.232–243) paragraph divisions as much as possible.[88] Second, I have punctuated the texts. Most of the punctuation signs used correspond to coordinating conjunctions such as *wa-* and *fa-*.[89]

III.2 Translation

I have tried to strike a balance between literalness and readability by translating the technical terms as literally and consistently as possible, but I have used idiomatic English in translating Arabic sentence structures. Whenever I added English words for the sake of producing a readable translation, I have enclosed these words in square brackets. As an extension of this practice, I have in many cases refrained from translating Arabic suffixed pronouns literally; instead, I have added the words to which these pronouns refer, when they were clear from the context, in square brackets. For example, the feminine pronominal suffix in *muḥīṭihā* might refer to a circle (*dāʾira*; feminine in Arabic), but translating that literally as ungendered "its" would have lost that reference and would have been confusing to the English reader. In that case, instead of "its perimeter," I translate "the perimeter of [the circle]."

I have translated Arabic numbers, regardless of how they are written in the manuscripts (with number words, Arabic *abjad* numerals, Arabic positional numerals, or mixed sexagesimal-decimal numerals), and fractions, with modern Western numerals. I have followed the convention of dividing sexagesimal places with commas and denoting the sexagesimal point with a semicolon in the translation. Diagram letters are translated according to the correspondence in Table 2.

[87] This makes my numbering the same as Knorr's (1989, 552–561) with one minor difference. See note 155.

[88] See Netz (2012, 191–195) for a discussion of the differences between Heiberg's layout and the layout of the Byzantine manuscripts of the works of Archimedes.

[89] Al-Dallāl (1997, 90) argues for using punctuation in the edition of Arabic scientific texts, on the grounds that since medieval Arabic manuscripts do not have punctuation, and the modern languages into which they are translated do, the readings adopted by the editor may depend on how one places the punctuation marks. A good example of this is provided by the emendation of *wa-lladhī* to *fa-lladhī*, and the placement of a period right before that emendation, toward the end of *Fatih* 1. It would certainly be possible to use a comma and keep the *wa-lladhī* but this would have made for an excessively long sentence and a less smooth reading.

Footnotes in the translation are for mathematical clarifications,[90] explanations of difficult choices in the translation, explanations of textual difficulties, discussion of important agreements and disagreements with or criticism of Knorr (1989), and explanations that are pointed to in the critical apparatus. The footnotes to the proposition numbers give the folio and line ranges of the propositions in the manuscripts; these ranges do not include the lines for titles, pious invocations, or colophons.

Punctuation of the translations generally follows that of the Arabic texts; on occasion, I have used extra commas in the translations to produce a smoother reading. Paragraph divisions of the translations follows that of the Arabic texts as well.

III.3 Diagrams

Diagram letters in Arabic scientific manuscripts are often inconsistently pointed just as the text is. However, in the case of the present texts, the diagram letters can, for the most part, be clearly read, in view of the following considerations: First, the Greek text provides clues as to what the diagram letters should be. Second, even where a diagram letter does not correspond to a Greek letter, it can often still be read if one considers the *abjad* order.[91] Accordingly, it is possible to adopt a minimalist policy on the reporting of variations in diagram letters in the critical apparatus, in much the same way as for the text. Exceptions to this policy include, first, where the skeleton of the letters is in question, and second, where the stacking of letters (especially *jīm* and *ḥāʾ*) and the unclear placement of dots makes an unambiguous reading difficult. In such cases, mathematical sense and the use of the letters in the text have dictated the reading adopted in the Arabic text, and the manuscript readings have been recorded in the critical apparatus.

Diagram captions in the Arabic texts report differences between the established diagram and the manuscript diagrams, and differences between manuscript diagrams, where applicable. They also report uncertainties in reading letters, in both the manuscript diagrams and the text.

For the *Fatih* version, manuscript diagrams in **F** provided the basis for the diagrams established in the text and translation. The diagrams in the texts have generally been put at the end of the relevant text blocks.

[90] In propositions where a sequence of steps is used more than once I have, for brevity, avoided repeating the mathematical explanations.

[91] Most letters in the manuscript diagrams are taken from the beginning of the *abjad* sequence, which makes the difference between the Western and Eastern variants much less significant.

III.4 Scholia

Scholia to a version are found after the text and translation, with pointers to scholia provided in both the critical apparatus and the translation. Footnotes to the scholia numbers indicate the folio and location of the scholia. For the *Columbia* version, whose scholia have been translated by Knorr (1989), I have generally used the same points in the text and translation for the pointers of the scholia as his choices are correct.

III.5 Transliteration of the Names of Geometrical Points

Table 2: Transliteration of Arabic Letters Denoting Geometrical Points

Arabic	English	Arabic	English
ا	A	س	S
ب	B	ع	Q
ج	G	ف	F
د	D	ص	U
ه	E	ق	C
ز	Z	ر	R
ح	H	ش	O
ط	T	ت	P
ي	I	ث	Y
ک	K	خ	X
ل	L	ظ	Ẓ
م	M	ض	Ḍ
ن	N		

The Arabic letters ذ, غ, and و have been omitted from this table since they do not occur in the geometrical diagrams of the Arabic texts.

IV Texts and Translations

Abbreviations Used in the Critical Apparatus

corr.	editorial correction to the consonantal skeleton
illeg.	partly or completely illegible (with the reason in parentheses)
(dam.)	physical damage to the manuscript
(skel.)	illegible skeleton
(Knorr)	changes suggested by Knorr's (1989) translations
mg.	margin
om.	omitted
read.	editorial reading of a word by supplying pointing
sup.	above the line
voc.	editorial vocalization of a word by supplying vowel signs

+	When a manuscript reading has to be broken apart (generally due to parts of it being written above the line), a plus sign is used. What comes after the plus sign is at the same spot on the manuscript as what comes before it.
⟨ ⟩	editorial addition
{ }	editorial restoration
†	Obeli indicate corrupt text that could not be emended. A single obelus is used before one corrupt word; two obeli enclose text where corruption is suspected.

Sigla

F	İstanbul, Süleymaniye Manuscript Library, Fatih 3414, 1286 (AH 684)
F^{i}	same hand, different ink
H	Bursa, İnebey Manuscript Library, Haraççıoğlu 1174, possibly 14th century
H^{i}	same hand, different ink
C	New York, Columbia University Rare Book and Manuscript Library, Or. 45, possibly the 13th or 14th centuries
C^{2}	second hand
C^{3}	third hand
R	Mashhad, Central Library of Āstān-i Quds-i Riżavī, 5634, date unknown

F*	A superscript on a siglum indicates an exact manuscript reading (that is, with no implicit correction of diacritics). (Used only with “read.,” “voc.,” or “corr.”)

IV.1 The *Fatih* Version

F 2v, H 1v

بسم الله الرحمن الرحيم

كتاب أرشميدس في مساحة الدائرة

١

كلّ دائرة فإنّها مساوية للمثلّث القائم الزاوية الذي أحد ضلعيه المحيطين بالزاوية القائمة مساوٍ لنصف قطر الدائرة والضلع الآخر منهما مساوٍ للخطّ المحيط بالدائرة.

فلتكن دائرة ا ب ج د قد ساوت مثلّث ة في الأشياء التي ذكرناها آنفًا في الخبر. فأقول إنّ مساحتها مساوية لمساحته.

فإن لم يكن كذلك فإنّ الدائرة أعظم أو أصغر أصغر منه. فلتكن أوّلًا أعظم منه. ونعمل في الدائرة مربّع ا ج. فقد انفصل من دائرة ا ب ج د أعظم من نصفها، وهو مربّع ا ج. ونقطع قوس ا ب ونظائرها من القسيّ بنصفين نصفين على نقطة ف ونظائرها من النقط. ونصل ا ف ف ب ونظائرهما. فقد انفصل أيضًا من بقيّة قطع دائرة ا ب ج د أعظم من نصفها، وهو ا ف ب ونظائره. فإذا فعلنا ذلك على ما يتلو، فسوف تبقى قطع هي أصغر من مقدار زيادة الدائرة على مثلّث ة. فالشكل حينئذٍ المستقيم الخطوط الكثير الزوايا الذي تحيط به الدائرة هو أعظم من المثلّث. ونجعل مركز الدائرة ن، ونخرج عمود ن س. فخطّ ن س أقلّ من أحد ضلعي المثلّث المحيطين بالزاوية القائمة، ومحيط الشكل الكثير الزوايا أقلّ من الضلع الباقي منهما، لأنّه أيضًا أقلّ من الخطّ المحيط بالدائرة. فالذي يكون من ضرب أحد ضلعي المثلّث المحيطين بالزاوية القائمة في الآخر، وهو ضعف ⟨تكسير⟩ المثلّث، أكثر من المجتمع من ضرب ن س في محيط الكثير | الزوايا، وهو ضعف تكسير الكثير الزوايا. وأنصاف ذلك أيضًا كذلك. F 3r
فالمثلّث أعظم من الكثير الزوايا، وقد كان أصغر منه. هذا خلف لا يمكن.

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2 كتاب أرشميدس في مساحة الدائرة] H om. 4 لنصف] F illeg. (dam.) 5 ذكرناها] ذكرنا F 7 أصغر منه] H om. 9 ا ب ونظائرها] F illeg. (dam.)، ا ف ب بنصفين في نظائرها H نصفين] H om. 10 ف ب] وب H ونظائرهما] ونظائرها FH 11 ونظائره] ونظائرها H ذلك] كذلك F 16 فالذي] والذي FH 17 وهو] هو H ضعف] F illeg. (dam.) المثلّث] F illeg. (dam.) 19 الكثير] كثير H

ولتكن أيضًا الدائرة أصغر من مثلّث ه إن كان يمكن ذلك. ونخطّ عليها مربّعًا يحيط بها، وهو مربّع ع ق. فقد انفصل من مربّع ع ق أكثر من نصفه، وهو الدائرة. ونقسم قوس ب ا بنصفين على ف، | **H** 2r ونظائرها من القسيّ بنصفين نصفين، ولتمرّ بنقط الأقسام خطوط مماسّة للدائرة. فخطّ ر ط قد انقسم بنصفين على نقطة ف، وخطّ ن ق عمود على ر ط، وكذلك نظائره من الخطوط. ولأنّ ق ر وق ط أعظم من ط ر، يكون نصفهما أعظم من نصفه. فخطّ ق ط أعظم من ط ف الذي هو مثل ط ب. فمثلّث ق ف ط أعظم من نصف مثلّث ق ف ب، وبأكثر من ذلك يكون أعظم من نصف شكل ق ف ي ب الذي تحيط به خطّا ب ق ق ف وقوس ب ي ف. وكذلك يكون مثلّث ق ف ر أعظم من ف ص ا ر. فجميع ط ق ر أعظم من نصف شكل ا ص ف ي ب ق، وكذلك تكون نظائره من المثلّثات أكثر من النصف من نظائر القطع الأخرى. فإذا فعلنا ذلك فيما يتلو، فستبقى قطع تفضل على الدائرة، وتكون إذا اجتمعت أقلّ من زيادة مثلّث ه على دائرة ا ب ج د. فلتبق قطعة ف ر ا ونظائرها من القطع. فالشكل حينئذٍ المستقيم الخطوط الذي يحيط بالدائرة أصغر من مثلّث ه. هذا غير ممكن لأنّه أعظم منه، وذلك أنّ ن ا مساوٍ لعمود المثلّث، ومحيط الشكل الكثير الزوايا أعظم من ضلع المثلّث الآخر الذي يحيط بالزاوية القائمة، لأنّه أعظم من الخطّ المحيط بالدائرة. فالذي يكون من ضرب ا ن في محيط الكثير الزوايا | **F** 3v أعظم من ضرب أحد ضلعي المثلّث المحيطين بالزاوية القائمة في الآخر. فليست الدائرة بأصغر من مثلّث ه. وقد تبيّن فيما تقدّم أنّها ليست بأعظم منه. فدائرة ا ب ج د إذًا | **H** 2v مساوية لمثلّث ه.

وأيضًا فإنّ مساحة مثلّث ه مساوية للذي يكون من ضرب عموده في نصف قاعدته، وعموده مساوٍ لنصف قطر دائرة ا ب ج، وقاعدته مساوية لمحيط دائرة ا ب ج. فالذي

[2–1] مربّعًا يحيط بها، وهو] .om **H** [3] قوس] .om **H** ولتمرّ] .read، وَلتمرّ **F***، وليمر **H*** [4] خطوط] خطوطًا **H** [5] وكذلك] وكذلك أيضًا **H** ط ر] .illeg. (skel.) **H** [6] نصفهما] نصفها **H** [7] ق ف ب] .illeg. (dam.) **F** [8] نصف] .om **H** ق ف ي ب] .illeg. (skel.) **Hⁱ** خطّا] .illeg. (dam.) **F** ب ق ق ف] ب ط ط ف **Hⁱ** [9] ط ق ر] .illeg. (skel.) **Hⁱ** [10] اص ف ي ب ق] اص ف ب ي ق **H** نظائره] نظائره من نظائره **H** [11] الأخرى] الأخر **FH** ذلك] كذلك **F** [12] اجتمعت] جمعت **H** فلتبق] فلتبقى **H** [14] ن ا] ر ا **H** [16] فالذي] والذي **F**، الذي **H** ان في محيط] .illeg. (dam.) **F**، ان في محيط الدائرة **H** [17] بأصغر] أصغر **H** [21] ا ب ج] ا ب ج د **H**، see note 102 مساوية] لمساوية **H** ا ب ج] ا ب ج د **H**، see note 102

يكون من ضرب نصف القطر في نصف الخطّ المحيط بدائرة اب ج مساوٍ لتكسير مثلّث ه. وذلك ما أردنا أن نبيّن.

ومن أجل ذلك يكون ضرب نصف القطر في نصف قطعة من المحيط هو تكسير الشكل الذي تحيط به تلك القطعة والخطّان المخرجان من طرفي القطعة إلى المركز.

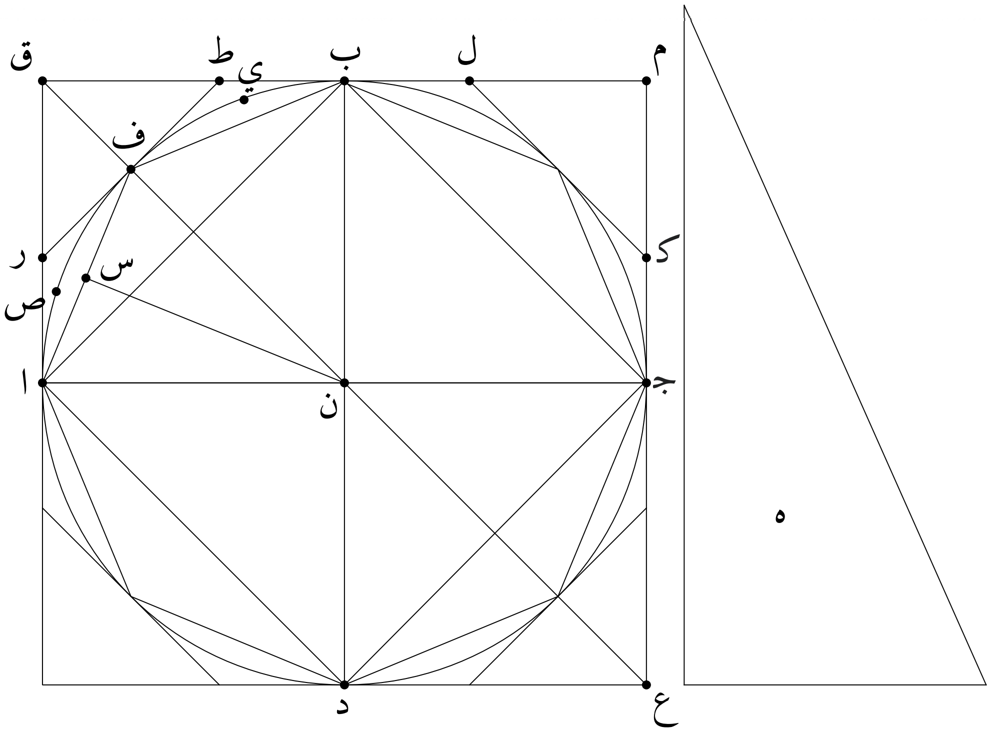

Figure 3: Diagram for *Fatih* 1. **F:** *Yāʾ* is written without dots in the diagram. There is a line of text at the bottom of the diagram but it cannot be read due to water damage. **H:** *Yāʾ* is written without dots in the diagram and in the text. *Rāʾ* is written as *zāʾ* in the diagram and the text. *Kāf* in the diagram is indistinct but looks like a *ṭāʾ*. *Ṭāʾ* is written as *ẓāʾ* in the diagram. Finally, *sīn* is misplaced in the diagram—at the intersection of the perpendicular to the side of the polygon and the line *alif bāʾ*—and it is sometimes written as *shīn* in the text.

F 4r **ب** نسبة تكسير كلّ دائرة إلى مربّع قطرها كنسبة الأحد عشر إلى الأربعة عشر.

فليكن خطّ اب قطر الدائرة، ولنعمل عليه مربّع ج ح، وليكن د ج نصف خطّ د ه، وليكن خطّ ه ز سبع ج د. فلأنّ نسبة مثلّث ا ج ه إلى مثلّث ا ج د كنسبة

[1] اب ج] اب ج د H، see note 102 [4] والخطّان المخرجان] والخطّين المخرجين H المركز] المركز وذلك ما أردنا أن نبيّن H

الأحد والعشرين إلى السبعة، ونسبة اجد إلى اهز كنسبة السبعة إلى الواحد، فلذلك تصير نسبة مثلّث اجز إلى مثلّث اجد كنسبة الاثنين والعشرين إلى السبعة. ولكنّ مربّع جح أربعة أضعاف ادج، ومثلّث اجز مساوٍ لدائرة اب، لأنّ عمود اج مساوٍ للخطّ الذي يُخْرَجُ من مركز هذه الدائرة إلى الخطّ المحيط بها، وقاعدة جز مساوية للخطّ المحيط بها، لأنّ الخطّ المحيط بالدائرة أكثر من ثلاثة أضعاف قطرها بسبع القطر بالتقريب. فقد وضح ممّا قلنا أنّ نسبة دائرة اب إلى مربّع جح كنسبة الأحد عشر إلى الأربعة عشر. وذلك ما أردنا أن نبيّن.

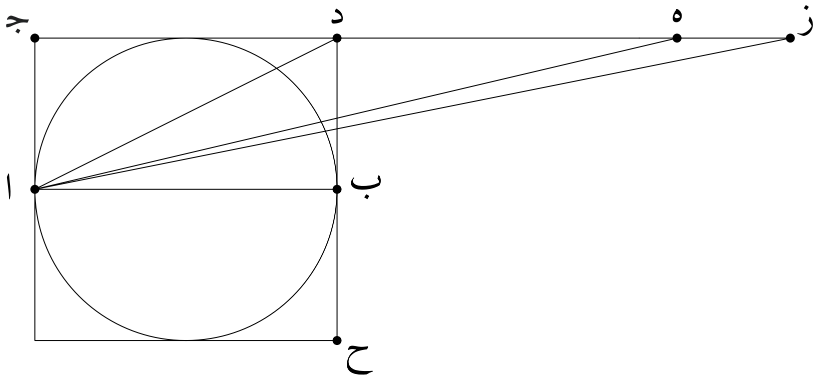

Figure 4: Diagram for *Fatih* 2. **H:** In **H**, the diagram above appears rotated by 180 degrees about the center of the circle. *Dāl* is placed between the corner of the square and *hāʾ*; the corner of the square is then labeled *ṭāʾ*. *Zāʾ* resembles a *nūn* in the diagram and it is often written without a dot in the text.

ج كلّ خطّ محيط بدائرة فإنّه زائد على ثلاثة أضعاف قطرها بأقلّ من سبع القطر وبأكثر من عشرة أجزاء من أحد وسبعين جزءًا من القطر.

فليكن | اج قطر دائرة، ومركزها ه، وخطّ دز مماسًّا للدائرة، وزاوية زهج ثلث **F** 4v
زاوية | قائمة. فنسبة هز إلى زج كنسبة الثلاثمائة والستّة إلى المائة والثلاثة والخمسين، **H** 3r
ونسبة هج إلى زج أعظم من نسبة المائتين والخمسة والستّين إلى المائة والثلاثة والخمسين. ونقسم زاوية زهج بنصفين بخطّ هح. فنسبة زه إلى هج كنسبة زح إلى حج. فنسبة زه وهج مجموعين إلى زج كنسبة هج إلى جح. فتصير نسبة جه إلى جح

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1 اهز] هز **H**   4 يُخْرَجُ] voc.، يخرّج **F***، يخرح **H***، see note 106   4–5 وقاعدة جز مساوية للخطّ المحيط بها] **H** om.   5 أضعاف] أمثال **H**   بسبع] وسبع **H**   6 أنّ] **H**ⁱ illeg. (skel.)   نسبة] **H**ⁱ mg.   8 كلّ] كلّ ~~دائرة~~ **H**   9 وبأكثر] وأكثر **F**   وسبعين جزءًا من] **F** illeg. (dam.)   10 دز] هز **H**   مماسًّا] مماسّ **H**   13 هح] جه **H**   حج] جح **H**

أعظم من نسبة الخمسمائة والأحد والسبعين إلى المائة والثلاثة والخمسين. فنسبة ه ح في القوّة إلى ح ج في القوّة كنسبة التسعة آلاف والأربعمائة والخمسين إلى الثلاثة آلاف والأربعمائة والتسعة. فأمّا نسبته إليه في الطول فأعظم من نسبة الخمسمائة والأحد والتسعين إلى المائة والثلاثة والخمسين. وأيضًا فلنقسم زاوية ح ه ج بنصفين بخطّ ه ط. فبمثل ما قلنا يتبيّن أنّ نسبة ه ج إلى ج ط أعظم من نسبة الألف والمائة والاثنين والستّين والثمن إلى المائة والثلاثة والخمسين. فنسبة ط ه إلى ط ج أعظم من نسبة الألف والمائة والاثنين والسبعين والربع إلى المائة والثلاثة والخمسين. وأيضًا فلنقسم زاوية ط ه ج بنصفين بخطّ ه ك. فنسبة ه ج إلى ج ك أعظم من نسبة الألفين والثلاثمائة والأربعة والثلاثين والربع إلى المائة والثلاثة والخمسين. فنسبة ه ك إلى ج ك أعظم من نسبة الألفين والثلاثمائة والتسعة والثلاثين والربع إلى المائة والثلاثة والخمسين. وأيضًا فلنقسم زاوية ك ه ج بنصفين بخطّ ل ه. فنسبة ه ج إلى ج ل في الطول | [F 5r] أعظم من نسبة الأربعة آلاف والستّمائة والثلاثة والسبعين والنصف إلى المائة والثلاثة والخمسين. فلأنّ زاوية ز ه ج قد كانت ثلث زاوية قائمة، يجب أن تكون زاوية ل ه ج جزءًا من ثمانية وأربعين جزءًا من زاوية قائمة. ونعمل على نقطة ه زاوية مساوية لزاوية ل ه ج، وهي زاوية ج ه م. فزاوية ل ه م هي جزء واحد من أربعة وعشرين جزءًا من زاوية قائمة. فخطّ ل م المستقيم هو ضلع الشكل الكثير الزوايا المحيط بالدائرة ذي الستّ والتسعين زاوية متساوية. ولأنّا قد كنّا بيّنّا أنّ نسبة ه ج إلى ج ل أعظم من نسبة الأربعة آلاف والستّمائة والثلاثة والسبعين والنصف إلى المائة والثلاثة والخمسين، وضعف ه ج خطّ ا ج، وضعف ج ل | [H 3v] خطّ ل م، يلزم أن تكون نسبة ا ج إلى محيط الشكل الكثير الزوايا ذي الستّ والتسعين زاوية أعظم من نسبة الأربعة آلاف والستّمائة والثلاثة والسبعين والنصف إلى الأربعة عشر ألفًا والستّمائة والثمانية والثمانين. وذلك أكثر من

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1 والسبعين] والتسعين **H** 2 ح ج] ج ح **H** كنسبة] كنسبة + لا sup. **F**ⁱ، see note 111 آلاف] الألف **H** 3 والتسعة] والتسعة + إلى sup. **F**ⁱ، see note 112 4 المائة] الثلاثمائة **H** 5 يتبيّن] illeg. (skel.) **H** 6 والثمن] والثمن + illeg. (skel.) sup. **F**ⁱ، see note 115 ط ه] ه ط **H** 8 ج ك] ح ك **H** 10–11 فلنقسم زاوية] illeg. (dam.) **F** 11 ج ل] ج ط **H** 12 آلاف] الألف **H** 13 تكون زاوية] زاوية .om **H** 14 ل ه ج] ا ه ج **H** 15 جزء] جزء من **H** 16 الزوايا] الزوايا في الستّ] الستّة **H** 17 زاوية متساوية] الزاوية المساوية **H** قد كنّا] كنّا قد **H** آلاف] الألف **H** 20 زاوية] الزاوية **H** آلاف] الألف **H** 21 ألفًا] ألف **F**، الألف **H**

ثلاثة أضعافه بستّمائة وسبعة وستّين ونصف التي نسبتها إلى الأربعة آلاف والستّمائة والثلاثة والسبعين والنصف أقلّ من السبع. فيجب أن يكون الشكل الكثير الزوايا المحيط بالدائرة | أكثر من ثلاثة أضعاف قطرها بأقلّ من سبع القطر. فما أكثر نقصان الخطّ المحيط بالدائرة من ثلاثة أضعاف قطرها وسبعه. F 5v

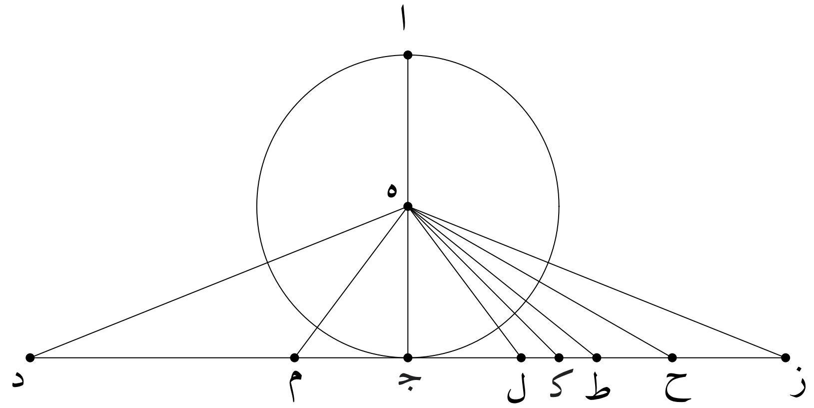

Figure 5: First diagram for *Fatih* 3. **F:** *Dāl* can be read with difficulty due to water damage; it can be identified from the text. There is something written above the line *dāl mīm* but it cannot be read due to water damage. **H:** In **H**, the diagram above has the line *dāl zāʾ* vertical and on the left side, with *zāʾ* at the top and *dāl* at the bottom. *Zāʾ* is written as a *rāʾ* in the diagram. **FH:** *Zāʾ* and *jīm* are often written as *rāʾ* and *ḥāʾ* in the text.

ولتكن دائرة على قطرها اج، وزاوية باج ثلث قائمة. فنسبة اب إلى بج أقلّ من نسبة الألف والثلاثمائة والأحد والخمسين إلى السبعمائة والثمانين. فأمّا نسبة اج إلى جب فمثل نسبة الألف والخمسمائة والستّين إلى السبعمائة والثمانين، لأنّ اج ضعف جب. ونقسم زاوية باج بنصفين بخطّ اح. فلأنّ زاوية باح مساوية لزاوية حجب، وزاوية باج قد قسمت بنصفين بخطّ اح، يجب أن تكون زاوية حجب مساوية لزاوية حاج. وزاوية احج مشتركة. فزوايا مثلّث احج مساوية لزوايا مثلّث حجز. فنسبة اح إلى حج كنسبة جح إلى حز، وكنسبة اج إلى جز، وكنسبة جا اب جميعًا إلى بج. ونسبة جا اب جميعًا إلى بج كنسبة اح

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[1] أضعافه] أمثال H   آلاف] الألف H   [6] والثلاثمائة] والمائة H   [8] اح] اج H   [9] حجب] جحب H   باج] باح H   [10] حجب] جحب H   [11] حجز] حجز or جحز   حج] حج F (uncertain placement of dot)   حز] جز H   [12] جز، وكنسبة] جز كنسبة H   جا اب] اب حا H   جا] حا H   بج] بح H

إلى ح جـ. ومن ذلك يتبيّن أنّ نسبة ا ح إلى ح جـ أقلّ من نسبة الألفين والتسعمائة والأحد عشر إلى السبعمائة والثمانين، وأنّ نسبة ا جـ إلى جـ ح أقلّ من نسبة الثلاثة آلاف والثلاثة عشر والنصف والربع إلى السبعمائة والثمانين. فلنقسم زاوية جـ ا ح بنصفين بخطّ ا ط. فيتبيّن ممّا قلنا أنّ نسبة ا ط إلى ط جـ أقلّ من نسبة الخمسة آلاف

H 4r والتسعمائة والأربعة والعشرين | والنصف والربع إلى السبعمائة والثمانين، وذلك كنسبة الألف والثمانمائة والثلاثة والعشرين إلى المائتين والأربعين، لأنّ نسبة كلّ واحد من العددين الأوّلين إلى نظيره من العددين الأخيرين كنسبة الثلاثة والربع إلى الواحد.

F 6r فتصير نسبة | ا جـ إلى جـ ط أقلّ من نسبة الألف والثمانمائة والثمانية والثلاثين والتسعة الأجزاء من أحد عشر جزءًا من الواحد إلى المائتين والأربعين. وأيضًا فإنّا نقسم زاوية ط ا جـ بنصفين بخطّ ا ک. فنسبة ا ک إلى ک جـ أقلّ من نسبة الثلاثة آلاف والستّمائة والأحد والستّين والتسعة الأجزاء من الأحد عشر جزءًا من الواحد إلى المائتين والأربعين. وذلك كنسبة الألف والسبعة إلى الستّة والستّين، لأنّ نسبة كلّ واحد من العددين الأوّلين إلى نظيره من العددين الأخيرين كنسبة الأربعين إلى الأحد عشر. فنسبة ا جـ إلى ک جـ كنسبة الألف والتسعة والسدس إلى الستّة والستّين. وأيضًا فلنقسم زاوية ک ا جـ بنصفين بخطّ ل ا. فنسبة ا ل إلى ل جـ أقلّ من نسبة الألفين والستّة عشر والسدس إلى الستّة والستّين. فنسبة ا جـ إلى جـ ل أقلّ من نسبة الألفين والسبعة عشر والربع إلى الستّة والستّين. وإذا قلبنا، صارت نسبة محيط الشكل الكثير الزوايا الذي كلّ ضلع من أضلاعه مساوٍ لخطّ جـ ل إلى القطر أعظم من نسبة الستّة آلاف والثلاثمائة والستّة والثلاثين إلى الألفين والسبعة عشر والربع. ولكنّ الستّة آلاف والثلاثمائة والستّة والثلاثين هي أكثر من ثلاثة أضعاف الألفين والسبعة عشر والربع بأكثر من عشرة أجزاء من أحد وسبعين جزءًا من واحد. فمحيط الشكل الكثير الزوايا ذي الستّ والتسعين زاوية الذي تحيط به الدائرة يزيد على ثلاثة أضعاف قطرها بأكثر من عشرة

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1 يتبيّن] (.skel) .H illeg   والتسعمائة] والسبعمائة H   2 السبعمائة] سبعمائة H   إلى] .H om   3 آلاف] الألف H   4 فيتبيّن] .corr، فسس *F، فسن *H   ممّا] ما H   إلى] .H om   آلاف] الألف H   7 العددين الأخيرين كنسبة الثلاثة والربع] (.F illeg. (dam   8 جـ ط] ح ط H   9–8 والتسعة الأجزاء من أحد] والسبعة والأحد H   10 آلاف] الألف H   11 والتسعة] والسبعة H   14 والتسعة] والسبعة H   15 ل ا] ا ل H   ل جـ] ا جـ H   17 قلبنا] قلنا H، see Scholium 1   كلّ] جـ ل H   18 مساوٍ لخطّ جـ ل] .H om   آلاف] الألف H   19 آلاف] الألف H   20 والثلاثين] والثلاثين إلى H   21 من واحد] .H om   السّت] الستّة H   22 زاوية] الزاوية H

أجزاء من أحد {وسبعين جزءًا. فيصير الخطّ المحيط} بالدائرة | أكثر من ثلاثة أضعاف قطرها بأكثر من عشرة أجزاء من أحد وسبعين جزءًا، وتكون زيادتها على هذا المقدار أكثر من زيادة أضلاع الشكل الكثير الزوايا. **F** 6v

فالخطّ المحيط بالدائرة يزيد على ثلاثة أضعاف قطرها بأقلّ من سبعه وبأكثر من عشرة أجزاء من أحد وسبعين جزءًا. وذلك ما أردنا أن نبيّن.

[**F**] تمّ كتاب أرشميدس في مساحة الدائرة. الحمد لله وصلواته على خيرته من خلقه محمّد نبيّه وآله وصحبه وسلامه.

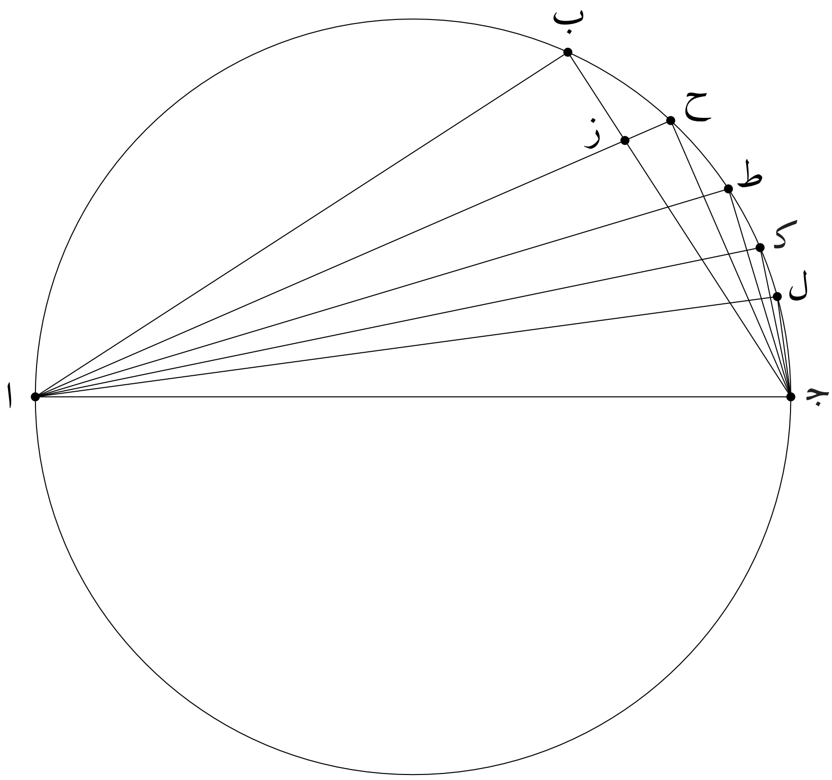

Figure 6: Second diagram for *Fatih* 3. **F:** *Zāʾ* is written as a *rāʾ* in the diagram and the text. **H:** *Zāʾ* is unmarked in the diagram and written as a *rāʾ* in the text.

2–1 {وسبعين جزءًا. فيصير الخطّ المحيط} بالدائرة أكثر من ثلاثة أضعاف قطرها بأكثر من عشرة أجزاء من أحد] **H** om. 1 {وسبعين جزءًا. فيصير الخطّ المحيط}] **F** illeg. (dam.)، see note 127 3 أضلاع] الأضلاع **H** 4 وبأكثر] وأكثر **FH**

In the name of God, the Most Gracious, the Most Merciful

The Book of Archimedes on the Measure of the Circle

1[92] Every circle is equal to the right-angled triangle one of whose sides surrounding the right angle is equal to half of the diameter of the circle and [whose] other side from the two [sides surrounding the right angle] is equal to the line surrounding the circle.

Let the circle *ABGD* be [set] equal to the triangle *E* in the properties[93] we mentioned earlier in the notification.[94] Then I say that the measure of [the circle] is equal to the measure of [the triangle].

For if it is not so, the circle is either greater or smaller than [the triangle]. First, let it be greater than [the triangle]. We construct the square *AG* in the circle. So [something] greater than its half, which is the square *AG*, has been removed from the circle *ABGD*. We cut the arc *AB* and its counterpart arcs in halves at the point *F* and its counterpart points. We join *AF*, *FB*, and their counterparts. So, also, [something] greater than their halves, which is *AFB* and its counterparts, has also been removed from the remainder of the segments of the circle *ABGD*.[95] If we do that repeatedly,[96] there will remain segments smaller than the amount of the excess of the circle over the triangle *E*. So, then, the rectilinear polygonal figure that the circle surrounds is greater than the triangle [*E*]. We make *N* the center of the circle, and we draw the perpendicular *NS*. So the line *NS* is less than one of the two sides of the triangle surrounding the right angle, and the perimeter of the polygonal figure is less than the remaining side from the two [sides surrounding the right angle], since it is also less than the line surrounding the circle. So that which ensues from the product of one of the two sides of the triangle surrounding the right angle by the other [side surrounding the right angle], which is the double of the ⟨area of the⟩ triangle, is more than the result of the product of *NS* and the perimeter of the polygon, which is the double of the area of the polygon. And their halves are also thus.[97] So the triangle is greater than the polygon, even though it was smaller than [the polygon]. This is a contradiction that is not possible.

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[92] **F** 2v.3–3v.9. **H** 1v.2–2v.7. Greek text in Heiberg (1972, I.232.1–234.17).

[93] Literally "things" (*ashyā*ʾ).

[94] "Notification" (*khabar*) is a common term for the enunciation. See Sidoli and Isahaya (2018, 212–213).

[95] The segments in question are the segment bounded by the arc *AB* and the line *AB*, and the counterparts of that segment. "*AFB*" refers to the triangle *AFB*.

[96] A nonliteral translation of the Arabic *ʿalā mā yatlū*. Knorr (1989, 436) translates literally as "according to what follows."

[97] That is, they satisfy the same inequality.

Now, let the circle be smaller than the triangle *E* if that were possible. We draw on [the circle] a square that surrounds it, which is the square *QC*. So [something] greater than its half, which is the circle, has been removed from the square *QC*. We divide the arc *BA* in two halves at *F*, and its counterpart arcs in halves, and let there pass lines tangent to the circle through the points of the division. So the line *RT* has been divided in two halves at the point *F*, the line *NC* is perpendicular to *RT*, and similarly its[98] counterpart lines. Since *CR* and *CT* are greater than *TR*, their halves are greater than its half. So the line *CT* is greater than *TF*, which is equal to *TB*. So the triangle *CFT* is greater than half of the triangle *CFB*, and all the more is it greater than half of the figure *CFIB*, which the lines *BC* and *CF* and the arc *BIF* surround. Similarly, the triangle *CFR* is greater than *FUAR*.[99] So the whole of *TCR* is greater than half of the figure *AUFIBC*,[100] and similarly its counterpart triangles are more than half of the counterparts of the other segments. If we do that repeatedly,[101] there will remain segments that are left over from the circle, and when added together become less than the excess of the triangle *E* over the circle *ABGD*. Let there remain the segment *FRA* and its counterpart segments. So, then, the rectilinear figure that surrounds the circle is smaller than the triangle *E*. This is not possible since it is greater than [the triangle], that is, *NA* is equal to the perpendicular of the triangle, and the perimeter of the polygonal figure is greater than the other side of the triangle that surrounds the right angle, since it is greater than the line surrounding the circle. So that which ensues from the product of *AN* and the perimeter of the polygon is greater than the product of one of the two sides of the triangle surrounding the right angle and the other. So the circle is not smaller than the triangle *E*. And it was proved in what preceded that it is not greater than [the triangle]. The circle *ABGD* is therefore equal to the triangle *E*.

Also, the measure of the triangle *E* is equal to that which ensues from the product of its perpendicular and half of its base, its perpendicular is equal to half of the diameter of the circle *ABG*,[102] and its base is equal to the perimeter of the circle

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[98] It is not completely clear what the Arabic *naẓāʾiruhu* refers to. Presumably it refers to *RT*.

[99] Note that the two figures *CFIB* and *FUAR* are asymmetric with respect to the line *CN*. Consequently, the inequalities are also different: one triangle is greater than half of one figure; another triangle is greater than the other figure.

[100] The conjecture of Knorr (1989, 428, 440, 452, n. 24), based on the evidence of **LG** and **HA**, that there might be a gap here that contained something like "contained by lines *AQ*, *QB* [that is, *AC* and *CB*] and arc *AFB*," is supported neither by **LP** (as he himself notes), nor by the evidence of **H**, nor by *Columbia*, nor by the *taḥrīr* of al-Ṭūsī.

[101] A nonliteral translation of the Arabic *fīmā yatlū*. Knorr (1989, 437) translates literally as "in what follows."

[102] I have kept *ABG* here and in the other two occurrences in this paragraph in accordance with the principle of *lectio difficilior*, even though **H** and the Hebrew and Latin translations all use the

ABG. So that which ensues from the product of half of the diameter and half of the line surrounding the circle *ABG* is equal to the area of the triangle *E*. And that is what we wanted to prove.

And because of that, the product of half of the diameter and half of a segment of the perimeter is the area of the figure that that segment and the two lines drawn from the two ends of the segment to the center surround.[103]

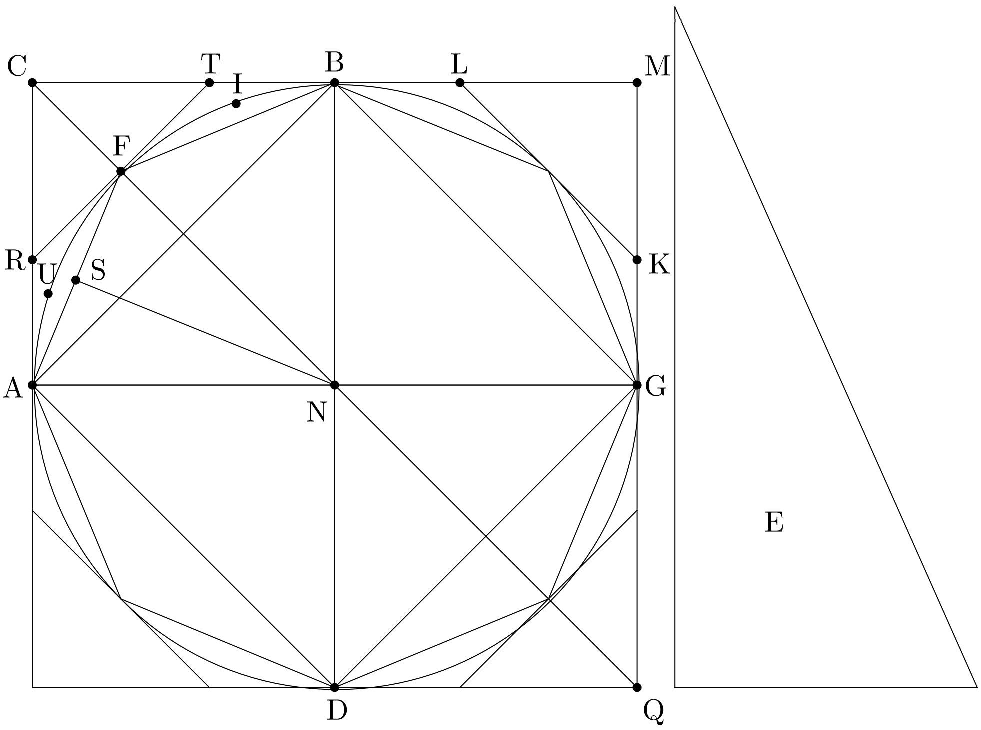

Figure 7: Diagram for *Fatih* 1.

2[104] The ratio of the area of every circle to the square of its diameter is as the ratio of 11 to 14.

Let the line *AB* be the diameter of the circle, let us construct the square *GH* on [the diameter], let *DG* be half of the line *DE*, and let the line *EZ* be a seventh of *GD*. Since the ratio of the triangle *AGE* to the triangle *AGD* is as the ratio of 21 to

four letters *ABGD*. That these other sources all have *ABGD* can be explained by the presence of the phrase *dā'irat alif bā' jīm dāl* numerous times in this proposition before this point. Knorr (1989, 438) erroneously has "*ABGD*" for the first two mentions of the circle in this paragraph, but the manuscript image he (1989, 457) provides shows clearly that the letters in question are *ABG* for both cases. The consistent use of *ABG* to denote the circle in this paragraph instead of *ABGD* strengthens the supposition that this paragraph is an interpolation, against Knorr (1989, 430–431).

103 The figure in question is a sector. This sentence is also likely to be an interpolation.

104 **F** 4r.1–11. **H** 2v.8–20. Greek text in Heiberg (1972, I.234.18–236.6).

7, and the ratio of AGD to AEZ is as the ratio of 7 to 1, therefore the ratio of the triangle AGZ to the triangle AGD becomes as the ratio of 22 to 7. But the square GH is four times ADG, and the triangle AGZ is equal to the circle AB[105] since the perpendicular AG is equal to the line that is drawn[106] from the center of this circle to the line surrounding [the circle], and the base GZ is equal to the line surrounding [the circle], as the line surrounding the circle is greater than three times its diameter by approximately a seventh of the diameter. So it has become clear from what we have said that the ratio of the circle AB to the square GH is as the ratio of 11 to 14. And that is what we wanted to prove.

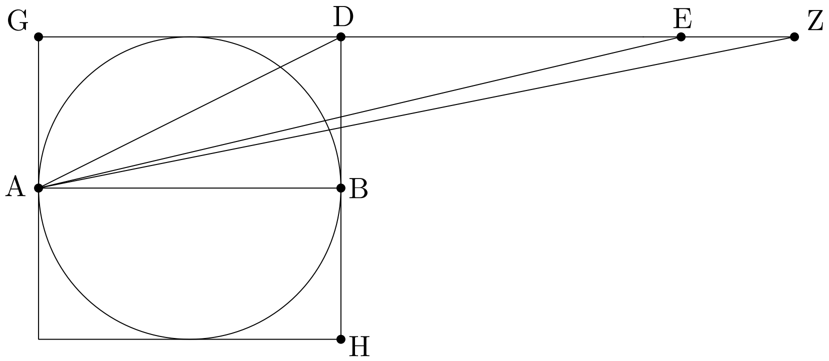

Figure 8: Diagram for *Fatih* 2.

3[107] Every line surrounding a circle exceeds three times its diameter by [something] less than a seventh of the diameter and more than 10/71 of the diameter.

Let AG be the diameter of a circle whose center is E, [let] the line DZ [be] tangent to the circle, and [let] the angle ZEG [be] a third of a right angle. So the ratio of EZ to ZG is as the ratio of 306 to 153, and the ratio of EG to ZG is greater than the ratio of 265 to 153.[108] We divide the angle ZEG in two halves by the line EH. So the ratio of ZE to EG is as the ratio of ZH to HG.[109] So the ratio of ZE and EG

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[105] This equality follows from *MC* 1 and *MC* 3. The implausibility of *MC* 2 preceding *MC* 3, which it requires, has been noted in the literature. See, for example, Knorr (1989, 477–478).

[106] I have vocalized this verb as *yukhraju*, a passive imperfect of the Form IV verb *akhraja*. Knorr (1989, 484) seems to have vocalized it as the Form I verb *yakhruju* since he translated the verb as "goes." Both vocalizations are equally plausible. The alternative reading *nukhriju* is less likely since it does not have a suffixed object pronoun. The Greek τῇ ἐκ τοῦ κέντρου, which does not contain a verb, is of no help in determining the reading of the Arabic.

[107] **F** 4r.12–6v.4. **H** 2v.21–4r.22. Greek text in Heiberg (1972, I.236.7–242.21).

[108] If one takes $ZG = 153$, then by *Elements* I.47, one has $EG^2 = EZ^2 - ZG^2 = 70227$, whose square root is slightly greater than 265.

[109] By *Elements* VI.3.

together to ZG is as the ratio of EG to GH.[110] So the ratio of GE to GH becomes greater than the ratio of 571 to 153. So the ratio of EH in power to HG in power is as the ratio of[111] 9450 to 3409.[112] As for its ratio to it in length, it is greater than the ratio of 591[113] to 153.[114] And also, let us divide the angle HEG in two halves by the line ET. So, similarly to what we said, it is proved that the ratio of EG to GT is greater than the ratio of 1162 1/8[115] to 153. So the ratio of TE to TG is greater than the ratio of 1172 1/4[116] to 153. And also, let us divide the angle TEG in two halves by the line EK. So the ratio of EG to GK is greater than the ratio of 2334 1/4 to 153. So the ratio of EK to GK[117] is greater than the ratio of 2339 1/4 to 153. And also, let us divide the angle KEG in two halves by the line LE. So the ratio of EG to GL in length is greater than the ratio of 4673 1/2 to 153. Since the angle ZEG was a third of a right angle, the angle LEG must be 1/48 of a right angle. On the point E we construct an angle equal to the angle LEG, namely GEM. So the angle LEM is 1/24 of a right angle. So the straight line LM is the side of the polygonal figure of 96 equal angles surrounding the circle. And since we had proved that the ratio of EG to GL is greater than the ratio of 4673 1/2 to 153, the line AG is the double of EG, and the line LM is the double of GL, it is necessary that the ratio of AG to the perimeter of the polygonal figure of 96 angles be greater than the ratio of 4673 1/2 to 14688. And that is more than three times it by 667 1/2 whose ratio to 4673 1/2 is less than a seventh.[118] So the polygonal figure surrounding the circle must be more than three times the diameter of [the circle] by less than a seventh of

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[110] Since $ZE : EG = ZH : HG$, by composition and alternation, $ZE + EG : ZG = EG : HG$.

[111] The word *lā* ("no"), written on top of the word *ka-nisbat* in red ink by the hand of Ibn Abī Jarāda ($\mathbf{F^i}$), shows that he realized that the numbers 9450 and 3409 were erroneous.

[112] The word *ilā* ("up to"), written on top of the word *al-tisʿa* in red ink by the hand of Ibn Abī Jarāda ($\mathbf{F^i}$), indicates the bound of the stretch of text containing the erroneous numbers. In fact, if one takes $GH = 153$ and hence $GH^2 = 23409$, then $GE > 571$, hence $GE^2 > 326041$, and by *Elements* I.47, $EH^2 = GE^2 + GH^2 > 349450$. It is clear that these erroneous values are due to errors in transmission, as I argue in Section I.3.1, and for this reason I have kept them in the Arabic text, against Knorr (1989, 485), who produced the correct values in his translation.

[113] The absence of the expected 1/8 (*wa-l-thumn*) here is possibly a corruption specific to **FH**.

[114] Since $\sqrt{349450} > 591 1/8$.

[115] On top of the word *wa-l-thumn*, there is written something whose meaning I cannot discern, in red ink by the hand of Ibn Abī Jarāda ($\mathbf{F^i}$). What is written looks like an initial *mīm* on the right, followed by short vertical strokes in the middle, and the mirror image of the initial *mīm* on the left.

[116] Instead of the expected 1/8 (*wa-l-thumn*), we have 1/4 (*wa-l-rubʿ*) here; this must be, again, a corruption specific to **FH**.

[117] Knorr (1989, 486) mistranslates as "*GK* to *HK*."

[118] That is, $14688 = 3 \cdot 4673\ 1/2 + 667\ 1/2$ and $667\ 1/2 : 4673\ 1/2 < 1 : 7$.

the diameter. All the more is the line surrounding the circle less than three times the diameter of [the circle] and a seventh of [the diameter].

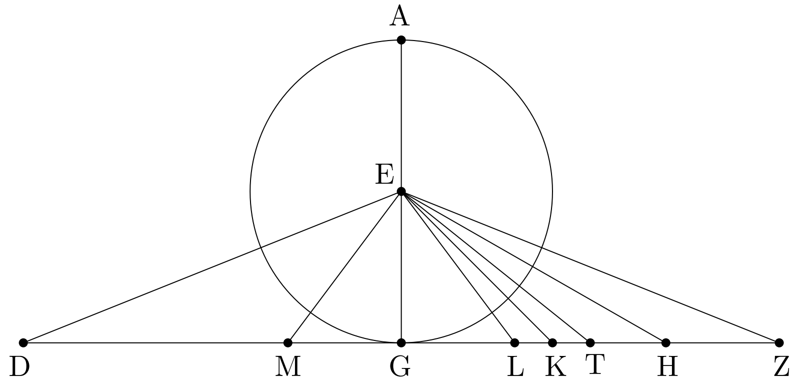

Figure 9: First diagram for *Fatih* 3.

Let[119] there be a circle on its diameter AG, and [let] the angle BAG [be] a third of a right [angle]. So the ratio of AB to BG is less than the ratio of 1351 to 780. As for the ratio of AG to GB, it is equal to the ratio of 1560 to 780, since AG is the double of GB.[120] We divide the angle BAG in two halves by the line AH. So since the angle BAH is equal to the angle HGB, and the angle BAG has been divided in two halves by the line AH, the angle HGB must be equal to the angle HAG. And the angle AHG is common. So the angles of the triangle AHG are equal to the angles of the triangle HGZ.[121] So the ratio of AH to HG is as the ratio of GH to HZ, as the ratio of AG to GZ, and as the ratio of GA and AB together to BG.[122] And the ratio of GA and AB together to BG is as the ratio of AH to HG. From that, it is proved that the ratio of AH to HG is less than the ratio of 2911 to 780,[123] and that the ratio of AG to GH is less than the ratio of 3013 1/2 1/4 to 780.[124] Let us divide the angle GAH in two halves by the line AT. So it is proved from what we said that the ratio of AT to TG is less than the ratio of 5924 1/2 1/4 to 780, and that is as the ratio of 1823 to 240, since the ratio of every one of the two former numbers to

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[119] At this point, **F** labels the text as the fourth proposition with *dāl* in red ink (5v, right margin).

[120] If one takes $GB = 780$ and hence $AG = 1560$, then by *Elements* I.47, one has $AB^2 = AG^2 - GB^2 = 1825200$, whose square root is slightly less than 1351.

[121] Hence, the triangles AHG and GHZ are similar.

[122] Since the triangles AHG and GHZ are similar, one has $AH : HG = GH : HZ = AG : GZ$. From *Elements* VI.3, one has $AB : AG = ZB : ZG$. By composition and alternation, $AG + AB : BG = AG : ZG$.

[123] If one takes $GB = 780$, then $GA = 1560$ and $AB < 1351$.

[124] If one takes $GH = 780$, then $AH < 2911$, and by *Elements* I.47, $AG = \sqrt{AH^2 + HG^2} < 3013$ 1/2 1/4.

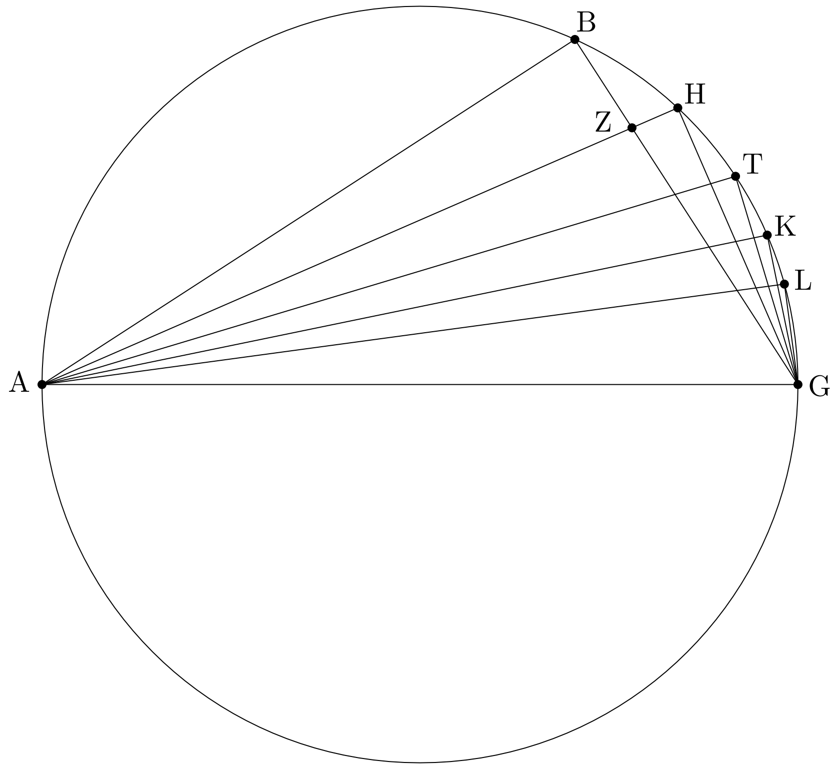

Figure 10: Second diagram for *Fatih* 3.

its counterpart among the two latter numbers is as the ratio of 3 1/4 to 1. So the ratio of AG to GT becomes less than the ratio of 1838 9/11 to 240. And also, we divide the angle TAG in two halves by the line AK. So the ratio of AK to KG is less than the ratio of 3661 9/11 to 240. And that is as the ratio of 1007 to 66, since the ratio of every one of the two former numbers to its counterpart among the two latter numbers is as the ratio of 40 to 11. So the ratio of AG to KG is as the ratio of 1009 1/6 to 66.[125] And also, let us divide the angle KAG in two halves by the line LA. So the ratio of AL to LG is less than the ratio of 2016 1/6 to 66. So the ratio of AG to GL is less than the ratio of 2017 1/4 to 66. If we invert (see Scholium 1), the ratio of the perimeter of the polygonal figure every one of whose sides is equal to the line GL to the diameter becomes greater than the ratio of 6336 to 2017 1/4. But 6336 is more than three times 2017 1/4 by more than 10/71 of 1.[126] So the perimeter of the polygonal figure of 96 angles that the circle surrounds exceeds three times the diameter of [the circle] by more than 10/{7}1. {So the line surrounding}

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[125] In fact, $AG : KG < 1009\ 1/6 : 66$.

[126] In other words, $6336 > (3\ 10/71) \cdot (2017\ 1/4)$.

the circle {becomes}[127] more than three times the diameter of [the circle] by more than 10/71, and the excess of [the circle] over this amount is more than the excess of the sides of the polygonal figure.[128]

So the line surrounding the circle exceeds three times the diameter of [the circle] by [something] less than a seventh of [the diameter] and more than 10/71. And that is what we wanted to prove.

[**F**] Archimedes's book on the measurement of the circle is complete. Praise be to God, his blessings and his peace upon the best of his creation, Muhammad his prophet, upon his family, and upon his companions.

IV.1.1 The Fatih *Version: Scholium*

قلت: المراد بالقلب ههنا العكس، لا نسبة المقدّم إلى زيادته على التالي.

Scholium 1.[129] I say: what is meant by [the word] *qalb* here is inversion, not the ratio of the antecedent to its excess over the consequent.[130]

[127] The upper parts of the words *wa-sabʿīn juzʾan fa-yaṣīr* are visible above the part of the paper damaged by water. As to *al-khaṭṭ al-muḥīṭ*, not only is the dot in the *khāʾ* visible, but the phrase is suggested by the text of the proposition itself, where it appears several times.

[128] Since the perimeter of the circle is greater than the perimeter of the inscribed 96-gon. The part of the sentence after the comma is quite possibly an interpolation since the perimeter of the polygon is described as "sides" (*aḍlāʿ*), which is never seen anywhere else in *Fatih*.

[129] **F**[i] 6r, middle of left margin. The placement of this scholium is indicated in the manuscript by a *signe de renvoi* just before the word *qalabnā*.

[130] In other words, *al-qalb* designates the inversion of a ratio (ἀνάπαλιν λόγος) and not the conversion of a ratio (ἀναστροφὴ λόγου), as it normally does (Rashed 2017, 557, 672).

IV.2 *Columbia Preliminaries* and the *Columbia* Version

C 24r

أشكال نافعة في كتاب أرشميدس لأبي الرشيد عبد {الهادي}

⟨ا⟩ كلّ مربّع متساوي الأضلاع في دائرة، فهو أكبر من نصفها، لأنّ ربع كلّ المربّع الأعظم، أعني مثلّث اه ج، أصغر من ربع الدائرة †بقوس، وهو مع ازج ضعفه، فجميع المربّع الأعظم أكبر من نصف الدائرة | بمقدار أربعة أمثال ازج.† C 24v

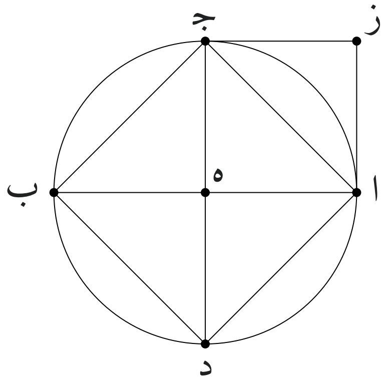

Figure 11: Diagram for *Columbia Preliminary* 1. **C:** *Zāʾ* is written as a *rāʾ* in the diagram and the text.

ب †وإذا نقصنا من القوس التي على ضلع المربّع ما هو ⟨على⟩ ضلع المثمّن، فهو أكبر من نصف ما بقي من الدائرة بعد المربّع،† كما أنّا إذا عملنا على ضلع دب، فدم ط أكبر من قوس د ط بمقدار قطعة د ز ط، لأنّ مثلّث د م ط نصف د ز ط م. فهو أكبر من نصف قوس د ط م.

2 {الهادي}] illeg. (skel.) + ابى C، see note 131   4 وهو] هو C   ا ز ج] ا ز C (Knorr)، see note 136
5 المربّع] المربّع له C   6 ب] ب + ب C sup.   التي] الذي C   9 د ط م] د ط ج C (Knorr)

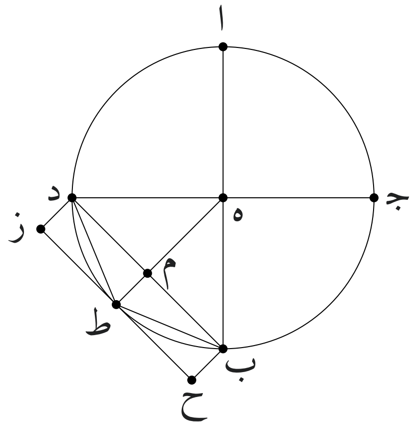

Figure 12: Diagram for *Columbia Preliminary* 2. **C:** *Zāʾ* is written as a *rāʾ* in the diagram and the text.

⟨ج⟩ وكلّ دائرة في مربّع متساوي الأضلاع فهي أكبر من نصف المربّع. ولتكن الدائرة ي ز ح ط، والمربّع ا ب ج د. فمربّع ا ب ج د قد انقسم بأربعة مثلّثات في داخلها متساويات ومساويات للأربعة التي بعضها بخارج من الدائرة. والمثلّثات الأربعة الداخلة نصف المربّع الأعظم، والأربعة في داخل الدائرة، فالدائرة أعظم من نصف المربّع الذي عليها.

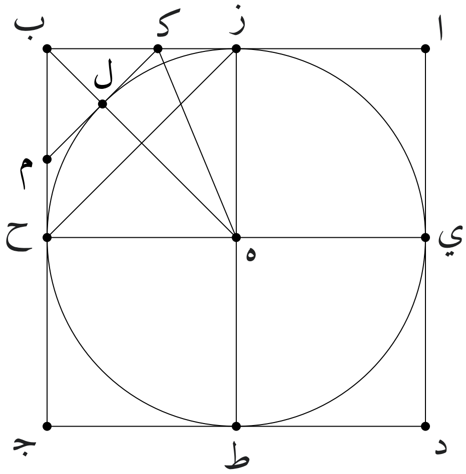

Figure 13: Diagram for *Columbia Preliminary* 3. **C:** *Yāʾ* and *zāʾ* are written without dots in the diagram and the text. *Hāʾ* is not marked in the diagram.

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[1] متساوي] متوازي C (Knorr) [2] بأربعة] بأربع C [3] للأربعة] للأربع C بخارج] خارج C (*bi-* added with darker ink) [4] في] sup. C داخل] الداخل C الدائرة] الدائرة أعظم C

ولنخرج أيضًا ⟨خطًّا⟩ من ه إلى ب، وهو ه ل ب، ومن ل نقطة التماسّ خطًّا في جهتيه، وهو خطّ م ل ك. فأقول ⟨إنّ⟩ ز ك مساوٍ لك ل. برهانه: فلنصل ه ك. فلأنّ ه ز مساوٍ له ل، وزاويتي ه ز ب ه ل ك قائمتان، وخطّ ه ك مشترك، فإذا نقصنا مربّع ه ز من مربّع ه ك، | بقي مربّع ك ز. وه ل مثل ه ز، فزك مثل ك ل، كما أردنا. C 25r

د ا ج وتر الزاوية القائمة من مثلّث ا ب ج مثلا وتر الزاوية التي هي ثلث قائمة. مثاله هذا: ا د مثل ا ب، ود ج مثل ا ب. فا ج وتر الزاوية القائمة مثلا ا ب وتر ثلث القائمة بالفرض. ويتبيّن من هذا الشكل أنّ ب ج وتر الثلثين. فهو في القوّة ثلاثة أمثال ا ب وتر الثلث من القائمة، لأنّه إن عمل مربّع على ا ج القطر، فهو أربعة أمثال الذي على نصف القطر، أعني ا د، أعني ا ب، وإذا نقص مربّع ا ب من مربّع ا ج تبقى منه ثلاثة أمثال مربّع ا ب. والله أعلم.

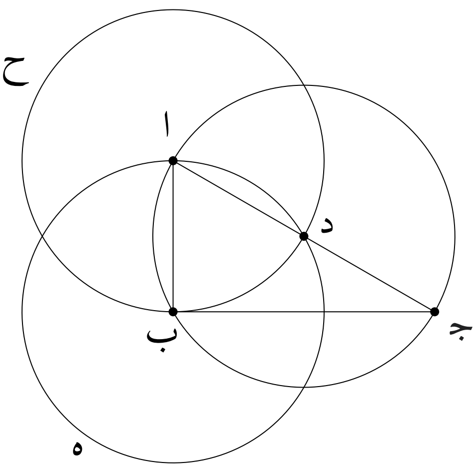

Figure 14: Diagram for *Columbia Preliminary* 4. **C:** All the letters have been written in darker ink but the same hand.

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1 ه ل ب] ل ب C (*hā*ʾ added with darker ink) 2 جهتيه] corr., حهه C*, see note 153 3 وزاويتي] وزاويتا C 4 ك ز] corr., ك ل C* (Knorr) 5 وتر الزاوية] corr., وترا لزاويه C* (line break between *alif* and *lām*) مثلّث] المثلّث C 7 ويتبيّن] corr., وسن C* وتر الثلثين] corr., وترا اللبن C* (line break after the struck-out *alif*), see note 157 8 اب] اب ج C, see note 158 وتر الثلث] corr., ورا للث C* (line break between *alif* and *lām*) القطر] للقطر C 10 ثلاثة] أربعة C sup., see note 159

بسم الله الرحمن الرحيم C 25r

قول منسوب إلى أرشميدس في مساحة الدائرة

⟨ا⟩ كلّ دائرة فإنّها مساوية لمثلّث قائم الزاوية يكون أحد ضلعيه المحيطين بالزاوية القائمة مساويًا لمحيط الدائرة والضلع الآخر مساويًا لنصف قطر الدائرة.

فلتكن دائرة عليها ا ب ج د، ومثلّث قائم الزاوية عليه ه ز ح، والزاوية التي عند نقطة ز قائمة، ويكون ه ز مثل نصف قطر الدائرة، وز ح مثل محيط تلك الدائرة. وبيّن أنّ دائرة ا ب ج د مساوية لمثلّث ه ز ح.

فإن لم يكن كذلك، فإنّها أعظمهما أو أصغرهما. فلتكن أوّلًا دائرة ا ب ج د أعظم من مثلّث ه ز ح. ونجعل في داخل الدائرة مربّعًا عليه ا ب ج د. فقد انفصل | من دائرة ا ب ج د أعظم من نصفها، وهو مربّع ا ب ج د. ونقطع قسيّ ا ط ب ب ك ج ج ل د د م ا بنصفين نصفين على نقط ط ك ل م. ونصل ا ط ط ب ب ك ك ج ج ل ل د د م م ا. فقد انفصل أيضًا من بقيّة قطع دائرة ا ب ج د أعظم من نصفها، وهو ا ط ب ب ك ج ج ل د د م ا. وإذا فعلنا ذلك على ما يتلو، ستتقاطع بقايا أصغر من زيادة دائرة ا ب ج د على مثلّث ه ز ح. فلتبق قطع ا ن ط ط ص ب ب ع ك ك ف ج ج ق ل ل ر د د ش م م ث ا أصغر من زيادة دائرة ا ب ج د على مثلّث ه ز ح. فالكثير الزوايا الذي عليه ا ط ب ك ج ل د م أعظم من مثلّث ه ز ح. ونجعل مركز دائرة ا ب ج د نقطة ت، ونخرج من مركز ت عمودًا إلى أحد أضلاع الكثير الزوايا عليه ت خ. ولأنّ خطّ ز ح مساوٍ لمحيط دائرة ا ب ج د الذي هو أعظم من محيط الكثير الزوايا الذي عليه ا ط ب ك ل ج د م، فخطّ ز ح أعظم من محيط ا ط ب ك ج ل د م الكثير الزوايا. وأيضًا لأنّ خطّ ه ز مساوٍ لنصف قطر دائرة ا ب ج د، | فهو أعظم من خطّ ت خ. فالذي يكون من ضرب ه ز في ز ح أعظم من الذي يكون من ضرب ت خ في محيط ا ط ب ك ل ج د م الكثير الزوايا. وأنصاف

(C 25v at the line beginning «| من دائرة»; C 26r at the line beginning «ا ب ج د، | فهو»)

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4 الآخر مساويًا] الآخر مساوٍ C 5 دائرة] الدائرة C والزاوية] وزاوية C 7 ه ز ح] ز ح C (*hā*ʾ added with darker ink) 8 أعظمهما أو أصغرهما] أعظمها وأصغرها C 9 مربّعًا] مربّع C 13 ستتقاطع] ستقاطع C 14 بقايا] corr.، بقى C* 18 دائرة] الدائرة C 20 مساوٍ] مساويًا C دائرة] الدائرة C 21 فالذي] والذي C

ذلك أيضًا كذلك. فمثلّث ه ز ح أعظم من ا ط ب ك ج ل د م الكثير الزوايا. وذلك ما لا يمكن لأنّه قد تبيّن أنّه أصغر منه. فليست دائرة ا ب ج د أعظم من مثلّث ه ز ح. وذلك ما أردنا أن نبيّن.

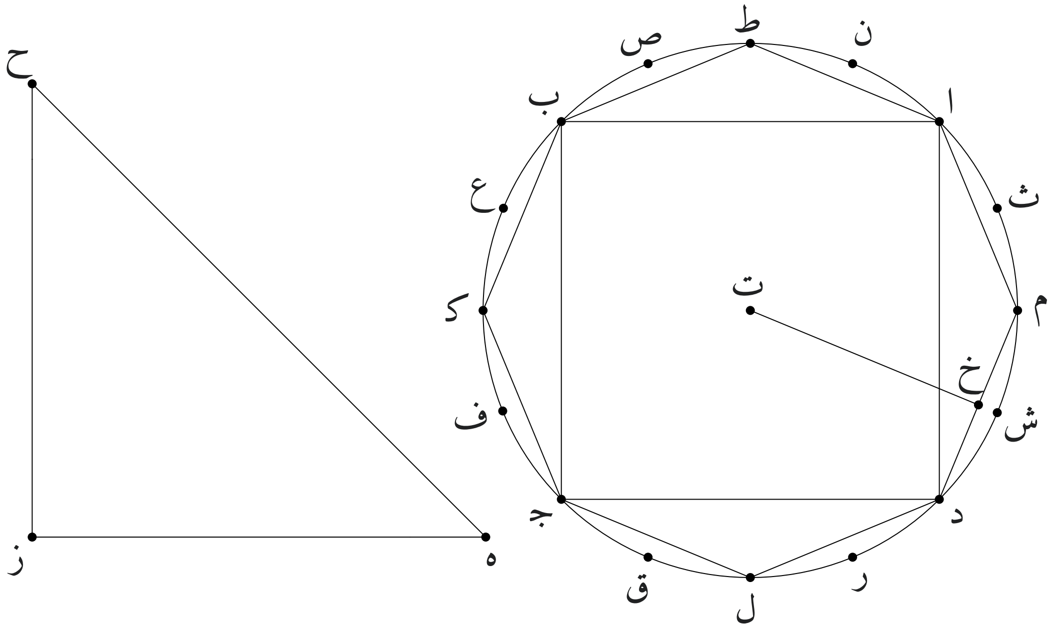

Figure 15: Diagram for *Columbia* 1, corresponding to the first part of *Fatih* 1. **C:** All the diagram letters are written in darker ink, but in the same hand. In the diagram, *zāʾ* is written like a *bāʾ* without a dot; in the text it is written without a dot. *Khāʾ* is written without a dot in the diagram and the text. Finally, the line *tāʾ khāʾ* extends to the point *shīn* in the diagram.

⟨ب⟩ وإن أمكن فلتكن دائرة د ب ك أصغر من مثلّث ا ب ج. ونجعل على دائرة د ب ك مربّعًا يحيط بها عليه ح ط ل ه. فقد انقطع من مربّع ح ط ل ه أعظم من نصفه، وهو دائرة د ب ك. ونخرج من مركز ج خطًّا عليه ج ت ه. ونخرج من نقطة ت خطًّا مماسًّا للدائرة عليه ن م. وكذلك أيضًا ص ع ف ق ر س. فخطّ ن م قد انفصل بنصفين على نقطة ت، وخطّ ج ت عمود على ن م، وكذلك أيضًا الخطوط الباقية. ونصل ت د. فلأنّ ه ن ه م أعظم من م ن، تكون كذلك نصفاهما أيضًا، فخطّ ه م أعظم من م ت الذي هو مثل د م. فمثلّث ه ت م أعظم من نصف مثلّث

2 فليست] فليس C   6 خطًّا] خطّ C   9 نصفاهما] أنصافها C

ه ت د، فيكون أعظم كثيرًا من نصف قطع ت د ه، وكذلك ه ت ن أعظم من نصف قطع ب ي ت ه. فجميع م ه ن أعظم من نصف ب د ه. وكذلك يكون كلّ واحد من ص ح ع ف ط ق ر ل س يقطع من ⟨كلّ واحد من⟩ ب ح خ خ ط ک ک ل د أعظم من نصفه. فإذا فعلنا ذلك فيما يتلو، †يتقاتعان من المربّع ⟨الذي⟩ يكون أصغر | من الموضوع.† فلتبق قطع د م ت ت ن ب ب ص ث ث ع خ خ ف ظ ظ ق ک ک ر ض ض س د أصغر من نقصان دائرة د ب ک عن مثلّث ا ب ج. فالكثير الزوايا الذي عليه م ن ص ع ف ق ر س أصغر من مثلّث ا ب ج. ولأنّ خطّ ب ج مساوٍ لمحيط دائرة د ب ک، ولكنّ محيط م ن ص ع ف ق ر س أعظم من محيط دائرة د ب ک، فمحيط م ن ص ع ف ق ر س الكثير الزوايا أعظم من خطّ ب ج. ولكنّ ا ب مساوٍ لخطّ ت ج، فالذي يكون من ضرب محيط م ن ص ع ف ق ر س في خطّ ت ج أعظم من ضرب ا ب في ب ج. وكذلك أيضًا نصفاهما. فم ن ص ع ف ق ر س الكثير الزوايا أعظم من مثلّث ا ب ج. وذلك ما لا يمكن لأنّه تبيّن أنّه أصغر. فليست دائرة د ب ک بأصغر من مثلّث ا ب ج. وقد تبيّن فيما مضى أنّها ليست بأعظم. فدائرة د ب ک مساوية لمثلّث ا ب ج. ولكنّ تكسير ا ب ج مساوٍ للذي يكون من ⟨ضرب⟩ خطّ ا ب في نصف ب ج، وخطّ ب ج مساوٍ لمحيط دائرة د ب ک. فالذي يكون من ضرب نصف القطر في نصف | محيط الدائرة مساوٍ لتكسير مثلّث ا ب ج.

C 26v

C 27r

ومن أجل هذه العلّة نضرب نصف القطر في نصف المحيط، فيكون من ذلك تكسير الدائرة المفروضة. وذلك ما أردنا أن نبيّن.

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4 ⟨الذي⟩] (Knorr) يكون] ~~يكون~~ يكون C أصغر] أصغر هي C، see note 173 8 ولكنّ] خ + ولكنّ C sup.، see note 174 د ب ک] د ر ک C 10 فالذي] والذي C 11 نصفاهما] أنصافها C 12 فليست] فليس C 14 ⟨ضرب⟩] (Knorr)

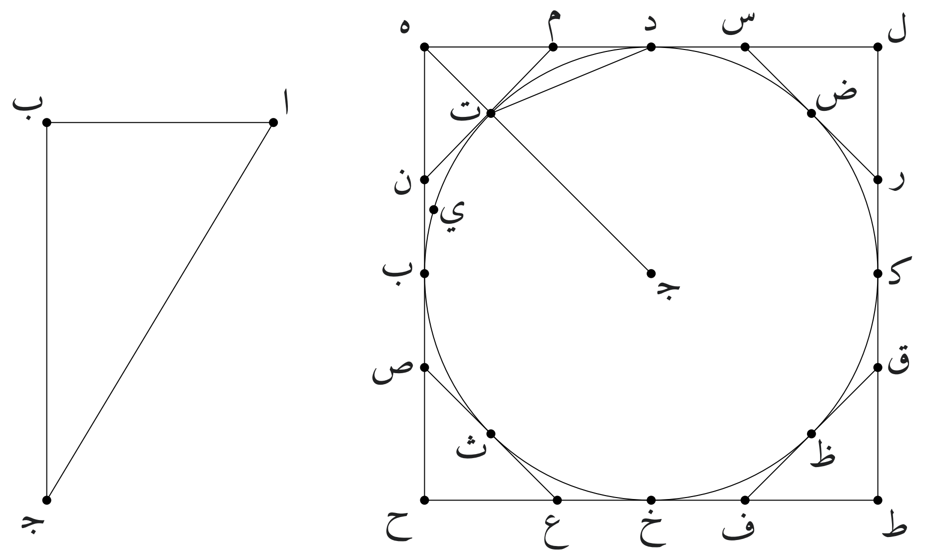

Figure 16: Diagram for *Columbia* 2, corresponding to the second part of *Fatih* 1. **C:** In the diagram, *bā*ʾ (on the circle) and *yā*ʾ are written without dots. In the text, *bā*ʾ, *tā*ʾ, and *khā*ʾ are sometimes written without dots.

ج نسبة الدائرة إلى المربّع الذي يكون من ضرب قطرها في نفسه كنسبة أحد عشر إلى أربعة عشر.

فلتكن دائرة قطرها اب، يحيط بها ⟨مربّع⟩ جح، ونجعل خطّ ده مثلي خطّ جد، ونجعل هز مثل سبع جد. ونصل اه اد از. فلأنّ نسبة مثلّث اجه إلى مثلّث اجد كنسبة واحد وعشرين إلى سبعة، ونسبة مثلّث اجد إلى مثلّث اهز كنسبة سبعة إلى واحد، تكون نسبة مثلّث اجز إلى مثلّث اجد كنسبة اثنين وعشرين إلى سبعة. ولكنّ مربّع جح أربعة أمثال مثلّث اجد، ومثلّث اجز مساوٍ لدائرة اب، لأنّ عمود اج مساوٍ لنصف القطر، وقاعدة جز مساوية لمحيط الدائرة، لأنّ المحيط ثلاثة أمثال القطر ومثل سبعه بالتقريب كما سنبيّن ذلك. فنسبة الدائرة إلى مربّع جح كنسبة أحد عشر إلى أربعة عشر.

لأنّ أربعة أمثال سبعة تكون ثمانية وعشرين، فنسبة مثلّث اجز، أعني الدائرة، إليها
C 27v ⟨كنسبة⟩ اثنين وعشرين | إلى ثمانية وعشرين. وذلك نسبة أحد عشر إلى أربعة عشر كما بيّنه. وذلك ما أردنا أن نبيّن.

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3 ⟨مربّع⟩] (Knorr)   10 أربعة عشر] أربعة وعشرون ظ C، see note 178   12 أحد] إحدى C   13 بيّنه] read.، بنه C*

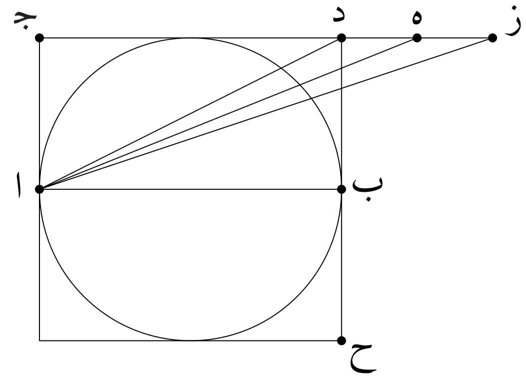

Figure 17: Diagram for *Columbia* 3, corresponding to *Fatih* 2. **C:** All the letters have been written in darker ink but in the same hand. *Zāʾ* resembles a *lām* in the diagram; it is written as a *rāʾ* in the text.

د محيط كلّ دائرة ثلاثة أمثال قطرها، ويزيد أيضًا بأقلّ من سبع القطر وبأكثر من عشرة أجزاء من واحد وسبعين منه.

فلتكن دائرة عليها ب ج ك، وقطرها ج ك، ومركزها د. ونخرج ج ه على زوايا قائمة من القطر، ونخرج ك ه حتّى تكون الزاوية التي على خطّ ك ج ثلث قائمة. فنسبة ك ه إلى ه ج كنسبة ثلاثمائة وستّة إلى مائة وثلاثة وخمسين. فخطّ ك ج في القوّة ثلاثة أمثال خطّ ج ه. وكذلك تكون نسبة ك ج إلى ج ه أعظم من نسبة مائتين وخمسة وستّين إلى مائة وثلاثة وخمسين. ولكنّ ك ه مثلا ج ه. فنسبة ك ج ك ه إلى ج ه أعظم من نسبة خمسمائة وأحد وسبعين إلى مائة وثلاثة وخمسين. ونخرج خطّ ط ك ⟨الذي⟩ يقطع الزاوية التي يحيط بها خطّا ك ه ك ج بنصفين. فنسبة كلا خطّي ك ه ك ج إلى ج ه كنسبة ك ج إلى ج ط. وكذلك تكون نسبة ك ج إلى ج ط أعظم من نسبة خمسمائة وأحد وسبعين إلى مائة وثلاثة وخمسين. فبالمقدار الذي يكون ⟨به⟩ مربّع ج ط ثلاثة وعشرين ألفًا وأربعمائة وتسعة، يكون به مربّع ك ج أكثر من ثلاثمائة ألف وستّة وعشرين ألفًا وأحد وأربعين، وكلا | مربّعي ك ج ج ط أكثر من ثلاثمائة C 28r

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[1] بأقلّ] أقلّ C [2] وبأكثر] وأكثر C [3] ب ج ك] ا ب ج ك C [5] وستّة] وستّ C وخمسين] see Scholium 1 [6] خطّ ج ه] see Scholium 2 مائتين] مائتي C [7] وستّين] وستّين لأنّ مربّع مائتي وخمسة وستّين ~~٧٠٢٢٥ ومربّع مائة وثلاث وخمسين مضروبًا في ثلاثة ٧٢٢٧ وإذا نسب شيء إلى شيئين فنسبته إلى الأوّل أعظم هذا بيّنه أقليدس في~~ C + حاشية C³ sup. (twice, one on each line)، see note 183 مثلا] مثلي C [8] وخمسين] see Scholium 3 [9] ⟨الذي⟩] (Knorr) كلا] كلي C [10] كنسبة ك ج إلى ج ط] see Scholium 4 [11] وسبعين] ~~وخمسين~~ وسبعين C [12] ك ج] ك ج ~~ج~~ C [13] وستّة] وستّ C وأربعين] see Scholium 5

ألف وتسعة وأربعين ألفًا وأربعمائة وخمسين. وكذلك يكون طول كط أعظم من
خمسمائة وواحد وتسعين وثمن بالمقدار الذي يكون به خطّ جط مائة وثلاثة وخمسين.
فنسبة كلا خطّي كط كج إلى جط أعظم من نسبة ألف ومائة واثنين وستّين وثمن
إلى مائة وثلاثة وخمسين. ونخرج أيضًا خطّ كي ⟨الذي⟩ يقطع الزاوية التي يحيط بها
خطّا كط كج بنصفين. فتكون نسبة كج إلى جي أعظم من نسبة ألف ومائة
واثنين وستّين وثمن إلى مائة وثلاثة وخمسين.

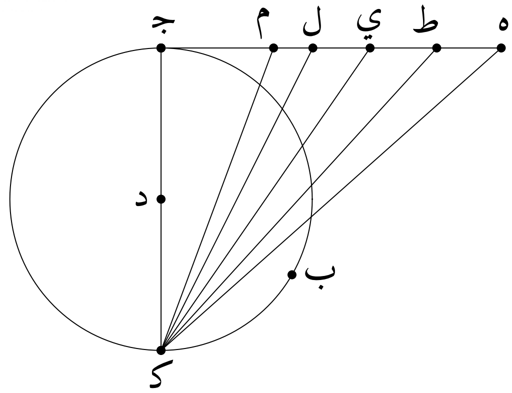

Figure 18: Diagram for *Columbia* 4, corresponding to the first part of *Fatih* 3. **C:** *Bāʾ* and *yāʾ* are written without dots in the diagram and the text. The line *jīm kāf* is tilted. On the manuscript diagram, see also Scholium 8.

فبالمقدار الذي به يكون مربّع جي ثلاثة وعشرين ألفًا وأربعمائة وتسعة، يكون به
مربّع كج أعظم من مائة ربوة وخمسة وثلاثين ربوة وخمسمائة وأربعة وثلاثين ونصف
وجزء من أربعة وستّين، وكلا مربّعي كج جي أعظم من مائة ربوة وسبعة وثلاثين
ربوة وثلاثة آلاف وتسعمائة وثلاثة وأربعين ونصف وجزء من أربعة وستّين. وكذلك
C 28v يكون خطّ كي في الطول أعظم من ألف ومائة واثنين | وسبعين وثمن بالمقدار الذي
به يكون خطّ جي مائة وثلاثة وخمسين. فنسبة كلا كي كج ⟨إلى جي⟩ أعظم
من نسبة ألفين وثلاثمائة وأربعة وثلاثين وربع إلى مائة وثلاثة وخمسين. ونخرج أيضًا

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2 خمسمائة] خمسين مائة **C** وخمسين] see Scholium 6 3 كلا] كلي **C** 4 وخمسين] see Scholium 7
⟨الذي⟩] (Knorr) 5 خطّا] خطّ **C** (Knorr) ومائة] sup. **C** 8 أعظم من] من أعظم **C** (Knorr)
وأربعة] وأربع ~~الربوة~~ **C** + عالت الربوة .mg ³C، see note 187 10 آلاف] ألف **C** 12 كلا] كلي **C**
⟨إلى جي⟩] (Knorr) 13 وأربعة] وأربع **C**

خطّ ك ل ⟨الذي⟩ يقطع الزاوية التي يحيط بها خطّا ك ي ك ج بنصفين. فتكون نسبة ك ج إلى ل ج أعظم من نسبة ألفين وثلاثمائة وأربعة ⟨وثلاثين وربع⟩ إلى مائة وثلاثة وخمسين. فبالمقدار الذي به يكون مربّع ج ل ثلاثة وعشرين ألفًا وأربعمائة وتسعة، يكون به مربّع ج ك أعظم من خمسمائة ربوة و٤٤ ربوة و٨٧٢٣، وكلا مربّعي ج ك ل ج أعظم من ٥٠٠ ربوة و٤٧ ربوة و٢١٣٢. نخطّ ك ل في الطول أعظم من ٢٣٣٩ وربع بالمقدار الذي ⟨يكون⟩ به ج ل ١٥٣. فنسبة كلا خطّي ك ج ك ل إلى ج ل أعظم من ⟨نسبة⟩ ٤٦٧٣ ونصف إلى ١٥٣. ونخرج أيضًا خطّ ك م ⟨الذي⟩ يقطع الزاوية التي يحيط بها خطّا ك ج ك ل بنصفين. ولكنّ نسبة ك ج إلى ج م أعظم من نسبة ٤٦٧٣ ونصف إلى ١٥٣. وخطّ ك ج مثل قطر الدائرة، وخطّ ج م ضلع الشكل الكثير الزوايا المحيط بالدائرة ذي ٩٦ ضلعًا. فبالمقدار الذي يكون به ضلع الشكل ذي ٩٦ زاوية ١٥٣، يكون جميع محيطه ١٤٦٨٨، | ويكون قطر الدائرة أعظم من أربعة آلاف وستّمائة وثلاثة وسبعين ونصف. فتبيّن من ذلك أنّ محيط الشكل ذي ستّة وتسعين ضلعًا المعمول على الدائرة أعظم من ثلاثة أمثال قطر الدائرة، ويزيد بأقلّ من سبعه. وكذلك يكون محيط الدائرة أعظم من ثلاثة أمثال قطرها، ويزيد بأقلّ من سبعه كثيرًا. وذلك ما أردنا أن نبيّن. C 29r

٥ ولتكن دائرة عليها ا ب ج، وقطرها ا ج، ولتكن زاوية ب ا ج أيضًا ثلث قائمة. ونصل ج ب. نخطّ ا ج مثلا خطّ ج ب، ونسبته إليه كنسبة ألف وخمسمائة

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[1] ⟨الذي⟩] (Knorr) [2] وأربعة] وأربع C ⟨وثلاثين وربع⟩] (Knorr) [3] ألفًا] ألف C [4] خمسمائة] .corr، خمس ن ماىه C^* و٤٤] ٤٤ C [6] فنسبة كلا] ونسبة كلي C [7] ⟨الذي⟩] (Knorr) [10–11] ضلع الشكل الكثير الزوايا المحيط بالدائرة ذي ٩٦ ضلعًا. فبالمقدار الذي يكون به ضلع الشكل ذي ٩٦] نصف قطر + ~~ضلع الشكل الكثير الزوايا المحيط بالدائرة ح نصف قطر~~ + C^3 sup. لأنّ C + من الحاشية C^3 sup. + ج م وتر زاوية هي نصف نصف نصف زاوية وهي ثلث قائمة فتكون جزء من ستّة عشر من هذه الزاوية وتكون في الدائرة أربع قوائم فتكون كلّ قائمة ثلاثة أمثال هذه التي ج م وتر جزء من ستّة عشر منه فإذا ضرب ستّة عشر في اثني عشر بلغ ١٩٦ ضلعًا فبالمقدار الذي يكون به ضلع ذي ٩٦ C + إلى ههنا C^3 sup. + ضلع الشكل الكثير الزوايا المحيط الدائرة ذو ٩٦١ ضلعًا فبالمقدار الذي يكون به ضلع ذي ٩٦ C^3 mg.، see note 188 [11] قطر] قطر لأنّ ج ك C + من الحاشية C^3 sup. + قطر الدائرة كما أنّ ج م ضلع شكل والله أعلم C + إلى ههنا C^3 sup.، see note 189 [12] الدائرة] للدائرة C آلاف] ألف C وثلاثة] وثلاث C [13] ذي] ذو C على الدائرة] على دائرة C (Knorr) [14] بأقلّ] أقلّ C [15] بأقلّ] أقلّ C [16] ا ب ج] ا ب ج د C ولتكن] وتكون C أيضًا] C^3، see note 191 [17] مثلا] مثلي C ونسبته] ونسبته + هي C^3 sup.

وستّين إلى سبعمائة وثمانين. وكذلك تكون نسبة اب إلى ب ج أصغر من نسبة ألف وثلاثمائة وأحد وخمسين إلى سبعمائة وثمانين. فنسبة كلا اب اج إلى ب ج أصغر من نسبة ٢٩١١ إلى سبعمائة وثمانين. ونخرج خطّ اه ⟨الذي⟩ يقطع زاوية ج ا ب بنصفين. فتكون نسبة اه إلى ه ج كنسبة ٢٩١١ إلى ٧٨٠. فبالمقدار الذي به يكون مربّع ه ج ستّين ربوة و٨٤٠٠، يكون به مربّع اه أصغر من ثمانمائة ربوة و٤٧ ربوة و٣٩٢١، وجميع مربّعي اه ه ج يكون ٩٠٨ ربوة و٢٣٢١. وكذلك يكون اج في الطول أصغر من ٣٠١٣ وثلاثة أرباع بالمقدار الذي به يكون ه ج ٧٨٠. فنسبة كلا اج ه ا إلى ه ج أصغر من نسبة ٥٩٢٤ وثلاثة أرباع إلى ٧٨٠، التي هي كنسبة ١٨٢٣ إلى ٢٤٠، لأنّ كلّ واحد من هذين | العددين الأخيرين يكون أربعة أجزاء من ثلاثة عشر جزءًا من العددين اللذين قبلهما، كلّ واحد لنظيره. فتكون نسبة كلا اه اج إلى ه ج أصغر من نسبة ١٨٢٣ إلى ٢٤٠. ونخرج أيضًا خطّ اط ⟨الذي⟩ يقطع زاوية ه ا ج بنصفين. فنسبة اط إلى ط ج كنسبة كلا ه ا اج إلى ه ج. فنسبة ط ا إلى ط ج أصغر من نسبة ١٨٢٣ إلى ٢٤٠. فبالمقدار الذي به يكون مربّع ج ط خمس ربوات و٧٦٠٠، يكون به مربّع اط أصغر من ثلاثمائة ربوة و٣٢ ربوة و٣٣٢٩، وكلا مربّعي اط ط ج أصغر من ثلاثمائة ربوة وثمان وثلاثين ربوة و٩٢٩. وكذلك يكون خطّ اج في الطول أصغر من ألف و٨٣٨ و٩ أجزاء من ١١ جزءًا بالمقدار الذي به يكون ج ط مائتين وأربعين، ونسبة كلا اط اج إلى ط ج أصغر من نسبة ٣٦٦١ وتسعة أجزاء من ١١ إلى ٢٤٠. ولكنّ نسبة ٣٦٦١ وتسعة أجزاء من ١١ إلى ٢٤٠ هي نسبة ١٠٠٧ إلى ٦٦، لأنّ كلّ واحد من هذين العددين يكون ١١ من ٤٠ جزءًا من العددين اللذين قبلهما، كلّ واحد لنظيره. فنسبة اط اج إلى ط ج أصغر من نسبة ١٠٠٧ إلى ٦٦. ونخرج أيضًا خطّ اك ⟨الذي⟩ يقطع زاوية ط ا ج بنصفين. ولتكن نسبة اك إلى ك ج كنسبة ١٠٠٧ إلى ٦٦. فبالمقدار الذي به

C 29v

[1] وثمانين] وثمانين ﺣ وخطّ C + من حاشية .C³ sup + اب في القوّة ثلاثة أمثال ب ج C، see note 192
[2] وخمسين] خمسين C وثمانين] وثمانين C + إلى .C³ sup كلا] كلّ C (Knorr) [3] ⟨الذي⟩] (Knorr)
[4] اه إلى] اه إلى كنسبة C + ه ج من حاشية .C³ sup + كلي اب ج فنسبة اه إلى ه ج C + إلى .C³ sup فبالمقدار] وبالمقدار C [7] كلا] كلي C [9] الأخيرين] .corr، الاخرن *C (Knorr) [10] اللذين] .corr، الدن *C كلا] كلي C [11] ⟨الذي⟩] (Knorr) [12] كلا] كلي C [15] وكلا] وكلي C [17] كلا] كلي C [20] جزءًا] جزء C العددين] عددين C (Knorr) اللذين] .corr، الدن *C قبلهما] قبله C (Knorr) [21] ⟨الذي⟩] (Knorr)

يكون مربّع كج ٤٣٥٦، به يكون مربّع اك أصغر من ١٠١ ربوة و٤٠٤٩، وكلا مربّعي كا كج يكون أقلّ من ١٠١ ربوة و٨٤٠٥. وكذلك يكون خطّ اج أصغر في الطول من ١٠٠٩ وسدس، وكلا خطّي اج اك عند كج أقلّ من نسبة ٢٠١٦ وسدس | C 30r إلى ٦٦. ونخرج أيضًا خطّ ال ⟨الذي⟩ يقطع زاوية كاج بنصفين. فتكون نسبة ال إلى لج أصغر من نسبة ٢٠١٦ وسدس إلى ٦٦. فبالمقدار الذي يكون به مربّع لج ٤٣٥٦، فبذلك المقدار يكون مربّع ال أصغر من ٤٠٦ ربوة و٤٩٢٨ وجزء من ٣٦. فكلا مربّعي لا لج أصغر من ٤٠٦ ربوة و٩٢٨٤ وجزء من ٣٦. وكذلك يكون طول خطّ اج أقلّ من ٢٠١٧ وَتُسْعَيْنِ بالمقدار الذي يكون به خطّ جل ٦٦. ولكنّ خطّ جل هو ضلع الكثير الزوايا الذي يكون في الدائرة ذي ٩٦ زاوية. فبالمقدار الذي يكون به جل ٦٦، يكون جميع محيط الكثير الزوايا ذي ٩٦ ⟨زاوية⟩ المرسوم في داخل دائرة ابج ٦٣٣٦، وقطر الدائرة أصغر من هذا ٢٠١٧ وَتُسْعَيْنِ. فيكون كذلك محيط ⟨الكثير الزوايا⟩ ذي ٩٦ زاوية الذي في داخل الدائرة أكثر من ثلاثة أمثال القطر بمائتين وأربعة وثمانين وثلث التي هي أكثر من ١٠ أجزاء من ٧١. وكذلك يكون أيضًا محيط الدائرة أكثر كثيرًا من ثلاثة أمثال القطر و١٠ من ٧١. فقد تبيّن من ذلك أنّ قدر محيط الدائرة عند قطرها أقلّ من ثلاثة أمثال | C 30v وسبعه، وأكثر من ثلاثة أمثال وعشرة أجزاء من واحد وسبعين منه. وذلك ما أردنا أن نبيّن.

تمّ القول المنسوب إلى أرشميدس في مساحة الدائرة ونسبة القطر إلى المحيط. والحمد لله حمدًا كثيرًا وعلى محمّد {السلام}.

[1] وكلا] وكلي C [3] وكلا] وكلي C [4] ⟨الذي⟩] (Knorr) [7] فكلا] فكلي C [8] وَتُسْعَيْنِ] .voc، تسعن C* + خ + mg. illeg. (skel.) C³، see note 198 [9] ذي] ذي هي ذي C، see note 200 [11] المرسوم] .corr، الموسومة C* [12] وَتُسْعَيْنِ] .voc، تسعين C* + خ + mg. illeg. (skel.) C³، see note 198 ⟨الكثير الزوايا⟩] (Knorr) الذي] التي C [13] أجزاء] جزءًا C [19] {السلام}] illeg. (skel.) C

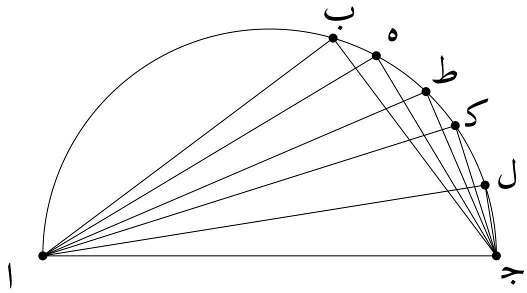

Figure 19: Diagram for *Columbia* 5, corresponding to the second part of *Fatih* 3. **C:** *Bāʾ* is often written without a dot in the diagram and the text. The diagram has a complete circle.

Propositions Useful for the Book of Archimedes, of Abū al-Rashīd ʿAbd {al-Hādī}[131]

⟨Preliminary 1⟩[132] Every equilateral quadrilateral[133] in a circle is greater than the half of [the circle], since a quarter of the whole of the greatest quadrilateral[134]—that is, the triangle AEG—is smaller than a quarter of the circle †by an arc[135] which, with AZG,[136] is its equal,[137] the whole of the greatest quadrilateral is greater than half of the circle by the amount of four times[138] AZG.†

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[131] *Al-Hādī* is Knorr's (1989, 543, 552) reading. Unfortunately, the word after *ʿabd* in the manuscript is almost completely illegible since perhaps another scribe attempted to redraw part of it in darker ink; the only legible feature I can discern is a final *yāʾ* that is not connected to the preceding letter. *Al-Bāriʾ* is, to my mind, equally plausible. In any case, Knorr's (1989, 543) suggestion that the author of the *Columbia* version is one Abū al-Rashīd Mubashshir ibn Aḥmad ibn ʿAlī is almost certainly wrong. See Section I.3.2.

[132] **C** 24r.17–24v.1. The text of the proof of the proposition is unclear, but the main thrust of the argument is obvious enough; namely, the square $AGBD$ is greater than the semicircle by twice the area bounded by the arc AG and the lines AZ and ZG. Knorr (1989, 552) attempts to correct the deficiencies in the proof by reinterpreting some of the expressions, some of which are pointed out below. For my part, I suspect textual corruption in the indicated range.

[133] That is, a square (Knorr 1989, 552).

[134] Here and below, I translate *murabbaʿ* as "quadrilateral."

[135] That is, the segment of the circle on the chord AG (Knorr 1989, 552).

[136] This correction, which is suggested by Knorr (1989, 552), is justified by the fact that the author of these propositions uses the letters AZG for the segment of the triangle AZG outside the circle at the very end of this proposition.

[137] That is, the equal of the triangle AEG. One way to interpret the passage *aṣghar min rubʿ al-dāʾira bi-qaws, wa-huwa maʿa alif zāʾ jīm ḍiʿfuhu*, suggested to me by Nathan Sidoli and which I have adopted in the translation, is to assume that *wa-huwa* refers to *qaws* (*qaws* having masculine gender being admittedly rare) and *ḍiʿf* is used in the sense of "equal." Another solution would be to take *wa-huwa* to refer to *rubʿ al-dāʾira* and *ḍiʿfuhu* to mean the double of the triangle AEG. In that case, the text would be stating that the quarter circle, together with AZG, would be equal to twice the triangle AEG. Both solutions are mathematically correct.

[138] The correct multiple should be two; Knorr (1989, 552) suggests a correction to "twice."

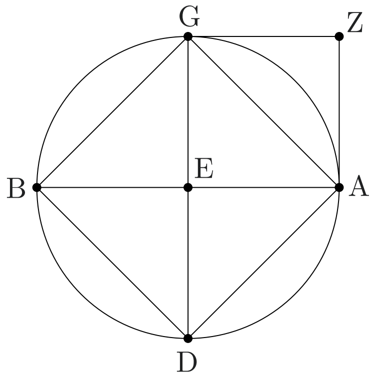

Figure 20: Diagram for *Columbia Preliminary* 1.

⟨Preliminary⟩ 2[139] †If we remove from the arc[140] that is on the side of the square that which is ⟨on⟩ the side of the octagon,[141] it[142] is greater than half of what remains from the circle after[143] the square,†[144] for if we construct on the side *DB*, then *DMT* is greater than the arc *DT*[145] by the amount of the segment *DZT*,[146] since the triangle *DMT* is half of *DZTM*. So it is greater than half of arc *DTM*.[147]

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[139] **C** 24v.1–24v.4. Again, there is reason to suspect textual corruption in the indicated range on account of the unclear meaning.

[140] That is, the segment of the circle on the side *DB* of the square (Knorr 1989, 552).

[141] That is, the two segments of the circle on the sides *DT* and *TB* of the octagon. Adding the word *ʿalā* to the text clarifies the meaning considerably. Knorr (1989, 552), not having made that addition, thinks that what is meant here is "the triangle bounded by a side of the square and the corresponding two sides of the octagon," that is, the triangle *DTB*. But it makes for a smoother reading if we are removing the two smaller segments of the circle from the greater segment of the circle, and then stating a conclusion about the remainder, which is the triangle *DTB*.

[142] That is, the triangle *DTB* (Knorr 1989, 552).

[143] That is, minus (Knorr 1989, 552).

[144] Again, the segment of the circle on the side *DB* of the square is meant.

[145] That is, the segment of the circle on the side *DT* of the octagon (Knorr 1989, 552).

[146] That is, the area bounded by the arc *DT* and the lines *DZ* and *ZT*.

[147] That is, the triangle *DMT* is greater than half of the area bounded by the arc *DT* and the lines *DM* and *MT*. Extending this result by symmetry to the triangle *MTB* yields the statement of the proposition, namely that the triangle *DTB* is greater than half of the area bounded by the arc *DB* and the line *DB*.

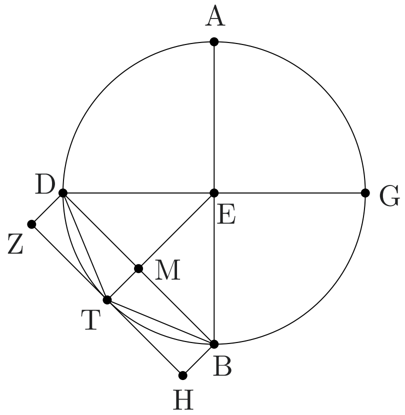

Figure 21: Diagram for *Columbia Preliminary* 2.

⟨Preliminary 3⟩[148] Every circle in an equilateral quadrilateral[149] is greater than half of the quadrilateral. Let the circle be *IZHT*, and [let] the quadrilateral [be] *ABGD*. So the quadrilateral *ABGD* is divided by four triangles inside [the circle] [that are] equal to each other and equal to the four [triangles] that are partially outside the circle.[150] The four interior triangles are half of the greatest quadrilateral,[151] and the four[152] are inside the circle, so the circle is greater than half of the quadrilateral that is [constructed] on it.

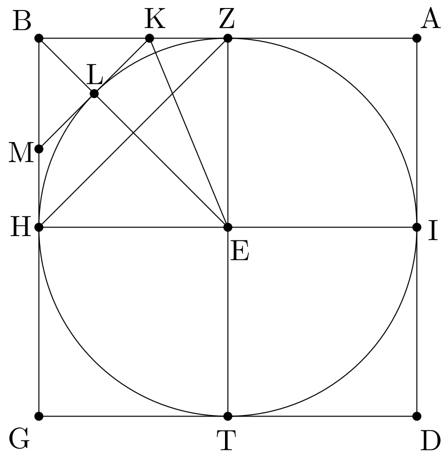

Figure 22: Diagram for *Columbia Preliminary* 3.

[148] **C** 24v.5–25r.1. Knorr (1989, 553, 561, n. 4) himself notes that his Propositions 3 and 4 use the same figure. I have joined his two propositions into one. See also note 155.

[149] That is, a square.

[150] The interior triangles are *ZEH* and its counterparts, and the triangles partially outside the circle are *ZBH* and its counterparts.

[151] That is, the square *ABGD* (Knorr 1989, 553).

[152] That is, the four interior triangles.

Let us also draw ⟨a line⟩ from *E* to *B*, which is *ELB*, and from *L*, the point of tangency [let us draw] a line on two sides of [the line *ELB*],[153] which is the line *MLK*. Then I say ⟨that⟩ *ZK* is equal to *KL*. Its proof: Let us join *EK*. Since *EZ* is equal to *EL*, the angles *EZB* and *ELK* are right, and the line *EK* is common, if we remove the square of *EZ* from the square of *EK*, there remains the square of *KZ*. And *EL* is equal to *EZ*, so *ZK* is equal to *KL*, as we wanted.[154]

⟨Preliminary⟩ 4[155] *AG*, the chord of the right angle from the triangle *ABG*, is twice the chord of the angle that is a third of a right angle. Its instantiation is this: *AD* is equal to *AB* and *DG* is equal to *AB*.[156] So *AG*, the chord of the right angle, is twice *AB*, the chord of a third of a right [angle], by assumption. And it is clear from this diagram that *BG* is the chord of two-thirds [of a right angle].[157] So it is, in power, three times *AB*,[158] the chord of a third of a right angle, since if a square is constructed on *AG*, the diameter, then it is four times that which is on half of the diameter, namely *AD*, namely *AB*, and if the square of *AB* is removed from the square of *AG*, then there remains from it three[159] times the square of *AB*. God knows best.

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[153] Knorr (1989, 553) presumably reads حهه of the manuscript as *jihatihi* since he translates this word as "its direction." The correction to *jihatayhi* is necessitated by the fact that *KLM* extends on both sides of *ELB*.

[154] Elements of this paragraph—the specification marked with *fa-aqūl*, the proof with *burhānuhu*, ending with *kamā aradnā*—as well as the fact that it is unrelated to the statement of *Columbia Preliminary* 3 even though it uses the same diagram, indicate that it could be an interpolation.

[155] **C** 25r.1–25r.9. Knorr (1989, 554) labels this proposition as the fifth. However, there is a large *dāl* above the first line in f. 25v that shows that it must be the fourth. Since the proof of this proposition seems confused and incomplete, again, there is probably a fair amount of corruption in the text.

[156] Knorr (1989, 554) thinks this could be *AD* or *AB*. However, the manuscript clearly has a *bā'*.

[157] It appears that the scribe who copied the text broke the definite article across two lines (he did this on two other occasions in this proposition), and this was corrected later, by a hand using the same ink as the text. Yet another hand, possibly different this time since he used black ink, put two dots arranged vertically on top of each *thā'* of the word *thulthayn*.

[158] Knorr (1989, 554, 561, n. 6) reads the last letter ح as a *ḥā'*, standing for *ḥīna'idhin*, which he translates "at the same time." That makes for an awkward reading by introducing an extra word in the middle of the apposition. It seems simpler to assume that the last letter is a *jīm* without its dot.

[159] This is the correct value, as pointed out by Knorr (1989, 554).

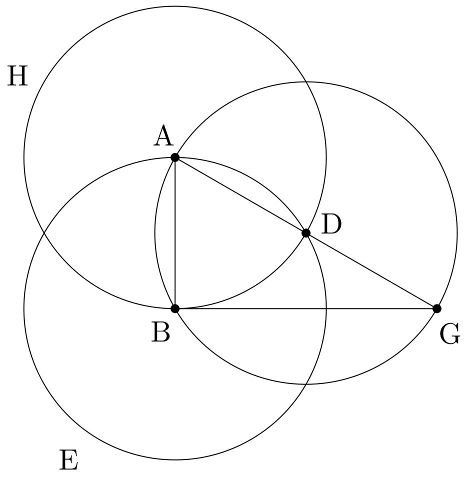

Figure 23: Diagram for *Columbia Preliminary* 4. Knorr (1989, 554) does not reproduce the circles H and E in their entirety.

In the name of God, the Most Gracious, the Most Merciful

Treatise Attributed to Archimedes on the Measure of the Circle

⟨1⟩[160] Every circle is equal to a right-angled triangle one of whose sides surrounding the right angle is equal to the perimeter of the circle and [whose] other side is equal to half of the diameter of the circle.

Let there be a circle on which are *ABGD*, [let there be] a right-angled triangle on which are *EZH*, [let] the angle which is at the point *Z* [be] right, *EZ* is equal to half of the diameter of the circle, and *ZH* is equal to the perimeter of that circle. It is clear that the circle *ABGD* is equal to the triangle *EZH*.

For if it is not so, [the circle] is the greater of the two or the smaller of the two. First, let the circle *ABGD* be greater than the triangle *EZH*. We make inside the circle a square on which are *ABGD*. So [something] greater than its half, which is the square *ABGD*, has been removed from the circle *ABGD*.[161] We cut the arcs *ATB*, *BKG*, *GLD*, and *DMA* in halves at the points *T*, *K*, *L*, and *M*. We join *AT*, *TB*, *BK*, *KG*, *GL*, *LD*, *DM*, and *MA*. So [something] greater than their half, which is *ATB*, *BKG*, *GLD*, and *DMA*,[162] has also been removed from the remainder of the segments of the circle *ABGD*.[163] And if we do that repeatedly,[164] there will be cut off remainders smaller than the excess of the circle *ABGD* over the triangle *EZH*. So let there remain the segments *ANT*, *TUB*, *BQK*, *KFG*, *GCL*, *LRD*, *DOM*, and *MYA* smaller than the excess of the circle *ABGD* over the triangle *EZH*. So the polygon on which are *ATBKGLDM* is greater than the triangle *EZH*. We make the center of the circle *ABGD* the point *P*, and we draw from the center *P* a perpendicular to one of the sides of the polygon, on which are *PX*. Since the line *ZH* is equal to the perimeter of the circle *ABGD*, which is greater than the perimeter of the polygon, on which are *ATBKLGDM*, the line *ZH* is greater than the perimeter of *ATBKGLDM* the polygon. Also, since the line *EZ* is equal to half of the diameter of the circle *ABGD*, it is greater than the line *PX*. So that which ensues from the product of *EZ* and *ZH* is greater than that which ensues from the product of *PX* and the perimeter of *ATBKLGDM* the polygon. And their halves are also thus.[165] So the triangle *EZH* is greater than *ATBKGLDM* the polygon. And that is impossible

[160] **C** 25r.11–26r.5. *Columbia* 1 corresponds to the first part of *Fatih* 1.

[161] By *Columbia Preliminary* 1.

[162] That is, the triangles with these vertices (Knorr 1989, 554).

[163] By *Columbia Preliminary* 2.

[164] A nonliteral translation of the Arabic *ʿalā mā yatlū*. Knorr (1989, 555) translates literally as "over what follows."

[165] That is, they satisfy the same inequality.

since [the triangle] was proved to be smaller than [the polygon]. Therefore the circle *ABGD* is not greater than the triangle *EZH*. And that is what we wanted to prove.

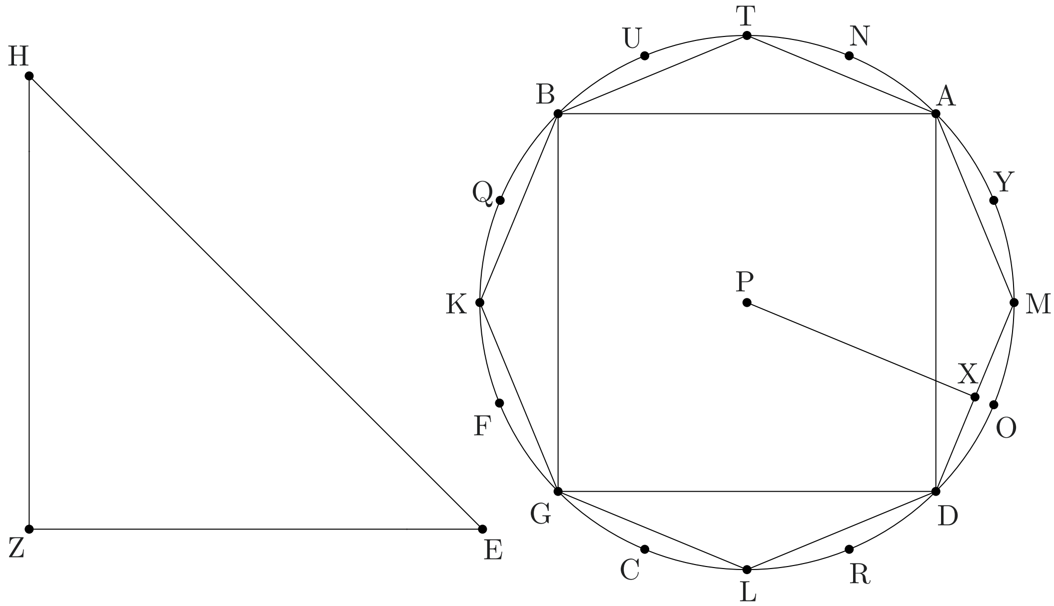

Figure 24: Diagram for *Columbia* 1, corresponding to the first part of *Fatih* 1.

⟨2⟩[166] If possible let the circle *DBK* be smaller than the triangle *ABG*. We make on the circle *DBK* a square that surrounds it, on which are *HTLE*. So [something] greater than its half has been cut from the square *HTLE*, which is the circle *DBK*.[167] We draw from the center *G* a line on which are *GPE*. We draw from the point *P* a line tangent to the circle on which are *NM*. And thus also *UQ*, *FC*, and *RS*. So the line *NM* has been separated in two halves at the point *P*, the line *GP* is perpendicular to *NM*, and thus also the remaining lines. We join *PD*. Since *EN* and *EM* are greater than *MN*, their halves are also thus, so the line *EM* is greater than *MP*, which is equal to *DM*.[168] So the triangle *EPM* is greater than half of the triangle *EPD*, and all the more is it greater than half of the segment *PDE*,[169] and thus *EPN* is greater than half of the segment *BIPE*. So all of *MEN* is greater than half of *BDE*.[170] And thus is it that every one of *UHQ*, *FTC*, and *RLS* cuts from ⟨every one of⟩ *BHX*, *XTK*, and *KLD*[171] [something] greater than its half. And if we

[166] **C** 26r.6–27r.3. *Columbia* 2 corresponds to the second part of *Fatih* 1.

[167] By *Columbia Preliminary* 3.

[168] That $MP = DM$ follows from *Columbia Preliminary* 3.

[169] That is, the region bounded by the lines *PE*, *ED*, and the arc *PD*. Similar explanations apply to the other segments mentioned in the proof.

[170] The segment *BDE* is the region bounded by the lines *BE*, *ED*, and the arc *BD*.

[171] Again, the respective segments are meant.

do that repeatedly,[172] †there will be cut off from the square ⟨that which⟩ is smaller than the supposed [thing].†[173] So let there remain the segments *DMP*, *PNB*, *BUY*, *YQX*, *XFẒ*, *ẒCK*, *KRḌ*, and *ḌSD* smaller than the deficit of the circle *DBK* from the triangle *ABG*. So the polygon, on which are *MNUQFCRS*, is smaller than the triangle *ABG*. And since the line *BG* is equal to the perimeter of the circle *DBK*, but[174] the perimeter of *MNUQFCRS* is greater than the perimeter of the circle *DBK*, the perimeter of *MNUQFCRS* the polygon is greater than the line *BG*. But *AB* is equal to the line *PG*, so that which ensues from the product of the perimeter of *MNUQFCRS* and the line *PG* is greater than the product of *AB* and *BG*. And their halves are also thus. So *MNUQFCRS* the polygon is greater than the triangle *ABG*. And that is impossible since it was proved that it was smaller. So the circle *DBK* is not smaller than the triangle *ABG*. But it was proved in what preceded that it was not greater. So the circle *DBK* is equal to the triangle *ABG*. But the area of *ABG* is equal to that which ensues from ⟨the product of⟩ the line *AB* and half of *BG*, and the line *BG* is equal to the perimeter of the circle *DBK*. So that which ensues from the product of half of the diameter and half of the perimeter of the circle is equal to the area of the triangle *ABG*.

And for this reason we multiply half of the diameter by half of the perimeter, so there ensues from that the area of the assumed circle. And that is what we wanted to prove.

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[172] A nonliteral translation of the Arabic *fīmā yatlū*. Knorr (1989, 556) translates literally as "in what follows."

[173] The *hiya* after *aṣghar* (on the last line of f. 26r) probably stands for *intihāʾ* ("end") (Gacek 2001, 146, s.v. "intihāʾ"). The reason why the text is supposed to end here is unclear and the text between the obeli is in all likelihood corrupt. But the next sentence may provide a clue as to how the extant words should be interpreted: *alladhī* ("that which") refers to the segments *DMP*, *PNB*, etc. and *al-mawḍūʿ* ("the supposed [thing]") refers to the difference of the areas of the triangle *ABG* and the circle *DBK*. Then, the polygon *MNUQFCRS* that is circumscribed about the circle is smaller in area than the triangle *ABG*, just like the text states. However, this interpretation is problematic in that the verb *yataqāṭaʿu* seems awkward to use for the aforementioned segments.

[174] The *khāʾ* probably stands for *nuskha* ("copy" or "variant reading") (Gacek 2001, 140, s.v. "nuskhah"), which may imply that the scribe corrected the word *wa-lākinna* from another manuscript.

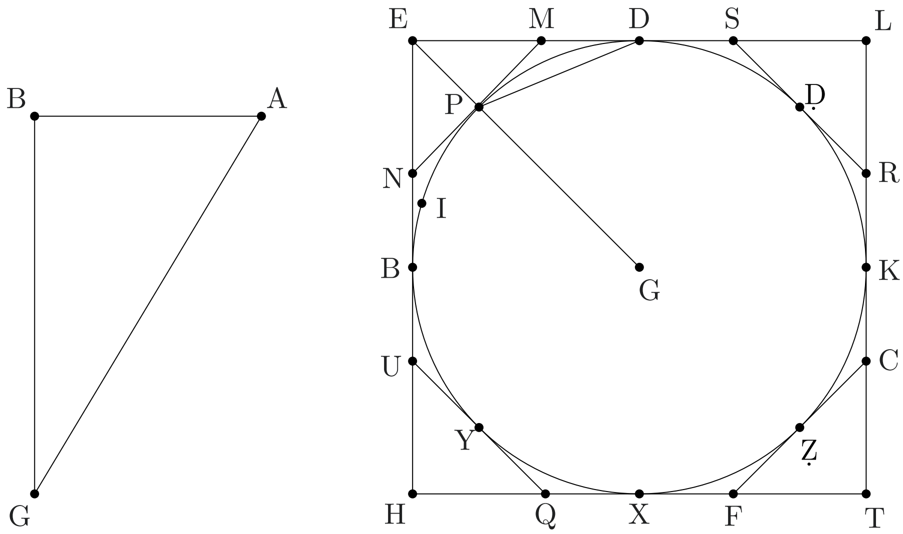

Figure 25: Diagram for *Columbia* 2, corresponding to the second part of *Fatih* 1. Knorr (1989, 555–556, 561, n. 8) labels the *bāʾ* in the square as "*Z*" and the *jīm* in the center as "*k*." The *yāʾ* is not shown in his diagram, even though it appears in his text.

3[175] The ratio of the circle to the square that ensues from the product of its diameter by itself is as the ratio of 11 to 14.

Let there be a circle whose diameter is *AB*, and which ⟨the square⟩ *GH* surrounds, we make the line *DE* equal to twice the line *GD*, and we make *EZ* equal to a seventh of *GD*. We join *AE*, *AD*, and *AZ*. Since the ratio of the triangle *AGE* to the triangle *AGD* is as the ratio of 21 to 7, and the ratio of the triangle *AGD* to the triangle *AEZ* is as the ratio of 7 to 1, the ratio of the triangle *AGZ* to the triangle *AGD* is as the ratio of 22 to 7. But the square *GH* is four times the triangle *AGD*, and the triangle *AGZ* is equal to the circle *AB*,[176] since the perpendicular *AG* is equal to half of the diameter, and the base *GZ* is equal to the perimeter of the circle, for the perimeter is three times the diameter and a seventh of [the diameter] approximately as we shall prove that.[177] So the ratio of the circle to the square *GH* is as the ratio of 11 to 14.[178]

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175 **C** 27r.3–27v.2. *Columbia* 3 corresponds to *Fatih* 2.

176 Again, this equality follows from *MC* 1 (*Columbia* 1 and 2) and *MC* 3 (*Columbia* 4 and 5). See note 105.

177 Knorr (1989, 557) probably reads سنبين of the manuscript as *sa-yubayyanu* since he translates the relevant part as "as that shall be proved."

178 Knorr (1989, 557, 561, n. 10) reads وعشرون ظ of the manuscript as *wa-ʿashar* followed by *wāw*, *nūn*, and *ṭāʾ*; unable to translate the final *ṭāʾ*, he notes that the construal of the *wāw* and the *nūn* with the preceding *ʿashar* would yield an incorrect value of 24. In fact, the last letter is a *ẓāʾ*,

Since 4 times 7 is 28, the ratio of the triangle *AGZ*—that is, the circle—to it[179] is ⟨as the ratio of⟩ 22 to 28. And that is the ratio of 11 to 14, as he proved.[180] And that is what we wanted to prove.

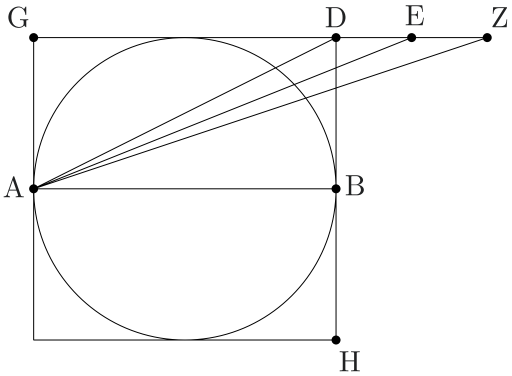

Figure 26: Diagram for *Columbia* 3, corresponding to *Fatih* 2.

4[181] The perimeter of every circle is three times its diameter, and also exceeds [it] by [something] less than a seventh of the diameter and more than 10/71 of [the diameter].

Let there be a circle on which are *BGK*, whose diameter is *GK*, and whose center is *D*. We draw *GE* at right angles to the diameter, and we draw *KE* so that the angle that is on the line *KG*[182] becomes a third of a right [angle]. So the ratio of *KE* to *EG* is as the ratio of 306 to 153 (see Scholium 1). So the line *KG* is, in power, three times the line *GE* (see Scholium 2). Similarly, the ratio of *KG* to *GE* is greater than the ratio of 265 to 153.[183] But *KE* is twice *GE*. So the ratio of *KG* and *KE* to *GE* is greater than the ratio of 571 to 153 (see Scholium 3). We draw the line *TK*, ⟨which⟩ cuts the angle that the lines *KE* and *KG* surround in halves.

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which is an abbreviation indicating a conjecture (Gacek 2001, 96, s.v. "ẓann"). Apparently the scribe had doubts about the correctness of the value 24.

[179] That is, the number 28.

[180] Knorr (1989, 557) probably reads ٮنه of the manuscript as *bayyin* since he translates the relevant part as "as is evident." This sentence together with the preceding one are probably an interpolation.

[181] **C** 27v.2–29r.6. *Columbia* 4 corresponds to the first part of *Fatih* 3. In all probability, both the text and the diagram of this proposition have been corrupted to some extent.

[182] That is, the angle *EKG*.

[183] Knorr (1989, 561, n. 13) is surely right in supposing that the manuscript text from *li-anna* to *fī*, most of which is struck through and then marked with the word *ḥāshiya* ("margin") twice, was a scholium that was inserted by the copyist into the main text by error; note also that there is a *signe de renvoi* on top of *wa-sittīn*, possibly intended to indicate a correction. For *KG* : *GE*, one can use *Elements* I.47 to find that the square of *KG* is 70227, which is slightly larger than the square of 265.

So the ratio of [the sum of] both lines *KE* and *KG* to *GE* is as the ratio of *KG* to *GT* (see Scholium 4).[184] Thus, the ratio of *KG* to *GT* is greater than the ratio of 571 to 153. So by the amount ⟨by⟩ which the square of *GT* is 23409, the square of *KG* is more than 326041 (see Scholium 5), and [the sum of] the squares of both *KG* and *GT* are more than 349450.[185] Thus, the length of *KT* is greater than 591 1/8 by the amount by which the line *GT* is 153 (see Scholium 6). So the ratio of [the sum of] both lines *KT* and *KG* to *GT* is greater than the ratio of 1162 1/8 to 153 (see Scholium 7).

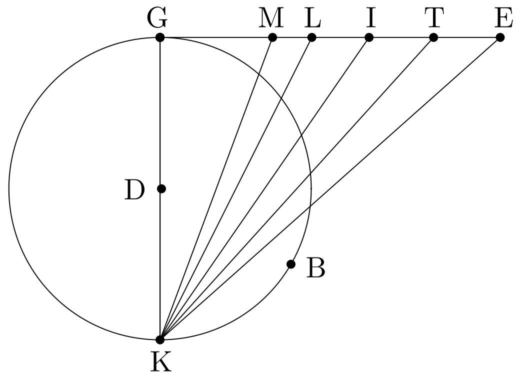

Figure 27: Diagram for *Columbia* 4, corresponding to the first part of *Fatih* 3. Since the text states that *GK* is the diameter, I have kept the manuscript diagram as it is. Knorr (1989, 545, 558, 562, n. 23) thinks that the manuscript diagram is mistaken in taking *GK* for the diameter of the circle, and he produces a different ("corrected") version of the diagram where *GK* is the radius. He then extends *GE* in the other direction and measures *GM′* equal to *GM*. Then *MM′* is the side of a regular 96-gon circumscribed about the circle with the required side length and the proof works. The disadvantage of this correction is that, in the text, there is no indication of *GE* being extended in the other direction (see Scholium 8).

And also, we draw the line *KI*, ⟨which⟩ cuts the angle that the lines *KT* and *KG* surround in halves. So the ratio of *KG* to *GI* is greater than the ratio of 1162 1/8 to 153.[186] So by the amount by which the square of *GI* is 23409, the square of *KG* is greater than 1350534[187] 1/2 1/64, and [the sum of] the squares of both *KG* and

[184] By Scholium 4, $KE : KG = ET : TG$ (via *Elements* VI.3). By composition and alternation, $KE + KG : GE = KG : TG$.

[185] Redefining $GT = 153$ and hence $GT^2 = 23409$ by some other measure forces $KG > 571$, where $571^2 = 326041$. Then, $KG^2 + GT^2 > 349450$.

[186] *Elements* VI.3 gives $KT : KG = IT : GI$. By composition and alternation, $KG : GI = KT + KG : GT$.

[187] There is a smudge of red ink diagonally across the word *al-ribwa* in the manuscript, which I have taken to be a deliberate erasure since the word is out of place there.

GI are greater than 1373943 1/2 1/64. Thus, the line *KI* is greater in length than 1172 1/8 by the amount by which the line *GI* is 153. So the ratio of [the sum of] both *KI* and *KG* ⟨to *GI*⟩ is greater than the ratio of 2334 1/4 to 153. And also, we draw the line *KL*, ⟨which⟩ cuts the angle that the lines *KI* and *KG* surround in halves. So the ratio of *KG* to *LG* is greater than the ratio of 23⟨3⟩4 ⟨1/4⟩ to 153. So by the amount by which the square of *GL* is 23409, the square of *GK* is greater than 5448723, and [the sum of] the squares of both *GK* and *LG* are greater than 5472132. Thus, the line *KL* is greater in length than 2339 1/4 by the amount by which the line *GL* ⟨is⟩ 153. So the ratio of [the sum of] both lines *KG* and *KL* to *GL* is greater than ⟨the ratio of⟩ 4673 1/2 to 153. And also, we draw the line *KM*, ⟨which⟩ cuts the angle that the lines *KG* and *KL* surround in halves. But the ratio of *KG* to *GM* is greater than the ratio of 4673 1/2 to 153. The line *KG* is equal to the diameter of the circle, and the line *GM* is the side of the polygonal figure of 96 sides that surrounds the circle.[188] So by the amount by which the side of the figure of 96 angles is 153, the whole of its perimeter is 14688, and the diameter[189] of the circle is greater than 4673 1/2. So it has become clear from that that the perimeter of the figure with 96 sides [that is] constructed on the circle is greater than three times the diameter of the circle, and exceeds [it] by [something] less than a seventh of [the diameter]. All the more is the perimeter of the circle greater than three times its diameter, and exceeds [it] by [something] less than a seventh of [the diameter]. And that is what we wanted to prove.

5[190] Let there be a circle on which are *ABG*, whose diameter is *AG*, and let the angle *BAG* also[191] be a third of a right [angle]. We join *GB*. So the line *AG* is twice the line *GB*, and its ratio to it is as the ratio of 1560 to 780.[192] Similarly the ratio of *AB* to *BG* is smaller than the ratio of 1351 to 780. So the ratio of [the sum of]

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188 Since several lines of text at the bottom of f. 28v have been marked by a corrector (**C³**) as an interpolation with the words "from the margin" (*min al-ḥāshiya*) and "up to here" (*ilā hāhunā*), and another line containing the letter *ḥāʾ* for *ḥāshiya* ("margin") has been struck out (Gacek 2001, 33, s.v. "ḥāshiyah"), blocks [8] and [9] in Knorr (1989, 559) disappear from this edition and translation. Knorr (1989, 562, n. 26) erroneously reads *min al-ḥāshiya* as *min al-ḥāsib* ("from the calculator").

189 Again, most of the first line on f. 29r has been marked by a corrector (**C³**) as an interpolation with the words "from the margin" (*min al-ḥāshiya*) and "up to here" (*ilā hāhunā*).

190 **C** 29r.6–30v.2. *Columbia* 5 corresponds to the second part of *Fatih* 3.

191 The word *ayḍan* is written in a third hand (**C³**) over a word that is now illegible except for the *lām* and *thāʾ* at the end; the three dots of the initial *thāʾ* of the word *thulth* have also been marked in this hand.

192 Of the segment of the text, contained in lines 9–11 of f. 29r, starting with the letter *ḥāʾ* for *ḥāshiya* ("margin") (Gacek 2001, 33, s.v. "ḥāshiyah") and which is marked with "from a margin" (*min ḥāshiya*) and "up to" (*ilā*) by a corrector (**C³**), I have removed only the part saying *AB* is

both AB and AG to BG is smaller than the ratio of 2911 to 780. We draw the line AE, ⟨which⟩ cuts the angle GAB in halves. So the ratio of AE to EG is as the ratio of 2911 to 780.[193] So by the amount by which the square of EG is 608400, the square of AE is smaller than 8473921, and the whole of the squares of AE and EG is 9082321.[194] Thus, AG is smaller than 3013 3/4 in length by the amount by which GE is 780.[195] So the ratio of [the sum of] both AG and EA to EG is smaller than the ratio of 5924 3/4 to 780, which is as the ratio of 1823 to 240, since each one of these two latter numbers is 4/13 of the two numbers that are before them, each one to its counterpart. So the ratio of [the sum of] both AE and AG to EG is smaller than the ratio of 1823 to 240. And also, we draw the line AT, ⟨which⟩ cuts the angle EAG in halves. So the ratio of AT to TG is as the ratio of [the sum of] both EA and AG to EG. So the ratio of TA to TG is smaller than the ratio of 1823 to 240. So by the amount by which the square of GT is 57600, the square of AT is smaller than 3323329, and [the sum of] the squares of both AT and TG are smaller than 3380929. Thus, the line AG is smaller than 1838 9/11 in length by the amount by which GT is 240, and the ratio of [the sum of] both AT and AG to TG is smaller than the ratio of 3661 9/11 to 240. But the ratio of 3661 9/11 to 240 is the ratio of 1007 to 66, since each one of these two numbers is 11/40 of the two numbers that are before them, each one to its counterpart. So the ratio of AT and AG to TG is smaller than the ratio of 1007 to 66. And also, we draw the line AK, ⟨which⟩ cuts the angle TAG in halves. So let the ratio of AK to KG be as the ratio of 1007 to 66.[196] So by the amount by which the square of KG is 4356, the square of AK is smaller than 1014049, and [the sum of] the squares of both KA and KG are less than 1018405. Thus, the line AG is smaller than 1009 1/6 in length, and [the sum of] both of the lines AG and AK relative to[197] KG are less than the ratio of 2016 1/6 to 66. And also, we draw the line AL, ⟨which⟩ cuts the angle KAG in halves. So the ratio of AL to LG is smaller than the ratio of 2016 1/6 to 66. So by the amount by which the square of LG is 4356, by that amount the square of AL is smaller than 4064928 1/36. So [the sum of] the squares of both LA and LG

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three times BG in power because the ratio $AB : BG$ itself is necessary for calculating $AB + AG : BG$ below.

[193] Labeling the intersection of BG and AE as P, *Elements* VI.3 gives $AB : BP = AG : GP$. Since $AE : EG = AB : BP$ by similarity of the triangles ABP and AEG, $AE : EG = AB + AG : BG$. This ratio is smaller than $2911 : 780$.

[194] The sum of these two squares is less than 9082321.

[195] By *Elements* I.47, the square of AG is equal to the sum of the squares of AE and EG, which is smaller than 9082321. Taking the square root of this number yields the statement.

[196] The stated ratio is smaller than $1007 : 66$.

[197] Here and in other instances of *ʿinda*, I have followed Knorr's (1989, 560–561) translation as "relative to."

are smaller than 4069284 1/36. Thus, the length of the line AG is less than 2017 2/9[198] by the amount by which the line GL is 66.[199] But the line GL is the side of the polygon of[200] 96 angles that is in the circle. So by the amount by which GL is 66, the whole of the perimeter of the polygon of 96 ⟨angles⟩ drawn inside the circle ABG is 6336, and the diameter of the circle is smaller than this 2017 2/9.[201] Thus, the perimeter ⟨of the polygon⟩ of 96 angles that is inside the circle is more than three times the diameter by 284 1/3, which is greater than 10/71.[202] All the more is the perimeter of the circle more than three times the diameter and 10/71. So it has become clear from that that the size[203] of the perimeter of the circle relative to its diameter is less than three times and a seventh of [the diameter], and more than three times and 10/71 of [the diameter]. And that is what we wanted to prove.

The treatise attributed to Archimedes on the measurement of the circle and the ratio of the diameter to the perimeter is complete. Much praise to God and on Muhammad {peace}.

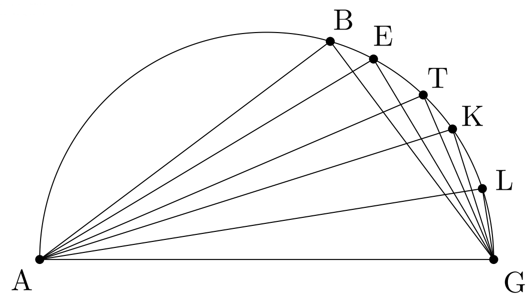

Figure 28: Diagram for *Columbia* 5, corresponding to the second part of *Fatih* 3.

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[198] Perhaps a corrector ($\mathbf{C^3}$) read the word as *tisʿīn* by error. The abbreviation *khāʾ* in the margin might stand for *nuskha* (“copy” or “variant reading”) (Gacek 2001, 140, s.v. “nuskhah”), or perhaps *khaṭaʾ* (“error”). The marginal correction is illegible. See also the discussion by Knorr (1989, 546).

[199] In fact, the square root of 4069284 1/36 is slightly greater than 2017 2/9, so the conclusion does not hold. One can state instead that AG is less than 2017 1/4.

[200] The *hiya* after *dhī* (on line 7 of f. 30r) probably stands for *intihāʾ* (“end”) (Gacek 2001, 146, s.v. “intihāʾ”). The reason why the sign is used here is not clear: it is followed by the diagram of the proposition, after which there is another *dhī* (on line 8) and the text continues without any noticeable disruption in meaning.

[201] See note 198.

[202] With the observation that AG is less than 2017 1/4, the assertion that the perimeter of the polygon exceeds 3 10/71 times the diameter is true.

[203] I follow Knorr (1989, 561, 562 n. 38) in translating *qadr* as “size.”

IV.2.1 The Columbia *Version: Scholia*

لأنّ وتر الزاوية القائمة مثلا وتر الزاوية التي هي ثلث القائمة لما عرّف في موضعه.

Scholium 1.[204] Since the chord[205] of a right angle is twice the chord of the angle that is a third of a right angle according to what has been explained in its place.

لأنّ وتر ثلثي القائمة في القوّة ثلاثة أمثال وتر ثلث القائمة لما عرّف أيضًا.

Scholium 2.[206] Since the chord[207] of two-thirds of a right angle, in power, is three times the chord of a third of a right angle according to what has been explained, as well.

†لأنّ بالفرض، فنسبته إلى مائة وثلاثة وخمسين، وضعفه ٣٠٦.† وإذا زيد عليه ما عرّفنا أنّه أقلّ من كـ ج، أعني ٢٦٥، بلغ الكلّ ٥٧١.

Scholium 3.[208] †Since by assumption, so its ratio to 153, whose double is 306.† And if that which we have explained that it is less than *KG* is added to it, namely 265, the total reaches 571.

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[204] **C** 27v, top of right margin. The placement of this scholium is indicated in the manuscript by a *signe de renvoi* just before the word *fa-nisbat* at the beginning of the sentence.

[205] In Scholia 1, 2, and 6, the Arabic word *watar* is used in the sense of the side of a triangle subtending a given angle.

[206] **C²** 27v, middle of right margin. The placement of this scholium is not indicated in the manuscript text.

[207] See note 205.

[208] **C²** 27v, bottom of right margin. The placement of this scholium is not indicated in the manuscript text.

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1 مثلا وتر] مثلا وترا **C** 3 بالفرض] corr.، illeg. (skel.) + بالفرض + illeg. (skel.) **C***

لأنّ أقليدس بيّن في شكل ج من المقالة و أنّ كلّ زاوية قسمت بنصفين من مثلّث مثل زاوية ك في هذه الصورة، فإنّ ك ه ك ج ه ط ط ج متناسبة.

Scholium 4.[209] Since Euclid showed in Proposition 3 of Book VI that if, in a triangle, an angle is cut in halves, such as angle K in this diagram, then KE, KG, ET, and TG are in continuous proportion.[210]

†لأنّ ج ط في هذه الصورة نظير ج ه، الذي هو ١٥٣، ومربّعه ما ذكر ٢٣٤٠٩، وك ج نظير الضلعين اللذين كانا ٥٧١، الذي مربّعه ما {ذكر} ٣٢٦٠٤١.†

Scholium 5.[211] †Since GT in this diagram is the counterpart of GE, which is 153, and whose square is what has been mentioned, [namely] 23409, and KG is the counterpart of the two sides that are 571, whose square is what has been mentioned, [namely] 326041.†

طول ك ط وتر الزاوية القائمة يكون مثل جذر المربّعين اللذين ⟨هما⟩ {ط ج} وك ج، ومربّعاهما ٣٤٩٤٥٠، وجذره ٥٩١ وهذا الكسر ⟨الذي هو⟩ أكثر من الثمن.

Scholium 6.[212] The length of KT, the chord[213] of a right angle, is equal to the root of the two squares that are TG[214] and KG, whose squares are 349450,[215] whose root is 591 and that fraction ⟨that is⟩ more than 1/8.

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[209] $\mathbf{C^2}$ 27v, bottom margin. The placement of this scholium is not indicated in the manuscript text.

[210] That is, $KE : KG = ET : TG$.

[211] $\mathbf{C^2}$ 28r, top margin. The placement of this scholium is not indicated in the manuscript text.

[212] $\mathbf{C^2}$ 28r, top of left margin. The placement of this scholium is not indicated in the manuscript text.

[213] See note 205.

[214] Knorr (1989, 562, n. 18) reads this as "*EG*," probably due to the resemblance of the initial letter to a *hāʾ*.

[215] In fact, since $KG > 571$, once one assumes that GT is 153, $TG^2 + KG^2 > 349450$.

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1 زاوية] corr., رواىه **C***   4 {ذكر}] **C** illeg. (skel.)   5 الزاوية] زاوية **C**   {ط ج}] **C** illeg. (skel.), see note 214   6 ⟨الذي هو⟩] (Knorr)

لأنّ طول كـ ط أكثر من خمسمائة وأحد وتسعين وثمن ومجموعهما ما ذكر ١١٦٢.

Scholium 7.[216] Since the length of KT is more than 591 1/8 and the sum of the two is what has been mentioned, [namely] 1162.

وما بعد هذا من الصورة تنصيفات الزاوية، ويعود البرهان على ما سبق بزيادة الأعداد.

Scholium 8.[217] What is after this [point] from the diagram is the halvings of the angle, and the proof reverts to that which preceded, with increase of numbers.

[216] **C²** 28r, middle of left margin. The placement of this scholium is not indicated in the manuscript text.

[217] **C²** 28r, bottom of left margin. The placement of this scholium is not indicated in the manuscript text.

² تنصيفات] corr.، بصمات **C***

IV.3 The *Riżā* Version

R 1v

بسم الله الرحمن الرحيم

رسالة أرشميدس في مساحة الدائرة ونسبة محيطها إلى قطرها ونسبة بسيطها إلى مربّع قطرها

⟨ا⟩ قال: كلّ دائرة فإنّ بسيطها كالمثلّث القائم الزاوية الذي أحد ضلعيه المحيطين بالقائمة كنصف قطرها والآخر كمحيطها.

مثاله: لتكن دائرة ا ب ج د مركزها ه وقطراها يتقاطعان على قوائم، وهما ا ج ب د. وليكن مثلّث قرّ بالشرط المذكور، فهو كما ذكر أوّلًا.

فهي أعظم منه: فمربّع ا ب ج د أعظم من نصفها. وننصّف أرباعها، ونصل أوتارها. فمثلّث ب ر ا أعظم من نصف قطعته، وكذا القول في باقي المثلّثات. ولا نزال نفعل كذلك إلى أن تبقى من الدائرة قطع أصغر من فضل الدائرة على المثلّث. ولتكن قطع ا ر ر ب ونظائرهما. فيبقى الشكل الكثير الزوايا الواقع في الدائرة أعظم من المثلّث. ونخرج عمود ه ش. فهو أصغر من أحد ضلعي المثلّث المحيطين بالقائمة، ومعلوم أنّ محيط الشكل أصغر من الضلع الآخر. ومساحة الشكل من ضرب ه ش في نصف أضلاعه، ومساحة المثلّث من ضرب أحد ضلعيه في نصف الآخر. فالمثلّث أعظم من الشكل، وقد كان أصغر منه. هذا خلف. فالدائرة ليست بأعظم من المثلّث.

أصغر: عملنا على الدائرة مربّعًا يحيط بها. فهي أعظم من نصفه. ونخرج خطوطًا تماسّ الدائرة على منتصف أرباعها كما في هذه الصورة. وليكن قطر المربّع الأعظم ك م. فخطّ ك ع أعظم من ع ر المساوي لخطّ ا ع. فمثلّث ك ر ع أعظم من مثلّث ر ع ا. فهو أعظم من الشكل الذي تحيط به ر ع ع ا وقوس ا ر. وهكذا القول على المثلّثات الباقية إنّها أعظم من القطع الداخلة على محيط الدائرة. ولا نزال نفعل كذلك إلى أن يبقى من القطع الفاضلة على الدائرة أصغر من فضل المثلّث على الدائرة. فالمثلّث أعظم من الشكل المحيط بالدائرة. ومعلوم أنّ محيطه أعظم من محيطها، ومساحته من

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7 أوّلًا] corr., والا R* 10 نزال نفعل] read., ىرال ىفعل R* 12 المحيطين] المحيط R 20 نزال نفعل] read., يزال يفعل R*

ضرب عمود هر في نصف أضلاعه الذي هو أعظم من الضلع الأعظم من المثلّث الذي مساحته من ضرب ضلعه الأصغر المساوي لخطّ هر في نصف ضلعه الآخر. فالشكل المحيط بالدائرة أعظم من المثلّث، وقد {كان} أصغر منه. هذا خلف. فسطح الدائرة كسطح المثلّث، وهو المطلوب.

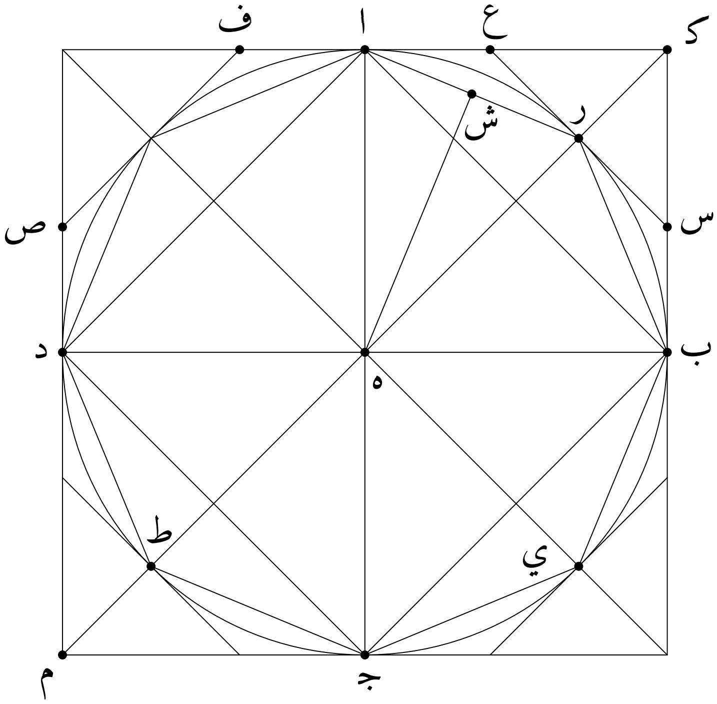

Figure 29: Diagram for *Riżā* 1, corresponding to *Fatih* 1. **R:** *Shīn* and *yā'* are written without dots in the diagram; *shīn* is written with dots in the text. There are two more letters in the manuscript diagram, *alif* and *hā'*, at the top left and bottom right corners of the big square, respectively. Since they do not appear in the text, I have removed them.

⟨**ب**⟩ وأمّا استخراج نسبة محيط الدائرة إلى قطرها، فهو كما أصف.

| لتكن دائرة قطرها اج، مركزها ب، ود ه ضلع المسدّس المحيط بها يماسّها على ج. **R** 2r
ونصل ب د ب ه. فمثلّث د ب ه متساوي الأضلاع، وزاوية د ب ج ثلث قائمة، وخطّ ب د ضعف د ج، ونسبته إليه نسبة ٣٠٦ إلى ١٥٣. مربّع الأوّل ٩٣٦٣٦، ومربّع الثاني ٢٣٤٠٩، فضل ما بينهما ٧٠٢٢٧ جذره ٢٦٥، وهو خطّ ب ج وأعظم من

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3 {كان}] illeg. (skel.) **R**   منه] من **R**   9 ب ج] ب ط **R**

الجذر بشيء يسير لا يُدْرِكُ الحسّ. ونسبته إلى جـ د أعظم من نسبة الجذر إلى ١٥٣. ثمّ ننصّف زاوية د ب جـ بخطّ ب ز. فنسبة د ب إلى ب جـ كنسبة د ز إلى ز جـ. فبالتركيب نسبة د ب ب جـ معًا إلى جـ ب كنسبة د جـ إلى جـ ز. حسابه أنّه مسطّح ب جـ في جـ د هو ٤٠٥٤٥، ⟨فإذا⟩ قسمناه على مجموع عددي د ب ب جـ الذي هو ٥٧١، خرج خطّ جـ ز ٧١ جزءًا. فإذا جعلناه ١٥٣، صار خطّ ب جـ بهذا المقدار ٥٧١، ونسبته إلى جـ ز أعظم من نسبة هذا العدد إلى ١٥٣. وأيضًا فلأنّ مربّع ب ز كمربّعي ب جـ جـ ز، لكنّ مربّع ب جـ ٣٢٦٠٤١، ومربّع جـ ز ٢٣٤٠٩، مجموعهما ٣٤٩٤٥٠ جذره ٥٩١ ح لد، وهو خطّ ب ز. فنسبته إلى جـ ز أعظم من نسبة هذا الجذر إلى ١٥٣. وأيضًا ننصّف زاوية ز ب جـ بخطّ ب ح. فعلى النسبة المذكورة يصير جـ ح معلومًا. فإذا جعلناه ١٥٣، يصير ب جـ بهذا المقدار ١١٦٢ ح لد، مربّعه و يه ط له ن ومربّعا جـ ح وب جـ مجموعهما و كا لط مد ن جذره ١١٧٢ ي يو، وهو خطّ ب ح. فنسبة ب ح إلى جـ ح أعظم من نسبة هذا الجذر إلى ١٥٣. وأيضًا ننصّف زاوية ح ب جـ بخطّ ب ط. فعلى النسبة المذكورة، أعني نسبة ح ب ب جـ معًا إلى جـ ب كنسبة ح جـ الى جـ ط، فيصير جـ ط معلومًا. فإذا جعلناه ١٥٣، يصير ب جـ بهذا المقدار معلومًا، وهو ٢٣٣٤ يح ن، مربّعه كه يح لز ا ك زيادة على مربّع جـ ط بلغ كه ك ز ي ك جذره ٢٣٣٩ يط. فنسبة ب ط إلى جـ ط أعظم من نسبة هذا الجذر إلى ١٥٣. وأيضًا ننصّف زاوية ط ب جـ بخطّ ب ي. فعلى النسبة المذكورة يصير جـ ي معلومًا. فإذا جعلناه ١٥٣، يصير ب جـ بهذا المقدار ٤٦٧٣ لح. فنسبة ب جـ إلى جـ ي أعظم من نسبة هذا العدد إلى ١٥٣. وأيضًا لمّا كانت زاوية د ب جـ ثلث قائمة، وزاوية ي ب جـ ربع ربعها، فهي جزء من يو جزءًا منها، وجزء من مح ⟨جزءًا⟩ من قائمة. ثمّ لتكن زاوية ك ب جـ كزاوية جـ ب ي. فزاوية ي ب ك

1 يُدْرِكُ [.voc، يدرك R* 7 كمربّعي [كمربّع R 11–10 و يه ط له ن [.corr، و به ط له حـ نه R*، see note 237 11 ومربّعا [ومربّع R وب جـ [وب ط R و كا لط مد ن [.corr، و كا لط مد نه or و كا لط ىد نه R* 14 ح جـ الى جـ ط [.corr، حـ الى خط R* 15 ٢٣٣٤ يح ن [.corr، ٢٣٣٤ يح نه or ٢٣٣٤ مح نه R* كه يح لز ا ك [.corr، كه ىحـ لر ا ك or كه محـ لر ا ك R* 16 ٢٣٣٩ يط [.corr، ٢٣٣٩ ىط or ٢٣٣٩ لط R* ب ط [ب جـ R جـ ط [.read، خط R* 18 ٤٦٧٣ لح [.corr، ٤٦٧٣ يح or ٤٦٧٣ لح R* 20 ي ب جـ [ي ز جـ R جزء [جزءًا R

جزء من كد جزءًا من قائمة. فهي جزء من ٩٦ جزءًا من أربع قوائم عند المركز. فخطّ
ي ك ضلع من أضلاع الشكل الكثير الزوايا ذي الستّ والتسعين ضلعًا المحيط بالدائرة.
وقد كانت نسبة | {ب ج} إلى ج ي أعظم من نسبة {٤٦٧٣ لح} إلى ١٥٣، وا ج **R** 2v
ضعف ج ب، وي ك ضعف {ج ي}. فنسبة ا ج إلى محيط أضلاع الشكل المحيط
بالدائرة أعظم من نسبة هذا العدد إلى مسطّح ١٥٣ في ٩٦، أعني تكسير أضلاع
الشكل، الذي هو ١٤٦٨٨، وهو أقلّ من ثلاثة أمثال العدد المذكور ومن سبعه بأكثر
من نصف جزء. فمحيط الدائرة، الذي هو أصغر من محيط الشكل، أقلّ من ثلاثة
أمثال قطرها ومن سبعه.

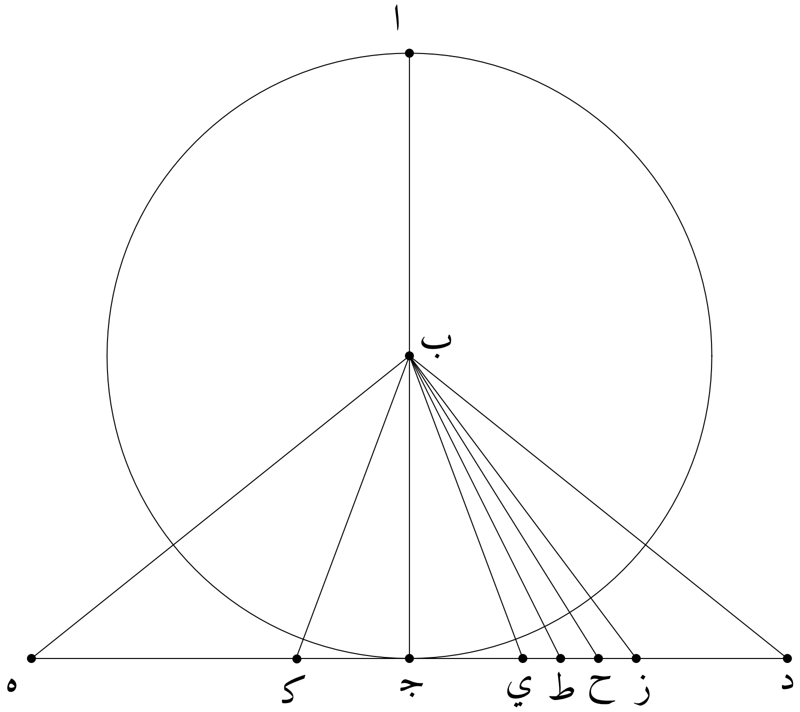

Figure 30: First diagram for *Riżā* 2, corresponding to the first diagram for *Fatih* 3. **R:** The diagram includes two other lines, from *bā*ʾ to two other points, one between *jīm* and *yā*ʾ, the other between *jīm* and *kāf*. Since their endpoints are not labeled, and the text does not mention them, they seem to have been drawn by mistake; I have removed them. *Zā*ʾ and *yā*ʾ are written without dots in the diagram and text. Finally, *hā*ʾ cannot be seen (or is not labeled) in the diagram, but the text provides its identification.

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1 جزء] جزءًا **R**   جزء] جزءًا **R**   3 كانت] كان **R**   {ب ج}] **R** illeg. (dam.)   {٤٦٧٣ لح}] **R** illeg. (dam.)   4 {ج ي}] **R** illeg. (dam.)   8 قطرها] قطر **R**

وأقول أيضًا إنّ نسبة محيط الدائرة إلى قطرها أعظم من ثلاثة أمثال بأكثر من نسبة عشرة إلى إحدى وسبعين جزءًا. لتكن دائرة اب جـ، وقطرها اجـ، وجـ ب ضلع مسدّسها. ونصل اب. فزاوية ا ثلث قائمة. ولنضع بإزاء قطر اجـ ١٥٦٠ جزءًا من العدد، وبإزاء جـ ب ٧٨٠. وأيضًا فإنّ مربّع العدد الأوّل ٢٤٣٣٦٠٠، ومربّع جـ ب ٦٠٨٤٠٠، فضل ما بينهما ١٨٢٥٢٠٠ جذره ١٣٥١، وهو خطّ اب. ونسبته إلى ب جـ أقلّ من نسبة الجذر ⟨إلى⟩ ٧٨٠. وأيضًا ننصّف زاوية ا بخطّ اد. ونصل جـ د. فزاوية د جـ ب كزاوية جـ اد، أعني زاوية داب. وزاوية د مشتركة. فنسبة اد إلى د جـ كنسبة جـ د إلى د ه، وكنسبة اجـ إلى جـ ه، وكنسبة اب إلى ب ه. فبالتبديل نسبة جـ ا اب معًا إلى ب جـ كنسبة اب إلى ب ه، أعني اد إلى د جـ لتشابه مثلّثي اب ه اد جـ. فنسبة اد إلى د جـ أقلّ من نسبة ٢٩١١ ⟨إلى⟩ ٧٨٠. فإذا جعلنا د جـ ٧٨٠، يصير خطّ اد ٢٩١١، مربّعه لط يج نب ا، ومربّع د جـ ب مط ٠ ٠، مجموعهما مب ب نب ا جذره ٣٠١٣ ما ك، وهو خطّ اجـ. فنسبته إلى جـ د أصغر من نسبة هذا الجذر إلى ٧٨٠. وأيضًا ننصّف زاوية داجـ بخطّ از. ونصل جـ ز. وعلى النسبة المذكورة تكون نسبة جـ ا اد معًا إلى د جـ كنسبة از إلى ز جـ. فإذا جعلناه ٧٨٠، يصير خطّ از بهذا المقدار ٥٩٢٤ ما ك. فنسبة از إلى ز جـ أصغر من نسبة هذا العدد إلى ٧٨٠. ونسبة الأكثر إلى الأقلّ هي نسبة ١٨٢٣ بالتقريب إلى ٢٤٠ لأنّ نسبة كلّ واحد منهما إلى نظيره كنسبة ثلاثة وربع إلى الواحد. وأيضًا مربّع هذا العدد الأكثر يه كج ح مط، ومربّع الأقلّ يو ٠ ٠، مجموعهما يه لط ح مط

R 3r | جذره ١٨٣٨ مج مط، وهو خطّ اجـ. فنسبة اجـ إلى جـ ز أصغر من نسبة هذا العدد إلى ٢٤٠. وأيضًا ننصّف زاوية زاجـ بخطّ اح. ونصل ح جـ. وعلى النسبة المذكورة إذا جعلنا خطّ جـ ح ٢٤٠، يصير اح بهذا المقدار ٣٦٦١ مج مط. فنسبة اح إلى ح جـ أصغر من نسبة هذا العدد إلى ٢٤٠. مربّع العدد الأكثر ا ب د لا ح لح، ومربّع الأقلّ يو ٠ ٠، مجموعهما ا ب ك لا ح لح جذره ٣٦٦٩ له يج، وهو خطّ اجـ. ونسبة هذا الجذر إلى ٤٠ كنسبة ١٠٠٧ إلى يا، وكذا نسبة ٢٤٠ إلى ٤٠

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[1] من نسبة] من نسبة صلح (last letter uncertain) **R**، see note 244 [3] بإزاء] corr.، بارا **R*** [4] وبإزاء] corr.، وبارا **R*** [5] وهو] أو هو **R** [8] جـ د] د جـ د **R** [12] مب ب نب ا] corr.، مب ٮ نب ا **R*** [18] يه لط ح مط] corr.، يه لط مط (gap between *lām ṭāʾ* and *mīm ṭāʾ*) **R*** [19] ١٨٣٨ مج مط] corr.، ١٨٣٨ مح مه **R*** [20] ٢٤٠] ٢٤ **R**

كنسبة ٦٦ إلى يا، وذلك بالتقريب لِحِفْظِ النسبة. مربّع العدد الأكثر د ما م مط، ومربّع الأقلّ ا يب لو، مجموعهما د مب نج كه جذره ١٠٠٩ ط لو، وهو خطّ اج. فنسبته إلى جح أصغر من نسبة هذا العدد إلى ٦٦. وأيضًا ننصّف زاوية حاج بخطّ اط. ونصل طج. وعلى النسبة المذكورة إذا جعلنا خطّ جط ٦٦ جزءًا، يصير خطّ اط بهذا المقدار ٢٠١٦ ط لو. فنسبة اط إلى طج أصغر من نسبة هذا العدد إلى ٦٦. مربّع الأكثر يح مط ح كا ط، ومربّع الأقلّ ا يب لو، مجموعهما يح ن ك نز ط جذره ٢٠١٧ يا، وهو خطّ اج. فنسبته إلى جط أصغر من نسبة هذا العدد إلى ٦٦. وأيضًا فلأنّ زاوية طاج جزء من مح جزءًا من قائمة، فضعفها التي على المركز جزء من كد جزءًا من قائمة، فهي جزء من ٩٦ جزءًا من أربع قوائم. فخطّ جط ضلع الشكل ذي الستّ والتسعين ⟨ضلعًا⟩ الذي تحيط به الدائرة. وتكسير محيطه من العدد ٦٣٣٦، أعني الحاصل من ضرب ٦٦ في ٩٦. فنسبة محيط أضلاع الشكل إلى قطر اج أعظم من ⟨نسبة⟩ تكسير أضلاعه إلى ٢٠١٧ يا الموضوع بإزاء القطر. لكنّ تكسير محيط الشكل أعظم من ثلاثة أضعاف هذا العدد بما مقداره ٢٨٤ كز. ⟨فإذا ضربنا هذا العدد في عا، يبلغ ٢٠١٩٥ نز⟩، وإذا ضربنا العدد الآخر في عشرة، يبلغ ٢٠١٧٢. ولمّا كان هذا العدد أقلّ من الآخر، وجب أن تكون نسبة الفضلة إلى ٢٠١٧ يا أعظم من نسبة ي إلى عا. ومحيط الدائرة أعظم من محيط الشكل المذكور. فنسبة محيطها إلى قطرها أعظم من ثلاثة أضعاف قطرها بأعظم من نسبة عشرة إلى ٧١. وقد كان أصغر من ثلاثة أضعاف قطرها وسبعه بالتقريب. وذلك ما أردنا أن نبيّن.

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1 لِحِفْظِ] .voc، لحفظ *R 7–6 يح ن ك نز ط] .corr، يح نه ك نو ط *R 8 طاج جزء] طاج جزءًا R 10 الشكل] شكل R 12 بإزاء] .corr، مارا or بارا *R 13 ٢٨٤ كز] .corr، ٢٨٤ كو *R 14 ⟨فإذا ضربنا هذا العدد في عا، يبلغ ٢٠١٩٥ نز⟩] see note 259 وإذا] فإذا R 16 عا] .corr، عا ومه *R، see note 260

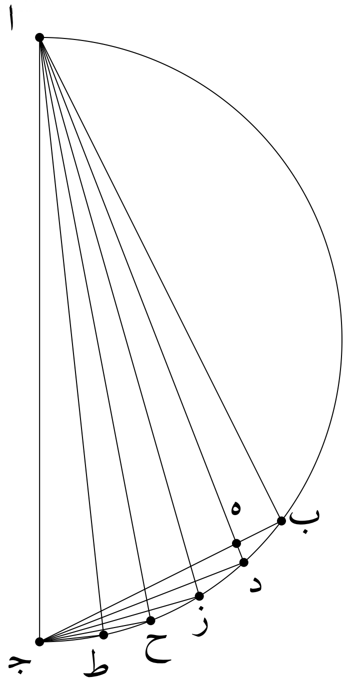

Figure 31: Second diagram for *Riżā* 2, corresponding to the second diagram for *Fatih* 3. **R:** *Zā*ʾ is written without a dot in the diagram and the text. The diagram has *ʿayn* in place of *ḥā*ʾ and *ḥā*ʾ in place of *jīm*; the text provides the identification of these two letters. Finally, no line is drawn between *ṭā*ʾ and *jīm* in the diagram.

⟨ج⟩ وأقول أيضًا إنّ نسبة المربّع المحيط بالدائرة إليها كنسبة الأربعة عشر إلى الإحدى عشر.

فليكن الدائرة والمربّع على هذه الصورة، وليكن خطّ ب د كخطّ د ه. فلأنّ د ب ضعف ب ج | المساوي لخطّ ا ج، فمسطّح ه ج في ج ب مع مربّع ج د، أعني مربّع (R 3v) ج ب، كمربّع د ب. ومسطّح ه ج في ج ب مع مربّع ج ب كمسطّح ه ب في ب ج. فمسطّح ه ب في ب ج كمربّع ب د المحيط بالدائرة. وأيضًا فإنّه قد تقدّم أنّ مساحة الدائرة من ضرب نصف قطرها في نصف محيطها. ونسبة ⟨قطرها⟩ إلى محيطها كنسبة السبعة إلى الاثنين والعشرين. فنصف محيطها أحد عشر جزءًا، وليكن ب ز. فمسطّح ز ب في ب ج كسطح الدائرة. ونسبة مسطّح ه ب في ب ج إلى مسطّح ز ب في ب ج كنسبة ه ب الأربعة عشر إلى ب ز الإحدى عشر. فنسبة المربّع المحيط بالدائرة إليها كنسبة الأربعة عشر إلى الإحدى عشر.

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[1] إليها] إليهما R [8] السبعة] سبعة R

وأقول أيضًا: إن كانت مساحة الدائرة معلومة، فمساحة مربّع قطرها معلومة . قيل إنّ نسبة الثلاثة التي بين الإحدى عشر والأربعة عشر إليه سبعه ونصف سبعه. فإن كانت مساحة الدائرة معلومة، ضربنا في يد، وقسمنا المبلغ على يا، تَخْرُجُ مساحة مربّع قطرها معلومة. أو نزيد على مساحتها سبعه ونصف سبعه، تحصل مساحة مربّعها معلومة. وكذا إن ضربنا مساحته في يا، وقسمنا على يد، تَخْرُجُ مساحة الدائرة معلومة، أو ننقص من مساحته سبعه ونصف سبعه. والله أعلم بالصواب.

تمّت بحمد الله وتوفيقه.

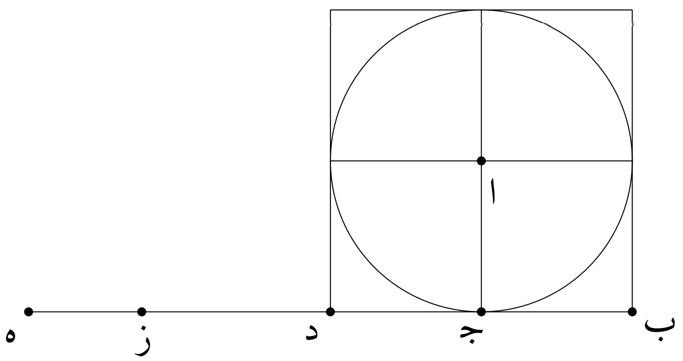

Figure 32: Diagram for *Rižā* 3, corresponding to *Fatih* 2. **R:** As the text mentions lines that connect to two points, *hā'* and *zā'*, that do not appear in the manuscript diagram, the manuscript diagram is corrupt. The line segment *hā' dāl* is my restoration based on the text.

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1 كانت] كان **R** قطرها معلومة] قطرها معلومًا **R** 3 كانت] كان **R** تَخْرُجُ] voc.، بحرح **R***
4 معلومة] معلومًا **R** معلومة] معلومًا **R** 5 تَخْرُجُ] voc.، بحرح **R***

In the name of God, the Most Gracious, the Most Merciful

Treatise of Archimedes on the Measure of the Circle, on the Ratio of its Perimeter to its Diameter, and on the Ratio of its Surface to the Square of its Diameter

⟨1⟩[218] He said: The surface of every circle is as the right-angled triangle one of whose sides surrounding the right [angle] is as half of the diameter of [the circle] and whose other [side] is as the perimeter of [the circle].

Its instantiation: Let there be the circle *ABGD* whose center is *E* and whose diameters, which are *AG* and *BD*, intersect at right [angles]. And let there be a triangle[219] satisfying[220] the stated condition, so that it[221] is as was mentioned before.

[The circle] is greater than [the triangle]: The square *ABGD* is greater than half of [the circle]. We divide the quarters of [the circle] in halves, and we join the chords of [the circle]. So the triangle *BRA* is greater than half of its segment,[222] and similarly the argument [goes] for the remaining triangles. We continue doing thus until there remain from the circle segments smaller than the excess of the circle over the triangle. Let there be the segments *AR*, *RB*, and their counterparts. So the polygonal figure falling in the circle is greater than the triangle. We draw the perpendicular *EO*. So it is smaller than one of the two sides of the triangle surrounding the right [angle], and [it is] known that the perimeter of the figure is smaller than the other side. And the measure of the figure [is obtained] from the product of *EO* and half of its sides, and the measure of the triangle [is obtained] from the product of one of its two sides and half of the other. So the triangle is greater than the figure, even though it was smaller than [the figure]. This is a contradiction. Therefore, the circle is not greater than the triangle.

Smaller: We have constructed on the circle a square that surrounds it. So [the circle] is greater than half of [the square]. We draw lines that are tangent to the circle at the middle of its quarters as in this picture. And let the diagonal[223] of the greater square be *KM*. The line *KQ* is greater than *QR*, which is equal to the line *AQ*. So the triangle *KRQ* is greater than the triangle *RQA*. Therefore it[224]

---

[218] **R** 1v.2–1v.22. *Riżā* 1 corresponds to *Fatih* 1.

[219] This triangle is both absent from the diagram and unnamed throughout the proof.

[220] Literally "abiding by" (*qarra bi-*).

[221] The pronoun *huwa* ("it") could refer to both *muthallath* ("triangle") and *sharṭ* ("condition").

[222] Namely, the segment of the circle bounded by the chord *BA* and the arc *BA*. Similar remarks apply to the other segments mentioned in the proof.

[223] Literally "diameter" (*quṭr*).

[224] That is, the triangle *KRQ*.

is greater than the figure that RQ, QA, and the arc AR surround. And thus the argument [goes] for the remaining triangles, [namely] that they are greater than the interior segments on the perimeter of the circle. We continue doing thus until there remain from the segments left over from the circle [something] smaller than the surplus of the triangle over the circle. So the triangle is greater than the figure surrounding the circle. And [it is] known that the perimeter of [the figure] is greater than the perimeter of [the circle], and the measure of [the figure] [is obtained] from the product of the perpendicular ER and half of its sides,[225] which are greater than the greater side of the triangle, whose measure [is obtained] from the product of its smaller side, which is equal to the line ER, and half of its other side. So the figure surrounding the circle is greater than the triangle, even though it {was} smaller than [the triangle]. This is a contradiction. So the surface of the circle is as the surface of the triangle, which is the desired [result].

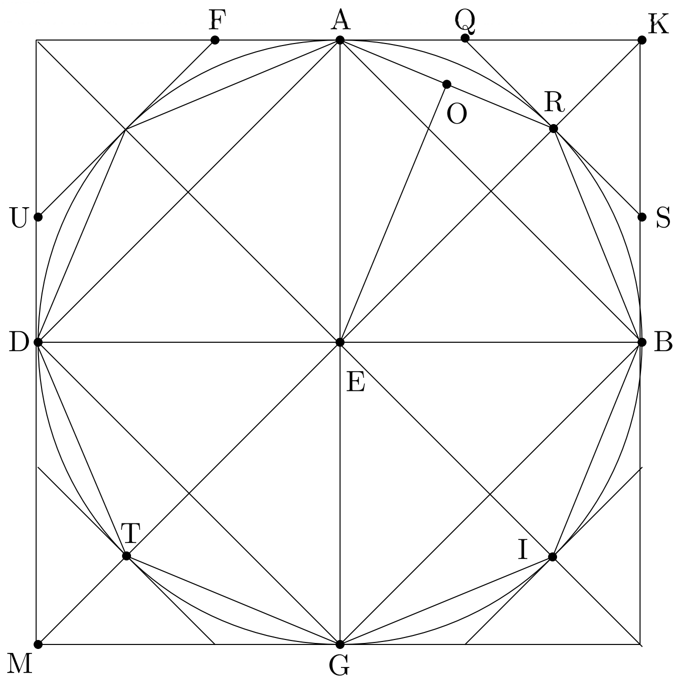

Figure 33: Diagram for *Rižā* 1, corresponding to *Fatih* 1.

⟨2⟩[226] As for the determination of the ratio of the perimeter of the circle to the diameter of [the circle], it is as I describe.

Let there be a circle whose diameter is AG, whose center is B, and [let] DE [be] the side of a hexagon[227] surrounding [the circle] and touching [the circle] at G. We

[225] That is, the perimeter of the polygonal figure circumscribed around the circle.

[226] **R** 1v.23–3r.20. *Rižā* 2 corresponds to *Fatih* 3.

[227] The Arabic is definite (*al-musaddas*). Evidently a regular hexagon is meant.

join BD and BE. So the triangle DBE is equilateral, the angle DBG is a third of a right angle, the line BD is twice DG, and its ratio to it is the ratio of 306 to 153. The square of the first is 93636, the square of the second is 23409, their difference is 70227 whose root is 265,[228] which is the line BG and [it is] greater than the root by an insignificant amount imperceptible to the senses.[229] And its ratio to GD is greater than the ratio of the root to 153.[230] And also, we divide the angle DBG in halves by the line BZ. So the ratio of DB to BG is as the ratio of DZ to ZG.[231] So by composition the ratio of DB and BG together to GB is as the ratio of DG to GZ. Its calculation is: the product of BG and GD is 40545, ⟨so if⟩ we divide it by the sum of the numbers DB and BG, which is 571, the line GZ results as 71 parts.[232] So if we make it 153, the line BG becomes, by that amount, 571, and its ratio to GZ is greater than the ratio of this number to 153.[233] Also, since the square of BZ is as the squares of BG and GZ, but the square of BG is 326041, and the square of GZ is 23409, their sum is 349450 whose root is 591;8,34, which is the line BZ. So its ratio to GZ is greater than the ratio of this root to 153.[234] And also, we divide the angle ZBG in halves by the line BH. So based on the mentioned ratio,[235] GH becomes known. So if we make it 153, BG becomes, by that amount, 1162;8,34,[236] whose square is 6,15,9,35;50[237] and the sum of the squares of GH and BG is 6,21,39,44;50 whose root is 1172;10,16, which is the line BH. So the ratio of BH to GH is greater than the ratio of this root to 153.[238] And also, we divide the angle HBG in halves by the line BT. So based on the mentioned ratio, I mean the ratio of HB and BG together to GB [which is] as the ratio of HG to GT, GT becomes known. So if

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[228] The correct value of $\sqrt{70227}$ is 265;0,13,35.

[229] Literally, "by an insignificant thing that the sense does not perceive" (*bi-shay' yasīr lā yudrik al-ḥiss*).

[230] That is, $BG : GD > 265 : 153$.

[231] By *Elements* VI.3.

[232] The correct value of 40545/571 is 71;0,25,13.

[233] GZ, which was 71 units, is redefined to be 153 units by some other measure. With this redefinition, BG becomes $265 \cdot (153/71)$=571;3,22,49 units. Then, $BG : GZ > 571 : 153$.

[234] The correct value of $\sqrt{349450}$ is 591;8,34,39,30.

[235] The author probably means $ZB + BG : BG = ZG : GH$.

[236] $ZB + BG : BG =$ 1162;8,34 : 571 and $ZG = 153$. Since $ZB + BG : BG = ZG : GH$, 1162;8,34 : 571 = 153 : GH. This gives $GH = (571 \cdot 153)/$1162;8,34. If now GH is redefined to be 153 units by some other measure, BG becomes 1162;8,34.

[237] The insertion of a *jīm* seems to be a scribal error. As to the last sexagesimal place, even though it seems to be written with a *hā'* in the manuscript, calculation of the square of 1162;8,34 shows it to be 50.

[238] The correct value of $\sqrt{6,21,39,44;50}$ is 1172;10,15,34 < 1172;10,16, so the stated inequality is incorrect.

we make it 153, *BG* becomes, by that amount, known, which is 2334;18,50, whose square is 25,13,37,1;20 [which when] added to the square of *GT* reaches 25,20,7,10;20 whose root is 2339;19. So the ratio of *BT* to *GT* is greater than the ratio of this root to 153. And also, we divide the angle *TBG* in halves by the line *BI*. So based on the mentioned ratio *GI* becomes known. So if we make it 153, *BG* becomes, by that amount, 4673;38.[239] So the ratio of *BG* to *GI* is greater than the ratio of this number to 153.[240] And also, since the angle *DBG* was a third of a right [angle], and the angle *IBG* is a fourth of a fourth of [the angle *DBG*], it is 1/16 of [the angle *DBG*], and 1/48 of a right [angle]. And also, let the angle *KBG* be as the angle *GBI*. So the angle *IBK* is 1/24 of a right [angle]. So it is 1/96 of four right [angles] about the center. So the line *IK* is 1 side from the sides of the polygonal figure of 96 sides surrounding the circle.[241] And the ratio of {*BG*} to *GI* was greater than the ratio of {4673;38} to 153, *AG* is twice *GB*, and *IK* is twice {*GI*}. So the ratio of *AG* to the perimeter of the sides of the figure surrounding the circle is greater than the ratio of this number to the product of 153 and 96, I mean the length[242] of the sides of the figure, which is 14688, and it is less than three times the mentioned number[243] and from a seventh of it by more than half a part. So the perimeter of the circle, which is smaller than the perimeter of the figure, is less than three times the diameter of [the circle] and a seventh of [the diameter].

And I also say that the ratio of the perimeter of the circle to the diameter of [the circle] is greater than three times by more than the ratio[244] 10 : 71. Let there be the circle *ABG*, whose diameter is *AG*, and [let] *GB* [be] a side of the hexagon of [the circle].[245] We join *AB*. So the angle *A* is a third of a right [angle]. Let us posit, corresponding to the diameter *AG*, 1560 as a number, and corresponding to *GB* 780. Also, the square of the first number is 2433600, and the square of *GB* is 608400, their

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239 The correct value is 4673;37,50, which the author rounds up to 4673;38.

240 Since the correct value of *BG* is 4673;37,50 and the author rounds this up, the stated inequality is incorrect by a very small amount.

241 Again, the regular 96-gon is meant.

242 The Arabic word used here, *taksīr*, typically means "area" or "volume."

243 That is, 4673;38. Indeed, $4673;38 \cdot (3 + 1/7) = 14688;33,42,51$, which is greater than 14688 by 0;33,42,51.

244 The abbreviation after the word "ratio" (*nisba*), whose last letter is probably a *ḥāʾ* despite looking more like a *jīm*, most likely indicates a correction (Gacek 2001, 85, s.v. "iṣlāḥ"), though it is not clear what it is supposed to correct.

The use of the word "ratio" (*nisba*) here is in contrast to the usage in Greek mathematical texts where the difference of two ratios, considered as another ratio, is never expressed in terms of a number.

245 The Arabic *ḍilʿ musaddasihā* is definite. Evidently a regular hexagon inscribed in the circle is meant.

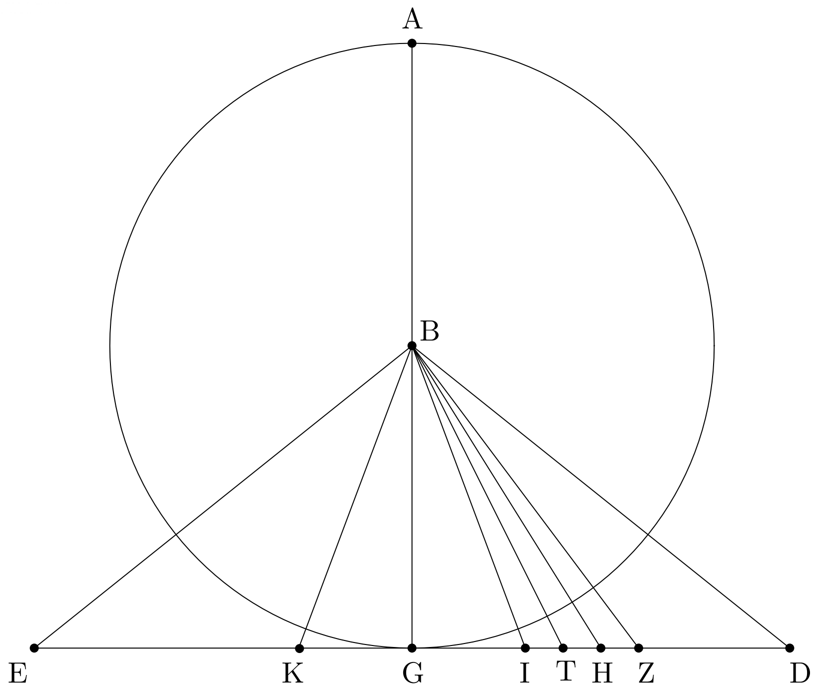

Figure 34: First diagram for *Riżā* 2, corresponding to the first diagram for *Fatih* 3.

difference is 1825200 whose root is 1351,[246] which is the line *AB*. And its ratio to *BG* is less than the ratio of the root ⟨to⟩ 780.[247] And also, we divide the angle *A* in halves by the line *AD*. We join *GD*. So the angle *DGB* is as the angle *GAD*, I mean the angle *DAB*. And the angle *D* is common.[248] So the ratio of *AD* to *DG* is as the ratio of *GD* to *DE*, as the ratio of *AG* to *GE*, and as the ratio of *AB* to *BE*.[249] By alternation, the ratio of *GA* and *AB* together to *BG* is as the ratio of *AB* to *BE*,[250] I mean *AD* to *DG* because of the similarity of the triangles *ABE* and *ADG*.[251] So

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[246] The correct value is 1350;59,58,40.

[247] That is, $AB : BG = 1350;59,58,40 : 780 < 1351 : 780$.

[248] Assuming there is no textual corruption here, the author probably wants to assert that since the angle *DGB*, equal to the angle *DGE*, is equal to the angle *GAD*, and since the angle at *D* is common to the triangles *ADG* and *GDE*, it follows that these triangles are similar to each other.

[249] The first two equalities of ratios follow from the similarity of the triangles *ADG* and *GDE*. The third follows from *Elements* VI.3.

[250] It takes more than one alternation to get this equality of ratios. First, alternation gives $AG : AB = GE : BE$. Next, composition gives $AG + AB : AB = BG : BE$. Finally, alternation gives the desired equality.

[251] It should be noted that the fact that the angle *A* is a third of a right angle is not used to deduce this proportion. So the author uses the same line of reasoning implicitly in the remainder of the

the ratio of AD to DG is less than the ratio of 2911 ⟨to⟩ 780. So if we make DG 780, the line AD becomes 2911, whose square is 39,13,52,1, the square of DG is 2,49,0,0, their sum is 42,2,52,1 whose root is 3013;41,20, which is the line AG. So its ratio to GD is smaller than the ratio of this root to 780.[252] And also, we divide the angle DAG in halves by the line AZ. We join GZ. Based on the mentioned ratio, the ratio of GA and AD together to DG is as the ratio of AZ to ZG. So if we make it 780, the line AZ becomes, by that amount, 5924;41,20. So the ratio of AZ to ZG is smaller than the ratio of this number to 780. And the ratio of the greater [number] to the lesser [number] is the ratio of approximately 1823 to 240 since the ratio of every one of them to its counterpart is as the ratio of 3 1/4 to 1.[253] Also, the square of this greater number is 15,23,8,49, the square of the lesser [number] is 16,0,0, their sum is 15,39,8,49 whose root is 1838;43,49, which is the line AG. So the ratio of AG to GZ is smaller than the ratio of this number to 240. And also, we divide the angle ZAG in halves by the line AH. We join HG. Based on the mentioned ratio, if we make the line GH 240, AH becomes, by that amount, 3661;43,49. So the ratio of AH to HG is smaller than the ratio of this number to 240. The square of the greater number is 1,2,4,31,8;38, the square of the lesser [number] is 16,0,0, their sum is 1,2,20,31,8;38 whose root is 3669;35,13, which is the line AG. And the ratio of this root to 40 is as the ratio of 1007 to 11,[254] and similarly the ratio of 240 to 40 is as the ratio of 66 to 11, and that is for the approximate preservation of the ratio.[255] The square of the greater number is 4,41,40,49, and the square of the lesser [number] is 1,12,36, their sum is 4,42,53,25 whose root is 1009;9,36, which is the line AG. So its ratio to GH is smaller than the ratio of this number to 66.[256] And also, we divide the angle HAG in halves by the line AT. We join TG. Based on the mentioned ratio, if we make the line GT 66 parts, the line AT becomes, by that amount, 2016;9,36. So the ratio of AT to TG is smaller than the ratio of this number to 66. The square of the greater [number] is 18,49,8,21;9, the square of the lesser [number] is 1,12,36, their sum is 18,50,20,57;9 whose root is 2017;11, which is the line AG. So its ratio to GT is smaller than the ratio of this number to 66.[257] Since the angle TAG is 1/48 of a

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proof, marked with the words "based on the mentioned ratio" (*ʿalā al-nisba al-madhkūra*), to assert the validity of other, similar, proportions.

[252] In fact, $\sqrt{42{,}2{,}52{,}1} > 3013;41{,}20{,}10$, so the stated inequality is incorrect.

[253] 5924;41,20 : 780 is in fact slightly smaller than 1823 : 240.

[254] 3669;35,13 : 40 is in fact slightly greater than 1007 : 11.

[255] As the following sentences reveal, the author intends to consider 1007 : 66 as an approximation to $AG : GH$.

[256] It is clear that the greater number is 1007 and the lesser number is 66. $\sqrt{4{,}42{,}53{,}25}$ is slightly greater than 1009;9,36, so the stated inequality is incorrect.

[257] The correct value of $\sqrt{18{,}50{,}20{,}57;9}$ is 2017;15, which is greater than 2017;11, so the stated inequality is incorrect.

right [angle], its double that is at the center is 1/24 of a right [angle], so it is 1/96 of four right [angles]. So the line GT is a side of the figure with 96 ⟨sides⟩ that the circle surrounds.[258] And the length of its perimeter is 6336 as a number, that is, the result from the product of 66 and 96. So the ratio of the perimeter of the sides of the figure to the diameter AG is greater than ⟨the ratio of⟩ the length of the sides of [the figure] to 2017;11, the [value] posited corresponding to the diameter. But the length of the perimeter of the figure is greater than three times this number by something whose amount is 284;27. ⟨So if we multiply this number by 71, 20195;57 results⟩,[259] and if we multiply the other number by 10, 20172 results. Since this number is less than the other [number], it is necessary that the ratio of the surplus to 2017;11 be greater than the ratio of 10 to 71.[260] And the perimeter of the circle is greater than the perimeter of the mentioned figure.[261] So the ratio of the perimeter of [the circle] to the diameter of [the circle] is greater than three times the diameter of [the circle] by [an amount] greater than the ratio of 10 to 71. And it was smaller than three times the diameter of [the circle] and a seventh of [the diameter] approximately. And that is what we wanted to prove.

⟨3⟩[262] And I also say that the ratio of the square surrounding the circle to [the circle] is as the ratio of 14 to 11.

Let the circle and the square be as in this picture, and let the line *BD* be as the line *DE*. Since *DB* is the double of *BG*, which is equal to the line *AG*,[263] the product of *EG* and *GB* together with the square of *GD*—that is, the square of *GB*—is as the square of *DB*.[264] And the product of *EG* and *GB* together with the square of *GB* is as the product of *EB* and *BG*. So the product of *EB* and *BG* is as the square of *BD*, which surrounds the circle. Also, it [was proved] before[265] that the measure of the circle is from the product of half of its diameter and half of its perimeter.[266] And the ratio of ⟨its diameter⟩ to its perimeter is as the ratio of 7 to 22. So half of its perimeter is 11 parts, and let it be *BZ*. So the product of *ZB* and *BG* is as the

[258] Literally "the side of the figure" (*ḍilʿ al-shakl*). Evidently, the 96-gon considered here is regular.

[259] This addition, whose disappearance from the manuscript is easily explained by *homoeoarchon*, is necessary so that the expressions "this number" and "other number" in the next sentence make sense.

[260] The letters *mīm hāʾ* in the manuscript seem to be simply a scribal error—perhaps an indication that the scribe was not a native Arabic speaker—for the first two letters of the following *muḥīṭ*, which the scribe did not then bother to erase.

[261] That is, the 96-gon inscribed in the circle.

[262] **R** 3r.20–3v.23. *Riżā* 3 corresponds to *Fatih* 2.

[263] That is, to the radius of the circle.

[264] $EG \cdot GB + GD^2 = GD^2$ by *Elements* II.5. It is not clear why there is a "since" (*li-anna*).

[265] Literally, "it preceded" (*taqaddama*).

[266] By *Riżā* 1.

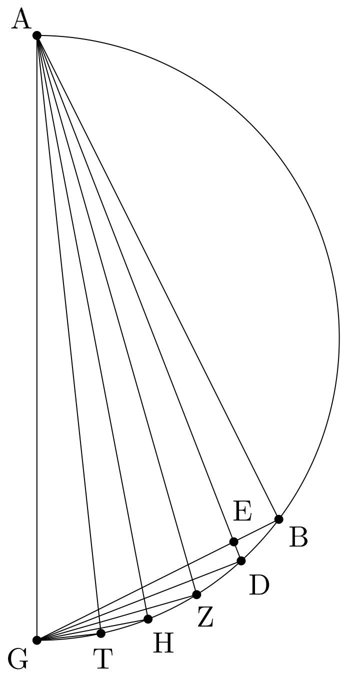

Figure 35: Second diagram for *Riżā* 2, corresponding to the second diagram for *Fatih* 3.

surface of the circle. And the ratio of the product of *EB* and *BG* to the product of *ZB* and *BG* is as the ratio of *EB*, [which is] 14, to *BZ*, [which is] 11. So the ratio of the square surrounding the circle to [the circle] is as the ratio of 14 to 11.

And I also say:[267] if the measure of the circle is known, then the measure of the square of its diameter is known. It was said that the ratio of 3, which is the difference of 14 and 11, to it,[268] is a seventh of it and a half of a seventh of it.[269] So if the measure of the circle is known, we multiply [it] by 14,[270] and we divide the result by 11, the measure of the square of the diameter of [the circle] ensues as known. Or [if] we add to the measure of [the circle] a seventh of it and a half of a

[267] These words, together with the mathematical mistake indicated in note 273, probably indicate that all that comes after this point is an interpolation.

[268] That is, to 14.

[269] The number 3 is referred to as feminine (*allatī*) but 14 is referred to as masculine (*ilayhi*, *subʿuhu*, and *niṣf subʿihi*), in accordance with Berggren's remark (2007, 537).

[270] From this point on, the numbers 11 and 14 are written in *abjad* numerals.

seventh of it,[271] the measure of its square[272] results as known.[273] And similarly if we multiply the measure of [the square] by 11, and divide [it] by 14, the measure of the circle ensues as known, or if we subtract from the measure of [the square] a seventh of it and a half of a seventh of it. God knows best what is right.

Finished by the praise of God and the success granted by him.

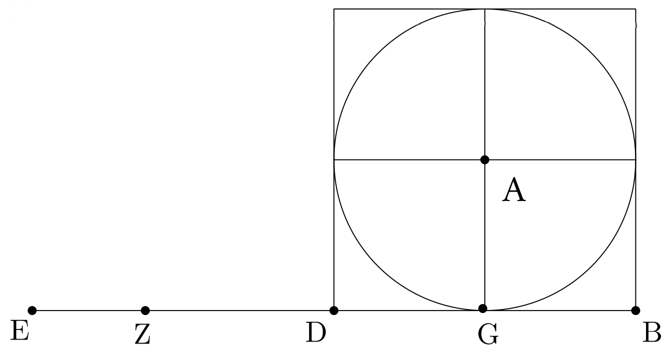

Figure 36: Diagram for *Riżā* 3, corresponding to *Fatih* 2.

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[271] In this instance of "a seventh of it and a half of a seventh of it" (*subʿahu wa-niṣf subʿihi*) and the next, the word "measure" (*misāḥa*) is referred to by a masculine suffixed pronoun. Again, in view of Berggren's remark (2007, 537), this probably indicates that the word *misāḥa* is being construed as a number.

[272] That is, the square surrounding the circle.

[273] This is a serious mathematical mistake. Apparently, the author is under the impression that if multiplying by 11 and then dividing by 14 is equivalent to subtracting a seventh and a half of a seventh, as is done in the next sentence, then multiplying by 14 and then dividing by 11 must be equivalent to adding a seventh and a half of a seventh.

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