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AN INVESTIGATION INTO THE ALGEBRAIC THINKING SKILLS OF FIFTH, SIXTH, AND SEVENTH GRADE STUDENTS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
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BY

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FOR
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ABSTRACT

AN INVESTIGATION INTO THE ALGEBRAIC THINKING SKILLS OF FIFTH, SIXTH, AND SEVENTH GRADE STUDENTS

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Master of Science, Mathematics Education in Mathematics and Science Education
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This study investigated the algebraic thinking skills of middle school students in grades 5, 6, and 7, focusing on their developmental levels and factors influencing these skills. It examined the effects of grade level and gender on algebraic thinking and assessed students' proficiency in generalized arithmetic, functional thinking, and modeling language. A specially designed test was administered to 487 students at the beginning of the spring semester for the 2023-2024 academic year.

Quantitative analysis revealed that students' algebraic thinking skills were generally low but improved significantly as the grade progressed. While all grade levels showed average success in generalized arithmetic questions, significant differences were observed in the functional thinking and modeling language subsections, with higher grades performing better. Research indicates that gender does not play a significant role in determining achievement levels.

Qualitative analysis provided deeper insights into students' reasoning processes and strategies, highlighting the influence of individual learning differences. The results of the study indicated that algebraic thinking progresses step-by-step and can be fostered from an early age, emphasizing the importance of incorporating algebra-focused resources into school curricula early on.

Keywords: Algebraic thinking skills, Early algebra instruction, Generalized arithmetic, Functional thinking, Modeling language.

ÖZ

BEŞ, ALTI VE YEDİNCİ SINIF ÖĞRENCİLERİNİN CEBİRSEL DÜŞÜNME BECERİLERİNİN İNCELENMESİ

Doğru, Tuba İpek
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Bu çalışma, ortaokul 5., 6. ve 7. sınıf öğrencilerinin cebirsel düşünme becerilerini, gelişim düzeyleri ve bu becerileri etkileyen faktörler bağlamında incelemiştir. Araştırmada, sınıf düzeyi ve cinsiyetin cebirsel düşünme üzerindeki etkileri değerlendirilmiş; öğrencilerin genelleştirilmiş aritmetik, işlevsel düşünme ve modelleme dili becerileri incelenmiştir. Bu amaçla, 2023-2024 akademik yılı bahar döneminin başında 487 öğrenciye özel olarak tasarlanmış bir test uygulanmıştır. Nicel analiz sonuçları, öğrencilerin cebirsel düşünme becerilerinin genel olarak düşük olduğunu, ancak sınıf düzeyi arttıkça önemli ölçüde geliştiğini göstermiştir. Genelleştirilmiş aritmetik görevlerinde tüm sınıf düzeylerinde orta düzeyde başarı gözlemlenirken, işlevsel düşünme ve modelleme dili gerektiren görevlerde üst sınıflar lehine belirgin farklılıklar tespit edilmiştir. Ayrıca, cinsiyetin cebirsel düşünme üzerindeki etkisinin istatistiksel olarak anlamlı olmadığı belirlenmiştir. Nitel analiz bulguları, öğrencilerin akıl yürütme süreçleri ve stratejileri hakkında daha derinlemesine bilgi sunarak bireysel öğrenme farklılıklarının önemini ortaya koymuştur. Araştırmanın sonuçları, cebirsel düşünmenin kademeli bir gelişim gösterdiğini ve bu becerinin erken yaşlardan itibaren desteklenebileceğini ortaya koyarak cebir odaklı kaynakların okul müfredatına erken yaşlarda dahil edilmesinin gerekliliğini vurgulamaktadır.

Anahtar Kelimeler: Cebirsel düşünme becerileri, Erken cebir eğitimi, Genelleştirilmiş aritmetik, İşlevsel düşünme, Modelleme dili.

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INTRODUCTION

During the early periods of humanity, mathematics was a field of knowledge that only the elite class should know. In these periods, mathematics was used for basic needs such as tax collection, land measurement, trade, astronomy, and calendar making. The ruling class used this knowledge to maintain order in society, control economic activities, and organize religious rituals. Therefore, mathematical knowledge was concentrated in a minority group with power and authority (Katz,1997).

The first mathematical records are from Mesopotamia and ancient Egypt, where mathematics was used in agriculture, trade, construction, and astronomy. In ancient Greece and Rome, mathematics was the basis of philosophy and scientific thought, and critical studies were carried out in geometry and arithmetic: abstract thinking and proof methods of mathematics developed in this period. Harezmi from the Islamic world gave his name to algebra and made essential studies on algebra and number theory (Berggren, 2016; Katz, 1997). Mathematics became a fundamental tool for scientific and technological advances, and the need for mathematics increased as societies became more complex and progress was made in various fields. During the Renaissance, scientists used mathematics to explain natural phenomena. The development of mathematics led to significant advances in physics and engineering. With this came the Industrial Revolution; statistics and probability theory became essential social sciences and economics tools. In the last century, the development of computers and digital technology has further increased the importance of mathematics. Cryptography, data analytics, artificial intelligence, and many other modern technologies require advanced mathematical knowledge. The use of mathematics in various fields during these periods has continuously increased

the need for mathematical knowledge and made it an indispensable tool in all areas of life.

A thorough understanding of society, politics, and economics requires mathematical skills and the ability to think critically, mainly to evaluate critically (Steen, 2001). Therefore, mathematics in other disciplines has dramatically increased due to its capacity to represent and communicate ideas shortly. In addition, technology usage has increased rapidly and cannot be overlooked. The growing use of technology in society necessitates that students learn to communicate using computers and their skills as communication tools. Students' mathematical understanding will significantly improve by developing strong reading, writing, listening, creative thinking, and problem-solving communication skills.

Mathematics education is essential for educating intelligent citizens and critically evaluating economic decisions (Steen, 2001). The field of mathematics, which can be defined in various forms, examines the relationships between mathematical entities, including numbers and geometric shapes. In the early stages of the educational process, mathematics explores the relationships between concrete objects. As the stage progresses, it involves acquiring the concept of variables, where it takes care of abstract objects. This perspective also has an impact on the curriculum. Many students who excel in mathematics until the sixth grade have difficulty understanding and using mathematical ideas when introduced to algebraic concepts. In this period, students learn basic arithmetic, geometry, and more abstract mathematical ideas such as algebraic concepts, equations, and inequalities. These concepts can often be new and complex for students. For many students, this is the first experience of failure in mathematics, which can seriously impact their attitudes towards mathematics. As a result, interest in mathematics may decrease, and some students begin to dislike mathematics. In other cases, this difficulty may repeat later, for example, in ninth-grade or first-year university mathematics (Kutluca, 2009). Students must enhance their abstract thinking and problem-solving skills at this stage.

Algebra is a critical branch of mathematics that requires abstract thinking. In this aspect, the nature of mathematics as a science of abstraction finds its whole meaning in algebraic expressions (Altun, 2018). Algebra, starting from 'symbolic algebra,' empowers the manipulation of abstract concepts and their practical application to real-life scenarios (Kieran, 1992). Algebraic thinking skills are considered as a defining component of algebra, involving procedural skills encompassing the manipulation of algebraic expressions and mathematical functions (Trybulski, 2007). The development of algebraic thinking relies on the increase in knowledge and skills within the field of algebra. With the introduction of algebra topics in mathematics classes, it has been observed that students face difficulties in learning algebra (Erbaş, 1999). In dealing with these difficulties, knowing students' level of algebraic thinking plays a vital role before teaching algebra.

Success in algebra is related to future achievement in high school math courses, college, and careers and is associated with proficiency in everyday life skills. Algebra preparation is an essential gateway to future success in mathematics (National Mathematics Advisory Panel [NMAP], 2008). To increase students' algebraic readiness, elementary school students should be shown how to connect with algebraic concepts during their arithmetic learning; this approach is defined as "early algebra." For example, recognizing the mathematical concepts, principles, and structures that underlie the mathematical strategies of operations can help students build a solid foundation for their algebra learning later on (Blanton & Kaput, 2005; Carpenter et al., 2003; Kieran et al., 2016). For this reason, the mathematics program implemented in schools should be organized carefully regarding the selection of topics and the distribution of topics across grade levels. Therefore, this thesis study will examine whether the algebra course taught in the first grades of middle school provides students with the necessary intellectual infrastructure for algebra subjects in advanced stages.

Algebra is a form of abstract thinking that utilizes variables and symbols, enabling students to develop the ability to understand numbers, relationships, structures, and generalizations. It involves processing numbers, variables, equations, and

mathematical relationships in abstract and generalized forms. The use of algebra facilitates a deeper understanding of arithmetic operations, thereby supporting students' comprehension of more complex mathematical structures. Algebraic thinking encompasses making generalizations, recognizing patterns, establishing relationships, and expressing variability (Blanton & Kaput, 2005; Kieran, 2006). This process lays an important foundation for transitioning to more advanced mathematical concepts such as modeling, problem-solving, and analytical thinking in higher levels of mathematics.

Students should engage in algebra through activities that foster the identification and presentation of mathematical structures and connections (Kieran et al., 2016). In this context, algebraic thinking plays a crucial role in developing mathematical modeling and problem-solving skills, both in daily life and in academic settings (Van de Walle, 2007). When children begin their school life, they step into this process with the experiences of the preschool period. Various aspects of algebraic thinking, such as generalization and pattern recognition, are constantly encountered by children in daily life (Levin & Walkoe, 2022). Additionally, algebraic thinking skills help children understand mathematical concepts and develop more advanced mathematical thinking skills such as critical thinking, problem-solving, logical reasoning, abstraction, generalization, and the ability to make and test conjectures. These skills form the foundation for tackling more complex mathematical tasks and concepts in later education.

In school, the objectives of algebra instruction extend beyond teaching the subject itself; they also aim to prepare students with thinking skills that can be applied in real-life situations (Ministry of National Education [MoNe], 2018). In accordance with elementary mathematics curriculum (MoNe, 2018), students initiate their algebraic journey by contemplating object patterns and numerical sequences. Then, they progress toward establishing connections between numbers, variables, and the concept of equations. In teaching algebra, the primary goal is to instruct students in utilizing symbols and algebraic expressions to solve various problems like mixtures, age-related challenges, and rate problems. Mastering concepts related to variables

and equations paves the way for more advanced algebraic topics in high school and university education.

Some educational research emphasizes that algebraic thinking should be introduced early. In particular, Blanton and Kaput (2011) and Kieran et al. (2016) suggest that algebraic concepts should be introduced in parallel with arithmetic by taking an "early algebra" approach. These studies emphasize the importance of supporting students' abilities, such as recognizing algebraic relationships, generalizing, and modeling while developing their arithmetic skills. Evidence indicates that no developmental restrictions would prevent the development of these skills at an early age; using case studies, Carraher, Schliemann, Brizuela and Earnest (2006) found that nine and ten-year-old students could do algebraic thinking. This approach uses children's initial exploration of natural curiosity and number concepts as a foundation for developing algebraic thinking (Kaput, 2008). In this way, students who are introduced to mathematical structure and general expressions at an early age take steps toward grasping more abstract algebraic concepts.

In the early twenty-first century, researchers and educators began to draw attention to the problems of teaching arithmetic and algebra separately. They stressed the importance of a more comprehensive and ongoing approach starting from elementary school (Kaput, 2008). In particular, the Principles and Standards for Mathematics Education (NCTM, 2000) recommended that algebra be mainstreamed in K12 education, pointing out that algebra should be treated as a method of thinking rather than a separate subject (Kaput, 2008). In this context, mathematics education in the Turkish Century Education Model aims to develop students' mathematical knowledge and skills as well as their higher-order thinking skills. The new curriculum identifies five core mathematical skills: mathematical reasoning, problem-solving, representation, data analysis, and the use of technology. These skills support students' application of mathematics to everyday life, critical thinking, and decision-making (MoNe, 2024).

In particular, from primary school onwards, algebraic thinking skills are gradually introduced with the theme "*from operations to algebraic thinking*" (MoNe, 2024; p.16). In this way, students are prepared to transition to abstract algebraic concepts by exploring the relationships between numbers and operations. Within the scope of this theme, students are first introduced to the processes of making operations with numbers and making sense of them. Then, they are provided with opportunities to develop their algebraic thinking skills. At the middle school level, mathematics teaching takes a more holistic approach, aiming to help students establish interdisciplinary relationships and acquire skills to solve real-life problems. One of the most essential features of the new curriculum is that it aims for students to understand and use mathematics rather than simply memorizing mathematical knowledge. This approach will help students develop positive attitudes toward mathematics and use mathematical thinking skills throughout their lives (MoNe, 2024).

The power of algebra, both in terms of knowledge production and social mobility, is undeniable. Algebra serves as a gateway to STEM (Science, Technology, Engineering, Mathematics) fields and higher education. This dual perspective on the power of algebra underscores the importance of identifying factors that enhance students' access to algebra (Matthews & Fuchs, 2020). However, findings in the literature (e.g., Altun, 2018; Baş et al., 2011; Erbaş, 1999; Radford, 2008) reveal that students experience significant difficulties in learning algebra. The factors affecting students' algebra learning processes must be examined in depth in this context. One of these factors is early algebra instruction.

International researchers have long highlighted the significance of early algebra instruction (Blanton & Kaput, 2011; Carraher et al., 2006; Kaput, 2008; Kieran et al., 2016; National Council of Teachers of Mathematics [NCTM], 2000). These studies show that acquiring algebraic thinking skills early on affects students' later mathematics learning (NMAP, 2008). However, when the 2018 Turkish National Curriculum is analyzed, it is seen that although algebraic thinking skills are included as implicit learning outcomes in primary school, systematic algebra instruction starts

in the sixth grade (MoNe, 2018). This raises the concern that there may be a disconnect in developing students' algebraic thinking skills. In addition to the studies conducted in the world, there have been studies in Turkey examining students' algebra achievement in early middle school periods (e.g., Atasoy, 2020; Birgin & Demirören, 2020; Çağdaşer, 2008; Çıkla Akkuş, 2004; Kaya, 2017) however, these studies did not address the algebraic thinking skills that fifth graders bring from primary school. In addition, most of the studies examined algebraic thinking skills at different levels but not in different categories (e.g., Birgin & Demirören, 2020; Çağdaşer, 2008; Gülpek, 2016; Kaya, 2017).

This study fills this gap in the existing literature on early algebra teaching. It aims to evaluate students' algebraic thinking skills through the three basic algebraic thinking domains identified by Kaput (2008). This study, conducted with the 2018 curriculum on fifth, sixth, and seventh-grade students, examined the effect of the education provided at school on students' algebraic thinking skills and significantly contributed to the existing body of knowledge in this field.

1.1 Statement of The Problem

Algebra is a fundamental skill for students to succeed in high school, college, and professional life (NMAP, 2008). Therefore, K-8 teachers need to identify deficits in students' algebraic thinking skills early. However, these deficits are not limited to not understanding or having difficulty solving algebraic equations. Algebraic thinking is based on a broader network of concepts, and often students' abstract thinking, generalization, and modeling skills may also be deficient. Early intervention can help students overcome algebraic learning difficulties (Levin & Walkoe, 2022), because such interventions provide students with a more solid understanding of basic algebraic concepts. Planning early interventions requires instructional strategies that are appropriate for students' current level, targeted, and provide ongoing support. These interventions should include structured and individualized approaches to strengthen students' algebraic thinking skills.

Early identification and intervention improve students' existing algebraic knowledge and strengthen their mathematical thinking skills and self-confidence, making it easier to grasp more complex mathematical concepts they will encounter in the following academic years. In this way, students acquire skills such as abstract thinking, problem-solving, and modeling earlier, providing the necessary cognitive and analytical foundation for future careers in STEM fields (Blanton & Kaput, 2011; Kieran et al., 2016). For this reason, this study examined the algebraic thinking skills of early middle school students.

By the time of data collection for the current study, students are first introduced to formal algebra in the sixth grade (MoNE, 2018). The algebra instruction in the sixth grade includes significant achievements aimed at developing students' basic algebraic thinking skills. At this stage, the aim is to understand the concept of variables, form algebraic expressions, and gain the ability to perform operations with these expressions. In addition, some of the elementary school mathematics curriculum objectives are essential for developing students' algebraic thinking foundations from an early age. For example, the learning outcome *"Recognizes that the sum remains the same when the places of the numbers being added change* ‘ in the first grade (MoNE, 2018, p. 27) and the learning outcome *’Recognizes number patterns whose difference is constant, find the rule of the pattern, and completes the pattern by identifying the missing element* ‘ in the second grade (MoNE, 2018, p. 32) and the explanation of the outcome *"Understands the meaning of ‘equality’ between mathematical expressions by using an equality sign* ‘ as *’The equal sign does not always mean the result of the operation, it shows the equilibrium state of the mathematical expressions on both sides of the equality* ” (MoNE, 2018, p. 33) show that the roots of algebraic thinking are laid as early as primary school. However, comprehensive research is needed to show whether these objectives yielded the desired result. To what extent do students acquire algebraic thinking skills by primary school and develop further through middle school? This constitutes the main problem area of this research.

This thesis fills the gap in the literature by examining the development of algebraic thinking skills of early middle school students who are educated with the 2018 curriculum and contributes to the field by measuring students' achievement in algebra-related content domains by grade level. Students' algebraic thinking abilities were assessed with a test covering three generally accepted algebraic content areas: general arithmetic, functional thinking, and the use of modeling languages (Kaput, 2008; Kieran et al., 2016). In addition, the study also focused on the acquisition of the basic concepts in the NCTM 2000 principles. The study's overarching goal is to consider the following three main research questions and related subquestions:

Research Question 1: Is there a significant difference in students' performance on algebra content strand tasks based on grade level and gender?

- Is there a significant difference between fifth, sixth, and seventh-grade students' overall performance on the algebraic thinking test in terms of grade and gender?
- Is there a significant difference between fifth, sixth, and seventh-grade students' generalized arithmetic (GA) performance in the algebraic thinking test regarding grade and gender?
- Is there a significant difference between fifth, sixth, and seventh-grade students' functional thinking (FT) performance in the algebraic thinking test regarding grade and gender?
- Is there a significant difference between fifth, sixth, and seventh-grade students' modeling language (ML) performances in the algebraic thinking test regarding grade and gender?

Research Question 2: Which basic algebraic concepts are better understood by fifth-, sixth-, and seventh-grade students?

- Which of equality, equations, properties of numbers and operations, variables, inequality, co-variation, and correspondence have fifth-grade students grasped?

- Which of equality, equations, properties of numbers and operations, variables, inequality, co-variation, and correspondence have sixth-grade students grasped?
- Which of equality, equations, properties of numbers and operations, variables, inequality, co-variation, and correspondence have seventh-grade students grasped?

Research Question 3: What common strategies and mistakes influence students' algebraic processes when solving tasks?

- Are there similarities in the answers given by students at different grades to specific questions?

1.2 Significance of The Study

Algebraic thinking is defined as "A process in which students generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways" (Blanton & Kaput, 2005, p. 99). Radford (2000) argued that algebraic thinking is a multifaceted process beyond symbolic language. Individuals can express their algebraic thinking using gestures, shapes, graphs, words, and symbols (Radford, 2014). This diversity shows that algebraic thinking is a creative and flexible process. The ability to form generalizations, interact with the world, distinguish similarities and differences, classify and model objects, and transfer meaning from familiar situations to new and different situations are among the requirements of algebraic thinking skills. Accordingly, algebraic thinking manifests itself when students become aware of these generalizations and express them with various symbols (Radford, 2014).

Teaching algebraic thinking enables students to develop transferable problem-solving skills within mathematics and across disciplines (Ketterlin-Geller & Chard, 2011). Research has shown that students who learn algebra early in eighth and tenth

grades show higher achievement than their peers who learn algebra later. In addition, students who completed algebra by eighth grade scored higher on twelfth-grade math tests (Lee & Mao, 2021). In this context, the arguments about the necessity of implementing the algebraic approach at all levels of education are no longer a matter of discussion (Freiman & Fellus, 2021). The necessity to include algebra in elementary mathematics has led to many studies that reveal specific features of algebraic thinking at all grade levels (e.g., Blanton & Kaput, 2011; Chimoni et al., 2018; Irwin & Britt, 2008; Lee & Mao, 2021; Radford, 2014). Many researchers have examined the integration of algebraic thinking into the educational system by defining algebraic thinking in terms of arithmetic and symbolization, numerical and geometric patterns, and tables and graphs (e.g., Carpenter et al., 2003; Carraher et al., 2006; Radford, 2014;).

Many studies in Turkey show that students struggle to learn algebra (e.g., Dede & Argün, 2003; Erbaş, 1999; Ersoy & Erbaş, 2005). In order to overcome these difficulties, different educational methods and algebra teaching approaches have been tried (e.g., Atasoy, 2020; Çağdaşer, 2008; Çıkla Akkuş, 2004; Kaş, 2010; Şengül & Tekcan, 2023; Yenilmez & Teke, 2008; Yıldırım, 2016) and teachers' opinions on the subject have been consulted (Baş et al., 2011; Işçi & Obay, 2022). Some studies have leveled students' algebraic performance by grade level (e.g., Gülpek, 2016; Kaya, 2017; Usta & Özdemir, 2018). However, most of these studies were limited to seventh and eighth-grade students. Students acquire the foundations of algebraic thinking through implicit learning outcomes starting from primary school (MoNE, 2018). Therefore, it is necessary to include fifth-grade students in the studies. In addition, the studies categorized algebraic thinking into stages, and students were leveled according to these stages. Literature reveals a gap in research that examines student achievement in various categories of algebraic thinking without stratifying by level. Additionally, there is a lack of studies investigating the algebraic thinking skills of early middle school students, beginning from the fifth grade.

This thesis fills the gap in the literature by covering early middle school students from the fifth-grade level onwards. It will contribute to the field by dividing algebraic thinking into three categories: generalization, functional thinking, and modeling languages, with an approach generally accepted in the literature (Kaput, 2008; Kieran et al., 2016). In this thesis, in line with the 2018 curriculum, students' algebraic thinking skills in early middle school were examined qualitatively, and their responses to algebraic thinking questions were analyzed in detail.

1.3 Definitions of The Important Terms

***Algebraic Thinking Skills*:** Algebraic Thinking Skills (ATS) are essential for students to develop to effectively tackle complex and non-routine problems (Jahudin & Siew, 2023). Algebraic thinking can be defined as the ability to perform operations on unknown quantities, to develop an abstract perspective on variables and their relationships, and to recognize mathematical structures. Students can demonstrate this way of thinking even if they do not use algebraic symbols (Carraher & Scliemann, 2020)

Algebraic Content Strands: Introduced by Kaput in 2008, algebraic content strands refer to specific areas of knowledge and skills that foster the development of algebraic thinking starting in elementary school.

Generalized Arithmetic: The teaching domain is a fundamental link for students transitioning to algebra. Its primary goal is to enhance students' algebraic thinking by presenting arithmetic operations and principles within a broader conceptual framework. Building on their arithmetic knowledge encourages the development of algebraic reasoning skills (Kaput, 2008).

Functional Thinking: This domain systematically enhances students' functional thinking skills by emphasizing functional generalization. Functions are viewed as mathematical structures representing relationships between variables and underpin algebraic thinking. Beginning with fundamental pattern activities fosters students' ability to make mathematical generalizations in teaching (Kaput, 2008).

***Modeling Language*:** Modeling language emphasizes the adaptability of algebraic modeling in tackling different kinds of problems and scenarios, ranging from specific numerical limitations to wide-ranging generalizations and pattern identification (Kaput, 2008).

CHAPTER 2

LITERATURE REVIEW

The literature review section aims to provide an overview of the literature on algebraic thinking education. First, the importance of algebra and the general characteristics of algebraic thinking are introduced. Then, the place of algebra in the curriculum is explained. In the last section, students' achievement in algebra will be discussed.

2.1 Importance of Algebra and Early Algebra Teaching

Algebra is crucial in K-12 math programs (Leung et al., 2014). It is an essential part of the mathematics curriculum. Then what is algebra? Usiskin 1995 defined algebra as a language of generalization. He argues that algebra allows a person to answer a specific question while considering multiple aspects of it at the same time; it is the language of relationships between quantities and solving some numerical issues. Algebra is a branch of mathematics that studies the relationships between numbers, symbols, and variables. Algebra aims to perform operations on unknowns by generalizing arithmetic operations and the rules between these operations. Algebra involves the decomposition of mathematical structures such as equations, inequalities, functions, and polynomials, and the modeling of these structures is based on logical rules (Kieran, 2006).

Algebra has an essential place in the history of mathematics and dates back to the early periods of human history. The ancient Babylonian and Egyptian civilizations show the first traces of algebraic thinking. Babylonian mathematicians developed algebraic-like techniques through arithmetical manipulations, although they did not use symbolic algebraic expressions to solve the relationship between numbers and figures. In Babylonia, for example, solving equations was often considered in a

geometric context. Some of the problems solved by Babylonian mathematicians resemble problems that can be expressed in equations in modern mathematical language, but the methods used are more arithmetical and geometric approaches (Corry, 2024). Similarly, the ancient Egyptians developed algebraic methods based on verbal expressions to solve linear equations. The algebraic methods used in this period were mainly used to solve problems related to daily life. Egyptian mathematics, especially in solving algebraic problems, used a method called “false assumption.” This method involved guessing an unknown value and using this guess to obtain a solution. For example, the Rhind Papyrus, dated 1650 B.C., was written by the Egyptian A'h-mose and contains problems related to linear equations with expressions similar to modern algebraic equations (Katz, 1997). However, ancient solutions lack symbolic representations; instead, they rely on verbal explanations, algebraic thinking, and rational thinking styles (Radford, 2000).

Berggren (2016) describes Al-Khwarizmi's contributions to algebra as revolutionary, emphasizing his systematic problem-solving methods. One of his most important works, *Al-Kitab al-mukhtasar fi hisab al-jabr wa'l-muqabala* (The Short Book on Completion and Balancing in Computation), laid the foundations of modern algebra. His algebraic methods include techniques such as “completion” (moving terms from one side of the equation to the other and changing their signs) and “balancing” (treating both sides equally), usually to solve quadratic equations. Berggren points out that Khwarezmi did not use symbolic algebraic notation, but his methods offer a more general approach to solving mathematical problems. This methodology paved the way for the development of algebra as a symbolic and systematic discipline and had a lasting impact on the Islamic world and, later on, European mathematics.

Diophantus, one of the Greek mathematicians, played an essential role in the development of algebra. In his work "Arithmetica," Diophantus included problems involving working with unknowns. Diophantus' algebraic approach laid the foundations of modern algebra, and he was named the "Father of Algebra." Greek mathematicians' understanding of algebra was based on a geometric approach (Katz, 1997).

Until the Renaissance, using symbols in algebraic expressions was uncommon. Mathematicians mostly performed their calculations with verbal expressions and geometric methods. However, in the 16th century, the French mathematician François Viète paved the way for expressing general solutions using letters in mathematical expressions and using algebra as a calculation tool. Using letters as symbols for unknowns, Viète made it possible to solve algebraic equations more systematically. In the 17th century, René Descartes developed analytic geometry by combining algebra and geometry. Descartes' “La Géométrie” offered a new perspective on solving mathematical problems by relating algebraic equations to geometric figures. During this period, the applicability of algebraic thinking and abstract concepts greatly increased, thus expanding the range of mathematical analysis and analysis tools (Editors of Encyclopaedia Britannica, 2024).

In the modern period, algebra has become much more abstract and divided into various branches. Group theory, ring theory, and linear algebra are among the modern applications of algebra. These fields are of great importance not only in theoretical but also in practical areas. For example, group theory is fundamental in physical symmetries and particle physics. Ring theory is used in cryptographic systems to ensure data security. Linear algebra is an indispensable tool for analyzing data in many fields, such as engineering and economics (Judson, 2020).

In today's mathematics education, algebra is not just about number operations. It's a powerful tool that fosters abstract thinking, problem-solving skills, and logical inferences. Learning algebra not only hones mathematical skills but also strengthens analytical thinking, reasoning, and problem-solving skills. These skills are useful not only in solving mathematical problems but also in tackling real-life challenges. For instance, identifying and solving unknowns through equations, a key aspect of algebraic thinking can make individuals more effective in making financial decisions such as budget management, healthcare costs, and future investments. Moreover, algebraic thinking is a fundamental tool for scientific research, engineering applications, and even research in the social sciences. Thinking abstractly and discovering general rules allows individuals to solve complex problems more

efficiently (Judson, 2020; Kieran, 2006). In this context, algebra is not just an educational tool but a valuable life skill at both individual and societal levels.

Algebra is crucial for several reasons, particularly in shaping a student's educational pathway. Early exposure to algebra significantly impacts academic success, as students who take algebra before high school are more likely to achieve higher math scores, enroll in advanced math courses, and graduate at higher rates. Furthermore, algebra has been recognized as a "new civil rights issue," highlighting the unequal access to algebra among student populations. Ensuring access to early algebra helps address systemic educational disparities and opens doors to advanced mathematical learning. Regarding long-term academic preparation, early algebra education fosters algebraic thinking, enhances readiness for advanced courses, and improves overall mathematical competency. Recognizing these benefits, educators advocate for stronger middle school math curricula, promote early algebraic thinking in elementary education, and prepare students for algebraic concepts at earlier stages (Lee & Mao, 2021).

Kaput (1999) emphasizes that it is crucial to understand the basic concepts of algebraic thinking and generalize them. According to him, students should have in-depth knowledge of variables, equations, and algebraic structures. Education should enable students to understand basic algebraic concepts. Sound teaching here includes understanding variables, equations, and algebraic expressions. Students should understand these concepts and be able to generalize them through applied problems. Indeed, in their 2008 study, Britt and Irvin observed that children who can generalize at an early age are more successful later on.

Various approaches to teaching algebra are designed to develop students' algebraic thinking and problem-solving skills. These approaches offer different ways to help students understand and apply algebraic concepts. In their 1996 book, Bednarz, Kieran, and Lee present a variety of perspectives on approaches to algebra instruction. For example, providing students with a deep understanding of algebraic concepts is essential rather than simply teaching algebraic operations. This approach

encourages students to see algebra as a meaningful system of thought rather than as a set of procedural rules. Developing conceptual understanding allows students to solve algebraic problems more creatively and critically. To help students better understand algebraic concepts, the connections between arithmetic knowledge and algebraic thinking are emphasized. It is argued that making connections between arithmetic and algebra can help students learn and understand algebra more easily. Making this connection, they place problem-solving at the center of algebra education. This approach aims to give students the skills to analyze and solve algebraic problems. Through problems, students are encouraged to explore algebraic concepts, which supports their active learning process.

In addition, Bednarz, Kieran, and Lee (1996) argue that teaching algebraic symbols and notation to students is crucial for understanding algebraic expressions. The educational literature frequently discusses the difficulties encountered in teaching symbolism and notation and the strategies that can be used to overcome these obstacles (e.g., Sfard, 2008; Zazkis et al., 2009). It is emphasized in the book (Bednarz et al., 1996) that for students to work effectively with algebraic symbols, they should have a deep understanding of the meaning of symbols. Moreover, using computer-assisted instructional tools and other technologies in algebra instruction is discussed as a factor that enhances the learning process. Dynamic mathematics software and visual aids can help to concretize algebraic concepts, which allows students to better understand them. According to another approach in the book, algebra education can also be approached from a historical and cultural perspective; by teaching students the historical development of algebraic thinking, interest in this discipline can be increased, and a broader perspective on mathematical thinking can be gained.

2.2 Arithmetic to Algebra

Arithmetic and algebra are two fundamental branches of mathematics, strongly related to numbers and each other. While arithmetic lays the foundation for algebraic thinking, algebra builds on this foundation to develop more abstract and general

mathematical concepts. As students transition from arithmetic to algebra, they must generalize the concepts learned in arithmetic and develop the abstract thinking skills required by algebra. There is an indispensable partnership between these two branches. Arithmetic and algebra use the same symbols for basic mathematical operations such as addition, subtraction, multiplication, and division. The equality sign ("=") is essential in both branches. However, the meanings and uses of these symbols have a broader scope in algebra. For example, in arithmetic, the equality sign indicates the result of an operation, while in algebra, it expresses a symmetrical and transitive relationship (Kaput, 2008; Matthews & Fuchs, 2020). At the same time, arithmetic forms the basis for students to transition to algebra. Arithmetic operations form the basis of numbers and number systems, and these concepts become more abstract with variables, equations, and functions in algebra. The numbers in arithmetic can be seen as a precursor to the variables in algebra.

Transitioning from arithmetic to algebra requires students to move from concrete mathematical operations to more abstract thought processes. This process is a critical stage in mathematics education and forms the basis for students to develop algebraic thinking skills. In their (1997) article, MacGregor and Stacey argued that to support students in making the transition from arithmetic to algebra more effective, teachers should introduce algebraic concepts more clearly and use various teaching methods to help students understand them. Many studies have discussed transition methods from arithmetic to algebra (e.g., Bednarz et al., 1996; Kieran, 1992; MacGregor & Stacey, 1997; Tall, 1992; Usiskin, 1995).

Usiskin (1995) states that the effort to transfer arithmetic strategies to algebraic problems results in students continuing their tendency to solve problems with concrete thinking and having difficulty abstractly understanding algebraic expressions. In particular, while arithmetic operations are simple and concrete (e.g., $4 + 3 = 7$), algebraic problems require an abstract solution using symbols ($x + 3 = 7$ or $x + y = z$). From a broader perspective, it has been observed that when students develop algebraic thinking skills, they often look for quick solutions rather than step-by-step strategies. This approach could lead to inaccuracies. In arithmetic, operations

between numbers yield clear results, whereas students try to solve more abstract equations in algebraic equations (Usiskin, 1995). In addition, in arithmetic, only a numerical solution is sufficient, whereas in algebraic questions, where values are expressed in letters instead of numbers, results that need to be expressed in general terms and the operation with unknowns are required (Bednarz et al., 1996).

Another difficulty is the equality sign; students often see it only as a sign to indicate a result, as in arithmetic, and fail to grasp its meaning in an algebraic equation, which expresses the equilibrium state of both sides. Moreover, the role of letters (variables) was confusing for students. In arithmetic, letters are usually used as units or labels, whereas in algebra, letters are supposed to represent unknown values, which confused students. For example, the expression '12 m' represents 12 meters in arithmetic and 12 multiplied by an unknown value (m) in algebra (Kieran, 1992).

Historically, algebra instruction began in adolescence based on symbol manipulation after students acquired arithmetic skills. This traditional teaching method usually focused on symbol-oriented topics such as "unknowns," "variables," and "solving equations." However, some difficulties were observed with this approach. Students have difficulties in traditional algebra lessons due to the abstract nature of algebra and the difficulties in the transition from arithmetic to algebraic thinking. Therefore, to increase access and understanding in algebra education, the basic assumptions of teaching methods need to be reconsidered (Levin & Walkoe, 2022). Additionally, one should not wait until adolescence to start thinking algebraically; instead, an approach should be adopted in which students can develop algebraic thinking skills from an early age. There is a growing acceptance of the view that it is possible to acquire algebraic thinking skills without fully understanding arithmetic (Levin & Walkoe, 2022).

The transition from arithmetic to algebra was generally defined as a transition from concrete to abstract in pre-2000s research. Based on the assumption that algebraic thinking is more abstract, studies in this period argued that the full mastery of arithmetic skills is a prerequisite for algebra learning (Mason, 1996). Sfard (2008)

emphasizes that although algebra is often perceived as abstract, it is no more abstract than arithmetic and only offers students a different thinking approach. Similarly, Kieran (2006) argues that algebra is a natural extension of arithmetic, while Kaput (2008) argues that algebra offers a mental framework different from arithmetic but based on existing mathematical knowledge. These perspectives support the idea that algebraic thinking should be developed early and that students can learn algebra more concretely (Kieran, 2006; Levin & Walkoe, 2022).

When students begin to learn algebra, they bring the concepts and skills acquired in arithmetic. However, these concepts need to be extended and sometimes even modified to meet the needs of algebra. However, algebra does not form a complete continuum with the methods and symbols learned in primary school arithmetic. One of the discontinuities between these two domains is that problems previously solved intuitively begin to be addressed in algebra through formal representations and methods. Britt and Irwin found that students participating in the Numeracy Project, published in 2008, who were aware of the generality of numerical processing strategies could extend algebraic thinking to standard algebraic symbols. According to them, young children need to work with several layers of general awareness before being introduced to the semiotic elements of algebra.

Developed number sense is critical for students, especially in middle school, to grasp the basic properties of number systems and to use them effectively in problem-solving (NMAP, 2008). Understanding these properties accelerates students' arithmetic operations and enables them to recognize and correct mathematical errors, apply algorithms appropriately to contexts, and generalize their knowledge to new situations (Geary, 2006). Basic number properties are key in developing students' algebraic thinking skills. Understanding concepts such as the distributive property, commutative and associative properties of addition and multiplication, unit elements for addition and multiplication, inverse properties, and mathematical equality increases the ability to perform regular operations on numbers and symbols (Milgram, 2005). These features strengthen students' mathematical reasoning

capacities and provide a solid foundation for the transition to algebra and advanced mathematics.

In particular, mathematical equality is a fundamental element of algebraic thinking. Understanding the equals sign as a bridge between numerically equivalent expressions is critical for students to grasp the relationships between mathematical expressions (Matthews & Fuchs, 2020). This understanding allows students to solve mathematical problems creatively by manipulating numbers and symbols. Moreover, the flexible manipulation of numbers in composition and decomposition processes enables students to understand the relationships between numbers (Britt & Irwin, 2008). Such an approach supports algebraic reasoning and contributes to students' development of broader mathematical thinking skills.

The central role of the concept of equality in students' mathematical thinking processes is a critical foundation for the transition from arithmetic to algebra. In this context, the study conducted by Xie and Cai (2022) examined the effect of early arithmetic strategies on fifth-grade students' solving equations involving two different representations of unknowns. In the study, 126 fifth-grade students were administered a pre-test and a post-test in a regular classroom setting, and the results revealed significant differences between students' success rates and the strategies they used. According to the study, most students adopted the reversal strategy (arithmetic operations) after receiving formal training in solving equations. However, success rates dropped significantly in cases where the unknown was a minuend or divisor, and students tended to use formal solution methods in these cases.

The study's findings emphasize that teachers should adopt an approach sensitive to this diversity, taking into account that students may interpret the unknown concept in different ways. It also points to the importance of an instructional approach based on students' prior meaning-making experiences in the transition from arithmetic to algebra. In this context, ensuring that students have a deep understanding of the

processes of solving equations and the representations of unknowns is critical for developing algebraic thinking skills.

2.3 Algebraic Thinking

As students progress from 5th grade to 8th, their abstract thinking skills develop significantly. Their ability to do mathematics matures, along with their ability to abstract, predict, and generalize common elements from situations. Their ability to express and communicate mathematics in this process should also become increasingly sophisticated. However, this development is not possible without conscious and attentive learning of the language of mathematics. The language of mathematics is hidden within the structure in which algebra builds words and sentences through specific symbols and letters. Furthermore, we express everyday life in mathematics by algebraic modeling. Therefore, algebra has a special place in mathematics, especially in the language of mathematics.

Algebraic thinking is acknowledged to exist independently from the structured language it uses to communicate. However, simplifying it to exist solely at this level would reduce everything to a manipulative system, often ineffective when handled by students (Radford, 2000). From the other perspective, algebraic reasoning is generally understood as operating unknowns, thinking about variables and relationships, and recognizing algebraic structure as a combination. Students can engage in algebraic reasoning regardless of whether they use algebraic notation (Carraher & Schliemann, 2020). Schmittau (2005) takes algebraic thinking from a Vygotskian perspective; he suggested that arithmetic should be taught after algebra, which requires different learning activities to teach algebraic thinking. His approach includes three essential characteristics: initial development from the most generalized conceptual base, ascent from the abstract to the concrete, and appropriation of psychological tools; in this system, he recommended beginning with general and going through particular. On the other hand, Britt and Irwin (2008) foresaw the integration of algebra and arithmetic within their studies; their studies demonstrated that students who had acquired sophisticated mental approaches to

handle addition, multiplication, and proportional operations were the ones able to utilize the alphanumeric symbols of algebra effectively. Kieran's (2018) view that algebraic thinking develops due to students' interactions with concepts such as properties of numbers, equality and equivalence, unknowns, variables, and functions is consistent with Irwin and Britt's findings.

These additional methods demonstrate the confusion in defining initial algebraic thought, distinguishing it from arithmetic thinking, and lightening its developmental process. Development of the level of algebraic thinking accelerates as students transition into abstract thinking (Altun, 2018). Hart et al. (1981) expressed the levels of algebraic thinking in four stages: the first level has basic symbolic structures that require specific unknowns that students were likely to evaluate the letter, the second level requires more complicated structures, the third level has basic structures where letters are perceived and used as an unknown, and the fourth level has complicated structures require letters as unknowns.

Algebra and algebraic thinking are different things, but they are closely related. Algebra focuses more on manipulating symbols and formulae, during which students solve equations within specific rules and procedures. Algebraic thinking, on the other hand, is not limited to symbols but also includes generalization and abstraction skills as part of mathematical thinking (Kaput, 1999). In terms of problems, algebra generally follows a specific solution process to solve mathematical problems. This solution process means applying a structured and logical sequence of steps to solve mathematical problems. This typically involves identifying the given information, representing the problem using algebraic expressions or equations, manipulating these equations through standard operations (such as addition, subtraction, multiplication, or division), and systematically working toward the solution. This process ensures consistency, accuracy, and clarity in solving various types of mathematical problems.

Algebraic thinking, on the other hand, refers to a more general mathematical understanding and develops the ability to solve different types of problems. This way

of thinking involves the ability to construct mathematical models and apply these models to different contexts. Kieran (2006) states that algebraic thinking can be defined as students' ability to understand mathematical relationships and apply these relationships to different situations. This reveals that algebraic thinking is a concept beyond algebraic operations.

Carraher and Schliemann (2020) argue that algebraic reasoning can occur without using algebraic notation; algebraic thinking is a broader concept than symbolic algebra. Developing this way of thinking often requires students to engage more deeply in problem-solving processes, and therefore, a more exploratory and inquiry approach to teaching is adopted. Algebraic thinking starts with concrete experiences with numbers and evolves towards generalization and abstract thinking skills through activities (Mason, 2005). Mason (2005) states that when children begin to explore numbers, teachers can guide this natural curiosity and transform the meaning-making process into algebraic thinking. In this context, algebraic thinking develops as children recognize number patterns and begin to generalize these patterns. Over time, with systematic and targeted instruction, the algebraic thinking of young learners becomes more complex. Research shows that many long-term studies of early algebra instruction have examined how students solve increasing algebraic pattern tasks through their discourse, work samples, and assessment results (Lentz, 2018).

There are many studies on algebraic thinking in the literature. It is seen that the number of studies focusing on early algebraic thinking is higher than others (Blanton et al., 2019; Chimoni et al., 2018). In addition, there are also studies on the measurement of algebraic thinking and scale development in this context (Chimoni et al., 2019; Ralston et al., 2018; Rittle-Johnson et al., 2013). Some of these studies were conducted at the preschool level (e.g., Mulligan et al., 2020; Rittle-Johnson et al., 2013), while a significant number of them focused on primary and secondary school levels (e.g., Chimoni et al., 2018; Chimoni et al., 2019; Irwin & Britt, 2008).

2.4 Algebra in School Mathematics: Lectures and Curricula

Algebra is taught in various ways across different education systems around the world. In some education systems, algebra is taught separately from mathematics, especially in middle and high school. This approach allows for a deeper understanding of algebraic concepts and problem-solving skills. In this way, students can specialize in algebraic operations, equations, functions, and inequalities. Teaching algebra can be taught as a separate discipline that contributes to developing students' algebraic thinking skills. In many countries, algebra is taught as part of the mathematics curriculum. This integrated approach teaches algebraic concepts with arithmetic, geometry, and other mathematical topics. Teaching algebra as part of the mathematics curriculum helps students to see the connections between different mathematical areas (Stacey & Chick, 2004). This method enables students to develop their mathematical thinking skills in a broader framework. Which approach is adopted in a country usually depends on the country's educational philosophy and curriculum structure. In both cases, however, algebra is recognized as an essential component of mathematics education.

Algebra has a central role in mathematics education. Algebraic thinking, which involves understanding and manipulating symbols and abstract concepts, helps students understand more complex mathematical concepts and develop problem-solving skills. Due to the abstract nature of algebra, algebra curricula must be designed to support students' acquisition of these skills and include strategies and teaching methods to alleviate the difficulties encountered. In addition, the lecture process should include problem-solving approaches in which students actively participate to understand algebraic concepts (Bednarz et al., 1996). Algebraic thinking is necessary for students to understand mathematical concepts at an abstract level, and various teaching strategies should be used to support the development of algebraic thinking and to overcome the difficulties students experience in the transition from arithmetic thinking to algebraic thinking. Moreover, Kaput (1999)

encourages students to understand algebraic concepts and structures and suggests that algebraic thinking should develop naturally.

Kieran (2006) emphasizes that algebraic thinking requires abstraction skills and understanding general concepts. Abstraction enables students to make the transition from specific problems to general concepts. Algebraic thinking instruction should help students transition from concrete examples to abstract concepts, enabling them to understand general mathematical structures and relationships. Algebraic notations and representations must be considered when acquiring abstraction skills. Students should be able to represent and understand mathematical relationships and structures accurately. Effective algebraic thinking instruction aims to help students understand algebraic concepts, develop problem-solving skills, think abstractly, use algebraic representations correctly, and increase their mathematical communication skills. These goals are usually achieved through interactive teaching methods, hands-on problems, and strategy development.

Early algebra instruction is becoming increasingly important in school mathematics programs. Developing algebraic thinking skills early provides a solid foundation for students to understand advanced mathematics topics and strengthen their problem-solving skills. In this context, Akın and Işler-Baykal (2024) examined the effects of a functional thinking intervention on the functional thinking skills of fifth-grade students. The experimental group received a 12-hour functional thinking intervention in the study, while the control group received a placebo program. According to the results of the Functional Thinking Test, although there was no significant difference between the two groups, it was found that the experimental group significantly improved from the pre-test to the post-test and showed significant progress in using variables to define function rules.

The study focuses on key areas of functional thinking, such as generalizing relationships between variable quantities, expressing and justifying these generalizations through natural language, variable notation, drawings, tables, and graphs, and reasoning with these generalizations. These findings suggest that early

algebra instruction is not limited to teaching mathematical procedures but has a critical role in helping students develop a deep understanding of mathematical thinking processes. Functional thinking involves understanding the relationships between variables and how these relationships can be expressed and reasoned about in various forms, such as through language, notation, and visual representations.

Lee and Mao (2021) state that early algebra is not only a matter of instructional timing but also aims to develop a holistic approach to mathematics education to support students' future academic success. However, the authors emphasize that the benefits of this approach are different for some students and may vary depending on individual student performance and the support opportunities offered. This approach suggests that integrating algebra into the curriculum at the right time and proper form is essential. Ding, Byrnes, and Ke (2023) found that the quality of instruction not only directly affects student achievement but also has indirect effects through students' prior knowledge. In particular, teachers asking deep questions is essential in developing students' algebraic thinking skills. (Ding et al., 2023).

To summarize, in order to develop algebraic thinking skills, the curriculum should be designed in such a way that students gradually adapt to algebraic concepts, both conceptual and practical aspects of algebra should be addressed, and teaching methods should be used to facilitate students to overcome the difficulties they encounter when learning algebra. This study will investigate whether the curriculum develops students' algebraic thinking ability in the early years of middle school.

2.5 Algebra in 2018 and 2024 Turkish National Education Curricula

Turkey's most recent mathematics curriculum is the updated 2024-2025 academic year curriculum (MoNe, 2024). This curriculum was prepared by the Ministry of National Education and put into practice as of the 2024-2025 academic year, starting from the first, fifth, and ninth grades and gradually widening to other grades each year. The most recent mathematics curriculum before 2024 is 2018. The updates made in 2018 covered the primary and secondary school mathematics curricula and

included some changes and innovations compared to the previous curricula. Since this study was conducted during the implementation of the 2018 curriculum, it compares this curriculum with the 2024 curriculum.

2.5.1 Primary School Level

This level is mainly based on arithmetic in the 2018 curriculum. Algebraic thinking starts with patterns of shapes and numbers. Students are introduced to basic algebraic concepts by discovering patterns and relationships between numbers and are guided to understand the underlying rules and relationships by studying number sequences and patterns. Patterns at this level are generally shape patterns. In contrast, the objectives of "*to complete a repetitive geometric pattern by identifying the missing ele*ments" (MoNe, 2018; M.2.2.3.1) and "*to create new patterns with the same relationship with different materials by using the relationship in a geometric pattern*" (MoNe, 2018; M.2.2.3.2) are processed in Grade 2, the number pattern is started in Grades 3 and 4. However, it is ensured that the difference between the numbers is constant and the pattern is expanded at most a certain number of steps: Grade 3 "*to expand and create number patterns with a fixed difference between them*" (MoNe, 2018; M.3.1.1.7); Grade 4 "*to create increasing or decreasing number patterns according to a certain rule and explains the rule*" (MoNe, 2018; M.4.1.1.6).

However, the concept of equality is emphasized in Grade 2. The objective of this subject is "*to recognize the meaning of the equal sign as 'equality' between mathematical expressions*" (MoNe, 2018; M.2.1.3.5). It is emphasized that the equal sign does not always mean the result of the operation but also indicates the equilibrium state (equality) of the mathematical expressions on both sides of the equality. The following operations are given as an example.

$5 + 6 = 10 + 1$; $15 – 3 = 18 – 6$; $8 + 7 = 20 – 5$; $18 = 16 + 2$

In addition, while the Grade 2 and Grade 3 learning outcomes include finding the ungiven sum, the Grade 4 learning outcome on this subject is "*to determine the ungiven value in one of the two mathematical expressions with equality between them*

and explains that equality is achieved" (MoNe, 2018; M.4.1.5.7). The following operations are given as an example of the fourth-grade outcome.

7 + ☐ = 15 – 3; 12 : 4 = ☐ + 1; 6 x ☐ = 48 – 12

Another fourth-grade outcome that can develop algebraic thinking is "*to explain the operations that need to be done to make two mathematical expressions that are not equal*" (MoNe, 2018; M.4.1.5.8); an explanation is made to focus on the operations that can be done to ensure equality in the expression 8+5 ≠ 12-3. The concept of inequality and the greater than and less than symbols are first introduced in Grade 3.

In the 2024 curriculum, algebraic outcomes, which are implicit in the 2018 curriculum, are integrated at the primary school level with the theme "*From Operations to Algebraic Thinking*" (MoNe, 2024; p.13). In the new curriculum, algebra at the primary school level has an essential place in all levels of mathematics education, accounting for approximately 30% at each grade level. The Primary School Mathematics Curriculum is structured based on a thematic approach. This approach considers each theme a holistic structure related to mathematical skills, topics, ideas, concepts, and operations. Therefore, in the thematic structure of the Primary School Mathematics Curriculum, topics that reflect the integrity of skills and knowledge more effectively come to the fore instead of traditional subfields or learning areas of mathematics.

The theme "*From Operations to Algebraic Thinking*" (MoNe, 2024; p.13) aims to guide students toward algebraic thinking skills, starting with operations, which are the cornerstones of mathematics. Within the framework of this theme, students are first introduced to the processes of making operations with numbers and making sense of them. Then, they are given opportunities to develop algebraic thinking skills. Thus, students can perceive mathematical thinking as a set of rules and calculations and an actional, imaginative, and symbolic process that will facilitate their daily lives. The content provided under this theme also contributes to students' preparation for mathematics subjects at the middle school level (MoNe, 2024). This theme focuses on the four operations and the relationships between these operations:

problem-solving, problem-posing, mental operations, estimation, and equality, and aims to develop students' procedural fluency skills. Arithmetic operations and the connections between these operations and the concept of equality are addressed at each grade level, laying the foundations for algebraic thinking.

Knowledge of the equal sign is recognized as a fundamental and critical component of early algebraic thinking (Carpenter et al., 2003). Early understanding of the equal sign is a powerful predictor of students' future algebraic proficiency. In this context, given that algebra plays a key role in students' future academic and economic success, knowledge of the equal sign is seen as a critical element for achieving this success (Matthews & Fuchs, 2020). While the 2018 curriculum includes gains related to the equality sign, in the 2024 curriculum, spreading these gains to all grade levels at the primary school level will make significant contributions to the development of students' algebraic thinking skills.

2.5.2 Middle School Level

Objectives in algebra begin in the 6th grade in the 2018 curriculum. Students are expected to find specific terms in number patterns and understand algebraic expressions during this period. In Grade seven, there are two main sub-learning areas: algebraic expressions and equations and equations. In this grade, students are expected to add and subtract algebraic expressions, understand the concept of equality, and solve equations with first-order unknowns. In Grade eight, algebra is covered in a much broader way. At this level, algebraic expressions, identities, linear equations, and inequalities are emphasized. Students should understand algebraic expressions and identities, factor algebraic expressions, analyze the linear relationship between two variables, and solve equations. Secondary school algebra education is completed by studying inequalities with one unknown (MoNe, 2018).

Students recognize the relationship between numbers and variables and express this relationship through equations. Basic equations are introduced in the middle school curriculum, and students learn how to analyze mathematical problems by solving

these equations. During this period, algebraic expressions and equations are taught by relating them to everyday problems. This helps students understand how to apply mathematical concepts in real life. Students develop skills for solving various problems such as mixture, age, and ratio problems using algebraic concepts.

In the 2024 curriculum, algebra at the middle school level is covered as a theme in each grade, with algebraic content rates of 11% in the fifth, 18% in the sixth, 21% in the seventh, and 23% in the eighth grades. The themes of "*Numbers and Quantities*" and "*Algebraic Thinking and Changes with Operations*" (MoNe, 2024; p.14) were structured together to enable students to extend their number skills to algebra and function concepts. Within the scope of these themes, the content built on the concepts acquired in the fourth grade of primary school shows a comprehensive expansion from natural numbers to real numbers, from operation properties to algebraic expressions, and from patterns to functions. Developing students' mathematical domain skills, such as problem-solving, mathematical representation, and mathematical reasoning skills, also stands out as a primary goal (MoNe, 2024).

In addition, in these themes, special emphasis was placed on thinking processes such as generalizing, analyzing, interpreting, inferring, constructing, evaluating, reflecting and inductive reasoning in order to support the development of conceptual skills. In this way, it is aimed at students deepening their conceptual understanding and acquiring mathematical thinking in a holistic manner.

In the fifth-grade curriculum in 2018, there is an outcome that "*to form the desired steps of number and shape patterns given the rule*" (MoNe, 2018; M.5.1.1.3), but this outcome is limited to patterns in which the difference between the steps is constant, and therefore no generalization is made. For example, there are activities to find out how many squares or triangles there will be in the desired step of the shape pattern seen in Figure 2.1 (MoNe, 2018; p.51);

Figure 2.1. Shape pattern example from MoNe 2018 curriculum

In the 2024 curriculum, the “Algebraic Thinking with Operations” theme at the fifth-grade level covers the topics of conservation of equality, properties of operations, precedence of operations, and number and shape patterns. Developing reasoning skills related to the conservation of equality and properties of operations at this level is important to build a foundation for algebraic expressions, equations, and inequalities in later grades. At the same time, the ability to reason about the rule of numbers and shape patterns is also developed within the scope of this theme; thus, increasing the ability to think about interconnected variable quantities provides a preliminary preparation for the concept of function (MoNe, 2024).

Under this theme, the newly added topic of algorithms is also included. In this context, it is aimed at students making sense of algorithms created with basic arithmetic operations in their forms expressed using pseudocode, flowcharts, and natural language. This process aims to contribute to developing students' algorithmic thinking skills.

In sixth grade, letters, previously used in unit measurements, started to be used instead of unknown numbers in the 2018 curriculum. It is stated that the letters used in algebraic expressions represent numbers and are called ‘variables.’ Algebraic expressions are defined, and students are asked to construct ‘algebraic expressions’ containing at least one variable and an operation and to attribute meaning to the constructed algebraic expressions. In addition, with the objective of "*to calculate the value of the algebraic expression for different natural number values that the variable will take,*" students switch between letters and numbers (MoNe, 2018; M.6.2.1.2).

Algebraic expressions and algorithms are addressed in the theme “Algebraic Thinking and Variations with Operations” at the sixth-grade level in the 2024 curriculum. In sixth grade, students who interpreted the properties of operations in fifth grade are expected to develop these inferences by reasoning about algebraic expressions and interpreting patterns. In this context, the algorithm interpretation skills acquired in the previous grade are further reinforced through algebraic expressions (MoNe, 2024).

In Grade 7, equations and problems are introduced in the 2018 curriculum. They start with "*to add and subtract with algebraic expressions*" (MoNe, 2018; M.7.2.1.1) and continue with "*to multiply an algebraic expression by a natural number*" (MoNe, 2018; M.7.2.1.2). However, the product of two algebraic expressions is in eighth grade. In the transition from pattern to equation, first, "*to understand the principle of conservation of equality*" (MoNe, 2018; M.7.2.2.1) and the basic concepts in this subject are given. The solution to equality has progressed in stages. In addition, the process of finding the rule of the pattern is used for the transition to equations. "*to express the rule of number patterns with a letter, finds the desired term of the pattern whose rule is expressed with a letter*” (MoNe, 2018; M.7.2.1.3). With the generalization learned here, students are expected to express the situations in the problems with algebraic expressions to form equations and solve these equations in stages.

In the 2018 curriculum, the student introduced to generalization almost for the first time in the seventh grade, and is asked to generalize in every mathematics subject starting from primary school in the 2024 curriculum (MoNe, 2024). In addition, the new curriculum emphasizes operations with algebraic expressions, equations, inequalities, proofs, and algorithms in the seventh grade. In the interpretation of operations with algebraic expressions, it is ensured that previous learning outcomes related to whole numbers and operation properties are utilized. While problem-solving skills are prioritized in equations and inequalities, mathematical representation and reasoning skills are also used effectively. Another goal of the theme is for students to gain mathematical proof skills by carrying mathematical

reasoning further. In developing this skill, the experiences built on the inferences about numbers and number properties in previous grades are utilized, and these properties are used as a tool in the proof process. Finally, this theme also covers the construction of algorithms. Students who interpret algorithms given with different expression methods in the 5th and 6th grades of middle school are expected to be able to construct algorithm structures by using algebraic expressions and operations as tools at this stage.

In eighth grade, the concept of identity is introduced with the multiplication of algebraic expressions in the 2018 curriculum, and students are expected "*to multiply algebraic expressions*" (MoNe, 2018; M.8.2.1.2) and "to *explain identities with models*" (MoNe, 2018; M.8.2.1.3). At this point, forming quadratic expressions that are not perfect squares is not attempted. In the eighth grade, linear equations are built on the ability to solve problems with equations learned in the seventh grade. Coordinate systems, tables, equations, and graphs are started to be used at this grade level. In explaining how to draw linear equation graphs, the points at which the line crosses the axes, its parallelism to the axes, and whether it passes through the origin are discussed (MoNe, 2018). In addition, linear equations are used "*to construct and interpret equations, tables, and graphs of real-life situations involving linear relationships*" (MoNe, 2018; M.8.2.2.5). Alternatively, "*to explain the slope of the line with models and associates linear equations and graphs with slope"* (MoNe, 2018; M.8.2.2.6). Finally, the objective of "*to write mathematical sentences suitable for daily life situations involving inequalities with one unknown in the first degree*" (MoNe, 2018; M.8.2.3.1) is included in this level. In addition, students are expected to show and solve these inequalities on the number line.

The 2024 curriculum is aimed to actively use the skills of analysis, mathematical representation, and mathematical reasoning in the development of concepts related to linear functions within the scope of the "Algebraic Thinking and Changes" theme at the eighth-grade level. How to represent linear relationships with linear functions using the orthogonal coordinate system, as well as the positions of the graphs of two linear functions with respect to each other, are addressed through these skills. Change

and algebraic operations covered in previous grades play an important role in representing the concept of function at this stage. In addition, the construction of algorithms based on problems related to linear functions is also an important part of this theme (MoNe, 2024).

In the eighth-grade curriculum, unlike previous years, linear functions were adopted instead of linear equations. In the historical process, students were expected to research how Descartes discovered the orthogonal coordinate system and in which areas this system found application. Students were asked to develop different algorithms and test their functioning by converting them into computer coding language. In addition, activities such as cycling, skiing, mountain climbing, ramp climbing, and hill climbing, frequently encountered in daily life, were associated with the concept of slope. In this context, the difficulties encountered by different cyclists while climbing slopes with various slopes were designed as in-class activities to be discussed.

Algorithms for problems related to linear functions were expected to be structured to enable students to develop at their own pace, and visual tools were expected to be utilized in this process. In teaching equations and inequalities, problem-solving skills were emphasized, and mathematical representation and reasoning skills were used in connection with this skill. Enabling students to solve real-life problems involving algebraic expressions, equations, and inequalities aimed to increase their awareness of the use of mathematics in various disciplines.

The curriculum aims to develop students' critical and analytical thinking skills through algebraic concepts. This process enables students to understand and solve mathematical problems in greater depth. For details on the 2018 curriculum algebra objectives, please refer to Appendix D. In order to study advanced algebra, students need to have basic algebraic thinking skills. These skills play an important role in understanding algebraic concepts, problem-solving, and developing abstract thinking skills. Basic skills to support the transition to advanced algebra include understanding the concept of variable (Carpenter et al., 2003), understanding the

meaning of the equality sign, and the difference between the concepts of equality and equivalence (Kieran, 1992; Mason, 1996), recognizing patterns and transforming these patterns into general rules and generalizing (Kaput, 2008; Kieran et al., 2016; Mason, 1996), performing arithmetic calculations and understanding the relationship between numbers and operations (Rittle-Johnson et al., 2013). This study examined whether the 2018 curriculum improves students' algebraic thinking skills in early middle school through a test prepared on this subject.

2.6 Theoretical Framework

2.6.1 Algebra Content Strands

In his study "Algebra and Algebraic Thinking in School Mathematics," which is a part of the "Second Handbook of Research on Mathematics Teaching and Learning," Kaput (2008) emphasizes how critical algebraic thinking is in mathematical thinking processes. He states that algebraic thinking will improve students' understanding of the relationships between numbers, make generalizations, and analyze mathematical structures. It provides information on how algebraic thinking can be encouraged earlier and how students can learn and develop these skills. The book offers practical suggestions for mathematics teachers and provides information on overcoming the problems encountered in teaching algebraic thinking. Additionally, he refers to specific areas of knowledge and skills that develop algebraic thinking starting in primary school. Kaput argued that algebraic thinking could be developed not only at the high school level but also at an earlier age. For this purpose, he categorized the algebraic content that students should acquire in different age groups. This thesis will use Kaput's categories as its framework text.

These content strands offer a structured approach that allows students to develop algebraic thinking skills from primary school onwards. He examines algebraic thinking from two core aspects: (A) making generalizations and conveying these generalizations using progressively structured and conventional systems and (B) manipulating symbols within structured symbol systems guided by syntax.

Generalization is defined as “syntactically guided reasoning and actions on generalizations expressed in conventional symbol systems” (Kaput, 2008; p. 11), while symbolization is explained as “systematically symbolizing generalizations of regularities and constraints” (Kaput, 2008; p. 11). Core Aspect B is commonly believed to mature later than Core Aspect A because actions based on rules concerning symbols rely on understanding the tolerable combinations and their interrelationships, particularly in determining equivalences among combinations.

In addition, Kaput (2008), based on these two basic components as seen in Table 2.1, considers algebra in three aspects: the study of structures and systems abstracted from computation and relations, the analysis of functions and their relations and their combinations, and the application of algebra as a language for mathematical modeling.

Table 2.1 Algebraic Thinking Framework Adapted from Kaput (2008)

The two core aspects	
A	Algebra involves the structured representation of patterns and limitations in a generalized manner using symbols.
B	Algebra involves employing syntactically guided reasoning and operations on generalizations presented within conventional systems of symbols.
The three strands emerge from two core aspects	
1.	Algebra examines abstracted structures and systems independent from specific computations and relationships, encompassing those found in arithmetic and quantitative reasoning.
2.	Algebra as functional thinking involves exploring functions, relationships, and variations.
3.	Algebra as modeling languages involves utilizing a set of modeling languages within and beyond the scope of mathematics.

Similarly, Radford (2000) argued that algebraic thinking is not merely a matter of learning and applying a precise symbolic language. According to him, algebraic thinking involves individuals' efforts to represent generalities, and these representational processes are realized through various means such as gestures, verbal expressions, drawings, and alphanumeric symbols.

Similar to Kaput's approach, Kieran et al. (2016) defined algebra in terms of three main activities. First, generalization activities involve deriving general expressions from shape patterns or numerical sequences and modeling quantitative problem situations with algebraic expressions. Moreover, according to Kieran, generalization involves three types of algebraic expressions: (i) equations representing quantitative problem situations, (ii) general expressions derived from shape patterns or numerical sequences, and (iii) expressions of rules describing numerical relationships.

Transformational activities are the second main activity in Kieran's (2006) algebra definition. These activities involve algebraic manipulations of equations and expressions, such as combining like terms, opening parentheses, simplifying expressions, exponentiating with polynomials, and solving equations. The third activity is meta-level activities, which involve problem-solving processes such as prediction, modeling, structure analysis, and generalization. These activities, in which algebra is used as a tool, include modeling, predicting, examining structure and change, analyzing relationships, generalizing, and proving.

In this context, algebraic thinking involves logical reasoning about unknowns, representing relationships between variables with mathematical expressions, and problem-solving through symbolic tools (Kriegler, 2007). Kriegler (2007) examines algebraic thinking in two main components: developing mathematical thinking tools and fundamental algebraic ideas. She discusses the tools of mathematical thinking in three main categories: problem-solving skills, representational skills, and quantitative reasoning skills. Similar to Kaput (2008) and Kieran et al. (2016), she explains the basic principles of algebra as being understood as an arithmetic

generalization, used as the language of mathematics, and operating as a functional and modeling tool of mathematics.

Symbolization and generalization are essential components of algebraic thinking and can have different weights. While generalization is more prominent in situations such as expressing number patterns and relationships, symbolization plays a more prominent role in transforming real-life situations into mathematical models. Kaput's categories show that these skills are intertwined and complementary (Ralston, 2013).

2.6.1.1 Generalized Arithmetic

This teaching area is a basic bridge for students' transition to algebra. It aims to develop students' algebraic thinking skills by addressing the operations and rules known in arithmetic in a more general framework. It helps students develop algebraic thinking skills based on their knowledge of arithmetic. It aims to go beyond arithmetic operations and properties to discover more general mathematical relationships and patterns, to understand that we can write a mathematical expression in different ways and that these different ways have the same meaning (Kaput, 2008). For this purpose, conceptually based calculation strategies include ratio and proportion, estimation and understanding, and using the properties of the number system processes (Kreigler, 2007). For example, discovering and generalizing the properties of zero, facilitating calculations by using the properties of operations and different strategies, finding order in mathematical relationships, solving a problem differently, and choosing the most appropriate solution (Chimoni et al., 2018).

2.6.1.2 Functional Thinking

This teaching area aims to systematically develop students' functional thinking skills by focusing on functional generalization. Functions are mathematical structures that model the relationships between variables and form the basis of algebraic thinking (Kaput, 2008). In the teaching process, students' ability to make mathematical generalizations is supported by starting with basic pattern activities. These activities include the meaning of variables and variable expressions, reading, writing, and

manipulating numbers and symbols using algebraic rules, using algebraic expressions to express patterns, and developing skills in graphing (Kriegler, 2007). Students gain skills in expressing and analyzing functions using different representations such as tables, graphs, and symbolic representations. This contributes significantly to the development of students' mathematical thinking skills.

2.6.1.3 Modeling Language

Emphasizes the versatility of algebraic modeling to address various problems and situations, from specific numerical constraints to broader generalizations and pattern recognition (Kaput, 2008). Algebra is applied to generalize and express patterns in situations or phenomena, which can be adapted to contexts applicable to mathematics or real-life problems. Modeling language includes understanding, expressing, and generalizing patterns and rules in real-world contexts, representing mathematical ideas with equations, tables, graphs, or verbal explanations, and working with input/output patterns (Kriegler, 2007). The language of modeling is a broad and rich domain that enables the use of concepts such as linearity and proportion in real-world scenarios.

In her 2013 dissertation on the development and validation of a diagnostic assessment tool to assess elementary school students' algebraic thinking skills, Ralston used Kaput's model of three intertwined circles to express Kaput's categories of algebraic thinking more effectively visually (Figure 2.2). This model provides a more explicit understanding of the interconnected structure and relationships between the different components of algebraic thinking.

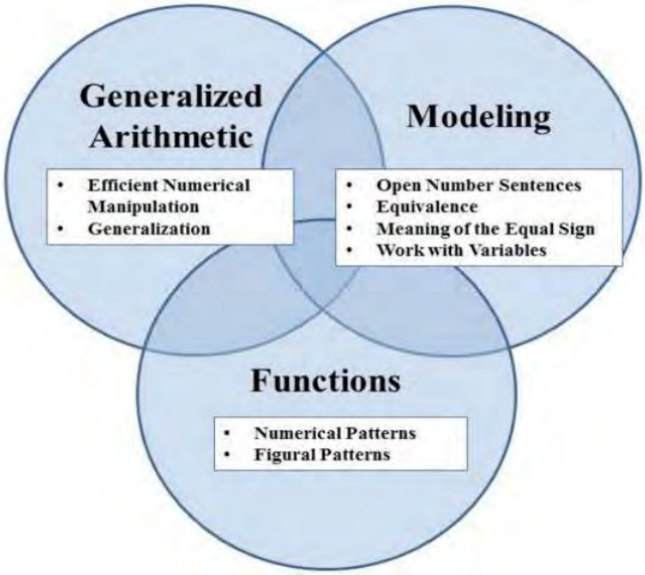

Figure 2.2. Kaput's (2008) algebraic thinking framework (Ralston, 2013, p. 11)

2.6.2 Basic Algebraic Concepts

The Principles and Standards for Mathematics Education describe algebra as emphasizing "relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change" (NCTM, 2000, p. 37) and has established several principles of algebraic reasoning. These principles are considered one of the cornerstones of mathematics education and help students develop mathematical thinking skills. The 2008 article "Algebra in Early Grades" emphasized that students should develop the ability to manipulate and simplify algebraic expressions. This article includes problem-solving principles, simplifying algebraic expressions, and connecting between equations. In 2011, in its publication "Developing Essential Understanding of Algebraic Thinking for Teaching Mathematics in Grades 3-5", NCTM mentioned the importance of developing the ability to create structural reasoning, recognizing the structures of algebraic expressions and equations, and drawing logical conclusions from these structures, and in 2014 it emphasized the importance of algebraically modeling real-world

problems. "Principles to Actions: Ensuring Mathematical Success for All" addresses algebraic modeling of problems as one of the basic principles required for mathematical success.

The NCTM (2000) underscores the significance of fostering students' comprehension of algebraic relationships and enhancing their ability to represent them. This includes a focus on understanding the interconnections among algebraic expressions, equations, and functions. These principles and standards highlight the need for students to grasp how algebra functions as a cohesive system, which is vital for problem-solving and real-world application in mathematics. Developing such knowledge is fundamental for students' success in algebra and other areas of mathematics. They explained the fundamental components of algebraic thinking in four steps. In the following headings, these items will be clarified.

2.6.2.1 Understanding Patterns, Relations, and Functions

Young children are interested in patterns in daily life; they observe, describe, repeat, extend, compare, and craft patterns. Additionally, they categorize, classify, and arrange items based on diverse attributes. They forecast forthcoming elements and recognize absent parts within patterns. Furthermore, they develop the ability to differentiate between different pattern types, like those that grow or repeat. In elementary grades, they like verbal representations; they present patterns numerically, graphically, and symbolically. They use relationships in patterns to make predictions and generalizations. They learn slope and distinguish linear from non-linear patterns.

2.6.2.2 Representing and Analyzing Mathematical Situations and Structures Using Algebraic Symbols

Young children can demonstrate mathematical properties (e.g., the commutativity of addition) using objects or specific numerical values. Additionally, students' comprehension of the equals sign evolves across various stages. Initially, they

perceive the equals sign operationally in equations like a + b = c. As students progress to the advanced stage, they interpret the equals sign relationally. For instance, they can compare expressions on both sides of an equation (e.g., 45 + 86 = 46 + 85) without executing any operations (Matthew et al., 2012).

When children are encouraged to represent quantities using varied methods, they acquire the skill to identify equivalent representations and enhance their capacity to utilize symbols for expressing their thoughts. In the latter stages of primary education, kids explore, indicate, and explain mathematical properties. They formulate general concepts about relationships and employ them in whole-number calculations. They often use a box, letter, or symbol to represent a placeholder as an idea of variables. Additionally, they grasp how to utilize variables to articulate a principle linking two quantities or to articulate connections through equations.

In the middle grades, students are introduced to broader applications of variables as changing quantities in generalized patterns, formulas, identities, and expressions representing mathematical properties, equations, and inequalities. They explore the concepts of dependence and independence among variables as they alter concerning one another. Additionally, they enhance their ability to identify similar mathematical representations, enabling them to manipulate expressions, solve problems, and establish connections among graphical, tabular, and symbolic representations.

2.6.2.3 Using Mathematical Models to Represent and Understand Quantitative Relationships

At a very young age, children start by using objects, images, and symbols to act out stories or simulate scenarios involving adding or subtracting whole numbers. As they move up to higher elementary levels, children start recognizing that mathematics serves as a tool to represent numerical and geometric patterns, scientific experiments, and real-world situations. They realize the capability of mathematical models not only to describe but also to predict outcomes. Using graphs, tables, and equations to illustrate connections and draw conclusions from their models, students compare

different models and explore whether diverse models of the same scenario produce the same results.

2.6.2.4 Analyzing Change in Various Contexts

Children recognize changes in the environment at an early age, such as taller, shorter, and lighter; they make these comparisons quantitatively after measuring and comparing quantities. In middle school, they demonstrate change in numerical, tabular, or graphical form; later, they learn that change can involve more than one quantity; for example, when they get tall, they gain weight.

Students in the higher grades initiate the examination of differences within evolving patterns. They contrast changes that transpire consistently, like the accumulation of money deposited weekly into a piggy bank without any change.

As a summary of NCTM (2000) principles, children in elementary grades should be able to,

- Understand patterns, relationships, and functions
- Express and examine mathematical scenarios and structures using letters
- Use mathematical representations to represent and comprehend relationships
- Analyze change in various contexts.

As outlined in Section 2.6 of this thesis, algebraic thinking will be examined under two main categories:

1. Exploring the arrangement and connections between three algebra strands: generalized arithmetic, functional thinking, and modeling languages (Kaput, 2008; Kieran et al., 2016; Kriegler, 2007).
2. Comprehending basic algebraic principles, including the basic concepts of the equal sign, equality, equations, properties of numbers and operations, variables, unknown values, symbols, inequality, co-variation, and correspondence (Chimoni et al., 2018; Kriegler, 2007; NCTM, 2000).

Equality: It is considered one of the cornerstones of algebraic thinking. Students use equality to understand the concept of equality and solve equations.

Equations: Equations are one of the basic structures of algebraic expressions. NCTM emphasizes equations as an important skill students need to solve and analyze.

Properties of Numbers and Operations: Understanding the basic properties of numbers and operations. NCTM states that these properties are essential in manipulating algebraic expressions and equations.

Variables: Variables represent unknowns in algebra and quantities that can take on different values. NCTM describes how variables are used to develop algebraic thinking.

Inequality: Inequality plays an important role in students' understanding of mathematical relationships and comparisons. Inequalities are treated like equations, but this time, instead of equality, inequalities are examined as greater than or less than. This concept is one of the mathematical tools that students can use to model and solve real-world problems.

Co-variation: This concept involves examining how two or more variables change together. NCTM recognizes this concept as an essential component of understanding functions.

Correspondence: This involves understanding how elements match between two sets. It plays a vital role in functions and relations.

This study aims to analyze the early stages of middle school students' algebraic thinking processes in depth within the framework of the dimensions mentioned in section 2.6. For this purpose, students' solution strategies in various algebraic tasks are analyzed through three algebra content strands and the basis of the basic concepts of algebra. The focus of the study is to identify the differences between the early algebraic thinking skills of students in three different age groups of students and to reveal their understanding of basic algebraic concepts. In order to visually support

the theoretical background of this thesis, Figure 2.3, adapted from Ralston (2013) and Chimoni et al. (2018), is presented. This figure visualizes the basic assumptions of the study and the relationships between these assumptions.

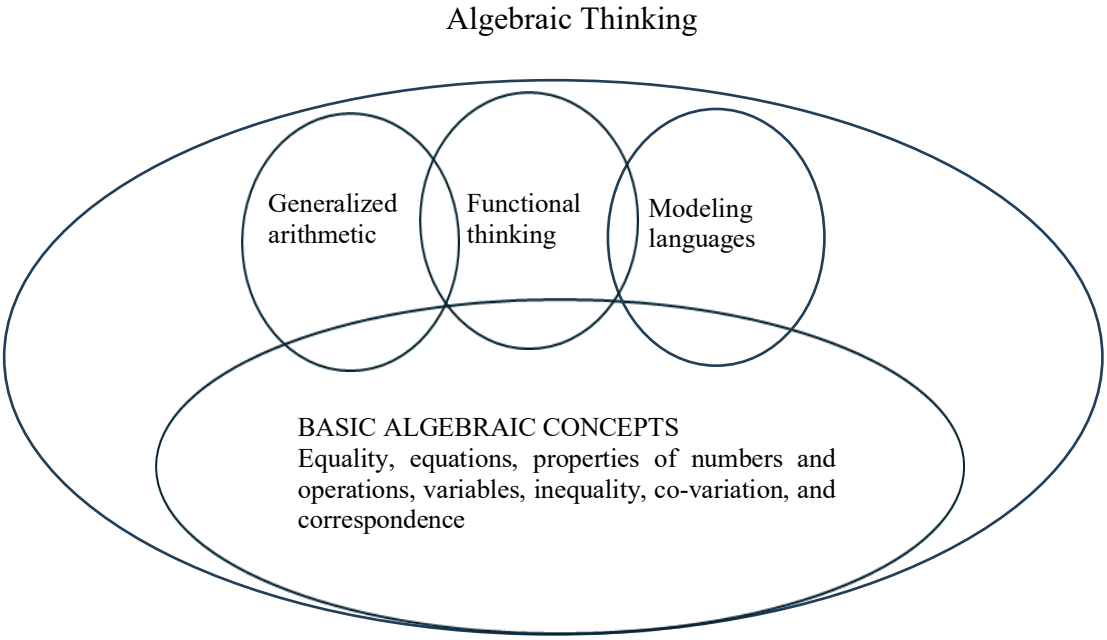

Figure 2.3. Algebraic thinking framework of this thesis study

2.7 Literature Review on Student Achievement in Algebra

This section will share some studies on students' algebraic thinking structures and algebraic thinking levels in Turkey and abroad. The selection of the shared studies is based on recent studies and their compatibility with the subject integrity of this thesis.

In their book "Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School," Carpenter et al. (2003) discuss how to develop children's arithmetic and algebraic thinking skills at an early age based on examples and case studies from natural classroom settings in mathematics education. The authors explain how algebraic thinking deepens children's mathematical understanding and what role arithmetic plays in this process. They examine the integration of arithmetic

and algebra and how this integration develops students' mathematical thinking abilities and suggest various strategies for developing students' mathematical thinking skills. In particular, it provides practical suggestions and teaching methods to improve students' mathematical problem-solving abilities.

In his study, Kaput (2008), discussed in detail in the previous section, defined the algebraic thinking levels of students and examined how these levels can be developed with teaching strategies. This study aims to understand students' algebraic problem-solving and generalization abilities.

"Adding It Up," a report prepared by the National Research Council in 2001 as a literature review and analysis, reviewed various research studies on mathematics learning and teaching and compiled the findings from these studies. It recommends improving educational policies and teaching practices based on general literature trends and current practice examples. This study considers algebraic thinking a vital topic and emphasizes that algebraic thinking is central to mathematical learning. It states that algebraic thinking has essential components such as understanding number and operation relationships, working with symbols, and making mathematical generalizations, and provides information on how children can develop algebraic thinking at an early age. It emphasizes how early algebraic thinking skills can be encouraged and how these skills should be supported. In addition, the report provides information about teaching strategies that support algebraic thinking, the integration of algebraic thinking with other mathematical concepts, the difficulties that children may encounter in the algebraic thinking process, and which approaches can be used to overcome these difficulties.

Chimoni et al.'s 2018 empirical study of 684 fourth, fifth, sixth, and seventh graders from 10 schools identified four distinct groups of students based on their performance on generalized arithmetic, functional thinking, and modeling tasks with algebra content strands. Group 1 struggled on all tasks, while Group 2 was able to solve some generalized arithmetic tasks. Group 3 solved more generalized arithmetic and functional thinking tasks than Group 2. They better understood symbolic

representation, but their generalization was still limited in certain contexts. Group 4 students exhibited improved early algebraic thinking and were able to solve all task types successfully. In conclusion, the study suggests a developmental progression in which students first develop generalized arithmetic skills, then functional thinking skills, after which they can move on to more advanced modeling tasks.

In their 2008 article, Irwin and Britt reported on a longitudinal study examining the algebraic thinking of students who had participated in the Numeracy Project, a mathematics education program in New Zealand, compared to students who had not participated. The study examined students' algebraic thinking in the Numeracy Project, a mathematics education program in New Zealand. The project sought to improve students' ability to express generalizations using algebraic symbols and representations. The study followed the students for three years and analyzed how their algebraic thinking developed. It found that students who participated in the Numeracy Project showed higher scores on measures of algebraic thinking than those who did not. They concluded that understanding arithmetic relationships and working comfortably with arithmetic operations are essential prerequisites for learning formal algebra.

Some studies published in Turkey on algebraic thinking have focused on improving mathematics education and student understanding. We will focus primarily on those who investigate the impact of a constructivist approach to mathematics education on algebraic thinking. They emphasize the need to move away from traditional behaviorist methods in favor of innovative education models. They argue that algebra and algebraic concepts must be better integrated into the curriculum, starting at earlier grade levels. They also stress the importance of students developing algebraic and mathematical modeling skills rather than simply memorizing procedures. They recommend using various assessment methods such as projects, group work, and portfolios rather than just tests. They also recognize students' difficulties in developing algebraic thinking and the need for targeted intervention and support.

In their article published in 2023, Şengül and Tekcan examined the effect of a complete learning-centered enriched learning environment on the algebraic thinking skills of fifth-grade students. In the main application, which was applied to 12 students selected with the maximum diversity method after the pilot application with 69 students, the "Algebraic Thinking Skills Test [CDBT]" prepared by the researchers as a pre-test and post-test, follow-up tests, and parallel tests conducted during the application were evaluated within the framework of the five themes determined by Kaput (1999) on algebraic thinking. It was determined that the implementation moderately affected students' algebraic thinking skills. It was determined that the students did not achieve the desired level of success, especially in the themes of "meaningful use of symbols" and "mathematical modeling."

Çağdaşer conducted an experimental study with 55 sixth-grade students in 2008. He evaluated the algebraic thinking levels of the students according to Altun's (2018) algebraic thinking levels with the "Algebraic Thinking Levels Test" that he applied at the end of 10 lesson hours with the activities he prepared with a constructivist approach. As a result, it was seen that teaching algebra with a constructivist approach significantly increased the algebraic thinking levels of sixth-grade students.

Kaş (2010) investigated the effect of teaching worksheets on eighth-grade students' algebra problem-solving and algebraic thinking skills. In addition, he clarified the differentiation questions according to the student's characteristics and which students had which characteristics the teaching with worksheets was more effective. He used the algebra test developed by Altun (2018) to measure the algebraic thinking levels of the students and determined the results according to Altun's algebraic thinking levels. Applying pre-tests and post-tests to sixty-three students showed that teaching with worksheets positively affected students' algebraic problem-solving and thinking skills.

In her dissertation in 2004, Çıkla Akkuş focused on teaching algebra and line graphs to seventh-grade students using multiple representations. Based on Lesh and Janvier's model of transformations between modes of representation, the study was

conducted over an 8-week instructional unit on “Equations and Line Graphs.” The study examined the effects of multiple representations on students' algebra performance, attitudes toward mathematics, and representation preferences. In addition, the students' pre- and post-instructional representation preferences, how they used multiple representations in algebraic word problems, and the reasons for preferring these representations were investigated. The results showed that the experimental group scored higher than the control group, some students created their own representations to solve problems, and the use of multiple representations extended to other mathematical contexts. The study evaluated its results using both quantitative and qualitative data analysis methods.

In a similar study, Girit and Akyüz (2016) examined the effects of a multiple representations-based intervention on seventh-grade students' algebra performance, attitudes towards mathematics, and representational preferences and found that this intervention improved students' algebra performance and helped them develop more positive attitudes towards mathematics compared to the control group. In addition, students in the experimental group showed a greater tendency to create and use their own representations to solve algebraic problems. This suggests that students' ability to use multiple representations resulted from the intervention. The study emphasizes that including multiple representations in algebra instruction effectively improves student understanding and problem-solving skills.

In addition, Türkmen and Tanışlı (2019) aimed to determine the level of generalization of functional relations in 3rd, 4th, and 5th-grade students. Emphasizing that the participants were in the pre-algebra period, it was stated that the development of functional thinking is important from an early age. The data were collected with open-ended questions and evaluated by thematic analysis. The results showed that students had many indicators of functional thinking in general but had difficulty in generalization and symbolic representation. These findings suggest that educational programs should be supported from an early age to develop functional thinking skills.

In addition, Kose and Kızıltoprak (2020) conducted a teaching experiment to improve fifth-grade students' relational thinking skills. In eight weeks of lessons, students were helped to understand the relationships between number sentences and operations. Students used relational thinking in addition, subtraction, multiplication, and division and learned the basic properties associated with equality. The results show that students evaluated number sentences without performing operations using equality axioms and developed their relational thinking skills.

Secondly, studies published in Turkey that directly investigate students' algebraic thinking skills are focused on. These studies have found that students often struggle with transitioning from arithmetic to algebra, especially with understanding the meaning of variables and using letters in algebraic expressions. Many students view algebraic equations as mere procedures for arriving at a result rather than understanding the relational nature of equality. Students often find it challenging to move from the concrete to the abstract in algebra and have difficulty representing verbal problems with algebraic expressions and equations. Studies have shown that students struggle most with higher-level algebraic thinking skills, such as solving verbal problems and working with more complex algebraic expressions. The results of different studies reveal a similar pattern, showing that students are stuck at lower levels of algebraic thinking and have difficulty progressing to higher levels.

Yenilmez and Teke in 2008 and Özbayar in 2017 investigated the effect of the mathematics curriculum on the algebraic thinking levels of sixth-grade students. In both studies, the "Development of Algebraic Thinking" test, as Altun (2018) reported, was applied as a pre-test and post-test. The evaluations were made according to Altun's algebraic thinking levels. According to the findings obtained from the pre-test-post-test data in both studies, there was a significant difference in favor of the post-test averages in all four levels of algebraic thinking.

In his 2017 article on examining the algebraic thinking levels and skills of seventh-grade students, Kaya conducted a survey model study with a total of 185 seventh-grade students. She used the "Algebraic Thinking Level Determination Tool

(CDDBA)" as a data collection tool. For the evaluations, students were asked to give their answers in three stages, and according to these stages, their algebra levels were analyzed as positive and negative skill deficiencies in the algebra learning area. According to the results of the descriptive analysis, it was determined that the positive skill deficiencies of the students related to the algebra learning domain decreased significantly from the first stage to the last stage.

Moreover, Kaya analyzed the relationship between eighth-grade students' mathematical epistemological beliefs and algebraic thinking skills in 2019. In her study with 162 students, she used the Pearson correlation technique and regression analysis to examine the effects of beliefs on algebraic thinking skills. The findings revealed significant positive relationships between students' mathematical beliefs and their algebraic thinking abilities, and different levels of significance were identified. Multiple regression analysis showed that beliefs about effort, ability, and the existence of a single correct answer explained 65% of the variance in algebraic thinking skills.

In Gülpek's (2016) thesis study, the test used by the Concepts in Secondary Mathematics and Science research project was adapted to Turkish and applied to seventh-grade students in the first stage, and in the second stage, it was applied to the same students one year later. In the evaluation, a level was determined according to the frequency of correct answers to the questions in the algebra test, and four levels emerged. It was observed that most of the seventh-grade students were at level 0 and level 1, while the eighth-grade students were more evenly distributed. In addition, there was a significant increase in the rate of solving level 3 and level 4 questions of eighth-grade students compared to seventh-grade students.

In their article published in 2018, Usta and Özdemir examined the algebraic thinking levels of middle school students. The Algebraic Thinking Level Detection Form, consisting of eight questions and two questions at each of the four levels, was applied to 12 students selected from the sixth, seventh, and eighth grades by purposive sampling method in clinical interviews. The evaluations were made according to the

algebraic thinking levels mentioned by Altun. All 12 students correctly answered the first two questions in Level 1. In the level 1 questions, the students treated the letters as objects and completed the operations without assigning any numerical value to these letters. In the fourth question at level 2, some students tried to find the perimeter of the given shape by assigning numerical values to the sides of unknown lengths. The fifth and sixth questions belonging to the third level of algebraic thinking posed difficulties for the seventh-grade students. At this level, while letters were expected to be perceived as unknowns, students tended to perform operations by assigning numerical values to letters. When the answers of some students who could not answer the questions at the fourth level with sufficient accuracy the sixth question were analyzed, it was observed that they had a wrong perception that multiplication would increase the value of the algebraic expression.

Park (2021) examined the algebraic thinking perception levels of middle school students in the Afyon sample in terms of various variables such as gender, grade level, mother's education level, father's education level, family income level, and the frequency of solving puzzles/playing mind games. As a result of the data analysis, it was determined that there was no statistically significant difference in the algebraic thinking perception levels of middle school students according to gender in the overall scale and sub-factors. However, statistically significant differences were found according to variables such as grade level, parental education level, family income level, and the frequency of solving puzzles/playing mind games. These differences favored students in the 6th grade, those whose parents have a university education level, family income level of 4000 TL or above, and those who frequently solve puzzles/play mind games.

In their article published in 2020, Birgin and Demirören examined the successful performance of seventh and eighth-grade middle school students on the topic of algebraic expressions. As a result of their study, conducted with 180 middle school students using a survey method, it was found that the algebra performance of eighth-grade students was higher than that of seventh-grade students, but there was no significant difference according to gender. It was also determined that students'

successful performance in algebraic expressions positively influenced their overall mathematics achievement.

In their article "An Examination of Seventh-Grade Students' Skills in Converting Verbal and Algebraic Expressions," Yağız et al. (2024) employed a qualitative research method involving 20 seventh-grade students. The study utilized a "Test of Algebraic Verbal Expressions" developed by the researcher, which consisted of two sections. The first section examined students' ability to convert verbal expressions into algebraic expressions, while the second section focused on their ability to convert algebraic expressions into verbal expressions. Descriptive analysis was used to analyze the data. The study concluded that while students successfully converted verbal expressions into algebraic ones, they struggled with converting algebraic expressions into verbal expressions.

Taking a broad perspective on studies related to algebraic thinking, Türkoğlu and Cihangir (2017) examined research on algebraic thinking skills. Their study assessed the prerequisite competencies for algebraic thinking, the critical stages of acquiring these skills, and the grade levels at which algebraic concepts are taught in different countries' curricula. The research findings indicated that the prerequisite skills for algebraic thinking are related to pattern generalizations and that the critical acquisition period occurs between the ages of 4 and 12. It was concluded that deficiencies in pattern-related topics negatively affect algebraic thinking, and thus, teaching patterns at an earlier age would be more beneficial. Additionally, supporting this view, it was observed that algebraic concepts are introduced at much earlier grade levels in the curricula of some countries. The study, which compared the grade levels at which algebraic concepts are introduced in the elementary mathematics curricula of China, Singapore, South Korea, Russia, and Turkey, revealed that the relationship between arithmetic and algebra is approached with a focus on arithmetic operations, generalization, and representation strategies.

Atasoy (2020) compared the performance of a group of children of public employees with medium-high socioeconomic status in an urban school and another group of

children of small business owners and low-income families with low socioeconomic status in a school far from the city center in solving algebraic verbal problems. Students' problem-solving strategies and errors were analyzed using a data collection tool previously developed by the researchers. The results revealed that student's ability to translate verbal problems into mathematical language and appropriate reasoning skills were not at an adequate level and that parents' level of education and higher socioeconomic status contributed positively to students' achievement.

In their 2003 article, Yaman, Toluk, and Olkun aimed to examine how 2nd, 3rd, 4th, 5th, and 6th grade students perceive the concept of equality and the equal sign. The results showed that students perceived the equal sign as an "operation sign" rather than a "relational symbol." Moreover, it was found that this perception did not change across age groups and that many students perceived equality only as the result of an operation. The study emphasizes the importance of correctly understanding the concept of equality in developing students' algebraic thinking.

2.8 Summary of The Literature

According to the literature review, many research results show that students have difficulty understanding algebraic concepts and have conceptual misunderstandings (Dede & Argün, 2003; Ersoy & Erbaş, 2005; Kaput, 1999; Kieran, 1992). This difficulty in learning algebraic concepts has been the subject of numerous studies, which have also revealed that students have difficulty in developing algebraic thinking skills. Indeed, international studies emphasize the importance of algebraic thinking and present various approaches to its teaching (e.g. Blanton & Kaput, 2011; Kaput, 2008; Kieran, 2007; Matthews & Fuchs, 2020; NCTM, 2000).

The teaching methods chosen contribute significantly to the meaningful and lifelong development of algebraic thinking and reasoning skills. Teachers should train their students to maximize their level of understanding and retention of algebra (Kaya & Keşan, 2014). In this context, activities in early age groups support the development of students' algebraic thinking skills (Chimoni et al., 2018; Mulligan et al., 2020). In

Turkey, many studies have been conducted on the effects of different teaching methods on middle school students' algebraic thinking skills (e.g., Çağdaşer, 2008; Çıkla Akkuş, 2004; Girit & Akyüz, 2016; Kaş, 2010; Kose & Kızıltoprak, 2020; Şengül & Tekcan, 2023; Türkmen & Tanışlı, 2019; Yıldırım, 2016). In addition, studies have also been conducted to obtain teachers' views on this issue (e.g., Baş et al., 2011; Işçi & Obay, 2022). In addition, studies have measured middle school students' algebraic thinking levels using Hart's four levels of algebraic thinking. For example, Gülpek (2016) measured the development of algebraic thinking levels of 7th and 8th-grade students; Usta and Özdemir (2018) used 6th, 7th, and 8th grades as a sample to investigate the algebraic thinking levels of middle school students; Kaya (2017) aimed to examine the positive and negative skill deficiencies of 7th-grade students in the field of algebra learning; and Kaya (2019) tried to explain the algebraic thinking skills of 8th-grade students in the context of mathematics-oriented epistemological beliefs.

While some studies on students' algebraic thinking levels were conducted at a single grade level (e.g., Kaya, 2017; Özbayar, 2017; Yenilmez & Teke, 2008), some other studies compared seventh and eighth grades (e.g., Birgin & Demirören, 2020; Gülpek, 2016). However, the number of studies analyzing the algebraic thinking skills of fifth-grade students in detail, especially during the transition from elementary to middle school, is limited. This situation reveals an important gap in knowledge in terms of assessing the level of preparation for advanced algebraic content at the eighth-grade level. The current study aims to fill this gap and examine the development of algebraic thinking skills of students in early middle school more closely, and in this context, it is thought to make a significant contribution to the literature.

CHAPTER 3

METHODOLOGY

This section provides comprehensive information about the research design of the study. First, the study participants, data collection tools, data collection process, and the methods used to analyze the data are explained in detail. The final section discusses potential threats to the validity and reliability of the study and its overall limitations.

3.1 Research Design

Causal-comparative research is a research design that aims to determine the relationships between independent and dependent variables after an event or situation has occurred. In this method, the researcher evaluates the effect of the independent variable on the dependent variable by comparing two or more groups. This approach, also known as ex post facto research, is based on the researcher analyzing the current situation without any intervention (Silva, 2010).

This study aims to examine the algebraic thinking skills of early middle school students. In the study, whether students' algebraic thinking skills were affected by independent variables such as gender or grade level was observed in the natural process. Therefore, ex post facto design was used in the study. In this type of research, there is no prior intervention or control over the variable as in experimental studies. Instead, naturally occurring situations are observed and the possible relationships between the dependent variable and independent variables are analyzed by examining the data obtained. The researcher takes the dependent variable (algebraic thinking skills) and evaluates the factors that may cause this situation to occur with a retrospective approach (Brewer et al., 2010).

In addition, the survey research, which is applied to “describe the characteristics of a population” (Fraenkel et al., 2012, p. 419), was also included in the study. This design was used to describe the participants' responses to the algebraic thinking test in detail. In order to investigate the algebraic thinking skills of fifth, sixth, and seventh-grade students, data were collected through the 'Algebraic Thinking Skills Test' (ALTS). The scores obtained by the students from the test were evaluated using the quantitative analysis method. At the same time, their answers and explanations about these answers were subjected to the qualitative analysis process.

The researcher found that the students in her school had difficulty especially with skill-based questions and problems involving variables. As stated in the literature, although the mathematics curriculum is divided into two separate categories, arithmetic and algebra, a deep understanding of arithmetic requires generalization skills, which is one of the basic elements of algebra (Carraher & Schliemann, 2020). Accordingly, the researcher concluded that students' lack of algebraic thinking skills was one of the main reasons for their academic difficulties, especially in skill-based questions and algebra topics. Similar problems were discussed in the mathematics department meetings, and as a result of the interviews with stakeholders, it was decided to investigate the deficiencies in students' algebraic thinking skills. The findings of the research were shared with the relevant mathematics teachers and necessary improvement activities were planned.

3.2 Research Question

This study aimed to answer the following three research questions and related subquestions.

Research Question 1: Is there a significant difference in students' performance on algebra content strand tasks based on grade level and gender?

- Is there a significant difference between fifth, sixth, and seventh-grade students' overall performance on the algebraic thinking test in terms of grade and gender?

- Is there a significant difference between fifth, sixth, and seventh-grade students' generalized arithmetic (GA) performance in the algebraic thinking test regarding grade and gender?
- Is there a significant difference between fifth, sixth, and seventh-grade students' functional thinking (FT) performance in the algebraic thinking test regarding grade and gender?
- Is there a significant difference between fifth, sixth, and seventh-grade students' modeling language (ML) performances in the algebraic thinking test regarding grade and gender?

Research Question 2: Which basic algebraic concepts are better understood by fifth-, sixth-, and seventh-grade students?

- Which of equality, equations, properties of numbers and operations, variables, inequality, co-variation, and correspondence have fifth-grade students grasped?
- Which of equality, equations, properties of numbers and operations, variables, inequality, co-variation, and correspondence have sixth-grade students grasped?
- Which of equality, equations, properties of numbers and operations, variables, inequality, co-variation, and correspondence have seventh-grade students grasped?

Research Question 3: What common strategies and mistakes influence students' algebraic processes when solving tasks?

- Are there similarities in the answers given by students at different grades to specific questions?

3.3 Participants and Research Context

The study participants were fifth, sixth, and seventh-grade students in a public middle school in the Mudanya district of Bursa. A convenient sampling method was

used to ensure easy access to the participants and consider accessibility (Fraenkel et al., 2012). The demographic characteristics of the participants are presented in Table 3.1.

The target group of the study was all fifth, sixth, and seventh-grade students at Mudanya Atatürk Secondary School. The researcher administered the tests to the classes taught by the researcher during her class hours and to the other classes during the class hours arranged by the researcher and in cooperation with the teachers of the other classes. Students who did not come to school on the day of the test were administered the test in the library or support education rooms under the supervision of the researcher. The target group was reached, and 487 students were administered the test. As seen in Table 3.1, the distribution of students in terms of gender at grade level and school-wide is approximate.

Table 3.1 Demographic Characteristics of the Sample

	Demographic variable		**(*f*)**		**%**
Students	Gender	Male	241		49
		Female	246		51
	Grade	5th	173	84 (male) 89 (female)	36
		6th	162	79 (male) 83 (female)	33
		7th	152	78 (male) 74 (female)	31

To assess the mathematics proficiency of the students, Table 3.2 displays the average achievement at each grade level for the six mathematics exams conducted at the school during the year of the study. Each exam comprised 20 skill-based questions.

Table 3.2 Average Mathematics Scores of the Sample

Grade	**Average**
5th	3.6
6th	2.7
7th	2.8

The school was located in a neighborhood inhabited by the indigenous population of the district. The school has seven classes of each grade level, with class sizes ranging from 20 to 30 students. The school is located in a garden between housing estates and there are different sports activity areas for physical education in the garden. The school has a computer lab, library and support education rooms.

The socio-economic status of the students' families is in line with the general socio-economic profile of the district. The majority of the families consist of middle class individuals who do not suffer from subsistence problems but cannot be considered economically rich. Although the family structure generally consists of mother, father and children, some students are children of divorced families. In some households, grandparents also live with the family.

While a significant portion of the families were interested in their children's academic achievement and school behavior, some families showed a lower level of interest in this regard. When the general family structure in the district is analyzed, it is found that families do not exhibit a distinctly different approach towards boys and girls. These data were obtained from school records, interviews with the guidance counselor, and information received from the school administration.

The academic achievement level of the school was found to be average when compared to other schools in the district. This assessment was based on students' course grades and the scores they obtained in the common exams held in the district.

The fact that two guidance counselors work at the school ensures that guidance services are provided at an adequate level. A total of 23 students from the 5th, 6th, and 7th grades who participated in the study were evaluated in the special case category because they had a report from the Guidance and Research Center. While some of these students have specific learning disabilities, others have difficulty performing certain psychomotor skills. These students were not considered separately from other students within the scope of the thesis, and their exam papers were evaluated in the same way as those of other students during the evaluation process.

3.4 Instruments

Considering the problem and sub-problems described in this study, an ‘algebraic thinking test’ (ATST) was developed to measure students' performance in algebraic thinking skills. The ATST was designed as an open-ended paper-and-pencil test compiled from recent studies and textbooks to cover three algebra content strands of Kaput (2008), which was explained in Chapter 2. Although it is a 10-question test, some questions are itemized to include a solution phase for the students, and some questions require additional explanation of the solution. Considering all these, the test consists of 20 items, and the answer key was evaluated over 20 items; each item was given 5 points so that when all correct answers were calculated, 100 points would be awarded; 2.5 points were awarded for incomplete solutions, and 0 points were awarded for incorrect answers. In Table 3.3, brief descriptions of each item, the corresponding algebraic content strands, and basic concepts are given, and the ATST is also available in Appendix A. In Appendix B, there is a specification table that clearly shows which objective each test item belongs to.

Generalized arithmetic questions generally include the general concepts of equality/inequality, unknown quantities, symbols, properties of numbers, and properties of operations. Additionally, the objectives expected from the students in these questions are detailed below.

In item 1, the student is asked to use the mental addition strategy by using operational properties and to generalize this strategy. In item 1a, the student was asked to capture the given strategy or create his/her own strategy; in item 1b, the student was asked to generalize the strategy s/he applied and express it with a letter. The question was taken from Irwin & Britt's study published in 2008.

In item 2, adapted from the third-grade mathematics textbook used in schools by the Ministry of National Education, the student is asked to determine the missing values in equations through an interpretive understanding of the relational nature of equal signs. The item will be evaluated in 2 stages; the student is asked to solve a problem and express the given-ungiven relationship in the problem with an equation.

Although item 2 was a generalized arithmetic question, we evaluated item 2e in the functional thinking section because this step added to the question asked the student to construct equations.

In item 3 (adopted from Çıkla Akkuş, 2004), the student is anticipated to recognize potential values for the variables as symbols in an inequality, demonstrating comprehension of using symbols that represent various values rather than a singular one. Moreover, the student must know that natural numbers begin with zero. Half-point was awarded in the solution of this question if the students did not include '0' in the answer and solved the problem by starting only from 1.

In item 4 (adapted from Altun, 2018), the student is expected to comprehend that the equal sign does not always signify the result of an operation. However, it also indicates the balance between the mathematical expressions on both sides of the equation. In 4a, students are expected to transfer the variables whose values they know to another equation.

Table 3.3 Description of the questions in the Algebraic Thinking Test

Items and Their Contexts	**Content strands**	**Basic concepts**
1a: Mental addition strategies 1b: Generalization of mental addition strategies 2: Identifying missing quantities in problems 3: Identifying variables in an inequality 4a/4b/4c: Comprehension of equal sign	Generalized arithmetic (ga)	Properties of numbers and operations; variables, unknown quantity, equality, inequality
2e: Writing an equation to a verbal problem 5: Choosing the appropriate graph for representing a correspondence relationship 6a/6b/6c: Creating the perimeter relationships of geometric shapes 6d: Generalizing the perimeter relationship	Functional thinking (ft)	Co-variation, correspondence, variable
7a/7b/7c: Identifying the far and next missing terms of a figural pattern 8: Increasing reduction 9: Examining correlational relationships 9b: Generalizing the correlational relationship 10: Examining correlational relationships	Modeling language (mL)	Variable, correspondence Equation, correlation

In contrast, in items 4b and 4c, s/he is required to determine the value for an absent quantity in a single-variable equation, wherein students were expected to employ the concept of "undoing."

Functional thinking questions emphasize the concepts of variables and functional relationships.

Item 5 is a graph construction question adapted from Chester (2012). The student must read the graph correctly and select the appropriate one according to the given problem. This question was the only multiple-choice question in the test. To eliminate the possibility of students getting it right by chance, students were asked for an explanation, which was required for scoring.

Item 6 (adopted from Hart, 1981) is a variable usage question. The student is expected to create the perimeter relationship of geometric shapes using variables and, at last, to generalize the formula s/he created in algebraic expression form. In the first three items of this question, the student is expected to write an algebraic expression using letters, while in the last item, s/he is expected to generalize.

Modeling language questions involved problems that required representing a situation through a model, using algebraic processes to reveal the situation's general structure.

Item 7 (adopted from Becker & Rivera, 2006) is a pattern question. The student is asked to find a given figural pattern's missing next and far terms, which require a general term in letters, and then find the far term using this generalization. The itemization in this question enables students to draw the pattern in stages, discover how it is formed, and then find the first steps in calculations through the visual.

In item 8 (adopted from Mason et al. 2005), the student is expected to evaluate the options by expressing each offer in mathematical terms or specifying them with numbers. The aim is to rank the statements about price reductions in order of increasing discount. One approach to solving this problem involves evaluating all options out of 100 points, while another calculates how much the original price was

paid. The principle behind these arithmetic operations is that they can be applied to any price since no specific price is mentioned. So, the expression 'buy one' and 'buy two' can be generalized to 'buy n.'

In item 9 (adopted from Chester, 2012), the student is expected to grasp the relationship between two variables and express this relationship using words or through the use of tables or graphs. The student is expected to comprehend the variables in real-life situations and identify the missing ones. Also, the student needs to replace the variable in the question with a symbol and translate the given real-life situation into an equation in item 9b.

Item 10 is adapted from Alteri et al. (2008) cited in the article by Chimoni et al. (2018). In this question, students are expected to compare two campaigns offered at the gym by translating them into a mathematical expression to calculate the total cost for any time period. As another solution, students can continue both campaigns as a table for a certain period and determine how much will be paid for both.

Fraenkel et al. (2012) state that Cronbach's alpha coefficient is a widely used reliability indicator to assess the internal consistency of a measurement tool. This coefficient determines the overall reliability level of the measurement tool by analyzing how consistently each test item is related to the total score. In the literature, Cronbach's alpha coefficient values of 0.70 and above generally indicate an acceptable level of reliability, values of 0.80 and above indicate high reliability, and values of 0.90 and above indicate very high reliability.

The reliability of the ATST was evaluated by calculating Cronbach's alpha coefficient, derived from the data obtained in the main study. The coefficient was found to be 0.817, indicating a high level of reliability for the test. Accordingly, the ATST was accepted as a valid and reliable measurement tool (Fraenkel et al., 2012).

3.5 Ethical Issues in The Study

With the permission of the Middle East Technical University (METU) Human Experiments Ethics Committee (see Appendix F), and following the submission of

the petition to the school principal, the tests were administered with the principal's verbal approval, as the researcher conducted the study at their own school. All participants were informed about the survey and signed a consent form (see Appendix G). In addition, all participants voluntarily participated in the study.

3.6 Data Collection Procedure

The test was administered at the beginning of the second semester of the 2023-2024 academic year, before the start of class sessions The implementation was carried out in each classroom with the researcher's presence to prevent differences and biases. The researcher administered the test in her own classes and, with permission from other teachers, entered those classes to conduct the test over two weeks. Before the test was administered, algebraic expressions were taught in the sixth and seventh grades. In addition, the researcher observed the students while they were solving the tests to ensure that they gave honest answers to the questions. Before the students started the test, each class was told that this test would not affect their grades, that it was only for a research study in our school, that their information and names would not be shared with third parties, and a voluntary consent form was distributed. All students completed the test in one class hour (40 minutes).

3.7 Data Analysis Procedure

In this study, data were analyzed using quantitative and qualitative methods. Within the scope of quantitative data analysis, descriptive statistics and inferential statistics were applied. Two independent variables (grade level and gender) and one dependent variable (students' scores on the test) were analyzed. In addition, the similarity of students' answers to the test was evaluated as a qualitative variable.

To examine the general characteristics of the sample, mean, median, standard deviation, skewness, and kurtosis values were calculated from descriptive statistics. The quantitative data used in the study were collected through the Algebraic Thinking Skills Test (ATST), and the Statistical Package for Social Science (SPSS) 29.0 program was used for analysis.

The dependent variable, students' scores from each subsection of the ATST, was evaluated in relation to the independent variables using a two-way analysis of variance (ANOVA). Assumptions (normality and homogeneity of variances) were tested before applying ANOVA. If a significant difference was found as a result of ANOVA, the Tukey test, one of the post-hoc tests, was performed to see in more detail which pairs of groups produced this difference.

As explained in the Instruments section, the ATST is a 20-item test. All open-ended responses were first analyzed using qualitative analysis methods. In this analysis process, participants' responses were classified in terms of common expressions, word usage, and sentence structures. In the analysis of qualitative data, an independent coding approach was adopted (Chang et al., 2009). Then, as a result of a quantification process carried out through these codes, qualitative data were transformed into quantitative data and analyzed. Each item in the test is equally weighted, and each item is scored out of 5 points. If the answer to an item was correct, 5 points were given, and if the answer was incorrect, 0 points were given. However, if the student's thinking was correct but a calculation error was made, or if a question requiring an explanation was only answered and the required explanation was not given, 2.5 points were given. The student's overall achievement scores from the test were calculated out of 100. In addition, the items in the ATST were grouped into three main sections, and the total score distribution for each section was as follows: Generalized Arithmetic, 35 points; Functional Thinking, 30 points; Modeling Language, 35 points. This score distribution was used to analyze the contribution of the items in each sub-dimension to the students' total scores.

The students' answers to the open-ended questions were scored using the structural coding approach in qualitative data analysis. While scoring the answers to the questions, categories were carefully created, and codes were defined accordingly to ensure the accuracy and validity of the coding.

Fully correct answer (FCA, 5 points): This is when the student gives a correct and complete answer, uses the concept correctly, and proposes a completely correct solution to the question.

Semi-correct answer (SCA, 2.5 points): Covers cases where the learner takes some correct steps but gives an incomplete or incorrect answer.

Wrong answer (WA, 0 points): Refers to the cases where the student provides an incorrect or meaningless solution to the question.

After completing the coding process, the distribution of student responses across each category was analyzed. Table 3.4 presents a sample coding from the first page of the test. Subsequently, a scoring rubric was developed, and student responses were evaluated based on this rubric (see Appendix C for the ATST scoring rubric).

Table 3.4 Sample Coding for The First Page of ATST

1a	Performs mental addition strategy using the method in the question (FCA), performs mental addition strategy using a different method (FCA), performs rounding instead of mental addition (WA), performs mental addition without using the digits (WA).
1b	Generalizes the mental process by using variables (FCA).
2	Writes the given and ungiven facts in the problem correctly, solves the problem correctly (FCA), writes the given and ungiven facts correctly but makes a mistake in the application (SCA), uses the correct method in problem-solving but makes an operation error (SCA), cannot solve the problem (WA).
2e	Expresses the equation correctly (FCA), substitutes names, figures, etc. for variables in the equation (FCA), cannot express the equation (WA).
3	Interprets the inequality correctly (FCA), knows that different numbers can replace the variable in the inequality (FCA), does not include '0' in the solution set (SCA1), uses a single number in the solution of the inequality (SCA2), does not comprehend the inequality (WA).
4a	Transfer the result in the figures to addition and solve the other operation (FCA), assign numbers to the figures and find the correct result (FCA), cannot transfer the result in the figures to addition (WA).
4b	Recognizes that the equal sign does not only indicate the result and finds what is not given (FCA), makes an operation error (SCA), ignores the equal sign, and adds all the numbers (WA).
4c	Finds the sum not given in addition (FCA), makes an operation error (SCA), and adds all numbers (WA).
5	Finds the correct graph by annotating (FCA), annotates correctly but marks the wrong graph (SCA), marks the correct graph without annotating (SCA), selects the wrong graph and does not explain (WA).

3.8 Internal and External Validity of The Study

3.8.1 Internal Validity

Fraenkel et al. (2012) explain internal validity as the clear identification of relationships between two or more variables without the influence of any other factor. This "other factor" that threatens internal validity arises primarily from participant characteristics. In particular, demographic characteristics of the participants such as age and gender may harm internal validity. However, this study minimized this threat by selecting participants from the same school.

Fraenkel et al. (2012) also considered differences in the location where the test was administered as a threat to internal validity. To eliminate this threat, the test was administered to all participants in this study without changing the classroom environment and in a standardized test setting.

Changes in measurement tools or scoring are also among the factors that may threaten internal validity (Fraenkel et al., 2012). In this study, the measurement tool was ensured to remain unchanged during the data collection process. In order to prevent possible biases that may arise from the characteristics of the data collector, the data collection process was carried out directly by the researcher in all classes. In order to ensure consistency in the scoring process, a rubric was created (see Appendix C) and all responses were evaluated according to this rubric. Each answer sheet was examined separately by the researcher and an independent teacher, and a common evaluation process was carried out by comparing the scores. Thus, the objectivity of the scoring and the reliability of the evaluation process were achieved.

In addition to internal validity, Fraenkel et al. (2012) define content validity as a criterion determining how comprehensively and accurately the instrument measures the targeted concept or skill. Content validity is usually established through expert judgments, which assess whether the test covers all components in the targeted domain. In this context, it is more appropriate to talk about the validity of a test for a specific application.

In this study, the content validity of the ATST was strengthened by systematically analyzing the task domain to ensure comprehensive coverage of essential elements. The researcher carefully selected the items regarding training and content after a thorough literature review and a detailed examination of the Ministry of National Education's descriptions and subject content. In addition, four mathematics teachers, experts in the field, were asked about the questions' comprehensibility and relevance to the learning outcomes. As a result of these expert teachers' recommendations, the test numbers were chosen to be smaller and more accessible to process, the questions were reorganized with shorter statements, item 7a was added to the questions, and the basic algebraic concepts measured by each question were determined.

At this stage, a specification table showing the subject distribution of the questions was prepared (see Appendix B for the specification table). This table and the test were revised in line with the expert opinions received from Prof. Dr. Ayhan Kürşat ERBAŞ, a highly respected advisor of the thesis study. His valuable insights led to certain adjustments, including the addition of the distribution of the questions to the objectives in the table, the inclusion of a stage for students to express the second item with an equation, the improvement of the drawings of the items of the 6th question, the simplification of the options of the 8th item, and the removal of items that were thought to be difficult for students. It was decided to add the 10th question when it was seen that there were few real-world problems. Opinions were taken on the similarity of the eighth and ninth items to real-world problems, and corrections were made to these questions.

3.8.2 External Validity

According to Fraenkel et al. (2012), external validity, defined as the generalizability of research results, is of two types. The first is 'population generalizability,' which is related to how representative the sample is of the population. Population generalizability can be defined as the extent to which the sample reflects the population. In this study, convenience sampling was used to measure population representativeness. Although this poses a threat to external validity, as convenience

sampling may not always accurately represent the entire population, efforts were made to include all students at the school, thereby minimizing this threat to external validity.

Ecological generalizability is the ability to generalize the study's results to other settings and conditions (Fraenkel et al., 2012). The fact that the study was conducted in a public school may threaten external validity. To minimize this threat, the conditions and nature of the study were described in detail. In this way, the aim was to obtain more reliable results regarding the generalizability of the findings to different settings and conditions.

3.9 Assumptions and Limitations

In this study, the test was based on student responses. The researcher assumed that all participants completed the test individually with their knowledge and that there was no cheating among the participants. It was also also assumed that all participants answered the questions with the same understanding under equal conditions during the data collection process.

The researcher assumed that the test used was a valid and reliable measurement tool. In this context, the researcher accepted that the test accurately measured the skills it aimed to measure and provided consistent results. It was assumed that the participants had high intrinsic motivation to participate in the test and did not experience any external obstacles or distractions while completing it. In addition, the researcher assumed that the classroom environment did not negatively affect the participants' performance during the data collection phase and that the data collection was carried out under standardized conditions. In addition, it was assumed that all participants honestly used the allotted test time and that the test was terminated within the time limits to ensure test completion.

The limitations of this study are as follows: The convenience sampling method may limit the generalizability of the results. The sample was selected only from public schools, and there may be different results and findings from private

schools. In addition, the study may give misleading results when the characteristics of the measurement tool based on the researcher's declaration are changed. The study's theoretical framework, methodology, and sampling are specific to the study, and the selection of these elements in a different way may lead to different results.

CHAPTER 4

RESULTS

The primary purpose of this study is to examine the developmental levels of algebraic thinking skills of middle school students, especially in terms of grade level and gender variables. The quantitative data obtained in the study were analyzed through the students' performances in the algebraic thinking test by using the SPSS 29.0 version. The student's scores on the dependent variables (Algebraic Thinking Skills Test 'ATST') will be presented according to the independent variables: grade level and gender. Additionally, the percentages of questions solved have been calculated for the sub-problem findings. The findings are presented in tables and graphs. Detailed raw data (percentage distribution of student responses per item) are included in Appendix E.

4.1 Findings Related to The First Research Question

In this section, the differences between students' scores by grade level and by gender are analyzed. Initially, the descriptive statistics of the collected data were analyzed. The results of these analyses are presented in Table 4.1.

The average score of sixth-grade students on the Algebraic Thinking Skills Test (ATST) was 1.06, the average score of sixth-grade students was 1.53, and the average score of seventh-grade students was 1.99. According to these data, as the grade level increases, the average scores of the students on the ATST increase.

When the standard deviation (SD) values are analyzed, it is seen that the distribution of scores widens as the grade level increases. In particular, it was found that the performance differences of seventh-grade students (SD = 1.022) were higher than those of fifth-grade students (SD = .629) and sixth-grade students (SD = .778).

In addition, the low standard error (SE) values indicate that the mean scores at each grade level are reliable. ANOVA test was applied to determine whether there was a statistically significant difference between the grade levels.

Table 4.1 Descriptive Analysis of Overall Average Scores

Variables	**Grade**	***n***	**x̄**	***SD***	***SE***
ATST	5th	173	1.06	.629	.047
	6th	162	1.53	.778	.061
	7th	152	1.99	1.022	.082
	Gender				
ATST	F	246	1.48	.84	.05
	M	241	1.54	.94	.06

Assumptions of ANOVA

ANOVA (Analysis of Variance) is a method for analyzing differences between groups. However, some assumptions must be met for this method to give accurate and reliable results.

i. Independence Assumption: Observations must be independent of each other. This means that the measurement of an individual within a group should not be related in any way to the measurements of other individuals. In this study, the researcher administered all the tests herself and observed all the students. In addition, the students answered all of the tests on their own.

ii. Normal Distribution Assumption: The dependent variable values of each group should show an approximately normal distribution. According to Fraenkel et al. (2012), skewness and kurtosis values should be in the range of -1 to +1 for conformity to a normal distribution. In this study, when the test's normality status was evaluated, the skewness and kurtosis values were 0.188 and 0.785, respectively. These values that can be seen in Table 4.2 suggest that the test shows a normal distribution.

Table 4.2 Descriptive Analysis of ATST

	Descriptive Statistics				Normality		Reliability
Variables	***n***	**x̄**	***SD***	***SE***	**Kurtosis**	**Skewness**	**Cronbach alpha**
ATST	487	1.51	.89	.04	.188	.785	.817

iii. Homoscedasticity (Homogeneity of Variances): According to Fraenkel et al. (2012), Levene's test tests whether the variances are equal between groups, i.e., the homogeneity assumption. In this study, Levene's test was applied within the scope of the analysis, and it was observed that the significance values for both groups were more outstanding than 0.05, seen in Table 4.3. This result shows no significant difference between the variances; therefore, the homogeneity assumption is satisfied.

Table 4.3 Levene's test for ATST

	df1	**df2**	***F***	***p***
ATST	2	487	1.543	.215

iv. Continuous Dependent Variable: The ATST results are scaled as continuous measures, so all test scores are proportional.

This study aims to examine students' algebraic thinking skills and how these skills differ according to factors such as grades (fifth, sixth, and seventh grades) and gender (boys and girls). In the research, answers to the following questions were sought:

Is there a significant difference between fifth, sixth, and seventh grade students' overall performance on the algebraic thinking test regarding grade and gender?

Is there a significant difference between fifth, sixth, and seventh grade students' generalized arithmetic (GA) performance in the algebraic thinking test regarding grade and gender?

Is there a significant difference between fifth, sixth, and seventh grade students' functional thinking (FT) performance in the algebraic thinking test regarding grade and gender?

Is there a significant difference between 5th, 6th and 7th grade students' modeling language (ML) performances in the algebraic thinking test regarding grade and gender?

4.1.1 Comparison of Grade Levels and Gender According to Overall Average Scores

Two-way analysis of variance (ANOVA) tests was conducted to determine the effect of the participants' grade levels and gender on ATST.

Table 4.4 Two-way Analysis of Variance for ATST

Variables	***SS***	**df**	***MS***	***F***	***p***	***η²***
Gender	.259	1	.259	.386	.535	
Grade	68.751	2	34.375	51.341	<.001	.176
Gender*Grade	1.223	2	.616	.921	.399	

As shown in Table 4.4, gender did not have a significant effect on students' algebraic thinking skills F (1,487) = .39, p =.54. Grade level, on the other hand, had a statistically significant effect on students' algebraic thinking skills F (2,487) = 51.34, p =<.001, η^2 = .18. The eta squared value of 0.18 indicates a large effect size. In addition, the interaction between gender and grade did not have a significant effect on algebraic thinking skills F (2,487) = .92, p =.40.

After it was determined that the grade levels significantly affected the overall average of the test, the Tukey test was applied as a post-hoc analysis to determine whether the pairwise differences between the grades were statistically significant.

Table 4.5 Class Comparison Across the ATST

Grades (I)	**Grades (J)**	**Mean Difference (I-J)**	***SE***	***p***	**%95 CI**	
					Lower bound	**Upper Bound**
5th	6th	-.471	.089	<.001	-.6821	-.2618
	7th	-.924	.090	<.001	-1.1379	-.7105
6th	5th	.471	.089	<.001	.2618	.6821
	7th	-.452	.092	<.001	-.6693	-.2352
7th	5th	.924	.090	<.001	.7105	1.1379
	6th	.452	.092	<.001	.2352	.6693

As grade levels increase, the results show a clear progression in algebraic thinking performance, as seen in Table 4.5. Fifth-grade students consistently scored lower than sixth and seventh-grade students, while seventh-grade students achieved the highest mean scores. These findings highlight the impact of grade level on students' development of algebraic thinking skills and suggest that students in higher grades have more advanced capabilities in this area.

4.1.2 Comparison of Grade Levels and Gender According to Generalized Arithmetic Scores

In order to answer one of the sub-questions of the first problem, “Is there a significant difference between the GA performances of fifth, sixth, and seventh-grade students in the algebraic thinking test in terms of grade level and gender?”, the scores of the students in the sub-sections of the test were compared according to their grade level

and gender. This comparison was evaluated primarily by the descriptive analysis method, results are given in Table 4.6.

Table 4.6 Descriptive Analysis of GA Items

Variables	Grade	*n*	x̄	*SD*	*SE*
"GA" items	5th	173	1.93	1.171	.089
	6th	162	2.24	1.094	.086
	7th	152	2.84	1.176	.095

According to the Table 4.6, it is observed that 5th-grade students scored the lowest (1.93), and 7th-grade students scored the highest (2.84) on the GA items. This finding shows that students' achievement in the GA items increases as the grade level increases. According to the standard deviation data, 7th-grade students' results have the widest distribution (*SD* = 1.176), while 5th-grade students' results are close to that of 7th-grade students. On the other hand, the results of 6th-grade students show a narrower distribution (*SD* = 1.094). This suggests that there is more variation in the performance of 7th graders, while 6th graders show more similar results.

An ANOVA test was applied to examine whether grade level and gender had a significant effect on the GA sub-dimension of the test.

Table 4.7 Two-way analysis of variance table for the GA subscale of the ATST

Variables	*SS*	df	*MS*	*F*	*p*	η^2
Gender	.425	1	.425	.323	.570	
Grade	67.704	2	33.852	25.761	<.001	.096
Gender*Grade	5.844	2	2.922	2.223	.109	

As shown in Table 4.7, grade levels have a significant effect on the GA sub-dimension F (2,487) = 25.8, p =<.001, η^2 = .096. The eta squared value of 0.096 indicates a medium effect size. However, gender F (1,487) = .32, p=.57 and

gender*grade factors F (2,487) = 2.22, p =.11 did not have a significant effect on the test. After it was determined that the grade levels significantly affected the GA scores, the Tukey test was applied as a post-hoc analysis to identify which grade pairs had statistically significant differences, as shown in Table 4.8.

Table 4.8 Grade Comparison for the GA Subscale of the ATST

Grade (I)	**Grade (J)**	**Mean Difference (I-J)**	***SE***	**p**	**%95 CI**	
					Lower Bound	**Upper Bound**
5th	6th	-.303	.125	.042	-.5987	-.0083
	7th	-.905	.127	<.001	-1.2061	-.6058
6th	5th	.303	.125	.042	.0083	.5987
	7th	-.602	.129	<.001	-.9073	-.2976
7th	5th	.905	.127	<.001	.6058	1.2061
	6th	.602	.129	<.001	.2976	.9073

The scores of fifth-grade students were significantly lower than those of sixth-grade students (mean difference =.303, p =.042). The mean score of sixth-grade students was also significantly lower than that of seventh-grade students (mean difference =.602, p <.001). The mean score of fifth-grade students was also significantly lower than that of seventh-grade students (mean difference = 0.905, p <.001). Confidence intervals support that these differences are statistically significant.

4.1.3 Comparison of Grade Levels and Gender According to Functional Thinking Scores

To answer one of the first problem's sub-questions, “Is there a significant difference between the FT performances of fifth, sixth, and seventh-grade students in the algebraic thinking test in terms of grade level and gender?”, the students' scores in

the sub-sections of the test were compared according to their grade level and gender. This comparison was evaluated primarily by descriptive analysis.

Table 4.9 Descriptive Analysis of FT Items

Variables	Grade	*n*	x̄	*SD*	*SE*
"FT" items	5th	173	.65	.795	.060
	6th	162	1.54	1.224	.096
	7th	152	1.92	1.534	.124

According to Table 4.9, it is observed that 5th-grade students received the lowest score (.65) in the FT items, while 7th-grade students received the highest score (1.534). This finding shows that students' achievement in FT items increases as the grade level increases. According to the standard deviation data, 7th-grade students' results have the widest distribution (SD = 1.534), while 6th-grade students' results are close to that of 7th grade. On the other hand, the results of 5th-grade students show a narrower distribution (SD =.795). This suggests there is more diversity in the performance of 7th-grade students, while 5th-grade students show more similar results.

ANOVA test was applied to examine whether grade level and gender significantly affected the FT sub-dimension of the test.

Table 4.10 Two-way analysis of variance table for the FT subscale of the ATST

Variables	*SS*	df	*MS*	*F*	*p*	η^2
Gender	1.225	1	1.225	.838	.360	
Grade	138.734	2	69.367	47.480	<.001	.165
Gender*Grade	1.350	2	.675	.462	.630	

According to Table 4.10, grade levels have a significant effect on the FT sub-dimension F (2,487) = 47.48, p =<.001, η^2 = .165. The eta squared value of 0.165

indicates a large effect size. However, gender F (1,487) = .39, p =.36 and interaction between gender and grade factors F (2,487) = .46, p =.63 did not have a significant effect on the test. After it was determined that the grade levels significantly affected the FT scores, the Tukey test was applied as a post-hoc analysis to determine which differences between the grades were statistically significant.

Table 4.11 Grade Comparison for the FT Subscale of the ATST

Grade (I)	**Grade (J)**	**Mean Difference (I-J)**	***SE***	***p***	**%95 CI**	
					Lower Bound	**Upper Bound**
5th	6th	-.888	.131	<.001	-1.1986	-.5780
	7th	-1.269	.134	<.001	-1.5851	-.9540
6th	5th	.888	.131	<.001	.5780	1.1986
	7th	-.381	.136	.015	-.7018	-.0608
7th	5th	1.269	.134	<.001	.9540	1.5851
	6th	.381	.136	.015	.0608	.7018

As seen in Table 4.11 mean score of fifth-grade students was significantly lower than that of sixth-grade students (mean difference = 0.888, $p < 0.001$). The mean score of sixth-grade students was also significantly lower than that of seventh-grade students (mean difference = 0.381, $p = 0.015$). The mean score of fifth-graders was also significantly lower than that of seventh-graders (mean difference = 1.269, $p < 0.001$). Confidence intervals support that these differences are statistically significant.

4.1.4 Comparison of Grade Levels and Gender According to Modeling Language Scores

To answer one of the first problem's sub-questions, “Is there a significant difference between the ML performances of fifth, sixth, and seventh-grade students in the algebraic thinking test in terms of grade level and gender?” the student's scores in

the sub-sections of the test were compared according to their grade level and gender. This comparison was evaluated primarily by descriptive analysis.

Table 4.12 Descriptive Analysis of ML Items

Variables	Grade	*n*	x̄	*SD*	*SE*
"ML" items	5th	173	.55	.619	.047
	6th	162	.83	.798	.062
	7th	152	1.19	1.072	.086

According to Table 4.12, it is observed that 5th-grade students received the Lowest score (.55) on ML items, while 7th-grade students received the highest score (1.19). This finding shows that students' achievement in ML items increases as the grade level increases. According to the standard deviation data, 7th grade students' results have the widest distribution (SD = 1.072), while 6th grade students' results are close to that of 5th grade students' results. In contrast, the results of Grade 5 students show a narrower distribution (SD =.619). This suggests that 7th graders show more variation in their performance, while 5th graders show more similar results.

ANOVA test was applied to examine whether grade level and gender significantly affected the ML subscale of the test.

Table 4.13 Two-way analysis of variance table for the ML subscale of the ATST

Variables	*SS*	df	*MS*	*F*	*p*	η^2
Gender	1.361	1	1.361	1.922	.166	
Grade	33.352	2	16.676	23.547	<.001	.090
Gender*Grade	.027	2	.014	.019	.981	

According to Table 4.13, it is seen that grade levels significantly affect the ML sub-dimension F (2,487) = 23.55, p =<.001, η^2 = .09. The eta squared value of 0.09 indicates a medium effect size. However, gender F (1,487) = 1.92, p =.17 and

interaction between gender and grade factors F (2,487) = .019, p =.981 did not significantly affect the test results. After it was determined that the grade levels significantly affected ML scores, the Tukey test was applied as a post-hoc analysis to determine which grades had statistically significant differences.

Table 4.14 Grade Comparison for the ML Subscale of the ATST

Grade (I)	**Grade (J)**	**Mean Difference (I-J)**	***SE***	***p***	**%95 CI**	
					Lower Bound	**Upper Bound**
5th	6th	-.282	.091	.006	-.4982	-.0661
	7th	-.644	.093	<.001	-.8645	-.4251
6th	5th	.282	.091	.006	.0661	.4982
	7th	-.362	.094	<.001	-.5858	-.1395
7th	5th	.644	.093	<.001	.4251	.8645
	6th	.362	.094	<.001	.1395	.5858

As seen in Table 4.14 mean score of fifth-grade students was significantly lower than that of sixth-grade students (mean difference = 0.282, p = 0.006). The mean score of sixth-grade students was also significantly lower than that of seventh-grade students (mean difference = 0.362, $p < 0.001$). The mean score of fifth-grade students was also significantly lower than that of seventh-grade students (mean difference = 0.644, $p < 0.001$). Confidence intervals support that these differences are statistically significant.

4.2 Findings Related to The Second Research Question

The study's second research question aimed to determine which algebraic concepts were better grasped by students at different grade levels. For this purpose, a content analysis was conducted to determine which algebraic concepts the questions in the measurement tool measured. Of course, it is not always possible for a question to

measure a single concept, but a classification was made based on the dominant features of the questions. The classification is given in Table 4.15.

Tablo 4.15 Description of items to algebraic concepts

Algebraic concepts	**Items**
Equality& equation	2e, 4a, 4b, 4c,7b,7c
Inequality	3
Properties of number and operations	1a, 2, 3
Variables	1b, 6a, 6b, 6c, 6d, 9b
Co-variation	5, 8
Correspondence	9, 10

To better understand the student's level of knowledge about algebraic concepts, the correct, incorrect, and semi-correct answer rates obtained in the questions involving each concept were analyzed in detail. It is important to note here that in the concept of 'inequality,' due to the structure of the third question, the students who gave semi-correct answers were evaluated in the same category as those who gave correct answers. The reason for this is that although these students could solve inequalities, they were considered to have solved the question semi-correctly because they did not include '0' in the set of natural numbers. However, semi-correct answers on the third question with the concept of 'properties of numbers and operations' indicated that students did not fully understand the concept.

The average scores by concept were calculated considering students' average scores for each item and the description of items related to algebraic concepts, and the results are presented in Table 4.16. Similarly, Table 4.17 shows the percentages of students' correct and partially correct answers.

Table 4.16 Average points of correct answers to related items

Algebraic concepts	5th	6th	7th	Overall mean
Equality and equation	1.5	1.7	2.4	1.9
Inequality	1.5	2.5	2.7	2.2
Properties of numbers and operations	1.5	1.8	2.2	1.8
Variables	0.2	1	1.4	0.5
Co-variation	1.5	2.1	2.6	3
Correspondence	1	1.3	1.5	3.2

Table 4.17 Percentages of students' correct (C) and semi-correct (SC) answers to related items

Concepts	Equality and equation		Inequality		Properties of numbers and operations		Variables		Co-variation		Correspondence	
Grades	C	SC	C	SC	C	SC	C	SC	C	SC	C	SC
5th	29.5	0	29	X	30.3	0	5.8	0	31	0	19.3	0
6th	33.5	0	47	X	36.7	0.3	20.7	0	36.5	0	26.3	0
7th	47.3	0	54	X	45	0	41	0	42	18.5	30.1	0

According to the findings, students understood the concept of 'inequality' better than the other concepts. On the other hand, the concept of 'variables' posed the most significant difficulty for students. Interestingly, the achievement levels of fifth-grade students in the concepts of 'equality and equation,' 'inequality,' 'properties of numbers and operations,' and 'co-variation' were quite close to each other. These four concepts stand out as the best-acquired concepts for fifth-grade students. However, while the concept of 'correspondence' was fully understood by only 19.3% of the students, most students did not grasp the concept of 'variables' concept. The average score of fifth-grade students on the concept of 'variables' was only 0.2, and the percentage of students who succeeded in the questions related to this concept was very low at 5.8%.

This can be explained by the fact that variables are an abstract concept, and students cannot quickly establish their counterparts in daily life, especially since the fifth-grade students did not see the concept of variable as an acquisition in the lesson; it may have been a concept they did not recognize at all.

In the sixth grade, the best achievement was in the concept of inequality. In contrast, the concepts of equality and equation, properties of numbers and operations, and co-variation had close averages. Approximately 35% of the students succeeded in these concepts. In the concepts of variables and correspondence, 20.7% and 26.3% success were achieved, respectively.

In the seventh grade, as in the sixth grade, inequality was the most successful concept. Students at this level also showed close success in equality and equation, properties of numbers and operations, and co-variation and inequality. While 54% of the students got the concept of inequality correct, about 45% succeeded in the other three concepts. The fact that the averages of the concepts of variables and correspondence were very close in the seventh grade is a strange finding because while correspondence is a more advanced concept, the concept of variables is a concept that has been in the lesson plan since the sixth grade.

In summary, the concept of inequality, the area where the students were most successful, was probably better understood by associating the subject with more concrete examples close to daily life. The fact that the student's achievement levels are close to each other in the concepts of “equality and equation,” “properties of numbers and operations,” and “co-variation” shows the fundamental connections between these concepts and the student's ability to establish these connections. For example, equality and equations are closely related to properties of numbers and operations. Teaching these concepts together may have helped students to achieve better integration in these subjects.

The lower success rates in variables and correspondence concepts compared to other concepts may be due to their more abstract nature and the requirement for generalization. The concept of variable especially stands out as one of the most

challenging concepts for students. This can be explained by the fact that variables can have different meanings in different situations, and expressing them with symbols requires abstract thinking. On the other hand, the concept of correspondence may create a complex structure for students as it expresses the relationship between two sets.

In conclusion, the analysis found a consistent developmental progression between grade levels in the process of students' learning algebraic concepts, and it is also seen that the acquisition process of concepts progresses gradually. The analysis presented in this section quantitatively compared students' performance on different algebraic concepts. However, it generally reveals significant deficiencies in students' algebraic thinking skills. The highest mean score for the concepts is 2.7, and the success percentages do not exceed 50%, indicating that students do not sufficiently grasp basic algebraic concepts. The low scores indicate that the current teaching methods and curriculum do not meet the needs of the students. This makes it imperative for school administrators and teachers to develop effective strategies to improve students' algebraic achievement.

4.3 Findings Related to The Third Research Question

In this study, the data collected to understand students' algebraic thinking processes were analyzed in detail using content analysis. In content analysis, the data obtained from students' written responses were analyzed through coding schemes, themes were given in the 3.7 Data Analysis Procedure section, and categories were created. In this way, students at different grade levels were presented in a thematic framework, such as their understanding of algebraic concepts, their preferred solution strategies, and the mistakes they frequently made. The findings for each question were explained in detail under algebraic content strands to shed light on the complexity of students' algebraic thinking processes. See Appendix E for the quantitative analysis of the student's responses to the questions as percentages table.

4.3.1 Generalized arithmetic

This section expects students to extend their prior knowledge to new situations and problem contexts through generalization. It frequently employs symbolic language to represent numerical relationships, manipulate operational properties, and formulate equalities and equations. This process encompasses fundamental mathematical concepts, including numbers, operations, equalities, equations, expressions, and variables. In the functional thinking section, the outcomes of generalization will be examined in detail through related questions.

The analysis reveals that students' performances on number and operation properties were not consistent. In Item 1a, approximately half of the students were able to perform mental calculations. While some students employed strategies similar to the example provided in the question, others utilized different mental strategies, as illustrated in Figure 4.1.

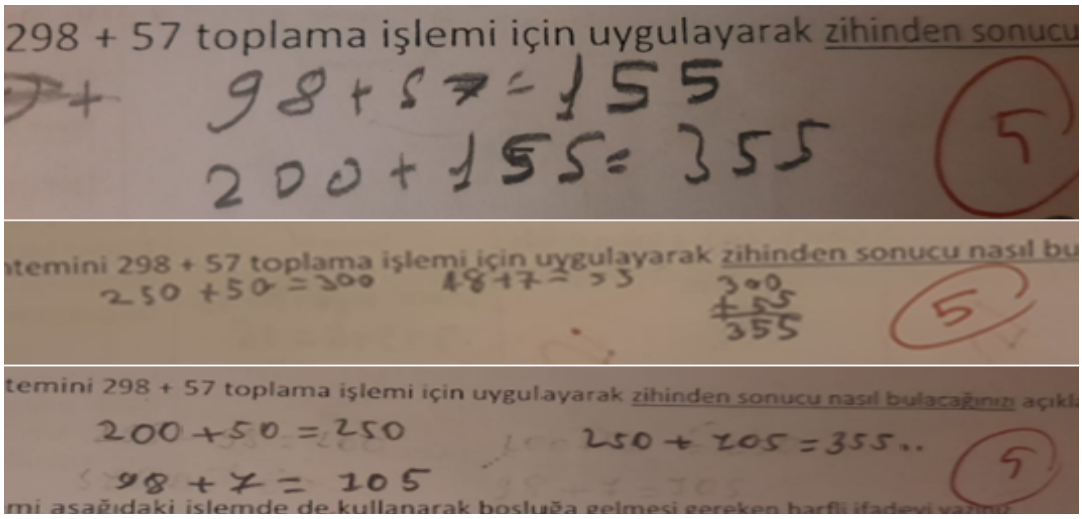

Figure 4.1 Students' alternative mental strategies for item 1a differ from the given strategy

In contrast, some students who provided incorrect responses confused rounding to the nearest ten with mental addition strategies. Figure 4.2 presents a sixth-grade student’s answer incorrect response resulting from this confusion.

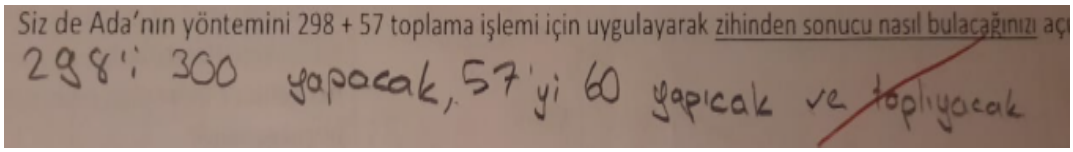

Figure 4.2 An incorrect response from a sixth-grade student

In the second item, students demonstrated basic computational skills by solving the problem and identifying the missing element. However, none of the fifth and sixth-grade students expressed the given verbal statement in the form of an algebraic expression or equation. In contrast, a small number of seventh-grade students employed two different equation representations. Figure 4.3 illustrates two distinct solution strategies, both based on arithmetic reasoning, while Figure 4.4 presents the two common equation forms used by seventh-grade students.

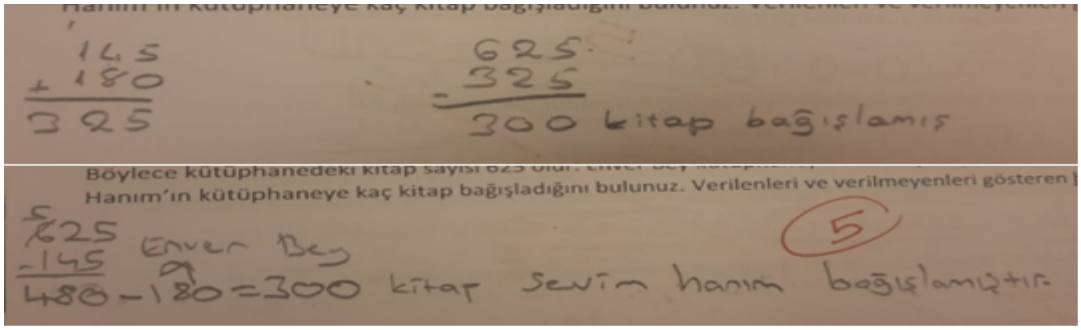

Figure 4.3 Mostly used two arithmetic solution strategies in item 2

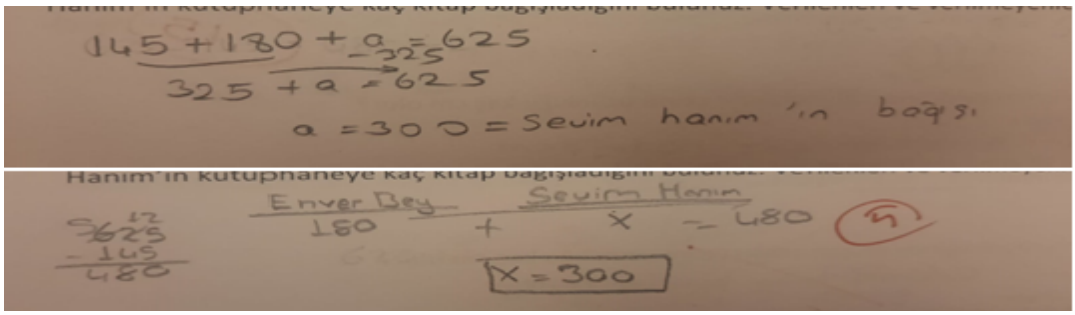

Figure 4.4 Two common equation forms used by some of the seventh-grade students

However, in the third item, although students successfully addressed the inequality aspects of it, they failed to recognize that zero is a natural number. Several students who provided incorrect answers made the mistake of expressing the solution set of

the inequality as a single natural number. In evaluating the third question regarding inequality outcomes, it is thought that students who made a mistake in taking zero in natural numbers should also be given full credit. This is because the error made by these students stemmed from their lack of knowledge about whether zero is in the set of natural numbers.

Within the framework of the generalized arithmetic domain, students are expected to understand the balance relationship represented by the equality sign and recognize the equivalence between mathematical expressions. Additionally, they are expected to demonstrate this understanding by determining the unknown value in a given equation, where empty boxes are used in place of letters for the missing addend, in the fourth questions' 'a,' 'b,' and 'c' items. The majority of students responded correctly to these items (see Figure 4.5).

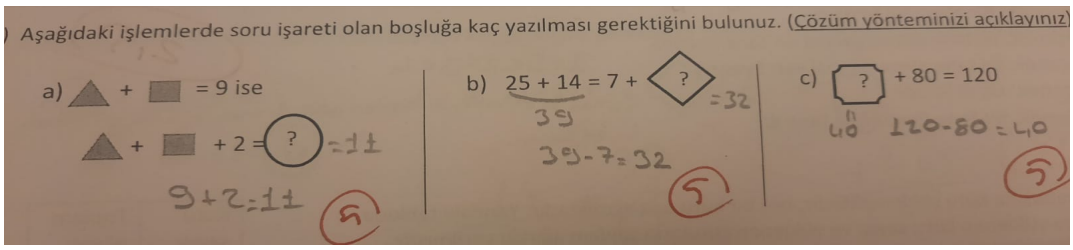

Figure 4.5 one seventh grade student's correct response to item 4 'a, b, and c'

However, a notable decline in student performance was observed in option (b), despite the use of boxes instead of letters. This decrease can be attributed to the different placement of the unknown compared to the other questions, as well as the involvement of more complex numbers. Additionally, students struggled to recognize the need to double the 7 on the right side of the equation to equate it with 14 on the left side. Figure 4.6 presents an example of this misconception from a sixth-grade student's response.

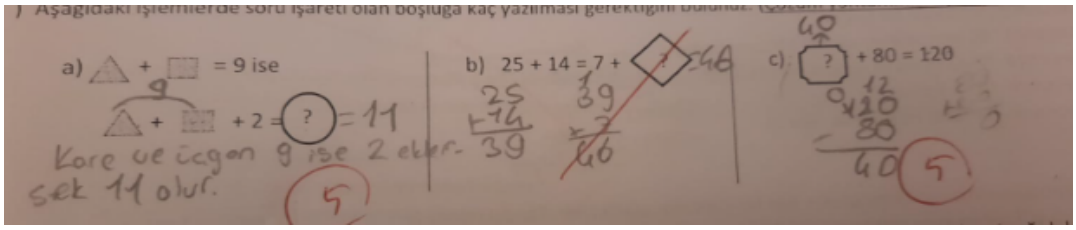

Figure 4.6 One sixth grade student's response to the 4a/4b and 4c items

4.3.2 Functional thinking

This section discusses the generalization of relationships between quantities that vary together at an abstract level; the capacity to represent numerical and geometric patterns using functions and algebraic expressions is central to this content strand. Basic mathematical concepts such as variable, equation, co-variation, and correspondence are treated in this context in a more abstract and structured framework than their more concrete use in generalized arithmetic.

In this section, we will first focus on the fifth question, which addresses the concept of 'co-variation.' Based on the analysis results, the majority of students answered this question correctly. As an explanation, some students indicated that the total value increased daily, while a few others selected option 'b' by identifying the errors in the other choices. An example of this is presented in Figure 4.7.

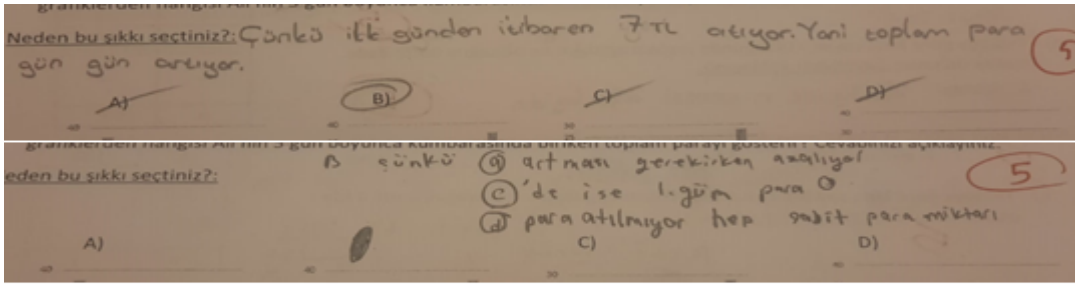

Figure 4.7 Students' two different solution strategies for item 5

However, some chose option 'd,' focusing on the fixed amount of money discarded daily, although the question was asked for the total amount of money accumulated. Figure 4.8 supports this with a concrete example.

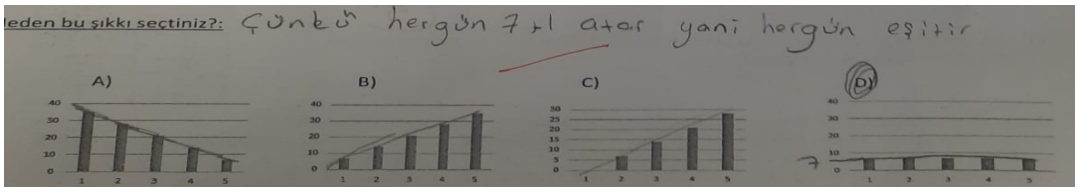

Figure 4.8 One sixth grade student's response to the graph question

Secondly, the sixth question for the concept of variables will be discussed. The 'a', 'b,' 'c', and 'd' options of the question are examined in two categories regarding variable usage and generalization skills. While options 'a', 'b,' and 'c' basically require understanding the concept of variable, option 'd' aims to measure the ability to generalize using variables. The higher success rates of 6th and 7th-grade students in this question may be attributed to their prior exposure to algebraic education. Figure 4.9 shows an example of seventh grade student's correct answer to the all items of the sixth question.

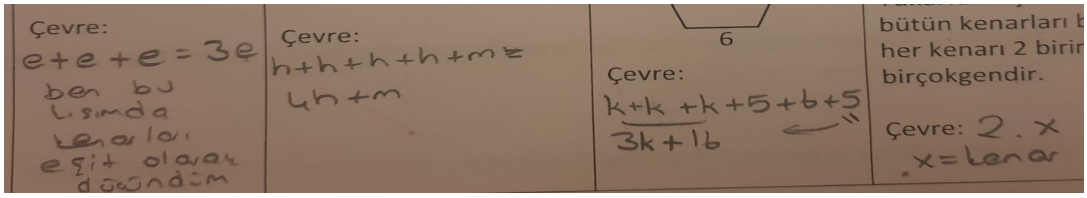

Figure 4.9 A seventh-grade student's correct response to the sixth question

The data obtained in this question provides important information about how students perceive letters. For example, error analysis among fifth-grade students revealed that many students tended to substitute numerical values for letters. Some students used random numbers, while others assigned numerical data from shapes to letters. Some students even matched the alphabetical order of the letters with numerical values. Figure 4.10 is presented as an example for both those who matched the order of the letters in the alphabet with numerical values and those who assigned the numbers in the figure to the letters in option 'c.'

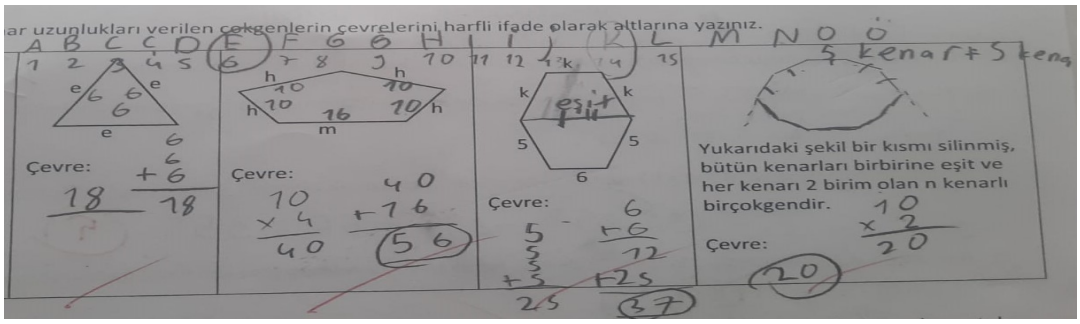

Figure 4.10 One fifth grade student's response to the variable question

Most of the seventh-grade students answered the first three choices of the question correctly, however, some of them who answered incorrectly tried to form an equation instead of giving the result and used expressions such as '3e=?'. This leads a confusion of algebraic expressions with equations.

Finally, the topic of generalization will be discussed comprehensively, independent of the other content strands. To address this, questions from all three strands will be considered. Firstly, in the second part of Item 2, none of the fifth and sixth-grade students were able to convert the problem into an equation, while a small number of seventh-grade students were able to form an equation as outlined in the generalized arithmetic section. Additionally, none of the fifth-grade students answered item 1b correctly. Only two sixth-grade students provided the correct answer, and just 8% of the seventh-grade students answered correctly. Figure 4.11 shows a fully correct response from a seventh-grade student, using the example provided in Item 1a.

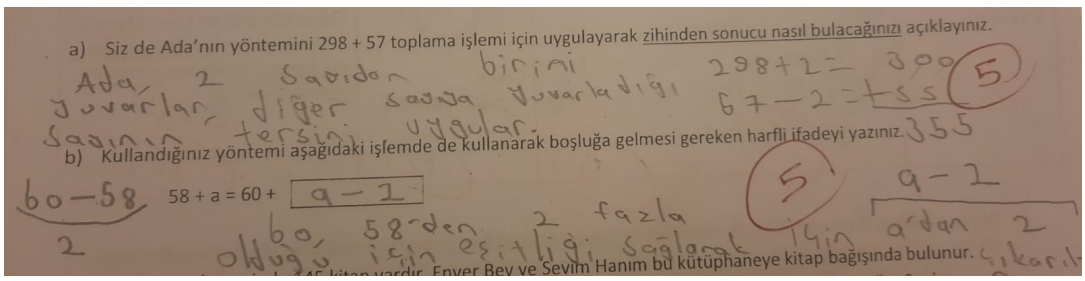

Figure 4.11 Correct response from a seventh-grader for items 1a and 1b

However, one fifth-grade student showed a correct approach to solving the problem but could not express the answer completely. This student response is presented visually in Figure 4.12.

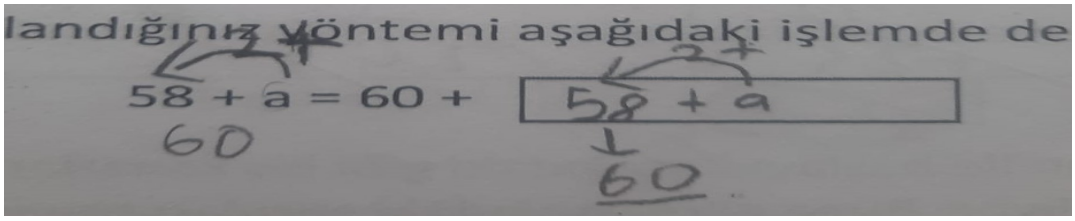

Figure 4.12 One fifth grader's incorrect response to generalization using variables

Additionally, the success rates of students were found to be relatively low in items 7b and 7c, which required identifying distant or near terms in figural patterns. Notably, the success rate in 7a and 7b which had the highest solution rate in seventh question, was only 12%, indicating that students encountered significant difficulties with such questions. In these items, students predominantly drew the terms of the figural patterns and then answered correctly, as demonstrated in Figure 4.13.

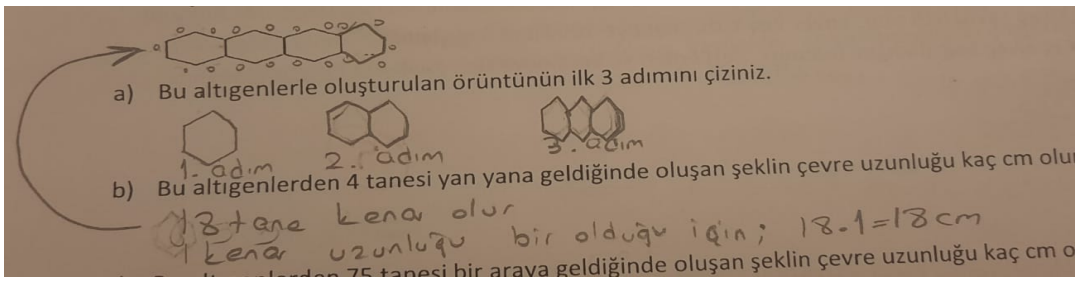

Figure 4.13 drawing strategy for items 7a and 7b

Only some students who correctly solved option C followed an algebraic solution by expressing the rule of the figural pattern with letters (see figure 4.14).

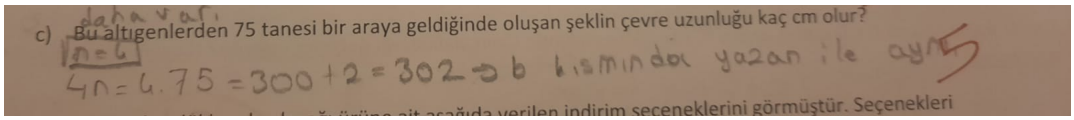

Figure 4.14 Algebraic expression strategy of seventh graders for item 7c

The other students reached the correct solution by performing arithmetic operations. These findings reveal that students have significant deficiencies in their ability to

symbolize and generalize regularities in figural patterns. One of example of the arithmetic solution of item 7c can be seen in Figure 4.15.

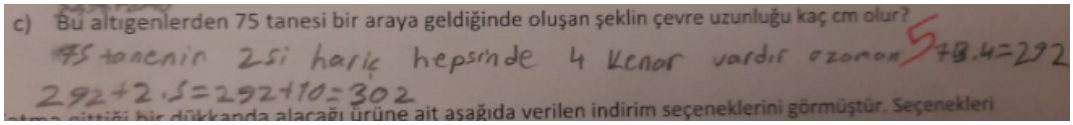

Figure 4.15 Arithmetic strategy of students for item 7c

Findings on generalization of this study show a significant decline, especially in items 6d and 9b. Although students showed a certain level of success in the first parts of these questions, their success rates were quite low in the last parts of the questions, requiring generalization. In item 6d, which requires expressing the perimeter of a polygon without a given number of sides, no fifth-grade student could give the correct answer, while the success rate remained below 10% in the sixth and seventh grades. Similarly, in item 9b, which required students to generalize the pattern they solved in item 9a, almost no students in the fifth and sixth grades could give the correct answer, and even in the seventh grade, the success rate was only 12%. These findings indicate that there are significant deficiencies in students' ability to generalize in different mathematical situations.

4.3.3 Modeling languages

This section provides an essential tool for solving complex real-world problems by transforming them into more straightforward and more understandable mathematical structures. This domain enables the application of the learning outcomes in GA and FT domains to real-life problem situations, generalizes these situations, and expresses them in an abstract framework.

Item 9a aimed to assess students' ability to identify subsequent terms in a numerical pattern within a real-world context. Interestingly, the majority of students were more successful with numerical patterns, despite having difficulty with figural patterns in the seventh question. Some students solved the problem by determining the weight of one box and then calculating the requested quantity, while others used proportion.

Figure 4.16 illustrates both strategies for Item 1a. However, when students were asked to express the pattern with a general rule in Item 9b, the success rate dropped significantly (see one correct response in Figure 4.16 for Item 9b). These findings suggest that, although students generally performed well in recognizing and extending numerical patterns, they exhibited considerable weaknesses in identifying regularities in figural patterns, symbolizing these regularities, and making generalizations.

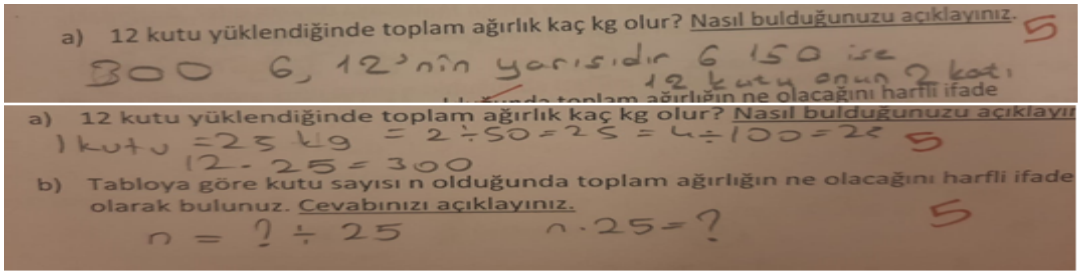

Figure 4.16 Two different solution strategies for 9a, a correct answer for 9b

In items 8 and 10, the concept of 'covariation' was examined in the context of the objectives of 'making comparisons by assigning a fixed value or establishing a proportion between values when a certain number is not given' and 'presenting the data obtained with different representations.' In item 8, students were requested to diagnose the changes made to an unspecified value. Approximately half of sixth and seventh-grade students received half and full points for trying to express the relationship between variables. This finding shows that many students in this age group try solving problems using algebraic thinking skills even when missing numerical data. Item 8 could be solved by students using three distinct strategies: one involves drawing shapes, another assigns a specific number, and the third utilizes verbal reasoning. All three strategies are illustrated in Figure 4.17.

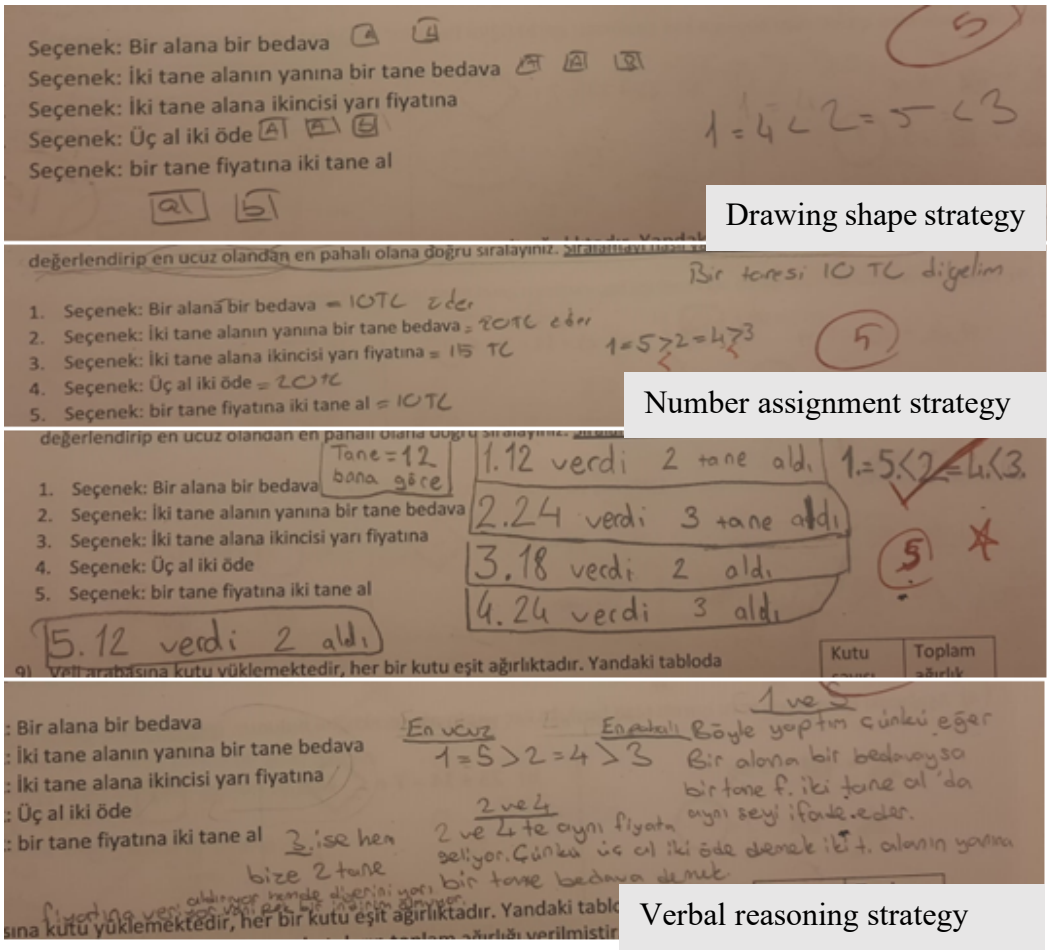

Figure 4.17 Sixth and seventh grade students' solution strategies for item 8

However, this rate decreased significantly in fifth-graders. This shows that most fifth-grade students have not yet developed their problem-solving skills using abstract mathematical concepts. Despite the majority, some fifth graders tried to solve the problem by assigning numerical data, as did sixth and seventh graders, although one student solved the problem using visual techniques and one student solved the problem using fractions. Figure 4.18 represents two fifth grade students' drawing shape and fraction strategies.

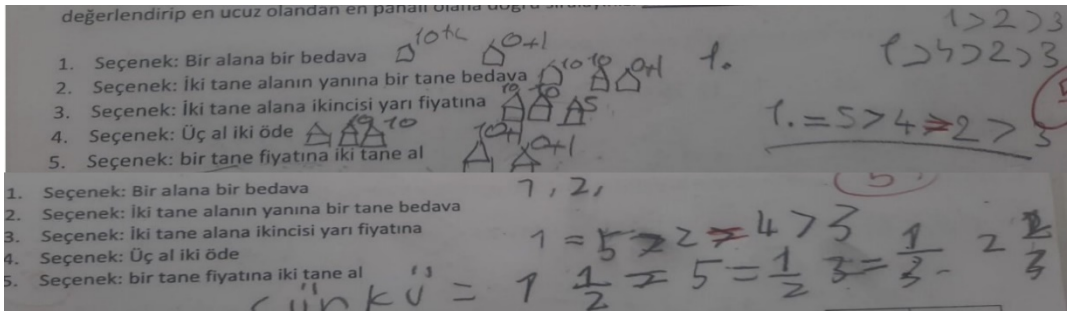

Figure 4.18 Two fifth grade students' responses to the covariation question

Students mostly used the tabulation method instead of symbolically expressing the relationship between two variables in item 10. The fact that one quarter of the students from sixth and seventh-grade reached a success in this question shows that they exhibited a more sophisticated problem-solving approach than the fifth-grade students, whose success rate was around 9%. In addition, most of the students who made mistakes tried to find a solution by using only the initial data without considering all the problem conditions. This shows that students have difficulty demonstrating a systematic approach to problem-solving and lack the ability to evaluate problems from a broad perspective and predict long-term outcomes. Figure 4.19 shows an example of predicting long-term outcomes strategy.

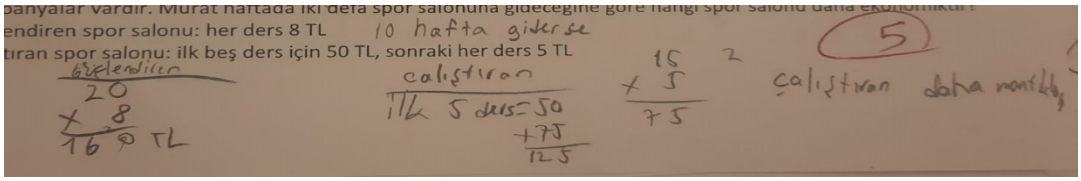

Figure 4.19 Predicting long-term outcomes strategy of a seventh-grade student

To present a summary of the findings related to Research Question 3, Table 4.18 was prepared to outline the common strategies used by students in problem-solving and the most frequently observed mistakes.

Table 4.18 Students' strategies and mistakes

Q	Common Strategies	Common Mistakes
1	Rounding and adjusting Breaking apart Adding tens and ones seperately	Rounding but then not adjusting numbers (1a) Mostly not generalize using letters (1b)
2	Artihmetic solutions Seventh graders used algebraic expressions	Artihmetic errors Procedural errors
3	Finding all pairs that sum to 10	Finding only one pair that sum to 10 Excluding 0 as a natural number
4	Balancing the equation Assigning numbers to boxes	Taking the equality sign as the result of the operation
5	Calculating the total amount of money per each day	Represent a constant increase of 7 as 7 each day on the graph
6	Writing letters together	Assigning numbers to the letters Count the lines in the figure and multiply by 2 (6d) Not generalize using letters (6d)
7	Drawing the required step and calculate (4a/4b) Arithmetic solution (7c) Rarely using algebraic expression (7c)	Drawing only the third step instead of the first three steps (7a) Multiply the perimeter of the hexagon by the given step (7b/7c)
8	Drawing figure Assigning number for ungiven value Verbal reasoning	Confusion with verbal explanations
9	Calculating by finding the value of one box (9a) Proportion (9a)	Not generalize using letters (9b)
10	Predicting long-term situations	Not demonstrate a systematic approach

4.4 Summary of The Results

The initial research problem of this study was to examine the impact of grade level on algebraic thinking skills. In order to find the answer to this question, the overall average achievement of the students from ATST was analyzed. The results indicate that although the overall average achievement is low across all students, the level of achievement improves as the grade level increases.

In the sub-problems of the first problem, it was investigated which areas students showed higher achievement in algebraic content strands according to grade level. In this context, the mean scores of students from each grade level on the test sections were evaluated. It was observed that as the grade level increased, the average performance of the students in all algebraic content strands also increased. This shows that students at all three grade levels were successful in the GA section, but the number of fifth-grade students among the successful students in the FT and ML sections decreased significantly. In addition, it was observed that the achievement of seventh-grade students was higher than that of sixth-grade students in the FT and ML sections.

Moreover, it should also be noted that there were students from different grade levels in each strand. This finding reveals that students exhibit different characteristics on different tasks regardless of their grade level. It was observed that some students who showed advanced algebraic thinking skills at a younger age had a similar level of thinking to that of older students.

On the other hand, seventh- and sixth-grade students also performed poorly on questions other than general arithmetic. This suggests that some older students exhibit a profile similar to younger students regarding early algebraic thinking skills. However, the fact that the overall average increased as the grade level increased and students performed better in Functional Thinking (FT) and Modeling Language (ML) suggests that students' ability to solve these tasks improves as they get older.

One of the sub-problems of the first research problem is whether there is a difference in algebraic thinking according to gender. To find an answer to this question, the overall grade point averages of male and female students and the grade point averages of male and female students at each grade were compared. The comparison results showed no significant difference between male and female students in the overall average.

In addition, the analysis of this study's second research problem shows that students' learning of algebraic concepts progresses consistently across grades and that the process of concept acquisition is gradual. The analysis in this section quantitatively compares students' performance on different algebraic concepts, but in general, there appear to be gaps in students' acquisition of basic algebraic concepts.

Finally, it is seen in the students' responses that they had significant deficiencies in making generalizations using variables and generalizing patterns using variables. Students had significant difficulties, especially in questions such as items 1b, 6d, and 9b, which require establishing a general rule or relationship through variables. This shows that students have trouble establishing abstract mathematical relationships with symbols rather than understanding the symbolic representation of variables. In other words, students exhibit a more procedural approach to understanding the concept of variable but have difficulty in understanding it more deeply and reaching generalizations.

CHAPTER 5

CONCLUSION

This thesis study examined the algebraic thinking skills of fifth, sixth, and seventh-grade students. The study test was conducted at the end of February of the 2023-2024 academic year, following the period when algebra topics were taught at a school in Mudanya. A total of 487 students, including all fifth, sixth, and seventh graders at the school, participated in the study, which utilized the "Algebraic Thinking Skills Test" developed by the researcher. Three main research questions were identified for the purpose of this study:

Problem 1: Is there a significant difference in students' performance on algebra content strand tasks based on grade level and gender?

Problem 2: Which basic algebraic concepts are better understood by fifth, sixth, and seventh-grade students?

Problem 3: What common strategies and mistakes influence students' algebraic processes when solving tasks?

Based on the data collected from the students, these questions were analyzed in detail, and some sub-questions related to the research problems in the previous sections were answered. In light of the research findings, some conclusions have been drawn, and some unanswered questions require further research. These conclusions and questions will be summarized in the following sections.

5.1 The Impact of Grade Level and Gender on Algebraic Thinking

Data analysis revealed that students' algebraic thinking skills improved with higher grade levels. This finding is also supported by the studies conducted by Gülpek

(2016) and Birgin and Demirören (2020), which found that the algebraic performance of eighth-grade students was higher than that of seventh-grade students. A significant number of students across all grade levels succeeded in the "Generalized Arithmetic" (GA) section, one of the algebraic content strands included in the test. However, it was noted that the number of fifth-grade students who performed well in the "Functional Thinking" (FT) and "Modeling Languages" (ML) sections was extremely low. While sixth-grade students are included in all sections, their success rates appear to decline from GA to ML. Accordingly, the highest number of successful students in each section belongs to seventh graders. In their study, Chimoni et al. (2018) suggest that students can move on to more advanced modeling tasks by first developing their generalized arithmetic and functional thinking skills. This supports the findings of this thesis. In a similar study conducted in Turkey, Şengül and Tekcan (2023) divided their test into five sub-sections according to Kaput (1999) and found that students did not show the expected success, especially in the themes of “meaningful use of symbols” and “mathematical modeling,” as a result, students had difficulty in advanced algebraic thinking skills.

However, some of the fifth-grade students achieved success in all sections of the test, some seventh- and sixth-grade students failed in sections other than the GA shows that younger students can also develop higher-order thinking skills and that older students may sometimes not have these skills as much as younger students. This suggests that students' individual differences also play an important role in this process (Chimoni et al., 2018). The main reason for the success of the upper grades can be attributed to their algebraic training, as algebraic training starts in the sixth grade, and there were almost no fifth-grade students in the FT and ML sections of the test. This shows that algebraic thinking skills improve as algebra education is provided (Çağdaşer, 2008; Kaş, 2010; Özbayar, 2017; Yenilmez & Teke, 2008). Xie and Cai (2022) found that the majority of students used the reversal strategy based on arithmetic operations after receiving formal training on solving equations. Similarly, Akın and Işler Baykal (2024) found that after the functional thinking-focused intervention training given to the experimental group, this group was more

successful in functional thinking tests. These findings support the higher achievement of 6th and 7th grade students compared to 5th grade students in this thesis.

These findings suggest that algebraic education should be introduced earlier and with more effective methods. The results in this study may be influenced by the students' cognitive development as they grow older. However, in many questions, particularly those that required the use of variables, sixth and seventh-grade students—who had already begun algebraic education—demonstrated significantly higher achievement levels compared to other grade levels. This was particularly evident in the GA section, where the gap in performance was much larger than the average difference observed in other sections. This result supports the notion that algebraic education enhances algebraic thinking, and it is further corroborated by existing research, which emphasizes the necessity of early algebraic education to foster such cognitive development. The studies of Carpenter et al. (2003) and NCTM (2000) provide information on how children can develop algebraic thinking at an early age.

In addition, Kaya (2019) predicted that students' starting algebra education early and being successful in it can positively affect their mathematical beliefs and increase their algebraic thinking levels. Türkmen and Tanışlı (2019) also argued in their study that functional relationships could be developed in the 5th grade in the pre-algebra period and that teaching tools for this should be increased. Türkoğlu and Cihangir (2017) found that the critical acquisition process of algebraic thinking occurs between the ages of 4-12 and that algebra concepts are introduced at very early grade levels in the curricula of some countries.

Finally, since there may be differences between male and female students' learning styles, motivation and problem-solving approaches, it was observed that the gender variable did not have a significant effect on algebraic thinking skills in the analysis conducted to understand the potential effects of gender on mathematical achievement and to improve teaching methods by taking these differences into consideration. This suggests that the effect of gender on algebraic thinking skills is limited in evaluations

aimed at understanding various factors that may affect students' achievement (Birgin & Demirören, 2020; Erbaş, 1999; Park, 2021; Yenilmez & Teke, 2008).

5.2 Students' Algebraic Acquisitions and Responses

In this section, the quantitative data on algebraic concept knowledge obtained in the second problem are analyzed and interpreted in depth together with the qualitative data obtained from the responses of the students in the test, which reveal the solution strategies and thought processes of the students within the scope of the third problem.

5.2.1 Inequality

The analyses revealed that the students correctly understood the use of greater-than and less-than signs in inequality questions and realized that variables in inequality expressions can be replaced by more than one possible value, not just a single number. However, the fact that many students did not include ‘0’ in their results even though the question asked for a natural number indicates that they had a conceptual deficiency in the concept of number sets. This deficiency resulted in students' inability to express correct results even though they understood the concept of inequality. This finding is consistent with Kaput (2008), who emphasizes that algebraic thinking is based on accurately understanding the relationships between numerical and symbolic systems and the importance of number sets in this context.

5.2.2 Equality & Equations

The principle of conservation of equality is a fundamental mathematical concept that students acquire from an early age. In the study, it was observed that students were generally successful in conservation of equality questions when boxes were utilized instead of letters. This finding is also confirmed by Xie and Cai (2022). They also state that students encounter difficulties in problems related to the transformation of the unknown from parentheses (·) to letters, x.

However, as emphasized by Yaman, Toluk, and Olkun (2003), some students perceive equality only as a symbol indicating the result of an operation. This was observed in item 4b in this thesis, because in item ‘b’ the box is on the left-hand side where the result of the operation should become. However, although such solutions were also found in this study, students' success in expressing equality as a balance in items 4a and 4c shows that students have understood the concept of equality to some extent, which they have developed since elementary school. This suggests that when elementary students grasp the concept of mathematical equivalence, it lays a solid foundation for their algebraic thinking as they transition into middle school. These findings are consistent with many studies (e.g., Blanton & Kaput, 2005; Blanton et al., 2019; Carpenter et al., 2003; Carraher et al., 2006; Hornburg, Devlin, & McNeil, 2022).

However, the deficiencies in students' ability to form equations appropriate for problem situations in the second question are in line with the finding of Erbaş (1999) that students have difficulty in organizing formulas using variables. Moreover, these findings are in line with other studies (Akın & Işler Baykal, 2024; Türkmen & Tanışlı, 2019). However, it contradicts the finding of Yağız et al. (2024) that students were successful in converting verbal expressions into algebraic expressions.

In the new curriculum, the topic of conservation of equality is addressed in the same acquisition as the ability to make inferences about the properties of operations in the fifth grade, which allows both the reinforcement of knowledge from primary school without wasting time and deepening it by associating it with the properties of operations. This approach contributes to strengthening early algebra education (see MoNE, 2024, M.5.2.1).

5.2.3 Properties of Numbers and Operations

This study's key finding is that students' numerical processing and algebraic thinking skills develop in parallel, as evidenced by their similar levels of achievement in equality and equation acquisitions, as well as number and operation properties

acquisitions. A long-term study by Irwin and Britt (2008) found that understanding arithmetic relationships and performing arithmetic operations efficiently is an essential foundation for learning formal algebra. Similarly, Carpenter et al. (2003) emphasized that integrating arithmetic and algebra develops students' mathematical thinking skills at an early age. These findings support the results of our study. In their study with fifth-grade students, Kose and Kızıltoprak (2020) showed a strong relationship between algebraic thinking and the development of areas such as relational thinking and properties of the number system (change, combination, dispersion). Türkoğlu and Cihangir (2017) compared the grade levels at which algebraic concepts are introduced in different countries' primary school mathematics curricula, emphasizing the relationship between arithmetic and algebra and focusing primarily on arithmetic operations, generalization, and representation strategies.

These findings have significant implications for the new curriculum, particularly in the sequential treatment of the law of conservation of equality and properties of operations in fifth grade, followed by the inclusion of pattern rules and algorithms in basic arithmetic operations. This approach indicates that arithmetic operations and algebraic thinking are being integrated, which in turn supports early algebraic education.

5.2.4 Variables

This study encountered that most students lacked a full understanding of variables, often producing solutions by using numerical values instead of letters. This situation is in the first step of Kuchemann's (1981) steps in students' letter learning, in which students assign a numerical value to letters. It was observed that students used different strategies, such as assigning arbitrary numbers instead of letters, choosing another number in the question, or assigning numerical values according to the alphabetical order of the letters. Especially in seventh-grade students, the misconception that the result of algebraic expressions must be equal to a number was widely observed. This suggests that students do not see the letters in equations as unknown numbers but as objects whose results must be found. Studies conducted by

Girit and Akyüz (2016) and Usta and Özdemir (2018) also revealed similar results. In these studies, it was stated that most of the middle school students did not fully understand the concept of variables and tended to perform operations by assigning numerical values to letters.

However, in the inequality question that was answered correctly at the highest rate in this study, it is found that the students were in Kuchemann's fifth step, where the letter is considered to represent more than one value, not just one, because in this question there were five values that the letter could take. The students who answered as a single value were considered incorrect. This shows that students can interpret different questions differently.

5.2.5 Generalization

The study results show that students have serious difficulties generalizing using variables. Especially in pattern questions, students resorted to arithmetic methods and avoided generalizing using an algebraic expression. This shows that students depend more on arithmetic than algebraic thinking patterns. As Bednarz et al. (1996) stated, since students are more comfortable with arithmetic operations, they struggle to transition to algebraic thinking, especially in expressing the result in general terms.

Studies support this finding Carraher et al. (2006), which reveals that students often encounter challenges when transitioning from arithmetic to algebra. and have deficiencies in generalization skills, one of the most critical components of this process. It was observed that students had significant difficulties, especially in forming algebraic expressions and generalizing numerical data with abstract symbols. Radford's (2014) study also supports these findings and shows that students are weak in algebraic thinking skills and fail, significantly when solving problems that require generalization and abstraction. The results emphasize that acquiring generalization as a form of mathematical thinking is complex and that teaching strategies should focus more on this area.

Similar results were obtained in the study conducted by Atasoy (2020), which stated that sixth-grade students' ability to solve algebraic word problems and make generalizations was insufficient. This situation supports this thesis and points to a general problem in developing students' algebraic thinking skills.

In the new curriculum, including generalization skills in every topic from primary school onwards encourages students to think about phenomena from a broader perspective. This approach allows students to think not only in terms of specific examples, but also in terms of more general concepts and relationships. As a result, when variables are introduced into students' lives, their generalization skills will be more grounded and they will not have difficulty making generalizations accurately using variables. This suggests that generalization skills acquired early on will contribute significantly to developing algebraic thinking skills in later years.

5.2.6 Expressing the Relationship Between Two Variables

This study found that students did not show the expected success in the questions involving graphs and patterns. The abstract and generalizing nature of these concepts can explain this. Zazkis and Liljedahl's 2002 study supports this finding; they examined students' ability to make generalizations based on number patterns and relationships. The results showed that many students did not generalize from numerical patterns and did not express these generalizations with algebraic symbols. At the beginning of the study conducted by Çıkla Akkuş (2004), it was determined that students could not solve the questions. After the multiple representation training, it was observed that students' success, especially in pattern questions, increased. This finding emphasizes the importance of using different representations in teaching abstract and generalizing concepts. Moreover, Girit and Akyüz (2016) found that students in the experimental group tended to create and use their representations to solve algebraic problems. This finding confirms that students have difficulty understanding the relationships between variables when effective educational methods are not used.

In the covariation questions in this study, although questions involving close terms were solved by most students at all grade levels, the correct solution rate dropped to deficient levels when the question was asked to be generalized using algebraic expressions. For example, in item 10 of the test, students were able to compare two recent campaigns, but they could not identify the reversal of fortunes over a long period. In a study by Akın and Işler Baykal (2024), it was found that most of the students in the experimental group successfully identified covariant and functional relations with the training given to fifth grade students. This finding proves the positive effect of early algebraic education on students' algebraic thinking skills.

Moreover, students had difficulty in expressing the relationship between two variables algebraically. However, some students tried to solve them using different representations, such as fractions, proportions, or tables. This shows that algebraic thinking is not limited to letter symbols and that different methods can reach the unknown. Kaput (1999) stated that algebraic thinking includes letter-symbolic algebra and different representations and thinking processes, supporting this finding.

The results of this study show that algebra is not limited to the use of symbols but has a broader meaning. Kaput (2008) emphasizes that algebraic symbols can be used as a tool in introductory algebra, but what is more important is that students develop more general thinking skills, such as analyzing relationships between quantities, recognizing structures, and examining change.

5.3 Implications

This study aims to determine the development of algebraic thinking skills that prepare the ground for students' understanding of abstract algebraic concepts at the eighth-grade level and to reveal the effect of mathematics education in the first years of middle school on these skills. The study results showed that although students improved algebraic thinking skills with increasing grade levels, they had deficiencies, especially in symbolic manipulation and generalization. These findings provide a basis for designing intervention programs to improve students' algebraic

thinking skills. In this context, it has important implications for teachers, educators, and curriculum writers.

Findings from this study and other studies in the literature support the strong relationship between arithmetic and algebraic thinking skills. Students' in-depth knowledge of numbers and operation properties helps them understand algebraic concepts more quickly. For this reason, it is vital that arithmetic and algebra topics are addressed about each other at an early age in mathematics teaching and that students' numerical and symbolic thinking skills are developed together. In particular, although students have a general knowledge of numbers and operation properties, they have difficulty transferring this knowledge to different problem situations and solving complex problems. Their mental processing skills and knowledge of basic concepts need to be reinforced. Activities that encourage students to use different solution strategies and help them understand the relationships between concepts should be included in the teaching process.

In addition, although there is a general improvement in students' algebraic thinking skills as the grade level increases, individual differences in student achievement show that students have different starting points in the development of algebraic thinking skills and progress at different learning rates. This situation emphasizes the importance of adopting differentiated teaching approaches that consider students' differences. Therefore, in-service training programs for teachers should be organized to ensure that teachers have up-to-date knowledge and skills in algebra instruction in primary and early middle school. These programs should include teaching methods that consider students' differences, effective use of materials, and assessment methods.

5.4 Recommendations for Further Research

This study investigates the algebraic thinking skills and algebraic concept knowledge of fifth, sixth, and seventh-grade students exposed to the Ministry of National Education (MoNE) 2018 curriculum in the 2023-2024 academic year. The study

aims to determine the current state of the algebraic thinking skills of this group of students and to provide essential data about early algebra teaching. The findings are supported by the data obtained from the last group of students who were educated within the scope of the MoNE 2018 curriculum and the study offers important insights into how effective the curriculum is. In the next phase of the study, conducting a long-term study by following the same group of students will allow for a more precise observation of the development of students' algebraic thinking skills and a better understanding of the long-term effects of curriculum changes. In this way, it will be possible to evaluate current practices in algebra instruction and develop more effective teaching strategies for the future.

The new mathematics curriculum published by MoNE in 2024 emphasizes the ability to generalize all subjects from the primary school level onwards. This curriculum aims to enable students to generalize mathematical relationships without using letter symbols. In addition, algebraic thinking was introduced in the first grade, along with basic concepts such as number and operation properties. In this context, the algebraic thinking skills of the students to whom the new curriculum is applied can be evaluated by comparing them with the findings obtained in this thesis. In this way, the impact of the 2024 curriculum on students' algebraic thinking skills can be revealed more clearly, and more comprehensive conclusions can be drawn about early algebra teaching.

However, further research is needed on how algebraic thinking skills support the transition to advanced algebraic concepts, particularly how algebraic thinking skills contribute to students' understanding of abstract algebraic concepts and reduce learning difficulties. In this context, developing teaching methods and materials to facilitate the transition from early algebra to advanced algebra skills is a vital research topic in mathematics education.

This study found that the algebraic thinking skills of fifth, sixth, and seventh graders showed increasing development as they encountered algebra. In addition, significant individual differences were observed between the algebraic thinking skills of

students at these grade levels. These findings suggest that students' algebraic thinking skills vary by grade level, individual characteristics, and prior mathematical experiences. This suggests that grade-level and individual factors shape the development of algebraic thinking and that instructional strategies should be designed to consider student differences. Designing activities that support students' strengths while strengthening their weaknesses will contribute positively to the mathematics achievement of all students. In this context, teachers' adequate knowledge and skills in early algebra teaching are critical in developing students' algebraic thinking skills.

The way students grasp the structure of numerical terms during their elementary years plays a crucial role in shaping their algebraic thinking skills later on. This study found that students improved their algebraic thinking skills as they were introduced to algebra. Algebraic thinking is a skill that needs continuous development, and teachers' support can guide students' progress in their individual development. These findings call for an examination of teachers' competence in teaching algebraic thinking skills. Such a study could be a significant contribution for teacher educators and curriculum designers.

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APPENDICES

A. Algebraic Thinking Skills Test

CEBİRSEL DÜŞÜNME BECERİLERİ TESTİ

Ad-Soyad: Sınıf:

1) Ada 27 + 15 ve 34 + 19 gibi sayıları zihinden toplamak için aşağıdaki zihinden toplama stratejisini kullanıyor.

$$27 + 15 = 30 + 12 = 42$$

$$34 + 19 = 33 + 20 = 53$$

a) Siz de Ada'nın yöntemini 298 + 57 toplama işlemi için uygulayarak zihinden sonucu nasıl bulacağınızı açıklayınız.

b) Kullandığınız yöntemi aşağıdaki işlemde de kullanarak boşluğa gelmesi gereken harfli ifadeyi yazınız.

$58 + a = 60 +$ ☐

2) Bir okulun kütüphanesinde 145 kitap vardır. Enver Bey ve Sevim Hanım bu kütüphaneye kitap bağışında bulunur. Böylece kütüphanedeki kitap sayısı 625 olur. Enver Bey kütüphaneye 180 kitap bağışlamıştır. Buna göre Sevim Hanım'ın kütüphaneye kaç kitap bağışladığını bulunuz. Verilenleri ve verilmeyenleri gösteren bir eşitlik yazınız.

3) C ve D doğal sayı ve C < D olmak üzere, C + D = 10 eşitliğinde C'nin alabileceği değerler nelerdir? (Çözüm yönteminizi açıklayınız)

4) Aşağıdaki işlemlerde soru işareti olan boşluğa kaç yazılması gerektiğini bulunuz. (Çözüm yönteminizi açıklayınız)

a) △ + □ = 9 ise

△ + □ + 2 = ?

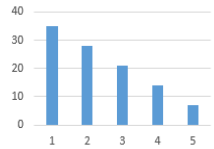
b) 25 + 14 = 7 + ?

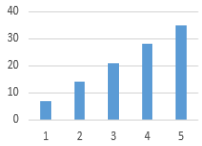
c) ? + 80 = 120

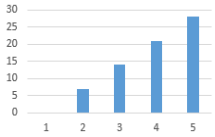
5) Yeni bir kumbara alan Ali kumbarasını aldığı gün 7 TL atmaya başlar ve her gün 7 TL atmaya devam eder. Aşağıdaki grafiklerden hangisi Ali'nin 5 gün boyunca kumbarasında biriken toplam parayı gösterir? Cevabınızı açıklayınız.

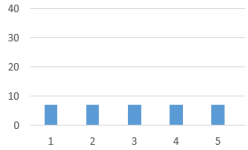
Neden bu şıkkı seçtiniz?:

A) B) C) D)

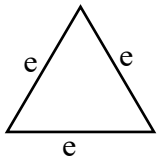
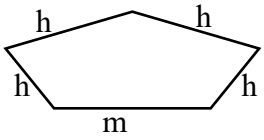
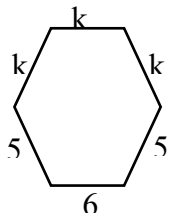
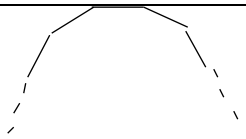




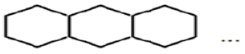




6) Aşağıdaki kenar uzunlukları verilen çokgenlerin çevrelerini harfli ifade olarak altlarına yazınız.

Örneğin:  Çevre: 2+3+4=9	 Çevre:	 Çevre:
 Çevre:	 Yukarıdaki şekil bir kısmı silinmiş, bütün kenarları birbirine eşit ve her kenarı 2 cm olan bir çokgendir. Çevre:	

7) Kenar uzunluğu 1 cm olan düzgün altıgenler, başlangıçta tek altıgen ile başlayıp aşağıdaki gibi bir kenarları ortak olacak şekilde her adımda bir altıgen eklenerek yan yana yerleştiriliyor. Buna göre aşağıdaki soruları cevaplayınız.



a) Bu altıgenlerle oluşturulan örüntünün ilk 3 adımını çiziniz.

b) Bu altıgenlerden 4 tanesi yan yana geldiğinde oluşan şeklin çevre uzunluğu kaç cm olur?

c) Bu altıgenlerden 75 tanesi bir araya geldiğinde oluşan şeklin çevre uzunluğu kaç cm olur?

8) Fatma gittiği bir dükkanda alacağı ürüne ait aşağıda verilen indirim seçeneklerini görmüştür. Seçenekleri değerlendirip en ucuz olandan en pahalı olana doğru sıralayınız. Sıralamayı nasıl yaptığınızı açıklayınız.

1. Seçenek: Bir alana bir bedava
2. Seçenek: İki tane alanın yanına bir tane bedava
3. Seçenek: İki tane alana ikincisi yarı fiyatına
4. Seçenek: Üç al iki öde
5. Seçenek: bir tane fiyatına iki tane al

9) Veli arabasına kutu yüklemektedir, her bir kutu eşit ağırlıktadır. Yandaki tabloda arabaya yüklenen kutu sayısı ve yüklenen kutuların toplam ağırlığı verilmiştir.

a) 12 kutu yüklendiğinde toplam ağırlık kaç kg olur? Nasıl bulduğunuzu açıklayınız.

b) Tabloya göre kutu sayısı n olduğunda toplam ağırlığın ne olacağını harfli ifade olarak bulunuz. Cevabınızı açıklayınız.

Kutu sayısı	Toplam ağırlık (kg)
2	50
4	100
6	150
…	…
n	?

10) Murat spor salonuna kaydolmayı düşünmektedir. İki farklı spor salonunda aşağıdaki kampanyalar vardır. Murat haftada iki defa spor salonuna gideceğine göre hangi spor salonu daha ekonomiktir?
Güçlendiren spor salonu: her ders 8 TL
Çalıştıran spor salonu: ilk beş ders için 50 TL, sonraki her ders 5 TL

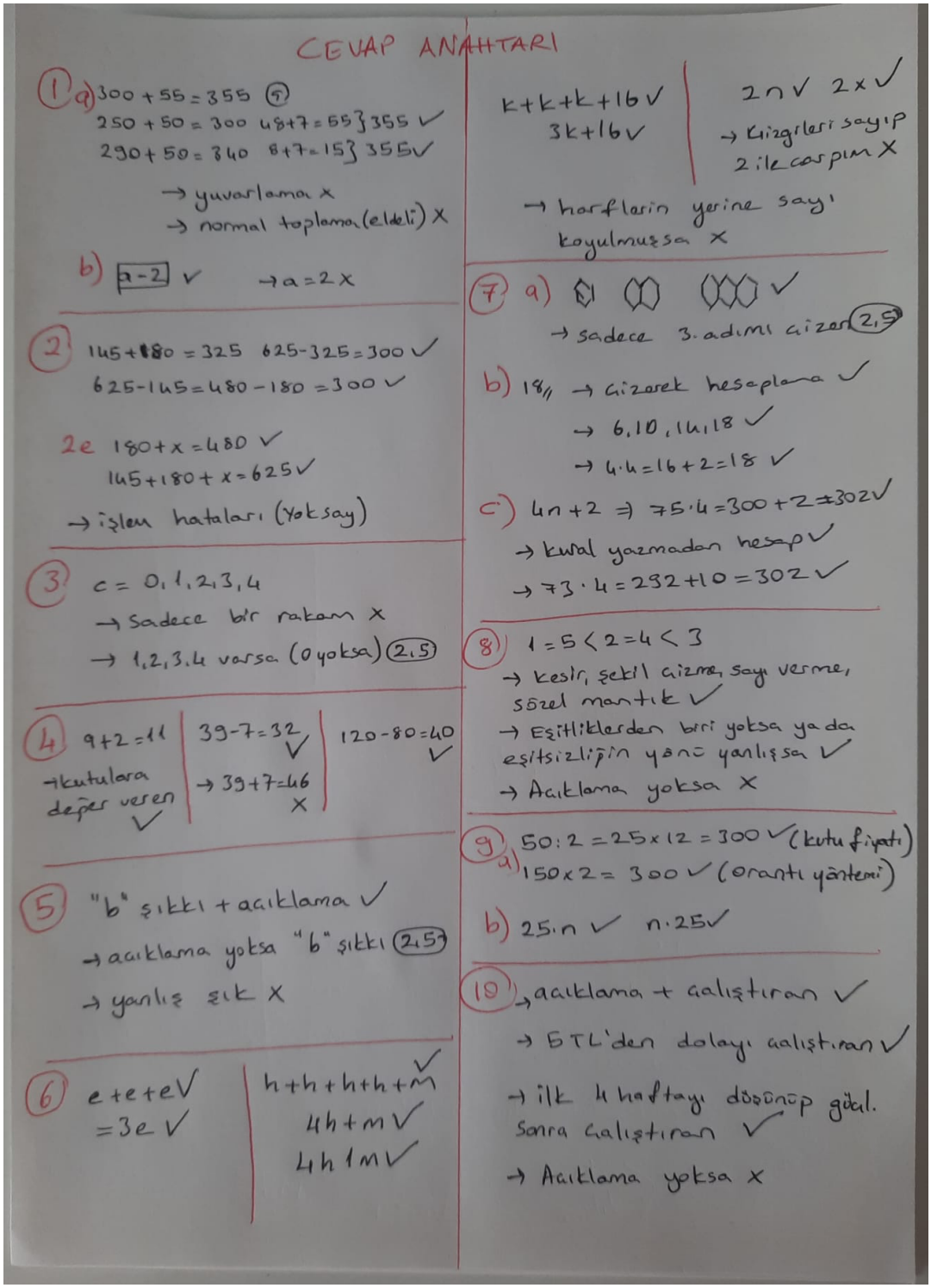
B. Specification Table of The Algebraic Thinking Skills Test

Content strand	Q	Objects from MEB 2018 Curriculum
Generalized arithmetic	1	5th grade: Identifies and uses strategies in mental addition and subtraction operations with two-digit natural numbers. (İki basamaklı doğal sayılarla zihinden toplama ve çıkarma işlemlerinde strateji belirler ve kullanır.) Generalizes the mental calculation strategy used to a new situation (Kullandığı zihinden işlem yapma stratejisini yeni bir duruma geneller.)
	2	3rd grade: Finds the missing addend in an addition operation. (Bir toplama işleminde verilmeyen toplananı bulur.) 6th grade: Writes an algebraic expression suitable for a given verbal situation and a verbal situation suitable for a given algebraic expression. (Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar.)
	3	4th grade: Sorts natural numbers with at most six digits using the greater than/small than symbol. (En çok altı basamaklı doğal sayıları büyük/küçük sembolü kullanarak sıralar.) Doğal sayıları sayar. (Counts natural numbers.) Writes natural numbers up to 20 as the sum of two natural numbers are included. (20'ye kadar olan doğal sayıları iki doğal sayının toplamı biçiminde yazar.)
	4	2nd grade: Recognizes the mathematical meaning of the equal sign as 'equality' between mathematical expressions. (Eşit işaretinin matematiksel ifadeler arasındaki "eşitlik" anlamını fark eder.) 3rd grade: Finds the ungiven sum in an addition operation. (toplama işleminde verilmeyen toplananı bulur.)

		4th grade: Determines the value not given in one of two mathematical expressions between which there is equality and explains that equality is achieved. (Aralarında eşitlik durumu olan iki matematiksel ifadeden birinde verilmeyen değeri belirler ve eşitliğin sağlandığını açıklar.) 7th grade: Understands the principle of conservation of equality. (Eşitliğin korunumu ilkesini anlar.)
Functional thinking	5	4th grade: Examines the column graph, makes comments and predictions on the graph. / Creates the column graph. (Sütun grafiğini inceler, grafik üzerinde yorum ve tahminler yapar. / Sütun grafiğini oluşturur.)
	6	4th grade: Solves problems related to calculating the perimeter lengths of shapes. (Şekillerin çevre uzunluklarını hesaplamayla ilgili problemleri çözer.) 6th grade: Writes an algebraic expression suitable for a given verbal situation and a verbal situation suitable for a given algebraic expression. (Sözel olarak verilen bir duruma uygun cebirsel ifade ve verilen bir cebirsel ifadeye uygun sözel bir durum yazar.)
	7	5th grade: Forms the desired steps of the number and shape patterns whose rules are given. (Kuralı verilen sayı ve şekil örüntülerinin istenen adımlarını oluşturur.) 7th grade: Expresses the rule of numerical and figural patterns with letters and finds the desired term of the pattern expressed with letters. (Sayı örüntülerinin kuralını harfle ifade eder, kuralı harfle ifade edilen örüntünün istenilen terimini bulur.)
Modeling language	8	5th grade: Calculates the desired simple fraction of a multiplicity and the whole of a multiplicity given a simple fraction using unit fractions. (Bir çokluğun istenen basit kesir kadarını ve basit kesir kadarı verilen bir çokluğun tamamını birim kesirlerden yararlanarak hesaplar.

		6th grade: Uses ratio in comparing multiplicities and shows the ratio in different forms. (Çoklukları karşılaştırmada oran kullanır ve oranı farklı biçimlerde gösterir.) Can make comparisons by substituting a number for a constant value or by using the ratio between values when a specific number is not given. (Belirli bir sayı verilmediğinde sabit değer yerine sayı koyarak ya da değerler arasında orantı kullanarak karşılaştırma yapabilir.)
	9	5th grade: Forms the desired steps of the number and shape patterns whose rules are given. (Kuralı verilen sayı ve şekil örüntülerinin istenen adımlarını oluşturur.) 7th grade: Expresses the rule of number patterns with a letter and finds the desired term of the pattern where the rule is expressed with a letter. (Sayı örüntülerinin kuralını harfle ifade eder, kuralı harfle ifade edilen örüntünün istenilen terimini bulur.)
	10	4th grade: Uses different representations to present the data obtained. (Elde ettiği veriyi sunmak amacıyla farklı gösterimler kullanır.) 4th grade: Solves real-life problems using information presented in bar graphs, tables, and other charts. Sütun grafiği, tablo ve diğer grafiklerle gösterilen bilgileri kullanarak günlük hayatla ilgili problemler çözer.

C. Scoring Rubric for The Algebraic Thinking Skills Test



CEVAP ANAHTARI

1) a) 300+55=355 (5)
250+50=300 48+7=55 } 355 ✓
290+50=340 8+7=15 } 355 ✓
→ yuvarlama ✗
→ normal toplama (elde) ✗

b) a-2 ✓ → a=2x

2) 145+180=325 625-325=300 ✓
625-145=480-180=300 ✓
2e 180+x=480 ✓
145+180+x=625 ✓
→ işlem hataları (Yoksay)

3) c = 0, 1, 2, 3, 4
→ sadece bir rakam ✗
→ 1, 2, 3, 4 varsa (0 yoksa) (2,5)

4) 9+2=11 → kutulara değer veren ✓
39-7=32 ✓ → 39+7=46 ✗
120-80=40 ✓

5) "b" şıkkı + açıklama ✓
→ açıklama yoksa "b" şıkkı (2,5)
→ yanlış şık ✗

6) e+e+e ✓ = 3e ✓
k+k+k+16 ✓ 3k+16 ✓
2n ✓ 2x ✓ → kirgileri sayıp 2 ile çarpım ✗
→ harflerin yerine sayı koyulmuşsa ✗

7) a) ✓ → sadece 3. adımı çizer (2,5)
b) 18 // → çizerek hesaplama ✓
→ 6, 10, 14, 18 ✓
→ 4·4=16+2=18 ✓
c) 4n+2 ⇒ 75·4=300+2=302 ✓
→ kural yazmadan hesap ✓
→ 73·4=292+10=302 ✓

8) 1=5 < 2=4 < 3
→ kesir, şekil çizme, sayı verme, sözel mantık ✓
→ Eşitliklerden biri yoksa ya da eşitsizliğin yönü yanlışsa ✓
→ Açıklama yoksa ✗

9) a) 50:2=25×12=300 ✓ (kutu fiyatı)
150×2=300 ✓ (oranti yöntemi)
b) 25·n ✓ n·25 ✓

10) → açıklama + çalıştıran ✓
→ 5 TL'den dolayı çalıştıran ✓
→ ilk 4 haftayı düşünüp gözl. sonra çalıştıran ✓
→ Açıklama yoksa ✗

D. Algebra Objectives from Ministry of National Education 2018 Curriculum

Learning Area	Objectives
	Grade 6
Algebraic Expressions	To write an algebraic expression appropriate to a given verbal situation and a verbal situation appropriate to a given algebraic expression (MoNe, 2018; M.6.2.1.1). To calculate the value of an algebraic expression for different natural number values of the variable (MoNe, 2018; M.6.2.1.2). To explain the meaning of simple algebraic expressions (MoNe, 2018; M.6.2.1.3).
	Grade 7
Algebraic Expressions	To add and subtract with algebraic expressions (MoNe, 2018; M.7.2.1.1). To multiply an algebraic expression by a natural number (MoNe, 2018; M.7.2.1.2). To express the rule of number patterns with letters, find the desired term of the pattern whose rule is expressed with letters (MoNe, 2018; M.7.2.1.3).
Equality and equations	To understand the principle of conservation of equality (MoNe, 2018; M.7.2.2.1). To recognize the first order equation with one unknown and construct a first order equation with one unknown in real life situations (MoNe, 2018; M.7.2.2.2). To solve equations of first order with one unknown (MoNe, 2018; M.7.2.2.3). To solve problems requiring the construction of an equation of first order with one unknown (MoNe, 2018; M.7.2.2.4).

Grade 8	
Algebraic expressions and identities	To understand simple algebraic expressions and writes them in different forms (MoNe, 2018; M.8.2.1.1). To perform the multiplication of algebraic expressions (MoNe, 2018; M.8.2.1.2). To explain identities using models (MoNe, 2018; M.8.2.1.3). To factorize algebraic expressions (MoNe, 2018; M.8.2.1.4).
Linear equations	To solve first-degree equations with one unknown (MoNe, 2018; M.8.2.2.1). To recognize the coordinate system and its properties, and represent ordered pairs (MoNe, 2018; M.8.2.2.2). To express how one of two variables with a linear relationship change depending on the other using tables and equations (MoNe, 2018; M.8.2.2.3). To draw the graph of linear equations (MoNe, 2018; M.8.2.2.4). To create and interpret equations, tables, and graphs related to real-life situations involving linear relationships (MoNe, 2018; M.8.2.2.5). To explain the slope of a line using models, and associate linear equations and graphs with the slope (MoNe, 2018; M.8.2.2.6).
Inequalities	To write mathematical statements appropriate for real-life situations involving first-degree inequalities with one unknown. To represent first-degree inequalities with one unknown on a number line (MoNe, 2018; M.8.2.3.1). To solve first-degree inequalities with one unknown (MoNe, 2018; M.8.2.3.2).

E. Percentage Distribution of Students' Responses per Item

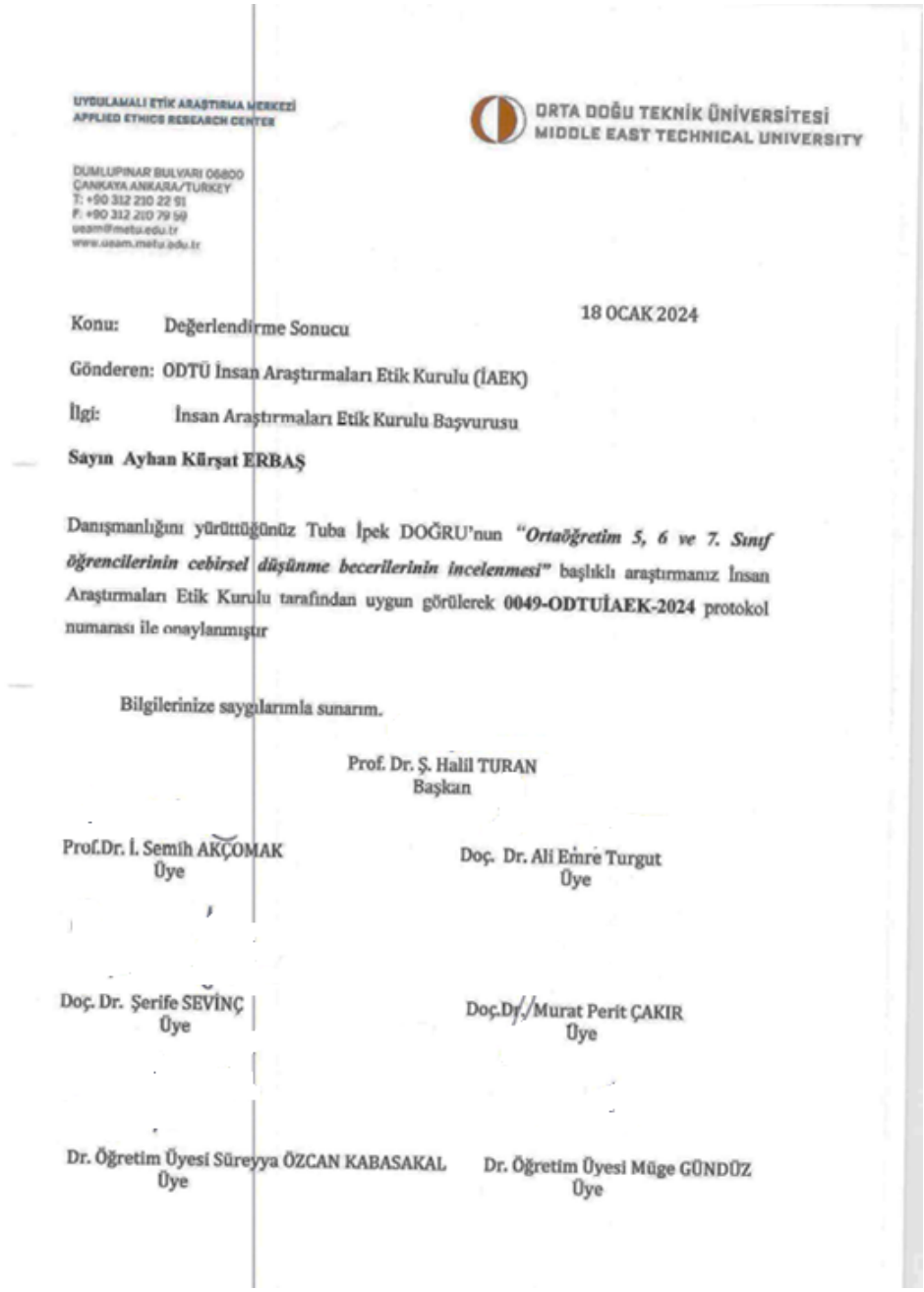
The average percentage of correct/semi-correct answers among all students					
Item	**1a**	**1b**	**2**	**2e**	**3**
Correct	39	3	63	6	10
Semi-Correct	0	0	0	0	27

Item	**4a**	**4b**	**4c**	**5**	**6a**
Correct	75	38	84	60	37
Semi-Correct	0	0	0	4	0

Item	**6b**	**6c**	**6d**	**7a**	**7b**
Correct	33	22	4	5	12
Semi-Correct	0	0	0	0	0

Item	**7c**	**8**	**9a**	**9b**	**10**
Correct	6	13	54	5	18
Semi-Correct	0	17	0	0	0

F. Middle East Technical University Human Research Ethics Commettee Approval



UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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18 OCAK 2024

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Ayhan Kürşat ERBAŞ

Danışmanlığını yürüttüğünüz Tuba İpek DOĞRU'nun ***"Ortaöğretim 5, 6 ve 7. Sınıf öğrencilerinin cebirsel düşünme becerilerinin incelenmesi"*** başlıklı araştırmanız İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek **0049-ODTUİAEK-2024** protokol numarası ile onaylanmıştır

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan

Prof.Dr. İ. Semih AKÇOMAK
Üye

Doç. Dr. Ali Emre Turgut
Üye

Doç. Dr. Şerife SEVİNÇ
Üye

Doç.Dr. Murat Perit ÇAKIR
Üye

Dr. Öğretim Üyesi Süreyya ÖZCAN KABASAKAL
Üye

Dr. Öğretim Üyesi Müge GÜNDÜZ
Üye

G. Consent Form for Participants

Araştırmaya Gönüllü Katılım Formu

Bu araştırma, ODTÜ Matematik Eğitimi Bölümü Yüksek Lisans öğrencisi Tuba İpek DOĞRU tarafından Prof. Dr. Ayhan Kürşat ERBAŞ danışmanlığındaki yüksek lisans tezi kapsamında yürütülmektedir. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir?

Araştırmanın amacı cebirsel düşünme becerilerinizin incelenmesidir. Araştırmaya katılmayı kabul ederseniz, sizden beklenen, çalışma kapsamında oluşturulan testi çözmenizdir. Bu çalışmaya katılım ortalama olarak bir ders saati (40 dakika) sürmektedir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz?

Çalışma kapsasında oluşturulan testi bireysel olarak doldurmanız beklenmektedir. Tamamlanması ortalama 40 dk süren ölçekte cebirsel düşünme becerileri ölçen çoktan seçmeli ve açık uçlu sorular bulunmaktadır. Tüm sorular için çözüm yolunuzu kağıtta göstermeniz gerekmektedir. Çalışma sonunda cevaplarınız ilgili ölçek sizden toplanacak ve analiz edilecektir.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız?

Araştırmaya katılımınız tamamen gönüllülük temelinde olmalıdır. Ankette, sizden kimlik veya kurum belirleyici hiçbir bilgi istenmemektedir. Cevaplarınız tamamıyla gizli tutulacak, sadece araştırmacılar tarafından değerlendirilecektir. Katılımcılardan elde edilecek bilgiler toplu halde değerlendirilecek ve bilimsel yayımlarda kullanılacaktır. Sağladığınız veriler gönüllü katılım formlarında toplanan kimlik bilgileri ile eşleştirilmeyecektir, kesinlikle ders notunuzu etkilemeyecektir.

Katılımınızla ilgili bilmeniz gerekenler:

Katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz cevaplama işini yarıda bırakıp çalışmayı bırakmakta serbestsiniz. Böyle bir durumda çalışmayı uygulayan kişiye, çalışmadan çıkmak istediğinizi söylemek yeterli olacaktır. Çalışma sonunda, bu araştırmayla ilgili sorularınız cevaplanacaktır

Araştırmayla ilgili daha fazla bilgi almak isterseniz:

Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Araştırma hakkında daha fazla bilgi almak için ODTÜ Matematik Eğitimi Bölümü Yüksek Lisans öğrencisi, okulumuzun matematik öğretmeni Tuba İpek DOĞRU ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum. (Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyisim	Tarih	İmza
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