THE MATHEMATICAL ABILITIES OF YOUNG CHILDREN IN THE CONTEXT OF HEMODYNAMIC CHANGES, EXECUTIVE FUNCTION, HOME AND SCHOOL ENVIRONMENT

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

BEYZA HAMAMCI

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
THE DEPARTMENT OF ELEMENTARY AND EARLY CHILDHOOD EDUCATION, EARLY CHILDHOOD EDUCATION

MARCH 2025

Approval of the thesis:

THE MATHEMATICAL ABILITIES OF YOUNG CHILDREN IN THE CONTEXT OF HEMODYNAMIC CHANGES, EXECUTIVE FUNCTION, HOME AND SCHOOL ENVIRONMENT

submitted by **BEYZA HAMAMCI** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Elementary and Early Childhood Education, Early Childhood Education, the Graduate School of Social Sciences of Middle East Technical University** by,

Prof. Dr. Sadettin KİRAZCI
Dean
Graduate School of Social Sciences

Prof. Dr. Feyza TANTEKİN ERDEN
Head of Department
Department of Elementary and Early Childhood Education

Assoc. Prof. Dr. Hasibe Özlen DEMİRCAN
Supervisor
Department of Elementary and Early Childhood Education

Assoc. Prof. Dr. Yusuke MORIGUCHI
Co-Supervisor
Kyoto University
Department of Psychology

Examining Committee Members:

Prof. Dr. Refika OLGAN (Head of the Examining Committee)
Middle East Technical University
Department of Elementary and Early Childhood Education

Assoc. Prof. Dr. Hasibe Özlen DEMİRCAN (Supervisor)
Middle East Technical University
Department of Elementary and Early Childhood Education

Prof. Dr. Feyza TANTEKİN ERDEN
Middle East Technical University
Department of Elementary and Early Childhood Education

Prof. Dr. İbrahim Hakkı ACAR
Özyeğin University
Department of Psychology

Prof. Dr. Ata AKIN
Acıbadem Mehmet Ali Aydınlar University
Department of Biomedical Engineering

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: Beyza HAMAMCI

Signature:

ABSTRACT

THE MATHEMATICAL ABILITIES OF YOUNG CHILDREN IN THE CONTEXT OF HEMODYNAMIC CHANGES, EXECUTIVE FUNCTION, HOME AND SCHOOL ENVIRONMENT

HAMAMCI, Beyza
Ph.D., The Department of Elementary and Early Childhood Education, Early Childhood Education
Supervisor: Assoc. Prof. Dr. Hasibe Özlen DEMİRCAN
Co-supervisor: Assoc. Prof. Dr. Yusuke MORIGUCHI

March 2025, 216 pages

Early mathematics abilities predict later life success and are influenced by biological, cognitive, and environmental factors. This study aims to investigate the relationship between children's mathematical abilities and hemodynamic changes in the parietal lobe related to math performances, child's executive function, and home and school math environments. Additionally, it aims to build a model of children's mathematical abilities. Data were obtained from 239 children (aged 51-74 months), their parents, and teachers across 81 schools in Istanbul during the 2023-2024 academic year. Test of Early Mathematics Abilities-3rd Edition, fNIRS math task, EF-Touch, Early Mathematics Questionnaire, Mathematics Activities in the Classroom Scale, and Mathematical Development Belief Scale were used as data collection tools. Results indicated the mean level of oxyhemoglobin in the left intraparietal sulcus during addition tasks was associated with children's addition performance. Additionally, a moderate positive relationship was found between children's executive function performance and their math performances. The home math environment (i.e., parent-child activities and beliefs about math) showed a low positive correlation with children's math abilities. However, school-related mathematical factors were not directly associated with children's mathematical abilities. The structural equation model, which incorporated children’s executive functions as well as home and school environments to predict mathematical abilities, demonstrated good fit indices. Furthermore, working memory, inhibitory control, and cognitive flexibility combined with mean oxyhemoglobin levels in the left intraparietal sulcus, accounted for 22% of the variance in math abilities. Overall, these findings emphasize the combined influence of biological, cognitive, and environmental factors, especially the home environment, in shaping children’s mathematical abilities.

Keywords: math abilities, hemodynamic changes, executive function, home math environment, school math environment

ÖZ

HEMODİNAMİK DEĞİŞİKLİKLER, YÜRÜTÜCÜ İŞLEV, EV VE OKUL İKLİMİ BAĞLAMINDA KÜÇÜK ÇOCUKLARIN MATEMATİK BECERİLERİ

HAMAMCI, Beyza
Doktora, Temel Eğitim, Okul Öncesi Eğitimi Bölümü
Tez Yöneticisi: Doç. Dr. Hasibe Özlen DEMİRCAN
Ortak Tez Yöneticisi: Doç. Dr. Yusuke MORIGUCHI

Mart 2025, 216 sayfa

Biyolojik, bilişsel ve çevresel faktörlerin etkisi ile şekillenen erken matematik becerileri yaşam başarısını öngörmektedir. Bu çalışmanın amacı, matematik performansıyla ilişkili parietal lobdaki hemodinamik değişiklikleri, yürütücü işlev, ev ve okul matematik iklimlerini inceleyerek çocukların matematik yeteneklerine ilişkin bir model oluşturmaktır. Veriler, 2023-2024 eğitim-öğretim yılı boyunca İstanbul'daki 81 okulda, 239 çocuktan (ranj, 51-74 ay), ebeveynlerinden ve öğretmenlerinden toplanmıştır. Veri toplama araçları olarak Erken Matematik Testi-3, fNIRS matematik görevi, EF-Touch, Erken Matematik Ölçeği, Sınıf İçi Matematik Etkinlikleri Ölçeği ve Matematiksel Gelişim İnanç Ölçeği kullanılmıştır. Sonuçlar, toplama görevi sırasında sol intraparietal sulkustaki ortalama oksihemoglobin seviyesinin çocukların toplama işlemi becerileri ilişkili olduğunu göstermiştir. Ayrıca, çocukların yürütücü işlev performansı ile matematik becerileri arasında orta düzeyde pozitif bir ilişki bulunmuştur. Ebeveyn-çocuk etkileşimleri ve matematikle ilgili inançlardan oluşan ev matematik iklimi çocukların matematik becerileriyle düşük düzeyde pozitif yönlü korelasyona sahiptir. Bununla birlikte, okulla ilgili matematiksel faktörler, çocukların matematik becerileriyle doğrudan ilişkili bulunmamıştır. Matematiksel becerileri tahmin etmek üzere çocukların yürütücü işlevleri ile ev ve okul ortamlarını da içeren yapısal eşitlik modeli, iyi uyum indeksleri sergilemiştir. Bunun yanı sıra, işleyen bellek, ketleyici kontrol, bilişsel esneklik ve sol intraparietal sulkustaki ortalama oksihemoglobin düzeyi çocukların matematik yeteneklerindeki varyansın %22'sini açıklamaktadır. Genel olarak, bu bulgular, çocukların matematiksel becerilerini şekillendirmede biyolojik, bilişsel ve çevresel faktörlerin, özellikle de ev ortamının, birleşik etkisini vurgulamaktadır.

Anahtar Kelimeler: matematik becerileri, hemodinamik değişiklikler, yürütücü işlevler, ev matematik iklimi, okul matematik iklimi

To my family, who always surrounded me with their love

ACKNOWLEDGMENTS

I accomplished this thesis as a result of a long process involving many collaboration and travel from Japan to Türkiye, from Ankara to Istanbul, so there are many people whom I owe a debt of gratitude. First of all, I would like to thank my advisor Assoc. Prof. Hasibe Özlen Demircan, who has been with me throughout this process, in all the ups and downs, and guided me with her knowledge and experience. Next, I would like to thank my co-advisor Assoc. Prof. Yusuke Moriguchi (sensei), who allow me to conduct fNIRS studies by welcoming my desire to learn, contributed to my improvement by supporting both my academic and cultural aspects, and whom I am honored to be accepted as his student.

I would like to thank committee members Professor Feyza Tatekin Erden and Professor İbrahim Hakkı Acar for their inspiring comments, improving suggestions, thought-provoking discussions during this period. In addition, I would like to thank jury members, Professor Ata Akın and Professor Refika Olgan for their contribution.

I would like to thank Jue Wang, who helped me learn the fNIRS technique and its implementation, as well as Yuko Takahashi, Sakata Chifumi, Nozomi Yamamoto, and Ryoichi Watanabe for their friendship and interest during my visit at the Moriguchi Child Development Lab at Kyoto University.

I thank Professor Ata Akın, Assoc. Prof. Dr. Sinem Burcu Erdoğan and Research Assistant Gülnaz Yükselen, for their support and positive attitudes, who enabled me to use this technique in Türkiye and supported my learning during the study. Without their support, this study would not have been possible.

Furthermore, this study was supported by the Scientific and Technological Research Council of Türkiye (TUBITAK) under Project No. 223K757. I would like to thank TUBITAK for its support for the project and for the scholarship support (Program

No. 2211-A) during my PhD. During this project, I would like to thank our project scholars Derin Sağlar and Berfin Şencan for all their dedication, labor, time and energy.

Special thanks to my office mates, Canan Peker, Eda Sevinç-Özgen, Fatma Demirci, Sümeyra Eryiğit and Tuğçe Esra Terzioğlu, who made me forget my exhaustion and stress with their energy and motivation, and nourished me with their knowledge, as well as whose joy gave me peace of mind.

Many thanks to my neighbors M. Candan Akyıldız, Sanem Yapar, and Elif Özbey, who turned my room into home and turned my ordinary days into "Perfect Days" with their kindness, friendship, sharing and conversations that were like a kind of therapy in Ankara, where I came without knowing anyone.

Additionally, I would like to thank İbrahim Çirozlar, who helped me when I was in trouble and supported me in solving my problems. In other words, "hontō ni arigatō gozaimashita".

I would like to thank my cozy family, who have been with me every day of these four years, which have been like an emotional roller coaster, and whose love I nourished, whose trust I grew up with, and who have contributed to this thesis beyond moral uplift. I would like to thank my father who worked with me during the data collection process of my thesis and worked full time with me even though he was retired. I would like to thank my mother, who has been my greatest strength in overcoming all my difficulties, with whom I shared all my tensions, disappointments, sleepless days, successes and "joy" in my isolated life squeezed between Ankara-Istanbul, work and thesis studies. Thank you to my dear brother, the happiness of my home, my companion of learning and fun, the one with whom I feel I breathe, for every day we have spent and will spend together. I would like to thank my angel sister, who was my best friend from birth but whom I lost in this process, for all her faith in me and what she contributed to my life. I am sure that if she saw this day, she would turn every day into a celebration.

TABLE OF CONTENTS

CHAPTERS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

AÇEV	Mother Child Education Foundation
ANOVA	Analysis of Variance
ANS	Approximate Number System
EEG	Electroencephalography
EF	Executive Function
ERI	Education Reform Initiative
fMRI	functional Magnetic Resonance Imaging
fNIRS	functional Near-Infrared Spectroscopy
IPS	Intraparietal Sulcus
MoNE	Ministry of National Education
OECD	Organization for Economic Cooperation and Development
SEM	Structural Equation Modeling
UNICEF	United Nations International Children’s Emergency Fund

CHAPTER 1

INTRODUCTION

Early childhood education years play an essential role in building the basis for cognitive development and learning. This crucial period marks the blossoming nature of the brain since the most dynamic and elaborative anatomical and physiological changes occur during this period (Brown et al., 2012). In this period, it is essential to promote key cognitive developmental goals, including executive function, problem-solving skills, and early math abilities (Diamond, 2013). Environmental factors play a crucial role in supporting cognitive development during this critical period. Cognitive abilities are shaped by a dynamic interplay of factors, including the child, the family, and the educational environment (Bronfenbrenner et al., 2006). During this process, the partnership of family and school is invaluable to children's development. This partnership provides children with opportunities to enhance their development to a higher level through the mechanism of scaffolding (Vygotsky, 1978). From this perspective, integrating the biological, cognitive, and environmental aspects through a holistic approach is beneficial for developing qualified individuals.

Previous studies suggest that brain development is influenced by cognitive and environmental factors (Westermann et al., 2010). This indicates that brain development is a dynamic and context-dependent process, where cognitive structures emerge through the interaction of environmental and biological factors. This perspective also applies to the mathematical domain. While mathematical abilities are inherent and rely on specific brain modules, children's performance in mathematics is influenced by a combination of cognitive skills, environmental context, and underlying biological factors (Butterworth et al., 2011). Therefore, biology, cognitive skills, and environmental resources play a crucial role in shaping mathematical abilities.

Children's mathematical abilities play a crucial role not only in their academic success but also in shaping their future life outcomes. In fact, early mathematical skills are strongly associated with later achievements in both reading and mathematics (Balladares et al., 2020; Davis-Kean et al., 2021; Nguyen et al., 2016; Wang et al., 2021; Watts et al., 2014). This success extends into adulthood and is influenced by both socioeconomic background and the level of socioeconomic status attained later in life (Ritchie et al., 2013). Given the strong link between mathematics and both academic and life success, it is significant to focus on developing mathematical abilities from an early age and explore the factors that influence them.

Regarding the development of mathematics, the first step begins with the concept of number sense, which is a non-symbolic but approximate understanding of quantity (Dehaene, 2011). This concept progresses to include naming numbers, recognizing their quantity, counting them in order, making comparisons between numbers, and distinguishing between more and less (National Research Council, 2009; Sarama et al., 2009; Westwood, 2021). Building on this non-symbolic foundation, the symbolic mathematical process follows (Starr et al., 2013; Sasanguie et al., 2013). Around the age of four to five, as children begin using numbers and symbols, they come to understand that addition involves combining quantities, while subtraction involves separating quantities, the opposite of addition (Harris, et al., 2017).

The transition from non-symbolic to symbolic mathematical abilities during development not only reflects cognitive evolution but also highlights the complexity of brain structures. Specifically, Dehaene (1992) defines numerical knowledge through three distinct representations: the analog magnitude code, the verbal form, and the visual Arabic form. These different numerical representations engage unique cognitive processes, which have been extensively studied in neuroscience research. Recent educational neuroscience studies have gained momentum, in understanding how mathematics is processed in the brain (see; Arsalidou, et al., 2011; Emerson et al., 2015; Hyde, 2021; Hyde et al., 2010; Szũcs et al., 2007; Zamarian et al., 2009). Understanding how mathematics works in the brain is important for comprehending children’s learning processes, which directly informs teaching methods (Howard-Jones et al., 2016). It also aids in identifying the needs of children in mathematics learning, whether or not these needs stem from impairments in the functioning of the necessary biological infrastructure, and in creating appropriate educational opportunities (Butterworth et al., 2011). Three physiological measurement tools, functional near-infrared spectroscopy (fNIRS), functional magnetic resonance imaging (fMRI), and electroencephalography (EEG), were used to answer this question (Dick et al., 2014). The first tool, fNIRS, measures oxygenation and deoxygenation levels of the hemoglobin in the brain. It is non-invasive, easy to use, and portable (Barreto et al., 2022). This tool is commonly employed in educational neuroscience studies to explore how we learn and to investigate the neural mechanisms underlying learning, going beyond behavioral data (Barreto et al., 2022). The second tool, fMRI, is another non-invasive neuro-imaging tool that requires participants to lie still in a supine position on a bed (Bartels et al., 2012). It is used to determine where information is processed in the brain and identify brain regions that work together as networks. This is achieved by analyzing the degradation time constants of hydrogen nuclei, which are influenced by the concentration of deoxyhemoglobin and brain tissue (Bartels et al., 2012). Lastly, EEG is a portable technique used to detect the temporal dynamics of the brain during information processing through direct measurement of neural activity (Dalenberg et al., 2018).

Based on the aforementioned measurement tools, studies on how mathematics is processed in the brain have highlighted specific brain areas associated with various mathematical skills. A meta-analysis emphasized that however there is an overlapping activation in the inferior parietal lobule, the numbers and calculations are processed in divergent parts of the prefrontal cortices (Arsalidou et al., 2011). Furthermore, different calculation processes such as addition, subtraction, and multiplication are reflected differently in the brain (Arsalidou et al., 2011). Notably, a review study indicated that the right parietal regions are involved in the representation of numerical quantity, while the left parietal regions are associated with the symbolic number system acquired through cultural transmission (Hyde, 2021). In addition, a study involving preschool-aged children emphasized that the left intraparietal sulcus is responsible for numerical discrimination, whereas the right intraparietal sulcus is consistently involved in number processing over a two-year period (Emerson et al., 2015). More specifically, regarding arithmetic operations, activation shifts from the frontal lobule to the parietal lobule and from the intraparietal sulcus to the left angular gyrus within the parietal region (for review, Zamarian et al., 2009). These studies demonstrate that the development of mathematical abilities in the brain evolves over time through different pathways. As the educational neuroscience literature continues to evolve and brain imaging methods have only recently been applied to young children, the way in which the brain forms networks during the development of mathematical abilities, especially in the preschool years, is not yet fully understood (Soltanlou et al., 2018; Zhan et al., 2024).

In addition to studies investigating the neurological pathways of mathematics research has also shown that mathematics performance is influenced by cognitive and environmental factors, which help explain the varying levels of mathematical ability among children (see, Butterworth et al., 2014; Silver et al., 2022; Kilday, 2011). Emerging literature suggests that executive function (for meta-analysis, Emslander et al., 2022), the home math environment, involving home math activities and parental factors (for meta-analysis, Daucourt et al., 2021), and the school math environment, including math activities and teacher-related factors, all contribute to children's mathematical abilities (see, Silver et al., 2022).

Executive function, which is a cognitive construct and an umbrella term encompassing the dimensions of working memory (holding and manipulating information), inhibitory control (suppressing the irrelevant information), and cognitive flexibility (shifting between information) (Miyake et al. 2000), contributes to mathematics by facilitating mathematical knowledge, recognizing inverse operations, and utilizing attention during operations (Cragg et al., 2014). Meta-analyses examining the overall effect of the relationship between each executive function dimension (i.e., inhibitory control, working memory, and shifting) and mathematics indicate that inhibitory control, working memory, and cognitive flexibility are important predictors of mathematical abilities (Allan et al., 2014; Cortés Pascual et al., 2019; Emslander et al., 2022; Friso-van den Bos et al., 2013; Peng et al., 2016; Yeniad et al., 2013). Focusing specifically on inhibitory control, Allan and colleagues (2014) found a moderate effect size (r = .27) and argued that this result is due to the relationship between problem-solving and inhibitory control, both of which involve the prefrontal cortex. In studies examining the general effect of the relationship between working memory and mathematics, moderate effect sizes were found (r = .37; r = .38; r = .35), and the relationship was explained by the role of working memory in the repositioning of knowledge (Cortés Pascual et al., 2019; Friso-van den Bos et al., 2013; Peng et al., 2016). Yeniad et al. (2013) conducted a study examining the overall impact of attentional shifting on mathematics. It was found that children with a high capacity to shift conceptual representations to new ones demonstrated improved mathematical abilities, with a moderate effect size (r = .26). Emslander and colleagues (2022) investigated all executive function dimensions together, and it found that the overall effect size of their relationship with mathematics is also moderate (r = .30). Given these findings, it is evident that executive function is an important cognitive factor in explaining young children's mathematical abilities. Although these studies examine the executive function as an independent cognitive factor, they also emphasize its environmental influences (Korucu et al., 2019; Soltani Kouhbani et al., 2023) and the crucial role of the environment in shaping other cognitive abilities, including mathematical skills.

Beyond biological and cognitive factors, children’s mathematical abilities are also shaped by environmental influences (Phair, 2021). Key environmental factors include language, culture, socioeconomic status, adults’ beliefs about early mathematics, and children’s engagement in math activities with adults (Silver et al., 2022). Among these, beliefs and activities play a particularly significant role in shaping children’s mathematical abilities in early childhood, with a strong connection between the two (Silver et al., 2022). As an individual factor, belief is at the core of providing enriching math experiences, as it serves as a driving force behind individuals’ behaviors (Fishbein et al., 1975). Moreover, beliefs influence child-adult interactions, instructional structure, and assessment practices, aligning with broader educational practices and ultimately impacting children’s achievement (Kagan, 1992). Therefore, beliefs and activities are not only interconnected but also key determinants of children’s mathematical abilities. Based on this, home and school environments are examined through adults’ (i.e., parents’ and teachers’) beliefs about early mathematics and their engagement in math-related activities with children.

The home math environment, encompassing family characteristics (i.e., socioeconomic status, home language) (Galindo et al. 2015; Kluczniok, 2017), parent-child activities (i.e. direct parent-child practices, which include sorting, singing, and counting) (Blevins-Knabe et al., 1996; Chan et al., 2021; Huntsinger et al., 2016; Manolitsis et al., 2013; Missal et al., 2017; Niklas et al., 2017; Pardo et al., 2020; Soto-Calvo et al., 2020; Thompson et al., 2016), and parental characteristics (i.e. educational background, beliefs, and competence) (Blevnis-Knabe et al., 2000; Claire-Son et al., 2020; DeFlorio et al., 2015; Foster et al., 2016; Huang et al., 2017; Sonnenschein et al., 2012; Zippert et al., 2020), has been shown to predict children’s mathematical abilities. The literature emphasizes that parents who engage in more mathematics-related activities with their children, have higher socioeconomic status, believe in the importance of mathematics, and possess greater mathematical competence tend to foster better outcomes in children’s mathematical performance. Taken together, these findings suggest that the home environment plays a fundamental role in shaping children’s mathematical abilities. Nevertheless, it is important to note that the home environment is not the only environmental factor influencing children's mathematical abilities; the school environment also plays a significant role (Silver et al., 2022).

The school environment is another environmental-level factor influencing children’s mathematical abilities. In this context, both the structural quality and process quality are investigated in the literature. Structural quality encompasses elements such as teacher-child ratios, program characteristics, staff characteristics, and physical structure (Harms et al., 1980). Process quality, on the other hand, refers to aspects of the daily routine, activities, and social interactions such as child-child interactions and teacher-child interactions (Pianta et al., 2005; Pianta et al., 2008). These factors have been found to be associated with children's mathematical abilities (Finders et al., 2021; Grammatikopoulos et al., 2018; Lehrl et al., 2016; Li et al., 2019; Mashburn et al., 2008; Schmitt et al., 2020; Schmerse, 2020). However, some studies suggest that children's mathematical abilities do not directly correlate with school quality (Abreu-Lima et al., 2013; Brunsek et al., 2017; Francis et al., 2019; Guerrero-Rosada et al., 2021). These differing findings are likely due to variations in how classroom quality is conceptualized and measured. For example, teacher-child interactions during instructional activities have been linked to children's academic skills, whereas teachers' emotional communication with children has been associated with social development (Mashburn et al., 2008). Furthermore, teachers' beliefs about early mathematics play a significant role in determining the impact of classroom quality on children's mathematical abilities. Teachers who recognize the importance of early mathematics tend to design their activities accordingly (Charlesworth et al., 1991; Charlesworth et al., 1993; Hofer, 2001; Muis et al., 2006; Karataş et al., 2017; Pajares, 1992, Stipek et al., 1997). Therefore, it is important to consider which specific characteristics are being evaluated when assessing classroom quality. Since this study focuses on mathematics, the aspects related to mathematics instruction and teacher characteristics are examined.

Overall, the literature highlights that early childhood is a critical period for development, particularly for brain and cognitive growth, which occurs rapidly during these years (e.g., Brown et al., 2012). Additionally, mathematical abilities developed during this time are identified as significant predictors of later academic achievement and life success in adulthood (Davis-Kean et al., 2021; Nguyen et al., 2016; Wang et al., 2021; Watts et al., 2014). Thus, investigating the determinants of mathematical abilities in early childhood becomes increasingly important. In this context, the literature explores biological, cognitive, and environmental factors to explain variations in individuals' math abilities (i.e., Butterworth et al., 2014; Silver et al., 2022; Kilday, 2011). Studies within the biological domain suggest that the parietal lobe is an important part of the brain for an individual to perform mathematical tasks (as shown in review by Zamarian et al., 2009). Additionally, executive function has been identified as one of the strongest cognitive predictors of mathematics performance (Allan et al., 2014; Cortés Pascual et al., 2019; Emslander et al., 2022; Friso-van den Bos et al., 2013; Peng et al., 2016; Yeniad et al., 2013). However, biological and cognitive factors do not function in isolation; instead, they are shaped by environmental interactions (Westermann et al., 2010). In line with this, environmental factors significantly influence mathematical abilities. Among these, the home and school environments emerge as the most impactful. When focusing on these environments, it becomes clear that the practices of adults (teachers and parents) and their beliefs play a crucial role in explaining children's mathematical skills (Silver et al., 2022). Therefore, the present study adopts an integrated approach to examine the influence of biological (i.e., hemodynamic responses in the parietal lobe), cognitive (i.e., executive function), and environmental factors (i.e., home and school environments) on young children's mathematical abilities.

1.1. The Study's Aim and Research Questions

The aim of the present research is to examine young children's math abilities by integrating biological, cognitive, and environmental factors. To this end, a model (see the proposed model in Figure 1) is developed to investigate the relationships among children's math abilities, hemodynamic changes in the parietal lobe, executive function performance, and home and school environments. The home and school environments are defined by variables related to children's exposure to direct math practices and adults' (i.e., parents and teachers) beliefs about early mathematics.

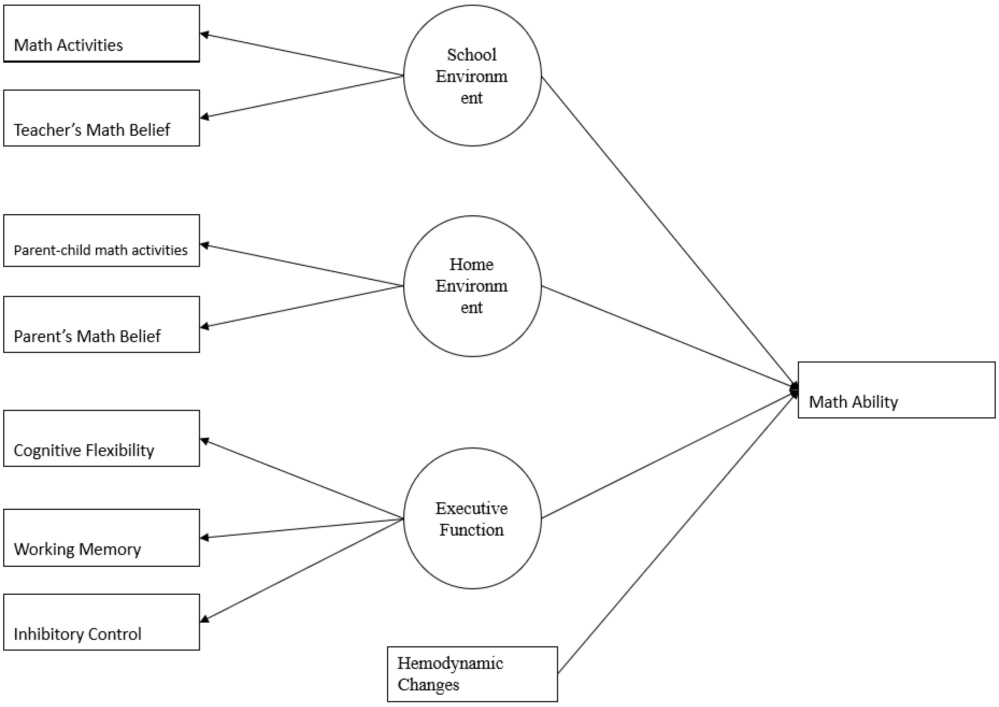


Figure 1. Hypothesized model

The proposed model includes three latent variables: school environment, home environment, and executive function. The first latent variable, school environment, consists of two observed variables: mathematical activities in the classroom and the teacher's beliefs about mathematics. The second latent variable, home environment, includes two observed variables: parent-child mathematical activities and parents' mathematical beliefs. The third latent variable, executive function, is composed of three observed variables: cognitive flexibility, working memory, and inhibitory control.

The research questions of the study, addressing the purpose, are presented below:

1. Are young children's math abilities predicted by hemodynamic responses in the parietal lobe during math tasks, their executive function performances, home math environment and school math-related environment?
    - 1.1. Which substructure of parietal lobe of young children is oxygenated during math tasks?
    - 1.2. Are young children's hemodynamic responses in the parietal lobe correlated with their mathematical abilities?
    - 1.3. Are young children's mathematical abilities correlated with their executive function performance?
    - 1.4. Are young children's mathematical abilities correlated with the home math environment (parent-child math activities and parental beliefs about early math)?
    - 1.5. Are young children's mathematical abilities correlated with the school math-related environment (classroom math activities and teacher–beliefs about early math)?

Based on the main research question and sub-questions, the following hypotheses are proposed:

H_1: Children's mathematical abilities are significantly predicted by hemodynamic responses in the parietal lobe, executive function, and the home and school environment. Specifically, higher oxygenation levels in the parietal lobe, higher executive function scores, and a math-enriched home and school environment are positively related to children's mathematical abilities.

$H_{1.1}$: The intraparietal sulcus and angular gyrus exhibit higher oxygen levels when children are performing mathematical tasks.

$H_{1.2}$: Children's mathematical abilities are positively associated with the oxygenation levels in the intraparietal sulcus and angular gyrus, such that the higher oxygenation levels in these regions are related to better math performance.

$H_{1.3}$: Children's mathematical abilities are significantly associated with their executive function, with higher executive function performances linked to better math abilities.

$H_{1.4}$: Children's mathematical abilities are significantly associated with their home environment, such that more frequent math-related activities at home and higher parental beliefs in the importance of mathematics are positively related to children's math abilities.

$H_{1.5}$: Children's mathematical abilities are significantly associated with their school math environment, such that more frequent math-related activities in the classroom and stronger teacher beliefs in the importance of mathematics are positively related to children's math abilities.

1.2. Significance of the Study

Studies have shown that the early years, especially the preschool years, play a crucial role in children's development. The first five years of life are characterized by the most rapid development of fundamental life skills (Institute of Medicine and National Research Council, 2000). During this period, high-quality early learning experiences, including mathematics provided by the child's closest context, positively predict general well-being, physical and mental health, educational attainment and employment in adulthood (OECD, 2024; Phair, 2021; Shuey et al., 2018). Moreover, these experiences help reduce inequality between children and lessen the effects of differences related to socioeconomic status (OECD, 2016; UNICEF, 2021). In addition, the latest report by the Education Reform Initiative (ERI) and the Mother Child Education Foundation (AÇEV) (2016), which aims to reveal the state of early childhood education in Türkiye and provide recommendations for future studies, reveals that the mathematics achievement of children who attend preschool, even for just half a semester, improves linearly. This finding aligns with the data from the Organization for Economic Cooperation and Development (OECD), which reports that children’s mathematics scores increase by 25 points if they attend preschool education, with SES-related factors controlled for. This alone underscores the importance of mathematics education at this age.

Moreover, a new body of literature, conceptualized as educational neuroscience, has emerged. This field investigates the sources of individual variations in learning and identifies optimal contexts for learners (Mareschal et al., 2014). It aims to support teaching and learning by exploring the brain functions underlying the learning process (Howard-Jones et al., 2016) and relies on an evidence-based approach to understand why some educational opportunities promote brain-based learning while others do not (Stern, 2005). Pioneering studies in this field, particularly those on mathematical abilities and their underlying brain mechanisms, key brain areas involved in these processes, such as the parietal lobe and frontal lobe (Arsalidou, et al., 2011; Emerson et al., 2015; Hyde, 2021; Hyde et al., 2010; Zamarian et al., 2009).

On the other hand, studies aimed at explaining individual differences in mathematical abilities have focused on cognitive factors (i.e., executive function), and environmental influences, such as direct interactions between adults and children, as well as adult’s beliefs (Silver et al., 2022; Kilday, 2011). Non-experimental research has highlighted that children’s mathematical abilities are predicted by both executive function (as shown in meta-analyses by Allan et al., 2014; Cortés Pascual et al., 2019; Emslander et al., 2022; Friso-van den Bos et al., 2013; Peng et al., 2016; Yeniad et al., 2013) and environmental factors, such as the home learning environment, the quality of schooling (especially math-related interactions), and teachers' beliefs (for reviews, see Mutaf-Yıldız et al., 2020; Soliday Hong et al., 2019).

Although brain research and cognitive-environmental factors are considered separate fields, theoretical studies suggest that they work in collaboration (see, Bickhard, 2009; Butterworth et al., 2011; Westermann et al., 2007). Therefore, a methodological and theoretical bridge needs to be established by integrating the fields of educational neuroscience, developmental psychology, and education to explain children's mathematical abilities. These factors, in the context of mathematical abilities, have yet to be integrated. While the relationship between mathematical ability and the hemodynamic responses in brain regions is often examined in isolation, key cognitive, behavioral, and environmental factors such as home environment, school experience, and executive function are typically addressed separately when discussing their influence on mathematical ability. With the integration of these two fields, the relationship between the factors predicting mathematical ability and the hemodynamic responses in brain regions has yet to be explored within a single mechanism. This study aims to bridge this gap in the literature by explaining mathematical ability through biological, cognitive, and environmental origins. Therefore, it explore how factors such as the educational environment, home environment, a child's executive function, and hemodynamic responses influence the development of mathematical ability in children.

In addition, an examination of studies in the Turkish literature reveals that research conducted within the context of educational neuroscience is relatively limited (e.g., Alkan, 2006; Coşkun, 2019; Ozcelik et al., 2009; Yılmaz, 2019). Furthermore, studies investigating the quality of early childhood education are primarily descriptive (Güçhan Özgül, 2011; Güleş, 2013; Oturakdaş, 2019; Tovim, 1996), with few examining the effects of quality on children's academic and developmental outcomes (Canbeldek, 2015). Similarly, national studies exploring the relationship between the home learning environment and children's mathematics abilities are scarce (e.g., Gürgah-Oğul, 2020; İvrendi et al., 2009; Okur-Ataş et al., 2022). Moreover, there is a notable lack of research investigating the combined influence of home and school quality on children's development within the national context. The current study aims to build on these national studies by advancing beyond descriptive research and incorporating multiple variables within a Turkish sample. In this context, it offers a valuable framework for further research that employs a multi-variable approach. The findings may also serve as an essential reference for national systems such as the Ministry of National Education (MoNE). In addition, the results obtained from this study could provide a foundation for cross-cultural studies. By presenting data on early childhood education quality, the home learning environment, home-school partnerships, and neuroscience approaches within the

Turkish context, this study contributes to comparative analyses across different countries. Finally, by examining the connections between brain, cognition, behavior, and the environment, the study offers insights into bridging physiological and behavioral data in the fields of child development and education.

Taken together, the current study aims to explore the relationship between Turkish children’s mathematical abilities and their executive function performance, as well as the influence of the home environment and school environments. It contributes to the literature by bridging the biological foundations of mathematical abilities and individual factors that affect children’s mathematical performance. In the Turkish context, the study examines the role of families in shaping children’s math abilities by investigating how the home environment impacts mathematics development. Similarly, understanding the influence of school-related factors sheds light on the role of classroom activities and teacher practices. By integrating these two critical components of a child’s immediate environment, the study investigates how to support children in achieving their full potential. Furthermore, the study combines biological, cognitive, and environmental dimensions to build a comprehensive understanding of the mechanisms linking environmental and cognitive factors to hemodynamic changes observed during children’s mathematics performance. This multidimensional approach highlights the value of assessing young children’s mathematical abilities from multiple perspectives and provides explanatory models for predicting mathematical performance. This perspective enables future researchers to study physiological, cognitive, and environmental factors in greater depth and formulate research questions that integrate these domains. By focusing on home and school environments, the study also examines how parental and teacher practices foster children’s mathematical abilities. In addition, by addressing the interaction of these two environments, understanding how these environments interact provides valuable clues about the impact of parent-teacher collaboration on young children’s mathematical development. The study’s findings inform educational programs, family guidance services, and R&D efforts at the policymaking level. For instance, math-related classroom practices, instructional strategies, and educational goals can be reconsidered in this context. The neurodevelopmental aspects of education are also explored, and this opens the door to projects that investigate the impact of education on children’s neural connections. Additionally, guidance services develop parenting programs aimed at enhancing early mathematical skills, fostering positive beliefs toward mathematics, and improving parental support for their children’s mathematical development.

1.3. Definitions of the Terms

Belief: It refers to the object-attribute relationship established by a person, which forms the basis of attitudes, intentions and therefore behaviors that differ in severity from person to person (Fishbein et al., 1975).

Math ability: According to Cambridge Dictionary (2022) the term ‘ability’ refers to “*the physical or mental power or skill needed to do something*”. In the context of the present data collection tool, ability is assessed using a criterion that considers children’s performance in mathematics, including both formal and informal components of mathematics, as well as age-appropriate expectations (Ginsburg & Baroody, 2003).

Executive function: This refers to the ability to coordinate thought and action and to direct them toward the pursuit of goals (Miller et al., 2009), encompassing working memory, inhibitory control, and cognitive flexibility (Miyake et al., 2000).

Hemodynamic responses: These are changes in the physiological blood circuitry of the brain (APA Dictionary of Psychology, 2024).

Home math environment: This encompasses direct and indirect interactions between parents and children in respect to mathematics, spatial activities, parental attitudes, beliefs, and expectations about mathematics, and “math talk” (Daucourt et al., 2021).

fNIRS: Functional near-infrared spectroscopy (fNIRS) is a neuroimaging technology used to map the functioning human cortex by measuring oxygenation and hemodynamic changes, based on the principles of near-infrared (NIR) spectroscopy (NIRS) (Ferrari et al., 2012).

CHAPTER 2

LITERATURE REVIEW

This section reviews the literature on math abilities of young children, focusing on biological, cognitive, and environmental factors studied to date. In this context, the theoretical underpinnings of the study, math abilities in early childhood education, brain imaging studies examining these abilities, and on executive functioning, as well as home and school environments related to math abilities are presented respectively.

2.1. Theoretical Background

This research aims to integrate biological, cognitive, and environmental factors in the context of children's math abilities, and the theoretical background has been specified accordingly. With an integrated approach to explaining young children's math abilities, this study is grounded on five theoretical frameworks that incorporate biology, cognition, and environment. The first framework is Urie Bronfenbrenner's Bioecological Model, followed by Mark Bickhard's Interactivsm, and Annette Karmiloff-Smith's Neuroconstructivism. Additionally, the two models specified for neuroscience and math are introduced. In this context, Stanislas Dehaene's Triple Code Model, and Butterworth and colleagues' Causal Model are presented. The Bioecological Model explains how the environment influences human development within a systemic framework, while Interactivism and Neuroconstructivism focus on the relationship between the brain and cognition and the environment. The last two models are neuroscience-based models and focus directly on mathematics. These five theoretical underpinnings are described separately below. The final section provides a synthesis of the five models and examines how they relate to the present study.

2.1.1. Bioecological Model

This model, based on continuity and change over the lifespan and across the generations in human biopsychological characteristics, explains the role of the properties of process, person, context, and time in this evolution (Bronfenbrenner et al., 2006). It focuses on the interrelation and embodiment of these properties (Bronfenbrenner, 1993). In this way, nested systems are created with the individual at the center. These systems are modeled as a microsystem, mesosystem, exosystem, macrosystem, and chronosystem (Bronfenbrenner, 1994). In other words, this model, centered around the child, explains the influences of the environment on children's development, from the most immediate circumstances to more distant areas, by enhancing relationships among the layers (Bronfenbrenner, 1994; see Figure 2).

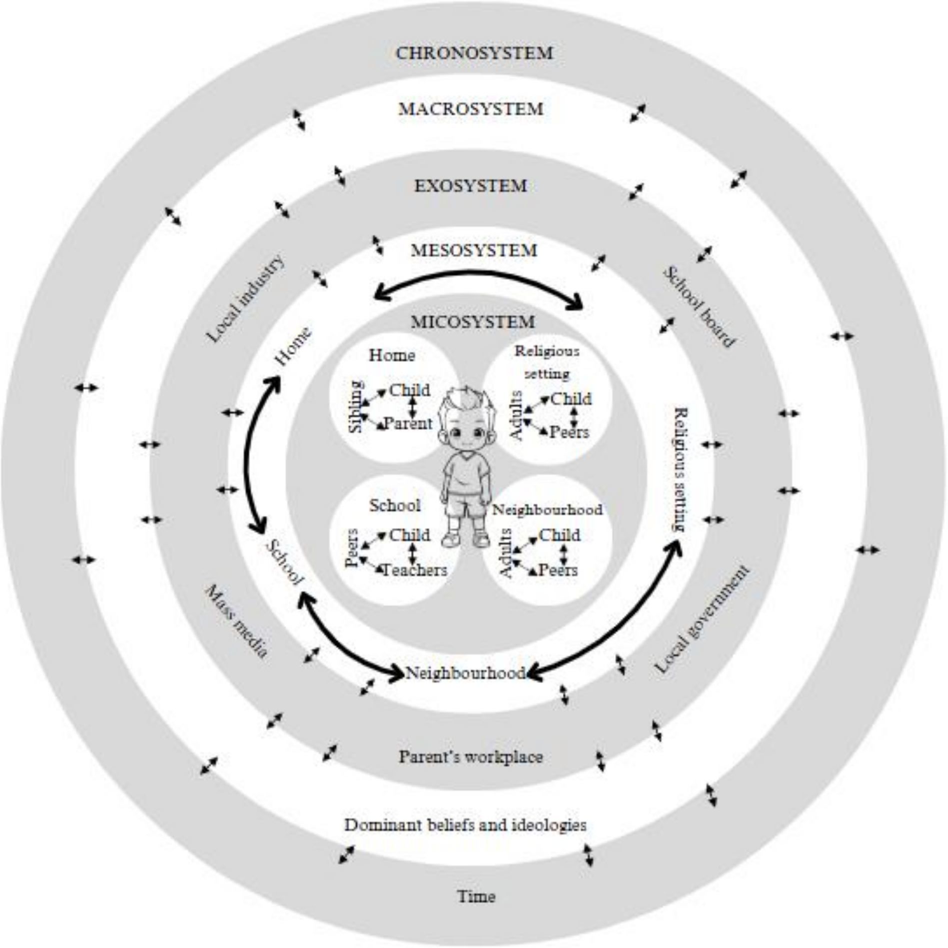


Figure 2. The Ecological Systems Theory of human development **(**received and adapted from Härkönen, 2008)

The microsystem comprises activities, social roles, and interpersonal relationships that the developing individual experiences in a face-to-face milieu. These have physical, social, and symbolic characteristics that enable, allow, or inhibit the individual from engaging in continuous and increasingly complex interactions with their proximal environment (Bronfenbrenner, 1979; 1994). Specifically, this system includes settings such as family, school, peer groups and workplace. The mesosystem, or the system of the microsystem, involves the connection of two or more milieus, such as home, school, or workplace. Specifically, it refers to these milieus' role in the development of the individual. This includes the developmental influence of the engagement and two-way communication between parents and teachers for the benefit of child development, as proposed by Epstein (Bronfenbrenner, 1994; Epstein, 2010). The exosystem includes relationships and processes in the convergent environment, such as the social environment of the family or neighborhood-community contexts. These relationships do not directly affect the individual’s development. For example, the influence of a parent's work life on the home environment can be considered (Bronfenbrenner, 1994). The macrosystem is the social schema of culture that embodies the cultural characteristics of the micro, meso and exosystems, particularly belief systems, knowledge, physical resources and lifestyle (Bronfenbrenner, 1994). The chronosystem refers to the time dimension and its effect on children’s development. This dimension encompasses particular forms of interaction between the organism and the environment, referred to as proximate processes, which operate over time and are assumed to be the primary mechanisms that produce child development (Bronfenbrenner et al., 2006).

Bronfenbrenner's theory provides a crucial foundation in the context of education (Tong et al., 2024). It is particularly significant in early childhood education, as it considers children's development not only in terms of individual factors but also within the broader environmental context in which they grow (Navarro et al., 2020; Tudge et al., 2017). Specifically, the “Process-Person-Context-Time” dimensions emphasized in the model highlight that proximal processes, which are the activities through which children interact with individuals, are central to their development, with these interactions being described as the “primary engines of development” (Xia et al., 2020). In the classroom setting, these processes involve children’s interactions with teachers and learning materials, while activities with parents at home help prepare children for these interactions in the classroom (Tudge et al., 2017).

Since the present research focuses on the home and school environments in the context of children’s math abilities, the microsystem at the core of the bioecological model corresponds to the link between children's development and immediate environments. Operationally, the bioecological model sheds light on the current study–by explaining the role of the most crucial circumstances, such as home and school, in children’s math abilities and emphasizing the interaction of children with their immediate environment. In other words, the nature of the child and its interaction with environmental conditions are key components of this study, with the microsystem forming one of the foundational elements of the theoretical background by capturing unique variables related to these cornerstones.

2.1.2. Interactivism and Neuroconstructivism

This section explains two cognition theories focusing on the environment-organism relationship to clarify how cognitive factors and brain activity are constructed. The first theory outlines the role of interaction in the general development of the mind, while the second theory is more specific, emphasizing interactive and continuous development by addressing multiple elements, such as genes, neural mechanisms, and the environment.

2.1.2.1. Interactivism

Interactivism is a complex system of theories rooted in strict naturalism, related to Pierce’s model of representation and Piaget’s genetic epistemology. In naturalism, reality is neither independent nor isolated; rather, the world consists of a thinking entity and an extended entity, as in Cartesian thought. In line with naturalism, this theory adopts a process-based conceptualization based on the principle that the world is based on fundamental process organizations. These processes are not deductive; instead, they involve nested patterns of hierarchy or constraint, beginning with what is considered and gradually deepening and specializing. Interactivism addresses various mental and social phenomena, including learning, emotions, consciousness, language, perception, memory, motivation, neural realizations of mental phenomena, the nature and emergence of social reality, human sociality and the social ontology of the person, development, personality and psychopathology, and rationality (Bickhard, 2009).

2.1.2.2. Neuroconstructivism

Neuroconstructivism provides more detailed information in explaining the neural structures of the developing brain based on multiple interactions, compared to interactivism. According to this theory, the brain's organization is shaped through interactions with the environment rather than being solely determined by genetics. The consistent emergence of specialized brain functions is primarily driven by shared experiential structures and certain inherent biases in the brain's receptivity to different–types of information (Westermann et al., 2010). Central to cognitive development is the understanding of the complex relationships between constraints emphasized in the neuroconstructivist approach, including the interaction between experience and genes, experience-dependent amplification of small-scale neural structures, the interaction of different brain regions in the construction of functional brain development, embodiment, proactive knowledge acquisition, social environment (see Figure 3, Westermann et al., 2007).

In detail, this figure presents four constraints (i.e., genes, body, environment, and brain) and their relationships in understanding the neural structure underlying cognitive development. Based on this the genes shape the first structure of the brain but they consistently change by environmental stimuli and experiences. For example, the environmental experiences or behaviors can trigger the gene expression. Thus, the genes do not only serve as a determinant of baseline but also show flexibility as a response of the environmental stimuli. Followingly, body plays the role of a filter and tool for the interaction. It is seen as a filter due to the restriction of the environmental information by the sensory organs. However, it creates experiences via interaction with the environment and these experiences trigger neural activity. On the other hand, the environment affects the emerging neural networks in social contexts such as social interactions and stimuli in the physical world. As well as it can also directly or indirectly affect neural representations, such as the experiences of a child in the social context can change the gene expression or the density of the neural activity. Lastly, the brain develops in a loop of multiple feedbacks, and it is affected by the other brain regions and environmental stimuli. The different brain regions (such as X and Y) collaborate to develop the functions of them. In addition, the neural activity creates a novel representation, and these continuously interact with genes, the environment, and the body.

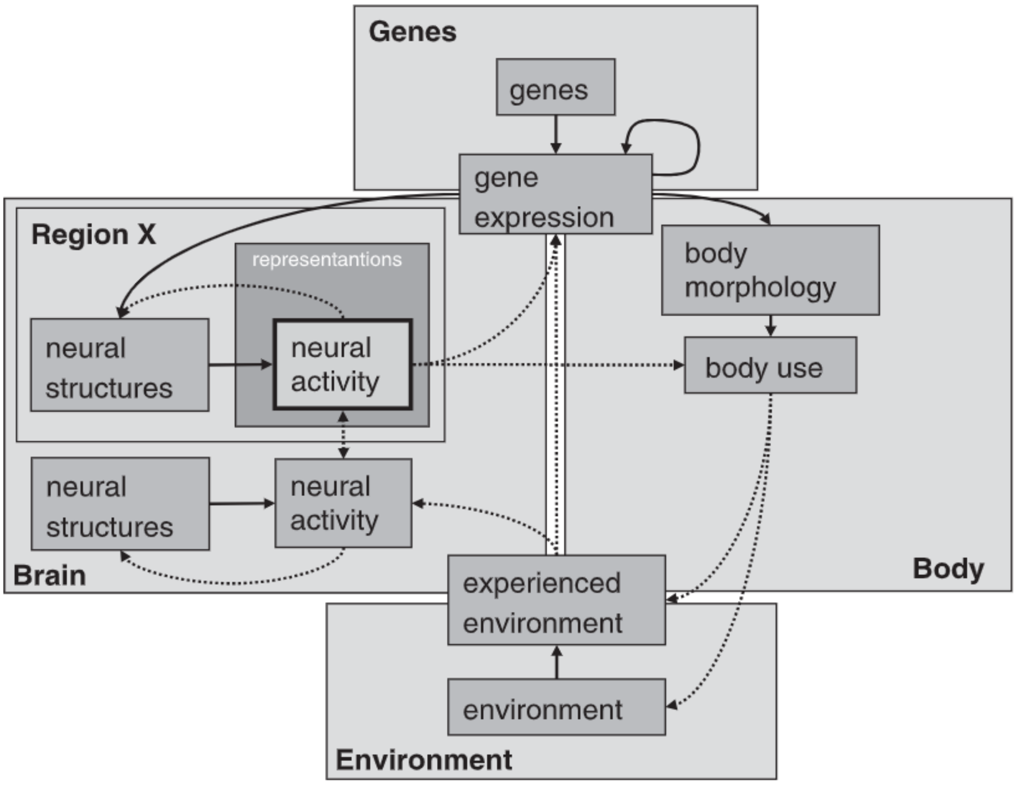


Figure 3. Neuroconstructivism: Multiple interactions among constraints **(**received from Westermann et al., 2007)

2.1.3. The Theoretical Models Specified for Neuroscience and Math

Thus far, the influence of environmental and other factors on the development of cognitive and biological structures has been discussed in broad terms. However, the specific context of mathematics remains unexplored. When examining studies that explore the development of math abilities as a cognitive skill are examined, the current study addresses two models. The first model, the Triple Code Model (Dehaene, 1992), emphasizes numerical abilities, while the second model is a causal framework that encompasses all arithmetic skills and explores their interrelationship with biological, cognitive, behavioral, and educational contexts (Butterworth et al., 2011).

2.1.3.1. Triple Code Model

The Triple Code Model, a leading approach in numerical cognition, was suggested by Dehaene (1992). Overall, this model explains that numbers are mentally represented by three different but communicated codes (see, Figure 4). According to this model, the auditory-verbal code (auditory verbal word frame), the visual Arabic code (visual Arabic number form), and the analog magnitude code are used to represent numbers in the human mind. This model emphasizes that mathematical abilities can be represented in the pre-verbal stage as shown in the analog magnitude representation code. Also, with the processing of language, numbers can be expressed and recognized by the auditory-verbal code. Furthermore, numbers can be represented as symbolic notation in the visual Arabic code. These three codes are associated with different cognitive processes but transform each other's representation. The pathways in Figure 4 represent complex transformations involving syntactic and semantic rules. Detailed information about the codes is presented below.

1. The auditory verbal code, generated and manipulated through general-purpose language modules, involves the mental manipulation of numbers similar to word sequences. This code includes both written and auditory inputs, with outputs in writing and speaking for the numbers. Therefore, it is important for counting and early mathematical operations.
2. The visual Arabic code refers to the processing of numbers in Arabic in a spatially extended representational environment. Reading numbers serves as the input of this code, while writing numbers is the output. During the number-reading process, number sequences are categorized for visual representation, whereas in the writing process, the code converts the writing

gestures into a motor program. This code is important for multi-digit operations and equality.

3. In the analog magnitude code, numerical quantities are naturally represented as variable activation distributions on a directed analog number line. It assumes input from visual numerosity estimation and subitizing recognition. This code is important for comparison and approximation.

Considering the inter-code communication represented by the arrows in Figure 4, three pathways are identified: A-B, C-D and C'-D' (Dehaene, 1992). The A-B pathway represents the verbal sequence corresponding to the digit representation, including syntactic organization and lexical retrieval, and vice versa (Dehaene, 1992; McCloskey et al, 1986). The C and C' pathways provide access to the quantity code from both numerical and verbal representations by approximating the input digit and activating the number line (Dehaene, 1992). The D and D' pathways return the name of the approximate number for the given quantifier by operationalizing the numerical and verbal categories assigned to specific lengths on the number line (Dehaene et al, 1992).

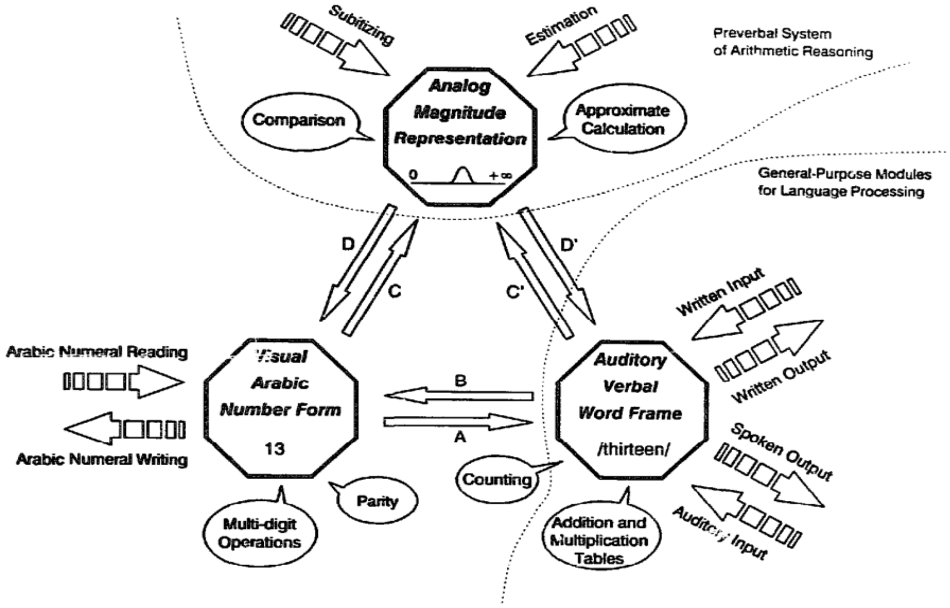


Figure 4. Triple-code Model’s schematic representation (received from Dehaene, 1992)

This model is supported by empirical studies, which show that different math abilities interact with distinct codes and communicate within the brain. For example, while exact arithmetic skills are linked to language-specific processes, they also establish a network with word-association mechanisms. In contrast, approximate arithmetic focuses on magnitude representation independent of language and involves visuo-spatial processes in the bilateral regions of the parietal lobe (Dehaene et al., 1999). Brain imaging studies have further shown that the codes for approximate numbers, dots, digits, and number words are activated in the horizontal segment of the intraparietal sulcus (Piazza, et al., 2007).

2.1.3.2. The Causal Model for Mathematical Development

The Triple Code Model (Deheane, 1992), one of the pioneering models in the field, emphasized the transitive relationships between mathematical abilities, starting from pre-linguistic math abilities to arithmetic, and highlighted the biological basis of these abilities, particularly in the parietal lobe. A more recent study by Butterworth and colleagues (2011) expanded this framework and developed a model that addresses the relationships of math abilities at different levels (see Figure 5). In this model, math abilities are explained based on scientific findings across three levels (i.e., biological, cognitive and behavioral) and one environmental factor (i.e., educational context).

In the first level of the model, the biological layer, genetics, and specific brain regions (i.e., the fusiform gyrus in the occipitotemporal lobe, the intraparietal sulcus, and angular gyrus in the parietal lobe, and the prefrontal cortex in the frontal lobe) are considered. Based on research on brain areas, it has been explained that there shifts occur between the frontal lobe, parietal lobe, and occipito-temporal lobe when the complexity of math abilities and the executive functions that need to be utilized are taken into account. Biology is thus explained in the model as the substructures in which mathematics is processed.

The second level describes two fundamental math abilities: numerosity representation and manipulation, and spatial abilities, along with the sub-mathematical abilities derived from them as the cognitive layer. Spatial skills, the first mathematical ability, are explained in terms of their relationship with concepts, principles and procedures. Subsequently, number symbols, the recall of arithmetic facts, and the concepts, principles and procedures associated with numerosity representation and manipulation, which are also influenced by spatial abilities, are addressed within the cognitive layer. It is further explained that this ability is directly linked to simple number tasks at the behavioral level.

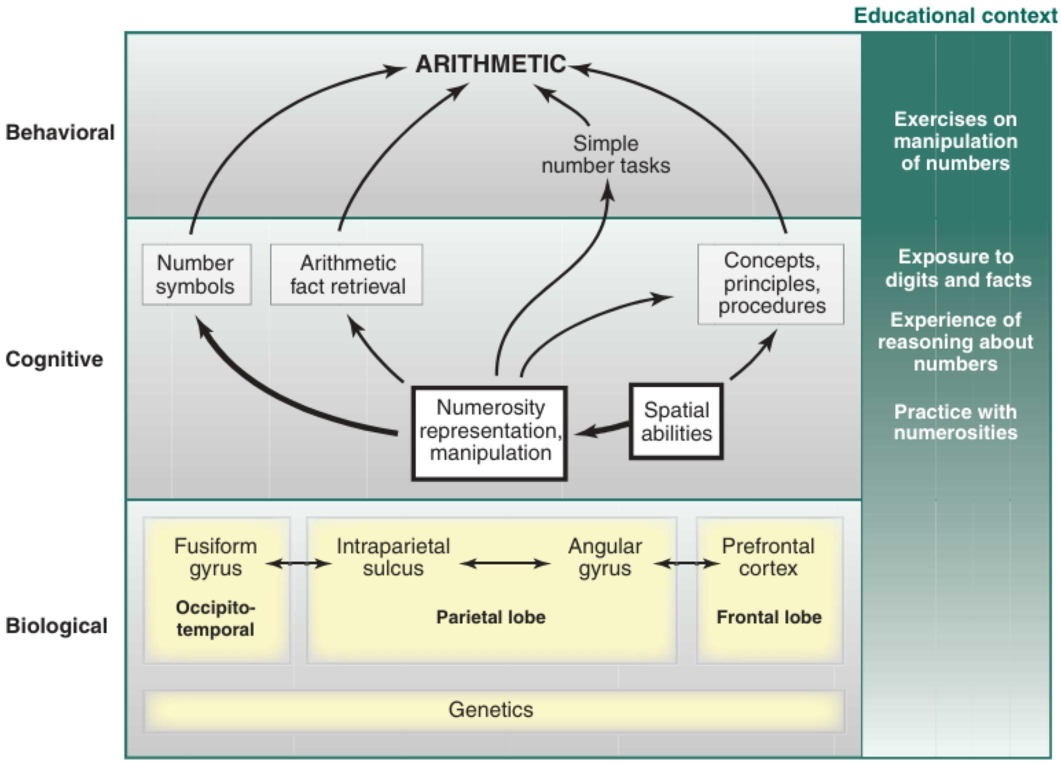


Figure 5. The causal model suggests potential connections between basic behavioral, biological, and cognitive levels

(received from Butterworth et al., 2011)

The third level directly addresses behavioral arithmetic production. This level presents how arithmetic and number task performances are transformed into behavioral outputs through the transitivity of cognitive skills. At this level, the simple number tasks are directly related to the cognitive ability of numerosity representation and the manipulation.

The only environmental factor included in this model is the educational context. This context is explored through the application of math abilities, exposure to numerical and factual elements, experiences with number reasoning, and practices involving numerosities. All of which highlight the areas that educational scientists should prioritize.

2.1.4. Synthesizing the Models Within the Scope of the Current Study

Taking into account these backgrounds, which can be interpreted as the interdisciplinary product of education, psychology, and cognitive sciences, it becomes evident that an individual's development occurs within the framework of various interconnected contexts. This thesis focuses on children’s math abilities in the context of biological, cognitive, and environmental factors. In particular, the study is closely aligned with Bronfenbrenner's Bioecological Model, as it explores how children's math abilities are influenced by their home and school environments. In this model, the home and school contexts, along with parents and teachers, are identified as key convergent factors in an individual’s development. Therefore, this model provides valuable insight into the core environmental sources that shape children’s math abilities.

On the other hand, this thesis examines how changes occur in the hemodynamic structure of the brain during mathematical operations. Numerical cognition and educational neuroscience specifically study the changes in brain structure associated with mathematical processes. According to educational neuroscience field, two models, the Triple Code Model and the causal model, indicate that specific brain regions are responsible for specific math abilities and interact with each other during mathematical processes. Variations in brain activity are influenced by an individual's level of competence. According to the “neural efficiency hypothesis” (Haier et al., 1988) individuals with higher cognitive abilities lower levels of brain activation, while those with lower cognitive abilities exhibit increased activation. From a biological perspective, math abilities are processed within interconnected brain regions, with the individual’s cognitive capabilities determining the extent of biological resources required for this process.

Lastly, current thesis examines the relationship between math abilities and executive function, a cognitive ability, as well as the neural basis of mathematical processes in specific brain regions. The study aims to explain how mathematics is associated with both children's executive function, home and school environments and how these associations relate to the hemodynamic responses in the brain. The interactivism approach and neuroconstructivism, as outlined earlier in this section, emphasize that cognitive processes are not merely outcomes of an individual's characteristics or heredity; rather, these processes are significantly shaped through interaction with the environment. In line with this perspective, the thesis integrates insights from three models and two theories: Bronfenbrenner's Bioecological Model, Bickhard's Interactivism Theory, Karmiloff-Smith's Neuroconstructivism, Dehaene's Triple Code Model, and the causal model introduced by Butterworth and colleagues.

2.2. Mathematical Abilities in Early Childhood

Mathematics, in its earliest sense, is characterized by the quantification of measurable aspects of the physical world and the number symbols created though their mental representation (Sophian, 2007). In the early years, children demonstrate fundamental mathematical concepts, such as the approximate number system, counting (e.g., one, two, three), quantity (e.g., more, less), shapes (e.g., triangles, squares, circle), spatial relationships (e.g., above, below), measurement (e.g., lenght, size), and patterns (e.g., ABAB) (Copley, 2000; National Research Council, 2009; Sarama et al., 2009). These abilities develop not only within formal school contexts but also outside of school through informal learning experiences (Geary, 1994). This section of the thesis systematically examines young children's math abilities, beginning with foundational constructs such as the approximate number system and subitizing, and progressing to more advanced arithmetic operations.

2.2.1. Approximate Number System (ANS)

Math abilities begin to emerge as early as infancy. Research has shown that, even at this early stage, humans exhibit sensitivity to quantitative differences (see, Gao et al., 2000). Prior to the development of language, the primary cognitive mechanism for estimating the cardinal value of a set of objects is conceptualized as the Approximate

Number System (ANS) (Gallistel et al., 1992). The ANS is intricately linked to the concepts of number sense and subitizing, as described in the literature.

The brain is specialized for number processing and the mechanism that enables organisms to perceive the cardinal value of a set of objects through sensory perception–referred to as number sense (Dehaene, 2011). In addition, prior to the development of language, infants possess an innate ability to quickly recognize the quantity of a group of objects, known as subitizing, which serves as the foundation for the acquisition of numerical knowledge (Sarama et al., 2009). Clements (1999) distinguishes two types of subitizing: perceptual and conceptual. Perceptual subitizing refers to the ability to determine the exact number of objects in a group without mathematical processing, whereas conceptual subitizing involves the awareness of the number by recognizing the pattern of combination in the parts of a whole (Clements, 1999).

In general, ANS, which refers to the immediate understanding of quantity before the use of language, forms the basis for concepts such as cardinality, part-whole relationships and addition (Clements et al. 2009). In children, this skill emerges prior to the ability to count and forms the basis for the understanding of the concept of number (Feigenson et al., 2004).

2.2.2. Counting

Around the age of two children become able to count (Geist, 2018). In this part children’s counting abilities are presented. Specifically, the verbal counting, object counting and principles for counting are demonstrated.

Counting is one of the primary tool children use to develop the concept of numbers and other related mathematical concepts (Baroody et al., 1998). It involves the ability to order objects using a sequence of numeral names and understanding that the final numeral represents the total cardinal value of the set (Clements et al., 2007; Gelman et al., 1978; Sarnecka et al., 2008). There are three basic skills involved in counting: verbal counting, object counting and comparing quantities (Baroody et al., 1998).

Verbal counting involves reciting the counting sequence (e.g., one, two, three...), and, when children count forward verbally, they must achieve certain abilities. Such as they need to memorize the single-digit sequence from one to nine. Then they need to recognize the pattern, which indicates that nine is in the last and needs to initiate the new series. They need to master the decade that starts each new series while memorizing the terms (i.e., ten, twenty, thirty…) and recognizing the pattern (i.e., adding -ty to the end of digits; sixty, seventy, eighty….). Finally, they need to comprehend the pattern where each new series incorporates combinations of decades and digit sequences (e.g., thirty, thirty-one, thirty-two….) (Baroody et al., 1998).

Object counting involves determining the total number of items in a series (enumeration) and counting a set of selected objects (set production) (Baroody et al., 1998). To perform this accurately, children must recognize the relevant section of the sequence for counting. They must assign a single number word to each object, and also keep track of which objects have been counted and which remain to be counted (Gelman et al., 1978).

Although counting comprises various skills, Gelman and Gallistel (1978) explain the general principles of “how to count” (i.e., the one-to-one correspondence principle, stable order principle, and cardinal principle) and “what to count” (i.e., order-irrelavence principle and abstraction principle) as five points. The one to one correspondence principle refers to each individual item in a set should be assigned one and only one number. The stable order principle refers to the digits used in counting should follow the same order in every count. The cardinal principle refers to the number attached to the last item in the group represents the number of items in the set. The order irrelevance principle refers to the order in which items are counted does not affect the final count. The abstraction principle refers to understanding everythings can be counted.

2.2.3. Arithmetic Operations

Arithmetic, derived from the Greek word *arithmos* meaning ‘number’, refers to the solution of problems involving numbers and quantities. The combination of at least two numbers to produce a third number is known as an operation (Gladle, 2015). These arithmetic operations, primarily addition and subtraction, which expand from non-verbal operations at ages 2-3 to verbal problems by ages 4-5 and number problems by age 7 (Clements et al., 2004).

Addition refers to combining two different sets of objects with no common members to form a union of the two sets, with the cardinal number of this new set being calculated (Haylock et al., 2008). Geary (1994) explains young children's strategies for simple addition operation as follows:

Counting manuplatives: Objects represent the augend and addend of the problem. The objects are then counted, starting from 1. For example, to solve 3 + 4, three blocks are counted aloud first, followed by four more blocks, resulting in a total of seven blocks.

Counting fingers: Fingers represent the addend and augend of the problem. The child counts the fingers, starting from 1. For example, to solve 3 + 4, three fingers are raised on one hand, and four fingers on the other hand. The child then moves each finger as they count them.

Verbal counting / counting all (sum): The child counts each augend and addend sequentially, starting from 1. For example, to solve 3 + 4, the child counts aloud: “one, two, three, four, five, six, seven,” with the final sum being seven.

Verbal counting / counting on first: The child counts until the total number of counts equals the value of the addend after first stating the value of the addend (e.g., to solve 3 + 4, the child counts “three, four, five, six, seven,” then concludes the answer is seven).

Verbal counting / counting on larger (min): After stating the value of the larger addend, the child counts until the value of the smaller addend is reached (e.g., to solve 3 + 4, the child counts “four, five, six, seven,” then concludes the answer is seven).

Derived facts (decomposition): achieve a sum of 10, one of the addends is decomposed into smaller numbers, which are then combined with the other addend. This method involves two addends. For instance, to solve 9 + 7, the first step is to decompose 7 into 6 + 1. Then, 9 is added to 1, resulting in 9 + 1 = 10. Finally, 10 is added to 6, resulting in a total of 16.

Fact retrieval: This strategy involves directly retrieving fundamental facts from long-term memory. For example, recalling that 3 + 4 equals 7 is an instance of fact retrieval based on memorization.

Subtraction is defined as the opposite of addition (Clements et al., 2009) and refers to the process of partitioning or taking away a certain amount of objects from a set (Haylock et al., 2008). Geary (1994) explains young children’s subtraction strategies in simple addition operations as follows:

Manipulatives / separating from: The object represents the value of minuend. The subtrahend's representation is removed from the set. The remaining objects represent the solution. For example, to solve 4 - 2, four blocks are counted aloud, and two are removed. The remaining blocks (two) represent the answer.

Manipulatives / adding on: The objects represent the values of the subtrahend. Additional blocks are added until the minuend’s value is reached. The number of blocks added to the subtrahend represents the solution. For example, to solve 4 - 2, two blocks are counted aloud, and two more are added while the child counts, “one, two,” to reach the answer.

Manipulatives / matching: One row of objects represents the minuend and another represents the subtrahend, with a one-to-one correspondence between the items. The number of unmatched objects indicates the solution. For example, to solve 4 - 2, one row consists of four blocks and another row of two blocks. The two unmatched blocks represent the answer.

Counting fingers: The correct number of fingers is raised to represent the minuend. Fingers corresponding to the subtrahend are folded down. The remaining raised fingers indicate the solution. For example, to solve 4 - 2, four fingers are raised, and two are folded down. The remaining raised fingers (two) are counted to get the answer.

Verbal counting / counting up: The child counts up from the subtrahend to the minuend. The number of counts represents the answer. For instance, to solve 4 - 2, the child counts “three, four” to reach the answer.

Verbal counting / counting down: The child counts from the minuend until the subtrahend’s value is reached. The number of counts represents the answer. For example, to solve 4 - 2, the child counts “three, two” to find the answer.
Retrieval: The child retrieves the answer from long-term memory. from long-term memory. For instance, recalling that 4 - 2 equals 2 without the need for manipulatives or counting.

2.2.4. Development of the mathematical abilities

Children exhibit a variety of math abilities from an early age, as detailed above. This process begins with approximate quantitative knowledge in infancy and progresses to foundational mathematical skills such as counting, number recognition, and arithmetic operations utilizing both verbal and symbolic systems. While each skill develops–through specific stages and principles, there is an overarching pattern of progression among these skills when viewed from a broader developmental perspective. Geist (2018) outlines the general developmental stages of math abilities in children, and this emphasizes a consistent and sequential progression.

Focusing on the first three years of life, children display a range of mathematical abilities, from gaining object permenance to sorting objects based on their features. Between 6-12 months, they acquire object permanence (conceptualized by Piaget, 1954), understanding that objects exist even when out of sight and develop distance judgment to distinguish between near and far. Between 12-18 months, children demonstrate the concept of “more”, recognizing without explanation that a group of five objects is greater than a group of three, and begin matching objects by color and shape. From 12-24 months, children use the concept of “more” to compare quantities and measure quantities. They can also group identical objects, an ability known as sorting. Between 18-24 months, children make multiple classifications by grouping objects based on one or more characteristics, though not all characteristics simultaneously. They can also recognize, replicate, and generate patterns. Between 24-30 months, children sort objects based on arbitrary or non-arbitrary properties, a skill known as sequence. Between 24-36 months, children demonstrate one-to-one matching, pairing one object with another. Additionally, they comprehend that objects have discrete quantities and can be counted. They also count by rote, even if the sequence is not always correct. At this stage, children can build towers using objects of different sizes and make size comparisons, such as identifying which objects are bigger or smaller. Between 30-36 months, children begin sorting objects from small to large based on numerical order, a skill known as seriation (Geist, 2018).

As children develop, their abilities become more specific, expanding to include counting to arithmetic operations and measurement. Between 3-4 years old, children can count a group of up to four objects. They understand that the last number indicates “how many,” even though their numerical understanding remains limited. While they grasp sequence, they do not fully comprehend quantity. At this stage, children begin associating the names of two- and three-dimensional shapes with their corresponding dimensions and orientations. They can identify and describe measurable characteristics of objects, such as length and weight. For example, they can replicate the length an object using a stick longer than the object but are unable to do so with a shorter stick. They sort items into groups and are able to count and compare the groups they create (Geist, 2018).

Between $4\text{-}4^{1/2}$ years old, children can add by “counting all,” which involves counting each number separately (e.g., for 3 + 4, they count “one, two, three” and then “four, five, six, seven”). They can recognize two-dimensional and three-dimensional shapes despite changes in size or orientation (e.g., a triangle is still a triangle even if it is upside-down). They begin measuring using non-standard units. They may need several of these units, but they can use an object that is shorter than the thing they want to measure (e.g., counting the length of a table with five shoes). They also start creating classifications based on intricate features (e.g., “things that cut,” which might include knives, scissors, and saws). Between $4^{1/2}$-5 years old, children develop the ability to “count on” during addition tasks. For instance, when rolling two dice, if one shows three dots and the other shows four, they look at the first die and say “three,” then count on with the second die, “four, five, six, seven.” They can create new shapes by combining existing ones, such as with tangrams. Children also begin measuring objects using standard tools like rulers and apply "unit iteration," such as measuring a 3-foot desk with a 1-foot ruler. Lastly, they start arranging and presenting data using basic numerical displays, such as bar graphs, and can count the items within each group (Geist, 2018).

2.2.5. Summary

This section focuses specifically on early childhood and explains the development of young children's mathematical abilities, how mathematics emerges and progresses, the mathematical skills children exhibit, and the general principles and strategies behind these abilities. These compiled sources show that mathematics develops both in school and outside of school (Geary, 1994) and begins with the formation of non-verbal, non-numerical and non-symbolic quantity concepts and gradually progresses as children acquire number symbols and develop arithmetic operations (Dehane,2011; National Research Council, 2009; Sarama et al., 2009; Sasanguie et al., 2013; Starr et al., 2013; Westwood, 2021).

So far, models and theories explaining mathematical skills have been described to identify the various math abilities exhibited by young children. Taken together, it observed that the math abilities that develop both within and outside of the school context, interact with different variables (such as biological, cognitive, and environmental factors) and are the focus of this study. Therefore, the following sections examine these factors in relation to mathematics, based on the existing literature. The next section presents brain imaging studies that contribute to understanding mathematics and explains how mathematical processes function in the brain. Then the literature related to association between children’s math abilities, executive function, home math environment, and school math-related environment are presented.

2.3. Neuro-imaging and Key Studies

The structure of mathematics in the brain has been studied across various disciplines, particularly cognitive neuropsychology, psychology, neuroscience, and educational neuroscience. Theoretical research in these fields indicates that mathematical skills elicit responses in specific brain regions, and the strength of these responses varies depending on individuals' mathematical performance and contextual factors (Bickhard, 2009; Butterworth et al., 2014; Dehaene, 1992; Westermann et al., 2007). Based on this, this section presents two major issues. The first one is the brain imaging techniques, which are used in the educational studies and then the brain imaging studies related to mathematics. These studies involve human participants and use techniques such as functional magnetic resonance imaging (fMRI), functional near-infrared spectroscopy (fNIRS), and electroencephalogram (EEG) to explore changes in the brain's physiological structure associated with math abilities.

These techniques have distinct characteristics but share the goal of examining the relationships between nervous system structures and processes to better understand human cognitive functions. Specifically, EEG measures the brain’s electrical activity by applying electrodes to the scalp. It records signals with high temporal resolution, typically measured in milliseconds (Dalenberg et al., 2018; Xu et al., 2018). This electrical activity is thought to reflect the synchronized activity of numerous neurons located in the upper layers of the brain's gray matter, which are geometrically parallel to one another (Buzsaki et al., 2012). EEG is frequently used to study rapid neurological processes, such as oscillatory brain activity and event-related potentials (ERPs) (Dalenberg et al., 2018; Xu et al., 2018). Due to its non-invasive nature, affordability, and portability, EEG is particularly suited for real-time monitoring. However, it can be affected by external electrical interference and artifacts from muscle activation (Dalenberg et al., 2018; Xu et al., 2018). EEG has been applied in educational studies examining motor skill acquisition, reading, mathematics, programming, and physics (Xu et al., 2018).

Similarly, fMRI is another non-invasive neuroimaging tool, although it requires participants to remain static, lying in a supine position (Bartels et al., 2012). fMRI uses strong magnetic fields to visualize biological tissues and detect neural activity by measuring the blood oxygenation level-dependent (BOLD) signal in brain vessels (Huettel et al., 2009). This technique is employed in educational research to determine where information is processed in the brain and which parts function together as a network. It considers degradation time constants of hydrogen nuclei influenced by the concentration of deoxyhemoglobin and brain tissue (Bartels et al., 2012).

Compared to EEG and fMRI, fNIRS is a more recent technique (Boas et al., 2014). It measures the oxygenation and deoxygenation of hemoglobin in the brain using non-invasive, portable, and user-friendly features (Barreto et al., 2022). Neural activation increases oxygen and glucose levels in the blood vessels surrounding the activated neurons (Phillips et al., 2016). fNIRS indirectly measures these changes through near-infrared light. Additionally, fNIRS is used in educational neuroscience studies to investigate questions beyond behavioral data, such as understanding the neural mechanisms underlying learning (Barreto et al., 2022).

When studies on different mathematical abilities were analyzed, it was found that various regions of the brain responded to these tasks depending on their complexity (Butterworth et al., 2011). In a quantitative meta-analysis, Arsalidou and Taylor (2011) examined 93 studies on numbers and calculation using fMRI. The analysis revealed that the parietal lobes, especially the inferior and superior parietal lobes, were activated during number tasks. In addition to these regions, the middle and superior frontal gyri in the prefrontal cortex were also activated during calculation tasks. Addition, subtraction, and multiplication tasks were analyzed separately. Results for addition indicated activation in the visual, parietal, frontal, and prefrontal regions, as well as the bilateral thalamus, cerebellum, right insula, and claustrum. Subtraction tasks showed activation in the occipito-temporal visual, parietal, frontal, and prefrontal regions, as well as the bilateral insula and right cerebellum. Multiplication tasks activated the occipito-temporal visual, parietal, temporal, frontal, and prefrontal regions, as well as the bilateral cingulate gyrus, bilateral thalamus, left claustrum, right insula, right caudate body, and right cerebellum (Arsalidou et al., 2011).

In a more recent review, Peters and Smedt (2018) focused on brain imaging studies examining children's arithmetic development, particularly neuronal alterations, and strategy use. This study investigated the development of the arithmetic network using fMRI and other imaging techniques. Regression models were applied to predict arithmetic performance, while voxel-based morphometry and ANOVA were used to evaluate developmental trends. The voxel-based morphometry is used to examine how anatomical characteristics of the related regions of the arithmetic brain network are associated with performance. Results demonstrated that the prefrontal, parietal, and hippocampal regions are critical for mathematical activities. Additionally, the connectivity of these networks among the prefrontal, parietal, and hippocampal regions, increases with age. Specifically, the intraparietal sulcus (IPS) was shown to facilitate the development of symbolic numerical processing. The study concluded that both domain-specific factors (e.g., numerical magnitude) and domain-general factors (e.g., working memory) influence arithmetic development.

Similarly, Visibelli and colleagues (2024) conducted a systematic review to investigate the brain processes underlying early numerical cognition and its developmental trajectory. This review included 21 studies involving a total of 732 participants, ranging in age from 30 weeks gestation to six years. The studies employed EEG, fMRI, and fNIRS to examine brain activity patterns related to numerosity. Statistical analyses, including ANOVA and correlation analyses, were used to assess brain responses in various conditions and to explore the relationship between age, behavioral performance, and neural activity. The findings revealed that by six months of age, distinct neural signatures for small and large numerical sets emerge, with the parietal, frontal, and occipital cortices already showing sensitivity to numerical aspects even before birth. This review concluded that although numerical abilities begin developing at an early age, individual and environmental factors are critical to understanding their further development. Despite the fact that review and meta-analysis studies have provided insights into how math abilities are associated with responses in specific brain regions, variations in these responses have been further elaborated in subsequent studies. In general, Deheane and colleagues (2003) sought to define the functional organization of the parietal lobe in numerical cognition by suggesting three different circuits for number processing. These circuits were determined and described using behavioral tests, neuropsychological findings, and fMRI data. These three circuits are the posterior superior parietal lobule involved in attentional processes on the mental number line, the left angular gyrus responsible for verbal manipulation of numbers, and the horizontal segment of the intraparietal sulcus linked to numerical quantity representation. Statistical methods, including comparisons of lesion localization in neuropsychological cases, overlap mapping for fMRI contrasts, and parametric modulation to evaluate activation strength under different task demands, have been employed. These three circuits aid in comprehending the ontogeny of arithmetic skills, the neural underpinnings of developmental dyscalculia, and numerical deficits in patients with brain damage.

Cantlon and colleagues (2006) aimed to determine the neural correlates of number processing in adults and young children aged 4 years. To measure brain activity in the intraparietal sulcus, participants were shown numerical stimuli using the fMRI technique. The General Linear Model (GLM) was utilized to analyze the fMRI data. Additionally, ANOVA and t-tests were employed to compare patterns between age groups and to determine significant neural activation. Neural activation similarities between children and adults were investigated using correlation analysis. In response to the number deviants, both age groups showed IPS activation, indicating that early-developing brain areas are important for numerical cognition. The results highlight the importance of the IPS for numerical comprehension even before symbolic learning experiences.

Additionally, Libertus and colleagues (2011) investigated the neural responses to numerosity changes in 7-month-old infants and adults, employing EEG with a steady-state visual paradigm. Neural oscillatory responses were analyzed to examine entrainment differences following numerosity shifts. Statistical analyses included time-frequency analyses of EEG data, ANOVA to compare responses across ratio conditions, and correlation analyses to tie the neural activity to later behavioral performance. The results indicated ratio-dependent changes in neural responses consistent with Weber’s Law, indicating that infants and adults share an Approximate Number System. The study provided evidence of ontogenetic continuity in numerical cognition.

Bugden and colleagues (2012) investigated the relationship between brain activation during symbolic number comparison and individual differences in arithmetic fluency in children with an average age of 8.8 years in their fMRI study. Seventeen typically developed children participated to the study. To measure children’s math abilities, the subtest from the Woodcock-Johnson-III was implemented, and a correlation was computed between children’s math abilities and their neural responses in the intraparietal sulcus. According to the results, activation in the left intraparietal sulcus was associated with higher arithmetics scores. This suggests that symbolic numbers are linked to the left intraparietal sulcus.

Similarly, Ben-Shalom and colleagues (2013) investigated the use of event-related potentials (ERPs) in preschoolers to process numerical values using a numerical Stroop task. The participants were between the ages of five and six and compared the physical and numerical sizes of digits. Statistical analyses included ANOVA applied to ERP waveforms for specific time windows, while repeated-measures ANOVA was used for behavioral data (i.e., accuracy and reaction times). According to the results, neural responses showed a mental number line as young as five years old, demonstrating automatic numerical processing. In both congruent and incongruent situations, different brain patterns were observed in the frontal and parietal areas. The results imply that early numerical cognition involves the integration of numerical meaning into cognitive processes, engaging both parietal and frontal networks.

Vogel and colleagues (2015) aimed to examine the age-related changes in neural correlates of symbolic numerical magnitude representation with a functional magnetic resonance adaptation (fMR-A) study with 33 typically developed children aged between 6-14 years. According to the GLM analysis, activation in the left intraparietal sulcus during the symbolic numerical magnitude task was found to increase with age, considering the association between age and numerical ratio in this region. Similarly, Emerson et al. (2015) conducted a neuroimaging study using fMRI to examine developmental neural activation changes in children aged 4 to 9 years in a matching task between symbolic and non-symbolic representations of number. Twenty-two typically developed children and twenty adults participated in the study. According to regression analysis, age-related changes were found in the left intraparietal sulcus. Additionally, the acuity of children's number skills is correlated with longitudinal changes in the left IPS.

On the other hand, the children's proficiency level of mathematics is related to the responses in the brain areas. Zamarian et al. (2009) conducted a review of brain imaging studies to provide a systematic analysis of the functional and structural changes in the brain associated with arithmetic acquisition and their relationship to experience and practice. According to the results, there was a general shift from frontal regions to the parietal lobe, and a shift from the intraparietal sulci to the left angular gyrus in the parietal lobe, considering both children's and adults' proficiency in math.

Rivera and colleagues (2005) aimed to investigate the age-related brain alterations between children and adolescents ages 8 to 19 during arithmetic reasoning tasks. Seventeen participants completed math-related activities while undergoing fMRI. A variety of statistical methods were used: t-tests were employed to compare activation under various task situations, correlation analyses were used to investigate the link between age and brain activity, and GLM analysis was applied to detect notable changes in brain activation. The findings demonstrated that younger children relied more on prefrontal and hippocampus regions, indicating higher demands on memory and attention, while older children showed increased activation in the left parietal cortex, linked to arithmetic reasoning. The results suggest a developmental shift away from memory systems and toward greater specialization in areas involved in numerical processing.

In a study examining children's math skills using the fNIRS technique, Dresler and colleagues (2009) investigated children's hemodynamic responses while they read and calculated arithmetic problems. A total of ninety typically developing children in fourth and eighth grades participated in the study. According to the ANOVA results, it was found that the parietal and posterior frontal regions of children’s brain were activated during the calculation task and the oxygenation level in these regions was not affected by either the type of problem or the age.

In another study, Kawashima et al. (2004) investigated fMRI detection of brain regions activated during an arithmetic test in children and adults aged 9-14, comparing age groups with a total of 16 participants. According to the ANOVA results, the left middle frontal, bilateral inferior temporal, and bilateral lateral occipital cortices were activated in both age groups. However, the intraparietal cortex was activated only in adults, and the right frontal cortex was activated specifically during addition and multiplication tasks.

In a similar direction, Kucian and colleagues (2008) aimed to examine brain images of children and adults using fMRI in approximate calculation, exact calculation, and magnitude comparison tasks. Twenty-two healthy adults and twenty-six typically developed children between third and sixth grades participated in the study. They found differences between children and adults in approximate and exact calculation tasks, but not in magnitude comparison. Children had weaker brain activation in the intraparietal sulcus and left inferior frontal gyrus during the exact calculation task, and in occipital regions during the magnitude comparison task. In contrast, children had more activation in the anterior cingulate gyrus.

Rickard and colleagues (2000) aimed to examine the fMRI output of college students during simple arithmetic, numerical magnitude judgment, and perceptual-motor control tasks. Eight typically developing adults aged twenty to twenty-four participated in the study. The results indicated activation in Brodmann area 44, dorsolateral prefrontal cortex, inferior and superior parietal regions, and lingual and fusiform gyri during the arithmetic task. This activation was more intense on the left side.

Artemenko and colleagues (2018) aimed to examine, longitudinally, the brain activation of 12- to 14-year-old children during addition, subtraction, multiplication, and division tasks in order to understand the development of processing during these four arithmetical abilities. Twenty-six adolescents, between grades six and seven, participated in the study, and the measurements were conducted twice with a one-year interval. Activation was found in the bilateral fronto-parietal network for all operations. Longitudinal results showed a decrease in the activation of the inferior frontal gyri during subtraction, while an increase was observed in the angular and middle temporal gyri during multiplication.

These studies suggest that there are common and differentiated brain networks in various mathematical tasks ranging from quantitative knowledge to arithmetic operations (see Arselidou et al., 2011; Artemenko et al., 2018; Bugden et al., 2012; Dresler et al., 2009; Emerson et al., 2015; Kawashima et al., 2004; Rickard et al., 2000; Vogel et al., 2015; Zamarian et al., 2019). In addition, although there are relatively few studies with young children in the literature, it is evident that math abilities are processed in similar brain regions in childhood as in adulthood, but the level of activation may vary with age, competence, and experience (see Artemenko et al., 2018; Emerson et al., 2015; Kawashima et al., 2004; Vogel et al., 2015; Zamarian et al., 2019).

The current thesis focuses on children’s mathematical abilities in early childhood and aims to explain how these abilities are biologically shaped through a neuroconstructivist and interactivist approach, incorporating both cognitive and environmental factors. Therefore, in the following section, executive function skills, which are one of the best cognitive predictors of children's math abilities, and their relationship with math abilities are presented.

2.4. Executive Function

Albeit biological factors form the baseline for the math abilities the cognitive aptitudes (i.e., executive function) represent another key individual factor influencing children's math achievement (Kilday, 2011). As a cognitive aptitude, executive function predicts children's outcomes, such as math and language, in learning more effectively than IQ (Zelazo et al., 2016). Executive function, which consists of three components: working memory, which refers to the retention and functional use of information; inhibitory control, which refers to the suppression of distractions and inappropriate responses; and cognitive flexibility, which enables flexible thinking, has a specific association with mathematics domains (Miller et al., 2009; Miyake et al., 2000). This tripartite structure supports one another's processes by working in unison, akin to an orchestra during high-level mental tasks such as mathematics (Diamond, 2013).

On the other hand, each executive function component has specific and specialized roles in the mathematical process (Bull et al., 2014; Cragg et al., 2014). For instance, working memory helps in retaining relevant information, storing, and retrieving results during problem-solving. Inhibitory control aids in suppressing inappropriate strategies, dominant number representations, remembering number bonds, or disregarding information from a word problem that is irrelevant to the solution. Cognitive flexibility skills assist in switching between operations, solution strategies, quantity ranges, and different representations, as well as between steps in complex multi-step problems (Bull et al., 2014).

Given all these points, mathematics involves various processes and knowledge demands. Three aspects of mathematical knowledge are presented: factual, procedural, and conceptual (Cragg et al., 2014; Hiebert et al., 2013). The factual concept involves facts, the procedural concept pertains to knowing how to perform tasks, and the conceptual concept relates to understanding why. (Hiebert et al., 2013). In more detail, procedural knowledge refers to the formal language or symbolic representation of mathematical systems, along with the algorithms and rules necessary to solve math tasks. Conceptual knowledge involves the ability to link discrete pieces of information and establish connections between stored information. (Hiebert et al., 2013). Cragg and colleagues (2014) proposed a theoretical model for these knowledge levels' predictive role of executive function in the context of mathematics. According to this model, working memory is closely linked to factual and procedural knowledge, inhibition is directly associated with conceptual knowledge, but is also indirectly linked to factual and procedural knowledge, and cognitive flexibility connects procedural and conceptual knowledge in mathematics (Cragg et al., 2014).

2.4.1. Working memory and math

The working memory, which provides temporary storage and enables the manipulation of information in complex cognitive tasks, is at the center of executive function by controlling attention (Baddeley, 1992). It includes visual spatial dimension through the manipulation of visual images, as well as the phonological loop, which keeps speech-based information and rehearses it (Baddeley, 1992). In general, this skill is related to mathematics as it supports processes such as keeping relevant information in the problem-solving process and storing and recalling results (Bull et al., 2014).

A meta-analysis was conducted of one hundred and eleven studies examining the correlation between working memory and mathematics achievement in typically developing children aged 4-12 years (Friso-van den Bos et al., 2013). This meta-analysis evaluated the relationships between different dimensions of working memory and mathematics achievement. As a result of the analysis, it was found that especially the verbal updating dimension of working memory showed the strongest correlation with children's math abilities (Friso-van den Bos et al., 2013).

Gordon and colleagues (2021) conducted a study to investigate the relationship between children's number knowledge, working memory, and spontaneous unprompted gesture use. Also, they consider the moderation of gesture use in the relationship between working memory and their cardinality performance. The study was conducted with 81 preschool children, aged between three to five. The results showed that children's gesture use (i.e., counting by pointing) was associated with children's working memory performance and helped them to better math knowledge.

Passolunghi and colleagues (2014) aimed to compare the effect of two types of intervention: the working memory training and early numeracy training, on five-year-old children's math performance. Intervention and control groups were defined.

Fifteen children were randomly assigned to each intervention group, and eighteen children to the control group. The training programs, provided as game-based sessions, consisted of a total of 10 sessions, with two sessions per week lasting one hour each over five weeks. The results indicated that the early numeracy training sessions only enhanced children’s math scores, whereas working memory training improved not only children’s math scores but also their working memory abilities.

In another intervention study, Kyttälä and colleagues (2015) provided both working memory and mathematics training simultaneously to one group, while the other group received only counting training. A total of sixty-one preschool-aged children, ranging from five to six years old, participated in eight sessions lasting 30 minutes each. As a result, it was found that children’s math abilities increased in both those who received only mathematics training and those who received mathematics training combined with working memory. Therefore, there is a causal relationship between working memory and math ability. As such, working memory is one of the leading cognitive factors for math abilities.

Taking into consideration all these studies focusing on the relationship between working memory and young children’s math abilities, it is suggested that working memory fosters the children’s math abilities. Specifically, it enables remembering and using information in a manipulative way. Thus, children use working memory in numeracy skills such as counting numbers (i.e., verbally enumerating numbers in sequence) and performing mathematical operations. There is a clear relationship between working memory and math ability. Beyond these correlational studies, it is also seen that when children are given working memory training, their math abilities also improve.

2.4.2. Inhibitory control and math

Subsequent to working memory, another dimension that predicts math ability is inhibitory control. Inhibitory control is characterized by behavioral and attentional inhibition, which refers to the suppression of the stimuli that compete for the primary response, thereby suppressing internal distractions that disrupt the current process of working memory (Nigg, 2000). Based on a meta-analysis of 75 studies, conducted by Allan and colleagues (2014), it was found that the relationship between inhibitory control and young children’s academic ability had an overall effect size of .27 and inhibitory control predicted math ability better than language skills.

Studies have shown that inhibitory control predicts preschool children's mathematics abilities both simultaneously and longitudinally. Ng and colleagues (2015) aimed to investigate the relationship between children’s inhibitory control and math achievement over two years. A total of 225 typically developed children from four ethnic groups (Chinese, African American, Dominican, and Mexican) participated in the study. The results revealed that inhibitory control predicts both the math abilities of four-year-old and six-year-old children.

Laski and colleagues (2015) investigated the relationship between ordinal number estimation skills and inhibitory control in a study with 53 adults and 42 kindergarten children. They found that individuals with better inhibitory control showed improvement in estimation, explaining that those with this skill are better at suppressing prior knowledge, which aids in accurate practice.

Overall, inhibitory control involves focusing attention on the selected stimulus, ignoring distractions, suppressing irrelevant cognitive representations, and delaying or suppressing behaviorally inappropriate responses (Diamond, 2013). This ability aids in mathematical operations by directing attention to the task at hand, considering relevant cognitive representations, filtering out irrelevant information, and facilitating effective problem-solving behavior.

2.4.3. Cognitive flexibility and math

Studies on cognitive flexibility, another component of executive function, and its relationship with mathematics are less common compared to the other two components. Cognitive flexibility refers to the ability to switch between multiple tasks, operations, or mental sets, characterized by shifting attention or task switching (Monsell, 1996). In a recent meta-analysis, Santana and colleagues (2022) found that the overall effect size between cognitive flexibility and mathematics was 0.40, based on a total of 23 studies. When specialized mathematical abilities were examined, they found that conceptual mathematics was associated with mathematics, with an overall effect size of 0.34, and procedural mathematics with an overall effect size of 0.33. The relationship between cognitive flexibility and mathematics has generally been explored in the context of executive function and mathematics.

In a study by Arán Filippetti and colleagues (2017), Structural Equation Modeling (SEM) was used to examine the relationship between number production, mental calculation, and arithmetic problem-solving in 8-12-year-old children, alongside executive function and intelligence components. The results highlighted that cognitive flexibility is the only component that predicts arithmetic operations, as it helps inhibit learned strategies during calculations and shift to new ones, including multi-digit operations.

In another study, Van der Ven and colleagues (2012) aimed to investigate the factor structure of the EF, its and math abilities' development, and the relationship between EF and children's math abilities. The sample consisted of 211 elementary school children across 10 schools. It was found that math ability was related to all executive function dimensions in their. However, through factor analysis, they determined that inhibitory control and cognitive flexibility were key predictors of mathematics ability as a combined factor.

2.4.4. Summary

The general analysis of the studies show that executive function of preschool-aged children make a unique contribution to their mathematics abilities both concurrently and longitudinally. Studies generally show that working memory plays the most significant role in mathematics, while inhibitory control and cognitive flexibility, either individually or in combination, also predict mathematics performance. Early math abilities encompass various components, such as number knowledge and arithmetic calculations, and require skills like retaining and recalling information, adaptively using new information, thinking flexibly to connect different ideas, and focusing on relevant details while inhibiting distractions. With its multidimensional structure, executive function is one of the most important cognitive factors that contribute significantly to math abilities, both through its subcomponents and through the integrated use of these components.

2.5. Environmental factors

In addition to biological and cognitive abilities, several environmental factors play a crucial role in a child’s mathematical development (Phair, 2021). Silver and colleagues (2022) conceptualized these environmental factors as community-level and individual-level factors. Community-level factors refer to the exposure children have to language, societal attitudes, and beliefs (i.e., gender stereotypes, gender equality). Individual-level factors include socioeconomic status (SES), parental education, children’s math-related activities with adults (i.e., parents and teachers), and adults’ (i.e., parents and teachers) attitudes and beliefs about early math. Notably, adults’ beliefs and adult-child activities play an instrumental role in shaping children's mathematical abilities in early childhood (Silver et al., 2022). Additionally, there is a strong link between adults’ early mathematics beliefs and activities they provided to their children. Since adults’ beliefs serve as a driving force that initiates behavior at an individual level, they are central to providing stimulating mathematics activities (Fishbein et al., 1975). These beliefs also influence child-adult interactions, the structure of instruction and assessment, and are aligned with practices in the educational environment, ultimately impacting children's achievement (Kagan, 1992). Hence, beliefs and activities are not only interrelated but also determinants of children’s mathematical abilities. Accordingly, the home and school environments are examined in terms of adults’ (i.e., parents’ and teachers’) beliefs about early mathematics and adult-child mathematics activities, which are presented in this section.

2.5.1. Home math environment

During early childhood, children spend a significant portion of their time in the home environment, where their experiences and social interactions play a crucial role in their development. In the context of mathematical development, the home environment encompasses the interactions between the child and the adult, as well as the individual characteristics of the parent regarding mathematics (Daucourt et al., 2021). Parents' beliefs about early mathematics significantly shape their children's mathematical abilities, with this influence directly tied to the extent to which they engage their children in math-related activities (Silver et al., 2024). In this section, the literature on home-related mathematics activities and parents' beliefs about mathematics, key environmental influences on children's mathematical abilities, is presented respectively.

2.5.1.1. Home math practices

In this part, the home math activities (Daucourt et al., 2021; Skwarkchuk et al., 2014), which refer to direct mathematical interactions between parent and child, such as discussing math concepts, sorting, singing songs, and counting, and the relationship of these practices to children's math abilities are presented.

The relationship between math activities at home and children's math skills has been explored in many studies. Blevins-Knabe and Musun-Miller (1996) aimed to determine the frequency and variety of home numeracy activities and their impact on children's math performance. To do this, 40 participants were contacted by phone and asked questions about math and reading activities involving their 4-6-year-old child. Math activities were assessed using a 7-point Likert-type checklist consisting of 13 questions. Children's math abilities were measured using the Test of Early Mathematics Ability - 2 (TEMA-2). The results indicate that mothers of daughters were more actively involved in counting and often used phrases like "same number." Additionally, there was a significant positive correlation between the frequency of these activities and their children's math scores.

Beyond the descriptive study conducted by Blevins-Knabe and Musun-Miller (1996), Levine and colleagues (2010) focused on parent-child interactions in the context of math talk. They investigated whether parents' conversations about numbers with their children during the early years predict children's cardinality performance later on. For this purpose, 44 child-parent dyads were included in the study. Over a 16-month period, these dyads were visited monthly in their homes, video-recorded for 90 minutes, and children's cardinality skills were measured using the Point to X test. When the relationship between transcribed and coded videos and children's cardinality scores was analyzed, a positive predictive relationship was found.

While Levine and colleagues (2010) provide evidence of direct activities and their relationship to children's math abilities through observations, Manolitsis and colleagues (2013) strengthened this finding by using a standardized questionnaire. Their study aimed to examine the impact of home numeracy and literacy activities on the acquisition of math and literacy in grade one. The study included 99 typically developed preschool children, selected through stratified random sampling. To measure the home math environment, a 5-point Likert-type scale was used, while children's preschool mathematics abilities were assessed using the Test of Early Mathematics Ability - 3 (TEMA-3), and their math abilities in first grade were measured in terms of math fluency. The results showed that both home literacy and numeracy activities predicted children's acquisition of academic abilities.

However, while Manolitsis and colleagues (2013) found a relationship between home math activities and children's outcomes in a cross-sectional study, Huntsinger and colleagues (2016) examined this relationship using a longitudinal design. Specifically, they aimed to determine the long-term impact of parent-provided practices on children's math and reading abilities. The sample of the study consisted of 200 preschool children. Likert-type measures of experiences, based on parental reports, were used to assess parent-related factors. Children's math abilities were measured using TEMA-2. According to the results, formal math activities were the strongest predictor of children's academic achievement, and parent-provided math activities had a longitudinal effect on children's academic achievement.

In addition to the study by Huntsinger and colleagues (2016), Thompson and colleagues (2016) also investigated the longitudinal relationship between home numeracy activities and children's math outcomes, focusing on 3-4-year-olds. Particularly, they aimed to explore the longitudinal relationship between formal home numeracy practices and children’s arithmetic abilities. The study was conducted with 184 participants, including 3-4-year-old children. A 5-point Likert-type scale was used to measure the home numeracy environment. Additionally, “The Preschool Early Numeracy Skills Test - Brief Version” (PENS-B) was used to assess children’s early numeracy skills. The findings suggested that parental engagement in home numeracy activities was higher for older children (i.e., 4 years) compared to younger ones (i.e., 3 years). Moreover, more complex activities were correlated with the numeracy performance of four-year-olds, whereas basic activities did not show significant associations with either age group.

Similar to Thompson and colleagues’ (2016) study, Missal and colleagues (2017) investigated young children’s math abilities in the context of parent-math activities using a cross-sectional design. They aimed to determine the association between parent-reported home math activities and observed parent-child math interactions. The study, conducted with 72 parent-child dyads from an early education center, examined parent-child mathematics-related activities using a 5-point Likert-type scale based on parent reports. In the study, the Bracken Basic Concepts Scale - Third Edition: Receptive (BBCS-3:R) and Individual Growth and Development Indicators of Early Numeracy (IGDIs-EN) were used to measure children’s math abilities. According to the results, there was a lack of relation between parent-reported activities and observed activities.

Different from the previous studies, Niklas and Schneider (2017) did not focus solely on the direct relationship between the home environment and children’s outcomes. Instead, they considered the mediation of academic precursors. Based on this, they aimed to determine the longitudinal impact of the home learning environment on children’s math and literacy abilities, while examining the mediating roles of academic precursors. There were 920 children aged 6-7 years who participated in this large-scale study. A test battery was used to assess children’s mathematical abilities, including rhythmic counting, calculation, matching quantities, and comparing quantities. The home learning environment was measured using a scale-type instrument. The results showed that the home learning environment was a strong predictor of both early intelligence and academic achievement, including language and math, at the end of elementary school, even after controlling for precursors, formal academic success, and demographic features.

Following the approach of Niklas and Schneider (2017), more recently, Pardo and colleagues (2020) focused on the same age group (6-7 years) by assessing math abilities in greater detail using three different tasks. Their aim was to explore the relationships between formal and informal home numeracy activities and children's basic number processing skills. The study involved 212 children aged six to seven and their parents. Direct parent-child activities were measured using a parent-report survey, while children's math abilities were assessed through performance-based tasks, and their intelligence was evaluated using a cognitive performance test. The results of the regression analysis indicated that both formal home numeracy practices and general cognitive abilities (i.e., intelligence) significantly contribute to predicting children's number processing skills.

In another study conducted with a similar aim, Claire-Son and Hur (2020) utilized direct observations and a longitudinal design to examine this relationship in detail. Specifically, they aimed to determine the longitudinal effect of parental math talk during cooking on children's math abilities. The sample of the study consisted of 48 preschool children. Home visits were conducted to evaluate parent-child interaction during cooking activities. During these visits, video recordings were taken and subsequently coded by the researchers. The verbal behaviors of the caregivers, including math talk and task talk, were systematically coded from the videos. In addition, the Woodcock Johnson Test of Achievement III (WJ-III) was used to measure children's math abilities. The results showed that the number of parent-child interactions during cooking was associated with children's math scores. Furthermore, an interaction effect was found between high task-oriented talk and low task-oriented talk.

Similar to Claire-Son and Hur (2020), Soto-Calvo and colleagues (2020) conducted a longitudinal study during the preschool period. In that study, they aimed to determine the longitudinal associations between home learning practices and early number abilities. The study included 274 preschool children, with measurements taken across three time periods (the spring and summer terms of preschool, and the summer term of the reception year). The home environment was assessed using a parent-reported 6-point Likert-type scale. Children's mathematics abilities were evaluated using both the British Ability Scales III (BAS-3) and a battery of early number skills (e.g., counting, number transcoding, and calculation). The results showed a significant relationship between home letter-sound interaction practices and children’s counting and number transcoding, from preschool to the first year of primary school. In contrast to other studies, Chan and colleagues (2021) conducted research to explore the relationship between home practices and school outcomes. Specifically, they compared a study of parent-engaged home activities with two early childhood programs: Care for Child Development (C4CD) and Early Childhood Care and Development (ECCD). The study included 245 parents with children aged 3-5 years. Children's mathematics abilities were assessed using items from the learning domain of The Early Childhood Development Index (ECDI). Home-based activities were measured using a list drawn from UNICEF's Multiple Indicators Cluster Survey 6 (MICS6). The results indicate that home-based activities were associated with C4CD, and there were interaction effects of age and C4CD. Additionally, home-based activities moderated the relationship between child development and C4CD.

Overall, the area of the home environment, which investigates home activities and individual factors (e.g., beliefs), and their association with children’s math achievement, is one of the oldest areas of study. Cumulative research in this area has shown that the frequency of these activities is positively correlated with children’s achievement. These activities play a promotive role in the acquisition of academic abilities, and their positive effects persist over the years on children’s academic outcomes. In addition, the parent’s intrusive and controlling interactions during these activities has been found to negatively impact children’s performance. Furthermore, home activities contribute to the positive association between educational programs and child development. In terms of child characteristics, parents are more likely to engage in activities with girls and older children.

2.5.1.2. Parental beliefs about math

The beliefs that an individual attributes to the importance of mathematics affect their intentions, attitudes, and thus behaviors. Similarly, the level of importance that parents attribute to mathematics in the home environment follows the same mechanism and contributes to children's mathematical skills by influencing their attitudes and behaviors toward mathematics activities with their children (Elliot et al., 2018; Fishbein et al., 1975; Silver et al., 2022; Taylor et al., 2004). Research on parental beliefs about mathematics operationally defines these beliefs as a range of values, experiences, and self-efficacy that reflect how parents perceive and feel about mathematics (Missall et al., 2015).Studies investigating the connection between parental beliefs and children’s math outcomes indicate that when parents hold strong beliefs about academic and cognitive abilities, children tend to display better performance in mathematics (Furnham et al., 2002; Phillipson et al., 2007).

In a study conducted by Blevins-Knabe and colleagues (2000), researchers aimed to examine parents’ beliefs and attitudes toward mathematics, as well as their engagement in math activities with their children. The study also explored the relationship between these beliefs, activities, and children's math abilities, which were measured using the TEMA-2 assessment. The participants included 40 parents/caregivers with children between the ages of four and six. The findings revealed that when parents held positive beliefs about mathematics and found math activities enjoyable, they were more involved in mathematics activities with their children at home. Moreover, the frequency of engaging in math activities at home was higher ($r = .55$; $p < .008$).

Similarly, Sonnenschein and her colleagues (2012) aimed to determine the association between parents’ beliefs about mathematics development and children’s math activities at home. A total of 73 parents of six-year-old children from different ethnic groups (i.e., African American, Chinese, Latino, and Caucasian) participated in the study. According to the results, it was found that parents’ beliefs about mathematics development were associated with children’s engagement in mathematics activities, and this association was independent of ethnicity. This means that as parents’ beliefs increased, their engagement in math activities also increased.

Likewise, DeFlorio and colleagues (2015) studied the relationship between children’s frequency of math activities and their math ability scores. They also examined differences in children’s math knowledge in relation to parental beliefs and socioeconomic status. This study was conducted with 178 families across different socioeconomic levels (i.e., low and middle) and different class levels (i.e., two years before the first grade and one year before it). The results revealed a positive, moderately significant relationship ($r = .39$, $p = .01$) between five-year-old children’s mathematics abilities and parental beliefs about early mathematics development. In addition, the middle SES families’ beliefs were higher in both class levels.

Similarly, Zippert and Rittle-Johnson (2020) examined the level of emphasis parents place on different math topics, the relationship between parental beliefs and the overall math environment at home, and the connection between the home math environment and children’s math abilities. The study involved sixty-three primary caregivers and their preschool-aged children from diverse preschool programs and various ethnic backgrounds. The results indicated that parents preferred engaging in activities that enhance their children’s numeracy skills. Additionally, parental academic beliefs, as a component of the home math environment, contributed to greater parental support for math, which was positively associated with children’s math abilities.

In general, studies investigating parents’ mathematical beliefs and children’s math abilities have emphasized that parents’ beliefs influence both the frequency of mathematical activities at home and their support for children's mathematical development. Furthermore, children’s mathematical abilities are positively affected by these beliefs. To summarize, when parents hold positive beliefs about early mathematics, their support, such as encouraging their children’s mathematical practices and engaging in math activities with them, increases correspondingly, ultimately resulting in improved math performance in children.

2.5.2. School math-related environment

Subsequent to the home environment, another significant individual-level factor affecting an individual’s math abilities is the school community. The frequency of mathematics activities in schools and the beliefs that teachers hold about mathematics are closely connected to children’s math abilities (Silver et al., 2022). This suggests that the more frequently mathematics activities are conducted, the faster children learn mathematical concepts. Furthermore, positive attitudes toward mathematics can enhance children’s math abilities. In this section, the literature on classroom math activities, often investigated under the term “quality in early childhood education”, and teachers’ beliefs about mathematics are presented, respectively.

2.5.2.1. Early Childhood Education Quality in Relation to Math Ability of Children

The quality in early childhood education, aimed at fostering well-being and developmental outcomes of children (Layzer et al., 2006), encompasses four key elements: the program’s structural components (e.g., length of the school day, teacher qualifications), general features of the classroom environment (e.g., playground equipment, staff, and parental involvement), teacher-child interactions directly experienced by children, and aggregate indices involving various program elements (Pianta et al., 2016). While the concept of quality is broad, aspects that are standardized and regulated by legislation, policies, and funding (e.g., child-to-staff ratio, group size, staff training or qualifications, etc.) are referred to as structural quality. On the other hand, the intimate processes in children’s daily experiences (e.g., in-person interactions, activities, routines, etc.) are referred to as process quality (for review, Slot, 2018). To assess structural quality, researchers frequently use the Early Childhood Environment Rating Scale (ECERS) (Harms et al., 1998), and to determine process quality, they use the Classroom Assessment Scoring System (CLASS) (Pianta et al., 2008). In the present section, studies from both perspectives are presented, and their relationship to children’s math abilities is discussed accordingly.

In considering the literature, studies on children’s math abilities and their association with early childhood quality show contradictory results depending on the dimensions measured. The review and meta-analysis conducted by Brunsek and colleagues (2017) examine the association between preschool quality and child outcomes, including language, math, and social behavior. The quality aspects were measured using the Early Childhood Environment Rating Scale (ECERS), which assesses structural quality in preschool settings. In total, 73 studies were reviewed, and a meta-analysis was performed on 16 of these studies, with results calculated at a 95% confidence interval. The meta-analysis concerning math achievement, based on eight studies that used specific tasks such as the Woodcock-Johnson Applied Problems, indicated no significant association. However, it was found that the quality of learning and teaching, as well as the interaction components within the preschool environment, were important factors influencing children's math achievement. Furthermore, in the meta-analysis of nine studies that directly examined the relationship between math quality and preschool quality, no significant relationship was found.

In the same vein, Abreu-Lima and Leal (2013) aimed to determine the relationship between classroom quality and the outcomes of literacy, math, and behavior in four- and five-year-old children in Portugal, involving 215 children from 60 preschool classrooms. According to the hierarchical linear model analyses (HLM) with significance levels at .05 or below, global classroom quality predicted children's literacy and behavior, but did not predict math scores. Additionally, maternal educational level was associated with all developmental variables. The non-significant result regarding children's math scores was explained by the socio-economic characteristics of the children.

Aligned with the results of Abreu-Lima and Leal's (2013) study, Grammatikopoulos and colleagues (2018) aimed to evaluate the structural quality of early childhood education and its association with children's outcomes (i.e., literacy and math) in 402 five-year-old children from 51 institutions in Greece. Due to nested variances (i.e., the number of classrooms), a multilevel approach was used for data analysis, and it was found that structural early childhood quality scores were significantly related to children's math and literacy skills.

Similarly, Lehrl and colleagues (2016) aimed to investigate the long-term association between preschool structural quality and children's math ability from preschool to elementary school. as well as the interaction between preschool quality and the home environment. The study involved 554 children, aged between seven to nine, from 97 preschools in Germany. According to the latent linear growth curve model analysis, preschool-aged math scores and SES (socioeconomic status) predicted first-grade math achievement. Additionally, preschool quality was associated with first-grade math abilities after considering the moderation of the home environment during middle childhood.

Another study by Schmerse (2020) aimed to determine whether children's academic achievement in math and reading is associated with preschool structural quality and the experiences they encountered during the transition process to formal schooling. In this study, children's learning behavior and socioeconomic status played a moderating role. The study included 435 children, aged three to eight, from 142 classrooms in 87 schools. Data analysis was conducted using path analysis, after controlling for children's self-regulation scores, working memory capacity, socioeconomic status, parent-child relationships, home learning environment, and the ratio of staff to children. Results indicated that children's learning behavior mediated the association between classroom quality and their grade-two achievement, and SES varied these relationships, meaning children from disadvantaged backgrounds benefited more than others.

In the same context, Li and colleagues (2019) examined preschool quality and its association with children's outcomes (i.e., language, math, social cognition, and physical movement) in the Chinese context. The study involved 2,210 children, aged three to six, from 428 classrooms across a total of 193 preschools. Descriptive analysis, hierarchical linear model, and piecewise regression models were computed, and the findings suggested that higher preschool quality strongly predicted children's outcomes, including math abilities, particularly in schools located in rural areas.

Hu and colleagues (2021) aimed to determine the relationship between teachers' strategies, a process quality aspect, and children's math thinking in the Chinese context. The study included 69 teachers, with each teacher supervising approximately 14 children, aged between three to six. Data were collected through observations using the CLASS tool, which assesses process quality in early childhood, and coding children’s responses during math lessons. Both qualitative and quantitative analyses were conducted, and the results indicated that teachers used similar strategies, but scaffolding and questioning during math lessons were often ineffective.

In a study conducted by Finders and colleagues in 2021, researchers examined the quality of teacher-child interactions and how variability in classroom quality, influenced by the duration of observations, affected children’s school readiness in areas such as literacy, language, and math. The study involved 684 children, all aged at least four years old, from 180 preschool classrooms across the United States. Multilevel analysis was employed to account for nested variables, and SEM was used to determine variability. The results showed that teacher-child interactions were moderately stable across observation times, and classroom organization was positively related to children’s math achievement.

In the same direction, Guerrero-Rosada and colleagues (2021) conducted research to test the relationship between classroom quality and and its benefits to the kindergarten age children. The major issues considred were language abilities, math abilities, and executive function. The regression analysis was computed to determine associations between classroom quality and their academic and cognitive abilities with 307 children from 42 classrooms in 20 schools in the United States. According to the results, none of the models provided a significant prediction of quality in children's gains involved in math achievement.

Similarly, Schmitt and colleagues (2020) investigated the relationship between classroom process quality and children’s math achievement, with the moderation of children’s behavioral self-regulation (i.e., executive function). The study included 102 children, with an average age of four years, from 40 classrooms across the United States. Results from the multilevel analysis indicated that children’s math achievement was affected both directly by classroom process quality and when moderated by executive function.

Francis and Barnett (2019) investigated the effect of class size on classroom quality and the academic achievement of preschool-aged children. The study involved 21 teachers and 354 children (with a mean age of 55.2 months) and utilized randomized control trials. In the intervention group, class sizes were reduced from 20 to 15 children. The researchers assessed children's literacy, vocabulary, and math outcomes compared to those in regular-sized classes. To analyze the impact, a multilevel analysis was performed, controlling for variables such as pre-test scores, age, gender, ethnicity, home language, income, and parental educational levels. The results indicated that children's literacy scores improved significantly, though the effect size was small. However, there were no significant differences in language or math scores, and the effect sizes for vocabulary and math remained small.

Similarly, Guerrero-Rosada and colleagues (2021) conducted research to explore the relationship between classroom quality and its impact on kindergarten-aged children. The key aspects examined included language abilities, math abilities, and executive function. Regression analysis was used to identify associations between classroom quality and children's academic and cognitive outcomes, involving 307 children from 42 classrooms across 20 schools in the United States. According to the results, none of the models provided a significant prediction of classroom quality on children's gains in math achievement.

Taking into account these studies, it is evident that there is disagreement regarding the associations between classroom quality and children's math achievement. Some studies found a relationship between them (Finders et al., 2021; Grammatikopoulos et al., 2018; Hu, 2021; Lehrl et al., 2016; Li et al., 2019; Schmitt et al., 2020; Schmerse, 2020), while others did not find any associations (Abreu-Lima & Leal, 2013; Brunsek et al., 2017; Francis & Barnett, 2021; Guerrero-Rosada et al., 2021). The key point of these studies is that classroom quality, regardless of whether it focuses on structural or process aspects, has the potential to enhance children's math achievement. However, several findings suggest the opposite. These studies were conducted in different countries, yet they show consistency in using similar data collection tools. Therefore, these results provide a meaningful pattern in terms of methodology, although there is cultural diversity among the studies. This suggests that these findings may have broader relevance, and teacher characteristics could be factors contributing to the differences observed across the studies. Supporting this, the general discussion of the negative results suggests that the null impact may be due to characteristics of the subjects or the instrumentation used. Furthermore, Guerrero-Rosada and colleagues (2021) suggest that these results could be explained by teacher-related factors, such as teacher beliefs, which are not captured by the quality instruments.

Considering this, the current study focuses on school quality in the context of classroom mathematics activities, which is a component of process quality. Additionally, conceptual (e.g., Silver et al., 2022) and theoretical (Bronfenbrenner, 1994) studies suggest that teachers' beliefs, within the school setting, are another key factor influencing young children's math abilities. Thus, teachers' beliefs about early mathematics should be considered as an important element in the school math-related environment.

2.5.2.2. Teachers' Beliefs About Early Math

Early childhood educators play a crucial role in supporting young children's development and learning. Reports from the United States, along with various researchers, show that to help children's math abilities, teachers often use integrated and whole-group activities (National Research Council (NRC), 2009; cited in Hyson et al., 2014). However, small-group activities are usually considered as more effective for teaching math (Wasik, 2008). Additionally, theoretical frameworks suggest that instructional scaffolding can be helpful in math instruction (Bodrova et al., 2007; Copple et al., 2009), though teachers use this strategy less often (Hyson et al., 2014). In general, teachers' practices in the classroom show differences between what is expected and the actual practices that happen in schools to promote math learning. To understand these differences, researchers have looked into various teacher-related factors, such as math knowledge, beliefs, attitudes, and educational backgrounds (Benz, 2012; Chen et al., 2013; Prewett et al., 2021; Takunyacı et al., 2012). Among these, teachers' beliefs and competence are often found to be major reasons for the ineffective use of teaching strategies. Further elaboration is provided in the subsequent paragraphs.

Early childhood educators’ beliefs are closely tied to their teaching experiences, providing a structured framework for their interactions and instructional approaches (Charlesworth et al., 1991; Charlesworth et al., 1993; Hofer, 2001; Muis et al., 2006; Karataş et al., 2017; Pajares, 1992; Stipek et al., 1997). Charlesworth and colleagues (1991, 1993), as well as Stipek and Byler (1997), highlight the importance of understanding early childhood educators’ beliefs. In both studies, the researchers developed a scale to measure teachers’ math-related beliefs and examined their relationship with teaching practices. Their correlational analysis revealed a strong connection between teachers’ beliefs and their practices. Specifically, teachers who hold developmentally appropriate beliefs tend to align their teaching practices accordingly, which positively influences classroom outcomes.

Similarly, Karataş and colleagues (2017) conducted a study to determine the early childhood educators’ beliefs about teaching math. The participants of the study were 139 early childhood educators of varying seniority and educational levels to compare their beliefs. The results show that teachers with at least ten years of experience place greater emphasis on children’s thoughts and adjust their practices accordingly.

Teachers’ beliefs have the potential to positively or negatively impact children’s math achievement. To address these dynamics, Chen and McCray (2013) introduced a framework called the “Whole Teacher.” This framework focuses on the professional development of teachers by integrating their math knowledge, practices, and attitudes. Based on this framework, they conducted a pre-test and post-test experimental study, which included coaching and learning sessions aimed at enhancing the “whole teacher” characteristics, specifically knowledge, practice, and attitudes, of early childhood educators regarding math. The study involved 80 teachers and 154 preschool-aged children, with the intervention group (91 children, 12 classrooms) and the control group (63 children, number of classrooms not mentioned) assigned through randomization. Hierarchical linear modeling was used to assess the intervention’s impact on children’s math achievement. The results showed that children in the intervention group demonstrated greater growth in math achievement compared to the control group. Notably, children with lower initial achievement benefited more than their peers with regular performance levels.

Further studies highlighted that educators possess varying beliefs, perspectives, and characteristics when teaching math to young children. Chen and colleagues (2013) conducted a survey to explore early childhood educators' confidence and beliefs about teaching math. The study included 346 teachers from the Midwest region of the United States. Confirmatory factor analysis was employed to assess how well the data aligned with a hypothetical model, and an independent t-test was used to compare teachers' confidence and commitment to their own math abilities. The results of the confirmatory factor analysis indicated that teachers' beliefs and confidence were influenced by their views on preschoolers and mathematics, their confidence in promoting math learning, and their self-confidence in their own math skills. Moreover, teachers believed their attitudes were well-suited for young children, acknowledging the importance of cognitive development and the role of early math instruction in school readiness. They also expressed confidence in setting math goals for young children and felt they had the necessary knowledge to teach math concepts effectively. However, the majority of teachers reported that math was not their best subject during their own schooling.

Similarly, Benz (2012) conducted a survey study in Germany to explore kindergarten teachers' attitudes toward mathematics, involving 589 in-service educators. The researcher developed a survey that included multiple-choice questions, Likert-type items, and open-ended questions. The findings revealed that teachers had a positive perception of the importance and usefulness of math. However, they also viewed math as challenging, confusing, and difficult to comprehend. They believe that teaching math is important in the early years but should be confined to the school setting. At the same time, they acknowledge that math is an integral part of daily life.

In another study, Takunyacı and Takunyacı (2014) aimed to examine early childhood educators' self-efficacy beliefs about teaching math. Data were collected from 95 early childhood educators working under the Ministry of National Education

(MoNE). The findings indicated that teachers believed their efforts in teaching math positively influenced children’s achievement. However, a significant portion of participants (42.1%) expressed uncertainty about their ability to teach math effectively. Additionally, the results underscored a connection between children achievement and the characteristics of the teachers. Despite this, most teachers did not agree with the notion that they taught math ineffectively.

In another study, Prewett and Whitney (2021) aimed to examine the direct effects of teachers’ self-efficacy and their negative emotions (e.g., anger, worry, fear, hate, stress, or sorrow) on eighth-grade children’s reading and math achievement. Structural Equation Modeling was conducted with a sample of 9,725 children and 9,128 teachers. The findings revealed that teachers’ self-efficacy did not significantly contribute to children’s outcomes, whereas teachers’ negative emotions had a negative impact on children’s math and reading performance. Furthermore, self-efficacy and negative emotions were found to be negatively correlated. These four survey studies (i.e., Benz, 2012; Chen et al., 2013; Prewett & Whitney, 2021; Takunyacı & Takunyacı, 2012) emphasized that teachers held varying perspectives on the value of math and its instruction, which, in turn, influenced their teaching practices.

Arby and colleagues (2015) examined these differences in teachers' characteristics and investigated early childhood educators' beliefs regarding the importance of school competencies, such as self-regulation, academic achievement, and social skills, particularly in relation to math. Their study focused on how misalignments in educators' beliefs about children's school competencies affected children's school adjustment. Additionally, they explored the relationship between teachers' misalignments and children's adjustment outcomes, considering the children’s socio-economic backgrounds. A total of 2,650 children and their teachers participated in the study. The regression analysis controlled for various child characteristics, including age, gender, socioeconomic status (SES), parental educational background, developmental status, and chronic health conditions. It also accounted for institutional factors, such as teachers' age, gender, ethnicity, years of experience, education level, class size, and the proportion of children with special needs.The results revealed that teachers' beliefs about the significance of academic competence were the most misaligned. Children from socioeconomically disadvantaged backgrounds were more susceptible to the negative effects of these misalignments across all adjustment outcomes compared to their more advantaged peers. A higher degree of misalignment in beliefs across all three domains of competence was associated with lower math achievement, poorer learning approaches, and weaker social skills.

Focusing on the results of this studies there are few keypoints. The main trend in studies on teachers’ beliefs and children’s math outcomes illustrates a pattern characterized by the diversity of teachers’ beliefs and their positive or negative attitudes toward early-year math. Overall, children’s math achievement tends to be higher when teachers hold positive math beliefs, whereas it is lower when teachers’ math beliefs are negative.

2.6. Summary of the literature review

In this chapter, the theoretical background of the study and the results of the studies in the literature are comprehensively introduced. The current study aims to explain young children's mathematical abilities through focusing on their biology, cognitive abilities and environmental factors. In this context, children’s development is addressed with a multidimensional assessment approach. Due to this multifaceted nature, the theoretical foundations of the study include studies that explain the interaction of children with the environment, the constructive interaction of brain structure with the environment, as well as studies integrated biology and environment to explain specifically mathematics.

Synthesizing these backgrounds, the factors that the current study focuses on hemodynamic responses in the parietal lobe of the brain as a biological factor, executive function as a cognitive factor, and home and school environment as environmental factors. Building on this synthesis, the literature on young children's math abilities, brain imaging studies, executive function research, and home and school environments is introduced. In this context, home environment was defined by parent-child math practices and parental beliefs about early math. Similarly, the school environment is constituted by math activities in the classroom and teachers' beliefs about early mathematics.

CHAPTER 3

METHOD

In this section, the design of the study, sample, data collection instruments, data collection procedure, and data analysis are explained respectively. In the first part, the design of the study, the internalized research design is considered with the objective (descriptive, predictive or explanatory) and the time characteristics (retrospective, cross-sectional or longitudinal) of the study are given. The information regarding the participants is presented in the sample section, describing the individuals who participated in the behavioral and brain data collection process.

Then, the characteristics of the instruments used for data collection are explained and introduced in the data collection instruments section. The protocol for the collection of the data is explained under the title of the data collection procedure section. Finally, the analysis method of the obtained data is explained under the heading of the data analysis.

3.1. Design of the Study

The aim of this study is to examine preschool children's math abilities in the context of its relationship with hemodynamic responses in brain areas during mathematics, measurement executive function, the home environment, and the school environment. Aligning with this purpose, the research questions of the study are set as follows:

1. Are young children's math abilities predicted by hemodynamic responses in the parietal lobe during math tasks, their executive function performances, home math environment and school math-related environment?

1.1. Which substructure of parietal lobe of young children is oxygenated during math tasks?
1.2. Are young children’s hemodynamic responses in the parietal lobe correlated with their mathematical abilities?
1.3. Are young children’s mathematical abilities correlated with their executive function performance~~s~~?
1.4. Are young children’s mathematical abilities correlated with the home math environment (parent-child math activities and parental beliefs about early math)?
1.5. Are young children’s mathematical abilities correlated with the school math-related environment (classroom math activities and teacher–beliefs about early math)?

The non-experimental research designs in educational research are classified by Johnson (2001) by examining both the objective and time dimensions of the research. According to these categories, the purposes of research can be divided into three types: descriptive, predictive, and explanatory. Additionally, he described the time dimension in three categories: retrospective, cross-sectional, and longitudinal. As a result, he presents each intersection of these dimensions as a distinct research design.

Regarding both the objective and time dimensions of the current non-experimental study, it is planned to conduct a predictive cross-sectional design, which was proposed by Johnson (2001). Predictive studies describe the examination of the relationship between predictor and criterion variables (Pedhazur et al., 1991), and the present study fits this type since the relationship between children’s math abilities and brain areas’ hemodynamic responses during math test, executive function, home and school climate was determined. In this regard, the predictor variables of the study are hemodynamic responses in parietal lobe, executive function, the home environment (i.e., parent-child math activities and parental beliefs about early mathematics) and the school environment (i.e., classroom math activities and teacher’s beliefs about early mathematics), while the criterion variable is children's math abilities.

The variables in this study are presented as behavioral data and brain data. Behavioral data encompasses the characteristics of the child and their environment, which are assessed using only standardized measurement tools. In contrast, brain data consist of information obtained through physiological measurements. In this context, behavioral data includes children’s math abilities, performance in executive function, and aspects of the mathematics-related home and school environment. Brain data focus on the hemodynamic responses in the child’s parietal lobe during a math task. This study describes the sample, data collection tools, procedures, data analysis, and results related to both behavioral and brain data categories.

3.2. Sample

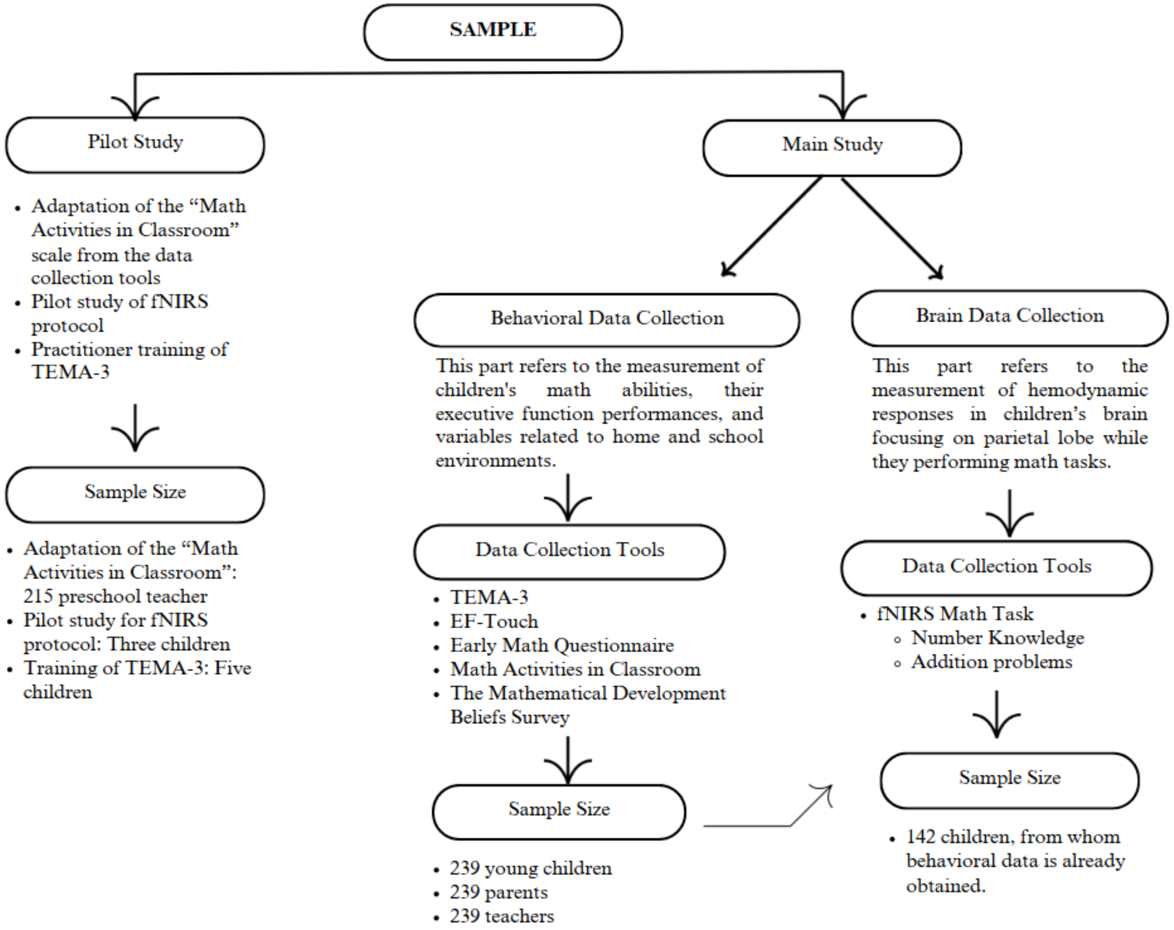
In this study, data were collected from three sources: children, parents and early childhood educators. The sample includes children in the direct measurement of skills, parents providing home-related assessments and teachers providing school-related assessments. The equal-sized stratified random sampling, which is one of the probability sampling methods referring to representation of each subgroup’s equally via randomly selection of an equal number of individuals from subgroups (Mills et al., 2016), is employed. In accordance with this sampling method, typically developing children, between 51-74 months, attended kindergartens and preschools within primary schools in İstanbul province (see Table 1). Their classroom teachers and parents were identified. Istanbul province was chosen for data collection because it has the highest population density of preschool children (Turkish Statistical Institute (TURKSTAT), 2023) and the fNIRS device used in the research is conveniently accesible in İstanbul, as it belongs to Acıbadem Mehmet Aydınlar University Faculty of Engineering and Natural Science. Since the study examines variables related to teacher features and classroom practices, one child from each classroom in each school, along with their parents, is included. This ensures that each sample group contributes uniquely to the data, preventing repeated data collection (i.e., matching teachers’ characteristics with all child data).

To determine an adequate sample size, the literature was reviewed. Since the current study aims to model a young children’s math abilities using multivariate analysis and includes latent variables, SEM is employed. Previous studies suggest that a sample size of at least 200 participants is necessary to conduct SEM (Barrett, 2007; Boomsma, 1982; Kline, 2023). A sample size of 200 or more participants is considered reliable because it minimizes bias in parameter estimation and standard errors, and the 95% confidence interval for parameters aligns closely with theoretical expectations when covariance matrices are taken into account (Boomsma, 1982).

Table 1. The descriptive statistics for districts, number of schools and number of participants

District	Number of schools	Number of classrooms & participants
Beşiktaş	17	37
Kadıköy	16	34
Kağıthane	18	77
Şişli	13	44
Üsküdar	17	47
Total	81	239

The determined sample size is applied at various stages of the data collection procedure. Since the study requires pilot studies and two main analysis steps, the sampling process follows a step-by-step approach (see, figure 6). Initially, pilot studies are conducted to adapt the data collection tool (Math Activities in Classroom), determine the fNIRS protocol, and train data collection tools using separate samples from the main study. The data gathered during the main study is also divided into two parts due to consent requirements. In the first part of the data collection, behavioral data are gathered from 239 young children, along with their teachers and parents. This phase includes paper-pencil-based questionnaires and surveys for teachers and parents, as well as performance-based assessments of children’s math abilities and executive function. Following participants’ consent, brain data collection is conducted. In this phase, the same children who provided behavioral data in the first part participated in the study. However, due to consent considerations 142 children completed the brain data collection process.



- Adaptation of the “Math Activities in Classroom” scale from the data collection tools
- Pilot study of fNIRS protocol
- Practitioner training of TEMA-3

- Adaptation of the “Math Activities in Classroom”: 215 preschool teacher
- Pilot study for fNIRS protocol: Three children
- Training of TEMA-3: Five children

This part refers to the measurement of children's math abilities, their executive function performances, and variables related to home and school environments.

- TEMA-3
- EF-Touch
- Early Math Questionnaire
- Math Activities in Classroom
- The Mathematical Development Beliefs Survey

- 239 young children
- 239 parents
- 239 teachers

This part refers to the measurement of hemodynamic responses in children’s brain focusing on parietal lobe while they performing math tasks.

- fNIRS Math Task
  - Number Knowledge
  - Addition problems

- 142 children, from whom behavioral data is already obtained.

Figure 6. The sampling and data collection procedures

Given the required sample sizes, for both the adaptation study and the main study, large samples were necessary. Specifically, the adaptation of the “Math Activities in the Classroom” questionnaire was completed by 215 early childhood educators. In the main study, behavioral data, which included questionnaires and surveys from parents and teachers, as well as performance-based assessments of children’s math abilities and executive function, were collected from 239 participants in each group, including teachers, parents, and children.

3.2.1. Sample of Behavioral Data

For the study’s behavioral part, data were obtained from teachers of 239 preschool classroom teachers (see Table 2), one selected child from each classroom (see Table 3), and the child’s parents (see Table 4) in the 2023-2024 academic year. The teachers who participated in the study were 97.5% female with a mean age of 36.39 years (SD=8.9). In addition, 94.6% of the teachers were working in public schools and the average length of service was 13.07 years (SD=8.17). On the other hand, 79% of the teachers had graduated from bachelor's degree programs.

Table 2. Descriptive statistics of teachers

Variables		**$\bar{X}$**	**Sd**	***F***	**$\%_{cum}$**
Age		36.39	8.9		
Years of working in current school		13.06	8.17		
Years of total working		13.07	8.17		
Income (₺)		30686.31	11534.44		
Sex	Female			233	97.5
	Male			6	2.5
	Total			239	
School type	Public			226	94.6
	Private			13	5.4
	Total			239	100
Educational level	High school			1	.5
	Associate degree			17	7.9
	Bachelor's degree			169	79
	Master degree			24	12.6
	Missing			15	
	Total			214	100

The study included 127 girls and 112 boys with typical development. The mean age of the children was 66.58 (SD=4.78) months, 50.6% were in their second year of preschool education and 70.4% had siblings.

Table 3. Descriptive statistics of children

Variable	**Groups**	$\bar{X}$	**Sd**	***F***	$\%_{cum}$
Age (in month)		66.58	4.78		
Sex	Girl			127	53.1
	Boy			112	46.9
	Total			239	100
Duration of preschool attendance	1 year			85	36.5
	2 years			118	50.6
	3 years and above			30	12.9
	Missing			6	
	Total			239	100
Number of children in the family	1 child			68	29.7
	2 children			133	58.1
	3 children and above			28	12.3
	Missing			10	
	Total			239	100

Regarding the parents who participated in the study, the average age of the mothers was 36.23 (SD=5.54), 29.8% had a bachelor’s degree, and 62.1% were unemployed. The mean age of the fathers was 39.28 (SD=5.50), 34.8% had high school education, and 88.8% were employed full-time.

Table 4. Descriptive statistics of parents

	Variable	**Groups**	$\bar{X}$	**Sd**	***F***	$\%_{cum}$
Mothers	Age		36.23	5.54		
	Educational level	Illiterate			2	.9
		Literate/non-degree			4	1.7
		Primary school			7	3.0
		Secondary school			30	12.8
		High school			66	28.1
		Associate degree			37	15.7
		Bachelor's degree			70	29.8
		Master’s degree			18	7.7
		PhD			1	.4
		Missing			4	
		Total			239	100
	Employment status	Full time			64	27.6
		Part-time			24	10.3
		Unemployed			144	62.1
		Missing			7	
		Total			239	100

Table 4. (continued)

Fathers	Age		39.28	5.50		
	Educational level	Primary school			13	5.6
		Secondary school			20	8.7
		High school			80	34.8
		Associate degree			27	11.7
		Bachelor's degree			71	30.9
		Master’s degree			19	8.3
		Missing			9	
		Total			239	100
	Employment status	Full time			199	88.8
		Part-time			22	9.8
		Unemployed			3	1.3
		Missing			15	
		Total			239	

3.2.2. Sample of Brain Data

The researcher visited the schools a second time after collecting behavioral data in order to gather brain data. Brain data were collected from 141 children whose behavioral data had already been gathered. Of these 141 typically developing children, 74 were girls and 68 were boys. Their average age was 66.98 (SD=4.86) months, 49.3% of them had been attending preschool for two years and 70% had siblings. Regarding the parents of these 141 children, the average age of the mothers was 35.89 (SD=5.71) years, while the average age of the fathers was 38.49 (SD=6.16) years. The 33.3% of the mothers completed their bachelor’s degree and 36% of the fathers completed high school. 65.5% of the mothers were unemployed and 86.6% of the fathers were employed full-time. Taking into account the teachers, 97.2% of them were female with an average age of 37.59 (SD=8.09). 93.7% of the teachers worked in public schools and their mean years of experience was 13.52 years (SD=8.05). 73.2% of the teachers were bachelor’s degree graduates.

3.3. Data Collection Instruments

In this section, the data collection tools used in the research are introduced. Since data are obtained from three sources, children, parents and teachers, the tools are grouped accordingly. Firstly, the tools used to measure children’s math abilities and executive function are introduced under the heading “Child-Oriented Performance-Based Tools.” Secondly, the tools completed by parents, including the “Child and Family Demographic Information Form” and the “Early Math Questionnaire,” are presented under the heading “Home Math Environment,” as these tools align with the conceptualization of variables in the current study according to the literature. Thirdly, the tools completed by teachers, including “Teacher Demographic Information Form”, “Math Activities in Classroom”, and “The Mathematical Development Beliefs Survey” are introduced under the heading “School Math-Related Environment,” based on the relevant literature.

In the context of the current study, the mathematics abilities of young children, their executive function performances, activities conducted at home and school, and the beliefs of adults are measured. For this purpose, the skill being assessed (i.e., math ability, home math activities, parental beliefs, classroom math activities, and teachers’ beliefs), age appropriateness, and the validity and reliability studies conducted for the Turkish culture were key criteria in selecting the measurement tools. Based on this, the “Test of Early Mathematics Ability-Third Edition,” which allows for a comprehensive measurement of math abilities and has validity and reliability studies, was used to measure children’s math abilities. The “Early Math Questionnaire” was used to assess home activities and beliefs, as it has been validated and reliable. The “Math Activities in Classroom” scale was used to evaluate classroom activities, and the “The Mathematical Development Beliefs Survey” was used to measure teachers’ beliefs, both of which have been validated and proven reliable through previous studies. Lastly, the “EF Touch” tool was used to measure executive function, including all its sub-dimensions, with corresponding validity and reliability studies. Detailed information about these measurement tools is provided below (see Table 5).

Table 5. Data collection tools

Name of the Tool	Source	Type	Measured feature	Data Type	Related Research Question(s)(RQ)
Test of Early Mathematics Ability-Third Edition (TEMA-3)	Child	Performance-based task	Math abilities of children	Behavioral	RQ1, RQ1.3, RQ1.4, and RQ1.5
fNIRS Math Task			Hemodynamic activities in parietal lobe	Brain	RQ1, RQ1.1, and RQ1.2
Executive Function Touch (EF Touch)			EF component: working memory, inhibitory control, cognitive flexibility	Behavioral	RQ1 and RQ1.3
Child and Family Demographic Information Form	Parent	Parent-report survey	Child and parent demographic characteristics	Behavioral	RQ1.4
Early Math Questionnaire (EMQ)		Parent-report questionnaire	Parent-child math related activities, and parental beliefs about math	Behavioral	RQ1 and RQ1.4
Teacher Demographic Information Form	Teacher	Teacher-report survey	Teachers demographic characteristics	Behavioral	RQ1.5
Math Activities in Classroom			The implemented math activities in classroom by teacher	Behavioral	RQ1 and RQ1.5
The Mathematical Development Beliefs Survey			Teacher’s beliefs about math	Behavioral	RQ1 and RQ1.5

3.3.1. Child-oriented performance-based tools

In this section of the the thesis, the data collection tools directly implemented with children are presented. Specifically, the tools used to measure their math abilities and executive function performances are explained in detail. The “Test of Early Mathematics Ability-Third Edition” (TEMA-3), a standardized instrument, was used to measure young children’s math abilities. Additionally, the “fNIRS Math Task,” a specifically designed protocol focusing on the number knowledge and addition items from the TEMA-3, was utilized for fNIRS measurement. Furthermore, three subtests from the “EF-Touch” battery were employed to assess children’s executive function performance. The features of these measurement tools are described below.

3.3.1.1. Test of Early Mathematics Ability-Third Edition (TEMA-3)

TEMA-3, developed by Ginsburg and colleagues (2003), is a norm-referenced and standardized, person-based test designed to assess both informal (e.g., counting and relative quantity awareness) and formal mathematics abilities (e.g., addition and subtraction) in children aged 3–8 years. The psychometric properties of the test were examined by Bliss (2006), and the following information is based on that study. The internal validity of TEMA-3 was calculated using the alpha coefficient, with values ranging between .92 and .96. Content validity was measured using discriminant analysis, with scores ranging from .45 to .68, and the correlation between other mathematics tasks (KeyMathR, Basic Concepts subtest) ranged from .54 to .91.

The Turkish adaptation of TEMA-3 was conducted by Erdoğan (2006). In this study, the researcher translated the A and B forms of the test and had the translations evaluated by an expert fluent in both languages. Subsequently, the Turkish version was back-translated into English and reviewed again by the language expert. Reliability values for the test-retest analysis were found to range between .88 and .90. Additionally, the Kuder Richardson-20 coefficient was calculated to determine test reliability, with values ranging from .92 to .96 (as cited in Avcı, 2015). In the current study, the correlation between TEMA-3 scores and the fNIRS Math Task, which consists of the number knowledge and addition problem items from TEMA-3, was computed. The correlation values were found to be .718 for raw scores and .564 for standardized scores.

3.3.1.2. fNIRS Math Task

One of the aims of the study is to examine hemodynamic changes in children's parietal lobe during mathematical operations. To achieve this, a special setup was prepared. First, the researcher designed a set of number knowledge and addition problems based on TEMA-3. Building on previous studies with young children, which focused on operations and number identification tasks (see Artemenko et al., 2018; Artemenko et al., 2022; Hyde et al., 2010), the experiment was designed to ensure the tasks were developmentally appropriate. Also, it is presented to children in the video format.

The first part of the experiment consisted of three task phases of increasing difficulty, with five numbers in each set. At the beginning of this task, children were expected to recite the names of the numbers. In the first phase, children were shown single-digit numbers (e.g., 1, 3, 4, 8, 9). In the second phase, they were shown two-digit numbers (e.g., 10, 27, 39, 56, 94). In the third phase, they were presented with three-digit numbers (e.g., 107, 164, 270, 326, 589).

The second part of the experiment included two task phases, each with five addition operations. During the first task of this section, children were presented with quantitative expressions (i.e., number of marbles representing the digit expression) for addition operations with one-digit outcomes (e.g., 1 + 2 = ?, 2 + 2 = ?, 3 + 2 = ?, 4 + 3 = ?, 5 + 2 = ?). In the second phase, addition problems with two-digit results were presented (e.g., 8 + 2 = ?, 7 + 4 = ?, 9 + 3 = ?, 8 + 5 = ?, 9 + 7 = ?). These problems also included both numerical and quantitative representations. Children were instructed verbally (e.g., "How many marbles does one marble plus two more marbles equal?") and were asked to say their answers aloud. The administrator recorded the children's responses.

The experiment was conducted using a block design, a specialized comparison paradigm that maintains cognitive engagement by presenting stimuli sequentially within a condition and alternating these with other moments (epochs) featuring a different condition (Amaro et al., 2006). In this study, the rest and task phases followed a two-state cycle known as the "AB block" design (see Figure 7). During the rest phase, an image of a meditating child was presented as a child-friendly fixation point, and the child was instructed to focus on this image. The design and task order are detailed below.

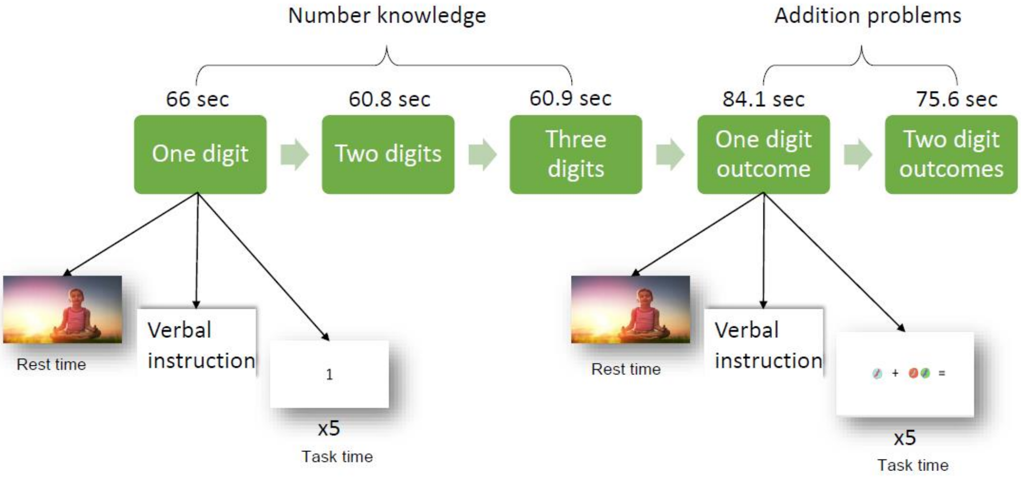


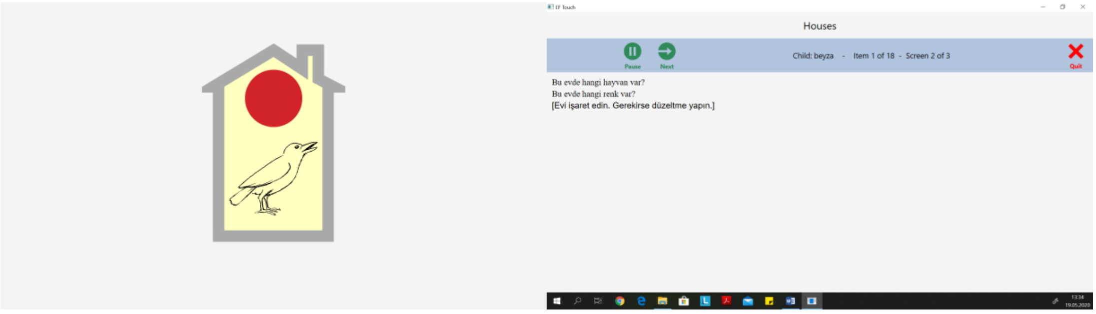
Figure 7. The sample of the AB block design

3.3.1.3. Executive Function-Touch

This measurement tool, originally designed by Willoughby and colleagues (2012) as a paper-and-pencil test known as the "Executive Function Battery," was later computerized by Willoughby and colleagues in 2016. The battery consists of three working memory tasks, three inhibitory control tasks, one cognitive flexibility task, and one reaction time task, all of which demonstrated a good fit based on the results of SEM (Structural Equation Modeling). The reliability coefficients for the scale were found to be .99 and .76 in test-retest results. According to a study on maximal reliability, the Houses, Something's the Same, and Pig tasks are the most effective for measuring executive function at ages three to five (Willoughby et al., 2013).

This measurement tool was adapted into Turkish by Hamamcı et al. (2023). In SEM analysis conducted according to the model from the original study, it was found that the Turkish version of the tool had a good fit. For reliability analysis, the composite reliability coefficient was calculated as .80. In the present study, Cronbach's Alpha values for the Houses, Pig, and Something's the Same tasks were computed as .865, .611, and .736, respectively.

Working memory task: The Houses task (see Figure 8) requires holding multiple representations of an object in mind simultaneously and selecting the correct one. Children are shown houses featuring a colored circle and a drawing of an animal, with both elements placed within the house frame. First, the administrator asks the child which animal and color are inside the house. When the screen advances, only the house frame remains, and the child must either identify which animal lives in the house or which color is present. Since two pieces of information must be retained, the child must select the appropriate response and deactivate irrelevant details. As the task progresses, the number of houses on each page increases according to the difficulty level. The first 11 items of the 18-item task are suitable for three-year-olds, while all items are appropriate for four- and five-year-olds.

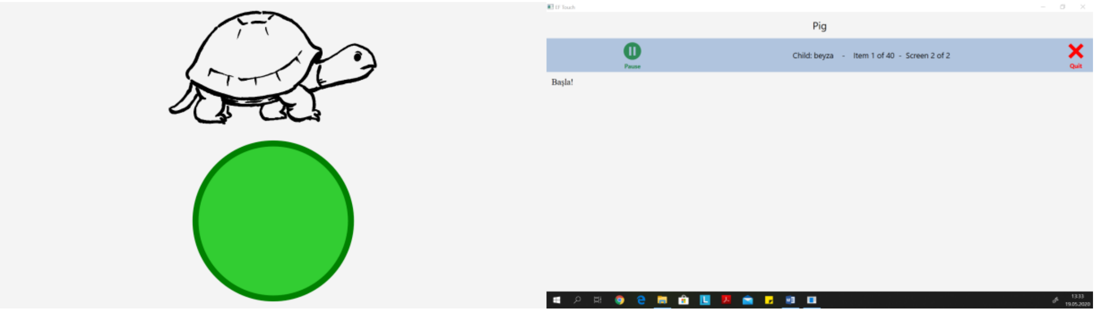


Child: beyza - Item 1 of 18 - Screen 2 of 3

Bu evde hangi hayvan var?
Bu evde hangi renk var?
[Evi işaret edin. Gerekirse düzeltme yapın.]

Figure 8. The houses task

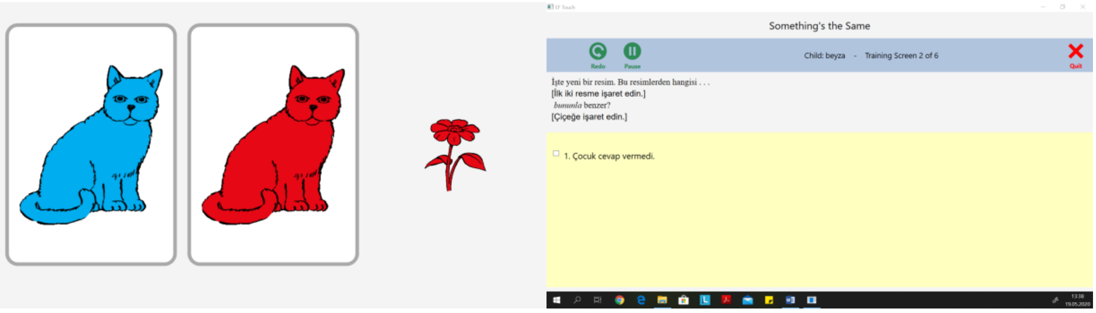
Inhibitory control task: In the Pig task (see Figure 9), a standard go-no-go application, children are shown a green circle and an animal image. Children are instructed to touch the circle every time they see an animal, unless the animal on the screen is a pig. All animals are introduced to the child before starting the task. The task alternates between go trials (presenting non-pig animals) and no-go trials (presenting pigs and other animals). This 40-item task can be used with children aged three, four, and five.



Başla!

Figure 9. The pig task

Cognitive flexibility task: In the "Something's the Same" (STS) task (see Figure 10), the child is shown a page with two similar pictures in one dimension (context, color, or size). The practitioner clarifies the dimension of similarity (e.g., "Here is a cat and a chair. They are similar in one way. They are both yellow"). On the next page, in addition to the two pictures, a third new picture is presented. The third picture is similar to one of the first two, but in a different dimension. For example, if the first two pictures share the same dimension of color, the third picture should match in a dimension like shape or size. The child is asked to choose which of the original pictures is similar to the new one. This 30-item task can be used with children aged three, four, and five.



İşte yeni bir resim. Bu resimlerden hangisi . . .
[İlk iki resme işaret edin.]
bununla benzer?
[Çiçeğe işaret edin.]

☐ 1. Çocuk cevap vermedi.

Figure 10. The something's the same task

3.3.2. Home math environment

In this section of the study, the data collection tools related to the home math environment are introduced. In this regard, the "Child and Family Demographic Information Form" and the "Early Math Questionnaire" are presented. The first form aims to gather information to determine the demographic characteristics of the child and the family. The second questionnaire is used to measure parent-child math activities in the home environment and parental beliefs about math, which are key variables of the study. The features of these measurement tools are explained below.

3.3.3. Child and Family Demographic Information Form

This form includes demographic information about the family and the child. The family-related questions cover the age, education level, employment status, and household income of the parents. The child-related questions inquire about the child’s chronological age, developmental characteristics, special needs (e.g., dyscalculia, etc.), ongoing illnesses, accidents, and medication use.

3.3.4. Early Math Questionnaire (EMQ)

The questionnaire developed by Missall et al. (2015) is administered in a paper-and-pencil format, with parents providing the responses. The original scale consists of three parts: a questionnaire that includes personal/demographic information, a 5-point Likert-type questionnaire assessing mathematics-related activities, and a 4-point Likert-type questionnaire measuring parental beliefs about mathematics. In the first section, which covers demographic characteristics, 19 questions are asked to gather information about the age, gender, and educational background of the parent, as well as the age, gender, mother tongue, developmental characteristics, and child care/preschool participation of the child. The second section contains questions that reflect early mathematics content in daily parent-child interactions, covering areas such as numbers and operations (19 items; sample item: “I encourage or help my child to count out a number of items from a larger group”), geometry (9 items; sample item: “I encourage or help my child to put shapes together to make a larger shape”), measurement (5 items; sample item: “I encourage or help my child to use measuring cups and spoons to measure and discuss amount”), and algebra/pattern (3 items; sample item: “I encourage or help my child to recognize patterns or repeating sequences of things in everyday settings and activities”).The third section consists of 13 questions focused on parents’ beliefs about mathematics (sample item: “Young children should learn about mathematics in the preschool setting”).

The adaptation study of the scale into Turkish was conducted by Karakuş (2022). In this adaptation, the second and third sections were translated and culturally adjusted. In the confirmatory factor analysis, which preserved the original item structure, it was found that the factors related to numbers, geometry, measurement, and patterns significantly loaded in the mathematics activities section and showed good fit with the model. Similarly, in the mathematics beliefs section, the confirmatory factor analysis, which maintained the number of items, indicated that the data fit well with the model.

For reliability, the Cronbach's Alpha values of the questionnaire's total score (α = .98), as well as its subcomponents, were calculated. These include parent-child math activities (numbers and operations [α = .96], geometry [α = .94], measurement [α = .87], and pattern [α = .96]), as well as individual math beliefs (child beliefs [α = .77] and parent beliefs [α = .80]).

3.3.5. School math-related environment

In this section of the thesis, the teacher-reported measurement tools used to assess the school math environment are presented. To determine teachers' demographic characteristics, the "Teacher Demographic Information Form" is employed. To measure in-class math activities, the "Math Activities in Classroom" survey is used. Additionally, the "Mathematical Development Beliefs Survey" is completed by teachers to measure their beliefs about early mathematics. Detailed information on these tools is provided below.

3.3.5.1. Teacher Demographic Information Form

The form consists of eight questions that gather information about teachers' demographic and professional characteristics. The questions include the teachers' age, gender, the type of institution they work for, the year they began working at their current institution, their total years of teaching experience, income, educational level, and any professional courses they have attended.

3.3.5.2. Math Activities in Classroom

The questionnaire developed by Choi and Dobbs-Oates (2014) measures math-related activities provided in the classroom, with teacher ratings. These activities include counting, basic operations, shapes and patterns, measurements, and manipulatives. The instrument consists of 10 items (sample item: “How often do you provide opportunities in your classroom for children to engage in the following activities? Work with counting manipulatives (things for children to count) to learn basic operations (adding or subtracting)”). It uses a Likert-type scale, with responses ranging from 1 (never) to 6 (every day).

The translation of this scale into Turkish was conducted as part of the current thesis. Data were collected from 215 early childhood educators during the 2022-2023 academic year (see Table 6). According to the literature, the minimum sample size for Confirmatory Factor Analysis (CFA) is 200 subjects (Barrett, 2007; Boomsma, 1982; Kline, 2023), and it is also recommended that there be at least 20 participants per item based on the number of items and sample size (Jackson, 2003). Since this measurement tool in the current pilot study consists of 10 items, a sample size of 200 participants is sufficient for ratio-based calculations. The data from 215 participants thus constitute an acceptable sample for analysis.

CFA was applied to test validity, and Cronbach’s Alpha coefficient was calculated to determine reliability. The CFA results (see Table 7 and Figure 9) showed that the data fit the model well (χ^2 (35) = 99.125, $p < .001$, CFI = .94, TLI = .92, NFI = .91, RMSEA = .94 [90% CI: .07 – .11], SRMR = .043, GFI = .99). The Cronbach’s Alpha coefficient was .90, indicating acceptable reliability (see Kline, 1999), and the composite reliability coefficient was found to be .92.

Table 6. Descriptive statistics of pilot sample

Variables	**x̄**	**SD**	***f***	**%**
Age	38.785	8.658		
Years of Working in Current School	6.718	6.855		
Years of Total Working	12.790	8.557		

Table 6. (continued)

Income		18408.567	13518.081		
Sex	Female			207	96.279
	Male			8	3.721
	Total			215	100.000
School Type	Public			110	51.163
	Private			105	48.837
	Total			215	100.000
Educational Level	HighSchool			7	3.256
	Bachelor's degree			165	76.744
	Master Degree			42	19.535
	PhD			1	0.465
	Total			215	100.00

Table 7. CFA Results

α =.90 *N*= 215	CFA Loadings	Total Item Correlations	Lower 27%			Higher 27%			*t*
			N	*M*	*SD*	*n*	*M*	*SD*	
Item 1	.81	.68	58	3.81	1.16	58	5.77	.42	-12.11**
Item 2	.72	.61	58	3.84	1.13	58	5.62	.52	-10.80**
Item 3	.83	.70	58	3.19	.87	58	5.30	.67	-14.56**
Item 4	.90	.76	58	3.27	.83	58	5.50	.57	-16.77**
Item 5	.89	.65	58	2.75	.92	58	4.98	.78	-13.98**
Item 6	.85	.73	58	3.18	.68	58	5.15	.74	-14.77**
Item 7	.84	.70	58	2.55	.73	58	4.65	.76	-15.18**
Item 8	.88	.58	58	2.19	.88	58	4.72	1.18	-13.06**
Item 9	.92	.69	58	2.77	.77	58	5.08	.82	-15.58**
Item 10	.95	.59	58	2.46	1.07	58	5.25	.98	-14.56**

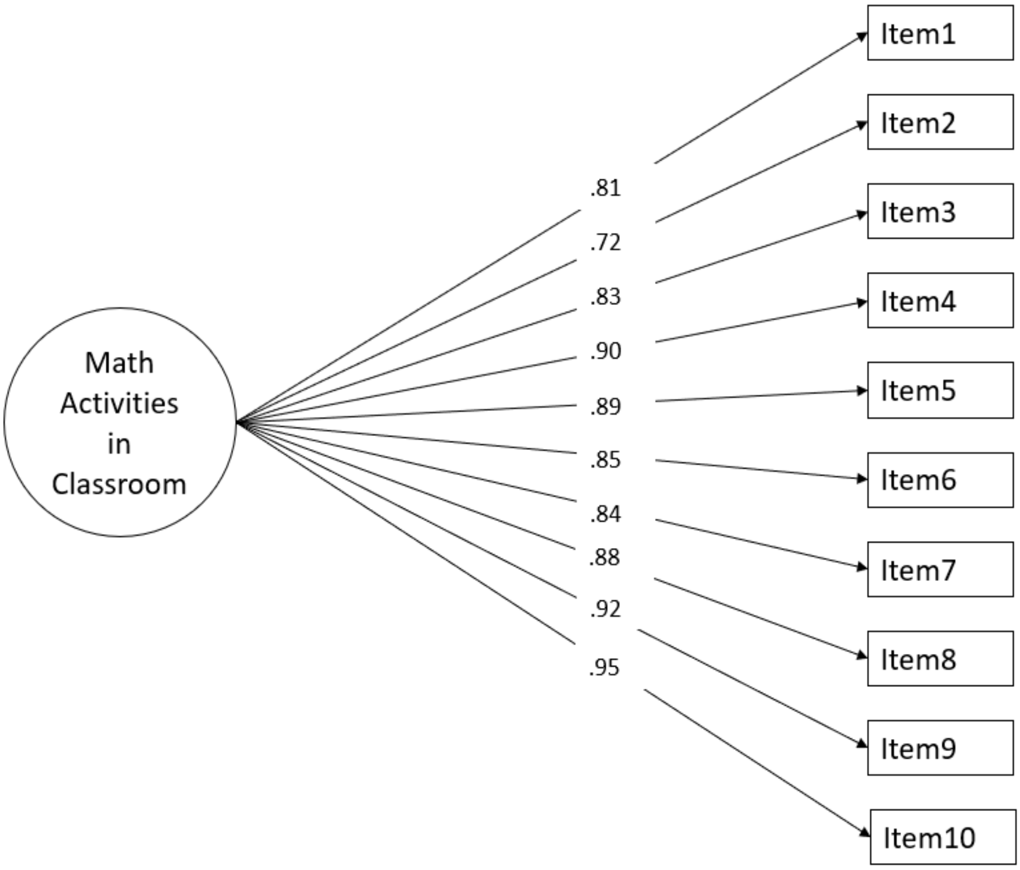


Figure 11. Path diagram of the CFA

3.3.5.3. The Mathematical Development Beliefs Survey

“The Mathematical Development Beliefs Survey," developed by Platas (2015), measures early childhood educators’ beliefs about mathematics, focusing on mathematical development as a primary goal of preschool education, the age-appropriateness of mathematics instruction, their confidence in providing mathematics instruction, and the classroom’s role in generating mathematical knowledge. The reliability values for the sub-dimensions of the scale were .85, .92, .89, and .83, respectively. The adaptation of the measurement tool into Turkish was conducted by Karakuş et al. (2018), and in this study, the consistency values for the sub-dimensions were found to be .82, .88, .84, and .88, respectively. The Cronbach’s Alpha values for the total score of the survey were found to be .79, while the subcomponents yielded the following results: optimum age for mathematics teaching (sample item: “Mathematical activities are age-appropriate for preschoolers”) scored

.74, classroom locus for the generation of mathematical knowledge (sample item: “Preschoolers learn mathematics best through direct teaching”) scored .46, mathematical development as a primary aim of preschool education (sample item: “Development in academics such as math is a goal of preschool”) scored .61, and confidence in mathematics education (sample item: “Math would be easy for me to incorporate into preschool curricula”) scored .71

3.4. Procedure

As the first step of the implementation, ethical permission was obtained from the Middle East Technical University Human Subjects Ethics Committee (see Appendix 1), as well as from the Ankara and İstanbul Directorate of National Education (see Appendix 2). Following this, pilot studies were conducted, which included a validity and reliability study of the “Math Activities in Classroom” tool, training for implementing the TEMA-3, and the standardization of the fNIRS procedure. Subsequently, the main study was carried out, where the data collection procedures for both behavioral and brain data were followed. Each of these steps is introduced below.

3.4.1. Pilot Studies

Prior to the main study, the researcher conducted pilot studies on the measurement tools to be used in the study. Since a validity and reliability study for one of the instruments had not been previously conducted in the Turkish context, a validity and reliability study for this instrument was carried out during the spring semester of 2022-2023. The processes of this study are explained in detail in the heading “3.3.5.2. Math Activities in Classroom,” under the section “3.3.3. Data Collection Instruments.”

Additionally, a training process and pilot study were conducted to fulfill the criteria for becoming a practitioner of the measurement tool titled “Test of Early Mathematics Ability - Third Edition,” which is used to assess children's mathematics skills. Finally, a pilot study was carried out on the use of the fNIRS device and the design of the task. The details of these processes are explained below.

3.4.1.1. Test of Early Mathematics Ability - Third Edition

To implement this test, a five-hour theoretical training session was conducted by the researcher and her team, who adapted the test to the Turkish context. Pilot studies were carried out with five children, and feedback was collected during this process as part of the supervision. The researcher attended a five-hour theoretical training session on April 15, 2023, to gain a comprehensive understanding of the test's structure, scoring, application process, instructions, and key considerations. Following this training, the researcher administered the test to five volunteer children in Ankara. One of these children was videotaped with the permission of both the parents and the child. This video recording was shared with the researchers who provided the theoretical training, and supervision was obtained regarding the proper implementation of the test. After completing this practice, the researcher was awarded a test practitioner certificate on June 5, 2023 (certificate no: 03230415).

3.4.1.2. fNIRS Tasks

To design the task used in fNIRS measurement, tools, device modifications, and pilot implementations with children, the Acıbadem Mehmet Ali Aydınlar University Bioengineering Department fNIRS laboratory was visited for five days during the spring semester of the 2022-2023 academic year.

During the task design process, three field experts were consulted, and the questions from TEMA-3 were videotaped to ensure consistency between behavioral and biological data. This videotaped task was transferred to the experimental environment using the PsychoPy (Peirce et al., 2019) program and coordinated with the NIRSport (NIRx, 2024) machine product. In the NIRX application (NIRStar 15.3), a parietal lobe setup with 8 detectors and 8 sources (scan rate: 7.81) was implemented, and the task was made ready for implementation. To test this task design, applications were conducted in the university's laboratory with one girl and one boy under the supervision of the researcher and two field experts. During these sessions, the video content was first explained to the children, the device and cap were introduced, and the children were allowed to examine the cap. The cap was then placed on the children after obtaining consent from both the children and their parents.

Once the participants, cap, and devices were ready for application, calibration tests were performed, and data quality from the channels was checked to ensure the device received high-quality data. For channels where quality data could not be obtained, the optodes were removed, the hair in the area was cleaned with a cotton swab, and the optodes were repositioned. Once calibration tests indicated appropriate data quality, data acquisition began. To synchronize the NIRX data with the PsychoPy data, video start markers were added in the NIRX data using a trigger. At the end of the experiment, data acquisition was stopped, the children were thanked, and a gift was given for their participation. Following these sessions, the task design was updated based on feedback from the field experts to improve its effectiveness. The field experts consisted of one early childhood education expert and two biomedical engineer experts. In order to do this the early childhood educator find and organize the child appropriate questions and two biomedical engineers assess the sequence and presentation of the stimuli in order to gather valid data. In this updated task design, the video block was revised to include three number knowledge tasks and two addition tasks, and this ensures alignment with the overall structure and preparation for the main study. For more information on this task design and its relaibility, please refer to “3.3.2. fNIRS Math Task” in the “3.3.3. Data Collection Instruments” section.

3.4.2. Main Study

The main study was conducted in two phases during the 2023-2024 academic year. In the first phase, the researcher collected behavioral data, including TEMA-3, EF-Touch, parent reports, and teacher reports. In the second phase, fNIRS application was conducted with children who participated in the first application and agreed to take part in the second phase.

3.4.2.1. Behavioral Data Collection Process

The researcher contacted the school administrations determined through randomization and sought their permission. The randomization process was based on the list of preschool education institutions in Istanbul province, available on the website mebbis.meb.gov.tr/KurumListesi.aspx. After making contact, cooperation was established with the teachers in the institutions that accepted participation. At this stage, another randomization process was carried out to select children from the class, aiming to ensure the participation of the child ranked 7th or a multiple of 7th on the class list. Then, consent forms were sent to the teachers and parents selected through randomization. After receiving the consent form, a demographic information form and questionnaire (i.e., Early Math Questionnaire) were sent to the parents to determine the developmental and socio-economic status of the children. Simultaneously, demographic forms and questionnaires (i.e., Math Activities in Classroom and The Mathematical Development Beliefs Survey) were sent to the teachers.

Subsequently, the researcher visited the schools to administer math ability and executive function measurements to the children whose forms had been filled out by parents and teachers. The researcher created a child-friendly atmosphere by placing TEMA-3 materials and equipment to implement EF Touch in a quiet and bright room, as directed by the school principal. The researcher then introduced herself to the designated classrooms and asked individual children (those who had received permission from their parents to participate in the study). Once in the implementation room, the researcher provided details about the activity and asked for the children's consent to participate. If the children gave consent, the implementation began. During the implementation, the researcher strictly followed the instructions to avoid any differences between the results. To this end, the researcher would say, "Remember, we are completing the task now; let’s talk about this after we finish. Okay?" If the child needed to use the restroom or drink water, the researcher would pause the task, note this in a notebook, and accompany the child for their needs. At the end of the task, the researcher said, "Thank you! I am grateful that you agreed to participate in my study and complete this task. I would like to give you a sticker to commemorate our work, and then let’s return to the classroom."

3.4.2.2. Brain data collection process

NIRSport2 device of NIRX company was used to measure hemodynamic changes in the brain. This device with 8 sources and 8 detectors was modified for the parietal lobe (see Figure 12a). In the experiment design for determining the Psychopy settings, the trigger (see Figure 12b) was used to mark the start of the video and to record the block times.

a) Sources and detectors **b)** Trigger, amplifier and cap **c)** Setup

Figure 12. The NIRX device

Besides measuring the parietal lobe, the modification settings for the NIRSport2 were uploaded into the system from the software library, and the placement of the optodes was determined accordingly (see Figure 13). In this setup, the red dots represent the source optodes, the blue dots indicate the detector optodes, and the purple lines signify the channels (see channel information in Table 8). The distance between each source and detector is 2-3 cm. Once the fNIRS measurement is taken, light in the near-infrared wavelength is transmitted from the sources to the tissues, where it is absorbed. The unabsorbed light is scattered and detected by the detectors. This area of photon scattering from the source to the detector is referred to as a channel. This modification includes 20 channels. Additionally, the substructures (i.e., postcentral gyrus, superior parietal lobe, supramarginal gyrus, angular gyrus, and intraparietal gyrus) (see Figure 14) of the parietal lobe were computed based on the reference from previous studies (see, https://github.com/nirx/fOLD-public; Artemenko et al., 2020) (see Table 9).

Table 8. The channel information accordingly the numbers of sources and detectors

Channel Number	Source Number	Detector Number
1	1	1
2	1	2
3	2	1
4	2	3
5	3	1

Table 8. (continued)

6	3	2
7	3	3
8	3	4
9	4	3
10	4	4
11	5	5
12	5	6
13	6	5
14	6	7
15	7	5
16	7	6
17	7	7
18	7	8
19	8	7
20	8	8

Table 9. The substructure information on parietal lobe

Substructure	Hemisphere	Optodes	Channels
Postcentral gyrus	Left	CP1-CP3, CP1-P1	1 and 2
	Right	CP2-CP4, CP2-P2	11 and 12
Superior parietal gyrus	Left	CP1-P1, CP1-CP3, P3-P1, P3-CP3	2, 1, 6, and 5
	Right	CP2-P2, CP2-CP4, P4-P2, P4-CP4	12, 11, 16, and 15
Supramarginal gyrus	Left	CP5-CP3, CP1-CP3, CP5-P5, P3-CP3	3, 1, 4, and 5
	Right	CP6-CP4, CP2-CP4, CP6-P6, CP4-P4	13, 11, 14, and 15
Angular gyrus	Left	P3-CP3, P3-P1, P3-P5, CP5-P5, CP5-CP3, CP1-CP3, CP1-P1, P7-P5	5, 7, 6, 4, 3, 1, 2, 9
	Right	CP6-CP4, P4-CP4, CP6-P6, CP2-CP4, P4-P2, P4-P6, CP2-P2, P8-P6	13, 15, 14, 11, 16, 17, 12, 19
Intraparietal Sulcus	Left	CP3-CP1	1
	Right	CP2-CP4	11

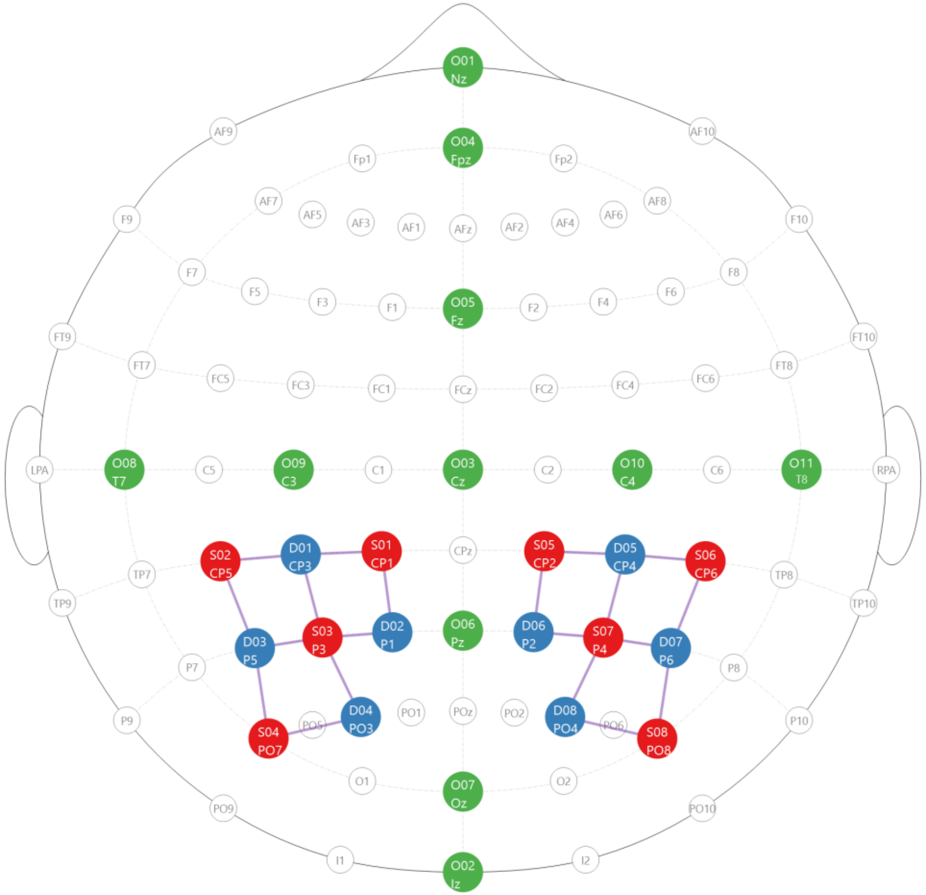


Figure 13. Optode placement and channels (see, Oostenveld et al., 2001)

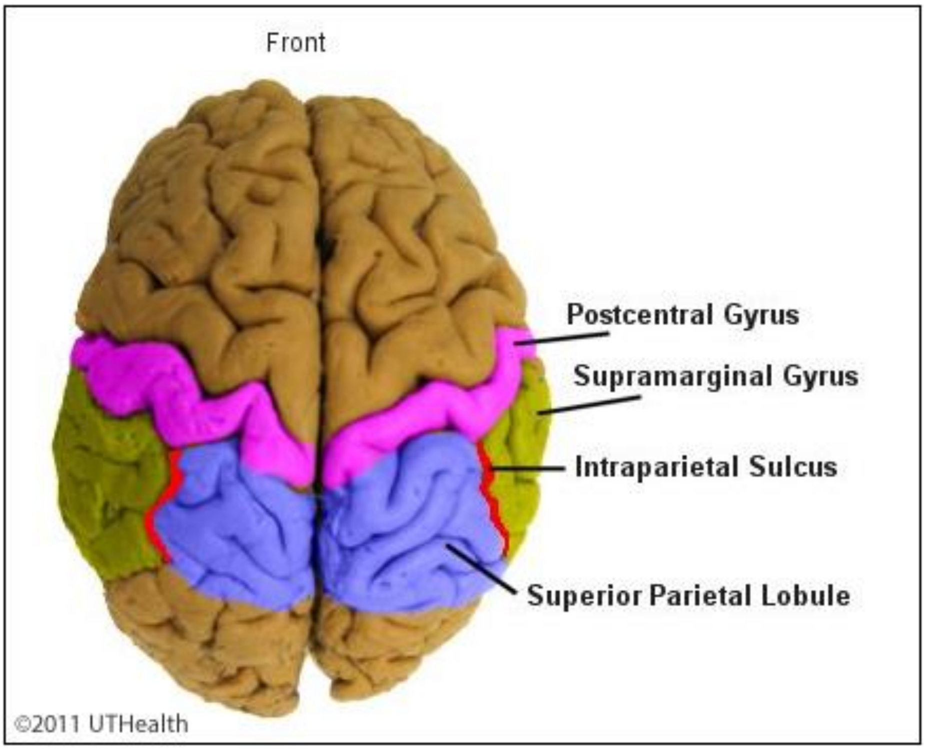


Figure 14. Substructures of parietal lobe
(recieved from, https://nba.uth.tmc.edu/neuroanatomy/L1/Lab01p14_index.html)

Prior to the implementation, a brief reminder and introduction were given to the child. The researcher began by saying, “Hello X! In our last meeting, I asked you some math questions using this booklet, and we played games on the computer. Do you remember what we did?”. Then, the researcher continued, “Today, we will look at some math problems again. I’m going to show you the math questions on this computer, but this time I’ve brought a cap, and you’re going to wear it while we work on our exercises. Here’s our cap; feel free to examine it. If you allow me, I would like to put this cap on your head.” If the child agreed, researcher responded with “thank you for your help,” and the cap was placed on the child’s head. At this stage, any questions from the child were addressed. If the child did not want to participate, they were thanked and sent back to the classroom. During the implementation, the hair in regions where the optodes were placed was loosened using a cotton swab in order to ensure clear data collection in areas with dense hair. Before the main application, a calibration measurement was taken to assess the quality of the data, and the main study was started after data acquisition from the channels was confirmed (see Figure 15). At the end of the session, the researcher thanked the children and asked their feedback regarding the task. After a brief chat, the child was sent back to the classroom.

a)Calibration phase **b)**Rest time **c)**Task time

Figure 15. The implementation environment and phases

3.5. Data Analysis

In this section of the thesis, the analysis steps for the data are presented. This part consists of two main components: behavioral data and the analysis of brain data. Firstly, the analysis of the behavioral data obtained is explained. In this context, information regarding bivariate correlation, SEM, and multiple regression are presented. Secondly, the data from the brain, obtained using the fNIRS technique, underwent a series of pre-processing steps before the analysis. The method for obtaining the mean value of oxygenation levels to be used in the analysis is explained. Lastly, the appropriate statistical method for analyzing brain data related to this variable is described.

3.5.1. Behavioral Data Analysis

This study examines the relationships between children’s math abilities, the substructure of the activated parietal lobe, children’s executive function performances, home math environments, and school mathematics-related environments within the scope of behavioral data analysis. In this context, the Pearson correlation coefficient was calculated to determine the bivariate relationships. Additionally, multiple regression and SEM were used to explain children’s math abilities.

3.5.1.1. Correlation

The relationship between child demographic characteristics, home math activities, school mathematics-related environment, executive function scores, and brain regions was found significant, as evaluated using the Pearson correlation coefficient. The assumptions for correlation, including normality (≤±2 for kurtosis and skewness; Tabachnick & Fidel, 2013) and the independence of observations, were checked.

3.5.1.2. Multiple Regression

The multiple regression refers to the statistical method used to predict the values of a criterion (dependent) variable based on a set of predictor (independent) variables. It aids researchers in scientific inquiry by providing a multifaceted approach, taking into account the combined and unique influence of numerous predictors on the dependent variable (Cohen et al., 2003). This prediction is mathematically expressed through the creation of a regression equation (Gravetter et al., 2016). The equation presents the association between a dependent variable (i.e., Y) and multiple independent variables (i.e., X_1, X_2, …, X_n) within a linear model. The equation is constructed as *"*$Y = B_1X_1 + B_2X_2 + B_3X_3 + \ldots + B_kX_k + B_0 + e$*"* (Cohen et al., 2003). In this equation, the notations B_1, B_2, B_3 represent the unique contribution of each variable (i.e., X_1, X_2, X_3) to Y. B_0 refers to the baseline value of the dependent variable (i.e., *Y)* when the predictors are zero; this is also called the intercept. The e represents the error term, which accounts for the unexplained variability in the dependent variable (i.e., Y).

To effectively employ multiple regression beyond simple correlation analysis, certain assumptions must be met. First, the sample size should be calculated using the formula $N > 50 + 8m$, where N is the sample size and m is the number of predictors (Tabachnick et al., 2007). Next, to ensure multicollinearity and singularity, there should be no high correlation ($r = .9$ or above) between the variables. Additionally, the independent variables should not include both sub-dimensions and the total factor. Furthermore, the assumptions of normal distribution, linearity, and homoscedasticity must also be satisfied (Pallant, 2011).

3.5.1.3. Structural Equation Modeling (SEM)

Structural equation modeling (SEM), also known as covariance structure analysis, analysis of covariance structures, or covariance structure modeling, refers to a family of statistical techniques, including factor analysis and regression (Kline, 2023; Sümer, 2000). Fundamentally, SEM tests theories by identifying a model that represents the predictions between plausible constructs measured through suitable observed variables (Hayduk et al., 2007). SEM has two key features: observed and latent variables. Observed (manifest) variables refer to directly collected categorical or continuous scores (Byrne, 2001), whereas latent variables represent hypothetical constructs or explanatory factors that are presumed to reflect a continuum, which cannot be directly observed (Kline, 2023). Additionally, latent variables are structured based on observed variables, which share common causes (Brown, 2006). In this sense, SEM is a comprehensive statistical approach used to test models that include both causal and bidirectional relationships between observed and latent variables. Furthermore, since the validity of the proposed model is tested (Cudeck et al., 2001), the fit of the model is determined using specific indices.

The present model is based on the HbO level in the related substructure of the parietal lobe (observed variable), children’s executive function performance (latent variable), home math environment (latent variable), and school math-related environment (latent variable) to explain children’s math abilities (observed variable). The model is evaluated using chi-square ($\chi 2$), goodness-of-fit index (GFI), root mean square error of approximation (RMSEA), comparative fit index (CFI), and standardized root mean square residual (SRMR). The cut-off criteria values for these indices used in the evaluation are provided in Table 10.

Table 10. The criteria values for model fit indices

Model fit index	**Criteria for acceptance**
CFI	≥0.95 (Hu&Bentler,1999)
TLI	≥.95 (Hooper et al., 2008)
NFI	≥0.90 (Thompson, 2004)
RMSEA	≤0.05 (Sümer, 2000)
SRMR	≤0.05 (Hu&Bentler,1999)
GFI	≥0.95 (Hooper et al., 2008)

3.5.2. Brain Data Analysis Steps

The analysis of the brain data collected via fNIRS consists of two parts: data preprocessing and data analysis (Schroeder et al., 2023; Yücel et al., 2021). The preprocessing part involves controlling the quality of the acquired signals, eliminating motion- and physiology-related noise, performing signal corrections, and converting the signals into oxyhemoglobin and de-oxyhemoglobin values. The data analysis part involves statistical analysis and interpretation, which is explained in detail below.

3.5.2.1. Data Preprocessing

The data preprocessing involves multiple steps. First, the Modified Beer-Lambert Law (mBLL) was applied. Next, temporal filtering, motion artifact correction, and signal quality assessment were carried out.

First, the signal levels of the raw data were checked to determine if they could be included in the analysis. Following this, undefined values in the acquired data were calculated using interpolation. Then, the raw light intensity readings were transformed into optical density (OD), oxyhemoglobin (HbO), and deoxyhemoglobin (HbR) values using the Modified Beer-Lambert Law (mBLL) procedure (Delpy et al., 1988). HbO and HbR are fundamental for measuring hemodynamic responses in the brain and detecting activation. Neurons in the brain require sufficient oxygen and glucose to meet their metabolic needs, and this relationship between neural activity and blood flow is referred to as neurovascular coupling (Phillips et al., 2016).

Secondly, the Signal-to-Noise Ratio (SNR) was computed to assess signal quality (Yücel et al., 2021). Third, the signal quality was enhanced by removing noise from the data. Since the data includes not only physiological responses related to the task but also motion-related responses and systematic noise (e.g., physiological noise like heartbeat or machine noise), cleaning irrelevant responses is crucial. Motion-related noise was reduced via Motion Artifact Correction (Brigadoi et al., 2014). Finally, filtering was performed using infinite impulse response (IIR) filtering (Yücel et al., 2021). These steps were carried out using HOMER, a MATLAB-based software package (Huppert et al., 2009).

3.5.2.2. Data Analysis

At this stage, the data processed through the preprocessing steps is analyzed using Analysis of Variance (ANOVA). ANOVA was employed to determine differences in hemodynamic responses in brain areas during task and rest periods for each math ability (i.e., number recognition and addition). After verifying the assumptions, a Three-Way ANOVA was applied, along with p-value ($p < .05$) and post-hoc analysis using the Bonferroni method (for results, see section 4.1. The determination of the math-related activation in the parietal lobe).

3.6. Summary of the Method Chapter

The methodological procedures are outlined in this chapter. The design of the study, sample features, data collection tools, procedure, and the data analysis methods are covered in this section. Hereby, the entire procedure, from design to analysis, is explained respectively.

In the first part of the chapter the design of the study is explained by considering the study's objectives and time dimensions. In this sense, the study is structured as a predictive cross-sectional study. This design was deemed suitable for the current study as it allows for examining relationships between variables and collecting data within a single time period.

The second part of the chapter covers the sample features, including the sampling technique and sample size. Since the variables in the current study are gathered from children, teachers, and parents, an equal-sized stratified random sampling technique is employed. This technique is particularly useful as it ensures proportional representation of each group. In line with this technique, participants were recruited from schools included in the Ministry of National Education's school list. The number of participants was determined according to the statistical method applied. In this context, more than two hundred participants were involved in the scale adaptation part during the pilot study and in the modeling study of the main study. In the scale adaptation study, the participants consisted of 215 early childhood educators. In the modeling part of the main study, 239 child-parent-teacher triads were included. After collecting data from these 239 children, data were further collected from 142 children to examine hemodynamic changes.

The third part of the chapter includes the data collection instruments. The “Test of Early Mathematics Abilities-3 (TEMA-3)”, “fNIRS math task”, “Executive Functions Battery (EF Touch)”, “Early Mathematics Questionnaire (EMQ)”, “Mathematics Activities in the Classroom Scale” and “Mathematical Development Belief Scale” were used to gather data. Each of the data collection instruments has been validated for use with Turkish sample, and their reliability values are acceptable. Information about each tool, including its type (e.g., survey or questionnaire), components, rating type, and sample items from both the original and adaptation studies, is provided in this part.

The fourth section of this chapter describes the data collection procedure. Data for the study were collected in two phases following a series of pilot studies. During the pilot studies, the training and supervision service received for the implementation of the mathematics test, the scale adaptation study and the fNIRS application study were explained. The main data collection involved first gathering behavioral data and then brain data.

The final section of this chapter outlines the analysis steps for the obtained data. The data required analysis in two separate procedures, as it includes both brain data and behavioral data. In this context, the steps for analyzing the data from fNIRS measurements are introduced, starting with the preprocessing steps, which include signal processing to remove noise. Following this, the statistical analysis of the fNIRS data is explained. Lastly, the behavioral data analysis is introduced, and the corresponding statistical methods are outlined.

CHAPTER 4

RESULTS

The current study aims to explain children’s math abilities in relation to hemodynamic changes in the parietal lobe, executive function performances, and the home and school math environments. In this section the findings are presented sequentially according to the research questions. First, the analysis is introduced, focusing on identifying which brain regions are activated while children perform mathematical tasks. In this context, the results of the preprocessing steps, which clean and filter the brain data, are presented. Then, the statistical analyses that determine the hemodynamic responses are introduced, including three-way ANOVA and bivariate correlation results that investigate the association between these hemodynamic responses and children's math abilities. This is followed by correlation analyses that explore the relationships between children’s math abilities and various factors (i.e., EF, home math environment, and school math-related environment). Next, the model studies are explained to demonstrate the roles of these factors in explaining children's math abilities. In this context, the results of structural equation models and multiple regression analyses conducted for the model studies are introduced.

4.1. The determination of the math-related hemodynamic responses in the parietal lobe

In this section of the study, where brain data is taken into account, preprocessing steps, feature extraction, and ANOVA are followed. Firstly, the processing phase is introduced, which includes cleaning the data and preparing it for ANOVA. Then, the parametric results, including ANOVA and correlation analyses, are described.

4.1.1. Processing of fNIRS data

Data were collected using the NIRx NIRSport system with 20 channels. Each recording lasted approximately 400 seconds, with a sampling rate of 7.18 Hz. Before the main analysis, a preprocessing step was conducted (see Figure 16). The raw signal, which was contaminated with noise, needed to be cleaned to ensure accurate results. The data were then preprocessed and segmented according to the tasks. During data collection, video recordings were made for each task, and this allowed the timing of each task to be determined. Consequently, the data were segmented from the onset of each task. Finally, features were extracted from the data for each channel under different conditions: rest, number knowledge, and addition. The features included: 1) the maximum signal value (MAX), 2) the mean signal value during the condition (MEAN), 3) the area under the curve (AUC) for each time period, and 4) the range of the signals in each condition (RANGE). These values were calculated for 20 channels per participant across the three conditions.

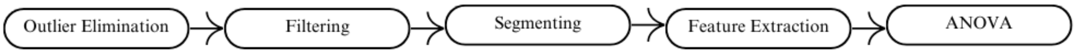


Figure 16. Preprocessing and feature extraction steps

4.1.2. The Investigation of the hemodynamic responses in the substructure of parietal lobe regarding children's math performances

The findings from the Three-Way ANOVA test, conducted during the initial examination of brain data collected while young children were performing math tests, are presented in this section. Given the block design, which included rest-time, number-knowledge task time, and addition task time (as detailed in section 3.3.1.2, fNIRS Math Task), the Three-Way ANOVA was employed to determine differences across these time periods. The primary objective of this analysis is to ascertain how children's brain activity varies before the task and to identify which brain regions become active during the math tasks. It is crucial to understand the level of brain activation elicited by the math test at this stage.

Table 11. The descriptive statistics of fNIRS measurement

Group	Laterality	Region	Mean	SD	SE	Coefficient of variation
Addition	Left	Angular Gyrus	-1.346×10^{-5}	2.720×10^{-5}	2.424×10^{-6}	-2.021
		Intraparietal Sulcus	4.291×10^{-6}	5.922×10^{-5}	5.255×10^{-6}	13.801
		Postcentral Gyrus	-1.418×10^{-5}	3.527×10^{-5}	3.130×10^{-6}	-2.487
		Superior Parietal Gyrus	-2.111×10^{-5}	2.756×10^{-5}	2.446×10^{-6}	-1.306
		Supramarginal Gyrus	-1.107×10^{-5}	3.247×10^{-5}	2.882×10^{-6}	-2.933
	Right	Angular Gyrus	-2.392×10^{-5}	2.247×10^{-5}	1.994×10^{-6}	-0.939
		Intraparietal Sulcus	-2.479×10^{-5}	3.059×10^{-5}	2.715×10^{-6}	-1.234
		Postcentral Gyrus	-2.923×10^{-5}	2.496×10^{-5}	2.215×10^{-6}	-0.854
		Superior Parietal Gyrus	-2.478×10^{-5}	2.456×10^{-5}	2.179×10^{-6}	-0.991
		Supramarginal Gyrus	-1.985×10^{-5}	2.589×10^{-5}	2.297×10^{-6}	-1.305
Number Knowledge	Left	Angular Gyrus	-1.081×10^{-5}	2.023×10^{-5}	1.802×10^{-6}	-1.871
		Intraparietal Sulcus	3.340×10^{-6}	4.476×10^{-5}	3.972×10^{-6}	13.400
		Postcentral Gyrus	-5.941×10^{-6}	3.268×10^{-5}	2.900×10^{-6}	-5.501
		Superior Parietal Gyrus	-1.509×10^{-5}	2.073×10^{-5}	1.839×10^{-6}	-1.373
		Supramarginal Gyrus	-8.880×10^{-6}	2.351×10^{-5}	2.086×10^{-6}	-2.647
	Right	Angular Gyrus	-8.417×10^{-6}	1.571×10^{-5}	1.394×10^{-6}	-1.867
		Intraparietal Sulcus	-1.798×10^{-5}	1.936×10^{-5}	1.718×10^{-6}	-1.077
		Postcentral Gyrus	-5.364×10^{-6}	2.190×10^{-5}	1.943×10^{-6}	-4.083
		Superior Parietal Gyrus	-9.259×10^{-6}	1.677×10^{-5}	1.488×10^{-6}	-1.811
		Supramarginal Gyrus	-9.003×10^{-6}	1.715×10^{-5}	1.521×10^{-6}	-1.904

Table 11. (continued)

Rest	Left	Angular Gyrus	-9.481×10^{-7}	1.407×10^{-5}	1.254×10^{-6}	-14.845
		Intraparietal Sulcus	-1.071×10^{-5}	3.091×10^{-5}	2.742×10^{-6}	-2.884
		Postcentral Gyrus	6.700×10^{-7}	1.692×10^{-5}	1.502×10^{-6}	25.259
		Superior Parietal Gyrus	2.161×10^{-6}	1.415×10^{-5}	1.256×10^{-6}	6.549
		Supramarginal Gyrus	-3.081×10^{-6}	1.746×10^{-5}	1.549×10^{-6}	-5.666
	Right	Angular Gyrus	2.828×10^{-6}	9.697×10^{-6}	8.605×10^{-7}	3.429
		Intraparietal Sulcus	5.762×10^{-6}	1.062×10^{-5}	9.428×10^{-7}	1.844
		Postcentral Gyrus	2.157×10^{-6}	1.346×10^{-5}	1.195×10^{-6}	6.241
		Superior Parietal Gyrus	1.448×10^{-6}	1.033×10^{-5}	9.169×10^{-7}	7.134
		Supramarginal Gyrus	4.712×10^{-6}	1.150×10^{-5}	1.021×10^{-6}	2.441

In this context, the mean values from five key components of the parietal lobe, which are postcentral gyrus, superior parietal gyrus, supramarginal gyrus, angular gyrus, and intraparietal sulcus, were analyzed in both the left and right hemispheres. These mean values were calculated during the addition, number-knowledge, and rest-time tasks. Table 11 provides the descriptive data for these values.

In the second step, the mean values of these five parietal lobe structures in the two hemispheres were compared between groups using the Three-Way ANOVA. A total of thirty different values corresponding to the three conditions (rest, number recognition task, and addition task) were tested (see Table 12). As a result, there was a significant difference ($F(9) = 9.155$; $p < .001$) between the groups.

Table 12. The Three-Way ANOVA Results

Cases	**Sum of Squares**	**df**	**Mean Square**	**F**	**p**	**η^2**	**η^2_p**
Group	2.127×10^{-7}	2	1.064×10^{-7}	164.816	< .001	0.075	0.080
Laterality	1.094×10^{-8}	1	1.094×10^{-8}	16.949	< .001	0.004	0.004
Region	8.185×10^{-9}	4	2.046×10^{-9}	3.171	0.013	0.003	0.003
Group * Laterality	5.865×10^{-8}	2	2.933×10^{-8}	45.446	< .001	0.021	0.023
Group * Region	2.760×10^{-8}	8	3.450×10^{-9}	5.346	< .001	0.010	0.011
Laterality * Region	1.744×10^{-8}	4	4.359×10^{-9}	6.755	< .001	0.006	0.007
Group * Laterality * Region	4.726×10^{-8}	8	5.908×10^{-9}	9.155	< .001	0.017	0.019
Residuals	2.437×10^{-6}	3777	6.453×10^{-10}				

Note. Type III Sum of Squares

Finally, in the third phase, post-hoc analyses were conducted using the Bonferroni method to identify where significant differences existed between the groups (see Table 13).

Table 13. The posthoc comparisons of the groups

		Mean Difference	**SE**	**t**	**Cohen's d**	**$p_{bonferroni}$**
Rest Left Angular Gyrus	Number Knowledge Left Angular Gyrus	9.866×10-6	3.200×10-6	3.083	0.388	0.899
	Addition Left Angular Gyrus	1.251×10-5	3.200×10-6	3.910	0.493	0.041*
Rest Right Angular Gyrus	Number Knowledge Right Angular Gyrus	1.125×10-5	3.188×10-6	3.528	0.443	0.184
	Addition Right Angular Gyrus	2.675×10-5	3.188×10-6	8.392	1.053	< .001***
Rest Left Intraparietal Sulcus	Number Knowledge Left Intraparietal Sulcus	-1.405×10-5	3.188×10-6	-4.409	-0.553	0.005**
	Addition Left Intraparietal Sulcus	-1.501×10-5	3.188×10-6	-4.707	-0.591	0.001**
Rest Right Intraparietal Sulcus	Number Knowledge Right Intraparietal Sulcus	2.374×10^{-5}	3.188×10^{-6}	7.448	0.935	< .001***
	Addition Right Intraparietal Sulcus	3.055×10^{-5}	3.188×10^{-6}	9.583	1.203	< .001***
Rest Left Postcentral Gyrus	Number Knowledge Left Postcentral Gyrus	6.611×10^{-6}	3.188×10^{-6}	2.074	0.260	1.000
	Addition Left Postcentral Gyrus	1.485×10^{-5}	3.188×10^{-6}	4.659	0.585	0.001**
Rest Right Postcentral Gyrus	Number Knowledge Right Postcentral Gyrus	7.521×10^{-6}	3.188×10^{-6}	2.359	0.296	1.000
	Addition Right Postcentral Gyrus	3.139×10^{-5}	3.188×10^{-6}	9.846	1.236	< .001***
Rest Left Superior Parietal Gyrus	Number Knowledge Left Superior Parietal Gyrus	1.725×10^{-5}	3.188×10^{-6}	5.412	0.679	< .001***
	Addition Left Superior Parietal Gyrus	2.327×10^{-5}	3.188×10^{-6}	7.299	0.916	< .001***
Rest Right Superior Parietal Gyrus	Number Knowledge Right Superior Parietal Gyrus	1.071×10^{-5}	3.188×10^{-6}	3.359	0.422	0.344
	Addition Right Superior Parietal Gyrus	2.623×10^{-5}	3.188×10^{-6}	8.227	1.032	< .001***
Rest Left Supramarginal Gyrus	Number Knowledge Left Supramarginal Gyrus	5.799×10^{-6}	3.188×10^{-6}	1.819	0.228	1.000
	Addition Left Supramarginal Gyrus	7.990×10^{-6}	3.188×10^{-6}	2.506	0.315	1.000
Rest Right Supramarginal Gyrus	Number Knowledge Right Supramarginal Gyrus	1.371×10^{-5}	3.188×10^{-6}	4.302	0.540	0.008**
	Addition Right Supramarginal Gyrus	2.456×10^{-5}	3.188×10^{-6}	7.704	0.967	< .001***

Note. P-value adjusted for comparing a family of 30

* $p < .05$, ** $p < .01$, *** $p < .001$

The 10 groups in this table display comparisons of mean values during rest times and the other task times, examined in this specific context:

1. There is no significant difference between the mean value of the left-angular gyrus at rest time (M = -9.481×10^{-7}) and the mean value of the left-angular gyrus in the number recognition task (M = -1.081×10^{-5}), whereas there is a significant difference between the mean value of the left-angular gyrus in the addition task (M = -1.346×10^{-5}). When this difference is analyzed, it is seen that the rest time HbO level is higher than the level in the addition task ($M_{difference} = 1.251\times10^{-5}$, $p<.05$, Cohen's d = 0.493) (see, Figure 17).
2. There is no significant difference between the mean value of the right-angular gyrus at rest-time (M = 2.828×10^{-6}) and the mean value of the right-angular gyrus in the number recognition task (M = -8.417×10^{-6}), whereas there is a significant difference between the mean value of the right-angular gyrus in the addition task (M = -2.392×10^{-5}). When this difference is analyzed, it is seen that the rest time HbO level is higher than the level in the addition task ($M_{difference} = 2.675\times10^{-5}$, $p<.001$, Cohen's d = 1.053) (see, Figure 17).

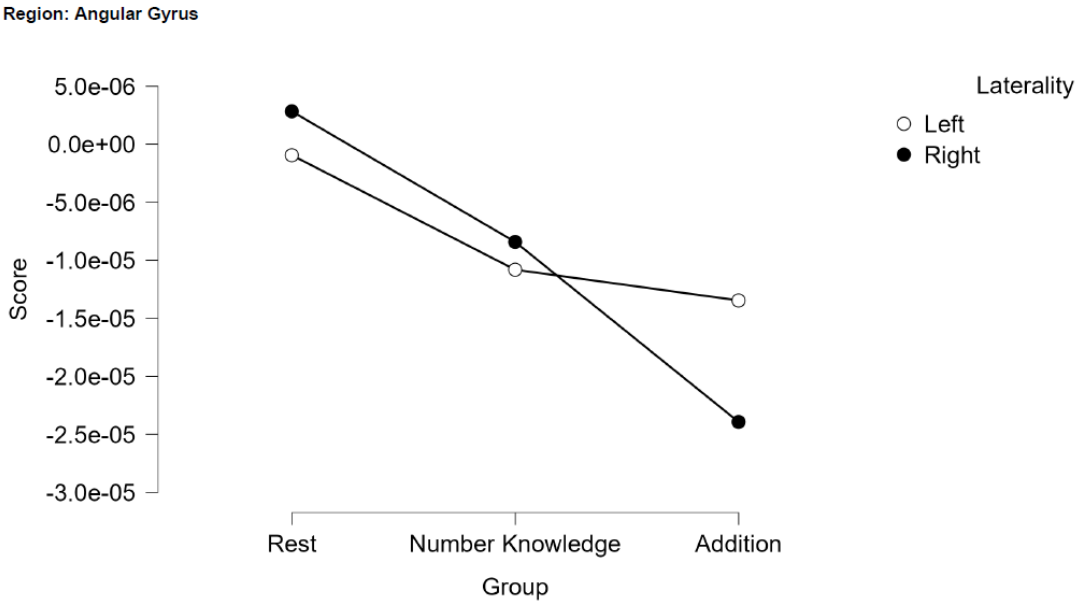


Figure 17. The comparison in the angular gyrus

3. There is a significant difference between the mean value of the left-intraparietal sulcus gyrus at rest time (M = -1.071×10^{-5}) and the mean value of the left-intraparietal sulcus gyrus in the number recognition task (M =

3.340×10^{-6}) and the mean value of the left-intraparietal sulcus gyrus in the addition task (M = 4.291×10^{-6}). When this difference is analyzed, it is seen that the rest time HbO level is lower than the level in both the number knowledge task ($M_{difference} = -1.405 \times 10^{-5}$, p<.01, Cohen's d = −0.553) and addition task ($M_{difference} = -1.501 \times 10^{-5}$, Cohen's d = −0.591) (see, Figure 18).

4. There is a significant difference between the mean value of the right-intraparietal sulcus at rest time (M = 5.762×10^{-6}) and the mean value of the right-intraparietal sulcus gyrus in the number recognition task (M = -1.798×10^{-5}) and the mean value of the right-intraparietal sulcus gyrus in the addition task (M = -2.479×10^{-5}). When this difference is analyzed, it is seen that the rest time HbO level is lower than the level in both the number knowledge task ($M_{difference} = -2.374 \times 10^{-5}$, p<.001, Cohen's d = -0.935) and addition task ($M_{difference} = -3.055 \times 10^{-5}$, p<.001, Cohen's d = -1.203) (see, Figure 18).
5.

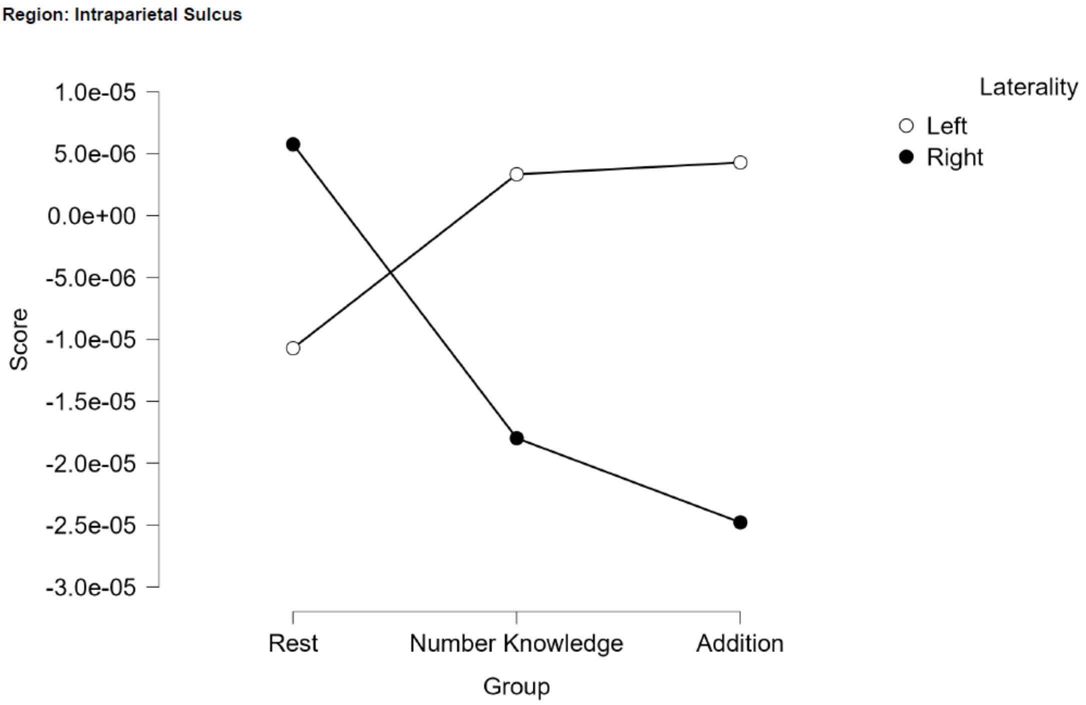


Figure 18. The comparison in the intraparietal sulcus

6. There is no significant difference between the mean value of the left-postcentral gyrus at rest time (M = 6.700×10^{-7}) and the mean value of the left-postcentral gyrus in the number recognition task (M = -5.941×10^{-6}),

whereas there is a significant difference between the mean value of the left-postcentral gyrus in the addition task (M = -1.418×10^{-5}). When this difference is analyzed, it is seen that the rest time HbO level is higher than the level in the addition task ($M_{difference} = 1.485\times10^{-5}$, $p<.001$, Cohen's d = 0.585) (see, Figure 19).

7. There is no significant difference between the mean value of the right-postcentral gyrus at rest time (M = 2.157×10^{-6}) and the mean value of the right-postcentral gyrus in the number recognition task (M = -5.364×10^{-6}), whereas there is a significant difference between the mean value of the right-postcentral gyrus in the addition task (M = -2.923×10^{-5}). When this difference is analyzed, it is seen that the rest time HbO level is higher than the level in the addition task ($M_{difference} = 3.139\times10^{-5}$, $p<.001$, Cohen's d = 1.236) (see, Figure 19).

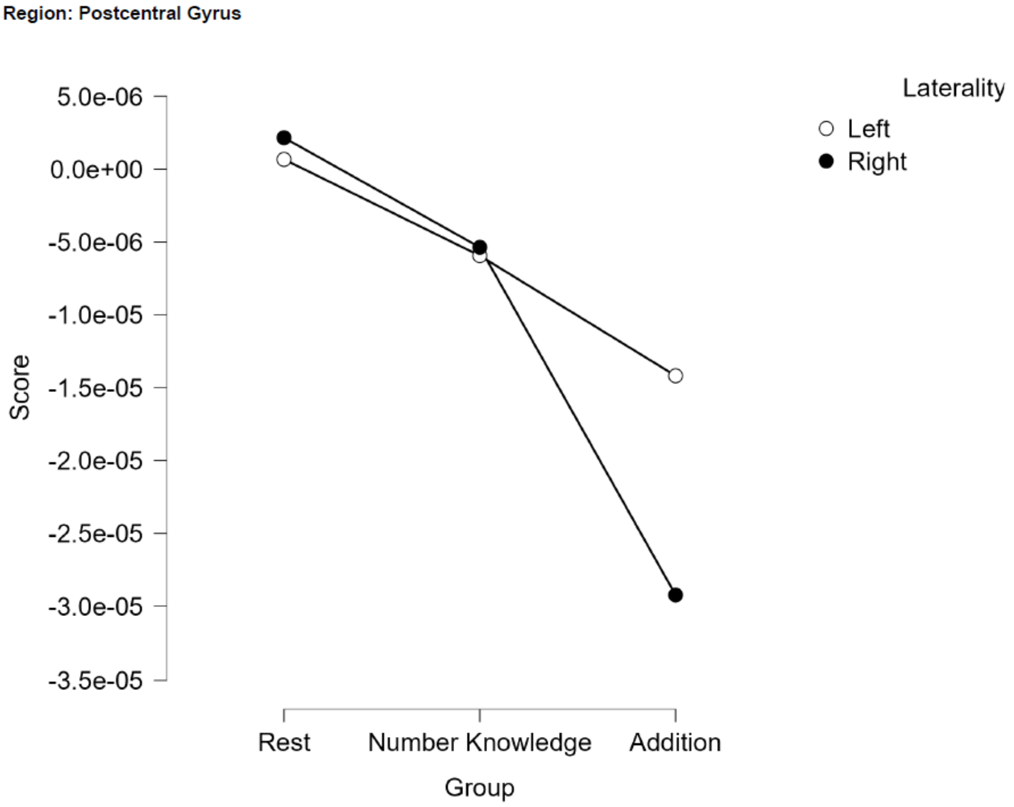


Figure 19. The comparison in the postcentral gyrus

8. There is no significant difference between the mean value of the left-supramarginal gyrus at rest (M = -3.081×10^{-6}) and the left-angular gyrus in

the number recognition task ($M = -8.880\times10^{-6}$), also there is no significant difference between the mean value of the left-supramarginal gyrus in the addition task ($M = -1.107\times10^{-5}$) (see, Figure 21).

9. There is no significant difference between the mean value of the right-superior parietal gyrus at rest time ($M = 1.448\times10^{-6}$) and the mean value of the right-superior parietal gyrus in the number recognition task ($M = -9.259\times10^{-6}$), whereas there is a significant difference between the mean value of the right-superior parietal gyrus in the addition task ($M = -2.478\times10^{-5}$). When this difference is analyzed, it is seen that the rest time HbO level is higher than the level in the addition task ($M_{difference} = 2.623\times10^{-5}$, $p<.001$, Cohen's d = 1.032) (see, Figure 20).

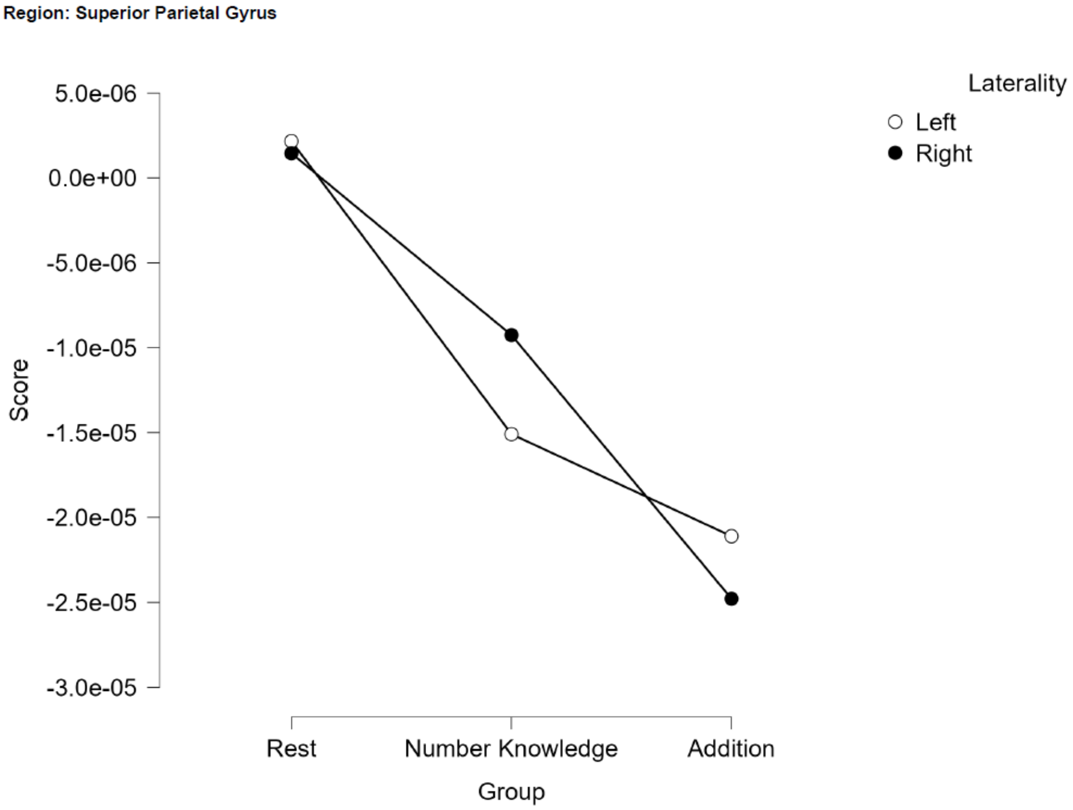


Figure 20. The comparison in the superior parietal gyrus

10. There is no significant difference between the mean value of the left-supramarginal gyrus at rest ($M = -3.081\times10^{-6}$) and the left-angular gyrus in the number recognition task ($M = -8.880\times10^{-6}$), also there is no significant

difference between the mean value of the left-supramarginal gyrus in the addition task (M = -1.107×10^{-5}) (see, Figure 21).

11. There is a significant difference between the mean value of the right supramarginal gyrus at rest time (M = 4.712×10^{-6}) and the mean value of the right supramarginal gyrus in the number recognition task (M = -9.003×10^{-6}) and the mean value of the right supramarginal gyrus in the addition task (M = -1.985×10^{-5}). When this difference is analyzed, it is seen that the rest time HbO level is higher than the level in both the number knowledge task ($M_{difference}$ = 1.371×10^{-5}, p<.01, Cohen's d = 0.540) and addition task ($M_{difference}$ = 2.456×10^{-5}, p<.001, Cohen's d = 0.967) (see, Figure 21).

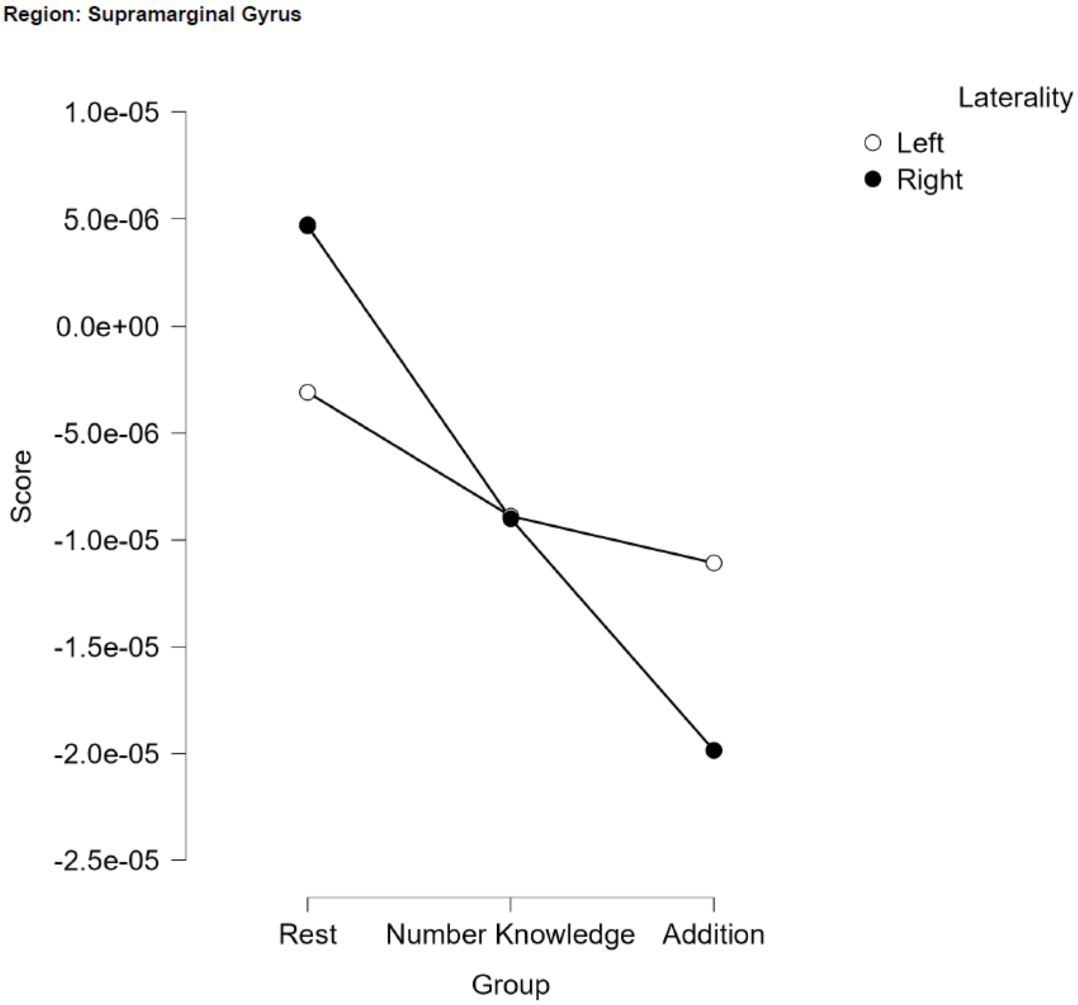


Figure 21. The comparison in the supramarginal gyrus

Table 14. The bivariate correlations between children’s math scores and the oxyhemoglobin level in the parietal lobe

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Number Knowledge	MathCorrect	-																				
	LPostcentralgyrus	-.028	-																			
	RPostcentralgyrus	-.022	.152	-																		
	LSuperiorparietalgyrus	.135	.587**	.166	-																	
	RSuperiorparietalgyrus	-.035	.207*	.564**	.279**	-																
	LSupramarginalgyrus	.098	.855**	.189*	.758**	.318**	-															
	RSupramarginalgyrus	.000	.119	.810**	.273**	.680**	.230**	-														
	LAngulargyrus	.130	.691**	.276**	.832**	.363**	.874**	.322**	-													
	RAngular gyrus	-.034	.154	.651**	.328**	.870**	.317**	.831**	.413**	-												
	LIntraparietalsulcus	.006	.875**	.093	.523**	.128	.704**	.030	.491**	.043	-											
	RIntraparietalsulcus	-.020	.084	.481**	.133	.329**	.064	.361**	.141	.235**	.048	-										
	LPostcentral gyrus	.094	.123	.146	.273**	.068	.144	.134	.196*	.122	.194*	.076	-									

Table 14. (continued)

RPostcentral gyrus	.064	-.044	.120	.117	.045	-.001	.127	.069	.104	.005	.057	.113	-								
LSuperiorparietal gyrus	.084	.160	.090	.362**	.137	.236**	.128	.330**	.189*	.200*	.028	.758**	.165	-							
RSuperiorparietal gyrus	.026	.065	.189*	.201*	.227*	.142	.217*	.192*	.272**	.078	.091	.246**	.698**	.346**	-						
LSupramarginal gyrus	.150	.250**	-.011	.380**	.056	.322**	.038	.355**	.144	.267**	-.051	.564**	.068	.775**	.295**	-					
RSupramarginal gyrus	-.085	.030	.275**	.129	.163	.102	.270**	.163	.272**	.004	.045	.143	.545**	.290**	.645**	.135	-				
LAngulargyrus	.139	.242**	.041	.381**	.085	.315**	.087	.382**	.166	.247**	-.056	.586**	.115	.850**	.394**	.910**	.242**	-			
RAngulargyrus	-.034	.148	.252**	.213*	.212*	.195*	.261**	.253**	.305**	.129	.030	.261**	.581**	.411**	.858**	.321**	.810**	.425**	-		
LIntraparietalsulcus	.216*	.171	.163	.316**	.126	.227*	.193*	.266**	.192*	.212*	.149	.783**	.109	.568**	.263**	.640**	.010	.543**	.217*	-	
RIntraparietalsulcus	.061	-.147	.103	-.066	-.101	-.142	.046	-.095	-.028	-.106	.068	-.013	.666**	-.023	.260**	-.130	.423**	-.127	.191*	-.050	-

*Correlation is significant at the 0.05 level (2-tailed).

**Correlation is significant at the 0.01 level (2-tailed).

4.1.3. The relationship between hemodynamic responses and children’s math abilities

Subsequently, after determining the brain regions that were active during the math tasks, a bivariate correlation analysis was conducted to examine the relationship between these hemodynamic responses and the children’s math performances. The only significant correlation found is between the children’s math scores and the left intraparietal sulcus ($r=.216$; $p<.05$) (see Table 14).

4.2. The relationships between children’s math abilities and the variables

The analysis related to cognitive and behavioral data obtained for the study is explained in this section via describing the preliminary analysis and main analysis. The cognitive variable of the present study is children's executive function performances. The information gathered in the setting of the home and school is referred to as an environmental variable. Direct parent-child math activities and parental beliefs about early math proficiency form the home environment variable. Meanwhile, the school environment includes data on the frequency of math activities in the classroom and teachers' beliefs about early math instruction. The preliminary analysis, conducted prior to addressing the central research questions, is outlined step-by-step below.

4.2.1. Preliminary analysis

Prior to the main analysis of the data, descriptive statistics, graphs, missing data analysis, and the reliability of the scales were tested to assess the distribution of variables in the dataset and ensure the data’s suitability for further analysis. In the descriptive analysis, skewness and kurtosis values were examined to determine the distribution of the data, and outliers were identified. Cronbach’s Alpha values were calculated to evaluate the reliability of the measurement tools. Additionally, Little’s MCAR test was applied during the missing data analysis, and the appropriate techniques for handling missing data were determined. Finally, correlation analysis was conducted to examine the relationships between variables prior to proceeding with SEM.

4.2.1.1. Outliers and normal distribution

Descriptive statistics, histograms, and normality tests were performed to assess the data distribution and detect any outliers. To quantitatively analyze the distribution of the data, skewness and kurtosis values were utilized. The skewness values for the scales ranged between -1.88 and .684, while the kurtosis values ranged between -.989 and 2.92 (see Table 15).

4.2.1.2. The reliability values of the scales

The reliability values of the measurement tools were determined by considering Croanbach's Alpha values. The alpha values of the total scores of the scales ranged between .786 and .980 (see, Table 15). Specifically, the relaibilty scores of TEMA-3 and fNIRS Math Task were calculated using equivalent forms/retest methods, with correlation values of .718 for raw scores and .564 for standardized scores. Regarding the reliability scores of the EF-Touch task, the Croanbach’s Alpha values for the working memory, inhibitory control, and cognitive flexibility tasks are .865, .611, and .736, respectively. Additionally, the Cronbach’s Alpha values for the Early Math Questionnaire show strong reliability: total score (α = .98), subcomponents (numbers and operations [α = .96], geometry [α = .94], measurement [α = .87], and pattern [α = .96]), and individual math beliefs (child beliefs [α = .77] and parent beliefs [α = .80]). Furthermore, the Math Activities in Classroom survey yielded a Cronbach’s Alpha of .92. Furthermore, the Math Activities in Classroom survey yielded a Cronbach’s Alpha of .92.

Lastly, Cronbach’s Alpha values for the total score of The Mathematical Development Beliefs Survey were found to be 0.79, and for the subcomponents, such as optimum age for mathematics teaching, classroom locus of mathematical knowledge generation, mathematical development as a primary aim of preschool education, and confidence in mathematics education, the values were 0.74, 0.46, 0.61, and 0.71, respectively. Overall, all data collection tools demonstrate acceptable reliability.

Table 15. The descriptive statistics of data collection tools

Data source	Questionnaire/task	Division	Sub-dimension	X̄	SD	Skewness	Kurtosis	Croanbach's Alpha
Child	TEMA-3	Math Ability	Raw scores	22.46	8.179	.736	.047	.718^{+}
			Ability scores	95.742	12.250	.309	.410	.564^{+}
	fNIRS Math Task	Math Abilities		12.887	5.442	.300	-.677	.718^{+}
	EF Touch	Inhibitory control		.933	.101	-2.244	5.565	.865
		Working memory		.676	.176	-.246	-.125	.611
		Cognitive flexibility		.742	.139	-.428	-.588	.736
Parent	Early math questionnaire	Total Score		2.406	.987	-.176	-1.084	.980
		Parent-child math activities	Numbers and operations	2.332	.981	-.024	-1.106	.958
			Geometry	2.588	1.110	-.404	-1.021	.945
			Measurement	2.336	1.074	-.081	-1.002	.867
			Pattern	2.476	1.289	-.384	-1.148	.956
		Individual Math Beliefs	Child beliefs	3.353	.551	-.841	.943	.770
			Parent beliefs	3.097	.511	-.750	1.020	.796
Teacher	The mathematical development beliefs scale	Total score		4.822	.358	-.656	1.791	.786
		Optimum age of mathematics teaching		5.115	.534	-.500	.657	.738
		Classroom focus of mathematical knowledge production		3.968	.590	.381	1.521	.464
		Mathematical development as the main aim of pre-school education		5.178	.481	-.797	1.218	.610
		The confidence in mathematics education		5.246	.528	-.939	2.512	.712
	Math activities in classroom			4.169	.652	-.080	.077	.776

Note: $^{+}$Refers to the equivalent forms/retest method

4.2.1.3. Missing data analysis

Little's MCAR test (Little, 1988) was performed to determine whether the missing data in the dataset were missing completely at random, and if the missing data were independent from the variables in the dataset. This test assesses the random distribution of missing data patterns by calculating the sum of squares of the standardized discrepancies between the expected population means and the sub-sample means, weighted by the estimated variance-covariance matrix and the number of observations in each sub-sample (Enders, 2010). The IBM SPSS Statistics (Version 23) was used to conduct this test on the available data. According to the results, the missing data is distributed completely at random (Little's MCAR test, χ^2 (63) = 71.166, $p = .225$) (criteria; $p > .05$; Little, 1988). In addition, the "exclude case pairwise" option was used in the correlation analysis of missing data. In this method, the participant is excluded only when necessary for specific analyses, but not from the total dataset (Pallant, 2011). In the SEM analysis, the "full information maximum likelihood (FIML)" method was employed.

4.2.2. The associations among children's math abilities and the variables

The Pearson Correlation coefficient values were taken into consideration to determine the relationship between the variables and children's math abilities. Regarding the relationships between parent-related factors and children's math abilities, significant associations were found between parents' beliefs about mathematics, their math-related activities with children, and various demographic characteristics (see Table 17). Regarding the relationship between children's mathematics abilities and teacher-related factors, significant associations were found between teachers' beliefs about the developmental appropriateness of mathematics and their competence in teaching mathematics. However, no relationship was observed between classroom mathematics activities and children's mathematics skill scores (see Table 18). Additionally, a significant relationship was found between children's math abilities and executive function, with each component of executive function showing a notable impact (see Table 16).

Table 16. Bivariate correlations between math scores and cognitive factors

Variables	1	2	3	4
Tema-3	-			
Inhibitory control	.346**	-		
Working memory	.352**	.157*	-	
Cognitive flexibility	.376**	.296**	.267**	-

*Correlation is significant at the 0.05 level (2-tailed).
**Correlation is significant at the 0.01 level (2-tailed).

The examination of the bivariate correlations between children’s math scores and their executive function revealed that there were moderate, significant, and positive link with the mathematical outcomes of the children. Specifically, the math scores of children had a moderate relationship with children’s inhibitory control performance ($r = .346$; $p < .01$), working memory performance ($r = .352$; $p < .01$), and cognitive flexibility performance ($r = .376$; $p < .01$).

In considering the bivariate correlations among the children’s math outcomes and the home math environment as well as the charecteristics of familyi there were low and modarete relationships. The relationship between children’s math outomes and parent-child activities ($r = .161$; $p < .05$), activities on numbers and operations ($r = .176$; $p < .05$), activities on pattern ($r = .188$; $p < .05$), parents’ math beliefs ($r = .200$; $p < .01$), children’s sex ($r = .206$; $p < .01$), and mothers’ educational degree ($r = .223$; $p < .01$) was found at the low-level, positive direction but the mothers’ working status ($r = -.140$; $p < .05$) had low-level negative direction. Also, the relationship between children’s age and their math scores was found to be moderate in a positive direction ($r =.349$; $p < .01$). The other variables did not significantly correlate ($p > .05$).

When considering the bivariate correlations between children's math outcomes and school as well as teacher-related factors, there were no significant correlations ($p > .05$).

Table 17. Bivariate correlations between math scores and home math environment

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tema-3	-														
Parent’s math activity	.161*	-													
Numbers and operations	.176*	.978**	-												
Geometry	.102	.944**	.874**	-											
Measurement	.135	.870**	.806**	.807**	-										
Pattern	.188**	.842**	.775**	.783**	.712**	-									
Child math belief	.139	.336**	.323**	.306**	.268**	.344**	-								
Parent’s math belief	.200**	.365**	.358**	.309**	.275**	.404**	.868**	-							
Child age	.349**	-.035	-.030	-.036	-.033	-.033	.044	.059	-						
Child sex$^+$	.206**	.027	.025	.024	.058	-.030	.004	.016	-.069	-					
Attendance to PreK$^+$	.095	.141*	.140*	.136*	.134*	.093	.124	.118	-.019	.026	-				
Mother’s educational degree$^+$	.223**	.073	.065	.039	.045	.105	.249**	.334**	-.109	.066	.330**	-			
Mother’s working status$^+$	-.140*	-.036	-.029	-.064	-.031	-.060	-.095	-.139*	.104	-.104	-.354**	-.343**	-		

Table 17. (continued)

Father's educational degree$^+$	.207**	.043	.032	.055	.011	.074	.157*	.241**	-.066	-.008	.304**	.660**	-.318**	-	
Father's working status$^+$	.077	.023	.041	.032	.024	-.092	-.064	-.086	.025	.142*	-.015	-.226**	.028	-.251**	-

*Correlation is significant at the 0.05 level (2-tailed). **Correlation is significant at the 0.01 level (2-tailed). $^+$ Standardized scores

Table 18. Bivariate correlations between math scores and school math-related factors

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tema-3	-														
Teacher's developmental belief	.110	-													
Optimum age	.078	.729**	-												
Classroom focus	.031	.499**	-.063	-											
Math dev main aim	.048	.707**	.587**	.072	-										
Teacher’s confidence	.133	.755**	.594**	.041	.468**	-									
Math activities in classroom	.039	.248**	.174**	.009	.232**	.317**	-								
Age of teacher	.114	.016	.001	.029	.054	-.032	.238**	-							
Teacher’s sex$^+$	.042	.123	.100	.061	.080	.092	-.072	-.057	-						
Teacher’s educational degree$^+$	.030	.011	.167*	-.170*	.043	.054	-.023	.205**	-.013	-					
Working year in institute$^+$	.022	.014	.064	-.046	.071	-.015	-.052	.004	-.019	.074	-				

Table 18. (continued)

Total working year$^+$	.143	.061	-.007	.105	.019	.019	.214*	.872**	-.033	.169*	-.098	-			
Income$^+$	.068	-.125	-.006	-.148	-.102	-.043	-.089	.354**	-.038	.352**	.073	.315**	-		
Institution type$^+$	.034	.097	-.055	.208**	.019	.025	.033	-.159*	-.035	-.0312**	-.025	-.052	-.189*	-	
School$^+$	.096	-.171**	-.083	-.154*	-.099	-.114	-.163*	-.101	-.080	.117	.092	-.186**	.334**	-.294**	-

*Correlation is significant at the 0.05 level (2-tailed). **Correlation is significant at the 0.01 level (2-tailed). $^+$ Standardized scores

4.3. The prediction of children’s math abilities by multifaceted factors

To understand the multifaceted factors that affect children’s mathematics abilities, SEM analysis was performed. SEM is an extended regression analysis in which multiple relationships between variables are simultaneously considered in a single integrated model. In this study, a model was developed that incorporates biological, cognitive, and environmental factors to explain children's math abilities, based on the literature explaining children’s math abilities. The home and school were considered within the context of environmental factor. In this model, there are three latent factors: EF, home environment, and school environment. The home factor consists of hands-on parent-child math activities and the parents’ beliefs in mathematics, while the school factor consists of the teacher’s beliefs in mathematics and in-class math activities. This analysis was run using JASP (2024) version 0.17.1. Initially, latent variables were created, and then the full model was run by writing lavaan syntax. The hypothesized model, which included the home environment, school environment, EF, and hemodynamic changes did not converge. Thus, nested models and multiple regression were utilized instead.

4.3.1. Results of Nested Models

In the present section, the nested models are introduced. The nested models were used to determine which model best explains children’s math abilities. Consequently, the following models are employed to characterize children's math abilities: 1) home, school, and EF model; 2) home and EF model; 3) school and EF model; 4) home and school model; 5) home model; 6) school model; and 7) EF model. The results of each model are presented below.

Table 19. Model 1: Home, School and EF

			Baseline test			Difference test		
AIC	BIC	n	$\chi2$	df	p	$\Delta\chi2$	Δdf	p
2260.681	2361.499	239	15.122	15	0.443	15.122	15	0.443
Index						Value		
Comparative Fit Index (CFI)						0.999		
Tucker-Lewis Index (TLI)						0.998		

Table 19. (continued)

Bentler-Bonett Normed Fit Index (NFI)	0.911
Root mean square error of approximation (RMSEA)	0.006
RMSEA 90% CI lower bound	0.000
RMSEA 90% CI upper bound	0.062
RMSEA p-value	0.872
Standardized root mean square residual (SRMR)	0.035
Goodness of fit index (GFI)	1.00

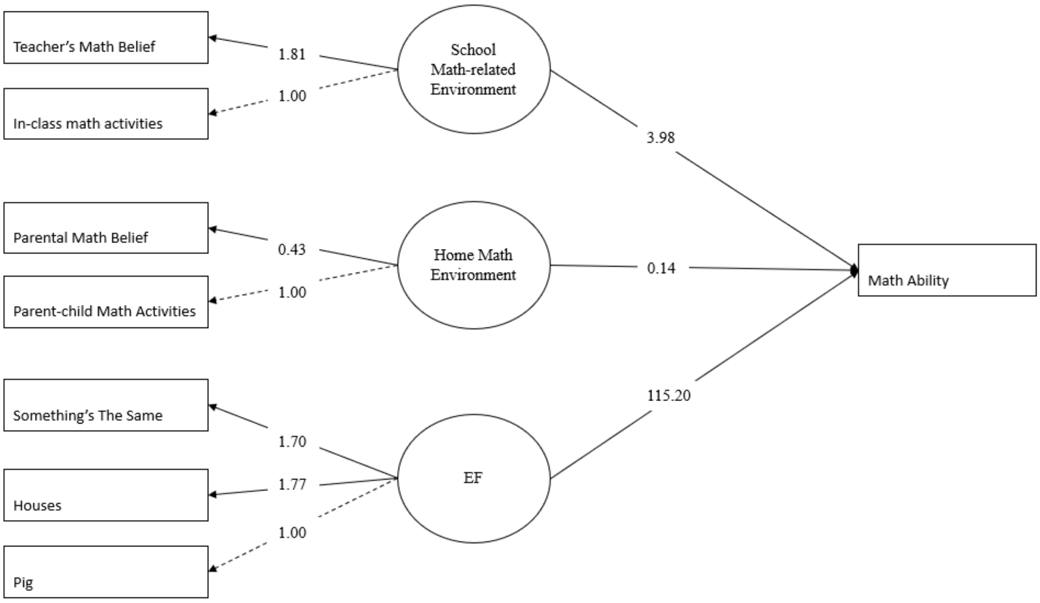


Figure 22. Path diagram of first model: EF + Home Factors + School Factors

The first model assesses the integrated influence of EF, school environment, and home environment on children's math achievement. The model fit to the data is excellent, as indicated by the fit indices: χ2 (15) = 15.122, p > .05, CFI = .99, TLI = .83, NFI = .91, RMSEA = .006 [90% CI: .00–.06], SRMR = .035. According to this model, EF, family, and school contexts all play critical roles in predicting children's math scores. In particular, moderate to significant loadings are observed in the latent components related to both the home (i.e., parent-child math activities and parents' beliefs on early math) and the school (i.e., teacher beliefs about math and math activities in the classroom). In addition, EF significantly improves the children's math abilities (Estimate = 115.201, p < .001), and this emphasizes the crucial role of cognitive regulation in math achievement. On the other hand, there were no significant covariances between home and school environments, indicating that their contributions to children's math abilities are relatively limited.

Table 20. Model 2: Home and EF

			Baseline test			Difference test		
AIC	BIC	n	χ2	df	p	Δχ2	Δdf	p
1602.993	1672.354	237	8.362	7	0.302	8.362	7	0.302
Index						Value		
Comparative Fit Index (CFI)						0.990		
Tucker-Lewis Index (TLI)						0.978		
Bentler-Bonett Normed Fit Index (NFI)						0.942		
Root mean square error of approximation (RMSEA)						0.029		
RMSEA 90% CI lower bound						0.000		
RMSEA 90% CI upper bound						0.088		
RMSEA p-value						0.650		
Standardized root mean square residual (SRMR)						0.035		
Goodness of fit index (GFI)						1.00		

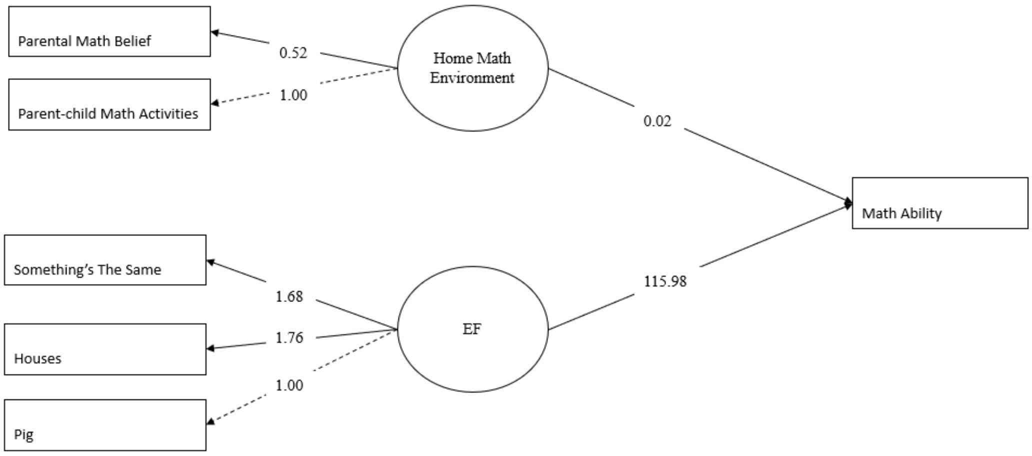


Figure 23. Path diagram of second model: EF + Home Factors

The second model, in which the school environment factor is eliminated, examines the predictive role of the home environment and EF on young children's math abilities. The fit statistics indicate a good model fit: χ2 (7) = 8.362, p > .05, CFI = .99, TLI = .97, NFI = .94, RMSEA = .029 [90% CI: .00–.08], SRMR = .035. The home environment variable, consisting of parents' math beliefs and parent-child math activities, continues to show significant loadings, though its predictive value is weaker compared to the first model. Consistent with the previous model, EF remains the strongest and most significant predictor of children's math abilities (Estimate = 115.983, p <.001), confirming its role in math achievement. Overall, this model emphasizes the importance of the home environment when combined with EF, but its predictive power is diminished by the absence of school-related factors.

Table 21. Model 3. School and EF

			Baseline test			Difference test		
AIC	BIC	n	χ2	df	p	Δχ2	Δdf	p
1304.815	1374.091	236	7.822	7	0.349	7.822	7	0.349
Index						Value		
Comparative Fit Index (CFI)						0.992		
Tucker-Lewis Index (TLI)						0.983		
Bentler-Bonett Normed Fit Index (NFI)						0.933		
Root mean square error of approximation (RMSEA)						0.022		
RMSEA 90% CI lower bound						0.000		
RMSEA 90% CI upper bound						0.085		
RMSEA p-value						0.692		
Standardized root mean square residual (SRMR)						0.029		
Goodness of fit index (GFI)						1.000		

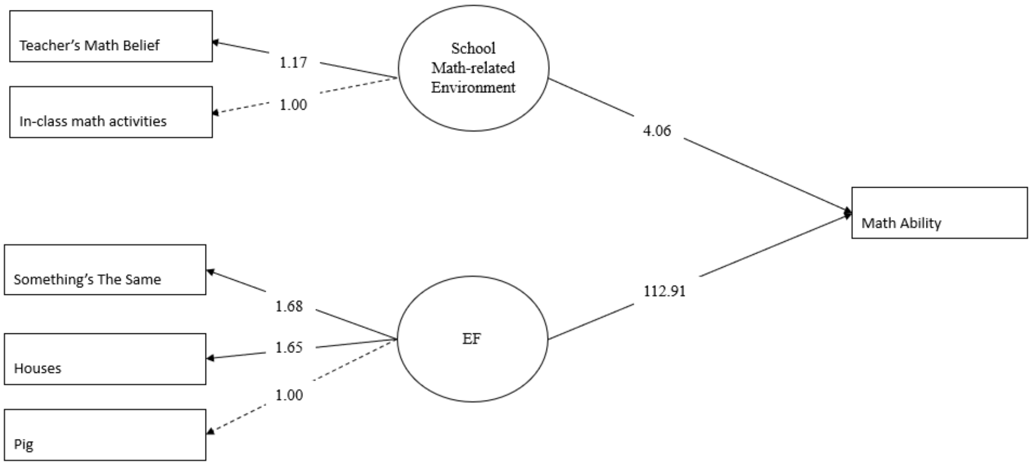


Figure 24. Path diagram of third model: EF + School Factors

The third model focuses on EF and school-related variables to explain young children's math abilities. This model also demonstrates good fit indices: χ2 (7) = 7.822, p > .05, CFI = .99, TLI = .98, NFI = .93, RMSEA = .022 [90% CI: .00–.08], indicating a strong alignment with the data. The school-related math environment exhibits moderate to high factor loadings, particularly regarding classroom math activities and teacher beliefs. There is a robust and statistically significant relationship between EF and children's math performance (Estimate = 112.914, p < .001), emphasizing the critical role of cognitive regulation in learning environments. However, compared to the first model, the exclusion of home factors leads to somewhat lower predictive power for various outcomes, suggesting that the contributions of the home and school environments are complementary.

Table 22. Model 4: Home and school

			Baseline test			Difference test		
AIC	BIC	n	χ2	df	p	Δχ2	Δdf	p
3014.093	3073.193	239	0.403	3	0.940	0.403	3	0.940

Index	Value
Comparative Fit Index (CFI)	1.00
Tucker-Lewis Index (TLI)	NaN
Bentler-Bonett Normed Fit Index (NFI)	0.993
Root mean square error of approximation (RMSEA)	0.000
RMSEA 90% CI lower bound	0.000
RMSEA 90% CI upper bound	0.024
RMSEA p-value	0.974
Standardized root mean square residual (SRMR)	0.011
Goodness of fit index (GFI)	1.000

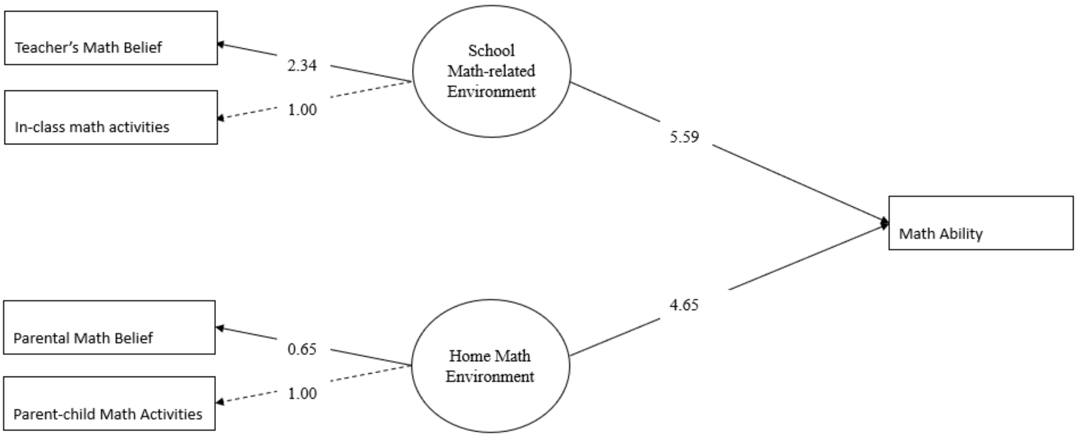


Figure 25. Path diagram of fourth model: Home Factors + School Factors

In the fourth model, EF was removed, and the model consisting of home and school factors was used to evaluate their combined predictive roles. The model fit results are as follows: χ2 (3) = 0.403, p > .05, CFI = 1.0, NFI = .99 , RMSEA = .000 [90% CI: .00–.02], SRMR = .011, indicating that this model is well fitted. Furthermore, evaluating the outcomes for children's math abilities, the home factors (Estimate = 4.651, p = 0.004) show a meaningful, albeit, reduced, predictive value, while the school factors exhibit no significant relationship. The lack of EF leads to a reduced explanatory power for this model. Furthermore, the absence of statistical significance in the covariance between home and school factors suggests that these settings largely influence outcomes independently.

Table 23. Model 5: EF

			Baseline test			Difference test		
AIC	BIC	n	χ2	df	p	∆χ2	∆df	p
648.668	688.426	203	2.391	2	0.302	2.391	2	0.302

Index	Value
Comparative Fit Index (CFI)	0.996
Tucker-Lewis Index (TLI)	0.987
Bentler-Bonett Normed Fit Index (NFI)	0.975
Root mean square error of approximation (RMSEA)	0.031
RMSEA 90% CI lower bound	0.000
RMSEA 90% CI upper bound	0.146
RMSEA p-value	0.468
Standardized root mean square residual (SRMR)	0.022
Goodness of fit index (GFI)	1.000

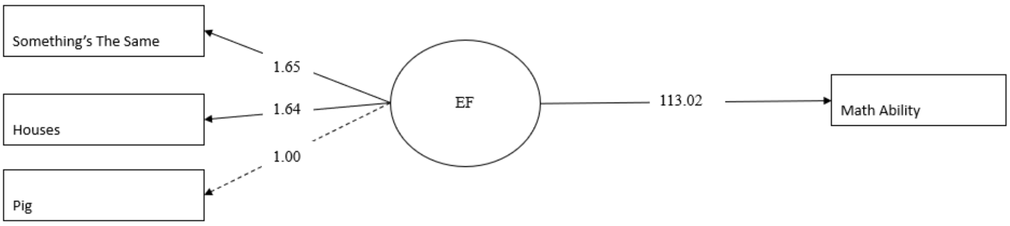


Figure 26. Path diagram of fifth model: EF

The most strongest model fit is provided by the fifth model, which focuses solely on EF, excluding home and school factors: χ2 (2) = 2.391, p > .05, CFI = .99, TLI = .98, NFI = .97, RMSEA = .031 [90% CI: .00–.146], SRMR = .022. A considerable proportion of the variance in the children's math ability is explained solely by EF ($R^2 = 0.513$). EF has a highly significant predictive role on children's math abilities (Estimate = 113.025, p <.001). This model highlights the vital role that EF plays in math achievement by indicating that children's academic performance is strongly predicted by cognitive control processes, independent of home or school contexts.

Table 24. Model 6: Home

			Baseline test			Difference test		
AIC	BIC	n	χ2	df	p	∆χ2	∆df	p
2358.226	2389.439	237	0.000	0	1.000	0.000	0	

Index	Value
Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	0.596
Bentler-Bonett Normed Fit Index (NFI)	1.000

Table 24. (continued)

Root mean square error of approximation (RMSEA)	0.000
RMSEA 90% CI lower bound	0.000
RMSEA 90% CI upper bound	0.000
RMSEA p-value	
Standardized root mean square residual (SRMR)	2.874×10^{-8}
Goodness of fit index (GFI)	1.000

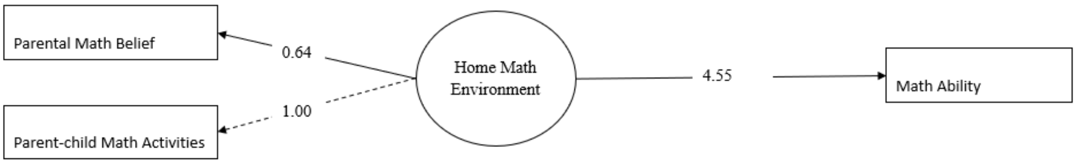


Figure 27. Path diagram of sixth model: Home Factors

The sixth model evaluates the predictive role of the home environment on math success, excluding the influences of EF and the school environment. Although the model fits very well, the baseline test indicates that this is just-identified/saturated model (df=0), thus the further examination was not considered.

Table 25. Model 7: School

			Baseline test			Difference test		
AIC	BIC	n	$\chi2$	df	p	$\Delta\chi2$	Δdf	p
2057.651	2088.825	236	0.000	0	1.000	0.000	0	
Index						Value		
Comparative Fit Index (CFI)						1.000		
Tucker-Lewis Index (TLI)						1.000		
Bentler-Bonett Normed Fit Index (NFI)						1.000		
Root mean square error of approximation (RMSEA)						0.000		
RMSEA 90% CI lower bound						0.000		
RMSEA 90% CI upper bound						0.000		
RMSEA p-value								
Standardized root mean square residual (SRMR)						1.194×10^{-7}		
Goodness of fit index (GFI)						1.000		

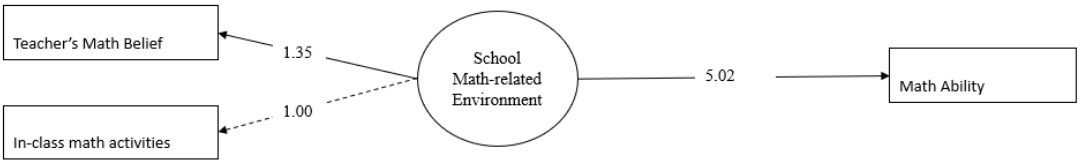


Figure 28. Path diagram of seventh model: School Factors

The seventh model evaluates how school-related factors affect children’s math outcomes. The baseline test indicates that this model is a just-identified/saturated model (df=0), although the fit indices, as observed in the sixth model, are appropriate.

4.3.1.1. General Results of the Nested Models

The results from the models explaining young children’s math abilities can be summarized as follows:

1. The findings indicate that the strongest predictor of children’s mathematical proficiency is their performance in EF.
2. The second most important finding is that EF enhances the predictiveness of children’s math scores when combined with environmental contexts. This is evident in the models that include EF alongside environmental factors: the first model, which integrates EF with both the home and school environments; the second model, which combines EF with the home factor; and the third model, which pairs EF with the school factor. Among these three, the first model demonstrates the strongest fit, followed by the combination of EF and the home factors, and finally, the integration of EF and the school factors.
3. Lastly, an examination of the models that explain children’s math abilities solely in terms of environmental factors reveals that the fourth model, which incorporates both home and school environments, is the only meaningful one. Nevertheless, compared to the models that integrate EF, this result offers a weaker predictive power. Furthermore, an analysis of the sixth model, which focuses solely on the home factor, and the seventh model, which examines only the school factor, indicates that it is not feasible to assess the fit of these models due to their limitations.

4.3.2. Result of Multiple Regression

Based on the findings from the previous correlation analysis, a multiple regression was performed using variables that demonstrated significant relationships with children's mathematical performance. In this analysis, the dependent variable is the children's accuracy on math tasks, specifically those modified for fNIRS, while the predictor variables include young children's performances in working memory, inhibitory control, and cognitive flexibility, as well as the mean activation in the left intraparietal region during the addition task. Since the values originate from different sources, they were standardized using z-scores.

Multiple regression analysis requires testing various assumptions, including sample size, outliers, multicollinearity, normality, linearity, homoscedasticity, and residual independence (Tabachnick et al., 2007).

1. *Sample Size:* According to Tabachnick and Fidell (2007), a minimum sample size of $N \geq 50 + 8m$ (where m is the number of predictors) is necessary. With the present sample size, this criterion is comfortably satisfied, as at least 90 participants are required.
2. *Multicollinearity and Singularity:* Multicollinearity occurs when predictor variables are highly correlated. Bivariate correlation analysis (Tabachnick et al., 2007), along with Tolerance and VIF (Variance Inflation Factor) values were evaluated (Pallant, 2011). The bivariate correlation results indicated no significant violations (see Table 26). According to Pallant (2011), VIF values should ideally be below 10, and the current dataset confirms this. To avoid singularity, only subcomponents of the EF-Touch were included as individual variables, ensuring that none of the predictors contained variations from others.
3. *Outliers:* The assumption of outliers was checked using Mahalanobis distance and Cook's distance. With four independent variables, the critical value for evaluating Mahalanobis distance is 18.47 at .001 (Tabachnick et al., 2007). One participant had a Mahalanobis distance value of 27.13, exceeding this threshold, so that participant was excluded from the analysis. After this exclusion, Mahalanobis distances ranged from 0.442 to 15.477, which are acceptable.
4. *Normality*: The skewness and kurtosis values were evaluated, and they fall within acceptable ranges. Additionally, the assumption of linearity was verified using a Normal P-P Plot (see Figure 29), and homoscedasticity was

assessed through scatterplots (see Figure 30), both of which indicated appropriate data distribution.

Table 26. The bivariate correlation for Multiple Regression Analysis

	1	2	3	4	5
Math accuracy	-				
Inhibitory control	.360**	-			
Working memory	.284**	.174*	-		
Cognitive flexibility	.355**	.324**	.292**	-	
Left Intraparietal Sulcus Activation	.217*	.178*	-.143	.156*	-

**Correlation is significant at the 0.01 level (2-tailed).
*Correlation is significant at the 0.05 level (2-tailed).

Table 27. The collinearity statistics for Multiple Regression Analysis

	Tolerance	VIF
Inhibitory control	.866	1.155
Working memory	.866	1.154
Cognitive flexibility	.817	1.224
Left Intraparietal Sulcus Activation	.914	1.094

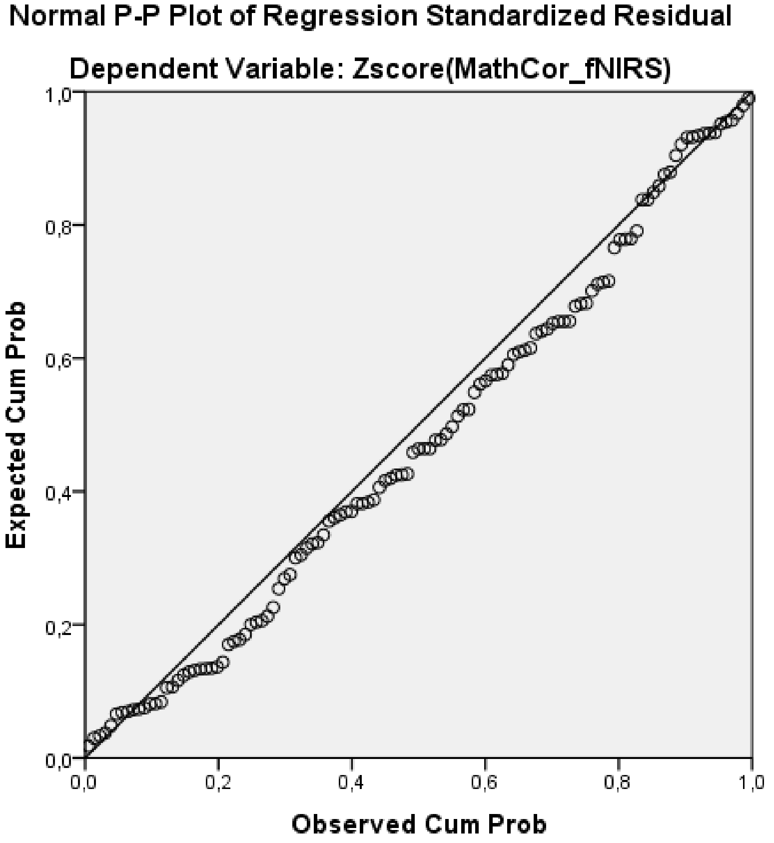


Figure 29. Normal P-P Plot of Regression Standardized Residual

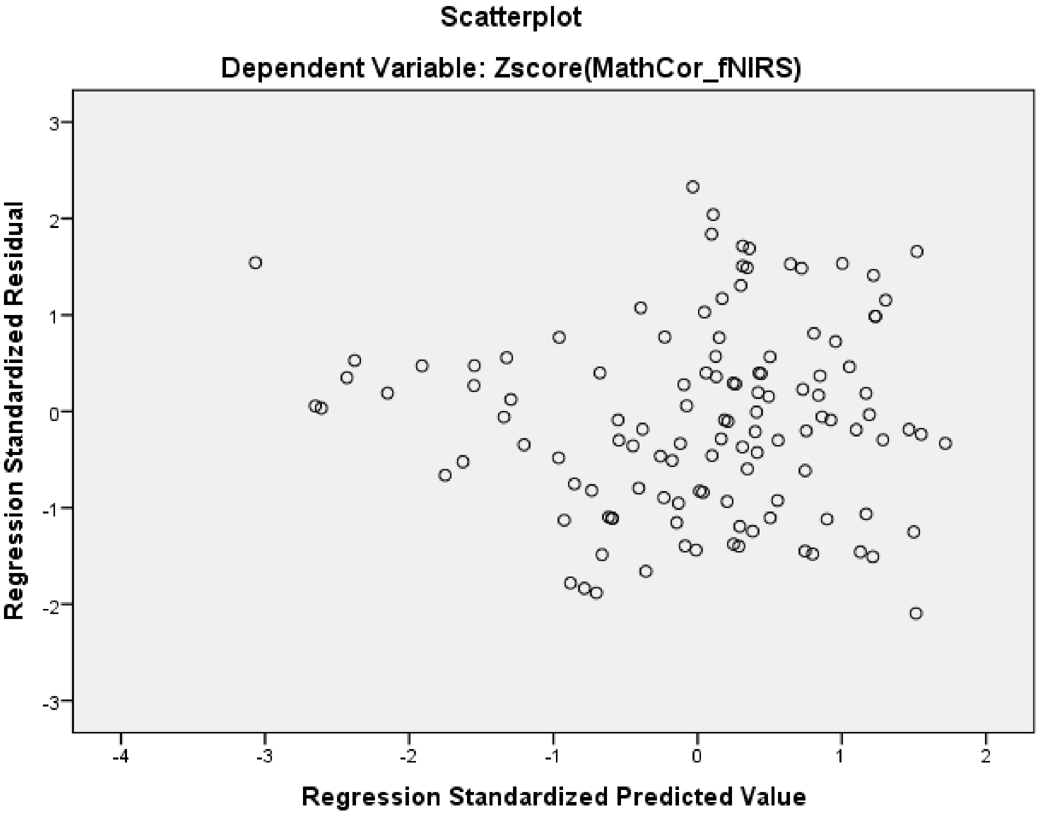


Figure 30. Scatterplot

Subsequently, after verifying the assumptions, the results of the model were evaluated. The model's significance was assessed using ANOVA results (see, Table 28). The findings indicated that the model is significant (p<.05), with the independent variables (inhibitory control, working memory, cognitive flexibility, and the hemodynamic response in left intraparietal sulcus) explaining 22.3% of the variance in children's math performance (see, Table 29).

Table 28. ANOVA results for the Multiple Regression Analysis

Source	SS	df	MS	F	p
Regression	29,855	4	7.464	9.621	<.001
Residual	89,992	116	0.776		
Total	119,847	120			

Table 29. Model summary

Model	R	R^2	Adjusted R^2	SE of Estimate
1	0.499	0.249	0.223	0.880

To ascertain which variable had the greatest impact on children's math performance, regression coefficients were analyzed (see Table 30). It was found that children's hemodynamic response in the left intraparietal sulcus (β = .17), working memory performance (β = .21), cognitive flexibility (β = .19), and inhibitory control (β = .23) all contributed to their math proficiency. Among these, the hemodynamic response in the left intraparietal sulcus had the smallest contribution, while inhibitory control had the greatest contribution.

Table 30. Prediction of young children's math abilities

Predictor	B	SE	β	t	p	95% CI [LL, UL]
(Constant)	.000	.080	—	-.003	.998	[-0.159, 0.158]
Inhibitory control	.250	.094	.230	2.656	.009	[0.064, 0.436]
Working memory	.214	.087	.213	2.465	.015	[0.042, 0.386]
Cognitive flexibility	.191	.089	.191	2.144	.034	[0.015, 0.367]
Left intraparietal sulcus activation	.176	.084	.177	2.098	.038	[0.010, 0.342]

4.4. Summary of Results

The major findings of the analysis are explained in this section. Below, the results of the analysis are presented in reference to each research question:

> *1.* Which substructure of parietal lobe of young children is oxygenated during math tasks?

To address this question, a Three-Way ANOVA was conducted (see section 4.1.2. The Investigation of the hemodynamic responses in the substructure of parietal lobe regarding children's math performances). The results showed that while children completed the number knowledge task, the left superior parietal gyrus, right supramarginal gyrus, and both the left and right intraparietal lobes were showed hemodynamic changes compared to rest time activation. Additionally, while children completed the addition task, both the left and right postcentral gyri, the superior parietal gyri, the right supramarginal gyrus, and the angular gyri in both hemispheres, along with the left and right intraparietal sulci, showed hemodynamic responses compared to rest time.

2. Are young children’s hemodynamic responses in the parietal lobe correlated with their mathematical abilities?

This question was examined through bivariate correlation analysis (see section 4.1.3. The relationship between hemodynamic responses and children’s math abilities). In this context, the relationships between the accuracy scores obtained from the fNIRS math task and the mean scores of the hemodynamic responses of the five components of the parietal lobe (postcentral gyrus, superior parietal gyrus, supramarginal gyrus, angular gyrus, and intraparietal sulcus), in both hemispheres (left and right) were examined. The only significant relationship between children’s math performance and the hemodynamic responses was with the left intraparietal sulcus. This indicates that as children’s math performance increases, the response in the left intraparietal sulcus also increases.

3. Are young children’s mathematical abilities correlated with their executive function performance~~s~~?

To address this question, bivariate correlation analysis was computed among children’s scores on mathematical ability and the three tasks for executive function: inhibitory control, working memory, and cognitive flexibility (see section 4.2.2., The associations among children’s math abilities and the variables). The findings revealed that each component is significantly related to children’s math scores, indicating that as children’s executive function performances increase, their math scores also improve in a similar manner.

4. Are young children’s mathematical abilities correlated with the home math environment (parent-child math activities and parental beliefs about early math)?

The results showed that parent-child math activities, particularly those involving numbers, operations, and patterning, are associated with young children’s math performance. This suggests that an increase in the frequency of these activities leads to greater math competence in children. Additionally, there was a relationship between parents' beliefs about early mathematics and young children’s math task scores. This implies that children whose parents have higher beliefs in the importance of early math perform better in math tasks.

Bivariate correlation analysis was conducted to investigate the relationship between children’s math performance and the home math environment (see section 4.2.2., The associations among children’s math abilities and the variables). The results showed that parent-child math activities, particularly those involving numbers, operations, and patterning, are associated with young children’s math performance. This suggests that an increase in the frequency of these activities leads to greater math competence in children. Additionally, there was a relationship between parents’ beliefs about early mathematics and young children’s math task scores. This implies that children whose parents have higher beliefs in the importance of early math perform better in math tasks.

5. Are young children’s mathematical abilities correlated with the school math-related environment (classroom math activities and teacher–beliefs about early math)?

Bivariate correlation analysis was employed to investigate this question (see section 4.2.2., The associations among children’s math abilities and the variables). The results showed no correlation between in-class math activities and children’s math performance. Furthermore, there was no significant relationship between teachers’ beliefs on early math and children’s math performance.

6. Are young children’s math abilities predicted by hemodynamic responses in the parietal lobe during math tasks, their executive function performances, home math environment and school math-related environment?

To address this question, both structural equation modeling and multiple regression analyses were employed (see section 4.3. The prediction of children’s math abilities by multifaceted factors). When the model proposed in the main hypothesis was tested, the results showed that the model did not converge. Therefore, nested models were constructed and structural equation models were evaluated. These models indicate that children’s math outcomes are predicted by the integration of executive function, home, and school environments. Additionally, executive function plays the most significant role in these models, and the predictive power of environmental factors increases with executive function. Furthermore, multiple regression analysis revealed that children’s math abilities are best explained by components of executive function and the hemodynamic response in the left intraparietal sulcus.

CHAPTER 5

DISCUSSION

The discussion section is divided into five parts. First, the discussions based on specific research questions are explained. Then an exploration of implications is presented for researchers, policymakers, parents, and teachers. Subsequently, the limitations of the present study are introduced, considering the nature of the current sudy. Following this, recommendations for further investigations are made based on the findings. Finally, the general conclusions from the study are provided.

In this context, the results of the current study are discussed in light of the existing literature. To do this, the current findings are reviewed and analyzed by comparing them with previous studies and offering explanations. Initially, the results regarding main research question (i.e., Are young children's math abilities predicted by hemodynamic responses in the parietal lobe during the math task, their executive function performances, home math environment and school math-related environment?) are discussed, explaining the hypothesized model's fit and comparing these results with the literature. Secondly the results from the second and third research questions on hemodynamic changes in the parietal lobe and their association with children's math outcomes are discussed within the scope of the current literature. Next, the connection between cognitive and environmental data, including children's executive function, their proximal environments (home and school), and children's math outcomes, is discussed, building bridges with the literature.

5.1. Discussion on the prediction of children's math abilities by the multifaceted factors

The main goal of the current study is to explore how children's biological, cognitive, and environmental factors are related to their mathematical abilities. In this context, the mean oxygenation level in the parietal lobe of children was considered a biological factor, children’s executive function performances were assessed as the primary cognitive ability, and the home and school environments of children were regarded as proximal environmental factors affecting their math abilities. The findings from the analysis indicate that the predictive power of children’s math abilities is enhanced when these factors are considered together. Specifically, children’s executive function performances emerged as the most significant variable, strengthening the impact of both the environmental factors and the biological factor, particularly in partial analyses. Furthermore, executive function performances were found to be a vital factor that can be evaluated alongside both environmental and biological factors, and it increases the predictive power in explaining young children’s mathematical abilities.

These results are grounded a theoretical level supported by the Bioecological Model (Bronfenbrenner, 1994), Interactivism (Bickhard, 2009), Neuroconstructivism (Westermann et al., 2010), the Triple Code Model (Dehaene, 1992) and the Causal Model (Butterworth et al., 2011). According to the underlying mechanisms of the present study, numerous integrated components, rather than a single factor, should be used to explain the development and mathematical abilities of young children (Bickhard et al., 2007; Butterworth et al., 2011; Westermann et al., 2007). Given this multifaceted structure, it is evident that the primary areas of research in young children’s mathematical abilities literature include children’s proximal environment (Silver et al., 2022), brain structure (Butterworth et al., 2011; Dehaene, 1992), and cognitive abilities (Bull et al., 2014; Cragg et al., 2014; Kilday, 2011). In line with this background, the findings of the current study suggest that a partial combination of biological factors (the activation in the left intraparietal sulcus during the addition task), cognitive factors (working memory, inhibitory control, mental flexibility), and environmental factors (home environment, including parent-child math activities and parental beliefs about early mathematics, and school environment, involving classroom math activities and teacher beliefs about early mathematics) can account for young children’s mathematical skills. Thus, it is clear that the numerous elements contributes to the development of mathematical abilities.

When the literature focusing on mathematical abilities and their projection onto the brain is analyzed, it becomes evident that the measurement of mathematical abilities and the difficulty of the tasks implemented are important factors in this regard (Butterworth et al., 2011). Meta-analyses and empirical studies show that while various lobes, including the parietal lobe, are activated during addition, subtraction, and multiplication operations, the parietal lobe, particularly its inferior and superior parts, is consistently activated in number tasks (Arsalidou et al., 2011; Artemenko et al., 2018; Bugden et al., 2012; Dresler et al., 2009; Emerson et al., 2015; Kawashima et al., 2004; Kucian et al., 2008; Rickard et al., 2000; Vogel et al., 2015). Reviewing this literature suggests that brain activation related to math abilities shifts from the frontal lobe to the parietal lobe, and within the parietal lobe, from the intraparietal sulcus to the angular gyrus (Zamarian et al., 2009). Additionally, damage to the parietal lobe, particularly the left angular gyrus and left intraparietal sulcus, impairs mathematical operations (Butterworth, 2020).

Regarding the findings of the main question in this study, it can be suggested that children’s math performance is predicted by the activation in the left intraparietal sulcus during the addition task. Consequently, it is evident that the activation in the left intraparietal sulcus not only predicts children’s mathematical abilities but also interacts with cognitive abilities in this context. In other words, this finding represents a significant advancement in connecting physiological brain activity to children’s math abilities. The focus on the left intraparietal sulcus as a key predictive region is supported by existing literature. Additionally, the association between neural activation and cognitive abilities provides a more detailed perspective. However, further research is necessary to establish causality, extend these findings to other mathematical tasks, and examine the influence of individual and contextual factors.

Research on EF, viewed as the primary cognitive component in this study, demonstrates that both the overall and specific components of executive function are strong predictors of mathematical ability (Kilday, 2011; Zelazo et al., 2016). The literature highlights that mathematical operations require a set of cognitive processes, and executive function skills are closely linked to cognitive abilities such as information retention and storage, disregarding inappropriate strategies, recognizing boundaries in numerical knowledge, and shifting between multiple operations (Bull et al., 2014). Furthermore, Craigg and colleagues (2014) emphasize that each level of knowledge is linked to EF by showing that mathematics at the knowledge level includes procedural and conceptual knowledge alongside factual knowledge. Consistent with this, the current study reaffirms the significance of executive function. It shows that executive function not only predicts children’s math abilities independently but also works in conjunction with environmental and biological factors. This underscores the crucial role of executive function in children’s mathematical development, highlighting how it supports other contributing elements.

Concerning the environmental factors in the main model, the literature suggests that family and school environments are key sources influencing children's math abilities (Bronfenbrenner et al., 1979, 1994; Silver et al., 2022). Drawing attention to these environments, it is found that antecedent influences include the beliefs of parents and teachers toward early mathematics, as well as the direct math activities they engage in with their children (Silver et al., 2022). Empirical studies indicate that direct interactions between parents and children during math activities significantly enhance children’s math abilities and improve their performance (Blevins-Knabe et al., 1996; Blevnis-Knabe et al., 2000; Chan et al., 2021; Claire-Son et al., 2020; DeFlorio et al., 2015; Foster et al., 2016; Huang et al., 2017; Huntsinger et al., 2016; Manolitsis et al., 2013; Missal et al., 2017; Niklas et al., 2017; Pardo et al., 2020; Sonnenschein et al., 2012; Soto-Calvo et al., 2020; Thompson et al., 2016; Zippert et al., 2020). Additionally, when parents hold a strong positive belief in the importance of early mathematics, it further boosts young children's math scores and overall performance. In contrast to home environment literature, investigations into the school environment present contradictory results. While there are evidence indicating that children’s math abilities are related to their school environment (Finders et al., 2021; Grammatikopoulos et al., 2018; Lehrl et al., 2016; Li et al., 2019; Mashburn et al., 2008; Schmitt et al., 2020; Schmerse, 2020), there are also studies indicate no relationship between them (Abreu-Lima et al., 2013; Brunsek et al., 2017; Francis et al., 2019; Guerrero-Rosada et al., 2021). In alignment with this background, the present study’s results indicate a positive predictive role of the environment when these environmental factors are considered in collaboration with children's executive function performances. Otherwise, these components did not have a significant direct prediction on their own or in combination. Thus, the findings for the main research question suggest that the considering multiple factors is important when evaluating the math abilities of young children. In other words, this finding provides valuable insights into how environmental and cognitive factors interact to predict mathematics performance. It is particularly noteworthy that environmental factors alone do not show a direct relationship, highlighting the importance of adopting a systemic approach to understanding and developing children’s math abilities. While the study offers potential, it could be improved by providing greater specificity in investigating the environmental factors, explaining their interaction mechanisms, and considering contextual variability.

5.2. Discussions on math-related activation in the parietal lobe

One of the sub-questions of the current study focuses on which specific areas in the parietal lobe are activated when young children complete number knowledge and addition tasks. The findings demonstrate that no singular region is activated; rather, multiple regions show hemodynamic responses. Specifically, while children are completing the addition task, both the left and right postcentral gyrus, both the left and right superior parietal gyrus, the right supramarginal gyrus, both the left and right angular gyrus, and both the left and right intraparietal sulcus regions display hemodynamic responses. On the other hand, during the number knowledge task, specifically the left superior parietal gyrus, the right supramarginal gyrus, and both the left and right intraparietal lobe regions show hemodynamic responses. This highlights the fact that multiple brain regions are engaged during these tasks, rather than indicating that a singular brain structure is responsible for them. In further analysis, it is emphasized that only the left intraparietal region showed a significant association with children’s accuracy rates in math tasks, as measured using fNIRS.

When the literature in this area is reviewed, it is demonstrated that multiple regions are activated, similar to the current findings. Arsalidou and colleagues (2011) indicated in a large meta-analysis study that activations were observed in the parietal lobe during number tests, as well as in the visual, parietal, frontal, and prefrontal regions, in addition to the bilateral thalamus and cerebellum, the right insula, and claustrum regions. Among these regions, studies have shown that the parietal lobe is crucial for number discrimination, and it is also sensitive to individual differences (Libertus et al., 2011; Cantlon et al., 2006). Furthermore, investigations in the literature have demonstrated that during number knowledge tests, the left intraparietal sulcus is engaged (Emerson et al., 2015; Vogel et al., 2015). More specifically, these studies suggest that the intraparietal sulcus plays a role in number manipulation, regardless of whether the stimuli involve numerals or quantities (Ben-Shalom et al., 2013; Dehaene et al., 2003).

Additionally, research on mathematical operations (i.e., addition, subtraction, multiplication, and division) using brain imaging reveals that different parts of the brain, such as the left middle frontal gyrus, bilateral inferior temporal cortices, bilateral occipital cortices, intraparietal sulcus, left inferior frontal gyrus, and bilateral frontoparietal regions, become active related to the math activities (Artemenko et al., 2018; Dresler et al., 2009; Kawashima et al., 2004; Kucian et al., 2008). Arsalidou and colleagues (2011) highlighted that activation in the parietal lobe, prefrontal lobe, frontal lobe, visual areas, as well as in the bilateral thalamus and cerebellum, and both right insula and claustrum regions is linked to the addition task. Focusing on the parietal lobe, studies emphasize the critical function of the IPS (intraparietal sulcus) in the addition task due to its role in representing numerical quantities (Peters et al., 2018; Rivera et al., 2005; Zamarian et al., 2009).

This information from the literature aligns with the present findings of the study, which suggest that numerous brain regions in the parietal lobe are activated concerning children’s performances on the number knowledge task and the addition task. However, according to the present study, which ties these activated regions to the behavioral data of the children in mathematics, only the mean activation in the left intraparietal sulcus during the addition task was associated with the children’s math scores. When considering this in the context of the literature, this scenario is supported. According to the review study by Zamarian and colleagues (2009), in the case of the parietal lobe, there is a transition from the left intraparietal sulcus to the left angular gyrus. This highlights the role of the left intraparietal region and the continuous shifting in the brain regarding math abilities. In other words, this finding contributes to the field by investigating preschoolers' math abilities in the context of number knowledge and addition. It is particularly important to link brain data to behavioral outcomes (children's math abilities). Previous studies investigated brain areas in isolation from behavioral outcomes, and this finding indicates that intraparietal sulcus activation is associated with math performances. Additionally, the activated brain areas suggest that the parietal lobe function is significant for math performance.

5.3. Discussion related to association among children's math abilities and the predictive variables

One of the aims of the study was to examine how children's mathematical performance relates to environmental and cognitive factors. The primary cognitive component in this context was the children's performance on the executive function scale, whereas the home and school environments, including direct adult-child math activities and adults' (parents and teachers) beliefs about early math, were the focus of the environmental factors analysis. In light of the results from SEM, it was indicated that these factors complement one another in explaining children's proficiency in mathematics, and this integrated structure fits the data well.

5.3.1. Discussion on the relationship between executive function and children's math outcomes

The findings indicated that the most important indicator of children's math abilities was their performances in executive function. Furthermore, executive functions were involved in boosting the prediction level of other variables in nested models. In addition to the general role of executive function in predicting young children's math abilities, the degree to which each component of executive function component contributed to children's mathematical proficiency was investigated by further analyses. As a result, it was found that working memory, inhibitory control, and mental flexibility all had a moderate effect on predicting children's math scores.

This finding aligns with the literature in that executive function is one of the best indicators of individual aptitude for mathematics (Kilday, 2011; Zelazo et al., 2016). Research suggests that mathematical knowledge, which includes factual, procedural, and conceptual understanding, is closely linked to executive function (Cragg et al., 2014). Additionally, executive function plays a crucial role in mathematical processes by aiding individuals in retaining and storing information, choosing effective strategies, filtering out irrelevant details, and switching between tasks when necessary (Bull et al., 2014). Moreover, empirical research highlights the strong correlation between children's achievement in mathematics and executive function (Allan et al., 2014; Cortés Pascual et al., 2019; Emslander et al., 2022; Friso-van den Bos et al., 2013; Peng et al., 2016; Yeniad et al., 2013). This is consistent with the current findings. Compared to other factors (i.e., home and school environments), executive function, as a latent factor, displayed the strongest predictor of children's mathematical abilities in this sample. It also highlights the individual contributions made by each dimension of executive function to children's mathematical outcomes. In other words, this study reinforces the critical role of EF in preschoolers' mathematics performance and adds to the growing body of evidence emphasizing EF as a significant determinant of academic achievement. By examining the individual contributions of various EF dimensions and comparing them with contextual factors, the research provides valuable insights for researchers. Overall, this finding is a significant step toward a better understanding of the cognitive foundations of mathematical learning.

Upon reviewing the literature, studies on the relationship between math abilities and executive function reveal that working memory plays a significant role in the mathematical process by aiding in the retrieval and storage of pertinent information, as well as accessing outcomes during problem-solving phases (Bull et al., 2014). Additionally, experimental research highlights that preschoolers who receive support for their working memory skills tend to perform better in mathematics (Gordon et al., 2021; Kyttälä et al., 2015; Passolunghi et al., 2014). Furthermore, working memory is one of the strongest indicators of typically developing children's success, according to the findings of a large-scale meta-analysis (Friso-van den Bos et al., 2013). This study aligns with existing literature, emphasizing the importance of working memory in children's math abilities. Particularly, it indicates that working memory has a moderate predictive role in understanding how children perform in mathematics. Thus, this finding underscores the crucial role of working memory in supporting children’s recall of information and applying it effectively during mathematical tasks. Therefore, it reinforces the idea that mathematical operations involve multiple cognitive processes and rely on complementary cognitive abilities.

Inhibitory control, one of the aspects of executive function, is involved in mathematical processes by suppressing irrelevant strategies, prioritizing numerical representations, recalling numerical boundaries, and applying pertinent information while filtering out distractions in verbal problems (Bull et al., 2014). Upon reviewing empirical research in the literature, cross-sectional studies (Ng et al., 2015) indicate that children who perform better in inhibitory control also tend to perform better in mathematics, and longitudinal research (Laski et al., 2015) highlights the persistence of this association over time. A meta-analysis conducted on this link also revealed a low overall effect size between inhibitory control and children's outcomes (Allan et al., 2014). These findings from the literature (i.e., Allan et al., 2014; Bull et al., 2014; Laski et al.,2015; Ng et al., 2015) align with the current study and show a moderate link between children’s math performance and inhibitory control. In other words, the present finding shows that preschoolers’ inhibition control, which they use to discriminate between relevant and irrelevant information, play a significant role in their mathematics performance. This supports the idea that not only recall and information utilization but also the ability to direct attention are important for success in mathematics.

Cognitive flexibility plays a key role in mathematical processes by facilitating the switching between different strategies for performing operations, paying attention to number representations and ranges, and following steps in solving problems (Bull et al., 2014). Additionally, empirical research indicates that children's performance on math problems increases in tandem with increases in their cognitive flexibility scores (Arán Filippetti et al., 2017; Van der Ven et al., 2012). Meta-analyses of the relevant literature suggest that the association between mathematical skills and cognitive flexibility has a moderate overall effect size (Martin et al., 2024; Santana et al., 2022). In alignment with this literature, the current findings also demonstrate that children’s cognitive flexibility predicts their math performance to a moderate degree. While this effect is not as strong as working memory and inhibitory control, it highlights the importance of shifting ideas during math tasks. In other words, children need to engage in cognitive shifts while performing math, which involves both memory and the selection of relevant information.

5.3.1.1. Discussion on the relationship between home math environment and children’s math outcomes

The SEM results indicated that the home math environment, which includes parent-child math activities and parental math beliefs, was not a significant factor on its own unless integrated with both school and executive function variables. That suggests that the predictive role of home math environment becomes significant only through collaboration with both school environment and executive function. Furthermore, bivariate correlation analysis revealed a weak relationship between children's math abilities and parent-child math activities and parental math beliefs. This emphasizes the following points: 1) children’s math abilities improve with the frequency of parent-child activities at home; 2) they improve with the strength of parental beliefs about the importance of early mathematics.

The literature demonstrates that the frequency of activities between parents and children is positively associated with children’s math outcomes, promotes the development of math abilities, and has a favorable impact that persists over time on young children’s academic achievement (Blevins-Knabe et al., 1996; Chan et al., 2021; Claire-Son et al., 2020; Huntsinger et al., 2016; Levine et al., 2010; Manolitsis et al., 2013; Missal et al., 2017; Niklas et al., 2017; Pardo et al., 2020; Soto-Calvo et al., 2020; Thompson et al., 2016). Consistent with the literature, the current findings also reveal that children's math scores increase alongside the frequency of parent-child activities. Moreover, the findings emphasize that activities involving numbers, operations, and patterns, in particular, are crucial to this relationship.

In other words, these results highlight the association between parent-child math activities and children’s math abilities, specifically emphasizing the critical role of activities that include numbers, operations, and patterns. This alignment with existing literature refines the understanding by underscoring the stimulative and scaffolding role of parental engagement in preschoolers’ math outcomes. However, questions regarding causality, the quality versus quantity of interactions, and developmental differentiation remain. On the other hand, these findings make an important contribution by identifying feasible ways to improve children’s math abilities through meaningful and focused parent-child engagement.

Another important factor that shapes the home math environment is parental beliefs about early mathematics. A review of recent literature on this topic shows that since individual beliefs form the foundation of attitudes and behaviors, it is important to consider that parental beliefs in early mathematics, and the increasing value placed on it, translate into higher scores in children’s mathematics outcomes (Elliot et al., 2018; Fishbein et al., 1975; Furnham et al., 2002; Missall et al., 2015; Phillipson et al., 2007; Silver et al., 2022; Taylor et al., 2004). The current findings, consistent with the literature, also demonstrate a similar trend: as parents’ beliefs in the importance of mathematics grow, their children’s arithmetic test scores improve accordingly.

Furthermore, research has highlighted the influence of parental traits on young children's success in mathematics. Specifically, studies have shown a correlation between children's mathematical abilities and maternal characteristics. For instance, children’s math achievement tends to improve as the mother's level of education increases (Harding 2015; Magnuson, 2007). Recent findings also show that children’s math scores improve concurrently with an increase in their mothers’ educational attainment. Furthermore, the current study highlights a relationship between children’s math outcomes and their mothers’ employment status, as well as an inverse correlation between children’s math performance and the number of hours their mothers work.

5.3.1.2. Discussion on the relationship between school math related environment and children’s math outcomes

The school math-related environment, composed of in-class mathematics activities and teachers’ beliefs about early mathematics, was not found to be a significant factor in predicting children’s mathematics outcomes on its own unless integrated with the home math environment and executive function factors. This finding parallels the results regarding the predictive role of the home environment in young children’s math outcomes. It underscores the idea that when the school math-related environment, executive function, and home math environment are considered collectively, their combined influence plays a more critical role in forecasting children's mathematical achievements. However, unlike the home math environment, no school characteristic was found to be directly correlated with children’s mathematical outcomes when bivariate correlations were examined.

Some studies highlight varying perspectives on the association between school-related variables and children’s math abilities. For instance, mathematics activities conducted by teachers in the classroom have been studied in the context of broader concepts such as quality. However, quality is a multifaceted construct encompassing various aspects of the school environment, from physical infrastructure to interpersonal interactions (Slot, 2018). Reflecting this complexity, the literature explores relationships between different dimensions of quality, such as in-class math activities, and children’s math abilities. Examining the literature within this framework, the majority of studies reveal a positive relationship between school quality and children's mathematical outcomes, with both cross-sectional and longitudinal studies demonstrating improvements in children's mathematics scores as quality scores increase (Finders et al., 2021; Grammatikopoulos et al., 2018; Lehrl et al., 2016; Li et al., 2019; Mashburn et al., 2008; Schmitt et al., 2020; Schmerse, 2020). However, some studies find no significant relationship between school quality and children’s math abilities, instead highlighting a connection with language skills (Abreu-Lima et al., 2013; Francis et al., 2019; Guerrero-Rosada et al., 2021). Furthermore, a meta-analysis concluded that there is no significant relationship between math outcomes and overall quality assessments, considering the combined effect size from the literature (Brunsek et al., 2017). The present study specifically focuses on in-class math activities as an element of school quality. Its findings also indicate no significant relationship between children’s mathematics abilities and the frequency of in-class math activities.

Teachers' beliefs about mathematics represent another environmental component linked to young children's mathematical abilities (Silver et al., 2022). The literature emphasizes that these beliefs significantly influence teachers' instructional practices, shaping their interactions with children and the quality of the instruction provided (Charlesworth et al., 1991; Charlesworth et al., 1993; Hofer, 2001; Muis et al., 2006; Karataş et al., 2017; Pajares, 1992; Stipek et al., 1997). Research on teachers' beliefs about mathematics has revealed that enhancing these beliefs can lead to improved children math performance. However, children performance also improved in classrooms where teachers did not receive targeted belief-based training (Chen et al., 2013).

Correlational studies have further demonstrated a link between teachers' beliefs and children's mathematical success, indicating that stronger teacher beliefs are associated with higher children achievement (Takunyacı et al., 2014). Nevertheless, other correlational research has found no significant relationship between teachers' beliefs and children's math performance (Prewet et al., 2021). The findings of the current study align with the latter perspective, and show no direct relationship between teachers' beliefs about the importance of early mathematics and children's mathematical performance. These findings contribute to the literature by exploring the association between teachers' beliefs about early mathematics and children's math abilities. While the majority of existing studies focus on beliefs as an individual factor, this study underscores the importance of considering beliefs within the broader context of environmental influences. Although teachers' beliefs did not directly impact children's math outcomes in this study, they gain significance when integrated with other factors to explain children's math abilities. When the children' math abilities are investigated it is beneficial to consider the school quality in broader perspective. Ultimately, the key takeaway from these findings is that children's math abilities are best understood through a multifaceted approach that considers multiple factors rather than isolating a single influence.

To sum up, the findings align closely with theoretical studies. Bringing all the findings together, it is evident that children's math abilities are shaped by a variety of interactive factors. As interactivism (see Bickhard, 2009) suggests, the development of cognition, in this context, children's math abilities, does not occur in isolation but is deeply influenced by environmental interactions. Furthermore, based on the neuroconstructivist approach (Westermann et al., 2007), these interactions also shape the individual's brain. While mathematical abilities are distributed across various brain structures, the parietal lobe plays a particularly important role (Butterworth et al., 2011; Dehaene et al., 1999). Additionally, the immediate environments, such as home and school, serve as primary scaffolds for children's development (Bronfenbrenner, 1979, 1994). Within these environments, direct adult-child math activities and adults' beliefs about math emerge as key variables for fostering children's mathematical abilities (Silver et al., 2022). The findings suggest that children's math skills are constructed on this foundation. Specifically, children's math abilities are influenced by a combination of brain activation, cognitive skills, and home and school environments. In detail, the left intraparietal sulcus, executive functions (i.e., working memory, inhibitory control, and cognitive flexibility), the home math environment (i.e., parent-child math activities and parental beliefs about math), and the school math-related environment (i.e., in-class math activities and teachers' beliefs about math) work together like an orchestra to support children's math abilities. In other words, math abilities are built through the interactions of biology, cognition, and environment. Thus, an individual's biological and cognitive characteristics, combined with environmental resources, form an integrated framework that underpins mathematical performance.

5.4. Implications

This section of the study presents a two-stage discussion of implications based on the findings. First, the theoretical and research-related implications derived from the applied research methodology are explained. Second, the practical implications of these results are discussed.

5.4.1. Implications for Theory and Research

The current findings have significant implications for theories and research perspectives related to young children. However, existing theoretical studies on the factors influencing young children’s math abilities lack a comprehensive framework that integrates biology, cognition, and the environment. As a result, children’s math abilities are often examined within isolated contexts. The current findings emphasize the importance of an interegrated assessment, considering the multifaceted aspects of biology, cognitive processes, and environmental elements to better understand children’s math abilities. Thus, a dynamic approach encompassing ecology, cognition, and neural factors can provide a holistic framework for explaining young children’s mathematical development.

This study also underscores the importance of future research combining behavioral and physiological data to explore young children’s math abilities. These findings highlight the relevance of understanding the behavior-brain connection in math performance as a product of multiple interacting factors, rather than isolated influences. Researchers could adopt an integrated approach that considers the brain-behavior association to explain children’s development. For instance, this could involve designing educational programs, individualized support strategies for learning, or more effective interventions for children with special needs, taking into account biological, cognitive, and environmental factors. Additionally, studies could explore how environmental factors act as stimulative sources for brain activation, and ultimately impact children’s academic achievement. Longitudinal studies could further examine whether these effects persist over time or lead to changes in function.

5.4.2. Implications for Practice

This study presents findings related to both the school and home environments, which inform the practical implications of the research. The current results indicate that children’s math abilities are better understood when these environments are supported by their cognitive abilities. Therefore, the first part of the implications focuses on practices within both the school and home contexts. It is crucial to provide stimulating opportunities that enhance children’s cognitive processes while also supporting the development of their math abilities. These environments, home and school, are the immediate settings where children are exposed and engaged in learning. Additionally, parents and teachers who engage in practices with children should be made aware of their beliefs about early mathematical abilities. Their attitudes and behaviors toward these activities should also be supported to ensure effective learning outcomes.

Focusing specifically on the educational implications, the current study found that the frequency of mathematics activities in the classroom and teachers' beliefs about the importance of early mathematics are not directly linked to young children's mathematical abilities.

This suggests that the learning environment is influenced not only by the frequency of activities and teacher beliefs but also by the quality of the activities and the interactions with children, both of which can potentially impact children's outcomes. Therefore, it is important to consider teaching strategies, the pedagogical quality of activities, children's engagement levels, and classroom interactions when examining the school environment. Additionally, the findings highlight that the school environment can play a role when combined with children's cognitive abilities. Consequently, teachers must consider children's cognitive characteristics when planning and implementing mathematics activities, as this is crucial for transforming these activities into effective learning experiences. Moreover, teachers' individual characteristics and pedagogical approaches also significantly influence the quality of the activities. To improve learning environments, it is essential to promote supportive training programs that enhance teachers' pedagogical competencies, provide guidance to improve activity quality, and encourage child-centered teaching strategies.

The second part of the implications addresses policymakers responsible for developing national education and parenting strategies. In this regard, policymakers can play a significant role by encouraging the enhancement of direct classroom practices. This includes promoting activities focused on math and integrating them with other subjects, while also incorporating objectives that target the development of executive function within the education curriculum. Such approaches would support children's cognitive processes alongside their math abilities.

In addition, training programs for parents and guidance services for families can be provided to ensure the active involvement of parents in supporting their children's math abilities as part of the educational system. These trainings can be designed to raise awareness among parents about effective ways they can contribute to their children's mathematics education, and to provide them with the necessary information. Based on previous studies, these training sessions can cover topics such as math concepts, incorporating mathematics into daily life, providing stimulating manipulatives related to math, utilizing technology, and addressing individual differences in how children learn math. Furthermore, guidance services can offer direct support to families facing challenges in helping their children with math. These services can share valuable resources on promoting math learning through play, organize workshops where parents can exchange experiences, and provide access to expert advice and guidance.

Finally, programs and projects that encourage neurodevelopmental practices while considering children's math abilities in the context of education and family can be developed. In this context, innovative applications can be designed to use neurodevelopmental approaches based on the functioning and development of the brain during the mathematical learning process. The aim of such programs and projects can be focused on strengthening children's cognitive processes related to mathematics (e.g. EF, memory and attention), to making the learning experience more efficient. Additionally, these initiatives can tailor strategies to each child's individual developmental needs.

Overall, the development of young children's math abilities can be achieved through a holistic approach that integrates biological, cognitive, environmental, and neurodevelopmental factors. In this framework, children's math abilities are shaped not by a single element, but by a multifaceted dynamic interaction where behavioral, physiological, and environmental factors work together. Both the school and home environments play crucial roles in this development by providing stimulating experiences that foster children's cognitive processes. At home, parents' involvement in teaching daily life skills and cultivating positive attitudes towards mathematics is significant to this process. Therefore, it is essential to ensure their active participation through guidance and training programs for parents. Similarly, adopting a curriculum that emphasizes executive functioning in schools and raising teachers’ awareness of pedagogical strategies that support early mathematical skills significantly contributes to children's development. Furthermore, integrating neurodevelopmental practices into the educational and family environments can strengthen children’s cognitive abilities, particularly in attention, memory, and problem-solving. These practices can enhance learning by incorporating technology and gamified tools. Lastly, to support this integrated approach, policymakers should focus on strengthening cognitive outcomes in curriculum development and encourage neurodevelopmental projects. This framework considers individual differences among children and ensures that both family and educational experiences have a direct impact on the development of their math abilities.

5.5. Limitations of the Study

While the current study provides valuable findings on children's math skills, it is important to acknowledge some limitations that frame the interpretation of the results. These limitations pertain to the study design, data collection process, and tools used.

The first limitation is related to the study’s design. In the current study a cross-sectional study is employed. It refers to the data was collected at a single point in time. As a result, it only provides information on the variables at that specific moment and does not offer conclusions regarding the sustainability or long-term impacts of the observed relationships.

Moreover, there are limitations regarding the data collection process and data collection tools. The data were collected through one-on-one interactions with the child, taking place outside the classroom environment. As a result, the study was not conducted in the child’s natural environment, but rather in a familiar room within the school. Additionally, child-adult mathematics activities and adults’ beliefs were not obtained through direct observation, so it is assumed that the respondents’ answers were objective and sincere. Another limitation in the data collection process is that fNIRS measurements were limited to the parietal lobe. This provides information solely on activation in that specific region. Consequently, hemodynamic changes in other regions of the brain could not be captured.

5.6. Recommendations for Further Studies

These results emphasize the importance of multidimensional assessment to explain children’s math abilities. For this reason, it is important to consider multiple factors when evaluating these factors in future research. Among these factors, executive function emerges as a key factor in explaining children’s math abilities. Given that EF, a frequently investigated area in the literature, can be addressed together with both environmental and biological factors, and it provides a valuable contribution to the field. In future studies, biological-cognitive structures in explaining mathematics abilities can be examined in detail and contribute to the literature.

Moreover, these results give us a valuable insight into the influence of environmental factors in the early childhood years. However, the current results suggest that there is no direct relationship between a child's math ability and school-related factors, so further research would be useful to understand the underlying mechanism. Thus, studies could examine the teacher-child relationship, mathematics-related materials in the classroom, how mathematics activities are conducted in the classroom, and children's participation in these activities. Qualitative studies could also be conducted to examine the nature of these activities in more depth.

Additionally, mathematics anxiety, defined as the discomfort or stress children experience while performing math, is also related to children’s mathematics skills (Devine et al., 2012). Physiologically, this situation affects factors such as stress levels and blood pressure (Hunt et al., 2017). From this perspective, it is important to explore the emotional aspects, such as anxiety and stress, in preschool-aged children and how these emotional states relate to their mathematical performance. Furthermore, research could be conducted on the impact of educational interventions by measuring brain activity during classroom instruction to determine how direct interactions in the learning environment influence brain function.

5.7. Conclusion

The aim of this study was to examine young children’s math abilities by integrating biological, cognitive and environmental factors. From this perspective, the relationships between children’s math abilities, hemodynamic changes in the parietal lobe, child’s executive function performance, and home and school environments were investigated. Home and school environments were constructed from the variables of direct mathematical practices to which children were exposed and adults’ (i.e., parent and teacher) beliefs about early mathematics. To achieve this aim, a variety of statistical techniques were employed.

Overall, the results demonstrated that children’s math abilities can be explained through the interaction of biology, cognition, and environment. More specifically, the most influential factor was children’s executive function performance in explaining their math abilities. While the parietal lobe is activated during mathematical tasks, it was found that only the intraparietal sulcus is associated with children’s math achievement. On the other hand, school and home environments alone did not explain math outcomes, but their impact became evident when combined with other factors. Additionally, when EF interacts with home and school environment or hemodynamic responses in the intraparietal sulcus, the explanation of math abilities is further strengthened. From this perspective, this study contributes to the literature by highlighting the importance of a holistic approach in understanding young children’s math abilities, and establishes connections between brain activity and behavior.

REFERENCES

Ability. Cambridge Dictionary. (2022). Retrieved January 5, 2022, from https://dictionary.cambridge.org/dictionary/english/ability.

Abreu-Lima, I.M.P., Leal, T.B., Cadima, J. et al. (2013). Predicting child outcomes from preschool quality in Portugal. *Eur J Psychol Educ 28,* 399–420. https://doi.org/10.1007/s10212-012-0120-y

Abry, T., Latham, S., Bassok, D., & LoCasale-Crouch, J. (2015). Preschool and kindergarten teachers’ beliefs about early school competencies: Misalignment matters for kindergarten adjustment. *Early Childhood Research Quarterly, 31*, 78–88. https://doi.org/10.1016/j.ecresq.2015.01.001

Alkan, S. (2006). *Using eye tracking data to analyze a computer game learning experience* (172254). [Unpublished Master’s Thesis, Middle East Technical University], YÖK Tez

Allan, N. P., Hume, L. E., Allan, D. M., Farrington, A. L., & Lonigan, C. J. (2014). Relations between inhibitory control and the development of academic skills in preschool and kindergarten: A meta-analysis. *Developmental Psychology, 50*(10), 2368–2379. https://doi.org/10.1037/a0037493

Amaro, E., & Barker, G. J. (2006). Study design in fMRI: Basic principles. *Brain and Cognition, 60*(3), 220-232. https://doi.org/10.1016/j.bandc.2005.11.009

APA Dictionary of Psychology. (2024). *Hemodynamic response.* Retrieved May 28, 2024, from https://dictionary.apa.org/hemodynamic-response

Arán Filippetti, V., & Richaud, M. C. (2017). A structural equation modeling of executive functions, IQ and mathematical skills in primary students: Differential effects on number production, mental calculus and arithmetical problems. *Child Neuropsychology, 23*(7), 864-888. https://doi.org/10.1080/09297049.2016.1199665

Arsalidou, M., & Taylor, M. J. (2011). Is 2+2=4? Meta-analyses of brain areas needed for numbers and calculations. *NeuroImage, 54*(3), 2382–2393. https://doi.org/10.1016/j.neuroimage.2010.10.009

Artemenko, C., Nuerk, H.-C., Dresler, T., Moeller, K., Wortha, S. M., Frey, M., & Barrocas, R. (2022). Finger-Based Numerical Training Increases Sensorimotor Activation for Arithmetic in Children—An fNIRS Study. *Brain Sciences, 12*(5). https://doi.org/10.3390/brainsci12050637

Artemenko, C., Sitnikova, M. A., Soltanlou, M., Dresler, T., & Nuerk, H. C. (2020). Functional lateralization of arithmetic processing in the intraparietal sulcus is associated with handedness. *Scientific Reports, 10*(1), 1775. https://doi.org/10.1038/s41598-020-58477-7

Artemenko, C., Soltanlou, M., Dresler, T., Ehlis, A.-C., & Nuerk, H.-C. (2018). The neural correlates of arithmetic difficulty depend on mathematical ability: evidence from combined fNIRS and ERP. *Brain Structure & Function, 223*(6), 2561–2574. https://doi.org/10.1007/s00429-018-1618-0

Artemenko, C., Soltanlou, M., Ehlis, AC. et al. (2018) The neural correlates of mental arithmetic in adolescents: a longitudinal fNIRS study. *Behav Brain Funct 14*, 5. https://doi.org/10.1186/s12993-018-0137-8

Avcı, K. (2015). *An investigation of early mathematics abilities among 48-66 month old preschool children in relation of some variables* (381204) [Unpublished Master’s Thesis, Çanakkale Onsekiz Mart University], YÖK Tez.

Baddeley, A. (1992). Working memory. *Science, 255*(5044), 556–559. https://doi.org/10.1126/science.1736359

Balladares, J., & Kankaraš, M. (2020). *Attendance in early childhood education and care programmes and academic proficiencies at age 15*. OECD Education Working Papers, No. 214. OECD Publishing. https://doi.org/10.1787/f16c7ae5-en.

Baroody, A. J. & Coslick, R. T. (1998). *Fostering children’s mathematical power: An investigative approach to K-8 mathematics instruction.* Lawrence Erlbaum Associates.

Barreto, C., & Soltanlou, M. (2022). Functional near-infrared spectroscopy as a tool to assess brain activity in educational settings: An introduction for educational researchers. *South African Journal of Childhood Education, 12*(1), 1-10.

Barrett, P. (2007). Structural equation modelling: Adjudging model fit. *Personality and Individual Differences, 42*(5), 815–824. https://doi.org/10.1016/j.paid.2006.09.018

Bartels, A., Goense, J., & Logothetis, N. (2012). Functional magnetic resonance imaging. In R. Brette & A. Destexhe (Eds.), *Handbook of Neural Activity Measurement* (pp. 410-469). Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511979958.011

Ben-Shalom, T., Berger, A., & Henik, A. (2013). My brain knows numbers! - an ERP study of preschoolers' numerical knowledge. *Frontiers in Psychology, 4*, 716. https://doi.org/10.3389/fpsyg.2013.00716

Benz, C. (2012). Attitudes of kindergarten educators about math. *Journal Für Mathematik-Didaktik, 33*(2), 203–232. https://doi.org/10.1007/s13138-012-0037-7

Bickhard, M. H. (2009). Interactivism: A manifesto. *New Ideas in Psychology, 27*(1), 85-95. https://doi.org/10.1016/j.newideapsych.2008.05.001

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development & Parenting, 5*(1), 35–45. https://doi.org/10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

Blevins-Knabe, B., Austin, A. B., Musun, L., Eddy, A., & Jones, R. M. (2000). Family home care providers' and parents' beliefs and practices concerning mathematics with young children. *Early Child Development and Care, 165*(1), 41-58. https://doi.org/10.1080/0300443001650104

Bliss, S. (2006). Test reviews: Ginsburg, H., & Baroody, A. (2003). J*ournal of Psychoeducational Assessment, 24*(1), 85–98. https://doi.org/10.1177/0734282905282839

Boas, D. A., Elwell, C. E., Ferrari, M., & Taga, G. (2014). Twenty years of functional near-infrared spectroscopy: Introduction for the special issue. *NeuroImage, 85*(Part 1), 1–5. https://doi.org/10.1016/j.neuroimage.2013.11.033

Bodrova, E., & Leong, D. J. (2007). *Tools of the Mind: The Vygotskian approach to early childhood education* (2nd Ed.). Columbus, OH; Merrill/Prentice Hall.

Boomsma, A. (1982) Robustness of LISREL against small sample sizes in factor analysis models. In: K.G. Joreskog; H. Wold (Eds). *Systems under indirection observation: Causality, structure, prediction (Part I).* (pp. 149-173) Amsterdam, Netherlands: North Holland.

Brigadoi, S., Ceccherini, L., Cutini, S., Scarpa, F., Scatturin, P., Selb, J., Gagnon, L., Boas, D. A., & Cooper, R. J. (2014). Motion artifacts in functional near-infrared spectroscopy: a comparison of motion correction techniques applied to real cognitive data. *NeuroImage, 85* Pt1(01), 181–191. https://doi.org/10.1016/j.neuroimage.2013.04.082

Bronfenbrenner, U. (1979). *The ecology of human development: Experiments by nature and design*. Harvard University Press.

Bronfenbrenner, U. (1993). The ecology of cognitive development: Research model and fugitive findings. In R. H. Wozniak, & K. W. Fischer (Eds.), *Development in the context: Acting and thinking in specific environments*. Lawrence Erlbaum Associates, Inc.

Bronfenbrenner, U. (1994). Ecological models of human development. In T. Husen & T. N. Postlethwaite (Eds.), *International encyclopedia of education* (2nd ed., Vol. 3). Oxford, England: Pergamon Press/Elsevier Science.

Bronfenbrenner, U., & Morris, P. (2006) The bioecological model of human development. In W. Damon & R. M. Lerner (Series Eds.) & R. M. Lerner (Vol. Ed.) *Handbook of child psychology: Vol. 1. Theoretical models of human development (6th ed. Pp.793-828)* New York: John Wiley.

Brown, T. T., & Jernigan, T. L. (2012). Brain development during the preschool years. *Neuropsychology review, 22*(4), 313–333. https://doi.org/10.1007/s11065-012-9214-1

Brunsek, A., Perlman, M., Falenchuk, O., McMullen, E., Fletcher, B., & Shah, P. S. (2017). The relationship between the Early Childhood Environment Rating Scale and its revised form and child outcomes: A systematic review and meta-analysis. *PloS One, 12*(6), e0178512. https://doi.org/10.1371/journal.pone.0178512

Bugden, S., Price, G. R., McLean, D. A., & Ansari, D. (2012). The role of the left intraparietal sulcus in the relationship between symbolic number processing and children's arithmetic competence. *Developmental Cognitive Neuroscience, 2*(4), 448-457. https://doi.org/10.1016/j.dcn.2012.04.001

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives, 8*(1), 36-41. https://doi.org/10.1111/cdep.12059

Butterworth, B. (2020). The biological roots of human numerical abilitiescA Brain for Numbers: The Biology of the Number Instinct Andreas Nieder (The MIT Press, Cambridge, MA; 2019) ISBN: 978-0-262-04278-9. *Current Biology, 30* (6), R242-R244. https://doi.org/10.1016/j.cub.2020.02.022

Butterworth, B. & Varma, S. (2014). Mathematical development. In D. Mareschal, B. Butterworth & A. Tolmie (Eds.). *Educational neuroscience* (pp. 201-236). Wiley.

Butterworth, B., & Walsh, V. (2011). Neural basis of mathematical cognition. *Current Biology, 21*(16), R618–R621.

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: from brain to education. *Science (New York, N.Y.), 332*(6033), 1049–1053. https://doi.org/10.1126/science.1201536

Buzsáki, G., Anastassiou, C. & Koch, C. The origin of extracellular fields and currents — EEG, ECoG, LFP and spikes. *Nat Rev Neurosci 13*, 407–420 (2012). https://doi.org/10.1038/nrn3241

Brown, T. A. (2006). *Confirmatory factor analysis for applied research.* The Guilford Press.

Byrne, B. M. (2001). *Structural equation modeling with AMOS: Basic concepts, applications, and programming*. Lawrence Erlbaum Associates Publishers.

Canbeldek, M. (2015). *Determination of the relation between preschool educational institution quality on the developmental levels of preschool children* (406996) [Master's Thesis, Pamukkale University]. YÖK Tez.

Cantlon JF, Brannon EM, Carter EJ, Pelphrey KA (2006) Functional Imaging of Numerical Processing in Adults and 4-y-Old Children. *PLOS Biology 4*(5): e125. https://doi.org/10.1371/journal.pbio.0040125

Chan, S. W. Y., Rao, N., Cohrssen, C., & Richards, B. (2021). Predicting child outcomes in Bhutan: Contributions of parenting support and early childhood education programmes. *Children and Youth Services Review, 126,* 1-10. https://doi.org/10.1016/j.childyouth.2021.106051

Charlesworth, R., Hart, C.H., Burts, D., & Hernandez, S. (1991). Kindergarten teachers’ beliefs and practices. *Early Development and Care, 70*(1), 17–35. https://doi.org/10.1080/0300443910700103

Charlesworth, R., Hart, C.H., Burts, D., Thomasson, R.H., Mosley, J., & Fleege, P.O. (1993). Measuring the developmental appropriateness of kindergarten teachers’ beliefs and practices. *Early Childhood Research Quarterly, 8*(3), 255–276. https://doi.org/10.1016/S0885-2006(05)80067-5

Chen, J.Q. & McCray, J. (2013). *Focusing on the whole teacher in early math teacher professional development*. (Early Math Collaborative Working Paper No. 2013-1). Retrieved from the Erikson Institute Early Math Collaborative website: earlymath.erikson.edu

Chen, J.-Q., McCray, J., Adams, M., & Leow, C. (2013). A survey study of early childhood teachers' beliefs and confidence about teaching early math. *Early Childhood Education Journal, 42*(6), 367–377. https://doi.org/10.1007/s10643-013-0619-0

Choi, J. Y., & Dobbs-Oates, J. (2014). Childcare quality and preschoolers' math development. *Early Child Development and Care, 184*(6), 915-932. https://doi.org/10.1080/03004430.2013.829822

Claire Son, S. H. & Hur, J. H. (2020) Parental math talk during home cooking and math skills in head start children: The role of task management talk, *Journal of Research in Childhood Education, 34*(3), 406-426, https://doi.org/10.1080/02568543.2019.1704318

Clements, D. H. (1999). Subitizing: What Is It? Why Teach It? *Teaching Children Mathematics*, *5*(7), 400–405. http://dx.doi.org/10.5951/TCM.5.7.0400

Clements, D. H., & Sarama, J. (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education.* Lawrence Erlbaum Associates

Clements, D. H., & Sarama, J. (2007). *Early childhood mathematics learning. Second Handbook of Research on Mathematics Teaching and Learning, 1,* 461–555.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The Learning Trajectories Approach* (2nd ed.). Routledge.

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences* (3rd ed.). Lawrence Erlbaum Associates, Publishers, London.

Coople, C. & Bredekamp, S. (2009). *Developmentally Appropriate Practice in Early Childhood Programs Serving Children from Birth through Age 8* (3rd Ed.) Washington DC: National Association for the Education of Young Children.

Copley, J. V. (2000). *The young child and mathematics.* National Association for the Education of Young Children.

Cortés Pascual, A., Moyano Muñoz, N., & Quílez Robres, A. (2019). The relationship between executive functions and academic performance in primary education: Review and meta-analysis. *Frontiers in Psychology, 10,* Article 1582. https://doi.org/10.3389/fpsyg.2019.01582

Coşkun, A. (2019). *Investigation of classroom management skills by using eye-tracking technology* (575068). [Unpublished Master's Thesis, Middle East Technical University], YÖK Tez.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3,* 63-68. https://doi.org/10.1016/j.tine.2013.12.001

Cudeck, R.A., Toit, S.D., & Sörbom, D. (2001). *Structural equation models: Present and Future.* (First Eddition). Lincolnwood: Scientific Software International, Inc.

Dalenberg, J. R., Hoogeveen, H. R., & Lorist, M. M. (2018). Physiological measurements: EEG and fMRI. EEG and FMRI. *Methods in consumer research, volume 2: Alternative approaches and special applications* (pp. 253-277) https://doi.org/10.1016/B978-0-08-101743-2.00011-X

Daucourt, M. C., Napoli, A. R., Quinn, J. M., Wood, S. G., & Hart, S. A. (2021). The home math environment and math achievement: A meta-analysis. *Psychological Bulletin, 147*(6), 565–596. https://doi.org/10.1037/bul0000330

Davis-Kean, P., Domina, T., Kuhfeld, M., Ellis, A., & Gershoff, E. T. (2021). It matters how you start: Early numeracy mastery predicts high school math course-taking and college attendance. *Infant and Child Development, 31,* e2281. https://doi.org/10.1002/icd.2281

DeFlorio, L., & Beliakoff, A. (2015). Socioeconomic status and preschoolers' mathematical knowledge: The contribution of home activities and parent beliefs. *Early Education and Development, 26*(3), 319-341. https://doi.org/10.1080/10409289.2015.968239

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*(1-2), 1-42. https://doi.org/10.1016/0010-0277(92)90049-n

Dehaene, S. (2011). *The number sense: How the mind creates mathematics* (Revised & Expanded Edition). Oxford University Press.

Dehaene, S., & Mehler, J. (1992). Cross-linguistic regularities in the frequency of number words. *Cognition, 43*(1), 1-29. https://doi.org/10.1016/0010-0277(92)90030-L

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*(3), 487–506. https://doi.org/10.1080/02643290244000239

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284*(5416), 970-974. https://doi.org/10.1126/science.284.5416.970

Delpy, D. T., Cope, M., van der Zee, P., Arridge, S., Wray, S., & Wyatt, J. (1988). Estimation of optical pathlength through tissue from direct time of flight measurement. *Physics in Medicine and Biology, 33*(12), 1433–1442. https://doi.org/10.1088/0031-9155/33/12/008

Devine, A., Fawcett, K., Szűcs, D. et al. (2012). Gender differences in mathematics anxiety and the relation to mathematics performance while controlling for test anxiety. *Behav Brain Funct 8*, 33. https://doi.org/10.1186/1744-9081-8-33

Diamond A. (2013). Executive functions. *Annual Review of Psychology, 64*, 135–168. https://doi.org/10.1146/annurev-psych-113011-143750

Dick F., Llyod-Fox S., Blasi A., Elwell C., & Mills D. (2014). Neuroimaging methods In. D. Mareschal, B. Butterworth, & A. Tolmie (Eds.). (2014). *Educational neuroscience*. Wiley

Dresler, T., Obersteiner, A., Schecklmann, M., Vogel, A. C., Ehlis, A. C., Richter, M. M., Plichta, M. M., Reiss, K., Pekrun, R., & Fallgatter, A. J. (2009). Arithmetic tasks in different formats and their influence on behavior and brain oxygenation as assessed with near-infrared spectroscopy (NIRS): a study involving primary and secondary school children. *Journal of Neural Transmission (Vienna, Austria:1996), 116*(12), 1689–1700. https://doi.org/10.1007/s00702-009-0307-9

Education Reform Initiative (ERI) & Mother Child Education Foundation (AÇEV) (2016) *Her çocuğa eşit fırsat: Türkiye'de erken çocukluk eğitiminin durumu ve öneriler.* Retrieved January 19, 2023, from http://www.egitimreformugirisimi.org/wp-content/uploads/2017/03/ERG_HERKES-%C4%B0C%C4%B0N-ESIT-FIRSAT-TURKIYEDE-ERKEN-COCUKLUK-EGITIMININ-DURUMU-VE-ONERILER.web_.pdf

Elliott, L., & Bachman, H. J. (2018). How do parents foster young children's math skills? *Child Development Perspectives, 12*(1), 16-21. https://doi.org/10.1111/cdep.12249

Emerson, R. W., & Cantlon, J. F. (2015). Continuity and change in children's longitudinal neural responses to numbers. *Developmental science, 18*(2), 314-326. https://doi.org/10.1111/desc.12215

Emslander, V., & Scherer, R. (2022). The relation between executive functions and math intelligence in preschool children: A systematic review and meta-analysis. *Psychological Bulletin, 148*(5-6), 337-369. https://doi.org/10.1037/bul0000369

Enders, C. K. (2010). *Applied missing data analysis*. Guilford Press.

Epstein, J. L. (2010). School/family/community partnerships: Caring for the children we share. *Phi Delta Kappan, 92*(3), 81–96. https://doi.org/10.1177/003172171009200326

Erdoğan, S. (2006). *A study on the effects of mathematics education given with drama method to six years old children mathematics ability* (182715) (Doctoral Dissertation, Ankara University). YÖK Tez

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002

Ferrari, M., & Quaresima, V. (2012). A brief review on the history of human functional near-infrared spectroscopy (fNIRS) development and fields of application. *NeuroImage, 63*(2), 921–935. https://doi.org/10.1016/j.neuroimage.2012.03.049

Finders, J. K., Budrevich, A., Duncan, R. J., Purpura, D. J., Elicker, J., & Schmitt, S. A. (2021). Variability in preschool class scores and children's school readiness. *AERA Open, 7*(1), 1-19. https://doi.org/10.1177/23328584211038938

Fishbein, M., & Ajzen, I. (1975). *Belief, Attitude, Intention, and Behavior: An Introduction to Theory and Research*. Reading, MA: Addison-Wesley.

Foster, T.D., Froyen, L.C., Skibbe, L.E. et al. (2016) Fathers' and mothers' home learning environments and children's early academic outcomes. *Read Writ* 29, 1845–1863. https://doi.org/10.1007/s11145-016-9655-7

Francis, J., & Barnett, W. S. (2019). Relating preschool class size to classroom quality and student achievement. *Early Childhood Research Quarterly, 49,* 49–58. https://doi.org/10.1016/j.ecresq.2019.05.002

Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review, 10,* 29–44. https://doi.org/10.1016/j.edurev.2013.05.003

Furnham, A., Rakow, T., & Mak, T. (2002). The determinants of parents' beliefs about the intelligence of their children: A study from Hong Kong. *International Journal of Psychology, 37*(6), 343–352. doi:10.1080/00207590244000151

Galindo, C.L., & Sonnenschein, S. (2015). Decreasing the SES math achievement gap: Initial math proficiency and home learning environments. *Contemporary Educational Psychology, 43,* 25-38. https://doi.org/10.1016/J.CEDPSYCH.2015.08.003

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*(1-2), 43–74. https://doi.org/10.1016/0010-0277(92)90050-r

Gao, F., Levine, S. C., & Huttenlocher, J. (2000). What Do Infants Know about Continuous Quantity? *Journal of Experimental Child Psychology, 77*(1), 20–29. https://psycnet.apa.org/doi/10.1006/jecp.1999.2556

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications.* American Psychological Association. https://doi.org/10.1037/10163-000

Geist, E. (2008). *Children are born mathematicians: Supporting mathematical development, birth to age eight* (1st ed.). Pearson Merrill Prentice Hall.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Harvard University Press.

Ginsburg, H., & Baroody, A. (2003). *Test of early mathematics ability* (3rd Ed.). Austin, TX: Pro-Ed.

Gladle, G. (Ed.). (2015). Arithmetic: *The Foundation of Mathematics.* Britannica Educational Publishing: Rosen Educational Services.

Gordon, R., Scalise, N. R., & Ramani, G. B. (2021). Give yourself a hand: The role of gesture and working memory in preschoolers' numerical knowledge. *Journal of Experimental Child Psychology, 208,* Article 105145. https://doi.org/10.1016/j.jecp.2021.105145

Grammatikopoulos, V., Gregoriadis, A., Tsigilis, N., & Zachopoulou, E. (2018). Evaluating quality in early childhood education in relation with children outcomes in Greek context. *Early Child Development and Care, 188*(12), 1816–1825. https://doi.org/10.1080/03004430.2017.1289192

Gravetter, F. J., & Wallnau, L. B. (2017). *Statistics for the behavioral sciences* (10th ed.). Cengage Learning.

Guerrero-Rosada, P., Weiland, C., McCormick, C., Hsueh, J., Sachs, J., Snow, C., & Maier, M. (2021). Null relations between CLASS scores and gains in children's language, math, and executive function skills: A replication and extension study. *Early Childhood Research Quarterly, 54*, 1-12. https://doi.org/10.1016/j.ecresq.2020.07.009

Güçhan Özgül, S. (2011). *Evaluation of early childhood education environments in terms of quality variables* (280612) [Master's Thesis, Balıkesir University]. YÖK Tez.

Güleş, F. (2013). *Determining quality standards of physical environment in pre-school education* (337677) [Doctoral Dissertation, Selçuk University]. YÖK Tez.

Gürgah Oğul, İ., & Aktaş Arnas, Y. (2020). Role of home mathematics activities and mothers' maths talk in predicting children's maths talk and early maths skills. *European Early Childhood Education Research Journal, 29,* 501 - 518. https://doi.org/10.1080/1350293X.2020.1858128

Haier, R. J., Siegel Jr, B. V., Nuechterlein, K. H., Hazlett, E., Wu, J. C., Paek, J., Browning, H. L., & Buchsbaum, M. S. (1988). Cortical glucose metabolic rate correlates of abstract reasoning and attention studied with positron emission tomography. *Intelligence (Norwood), 12*(2), 199-217. https://doi.org/10.1016/0160-2896(88)90016-5

Hamamcı, B., Acar, I. & Uyanik, G. (2023). Association between performance-based and ratings of Turkish children’s executive function. *Current Psycholo*gy. https://doi.org/10.1007/s12144-021-02307-0

Härkönen, U. (2008). The Bronfenbrenner ecological systems theory of human development.

Harding, J. F. (2015). Increases in maternal education and low-income children's cognitive and behavioral outcomes. *Developmental Psychology*, 51(5), 583–599. https://doi.org/10.1037/a0038920

Harms, T., Clifford, R. M., &Ceyer, D. (1980). *Early childhood environment rating scale.* New York: Teachers College Press, Columbia University.

Harms, T., R. Clifford & D. Cryer (1998), *Early Childhood Environmental Rating Scale-Revised*, Teachers College Press, New York, NY

Harris, B., & Petersen, D. (2017). *Developing math skills in early childhood*. Issue Brief. Mathematica Policy Research, Inc

Hayduk, L., Cummings, G., Boadu, K., Pazderka-Robinson, H., & Boulianne, S. (2007). Testing! Testing! One, two, three--Testing the theory in structural equation models! *Personality and Individual Differences, 42*(5), 841–850. https://doi.org/10.1016/j.paid.2006.10.001

Haylock, D. & Cockburn, A. (2008). *Understanding mathematics for young children: a guide for foundation stage and lower primary teachers.* SAGE Publications.

Hiebert, J., & Lefevre, P. (2013). Conceptual and procedural knowledge in mathematics: An introductory analysis. *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-28) https://doi.org/10.4324/9780203063538

Hofer, B.K. (2001). Personal epistemology research: Implications for learning and teaching. *Educational Psychology Review, 13,* 353–383. https://doi.org/10.1023/A:1011965830686

Hooper, D., Coughlan, J., & Mullen, M. R. (2008). Structural equation modelling: Guidelines for determining model fit. Electronic *Journal of Business Research Methods, 6*(1), 53-60.

Howard-Jones, P. A., Varma, S., Ansari, D., Butterworth, B., De Smedt, B., Goswami, U., Laurillard, D., & Thomas, M. S. C. (2016). The principles and practices of educational neuroscience: Comment on Bowers (2016). *Psychological Review, 123*(5), 620–627. https://doi.org/10.1037/rev0000036

Hu, B. Y., Li, Y., Zhang, X., Roberts, S. K., & Vitiello, G. (2021). The quality of teacher feedback matters: Examining Chinese teachers' use of feedback strategies in preschool math lessons. *Teaching and Teacher Education, 98*. https://doi.org/10.1016/j.tate.2020.103253

Hu, L., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling, 6*(1), 1-55. https://doi.org/10.1080/10705519909540118

Huang, Q., Zhang, X., Liu, Y., Yang, W., & Song, Z. (2017). The contribution of parent-child numeracy activities to young Chinese children's mathematical ability. *The British Journal of Educational Psychology, 87*(3), 328–344. https://doi.org/10.1111/bjep.12152

Huettel., S.A., Song, A. W., & McCarthy, G. (2009). *Functional magnetic resonance imaging* (2nd Ed.). Sinauer Associates, Inc.

Hunt, T. E., Bhardwa, J., & Sheffield, D. (2017). Mental arithmetic performance, physiological reactivity and mathematics anxiety amongst U.K. Primary school children. *Learning and Individual Differences, 57,* 129–132. https://doi.org/10.1016/j.lindif.2017.03.016

Huntsinger, C. S., Jose, P. E., & Luo, Z. (2016). Parental facilitation of early mathematics and reading skills and knowledge through encouragement of home-based activities. *Early Childhood Research Quarterly, 37,* 1–15. https://doi.org/10.1016/j.ecresq.2016.02.005

Huppert, T. J., Diamond, S. G., Franceschini, M. A., & Boas, D. A. (2009). HomER: a review of time-series analysis methods for near-infrared spectroscopy of the brain. *Applied Optics, 48*(10), D280–D298. https://doi.org/10.1364/ao.48.00d280

Hyde, D. C. (2021). The emergence of a brain network for numerical thinking. *Child Development Perspectives, 15*(3), 168-175. http://dx.doi.org/10.1111/cdep.12418

Hyde, D. C., Boas, D. A., Blair, C., & Carey, S. (2010). Near-infrared spectroscopy shows right parietal specialization for number in pre-verbal infants. *Neuroimage, 53*(2), 647-652. https://psycnet.apa.org/doi/10.1016/j.neuroimage.2010.06.030

Hyson, M., Biggar Tomlinson, H., & Jones, J. (2014). *The early years matter : education, care, and the well-being of children, birth to 8 / Marilou Hyson and Heather Biggar Tomlinson ; foreword by Jacqueline Jones*. Teachers College Press.

IBM Corp. (2015). SPSS Windows, 23.0. Armonk, NY: IBM Corp. [Computer Software]

Institute of Medicine and National Research Council (2000). *From Neurons to Neighborhoods: The Science of Early Childhood Development*. Washington, DC: The National Academies Press. https://doi.org/10.17226/9824.

İvrendi, A., & A. Wakefield. 2009. “Mothers’ and fathers’ participation in mathematical activities of their young children.” In *Education in Balkans Today–Proceedings of the 5th International Balkan Education and Science Congress*, edited by H. Asutay and E. Bayındır, 50–54. Edirne: Trakya University Publication.

Jackson, D. L. (2003). Revisiting sample size and number of parameter estimates: Some support for the N:Q hypothesis. *Structural Equation Modeling: A Multidisciplinary Journal, 10*(1), 128-141. https://doi.org/10.1207/S15328007SEM1001_6

JASP Team (2024). JASP (Version 0.17.1)[Computer software].

Johnson, B. (2001). Toward a new classification of nonexperimental quantitative research. *Educational Researcher, 30*(2), 3-13. https://doi.org/10.3102/0013189x030002003

Kagan, D. M. (1992). Implications of research on teacher beliefs. *Educational Psychologist*, 27(1), 65–75. https://doi.org/10.1207/s15326985ep2701_6

Karakuş, H. (2022). Erken Matematik Ölçeği'nin Türkçeye uyarlanması: Geçerlik ve güvenirlik çalışması. *Kocaeli Üniversitesi Eğitim Dergisi, 5*(1), 197-220. http://doi.org/10.33400/kuje.1059336

Karakuş, H., Akman, B., & Ergene, Ö. (2018). The Turkish adaptation study of The Mathematical Development Beliefs Scale. *Pegem Eğitim ve Öğretim Dergisi, 8*(2), 211-218, http://dx.doi.org/10.14527/pegegog.2018.009

Karatas, I., Guven, B., Öztürk, Y., Arslan, S., & Gürsöy, K. (2017). Investigation of pre-school teachers' beliefs about mathematics education in terms of their experience and structure of their education. *EURASIA Journal of Mathematics, Science & Technology Education, 13*(3), 673–689. https://doi.org/10.12973/eurasia.2017.00638a

Kawashima, R., Taira, M., Okita, K., Inoue, K., Tajima, N., Yoshida, H., Sasaki, T., Sugiura, M., Watanabe, J., & Fukuda, H. (2004). A functional MRI study of simple arithmetic--a comparison between children and adults. Brain research. *Cognitive Brain Research, 18*(3), 227–233. https://doi.org/10.1016/j.cogbrainres.2003.10.009

Kilday, C. R. (2010). *Factors affecting children's math achievement scores in preschool* (3442301). [Unpublished Doctoral Thesis, University of Virginia], ProQuest Dissertations & Theses Global

Kline, P. (1999). *The handbook of psychological testing* (2nd ed.). London: Routledge.

Kline, R. B. (2023). *Principles and Practice of Structural Equation Modeling* (5th ed.). New York: Guilford Press

Kluczniok, K. (2017). Early family risk factors and home learning environment as predictors of children's early numeracy skills through preschool. *SAGE Open, 7*(2), 1-13. https://doi.org/10.1177/2158244017702197

Korucu, I., Rolan, E., Napoli, A. R., Purpura, D. J., & Schmitt, S. A. (2019). Development of the Home Executive Function Environment (HEFE) Scale: Assessing its relation to preschoolers' executive function. *Early Childhood Research Quarterly, 47*, 9–19. https://doi.org/10.1016/j.ecresq.2018.09.001

Kucian, K., von Aster, M., Loenneker, T., Dietrich, T., & Martin, E. (2008). Development of neural networks for exact and approximate calculation: a FMRI study. *Developmental Neuropsychology, 33*(4), 447–473. https://doi.org/10.1080/87565640802101474

Kyttälä, M., Kanerva, K., & Kroesbergen, E. (2015). Training counting skills and working memory in preschool. *Scandinavian journal of psychology, 56*(4), 363–370. https://doi.org/10.1111/sjop.12221

Laski, E. V., & Dulaney, A. (2015). When prior knowledge interferes, inhibitory control matters for learning: The case of numerical magnitude representations. *Journal of Educational Psychology, 107*(4), 1035-1050. https://doi.org/10.1037/edu0000034

Layzer, J. & B. Goodson (2006), The ‘quality’ of early care and education settings: Definitional and measurement issue, *Evaluation Review,* 30/5, pp. 556-576, http://dx.doi.org/10.1177/0193841X06291524.

Lehrl, S., Kluczniok, K., & Rossbach, H.-G. (2016). Longer-term associations of preschool education: The predictive role of preschool quality for the development of mathematical skills through elementary school. *Early Childhood Research Quarterly, 36,* 475–488. https://doi.org/10.1016/j.ecresq.2016.01.013

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What Counts in the Development of Young Children’s Number Knowledge? *Developmental Psychology, 46*(5), 1309–1319.

Li, K., Zhang, P., Hu, B. Y., Burchinal, M. R., Fan, X., & Qin, J. (2019). Testing the ‘thresholds’ of preschool education quality on child outcomes in China. *Early Childhood Research Quarterly, 47,* 445–456. https://doi.org/10.1016/j.ecresq.2018.08.003

Libertus, M. E., Brannon, E. M., & Woldorff, M. G. (2011). Parallels in Stimulus-Driven Oscillatory Brain Responses to Numerosity Changes in Adults and Seven-Month-Old Infants. *Developmental Neuropsychology, 36*(6), 651–667. https://doi.org/10.1080/87565641.2010.549883

Little, R. J. A. 1988. A test of missing completely at random for multivariate data with missing values. *Journal of the American Statistical Association 83:404,* 1198–1202 https://doi.org/10.1080/01621459.1988.10478722

Magnuson, K. (2007). Maternal education and children's academic achievement during middle childhood. *Developmental Psychology*, 43(6), 1497–1512. https://doi.org/10.1037/0012-1649.43.6.1497

Manolitsis, G., Georgiou, G. K., & Tziraki, N. (2013). Examining the effects of home literacy and numeracy environment on early reading and math acquisition. *Early Childhood Research Quarterly, 28*(4), 692–703. https://doi.org/10.1016/j.ecresq.2013.05.004

Mareschal, D., Butterworth, B., & Tolmie, A. (Eds.). (2014). *Educational neuroscience*. Wiley.

Martin, T., Hamamcı B., Sherlock, P. & Zelazo, P. (2024) Meta-analysis of the relations between the dimensional change card sort (DCCS) and measures of math and reading. *JPS Conference 2024*. Toronto

Mashburn, A. J., Pianta, R. C, Hamre, B. K., Downer, J. T., Barbarin, O., Bryant, D., ... Howes, C. (2008). Measures of classroom quality in prekindergarten and children's development of academic, language, and social skills. *Child Development, 79*(3), 732-749. https://doi.org/10.1111/j.1467-8624.2008.01154.x

McCloskey, M., Sokol, S. M., & Goodman, R. A. (1986). Cognitive processes in verbal-number production: Inferences from the performance of brain-damaged subjects. *Journal of Experimental Psychology. General, 115*(4), 307-330. https://doi.org/10.1037/0096-3445.115.4.307

Miller, E. K. & Wallis J D (2009) Executive function and higher-order cognition: Definition and neural substrates. In: Squire LR (ed.) *Encyclopedia of Neuroscience, 4,* 99-104. Oxford: Academic Press.

Mills, G. E., & Gay, L. R. (2016). *Educational research: Competencies for analysis and applications.* Pearson. One Lake Street, Upper Saddle River, New Jersey 07458.

Missall, K. N., Hojnoski, R. L. & Moreano, G. (2017) Parent–child mathematical interactions: examining self-report and direct observation, *Early Child Development and Care, 187*(12), 1896-1908, https://doi.org/10.1080/03004430.2016.1193731

Missall, K., Hojnoski, R. L., Caskie, G. I. L., & Repasky, P. (2015). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Education and Development, 26*(3), 356–376. https://doi.org/10.1080/10409289.2015.968243

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A. & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology, 41*, 49-100. https://doi.org/10.1006/cogp.1999.0734

Monsell, S. (1996). Control of mental process. In V. Bruce (Ed.), *Unsolved mysteries of the mind: Tutorial essays in cognition* (pp. 93-148). Hove, UK: Erlbaum.

Muis, K.R., Bendixen, L.D. & Haerle, F.C.(2006) Domain-Generality and Domain-Specificity in Personal Epistemology Research: Philosophical and Empirical Reflections in the Development of a Theoretical Framework. *Educ Psychol Rev, 18*, 3–54. https://doi.org/10.1007/s10648-006-9003-6

Mutaf-Yıldız, B., Sasanguie, D., De Smedt, B., & Reynvoet, B. (2020). Probing the relationship between home numeracy and children's mathematical skills: A Systematic review. *Frontiers in psychology, 11*, 2074. https://doi.org/10.3389/fpsyg.2020.02074

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519

Navarro, J., Doucet, F., & Tudge, J. (2020). Bioecological systems influences on early childhood education. In *Scientific influences on early childhood education* (pp. 55-68). Routledge.

Ng, F. F., Tamis-LeMonda, C., Yoshikawa, H., & Sze, I. N. (2015). Inhibitory control in preschool predicts early math skills in first grade: Evidence from an ethnically diverse sample. *International Journal of Behavioral Development, 39*(2), 139-149. https://doi.org/10.1177/0165025414538558

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J., Wolfe, C. B., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly, 36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

NIRx (2024). NIRSport2 [Machine]. https://nirx.net/nirsport

Nigg, J. T. (2000). On inhibition/disinhibition in developmental psychopathology: Views from cognitive and personality psychology and a working inhibition taxonomy. *Psychological Bulletin, 126*(2), 220–246. https://doi.org/10.1037/0033-2909.126.2.220

Niklas, F., & Schneider, W. (2017). Home learning environment and development of child competencies from kindergarten until the end of elementary school. *Contemporary Educational Psychology, 49*, 263-274. https://doi.org/10.1016/J.CEDPSYCH.2017.03.006

OECD. (2024). *Do adults have the skills they need to thrive in a changing world?*: Survey of Adult Skills 2023. OECD Skills Studies. OECD Publishing. Paris. https://doi.org/10.1787/b263dc5d-en

OECD (2016), *"What are the benefits from early childhood education?", Education Indicators in Focus*, No. 42, OECD Publishing, Paris, https://doi.org/10.1787/5jlwqvr76dbq-en.

Okur-Atas, S., & Berument, S. K., (2022). School readiness of Turkish children living in poverty: The mediating roles of home environment and maternal behaviors. *Early Education And Development*. https://doi.org/10.1080/10409289.2022.2127291

Oostenveld, R., & Praamstra, P. (2001). The five percent electrode system for high-resolution EEG and ERP measurements. *Clinical neurophysiology : official journal of the International Federation of Clinical Neurophysiology, 112*(4), 713–719. https://doi.org/10.1016/s1388-2457(00)00527-7

Oturakdaş, S. (2019). *Analysis of the structural and functional quality traits of preschool education institutions* (598093) [Master's Thesis, Bolu Abant İzzet Baysal University]. YÖK Tez.

Ozcelik, E., Karakus, T., Kursun, E., & Cagiltay, K. (2009). An eye-tracking study of how color coding affects multimedia learning. *Computers & Education, 53*(2), 445-453. https://doi.org/10.1016/j.compedu.2009.03.002

Pajares, M. F. (1992). Teachers’ beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332. https://doi.org/10.3102/00346543062003307

Pallant, J. (2011). *SPSS survival manual : a step by step guide to data analysis using SPSS.* 4th Ed., Allen&Unwin, Berkshire.

Pardo, J. R., Baz, M. R., Gonzalez, A. J., & Herrera, R. D. S. (2020) Home numeracy activities in relation to basic number processing in kindergartners. *Revista de Education, 389*(2), 45-66. https://doi.org/10.4438/1988-592X-RE-2020-389-454

Passolunghi, M. C., & Costa, H. M. (2016). Working memory and early numeracy training in preschool children. Child neuropsychology : A journal on normal and abnormal development in childhood and adolescence, 22(1), 81–98. https://doi.org/10.1080/09297049.2014.971726

Pedhazur, E. J., & Schmelkin, L. P. (1991). *Measurement, design, and analysis: An integrated approach.* Hillsdale, NJ: Lawrence Erlbaum.

Peirce, J. W., Gray, J. R., Simpson, S., MacAskill, M. R., Höchenberger, R., Sogo, H., Kastman, E., Lindeløv, J. (2019). PsychoPy2: experiments in behavior made easy. *Behavior Research Methods*. https://doi.org/10.3758/s13428-018-01193-y

Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics. *Journal of Educational Psychology, 108*(4), 455–473. https://doi.org/10.1037/edu0000079

Peters, L., & De Smedt, B. (2018). Arithmetic in the developing brain: A review of brain imaging studies. *Developmental Cognitive Neuroscience, 30*, 265–279. https://doi.org/10.1016/j.dcn.2017.05.002

Phair, R. (2021). *International early learning and child well-being study assessment framework*. OECD Education Working Papers, No. 246. OECD Publishing, Paris. https://doi.org/10.1787/af403e1e-en.

Phillips, A. A., Chan, F. H., Zheng, M. M., Krassioukov, A. V., & Ainslie, P. N. (2016). Neurovascular coupling in humans: Physiology, methodological advances and clinical implications. *Journal of cerebral blood flow and metabolism : official journal of the International Society of Cerebral Blood Flow and Metabolism, 36*(4), 647–664. https://doi.org/10.1177/0271678X15617954

Phillipson, S., & Phillipson, S. N. (2007). Academic expectations, belief of ability, and involvement by parents as predictors of child achievement: A cross-cultural comparison. *Educational Psychology (Dorchester-on-Thames), 27*(3), 329-348. https://doi.org/10.1080/01443410601104130

Piaget, J. (1954). *The construction of reality in the child*. New York: Basic.

Pianta, R. C., La Paro, K. M., & Hamre, B. K. (2008). *Classroom assessment scoring system™: Manual k-3*. Paul H Brookes Publishing.

Pianta, R., Downer, J., & Hamre, B. (2016). Quality in Early Education Classrooms: Definitions, Gaps, and Systems. *Future of Children, 26*(2), 119–137. https://doi.org/10.1353/foc.2016.0015

Pianta, R., Howes, C, Burchinal, M., Bryant, D., Clifford, R., Early, D., & Barbarin, O. (2005). Features of pre-kindergarten programs, classrooms and teachers: Do they predict observed classroom quality and child-teacher interactions. *Applied Developmental Science, 9*(3), 144-159. https://doi.org/10.1207/s1532480xads0903_2

Piazza, M., Pinel, P., Le Bihan, D., & Dehaene, S. (2007). A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron, 53*(2), 293-305. https://doi.org/10.1016/j.neuron.2006.11.022

Platas, L. M. (2015). The Mathematical Development Beliefs Survey: Validity and reliability of a measure of preschool teachers' beliefs about the learning and teaching of early mathematics. *Journal of Early Childhood Research, 13*(3), 295–310. https://doi.org/10.1177/1476718X14523746

Prewett, S., & Whitney, S.D. (2021). The relationship between teachers' teaching self-efficacy and negative affect on eighth grade U.S. students' reading and math achievement. *Teacher Development, 25,* 1 - 17. https://doi.org/10.1080/13664530.2020.1850514

Rickard, T. C., Romero, S. G., Basso, G., Wharton, C., Flitman, S., & Grafman, J. (2000). The calculating brain: an fMRI study. *Neuropsychologia, 38*(3), 325–335. https://doi.org/10.1016/s0028-3932(99)00068-8

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science, 24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Rivera, S. M., Reiss, A. L., Eckert, M. A., & Menon, V. (2005). Developmental changes in mental arithmetic: evidence for increased functional specialization in the left inferior parietal cortex. *Cerebral Cortex (New York, N.Y. : 1991), 15*(11), 1779–1790. https://doi.org/10.1093/cercor/bhi055

Santana, A. N. d., Roazzi, A., & Nobre, Alena Pimentel Mello Cabral. (2022). The relationship between cognitive flexibility and mathematical performance in children: A meta-analysis. *Trends in Neuroscience and Education, 28,* 100179-100179. https://doi.org/10.1016/j.tine.2022.100179

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108*(3), 662–674. https://doi.org/10.1016/j.cognition.2008.05.007

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number–space mappings: What underlies mathematics achievement? *Journal of Experimental Child Psychology, 114*(3), 418-431. https://doi.org/10.1016/j.jecp.2012.10.012

Schmerse D. (2020). Preschool quality effects on learning behavior and later achievement in Germany: Moderation by socio-economic status. *Child development, 91*(6), 2237–2254. https://doi.org/10.1111/cdev.13357

Schmitt, S. A., Duncan, R. J., Budrevich, A., & Korucu, I. (2020). Benefits of behavioral self-regulation in the context of high classroom quality for preschoolers' mathematics. *Early Education and Development, 31*(3), 323-334. https://doi.org/10.1080/10409289.2019.1660555

Schroeder, P. A., Artemenko, C., Kosie, J. E., Cockx, H., Stute, K., Pereira, J., Klein, F., & Mehler, D. M. A. (2023). Using preregistration as a tool for transparent fNIRS study design. *Neurophotonics, 10*(2), 023515. https://doi.org/10.1117/1.NPh.10.2.023515

Shuey, E. and M. Kankaraš (2018), "The Power and Promise of Early Learning", *OECD Education Working Papers*, No. 186, OECD Publishing, Paris, https://doi.org/10.1787/f9b2e53f-en.

Silver, A. M., & Libertus, M. E. (2022). Environmental influences on mathematics performance in early childhood. *Nature Reviews Psychology, 1*(7), 407–418. https://doi.org/10.1038/s44159-022-00061-z

Silver, A. M., Miller, P., Votruba-Drzal, E., Libertus, M. E., & Bachman, H. J. (2024). Parent predictors of the home math environment and associations with toddlers’ math skills. *Early Childhood Research Quarterly, 69*, 88–100. https://doi.org/10.1016/j.ecresq.2024.07.007

Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: the development of a home numeracy model. *Journal of Experimental Child Psychology, 121*, 63–84. https://doi.org/10.1016/j.jecp.2013.11.006

Slot, P. (2018). Structural characteristics and process quality in early childhood education and care: A literature review, *OECD Education Working Papers, 176*, OECD Publishing, Paris, https://doi.org/10.1787/edaf3793-en.

Soliday Hong, S. L., Sabol, T. J., Burchinal, M. R., Tarullo, L., Zaslow, M., & Peisner-Feinberg, E. S. (2019). ECE quality indicators and child outcomes: Analyses of six large child care studies. *Early Childhood Research Quarterly, 49*, 202-217. https://doi.org/10.1016/j.ecresq.2019.06.009

Soltani Kouhbanani, S., Arabi, S.M. Home executive function environment and executive functions in children: The mediating role of brain electrical activity. *Curr Psychol 42*, 7081–7089 (2023). https://doi.org/10.1007/s12144-021-02044-4

Soltanlou, M., Sitnikova, M. A., Nuerk, H. C., & Dresler, T. (2018). Applications of Functional Near-Infrared Spectroscopy (fNIRS) in Studying Cognitive Development: The Case of Mathematics and Language. *Frontiers in Psychology, 9*, 277. https://doi.org/10.3389/fpsyg.2018.00277

Sonnenschein, S., Galindo, C., Metzger, S. R., Thompson, J. A., Huang, H. C., & Lewis, H. (2012). Parents' beliefs about children's math development and children's participation in math activities. *Child Development Research, 2012,* 1-13. https://doi.org/10.1155/2012/851657

Sophian, C. (2007). *The origins of mathematical knowledge in childhood.* Lawrence Erlbaum Associates. https://doi.org/10.4324/9781315085654

Soto-Calvo, E, Simmons, FR, Adams, AM, Francis, H, Patel, H and Giofre, D (2020) Identifying the preschool home learning experiences that predict early number skills: Evidence from a longitudinal study. *Early Childhood Research Quarterly, 53,* 314-328. https://doi.org/10.1016/j.ecresq.2020.04.004

Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences PNAS, 110*(45), 18116-18120. https://doi.org/10.1073/pnas.1302751110

Stern, E. (2005). Pedagogy meets neuroscience. *Science (New York, N.Y.), 310*(5749), 745. https://doi.org/10.1126/science.1121139

Stipek, D., & Byler, P. (1997). Early childhood education teachers: Do they practice what they preach? *Early Childhood Research Quarterly, 12*(3), 305-325. https://doi.org/10.1016/S0885-2006%2897%2990005-3

Sümer, N. (2000). Yapisal Eşitlik Modelleri: Temel Kavramlar ve Örnek Uygulamalar [Structural Equation Modeling: Basic Concepts and Applications]. *Türk Psikoloji Yazilari, 3*(6), 49–74.

Szũcs, D., Soltész, F., Csépe, V., & Jármi, E. (2007). The speed of magnitude processing and executive functions in controlled and automatic number comparison in children: An electro-encephalography study. *Behavioral and Brain Functions, 3*. https://doi.org/10.1186/1744-9081-3-23

Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics* (5th ed.). Boston: Pearson Education.

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics* (6th ed.). Boston: Pearson Education.

Takunyaci, M., Takunyaci, M. (2014). Preschool teachers' mathematics teaching efficacy belief. *Procedia - Social and Behavioral Sciences, 152*, 673 – 678. https://doi.org/10.1016/j.sbspro.2014.09.261

Taylor, L. C., Clayton, J. D., & Rowley, S. J. (2004). Academic socialization: Understanding parental influences on children's school-related development in the early years. *Review of General Psychology, 8*(3), 163-178. https://doi.org/10.1037/1089-2680.8.3.163

Thompson, B. (2004). *Exploratory and confirmatory factor analysis: Understanding concepts and applications*. American Psychological Association. https://doi.org/10.1037/10694-000

Thompson, R. J., Napoli, A. R., & Purpura, D. (2016) Age- related differences in the relation between the home numeracy environment and numeracy skills. *Inf. Child Dev, 26*(5), 1-13. https://doi.org/10.1002/icd.2019

Tong, P., & An, I. S. (2024). Review of studies applying Bronfenbrenner's bioecological theory in international and intercultural education research. *Frontiers in psychology, 14*, 1233925. https://doi.org/10.3389/fpsyg.2023.1233925

Tovim, K. K. (1996). The *Turkish adaptation of the early childhood environment rating scale* (52123) [Master's Thesis, Boğaziçi University]. YÖK Tez.

Tudge, J. R. H., Merçon-Vargas, E. A., Liang, Y., & Payir, A. (2017). The importance of Urie Bronfenbrenner's bioecological theory for early childhood educators and early childhood education. In L. Cohen & S. Stupiansky (Eds.), *Theories of early childhood education: Developmental, behaviorist, and critical* (45–57). New York: Routledge. https://doi.org/10.4324/9781315641560

Turkish Statistical Institute (TURKSTAT). *Adrese Dayalı Nüfus Kayıt Sistemi Sonuçları*, 2023. Retrieved November, 24, 2024, from https://data.tuik.gov.tr/Bulten/Index?p=Adrese-Dayali-Nufus-Kayit-Sistemi-Sonuclari-2023-49684

UNICEF. (2021). Early childhood development, stimulation, and responsive care (2014-2021). *UNICEF*. https://www.unicef.org/evaluation/reports#/detail/16868/early-childhood-development-stimulation-and-responsive-care-2014-2021

Van der Ven, Sanne H. G., Kroesbergen, E. H., Boom, J., & Leseman, P. P. M. (2012). The development of executive functions and early mathematics: A dynamic relationship: Development executive functions mathematics. *British Journal of Educational Psychology, 82*(1), 100-119. https://doi.org/10.1111/j.2044-8279.2011.02035.x

Visibelli, E., Porru, A., Lucangeli, D., Butterworth, B., & Benavides-Varela, S. (2024). Neural indicators of numerical abilities in the infant human brain: A systematic review. *Developmental Review*. https://doi.org/10.1016/j.dr.2024.101150

Vogel, S. E., Goffin, C., & Ansari, D. (2015). Developmental specialization of the left parietal cortex for the semantic representation of arabic numerals: An fMR-adaptation study. *Developmental Cognitive Neuroscience*, *12*, 61–73. https://doi.org/10.1016/j.dcn.2014.12.001

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wang, S., Hu, B. Y., & Zhang, X. (2021). Kindergarteners’ spatial skills and their reading and math achievement in second grade. *Early Childhood Research Quarterly, 57*(4), 156-166. https://doi.org/10.1016/j.ecresq.2021.06.002

Wasik, B. (2008). When fewer is more: Small groups in early childhood classrooms. *Early Childhood Education Journal, 35*(6), 515–521. https://doi.org/10.1007/s10643-008-0245-4

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What’s past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher, 43*(7), 352–360. https://doi.org/10.3102/0013189X14553660

Westermann, G., Mareschal, D., Johnson, M. H., Sirois, S., Spratling, M. W., & Thomas, M. S. (2007). Neuroconstructivism. *Developmental science, 10*(1), 75–83. https://doi.org/10.1111/j.1467-7687.2007.00567.x

Westermann, G., Thomas, M. S. C., & Karmiloff-Smith, A. (2010). Neuroconstructivism. In U. Goswami (Ed.) *The wiley‐ blackwell handbook of childhood cognitive development* (pp.723-748). Oxford: Wiley-Blackwell.

Westwood, P.S. (2021). Numeracy in preschool and kindergarten years. *In: Teaching for numeracy across the age range*. Springer Briefs in Education. Springer, Singapore. https://doi.org/10.1007/978-981-16-3761-2_

Willoughby, M. T., Kuhn, L. J., Blair, C. B., Samek, A. & List, J. A. (2016). The test–retest reliability of the latent construct of executive function depends on whether tasks are represented as formative or reflective indicators. *Child Neuropychology, 23*(7), 822-837. https://doi.org/10.1080/09297049.2016.1205009

Willoughby, M. T., Pek, J., & Blair, C. B. (2013). Measuring executive function in early childhood: a focus on maximal reliability and the derivation of short forms. *Psychological assessment, 25*(2), 664–670. https://doi.org/10.1037/a0031747

Willoughby, M. T., Wirth, R. J. & Blair, C. B. (2012). Executive function in early childhood: Longitudinal measurement invariance and developmental change. *Psychological Assessment, 24*(2), 418-431. https://doi.org/10.1037%2Fa0025779

Xia, M., Li, X., & Tudge, J.R. (2020). Operationalizing Urie Bronfenbrenner’s Process-Person-Context-Time Model. *Human Development, 64*(1), 10–20. https://doi.org/10.1159/000506753

Xu, J., & Zhong, B. (2018). Review on portable EEG technology in educational research. *Computers in Human Behavior, 81*, 340–349. https://doi.org/10.1016/j.chb.2017.12.037

Yeniad, N., Malda, M., Mesman, J., van IJzendoorn, M. H., & Pieper, S. (2013). Shifting ability predicts math and reading performance in children: A meta-analytical study. *Learning and Individual Differences, 23,* 1–9. https://doi.org/10.1016/j.lindif.2012.10.004

Yılmaz, N. (2019). *The effect of the hypothetical learning trajectories and the contribution of the eye-tracking technology in understanding the young children's mathematical patterning recognition and generalization* (602294). [Unpublished Doctoral Thesis, Middle East Technical University], YÖK Tez

Yücel, M. A., Lühmann, A. V., Scholkmann, F., Gervain, J., Dan, I., Ayaz, H., Boas, D., Cooper, R. J., Culver, J., Elwell, C. E., Eggebrecht, A., Franceschini, M. A., Grova, C., Homae, F., Lesage, F., Obrig, H., Tachtsidis, I., Tak, S., Tong, Y., Torricelli, A., … Wolf, M. (2021). Best practices for fNIRS publications. *Neurophotonics, 8*(1), 012101. https://doi.org/10.1117/1.NPh.8.1.012101

Zamarian, L., Ischebeck, A., & Delazer, M. (2009). Neuroscience of learning arithmetic evidence from brain imaging studies. *Neuroscience and Biobehavioral Reviews, 33*(6), 909–925. https://doi.org/10.1016/j.neubiorev.2009.03.005

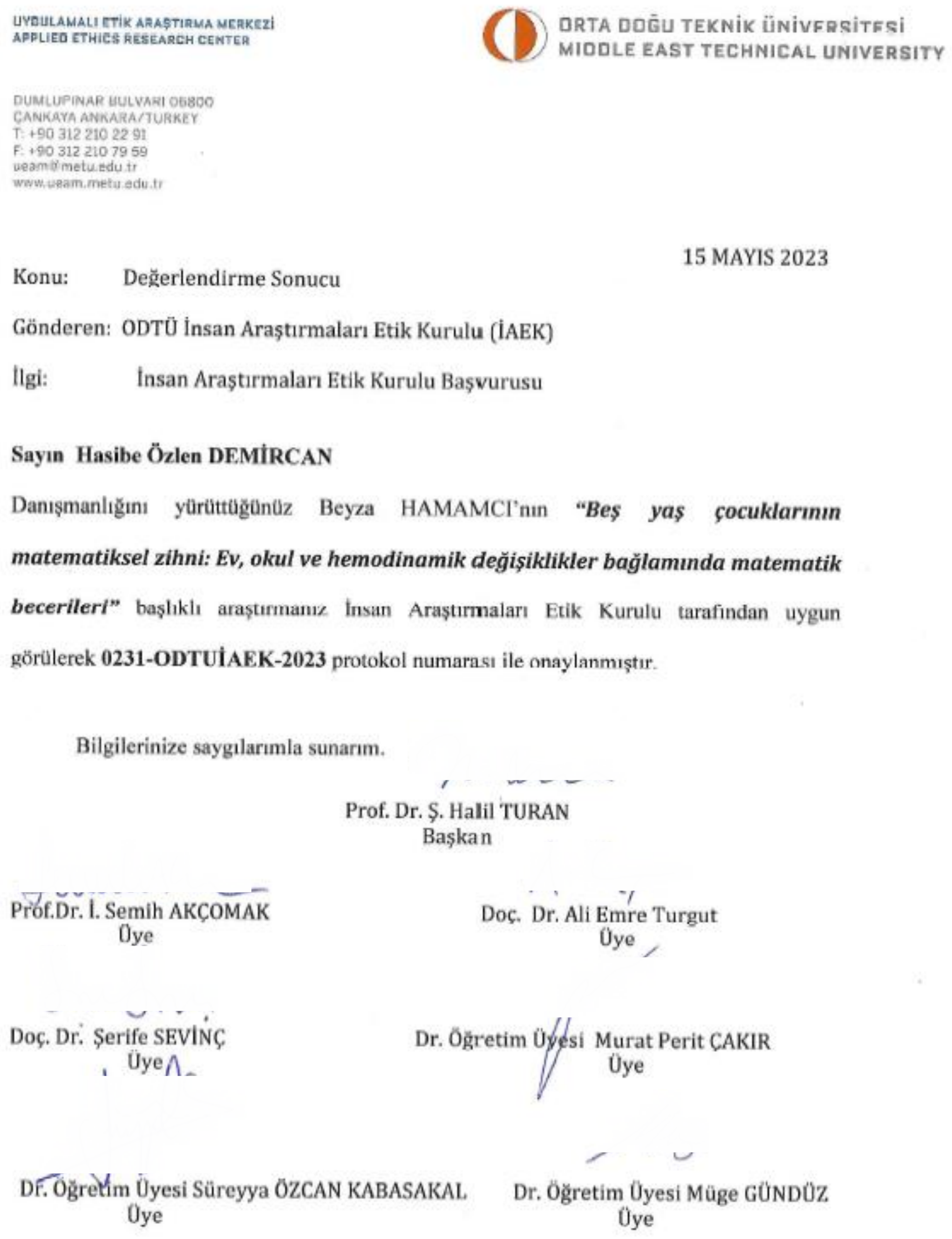
Zhan, Z., Yang, Q., Luo, L. et al. (2024). Applying functional near-infrared spectroscopy (fNIRS) in educational research: a systematic review. *Current Psychology, 43*, 9676–9691. https://doi.org/10.1007/s12144-023-05094-y

Zelazo, P. D., Blair, C. B. & Willoughby, M. T. (2016). *Executive function: Implications for education.* NCER 2017-2020: National Center for Education Research.

Zippert, E. L., & Rittle-Johnson, B. (2020). The home math environment: More than numeracy. *Early Childhood Research Quarterly, 50*, 4-15. https://doi.org/10.1016/j.ecresq.2018.07.009

APPENDICES

A. APPROVAL OF THE METU HUMAN SUBJECTS ETHICS COMMITTEE



UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

15 MAYIS 2023

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Hasibe Özlen DEMİRCAN

Danışmanlığını yürüttüğünüz Beyza HAMAMCI'nın ***"Beş yaş çocuklarının matematiksel zihni: Ev, okul ve hemodinamik değişiklikler bağlamında matematik becerileri"*** başlıklı araştırmanız İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek **0231-ODTUİAEK-2023** protokol numarası ile onaylanmıştır.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN
Başkan

Prof.Dr. İ. Semih AKÇOMAK
Üye

Doç. Dr. Ali Emre Turgut
Üye

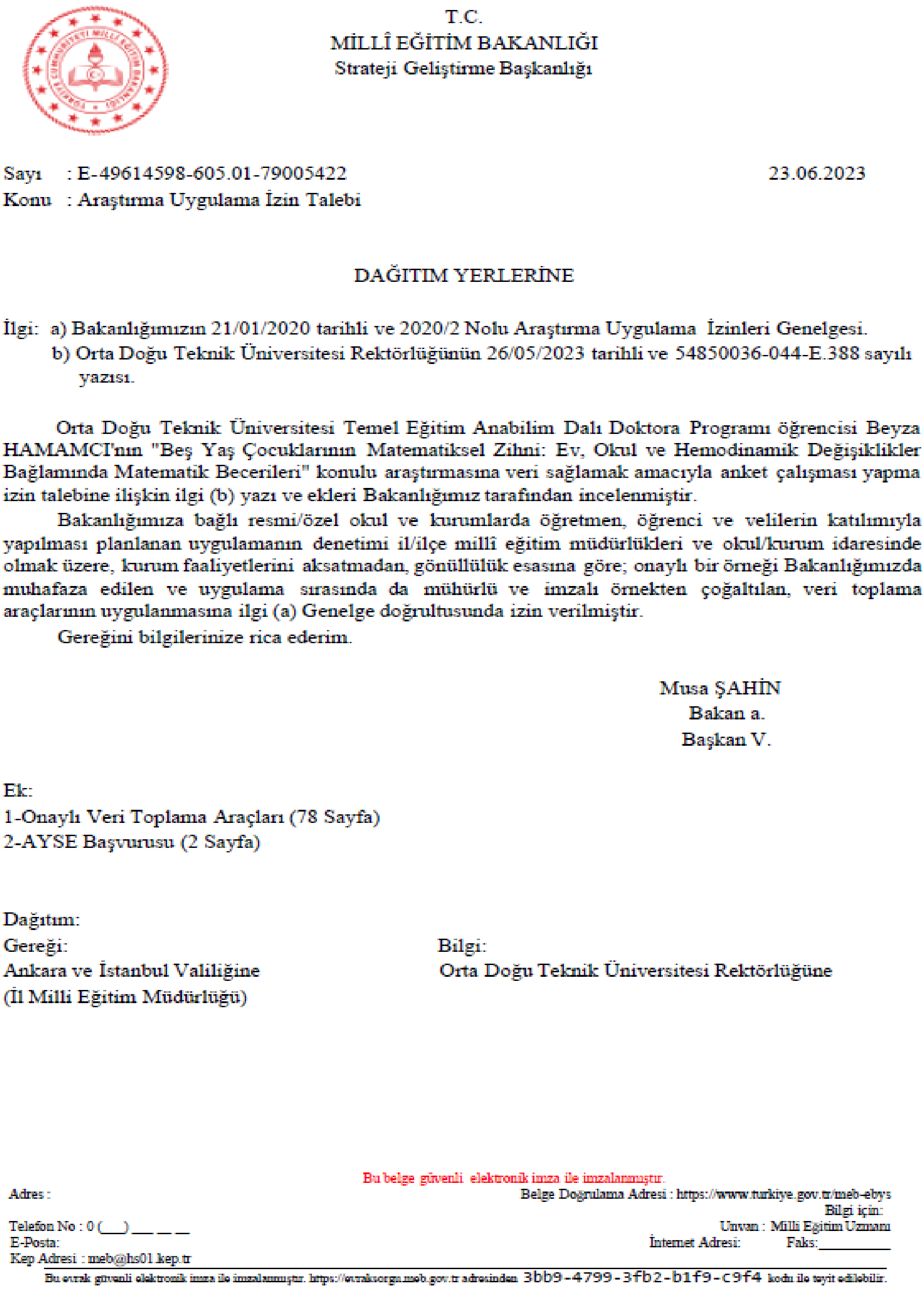
Doç. Dr. Şerife SEVİNÇ
Üye

Dr. Öğretim Üyesi Murat Perit ÇAKIR
Üye

Dr. Öğretim Üyesi Süreyya ÖZCAN KABASAKAL
Üye

Dr. Öğretim Üyesi Müge GÜNDÜZ
Üye

B. MONE RESEARCH APPLICATION APPROVAL



T.C.
MİLLÎ EĞİTİM BAKANLIĞI
Strateji Geliştirme Başkanlığı

Sayı : E-49614598-605.01-79005422 23.06.2023
Konu : Araştırma Uygulama İzin Talebi

DAĞITIM YERLERİNE

İlgi: a) Bakanlığımızın 21/01/2020 tarihli ve 2020/2 Nolu Araştırma Uygulama İzinleri Genelgesi.
b) Orta Doğu Teknik Üniversitesi Rektörlüğünün 26/05/2023 tarihli ve 54850036-044-E.388 sayılı yazısı.

Orta Doğu Teknik Üniversitesi Temel Eğitim Anabilim Dalı Doktora Programı öğrencisi Beyza HAMAMCI'nın "Beş Yaş Çocuklarının Matematiksel Zihni: Ev, Okul ve Hemodinamik Değişiklikler Bağlamında Matematik Becerileri" konulu araştırmasına veri sağlamak amacıyla anket çalışması yapma izin talebine ilişkin ilgi (b) yazı ve ekleri Bakanlığımız tarafından incelenmiştir.

Bakanlığımıza bağlı resmi/özel okul ve kurumlarda öğretmen, öğrenci ve velilerin katılımıyla yapılması planlanan uygulamanın denetimi il/ilçe millî eğitim müdürlükleri ve okul/kurum idaresinde olmak üzere, kurum faaliyetlerini aksatmadan, gönüllülük esasına göre; onaylı bir örneği Bakanlığımızda muhafaza edilen ve uygulama sırasında da mühürlü ve imzalı örnekten çoğaltılan, veri toplama araçlarının uygulanmasına ilgi (a) Genelge doğrultusunda izin verilmiştir.

Gereğini bilgilerinize rica ederim.

Musa ŞAHİN
Bakan a.
Başkan V.

Ek:
1-Onaylı Veri Toplama Araçları (78 Sayfa)
2-AYSE Başvurusu (2 Sayfa)

Dağıtım:

Gereği:	Bilgi:
Ankara ve İstanbul Valiliğine (İl Milli Eğitim Müdürlüğü)	Orta Doğu Teknik Üniversitesi Rektörlüğüne

Bu belge güvenli elektronik imza ile imzalanmıştır.

Adres :
Belge Doğrulama Adresi : https://www.turkiye.gov.tr/meb-ebys
Bilgi için:
Telefon No : 0 (___) ___ __ __
Unvan : Milli Eğitim Uzmanı
E-Posta:
İnternet Adresi:
Faks:__________
Kep Adresi : meb@hs01.kep.tr

Bu evrak güvenli elektronik imza ile imzalanmıştır. https://evraksorgu.meb.gov.tr adresinden 3bb9-4799-3fb2-b1f9-c9f4 kodu ile teyit edilebilir.

C. CURRICULUM VITAE

Beyza Hamamcı
Department of Early Childhood Education
Middle East Technical University
Üniversiteler Mh. Faculty of Education
First Floor, EF-A37 Çankaya
Ankara, Türkiye

Education:

Schools	Dates	Degree	Major
Middle East Technical University (METU)	2020 - 2025	PhD	Early Childhood Education
Marmara University	2018 - 2020	MA	Preschool Education
Yildiz Technical University	2014 - 2017	BSc	Preschool Education

Employment:

2021 – Cont	Research and Teaching Assistant, Middle East Technical University
2022 - 2022	Guest Research Associate, Kyoto University
2019 - 2021	Research and Teaching Assistant, Istanbul University - Cerrahpaşa
2018 - 2019	Preschool Teacher, KEP Preschool
2018 - 2018	Preschool Teacher, Eyüboğlu Schools
2017 - 2018	Preschool Teacher, Ortaköy Burak Reis Primary School

Courses assisted:

ECE 112	Motor Development	Spring 2024
ECE 130	Practicum in Early Childhood Education	Spring 2022
ECE 240	Statistics for Early Childhood Teachers	Spring 2022 – 2024
ECE 248	Community Service	Fall 2021
ECE 301	Research Methods for Early Childhood Teachers	Fall 2021 – 2024
ECE 303	School Experience	Fall 2021
ECE 310	Teaching Mathematics in Early Childhood	Spring 2023
ECE 325	Parent Involvement and Education	Fall 2022
ECE 411	Practice Teaching I	Fall 2023 – 2024
ECE 430	Practice Teaching II	Spring 2023 – 2024
ECE 603	Advanced Educational Research & Ethics	Fall 2024

Memberships in Professional Organizations:

Society for Research in Child Development

International Mind Brain and Education Society

Major Awards and Honors:

High Honors Degree Yıldız Technical University, Istanbul, 2017

Honored Degree, Marmara University, Istanbul, 2020

Project Scholarship, Turkish Scientific and Technological Research Council (TUBITAK), 2020

Scholar of Psychology Summer School, The Science Academy Society of Türkiye, 2020

International Scientific Publications Incentive Program, TUBITAK, 2022

Grant for Attending International Conference, TUBITAK, 2023

General Domestic Doctoral Fellowship Program, TUBITAK, 2023-Present

IMBES Travel Grant - Hardship Fund Award, IMBES Trainee Board & Jacob Foundation, 2024

Publications:

Dissertations

Hamamcı, B. (2020). *Assessment of preschool children's executive function skills.* (635388) (Master Dissertation, Marmara University). YÖK Tez

Hamamcı, B. (2025). *The mathematical abilities of young children in the context of hemodynamic changes, executive function, home and school environment.* (Doctoral Dissertation, Middle East Technical University)

Peer-Reviewed Journal Articles

Balaban Dağal, A., Hamamcı, B. & Yayla, K., & (2020). The relationship between metacognitive awareness and emotional intelligence of preschool teacher candidates. *Journal of Early Childhood Studies, 4*(3), 583-608. http://doi.org/10.24130/eccd-jecs.1967202043222

Güven, Y., Hamamcı, B., & Yayla, K. (2021). The relationship between patterning skills and language development of preschool children. *Elementary Education Online, 20*(1), 999-1009. http://doi.org/10.17051/ilkonline.2021.01.68.

Hamamcı, B., Acar, İ. H., & Uyanık, G. (2021). The validity and reliability study of parent-report of Childhood Executive Functioning Inventory for preschoolers. *Journal of Theory and Practice in Education, 17*(2), 1-10. http://doi.org/10.17244/eku.884918.

Hamamcı, B. & Balaban Dağal, A. (2022). Self-regulation and play: How children's play directed with executive function and emotion regulation. *Early Child Development and Care, 192*(13), 2149-2159. https://doi.org/10.1080/03004430.2021.1990906

Hamamcı, B., Acar, İ. H., & Uyanık, G. (2023). Association between performance-based and ratings of Turkish children’s executive function. *Current Psychology, 42,* 10053–10062. https://doi.org/10.1007%2Fs12144-021-02307-0.

Book Chapters

Acar, I. H. & Hamamcı, B. (2023). Öğretmen çocuk ilişkisi ve öz-düzenleme [Teacher-child relationship and self-regulation]. In M. Ekerim Akbulut (Ed.).

Sosyal zihin: Bilişsel gelişimde sosyal etkileşimin rolü [Social mind: The role of social interaction on cognitive development] (pp. 21-48). Vakıfbank Kültür Yayınları

Hamamcı, B. (2023) Yaratıcılık ve bilişsel beceriler [Creativity and cognitive skills] In A. Balaban Dağal & Ş. Değirmenci (Eds.). *Erken çocuklukta yaratıcılığa genel bakış [Overview of creativity in early childhood]* (pp. 139-157). Eğiten Kitap

Ateş, B. & Hamamcı, B. (2024). Paylaşımlı kitap okumanın çocukların dil gelişimi üzerine etkisi [The effect of shared book reading on children's language development]. In B. Ateş (Ed.). *Erken Çocuklukta Okuryazarlık Deneyimi [Early Childhood Literacy Experience]* Koç Üniversitesi Yayınları

Peer-Reviewed Conference Papers

Hamamcı, B. & Öztürk, G (2021). The experiences of teachers during COVID-19. *VIIIth International Eurasian Educational Research Congress Online.* Aksaray University

Hamamcı, B. (2022) The development of metacognition in early years: A short-term longitudinal analysis. *IXth International Eurasian Educational Research Congress.* Ege University

Hamamcı, B., Acar, İ. H., Bostancı, S. (2023). Pathways to children’s social competence: Household chaos, parent-child relationships, sibling relationships, and child executive function. *SRCD 2023 Biennial.* Utah Salt Lake City (Poster)

Hamamcı, B. & Demircan, H. Ö. (2023) Home environment to foster math development of children: A review study. *Xth International Eurasian Educational Research Congress Online.* TED University

Hamamcı, B., Demircan, H. Ö. & Tantekin Erden, F. (2023) The validity and reliability of home learning environment index. *Xth International Eurasian Educational Research Congress Online.* TED University

Altun, D., Hamamcı, B. & Tantekin Erden, F. (2023). Investigation of pre-service preschool teachers' satisfaction levels and efficacy perceptions of digital-assisted education. *International Education Congress 2023.* Ankara University

Martin, T., Hamamcı B., Sherlock, P. & Zelazo, P. (2024) Meta-analysis of the relations between the dimensional change card sort (DCCS) and measures of math and reading. *JPS Conference 2024*. Toronto (Poster)

Hamamcı, B. & Sevinç Özgen, E. (2024) Cognitive foundations of early mathematics: Attention and executive function. *IMBES 2024*. Leuven (Poster)

Hamamcı, B. & Peker, C. (2024) Diving into preschoolers' coping strategies with emotions: A mixed method study. *EARLI SIG 5 and SIG 28 Conference 2024*. Warsaw (Poster)

Research Projects

2020 - Examining the effects of parent's individual and environmental stressors on children's psychological health in the context of Covid-19 pandemic. TUBITAK SOBAG 1001 Project (No: 120K387) PI: İbrahim H. Acar

2024 - The mathematical mind of five-year-old children: Mathematical abilities in the context of home, school and hemodynamic changes. TUBITAK SOBAG 1002-A (No: 223K757) PI: Beyza Hamamcı

Professional Work:

Reviews

Journals

Behaviour & Information Technology (SSCI), 2022

Environment and Social Psychology (Scopus), 2023

Frontiers in Psychology (SSCI), 2024

Journal of Cognition and Development (SSCI), 2025

Conferences

"JURE 2022 Conference", EARLI, Porto, Portugal, July 18th - 22nd 2022

"SRCD 2023 Biennial Meeting", Utah, USA, March 23rd - 25th 2023

“SRCD 2025 Biennial Meeting”, Minnesota, USA, May 1st - 3rd 2025

D. TURKISH SUMMARY / TÜRKÇE ÖZET

I. GİRİŞ

Çocukların matematik becerilerinin ileri yıllardaki akademik başarıları ve yetişkinlikteki yaşam koşulları üzerinde belirleyici bir rolü vardır. Erken matematik becerileri gelecekteki okuma becerileri ve matematik becerileri ile ilişkilidir (Nguyen vd., 2016; Wang vd., 2021; Watts vd., 2014). Bu başarı yaşam boyu devam ederek hem sosyoekonomik düzeyden etkilenir hem de kişinin yetişkinlikteki sosyoekonomik düzeyini etkiler (Ritchie vd.,2013). Matematik becerilerinin hem akademik başarı hem de yaşam başarısı ile olan bu ilişkisi göz önünde bulundurulduğunda erken yaşlardaki matematik becerilerine odaklanarak bunu etklileyen faktörler ile ilişkilerinin belirlenmesi önemli görülmektedir.

Matematiğin gelişimi incelendiğinde ilk aşama olarak sayı hissi kavramı (niceliğin algılandığı sembolik olmayan yaklaşık duyum; Dehaene, 2011) gelişmekte sonrasında sayıları tanıma, niceliği fark etme, sıra ile sayma, sayılar arasında karşılaştırmalar yapma ve az ile çok ayrımını yapma ile devam etmektedir (National Research Council, 2009; Sarama vd.,2009; Westwood, 2021). Bu sembolik olmayan aşamaya dayanarak sembolik matematiksel süreçler yapılanmaktadır (Starr vd.,2013; Sasanguie vd.,2013). Bunun yanı sıra dört-beş yaşlarında çocuklar sayıları ve sembolleri kullanarak toplamanın nicelikleri birleştirmek olduğunu ve çıkarmanın toplamanın tersi olarak nicelikleri ayrıştırmak olduğunu kavrayabilir (Harris, vd.,2017).

Gelişimsel süreçte izlenen bu seyirde sadece sembolik olmayan matematiksel süreçlerden sembolik olanlara geçiş sadece bilişsel dönüşümü yansıtmakla kalmaz, aynı zamanda karmaşık beyin yapılarını da içermektedir. Bu bağlamda Dehaene (1992) sayısal bilgiyi analog büyüklük kodu, sözel ve görsel Arapça form olmak üzere üç temsilde tanımlamıştır. Sayıların temsili arasındaki bu ayrım, sinirbilim çalışmaları tarafından araştırılan farklı ve benzersiz bilişsel süreçlere hizmet etmektedir. Bu anlamda eğitimsel sinirbilim araştırmaları ivme kazanarak matematiğin beyinde nasıl geliştiğini inceleyen güncel araştırmalar yapılmaktadır (bkz; Arsalidou, vd., 2011; Emerson vd., 2015; Hyde, 2021; Hyde vd., 2010; Zamarian vd., 2009). Bu konuda yürütülmekte olan araştırmalar, çocuğun öğrenme sürecinin anlaşılması ve öğretimin düzenlenmesinde önemli görülürken (Howard-Jones vd., 2016) çocuğun matematik öğrenmedeki ihtiyaçlarının tespiti ve uygun öğrenme fırsatlarının oluşturulması için kritik önem taşımaktadır (Butterworth vd., 2011).

Bu alanda yapılan çalışmalarda fNIRS, fMRI ve EEG gibi beyin görüntüleme teknikleri kullanılmaktadır. fNIRS beyindeki hemoglobinin oksijenlenme ve deoksijenlenmesini ölçen, invaziv olmayan, kolay kullanımlı, taşınabilir bir teknik olup eğitimsel sinirbilim çalışmalarında öğrenmenin nasıl gerçekleştiğini ve öğrenmenin davranışsal verinin ötesinde, temelinde yer alan nöral mekanizmayı araştırmaktadır (Barreto vd., 2022). fMRI ise kişinin yatar sabit pozisyonda olmasını gerektiren diğer bir invazif teknik olarak bilgilerin beyinde nerede işlendiğini ve beynin hangi bölgelerinin birlikte ağ olarak çalıştığını deoksihemoglobin konsantrasyonu ve beyin dokusundan etkilenen hidrojen çekirdeklerinin bozunma zaman sabitleri aracılığı ile yanıtlamak için kullanılmaktadır (Bartels vd., 2012). Diğer bir taşınabilir fizyolojik ölçüm olan EEG ise bilgiyi işlerken doğrudan nöral aktiviteyi ölçerek beyindeki zamansal dinamikleri tespit etmektedir (Dalenberg vd., 2018).

Yukarıda açıklanan teknikler kullanılarak yapılan çalışmalar matematiğin beyinde nasıl işlediğini belirli beyin bölgelerine dikkat çekerek açıklamıştır. Matematiksel işlemlerin beyindeki yansımalarını açıklamak üzere yapılan meta-analiz çalışması, her ne kadar toplama, çıkarma ve bölmenin prefrontal kortekste farklı bölgelerde işlediğini gösterse de sayılar ve hesaplamalar için inferior parietal lobun ortak olduğunu vurgulamaktadır (Arsalidou vd., 2011). Öte yandan özellikle sağ parietal bölgelerde sayı niceliğinin temsil edildiğini, sol parietal bölgelerin kültürel aktarımla sembolik sayı sistemine ait olduğu vurgulanmaktadır (Hyde vd., 2020; Hyde, 2021). Bunun yanı sıra okul öncesi dönem çocukları ile yapılan boylamsal çalışmada sol intraparietal sulkusun sayıların ayrımından sorumlu olarak görülürken sağ intraparietal sulkusun sayı işemesinden sorumlu olduğu belirtilmiştir (Emerson vd., 2015). Aritmetik işlemler göz önünde bulundurlduğunda ise frontal lob ve parietal lob arasında ve parietal lobun içinde intraparietal sulkus ile sol angular girus arasında geçişler olduğu görülmektedir (for review, Zamarian vd., 2009). Çalışmalar göstermektedir ki beyinde matematiğin gelişimi süreç içinde farklı yollar ile değişmektedir. Bununla beraber beyin görüntüleme tekniklerinin bu alandaki uygulamalarının görece yeni olması sebebi ile özellikle okul öncesi dönemde matematik becerileri gelişiminin beyinde nasıl ağ kurduğu henüz tamamıyla anlaşılmış değildir.

Alanyazında matematiğin nörolojik yollarının incelenmesinin yanı sıra yapılan çalışmalar çocukların matematiksel becerilerindeki farklılaşmaları anlamak için matematiğin bilişsel ve çevresel faktörlerin önemine dikkat çekmektedir (bkz., Silver vd., 2022; Kilday, 2011). Bu bağlamda mevcut alanyazın yürütücü işlevlerin (meta-analiz için bkz, Emslander vd., 2022), evdeki matematik etkinlikleri ve ebeveynsel faktörleri içeren ev matematik ikliminin (meta-analiz için bkz, Daucourt vd., 2021) ve sınıf içi matematik etkinliklerini ve öğretmene bağlı faktörleri içeren okul süreç kalitesinin (bkz, Silver vd., 2022) çocukların becerilerine katkıda bulunduğunu göstermektedir.

İşleyen belleği (bilgiyi saklama ve manipüle etme), ketleyici kontrolü (ilgisiz bilgileri bastırma) ve bilişsel esnekliği (bilgiler arasında geçiş yapa) kapsayan bir çatı terim ve bilişsel yapı olan yürütücü işlevler (Miyake, 2000) matematiksel bilginin bilinmesi, işlemlerin birbirinin tersi olduğunun farkına varılması ve işlemler boyunca dikkatin kullanılması ile matematiksel becerilere katkıda bulunmaktadır (Cragg vd., 2014). Bunun ötesinde matematik ile yürütücü işlevlerin alt boyutları (ketleyici kontrol, işleyen bellek ve zihinsel esneklik) ile ilişkisinin genel etkisini belirlemeyi amaçlayan meta-analiz çalışmaları bu becerilerin matematik becerisi için önemli yordayıcılar olduğunu göstermektedir (Allan vd., 2014; Cortés Pascual vd., 2019; Emslander vd., 2022; Friso-van den Bos vd., 2013; Peng vd., 2016; Yeniad vd., 2013). Allan ve meslektaşları (2014) ketleyici kontrole odaklanan meta-analiz çalışmasında orta düzeyde etki büyüklüğü (r = .27) bulmuşlardır ve bu sonucu hem problem çözmenin hem de ketleyici kontrolün prefrontal korteks ile ilişki olması ile açıklamışlardır. İşleyen bellek ve matematik arasındaki ilişkinin genel etkisini inceleyen meta-analiz çalışmaları etki büyüklüğünün orta düzeyde olduğunu (r = .37; r = .38; r = .35) ve bunun nedenini işleyen belleğin bilginin yeniden yapılandırılmasına olanak sağlanmasıyla açıklamaktadır (Cortés Pascual vd., 2019; Friso-van den Bos vd., 2013; Peng vd., 2016). Yeniad ve arkadaşları (2013) tarafından yapılan bilişsel esneklik ile matematik arasındaki ilişkinin genel etkisini açıklamayı hedefleyen meta-analiz çalışmasında orta düzeyli bir ilişki (r = .24) bulunurken; yeni kavramsal temsillere yüksek geçişkenlik kapasitesine sahip çocukların matematik becerilerinin de gelişkin olduğu vurgulanmaktadır. Bir başka çalışmada ise tüm yürütücü işlev boyutları birlikte değerlendirilmiş, matematik ile yürütücü işlevlerin ilişkisinin orta düzeyli olduğu (r = .30) bulunmuştur (Emslander vd., 2022).

Ev iklimi göz önünde bulundurulduğunda ise aile özelliklerinin (sosyoekonomik düzey, dil) (Galindo vd., 2015; Kluczniok, 2017), ebeveyn-çocuk etkinliklerinin (gruplama, şarkı söyleme ve sayma gibi doğrudan ebeveyn-çocuk alıştırmaları) (Blevins-Knabe vd., 1996; Chan vd., 2021; Huntsinger vd., 2016; Manolitsis vd., 2013; Missal vd., 2017; Niklas vd., 2017; Pardo vd., 2020; Soto-Calvo vd., 2020; Thompson vd., 2016) ve ebeveynin özelliklerinin (eğitimsel geçmişi, inançları ve yetkinliği) (Blevnis-Knabe vd., 2000; Claire-Son vd., 2020; DeFlorio vd., 2015; Foster vd., 2016; Huang vd., 2017; Sonnenschein vd., 2012; Zippert vd., 2018) çocukların matematik becerilerini yordadığı görülmektedir. Uluslarası alanyazındaki bu çalışmalar ebeveynlerin çocuklarla daha fazla matematik ile ilişkili etkinliklere dahil olmasının, yüksek sosyoekonomik düzeyin, matematiğin önemli olduğu inancına sahip olmanın ve matemetikte yetkin olmanın çocukların matematik görevlerindeki performanslarına olumlu etkileri olduğunu vurgulamaktadır.

Bir diğer yandan okul iklimi çocukların matematik becerileri ile ilişkili görülen diğer bir faktördür. İlgili alanyazında okul iklimi; öğretmen-çocuk oranı, program özellikleri, çalışan özellikleri ve fiziksel yapıyı ifade eden yapısal kalite (Harms & Clifford, 1980) ve çocuklar arası etkileşimler ve öğretmen-çocuk etkileşimlerini kapsayan günlük akış ve etkinliklerdeki sosyal etkileşimleri ifade eden süreç kalitesi

(Pianta vd., 2005; Pianta vd., 2008) olmak üzere iki alanda ele alınarak bunların çocukların matematik becerileri ile ilişkili olduğu bulunmuştur (Finders vd., 2021; Grammatikopoulos vd., 2018; Lehrl vd., 2016; Li vd., 2019; Mashburn vd., 2008; Schmitt vd., 2020; Schmerse, 2020). Ayrıca okul kalitesinin doğrudan çocukların matematik becerileri ile ilişkili olmadığını gösteren çalışmalar da bulunmaktadır (Abreu-Lima vd., 2013; Brunsek vd., 2017; Francis vd., 2019; Guerrero-Rosada vd., 2021). Araştırmalar arasındaki bu farklı sonuçlar ölçülen okul kalitesi kavramının farklı perspektiflere dayanması ile ilişkilidir. Örneğin, öğretmenin öğretim sırasında çocuklar ile etkileşimi onların akademik becerileri ile ilişkili bulunurken çocuk ile kurduğu duygusal iletişim onların sosyal gelişimleri ile ilişkili bulunmuştur (Mashburn vd., 2008). Bir diğer yandan öğretmenlerin erken matematiğe ilişkin inançları okul kalitesinin çocukların matematik becerileri üzerindeki etkisini değiştirmektedir. Öğretmenler erken yıllarda matematiğin önemli olduğu inancına sahip oldukça, etkinliklerini de bu yönde organize etmektedir. Bu nedenle kalite ölçümlerinde öğretmenin inanışları ile ilgili faktörlerin de göz önünde bulundurulması önemlidir. Mevcut çalışmanın kapsamı erken matematik becerileri olduğu için matematiğe ilişkin sınıf içi etkinlikler ve öğretmenin inançları göz önünde bulundurulmuştur.

Yukarıda anlatılan alanyazın göz önünde bulundurulduğunda çocukların matematik becerilerinin gelişiminde erken yılların, özellikle okul öncesi dönemin, kritik olduğu ve bu becerilerin biyolojik, bilişsel ve çevresel faktör ile şekillendiği görülmektedir. Bu çalışmalar beyindeki belirli bölgelerin, çocukların yürütücü işlev performanslarının, ev ve okuldaki matematik etkinliklerinin ve yetişkinlerin matematiğe ilişkin inançlarının çocukların matematik becerileri ile ilişkili olduğunu göstermektedir.

Araştırmanın Önemi

Çalışmalar erken yılların, özellikle okul öncesi dönemin, çocukların gelişiminde önemli rol oynadığını göstermektedir. Yaşamın ilk beş yılı temel yaşam becerilerinin gelişiminde en hızlı ilerlemenin kaydedildiği dönemdir (Institute of Medicine and National Research Council, 2000). Bu dönemde edinilen yakın çevreden sağlanan matematiği de içeren yüksek nitelikli erken öğrenme deneyimleri, yetişkinlikteki genel iyi oluşu, sağlığı (fiziksel ve zihinsel), eğitimsel başarıyı ve iş durumunu olumlu yönde etkilemektedir (Shuey vd., 2018) ve sosyoekonomik seviye farkından kaynaklanan eşitsizliği azaltmaktadır (OECD, 2016; UNICEF, 2021). Bunun yanı sıra, Türkiye'deki erken çocukluk durumunu incelemeyi ve ileriki çalışmalara tavsiyelerde bulunmayı amaçlayan Eğitim Reformu Girişimi (ERG) ve Anne Çocuk Eğitim Vakfı (AÇEV) tarafından yayınlanan son raporda (2016) yarım dönem bile olsa okul öncesi eğitime katılan çocukların matematik başarılarının linear bir şekilde arttığı görülmektedir. Ekonomik Kalkınma ve İş Birliği Örgütü'nün (OECD) değerlendirmesine göre sosyoekonomik faktörler kontrol edildiğinde okul öncesi dönem eğitimine katılan çocukların matematik puanları 25 puan yükseldiğini göstererek okul öncesi dönemin çarpıcı etkisini göstermektedir. Tüm bunlar okul öncesi dönemin önemine vurgu yapmaktadır.

Son yıllarda, eğitimsel sinirbilim olarak tanımlanan yeni bir literatür ortaya çıkmıştır. Bu alanda araştırmacılar, öğrenmedeki bireysel farklılıkların kaynaklarını incelemekte ve öğrenciler için en uygun öğrenme bağlamlarını belirlemeye çalışmaktadır (Mareschal vd., 2014). Ayrıca, öğrenme sürecinin temelinde yatan beyin fonksiyonlarını keşfederek öğretme ve öğrenmeyi daha etkili hale getirmeyi amaçlamaktadırlar (Howard-Jones vd., 2016). Eğitim fırsatlarının bazılarının beyin temelli öğrenmeyi nasıl teşvik ettiğini, bazılarının ise etkilemediğini anlamak için kanıta dayalı yaklaşımlar kullanılmaktadır (Stern, 2005). Bu alandaki öncü çalışmalar, özellikle matematiksel yetenekler ve bu yeteneklerin arkasındaki beyin mekanizmalarına odaklanmakta, parietal lob ve frontal lob gibi matematiksel süreçlerde rol oynayan beyin bölgelerini incelemektedir (Arsalidou, vd., 2011; Emerson vd., 2015; Hyde, 2021; Hyde vd., 2010; Zamarian vd., 2009).

Öte yandan matematik becerilerindeki bireysel farklılıklar literatürede bilişsel (yürütücü işlevler) ve çevresel etmenlere (yetişkin-çocuk etkileşimi ve yetişkin inançları) odaklanarak açıklanmaya çalışılmıştır (Silver vd., 2022; Kilday, 2011). Bu alandaki ilişkisel çalışmalar matematik becerilerinin yürütücü işlevler (meta-analiz için bkz; Allan vd., 2014; Cortés Pascual vd., 2019; Emslander vd., 2022; Friso-van den Bos vd., 2013; Peng vd., 2016; Yeniad vd., 2013) ve çevresel uyaranlar (ev öğrenme ortamı, okul iklimi ve yetişkin inançları) (inceleme için bkz.; Mutaf-Yıldız vd., 2020; Soliday Hong vd., 2019) tarafından yordandığını vurgulamaktadır.

Literatür incelendiğinde beyin araştırmaları ve bilişsel-çevresel faktörler ayrı bağlamlarda ele alınsa da teorik çalışmalar beyin, biliş ve çevrenin iş birliği içinde çalıştığını vurgulamaktadır (see, Bickhard, 2009; Westermann vd., 2007). Bu nedenle çocukların matematiksel becerilerini açıklamakta eğitimsel sinirbilim, gelişim psikolojisi ve eğitim alanlarının entegrasyonu ile metodolojik ve teorik bir köprünün kurulması teorik alt yapı ile uyumlu olacaktır. Mevcut alan yazında matematik becerileri bağlamında ele alınan bu faktörler henüz eş zamanlı olarak incelenmemiştir. Bu nedenle beyin bölgelerindeki hemodinamik değişiklikler bir bağlamda ele alınırken yürütücü işlevler, ev ve okul iklimi bir diğer alanda ele alınarak tartışılmaktadır. Bu iki alanın birleşimi ile matematiği yordayan beceriler ve beyin bölgelerindeki hemodinamik yanıtlar tek bir mekanizma olarak henüz çalışılmamıştır. Bu nedenle mevcut çalışma ile teori ve metodoloji arasında bir köprü kurularak çocukların matematik becerileri biyolojik kökenler, bilişsel beceriler ve çevresel (ev ve okul) etmenler çerçevesinde açıklanmaktadır. Bu nedenle, mevcut araştırmada okul ve ev ortamları, çocuğun yürütücü işlevleri ve hemodinamik tepkiler gibi faktörlerin, çocukların matematiksel yeteneklerini nasıl etkilediği incelenmiştir.

Ulusal alanyazın incelendiğinde ise eğitsel sinirbilim araştırmalarının nadir olduğu görülmektedir (örn., Alkan, 2006; Coşkun, 2019; Özçelik vd., 2009; Yılmaz, 2019). Ayrıca erken çocukluk dönemindeki kalite üzerine yapılan çalışmalar betimleyici nitelikte olup (Güçhan Özgül, 2011; Güleş, 2013; Oturakdaş, 2019; Tovim, 1996), kalitenin çocukların akademik ve gelişimsel çıktıları üzerindeki etkisini açıklayan çalışmalar görece az sayıdadır (örn., Canbeldek, 2015). Benzer şekilde ev öğrenme iklimi ve matematik başarısını konu alan araştırmalar incelendiğinde ise az sayıda çalışmanın olduğu görülmektedir (örn., Gürgah-Oğul, 2020; İvrendi vd., 2009; Okur-Ataş vd., 2022). Bunun yanı sıra bu faktörlerin birlikte ele alınmadığı görülmektedir.

Tüm bunlar göz önünde bulundurulduğunda, bu çalışma matematik yeteneğinin biyolojik temelleri ile çocukların matematik performanslarını etkileyen bireysel faktörler arasında köprü kurarak uluslararası literatüre de katkı sağlamaktadır. Ulusal katkı göz önünde bulundurulduğunda, matematik yeteneği ile çocukların yürütücü işlev performansı, ev ortamı ve okul ortamı arasındaki ilişki belirlenerek alanyazına katkı sağlanmıştır. Öte yandan, ev ortamının matematikle nasıl bir ilişki içinde olduğu belirlenerek ailelerin bu becerideki rolü incelenmektir. Ayrıca, okul ile ilgili faktörler ve çocuğun matematik becerileri arasındaki ilişki belirlendiğinde, hem programdaki matematik etkinliklerinin hem de öğretmenin rolünün anlaşılmasına destek olmaktadır. Böylelikle, çocukların en yüksek potansiyellerine ulaşmalarında yakın çevre sistemindeki iki önemli faktörün rolü göz önünde bulundurulmaktadır. Bir diğer taraftan, çocukların matematik becerilerini etkileyen biyolojik, bilişsel ve çevresel boyutları birlikte ilişkilendirerek, çocukların matematik performansları üzerinden çevresel ve bilişsel faktörlerden hiyodinamik değişimlere uzanan işbirliği mekanizmasını incelemek, teorik ve metodolojik arka planları ve uygulamaları anlamak için bir köprü kurulmasına yardımcı olmaktadır.

Araştırmanın Soruları ve Hipotezler

1. Küçük çocukların matematik becerileri, matematik görevi sırasında parietal lobdaki hemodinamik tepkiler, yürütücü işlev performansları, evdeki matematik ortamı ve okuldaki matematikle ilgili ortam tarafından öngörülüyor mu?

1.1. Küçük çocukların parietal lobunun hangi alt yapısı matematik görevi sırasında oksijenlenir?

1.2. Küçük çocukların parietal lobundaki hemodinamik tepkiler çocukların matematik becerisi ile ilişkili midir?

1.3. Küçük çocukların matematik becerileri yürütücü işlev performansları ile ilişkili midir?

1.4. Küçük çocukların matematik becerileri evdeki matematik ortamıyla (ebeveyn-çocuk matematik aktiviteleri ve ebeveynin erken matematik hakkındaki inancı) ilişkili midir?

1.5. Küçük çocukların matematik becerileri okuldaki matematikle ilgili ortamla (sınıf içi matematik etkinlikleri ve öğretmenin erken matematikle ilgili inançları) ilişkili midir?

Ana araştırma sorusu ve alt sorulara dayalı olarak, aşağıdaki hipotezler önerilmektedir:

H1: Çocukların matematiksel yetenekleri, paryetal lobdaki hemodinamik yanıtlar, yürütücü işlev ve ev ve okul ortamı tarafından anlamlı bir şekilde tahmin edilmektedir. Özellikle, paryetal lobdaki daha yüksek oksijen seviyeleri, daha yüksek yürütücü işlev puanları ve matematik zenginleştirilmiş ev ve okul ortamı, çocukların matematiksel yetenekleri ile pozitif ilişkilidir.

H1.1: İntraparyetal sulkus ve angular girus, çocuklar matematiksel görevleri yerine getirirken daha yüksek oksijen seviyeleri gösterir.

H1.2: Çocukların matematik yetenekleri, intraparietal sulkus ve angular girustaki oksijen seviyeleriyle pozitif ilişkilidir; bu bölgelerdeki daha yüksek oksijen seviyeleri, daha iyi matematik performansı ile ilişkilidir.

H1.3: Çocukların matematik yetenekleri yürütme işlevleri ile anlamlı şekilde ilişkilidir; yüksek yürütme işlevi performansları, daha iyi matematik yetenekleriyle bağlantılıdır.

H1.4: Çocukların matematik yetenekleri, ev ortamları ile anlamlı bir şekilde ilişkilidir; evde daha sık yapılan matematikle ilgili etkinlikler ve ebeveynlerin matematiğin önemi konusundaki inançları, çocukların matematiksel yetenekleriyle pozitif ilişkilidir.

H1.5: Çocukların matematik becerileri, okul ortamlarıyla anlamlı bir şekilde ilişkilidir; sınıfta daha sık yapılan matematikle ilgili etkinlikler ve öğretmenlerin matematiğin önemi konusundaki inançları, çocukların matematiksel yetenekleriyle pozitif ilişkilidir.

II. YÖNTEM

Araştırmanın Deseni

Bu çalışmanın amacı, okul öncesi dönemdeki çocukların matematik becerilerini ve matematik ölçümü sırasında beyin bölgelerinde gerçekleşen hemodinamik tepkileri, yürütücü işlev, ev ortamı ve okul ortamı ile ilişkisi bağlamında incelemektir. Deneysel olmayan bu çalışmanın hem amaç hem de zaman boyutları göz önüne alındığında, Johnson (2001) tarafından önerilen yordayıcı kesitsel tasarıma uygun olduğu görülmektedir. Yordayıcı çalışmalar, yordayıcı ve ölçüt değişkenler arasındaki ilişkinin incelenmesini ifade etmektedir (Pedhazur vd., 1991). Bu çalışma, çocukların matematik becerileri ile matematik testi sırasında beyin bölgelerinin hemodinamik tepkileri ve yürütücü işlev, ev ve okul iklimi arasındaki ilişkiyi belirlediği için bu türe uygun görülmektedir. Bu bağlamda, çalışmanın yordayıcı değişkenleri parietal lobdaki hemodinamik tepkiler, yürütücü işlev, ev ortamı (ebeveyn-çocuk matematik etkinlikleri ve ebeveynlerin erken matematiğe yönelik inançları) ve okul ortamı (sınıf matematik etkinlikleri ve öğretmenin erken matematiğe yönelik inançları) iken, ölçüt değişkenleri çocukların matematik becerileridir.

Örneklem

Bu çalışmada veriler çocuklar, ebeveynler ve öğretmenler olmak üzere üç kaynaktan toplanmıştır. Dolayısıyla örneklem, becerilerin doğrudan ölçümünde çocukları, ev ile ilgili değerlendirmeleri sağlayan ebeveynleri ve okul ile ilgili değerlendirmeleri sağlayan öğretmenleri kapsamaktadır.

Bu çalışmada alt gruplardan eşit sayıda bireyin rastgele seçilmesi yoluyla her bir alt grubun eşit olarak temsil edilmesini ifade eden olasılıklı örnekleme yöntemlerinden biri olan oranlı tabakalı rastgele örnekleme kullanılmıştır (Mills vd., 2016). Bu örnekleme yöntemine uygun olarak, İstanbul ilindeki bağımsız anaokulu ve ilkokullar bünyesindeki anasınıflarına devam eden çocuklar, sınıf öğretmenleri ve ebeveynleri belirlenmiştir. Araştırmanın değişkenlerinden birinin öğretmenin özelliklerini ve sınıf içinideki uygulamalarını içermesi nedeniyle her okulun her sınıfından bir çocuk ve o çocuğun velisi araştırmaya dahil edilmiştir. Bu şekilde, her örneklem grubundan bir katılımcı çalışmaya dahil edilerek veri tekrarından (bir öğretmenin özelliklerinin tüm çocuk verileriyle eşleştirilmesi) kaçınılmıştır.

Yeterli örneklem büyüklüğünü belirlemek için ilgili literatür dikkate alınmıştır. Mevcut çalışma beş yaşındaki bir çocuğun matematiksel becerilerini çok değişkenli analiz kullanarak modellemeyi amaçladığından ve gizil değişkenler içerdiğinden veri analizinde Yapısal Eşitlik Modellemesi (YEM) kullanılmıştır. Araştırmalar YEM yapabilmek için en az 200 katılımcı olması gerektiğini vurgulamaktadır (Barrett, 2007; Boomsma, 1982; Kline, 2023). Bu analizde 200 ve üzeri katılımcı sayısı, parametre tahminindeki yanlılığı ve standart hataları en aza indirdiği ve kovaryans matrisleri dikkate alındığında parametreler için %95 güven aralığının teorik beklentiye yakın olduğu için güvenilir kabul edilmektedir (Boomsma, 1982).

Ana çalışma için veriler, 2023-2024 eğitim-öğretim yılında 239 okul öncesi sınıfının öğretmenlerinden ve her sınıftan seçilen bir çocuktan ve çocuğun ebeveynlerinden elde edilmiştir. Araştırmaya katılan öğretmenlerin %97,5'i kadındır ve yaş ortalamaları 36,39'dur (SS=8,9). Ayrıca, öğretmenlerin %94,6'sı devlet okullarında görev yapmaktadır ve ortalama hizmet süreleri 13,07 (SS=8,17) yıldır. Öte yandan, öğretmenlerin %79'u lisans düzeyindeki programlardan mezun olmuştur.

Çalışmaya tipik gelişim gösteren 127 kız ve 112 erkek çocuk dahil edilmiştir. Çocukların yaş ortalaması 66,58 (SS=4,78) ay olup, %50,6'sı okul öncesi eğitimin ikinci yılındadır ve %70,4'ünün kardeşi vardır.
Araştırmaya katılan ebeveynlere bakıldığında, annelerin yaş ortalamasının 36,23 (SS=5,54) olduğu, %29,8'inin lisans mezunu olduğu ve %62,1'inin çalışmadığı bildirilmiştir. Babaların yaş ortalaması 39,28 (SS=5,50), %34,8'i lise mezunu ve %88,8'i tam zamanlı çalışmaktadır.

Veri Toplama Araçları

Bu çalışma kapsamında, beş yaşındaki çocukların matematik becerileri, parietal lobdaki hemodinamik değişiklikler, çocukların yürütücü işlevleri, ev ve okul bağlamında gerçekleştirdikleri etkinlikler ve yetişkinlerin inançları ölçülmektedir. Bu amaçla ölçme araçlarının seçiminde temsil ettiği beceri, yaşa uygunluğu, Türk kültürü için geçerlik ve güvenirlik çalışmaları temel kriterler olmuştur. Buradan hareketle matematik becerilerinin ölçümünde, matematik becerilerinin kapsamlı bir şekilde ölçülmesine olanak sağlayan, geçerlik ve güvenirlik çalışmaları yapılmış "Erken Matematik Yeteneği Testi (TEMA-3)", ev etkinlikleri ve inançların ölçümünde ise geçerlik ve güvenirlik çalışmaları yapılmış "Erken Matematik Ölçeği" kullanılmıştır, "Sınıf İçi Etkinlik Ölçeği" ölçeği sınıf içi etkinlikleri değerlendirdiği için, "Matematiksel Gelişim İnançları Ölçeği" öğretmen inançlarını ölçtüğü ve geçerlilik ve güvenilirlik çalışmaları yapıldığı için, son olarak "EF Touch" yürütücü işlevleri tüm alt boyutlarıyla ölçtüğü ve geçerlilik ve güvenilirlik çalışmaları yapıldığı için kullanılmıştır. Ölçme araçları hakkında detaylı bilgi aşağıda verilmiştir.

Erken Matematik Yeteneği Testi (TEMA-3)

Ginsburg ve arkadaşları (2003) tarafından geliştirilen TEMA-3, 3-8 yaş arası çocukların hem informal (örn. sayma ve bağıl miktar farkındalığı) hem de formal matematik becerilerini (örn. toplama ve çıkarma) değerlendirmek üzere tasarlanmış, norm referanslı ve standardize edilmiş kişi temelli bir testtir. Testin psikometrik özellikleri Bliss (2006) tarafından incelenmiştir ve aşağıdaki bilgiler bu çalışmaya dayanmaktadır. TEMA-3'ün iç geçerliliği alfa katsayısı ile hesaplanmış ve değerler .92 ile .96 arasında değişmiştir. Kapsam geçerliği diskriminant analizi ile ölçülmüş ve .45 ile .68 arasında, diğer matematik görevleri (KeyMathR, Temel Kavramlar alt testi) arasındaki korelasyon ise .54 ile .91 arasında değişmiştir.

TEMA-3'ün Türkçe uyarlaması Erdoğan (2006) tarafından yapılmıştır. Bu çalışmada araştırmacı testin A ve B formlarını çevirmiş ve her iki dili de iyi bilen bir uzmandan çeviriyi değerlendirmesini istemiş, daha sonra Türkçe form İngilizceye çevrilmiş ve dil uzmanı tarafından değerlendirilmiştir. Test-tekrar test analizinde güvenirlik değerleri .88 ile .90 arasında bulunmuştur. Ayrıca test güvenirliğini belirlemek için Kuder Richardson-20 hesaplanmış ve değerler .92 ile .96 arasında bulunmuştur (aktaran Avcı, 2015). Mevcut çalışmada TEMA-3 puanları ile TEMA-3'ün sayı bilgisi ve toplama problemi maddelerinden oluşan bir görev olan fNIRS Matematik Görevi arasındaki korelasyon değeri hesaplanmıştır. İlişki değerleri ham puanlar için .718 ve standartlaştırılmış puanlar için .564 olarak bulunmuştur.

fNIRS Matematik Görevi

Çalışmanın amaçlarından bir diğeri de matematik işlemleri sırasında çocukların beyinlerindeki hemodinamik değişiklikleri incelemek olduğu için bunun ölçülmesinde özel bir deney düzeneği hazırlanmıştır. Öncelikle araştırmacı TEMA-

3’ün maddeleri ile uyumlu olarak bir dizi sayı bilgisi ve toplama problemi hazırlamıştır. Küçük çocuklarla yapılan önceki çalışmalar aritmetik işlemlere ve sayı tanıma görevlerine odaklandığından (bkz. Artemenko vd., 2018; Artemenko vd., 2022; Hyde vd., 2010), mevcut araştırmada da gelişimsel olarak uygun hale getirilerek matematik soruları buna göre tasarlanmıştır. Deneyin ilk bölümü, her sette beş sayı olmak üzere, zorluk derecesi artan üç görev zamanından oluşmuştur. İlk görev zamanında çocuklara rakamlar (örn. 1, 3, 4, 8, 9), ikinci görev zamanında çocuklara iki basamaklı sayılar (örn. 10, 27, 39, 56, 94) ve üçüncü görev zamanında çocuklara üç basamaklı sayılar (örn. 107, 164, 270, 326, 589) gösterilmiştir. Deneyin ikinci bölümünde, çocuklara her sette beş toplama işleminin yer aldığı iki görev zamanı sunulmuştur. Bu bölümün ilk görev zamanında, çocuklara tek basamaklı sonuçlara sahip toplama işlemleri (örn. 1+2= , 2+2= , 3+2= , 3+2= , 4+3= , 5+2=) için nicel ifadeler (örn. basamak ifadesini temsil eden bilye sayısı) ile sunulmuştur. İkinci görev bölümünde de çocuklara iki basamaklı sonuçları olan toplama işlemleri için (örneğin, 8+2= , 7+4= , 9+3= , 8+5= , 9+7=) niceliksel ifadeler (yani, rakam ifadesini gösteren bilyeler) ile sunulmuştur.

Çalışma blok tasarımına göre tasarlanmıştır. Bu tasarım, uyarıcıları bir koşul içinde sırayla sunarak (uyarıcı sunum stratejisi) ve bunu farklı bir koşulun sunulduğu diğer anlarla (epoklar) değiştirerek bir göreve bilişsel katılımı sürdürmeye dayanan özel bir karşılaştırma paradigması kategorisidir (Amaro ve ark., 2006). Bu çalışmada, dinlenme ve görev süreleri "AB bloğu" olarak bilinen iki durumlu bir döngü oluşturularak tasarlanmıştır. Dinlenme süresi boyunca, çocuk dostu sabitlemeyi kullanmak için meditasyon yapan bir çocuk görüntüsü sunulmuş ve çocuktan bu görüntüye odaklanması istenmiştir.

EF Touch

İlk olarak Willoughby ve arkadaşları (2012) tarafından "EF Touch" adıyla kağıt-kalem testi olarak tasarlanan bu ölçüm aracı, 2016 yılında Willoughby ve arkadaşları tarafından bilgisayar ortamına aktarılmıştır. Üç çalışma belleği, üç ketleyici kontrol, bir bilişsel esneklik ve bir reaksiyon zamanı görevinden oluşan batarya, YEM sonuçlarına göre iyi bir uyum göstermiş ve ölçeğin güvenirlik katsayıları test-tekrar test sonuçlarında .99 ve .76 olarak bulunmuştur. Maksimum güvenirlik çalışmasına göre evler, bir şeyler aynı ve domuz üçten beşe kadar her yaş için yürütme işlevini ölçmek için en etkili görevler olarak bulunmuştur (Willoughby vd., 2013).

Bu ölçme aracı Hamamcı ve diğerleri (2023) tarafından Türkçeye uyarlanmıştır. Orijinal çalışmada oluşturulan modele uygun olarak yapılan YEM analizinde ölçme aracının iyi uyum verdiği görülmüştür. Güvenirlik analizi için bileşik güvenirlik katsayısı .80 olarak hesaplanmıştır. Mevcut çalışmada evler, domuzcuk ve bir şeyler benziyor görevleri kullanılmış ve bu görevlerin Croanbach's Alpha değerleri sırasıyla .865, .611 ve .736 olarak hesaplanmıştır.

Ev İle İlgili Faktörler

Çocuk ve Aile Demografik Bilgi Formu

Bu form ailenin ve çocuğun demografik bilgilerini edinmeyi amaçlayan soruları içermektedir. Aile ile ilgili sorular ebeveynlerin yaşı, eğitim düzeyi, çalışma durumu ve hane gelirini kapsamaktadır. Çocukla ilgili sorular ise çocuğun kronolojik yaşını, gelişimsel özelliklerini, özel ihtiyaçlarını (örn. diskalkuli vb.), süregelen hastalıklarını, kazalarını ve ilaç kullanımını içermektedir.

Erken Matematik Anketi (EMQ)

Missall ve diğerleri (2015) tarafından geliştirilen anket, ebeveyn raporlaması ile kağıt-kalem formatında uygulanmaktadır. Orijinal ölçek üç bölümden oluşmaktadır: kişisel/demografik bilgileri içeren bir anket, matematikle ilgili etkinlikleri kapsayan 5'li Likert tipi bir anket ve ebeveynlerin matematik hakkındaki inançlarını ölçen 4'lü Likert tipi bir anket. Demografik özellikleri kapsayan ilk bölümde, ebeveynin yaşı, cinsiyeti ve eğitim durumu ile çocuğun yaşı, cinsiyeti, anadili, gelişim özellikleri ve çocuk bakımı/okul öncesi katılımı hakkında bilgi edinmek için 19 soru sorulmuştur. İkinci bölümde, günlük ebeveyn-çocuk etkileşimine dayalı sorular sayılar ve işlemler (19 madde), geometri (9 madde), ölçme (5 madde) ve cebir (3 madde) alanlarında erken matematik içeriğini yansıtmaktadır. Üçüncü bölüm, ebeveynlerin matematikle ilgili inançlarına ilişkin 13 sorudan oluşmaktadır.

Ölçeğin Türkçeye uyarlama çalışması Karakuş (2022) tarafından yapılmıştır. Bu uyarlama çalışmasında ikinci ve üçüncü bölümler uyarlanmıştır. Madde sayısı korunarak yapılan doğrulayıcı faktör analizinde matematik etkinlikleri bölümünde sayı, geometri, ölçme ve örüntü boyutlarına ait faktörlerin anlamlı yük aldığı ve modelle iyi uyum gösterdiği bulunmuştur. Matematik inançları bölümünde ise madde sayısı korunarak doğrulayıcı faktör analizi yapılmış ve elde edilen verilerin modele uygun olduğu görülmüştür. Güvenilirlik kontrolü için, anketin toplam puanı (α=.98) ve alt bileşenleri olan ebeveyn çocuk matematik etkinlikleri (yani, sayılar ve işlemler [α=.96], geometri [α=.94], ölçme [α=87] ve örüntü [α=.96]) ile bireysel matematik inançlarının (yani, çocuk inançları [α=.77] ve ebeveyn inançları [α=.80]) Croanbach's Alpha değerleri hesaplanmıştır.

Matematik İle İlgili Okul Faktörleri

Öğretmen Demografik Bilgi Formu

Bu form, öğretmenlerin demografik ve mesleki özelliklerine ilişkin sekiz sorudan oluşmaktadır. Sorular öğretmenlerin yaşı, cinsiyeti, çalıştığı kurum türü, mevcut kurumda çalışma yılı, toplam kıdem yılı, geliri, eğitim düzeyi ve katıldığı mesleki kursları içermektedir.

Sınıfta Matematik Etkinlikleri

Choi ve Dobbs-Oates (2014) tarafından geliştirilen anket, öğretmen derecelendirmeleri ile sınıfta sağlanan matematikle ilgili etkinlikleri ölçmektedir. Bu bağlamda maddeler; sayma, temel işlemler, şekiller ve örüntüler, ölçümler ve manipülatifler ile ilgilidir. 10 maddeden oluşan ölçek, 1 (hiçbir zaman) ile 6 (her gün) arasında değer alabilen Likert tipindedir.

Bu ölçeğin Türkçeye çevirisi mevcut tez kapsamında gerçekleştirilmiştir. Bu amaçla, 2022-2023 eğitim-öğretim yılında 215 okul öncesi öğretmeninden veri toplanmıştır. Örneklem büyüklüğü için Doğrulayıcı Faktör Analizi (DFA) uygulamasında minimum örneklem büyüklüğü 200 katılımcı olarak belirtilirken (Barrett, 2007; Boomsma, 1982; Kline, 2023), literatürde bu tür analizler için madde sayısı ve örneklem büyüklüğü dikkate alındığında madde başına 20 katılımcı olması gerektiği de belirtilmektedir (Jackson, 2003). Mevcut pilot çalışmadaki ölçme aracının 10 maddeden oluşması, orana dayalı hesaplamada 200 katılımcıya karşılık gelmektedir ve bu sayı DFA için minimum sınırdır. Buna dayanarak 215 katılımcıdan elde edilen verilerin analiz için kabul edilebilir bir örneklem oluşturduğu söylenebilir.

Geçerliliği test etmek için DFA uygulanmış ve güvenilirliği belirlemek için Cronbach Alpha katsayısı hesaplanmıştır. DFA sonuçlarına göre, elde edilen verilerin modele iyi uyum sağladığı ($\chi 2$ (35) = 99.125, $p < .001$, CFI = .94, TLI = .92, NFI= .91, RMSEA = .94 [%90 GA: .07 -.11], SRMR =. 043, GFI = .99) ve Cronbach Alpha katsayısının .90 olması ölçeğin güvenilirliğinin kabul edilebilir olduğunu göstermektedir (bkz. Kline,1999). Ayrıca, bileşik güvenirlik katsayısı da .92 olarak bulunmuştur.

Öğretmenin Matematik İnançları

Platas (2015) tarafından geliştirilen "Matematiksel Gelişim İnançları Ölçeği" erken çocukluk öğretmenlerinin matematik hakkındaki inançlarını okul öncesi eğitimin öncelikli hedefi olarak matematiksel gelişim, matematik öğretiminin yaşa uygunluğu, matematik öğretimi sağlamadaki güven düzeyi ve matematiksel bilgi üretiminin sınıf odağı (öğretmene karşı çocuk) bağlamında ölçmektedir. Ölçeğin alt boyutlarının güvenirlik değerleri sırasıyla .85, .92, .89 ve .83'tür. Ölçme aracının Türkçeye uyarlama çalışması Karakuş ve diğerleri (2018) tarafından yapılmış ve bu çalışmada alt boyutların tutarlılık değerleri sırasıyla .82, .88, .84 ve .88 olarak bulunmuştur. Ölçeğin toplam puanı için Croanbah's Alpha değeri .79, matematik öğretiminin optimum yaşı, matematiksel bilgi üretiminin sınıf odağı, okul öncesi eğitimin temel amacı olarak matematiksel gelişim ve matematik eğitimine duyulan güven alt boyutları için ise sırasıyla .74, .46, .61 ve .71 olarak bulunmuştur.

Veri Analizi

ANOVA

Varyans Analizi (ANOVA) testi, her bir matematik becerisi (sayı tanıma ve toplama) için görev ve dinlenme süresi boyunca beyin bölgelerindeki hemodinamik tepkilerdeki farklılıkları belirlemek için uygulanmıştır. Bunun için ön analizler kontrol edildikten sonra Üç Yönlü ANOVA uygulanmış ve p-değeri ($p < .05$) ve eta-kare etki büyüklüğü dikkate alınmıştır. Ayrıca bölgeler arası farkın belirlenmesi için Bonferroni metodu kullanılarak Post-hoc analizler yapılmıştır.

Korelasyon

Çocukların demografik özellikleri, evdeki matematik etkinlikleri, sınıftaki matematikle ilgili süreç kalitesi, yürütücü işlev puanları ve anlamlı bulunan beyin bölgelerinin çocukların matematik becerileriyle arasındaki ilişki Pearson korelasyon katsayısına göre değerlendirilmiştir. Korelasyon varsayımları için normallik (≤+/-2; basıklık ve çarpıklık) ve gözlemlerin bağımsızlığı kontrol edilmiştir.

SEM

Model ki-kare ($\chi 2$), iyilik uyum indeksi (GFI), kök ortalama kare yaklaşım hatası (RMSEA), karşılaştırmalı uyum indeksi (CFI) ve standardize edilmiş kök ortalama kare artık (SRMR) ile değerlendirilmiştir.

Çoklu Regresyon

Çoklu regresyon, bir dizi yordayıcı (bağımsız) değişkene dayalı olarak bir ölçüt (bağımlı) değişkenin değerlerini tahmin etmek için kullanılan istatistiksel yöntemidir. Bu yöntem çok sayıda yordayıcının bağımlı değişken üzerindeki birleşik ve bağımsız etkisini dikkate alan çok yönlü bir yaklaşım sağlayarak bilimsel sorgulamada araştırmacılara yardımcı olur (Cohen vd., 2003). Mevcut çalışmada, çocukların matematik becerilerini etkileyen çoklu faktörlerin belirlenmesinde bu analiz kullanılmıştır. Bu analizi gerçekleştirebilmek için, basit korelasyon analizinin ötesinde, belirli varsayımların karşılanması gerekmektedir. İlk olarak, örneklem büyüklüğü $N > 50 + 8m$ formülü kullanılarak hesaplanmalıdır; burada N örneklem büyüklüğü ve m yordayıcı sayısını temsil etmektedir (Tabachnick vd., 2007). Ayrıca, çoklu doğrusal ilişkiyi ve tekilliği sağlamak için değişkenler arasında yüksek korelasyon ($r = .9$ veya üzeri) olmamalıdır. Bağımsız değişkenler, hem alt boyutları hem de toplam faktörü içermemelidir. Bunun yanı sıra, normal dağılım, doğrusal ilişki ve homoscedasticity (varyansın sabitliği) varsayımlarının da karşılanması gerekmektedir (Pallant, 2011). Modelin değerlendirilmesinde ANOVA sonuçları, Adjusted R^2 ve beta değerleri dikkate alınmıştır.

III. BULGULAR VE TARTIŞMA

Bu bölümde ilk olarak analizin temel bulguları açıklanmış sonrasında ilgili literatür ışığında bu bulgular tartışılmıştır. Aşağıda her bir araştırma sorusuna atıfta bulunularak analiz bulguları açıklanmakta ve literatür ile tartışması yapılmaktadır.

1. Matematik görevi sırasında küçük çocukların parietal lobunun hangi alt yapısı oksijenlenir?

Bu soruyu yanıtlamak için Üç Yönlü ANOVA uygulanmış ve sonuçlar, çocuklar sayı bilgisi görevini tamamlarken sol superior parietal girusta, sağ supramarginal girusta ve hem sol hem de sağ intraparietal sulkusta dinlenme zamanı aktivasyonuna kıyasla hemodinamik yanıt olduğunu göstermektedir. İkinci olarak, çocuklar toplama görevini tamamlarken hem sol hem de sağ postcentral girusta, hem sol hem de sağ superior parietal girusta, sağ supramarginal girusta, hem sol hem de sağ angular girusta ve hem sol hem de sağ intraparietal sulkusta dinlenme zamanına kıyasla hemodinamik yanıt olduğunu göstermektedir. Fakat sadece sol intraparietal sulkustaki oksijen düzeyi dinlenme zamanına kıyasla yüksek miktarda bulunmuştur.

2. Küçük çocukların parietal lobdaki hemodinamik yanıtları çocukların matematik becerileri ile ilişkili midir?

Bu soru iki değişkenli korelasyon analizi ile gerçekleştirilmiştir. Bu bağlamda, fNIRS matematik görevinden elde edilen doğruluk puanları ile her iki hemisferdeki (sol ve sağ) parietal lobun beş bileşeninin (postcentral gyrus, superior parietal gyrus, supramarginal gyrus, angular gyrus ve intraparietal sulcus) hemodinamik yanıtlarının ortalama puanları arasındaki ilişkiler incelenmiştir. Çocukların matematik performansları ile bu bölgelerdeki hemodinamik tepkiler arasındaki tek anlamlı ilişki sol intraparietal sulkus ile olmuştur. Bu, çocukların matematik performansları arttığında sol intraparietal sulkustaki oksijenlenme yanıtının da arttığını göstermektedir.

3. Küçük çocukların matematik becerileri yürütücü işlev performanslarıyla ilişkili midir?

Bu soruyu ele almak için çocukların TEMA-3 ve EF-Touch'ın üç görevindeki (işleyen bellek, ketleyici kontrol ve zihinsel esneklik) puanları arasında iki değişkenli korelasyon hesaplanmıştır. Bulgular, yürütücü işlev bileşenlerinin her birinin çocukların matematik puanlarıyla ilişkili olduğunu ortaya koymuş ve çocukların yürütücü işlev performansları arttığında matematik puanlarının da aynı şekilde arttığını vurgulamıştır.

4. Küçük çocukların matematik yetenekleri evdeki matematik ortamıyla (ebeveyn-çocuk matematik aktiviteleri ve ebeveynin erken matematik hakkındaki inancı) ilişkili midir?

İki değişkenli korelasyon, çocukların matematik performansları ile evdeki matematik ortamı arasındaki ilişkiyi araştırmak için kullanılmıştır. Bu bağlamda, ebeveyn-çocuk matematik etkinlikleri, özellikle de sayılar, işlemler ve örüntü oluşturma etkinlikleri, küçük çocukların matematik performanslarıyla ilişkilidir; bu da etkinliklerin sıklığı arttıkça çocukların matematik görevlerinde daha fazla yeterlilik gösterdikleri anlamına gelmektedir. Bunun yanı sıra ebeveynlerin erken dönem matematiğe yönelik inançları ile küçük çocukların matematik görevindeki puanları arasında bir ilişki bulunmuştur. Bu durum, erken matematiğin önemine ilişkin daha yüksek inançlara sahip ebeveynleri olan çocukların matematik görevinde daha iyi performans gösterdikleri anlamına gelmektedir.

6. Küçük çocukların matematik becerileri okuldaki matematikle ilgili ortamla (sınıf içi matematik etkinlikleri ve öğretmenin erken matematikle ilgili inançları) ilişkili midir?

Bu soruyu araştırmak için iki değişkenli korelasyon kullanılmıştır. Sonuçlar, sınıf içi matematik etkinlikleri ile çocukların matematik performansları arasında bir ilişki olmadığını göstermiştir. Benzeri şekilde, öğretmenlerin erken matematik hakkındaki inançları ile çocukların matematik performansları arasında bir ilişki olmadığını göstermiştir.

8. Beş yaşındaki çocukların matematik becerileri, matematik görevi sırasında parietal lobdaki hemodinamik tepkiler, yürütücü işlev performansları, evdeki matematik ortamı ve okuldaki matematikle ilgili ortam tarafından yordanıyor mu?
Bu soruyu ele almak için hem yapısal eşitlik modellemesi hem de çoklu regresyon analizleri kullanılmıştır. Ana hipotezde önerilen model çalıştırıldığında, sonuçlar modelin yakınsamadığını/birleşmediğini göstermiştir. Bu nedenle, kısmi modeller oluşturularak yapısal eşitlik modelleri değerlendirilmiştir. Bu modeller, çocukların matematik çıktılarının yürütücü işlev, ev ve okul ortamlarının entegrasyonu tarafından yordandığını göstermektedir. Ayrıca, yürütücü işlevin bu modellerde en önemli rolü oynadığı ve çevresel faktörlerin yordama gücünün yürütücü işlev performansları ile bir araya geldiğinde arttığı bulunmuştur. Buna ek olarak, çoklu regresyon analizi çocukların matematik becerilerinin en iyi yürütücü işlev bileşenleri ve sol intraparietal sulkustaki hemodinamik tepki ile açıklandığını ortaya koymuştur.

Elde edilen bu sonuçlar literatür ile birlikte değerlendirildiğinde mevcut sonuçlar teorik düzeyde açıklanmaktadır. Bu çalışmanın altında yatan mekanizmaya göre, küçük çocukların gelişimini ve matematiksel becerilerini açıklamak için tek bir faktör yerine çok sayıda ve bütünleşik bileşenler kullanılmalıdır (Bickhard vd., 2007; Butterworth vd., 2011; Westermann vd., 2007). Bu çok yönlü yapı göz önüne alındığında, küçük çocukların matematik becerileri literatüründe öncelikli araştırma konularının çocukların yakın çevresi (Silver vd., 2022), beyin yapısı (Butterworth vd., 2011; Dehane, 1992) ve bilişsel yetenekler (Bull vd., 2014; Cragg vd.,2014; Kilday, 2011) olduğu görülmektedir. Bu teorik altyapı doğrultusunda mevcut çalışmanın bulguları; biyolojik boyutun (toplama görevi sırasında sol intraparietal sulkustaki aktivasyon), bilişsel boyutun (işleyen bellek, ketleyici kontrol, zihinsel esneklik) ve çevresel boyutların (ebeveyn-çocuk matematik etkinlikleri ve erken matematik hakkındaki ebeveyn inançlarından oluşan ev ortamı ve sınıf içi matematik etkinlikleri ve erken matematik hakkındaki öğretmen inançlarını içeren okul ortamı) kısmi kombinasyonunun küçük çocukların matematik becerilerini açıklayabileceğine işaret etmektedir. Dolayısıyla, çok sayıda unsurun karşılıklı etkileşiminin matematiğin gelişimiyle sonuçlandığı görülmektedir.

Alanyazında matematiksel yeteneklere ve bunun beyindeki izdüşümüne odaklanıldığında, ölçülen matematiksel becerilerin ve uygulanan görevlerin zorluğunun bu konuda belirleyici olduğu görülmektedir (Butterworth vd., 2011). Hem meta-analizler hem de amprik çalışmalar incelendiğinde, toplama, çıkarma ve çarpma işlemlerinde parietal lob da dahil olmak üzere birçok farklı lobun aktive olduğu görülürken, sayı görevlerinde parietal lobun, özellikle de alt ve üst kısımlarının aktive olduğu vurgulanmaktadır (Arsalidou vd, 2011; Artemenko vd., 2018; Bugden vd., 2012; Dresler vd., 2009; Emerson vd., 2015; Kawashima vd., 2004; Kucian vd., 2008; Rickard vd., 2000; Vogel vd., 2015). Bunun yanı sıra, matematiksel yeteneklerle ilgili beyin aktivasyonunun frontal lobdan parietal loba, parietal lobda ise intraparietal sulkustan angular girusa kaydığı öne sürülmektedir (Zamarian vd., 2009). Ayrıca, insan beyninin parietal lobunda, özellikle sol angular girus ve sol intraparietal sulkusta meydana gelen hasar matematiksel işlemlerde bozulmalara neden olmaktadır (Butterworth, 2020). Mevcut sonuçlar da çalışmanın ana sorusu ile ilgili olarak, çocukların matematik performansının, toplama işlemini gerçekleştirirken sol intraparietal sulkustaki aktivasyonun ortalaması tarafından yordandığını göstermektedir. Sonuç olarak, sol intraparietal sulkus aktivasyonunun çocukların matematik becerilerini yordamanın ötesine geçtiği ve bunu yordarken bilişsel becerilerle birleşebildiği görülmektedir.

Ana modelin bilişsel bileşeni olarak çocukların yürütücü işlev performasları değerlendirildiğinde, yürütücü işlevinin tek bir faktör olarak ve her bir alt bileşeninin ayrı ayrı matematiğin en iyi yordayıcıları olduğu görülmüştür (Kilday, 2011; Zelazo vd., 2016). Literatür, matematiksel işlemlerin bir dizi bilişsel süreç gerektirdiğini ve yürütücü işlev becerilerinin bilgiyi tutma ve depolama, uygun olmayan stratejileri göz ardı etme, sayı bilgisindeki sınırları fark etme ve çoklu işlemlerde dönüşüm sağlama gibi bilişsel becerilerle bağlantılı olduğunu vurgulamaktadır (Bull vd., 2014). Ayrıca, bilgi düzeyindeki matematiğin ve olgusal bilginin yanı sıra işlemsel ve kavramsal bilgiyi de kapsadığını göstererek her bir bilgi düzeyinin yürütücü işlev ile bağlantılı olduğunu vurgulamaktadır (Craigg vd., 2014). Literatürle uyumlu olarak bu çalışmanın sonucunda da yürütücü işlevin önemi, çocukların yürütücü işlev performanslarının onların matematik becerilerini bağımsız olarak, çevresel faktörlerle birlikte ve biyolojik faktörlerle birlikte açıkladığını görülmüştür. Bu durum, çocukların yürütücü işlev performanslarının onların matematik becerileri için kritik rolünü vurgularken, diğer unsurlar ile bir araya gelerek onların matematiği yordamadaki gücünü de desteklediğini göstermiştir.

Ana modeldeki çevresel faktörlere odaklanıldığında alanyazın, aile ve okul ortamlarının çocukların matematiksel becerilerini etkileyen birincil çevresel kaynaklar olarak kavramsallaştırıldığını göstermektedir (Bronfenbrenner vd., 1979, 1994; Silver vd., 2022). Bu ortamların işlevine dikkat çekildiğinde, matematiği yordayan temel unsurların ebeveynlerin ve öğretmenlerin erken dönem matematiğe yönelik inançlarının yanı sıra çocuklarıyla/öğrencileri ile birlikte gerçekleştirdikleri doğrudan matematik etkinliklerini kapsadığı öne sürülmektedir (Silver vd., 2022). Bunun yanı sıra, alanyazındaki amprik çalışmalar matematik etkinliklerinde ebeveyn ile çocuğun doğrudan etkileşiminin çocukların matematik becerilerini geliştirdiğini ve çocukların matematik puanlarını arttığını ayrıca ebeveynlerin erken matematiğin önemine dair daha yüksek olumlu inanca sahip olmalarının, küçük çocukların matematik puanlarını artırarak daha iyi performanslar sergilediklerini göstermektedir (Blevins-Knabe vd., 1996; Blevnis-Knabe vd., 2000; Chan vd., 2021; Claire-Son vd., 2020; DeFlorio vd., 2015; Foster vd., 2016; Huang vd., 2017; Huntsinger vd., 2016; Manolitsis vd, 2013; Missal vd., 2017; Niklas vd., 2017; Pardo vd., 2020; Sonnenschein vd., 2012; Soto-Calvo vd., 2020; Thompson vd., 2016; Zippert vd., 2020). Bu durum, mevcut çalışmanın bulgularıyla da uyumludur. Mevcut çalışmada, çocukların matematiksel becerileri ile ev ortamındaki ebeveyn-çocuk matematik etkinlikleri ve ebeveynin matematiğin önemine olan yüksek inancı arasında bir ilişki olduğu tespit edilmiştir. Bu bulgu, literatürde yer alan ebeveyn-çocuk matematik etkinliklerinin artmasının ve ebeveynin matematiğin önemine dair yüksek inancının, çocukların daha iyi matematiksel performans sergilemesine katkı sağladığına dair mevcut bulguları desteklemektedir. Ev ortamı literatürünün aksine, okul ortamına ilişkin araştırmalar çelişkili sonuçlara sahiptir. Çocukların matematik becerilerinin okul ortamlarıyla ilişkili olduğunu gösteren bulgular varolmak ile beraber (Finders vd., 2021; Grammatikopoulos vd., 2018; Lehrl vd., 2016; Li vd., 2019; Mashburn vd, 2008; Schmitt vd., 2020; Schmerse, 2020) okul iklimi ile çocukların matematik becerileri arasında bir ilişki olmadığını gösteren çalışmalar da bulunmaktadır (Abreu-Lima vd., 2013; Brunsek vd., 2017; Francis vd., 2019; Guerrero-Rosada vd., 2021). Mevcut çalışmanın sonuçları da öğretmenlerin sınıf içinde uyguladığı matematik etkinlikleri ve onların erken matematiğe yönelik tutumlarının çocukların matematik becerileri ile ilişkili olmadığını göstermektedir. Bu durum göz önünde bulundurulduğunda, araştırmalarda ölçülen okul iklimi değişkeninin önemli bir rol oynadığı söylenebilir. Mevcut çalışmada, sınıf içindeki matematik etkinlikleri sıklık boyutu ile ele alınmıştır. Bu durum nedeniyle, çocukların matematik etkinliklerine katılımı, matematik etkinliklerinin niteliği ve kullanılan öğretim teknikleri hakkında bilgi barındırmamaktadır. Benzer şekilde, öğretmenin matematiğe yönelik inancının değerlendirilmesinde, öğretmenin çocuklarla olan ilişkisi ve bireysel özellikleri göz önünde bulundurulmamıştır. Literatürdeki çalışmalar, bu faktörlerin önemine dikkat çekerek, ölçülen özelliklerle ilişkili olabileceğini vurgulamış ve bu nedenle öğretim sırasındaki etkileşimlerin de göz önünde bulundurulması gerektiğini önermektedir (Mashburn vd., 2008).

Bu arka planla uyumlu olarak, mevcut çalışmanın sonuçları, çocukların yürütücü işlev performansları ile işbirliği içinde ele alındığında çevrenin olumlu etkisi ile tutarlıdır. Aksi takdirde, bu bileşenler tek başlarına veya birbirleriyle kombinasyon halinde anlamlı bir doğrudan yordamaya sahip değildir. Dolayısıyla, ana soruya ilişkin sonuçlar, küçük çocukların matematik becerilerini değerlendirirken işbirlikçi yaklaşımın önemli olduğunu göstermektedir.

E. THESIS PERMISSION FORM / TEZ İZİN FORMU

(Please fill out this form on computer. Double click on the boxes to fill them)

ENSTİTÜ / INSTITUTE

Fen Bilimleri Enstitüsü / Graduate School of Natural and Applied Sciences ☐

Sosyal Bilimler Enstitüsü / Graduate School of Social Sciences ☒

Uygulamalı Matematik Enstitüsü / Graduate School of Applied Mathematics ☐

Enformatik Enstitüsü / Graduate School of Informatics ☐

Deniz Bilimleri Enstitüsü / Graduate School of Marine Sciences ☐

YAZARIN / AUTHOR

Soyadı / Surname : Hamamcı
Adı / Name : Beyza
Bölümü / Department : Temel Eğitim, Okul Öncesi Eğitimi / Early Childhood Education

TEZİN ADI / TITLE OF THE THESIS (**İngilizce** / English): The mathematical abilities of young children in the context of hemodynamic changes, executive function, home and school environment

TEZİN TÜRÜ / DEGREE: **Yüksek Lisans** / Master ☐ **Doktora** / PhD ☒

1. **Tezin tamamı dünya çapında erişime açılacaktır. /** Release the entire work immediately for access worldwide. ☒
2. **Tez iki yıl süreyle erişime kapalı olacaktır.** / Secure the entire work for patent and/or proprietary purposes for a period of **two years. *** ☐
3. **Tez altı ay süreyle erişime kapalı olacaktır.** / Secure the entire work for period of **six months**. ***** ☐

***** *Enstitü Yönetim Kurulu kararının basılı kopyası tezle birlikte kütüphaneye teslim edilecektir. / A copy of the decision of the Institute Administrative Committee will be delivered to the library together with the printed thesis.*

Yazarın imzası / Signature .... **Tarih** / Date ....
(Kütüphaneye teslim ettiğiniz tarih. Elle doldurulacaktır.)
(Library submission date. Please fill out by hand.)

Tezin son sayfasıdır. / This is the last page of the thesis/dissertation.