CONSTRAINT ON THE NUMBER OF QUARK GENERATIONS THROUGH MASS MATRICES

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ABSTRACT

CONSTRAINT ON THE NUMBER OF QUARK GENERATIONS THROUGH MASS MATRICES

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Kobayashi-Maskawa mixing, flavor mixing through mass matrices and the relation between them in the general case, in Glashow-Salam-Weinberg model and in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models are reviewed. It is shown that the number of quark generations must be three in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models with one fermion mass generating Higgs multiplet through analysis of quark mass and flavor mixing matrices under the assumption no zero mass quark exist.

Keywords: Flavor Mixing, Quark Generations, Standard Model, Left-Right Symmetric Models, Electroweak Models

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ÖZET

KÜTLE MATRİKSLERİ YOLUYLA KUARK NESİLLERİNE SINIRLAMA

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Genel halde, Glashow-Salam-Weinberg modelinde ve $SU(2)_L \otimes SU(2)_R \otimes U(1)$ modellerinde Kobayashi-Maskawa karışması, kütle matriksleri yoluyla kuark çeşni karışması ve aralarındaki ilişki gözden geçirildi. Fermiyonlara kütle veren Higgslerden bir tane kullanılan $SU(2)_L \otimes SU(2)_R \otimes U(1)$ modellerinde kütle matriksleri ve çeşni karışma matriksleri incelenerek, hiç bir kuarkın kütlesinin sıfır olamıyacağı varsayımı altında, kuark nesilleri sayısının üç tane olması gerektiği gösterildi.

Anahtar kelimeler: Çeşni Karışması, Kuark Nesilleri, Standart Model, Sol-Sağ Simetrik Modeller, Elektro-zayıf Modeller

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INTRODUCTION

The standard model of electroweak interactions and its extensions (e.g. left-right symmetric models) make use of the fermions whose, at least, left banded components are doublets of the internal group. Each such doublet is called a lepton or quark generation. The standard model does not predict the number of fermion generations. It works equally well for all number of quark and lepton generations although we know that experiments fix the number of lepton generations as three thus requiring the number of quark generations as well to be three through the absence of gauge anomaly. All generations couple to gauge bosons with the same coupling constant. Hence we may say that all generations are equivalent under all known interactions (e.g.weak interactions). The differences between different generations are due to different masses and in the case of quarks, also due to their different flavor mixings with the other generations. Both fermion masses and flavor mixings result from the part of the Lagrangian for fermion masses in the standard model and in its extensions. Hence the number of quark generations somehow must be related to the form of the fermion mass terms in the Lagrangian. This is our starting point to seek such a relation. We study the standard model and its most viable extension; $SU(2)_L \otimes SU(2)_R \otimes U(1)$. We see that altough the quark mass matrices in the standard model does not put a constraint on the number of quark generations the quark mass matrices in a class of left-right symmetric models with a simple plausible assumption require the number of quark generations to be three.

The necessity for a non-diagonal mass matrix follows from the flavor mixing in left handed charged weak currents which is known as Cabbibo mixing in the two generations case and as Kobayashi-Maskawa mixing in the three generations case. So first we must understand Kobayashi-Maskawa mixing. This is reviewed in chapter 2. Then we point out the relation between the mass matrices and Kobayashi-Maskawa mixing in the first section of chapter 3. The fermion mass terms in the Lagrangian and hence the fermion mass matrices result from Yukawa Lagrangians through spontaneous symmetry breaking in electro-weak models. These are found in the remaining part of chapter 3 for the standard model and in chapter 4 for $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models. Finally we derive a constraint on the number of quark generations in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models with one Higgs multiplet belonging to $(\frac{1}{2}, \frac{1}{2}, 0)$ representation of the internal group.

Chapter 2

KOBAYASHI-MASKAWA MIXING

2.1 Historical Background

The fact that $\Delta S = 1$ weak decays are suppressed compared to $\Delta S = 0$ weak decays together with a Fermi type maximally parity violating weak current-current theory was formulated by Cabbibo[1] based on an $SU(3)$ symmetry by making use of all three quarks known at that time. He wrote the the total hadronic charged current in the form

$$(J_\mu)_c = \cos\theta_c J_\mu(\Delta S = 0) + \sin\theta_c J_\mu(\Delta S = 1) \tag{2.1}$$

where

$$J_\mu(\Delta S = 0) = J_\mu{}^1 + iJ_\mu{}^2 \qquad J_\mu(\Delta S = 1) = J_\mu{}^4 + iJ_\mu{}^5 \tag{2.2}$$

with

$$J_\mu^i = \bar{\psi}\gamma_\mu(1+\gamma_5)\frac{1}{2}\lambda_i\psi \tag{2.3}$$

Where $\psi^T = (u, d, s)$ and λ_i $(i = 1, 2.....8)$ are the $SU(3)$ generators. The electric charge is given by $Q = I_3 + \frac{1}{2}(B + S)$. Here B and S are baryon and strangeness quantum numbers respectively. I_3 of u and d are 1/2 and -1/2 respectively and of s is zero. In order to calculate some cross section by using this scheme we first determine the corresponding current from (2.3) then we multiply it with $\cos\theta$ or $\sin\theta$ depending on whether it is a $\Delta S = 0$ or a $\Delta S = 1$ transition as in (2.1). When one tries to use this formulation in Glashow-Salam-Weinberg's $SU(2)_L \otimes U(1)$ electro-weak model (now known as the standard model of electro-weak interactions), which was originally developed only for leptons, the following observation becomes crucial: The total hadronic hadronic charged current in (2.1) may be written as

$$J_{c\mu} = \bar{u}\gamma_\mu(1+\gamma_5)d_c + h.c. \tag{2.4}$$

where

$$d_c = d\cos\theta_c + s\sin\theta_c \tag{2.5}$$

We notice from (2.4) that instead of treating u, d, s, as members of a triplet of $SU(3)$ we may treat them as members of an $SU(2)$ doublet (u, d_c) with $Q = I_3 + \frac{1}{2}Y$, $I_3(\mathrm{u}) = 1/2$, $I_3(d) = I_3(s) = -1/2$ $Y = 1/3$. This interpretation has the trouble of gauge anomaly because of lepton-quark asymmetry (two generation of leptons were known at that time so the number of quark and lepton doublets are not the same in this case). Moreover even if we did not take care of gauge anomaly it would introduce unacceptly large flavor changing neutral currents, through $\sin\theta\cos\theta\bar{d}\gamma^\mu(1+\gamma_5)s$ type of terms, which are dangerous as we see in section 4 of chapter 3. In 1970 Glashow, Illioupulos and Maiani[2] introduced the orthogonal combination $s_c = -d\sin\theta_c + s\cos\theta_c$ and the additional quark field c to solve the problem. In 1973 the extension of the scheme to three generations to incorporate CP violation (which is known as Kobayashi-Maskawa mixing) into the scheme was formulated by Kobayashi and Maskawa[3] while only two generations of quarks were known at that time. We shall study three most frequently used parametrizations of Kobayashi-Maskawa mixing namely, the original Kobayashi-Maskawa, Maiani[4] and Wolfenstein[5] parametrizations. In all these parametrizations we use 'quark phase convention' which assumes $K^0 \to 2\pi$; $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes being real to lowest order, which is easly achieved by choosing the mixing elements U_{ud} and U_{us} as real.

2.2 Kobayashi-Maskawa Parametrization

We may account for flavour changing charged weak currents as well as flavor conserving charged currents by assuming the lower element of quark doublet to be a mixture of left handed charge -1/3 quarks, that is,

$$q_i = \begin{pmatrix} u' \\ d'_c \end{pmatrix}_L , \quad \begin{pmatrix} c' \\ s'_c \end{pmatrix}_L , \quad \begin{pmatrix} t' \\ b'_c \end{pmatrix}_L \tag{2.6}$$

in the three generations case where d'_c, s'_c, b'_c are mixtures of d,s,b quarks. The Lagrangian giving rise to charged currents may be written as

$$\mathcal{L} = ig\bar{q}_i \frac{1}{2}\vec{\tau}\cdot\gamma_\mu \vec{A}^\mu (1+\gamma_5) q_i \tag{2.7}$$

The neutral current part of (2.7) is

$$\begin{aligned} \mathcal{L}_n &= ig\bar{q}_i \frac{1}{2}\tau_3 \gamma^\mu A_3^\mu (1+\gamma_5) q_i \\ &= \frac{i}{2} g\bar{u}'_i \gamma^\mu (1+\gamma_5) u'_i A_\mu - \frac{i}{2} g\bar{d}'_{ci} \gamma^\mu (1+\gamma_5) d'_{ci} A_\mu \end{aligned} \tag{2.8}$$

where $u'^T = (u', c', t')$, $d'^T_c = (d'_c, s'_c, b'_c)$. We know that flavor changing neutral currents(FCNC) are absent or highly suppressed[6]. So the terms of the form $\bar{d}_{ci}\gamma^\mu(1+\gamma_5)d_{ci}$ in Eqn.2.8 must be flavor diagonal. In other words

$$d'_c = U_{KM} d \tag{2.9}$$

where U_{KM} is a unitary matrix. After identifying Kobayashi-Maskawa mixing by a unitary matrix we must parametrize it. The form of mixing in two generations model is a rotation. We seek a similiar form for three generations. It must be reducible to a two generations case after setting some of the mixing angles to zero.

A general orthogonal continous tranformation may be constructed by applying three successive rotations, in other words $O = O_z(\theta_2)O_x(\theta_1)O_z(\theta_3)$. Similiarly we may parametrize a general unitary transformation as

$$U = D_2 O_z(\theta_2) D_1 O_x(\theta_1) D_3 O_z(\theta_3) D_4^\dagger = U_2 U_1 U_0 U_3 \tag{2.10}$$

where

$$U_2 = D_2 O_z(\theta_2) D_2^\dagger \quad U_1 = D_2 D_1 O_z(\theta_1)(D_2 D_1)^\dagger$$
$$U_0 = D_2 D_1 D_3 D_4^\dagger \quad U_3 = D_4 O_z(\theta_3) D_4^\dagger \tag{2.11}$$

$$O_z(\theta_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_i & s_i \\ 0 & -s_i & c_i \end{pmatrix}, \quad O_z(\theta_1) = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.12}$$
$$i = 2,3$$

Unitarity of U_2, U_1, U_3 imposes

$$D_2 = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\beta_2} \end{pmatrix}, \quad D_2 D_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\beta_1} & 0 \\ 0 & 0 & e^{i\rho} \end{pmatrix}$$
$$D_4 = \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\beta_4} \end{pmatrix} \tag{2.13}$$

with

$$U_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_i & s_i e^{i\beta_i} \\ 0 & -s_i e^{-i\beta_i} & c_i \end{pmatrix}, \quad U_1 = \begin{pmatrix} c_1 & s_1 e^{i\beta_1} & 0 \\ -s_1 e^{-i\beta_1} & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.14}$$
$$i = 2,3 \tag{2.15}$$

α, ρ, β_i are arbitrary real numbers. D_3 remains arbitrary. So $D_2 D_1 D_3 D_4^\dagger$ is an arbitrary diagonal matrix which can be parametrized as

$$U_0 = D_2 D_1 D_3 D_4^\dagger = exp[i(\alpha_0 + \alpha_3 \lambda_3 + \alpha_8 \lambda_8)] \tag{2.16}$$

where λ_3 and λ_8 are the diagonal $SU(3)$ generators.

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{2.17}$$

Now we redefine the phases of the mixing matrix U and the fermion fields d'_{c_i}, u' by

$$d'_{c_i} = D_n d_{c_i} \quad u' = D_p u \quad U' = D_p U D_n^\dagger \tag{2.18}$$

we rewrite U' as

$$U' = D_p U_2 D_p^\dagger D_p U_1 D_p^\dagger D_p U_0 D_n^\dagger D_n U_3 D_n^\dagger \tag{2.19}$$

We take

$$\begin{aligned} D_p &= exp[i(a_0 + a_3\lambda_3 + a_8\lambda_8)] & (2.20) \\ D_n &= exp[i(b_0 + b_3\lambda_3 + b_8\lambda_8)] & (2.21) \end{aligned}$$

We may express (2.19) and (2.21) as

$$D_p = \begin{pmatrix} exp[i(a_0 + a_3 + \frac{1}{\sqrt{3}}a_8)] & 0 & 0 \\ 0 & exp[i(a_0 - a_3 + \frac{1}{\sqrt{3}}a_8)] & 0 \\ 0 & 0 & exp[i(a_0 - \frac{2}{\sqrt{3}}a_8)] \end{pmatrix} \tag{2.22}$$

$$D_n = \begin{pmatrix} exp[i(b_0 + b_3 + \frac{1}{\sqrt{3}}b_8)] & 0 & 0 \\ 0 & exp[i(b_0 - b_3 + \frac{1}{\sqrt{3}}b_8)] & 0 \\ 0 & 0 & exp[i(b_0 - \frac{2}{\sqrt{3}}b_8)] \end{pmatrix} \tag{2.23}$$

After using (2.22) and (2.23) we obtain

$$D_p U_0 D_n^\dagger = exp[i(\alpha_0 + a_0 - b_0) + (\alpha_3 + a_3 - b_3)\lambda_3) + (\alpha_8 + a_8 - b_8)\lambda_8] \tag{2.24}$$

$$D_p U_1 D_p^\dagger = \begin{pmatrix} c_1 & s_1 exp[i(\beta_1 + 2a_3)] & 0 \\ -s_1 exp[-i(\beta_1 + 2a_3)] & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.25}$$

$$D_p U_2 D_p^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 exp[i(\beta_2 - a_3 + \sqrt{3}a_8)] \\ 0 & -s_2 exp[-i(\beta_2 - a_3 + \sqrt{3}a_8)] & c_2 \end{pmatrix} \tag{2.26}$$

$$D_s U_3 D_s^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 exp[i(\beta_3 - b_3 + \sqrt{3}b_8)] \\ 0 & -s_3 exp[-i(\beta_3 - b_3 + \sqrt{3}b_8)] & c_3 \end{pmatrix} \quad (2.27)$$

The parameters $a_0, a_3, a_8, b_0, b_3, b_8$ may be chosen such that

$$\beta_2 - a_3 + \sqrt{3}a_8 = 0 \quad (2.28)$$

$$\beta_1 + 2a_3 = 0 \quad (2.29)$$

$$\alpha_3 + a_3 - b_3 = 0 \quad (2.30)$$

$$\beta_3 - b_3 + \sqrt{3}b_8 = 0 \quad (2.31)$$

$$\alpha_0 + a_0 - b_0 = -\frac{1}{\sqrt{3}}(\alpha_8 + a_8 - b_8) \quad (2.32)$$

Using Eqn.2.30 and Eqn.2.31 in Eqn.2.24 we obtain

$$\alpha_0 + a_0 - b_0 + (\alpha_8 + a_8 - b_8)\lambda_8 = (\alpha_8 + a_8 - b_8)(\lambda_8 - \frac{1}{\sqrt{3}})$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \delta \end{pmatrix} = A \quad (2.33)$$

where $\delta = \alpha_8 + a_8 - b_8$. The only remained phase is δ. The others are removed by 2.28 - 2.32. Finally we obtain Kobayashi-Maskawa mixing matrix

$$U_{KM} = U'$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \times$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \quad (2.34)$$

where we used the identity

$$e^{iA} = 1 + \Sigma_{n=1}^{\infty} \frac{(iA)^n}{n!} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \tag{2.35}$$

2.3 Maiani Parametrization

We may derive other forms of Kobayashi-Maskawa mixing matrix as well by taking rotation matrices in different orders or through rotations around different axis. Although the resultant parametrizations are different they are equivalent. One of the most frequently used parametrizations is Maiani parametrization[4] which is especially convenient for B meson physics since CP violating phase δ' is written as a common factor in $(U_M)_{23}$ element of the mixing matrix.

One may take $s_1 s_3 = s_\beta$, $s_1 c_3 = c_\beta s_\theta$, $(c_1 c_2 c_3 + s_2 s_3 e^{i\alpha}) = s_\gamma c_\beta e^{i\delta'}$ and parametrize (2.34) accordingly. If we take

$$s_1 s_3 = s_\beta, \quad s_1 c_3 = c_\beta s_\theta \tag{2.36}$$

then

$$c_1 = c_\beta c_\theta, \quad s_1 = \sqrt{(1 - {c_\beta}^2 {c_\theta}^2)}, \quad c_\beta = \sqrt{(1 - {s_1}^2 {s_3}^2)} \tag{2.37}$$

The equation

$$(c_1 c_2 s_3 + s_2 c_3 e^{i\delta}) = s_\gamma c_\beta e^{i\delta'} \tag{2.38}$$

together with the relations

$$\begin{aligned} |s_\gamma s_\beta e^{i\delta'}|^2 &= s_\gamma^2 c_\beta^2 = c_1^2 c_2^2 s_3^2 + s_2^2 c_3^2 + 2 c_1 c_2 c_3 s_2 s_3 \cos\delta \\ |-c_1 s_2 s_3 + c_2 c_3 e^{i\delta}|^2 &= c_1^2 s_2^2 s_3^2 + c_2^2 c_3^2 - 2 c_1 s_2 s_3 c_2 c_3 \cos\delta \end{aligned}$$

and

$$\begin{aligned} s_\gamma^2 {c_\beta}^2 + |-c_1 s_2 s_3 + c_2 c_3 e^{i\delta}|^2 &= {c_1}^2 {s_3}^2 + {c_3}^2 \\ &= 1 - {s_1}^2 {s_3}^2 = {c_\beta}^2 \end{aligned}$$

gives us

$$-c_1 s_2 s_3 + c_2 c_3 e^{i\delta} = c_\gamma c_\beta e^{i\beta'} \qquad (2.39)$$

We may take $\beta' = 0$ after a suitable definition of c_β and δ'

$$-c_1 s_2 s_3 + c_2 c_3 e^{i\delta} = c_\gamma c_\beta \qquad (2.40)$$

and we take

$$c_1 c_2 c_3 + s_2 s_3 e^{i\delta} = s_\gamma e^{i\delta'} \qquad (2.41)$$

By using Eqn.2.40 and Eqn.2.41 we obtain

$$e^{i\delta'} = \frac{c_\gamma}{s_\gamma} \frac{c_1 c_2 c_3 + s_2 s_3 e^{i\delta}}{-c_1 s_2 s_3 + c_2 s_2 s_3 + c_2 c_3 e^{i\delta}} \qquad (2.42)$$

From Eqn.2.36

$$s_1 = (1 - c_\beta c_\theta)^{\frac{1}{2}}, \qquad c_1 = c_\beta c_\theta \qquad (2.43)$$

using Eqns.2.41 and 2.42

$$c_3 = \frac{c_\beta s_\theta}{(1 - c_\beta c_\theta)^{\frac{1}{2}}}, \qquad s_3 = \frac{s_\beta}{(1 - c_\beta c_\theta)^{\frac{1}{2}}} \qquad (2.44)$$

using Eqns.2.43, 2.44, 2.40, 2.41, 2.42

$$s_2 = \left(\frac{s_\gamma^2 s_\theta^2 + s_\beta^2 c_\gamma^2 c_\theta^2 - 2 s_\beta s_\gamma c_\gamma s_\theta c_\theta cos\delta'}{1 - c_\theta^2 c_\beta^2} \right)^{\frac{1}{2}} \qquad (2.45)$$

$$c_2 = \left(\frac{s_\gamma^2 s_\theta^2 + s_\beta^2 s_\gamma^2 c_\gamma^2 c_\theta^2 + 2 s_\beta c_\gamma c_\gamma s_\theta c_\theta cos\delta'}{1 - c_\beta^2 c_\beta^2} \right)^{\frac{1}{2}} \qquad (2.46)$$

$$e^{i\delta} = \frac{s_\beta c_\theta}{s_\theta} \frac{c_2}{s_2} = \frac{s_\gamma}{s_\theta} \frac{s_\beta s_\gamma s_\theta + s_\theta s_\gamma e^{i\delta'}}{s_2 c_2} \qquad (2.47)$$

Replacing these expressions in the other terms of U_{KM} we get Maiani parametrization of Kobayashi-Maskawa mixing.

$$U = \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -s_\beta s_\gamma c_\theta e^{i\delta'} - s_\beta s_\gamma & c_\gamma c_\theta - s_\beta s_\gamma s_\theta e^{i\delta'} & s_\gamma c_\beta e^{i\delta'} \\ -s_\beta c_\gamma c_\theta + s_\gamma s_\theta e^{i\delta'} & s_\beta s_\theta c_\gamma - s_\gamma c_\theta e^{i\delta'} & c_\beta c_\gamma \end{pmatrix} \qquad (2.48)$$

2.4 Wolfenstein Parametrization

The general form of Kobayashi-Muskawa mixing matrix may be expressed as

$$U = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \tag{2.49}$$

U_{KM} and U_M are some specific forms of U. Another form of Eqn.2.49 is Wolfestein's parametrization. It depends on the observation that the off-diagonal elements of Eqn.2.49 are small while the diagonal elements are of the order 1 and we can choose an approximately unitary form where we can parametrize all elements in terms of $\lambda = | U_{us} | = 0.220 \pm 0.002$ [7]. It is easily noticed that $| U_{cb} | = 0.050 + (-)0.010$[8] $\propto \lambda^2$, $| U_{ub} | \leq 0.009$ [9] $\propto \lambda^3$. We may treat either one of this parametrization and Kobayashi-Muskawa parametrization as an approximation of the other

$$\cos\theta_1 = \sqrt{1-\lambda^2} \cong 1 - \frac{1}{2}\lambda^2 \qquad \text{for } \lambda \text{ small} \tag{2.50}$$

Both $cos\theta$ and $1-\frac{1}{2}\lambda^2$ values are consistent with data. So we may use $1-\frac{1}{2}\lambda^2$ instead of $cos\theta_1$ of KM as U_{ud}. The other terms are determined by the approximate unitarity requirement

$$U_W = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & \lambda^2 A \\ \lambda^2 A(1-\rho-i\eta) & -\lambda A & 1 \end{pmatrix} \tag{2.51}$$

where A, ρ, η are determined by experimental data.

2.5 Kobayashi-Maskawa Mixing for n Generations

In this section we shall outline some essential points of a scheme for n generations[10]. Although experiments exclude the possibility of the number of fermion generations being greater than three and our analysis for a specific set of models supports this derivation of the general aspects of such a scheme will be useful for understanding some general arguments especially in chapter 5. Kobayashi-Maskawa mixing matrix for n generations is a n×n unitary matrix as can be seen from Eqn.2.8. So it has n^2 parameters corresponding to generators of $U(n)$. The number of mixing angles in the case of n generations is $\frac{n(n-1)}{2}$ since one may change a $U(n)$ tranformation to a $O(n)$ tranformation by setting all its phases to zero. Then we conclude that the number of the mixing angles of $U(n)$ must be equall to the the number of the parameters of $O(n)$ which is $\frac{n(n-1)}{2}$. We may find the number of phases by subtacting the number of angles from the total number of parameters. The result is $\frac{n(n+1)}{2}$. We may absorb $2n-1$ of these phases in the redefinition of up and down quarks as in Eqns.2.28-2.32. Hence $\frac{n(n+1)}{2} - (2n-1) = \frac{(n-1)(n-2)}{2}$ phases remain as the physically observable phases. Then we may express Kobayashi-Maskawa mixing for n generations as a general unitary trasformation constructed from application of $\frac{n(n-1)}{2}$ successive orthogonal transformations together with $\frac{n(n-1)}{2}$ diagonal tranformations (whose entries are phases and some of these phases are removed). The order of the application of the orthogonal tranformations and the selection of the rotation axis depends on the convention one chooses.

Chapter 3

FLAVOR MIXING AND

KOBAYASHI-MASKAWA MIXING IN

GLASHOW-SALAM-WEINBERG MODEL

3.1 Connection between Kobayashi-Maskawa Mixing Matrix to Mass Matrices and Flavor Mixing Matrices

The general form of the Lagrangian for mass terms is given by

$$\mathcal{L}_{fm} = \bar{u}'_{iL} M^u_{ij} u'_{jR} + \bar{d}'_{iL} M^d_{ij} d'_{jR} + h.c. \tag{3.1}$$

where u'_i and d'_i are up and down quarks of n quark generations in the weak interaction basis. M^u andM^d are mass matrices for up and down quarks. By the term weak interaction basis we mean the states coupling to weak gauge bosons in a gauge invariant form i.e as $\bar{\Psi}' D \Psi'$. We determine the quark masses after diagonalizing M^u andM^d. In the case of a non-singular Hermittian mass matrix M(where M corresponds to either M^u or M^d) we can always diagonalize M through a unitary matrix V as

$$V M V^{\dagger} = m \tag{3.2}$$

where m is a diagonal matrix whose non-vanishing elements are the quark masses. The corresponding quark mass states are

$$u_L = V_u u'_L \qquad d_L = V_L d'_L \tag{3.3}$$

$$u_R = V_u^{\prime} u_R^{\prime} \qquad d_R = V_d^{\prime} d_R^{\prime} \tag{3.4}$$

Here $u^T = (u_1, u_2, ..., u_n)$,$d^T = (d_1, d_2, ..., d_n)$. Then we may rewrite the charged left-handed weak current in terms of the physical quarks.

$$J_{L\mu}^{\dagger} = \bar{u}_{iL}^{\prime} \gamma_\mu d_{iL}^{\prime} = \bar{u} V_u \gamma_\mu V_d^{\dagger} d = \bar{u} \gamma_\mu U_{KM} d \tag{3.5}$$

we see that the Kobayashi-Maskawa mixing matrix is

$$U_{kM} = V_u V_d^{\dagger} \tag{3.6}$$

For a positive definite mass matrix of general form (i.e either Hermitean or not) we may derive Kobayashi-Maskawa mixing matrix as follows: We notice that $(M^\dagger M)^\dagger = M^\dagger M$ for any mass matrix M. Hence there exist a unitary transformation V which diagonalizes $M^\dagger M$. Then we have the relation

$$M^\dagger M = V^\dagger m^2 V \tag{3.7}$$

Here m^2 is a diagonal matrix with $tr(m^2) > 0$. Now we define a diagonal matrix m so that $mm = m^2$ and $trm = (trm^2)^{\frac{1}{2}} > 0$. We may write

$$M = V^\dagger m (m^{-1} V M) \tag{3.8}$$

We call $m^{-1} V M = V'$. It is easy to show that V' is unitary. Thus we may say that any positive definite complex mass matrices may be diagonalized as follows

$$m^u = V_u M^u V_u^{\prime\dagger} \quad , \quad m^d = V_d M^d V_d^{\prime\dagger} \tag{3.9}$$

We may write the charged left-handed weak current as

$$J_{L\mu}^{\dagger} = \bar{u}_{iL}^{\prime} \gamma_\mu d_{iL}^{\prime} = \bar{u} \gamma_\mu V_u V_d^{\dagger} d \tag{3.10}$$

and we identify

$$U_{kM} = V_u^{\dagger} V_d \tag{3.11}$$

In this and following chapters we shall mainly deal with Yukawa Lagrangians to derive the mass matrices and the flavor mixing matrices through vacuum expectation values of the Higgs field(s).

3.2 Minimal Glashow-Salam-Weinberg Model and Flavor Mixing

The standard model (GSW)[11] is the first and the simplest complete theory of electroweak interactions. At the present time it accounts for all experimental data very well[12]. This tells us that GSW must be at least the effective low energy theory if it is not the fundamental one. Therefore the illumination of the form and the origin of Kobayashi-Maskawa mixing(KMM) is necessary. In fact the question of the origin of Yukawa couplings which give rise to KMM is one of the motivations for extensions of GSW. So we first examine KMM in GSW.

We take three SU(2) gauge fields $\vec{B}_\mu$, a U(1) gauge field a_μ and a set of fermions whose left handed components tranform as doublets and right handed components as singlets under SU(2) namely

$$\Psi_{Li} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_e \\ \tau^- \end{pmatrix}_L \tag{3.12}$$

(3.13)

with hypercharge Y=-1

$$l_{Ri} = e_R, \mu^-{}_R, \tau^-{}_R \tag{3.14}$$

with hypercharge Y=-2

$$Q_{Li} = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t' \\ b' \end{pmatrix}_L \tag{3.15}$$

(3.16)

with hypercharge Y=1/3

$$q_{Ri} = u'_R, d'_R, c'_R, s'_R, t'_R, b'_R \tag{3.17}$$

with hypercharge Y=2/3, and a scalar SU(2) doublet

$$\Phi = \begin{pmatrix} \phi^{\dagger} \\ \phi^{\circ} \end{pmatrix} \tag{3.18}$$

with Y=1 to costruct an $SU(2)\otimes U(1)$ invariant Lagrangian,

$$\mathcal{L} = \mathcal{L}_G + L_Y - V(\Phi) \tag{3.19}$$

where

$$\begin{aligned} \mathcal{L}_G &= -\frac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}^{\mu\nu} - \frac{1}{4}a_{\mu\nu}a^{\mu\nu} - i\bar{\Psi}_L D_\mu\gamma^\mu\Psi_L - i\bar{e}_R D_\mu\gamma^\mu e_R \\ &\quad -i\bar{Q}_L D_\mu\gamma^\mu Q_L - i\bar{u}_R D_\mu\gamma^\mu u_R - id_R D_\mu\gamma^\mu d_R - (D_\mu\Phi)^\dagger(D_\mu\Phi) \quad (3.20) \\ \mathcal{L}_Y &= h_{ij}\bar{\Psi}_{iL}\Phi e_{jR} + h^d_{ij}\bar{Q}_{iL}\Phi d_{jR} + h^u_{ij}\bar{Q}_{iL}\tilde{\Phi}u_{jR} + h.c. \quad (3.21) \end{aligned}$$

here $D_\mu = \partial_\mu + ig\vec{\tau}\cdot\vec{B}_\mu + ig'a_\mu$, it must be under stood that $\vec{\tau}\cdot\vec{B}q_R(l_R) = 0$ and $\tilde{\Phi} = i\tau_2(\Phi)^*$. Finally

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \tag{3.22}$$

with the minimum

$$|<\Phi>| = \frac{v}{\sqrt{2}} \qquad v = \frac{\mu}{\sqrt{\lambda}} \tag{3.23}$$

which is the vacuum expectation value of Φ in tree order. So we must define Φ about the true vacuum defined by Eqn. 3.21

$$\Phi = exp\{i\vec{\tau}.\vec{\theta}\}\begin{pmatrix} 0 \\ v+\chi \end{pmatrix} \tag{3.24}$$

χ 's are the true Higgs fields. After an SU(2) transformation $\vec{\theta}$ are absorbed into gauge fields as longitudinal components. The result is three massive and one massless(photon) gauge bosons. But we are interested in fermion masses and flavor mixing. So we only deal with $\mathcal{L}_Y$.

We have no evidence for flavor mixing in lepton sector. Hence we take $h_{ij} = h_{ik}\delta_{kj}$ and $h_{ii}v$ are charged lepton masses as neutrinos remain massless. If we assume any flavor mixing in lepton sector we may use a similiar procedure as for quarks. Therefore we may assume flavor mixing only in quark sector without lack of generality . After spontaneous symmetry breaking (SSB) we may write the quark sector of eqn.3.21 as

$$\mathcal{L}_Y = h^d_{ij} v \bar{d}'_{iL} d'_{jR} + h^u_{ij} v \bar{u}'_{iL} u'_{jR}$$
$$+ h^d_{ij} \bar{d}'_{iL} \chi d'_{jR} + h^u_{ij} \bar{u}'_{iL} \chi u'_{jR} + h.c. \qquad (3.25)$$

We determine the quark masses through diagonalization of the mass matrices

$$M^d = h^d_{ij} v \qquad M^u = h^u_{ij} v \qquad (3.26)$$

as

$$m^u = V_u M^u V_u^{\dagger'} \qquad m^d = V_d M^d V_d^{\dagger'} \qquad (3.27)$$

with the corresponding (physical) mass eigenstates

$$u_L = V_u u'_L \qquad d_L = V_d d'_L \qquad (3.28)$$

$$u_R = V'_u u'_R \qquad d_R = V'_d d'_R \qquad (3.29)$$

which we derived in the previous section. $V = V'$ in the special case of M^d and M^u being Hermitean. The Kobayashi-Maskawa mixing matrix is given by $U_{KM} = V_u^{\dagger} V_d$ as in Eqn.3.11.

3.3 Multi-Higgs Extensions of GSW and Flavor Changing Neutral Currents[13]

The Yukawa interaction for charge $-\frac{1}{3}$ quarks with n Higgs doublets in the context of GSW may written as (a similiar expression may be written for charge $\frac{2}{3}$ quarks)

$$\mathcal{L}_Y = h^1_{ij} \bar{Q}_{iL} \Phi_1 q_{jR} + h^2_{ij} \bar{Q}_{iL} \Phi_2 q_{jR} + \cdots + h^n_{ij} \bar{Q}_{iL} \Phi_n q_{jR} + h.c. \qquad (3.30)$$

The corresponding mass matrix is

$$M^{ij}_d = h^1_{ij} v_1 + h^2_{ij} v_2 + \cdots + h^n_{ij} v_n \qquad (3.31)$$

$v_1 v_2 \cdots v_n$ being $|<\Phi_1>|$, $|<\Phi_2>|, \cdots, |<\Phi_n>|$, respectively. We may see from the inspection of Eqn.3.30 and Eqn.3.31 that the diagonalization of M^d does not lead to the diagonalization of Yukawa couplings; $h^1_{ij}, h^2_{ij}, \cdots, h^n_{ij}$ in general. Simultaneous diagonalizability of the mass matrices and Yukawa coupling matrix are a special case where h^k_{ij} are diagonalized by the same transformation for every k=1,2,···,n. In the general case the resulting Yukawa interactions are not flavor diagonal; they contain, for example, $\bar{d}s\Phi$ interactions. Experimental data[6] indicates no evidence for flavor changing neutral currents . The most stringent upper bound comes from $K_L - K_S$ mass difference. We may derive the lower limit on the mass of the flavor changing neutral Higgs bosons by using this bound. $K_L - K_S$ mass difference is given by[14]

$$\delta m_K = m_L - m_S = \frac{1}{m_K} Re[< \bar{K}^\circ | \mathcal{H}^{\Delta S=2}_{eff}(0) | K^\circ >] \tag{3.32}$$

which corresponds to the diagram

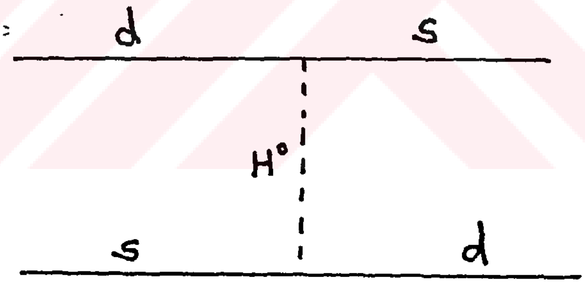

Figure 3.1: Neutral Higgs Contribution to Δm_K

The corresponding $\mathcal{L}$ is

$$\begin{aligned} \mathcal{L}^{\Delta S=2}_{Higgs} &= \sum_i (h^i_{sd})^2 (m_2)^{-2} (\bar{s}(1+\gamma_5)d)(\bar{s}(1+\gamma_5)d) \\ &= \sum_i \sqrt{2} G_F (M^i_{sd})^2 (m_H)_i^{-2} (\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d) \end{aligned} \tag{3.33}$$

Here $M^i_{sd} = h^i_{sd} v_i$. We dropped $\bar{s}d\bar{s}d$ term since it has a small contribution. The value of $< \bar{K}^0 | \bar{s}\gamma_5 d \bar{s}\gamma_5 d | K^0 >$ is determined by making the use of bag model results[14] to be

$$< \bar{K}^0 | \bar{s}\gamma_5 d \bar{s}\gamma_5 d | K^0 > = 8,5 \times 10^{-2} GeV^3 \tag{3.34}$$

We may set a mass scale $\bar{M}_{jk}$ for quark masses and m_H for Higgs masses as

$$\frac{\bar{M}_{jk}^2}{m_H^2} = \sum_i \frac{M_{jk}^{i^2}}{(m_H^2)_i} \tag{3.35}$$

If we set $\bar{M}_{sd}$ equal to the bottom quark mass, m_b, then

$$m_H^2 > \frac{2\sqrt{2} G_F m_b^2}{\Delta m_K} \times 8.5 \times 10^{-2} GeV^3 \tag{3.36}$$

If we set $\bar{M}_{sd}$ equal to $m_t = 100$GeV then

$$m_H^2 > \frac{2\sqrt{2} G_F (100GeV)^2}{\Delta m_K} \times 8.5 \times 10^{-2} GeV^3 > (10^4 TeV)^2 \tag{3.37}$$

where we have taken $\Delta m_K = 10^{-15} GeV$ and $G_F = 10^{-5} GeV$.

Chapter 4

FLAVOR MIXING IN $SU(2)_L \otimes SU(2)_R \otimes U(1)$

4.1 Motivation and Tests for $SU(2)_L \otimes SU(2)_R \otimes U(1)$

Although GSW accounts for all electroweak data very well it has some theoretical shortcomings[15]. Left-right symmetric[16] models eliminate some of them . The origin of parity violation is not evident in GSW since it was introduced by hand through the internal representations of the fermionic fields. This question becomes quite pertinent if to maintain lepton-quark symmetry one introduces the right handeded neutrinos into the standart model. Even if we assumed right handed fermions to be singlets under SU(2) it would cause other problems; it would cause neutrinos become massive since then lepton number conservation could not prevent neutrino to have mass and it would cause neutral currents conserve parity in contradiction with experiment. In $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models we can have naturally small neutrino masses (see Appendix C). Experiments put only an upper bound on neutrino masses[17]. All bounds are consistent with $SU(2)_L \otimes SU(2)_R \otimes U(1)$ provided that m_{W_R} is taken high enough. Direct kinematical limits on the electron neutrino comes from searches of spectral distortions at the endpoint energy in tritium beta decay. The strongest limit is due to Los Alamos group $m_{\nu_e} \leq 9.4$ eV[18]. This corresponds to $m_{W_R} \geq 25$GeV for $v_L = 0$GeV and $m_{W_R} \geq 10^5$GeV for $v_L = 1$GeV as shown in Appendix C. The direct kinematical limit on muon neutrino comes from the analysis of the endpoint spectrum of $\pi \to 2\mu$

and the limit on tau neutrino comes from the end point spectrum of decaying τ, produced in e^+e^- storage rings, into 5 charged pions. The results are $m_{\nu_\mu} \leq 250$ KeV[19] and $m_{\nu_\tau} \leq 35$ MeV[20]. We find the same lower bound on the right handed gauge boson masses from the bounds on masses of ν_μ and ν_τ if we take the Yukawa couplins accordingly. Thus we may say that $SU(2)_L \otimes SU(2)_R \otimes U(1)$ will be consistent with even smaller upperbounds on neutrino masses for higher m_{W_R}. Left-right symmetric models (e.g.$SU(2)_L \otimes SU(2)_R \otimes U(1)$) spontaneously break the parity so they elucidate the origin of parity violation. Moreover in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models hypercharge gains a physical significance as Y=B-L where B is baryon and L is lepton number. $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models are in agreement with data under suitable adjustment of parameters[17] as already shown in the case of neutrino mass. As we have shown in Appendix B the left-right symmetric model parameters which can directly be checked through experiments; ρ, the mixing angle of left handed and right handed gauge bosons η and the weak current at low energies, J_W, go to the GSW values in the limit of $m_{W_R} \rightarrow \infty$. Their deflection from the results of GSW are due to finite value of this mass, m_{W_R}. The deflections are very small but their effect may be observed in more precise experiments. Their nonobservance does not rule out the models; it only changes the lower bound on m_{W_R} and the upper bound on η as long as W_R remains unobserved. The search for right handed neutrinos (i.e. right handed currents) in 100 % stoped μ^+ polarized along the direction opposite to the direction of the product e^+ through the analysis of the energy spectrum near the endpoint gives a lower bound on the mass of W_R as discussed above and an upper bound on η (in GSW there exist no event at the end point of this process since charged weak currents are wholly left handed in GSW). More elaborate bounds are $m_{W_R} \geq 432$GeV for arbitrary η and $\eta \leq 0.35$ for $m_{W_R} \rightarrow \infty$ [17]. The most stringent bound comes from K_L-K_S mass difference. The K_L-K_S mass difference is almost wholly due to the $\Delta S=2$ box diagram where two gauge bosons are exchanged in the internal lines. In the case of

$SU(2)_L \otimes SU(2)_R \otimes U(1)$ right handed gauge bosons also contribute to the process. The leading contributions are due to two left handed gauge bosons and one left handed gauge boson, one right handed gauge boson. The result is $\Delta m_K = \Delta m_K^\circ [1 - 430r]$ where $r = \frac{W_L^2}{W_R^2}$. We see that unless $r \leq 2.3 \times 10^{-3}$ or $m_R \geq 1.6$TeV we would have wrong sign for Δm_K [22]. This bound is is independent of the mass of the neutrinos. Another bound on η is due to the analysis of y-distribution in deep inelastic scattering of antineutrinos off nuclei. The interaction of antineutrinos with valence quarks give terms proportional with $(1 - y^2)$ while the interaction of antineutrinos with sea quarks give terms proportional with y. If we include right handed neutrinos the coeficient of y increases. The analysis of y-distribution gives the upper bound $\eta < 0.1$ [23]. This result is model independent. Here we reviewed the main results on the bounds for the experimentaly observable parameters of $SU(2)_L \otimes SU(2)_R \otimes U(1)$. Briefly we may say that $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models are consistent with experimental data as long as we choose the parameters m_{W_R} and η in accordance with the experimental bounds. The left-right symmetric models have a drawback namely, the increase in the number of Higgs bosons[15]. However left-right symmetric models are viable models of electroweak interactions and we obtain interesting results in the final chapters in the context of a set of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models, we shall now review some of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models and the flavor mixing mechanism in them. First we analyze the minimal $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model.

4.2 Minimal $SU(2)_L \otimes SU(2)_R \otimes U(1)$ Model

We begin with the assumption of left-right symmetry (except in vacuum) as an extention of GSW. The immediate possibility is $SU(2)_L \otimes SU(2)_R \otimes U(1)$. Then fermionic sector consists of quark representations,

$$Q_{1L} = \begin{pmatrix} u' \\ d' \end{pmatrix}_L , \quad Q_{2L} = \begin{pmatrix} c' \\ s' \end{pmatrix}_L , \quad Q_{3L} = \begin{pmatrix} t' \\ b' \end{pmatrix}_L \tag{4.1}$$

$$Q_{1R} = \begin{pmatrix} u' \\ d' \end{pmatrix}_R , \quad Q_{2R} = \begin{pmatrix} c' \\ s' \end{pmatrix}_R , \quad Q_{3R} = \begin{pmatrix} t' \\ b' \end{pmatrix}_R \tag{4.2}$$

and the leptonic representations

$$\Psi_{1L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \quad \Psi_{2L} = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \quad \Psi_{3L} = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \tag{4.3}$$

$$\Psi_{1R} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_R , \quad \Psi_{2R} = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_R , \quad \Psi_{3R} = \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_R \tag{4.4}$$

The electric charge operator is defined as

$$Q_{el} = I_{3L} + I_{3R} + \frac{1}{2}Y \tag{4.5}$$

So the representation content of fermionic multiplets are

$$Q_{iL} \equiv (\frac{1}{2}, 0, \frac{1}{3}) \qquad Q_{iR} \equiv (0, \frac{1}{2}, \frac{1}{3}) \tag{4.6}$$

$$\Psi_{iL} \equiv (\frac{1}{2}, 0, -1) \qquad \Psi_{iR} \equiv (0, \frac{1}{2}, -1) \tag{4.7}$$

$$i = 1, 2, 3$$

To gauge the theory we must introduce three SU(2)$_L$,three SU(2)$_R$ and one U(1) gauge field in interaction with the fermion fields previously defined

$$\mathcal{L}_1 = -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} + \bar{Q}_{iL}DQ_{iL} + \bar{Q}_{iR}DQ_{iR} + \bar{\Psi}_{iL}D\Psi_{iL} + \bar{\Psi}_{iR}D\Psi_{iR} + \tag{4.8}$$

where

$$D_\mu = \partial_\mu + ig\vec{\tau}.\vec{B}^L_\mu + ig\vec{\sigma}\cdot\vec{B}^R_\mu + ig'a_\mu \tag{4.9}$$

$$\begin{aligned} f_{\mu\nu} &= [D_\mu, D_\nu] = \partial_\mu\vec{B}^L_\nu - \partial_\nu\vec{B}^L_\mu + g\vec{B}^L_\mu \times \vec{B}^L_\nu \\ &\quad +\partial_\mu\vec{B}^R_\nu - \partial_\nu\vec{B}^R_\mu + g\vec{B}^R_\mu \times \vec{B}^R_\nu + \partial_\mu a_\nu - \partial_\nu a_\mu \end{aligned} \tag{4.10}$$

We must add Higgs fields to break the symmetry spontaneously ; $\Delta_L \equiv (1,0,2)$, $\Delta_R \equiv (0,1,2)$ to suppress left and right handed currents in different magnitudes (i.e. spontaneous parity violation) and $\Phi \equiv (\frac{1}{2},\frac{1}{2},0)$ to give masses to fermions. We write the kinetic terms for Higgs fields and the coupling of Higgs fields to the other fields as

$$\begin{aligned} \mathcal{L}_2 &= tr[(D_\mu\Phi)^\dagger(D^\mu\Phi)] + [tr(D_\mu\Delta_L)]^\dagger[tr(D^\mu\Delta_L)] + [tr(D_\mu\Delta_R)]^\dagger[tr(D^\mu\Delta_R)] \\ &\quad + h'^1_{ij}\bar{\Psi}_{iL}\Phi\Psi_{jR} + h'^2_{ij}\bar{\Psi}_{iL}\tilde{\Phi}\Psi_{jR} + h^1_{ij}\bar{Q}_{iL}\Phi Q_{jR} + h^2_{ij}\bar{Q}_{iL}\tilde{\Phi}Q_{jR} \\ &\quad + h^m_{ij}(\Psi^T_{iL}c^{-1}\tau_2\tau_a\Psi_{jL}\Delta'^a_L + \Psi^T_{iR}c^{-1}\tau_2\tau_a\Psi_{jR}\Delta'^a_R) + h.c. \end{aligned} \tag{4.11}$$

where $\tilde{\Phi} = \tau_2\Phi^*\tau_2$ is the charge conjugate of Φ . One may induce the vacuum expectation values for Higgs fields to break the symmetry through the following Higgs potential

$$\begin{aligned} \mathcal{L}_3 &= \mu^2_{ij}tr(\Phi^\dagger_i\Phi_j) + \lambda_{ijkl}[tr(\Phi^\dagger_i\Phi_j)tr(\Phi^\dagger_k\Phi_l)] + \lambda'_{ijkl}tr[\Phi^\dagger_i\Phi_j\Phi^\dagger_k\Phi_l] \\ &\quad + \mu^2_2 tr(\Delta^\dagger_L\Delta_L + \Delta^\dagger_R\Delta_R) - \lambda_2[tr(\Delta^\dagger_L\Delta_L)]^2 - \lambda_3[tr(\Delta^\dagger_R\Delta_R)]^2 \end{aligned} \tag{4.12}$$

$$\begin{aligned} \Phi_1 &= \Phi, \quad \Phi_2 = \tilde{\Phi}, \quad \mu_{ij} = \mu_{ji}, \quad \lambda_{1212} = \lambda_{1221}, \\ &\quad \lambda_{iijk} = \lambda_{iikj}, \quad \lambda_{ijkk} = \lambda_{jikk}, \quad \lambda'_{ijkl} = \lambda'_{lijk} = \lambda'_{klij} = \lambda'_{jkli} \end{aligned} \tag{4.13}$$

with the minimum given by

$$< \Phi > = \begin{pmatrix} k & 0 \\ 0 & k'e^{i\delta'} \end{pmatrix} \quad \Phi = \begin{pmatrix} \chi^0_1 + k & \chi^+ \\ \chi^- & \chi^0_2 + k' \end{pmatrix}$$

$$< \Delta_{R(L)} > = \begin{pmatrix} 0 & 0 \\ v_{R(L)} & 0 \end{pmatrix} \Delta_{R(L)} = \begin{pmatrix} \Delta^+_{R(L)} & \Delta^{++}_{R(L)} \\ \Delta'^0_{R(L)} + v_{L(R)} & \Delta^+_{R(L)} \end{pmatrix}$$

The spontaneous symmetry breaking of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ leads to 1) six massive and one massless gauge bosons (see Appendix A) 2) masses for Higgs bosons which can be read from the lagrangian.

4.3 Flavor Mixing in the Left-Right Symmetric Models with One Φ Field

The fermion masses and flavor mixings result from only the coupling of $\Phi \equiv (\frac{1}{2}, \frac{1}{2}, 0)$ field(s) to fermions as can be seen from Eqn.4.11. Therefore the conclusions derived for the Yukawa Lagrangian of the minimal model is applicable to any $SU(2)_L \otimes SU(2)_R \otimes U(1)$ with only one Φ field. After the spontaneous symmetry breaking the Yukawa interaction part of the Lagrangian 4.8. takes the form

$$\mathcal{L}_Y = \mathcal{L}_m + \mathcal{L}_{YI} \tag{4.14}$$

$$\begin{aligned}
\mathcal{L}_m = \; & (h'^1_{ij} k + h'^2_{ij} k' e^{-i\delta'}) \bar{\nu}_{iL} \nu_{jR} + h.c. + h^m_{ij} (\nu_L \nu^T_{iL} c^{-1} \nu_{jL} + L \rightarrow R) \\
& + (h'^2_{ij} k + h'^1_{ij} k' e^{i\delta'}) \bar{e}_{iL} e_{jR} + h.c. \\
& + (h^1_{ij} k + h^2_{ij} k' e^{-i\delta'}) \bar{u}_{iL} u_{jR} + (h^2_{ij} k + h^1_{ij} k' e^{i\delta'}) \bar{d}_{iL} d_{jR} + h.c.
\end{aligned} \tag{4.15}$$

$$\begin{aligned}
\mathcal{L}_{YI} = \; & h'^1_{ij} \bar{\nu}_{iL} \chi^0_1 \nu_{jR} + h'^1_{ij} \bar{e}_{iL} \chi^0_2 e_{jR} \\
& + h'^2_{ij} \bar{\nu}_{iL} \chi^{0*}_2 \nu_{jR} + h'^2_{ij} \bar{e}_{iL} \chi^{0*}_1 e_{jR} \\
& + h'^m_{ij} (\nu^T_{iL} c^{-1} \nu_{jL} \Delta^*_L + \nu^T_{iR} c^{-1} \nu_{jR} \Delta^0_R) \\
& + h^1_{ij} \bar{d}_{iL} \chi^0_2 d_{jR} + h^1_{ij} \bar{u}_{iL} \chi^0_1 u_{iR} + h^2_{ij} \bar{d}_{iL} \chi^{0*}_1 d_{jR} \\
& + h^2_{ij} \bar{u}_{iL} \chi^{0*}_2 u_{jR} + h.c.
\end{aligned} \tag{4.16}$$

Since there is no evidence for flavor mixing in leptonic sector we assume $h'^{1(2)}_{ij} = h'^{1(2)}_{ii}$. We may read charged lepton masses directly from Eqn.4.16. The remarkable aspect of left-right symmetric models in lepton sector is that we get naturally small neutrino masses after diagonalizing Dirac and Majarona masses (see Appendix C) . In quark sector we diagonalize Eqn.5.1 to obtain quark masses and flavor mixing as in the standard model. Unlike minimal GSW there exist flavor changing neutral currents in the minimal $SU(2)_L \otimes SU(2)_R \otimes U(1)$.

This is due to the form of Yukawa coupling being , for example for d quarks,

$$\bar{d}_{iL}(h^1_{ij}\chi^0_2 + h^2_{ij}\chi^{0*}_1)d_{jR} + h.c. \tag{4.17}$$

whereas the mass term is

$$\bar{d}_{iL}(h^1_{ij}k + h^2_{ij}k'e^{i\delta_r})d_{jR} \tag{4.18}$$

The transformation diagonalizing the matrix $h^1_{ij}k + h^2_{ij}k'e^{i\delta_r}$ does not necessarily diagonalize both h^1_{ij} and h^2_{ij}. So in general there remains non-diagonal terms in Eqn.4.17 after mass diagonalization. Hence there exist flavor changing neutral currents even in minimal SU(2)$_L\otimes$ SU(2)$_R\otimes$ U(1) in general. The procedure to suppress these currents is similiar to the procedure in multi-Higgs extensions of the standard model.

Chapter 5

CONSTRAINT ON THE NUMBER OF QUARK GENERATIONS IN $SU(2)_L \otimes SU(2)_R \otimes U(1)$ MODELS WITH ONE Φ SCALAR

5.1 Preliminaries

The standart model does not predict the number of fermion generations. It works equally well with any number of generations[15]. Experimental results exclude the possiblity of light neutrino generations being more than three[24]. The results come from the experiments measuring decay width of Z bosons. One determines the number of light neutrino generations; N_ν by measuring the decay width corresponding to invisible decay modes and then setting this value equal to $N_\nu \Gamma_\nu$ where Γ_ν is the expected value of neutrino width. The latest experimental results for the number of light neutrino generations is due to L3 Collabration. Their result gives the number of light neutrino generations as $N_\nu = 3.01 \pm 0.11$. We may argue that if there exist three lepton generations the number of quark generations must be three as well due to absence of gauge anomaly. There are some studies requiring the number of fermion generations being three in superstring theories[25] and there are some others making the use of chirally confining dynamics[26]. An electro-weak model constraining the number of generations to three would be highly desirable. We shall show that under the assumption that there exist no zero mass quark, $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models of electroweak interactions with only one (1/2,1/2,0) Higgs field Φ necessarily require the number of quark generations to be three. First we shall study the case of the mass matrices for up and down quarks being non-Hermitean. Then we shall study the case where the mass matrices are Hermitean. Lastly we shall show that we can always find mass matrices consistent with our assumption for the number of generations being three.

5.2 The Case of Non-Hermitean Mass Matrices

The Yukawa Lagrangian for $SU(2)_L \otimes SU(2)_R \otimes U(1)$ with one Higgs field Φ for n generations may be written as (summation over dummy indices is implied)

$$\mathcal{L}_{YI} = h^1_{ij} \bar{Q}_{iL} \Phi Q_{jR} + h^2_{ij} \bar{Q}_{iL} \tilde{\Phi} Q_{jR} \tag{5.1}$$

where

$$Q_{iL} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_L \quad Q_{iR} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_R \quad i = 1,2....n \tag{5.2}$$

are (1/2,0,1/3) and (0,1/2,1/3) representations of $SU(2)_L \otimes SU(2)_R \otimes U(1)$. The Lagrangian in Eqn.5.1 induces the fermion mass terms through breaking the symmetry spontaneously (see Eqn.4.15)

$$\begin{aligned} \mathcal{L}_{fm} &= (h^1_{ij} k + h^2_{ij} k' e^{-i\delta'}) \bar{u}'_{iL} u'_{jR} \\ &\quad + (h^1_{ij} k' e^{i\delta'} + h^2_{ij} k) \bar{d}'_{iL} d'_{jR} \end{aligned} \tag{5.3}$$

After diagonalizing the mass matrices namely,

$$M^u_{ij} = (h^1_{ij} k + h^2_{ij} k' e^{-i\delta'}) \quad M^d_{ij} = (h^1_{ij} k' e^{i\delta'} + h^2_{ij} k) \tag{5.4}$$

we obtain

$$m^u = V^u_L M^u V^{u\dagger}_R \tag{5.5}$$

$$= \begin{pmatrix} m_1 & 0 & 0 & . & . & 0 \\ 0 & m_2 & 0 & . & . & 0 \\ 0 & 0 & m_3 & . & . & 0 \\ . & . & . & . & . & 0 \\ . & . & . & . & . & 0 \\ 0 & 0 & 0 & 0 & 0 & m_n \end{pmatrix} \tag{5.6}$$

$$m^d = V_L^d M^d V_R^{d\dagger} \tag{5.7}$$

$$= \begin{pmatrix} m'_1 & 0 & 0 & . & . & 0 \\ 0 & m'_2 & 0 & . & . & 0 \\ 0 & 0 & m'_3 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & m'_n \end{pmatrix} \tag{5.8}$$

$V_{L(R)}^{u(d)}$ are the diagonalization matrices, which are known as flavor mixing matrices, given by

$$(V_L^u)_{ij} = a_{ij} e^{i\alpha_j^i} \quad (V_R^u)_{ij} = b_{ij} e^{i\beta_j^i} \tag{5.9}$$

$$(V_L^d)_{ij} = a'_{ij} e^{i\alpha_j^{i\prime}} \quad (V_R^d)_{ij} = b'_{ij} e^{i\beta_j^{i\prime}} \tag{5.10}$$

nosummation over dummy indices

where a_{ij}, b_{ij}, a'_{ij}, b'_{ij} are the $\frac{n(n-1)}{2}$ angles and α_j^i, β_j^i, $\alpha_j^{i\prime}$, $\beta_j^{i\prime}$ are the $\frac{n(n+1)}{2}$ phases of the associated flavor mixing matrices (see section 2.5 and section 3.1). We may invert Eqns. 5.6 and 5.8 as

$$M_{ij}^u = (V_L^{u\dagger} m^u V_R^u)_{ij} \tag{5.11}$$

$$
\begin{aligned}
&= m_1 V^{u*}_{L1i} V^u_{R1j} \\
&\quad + m_2 V^{u*}_{L2i} V^u_{R2j} \\
&\quad + \ldots\ldots m_n V^{u*}_{Lni} V^u_{Rnj} \\
&= m_1 a_{i1} b_{1j} e^{i\alpha^1_{ij}} \\
&\quad + m_2 a_{i2} b_{2j} e^{i\alpha^2_{ij}} \\
&\quad + \ldots\ldots m_n a_{in} b_{nj} e^{i\alpha^n{}_{ij}} \qquad (5.12)
\end{aligned}
$$

$$
\begin{aligned}
M^d_{ij} &= (V^{d\dagger}_L m^d V^d_R)_{ij} \qquad (5.13) \\
&= m'_1 V^{d*}_{L1i} V^d_{R1j} \\
&\quad + m'_2 V^{d*}_{L2i} V^d_{R2J} \\
&\quad + \ldots\ldots m'_n V^{d*}_{Lni} V^d_{Rnj} \\
&= m'_1 a'_{i1} b'_{1j} e^{i\alpha'^1} \\
&\quad + m'_2 a'_{i2} b'_{2j} e^{i\alpha'^2_{ij}} \ldots\ldots \\
&\quad + m'_n a'_{in} b'_{nj} e^{i\alpha'^n_{ij}} \qquad (5.14)
\end{aligned}
$$

where m_k,m'_k are the eigenvalues(i.e. masses of the quarks) with $m_1 = m_u$, $m_2 = m_c$, $m_3 = m_t$ ....$m'_1 = m_d$, $m'_2 = m_s$, $m'_3 = m_b$ .... with the mass eigenstates

$$u_{kL} = V^u{}_{ki} u'{}_{iL} = a_{ki} e^{i\alpha^k}_i u'_{iL} \qquad (5.15)$$

$$d_{kL} = V^d_{ki} d'{}_{iL} = a'_{ki} e^{i\alpha'^k}_i d'_{iL} \qquad (5.16)$$

$$u_{kR} = V'_{ki} u'_{iR} = b_{ki} e^{i\beta^k_i} u'_{iR} \qquad (5.17)$$

$$d_{kR} = V'^u_{ki} d'_{kR} = b'_{ki} e^{i\beta'^k_i} d'_{iR} \qquad (5.18)$$

We observe that the quark mass correction graphs at the end of the chapter[27] require that

$$\alpha^k_i - \alpha'^k_j = \beta^k_i - \beta'^k_j \qquad (5.19)$$

since we have $U^{L*}_{ji} U^{R}_{ji}$ in the mass correction term with

$$\begin{aligned} U^{L*}_{ji} U^{R}_{ji} &= (V^{u}_{L} V^{d\dagger}_{L})^{*}_{ji} (V^{u}_{R} V^{d\dagger}_{R})_{ji} \\ &= V^{u*}_{Ljk} V^{d}_{Lik} V^{u}_{Rjr} V^{d*}_{Rir} \\ &= a_{jk} a'_{ik} b_{jr} b'_{ir} e^{-i(\alpha'^{k}_{i} - \alpha^{k}_{j})} e^{i(\beta'^{r}_{i} - \beta^{r}_{j})} \end{aligned} \tag{5.20}$$

where U^{L}_{ij} is the ij'th element of Kobayashi-Maskawa mixing matrix associated with the first vertex of the first graph and U^{R}_{ji} is the ij'th element of the right handed anolog of Kobayshi-Maskawa mixing matrix associated with the second vertex of the same graph. We see that in order to keep the fermion masses real we must cancel the phases in Eqn.5.20. We may take

$$\alpha'^{k}_{i} - \alpha^{k}_{j} = \beta'^{r}_{i} - \beta^{r}_{j} \tag{5.21}$$

For k=r Eqn.5.19 follows. We may express this relation more generally as Im(U^{L})= Im(U^{R}). This reduces the number of phases from $\frac{4n(n+1)}{2}$ to $\frac{3n(n+1)}{2}$. This conclusion is valid for all higher order mass correction graps (see Appendix D). The equations 5.12 and 5.14 are not independent since they are expressed through the same matrices ${h^{1}}_{ij}$ and h^{2}_{ij}. So we should solve them simultaneously. The two equations define $4n^2$ equations with $3\frac{n(n+1)}{2}$ phases instead of $4\frac{n(n+1)}{2}$ ones, $2n$ eigenvalues and $4\frac{n(n-1)}{2}$ mixing angles. Then we have $\frac{n(7n+3)}{2}$ unknowns and $4n^2$ equations. In order to determine all the parameters uniquely we must have

$$n(7n+3) = 8n^2 \tag{5.22}$$

Eqn.5.22 yields $n = 3$. When $n < 3$ the number of equations is less than the number of parameters. In other words the mixings can not be determined uniquely i.e. physical quarks can not be determined. When $n > 3$ the number of equations is more than the number of unknowns. Then either the extra equations are redundant i.e. the same quarks are repeated or the eigenvalues(i.e. masses) corresponding to the extra generations are zero which contradict with our assumption. So only $n = 3$ is possible in this framework.

We shall see if a similiar restriction applies to GSW. The Yukawa Lagrangian for GSW is the same as Eqn.5.1 as Φ changed from (1/2,1/2,0) representation of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ to (1/2,1) representation of $SU(2)_L \otimes U(1)$ with $< \Phi^T > = (0, v)$ and Q_R from (0,1/2,1/3) representation of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ to (0,2/3) representation of $SU(2) \otimes U(1)$. The related mass matrices are

$$M^u{}_{ij} = h^1{}_{ij} v, \quad M^d{}_{ij} = h^2{}_{ij} v \tag{5.23}$$

We see that Eqns.5.23 are wholly independent as M^u and M^d for $SU(2)_L \otimes SU(2)_R \otimes U(1)$ are not independent. This will be more obviously evident in the Hermitean case. A second reason for absence of any restriction on the number of generations in GSW is that the graphs given in Fig.2 do not exist in $SU(2)_L \otimes U(1)$ models because of the absence of right handed currents. Hence we do not have the constraint given in Eqn.5.19 in the case of $SU(2)_L \otimes U(1)$ models. Therefore no restriction arises on the number of fermion generations in GSW.

5.3 The Case of Hermitean Mass Matrices

Now we study the special case where the mass matrices are Hermitean. In this case we may write the mass matrices as

$$h^1_{ij} k + h^2_{ij} k' e^{-i\delta'} = m_1 a_{i1} a_{1j} e^{i\alpha^1_{ij}} + m_2 a_{i2} a_{2j} e^{i\alpha^2_{ij}} + \ldots\ldots m_n a_{in} a_{nj} e^{i\alpha^n_{ij}} \tag{5.24}$$

$$h^1_{ij} k' e^{i\delta'} + h^2_{ij} k = m'_1 a'_{i1} a'_{1j} e^{i\alpha'^1_{ij}} m'_2 a'_{i2} a'_{2j} e^{i\alpha'^2_{ij}} + \ldots\ldots\ldots m'_n a'_{in} a'_{nj} e^{i\alpha'^n_{ij}} \tag{5.25}$$

where $\alpha^k_{ij} = \alpha^k_i - \alpha^k_j$, $\alpha'^k_{ij} = \alpha'^k_i - \alpha'^k_j$. Eqn.5.19 is automatically satisfied since in the Hermitean case $\alpha^k_i = \beta^k_i$, $\alpha'^k_i = \beta'^k_i$ so it does not reduce the number of relevant phases. We have n^2 equations and n^2 unknowns. Hence we do not have any constraint on the number of quark generations for the case of Hermitean mass matrices. We develope another method for this case. Because V^u, V'^u, V^d, V'^d are Hermitean $\alpha^k_i = -\alpha^i_k$,

$\alpha_i'^k = \alpha_k'^i$, $\beta_i^k = -\beta_k^i$, $\beta_i'^k = \beta_k'^i$. Then we may rewrite Eqn.5.20 as

$$\alpha_k'^i - \alpha_k^j = \beta_r^i - \beta_r^j \qquad (5.26)$$

If we take $i = j$ Eqn.5.25 becomes

$$\alpha_k'^i - \alpha_k^i = \beta_r'^i - \beta_r^i \qquad (5.27)$$

We may rearrange Eqn.5.26 as

$$\alpha_k'^i - \beta_r'^i = \alpha_k^i - \beta_r^i \qquad (5.28)$$

In the Hermitean case Eqn.5.27 reduces to

$$\alpha_{kr}^i = \alpha_k^i - \alpha_r^i = \alpha_{kr}'^i = \alpha_k'^i - \alpha_r'^i \qquad (5.29)$$

Then we may say that we have a set of phases whose total number is $\frac{n(n+1)}{2}$, $\frac{2n(n-1)}{2}$ angels and $2n$ eigenvalues versus $2n^2$ equations i.e. we must set $\frac{3n(n+1)}{2} = 2n^2$ which again gives the result $n = 3$. This reasoning as well is not valid in the case of GSW since Eqn.5.28 does not apply to it.

5.4 Existence of Mass Matrices Consistent with the Required Form for n=3

Let us see that we can construct up and down quark mass matrices consistent with the form we require for $n = 3$ in the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models with one Φ field. We may write

$$kh_{ij}^2 e^{-i\delta'} = k \mid h_{ij}^2 \mid e^{i(\rho_{ij}' - \delta')} = k' \mid h_{ij}^1 \mid + g_{ij} e^{i\beta_{ij}} \quad \text{no summation} \qquad (5.30)$$

where ρ_{ij}' is the complex phase of h_{ij}^2. We have two equations and two unknowns so we can determine β_{ij} and g_{ij} uniquely. We may also write

$$k' \mid h_{ij}^1 \mid e^{i(\rho_{ij} + \delta')} = k_{ij} + h_{ij}' e^{i\rho_{ij}} \qquad (5.31)$$

$$k \mid h_{ij}^2 \mid e^{i\rho_{ij}'} = k_{ij}' + g_{ij}' e^{i\beta_{ij}} \qquad (5.32)$$

which fix h'_{ij}, f'_{ij} and g'_{ij}. Then the mass matrices may be expressed as

$$M^u_{ij} = f_{ij} + g_{ij}e^{i\beta_{ij}} + k \mid h^1_{ij} \mid e^{i\rho_{ij}} \tag{5.33}$$

$$M^d_{ij} = f'_{ij} + g'_{ij}e^{i\beta_{ij}} + h'_{ij}e^{i\rho_{ij}} \tag{5.34}$$

with $f'_{ij} = k'_{ij} + k_{ij}$, $f_{ij} = k' \mid h^1_{ij} \mid$. By taking $\mid h^1_{ij} \mid$, f_{ij}, g_{ij}, h_{ij}, g'_{ij}, ρ_{ij}, β_{ij} accordingly we may write

$$\begin{aligned} f_{ij} &= m_u a_{i1} a_{1j}, \quad g_{ij} = m_c a_{i2} a_{2j}, \quad k \mid h^1_{ij} \mid = m_t a_{i2} a_{2j} \\ f'_{ij} &= m_d a'_{i1} a'_{3i}, \quad g'_{ij} = m_s a'_{i2} a'_{2j}, \quad h'_{ij} = m_b a'_{i3} a'_{3j} \\ \beta_{ij} &= \alpha^2_i - \alpha^2_j = \alpha'^2_i - \alpha'^2_j \quad \rho_{ij} = \alpha^3_i - \alpha^3_j = \alpha'^3_i - \alpha'^3_j \end{aligned} \tag{5.35}$$

We notice that if we introduce more Φ's into the scheme there will be more parameters in the form of 5.33 in general. This means if we introduce more Φ's more generations exist in general. Then Eqns.5.23 and 5.24 suplemented with Eqn.5.28 exist for n=3 in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models with one Φ.

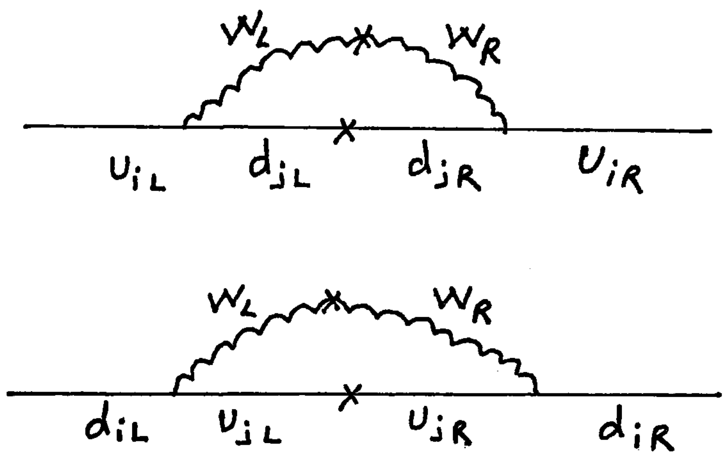

Figure 5.1: Second Order Quark Mass Correction Diagram

Chapter 6

CONCLUSION

After clarifying the basis and the form of Kobayashi-Maskawa mixing we pointed out the connection betweeen flavor mixing through fermion mass matrices and flavor mixing in left handed charged weak currents namely, Kobayashi-Maskawa mixing. Then we reviewed Yukawa interaction, fermion mass matrices, flavor mixing through fermion mass terms and Kobayashi- Maskawa mixing in the standard model of weak interactions and in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models of weak interactions. We used this review material to investigate existence of a constraint on the number of generations in the standard model and in $SU(2)_L \otimes SU(2)_R \otimes U(1)$. We found that under the assumption that there exist no zero mass quark , the mass matrices in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models with one fermion mass generating Higgs multiplet Φ restrict the number of quark generations to 3 on the other hand the standard model can not put such a constraint. This prediction is in agreement with experimental studies mentioned in the first section of Chapter 5. As we pointed out in Chapter 4 $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models are viable models of electroweak interactions. The prediction of the right number of quark generations is another point in favor of left-right symmetric models which the standard model can not provide.

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APPENDICES

Appendix A

GAUGE BOSON MASSES IN MINIMAL

$SU(2)_L \otimes SU(2)_R \otimes U(1)$

We start with

$$D_\mu = \partial_\mu + \frac{1}{2} ig\vec{\tau}.\vec{B}^L_\mu + \frac{1}{2} ig\vec{\sigma}\cdot \vec{B}^R_\mu + \frac{1}{2} ig' Y a_\mu \quad (A.1)$$

$$\Phi \equiv (\frac{1}{2},\frac{1}{2},0) \equiv \psi_L \bar{\psi}_R , \quad \psi_L \equiv (\frac{1}{2},0,1) \; \psi_R \equiv (0,\frac{1}{2},1) \quad (A.2)$$

$$\Delta_L \equiv (1,0,2) = \psi_L \bar{\psi}^c_L \quad \Delta_R \equiv (1,0,2) \equiv \psi_R \bar{\psi}^c_R \quad (A.3)$$

$$D_\mu \Phi = \partial_\mu \Phi + \frac{1}{2} ig(\vec{\tau}\cdot \vec{B}^L_\mu \Phi - \Phi \vec{\tau}\cdot \vec{B}^R_\mu) \quad (A.4)$$

$$D_\mu \Delta_{L(R)} = \partial_\mu \Delta_{L(R)} + \frac{1}{2} ig(\vec{\tau}.\vec{B}^{L(R)}_\mu \Delta_{L(R)} + \Delta_{L(R)} \vec{\sigma}\cdot \vec{B}^{L(R)}_\mu) + ig' B_\mu \quad (A.5)$$

The gauge invariant gauge bosons mass term is

$$\mathcal{L}_D = tr[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + tr[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)] + tr[D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)] + h.c. \quad (A.6)$$

From now on we will omit the Lorentz indices, μ's and lower the superindices, L and R for simplicity. The interaction Lagrangian for gauge boson masses corresponding to A.6 is

$$\mathcal{L}_{int} = \frac{1}{4}(g^2 tr[(\vec{\tau}\cdot \vec{B}^L_\mu \Phi - \Phi \vec{\tau}\cdot \vec{B}^R_\mu)^\dagger (\Phi \vec{\tau}\cdot \vec{B}^R_\mu - \vec{\tau}\cdot \vec{B}^L_\mu \Phi)]$$
$$+ tr[(g\vec{\tau}\cdot \vec{W}_L + g' a)\Delta_L]^\dagger [(g\vec{\tau}\cdot \vec{W}_L + g' a)\Delta_L] + L \to R \quad (A.7)$$

We make the use of

$$(\vec{\tau}\cdot \vec{B}^L_\mu < \Phi > - < \Phi > \vec{\tau}\cdot \vec{B}^R_\mu) =$$

$$\begin{pmatrix} W_L^3 & W_L^- \\ W_L^+ & -W_L^3 \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & k'e^{i\delta'} \end{pmatrix} - \begin{pmatrix} k & 0 \\ 0 & k'e^{i\delta'} \end{pmatrix}\begin{pmatrix} W_R^3 & W_R^- \\ W_R^+ & -W_R^3 \end{pmatrix}$$

$$= \begin{pmatrix} k(W_L^3 - W_R^3) & k'e^{i\delta'}W_L^- - kW_R^- \\ kW_R^+ - e^{i\delta'}W_R^+ & -k'e^{i\delta'}(W_L^3 - W_R^3) \end{pmatrix} \qquad \text{(A.8)}$$

$$(g\vec{\tau}\cdot\vec{W}_L + g'a) < \Delta_{L(R)} >= \begin{pmatrix} gW_{L(R)}^3 + g' & gW_{L(R)}^- \\ gW_R^+ & -gW_{L(R)} + g'a \end{pmatrix}\begin{pmatrix} 0 & 0 \\ v_{L(R)} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} gv_{L(R)}W_{L(R)}^- & 0 \\ v_{L(R)}(-gW_{L(R)}^3 + g'a) & 0 \end{pmatrix} \qquad \text{(A.9)}$$

then

$$tr[(\vec{\tau}\cdot\vec{B}_\mu^L < \Phi > - < \Phi > \vec{\tau}\cdot\vec{B}_\mu^R >)^\dagger(\vec{\tau}\cdot\vec{B}_\mu^L < \Phi > - < \Phi > \vec{\tau}\cdot\vec{B}_\mu^R)]$$

$$= k^2(W_L^3 - W_R^3)^2 + (kW_L^- - k'e^{-i\delta'}W_R^-)(kW_L^+ - k'e^{i\delta'}W_R^+) +$$

$$(k'e^{-i\delta'}W_L^+ - kW_R^+)(k'e^{i\delta'}W_L^- - kW_R^-) + k'^2(W_L^3 - W_R^3)^2 \qquad \text{(A.10)}$$

$$tr[(g\vec{\tau}\cdot\vec{W}_L + g'a) < \Delta_{L(R)} >]^\dagger[(g\vec{\tau}\cdot\vec{W}_L + g'a) < \Delta_{L(R)} >] =$$

$$g^2v_{L(R)}^2W_{L(R)}^+W_{L(R)}^- + v_{L(R)}^2(-gW_{L(R)}^3 + g'a)^2 \qquad \text{(A.11)}$$

By using $A.11$ and $A.10$ in $A.7$ we obtain the gauge boson mass Lagrangian

$$\begin{aligned} \mathcal{L}_{gm} = {} & \frac{1}{4}g^2(k^2 + k'^2)(W_L^3 - W_R^3)^2 + \frac{1}{4}g^2(k^2 + k'^2)W_L^+W_L^- + \\ & \frac{1}{4}g^2(k^2 + k'^2)W_R^+W_R^- - \frac{1}{2}g^2kk'\cos\delta'(W_R^+W_L^- + W_L^+W_R^-) \\ & + g^2v_L^2W_L^+W_L^- + g^2v_R^2W_R^+W_R^- + v_L^2(-gW_R^3 + g'a)^2 + h.c. \end{aligned} \qquad \text{(A.12)}$$

a) Charged Gauge Boson Masses: The mass matrix for charged gauge bosons may be written by making use of Eqn.A.12,

$$M_c = \begin{pmatrix} g^2[\frac{1}{4}(k^2 + k'^2) + v_L^2] & -\frac{1}{2}g^2kk'\cos\delta' \\ -\frac{1}{2}g^2kk'\cos\delta' & g^2[\frac{1}{4}(k^2 + k'^2) + v_R^2] \end{pmatrix} \qquad \text{(A.13)}$$

The eigenvalues of $A.13$ are the masses for W_L and W_R.They are the solutions of an equation of the form

$$\begin{vmatrix} m_{11}-\lambda & m_{12} \\ m_{12} & m_{22}-\lambda \end{vmatrix} = 0 \tag{A.14}$$

i.e.

$$\begin{aligned} \lambda_{1,2} &= \frac{m_{11}+m_{22}+(-)\sqrt{g^4(v_L^2-v_R^2)^2+g^4k^2k'^2\cos\delta'}}{2} \\ &\cong \frac{1}{4}g^2(k^2+k'^2+v_L^2+v_R^2)+(-)v_R^2) \end{aligned}$$

Then we have two sets of charged gauge bosons with masses

$$m_{W_1}^2 \cong \frac{1}{2}g^2v_R^2 \quad m_{W_2}^2 \cong \frac{1}{4}g^2v_L^2 \tag{A.15}$$

The eigenstates may be determined after diagonalization as

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \cos\eta & e^{i\delta'}\sin\eta \\ -e^{i\delta'}\sin\eta & \cos\eta \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}$$

$$W_1 = \cos\eta W_L + e^{i\delta'}\sin\eta W_R \tag{A.16}$$

$$W_2 = \cos\eta W_R - e^{-i\delta'}\sin\eta W_L \tag{A.17}$$

where η is determined from the off-diagonal elements of the diagonalized matrix,

$$-\sin\eta\cos\eta[\frac{1}{4}g^2(k^2+k\prime 2)+g^2v_L^2]+\frac{1}{2}g^2kk'\sin^2\eta\cos\delta' -$$
$$\frac{1}{2}g^2\cos^2\eta kk'\cos\delta'+g^2\sin\eta\cos\eta[\frac{1}{4}(k^2+k'^2)+v_R^2]=0$$

$$\tan\eta = \frac{kk'\cos\delta'}{v_R^2-v_L^2} \tag{A.18}$$

b)Neutral Gauge Boson Masses: The mass matrix for neutral gauge bosons is

$$M_0 = \frac{1}{4}\begin{pmatrix} g^2(k^2+k'^2)+4g^2v_L^2 & -g^2(k^2+k'^2) & -4gg'v_L^2 \\ -g^2(k^2+k'^2) & g^2(k^2+k'^2)+4g^2v_R^2 & -4gg'v_R^2 \\ -4gg'v_L^2 & -4gg'v_R^2 & 4g'^2(v_L^2+v_R^2) \end{pmatrix} \tag{A.19}$$

The eigenvalues of the equation A.19 are the masses of the neutral gauge bosons. The equation $|M_0-\lambda I|=0$ results in

$$-\lambda^3+b\lambda^2+c\lambda+d=0 \tag{A.20}$$

where

$$
\begin{aligned}
b &= \frac{1}{2}g^2(k^2+k'^2)+(g^2+g'^2)(v_L^2+v_R^2) \\
-4c &= [g^2(k^2+k'^2)+4g^2v_L^2] \\
&\quad \times[\frac{1}{4}g^2(k^2+k'^2)+g^2v_R^2] \\
&\quad +[g^2(k^2+k'^2)+4g^2v_L^2]g'^2(v_L^2+v_R^2) \\
&\quad +[g^2(k^2+k'^2)+4g^2v_R^2]g'^2(v_L^2+v_R^2) \\
&\quad +\frac{1}{4}g^4(k^2+k'^2)^2-4g^2g'^2v_L^4-4g^2g'^2v_R^4 \qquad (A.21) \\
d &= [\frac{1}{4}g^2(k^2+k'^2)+g^2v_L^2] \\
&\quad \times[\frac{1}{4}g^2(k^2+k'^2)+g^2v_R^2]g'^2(v_L^2+v_R^2) \\
&\quad -[\frac{1}{4}g^2(k^2+k'^2)+g^2v_L^2]g^2g'^2v_R^4 \\
&\quad +\frac{1}{4}g^2(k^2+k'^2)[-\frac{1}{4}g^2(k^2+k'^2) \\
&\quad \times g'^2(v_L^2+v_R^2)-g^2g'^2(v_L^2v_R^2] \\
&\quad -gg'v_L^2[\frac{1}{4}g^3g'(k^2+k'^2)v_R^2 \\
&\quad +gg'v_L^2[\frac{1}{4}(k^2+k'^2)+g^2v_R^2]]=0 \qquad (A.22)
\end{aligned}
$$

Since $d=0$ one of the eigenvalues is zero which corresponds to massless photon field. The masses of the other two gauge bosons is determined through

$$
\begin{aligned}
-\lambda^2+b\lambda+c &= 0 \\
b^2+4c &= -\frac{1}{4}g^4(k^2+k'^2)^2+(g^2+g'^2)^2(v_L^2+v_R^2)^2- \\
&\quad g^2g'^2(k^2+k'^2)(v_L^2+v_R^2)-4g^2(g^2+2g'^2v_L^2v_R^2 \\
&\quad m_{1,2}^2=\frac{1}{4}g^2(k^2+k'^2)+(g^2+g'^2)(v_L^2+v_R^2) \\
&\quad +(-)\frac{1}{2}\sqrt{(g^2+g'^2)(v_L^2+v_R^2)-\frac{g^2g'^2}{2(g^2+g'^2)}(k^2+k'^2)} \qquad (A.23)
\end{aligned}
$$

We may approximate the inside of the square root bracket since $(k^2+k'^2) << (v_L^2+v_R^2)$. The first order approximation gives

$$
m_1^2 = \frac{1}{4}g^2(k^2+k'^2)+\frac{g^2g'^2}{2(g^2+g'^2)}(k^2+k'^2)
$$

$$= \frac{1}{4}g^2(\frac{g^2+2g'^2}{g^2+g'^2})(k^2+k'^2) \quad (A.24)$$

$$m_2^2 = \frac{1}{4}g^2(\frac{g^2}{g^2+g'^2})(k^2+k'^2)+2(g^2+g'^2)(v_L^2+v_R^2) \quad (A.25)$$

$$(A.26)$$

We may write m_1 and m_2 in a more convinient way by defining weak mixing angle θ as $\sin\theta = \frac{g'}{\sqrt{g^2+2g'^2}}$. Then

$$m_{Z_1}^2 = \frac{m_{W_1}^2}{\cos^2\theta} \quad (A.27)$$

$$m_{Z_2}^2 = \frac{1}{4}(\frac{\cos 2\theta}{\cos^2\theta})m_{W_1}^2 + \frac{\cos^2\theta}{\cos 2\theta}m_{W_2}^2 \quad (A.28)$$

We see that in the first order approximation the relation between the masses of the usual charged and neutral weak gauge is the as GSW which is $\frac{m_{Z_1}}{m_{W_1}} = \cos\theta$. If we include the higher order contributions for m_{W_1} and m_{Z_1} we get

$$m_{W_1}^2 = \frac{1}{4}g^2(k^2+k'^2)(1+\frac{2v_L^2}{(k^2+k'^2)}-\frac{2k^2k'^2}{(k^2+k'^2)}+.....) \quad (A.29)$$

$$m_{Z_1}^2 = \frac{\frac{1}{4}g^2(k^2+k'^2)}{\cos^2\theta}[1+\frac{4v_L^2}{(k^2+k'^2)}-\frac{1}{4}(\frac{\cos^2 2\theta}{\cos^4\theta})\frac{(k^2+k'^2)}{v_R^2}+.......] \quad (A.30)$$

We see that the higher order contributions are highly suppressed since $v_L \ll (\sqrt{k^2+k'^2}) \ll v_R$.

The neutral gauge boson eigenststes may be determined by constructing the diagonalization matrix by the use of the eigenvectors of $A.19$. The result is (in the limit $k^2+k'^2 << v_R^2$)

$$A = \sin\theta(W_L^3+W_R^3)+\sqrt{\cos 2\theta}a \quad (A.31)$$

$$Z_1 = \cos\theta W_L^3 - \sin\theta\tan\theta W_R^3 - \sqrt{\cos 2\theta}\tan\theta a \quad (A.32)$$

$$Z_2 = \frac{\sqrt{\cos 2\theta}}{\cos\theta}W_R^3 - \tan\theta a \quad (A.33)$$

Appendix B

LOW ENERGY LIMIT OF

$SU(2)_L \otimes SU(2)_R \otimes U(1)$

In this appendix we shall show that low energy limit of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ almost reduces to that of GSW. From Appendix B we infer that $\sin\theta = \frac{g'}{\sqrt{g^2+2g'^2}}$ in this model as $\sin\theta = \frac{g'}{\sqrt{g^2+g'^2}}$ in GSW. We notice that if we eliminate all g' by the variable $g'' = \frac{gg'}{\sqrt{g^2+g'^2}}$ in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ the electroweak currents are the same as of GSW up to a high degree of approximation at the low energy limit. This result is not surprising. In $SU(2)_L \otimes SU(2)_R \otimes U(1)$ we first break $SU(2)_R \otimes U(1)$ down to $U(1)'$ through Higgs field Δ_R so that the generator of $U(1)'$ is Y' with

$$\frac{1}{2}Y' = I_3 + \frac{1}{2}Y \tag{B.1}$$

The coupling constant of $U(1)'$ is g'' where

$$\frac{1}{g''^2} = \frac{1}{g^2} + \frac{1}{g'^2} = \frac{\sqrt{g^2+g'^2}}{gg'} \tag{B.2}$$

The next step of symmetry breaking is the same as that of $SU(2)_L \otimes U(1)$ while $U(1)$ is replaced by $U(1)'$.

In order to see the situation more explicitly first we must derive the electroweak currents in $SU(2)_L \otimes SU(2)_R \otimes U(1)$. The fermion-gauge boson interaction Lagrangian of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ is given by

$$\mathcal{L}_{gf} = g\bar{\Psi}_L\gamma\vec{\tau}.\vec{W}_L\Psi_L + g\bar{\Psi}_R\gamma\vec{\tau}.\vec{W}_R\Psi_R + g'\frac{Y}{2}a\bar{\Psi}\gamma\Psi \tag{B.3}$$

where $\Psi^T = (\psi_1, \psi_2)_{L(R)}$ are $SU(2)_{L(R)}$ doublets and we ommited Lorentz indices as before. From $A.17$ we see that $W_L^{+(-)} = W_1^{+(-)}$ $W_R^{+(-)} = W_2^{+(-)}$ since $k^2 + k'^2 << v_R^2$. So the charged weak current of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ is almost the same as that of GSW as easily seen. The neutral weak current of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ may be written by using $B.3$ as

$$J^0 = g\bar{\Psi}_L\gamma\frac{1}{2}\tau_3^L\Psi_L W_L^3 + g\bar{\Psi}_L\gamma\tau_3^R\Psi_R W_R^3 + g'\bar{\Psi}_L\frac{Y}{2}\Psi_L a + g'\bar{\Psi}_R\frac{Y}{2}\Psi_R a \tag{B.4}$$

After using $A.31$-$A.33$ we express $W_{L(R)}^3$ and a in terms of the mass eigenstates

$$W_L^3 = \cos\theta Z_1 + \sin\theta A \tag{B.5}$$

$$W_R^3 = [\frac{\cos\theta}{\sqrt{\cos 2\theta}} - \frac{\sin^2\theta}{\sqrt{\cos 2\theta}}(\cos\theta + \sin\theta\tan\theta)]Z_2 + \sin\theta A - \sin\theta\tan\theta Z_1 \tag{B.6}$$

$$a = [\sqrt{\cos 2\theta}(A - \tan\theta Z_1) - \sin\theta(\cos\theta + \sin\theta\tan\theta)Z_2] \tag{B.7}$$

Then we may write $B.4$ as

$$\begin{aligned} J^0 = {} & g\bar{\Psi}_L\frac{1}{2}\tau_3^L\Psi_L(\cos\theta Z_1 + \sin\theta A) + g\bar{\Psi}_R\frac{1}{2}\tau_3^R\Psi_R\left[\frac{\cos\theta}{\sqrt{\cos 2\theta}}\right. \\ & \left. - \frac{\sin^2\theta}{\sqrt{\cos 2\theta}}(\cos\theta + \sin\theta\tan\theta)\right] Z_2 + \sin\theta A - \sin\theta\tan\theta Z_1 + \\ & g'\bar{\Psi}\frac{1}{2}Y\Psi\left[\sqrt{\cos 2\theta}(A - \tan\theta Z_1) - \sin\theta(\cos\theta + \sin\theta\tan\theta)Z_2\right] \end{aligned} \tag{B.8}$$

For we may take only Z_1 and A since the contribution of Z_2 is highly suppressed because of its large mass

$$\begin{aligned} J^0 = {} & [g\bar{\Psi}_L\frac{1}{2}\tau_3^L\Psi_L\cos\theta - g\bar{\Psi}_R\frac{1}{2}\tau_3^R\Psi_R\sin\theta\tan\theta - \\ & g'\bar{\Psi}\frac{1}{2}Y\Psi\sqrt{\cos 2\theta}\tan\theta]Z_1 + [g\bar{\Psi}_L\frac{1}{2}\tau_3^L\Psi_L\sin\theta + \\ & g\bar{\Psi}_R\frac{1}{2}\tau_3^R\Psi_R\sin\theta + g'Y\bar{\Psi}\frac{1}{2}\Psi\sqrt{\cos\theta}]A \end{aligned} \tag{B.9}$$

We make the folowing idetifications (as in GSW)

$$\sin\theta = \frac{g'}{\sqrt{g^2 + 2g'^2}} \quad g_Z = \frac{g}{\cos\theta} \quad e = \frac{gg'}{\sqrt{g^2 + 2g'^2}} \tag{B.10}$$

$$
\begin{aligned}
J^0 &= g_Z[\bar\Psi_L \frac{1}{2}\tau_3^L - \\
&\quad (\bar\Psi_R \frac{1}{2}\Psi_R + \bar\Psi_L \frac{1}{2}\tau_3^L \Psi_L)\sin^2\theta - \bar\Psi\frac{1}{2}Y\Psi\sin^2\theta]Z_1 \\
&\quad + e[\bar\Psi(\frac{1}{2}\tau_3^L + \frac{1}{2}\tau_3^R + \frac{1}{2}Y)\Psi]A \\
&= g_Z[\bar\Psi_L \frac{1}{2}\tau_3^L \Psi_L - Q sin^2\theta\bar\Psi\Psi]Z_1 + eQ\bar\Psi\Psi A \qquad \text{(B.11)}
\end{aligned}
$$

We arrived the expression for electro-weak current in GSW at the symmetry breaking scale of $SU(2)_L \otimes U(1)$. In other words for $v_R \to \infty$ J^0 of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ is just the electroweak current of GSW. This conclusion is independent of the representation of Higgses. The reason is that as $v_R \to \infty$ the mass eigenstates of gauge bosons are independent of Higgs sector as can be seen from Eqns.(A.17, A.31-A.33) although gauge boson masses carry information on Higgs sector as can be seen from Eqns.(A.15,A.27, A.28). Hovewer the conclusion $\rho = 1$ of GSW is maintained as $v_R \to \infty$ since

$$
\rho = \frac{tr[(I_\phi)^2 < \phi >< \phi >^\dagger + (I_\Delta)^2 < \Delta_L >< \Delta_L >^\dagger]}{2tr[(I_{\phi 3})^2 < \phi >< \phi >^\dagger + (I_{\Delta 3})^2 < \Delta_L >< \Delta_L >^\dagger]} \qquad \text{(B.12)}
$$

where I's are isospin representation of the corresponding Higgs fields with $I = (\vec{I})^2 - (I_3)^2$. We see that $\rho = 1$ in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ as well since ϕ are doublets under $SU(2)_L$ and $v_L^2 << (k^2 + k'^2$. Another parameter which can be checked through experiments is the mixing angle of left handed and right handed charged gauge bosons, η which was defined in A.17. We may relate it to the mass of the right handed gauge bosons through the observation

$$
k^2 + k'^2 >> kk' \qquad \text{(B.13)}
$$

which can be reexpressed in general as

$$
tr[(\tau_i)^2 < \phi >< \phi >^\dagger] >> | tr(\tau_- < \phi >^\dagger \tau_+ < \phi >) | \qquad \text{(B.14)}
$$

This means that

$$
m_{LL}^2 >> m_{LR}^2 \qquad \text{(B.15)}
$$

where m_{LL} and m_{LR} are the terms corresponding to the coefficients of $W_L^+W_L^-$ and $W_L^+W_R^-$ respectively in the charged boson mass matrix. This, in turn, implies

$$\mid \eta \mid \cong \frac{m_{LR}^2}{m_{W_R}^2} \leq (\frac{m_{W_L}^2}{m_{W_R}})^2 \qquad \text{(B.16)}$$

We again conclude that as $m_{W_R} \rightarrow \infty$ $\eta \rightarrow 0$ which is the situation in GSW.

Appendix C

NATURALLY SMALL NEUTRINO MASSES

IN $SU(2)_L \otimes SU(2)_R \otimes U(1)$

The neutrino mass Lagrangian is given in 3rd section of chapter 4 as

$$\mathcal{L}_m^{\nu} = (h_{ij}^{l1} k + h_{ij}^{l2} k' e^{-i\delta'}) \bar{\nu}_{iL} \nu_{jR} + h.c. + h_{ij}^{m} (\nu_{iL}^T c^{-1} \nu_{jL} v_L + \nu_{iR}^T c^{-1} \nu_{jR} v_R) \qquad (C.1)$$

The neutrino masses are determined after diagonalizing the the neutrino mass matrices whose entries are the Dirac and Majarona masses of neutrino which are given in C.1. For the ith neutrino,

$$M_{\nu}^{i} = \begin{pmatrix} h_{ii}^{m} v_L & (h_{ii}^{l1} k + h_{ii}^{l2} k' e^{-i\delta'}) \\ (h_{ii}^{l1} k + h_{ii}^{l2} k' e^{i\delta'}) & h_{ii}^{m} v_R \end{pmatrix} \qquad (C.2)$$

We assume $k << k'$

$$m_{1,2} = \frac{h_{ii}^{m}(v_L + v_R) + (-)\sqrt{h_{ii}^{m2}(v_L + v_R)^2 + 4 \mid h_{ii}^{l1} k + h_{ii}^{l2} k' e^{-i\delta'} \mid^2}}{2}$$

$$m_1 \cong h_{ii}^{m}(v_L + v_R) \qquad m_2 \cong \frac{(h_{ii}^{l1} k)^2}{h_i^{m}(v_L + v_R)} \cong \frac{g m_e^2}{h_{ii}^{m} m_{W_R}} \qquad (C.3)$$

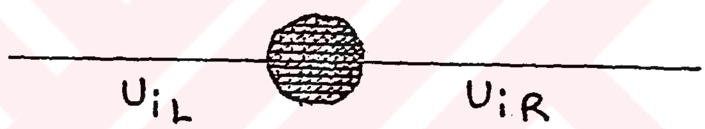
m_1 is a very heavy neutrino and m_2 is the usual neutrino mass whose smallness is related to the heaviness of m_{W_R}.

Appendix D

VALIDITY OF THE CONSTRAINT

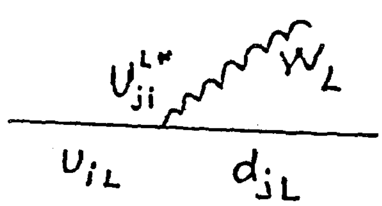
IN HIGHER ORDERS

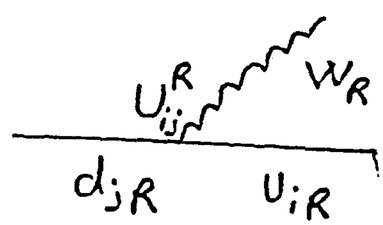
The general Feynman graph for up quark masses (a similiar graph may be drawn for down quarks) may be drawn as

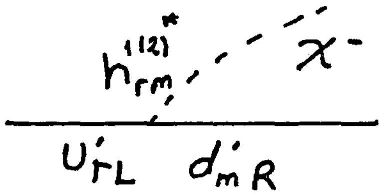


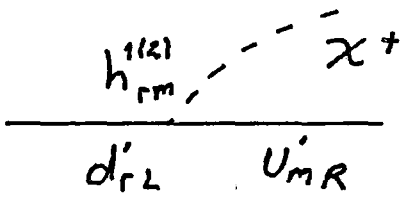
where all the mass terms are included in the blob. We require the corresponding terms to be real in order to keep quark masses real.

Flavor mixing angles and phases are introduced only through left handed and right handed charged currents i.e. through Kobayashi-Maskawa mixing matrix U^L, its right handed anologous matrix U^R, $(V^{d*}_{Rjk} V^{u}_{Lik})$, $(V^{u*}_{Ril} V^{d}_{Ljl})$. In other words through the vertices

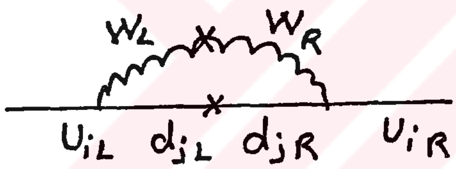


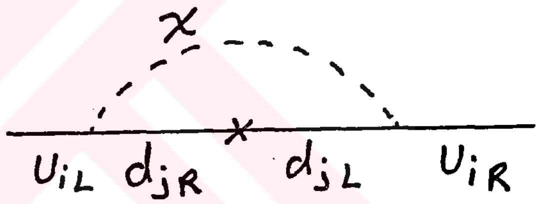


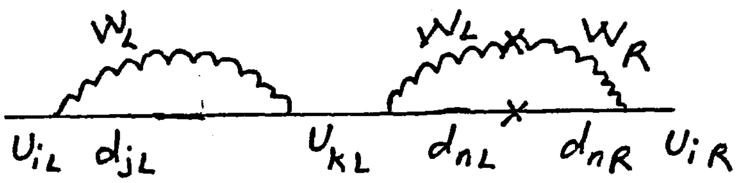


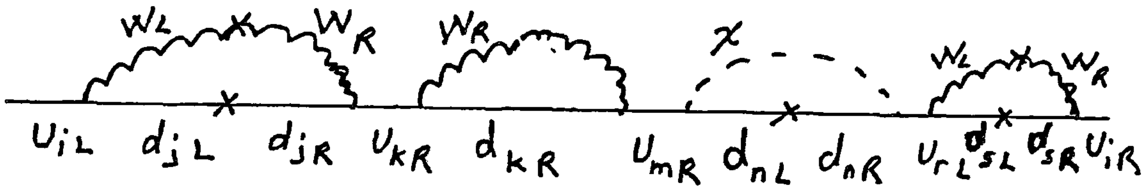


The only graphs we may construct out of these vertices for quark masses (which must have two external lines ending with u_{iL} and u_{iR}) must be of the form

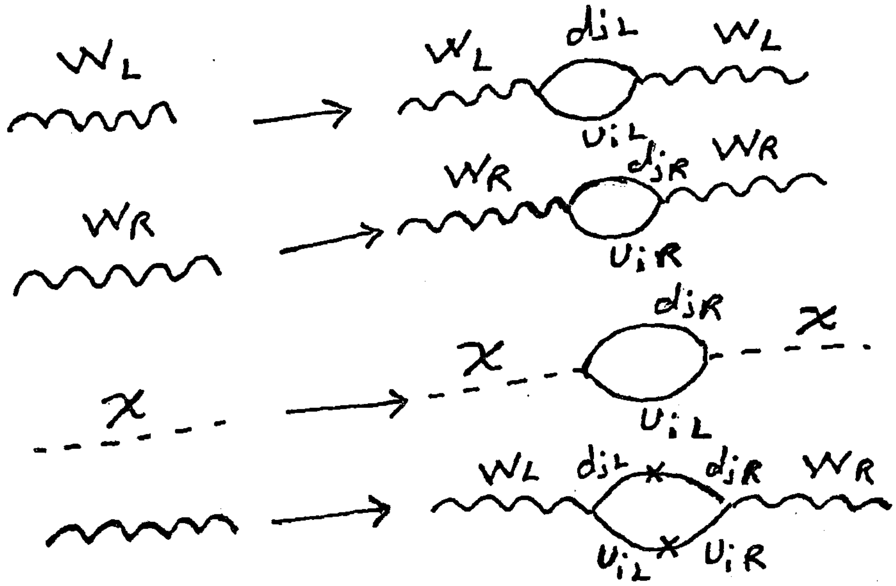




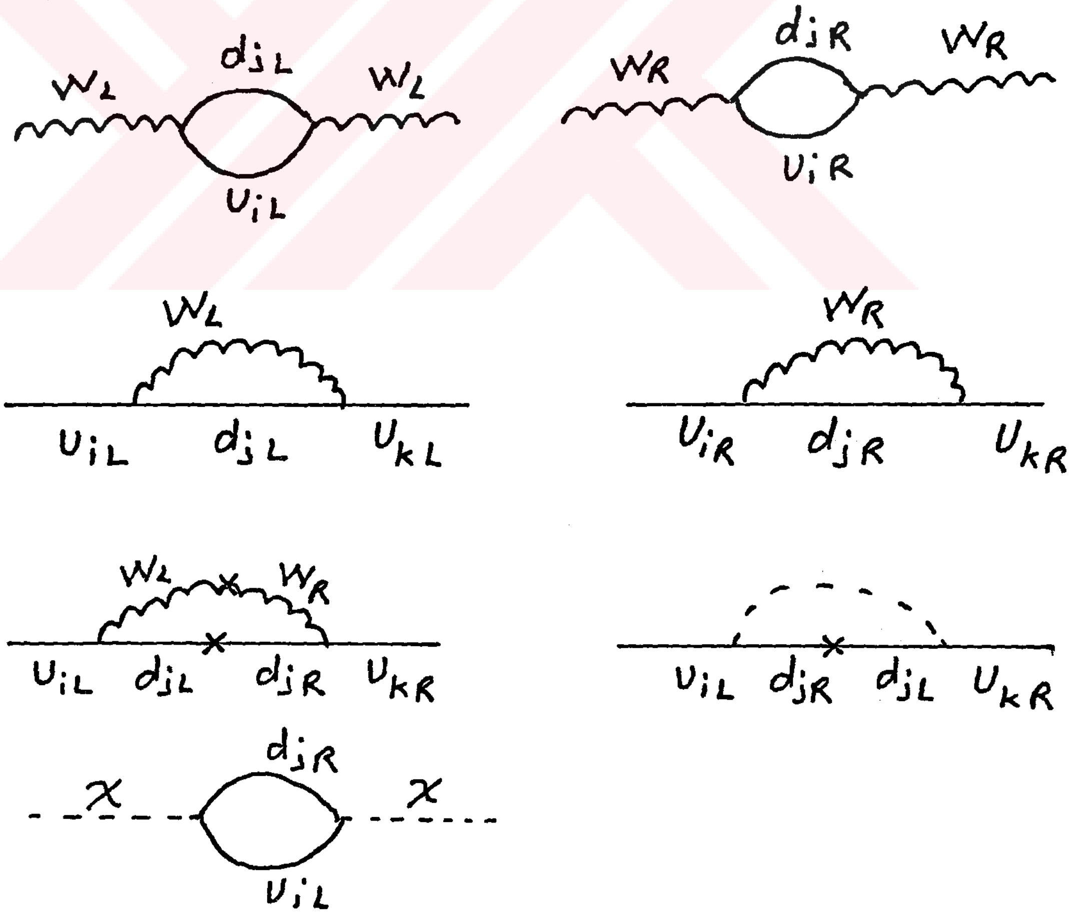




and the graphs with

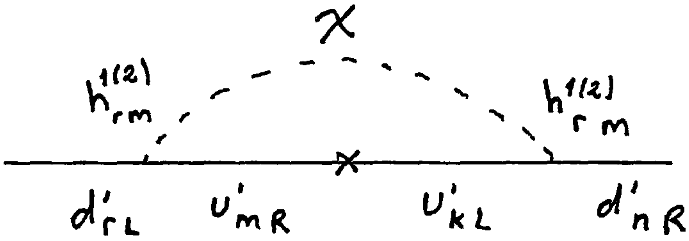


So the only subgraphs which flavor mixing contributes are



The flavor mixing terms for the first seven subgraphs are of the form $U_{ij}^{L*}U_{ij}^{L}$, $U_{ij}^{R*}U_{ij}^{R}$, $(V_{Rjk}^{d*}V_{Lik}^{u})^*(V_{Rjk}^{d*}V_{Lik}^{u})$, $(V_{Rir}^{u*}V_{Ljr}^{d})^*(V_{Rir}^{u*}V_{Ljr}^{d})$ which do not have imaginary parts, so no phases. Only the eighth and nineth subgraphs give the constraint in chapter 5 in order to keep quark masses real. When each such subgraph is repetead in the main graph the relation Im$[U^L(\alpha,\alpha')]$ =Im$[U^R(\beta,\beta')]$ or Im$[V_R^{d\dagger}V_L^{u}(\beta',\alpha)]$ =Im$[V_R^{u\dagger}V_L^{d}(\beta,\alpha')]$ is repeated. Thus one set of phases reduced. For example in the graph

the releation repeated twice as $\alpha_i^k - \alpha_j^k = \beta_i^r - \beta_j^r$ and $\alpha_i^k - \alpha_l^k = \beta_l^r - \beta_l^r$. and once as Im$[V_R^{d\dagger}V_L^{u}(\beta',\alpha)]$ =Im$[V_R^{u\dagger}V_L^{d}(\beta,\alpha')]$. Therefore we conclude that higher order contributions to quark masses can not change the relation given in Eqn.5.20. In fact the relation imposed by Higgs boson exchange is equivalent the one imposed by gauge boson exchange. Let us analyze the graph



The corresponding term is

$$
\begin{aligned}
&(h^{1(2)*}_{rm}\bar{u}'_{mR}d'_{rL})(h^{1(2)*}_{mn}\bar{d}'_{nR}u'_{kL})M^{u}_{mk}\bar{u}'_{kL}u'_{mR} \\
&= (h^{1(2)*}_{rm}\bar{u}_{jR}V^{uR}_{jm}V^{dL*}_{ir}d_{iL})(h^{1(2)*}_{kn}\bar{d}_{iR}V^{dR}_{in}V^{uL*}_{jk}u_{jL})m_j\bar{u}_{jL}u_{jR}
\end{aligned}
\tag{D.1}
$$

Then

$$
\begin{aligned}
\mathrm{Im}(V^{uR}_{jm}V^{dL*}_{ir}) &= -\mathrm{Im}(V^{dR}_{in}V^{uL*}_{jk}) \\
\beta^{j}_{m} - \alpha^{i}_{r} &= -(\beta^{ni}_{n} - \alpha^{j}_{k})
\end{aligned}
\tag{D.2}
$$

for $j = n = r$, $m = k$ we get

$$
\beta^{rj}_{i} - \beta^{j}_{k} = \alpha^{rj}_{i} - \alpha^{j}_{k} \tag{D.3}
$$

which is the as Eqn.5.15 where we used $\alpha^{ri}_{r} = -\alpha^{r}_{i}$ and $\beta^{ni}_{n} = -\beta^{n}_{i}$.

VITA

Author of this work was born in 1960 at Bumsuz; a village of Ankara, as a Turkish citizen. He recieved his BS degree in physics in 1982 at M.E.T.U and MS degree in physics in 1985 at M.E.T.U. Presently he works as a research assistant in the Physics Depertmant, METU Ankara.