# *On the Arabic Translations of Eutocius' Commentary on Book II of Archimedes'* Sphere and Cylinder

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Abstract: In the 9th century, parts of Eutocius' commentary on Book II of Archimedes' *Sphere and Cylinder* were translated into Arabic. Most extant manuscripts of these translations contain only fragments. However, one manuscript, Escorial, Árabe 960, contains the longest known Arabic text of Eutocius' commentary, at 41 folios long. Comparison with the Greek text reveals that the Arabic text in Escorial 960 is composed of the translations of two disjoint parts of Eutocius' commentary, on *Sphere and Cylinder* II.1 and II.4. A comparison of various mathematical terms used in the two parts, which I call Text A and Text B, shows that they differ systematically, and were therefore probably translated by different individuals, in different scholarly contexts.

Key Words: Eutocius, commentary, Archimedes, *Sphere and Cylinder*, translation movement.

Resum: Al segle ix, algunes parts del comentari d'Eutoci al segon llibre d'*Esfera i cilindre* d'Arquímedes van ser traduïdes a l'àrab. La major part de manuscrits d'aquestes traduccions només contenen fragments. Tanmateix, un manuscrit, Escorial árabe 960, conté el text àrab més llarg del comentari d'Eutoci, d'una extensió de 41 folis. La comparació amb el text grec revela que el text àrab d'aquest manuscrit està composat de traduccions de dues parts diferents del comentari d'Eutoci a *Esfera i cilindre* II.1 i II.4. Comparant diversos termes matemàtics usats en ambdues parts, que anomeno Text A i Text B, comprovo que difereixen sistemàticament, i que, per tant, probablement foren traduïdes per dos individus diferents que treballaven en diferents contextos intel·lectuals.

Paraules clau: Eutoci, comentari, Arquimedes, *Esfera i cilindre*, moviment de traducció.

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## 1. Introduction

The 8th–10th centuries[1] in Abbasid society witnessed the «translation movement», which refers to the activities of translation and study of classical philosophical and scientific texts, mostly in Greek but also in other languages such as Persian, Sanskrit, and Syriac.[2] The translated texts were studied, commented upon, criticized, and sometimes retranslated. During the translation movement itself and in later centuries, these texts formed much of the basis for original works by scholars in various Islamicate societies.[3]

It is clear from the sources that mathematical scientific texts have played an important role in the Abbasid translation movement from the very beginning. However, the details given by the sources are often contradictory and do not allow us to form an unambiguous account about the processes of translation. For example, according to a report transmitted by al-Mas'ūdī (wr. 943–956) in *Meadows of Gold and Mines of Jewels* (*Murūj al-Dhahab wa-Ma'ādin al-Jawhar*), Euclid's (3rd century BCE) *Elements*, Ptolemy's (ca. 100–ca. 170) *Almagest*, and Nicomachus of Gerasa's (fl. ca. 100) *Arithmetic*, together with some other foreign works, were translated as early as the caliphate of al-Manṣūr (Barbier de Meynard 1874, 291).[4] However, according to Ibn al-Nadīm (wr. 987–988), writing in

1. All dates are CE unless otherwise indicated.

2. For an overview of the translations of scientific texts into Arabic, with an extended bibliography, see Endress (1987). Gutas' *Greek Thought, Arabic Culture* (1998) is an excellent exposition of the social and political contexts in which the various phases of the translation movement took place. Sidoli (2022) provides a general survey of translations in mathematical sciences in the early Abbasid period. For an overview of mathematical sciences in Syriac during the same period, see Hugonnard-Roche (2014). For the role of the Barmakids in the translation of Sanskrit scientific texts in the early Abbasid period, see Van Bladel (2011). More recently, Van Bladel (2015) suggests that one reason for the early interest in Indian astronomy shown by Abbasid scholars was the presence of Indian astronomers in the Tang court in China, with which the Abbasids had diplomatic contacts during the reign of al-Manṣūr (r. 754–775) through numerous embassies that were sent there.

This article arose out of a talk I gave at the Delphi Symposium on Greek Mathematics in Delphi, Greece, 25–28 October 2023. I would like to thank the organizers of the symposium as well as the audience members for their questions and comments.

3. See Rashed (1989) for two case studies of the relation between translation and research, involving Diophantus' (fl. 250) *Arithmetica* and the subject of burning mirrors. Rashed points out that *Arithmetica* was translated using Arabic terminology that was already being used in algebra texts, and which would have been foreign to Diophantus himself; moreover, the translation itself was probably stimulated by interest in algebra.

4. The report in question is part of a conversation between the 10th-century Abbasid caliph al-Qāhir (r. 932–934) and one Muḥammad ibn'Alī al-'Abdī al-Khurāsānī al-Akhbārī («transmitter

*Kitāb al-Fihrist*, the *Elements* were translated first during the caliphate of Hārun al-Rashīd (r. 786–809) and then a second time during the caliphate of al-Ma'mūn (r. 813–833) (Flügel 1871, 265). Ibn al-Nadīm also reports that the *Almagest* was first translated into Arabic under the patronage of Yaḥyā ibn Khālid ibn Barmak (d. 805), the famous vizier of Hārūn al-Rashīd (r. 785–809) (Flügel 1871, 267).

Other geometrical texts, such as Apollonius' (2nd century BCE) *Conics*, Archimedes' (3rd century BCE) *Measurement of the Circle* and *Sphere and Cylinder*, as well as some other works attributed to these two Greek mathematicians, were also translated into Arabic in the 9th century, even though important pieces of contextual information such as dates, translators, patrons, the manner of obtaining the Greek originals, and the motives for the translations in the first place, often elude us.[5]

Even though Eutocius' (b. ca. 480) commentary on Archimedes' *Sphere and Cylinder* is also known to have been translated into Arabic, at least in part, this translation has so far received very little scholarly attention. Indeed, there are only two studies known to me that consider Eutocius' commentary on the *Sphere and Cylinder* in Arabic. The first of these is the publication of a Beirut fragment containing Eratosthenes' solution to the problem of two mean proportionals,[6] by Muwafi and Philippou (1981); there is also an appendix by Lorch on the technical terms used in the fragment. The second is an article on the transmission of the *Sphere and Cylinder*, where Lorch (1989, 106–108) dedicates a few pages to Eutocius' commentary. After listing some Arabic manuscripts of Eutocius' commentary on the *Sphere and Cylinder*, Lorch goes on to describe briefly the contents of these manuscripts, followed by a description of a Hebrew manuscript (Bodleian Heb. d. 4) containing a Hebrew translation of the commentary.

In this article, I shall focus on the Arabic versions of Eutocius' commentary on Book II of the *Sphere and Cylinder* (henceforth simply the *Commentary*). After a brief survey of some Arabic translations of texts related to the study of conic sections (Section 2), a summary of the contents of Propositions II.1 and II.4 of

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of reports»). Muḥammad al-Akhbārī does not name the author of the *Arithmetic* but it must be Nicomachus of Gerasa since no other Greek author is known to have written a work of that name. Similarly, Muḥammad mentions a «book of Euclid» (*kitāb Uqlīdis*) without a title but, again, he probably means the *Elements*. See Gutas (1998, 30–31) for more details on this report and a translation of the relevant passage.

5. Apollonius' *Conics* is a partial exception to this rule. See 2.1.

6. By «the problem of two mean proportionals» I refer to the problem of finding two mean proportionals to two given lines. In symbolic notation, if the given lines are $A$ and $D$, we want to find lines $B$ and $C$ such that $\frac{A}{B} = \frac{B}{C} = \frac{C}{D}$.

the *Sphere and Cylinder*, as well as Eutocius' commentaries on them (Section 3), and some of the existing fragments of the Arabic versions of the *Commentary* (Section 4), I shall examine some of the mathematical terms used in these Arabic versions (Section 5). After a comparison of these terms with those used in other Arabic mathematical texts, my main conclusions shall be that the largest extant Arabic text containing the *Commentary* is composed of two parts, relating to II.1 and II.4,[7] and that these two parts were translated in different scholarly contexts and most likely by different individuals. The reader should be aware that other possibilities exist; for example, both texts may have been translated by the same individual but then one of them may have been revised by a different individual. I stick to the hypothesis of two different translators as I find it the most economical explanation of the divergent terminology between the two texts. Hence, from now on, I shall be referring to the «translations» of the *Commentary* throughout.

## 2. Early Arabic Translations of Texts Related to the Study of Conic Sections

### *2.1 The* Conics *of Apollonius*

An eight-book work of which only the first four are extant in Greek, the *Conics* of Apollonius is the most important Greek mathematical text concerning conic sections to have been translated into Arabic.[8] According to the introduction to the *Preliminaries to the Conics* (*Muqaddamāt Kitāb al-Makhrūṭāt*) by the Banū Mūsā (Rashed, Decorps-Foulquier, and Federspiel 2008–2010, 1.1.504–507), the *Conics* was translated into Arabic under the patronage of Aḥmad ibn Mūsā. He assigned the translation of Books I–IV to Hilāl ibn Abī Hilāl al-Ḥimṣi[9] and of Books V–VII to Thābit ibn Qurra (826 or 836–901).[10] This case is valuable for

7. A fact that was already noticed by Lorch (1989, 107).

8. Books V–VII of the Arabic translation have been edited and translated by Toomer (1990). More recently, all seven books of the Arabic translation, as well as the extant first four books of the Greek, have been edited and translated by Rashed *et al*. (2008-2010) in several volumes.

9. This is the son of one Abū Hilāl al-Ḥimṣi, who wrote a book on medicine, from which only fragments remain (Ullmann 1970, 122). Ullmann gives the year of death of Hilāl ibn Abī Hilāl al-Ḥimṣi as around 883, but the source for this is unknown to me.

10. The three brothers, Muḥammad (d. 873), Aḥmad, and al-Ḥasan, sons of one Mūsā ibn Shākir, are known collectively as the Banū Mūsā. Rashed (1996, 1–7) provides a summary of their lives

the history of translations from Greek into Arabic in the Abbasid period since the patron himself is giving a detailed explanation on the processes of translation. However, this information is incomplete—perhaps deliberately so—in one very important respect: an examination of references to various propositions of the *Conics* made in Arabic mathematical texts by various authors from the 9th–11th centuries, and the discrepancies between such references, makes it clear that there was another Arabic version of Books I–IV of the *Conics* in circulation in these centuries (Rashed, Decorps-Foulquier, and Federspiel 2008–2010, 1.1.28–44), of which there is no mention in the introduction to the *Preliminaries to the Conics*.

A scholium to the Arabic text of the *Commentary* found on f. 63r of the manuscript Fatih 3414 (İstanbul, Süleymaniye Manuscript Library), in the hand of the 13th-century mathematician Muḥammad ibn 'Umar ibn Abī Jarāda, who is also the copyist of the whole manuscript, brings further confirmation on the presence of another translation as well as information on the time of its production.[11] Below I provide a transcription of the Arabic text of the scholium as well as a translation. As parts of the scholium suffered water damage, I have restored the damaged parts in angle brackets. An image is also available in Figure 1 to enable comparison with the text presented here.

*Hādhā 'aks al-thānī 'ashar min al-maqāla al-thā⟨niya⟩ min kitāb al-makhrūṭāt fī iṣlāḥ Banī Mūsā. Wa-qad waqa'tu 'alā nuskha qadīma bi-l-maqālāt al-arba' al-ūlā*[12] *'alayhā rashm al-Ma'mūn wa-rashm al-Wāthiq. Yaghlubu 'alā ẓannī annahā allatī dhakara Banū Mūsā annahā iṣlāḥ Ū⟨ṭūqi⟩ yus wajadtu ...*[13] *⟨al⟩-thānī 'ashar min al-maqāla ...*[14] *thāminan minhā. Fa-li-dhālika ja'ala Ūṭūqiyus hādhā 'aksan li-l-thāmin binā'an 'alā iṣlāḥihi lā 'alā iṣlāḥ Banī Mūsā. Wa-kataba Muḥammad ibn 'Umar ibn Abī Jarāda.*

This is the converse of the twelfth [proposition] of the second [book] from the *Conics* in the correction of the Banū Mūsā. I came across an old copy containing the first four books and upon which were the marks of al-Ma'mūn and al-Wāthiq (r. 842–

and mathematical works; his account has the merit of emphasizing their political and administrative activities, which were not without consequence for their scholarly production and their patronage of the scholarly activities of others. See also Sidoli (2022, 51–53) for a discussion of the translation of the *Conics* into Arabic under the patronage of the Banū Mūsā.

11. For the text of the *Commentary* on Fatih 3414, see 4.2.3.

12. Corrected from the manuscript reading *al-awwal*.

13. Water damage.

14. Water damage.

847). I think it likely that it is that which the Banū Mūsā mentioned as the correction of Eutocius, and I found ... the twelfth [proposition] from the book ... the eighth in it. For this reason, Eutocius calls this as the converse of the eighth, according to his correction and not according to the correction of the Banū Mūsā. Muḥammad ibn ʻUmar ibn Abī Jarāda wrote [it].

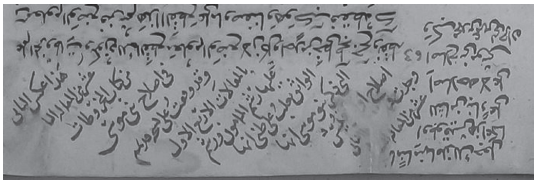

Figure 1: Scholium by Ibn Abī Jarāda, f. 63r, Fatih 3414.

The scholium is added at the point in the *Commentary* where the converse of Proposition II.12 of the *Conics* is used after being cited, but it is cited as the eighth proposition instead of the twelfth.[15] The numbering II.12 is what is found in the extant Greek text of the *Conics* as well as the extant Arabic translation of the Banū Mūsā (Heiberg 1974, 212–214; Rashed, Decorps-Foulquier, and Federspiel 2008–2010, 2.1.141–143). Since some portions of the scholium are made illegible by water damage it is impossible to read it completely, and further confusion is added to the matter by Ibn Abī Jarāda's comments as he seems to think that the «old copy» represented the «correction» (*iṣlāḥ*) of Eutocius. However, the basic sense of the scholium is clear enough: this «old copy» contained the extant proposition II.12 as the eighth proposition in the second book,[16] and it seems likely that it is the other translation whose existence has been shown by Rashed (Rashed, Decorps-Foulquier, and Federspiel 2008– 2010, 1.1.33–34). If the information given by Ibn Abī Jarāda is reliable, as I shall argue below, the time of the translation must be before 847, the year of death of the caliph al-Wāthiq.[17]

The manuscript found by Ibn Abī Jarāda is the only instance known to me where a mathematical manuscript is reported to bear the mark of a caliph. Since

15. For the Greek text and an English translation of the citation, see Heiberg (1881, 158.8–10) and Netz (2004a ,320), respectively.

16. I take the suffixed pronoun in *minhā* to refer to *nuskha qadīma*.

17. I refrain from pushing this date to 833, the year of death of al-Ma'mūn, since it is possible that different translations in a manuscript may have been made during the caliphates of different caliphs.

there were the marks of two Abbasid caliphs of the first half of the 9th century on it, could it be that the manuscript was once in the *Bayt al-Ḥikma* which, as Gutas (1998, 53–60) has plausibly argued, was an antiquarian library? The sources Gutas quotes on the contents of the *Bayt al-Ḥikma* are all very dubious and vague, as he himself points out; moreover, we do not have any other source suggesting that the books stored at the *Bayt al-Ḥikma* were marked by the caliphs. However, Ibn Abī Jarāda claims to have seen the «marks» (*rashm*) of al-Ma'mūn and al-Wāthiq on the manuscript.[18] I see no reason why, if Ibn Abī Jarāda wanted to lie about the antiquity of the manuscript, he would invent the presence of the mark of al-Wāthiq on it, since this caliph with a relatively short reign was not associated with translations from foreign languages into Arabic in the imagination of later writers the way al-Ma'mūn was. It is therefore reasonable to suppose that the manuscript Ibn Abī Jarāda saw indeed came from a library that belonged to al-Ma'mūn and al-Wāthiq.

### *2.2* On Burning Mirrors *of Diocles*

Another subject in Greek mathematics related to the study of conic sections is burning mirrors. One early Greek treatise on this subject, *On Burning Mirrors* (*Fī al-Marāyā al-Muḥriqa*) of Diocles (fl. probably early 2nd century BCE), was translated into Arabic and is extant on one manuscript. This Arabic translation has been edited by Toomer (1976) first, and more recently by Rashed (2002, 1–151).[19] Rashed (2002, 21–22) conjectures, based on the difference of the vocabulary used in the translation from that used in writings of al-Kindī (d. ca. 873) and Qusṭā ibn Lūqā (fl. 860–900), that the translation was made «relatively early». The translator is unknown; the conjecture of Toomer (1976, 21) that it was Qusṭā ibn Lūqā is unlikely.

The extant Arabic text is in fact an amalgamation of three parts, which probably were separate treatises originally: the first on burning mirrors (Propositions 1–5); the second on the problem, posed by Archimedes in *Sphere and Cylinder* II.4, of cutting a sphere by a plane so that the volumes of the two resulting segments have

18. Since the *shīn* is fully dotted in both occurrences of this word (see Figure 1), there is no doubt as to the reading.

19. I shall be referring to Toomer's edition in this article.

a given ratio (Propositions 7–8); and Diocles' solution to the problem of two mean proportionals (Propositions 10–16) (Sesiano 1988, 194).

All three parts of *On Burning Mirrors* outlined above involve the use of conic sections to some extent. Only the parabola, used in the construction of burning mirrors, appears in the first part. In Proposition 1, Diocles proves the reflective property of the parabola: if a ray parallel to the axis of the parabola is reflected from the parabola, it passes through the focus. This is then used for the construction of a burning mirror by rotating the parabola about its axis. In Proposition 4, Diocles constructs a parabola with a given focal length[20] by determining a number of points on the parabola and drawing a curved line through them with a ruler; the focus-directrix property of the parabola is used in the construction. Proposition 5 is a proof that the curve so constructed is in fact a parabola.[21] In the second part, the solution to the problem of cutting a sphere is found by the intersection of a hyperbola and an ellipse given in position (Proposition 8). The third part starts with Proposition 10, in which Diocles solves the problem of duplicating a given cube by intersecting two parabolas constructed pointwise with the focus-directrix property.[22] The remaining propositions solve the problem of two mean proportionals by constructing a curve pointwise, but this curve is not a conic section.

### *2.3* Burning Mirrors and the Digest of the Conics *of Dtrūms*

Just like Diocles' *On Burning Mirrors*, *Burning Mirrors and the Digest of the Conics* (*Al-Marāyā al-Muḥriqa wa-Jawāmi' al-Makhrūṭāt*), attributed to one Dtrūms (the name is corrupt), is a treatise on burning mirrors that was translated into Arabic. The treatise, extant on one manuscript, has been edited and translated by Rashed (2002, 153–213).[23] The translator is unknown. Based on the vocabulary, Rashed (2002, 158–159) suggests that *Burning Mirrors and the Digest of*

20. The distance between the vertex and the focus of the parabola.

21. For a more detailed examination of this first part, including Propositions 2 and 3, which concern spherical burning mirrors, see Sesiano (1988, 195–202).

22. Diocles' solution to the problem of cube duplication in Proposition 10 of *On Burning Mirrors* is in fact a special case of the alternative solution attributed to Nicomedes in the list of 11 solutions, transmitted by Eutocius, to the problem of two mean proportionals. For the 11 solutions and some questions about their attributions, see 3.1 and note 30 there.

23. According to Rashed (2002, 156–157), «Dtrūms» could be a corruption of Dīdūms, that is, Didymus.

*the Conics* may have been translated in the first half of the 9th century since the terms relating to conic sections differ from those used by the Banū Mūsā version of the *Conics*. The words *jawāmi' al-makhrūṭāt* were at first taken by Rashed (2002, 156) to refer to cones (geometric objects) but he has since changed his opinion and now suggests that these words refer to the work of Apollonius (Rashed, Decorps-Foulquier, and Federspiel 2008–2010, 1.1.38). I have followed his later opinion in translating the title.

*Burning Mirrors and the Digest of the Conics*, as it currently exists, consists of 15 propositions.[24] However, the text shows signs of editorial modifications, as after Proposition 9 the following note is inserted:

> Since we have first presented the things we have mentioned from the first book of Apollonius' *Conics*, after this there is nothing we need from the first and second books of Dtrūms' *On Burning*. (Rashed 2002, 193; my translation.)

A later, unknown, editor thus seems to have removed the first two books of *Burning Mirrors and the Digest of the Conics* of Dtrūms and replaced them with an exposition containing some definitions and nine propositions from Book I of Apollonius' *Conics* (Rashed 2002, 155–156). The definitions are those of the cone (including its vertex, base, and axis), the right cone, and the diameter of a curve in a plane. Propositions 1 through 4 of *Burning Mirrors and the Digest of the Conics*, which correspond to *Conics* I.1, I.3, I.4, I.6, respectively, are about various properties of cones. Proposition 5 is a proposition of purely algebraic content and thus has no counterpart in the *Conics*. Propositions 6 through 9 of *Burning Mirrors and the Digest of the Conics* investigate properties of parabolas; they correspond to *Conics* I.11, I.20, I.33, I.35, respectively.

The remaining propositions, which seem to have constituted the third book of *Burning Mirrors and the Digest of the Conics*, are about burning mirrors. In Proposition 10, it is proved that if a parabolic mirror is constructed and rays parallel to the axis fall on it, then these rays are reflected to the focus, which is the point on the axis whose distance from the vertex of the parabola is one-fourth of the *latus rectum*. After Proposition 11, again a purely algebraic proposition, comes Proposition 12, a technical result about tangents to a parabola, which is used in

24. The proposition numbers are not marked in the manucript; they are given by Rashed (2002).

Proposition 13 to construct a parabola pointwise. Finally, Propositions 14 and 15 are about spherical burning mirrors.

## 3. Eutocius' «Commentary» on «Sphere and Cylinder» II.1 and II.4

Since, as we shall see below, the translations of only some parts of the *Commentary* pertaining to Propositions II.1 and II.4 of the *Sphere and Cylinder* are extant in Arabic,[25] the contents of the *Commentary* on these two propositions are summarized in this section.

### *3.1* II.1

The problem proposed by Archimedes in *Sphere and Cylinder* II.1 is:[26]

> Given a cone or a cylinder, to find a sphere equal[27] to the cone or to the cylinder. (Netz 2004a, 187)

To solve this problem, Archimedes proceeds by using the method of analysis and synthesis. In the analysis, he assumes that the problem is already solved. Thus, let $A$ be the given cone or cylinder and $B$ a sphere equal to $A$. Let $D$ be the diameter of $B$. We take a cylinder $C_1$ such that $C_1 = \frac{3}{2}A$.[28] Let the diameter and height of $C_1$ be $D_1$ and $H_1$, respectively. We then take another cylinder $C_2$ with diameter and height $D_2 = D$ and $H_2 = D$, respectively. By the Corollary to *Sphere and Cylinder* I.34, we have $C_2 = \frac{3}{2}B$ and hence $C_1 = C_2$. Archimedes

25. See Table 1.

26. The Greek text and English translation of *Sphere and Cylinder* II.1 can be found in Heiberg (1972, 170–174) and Netz (2004a, 187–191); the latter also provides extensive comments.

27. That is, equal in volume.

28. This step, not explained by Archimedes, can be explained as follows: if $A$ is a cylinder, we take another cylinder with the same base as $A$ and with height $\frac{3}{2}$ times the height of $A$. If $A$ is a cone, we take a cylinder with the same base as $A$ and with height $\frac{1}{2}$ times the height of $A$; this cylinder is then equal to $\frac{3}{2}A$ by *Elements* XII.10.

now shows that if $X$ is such that $D_1 \cdot X = D_2^2$, then $D_2$ and $X$ are two mean proportionals between $D_1$ and $H_1$; in other words,

$$\frac{D_1}{D_2} = \frac{D_2}{X} = \frac{X}{H_1}$$

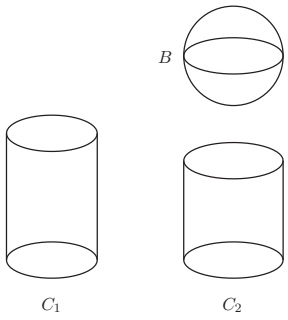


FIGURE 2: *Sphere and Cylinder* II.1

In the synthesis, Archimedes starts by considering a cylinder $C_1$ such that $C_1 = \frac{3}{2}A$, where $A$ is the given cone or cylinder. Let the diameter and height of this cylinder be $D_1$ and $H_1$, respectively. Let there be found two mean proportionals $D_2$ and $X$ between $D_1$ and $H_1$; in other words,

$$\frac{D_1}{D_2} = \frac{D_2}{X} = \frac{X}{H_1}$$

Archimedes now considers another cylinder $C_2$ with diameter and height both equal to $D_2$ and shows that $C_1 = C_2$. But, again by the Corollary to *Sphere and Cylinder* I.34, $C_2 = \frac{3}{2}B$, where $B$ is some sphere whose diameter is $D_2$. Therefore, $B = A$ and the required sphere has been found.

The key step of this solution is the finding of the two mean proportionals. After pointing out that Archimedes simply assumes that the two mean proportionals have been found without showing how it can be done,[29] Eutocius says that he has found

29. The Greek text and English translation of Eutocius' commentary to *Sphere and Cylinder* II.1 can be found in Heiberg (1881, 60–126) and Netz (2004a, 270–306), respectively.

solutions to this problem by a number of mathematicians and he will write them down next, with the exception of the solution by Eudoxus of Cnidus (ca. 400 BCE–ca. 347 BCE) (Heiberg 1881, 66; Netz 2004a, 272– 273). There follow 11 solutions to the problem of two mean proportionals, each attributed by Eutocius to a different author, and it is these solutions that have been translated into Arabic, but the orders of the solutions in the Greek and Arabic texts are different.[30]

The 11 solutions of the problem of two mean proportionals presented in Eutocius' commentary to *Sphere and Cylinder* II.1 can be classified into three main groups: those that use a pivoting ruler, those that involve a mechanical construction, and those that use curves other than circles. This classification shall be useful to us in the study of various mathematical terms in the Arabic translations of the *Commentary* that we shall undertake in Section 5. However, it should be noted that these groups neither exhaust all solutions nor are mutually exclusive. Indeed, one solution, the one by Nicomedes, falls into two groups since it first constructs a mechanical device with a total of three rulers (two fixed and one movable) that is then used to draw a «conchoid» line. Sporus' solution, by contrast, does not fall into any of the three groups: even though it, just like Pappus' solution, is based on drawing a certain line in such a way that two portions of this line taken between certain other lines are equal to each other, it differs from Pappus' solution in that the latter is phrased in terms of a ruler one of whose endpoints is fixed while the other is moving, Sporus' solution simply assumes the line to have been constructed without any mention of a ruler.

30. See Tables 1 and 2.

See Knorr (1986, 17–24, 50–66, 188–191, 210–213, 219–226, 240–246, 305–306) for discussions of various solutions of the problem of two mean proportionals. The reader should, however, be aware that Knorr calls them solutions to the «cube duplication» problem (since the latter problem can be reduced to the problem of two mean proportionals), whereas in Eutocius' commentary, the only place where the problem of cube duplication is mentioned as such is in the beginning of Eratosthenes' solution and the poem at the end of it.

Some scholars have questioned some of the attributions of these 11 solutions to their putative authors. One is the solution attributed to Plato, which is widely recognized not to be by him (Knorr 1986, 57–59). Another is the second, alternative, solution attributed to Menaechmus, which uses the intersection of two parabolas. According to Toomer (1976, 169–170), this alternative solution is in fact due to Diocles. His argument rests on the fact that Proposition 10 in Diocles' *On Burning Mirrors*, just like the second solution attributed to Menaechmus, uses the intersection of two parabolas, albeit using the focus-directrix property. Unfortunately, Toomer does not explain why Eutocius, who certainly had access to *On Burning Mirrors*, would have erased Diocles' name from the solution he put under Menaechmus' name.

(*i*) Solutions that use a ruler pivoting about a point: Hero, Philo, Apollonius, Pappus.
(*ii*) Solutions that involve mechanical constructions: Plato, Eratosthenes, Nicomedes.
(*iii*) Solutions that use curves other than circles: Diocles, Menaechmus, Archytas, Nicomedes.

### *3.2* II.4

The problem proposed by Archimedes in *Sphere and Cylinder* II.4 is:[31]

> To cut the given sphere so that the segments of the sphere have to each other the same ratio as the given. (Netz 2004a, 202)

To solve this problem, Archimedes again follows the method of analysis and synthesis. In the analysis, Archimedes assumes the problem solved and, using the result of *Sphere and Cylinder* II.2, which states the equality of a section of a sphere with a cone having the same base as the section, restates the problem in terms of the heights of the cones corresponding to the two segments. Thus, given two lines $B\Delta$ and $BZ$ (on the same straight line), where $B\Delta = 2BZ$, and given a point $\Theta$ on $BZ$, $\Delta B$ is to be cut at a point $X$ so that

$$\frac{(B\Delta)^2}{(\Delta X)^2} = \frac{XZ}{Z\Theta}$$

(Netz 2004a, 204). One could also formulate the problem in more general terms: given a line and an area, to cut another given line in such a way that the ratio of one of the resulting segments to the given line is equal to the ratio of the given area to the square on the remaining segment. This more general problem is solvable only under certain conditions, whereas the original form of the problem—related to the problem of cutting a sphere so that the segments have a given ratio—is always solvable. Archimedes promises to analyze and synthesize these problems

31. The Greek text and English translation with comments of *Sphere and Cylinder* II.4 can be found in Heiberg (1972, 186–194) and Netz (2004a, 202–208).

at the end. However, these vanished from the textual tradition of the *Sphere and Cylinder* early and were not available to Eutocius (Netz 2004a, 204, 206). *Sphere and Cylinder* II.4 ends with Archimedes giving the synthesis of the problem.

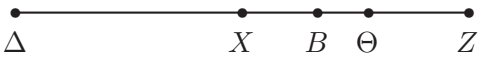


FIGURE 3: *Sphere and Cylinder* II.4, after Netz (2004a, 205)

The part of Eutocius' commentary to *Sphere and Cylinder* II.4 that was translated into Arabic starts with the problem, given above, of cutting the line $\Delta B$ at a point $X$ so that the desired equality of ratios obtains, and where Archimedes promises to analyze and synthesize these problems; it ends with the synthesis to II.4.[32] Eutocius remarks that in none of the manuscripts available to him was this promise of Archimedes fulfilled. As a result, Dionysodorus (fl. 3rd-2nd centuries BCE) and Diocles came up with solutions on their own. Eutocius reproduces these in his commentary. He also informs us that in his search, he came across a manuscript containing a text, written in the Doric dialect, that contained a solution—albeit full of mistakes—to the problem. So Eutocius adds that solution in his commentary, modernizing the terminology.[33]

After these remarks, Eutocius' exposition is concerned with the general form of the problem: given a line $AB$, another line $A\Gamma$, and an area $\Delta$, to cut $AB$ at a point, say $X$,[34] so that $\frac{XA}{A\Gamma} = \frac{\Delta}{(BX)^2}$. He then presents the analysis and synthesis of this problem. Since the general form of the problem is not always solvable, Eutocius also investigates the conditions for solvability. Let $E$ be on $AB$ such that $AE = \frac{AB}{3}$. If $\Delta \cdot A\Gamma > BE^2 \cdot EA$, the problem cannot be solved. If $\Delta \cdot A\Gamma = BE^2 \cdot EA$, $E$ itself solves the problem. And if $\Delta \cdot A\Gamma < BE^2 \cdot EA$, Eutocius shows that there are indeed two solutions to the problem. The solution as presented by Eutocius, as well as the proof that the quantity $BX^2 \cdot XA$ is maximized when

32. The Greek text and English translation of Eutocius' commentary to *Sphere and Cylinder* II.4 can be found in Heiberg (1881, 136–210) and Netz (2004a, 310–345), respectively. See Table 1 for the range of the Greek text that was translated into Arabic. See also Netz (2004b, 11–127) for an extended discussion of *Sphere and Cylinder* II.4 and Eutocius' commentary on it.

33. See Heiberg (1881, 152–154) and Netz (2004a, 318) for the Greek text and English translation of Eutocius' remarks, respectively.

34. The point $X$ does not appear in Eutocius' commentary; I have added it for clarity.

$X = E$, involve the intersection of a parabola and a hyperbola.[35] The solutions of Dionysodorus and Diocles as presented by Eutocius also involve the intersection of conic sections, a parabola and a hyperbola in the case of Dionysodorus and an ellipse and a hyperbola in the case of Diocles.

## 4. Biobibliographical Sources and Manuscripts

### *4.1 The Evidence for Eutocius in Arabic in the Biobibliographical Sources*

In the *Kitāb al-Fihrist*, Ibn al-Nadīm lists the following works of Eutocius on Archimedes:[36]

1. Commentary on Book I of Archimedes' *Sphere and Cylinder* (*Sharḥ al-Maqāla al-Ūlā min Kitāb Arshimīdis fī al-Kura wa-l-Usṭuwāna*),
2. On the two lines, and he proved all of this from the accounts of the geometer-philosophers. Thābit and Usṭāth translated it into Arabic (*Fī al-Khaṭṭayn wa-bayyana jamī' dhālika min aqāwīl al-falāsifa al-muhandisīn naqalahu Thābit wa-Usṭāth ilā al-'arabī*) (Sayyid 2014, 213).[37]

We shall only be concerned with the second title in this article. In the *Ta'rīkh al-Ḥukamā'*, Ibn al-Qifṭī (ca. 1172–1248) gives the title of that item with no mention of translator(s):

1. On the two lines, and he proved all of this from the accounts of the geometer-philosophers (*Fī al-Khaṭṭayn wa-bayyana jamī' dhālika min aqāwīl al-falāsifa al-muhandisīn*) (Lippert 1903, 73).

In view of Ibn al-Qifṭī's well-known tendency to copy passages from the *Kitāb al-Fihrist* wholesale, it is tempting to see here another act of such copying. However,

35. Indeed, this problem, being cubic, cannot be solved by the straightedge and compass.

36. For brevity, I have dropped the word «book» (*kitāb*) that Ibn al-Nadīm puts at the beginning of the titles in the list.

37. I have changed Sayyid's *Isṭāth* to *Usṭāth* since this word is the Greek name Eustathius. Flügel (1871, 267) has *naqalahu Thābit ilā al-'arabī wa-staṭāba* at the end of the entry, where the last verb («he deemed good») is an obvious corruption.

the reasons for the presence of the names of translators in the *Kitāb al-Fihrist* and their absence in the *Ta'rīkh al-Ḥukamā'* are unclear; perhaps Ibn al-Qifṭī had access to a manuscript of the *Kitāb al-Fihrist* where these names were absent and the names were added to the manuscripts of the *Kitāb al-Fihrist* at a later stage.

Even though it is not, strictly speaking, a biobibliographical source, the introduction of Naṣīr al-Dīn al-Ṭūsī (d. 1274) to his *taḥrīr* of Archimedes' *Sphere and Cylinder* might give a clue as to the translator of at least part of the *Commentary*.[38] In this introduction, al-Ṭūsī describes his search for the texts of the *Sphere and Cylinder*. According to his account, he first found the version of the *Sphere and Cylinder* that was amended by Thābit ibn Qurra. Later, he came across a copy containing the *Commentary*:

> *Fa-ẓafirtu bi-daftar 'atīq fīhi sharḥ Ūṭūqiyūs al-'Asqalānī li-mushkilāt hādhā al-kitāb alladhī naqalahu Isḥāq ibn Ḥunayn ilā al-'arabiyya naqlan 'alā baṣīra. Wa-kāna fī dhālika al-daftar ayḍan matn al-kitāb min ṣadrihi ilā ākhir al-shakl al-rābi' 'ashar min al-maqāla al-ūlā ayḍan min naql Isḥāq*. (al-Ṭūsī 1939, 134.17–20)

If we focus only on the first sentence of this passage, the suffixed pronoun in *naqalahu* may, with equal likelihood, be taken to refer to either the commentary of Eutocius or «this book» (that is, the *Sphere and Cylinder*). But the second sentence adds that the text of the *Sphere and Cylinder* from the beginning until the end of Proposition 14 of Book I was also contained in this codex, and al-Ṭūsī states explicitly that this part was «also from the translation of Isḥāq» (*ayḍan min naql Isḥāq*). It therefore seems that in the first sentence al-Ṭūsī is referring to the *Commentary* when he says that it was translated into Arabic by «Isḥāq ibn Ḥunayn with understanding» (*alladhī naqalahu Isḥāq ibn Ḥunayn ilā al-'arabiyya naqlan 'alā baṣīra*), since otherwise he would be referring to Isḥāq's translation of the *Sphere and Cylinder* twice. However, later on in the introduction, Naṣīr al-Dīn al-Ṭūsī states that there are 43 propositions in the version of Isḥāq, which contradicts the passage quoted above. This contradiction perhaps can be resolved by assuming al-Ṭūsī managed to find other copies of Isḥāq's translation later. Be that as it may, we shall see later that part of the *Commentary*, namely Text B, seems to have

38. For the full text of the introduction and a German translation, see al-Ṭūsī (1939, 134–135) and Sezgin (1974, 128–129).

been translated by someone close to the Banū Mūsā;[39] the hypothesis that Isḥāq translated parts of Eutocius' commentary on the *Sphere and Cylinder*, including Text B, is therefore consistent with the information provided by al-Ṭūsī, since Isḥāq is the son of Ḥunayn ibn Isḥāq (808–873), a famous translator of medical texts, many of which were commissioned by the Banū Mūsā.

### *4.2 Manuscripts*

For this article, I have consulted the following manuscripts containing the Arabic translations of the *Commentary*:

(*i*) San Lorenzo de El Escorial, Real Biblioteca del Monasterio, Árabe 960,
(*ii*) Paris, Bibliothèque Nationale de France, Arabe 2457,
(*iii*) İstanbul, Süleymaniye Manuscript Library, Fatih 3414.

This list was obtained from Lorch (1989, 106) and Sezgin (1974, 130), with two omissions that require a brief explanation. First, I have not taken into consideration the fragment in the Beirut manuscript St. Joseph 223 since it has already been published by Muwafi and Philippou (1981), as was mentioned in Section 1. Second, the Geniza fragment listed by Sezgin (1974, 130) is reported by Hogendijk (1984, 278) to not contain fragments of the *Commentary*;[40] it is therefore irrelevant to this article.[41]

Below I give a description of the portions of these manuscripts related to the *Commentary*; it should be noted that some of these observations were already made by Lorch (1989, 107–108).

39. See 5.7.

40. I was not able to confirm this myself.

41. I have also left out the *taḥrīr* of Naṣīr al-Dīn al-Ṭūsī to the *Sphere and Cylinder* from consideration since it does not make use of Text A. Indeed, after *Sphere and Cylinder* II.1, al-Ṭūsī (1939, 286) comments that «the ancients had methods for the finding of two mean proportionals to two given lines, most of which concern moving devices», thus showing that he had read Eutocius' commentary to that proposition. But he then suggests that the more theoretically-inclined should use Apollonius' *Conics* instead, and he gives a solution to the problem of finding two mean proportionals to two given lines that uses a hyperbola. This solution is not among the 11 given by Eutocius. See al-Ṭūsī (1939, 286–291) for his remarks and the text of the solution.

4.2.1 San Lorenzo de El Escorial, Real Biblioteca del Monasterio, Árabe 960

The colophon on f. 54r gives the date of completion as AH 11 Ramaḍān 742 (18 February 1342). Of the manuscripts considered, Escorial 960 contains the longest, and the most complete text of the Arabic translations of the *Commentary*, on ff. 22v–42v. The title at the top of f. 22v reads:

> *Jumal dhakarahā Ūṭūqiyūs fī tafsīrihi li-l-maqāla al-thāniya min kitāb Arshimīdis fī al-kura wa-l-usṭuwāna. Wa-ḥakā annahā fī istikhrāj khaṭṭayn bayna khaṭṭayn tatawāliya al-arba'a 'alā nisba wāḥida li-jamā'a min al-qudamā', wa-hum ...,*

followed by a list of the names of the 11 authors of the solutions to the problem of finding two mean proportionals to two given lines; the names of the authors are often corrupted.[42] No translator is given. Since this is the most complete text we have, and the contents of the other two manuscripts examined below are fragments of it, I shall be using it exclusively in the terminological study in Section 5. Since this study reveals that the text in Escorial 960 is actually composed of two parts, I shall call these parts Texts A and B.[43] Text A corresponds to the 11 solutions of the problem of two mean proportionals that Eutocius lists in his commentary on *Sphere and Cylinder* II.1 (Heiberg 1881, 66.21–126.3).[44] Text B corresponds to a part Eutocius' commentary on *Sphere and Cylinder* II.4, including the solutions of Dionysodorus and Diocles, up to the synthesis of II.4 Heiberg 1881, 152.12–208.6). The only indication of a change from Text A to Text B is the phrase *qāla Ūṭuqiyūs al-'Asqalānī* on f. 32r, line 26. Since, as I shall argue in Section 5, Texts A and B were translated by different individuals, it is likely that the two texts were joined together in the line of transmission to Escorial 960.

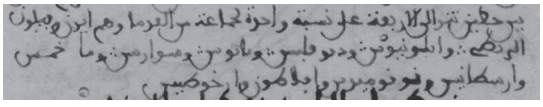

Figure 4: Names of the authors of the solutions to the problem of two mean proportionals, f. 22v, Escorial 960.

42. See Figure 4 and Tables 2 and 3.
43. For the breakdown of these parts according to folio and line numbers, see Table 1.
44. See Tables 2 and 3.

### 4.2.2 Paris, Bibliothèque Nationale de France, Arabe 2457

Most of this manuscript was copied by the 10th-century mathematician Abū Sa'īd Aḥmad ibn Muḥammad ibn 'Abd al-Jalīl al-Sijzī (ca. 945–ca. 1020). There are colophons at the end of many of the more than 40 treatises in this manuscript; the one on f. 180v states that al-Sijzī copied it in the Iranian city of Shiraz at the end of Khurdādmāh of year 338 of the Yazdegerd era (June 969). BnF 2457 contains a fragment of Text A on f. 191v, whose title reads:

> *Kitāb Ūṭūqiyus fī ḥikāyat mā istakhrajahu al-qudamā' min khaṭṭayn bayna khaṭṭayn ḥattā tatawāliya al-arba'a munāsiba naql Abī al-Ḥasan Thābit ibn Qurra, wa-fīhi thamāniyat 'ashar shaklan aḥad 'ashar muhandisan, wa-hum ...*

It is noteworthy that the title gives the name of the translator as Thābit ibn Qurra. The title is followed by a list of the names of the 11 authors; again with corruptions.[45] However, this list is different from the list of authors in Escorial 960; both lists are presented in Table 3. Since the fragment contains only the solution by Hero—and even that is incomplete—there is no way to know which order of authors was present in al-Sijzī's exemplar. An examination of the text of f. 191v reveals that it is identical, up to scribal errors, to the text of the solution by Hero in Escorial 960, ff. 22v–23r. We shall therefore not consider this manuscript in Section 5.

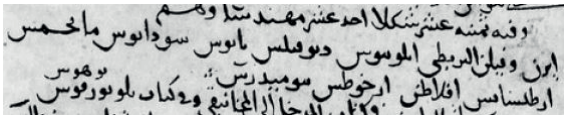

FIGURE 5: Names of the authors of the solutions to the problem of two mean proportionals, f. 191v, BnF 2457.

### 4.2.3 İstanbul, Süleymaniye Manuscript Library, Fatih 3414

This manuscript was copied by the 13th-century mathematician Muḥammad ibn 'Umar ibn Aḥmad ibn Abī Jarāda.[46] The last colophon, which is on f. 75v, gives

45. See Figure 5.
46. For more information on this manuscript, see Coşkun (2018, 61–63).

the date of completion as AH 2 Dhū al-Ḥijja 684 (29 January 1286). Fatih 3414 contains a fragment of the *Commentary* on ff. 61r–67v and 74r–74v. The text on ff. 74r–74v is a continuation of the text on ff. 61r–67v and seems to have been placed in the wrong spot due to a binding error. The title on f. 61r is *Min tafsīr kitāb Arshimīdis fī al-kura wa-l-usṭuwāna li-Ūṭūqiyūs*; it is repeated on f. 61v where the text begins. An examination of the text reveals that the text on ff. 61v–67v (up to *dal* on f. 67v) is identical to the beginning of Text B (Escorial 960, ff. 32r–36v, up to *ammā jumla*, written in larger letters). The rest of the fragment in Fatih 3414 is a fragment of the translation of Eutocius' commentary on Book I of *Sphere and Cylinder*. We shall therefore not consider this manuscript in Section 5.

Table 1: Texts A and B in Escorial 960

| Texts | Greek (Heiberg 1881) | Arabic (Escorial 960) |
|---|---|---|
| A | 66.21–126.3 | 22v.8–32r.25 |
| B | 152.12–208.6 | 32r.26–42v.29 |

Table 2: Correspondence of the 11 solutions to the problem of two mean proportionals as they appear in the Greek and Arabic (Text A). The order of the solutions given in the table is the same as in Text A.

| Solution by | Text A (Escorial 960) | Greek (Heiberg 1881, 66.21-126.3) |
|---|---|---|
| Hero | 22v.8-23r.6 | 70.3–72.21 |
| Philo | 23r.7-23v.21 | 72.22-76.21 |
| Apollonius | 23v.22-24r.6 | 76.22-78.18 |
| Diocles | 24r.7-24v.22 | 78.19-82.29 |
| Pappus | 24v.23-25v.26 | 84.1-90.3 |
| Sporus | 25v.27-26r.23 | 90.4-92.17 |
| Menaechmus | 26r.24-27v.5 | 92.18-98.17 |
| Eratosthenes[47] | 27v.5-29r.8 | 102.20-112.12 |
| Nicomedes | 29r.9-31r.13 | 114.9-126.3 |
| Plato | 31r.14-31v.8 | 66.21-70.2 |
| Archytas | 31v.9-32r.25 | 98.18-102.19 |

47. The poem at the end of Eratosthenes' solution is absent in the Arabic text (Heiberg 1881, 112.13–114.8). A translation is given by Netz (2004a, 298).

TABLE 3: Lists of authors of solutions to the problem of two mean proportionals as they appear in the Greek and Arabic (Text A)

| Greek (Heiberg 1881) | Text A (Escorial 960) | BnF 2457 |
|---|---|---|
| Plato | Hero | Hero |
| Hero | Philo | Philo |
| Philo | Apollonius | Apollonius |
| Apollonius | Diocles | Diocles |
| Diocles | Pappus | Pappus |
| Pappus | Sporus | Sporus |
| Sporus | Menaechmus | Menaechmus |
| Menaechmus | Eratosthenes | Eratosthenes |
| Archytas | Nicomedes | Plato |
| Eratosthenes | Plato | Archytas |
| Nicomedes | Archytas | Nicomedes |

## 5. TERMINOLOGICAL CONSIDERATIONS: TEXTS A AND B

In this section, I examine certain mathematical terms used by the translators of Text A and Text B into Arabic. This examination is not meant to be exhaustive; indeed, an exhaustive consideration of all the terms used in the two texts would also necessitate comparisons with many other early Arabic mathematical texts, including the translations of Euclid's *Elements*, which exceeds the scope of this article.

I have chosen to focus on terms and phrases that can be classified in the following four groups:

(*i*) introduction of the lines between which two mean proportionals are to be found,
(*ii*) nouns and adjectives describing parts of mechanical instruments that appear in more than one solution,
(*iii*) terms related to conic sections,
(*iv*) algebraic expressions.

This choice is guided by comparative concerns. More explicitly, my goals are, first, to examine the consistency of terminology inside of Text A and Text B and, second, to compare the terminological choices made in Text A and Text B, where

possible. The first and second groups on the list allow us to examine the consistency of terminology in Text A.[48] The third group is relevant to the goal of comparing Text A and Text B, and the fourth group is relevant to both goals.

For each group of terms, there is a dedicated subsection containing a table or tables, which are then followed by my remarks. For the third and fourth groups listed above, I also make comparisons to Diocles' *On Burning Mirrors* and Dtrūms' *Burning Mirrors and the Digest of the Conics*. Finally, the last subsection draws some broad conclusions from the results of the comparisons.

Numbers in parentheses indicate page or folio numbers, unless stated otherwise. For the Greek text, I have used the edition of Heiberg (1881). For the Arabic Texts A and B, I have used the manuscript Escorial 960.[49]

### *5.1 Text A: Introduction of the Lines between Which Two Mean Proportionals Are to Be Found*

Table 4: Greek and Arabic expressions for the introduction of the lines between which two mean proportionals are to be found

| Solution by | Greek | Arabic (Text A, Escorial 960) |
|---|---|---|
| Hero | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ ΑΒ, ΒΓ, ὧν δεῖ δύο μέσας ἀνάλογον εὑρεῖν (70) | *naj'alu al-khaṭṭayn al-ma'lūmayn al-ladhayn nurīdu an yaqa'a baynahumā khaṭṭān ḥattā tatawāliya mutanāsiba khaṭṭay alif bā' bā' jīm* (22v) |
| Philo | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ ΑΒ, ΒΓ, ὧν δεῖ δύο μέσας ἀνάλογον εὑρεῖν (72) | *naj'alu al-khaṭṭayn al-ma'lūmayn alif bā' bā' jīm wa-nurīdu an najida baynahumā khaṭṭayn munāsibayn lahumā* (23r) |
| Apollonius | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι, ὧν δεῖ δύο μέσας ἀνάλογον εὑρεῖν, αἱ ΒΑΓ ὀρθὴν περιέχουσαι γωνίαν τὴν πρὸς τῷ Α (76) | *naj'alu al-khaṭṭayn al-ma'lūmayn al-ladhayn nurīdu an najida baynahumā khaṭṭayn munāsibayn lahumā khaṭṭay alif bā' bā' jīm wa-l-takun al-zāwiya allatī baynahumā wa-hiya zāwiyat alif qā'ima* (23v) |

Continued on next page

48. We recall that, of the 11 solutions of the problem of two mean proportionals, a total of 7 use mechanical terms to some extent; see 3.1.

49. See 4.2.1.

Table 4 – continued from previous page

| Solution by | Greek | Arabic (Text A, Escorial 960) |
|---|---|---|
| Diocles | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι, ὧν δεῖ δύο μέσας ἀνάλογον εὑρεῖν, αἱ Α, Β (82) | *fa-innā naj'alu al-khaṭṭayn alladhayn nurīdu an najida baynahumā khaṭṭayn 'alā nisba khaṭṭay alif bā'* (24v) |
| Sporus | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι ἄνισοι αἱ ΑΒ, ΒΓ. δεῖ δὴ τῶν ΑΒ, ΒΓ δύο μέσας ἀνάλογον εὑρεῖν ἐν συνεχεῖ ἀναλογίᾳ (90) | *naj'alu al-khaṭṭayn al-ma'lūmayn ghayr al-mutasāwiyayn khaṭṭay alif bā' bā' jīm fa-idhā aradnā an najida bayna khaṭṭay alif bā' bā' jīm khaṭṭayn munāsibayn lahumā* (25v) |
| Menaechmus | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ Α, Ε. δεῖ δὴ τῶν Α, Ε δύο μέσας ἀνάλογον εὑρεῖν (92) | *naj'alu al-khaṭṭayn al-ma'lūmayn dāl hā' wa-nurīdu an nubayyina kayfa najidu baynahumā khaṭṭayn tatawāliya 'alā nisba* (26r) |
| Eratosthenes | δεδόσθωσαν δύο ἄνισοι εὐθεῖαι, ὧν δεῖ δύο μέσας ἀνάλογον εὑρεῖν ἐν συνεχεῖ ἀναλογίᾳ, αἱ ΑΕ, ΔΘ (106) | *naj'alu al-khaṭṭayn al-ma'lūmayn al-ladhayn nurīdu an najida baynahumā khaṭṭayn munāsibayn lahumā 'alā al-wilā' ghayr al-mutasawiyayn alif hā' dāl ṭā* (28r) |
| Nicomedes | δεδόσθωσαν δύο εὐθεῖαι αἱ ΓΛ, ΛΑ πρὸς ὀρθὰς ἀλλήλαις, ὧν δεῖ δύο μέσας ἀνάλογον κατὰ τὸ συνεχὲς εὑρεῖν (122) | *fa-innā naj'alu al-khaṭṭayn al-ma'lūmayn alladhayn nurīdu an najida baynahumā khaṭṭayn munāsibayn lahumā ḥatta takūna kulluhā tatawāliya mutanāsiba khaṭṭay dāl lām alif lām wa-l-yakun kull wāḥid minhumā qā'iman[50] 'alā ṣāḥibihi 'alā zawāyā qā'ima* (30r) |
| Plato | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ ΑΒΓ πρὸς ὀρθὰς ἀλλήλαις, ὧν δεῖ δύο μέσας ἀνάλογον εὑρεῖν (68) | *naj'alu al-khaṭṭayn al-ma'lūmayn alif bā' bā' jīm wa-l-yakun kull wāḥid minhumā qā'iman 'alā ṣāḥibihi 'alā zawāyā qā'ima wa-nurīdu an najida baynahumā khaṭṭayn munāsibayn lahumā* (31r) |
| Archytas | ἔστωσαν αἱ δοθεῖσαι δύο εὐθεῖαι αἱ ΑΔ, Γ. δεῖ δή τῶν ΑΔ, Γ δύο μέσας ἀνάλογον εὑρεῖν (98) | *naj'alu al-khaṭṭayn alif[51] dāl jīm hā' wa-naḥtāju an nakhuṭṭa bayna khaṭṭay alif dāl jīm hā' khaṭṭayn munāsibayn lahumā* (31v) |

50. Corrected from *qā'im* of the manuscript.

51. Corrected from the illegible word of the manuscript, which starts nevertheless with *alif lām fā'*. The correct form is suggested by the repetition of these diagram letters later.

Remarks:

(*i*) The translations of ἡ εὐθεῖα, ὁ λόγος, and ἡ γωνία as *khaṭṭ*, *nisba*, and *zāwiya*, respectively, are attested from the earliest translations of Euclid's *Elements*, in the beginning of the 9th century; *khaṭṭ* also occurs many times in the sense of «line» in al-Khwārizmī's *Kitāb al-Jabr wa-l-Muqābala* (Rashed 2017, 240, 318, 321, 789; Rashed 2009, 353). The translator of Text A is obviously thoroughly familiar with these terms.

(*ii*) The «givens», which are expressed in Greek with aorist and perfect passive forms of the verb δίδωμι, are consistently translated into Arabic using the passive participle *ma'lūm*. Diocles' solution, where δοθεῖσαι is not translated at all (perhaps through an oversight on the part of the translator), is an exception. In Pappus' solution, where a formulation similar to the ones in Table 4 is not found, δοθέντα and δεδομένον (84) are translated as *ma'lūm* and *ma'lūma*, respectively (24v). Again, this is consistent with the Arabic terminology of Euclid's *Elements* (Rashed 2017, 569).

(*iii*) The translator's style is according to sense, not literal. This is seen, for instance, in the translations of ἔστωσαν and δεδόσθωσαν as *naj'alu*; not only is the meaning of the verb different, but the Arabic text shows a tendency, seen throughout Text A, to use the first person plural active instead of the third person passive of the Greek. The tendency of the translator to translate according to sense is also seen in the way that ὧν δεῖ δύο μέσας ἀνάλογον εὑρεῖν, found in all solutions up to minor variants, is translated. This involves two lines (*khaṭṭayn*) we want to find (*nurīdu*) between the two given lines (*baynahumā*) that are proportional to them (*munāsibayn lahumā*) or be in continuous proportion with them (*ḫattā tatawāliya mutanāsiba*, *'alā nisba*, *tatawāliya 'alā nisba*, *munāsibayn lahumā 'alā al-wilā'*). The emphasis on the two lines being between the two given lines is difficult to explain solely by the fact that the Greek word used, μέσας, means «middle». It seems, rather, to betray an awareness of the fact that, mathematically, one is looking for two lines, say B and Γ, such that A :: B = B :: Γ = Γ :: Δ, where A and Δ are the given two lines, and thus that B and Γ are «between» A and Δ.[52]

52. It should be noted that the manuscript diagrams also depict the two sought lines as being between the two given lines in the first solution attributed to Menaechmus. See Netz (2004a, 288).

(*iv*) The translations listed in Table 4 use similar terminology, despite their variations. One partial exception is the phrase in Archytas' solution, which uses *naḥtāju an nakhuṭṭa* («we need to draw») for the two lines that are sought. This is unusual in that the choice of verbs differs from that of the remaining solutions (mostly *nurīdu an najida*), as well as the apparent suggestion that the two lines sought are supposed to be *drawn* between the two given lines. However, this exception does not seem to call into question the general suggestion that Text A was translated by one translator.

### *5.2 Text A: Parts of Mechanical Instruments*

TABLE 5: The Greek and Arabic words for nouns and adjectives describing parts of mechanical instruments. I have only included items that appear in more than one solution, as well as items similar in form or meaning that together appear in more than one solution.

| Solution by | Greek | Arabic (Text A, Escorial 960) |
|---|---|---|
| Hero | τὸ κανόνιον (70)<br>ὁ τύλος (70) | *misṭara* (22v)<br>*'amūd* (22v) |
| Philo | τὸ κανόνιον (74)<br>ὁ κανών (74, 76) | *misṭara* (23r)<br>*misṭara* (23r, 23v) |
| Apollonius | τὸ κανόνιον (78) | *misṭara* (24r) |
| Diocles | ὁ κανών (80) | *misṭara* (24v) |
| Pappus | τὸ κανόνιον (84, 86)<br>ὁ κανών (88)<br>τὸ τυλάριον (84)<br>τὸ τυλίον (84) | *misṭara* (25r)<br>*misṭara* (25v), untranslated (25v)[53]<br>untranslated (25r)<br>*'amūd* (25r) |
| Eratosthenes | ἡ χολέδρα (110) | *majran* (28v) |
| Nicomedes | ὁ κανών (114, 116, 118, 122)<br>πελεκινοειδής (114, 116)<br><br>συμφυής (116)<br>ὁ σωλήν (114, 116) | *misṭara* (29r, 29v, 30r)<br>untranslated/illegible, untranslated (29r, 29v)<br>*multaḥim* (29r), *thābit* (29r, 29v)<br>*majran*[54] (29r, 29v) |

Continued on next page

53. Folio numbers for untranslated or illegible words indicate where the translation would have appeared if the word in question had been translated or been legible.

54. The second occurrence of *majran*, with the definite article, is written in the ms. without an *alif maqṣūra*.

Table 5 – continued from previous page

| Solution by | Greek | Arabic (Text A, Escorial 960) |
|---|---|---|
| Plato | τὸ κανόνιον (68) | *misṭara* (31r) |
| | ὁ κανών (68) | *misṭara* (31r, 31v) |
| | πελεκινοειδής (68) | *shabīh* followed by transliteration (31r) |
| | συμφυής (68) | *muttaṣil* (31r), *mulṣaq* (31r) |
| | ὁ σωλήν (68) | *mīzāb* (31r) |
| | ὁ τύλος (68) | *miḥwar* (31r) |

Remarks:

(*i*) In solutions using a moving ruler, ὁ κανών and τὸ κανόνιον, which are the most commonly used words in Table 5, are translated as *misṭara* with one exception, where it is not translated at all. We note that the Arabic translation of Diocles' *On Burning Mirrors* also uses *misṭara* a total of five times in the sense of «ruler» (Toomer 1976, 229). However, only one of these is for drawing a straight line; the rest are uses of curved rulers for drawing curved lines. According to Rashed (2017, 365), in texts on burning mirrors written by al-Kindī and Qusṭā ibn Lūqā in the 9th century, *misṭara* is used in the sense of «template», which evidently differs from the sense in which the word is used in Text A.

(*ii*) In Hero's and Pappus' solutions, ὁ τύλος and τὸ τυλίον are both translated as *'amūd*, whereas in Plato's solution, ὁ τύλος is translated as *miḥwar*. This suggests a terminological inconsistency in Plato's solution compared to the solutions at the beginning of Text A.

(*iii*) Ἡ χολέδρα in Eratosthenes' solution and ὁ σωλήν in Nicomedes' solution are both translated as *majran*; the translation of ὁ σωλήν in Plato's solution is *mīzāb*. All of these translations are accurate—both *majran* and *mīzāb* meaning «a conduit through which water flows»—but we see a small change in terminology in Plato.

(*iv*) In Plato's solution, πελεκινοειδής is partly translated (-ειδής translated as *shabīh*) and partly transliterated; the latter seems to be a rendering of πελεκῖνος.[55] In Nicomedes' solution, where there are two occurrences, it

55. See Figure 6.

is either untranslated or illegible in the former case and left untranslated in the latter case. This suggests that Nicomedes' solution was translated before Plato's—in agreement with the order of solutions in Escorial 960—and that in any case the translator did not know the meaning of the word completely.

(*v*) There is no visible pattern to the translation of συμφυής in Nicomedes' and Plato's solution.

(*vi*) The translations of these terms are generally consistent, but with a degree less than that observed in 5.1. This might indicate that most of the terms considered in Table 5 had not been used in Arabic mathematical texts often by the time of the translation of Text A.

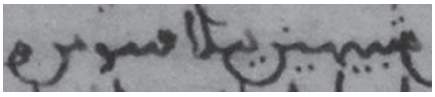

FIGURE 6: Transliteration of πελεκινοειδέσιν, f. 31r, Escorial 960

### *5.3 Text A: Terms Relating to Conic Sections in Menaechmus' Solution*

The only solution in Text A that uses conic sections is that of Menaechmus. Table 6 presents the Greek terms for conic sections and related objects used in Menaechmus' solution, along with their Arabic translations in Text A. Table 7 presents the Arabic terms used for conic sections and related objects in Menaechmus' solution in Text A and the extant Arabic translations of Diocles' *On Burning Mirrors* and Dtrūms' *Burning Mirrors and the Digest of the Conics*. I have avoided giving the Greek terms in Table 7 since the Greek texts of Diocles' *On Burning Mirrors* and Dtrūms' *Burning Mirrors and the Digest of the Conics* are not extant.

TABLE 6: Greek terms for conic sections and related objects, along with their Arabic translations in Text A

| Greek | Arabic (Text A, Escorial 960) |
|---|---|
| ὁ ἄξων[56] (94, 96, 98) | *miḥwar* (26v, 27r) |
| αἱ ἀσύμπτωτοι (94) | *al-khaṭṭān alladhān lā yalqayānihi*[57] (26v) |
| δύνασθαι (94, 98) | *qawiya ʿalā* (26v, 27r), omitted (27r) |
| αἱ καταγόμεναι (94, 96, 98) | *al-khuṭūṭ allatī taḥduthu minhu*[58] (26v, 27r), omitted (27r) |
| ἡ ὀρθία πλευρά (94) | *al-ḍilʿ al-muntaṣib min al-nisba* (26v) |
| ἡ παραβολή (94, 96, 98) | transliterated (26v, 27v) |
| ἡ ὑπερβολή (94) | transliterated (26v) |

TABLE 7: Arabic terms used for conic sections and related objects in Menaechmus' solution in Text A and the extant Arabic translations of Diocles' *On Burning Mirrors* and Dtrūms' *Burning Mirrors and the Digest of the Conics*. The numbers in parentheses for Diocles are the numbers for the divisions of the text established by Toomer (1976); they are not page numbers. The numbers in parentheses for Dtrūms are page numbers in Rashed (2002).

| Arabic (Text A, Escorial 960) | Arabic (Diocles) | Arabic (Dtrūms) |
|---|---|---|
| Asymptotes: *al-khaṭṭān alladhān lā yalqayānihi* | *al-khaṭṭān alladhān lā yalqayānihi*[59] (171) | not applicable |
| Axis of a parabola: *miḥwar* | *al-khaṭṭ alladhī yaqsimuhu bi-niṣfayn* (8, 38), *sahm* (9), *al-khaṭṭ alladhī yaqsimuhu*[60] (39) | *quṭr* (181,[61] 187, 190–192, 199, 200), *sahm* (194, 195, 197, 199, 202) |
| Hyperbola: transliterated | *qaṭʿ zāʾid* (171, 172) | not applicable |
| *Latus rectum*: *al-ḍilʿ al-muntaṣib min al-nisba* | *al-khaṭṭ alladhī taqwā ʿalayhi al-aʿmida allatī tukhraju ilā al-sahm* (9), *al-khaṭṭ alladhī taqwā ʿalayhi al-aʿmida* (38, 124) | *al-khaṭṭ al-muntaṣib* (189, 194–196), *al-muntaṣib* (197), *al-khaṭṭ alladhī yalī al-khaṭṭ alladhī yaqwā* (189, 190) |

Continued on next page

56. Axis of a parabola.

57. The suffixed pronoun in *yalqayānihi* refers to the hyperbola.

58. The suffixed pronoun in *minhu* refers to the conic section from which the ordinates are drawn.

59. The suffixed pronoun in *yalqayānihi* refers to the hyperbola.

60. In all these phrases, the suffixed pronoun in *yaqsimuhu* refers to the conic section.

61. This is the translation of the definition of the diameter of a curved line in a plane at the beginning of Book I of the *Conics* (Heiberg 1974, 6), but in Dtrūms' treatise it is applied exclusively to the parabola.

Table 7 – continued from previous page

| Arabic (Text A, Escorial 960) | Arabic (Diocles) | Arabic (Dtrūms) |
|---|---|---|
| Ordinates: *al-khuṭūṭ allatī taḥduthu minhu*[62] | *al-a'mida allatī tukhraju ilā al-sahm* (9), *al-a'mida* (38, 124), *'amūd* (39) | *khaṭṭ mustaqīm 'alā al-tartīb* (190), *khaṭṭ 'alā al-tartīb* (190, 200), *khaṭṭ al-tartīb* (191, 199, 201), *khaṭṭ murattab* (191, 192, 199), *khaṭṭ mustaqīm murattab* (192) |
| Parabola: transliterated | *qaṭ' al-makhrūṭ al-qā'im al-zāwiya* (8, 15, 32, 38, 103, 108), *al-qaṭ' al-qā'im al-zāwiya* (124), *qaṭ' makhrūṭ qā'im al-zāwiya* (207) | transliterated (188, 190–192, 195–197, 199–202, 204) |
| Power:[63] *qawiya 'alā* | *qawiya 'alā* (9, 38, 124) | *qawiya 'alā* (187, 188) |

Remarks:

(*i*) The names of the conic sections that appear in Menaechmus' solution—ἡ παραβολή and ἡ ὑπερβολή—are always transliterated.[64] This is in sharp contrast to the extant Arabic text of Diocles' *On Burning Mirrors*, where a parabola is *qaṭ' al-makhrūṭ al-qā'im al-zāwiya*, up to minor variations. As Toomer (1976, 141) points out, this is a translation of τομὴ κώνου ὀρθογωνίου, which is the term for a parabola that seems to have been current before Apollonius' *Conics*. As for hyperbolas and ellipses, which appear only in Proposition 8 of *On Burning Mirrors*, the Arabic translation has *qaṭ' zā'id* and *qaṭ' nāqiṣ*, respectively,[65] and Toomer (1976, 167) is surely right in supposing that they could only be the translations of ἡ ὑπερβολή and ἡ ἔλλειψις, as opposed to τομὴ κώνου ἀμβλυγωνίου or τομὴ κώνου ὀξυγωνίου. It is reasonable to assume that the translator of Diocles knew to translate ἡ παραβολή as *qaṭ' mukāfi'*, even though this term does not appear in the Arabic text.

62. Here, the suffixed pronoun in *minhu* would refer to the conic section from which the ordinates are drawn.

63. That is, to be equal in square. This is used to state that the squares of the ordinates for a given conic section are proportional to the *latus rectum*.

64. See Figures 7 and 8 for examples.

65. The latter in 170 and 172 in Toomer (1976); not shown in Table 7.

However, the name of the parabola is transliterated again in Dtrūms' *Burning Mirrors and the Digest of the Conics*, without exception. Even though there are many differences in terminology between the three texts considered in Table 7, the transliteration of the name of the parabola in Text A and Dtrūms' *Burning Mirrors and the Digest of the Conics*, and the use of the «modern» terminology for hyperbolas and ellipses[66] in Diocles' *On Burning Mirrors* suggests that Text A and Dtrūms' *Burning Mirrors and the Digest of the Conics* were translated in an earlier phase of the study of conic sections among mathematicians of the 9th century Abbasid society, whereas Diocles' *On Burning Mirrors* was translated later. It should also be added that, since the first part of Dtrūms' treatise, up to and including Proposition 9,[67] are a digest of a now lost translation of Apollonius' *Conics*, this lost translation very likely also had transliterations for the names of the conic sections.

(*ii*) The Arabic term for ἡ ὀρθία πλευρά, *al-ḍil' al-muntaṣib min al-nisba*, which occurs once in Text A, is similar to the corresponding expressions in Dtrūms' *Burning Mirrors and the Digest of the Conics*, and different from those used in Diocles' *On Burning Mirrors*.

(*iii*) The Greek terms ὁ ἄξων, αἱ ἀσύμπτωτοι, δύνασθαι, and αἱ καταγόμεναι are consistently translated in Text A. Of these, *al-khaṭṭān alladhān lā yalqayānihi* (αἱ ἀσύμπτωτοι) is identical to the Arabic terms used in the translation of Diocles' *On Burning Mirrors*. *Qawiya 'alā* is consistently used to translate δύνασθαι in all three texts.

(*iv*) In the Arabic translation of Diocles' *On Burning Mirrors*, an axis is *al-khaṭṭ alladhī yaqsimuhu bi-niṣfayn*, up to minor variations. Toomer (1976, 141) plausibly suggests that this translates ἡ διχοτομοῦσα; he notes in this connection that τῇ διχοτομοῦσῃ διαμέτρῳ occurs in Apollonius' *Conics* as the axis of a hyperbola. If this is correct, then the different Arabic terms for axis in Text A and *On Burning Mirrors* is without significance for the identities of the translator(s) of the two texts.

66. *Qaṭ' zā'id* and *Qaṭ' nāqiṣ*, respectively.
67. See 2.3.

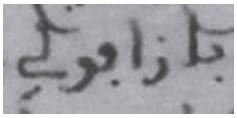

FIGURE 7: Transliteration of παραβολή, f. 26v, Escorial 960

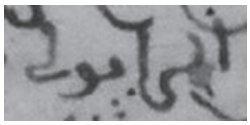

FIGURE 8: Transliteration of ὑπερβολή, f. 26v, Escorial 960

### *5.4 Text A: Algebraic Expressions*

Since algebraic expressions are used more than 100 times in Text A, Tables 8 and 9 only show the number of occurrences in Text A of terms indicating squares and products, with those used only once omitted for brevity.

TABLE 8: Number of occurrences of terms indicating squares in Text A

| Arabic (Text A, Escorial 960) | Number |
|---|---|
| *murabba' X*[68] | 54 |
| *X fī mithlihi* | 2 |

TABLE 9: Number of occurrences of terms indicating products in Text A

| Arabic (Text A, Escorial 960) | Number |
|---|---|
| *alladhī yakūnu min ḍarb X fī Y* | 22 |
| *alladhī yakūnu min X fī Y* | 4 |
| *al-mujtami' min X fī Y*[69] | 3 |
| *al-mujtami' min ḍarb X fī Y* | 8 |

68. Throughout this subsection and 5.6, *X* and *Y* denote quantities whose square or product is being taken. Typically they are pairs of points indicating the ends of a line or, less commonly, they are more complex phrases defining the quantity in question.

69. *Fī* is replaced once by *wa-min*.

Remarks:

(*i*) Throughout Text A, τὸ ἀπὸ AB, which designates the area of the square constructed on the given line segment AB, is translated, with very few exceptions, as *murabba‘ alif bā’*.

(*ii*) Exhibiting slightly less consistency than the previous item, τὸ ὑπὸ ΑΒΓ, which designates the area of the rectangle whose right sides are AB and ΒΓ, is translated most commonly as *alladhī yakūnu min ḍarb alif bā’ fī bā’ jīm*, but *alladhī yakūnu min alif bā’ fī bā’ jīm* and *al-mujtami‘ min ḍarb alif bā’ fī bā’ jīm* are also used. Shorter expressions derived from these are used much less frequently.

(*iii*) In Pappus’ solution, ὁ ἀπὸ τῆς AB κύβος, which designates the volume of the cube constructed on the given line segment AB, is always translated as *al-muka“ab alladhī ḍil‘uhu alif bā’*, a total of 10 times.

(*iv*) The algebraic expressions used in Text A are in almost complete agreement with those in Dtrūms’ *Burning Mirrors and the Digest of the Conics*, which uses *murabba‘ X* for the square of a line and *alladhī yakūnu min ḍarb X fī Y* for the product of one line by another. A few exceptions to these patterns are *al-murabba‘ alladhī yakūnu min X* (188) and *al-mujtami‘ min ḍarb X fī Y* (196, 200).

(*v*) The algebraic expressions used in Diocles’ *On Burning Mirrors* are different from those used in Text A and Dtrūms’ *Burning Mirrors and the Digest of the Conics*, and they exhibit more variation. The square of a line is most commonly expressed as *al-murabba‘ al-kā’in min X*; *murabba‘ X* is also used a few times. For the product of one line by another, *al-mujtami‘ min ḍarb X fī Y* is by far the most commonly used expression, with exceptions *mā yajtami‘u min ḍarb X fī Y* and *alladhī yakūnu min X fī Y*. For cubes, *On Burning Mirrors* uses *al-muka“ab al-kā’in min X*, in parallel to *al-murabba‘ al-kā’in min X* mentioned above, *al-muka“ab alladhī yakūnu min X* is a rare exception.

### *5.5 Text B: Conic Sections*

As I pointed out in 3.2, Eutocius’ commentary on *Sphere and Cylinder* II.4 uses conic sections extensively. Table 10 presents the Greek terms for conic sections and related objects in Eutocius’ commentary, along with their Arabic translations

used in Text B, as well as the Arabic terms used in the translation of Apollonius' *Conics*. Since the translation of Books I–IV of the *Conics* is due to Hilāl ibn Abī Hilāl al-Ḥimṣī and the translation of Books V–VII is due to Thābit ibn Qurra, I have given the Arabic terms in Apollonius' *Conics* from both Books I and V so that both translators may be represented, although revisions (*iṣlāḥ*) by the Banu Musa would presumably have a homogenizing effect on the terminology.

TABLE 10: Greek terms for conic sections and related objects, along with their Arabic translations in Text B and Books I and V of Apollonius' *Conics*

| Greek | Arabic (Text B, Escorial 960) | Arabic (Apollonius' *Conics* I & V) |
|---|---|---|
| ὁ ἄξων | *sahm* | *sahm* |
| αἱ ἀσύμπτωτοι | *al-khaṭṭān alladhān lā yaqa'āni 'alāyhi*[70] | *al-khaṭṭān alladhān lā yaqa'āni 'alāyhi*[71] |
| δύνασθαι | *qawiya 'alā* | *qawiya 'alā* |
| ἡ ἔλλειψις | *qaṭ' nāqiṣ* | *qaṭ' nāqiṣ* |
| αἱ καταγόμεναι | *khuṭūṭ tartībihi*,[72] *khuṭūṭ al-tartīb* | *khuṭūṭ al-tartīb*, *al-khuṭūṭ allatī tukhraju 'alā al-tartīb*, *khaṭṭ tartīb* |
| ἡ παραβολή | *qaṭ' mukāfi'* | *qaṭ' mukāfi'* |
| ἡ ὑπερβολή | *qaṭ' zā'id* | *qaṭ' zā'id* |

Remarks:

(*i*) The terms for the conic sections in Text B and the translations of Apollonius' *Conics* are in complete agreement, but the terms used in Text B stand in sharp contrast to those used in Text A, the transliterations of the Greek terms.

The terms for hyperbolas and ellipses used in Text B also agree with those used in the translation of Diocles' *On Burning Mirrors*. That the latter also uses *qaṭ' al-makhrūṭ al-qā'im al-zāwiya* does not indicate a divergence in terminology. Rather, it is a reflection of the underlying Greek term, which was probably ἡ τοῦ ὀρθογωνίου κώνου τομή, whereas for ellipses and hyperbolas ἔλλειψις and ὑπερβολή must have been used.

70. The suffixed pronoun in *'alāyhi* refers to the hyperbola; sometimes it is replaced by *al-qaṭ'*.
71. The suffixed pronoun in *'alāyhi* refers to the hyperbola; sometimes it is replaced by *al-qaṭ'*.
72. The suffixed pronoun in *tartībihi* refers to the conic section.

(*ii*) The terms «axis», «asymptotes», and «ordinates» are translated in Text B differently from Text A, but mostly identically to Apollonius' *Conics*.

(*iii*) δύνασθαι is consistently translated in all texts considered so far: Texts A and B, Apollonius' *Conics*, Diocles' *On Burning Mirrors*, and Dtrūms' *Burning Mirrors and the Digest of the Conics*. It might be conjectured that this is due to the fact that ἡ δύναμις, in the meaning of «square», is used widely in Euclid's *Elements* which, in all probability, was translated before any work considered in this article.

(*iv*) Despite the agreement in the translations of the conic sections between Text B and Diocles' *On Burning Mirrors*, there are important differences between the two, in the translations of «asymptotes» and «ordinates.»

(*v*) Comparison of Text B with Dtrūms' *Burning Mirrors and the Digest of the Conics* reveals similarities as well as differences: the latter translates παραβολή, as we have seen, which means it is unlikely to have been translated by the same individual as Text B. However, «axis» and «ordinates» are translated similarly in both texts.[73]

### *5.6 Text B: Algebraic Expressions*

As in 5.4, Tables 11 and 12 only show the number of occurrences in Text B of terms indicating squares and products, with those used at most 10 times omitted for brevity.

Table 11: Number of occurrences of terms indicating squares in Text B

| Arabic (Text B, Escorial 960) | Number |
|---|---|
| *murabba' X* | 60 |
| *al-murabba' alladhī yakūnu min X* | 51 |
| *al-murabba' al-kā'in min X* | 11 |

73. The use of *quṭr* for the axis of a parabola in Dtrūms' *Burning Mirrors and the Digest of the Conics* might reflect an underlying Greek ἡ διάμετρος and may not be meaningful in comparison.

TABLE 12: Number of occurrences of terms indicating products in Text B

| Arabic (Text B, Escorial 960) | Number |
|---|---|
| *alladhī yakūnu min ḍarb X fī Y*[74] | 70 |
| *al-saṭḥ alladhī yakūnu min ḍarb X fī Y*[75] | 13 |
| *al-mujtami' min ḍarb X fī Y* | 12 |

Remarks:

(*i*) For most of Text B, τὸ ἀπὸ AB is translated as *murabba' alif bā'* or *al-murabba' alladhī yakūnu min alif bā'*; the two expressions are roughly balanced. *Al-murabba' al-kā'in min alif bā'* occurs 11 times toward the end of Text B (ff. 41v–42v in Escorial 960).

(*ii*) To translate τὸ ὑπὸ ΑΒΓ, Text B most commonly uses *alladhī yakūnu min ḍarb alif bā' fī bā' jīm*, *al-saṭḥ alladhī yakūnu min ḍarb alif bā' fī bā' jīm*, and *al-mujtami' min ḍarb alif bā' fī bā' jīm*. Variants of these expressions are not used frequently enough to appear in Table 12.

The use in Text B of *saṭḥ* in the sense of «product», absent in Text A, is common in the treatises of the Banū Mūsā, as well as the extant Arabic translation of Books V–VII of Apollonius' *Conics*, made under the patronage of Aḥmad ibn Mūsā (Rashed 2017, 356, Toomer 1990, 867).

### *5.7 Conclusions*

Having compared some mathematical terms used in Texts A and B, in addition to those used in the Arabic translations of Apollonius' *Conics*, Diocles' *On Burning Mirrors*, and Dtrūms' *Burning Mirrors and the Digest of the Conics*, what conclusions can we draw from these comparisons? The following hypotheses seem to me to account for the observed differences in the most economical way:

(*i*) The separate transmission of some of the 11 solutions in Text A—those by Eratosthenes (Muwafi and Philippou 1981), or Archytas (Rashed 1996,

74. Only a critical edition of Text B would clarify whether the four instances of *alladhī yakūnu min X fī Y* are corruptions of this item.

75. Again, only a critical edition would clarify whether the three instances of *al-saṭḥ alladhī yakūnu min X fī Y* are corruptions.

116–121)—raises the question of whether different parts of Text A might have been translated by different individuals, possibly at different times and in different contexts. However, the uniformity of terminology observed in 5.1, 5.2, and 5.4 suggests that all 11 solutions contained in Text A were translated by the same individual, possibly as part of the same translation project. The same conclusion holds for Text B, in view of the uniformity of terminology examined in 5.5 and 5.6.

(*ii*) Texts A and B were almost certainly translated by different individuals. This is indicated by the differences in the translations of the names for the conic sections, other terms relating to the conic sections, and squares. One could perhaps hypothesize that Text B was translated by the same individual as Text A but revised by a different individual, but this implies that this reviser did such a thorough job of it that no term slipped through. In my view, this is certainly possible, but not very likely. Moreover, as I pointed out in 5.3, Text A may have been translated in an earlier phase of the study of conic sections among mathematicians of the 9th century, thus earlier than Text B.

(*iii*) The terms relating to conic sections in Text B are almost identical to those in the Apollonius translations. Hence, Text B is likely to have been translated, or revised, by an individual associated to the Banū Mūsā. As we have seen in 4.1, Naṣīr al-Dīn al-Ṭūsī's statement that Isḥāq ibn Ḥunayn translated the *Commentary* is consistent with this.

(*iv*) The terms relating to conic sections used in the translation of Diocles' *On Burning Mirrors*, Dtrūms' *Burning Mirrors and the Digest of the Conics*, and Texts A and B are different enough to conclude that the translators of Diocles or Dtrūms did not translate either Text A or Text B.

(*v*) The survey undertaken here has revealed no other parts of the *Commentary* known to have been translated into Arabic other than Texts A and B. And since these were translated (or revised) by different individuals, we might conjecture that the interests that led to the translations of the two texts were different. Moreover, the reason for the translation of Text B might have been related to the study of Book II of the *Sphere and Cylinder* itself (thus, Text B may have been translated after it).

(*vi*) Some of the information provided by the titles in the manuscripts and *Kitāb al-Fihrist* is false: Thābit ibn Qurra, an associate of the Banū Mūsā, is almost certainly not the translator of Text A, as claimed in BnF 2457; the title in Escorial 960 is relevant only to Text A and not Text B.

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