A COMPARATIVE TIME SERIES STUDY FOR MEDIUM TERM ELECTRICITY DEMAND FORECASTING FOR TURKEY

A Master's Thesis

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ABSTRACT

A COMPARATIVE TIME SERIES STUDY FOR MEDIUM TERM ELECTRICITY DEMAND FORECASTING FOR TURKEY

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In this study, a medium term Box-Jenkins time series model for electricty load demand for Turkey is developed and used to forecast for 24 month periods. Exponential Smoothing and Regression time series models are also developed and used for in-sample forecasting in order to evaluate the models' performances and compare them with each other. Box-Jenkins technique proved to be the best forecasting technique among the time series models for medium term horizon. The effect of forecast horizon on the forecast performance of the techniques is also analized and discussed. The motivation for this thesis is based on providing the best possible handy tool for intermediate term load forecasting which can be used by Turkish Electricity Authority and Energy and Natural Resources Ministry in Turkey .

Keywords: Forecasting, Time Series Modelling,

Science Code: 608.02.01

ÖZ

TÜRKIYE ORTA DÖNEM ELEKTRIK TALEBİ İÇİN KARŞILAŞTIRMALI ZAMAN SERİLERİ ÇALIŞMASI

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Bu tez çalışmasında, Türkiye'nin orta dönem elektrik talebi için bir Box-Jenkins zaman serisi modeli kurulmuş ve bu model 24 aylık peryod tahmini için kullanılmıştır. Eksponensiyel ve Regrasyon zaman serileri modelleri de kurularak, zaman serileri tekniklerinin performansının ölçülmesinde ve bu tekniklerin karşılaştırılmasında kullanılmak üzere veri içi tahminler yapılmıştır. Bu teknikler arasında Box-Jenkins tekniği en iyi sonucu vermiştir. Yine bu çalışmada, tahmin süresinin tahmin performansına olan etkisi analiz edilmiş ve tartışılmıştır. Bu tezde motivasyon, Türkiye Elektrik Kurumu ve Enerji ve Tabii Kaynaklar Bakanlığı tarafından orta dönem yük tahmini için kullanılabilecek mümkün olan en iyi araci sağlamaya yöneliktir.

Anahtar Kelimeler: Tahminleme, Zaman Serileri Modellemesi

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ACF	Autocorrelation Function
AR	Autoregression
ARMA	Autoregressive and Moving Average
ARIMA	Integrated Autoregressive and Moving Average
PACF	Partial Autocorrelation Function
MA	Moving Average
SARIMA	Seasonal Integrated Autoregressive and Moving Average
TEA	Turkish Electricity Authority

CHAPTER I

INTRODUCTION

1.1 Introduction

Load forecasting is an integral part of electric power system operations. Intermediate Term Forecasts are forecasts of a few months to 5 years ahead, which are needed for maintenance scheduling, coordination of power sharing arrangements and setting of prices, so that demand can be met with fixed capacity.

The focus of this thesis is the comparative time series study of medium term load forecasting for Turkish electric power demand using six different time series models.

The use of time series techniques in power engineering areas is not extensively explored in Turkey. The electric load forecasting in Turkey is made by the Ministry of Energy and Natural Sources. They are using a comprehensive software which handles all kinds of energy requirements like oil, coal, electricity, etc. On the other hand Turkish Electricity Authority (TEK) is responsible from the generation, transmission and distribution of electrical energy in Turkey.

The motivation for this thesis is based on providing the best possible handy tool for intermediate term load forecasting which can be used by

1. Turkish Electricity Authority can utilize this technique to obtain medium term forecasts and schedule its maintenance requirements, coordinate power sharing arrangements, and set prices.

2. Ministry of Energy and Natural Sources can utilize this technique to support its forecasts and also use this method in forecasts of demand for other energy sources.

Time series analysis has become popular over three decades because it has provided satisfactory solutions to a number of difficult problems. Some examples are forecasting, control, measuring the effects of policies, identifying leading indicators and using them efficiently, modelling systems with feedback in economics and engineering. In the twenty years since Box and Jenkins, and developments in computers made time series modelling a practical proposition. It has been applied to many problems in business, industry and the social and physical sciences.

The first time series technique to be widely applied was spectral analysis in the 1940's, followed by seasonal adjustment in the 1950's, exponential smoothing in the 1960's, Box-Jenkins modelling in the 1970 and 1980's. Today, Box-Jenkins and its derivatives are the most powerful time series techniques.

In this thesis work, Box-Jenkins method is used to forecast peak electricity load demand of Turkey for 24 month periods. This technique provided forecasts which has only 1.19 % mean absolute percentage error for two years forecast. In addition, the forecast performance of the Box Jenkins model is compared with other time-series models in Chapter IV, an Application of Box-Jenkins Time Seris Model. It proved that Box-Jenkins method is a powerful technique for peak load forecasting and useful for decision making.

1.2 Outline of the Study

In the first chapter, introductions to the load forecasting is given, the focus and motivation of this thesis are stated.

Second chapter summarizes the load forecasting techniques, time series techniques, the historical background of the subject, and gives drawbacks and advantages of available methods with reference.

In the third chapter, Box-Jenkins technique is described in detail and other time series methods are given.

Fourth chapter is about the application of the Box-Jenkins method. In this chapter, results of other time series techniques are also given and results are compared.

In the last chapter, the conclusions of the study are given.

CHAPTER II

LITERATURE SURVEY

Load forecasting can be classified according to the time for which forecasts are made:

1. Long Term Forecasts : They cover 5 to 20 years forecasts which are needed for scheduling construction of new generating capacity as well as the determination of prices and requlatory policy.

2. Intermediate Term Forecasts : They cover a few months to 5 years forecasts. In intermediate term capacity is fixed. These are used for maintenance scheduling, coordination of power sharing arrangements and setting of prices. In the ballance of this thesis we used intermediate term forecast for Turkish electricity demand.

3. Short Term Forecasts : They are forecasts of a few hours to a few weeks ahead, which are needed for economic scheduling of generating capacity, scheduling of fuel purchases, security analysis and short term maintanance scheduling.

4. Very Short Term Forecasts : They are forecasts of a few minutes to an hour ahead, which are needed for real time control and real time security evaluation.

Load forecasting can also be classified according to the methods:

1. Econometric Models : These models are also called multiple reqression models. Econometric models depends on relationships between electricity consumption and certain characteristics of the economy. Explanotary variables may be gross national product, per capita income, employment levels, manufacturing and construction indexes, electricity price elasticities, alternative cross elasticities, wholesale and retail trade volumes, etc.

2. End-Use Models: Econometric Models may be considered "top down aproaches to forecasting. End-Use models, whereas, follow a "bottom up" methodology. Typically, they examine usage and consumption patterns for individual types of appliances or equipment at the actual point of consumption. End-Use models may include a wide variety of appliances and equipment for residential, commercial, industrial, and government sectors. Individual data bases may be developed for each

appliance type and each sector or subsector, causing the logictics of data handling and updating to become formidable. More advanced models analyze the effects on usage and load of population changes, future industrial activity, equipment efficiencies, competitive fuel availability, appliance saturation levels, load shapes, conservation, load management techniques, and relationships between the various sectors.

3. Qualitative Models: These models do not use mathematical techniques but opinions of experts. Delphi method, Cross-Impact Matrices and The Juster Survey Approach are some examples. Among these, The Delphi Method is the most common one. In this approach the experts doing the forecasting form a panel or group of technical experts and then deal with specific questions. Unlike many forecasting methods, does not have to produce a single answer as its output. Instead of reaching a consensus, the Delphi approach can leave a spread of opinions. . The objective is to narrow down the quartile range as much as possible without pressuring the respondent. The Delphi method is by no means without disadvantages. The general complaints against it have been insufficient reliability, over sensitivity of results to ambiguity of questions, difficulty in assessing the degree of expertise and the impossibility of taking into account the unexpected.

Cross-Impact Matrices are closely related to the Delphi method. A Cross-impact matrix describes two types of data for a set of possible future developments. The first type estimates the probability that each development will occur within some specified time period in the future. The second estimates the probability that the occurences of any one of the potential developments would have an effect on the occurance of each of the others. The aim of cross-impact analysis is to refine the probabilities relating to the occurance of individual future developments and their interaction with other developments. Therefore, these probabilities can be used either as the basis for planning or forecasting.

The Juster Survey Approach tries to solve the difficulty with a required yes - no response to survey questions. It assigns the probabilites to the descriptive words and compiles the answers to get a probabilistic forecast.

4. State Space Models: State space models originally developed by control engineers. However they have also been found to be useful in many types of time series problem, such as short-term forecasting.

5. Time Series Models: A 'time series' is a time-ordered

sequence of observations that have been taken at regular intervals over a period of time (hourly, daily, weekly, monthly, quartely, and so on). Forecasting techniques that are based on time series data are derived on the assumption that future values of the series can be estimated from past values of the series. In spite of the fact that no attempt is made to identify variables that influence the series, these methods are widely used, often with very satisfactory results.

Analysis of time series data requires the analyst to identify the underlying behaviour of the series. This can often be accomplished by merely plotting the data and visually examinig the plot. One or more of the following behaviours might appear; trend, seasonal variations, cycles and variations around an average. In addition there can be random and irregular variations. These variations can be described as follows:

1. "Trend" refers to a gradual, long term movement in the data.

2. "Seasonality" refers to short term, fairly regular, periodic variations that are generally related to weather factors or to man made factors such as holidays and vacations.

3. "Cycles" are wave-like variations of more than one year's duration. These are often related to a variety of economic and political factors and even to severe weather conditions.

4. "Irreqular" varitions are due to unusual cirsumstances such as severe weather conditions, strikes, or a major change in a product or service. They do not reflect typical behaviour, and inclusion in the series can distort the overall picture.

5. "Random" variations are the residual variations that remain after all of the other behaviours have been accounted for.

The Naive Approach, Moving Averages, Simple Exponential Smoothing, Holt's Exponential Smoothing, Winter's Exponential Smoothing, Decomposition of a time - series, Simple Linear Regression, Curvilinear Regression, Exponential Regression and Box-Jenkins Approach are examples of the time series techniques.

Among these, Box-Jenkins is the most powerful time series technique used for forecasting. It is described and analyzed in detail in Chapter III. A Box-Jenkins model will be developed on the monthly peak load demand. By this model, 24 months forecasts will be obtained. In addition, the forecast performance of the Box Jenkins model will be compared with other time-series models.

In the beginning of this chapter, forecasts were classifed according to the time for which forecasts are made and methods applied. In the presentation of literature summary, we follow the classification according to the method by which forecast are made since some techniques can be applied to more than one forecast time frame. As an example, Box-Jenkins method can be applied to long term, intermediate term and short term forecasts. No forecast examples is given on qualitative forecasting.

2.1 Econometric Models

Econometric model, as stated in the introduction chapter, depends on relationships between electricity consumption and certain characteristics of the economy. Some of the works based on this method is summarized chronologically.

Houthakker (1951) studied the residential electricity consumption in the United Kingdom, using 1937 to 1938 datas. His model included income, price of electricity and gas, and average holdings of heavy domestic equipment per customer.

Hauthakker and Taylor (1970) estimated an equation for personel consumption expenditure for electricity. This model consists of two equations. The first one is a behavioral relationship that specifies consumption as a function of stocks, income and relative price. The second one is a relationship that expresses the rate of change in stocks to consumption and depreciation

Baxter and Rees (1968) used tree alternative models for electricity demand. The first one related output to capital, labor, oil, gas, coal and electricity. They used Cobb-Doglas production function for this model. The second model searched for the effects of changes in fuel technology. In this model, electric power consumption is related to output and a surrogate for technology in place of input prices. Since during the period studied, most of the substitution in enery had been against coal, coal consumption was employed as the surrogate. Finally, the third model was constructed on the hypothesis that there is a proportional relationship between changes in output and electricity consumption and that deviations from this relationship are induced by changes in relative prices and changes in labor and capital intensity. Baxten and Rees concluded from their analysis that relative price changes are not unambiguously an important determinant of growth in output and changes in technology.

Wilson (1971) analized the residential demand for electricity and also the residential demand for six different categories of household appliances.

Anderson (1971) analized the producer's demand for energy by the U.S. primary metals industry. It's data set covered 1958 to 1962 datas. The independent variables were price of coal, coke, oil, electricity and average wage rate. The price elasticity of demand for electricity was negative, substantial, and highly significant statistically.

Lyman (1973) analized the demand for electric power for the three major consumer classes; residential, commercial, and industrial. Lyman assumes that demand is related to price of electricity and gas, index of other prices, and vector of economic, demographic and climatic variables.

Lyman found that price elasticities of demand are typically elastic for each of the customer classes and, for residential demand, is positively correlated with income.

2.2 End-Use Models

Fisher and Kaysen (1962) analized the residential and industrial demand for electricity. They were the first to distinquish explicitly for residential demand, between the shortrun and the longrun. The demand for electricity in the short run is identified with choice of utilization rate for the existing stock of electricity - consuming capital goods, while demand in the long run is identified with choice of the size of the capital stock.

The equation was estimated for five classes of white goods, namely, electric washing machines, electric refrigerators, electric ironers, electric ranges, and electric water heaters using annual data.

They concluded that, in general, net changes in the stock of appliaences seem mainly to depend on changes in longrun income or changes population and in the number of wired households per capita. The price of electricity seems to have nearly no effect: the prices of appliances only relatively.

2.3 State Space Models

In state-space models, the actual observation is the sum of signal and noise. The signal is taken to be a linear combination of a set of variables, called state variables, which constitute what is called the state vector at time t. This vector describes the state of the system at time t, and is sometimes called "the state of the nature".

$$X_t = h_t^T \theta_t + \eta_t \tag{2.1}$$

where,

X_t = observation

θ_t = state vector (mx1)

h_t = assumed to be a know vector (mx1)

η_t = observation error.

The state vector θ_t which is of prime importance is unobservable. Therefore, the observations on X_t is used to make inferences about θ_t. θ_t is assumed to be changing through time:

$$\theta_t = G_t \, \theta_{t-1} + w_t \tag{2.2}$$

where

G_t : is assumed to be known matrix (mxm)

w_t : a vector of deviations.

The equations constitute the general form of the state-space model. First equation is called the observation equation and the last one is called the transition equation.

The state-spece model can be generalized to the case where X_t is a vector by making η_t a matrix of appropriate size and by making n_t a vector of appropriate length. It is also possible to add terms involving known linear combinations of explanatory variables to the rigth - hand side of the first equation. They include regression and Box-Jenkins Autoregressive Integrated Moving Average ARIMA models as well as the sort of trend-and-seasonal model of which exponential smoothing methods are thought to be appropriate. Bayesian forecasting (Kalman Filter) also relies on what is essentially a state-space representation, while some models with time varying coefficients can also be represented in this way.

Harvey (1984) has described a general class of trend-and-seasonal models which involve the classical decomposition of a time series into trend, seasonality and irregular variation, but which can also be represented as state-space models.

Note that the decomposition must be additive in order to get a linear state-space model. If for example the seasonal effect is thought to be multiplicative then logarithims must be taken in order to fit a structural model, altough this implicitly assumes that the 'error' terms are also multiplicative. A key feature of linear state-space models is that the observation equation involves linear function of the state variables. It does not restrict the model to be constant through time. Rather it allows local features, such as trend and seasonality, to be updated through time using the transition equation.

In general, state-space approaches concantrates on the short-term load forecasting. This approach is well suited for on-line applications. Another advantage of forecasting with this approach is that the Kalman filtering theory can be used to obtain forecasts.

Scheweppe and Wildes (1970), and Larson (1970) suggested that Kalman filter would be useful for short term load forecasting. Togoda, Chenn and Inouye (1970) made the first study of the application of the state estimation. They considered the state variables to be the system, the increment of the system load, and the short-term and long-term load patterns. They proposed different models for different time frames.

The main with the Kalman filtering theory is that the noise covariances are unknown. Mefira (1970) developed an algorithm to identifiy the noise covariances, which is based on calculating the autocorrelations of the observations.

Toyoda and Chen (1970) stutied on similar models for ten-minute, hourly, and yearly load forecasting. They used the adaptive state estimation technigues for the optimization of forecasting, and also examined the correlations between forecasting errors and weather conditions to improve the accuracy.

Sharma and Mahalanabis (1972) proposed a state-space approach for short-term load forecasting. They did not assume that the system matrix is known. In order to determine the system matrix, they assumed that the power load can be approximated by a finite order polynomial and the ratio of the change in the load demand to the actual load demand is approximately the same for all weekdays at any instant. Since the state and observation equations were known, adaptive estimation techniques were used for load predictions.

Gupta and Yamada (1972) proposed two sophisticated techniques to obtain estimates of the hourly load up to 24 hours in advance.

Singh, Biswas and Mahalanabis (1978) proposed a model which uses Kalman filtering theory. They assumed that the load demand can be represented by a time series model and they developed a predictor which identifies the coefficients of the time series in an on-line fashion. The load demand x(k) at any time can be expressed by an autoregressive model.

Galina, Hanschin and Fiecter (1974) proposed a methodology for obtaining load forecasts up to one week in advance. The total load was the sum of a periodic term and a residual term.

They expressed their model in state space form. The model parameters and order were estimated using a maximum likelihood approach.

Panuska and Koutchouk (1975) and Panuska (1977) used the same model proposed by Galiana and his friends. They tried to reduce the computation effort needed for model identification and forecasting.

2.4 Time Series Models

Time series models were classified in the beginning of this chapter Of these, spectral decomposition, exponential smoothing and Box-Jenkins approaches are worth to note in the literature since these time series techniques applied widely for forecasting purposes. Therefore they are described briefly in the following sections.

2.4.1 Spectral Decomposition

Time series many have one or more of the following patterns; trend, seasonality, cycles, irregular variations, and random variations.

Farmer (1968) divided the load demand into three components; a long term trend, a component varying with the day of the week, and a random component, in his short term forecasting load demand study. The load in the w th week of the year n on the d th day of the week and at time of the day (t) :

$$Ywd(t) = Aw(t) + Bd(t) + Xwd(t) \tag{2.3}$$

where

Aw(t) : trend term which is updated weekly

Bd(t) : a term dependent on the day of the week

Xwd(t): the residual component.

Lijesen and Arvanitidis (1970), and Lijesen and Rosing (1971) used a method similar to that of Farmer. They assumed that residual component depends on the significant weather variables, which differed from Farmer's.

2.4.2 Exponential Smoothing Method

Exponential Smoothing method is explained in detail in Chapter III. In this section, a brief literature summary of this method in the application of forecasting load demand is given.

In literature, the application of exponential smoothing method to load demand forecasting is mostly in short term. Christiaanse (1971) developed a model for the hourly loads over an interval of one week.

The weekly variations in hourly load were described as a cyclic function of time with a period of one week. The model was :

$$X(t) = c + \sum_{i=1}^{m} (a_i \; Sin \; w_i t + b_i \; Cos \; w_i t) \qquad (2.4)$$

where c is a constant and the rest is a Fourier series with "m" frequecies. All ws are of the form

$$w_i = \frac{2\pi}{168} K_i \; , \qquad (2.5)$$

where,

K_i: a positive integer less than 168, the Niquist limit.

Forecasts for lead time L are in the form:

$$X(t+1) = a(t) \; f \; (t+1) \qquad (2.6)$$

where,

$$f = \begin{bmatrix} Sinw1t \\ Cosw1t \\ . \\ . \\ Sinwmt \\ Coswmt \end{bmatrix}$$

and a is row vector containing the estimates of the parameters in the model. These parameters were estimated in such a way as to minimize the square of the residuals, using a weighted least-squares criterion :

$$\sum_{j=0}^{T} \beta^j \, [\, x(T-j) - a(T) \; F(-j) \,]^2 \qquad (2.7)$$

where a is a smoothing costant between zero and one.

The accuracy of the forecasts depends heavily on the smoothing constant. To improve the accuracy of the forecasts, Christiaanse suggested an extension to this method. He used an autoregressive model (with lags of 1 h and 24 h) to forecast the errors after studying the autocorrelation of the observed residuals for shorter lead times.

Sachdev and Ibrahim (1972) attempted to reduce the computing effort and memory. They proposed an on-line technique using the exponential smoothing models in two stages.

Phi, Austin, Mcllister, Thorne and Patterson (1978) used the same techniqe proposed by Cristiaanse to forecast hourly loads up to 1 week into the future. They proposed a simple method to adjust the load forecasts in order to account for temperature effect on the load. One year data was used to arrive at a megawatt per degree difference adjustment for each month. The effect of temperature on the load forecasts can be obtained with the help of forecasted temperatures.

2.4.3 Box-Jenkins Method

Box-Jenkins method, as other time series techniques, is explained in detail in Chapter III, Box-Jenkins Technique as the Proposed Method of Analysis, to provide a clear understanding.

Box-Jenkins model was first used by Stanton and Gupta (1969), Standon (1971) and Gupta (1971) to forecast the intermediate and long term load demand of a power system. Vemuri, Balasubramanian and Hill (1973) applied Box-Jenkins techniques to intermediate term load forecasts.

They used an Seasonal Autoregressive Integrated Moving Average SARIMA $(0,1,1)_1 x (0,1,1)_{12}$ model with a twelve month period to forecast monthly peak loads for lead times of up to 40 months. The same model with different parameter values was used by Uri (1974) to forecast monthly average loads for lead times of up to 2 years.

Keyhani and El-Abiad (1975) used Autoregressive Moving Average ARMA models for very short term forecasts. An ARMA (1,0) model is used on 1 minute load data. ARMA (1,1) and (2,1) models are used on 5 minute data, and an ARMA (2,0) model is used on hourly load data.

Keyhani and Rad (1977) included some weather inputs and trigonometric trend functions to Autoregressive AR models in order to forecast hourly loads up to one week ahead. The model was of the following form.

$$Z_t = 0.026 + 1.36\ Z_{t-1} - 0.45\ Z_{t-2} - 0.33\ Z_{t-167} + 0.0008\ Z_{t-168} + 0.289\ X_{t-4} \qquad (2.8)$$

where X_t is hourly temperature.

Hagan and Klein (1977) developed ARIMA models with a daily period to forecast hourly loads with one to four hour lead times. They later extended the models to include a weekly, a daily period and temperature inputs in a transfer function model.

A method for adaptively updating the parameters in an ARMA or a Transfer Function model was proposed by them. Then, it become possible for load models adapting continously to changing seasonal patterns. Hence Box-Jenkins models become well suited to on-line applications.

Meslier (1978) proposed ARIMA models $(1,0,0)_1 x (0,1,1)_7 x (0,1,1)_{365}$ to forecast daily energy consumption one day ahead. He also proposed using a transfer function model of the form

$$(1-\sigma_1 B)v_t = W_0(1-W_7 B)(1-W_{365} B)X_t + N_t \qquad (2.9)$$

$$(1-\phi_{365} B^{365})\nabla\nabla_7 N_t = (1-\theta_1 B)(1-\theta_7 B^7)a_t \qquad (2.10)$$

where v_t is consumption which has been corrected to account for holidays, and X_t is temperature.

Uri (1978) and Maybee and Uri (1979) used periodic ARIMA models to forecast load demand one week ahead and the load duration curve one year ahead.

Abu-El-Magd and Sinha (1981) used multivariate AR models for forecasting load demands of a multinode system. They proposed an ARI (6,1) model to forecast the load demand of four loading stations at five minute intervals and an AR(2) model with 24 hour differencing to forecast at 1 hour intervals.

Vemuri, Huang and Nelson (1981) developed a method for on-line identification of autoregressive models using sequential least squares. They used AR (10) models on 3 hour load data to forecast up to 21 hours ahead.

CHAPTER III

BOX-JENKINS TECHNIQUE AS THE PROPOSED METHOD OF ANALYSIS

In this chapter, Box-Jenkins technique, which is the proposed method, and other time series techniques are described. The reason that other time series are presented is that in chapter IV performance of some methods are evaluated and compared with that of Box-Jenkins'.

3.1 Naive Approach

In this approach, the latest observation in a sequence is used as the forecast for the next period. The naive method can also be extended to seasonal and trend variations. For instance, if seasonality is the primary factor, the forecast for the upcoming season would equal demand in the preceding season. The naive approach can serve a useful purpose by providing a standart against which other forecasting alternatives can be measured.

3.2 Moving Averages

Historical data typically contain amount of random variation or noise that tends to distort systematic movements in the data. This randomness arises from the combined influence of many relatively unimportant factors. Ideally, it would be desirable to be able to completely remove any randomness from the data and leave only "real" variations. Averaging techniques smooth out some of the fluctuations in a time series, because the individual highs and lows in the data 'moving avarage' is an average that is repeatedly updated: as new observations become available, the oldest values in the series are delated, thereby keeping the average current. Two common types of moving averages are the simple moving average and the weighted moving average.

Using a simple average approach involves computing the average value of a time series for a certain number of the most recent periods. Thus, 3-period moving average would require determining the average of the three previous values.

A second kind of moving average is the weighted average. This is similar to simple moving average. Weighted moving average generally weights the most recent observations more heavily than older ones whereas simple moving average assigns an equal weight to each obsevation.

Averaging techniques have two fundamental characteristics that users should be aware of. One is that they smooth variations in the data, and the other is that they do not react immidiately to changing patterns in the data. The extent of smoothing and lagging are a function of the number of periods included in the average: the more periods, the greater the smoothing and lagging, and vice versa. The choice of number of periods to include in a moving average will depend on such things as the cost of not reacting quickly to changes relative to the cost of reacting when no real changes are present.

3.3 Decomposition of a Time Series

In this classical approach to the analysis of historical data, a time series is viewed as being made up of four possible components trend, seasonality, cycles, and random plus irregular variations.

There are two models for describing how these components are combined in a series. The 'additive' model treats the series as the sum of components

$$Y=T+S+C+r \qquad (3.1)$$

where

T = Trend component

S = Seasonal

C = cyclical

r = Residual

The 'multiplicative' model treats the series as the product of the components :

$$Y=T*S*C*r \qquad (3.2)$$

A trend curve can be obtained by using a least squares appraoch. The curve may have the equation

$$yt=a+bt \quad \text{or,} \qquad (3.3)$$

$$yt=a+bt+ct^2 \qquad (3.4)$$

depending on the shape of trend.

For the multiplicative model, the observation values are divided by this trend value :

$$Y/T=\frac{T*S*C*r}{T}=S*C*r \qquad (3.5)$$

If there is no cyclic chages, it becomes

$$Y/T = \frac{T*S*r}{T} = S*r \qquad (3.6)$$

Then grouping the seasonals by quarters, months or weeks depending on the type of data, and averaging removes the inherent randomness.

In practice least square approach often is not used. Instead, centered moving average is used to eliminate trend from a time series for several reasons. One is that cycles may be present, which would distort the computation of seasonal relatives. Another reason is that a centered moving average can handle changes in trend.

For cyclical variations that are fairly reqular, the analysis of cycles is essentially identical to the analysis of seasonality, where the length of a "season" become the length of the cycles.

3.4 Exponential Smoothing

Exponential smoothing is a widely used method of forecasting. In actuality, it is a form of weighted moving average and it is superior to the methods described previously. It's superiority comes from the ease with which weighting patterns can be altered to meet the needs of a particular situation.

The name exponential smoothing is derived from the way weights are assigned to historical data. The most recent values receive most of the weight and weights fall off exponentially as the age of the data increases.

Each new forecast is based on the previous forecast plus a percentage of the difference between that forecast and the actual value of the series at that point. That is :

New forecast=Old forecast+α(Actual-Old forecast)

where α is a percentage and (Actual-Old Forecast) represents the forecast error. More concisely, we have

$$F_t=F_{t-1}+\alpha(A_{t-1}-F_{t-1})$$

where

α = smoothing constant

t = current period

A_{t-1}= Previous (actual) value of the time series

F_{t-1} = Previous Forecast

Expanding the equation :

$$F_t=\alpha A_{t-1}+(1-\alpha)F_{t-1}$$

$$=\alpha A_{t-1}+(1-\alpha)[\alpha A_{t-2}+(1-\alpha)F_{t-2}] \quad \text{and so on.}$$

$$=\alpha A_{t-1}+\alpha(1-\alpha)A_{t-2}+\alpha(1-\alpha)^2A_{t-3}+ \ldots \qquad (3.8)$$

The responsiveness of an exponential forecast to changes in a time series is totally dependent on the value of the smoothing constant. Small values of the smoothing constant cause a slow response to discrepancies between actual and forecast values.

The choice of a smoothing constant depends on the extent to which the forecaster feels future changes will reflect mainly random variations versus the extent to which "real" changes may occur. One important limitation of simple exponential smoothing is that it is ill-suited for data that include long-term upward or downward movements (ie, trend). Use of simple exponential smoothing in such instances would produce forecasts that were too low for upward movements and too high for downward movements. Therefore, when trend is present in time series data, simple exponential smmothing should not be used. Instead, exponential smoothing adjusted for trend should be used. Holt's two-parameter exponential smoothing method is an extension of simple exponential smoothing; it adds a growth factor (or trend factor) to the smoothing equation as a way of adjusting for the trend. Three equations and two smoothing constants are used in the model.

$$F_{t+1}=\alpha X_t+(1-\alpha)(F_t+T_t) \quad (3.9)$$

$$T_{t+1}=\beta(F_{t+1}-F_t)+(1-\beta)T_t \quad (3.10)$$

$$H_{t+m}=F_{t+1}+mT_{t+1} \quad (3.11)$$

where,

H_{t+m}= Holt's forecast value for period t+m

F_{t+1}= Smoothed value for period t+1

T_{t+1}= Trend estimate

α = Smoothing constant for the data

β = Smoothing constant for the trend estimate

m = number of periods ahead to be forecast

First equation adjust F_{t+1} for the growth of the previous period, T_t, by adding T_t to the smoothed value of the previous period, F_t. The trend estimate is calculated in second equation, where the difference of the last two smoothed values is calculated. Because these two values have already been smoothed, the difference between them is assumed to be an estimate of trend in the data. The second smoothing constant, β in second equation, is arrived at by using the same principle used in simple exponential smoothing. The most recent trend, $(F_{t+1} - F_t)$, is weighted by $(1-\beta)$. The sum of the weighted values is the new smoothed trend value T_{t+1}.

The last equation is used to forecast m periods into the future by adding the product of the trend component, T_{t+1}, and the number of periods to forecast, m, to the current value of the smoothed data F_{t+1}.

Winter's exponential smoothing model is the second extension of the basic smoothing model; it is used for data that exhibit both trend and seasonality; it is a three-parameter model that is an extension of Holt's model. An additional equation adjust the model for the seasonal component. The four equations necessery for Winter's model are:

$$F_t=\alpha X_t/S_{t-m}+(1-\alpha)(F_{t-1}+T_{t-1}) \quad (3.12)$$

$$S_t=\beta X_t/F_t+(1-\beta)S_{t-m} \quad (3.13)$$

$$T_t=\gamma(F_t-F_{t-1})+(1-\gamma)T_{t-1} \quad (3.14)$$

$$W_{t+m}=(F_t+mT_t)S_t \quad (3.15)$$

where

F_t = Smoothed value for period t

F_{t-1} = Forecast value for time period t-1

X_t = Actual value in period t

T_t = Trend estimate

S_t = Seasonality estimate

W_{t+m} = Winter's forecast for m periods into the future

α = Smoothing constant for the data

β = Smoothing constant for the trend estimate

γ = Smoothing constant for seasonality estimate

m = number of periods ahead to be forecast

First equation updates the smoothed series for both trend and seasonality. X_t is divided by S_{t-m} to adjust for seasonality; this operation deseasonalizes the data or removes any seasonal effects left in the data. The seasonality estimate is smoothed in the second equation and the trend estimate is smoothed in the third equation. Each of these processes is exactly the same as in simple exponential smoothing. The final equation is used to compute the forecast for m periods into the future.

3.5 Simple Linear Regression

The objective in linear regression is to obtain an equation of a straight line that minimizes the sum of squared vertical deviations of points around the line.

$$X_t = \alpha + \beta T + e_t \quad (3.16)$$

Ordinary Least Squares method is applied to obtain estimates of α and β. A brief description of this method can be found in Appendix A.

Use of simple regression analysis implies that certain assumptions have been satisfied. Basically these are:

1. Variations around the line are random. If they are, no patterns such as cycles should be apperent when the line and data are plotted

2. Deviations around the line should be normally distributed.

3.6 Curvilinear Regression Analysis

Simple linear regression may prove inadequate to handle certain problems because a linear model is inappropriate. When nonlinear relationships are present, curvilinear regression is used. As in linear regression, the explanotory variables should be tested for significance.

$$X_t=\alpha+\beta_1T+\beta_2T^2+e_t \tag{3.17}$$

In the estimation of α, β_1 and β_2, OLS method is used. The assumptions that should be satisfied for simple regression analysis are also valid for curvilinear regression analysis.

3.7 Exponential Regression Analysis

Sometimes the use of exponential regression analysis may yield better results than curvilinear regression analysis. This is due to the fact that the trend curve is closer to exponential function than the second degree polynomial function.

$$X_t=e^{(\alpha+\beta T)}+e_t \tag{3.18}$$

Ordinary Least Squares or Maximum Likelihood methods can be applied to obtain estimates of α and β. The assumptions for curvilinear regression analysis are valid for this technique as well.

3.8 Box-Jenkins Models

In this section, Box-Jenkins models are described in detail. Before giving the theory behind Box Jenkins models, some introductory definitions related with that approach are given.

3.8.1 Stationarity

Broadly speaking a time series is said to be stationary if there is no systematic change in mean (no trend), if there is no systematic change in variance, and if strictly periodic variations have been removed. The theory of Box-Jenkins models is concerned with stationary time-series. Therefore, if the series is nonstationary, then it should be turned into a stationary one.

A time series is said to be 'strictly stationary' if the joint distribution of $X(t_1),...,X(t_n)$ is the same as the joint distribution of $X(t_{1+T}),...,X(t_{n+T})$ for all $t_1,...,t_n,T$. In other words, shifting the time origin by an amount T has no effect on the joint distributions, which must therefore depend only on the intervals between $t_1,t_2,...t_n$. The above definition holds for any value of n.

In particular, if n=1, it implies that the distribution of X(t) must be the same for all t, so that

$$\mu(t) = \mu \tag{3.19}$$

$$\sigma^2(t) = \sigma^2 \tag{3.20}$$

are both constants which do not depend on the value of t.

A process is called second-order stationary (or weakly stationary) if its mean is constant and its autocorrelation function depends only on the lag, so that

$$E\,[X(t)]=\mu \tag{3.21}$$

and

$$Cov\,[X(t),X(t+\tau)]=\gamma(\tau). \tag{3.22}$$

To obtain stationary time series from a nonstationary time series, transformation techniques should be applied. Before applying these techniques, the first and most important step in any time-series analysis is to plot the observations against time. This graph should show up important features of the series such as trend, seasonality, outliers and discontinuities.

The three main reasons for moving a transformation are as follows:

1. To stabilize the variance:

If there is trend in the series and the variance appears to increase with the mean, then it may be advisable to transform the data. In particular, if the standart deviation is directly proportional to the mean, a logaritmic transformation is indicated.

2. To make the seasonal effect additive

If there is a trend in the series and the size of the seasonal effect appears to increase with the mean then it may be advisable to transform the data so as to make the seasonal effect constant from year to year. The seasonal effect is then said to be additive. In particular if the size of the seasonal effect is directly proportional to the mean, then the seasonal effect is said to be multiplicative and a logarithmic transformation is appropriate to make the effect additive.

3. To make the data normally distributed

Model building and forecast are usually carried out on the assumption that the data are normally distributed. In practice this is not necessarily the case, there may for example be evidence of skewness in that there tend to be 'spikes' in the time plot which are all in the same direction. This effect can be difficult to eliminate and it may be necessary to assume a different 'error' distribution.

3.8.2 Analysing Series Which Contain a Trend

The simplest trend is the familiar 'linear trend+noise', for which the observation at time t is a random variable Xt given by

$$X_t=\alpha+\beta_t+\varepsilon_t \qquad (3.23)$$

where α, β are constants and ε_t denotes a randon error term with zero mean. Alternatively, the trend may be of non-linear form such as quadratic growth. Exponential growth can be particularly difficult to handle, even if logarithims are taken to transform the trend to a linear form.

3.8.2.1 Filtering

A second procedure for dealing with a trend is to use a 'linear filter' which converts one time series, X_t, into another, Y_t, by the linear operation.

$$Y_t = \sum_{r=-q}^{+s} a_r \, X_{t+r} \tag{3.24}$$

where, a_r is a set of weights. In order to smooth out local fluctuations and estimate the local mean, one should clearly choose the weights so that $\Sigma a_r=1$ and then the operation is often referred to as a moving average. Moving averages are often symmetric with s=q and $a_j=a_{-j}$. The simplest example of a symmetric smoothing filter is the simple moving average, for which $a_r=1/(2q+1)$ for r=-q...+q, and the smoothed value of X_t is given by

$$Sm\ (X_t) = \frac{1}{2q+1} \sum_{r=-q}^{+q} X_{t+r} \tag{3.25}$$

There are other kinds of moving average techniques as described in this chapter. Another example would be exponential smoothing :

$$Sm(X_t)=\sum_{j=0}^{\infty} \alpha(1-\alpha)^j \; X_{t-j} \qquad (3.26)$$

where α is a constant such that $0 < \alpha < 1$. Here we note that the weights $a_j=\alpha(1-\alpha)$ decrease geometrically with j. Having estimated the trend, we can look at the local fluctuations by examining,

$Res(X_t)$ = residual from smoothed value

$$= X_t-S_m(X_t)$$

$$= \sum_{r=-q}^{+s} b_r \; X_{t+r} \qquad (3.27)$$

Choosing the appropriate filter requires considerable experience plus a knowledge of the frequency aspects of time-series analysis.

Very often a smoothing procedure is carried out in two or more stages so that one has in effect several linear filters in series. For ewample two filters in series may be represented as follows:

X_t [I] Y_t [II] Z_t

3.8.2.2 Differencing

A special type of filtering, which is particularly useful for removing a trend, is simply to difference a given time series until it becomes stationary.

This method is an integral part of the procedures advocated by Box and Jenkins for non-seasonal data, first order differencing is usually sufficient to obtain apparent stationarity, so that the new series $(Y_1,...,Y_{n-1})$ is formed from the original series $(1,..,X_n)$ by

$$Y_t=X_{t+1}-X_t=\nabla X_{t+1} \qquad (3.28)$$

First-Order differencing is widely used. Occasionally second order differencing is required using the operator ∇^2 , where

$$\nabla^2 X_{t+2}=\nabla X_{t+2}-\nabla X_{t+1}=X_{t+2}-2X_{t+1}+X_t \qquad (3.29)$$

3.8.3 Analysing Series Which Contain Seasonal Variation

Seasonal models may be in the form of additive or multiplicative for series showing little trend. It is usually adequate to estimate the seasonal effect for a particular period by finding the average of each particular observation minus the corresponding yearly average in the additive case, or the particular observation divided by the yearly average in the multiplicative case.

For series which contain a substantial trend, a more sophisticated approach may be required. With monthly data, the commonest way of eliminating the seasonal effect is to calculate

$$Sm(X_t)=\frac{0.5X_{t-6}+X_{t-5}+\ \ldots\ X_{t+5}+0.5X_{t+6}}{12} \quad (3.30)$$

Note that the sum of the coefficients is 1.

A seasonal effect can also be eliminated by differencing. For example with monthly data one can employ the operator ∇_{12}, where

$$\nabla_{12}X_t=X_t-X_{t-12} \quad (3.31)$$

3.8.4 Autocorrelation

An important guide to the properties of a time series is provided by a series of quantities called sample autocorelation coefficients, which measure the correlation between observations at different distances apart. These coefficients often provide insight into the probability model which generated the data. The correlation coefficient for N pairs of observations on two variables x and y is given by

$$r = \frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum (X_i - \bar{X})^2 \quad \sum (Y_i - \bar{Y})^2}} \qquad (3.32)$$

A similar idea can be applied to time series to see if successive observations are correlated. Given N observations $X_1, \ldots, X_n$ on a discrete time series we can form N-1 pairs of observation, (X_1, X_2), (X_2, X_3), $, (X_N, X_{N-1})$. Regarding the first equation in each pair as one variable, and the second observation as a second variable, the correlation coefficient between X_t and X_{t+1} is given by

$$r_1 = \frac{\sum_{t=1}^{N-1} (X_t - \bar{X}_{(1)})(X_{t+1} - \bar{X}_{(2)})}{\sqrt{\sum_{t=1}^{N-1} (X_t - \bar{X}_{(1)})^2 \quad \sum_{t=1}^{N-1} (X_{t+1} - \bar{X}_{(2)})^2}} \qquad (3.33)$$

where

$$\bar{X}_{(1)} = \sum_{t=1}^{N-1} X_t / (N-1) \qquad (3.34)$$

is the mean of the first N-1 observations and

$$\bar{X}_{(2)} = \sum_{t=2}^{N} X_t/(N-1) \qquad (3.35)$$

is the mean of the last N-1 observations. As the coefficient given by that equation measures correlation between successive observations it is called an autocorrelation coefficient or serial correlation coefficient.

In a similar way we can find the correlation between observations a distance k apart.

3.8.5 Stochastic Processes

This section describes several different types of stochastic processes which are useful in setting up a model for a time series.

3.8.5.1 Purely Random Process

A discrete-time process is called a purely random process if it consists of a sequence of random variables Z_t which are mutually independent and identically distributed. From the definition it follows that the process has constant mean and variance and that

$$\gamma(k)=Cov(Z_t,Z_{t+k})$$

$$=0 \qquad \text{for } k = \pm 1, 2, \ldots \qquad (3.36)$$

A purely random process is sometimes called white noise. Processes of this type are useful in many situations, particularly as building blocks for more complicated processes such as moving average processes.

3.8.5.2 Random Walk

Suppose that Z_t is a discrete, purely random process with mean μ and variance σ. A process X_t is said to be a random walk if

$$X_t=X_{t-1}+Z_t \qquad (3.37)$$

The process is customarily started at zero when t=0, so that

$$X_1=Z_1 \qquad (3.38)$$

and

$$X_t=\sum_{i=1}^{t} Z_i \tag{3.39}$$

Then we find that $E(X_t)=t\mu$ and that $Var(X_t)=t\sigma_z^2$. As the mean and variance change with t, the process is non-stationary.

The first differences of a random walk is given by,

$$X_t=X_t-X_{t-1}=Z_t \tag{3.40}$$

form a purely random process, which is therefore stationary.

3.8.5.3 Moving Average Processes

Suppose that Z_t is a purely random process with mean zero and variance σ_z^2. Then a process X_t is said to be a moving average process of order q (abbreviated to an MA(q) process) if

$$X_t=\beta_0 Z_t+\beta_1 Z_{t-1}+\ldots+ \beta_q Z_{t-q} \tag{3.41}$$

where β_i are constants. The Zs are usually scaled so that $\beta_o=1$ Then,

$$E(X_t)=0 \tag{3.42}$$

$$Var(X_t)=\sigma_z^2 \sum^{q} \beta_i^2 \tag{3.43}$$

since the Zs are independent

$$\gamma(k)=Cov(X_t,X_{t+k})$$

$$=Cov(\beta_o Z_t+...+\beta_q Z_{t-q} \ , \ \beta_o Z_{t+k}+...+\beta_q Z_{t+k-q})$$

$$= \begin{cases} 0 & k>q \\ \sigma_z^2 \sum_{i=o}^{q-k} \beta_i & k=0,1,...,q \\ \gamma(-k) & k<q \end{cases} \tag{3.44}$$

since

$$Cov(Z_s,Z_t)= \begin{cases} \sigma^2 & s=t \\ 0 & s\neq t \end{cases} \tag{3.45}$$

As $\gamma(k)$ does not depend on t, and the mean is constant, the process is second order stationary for all values of the β_i. Furthermore, if the Zs are normally distributed then so are the Xs, and we have a strictly stationary normal process.

The acf of the MA(q) process is given by

$$\rho(k)= \begin{cases} 1 & k=0 \\ \sum_{i=1}^{q-k} \beta_i \beta_{i+k} \Big/ \sum_{i=0}^{q} \beta_i^2 & k=1,\ldots,q \\ 0 & k>q \\ \rho & k<0 \end{cases} \tag{3.46}$$

Note that the acf 'cuts off' at lag q, which is a special feature of MA processes. In particular the MA(1) process with $\beta_0=1$ has an acf given by

$$\rho(k)= \begin{cases} 1 & k=0 \\ \beta_1/(1+\beta_1^2) & k=\pm 1 \\ 0 & \text{otherwise} \end{cases} \tag{3.47}$$

It is generally desirable to impose restrictions on the β_i to ensure that the process satisfies a condition called 'invertibility'. As an example, consider two series

A $\quad Z_t=X_t-\theta X_{t-1}+\theta^2 X_{t-2}-\ldots$ (3.48)

B $$Z_t=X_t-(1/\theta)X_{t-1}+(1/\theta^2)X_{t-2}-.... \quad (3.49)$$

If $|\theta|<1$, the series for A converges, whereas that for B does not. Thus if $|\theta|<1$, model A is said to be invertible whereas model B is not. The imposition of the invertibility condition ensures that there is a unique MA process for a given acf.

$$B^j X_t=X_{t-j} \quad \text{for all } j \quad (3.50)$$

where B is the backward shift orerator. Then,

$$X_t=(\beta_0+\beta_1 B+...+\beta_q B^q)\ Z_t$$

$$=\theta(B)Z_t \quad (3.51)$$

where $\theta(B)$ is a polynomial of order q in B. An MA process of order q is invertible if the roots of the equation

$$\theta(B)=\beta_0+\beta_1 B+...+\beta_q B^q=0 \quad (3.52)$$

all lie outside the unit circle.

MA processes have been used in many areas, particularly econometrics. For example economic indicators are affected by a variety of 'random' events such as strikes, government decisions, shortages of key materials and so on. Such events will not only have an immidiate effect but may also effect economic indicators in several subsequent periods. Thus, it is at least reasonable that an MA process may be appropriate.

3.8.5.4 Autoregressive Processes

Suppose that Z_t is a purely random process with mean zero and variance σ^2. Then a process X_t is said to be an autoregressive process of order p if

$$X_t=\alpha_1 X_{t-1}+\ldots+\alpha_p X_{t-p}+Z_t \qquad (3.53)$$

This is rather like a multiple regression model but X_t is regressed not on independent variables but on past values of X_t. Hence the prefix 'auto'. An autoregressive process of order p will be abbreviated to an AR(p) process.

a. First Order Process

For simplicity, we begin by examining the first-order case, where p=1. Then,

$$X_t=\alpha X_{t-1}+Z_t \qquad (3.54)$$

The AR(1) process is sometimes called the Markov process. By successive substitution it can be written

$$X_t = \alpha(\alpha X_{t-2} + Z_{t-1}) + Z_t$$

$$= \alpha(\alpha X_{t-3} + Z_{t-2}) + \alpha Z_{t-1} + Z_t \quad (3.55)$$

and eventually X_t may be expressed as an infinite-order MA process in the form (provided $-1<\alpha<+1$).

$$X_t = Z_t + \alpha Z_{t-1} + \alpha^2 Z_{t-2} + \ldots \quad (3.56)$$

using the backward shift operator B,

$$(1-\alpha B)X_t = Z_t \quad (3.57)$$

Then,

$$X_t = Z_t/(1-\alpha B)$$

$$= Z_t + \alpha Z_{t-1} + \alpha^2 Z_{t-2} + \ldots \quad (3.58)$$

And also,

$$E(X_t) = 0 \quad (3.59)$$

$$Var(X_t)=\sigma_z^2(1+\alpha^2+\alpha^4 \; ...) \qquad (3.40)$$

The acf is given by,

$$\gamma(k)=E[X_t X_{t+k}]$$

$$=E[(\sum \alpha^i Z_{t-i})(\sum \alpha^j Z_{t+k-j})]$$

$$=\sigma_z^2 \sum^{\infty} \alpha^i \alpha^{k+i} \qquad \text{for } k >= 0 \qquad (3.61)$$

which converges for $|\alpha|<1$ to

$$\gamma(k)=\alpha^k \sigma_z^2/(1-\alpha^2)$$

$$=\alpha^k \sigma_x^2 \qquad (3.62)$$

For $k<0$, $\gamma(k)=\gamma(-k)$. Since (k) does not depend on t, an AR process of order 1 is second-order stationary provided that $|\alpha|<1$. The acf is given by

$$\rho(k)=\alpha^k \qquad k = 0, 1, 2 \; ... \qquad (3.63)$$

b. General Order Case

An AR process of finite order can be expressed as an MA process of infinite order.

$$(1-\alpha_1 B-\ldots-\alpha_p B^p)X_t=Z_t \tag{3.64}$$

or

$$X_t=Z_t/(1-\alpha_1 B-\ldots-\alpha_p B^p)^{-1}$$

$$=f(B)Z_t \tag{3.65}$$

where

$$f(B)=(1-\alpha_1 B-\ldots-\alpha_p B^p)^{-1}$$

$$=(1+\beta_1 B+\beta_2 B^2+\ldots) \tag{3.66}$$

Having expressed X_t as an MA process, it follows that $E(X_t)=0$. The acf is given by

$$\gamma(k)=\sigma_z^2 \sum_{i=0}^{\infty} \beta_i \beta_{i+k} \tag{3.67}$$

where $\beta_0=1$.

A sufficient condition for this to converge, and hence for stationarity is that $\sum|\beta_i|$ converges. An equivalent way of expressing the stationarity condition is to say that the roots of the equation

$$\phi(B)=1-\alpha_1 B-\ldots-\alpha_p B^p=0 \qquad (3.68)$$

must lie outside the unit circle.

AR processes have been applied to many situations in which it is reasonable to assume that the present value of a time series depends on the immediate past values together with a random error. For non-zero means,

$$X_t-\mu=\alpha_1(X_{t-1}-\mu)+\ldots+\alpha_p(X_{t-p}-\mu)+X_t. \qquad (3.69)$$

3.8.5.5 Mixed ARMA Models

A mixed autoregressive/moving average process containing p AR terms and q MA terms is said to be an ARMA process of order (p,q). It is given by,

$$X_t=\alpha_1 X_{t-1}+\ldots+\alpha_p X_{t-p}+Z_t+\beta_1 Z_{t-1}+\ldots+\beta_q Z_{t-q} \qquad (3.70)$$

using the backward shift operator B, this equation may be written in the form

$$\phi(B)X_t=\theta(B)Z_t \qquad (3.71)$$

where $\phi(B)$, $\theta(B)$ are polynomials of order p, q respectively, such that

$$\phi(B)=1-\alpha_1 B-\ldots-\alpha_p B \qquad (3.72)$$

$$\theta(B)=1+\beta_1 B+\ldots+\beta_q B \qquad (3.73)$$

As for an AR process, the values of α_i which make the process stationary are such that the roots of

$$\phi(B)=0$$

lie outside the unit cicle. As for an MA process, the values β_i which make the process invertible are such that the roots of

$$\theta(B)=0$$

lie outside the unit circle.

The importance of ARMA processes lies in the fact that a stationary time series may often be described by an ARMA model involving fewer parameters than a pure MA or AR process by itself.

3.8.5.6 Integrated ARIMA Models

In practice most time series are non-stationary. In order to fit a stationary model, it is necessary to remove non-stationary sources of variation. If the observed time series is nonstationary in the mean, then we can difference the series. If X_t is replaced by $\nabla^d X_t$, then we have a model capable of describing certain types of nonstationary series. Such a model is called an 'integrated' model because the stationary model which is fitted to the differenced data has to be summed or 'integrated' to provide a model for the non-stationary data.

$$W_t=\nabla^d X_t=(1-B)X_t \qquad (3.74)$$

The general autoregressive integrated moving average process (ARIMA) is of the form,

$$W_t=\alpha_1 W_{t-1}+...+\alpha_p W_{t-p}+Z_t+...+\beta_q Z_{t-q} \qquad (3.75)$$

$$\phi(B)W_t=\theta(B)Z_t \qquad (3.76)$$

$$\phi(B)(1-B)^d X_t=\theta(B)Z_t \qquad (3.77)$$

Thus, we have an ARIMA process of order (p, d, q)

3.8.5.7 Seasonal SARIMA Models

In practice, many time series contain a seasonal periodic component which repeats every s observations. For example, with monthly observations where s=12 we may typically expect X_t to depend on terms such as X_{t-12}, and perhaps X_{t-24}, as well as terms such as X_{t-1}, X_{t-12},.... Box and Jenkins have generalized the ARIMA model to deal with seasonality, and define a general multiplicative seasonal ARIMA model (SARIMA) as

$$\phi_p(B)\Phi_P(B^s)W_t=\theta_q(B)\Theta_Q(B^s)Z_t \qquad (3.78)$$

where

$$W_t=\nabla^d\nabla^D_s X_t \qquad (3.79)$$

or in the full form,

$$\phi_p(B)\Phi_P(B^s)\nabla^d\nabla^D X_t=\theta_q(B)\Theta_Q(B^s)Z_t \qquad (3.80)$$

For d=D=1 and s=12, then

$$W_t=\nabla\nabla_{12}X_t=\nabla_{12}X_t-\nabla_{12}X_{t-1} \qquad (3.81)$$

The model is said to be a SARIMA model of order

$(p,d,q)_1 \times (P,D,Q)_s$. When fitting a seasonal model to data, the first task is to assess valves of d and D which reduce the series to stationarity and remove most of the seasonality. Then the values of p, P, q and Q need to be assessed by looking at the acf and partial acf of the differenced series and choosing a SARIMA model whose acf and partial acf are of similar form.

3.8.6 The Partial Autocorrelation Function

The description of the partial autocorrelation function is left to this section. Since the usefulness of it could be explained after AR and MA processes are described. Initially, we may not know which order of autoregressive process to fit to an observed time series. This problem is analogous to deciding on the number of independent variables to be included in a multiple regression.

The partial autocorrelation function is a device which exploits the fact that whereas an AR(p) process has an autocorrelation function which is infinite in extent it can by its very nature be described in terms of p non-zero functions of the autocorrelations. The j th coefficient in an autoregressive process of order k, denoted by ϕ_{kj}, so that ϕ_{kk} is the last coefficient. The

ϕ_{kj} satisfy the set of equations

$$\rho_j=\phi_{k-1}+P_{j-1}+\ldots+\phi_{k(k-1)}P_{j-k+1}+\phi_{kk}P_{j-k} \qquad j=1,2,\ldots,k. \tag{3.82}$$

leading to the equations

$$\begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \cdot & & & & \\ \cdot & & & & \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \phi_{k1} \\ \phi_{k2} \\ \\ \phi_{kk} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \\ \rho_3 \end{bmatrix} \tag{3.83}$$

or

$$P_k\phi_k=\rho_k \tag{3.84}$$

Solving these equations for k=1, 2, 3,..., successively, we obtain

$$\phi_{11}=\rho_1 \tag{3.85}$$

$$\phi_{22}=\frac{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & \rho_2 \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{vmatrix}}=\frac{\rho_2-\rho_1^2}{1-\rho_1} \tag{3.86}$$

$$\phi_{33} = \frac{\begin{vmatrix} 1 & \rho_1 & \rho_1 \\ \rho_1 & 1 & \rho_2 \\ \rho_2 & \rho_1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 & \rho_2 \\ \rho_1 & 1 & \rho_1 \\ \rho_2 & \rho_1 & 1 \end{vmatrix}} \tag{3.87}$$

In general, for ϕ_{kk}, the determinant in the numerator has the same elements as that in the denominator, but with the last column replaced by ρ_k. The quantity ϕ_{kk}, regarded as a function of the lag k, is called the partial autocorrelation function.

For an autoregressive process of order p, the partial autocorrelation function ϕ_{kk} will be nonzero for k less than or equal to p and zero for k greater then p. In other words, the partial autocorrelation function of a p th order autoregressive process has a cut off after lag p.

3.9 The Model Development Process

In the following a general methodology for model building process is given. This methodology is applied to the time series of electricity load demand of Turkey. After estimation of the model, it will be used to forecast 2 years load demand.

The flowchart of model building process is given in the Figure 3.1.

The first step in time series modelling is 'initial data analysis'. The aim of the initial data analysis is to put forward whether time series contains trends, seasonal effects and unusual observations.

The first, and most important, step in any time series analysis is to plot the observations against time. This graph should show up important features of the series such as trend, seasonality, outliers and discontinuties. The plot is vital, both to describe the data and to help in formulating a sensible model.

The autocorrelations of the series is calculated and called correlogram. In a correlogram the lags are printed on the horizantal axis and the correlation values on the vertical axis from -1 to +1. Correlograms are probably the easiest and most widely used method of displaying autocorrelations, since they allow one to visually determine the autocorrelation patterns.

Time plot and correlograms reveals that the time series is stationary or not. If the time series is

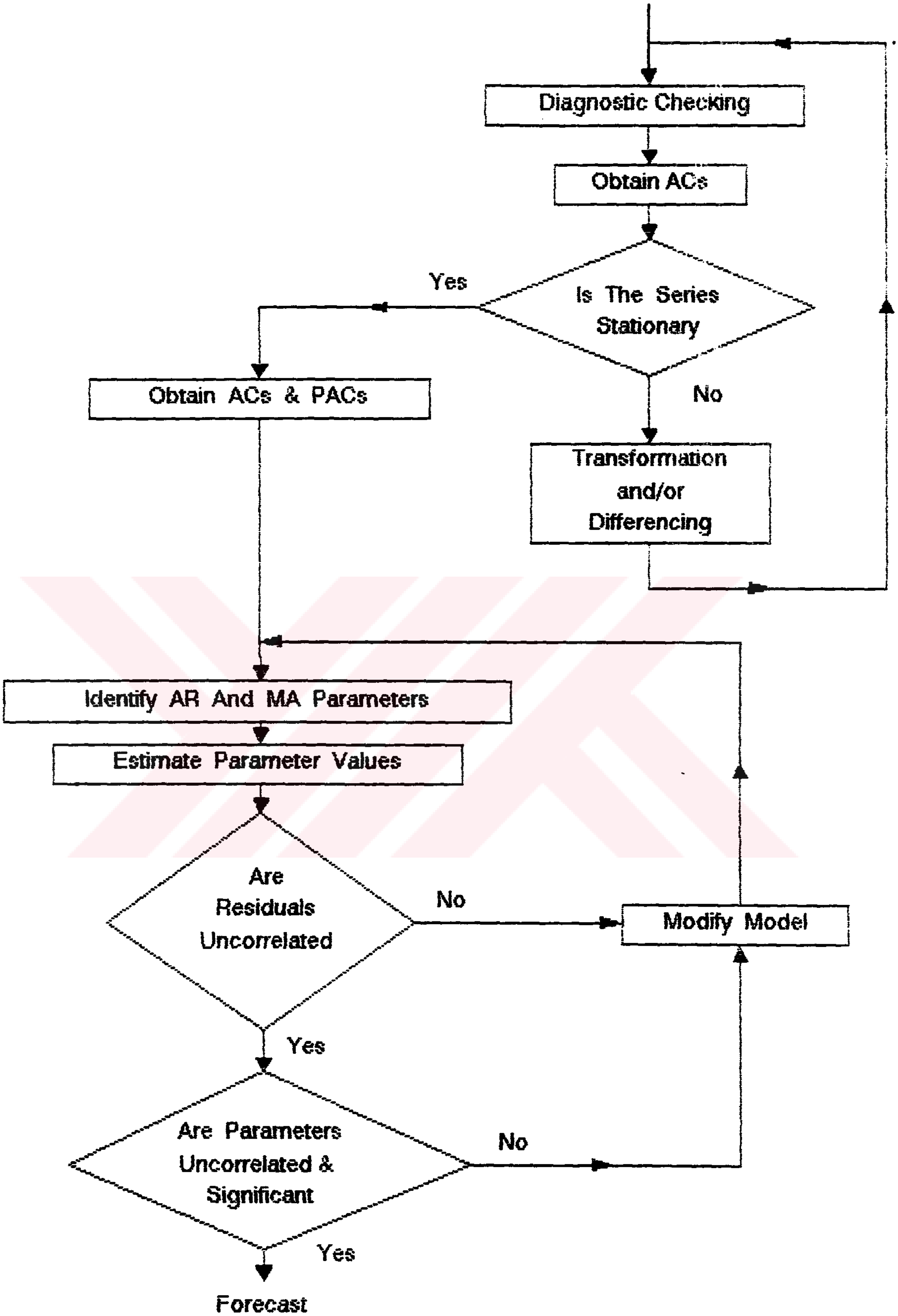

Figure 3.1 The Flowchart of Model Estimating Process

nonstationary, then transformation by taking logarithms or squarerooting and sufficient differencing is applied to data until the time series becomes stationary. After transformation, in general first differencing for nonseasonal data, and first and seasonal first differencing for sesonal data is sufficient to remove the trend and make the series stationary.

Once stationary series is obtained, autocorrelations and partial autocorrelations are calculated and plotted on a correlogram.

The behaviour of the ACs and PACs gives the clues for fitting the appropriate model. ACs and PACs of some basic theoratical processes are given in the Appendix B.

After identifying AR and MA parameters with the aids of ACs and PACs, model parameters are estimated.

The high-order rk may be unreliable as a guide to model identification, it is favorable taking the maximum lag to be at most n/4. In parameter estimation, the sample PACs is more useful for detecting AR schemes; conversely the sample ACs does a better job for MA schemes.

When a model has been fitted to a time series, it is advisable to check that the model really does provide an adequate description of the data. This is usually done by looking at the residuals, which are defined as

residual = fitted value - observation

If we have a 'good' model then we expect the residuals to be 'random' and 'close to zero', and model validation usually consists of plotting residuals.

Then residuals are plotted as a time plot and the correlogram of the residuals are calculated. The time plot will reveal any outliers and any obvious autocorrelation or cyclic effects. Let rk denote the autocorrelation coefficent at lag k of the Z_t. If we have fitted the true model, then the true errors form a purely random process and, their correlogram is such that each autocorrelation coefficient is approximately normally distributed with mean 0 and variance 1/N for reasonably large values of N. However, the correlogram of the residuals has somewhat different properties.

Box-Jenkins describe Portmanteau lack of fit test which looks at the first M values of the correlogram all at once. The test statistic

$$Q=N\sum_{k=1}^{M} rk^2 \quad (3.88)$$

where N is the number of terms in the differenced series and M is typically choosen in the range 15 to 30. If the fitted model is appropriate, then Q should be approximately distributed as Chi Square with (M - p - q) degrees of freedom, where p, q are the number of AR and MA terms respectively in the model.

However, a simple way of analysing residuals is to look at residual correlogram and see whether there are significant r_k or not. The model is not rejected even when there is significant r_k for higher lags, say k=5, or k=20 for seasonal data.

After deciding residuals are uncorrelated, estimated parameters are analysed whether they are uncorrelated and significant.

Estimated parameters are significant if their t ratios are higher from the table values in absolute terms for the corresponding degrees of freedom.

The model is accepted as adequate if there is no high correlation between estimated parameters.

If any of the above analysis prove that the model is not appropriate, the model is modified and parameters are reestimated.

CHAPTER IV

AN APPLICATION OF BOX-JENKINS TIME SERIES MODEL

In this chapter, Box-Jenkins time series model for electricity load demand of Turkey is developed. The monthly electricity load demand data was obtained from Turkish Electricity Authority (TEK). The time series covers the 144 months period from January 1979 to December 1990.

4.1 Model Estimation

As it is seen from the time plot in Figure 4.1, there is a trend, which is increasing in long term, and probably a seasonal variation. In Table 4.1, the computed autocorrelations are tabulated, and in Figure 4.2, they are displayed on a correlogram. Both the visual check and very slow decaying autocorrelation pattern suggest that the series is non-stationary.

In order to make the series stationary, first order differencing is done in the first step. The differenced series is plotted against time, Figure 4.3. The time plot of the first differenced series and

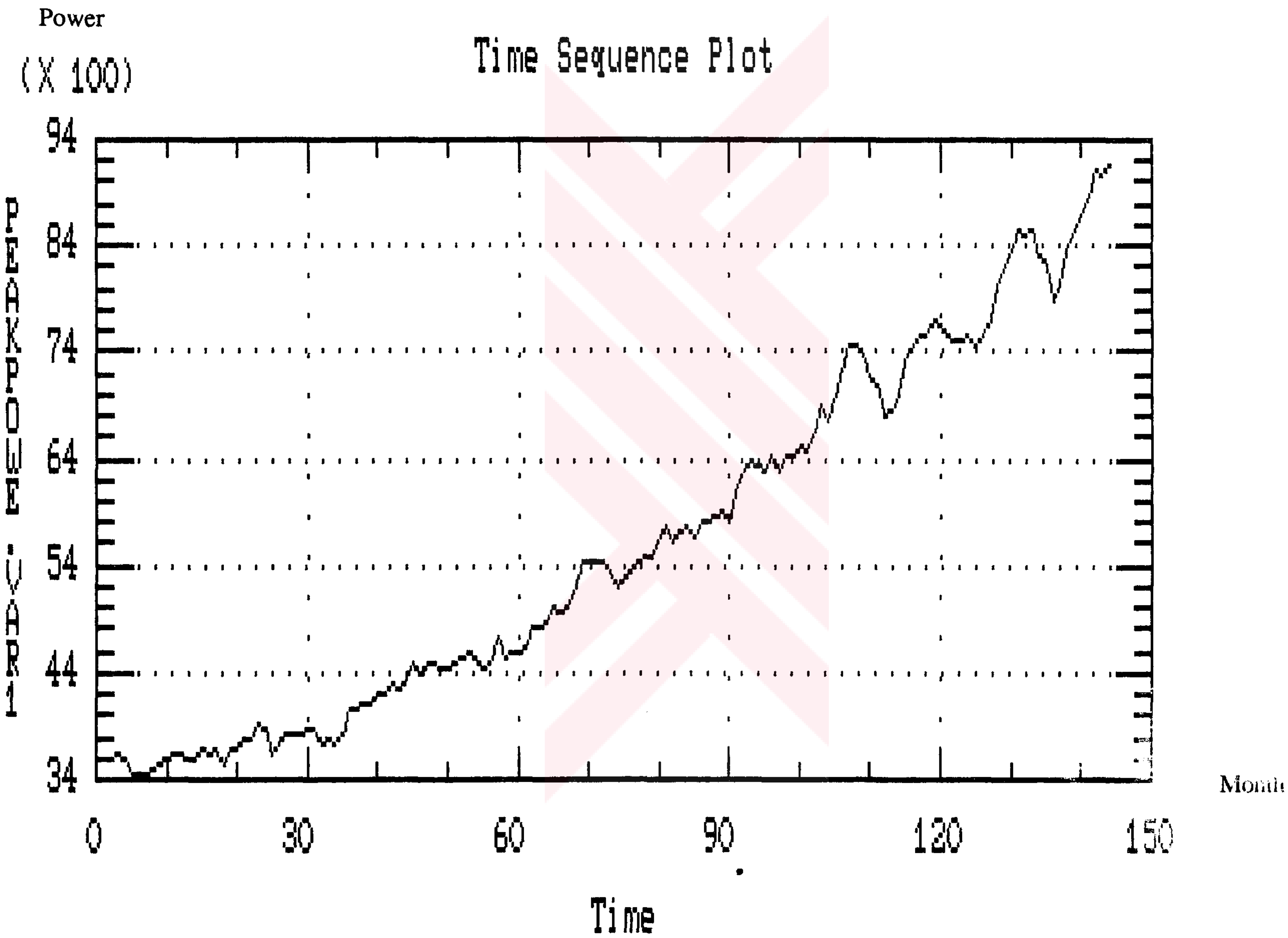

Figure 4.1 Time Plot of Peak Load Demand of Turkiye

Table 4.1 Autocorrelations of Peak Load Power

Estimated autocorrelations for PEAKPOWE.VAR1

Lag	Estimate	Stnd.Error	Lag	Estimate	Stnd.Error
1	.97598	.08333	2	.95185	.14204
3	.92659	.18099	4	.90283	.21138
5	.88023	.23665	6	.85961	.25839
7	.84154	.27754	8	.82675	.29473
9	.81340	.31041	10	.79618	.32488
11	.77817	.33816	12	.75694	.35037
13	.73398	.36155	14	.70927	.37175
15	.68568	.38103	16	.66259	.38951
17	.64064	.39726	18	.62138	.40437
19	.60431	.41095	20	.58851	.41707
21	.57292	.42280	22	.55756	.42816
23	.54180	.43317	24	.52418	.43785
25	.50232	.44219	26	.47984	.44613
27	.45784	.44970	28	.43515	.45293
29	.41340	.45582	30	.39333	.45842
31	.37586	.46076	32	.35937	.46288

Estimated autocorrelations for PEAKPOWE.VAR1

Lag	Estimate	Stnd.Error	Lag	Estimate	Stnd.Error
33	.34345	.46481	34	.32452	.46657
35	.30487	.46814	36	.28407	.46951
37	.26127	.47071	38	.23774	.47171
39	.21591	.47254	40	.19489	.47323

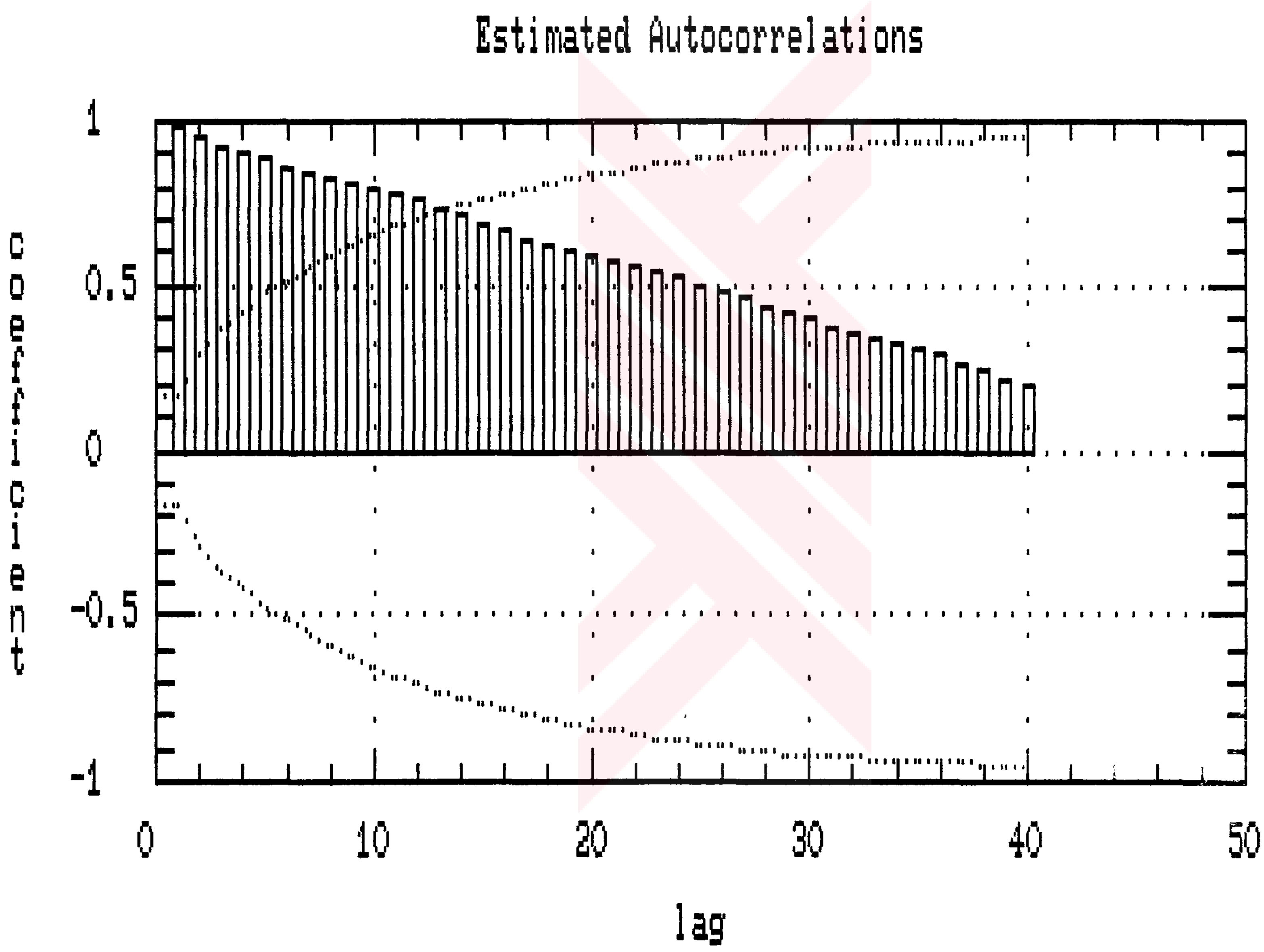

Figure 4.2 Correlagram of Peak Load Demand Time Series

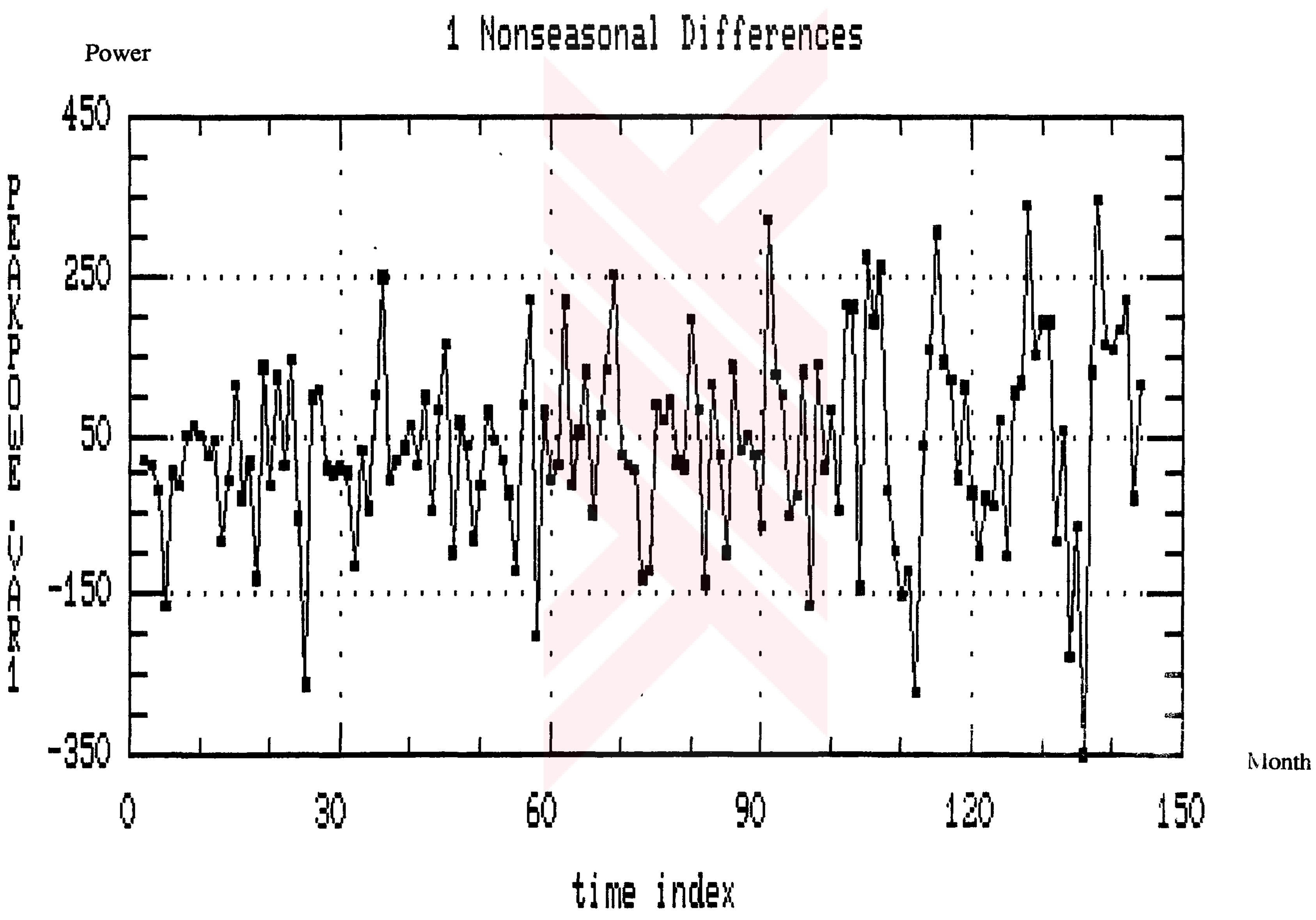

Figure 4.3 1 Nonseasonal Differenced Plot of Peak Load Demand Time Series

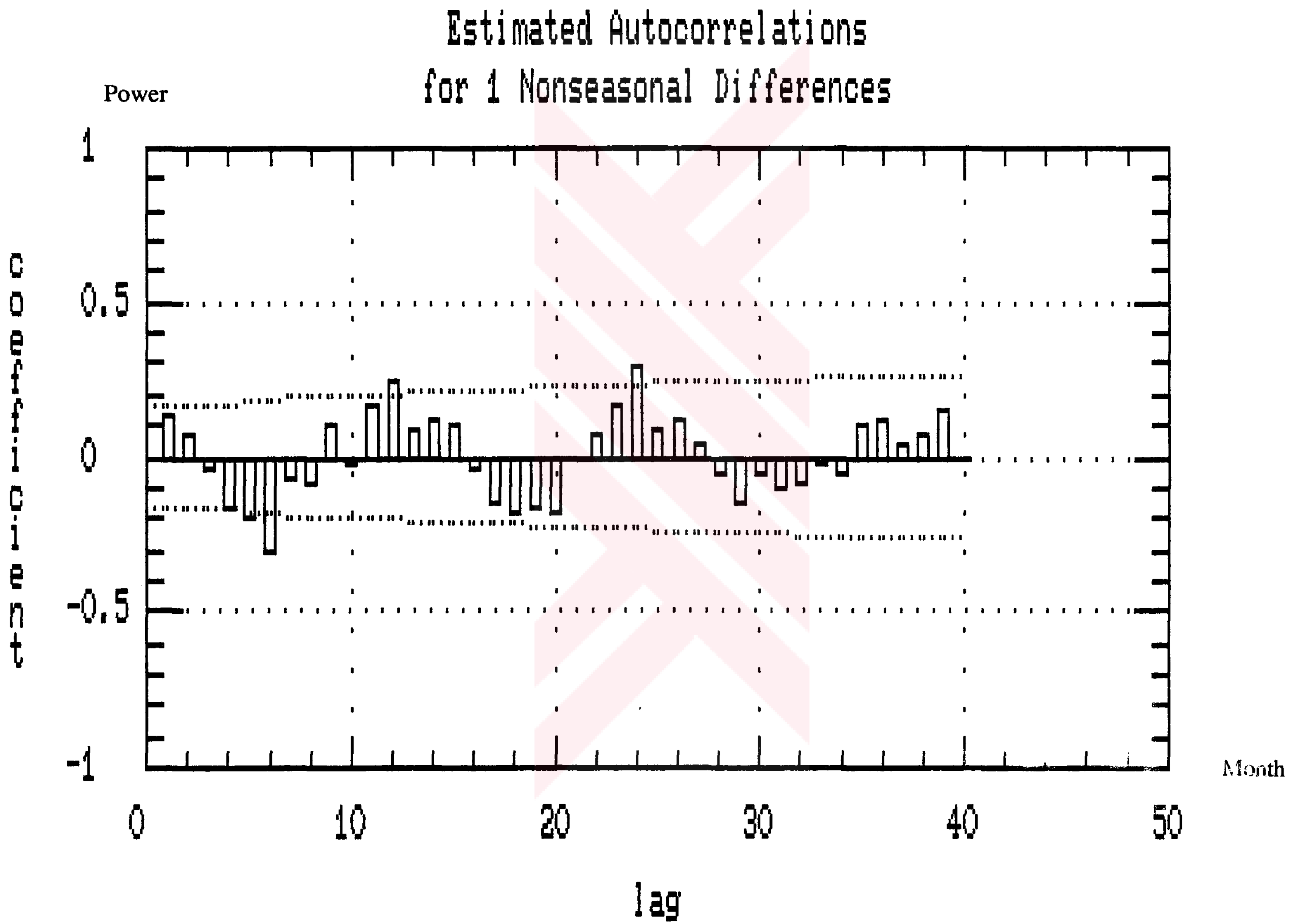

Figure 4.4 Correlagram of 1 Nonseasonal Differenced Peak Load Demand Time Series

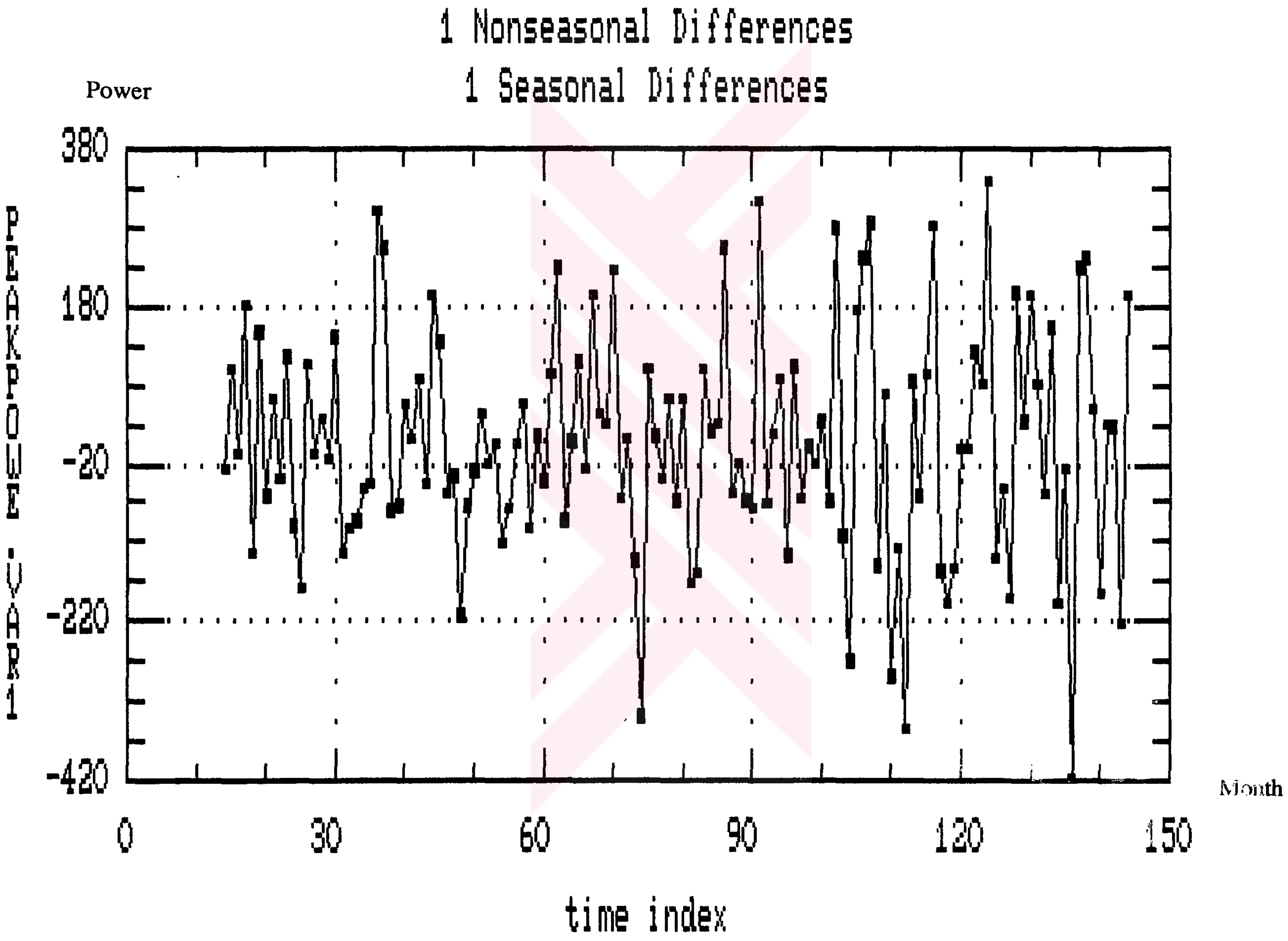

Figure 4.5 1 Nonseasonal and 1 Seasonal Differenced Peak Load Demand Time Series

auotocorrelation correlagram, Figure 4.4, reveal that the trend is removed but seasonal variation has 12 period effect. Seasonal differencing of 12 periods is applied to this series.

The time plot of the first and seasonallly differenced series, Figure 4.5, shows that there is no more trend and seasonal trend effect is removed.

To understand the behaviour of the series, it is decomposed into its components moving average trend, seasonal component and residual component. The estimated seasonal component is displayed in Figure 4.6. The pattern of the seasonal component is such that it has a peak in the November and has its lowest value in May.

The next step is to calculate autocorrelation and partial autocorrelation function of the stationary series and plot them on a correlagram. The correlagrams are depicted in Figures 4.7 and 4.8.

The ACs and PACs correlograms reveals that there is a significant spike at lag 12. Another interesting point is that within the confidence band, there is a cyclic variation of 6 months periods. This is due to the fact that each month is negatively correlated with the month which is 6 months ahead. Since these cyclic

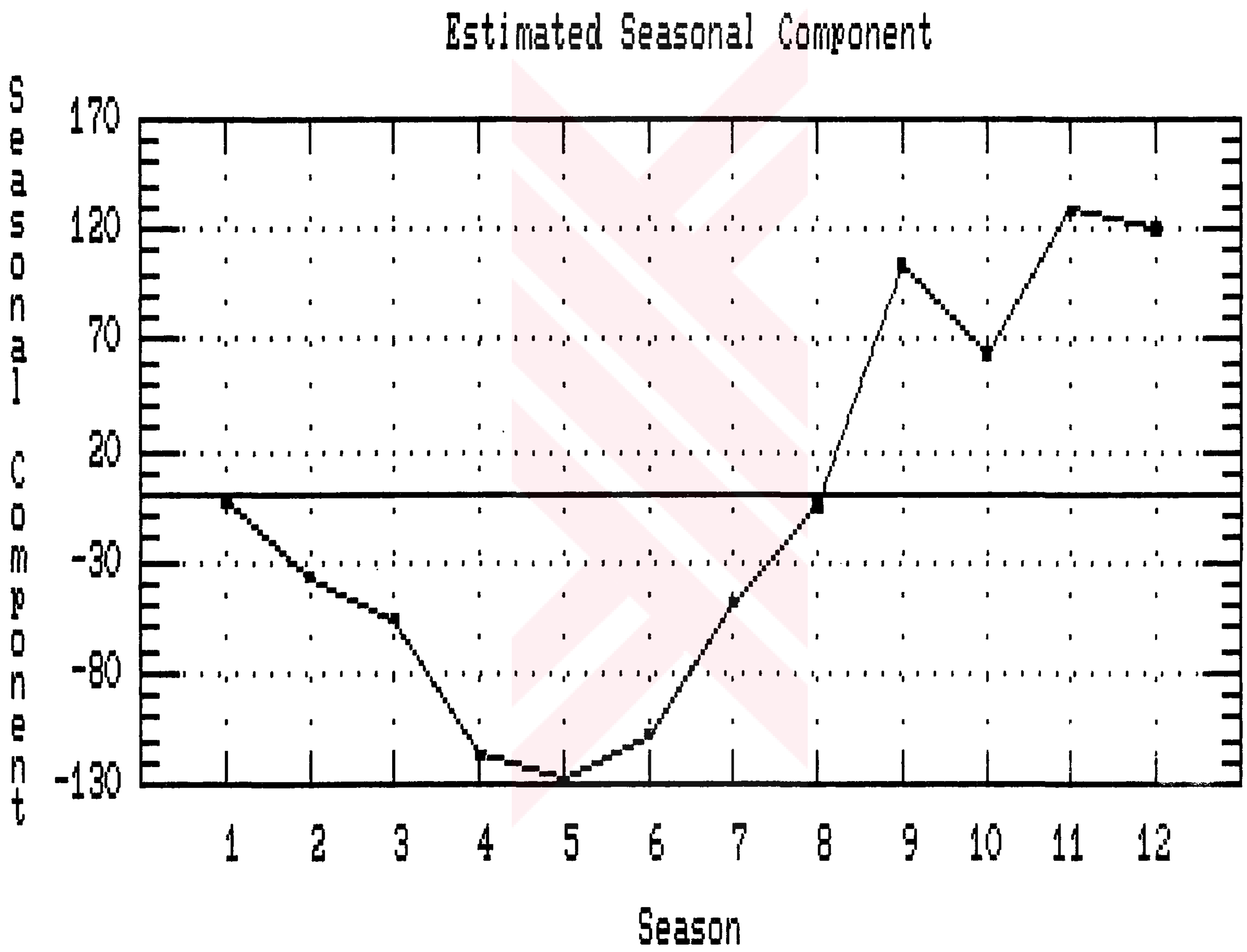

Figure 4.6 Seasonal Component of Time Series

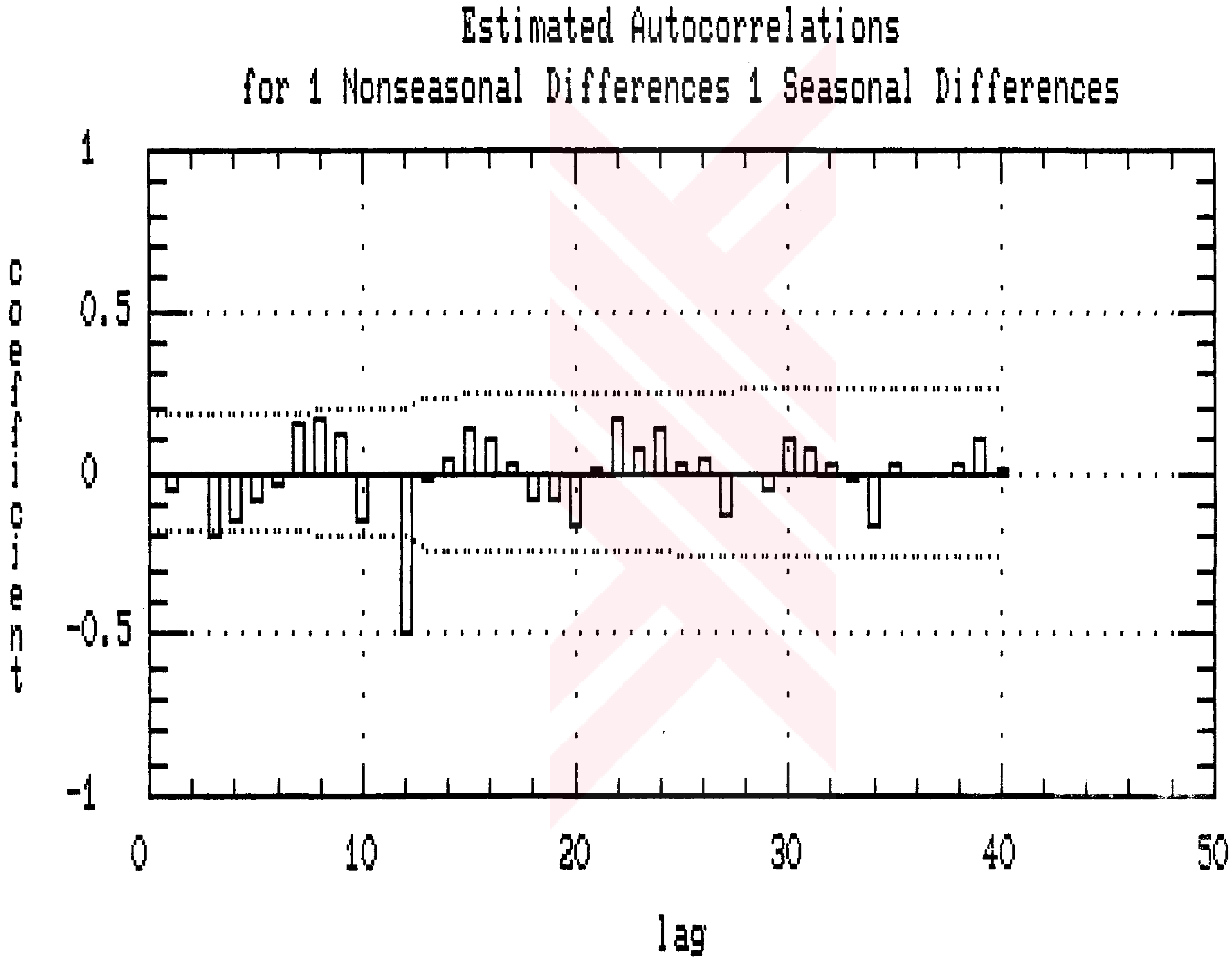

Figure 4.7 Correlagram of Stationary Series

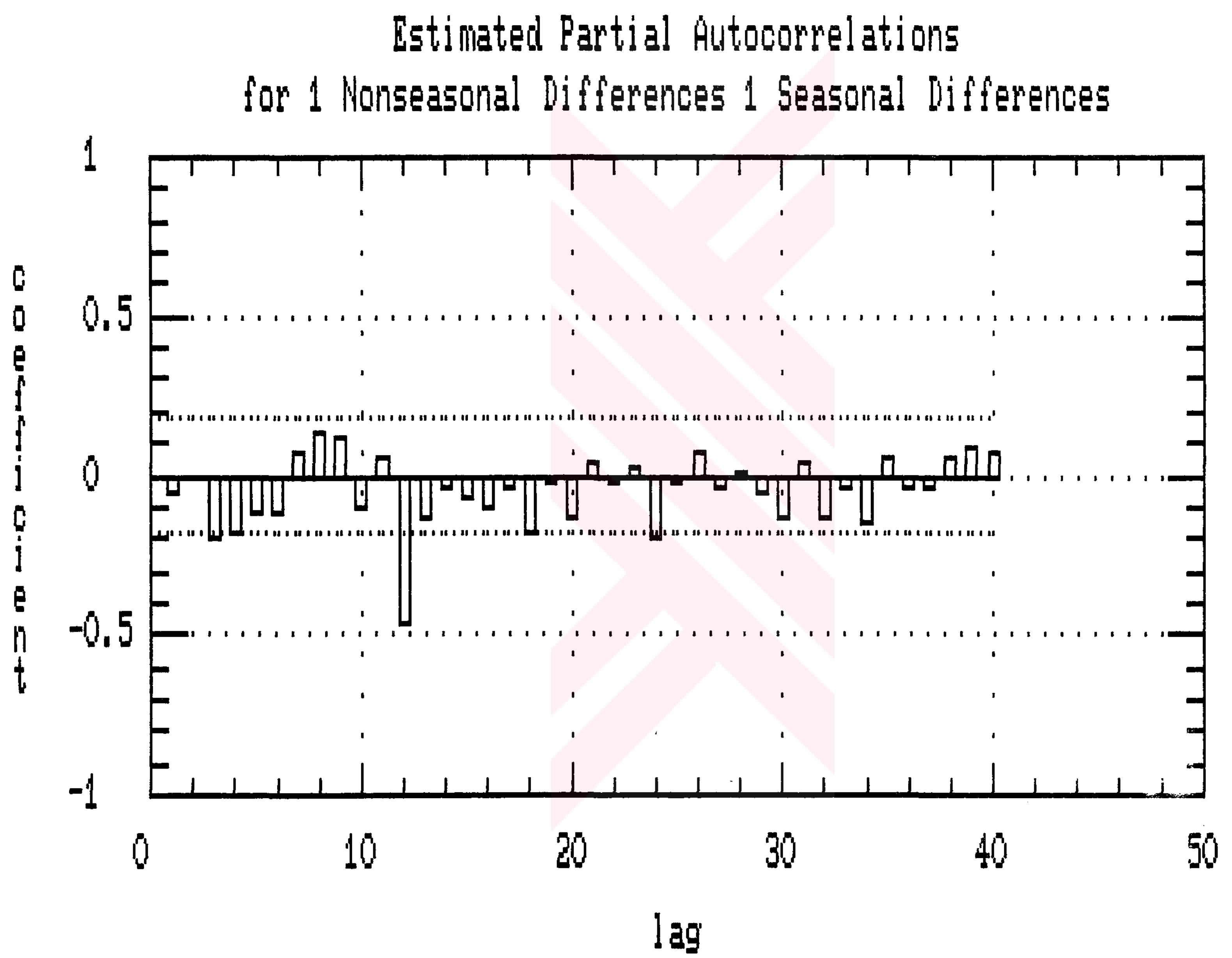

Figure 4.8 Partial Autocorrelation Correlagram of Stationary Series

variations are approximately in the confidence band, we can assume that these are insignificant, so that we will not consider them in model estimating. As a matter of fact after model estimating process completed, we will check the importance of this variation and conclude that whether it is significant or not.

As both ACs and PACs shows a significant spike at lag 12, this patterns suggests that a seasonal autoregressive model of order 1 SARIMA $(0,1,0)_1x(1,1,0)_{12}$ should be tried.

Beside this model, also seasonal moving average model of order 1 SARIMA $(0,1,0)_1x(0,1,1)_{12}$, seasonal autoregressive of order 1 and seasonal moving average of order 1 SARIMA $(0,1,0)_1x(1,1,1)_{12}$ should be tried.

These three models are estimated and resulting coefficents are obtained. Among them seasonal autoregressive model of order 1 SARIMA $(0,1,1)_1x(1,1,0)_{12}$ gave the best result. The portmanteau lack of fit test of the first model is the smallest among them. Another supporting data is that the autocorrelation and partial autocorrelation coefficents of this model, up to lag 40, is inside the confidence interval. In other words, there is no significant autocorrelation coefficent for a 95% confidence interval.

The portmenteau lack of fit test which is calculated by the computer programme is 20.2799 . The chi square statistic with 144-13-1=130 degrees of freedom and α =0.05 is greater than 124.342, which is far from the calculated value. Thus, residual correlation is very small and we can conclude that residuals shows no significant correlation pattern. In fact, as it is seen from the residual correlogram, Figure 4.9, there may be very small seasonal autocorrelation pattern, but it is inside the confidence interval and it is insignificant.

The estimated seasonal autoregressive parameter is -0.60727 with a standard error of 0.08087. The t test statistic is -7.80964 which is greater than 1.65 in absolute term. Therefore, the estimated SAR parameter is significant.

Since there is no other estimated parameter for SARIMA $(0,1,0)_1 x(1,1,0)_{12}$ model, no correlation problem exists.

The model in full form is

$$(1-\phi B^{12})\nabla\nabla^{12}X_t=e_t \tag{4.1}$$

$$(1+0.60727B^{12})(X_t-X_{t-1}-X_{t-12}+X_{t-13})=9.65+e_t \tag{4.2}$$

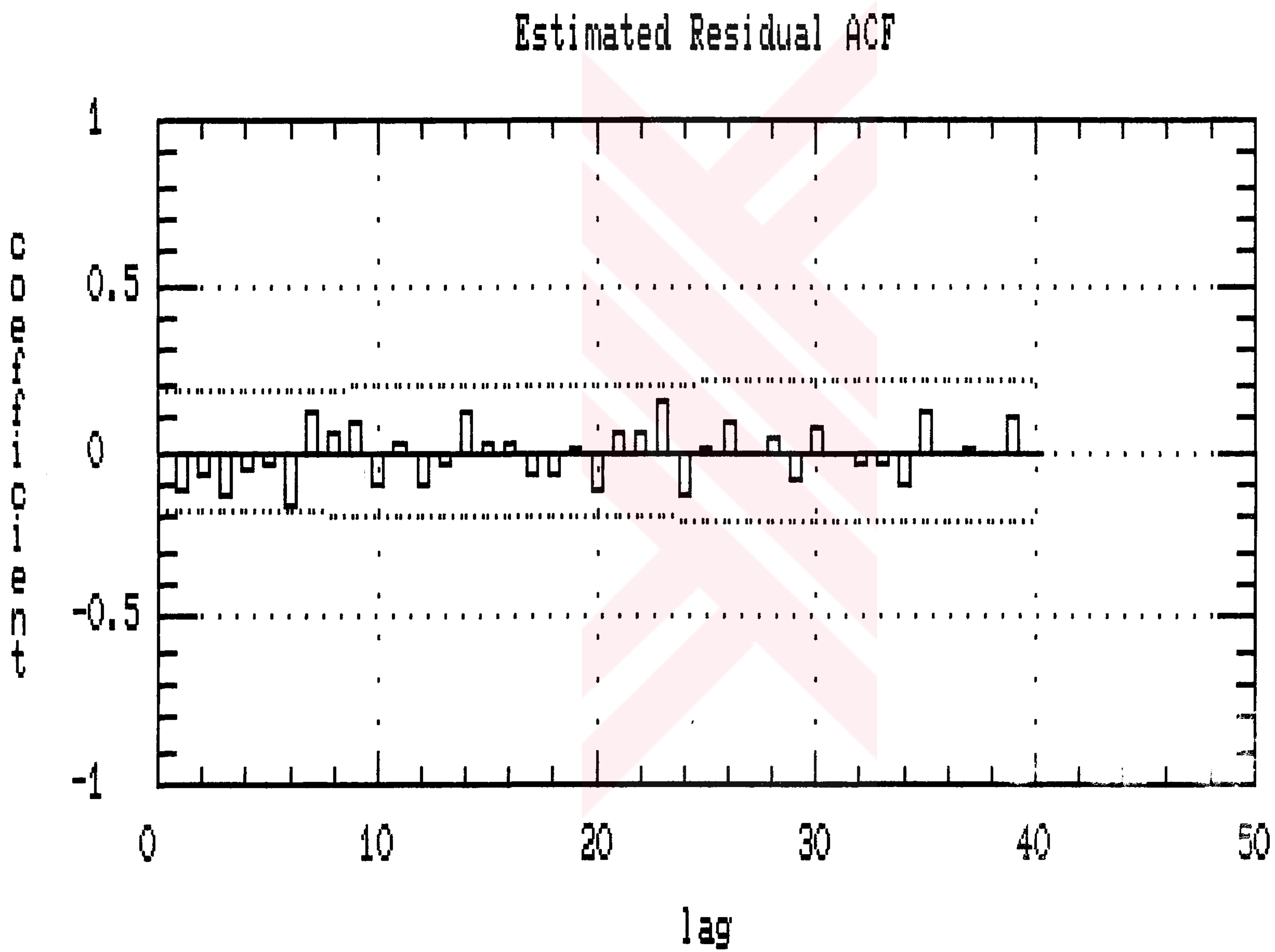

Figure 4.9 Residual Correlagram of Fitted SARIMA $(0,1,0)_1 x (1,1,0)_{12}$ Model

$$X_t = 9.65 + X_{t-1} + 0.39273X_{t-12} - 0.39273X_{t-13} + 0.60727X_{t-24} - 0.60727X_{t-25} + e_t \qquad (4.3)$$

The computer output is displayed in Table 4.2. There is a constant term , which is 9.65. The computer also calculated the mean which can be introduced to the formula, as 6.00. However, it is insignificant and it can be neglected from the formula. Statgraph software also didn't take this mean into account in forecasting and residual calculations.

The model is used to forecast the peak electricity power consumption of Turkey for 24 months period. The results and forecasting graph can be seen in Table 4.3 and Figure 4.10, respectively.

The residuals after model fitting is displayed in Table 4.4 and their probability plot can also be seen in Figure 4.11.

The tables and figures of the computer output for fitted model, residual ACF, forecast values, plot of forecast values and resuduals after model fitting of SARIMA $(0,1,1)_1 x(0,1,1)_{12}$ and SARIMA $(0,1,0)_1 x(1,1,1)_{12}$ are depicted in Appendix C.

Table 4.2 Summary of SARIMA $(0,1,0)_1 x (1,1,0)_{12}$ Model

Summary of Fitted Model for: PEAKPOWE.VAR1

Parameter	Estimate	Stnd.error	T-value	P-value
SAR(12)	-.60727	.08087	-7.50964	.00000
MEAN	6.00354	6.75864	.88828	.37604
CONSTANT	9.64934			

Model fitted to differences of order 1
Model fitted to seasonal differences of order 1 with seasonal length = 12
Estimated white noise variance = 14503.9 with 129 degrees of freedom.
Estimated white noise standard deviation (std err) = 120.432
Chi-square test statistic on first 20 residual autocorrelations = 20.2799
with probability of a larger value given white noise = 0.377914
Backforecasting: no   Number of iterations performed: 4

Residual Summary

Number of observations = 131
Residual average = -0.0447561
Residual variance = 14503.9
Residual standard error = 120.432

Coeff. of skewness = 0.0538851   standardized value = 0.251785
Coeff. of kurtosis = 0.455451   standardized value = 1.06407

Table 4.3 SARIMA (0,1,0)1x(1,1,0)12 Model Forecasts

SARIMA (0,1,0)x(1,1,0) FORECASTS

(1,1)	9153.07	(1,2)	8914.74	(1,3)	9391.4
(2,1)	9056.01	(2,2)	8718.96	(2,3)	9393.06
(3,1)	9019.02	(3,2)	8606.22	(3,3)	9431.83
(4,1)	8933.56	(4,2)	8456.89	(4,3)	9410.22
(5,1)	8934	(5,2)	8401.08	(5,3)	9466.93
(6,1)	9143.25	(6,2)	8559.46	(6,3)	9727.04
(7,1)	9287.14	(7,2)	8656.58	(7,3)	9917.71
(8,1)	9564.3	(8,2)	8890.19	(8,3)	10238.4
(9,1)	9737.92	(9,2)	9022.93	(9,3)	10452.9
(10,1)	9949.03	(10,2)	9195.36	(10,3)	10702.7
(11,1)	10064.4	(11,2)	9273.95	(11,3)	10854.9
(12,1)	10069.8	(12,2)	9244.2	(12,3)	10895.4
(13,1)	10103.1	(13,2)	9213.26	(13,3)	10992.9
(14,1)	9937.31	(14,2)	8987.59	(14,3)	10887
(15,1)	9894.61	(15,2)	8888.55	(15,3)	10900.7
(16,1)	9659.66	(16,2)	8600.26	(16,3)	10719.1
(17,1)	9748.31	(17,2)	8638.13	(17,3)	10858.5
(18,1)	10050.6	(18,2)	8891.81	(18,3)	11209.3
(19,1)	10216.3	(19,2)	9010.96	(19,3)	11421.7
(20,1)	10430.2	(20,2)	9179.99	(20,3)	11680.4
(21,1)	10619.1	(21,2)	9325.52	(21,3)	11912.6
(22,1)	10844.1	(22,2)	9508.62	(22,3)	12179.5
(23,1)	10882	(23,2)	9505.94	(23,3)	12258.1
(24,1)	10962.7	(24,2)	9547.17	(24,3)	12378.2

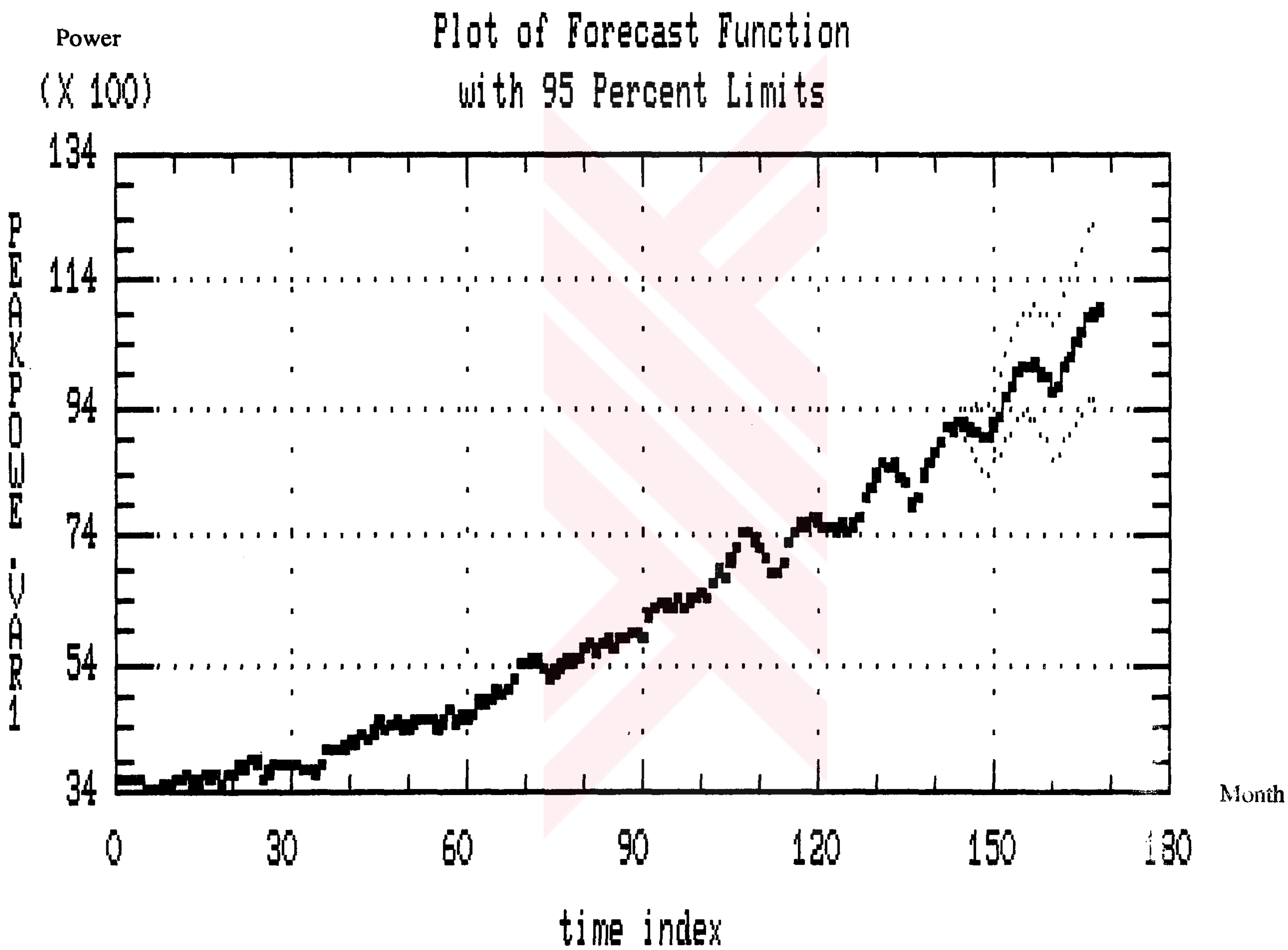

Figure 4.10 Plot of Forecasted Values of
SARIMA $(0,1,0)_1 x (1,1,0)_{12}$ Model with 95 Percent Limits

Table 4.4 SARIMA $(0,1,0)_1 x (1,1,0)_{12}$ Model Residuals

SARIMA (0,1,0)x(1,1,0) RESIDUALS

(1)		(19)	139.696	(37)	135.892	(55)	-112.659
(2)		(20)	-66.7035	(38)	-26.3249	(56)	114.03
(3)		(21)	54.1965	(39)	-85.7573	(57)	127.447
(4)		(22)	-44.0035	(40)	68.6056	(58)	-147.664
(5)		(23)	110.296	(41)	-6.85443	(59)	-17.0039
(6)		(24)	-103.804	(42)	162.765	(60)	-180.527
(7)		(25)	-184.604	(43)	-135.352	(61)	38.8019
(8)		(26)	78.6292	(44)	124.541	(62)	200.561
(9)		(27)	44.7816	(45)	68.2565	(63)	-74.833
(10)		(28)	22.6245	(46)	-98.5312	(64)	-11.9519
(11)		(29)	87.6562	(47)	-70.8835	(65)	102.587
(12)		(30)	48.4154	(48)	-38.3816	(66)	-106.276
(13)		(31)	-53.5697	(49)	71.898	(67)	138.402
(14)	-33.7035	(32)	-146.811	(50)	-87.3348	(68)	37.9085
(15)	94.4965	(33)	-65.7915	(51)	-9.93372	(69)	55.915
(16)	-15.1035	(34)	-85.2257	(52)	4.33286	(70)	153.359
(17)	173.496	(35)	17.7765	(53)	1.36267	(71)	-65.9082
(18)	-140.104	(36)	231.759	(54)	-75.4915	(72)	-22.4584

(73)	-96.2159	(91)	261.692	(109)	19.1317	(127)	-142.009
(74)	-207.873	(92)	-39.7519	(110)	-294.97	(128)	359.352
(75)	35.9245	(93)	-94.6894	(111)	-149.459	(129)	-70.8337
(76)	11.2805	(94)	-20.7665	(112)	-342.713	(130)	65.5717
(77)	18.9649	(95)	-87.2973	(113)	32.5776	(131)	-19.3832
(78)	36.8939	(96)	105.275	(114)	99.494	(132)	-69.2817
(79)	42.3009	(97)	-54.6526	(115)	17.2114	(133)	142.75
(80)	80.2136	(98)	147.502	(116)	110.698	(134)	-132.83
(81)	-160.838	(99)	-65.7748	(117)	-58.2802	(135)	14.2256
(82)	-31.6271	(100)	10.9481	(118)	-57.85	(136)	-219.141
(83)	48.9601	(101)	-118.076	(119)	13.6307	(137)	135.204
(84)	18.0452	(102)	221.183	(120)	-101.954	(138)	201.34
(85)	-66.5679	(103)	69.4948	(121)	28.2987	(139)	-75.5454
(86)	40.5033	(104)	-319.626	(122)	-62.976	(140)	-71.8773
(87)	-6.69331	(105)	176.085	(123)	-5.15519	(141)	41.2085
(88)	-20.0795	(106)	284.173	(124)	115.505	(142)	135.379
(89)	-99.4435	(107)	193.094	(125)	-97.4028	(143)	-180.296
(90)	-50.4235	(108)	-96.3609	(126)	-96.8643	(144)	148.664

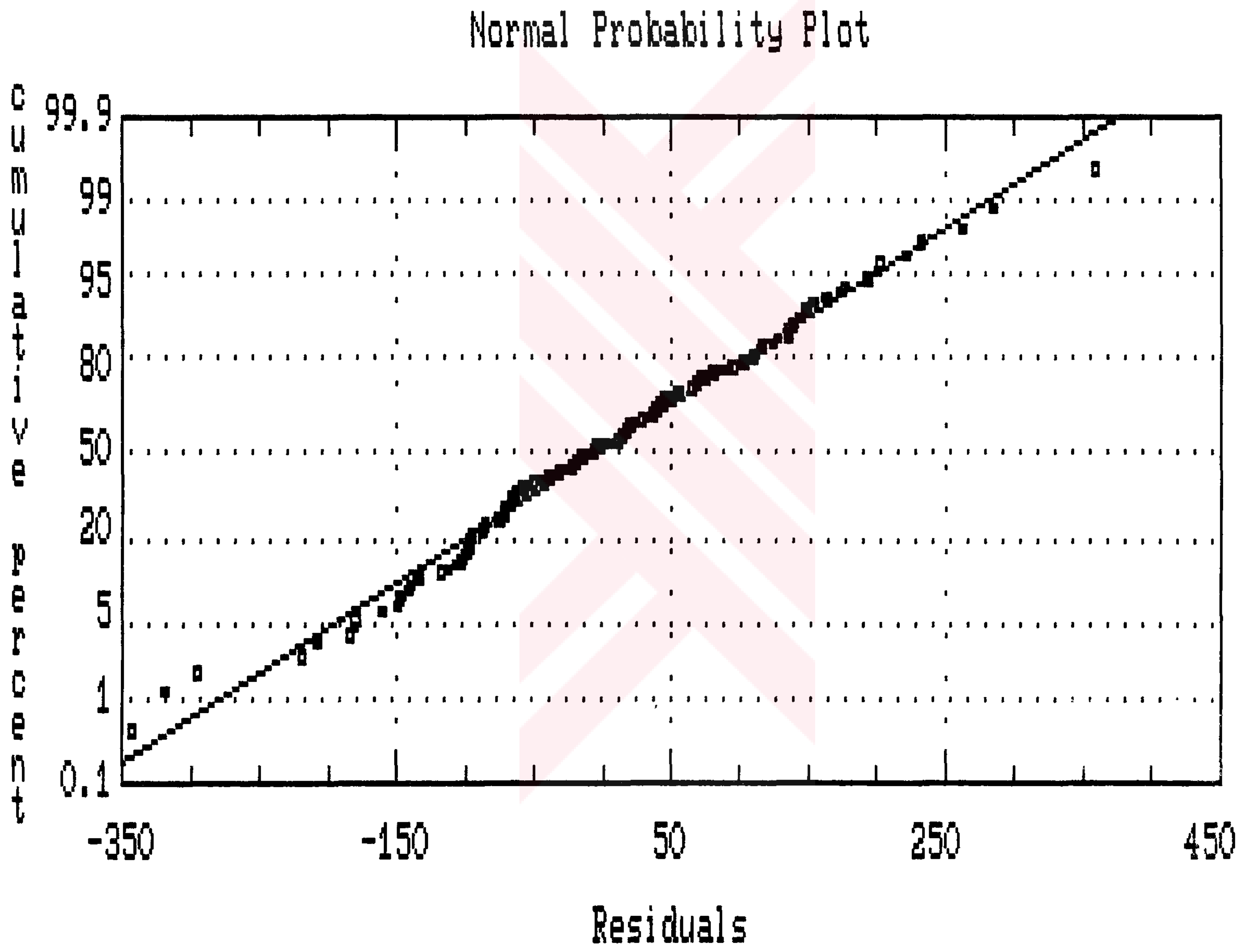

Figure 4.11 Plot of Residuals' Normal Probability Plot

In order to evaluate the forecasting performance of Box-Jenkins method, the model is reestablished with 120 month datas. The resulting model is used to forecast 24 months power consumption and compared with actual datas, for 1989 and 1990 years.

4.2 Comparison of the Methods

Mean Absolute Percent Error (MAPE) and Root Mean Square Error (RMSE) are used extensively to evaluate forecasting performance of the models.

The Simple Linear Regression, Curvilinear Regression, Exponential Regression, Moving Averages, Simple Exponential Smoothing, Holt's(Linear) Exponential Smoothing and Winter's Seasonal Exponential Smoothing methods are estimated, and these models are used to forecast 1989 January to 1990 December peak electricity power consumption.

The Curvilinear Regression and Exponential Regression is better than Simple Linear Regression model since the trend is of 2nd degree. Winter's Exponential Smoothing method is superior to Simple Exponential Smoothing and Holt's Exponential Smoothing since it allows both trend and seasonality adjustment.

In addition, moving averages is appropriate for short form forecasts and is beneficial only for 1 period and 1 season ahead forecasts.

The models that are estimated by Statgraph Software is summarized below.

1. The Simple Linear Regression

$$X_t = 2901.57 + 35.7818 * T \qquad (4.3)$$

2. The Curvilinear Regression

$$X_t = 3459.68 + 8.33366 * T + 0.226844 * T^2 \qquad (4.4)$$

3. The Exponential Regression

$$X_t = e^{(8.07565 + 0.00701084 * T)} \qquad (4.5)$$

The least squares estimation method is used to obtain these models by Statgraph software.

The simple exponential smoothing method is useless since trend and seasonality exist. Holt's exponential smoothing, as explained in CHAPTER III, does not handle seasonality effect but trend effect. These models are given in the following,

4. Holt's Exponential Smoothing

α and β constants are found to be 0.9 and 0.1, respectively.

5. Winter's Seasonal Exponential Smoothing

α, β and γ constants are obtained as 0.9, 0.1 and 0.4, respectively.

The forecast, actual and residual values for 24 month periods of these models and their plots can be seen in Appendix D.

The forecast performance criterias are summarized below.

	MAPE	RMSE	MPE	MAE	MSE
Sim. Lin. Regr.	6.81	666.43	-6.81	577.37	444122.45
Curv. Regr.	4.32	415.55	4.21	346.60	172683.51
Exp. Regr.	2.67	264.89	-0.71	222.55	70168.27
H. Exp. Sm.	3.07	293.17	1.79	246.09	85946.66
W. Exp. Sm.	4.56	458.54	-3.52	386.76	210259.26
Box-Jenkins	1.20	123.82	-0.68	100.50	15331.31

Box-Jenkins appeared to be the best forecasting technique according to the all evaluation criterias listed above. It's performance is better than Exponential Regression, which is the second best one, more than twice. Thus one can conclude that Box-Jenkins technique is superior to other forecasting techniques.

In order to analyse the the effect of forecasting horizon in forecast performance, the peak electricity power consumption datas for 132 month periods are used to estimate the Box-Jenkins and forecast 12 months peak electricity power consumption. Then the values obtained from forecasting procedure are compared with the other methods used before. In this comparison work the deviations from actual values are analised and compared with other techniques. The forecast evaluation criterias are used for comparison, as well.

The Statgraph software is rerun to estimate the models. In the following, estimated models are presented in brief.

1. Box-Jenkins Model

$$X_t=8.28+X_{t-1}+0.4517\ X_{t-12}-0.4517\ X_{t-13}+0.5483\ X_{t-24}-0.5483X_{t-25}+e_t \qquad (4.6)$$

2. Winter's Exponential Smoothing

$\alpha=0.9$, $\beta=0.1$ and $\gamma=0.4$

3. Holt's Exponential Smoothing

$\alpha=0.9$ and $\beta=0.1$

4. The Simple Linear Regression

$$X_t=2827.69+37.4983*T \qquad (4.7)$$

5. The Curvilinear Regression

$$X_t=3415.53+11.1771*T+0.197904*T^2 \qquad (4.8)$$

6. The Exponential Regression

$$X_t=e^{(8.0742+0.00704234*T)} \qquad (4.9)$$

The forecast evaluation criterias for each technique are listed below.

	MAPE	RMSE	MPE	MAE	MSE
Sim. Lin. Reg.	6.30	629.89	-6.24	553.00	396758.41
Curv. Reg.	2.93	322.86	2.38	241.28	104237.93
Exp. Reg.	2.46	267.19	0.28	209.58	71388.54
H. Exp. Sm.	5.32	509.54	5.32	446.41	259627.19
W. Exp. Sm.	2.59	303.21	-1.36	213.40	91938.71
Box-Jenkins	0.99	106.40	-0.27	83.65	11321.93

The maximum error dropped from -277.61 to -150.00 MW at period 142. The forecast evaluation criterias for each technique except Holt's Exponential Smoothing method decreased as well as maximum error. As an example, the MAPE and RMSE of Box-Jenkins technique reduced from 1.20 and 123.82 to 0.99 and 106.40, respectively. Other techniques also gave better results. In conclusion, as the forecast horizon is reduced, the forecast performance improves considerably.

The forecast, actual and residual values for 12 month periods of these models and their plots can be seen in Appendix D.

CHAPTER V

CONCLUSION

In this thesis, an intermediate term peak load forecasting time series model has been developed, its performance has been evaluated with forecast performance criterias and compared with other time series techniques.

Box-Jenkins technique proved that it is able to forecast peak load demand with a maximum error of 3.05 % for two years forecast. The average absolute percentage error turned out to be % 1.19 for the same horizon. It has been observed that Box-Jenkins technique is superior to other time series forecast techniques. Apart from complying with small errors of forecast criterias, Box-Jenkins technique has the advantage of being able to handle variability in the data. This feature of technique makes it possible to use with data possessing trend and seasonality. This is an important aspect while forecasting electricity demand because apart from growth, electricity demand is highly subject to variations which needs to be explicitly modelled.

Box-Jenkins method can be applied to short, medium or long term forecast horizons. With the intermediate term model developed in this thesis, Box-Jenkins technique has proven to be a powerfull tool for peak load forecasting. Therefore, it is suggested that it can be effectively used for maintenance scheduling, coordination of power sharing arrangements and setting of prices in order to meet the demand with fixed capacity.

It should, however, be noted that as the forecast horizon increases, the forecast performance decreases as with other forecasting techniques.

Furthermore because of the complexity of model identification, forecast preperation can take an extended period of time. Therefore, the preparer and users of Box-Jenkins forecasts need highly sophisticated technical background.

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APPENDICES

APPENDIX A

ORDINARY LEAST SQUARES ESTIMATION

In matrix form,

$$y = xb + e \quad (A.1)$$

where b is the vector of coefficients to be estimated, x is the vector of independent variables and y is the vector of dependent variables.

Error vector e is desired to be as small as possible:

$$y = xb \quad (A.2)$$

$$x'y = x'xb \quad (A.3)$$

$$b = (x'x)^{-1}x'y \quad (A.4)$$

APPENDIX B

BASIC THEORATICAL PROCESSES

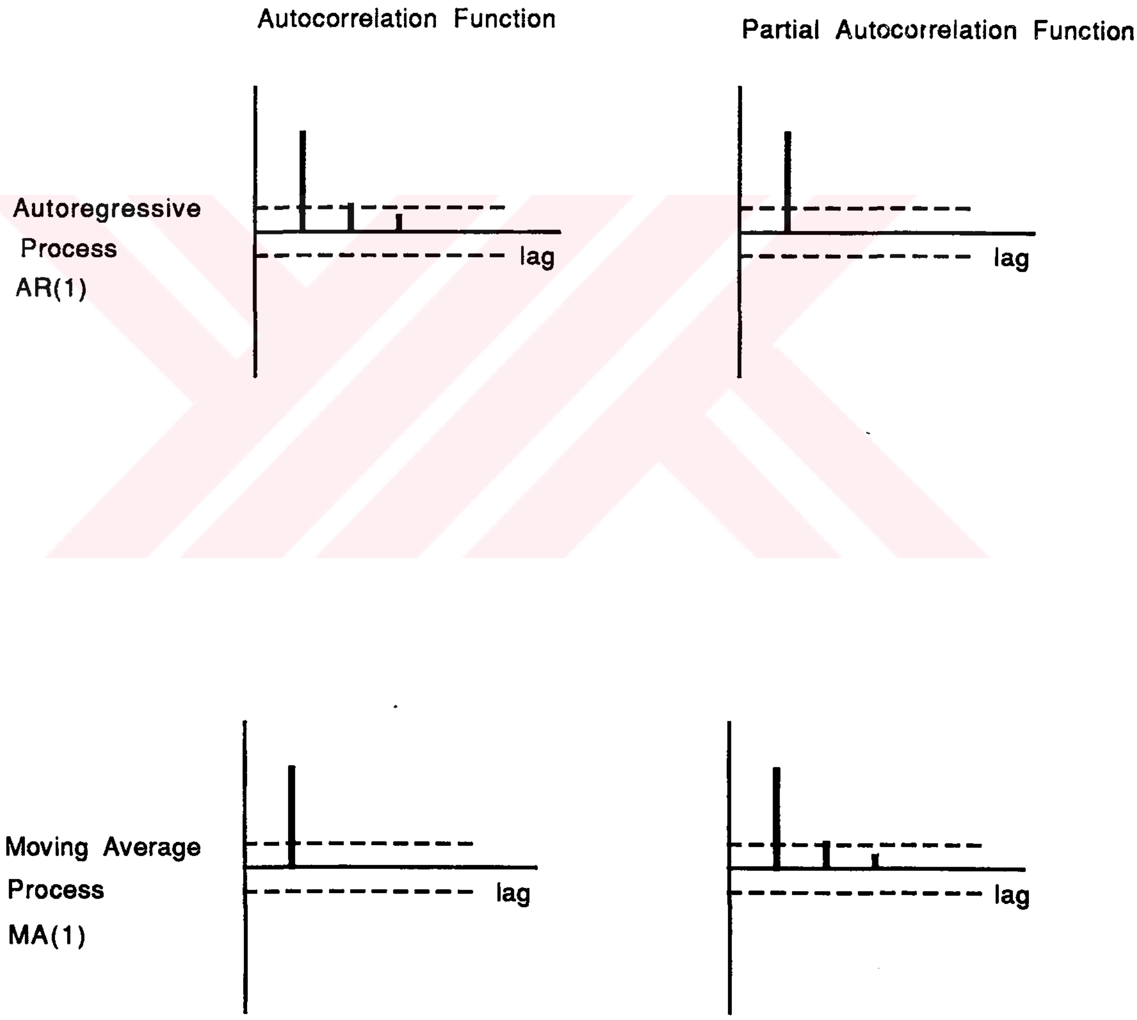

Figure B.1 Basic Theoratical Processes

APPENDIX C

ALTERNATIVE BOX-JENKINS MODELS TABLES AND FIGURES

Table C.1 Summary of SARIMA $(0,1,0)_1 x (0,1,1)_{12}$ Model

Summary of Fitted Model for: PEAKPOWE.VAR1

Parameter	Estimate	Stnd.error	T-value	P-value
SMA(12)	.72397	.07763	9.32614	.00000
MEAN	5.70963	3.65141	1.56368	.12034
CONSTANT	5.70963			

Model fitted to differences of order 1
Model fitted to seasonal differences of order 1 with seasonal length = 12
Estimated white noise variance = 13971.8 with 129 degrees of freedom.
Estimated white noise standard deviation (std err) = 118.202
Chi-square test statistic on first 20 residual autocorrelations = 24.4378
with probability of a larger value given white noise = 0.17989
Backforecasting: no   Number of iterations performed: 4

Residual Summary

Number of observations = 131
Residual average = 1.31322
Residual variance = 13971.8
Residual standard error = 118.202

Coeff. of skewness = -0.253862   standardized value = -1.1862
Coeff. of kurtosis = 0.537298   standardized value = 1.25529

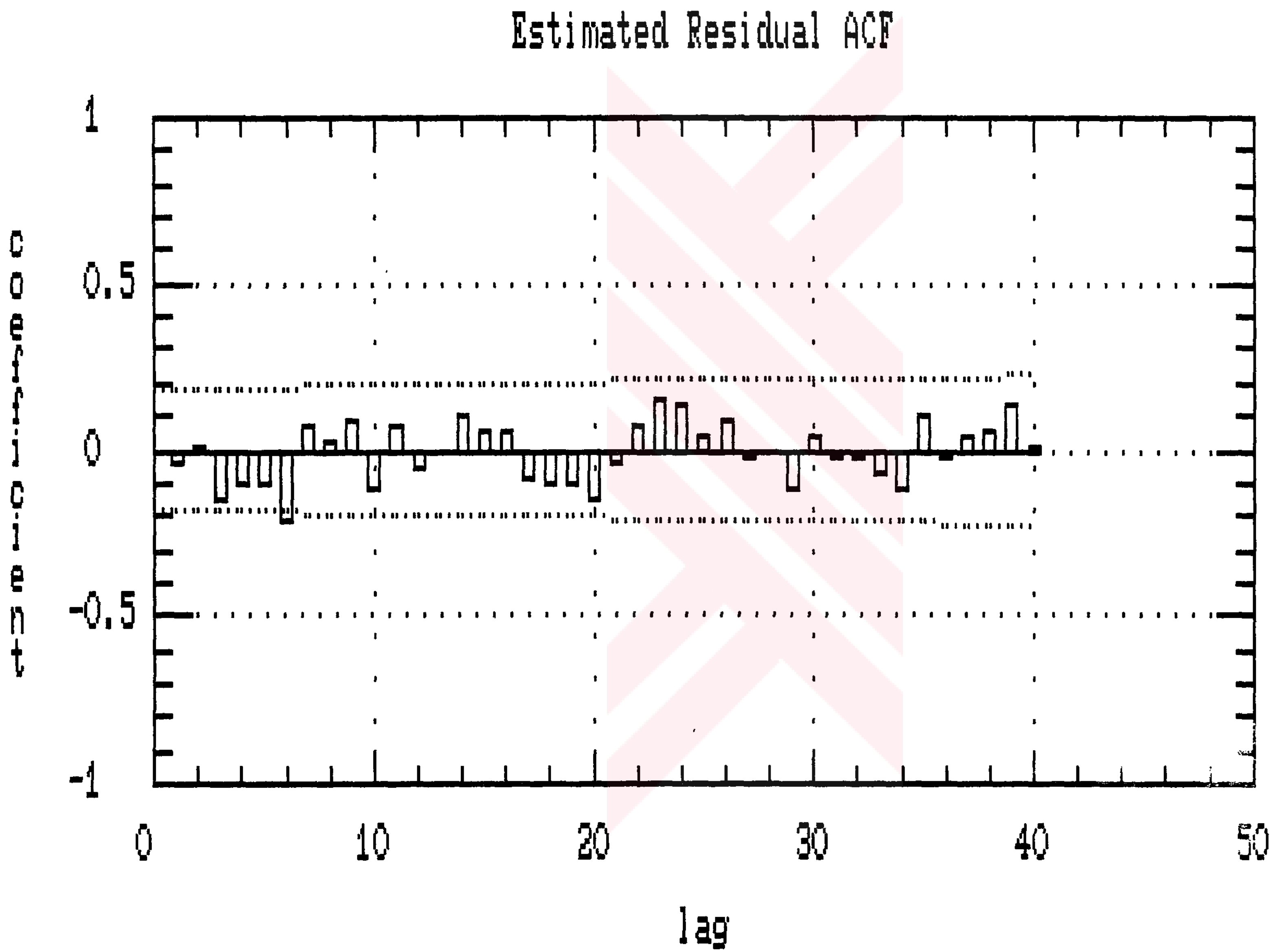

Figure C.1 Residual Correlagram of SARIMA $(0,1,0)_1 \times (0,1,1)_{12}$ Model

Table C.2 SARIMA (0,1,0)1x(0,1,1)12 Model Forecasts

SARIMA (0,1,0)x(0,1,1) FORECASTS

(1,1)	9142.52	(1,2)	8908.61	(1,3)	9376.44
(2,1)	9101.1	(2,2)	8770.29	(2,3)	9431.91
(3,1)	9093.48	(3,2)	8688.32	(3,3)	9498.64
(4,1)	9012.11	(4,2)	8544.28	(4,3)	9479.95
(5,1)	9057.73	(5,2)	8534.67	(5,3)	9580.78
(6,1)	9234.04	(6,2)	8661.06	(6,3)	9807.02
(7,1)	9413.25	(7,2)	8794.36	(7,3)	10032.1
(8,1)	9579.79	(8,2)	8918.17	(8,3)	10241.4
(9,1)	9761.96	(9,2)	9060.21	(9,3)	10463.7
(10,1)	9882.99	(10,2)	9143.27	(10,3)	10622.7
(11,1)	9989.55	(11,2)	9213.73	(11,3)	10765.4
(12,1)	10036.3	(12,2)	9225.95	(12,3)	10846.6
(13,1)	10004.1	(13,2)	9140.55	(13,3)	10867.6
(14,1)	9968.39	(14,2)	9054.71	(14,3)	10882.1
(15,1)	9966.47	(15,2)	9005.27	(15,3)	10927.7
(16,1)	9890.81	(16,2)	8884.34	(16,3)	10897.3
(17,1)	9942.14	(17,2)	8892.33	(17,3)	10991.9
(18,1)	10124.2	(18,2)	9032.74	(18,3)	11215.6
(19,1)	10309.1	(19,2)	9177.58	(19,3)	11440.6
(20,1)	10481.3	(20,2)	9311.13	(20,3)	11651.5
(21,1)	10669.2	(21,2)	9461.54	(21,3)	11876.9
(22,1)	10795.9	(22,2)	9551.93	(22,3)	12040
(23,1)	10908.2	(23,2)	9628.9	(23,3)	12187.5
(24,1)	10960.6	(24,2)	9646.96	(24,3)	12274.3

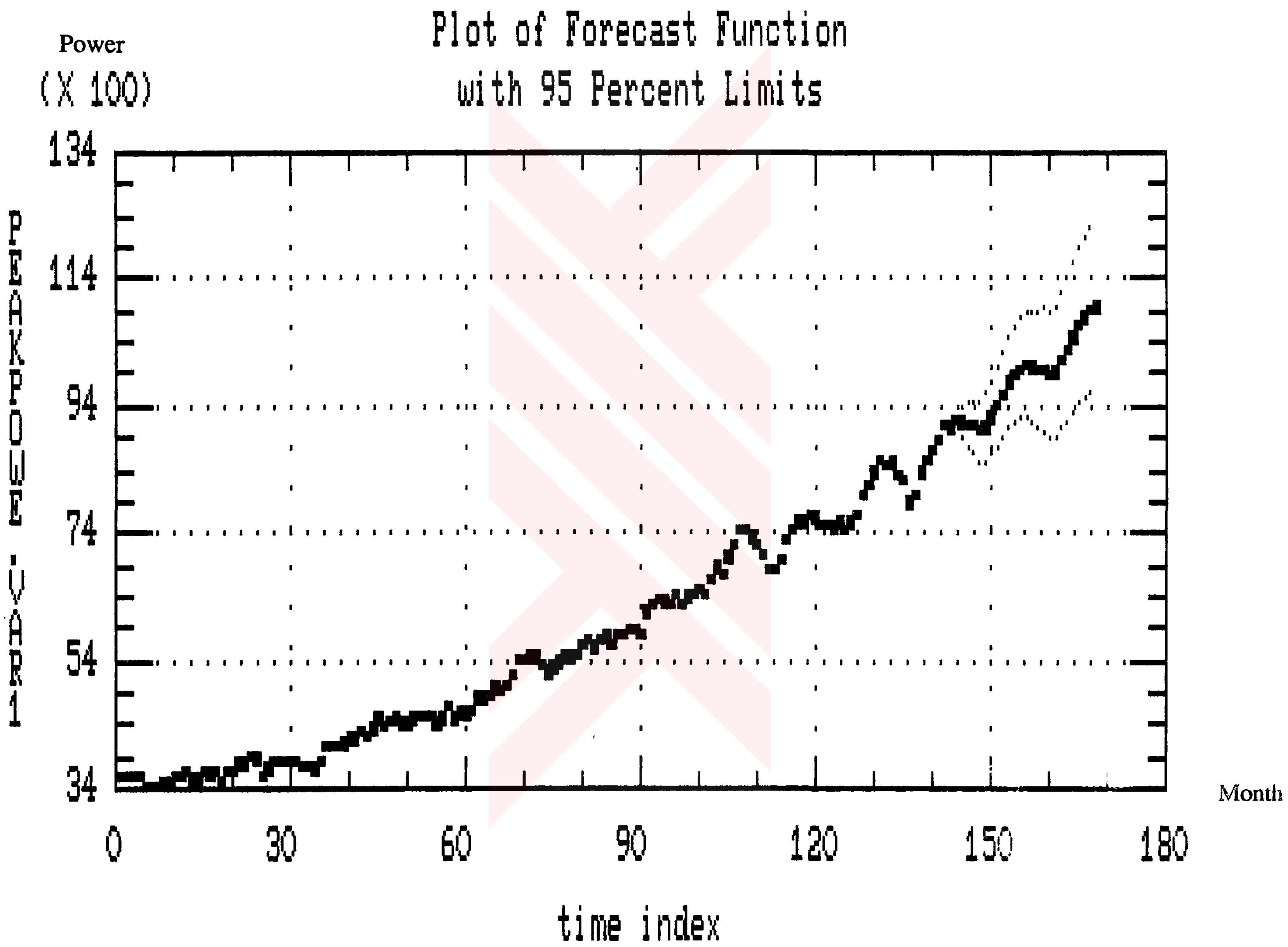

Figure C.2 Plot of Forecasted Values of SARIMA $(0,1,0)_1 x (0,1,1)_{12}$ Model with 95 Percent Limits

Table C.3 SARIMA $(0,1,0)_1 x (0,1,1)_{12}$ Model Residuals

SARIMA (0,1,0)x(0,1,1) RESIDUALS

(1)		(19)	139.99	(37)	114.857	(55)	-137.406
(2)		(20)	-66.4096	(38)	-31.7653	(56)	55.841
(3)		(21)	54.4904	(39)	-37.0393	(57)	112.01
(4)		(22)	-43.7096	(40)	65.0606	(58)	-201.604
(5)		(23)	110.59	(41)	82.6745	(59)	-4.95338
(6)		(24)	-103.51	(42)	105.572	(60)	-90.0016
(7)		(25)	-184.31	(43)	-77.6236	(61)	90.3238
(8)		(26)	75.203	(44)	77.836	(62)	180.358
(9)		(27)	56.3153	(45)	85.805	(63)	-89.3639
(10)		(28)	21.3687	(46)	-127.761	(64)	19.667
(11)		(29)	108.409	(47)	-18.1552	(65)	146.308
(12)		(30)	32.573	(48)	-57.7265	(66)	-65.0344
(13)		(31)	-36.7614	(49)	4.74256	(67)	88.7128
(14)	-33.4096	(32)	-154.088	(50)	-57.5066	(68)	79.1173
(15)	94.7904	(33)	-58.9605	(51)	10.9752	(69)	107.182
(16)	-14.8096	(34)	-89.8539	(52)	21.792	(70)	74.136
(17)	173.79	(35)	31.154	(53)	59.1438	(71)	-73.9957
(18)	-139.81	(36)	220.153	(54)	-48.3791	(72)	-57.8677

(73)	-83.1183	(91)	300.317	(109)	-7.3382	(127)	-77.9192
(74)	-211.637	(92)	8.86168	(110)	-244.769	(128)	251.871
(75)	30.794	(93)	-58.6025	(111)	-174.026	(129)	-22.8375
(76)	23.6286	(94)	-0.669485	(112)	-343.414	(130)	166.151
(77)	62.5127	(95)	-114.85	(113)	10.1357	(131)	66.2001
(78)	8.50772	(96)	77.0614	(114)	91.1179	(132)	-121.625
(79)	-7.28459	(97)	-94.9335	(115)	161.597	(133)	138.384
(80)	114.469	(98)	66.0816	(116)	83.8167	(134)	-247.205
(81)	-98.6136	(99)	-56.6553	(117)	-68.8263	(135)	-67.6344
(82)	-111.138	(100)	20.8515	(118)	-30.1668	(136)	-359.721
(83)	38.62	(101)	-93.312	(119)	-13.385	(137)	124.172
(84)	-27.8038	(102)	216.347	(120)	-79.169	(138)	242.964
(85)	-36.0844	(103)	101.809	(121)	-13.8222	(139)	-13.1204
(86)	95.5725	(104)	-267.794	(122)	-62.0138	(140)	-5.16388
(87)	-41.9159	(105)	126.364	(123)	-49.8985	(141)	8.75681
(88)	-8.20332	(106)	235.706	(124)	85.0706	(142)	141.978
(89)	-27.3526	(107)	197.143	(125)	-136.772	(143)	-177.983
(90)	-77.5503	(108)	-99.1199	(126)	9.35655	(144)	100.138

Table C.4 Summary of SARIMA $(0,1,0)_1 x(1,1,1)_{12}$ Model

Summary of Fitted Model for: PEAKPOWE.VAR1

Parameter	Estimate	Stnd.error	T-value	P-value
SAR(12)	-.26793	.14545	-1.84204	.06778
SMA(12)	.49743	.13538	3.67434	.00035
MEAN	5.55446	4.55024	1.22070	.22445
CONSTANT	7.04268			

Model fitted to differences of order 1
Model fitted to seasonal differences of order 1 with seasonal length = 12
Estimated white noise variance = 13889.9 with 128 degrees of freedom.
Estimated white noise standard deviation (std err) = 117.856
Chi-square test statistic on first 20 residual autocorrelations = 22.9511
with probability of a larger value given white noise = 0.192477
Backforecasting: no Number of iterations performed: 4

Residual Summary

Number of observations = 131
Residual average = 0.69891
Residual variance = 13889.9
Residual standard error = 117.856

Coeff. of skewness = -0.129502 standardized value = -0.605112
Coeff. of kurtosis = 0.388302 standardized value = 0.907193

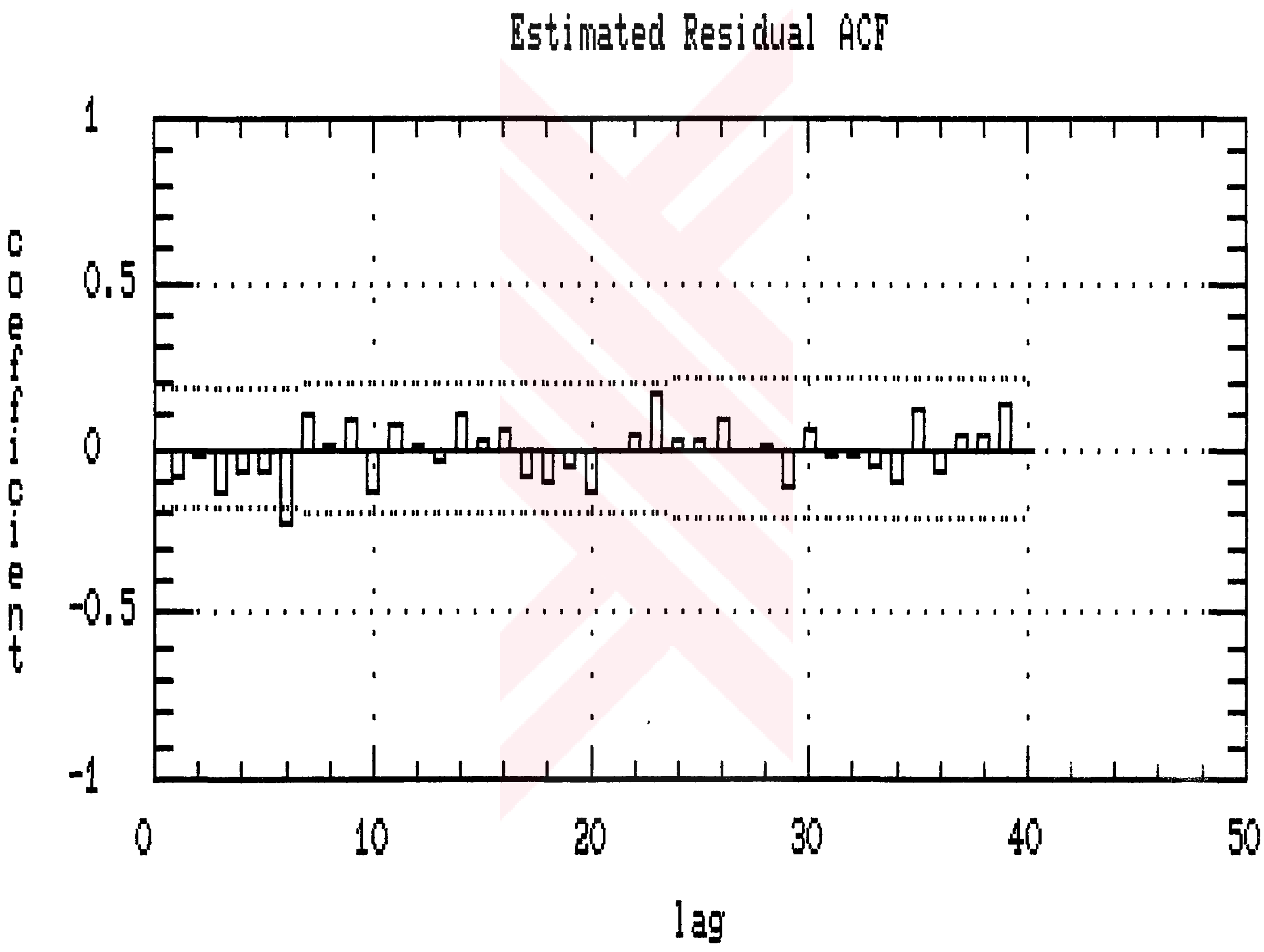

Figure C.3 Residual Correlagram of SARIMA $(0,1,0)_1 x (1,1,1)_{12}$ Model

Table C.5 SARIMA $(0,1,0)_1 x(1,1,1)_{12}$ Model Forecasts

SARIMA (0,1,0)x(1,1,1) FORECASTS

(1,1)	9128.17	(1,2)	8894.92	(1,3)	9361.42
(2,1)	9068.15	(2,2)	8738.29	(2,3)	9398.02
(3,1)	9035.85	(3,2)	8631.86	(3,3)	9439.85
(4,1)	8955.35	(4,2)	8488.85	(4,3)	9421.85
(5,1)	8969.83	(5,2)	8448.27	(5,3)	9491.39
(6,1)	9152.53	(6,2)	8581.19	(6,3)	9723.87
(7,1)	9343.05	(7,2)	8725.93	(7,3)	9960.17
(8,1)	9550.78	(8,2)	8891.05	(8,3)	10210.5
(9,1)	9726.37	(9,2)	9026.62	(9,3)	10426.1
(10,1)	9875.87	(10,2)	9138.27	(10,3)	10613.5
(11,1)	10005.8	(11,2)	9232.18	(11,3)	10779.4
(12,1)	10016.6	(12,2)	9208.63	(12,3)	10824.6
(13,1)	10000.6	(13,2)	9142.82	(13,3)	10858.4
(14,1)	9903.13	(14,2)	8998.29	(14,3)	10808
(15,1)	9869.83	(15,2)	8920.28	(15,3)	10819.4
(16,1)	9724.84	(16,2)	8732.57	(16,3)	10717.1
(17,1)	9777.26	(17,2)	8744.05	(17,3)	10810.5
(18,1)	10010.9	(18,2)	8938.3	(18,3)	11083.5
(19,1)	10201.3	(19,2)	9090.76	(19,3)	11311.9
(20,1)	10402.5	(20,2)	9255.24	(20,3)	11549.9
(21,1)	10587.1	(21,2)	9404.22	(21,3)	11770
(22,1)	10762	(22,2)	9544.59	(22,3)	11979.5
(23,1)	10856.7	(23,2)	9605.64	(23,3)	12107.7
(24,1)	10902.1	(24,2)	9618.32	(24,3)	12185.8

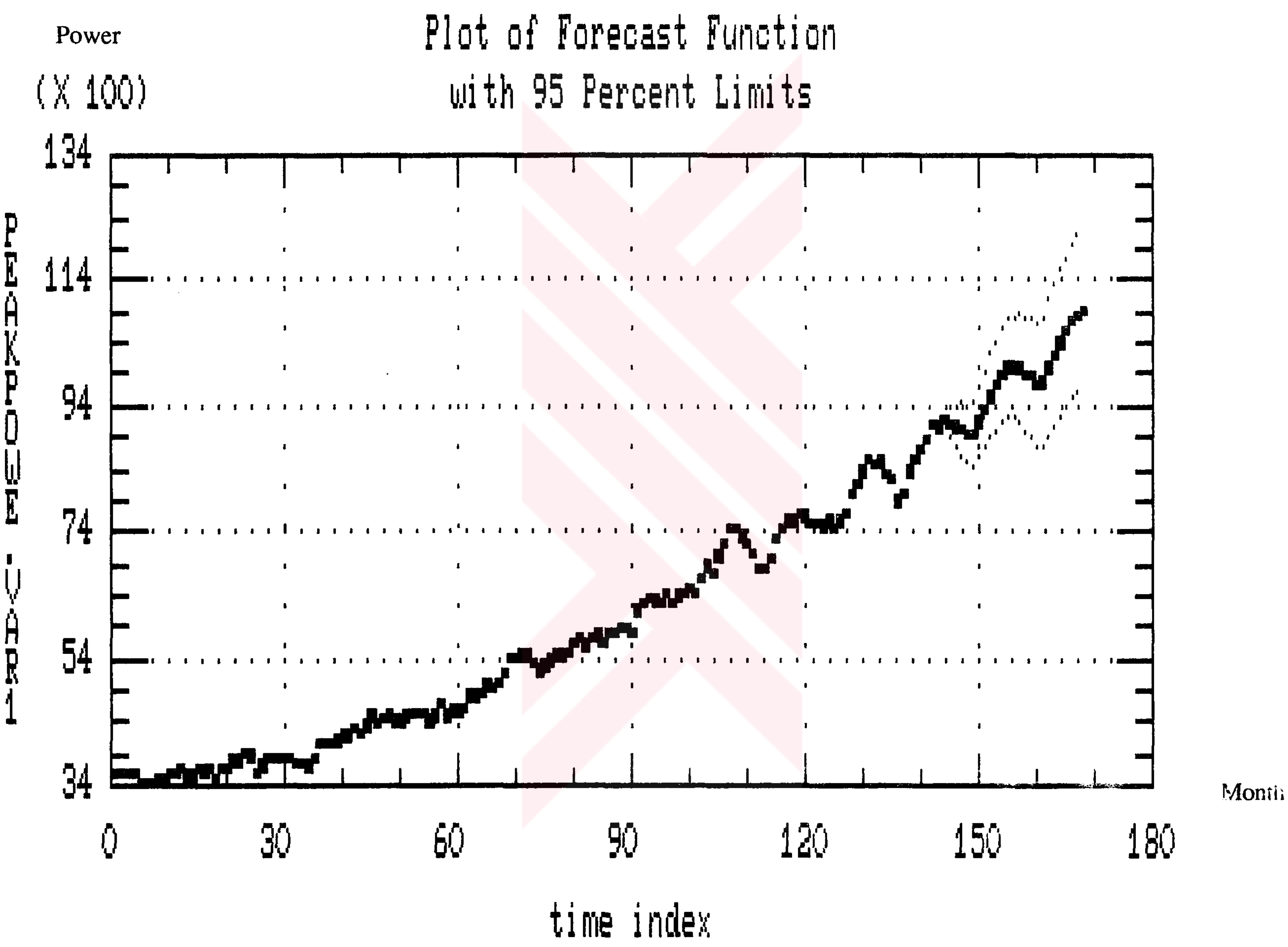

Figure C.4 Plot of Forecasted Values of SARIMA $(0,1,0)_1 x (1,1,1)_{12}$ Model with 95 Percent Limits

Table C.6 SARIMA (0,1,0)1x(1,1,1)12 Model Residuals

SARIMA (0,1,0)x(1,1,1) RASIDUALS

(1)		(19)	140.146	(37)	107.5	(55)	-145.957
(2)		(20)	-66.2545	(38)	-22.5264	(56)	91.8689
(3)		(21)	54.6455	(39)	-50.8095	(57)	121.448
(4)		(22)	-43.5545	(40)	68.8459	(58)	-187.182
(5)		(23)	110.746	(41)	57.3639	(59)	-20.2819
(6)		(24)	-103.354	(42)	131.494	(60)	-121.272
(7)		(25)	-184.154	(43)	-103.084	(61)	86.8645
(8)		(26)	74.0937	(44)	83.3039	(62)	178.809
(9)		(27)	60.5136	(45)	74.2496	(63)	-91.0342
(10)		(28)	21.0295	(46)	-123.569	(64)	8.45844
(11)		(29)	115.877	(47)	-35.7066	(65)	117.994
(12)		(30)	27.0589	(48)	-30.3327	(66)	-81.7756
(13)		(31)	-30.6919	(49)	41.7861	(67)	94.0243
(14)	-33.2545	(32)	-156.563	(50)	-68.6165	(68)	84.4493
(15)	94.9455	(33)	-56.4307	(51)	-8.13492	(69)	100.067
(16)	-14.6545	(34)	-91.3895	(52)	22.4202	(70)	97.9424
(17)	173.946	(35)	36.0063	(53)	29.1446	(71)	-78.1073
(18)	-139.654	(36)	216.142	(54)	-37.2356	(72)	-65.7545

(73)	-81.8229	(91)	299.501	(109)	-1.67613	(127)	-114.189
(74)	-193.589	(92)	-3.89075	(110)	-246.443	(128)	294.286
(75)	24.3313	(93)	-93.542	(111)	-167.923	(129)	-41.7658
(76)	14.837	(94)	7.51355	(112)	-344.849	(130)	130.778
(77)	43.2099	(95)	-100.667	(113)	-3.43133	(131)	41.6061
(78)	7.06864	(96)	92.9706	(114)	116.074	(132)	-110.728
(79)	25.8801	(97)	-90.1165	(115)	114.539	(133)	149.939
(80)	109.761	(98)	94.0896	(116)	57.2878	(134)	-213.831
(81)	-119.246	(99)	-56.4115	(117)	-52.2701	(135)	-32.2563
(82)	-56.9239	(100)	12.6367	(118)	-7.24963	(136)	-298.731
(83)	34.6689	(101)	-123.882	(119)	14.7671	(137)	122.694
(84)	-16.4681	(102)	217.875	(120)	-89.7194	(138)	213.148
(85)	-56.2048	(103)	115.446	(121)	7.30176	(139)	-65.5368
(86)	61.0006	(104)	-295.778	(122)	-85.6012	(140)	10.3009
(87)	-26.3248	(105)	125.883	(123)	-42.881	(141)	11.943
(88)	-15.2165	(106)	261.503	(124)	66.2915	(142)	137.309
(89)	-62.5496	(107)	192.149	(125)	-124.804	(143)	-184.683
(90)	-65.1023	(108)	-82.4258	(126)	-16.2258	(144)	116.077

Table C.7 SARIMA $(0,1,0)_1 x(0,1,1)_{12}$ Model, SARIMA $(0,1,0)_1 x(0,1,1)_{12}$ Model and SARIMA $(0,1,0)_1 x(0,1,1)_{12}$ Model Parameter Correlations

SARIMA (0,1,0)x(1,1,0) PARAMETER CORRELATIONS

(1,1) 0	(1,2) 1	(1,3) 2
(2,1) 1	(2,2) 1	(2,3) -6.81718E-3
(3,1) 2	(3,2) -6.81718E-3	(3,3) 1

SARIMA (0,1,0)x0,1,1) PARAMETER CORRELATIONS

(1,1) 0	(1,2) 1	(1,3) 2
(2,1) 1	(2,2) 1	(2,3) 0.0728356
(3,1) 2	(3,2) 0.0728356	(3,3) 1

SARIMA (0,1,0)x(1,1,1) PARAMETER CORRELATIONS

(1,1) 0	(1,2) 1	(1,3) 2	(1,4) 3
(2,1) 1	(2,2) 1	(2,3) 0.761876	(2,4) 0.0204256
(3,1) 2	(3,2) 0.761876	(3,3) 1	(3,4) 0.0328505
(4,1) 3	(4,2) 0.0204256	(4,3) 0.0328505	(4,4) 1

APPENDIX D

TIME SERIES TECNIQUES FORECAST EVALUATION

TABLES AND FIGURES

Table D.1 Summary of 24 Months Peak Load Demand Forecast Evaluation Criterias

	MAPE	RMSE	MPE	MAE	MSE
BOX-JENKINS METHOD	1.20	123.82	-0.68	100.50	15,331
WINTER'S EXPONENTIAL SMOOTHING	4.56	458.54	-3.52	386.76	210,259
HOLT'S EXPONENTIAL SMOOTHING	3.07	293.17	1.79	246.09	85,947
EXPONENTIAL REGRESSION	2.67	264.89	-0.71	222.55	70,168
CURVILINEAR REGRESSION	4.32	415.55	4.21	346.60	172,684
SIMPLE LINEAR REGRESSION	6.81	666.43	-6.81	577.37	444,122

Table D.2 Summary of 12 Months Peak Load Demand Forecast Evaluation Criterias

	MAPE	RMSE	MPE	MAE	MSE
BOX-JENKINS METHOD	0.99	106.40	-0.27	83.65	11,322
WINTER'S EXPONENTIAL SMOOTHING	2.59	303.21	1.36	213.40	91,939
HOLT'S EXPONENTIAL SMOOTHING	5.32	509.54	5.32	446.41	259,627
EXPONENTIAL REGRESSION	2.46	267.19	-0.28	209.58	71,389
CURVILINEAR REGRESSION	2.93	322.86	2.38	241.28	104,238
SIMPLE LINEAR REGRESSION	6.30	629.89	-6.24	553.00	396,758

Table D.3 24 Months Peak Load Demand Forecast Evaluation with Simple Linear Regression Method

t	Ft	At	e	%e	lel	%lel	e2
121	7,231.16	7,560.00	-328.84	-4.35	328.84	4.35	108,136
122	7,266.95	7,530.60	-263.65	-3.50	263.65	3.50	69,511
123	7,302.73	7,494.10	-191.37	-2.55	191.37	2.55	36,622
124	7,338.51	7,562.20	-223.69	-2.96	223.69	2.96	50,037
125	7,374.29	7,463.10	-88.81	-1.19	88.81	1.19	7,887
126	7,410.07	7,567.70	-157.63	-2.08	157.63	2.08	24,847
127	7,445.86	7,682.70	-236.84	-3.08	236.84	3.08	56,093
128	7,481.64	8,021.60	-539.96	-6.73	539.96	6.73	291,557
129	7,517.42	8,173.40	-655.98	-8.03	655.98	8.03	430,310
130	7,553.20	8,364.10	-810.90	-9.70	810.90	9.70	657,559
131	7,588.98	8,556.30	-967.32	-11.31	967.32	11.31	935,708
132	7,624.76	8,475.90	-851.14	-10.04	851.14	10.04	724,439
133	7,660.55	8,532.50	-871.95	-10.22	871.95	10.22	760,297
134	7,696.33	8,306.40	-610.07	-7.34	610.07	7.34	372,185
135	7,732.11	8,244.10	-511.99	-6.21	511.99	6.21	262,134
136	7,767.89	7,896.60	-128.71	-1.63	128.71	1.63	16,566
137	7,803.67	8,026.40	-222.73	-2.77	222.73	2.77	49,609
138	7,839.46	8,372.90	-533.44	-6.37	533.44	6.37	284,558
139	7,875.24	8,536.90	-661.66	-7.75	661.66	7.75	437,794
140	7,911.02	8,694.00	-782.98	-9.01	782.98	9.01	613,058
141	7,946.80	8,876.80	-930.00	-10.48	930.00	10.48	864,900
142	7,982.58	9,094.90	-1,112.32	-12.23	1,112.32	12.23	1,237,256
143	8,018.36	9,066.90	-1,048.54	-11.56	1,048.54	11.56	1,099,436
144	8,054.15	9,180.40	-1,126.25	-12.27	1,126.25	12.27	1,268,439
TOTALS			-13,856.77	-163.36	13,856.77	163.36	10,658,939
NUMBER OF OBSERVATION			24				

RMSE	MPE	MAE	MAPE	MSE
666.43	-6.81	577.37	6.81	444,122

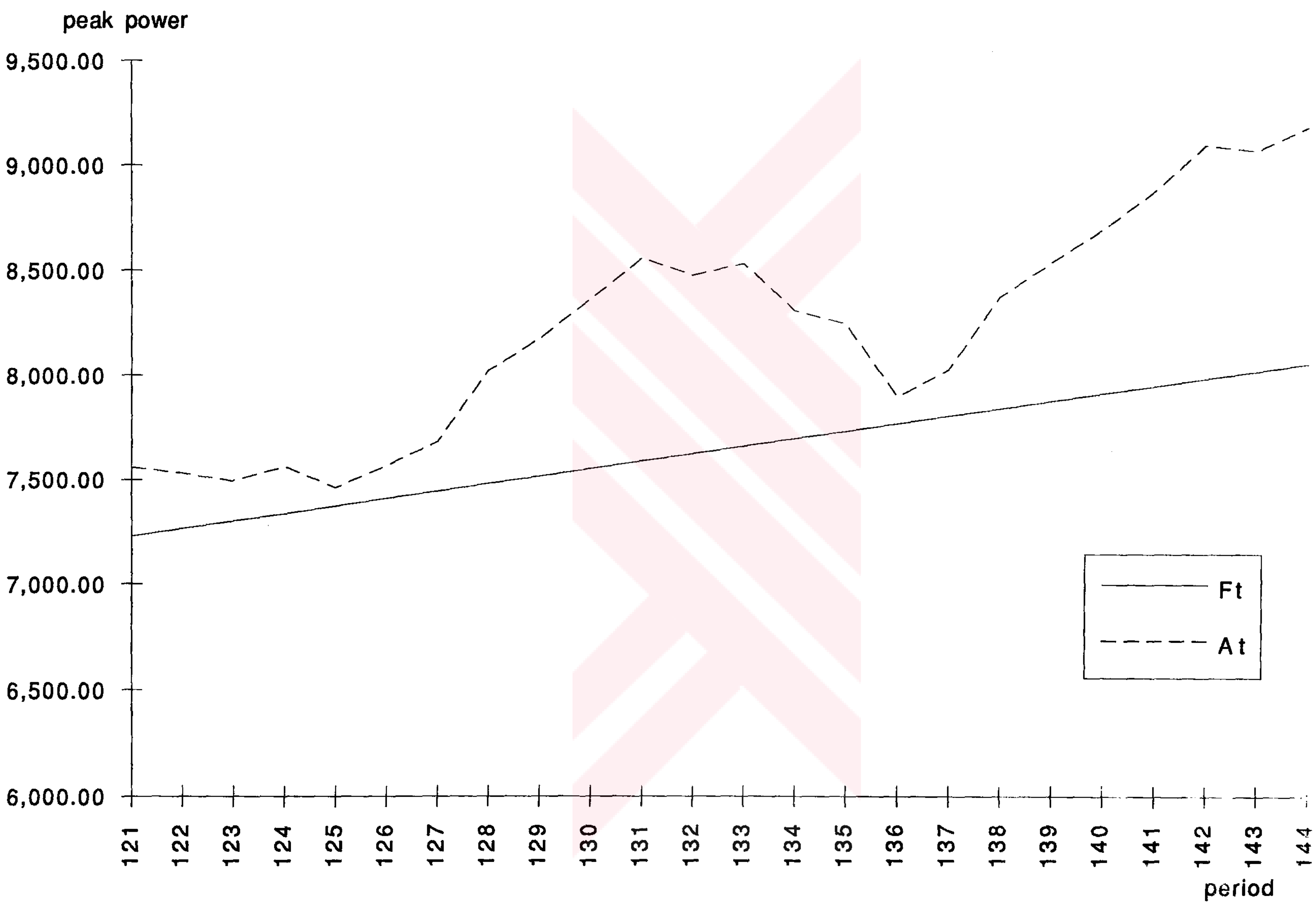

Figure D.1 Plot of 24 Months Peak Load Demand Forecast with Simple Linear Regression Method

Table D.4 24 Months Peak Load Demand Forecast Evaluation with Curvilinear Regression Method

t	Ft	At	e	%e	lel	%lel	e2
121	7,789.16	7,560.00	229.16	3.03	229.16	3.03	52,514
122	7,852.73	7,530.60	322.13	4.28	322.13	4.28	103,768
123	7,916.64	7,494.10	422.54	5.64	422.54	5.64	178,540
124	7,981.01	7,562.20	418.81	5.54	418.81	5.54	175,402
125	8,045.83	7,463.10	582.73	7.81	582.73	7.81	339,574
126	8,111.10	7,567.70	543.40	7.18	543.40	7.18	295,284
127	8,176.82	7,682.70	494.12	6.43	494.12	6.43	244,155
128	8,243.00	8,021.60	221.40	2.76	221.40	2.76	49,018
129	8,309.63	8,173.40	136.23	1.67	136.23	1.67	18,559
130	8,376.72	8,364.10	12.62	0.15	12.62	0.15	159
131	8,444.26	8,556.30	-112.04	-1.31	112.04	1.31	12,553
132	8,512.25	8,475.90	36.35	0.43	36.35	0.43	1,321
133	8,580.70	8,532.50	48.20	0.56	48.20	0.56	2,323
134	8,649.60	8,306.40	343.20	4.13	343.20	4.13	117,786
135	8,718.96	8,244.10	474.86	5.76	474.86	5.76	225,492
136	8,788.77	7,896.60	892.17	11.30	892.17	11.30	795,967
137	8,859.03	8,026.40	832.63	10.37	832.63	10.37	693,273
138	8,929.74	8,372.90	556.84	6.65	556.84	6.65	310,071
139	9,000.91	8,536.90	464.01	5.44	464.01	5.44	215,305
140	9,072.54	8,694.00	378.54	4.35	378.54	4.35	143,293
141	9,144.61	8,876.80	267.81	3.02	267.81	3.02	71,722
142	9,217.14	9,094.90	122.24	1.34	122.24	1.34	14,943
143	9,290.13	9,066.90	223.23	2.46	223.23	2.46	49,832
144	9,363.57	9,180.40	183.17	2.00	183.17	2.00	33,551
TOTALS			8,094.35	100.99	8,318.43	103.61	4,144,404
NUMBER OF OBSERVATION			24				

RMSE	MPE	MAE	MAPE	MSE
415.55	4.21	346.60	4.32	172,684

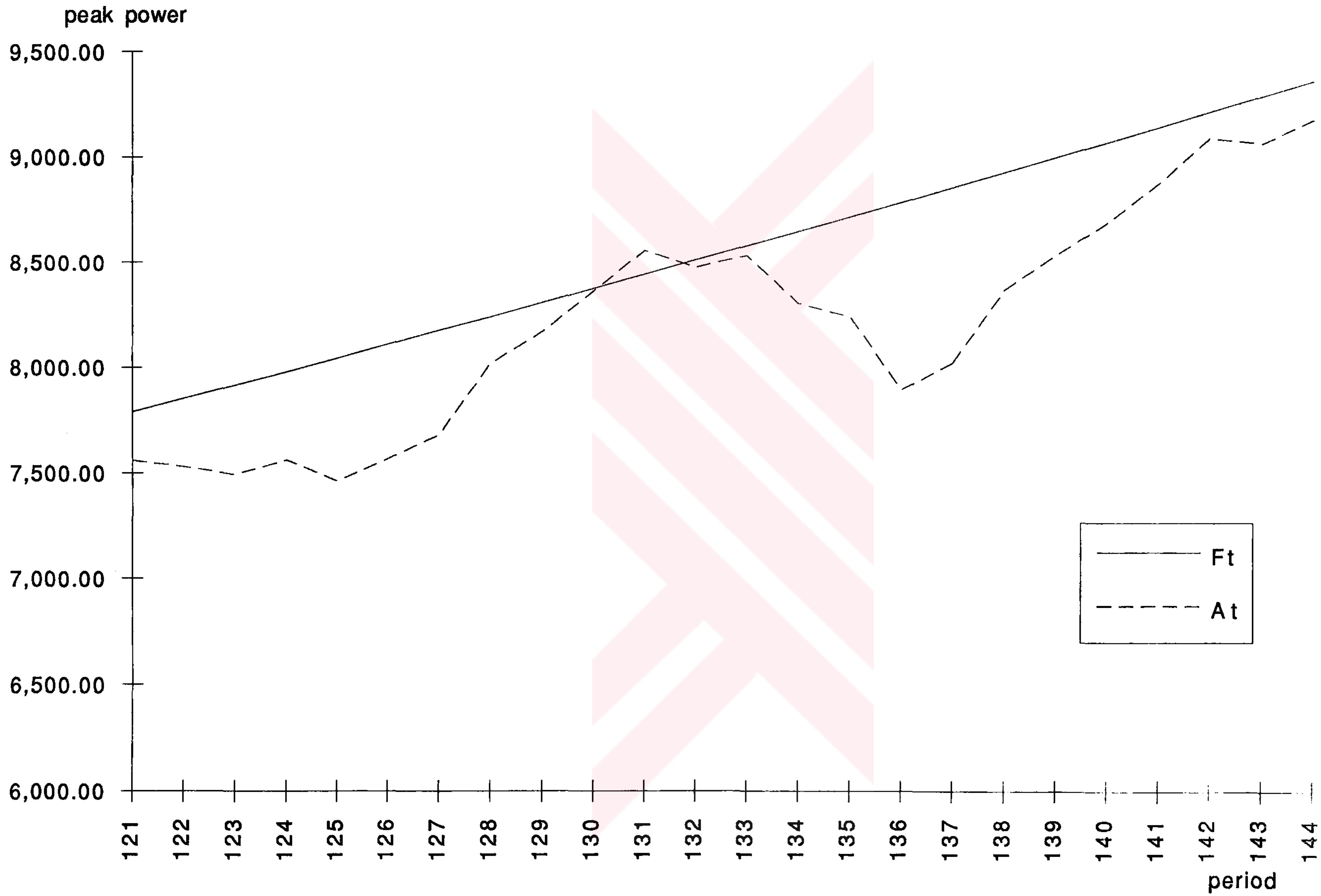

Figure D.2 Plot of 24 Months Peak Load Demand Forecast with Curvilinear Regression Method

Table D.5 24 Months Peak Load Demand Forecast Evaluation with Exponential Regression Method

t	Ft	At	e	%e	lel	%lel	e2
121	7,509.75	7,560.00	-50.25	-0.66	50.25	0.66	2,525
122	7,562.59	7,530.60	31.99	0.42	31.99	0.42	1,023
123	7,615.79	7,494.10	121.69	1.62	121.69	1.62	14,808
124	7,669.37	7,562.20	107.17	1.42	107.17	1.42	11,485
125	7,723.33	7,463.10	260.23	3.49	260.23	3.49	67,720
126	7,777.67	7,567.70	209.97	2.77	209.97	2.77	44,087
127	7,832.39	7,682.70	149.69	1.95	149.69	1.95	22,407
128	7,887.49	8,021.60	-134.11	-1.67	134.11	1.67	17,985
129	7,942.99	8,173.40	-230.41	-2.82	230.41	2.82	53,089
130	7,998.87	8,364.10	-365.23	-4.37	365.23	4.37	133,393
131	8,055.14	8,556.30	-501.16	-5.86	501.16	5.86	251,161
132	8,111.82	8,475.90	-364.08	-4.30	364.08	4.30	132,554
133	8,168.89	8,532.50	-363.61	-4.26	363.61	4.26	132,212
134	8,226.36	8,306.40	-80.04	-0.96	80.04	0.96	6,406
135	8,284.23	8,244.10	40.13	0.49	40.13	0.49	1,610
136	8,342.52	7,896.60	445.92	5.65	445.92	5.65	198,845
137	8,401.21	8,026.40	374.81	4.67	374.81	4.67	140,483
138	8,460.32	8,372.90	87.42	1.04	87.42	1.04	7,642
139	8,519.84	8,536.90	-17.06	-0.20	17.06	0.20	291
140	8,579.78	8,694.00	-114.22	-1.31	114.22	1.31	13,046
141	8,640.14	8,876.80	-236.66	-2.67	236.66	2.67	56,008
142	8,700.93	9,094.90	-393.97	-4.33	393.97	4.33	155,212
143	8,762.15	9,066.90	-304.75	-3.36	304.75	3.36	92,873
144	8,823.79	9,180.40	-356.61	-3.88	356.61	3.88	127,171
TOTALS			-1,683.14	-17.13	5,341.18	64.18	1,684,039
NUMBER OF OBSERVATION			24				

RMSE	MPE	MAE	MAPE	MSE
264.89	-0.71	222.55	2.67	70,168

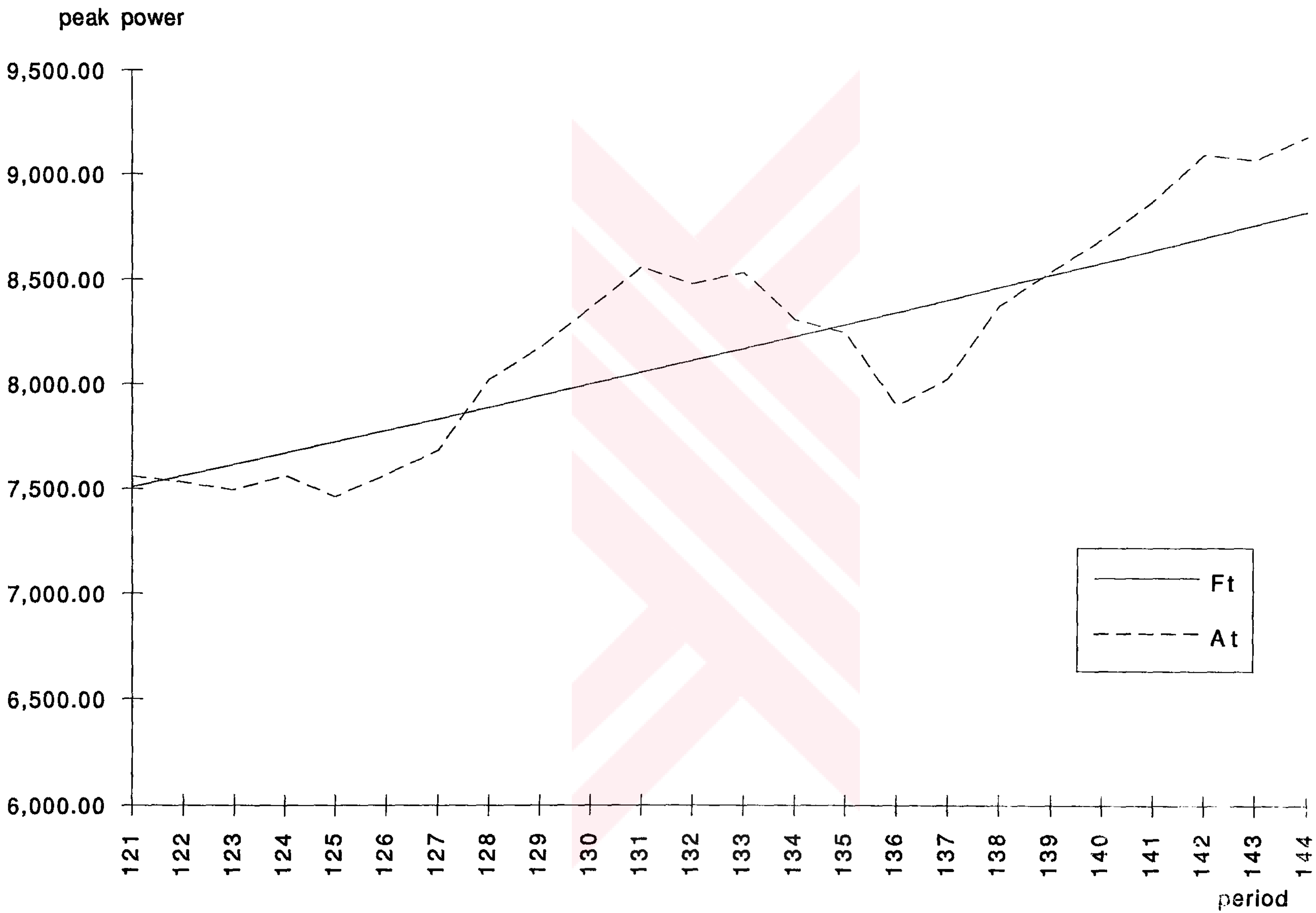

Figure D.3 Plot of 24 Months Peak Load Demand Forecast with Exponential Regression Method

Table D.6 24 Months Peak Load Demand Forecast Evaluation with Holt's Exponential Smoothing Method

t	Ft	At	e	%e	lel	%lel	e2
121	7,720.64	7,560.00	160.64	2.12	160.64	2.12	25,805
122	7,775.75	7,530.60	245.15	3.26	245.15	3.26	60,099
123	7,830.85	7,494.10	336.75	4.49	336.75	4.49	113,401
124	7,885.95	7,562.20	323.75	4.28	323.75	4.28	104,814
125	7,941.06	7,463.10	477.96	6.40	477.96	6.40	228,446
126	7,996.16	7,567.70	428.46	5.66	428.46	5.66	183,578
127	8,051.27	7,682.70	368.57	4.80	368.57	4.80	135,844
128	8,106.37	8,021.60	84.77	1.06	84.77	1.06	7,186
129	8,161.47	8,173.40	-11.93	-0.15	11.93	0.15	142
130	8,216.58	8,364.10	-147.52	-1.76	147.52	1.76	21,762
131	8,271.68	8,556.30	-284.62	-3.33	284.62	3.33	81,009
132	8,326.79	8,475.90	-149.11	-1.76	149.11	1.76	22,234
133	8,381.89	8,532.50	-150.61	-1.77	150.61	1.77	22,683
134	8,436.99	8,306.40	130.59	1.57	130.59	1.57	17,054
135	8,492.10	8,244.10	248.00	3.01	248.00	3.01	61,504
136	8,547.20	7,896.60	650.60	8.24	650.60	8.24	423,280
137	8,602.31	8,026.40	575.91	7.18	575.91	7.18	331,672
138	8,657.41	8,372.90	284.51	3.40	284.51	3.40	80,946
139	8,712.51	8,536.90	175.61	2.06	175.61	2.06	30,839
140	8,767.62	8,694.00	73.62	0.85	73.62	0.85	5,420
141	8,822.72	8,876.80	-54.08	-0.61	54.08	0.61	2,925
142	8,877.82	9,094.90	-217.08	-2.39	217.08	2.39	47,124
143	8,932.93	9,066.90	-133.97	-1.48	133.97	1.48	17,948
144	8,988.03	9,180.40	-192.37	-2.10	192.37	2.10	37,006

TOTALS	3,223.60	43.04	5,906.18	73.70	2,062,720
NUMBER OF OBSERVATION	24				

RMSE	MPE	MAE	MAPE	MSE
293.17	1.79	246.09	3.07	85,947

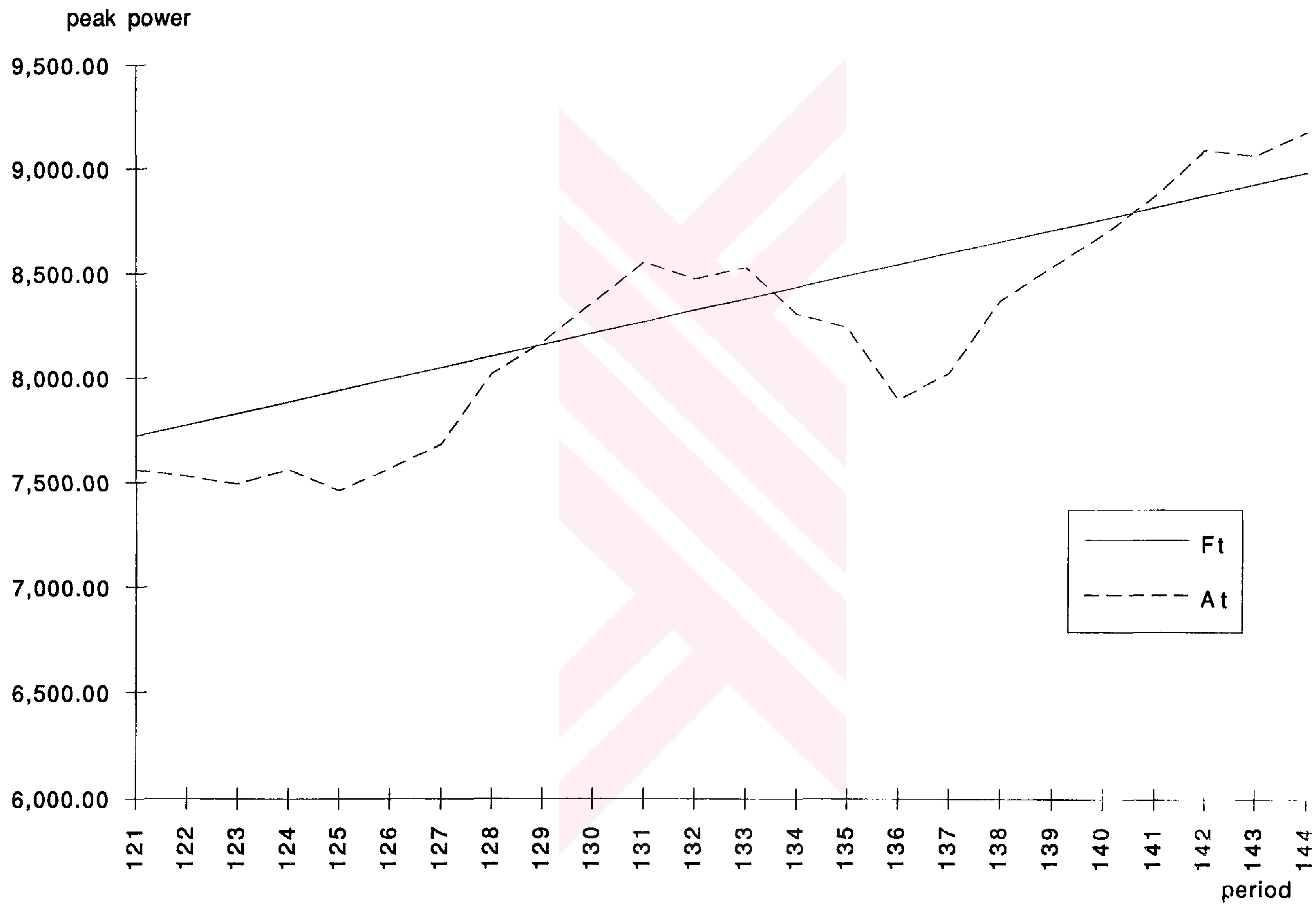

Figure D.4 Plot of 24 Months Peak Load Demand Forecast with Holt's Exponential Smoothing Method

Table D.7 24 Months Peak Load Demand Forecast Evaluation with Winter's Exponential Smoothing Method

t	Ft	At	e	%e	lel	%lel	e2
121	7,584.72	7,560.00	24.72	0.33	24.72	0.33	611
122	7,674.48	7,530.60	143.88	1.91	143.88	1.91	20,701
123	7,748.01	7,494.10	253.91	3.39	253.91	3.39	64,470
124	7,747.64	7,562.20	185.44	2.45	185.44	2.45	34,388
125	7,528.52	7,463.10	65.42	0.88	65.42	0.88	4,280
126	7,467.36	7,567.70	-100.34	-1.33	100.34	1.33	10,068
127	7,485.22	7,682.70	-197.48	-2.57	197.48	2.57	38,998
128	7,566.14	8,021.60	-455.46	-5.68	455.46	5.68	207,444
129	7,757.50	8,173.40	-415.90	-5.09	415.90	5.09	172,973
130	7,826.32	8,364.10	-537.78	-6.43	537.78	6.43	289,207
131	7,979.71	8,556.30	-576.59	-6.74	576.59	6.74	332,456
132	8,091.89	8,475.90	-384.01	-4.53	384.01	4.53	147,464
133	8,003.19	8,532.50	-529.31	-6.20	529.31	6.20	280,169
134	8,095.96	8,306.40	-210.44	-2.53	210.44	2.53	44,285
135	8,171.59	8,244.10	-72.51	-0.88	72.51	0.88	5,258
136	8,169.28	7,896.60	272.68	3.45	272.68	3.45	74,354
137	7,936.38	8,026.40	-90.02	-1.12	90.02	1.12	8,104
138	7,870.09	8,372.90	-502.81	-6.01	502.81	6.01	252,818
139	7,887.11	8,536.90	-649.79	-7.61	649.79	7.61	422,227
140	7,970.57	8,694.00	-723.43	-8.32	723.43	8.32	523,351
141	8,170.31	8,876.80	-706.49	-7.96	706.49	7.96	499,128
142	8,240.96	9,094.90	-853.94	-9.39	853.94	9.39	729,214
143	8,400.61	9,066.90	-666.29	-7.35	666.29	7.35	443,942
144	8,516.84	9,180.40	-663.56	-7.23	663.56	7.23	440,312

TOTALS	-7,390.10	-84.55	9,282.20	109.37	5,046,222
NUMBER OF OBSERVATION	24				

RMSE	MPE	MAE	MAPE	MSE
458.54	-3.52	386.76	4.56	210,259

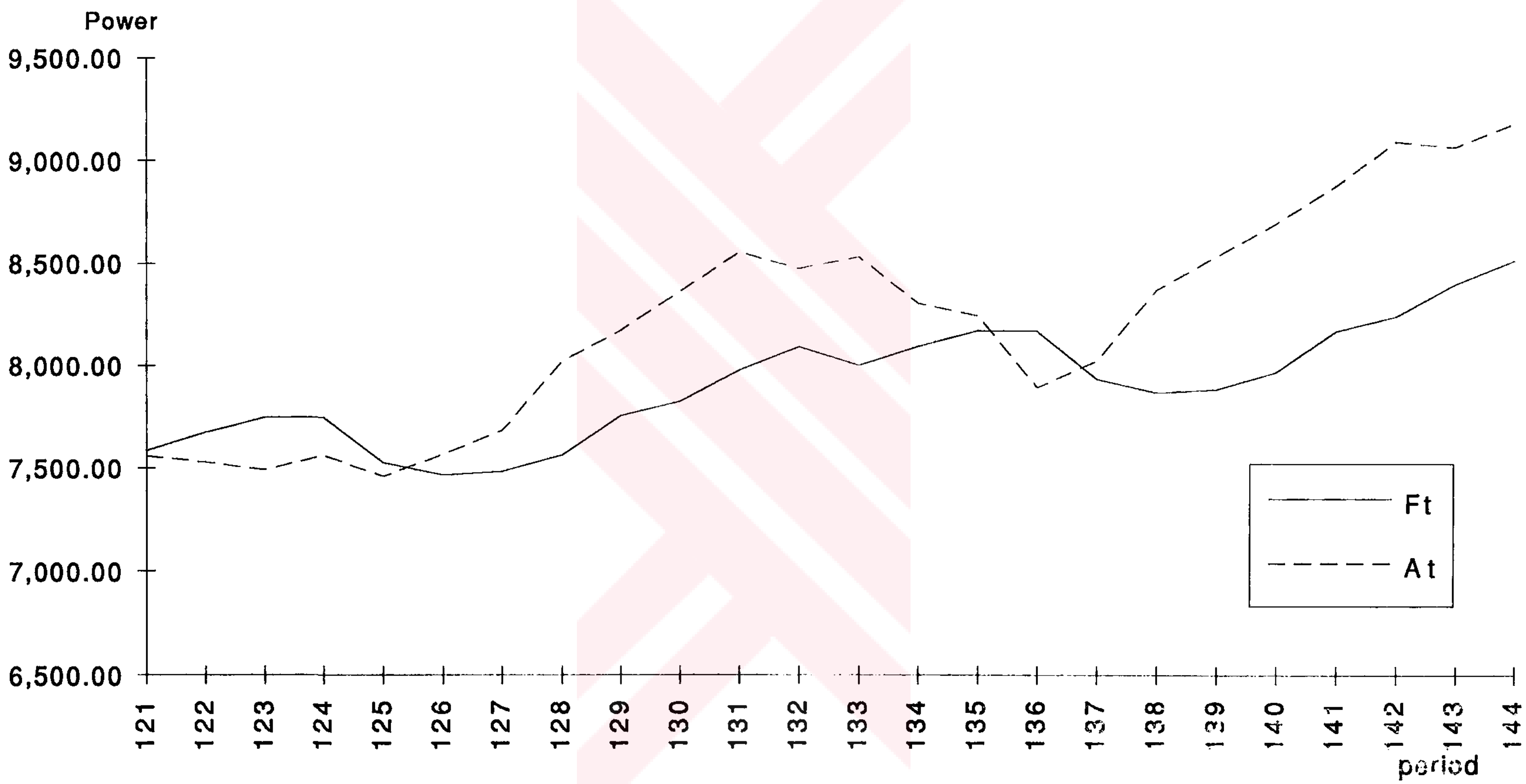

Figure D.5 Plot of 24 Months Peak Load Demand Forecast with Winter's Exponential Smoothing Method

Table D.8 Summary of SARIMA (0,1,0)$_1$x(1,1,0)$_{12}$ Model for 24 Months Peak Load Demand Forecasts

Summary of Fitted Model for: PEAKPOWE.VAR3

Parameter	Estimate	Stnd.error	T-value	P-value
SAR(12)	-.57543	.09494	-6.06126	.00000
MEAN	5.00583	7.45903	.67111	.50362
CONSTANT	7.88634			

Model fitted to differences of order 1
Model fitted to seasonal differences of order 1 with seasonal length = 12
Estimated white noise variance = 13679.4 with 105 degrees of freedom.
Estimated white noise standard deviation (std err) = 116.959
Chi-square test statistic on first 20 residual autocorrelations = 16.678
with probability of a larger value given white noise = 0.611675
Backforecasting: no Number of iterations performed: 4

Residual Summary

Number of observations = 107
Residual average = -0.0100665
Residual variance = 13679.4
Residual standard error = 116.959

Coeff. of skewness = -0.136236 standardized value = -0.575316
Coeff. of kurtosis = 0.507863 standardized value = 1.07234

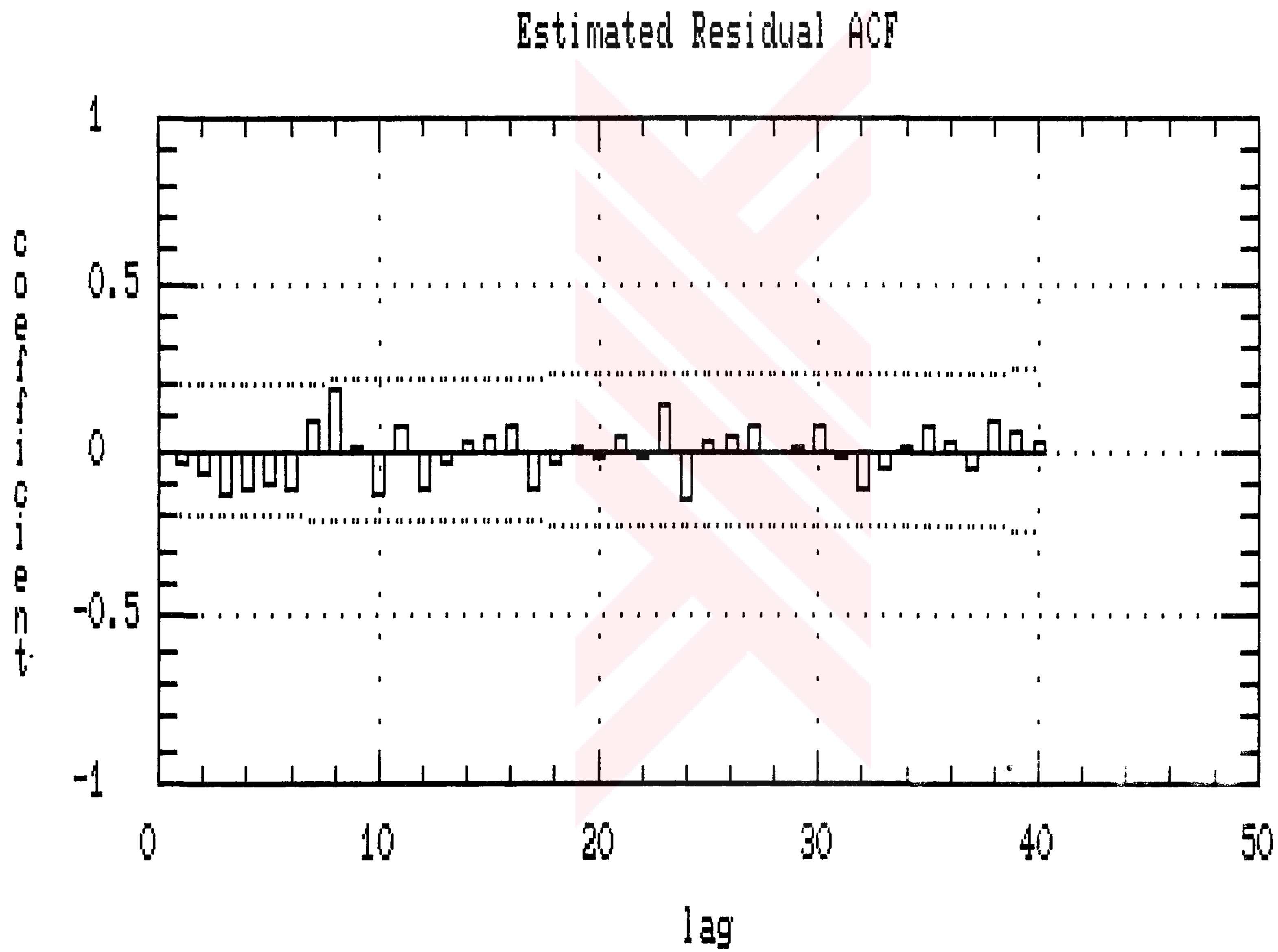

Figure D.6 Residual Correlagram of SARIMA $(0,1,0)_1 x (1,1,0)_{12}$ Model 'able D.9 24 Months Peak Load Demand Forecast Evaluation with Box-Jenkins Method

t	Ft	At	e	%e	lel	%lel	e2
121	7,532.16	7,560.00	-27.84	-0.37	27.84	0.37	775
122	7,554.72	7,530.60	24.12	0.32	24.12	0.32	582
123	7,517.55	7,494.10	23.45	0.31	23.45	0.31	550
124	7,457.14	7,562.20	-105.06	-1.39	105.06	1.39	11,038
125	7,456.32	7,463.10	-6.78	-0.09	6.78	0.09	46
126	7,654.10	7,567.70	86.40	1.14	86.40	1.14	7,465
127	7,912.30	7,682.70	229.60	2.99	229.60	2.99	52,716
128	7,899.09	8,021.60	-122.51	-1.53	122.51	1.53	15,009
129	8,115.02	8,173.40	-58.38	-0.71	58.38	0.71	3,408
130	8,232.16	8,364.10	-131.94	-1.58	131.94	1.58	17,408
131	8,437.18	8,556.30	-119.12	-1.39	119.12	1.39	14,190
132	8,424.23	8,475.90	-51.67	-0.61	51.67	0.61	2,670
133	8,324.35	8,532.50	-208.15	-2.44	208.15	2.44	43,326
134	8,255.25	8,306.40	-51.15	-0.62	51.15	0.62	2,616
135	8,179.26	8,244.10	-64.84	-0.79	64.84	0.79	4,204
136	8,005.36	7,896.60	108.76	1.38	108.76	1.38	11,829
137	8,035.50	8,026.40	9.10	0.11	9.10	0.11	83
138	8,216.82	8,372.90	-156.08	-1.86	156.08	1.86	24,361
139	8,509.36	8,536.90	-27.54	-0.32	27.54	0.32	758
140	8,593.33	8,694.00	-100.67	-1.16	100.67	1.16	10,134
141	8,761.41	8,876.80	-115.39	-1.30	115.39	1.30	13,315
142	8,817.29	9,094.90	-277.61	-3.05	277.61	3.05	77,067
143	8,975.84	9,066.90	-91.06	-1.00	91.06	1.00	8,292
144	8,965.67	9,180.40	-214.73	-2.34	214.73	2.34	46,109
TOTALS			-1,449.09	-16.30	2,411.95	28.81	367,951
NUMBER OF OBSERVATION			24				

RMSE	MPE	MAE	MAPE	MSE
123.82	-0.68	100.50	1.20	15,331

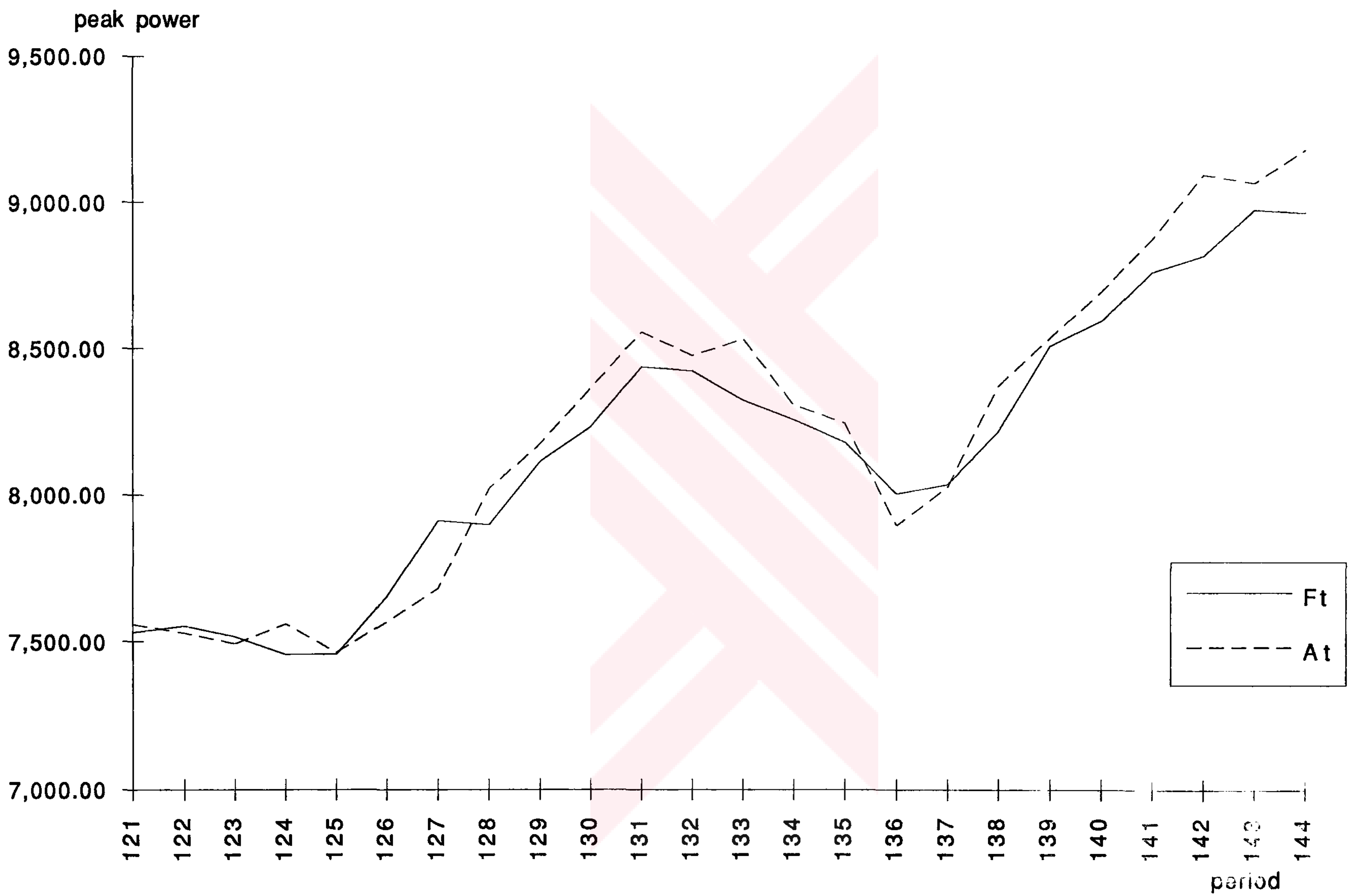

Figure D.7 Plot of 24 Months Peak Load Demand Forecast with Box-Jenkins Method

Table D.10 12 Months Peak Load Demand Forecast Evaluation with Simple Linear Regression Method

t	Ft	At	e	%e	lel	%lel	e2
133	7,814.97	8,532.50	-717.53	-8.41	717.53	8.41	514,849
134	7,852.47	8,306.40	-453.93	-5.46	453.93	5.46	206,052
135	7,889.97	8,244.10	-354.13	-4.30	354.13	4.30	125,408
136	7,927.46	7,896.60	30.86	0.39	30.86	0.39	952
137	7,964.96	8,026.40	-61.44	-0.77	61.44	0.77	3,775
138	8,002.46	8,372.90	-370.44	-4.42	370.44	4.42	137,226
139	8,039.96	8,536.90	-496.94	-5.82	496.94	5.82	246,949
140	8,077.46	8,694.00	-616.54	-7.09	616.54	7.09	380,122
141	8,114.96	8,876.80	-761.84	-8.58	761.84	8.58	580,400
142	8,152.45	9,094.90	-942.45	-10.36	942.45	10.36	888,212
143	8,189.95	9,066.90	-876.95	-9.67	876.95	9.67	769,041
144	8,227.45	9,180.40	-952.95	-10.38	952.95	10.38	908,114
TOTALS			-6,574.28	-74.88	6,636.00	75.66	4,761,101
NUMBER OF OBSERVATION			12				

RMSE	MPE	MAE	MAPE	MSE
629.89	-6.24	553.00	6.30	396,758

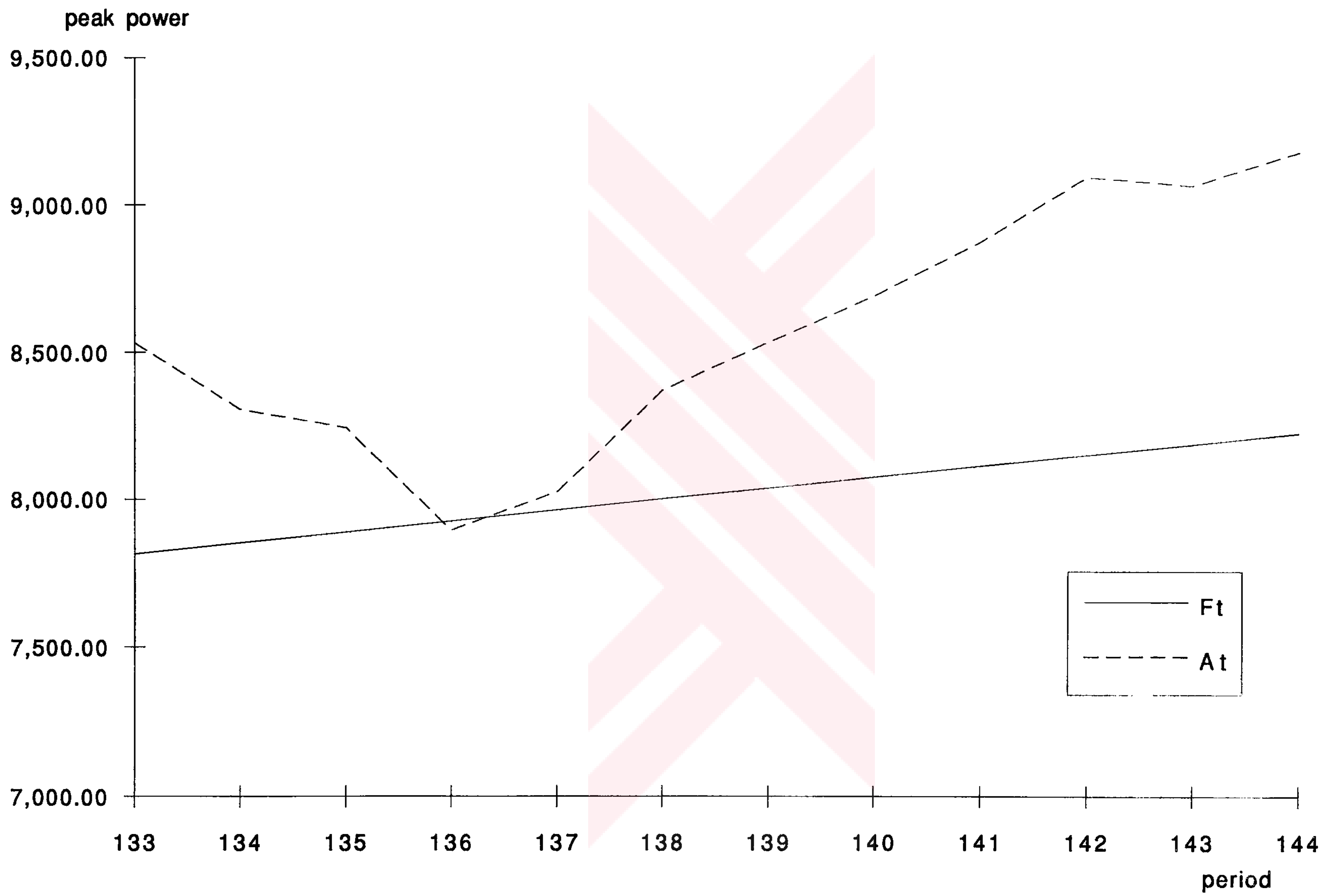

Figure D.8 Plot of 12 Months Peak Load Demand Forecast with Simple Linear Regression Method

Table D.11 12 Months Peak Load Demand Forecast Evaluation with Curvilinear Regression Method

t	Ft	At	e	%e	lel	%lel	e2
133	8,402.81	8,532.50	-129.69	-1.52	129.69	1.52	16,819
134	8,466.83	8,306.40	160.43	1.93	160.43	1.93	25,738
135	8,531.24	8,244.10	287.14	3.48	287.14	3.48	82,449
136	8,596.05	7,896.60	699.45	8.86	699.45	8.86	489,230
137	8,661.26	8,026.40	634.86	7.91	634.86	7.91	403,047
138	8,726.86	8,372.90	353.96	4.23	353.96	4.23	125,288
139	8,792.85	8,536.90	255.95	3.00	255.95	3.00	65,510
140	8,859.25	8,694.00	165.25	1.90	165.25	1.90	27,308
141	8,926.03	8,876.80	49.23	0.55	49.23	0.55	2,424
142	8,993.22	9,094.90	-101.68	-1.12	101.68	1.12	10,339
143	9,060.80	9,066.90	-6.10	-0.07	6.10	0.07	37
144	9,128.77	9,180.40	-51.63	-0.56	51.63	0.56	2,666

TOTALS	2,317.17	28.59	2,895.37	35.13	1,250,855
NUMBER OF OBSERVATION	12				

RMSE	MPE	MAE	MAPE	MSE
322.86	2.38	241.28	2.93	104,238

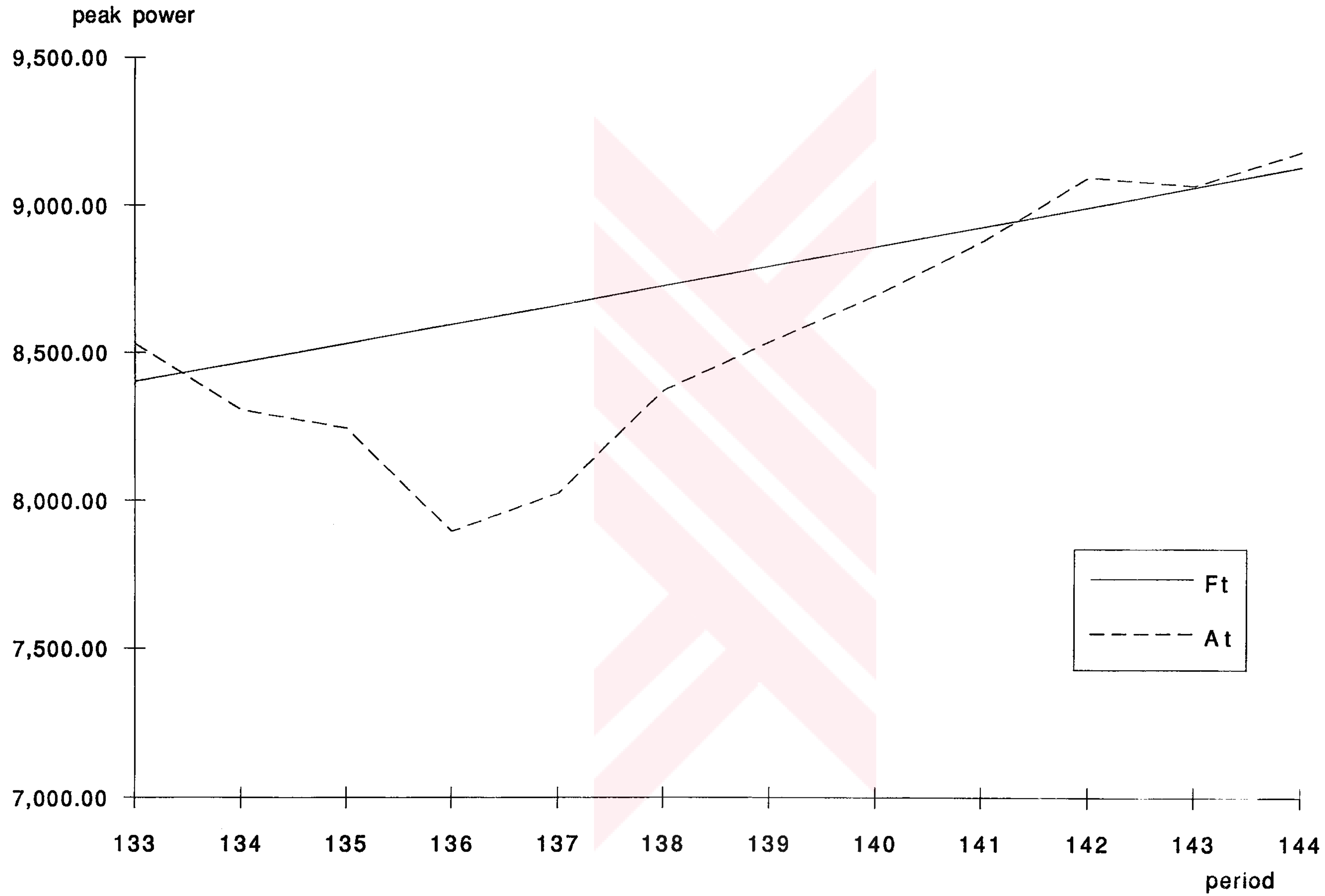

Figure D.9 Plot of 12 Months Peak Load Demand Forecast with Curvilinear Regression Method

Table D.12 12 Months Peak Load Demand Forecast Evaluation with Exponential Regression Method

t	Ft	At	e	%e	lel	%lel	e2
133	8,191.33	8,532.50	-341.17	-4.00	341.17	4.00	116,397
134	8,249.22	8,306.40	-57.18	-0.69	57.18	0.69	3,270
135	8,307.52	8,244.10	63.42	0.77	63.42	0.77	4,022
136	8,366.23	7,896.60	469.63	5.95	469.63	5.95	220,552
137	8,425.36	8,026.40	398.96	4.97	398.96	4.97	159,169
138	8,484.90	8,372.90	112.00	1.34	112.00	1.34	12,544
139	8,544.86	8,536.90	7.96	0.09	7.96	0.09	63
140	8,605.25	8,694.00	-88.75	-1.02	88.75	1.02	7,877
141	8,666.07	8,876.80	-210.73	-2.37	210.73	2.37	44,407
142	8,727.31	9,094.90	-367.59	-4.04	367.59	4.04	135,122
143	9,060.80	9,066.90	-6.10	-0.07	6.10	0.07	37
144	8,788.99	9,180.40	-391.41	-4.26	391.41	4.26	153,202
TOTALS			-410.96	-3.34	2,514.90	29.57	856,662
NUMBER OF OBSERVATION			12				

RMSE	MPE	MAE	MAPE	MSE
267.19	-0.28	209.58	2.46	71,389

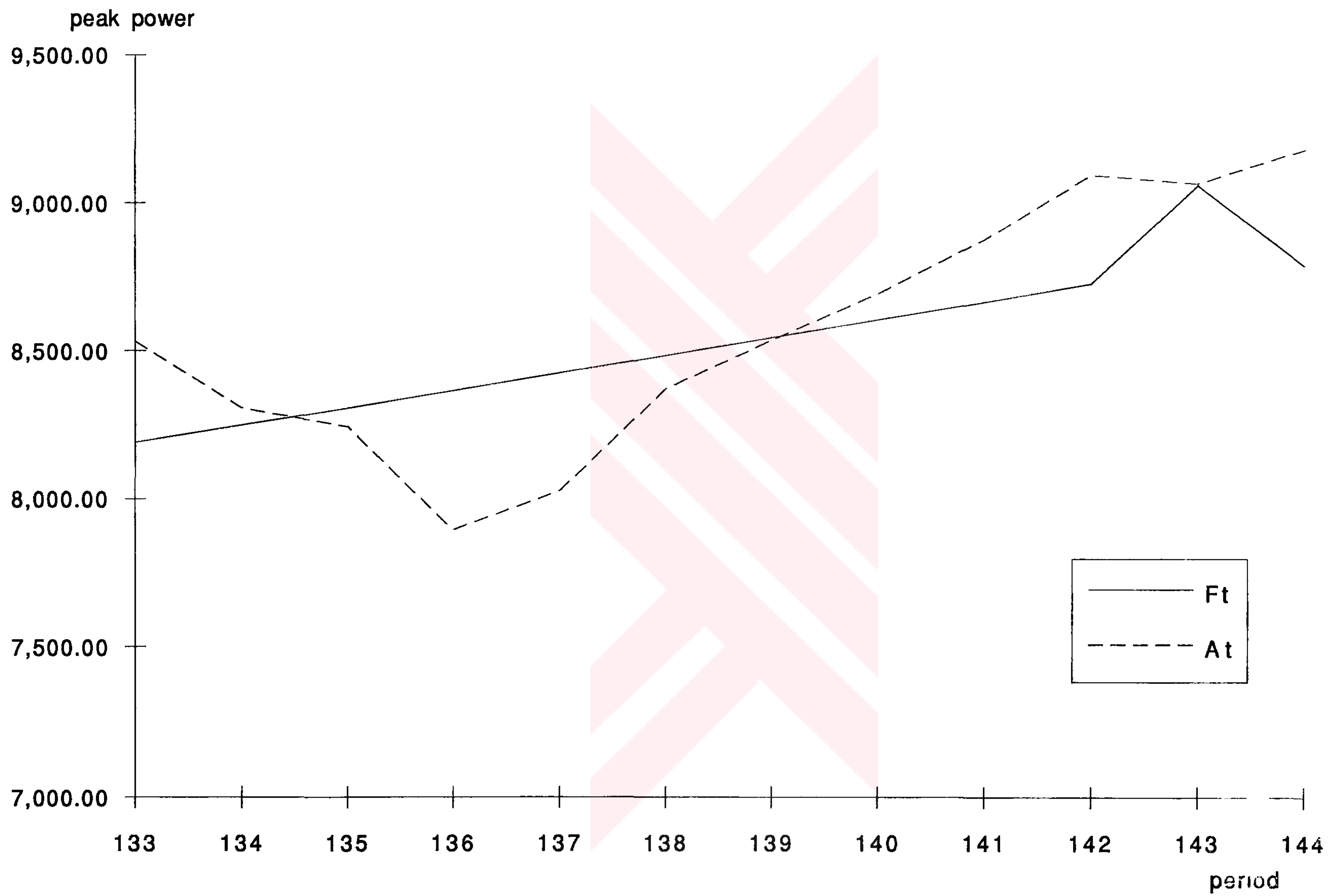

Figure D.10 Plot of 12 Months Peak Load Demand Forecast with Exponential Regression Method

Table D.13 12 Months Peak Load Demand Forecast Evaluation with Holt's Exponential Smoothing Method

t	Ft	At	e	%e	lel	%lel	e2
133	8,572.75	8,532.50	40.25	0.47	40.25	0.47	1,620
134	8,653.24	8,306.40	346.84	4.18	346.84	4.18	120,298
135	8,733.74	8,244.10	489.64	5.94	489.64	5.94	239,747
136	8,814.24	7,896.60	917.64	11.62	917.64	11.62	842,063
137	8,894.73	8,026.40	868.33	10.82	868.33	10.82	753,997
138	8,975.23	8,372.90	602.33	7.19	602.33	7.19	362,801
139	9,055.73	8,536.90	518.83	6.08	518.83	6.08	269,185
140	9,136.22	8,694.00	442.22	5.09	442.22	5.09	195,559
141	9,216.72	8,876.80	339.92	3.83	339.92	3.83	115,546
142	9,297.21	9,094.90	202.31	2.22	202.31	2.22	40,929
143	9,377.71	9,066.90	310.81	3.43	310.81	3.43	96,603
144	9,458.21	9,180.40	277.81	3.03	277.81	3.03	77,178
TOTALS			5,356.93	63.89	5,356.93	63.89	3,115,526
NUMBER OF OBSERVATION			12				

RMSE	MPE	MAE	MAPE	MSE
509.54	5.32	446.41	5.32	259,627

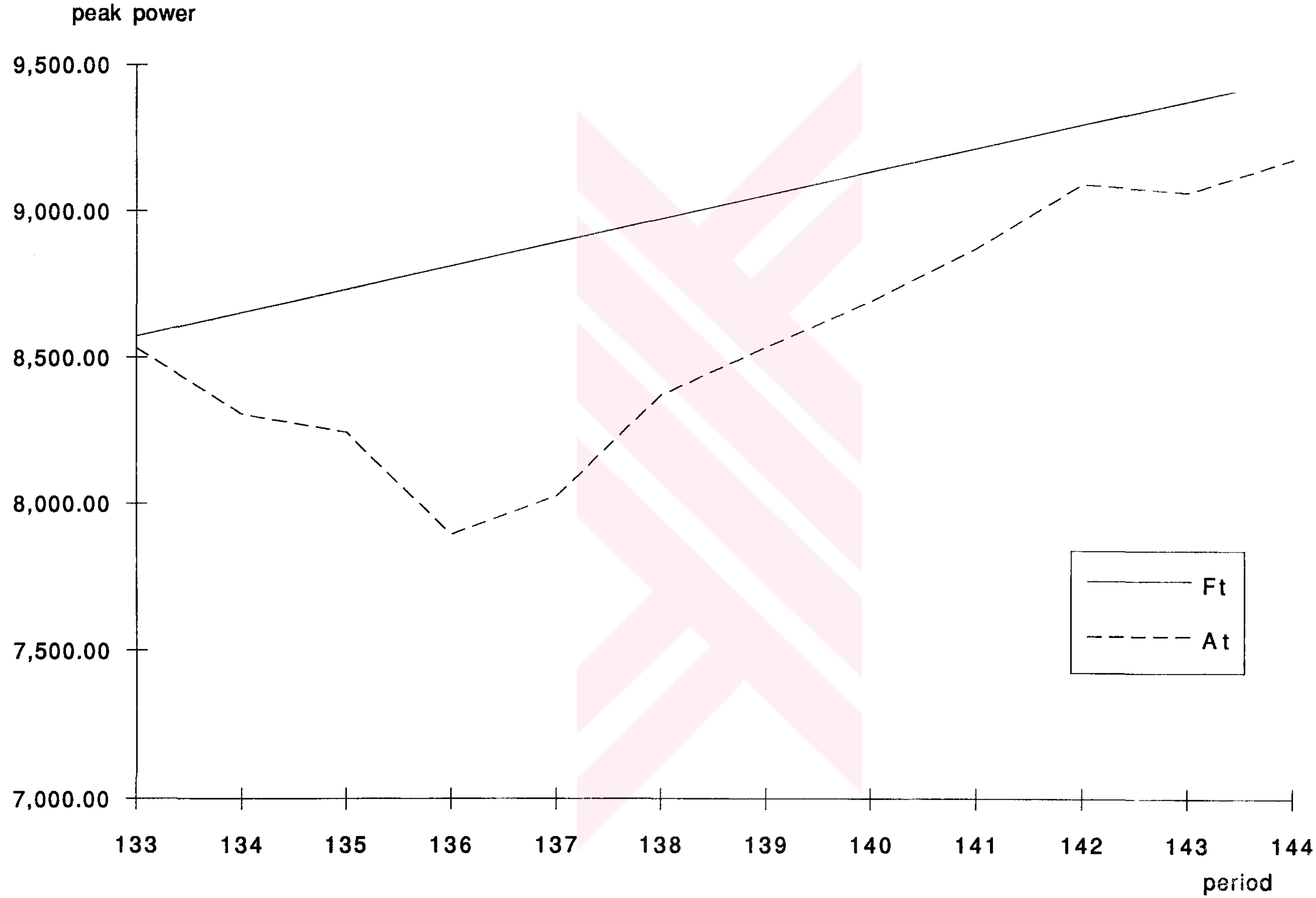

Figure D.11 Plot of 12 Months Peak Load Demand Forecast with Holt's Exponential Smoothing Method

Table D.14 12 Months Peak Load Demand Forecast Evaluation with Winter's Exponential Smoothing Method

t	Ft	At	e	%e	lel	%lel	e2
133	8,428.56	8,532.50	-103.94	-1.22	103.94	1.22	10,804
134	8,544.39	8,306.40	237.99	2.87	237.99	2.87	56,639
135	8,647.13	8,244.10	403.03	4.89	403.03	4.89	162,433
136	8,675.57	7,896.60	778.97	9.86	778.97	9.86	606,794
137	8,452.34	8,026.40	425.94	5.31	425.94	5.31	181,425
138	8,404.72	8,372.90	31.82	0.38	31.82	0.38	1,013
139	8,440.23	8,536.90	-96.67	-1.13	96.67	1.13	9,345
140	8,556.43	8,694.00	-137.57	-1.58	137.57	1.58	18,926
141	8,778.49	8,876.80	-98.31	-1.11	98.31	1.11	9,665
142	8,881.86	9,094.90	-213.04	-2.34	213.04	2.34	45,386
143	9,071.94	9,066.90	5.04	0.06	5.04	0.06	25
144	9,208.86	9,180.40	28.46	0.31	28.46	0.31	810
TOTALS			1,261.72	16.29	2,560.78	31.05	1,103,264
NUMBER OF OBSERVATION			12				

RMSE	MPE	MAE	MAPE	MSE
303.21	1.36	213.40	2.59	91,939

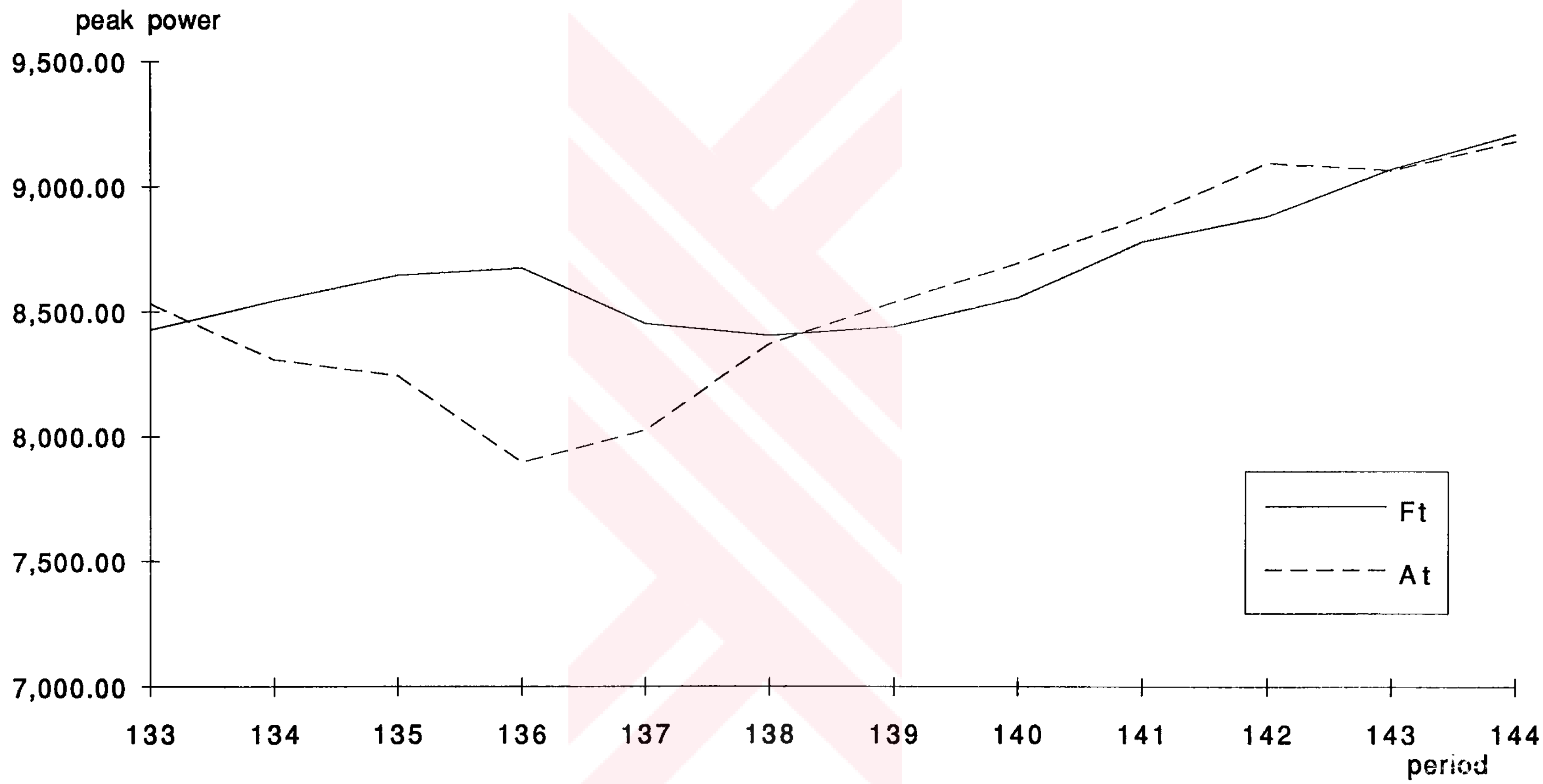

Figure D.12 Plot of 12 Months Peak Load Demand Forecast with Winter's Exponential Smoothing Method

Table D.15 Summary of SARIMA $(0,1,0)_1 x(1,1,0)_{12}$ Model for 12 Months Peak Load Demand Forecasts

Summary of Fitted Model for: PEAKPOWE.VAR2

Parameter	Estimate	Stnd.error	T-value	P-value
SAR(12)	-.54833	.08480	-6.46608	.00000
MEAN	5.34488	7.22958	.73931	.46120
CONSTANT	8.27566			

Model fitted to differences of order 1
Model fitted to seasonal differences of order 1 with seasonal length = 12
Estimated white noise variance = 13947.9 with 117 degrees of freedom.
Estimated white noise standard deviation (std err) = 118.101
Chi-square test statistic on first 20 residual autocorrelations = 18.7606
with probability of a larger value given white noise = 0.47229
Backforecasting: no Number of iterations performed: 4

Residual Summary

Number of observations = 119
Residual average = -0.0250785
Residual variance = 13947.9
Residual standard error = 118.101

Coeff. of skewness = 0.0817618 standardized value = 0.364123
Coeff. of kurtosis = 0.721808 standardized value = 1.60727

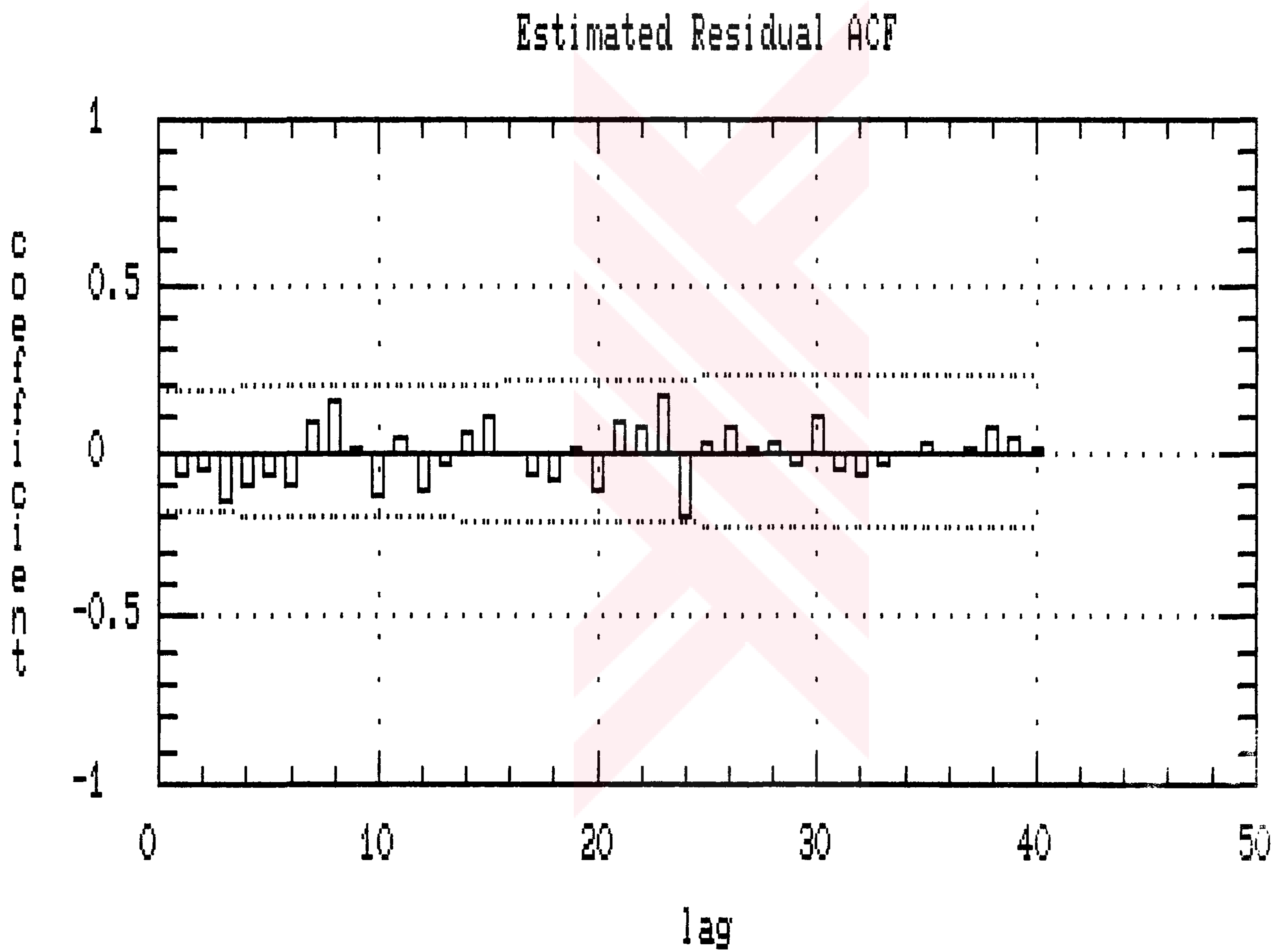

Figure D.13 Residual Correlagram of SARIMA $(0,1,0)_1 \times (1,1,0)_{12}$ Model

Table D.16 12 Months Peak Load Demand Forecast Evaluation with Box-Jenkins Method

t	Ft	At	e	%e	lel	%lel	e2
133	8,388.21	8,532.50	-144.29	-1.69	144.29	1 69	20,820
134	8,300.70	8,306.40	-5.70	-0.07	5.70	0.07	32
135	8,227.62	8,244.10	-16.48	-0.20	16.48	0.20	272
136	8,117.91	7,896.60	221.31	2.80	221.31	2.80	48,978
137	8,102.97	8,026.40	76.57	0.95	76.57	0.95	5,863
138	8,243.75	8,372.90	-129.15	-1.54	129.15	1.54	16,680
139	8,470.76	8,536.90	-66.14	-0.77	66.14	0.77	4,374
140	8,709.98	8,694.00	15.98	0.18	15.98	0.18	255
141	8,852.13	8,876.80	-24.67	-0.28	24.67	0.28	609
142	8,944.90	9,094.90	-150.00	-1.65	150.00	1.65	22,500
143	9,100.63	9,066.90	33.73	0.37	33.73	0.37	1,138
144	9,060.64	9,180.40	-119.76	-1.30	119.76	1.30	14,342
TOTALS			-308.60	13.67	2,643.82	10.30	137,256
NUMBER OF OBSERVATION			12				

RMSE	MPE	MAE	MAPE	MSE
106.95	1.14	220.32	0.86	11,438

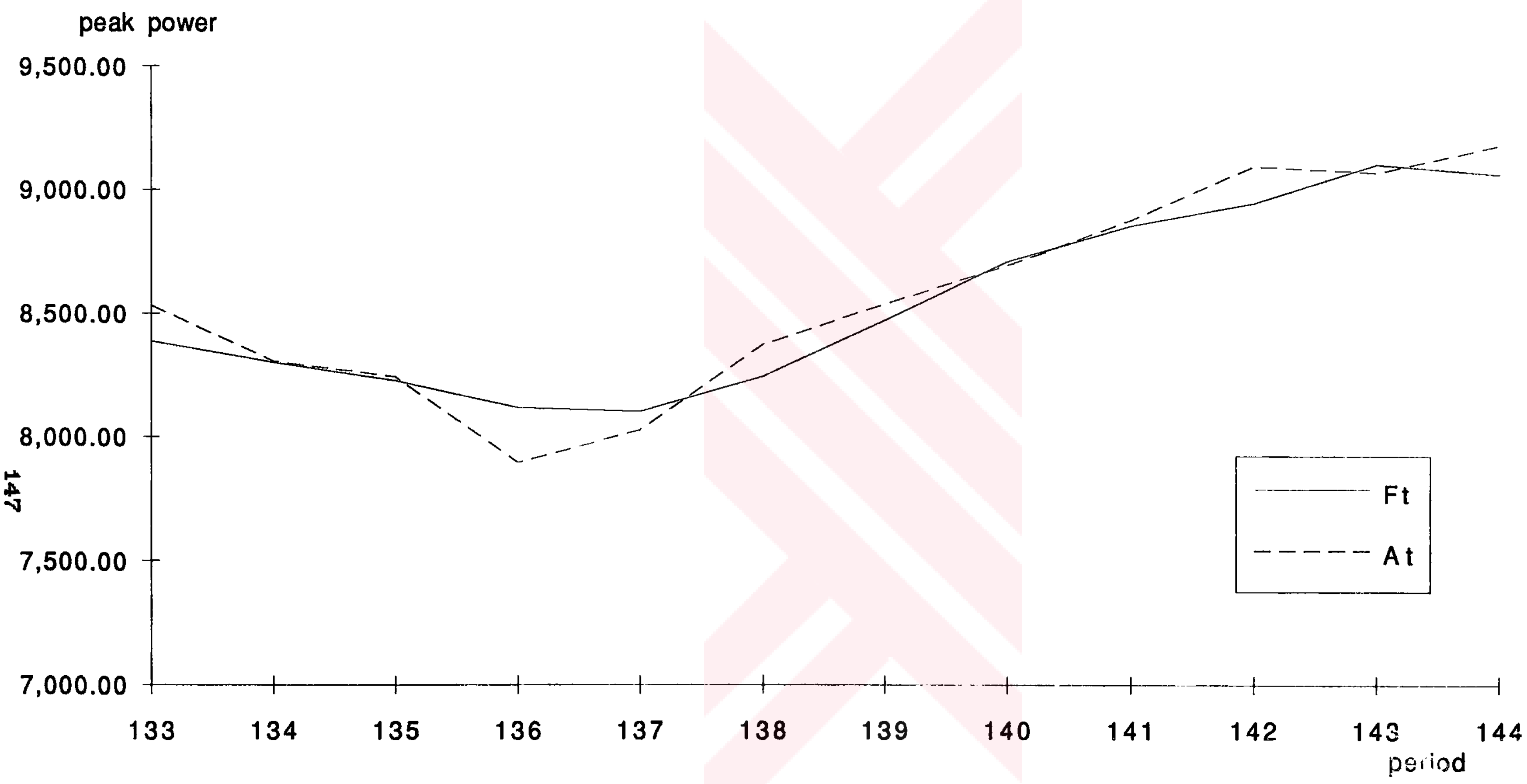

Figure D.14 Plot of 12 Months Peak Load Demand Forecast with Box-Jenkins Method

APPENDIX E

FORECAST EVALUATION CRITERIAS

In the following, a brief summary of forecasting performance measurement criterias is given.

1. Mean Absolute Percent Error (MAPE)

$$MAPE = \frac{\Sigma\ (|e_t|/a_t)}{T} \tag{E.1}$$

2. Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{\Sigma\ e_t^2}{T}} \tag{E.2}$$

3. Mean Percent Error (MPE)

$$MPE = \frac{\Sigma\ (e_t/a_t)}{T} \tag{E.3}$$

4. Mean Absolute Error (MAE)

$$MAE = \frac{\Sigma\ |e_t|}{T} \tag{E.4}$$

5. Mean Square Error (MSE)

$$MSE = \frac{\sum e_t^2}{T} \qquad (E.5)$$

where,

e_t=error at period t

a_t=actual value for period t

T=number of forecasts

APPENDIX F

STATISTICAL DISTRIBUTIONS

1. Normal Distribution

A continuous random variable X is said to be normally distributed if its Probability Density Function (PDF) has the following form:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right) \qquad -\infty<x<\infty \qquad (F.1)$$

where μ and σ^2, known as the parameters of the distribution, are the mean and the variance of the distribution. The properties of this distribution are as follows:

1. It is symmetrical around its mean value.
2. Approximately 68 percent of the area under the normal curve lies between the values of $\mu\pm\sigma$, about 95 percent of the area lies between $\mu\pm2\sigma$, and about 99.7 percent of the area lies between $\mu\pm3\sigma$.
3. Since the normal distribution depends on the two parameters μ and σ^2, once these are specified one can find out the probabilities of X lying within a certain interval by using the PDF of the normal distribution.

2. The Chi-Square Distribution

Let Z_1, Z_2,..., Z_k be independent normal variables with zero mean and unit variance. Then, the quantity

$$Z=\sum_{i=1}^{k} Z_i^2 \qquad \text{(F.2)}$$

is said to process the chi-square distribution with k degrees of freedom (df), where the term df means the number of independent quantities in the sum. The properties of this distribution is as follows:

1. The chi-square distribution is a skewed distribution, the degree of skewnes depending on the df. For comparatively few df, the distribution is highly skewed to the right; but as the df increase, the distribution becomes increasing symmetrical.
2. The mean of the chi-square distribution is k, and its variance is 2k, where k is the df.
3. If Z_1 and Z_2 are two independent chi-square variables with k_1 and k_2 df, then, the sum Z_1+Z_2 is also a chi-square variable with $df=k_1+k_2$.

3. Student's t Distribution

If Z_1 is a standardized normal variable and another variable Z_2 follows the chi-square distribution with k df and is distributed independently of Z_1, then the variable defined as

$$t=\frac{Z_1}{\sqrt{(Z_2/k)}}$$

$$=\frac{Z_1\sqrt{k}}{\sqrt{Z_2}} \qquad (F.3)$$

follows Student's t distribution with k df. The properties of t distribution is as follows:

1. The t distributionis symmetrical but flatter than the normal distribution. But as the df increase, the t distribution approximates the normal distribution.
2. The mean of the t distribution is zero, and its variance is k/(k-2).