ANALYSIS OF AN ASSEMBLER-DISTRIBUTOR NETWORK UNDER REVENUE SHARING AND WHOLESALE PRICE CONTRACTS

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ABSTRACT

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In this study, a decentralized assembler-distribution network with several suppliers is considered. There is a single product with random demand. The suppliers are of two kinds. Type 1 supplier (wholesaler) that operates with a wholesale price contract is either a monopoly or one of the firms in an oligopolistic market, and powerful enough to set the wholesale price. Type 2 suppliers are follower type companies, which are clustered around the assembler and usually their main customer is the assembler. There are revenue sharing contract between the type 2 suppliers and the assembler. The power structure is reflected to the contractual agreements that occur between the actors. In such an environment, the effect of the contracting schemes on the supply chain and the actors is analyzed.

In another setting, we extend our previous system with a distributor. Here, the assembler faces with an opportunity to utilize a distributor to serve its market or to enter a new market with a distributor. We specify a revenue sharing contract between the distributor and assembler. Different distribution alternatives are analyzed under different market structures from the assembler's point of view to provide managerial insight.

Keywords: Supply Chain Management, Revenue Sharing Contract, Wholesale Price Contract

ÖZ

BİR MONTAJ DAĞITIM ŞEBEKESİNİN GELİR PAYLAŞIMLI VE TOPTAN SATIŞ FİYATLI SÖZLEŞMELER ALTINDA İNCELENMESİ

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Bu çalışmada farklı tedarikçilerin yer aldığı merkezcil olmayan bir montaj-dağıtım şebekesi ele alınmıştır. Bu sistemde tek ürün rassal talep görmektedir ve iki çeşit tedarikçi tanımlanmıştır. Toptan satış sözleşmesiyle çalışan 1. tip tedarikçi toptan satış fiyatını belirleyebilecek kadar kuvvetli bir tedarikçiyi temsil eder. Öbür yanda 2. Tip tedarikçiler genellikle montaj firmasının etrafında toplanmış olan ve çoğunlukla tek alıcılarının montaj firması olduğu destekçi firmaları temsil etmektedir. Bu tip tedarikçilerle montaj firması arasında gelir paylaşımı sözleşmesi vardır. Sistemdeki güç dengeleri kontrat tiplerine yansıtılmıştır. Böyle bir kurgu içerisinde, kontrat yapılarının tedarik zinciri ve aktörler üzerindeki etkisi incelenmiştir.

Bir başka kurguda ise, eski sisteme birde dağıtıcı firma eklenerek sistem genişletilmiştir. Burada montaj firması bir dağıtıcı firma aracılığı ile kendi pazarına ulaşmak veya yeni bir pazara girme fırsatı yakalamıştır. Bu durumda montaj firmasıyla dağıtıcı firma arasında bir gelir paylaşımı sözleşmesi olduğu varsayılmıştır. Farklı dagıtım alternatifleri, montaj firması açısından incelenmiştir

Anahtar Kelimeler: Tedarik Zinciri Yönetimi, Gelir Paylaşımı Sözleşmesi, Toptan Satış Fiyatı Sözleşmesi

To My Family

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CHAPTER 1

INTRODUCTION

Nowadays, manufacturing-assembly firms have to work in coordination with different type of suppliers. While providing simple input from small suppliers, the assembly firm's production strategy is affected by the specialized input (specialized product with high value added, technology and knowhow) providers, which are powerful firms in their sector. In manufacturing sector, different practices are observed. Usually, assembly firms try to create a cluster of firms located close to their facility, by encouraging or forcing them in order to strengthen their control over the relatively small suppliers. The assembly firms look for cooperation strategies (like strategic alliances) with the specialized input producers, however usually, the characteristics of the cooperation is determined by the economic power of the specialized input producers.

Besides, the assembly firms experiment various distribution alternatives to serve their customers.

In this study, we analyze a decentralized assembly system consisting of an assembler, a number of small suppliers and a wholesaler where the assembler is supplied with complementary components by both small suppliers and wholesaler. The assembler performs assembly and kitting operations, and finally sells the product to the market. The small suppliers are complementary parts producers with low value added. The wholesaler is specialized input producer for the assembler, usually a monopoly or one of the firms in an oligopolistic market structure. In this setting, the wholesaler appears as the most powerful actor, followed by the assembler. The small suppliers are followers. The power structure of the supply chain is reflected by the contracts between these actors.

The way to reach the market appears to be another decision issue for the assembler. The assembler may prefer to utilize a distributor to serve all the market instead of selling directly or a combination of these strategies may be possible.

We consider two types of contracts. The first one is the revenue sharing contracts between the assembler and the suppliers and between the assembler and the distributor. According to this contracting scheme, the assembler determines the revenue shares given to the suppliers and the distributor. Then, the suppliers and the distributor decide on the quantity. The second contracting scheme is wholesale price contract between the assembler and the wholesaler. According to this contract, the wholesaler first determines the wholesale price and then the assembler decides on the order quantity. These are only suggested contract types. The study can be repeated by other contract types (as long as the analysis is tractable).

The modelling approach followed in this study satisfies the following characteristics:

- We model a decentralized system considering each actors' profit maximization objectives. We also model the centralized system, where maximizing the total supply chain profit is the main objective.
- We use single period problem framework.
- We assume that the demand is stochastic.
- In this study, we are interested in strategic level decisions rather than the operational level.

The main objectives of this study can be summarized as follows:

- Observing the performance of contracting mechanisms.
- Understanding the effect of the power structure on the system.
- Observing the effect of the different distribution strategies from the assemblers point of view.

In this study, we do not specially emphasize on full coordination issues (although a case is to be considered in Section 3.4).

In order to fulfill the objectives, four cases are analyzed in this study.

- Case I: In this case, we analyze the assembler-small suppliers system where the wholesaler's selling price is fixed and the assembler sells the product directly to the market. Here, we try to understand, how our contracting mechanism performs and the effect of power relations on the system.
- Case Ia: In case Ia, we extend the subsystem in case I with the inclusion of wholesaler, which decides on the wholesale price to offer having the full information on the rest of the system. In this case, we try to understand the effect of such a wholesaler on the system.
- Case II: In case II, the system examined consists of the assembler, small suppliers and the distributor, where only the distributor faces customer demand and the wholesaler's selling price is set at a fixed level as in case I. This case reflects an alternative distribution strategy for the assembler.
- Case III: In this case, we change the system in case II in the following way: Both the assembler and the distributor face customer demands. The demand for the assembler's own market and the demand of the distributor may not be independent. We assume that the assembler first serves its market and then sends the unsold product to the distributor to be sold in the distributor's market (secondary market). This case reflects a market extension for the assembler via a distributor.

The work done in this study can be summarized as follows:

- Mathematical formulation of the system in terms of contract structure.
- Obtaining first order and second order conditions.
- Discussion of concavity issues.
- Computational analysis.

The outline of the thesis is as follows. In Chapter 2, the motivation of the study, detailed description of the system and a literature review on four categories are introduced.

In Chapter 3, Case I and Case Ia are modelled and computational results are stated.

In Chapter 4, Case II and Case III are modelled and computational results are stated.

Finally, in Chapter 5, concluding remarks are made and directions for further research are discussed.

CHAPTER 2

MOTIVATION AND LITERATURE

2.1 Motivation of the study

Nowadays, it is realized that most of the supply chains are decentralized in nature. Rapidly changing market conditions and customer preferences together with increasing cost competition force the firms to appear in the market, producing diversified products with additional functions and attributes. Besides, the product life cycles become shorter. As the market competition becomes stronger, the companies should invest on R&D and product development activities to attract more customers and preserve their existence in the market. As the product turnover increases, the adjustment of manufacturing processes from one product to a newly developed one appears as a critical issue that should be handled by manufacturers. Consequently, it becomes a common trend to outsource some (most) of the small components and concentrate on (1) production of the parts that constitute the core competency of the manufacturing company and are not economical to outsource, and (2) the assembly of the product. Besides, manufacturing of some products may require high technology and knowhow, and it can be either easier and cheaper for the production companies to transfer technology and knowhow from outside, or it can be the only choice. Therefore, most of the production and assembly firms have to work in coordination with different firms (actors), which have conflicting objectives (individual profit maximization) and improved information technologies provide the necessary information transfer substructure for fast and accurate communication and data transfer (like demand information and orders given) between these actors. The power structure of the supply chain may vary, according to the kind of service or product (eg. knowhow, high technology product etc.) the actors provide to the manufacturing company, the market structure and economic conditions of their individual sectors.

In this work, we name the company that performs manufacturing and assembly operations, as the assembly firm or the assembler. Here, the assembler firm appears as the company that organizes the relations between other actors in the system.

To be more specific, consider a supply chain, where there are two types of suppliers. First type of suppliers represents producers of small complementary parts like small electronic cards, plastic components, etc. Usually, these types of suppliers are geographically clustered around the assembler. They either manufacture a specific component (input) for the assembler only or sometimes sell their product to other firms as well. We assume that the assembler is either their single or main customer, and hence the existence of this type of suppliers depends on the assembler. So, in this case the assembler is the powerful side in the assembler-supplier relationship and may dictate the supplier in arranging its production plan and processes applied. The suppliers manufacturing ordinary products with very low/value added can be considered in this type, as well. However, we are interested especially in the former case. In this work, the generic term "suppliers" is used to represent this type of suppliers.

On the other hand, there may be suppliers, say second type of suppliers, where the power relation may be reversed. A supplier of this type can either be a monopoly, or one of the firms in an oligopolistic market. For example, Intel is one of the biggest CPU producers in the computer electronics market, and a small manufacturer that uses Intels CPU's in their product can hardly negotiate for the CPUs selling price. We can extend examples to other manufacturers of special products that are either high technology or require special production processes (i.e. stainless steel manufacturers). Another example of a powerful supplier can be given in the case of a firm from which the assembler transfers technology or knowhow. According to the agreement between firms, the assembler has to make a transfer payment, that can be either fixed annually or based on per unit manufactured (or on another criteria). We are going to name such a supplier as the wholesaler in the rest of this work.

Strategic alliances are observed in some industries. To enter a domestic market or to invest in a country, a global firm prefers to build a strategic alliance with domestic firms to overcome governmental restrictions. In these alliances, usually, global firms delegate the management of production and marketing activities to domestic firms, and permit the local firm to manufacture products or services using global firms' technology. However, the global firm can place restrictions on the production volume per year by the domestic firm or on the markets where the product can be sold. Additionally, the global firm can be authorized to inspect the management and operations of the domestic firm, to check whether it obeys the stipulations of the agreement about the quality requirements of the product or not.

Another issue that the assembly firm should deal with is the means with which it is going to reach its customers. Nowadays, companies experiment various alternative distribution strategies: selling directly, selling through vertically-integrated retailers, selling through franchised retailers, selling through independent retailers, or using a combination of these alternatives (hybrid distribution strategies). While selling their product through their own retailer-distribution network (or directly), which is a powerful way with the rapid growth of the Electronic Commerce and use of internet, the firms may prefer entering into a new market or increasing their access ability to customers via an independent distributor. Additionally, selling the remaining products in a secondary market through an independent distributor at the end of the main selling period appears as an effective strategy to dispose the remaining products and increase the efficiency of the system, while reducing the risk of carrying excess inventory.

Especially in manufacturing industries, systems similar to those described above are present. For example a dishwasher manufacturer in Turkey encourages its small suppliers to be geographically located around the main plant as a cluster. The reason behind such a strategy is to maintain monitoring and control activities over the suppliers easily, and to apply some production policies (like JIT) that facilitate production planning and achieve quick response. Existence of these small firms depends on their cooperation with the dishwasher manufacturer, since the dishwasher manufacturer is their either single or main customer. Therefore, small suppliers perform necessary improvement activities to reach the quality and flexibility level as dictated by the dishwasher manufacturer.

On the other hand, the dishwasher manufacturer procures a special stainless steel used as an input for the main frame of dishwashers, from one of the main overseas producers. Additionally, the dishwasher manufacturer has license and know-how agreements with other dishwasher manufacturers abroad, all exemplifying typical wholesaler. Automobile industry also shows similar characteristics, where small suppliers clustered close to the main manufacturer and the manufacturer depends on other firms for procurement of specialized input (like engine, know-how, computer software, etc.).

Up to now, we draw a general picture of the supply chain considered in this study. Supply chains are environments in which there are multiple decision makers, and behavior that is partly rational for each actor can be inefficient for the whole supply chain. A mechanism to relieve the system of such inefficiencies is contracting. This includes the reallocation of decision rights, rules for sharing the costs of inventory and stockout, and policies governing pricing to the end-customer or between supply chain partners. Representation of the information structure and rules for information sharing are also important, since the assumption of full availability of information usually made in multi-echelon inventory theory may not be an accurate representation in real supply chains.

A list of typical contract types that are examined in the literature is given below (Lariviere, 1998). In the literature, the actors are named as supplier-manufacturer, manufacturer-retailers or supplier-retailer, where the contracts arrange the relations between these pairs. Here we use supplier-retailer terminology to represent the actors.

- Price-Only Contracts (Wholesale Price Contracts): According to this contracting scheme, the supplier offers the good at a per unit wholesale price w, and the retailer determines the order quantity. The retailer retains possession of any excess stock.

- Revenue Share Contracts: In revenue share contracts, the supplier sells its product at a per unit wholesale price $w \geq 0$ and additionally the retailer gives a revenue share α for each unit sold keeping $(1 - \alpha)$ for itself. In a manufacturer-retailer setting, w and α are determined by the manufacturer and then the retailer orders Q units.

- Return Policies (Buy Back Contracts): Here, the supplier offers the good at a per unit wholesale price w and ready to buy back any unsold stock from the retailer at a per unit rate b where $b < w$ to avoid trivial cases. Then the retailer determines the order quantity.

- Return Policies (Quantity Flexibility Contracts):According to this contract the supplier determines a per unit wholesale price w, a downward adjustment parameter $d \epsilon [0, 1)$ and an upward adjustment parameter $u \geq 0$. Then the retailer gives its initial order y. After the demand realization the retailer is allowed to change its initial order y within the range $[y(1 - d), y(1 + u)]$.

- Quantity Discounting Schemes: Here,the supplier applies different discount policies for distinct range of order quantities over its wholesale price w for encouraging the retailer to order more.

- Penalty Methods: Here, the supplier offers the good to the retailer at a wholesale price w with no return policy, but with a demand for a payment of p per unit for any missed sales. The difficulty of the monitoring missed demand restricts the real life application of this method.

- Sales-rebate contract: With this contract, the supplier sells its products at a per unit wholesale price w to the retailer, but then gives the retailer an r rebate per unit sold above a threshold t.

Besides these contract types, two policies dwelled upon the reallocation of decision rights and examined in the multi-echelon literature nowadays are the following:

- RMI (Retail managed inventory): Applying this policy, every actor in each echelon manage their own inventory. (Determination of reorder and order up to points or order quantities or etc. according to inventory policy used)
- VMI (Vendor managed inventory): According to this policy, upper echelon manages the inventory of lower echelon having an immediate access to its inventory level information.

The aim of this study is to analyze the behavior of the roughly described supply chain under a specific power structure reflected by the contracts.

2.2 Description of the System

In this study, we analyze a decentralized supply chain, where certain contract structures are assumed to exist among the actors of the chain. Contracts reflect the existing power structure available in the market, and hence similar studies can be repeated for different markets (with other contract structures), keeping the supply chain structure as is.

Although in this study, four slightly different cases are examined, we start with the common description of the system and relations between actors that are arranged with contracts.

The chain we analyzed consists of an assembler, a distributor, a powerful supplier that will be referred as the wholesaler, and a number of smaller suppliers,- to be referred as suppliers. The assembler is supplied with complementary components by the suppliers and performs assembly and kiting operations, and finally sells the product to the market directly and/or via a distributor. We model the interaction between the assembler and other actors as a Stackelberg game, where there are two different types of incentive schemes (contracts) in use between them. In Stackelberg game, every decision are taken successively (for example, first the wholesaler determines the wholesale price and then the assembler decides on the order quantity). The first type of contract is a revenue sharing contract between the assembler and suppliers. According to this incentive scheme, the assembler sets a revenue share, a percentage of selling price that will be paid to the supplier for each final product sold. Knowing the processing capability of the supplier, this sets a supply quantity. Actually, the decision mechanism performs as follows: Firstly, the assembler makes an offer to the supplier setting the revenue share and supply quantity. If the supplier accepts this offer, it decides on the capacity, which will be setup and/or allocated for the assembler. One important point to notice is that to avoid any mismatch between the allocated capacity and desired supply quantity, the assembler equalizes these quantities knowing the reaction of the supplier for every revenue share. This approach of the assembler also guaranties the acceptance of the offer by the supplier.

Another interpretation of this contract might be as follows: Consider that initially the assembler decides on the revenue share and then the supplier determines the capacity allocated, hence the supply quantity. This subsystem acts as a VMI system, which means that the suppliers manage the assembler inventory. Although VMI systems are more common in retail settings, this incentive scheme seems to be used even in assembly systems where the complementary products are delivered by different suppliers. One application of VMI in assembly systems appears in the industry as follows: All small suppliers have a shelf for their product in the assemblers production facility. The suppliers have the full responsibility of all controlling and management activities of these stocking points. Additionally, the assembler pays to suppliers only for the components used from the shelves. In our setting, determining an appropriate revenue share for the suppliers, the assembler actually controls the production capacity of the supplier allocated for the product of concern.

The second type of contract is wholesale-price contract between the assembler and the wholesaler. According to this contract, the wholesaler, being one of the few producers for this specialized product and powerful enough to set his selling price, sets its wholesale price and the assembler decides on the order quantity. Actually, this contract type is the traditional one that appears in many industries and used very commonly. Although this traditional way of doing business tends to replace themselves with other incentive schemes like revenue sharing contracts or VMI systems that increase the coordination between the actors in the supply chain, the most powerful suppliers in the industry still declares the terms of trade in the traditional way.

The type of contract between the assembler and the distributor is also a revenue sharing contract. According to this contract the assembler offers to the distributor a revenue share for a certain quantity of product that will be sent to the distributor for sale. The distributor accepts this offer only if the offered quantity is in accordance with the quantity that the distributor finds appropriate with respect to offered revenue share.

In our analysis, contracts reflect the power structure of the system. Using a revenue share contract, the assembler becomes more powerful than the small suppliers and distributor, because the power of determining the system's operating quantity is in assembler's hands by setting appropriate revenue shares. Small suppliers and the distributor are the followers that could only react according to a given revenue share. Similarly, using wholesale-price contract the wholesaler is more powerful than the assembler, because setting a wholesale price (which may or may not maximize its profit), the wholesaler limits profitable operating quantity range of the assembler. Hence, the wholesaler controls its subsystem.

In this study, we analyze revenue share contracts and wholesale price contracts simultaneously. In most supply chains, actors try to coordinate their operations via applying different incentive schemes that reallocate the operational risk through the supply chain. We describe above the contract schemes that appropriately reflect the power structure mentioned above. Here, we use our model to test how these contract schemes perform at a strategic level.

In this study, we examine four cases:

- Case I: We analyze the assembler and small suppliers subsystem where only the assembler faces customer demand and the wholesalers selling price (wholesale price) is set to a fixed value.

- Case Ia: In case Ia, we extend the subsystem in case I with the inclusion of the wholesaler. Here, the wholesaler decides on the wholesale price to offer having the full information of the system.
- Case II: In case II, the system examined consists of the assembler, small suppliers and the distributor, where only the distributor faces customer demand and the wholesalers selling price is set at a fixed quantity as in case I.
- Case III: In this case, we change the system in case II in the following way: Both the assembler and the distributor face with customer demands. The demand for the assembler's own market and the demand of the distributor may not be independent. We assume that the assembler first serves its market and then sends the unsold product to the distributor to be sold in the distributor's market (secondary market).

For each case, we have a number of questions to answer. These are:

- What are the optimal behavior of the assembler and suppliers under revenue share contracts for a given wholesale price? How far away is the system profit from the centralized solution? (Case I)
- What are the effects of the wholesaler on the system when it has full information? (Case Ia)
- How is the system affected when the assembler reaches its customers via a distributor? (Case II)
- How is the system affected when the assembler finds an opportunity to sell its unsold products through a distributor in a secondary market or to expend its market reaching another market via a distributor? (Case III)
- What is the reaction of the system to different parameters? (Variance, production costs, correlation between demands, etc.)

Through these questions, we aim to create a managerial insight and possibly observe some quantitative characteristics, which are also observed in real life.

2.3 Literature Review

Following Boyacı (2002), historical evolution of SCM can be categorized into six periods:

- Late 50's & Early 60's: The literature on supply chain modeling dates back to late 50's and early 60's. Multi-echelon inventory theory was born with the famous proof of optimality of (s,S) policies by Scarf (1960), and mainly mathematical properties of multi-echelon systems were focused on those days.

- 60's-Early 70's: Analysis of inventory systems under different settings took the focus and structure and properties of optimal policies and algorithms were analyzed

- 70's-Early 80's: The studies specially focused on computational aspects like heuristics, approximation and exact algorithms for single and multi-stage systems.

- Late 80's-Early 90's: Researches on practical issues gained importance,and the subjects like quantity discounts, joint ELSP and two-part tariffs were investigated.

- Late 90's: It is realized that Supply Chains are decentralized in nature,which means that Supply Chains are managed by independent owners or organizational units with conflicting objectives. Besides, IT developments enable transactional efficiency, accurate and timely information sharing-exchange. In those days industry leaders (eg. Wal-mart & P&G) initiated some studies to obtain creative partnerships whereby parties share benefits and risks through some trading agreement or contract, academic world tended to use existing inventory models to address practical issues and generate managerial insights.

- 2000's: Digital supply chain management issues become a common trend. The subjects like e-commerce, use of the internet are investigated.

This study is mostly related with literature in late 90's. We review the literature under four categories.

- Contract analysis under newsvendor problem framework.
- Cooperation strategies under multi period problem framework.
- Value of shared information.
- Others.

Common issues of the literature covered in this section are description of system dynamics and cooperation strategies.

2.3.1 Contract Analysis under Newsvendor Problem Framework

Through our literature review, we realized that the supply chain contracting literature extensively deals with two echelon structures (supplier-retailer) where the upper echelon monopolize the decision right that effects whole system's behavior. The lower echelon acts as a follower. However, our system behaves in a reverse manner. Additionally, we define two types of suppliers (small suppliers and a wholesaler), which operate differently, in the upper echelon. Despite this, the findings and remarks provide us the necessary insight to explain our system's tendencies.

Cachon (1998) reviews competitive supply chain management, where the topic is broadly covered, with emphasis given to game theory as the primary methodology. In this study, the basic system analyzed consists of a supplier and a retailer. The supplier sells a product to a retailer that faces a downward sloping demand curve as a function of selling price. The system is modelled as a single period problem and the effect of double marginalization (under stocking) is shown. Double marginalization appears when each actor only considers its own profit margin (not the supply chain's profit margin) in making its decision. To eliminate double marginalization effect, alternative strategies (like contracts and quantity discounts) are mentioned from the literature and especially focused on buy-back contracts. The coordination condition is derived for buy-back contracts. One important observation pointed out is that the suppliers earn essentially all of the supply chain profits when wholesale price is very close to selling price. Here, the retailers profit margin approaches zero, but the buy-back rate approaches wholesale price. In another setting, the same system is modelled as infinite horizon, stochastic demand inventory game between one supplier and one retailer. There are lead times for every actor's shipment, and every actor tries to minimize its cost using a base stock policy, where holding costs and back order costs are charged to every actor. Observing that there is no Nash equilibrium at the optimal solution, different transfer payment schemes that satisfy coordination are presented.

Ryzin and Mahanjan (1999) analyze a two-echelon supply chain model, where competing goods are supplied by manufacturers and held in inventory at the retail echelon. Here, two different systems are investigated. These are:

- First system consists of a monopolistic retailer trading with an oligopoly of n competing manufacturers.
- Second system consists of a monopolistic manufacturer trading with an oligopoly of n competing retailers.

These systems are modelled as a single period model in which manufacturers sell products at wholesale prices to retailers facing uncertain demand. The cost and demand information is known to all firms (full information), and the manufacturing cost and retail price are fixed. The analyzes are performed on wholesale price and inventory stocking decisions. The competition is reflected to the model as follows: A sequence of heterogenous customers dynamically substitutes among the goods or retailers based on the availability of retail stocks. To mitigate the negative effect of double marginalization, vendor managed inventory (VMI) and retail managed inventory (RMI) schemes are applied to the systems, and the performance of both systems are compared under VMI and RMI schemes with the first-best(coordinated) solution. In this study, it is shown that neither VMI or RMI achieves first-best profits. However, both tend to achieve first-best profits as the number of horizontally competing firms increases or as goods become closer substitutes. One related observation with our study pointed out in this study is that in both systems, the monopolist firms prefer to have the right of wholesale price decision instead of managing the inventory even if the reverse scheme performs better in terms of total system performance. In our study, the decision of the wholesale price and revenue shares are taken by the powerful actors (the wholesaler and the assembler, respectively), reflecting this tendency.

Cachon (2001) reviews and extends the literature on the management of incentive conflicts within a supply chain. A numerous number of supply chain models under different contracts that establish a transfer payment scheme is constructed and their benefits and drawbacks are illustrated. In all of the models, it is assumed that each firm has access to the information needed to determine the optimal actions (full information) and the optimal actions are feasible for each firm. In the first part of the study, a number of contracting schemes is analyzed on a single retailer and single supplier system for fixed retail price. The analyses are performed solving profit maximization problem of every actor taking production costs, back order cost and salvage value of the unsold product into consideration. Here, the game follows the following sequence of events: the supplier offers a contract and the retailer accepts or rejects the contract. If the retailer accepts the contract, it submits an order quantity, and the supplier produces and delivers to the retailer before the selling season. If the retailer rejects the contract, the game ends. In Cachon (2001), the performance of the contracts is investigated under voluntary and forced compliances. Under voluntary compliance, the supplier could deliver less than the retailer order quantity if that maximizes its profit, however under forced compliance the supplier has to deliver the order quantity of the retailer. One important issue to notice here is that in this study redefining the system parameters, back order cost and salvage value disappear and the analysis could be performed using redefined manufacturing cost and selling price. Therefore, we could extend our models adding backorder cost and salvage value and obtain similar results. Starting with the centralized system solution, following contracts are modelled and coordination characteristics are stated in Cachon (2001):

- Wholesale Price Contract,
- Buy-back contracts,
- Revenue sharing contract,
- Quantity-flexibility contracts,
- Sales-rebate contract.

Cachon and Lariviere (2001a) focus on revenue sharing contracts for one supplier and one retailer system using a similar modelling approach as in Cachon (2001) (using generic system parameters). Assuming full information, the following characteristic of revenue sharing contract is shown in the study:

- Assuming the retail price is also a decision variable, the revenue sharing contract can achieve coordination.
- Dropping the retail price as a decision variable, revenue sharing contracts satisfies coordination for n competing retailers' setting.

Considering the administrative cost of applying the revenue sharing contract (cost of monitoring real sales), a benefit analysis is performed comparing wholesale price contract's solution with coordinated solution. It is observed that the potential gain from coordination in a supply chain with a single retailer depends on the shape of the marginal revenue curve, while a convex marginal revenue curve leads to more benefit than the concave one using revenue sharing contract to satisfy coordination. One important aspect to observe is that for revenue sharing contract there are unique pairs of contract parameters, which satisfy coordination and reflect different revenue allocations for the actors.

An interesting study, which considers the effect of coordination mechanisms to each actor in the system, is Anupindi and Bassok (1999). It studies the effect of coordination at the retailer level to the manufacturer. The system consists of a manufacturer and two competing retailers. In the system, the customers search markets for available goods. According to market search, a fraction of unsatisfied customers at their local retailer due to a stock-out search for the good at the other retailer. It is shown that the manufacturer may not benefit from centralization of stock. The benefit depends on demand distribution, the service level and level of market search. As an extension, optimal incentive of the manufacturer (optimal wholesale price) is analyzed under decentralized and centralized stock cases. Here, the manufacturer and the retailers play a Stackelberg game with the manufacturer being the leader and the retailers followers. This extension provides insight for circumstances under which the manufacturer benefits from horizontal competition between retailers.

Corbett and DeCroix (2001), consider one indirect material supplier and one consumer, which consumes the indirect material in manufacturing system. The consumption of the indirect material depends on efficiency of usage and may decrease with additional effort of the supplier and consumer at a cost which varies with effort level taken. Here, a pair of different types of contracts (Joint investment contract, Shared-savings contract) are evaluated from a theoretical and practical perspective taking the incentives of the actors into account (Looking for a Nash equilibrium). In joint investment contract, every actor pays a fraction of all costs related to the indirect material and the consumer makes a fixed transfer payment to the supplier. And in shared savings contract, the customer pays a fixed transfer t and a for each indirect material used, to the supplier.

Emmons and Gilbert (1998), study the effect of buyback contracts on the manufacturer, retailer and the system performance in fashion industry. The demand is retail price-dependent and every actor tries to maximize its profit. One important observation is that for a range of wholesale prices, there exist a buy back value that increase both the manufacturer and the retailer profits.

One important paper related with our system is Gerchak and Wang (2000). This study considers a supply chain consisting of an assembler and a number of suppliers. In this study, the actor decides on the production quantities besides the contract parameters. Gerchak and Wang (2000) explores and compares two types of setting. One is a Vendor Managed Inventory (VMI) system with revenue sharing. This setting is nothing but Case I, where there are no wholesaler in the system. The other setting is a wholesale-price base system, where there are wholesale price contracts between the assembler and the suppliers. In this setting, the suppliers are the powerful actors in the system. In case Ia, we extend the system of Gerchak and Wang (2000) with a wholesaler, where revenue sharing contracts and wholesale price contract are analyzed simultaneously. Since our study shows similar characteristics, we refer to Gerchak and Wang (2000) in the following chapters, when it is the focus of concern.

Gerchak and Wang (2002), study the same system in Gerchak and Wang (2000). However, in Gerchak and Wang (2002), the actors decide on the capacity build-up before the selling period, taking the expected revenue into consideration. In this study, two types of setting are considered. The first one is the assembler-as-leader game and the other is the suppliers-as-leaders game.

The following papers consider a special case of the newsvendor problem (Two period newsvendor problems), where two demands occur in non-overlapping periods.

Cachon and Kok (2002) criticizes the truth of used input parameters in any decision model that affect the decision of optimal actions. The contradiction mentioned is explained with the following statement: "The inputted parameters may depend on the actions taken". This phenomenon is analyzed on classical newsvendor model where the discussion takes place around fixed salvage value assumption. A more realistic model is constructed as two period newsvendor problem. In the first period (regular season), the retailer chooses a procurement quantity and sells at a fixed price. At the end of the regular season, the retailer chooses a markdown price to liquidate the remaining quantity, if any. The demand in the second period depends on the markdown price and the realization of the first period demand. The solution of this model is compared with the solution of classical newsvendor models, where the salvage value is computed using different heuristics. This analysis reveals that the heuristic approaches always perform poorly than the realistic model (optimal model).

Petruzzi and Dada (2001) extend the realistic model of Cachon and Kok (2002) allowing the retailer to give an additional order at the beginning of the second period. In this study, observation of the first period demand provides the retailer additional information to give a more precise decision on second period selling price and procurement quantity.

Lee (2001) studies a three echelon system in a two-period news vendor problem framework, where there is one supplier at the first echelon, one retailer at the second echelon and a discount sales outlet (DSO) at the third echelon. The sequence of events occurs as follows: In the first period the retailer orders a quantity from the supplier and sells them in its market. At beginning of the second period, the DSO gives an order to the retailer (q) and the retailer sends its left-over inventory, I, from the sale in the first period if $I \leq q$, otherwise the retailer sends q to DSO and return $I - q$ to the supplier. There are wholesale price contracts between the echelons, and additionally the supplier buys back the unsold goods from the retailer and DSO with diminishing refunds (refund of DSO is less than the retailers). According to these contracts, the echelons take the following decisions: The supplier decides on wholesale price and corresponding refund. Then the retailer determines the order quantity from the supplier and the wholesale price charged from DSO for every unsold item delivered. Finally the DSO determines a markdown sale price $P\alpha$ where P is a fixed sale price of the retailer to customer and $0 \leq \alpha \leq 1$, and its order quantity from the retailer. This system is analyzed in four different coordination status as follows:

- Retailer-DSO Centralized Model: Here, the retailer and DSO act in coordination to maximize the system profit under fixed wholesale price and refund of the supplier.

- Retailer-DSO Decentralized Model: In this model, the retailer and DSO try to optimize their individual profits under fixed wholesale price and refund of the supplier.

- The supplier-retailer-DSO coordination model: Here, each echelon coordinates its decisions to maximize their joint profit.

- Supplier-retailer-DSO uncoordinated model: In this model, the retailer and

DSO acts like in Retailer-DSO centralized model and the supplier decides on the refund rate for the send back goods from the retailer. The supplier does not buy back unsold items from DSO. The wholesale price of the suppliers is assumed fixed.

In the decentralized and uncoordinated cases, some coordination policies are proposed. To achieve a fair profit sharing, these policies satisfy pareto conditions, i.e. conditions, which ensure that after the coordination the actors' share of joint profit will not decrease. Therefore, every actor will take exactly same share that they gain before coordination from the total system profit. Since coordination increases total system profit, every actor will benefit.

2.3.2 Cooperation Strategies under Multi Period Problem Framework

The literature, which concerned with cooperation strategies under multi period problem framework, usually deals with inventory management problem of the actors in the supply chain. The optimal decision, which minimizes the total cost, on the parameter of the inventory management policy is searched. The cooperation strategies are based on reallocation of costs in the supply chain with different transfer payment strategies.

Cachon (1999) studies the competitive and cooperative selection of inventory policies in a two-echelon supply chain with one supplier and N retailers. Minimizing their own holding costs and backorder penalty costs charged at the retailer level, every actor decides on its reorder point. Obtaining the Nash equilibrium solution under competition, three cooperation strategies are investigated. The first one is to change the actors' incentives so that the optimal solutions are a Nash equilibrium. This is done by creating two transfer payments via a contract (one from the retailer to the suppliers depending on the retailers' backorders and one from the supplier to the retailers for its backorders i.e., for late deliveries. In the second strategy, if there are multiple Nash equilibria, then the firms make sure that they choose the lowest cost equilibrium. This will not guarantee optimal performance, but it requires a minimal change to current operating procedure. In the third strategy, the responsibility of choosing policies at all locations in the supply chain is taken by the supplier (VMI). Here, VMI coordinates the supply chain as long as the firms are willing to use fixed transfer payment so that they can share the gains from VMI.

Cachon and Zipkin (1999) study a two echelon supply chain with one supplier and one retailer. The chain is modelled as multi period problem with stationary stochastic demand and fixed transportation times, where inventory holding costs and consumer backorder penalty cost are charged at each stage. Minimizing their cost, every actor chooses a base stock policy simultaneously, so a Nash equilibrium is searched. During the analysis, echelon inventory game (EI) and local inventory game (LI) are considered. In the EI game, every actor decides on echelon base stock level tracking echelon inventory, whereas in LI game only local inventory is considered. Observing the optimal solution is typically not a Nash equilibrium, a set of linear contracts are developed. Additionally, Stackelberg versions of the games are studied, where one dominant player chooses its strategy before the other.

Chen, Federgruen and Zheng (2001) consider a two-echelon system, where a supplier distributes a single product to multiple retailers who serve different markets. The demands are deterministic with a rate that depends on retailers' selling price. The system is analyzed in EOQ (Economic Order Quantity) framework, where the supplier and the retailers decide on the order frequency from the set of powers of two values. After the solution of the centralized system, an order quantity discount policy with fixed annual transfer payments is proposed that satisfy coordination. Fixed annual transfer payments are utilized to rearrange the profit shares of the actors as new profits are greater than the one before coordination. This makes the contract acceptable for all actors.

2.3.3 Value of Shared information

Cachon and Lariviere (2001b) study the effect of demand forecast sharing in one manufacturer, one supplier supply chain where the manufacturer offers the supplier a contract with an initial demand forecast to build capacity for supplier's product. Assuming that the contract is accepted, the supplier builds capacity and the manufacturer submits final order after the demand is realized. The offered contract consists of commitments (m) and options (o), that restrict the manufacturer's final order (q) as $m \leq q \leq m + o$. If the supplier accepts the contract, the manufacturer pays the supplier w_m per commitment and w_o per option. Additionally, the manufacturer pays w_e per option utilized in the final order. The system is analyzed for full information and asymmetric information cases under forced and voluntary compliance regime assumptions.

Cachon and Fisher (2000) study the value of information in a one supplier, n identical retailers system, where the retailers face stationary consumer demand with a known distribution. There are fixed transportation times between locations, and holding costs are charged at all levels where back-order penalty cost is charged only at the lowest level. The retailers and the supplier implement a reorder point policy. According to this policy, at the beginning of each period, they order the smallest integer multiple of batches that arises their inventory position above reorder point if inventory position is less than reorder point, otherwise they do not give any order. Here, two levels of information sharing are considered. With traditional information sharing the supplier only observes the retailers' order. With full information sharing the supplier has immediate access to retailers' inventory data. The level of information sharing changes the allocation policy of available shipments used by the supplier, when shipping the orders to the retailers. Using the additional inventory information of the retailers with full information sharing, the supplier can improve its order quantity and also its allocation decision shipping the available batch to the retailer with lowest inventory position in the period a batch is shipped. Besides, the necessary information technology to facilitate full information for the supplier, also contributes to the reduction of lead times and shipment frequency by reducing the time and cost to process orders. This effect of information technology is also investigated through computational analysis.

Lau and Lau (2001), study a manufacturer, a retailer system under wholesale price contract. The system is modelled as a single period problem and the effect of asymmetric market information is investigated. Three information structures are applied to the system. The first one is symmetric market information, where the manufacturer and the retailer have the same demand information. The asymmetric information case is extended into two situations. In known superiority model, the retailer has more accurate estimation of the demand distribution than the manufacturer and the manufacturer is aware of this fact. Differently, in hidden-superiority model, the manufacturer does not know that the retailer has better market information. The computational analysis reveals that the superior market-knowledge of the retailer increases the channel profit and the manufacturer's profit, however the retailer can increase its profit only if the manufacturer does not know that the retailer has superior knowledge. In other cases, the manufacturer takes a larger share of the channel profit.

2.3.4 Others

Eppen and Iyer (1997) study backup agreements between a catalog company and manufacturers. There are two selling periods for the goods. A backup agreement performs as follows: The catalog company gives an order of y units at the start of the first period. The manufacturer delivers $(1-\rho)y$ units immediately and holds ρy units for the second period. At the beginning of the second period, the catalog company is allowed to purchase any quantity within the range $[0, \rho y]$ at the original purchase cost and pays a penalty for each unit not purchased. The performance of the contract is analyzed from the catalog company's point of view. However, an additional study is performed to show the impact of the contract on the manufacturer and it reveals that there is a range of contract parameters for which the manufacturer's and catalog company's expected profits improve.

Schuster, Bassok and Anupindi (2000) extend Eppen and Iyer (1997) applying options that provide flexibility to the buyer to respond to market changes in the second period. Different from Eppen and Iyer (1997), the profit maximization problem of the supplier, which decides on the contract parameters, is also considered here. The coordination mechanism and its drawbacks are analyzed under linear prices, return policies and simple and bundle quantity discount schemes.

Lee, Padmanabhan, Taylor and Whang (2000), consider the effect of price protection policy in personal computer industry, where there are two selling periods, and the demand and the retail price are diminishing in the second period. The price protection policy is applied giving a rebate to the retailer for units unsold. Playing a Stackelberg game, the efficiency and drawbacks (from the actors and system point of view) of price protection policy is compared with the integrated channel and no price protection scenarios under one and two replenishment opportunity cases for the retailer.

2.3.5 Relation of the Literature with the System Analyzed

The literature reviewed, mainly focuses on two issues. The first issue is description of dynamics of the supply chain and the second one is cooperation strategies. In this study, we mainly focus on the dynamics of the system described, mentioning little about coordination issues. In this section, the dynamics of supply chain contracting and observations in the literature related with the system analyzed in this study is mentioned.

The literature mentions about three fundamental causes that prevent coordination in a decentralized supply chain. These are explained below.

- Information asymmetry: Information sharing among all stages of a supply chain appears as sharing demand information and sharing the cost parameters and contract parameter decisions. The bullwhip effect (see Lee, Padmanabhan and Whang, 1997, for detailed discussion), that is, the increase in variability in the supply chain due to lack of sharing demand information may be prevailed by Electronic Data Interchange (EDI). EDI allows the demand data occurring at the selling point to be shared immediately with all stages of the supply chain. Although there are many other reasons of bullwhip effect besides the lack of demand data sharing, in our system it would be sufficient to share demand data among the members of the supply chain to eliminate bullwhip effect. The other one, the sharing of production costs and other contract parameter decisions (revenue shares and wholesale price) of all actors within the supply chain, also affects the decisions of the actors and system efficiency. Usually, distorted information sharing increases the negative effect of horizontal and vertical competition. Therefore, we assume that all of the actors have the full knowledge demand, and know production cost of other actors.

- Horizontal Competition: Most of the past studies point out that horizontal competition among independent sellers of substitutable products causes over-stocking within the supply chain. Since in our system the suppliers supply the assembler with complementary products, there is no horizontal competition among the suppliers. In our setting, the profit of one supplier depends on the minimum quantity delivered by the other suppliers to the assembler to become a complete product. The product delivered over this minimum quantity can not assembled because of the lack of complementary products. However it does not create inefficiency since there is full information and the assembler equalizes the delivery quantities of the suppliers setting appropriate revenue shares for the suppliers.

- Vertical Competition: Vertical competition creates inefficiency because of the double-marginalization. Double-marginalization appears when each actor only considers its own profit margin (not the supply chain's profit margin) in making its decision (See Cachon, 1998). In our system every actor tries to maximize its profit, and therefore the system operates with quantities that are lower than the centralized one where every actor operates centrally to maximize the supply chain profit. We observe the negative effect of double-marginalization.

In the literature, supply chain contracting is usually modelled as single period models, since this modeling approach is considered to be sufficiently rich to explore the characteristics of system. Since we perform a strategic level analysis, we model our system as single period problem. In the below part, relation of the review literature with our system is mentioned.

Cachon (1998) observes that using buy back contracts the suppliers earn essentially all of the supply chain profit when wholesale price is very close to selling price. In Section 3.4.1, we analyze revenue sharing contracts with surplus subsidy. Similarly, it is observed that the assembler tries to take all system profit setting surplus subsidies that make the revenue shares of the suppliers to production cost of the suppliers. This tendency of the assembler results with zero profit for the suppliers. To obtain a fair allocation of system profit to the actors, we propose an alternative contract.

Ryzin and Mahanjan (1999) point out that the monopolist firms prefers to have the right of wholesale price decision instead of managing the inventory even if the reverse scheme performs better in terms of the total system performance. We model the system so that the powerful actors take the decision of the wholesale price and revenue shares (the wholesaler and the assembler, respectively), to reflect this tendency.

Cachon and Lariviere (2001b) study a two echelon supply chain, where the supplier decides on the capacity built for the manufacturer. The supplier could not send to the manufacturer more than this capacity even if the manufacturer orders more. The notion of building capacity simply fits the relation between the assembler and the suppliers in our model, where the assembler offers a revenue share and the suppliers decide on the capacity allocation. The allocated capacity determines the operating quantity of the system. Here, we assume that the capacity already exists.

Cachon (2001) studies the performance of the contracts under voluntary and force compliance regimes. The system we model, operates under voluntary compliance regime. The suppliers determine the capacity allocated for the assembler and the system can not operate for the quantities greater than the allocated capacities.

Lee (2001) considers a three echelon system, where the demand occurs in the last two echelons in non-overlapping selling periods. Although, the general view of this supply chain is similar the one described in Case III, Case III differs from this study in the following manner: In Case III, we apply revenue sharing contracts between the actors and the actor in the second echelon (assembler) is the powerful one instead of the actor in the first echelon (supplier) in Lee (2001). We also fix the selling price of the second period, whereas the clearance sale price influencing the demand is determined by discount sales outlet (last echelon) in Lee (2001). In Case III, actually the assembler determines the selling price although it is assumed fixed and gives a revenue share to the distributor to increase its willingness to sell the product.

In this study, we extend the study of Gerchak and Wang (2000) with a wholesaler, where revenue sharing contracts and wholesale price contract are analyzed simultaneously. Here, the wholesaler represents another type of supplier that the production and assembly firms operates with in real life. There is wholesale price contract between the assembler and the wholesaler. In Case I, we analyze a similar system in Gerchak and Wang (2000). Different from Gerchak and Wang (2000), in our system, there is an additional wholesaler that supplies the assembler with specialized product at a previously announced wholesale price. Since the wholesale price appears as an additional cost of production in the assembler's problem, all characteristics fit Gerchak and Wang (2000). As an other extension to Gerchak and Wang (2000), in Case Ia, we consider the wholesale price decision of the wholesaler, where the wholesaler has full information about the customer demand and the assembler's order decision that depends on the wholesale price. We analyze the effect of the wholesaler on the system performance, when the wholesaler choices the wholesale price that maximizes its expected profit. We refer to Gerchak and Wang (2000) in the following chapters, when it is the focus of concern. The other cases represent different extensions about the assembler's distribution alternatives, where the assembler has the opportunities to serve its market utilizing a distributor (Case II) and to enter another market via a distributor (Case III).

Through our analysis, the following general assumptions are made:

- Single Period Problem: We construct our model as a single period problem, since one period is enough to cover essential dynamics of the contracting mechanisms and single period models are applicable for strategic level analysis where contracts are long term agreements like in our case.

- Stochastic demands: We assume that demands are random to reflect the uncertainty of demand in a long planning horizon.

- Full information: There is full information in the system. All of the actors have the full knowledge about demand, and know production cost of other actors.

- No salvage value: Since we perform a strategic level analysis, we assume that the salvage value of unsold products is zero, and they become scrap at the end of the period.

- Stackelberg game: The game played in the system is a Stackelberg game, where every decision is taken successively. (Firstly, the wholesaler, then the assembler, and finally small suppliers and the distributor)

- Profit maximization for each actor: At every decision instant, responsible actor makes a decision by solving its profit maximization problem.

2.4 Solution Procedure

Because of the nature of the problem, we apply the following algorithmic solution procedure:

- Step 1: Formulation and solution of the centralized system (a benchmark for the system performance).

- Step 2: Formulation and solution of small suppliers' problem for given revenue shares. Derivation of the relation between the revenue shares and quantity decision of suppliers (suppliers' reaction function).

- Step 2a (For case II and III): Formulation and solution of the distributor's problem for given revenue shares. Derivation of the relation between the revenue shares and quantity decision of the distributor(distributor's reaction function).

- Step 3: Formulation and solution of the assembler's problem knowing the reaction of small suppliers (and the distributor for case II and III) for a given wholesale price. Determination of revenue shares. Derivation of the relation between the wholesale price and assembler's revenue shares decision (assembler's reaction function for case Ia).

- Step 4 (For case Ia): Formulation and solution of the wholesaler's problem knowing the reaction of the assembler. Determination of wholesale price.

In our analysis, solution of the centralized systems, where the systems are operated centrally, constitutes benchmarks to measure the performance of decentralized systems in all cases.

CHAPTER 3

ANALYSIS OF ASSEMBLER NETWORK WITHOUT A DISTRIBUTOR (CASE I AND CASE Ia)

In this chapter, we analyze Case I and Case Ia introduced in Chapter 2. The system consists of an assembler, a number of suppliers and a wholesaler. There are revenue share contracts between assembler and suppliers. In case I, the assembler-suppliers system is analyzed for an announced wholesale price. Here, the assembler is the most powerful actor deciding on the revenue shares. In case Ia, the former system is extended including the wholesaler's wholesale price decision (according to the wholesale price contract between the assembler and the wholesaler) under full information assumption for the wholesaler. Here, the wholesaler appears as the most powerful actor accessing the necessary information (demand information and production costs of other actors). Deciding on the wholesale price, the wholesaler limits the feasible actions of the assembler which is the second powerful actor of the system. The suppliers are the followers in each case. The system representation for each case can be seen in Figure 3.1 and Figure 3.2.

The solution procedure follows the below steps:

- Step 1: Formulation and solution of the centralized system.

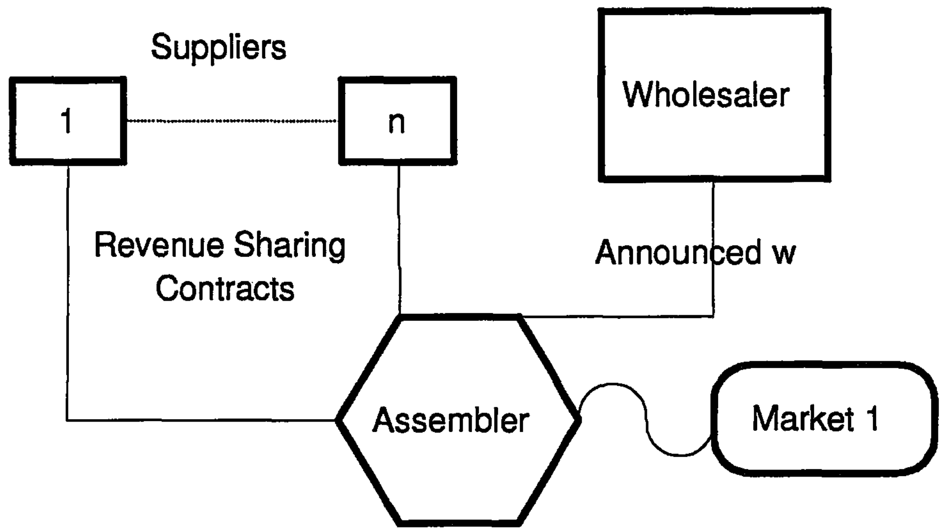

Figure 3.1: System Description Case I

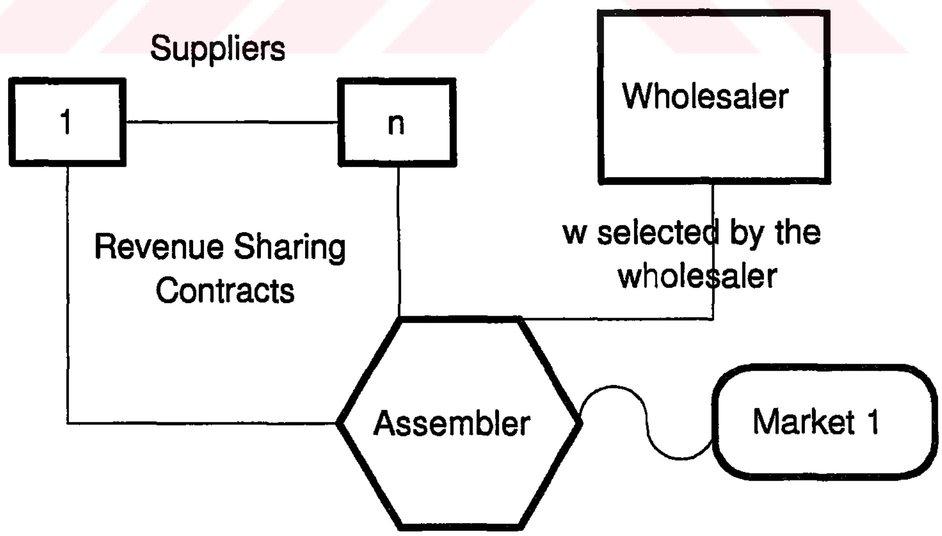

Figure 3.2: System Description Case Ia

- Step 2: Formulation and solution of suppliers' problem for given revenue shares. Derivation of the relation between the revenue shares and quantity decision of suppliers (suppliers' reaction functions).

- Step 3: Formulation and solution of the assembler's problem knowing the reaction of suppliers for a given wholesale price. Determination of revenue shares. End of case I. Derivation of the relation between the wholesale price and revenue shares decision of the assembler (assembler's reaction function for case Ia).

- Step 4 (For case Ia): Formulation and solution of the wholesaler's problem knowing the reaction of the assembler. Determination of the wholesale price.

These two cases consider an extension of Gerchak and Wang (2000) adding a wholesaler to the system.

Let D be the random demand of the final product, with cumulative distribution function F and probability density function f. Let n be the number of suppliers. Unit revenue of the assembler from selling the product, which consists of n+1 component or sets (one for each unit without loss of generality) supplied by independent suppliers, is set to 1 (one). Unit production costs of the component for small suppliers and wholesaler are c_i, $i = 1, ..., n$ and c_ω, respectively. The unit cost of mating the components for the assembler is c_0. Note that c_i, $i = 1, .., n$, c_ω, c_0 all are assumed to be less than one. Moreover, $\sum_{i=1}^{n} c_i + c_0 + c_\omega < 1$ holds. Q_i, $i = 1, ..., n$ are quantities (or capacity) allocated by the suppliers, and Q_0 is the assemblers order quantity from the suppliers and the wholesaler. The notation used throughout the chapter is summarized in table 3.

3.1 The Centralized System

In this section, we analyze the centralized system to evaluate the performance of the decentralized model. To be able to make a detailed analysis of our system performance we define one centralized system for each of Case I and Case Ia. In

Table 3.1: Notation for Chapter 3

D, f, F **and** $\bar{F}$:	the random demand of final product, PDF, CDF and complementary cumulative function, respectively.
c:	unit costs (for all actors),
π:	the expected profit function (for all actors),
Q:	quantity (for all actors),
Q_{dec}:	operating quantity of the decentralized system,
α:	the revenue share (for all other than the wholesaler)
w:	the selling price of the wholesaler (Wholesale price)
Superscripts:	For each of the cases, case number enters as the superscript (for all of the above other than the unit costs, and wholesale price)
Subscripts:	Subscripts denote the actors and centralized problem: 0 for the assembler, $1, ..., n$ for the suppliers, ω for the wholesaler and c for the centralized problem.

the first one, suppliers and assembler act centrally and the wholesale price of the wholesaler is set at a fixed level. The optimal solution of this system gives the maximum expected profit that the assembler-suppliers subsystem can gain after the wholesale price is determined. In the second one, consisting of wholesaler, suppliers and the assembler operate centrally, and the optimal solution gives the maximum expected profit that the system can achieve.

Starting with the first system (assembler-suppliers subsystem), where the wholesaler acts as an independent actor and only sets its wholesale price, we try to maximize the subsystem-wide expected profit taking this wholesale price as given. We set the selling price of the final product to 1. Here the subsystem expected profit is:

$$\begin{aligned}\pi_c^I(Q) &= E\left\{-[(\sum_{i=1}^{n} c_i) + c_0 + w]Q + min(Q, D)\right\} \\ &= -(\sum_{i=1}^{n} c_i + c_0 + w)Q + \int_0^Q xf(x)dx + Q\bar{F}(Q) \qquad (3.1)\end{aligned}$$

where w is the wholesale price. Since the expected profit function in 3.1 is concave in Q and the optimal production quantity Q_c^I is the one that satisfies the first order optimality condition,

$$\bar{F}(Q_c^I) = (\sum_{i=1}^{n} c_i) + c_0 + w \tag{3.2}$$

Hence, the optimal system-wide expected profit is given by

$$\pi_c^I(Q_c^I) = \int_0^{Q_c^I} \bar{F}(x)dx - (\sum_{i=1}^{n} c_i + c_0 + w)Q_c^I \tag{3.3}$$

In the second system, if the system is totally centralized, the assembler wants to maximize the expected system-wide revenue. And the system expected profit is:

$$\begin{aligned} \pi_c^{Ia}(Q) &= E\left\{-(\sum_{i=1}^{n} c_i + c_0 + c_\omega)Q + min(Q, D)\right\} \\ &= -(\sum_{i=1}^{n} c_i + c_0 + c_\omega)Q + \int_0^Q xf(x)dx + Q\bar{F}(Q) \end{aligned} \tag{3.4}$$

We assume that $\sum_{i=1}^{n} c_i + c_0 + c_\omega < 1$. Since the expected profit function in Equation 3.4 is concave, and therefore the optimal production quantity Q_c^{Ia} for the totally centralized system is the one that satisfies the first order condition,

$$\bar{F}(Q_c^{Ia}) = \sum_{i=1}^{n} c_i + c_0 + c_\omega \tag{3.5}$$

Hence, the optimal system-wide expected profit is given by

$$\pi_c^{Ia}(Q_c^{Ia}) = \int_0^{Q_c^{Ia}} \bar{F}(x)dx - (\sum_{i=1}^{n} c_i + c_0 + c_\omega)Q_c^{Ia} \tag{3.6}$$

3.2 Decentralized System

In the decentralized system, a wholesale price contract is used for the wholesaler, and suppliers obtain a certain share of the revenue. According to the revenue-share contract, the assembler pays each supplier a revenue share of $\alpha_i, 0 < \alpha_i <$

$1, i = 1, ..., n$ (unit selling price is assumed as 1, hence revenue obtained from sales per unit is 1) for each unit of final product sold and the supplier determines the quantity allocated $Q_i,\ i = 1, ..., n$. The part of the revenue kept by the assembler is $\alpha_0 = 1 - \sum_{i=1}^{n} \alpha_i$. According to wholesale price contract, the wholesaler sets a wholesale price w and the assembler determines the order quantity Q_0. Clearly, the necessary conditions for each party to stay in the business are

$$\begin{aligned} \alpha_0 &> c_0 + w && \text{for the assembler,} \\ \alpha_i &> c_i;\ i = 1, ..., n && \text{for suppliers,} \\ w &> c_w && \text{for the wholesaler,} \end{aligned}$$

In this system, we assume that each actor knows the production cost of others, and they all have the full information on the demand distribution. We analyze Case I and Case Ia in the following subsections. We start the analysis with the system comprised of suppliers and the assembler, where the assembler is powerful and makes the main decision of determining the revenue shares ($\alpha_i, i = 1, ..., n$), which affect the system behavior between suppliers and the assembler if the wholesale price is previously announced by the wholesaler (Case I). Then we will continue with the wholesaler's problem, where the wholesaler determines the wholesale price (Case Ia).

3.2.1 Revenue Shares Maximizing Assembler's Expected Profit

The system analyzed in this section is a special case of the system described in Gerchak and Wang (2000). The only difference is that there is an additional wholesale price for the assembler and it appears as an additional cost of production in the assembler's problem. Since it is fixed, all findings of Gerchak and Wang (2000) are also valid in this section.

In this section we analyze how the assembler determines the revenue share of suppliers ($\alpha_i, i = 1, ..., n$) for a given wholesale price w. Assume that first the assembler sets up the revenue share and an order quantity Q_0, and the suppliers determine the number of units Q_i to allocate for the assembler from the capacity (Stackelberg type game). For a given revenue-sharing scheme, the assembler and suppliers would solve their own newsvendor problem. Here, we drop the superscript denoting cases I and Ia, as there is no difference for these cases. The assembler's objective function and first order optimality condition for the quantity are given by Equation 3.7 and Equation 3.8, and for a typical supplier i in Equation 3.9 and 3.10.

$$\begin{aligned} Max\ \pi_0(Q) &= E\{-(c_0+w)Q+\alpha_0 min(Q,D)\} \\ &= -(c_0+w)Q+\alpha_0\int_0^Q xf(x)dx+\alpha_0 Q\bar{F}(Q) \qquad (3.7) \\ \bar{F}(Q_0) &= \frac{c_0+w}{\alpha_0} \qquad (3.8) \\ Max\ \pi_i(Q) &= E\{-c_i Q+\alpha_i min(Q,D)\} \\ &= -(c_i)Q+\alpha_i\int_0^Q xf(x)dx+\alpha_i Q\bar{F}(Q),\ i=1,...,n \qquad (3.9) \\ \bar{F}(Q_i) &= \frac{c_i}{\alpha_i},\ i=1,...,n \qquad (3.10) \end{aligned}$$

where Q_0 and Q_i are the order quantity of the assembler and the quantity allocated by the supplier i, respectively.

However, in our system the allocated quantities and revenues of suppliers are inter-dependent to each other, because the assembler can not assemble more than $Q_1 = min_i(Q_i)$ $i = 1,...,n$, which is the maximum amount of capacity allocated for the complementary products by the suppliers. Gerchak and Wang (2000) proves Proposition 1 for a similar system (Proposition 1 in Gerchak and Wang (2000)). In our case, for any of the solutions, which are Nash equilibrium, none of the suppliers could get more profit by deviating from the solution and Pareto optimal point is the point that maximizes all suppliers' expected profits. Formally, a solution (call it A) to a multiple-objective problem is Pareto optimal if no other feasible solution is at least as good as A with respect to every objective and strictly better than A with respect to at least one objective.

Proposition 1 *If* $\frac{c_1}{\alpha_1} = max(\frac{c_i}{\alpha_i} : i = 1,...,n)$*, all points in* $[0, Q_1]$ *are Nash equilibria, and* $Q_{dec} = Q_1$ *is the Pareto optimal among them.*

Proof: Suppose that $\frac{c_1}{\alpha_1} = max(\frac{c_i}{\alpha_i} : i = 1,...,n)$, $Q_1 = min_i(Q_i)$ and let S_1 be the critical supplier. Here the optimal amount for S_1 is Q_1, and S_1 does not want to allocate more than Q_1. And for the other suppliers, they would like to allocate more than Q_1, but they will prefer to allocate Q_1 units because they are paid only for the final product sold and the extra units sent out of Q_1 will become directly scrap since they can not be assembled. Thus at equilibrium, all suppliers will deliver, and the assembler will produce, no more than Q_1. So, we can say that any amount chosen by all of the suppliers in $[0, Q_1]$ is a Nash equilibrium and $Q_1 = Q_{dec}$ is the quantity, which maximizes all suppliers' profits.□

Here it is convenient to assume that the assembler tries to maximize its own expected profit function $\pi_0(\alpha_1, ..., \alpha_n)$. So:

$$Max\ \pi_0(\alpha_1, ..., \alpha_n) = E\{-(c_0 + w)Q_{dec} + (1 - \sum_{i=1}^{n} \alpha_i)min(Q_{dec}, D)\} \quad (3.11)$$

where $Q_{dec} = min_i(Q_i : i = 0, .., n)$ is the operating quantity of the system for given $\alpha_1, ..., \alpha_n$.

Gerchak and Wang (2000) shows that the assembler always sets $\alpha_i, i = 1, ..., n$ according to the following property in the optimal solution (Proposition 3 in Gerchak and Wang (2000)).

Proposition 2 *The Assembler will always set $\alpha_1, ..., \alpha_n$ such that*

$$\frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n} \geq \frac{c_0 + w}{\alpha_0} \quad (3.12)$$

Proof: We start with the first part, that is, the assembler sets $\alpha_i, i = 1, ..., n$ such that $\frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n}$. Assume that $\frac{c_1}{\alpha_1} = max(\frac{c_i}{\alpha_i} : i = 1, ..., n)$, S_1 is the critical supplier and $\frac{c_1}{\alpha_1} > \frac{c_i}{\alpha_i}$ for some $i \geq 2$. The optimal quantities to deliver for the suppliers are $Q_1 < Q_i, i = 2, ..., n$. The assembler can produce only Q_1 complete units. Thus, by reducing α_i to a value such that $\frac{c_1}{\alpha_1} = \frac{c_i}{\alpha_i}$, the assembler will increase its own share α_0 and expected profit $\pi_0(\alpha_1, ..., \alpha_n)$ in Equation 3.11 without reducing the delivery quantity of complete sets of components.

For the second part, suppose that $\frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n} < \frac{c_0+w}{\alpha_0}$. Here, from Equation 3.8 and Equation 3.10 we know that the suppliers would like to deliver more (Q_1) than the assembler is willing to assemble (Q_0). Therefore, by reducing the revenue share allocated to each supplier at least to a value such that $\frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n} = \frac{c_0+w}{\alpha_0}$, the assembler can improve its own expected profit $\pi_0(\alpha_1, ..., \alpha_n)$ in (3.11).□

Following the above, since the assembler can not produce more that Q_1 even if $Q_0 > Q_1$, at the optimal solution, all suppliers will become critical and the decentralized production quantity Q_{dec} (system's actual operation quantity) will be equal to Q_1.

From Equation 3.12, we know that $c_1\alpha_i = \alpha_1 c_i$, for $i = 1, ..., n$, so $\sum_{i=1}^{n} \alpha_i = \alpha_1(\sum_{i=1}^{n} c_i)/c_1$ and moreover $\alpha_1 \leq c_1/(\sum_{i=1}^{n} c_i + c_0 + w)$ since $c_1\alpha_0 \geq \alpha_1 c_0$ and $\sum_{i=1}^{n} \alpha_i + \alpha_0 = 1$. Then it can be said that $Q_{dec} \leq Q_c^I$ and will be equal if and only if $\alpha_0 = (c_0 + w)/(\sum_{i=1}^{n} c_i + c_0 + w)$ and $\alpha_i = c_i/(\sum_{i=1}^{n} c_i + c_0 + w)$ for $i = 1, ..., n$. This creates a natural upper bound to our optimization problem (Equation 3.11). Next proposition follows (Proposition 2 in Gerchak and Wang (2000)).

Proposition 3 1. *The optimal decentralized production quantity can not be larger than the centralized quantity of Case I. That is,* $Q_{dec} \leq Q_c^I$.

2. *The decentralized production quantity will be the same as the centralized quantity of Case I if and only if the revenue share of each party equals to its cost share. That is,* $Q_{dec} = Q_c^I$ *iff* $\alpha_0 = (c_0 + w)/(\sum_{i=1}^{n} c_i + c_0 + w)$ *and* $\alpha_i = c_i/(\sum_{i=1}^{n} c_i + c_0 + w)$ *for* $i = 1, ..., n$.

Substituting $\sum_{i=1}^{n} \alpha_i$ with $\alpha_1(\sum_{i=1}^{n} c_i)/c_1$ in Equation 3.11, the assembler's n dimensional problem can be reduced to one dimensional one. That is,

$$
\begin{aligned}
Max\ \pi_0(\alpha_1) \;&=\; E\{-(c_0 + w)Q_1 \\
&+ \left[1 - \left(\frac{\sum_{i=1}^{n} c_i}{c_1}\right)\alpha_1\right] min(Q_1, D)\Big\}
\end{aligned}
\tag{3.13}
$$

Since the behavior of the suppliers are known as

$$
\alpha_1 = \frac{c_1}{\bar{F}(Q_1)}, \tag{3.14}
$$

a monotone increasing function, we can perform the optimization over Q_1 rather than over α_1. So, suppressing the super/subscripts on Q our problem (Equation

3.13) becomes

$$\begin{aligned} Max\ \pi_0(Q) &= E\left\{-(c_0+w)Q + \left[1 - \frac{\sum_{i=1}^n c_i}{\bar{F}(Q)}\right] min(Q,D)\right\} \\ &= -(c_0+w)Q + \left[1 - \frac{\sum_{i=1}^n c_i}{\bar{F}(Q)}\right] \int_0^Q \bar{F}(x)dx \end{aligned} \tag{3.15}$$

As can be observed from Equation 3.15, we are dealing with the same problem where there is only one supplier and its production cost is equal to $\sum_{i=1}^n c_i$ since the total cost $\sum_{i=1}^n c_i$ appears as a single parameter in Equation 3.15. Hence, the number of suppliers does not affect the production quantity decision of the assembler if the total production cost of suppliers $\sum_{i=1}^n c_i$ is constant. So, we have the following result (corollary 1 in Gerchak and Wang (2000)).

Corollary 1 *For a given total components production cost $\sum_{i=1}^n c_i$, the optimal decentralized production quantity Q_{dec} and, hence total channel profit $\pi_c^I(Q_{dec})$ are not affected by the number of suppliers and the allocation of the total cost among them.*

The first order condition of optimality for the problem (Equation 3.15) is

$$\frac{d\pi_0(Q)}{dQ} = -(c_0+w) + \bar{F}(Q) - \sum_{i=1}^n c_i \left[1 + \frac{f(Q)}{[\bar{F}(Q)]^2} \int_0^Q \bar{F}(x)dx\right] \tag{3.16}$$

To analyze the concavity of the problem (Equation 3.15), we investigate the second order of the profit function

$$\begin{aligned} \frac{d^2\pi_0(Q)}{dQ^2} &= -f(Q) - \sum_{i=1}^n c_i \left[\frac{[f'(Q)\int_0^Q \bar{F}(x)dx + f(Q)\bar{F}(Q)]\bar{F}(Q)^2}{\bar{F}(Q)^4}\right] \\ &- \sum_{i=1}^n c_i \left[\frac{2\bar{F}(Q)f(Q)^2 \int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^4}\right] \\ &= -f(Q) - \sum_{i=1}^n c_i \left(\frac{f(Q)\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^2}\right)\left[\frac{f'(Q)}{f(Q)} + \frac{\bar{F}(Q)}{\int_0^Q \bar{F}(x)dx} + 2\frac{f(Q)}{\bar{F}(Q)}\right] \end{aligned}$$

Because we know that $f(Q)$, $\int_0^Q \bar{F}(x)dx$ and $\bar{F}(Q)$ are all positive, the only condition that should be satisfied to prove the concavity of the function is the following

$$\left[\frac{f'(Q)}{f(Q)} + \frac{f(Q)}{\bar{F}(Q)}\right] > 0 \tag{3.17}$$

This condition in Equation 3.17 is satisfied if our demand distribution has increasing failure rate (IFR) property, which means that the derivative of $\frac{f(Q)}{\bar{F}(Q)}$ is always positive, ie.

$$\begin{aligned} Y &= \frac{f(Q)}{\bar{F}(Q)} \\ \frac{dY}{dQ} &= \left[\frac{f'(Q)\bar{F}(Q) + f(Q)^2}{\bar{F}(Q)^2}\right] > 0 \\ &= \left[\frac{f(Q)}{\bar{F}(Q)}\right]\left[\frac{f'(Q)}{f(Q)} + \frac{f(Q)}{\bar{F}(Q)}\right] > 0 \end{aligned}$$

Since $\bar{F}(Q)$ and $f(Q)$ are positive

$$\left[\frac{f'(Q)}{f(Q)} + \frac{f(Q)}{\bar{F}(Q)}\right] > 0$$

Thus the following proposition follows immediately.

Proposition 4 *If*

$$\frac{f(Q)}{\bar{F}(Q)} \tag{3.18}$$

is increasing, then $\pi_0(Q)$ is concave and has a unique maximum (Q_{dec}) with in range $[0, Q_c^I]$ (from Proposition 3.1), which can be found by solving $\frac{d\pi_0(Q)}{dQ} = 0$

Proposition 5 *For a given wholesale price the optimal decentralized production quantity Q_{dec} and, hence total system profit $\pi_c^I(Q_{dec})$ is*

1)decreasing in $\sum_{i=1}^n c_i$ and $c_0 + w$

2)increasing in the ratio of $\frac{c_0+w}{c_0+w+\sum_{i=1}^n c_i}$ for any given $c_0 + w + \sum_{i=1}^n c_i$

Proof: Can be proved easily from Equation 3.16 and 3.1.□

Proposition 6 *The supplier's expected profit is proportional with their production costs c_i, $i = 1, ..., n$.*

Proof: From Proposition 2, we know that the assembler will always set α_is such that $\frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n}$ so the expected profit of supplier i is

$$\begin{aligned}
\pi_i(Q_{dec}) &= E\{-c_iQ_i + \alpha_i min(Q_{dec}, D)\} \\
&= -c_iQ_{dec} + \alpha_i \int_0^{Q_{dec}} xf(x)dx + \alpha_i Q_{dec}\bar{F}(Q_{dec}) \\
&= -c_iQ_{dec} + \frac{c_i}{\bar{F}(Q_{dec})} \int_0^{Q_{dec}} xf(x)dx + \frac{c_i}{\bar{F}(Q_{dec})} Q_{dec}\bar{F}(Q_{dec}) \\
&= -c_iQ_{dec} + \frac{c_i}{\bar{F}(Q_{dec})} \int_0^{Q_{dec}} xf(x)dx + c_iQ_{dec} \\
&= \frac{c_i}{\bar{F}(Q_{dec})} \int_0^{Q_{dec}} xf(x)dx \quad i = 1, ..., n
\end{aligned}$$

which completes the proof.□

3.2.2 Wholesale Price that Maximizes Wholesaler's Expected Profit (Case Ia)

In the previous section, we analyzed the system consisting of the suppliers and the assembler for a given wholesale price under the condition that they all have full information on demand. In this section, we investigate how the wholesaler sets its wholesale price if it has all the information about the demand, costs of all suppliers and the cost of the assembler and the sale price of the product. From the nature of the problem, it can be said that if the wholesaler has access to all the information (including the reaction behavior of other actors), setting the wholesale price accordingly, it can control all the supply chain. Assume that the wholesaler tries to maximize its profit function

$$Max\ \pi_\omega^{Ia}(w) = (w - c_\omega)Q_{dec} \tag{3.19}$$

where (Q_{dec}), found by setting Equation 3.16 equal to zero, is the order quantity of the assembler for a given w

From Equation 3.16, the wholesaler knows that the assembler determines the order quantity (Q_{dec}) according to the following equation:

$$w = -c_0 + \bar{F}(Q_{dec}) - \sum_{i=1}^{n} c_i \left[1 + \frac{f(Q_{dec})}{[\bar{F}(Q_{dec})]^2} \int_0^{Q_{dec}} \bar{F}(x)dx \right] \tag{3.20}$$

This equation is actually representing nothing but the price-demand relation between the wholesaler and the rest of the chain. Analyzing Equation 3.20 one can observe that (given the demand distribution has IFR property) the order quantity of the assembler Q_{dec} is decreasing in wholesale price w, satisfying a condition of a typical price-demand curve.

Substituting Equation 3.20 into Equation 3.19 the problem of the wholesaler becomes

$$Max\ \pi_w^{Ia}(Q) = -c_0Q - c_wQ + \bar{F}(Q)Q - \sum_{i=1}^{n} c_iQ \left[1 + \frac{f(Q)}{[\bar{F}(Q)]^2} \int_0^{Q} \bar{F}(x)dx \right] \tag{3.21}$$

From now on, we try to derive a concavity condition of the problem and obtain the optimal quantity of w.

Now, the first order condition obtained from Equation 3.21 is

$$\begin{aligned} \frac{d\pi_w^{Ia}(Q)}{dQ} &= -(c_0 + c_w) + \bar{F}(Q) - f(Q)Q - \sum_{i=1}^{n} c_i \left[1 + \frac{f(Q)}{[\bar{F}(Q)]^2} \int_0^{Q} \bar{F}(x)dx \right] \\ &- Q\sum_{i=1}^{n} c_i \left[\frac{(f'(Q)\int_0^Q \bar{F}(x)dx + f(Q)\bar{F}(Q))}{\bar{F}(Q)^2} \right] \\ &- Q\sum_{i=1}^{n} c_i \left[\frac{2\bar{F}(Q)f(Q)^2 \int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^4} \right] = 0 \end{aligned} \tag{3.22}$$

And the second order condition is

$$\begin{aligned} \frac{d^2\pi_w^{Ia}(Q)}{dQ^2} &= -f(Q) - f'(Q)Q \\ &- 2\sum_{i=1}^{n} c_i \left(\frac{f(Q)\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^2} \right) \left[\frac{f'(Q)}{f(Q)} + \frac{\bar{F}(Q)}{\int_0^Q \bar{F}(x)dx} + 2\frac{f(Q)}{\bar{F}(Q)} \right] \end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=1}^{n} c_i Q \left(\frac{f(Q) \int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^2} \right) \left[\frac{f''(Q)}{f(Q)} + 2\frac{f'(Q)\bar{F}(Q)}{f(Q)\int_0^Q \bar{F}(x)dx} \right. \\
&+ \left. 3\frac{f(Q)}{\int_0^Q \bar{F}(x)dx} + 6\frac{f(Q)^2}{\bar{F}(Q)^2} + 6\frac{f'(Q)}{\bar{F}(Q)} \right]
\end{aligned} \tag{3.23}
$$

Proposition 7 *Equation 3.21 is not a concave function in general.*

Proof: If the demand is normally distributed with $N(30000, 3800)$, Equation 3.21 is not concave when $c_0 = 0.06$ and $\sum_{i=1}^{n} c_i = 0.0008$. □

However, the following corollaries give sufficient conditions for quasi-concavity of Equation 3.21, under some restrictions.

Corollary 2 *If demand is normally distributed with $N(\mu, \sigma)$ and for $\mu > 1.25\sigma$ the optimal point of $\pi_{\omega}^{I_a}(Q)$ is always less than or equal to μ.*

Proof: Consider Equation 3.22.

$$
\begin{aligned}
\frac{d\pi_{\omega}^{I_a}(Q)}{dQ} &= -(c_0 + c_\omega) + \bar{F}(Q) - f(Q)Q - \sum_{i=1}^{n} c_i \left[1 + \frac{f(Q)}{[\bar{F}(Q)]^2} \int_0^Q \bar{F}(x)dx \right] \\
&- Q\sum_{i=1}^{n} c_i \left[\frac{(f'(Q)\int_0^Q \bar{F}(x)dx + f(Q)\bar{F}(Q))}{\bar{F}(Q)^2} + \frac{2\bar{F}(Q)f(Q)^2\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^4} \right]
\end{aligned}
$$

Here, two terms that might be positive are $\bar{F}(Q)$ and $-Q\sum_{i=1}^{n} c_i \left[\frac{(f'(Q)\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^2} \right]$.

From IFR property it is known that $-Q\sum_{i=1}^{n} c_i \left[\frac{f'(Q)\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^2} \right]$

$- Q\sum_{i=1}^{n} c_i \left[\frac{\bar{F}(Q)f(Q)^2\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^4} \right]$ is always negative. Hence the only term that can make Equation 3.22 positive is $\bar{F}(Q)$.

Let

$$
H(Q) = \bar{F}(Q) - f(Q)Q
$$

and we know that

$$G(Q) = \frac{d\pi_{\omega}^{Ia}(Q)}{dQ} - H(Q) < 0 \; for \; every \; Q$$

Working on $H(Q)$, we obtain the following.

$$\begin{aligned}
H(Q) &= \bar{F}(Q) - f(Q)Q \\
&= \bar{F}(Q) - \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{1}{2}\left(\frac{Q-\mu}{\sigma}\right)^2} Q \\
H(\mu) &= \bar{F}(\mu) - \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{1}{2}\left(\frac{\mu-\mu}{\sigma}\right)^2} \mu \\
&= 0.5 - \frac{\mu}{2.5\sigma} \leq 0 \; for \; \mu \geq 1.25\sigma \\
\frac{dH(Q)}{dQ} &= \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{1}{2}\left(\frac{Q-\mu}{\sigma}\right)^2} - \frac{Q-\mu}{\sigma^2} \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{1}{2}\left(\frac{Q-\mu}{\sigma}\right)^2} - \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{1}{2}\left(\frac{Q-\mu}{\sigma}\right)^2} \\
&= -\frac{Q-\mu}{\sigma^2} f(Q) \leq 0 \; for \; Q \geq \mu
\end{aligned}$$

So for $Q \geq \mu$ where $\mu \geq 1.25\sigma$, $G(Q) + H(Q) = \frac{d\pi_{\omega}^{Ia}(Q)}{dQ}$ is always negative and the optimal point of $\pi_w(Q)$ can not be greater than μ.□

Note that $\mu \geq 1.25\sigma$ is reasonable for the case of normal distribution, as we do not expect to have negative demand (i.e. we expect to have $\mu \geq 3\sigma$).

Corollary 3 *If demand is normally distributed $N(\mu, \sigma)$, $\pi_w(Q)$ is concave in the interval* $0 \leq Q \leq \mu - \sigma$.

Proof: Consider Equation 3.23, since $f'(Q)$, $f''(Q)$, $f(Q)$, $\bar{F}(Q)$, $\int_0^Q \bar{F}(x)dx$ and Q are all positive in the interval $0 \leq Q \leq \mu - \sigma$, then $\frac{d^2\pi_0(Q)}{dQ^2} < 0$ and $\pi_w(Q)$ is concave for $0 \leq Q \leq \mu - \sigma$.□

Corollary 4 *If demand is normally distributed $N(\mu, \sigma)$, $\pi_w(Q)$ is concave in the interval $\mu - \sigma \leq Q \leq \mu$ for $\frac{\mu}{\sigma} > 0.38$.*

Proof: Consider Equation 3.23, since $f'(Q)$, $f(Q)$, $\bar{F}(Q)$, $\int_0^Q \bar{F}(x)dx$ and Q are all positive in the interval $\mu - \sigma \leq Q \leq \mu$, the only term that can be negative is $f''(Q)$. Therefore, it is sufficient to show the following inequality to prove the concavity of $\pi_w(Q)$ in the interval $\mu - \sigma \leq Q \leq \mu$.

$$\frac{f''(Q)}{f(Q)} + \frac{f(Q)}{\int_0^Q \bar{F}(x)dx} + 6\frac{f(Q)^2}{\bar{F}(Q)^2} > 0 \; for \; \frac{\mu}{\sigma} > 0.38 \tag{3.24}$$

Consider the worst case, the minimum value that $\frac{f''(Q)}{f(Q)}$ (the only negative term in the inequality) can take is $-\frac{1}{\sigma^2}$ for $\mu - \sigma \leq Q \leq \mu$ and for the other positive terms the minimum value that they can take are (Note that all constants are rounded up.)

$$\begin{aligned}
min\left(\frac{f(Q)}{\int_0^Q \bar{F}(x)dx}\right) &= \frac{f(\mu-\sigma)}{\int_0^{\mu-\sigma} \bar{F}(x)dx} \\
&= \frac{\exp^{-1/2}\frac{1}{(\sigma\sqrt{2\pi})}}{\mu F(\mu-\sigma) - \frac{\sigma\exp^{-1/2}}{\sqrt{2\pi}} + (\mu-\sigma)\bar{F}(\mu-\sigma)} \\
&= \frac{0.24\frac{1}{\sigma}}{\mu - 0.40\sigma} \\
min\left(6\frac{f(Q)^2}{\bar{F}(Q)^2}\right) &= 6\frac{f(\mu-\sigma)^2}{\bar{F}(\mu-\sigma)^2} \\
&= 6\frac{\left(\exp^{-1/2}\frac{1}{\sigma\sqrt{2\pi}}\right)}{(1-0.84)^2} \\
&= \frac{13.73}{\sigma^2}
\end{aligned}$$

So for the worst case

$$\begin{aligned}
-\frac{1}{\sigma^2} + \frac{0.24\frac{1}{\sigma}}{\mu - 0.40\sigma} + \frac{13.73}{\sigma^2} &> 0 \\
\frac{1}{\sigma^2} &> \frac{0.24\frac{1}{\sigma}}{\mu - 0.40\sigma} + \frac{13.73}{\sigma^2} \\
-\frac{12.73}{\sigma^2} &< \frac{0.24\frac{1}{\sigma}}{\mu - 0.40\sigma} \\
-\frac{12.73}{\sigma} &< \frac{0.24}{\mu - 0.40\sigma} \\
-(\mu - 0.40\sigma) &> 0.02\sigma \\
\mu &> 0.38\sigma \\
\frac{\mu}{\sigma} &> 0.38 \quad \square
\end{aligned}$$

Corollary 5 *If demand is normally distributed $N(\mu,\sigma)$, $\pi_w^{Ia}(Q)$ is quasi-concave for $\mu > 1.25\sigma$ and the optimal point can be found by solving $\frac{d\pi_w^{Ia}(Q)}{dQ} = 0$.*

Proof: From Corollary 3 and 4, we show that $\pi_w^{Ia}(Q)$ is concave for $(0 \leq Q \leq \mu)$ and in Corollary 2 it is shown that $\pi_w^{Ia}(Q)$ is always negative for $Q \geq \mu$ where $\mu \geq 1.25\sigma$. So $\pi_w^{Ia}(Q)$ is quasi-concave for $\mu > 1.25\sigma$. □

3.3 Sensitivity Analysis and Computational Results

In this section, we perform sensitivity analyses to analyze the effect of system parameters on the performance of the profits of the whole system and the actors, hence our contracting schemes for Case I and Case Ia. We introduce the analyses on Case I and Case Ia separately.

3.3.1 Parameter Sets for Case I

In this analysis, we assume that the demand is normally distributed with $N(\mu,\sigma)$. We perform our analysis on $\sum_{i=1}^{n} c_i$, c_0 and σ for three different $w = (0.3, 0.2, 0.1)$. Since in the assembler's profit function (Equation 3.15) the suppliers appears as one supplier with production cost $(\sum_{i=1}^{n} c_i)$, we analyze the effect of $\sum_{i=1}^{n} c_i$ in the computations. We define three basic systems, which differ in w, with parameters presented in Table 3.2.

Table 3.2: Parameters for Base Systems (Case I)

μ	σ	w	c_0	$\sum_{i=1}^{n} c_i$
1500	400	0.3	0.2	0.2
1500	400	0.2	0.2	0.2
1500	400	0.1	0.2	0.2

To analyze the effect of each parameter, we change the concerned parameter within the range presented in Table 3.3, maintaining others as in each base system.

Table 3.3: Range of Parameters (Case I)

Parameter	Start	End	Step size
σ	500	200	10
c_0	0.3	0.05	0.01
$\sum_{i=1}^{n} c_i$	0.3	0.05	0.01

We do not investigate the effect of μ, since for every demand distribution $N(\mu, \sigma)$, where $\frac{\mu}{\sigma} = A$, the system operates for quantities that correspond to the same fractile on the demand distribution. All computations are performed in Excell.

3.3.2 Results (Case I)

In this analysis, we are especially interested in, how the system and actors are affected with an improvement in the system parameters. Note that, by an improvement in a parameter, we mean a decrease in the value of σ (or c_0 or $\sum_{i=1}^{n} c_i$), which is expected to improve the performance of all actors. Table 3.4 summarizes the results of computations. Here, π_{dec}^{I} is the total profit of the decentralized system, which is the sum of assembler's and suppliers' profits and is also equal to $\pi_c^I(Q_{dec}^I)$, and π_s^I is the sum of suppliers' profits.

Table 3.4: Results (Case I)

	π_c^I	Q_c^I	π_{dec}^I	Q_{dec}^I	π_0^I	π_s^I
$\sigma \Downarrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Downarrow$
$c_0 \Downarrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$
$\sum_{i=1}^{n} c_i \Downarrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$, $\Downarrow$

The detailed computational results can be found in Appendix A.

Before commenting on the computational results, we should consider the effect of power structure between the assembler and the suppliers, reflected with the revenue sharing contracts, on the expected profits of the actors. Since in this setting the assembler is the powerful actor that manages the system and the suppliers are followers, we do not expect any decrease in the expected profit of the assembler when there is an improvement in the system parameters. The computational results also prove our expectation. But, the situation for the suppliers is very interesting.

The results shown in Table 3.4 are mostly quite straightforward. When there is an improvement in the system parameters, the centralized system, the decentralized system and the assembler all increase their expected profits. The only exception appears for the suppliers. Lets have a detailed look to this situation .

- Decreasing σ: Smaller standard deviation of demand can be interpreted as an optimistic expectation of demand or a mature market where there is less change in customers choices (demand becomes more deterministic). From Equation 3.10, we know that the assembler should offer the suppliers higher α_i for the same quantity of capacity allocated by the suppliers, when the standard deviation of the demand increases. This reveals that the suppliers operate only for higher expected profits when the demand uncertainty increases (increasing risk of not-selling) with higher standard deviation. Since risk of not-selling decreases with decreasing standard deviation of demand, the expected profit of the suppliers are also decreasing.

- Decreasing c_0: Since the risk of not-selling is increasing with larger operating quantities (Q^I_{dec}), the assembler should offer larger α_i to increase the suppliers' expected profit. Since Q^I_{dec} is increasing with decreasing c_0, the suppliers' expected profit is increasing in this case.

- Decreasing $\sum_{i=1}^{n} c_i$: Table (3.4) shows that the expected profit of the suppliers is first increasing and then decreasing, when $\sum_{i=1}^{n} c_i$ is decreasing. This phenomenon can be explained as follows. We observed that Q^I_{dec} is increasing with decreasing $\sum_{i=1}^{n} c_i$ and we know that the risk of not-selling is increasing with Q^I_{dec} and decreasing with decreasing $\sum_{i=1}^n c_i$. Since in our system, Q^I_{dec} is also dependent on $\sum_{i=1}^n c_i$, the marginal effect of $\sum_{i=1}^n c_i$ determines the increase or decrease of risk. The computations reveal that the marginal of $\sum_{i=1}^n c_i$ is decreasing with decreasing $\sum_{i=1}^n c_i$ and the marginal becomes negative starting form a positive one, hence the expected profit increases first and then starts decreasing. This interesting observation can reveal that the suppliers with very low value added should look for new markets to increase their expected profits. With the improvement on production processes that causes very low production costs, the firms should be able to sell in additional markets in order to keep up with their expected profit levels.

Since w appears as a production cost in the assemblers profit function, its effect is similar to c_0.

Through our computations, we do not observe a gap that exceeds 10% between the centralized solution and the decentralized solution. The revenue share contract performs quite well for the markets considering the total system performance.

3.3.3 Parameter Sets for Case Ia

For Case Ia, we perform similar analysis as in Case I. We assume that the demand is normally distributed with $N(\mu, \sigma)$. We perform our analysis on $\sum_{i=1}^n c_i$, c_0, c_ω and σ. Since in the assembler's profit function (Equation 3.15), the suppliers appears as one supplier with production cost $(\sum_{i=1}^n c_i)$, we analyze the effect of $\sum_{i=1}^n c_i$ in the computations. We define a basic system with the parameters presented in Table 3.5.

To analyze the effect of each parameter, we change the concerned parameter within the range given in Table 3.6, maintaining others as in the base system. We decrease the concerned parameter, each decrease reflecting an anticipated improvement in the performance.

We do not investigate the effect of μ, since for every demand distribution

Table 3.5: Parameters for the Base System (Case Ia)

μ	σ	c_ω	c_0	$\sum_{i=1}^{n} c_i$
1500	500	0.2	0.2	0.2

Table 3.6: Range of Parameters (Case Ia)

Parameter	Start	End	Step of Decrease
σ	500	200	10
c_ω	0.3	0.01	0.01
c_0	0.3	0.01	0.01
$\sum_{i=1}^{n} c_i$	0.3	0.01	0.01

$N(\mu, \sigma)$, where $\frac{\mu}{\sigma} = A$, the system operates for quantities that correspond to the same fractile on the demand distribution. All computations are performed in Excell.

3.3.4 Results (Case Ia)

In this case, we observe that the wholesaler dominates the system, when it has full information of the system. Figure 3.3 shows the expected profit of the wholesaler, the assembler and suppliers versus Q for the base case in Table (3.5). Notice that, for every Q, w and α_i have different values.

The detailed computational results can be found in Appendix B.

As seen in Figure 3.3, the wholesaler, which is the most powerful actor of the supply chain, dominates the system, taking a larger portion of the profit. As a second powerful actor, the assembler takes the second highest expected profit.

In the sensitivity analysis, we are also interested in, how the system and actors are affected with an improvement in the system parameters. Table 3.7 summarizes the results of computations. Here, π_{dec}^{Ia} is the total profit of the

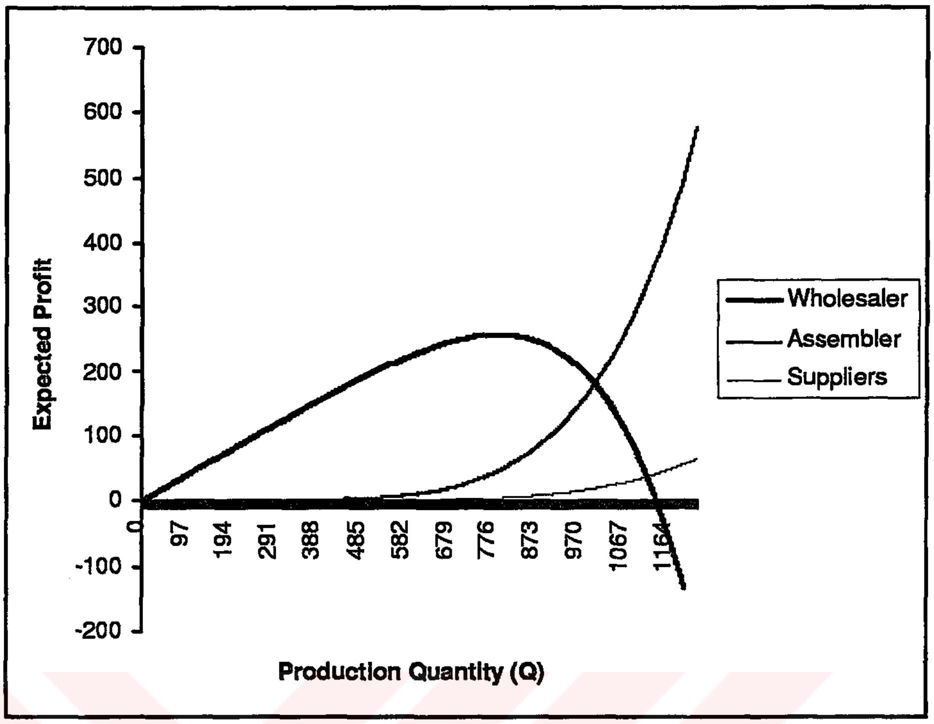

Figure 3.3: Expected Profits of The Actors versus Q (Case Ia)

decentralized system, which is the sum of assembler's, wholesalers and suppliers' profits and is also equal to $\pi_c^{Ia}(Q_{dec}^{Ia})$, and π_s^{Ia} is the sum of suppliers' profits.

As shown in Table 3.7, the improvement in different system parameters have different effect on actors of the supply chain. As we expected, the powerful actor (the wholesaler) always benefits with any improvement. For the other actors, we interpret the results as follows:

- Decreasing σ: As in Case I, with a decrease in the risk of not-selling , when the demand becomes less variable, the less powerful actors' expected profits are diminishing.
- Decreasing cost parameters c_ω and c_0: All actors benefit from any improvement in the wholesaler and assembler level.
- Decreasing $\sum_{i=1}^{n} c_i$: For the suppliers, we observe similar effect of improvement as in Case I. Their expected profit is first increasing and then starts

Table 3.7: Results (Case Ia)

	π_c^{Ia}	Q_c^{Ia}	π_{dec}^{Ia}	Q_{dec}^{Ia}	π_ω^{Ia}	π_0^{Ia}	π_s^{Ia}
$\sigma \Downarrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Downarrow$	$\Downarrow$
$c_\omega \Downarrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$
$c_0 \Downarrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$
$\sum_{i=1}^n c_i \Downarrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$	$\Uparrow$, $\Downarrow$

decreasing. And other powerful actors (the wholesaler and the assembler) benefit from this improvement. This observation strengthen our interpretation about the firms with low value added.

In our set of computation, the gap between the centralized solution and the decentralized solution fluctuates between 37% and 23%. The system performance deteriorates when the wholesaler has more information about the reaction of the assembler.

3.4 Revenue Sharing Contracts with Surplus Subsidy

In the literature, it is shown that incentive schemes, like buy-back contracts that decrease the risk of over stocking, could increase system efficiency while improving system's total profit. In our previous system, there is no incentive scheme that reduces the production cost of unsold products for the suppliers. This tendency reduces the optimal operating quantity of the system and creates inefficiency. In this section, using the same demand and information structure, a new type of contract is proposed between n suppliers and the assembler, that reduces the risk of over-producing for the suppliers. We assume that the wholesale price of the wholesaler is announced and known by the assembler. According to this new contract scheme, the assembler gives a share α_i of revenue for every final product sold as in the previous analysis and additionally pays suppliers s_i per unit for their delivered components that are not sold. Given the assembler's choices, the suppliers determine the allocated quantity (hence the delivery quantity) of components. Gerchak and Wang (2000) also analyzed a similar system where there is no wholesaler.

To avoid trivial cases, it is assumed that $\alpha_i > c_i > s_i$, for $i = 1, ..., n$ and $\sum_{i=1}^{n} \alpha_i \leq 1 - (c_0 + w)$. Clearly, the necessary conditions for each party to stay in the business are

$$\begin{aligned} \alpha_0 > c_0 + w & \quad \text{for the assembler,} \\ \alpha_i > c_i; i = 1, n & \quad \text{for small suppliers,} \end{aligned}$$

For given α_i and s_i, the assembler and suppliers determine the production and deliver quantities, respectively.

The assembler's objective function and the first order optimality conditions for the quantity to produce is given by Equation 3.25 and 3.26.

$$\begin{aligned} Max\ \pi_0(Q) &= E\left\{-(c_0+w)Q + (1-\sum_{i=1}^{n}\alpha_i)min(Q,D) - \sum_{i=1}^{n} s_i[Q-D]^+\right\} \\ &= -(c_0+w)Q + (1-\sum_{i=1}^{n}\alpha_i)\int_0^Q \bar{F}(x)dx - \sum_{i=1}^{n} s_i \int_0^Q F(x)dx \quad (3.25) \\ \bar{F}(Q_0) &= \frac{\sum_{i=1}^{n} s_i + c_0 + w}{1 - \sum_{i=1}^{n}\alpha_i + \sum_{i=1}^{n} s_i} \quad (3.26) \end{aligned}$$

Similarly, the suppliers' objective function and the first order optimality conditions for the quantity to allocate are given by Equation 3.27 and Equation 3.28.

$$\begin{aligned} Max\ \pi_i(Q) &= E\left\{-c_iQ + \alpha_i min(Q,D) + s_i[Q-D]^+\right\} \\ &= -(c_i)Q + \alpha_i \int_0^Q \bar{F}(x)dx + s_i \int_0^Q F(x)dx,\ i = 1, ..., n \quad (3.27) \\ \bar{F}(Q_i) &= \frac{c_i - s_i}{\alpha_i - s_i}\ i = 1, ..., n. \quad (3.28) \end{aligned}$$

As in the previous system, the assembler sets the contract parameters α_i's and s_i's as in the following proposition.

Proposition 8 *The assembler will always set $\alpha_1, ..., \alpha_n$ and $s_1, ..., s_n$ such that*

$$\frac{c_1 - s_1}{\alpha_1 - s_1} = ... = \frac{c_n - s_n}{\alpha_n - s_n} \geq \frac{\sum_{i=1}^n s_i + c_0 + w}{1 - \sum_{i=1}^n \alpha_i + \sum_{i=1}^n s_i} \tag{3.29}$$

Proof: Assume that the assembler sets $\alpha_1, ..., \alpha_n$ and $s_1, ..., s_n$ such that $\frac{c_1-s_1}{\alpha_1-s_1} = ... > \frac{c_n-s_n}{\alpha_n-s_n}$. Here, some suppliers will deliver more components that others. So by reducing either α_n or s_n, the assembler could gain more revenue while the number of completely assembled product does not change. For the inequality part assume that $\frac{c_1-s_1}{\alpha_1-s_1} = ... = \frac{c_n-s_n}{\alpha_n-s_n} < \frac{\sum_{i=1}^n s_i+c_0+w}{1-\sum_{i=1}^n \alpha_i+\sum_{i=1}^n s_i}$. Here the assembler willing to assembler less product than the suppliers deliver, so reducing α_i or s_i the assembler can increase its profit.

Therefore, the assembler makes sure that all suppliers will deliver the same quantity Q_1.□

In Proposition 8, it is shown that at the feasible points the suppliers' allocated quantity is Q_1 and the assemblers order quantity is $Q_0 \geq Q_1$. Since the assembler can not produce more than Q_1 units, the operating quantity of the system is $Q_{dec} = Q_1$.

Gerchak and Wang (2000) pointed out that the system can be coordinated if the assembler sets the revenue shares and surplus subsidies as in the following proposition.

Proposition 9 *To coordinate the system, the assembly firm only needs to set (α_i, s_i) for each i, $i = 1, ..., n$, such that*

$$\begin{aligned} \frac{c_i - s_i}{\alpha_i - s_i} &= \sum_{i=0}^{n} c_i + w, \\ or\ s_i &= \frac{c_i - \alpha_i(\sum_{i=0}^n c_i + w)}{1 - (\sum_{i=0}^n c_i + w)} \end{aligned}$$

so

$$\sum_{i=1}^{n} s_i = \frac{\sum_{i=1}^n c_i - \sum_{i=1}^n \alpha_i(\sum_{i=0}^n c_i + w)}{1 - (\sum_{i=0}^n c_i + w)}$$

Proof: See Proposition 6 in Gerchak and Wang (2000). □

Gerchak and Wang (2000) do not consider assemblers objective of profit maximization. In the following section, we analyzed how the assembler's profit maximization objective influences the system.

3.4.1 Assembler's Profit Maximization Problem under Revenue Sharing Contracts with Surplus Subsidy

The assembler's profit function if written more explicitly is,

$$\begin{aligned} Max\ \pi_0(\sum_{i=1}^{n}\alpha_i, \sum_{i=1}^{n}s_i) &= E\left\{-(c_0+w)Q_{dec} + (1-\sum_{i=1}^{n}\alpha_i)min(Q_{dec}, D)\right. \\ &- \left.\sum_{i=1}^{n}s_i[Q_{dec}-D]^+\right\} \\ &= -(c_0+w)Q_{dec} + (1-\sum_{i=1}^{n}\alpha_i)\int_0^{Q_{dec}}\bar{F}(x)dx \\ &- \sum_{i=1}^{n}s_i\int_0^{Q_{dec}}F(x)dx \end{aligned}$$

The suppliers' behavior is known by the assembler from (3.29) and (3.28).

$$\bar{F}(Q_1) = \frac{c_1-s_1}{\alpha_1-s_1} = \ldots = \frac{c_n-s_n}{\alpha_n-s_n}$$

One can also write the condition as

$$\bar{F}(Q_1) = \frac{\sum_{i=1}^{n}c_i - \sum_{i=1}^{n}s_i}{\sum_{i=1}^{n}\alpha_i - \sum_{i=1}^{n}s_i}$$

or

$$\sum_{i=1}^{n}\alpha_i = \frac{\sum_{i=1}^{n}c_i - \sum_{i=1}^{n}s_i}{\bar{F}(Q_1)} + \sum_{i=1}^{n}s_i$$

Since $Q_{dec} = Q_1$, replacing $\sum_{i=1}^{n}\alpha_i$, and dropping the subscript on Q, the assembler problem becomes

$$\begin{aligned} Max\ \pi_0(Q, \sum_{i=1}^{n}s_i) &= -(c_0+w)Q + \left(1-\left[\frac{\sum_{i=1}^{n}c_i - \sum_{i=1}^{n}s_i}{\bar{F}(Q)}\right.\right. \\ &+ \left.\left.\sum_{i=1}^{n}s_i\right]\right)\int_0^{Q}\bar{F}(x)dx - \sum_{i=1}^{n}s_i\int_0^{Q}F(x)dx \\ &= -(c_0+w)Q + \left(1-\left[\frac{\sum_{i=1}^{n}c_i - \sum_{i=1}^{n}s_i}{\bar{F}(Q)}\right.\right. \\ &+ \left.\left.\sum_{i=1}^{n}s_i\right]\right)\int_0^{Q}xf(x)dx \\ &+ \left(1-\left[\frac{\sum_{i=1}^{n}c_i - \sum_{i=1}^{n}s_i}{\bar{F}(Q)} + \sum_{i=1}^{n}s_i\right]\right)\int_Q^{\infty}Qf(x)dx \end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=1}^{n} s_i \int_0^Q (Q-x) f(x) dx \\
&= -(c_0+w)Q + \left(1 - \left[\frac{\sum_{i=1}^n c_i - \sum_{i=1}^n s_i}{\bar{F}(Q)} + \sum_{i=1}^n s_i\right]\right) Q \\
&+ \left(1 - \left[\frac{\sum_{i=1}^n c_i - \sum_{i=1}^n s_i}{\bar{F}(Q)} + \sum_{i=1}^n s_i\right]\right) \int_0^Q x f(x) dx \\
&- \left(1 - \left[\frac{\sum_{i=1}^n c_i - \sum_{i=1}^n s_i}{\bar{F}(Q)} + \sum_{i=1}^n s_i\right]\right) \int_0^Q Q f(x) dx \\
&- \sum_{i=1}^{n} s_i \int_0^Q (Q-x) f(x) dx \\
&= -(c_0+w)Q + \left(1 - \left[\frac{\sum_{i=1}^n c_i - \sum_{i=1}^n s_i}{\bar{F}(Q)} + \sum_{i=1}^n s_i\right]\right) Q \\
&+ \left(1 - \left[\frac{\sum_{i=1}^n c_i - \sum_{i=1}^n s_i}{\bar{F}(Q)}\right]\right) \int_0^Q x f(x) dx \\
&- \left(1 - \left[\frac{\sum_{i=1}^n c_i - \sum_{i=1}^n s_i}{\bar{F}(Q)}\right]\right) \int_0^Q Q f(x) dx \\
&= -(c_0+w)Q - \sum_{i=1}^n s_i Q + \left(1 - \left[\frac{\sum_{i=1}^n c_i}{\bar{F}(Q)}\right.\right. \\
&- \left.\left.\frac{\sum_{i=1}^n s_i}{\bar{F}(Q)}\right]\right) \int_0^Q \bar{F}(x) dx \\
&= -(c_0+w)Q - \sum_{i=1}^n s_i Q + \left(\frac{\sum_{i=1}^n s_i}{\bar{F}(Q)}\right) \int_0^Q \bar{F}(x) dx \\
&+ \left(1 - \left[\frac{\sum_{i=1}^n c_i}{\bar{F}(Q)}\right]\right) \int_0^Q \bar{F}(x) dx \qquad (3.30)
\end{aligned}
$$

The first orders of $\pi_0(Q, \sum_{i=1}^n s_i)$ are obtained as follows

$$
\begin{aligned}
\frac{\partial \pi_0(Q, \sum_{i=1}^n s_i)}{\partial Q} &= -(c_0+w) - \sum_{i=1}^n s_i + \bar{F}(Q) - \frac{(\sum_{i=1}^n c_i - \sum_{i=1}^n s_i)\bar{F}^2(Q)}{\bar{F}^2(Q)} \\
&- (\sum_{i=1}^n c_i - \sum_{i=1}^n s_i) \frac{f(Q)}{\bar{F}^2(Q)} \int_0^Q \bar{F}(x) dx \\
&= -(c_0+w) - \sum_{i=1}^n c_i + \bar{F}(Q) \\
&- (\sum_{i=1}^n c_i - \sum_{i=1}^n s_i) \frac{f(Q)}{\bar{F}^2(Q)} \int_0^Q \bar{F}(x) dx \qquad (3.31)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \pi_0(Q, \sum_{i=1}^n s_i)}{\partial (\sum_{i=1}^n s_i)} &= -Q + \frac{\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)} \\
&= -Q + \frac{\int_0^Q xf(x)dx}{\bar{F}(Q)} + \frac{Q\bar{F}(Q)}{\bar{F}(Q)} \\
&= \frac{\int_0^Q xf(x)dx}{\bar{F}(Q)}
\end{aligned}
\tag{3.32}
$$

Although, we can not show the concavity of Equation 3.30, the assembler's problem can be solved in the following way. Since the function in Equation 3.32 is always positive for every Q, the assembler is willing to equalize $\sum_{i=1}^n s_i$ to its upper bound $\sum_{i=1}^n c_i$ to make maximum profit in its optimal solution. Replacing $\sum_{i=1}^n s_i$ with $\sum_{i=1}^n c_i$ in Equation 3.30, the assembler's profit function becomes

$$
Max\ \pi_0(Q) = -(c_0 + w)Q - \sum_{i=1}^n c_i Q + \int_0^Q \bar{F}(x)dx. \tag{3.33}
$$

This is nothing but the centralized problem of case I, it is concave and the solution of the problem is Q_c^I where the first order of (3.33) is equal to zero, i.e.,

$$
\bar{F}(Q_c^I) = (\sum_{i=1}^n c_i) + c_0 + w. \tag{3.34}
$$

Accordingly, the assembler's optimal solution is $\sum_{i=1}^n s_i = \sum_{i=1}^n c_i$, $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n c_i$ and operating with Q_c^I. However, this is not a fair game for the suppliers. At the assembler's optimal solution $\sum_{i=1}^n s_i = \sum_{i=1}^n c_i$, $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n c_i$, suppliers' profit function becomes $\pi_i(Q) = 0$ for every $Q \geq 0$, so the suppliers are indifferent between zero production and a positive one, and make zero profit in every case while the assembler gains full system profit.

Every $(\sum_{i=1}^n \alpha_i, \sum_{i=1}^n s_i)$ pair that satisfies Proposition 8, hence coordination, there is a different allocation of system profit between the assembler and suppliers. However, the assembler would like to take all system profit, setting $\sum_{i=1}^n s_i = \sum_{i=1}^n c_i$, $\sum_{i=1}^n \alpha_i = \sum_{i=1}^n c_i$. Hence, the game becomes unfair for the suppliers.

Similarly, for each $(\sum_{i=1}^n \alpha_i, \sum_{i=1}^n s_i)$ pair that satisfies coordination, there are infinitely many solutions of α_i and s_i, which allocates the total revenue of suppliers differently among the suppliers.

The revenue sharing contract with surplus subsidy has many solutions that satisfy coordination, and when we consider the profit maximization objective of the assembler, the game becomes unfair for the suppliers. In the following section, we propose an alternative contract, which gives a specific solution among the alternative solutions of revenue sharing contract with surplus subsidy. The efficiency of the alternative contract is tested performing a comparison with revenue sharing contract in Case I.

3.4.2 An Alternative Contracting Scheme

In this section, we consider an alternative contract scheme for the assembler-suppliers subsystem where the wholesale price is set and known. According to the proposed alternative contract scheme, the assembler sets revenue shares of the suppliers as in Section 3.2.1, and additionally propose to make an additional payment (surplus subsidy) s_i for every unsold unit to every supplier i to let them deliver the centralized quantity of Case I (Q_c^I).

The aim of the assembler is letting himself and the suppliers to benefit of working with centralized quantities of case I if it is possible. Briefly proposing to give an additional surplus subsidy taking the revenue shares as in Section 3.2.1, we are trying to get a better solution for each actor in the subsystem, that means that we are looking for a win-win situation where assembler and suppliers making more profit than they gain in Section 3.2.1 (revenue sharing contract).

Here, α_i's (obtained from the solution of Equation 3.15) are optimal revenue shares of the system where revenue share contract is in use, Q_c^I (obtained from the solution of (3.1) is the centralized quantity of case I that we are trying to reach with the alternative contract scheme.

The offered surplus subsidies that satisfy the suppliers to produce Q_c^I units are calculated from the following equation, which is the solution of Equation 3.28

$$s_i = \frac{c_i - \bar{F}(Q_c^I)\alpha_i}{1 - \bar{F}(Q_c^I)} \tag{3.35}$$

First, we analyze the suppliers' condition of benefiting from this offer. $\pi_i^I(Q_{dec})$

is the expected profit of the supplier i where revenue sharing contract is in use.

$$\pi_i^I(Q_{dec}) = -(c_i)Q_{dec} + \alpha_i \int_0^{Q_{dec}} \bar{F}(x)dx, \; i = 1, ..., n \tag{3.36}$$

and $\pi_i^I(Q_{dec})$ is the expected profit of supplier i when the alternative contract scheme is in use. The superscript A denotes the alternative contracting scheme.

$$\pi_i^A(Q_c^I) = -(c_i)Q_c^I + \alpha_i \int_0^{Q_c^I} \bar{F}(x)dx + s_i \int_0^{Q_c^I} F(x)dx, \; i = 1, ..., n \tag{3.37}$$

So the suppliers can advantage with this new offer only if the following inequality is satisfied:

$$\pi_i^A(Q_c^I) > \pi_i^I(Q_{dec}). \tag{3.38}$$

Proposition 10 *All suppliers increase their profit with the alternative contract scheme defined above.*

proof: Consider a typical supplier i. The required condition is

$$\pi_i^A(Q_c^I) > \pi_i^I(Q_{dec}). \tag{3.39}$$

$$\begin{aligned}
-(c_i)Q_c^I + \alpha_i \int_0^{Q_c^I} \bar{F}(x)dx + s_i \int_0^{Q_c^I} F(x)dx &> -(c_i)Q_{dec} + \alpha_i \int_0^{Q_{dec}} \bar{F}(x)dx \\
-(c_i)Q_c^I + \alpha_i \int_0^{Q_c^I} \bar{F}(x)dx + s_i \int_0^{Q_c^I} F(x)dx + (c_i)Q_{dec} - \alpha_i \int_0^{Q_{dec}} \bar{F}(x)dx &> 0 \\
-(c_i)Q_c^I + \alpha_i \int_0^{Q_c^I} \bar{F}(x)dx + s_iQ_c^* - s_i \int_0^{Q_c^I} \bar{F}(x)dx + (c_i)Q_{dec} - \alpha_i \int_0^{Q_{dec}} \bar{F}(x)dx &> 0 \\
(s_i - c_i)Q_c^I + (\alpha_i - s_i) \int_0^{Q_c^I} \bar{F}(x)dx + (c_i)Q_{dec} - \alpha_i \int_0^{Q_{dec}} \bar{F}(x)dx &> 0
\end{aligned} \tag{3.40}$$

since

$$\int_0^{Q_c^I} \bar{F}(x)dx = \int_0^{Q_{dec}} \bar{F}(x)dx + \int_{Q_{dec}}^{Q_c^I} \bar{F}(x)dx.$$

So

$$s_i Q_{dec} - (c_i - s_i)(Q_c^I - Q_{dec}) - s_i \int_0^{Q_{dec}} \bar{F}(x)dx + (\alpha_i - s_i) \int_{Q_{dec}}^{Q_c^I} \bar{F}(x)dx > 0. \quad (3.41)$$

From the solution of the suppliers problem (Equation 3.27), we know that

$$\begin{aligned} \bar{F}(Q_c^I) &= \frac{c_i - s_i}{\alpha_i - s_i}, \; i = 1, ..., n \\ \alpha_i &= \frac{c_i - s_i + s_i \bar{F}(Q_c^I)}{\bar{F}(Q_c^I)}, \; i = 1, ..., n \end{aligned}$$

Replacing α_i, the condition in Equation 3.41 becomes

$$s_i Q_{dec} - (c_i - s_i)(Q_c^I - Q_{dec}) - s_i \int_0^{Q_{dec}} \bar{F}(x)dx + \left(\frac{c_i - s_i}{\bar{F}(Q_c^I)}\right) \int_{Q_{dec}}^{Q_c^I} \bar{F}(x)dx > 0$$

$$s_i(Q_{dec} - \int_0^{Q_{dec}} \bar{F}(x)dx) + (c_i - s_i)\frac{\int_{Q_{dec}}^{Q_c^I} \bar{F}(x)dx - (Q_c^I - Q_{dec})\bar{F}(Q_c^I)}{\bar{F}(Q_c^I)} > 0$$

$$s_i \int_0^{Q_{dec}} (1 - \bar{F}(x)dx) + (c_i - s_i)\frac{\int_{Q_{dec}}^{Q_c^I} (\bar{F}(x)dx - \bar{F}(Q_c^I))}{\bar{F}(Q_c^I)} > 0$$

Since $1 - \bar{F}(x)dx$ and $\int_{Q_{dec}}^{Q_c^I} (\bar{F}(x)dx - \bar{F}(Q_c^I))$ are always positive and hence, the condition in Equation 3.39 is always satisfied. This completes the proof. □

On the other hand, the assembler can benefit from the alternative contract scheme if $[\pi_c^I(Q_c^I) - \pi_c^I(Q_{dec})] - [\sum_{i=1}^n \pi_i^A(Q_c^I) - \sum_{i=1}^n \pi_i^R(Q_{dec})]$ is positive. Here $\pi_c^I(Q_c^I)$ and $\pi_c^I(Q_{dec})$ are expected total system profits working with Q_c^I and Q_{dec}, respectively. So the expression above gives the increase in the expected total system profit minus the increase in expected suppliers' profit using the alternative contracting scheme which is nothing but the increase in assemblers expected profit using the alternative contracting scheme.

$$\begin{aligned} K &= \left[\pi_s(Q_c^I) - \pi_s(Q_{dec})\right] - \left[\sum_{i=1}^n \pi_i^A(Q_c^I) - \sum_{i=1}^n \pi_i^R(Q_{dec})\right] \\ K &= -(c_0 + w + \sum_{i=1}^n c_i)Q_c^I + \int_0^{Q_c^I} \bar{F}(x)dx \end{aligned}$$

$$
\begin{aligned}
- & \quad (c_0 + w + \sum_{i=1}^{n} c_i)Q_{dec} + \int_{0}^{Q_{dec}} \bar{F}(x)dx \\
- & \quad (\sum_{i=1}^{n} s_i Q_{dec} - (\sum_{i=1}^{n} c_i - \sum_{i=1}^{n} s_i)(Q_c^I - Q_{dec}) \\
- & \quad \sum_{i=1}^{n} s_i \int_{0}^{Q_{dec}} \bar{F}(x)dx + (\sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} s_i) \int_{Q_{dec}}^{Q_c^I} \bar{F}(x)dx
\end{aligned}
$$

Although we can not explicitly show that the alternative contracting scheme is also always beneficial for the assembler, we did not encounter a counter example in our computational analysis. Computational results can be found in Appendix C.

The percentage increases of the suppliers' and assembler's expected profit functions with applying alternative contracting can be observed in Appendix C. Although, the percentages are very low, especially for higher demands, actual profit increase of the actors may correspond to high absolute values.

CHAPTER 4

ANALYSIS OF ASSEMBLER NETWORK WITH A DISTRIBUTOR (CASE II AND III)

In this chapter, we introduce and analyze two cases (Case II and Case III). Basically, we extend the system that is examined in Chapter 3 by adding a distributor, and each case reflects different scenarios for the assembler, while selling its product. In Case II, the assembler reaches its customers via a distributor, using available supply structure as in Case I in Chapter 3. In this case, all the demand appears at the distributor level. The assembler supplies small complementary products from suppliers applying revenue-sharing contract and a specialized input from the wholesaler at a fixed wholesale price. In case III, the assembler sells its product directly to the customers and finds an opportunity to enter a new market via a distributor where the selling is performed in non-overlapping periods. So the assembler sends the unsold products in its market at the end of the selling period to the distributor to be sold in distributor's market. Note that, it is possible to argue that the distributor's market reflects a clearance sale market, where the unsold products in the main selling period is introduced with a lower price.

4.1 Case II: Distributor is the only market for the product

In this section, we consider an assembler network with distributor as depicted in Figure 4.1.

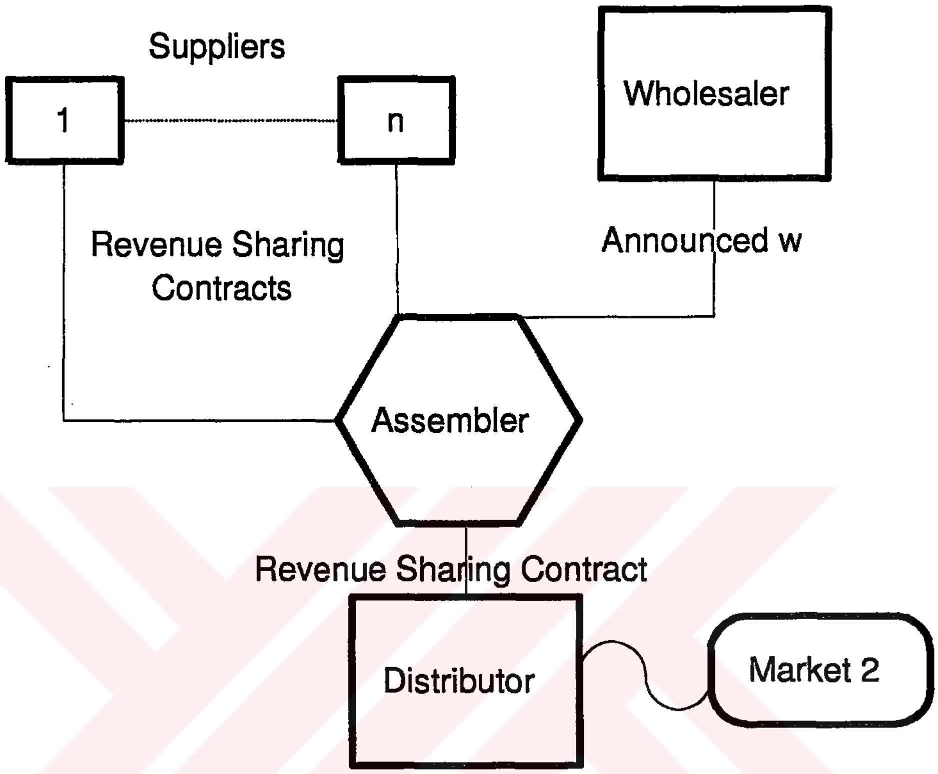

Figure 4.1: System Description Case II

As seen in the Figure 4.1, the system that we examined in Chapter 3 is extended by adding a distributor, where demand appears at distributor level and the selling price of the complete product is p_2. In this case, a generic selling price (p_2), which is around 1, is used to be able to make a detailed comparison with the previous cases in Chapter 3. Because of the additional operation cost (c_d) incurred to the system, the assembler could prefer to change the selling price, which is set to 1 in the model described in Chapter 3. There are n suppliers, each having revenue sharing contract with the assembler, a distributor that operates also with revenue sharing contract and an assembler which is supplied with some special complementary product by a wholesaler who sells its product at a previously determined price w. The assembly firm assembles the product and sells it via the distributor in the market. According to the contracting scheme between the assembler and the distributor (revenue sharing contract) the assembler determines a revenue share (α_d) to pay to the distributor for each product sold by the distributor. Then the distributor determines the order quantity of the assembled product from the assembler. The revenue sharing contract between the assembler and suppliers is same as the one discussed in Chapter 3. The assembler sets the revenue shares α_i for supplier i, and the supplier determines the production quantity to deliver to the assembler. In this setting, the suppliers and the distributor are followers according to the Stakelberg game played between actors.

The solution procedure follows the following steps:

- Step 1: Formulation and solution of the centralized system (a benchmark for the system performance.)

- Step 2: Formulation and solution of suppliers' problem for given revenue shares. Derivation of the relation between the revenue shares and quantity decision of suppliers (suppliers' reaction functions).

- Step 2a: Formulation and solution of the distributor's problem for given revenue share. Derivation of the relation between the revenue share and quantity decision of the distributor (distributor's reaction function).

- Step 3: Formulation and solution of the assembler's problem knowing the reaction of suppliers and the distributor for a given wholesale price. Determination of revenue shares.

Here p_2 is the selling price of the product, c_0 is the production cost of the assembler, c_i is the production cost of supplier i, and c_d is the transfer and other operations cost of the distributor paid for each product. Let $\pi_d^{II}(Q)$, $\pi_0(\alpha_1, ..., \alpha_n, \alpha_d)$ and $\pi_i^{II}(Q)$ $i = 1, ..., n$ be the expected profit functions of the distributer, the assembler and the supplier i respectively where Q_d is the order quantity of the distributor which will be sold in the market. $f_d(Q)$ is the proba-

Table 4.1: Notation for Chapter 4.1

D, f, F and $\bar{F}$:	the random demand of final product, PDF, CDF and complementary cumulative function respectively.
c:	unit costs (for all actors),
p_2:	the selling price of the final product,
π:	the expected profit function (for all actors),
Q:	quantity (for all actors),
Q_{dec}:	operating quantity of the decentralized system,
α:	the revenue share (for all other than the wholesaler)
w:	the selling price of the wholesaler(Wholesale price)
Superscripts:	Case number II enters as the superscript (for all of the above other than the unit costs, wholesale price and selling price of final product)
Subscripts:	Subscripts denote the actors and centralized problem: 0 for the assembler, $1, ..., n$ for the small suppliers, d for the distributor, ω for the wholesaler and c for the centralized problem.

bility density function of the demand. The notation used throughout the section is summarized in Table 4.1.

4.1.1 Centralized System

Consider the centralized system where w is previously determined. Here, the system operates centrally and the objective is to maximize total system profit.

$$\begin{aligned}\pi_c^{II}(Q) &= E\left[p_2 min(Q,D) - (\sum_{i=1}^{n} c_i + c_d + c_0 + w)Q\right] \\ &= p_2 \int_0^Q \bar{F}(x)dx - (\sum_{i=1}^{n} c_i + c_d + c_0 + w)Q \;\; i = 1, ..., n \qquad (4.1)\end{aligned}$$

Since $\pi_c^{II}(Q)$ is a concave function, the optimal solution can be easily found by checking the first order condition.

$$\frac{d\pi_c^{II}(Q)}{dQ} = p_2\bar{F}(Q) - (\sum_{i=1}^{n} c_i + c_d + c_0 + w) = 0$$

$$\bar{F}_d(Q_c^{II}) = \frac{\sum_{i=1}^{n} c_i + c_d + c_0 + w}{p_2} \quad (4.2)$$

where Q_c^{II} is the optimal operating quantity of the centralized system for given w.

4.1.2 Decentralized System

In the decentralized system, every actor tries to maximize its own profit. The assembler first decides on the revenue shares (α_i, α_d given to suppliers and the distributor knowing the reactions of these actors). The part of revenue kept by the assembler is $p_2 - \sum_{i=1}^{n} \alpha_i - \alpha_d$. Then the suppliers and the distributor determine the supply quantities and the order quantity, respectively. Here, the assembler coordinates the supply chain, where the supply quantity and the order quantity may be different according to the revenue shares chosen. By coordination we mean that the supply quantity and the order quantity is equalized, the assembler actually benefiting from the coordination. Proposition 11 discusses that issue.

Clearly, the necessary conditions for each party to stay in the business are

$$(p_2 - \sum_{i=1}^{n} \alpha_i - \alpha_d) > c_0 + w \quad \text{for the assembler,}$$

$$\alpha_i > c_i; \; i = 1, n \quad \text{for supplier i,}$$

$$\alpha_d > c_d \quad \text{for the distributor.}$$

The profit function of a typical supplier is

$$\pi_i(Q) = E\left[\alpha_i min(Q, D) - c_i Q\right]$$

$$= \alpha_i \int_0^Q \bar{F}(x)dx - c_i Q \; i = 1, ..., n \quad (4.3)$$

It is a typical newsvendor problem and concave, so the suppliers problem can be solved for a given α_i.

$$\bar{F}(Q_i^{II}) = \frac{c_i}{\alpha_i} \; i = 1, ..., n \quad (4.4)$$

Here the profit function of the distributor is

$$\begin{aligned}\pi_d(Q) &= E\{\alpha_d min(Q,D) - c_d Q\} \\ &= \alpha_d \int_0^Q \bar{F}(x)dx - c_d Q \end{aligned} \tag{4.5}$$

Similarly, the distributor determines its order quantity for a given α_d as follows

$$\bar{F}_d(Q_d^{II}) = \frac{c_d}{\alpha_d} \tag{4.6}$$

And finally the assembler's profit function is

$$\begin{aligned}\pi_0^{II}(\alpha_1,...,\alpha_n,\alpha_d) &= E\left[(p_2 - \sum_{i=1}^{n}\alpha_i - \alpha_d)min[Q_{dec}^{II}, D] - (c_0+w)Q_{dec}^{II}\right] \\ &= (p_2 - \sum_{i=1}^{n}\alpha_i - \alpha_d)\int_0^{Q_{dec}^{II}} \bar{F}_d(x)dx - (c_0+w)Q_{dec}^{II} \end{aligned} \tag{4.7}$$

where the operating quantity of the system (Q_{dec}^{II}) appears as $Q_{dec}^{II} = min(Q_1^{II},..,Q_n^{II},Q_d^{II})$, because the assembler could not send more to the distributor than the quantity that the distributor orders and the suppliers deliver.

As seen from the assembler's expected profit function our problem is similar to the one where there are $(n+1)$ suppliers in the system and the assembler sells its product directly. Therefore the characteristics of this problem are similar to the one provided in Section 3.1.

Proposition 11 *The Assembler will always set $\alpha_1,...,\alpha_n,\alpha_d$ such that*

$$\frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n} = \frac{c_d}{\alpha_d} \tag{4.8}$$

Proof: Similar to Proposition 2.□

In Proposition 11, it is shown that the assembler determines α_i's and α_d such that every supplier and the distributor select same quantity to deliver and order, respectively. So

$$\begin{aligned}\bar{F}_d(Q_{dec}^{II}) = \frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n} = \frac{c_d}{\alpha_d} \\ &= \frac{\sum_{i=1}^{n} c_i + c_d}{\sum_{i=1}^{n}\alpha_i + \alpha_d} \\ \sum_{i=1}^{n}\alpha_i + \alpha_d &= \frac{\sum_{i=1}^{n} c_i + c_d}{\bar{F}(Q_{dec}^{II})}\end{aligned}$$

Replacing $\sum_{i=1}^{n} \alpha_i + \alpha_d$ with $\frac{\sum_{i=1}^{n} c_i + c_d}{\bar{F}(Q_{dec}^{II})}$, the assembler's problem (Equation 4.7) becomes over Q (dropping super and subscript from Q_{dec}^{II}):

$$\pi_0^{II}(Q) = (p_2 - \frac{\sum_{i=1}^{n} c_i + c_d}{\bar{F}(Q)}) \int_0^Q \bar{F}(x)dx - (c_0 + w)(Q) \qquad (4.9)$$

Comparing Equation 4.9 with Equation 3.15, the assembler's problem is similar to the assembler's problem in case I. Only difference is the additional cost of the distributor (c_d).

Now, the first order condition of optimality for the problem is

$$\frac{d\pi_0(Q)}{dQ} = -(c_0+w)+p_2\bar{F}(Q)-(\sum_{i=1}^{n} c_i+c_d)\left[1+\frac{f(Q)}{[\bar{F}(Q)]^2}\int_0^Q \bar{F}(x)dx\right] = 0 \quad (4.10)$$

To analyze the concavity of the problem, we investigate the second order of assemblers profit function

$$\begin{aligned}\frac{d^2\pi_0(Q)}{dQ^2} &= -p_2 f(q) - (\sum_{i=1}^{n} c_i + c_d)\left[\frac{(f'(Q)\int_0^Q \bar{F}(x)dx + f(Q)\bar{F}(Q))\bar{F}(Q)^2}{\bar{F}(Q)^4}\right.\\ &+ \left.\frac{2\bar{F}(Q)f(Q)^2\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^4}\right]\\ &= -p_2 f(q)\\ &- (\sum_{i=1}^{n} c_i + c_d)\left(\frac{f(Q)\int_0^Q \bar{F}(x)dx}{\bar{F}(Q)^2}\right)\left[\frac{f'(Q)}{f(Q)} + \frac{\bar{F}(Q)}{\int_0^Q \bar{F}(x)dx} + 2\frac{f(Q)}{\bar{F}(Q)}\right]\end{aligned}$$

Because we know that $f(Q)$, $\int_0^Q \bar{F}(x)dx$ and $\bar{F}(Q)$ are all positive, the only condition that should be satisfied to prove the concavity of the function is the following

$$\left[\frac{f'(Q)}{f(Q)} + \frac{f(Q)}{\bar{F}(Q)}\right] > 0 \qquad (4.11)$$

This condition in Equation 4.11 is satisfied if our demand distribution has IFR property (increasing $\frac{f}{\bar{F}}$). Similar to Proposition 4, the following proposition considers the condition of concavity of assembler's profit function shown in Equation 4.9.

Proposition 12 *If*

$$\frac{f(Q)}{\bar{F}(Q)} \tag{4.12}$$

is increasing, then $\pi_0(Q)$ *is concave and has a unique maximum* (Q^{*II}_{dec} *within the range* $[0, Q^{II}_c]$, *which can be found by solving* $\frac{d\pi_0(Q)}{dQ} = 0$.

4.1.3 Findings and Remarks

From Equation 4.7, we know that the distributor appears as an additional supplier in Equation 4.7 and hence, all of the observations made for Case I are also valid for Case II. In this section, we analyze the choice of the assembler to reach the customers via a distributor. From Case I, it is known that the assembler's profit is diminishing with the increase of the suppliers' production costs. Since the distributor acts as an additional supplier for the assembler, the assembler prefers selling the market via a distributor, if it can expand the market (increase in μ) or obtain more reliable information about demand (decrease in σ). Consider that the assembler is selling directly to the market (Case I) and the system parameters are as given in Table 4.2. The demand is assumed Normally distributed with $N(\mu, \sigma)$.

Table 4.2: Scenario Parameters

w	c_0	$\sum_{i=1}^{n} c_i$
0.2	0.2	0.2

Then the assembler finds an opportunity to utilize a distributor with $c_d = 0.05$. We try to answer the following questions: What is the needed change (increase) in the expected demand so that the assembler prefers to use the distributor? And what is the needed change (decrease) in the variance of demand so that the assembler prefers to use the distributor? Therefore, we compare the assembler's profit in Case I and Case II, changing μ and σ, while the other parameters are held constant as in Table 4.2. The results can be seen in Figure 4.2 and 4.3.

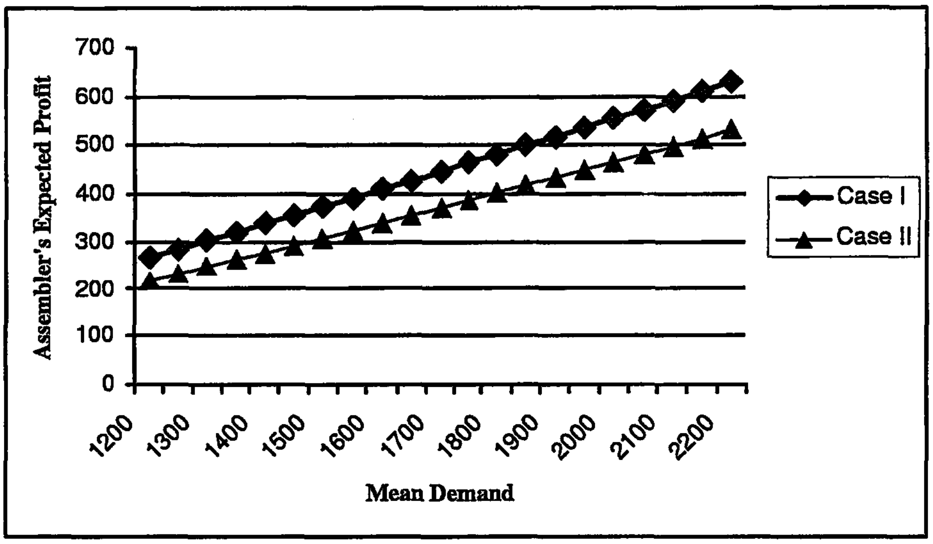

Figure 4.2: Assembler's Profit versus μ given that $\sigma = 400$(Case I and Case II)

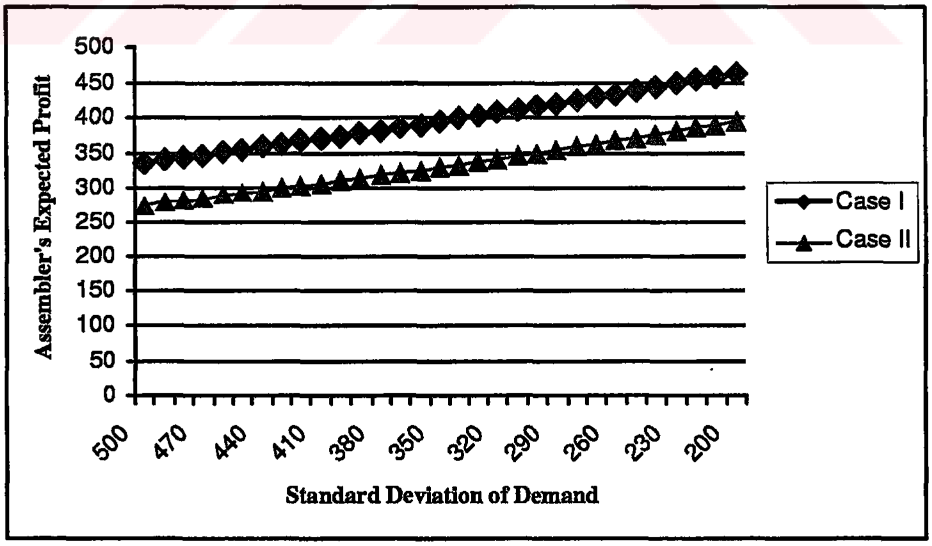

Figure 4.3: Assembler's Profit versus σ given that $\mu = 1500$(Case I and Case II)

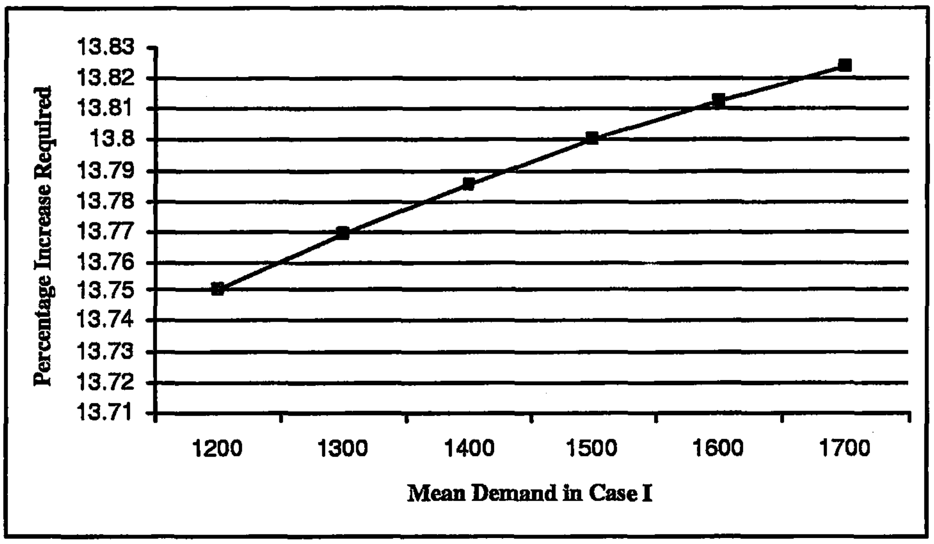

Figure 4.4: Percentage increase in μ required to select Case II instead of Case I $\sigma = 400$

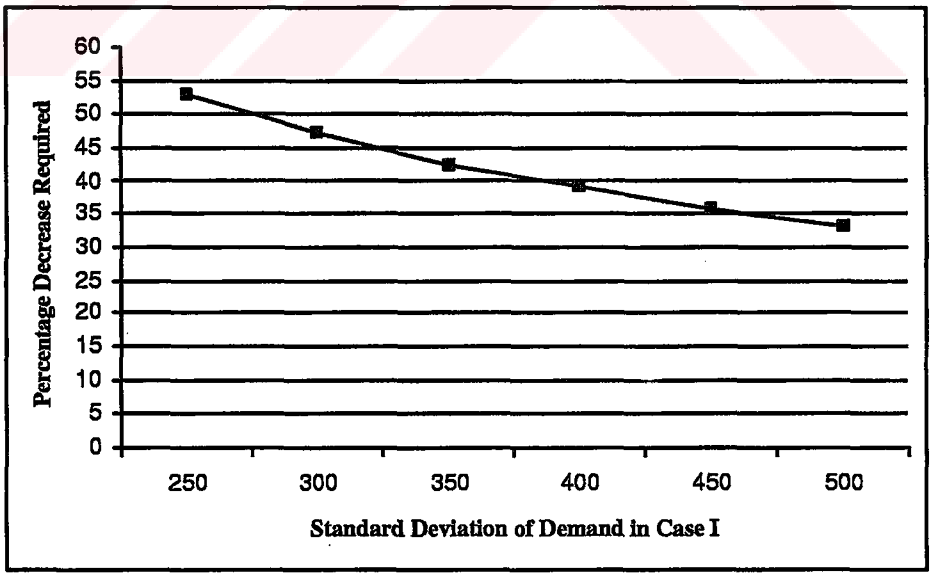

Figure 4.5: Percentage decrease in σ required to select Case II instead of Case I $\mu = 1500$

The horizontal grids in Figure 4.2 and 4.3 show μ and σ threshold values, where the expected profits of the assembler are equal in Case I and Case II. The assembler can only benefit from choice of utilizing the distributor (Case II), if the expected μ or σ becomes higher or lower than these values for Case II, respectively.

In Figure 4.4 and 4.5, percentage increase of μ and percentage decrease of σ required for the assembler to choice the distributor to serve the market (Case II)are shown, respectively. For the case $\mu = 1500$ $\sigma = 400$, we calculate the needed minimum percentage increase in mean of the demand as 13.8% and the needed minimum percentage decrease in standard deviation of the demand as 39%.

4.2 Case III: Distributor is the secondary market for the product

In this section, we consider another setting (depicted in Figure 4.6)where there are n suppliers, each having revenue sharing contract with the assembler, a distributor that operates with a revenue sharing contract as well, and an assembler, which is supplied with some special complementary product by a wholesaler selling its product at a previously determined price w.

As seen in Figure 4.6, different from the setting considered in Section 4.1, we consider the product to be marketed by both the assembler and the distributor in non overlapping selling periods. In the first period, suppliers deliver the total quantity of complementary products that will be sold in both markets to the assembler. Then the assembler performs assembly operations and sells the complete products in its market with a selling price p_1 ($p_1 = 1$). In the second period, the assembler makes an agreement with the distributor to sell unsold products into a secondary market possibly with a lower price p_2.

According to the revenue sharing contract between the assembler and suppliers, the assembler sets the revenue shares, α_i, of the suppliers, this is the suppliers share of revenue gained from sale of complete products in both markets and the

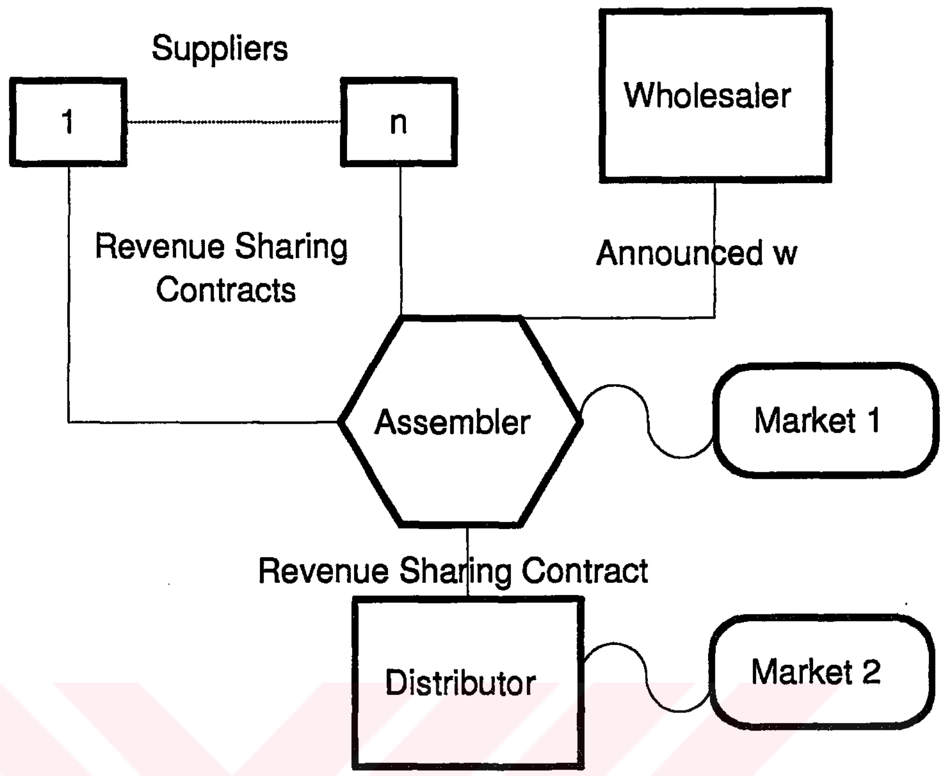

Figure 4.6: System Description Case III

suppliers determine the production quantity to deliver to the assembler. For the contracting scheme between the assembler and distributor (a revenue sharing contract), after the first period's demand is realized the assembler offers a revenue share α_d to pay to the distributor for each product sold by the distributor and delivery quantity, which is nothing but the quantity of unsold product in the first period.

The solution procedure follows the following steps:

- Step 1: Formulation and solution of the centralized system (a benchmark for the system performance).
- Step 2: Formulation and solution of suppliers' problem for given revenue shares. Derivation of the relation between the revenue shares and quantity decision of suppliers (suppliers' reaction functions).

- Step 2a: Formulation and solution of the distributor's problem for given revenue shares. Derivation of the relation between the revenue shares and quantity decision of the distributor (distributor's reaction function).

- Step 3: Formulation and solution of the assembler's problem knowing the reaction of suppliers and the distributor for a given wholesale price. Determination of revenue shares.

Let c_0 be the production cost of the assembler, c_i be the production cost of the supplier i and c_d is the transfer and other operations cost of the distributor paid for each product. p_1 and p_2 are the selling price of the product in the main and secondary markets, respectively. Let $\pi_d^{III}(z)$, $\pi_0^{III}(\alpha_1, ..., \alpha_n, \alpha_d)$ and $\pi_i^{III}(Q)$, $i = 1, ..., n$, be the expected profit functions of the distributor, the assembler and the supplier i, respectively, where z is the quantity sent to the distributor. D_1 and D_2 are the random variables denoting demands in the assembler's and distributor's markets, respectively, and $f_1(x)$ and $f_2(x)$ are the marginal probability density functions, respectively. We assume that D_1 and D_2 can be correlated, $f_{1,2}(x, y)$ being the joint probability density function and $f_{2|1}(y)$ being the conditional probability density function of the secondary market's demand.

In this section, we assume that the suppliers know the distribution of the total demand which appears in assemblers and distributors market. $D_t = D_1 + D_2$ and $f_t(x)$ is probability density function of the total demand. The following are the other quantities of interest:

$$f_{2|1}(y) = \frac{f_{1,2}(x, y)}{f_1(x)}$$

$$\bar{F}_{2|1}(y) = \frac{\bar{F}_{1,2}(x, y)}{f_1(x)}$$

The notation used throughout the section is summarized in Table 4.3.

Before starting our modelling approach, important issues that should be mentioned about Case III are:

- The existence of two, possibly overlapping, markets is a frequently encountered phenomena. However, the assumption we make with respect to the

Table 4.3: Notation for Chapter 4.2

D_j, f_j:	the random demand of final product and marginal probability distribution function of market j $j = 1, 2$,
$f_{1,2}$:	joint probability function,
$f_{2\|1}$:	conditional probability function of distributor's demand,
D_t, f_t:	random demand of total demand and probability density function of total demand seen by the suppliers,
c:	unit costs (for all actors),
p_1,p_2:	the selling prices of the final product in assembler's and distributor's market, respectively ,
π:	shows the expected profit function (for all actors),
Q:	quantity (for all actors),
Q_{dec}:	operating quantity of the decentralized system,
α:	the revenue share (for all other than the wholesaler)
w:	the selling price of the wholesaler(Wholesale price)
Superscripts:	Case number III enters as the superscript (for all of the above other than the unit costs, wholesale price and selling prices of final product)
Subscripts:	Subscripts denote the actors and centralized problem: 0 for the assembler, $1, ..., n$ for the suppliers, d for the distributor, ω for the wholesaler and c for the centralized problem.

availability of information to all parties should be challenged, if the model is planned for a tactical level use. One should utilize the results with this caution and only for strategic level purposes.

- The existence of two non-overlapping periods is a reality in secondary markets. However, under more general settings where both markets operate in the same period, our approach becomes an approximation (or a bound). Note that by creating a second period, we eliminate all inefficiencies that may be created by the allocation of the manufactured quantity into two markets for the sales in the same period.

- We do not restrict the revenue share paid to the distributor even if it exceeds p_2, when the remaining quantity for the sales in the secondary market is very large. We implicitly assume that the probability of occurrence of such a case is negligible. More is discussed on this issue at the end of Section 4.2.2.

4.2.1 Centralized System

Let's start our analysis with the centralized system where w is previously determined. Here, the system operates centrally and the objective is to maximize total system profit.

$$\begin{aligned}
\pi_c^{III}(Q) &= E\left[p_1 min(Q, D_1)\right] + E\left[G_c(z)\right] - (\sum_{i=1}^{n} c_i + c_0 + w)Q \\
&= p_1 \int_0^Q \bar{F}_1(x)dx + \int_0^Q G_c(Q-y)f_0(x)dx \\
&- (\sum_{i=1}^{n} c_i + c_d + c_0 + w)Q
\end{aligned} \tag{4.13}$$

where $G_c(z)$ is the expected profit from the sale in the second period. z is the quantity sent to the distributor, i.e., $z = max(0, Q - D_1)$.

$$G_c(z) = E\left[p_2 min(z, D_2)\right] - c_d z$$

$$= p_2 \int_0^z \bar{F}_{2|1}(y)dy - c_d z$$

Replacing $G_c(z)$ into (4.13), the centralized problem becomes

$$\begin{aligned}
\pi_c^{III}(Q) &= p_1 \int_0^Q \bar{F}_1(x)dx + p_2 \int_0^Q \int_0^{Q-x} \bar{F}_{2|1}(y)dy f_1(x)dx - c_d(Q_c - \int_0^{Q_c} \bar{F}_1(x)dx) \\
&- (\sum_{i=1}^n c_i + c_d + c_0 + w)Q
\end{aligned}$$

The first order and second order optimality conditions are given below.

$$\begin{aligned}
\frac{d\pi_c(Q)}{dQ} &= (p_1 + c_d)\bar{F}_1(Q) + p_2 \int_0^Q \bar{F}_{2|1}(Q-x) f_1(x)dx \\
&- (\sum_{i=1}^n c_i + c_d + c_0 + w) = 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{d^2\pi_c(Q)}{d^2Q} &= -(p_1 + c_d)f_1(Q) + p_2 \left[f_1(Q)\bar{F}_{2|1}(0) - \int_0^Q f_{2|1}(Q-x) f_1(x)dx \right] \\
&= \left[-p_1 - c_d + p_2 \bar{F}_{2|1}(0) \right] f_1(Q) - p_2 \int_0^Q f_{2|1}(Q-x) f_1(x)dx \qquad (4.14) \\
&\leq 0
\end{aligned}$$

Proposition 13 *If $p_2 \leq p_1 + c_d$, $\pi_c(Q)$ is concave and has a unique maximum Q_c^{III} which can be found by solving $\frac{d\pi_c(Q)}{dQ} = 0$.*

Proof: The last term in (4.14) is always negative. Since $\bar{F}_{2|1}(0)$ can not be greater than 1, the first term in (4.14) is also negative for $p_2 \leq p_1 + c_d$. □

4.2.2 Decentralized System

In the decentralized system, every actor tries to maximize its individual expected profit. The following sequence of events occurs in case III: First the assembler determines the revenue shares of each supplier i (α_i). Then the suppliers follows with delivery quantities. We assume that the suppliers know the distribution of the total demand ($D_t = D_1 + D_2$) which appears in assembler's and distributor's market. At the end of the first selling period, the assembler sends the unsold products to the distributor and determines the revenue share α_d given to the distributor for each product sold in the second period (in distributor's market). While determining α_d, the assembler considers the distributor's profit maximization problem to make an acceptable offer to the distributor. So the offered α_d is nothing but the revenue share which maximizes the distributor's expected profit for the now known delivery quantity (unsold products in the first period).

Here, the probability density function of the total demand (f_t) is the convolution of $f_{1,2}$, that is:

$$\begin{aligned} f_t(Q) &= \int_0^\infty f_{1,2}(x, Q-x)dx \\ \bar{F}_t(Q) &= \int_Q^\infty \int_0^\infty f_{1,2}(x, v-x)dxdv \end{aligned}$$

Clearly, the necessary conditions for each party to stay in the business are

$$\begin{aligned} &(p_1 - \textstyle\sum_{i=1}^n \alpha_i) > c_0 + w && \text{for the assembler to serve its market ,} \\ &\alpha_i > c_i; i = 1, n && \text{for suppliers,} \\ &\alpha_d > c_d && \text{for the distributor,} \end{aligned}$$

The profit function of a typical supplier i is

$$\begin{aligned} \pi_i^{III}(Q) &= E\left[\alpha_i min(Q, D_t) - c_i Q\right] \\ &= \alpha_i \int_0^Q \bar{F}_t(x)dx - c_i Q \;\; i = 1, ..., n \end{aligned} \tag{4.15}$$

So the supplier's problem can be solved for a given α_i.

$$\bar{F}_t(Q_i^{III}) = \frac{c_i}{\alpha_i} \;\; i = 1, ..., n \tag{4.16}$$

As in the previous problems, we know that the optimal solution satisfies the following proposition.

Proposition 14 *The Assembler will always set $\alpha_1, ..., \alpha_n$ such that*

$$\frac{c_1}{\alpha_1} = ... = \frac{c_n}{\alpha_n} = \bar{F}_t(Q_{dec}^{III}) \tag{4.17}$$

This proposition results that at the optimal solution $Q_1^{III} = Q_2^{III} = ... = Q_n^{III} = Q_{dec}^{III}$.

Here the profit function of the distributor for a given α_d and z is

$$\begin{aligned}
\pi_d^{III}(z) &= E\left\{\alpha_d min(z, D_{2|1}) - c_d z\right\} \\
&= \alpha_d \int_0^z \bar{F}_{2|1}(x)dx - c_d z
\end{aligned} \tag{4.18}$$

We assume that the distributor accepts the offer of the assembler only if the following is satisfied.

$$\bar{F}_{2|1}(z) = \frac{c_d}{\alpha_d} \tag{4.19}$$

Now the assembler's profit function becomes

$$\begin{aligned}
\pi_0^{III}(\alpha_1, ..., \alpha_n) &= E\left[(p_1 - \sum_{i=1}^{n} \alpha_i) min(Q_{dec}^{III}, D_1)\right] \\
&+ E\left[G(z)\right] - (c_0 + w)Q_{dec}^{III} \\
&= (p_1 - \sum_{i=1}^{n} \alpha_i) \int_0^{Q_{dec}^{III}} \bar{F}_1(x)dx \\
&+ \int_0^{Q_{dec}^{III}} G(Q_{dec}^{III} - x) f_1(x)dy - (c_0 + w)Q_{dec}^{III}
\end{aligned} \tag{4.20}$$

where $G(z)$ is the expected profit of the assembler from the sale in the second period and z is the quantity sent to the distributor $z = max(0, Q_{dec}^{III} - D_1)$.

Writing $G(z)$ explicitly, we obtain

$$\begin{aligned}
G(z) &= E\left[(p_2 - \sum_{i=1}^{n} \alpha_i - \alpha_d) min(z, D_{2|1})\right] \\
&= (p_2 - \sum_{i=1}^{n} \alpha_i - \alpha_d) \int_0^z \bar{F}_{2|1}(y)dy
\end{aligned}$$

Replacing $G(z)$ into (4.20), the assembler's problem becomes

$$\begin{aligned}
\pi_0^{III}(\alpha_1, ..., \alpha_n) &= (p_1 - \sum_{i=1}^{n} \alpha_i) \int_0^{Q_{dec}^{III}} \bar{F}_1(x)dx \\
&+ \int_0^{Q_{dec}^{III}} (p_2 - \sum_{i=1}^{n} \alpha_i - \alpha_d) \int_0^{Q_{dec}^{III} - x} \bar{F}_{2|1}(y)dy \\
&- (c_0 + w)Q_{dec}^{III}
\end{aligned} \tag{4.21}$$

From the solution of the suppliers' problem, distributor's problem and Proposition 14, the following relations can be shown for α_d and $\sum_{i=1}^{n}\alpha_i$.

$$\begin{aligned}
\bar{F}_{2|1}(z) &= \frac{c_d}{\alpha_d} \\
or \;\; \alpha_d &= \frac{c_d}{\bar{F}_{2|1}(z)} \\
\bar{F}_t(Q_{dec}) &= \frac{c_1}{\alpha_1} = \ldots = \frac{c_n}{\alpha_n} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n}\alpha_i} \\
or \;\; \sum_{i=1}^{n}\alpha_i &= \frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(Q_{dec})}
\end{aligned}$$

Replacing α_d and $\sum_{i=1}^{n}\alpha_i$ in (4.21) with $\frac{c_d}{\bar{F}_{2|1}(z)}$ and $\frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(Q_{dec}^{III})}$, respectively, the assembler's profit function (with respect to Q) becomes

$$\begin{aligned}
\pi_0^{III}(Q) &= (p_1 - \frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(Q)})\int_0^Q \bar{F}_1(x)dx \\
&+ \int_0^Q \left[(p_2 - \frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(Q)} - \frac{c_d}{\bar{F}_{2|1}(Q-x)})\int_0^{Q-x} \bar{F}_{2|1}(y)dy\right] f_1(x)dx - (c_0+w)Q \\
&= (p_1 - \frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(Q)})\int_0^Q \bar{F}_1(x)dx + (p_2 - \frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(Q)})\int_0^Q\int_0^{Q-x} \bar{F}_{2|1}(y)dy f_1(x)dx \\
&- \int_0^Q\int_0^{Q-x} \frac{c_d}{\bar{F}_{2|1}(Q-x)}\bar{F}_{2|1}(y)dy f_1(x)dx - (c_0+w)Q.
\end{aligned}$$

To handle computational complexity while proving the concavity of the assembler's profit function, we rewrite the assembler's expected profit function as

$$\begin{aligned}
\pi_0^{III}(Q) &= (p_1 - \frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(Q)})\int_0^Q \bar{F}_1(x)dx \\
&+ \int_0^Q G(z(Q,x))f_1(x)dy - (c_0+w)Q
\end{aligned}$$

where $z = Q - x$ and

$$G(z(Q,x)) = (p_2 - \frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(z+x)} - \frac{c_d}{\bar{F}_{2|1}(z)})\int_0^z \bar{F}_{2|1}(y)dy$$

Now, using this form, one can take the first and second derivative of $G(z)$ with respect to z.

$$\begin{aligned}\frac{dG(z(Q,x))}{dz} &= p_2\bar{F}_{2|1}(z) - \sum_{i=1}^{n} c_i \left[\frac{\bar{F}_{2|1}(z)}{\bar{F}_t(z-x)} + \frac{f_t(z+x)}{\left[\bar{F}_t(z+x)\right]^2}\int_0^z \bar{F}_{2|1}(y)dy\right] \\ &- c_d\left[1 + \frac{f_{2|1}(z)}{\left[\bar{F}_{2|1}(z)\right]^2}\int_0^z \bar{F}_{2|1}(y)dy\right]\end{aligned}$$

and

$$\begin{aligned}\frac{d^2G(z(Q,x))}{d^2z} &= -p_2 f_{2|1}(z) - \sum_{i=1}^{n} c_i \left[-\frac{f_{2|1}(z)}{\bar{F}_t(z+x)} + \frac{f_t(z+x)\bar{F}_{2|1}(z)}{\left[\bar{F}_t(z+x)\right]^2}\right] \\ &- \sum_{i=1}^{n} c_i \left[\frac{f_t'(z+x)\int_0^z \bar{F}_{2|1}(y)dy + f_t(z+x)\bar{F}_{2|1}(z)}{\left[\bar{F}_t(z+x)\right]^2}\right] \\ &- \sum_{i=1}^{n} c_i \left[\frac{2\bar{F}_t(z+x)f_t^2(z+x)}{\left[\bar{F}_t(z+x)\right]^4}\int_0^z \bar{F}_{2|1}(y)dy\right] \\ &- c_d\left[\frac{f_{2|1}'(z)\int_0^z \bar{F}_{2|1}(y)dy + f_{2|1}(z)\bar{F}_{2|1}(z)}{\left[\bar{F}_{2|1}(z)\right]^2}\right] \\ &- c_d\left[\frac{2\bar{F}_{2|1}(z)f_{2|1}^2(z)}{\left[\bar{F}_{2|1}(z)\right]^4}\int_0^z \bar{F}_{2|1}(y)dy\right] \qquad (4.22)\end{aligned}$$

Proposition 15 *$G(z(Q,x))$ is concave in z if $\frac{f_{2|1}(z)}{\bar{F}_{2|1}(z)}$ and $\frac{f_t(z)}{\bar{F}_t(z)}$ are increasing.*

Proof: Consider Equation 4.22. If $\frac{f_{2|1}(y)}{\bar{F}_{2|1}(y)}$ and $\frac{f_t(Q)}{\bar{F}_t(Q)}$ are increasing, all terms in Equation 4.22 are negative except $-\sum_{i=1}^{n} c_i\left[-\frac{f_{2|1}(z)}{\bar{F}_t(z+x)}\right]$, where $\frac{\sum_{i=1}^{n} c_i}{\bar{F}_t(z+x)}$ is the total revenue share given to suppliers ($\sum_{i=1}^{n}\alpha_i$). Since $\sum_{i=1}^{n}\alpha_i$ can not be greater than p_2, $-p_2 f_{2|1}(z)$ makes the total negative. □

Now, the first order condition of optimality for the assembler's problem is

$$\begin{aligned}\frac{d\pi_0^{III}(Q)}{dQ} &= -(c_0+w) + p_1\bar{F}_1(Q) - (\sum_{i=1}^{n} c_i)\left[\frac{\bar{F}_1(Q)}{\bar{F}_t(Q)} + \frac{f_t(Q)}{[\bar{F}_t(Q)]^2}\int_0^Q \bar{F}_1(x)dx\right] \\ &+ \int_0^Q \frac{dG(z(Q,x))}{dz} f_1(x)dx\end{aligned}$$

To analyze the concavity of the problem, we investigate the second order condition of the assemblers profit function

$$
\begin{aligned}
\frac{d^2\pi_0^{III}(Q)}{dQ^2} &= -p_1 f_1(Q) - (\sum_{i=1}^{n} c_i)\left[-\frac{f_1(Q)}{\bar{F}_t(Q)} + \frac{f_t(Q)\bar{F}_1(Q)}{\left[\bar{F}_t(Q)\right]^2}\right] \\
&- (\sum_{i=1}^{n} c_i)\left[\frac{f_t'(Q)\int_0^Q \bar{F}_1(x)dx + f_t(Q)\bar{F}_1(Q)}{\left[\bar{F}_t(Q)\right]^2}\right. \\
&+ \left.\frac{2\bar{F}_t(Q)f_t^2(Q)}{\left[\bar{F}_t(Q)\right]^4}\int_0^Q \bar{F}_1(x)dx\right] \\
&+ \int_0^Q \frac{d^2G(z(Q,x))}{d^2z} f_1(x)dx + \frac{dG(0)}{dz} f_1(Q) \\
&= -p_1 f_1(Q) - (\sum_{i=1}^{n} c_i)\left[\frac{f_t(Q)\bar{F}_1(Q)}{\left[\bar{F}_t(Q)\right]^2}\right] \\
&- (\sum_{i=1}^{n} c_i)\left[\frac{f_t'(Q)\int_0^Q \bar{F}_1(x)dx + f_t(Q)\bar{F}_1(Q)}{\left[\bar{F}_t(Q)\right]^2}\right. \\
&+ \left.\frac{2\bar{F}_t(Q)f_t^2(Q)}{\left[\bar{F}_t(Q)\right]^4}\int_0^Q \bar{F}_1(x)dx\right] \\
&+ \int_0^Q \frac{d^2G(z(Q,x))}{d^2z} f_1(x)dx + \left[p_2\bar{F}_{2|1}(0) - c_d\right] f_1(Q) \qquad (4.23)
\end{aligned}
$$

Proposition 16 *If $\frac{f_{2|1}(y)}{\bar{F}_{2|1}(y)}$ and $\frac{f_t(Q)}{\bar{F}_t(Q)}$ are increasing, $\pi_0^{III}(Q)$ is concave for $p_2 \leq p_1 + c_d$.*

Proof: Consider (4.23), if $\frac{f_{2|1}(y)}{\bar{F}_{2|1}(y)}$ and $\frac{f_t(Q)}{\bar{F}_t(Q)}$ are increasing (from IFR property), all terms are negative in (4.23) except $\left[p_2\bar{F}_{2|1}(0) - c_d\right] f_1(Q)$. Since $\bar{F}_{2|1}(0)$ can not be greater than 1, $-p_1 f_1(Q)$ makes the total negative for $p_2 \leq p_1 + c_d$. Since $G(z(Q,x))$ is concave, $\int_0^Q \frac{d^2G(z(Q,x))}{d^2z} f_1(x)dx$ is always negative.□

According to the revenue share contract between the assembler and the distributor, the assembler offers α_d which satisfies Equation 4.19 for z units of delivery. Therefore, for some z values, α_d becomes greater than p_2, and the assembler worsens off from the sales in the secondary market. To overcome this inconsistency,

Table 4.4: Set I for Case III

μ_1	μ_2	μ_t	σ_t	ρ	p_1	p_2	w	c_0	$\sum_{i=1}^{n} c_i$
1000	500	1500	300	0	1	1.05	0.2	0.15	0.15
				0.3		1	0.1		
				-0.3		0.8			

we try to restrict α_d with p_2, but we can not show the concavity of the assembler's profit function for this case. Since the probability of facing such z values corresponding $\alpha_d \geq p_2$ is diminishing with relatively higher mean of the secondary market demand, we expect this model to provide us acceptable solutions.

4.3 Computational Analysis

In this section, we analyze the effect of system parameters on the assembler's expected profit in Case III. Additionally, we try to answer, how the assembler is affected, when it finds an opportunity to sell its unsold products through a distributor in a secondary market or to expand its market reaching another market via a distributor. To be able to do this analysis, we compare Case I and Case III. First, we introduce the sets used for our computations. Then, we continue with the presentation of the results and comments.

4.3.1 Parameter Sets for Case III

Through our computational analysis, we assume that demands (D_1) and (D_2) have bivariate normal distribution, with marginal distributions $N(\mu_1, \sigma_1)$ and $N(\mu_2, \sigma_2)$, respectively. ρ is the correlation of D_1 and D_2. If D_1 and D_2 have bivariate normal distribution, the total demand ($D_t = D_1 + D_2$) is also normally distributed with $N(\mu_t = \mu_1 + \mu_2, \sigma_t = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2})$.

We perform our analysis on three different sets presented in Table 4.4, Table 4.5 and Table 4.6.

Table 4.5: Set II for Case III

μ_1	μ_2	μ_t	σ_t	ρ	p_1	p_2	w	c_0	$\sum_{i=1}^{n} c_i$
500	1000	1500	300	0	1	1.05	0.2	0.15	0.15
				0.3		1	0.1		
				-0.3		0.8			

Table 4.6: Set III for Case III

μ_1	μ_2	μ_t	σ_t	ρ	p_1	p_2	w	c_0	$\sum_{i=1}^{n} c_i$
1500	750	2250	450	0	1	1.05	0.2	0.15	0.15
				0.3		1	0.1		
				-0.3		0.8			

For every data set, 18 computations are performed by changing ρ, p_2 and w. While changing ρ, we use σ_1 and σ_2 that correspond to a standard deviation of total demand (σ_t) presented in the fourth columns of Tables 4.4, 4.5 and 4.6.

In order to see the effect of standard deviations (σ_1) and (σ_2) that correspond to same standard deviation of total demand (σ_t), additional computations for data set I is performed using different σ_1 and σ_2 values for each ρ. In these computations, σ_t is fixed to 300 and w to 0.2. These cases correspond to 9 additional computations.

To be able to analyze the opportunities created by the distributor from the assembler's point of view, we compare the profit of the assembler in Case III with the one in Case I. The scenario is created as follows: The assembler operates as in Case I (without a distributor) and faces a demand which is normally distributed with $N(\mu_1, \sigma_1)$. Then, the assembler finds an opportunity to sell its unsold product in a secondary market at a price p_2 via a distributor with operating cost $c_d = 0.05$. The demand in the secondary market is also normally distributed with $N(\mu_2, \sigma_2)$. The secondary market demand can be correlated with the first market. So we try to answer how the assembler benefits from this opportunity. We carry out the comparison for $p_2 = 1.05, 1.0, 0.8$, $\rho = 0.3, 0.0, -0.3$ and $w = 0.2, 0.1$. While performing our analysis, we assume that the first market's μ_1 and σ_1 do not change. We use the solutions of data set I and data set II.

The computations for Case III are carried out using MAPLE V. The code can be found in Appendix E. The computational results can be seen in Appendix D.

4.3.2 Results and Comments

Through our computations, we first analyze the effect of p_2, ρ and w on the assembler. The results can be summarized as follows:

- p_2: As expected the diminishing second period selling price p_2 has a negative effect on assembler's profit. In Figure 4.7, the percentage deviation of the assembler's expected profit with respect to p_2 ($p_2 = 1$ is the base) is shown. The system parameters of the computations are given in Appendix D.

- ρ: The assembler's profit is increasing with negative correlation between markets and decreasing with positive one. This is also an expected result. If the correlation is negative, the probability of selling the unsold products in the second period is higher when the first period demand is realized low. So the assembler can sell the product either in the first or in the second period.In Figure 4.8, the percentage deviation of the assembler's expected profit with respect to ρ ($\rho = 0$ is the base) is shown. The system parameters of the computations are given in Appendix D.

- w: As w decreases the profit of the assembler is increasing since the profit margin of the assembler is increasing.

The computations reveal that the assemblers profit is increasing in decreasing standard deviation of the first market demand (σ_1) and increasing in standard deviation of the secondary market demand (σ_2), when the standard deviation of total demand (σ_t), μ_1 and μ_2 are kept constant. Besides, the suppliers' and the

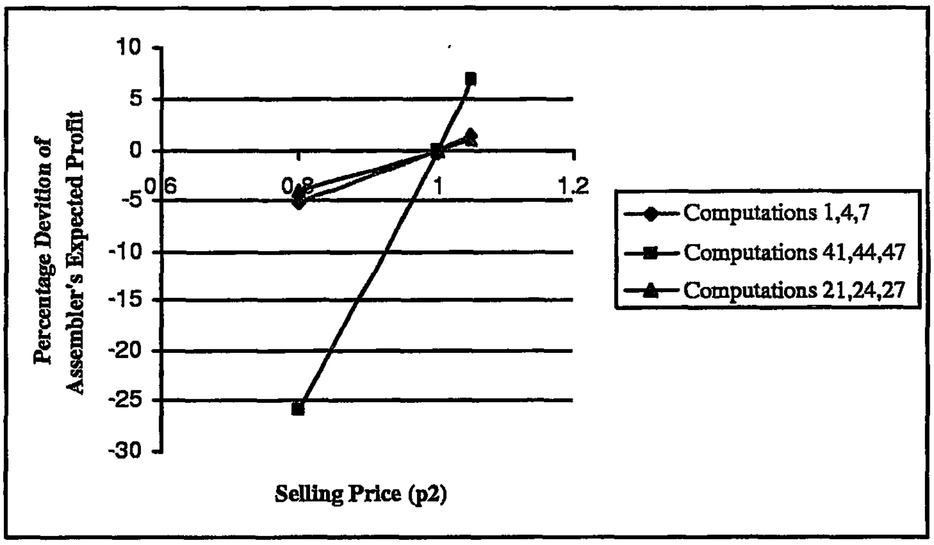

Figure 4.7: Percentage deviation of assembler's expected profit versus p_2

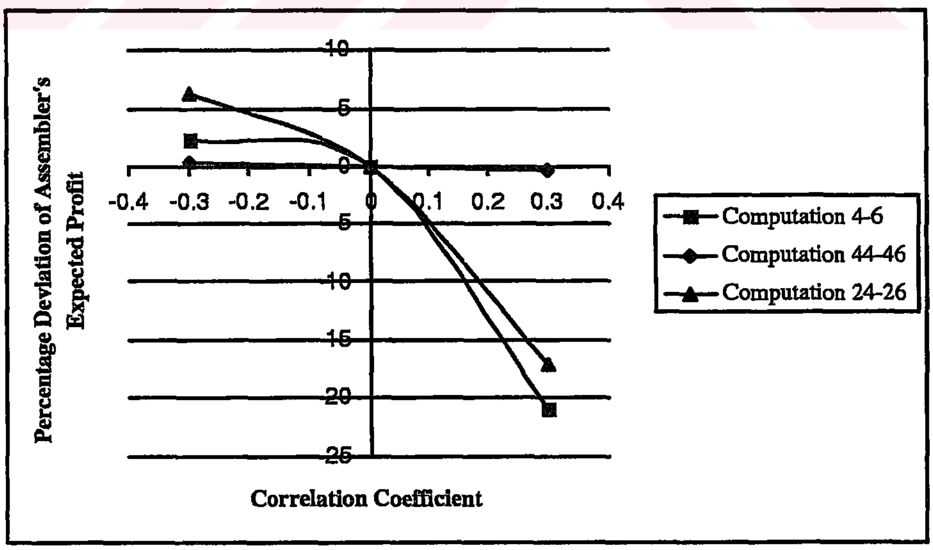

Figure 4.8: Percentage deviation of assembler's expected profit versus ρ

distributor's expected profits are also increasing in these cases. More numerical studies are required to generalize this observation.

Comparisons between the results of data set I and data set III, reveal that the correlation effect on the expected profit of the assembler is diminishing when the expected value of the secondary market demand increases. This observation can be also seen in Figure 4.8. In Figure 4.8, computations 4-6 and 44-46 are the systems, where $\mu_1 = 1000$, $\mu_2 = 500$, and $\mu_1 = 500$, $\mu_2 = 1000$, respectively. The slope of the curve of computations 44-46 is less than other, which means the effect of ρ is less when the second market becomes the biggest market.

Similarly, we consider the effect of the selling price of the second market on the assembler's expected profit. As seen in Figure 4.7, the slope of the computations 41-44 is higher, which means the selling price effect of second market becomes significant, when the second market becomes the biggest market.

Additionally, the assembler's and other actors' expected profits increase, when the expected value of the second period demand increases, if the selling price of second period (p_2) is not less than the first period selling price (p_1). Again, in this analysis, the mean of total demand (μ_t) is kept constant.

Consider the following scenario. The assembler has an opportunity to enter in a secondary market via a distributor. To analyze this situation, we compute the percentage increase of assembler's profit when it applies this strategy. The percentage increases are calculated using the following formula $\frac{\pi_0^{III} - \pi_0^{I}}{\pi_0^{I}} * 100$. We also consider the percentage increase on the assembler's expected profit when the assembler is assumed to enter in the secondary market directly with a selling price 1. As expected, the percentage increase in assembler's profit is greater when it serves the secondary market directly, as well. Note that, this case may not be realizable, so it only acts as a bound on the possible expected profits.

The computations reveal that the percentage increase of the assembler expected profit is affected by the following factors:

- The effect of p_2: As expected for higher p_2 the assembler is benefiting more from the secondary market.

- The effect of ρ: The assembler increases its expected profits more when the secondary market demand is negatively correlated with the first period demand. For the positive $\rho = 0.3$, the percentage increase diminish to 10% even for the case where the expected demand of the total market is increased by 50%.

CHAPTER 5

CONCLUSION

In this thesis, we consider a supply chain, where the assembler provides complementary parts from two different types of suppliers (a number of small suppliers and the wholesaler). The suppliers and the wholesaler differ with respect to their bargaining strength with the assembler. The wholesaler is very powerful in the sector, because it is a monopoly. The suppliers are followers of the assembler, since the assembler is their main customer. The power structure is reflected to the contractual agreements that occur between the actors. We assume that there is wholesale price contract between the wholesaler and the assembler, where the wholesaler declares (or determines) the wholesale price. There are revenue share contracts between the assembler and the suppliers, where the revenue shares given to the suppliers are determined by the assembler. We analyze this environment for two cases. In the first one (Case I), we fix the wholesale price (ie. wholesaler announces a price) and consider the assembler-suppliers system. In the second one (Case Ia), we consider the wholesaler to determine its wholesale price having the full information about assembler's reaction to the wholesale price.

In this study, we also consider two distribution alternatives of the assembler by adding a distributor to the system. For these alternatives, we assume that there is a revenue share contract between the assembler and the distributor. According to the contract, the assembler determines the revenue share given to the distributor. In the first alternative (Case II), we consider the choice of the assembler to utilize a distributor to serve its market. In the second alternative (Case III), we analyze the opportunity of the assembler to serve to a secondary market via the distributor, as well as its own market.

The first two cases (Case I and Case Ia) provide us an understanding about the effect of power structure on the supply chain. In the other two cases (Case II and Case III), we consider alternative distribution strategies from the assembler's point of view.

The literature reviewed in the thesis, mainly considers manufacturer-retailer or supplier-manufacturer systems where the upper echelon determines the contract parameters and the lower echelon then decides on the quantity. In this study, we analyze a supply chain where the power structure is more complex. Another focus of the literature is the existence of coordination mechanisms. In this study, we consider the dynamics of the supply chain under the power structure defined, and give less emphasis to coordination mechanisms.

The literature mainly examined two period newsvendor problem, where the newsvendor has an unlimited supply at a fixed cost. Our study (Case III) differs with the literature on the structure. We consider the two period problem of the assembler on a more complex environment. Lee (2001) considers a similar system with Case III. However, the contracting mechanisms and power structure are different in Case III. Another related study is Gerchak and Wang (2000). We extend the system of Gerchak and Wang (2000) with a wholesaler, analyzing the revenue sharing and wholesale price contracts simultaneously (Case Ia). Case II and Case III are other extension of Gerchak and Wang (2000).

We can summarize the findings from our computational results as follows:

- In Case I: We do not observe a gap that exceeds 10% between the centralized solution and the decentralized solution. The revenue sharing contract performs quite well considering the total system performance. One important observation is that for the suppliers when the production cost of the product is decreasing, the suppliers' expected profit starts to decrease for the low production costs. The observation can be interpreted as follows: A supplier firm is going to benefit from a decrease in their production costs up to a certain extend. At some point, any decrease in the production cost will result in a lower value-added to the final product, hence decreasing the expected profits. Such firms need to establish themselves in other markets for survival. Additionally, the suppliers' expected profits are decreasing and the expected profit of the assembler increases, when the standard deviation of the demand decreases. When there is an decrease in the assembler's production cost (c_0), the assembler and the suppliers increases their expected profits.

- In Case Ia: We observe that the gap between the centralized solution and the decentralized solution fluctuates between 37% and 23%. The system performance deteriorates when the wholesaler has full information about the reaction of the assembler. We also observe the similar behavior (the expected profit of the suppliers first increases then starts decreasing) described above for the small suppliers when their production costs decrease. Another interesting finding is both improvements on the wholesaler's cost (c_ω) and assembler's cost (c_0) have the same effect (increasing expected profits) on the expected profits of all the actors.

- In Case II: When the assembler utilizes the distributor to sell the products, the system behaves as in Case I with additional supplier cost c_d. The system is similar to Case I with an additional supplier that has a production cost c_d.

- In Case III: The profit of the assembler increases as p_2 increases, ρ decreases and w decreases. The contracting schemes perform better for all of the actors when the expected second period demand is increasing. Two important factors, that should be considered by the assemblers while making the choice of entering a secondary market via a distributor are the values of p_2 and ρ. Our comparisons (Case I with Case III) reveal that increasing

p_2 has a positive effect on the assembler's profit, as expected. Besides, the assembler benefits more from entering to a secondary market which has a negatively correlated demand with the first market demand.

Because of the nature of the models we created, the concavity of the profit functions depend on the IFR property of the demand distribution. But this does not create much problem, since usually used distributions (like normal distribution, exponential distribution and uniform distribution) satisfy this property.

We believe that this study provides a framework for further studies in this area, especially on power relations and coordination strategies. We believe that this thesis is instrumental in creating a managerial insight on the system performance under different possible situations.

In the thesis, the power structure of the system is reflected by the revenue sharing contracts and wholesale price contract. This study can be repeated with other contracting schemes and allocation of decision rights.

In Case III, we do not consider the flexibility of given additional order for the second period demand. This can be a reasonable extension for further studies.

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APPENDIX A

COMPUTATIONAL RESULTS FOR CASE I

Table A.1: (Case I) The Effect of c_0 (w=0.3)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_{1}^{n} \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	400	0.3	0.3	0.2	160.804	982	0.222	0.778	17.173	187.987	1163	0.053
1500	400	0.29	0.3	0.2	170.678	993	0.223	0.777	18.328	199.690	1177	0.054
1500	400	0.28	0.3	0.2	180.653	1002	0.224	0.776	19.322	211.533	1191	0.055
1500	400	0.27	0.3	0.2	190.724	1012	0.225	0.775	20.481	223.510	1204	0.055
1500	400	0.26	0.3	0.2	200.890	1021	0.226	0.774	21.574	235.619	1217	0.056
1500	400	0.25	0.3	0.2	211.145	1030	0.227	0.773	22.718	247.857	1230	0.056
1500	400	0.24	0.3	0.2	221.488	1039	0.228	0.772	23.914	260.221	1243	0.057
1500	400	0.23	0.3	0.2	231.916	1047	0.230	0.770	25.023	272.708	1255	0.058
1500	400	0.22	0.3	0.2	242.426	1055	0.231	0.769	26.176	285.316	1267	0.059
1500	400	0.21	0.3	0.2	253.016	1063	0.232	0.768	27.374	298.044	1279	0.059
1500	400	0.2	0.3	0.2	263.684	1071	0.233	0.767	28.620	310.888	1290	0.060
1500	400	0.19	0.3	0.2	274.427	1078	0.234	0.766	29.750	323.847	1302	0.061
1500	400	0.18	0.3	0.2	285.244	1085	0.235	0.765	30.919	336.919	1313	0.062

Table A.1 (Continued)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_{1}^{n} \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	400	0.17	0.3	0.2	296.133	1092	0.236	0.764	32.127	350.104	1324	0.062
1500	400	0.16	0.3	0.2	307.091	1099	0.238	0.762	33.376	363.399	1335	0.063
1500	400	0.15	0.3	0.2	318.118	1106	0.239	0.761	34.668	376.803	1346	0.064
1500	400	0.14	0.3	0.2	329.212	1113	0.240	0.760	36.002	390.315	1357	0.064
1500	400	0.13	0.3	0.2	340.371	1119	0.241	0.759	37.181	403.934	1367	0.065
1500	400	0.12	0.3	0.2	351.594	1125	0.242	0.758	38.394	417.659	1378	0.066
1500	400	0.11	0.3	0.2	362.880	1132	0.244	0.756	39.853	431.489	1388	0.067
1500	400	0.1	0.3	0.2	374.226	1138	0.245	0.755	41.141	445.424	1399	0.067
1500	400	0.09	0.3	0.2	385.633	1144	0.246	0.754	42.466	459.462	1409	0.068
1500	400	0.08	0.3	0.2	397.098	1149	0.247	0.753	43.598	473.603	1419	0.069
1500	400	0.07	0.3	0.2	408.621	1155	0.248	0.752	44.992	487.846	1429	0.070
1500	400	0.06	0.3	0.2	420.201	1161	0.249	0.751	46.424	502.191	1440	0.071
1500	400	0.05	0.3	0.2	431.836	1166	0.251	0.749	47.649	516.638	1450	0.072

Table A.2: (Case I) The Effect of c_0 (w=0.2)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	400	0.3	0.2	0.2	263.684	1071	0.233	0.767	28.62	310.888	1290	0.06
1500	400	0.29	0.2	0.2	274.427	1078	0.234	0.766	29.75	323.847	1302	0.061
1500	400	0.28	0.2	0.2	285.244	1085	0.235	0.765	30.919	336.919	1313	0.062
1500	400	0.27	0.2	0.2	296.133	1092	0.236	0.764	32.127	350.104	1324	0.062
1500	400	0.26	0.2	0.2	307.091	1099	0.238	0.762	33.376	363.399	1335	0.063
1500	400	0.25	0.2	0.2	318.118	1106	0.239	0.761	34.668	376.803	1346	0.064
1500	400	0.24	0.2	0.2	329.212	1113	0.24	0.760	36.002	390.315	1357	0.064
1500	400	0.23	0.2	0.2	340.371	1119	0.241	0.759	37.181	403.934	1367	0.065
1500	400	0.22	0.2	0.2	351.594	1125	0.242	0.758	38.394	417.659	1378	0.066
1500	400	0.21	0.2	0.2	362.88	1132	0.244	0.756	39.853	431.489	1388	0.067
1500	400	0.2	0.2	0.2	374.226	1138	0.245	0.755	41.141	445.424	1399	0.067
1500	400	0.19	0.2	0.2	385.633	1144	0.246	0.754	42.466	459.462	1409	0.068
1500	400	0.18	0.2	0.2	397.098	1149	0.247	0.753	43.598	473.603	1419	0.069
1500	400	0.17	0.2	0.2	408.621	1155	0.248	0.752	44.992	487.846	1429	0.07
1500	400	0.16	0.2	0.2	420.201	1161	0.249	0.751	46.424	502.191	1440	0.071
1500	400	0.15	0.2	0.2	431.836	1166	0.251	0.749	47.649	516.638	1450	0.072
1500	400	0.14	0.2	0.2	443.525	1172	0.252	0.748	49.155	531.186	1460	0.072
1500	400	0.13	0.2	0.2	455.268	1177	0.253	0.747	50.442	545.834	1470	0.074
1500	400	0.12	0.2	0.2	467.064	1182	0.254	0.746	51.759	560.583	1480	0.074
1500	400	0.11	0.2	0.2	478.911	1187	0.255	0.745	53.105	575.433	1490	0.075
1500	400	0.1	0.2	0.2	490.809	1192	0.257	0.743	54.483	590.383	1500	0.076

Table A.3: (Case I) The Effect of c_0 (w=0.1)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_{1}^{n} \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	400	0.3	0.1	0.2	374.226	1138	0.245	0.755	41.141	445.424	1399	0.067
1500	400	0.29	0.1	0.2	385.633	1144	0.246	0.754	42.466	459.462	1409	0.068
1500	400	0.28	0.1	0.2	397.098	1149	0.247	0.753	43.598	473.603	1419	0.069
1500	400	0.27	0.1	0.2	408.621	1155	0.248	0.752	44.992	487.846	1429	0.07
1500	400	0.26	0.1	0.2	420.201	1161	0.249	0.751	46.424	502.191	1440	0.071
1500	400	0.25	0.1	0.2	431.836	1166	0.251	0.749	47.649	516.638	1450	0.072
1500	400	0.24	0.1	0.2	443.525	1172	0.252	0.748	49.155	531.186	1460	0.072
1500	400	0.23	0.1	0.2	455.268	1177	0.253	0.747	50.442	545.834	1470	0.074
1500	400	0.22	0.1	0.2	467.064	1182	0.254	0.746	51.759	560.583	1480	0.074
1500	400	0.21	0.1	0.2	478.911	1187	0.255	0.745	53.105	575.433	1490	0.075
1500	400	0.2	0.1	0.2	490.809	1192	0.257	0.743	54.483	590.383	1500	0.076
1500	400	0.19	0.1	0.2	502.757	1197	0.258	0.742	55.892	605.433	1510	0.077
1500	400	0.18	0.1	0.2	514.755	1202	0.259	0.741	57.333	620.583	1520	0.078
1500	400	0.17	0.1	0.2	526.8	1207	0.26	0.740	58.807	635.834	1530	0.079
1500	400	0.16	0.1	0.2	538.894	1212	0.262	0.738	60.314	651.186	1540	0.08
1500	400	0.15	0.1	0.2	551.034	1216	0.263	0.737	61.545	666.638	1550	0.081
1500	400	0.14	0.1	0.2	563.22	1221	0.264	0.736	63.114	682.191	1560	0.082
1500	400	0.13	0.1	0.2	575.452	1225	0.265	0.735	64.394	697.846	1571	0.083
1500	400	0.12	0.1	0.2	587.729	1230	0.267	0.733	66.028	713.603	1581	0.084
1500	400	0.11	0.1	0.2	600.05	1234	0.268	0.732	67.361	729.462	1591	0.085
1500	400	0.1	0.1	0.2	612.415	1239	0.269	0.731	69.061	745.424	1601	0.086

Table A.4: (Case I) The Effect of sum of c_i (w=0.3)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	400	0.2	0.3	0.3	153.342	936	0.326	0.674	19.496	187.987	1163	0.081
1500	400	0.2	0.3	0.29	163.454	950	0.317	0.683	20.537	199.690	1177	0.079
1500	400	0.2	0.3	0.28	173.761	964	0.308	0.692	21.586	211.533	1191	0.077
1500	400	0.2	0.3	0.27	184.267	977	0.299	0.701	22.503	223.510	1204	0.075
1500	400	0.2	0.3	0.26	194.973	991	0.289	0.711	23.547	235.619	1217	0.073
1500	400	0.2	0.3	0.25	205.884	1004	0.280	0.720	24.436	247.857	1230	0.071
1500	400	0.2	0.3	0.24	217.004	1017	0.271	0.729	25.299	260.221	1243	0.069
1500	400	0.2	0.3	0.23	228.337	1030	0.261	0.739	26.126	272.708	1255	0.067
1500	400	0.2	0.3	0.22	239.891	1044	0.252	0.748	27.062	285.316	1267	0.064
1500	400	0.2	0.3	0.21	251.670	1057	0.243	0.757	27.794	298.044	1279	0.062
1500	400	0.2	0.3	0.2	263.684	1071	0.233	0.767	28.620	310.888	1290	0.060
1500	400	0.2	0.3	0.19	275.940	1084	0.223	0.777	29.212	323.847	1302	0.058
1500	400	0.2	0.3	0.18	288.450	1098	0.214	0.786	29.876	336.919	1313	0.055
1500	400	0.2	0.3	0.17	301.223	1112	0.204	0.796	30.437	350.104	1324	0.053
1500	400	0.2	0.3	0.16	314.274	1126	0.194	0.806	30.880	363.399	1335	0.050
1500	400	0.2	0.3	0.15	327.617	1141	0.184	0.816	31.349	376.803	1346	0.047
1500	400	0.2	0.3	0.14	341.269	1156	0.174	0.826	31.660	390.315	1357	0.045
1500	400	0.2	0.3	0.13	355.250	1171	0.164	0.836	31.786	403.934	1367	0.042
1500	400	0.2	0.3	0.12	369.583	1187	0.153	0.847	31.863	417.659	1378	0.039
1500	400	0.2	0.3	0.11	384.295	1203	0.143	0.857	31.694	431.489	1388	0.036
1500	400	0.2	0.3	0.1	399.418	1220	0.132	0.868	31.399	445.424	1399	0.033
1500	400	0.2	0.3	0.09	414.989	1238	0.121	0.879	30.923	459.462	1409	0.029
1500	400	0.2	0.3	0.08	431.055	1257	0.110	0.890	30.201	473.603	1419	0.026
1500	400	0.2	0.3	0.07	447.673	1276	0.098	0.902	29.008	487.846	1429	0.023
1500	400	0.2	0.3	0.06	464.914	1298	0.087	0.913	27.667	502.191	1440	0.019
1500	400	0.2	0.3	0.05	482.869	1321	0.074	0.926	25.751	516.638	1450	0.016

Table A.5: (Case I) The Effect of sum of c_i (w=0.2)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_{1}^{n} \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	400	0.2	0.2	0.3	251.364	1020	0.339	0.661	32.176	310.888	1290	0.088
1500	400	0.2	0.2	0.29	262.723	1031	0.33	0.670	33.13	323.847	1302	0.086
1500	400	0.2	0.2	0.28	274.272	1043	0.321	0.679	34.248	336.919	1313	0.084
1500	400	0.2	0.2	0.27	286.015	1054	0.311	0.689	35.139	350.104	1324	0.083
1500	400	0.2	0.2	0.26	297.959	1066	0.302	0.698	36.186	363.399	1335	0.08
1500	400	0.2	0.2	0.25	310.11	1077	0.292	0.708	36.983	376.803	1346	0.079
1500	400	0.2	0.2	0.24	322.474	1089	0.283	0.717	37.925	390.315	1357	0.077
1500	400	0.2	0.2	0.23	335.058	1101	0.274	0.726	38.802	403.934	1367	0.074
1500	400	0.2	0.2	0.22	347.872	1113	0.264	0.736	39.602	417.659	1378	0.072
1500	400	0.2	0.2	0.21	360.924	1125	0.254	0.746	40.314	431.489	1388	0.07
1500	400	0.2	0.2	0.2	374.226	1138	0.245	0.755	41.141	445.424	1399	0.067
1500	400	0.2	0.2	0.19	387.789	1150	0.235	0.765	41.637	459.462	1409	0.065
1500	400	0.2	0.2	0.18	401.627	1163	0.225	0.775	42.22	473.603	1419	0.063
1500	400	0.2	0.2	0.17	415.755	1176	0.215	0.785	42.655	487.846	1429	0.06
1500	400	0.2	0.2	0.16	430.189	1190	0.205	0.795	43.143	502.191	1440	0.057
1500	400	0.2	0.2	0.15	444.951	1204	0.195	0.805	43.439	516.638	1450	0.055
1500	400	0.2	0.2	0.14	460.061	1218	0.184	0.816	43.518	531.186	1460	0.052
1500	400	0.2	0.2	0.13	475.547	1233	0.174	0.826	43.566	545.834	1470	0.049
1500	400	0.2	0.2	0.12	491.439	1249	0.163	0.837	43.546	560.583	1480	0.046
1500	400	0.2	0.2	0.11	507.772	1265	0.152	0.848	43.193	575.433	1490	0.043
1500	400	0.2	0.2	0.1	524.589	1282	0.141	0.859	42.67	590.383	1500	0.039

Table A.6: (Case I) The Effect of sum of c_i (w=0.1)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	400	0.2	0.1	0.3	356.59	1083	0.352	0.648	45.871	445.424	1399	0.096
1500	400	0.2	0.1	0.29	369.036	1093	0.343	0.657	46.839	459.462	1409	0.095
1500	400	0.2	0.1	0.28	381.678	1103	0.334	0.666	47.753	473.603	1419	0.093
1500	400	0.2	0.1	0.27	394.522	1114	0.324	0.676	48.865	487.846	1429	0.091
1500	400	0.2	0.1	0.26	407.576	1125	0.315	0.685	49.913	502.191	1440	0.089
1500	400	0.2	0.1	0.25	420.847	1135	0.305	0.695	50.616	516.638	1450	0.087
1500	400	0.2	0.1	0.24	434.345	1146	0.296	0.704	51.499	531.186	1460	0.085
1500	400	0.2	0.1	0.23	448.078	1158	0.286	0.714	52.559	545.834	1470	0.083
1500	400	0.2	0.1	0.22	462.059	1169	0.276	0.724	53.236	560.583	1480	0.081
1500	400	0.2	0.1	0.21	476.298	1180	0.266	0.734	53.79	575.433	1490	0.079
1500	400	0.2	0.1	0.2	490.809	1192	0.257	0.743	54.483	590.383	1500	0.076
1500	400	0.2	0.1	0.19	505.607	1204	0.247	0.753	55.023	605.433	1510	0.074
1500	400	0.2	0.1	0.18	520.709	1217	0.237	0.763	55.67	620.583	1520	0.071
1500	400	0.2	0.1	0.17	536.134	1230	0.227	0.773	56.124	635.834	1530	0.069
1500	400	0.2	0.1	0.16	551.902	1243	0.216	0.784	56.359	651.186	1540	0.066
1500	400	0.2	0.1	0.15	568.04	1257	0.206	0.794	56.626	666.638	1550	0.063
1500	400	0.2	0.1	0.14	584.575	1271	0.195	0.805	56.614	682.191	1560	0.06
1500	400	0.2	0.1	0.13	601.54	1285	0.185	0.815	56.286	697.846	1571	0.057
1500	400	0.2	0.1	0.12	618.974	1301	0.174	0.826	56.142	713.603	1581	0.054
1500	400	0.2	0.1	0.11	636.924	1317	0.163	0.837	55.578	729.462	1591	0.051
1500	400	0.2	0.1	0.1	655.443	1334	0.151	0.849	54.795	745.424	1601	0.047

Table A.7: (Case I) The Effect of σ (w=0.3)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	500	0.2	0.3	0.2	230.558	1010	0.239	0.761	29.142	276.110	1238	0.059
1500	490	0.2	0.3	0.2	233.783	1016	0.239	0.761	29.201	279.587	1243	0.059
1500	480	0.2	0.3	0.2	237.025	1022	0.238	0.762	29.236	283.065	1248	0.059
1500	470	0.2	0.3	0.2	240.286	1027	0.237	0.763	29.101	286.543	1253	0.060
1500	460	0.2	0.3	0.2	243.565	1033	0.237	0.763	29.080	290.021	1259	0.060
1500	450	0.2	0.3	0.2	246.865	1039	0.236	0.764	29.029	293.499	1264	0.060
1500	440	0.2	0.3	0.2	250.185	1046	0.236	0.764	29.098	296.976	1269	0.060
1500	430	0.2	0.3	0.2	253.526	1052	0.235	0.765	28.989	300.454	1274	0.060
1500	420	0.2	0.3	0.2	256.889	1058	0.234	0.766	28.848	303.932	1280	0.059
1500	410	0.2	0.3	0.2	260.274	1064	0.234	0.766	28.672	307.410	1285	0.060
1500	400	0.2	0.3	0.2	263.684	1071	0.233	0.767	28.620	310.888	1290	0.060
1500	390	0.2	0.3	0.2	267.118	1077	0.232	0.768	28.374	314.365	1295	0.060
1500	380	0.2	0.3	0.2	270.578	1084	0.232	0.768	28.253	317.843	1301	0.060
1500	370	0.2	0.3	0.2	274.064	1090	0.231	0.769	27.931	321.321	1306	0.060
1500	360	0.2	0.3	0.2	277.578	1097	0.230	0.770	27.735	324.799	1311	0.060
1500	350	0.2	0.3	0.2	281.121	1104	0.230	0.770	27.499	328.277	1316	0.060

Table A.7 (Continued)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	340	0.2	0.3	0.2	284.694	1111	0.229	0.771	27.222	331.754	1322	0.060
1500	330	0.2	0.3	0.2	288.298	1118	0.228	0.772	26.902	335.232	1327	0.060
1500	320	0.2	0.3	0.2	291.935	1125	0.227	0.773	26.535	338.710	1332	0.060
1500	310	0.2	0.3	0.2	295.607	1132	0.227	0.773	26.121	342.188	1337	0.060
1500	300	0.2	0.3	0.2	299.314	1140	0.226	0.774	25.840	345.666	1343	0.059
1500	290	0.2	0.3	0.2	303.060	1147	0.225	0.775	25.324	349.144	1348	0.059
1500	280	0.2	0.3	0.2	306.844	1155	0.224	0.776	24.941	352.621	1353	0.059
1500	270	0.2	0.3	0.2	310.671	1162	0.224	0.776	24.311	356.099	1358	0.059
1500	260	0.2	0.3	0.2	314.541	1170	0.223	0.777	23.814	359.577	1364	0.059
1500	250	0.2	0.3	0.2	318.457	1178	0.222	0.778	23.257	363.055	1369	0.059
1500	240	0.2	0.3	0.2	322.422	1187	0.221	0.779	22.838	366.533	1374	0.058
1500	230	0.2	0.3	0.2	326.438	1195	0.220	0.780	22.149	370.010	1379	0.058
1500	220	0.2	0.3	0.2	330.508	1204	0.220	0.780	21.597	373.488	1385	0.057
1500	210	0.2	0.3	0.2	334.635	1213	0.219	0.781	20.973	376.966	1390	0.057
1500	200	0.2	0.3	0.2	338.823	1222	0.218	0.782	20.272	380.444	1395	0.056

Table A.8: (Case I) The Effect of σ (w=0.2)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	500	0.2	0.2	0.2	335.997	1096	0.253	0.747	43.105	406.780	1373	0.068
1500	490	0.2	0.2	0.2	339.701	1099	0.252	0.748	42.845	410.644	1376	0.068
1500	480	0.2	0.2	0.2	343.429	1103	0.251	0.749	42.740	414.509	1378	0.068
1500	470	0.2	0.2	0.2	347.182	1107	0.250	0.750	42.600	418.373	1381	0.068
1500	460	0.2	0.2	0.2	350.961	1112	0.250	0.750	42.624	422.237	1383	0.068
1500	450	0.2	0.2	0.2	354.766	1116	0.249	0.751	42.411	426.102	1386	0.068
1500	440	0.2	0.2	0.2	358.599	1120	0.248	0.752	42.158	429.966	1388	0.068
1500	430	0.2	0.2	0.2	362.460	1124	0.247	0.753	41.862	433.831	1391	0.068
1500	420	0.2	0.2	0.2	366.351	1129	0.246	0.754	41.735	437.695	1394	0.068
1500	410	0.2	0.2	0.2	370.273	1133	0.246	0.754	41.353	441.559	1396	0.068
1500	400	0.2	0.2	0.2	374.226	1138	0.245	0.755	41.141	445.424	1399	0.067
1500	390	0.2	0.2	0.2	378.213	1142	0.244	0.756	40.665	449.288	1401	0.068
1500	380	0.2	0.2	0.2	382.234	1147	0.243	0.757	40.360	453.153	1404	0.067
1500	370	0.2	0.2	0.2	386.290	1152	0.242	0.758	40.007	457.017	1406	0.067
1500	360	0.2	0.2	0.2	390.384	1157	0.241	0.759	39.603	460.881	1409	0.067
1500	350	0.2	0.2	0.2	394.517	1162	0.240	0.760	39.146	464.746	1411	0.067

Table A.8 (Continued)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	340	0.2	0.2	0.2	398.690	1167	0.239	0.761	38.633	468.610	1414	0.067
1500	330	0.2	0.2	0.2	402.905	1172	0.238	0.762	38.062	472.475	1416	0.067
1500	320	0.2	0.2	0.2	407.164	1178	0.237	0.763	37.671	476.339	1419	0.066
1500	310	0.2	0.2	0.2	411.470	1183	0.236	0.764	36.975	480.203	1421	0.066
1500	300	0.2	0.2	0.2	415.823	1189	0.235	0.765	36.461	484.068	1424	0.066
1500	290	0.2	0.2	0.2	420.227	1194	0.234	0.766	35.628	487.932	1427	0.066
1500	280	0.2	0.2	0.2	424.685	1200	0.233	0.767	34.975	491.797	1429	0.065
1500	270	0.2	0.2	0.2	429.198	1206	0.232	0.768	34.249	495.661	1432	0.065
1500	260	0.2	0.2	0.2	433.769	1212	0.231	0.769	33.445	499.525	1434	0.065
1500	250	0.2	0.2	0.2	438.402	1219	0.230	0.770	32.828	503.390	1437	0.064
1500	240	0.2	0.2	0.2	443.101	1225	0.229	0.771	31.857	507.254	1439	0.064
1500	230	0.2	0.2	0.2	447.868	1232	0.228	0.772	31.069	511.119	1442	0.063
1500	220	0.2	0.2	0.2	452.707	1239	0.227	0.773	30.190	514.983	1444	0.062
1500	210	0.2	0.2	0.2	457.624	1246	0.226	0.774	29.212	518.847	1447	0.062
1500	200	0.2	0.2	0.2	462.623	1253	0.224	0.776	28.129	522.712	1449	0.061

Table A.9: (Case I) The Effect of σ (w=0.1)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_1^n \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	500	0.2	0.1	0.2	449.146	1165	0.267	0.733	58.173	550.478	1500	0.078
1500	490	0.2	0.1	0.2	453.161	1168	0.266	0.734	58.089	554.469	1500	0.078
1500	480	0.2	0.1	0.2	457.206	1170	0.265	0.735	57.710	558.459	1500	0.078
1500	470	0.2	0.1	0.2	461.283	1173	0.264	0.736	57.541	562.450	1500	0.078
1500	460	0.2	0.1	0.2	465.393	1175	0.263	0.737	57.069	566.440	1500	0.078
1500	450	0.2	0.1	0.2	469.537	1178	0.262	0.738	56.809	570.430	1500	0.077
1500	440	0.2	0.1	0.2	473.716	1181	0.261	0.739	56.502	574.421	1500	0.077
1500	430	0.2	0.1	0.2	477.931	1183	0.260	0.740	55.877	578.411	1500	0.077
1500	420	0.2	0.1	0.2	482.184	1186	0.259	0.741	55.466	582.402	1500	0.077
1500	410	0.2	0.1	0.2	486.477	1189	0.258	0.742	55.003	586.392	1500	0.077
1500	400	0.2	0.1	0.2	490.809	1192	0.257	0.743	54.483	590.383	1500	0.076
1500	390	0.2	0.1	0.2	495.184	1195	0.255	0.745	53.905	594.373	1500	0.076
1500	380	0.2	0.1	0.2	499.602	1199	0.255	0.745	53.552	598.363	1500	0.076
1500	370	0.2	0.1	0.2	504.066	1202	0.253	0.747	52.853	602.354	1500	0.075
1500	360	0.2	0.1	0.2	508.576	1206	0.252	0.748	52.380	606.344	1500	0.075
1500	350	0.2	0.1	0.2	513.137	1209	0.251	0.749	51.547	610.335	1500	0.075

Table A.9 (Continued)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	π^I_0	Q^I_{dec}	$\sum_{1}^{n} \alpha_i$	α_0	π^I_s	π^I_c	Q^I_c	$(\pi^I_c-\pi^I_0-\pi^I_s)/\pi^I_c$
1500	340	0.2	0.1	0.2	517.748	1213	0.250	0.750	50.941	614.325	1500	0.074
1500	330	0.2	0.1	0.2	522.412	1217	0.249	0.751	50.266	618.316	1500	0.074
1500	320	0.2	0.1	0.2	527.132	1220	0.247	0.753	49.208	622.306	1500	0.074
1500	310	0.2	0.1	0.2	531.911	1224	0.246	0.754	48.379	626.297	1500	0.073
1500	300	0.2	0.1	0.2	536.750	1229	0.245	0.755	47.784	630.287	1500	0.073
1500	290	0.2	0.1	0.2	541.654	1233	0.243	0.757	46.790	634.277	1500	0.072
1500	280	0.2	0.1	0.2	546.624	1237	0.242	0.758	45.705	638.268	1500	0.072
1500	270	0.2	0.1	0.2	551.665	1242	0.241	0.759	44.854	642.258	1500	0.071
1500	260	0.2	0.1	0.2	556.779	1247	0.240	0.760	43.909	646.249	1500	0.071
1500	250	0.2	0.1	0.2	561.971	1252	0.238	0.762	42.865	650.239	1500	0.070
1500	240	0.2	0.1	0.2	567.246	1257	0.237	0.763	41.716	654.230	1500	0.069
1500	230	0.2	0.1	0.2	572.607	1262	0.235	0.765	40.453	658.220	1500	0.069
1500	220	0.2	0.1	0.2	578.060	1267	0.234	0.766	39.069	662.210	1500	0.068
1500	210	0.2	0.1	0.2	583.611	1273	0.233	0.767	37.917	666.201	1500	0.067
1500	200	0.2	0.1	0.2	589.264	1279	0.231	0.769	36.636	670.191	1500	0.066

APPENDIX B

COMPUTATIONAL RESULTS FOR CASE Ia

Table B.1: (Case Ia) The Effect of σ

System Parameters										
μ	c_0	c_w	$\sum_{1}^{n} c_i$							
1500	0.2	0.2	0.2							
For	Results									
σ	Q^{Ia}_{dec}	Q^{Ia}_{c}	α_0	w1	$\sum_{1}^{n} \alpha_i$	π^{Ia}_{w}	π^{Ia}_{c}	$\pi^{Ia}_{c}(Q^{Ia}_{dec})$	π^{Ia}_{0}	π^{Ia}_{s}
500	719	1373	0.787	0.503	0.213	218.040	406.780	274.882	50.506	6.336
490	725	1376	0.788	0.506	0.212	221.671	410.644	278.095	50.206	6.218
480	731	1378	0.788	0.508	0.212	225.384	414.509	281.285	49.813	6.087
470	738	1381	0.789	0.511	0.211	229.184	418.373	284.800	49.636	5.980
460	744	1383	0.789	0.513	0.211	233.072	422.237	287.945	49.052	5.822
450	751	1386	0.790	0.516	0.210	237.050	426.102	291.419	48.680	5.688
440	758	1388	0.790	0.518	0.210	241.122	429.966	294.871	48.208	5.541
430	765	1391	0.791	0.521	0.209	245.289	433.831	298.303	47.632	5.381
420	773	1394	0.791	0.523	0.209	249.556	437.695	302.070	47.270	5.244
410	780	1396	0.792	0.526	0.208	253.924	441.559	305.459	46.476	5.059
400	788	1399	0.792	0.528	0.208	258.397	445.424	309.188	45.893	4.897
390	796	1401	0.793	0.530	0.207	262.978	449.288	312.896	45.195	4.723
380	805	1404	0.793	0.533	0.207	267.671	453.153	316.951	44.710	4.571
370	813	1406	0.793	0.535	0.207	272.478	457.017	320.620	43.769	4.373
360	822	1409	0.794	0.537	0.206	277.405	460.881	324.639	43.037	4.197
350	831	1411	0.794	0.540	0.206	282.454	464.746	328.639	42.176	4.009
340	840	1414	0.795	0.542	0.205	287.629	468.610	332.620	41.179	3.811
330	850	1416	0.795	0.545	0.205	292.936	472.475	336.957	40.387	3.634

Table B.1: (Continued)

System Parameters			
μ	c_0	c_w	$\sum_1^n c_i$
1500	0.2	0.2	0.2

For	Results									
σ	Q^{Ia}_{dec}	Q^{Ia}_{c}	α_0	w1	$\sum_1^n \alpha_i$	π^{Ia}_{w}	π^{Ia}_{c}	$\pi^{Ia}_{c}(Q^{Ia}_{dec})$	π^{Ia}_{0}	π^{Ia}_{s}
320	860	1419	0.795	0.547	0.205	298.378	476.339	341.279	39.453	3.447
310	870	1421	0.796	0.549	0.204	303.960	480.203	345.583	38.373	3.250
300	881	1424	0.796	0.552	0.204	309.688	484.068	350.250	37.489	3.073
290	892	1427	0.796	0.554	0.204	315.566	487.932	354.903	36.451	2.887
280	903	1429	0.797	0.556	0.203	321.599	491.797	359.540	35.249	2.692
270	915	1432	0.797	0.558	0.203	327.795	495.661	364.547	34.236	2.516
260	927	1434	0.797	0.560	0.203	334.158	499.525	369.541	33.050	2.333
250	940	1437	0.797	0.562	0.203	340.696	503.390	374.908	32.044	2.167
240	953	1439	0.798	0.565	0.202	347.416	507.254	380.264	30.854	1.995
230	966	1442	0.798	0.567	0.202	354.324	511.119	385.608	29.468	1.816
220	980	1444	0.798	0.569	0.202	361.429	514.983	391.331	28.247	1.655
210	994	1447	0.798	0.571	0.202	368.739	518.847	397.044	26.816	1.488
200	1009	1449	0.799	0.573	0.201	376.263	522.712	403.139	25.537	1.339

Table B.2: (Case Ia) The Effect of c_0

System Parameters			
μ	σ	c_w	$\sum_1^n c_i$
1500	500	0.2	0.2

For	Results									
c_0	Q^{Ia}_{dec}	Q^{Ia}_c	α_0	w1	$\sum_1^n \alpha_i$	π^{Ia}_w	π^{Ia}_c	$\pi^{Ia}_c(Q^{Ia}_{dec})$	π^{Ia}_0	π^{Ia}_s
0.3	661	1238	0.790	0.425	0.210	148.948	276.110	188.642	35.248	4.446
0.29	667	1252	0.790	0.433	0.210	155.588	288.559	196.828	36.624	4.617
0.28	673	1266	0.790	0.441	0.210	162.291	301.149	205.127	38.044	4.793
0.27	680	1280	0.789	0.449	0.211	169.056	313.880	213.818	39.757	5.005
0.26	686	1294	0.789	0.456	0.211	175.882	326.748	222.351	41.276	5.193
0.25	691	1307	0.789	0.464	0.211	182.768	339.754	230.700	42.578	5.354
0.24	697	1321	0.789	0.472	0.211	189.711	352.894	239.448	44.184	5.553
0.23	703	1334	0.788	0.480	0.212	196.711	366.168	248.309	45.840	5.758
0.22	708	1347	0.788	0.488	0.212	203.767	379.574	256.959	47.258	5.934
0.21	714	1360	0.788	0.495	0.212	210.877	393.112	266.035	49.008	6.150
0.2	719	1373	0.787	0.503	0.213	218.040	406.780	274.882	50.506	6.336
0.19	724	1386	0.787	0.511	0.213	225.255	420.577	283.823	52.042	6.526
0.18	729	1399	0.787	0.519	0.213	232.521	434.503	292.858	53.615	6.721
0.17	734	1412	0.787	0.527	0.213	239.838	448.558	301.987	55.228	6.921
0.16	739	1424	0.786	0.535	0.214	247.204	462.739	311.210	56.880	7.126
0.15	744	1437	0.786	0.542	0.214	254.618	477.047	320.527	58.573	7.336

Table B.2: (Continued)

System Parameters			
μ	σ	c_w	$\sum_1^n c_i$
1500	500	0.2	0.2

For	Results									
c_0	Q^{Ia}_{dec}	Q^{Ia}_c	α_0	w1	$\sum_1^n \alpha_i$	π^{Ia}_w	π^{Ia}_c	$\pi^{Ia}_c(Q^{Ia}_{dec})$	π^{Ia}_0	π^{Ia}_s
0.14	749	1450	0.786	0.550	0.214	262.080	491.482	329.937	60.306	7.551
0.13	753	1462	0.786	0.558	0.214	269.588	506.043	339.038	61.723	7.727
0.12	758	1475	0.785	0.566	0.215	277.143	520.729	348.627	63.533	7.951
0.11	762	1487	0.785	0.574	0.215	284.743	535.541	357.889	65.012	8.135
0.1	767	1500	0.785	0.581	0.215	292.387	550.478	367.656	66.900	8.369
0.09	771	1513	0.784	0.589	0.216	300.075	565.541	377.078	68.442	8.561
0.08	775	1525	0.784	0.597	0.216	307.806	580.729	386.576	70.014	8.756
0.07	779	1538	0.784	0.605	0.216	315.579	596.043	396.149	71.615	8.955
0.06	784	1550	0.784	0.612	0.216	323.395	611.482	406.262	73.658	9.209
0.05	788	1563	0.783	0.620	0.217	331.252	627.047	415.995	75.327	9.417
0.04	792	1576	0.783	0.628	0.217	339.149	642.739	425.804	77.027	9.628
0.03	796	1588	0.783	0.636	0.217	347.086	658.558	435.688	78.758	9.844
0.02	800	1601	0.782	0.644	0.218	355.063	674.503	445.647	80.521	10.063
0.01	804	1614	0.782	0.652	0.218	363.079	690.577	455.681	82.315	10.287

Table B.3: (Case Ia) The Effect of c_w

System Parameters			
μ	σ	c_0	$\sum_1^n c_i$
1500	500	0.2	0.2

For	Results									
c_w	Q^{Ia}_{dec}	Q^{Ia}_c	α_0	w1	$\sum_1^n \alpha_i$	π^{Ia}_w	π^{Ia}_c	$\pi^{Ia}_c(Q^{Ia}_{dec})$	π^{Ia}_0	π^{Ia}_s
0.3	661	1238	0.790	0.525	0.210	148.948	276.110	188.642	35.248	4.446
0.29	667	1252	0.790	0.523	0.210	155.588	288.559	196.828	36.624	4.617
0.28	673	1266	0.790	0.521	0.210	162.291	301.149	205.127	38.044	4.793
0.27	680	1280	0.789	0.519	0.211	169.056	313.880	213.818	39.757	5.005
0.26	686	1294	0.789	0.516	0.211	175.882	326.748	222.351	41.276	5.193
0.25	691	1307	0.789	0.514	0.211	182.768	339.754	230.700	42.578	5.354
0.24	697	1321	0.789	0.512	0.211	189.711	352.894	239.448	44.184	5.553
0.23	703	1334	0.788	0.510	0.212	196.711	366.168	248.309	45.840	5.758
0.22	708	1347	0.788	0.508	0.212	203.767	379.574	256.959	47.258	5.934
0.21	714	1360	0.788	0.505	0.212	210.877	393.112	266.035	49.008	6.150
0.2	719	1373	0.787	0.503	0.213	218.040	406.780	274.882	50.506	6.336
0.19	724	1386	0.787	0.501	0.213	225.255	420.577	283.823	52.042	6.526
0.18	729	1399	0.787	0.499	0.213	232.521	434.503	292.858	53.615	6.721
0.17	734	1412	0.787	0.497	0.213	239.838	448.558	301.987	55.228	6.921
0.16	739	1424	0.786	0.495	0.214	247.204	462.739	311.210	56.880	7.126
0.15	744	1437	0.786	0.492	0.214	254.618	477.047	320.527	58.573	7.336

Table B.3: (Continued)

System Parameters			
μ	σ	c_0	$\sum_1^n c_i$
1500	500	0.2	0.2

For	Results									
c_w	Q^{Ia}_{dec}	Q^{Ia}_c	α_0	w1	$\sum_1^n \alpha_i$	π^{Ia}_w	π^{Ia}_c	$\pi^{Ia}_c(Q^{Ia}_{dec})$	π^{Ia}_0	π^{Ia}_s
0.14	749	1450	0.786	0.490	0.214	262.080	491.482	329.937	60.306	7.551
0.13	753	1462	0.786	0.488	0.214	269.588	506.043	339.038	61.723	7.727
0.12	758	1475	0.785	0.486	0.215	277.143	520.729	348.627	63.533	7.951
0.11	762	1487	0.785	0.484	0.215	284.743	535.541	357.889	65.012	8.135
0.1	767	1500	0.785	0.481	0.215	292.387	550.478	367.656	66.900	8.369
0.09	771	1513	0.784	0.479	0.216	300.075	565.541	377.078	68.442	8.561
0.08	775	1525	0.784	0.477	0.216	307.806	580.729	386.576	70.014	8.756
0.07	779	1538	0.784	0.475	0.216	315.579	596.043	396.149	71.615	8.955
0.06	784	1550	0.784	0.472	0.216	323.395	611.482	406.262	73.658	9.209
0.05	788	1563	0.783	0.470	0.217	331.252	627.047	415.995	75.327	9.417
0.04	792	1576	0.783	0.468	0.217	339.149	642.739	425.804	77.027	9.628
0.03	796	1588	0.783	0.466	0.217	347.086	658.558	435.688	78.758	9.844
0.02	800	1601	0.782	0.464	0.218	355.063	674.503	445.647	80.521	10.063
0.01	804	1614	0.782	0.462	0.218	363.079	690.577	455.681	82.315	10.287

Table B.4: (Case Ia) The Effect of Sum of c_i

System Parameters			
μ	σ	c_0	c_w
1500	500	0.2	0.2

For	Results									
$\sum_1^n c_i$	Q^{Ia}_{dec}	Q^{Ia}_c	α_0	w1	$\sum_1^n \alpha_i$	π^{Ia}_w	π^{Ia}_c	$\pi^{Ia}_c(Q^{Ia}_{dec})$	π^{Ia}_0	π^{Ia}_s
0.3	625	1238	0.687	0.425	0.313	140.798	276.110	179.402	33.311	5.294
0.29	634	1252	0.697	0.433	0.303	147.841	288.559	188.074	34.807	5.426
0.28	644	1266	0.707	0.441	0.293	155.028	301.149	197.188	36.573	5.588
0.27	653	1280	0.717	0.449	0.283	162.359	313.880	206.199	38.134	5.706
0.26	662	1294	0.727	0.457	0.273	169.839	326.748	215.375	39.720	5.816
0.25	672	1307	0.737	0.464	0.263	177.470	339.754	225.016	41.592	5.954
0.24	681	1321	0.747	0.472	0.253	185.257	352.894	234.528	43.228	6.043
0.23	690	1334	0.757	0.480	0.243	193.202	366.168	244.202	44.881	6.120
0.22	700	1347	0.767	0.488	0.233	201.311	379.574	254.365	46.834	6.220
0.21	709	1360	0.777	0.496	0.223	209.588	393.112	264.372	48.516	6.268
0.2	719	1373	0.787	0.503	0.213	218.040	406.780	274.882	50.506	6.336
0.19	729	1386	0.798	0.511	0.202	226.671	420.577	285.568	52.512	6.385
0.18	739	1399	0.808	0.519	0.192	235.489	434.503	296.430	54.527	6.413
0.17	749	1412	0.818	0.526	0.182	244.502	448.558	307.467	56.547	6.418
0.16	759	1424	0.828	0.534	0.172	253.717	462.739	318.678	58.564	6.397
0.15	769	1437	0.838	0.542	0.162	263.143	477.047	330.062	60.571	6.348

Table B.4: (Continued)

System Parameters			
μ	σ	c_0	c_w
1500	500	0.2	0.2

For	Results									
$\sum_1^n c_i$	Q^{Ia}_{dec}	Q^{Ia}_{c}	α_0	w1	$\sum_1^n \alpha_i$	π^{Ia}_{w}	π^{Ia}_{c}	$\pi^{Ia}_{c}(Q^{Ia}_{dec})$	π^{Ia}_{0}	π^{Ia}_{s}
0.14	780	1450	0.849	0.550	0.151	272.791	491.482	342.004	62.910	6.304
0.13	791	1462	0.859	0.557	0.141	282.672	506.043	354.132	65.237	6.224
0.12	802	1475	0.869	0.565	0.131	292.798	520.729	366.445	67.542	6.105
0.11	813	1487	0.880	0.573	0.120	303.185	535.541	378.941	69.813	5.943
0.1	825	1500	0.890	0.580	0.110	313.848	550.478	392.031	72.417	5.765
0.09	837	1513	0.901	0.588	0.099	324.807	565.541	405.315	74.975	5.533
0.08	849	1525	0.911	0.596	0.089	336.081	580.729	418.791	77.469	5.242
0.07	863	1538	0.922	0.603	0.078	347.695	596.043	433.316	80.684	4.936
0.06	876	1550	0.933	0.611	0.067	359.680	611.482	447.618	83.411	4.527
0.05	890	1563	0.944	0.618	0.056	372.067	627.047	462.556	86.436	4.054
0.04	906	1576	0.955	0.625	0.045	384.897	642.739	478.586	90.172	3.517
0.03	922	1588	0.966	0.632	0.034	398.220	658.558	494.836	93.758	2.858
0.02	939	1601	0.977	0.639	0.023	412.093	674.503	511.749	97.583	2.073
0.01	957	1614	0.988	0.646	0.012	426.594	690.577	529.332	101.606	1.132

APPENDIX C

COMPUTATIONAL RESULTS FOR ALTERNATIVE CONTRACTING SCHEME

Table C.1: Computations (Alternative Contract)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	Q^I_{dec}	Q^I_c	π^I_0	π^I_c	$\pi^I_0+\pi^I_s$	π^I_s	π^A_0	π^A_s
1500	500	0.2	0.2	0.2	1096	1373	335.997	406.780	379.103	43.105	352.874	53.906
1500	450	0.2	0.2	0.2	1116	1386	354.766	426.102	397.177	42.411	373.921	52.181
1500	400	0.2	0.2	0.2	1138	1399	374.226	445.424	415.367	41.141	395.617	49.807
1500	350	0.2	0.2	0.2	1162	1411	394.517	464.746	433.663	39.146	418.137	46.608
1500	300	0.2	0.2	0.2	1189	1424	415.823	484.068	452.285	36.461	441.374	42.694
1500	250	0.2	0.2	0.2	1219	1437	438.402	503.390	471.230	32.828	465.605	37.785
1500	200	0.2	0.2	0.2	1253	1449	462.623	522.712	490.752	28.129	490.937	31.775
1500	400	0.3	0.3	0.2	982	1163	160.804	187.987	177.977	17.173	167.647	20.340
1500	400	0.25	0.3	0.2	1030	1230	211.145	247.857	233.864	22.718	220.823	27.034
1500	400	0.2	0.3	0.2	1071	1290	263.684	310.888	292.304	28.620	276.663	34.225
1500	400	0.15	0.3	0.2	1106	1346	318.118	376.803	352.786	34.668	335.079	41.724
1500	400	0.1	0.3	0.2	1138	1399	374.226	445.424	415.367	41.141	395.617	49.807
1500	400	0.05	0.3	0.2	1166	1450	431.836	516.638	479.484	47.649	458.556	58.082
1500	400	0.2	0.3	0.3	936	1163	153.342	187.987	172.838	19.496	163.717	24.270
1500	400	0.2	0.3	0.25	1004	1230	205.884	247.857	230.320	24.436	218.012	29.845
1500	400	0.2	0.3	0.2	1071	1290	263.684	310.888	292.304	28.620	276.663	34.225
1500	400	0.2	0.3	0.15	1141	1346	327.617	376.803	358.966	31.349	340.210	36.593
1500	400	0.2	0.3	0.1	1220	1399	399.418	445.424	430.817	31.399	409.872	35.552
1500	400	0.2	0.3	0.05	1321	1450	482.869	516.638	508.620	25.751	488.709	27.929
1500	400	0.3	0.2	0.2	1071	1290	263.684	310.888	292.304	28.620	276.663	34.225
1500	400	0.25	0.2	0.2	1106	1346	318.118	376.803	352.786	34.668	335.079	41.724
1500	400	0.2	0.2	0.2	1138	1399	374.226	445.424	415.367	41.141	395.617	49.807
1500	400	0.15	0.2	0.2	1166	1450	431.836	516.638	479.484	47.649	458.556	58.082
1500	400	0.1	0.2	0.2	1192	1500	490.809	590.383	545.292	54.483	523.523	66.860
1500	400	0.05	0.2	0.2	1216	1550	551.034	666.638	612.578	61.545	590.577	76.061

Table C.1: (Continued)

System Parameters					Results							
μ	σ	c_0	w	$\sum_{i=1}^{n} c_i$	Q^I_{dec}	Q^I_c	π^I_0	π^I_c	$\pi^I_0+\pi^I_s$	π^I_s	π^A_0	π^A_s
1500	400	0.2	0.2	0.3	1020	1290	251.364	310.888	283.539	32.176	270.469	40.419
1500	400	0.2	0.2	0.25	1077	1346	310.110	376.803	347.093	36.983	331.102	45.701
1500	400	0.2	0.2	0.2	1138	1399	374.226	445.424	415.367	41.141	395.617	49.807
1500	400	0.2	0.2	0.15	1204	1450	444.951	516.638	488.390	43.439	465.292	51.346
1500	400	0.2	0.2	0.1	1282	1500	524.589	590.383	567.259	42.670	541.478	48.905
1500	400	0.2	0.2	0.05	1385	1550	618.252	666.638	653.084	34.833	628.404	38.233
1500	400	0.4	0.1	0.2	1071	1290	263.684	310.888	292.304	28.620	276.663	34.225
1500	400	0.35	0.1	0.2	1106	1346	318.118	376.803	352.786	34.668	335.079	41.724
1500	400	0.3	0.1	0.2	1138	1399	374.226	445.424	415.367	41.141	395.617	49.807
1500	400	0.25	0.1	0.2	1166	1450	431.836	516.638	479.484	47.649	458.556	58.082
1500	400	0.2	0.1	0.2	1192	1500	490.809	590.383	545.292	54.483	523.523	66.860
1500	400	0.15	0.1	0.2	1216	1550	551.034	666.638	612.578	61.545	590.577	76.061
1500	400	0.1	0.1	0.2	1239	1601	612.415	745.424	681.476	69.061	659.473	85.950
1500	400	0.05	0.1	0.2	1259	1654	674.871	826.803	751.121	76.250	731.074	95.729
1500	400	0.2	0.1	0.4	981	1290	241.832	310.888	275.974	34.142	266.251	44.636
1500	400	0.2	0.1	0.35	1032	1346	297.108	376.803	337.322	40.214	324.929	51.874
1500	400	0.2	0.1	0.3	1083	1399	356.590	445.424	402.461	45.871	387.109	58.314
1500	400	0.2	0.1	0.25	1135	1450	420.847	516.638	471.463	50.616	453.302	63.336
1500	400	0.2	0.1	0.2	1192	1500	490.809	590.383	545.292	54.483	523.523	66.860
1500	400	0.2	0.1	0.15	1257	1550	568.040	666.638	624.666	56.626	598.800	67.838
1500	400	0.2	0.1	0.1	1334	1601	655.443	745.424	710.238	54.795	681.697	63.726
1500	400	0.2	0.1	0.05	1442	1654	759.676	826.803	805.008	45.332	776.354	50.449

APPENDIX D

COMPUTATIONAL RESULTS FOR CASE III

Table D.1: Computations of Set I (Case III)

Computation Number		1	2	3	4	5	6	7	8	9
System Parameters	μ_1	1000	1000	1000	1000	1000	1000	1000	1000	1000
	μ_2	500	500	500	500	500	500	500	500	500
	μ_t	1500	1500	1500	1500	1500	1500	1500	1500	1500
	σ_1	254.951	244.949	304.302	254.951	244.949	304.302	254.951	244.949	304.302
	σ_2	158.114	114.891	167.332	158.114	114.891	167.332	158.114	114.891	167.332
	σ_t	300.000	300.142	300.080	300.000	300.142	300.080	300.000	300.142	300.080
	p_1	1	1	1	1	1	1	1	1	1
	p_2	1.05	1.05	1.05	1	1	1	0.8	0.8	0.8
	ρ	0	0.3	-0.3	0	0.3	-0.3	0	0.3	-0.3
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	c_d	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	w	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Results	π^{III}_c	624.571	624.491	624.258	606.097	606.057	605.552	537.830	538.104	535.614
	Q^{III}_c	1482	1482	1482	1464	1463	1465	1372	1370	1381
	π^{III}_0	494.547	387.598	508.251	488.108	386.148	499.655	463.315	380.409	466.646
	Q^{III}_{dec}	1069	807	1115	1065	807	1111	1050	804	1089
	π^{III}_s	11.433	1.118	16.124	11.085	1.118	15.654	9.863	1.084	13.277
	π^{III}_d	17.874	13.653	17.955	16.591	13.653	16.895	12.626	12.311	12.186
	α_d	0.162	0.152	0.167	0.162	0.152	0.166	0.161	0.152	0.164
	$(\pi^{III}_0 - \pi^{I}_0)/\pi^{I}_0$	0.445	0.119	0.580	0.426	0.114	0.553	0.354	0.098	0.451

Table D.1 (Continued)

Computation Number		10	11	12	13	14	15	16	17	18
System Parameters	μ_1	1000	1000	1000	1000	1000	1000	1000	1000	1000
	μ_2	500	500	500	500	500	500	500	500	500
	μ_t	1500	1500	1500	1500	1500	1500	1500	1500	1500
	σ_1	254.951	244.949	304.302	254.951	244.949	304.302	254.951	244.949	304.302
	σ_2	158.114	114.891	167.332	158.114	114.891	167.332	158.114	114.891	167.332
	σ_t	300.000	300.142	300.080	300.000	300.142	300.080	300.000	300.142	300.080
	p_1	1	1	1	1	1	1	1	1	1
	p_2	1.05	1.05	1.05	1	1	1	0.8	0.8	0.8
	ρ	0	0.3	-0.3	0	0.3	-0.3	0	0.3	-0.3
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	c_d	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	w	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Results	π^{III}_c	776.368	776.230	776.059	756.192	756.072	755.745	679.516	679.637	678.048
	Q^{III}_c	1554	1554	1554	1538	1539	1539	1461	1460	1468
	π^{III}_0	601.995	468.583	620.544	595.255	467.081	611.535	569.077	461.125	576.605
	Q^{III}_{dec}	1080	812	1130	1077	812	1126	1065	810	1109
	π^{III}_s	12.437	1.176	17.996	12.156	1.176	17.479	11.085	1.153	15.424
	π^{III}_d	22.021	16.253	22.650	20.791	16.253	21.275	16.591	15.154	16.392
	$(\pi^{III}_0 - \pi^{I}_0)/\pi^{I}_0$	0.413	0.088	0.533	0.397	0.084	0.511	0.335	0.071	0.424

Table D.2: Computations of Set II (Case III)

Computation Number		21	22	23	24	25	26	27	28	29
System Parameters	μ_1	500	500	500	500	500	500	500	500	500
	μ_2	1000	1000	1000	1000	1000	1000	1000	1000	1000
	μ_t	1500	1500	1500	1500	1500	1500	1500	1500	1500
	σ_1	158.114	114.891	167.332	158.114	114.891	167.332	158.114	114.891	167.332
	σ_2	254.951	244.949	304.302	254.951	244.949	304.302	254.951	244.949	304.302
	σ_t	300.000	300.142	300.080	300.000	300.142	300.080	300.000	300.142	300.080
	p_1	1	1	1	1	1	1	1	1	1
	p_2	1.05	1.05	1.05	1	1	1	0.8	0.8	0.8
	ρ	0	0.3	-0.3	0	0.3	-0.3	0	0.3	-0.3
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	c_d	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	w	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Results	π^{III}_c	623.775	624.494	622.774	580.516	581.199	579.570	414.450	414.991	413.737
	Q^{III}_c	1482	1482	1482	1462	1462	1462	1353	1353	1353
	π^{III}_0	536.539	535.057	538.560	502.040	500.841	503.767	371.853	371.578	372.527
	Q^{III}_{dec}	1226	1218	1235	1211	1204	1219	1127	1122	1133
	π^{III}_s	34.999	33.325	37.026	31.639	30.322	33.262	17.524	17.030	18.194
	π^{III}_d	9.825	11.194	7.659	8.755	9.963	6.818	4.497	4.980	3.535

Table D.2 (Continued)

Computation Number		30	31	32	33	34	35	36	37	38
System Parameters	μ_1	500	500	500	500	500	500	500	500	500
	μ_2	1000	1000	1000	1000	1000	1000	1000	1000	1000
	μ_t	1500	1500	1500	1500	1500	1500	1500	1500	1500
	σ_1	158.114	114.891	167.332	158.114	114.891	167.332	158.114	114.891	167.332
	σ_2	254.951	244.949	304.302	254.951	244.949	304.302	254.951	244.949	304.302
	σ_t	300.000	300.142	300.080	300.000	300.142	300.080	300.000	300.142	300.080
	p_1	1	1	1	1	1	1	1	1	1
	p_2	1.05	1.05	1.05	1	1	1	0.8	0.8	0.8
	ρ	0	0.3	-0.3	0	0.3	-0.3	0	0.3	-0.3
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	c_d	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	w	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Results	π^{III}_c	775.570	776.291	774.551	730.515	731.199	729.549	554.844	555.384	554.097
	Q^{III}_c	1554	1554	1554	1538	1538	1538	1453	1453	1453
	π^{III}_0	660.848	658.485	663.777	624.973	622.949	627.569	487.396	486.542	488.758
	Q^{III}_{dec}	1259	1249	1269	1247	1237	1256	1182	1175	1189
	π^{III}_s	43.516	40.922	46.289	40.229	37.817	42.531	25.933	24.845	27.096
	π^{III}_d	12.621	14.469	9.757	11.528	13.102	8.902	6.986	7.816	5.456

Table D.3: Computations of Set III (Case III)

Computation Number		41	42	43	44	45	46	47	48	49
System Parameters	μ_1	1500	1500	1500	1500	1500	1500	1500	1500	1500
	μ_2	750	750	750	750	750	750	750	750	750
	μ_t	2250	2250	2250	2250	2250	2250	2250	2250	2250
	σ_1	400.000	370.810	461.519	400.000	370.810	461.519	400.000	370.810	461.519
	σ_2	206.155	167.332	232.379	206.155	167.332	232.379	206.155	167.332	232.379
	σ_t	450.000	450.254	450.168	450.000	450.254	450.168	450.000	450.254	450.168
	p_1	1	1	1	1	1	1	1	1	1
	p_2	1.05	1.05	1.05	1	1	1	0.8	0.8	0.8
	ρ	0	0.3	-0.3	0	0.3	-0.3	0	0.3	-0.3
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	c_d	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	w	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Results	π^{III}_c	936.782	936.578	936.267	909.003	908.897	908.183	806.061	806.754	803.106
	Q^{III}_c	2223	2224	2223	2196	2196	2197	2061	2058	2072
	π^{III}_0	698.226	573.944	745.299	691.055	571.885	733.726	663.090	563.727	688.974
	Q^{III}_{dec}	1485	1194	1619	1482	1194	1614	1468	1190	1588
	π^{III}_s	9.089	1.498	18.498	8.938	1.498	18.032	8.262	1.458	15.767
	π^{III}_d	26.487	20.031	27.916	25.320	20.031	26.369	20.583	18.229	19.733
	$(\pi^{III}_0 - \pi^{I}_0)/\pi^{I}_0$	0.380	0.107	0.551	0.366	0.103	0.527	0.311	0.088	0.434

Table D.3 (Continued)

Computation Number		50	51	52	53	54	55	56	57	58
System Parameters	μ_1	1500	1500	1500	1500	1500	1500	1500	1500	1500
	μ_2	750	750	750	750	750	750	750	750	750
	μ_t	2250	2250	2250	2250	2250	2250	2250	2250	2250
	σ_1	400.000	370.810	461.519	400.000	370.810	461.519	400.000	370.810	400.000
	σ_2	206.155	167.332	232.379	206.155	167.332	232.379	206.155	167.332	206.155
	σ_t	450.000	450.254	450.168	450.000	450.254	450.168	450.000	450.254	450.000
	p_1	1	1	1	1	1	1	1	1	1
	p_2	1.05	1.05	1.05	1	1	1	0.8	0.8	0.8
	ρ	0	0.3	-0.3	0	0.3	-0.3	0	0.3	0
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	c_d	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	v	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Results	π^{III}_c	1164.476	1164.347	1163.964	1134.175	1134.105	1133.479	1018.853	1019.252	1018.853
	Q^{III}_c	2331	2332	2331	2308	2308	2309	2194	2191	2194
	π^{III}_0	847.416	693.766	908.176	839.963	691.635	896.114	810.755	683.175	810.755
	Q^{III}_{dec}	1498	1202	1637	1495	1201	1633	1484	1198	1484
	π^{III}_s	9.770	1.583	20.265	9.609	1.572	19.860	9.039	1.540	9.039
	π^{III}_d	32.289	24.243	34.394	30.834	23.667	32.820	26.091	22.028	26.091
	$(\pi^{III}_0 - \pi^{I}_0)/\pi^{I}_0$	0.342	0.076	0.501	0.330	0.073	0.481	0.284	0.060	0.340

Table D.4: Additional Computations (Case III)

Computation Number		61	62	63	64	65	66	67	68	69
System Parameters	μ_1	1000	1000	1000	1000	1000	1000	1000	1000	1000
	μ_2	500	500	500	500	500	500	500	500	500
	μ_t	1500	1500	1500	1500	1500	1500	1500	1500	1500
	σ_1	273.861	260.768	313.050	273.861	260.768	313.050	273.861	260.768	313.050
	σ_2	122.474	90.000	126.491	122.474	90.000	126.491	122.474	90.000	126.491
	σ_t	300.000	300.302	300.402	300.000	300.302	300.402	300.000	300.302	300.402
	p_1	1	1	1	1	1	1	1	1	1
	p_2	1	1	1	1.05	1.05	1.05	0.8	0.8	0.8
	ρ	0	0.3	-0.3	0	0.3	-0.3	0	0.3	-0.3
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	c_d	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	w	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Results	π^{III}_c	605.947	605.920	605.241	624.498	624.409	623.987	537.083	537.512	535.020
	Q^{III}_c	1464	1464	1465	1482	1482	1482	1376	1373	1382
	π^{III}_0	438.909	351.461	458.123	442.686	352.449	463.732	424.086	347.542	436.161
	Q^{III}_{dec}	932	729	985	934	730	987	926	728	976
	π^{III}_s	3.669	0.486	5.784	3.734	0.492	5.883	3.477	0.481	5.357
	π^{III}_d	16.062	11.037	17.032	16.915	11.520	17.822	13.777	10.575	13.935

Table D.5: Comparable Scenarios in Case I

System Parameters	μ_1	1000	1000	1000	1000	1000	1000	1500	1500	1500	1500	1500	1500
	σ_1	254.951	244.949	304.303	254.951	244.949	304.303	400.000	370.810	461.519	400.000	370.810	461.519
	w	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.1
	c_0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	$\sum_{i=1}^{n} c_i$	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Results	π^1_0	342.205	346.515	321.670	426.175	430.690	404.813	505.838	518.306	480.457	631.459	644.497	605.100
	Q^I_{dec}	824	827	812	855	856	850	1231	1239	1217	1280	1284	1274
	π^1_s	32.747	32.337	34.447	41.572	40.753	44.304	49.769	48.529	51.815	63.384	61.528	66.494
Compared with computation :		1-4-7	2-5-8	3-6-9	10-13-16	11-14-17	12-15-18	41-44-47	42-45-48	43-46-49	50-53-56	51-54-57	52-55-58

APPENDIX E

MAPLE CODE

```
mu1:=1000; mu2:=500; mu3:=evalf(mu1+mu2);
sigma1:=evalf(sqrt(60000)); sigma2:=evalf(sqrt(13200)); cor:=0.3;
sigma3:=evalf(sqrt(sigma1^2+sigma2^2+2*cor*sigma1*sigma2));
c0:=0.15; w:=0.2; c:=0.15; cd:=0.05; p1:=1; p2:=1.05; a0:=0; l0:=0;

fl12:=(x1,x2)->1/(2*Pi*sigma1*sigma2*(sqrt(1-cor^2)))*exp(-1/(2*(1-cor^2))*((x1-
mu1)^2/sigma1^2-2*cor*(x1-mu1)/sigma1*(x2-mu2)/sigma2+(x2-
mu2)^2/sigma2^2));
fl1:=x->1/(sqrt(2*Pi*sigma1^2))*exp(-0.5/(sigma1^2)*(x-mu1)^2);
fl11:=x->int(fl1(t1),t1=0..x);
cfl1:=x->int(fl1(t1),t1=x..infinity);
fl2:=(x1,x2)->fl12(x1,x2)/fl1(x1);
cfl2:=(x1,x2)->int(fl2(x1,t2),t2=x2..infinity);
fl3:=x->1/(sqrt(2*Pi*sigma3^2))*exp(-0.5/(sigma3^2)*(x-mu3)^2);
cfl3:=x->int(fl3(t1),t1=x..infinity);

sel:=x->int(cfl1(t4),t4=0..x); sel3:=x->int(cfl3(t50),t50=0..x);

p2:=x->int(int(t11*fl12(t6,t11),t11=0..x-t6),t6=0..x)+int(int(x*fl12(t14,t15),t15=x-
t14..infinity),t14=0..x)-int(int((t20)*fl12(t20,t21),t21=x-t20..infinity),t20=0..x);

p4:=x->int(cd/int(fl12(t9,t12),t12=x-t9..infinity)*fl1(t9)*(int(y*fl12(t9,y),y=0..x-
t9)+int((x-t9)*fl12(t9,t17),t17=x-t9..infinity)),t9=0..x);
profit:=x->(p1-c/cfl3(x))*sel(x)+(p2-c/cfl3(x))*p2(x)-p4(x)-(c0+w)*x;
a:=x->c/cfl3(x); supplyprof:=x->-c*x+a0*sel(x)+a0*p2(x);
supplyprof3:=x->-c*x+a0*sel3(x); disprof:=x->p4(x)-cd*(x-sel(x));
centprof:=x->(p1+cd)*sel(x)+p2*p2(x)-(c0+w+c+cd)*x;
```

```
z1:=evalf(profit(l0)); l0:=l0+10; z2:=evalf(profit(l0)); if (z2>z1)
then while (z2>z1) do l0:=l0+10; z1:=z2; z2:=evalf(profit(l0)); od;
fi; l0:=l0-11; z2:=evalf(profit(l0)); if (z2>z1)then while (z2>z1)
do l0:=l0-1; z1:=z2; z2:=evalf(profit(l0)); od;

l0:=l0+1;
z2:=evalf(profit(l0)); else l0:=l0+2; z2:=evalf(profit(l0)); while
(z2>z1) do l0:=l0+1; z1:=z2; z2:=evalf(profit(l0)); od; l0:=l0-1;
z2:=evalf(profit(l0)); fi; a0:=evalf(a(l0));
z3:=evalf(supplyprof(l0)); z4:=evalf(disprof(l0));

l1:=l0;
z5:=evalf(centprof(l1));
l1:=l1+10;
z6:=evalf(centprof(l1));
if (z6>z5) then
while (z6>z5) do
l1:=l1+10;
z5:=z6;
z6:=evalf(centprof(l1));
od;
fi;
l1:=l1-11;
z6:=evalf(centprof(l1));
if (z6>z5)then
while (z6>z5) do
l1:=l1-1;
z5:=z6;
z6:=evalf(centprof(l1));
od;
l1:=l1+1;
z6:=evalf(centprof(l1));
else
l1:=l1+2;
z6:=evalf(centprof(l1));
while (z6>z5) do
l1:=l1+1;
z5:=z6;
z6:=evalf(centprof(l1));
od;
l1:=l1-1;
z6:=evalf(centprof(l1));
fi;
```

```
f:=fopen(`crun2.txt`,WRITE); fprintf(f,`%f\n`,mu1);
fprintf(f,`%f\n`,mu2); fprintf(f,`%f\n`,mu3);
fprintf(f,`%f\n`,sigma1); fprintf(f,`%f\n`,sigma2);
fprintf(f,`%f\n`,sigma3); fprintf(f,`%f\n`,p1);
fprintf(f,`%f\n`,p2); fprintf(f,`%f\n`,cor); fprintf(f,`%f\n`,c0);
fprintf(f,`%f\n`,c); fprintf(f,`%f\n`,cd); fprintf(f,`%f\n`,w);
fprintf(f,`%f\n`,z6); fprintf(f,`%f\n`,l1); fprintf(f,`%f\n`,z2);
fprintf(f,`%f\n`,l0); fprintf(f,`%f\n`,z3); fprintf(f,`%f\n`,z4);
fprintf(f,`%f\n`,a0); fclose(f);
```