INVESTIGATION OF STUDENTS' PROBABILISTIC MISCONCEPTIONS

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ALI IHSAN MUT

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Prof. Dr. Canan ÖZGEN
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Ömer GEBAN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Safure BULUT
Supervisor

Examining Committee Members

Prof. Dr. Ömer GEBAN

Assoc. Prof. Dr. Safure BULUT

Assist. Prof. Dr. Ahmet ARIKAN

Assist. Prof. Dr. Oya YERIN GUNERI

Assist. Prof. Dr. Soner YILDIRIM

ABSTRACT

INVESTIGATION OF STUDENTS' PROBABILISTIC MISCONCEPTIONS

MUT, Ali Ihsan

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The purpose of the study was to investigate the students' probabilistic misconceptions with respect to grade level, previous instruction on probability and gender.

The sample of the study was 885 students from different types of the schools (general high schools, private collages, Anatolian high school, vocational high schools, and elementary schools) and from grade levels (5, 6, 7, 8, 9 and 10). The sample represented a range of students with respect to socio-economic level and cultural background.

To collect data for the study Probabilistic Misconception Test (PMT) and a questionnaire were administered. The test consisted of 14 problems from 8 probabilistic misconception types. Its content validity was tested.

The data of the study were analyzed by means of SPSS. Each misconception type is investigated with respect to all variables. The results of the study revealed that:

(a) The frequencies of all misconception types varied across grade levels.

(b) The percentages of students who received instruction on probability in the school were higher than those who did not received instruction in terms of misconceptions on Effect of Sample Size and Time Axis Fallacy. In addition, the other misconception types were more frequent among the students who did not receive a certain instruction on probability than the students who received a certain instruction probability before the study;

(c) The frequencies of all misconception types varied across gender.

Keywords: Probability, Misconception, Grade Level, Gender

OZ

OGRENCILERIN OLASILIK KONUSUNDAKI KAVRAM YANILGILARININ INCELENMESI

MUT, Ali Ihsan
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Bu calismanin amaci olasilik konusunda ogrencilerin gosterdikleri kavram yanilgilarini, ogrencilerin sinif seviyelerine, onceden olasilik konusu ile ilgili ogretim alip almadiklarina ve cinsiyetlerine gore incelemektir.

Calismanin orneklemini degisik okul cesidi (genel liseleri, ozel liseler, Anadolu liseleri, meslek liseleri, ve orta okullar) ve degisik sinif seviyesinden (5.sinif, 6.sinif, 7.sinif, 8.sinif, 9.sinif ve 10.sinif) secilmis, sosyo-ekonomik ve kulturel bakimindan farklilik gosteren 885 ogrenci olusturmustur.

Gerekli verileri toplamak amaciyla Olasilik Kavram Yanilgisi Testi (OKYT) ve bir anket uygulanmistir. Test 8 farkli kavram yanilgisi cesidini iceren ve 14 olasilik probleminden olusmustur. Testin kapsam gecerliligi test edilmistir.

Arastirmanin verileri SPSS paket programi kullanilarak analiz edilmistir. Her kavram yanilgi cesidi butun degiskenlere gore incelenecektir. Calismanin sonuclari sunlari ortaya cikarmistir:

a) Ogrencilerin kavram yanilgisi cesitlerinin sikligi sinif seviyelerine gore degismektedir.
b) Orneklem Buyuklugunun Etkisi Yanilgisi'nda ve Zaman Etkisi Yanilgisi'nda onceden olasilik ogretimi almis ogrencilerin yuzdesinin ogretim almamis ogrencilerin yuzdesinden daha yuksek oldugu gozlenmistir. Buna ek olarak, diger olasilik kavram yanilgisi

cesitlerinde ise olasilik konusunda ogretim almamis ogrencilerin yuzdelerinin, ogretim almis ogrencilere gore daha yuksektir

c) Cinsiyete gore tum olasilik kavram yanilgisi cesitlerinin sikligi degismistir.

Anahtar Kelimeler: Olasilik, Kavram Yanilgisi, Sinif Seviyesi, Cinsiyet

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LIST OF ABBREVIATIONS

C	: Correct
Cat	: Category
F	: Female
GL	: Grade Level
I	: Incorrect
M	: Misconception
NA	: No Answer
n	: Number of Cases
OKYT	: Olasilik Kavram Yanilgisi Testi
PIoP	: Previous Instruction on Probability
PMT	: Probability Misconception Test
w.r.t.	: with respect to

CHAPTER 1

INTRODUCTION

Students are sometimes presented with the information in the schools as well as through the media that is expressed in probabilistic terms. They may be told that smoking increases the risk of health problems such as lung cancer, emphysema, or hearth disease or that using seat belts when riding a car reduces the risk of severe injury in case of an accident. They are told that the probability of winning a lottery is small but they are reminded that it can be done by seeing pictures of happy winners (Madsen, 1995).

In the past two decades several influential organizations, including National Council of Supervisors of Mathematics in 1978, NACOME in 1975, UNESCO in 1972, CEEB in 1959, and the Cambridge Conference on School Mathematics in 1963 have acknowledged the role that probability and statistics play in our society (Hope & Kelly, 1983).

In recent years, there have been many developments in science and technology. In order to be able to follow these developments, teaching mathematics and probability that is related to real life have been getting importance (Bulut, Ekici & Iseri, 1998; Cockcroft, 1982; Department of Education and Science and the Welsh Office, 1991; National Council of Teachers of Mathematics, 1989; Nemetz, 1980). However; there are difficulties in teaching and learning probability. According to the research literature, students have difficulty in learning probability concepts, for the following reasons:

- They have difficulty with prerequisite concepts including fractions, decimals, percents (Carpenter et al., 1981), or operations on sets (Bar-on & Or-Bach, 1988).
- They have difficulty in interpreting the problems (Carpenter et al., 1981; Mosteller, 1967).

- There is a conflict between probability ideas and students' experiences and how they view the world (Hope & Kelly, 1983).
- They develop distaste for probability because it is taught in a highly abstract and formal way (Garfield & Ahlegen, 1988).
- They perceive interdependence between unrelated events, i.e. Gambler' s Fallacy (Hope & Kelly, 1983).
- The less able pupils have difficulty with thinking of the probability of occurrence of events as a continuum ranging from "certain" to “impossible" (Baron & Or-Bach, 1988).
- They cannot understand the idea of "Conditional Probability" (Bar-on & Or-Bach, 1988).
- They have difficulty in determining the probability of compound events (Carpenter et al, 1981).

Adults as well as children confuse probability concepts and have some difficulties in problem solving. For example, Hope and Kelly (1983) pointed out that people:

- are unaware of highly ambiguous everyday expressions of probability;
- have undue confidence in the reliability of small samples;
- have a tendency to confuse the categories of unusual events with those of low probability events;
- have difficulty estimating the frequency of many salient or memorable events.

As seen in the reasons, probabilistic misconceptions are very important to increase the efficiency of the instruction on probability. That is, it is important to know how the students interpret the statements about probability; our teaching may be more effective if we know what the misconceptions are so that they may be addressed directly (Madsen, 1995).

Stochastics as a scientific discipline is usually first taught at the college level. (We will use the term stochatics to refer to the study of probability and statistics, as is common in Europe). Several recommendations from teachers for overcoming difficulties in learning stochastics can be generalized from the literature. (e.g. Garlfield & Ahlgren, 1988)

Therefore teachers should-

1. recognize and confront common errors in students' probabilistic thinking;
2. create situations requiring probabilistic reasoning that correspond to the students' views of the world.

There are several studies done about probabilistic misconceptions of students. For example, in the study of Fischbein and Schnarch (1997), the misconceptions are categorized into 7 topics namely representatives, negative and positive regency effects, compound and simple events, conjunction fallacy, sample size, heuristic of availability, and time axis fallacy. The subjects of the study were, the students in grade levels 5, in grade level 7, in grade level 9, in grade level 11 and college students who were prospective teachers specializing in mathematics. They did not previously receive any instruction in probability. They found a relation between age and understanding the probability. Some misconceptions grew stronger with age. Only one misconception was stable across ages. However, they could not give any theoretical explanation to this result.

Madsen (1995) also conducted research study on secondary students' concepts of probability. He categorized them into three groups: representativeness, outcome approach and availability.

Although there are a few research studies on probability in Turkey (e.g. Bulut, 1994; Bulut, Ekici & Iseri, 1998; Bulut & Sahin, 2003; Bulut, Kazak & Yetkin, 1999; Bulut, Yetkin, & Kazak, 2002; Cankoy, 1989), I could not reach any research study published on determining the misconceptions of the learners.

The purpose of the present study is to investigate the basic misconceptions of the students on probability with respect to grade level, gender and previous instruction on probability. In the present study we examine eight misconception types: Representatives, Negative and Positive Regency Effects, Compound and Simple Events, Effect of Sample Size, Conjunction Fallacy, Heuristic of Availability, Time Axis Fallacy and Equiprobability Bias. In the questions, the students were required to select the right choice according to their opinion.

Probability is one of the most important subjects in mathematics. In spite of the importance of probability, there are fewer studies on this area not only in our country but also in other world countries. Moreover, not only students but also the teachers have difficulty in probability. Understanding the nature and the types of misconceptions in probability, frequent among students will be helpful to teachers and mathematics educators to overcome these difficulties.

CHAPTER 2

REVIEW OF RELATED LITERATURE

In this chapter, theoretical bases of the present study will be explained, and related studies done will be reviewed. Theoretical background, probabilistic thinking, the concept of intuition, the concept of randomness, mathematics learning and instruction on "Probability", and gender differences in Mathematics are presented in this chapter.

2.1 Theoretical Background

There is much psychological theory to suggest that Man capable of probabilistic judgments from every earliest stages of cognitive development. For example, Information Theory models of Man often presuppose that either the information itself will be stochastic (predictable with varied degrees of certainty) or the processing of that information will be probabilistic (Hawkins & Kapadia, 1984)

Piaget and Inhelder analyzed children's thinking about probability into the usual stages (pre-operational, concrete, operational, etc.), culminating in a formal understanding of probability through combinatorics (Amir and Williams). Piaget and Inhelder, in their book, have concluded that during the intuitive period (before the age of 6-7), the child is not able to distinguish clearly between chance and necessary phenomena. That distinction appears during the concrete-operational period together with elementary forms of probabilistic estimations. The concept of probability, as a formal construct, develops only during the formal operational stage and it represents a synthesis between necessary and the possible (Piaget and Inhelder, 1975; original in French, 1951).

Piaget and Inhelder suggested that children in the primary grades were able to identify all possible outcomes in a one-stage experiment; Jones (1974) concluded that significant numbers of grades one through three children were not able to list the outcomes of one stage experiment. Consistent with Jones's finding, Borovcnik and Bentz (1991) observed that more than 62 percent of 11-year-olds in Green's study (1982) students were not able to solve both of one stage sample space items.

Fischbein (1991) have suggested that the concept of "possible" may develop before the concept of "certainty". According to results of his study, some children develop mathematically mature language for certain and impossible events before they can use it for possible events.

In his study, Green concluded that most English pupils finish secondary school without achieving the level of formal operations. According to him, the reasoning ability was the main significant factor associated with students' level of understanding, explaining 44 percent of variance.

Parallel to Green's conclusion, that is, with the fact that, most English 16 year old do not reach Piaget's formal stage, Kahnemann, Slovic and Tversky showed that even adults reason in situations of uncertainty using heuristics.

Fischbein (1975) showed that some intuitions in young children's thinking are important in helping their pre-formal probabilistic thinking. These intuitions are a product of personal experience (Fischbein, 1987).

2.2 Misconceptions

Many educators, researchers and mathematics teachers have become concerned about students' misconceptions. Therefore, there are increasing numbers of studies in mathematics education that have focused on students' misconceptions in the different fields of mathematics.

According to the results of studies reported in recent years, children constructed an impressive body of informal knowledge from their environment before their formal education in schools. Their informal knowledge constitutes the child's total belief system about the world and how it works (Gilbert & Watts, 1983). These beliefs can be thought as pre-instructional conceptions which affect their learning. Misconception is the subcategory of pre-instructional conception, which is contradictory with the mathematical concepts.

Misconceptions do not simply signify a lack of knowledge, factual errors, or incorrect definitions. They represent explanations of phenomena constructed by students in response to their prior knowledge and experience.

Misconceptions can be defined also, as the ideas that students have about natural phenomena that are inconsistent with mathematical conceptions, are pervasive, stable and often resistant to change at least through traditional instruction. Misconceptions are seen in: **(a)** people's tendency to be more confident when making predictions of the basis of redundant information than with independent information although the latter has greater predictive validity (Kahnemann & Tversky, 1973) and **(b)** in people's readiness to find interpretable patterns in random sequences (Furby, 1973; Fama, 1965) and significant relationships in mere coincidence (Chapman & Chapman, 1969). From another perspective, Meyer (1993) stated that misconceptions might arise from two sources: (a) from errors in understanding new information or (b) from previous misunderstanding remaining a part of the newly formed knowledge. Therefore, the mathematics teachers try to reconcile new knowledge with students' existing conceptions, because students' prior knowledge is an important factor that affects students' learning of a new concept.

In the lights of given information above, remediation of misconceptions could be achieved if students' misconceptions and their sources are taken into account.

2.3 Probabilistic Thinking

There has been considerable research into students' probabilistic thinking (e.g. Fischbein, Nello & Marino, 1991; Fischbein & Schnarch, 1997; Jones, Langrall, Thornton & Mogill, 1997; Piaget & Inhelder, 1975; Shaughnessy, 1992).

In the present study, the term *Probabilistic Thinking* is used to describe children's thinking in response to any probability situation. In particular, researchers advocated the use of a general instructional model in which research-based knowledge of students' thinking is used to inform classroom instruction (Jones, Langrall, Thornton, Mogill, 1999).

There have been various studies published concerning the developmental aspects of probabilistic thinking, starting especially with the book of Piaget and Inhelder: *La Genése de l'Idée de Hasard chez l'Enfant (1951).* The studies done by Fischbein (1975); Fischbein and Gazit (1984); Fischbein, Nello and Marino (1991); Garfield and Ahlgren (1988); Green (1983); Hawkins and Kapadia (1984) and Shaughnessy (1992) could be useful to see the developmental aspects of probabilistic intuitions. The main work for misconceptions in statistical and probabilistic reasoning was edited by Kahnemann, Slovic and Tversky (1982).

According to the study of Jones, Langrall, Thornton and Mogill (1997) for the children to exhibit probabilistic thinking, there is need for them to understand probability concepts, which are *multifaceted* and *develop* over time. In order to capture the manifold nature of probabilistic thinking, their framework incorporates four key constructs. Three of these -sample space, probability of an event, and probability comparisons-have been investigated by several researchers. Few studies on the fourth construct, *conditional probability*, have been directed to young children although interpretations have been drawn from data involving conditional probability.

One technique used in the assessment of children's understanding of probability has been to present to a subject, two urns containing different proportions of balls of two colors and to ask which urn would be better to choose if one wanted to draw out at random a ball of a specified color. This technique was used by Piaget and Inhelder (1951), Siegler (1981), Green (1982), and Singer and Resnick (1992). Others, such as Hoemann and Ross (1972), have done similar work involving the comparison of sectors of spinners. It has been argued by some researchers (e.g., Fischbein, 1975, pp.82-89) that in such experiments children may be making perceptual rather than probabilistic judgments.

Although there has been considerable research into students' probabilistic thinking, there has not been sufficient research on the development and evaluation of instructional program on probability. Fischbein's book potentially offers more guidance on how to modify the development of probability concepts. Piaget, in contrast, tends to define the "status quo", providing reasons for inaction or delay on the part of teacher. In the recent years, the researchers have pointed out the ways to bridge the gap between learning and teaching.

A different approach to misconceptions in probability was made by Amir and Williams. They stated that some intuitions, inclinations and biases might be affected by cultural factors. As a result of their study, they found that certain intuitions, approaches, biases and heuristics noted in the literature take a strong and common forming 11 year old thinking, e.g. the outcome approach, representativeness, availability, equiprobability. In addition, Fischbein, Nello and Marino also indicate that cultural influences on probabilistic reasoning might be important.

The history of probability is full of mathematicians making errors. Some maintain that probability was initiated by a chance correspondence from Chevalier de Meré to Blaise Pascal about the occurrence of a double six in 24 tosses of a pair of dice. De Meré tried to apply the rule of three: if a bet on one specific result out of six in four tosses is worthwhile, so is a bet on one specific result out of thirty-six in twenty-four tosses because 6:4 = 36:24. Another famous example relates to the possibility of a head and a tail in tossing two coins. Here, a number of mathematicians have assigned a probability of 1/3 as they have erroneously assumed an equally likely sample space of three possibilities (two heads and two tails or a head and a tail).

There is a growing movement to introduce elements of statistics and probability into the secondary and even elementary school curriculum, as a part of basic literacy in mathematics (Scheaffer, 1984; Swift, 1982). Moreover, statistics and probability were major themes in recent publications of the National Council of Teachers of Mathematics (NTCM) (Shufelt, 1983; Shulte, 1981).

In modern, technologically advanced societies, being about to cope with a vast amount of information is crucial to many aspects of our daily life. Most of the information on which we base our daily judgments is *uncertain*. In other words, we routinely have to reason about and act on the basis of *probabilities*. (Sedlmeier, 2000)

Moreover, in recent recommendations, the importance of having all students develops an awareness of probability concepts and applications have been recognized (National Council of Teachers of Mathematics, 1989). Because of this emphasis on probability in the school curriculum, there is need for further, ongoing research into learning and teaching of probability (Shaughnessy, 1992). It is understood that the future for research in stochastic, looks very bright.

There are two types of studies in the literature on probabilistic thinking. The first type describes how people think; the second type is concerned with influencing how people think. The first type investigates primitive conceptions or intuitions of probability, misconceptions fallacies in thinking, judgmental biases, and so forth. The second type is concerned with influencing beliefs or conceptions, even changing them if possible. It is true that the first type has been carried out by primarily by *psychologists* and the second type primarily by *mathematics educator* (Shaughnessy, 1992).

Garfield and Ahlgren (1988a, 1988b) suggest that cooperative research endeavors between *psychologists* and *mathematics educators* will accomplish research goals much more effectively than the isolated efforts that we have seen so far in stochastic. The work of Scholz and Bentrup (1984) and Konold (1989a) are examples of cooperative research efforts between psychologists and mathematics educators that we need to encourage.

The relationship between the natural, intuitive approaches individuals' hold with regard to probabilistic situations and the formal, mathematically based solutions was the central issue revealed by the most of these studies. To illustrate, Tversky and Kahnemann studied about heuristics including availability, representativeness and anchoring (1973). Fischbein (1975) mentioned about the negative recency effect, and also the notions of certain, possible and impossible events studied by Fischbein and Gazit (1984). "Comp ound Events" was mentioned by Lecoutre and Durand (1988).

Because of increasing importance of probability in all around world, new mathematics curricula for elementary and secondary school are being introduced in countries all around the world. These curricula reflect a change in beliefs about how probability should be taught. As Shaughnessy (1993) stated, Probability is often included in the secondary school curriculum only as a short unit inside a course. As a result, many students do not have the opportunity to study these topics and teachers often skip them. This situation is not different in our country. The teachers in some semesters can not complete the curriculum on time. Since, probability subject takes place towards to the end of semester at grade level 8; teachers may have not an opportunity to teach some topics such as probability because of lack of time. This situation is very unfortunate because perhaps no other branch of the mathematical sciences is as important as probability for all students.

2.4 The Concept of Intuition

The concept of *Intuition* can be defined as a cognition that appears subjectively as self-evident, directly acceptable, holistic, coercive, and extrapolative (Fischbein, 1987). An intuitive cognition is distinguished from an analytically and logically based cognition by the feeling of obviousness, of intrinsic certainty. To illustrate, we are all sure that, the sum of interior angles of a triangle is 180^0 because we have been taught this or because we can prove it. However, it is not obvious, it must be so. On the other hand, the fact that the shortest distance between two points is a straight line subjectively appears to be absolutely true without need any formal or empirical proof. In the first case we deal with a nonintuitive cognition and in the second with an intuitive cognition (Fischbein & Schnarch, 1997).

The evolution of probabilistic intuitions with age has not been extensively studied. However, various intuitively based misconceptions related to the notion of infinity were relatively stable across ages, beginning at the formal operational period, according to the study of Fischbein, Tirosh and Hess (1979).

2.5 The Concept of Randomness

Randomness has been interpreted in various ways during different periods in history (Battanero, Serrano, & Green, 1998; Bennett, 1993). For example; the concept of randomness is something we all feel we understand, but to provide an acceptable definition is by no means straightforward. Randomness to us is more of an intuition and we all know that intuition can let us down (Green, 1997).

From ancient times to the beginning of the Middle Ages, randomness was considered to be the opposite of something that had known causes. With first theoretical developments of probability, for example in the Liber de Ludo Alea by Cardano, randomness was related to equiprobability because these developments were closely linked to games of chance for which the principle of equal probabilities is reasonable. In later conceptions randomness was lined to the frequentist and to the subjective approaches to probability (Bennett, 1993).

It is seen that, research into children's perceptions of randomness started with Piaget and Inhelder (1951). They investigated children's understanding of patterns in two-dimensional random distributions. They prepared a mechanism to stimulate raindrops falling on paving stones. When asked where the next raindrop would fall, young children (6-9 years old) allocated the raindrops in approximately equal numbers on each pavement square, thereby producing a uniform distribution.

In Green's study in which he used paper -and-pencil version of Piaget's task, showed that ability to recognize randomness does not improve with age.

2.6 Mathematics Learning and Probability Teaching

Mathematics learning is a process in which students reorganize their mathematical thinking to resolve situations that are problematic for them (Jones, Langrall, Thornton & Mogill 1997). Mathematical learning is an interactive as well as a constructive process. (Cobb et al., 1993).

Research-based knowledge of students' thinking is increasingly being identified as an important component of instruction because it has been shown that this kind of knowledge is useful to teachers as they plan and implement instruction in class sessions. (Fennema et al., 1996; Fennema, Franke, Carpenter & Carey, 1993).

The introduction of a new topic must always preceded by a systematic psycho-didactical investigation. This is true for mathematics especially true for probabilities. The cultural environment, the ensemble of existing curricula concerning other domains, the socio-economic level of population, the philosophy behind the didactical methodology etc., may have a certain impact on the children's receptivity topic. This assertion is true for probabilities (Fischbein, Nello & Marino 1991).

Are there optimum teaching and learning techniques which take account of the child's spontaneous conceptions of probabilistic notions while developing his understanding of the formal knowledge of probability? (Hawkins & Kapadia, 1984). According to Hawkins and Kapadia (1984), the present authors feel that these questions have not yet been answered by the available research findings. We still have little idea about the conceptions of probability children of various ages have.

After this study, it can be understood that, in the psychological meaning, the concept of probability seems to be much more complex than it is usually considered. At this point one can ask that "Is there a right way to teach probability?" As Shaughnessy (1993) stated students' conceptions are often deeply entrenched in past experiences and are very difficult to change. It is important to make students aware of how beliefs and conceptions can affect decisions under uncertainty.

There has been an expanding body of research on students' thinking about whole numbers (e.g., Carpenter, Fennema, Peterson, Chiang and Loef), geometry (e.g., Fuys, Geddes and Tischler, 1988), fractions (e.g. Mack, 1995) and ratio and proportion (e.g., Lamon, 1993) used to guide in instruction. In the light of these studies, a program could be developed and to implemented on probability in classes.

A teaching program can bring significant improvement (Fischbein & Gazit, 1984). Introducing real-world applications to mathematics classes is one of the best educational ways of motivating students in classes. Probability is an area of mathematics with many interesting applications and it is the branch of mathematics concerned with making rational statements about phenomena that possess an element of uncertainty. When mathematical probability is used correctly, it is an effective tool in legal decision-making. Most students in probability classes will find a discussion of its possible uses and misuses in the legal profession interesting and stimulating (Halpern, 1987).

In the study of Fischbein and Gazit (1983), they taught 9 classes of older children (10-13 years) probability up to simple and compound events. They found a clear improvement with age and found that two biased intuitions were improved by teaching- the representativeness tendency and the negative regency tendency.

Madsen (1997) stated that in teaching about probability, the teaching might be more effective if we know what the misconceptions are so that they may be addressed directly. If the teaching program does not care about possible intuitive biases the students will continue to mislead the learner despite the conceptual structures he has been taught. Moreover, to improve students' conceptions of randomness, and probability, Green advocated experimental activities and encouraged explicit classroom discussion of the language of probability. Because Green found that the students' verbal abilities were inadequate for describing probabilistic situations. In addition, he concluded that (a) the ratio concept is crucial to a conceptual understanding of probability and is not well understood (b) students are weak when it comes to understanding and using the common language of probability such as "at least" or "certain" or impossible; (c) students are particularly are weak in their concepts of randomness, stability of frequencies and inference. (Shaugnessy, 1993)

In addition, so as to construct such a model in which students do not have any difficulty in understanding probability concept; misconceptions, biases, emotional tendencies and a large variety of misunderstandings should be considered. Moreover, in the study, there were instances where students' probabilistic reasoning appear to be related to their beliefs about the world and the events. Therefore, a few stereotypes of religious, superstitious, casual and suspicious thinkers emerged.

Newer approaches suggest an active learning format where students first make predictions about the chance of occurrence for different outcomes, then do experiments with random devices such as spinners, dice and coins, record their results and compare the experimental probabilities generated to their original predictions. Indeed, several researchers have recommended this method as a way to encourage students to confront and correct their misconceptions about chance events (e.g. Batanero, Serrano & Garfield; Godino et al, 1987; delMas & Bart, 1989; Shaughnessy, 1992). Since the students have misconceptions and have incorrect views about probability and randomness, Garfield (1995) suggests that effective teaching be based on the knowledge of students' preconceptions. Garfield also stated that when students learn something new, they construct their own meaning.

According to Fischbein and Gazit, new intuitive attitudes can be developed only through personal involvement of the learner in a *practical activity*. Intuitions (cognitive beliefs) cannot be modified by verbal explanations. Therefore, a teaching program, which intends to develop an improved and efficient intuitive background for probability concepts and strategies along with the corresponding formal knowledge, must provide the learner with frequent opportunities to live actively, even emotionally stochastic situations. In such situations, he will confront his plausible expectations with empirical obtained outcomes. To illustrate, in the study of Jones, Langrall, Thornton, and Mogill (1997), they aimed to teach some concepts such as sample space, probability of an event, probability comparisons and conditional probability by means of some kind of games such as colored spinners.

However, according to Konold's research on probabilistic reasoning (1985), merely having students make predictions and compare these to experimental data is not sufficient to make students to change their conceptions, because enough data are rarely collected to reveal the correct patterns of outcomes, students' attention are limited and data variability is typically ignored.

Children do have some probabilistic understanding when comparing urns and that at least some children develop sound mathematical language to describe impossible and certain events. This suggests that the teaching of probability measures with a number line may be effective and that it may also be effective in providing helpful strategies for comparing proportions in general.

However, although the pedagogical features of instructional program had a clearly developed theoretical rationale, the major focus of the evaluation was on students learning, not on instructional practice.

In the study, there exists another important result, which should be treated carefully by teachers or educators. The students (even adolescents) believe that an outcome of a stochastic experiment depends only on chance, no matter what the given conditions. This fact is another important factor, which influences the students' reactions.

Green (1979; 1983) and Truran (1994) have discussed students' understanding of probability concepts for students. Konold (1991) reports on beliefs of college students about probabilities. In his study, he bases much of his work, on direct interviews with students who explain to him their thought process when addressing certain probability problems. There are several models discussed in the study of Kahneman, Slovic, and Tversky (1982), Garfield and Ahlgren (1988) or Konold (1989).

Research in stochastic has found that misconceptions of probability are difficult to remove (at least for some of the students) despite best efforts at instruction. Similarly, research in problem solving has determined that instruction in problem solving heuristics and strategies is not sufficient to improve some students' problem solving abilities. The problem-solving researcher has begun to investigate the role of metacognition. Garfalo and Lester (1985) identify two primary aspects of metacognition: knowledge of cognition and regulation of cognition. Knowledge of cognition includes knowledge of strategies and heuristics, but also includes self knowledge, such as beliefs and attitudes. Regulation of cognition includes our monitoring and decision making mechanism as we mentally step outside ourselves and reflect on the processes and progress of solving a problem. We must begin to pay attention to the meta-cognitive aspects of thinking under uncertainty, both in our teaching and in our research in stochastic (Shaughnessy, 1992).

In Turkey, there are few studies on teaching probability (e.g. Bulut, 1994; Cankoy, 1989). Bulut (1994) and Cankoy (1989) conducted study on 8^{th} grade level. Bulut found that students taught by cooperative learning method scored significantly better on the Probability Achievement Test than those taught by traditional lecture method. However, there were no statistically significant mean differences on PAT scores among any other pairs of groups. Cankoy (1989) found that there was a significant mean difference in the favor of the mathematics laboratory group over those taught traditionally.

2.7 Gender Differences in Mathematics

There have been many studies on gender differences in mathematics achievement to investigate the reasons and to overcome these differences on mathematics (Gallagher & Delisi, 1994; Leder, 1992; Levi, 2000; Marshall, 1984). In the present study the results of some of the studies on gender difference are given.

According to Leder (1992), male students express the need for mathematics for better occupational opportunities more commonly than female students. The commonly supported societal belief that mathematics is male oriented domain demonstrates that the differences in mathematics disadvantageous to girls arise from the social and cultural reasons (Damarin, 1995; Leder, 1992). Researchers pointed out that the reasons for females not achieving in mathematical and scientific fields have focused on difference in cognitive abilities, personality characteristics and differences between in school and out of school experiences.

Casey et al (1995) found that boys outperform girls in mental rotation and SAT-M scores and when mental rotation ability was adjusted, the gender difference in SAT-M scores was eliminated. However, Marshall (1984) found that girls were better than boys in solving computations, whereas boys were better than girls in solving story problems. According to the investigation of the results of International Assessment of Educational Progress on 1988 and 1991 by Beller and Gafli (2000), they have found that boys performed better than girls. There was a correlation between gender effect size and item difficulty. Kimball (1989) also focused on the differences between girls' and boys' approaches to mathematics learning. The researcher tried to explain boys' greater performance in mathematics with their autonomous learning style that was based on boys' rebellion to teachers' solutions and tendency to develop their independent solutions. On the other hand, girls more dependency to teachers and their greater focus on classroom behavior led to rote learning style.

In spite of the fact that the female students were successful in computations in algorithms, the female students did not understand the structure of the problem and to select the appropriate solution algorithms (Low & Over, 1993). In the study of Ferrini and Mundy (1987), the female students were better than male students on calculus test which was administered after eight-week calculus course.

These findings are consistent with some other studies that dealt with individual differences that can be let to gender differences in mathematics achievement. Gallagher and DeLisi (1994) investigated the gender differences in students' use of problem solving strategies. The researchers hypothesized that male students were mere able to use unconventional strategies than female students mathematics. According to the study of Secil (2002), boys outperform girls in unconventional strategy use in geometry test.

However, according to Kimball and Marshal (1984), gender differences in mathematics achievement do not appear in early ages of education, however the differences are seen after junior high school years.

Fennema (1974) concluded was that there were no gender related differences in elementary school children' s mathematics achievement and little evidence that such differences exist in high school learners. However, there was some indication that males got better scores in higher level cognitive tasks and females got higher scores in lower level cognitive tasks.

We could reach several international studies on gender difference on Probability (e.g. Dusek & Hill, 1970; Hanna, 1986; Krietler, Zigler & Krietler, 1983; Moran & McCullars, 1979). Hanna (1986) stated that there was no significant mean difference with respect to 8th grade students' probability achievement. In addition, Moran and McCullars (1979) found that female 1st year university students had significantly higher mean scores than males had. Dusek and Hill (1970) and Krietler, Zigler and Krietler (1983) found that males outperformed significantly than females. While Dusek and Hill who studied 4th and 5th grade level students, Krietler, Zigler and Krietler conducted research on 10th grade students.

In our country there is a few study on gender difference with respect to probability achievement (Bulut, Yetkin & Kazak, 2002; Bulut, 1994). For example Bulut, Yetkin and Kazak (2002) stated that there was a statistically significant mean difference on preservice secondary mathematics teachers' probability achievement in the favor of male. In addition Bulut (1994) found that 8th grade female students had significantly higher mean score on probability achievement than males had.

As a summary, although male students were better in some topics taught in school mathematics, the male students performed better than female students in other topics of mathematics.

CHAPTER 3

METHOD

In this chapter, research design of the study, main and sub-problems, definition of terms, variables, subject of the study, instruments, procedure, analysis of the data and assumptions are included.

3.1 Research Design of the Study

The purpose of the study was to investigate the students' probabilistic misconceptions with respect to grade level, gender and previous instruction on probability.

To achieve the purposes, the students were applied to Probabilistic Misconception Test. The test is in multiple-choice formats. Each problem was related to a well-known probabilistic misconception. The test was administered to each group of students during a regularly scheduled class, under usual classroom conditions, in a session lasting about one-lesson hour (40-45 minutes).

For this study the survey research techniques were utilized. Three major characteristics of a survey research can be found in this study (Fraenkel & Wallen, 1996):

1. Information was collected from a group of students in different grade level in order to determine their misconceptions on probability.
2. The main way used in collecting the information was asking questions. The answers to these questions by the students constituted the data of the study.
3. Information was collected from a sample rather than from every member of the population.

Of the two different types of survey researches, the present study was a "cross - sectional survey". Information was obtained from a sample that has been drawn from a predetermined population and the information is collected at just one point in time (Fraenkel & Wallen, 1996).

3.2 Main Problem and Sub-problems of the Study

The main research problem of the present study is the following:

"What are the students' misconceptions (*Representativeness, Positive and Negative Recency Effects, Simple and Compound Events, Effect Sample Size, Conjunction Fallacy, Heuristic Availability, The Time Axis Fallacy and Equiprobability Bias*) on probability with respect to grade level, previous instruction on probability and gender?"

The sub-problems related with the main problem are as follows:

<u>*Sub-problems*</u>:

I. What are the students' probabilistic misconceptions (*Representativeness, Positive and Negative Recency Effects, Simple and Compound Events, Effect Sample Size, Conjunction Fallacy, Heuristic Availability, The Time Axis Fallacy and Equiprobability Bias*) with respect to grade levels?

II. What are the students' probabilistic misconceptions (*Representativeness, Positive and Negative Recency Effects, Simple and Compound Events, Effect Sample Size, Conjunction Fallacy, Heuristic Availability, The Time Axis Fallacy and Equiprobability Bias*) with respect to previous instruction on probability?

III. What are the students' probabilistic misconceptions (*Representativeness, Positive and Negative Recency Effects, Simple and Compound Events, Effect Sample Size, Conjunction Fallacy, Heuristic Availability, The Time Axis Fallacy and Equiprobability Bias*) with respect to gender?

A questionnaire was administered to the subject of the present study to gather their personal information: Gender, grade level, name of the school and whether or not they learnt probability before this study.

3.3 Definition of Terms:

In this part, some terms that are used in the present study are defined to be clear to prevent any misunderstanding.

1. *Represantativeness* (Fallacy of Regression): In this misconception people will predict the likelihood of events based on how well on outcome represents some aspect of its parent population.
2. *Negative and Positive Recency Effects*: Negative recency effect is the tendency to predict an outcome, which has not appeared for some time in a series of trials. It has been called the 'Gambler's Fallacy". This is related to heuristic of representativeness. After the repeated occurrence of one outcome, a gambler comes to believe that probability of the alternative outcome is increasing even though successive events are independent. The converse of this is a tendency to predict an outcome, which has repeatedly occurred.
3. *Simple and Compound Events*: Students confuse or do not separate these events.
4. *Conjunction Fallacy:* The probability of an event appears under certain conditions, to be higher than the probability of the intersection of the same event with another event.
5. *Effect of Sample Size:* Students tend to neglect the influence of the magnitude of a sample when estimating probabilities. Represantativeness also occur when students neglect the sample size.
6. *Availability:* The tendency to make predictions based on how accessible instances of an event are to the memory or on how easy it is to construct particular instances of events. The judgmental heuristic can induce significant bias because of one's own narrow experience or personal perspective. We all have egocentric impressions of the frequency of events based on our own experiences. Often these impressions are biased.
7. *Time Axis Fallacy (Effect of the Time Axis):* Children may assign a role in chance events to the personal qualities of the player though objectively such

an effect does not exist. An inversion of the time axis of cause implying effect contradicts one of our basic intuitions.

8. *Misconception:* A misconception is an underlying belief that governs an error.
9. *Probabilistic Thinking:* It is used to describe children's thinking in response to any probability situation.
10. *Previous instruction on probability:* It refers to the condition of students being whether they had some instruction on probability or not before this study.

3.4 Variables

There are eight dependent variables in the study where each dependent variable is related to a misconception type. For each dependent variable, there are three independent variables: grade level, previous instruction on probability and gender. In other words, each misconception type as a dependent variable is investigated with respect to grade level, PIoP and gender.

At this point, the independent variables should be defined clearly;

1. *Grade Level* includes students from the grade levels 5, 6, 7, 8, 9 and 10.
2. *Previous instruction on probability:* By means of this variable we understand that whether a student had any instruction on probability in schools or not.
3. *Gender* shows whether a student is male or female.

3.5 Subjects of the Study

Six groups of the students were investigated: 34 students in Grade 5 (ages10-11), 36 students in Grade 6 (ages 11-12), 288 students in Grade 7 (ages 12-13), 143 students in Grade 8 (ages13-14), 201 students in Grade 9 (ages14-15) and 183 students in Grade 10 (ages15-16). There were 885 students totally in the study. Convenience sampling method was used in the study.

The sample represented a range of students with respect to socio economic level and cultural background. There were students from Ankara and Karadeniz

Eregli. 53 percent of the subjects were female students and 47 percent of the subjects were male students. Since the Probability subject is lectured at grade levels 8, 9 and 10 in schools, the students were selected from these grade levels for the third variable 'PIoP".

The tables, which express the distribution with respect to Grade Levels, Gender, and the PIoP, are shown as the following:

Table 3.1 Distribution of students' with respect to Grade Levels

Grade Levels	n	Percent
5	34	3.8
6	36	4.1
7	288	32.5
8	143	16.2
9	201	22.7
10	183	20.7
Total	885	100.0

Table 3.2 Distribution of students' with respect to Gender

Gender	n	Percent
Female	469	53
Male	416	47
Total	885	100.0

Table 3.3 Distribution of students' with respect to PIoP

Levels	PIoP		Total
	No	Yes	
8	34 21.4%	109 29.6%	143 27.1%
9	121 76.1%	80 21.7%	201 38.1%
10	4 2.5%	179 48.6%	183 34.7%
Total	159 30%	368 70%	527 100.0%

3.6 Instruments

As it was previously stated, in order to collect data for the study, Probabilistic Misconception Test (PMT), consisting of 14 well-known probability questions, was administered. They were related to misconception types on *Representativeness, Positive and Negative Recency Effects, Simple and Compound Events, Effect Sample Size, Conjunction Fallacy, Heuristic Availability, The Time Axis Fallacy and Equiprobability Bias.* Several sources were reviewed to form the PMT. For this, an item pool consisting of forty probability questions was formed and fourteen of them were selected for this study. The questions were related to well-known probabilistic misconceptions types and all of the items used in PMT were taken from the studies in the literature.

The questions were translated from English to Turkish and then they were translated into English from Turkish again. To check the content validity of the test, three instructors from Mathematics Department in Middle East Technical University revised and controlled the questions in the test in terms of mathematical structure of the test. In addition, a Turkish teacher controlled for grammar of the test, and a specialist in measurement and evaluation, check the appropriateness of the test with the curriculum program followed in schools. Their recommendations were taken into account and some changes were made on some items and alternatives. The test is presented in the Appendix.

The test was in multiple-choice format. The questions were coded according to the name of alternatives. For example, we assume that the item had 4 alternatives: a, b, c and d. They were coded as 1, 2, 3, and 4 respectively. One of them was misconception, another was correct and others were incorrect. In the SPSS program alternatives were labeled as misconception, correct and incorrect. The percentages and frequencies of each alternative were computed.

The questions and misconception types are given as following:

Questions 1and 2 tested for the misconception on Representativeness. They were also used also in the studies done by Kahneman and Tversky (1972); Shaughnessy (1992); Tversky and Kahneman (1982). The questions are stated below:

Question 1: Say you flip an ordinary quarter several times in successions with H representing a Head coming up and T representing a Tail. The notation HT means in two successive flips a Head occurred followed by a Tail. If you flip a quarter 5 times in succession, which of the following sequences are you most likely to observe:

a) TTTHH (*incorrect*)
b) THHTH (*incorrect*)
c) HTHHH (*incorrect*)
d) THTHT (*main misconception*)
e) Among (a)-(d) one is likely as the other (*correct*)

Question 2: In a lotto game, one has to choose 6 numbers from a total of 40. Ahmet has chosen 1, 2, 3, 4, 5 and 6, and Nuray has chosen 39, 1, 17, 33, 8 and 27. Who has a greater chance of winning?

a) Ahmet (*incorrect*)

b) Nuray (*main misconception*)

c) Ahmet and Nuray have the same chance of winning (*correct*)

Questions 3 and 4 tested for the misconception on Negative and Positive Recency Effects. They were also used also in the studies done by Cohen, 1957; Fischbein, 1975; Fischbein, Nello and Marino (1991). The questions are given below:

Question 3: When tossing a coin, there are two possible outcomes: either heads or tails. Özge flipped a fair coin three times and in all cases tails came up. Özge intends to flip the coin again. What is the chance of getting heads at the fourth time?

a) Equal to the chance of getting tails (*correct*)

b) Greater than the chance of getting tails (*main misconception*)

c) Smaller than the chance of getting tails (*incorrect*)

Question 4: A father plays the following game with his son: The father hides a coin in one of his hands behind his back, and if his son knows in which hand he hides the coin, he wins the coin. The past 14 days (or hands), the son wins 5 times and looses 9 times. Which of the following options would you expect to happen the next 14 days (or hands)?

a) The son wins more than he looses (*main misconception*)

b) The son looses more than he wins (*incorrect*)

c) The number of the games he looses is equal to the number of the games he wins (*correct*)

Questions 5 and 6 tested for the misconception on Simple and Compound Events. They were also used also in the studies done by Lecoutre and Durant (1988). The questions are stated below:

Question 5:Suppose one rolls a dice simultaneously. Which of the following has a greater chance of happening?

a) Getting the pair of 6-6 (*incorrect*)

b) Getting the pair of 5-6 (*correct*)

c) Both have the same chance (*main misconception*)

Question 6: The letters in the word "**CICEK** " are written one by one on the cards and then these cards are placed in a bag. What is the probability of getting the letter "C " from this box at random?

a) $\frac{2}{5}$ (*correct*) b) $\frac{2}{3}$ (*incorrect*) c) $\frac{1}{4}$ (*main misconception*)

Questions 7 and 8 tested for misconception on Effect of Sample Size. They were also used also in the studies done by Tversky and Kahneman (1982). The questions are given below:

Question 7: A doctor keeps the records of newborn babies. According to his records, the probability of which of the following options is higher?

a) Out of the first 10 babies, the gender of 8 or more of them is female. (*correct*)

b) Out of the first 100 babies, the gender of 80 or more of them is female. (*incorrect*)

c) The probability of events (a) and (b) is the same.

(*misconception*)

Question 8: The likelihood of getting tails at least twice when tossing three coins is:

a) Smaller than (*incorrect*)

b) Greater than (*correct*)

c) Equal to (*main misconception*)

the likelihood of getting tails at least 200 times out of 300 times.

Question 9 tested for the misconception on Conjunction Fallacy. It was also used also in the studies done by Shaughnessy, 1992; Tversky and Kahneman (1983). The question is written below:

Question 9: Fatih dreams of becoming a doctor. He likes to help people. When he was in high school, he volunteered for Kizilay organization .He accomplished his studies with high performance and served in the army as a medical attendant. After ending his army service, Fatih registered at the university. Which seems to you to be more likely?

a) Fatih is a student of the medical school (*main misconception)*

b) Fatih is a student (*correct*)

Question 10 tested for the misconception on Heuristics Availability. It was also used also in the studies done by Tversky and Kahneman (1973). The question is stated below:

Question 10: **K:** The number of groups composed of 2 members from among 10 candidates.

L: The number of groups composed of 8 members from among 10 candidates.

According to the given information above, which of the following is correct?

a) K is greater than L (*main misconception*)

b) K is smaller than L (*incorrect*)

c) K is equal to L (*correct*)

Questions 11 and 12 together tested for the misconception on The Time Axis Fallacy (also called the Falk Phenomenon). In these questions, people are likely to answer 11th question correctly, then answer 12th question differently on the basis of the principle that an event cannot retroactively on its cause. The responses for the questions 11 and 12 are divided into three categories: In category I, both responses are correct; in category II, the 11th response is correct while the 12th is incorrect; and in category III both responses are incorrect. Category II represents the *main misconception*. An inversion of the time axis of cause implying effect contradicts one of our basic intuitions. They were also used also in the studies done by Falk, 1979, Shaughnessy (1992) and Fischbein (1997). The questions are stated below:

Question 11: Dilek receive a box containing two white marbles and two black marbles. Dilek extracts a marble from her box and finds out that it is a white one. Without replacing the first marble, she extracts a second marble. According to this information which of the following is correct?

a) The likelihood that the second marble is also white equal to the likelihood that it is a black marble.

b) The likelihood that the second marble is also white greater than the likelihood that it is a black marble.

c) The likelihood that the second marble is also white smaller than the likelihood that it is a black marble.

Question 12: Ahmet receive a box containing two white marbles and two black marbles. Ahmet extracts a marble from his box and puts it aside without looking at it. He then extracts a second marble and sees that it is white. According to this information which of the following is correct?

a) The likelihood that the first marble he extracted is white greater than the likelihood that it is a black.

b) The likelihood that the first marble he extracted is white smaller than the likelihood that it is a black.

c) The likelihood that the first marble he extracted is white equal to the likelihood that it is a black.

Questions 13 and 14 tested for the misconception on Equiprobability Bias. They were also used also in the studies done by Green (1983). The questions are given below:

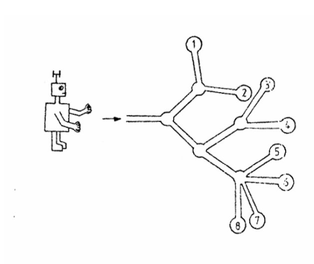
Question 13: There are six fair dies each of which is an ordinary cube with one face painted white and the other faces painted black. If these dies are tossed which of the following would be more likely?

a) You would observe 5 black and 1 white (*correct*)

b) You would observe 6 black (*incorrect*)

c) One is as likely as the other (*misconception*)

Question 14: A robot, which is placed in a labyrinth with eight same types of traps in it, is programmed to always go forward and never to come back. In every cross road, the robot chooses the road that he is going to follow at random. By which trap is this robot's possibility of being catched the most possible?



a) The first trap (*correct*)

b) The third trap (*incorrect*)

c) The fifth trap (*incorrect*)

d) One is as likely as the other (*misconception)*

3.7 Procedure

This study started with a review of literature related to the intended components of the main research question. An item pool consisting of forty probability questions was formed and fourteen of them were selected for this study. After that, data collection instrument were developed. Probabilistic Misconception Test was developed and piloted with eighth grade students in January 2001. The pilot study was conducted to determine the validity of the test. According to results of the pilot study, the test was revised and then the test was administered to the subjects of the study.

There were 885 students in the sample of the study. Convenience sampling method was used in the study. The subjects were from six different grade levels (grade levels 5, 6, 7, 8, 9 and 10) and the subjects were from different schools in Ankara and Karadeniz Eregli. Moreover, the students in the study were from different socio economic level and cultural background.

Before the pilot study three interviewees were selected in each grade level with respect to their performance on the test in order to determine the reasons underlie the misconceptions of probability. It was examined whether the questions are answered or not.

After that, the place where the test was applied was decided. The necessary permissions were obtained for the administration of the test in the schools before the main application of the test.

The researcher for the sample selected in Ankara conducted all of the administrations of the tests. However, an instructor for the sample selected in Karadeniz Eregli conducted the administrations of the tests.

There were 885 students participated in the study. As stated in methodology part of the study, answers were to be selected with the reasons. The test was administered to each group of students during a regularly scheduled class, under usual classroom conditions, in a session lasting about one-lesson hour (40-45 minutes). The time allocated for the study was sufficient to complete the test. Before the administration of the test, the researcher read all of the necessary instructions and explained the purpose of the study to the participants. In order to prevent the students be effected by the others, the researcher helped the students one by one.

3.8 Analysis of Data

The following steps were followed for the data analysis procedure of the study:

a) The data collected from the subjects were coded and then transferred to computer with SPSS package program.
b) Descriptive statistic, especially, frequency of dependent variables was calculated by levels of all of the independent variables used in the present study.
c) The dependent variables were tabulated with respect to independent variables by means of frequency tables in SPSS package program.

3.9 Assumptions and Limitations

There are several assumptions and limitations of the present study as in other studies. The main assumptions of the present study are the following:

a) The participants of the study were assumed to answer the questions in the test and in the pilot study sincerely and accurately.
b) It is assumed that the test was completed under standard conditions.
c) There was no interaction between the subjects to affect the results of the present study.
d) There was not any event occurred outside during the study to affect misconceptions of the subjects.

The limitations of the present study are as listed below.

a) This study was limited to students in the grade levels 5 through 10 in Ankara and Karadeniz Eregli during the fall semester of the 2000-2001 academic year.

b) Self-report techniques, which require the subject to respond truthfully and willingly, were used.

c) The use of questionnaire alone could not give a complete picture of students' misconceptions on probability but it gave access to their surface misconceptions. For reaching the subjects' misconceptions, the collection of data should also include other methods like interviews.

CHAPTER 4

RESULTS AND DISCUSSION

In this chapter, the results of the analyses will be presented and discussed. In the previous chapters, theoretical basis of the study, the review of the previous studies related to the present study and methodology part of the present study were stated in order. This chapter contains three sections. The first section contains the effect of grade level on the misconception types in probability. In the second section, the changes of probabilistic misconception types with respect to previous knowledge of students, is included. The effect of gender on misconception types is included in the third section.

The purpose of the study was to investigate the students' probabilistic misconceptions with respect to grade level, previous instruction on probability and gender. In the present study there are 8 misconception types: Representatives, Negative and Positive Regency Effects, Compound and Simple Events, Effect of Sample Size, Conjunction Fallacy, Heuristic of Availability, Time Axis Fallacy and Equiprobability Bias.

4.1 Misconception Types with respect to Grade Levels

In this section, the misconception types will be analyzed with respect to grade levels.

4.1.1 Misconception on Representativeness w.r.t. GL

The first and the second questions related to misconception on "Representativeness" are examined by taking into account GL. The results are given in Table 4.1 and Table 4.2.

Table 4.1 Number and Percentages of Students' Answers for the Question 1 w.r.t.GL

Answers	Grade levels						Total
	5	6	7	8	9	10	
No answer	0 0%	0 0%	16 5.6%	3 2.1%	5 2.5%	3 1.6%	27 3.1%
Incorrect	1 2.9%	0 0%	17 5.9%	0 0%	11 5.5%	2 1.1%	31 3.5%
Incorrect	0 0%	2 5.6%	21 7.3%	5 3.5%	4 2.0%	3 1.6%	35 4.0%
Incorrect	2 5.9%	4 11.1%	12 4.2%	1 .7%	8 4.0%	2 1.1%	29 3.3%
Misconception	3 8.8%	9 25.0%	52 18.1%	17 11.9%	22 10.9%	12 6.6%	115 13.0%
Correct	28 82.4%	21 58.3%	170 59.0%	117 81.8%	151 75.1%	161 88.0%	648 73.2%

In the question 1, 73.2 percent of the students answered the question correctly and 13 percent of them had a misconception. This misconception type "Represantativeness" *decreased with* grade level as seen in the study of Fischbein and Schnarch (1997). It is seen that the percentage for the correct answer was higher at the grade levels 8 and 10 rather than the other levels.

Table 4.2 Numbers and Percentages of Students' Answers for the Question 2 w.r.t. GL

Answers	Grade levels						Total
	5	6	7	8	9	10	
No answer	1 2.9%	0 0%	6 2.1%	6 4.2%	2 1.0%	4 2.2%	19 2.1%
Incorrect	0 0%	0 0%	9 3.1%	1 .7%	7 3.5%	0 0%	17 1.9%
Misconception	18 52.9%	16 44.4%	170 59.0%	48 33.6%	89 44.3%	36 19.7%	377 42.6%
Correct	15 44.1%	20 55.6%	103 35.8%	88 61.5%	103 51.2%	143 78.1%	472 53.3%

Although, the second question investigates the same misconception type with the first question, the misconception was stronger in the second question. As a reason, it can be stated that there is a more definite variation in the alternatives for the second question. In other words, there is a definite diversity in the alternatives for the first question. As a result 42.6 percent of the students had misconception in this question. However, the misconception varied across grade level. However, there were a decreasing in this misconception as students gets older in secondary school level and high school level separately. While the effects of the students' grade level are examined, it is seen that there was a decreasing in the frequency of the misconceptions at grade levels 8 and 10. If mathematics curriculum is investigated, it will be seen that the probability subject is taught at these levels.

4.1.2 Misconception on Positive and Negative Recency Effects w.r.t. GL

The third and the fourth questions related to misconception on "Positive and Negative Recency Effects" are examined. The results are given in Table 4.3 and Table 4.4.

Table 4.3 Number and Percentages of Students' Answers for the Question 3 w.r.t. GL

Answers	Grade levels						Total
	5	6	7	8	9	10	
No answer	1 2.9%	0 0%	10 3.5%	4 2.8%	1 .5%	0 0%	16 1.8%
Correct	24 70.6%	24 66.7%	159 55.2%	117 81.8%	134 66.7%	139 76.0%	597 67.5%
Misconception	7 20.6%	4 11.1%	40 13.9%	6 4.2%	25 12.4%	10 5.5%	92 10.4%
Incorrect	2 5.9%	8 22.2%	79 27.4%	16 11.2%	41 20.4%	34 18.6%	180 20.3%

In this misconception type, the fallacy is assuming that outcome of independent events is dependent on previous outcomes. The Gambler's fallacy is committed, for example, when one contends that the probability of obtaining a head is increased on the next toss after a run of five tails with a fair coin. According to the results above, 67.5 percent of the students could manage to answer the 3rd question correctly. However, 10.4 percent of the students had negative regency effect (main misconception) and 20.3 percent of the students had positive regency effect. Positive-Negative Regency Effect misconception type did not change across grade levels according to the data obtained. However, it was seen that the students had this misconception, which is called also as 'Gambler Fallacy" rarely at the grade levels 8 and 10 in which the probability subject is taught, rather than the other levels. This fact stresses the importance of teaching probability in classes effectively. According to Konold (1993) there are two approaches; the first one is some of the students think that they are required to guess a certain outcome, whereas others consider that they have to evaluate the probability of a string of outcome. There may be conflicts that lead to inconsistencies between the two approaches, even for the same subject.

Table 4.4 Number and Percentages of Students' Answers for the Question 4 w.r.t. GL

Answers	Grade levels						Total
	5	6	7	8	9	10	
No answer	1 2.9%	2 5.6%	23 8.0%	19 13.3%	5 2.5%	20 10.9%	70 7.9%
Misconception	11 32.4%	8 22.2%	113 39.2%	25 17.5%	67 33.3%	41 22.4%	265 29.9%
Incorrect	3 8.8%	10 27.8%	75 26.0%	38 26.6%	38 18.9%	36 19.7%	200 22.6%
Correct	19 55.9%	16 44.4%	77 26.7%	61 42.7%	91 45.3%	86 47.0%	350 39.5%

Positive-Negative Regency Effect varied across grade levels. The results of the 4th question were closure to the results of the 3rd question. Both of them investigate Positive-Negative Regency effect. 39.5 percent of the students solved the question correctly. However this ratio was higher for the 3rd question. This could be as a result of the facts in the 4th question being more complex than 3rd question. 29.9 percent of the students had negative regency effect (main misconception) and 22.6 percent of the students had positive regency effect misconception.

As in the 3rd question there was not effect of grade level on Positive-Negative Regency Effect. In other words, positive-negative regency effect was almost absent with respect to 3rd and 4th questions. However, there was a significant decreasing for the misconception type at the 8th and 10th grade level in which probability subject takes a place in the mathematics curriculum.

4.1.3 Misconception on Simple and Compound Events w.r.t. GL

The fifth and the sixth questions related to misconception on 'Simple and Compound Events" are examined in terms of the GL. The results are given in Table 4.5 and Table 4.6.

Table 4.5 Number and Percentages of Students' Answers for the Question 5 w.r.t. GL

Answers	Grade levels						Total
	5	6	7	8	9	10	
No answer	3 8.8%	0 0%	23 8.0%	7 4.9%	6 3.0%	6 3.3%	45 5.1%
Incorrect	1 2.9%	3 8.3%	20 6.9%	4 2.8%	13 6.5%	9 4.9%	50 5.6%
Correct	2 5.9%	3 8.3%	55 19.1%	17 11.9%	27 13.4%	13 7.1%	117 13.2%
Misconception	28 82.4%	30 83.3%	190 66.0%	115 80.4%	155 77.1%	155 84.7%	673 76.0%

It can be clearly seen that only 13.2 percent of the students solved the question correctly. In other words, only a small proportion of answers indicate that the probabilities are different (which is the correct answer). In this question, it can be supposed that some of the students have considered the pairs as ordered pairs and then the probability of the pairs (5,6) and (6,6) is the same, however this was never the case. According to the justifications, it became clear that the students did not consider the order and "equiprobable" type of misconception was mostly justified by the effect of chance or by the effect of considering separately the two elements 5 and 6.

The percentage of correct answer was almost equal (stable) with respect to the grade levels. This well known misconception that is the two outcomes (5,6) and (6,6) were considered equivalent at all grade levels. To illustrate, the highest percentage of the misconception was seen among the students in grade level 10 with 84 percent. Similarly, the percentages of Simple and Compound Events misconceptions were 82.4 and 83.3 in grade level 5 and 6 respectively. The results for the question 5th were very close to the results obtained in the study of Lecoture and Durand (1988) and Fischbein, Nello and Marino (2000).

For this type of misconception there is no natural intuition for evaluating the probability of a compound event according to Fischbein, Nello and Marino (2000). The students do not count separately various possible orders of a set of elementary results (to illustrate 5-6 and 6-5 or H-T and T-H) when defining the magnitude of sample space. Some explanations offered by the students were related to chance events. For example, "The probability is the same because one can obtain 6 -6 and 5-6 or none of these results"; "The probability is the same because the obtained number is surprise"; "There is the same number of 5 and 6 in both d ice; therefore, the probability is the same". An interesting justification was: "Each die is independent to each other. The probability that with one die, one will obtain a certain number is 1/6 and it is the same probability that one will obtain the same number with the other". The students who received a certain instruction on probability made this explanation.

The main idea expressed to justify the equality of the probabilities of getting 6-6 and 5-6 is "both events are the effect of chance and therefo re there is no reason to expect one more than the other.

Some of the students, on the other hand, stated that (6-6) is less probable because of the fact that they think that identical results appear less often than different results. For instance, "the pro bability of getting (6-6) is small. It is more likely to have 6 and 5".

For the correct answer, the pupils who had instruction stated that "the pair (5 - 6) is more probable because there are two possibilities (5-6) and (6-5) in sample space whereas 6-6 represents only one possibility".

In brief, Simple and Compound Event was frequent and stable across grade levels.

Table 4.6 Number and Percentages of Students' Answers for the Question 6 w.r.t. GL

Answers	Grade levels						Total
	5	6	7	8	9	10	
No answer	1 2.9%	0 0%	21 7.3%	6 4.2%	6 3.0%	4 2.2%	38 4.3%
Correct	24 70.6%	23 63.9%	162 56.3%	123 86.0%	115 57.2%	152 83.1%	599 67.7%
Incorrect	3 8.8%	4 11.1%	49 17.0%	7 4.9%	44 21.9%	13 7.1%	120 13.6%
Misconception	6 17.6%	9 25.0%	56 19.4%	7 4.9%	36 17.9%	14 7.7%	128 14.5%

67.7 percent of the students answered the question correctly and it can be concluded that students do not confuse the concepts; sample space and sets with a high proportion. The percentage of correct answer was higher in the grade levels 5, 8, and10. However, 14.5 percent of the students had misconception in Simple and Compound Events. There was no effect of grade level on this misconception type as in the 6th question, however, the students in grade level 8 and those in grade level 10 had little tendency in Simple and Compound Events misconception type because the students had course on probability in these levels in schools.

4.1.4 Misconception on Effect of Sample Size w.r.t. GL

The seventh and the eighth questions related to misconception on ‘Effect of Sample Size" are examined by taking into account the GL. The results are given in Table 4.7 and Table 4.8.

Table 4.7 Number and Percentages of Students' Answers for the Question 7 w.r.t. GL

Answers	Grade levels						Total
	5	6	7	8	9	10	
No answer	1 2.9%	1 2.8%	29 10.1%	8 5.6%	8 4.0%	5 2.7%	52 5.9%
Correct	0 0%	3 8.3%	29 10.1%	12 8.4%	14 7.0%	4 2.2%	62 7.0%
Incorrect	1 2.9%	2 5.6%	19 6.6%	4 2.8%	10 5.0%	8 4.4%	44 5.0%
Misconception	32 94.1%	30 83.3%	211 73.3%	119 83.2%	169 84.1%	166 90.7%	727 82.1%

In the question 7, only 7 percent of the students answered the question correctly, whereas, 82.1 percent of the students had misconception on this question. This is a very high percentage and shows that the students have confusion between Ratio and Proportion, and Probability subject. It can be concluded from the students' explanation that they stated as a reason. None of the students from grade level 5 solved the question correctly. This misconception type developed for the students from grade 7 to 10. The frequencies of the main misconception increased with the grade level.

How the misconception type "Effect of Sample Size" changes with "school type" is rather difficult and complex to explain. However, it is seen that the students in Anatolian High School and in Private High Schools had this misconception type more frequently than the students in the other school types. This is a surprising situation.

Table 4.8 Number and Percentages of Students' Answers for the Question 8 w.r.t. GL

Answers	Grade Level						Total
	5	6	7	8	9	10	
No answer	1 2.9%	0 0%	21 7.3%	3 2.1%	9 4.5%	3 1.6%	37 4.2%
Incorrect	1 2.9%	2 5.6%	30 10.4%	16 11.2%	32 15.9%	17 9.3%	98 11.1%
Correct	10 29.4%	9 25.0%	79 27.4%	21 14.7%	47 23.4%	23 12.6%	189 21.4%
Misconception	22 64.7%	25 69.4%	158 54.9%	103 72.0%	113 56.2%	140 76.5%	561 63.4%

The results of question 8 are parallel to results of question 7. 21.4 percent of the students replied the question 8 correctly. However, 63.4 percent of the students had misconception on "Effect of Sample Size" misconception type w.r.t. grade levels. There was not a contingency of "Effect of Sample Size" misconception type. It is interesting that the misconception was most frequent among the students at grade level 10 with 76.5 percent. As stated in question 7, the students had confusion on Ratio and Proportion subject with Probability of events. It is also surprising that the percentage of correct answer was highest in grade level 5.

Although the students have a course on probability at grade levels 8 and 10, there was increasing for this type of misconception at these levels. Therefore, this fact should not be ignored while probability subject is taught. Moreover, it is obvious that the misconception type was frequent among the students in Anatolian High School and Private High Schools. This fact is also surprising.

4.1.5 Misconception on Conjunction Fallacy w.r.t. GL

The ninth question related to misconception on "Conjunction Fallacy" is examined in terms of the GL. The result is given in Table 4.9.

Table 4.9 Number and Percentages of Students' Answers for the Question 9 w.r.t. GL

Answers	Grade Level						Total
	5	6	7	8	9	10	
No answer	1 2.9%	1 2.8%	14 4.9%	2 1.4%	8 4.0%	8 4.4%	34 3.8%
Misconception	11 32.4%	21 58.3%	168 58.3%	65 45.5%	134 66.7%	70 38.3%	469 53.0%
Correct	22 64.7%	14 38.9%	106 36.8%	76 53.1%	59 29.4%	105 57.4%	382 43.2%

In the misconception type "Conjunction Fallacy" 53 percent of the students had misconception and 43.2 percent of the students solved the question correctly. There were not any changes in Conjunction Fallacy with respect to grade level. In particular, however, the frequency of the misconception was very frequent in grade level 9 as Fischbein and Schnarch found (1997). It is surprising that the percentage of correct answer was highest in grade level 5. Parallel to that fact, the percentage of misconception was lowest in grade level 5.

4.1.6. Misconception on Heuristic Availability w.r.t. GL

The tenth question related to misconception on "Heuristic Availability" is examined by taking into account the GL. The result is given in Table 4.10.

Table 4.10 Number and Percentages of Students' Answers for the Question 10 w.r.t. GL

Answers	Grade Level						Total
	5	6	7	8	9	10	
No answer	2 5.9%	0 0%	34 11.8%	10 7.0%	16 8.0%	5 2.7%	67 7.6%
Misconception	16 47.1%	24 66.7%	132 45.8%	91 63.6%	78 38.8%	61 33.3%	402 45.4%
Incorrect	11 32.4%	5 13.9%	69 24.0%	29 20.3%	41 20.4%	33 18.0%	188 21.2%
Correct	5 14.7%	7 19.4%	53 18.4%	13 9.1%	66 32.8%	84 45.9%	228 25.8%

The "Heuristics Availability" misconception occurred in the students with the 45.4 percent. The students could not manage to solve this problem correctly and only 25.8 percent of the students solved the question correctly. They were not able to identify possible combinations. In other words, the students could not understand intuitively the complementarity and subsequent equality of the two groups. The students could not identify possible combination, as the subjects grew older. As stated in the study of Fischbein and Scnarch (1997), because of the fact that, it is easier to produce various combinations of two elements than combinations of eight elements (availability), two elements is selected as an answer.

The subjects "Permutation and Combination" are taught at grade level 10 therefore this misconception was less stronger in grade level 10 rather than the other grade levels. Finally; although the frequency of the misconception type was getting decreasing from grade level 8 to 10, there was not a clear effect of grade level on the misconception type.

4.1.7 Misconception on Time Axis Fallacy w.r.t. GL

The eleventh and twelfth question related to misconceptions on "Time Axis Fallacy" are examined in terms of the GL. The results are given in Table 4.11.

Table 4.11 Number and Percentages of Categories for the Question 11and 12 w.r.t. GL

Categories	Grade Level						Total
	5	6	7	8	9	10	
	0	1	17	4	8	2	32
No answer	0%	2.8%	5.9%	2.8%	4.0%	1.1%	3.6%
	11	8	96	49	32	39	235
Category I	32.4%	22.2%	33.3%	34.3%	15.9%	21.3%	26.6%
	9	9	46	57	53	73	247
Category II	26.5%	25.0%	16.0%	39.9%	26.4%	39.9%	27.9%
	6	12	88	21	91	57	275
Category III	17.6%	33.3%	30.6%	14.7%	45.3%	31.1%	31.1%
	8	6	41	12	17	12	96
Other	23.5%	16.7%	14.2%	8.4%	8.5%	6.6%	10.8%

Questions 11 and 12 tested for the "effect of the time axis which is also called as the Falk Phenomenon. As in the study of Fischbein (1997), the responses for the questions 11 and 12 are divided into three categories: In category I, both responses are correct; in category II, the 11th response is correct while the 12th is incorrect; and in category III both responses are incorrect. Category II represents the *main misconception*.

According to the results obtained in the study, the students almost equally distributed with respect to all of the three categories. The frequency of misconception was higher in grade level 8 and level 10 with 39.9 percent rather than the other levels. This misconception decreased with grade level at secondary school level. The percentages of misconception decreased from grade level 5 to 7, however, the percentage increased at grade level 8. There was a decreasing in this misconception at grade level 9 again. However, the percentage increased at grade level 10 surprisingly. In spite of the fact that the students had probability course at grade levels 8 and 10, the percentage of misconception increased at these levels. This is an interesting point, which will be investigated at the other section of this study.

4.1.8 Misconception on Equiprobability Bias w.r.t. GL

The thirteenth and the fourteenth question related to misconceptions on "Equiprobability Bias" are examined by taking into account the GL. The results are given in Table 4.12 and Table 4.13.

Table 4.12 Number and Percentages of Students' Answers for the Question 13 w.r.t. GL

Answers	Grade level						Total
	5	6	7	8	9	10	
	1	3	31	11	9	7	62
No answer	2.9%	8.3%	10.8%	7.7%	4.5%	3.8%	7.0%
	12	9	95	36	64	66	282
Correct	35.3%	25.0%	33.0%	25.2%	31.8%	36.1%	31.9%
	4	10	76	61	36	53	240
Incorrect	11.8%	27.8%	26.4%	42.7%	17.9%	29.0%	27.1%
	17	14	86	35	92	57	301
Misconception	50.0%	38.9%	29.9%	24.5%	45.8%	31.1%	34.0%

The students had a difficulty in solving 13th question. Only 31.9 percent of the students could solve the question correctly. 34 percent of the students had misconception. The frequency of Equiprobability Bias misconception decreased from grade level 5 to grade level 8. However, the frequency was higher in grade 9 rather than in grade level 8. There was a decreasing again from level 9 to level 10. The percentage of correct answers was approximately equal to each other in all grade levels.

Table 4.13 Number and Percentages of Students' Answers for the Question 14 w.r.t. GL

Answers	Grade Level						Total
	5	6	7	8	9	10	
No answer	0 0%	1 2.8%	21 7.3%	11 7.7%	12 6.0%	9 4.9%	54 6.1%
Correct	12 35.3%	9 25.0%	79 27.4%	37 25.9%	22 10.9%	44 24.0%	203 22.9%
Incorrect	1 2.9%	0 0%	12 4.2%	3 2.1%	3 1.5%	1 0.5%	20 2.3%
Incorrect	2 5.9%	3 8.3%	20 6.9%	4 2.8%	22 10.9%	9 4.9%	60 6.8%
Misconception	19 55.9%	23 63.9%	156 54.2%	88 61.5%	142 70.6%	120 65.6%	548 61.9%

Equiprobability Bias misconception type was strong among students. The frequency of this misconception varied across grade level according to this question. The pupils had a difficulty in solving the question correctly although it seems to be an easy question. Only 22.9 percent of the students could solve the question correctly. 61.9 percent of the students had misconception according to the question. The Equiprobability Bias misconception was most frequent among the students in grade level 9. Moreover, it is surprising that the percentage of correct answer for this question was the highest in grade level 5.

It is well for the reader to be aware of those topics which are of perceived importance so that he may better evaluate the contributions made so far;

1. What conceptions of probability do children of various ages have?
2. How might these conceptions be changed?

3. Is there an optimum age at which to introduce a child to formal probability?

According to Hawkins and Kapadia (1984), the researchers feel that these questions have not yet been answered by the available research findings. We still have little idea about the conceptions of probability children of various ages have.

4.2 Misconception Types with respect to Previous Instruction on Probability

In this section, misconception types are analyzed with respect to the independent variable, previous instruction on probability (PIoP), which represents whether the subjects had instruction on probability before the study or not. The misconception types; Representativeness, Positive and Negative Recency Effects, Simple and Compound Events, Effect Sample Size, Conjunction Fallacy, Heuristic Availability, The Time Axis Fallacy and Equiprobability Bias are analyzed with respect to PIoP separately.

4.2.1 Misconception on Representativeness w.r.t. PIoP

The first and the second questions related to misconception on 'Representativeness" are exami ned by taking into account PIoP. The results are given in Table 4.14 and Table 4.15.

Table 4.14 Number and Percentages of Students' Answers for the Question 1 w.r.t. PIoP.

Answers	PIoP No	PIoP Yes	Total
No answer	3 1.9%	8 2.2%	11 2.1%
Incorrect	9 5.7%	4 1.1%	13 2.5%
Incorrect	4 2.5%	8 2.2%	12 2.3%
Incorrect	5 3.1%	6 1.6%	11 2.1%
Misconception	21 13.2%	30 8.2%	51 9.7%
Correct	117 73.6%	312 84.8%	429 81.4%

The students who had instruction in probability had lower level of misconception than the students who had not any instruction before the study. This fact, stresses the importance of effective teaching of probability in schools.

Table 4.15 Number and Percentages of Students' Answers for the Question 2 w.r.t. PIoP

Answers	PIoP No	PIoP Yes	Total
No answer	3 1.9%	9 2.4%	12 2.3%
Incorrect	5 3.1%	3 .8%	8 1.5%
Misconception	70 44.0%	103 28.0%	173 32.8%
Correct	81 50.9%	253 68.8%	334 63.4%

The frequency of the misconception for the students who had no instruction before the study was 44 percent and for the students who had instruction before the study was 28 percent. Both of the questions 1 and 2, which investigate

‘Represantativeness”, show that there is a decreasing in this misconception type with instruction.

4.2.2 Misconception on Positive and Negative Regency Effect w.r.t. PIoP

The third and the fourth questions related to misconception on “Positive and Negative Regency Effect” are examined by taking into account PIoP. The results are given in Table 4.16 and Table 4.17

Table 4.16 Number and Percentages of Students’ Answers for the Question 3 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	3 1.9%	2 .5%	5 .9%
Correct	106 66.7%	284 77.2%	390 74.0%
Misconception	23 14.5%	18 4.9%	41 7.8%
Incorrect	27 17.0%	64 17.4%	91 17.3%

It is seen that, 4.9 percent of the students who took instruction and 14.5 percent of the students who hadn’t taken instruction before the study had misconception according to 3rd question. Therefore, it can be stated that instruction on probability decreases ‘Positive –Negative Regency Effect” misconception type.

Table 4.17 Number and Percentages of Students' Answers for the Question 4 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	4 2.5%	40 10.9%	44 8.3%
Misconception	47 29.6%	86 23.4%	133 25.2%
Incorrect	36 22.6%	76 20.7%	112 21.3%
Correct	72 45.3%	166 45.1%	238 45.2%

Although 3rd and 4th question investigate the same kind of misconception, the effect of the instruction was clearer at the 3rd question. This could be the reason of the fact that, the 3rd question is easier to understand than the 4th question. However, it can be stated that this misconception was frequent among the students who had not instruction before the study in the 4th question as seen in the 3rd question. Therefore, we can conclude that instruction decreases "Positive -Negative Regency Effect" misconception type.

4.2.3 Misconceptions on Simple and Compound Events w.r.t. PIoP

The fifth and the sixth questions related to misconception on "Simple and Compound Events" are examined by taking into account PIoP. The results are given in Table 4.18 and Table 4.19.

Table 4.18 Number and Percentages of Students' Answers for the Question 5 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	6 3.8%	13 3.5%	19 3.6%
Incorrect	9 5.7%	17 4.6%	26 4.9%
Correct	25 15.7%	32 8.7%	57 10.8%
Misconception	119 74.8%	306 83.2%	425 80.6%

There were fewer correct answers in children who received a certain instruction than in children who did not receive any instruction in probabilities according to the results. This result is same with the results of the study of Fischbein, Nello and Marino (2000).

Table 4.19 Number and Percentages of Students' Answers for the Question 6 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	3 1.9%	13 3.5%	16 3.0%
Correct	100 62.9%	290 78.8%	390 74.0%
Incorrect	35 22.0%	29 7.9%	64 12.1%
Misconception	21 13.2%	36 9.8%	57 10.8%

The students who had instruction in probability had little tendency in Simple and Compound Events misconception type. Parallel to this fact, this misconception type was more frequent among the students who had not had any course on probability before the study. Although, the 5th and 6th questions investigate the Simple and Compound Events misconception type, while instruction on probability increased this misconception type with respect to 5th question, there was a decreasing with respect to 6th question.

4.2.4 Misconception on Effect of Sample Size w.r.t. PIoP

The seventh and the eighth questions related to misconception on "Effect of Sample Size" are examined by taking into account PIoP. The results are given in Table 4.20 and Table 4.21.

Table 4.20 Number and Percentages of Students' Answers for the Question 7 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	7 4.4%	14 3.8%	21 4.0%
Correct	11 6.9%	19 5.2%	30 5.7%
Incorrect	8 5.0%	14 3.8%	22 4.2%
Misconception	133 83.6%	321 87.2%	454 86.1%

Although at the 8th and 10th grades the probability subjects taught, there was an increase for this type of misconception at these grade levels. This result is verified by the result as, 87.2 percent of the students who had instruction before the study and 83.6 percent of the students who had no instruction before the study, had misconception on 'Effect of Sample Size". The misconception developed with instruction.

Table 4.21 Number and Percentages of Students' Answers for the Question 8 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	6 3.8%	9 2.4%	15 2.8%
Incorrect	30 18.9%	35 9.5%	65 12.3%
Correct	36 22.6%	55 14.9%	91 17.3%
Misconception	87 54.7%	269 73.1%	356 67.6%

73.1 percent of the students who had instruction before the study and 54.7 percent of the students who had no instruction before the study had misconception on 'Effect of Sample Size". It is understood that the students who had instruction on probability had this misconception more frequently than the students with no instruction on probability.

It is understood that the students confused "Ratio and Proportion" subject with "Probability of Events". It is obvious that these students had been taught probability. They used basic notions they had been taught: the concept of sample space and probability of an event. However, they confused probability of an event with ratio and proportion. The main idea for this inference is "the students wrote down equality of proportions $\frac{8}{10}=\frac{80}{100}$ and $\frac{200}{300}=\frac{2}{3}$ as equality of probability for both of 7^{th} and 8^{th} questions.

4.2.5 Misconception on Conjunction Fallacy w.r.t. PIoP

The ninth question related to misconception on "Conjunction Fallacy" is examined by taking into account PIoP. The results are given in Table 4.22.

Table 4.22 Number and Percentages of Students' Answers for the Question 9 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
	4	14	18
No answer	2.5%	3.8%	3.4%
	113	156	269
Misconception	71.1%	42.4%	51.0%
	42	198	240
Correct	26.4%	53.8%	45.5%

There were fewer correct answers in children who did not receive a certain instruction than in children who received any instruction in probabilities according to the results. 71.1 percent of the students who did not have instruction and 42.4 percent of the students who had instruction had "Conju nction Fallacy"

4.2.6 Misconception on the Heuristic of Availability w.r.t. PIoP

The tenth question related to misconception on "Heuristic of Availability" is examined by taking into account PIoP. The results are given in Table 4.23.

Table 4.23 Number and Percentages of Students' Answers for the Question 10 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	10 6.3%	21 5.7%	31 5.9%
Misconception	75 47.2%	155 42.1%	230 43.6%
Incorrect	33 20.8%	70 19.0%	103 19.5%
Correct	41 25.8%	122 33.2%	163 30.9%

The students who had instruction on probability had lower level of misconception Heuristics Availability. It is obvious that these students had been taught probability. They used basic notions they had been taught about probability subject.

4.2.7. Misconception on Effect of Time Axis w.r.t. PIoP

The eleventh and twelfth questions related to misconception on "Effect of Time Axis" (The Falk Phenomenon) are examined by taking into account PIoP. The results are given in Table 4.24.

Table 4.24 Number and Percentages of Students' Answers for the question 11and 12 w.r.t. PIoP

Categories	PIoP		Total
	No	Yes	
No Answer	1 .6%	13 3.5%	14 2.7%
Category I	37 23.3%	83 22.6%	120 22.8%
Category II	37 23.3%	146 39.7%	183 34.7%
Category III	70 44.0%	99 26.9%	169 32.1%
Other	14 8.8%	27 7.3%	41 7.8%

39.7 percent of the students who had instruction before the study and 23.3 percent of the students who had no instruction before the study had misconception on 'The Effect of Time Axis". It is understood that the students who had instruction on probability had this misconception more frequently than the students with no instruction on probability. Therefore, we can conclude that there is a development on 'The Effect of Time Axis" misconception as an effect of instruction.

4.2.8. Misconception on Equiprobability Bias w.r.t. PIoP

The thirteenth and fourteenth questions related to misconception on 'Equiprobability" are examined by taking into account PIoP. The results are given in Table 4.25 and Table 4.26.

Table 4.25 Number and Percentages of Students' Answers for the Question 13 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	3 1.9%	24 6.5%	27 5.1%
Correct	59 37.1%	107 29.1%	166 31.5%
Incorrect	38 23.9%	112 30.4%	150 28.5%
Misconception	59 37.1%	125 34.0%	184 34.9%

The students who had instruction in probability had little tendency in Equiprobability Bias misconception type. Parallel to this fact, this misconception type was more frequent among the students who had not had any course on probability before the study.

Table 4.26 Number and Percentages of Students' Answers for the Question 14 w.r.t. PIoP

Answers	PIoP		Total
	No	Yes	
No answer	4 2.5%	28 7.6%	32 6.1%
Correct	17 10.7%	86 23.4%	103 19.5%
Incorrect	2 1.3%	5 1.4%	7 1.3%
Incorrect	20 12.6%	15 4.1%	35 6.6%
Misconception	116 73.0%	234 63.6%	350 66.4%

The students who did not any instruction before the study had misconception more frequently than the students who received a certain instruction in probabilities according to the results. Therefore, there was decreasing in Equiprobability Bias misconception type with the effect of instruction.

Table 4.27 Comparison of Percentages of Students in all of Misconception Types w.r.t. PIoP.

Misconception type	Questions	PIoP No	PIoP Yes
Representativeness	1	13.20	8.20
	2	44.00	28.00
Positive and Negative Regency Effect	3	14.50	4.90
	4	29.60	23.40
Simple and Compound Events	5	74.80	83.20
	6	13.20	9.80
Effect of Sample Size	7	83.60	87.20
	8	54.70	73.10
Conjunction Fallacy	9	71.10	42.40
Heuristic of Availability	10	47.20	42.10
Effect of Time axis	11&12	23.30	39.70
Equiprobability Bias	13	37.10	34.00
	14	73.00	63.60

Probabilistic reasoning appears to increase slightly for older students, which is not surprising given that these students had some formal study of probability. In brief, as a result of instruction on probability, there was a decreasing in the misconception types; Represantativeness (by 1st and 2nd questions), Positive and Negative Regency Effect (by 3rd and 4th questions), Conjunction Fallacy (by 9th question), Heuristic of Availability (by 10th question) and Equiprobability Bias (by 13th and 14th questions). Because of the teaching of probability, there was a development on the misconception types; Effect of Sample Size (by 7th and 8th questions) and Effect of Time Axis (by 11 and 12th questions together). There was a decreasing on Simple and Compound Events with instruction with respect to 6th question; however, there is a development in Simple and Compound Event misconception type with instruction on probability with respect to 5th question. It is necessary to state that 5th question is rather difficult to be understood by the students as they stated in their answers.

4.3 Misconception Types with respect to Gender

How misconception types change with respect to gender was another purpose of the research. As stated in previous chapters of the present study, there were several studies that have found gender differences in different field of mathematics (Gallegher and De Lisi 1994; Halpern, 1997; Kimball, 1989). However, the effect of gender difference on probability has not been studied sufficiently not only in Turkey but also in the other countries.

In this section, the change in misconception types with respect to gender, will be analyzed. Each misconception type is examined with respect to gender at all grades separately.

4.3.1 Misconception on Represantativeness w.r.t. Gender

The distribution of the misconception type Representativeness with respect to gender at all grade levels is given in Table 4.28

The 1st and 2nd questions investigated Represantativeness misconception type. The misconception was frequent among male students at grade levels 8 and 10, whereas female students have much tendency to have this kind of misconception in grade level 9. The frequency was equal for males and females in grade level 6 for the first question. Finally, it can be stated that Representativeness did not across gender.

4.3.2 Misconception on Positive and Negative Regency Effect w.r.t. Gender

The distribution of the misconception type 'Positive and Negative Regency Effect" with respect to gender at all grade levels is given in Table 4.29.

The 3rd and 4th questions investigate the misconception type 'Positive - Negative Regency Effect". While female students had less frequent misconception in question 3, the misconception was frequent among female students according to 4th question in grade level 5. As in grade level 5, the misconception was frequent among males w.r.t. 3rd question, however, the misconception was frequent among females w.r.t. 4th question at grade level 8. Positive-Negative Regency Effect was frequent among boys for both of the questions in grade level 6. In grade level 7, 9, and 10, the misconception was stronger among girls than boys with respect to 3rd and 4th question. However, the misconception was frequent among female students at all grade levels except grade level 6 according to 4th question. At the end, it can be stated that Positive-Negative Regency Effect was frequent among female students.

4.3.3 Misconception on Simple and Compound Events w.r.t. Gender

The distribution of the misconception type Simple and Compound Events with respect to gender at all grade levels is given in Table 4.30.

5th and 6th questions investigate "Simple and Compound Events" misconception type. The misconception was more frequent among females than males in grade levels 5, 6 and 7. However, the misconception was more frequent among male students in grade level 8. Female students had little tendency in Simple and Compound Events misconception with respect to 5th question however; the misconception was frequent among female students with respect to 6th question in grade level 9. The situation is reverse in grade level 10, that is male students had little tendency in Simple and Compound Events misconception with respect to 5th question; however, the misconception was frequent among male students with respect to 6th question in grade level 10. At last, Simple and Compound Events varied across gender.

4.3.4 Misconception on Effect of Sample Size w.r.t. Gender

The distribution of the misconception type Effect of Sample Size with respect to gender at all grade levels is given in Table 4.31.

Questions 7 and 8 investigate "Effect of sample Size" misconception type. In grade level 5, the misconception was more frequent among male students than female students. However, the frequency of the misconception was higher for female students in grade level 6, level 7, level 8 and level 10. Although female students had more misconception in 7th question, they had less frequent misconception with respect to male students in question 8 at grade level 9. Finally, although misconception types varied across gender definitely, it seems that the misconception types were more frequent among females than males.

Table 4.28 Number and Percentages of Students' Answers for the Questions 1 and 2 w.r.t. Gender and GL

			Question 1						Question 2			
GL	G		NA	I	I	I	M	**C**	NA	I	M	C
5	F	n	--	--	--	2	2	13	--	--	7	10
		%	--	--	--	11.8	11.8	76.5	--	--	41,2	58,8
	M	n	--	1	--	--	1	15	1	--	11	5
		%	--	5,9	--	--	5,9	88.2	5,9	--	64,7	29,4
6	F	n	--	--	--	--	3	9	--	--	5	7
		%	--	--	--	--	25.5	75.5	--	--	41,7	58,3
	M	n	--	--	2	4	6	12	--	--	11	13
		%	--	--	8.3	16,7	25	50	--	--	45,8	54,2
7	F	n	9	10	10	6	8	75	1	3	79	35
		%	7,6	8.5	8.5	5.1	6.8	63.6	0,8	2,5	66,9	29,7
	M	n	7	7	11	6	44	95	5	6	91	68
		%	4,1	4.1	6.5	3.5	25,9	55.9	2,9	3,5	53,5	40
8	F	n	--	2	1	--	5	57	6	1	21	37
		%	--	3,1	1,5	--	7,7	87,7	9,2	1,5	32,3	56,9
	M	n	--	1	4	1	12	60	--	--	27	51
		%	--	1,3	5,1	1,3	15,4	76,9	--	--	34,6	65,4
9	F	n	3	11	4	4	16	108	1	4	71	70
		%	2,1	7,5	2,7	2,7	11	74	0,7	2,7	48,6	47,8
	M	n	2	--	--	4	6	43	1	3	18	33
		%	3,6	--	--	7,3	10,9	78,2	1.8	5,5	32,7	60
10	F	n	1	1	2	1	7	99	3	--	24	84
		%	0,9	0,9	1,8	0,9	6,3	89,2	2,7	--	21,6	75,5
	M	n	2	1	1	1	5	62	1	--	12	59
		%	2,8	1,4	1,4	1,4	6,9	86,1	1,4	--	16,7	81,9

Table 4.29 Number and Percentages of Students' Answers for the Que stions 3 and 4 w.r.t. Gender and GL

			Question 3				Question 4			
GL	G		NA	C	M	I	NA	M	I	C
5	F	n	--	12	3	2	--	7	2	8
		%	--	70,6	17,6	11,8	--	41,2	11,8	47,1
	M	n	1	12	4	--	1	4	1	11
		%	5,9	70,6	23,5	--	5,9	23,5	5,9	64,7
6	F	n	--	10	0	2	1	2	3	6
		%	--	83,3	0	16.7	8,3	16,7	25	50
	M	n	--	14	4	6	1	6	7	10
		%	--	58,3	16,7	25	1,4	25	29,2	41,7
7	F	n	3	61	21	33	9	52	27	30
		%	2,5	51,7	17,8	28	7,9	44,1	22,9	25,4
	M	n	7	98	19	46	14	61	48	47
		%	4,1	57,6	11,2	27,1	8,2	35,9	28,2	27,6
8	F	n	4	48	2	11	9	15	21	20
		%	6,2	73,8	3,1	16,9	13,8	23,5	32,3	30,8
	M	n	--	69	4	5	10	10	17	41
		%	--	88,5	5,1	6,4	12,8	12,8	21,8	52,6
9	F	n	1	98	19	28	2	56	25	63
		%	0,7	67,1	13,0	19,2	1,4	38,4	17,1	43,2
	M	n	--	36	6	13	3	11	13	28
		%	--	65,5	10,9	23,6	5,5	20	23,6	50,9
10	F	n	--	78	7	26	11	28	20	52
		%	--	70,3	6,3	23,4	9,9	25,2	18	46,8
	M	n	--	61	3	8	9	13	16	34
		%	--	84,7	4,2	11,1	12,5	18,1	22,2	47,2

Table 4.30 Number and Percentages of Students' Answers for the Questions 5 and 6 w.r.t. Gender and GL

			Question 5				Question 6			
GL	G		NA	I	C	M	NA	C	I	M
5	F	n	1	--	1	15	--	12	1	4
		%	5,9	--	5,9	88,2	--	70,6	5,9	23,5
	M	n	2	1	1	13	1	12	2	2
		%	11,8	5,9	5,9	76,5	5,9	70,6	11,8	11,8
6	F	n	--	1	--	11	--	6	2	4
		%	--	8,3	--	91,7	--	50,0	16,7	33,3
	M	n	--	2	3	19	--	17	2	5
		%	--	8,3	12,5	79,2	--	70,8	8,3	20,8
7	F	n	5	10	24	79	8	61	25	24
		%	4,2	8,5	20,3	66,9	6,8	51,7	21,2	20,3
	M	n	18	10	31	111	13	101	24	32
		%	10,6	5,9	18,2	65,3	7,6	59,4	14,1	18,8
8	F	n	6	3	7	49	5	56	1	3
		%	9,2	4,6	10,8	75,4	7,7	86,2	1,5	4,6
	M	n	1	1	10	66	1	67	6	4
		%	1,3	1,3	12,8	84,6	1,3	85,9	7,7	5,1
9	F	n	4	10	20	112	5	72	41	28
		%	2,7	6,8	13,7	76,7	3,4	49,3	28,1	19,2
	M	n	2	3	7	43	1	43	3	8
		%	3,6	5,5	12,7	78,2	1,8	78,2	5,5	14,5
10	F	n	3	4	6	98	3	92	9	7
		%	2,7	3,6	5,4	88,3	2,7	82,9	8,1	6,3
	M	n	3	5	7	57	1	60	4	7
		%	2,7	6,9	9,7	79,2	1,4	83,3	5,6	9,7

Table 4.31 Number and Percentages of Students' Answers for the Questions 7 and 8 w.r.t. Gender and GL

			Question 7				Question 8			
GL	G		NA	C	I	M	NA	I	C	M
	F	n	1	--	1	15	--	--	7	10
5		%	5,9	--	5,9	88,2	--	--	41,2	58,8
	M	n	--	--	--	17	1	1	3	12
		%	--	--	--	100	5,9	5,9	17,6	70,6
	F	n	--	--	--	12	--	--	3	9
6		%	--	--	--	100	--	--	25	75
	M	n	1	3	2	18	--	2	6	16
		%	4,2	12,5	8,3	75	--	8,3	25	66,7
	F	n	7	13	6	92	9	11	33	65
7		%	5,9	11	5,1	78	7,6	9,3	28	55,1
	M	n	22	16	13	119	12	19	46	93
		%	12,9	9,4	7,6	70	7,1	11,2	27,1	54,7
	F	n	7	4	--	54	3	7	4	51
8		%	10,8	6,2	--	83,1	4,6	10,8	6,2	78,5
	M	n	1	8	4	65	--	9	17	52
		%	1,3	10,3	5,1	83,3	--	11,5	21,8	66,7
	F	n	4	10	7	125	7	26	35	78
9		%	2,7	6,8	4,8	85,6	4,8	17,8	24	53,4
	M	n	4	4	3	44	2	6	12	35
		%	7,3	7,3	5,5	80	3,6	10,9	21,8	63,6
	F	n	2	1	6	102	2	10	11	88
10		%	1,8	0,9	5,4	91,9	1,8	9	9,9	79,3
	M	n	3	3	2	64	1	7	12	52
		%	4,2	4,2	2,8	88,9	1,4	9,7	16,7	72,2

4.3.5 Misconception on Conjunction Fallacy w.r.t. Gender

The distribution of the misconception type Conjunction Fallacy with respect to gender at all grade levels is given in Table 4.32

Table 4.32 Number and Percentages of Students' Answers for the Question 9 w.r.t. Gender and GL

			Q 9		
GL	G		NA	M	C
5	F	n	--	8	9
		%	--	47,1	52,9
	M	n	1	3	13
		%	5,9	17,6	76,5
6	F	n	1	4	7
		%	8,3	33,3	58,3
	M	n	--	17	7
		%	--	70,8	29,2
7	F	n	3	70	45
		%	2,5	59,3	38,1
	M	n	11	98	61
		%	6,5	57,6	35,9
8	F	n	--	35	30
		%	--	53,8	46,2
	M	n	2	30	46
		%	2,6	38,5	59,0
9	F	n	4	106	36
		%	2,7	72,6	24,7
	M	n	4	28	23
		%	7,3	50,9	41,8
10	F	n	3	48	60
		%	2,7	43,2	54,1
	M	n	5	22	45
		%	6,9	30,6	62,5

Conjunction Fallacy was investigated by question 9. The misconception was more frequent among female students in grade levels 5, 7, 8, 9 and 10; however, the reverse was true for the students in grade level 6. The conjunction fallacy was more frequent among female students than male students.

4.3.6 Misconception on the Heuristic of Availability w.r.t. Gender

The distribution of the misconception type The Heuristic of Availability with respect to gender at all grade levels, is given in Table 4.33.

Table 4.33 Number and Percentages of Students' Answers for the Question 10 w.r.t. Gender and GL

			Q 10			
GL	G		NA	M	I	C
	F	n	1	9	6	1
5		%	5,9	52,9	35,3	5,9
	M	n	1	7	5	4
		%	5,9	41,2	29,4	23,5
	F	n	--	8	2	2
6		%	--	66,7	16,7	16,7
	M	n	--	16	3	5
		%	--	66,7	12,5	20,8
	F	n	17	52	30	19
7		%	14,4	44,1	25,4	16,1
	M	n	17	80	39	34
		%	10	47,1	22,9	20
	F	n	8	38	15	4
8		%	12,3	58,5	23,1	6,2
	M	n	2	53	14	9
		%	2,6	67,9	17,9	11,5
	F	n	10	59	31	46
9		%	6,8	40,4	21,2	31,5
	M	n	6	19	10	20
		%	10,9	34,5	18,2	36,4
	F	n	2	39	19	51
10		%	1,8	35,1	17,1	45,9
	M	n	3	22	14	33
		%	4,2	30,6	19,4	45,8

'The Heuristic of Availability" misconception type was investigated by 10 th question. This misconception was stronger for female students than male students in grade levels 5, 9 and 10. The frequency was equal for both groups in grade 6. The Heuristic of Availability was frequent among male students in grade levels 7, 8.

4.3.7 Misconception on Effect of Time Axis (The Falk Phenomenon) w.r.t. Gender

The distribution of the misconception type *The Falk Phenomenon* with respect to gender at all grade levels is given in Table 4.34.

Table 4.34 Number and Percentages of Students' Answers for the Questions 11 and 12 w.r.t. Gender and GL

				Category			
				Cat1	Cat2	Cat3	Other
GL	G		NA	C	M	I	I
5	F	N	--	5	17,6	4	5
		%	--	29,4	35,3	23,5	29,4
	M	N	--	3	6	2	3
		%	--	17,6	35,3	11,8	17,6
6	F	N	--	2	5	3	2
		%	--	16,7	41,7	25	16,7
	M	N	1	6	4	9	4
		%	4,2	25	16,7	37,5	16,7
7	F	N	7	28	15	47	21
		%	5,9	23,7	12,7	39,8	17,8
	M	N	10	68	31	41	20
		%	5,9	40	18,2	24,1	11,8
8	F	N	2	19	25	13	6
		%	3,1	29,2	38,5	20	9,2
	M	N	2	30	32	8	6
		%	2,6	38,5	41	10,3	7,7
9	F	N	5	18	39	71	13
		%	3,4	12,3	26,7	48,6	8,9
	M	N	3	14	14	20	4
		%	5,5	25,5	25,5	36,4	7,3
10	F	N	1	20	43	41	6
		%	0,9	18	38,7	36,9	5,4
	M	N	1	19	30	16	6
		%	1,4	26,4	41,7	22,2	8,3

Questions 11 and 12 together tested the "Effect of the time axis" which is also called the "Falk Phenomenon". The 2nd category represents the main misconception in which, a student solved the 11th question correctly whereas 12th

question was solved incorrectly. Female students had little tendency to have Falk Phenomenon than males at all grade levels. In other words male students had this misconception more frequently than female students at all grade levels.

4.3.8 Misconception on Equiprobability Bias w.r.t. Gender

The distribution of the misconception type Equiprobability Bias with respect to gender at all grade levels is given in Table 4.35.

Question 13 and 14 investigate Equiprobability Bias misconception type. The frequency of Equiprobability Bias among females was clearly higher than males at grade levels 5, 7, 8, 9 and 10. In other words, Equiprobability Bias was frequent among female students in these levels. The misconception was also stronger for females for 13th question however males had much tendency to have this misconception in grade level 6. It can be stated that Equiprobability Bias was frequent among female students.

Table 4.35 Number and Percentages of Students' Answers for the Questions 13 and 14 w.r.t. Gender and GL

			Question13				Question 14				
GL	G		NA	C	I	M	NA	C	I	I	M
	F	n	--	2	3	12	--	3	1	1	12
5		%	--	11,8	17,6	70,6	--	17,6	5,9	5,9	70,6
	M	n	1	10	1	5	--	9	--	1	7
		%	5,9	58,8	5,9	29,4	--	52,9	--	5,9	41,2
	F	n	1	1	4	6	1	2	--	2	7
6		%	8,3	8,3	33,3	50	8,3	16,7	--	16,7	58,3
	M	n	2	8	6	8	--	7	--	1	16
		%	8,3	33,3	25	33,3	--	29,2	--	4,2	66,7
	F	n	15	41	25	37	7	27	2	12	70
7		%	12,7	34,7	21,2	31,4	5,9	22,9	1,7	10,2	59,3
	M	n	16	54	51	49	14	52	10	8	86
		%	9,4	31,8	30	28,8	8,2	30,6	5,9	4,7	50,6
	F	n	5	19	22	19	4	8	2	--	51
8		%	7,7	29,2	33,8	29,2	6,2	12,3	3,1	--	78,5
	M	n	6	17	39	16	7	29	1	4	37
		%	7,7	21,8	50	20,5	9	37,2	1,3	5,1	47,4
	F	n	7	46	23	70	10	14	2	15	105
9		%	4,8	31,5	15,8	47,9	6,8	9,6	1,4	10,3	71,9
	M	n	2	18	13	22	2	8	1	7	37
		%	3,6	32,7	23,6	40	3,6	14,5	1,8	12,7	67,3
	F	n	4	43	29	35	5	17	--	4	85
10		%	3,6	38,7	26,1	31,5	4,5	15,3	--	3,6	76,6
	M	n	3	23	24	22	4	27	1	5	35
		%	4,2	31,9	33,3	30,6	5,6	37,5	1,4	6,9	48,6

The frequencies of students' misconception types varied across the gender. In the literature we could not reach any study on examining probabilistic misconceptions with respect to gender. Hence, we could not compare the results of the present study on gender with the results of the other studies on the same topic. When it was desired to compare the results of this study indirectly with studies on gender difference in mathematics achievement and especially probability achievement. Each area had mixed results. In other words, some studies (e.g. Bulut, 1994; Moran & McCullars, 1979) found higher probability achievement for females, some (e.g. Dusek & Hill, 1977; Krietler, Zigler & Krietler, 1983) found higher achievement for males, and some found no significant differences (e.g. Hanna, 1986).

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

In this chapter, the conclusions obtained from the results of the study and some recommendations for mathematics teachers and for further studies will be included. The conclusions of the study will be stated at the first section of this chapter. Secondly, there are some recommendations given for mathematics teachers and for further studies. Internal and external validity for the study will be included in the third section of this chapter.

5.1 Conclusions

In this section, conclusions based on the present study are stated. The purpose of the study was to investigate the students' probabilistic misconceptions: Representativeness, Positive and Negative Recency Effects, Simple and Compound Events, Effect Sample Size, Conjunction Fallacy, Heuristic Availability, The Time Axis Fallacy and Equiprobability Bias with respect to grade levels, gender and previous instruction on probability. A set of well-known probability problems that have been described as leading to intuitively based misconceptions, were selected. After that the Probability Misconception Test (PMT) was administered to the subject of the study.

5.1.1 Misconception Types with respect to Grade Level

The results obtained from the study shows that, the general assumption about the stability of intuition is not true. In other words, the frequencies of misconceptions varied across grade levels. To explain the change in misconceptions on probability w.r.t. grade levels was rather complex: some misconceptions diminished with age, some stable and some gained greater influence. However, it should not be ignored that at grade levels 8 and 10, the students have little tendency against misconceptions with respect to preceding levels. It was expressed in the study that the curriculum program includes probability subject at these levels.

5.1.2 Misconception Types with respect to Previous Instruction on Probability

The students tried to reply almost all of the questions in the study. Although, some of the participants had not any instruction on probability, they used their own idea to solve the questions. In other words, it can be said that the students tried to solve the questions intuitively. However, it is understood from the results of the present study that the students received formal teaching on probability were more successful almost all part of the study. Except for the misconception types named "Effect of Sample Size" (by 7 th and 8th questions) and "Effect of Time Axis" (by 11&12th questions together), the instruction on probability has a *positive effect* on students to prevent from the misconceptions on probability in the study. From that point, the teachers should consider some difficulties that students have inconsistencies on probability. This fact will be included deeply at section of "recommendations for mathematics teachers."

5.1.3 Misconception Types with respect to Gender

How misconception types vary across respect to gender was another purpose of the research. At the end of the present study, it is understood that explaining the change in misconception types in probability w.r.t. gender is rather difficult and complex according to the results. In spite of the fact that the misconception types in probability did not vary across gender definitely, female students had more tendencies in misconception on probability than males. Differently, there was not encountered any situation in which the misconception types were more frequent among male students than female students. At the present study, in order to see how misconception types vary across gender, grade levels were kept constant in other words, each misconception type was investigated with respect to gender at all grade levels separately. Briefly; the change in misconception types with respect to gender can be stated as following;

a) The misconception types were frequent among females in grade levels 7, 9 and 10, however, the frequency of misconception types almost equal for both females and males in grade levels 5, 6 and 8.
b) The misconception types named 'Simple and Compound Events" (4^{th} and 5^{th} questions), 'Effect of Sample Size" (7^{th} question), "The conjunction Fallacy" (9^{th} question) and 'Equiprobability Bias" (13^{th} and 14^{th} questions) were more frequent among female students than male students. The frequency of other misconception types was equal for females and males.

5.2 Recommendations

In this section, some recommendations for mathematics teachers and some recommendations for further studies are presented. In the present study it is intended to provide an idea about the misconceptions as mentioned before, a map of misconception.

5.2.1 Recommendations for Further Studies

According to the results of the present study, the students who had instruction on probability had more tendencies in the misconception types; Effect of Sample Size and Effect of Time Axis than the students who had not any instruction on probability. Therefore, in this study, the teachers must consider Effect of Sample Size and Effect of Time Axis misconception types while teaching probability in classes. The teacher should stress problematic parts of probability to prevent misunderstandings. Moreover, the students have contradictions on some type of questions such as the 5^{th} question in this study. In addition to these misconception types almost all students had different types of probabilistic misconceptions so teacher must be aware of them and overcome these misconceptions while teaching their subject. For instance, many examples should be solved in classes to make the instruction effective.

If one wants to develop probabilistic thinking or probabilistic reasoning by means of instruction; the mathematics teachers should develop a strong, correct, coherent and formal instruction model in schools. In addition, this model should include intuitive background for probabilistic thinking. Moreover, in order to teach probability concept effectively in classes, the teachers may be able to show the students where their ideas are inconsistent by having some idea as to what preconceived ideas of probability are.

It is seen that the students with no formal teaching do indeed have some notion of probability according the answers that the students stated. In spite of the fact that the students do not have any course on probability, most of them gave answer to the questions in the study. From the reasons they stated for the questions, it will be understood that the students believe that their notion of probability is correct, although their choices of responses indicate the wrong answer.

Furthermore I believe that the problems analyzed in this study could be useful to consider and discuss during instruction on probability in classes. Such problems could be found in the studies by reviewing the literature. Besides presenting such problems and their solutions, it could be effective to analyze the structure of corresponding misconceptions psychologically. Because Probability subject requires a way of thinking, it does not consist of only technical information and procedures leading to solution. The teachers should make students create new intuitions in teaching probability. If the students can learn to analyze the causes underlying conflict and mistakes, they may be able to overcome these difficulties. Therefore, the instruction on probability should lead students gain experience about the conflicts between their own intuition and the particular types of thinking in stochastic situations.

Finally, while preparing the curriculum program on probability, these difficulties should be considered by program makers to help the teachers. Another point could be pre-service and in-service mathematics teachers should be educated how to diagnosis students' and even their own misconceptions and overcome them. Parallel to curriculum in schools, there should be some courses in Mathematics Education Program in universities about how to teach probability and statistics in schools for mathematics teacher candidates.

5.2.2 Recommendations for Further Studies

This subject requires more detailed researches. First of all the sample size must be increased in further studies. To be able to talk about Turkey overall, subjects from different schools of different geographical regions should be selected. As a second issue, as mentioned in the present study before, using scales, questionnaires may provide an idea on misconceptions of subjects but for a "deep" investi gation of misconceptions, qualitative methods of research seem more appropriate. In other words, to have a more detailed opinion of students' on probability subject, a one -to-one observation of students could be conducted. Case studies, interviews should also be conducted and the investigation process should be laid in time. By employing these techniques, the research will provide more accurate information a instead of a snapshot picture of the beliefs.

The present study contributes to be aware of students probabilistic misconceptions. Further research is needed to continue the investigation begun here.

5.3 Internal and External Validity

In this section internal and external validity of the study will be discussed and how the threats to internal and external validity are controlled will be explained.

5.3.1 Internal Validity of the Study

By internal validity, it is meant that observed differences on the dependent variable are directly related to the independent variable, and due to some unintended variable (Fraenkel & Wallen, 1996). There were possible threats to internal validity in this study. Subject characteristics, location, data collector characteristics, data collector bias were such threats. The ways of controlling these treats were discussed.

The number of boys and girls was not equal but difference with respect to gender was considered as a variable and examined as a sub-problem. Hence, those characteristics did not affect research results intentionally.

Data collector's characteristics and bias should not be threats for this study since the data collector (the researcher) followed the same procedure while administering the test and questionnaire.

5.3.2 External Validity of the Study

5.3.2.1 Population Validity

In the present study, convenience sampling was utilized. Because of this, generalizations of the findings of the study were limited. However, generalizations can be done on subjects having the same characteristics mentioned in the "subjects of the study" section.

5.3.2.2 Ecological Validity

The ecological validity is the degree to which results of a study can be extended to other setting or conditions (Fraenkel &Wallen, 1996). Since the study is on students from grade level 5 through 10, the results of the present study can be generalized to similar settings to this study.

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APPENDIX

OLASILIK KAVRAM YANILGISI TESTI

Sevgili Ögrenciler **:**

Bu test sizin olasilik konusu uzerine nasil dusundugunuzu ölcmek icin hazirlanmistir. Bu test sonuclari **sadece arastirma amacli kullanilacaktir**. Testten alacaginiz puanlar ve bilgiler kesinlikle **aciklanmayacaktir** ve **not verilmeyecektir**. Butun sorulari dikkatli okumaya ve kendiniz cevaplamaya özen gösteriniz. Her soru icin **en uygun** buldugunuz secenegi isaretlemeniz ve altina o secenegi **neden** isaretlediginizi aciklamaniz gerekmektedir.Test toplam **14** sorudan ve **4** sayfadan olusmaktadir. Sinav suresi 35 dakikadir. Tesekkur eder, basarilar dileriz.

Ali Ihsan MUT & Doc. Dr. Safure BULUT
O.D.T.U Egitim Fakultesi

SORULAR

1.) Hilesiz bir madeni para 5 defa arka arkaya havaya atiliyor. **Y** *yaziyi* **T** *turayi* temsil ettigine göre bu atislarda sirasiyla asagidakilerden hangisinin gelme olasiligi en buyuktur?

a) YYYTT **b)** YTTYT **c)** TYTTT
d) YTYTY **e)** a, b, c ve d siklarinin gelme olasiliklari esittir.

2.) Bir sayisal loto oyununda bir kisi **1**'den **40**'a kadar olan sayilardan 6 tanesini secmek zorundadir. Ahmet 1, 2, 3, 4, 5 ve 6 sayilarini, Nuray 39, 1, 17, 33, 8 ve 27 sayilarini secmistir. Sizce kimin kazanma olasiligi daha buyuktur?

a) Ahmet **b)** Nuray **c)** Ikisinin kazanma olasiliklari esittir.

3.) Özge, hilesiz bir madeni parayi uc kez havaya atmis ve hepsinde yazi gelmistir. Özge, 4.kez parayi havaya attiginda asagidakilerden hangisi dogru olur?

a) Yazi gelme olasiligi, tura gelme olasiligina esittir.
b) Yazi gelme olasiligi, tura gelme olasiligindan kucuktur.
c) Yazi gelme olasiligi, tura gelme olasiligindan buyuktur.

4.) Bir baba ve ogul her gun bir oyun oynuyorlar. Oyunda baba, eline bir madeni para alir ve ellerini arkasina saklar.Eger cocuk paranin babasinin hangi elinde oldugunu bilirse parayi kazanir. Gecen 14 gun icinde cocuk 5 defa dogru, 9 defa yanlis tahminde bulunmustur. Gelecek 14 gunde asagidakilerden hangisinin olmasini beklersiniz?

a) Cocugun dogru tahmin sayisinin yanlis tahmin sayisindan fazla olmasini
b) Cocugun dogru tahmin sayisinin yanlis tahmin sayisindan az olmasini
c) Cocugun dogru tahmin sayisinin yanlis tahmin sayisina esit olmasini

5.) Hilesiz iki zar ayni anda havaya atiliyor.Asagidakilerden hangisinin olma olasiligi daha buyuktur?

a) 6 ve 6 rakamlarinin gelmesi (baska bir deyisle 6-6 ciftinin gelmesi)
b) 5 ve 6 rakamlarinin gelmesi (baska bir deyisle 5-6' n in gelmesi)
c) "a" ve "b" siklarinin olma olasiliklari esittir.

6.) " **CICEK** " kelimesini olu sturan harfler kagitlara yazilip bir torbaya atiliyor.Bu torbadan rasgele secilen harfin "**C**" olmas i olasiligi asagidakilerden hangisidir?

a) $\frac{2}{5}$ **b)** $\frac{2}{3}$ **c)** $\frac{1}{4}$

7.) Bir hastanede yeni doganlarin kayitlari tutuluyor. Buna göre asagidakilerden hangisinin olma olasiligi daha buyuktur?

a) Ilk dogan 10 bebekten 8 veya daha fazlasinin kiz olmasi.

b) Ilk dogan 100 bebekten 80 veya daha fazlasinin kiz olmasi.

c) a ve b siklarinin olma olasiliklari esittir.

8.) **Olay 1:** Hilesiz bir madeni paranin **300** kez havaya atilmasi deneyi sonucunda en az **200** kez yazi gelmesi.

Olay 2: Hilesiz bir madeni paranin **3** kez havaya atilmasi deneyi sonucunda en az **2** kez yazi gelmesi.

Yukaridaki deneylerden hangisinin sonucunun olma olasiligi daha buyuktur?

d) Olay 1.

e) Olay 2.

f) Olay 1 ve Olay 2' nin olma olasiliklari esittir.

9.) Fatih insanlara yardim etmeyi sevmekte ve doktor olmayi istemektedir.Lisedeyken Kizilay Kolu'nda görev almis ve yaz kamplarinda saglik hizmetlerinde calismistir. Su anda bir universiteye kayitlidir. Buna göre asagidakilerden hangisi daha olasi görunmektedir?

a) Fatih Tip Fakultesinde ögrencidir. **b)** Fatih ögrencidir.

10.) **K:** 10 kisilik bir topluluk icinden olusturulacak 2 kisilik guruplarin sayisi,

L: 10 kisilik bir topluluk icinden olusturulacak 8 kisilik guruplarin sayisi,

olduguna göre **K** ve **L** sayilari arasinda nasil bir iliski vardir?

a) K, L' den buyuktur.

b) K, L' den kucuktur.

c) K, L' ye esittir.

11.) Dilek 'in elinde, i cinde iki siyah ve iki beyaz bilye bulunan bir torba var. Dilek torbadan bir bilye cekiyor ve bilyenin beyaz oldugunu göruyor. Elindeki bilyeyi geri koymadan bir bilye daha cekiyor. Buna göre asagidakilerden hangisi dogrudur?

d) Ikinci bilyenin beyaz olma olasiligi, siyah olma olasiligina esittir.
e) Ikinci bilyenin beyaz olma olasiligi, siyah olma olasiligindan buyuktur.
f) Ikinci bilyenin beyaz olma olasiligi, siyah olma olasiligindan kucuktur.

12.) Ahmet'in elinde i cinde iki siyah ve iki beyaz top bulunan bir torba var. Ahmet torbadan bir top cekiyor ve bakmadan topu bir kenara koyuyor.Torbadan baska bir top daha cekiyor ve bunun beyaz oldugunu göruyor. Buna göre asagidakilerden hangisi dogrudur?

d) Ilk cektigi topun beyaz olma olasiligi, siyah olma olasiligindan buyuktur.
e) Ilk cektigi topun beyaz olma olasiligi, siyah olma olasiligindan kucuktur.
f) Ilk cektigi topun beyaz olma olasiligi, siyah olma olasiligina esittir.

13.) 5 yuzu siyaha, 1 yuzu de beyaza boyanmis 6 tane hilesiz zar atildiginda asagidakilerden hangisinin olma olasiligi daha buyuktur?

a) 5 zarin siyah, 1 zarin beyaz gelmesi.
b) 6 zarin siyah gelmesi.
c) "a" ve "b" siklarinin olma olasiliklari esittir.

14.) Sonunda 8 tane ayni cesit tuzak bulunan bir labirente birakilan robot devamli ileri gitmek uzere hic geri gelmeyecek sekilde programlanmistir. Robot her bir yol ayiriminda devam edecegi yolu rasgele secmektedir. Bu robotun hangi tuzaga yakalanma olasiligi daha buyuktur?

a) 1. tuzak
b) 3. tuzak
c) 5. tuzak
d) Butun tuzaklara yakalanma olasiligi esittir.