# THE INCLUSIVE SEMILEPTONIC DECAYS OF THE B-MESON IN A CP SOFTLY BROKEN TWO HIGGS DOUBLET MODEL

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

HİLAL ACAR

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

THE DEPARTMENT OF PHYSICS

JANUARY 2004

Approval of the Graduate School of Natural and Sciences.

Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Sinan Bilikmen
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Gürsevil Turan
Supervisor

Examining Committee Members

Prof. Dr. Hüseyin Koru

Assoc. Prof. Dr. Gürsevil Turan

Prof. Dr. Mustafa Savcı

Prof. Dr. Osman Yılmaz

Assoc. Prof. Dr. Meltem Serin Zeyrek

# ABSTRACT

THE INCLUSIVE SEMILEPTONIC DECAYS OF THE B-MESON IN A CP SOFTLY BROKEN TWO HIGGS DOUBLET MODEL

Acar, Hilal
M.S., Department of Physics
Supervisor: Assoc. Prof. Dr. Gürsevil Turan

January 2004, 53 pages.

In this work, the $B \to X_d \ell^+ \ell^-$decays are examined in the context of a CP softly broken two Higgs doublet model. The differential branching ratio, forward-backward asymmetry, CP-violating asymmetry, CP-violating asymmetry in the forward-backward asymmetry and polarization asymmetries of the final lepton in this decay are studied. The dependencies of these physical parameters on the model parameters are analyzed by paying a special attention to the effects of neutral Higgs boson (NHB) exchanges and possible CP violating effects. It has been found that NHB effects are quite significant for the $\tau$ mode and the above-mentioned observables seems to be promising as a testing ground for new physics beyond the SM, especially for the existence of the CP-violating phase in the theory.

Keywords: Two Higgs doublet model, Flavor Changing Neutral Current, Rare Decay, CP asymmetry.

# ÖZ

CP ZAYIFÇA KIRILAN İKİ HIGGS DUBLET MODELDE İNKLUSİF YARILEPTONİK B-MESON BOZUNMALARI

Acar, Hilal
Yüksek Lisans Tezi , Fizik Bölümü
Tez Yöneticisi: Assoc. Prof. Dr. Gürsevil Turan

Ocak 2004, 53 sayfa.

Bu çalışmada, CP simetrisinin zayıfça bozulduğu iki Higgs dublet modelinde $B \to X_d \ell^+\ell^-$bozunumu incelendi. Bu bozunumun difransiyel dallanma oranı, ileri-geri asimetrisi, CP bozulma asimetrisi, ileri-geri asimetrisindeki CP bozulma asimetrisi ve lepton polarizasyon asimetrisi çalışıldı. Bu fiziksel parametrelerin model parametrelerine bağlılıkları nötr Higgs bozon etkileri (NHB) ve CP bozulma etkileri özellikle dikkate alınarak incelendi. NHB etkilerinin $\tau$ modu için oldukça fazla olduğu, incelenen parametrelerin SM ötesi yeni modellerin test edilmesinde, özellikle teorideki CP bozan fazın varlığı konusunda umut vaat ettiği görüldü.

Anahtar Sözcükler: İki Higgs Dublet Modeli, Çeşni Değiştiren Nötr Akımlar, Nadir Bozunumlar, CP Asimetrisi.

TO MY FAMILY

## ACKNOWLEDGMENTS

I would like to thank my supervisor Assoc. Prof. Dr. Gürsevil Turan for her encouragement, support and help throughout this work.

I am also thankful to my family for constant support and patience.

# TABLE OF CONTENTS

# LIST OF FIGURES

# CHAPTER 1

# INTRODUCTION

The theory that currently describes all what is known about matter and the forces of nature is the Standard Model (SM). According to the SM, all the particles in the universe can be grouped into three "families" of particles: quarks, leptons, and force carrier particles. There are six types ('flavours') of quarks u,d,c,s,t,b, and also six flavours of leptons $e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$. The charge of each quark is a fraction of the charge of an electron. Leptons and quarks are classified in three generations. Each generation is made up of a charged lepton, its associated neutrino and two quarks, one quark with charge -1/3 and one with charge +2/3.

There appear to be four distinctly different types of forces in nature. The gravitational and electromagnetic forces are already known from classical physics. Gravitation is believed to play a negligible role in the phenomenology of elementary particle physics, since it is extremely weak between the individual particles. Whereas the gravitational and electromagnetic forces have an infinite range, i.e., they produce potentials which fall off as 1/r with distance r, there are also two very short range-forces: the weak and strong forces. The strong force has a range of about $10^{-13}$ cm, and is responsible for binding the quarks into the finite structures called hadrons. The weak force has a range of about $10^{-16}$ cm, and is responsible for the $\beta$-decay of nuclei.

All the known elementary particles are subject to the gravitational and weak interactions. Particles that are not influenced by the strong force are leptons. All hadrons and leptons, apart from the neutrinos, take part in the electromagnetic interactions. On the other hand, the strong interaction is only effective between hadrons. The quark scheme naturally accommodates the observed separation of hadrons into baryons and mesons. The baryons are bound states of three quarks; the mesons are composed of a quark and an antiquark.

All these forces are associated with elementary spin-1 bosons, the gauge bosons or force carriers. Consider for example the electromagnetic interaction. According to the quantum theory of electromagnetism, the so-called Quantum Electrodynamics (QED), the interaction is transmitted discontinuously by exchange of spin-1 photon, whereas in the classical theory the interaction between two charged particles is transmitted by electromagnetic waves which are continuously emitted and absorbed. The classical description is adequate at long distances, but at short distances the quantum nature of the interaction must be taken into account. The long-range nature of the electromagnetic force is related to the fact that the photons have zero mass.

The equivalent exchange particles for the strong interactions are called gluons, and are massless like the photon. In addition to electric charge, quarks have another property called color; each flavor of quark comes in three colors: red, green, blue. The theory of the strong force, which is modelled directly on QED, is called Quantum Chromodynamics (QCD) and acts not between the electric charges but between the color charges. Whereas QED has a single photon to transmit the electromagnetic interaction, QCD has eight gluons, which carry color charges too. By analogy with electromagnetism, the basic strong interaction between the quarks is long-range. However, the strong interaction between the quark bound states, namely hadrons, is short range. This is a result of the fact that the gluons carry color charges and they become asymptotically free at small separations.

For the weak interactions, the spin-1 exchange particles which transmit the interaction are called $W$ and $Z$ bosons, and are very massive. So the resulting force is short-range, and in many applications may be approximated by an interaction at a point. However, this phenomenological heavy boson exchange model of weak interactions ( Fermi model) is very singular and of no use for higher-order calculations.

Calculations of various phenomena in QED show that there are similar problems with this theory in that divergent results are obtained for physically measurable quantities such as mass and charge of the electron. However, these divergences can be controlled by a procedure known as renormalization, leading to successful estimates of quantities such as the anomalous magnetic moment of the electron and the Lamb shift in hydrogen. The crux of QED, which guarantees its renormalizability, is its gauge invariance. In the same way as the photon is massless, the gauge invariance requires the fundamental vector fields of any gauge theory to be massless. To resolve this dilemma, one should introduce the photon and the intermediate bosons $W^{\pm}$ and $Z^0$ on an equal basis, as massless gauge fields, and then give masses spontaneously to intermediate bosons leaving the photon massless. So the extension of the Fermi model to a renormalizable theory of weak interactions results in a unification of the electromagnetic and weak interactions. Today, the standard model for the unified electromagnetic and weak interactions is that of Glashow, Weinberg and Salam (GWS) [1]-[4].

The SM has been very successful phenomenologically; there is no confirmed experimental evidence against the SM with the exception of neutrino oscillations. [1] Nevertheless, there are some unsatisfactory features and unanswered questions of the SM that make physicists to think that it is not the final theory of nature. First of all, the SM contains at least 19 physical parameters that can not be computed in the context of the model: 3 gauge couplings, 6 quark and 3 charged-lepton masses with 3 charged weak mixing angles and 1 CP-violating phase and 2 parameters to characterize the Higgs sector and 1 CP-violating non-perturbative vacuum angle. Another point is that the SM does not unify all the fundamental interactions, which also gives rise to the problem known as the hierarchy problem. The latter is related to the instability of the Higgs' mass under radiative corrections in the presence of a high scale, say $\Lambda \approx (10^{-15} - 10^{-19})$, the scale

[1] There are some indications that there exits neutrino oscillation between different flavors. This implies nonzero neutrino masses which are not allowed in the SM.

where the quantum gravity becomes effective. Finally, the SM does not address the question of the replication of families and the observed mass spectra.

There are various new models beyond the SM that proposes new approaches to solve the open questions of the SM. Some of them are left-right symmetric models, the minimal supersymmetric model (MSSM), technicolor models, and the two Higgs doublet models (2HDM). Among them, the most economical extension of the SM is the 2HDM, which is obtained from the SM with the addition of one extra scalar $SU(2)_L$ doublet. We note that such a Higgs structure is also required in low energy supersymmetric models, which are the most popular models in the field of particle physics at present.

The weak decays are concerned with all the unanswered questions of the SM, as summarized above and their phenomenology is very rich. Among the weak interactions, the rare B-meson decays have a special place for providing the essential information about the higher structure of the SM, and also poorly studied aspects of it, particularly Cabibbo-Kobayashi-Maskawa matrix elements, the leptonic decay constants, etc. From the theoretical point of view, rare decays take place via flavor changing neutral currents (FCNC), that is, via the currents that change the flavor but not the charge of the quark. In the SM at tree level, unitarity implies that FCNC processes are absent. However, they may appear at one loop level through the so-called box and/or penguin diagrams in the SM , but they are very suppressed with respect to the processes that occur at tree level. Therefore, as far as FCNC processes are concerned, any deviations from the SM results would be a certain indication of the presence of the new physics beyond the SM.

A very important and distinct property of the weak interaction is that it is invariant under neither the parity P transformation (that changes the sign of spatial coordinates) nor the charge conjugation C transformation (that changes a particle into its antiparticle). In fact, the combination of C and P is also not conserved in weak interaction. It is this CP violation, first detected in the decay of $K^0$ mesons, which is recognized as one of the most important phenomena in particle physics. However, the problem of CP violation is still one of the least tested aspects of the the SM. Since its first observation in K-meson system, accurate measurements have taken place to determine its origin. However, in the K-mesons, the effects of strong interactions are too large to draw any conclusion about the CP violation. The expectation is that these effects will be less and better to determine in case of B-meson, which is much more heavier than the K-meson. Indeed, very recently, the first observation of CP violation in the B-meson system have been reported by the $e^+e^-$ B factories [5] providing the the first test of the SM CP violation. In the near future, more experimental tests will be possible at the B factories and possible deviations from the SM predictions will provide important clues about physics beyond it. This situation makes the search for CP violation in B decays highly interesting.

In this thesis, we investigate the rare inclusive $B \to X_d \ell^+ \ell^-$ decays with the emphasis on CP violation and NHB effects within the framework of a CP spontaneously broken 2HDM, which is called model IV in the literature. Being a FCNC process, $B \to X_{s,d}\, \ell^+ \ell^-$ decays provide the most reliable testing grounds for the SM at the loop level and they are also sensitive to new physics. In addition, $B \to X_d\, \ell^+ \ell^-$mode is especially important in the CKM phenomenology. In Chapter 3, after introducing the basic formulas of the double and differential decay rates, the physical observables such as CP violation asymmetry, $A_{CP}$, forward-backward asymmetry, $A_{FB}$, and CP violating asymmetry in forward-backward asymmetry $A_{CP}(A_{FB})$ for $B \to X_d\, \ell^+ \ell^-$decays are calculated. The last chapter of the thesis is devoted to the conclusion of the thesis. Before presenting the work outlined above, we will first give a brief summary of the SM and a CP spontaneously broken 2HDM in Chapter 2 and 3, respectively.

# CHAPTER 2

# THE STANDARD MODEL

The SM is a gauge theory based on a $SU(3)_C \times SU(2)_L \times U(1)_Y$ group for the description of the current view of elementary particle physics and three fundamental interactions of the nature.

## 2.1 The Gauge Theories

According to the Gauge principle, for a field whose Lagrangian is invariant under a global symmetry, if this global symmetry is turned to a local one then the original free theory transforms into an interacting theory [6]. The procedure in order to get the theory invariant under local transformations is to introduce new vector boson fields, the so-called gauge fields, that interact with the field in a gauge invariant way. The number of associated gauge boson fields is equal to the number of generators of the group. That is, $SU(N)$ has $N^2 - 1$ generators so it has the same number of gauge bosons.

### 2.1.1 Quantum Electrodynamics (QED) : The Paradigm of Gauge Theories

QED is the gauge theory of system of interacting electrons, positrons and photons. Its many predictions have been tested up to an extremely high level of precision so that it appears to give a completely satisfactory account of electrodynamic processes.

Starting with a free Dirac field $\psi$ with spin $s = \frac{1}{2}$ , mass m and electric charge eQ, the Lagrangian is given by

$$L = \overline{\psi}(x)(i\,\partial\!\!\!/ - m)\psi(x)\,, \tag{2.1}$$

which follows from the Dirac equation

$$(i\,\partial\!\!\!/ - m)\psi(x) = 0\,. \tag{2.2}$$

The Lagrangian in Eq.(2.1) is invariant under global $U(1)$ transformations

$$\psi \to e^{iQ\theta}\psi \;\;,\;\; \overline{\psi} \to e^{-iQ\theta}\overline{\psi} \;\;,\;\; \partial_\mu\psi \to e^{iQ\theta}\partial_\mu\psi\,. \tag{2.3}$$

By Noether's theorem, this invariance implies the conservation of electromagnetic current $J_\mu$ and charge eQ, which are given by

$$J_\mu = eQ\,\overline{\psi}\gamma_\mu\psi \;\;,\;\; eQ = \int d^3x J_0(x)\,. \tag{2.4}$$

Transformation becomes local if the parameter $\theta$ is allowed to depend on the space-time point $x$. The corresponding transformations are

$$\psi \to e^{iQ\theta(x)}\psi \;,\; \overline{\psi} \to e^{-iQ\theta(x)}\overline{\psi} \;,\; \partial_\mu\psi \to e^{iQ\theta(x)}\partial_\mu\psi + iQ(\partial_\mu\theta(x))e^{iQ\theta(x)}\psi\,. \tag{2.5}$$

Lagrangian is not invariant under this local transformation. So, introduce a gauge vector boson field, $A_\mu(x)$ (the photon field), which interacts with the field $\psi$ and transforms under $U(1)$ gauge transformation as

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta(x)\,. \tag{2.6}$$

The most economical way of building this gauge invariant Lagrangian is simply to replace the normal derivative $\partial_\mu$ by the so-called covariant derivative $D_\mu$ so that

$$D_\mu\psi = (\partial_\mu - ieQA_\mu)\psi\,, \tag{2.7}$$

which transforms as

$$D_\mu\psi \to e^{iQ\theta(x)}D_\mu\psi\,. \tag{2.8}$$

To include the propagation of photon field we add a so-called kinetic term which must be also gauge invariant and is given in terms of field strength tensors by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{2.9}$$

The total Lagrangian is Lorentz and $U(1)$ gauge invariant and is the well known Lagrangian of QED

$$L_{QED} = \overline{\psi}(x)(i \not{D} - m)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)\,. \tag{2.10}$$

The gauge group for electromagnetism is $U(1)_{em}$ with one generator Q and one parameter $\theta$.

## 2.2 The Structure of the Standard Model

The SM consists of three components:

1. The basic constituents of matter are leptons and quarks which are realized in three families:

$$\begin{array}{rccc} \underline{\text{leptons}}: & \nu_e & \nu_\mu & \nu_\tau \\ & e^- & \mu^- & \tau^- \\ \underline{\text{quarks}}: & u & c & t \\ & d & s & b \end{array}$$

2. Four different forces act between the leptons and quarks. These are the electromagnetic, strong, weak and gravitational forces. The electromagnetic and weak forces are unified in the SM. The fields associated with these forces and also with the strong force, are spin-1 fields, describing the photon $\gamma$, the electroweak gauge bosons $W^\pm$ and $Z^0$, and the gluon $g$. The gravitational interaction is mediated by a spin-2 field, describing the graviton $G$. The gravity sector is not yet formulated as a proper quantum field theory.

3. The third component of the SM is the Higgs mechanism [7]. Any unified theory of the weak and electromagnetic interactions must be broken, since the photon is massless while the $W^\pm$ and $Z^0$ bosons are not. The SM is defined with the simplest realization of the Higgs mechanism by adding one scalar doublet to the theory which interacts with each other in such a way that the ground state acquires a non-zero field strength, breaking the electroweak symmetries spontaneously. The interaction energies of electroweak

gauge bosons, leptons, and quarks with these field manifest themselves as non-zero masses of these particles.

The gauge group of the SM is $SU(3)_C \times SU(2)_L \times U(1)_Y$ of unitary gauge transformations. $SU(3)_C$ is the non-Abelian symmetry group of the strong interactions of quarks and gluons, which is described by the gauge theory called Quantum Chromodynamics (QCD). $SU(2)_L$ is the non-Abelian electroweak-isospin group, to which three $W$ gauge fields are associated. $U(1)_Y$ is the Abelian hypercharge group, where the hypercharge $Y$ is connected with electric charge $Q$ and the isospin $T_3$ by the by the Gell - Mann - Nishijima formula, $Q = T_3 + \frac{Y}{2}$ [8]. The associated $B$ field and the neutral component of the $W$ triplet field mix to form the photon field $A$ and the electroweak field $Z$. The gauge theory of the electroweak interactions based on the symmetry group $SU(2)_L \times U(1)_Y$ is known as the Glashow-Weinberg-Salam theory [1]-[4].

Before going into the details of the construction of the SM, let us briefly summarize the mathematical details of its third component, namely the Higgs mechanism.

## 2.3 Goldstone Theorem and Higgs Mechanism

The simple definition of the phenomenon of spontaneous symmetry breaking can be given as follows: A physical system has a symmetry that is spontaneously broken if the interactions governing the dynamics of the system possess such a symmetry but the ground state of this system does not. In addition, the so-called Goldstone theorem [9] states that if a theory has a global symmetry of the Lagrangian which is not a symmetry of the vacuum then there must exist one massless boson, scalar or pseudoscalar, associated to each generator which does not annihilate the vacuum and having its same quantum numbers. These modes are referred to as Goldstone bosons.

Suppose a complex scalar filed of the form

$$\Phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}, \tag{2.11}$$

described by the Lagrangian

$$L_{\Phi} = (\partial_{\mu}\Phi)^{\dagger}(\partial^{\mu}\Phi) - \mu^{2}\Phi^{\dagger}\Phi - \lambda(\Phi^{\dagger}\Phi)^{2}, \tag{2.12}$$

with

$$\Phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x)), \tag{2.13}$$

where $\mu$ and $\lambda$ are arbitrary real parameters. The Lagrangian is invariant under the group $SO(2)$ rotations in the plane

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{2.14}$$

We shall see that this symmetry is spontaneously broken in this model.

Depending on the sign of the mass parameter $\mu^2$, there are two possibilities for the vacuum expectation value $< \Phi >_0 \equiv < 0 \mid \Phi \mid 0 >$ that minimizes the potential part of the Lagrangian (2.12):

1. $\mu^2 > 0$. There is a unique vacuum at $< \phi_1 >_0 = < \phi_2 >_0 = 0$. The vacuum is symmetric and therefore no symmetry breaking occurs.

2. $\mu^2 < 0$. The minimum is at

$$< |\Phi|^2 >_0 = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2} \tag{2.15}$$

which shows a $SO(2)$ symmetry unless a choice of vacuum is made. Let us choose

$$\begin{pmatrix} < \phi_1 >_0 \\ < \phi_2 >_0 \end{pmatrix} = \begin{pmatrix} v \\ 0 \end{pmatrix}. \tag{2.16}$$

Then the Lagrangian becomes

$$L_{\Phi} = \frac{1}{2}\partial\phi_1' \, \partial\phi_1' - \frac{1}{2}(-2\mu^2)\phi_1'^2 + \frac{1}{2}\partial\phi_2' \, \partial\phi_2' + \text{other int. terms}, \tag{2.17}$$

where we shifted the fields so that

$$\begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \begin{pmatrix} \phi_1 - v \\ \phi_2 \end{pmatrix} . \tag{2.18}$$

Thus, after spontaneous breakdown of the $SO(2)$ symmetry, we get a scalar field $\phi_1'$ with real and positive mass $-2\mu^2$ and a massless scalar boson $\phi_2'$, as predicted by the Goldstone theorem.

The Goldstone theorem is for theories with spontaneously broken global symmetries but does not hold for gauge theories. When a spontaneous symmetry breaking takes place in a gauge theory, the Goldstone model is to be generalized to be invariant under local gauge transformations, and in this way the so called Higgs mechanism operates. There we require that the Lagrangian is invariant under local gauge transformation

$$\Phi \to e^{iQ\theta(x)}\Phi \, . \tag{2.19}$$

We introduce a gauge field $A_\mu$, replace the ordinary derivatives in the Goldstone Lagrangian (2.12) by the covariant derivatives

$$\partial_\mu \to D_\mu + iQA_\mu \, , \tag{2.20}$$

and add the Lagrangian of the free gauge field

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x).$$

In this way the Lagrangian (2.12) becomes

$$L_\Phi = (D^\mu\Phi)(D^\mu\Phi)^* - \mu^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2 - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x), \tag{2.21}$$

This Lagrangian is invariant under local gauge transformations (2.19) and under

$$A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\theta(x) \, . \tag{2.22}$$

For $\mu^2 < 0$, the vacuum state is not unique, leading to spontaneous symmetry breaking. Choosing the vacuum expectation value and the shifted fields as in the

Goldstone model (see Eqs.(2.16) and (2.18)), the Lagrangian becomes

$$\begin{aligned} L_\Phi &= \frac{1}{2}\partial\phi_1'\,\partial\phi_1' + \frac{1}{2}\partial\phi_2'\,\partial\phi_2' - \frac{1}{2}(-2\mu^2)\phi_1'^1 - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) \\ &+ +\frac{1}{2}Q^2v^2A_\mu A^\mu + QvA_\mu\partial^\mu\phi_2' + \text{other int. terms}\,. \end{aligned} \tag{2.23}$$

We can see from the above Lagrangian that after spontaneous symmetry breaking, the vector boson $A_\mu$ gains a mass of $m_A = Qv$. In addition, there is a scalar field $\phi_1'$ with mass $m_{\phi_1'} = \sqrt{-2\mu^2}$ and also a massless scalar field $\phi_2'$, which is identified as Goldstone boson.

## 2.4 Constructing the Standard Model

The matter fields of the SM, which are the leptons and quarks carrying spin-1/2, are classified as left-handed (LH) isospin doublets and right-handed (RH) isospin singlets:

$$\begin{aligned} \ell_L &= \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad \ell_R = e_R\,, \quad \mu_R\,, \quad \tau_R\,, \\ q_L &= \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad q_R = u_R\,, \quad d_R\,, \quad c_R\,. \end{aligned} \tag{2.24}$$

As the gauge sector, there are four vector bosons as carriers of the electroweak force, and the corresponding spin-1 gauge vector fields are the $SU(2)_L$ isotriplet, $W_\mu^1$, $W_\mu^2$,$W_\mu^3$ and $U(1)_Y$ hypercharge $B_\mu$.

For spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry leaving the electromagnetic gauge subgroup $U(1)_{em}$ unbroken, a single complex scalar doublet field with hypercharge $Y = 1$

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi_0(x) \end{pmatrix} \tag{2.25}$$

is coupled to the gauge fields.

The interactions of the SM are summarized by the three terms in the basic Lagrangian:

$$L_{SM} = L_{gauge} + L_{fermions} + L_{Higgs}\,. \tag{2.26}$$

The first term represents the self interactions of the gauge fields, and given by

$$L_{gauge} = -\frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.27}$$

where

$$\begin{aligned} W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk}W^j_\mu W^k_\nu\,, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\,. \end{aligned} \tag{2.28}$$

The tensor $\epsilon^{ijk}$ is the $SU(2)_L$ structure constant, and $g$ is the weak coupling constant.

The second term in (2.26) represents the fermion-gauge boson couplings

$$L_{fermions} = \sum \overline{f}_i \ \not{D} f\,, \tag{2.29}$$

with the sum running over the LH and RH field components of the leptons and quarks, and the covariant derivative is given by

$$D_\mu = \partial_\mu - ig\tau^i W^i_\mu - ig'\frac{Y}{2}B_\mu\,, \tag{2.30}$$

where the hypercharge coupling is denoted by $g'$ and $\tau^i$ are Pauli matrices.

Finally, the Higgs Lagrangian contains the Higgs-gauge boson interactions together with the Higgs self-interaction potential, $L_{HG}$ and Higgs-fermion Yukawa couplings, $L_{YW}$ :

$$L_{Higgs} = L_{HG} + L_{YW} \tag{2.31}$$

where

$$\begin{aligned} L_{HG} &= (D_\mu\Phi)^+(D_\mu\Phi) - \mu^2\Phi^+\Phi - \lambda(\Phi^+\Phi)^2 \\ L_{YW} &= \lambda_e\bar{\ell}_L\Phi e_R + \lambda_u\bar{q}_L\tilde{\Phi}u_R + \lambda_d\bar{q}_L\Phi d_R + \text{h.c} + 2^{nd}\text{ and }3^{rd}\text{families.} \end{aligned} \tag{2.32}$$

Next step is to apply the Higgs mechanism to $SU(2)_L \times U(1)_Y$ group to acquire mass for gauge bosons and fermions. The following steps summarize the procedure to get the spectrum from $L_{SM}$:

1. A non-symmetric vacuum must be fixed. Let's choose

$$< \Phi >_0 \equiv < 0 \mid \Phi \mid 0 >= \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} . \qquad (2.33)$$

2. The physical spectrum is built by performing small oscillations around this vacuum. These are parameterized by

$$\Phi(x) = \exp\left(\frac{i\vec{\xi}(x)\cdot\vec{\sigma}}{v}\right)\begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix} , \qquad (2.34)$$

where $\vec{\xi}(x)$ is a small field and $H(x)$ describes the neutral Higgs boson.

3. In order to eliminate the unphysical field $\vec{\xi}(x)$ we make the following gauge transformation

$$\Phi' = U(\xi)\Phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} , \qquad (2.35)$$

where

$$U(\xi) = \exp\left(-i\frac{\vec{\xi}\cdot\vec{\sigma}}{v}\right) . \qquad (2.36)$$

The fermion and the gauge fields are transformed accordingly;

$$\begin{aligned} \ell_L' &= U(\xi)\ell_L \ , \ e_R' = e_R \, , \\ q_L' &= U(\xi)q_L \ , \ u_R' = u_R \ , \ d_R' = d_R \, , \\ \frac{\vec{\sigma}\cdot\vec{W}_\mu'}{2} &= U(\xi)\frac{\vec{\sigma}\cdot\vec{W}_\mu}{2}U^{-1}(\xi) - \frac{i}{g\partial_\mu U(\xi)U^{-1}(\xi)} \, , \\ B_\mu' &= B_\mu \, , \end{aligned} \qquad (2.37)$$

and we rewrite the Lagrangian for them in a new gauge.

The physical bosons consist of the charged particles $W_\mu^\pm$ and the neutrals $Z_\mu$ and $A_\mu$ (the photon). The latter are taken as a linear combinations of $W_\mu^3$ and $B_\mu$. Thus, we can set

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2) \, , \\ Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2+g'^2}} \quad &, \quad A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}} \, . \end{aligned} \qquad (2.38)$$

It is possible to relate the coupling constants of $SU(2)_L$ and $U(1)_Y$ to the so-called the Weinberg angle $\theta_W$ by using the definition $g/g' = \tan\theta_W$,

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \;,\; \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \,. \tag{2.39}$$

The photon field $A_\mu$ couples via the electric charge $e = \sqrt{4\pi\alpha}$ to the electron, thus $e$ can be expressed in term of the gauge couplings in the following way

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} \quad \text{or} \quad e = g\sin\theta_W = g'\cos\theta_W \,. \tag{2.40}$$

It is now easy to read the masses from the following terms of $L_{SM}$:

$$\begin{aligned} D_\mu\Phi' D^\mu\Phi' &= \frac{g^2v^2}{4}W^+_\mu W^{\mu-} \\ &+ \frac{1}{2}\frac{(g^2+g'^2)v^2}{4}{}_\mu Z^\mu + .... \\ V(\Phi') &= \frac{1}{2}2\mu^2H^2 + .... \\ L_{YW} &= \lambda_e\frac{v}{\sqrt{2}}\bar{e}'_L e'_R + \lambda_u\frac{v}{\sqrt{2}}\bar{u}'_L u'_R + \lambda_d\frac{v}{\sqrt{2}}\bar{d}'_L d'_R + .... \end{aligned} \tag{2.41}$$

and get finally the three level predictions

$$\begin{aligned} m_W &= \frac{gv}{\sqrt{2}} \;,\; m_Z = \frac{\sqrt{g^2+g'^2}}{2}v \;,\; m_H = \sqrt{2}\mu \,, \\ m_e &= \lambda_e\frac{v}{\sqrt{2}} \;,\; m_u = \lambda_u\frac{v}{\sqrt{2}} \;,\; m_d = \lambda_d\frac{v}{\sqrt{2}} \,, \end{aligned} \tag{2.42}$$

where

$$v = \sqrt{\frac{\mu^2}{\lambda}} \,, \tag{2.43}$$

and photon remains massless, $m_A = 0$. The relations in Eq. (2.42) together with (2.39) allow the masses of the $W^\pm$ and $Z^0$ bosons to be determined in terms of three experimentally well known quantities: the fine structure constant $\alpha = e^2/4\pi = 1/137$, the Fermi coupling constant $G_F = 1.166 \times 10^{-5}\,GeV^{-2}$, and the weak mixing angle $\theta_W$, which is determined from neutrino scattering experiments and given by $\sin^2\theta_W = 0.231 \pm 0.014$. Since the Fermi Constant $G$ is related to $g/M_W$, one finds

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{1}{2v^2} \;\Rightarrow\; v = 2^{-1/4}G_F^{-1/2} = 246\,GeV \,. \tag{2.44}$$

Combining (2.40),(2.42) and (2.44) gives

$$m_W = \left(\frac{\alpha\pi}{G_F\sqrt{2}}\right)^{1/2} \frac{1}{\sin\theta_W} \quad , \quad m_Z = \left(\frac{\alpha\pi}{G_F\sqrt{2}}\right)^{1/2} \frac{2}{\sin 2\theta_W} \, , \tag{2.45}$$

and substituting the above values for $\alpha$, $G_F$ and $\theta_W$ leads to

$$m_W = 78.3^{+2.5}_{-2.3}\,\text{GeV} \quad , \quad m_Z = 89.0^{+2.1}_{-1.8}\,\text{GeV} \, . \tag{2.46}$$

These predictions of the electroweak theory are in a good agreement with the experimental masses of the $W^{\pm}$ and $Z^0$ bosons, which were measured for the first time in very high energy experiments at CERN [10].

This leaves only the parameter $\lambda$ in (2.43) to be determined. From Eqs. (2.42) and (2.43)

$$m_H \;=\; \sqrt{2v\lambda} \tag{2.47}$$

which can not be predicted in the SM since the coupling $\lambda$ is an unknown parameter. Nevertheless, its value can be constrained by the assumption that the SM is valid up to an energy scale $\Lambda$. If one demands that the SM remains as a perturbative theory up to the scale of the so-called GUT (Grand Unified Theory), which is $\mathcal{O}(10^{16})$ GeV, an upper bound of the Higgs mass is given by $\sim$ 200 GeV. For $\Lambda \sim$ 1 TeV and the constraint $m_H \leq \Lambda$ predict an upper bound of $\sim$ 700 GeV. A lower bound on the Higgs mass is given by the requirement of vacuum stability. With a top quark of mass 175 GeV, and $\Lambda \sim$ 1 TeV , the Higgs mass is given by $\sim$ 55 GeV. For $\Lambda \sim M_{GUT}$ the lower bound increases to 130 GeV.

The direct Higgs boson search in the $e^+ e^- \to H^0 Z^0$ process at CERNs LEP experiment indicates that $m_H > 114$ GeV. Search for Higgs particles will be the main goal of a new machine at CERN, namely Large Hadron Collider (LHC), which is expected to operate in the year 2005.

## 2.5 Unsatisfactory Features of the SM

Despite the SM has been very successful in describing most of the elementary particles phenomenology, there are several unsatisfactory features of the theory. Let us enumerate some them:

- The Higgs sector of the theory: It remains unknown so far, and there is not any fundamental reason to assume that this sector must be minimal i.e. only one Higgs doublet.

- There are too many free parameters: There are at least 19 physical parameters that can not be computed in the context of the SM model: 3 gauge couplings, 6 quark and 3 charged-lepton masses with 3 charged weak mixing angles and 1 CP-violating phase and 2 parameters to characterize the Higgs sector and 1 CP-violating non-perturbative vacuum angle.

- The mass scale $v$ is not "natural": The only scale in the SM is $v$; all masses are proportional to $v$. However, since gravity is not included in the SM, there is for sure another relevant scale, a scale $\Lambda \simeq M_{Planck} \simeq 10^{19}$ GeV $>> v$. Since the radiative corrections to the Higgs mass term is proportional to this scale, $\delta m_H^2 \sim \Lambda^2$, it qaudratically divergent since $\Lambda >> v$.

- Interactions are not unified: There is no unification of the fundamental forces in the SM, because a separate gauge group and coupling is introduced for each interaction.

- Gravity is not included in the SM: General relativity can be formulated as a classical field theory, but attempts to quantize it yield a non-renormalizable theory. The hope is to unify gravity with other forces in such a way that the infinities arising in different sectors cancel among themselves, yielding a combined renormalizable theory.

- Origin of CP violation: In the SM the only source of CP violation is the complex Cabibbo-Kobayashi-Maskawa (CKM) matrix elements which appears too weak to drive the observed asymmetry in the nature.

These and many other unsatisfactory features of the SM lead the physicists to search for new models beyond it. In the next chapter, we will give a brief summary of one these models, namely, the two Higgs doublet model, which is the most economical extension of the SM.

# CHAPTER 3

# THE TWO HIGGS DOUBLET MODEL

The SM has a minimal Higgs sector: there is one physical neutral Higgs scalar in the spectrum and its mass is a free parameter not fixed by the theory. However, experimental information concerning the Higgs sector is still very limited; it is therefore reasonable to explore the implications of more complicated Higgs models.

The two Higgs doublet version of the SM [11]-[13] is particularly attractive because

1. It is an extension of the minimal model which adds new phenomena ( e.g. physical charged Higgs bosons ).

2. It is a minimal extension in that it adds the fewest new arbitrary parameters.

3. It satisfies theoretical constraints of

$$\rho = \frac{m_W^2}{m_Z^2 cos^2\theta_W} \simeq 1\,. \tag{3.1}$$

   In the SM at tree level, this relation is exact. In addition, an infinite number of complicated Higgs representations satisfy this constraint. The simplest choice among them are $SU(2)$ doublets with $Y = 1$.

4. Such a Higgs structure is required in 'low energy' supersymmetric models.

5. It contains direct CP violating vertices.

To summarize, this model possesses five physical Higgs bosons: a charged pair ($H^{\pm}$); two neutral CP even scalars ($H^0$and $h^0$); and a neutral CP odd scalar ($A^0$), often called a pseduoscalar. Instead of one free parameter of the minimal model, this model has six free parameters: four Higgs masses, the ratio of the vacuum expectation values, $tan\beta$, and a Higgs mixing angle, $\alpha$. Note that $v_1^2 + v_2^2$ is fixed by the W mass $m_W^2 = g^2 \frac{(v_1^2+v_2^2)}{2}$, and the Goldstone bosons $G^\pm$ and $G^0$ are eaten by the $W^\pm$ and $Z^0$ bosons.

A possible problem in the 2HDM is the possibility of appearing flavor changing neutral currents (FCNC) at the tree level, which are automatically absent in the SM, because the same operations that diagonalize the mass matrix automatically diagonalize the Higgs-fermion coupling. To avoid this unwanted FCNCs one can impose an *ad hoc* discrete symmetry based on a theorem of Glashow and Weinberg [14] which states that the tree level FCNC's mediated by Higgs bosons will be absent if all fermions of a given electric charge couple to no more than one Higgs doublet. However this constraint on the coupling is not unique. For example, there are at least two ways to satisfy this theorem in 2HDM [13]: One possibility ( Model I ) is a model in which one Higgs doublet does not couple to fermions at all and the other Higgs doublet couples to fermions in the same way as in the minimal Higgs model. A second possibility ( Model II ) is a model in which one Higgs doublet couples to down quarks while the second Higgs doublet couples to up quarks. Then, the part of the Lagrangian that contains the interaction of the fermions and the scalars can be written for model I and II, respectively, as follows:

$$\begin{aligned}
\mathcal{L}_{YI} &= -\lambda_{ij}^d \, \overline{q}_{Li} \, \Phi_2 \, d_{Rj} - \lambda_{ij}^u \, \overline{q}_{Li}, \widetilde{\Phi}_2 \, u_{Rj} - \lambda_{ij}^\ell \, \overline{\ell}_{Li} \, \Phi_2 \, \ell_{Rj} + \text{h.c.}\,, \\
\mathcal{L}_{YII} &= -\lambda_{ij}^d \, \overline{q}_{Li} \, \Phi_2 \, d_{Rj} - \lambda_{ij}^u \, \overline{q}_{Li} \, \widetilde{\Phi}_1 \, u_{Rj} - \lambda_{ij}^\ell \, \overline{\ell}_{Li} \, \Phi_1 \, \ell_{Rj} + \text{h.c.}\,, \qquad (3.2)
\end{aligned}$$

where

$$\widetilde{\Phi} \equiv i\sigma_2 \Phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Phi^* \,, \qquad (3.3)$$

and $i, j$ label the three generations.

As explained above, in the 2HDM, the scalar sector contains two Higgs doublets with the same quantum numbers

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \;,\; \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} . \tag{3.4}$$

In general, both doublets could acquire vacuum expectation values (vev)

$$\langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}} \quad , \quad \langle \Phi_2 \rangle = \frac{v_2}{\sqrt{2}} e^{i\xi} . \tag{3.5}$$

so it is more convenient to parametrize the doublets in the following way

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+ig_1}{\sqrt{2}} \end{pmatrix} \;;\; \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2 e^{i\xi}+ig_2}{\sqrt{2}} \end{pmatrix} \tag{3.6}$$

The most general Higgs potential describing the interactions of scalar fields $\Phi_1$ and $\Phi_2$ which spontaneously breaks $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$ contains all possible hermitian bilinear and quartic interactions compatible with gauge invariance:

$$\begin{aligned} V_G(\Phi_1, \Phi_2) &= -\mu_1^2 \widehat{A} - \mu_2^2 \widehat{B} - \mu_3^2 \widehat{C} - \mu_4^2 \widehat{D} + \lambda_1 \widehat{A}^2 + \lambda_2 \widehat{B}^2 + \lambda_4 \widehat{C}^2 + \lambda_5 \widehat{D}^2 \\ &\quad + \lambda_3 \widehat{A}\widehat{B} + \lambda_6 \widehat{A}\widehat{C} + + \lambda_7 \widehat{B}\widehat{C} + \lambda_8 \widehat{A}\widehat{D} + \lambda_9 \widehat{B}\widehat{D} + \lambda_{10} \widehat{C}\widehat{D}, \end{aligned} \tag{3.7}$$

where

$$\begin{aligned} \widehat{A} &\equiv \Phi_1^\dagger \Phi_1 \;,\; \widehat{B} \equiv \Phi_2^\dagger \Phi_2 ,\; \widehat{C} \equiv \frac{1}{2}\left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) = \mathrm{Re}\left(\Phi_1^\dagger \Phi_2\right), \\ \widehat{D} &\equiv -\frac{i}{2}\left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1\right) = \mathrm{Im}\left(\Phi_1^\dagger \Phi_2\right), \end{aligned}$$

and $\lambda_i$'s are all real parameters because of hermiticity.

Let us investigate the conditions of CP invariance of the Higgs potential (3.7):

The CP violation is the violation of the combined conservation laws associated with parity P and charge conjugation C. The parity operation is the spatial inversion of the coordinates; $(x, y, z) \longrightarrow (-x, -y, -z)$ and it is a discrete transformation. Under P, left-handed (LH) components of fermions, $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ transform into right-handed (RH) ones, $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$, and vice-versa. Since weak interactions only involve the LH components, parity is not a good symmetry of the weak force. The charge conjugation operation reverses the sign of the charge and magnetic moment of a particle, leaving coordinates untouched. Thus, it converts each particle into its antiparticle. Charge conjugation implies that every charged particle has an oppositely charged antiparticle. The antiparticle of an electrically neutral particle may be identical to the particle, as in the case of the neutral pi meson, or it may be distinct, as the antineutron. Strong and electromagnetic interactions are found experimentally to be invariant under the C conjugation operation. On the other hand , it is not a symmetry of weak interactions, because when it is applied to a neutrino (LH) it gives a LH antineutrino which does not exist.

If we come back to the Higgs potential in Eq. (3.7), since all fields are scalars, CP invariance is equivalent to the charge conjugation invariance here. Under charge conjugation, a Higgs doublet $\Phi_i$ transforms as $\Phi_i \rightarrow e^{i\beta_i}\Phi_i^*$, where the parameters $\beta_i$ are arbitrary. Therefore, we get $\Phi_i^\dagger\Phi_j \rightarrow e^{i(\beta_j-\beta_i)}\Phi_j^\dagger\Phi_i$. In particular, if we choose $\beta_i = \beta_j$, the operator $\widehat{D}$ in Eq.(3.7) reverses sign under C-conjugation, while the others are invariant. Then, the number of parameters of the Higgs potential that holds a C-conjugation invariance reduces to ten

$$\begin{aligned} V_G'(\Phi_1, \Phi_2) &= -\mu_1^2\widehat{A} - \mu_2^2\widehat{B} - \mu_3^2\widehat{C} + \lambda_1\widehat{A}^2 + \lambda_2\widehat{B}^2 + \lambda_3\widehat{C}^2 + \lambda_4\widehat{D}^2 \\ &\quad + \lambda_5\widehat{A}\widehat{B} + \lambda_6\widehat{A}\widehat{C} + \lambda_7\widehat{B}\widehat{C}\,. \end{aligned} \tag{3.8}$$

However, potential (3.8) could induce spontaneous CP violation due to the complex phase $\xi$ in the vev of $\Phi_2$ [15]-[18]. It is possible to demand a $Z_2$ invariance where $\Phi_1 \rightarrow \Phi_1,\ \Phi_2 \rightarrow -\Phi_2$, under which vev is CP invariant. The resulting potential is given by

$$V_G'' = -\mu_1^2\widehat{A} - \mu_2^2\widehat{B} + \lambda_1\widehat{A}^2 + \lambda_2\widehat{B}^2 + \lambda_4\widehat{C}^2 + \lambda_5\widehat{D}^2 + \lambda_3\widehat{A}\widehat{B}\,, \tag{3.9}$$

and correspond to setting $\mu_3^2 = \lambda_6 = \lambda_7 = 0$ in Eq. (3.8). If we permit a soft breaking term of the form $-\mu_3^2\widehat{C}$, spontaneous $CP$ violation occurs, in that case the potential explicitly reads

$$V(\Phi_1,\Phi_2) = \sum_{i=1,2}[\mu_i^2\Phi_i^\dagger\Phi_i+\lambda_i(\Phi_i^\dagger\Phi_i)^2]+\mu_3^2\mathrm{Re}(\Phi_1^\dagger\Phi_2)+\lambda_3[(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)]$$
$$+ \lambda_4[\mathrm{Re}(\Phi_1^\dagger\Phi_2)]^2+\lambda_5[\mathrm{Im}(\Phi_1^\dagger\Phi_2)]^2 . \quad (3.10)$$

Comparing with the model I and II, the Higgs potential of such a model, which is called as model IV in the literature, has an additional linear terms of $\mathrm{Re}(\Phi_1^\dagger\Phi_2)$. So, model IV is the minimal among the extensions of that provide a new source of CP violation [17, 18].

The constraints on the $\lambda_i$'s can be obtained from the requirement that the vacuum is at least a stationary point of the potential, that is

$$\left.\frac{\partial V}{\partial\Phi_i}\right|_{min} = 0 , \quad (3.11)$$

where "min" means the vanishing expectation values of all components except the real parts of the neutral components of the doublets. From (3.11), we get the following conditions

$$m_1^2 = -[2\lambda_1 v_1^2+(\lambda_3+\lambda_5)v_2^2],$$
$$m_2^2 = -[2\lambda_2 v_2^2+(\lambda_3+\lambda_5)v_1^2],$$
$$m_3^2 = -2v_1v_2(\lambda_4-\lambda_5)\cos\xi. \quad (3.12)$$

From Eq. (3.12), one can see that the necessary condition to have spontaneously broken CP is $\lambda_4 \neq \lambda_5$ and $m_3^2 \neq 0$, i.e., the real and imaginary parts of $\phi_1^+\phi_2$ have different self-couplings and there exists a linear term of $\mathrm{Re}(\phi_1^+\phi_2)$ in the potential.

As has already been noted, after the spontaneous symmetry braking of the gauge symmetry five physical Higgs fields appear in the Higgs sector. The masses of these Higgs bosons can be calculated from the mass squared matrix defined by

$$M_{ij}^2 = \frac{1}{2}\left.\frac{\partial^2 V}{\partial\Phi_i\partial\Phi_j}\right|_{min} , \quad (3.13)$$

with $i=1,..,8$. We can write the potential at the stationary point as:

$$V = m_1^2v_1^2+m_2^2v_2^2+\lambda_1v_1^4+\lambda_2v_2^4+(\lambda_3+\lambda_5)v_1^2v_2^2$$
$$+(\lambda_4-\lambda_5)v_1^2v_2^2\left[(\cos\xi-\Delta)^2-\Delta^2\right], \quad (3.14)$$

with

$$\Delta = -\frac{m_3^2}{2v_1 v_2(\lambda_4 - \lambda_5)}.$$

In model IV, for charged components, the mass-squared matrix for negative states is given as

$$-\lambda_5 \begin{pmatrix} v_1^2 & -v_1 v_2 e^{i\xi} \\ -v_1 v_2 e^{-i\xi} & v_2^2 \end{pmatrix}, \tag{3.15}$$

Diagonalizing the mass-squared matrix results in one zero-mass Goldstone state:

$$G^- = e^{i\xi} \sin\beta \phi_2^- + \cos\beta \phi_1^-, \tag{3.16}$$

and one massive charged Higgs boson state:

$$H^- = e^{i\xi} \cos\beta \phi_2^- - \sin\beta \phi_1^-, \tag{3.17}$$

$$m_{H^-} = |\lambda_5| v^2, \tag{3.18}$$

where $tan\beta = v_2/v_1$ and $v^2 = v_1^2 + v_2^2$, which is determined by $2m_W^2/g^2$. The positive states $G^+$ and $H^+$ could be obtained similarly.

For neutral Higgs components, because CP-conservation is broken, the mass-squared matrix is $4\times4$, which can not be simply separated into two $2\times2$ matrices as usual. After rotating the would-be Goldstone boson $(v_1 \text{Im}\phi_1^0 + v_2 \text{Im}\phi_2^0)/v$ away and using the constraints in Eq. (3.12), the elements of the mass matrix of the three physical neutral Higgs bosons $\mu_{ij}$, in the basis of $\{\text{Re}\phi_1^0, \text{Re}\phi_2^0, (v_2 \text{Im}\phi_1^0 - v_1 \text{Im}\phi_2^0)/v\}$, can be written as

$$\begin{aligned}
\mu_{11} &= 4\lambda_1 v_1^2 + (\lambda_4 - \lambda_5) v_2^2 \cos^2\xi\,, \\
\mu_{12} &= v_1 v_2 [2\lambda_3 + \lambda_4 \cos^2\xi + \lambda_5 (1 + \sin_\xi^2)]\,, \\
\mu_{13} &= \frac{1}{2}(\lambda_4 - \lambda_5) v_2 v s \sin^2\xi\,, \\
\mu_{22} &= 4\lambda_2 v_2^2 + (\lambda_4 - \lambda_5) v_1^2 \cos^2\xi\,, \\
\mu_{23} &= \frac{1}{2}(\lambda_4 - \lambda_5) \sin^2\xi v_1 v\,, \\
\mu_{33} &= (\lambda_4 - \lambda_5) v^2 \sin^2\xi\,.
\end{aligned} \tag{3.19}$$

In eq. (3.19), the constraints in eq. (3.12) have been used. Diagonalizing the Higgs boson mass-squared matrix results in

$$\begin{pmatrix} H_1^0 \\ H_2^0 \\ H_3^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{Im}\phi_1^0 \\ \mathrm{Re}\phi_1^0 \\ \mathrm{Re}\phi_2^0 \end{pmatrix} \tag{3.20}$$

with masses

$$m^2_{H_1^0,H_2^0} = \frac{1}{2}\left(\mu_{11} + \mu_{33} \mp \sqrt{(\mu_{11} - \mu_{33})^2 + 4\mu_{13}^2}\right) \tag{3.21}$$

and the mixing angle

$$\tan(2\alpha) = \frac{2\mu_{13}}{\mu_{33} - \mu_{11}}. \tag{3.22}$$

In model IV, it is assumed that the fermions obtain masses in the same way as in model II 2HDM. That is, the up-type quarks get masses from Yukawa couplings to the Higgs doublet $\Phi_2$ and down-type quarks and leptons get masses from Yukawa couplings to the Higgs doublet $\Phi_1$. Then it is easy to obtain the couplings of neutral Higgs bosons to fermions

$$\begin{aligned} H_1^0 \bar{f} f : &\qquad -\frac{igm_f}{2m_W \cos\beta}(\sin\alpha + i\cos\alpha\gamma_5) \\ H_2^0 \bar{f} f : &\qquad -\frac{igm_f}{2m_W \cos\beta}(\cos\alpha - i\sin\alpha\gamma_5) \end{aligned} \tag{3.23}$$

where $f$ represents down-type quarks and leptons. The couplings of the charged Higgs bosons to fermions are the same as those in the CP-conservative 2HDM (model II).

# CHAPTER 4

# $B \to X_d \ell^+ \ell^-$ in a CP SOFTLY BROKEN TWO HIGGS DOUBLET MODEL

## 4.1 Introduction

Although CP violation is one of the most fundamental phenomena in particle physics it is still one of the the least tested aspects of the SM. Before the start of the B factories, CP violation has only been measured in the kaon system [19]. Very recently, the observation of CP violation in the B-meson system have been reported by the $e^+e^-$ B factories [5] providing the first test of the SM CP violation. In the near future, more experimental tests will be possible at the B factories and possible deviations from the SM predictions will provide important clues about physics beyond it. This situation makes the search for CP violation in B decays highly interesting.

Interest in CP violation is not limited to particle physics; it plays an important role in cosmology, too. One of the necessary conditions to generate the matter-antimatter asymmetry observed in the Universe is -in addition to baryon number violation and deviations from the thermal equilibrium- that the elementary interactions have to violate CP. In the SM the only source of CP violation is the complex Cabibbo-Kobayashi-Maskawa (CKM) matrix elements which appears too weak to drive such an asymmetry [20], giving a strong motivation to search for new physics. In many cases, extensions of the SM such as the 2HDM or the supersymmetric extensions of the SM are able to supply the new sources of CP violation, providing an opportunity to investigate the new physics by analyzing the CP violating effects.

Being a FCNC process, $B \to X_{s,d}\,\ell^+\ell^-$ decays provide the most reliable testing grounds for the SM at the loop level and they are also sensitive to new physics. In addition to, $B \to X_d\,\ell^+\ell^-$mode is especially important in the CKM phenomenology. In case of the $b \to s\ell^+\ell^-$ decays, the matrix element receives a combination of various contributions from the intermediate $t$, $c$ or $u$ quarks with factors $V_{tb}V_{ts}^* \sim \lambda^2$, $V_{cb}V_{cs}^* \sim \lambda^2$ and $V_{ub}V_{us}^* \sim \lambda^4$, respectively, where $\lambda = \sin\theta_C \cong 0.22$. Since the last factor is extremely small compared to the other two, we can neglect it and this reduces the unitarity relation for the CKM factors to read $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* \approx 0$. Hence, the matrix elements for the $b \to s\ell^+\ell^-$ decays involve only one independent CKM factor so that CP violation would not show up. On the other hand, as pointed out before [21, 22], for $b \to d\ell^+\ell^-$ decay, all the CKM factors $V_{tb}V_{td}^*$, $V_{cb}V_{cd}^*$ and $V_{ub}V_{ud}^*$ are at the same order $\lambda^3$ in the SM and the matrix element for these processes would have sizable interference terms, so as to induce a CP violating asymmetry between the decay rates of the reactions $b \to d\ell^+\ell^-$ and $\bar{b} \to \bar{d}\ell^+\ell^-$. Therefore, $b \to d\ell^+\ell^-$ decays seem to be suitable for establishing CP violation in B mesons.

We note that the inclusive $B \to X_s\ell^+\ell^-$ decays have been widely studied in the framework of the SM and its various extensions [23]-[40]. As for $B \to X_d\ell^+\ell^-$ modes, they were first considered within the SM in [21] and [22]. The general two Higgs doublet model and MSSM contributions to the CP asymmetries were discussed in refs. [41] and [42], respectively. Recently, CP violation in the polarized $b \to d\ell^+\ell^-$ decay has been also investigated in the SM [43] and also in a general model independent way [44].

In this work, we investigate $B \to X_d\,\ell^+\ell^-$decay with an emphasis on CP violation and NHB effects in a CP softly broken 2HDM ( model IV) [17, 18], whose main points are already summarized in Chapter 3. In summary, in model IV, up-type quarks get masses from Yukawa couplings to the one Higgs doublet, and down-type quarks and leptons get masses from another Higgs doublet. In such a 2HDM, all the parameters in the Higgs potential are real so that it is

CP-conserving, but one allows the real and imaginary parts of $\phi_1^+\phi_2$ to have different self-couplings so that the phase $\xi$, which comes from the expectation value of Higgs field, can not be rotated away, which breaks the CP symmetry. In model IV, interaction vertices of the Higgs bosons and the down-type quarks and leptons depend on the CP violating phase $\xi$ and the ratio $\tan\beta = v_2/v_1$, where $v_1$ and $v_2$ are the vacuum expectation values of the first and the second Higgs doublet respectively, and they are free parameters in the model. The constraints on $\tan\beta$ are usually obtained from $B-\bar{B}$, $K-\bar{K}$ mixing, $b \to s\,\gamma$ decay width, semileptonic decay $b \to c\,\tau\bar{\nu}$ and is given by [45]

$$0.7 \le \tan\beta \le 0.52(\frac{m_{H^\pm}}{1\ GeV})\,, \tag{4.1}$$

and the lower bound $m_{H^\pm} \ge 200$ GeV has also been given in [45]. As for the constraints on $\xi$, it is given in ref.[17] that $\sqrt{|\sin 2\xi|}\tan\beta < 50$, which can be obtained from the electric dipole moments of the neutron and electron.

For inclusive B-decays into lepton pairs, in addition to the CP asymmetry and the forward-backward asymmetry, there is another parameter, namely polarization asymmetry of the final leptons, which is likely to play an important role for comparison of theory with experimental data. It has been already pointed out [46] that together with the longitudinal polarization, $P_L$, the other two orthogonal components of polarization, transverse, $P_T$, and normal polarizations, $P_N$, are crucial for the $\tau^+\tau^-$ mode since these three components contain the independent, but complementary information because they involve different combinations of Wilson coefficients in addition to the fact that they are proportional to $m_\ell/m_b$.

The rest of the chapter is organized as follows: Following this brief introduction, in section 4.2, we first present the effective Hamiltonian. Then, we introduce the basic formulas of the double and differential decay rates, CP violation asymmetry, $A_{CP}$, forward-backward asymmetry, $A_{FB}$, and CP violating asymmetry in forward-backward asymmetry $A_{CP}(A_{FB})$ for $B \to X_d\,\ell^+\ell^-$decay. Section 4.3 is devoted to the numerical analysis and discussion.

## 4.2 The Effective Hamiltonian for $B \to X_d \ell^+ \ell^-$

It is well known that inclusive decay rates of the heavy hadrons can be calculated in the heavy quark effective theory (HQET) [47] and the important result from this procedure is that the leading terms in $1/m_q$ expansion turn out to be the decay of a free quark, which can be calculated in the perturbative QCD. On the other hand, the effective Hamiltonian method provide a powerful framework for both the inclusive and the exclusive modes into which the perturbative QCD corrections to the physical decay amplitude are incorporated in a systematic way. In this approach, heavy degrees of freedom, namely $t$ quark and $W^\pm, H^\pm, h^0, H^0$ bosons in the present case, are integrated out. The procedure is to take into account the QCD corrections through matching the full theory with the effective low energy one at the high scale $\mu = m_W$ and evaluating the Wilson coefficients from $m_W$ down to the lower scale $\mu \sim \mathcal{O}(m_b)$. The effective Hamiltonian obtained in this way for the process $b \to d\,\ell^+\ell^-$, is given by [37, 38]:

$$\begin{aligned}\mathcal{H}_{eff} &= \frac{4G_F\alpha}{\sqrt{2}} V_{tb}V_{td}^* \Bigg\{ \sum_{i=1}^{10} C_i(\mu)\,O_i(\mu) + \sum_{i=1}^{10} C_{Q_i}(\mu)\,Q_i(\mu) \\ &- \lambda_u\{C_1(\mu)[O_1^u(\mu) - O_1(\mu)] + C_2(\mu)[O_2^u(\mu) - O_2(\mu)]\}\Bigg\}, \end{aligned} \quad (4.2)$$

where

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}, \quad (4.3)$$

and we have used the unitarity of the CKM matrix i.e., $V_{tb}V_{td}^* + V_{ub}V_{ud}^* = -V_{cb}V_{cd}^*$. The operator basis in the 2HDM for the process under consideration is given by [48, 49]

$$\begin{aligned} O_1 &= (\bar{d}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}), \\ O_2 &= (\bar{d}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}), \\ O_3 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\beta}), \\ O_4 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}), \end{aligned}$$

$$
\begin{aligned}
O_5 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\beta}), \\
O_6 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}), \\
O_7 &= \frac{e}{16\pi^2}\bar{d}_\alpha \sigma_{\mu\nu}(m_b R + m_s L) b_\alpha \mathcal{F}^{\mu\nu}, \\
O_8 &= \frac{g}{16\pi^2}\bar{d}_\alpha T^a_{\alpha\beta}\sigma_{\mu\nu}(m_b R + m_s L) b_\beta \mathcal{G}^{a\mu\nu} \ , \\
O_9 &= \frac{e}{16\pi^2}(\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{l}\gamma^\mu l) \ , \\
O_{10} &= \frac{e}{16\pi^2}(\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{l}\gamma^\mu \gamma_5 l) \ , \\
Q_1 &= \frac{e^2}{16\pi^2}(\bar{d}^\alpha_L\, b^\alpha_R)\,(\bar{\tau}\tau)\,, \\
Q_2 &= \frac{e^2}{16\pi^2}(\bar{d}^\alpha_L\, b^\alpha_R)\,(\bar{\tau}\gamma_5\tau)\,, \\
Q_3 &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_L\, q^\beta_R)\,, \\
Q_4 &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_R\, q^\beta_L)\,, \\
Q_5 &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, b^\beta_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_L\, q^\alpha_R)\,, \\
Q_6 &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, b^\beta_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_R\, q^\alpha_L)\,, \\
Q_7 &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, \sigma^{\mu\nu}\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_L\, \sigma_{\mu\nu} q^\beta_R)\,, \\
Q_8 &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, \sigma^{\mu\nu}\, b^\alpha_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_R\, \sigma_{\mu\nu} q^\beta_L)\,, \\
Q_9 &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, \sigma^{\mu\nu}\, b^\beta_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_L\, \sigma_{\mu\nu} q^\alpha_R)\,, \\
Q_{10} &= \frac{g^2}{16\pi^2}(\bar{d}^\alpha_L\, \sigma^{\mu\nu}\, b^\beta_R) \sum_{q=u,d,s,c,b} (\bar{q}^\beta_R\, \sigma_{\mu\nu} q^\alpha_L)
\end{aligned} \tag{4.4}
$$

where $\alpha$ and $\beta$ are $SU(3)$ colour indices and $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively.

$O^u_1$ and $O^u_2$ are the new operators for $b \to d$ transitions which are absent in the $b \to s$ decays and given by

$$
\begin{aligned}
O^u_1 &= (\bar{d}_\alpha \gamma_{mu} P_L u_\beta)(\bar{u}_\beta \gamma^{mu} P_L d_\alpha)\,, \\
O^u_2 &= (\bar{d}_\alpha \gamma_{mu} P_L u_\alpha)(\bar{u}_\beta \gamma^{mu} P_L d_\beta).
\end{aligned}
$$

The initial values of the Wilson coefficients for the relevant process in the SM are [26]

$$\begin{aligned}
C^{SM}_{1,3,...6,11,12}(m_W) &= 0 \ , \\
C^{SM}_{2}(m_W) &= 1 \ , \\
C^{SM}_{7}(m_W) &= \frac{3x^3-2x^2}{4(x-1)^4}\ln x + \frac{-8x^3-5x^2+7x}{24(x-1)^3} \ , \\
C^{SM}_{8}(m_W) &= -\frac{3x^2}{4(x-1)^4}\ln x + \frac{-x^3+5x^2+2x}{8(x-1)^3} \ , \\
C^{SM}_{9}(m_W) &= -\frac{1}{sin^2\theta_W}B(x) + \frac{1-4\sin^2\theta_W}{\sin^2\theta_W}C(x) - D(x) + \frac{4}{9} \ , \\
C^{SM}_{10}(m_W) &= \frac{1}{\sin^2\theta_W}(B(x)-C(x)) \ , \\
C^{SM}_{Q_i}(m_W) &= 0 \quad i=1,..,10 \ . \qquad (4.5)
\end{aligned}$$

The initial values for the additional part due to charged Higgs bosons are

$$\begin{aligned}
C^{H}_{1,...6}(m_W) &= 0 \, , \\
C^{H}_{7}(m_W) &= \frac{1}{\tan^2\beta}\, F_1(y) \ + F_2(y) \ , \\
C^{H}_{8}(m_W) &= \frac{1}{\tan^2\beta}\, G_1(y) \ + G_2(y) \ , \\
C^{H}_{9}(m_W) &= \frac{1}{\tan^2\beta}\, H_1(y) \ , \\
C^{H}_{10}(m_W) &= \frac{1}{\tan^2\beta}\, L_1(y) \ , \qquad (4.6)
\end{aligned}$$

and due to the neutral Higgs bosons are [37]

$$\begin{aligned}
C^{H}_{Q_1}(m_W) &= \frac{m_b m_\ell \tan^2\beta\, x}{2\sin^2\theta_W}\left\{\sum_{i=H_1,H_2}\frac{A_i}{m_i^2}(f_1B_i+f_2E_i)\right\} \\
,C^{H}_{Q_2}(m_W) &= \frac{m_b m_\ell \tan^2\beta\, x}{2\sin^2\theta_W}\left\{\sum_{i=H_1,H_2}\frac{D_i}{m_i^2}(f_1B_i+f_2E_i)\right\} \\
,C^{H}_{Q_3}(m_W) &= \frac{m_b e^2}{m_\tau g^2}(C_{Q_1}(m_W)+C_{Q_2}(m_W)) \\
,C^{H}_{Q_4}(m_W) &= \frac{m_b e^2}{m_\tau g^2},(C_{Q_1}(m_W)-C_{Q_2}(m_W)) \\
,C_{Q_i}(m_W) &= 0 \ , \ i=5,..,10 \ , \qquad (4.7)
\end{aligned}$$

where

$$A_{H_1} = -\sin\xi \ , \ D_{H_1} = i\cos\xi \ , \ A_{H_2} = -iD_{H_1} \ , \ D_{H_2} = -iA_{H_1} \ ,$$

$$
\begin{aligned}
B_{H_1} &= \frac{i}{2}e^{i\xi} \ , \ B_{H_2} = \frac{1}{2}e^{i\xi} \, , \\
f_1 &= \frac{x \ln x}{x-1} - \frac{y \ln y - x \ln x}{y-x} \ , \ f_2 = \frac{x \ln x}{(x-1)(y-1)} - \frac{y \ln y}{(y-x)(y-1)} \, , \\
E_{H_1} &= \frac{1}{2}(c_2 \cos\xi - c_1 \sin\xi) \ , \ E_{H_2} = \frac{1}{2}(c_2 \sin\xi + c_1 \cos\xi) \, , \\
c_1 &= -y + 2B_{H_2} \cos\xi x_{H_1} - 2B_{H_1} \sin\xi x_{H_2} \, , \\
c_2 &= i(-y - 2B_{H_1} \sin\xi x_{H_1} + 2B_{H_2} \cos\xi x_{H_2}) \, , \qquad (4.8)
\end{aligned}
$$

with

$$
x = \frac{m_t^2}{m_W^2} \quad , \quad y = \frac{m_{H^\pm}^2}{m_W^2} \quad , \quad x_{H_i} = \frac{m_{H_i}^2}{m_W^2}. \qquad (4.9)
$$

The explicit forms of the functions $A(x), B(x), C(x), D(x)$ and $F_{1(2)}(y), G_{1(2)}(y)$, $H_1(y)$ and $L_1(y)$ are given as

$$
\begin{aligned}
A(x) &= \frac{x(8x^2+5x-7)}{12(x-1)^3} + \frac{x^2(2-3x)}{2(x-1)^4} \ln x \, , \\
B(x) &= \frac{x}{4(1-x)} + \frac{x}{4(x-1)^2} \ln x \, , \\
C(x) &= \frac{x(x-6)}{x(x-1)} + \frac{x(3x+2)}{8(x-1)^2} \ln x \, , \\
D(x) &= \frac{-19x^3+25x^2}{36(x-1)^3} + \frac{x^2(5x^2-2x-6)}{18(x-1)^4} \ln x - \frac{4}{9} \ln x \, , \\
F_1(y) &= \frac{y(7-5y-8y^2)}{72(y-1)^3} + \frac{y^2(3y-2)}{12(y-1)^4} \ln y \ , \\
F_2(y) &= \frac{y(5y-3)}{12(y-1)^2} + \frac{y(-3y+2)}{6(y-1)^3} \ln y \ , \\
G_1(y) &= \frac{y(-y^2+5y+2)}{24(y-1)^3} + \frac{-y^2}{4(y-1)^4} \ln y \ , \\
G_2(y) &= \frac{y(y-3)}{4(y-1)^2} + \frac{y}{2(y-1)^3} \ln y \ , \\
H_1(y) &= \frac{1-4sin^2\theta_W}{sin^2\theta_W} \ \frac{xy}{8} \left[ \frac{1}{y-1} - \frac{1}{(y-1)^2} \ln y \right] \\
&- \ y \left[ \frac{47y^2-79y+38}{108(y-1)^3} - \frac{3y^3-6y+4}{18(y-1)^4} \ln y \right] \ , \\
L_1(y) &= \frac{1}{sin^2\theta_W} \ \frac{xy}{8} \left[ -\frac{1}{y-1} + \frac{1}{(y-1)^2} \ln y \right] \ .
\end{aligned}
$$

(4.10)

Finally, the initial values of the coefficients in the 2HDM are given by

$$C_i^{2HDM}(m_W) \;=\; C_i^{SM}(m_W) + C_i^H(m_W)\,. \tag{4.11}$$

In Eq.(4.2), $C_i(\mu)$ are the Wilson coefficients calculated at a renormalization point $\mu$ and their evolution from the higher scale $\mu = m_W$ down to the low-energy scale $\mu = m_b$ is described by the renormalization group equation. Although this calculation is performed for operators $O_i$ in the next-to-leading order (NLO) the mixing of $O_i$ and $Q_i$ in NLO has not been given yet. Therefore we use only the LO results. $C_7^{eff}(\mu)$ is defined as [50]

$$C_7^{eff}(\mu) \;=\; C_7^{2HDM}(\mu) + Q_d\,(C_5^{2HDM}(\mu) + N_c\,C_6^{2HDM}(\mu))\ ,$$

where $N_c$ is the number of colors, $Q_d$ is the charge for down type quarks, and the leading order QCD corrected Wilson coefficients $C_7^{LO,2HDM}(\mu)$ are given by [48, 49, 34]:

$$\begin{aligned} C_7^{LO,2HDM}(\mu) &= \eta^{16/23}C_7^{2HDM}(m_W) + (8/3)(\eta^{14/23} - \eta^{16/23})C_8^{2HDM}(m_W) \\ &+ C_2^{2HDM}(m_W)\sum_{i=1}^{8} h_i \eta^{a_i}\ , \end{aligned} \tag{4.12}$$

and $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, $h_i$ and $a_i$ are the numbers which appear during the evaluation [34].

The Wilson coefficient $C_9(\mu)$ contains as well as a perturbative part, a part coming from long distance (LD) effects due to conversion of the real $\bar{c}c$ into lepton pair $\ell^+\ell^-$:

$$C_9^{eff}(\mu) = C_9^{pert}(\mu) + Y_{reson}(s)\ , \tag{4.13}$$

where

$$\begin{aligned} C_9^{pert}(\mu) &= C_9 + h(u,s)[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu) \\ &+ \lambda_u(3C_1 + C_2)] - \frac{1}{2}h(1,s)\,(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ &- \frac{1}{2}h(0,s)\,[C_3(\mu) + 3C_4(\mu) + \lambda_u(6C_1(\mu) + 2C_2(\mu))] \\ &+ \frac{2}{9}\,(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu))\ , \end{aligned} \tag{4.14}$$

and

$$
\begin{aligned}
Y_{reson}(s) &= -\frac{3}{\alpha^2}\kappa \sum_{V_i=\psi_i} \frac{\pi \Gamma(V_i \to \ell^+\ell^-) m_{V_i}}{m_B^2 s - m_{V_i}^2 + i m_{V_i}\Gamma_{V_i}} \\
&\times \left[(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \right. \\
&+ \left. \lambda_u (3C_1(\mu) + C_2(\mu))\right] . \qquad (4.15)
\end{aligned}
$$

In Eq.(4.14), $s = q^2/m_B^2$ where $q$ is the momentum transfer, $u = \frac{m_c}{m_b}$ and the functions $h(u,s)$ arise from one loop contributions of the four-quark operators $O_1 - O_6$ and are given by

$$
\begin{aligned}
h(u,s) &= -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln u + \frac{8}{27} + \frac{4}{9}y \qquad (4.16) \\
&-\frac{2}{9}(2+y)|1-y|^{1/2} \begin{cases} \left(\ln\left|\frac{\sqrt{1-y}+1}{\sqrt{1-y}-1}\right| - i\pi\right), & \text{for } y \equiv \frac{4u^2}{s} < 1 \\ 2\arctan\frac{1}{\sqrt{y-1}}, & \text{for } y \equiv \frac{4u^2}{s} > 1, \end{cases} \\
h(0,s) &= \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s + \frac{4}{9}i\pi \ . \qquad (4.17)
\end{aligned}
$$

The phenomenological parameter $\kappa$ in Eq. (4.15) is taken as 2.3 (see e.g. [51]).

Finally, the Wilson coefficients $C_{Q_1}(\mu)$ and $C_{Q_2}(\mu)$ at any scale are given by [37]

$$
C_{Q_i}(\mu) = \eta^{-12/23}\, C_{Q_i}(m_W) \ , \ i = 1, 2 \ . \qquad (4.18)
$$

Next we proceed to calculate the differential branching ratio $dBR/ds$, forward-backward asymmetry $A_{FB}$, CP violating asymmetry $A_{CP}$, CP asymmetry in the forward-backward asymmetry $A_{CP}(A_{FB})$ and finally the lepton polarization asymmetries of the $B \to X_d\, \ell^+\ell^-$decays. In order to find these physically measurable quantities we first need to calculate the matrix element of the $B \to X_d \ell^+\ell^-$ decay. The relevant one-loop diagrams contributing to this decay in the SM are given in Fig.(4.1). The additional contributions from the 2HDM can be obtained from Fig. (4.1) by replacement $W, \phi \to H^\pm$. When we take into account the contributions coming from NHB we get the diagrams depicted in Fig. (4.2).

Neglecting the mass of the $d$ quark, the effective short distance Hamiltonian in Eq.(4.2) leads to the following QCD corrected matrix element:

$$
\mathcal{M} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{td}^* \Big\{ C_9^{eff}\ \bar{d}\gamma_\mu (1-\gamma_5) b\, \bar{\ell}\gamma^\mu \ell + C_{10}\ \bar{d}\gamma_\mu(1-\gamma_5) b\, \bar{\ell}\gamma^\mu\gamma_5 \ell
$$

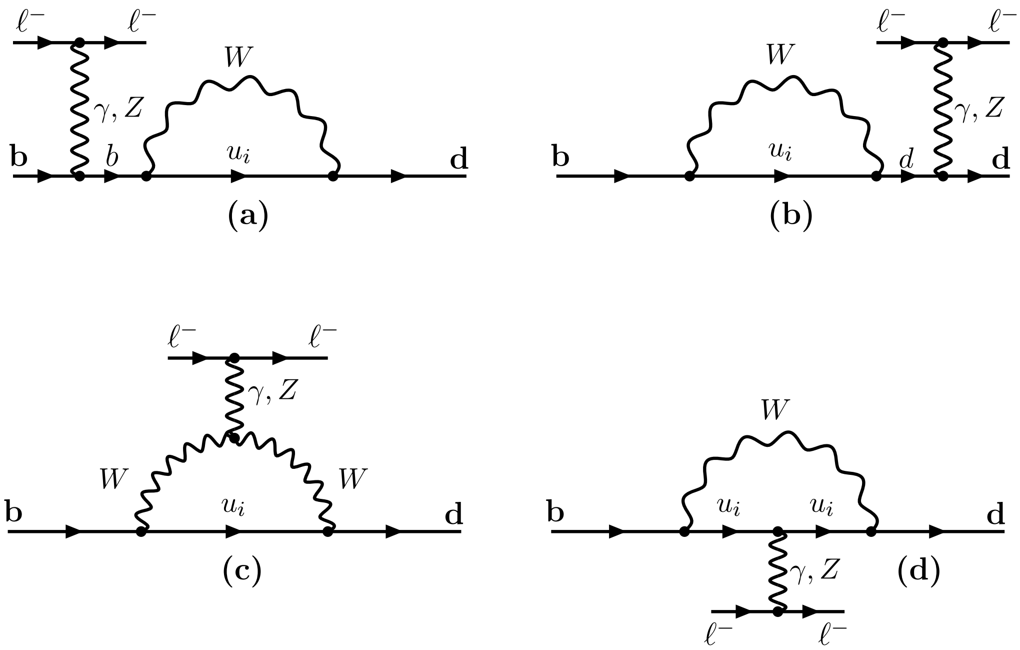


Figure 4.1: The one-loop Feynman diagrams contributing the decay $b \to d\ell^+\ell^-$ in the SM.

$$- \quad 2C_7^{eff}\ \frac{m_b}{q^2}\bar{d}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar{\ell}\gamma^\mu\ell + C_{Q_1}\ \bar{d}(1+\gamma_5)b\,\bar{\ell}\ell + C_{Q_2}\ \bar{d}(1+\gamma_5)b\,\bar{\ell}\gamma_5\ell\Big\}. \tag{4.19}$$

When the initial and final state polarizations are not measured, we must average over the initial spins and sum over the final ones, that leads to the following double differential decay rate

$$\begin{aligned}
\frac{d^2\Gamma}{ds\,dz} &= \Gamma(B\to X_c\ell\nu)\frac{3\alpha^2}{4\pi^2 f(u)k(u)}(1-s)^2\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2}\,v\,\Big\{2\,v\,z\,\mathrm{Re}(C_7^{eff}C_{10}^*) \\
&+ 2\Big(1+\frac{2t}{s}\Big)\mathrm{Re}(C_7^{eff}C_9^{eff\,*}) + v\,s\,z\,\mathrm{Re}(C_{10}C_9^{eff\,*}) \\
&+ v\sqrt{t}z\,\mathrm{Re}((2C_7^{eff}+C_9^{eff})C_{Q_1}^*) + \sqrt{t}\mathrm{Re}(C_{10}C_{Q_2}^*) \\
&+\frac{1}{4}\Big[(1+s)-(1-s)\,v^2z^2+4t\Big]|C_9^{eff}|^2 + \Big[\Big(1+\frac{1}{s}\Big)-\Big(1-\frac{1}{s}\Big)\,v^2z^2+\frac{4t}{s}\Big]|C_7^{eff}|^2 \\
&+\frac{1}{4}\Big[(1+s)-(1-s)\,v^2z^2-4t\Big]|C_{10}|^2 + \frac{1}{4}s|C_{Q_2}|^2 + \frac{1}{4}(s-4t)|C_{Q_1}|^2\Big\}.
\end{aligned} \tag{4.20}$$

Here, $v=\sqrt{1-4t/s}$, $t=m_\ell^2/m_b^2$ and $z=\cos\theta$, where $\theta$ is the angle between the momentum of the B-meson and that of $\ell^-$ in the center of mass frame of the

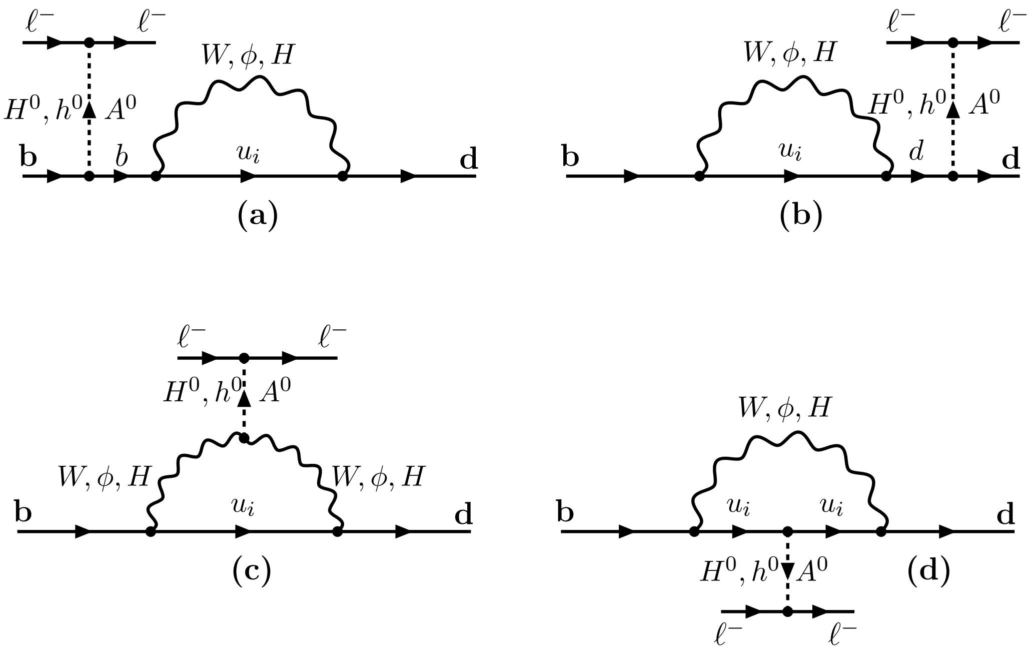


Figure 4.2: The one-loop diagrams contributing the process $b \to d\ell^+\ell^-$ within the framework of model IV by including NHB contributions

dileptons $\ell^-\ell^+$. In Eq. (4.20),

$$\Gamma(B \to X_c \ell\nu) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 f(u)k(u)\,, \tag{4.21}$$

where

$$f(u) = 1 - 8u + 8u^4 - u^8 - 24u^4 \ln(u) \tag{4.22}$$
$$k(u) = 1 - \frac{2\alpha_s(m_b)}{3\pi}\left[\left(\pi^2 - \frac{31}{4}\right)(1-u^2) + \frac{3}{2}\right], \tag{4.23}$$

are the phase space factor and the QCD corrections to the semi-leptonic decay rate, respectively, which is used to normalize the decay rate of $B \to X_d \ell^+\ell^-$ to remove the uncertainties in the value of $m_b$.

Having established the double differential decay rates, let us now consider the forward-backward asymmetry $A_{FB}$ of the lepton pair, which is defined as

$$A_{FB}(s) = \frac{\int_0^1 dz \frac{d^2\Gamma}{dsdz} - \int_{-1}^0 dz \frac{d^2\Gamma}{dsdz}}{\int_0^1 dz \frac{d^2\Gamma}{dsdz} + \int_{-1}^0 dz \frac{d^2\Gamma}{dsdz}}\;. \tag{4.24}$$

The $A_{FB}$'s for the $B \to X_d\, \ell^+\ell^-$decays are calculated to be

$$A_{FB}(s) = \frac{-3\ v}{\Delta(s)}\, \mathrm{Re}[C_{10}(2C_7^{eff} + s\, C_9^{eff\,*})] + \sqrt{t}\, \mathrm{Re}[C_{Q_1}(2C_7^{eff\,*} + C_9^{eff\,*})], \tag{4.25}$$

where

$$\begin{aligned}\Delta(s) &= \frac{(s + 2s^2 + 2t - 8st)}{s}|C_{10}|^2 + \frac{4}{s^2}(2+s)(s+2t)|C_7^{eff}|^2 \\ &+ (1+2s)(1+\frac{2t}{s})|C_9^{eff}|^2 + \frac{12}{s}(s+2t)\mathrm{Re}(C_7^{eff} C_9^{eff\,*}) + 6\sqrt{t}\mathrm{Re}(C_9^{eff} C_{Q_2}^*) \\ &+ \frac{3}{2}(s-4t)|C_{Q_1}|^2 + \frac{3}{2}s|C_{Q_2}|^2\,, \end{aligned} \tag{4.26}$$

which agrees with the result given by ref. [22], in case of switching off the NHB contributions and setting $m_\ell = 0$, but differs slightly from the results of [42].

We next consider the CP asymmetry $A_{CP}$ between the $B \to X_d \ell^+\ell^-$ and the conjugated one $\bar{B} \to \bar{X}_d \ell^+\ell^-$, which is defined as

$$A_{CP}(s) = \frac{\frac{d\Gamma}{ds} - \frac{d\bar{\Gamma}}{ds}}{\frac{d\Gamma}{ds} + \frac{d\bar{\Gamma}}{ds}}\,, \tag{4.27}$$

where

$$\frac{d\Gamma}{ds} = \frac{d\Gamma(B \to X_d \ell^+\ell^-)}{ds} \quad , \quad \frac{d\bar{\Gamma}}{ds} = \frac{d\Gamma(\bar{B} \to \bar{X}_d \ell^+\ell^-)}{ds}\,. \tag{4.28}$$

After integrating the double differential decay rate in Eq.(4.20) over the angle variable, we find for the $B \to X_d\, \ell^+\ell^-$decays

$$\frac{d\Gamma}{ds} = \Gamma(B \to X_c \ell\nu) \frac{\alpha^2}{4\pi^2 f(u) k(u)} (1-s)^2 \frac{|V_{tb} V_{td}^*|^2}{|V_{cb}|^2} \sqrt{1 - \frac{4t}{s}}\, \Delta(s)\,. \tag{4.29}$$

For the antiparticle channel, we have

$$\frac{d\bar{\Gamma}}{ds} = \frac{d\Gamma}{ds}(\lambda_u \to \lambda_u^*; \xi \to -\xi) \tag{4.30}$$

We have also a CP violating asymmetry in $A_{FB}$, $A_{CP}(A_{FB})$, in $B \to X_d\, \ell^+\ell^-$ decay. Since in the limit of CP conservation, one expects $A_{FB} = -\bar{A}_{FB}$ [22, 52], where $A_{FB}$ and $\bar{A}_{FB}$ are the forward-backward asymmetries in the particle and antiparticle channels, respectively, $A_{CP}(A_{FB})$ is defined as

$$A_{CP}(A_{FB}) = A_{FB} + \bar{A}_{FB}\ \ , \tag{4.31}$$

with

$$\bar{A}_{FB} = A_{FB}(\lambda_u \to \lambda_u^*; \xi \to -\xi)\,. \tag{4.32}$$

Finally, we would like to discuss the lepton polarization effects for the $B \to X_d\,\ell^+\ell^-$ decays. The polarization asymmetries of the final lepton is defined as

$$P_n(s) \;=\; \frac{(d\Gamma(S_n)/ds) - (d\Gamma(-S_n)/ds)}{(d\Gamma(S_n)/ds) + (d\Gamma(-S_n)/ds)}\,, \tag{4.33}$$

for $n = L,\ N,\ T$. Here, $P_L$, $P_T$ and $P_N$ are the longitudinal, transverse and normal polarizations, respectively. The unit vectors $S_n$ are defined as follows:

$$\begin{aligned}
S_L &= (0, \vec{e}_L) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), \\
S_N &= (0, \vec{e}_N) = \left(0, \frac{\vec{p} \times \vec{p}_-}{|\vec{p} \times \vec{p}_-|}\right), \\
S_T &= (0, \vec{e}_T) = \left(0, \vec{e}_N \times \vec{e}_L\right),
\end{aligned} \tag{4.34}$$

where $\vec{p}$ and $\vec{p}_-$ are the three-momenta of $d$ quark and $\ell^-$ lepton, respectively. The longitudinal unit vector $S_L$ is boosted to the CM frame of $\ell^+\ell^-$ by Lorentz transformation

$$S_{L,CM} = \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E_\ell\,\vec{p}_-}{m_\ell|\vec{p}_-|}\right). \tag{4.35}$$

It follows from the definition of unit vectors $S_n$ that $P_T$ lies in the decay plane while $P_N$ is perpendicular to it, and they are not changed by the boost.

After some algebra, we obtain the following expressions for the polarization components of the $\ell^-$ lepton in $B \to X_d\,\ell^+\ell^-$decays:

$$\begin{aligned}
P_L &= \frac{v}{\Delta}\mathrm{Re}\Big[2C_{10}(6\,C_7^{eff,*} + (1+2s)C_9^{eff,*}) - 3C_{Q_1}(2\sqrt{t}\,C_{10} + sC_{Q_2}^*)\Big]\,, \\
P_T &= \frac{3\pi\sqrt{t}}{2\sqrt{s}\Delta}\Bigg( -\frac{4}{s}|C_7^{eff}|^2 - s|C_9^{eff}|^2 + \mathrm{Re}\Big[2C_7^{eff*}(C_{10} - 2C_9^{eff*} + \frac{s}{2\sqrt{t}}C_{Q_2}^*) \\
&+ C_9^{eff}(C_{10} + \frac{s}{2\sqrt{t}}C_{Q_2}^*) + \frac{s-4t}{2\sqrt{t}}C_{10}C_{Q_1}^*\Big]\Bigg)\,, \qquad (4.36) \\
P_N &= \frac{3\pi v}{4\sqrt{s}\Delta}\mathrm{Im}\Big[C_{10}(sC_{Q_2}^* + 2\sqrt{t}(C_7^{eff*} + sC_9^{eff*})) + sC_{Q_1}(2C_7^{eff*} + C_9^{eff*})\Big]\,.
\end{aligned}$$

## 4.3 Numerical results and discussion

In this section we present the numerical analysis of the inclusive decays $B \to X_d \ell^+\ell^-$in model IV. We will give the results for only $\ell = \tau$ channel, which demonstrates the NHB effects more manifestly. The input parameters we used in this analysis are as follows:

$$
\begin{aligned}
&m_b = 4.8\,GeV\,,\ m_c = 1.4\,GeV\,,\ m_t = 175\,GeV\,,\ m_\tau = 1.78\,GeV\,,\\
&BR(B \to X_c e \bar{\nu}_e) = 10.4\%\,,\ m_{H^\pm} = 200\,GeV\,,\ m_{H^0} = 160\,GeV\,,\\
&m_{h^0} = 115\,GeV\,,\ \alpha^{-1} = 129\,,\ G_F = 1.17 \times 10^{-5}\,GeV^{-2}\,. \qquad (4.37)
\end{aligned}
$$

The Wolfenstein parametrization [53] of the CKM factor in Eq. (4.3) is given by

$$
\lambda_u = \frac{\rho(1-\rho) - \eta^2 - i\eta}{(1-\rho)^2 + \eta^2} + O(\lambda^2)\,, \qquad (4.38)
$$

and also

$$
\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2} = \lambda^2[(1-\rho)^2 + \eta^2] + \mathcal{O}(\lambda^4)\,. \qquad (4.39)
$$

The updated fitted values for the parameters $\rho$ and $\eta$ are given as [54]

$$
\begin{aligned}
\bar{\rho} &= 0.22 \pm 0.07\ (0.25 \pm 0.07)\,,\\
\bar{\eta} &= 0.34 \pm 0.04\ (0.34 \pm 0.04)\,, \qquad (4.40)
\end{aligned}
$$

with (without) including the chiral logarithms uncertainties. In our numerical analysis, we have used $(\rho,\, \eta) = (0.25;\, 0.34)$ .

The masses of the charged and neutral Higgs bosons, $m_{H^\pm}$, $m_{H^0}$, and $m_{h^0}$, and the ratio of the vacuum expectation values of the two Higgs doublets, $\tan\beta$, remain as free parameters of the model. The restrictions on $m_{H^\pm}$, and $\tan\beta$ have been already discussed in section 4.1. For the masses of the neutral Higgs bosons, the lower limits are given as $m_{H^0} \geq 115$ GeV and $m_{h^0} \geq 89.9$ GeV in [55].

In the following, we give results of our calculations about the dependencies of the differential branching ratio $dBR/ds$, forward-backward asymmetry $A_{FB}(s)$, CP violating asymmetry $A_{CP}(s)$, CP asymmetry in the forward-backward asymmetry $A_{CP}(A_{FB})(s)$ and finally the components of the lepton polarization asymmetries, $P_L(s)$, $P_T(s)$ and $P_N(s)$, of the $B \to X_d\,\tau^+\tau^-$decays on the invariant dilepton mass $s$. In order to investigate the dependencies of the above physical quantities on the model parameters, namely CP violating phase $\xi$ and $\tan\beta$, we eliminate the other parameter $s$ by performing the $s$ integrations over the allowed kinematical region so as to obtain their averaged values, $< A_{FB} >$, $< A_{CP} >$, $< A_{CP}(A_{FB}) >$, $< P_L >$, $< P_T >$ and $< P_N >$.

Numerical results are shown in Figs. (4.3)-(4.15) and we have the following line conventions: dashed lines, dot lines and dashed-dot lines represent the model IV contributions with $\tan\beta = 10, 40, 50$, respectively and the solid lines are for the SM predictions. The cases of switching off NHB contributions i.e., setting $C_{Q_i} = 0$, almost coincide with the cases of 2HDM contributions with $\tan\beta = 10$, therefore we did not plot them separately.

In Fig.(4.3), we give the dependence of the $dBR/ds$ on $s$. From this figure NHB effects are very obviously seen, especially in the moderate-s region.

In Fig. (4.4) and Fig. (4.5), $A_{FB}(s)$ and $< A_{FB} >$ as a function of $s$ and CP violating phase $\xi$ are presented, respectively. We see that $A_{FB}$ is more sensitive to $\tan\beta$ than the $dBR/ds$ and it changes sign with the different choices of this parameter. It is seen from Fig.(4.5) that $< A_{FB} >$ is quite sensitive to $\xi$ and between $(0.15, 0.28) \times 10^{-1}$. We also observe that $< A_{FB} >$ differs essentially from the one predicted by the CP-conservative 2HDM (model II), which is 0.028 and 0.023 for $\tan\beta = 40, 50$, respectively. In region $1 < \xi < 2$ change in $< A_{FB} >$ with respect to model II reaches 25%.

Figs. (4.6) and (4.7) show the dependence of $A_{CP}(s)$ on $s$ and $< A_{CP} >$ on $\xi$, respectively. We see that $A_{CP}(s)$ is also sensitive to $\tan\beta$ and its sign does not change in the allowed values of $s$ except in the resonance mass region. It follows from Fig. (4.7) that $< A_{CP} >$ is not as sensitive as $< A_{FB} >$ to $\xi$, and it varies in the range $(0.15, 0.33) \times 10^{-1}$.

$A_{CP}(A_{FB})(s)$ and $< A_{CP}(A_{FB}) >$ of $B \to X_d\,\tau^+\tau^-$as a function of $s$ and CP violating phase $\xi$ are presented in Fig. (4.8) and Fig. (4.9), respectively. We see that $A_{CP}(A_{FB})(s)$ changes sign with the different choices of $\tan\beta$. $< A_{CP}(A_{FB}) >$ is between $(0.010, 0.040)$ and differs essentially from the one predicted by model II, which is 0.038 and 0.027 for $\tan\beta = 40, 50$, respectively. In region $1.5 < \xi < 2.5$ change in $< A_{FB} >$ with respect to model II reaches 35%.

In Figs. (4.10)-(4.12), we present the $s$ dependence of the longitudinal $P_L$, transverse $P_T$ and normal $P_N$ polarizations of the final lepton for $B \to X_d\tau^+\tau^-$ decay. It is seen that NHB contributions changes the polarization significantly, especially when $\tan\beta$ is large. We also observe that except the resonance region, $P_T$ is negative for all values of $s$, but $P_L$ and $P_N$ change sign with the different choices of the values of $\tan\beta$. In Figs. (4.13)-(4.15), dependence of the averaged values of the longitudinal $< P_L >$, transverse $< P_T >$ and normal $< P_N >$ polarizations of the final lepton for $B \to X_d\tau^+\tau^-$decay on $\xi$ are shown. It is obvious from these figures that $< P_N >$ and $< P_T >$ are more sensitive to $\xi$ than $< P_L >$. In region $1.5 < \xi < 2.0$ change in $< P_N >$ with respect to model II reaches 25%.

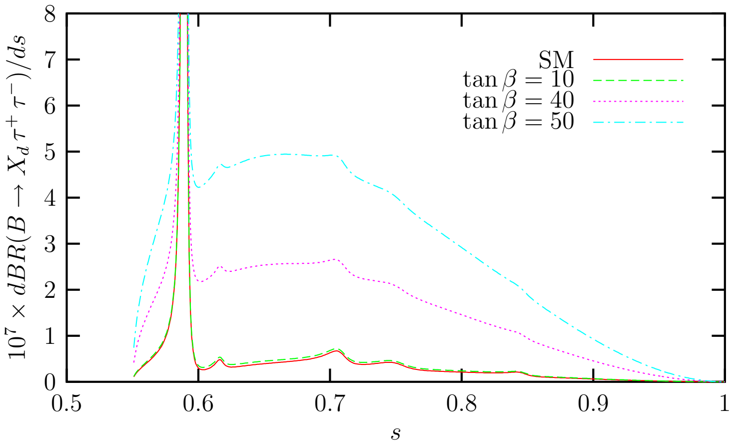


Figure 4.3: Differential branching ratio as a function of $s$, where $\xi = \pi/4$.

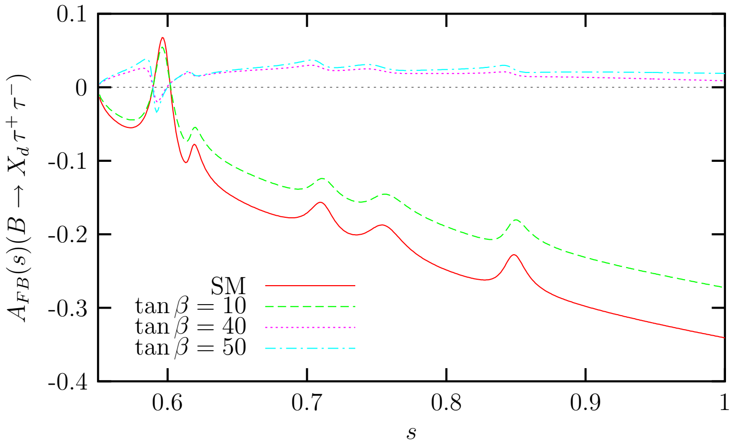


Figure 4.4: The forward-backward asymmetry as a function of $s$, where $\xi = \pi/4$.

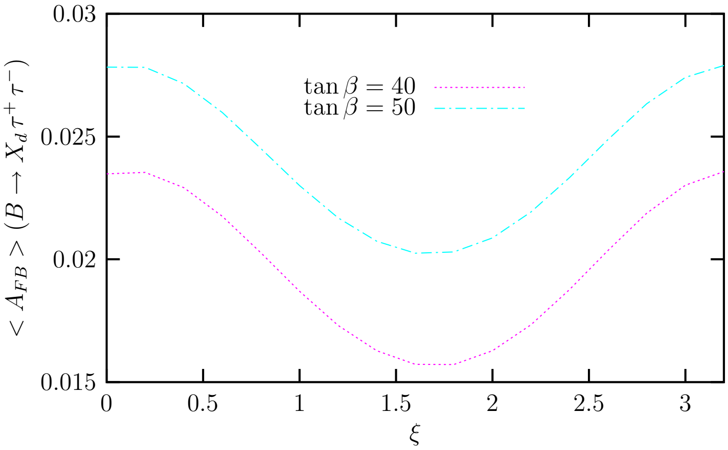


Figure 4.5: $< A_{FB} >$ as a function of $\xi$.

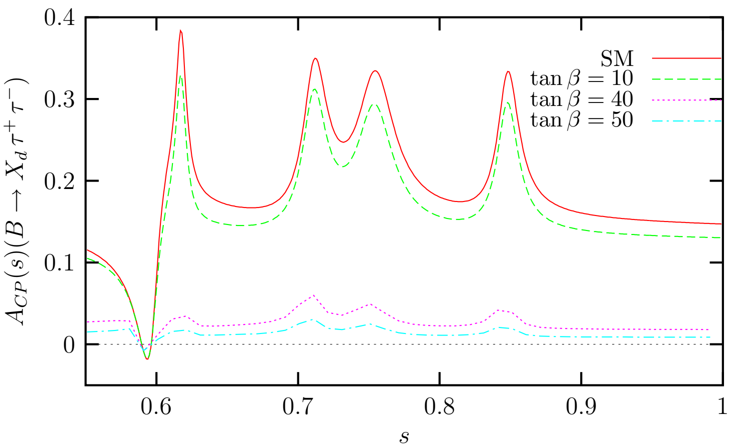


Figure 4.6: The CP asymmetry as a function of $s$, where $\xi = \pi/4$.

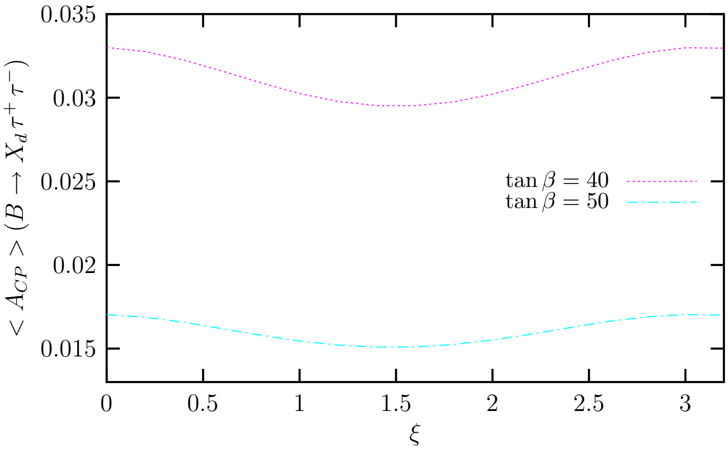


Figure 4.7: $< A_{CP} >$ as a function of $\xi$.

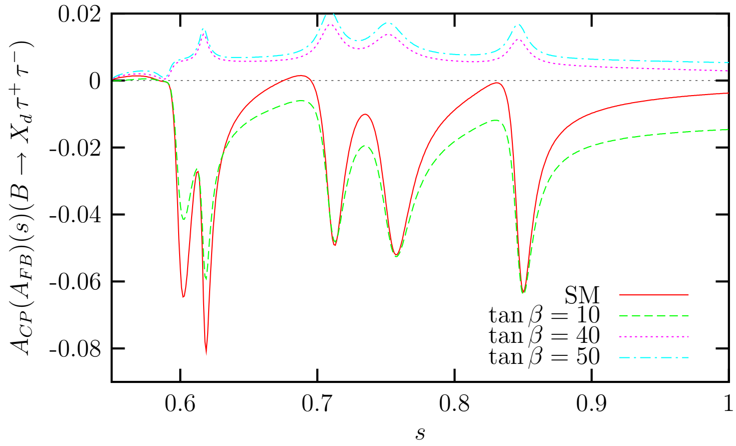


Figure 4.8: The CP asymmetry in the forward-backward asymmetry as a function of $s$, where $\xi = \pi/4$.

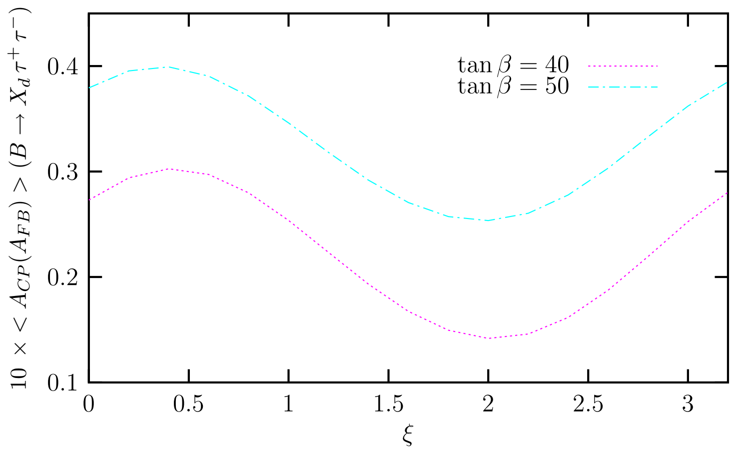


Figure 4.9: The CP asymmetry in the forward-backward asymmetry as a function of $\xi$.

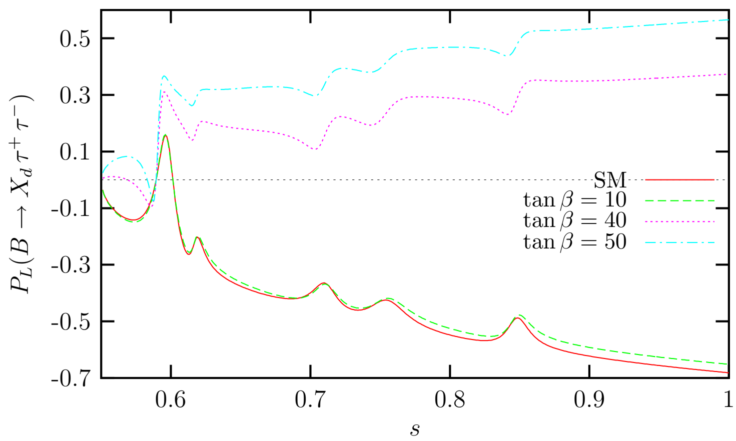


Figure 4.10: $P_L(s)$ as a function of $s$ , where $\xi = \pi/4$.

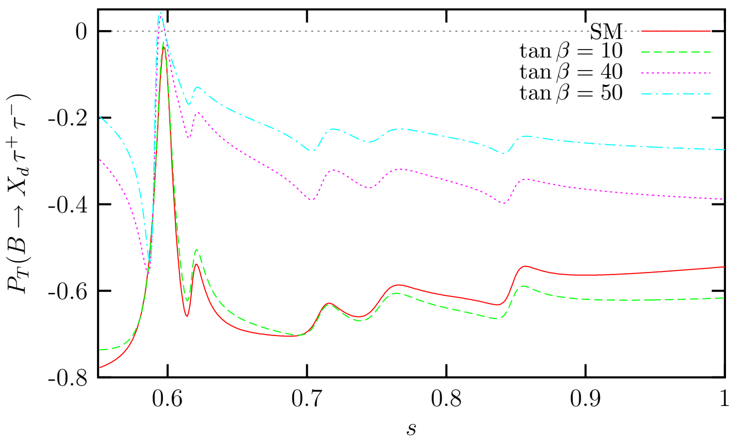


Figure 4.11: $P_T(s)$ as a function of $s$ , where $\xi = \pi/4$.

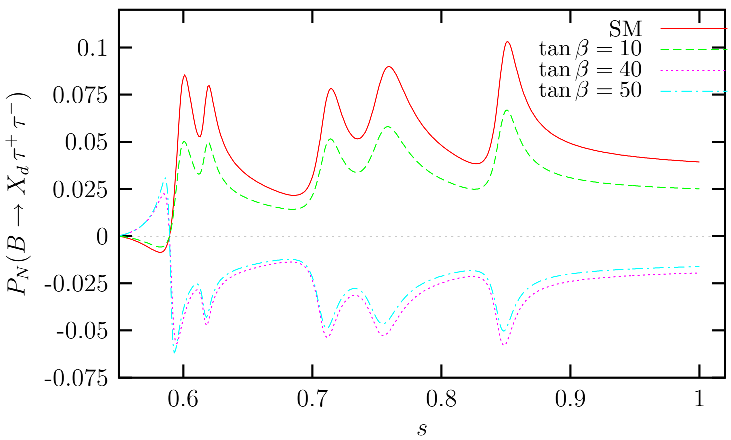


Figure 4.12: $P_N(s)$ as a function of $s$ , where $\xi = \pi/4$.

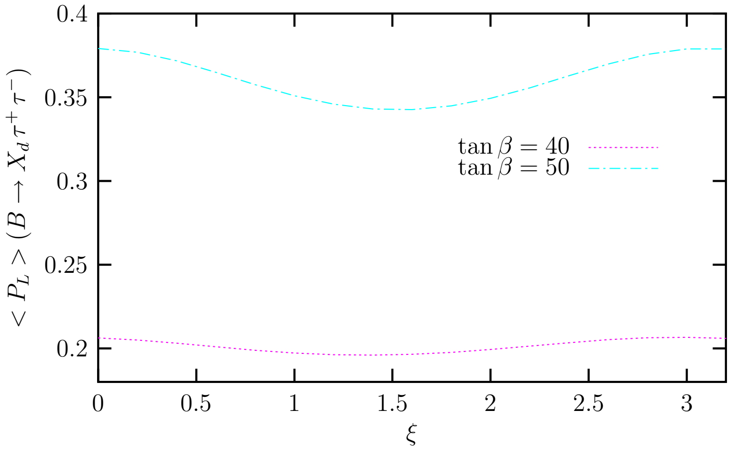


Figure 4.13: $< P_L >$ as a function of $\xi$.

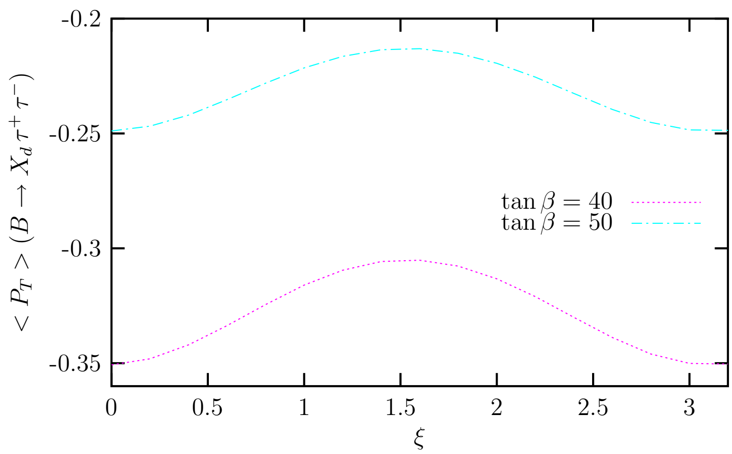


Figure 4.14: $< P_T >$ as a function of $\xi$.

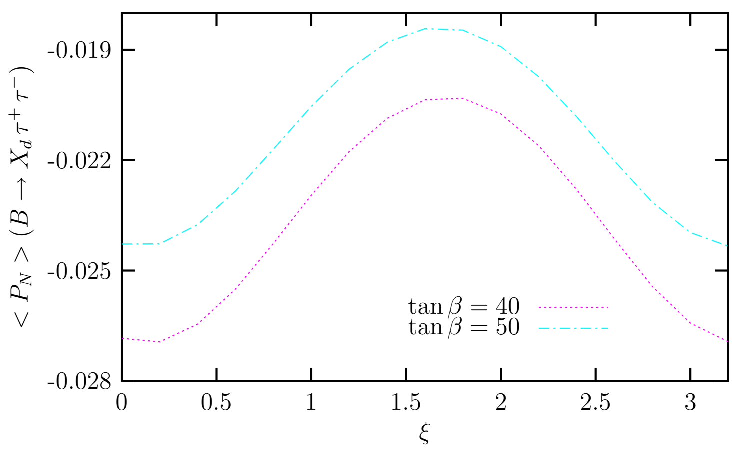


Figure 4.15: $< P_N >$ as a function of $\xi$. .

# CHAPTER 5

# CONCLUSION

The SM has been very successful as a theory for the quantitative description of all interactions of fundamental particles except gravity; all measurements are in agreement with the SM predictions with the exception of neutrino oscillations. However, there are several motivations for physicists to search for physics beyond it, such as having a totally unknown Higgs sector, hierarchy problem, origin of CP violation, etc. There are several classes of new approaches in this direction and among them the 2HDM, as being the most economical extensions of the SM, has been very attractive. To summarize, this model is obtained from the SM with the addition of one extra scalar $SU(2)_L$ doublet and possesses five physical Higgs bosons: a charged pair ($H^{\pm}$); two neutral CP even scalars ($H^0$and $h^0$); and a neural CP odd scalar ($A^0$), often called a pseduoscalar. Instead of one free parameter of the minimal model, this model has six free parameters: four Higgs masses, the ratio of the vacuum expectation values, $tan\beta$, and a Higgs mixing angle, $\alpha$.

In this thesis, we have examined the rare inclusive $B \to X_d \ell^+ \ell^-$ decay with emphasis on CP violation and neutral Higgs boson (NHB) effects within the framework of the model IV version of the 2HDM. Being a FCNC process, it is well known that $B \to X_{s,d}\, \ell^+ \ell^-$ decays provide reliable testing grounds for the SM at the loop level and they are also sensitive to new physics. In addition, $B \to X_d\, \ell^+ \ell^-$mode is especially important in the CKM phenomenology. Although CP violation is one of the most fundamental phenomena in particle physics it is still one of the the least tested aspects of the the Standard Model (SM). In the near future, more experimental tests will be possible at the B factories and possible deviations from the SM predictions will provide important clues about physics beyond it. This situation makes the search for CP violation in B decays highly interesting.

After presenting a brief summary of the SM and a CP spontaneously broken 2HDM in chapter 2 and 3, respectively, in Chapter 3, we have analyzed the double and differential decay rates, CP violation asymmetry, $A_{CP}$, forward-backward asymmetry, $A_{FB}$, and CP violating asymmetry in forward-backward asymmetry $A_{CP}(A_{FB})$ for $B \to X_d \, \ell^+ \ell^-$decay in detail. The important conclusions that can be pointed out from this work can be summarized as follows:

- NHB effects are seen to be quite significant on the differential branching ratio of the inclusive process $B \to X_d \, \ell^+ \ell^-$for the $\tau$ mode, especially in the moderate-s region.

- $A_{FB}$ is more sensitive to $\tan\beta$ than the $dBR/ds$ and it changes sign with the different choices of this parameter. It is also seen that $< A_{FB} >$ is quite sensitive to $\xi$ and between $(0.15, 0.28) \times 10^{-1}$. We also observe that $< A_{FB} >$ differs essentially from the one predicted by the CP-conservative 2HDM (model II), which is 0.028 and 0.023 for $\tan\beta = 40, 50$, respectively. In region $1 < \xi < 2$ change in $< A_{FB} >$ with respect to model II reaches 25%.

- $A_{CP}(s)$ is also sensitive to $\tan\beta$ and its sign does not change in the allowed values of $s$ except in the resonance mass region. $< A_{CP} >$ is not as sensitive as $< A_{FB} >$ to $\xi$, and it varies in the range $(0.15, 0.33) \times 10^{-1}$.

- $A_{CP}(A_{FB})(s)$ changes sign with the different choices of $\tan\beta$. $< A_{CP}(A_{FB}) >$ is between $(0.010, 0.040)$ and differs essentially from the one predicted by model II, which is 0.038 and 0.027 for $\tan\beta = 40, 50$, respectively. In region $1.5 < \xi < 2.5$ change in $< A_{FB} >$ with respect to model II reaches 35%.

- NHB contributions changes the polarization significantly, especially when

$\tan\beta$ is large. We also observe that except the resonance region, $P_T$ is negative for all values of $s$, but $P_L$ and $P_N$ change sign with the different choices of the values of $\tan\beta$. $< P_N >$ and $< P_T >$ are more sensitive to $\xi$ than $< P_L >$. In region $1.5 < \xi < 2.0$ change in $< P_N >$ with respect to model II reaches 25%. Thus, measurement of this component in future experiments may provide information about the model IV parameters.

Therefore, the experimental investigation of $A_{FB}$, $A_{CP}$, $A_{CP}(A_{FB})$ and the polarization components in $B \to X_d\, \ell^+\ell^-$decays may be quite suitable for testing the new physics effects beyond the SM.

## REFERENCES

[1] S. L. Glashow, *Nucl. Physc.* **B22** (1961) 579; S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264; A. Salam, in: *Proceedings of the 8th Nobel symposium*, p. 367, ed. N. Svartholm, Almqvist and Wiksell, Stockholm 1968.

[2] S. L. Glashow, I. Iliopoulos, L. Maiani, *Phys. Rev.* **D2** (1970) 1285.

[3] N. Cabbibo, *Phys. Rev. Lett.* **10** (1963) 531; M. Kobayashi, K. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.

[4] H. Y. Han, Y. Nambu, *Phys. Rev.* **139** (1965) 1006; C. Bouchiat, I. Iliopoulos, Ph. Meyer, *Phys. Lett.* **B138** (1972) 652.

[5] K. Abe, *et. al.*, Belle Collaboration, *Phys. Rev. Lett.* **87**(2001) 091802; B. Aubert, *et. al.*, BABAR Collaboration, *Phys. Rev. Lett.* **87**(2001) 091801.

[6] I. J. R, Aitchison, and J. G. Anthony, Gauge theories in particle physics : a practical introduction, 2nd edition, 1989, Bristol, England ; Philadelphia : A. Hilger in association with.

[7] P. W. Higgs, *Phys. Lett.* **12** (1964) 132; F. Englert and R. Brout, *Phys. Rev. Lett.* (1964) 321; G. S. Gurallnik, C. R. Hagen and T. W. B. Kibble, *Phys. Rev. Lett.* **13** (1964) 585; P. W. Higgs, *Phys. Rev.* **145** (1966) 1156; T. W. B. Kibble, *Phys. Rev.* **155** (1967) 1554.

[8] M. Gell-Mann, *Phys. Rev.* **92** (1953) 833; K. Nishijima and T. Nakano, *Prog. Theor. Phys.* **10** (1953) 581.

[9] Y. Nambu, *Phys. Rev. Lett.* **4** (1960) 380; J. Goldstone, *Nuovo Cimento* **19** (1961) 154; J. Goldstone, A. Salam and S. Weinberg, *Phys. Rev.* **127** (1962) 965.

[10] G. Arnison, *et. al.*, (UA1 Collaboration) *Phys. Lett.* **B122** (1983) 103; P. Bagnaia *et. al.*, (UA2 Collaboration) *Phys. Lett.* **B129** (1983) 130.

[11] S. Bertolini, *Nucl. Phys.* **B272** (1986) 77.

[12] C. Froggat, R. G. Moorehouse and I. G. Knowles, Prep. GUTPA/91/10/2, ANL-HEP-PR-92-01 (1992) .

[13] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *Higgs Hunter's Guide* (Addison-Wesley Publishing, 1990).

[14] S. L. Glashow and S. Weinberg, *Phys. Rev.* **D15** (1977) 1958.

[15] H. Georgi, *Hadronic Jour.* **1** (1978) 155.

[16] I. Vendramin, **hep-ph/9909291**.

[17] Chao-Shang Huang and Shou-Hua Zhu, *Phys. Rev.* **D61** (2000) 015011 [Erratum: **D61** (2000) 119903].

[18] Chao-Shang Huang, Wei Liao, Qi-Shu Yan and Shou-Hua Zhu, *Eur. Phys. J.* **C25** (2002) 103.

[19] J. H. Christenson, **et al,** *Phys. Rev. Lett.* **13** (1964) 138.

[20] See e.g., A. G. Cohen, D. B. Kaplan and A. E. Nelson, , *Ann. Rev. Nucl. Part. Sci.***43**(1993) 27.

[21] F. Krüger, L.M. Sehgal, *Phys. Rev.*, **D55**, (1997) 2799.

[22] S. Rai Choudhury, *Phys. Rev.*, **D56**, (1997) 6028.

[23] W. S. Hou, R. S. Willey and A. Soni, *Phys. Rev. Lett.* **58** (1987) 1608.

[24] N. G. Deshpande and J. Trampetic, *Phys. Rev. Lett.* **60** (1988) 2583.

[25] C. S. Lim, T. Morozumi and A. I. Sanda, *Phys. Lett.* **B218** (1989) 343.

[26] B. Grinstein, M. J. Savage and M. B. Wise, *Nucl. Phys.* **B319** (1989) 271; G. Buchalla, A. Buras, and M. Lautenbacher, *Rev. Mod. Phys.* **68** (1996) 1125.

[27] C. Dominguez, N. Paver and Riazuddin, *Phys. Lett.* **B214** (1988) 459.

[28] N. G. Deshpande, J. Trampetic and K. Ponose, *Phys. Rev.* **D39** (1989) 1461.

[29] P. J. O'Donnell and H. K. Tung, *Phys. Rev.* **D43** (1991) 2067.

[30] N. Paver and Riazuddin, *Phys. Rev.* **D45** (1992) 978.

[31] A. Ali, T. Mannel and T. Morozumi, *Phys. Lett.* **B273** (1991) 505.

[32] A. Ali, G. F. Giudice and T. Mannel, *Z. Phys.* **C67** (1995) 417.

[33] C. Greub, A. Ioannissian and D. Wyler, *Phys. Lett.* **B346** (1995) 145;
D. Liu *Phys. Lett.* **B346** (1995) 355;
G. Burdman, *Phys. Rev.* **D52** (1995) 6400:
Y. Okada, Y. Shimizu and M. Tanaka *Phys. Lett.* **B405** (1997) 297.

[34] A. J. Buras and M. Münz, *Phys. Rev.* **D52** (1995) 186.

[35] N. G. Deshpande, X. -G. He and J. Trampetic, *Phys. Lett.* **B367** (1996) 362.

[36] W. Jaus and D. Wyler, *Phys. Rev.* **D41** (1990) 3405.

[37] Y. B. Dai, C. S. Huang and H. W. Huang, *Phys. Lett.* **B390** (1997) 257; Erratum-ibid. **B513** (2001) 429.

[38] C. S. Huang, L. Wei, Q. S. Yan and S. H. Zhu, *Phys. Rev.* **D63** (2001) 114021; C. S. Huang, W. Liao and Q. S. Yan, *Phys. Rev.* **D59** (1999) 011701; Z. Xiong and J. M. Yang, *Nucl. Phys.* **B628** (2002) 193; C. S. Huang and X. H. Wu, *Nucl. Phys.* **B657** (2003) 304.

[39] H. E. Logan and U. Nierste, *Nucl. Phys.* **B586** (2000) 39.

[40] E. Iltan and G. Turan, *Phys. Rev.* **D63** (2001) 115007.

[41] T. M. Aliev and M.Savcı, *Phys. Lett.* **B452** (1999) 318.

[42] S. Rai Choudhury and N. Gaur, *hep-ph/0207353*.

[43] K. S. Babu, K. R. S. Balaji and I. Schienbein, *Phys. Rev.* **D68** (2003) 014021.

[44] T. M. Aliev, V. Bashiry, and M. Savcı, *hep-ph/0308069*.

[45] D. Buskulic, *et. al.*, ALEPH Collaboration, *Phys. Lett.* **B343** (1995) 444; J. Kalinowski, *Phys. Lett.* **B245** (1990) 201; A. K. Grant, *Phys. Rev.* **D51** (1995) 207.

[46] K. Krüger and L.M. Sehgal, *Phys. Lett.* **B380**(1996)199.

[47] I.I. Bigi, M. Shifman, N.G. Vraltsev and A.I. Vainstein, *Phys. Rev. Lett.* **71** (1993) 496; B. Blok, L. Kozrakh, M. Shifman and A.I. Vainstein, *Phys. Rev.* **D49** (1994) 3356; A.V. Manohar and M.B. Wise, *Phys. Rev.* **D49** (1994)1310; S. Balk, T.G. Körner, D. Pirjol and K. Schilcher, *Z. Phys.* **C64** (1994) 37; A.F. Falk, Z. Ligeti, M. Neubert and Y. Nir, *Phys. Lett.* **B326**(1994)145.

[48] B. Grinstein, R. Springer and M. Wise, **Nucl. Phys. B339** (1990) 269; R. Grigjanis, P. J. O'Donnell, M. Sutherland and H. Navelet, *Phys. Lett.* **B213** (1988) 355; Erratum: *ibid* **B286** (1992) 413; G. Cella, G. Curci, G. Ricciardi and A. Viceré, *Phys. Lett.* **B325** (1994) 227; *ibid Nucl. Phys.* **B431** ( 1994) 417.

[49] M. Misiak, *Nucl. Phys.* **B393** (1993) 23; Erratum: *ibid.* **B439** (1995) 461.

[50] T. M. Aliev and E. O. Iltan , *Phys. Rev.* **D58** (1998) 095014.

[51] Z. Ligeti, I. W. Steward, and M. B. Wise, *Phys. Lett.***B420**(1998)359.

[52] G. Buchalla, G. Hiller and G. Isidori, *Phys. Rev.* **D63** (2001) 014015.

[53] L. Wolfenstein, *Phys. Rev. Lett.* **51** (1983) 1945.

[54] A. Ali, and E. Lunghi, *Eur. Phys. J.* **C26** (2002) 195.

[55] A. Heister, *et al*, ALEPH Collaboration, CERN-EP/2001-095; L3 Collaboration, *Phys. Lett.* **B503** (2001) 21.

# VITA