OPTIMAL AIR DEFENSE STRATEGIES
FOR A NAVAL TASK GROUP

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ABSTRACT

OPTIMAL AIR DEFENSE STRATEGIES
FOR A NAVAL TASK GROUP

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We develop solution methods for the air defense problem of a naval task group in this dissertation. We consider two interdependent problems. The first problem is the optimal allocation of a set of defensive missile systems of a naval task group to a set of attacking air targets. We call this problem the Missile Allocation Problem (MAP). The second problem called the Sector Allocation Problem (SAP) is the determination of a robust air defense formation for a naval task group by locating ships in predefined sectors on the surface. For MAP, we present three different mixed integer programming formulations. MAP by its nature requires real time solution. We propose efficient heuristic solution procedures that satisfy the demanding time requirement of MAP. We also develop mathematical programming models for SAP. Proposed branch and bound solution scheme for SAP yields highly satisfactory solutions. We characterize the interaction between MAP and SAP and develop an integrated solution approach.

Keywords: Air Defense, Naval Task Group, Formation, Weapon Target Allocation Problem, Military Operations Research, Quadratic Assignment, Location.

ÖZ

BİR DENİZ GÖREV GRUBU İÇİN
OPTİMAL HAVA SAVUNMA STRATEJİLERİ

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Bu tezde, deniz görev gruplarının hava savunma problemlerinin çözümü için bir metodoloji geliştirilmiştir. Bu kapsamda, birbirine bağımlı iki problem ele alınmıştır. İlk problem, bir deniz görev grubunda bulunan gemiler üzerinde konuşlu hava savunma güdümlü mermilerinin tehdit hava hedeflerine optimal tahsisidir. Bu problemi Güdümlü Mermi Tahsis Problemi (MAP) olarak adlandırıyoruz. Sektör Tahsis Problemi (SAP) olarak adlandırdığımız ikinci problem, gemileri deniz üzerinde tanımlanmış sektörlere yerleştirmek suretiyle deniz görev grubu için etkin ve gürbüz bir hava savunma nizamının belirlenmesidir. MAP için üç ayrı güdümlü mermi tahsis modeli geliştirilmiştir. MAP çok hızlı reaksiyon ihtiyacı nedeniyle gerçek zamanlı çözümlere ihtiyaç duymaktadır. MAP, ihtiyacı karşılayacak şekilde çok kısa zaman içinde etkin çözümler üretebilen sezgisel yöntemler kullanılarak çözülmektedir. SAP için önerilen dal-sınır algoritması kabul edilen iyi çözümler üretmektedir. Son olarak, MAP ve SAP problemleri arasındaki etkileşim tanımlanmış ve her iki probleme bütünleşik bir çözüm yöntemi geliştirilmiştir.

Anahtar Kelimeler: Hava Savunma, Deniz Görev Grubu, Nizam, Silah Hedef Tahsis Problemi, Askeri Yöneylem Araştırması, Kuadratik Atama, Yer Seçimi.

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LIST OF ACRONYMS

AAD	Area Air Defense
AAW	Anti-Air Warfare
ASM	Anti-Ship Missile
BMD	Ballistic Missile Defense
C^2	Command and Control
C^4I	Command Control Communications Computers and Intelligence
CEC	Cooperative Engagement Capability
HVU	High Value Unit
IPP	Impact Point Prediction
MAP	Missile Allocation Problem
ND	No-Defense. i.e. a Ship Having No Air Defense Capability
NDP	Neuro-Dynamic Programming
OR	Operations Research
SAM	Surface-to-Air Missile
SAP	Sector Allocation Problem
SD	Self Defense. i.e. a Ship Having Only Self Air Defense Capability
SLS	Shoot-Look-Shoot
SSPK	Single Shot Kill Probability
TBM	Tactical Ballistic Missiles
TEWA	Threat Evaluation and Weapon Assignment
TG	Task Group
TPZS	Two Person Zero Sum
WTA	Weapon-Target Allocation

CHAPTER I

INTRODUCTION

1.1 MOTIVATION

Air defense has been an increasingly important problem for national authorities and armed forces. Substantial resources have been devoted to develop both defensive and offensive weapons and systems. The use of aircraft and air-dropped munitions in World War-I, the attack of German V-1 cruise missiles and V-2 ballistic missiles on London in World War-II, the sinking of the Israeli destroyer Eilat by Styx guided missiles in 1973, Exocet missiles in the Falklands, the Tomahawk cruise missiles and SCUD theatre ballistic missiles during the Gulf War, and the decisive allied air operation against Yugoslavia in 1999 are important benchmarks that trace the evolution of air power and the air threat for armed forces and nations. After the nuclear threat of the cold-war, the post-cold-war era witnesses the proliferation of weapons of mass destruction, tactical ballistic missiles and cruise missiles. Many nations still devote a substantial amount of their defense budget for acquisition of air defense weapons and systems. The effective use of and defense against these weapon systems is of the utmost importance for the armed forces.

The proliferation of anti-ship missiles (ASMs) and the increasing frequency of littoral operations (i.e. operations close to land and territorial waters) have increased the threat to the navies posed by the ASMs. Townsend (1999) reports that there are 13 nations (not including the NATO countries) having an ASM production capacity and an additional 15 nations developing this capability.

The competing technologies of ASMs and ASM defense systems force the navies to update the systems and to develop new tactics continuously. All modern navies devote considerable resources to ASM defense systems (Carus, 1992). The sinking of Israeli destroyer Eilat by four Styx ASMs by the Egyptian Navy in 1967 was a first in naval history and the demonstration of the potential ASM threat. Six years later in 1973, a total of 54 ASMs launched by the Syrian and Egyptian Navies failed to hit their intended targets due to the defensive tactics developed by the Israeli Navy (Carus, 1992). The Exocet ASMs sank the British destroyer HMS Sheffield during the Falklands War in 1982. The ASM attack on the US Navy frigate Stark in Persian Gulf in 1987 is another example of the fragility of ASM defense.

Although significant resources are allocated to technological development, planning for effective use of these systems in operation has not been paid equal attention. One particular aspect of planning is coordinated allocation of defense systems within a group of ships to attacking missiles, which we intend to tackle in this study. A second aspect we deal with is formation of ships on the surface prior to allocation.

1.2 STATEMENT OF THE GENERAL PROBLEM

A most generic form of the weapon-target allocation (WTA) problem is the following: given an existing weapon force and a set of targets, what is the optimal allocation of weapons to targets (Matlin, 1970)? WTA problem can be viewed both from an attacker's and a defender's perspective. We restrict ourselves to the defense of the friendly forces with surface-to-air (SAM) missiles, and call the problem defensive missile allocation problem (MAP). MAP can be stated as *the optimal allocation of a set of defensive missile systems to a set of attacking air targets*.

In 1997, Panel on Modeling and Simulation of Naval Studies Board identified air defense as one of the warfare areas for focused research. The Naval Studies Board (1997) states that: "*There has been relatively little recent investment in understanding the phenomenology of military operations at the mission and operational levels. Much of the basis for related modeling and simulation is still programmer hypothesis and qualitative opinions expressed by subject matter experts.*"

In this research, we further focus on the MAP of the navies. In particular we address the issue of allocating air defense missiles to incoming air targets in a coordinated way within a naval task group (TG) such that the available defense capability is used in the most effective manner. A TG is a collection of naval combatants and auxiliaries that are grouped together for the accomplishment of one or more missions.

Nations spend billions of dollars for their navies. However, it is still prohibitively expensive to equip all the platforms with adequate air defense systems. For many navies, equipping all the platforms with air defense systems is clearly not the best and the cost-effective solution. A number of NATO navies have plans for acquiring area air defense (AAD) platforms that can provide air defense support to the other ships that have limited or no effective air defense capability. The Canadian Command and Control, Area Air Defense Replacement (CADRE) project, and the Turkish Navy's Area Air Defense Frigate Project (TF-2000) are the two examples of these projects. The allocation of the capability of the AAD ship(s) to the other units in the TG is an immediate problem to be solved for effective use of these platforms.

The aim of this study is twofold. The first one is to develop a MAP model for TG air defense that captures the reality of ASM defense, generates an efficient allocation plan and measures the effectiveness of the air defense under a given scenario. A scenario is composed of the information on the attacking ASMs and the defensive SAM systems as well as the relative positions of the ships in TG, which is called the *formation* of the TG. Our second aim is to develop an approach for determining a robust air defense formation for a naval TG with known ships and air defense capabilities. We refer to this second problem as sector allocation problem (SAP) since we intend to locate ships in predefined sectors on the surface. A robust formation is the one that is very effective against a variety of attack scenarios (i.e. independent of the scenarios) but not necessarily the most effective one against any of the scenarios. The reason of seeking robustness is that formation takes much longer time compared to allocation. Given the available SAM systems and attacking

ASMs, allocation and engagement are almost instant whereas changing the formation may take hours.

We further develop integrated solution methods for the air defense problem of a naval TG. We first develop analytical solution methods for the TG MAP. Formation data will be used as an input to the model. Algorithms for MAP may be used in command and control systems of warships. We next consider the development of a solution procedure for SAP. Solving SAP will enable the naval tactician to evaluate the effectiveness of present formations, to develop new air defense tactics, and to use air defense systems at their best.

1.3 OVERVIEW AND CONTRIBUTION OF THE DISSERTATION

The purpose of this thesis is to develop air defense strategies for a naval TG. We identify two problems, MAP and SAP, that enable the TG to use its defensive resources at the maximum extent possible against air attacks under several assumptions.

MAP, which can be categorized as a weapon target allocation problem, is a new treatment of an emerging problem fostered by the rapid increase in the capabilities of ASMs and the different levels of air defense capabilities of the warships against the ASM threat. Area air defense missile systems can provide support to the other ships in TG and new technologies such as improved tactical data links and cooperative engagement capability (CEC) enable a fully coordinated air defense within a TG. In addition to allocating SAMs to ASMs, MAP also schedules launching of SAM rounds according to shoot-look-shoot tactic considering multiple

SAM and ASM types. Although we have developed mathematical programming models for this new variant of WTA problem, we did not explicitly use those models to solve MAP. We developed efficient heuristic algorithms to solve MAP. MAP solution can be used for both real time and non-real time applications. MAP can produce the best course of action for defending the TG against an immediate and simultaneous ASM threat. Using MAP to provide input for SAP is an example of non-real time use of MAP. MAP can also be used for off-line analysis of the air defense effectiveness of warships under different scenarios.

In SAP, we make use of the information on possible threat and decide on formation of the TG before the air attack by allocating the warships to sectors intelligently. Although SAP resembles the quadratic assignment problem in several ways, we do not use this type of formulation. We develop strong formulations that make use of the special properties of the problem. In SAP, locations of both facilities and demand points are unknown. To our knowledge, our formulations and the solution procedure for SAP are new in open literature.

We also integrated the two problems such that sector-to-sector coverage values produced by MAP for various attack scenarios are used as input parameters in solving SAP. This way, we can propose TG formations based on partial information concerning the expected threat.

The next chapter contains the detailed description of MAP and SAP. We discuss special properties, assumptions, and environments of the problems. We characterize the interaction between MAP and SAP.

In Chapter 3, we present the relevant literature on MAP and SAP. This chapter contains different WTA models and definition of a classification scheme for WTA models. Literature on SAP covers the relevant researches, which mainly focus on the geometric aspects of the problem rather than the optimum allocation using mathematical programs.

In Chapter 4, we formulate MAP for a naval TG. Three different formulations with several extensions are given in three different sections. Theoretical development of those models and possible solution approaches are discussed. However, we propose efficient heuristic solution algorithms for MAP in order to satisfy the demanding solution time requirement of the problem. Chapter 5 gives the details of the solution approach and the computational results.

We present sector allocation models in Chapter 6. We developed five different sector allocation models and several variations. We also investigated the validity of different objective functions. We identify the most suitable model for SAP by identifying the features and drawbacks of each model. We developed cuts for linear programming relaxation of the models and proposed branch and bound solution approaches. Solution algorithms and computational results for SAP are reported in Chapter 7.

In Chapter 8, we discuss an integrated solution approach to attain a robust sector allocation for a naval TG by using MAP results within SAP. Two different coverage aggregation procedures in the development of a robust formation are discussed. MAP and SAP interactions are presented using sample scenarios and sample problems.

We conclude the dissertation with the chapter on conclusion and directions for future research.

CHAPTER II

DEFINITION OF PROBLEMS

2.1 MISSILE ALLOCATION PROBLEM

Consider a naval TG, composed of several ships with variable air defense capabilities, defending itself against an air attack. These ships may either be equipped with one or more surface-to-air missile (SAM) systems or none at all. Their air defense capability may be limited to self-defense or may extend to area defense, i.e. a ship may defend the other ships within its effective weapon range. In a naval TG, the individual ships function together as a team to provide mutual support and defense against opposition to assigned missions. These ships are typically arrayed into a formation, called a screen, in which the most valuable and important units (termed high value unit or HVU) are surrounded and protected by the escorting vessels. Within the screen, the escort ships are stationed in sectors away from the HVU. Figure 2.1 depicts a generic naval TG composition and an air attack scenario. In this scenario a TG composed of four ships in formation, one HVU and three escort ships, is attacked by four ASMs. Ship 1 (HVU with no SAM system onboard) is targeted by ASM2 and ASM3, Ship 2 is targeted by ASM1 and Ship 4 is targeted by ASM4. There is no ASM threat to Ship 3. Ships 2, 3, and 4 have short-range self defense SAM systems (such as NATO Sea Sparrow SAM) and the effective ranges are depicted by the circular areas around each ship. Ship 2 also has a long-range area defense SAM system (such as SM-2 SAM), and part of its effective range is depicted by the dotted area and the arc drawn in dashed-line. ASM1 can be engaged by both SAM1 and SAM2. ASM2 and ASM3 can be engaged by only SAM2. Note that SAM4 cannot engage ASM3 even if some part of the ASM3's flight path falls into the effective range of SAM4, since SAM4 is a self defense system and can only engage the ASMs that are a direct threat to it. ASM4 can be engaged by both SAM2 and SAM4. A TG typically consists of 4 to8 warships, and the maximum number of warships hardly exceeds 10.

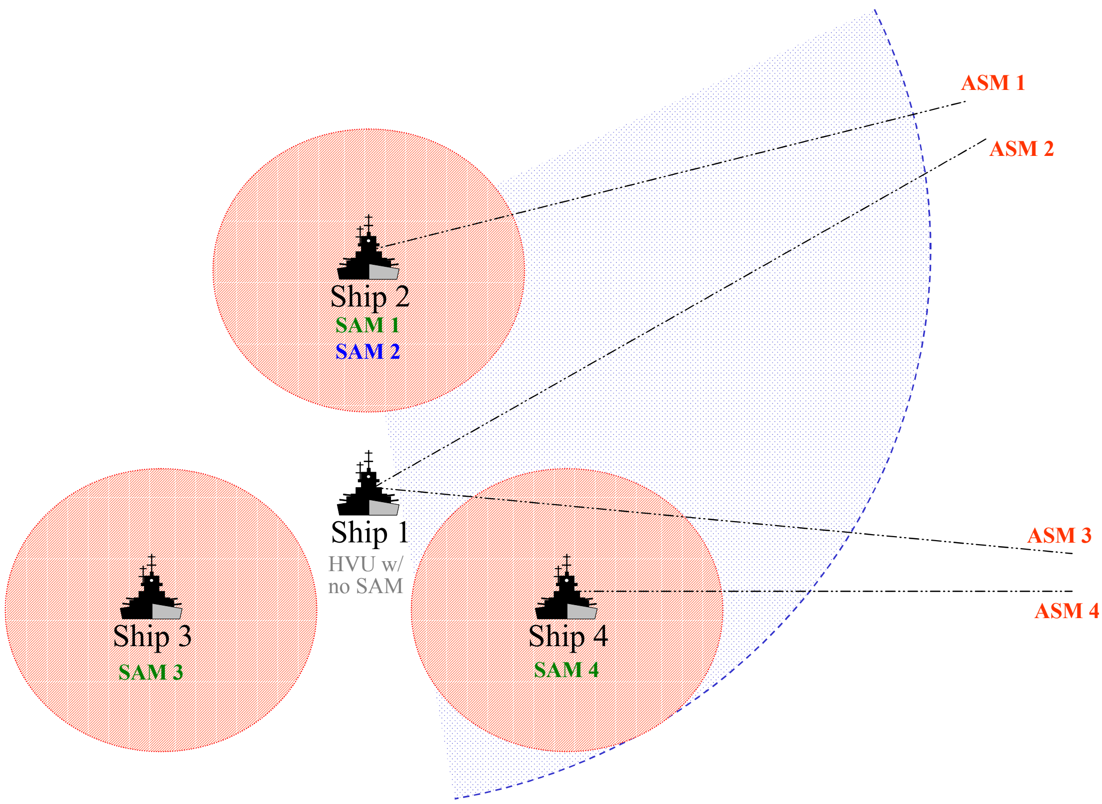

Figure 2.1. Composition of a Naval TG and an Air Attack Scenario.

The TG air defense commander will maintain the air picture for the TG and coordinate the response until the time when ships are forced to defend themselves. The air defense command and control ship will, in most cases, have to coordinate the TG response to an air threat to ensure maximum efficiency and probability of success. In this role a set of command decision tools is required to plan the air defense of the TG, and to schedule the force defense as an attack develops, allocating assets on a real-time basis.

Maximization of probability of the shooting down all the incoming ASMs is an important objective for a TG air defense commander at sea. However, saving the maximum number of the SAMs (for possible future attacks) from a limited number of SAMs in the magazines available onboard of the ships, and the high price tag of each missile have to be considered as well. The objective might be to use the SAM expenditure with the minimum cost subject to goal constraints for the minimum probability of neutralizing the incoming ASMs. Several missile engagement tactics have been developed to achieve a balance between these conflicting objectives. One of the missile engagement tactics employed by navies is called shoot-look-shoot (SLS). The SLS tactic requires shooting at the target first, then looking to see if it was killed, and shooting again if necessary to achieve the kill. In this research, we consider the case when the TG employs a SLS tactic.

Engagement process of a SAM system to an ASM can be divided into four phases. These are the tracking of the target illumination radar, the solution of the fire control problem, the launch delay (i.e. the system delay between receiving the launch signal from the fire control console and the missile leaving the launchers), and the flight time to the engagement. Note that this engagement process is for a generic semi-active SAM (i.e. the SAM is to be illuminated by the fire-control radar either throughout its flight or at intermitted time intervals during its flight). The engagement process for an active SAM (i.e., one that does not need an illumination radar) may be considered to have only three phases except the tracking phase of the target illumination radar. Each engagement of both active and semi-active SAM systems takes a constant setup time for the first three phases and a variable time for the last phase, which is the flight time to the engagement. Each engagement takes less time compared to the one before as the attacking ASM is approaching the TG.

The maximum distance at which an ASM intercept can take place is determined by either the maximum effective range of the SAM system or the radar horizon of the fire control radar against the incoming ASM, or the first detection range of the ASM if it is smaller than the above two.

When a SLS firing policy is used, there are few engagement opportunities (mostly less than 10) against each ASM. For example, an ASM with 300 m/sec velocity, which is detected at 30 km distance can be attacked at most four times by a SAM with 600 m/sec velocity using a SLS tactic, given that the target illumination radar track time is 5 sec, the fire control solution time is 2 sec, and the launch delay is 2 sec. In this calculation, we use a total of 9 sec setup time before each engagement. In reality each engagement does not take the same set-up time, since the target illumination radar may already be on track, or the fire control problem may have already been solved. However, we use a conservative approach and consider that each engagement takes a constant setup time.

In summary, MAP is concerned with allocation of different types of SAMs available to attacking ASMs and scheduling the SAM launches under SLS firing policy, so as to maximize the TG's air defense capability.

2.2 SECTOR ALLOCATION PROBLEM

Formation is the geographical order of the ships in TG. Ships in a TG operate together as a coherent unit. HVUs are usually located at the center of the formation. The escort ships station away from the center of the formation at a point designated by a bearing and range relative to the center of formation, or in a sector designated by two bearings and two ranges relative to the center. Figure 2.2 depicts such a generic formation in which ships are stationed in their assigned sectors.

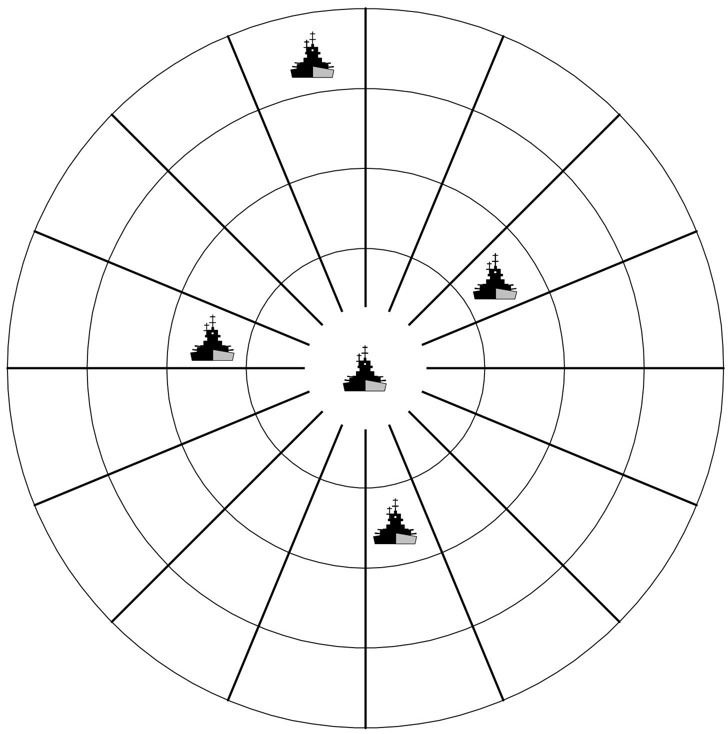

Figure 2.2. A Typical Formation of a TG with 5 Ships Allocated to Sectors.

In MAP, we assume that the formation information such as the relative bearings and the distances between ships is given in a scenario as well as the specific attack information. This approach is reasonable when TG operates in a formation and encounters an immediate air attack by ASMs. In this case, solving the MAP and fighting the war accordingly would optimize the effectiveness of air defense. However, we may ask ourselves a second question: can we define a formation that keeps the effectiveness of the TG against an air attack at a high level, independent of the specific attack scenario? We call this problem as the Sector Allocation Problem (SAP). Note that the speed of the ships is very small compared to the speed of the air attack. It may take from tens of minutes to several hours to change the formation from one to the other, while it takes tens of seconds from detection to time-on-target for an ASM. Thus, it is important to be in a suitable formation before a possible air attack. We may investigate this problem under two different assumptions:

1. No information is available about the possible attack direction, i.e. the attack is expected from any direction.
2. Information coming from intelligence and surveillance sources indicates the general direction of the attack, i.e. the attack is expected from a direction such as north or south or between bearings 120 and 180 of the TG.

2.3 INTERACTION BETWEEN THE MODELS

Consider an operational scenario for a naval TG that is on mission at sea under an immediate air threat. Assume that the TG has the information that an air attack is expected from a certain direction (i.e. no surprise air attack). The officer in tactical command would order his ships to form a formation that is most suitable for that situation. After getting ready for an attack in terms of the formation, he would use his SAM systems to counter the possible attack. Reality dictates us to make a decision on formation before the missile allocation decision for an immediate air attack. However this should not lead us to consider those problems independently. In general, the solution to MAP can be used in solving SAP and vice versa. We define two different interactions between those models.

Figure 2.3 depicts the Interaction Model-1. In Interaction Model-1, we solve MAP for a number of attack scenarios, and using aggregated results as input, we solve SAP. Scenarios are expected to reflect the possible threat to the TG at sea. A scenario typically involves size and type of ASMs, attack directions, detection distances of ASMs, defending SAM systems, number of available missiles in the magazines and TG formation. Information on the enemy inventory of warships and their weapon systems and the intelligence coming from different sources may enable the decision maker (or officer in tactical command of the ships within TG) to generate such representative scenarios. For each scenario, we can calculate the "coverage" provided by an AAD ship to another ship for all possible pairwise sector allocations. Then, we can aggregate the coverage values for each sector pair and use that information in solving SAP. Here, a robust formation that will satisfy all scenarios at a certain degree can be found. This type of interaction implies off-line use of the two models.

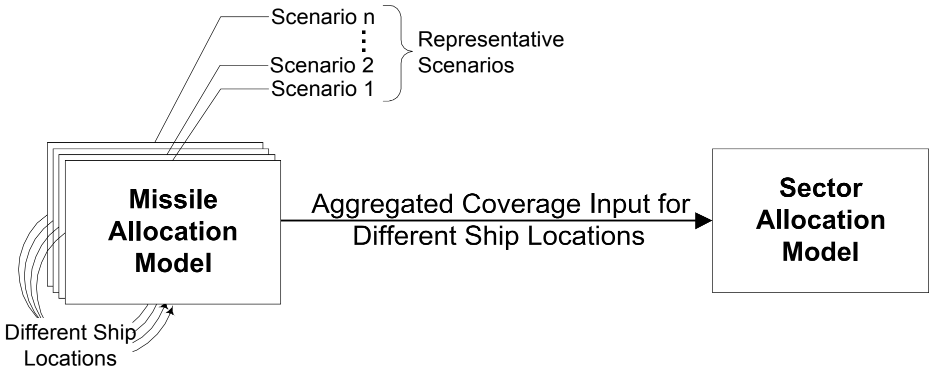

Figure 2.3. Interaction Model-1 Between MAP and SAP.

In Interaction Model-2, which is depicted in Figure 2.4, we assume that we have determined the formation of TG using the sector allocation model (or chosen one of those formations generated off-line), and we are operating at sea. Then, in the presence of an immediate ASM threat, we solve MAP to optimize our air defense against the threat. In this interaction model, MAP can be used on-line.

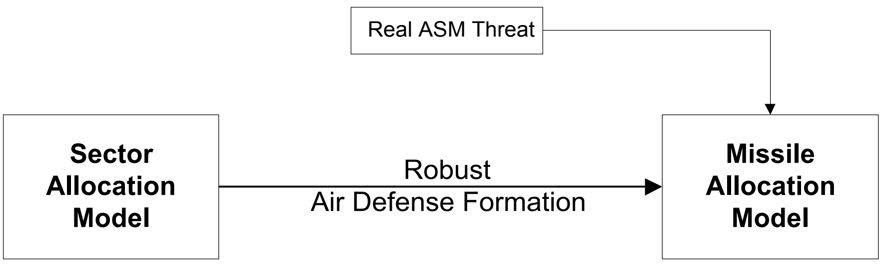

Figure 2.4 Interaction Model-2 Between MAP and SAP.

CHAPTER III

LITERATURE REVIEW

In this chapter, we review the relevant literature for MAP and SAP in separate sections. We start with background literature on MAP and continue with the literature review on SAP.

3.1 LITERATURE REVIEW ON WEAPON-TARGET ALLOCATION PROBLEM

MAP models use different parameters and assumptions depending on the requirement of the specific air defense scenario under consideration. Simplifying assumptions are generally used to reduce the problem to a level of suitable mathematical tractability. Even a simple MAP can be quite hard to solve in terms of the computational complexity. Indeed, Lloyd and Witsenhausen (1986) prove that the weapon allocation problem is NP-Complete even in its simplest form. Thus, using the simplifying assumptions in the modeling phase of the problem is a prerequisite for a successful solution of the problem.

MAP in general has many characteristics. However, an exhaustive categorization can be quite large, and therefore we focus on those aspects that provide for distinctive characteristics of the models. The important characteristics for the modeling and the solution process in the present applications are:

1. simultaneous or sequential attack,
2. command and control system capability,
3. point or area defense systems,
4. number of attacking and defending weapon types,
5. one or multiple layers of defense,
6. radar capability, and,
7. interceptor missile allocation policy.

In order to better understand how these aspects provide a characterization of air defense problem we shall consider each in further detail.

Simultaneous or Sequential Attack: A simultaneous attack is one for which the defense sees all of the air threat to intercept. The term "known attack size" is also used synonymously for simultaneous attack. A sequential attack is the case for which the defense does not know the number of attack groups and the number of attackers in each attack group. A mixture of simultaneous and sequential attack may exist for real world situations.

Command and Control System Capability: Point and area defenses may function in full coordination, partial coordination or autonomously. For example, Cooperative Engagement Capability is a new technology (at sea testing conducted in the last 10 years) that allows a fully coordinated air defense within a group of warships.

Point or Area Defense: Point defense systems are those designed for the defense of a single target such as a strategic facility, an air base, or a command control center. Point defense systems may intercept the attackers that are attacking their assigned targets. Area defense systems may intercept an attacker within the area of its effective range. Defense system may or may not be a collocated with the target of the attack.

Weapon Types: There may be identical attackers and defenders as well as a number of different weapon system types according to the scenario considered. Different weapon types usually complicate the problem.

Layered Defense: Defense systems of different types protecting the same target may have different effective ranges. These are then said to constitute the layers of defense. Layers typically overlap.

Radar Capability: Defense systems may or may not predict the eventual target of the attacking missile, i.e. either impact point prediction (IPP) or no IPP. Since such defense systems are typically radar controlled, we characterize this under radar capability.

Missile Allocation Policy: The defense's interceptor allocation policy such as salvo, shoot-look-shoot, shoot-shoot-look effects the performance and modeling of the air defense system. Defensive systems may have single or multiple engagement opportunities depending on the time-space conditions of the interception.

The problem and the solution procedure differ significantly depending on the assumptions made and parameters used. These may range from simulation based approaches to analytical methods. However, Bracken and Brooks (1985) argue that the MAP is not addressed much in the literature in an analytic sense after the 1972

Anti-Ballistic Missile Treaty. Those papers that address the MAP mostly consider the scenario of a defense against tactical ballistic missile (TBM) attack. Matlin (1970) and Ecker and Burr (1972) review the literature on missile allocation problem. However, Matlin focuses on the problem from the attacker's perspective.

The MAP models and solution methods differ significantly according to the assumptions made and the parameters used. There are a number of possible classification schemes for the OR literature on MAP, with the merits of classification fairly subjective. Our intent is to provide a classification scheme that identifies and delineates the major aspects of the problem. Thus, we propose a two-level classification, first according to inclusion or exclusion of the opponent's moves and second the identifying characteristics of the approach.

We use the first level of the classification scheme proposed by Matlin (1970) for our first level grouping of the MAP approaches. We classify the literature into three groups. The first class of approaches allocates the defensive sources to targets without taking into account the behavior of the opposing side. This is the group of **defense allocation models**. These methods generally use different versions of dynamic programming, integer programming, non-linear programming, and meta-heuristics. The actions of the opposing side are included in the scenario as a given input.

The second class of approaches takes into account the opposing side's moves as well as the defensive moves. These methods, namely **game models** employ the two-person-zero-sum-game concept from the game theory in the solution process. These methods reach the solution value of the game by assuming best defensive and offensive moves. The defense wants to minimize the maximum offense return while the offense acts to maximize the minimum expected return. This approach is more suitable when the inventories of the opposing sides are known to some degree.

The third class of techniques covers the rest of the literature that address different aspects and questions within the MAP context. Simulation models and layered defense models are included in this category.

The research and literature on MAP mainly focus on the attack or the defense of ballistic missiles. However, the methodologies used in these models have potential for use under different air defense scenarios. In Section 3.1.1, we review approaches that allocate defensive sources to offensive targets. In Section 3.1.2, we present approaches employing both defensive and offensive actions in a game theoretic context and we discuss methods that deal with different aspects of the MAP in Section 2.2.3.

3.1.1 Defense Allocation Models

The defense allocation models optimize the defense without explicit knowledge of the actions of the opposing side. The threat to the defense is considered as given.

Prim-Read Defense

Prim-Read defense has the general form of minimizing the total number of interceptors required for defending separated point targets against an attack by an unknown number of sequentially arriving tactical ballistic missiles (TBM). This method produces solutions both for the deployment and the firing doctrine of interceptors, while ensuring that the damage to the target is bounded by a linear function of the attack size. In a sense, it finds the optimum allocation of interceptors to keep the damage incurred by each attacking missile as low as possible. Thus, Prim-Read defense uses the allowable damage per attacker instead of the number of attacking missiles and builds the model without explicitly knowing the attack size. Burr et al. (1985) developed the optimal integer solutions for Prim-Read defense. They formulate the problem first as a multi-target problem and later reduce it to the single target problem. They show that the greedy algorithm optimally solves the multi-target problem through solving single target problems. They investigate both the perfect and the imperfect interceptor cases (An interceptor is called perfect, if its probability of successfully intercepting the attacker is 1. If the probability is less than 1, then the interceptor is called imperfect.).

The Prim-Read defense formulation is as follows: Define a defense strategy *d* to be a semi-definite matrix, where *d(i,j)* is the number of interceptors assigned at target *i* against the j^{th} incoming attacker (if there are any). An attack strategy is a vector ***a*** where *a(i)* is the number of attackers against target *i*. The total value of target destroyed, *V(d,a)*, for a given defense and attack strategy is;

$$V(d,a) = \sum_i v(i)\left[1 - \prod_{j=1}^{a(i)}\left(1 - q^{d(i,j)}\right)\right]$$

where, *v(i)* is the value of target *i* and *q* is the probability of the inceptor's failure to destroy attacker. Then, the problem Prim-Read defense addresses has the following form:

$$
\begin{aligned}
& Min_{d(i,j)} \sum_i \sum_{j=1}^{\infty} d(i,j) \\
& s.t. \\
& V(d,a) \le s \sum_i a(i)
\end{aligned}
$$

where *s* is the upper bound on the maximum damage per attacker. In order to solve this multi-target problem, Burr et al. (1985) solves all the single target problems of the type,

$$
\begin{aligned}
& Min \sum_{j=1}^{\infty} d(j) \\
& s.t. \\
& V(d,k) \le sk \quad (k = 1,2,...)
\end{aligned}
$$

where *k* is the number of attacking missiles and $V(d,k) = \left(1 - \prod_{j=1}^{k} \left(1 - q^{d(j)}\right)\right)$.

The greedy algorithm that solves the single target problem is as follows.

Let $\bar{d}(1) = \lceil \ln s / \ln q \rceil$ and

$$
\begin{aligned}
& \bar{d}(k) = \left\lceil \ln\left[1 - \left(\left(1 - sk\right) \middle/ \prod_{i=1}^{k-1} \left(1 - q^{\bar{d}(i)}\right)\right)\right] \middle/ \ln q \right\rceil \qquad \text{for } 2 \le k \le 1/s \\
& \bar{d}(k) = 0 \qquad \text{for } k \ge 1/s
\end{aligned}
$$

Prim-Read defense implicitly assumes that the defense can determine the attacker's firing schedule before making interceptor allocations. The defense may not be able to ascertain such information in many real life settings.

Proportional Defense

The proportional defense introduced by Shumate and Howard (1974) is based on the idea of preventing "cheap" kills of the attacker. The objective of the defense is to balance its resources in order to make sure that the offense will pay a price (which is proportional to the value of the target) greater than or equal to a fixed value for every unit of damage inflicted. Proportional defense assumes that the offense knows the allocation of interceptors and firing doctrine at each target. That is, the defense first decides upon its allocation of interceptors with no information on the planned size of attack at any target or the total attack size, and then the offense allocates its missiles to targets. Defensive allocation can leave some of the targets undefended depending on their values. The solution procedure includes the classification of the targets into three groups (small, medium, and large value targets) according to their values and implementation of a dynamic programming scheme for optimum (minimizing) interceptor allocation.

Known Attack Size

Soland (1987a) considers the defense of a single target against a simultaneous attack (known attack size) and assumes that the defense has interceptor missiles with a fixed number of engagements and shoot-look-shoot capability between engagements. The single-shot kill probability of interceptors may change between the engagements. Thus, this model captures the realm of change in kill probability depending on the range at which interception occurs. The objective of the defense is to minimize the expected fraction of target destroyed. Soland (1987a) determines the optimal allocation of interceptors using stochastic dynamic programming. The objective of minimizing the required number of interceptors while keeping the damage below a linear function of the attack size, as in the Prim-Read defense case, can be employed by minor modifications to the dynamic programming scheme.

A short description of the model proposed by Soland (1987a) is as follows. Let k be the number of engagements left after a previous engagement, p_k be the single-shot kill probability of one interceptor ($q_k=1-p_k$), d be the number of interceptors left, and a be the number of attackers left. We define the expected fraction of target destroyed when k engagements, d interceptors, and a attackers left, $S(a,d,k)$, using the following formula:

$$S(a,d,k) = \min_{i=0,\dots,d} \left\{ \sum_{j=0}^{a} P(j \mid a,i,d,k)\; S(j,d-i,k-1) \right\}$$

where $P(j|a,i,d,k)$ is the probability that j attackers survive the k^{th} engagement when there are a attackers, and i of the d remaining interceptors are used at that engagement.

Soland (1987a) calculates $P(j|a,i,d,k)$ by assuming the survival of each attacker at each engagement as a Bernoulli trial, and by using the quasi-uniform defense theorem through the following recursive formula.

Quasi-Uniform Defense Theorem: Let d be the number of interceptor missiles, each of which will kill an attacker with probability p, a be the number of attacking missiles. The defender's goal is to maximize the probability that the target survives, then he should distribute the interceptor missiles as evenly as possible. Soland (1987a) shows that such a quasi-uniform defense is optimal.

Let J_a be a random variable whose probability distribution is $P(j|a,i,d,k)$. Then J_a is the sum of a independent random variables. Hence, we can calculate the probability distributions of the J_s, $s=1,\dots,a$ successively from the recursion in the following way:

$$J_s = \sum_{l=1}^{s} X_l \quad \text{where } P(X_l = 1) = r_l \text{ and } P(X_l = 0) = 1 - r_l \text{ , } s = 1,\dots,a$$

$$r_l = \begin{cases} q_k^{\lfloor i/a \rfloor} & \textit{for } l = 1,\dots,a + a\lfloor i/a \rfloor - i \\ q_k^{\lceil i/a \rceil} & \textit{for } l = a + a\lfloor i/a \rfloor - i + 1,\dots,a \end{cases}$$

$$P(J_s = j) = \begin{cases} \prod_{l=1}^{s} (1 - r_l) & \textit{for } j = 0 \text{ and } s = 1,\dots,a \\ r_1 & \textit{for } j = 1 \text{ and } s = 1 \\ (1 - r_s)P(J_{s-1} = j) + r_s P(J_{s-1} = j - 1) & \textit{for } j = 1,\dots,s \text{ and } s = 2,\dots,a \end{cases}$$

This solution procedure is untractable as the size of the MAP gets larger. Bertsekas et al. (2000) propose a solution method for large MAPs with known attack size using neuro-dynamic programming (NDP). NDP is a class of reinforcement learning methods that deals with the complexity problem of the dynamic programming by using neural network based approximations of the optimal cost-to-go function (Bertsekas et al., 2000). The formulation of the MAP is cast to be a type of stochastic shortest path problem (Bertsekas and Tsitsiklis, 1991), which employs the different probability of kill values for each level of interceptor and attacker allocation and for each target type without explicitly calculating the values. The attacker's choice of attack wave against selected targets is selected probabilistically. Then, the defense optimizes the goal of maximizing the expected value of targets that are surviving at the end of the battle. Three different NDP algorithms, namely approximate policy iteration with neural network architecture, approximate policy iteration with linear architecture, and optimistic policy iteration with neural network architecture are investigated and the results are reported. The authors conclude that the NDP approach is promising for very large size problems.

Due to the computational complexity of the MAP, a number of heuristic approaches have been suggested. All of those models to our knowledge use a completely known attack scenario. Wacholder (1989) proposes a solution for a one sided many-on-many MAP using artificial neural networks combined with the Lagrange differential multipliers method. Jaiswal (1997) investigates a similar problem using simulated annealing, genetic algorithms and artificial neural networks in a layered defense context.

3.1.2 Game Models

MAP has been frequently treated using game theory. We refer to two-person-zero-sum (TPZS) games in the context of this review. TPZS games contain exactly two sides whose interests are in complete opposition. In this exposition, we include the min-max theorem, which is the keystone of the theory of finite TPZS games.

Min-Max Theorem (Karlin, 1959): If there exist strategies $x_0 \in X$, $y_0 \in Y$ and a real number v such that

$$f(x_0, y) \geq v \quad \textit{for all } y \in Y \quad \text{and} \quad f(x, y_0) \leq v \quad \textit{for all } x \in X \text{, then}$$

$$\bar{v} = \min_{y} \max_{x} f(x, y) = v = \max_{x} \min_{y} f(x, y) = \underline{v}$$, and conversely.

A strategy x is a vector that represents a point in polytope X. For example, the elements of the vector may represent the allocation of interceptors to attackers. $\bar{v}$ and $\underline{v}$ are the value of the minimax and maximin strategies of defense and offense respectively. All strategies x_0 and y_0 such that $f(x_0, y) \geq v$ for all y and $f(x, y_0) \leq v$ for all x is referred to as optimal strategies for defense and offense, and v is the value of the game.

An interested reader may refer to Jones (1980) for a comprehensive treatment of game theory.

Min-Max Defense

The min-max problem may be defined as

$$\begin{aligned}
&\underset{x}{Min}\ \underset{y}{Max} \sum_{i} f_i(x_i, y_i) \\
&s.t. \\
&x_i \in X \\
&y_i \in Y \\
&x_i \geq 0,\ y_i \geq 0 \quad \forall i
\end{aligned}$$

Randolph and Swinson (1969) describe the MAP as a discrete max-min problem. Their work uses dynamic programming to obtain the upper and the lower bounds on the value of the game and proposes a procedure for determining an optimal stopping rule that indicates the solution found is sufficiently close to the optimal value.

Soland (1973) uses 0-1 implicit enumeration scheme and a branch-and-bound procedure for solving a similar problem. The objective is the minimization of the damage done by an optimal offensive attack with a known number of attacking missiles of one type. Defensive allocation policy is further constrained by a certain budget level. It is assumed that no damage can be inflicted on a target unless its defense is first exhausted. O'Meara (1988) proposes a number of solutions for the MAP under different combinations of hit probability (perfect and imperfect weapons), with or without the defensive capability of identifying the eventual target of each attacker (which is called the impact point prediction, IPP), and different engagement rules (one-on-one or many interceptor-on-one attacker). Different settings are investigated in a min-max problem context. The defense seeks an allocation that maximizes the total expected survival value while the attacker seeks an allocation that minimizes the total expected survival value against the best defense. Both defender and attacker know the size of the opponent, and both have only one type of interceptors and attacking missiles. O'Meara (1988) presents the solution algorithms for each scenario under consideration. O'Meara and Soland (1990) investigate a very similar problem under full and partial defensive coordination conditions. Defensive setting contains full coordination with IPP, or full coordination without IPP, or partial coordination without IPP. O'Meara and Soland (1991) and O'Meara and Soland (1992) give detailed formulation of min-max MAP without IPP and with IPP respectively.

Preferential Strategies

Preferential strategies imply that the number of interceptors defending some targets can be higher than the number defending other targets. The problem addressed in preferential strategies context is the protection of a number of identically valued targets defended by identical interceptors against identical attacking weapons. Both attacker and defender know the total number of targets, interceptors and attacking weapons. Matheson (1967) uses the idea of mixed strategies of preferential defenses with imperfect weapons. He represents the scenario as a TPZS game by allowing the attacker and the defender to choose allocations independently. The objective function to be maximized by the defense and minimized by the offense is the expected fraction of surviving targets, which is a function of opposing strategies. The opponents preallocate their weapons (i.e. they allocate their weapons to the attack and defense of specific targets before the engagement; however, they are not informed about the specific allocation.).

Bracken, Brooks and Falk (1987) and Bracken, Falk and Tai (1987) introduce the issue of robustness for preallocated preferential defense under the assumption of perfect weapons and imperfect weapons respectively. The robust defense does not require the defender to assume the knowledge of the total attack size. A robust strategy is good for a predetermined attack range (any attack size falls within this range), but is not the best for any of a particular attack size within range.

Haaland and Wigner (1977) analyze the robust min-max allocation of the resources for the perfect interceptors and attackers case. They give an optimal allocation algorithm for defense independent of the attack size, provided that the total number of interceptors and targets are reasonably large.

Bracken and Brook (1985) consider the optimal allocation strategies for attack and defense of intercontinental ballistic missiles deceptively based in a number of identical sites in different areas. They consider the cases of uniform allocation of attackers, and either uniform or preferential allocation of interceptors within selected sites.

Lansdown (1989) implements the preferential defense strategy including decoys and a two-layered defense. Layered defense includes a probabilistic model of terminal defense (last defense layer) and a TPZS game model of second layer. Uniform defense and tapered defense (either a modified Prim-Read or user specified tapered defense) doctrines are also investigated.

Minimum Cost Defense

A number of researchers worked on the minimum cost defense as reported by Soland (1987b). Soland's paper is the only one in the open literature that we know of in this classification. The objective of the defense is to select the number of interceptors to minimize the cost while bounding the total expected value of target destroyed by a specified function. This approach is similar to the Prim-Read defense doctrine in the sense that defense keeps the damage, inflicted by an unknown number of attacking missiles, reasonable.

Soland (1987b) models the MAP as a three sequential move of a game. The defender first selects the minimum cost defense including the area interceptors and the point interceptors. After the defensive move, the attacker selects an allocation policy that maximizes the total expected value of targets destroyed against the known minimum cost defense. Finally, the defender allocates its interceptors against the known policy (simultaneous attack) of the attacker so as to minimize the total expected value of target destroyed. He assumes a superadditive damage function and an isotone increasing cost function. It is shown that the defender's first-move problem can be decomposed into smaller problems under certain conditions. This result is similar to that of Burr et al. (1985) on Prim-Read defense. Soland also shows that under certain conditions, low valued targets do not need any protection.

3.1.3 Special Feature Models

In this category, we investigate the MAP models that do not fall into the two preceding categories.

Effectiveness Evaluation

Nguyen et al. (1997) introduce the idea of using generating functions as a simple, consistent and easily applied tool for evaluating the effectiveness of an air defense system. This approach does not provide any interceptor allocation plan. The scenario considered is similar to that of Soland (1987a) discussed under "known attack size" sub-category. However, this method can accommodate both simultaneous and sequential attack scenarios by carefully selecting the parameters. The model described in Nguyen et al. (1997) is based on four parameters, such as the total number of available interceptors, the total number of attackers, the maximum number of engagement opportunities against each threat, and a constant probability of successful interception. Nguyen and Reding (1997) extend the model to include the case of incomplete damage assessment. Nguyen and Reding (1998) present a multi-layer air defense model using the same idea. Their model handles both perfect and imperfect kill assessment cases. Nguyen et al. (1999) developed a ballistic missile defense (BMD) evaluation model using generating functions. They use the generic defense decision making cycle (observe-orient-decide-act) as the underlying idea of their BMD model. This cycle is represented by a generating function sequence. The model evaluates important measures of effectiveness for BMD for both simultaneous and sequential attack scenarios.

Analysis of a Layered Defense

Nunn et al. (1982) propose a Markov chain formulation for analyzing a layered defense model. They assume that the layers are independent and produce attrition according to a Binomial distribution. Since each layer has a distinct probability of successful interception, the discrete Markov chain is non-homogeneous. However, a closed form solution is presented for analyzing the number of leakers at each layer. Orlin (1987) solves the layered defense problem from the attacker's perspective. His objective is to maximize the net value of the attack, which is the difference between the value of the damage inflicted on defense and the cost of the offensive weapons used. Comparisons between exhaustion and attrition algorithms are made, and the results of a hybrid algorithm are reported. Al-Mutairi et al. (1997) analyze the layered defense using Bayesian inference. They present the predictive distribution of the number of attackers surviving under two different priori assumptions, which are the independence of penetration probabilities and dependence of penetration probabilities under Dirichlet law.

Simulation Models

Simulation is one of OR tools frequently used to evaluate the effectiveness of the air defense systems. Hoyt (1985) reports a simple Monte-Carlo simulation model of BMD system. Hoyt argues that a simple model can identify important characteristics, salient features and the weaknesses of the BMD system in question. This model evaluates the probability of success of the BMD system with given interceptor inventory and a time frame.

Beare (1987) describes the use of linear programming to reduce the number of air defense weapon mixes that would be investigated in detail by a Monte-Carlo simulation model. The objective of the deterministic model is to choose the most effective defense in terms of robustness and maximizing attrition of the attacker.

Martin et al. (1995) communicates the use of simulation for evaluating and analyzing the performance of a ship air defense system, called SEAROADS. SEAROADS is a high-resolution Monte-Carlo simulation model that incorporates the important aspects of a modern air defense system, such as threat evaluation and weapon allocation, chaff, decoys, jamming etc. Bloemen and Witberg (2000) report that SEAROAD model is extended to include the evaluation of air defense effectiveness of a naval task group.

Smith et al. (1995) develop a low resolution simulation model, called JASMINE for estimating the effectiveness of maritime air defense systems.

JASMINE uses Nguyen et al. (1997)'s effectiveness evaluation model for calculating the effectiveness of the defense under a variety of scenarios. Layered defense and effectiveness of the soft-kill weapons can be investigated by JASMINE.

In the early 1990s, a medium resolution simulation model has been developed in the Directorate of Operational Research (see Ormrod and Carleton, 1991). The model, called the ship area air defense simulation (SAADS) provides a generalized overview of the defense capability of a group of ships defended by guns and missiles, and under attack by anti ship missiles. SAADS allows the evaluation of the air defense scenarios either deterministically (using binary trees) or stochastically (using Monte Carlo simulations).

The ship air defense model (SADM) developed by British Aerospace Australia Ltd. is a high resolution simulation model designed to evaluate the defense of a single ship against one or more antiship missiles. It simulates both soft-kill and hard-kill systems and the interactions between them. SADM has another version, which models the defense of multiple ships in a task group (see Chapman, 1999).

Other Models

A number of models that are closely related to the MAP are discussed shortly in this section. Nguyen (1996) studies the quantification of benefit from resource allocation for a naval task group having perfect coordination between its assets. The interceptors are assumed to cover all the other ships of the task group and are capable of defending the ships within range. A quasi-uniform defense algorithm is used to allocate resources.

Griffiths et al. (1991) studies a highly restricted scenario of a naval MAP. They consider an attack of a group of identical aircraft in line-ahead formation against a group of ships with identical anti-aircraft weapons. They present a difference equation for bivariate probability distribution of the attrition of both sides. They report that their model has been used to approximate more complex scenarios as a screening process for detailed simulations.

Almeida et al. (1995) present the impact of information on the effectiveness of air defense in a time-constrained context. They illustrate the expected payoff from a reduction in uncertainty by the utilization of the information gathered from the sensors and the C^4I (command, control, communications, computers and intelligence) capabilities in a scenario with a single defensive unit against a massed missile attack.

Sherali et al. (1995) present algorithms to schedule a set of illumination radars to engage incoming targets using surface-to-air missiles in a naval task group (TG). The problem is handled as a production shop floor scheduling problem of minimizing the total weighted flow time, subject to time-window job availability and machine downtime side constraints. It assumes a perfect coordination, such as CEC within the task group.

Kohlberg and Greer (1996) discuss the uncertainty issue in MAP inherent to the inventory and the allocation plan of the attacker. The objective is to find the minimum cost or maximum effective mix of interceptor inventory using statistical inference. They solve an unconstrained optimization problem using the method of Lagrange multipliers.

Friedman (1977) investigates the optimal strategies of survival for one-on-many engagements. He suggests a procedure for the optimal defensive order of engagement in the presence of varying fire effectiveness and vulnerability of offensive units. Manor and Kress (1997) consider the incomplete damage information case within similar settings. They show that a certain type of a shooting strategy, called "greedy strategy" is optimal when the objective is to maximize the expected number of killed targets.

3.2 DISCUSSION ON WEAPON-TARGET ALLOCATION MODELS

The related literature discussed in preceding section is summarized in Figure 3.1, where the incidence of each work with the set of features discussed above is shown. Our version of MAP, which will be discussed in detail in the next chapter, is included in the last line.

We conclude that there is a gap between the theory on air defense and the practice. Despite the fact that weapon technology development pace and air threat growth are fast, analytical research has not been evolved accordingly. While simulation techniques are well-developed, their ability to quickly evaluate a wide range of potential solutions is limited.

The theory developed so far can be applied to a wide range of MAPs. However, the assumptions and solutions are still required to be customized for the specific scenario under consideration. For example, a damage function for a TBM defense problem may not be that suitable for a ship MAP. Damage inflicted by the attacker is generally not linear. It may be plausible to use a linear damage function for TBM defense, but the similar linear function cannot be used for the damage of a ship.

Information technology driven integrated command and control systems for air defense bring up new problems for optimal use of resources under more complex environments as well as new capabilities. CEC is an example of this case. A C^2 system with CEC capability is expected to allocate the distributed defensive systems of a naval task group in a concerted way to optimize the effectiveness. Focused analytical research on this area is required to answer the practical problems.

The problem of solving formulations for MAP does not arise only once for a detached theoretical study or for off-line evaluation of engagement strategies. Indeed, supersonic and maneuvering anti-ship missiles and littoral operations bring about the need for development of on-line and near-real-time solutions for the allocation problem of the navy. Not only an optimal solution algorithm but a fast one is required to answer the question in a situation where seconds are vital to the "survivability" of the ship and crew.

Integration and evaluation of the soft-kill systems together with the hard-kill weapons is another area requiring focused research. Hard-kill encompasses the kinematic kill that destroys the threat physically either by collusion or by explosion. Soft-kill is aimed at the control and guidance subsystems of the threat and diverts it away from the ship through confusion, distraction, or seduction (The and Liem, 1992).

	WORK \ FEATURES	Simultaneous Attack	Sequential Attack	C2 Capability	Point Defense	Area Defense	Multiple Types of Weapons	One Layer of Defense	Multiple Layers of Defense	Radar Capability	Missile Allocation Policy
Defense Allocation Models	Burr et.al. (1985)		+		+		+	+			
	Shumate, Howard (1974)		+		+			+		+	
	Soland (1987a)	+			+				+	+	+
	Bertsekas et.al. (2000)	+			+	+	+		+		
	Watcholder (1989)	+		+	+		+	+		+	
	Jaiswal (1997)	+			+		+		+	+	
Game Models	Randolp, Swinson (1969)	+			+			+			
	Soland (1973)	+			+	+	+	+			
	O'Meara, Soland (1990)	+		+		+				+	
	O'Meara, Soland (1991)	+				+		+			
	O'Meara, Soland (1992)	+				+		+		+	
	Matheson (1967)	+			+			+			
	Haaland, Wigner (1977)	+			+			+			
	Bracken, Brooks (1985)	+			+			+			
	Bracken et.al. (1987)	+	+		+			+			+
	Lansdown (1989)	+	+		+		+		+	+	+
	Soland (1987b)		+		+	+		+			
Special Feature Models	Nguyen et.al.(1997)	+			+			+		+	+
	Nguyen, Reding (1997)	+		+	+			+		+	+
	Nguyen, Reding (1998)	+		+	+				+	+	+
	Nguyen et.al. (1999)	+		+	+			+		+	+
	Nunn et.al. (1982)	+			+	+			+	+	+
	Orlin (1987)	+			+		+		+	+	+
	Al-Mutairi et.al. (1997)	+			+		o		+	+	+
	Hoty (1985)	+			+				+		
	Beare (1987)	+			+		+	+			
	Nguyen (1996)	+		+	+						
	Griffiths et.al. (1991)	+			+			+			
	Sherali et.al. (1995)	+			+		o	+		+	
	Our MAP	+		+	+	+	+	+	+	+	+

+ Model has the correponding feature
o Not Applicable

Figure 3.1. A Summary of Surveyed Papers on WTA Problem.

Realistic modeling, problem specific environmental considerations, soft-kill and hard-kill integration, active and semi-active missiles, long range interception, overlapping coverage, different interceptor and attacker types, different probabilities of kill for each attacker-interceptor combination are a number of points required to be addressed.

3.3 LITERATURE REVIEW ON SECTOR ALLOCATION PROBLEM

There is not much research and literature on SAP that is known to us. None of the models produces a reasonable solution to our SAP.

Magonet-Neray (1983) presents an optimization model to maximize the survival probability of a carrier operating in a TG environment given anti-air warfare (AAW) and anti-submarine warfare (ASW) resources. This model is a static, probabilistic, 2-dimensional representation. The solution to the problem is the optimum location of the AAW and ASW ships with respect to the carrier; those locations that maximize the probability of survival of the carrier from the air and submarine threat.

Helmbold (1982) discusses mathematical programming formulations for the problem of optimizing the stationing and vectoring of aircraft employed in the air defense of a naval TG. He assumes that all aircraft and missiles move in straight lines and at constant speed. It is assumed that the threat moves directly toward the center of the TG. All actions are treated as taking place in plane.

Kelley (1991) addresses coordination between ships of a TG in AAW. Two coordination schemes are presented. One is based on earliest intercept time and the second scheme introduces a load sharing feature wherein current magazine inventories are considered.

Chouinard and Baker (1994) outline the basic requirements for an objective and quantifiable model of determining the operational effectiveness of a TG. They assume that the effectiveness of a naval TG can be divided into the achievement of objectives in each warfare area such as AAW and ASW.

Drezner (1988) investigates the problem of covering a given area by moving satellites in space. The locations of facilities that are moving in space are considered in this research. This problem may have some resemblance to SAP because of the moving facilities. However, we have moving demand points in SAP in addition to the moving facilities, i.e. both demand points and facilities need to be located. Moreover, we are maximizing the coverage of the demand points instead of maximizing the overall area coverage. Thus, SAP and Drezner's problem have substantial differences. SAP may be viewed as stationary location problem since both facilities and the demand points are moving at the same speed on the average. This exposition is included here in order to show the difference of SAP from the similar problems addressed in literature. Wolfe and Srensen (2000) address a problem similar to that of Drezner's in a scheduling context.

One of the SAP models resembles Quadratic Assignment Problem (QAP) in terms of the constraints. See Burkard (1990) for a detailed discussion on QAP.

CHAPTER IV

MISSILE ALLOCATION MODELS

The literature review on generic MAP shows that there are not many models that can be used to solve the TG air defense problem. The existing analysis methods mainly consist of computer models that simulate the defense against ASM attack. SEAROAD (Martin et al.,1995; Bloemen and Witberg, 2000), JASMINE (Smith et al., 1995) and SAADS (Ormrod and Carleton, 1991) are the examples of such models. The only analytical model known to us is Nguyen (1996). In his work, Nguyen studies the quantification of benefit from resource allocation for a naval TG having perfect coordination among its assets. The interceptors are assumed to cover all the other ships of the TG and are capable of defending the ships within range. Other geometric and defense system limitations are not considered.

Firstly, we state the basic assumptions that are needed to develop the missile allocation models.

1. The TG sees all of the air threats to intercept simultaneously. Thus, we investigate the case where the attack size is known. This is a reasonable assumption in a naval air defense scenario context providing that the TG has modern search and detection sensors and systems. However, there may be undetected or newly launched ASMs and these missiles may be

detected after the initial attack wave in a sequential order. Here we restrict our scope to the case of simultaneous attack size.

2. The ships in the TG are capable of coordinating the allocation of the air defense, i.e. C^2 capability is assumed.

3. The TG has both point and area air defense missile systems. A point defense system may intercept the attackers that are attacking its own ship. An area defense system may intercept attackers within the area of its effective range.

4. Both attacking ASMs and SAM systems onboard ships may be of different types.

5. Different SAM systems may have different effective ranges, i.e. layered defense is assumed. Layers may overlap.

6. Defense systems may predict the eventual target of the attacking ASMs, i.e. impact point prediction capability is assumed.

7. Missile allocation policy is SLS.

8. The incoming ASMs are assumed to be classified in terms of their speed (e.g. supersonic or subsonic) and attack profile (e.g. sea-skimmer, high diver). Thus, the single shot kill probability of each SAM against each ASM is known.

9. The relative positions of the ships within TG do not change as the air raid continues. The ships are thought to be stationary. This is a reasonable

assumption since the speed of the ships is very low compared to the speed of the ASMs.

10. There are no limitations on the number of SAMs in flight that are launched from the same SAM system. i.e. a SAM system may launch one missile right after the other.

Note that the first seven assumptions place our version of MAP in the classification scheme given in Section 3.1. We consider each SAM system distinct even if they are of the same type as long as they are onboard of different ships. This enables us to capture the geometric differences that need to be studied to develop the best stationing tactics for the TG.

In this chapter, we present three different missile allocation models. Each model has some features and drawbacks. First, we develop a missile allocation model with no time dimension. The second model solves MAP with a discretized time dimension. The last one uses a continuous time dimension.

Although solution procedures are also discussed and illustrated for these mathematical programming models that will be introduced in the following sections, they will not explicitly be used to solve MAP. Mathematical programming models do guarantee to find an optimal solution (without loss of generality), but they usually take more than a few seconds in which we have to find solution for MAP. Since MAP requires real time solution, we develop very fast solution approaches for MAP in the next chapter.

4.1 MAP1 - MISSILE ALLOCATION MODEL WITH NO TIME DIMENSION

In this section, we formulate a MAP with an implicit treatment of time and discuss the solution procedures for this problem. Although the formulation does not have an explicit time dimension, time is embedded inside the parameter of maximum possible number of engagements for each ASM.

4.1.1 Formulation of the Problem

Suppose that there are n incoming ASMs, indexed $i \in N = \{1,...,n\}$ and there are m SAM systems onboard of the warships composing the naval TG, indexed $j \in M = \{1,...,m\}$. Let V denote the set of valid combinations of the ASM and the SAM systems, i.e. $(i, j) \in V$ if SAM system j can engage ASM i. Each ASM i has a specified engageability duration Δ_{ij}, which depends on the location and the effective range of the SAM system j, and a successful engagement can be achieved only during this interval. As explained in Section 3.1, time taken by each feasible engagement is determined as the sum of a constant setup time and a variable flight time to the engagement. Thus, each engagement process takes a specified time according to the ASM and SAM combination $(i, j) \in V$ and the starting time of the engagement. The SLS tactic requires us to ensure that the SAMs allocated against each ASM are scheduled in non-overlapping intervals. Thus, we define u_{ij} as the maximum number of missiles that can be launched from SAM system j against ASM i, $(i, j) \in \mathrm{V}$ using a SLS tactic.

We need the following additional notation and variables to formulate the TG air defense problem:

x_{ij} : the number of the missiles (rounds) of SAM system j to be launched against ASM i, $(i, j) \in V$.

p_{ij} : the single shot kill probability of SAM system j against ASM i, $0 < p_{ij} < 1, \quad (i, j) \in V$.

d_j : the number of available rounds on SAM system j.

s_i : the maximum number of engagements that can be done against ASM i using a SLS tactic.

h_i : the minimum desired probability of shooting-down the ASM i, $0 < h_i < 1, \quad i \in N$.

The TG air defense problem MAP1 can be formulated as the following nonlinear integer programming model. Note that the objective function (4.1) is constant. Thus, this model just checks the feasibility of the constraints. If the model gives a feasible solution, it means that the desired probabilities set forth for each incoming ASM can be met within the limits of the defensive potential.

$$Min \quad 0 \tag{4.1}$$

subject to

$$\sum_{\{i \in N | (i,j) \in V\}} x_{ij} \leq d_j \quad \text{for all } j \in M \tag{4.2}$$

$$1 - \prod_{\{j \in M | (i,j) \in V\}} \left(1 - p_{ij}\right)^{x_{ij}} \geq h_i \quad \text{for all } i \in N \tag{4.3}$$

$$\sum_{\{j \in M | (i,j) \in V\}} x_{ij} \leq s_i \quad \text{for all } i \in N \tag{4.4}$$

$$0 \leq x_{ij} \leq u_{ij} \quad \text{for all } (i,j) \in V \quad \text{and} \quad x_{ij} \quad \text{is integer} \tag{4.5}$$

Constraint set (4.2) reflects the restriction on the number of rounds available for each SAM system. Constraints of type (4.3) require the allocation of enough SAMs that meet the desired probability of shooting down each ASM. Constraints of type (4.4) limit the total number of engagements that can be done against each ASM. Constraint set (4.5) imposes integer restriction and lower and upper limits on the decision variables. The upper limit is determined by the maximum number of engagements that can be done during the engageability duration of each valid ASM and SAM combination using a SLS tactic.

Constraint set (4.4) actually employs a loose upper bound on the total number of engagements that can be done against each ASM, when more than one SAM system can engage the ASM along its flight path. If more than one SAM system can engage an incoming ASM, calculating the maximum number of engagements against an ASM may be cumbersome. Figure 4.1 depicts an example of such a situation. Both SAM1 and SAM2 can engage ASM1 and their engageability durations are overlapping. Clearly $s_i \le \sum_{\{j \in M | (i,j) \in V\}} u_{ij}$. However this upper bound will not be tight when the overlap in engageability durations is large. Developing a tight bound for s_i is required in order to be able to have a feasible SLS allocation. Let k_i show the number of different SAMs that can engage ASM i, $k_i = \sum_{\{j \in M | (i,j) \in V\}} 1$. Then there are 2^{k_i} different combinations of SAM systems that can be used against ASM i. For a thorough control of the upper limit on possible engagements in a SLS tactic, we need to determine the upper limit for each combination since the speeds of the SAMs vary. This would require $2^{k_i} - (k_i + 1)$ additional constraints. (Note that we impose the upper bounds of single combinations through u_{ij}.) Instead of $2^{k_i} - (k_i + 1)$ constraints we propose an approximation by means of only one constraint. The engageable portion of the flight path of an ASM, l, can be divided into parts such that the number of SAMs that can engage the ASM is different compared to the neighboring parts. For example, in Figure 4.1 the flight path of ASM1 is divided into three parts l_1, l_2, l_3. SAM2 is the only one that can engage ASM1 in part l_1. Both SAM1 and SAM2 can engage in part l_2. In the last part, l_3, only SAM1 can be used against ASM1. In this way, the speed of the fastest SAM for each part of the flight path can be used to calculate s_i for each ASM i. If there are two SAM systems that can engage an ASM, then this approximation is exact. If more than two SAMs can engage one or more ASMs, then the allocation requires a feasibility check after solving the problem.

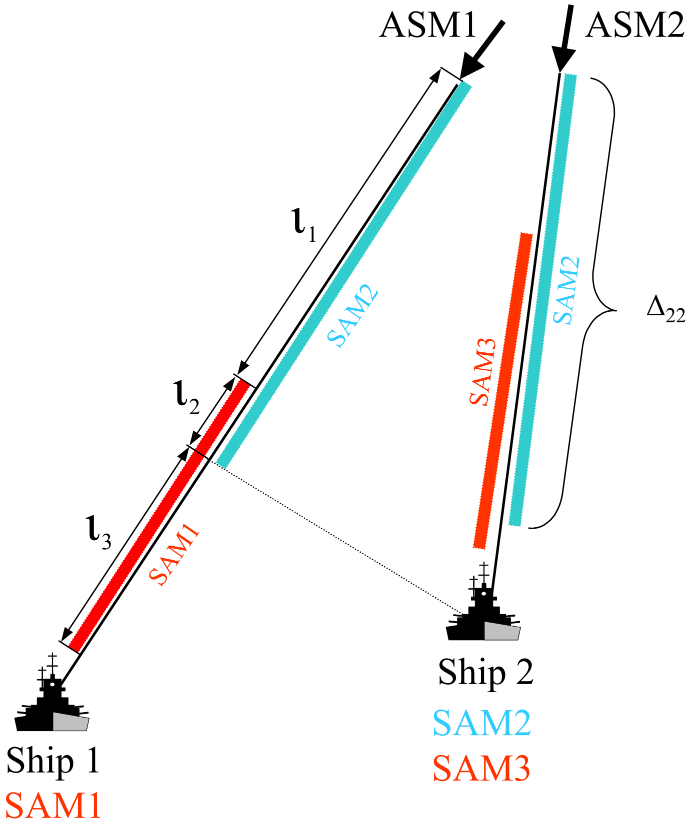

Figure 4.1. An Illustration of the ASM Engageability Durations for Different SAM Systems.

The non-linearity in problem MAP1 can be transformed into linearity by using logarithms as in Kwon et al. (1999). Since logarithm preserves normative order ($0 < a \leq b$ if and only if $\ln(a) \leq \ln(b)$ where $a, b \in \Re$), taking the logarithms of constraint set (4.3) does not affect the optimal solution. Then set (4.3) becomes $\sum_{\{j \in M | (i,j) \in V\}} \ln(1 - p_{ij}) x_{ij} \leq \ln(1 - h_i)$ for all $i \in N$. We may further continue our linearization process by scaling the constraint with a large number, say β, and then rounding down. This approximation is reasonable from a practical point of view since the values of the coefficients in the inequalities come from probabilistic estimates. This gives an approximation of the feasible region with integer coefficients and transforms the problem from a non-linear integer programming model into a linear integer programming model.

The resulting linear integer program is still not much of use in practical sense. It only gives us whether the desired probability levels are achievable or not. However, we can guarantee reaching a feasible solution by making a minor modification to the model. If we introduce an artificial SAM system that can engage every ASM and has a large inventory, then the model becomes a flexible one that reaches a feasible solution whatever the desired probability levels, h_i, are. Let j^* denote the artificial SAM. We revise the set definitions, M and V including the artificial SAM accordingly. If we penalize the use of the artificial SAM in the objective function, and set the desired probability levels, h_i for all $i \in N$ very high (say $h_i = 0.99$ for all $i \in N$) then the model will minimize the use of the artificial SAM and maximize the use of real SAMs to achieve the desired levels for the probability of shooting down each ASM. The under-achievement will be met by the artificial SAM.

The resulting elastic linear integer programming model MAP1.1 is:

$$Min \quad \sum_{i \in N} x_{ij^*} \tag{4.6}$$

subject to

$$\sum_{\{j \in M | (i,j) \in V\}} a_{ij} x_{ij} \geq b_i \text{ for all } i \in N, \tag{4.7}$$

(4.2), (4.4), and (4.5)

where $a_{ij} = \lfloor -\beta \ln(1 - p_{ij}) \rfloor$ and $b_i = \lfloor -\beta \ln(1 - h_i) \rfloor$.

Musman and Lehner (2001) state that an ideal weapon allocation solution is the one that maximizes the probability of shooting down each threat. Model MAP1.1 does not guarantee this solution. It minimizes the total number of artificial SAM engagements used to achieve the desired probability levels. In a sense, it will minimize the total deviation from the desired probability levels. However, MAP1.1 is not very sensitive to the individual deviations for each ASM. Thus it is possible to have a larger deviation from the desired probability level of one ASM and very small or no deviations for the rest. This may lead us to a second formulation. We can easily convert MAP1.1 to a model that minimizes the maximum number of artificial SAM engagements used to achieve the desired probability levels. In this new model, MAP1.2, we define a single elastic decision variable, e, instead of the artificial SAM of model MAP1.1. Let us define the sets M and V as in the original model excluding the artificial SAM. Then model MAP1.2 can be written as:

$$Min \quad e \tag{4.8}$$

subject to

$$\sum_{\{j \in M | (i,j) \in V\}} a_{ij} x_{ij} + e \geq b_i \text{ for all } i \in N, \tag{4.9}$$

$$e \geq 0 \tag{4.10}$$

(4.2), (4.4), and (4.5)

where e is the elastic decision variable that shows the maximum deviation from the right hand side values of constraint set (4.9).

MAP1.1 and MAP1.2 both minimize the number of engagements emphasizing the cost in the objective. From a TG perspective, maximizing the probability of no-leaker (i.e. shooting-down all the threats) may also be an important objective. One might ask why we do not consider it as the objective of the models. Note that using this objective turns the model into a non-linear integer programming problem. The objective function becomes a non-separable one and no efficient solution algorithm is readily available. Thus we choose to develop these models as linear integer programming problems, leaving the maximization of probability of no-leaker objective to be discussed later.

Both models can be solved using a standard mathematical programming package for reasonable size problems. The application of the models and comparison of the solutions are presented in the next section.

4.1.2 Solution Procedures

In the next section, we discuss the implementation of models MAP1.1 and MAP1.2 and present the result of a test problem. We develop a Lagrangean Relaxation approach for MAP1.1 in the second section.

Implementation of the Models

Models MAP1.1 and MAP1.2 have been implemented using GAMS (General Algebraic Modeling Language) mathematical programming package and solved using OSL Solver (Brooke et al., 1988).

We show the results of the proposed models MAP1.1 and MAP1.2 on a simple example. The example is depicted in Figure 4.2. Ship 1 has only a self-defense SAM system, and Ship 2 has both self-defense and area defense SAM systems (SAM2 is the area defense system). We assume that we did all the necessary calculations for the given input data.

The SAM allocation plans generated by the models are reported in Table 4.1. An allocation plan shows which SAMs should engage which ASMs and with how many missiles. For example, “4” in the last row and the last column of the Table 4.1 means that SAM3 is to engage ASM2 with 4 missiles.

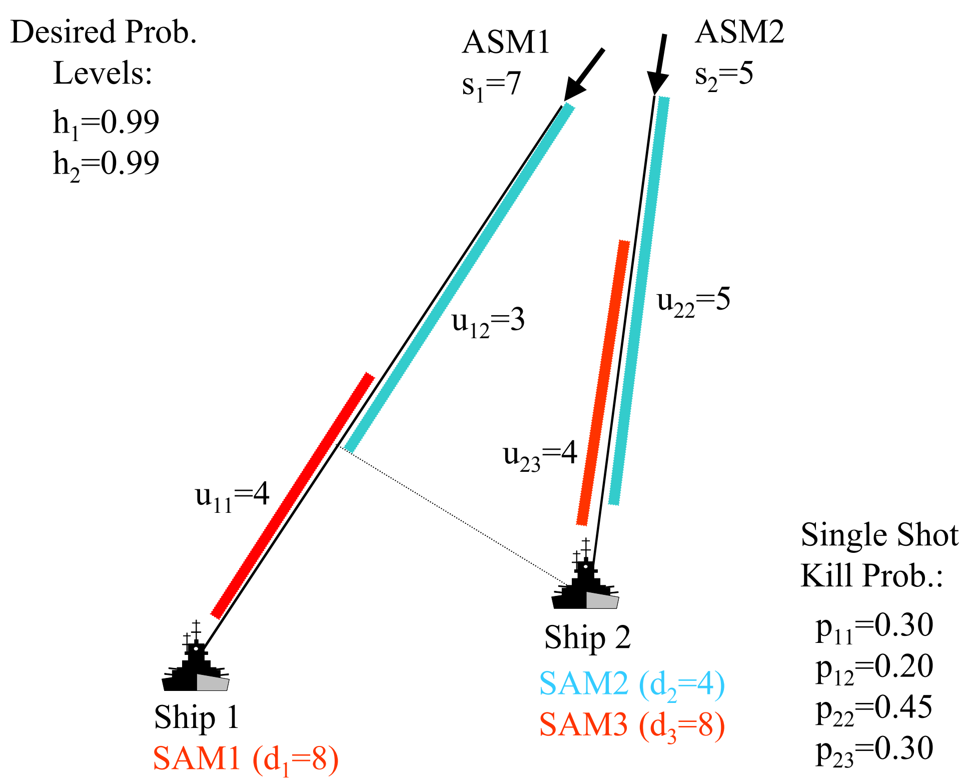

Figure 4.2. Example of an air defense scenario.

Table 4.1. Allocation Plans Generated by the Models.

MODEL		SAM1	SAM2	SAM3
MAP1.1	ASM1	4	1	
	ASM2		3	2
MAP1.2	ASM1	4	3	
	ASM2		1	4

Since the example is small, we can write down a subset of possible allocations that ensure maximum number of engagements against each ASM while keeping the allocation plans feasible. Table 4.2 shows these allocation plans and their respective probability measures. All the other allocation plans for this problem will have fewer engagements and will achieve lower levels for the probability of shooting down the ASMs and the probability of no-leaker for the TG.

Table 4.2. A Subset of the Possible Allocation Plans.

Allocation Plan No	ASM1		ASM2		Prob. of Shooting		Prob. of No-Leaker
	SAM1	SAM2	SAM2	SAM3	ASM1	ASM2	
1	3	4	0	4	0,860	0,760	0,653
2	4	3	1	4	0,877	0,868	0,761
3	4	2	2	3	0,846	0,896	0,759
4	4	1	3	2	0,808	0,918	0,742
5	4	0	4	1	0,760	0,936	0,711

The SAM allocation plan no 4 in Table 4.2 is the same as the plan generated by the model MAP1.1. The model MAP1.2 generates the plan no 2, which has the highest probability of no-leaker for the TG.

Both models generated highly efficient allocation plans in terms of the probability of no-leaker for the TG. However, in this example model MAP1.2 generated a more balanced allocation in the sense that the probability of shooting down each ASM is within 0.01 of each other.

A Lagrangean Relaxation Based Solution Procedure

The final form of model MAP1.1 is given below for convenience.

$$Min \quad \sum_{i \in N} x_{ij^*} \tag{4.6}$$

subject to

$$\sum_{\{i \in N | (i,j) \in V\}} x_{ij} \leq d_j \quad \text{for all } j \in M \tag{4.2}$$

$$\sum_{\{j \in M | (i,j) \in V\}} a_{ij} x_{ij} \geq b_i \text{ for all } i \in N \tag{4.7}$$

$$\sum_{\{j \in M | (i,j) \in V\}} x_{ij} \leq s_i \quad \text{for all } i \in N \tag{4.4}$$

$$0 \leq x_{ij} \leq u_{ij} \quad \text{for all } (i,j) \in V \text{ and } x_{ij} \text{ is integer} \tag{4.5}$$

Problem MAP1.1 can be solved using Lagrangean relaxation. There are several ways to relax the problem. One such way is relaxing constraint set (4.2) to obtain:

$$Min \quad \sum_{i \in N} x_{ij^*} + \sum_j \lambda_j \left[\sum_{\{i \in N | (i,j) \in V\}} x_{ij} - d_j \right] \tag{4.11}$$

subject to

(4.4), (4.5), and (4.7).

Objective function can be rewritten as:

$$Min \quad \sum_{i \in N} x_{ij^*} + \sum_{j \in M} \sum_{\{i \in N | (i,j) \in V\}} \lambda_j x_{ij} - \sum_{j \in M} \lambda_j d_j \tag{4.11'}$$

Then the problem can be decomposed into n sub-problems, each of which should be solved for one ASM.

$$Min \quad x_{ij^*} + \sum_{j \in M} \lambda_j x_{ij} - \sum_{j \in M} \lambda_j d_j \tag{4.11''}$$

subject to

$$\sum_{\{j \in M | (i,j) \in V\}} a_{ij} x_{ij} \geq b_i \tag{4.7'}$$

$$\sum_{\{j \in M | (i,j) \in V\}} x_{ij} \leq s_i \tag{4.4'}$$

$$0 \leq x_{ij} \leq u_{ij} \quad \text{for all } j \text{ where } (i,j) \in V \text{ and } x_{ij} \text{ is integer} \tag{4.5'}$$

Relaxing constraint set (4.2) left us with several smaller and easier problems. However, we end the discussion on solving MAP1 models at this point. We develop missile allocation models that can both allocate the missiles to targets and schedule the missiles for engagements in the following sections.

4.2 MAP2 - MISSILE ALLOCATION MODEL WITH DISCRETIZED TIME

In this section, we formulate a MAP with an explicit treatment of time and discuss the solution procedures for this problem.

4.2.1 Formulation of the Problem

Suppose that there are n incoming ASMs, indexed $i \in N = \{1,...,n\}$ and there are m SAM systems on board of warships composing the naval TG, indexed $j \in M = \{1,...,m\}$. Define t_i as the time taken by ASM i to reach its target. Letting $T = \max_{i \in N} \{\lceil t_i \rceil\}$ be the problem horizon given by the highest time-on-target, the interval $[0,T]$ may be divided into t non-overlapping slots each of unit duration Δ, indexed $k \in K = \{1,...,t\}$, and τ_k denotes the beginning time of slot k, $k \in K$. Let V denote the valid combinations of ASM and SAM systems, i.e. $(i,j) \in V$ if SAM system j can engage ASM i. Each ASM i has a specified engageability duration $[q_{ij}, r_{ij}]$, which depends on the location and capability of the SAM system j, and a successful engagement can be achieved only during this interval. We assume that the problem data related with time have been perturbed such that each value is a multiple of the unit time Δ. Time taken by each feasible engagement is again determined as the sum of a constant setup time and a variable flight time to the engagement. Thus, each engagement process takes a specified time according to the ASM and SAM combination $(i,j) \in V$ and the starting time of the engagement. This engagement period is denoted by Δ_{ijk}.

To formulate the problem, recalling that τ_k denotes the beginning time of slot $k \in K$, let us define for each valid combination of ASM $i \in N$ and SAM $j \in M$, a set

$$S_{ij} = \{k \in K : (i,j) \in V \text{ and } [\tau_k, \tau_k + \Delta_{ijk}] \subseteq [q_{ij}, r_{ij}]\}.$$

Note that S_{ij} denotes the slots for which SAM j can be scheduled to engage ASM i.

Accordingly we define the binary decision variable $x_{ijk} = 1$, if SAM j is scheduled to engage ASM i at the beginning of the slot k, and $x_{ijk} = 0$ otherwise. Furthermore, in order to ensure that the schedule of the SAMs against each ASM does not overlap in accordance with the SLS tactic, let us define for each slot $k \in K$ and for each ASM $i \in N$, the set

$$J_{ik} = \{(j, \rho) : (i, j) \in V,\ \rho \in S_{ij}, \text{and}\ [\tau_k, \tau_k + \Delta] \subseteq [\tau_\rho, \tau_\rho + \Delta_{ij\rho}]\}.$$

Note that for each $i \in N$ and $k \in K$, J_{ik} is the set of combinations (j, ρ) such that the slot k for ASM i will be occupied if $x_{ij\rho} = 1$.

Let us give an example to illustrate the sets S_{ij} and J_{ik}. Suppose that we divide the engageability duration $[q_{ij}, r_{ij}]$ into 9 slots as in Figure 4.3 and engagement period, $\Delta_{ijk} = 5$ slots. Note that the engagement period may vary depending on the slot, in which the engagement starts. Here, we kept the engagement period constant for simplicity.

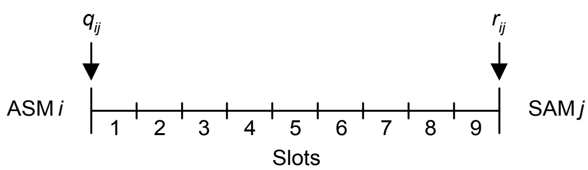

Figure 4.3. An Example of Set Definitions For MAP2.

$S_{ij} = \{1,2,3,4,5\}$. S_{ij} denotes that [1,5], [2,6], [3,6], [4,8], and [5,9] are possible engagement intervals. Thus;

$J_{i1} = \{(j,1)\}$,

$J_{i2} = \{(j,1),(j,2)\}$,

$J_{i3} = \{(j,1),(j,2),(j,3)\}$,

$J_{i4} = \{(j,1),(j,2),(j,3),(j,4)\}$,

$J_{i5} = \{(j,1),(j,2),(j,3),(j,4),(j,5)\}$,

$J_{i6} = \{(j,2),(j,3),(j,4),(j,5)\}$,

$J_{i7} = \{(j,3),(j,4),(j,5)\}$,

$J_{i8} = \{(j,4),(j,5)\}$,

$J_{i9} = \{(j,5)\}$.

For example, J_{i4} means that if an engagement starts at the beginning of slots 1, 2, 3, or 4, then slot 4 will be occupied.

We need the following additional notation and variables to formulate the TG air defense problem:

p_{ijk}: the single shot kill probability (SSPK) of SAM j against ASM i when the engagement begins at the beginning of slot k, $0 < p_{ijk} < 1$, $(i, j) \in V$ and $k \in S_{ij}$.

d_j: the number of available rounds on SAM system j.

u_{ij}: the upper bound on the number of engagements that can be done by SAM system j against ASM i, $(i, j) \in V$.

Then the TG air defense problem MAP2 can be formulated as the following nonlinear integer programming model.

$$Max \quad \prod_{i \in N} \left(1 - \prod_{\substack{\{k \in K\} \\ \{j \in M | (i,j) \in V\}}} \left(1 - p_{ijk}\right)^{x_{ijk}} \right) \tag{4.12}$$

subject to

$$\sum_{\substack{\{k \in K\} \\ \{i \in N | (i,j) \in V\}}} x_{ijk} \le d_j \quad \text{for all } j \in M \tag{4.13}$$

$$\sum_{(j,\rho) \in J_{ik}} x_{ij\rho} \le 1 \quad \text{for all } i \in N \text{ and } k \in K \tag{4.14}$$

$$\sum_{k \in S_{ij}} x_{ijk} \le u_{ij} \quad \text{for all } (i,j) \in V \tag{4.15}$$

$$x_{ijk} \in \{0,1\} \quad \text{for all } (i,j) \in V \text{ and } k \in S_{ij} \tag{4.16}$$

The objective function (4.12) maximizes the probability of no-leaker for the whole TG. Constraint set (4.13) reflects the restriction on the number of rounds available for each SAM system. The constraints of type (4.14) ensure that there is no overlap of the engagements against each ASM. The constraints of type (4.15) limit the total number of rounds that can be fired for each valid ASM and SAM combination. This constraint set tightens the feasible space of the problem. The constraint set (4.16) imposes binary restriction on the decision variables.

4.2.2 Solution Procedure

The non-linearity in the above model can be transformed into linearity by using logarithms. Taking the logarithm of equation (4.12) does not affect the optimal solution. Then equation (4.12) becomes: $Max \sum_{i \in N} \ln\left(1 - \prod_{\substack{\{k \in K\} \\ \{j \in M | (i,j) \in V\}}} \left(1 - p_{ijk}\right)^{x_{ijk}}\right)$.

Let $h_i = \left(1 - \prod_{\substack{\{k \in K\} \\ \{j \in M | (i,j) \in V\}}} \left(1 - p_{ijk}\right)^{x_{ijk}}\right)$ for all $i \in N$ and $0 < h_i < 1$. Equivalently we can write equation (4.12) as $Max \sum_{i \in N} \ln(h_i)$ (4.17) and introduce a new set of constraints into the problem as follows:

$1 - \prod_{\substack{\{k \in K\} \\ \{j \in M | (i,j) \in V\}}} \left(1 - p_{ijk}\right)^{x_{ijk}} \geq h_i$ for all $i \in N$. Taking the logarithm of both sides of the constraints leaves us with a simpler constraint set.

$$\sum_{\substack{\{k \in K\} \\ \{j \in M | (i,j) \in V\}}} a_{ijk} x_{ijk} \geq b_i \text{ for all } i \in N, \qquad (4.18)$$

where $a_{ijk} = -\ln(1 - p_{ijk})$ and $b_i = -\ln(1 - h_i)$.

We can further simplify the model by exploiting the relation between the term $\ln(h_i)$ in the objective function and b_i, and then removing the logarithms. If we have the same term in the objective function (4.17) and constraint set (4.18), we can replace the logarithms with a variable. Let $c_i = \frac{\ln(h_i)}{-\ln(1 - h_i)}$ for all $i \in N$. Then objective function (4.17) becomes $Max \sum_{i \in N} c_i b_i$. Note that c_i is the ratio between the objective function variable and the term at the right hand side of the constraint set (4.18). Figure 4.4 depicts the graph of $\ln(h_i)$ against $-\ln(1-h_i)$. c_i is a concave function that enables us to easily make a linear approximation. In Figure 4.4, we approximate the function with three line segments. Note that this is a rough and conservative approximation, and further investigation is needed to justify the quality of the approximation. Rosenthal et.al. (2001) propose several methods for attaining high quality piecewise linearization. However, the approximation in Figure 4.4 is sufficient for illustrating the approach.

Let c^1, c^2, c^3 be the slope of the line segments that approximate the function and b_i is represented as the sum of three different variables corresponding to those three line segments, $b_i = b_i^1 + b_i^2 + b_i^3$.

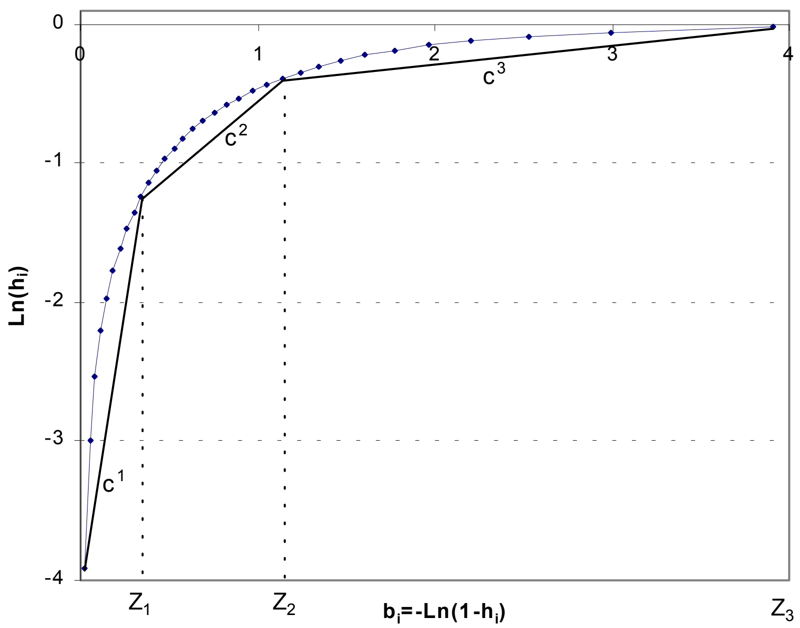

Figure 4.4. Relationship between $\ln(h_i)$ and b_i.

After introducing the following three simple upper bounding constraints for each ASM i into the model,

$$0 \le b_i^1 \le Z_1 \qquad \text{for all } i \in N \tag{4.19}$$

$$0 \le b_i^2 \le Z_2 - Z_1 \qquad \text{for all } i \in N \tag{4.20}$$

$$0 \le b_i^3 \le Z_3 - Z_2 \qquad \text{for all } i \in N \tag{4.21}$$

and replacing b_i with $b_i^1 + b_i^2 + b_i^3$, we can rewrite the objective function as $Max \sum_{i \in N} c_i b_i \cong Max \sum_{i \in N} \left(c^1 b_i^1 + c^2 b_i^2 + c^3 b_i^3 \right)$. This completes the linearization process. The resulting model MAP2 is as follows.

$$Max \sum_{i \in N} \left(c^1 b_i^1 + c^2 b_i^2 + c^3 b_i^3 \right) \tag{4.22}$$

subject to

$$\sum_{\substack{\{k \in K\} \\ \{j \in M | (i,j) \in v(i,j)\}}} a_{ijk} x_{ijk} \ge b_i^1 + b_i^2 + b_i^3 \quad \text{for all } i \in N, \tag{4.23}$$

and (4.13), (4.14), (4.15), (4.16), (4.19), (4.20), (4.21).

We illustrate solution of MAP2 on an example problem in Section 4.4, following the MAP3 formulations.

4.3 MAP3 - MISSILE ALLOCATION MODEL WITH CONTINUOUS TIME

In this section, we formulate a MAP with an explicit and continuous treatment of time and discuss the solution procedures for this problem.

4.3.1 Formulation of the Problem

Suppose that there are n incoming ASMs, indexed $i \in N = \{1,...,n\}$ and there are m SAM systems on board of warships composing the naval TG, indexed $j \in M = \{1,...,m\}$. Let s_i be the maximum number of engagements that can be done against ASM i using a SLS tactic. Let V denote the valid combinations of ASM and SAM systems, i.e. $(i,j) \in V$ if SAM system j can engage ASM i. Each ASM i has a specified engageability duration $[q_{ij}, r_{ij}]$, which depends on the location and capability of the SAM system j, and a successful engagement can be achieved only during this interval. Define k as the order of the shots against an ASM, i.e. k=1 denotes the 1st shot against an ASM. Let Δ_{ijk} be the time-to-engagement for the k^{th} shot against ASM i, if the missile is launched by SAM system j.

Accordingly we define the binary decision variable $x_{ijk} = 1$, if the k^{th} shot is fired against ASM i from SAM system j, and $x_{ijk} = 0$ otherwise.

We need the following additional notation and variables to formulate the continuous time MAP:

p_{ij}: single shot kill probability of SAM j against ASM i, $0 < p_{ij} < 1, \quad (i,j) \in V$.

d_j: number of available rounds on SAM system j.

t_{ik}: time of the k^{th} shot against ASM i.

L: a large number.

Then MAP3 can be formulated as the following nonlinear integer programming model:

$$Max \quad \prod_{i \in N} \left(1 - \prod_{\{j \in M | (i,j) \in V\}} \prod_{k=1}^{s_i} \left(1 - p_{ij}\right)^{x_{ijk}} \right) \tag{4.24}$$

subject to

$$\sum_{\{i \in N | (i,j) \in V\}} \sum_{k=1}^{s_i} x_{ijk} \le d_j \quad \text{for all } j \in M \tag{4.25}$$

$$\sum_{j \in M} x_{ij1} \le 1 \quad \text{for all } i \in N \text{ and } k = 1 \tag{4.26}$$

$$\sum_{j \in M} x_{ijk} \le \sum_{j \in M} x_{ij,k-1} \quad \text{for all } i \in N \text{ and } k = 2,3,\ldots,s_i \tag{4.27}$$

$$L\left(1 - \sum_{j \in M} x_{ij,k+1} \right) + t_{i,k+1} \ge t_{ik} + \sum_{j \in M} x_{ijk} \Delta_{ijk} \quad \text{for all } i \in N \text{ and } k = 1,2,\ldots,s_i - 1 \tag{4.28}$$

$$\sum_{j \in M} x_{ijk} q_{ij} \le t_{ik} \le \sum_{j \in M} x_{ijk} (r_{ij} - \Delta_{ijk}) \quad \text{for all } i \in N \text{ and } k = 1,2,\ldots,s_i - 1 \tag{4.29}$$

$$x_{ijk} \in \{0,1\} \quad \text{for all } (i,j) \in V \text{ and } k = 1,2,\ldots,s_i \tag{4.30}$$

$$t_{ik} \ge 0 \quad \text{for all } i \in N \text{ and } k = 1,2,\ldots,s_i \tag{4.31}$$

The objective function, (4.24) maximizes the probability of no-leaker for the whole TG. Constraint set (4.25) reflects the restriction on the number of rounds available for each SAM system. Constraints of type (4.26) ensure that there is only one first shot against each ASM if there is any. Constraints of type (4.27) ensure that the shots are counted in order and there is only one k^{th} shot against any ASM fired by any one of the valid SAMs. Constraint set (4.28) makes sure that the next engagement will start after the end of the previous engagement. Constraint set (4.29) restricts an engagement to be within the engageability duration and ensures that there is enough time for the last engagement. The constraint sets (4.30) and (4.31) impose binary restriction and non-negativity restriction on the decision variables.

4.3.2 Solution Procedure

In addition to the objective function, MAP3 has non-linearity in the constraints as well. Since Δ_{ijk} depends on the time of the k^{th} shot that is determined indigenously, there is non-linearity in the constraint sets (4.28) and (4.29). However, we may eliminate the non-linearity in the constraints by applying the following transformation process. Let

$$\Delta_{ijk} = \frac{D_i - v_i(\Delta_c + t_{ik})}{v_j + v_i} + \Delta_c$$

where D_i is the initial detection distance of ASM i, Δ_c is the constant setup time for an engagement, v_i is the speed of ASM i, and v_j is the speed of SAM j. For example, when, $D_i = 10{,}000\text{ m}$, $v_i = v_j = 300\text{ m/sec}$, $\Delta_c = 4\text{ sec}$, and time of the k^{th} shot is 15, the engagement would take 7.17 seconds. Then, we substitute Δ_{ijk} in the following equation.

$$\sum_{j \in M} x_{ijk}\Delta_{ijk} = \sum_{j \in M} x_{ijk}\left(\frac{D_i - v_i(\Delta_c + t_{ik})}{v_j + v_i} + \Delta_c\right)$$

$$= \sum_{j \in M}\left(\frac{D_i - v_i\Delta_c}{v_j + v_i}\right)x_{ijk} - \sum_{j \in M}\left(\frac{v_i}{v_j + v_i}\right)x_{ijk}t_{ik} + \sum_{j \in M}\Delta_c x_{ijk}$$

Defining $\alpha_{ij} = \dfrac{D_i - v_i \Delta_c}{v_j + v_i}$ and $\beta_{\text{ij}} = \dfrac{v_i}{v_j + v_i}$ leads us to the following equality.

$$\sum_{j \in M} x_{ijk} \Delta_{ijk} = \sum_{j \in M} \alpha_{ij} x_{ijk} - \sum_{j \in M} \beta_{ij} x_{ijk} t_{ik} + \sum_{j \in M} \Delta_c x_{ijk}$$

Let $y_{ijk} = x_{ijk} t_{ik}$. Then, when $x_{ijk} = 0$, y_{ijk} must be equal to 0, and when $x_{ijk} = 1$, y_{ijk} must be equal to t_{ik}. Then the constraint sets (4.28) and (4.29) become

$$L\left(1 - \sum_{j \in M} x_{ij,k+1}\right) + t_{i,k+1} \geq t_{ik} + \sum_{j \in M} \alpha_{ij} x_{ijk} - \sum_{j \in M} \beta_{ij} y_{ijk} + \sum_{j \in M} \Delta_c x_{ijk}$$

for all $i \in N$ and $k = 1,2,..., s_i - 1$ (4.28')

$$\sum_{j \in M} x_{ijk} q_{ij} \leq t_{ik} \leq \sum_{j \in M} x_{ijk} r_{ij} - \sum_{j \in M} \alpha_{ij} x_{ijk} + \sum_{j \in M} \beta_{ij} y_{ijk} - \sum_{j \in M} \Delta_c x_{ijk}$$

for all $i \in N$ and $k = 1,2,..., s_i - 1$ (4.29')

and we introduce the linking constraints for y_{ijk} as follows:

$y_{ijk} \leq L x_{ijk}$ for all $(i, j) \in V$ and $k = 1,2,..., s_i$ (4.32)

$y_{ijk} \leq t_{ik}$ for all $(i, j) \in V$ and $k = 1,2,..., s_i$ (4.33)

$y_{ijk} \geq 0$ for all $(i, j) \in V$ and $k = 1,2,..., s_i$ (4.34)

We need to modify the objective function in order to be able to force y_{ijk} to be equal to t_{ik} when $x_{ijk} = 1$. We add a small weight to the objective function as follows:

$$Max \quad \prod_{i \in N} \left(1 - \prod_{\substack{\{k \in K\} \\ \{j \in M | (i,j) \in V\}}} \left(1 - p_{ij}\right)^{x_{ijk}}\right) + \varepsilon \frac{1}{nKT} \sum_{ijk} y_{ijk} \qquad (4.24')$$

where ε is a very small positive number, $T = \max_{i,j}\{r_{ij}\}$, and $K = \max_i \{s_i\}$.

We remove the non-linearity of the constraint from MAP3 by using the above procedure. We may use the method described for model MAP2 for the transformation of the objective function. The implementation of MAP3 using the procedure described in Section 4.2.2 will be discussed in the following section.

4.4 DISCUSSION

In this section, we discuss and compare the MAP models on an example problem. The problem introduced in Section 4.1.2.1 (see Figure 4.2) will be used here with some modifications that ease the application and comparison of the models. Note that our primary aim is to demonstrate the applicability of the models.

MAP2 and MAP3 require more detailed scenario information compared to MAP1. Information required to describe a scenario is given in Table 4.3, Table 4.4, and Table 4.5 for the example problem.

Table 4.3. Task Group Formation Information.

Ship	Bearing*	Range (m)
1	Center	0
2	070	2000

* Relative bearing from the center of the formation.

Table 4.4. Attack Information.

ASM	Target Ship	Bearing	Range (m)	Speed (m/sec)
1	1	020	11000	300
2	2	015	10000	300

Table 4.5. Defense Information.

SAM System	Hosting Ship	Minimum Range (m)	Maximum Range (m)	Speed (m/sec)
SAM1	1	500	5000	300
SAM2	2	2000	20000	600
SAM3	2	500	5000	300

Implementations of the models MAP2 and MAP3 have also been done in GAMS by using OSL Solver (Brooke et al., 1988).

Sizes and solution times of the models for the example problem are reported in Table 4.6. For MAP2, we used a 3 seconds unit time for each slot. We implemented the piecewise linearization of the objective functions of MAP2 and MAP3 as described in Figure 4.4. We set the parameters as follows: $(c_1, c_2, c_3) = (7.74, 0.94, 0.16)$ and $(Z_1, Z_2, Z_3) = (0.35, 1.17, 3.91)$. For illustrative purposes, we only solved MAP1.2 of MAP1 models. MAP1.2 is the smallest in size. MAP2 has the largest number of integer variables. Solution times are all less than 1 sec. Although there is no significant difference between the solution times for the test problem, we expect that the computational time will increase with a higher rate for MAP2 and MAP3 compared to MAP1.

Table 4.6. Sizes and solution times of the models for the example problem.

	MAP1.2	MAP2	MAP3
Constraints	8	25	36
Continuous Variables	3	7	27
Discrete Variables	6	30	8
Non-zero Elements*	19	211	157
Solution Time (sec)**	0.11	0.33	0.21

* Decision variables that have nonzero coefficient values in generated problem.
** Runs carried out on a personal computer with 2.1 GHz CPU and 256 MB RAM.

We do not have an explicit control of time dimension in MAP1. MAP1 implicitly controls time by using two parameters, the maximum number of engagements against each ASM and the maximum number of engagements for each valid combination of SAM systems and ASMs. MAP1.2 allocates the defensive capacity whereas MAP2 and MAP3 schedule the engagements in addition to allocation at the expense of increased problem size. In MAP1, there is a chance of producing infeasible allocation. In MAP2, there is a trade-off between the resolution of the model and increased problem size. We can increase the resolution of the model by choosing the unit time of the discretized time dimension small. However, the problem size increases as the unit time decreases. If we increase the unit time, solutions may be unrealistic and unreasonable. Thus, we need to find a reasonable value of unit time for MAP2. In MAP3, we reduce the number of integer variables compared to MAP2. However, we need to find the correct objective function weight in order to solve the problem successfully. Summary of features and drawbacks of the models are given in Table 4.7.

Table 4.7. Summary of MAP Models.

Model	SAM, ASM Types	Allocation	Scheduling	Time Dimension	Size of the Formulation	Other
MAP1	Multiple	Yes	No	Not Explicit	Small	SLS may be violated
MAP2	Multiple	Yes	Yes	Discretized	Large	SSPK can be different for different time slots
MAP3	Multiple	Yes	Yes	Continuous	Medium	Fine tuning of ε is required.

Results of the example problem are depicted in Table 4.8. Models produce comparable and reasonable results. MAP1.2 and MAP2 produced exactly the same result. MAP3 result is different than the result of other two models. The difference in results is due to the parameter settings. Note that we need to fine-tune the objective function weight, ε in MAP3. We do not discuss the results and parameter settings more, since we will not directly use those models to solve MAP. These results show that all of the MAP models work and produce results as expected.

Table 4.8. Results of the Example Problem.

	Allocation			Prob. of Shooting		Prob. of
	SAM1	SAM2	SAM3	ASM1	ASM2	No-leaker
MAP1.2	1	2	1	0.920	0.875	0.805
MAP2	1	2	1	0.920	0.875	0.805
MAP3	0	3	0	0.800	0.938	0.750

Air defense of a TG requires very quick reaction. The duration of an air attack might range from tens of seconds to a few minutes at the most. Coordination of the air defense of the ships within the TG is prone to confusion. This may suggest allocating the SAM systems once at the beginning of the attack and then sticking to this allocation policy throughout the raid. However, we may also choose to improve the initial allocation plan autonomously, or cooperatively with the other TG units. By autonomously we mean that a TG unit acts independently as the situation warrants. Thus models developed in this research may be used in threat-evaluation and weapon-assignment (TEWA) module of a TG AAW command ship to allocate the air defense missiles to incoming air targets. However, the solution time for relatively larger size problems may suggest using other solution techniques such as heuristics instead of a standard mathematical programming package.

The models presented here may be used off-line to investigate the effectiveness of the air defense formations under different scenarios in an exploratory analysis setting.

The proposed solution procedures were applied to an example problem. The quality of the results represents the potential value and the use of the models. A more thorough investigation of the models using different test scenarios may secure a robust solution procedure for the TG air defense problem. However, we will develop solution algorithms that satisfy the demanding time requirements of a real time defense against ASMs in the next section. The findings of this research are expected to provide valuable insights to the decision-maker and the commander at sea.

CHAPTER V

SOLUTION OF THE MISSILE ALLOCATION PROBLEM (MAP)

In this chapter, we develop greedy construction and improvement heuristic solution procedures for MAP. We discuss our reasoning for using heuristics in the next section. We present an implicit enumeration algorithm in Section 2, which is used to measure the quality of the solutions produced by the heuristics. Section 3 contains the construction heuristics for MAP. We present the improvement heuristics in Section 4 and we conclude this chapter by reporting computational results. We also discuss scenario and the problem generation issues in the last section.

5.1 NATURE OF THE PROBLEM

On-line use of MAP requires real time solution and very fast implementation without even sacrificing a single second. Thus, any solution procedure has to produce reasonable and high quality solutions in no more than several seconds. This is a must feature of any solution algorithm that is eligible to be used in TEWA module of a warship.

Solving MAP for a large number of representative cases is a prerequisite for successfully solving SAP. Since this process requires running MAP many times for a single SAP solution, off-line use of MAP also requires fast and high quality solutions.

Mathematical programming models presented in the preceding chapter do not meet the solution time requirements for using MAP on-line or off-line. Thus, we focus on heuristic solution procedures for MAP in order to meet aforementioned requirements.

5.2 IMPLICIT ENUMERATION

In this section, we develop an implicit enumeration algorithm for MAP. In order to determine the quality of the solutions produced by the heuristics, we need to compare the heuristic solutions with the optimal solutions. Thus, finding the optimal solution for the problems with sizes as large as possible is desirable. Implicit enumeration does help to attain solutions of relatively larger problems compared to the complete enumeration. We first developed a complete enumeration scheme and then improved it to an implicit enumeration algorithm. Development of the implicit enumeration algorithm is presented below:

Let $A = (a_0, a_1, ..., a_m)$ and $B = (b_0, b_1, ..., b_m)$ be two SAM engagement vectors showing the number of missiles launched from SAM system $i \in M$.

Definition: A dominates B, if and only if $a_i \geq b_i$ for all $i \in M$, $a_i > b_i$ for at least one $i \in M$ and both A and B have at least one feasible engagement schedule against threat ASMs.

Let $S_A = \sum_{i=0}^{m} a_i$ and $S_B = \sum_{i=0}^{m} b_i$.

Proposition: If $S_A > S_B$ then the best engagement schedule using S_A number of SAM missiles is better than the best engagement schedule using S_B number of SAM missiles.

Proof: If $S_A > S_B$ then an engagement vector that dominates every specific engagement vector using S_B number of SAM missiles can be found. □

Implicit Enumeration Algorithm:

Step 0: Find the maximum number of engagements possible against each ASM.

- Find the fastest SAM system that can be used against each ASM.
- Find the maximum and minimum engagement ranges for each ASM using all SAM systems that can be used against the ASM.
- Calculate the maximum number of engagements for each ASM based on the speed of the fastest SAM system.
- Calculate the total number of SAMs that can be launched.

Step 1: Generate all possible engagement schedules for the given total number of SAMs and, if there are feasible engagements, find the best one. We generate the engagement schedules as follows:

- Given the total number of SAMs used, generate all combinations of SAM launches by different SAM systems; i.e. in each instance, we determine the number of missiles consumed from each SAM system.
- Given the number of missile launches by each SAM system, generate all combinations of missile launch sequences.

- According to the given SAM launch sequences, generate all combinations of target ASMs.

Step 2: If there is no feasible schedule then reduce the total number of SAMs by one and go to Step 1. Otherwise, stop. The best schedule is the optimal schedule.

5.3 CONSTRUCTION HEURISTICS FOR MAP

In this section, we present two greedy construction algorithms for MAP. First of those algorithms, best engagement construction heuristic, allocates SAM systems to incoming ASMs according to a measure, called engagement potential. In quasi-uniform construction algorithm, we aim to engage each threat ASM at least once. Thus, we give precedence to the ASM that has the lowest number of SAM systems that can engage it.

We present the notation and variables for the construction algorithms below:

Suppose that there are n incoming ASMs indexed $i \in N = \{1,...,n\}$ and there are m SAM systems on board of warships composing the naval task group, indexed $j \in M = \{1,...,m\}$.

s_i : maximum number of engagements possible against ASM i using a SLS tactic.

V : set of valid combinations of ASM and SAM systems, i.e. $(i, j) \in V$ if SAM system j can engage ASM i.

va_i : speed of ASM i,

vs_j : speed of SAM j

$\bar{r}_j$: maximum range of SAM j

$\underline{r}_j$: minimum range of SAM j

Δ_c : constant setup time for an engagement

f_i : initial detection distance of ASM i

pf_i : present distance of ASM i

d_j : number of available rounds on SAM system j

p_{ij} : single shot kill probability of SAM j against ASM i, $0 < p_{ij} < 1, \quad (i, j) \in V$

tot_i : time to reach the target (i.e. time-on-target (TOT)) for ASM i, $tot_i = f_i / va_i$.

5.3.1 Best Engagement Construction (BEC) Algorithm

In this algorithm, we allocate SAM rounds to ASMs according to *engagement potential*, which is a measure of defensive capability of a SAM system against a given ASM. We compare each SAM system with a hypothetical SAM, which has the best features such as maximum single shot kill probability, maximum speed, maximum effective range, and minimum effective range against a given ASM. We assign the SAM with the highest engagement potential to the closest ASM in terms of TOT at each step of the algorithm.

Step 0: Determine the *ideal* SAM for each ASM.

- Find the best features for each ASM using all SAM systems that can be used against the ASM.
- For each ASM, define a new SAM called *ideal SAM* with the best features such as largest maximum range (maximum range may be limited to the initial detection distance of ASM if the detection distance is smaller than the maximum effective range of SAM), smallest minimum range, largest speed and largest single shot kill probability, $vs_i^*, \bar{r}_i^*, \underline{r}_i^*, p_i^*$ respectively.

  $$vs_i^* = \max_j \left\{ vs_j : (i,j) \in V \right\}$$

  $$\bar{r}_i^* = \min\left\{ f_i ,\ \max_j \left\{ \bar{r}_j : (i,j) \in V \right\} \right\}$$

  $$\underline{r}_i^* = \min_j \left\{ \underline{r}_j : (i,j) \in V \right\}$$

  $$p_i^* = \max_j \left\{ p_j : (i,j) \in V \right\}$$

- Initialize present ASM distances to initial detection distances, $pf_i = f_i \quad \forall\, i \in N$.

Step 1: Determine the engagement potential, ep_{ij} of each SAM system against each ASM if the engagement is feasible.

- $$ep_{ij:(i,j)\in V} = w_1 \frac{vs_j}{vs_i^*} + w_2 \min\left\{ \frac{\bar{r}_j}{\bar{r}_i^*}, 1 \right\} + w_3 \frac{\underline{r}_i^*}{\underline{r}_j} + w_4 \frac{p_{ij}}{p_i^*}$$

  where w_1, w_2, w_3, w_4 are the weights of the components of the engagement potential.

- Let G_i be the set of engagement potentials of the SAMs that can be used against ASM i. $G_i = \{ep_{ij} : (i,j) \in V\}$

Step 2: Determine the TOT for each ASM and let $T = \{tot_i : i \in N\}$ be the set of TOTs of ASMs.

Step 3: If all ASMs have been engaged then start a new engagement wave.

If $T = \{\ \}$, then reinitialize $T = \{tot_i : i \in N\}$.

This step ensures that the final engagement schedule is as uniform as possible. ASMs have been engaged with more or less equal number of SAMs.

Step 4: Find the ASM with minimum TOT and remove its TOT from the engagement list, T.

$k = \arg\min_i T$, $T = T \setminus \{tot_k\}$.

Step 5: If there is no SAM missiles left that can be used against any of the ASMs, stop.

If $G_i = \{\ \} \quad \forall i \in N$, then STOP.

Step 6: If there is no SAM system that can be used against ASM k, then return to Step 3, otherwise find the SAM system with maximum engagement potential against the ASM in the engagement order.

If $G_k = \{\ \}$ then go to Step 3, otherwise $l = \arg\max_j G_k$.

Step 7: If there is at least one SAM round of type l and the intercept distance is larger than the minimum engagement range of SAM l, assign SAM l to ASM k.

Reduce the number of available rounds of SAM l by one and go to Step 3. Otherwise update the set of engagement potentials and go to Step 5.

If $d_l \geq 1$ and $\left(pf_k - va_k \Delta_c - \left[\frac{pf_k - va_k \Delta_c}{va_k + vs_j} \right] va_k \right) \geq \underline{r}_l$ then,

If intercept distance is larger than the maximum engagement range of SAM l, then reduce intercept distance to maximum engagement range of SAM l, i.e.

If $\left(pf_k - va_k \Delta_c - \left[\frac{pf_k - va_k \Delta_c}{va_k + vs_j} \right] va_k \right) \geq \overline{r}_l$, then $pf_k = \overline{r}_l$,

else $pf_k = pf_k - va_k \Delta_c - \left[\frac{pf_k - va_k \Delta_c}{va_k + vs_j} \right] va_k$.

Assign SAM l to ASM k and $d_l = d_l - 1$. Go to Step 3.

Otherwise $G_k = G_k \setminus \{ep_{kl}\}$ and go to Step 5.

5.3.2 Quasi-Uniform Construction (QUC) Algorithm

BEC algorithm assigns the SAM with the highest engagement potential to the closest ASM in terms of TOT. However, if the number of missiles in magazine or launcher is limited, assignment rule may produce unsatisfactory results. Note that probability of no-leaker will be zero by allocating anything less then one shot per ASM. This discontinuity, the jump from zero to positive probability of no-leaker value as the last ASM in the first engagement wave is shot at, causes difficulties for our construction algorithm. If there is an engagement schedule that has at least one shot per ASM, then it is desirable to find that one. This variation makes sure that we find the desirable engagement schedule if there is one. Step 3 of the previous algorithm is to be read as follows:

Step 3: If $T = \{\ \}$ and there exists at least one ASM with no interceptor assigned then disregard all assignments made so far and let TOTs be the cardinality of the corresponding set of engagement potentials, $T = \left\{ tot_i = |G_i| :\ i \in N \right\}$.

Else if $T = \{\ \}$, then reinitialize $T = \left\{ tot_i :\ i \in N \right\}$.

5.4 IMPROVEMENT HEURISTICS FOR MAP

In this section, we present two improvement algorithms for MAP. First of those algorithms, opt-change (OC) algorithm, improves the initial feasible engagement schedule by changing the target ASM or defending SAM system of an engagement in the engagement list. In 2-opt exchange (2OX) algorithm, we aim to exchange target ASMs of two engagements to improve the solution. For both algorithms, we choose the best move (change or exchange) in each iteration. Since both OC and 2OX algorithms are lengthy, we give summary of the algorithms here. We present the details of OC and 2OX algorithms in Appendix A and B respectively.

5.4.1 Opt-Change (OC) Algorithm

Our purpose in this algorithm is to find the engagements that would increase the objective function value by (1) changing the target ASM of an engagement under consideration and (2) simultaneously considering the enhancement of the effectiveness of defense by increasing the total number of SAM missiles launched against target ASMs. Changing the target ASM means that while an ASM will get one less shot, another ASM will get one more shot. The ASM that gets one less shot after change is considered for an additional shot observing the SLS tactic.

Summary of the Algorithm

Step 0: Select an initial feasible engagement list.

Step 1: For each engagement in the list, check the possibility of the change of target ASM. A change of target ASM will degrade defense against the target ASM before change, and will enhace the defense against the new target ASM. Thus, we simultaneously consider enhancing the defense against the previous target ASM of the engagement using remaining SAM rounds, if any, while enhancing the defense against the new target ASM by the change.

Step 2: Consider changing the defending SAM for the engagements in the list.

Step 3: Find the best change in Step 1 and 2. Update the engagement list, if it is needed. If there is an improvement, go back to Step 1. Otherwise, stop.

5.4.2 2-Opt-Exchange (2OX) Algorithm

Our purpose in this algorithm is to find the engagement pairs that would increase the objective function value by exchanging the target ASMs of the SAMs in the engagements. We also try to increase the number of engagements done against the ASMs under consideration with each exchange simultaneously.

Summary of the Algorithm

Step 0: Select an initial feasible engagement list.

Step 1: For each engagement in the list, check the possibility of the exchange of target ASMs with all the other engagements in the list. Simultaneously consider enhancing the defense against both target ASMs using remaining SAM rounds.

Step 2: Consider exchanging all the scheduled engagements of two ASMs.

Step 3: Find the best exchange in Step 1 and 2. Update the engagement list, if it is needed. If there is an improvement, go back to Step 1. Otherwise, stop.

5.5 COMPUTATIONAL RESULTS

We randomly generated test problems using the random number generator explained in Law and Kelton (1991). We defined seven different SAM systems, including four self-defense and three area air defense SAM systems, and seven ASMs. We created a sample single shot kill probability matrix for SAM and ASM systems using open sources. Those representative SAM and ASM systems are in use by the navies and are reported in Appendix C. For the examples in this section, we assume that the ships in TG are in close formation and the distances between the ships are negligible compared to the initial detection distances of the ASMs for simplicity. We discuss the sector allocation of ships in Chapter VI, VII, and VIII in detail.

We find the optimal solution to MAP by using the implicit enumeration algorithm. Implicit enumeration algorithm generates a fraction of solutions compared to complete enumeration. However, it is still very expensive to find the optimal solution in terms of computational time. Thus, we restrict the sample problem size to a maximum of five SAM systems with a total of nine missiles in the launchers and five ASMs. We generated five problem sets, each having SAM systems and ASMs from one to five composing a total of 125 problems. We used different random number streams for each problem set. Details of the sample problem generation are given in Appendix C.

We start computational experiments by comparing the solutions of implicit enumeration and BEC heuristic. Table 5.1 and Table 5.2 depict the results of implicit enumeration and the BEC heuristic for the first set of 25 problems. The 3 ASM and 4 SAM case produces zero probability of no-leaker since none of the SAMs engage the second ASM. This is a representative case where we need to use the QUC heuristic to produce a feasible engagement schedule.

In Table 5.3, we present the summary result of implicit enumeration and the best of construction heuristics. QUC heuristic produces the optimal solution for 3 ASM and 4 SAM case where BEC has 100% gap as well as for two other cases (2 ASM 2 SAM and 2 ASM 3 SAM). It also improves the solution for one case (5 ASM 4 SAM). Detailed results for the first 25 problems and the remaining 100 problems (2^{nd} – 5^{th} problem sets) are given in Appendix D.

Table 5.1. Comparison of Implicit Enumeration (IE) and BEC Heuristic.

ASM		SAM 1	2	3	4	5
1	IE Obj	0.640	0.874	0.874	0.874	0.927
	BEC Obj	0.640	0.874	0.874	0.874	0.927
	IE Sched.*	11 / 11	211 / 111	211 / 111	233 / 111	553 / 111
	BEC Sched.*	11 / 11	211 / 111	212 / 111	233 / 111	253 / 111
	IE Time**	0.00	0.00	0.00	0.00	0.00
	BEC Time**	0.00	0.00	0.00	0.00	0.00
2	IE Obj	0.160	0.416	0.559	0.416	0.602
	BEC Obj	0.160	0.316	0.506	0.416	0.602
	IE Sched.	11 / 11	211 / 122	33211 / 22111	332 / 112	5553 / 1121
	BEC Sched.	11 / 21	211 / 212	2131 / 2121	233 / 211	2553 / 2111
	IE Time	0.00	0.00	0.63	0.62	1.75
	BEC Time	0.00	0.00	0.00	0.00	0.00
3	IE Obj	0.164	0.120	0.307	0.166	0.452
	BEC Obj	0.164	0.120	0.307	0.000	0.452
	IE Sched.	11111 / 11223	211 / 312	33211 / 22311	3321 / 1123	55532 / 11213
	BEC Sched.	11111 / 32121	211 / 321	23131 / 32121	233 / 311	25553 / 32111
	IE Time	0	0	0.422	0.422	9.812
	BEC Time	0	0	0	0	0
4	IE Obj	0.051	0.339	0.096	0.118	0.383
	BEC Obj	0.051	0.284	0.096	0.065	0.383
	IE Sched.	11111 / 11234	2211111 / 3411122	33211 / 24311	443321 / 221143	555332 / 123114
	BEC Sched.	11111 / 32412	2211111 / 3241241	23311 / 32411	24433 / 32411	255533 / 324111
	IE Time	0.50	41.90	0.00	1.99	1294.61
	BEC Time	0.00	0.00	0.00	0.00	0.00
5	IE Obj	0.016	0.138	0.159	0.037	0.225
	BEC Obj	0.016	0.089	0.143	0.020	0.173
	IE Sched.	11111 / 12345	2211111 / 3411225	32211111 / 23411155	443321 / 221543	555332 / 234115
	BEC Sched.	11111 / 32415	2211111 / 3241524	22311111 / 3241515	24433 / 32415	255533 / 324151
	IE Time	0.22	228.77	7008.60	4.96	8503.29
	BEC Time	0.00	0.00	0.00	0.00	0.00

* IE or BEC Sched: SAM Engagement Order / Target ASM Order

** Elapsed time in seconds.

Table 5.2. % Gap Between Implicit Enumeration (IE) and BEC Heuristic Solutions.

ASM	SAM 1	2	3	4	5
1	0.0	0.0	0.0	0.0	0.0
2	0.0	24.0	9.6	0.0	0.0
3	0.0	0.0	0.0	100.0	0.0
4	0.0	16.0	0.0	44.9	0.0
5	0.0	35.7	9.7	44.9	22.9

Table 5.3. % Gap Between Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (First Set)

ASM	SAM				
	1	2	3	4	5
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	16.0	0.0	44.9	0.0
5	0.0	35.7	9.7	41.9	22.9

Table 5.4 presents the summary of all 125 sample MAPs in terms of minimum, average, and maximum % gaps between the optimal solution and the best construction heuristic solution. Construction heuristics failed to produce the optimal solution to MAP in 38 out of 125 cases. Although the construction heuristics attained the optimal solution in 70 % of the test cases, we may conclude that the construction algorithms can frequently produce unsatisfactory results.

Table 5.4. Minimum, Average and Maximum % Gap for Five Problem Sets.

ASM		SAM				
		1	2	3	4	5
1	Min[a]	0.0	0.0	0.0	0.0	0.0
	Ave[b]	0.0	0.0	0.0	0.0	0.0
	Max[c]	0.0	0.0	0.0	0.0	0.0
2	Min	0.0	0.0	0.0	0.0	0.0
	Ave	0.0	4.0	4.8	2.1	1.5
	Max	0.0	20.0	23.8	10.7	7.3
3	Min	0.0	0.0	0.0	0.0	0.0
	Ave	2.4	2.0	3.0	0.4	1.2
	Max	5.9	5.7	14.8	2.0	6.0
4	Min	0.0	0.0	0.0	0.0	0.0
	Ave	0.0	11.0	7.6	19.2	20.0
	Max	0.0	17.9	33.3	44.9	100.0
5	Min	0.0	0.0	0.0	0.0	0.0
	Ave	5.1	20.8	16.3	14.0	10.3
	Max	11.4	38.6	38.6	41.9	23.1

[a] Minimum % gap, [b] Average % gap, [c] Maximum % gap.

We run our improvement algorithms for those 38 cases, where the construction algorithms failed to produce the optimal solutions. Two different combinations of the improvement algorithms are also investigated. One of those combinations (OC+2OX) is running OC first and then 2OX. The other (2OX+OC) is running 2OX first and OC second. The summary results of improvement heuristics are given in Table 5.5. Detailed computational results are presented in Appendix E. Last column of Table 5.5 depicts the best results of the improvement heuristics. The best results may be viewed as another heuristic that runs OC, 20X, OC+2OX, and 2OX+OC in this order and takes the best solution. We call that heuristic "Best".

We provide some measures of accuracy for heuristics, OC+2OX, 2OX+OC, and "Best" in Table 5.6. OC+2OX dominates 2OX+OC with respect to five measures given in Table 5.6. "Best" provides a slight improvement on the OC+2OX results. OC+2OX attains the optimal solution in 33 out of 38 problems. In one out of five cases, where OC+2OX failed to achieve the optimal results, "Best" yields better result than OC+2OX. 2OX+OC is the worst one with respect to five measures given. We statistically compared "Best" and OC+2OX against 2OX+OC heuristic using Wilcoxon signed rank test as described in Golden and Stewart (1985). Detailed calculations for Wilcoxon tests are given in Appendix F. Wilcoxon tests showed that "Best" and OC+2OX heurisitics are statistically better than 2OX+OC heurisitic at $\alpha = 0.05$ significance level.

Table 5.5. % Gap Between Optimal Solution and the Improvement Heuristics for the Problems, Where Constructions Heuristics Failed to Find Optimal Solution.

* Problem Number	Best of Construction Heuristics	Improvement Algorithms				
		OC	2OX	OC+2OX	2OX+OC	Best
I.4.2	16.0	15.4	13.8	0.0	0.0	0.0
I.4.4	44.9	1.2	44.9	0.0	1.2	0.0
I.5.2	35.7	15.4	0.0	0.0	0.0	0.0
I.5.3	9.7	9.7	0.0	0.0	0.0	0.0
I.5.4	41.9	1.2	24.7	0.0	24.7	0.0
I.5.5	22.9	4.8	0.0	4.8	0.0	0.0
II.3.1	5.9	0.0	5.9	0.0	0.0	0.0
II.3.5	6.0	6.0	6.0	6.0	6.0	6.0
II.4.2	10.1	0.0	10.1	0.0	0.0	0.0
II.4.3	4.8	0.0	4.8	0.0	0.0	0.0
II.4.4	7.2	7.2	7.2	7.2	7.2	7.2
II.5.1	11.4	0.0	11.4	0.0	0.0	0.0
II.5.2	38.6	0.0	9.4	0.0	9.4	0.0
II.5.3	38.6	0.0	9.4	0.0	9.4	0.0
III.2.2	20.0	20.0	0.0	0.0	0.0	0.0
III.2.3	23.8	23.8	0.0	0.0	0.0	0.0
III.3.2	4.1	4.1	4.1	4.1	4.1	4.1
III.3.3	14.8	14.8	0.0	0.0	0.0	0.0
III.3.4	2.0	0.0	2.0	0.0	0.0	0.0
III.4.2	11.1	11.1	0.0	0.0	0.0	0.0
III.4.3	33.3	0.0	5.6	0.0	5.6	0.0
III.4.4	12.5	0.0	12.5	0.0	0.0	0.0
III.5.1	8.3	0.0	8.3	0.0	0.0	0.0
III.5.2	22.2	22.2	0.0	0.0	0.0	0.0
III.5.3	33.3	0.0	5.6	0.0	5.6	0.0
III.5.4	22.2	0.0	22.2	0.0	0.0	0.0
IV.4.4	31.3	0.0	0.0	0.0	0.0	0.0
IV.4.5	100.0	20.0	100.0	0.0	20.0	0.0
IV.5.1	5.9	0.0	5.9	0.0	0.0	0.0
IV.5.4	5.9	0.0	5.9	0.0	0.0	0.0
IV.5.5	23.1	23.1	0.0	0.0	0.0	0.0
V.2.4	10.7	0.0	10.7	0.0	0.0	0.0
V.2.5	7.3	7.3	7.3	7.3	7.3	7.3
V.3.1	5.9	0.0	5.9	0.0	0.0	0.0
V.3.2	5.7	0.0	5.7	0.0	0.0	0.0
V.4.2	17.9	17.9	0.0	0.0	0.0	0.0
V.5.2	7.7	7.7	0.0	0.0	0.0	0.0
V.5.5	5.6	0.0	5.6	0.0	0.0	0.0
No. of Optimal Found		19	12	33	27	34

* Problem Number: Roman numeral shows the problem set number, 2nd and 3rd numeral show the number of ASMs and SAM systems, respectively.

Table 5.6. Comparison of OC+2OX and 2OX+OC Heuristics with Best Results.

	OC+2OX	2OX+OC	Best
Number of times heuristic is best or tied for best	35	31	38
Average percentage below optimal value	0.77	2.64	0.65
Average rank among three results	1.05	1.37	1.00
Worst ratio of solution to optimal value	0.93	0.75	0.93
Number of times heuristic found the optimal solution	33	27	34

Improvement heuristics enhanced the quality of the solutions significantly. However, we do not specifically test our heuristics in terms of computation time, which is a very important issue for providing real time solutions. Up to this point, we investigated relatively small test problems in order to be able to compare the results of heuristics with optimal solutions. We generated large test problems in order to be able to test the performance of heuristics in terms of elapsed time. Table 5.7 depicts the results for those large test problems. The largest run time recorded is 1.170 seconds. We solved the problem with 15 ASMs and 20 SAM systems using 2OX+OC algorithm for that case. Run times of the improvement heuristics for most of the problems (44 out of 48 problems) are less than half a second.

Table 5.7. Performance of Heuristics for Large Problems in Terms of Elapsed Time.

# of ASMs	# of SAM Systems	*Elapsed Time (sec)					
		BEC	QUC	OC	2OX	OC+2OX	2OX+OC
10	10	0.000	0.000	0.010	0.020	0.020	0.020
10	15	0.010	0.000	0.020	0.020	0.020	0.020
10	20	0.000	0.000	0.010	0.020	0.040	0.030
15	10	0.000	0.000	0.100	0.090	0.141	0.160
15	15	0.010	0.000	0.050	0.040	0.090	0.080
15	20	0.000	0.000	0.080	0.101	0.110	1.170
20	10	0.000	0.000	0.120	0.100	0.241	0.180
20	15	0.000	0.000	0.080	0.110	0.170	0.200
20	20	0.000	0.000	0.161	0.180	0.320	0.330
25	10	0.000	0.000	0.141	0.300	0.441	0.350
25	15	0.010	0.000	0.200	0.291	0.390	0.411
25	20	0.000	0.010	0.561	0.410	0.881	0.961

* CPU time for the algorithms on a personal computer with AMD Athlon 2000+ CPU and 256 MB of RAM.

For all the test problems presented above, MAP solution procedure produced high quality solutions while satisfying the run time requirement for MAP.

We used small test problems in order to be able to compare the heuristic results with the optimal results. We restrict the number of total SAMs to 8. Thus, the average number of missiles available on the magazines for the problems with 5 SAM systems falls below 2 missiles per system. Since the average number of available missiles for each system is low, using this valuable asset against one ASM may prevent using it against another one more effectively at a later engagement. This argument is generally valid for construction algorithms. We expect that if we have had larger number of missiles per SAM system, construction algorithms would have produced better results. Although we intuitively state that argument, the formal investigation of the quality of the solutions for the large size problems should be investigated.

CHAPTER VI

SECTOR ALLOCATION MODELS

In this chapter, we present five different sector allocation models. Each model has some features and drawbacks that we discuss in detail. We start with a relatively simple one and continue with more developed ones.

In SAP, we would like to maximize the air defense effectiveness of the TG, i.e. the coverage level of each individual ship composing the TG. One may think that maximizing the area coverage does increase the effectiveness of the air defense shield around the TG since threat must pass through a longer defense layer in order to reach the TG, which is assumed to be stationed in the center of the area of defense. However, defending every square inch of the area at a relatively low level does not necessarily pay off. On the contrary, having multiple coverage over a ship increases her defensive potential and creates a stronger defense. An air defense ship may even provide a stronger defense when stationed between the threat and the target without physically covering the target. Thus, we focus on the air defense of individual ships in TG rather than defending the area around TG.

6.1 SAP1 - SECTOR ALLOCATION MODEL-I

Maximizing the air defense effectiveness of a TG may be represented by maximizing the probability of no-leaker as the objective function in a mathematical program. This yields a nonlinear objective function creating a need for futher treatment compared to that of a linear objective function. Thus, in our first formulation for SAP, we use an indirect treatment approach. We develop the model that incorporates probability of no-leaker function in the next section. In this section, we formulate a SAP using an expected value approach for objective function and discuss the solution procedures.

6.1.1 Formulation of the Problem

Suppose that there are n ships, indexed $i \in N = \{1,...,n\}$ and there are m sectors in which the warships composing the naval TG may be assigned, indexed $j \in M = \{1,...,m\}$. Let k be an alias for j. We further define a subset of ships, namely area air defense ships, indexed $a \in A = \{1,...,n_a\}$ and $A \subseteq N$. Let p_{jak} be the expected level of coverage provided to the ship at sector j by ship a at sector k. In this way, p_{jak} constitutes the link between MAP and SAP. We can calculate the coverage probabilities for a range of attack scenarios involving different area air defense ship types by using any of the MAP solution procedures. p_{jak} values can be calculated both for directional and omni-directional attack scenarios and can be used in SAP without any modification to the model. All of the SAP models presented in this chapter are based on the knowledge about this input parameter, the level of coverage provided by each AAD ship to all other ships in TG. The level of coverage depends on the distance between each pair of ships, direction of the attack and the bearing of the covered ship from the AAD ship. Note that this definition of p_{jak} characterizes the relationship between SAP and MAP. We will elaborate more on this relationship in Chapter VIII.

We need the following notation and variables to formulate the TG sector allocation problem:

w_i: the military value of ship i.

ps_i: the expected level of self-coverage of ship i.

$$x_{ij} = \begin{cases} 1, & \text{if ship } i \text{ is located at sector } j \\ 0, & \text{otherwise.} \end{cases}$$

$$Max \quad \sum_i w_i ps_i + \sum_i w_i \left(\sum_j \sum_a \sum_{\{k|k \neq j\}} p_{jak} x_{ij} x_{ak} \right) \tag{6.1}$$

subject to

$$\sum_j x_{ij} = 1 \quad \text{for all } i \in N \tag{6.2}$$

$$\sum_i x_{ij} \leq 1 \quad \text{for all } j \in M \tag{6.3}$$

$$x_{ij} \in \{0,1\} \quad \text{for all } i \in \mathrm{N} \text{ and } j \in M \tag{6.4}$$

The objective function (6.1) maximizes the total weighted expected level of coverage provided within the TG. Constraint set (6.2) ensures that every ship is assigned to a sector. Constraints of type (6.3) reflect that each sector can accommodate at most one ship. Constraint set (6.4) imposes binary restriction on the decision variables.

6.1.2 Solution Procedure

We have a quadratic term in the objective function. However we may remove the nonlinearity by introducing a new variable. Let $x_{ij} x_{ak} = y_{ijak}$. When both $x_{ij} = 1$ and $x_{ak} = 1$ then y_{ijak} is to be 1. y_{ijak} must take a value of zero for all the other cases. Since our objective function is of maximization type we need to force y_{ijak} to take a value of zero when required. We can guarantee y_{ijak} taking the correct values in two different ways among possible other ways.

First way:

$$y_{ijak} \le \frac{x_{ij} + x_{ak}}{2} \quad \text{for all } j,k \in M,\ i \in N \text{ and } a \in A \qquad (6.5)$$

$$y_{ijak} \in \{0,1\} \quad \text{for all } j,k \in M,\ i \in N \text{ and } a \in A \qquad (6.6)$$

Second way:

$$y_{ijak} \le x_{ij} \quad \text{for all } j,k \in M,\ i \in N \text{ and } a \in A \qquad (6.7)$$

$$y_{ijak} \le x_{ak} \quad \text{for all } j,k \in M,\ i \in N \text{ and } a \in A \qquad (6.8)$$

$$y_{ijak} \geq 0 \quad \text{for all } j,k \in M,\ i \in N \text{ and } a \in A \qquad (6.9)$$

In the first set, we introduce comparatively fewer constraints into the model. However we have to define y_{ijak} as a binary variable. In this case, even a small size problem instance may lead an intractable formulation because of the large number of binary variables. In the second set, we introduce twice as many constraints into the model as in the first case. However, we may relax the variable y_{ijak} to be defined as a continuous variable as a result of stronger constraints. We expect that the augmentation of the model with the second set of constraints will lead to a more tractable model. Thus, the resulting model can be written as follows:

$$Max \quad \sum_i w_i ps_i + \sum_i w_i \left(\sum_j \sum_a \sum_{\{k|k \neq j\}} p_{jak} y_{ijak} \right) \qquad (6.1')$$

subject to

(6.2), (6.3), (6.4), (6.7), (6.8), and (6.9).

Note that in this formulation we do not guarantee reaching an optimal solution in terms of the maximization of the coverage of the whole TG. Moreover we cannot control the coverage provided to each ship. That is, while one ship has a strong coverage, another ship may have relatively poor coverage. We may modify the model to maximize the minimum expected level of coverage provided to the ships. We will refer to the preceding model as SAP1.1 thereafter and define the model, SAP1.2 that has a maximin objective function as follows:

$$Max \quad \alpha \tag{6.10}$$

subject to

(6.2), (6.3), (6.4), (6.7), (6.8), and (6.9),

$$w_i\left(ps_i + \sum_j \sum_a \sum_{\{k|k \neq j\}} p_{jak}\, y_{ijak}\right) \geq \alpha \quad \text{for all } i \in N \tag{6.11}$$

$$\alpha \geq 0 \tag{6.12}$$

The objective function, equation (6.10) maximizes the minimum weighted expected level of coverage provided to any one of the ships in the TG. Constraint set (6.11) ensures that the objective function will be less than or equal to the minimum weighted expected level of coverage. Constraint (6.12), which is added for the sake of completeness imposes nonnegativity restriction on the decision variable.

SAP1.1 model resembles Quadratic Assignment Problem (QAP) in terms of the constraints. QAP is a NP-Hard problem. Enumeration algorithms, cutting plane algorithms for the linear transformation of the objective, and heuristic approaches are available to solve QAP in the literature. However, in the following sections, we develop stronger formulations that make use of special attributes of SAP such as having sister ships within the TG and classifying ships into three groups each having similar air defense capabilities.

6.2 SAP2 - SECTOR ALLOCATION MODEL-II

In previous section, we formulated SAP1 with an indirect treatment of probability of no-leaker objective. We used an expected value approach instead of a direct probabilistic one. In this section, we formulate a SAP with a probabilistic objective function and discuss the solution procedures for this problem.

6.2.1 Formulation of the Problem

Suppose that there are n ships, indexed $i \in N = \{1,...,n\}$ and there are m sectors in which the warships composing the naval TG may be assigned, indexed $j \in M = \{1,...,m\}$. Let k be an alias for j. We further define a subset of ships, namely area air defense ships, indexed $a \in A = \{1,...,n_a\}$ and $A \subseteq N$. Let p_{jak} be the probability that the ship at sector j is covered by ship a at sector k. Here, p_{jak} is defined as a probability measure different from the one in SAP1 models.

We need the following notation and variables to formulate SAP2:

ps_i : the self-defense probability of having no-leaker of ship i.

$$x_{ij} = \begin{cases} 1, & \text{if ship } i \text{ is located at sector } j \\ 0, & \text{otherwise.} \end{cases}$$

Then, SAP2 can be formulated as follows:

$$Max \quad \prod_i \left[1 - (1 - ps_i) \prod_j \prod_a \prod_{\{k|k \neq j\}} (1 - p_{jak})^{x_{ij} x_{ak}} \right] \tag{6.13}$$

subject to

(6.2), (6.3), and (6.4).

The objective function (6.13) maximizes the probability of no-leaker for the whole TG. We have a nonlinear objective function similar to that of MAP2 and MAP3 models. However, we have an additional quadratic term in the power. We can remove the quadratic term from the model as in SAP1 case. The revised model can be written as follows:

$$Max \quad \prod_i \left[1 - (1 - ps_i) \prod_j \prod_a \prod_{\{k | k \neq j\}} (1 - p_{jak})^{y_{ijak}} \right] \qquad (6.13')$$

subject to

(6.2), (6.3), (6.4), (6.7), (6.8), and (6.9).

SAP2 guarantees reaching an optimal solution in terms of the maximization of the coverage of the whole TG, whereas SAP1 does not have any explicit control over the coverage of the whole TG.

6.2.2 Solution Procedure

We may use the method described for model MAP2 for the linearization of the objective function. After taking the logarithms of equation (6.13'), the equation becomes: $Max \quad \sum_i \ln \left[1 - (1 - ps_i) \prod_j \prod_a \prod_{\{k | k \neq j\}} (1 - p_{jak})^{y_{ijak}} \right]$.

Equivalently we can write equation (6.13') as;

$$Max \quad \sum_{i \in N} \ln(h_i) \tag{6.14}$$

and introduce a new set of constraints into the problem as follows:

$1-(1-ps_i)\prod_j \prod_a \prod_{\{k|k \neq j\}} (1-p_{jak})^{y_{ijak}} \geq h_i$ for all $i \in N$. We can rewrite the constraint as follows after taking the logarithm of both sides:

$$\sum_j \sum_a \sum_{\{k|k \neq j\}} a_{jak} y_{ijak} + s_i \geq b_i \text{ for all } i \in N, \tag{6.15}$$

where $a_{jak} = -\ln(1-p_{jak})$, $b_i = -\ln(1-h_i)$, and $s_i = -\ln(1-ps_i)$.

Let $c_i = \dfrac{\ln(h_i)}{-\ln(1-h_i)}$ for all $i \in N$. Then objective function (6.14) becomes $Max \quad \sum_{i \in N} c_i b_i$.

Let c^1, c^2, c^3 be the slope of the line segments that approximate the function and b_i is defined as the sum of three different variables corresponding to those three line segments, $b_i = b_i^1 + b_i^2 + b_i^3$.

Then the resulting model SAP2 is as follows.

$$Max \sum_{i \in N} \left(c^1 b_i^1 + c^2 b_i^2 + c^3 b_i^3\right) \tag{6.16}$$

subject to

$$\sum_j \sum_a \sum_{\{k|k \neq j\}} a_{jak} y_{ijak} + s_i \geq b_i^1 + b_i^2 + b_i^3 \quad \text{for all } i \in N, \tag{6.17}$$

$$0 \leq b_i^1 \leq Z_1 \qquad \text{for all } i \in N \tag{6.18}$$

$$0 \leq b_i^2 \leq Z_2 - Z_1 \quad \text{for all } i \in N \tag{6.19}$$

$$0 \leq b_i^3 \leq Z_3 - Z_2 \quad \text{for all } i \in N \tag{6.20}$$

and (6.2), (6.3), (6.4), (6.7), (6.8), (6.9).

SAP1 and SAP2 formulations are similar to each other except the objective function. Both formulations resemble QAP in terms of constraints. We develop more tractable models in the following sections by using the special features of SAP.

6.3 SAP3 - SECTOR ALLOCATION MODEL-III

In this section, we formulate SAP as a location problem with nonlinear objective function. The constraints resemble those of a p-median formulation. We develop the model in two phases. First, we present a simple model with only one type of ship available in the TG. Second, we extend the simple model to include multiple types of ships in TG.

6.3.1 Formulation of the Problem

Suppose that there are P ships with identical air defense capabilities and there are m sectors in which the warships composing the naval TG may be assigned, indexed $j \in M = \{1, \ldots, m\}$. Let i be an alias for j. Let c_{ij} be the probability of coverage provided by the ship at sector j to sector i. c_{ij} parameters can be obtained by solving MAP, establishing the link between the two problems.

We need the following notation and variables to formulate the TG sector allocation problem:

$$y_j = \begin{cases} 1, & \text{if ship is assigned to sector } j \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if the demand at sector } i \text{ can be covered by a ship at sector } j \\ 0, & \text{otherwise.} \end{cases}$$

Then, SAP3.1 can be written as follows:

$$Max \quad \sum_i \left[1 - \prod_j \left(1 - c_{ij}\right)^{x_{ij}} \right] \tag{6.21}$$

subject to

$$\sum_j y_j = P \tag{6.22}$$

$$x_{ij} \le y_j \quad \text{for all } i, j \tag{6.23}$$

$$x_{ij} \le y_i \quad \text{for all } i, j \tag{6.24}$$

$$x_{ij} \in \{0,1\} \quad \text{for all } i, j \tag{6.25}$$

$$y_j \in \{0,1\} \quad \text{for all } j \tag{6.26}$$

The objective function (6.21) maximizes the sum of probabilities of no-leaker for the ships in TG. Maximizing the probability of no-leaker for the whole TG might have been a better objective for the TG commander. Here in this formulation, however, we have to use the summation of the probabilities of no-leaker for the ships, since the overall probability of no-leaker for the whole TG will always yield a value of zero because of the empty sectors. Note that x_{ij} is equal to 1 only when both sectors i and j accommodate ships. $\left[1-\prod_{j}\left(1-c_{ij}\right)^{x_{ij}}\right]$ is the probability of coverage for a ship at a sector, say i, by at least one ship in any other sector, say j. Constraint (6.22) enforces all of P ships to be allocated. Constraints (6.23) and (6.24) ensure that if there is no ship allocated to sector j then there can be no coverage provided from sector j, and if there is no demand (ship) at sector i then there can be no coverage provided to sector i. Constraints (6.25) and (6.26) enforce binary restrictions on the decision variables.

By using the correct c_{ij} parameters, we can accommodate both omni-directional and directional attack cases for SAP. When TG has no information about the direction of the attack, c_{ij} can be determined using MAP accordingly. In this case, the distance between any two sectors will be the primary factor affecting the coverage. When TG has information on the attack direction, c_{ij} can be determined using the distance and the relative bearing from sector j to sector i.

SAP3.1 can easily be extended to include different types of ships. We define a new index $k \in K$ denoting the ship types. We redefine the parameters and the decision variables to accommodate the ship types.

P_k: the number of ships of type k to assign sectors.

c_{ijk}: the probability of coverage provided by the ship of type k at sector j to sector i.

$$y_{jk} = \begin{cases} 1, & \text{if ship of type } k \text{ is assigned to sector } j \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ijk} = \begin{cases} 1, & \text{if the demand at sector } i \text{ can be covered by a ship of type } k \text{ at sector } j \\ 0, & \text{otherwise.} \end{cases}$$

Then, SAP3.2 can be written as follows:

$$Max \quad \sum_i \left[1 - \prod_j \prod_k \left(1 - c_{ijk}\right)^{x_{ijk}} \right] \tag{6.27}$$

subject to

$$\sum_j y_{jk} = P_k \quad \text{for all } k \tag{6.28}$$

$$\sum_k y_{jk} \leq 1 \quad \text{for all } j \tag{6.29}$$

$$x_{ijk} \leq \frac{1}{2}\left(y_{jk} + \sum_{l \in K} y_{il} \right) \quad \text{for all } i, j, k \tag{6.30}$$

$$x_{ijk} \in \{0,1\} \quad \text{for all } i, j, k \tag{6.31}$$

$$y_{jk} \in \{0,1\} \quad \text{for all } j, k \tag{6.32}$$

The objective function (6.27) maximizes the sum of probabilities of no-leaker for the ships in TG. Constraint (6.28) ensures that no more than the available ships are allocated. Constraint (6.29) enforces that each sector can accommodate at most one ship. Constraints (6.30) ensure that if there is no ship allocated to sector j, there can be no coverage provided from sector j, and if there is no demand (ship) at sector i, there can be no coverage provided to sector i. Constraints (6.31) and (6.32) enforce binary restrictions on the decision variables.

We can equivalently rewrite equation (6.28) as follows:

$$\sum_{j} y_{jk} \leq P_k \quad \text{for all } k \tag{6.28'}$$

Since objective function (6.27) forces x_{ijk} to take positive values, and x_{ijk} forces y_{jk} to be as large as possible, summation of y_{jk} over the sectors will be equal to the total number of ships of respective type. Same reasoning is also valid for equation (6.22) in SAP3.1 model.

SAP3 models have similarities in the constraints with the models in location literature. This may enable us to use similar solution approaches. Additionally, SAP3.2 model captures the reality of having multiple ships of the same type in TG and uses it as a simplifying assumption in modeling the problem. Therefore we prefer SAP3.2 to previous SAP formulations, which treat the ships individually.

6.3.2 Solution Procedure

SAP formulation has resemblance to maximal covering location problem in the constraints, and MAP2 and MAP3 in the objective function. Before developing any solution procedure, we need to get rid of the non-linearity in the objective function. We can use the same procedure as in MAP2 case.

We can write equation (6.27) as $Max \quad \sum_i h_i$ (6.33) and introduce a new set of constraints into the problem as follows:

$1-\prod_j \prod_k \left(1-c_{ijk}\right)^{x_{ijk}} \geq h_i$ for all $i \in M$. We can rewrite the constraint as follows after taking the logarithm of both sides:

$$\sum_j \sum_k a_{ijk} x_{ijk} \geq b_i \quad \text{for all } i \in M, \tag{6.34}$$

where $a_{ijk} = -\ln(1-c_{ijk})$, and $b_i = -\ln(1-h_i)$.

Let $c_i = \dfrac{h_i}{-\ln(1-h_i)}$ for all $i \in M$. Then objective function (6.33) becomes $Max \quad \sum_i c_i b_i$.

Since we develop SAP3.2 further as we proceed, we call original SAP3.2 as SAP3.2-P thereafter. The resulting linear formulation of SAP3.2-P model, SAP3.2-L is as follows.

$$Max \sum_i \left(c^1 b_i^1 + c^2 b_i^2 + c^3 b_i^3\right) \tag{6.35}$$

subject to

$$\sum_j \sum_k a_{ijk} x_{ijk} \geq b_i^1 + b_i^2 + b_i^3 \quad \text{for all } i \in M, \tag{6.36}$$

and (6.28'), (6.29), (6.30), (6.31), (6.32), (6.18), (6.19), (6.20).

Comparison of SAP3.2-L with the Original SAP3.2-P Formulation:

The resulting linear formulation of SAP, SAP3.2-L is an approximation of the original nonlinear programming formulation of SAP3.2. In the new formulation, we approximate the objective function value. Here, in this part of the section we verify the representativeness of the approximation.

We approximate the nonlinear objective function coefficients with three line segments. Taking a conservative approach, we make sure that the original function is greater than the approximate line segments. Thus, the approximation underestimates the objective function.

We relax the binary restriction on the decision variables in both of the formulations in order to be able to solve the models. Moreover, we need to show that the NLP model produces global optimum solutions. The following proof of concavity of the objective function shows that the NLP model will always produce a global optimum solution.

Proposition: $Z_i = 1 - \prod_{jk} \left(1 - c_{ijk}\right)^{x_{ijk}}$, $0 \le c_{ijk} < 1$, $0 \le x_{ijk} \le 1$ is concave if and only if

$$1 - \prod_{jk} \left(1 - c_{ijk}\right)^{\lambda x_{ijk}^{1} + (1-\lambda) x_{ijk}^{2}} \ge \lambda \left[1 - \prod_{jk} \left(1 - c_{ijk}\right)^{x_{ijk}^{1}} \right] + (1-\lambda) \left[1 - \prod_{jk} \left(1 - c_{ijk}\right)^{x_{ijk}^{2}} \right].$$

Proof:

$$\prod_{jk} \left[\left(1 - c_{ijk}\right)^{\lambda x_{ijk}^{1}} \quad \left(1 - c_{ijk}\right)^{(1-\lambda) x_{ijk}^{2}} \right] \le \lambda \left[\prod_{jk} \left(1 - c_{ijk}\right)^{x_{ijk}^{1}} \right] + (1-\lambda) \left[\prod_{jk} \left(1 - c_{ijk}\right)^{x_{ijk}^{2}} \right]$$

$$\left[\prod_{jk}\left(1-c_{ijk}\right)^{x_{ijk}^{1}}\right]^{\lambda}\left[\prod_{jk}\left(1-c_{ijk}\right)^{x_{ijk}^{2}}\right]^{(1-\lambda)} \leq \lambda\left[\prod_{jk}\left(1-c_{ijk}\right)^{x_{ijk}^{1}}\right]+(1-\lambda)\left[\prod_{jk}\left(1-c_{ijk}\right)^{x_{ijk}^{2}}\right]$$

Let $y_1 = \prod_{jk}\left(1-c_{ijk}\right)^{x_{ijk}^{1}}$ and $y_2 = \prod_{jk}\left(1-c_{ijk}\right)^{x_{ijk}^{2}}$, then $y_1^{\lambda} y_2^{(1-\lambda)} \leq \lambda y_1 + (1-\lambda) y_2$

$$\ln\left(y_1^{\lambda} y_2^{(1-\lambda)}\right) \leq \ln\left(\lambda y_1 + (1-\lambda) y_2\right)$$

$$\lambda \ln(y_1) + (1-\lambda)\ln(y_2) \leq \ln\left[\lambda y_1 + (1-\lambda) y_2\right]$$

The last equation implies that proving the concavity of the objective function is the same as proving the concavity of the logarithmic function, $\ln(y)$. From the second derivative of $\ln(y)$, we get $\ln''(y) = \frac{-1}{y^2} < 0$, which establishes the concavity of $\ln(y)$. ■

To have an idea about the quality of linear approximation, linear programming relaxation of SAP3.2-L and nonlinear programming solution of SAP3.2-P with relaxed binary restrictions on the decision variables have been solved for several test problems in the presence of 19 sectors. Results are presented in Table 6.1.

The gap between SAP3.2-P and SAP3.2-L solutions was less than 3% for all the cases even with this rough approximation.

Table 6.1. Results of the Test Problems.

Total Number of Ships	Number of Ships of Type (1,2,3)	Obj. Value of SAP3.2-L*	Obj. Value of SAP3.2-P**	Gap (%)
3	1,1,1	13.93	14.36	2.96
4	1,1,2	14.31	14.71	2.77
4	1,2,1	14.43	14.81	2.59
4	2,1,1	17.00	17.36	2.07
5	1,2,2	14.74	15.11	2.46
5	2,1,2	17.21	17.57	2.05
5	2,2,1	17.24	17.59	1.99
6	2,2,2	17.37	17.74	2.09
10	2,5,3	17.84	18.12	1.57

* Linear programming relaxation of SAP3.2-L

** Solution of SAP3.2-P with relaxed binary restrictions on decision variables

Lower Bounding Strategies

Since SAP3.2-P has a nonlinear objective function, we cannot solve it directly. Although we proved that the objective function of SAP3.2-P is concave, solving a nonlinear 0-1 integer programming problem is out of the scope of this research. Instead, we developed a linearization procedure for SAP3.2-P. Without a formal proof, we can say that SAP3.2-L is very hard to solve in terms of computational complexity. Kariv and Hakimi (1979) proved that the problem of finding a p-median of a network is NP-hard even when the network has a simple structure. SAP3.2-L has a complex objective function and additional constraints besides those similar to the p-median constraints. Thus, development of tight upper and lower bounds is very important for solving the problem successfully.

We developed a randomized heuristic (taking the best of a large number of randomly generated solutions) to establish a simple lower bound for SAP3.2-L. This is taken as a first step toward developing tighter lower bounds. Randomized heuristic performed well for the small test problems that we could solve to optimality. The results are depicted in Table 6.2. However, we expect that the quality of lower bounds will deteriorate as the size of the problem increases.

Table 6.2. Results of the Randomized Heuristic.

Total Ships	Ship Types[1]	Obj. Value SAP3.2-L	Obj. Value SAP3.2-P[2]	Number of Solutions Generated (best is choosen) 5000	10000	25000	50000	100000	200000
3	1,1,1	2.61	2.69	2.69	2.69	2.69	2.69	2.69	2.69
4	1,1,2	3.49	3.57	3.56	3.58	3.59	3.59	3.59	3.59
4	1,2,1	3.52	3.63	3.61	3.62	3.62	3.63	3.63	3.63
4	2,1,1	3.82	3.88	3.87	3.87	3.88	3.88	3.88	3.88
5	1,2,2	4.41	4.53	4.50	4.50	4.51	4.52	4.52	4.53
5	2,1,2	4.80	4.86	4.85	4.85	4.85	4.86	4.86	4.86
5	2,2,1	4.80	4.86	4.86	4.86	4.86	4.86	4.86	4.86
6	2,2,2	5.78	5.85	5.83	5.83	5.83	5.84	5.84	5.85
10	2,5,3	9.71	9.77	9.75	9.76	9.76	9.76	9.77	9.77

[1] Number of Ships of Type 1,2, and 3.

[2] SAP3.2-P Objective Calculated for SAP3.2-L Solution.

We also developed Lagrangean Relaxation of SAP3.2-L by relaxing constraint set (6.30). Lagrangean Relaxation scheme produced high quality lower bounds through the Lagrangean heuristic developed within the relaxation procedure. Lagrangean subproblems produce feasible solutions in terms of allocating ships to sectors. Then calculating the lower bound is a matter of finding the correct linking variables and substituting them in the original objective function. Since Lagrangean Relaxation scheme failed to produce reasonable upper bounds, which will be discussed next, we chose not to use this lower bounding strategy and stopped any further experimentation.

The last lower bounding scheme is achieved through modifying the objective function of SAP3.2-P as follows:

$$Max \quad \sum_{i=1}^{m}\left[1-\prod_{j}\prod_{k}\left(1-c_{ijk}\right)^{x_{ijk}}\right]=Min\sum_{i}\prod_{j}\prod_{k}\left(1-c_{ijk}\right)^{x_{ijk}}$$

Taking the logarithm of the objective function does not change the optimum solution to the problem. Then we have,

$$Min \ln\left[\sum_{i}\prod_{j}\prod_{k}\left(1-c_{ijk}\right)^{x_{ijk}}\right].$$

Since logarithm is a concave function, $\ln(a)+\ln(b)\geq\ln(a+b)$. This implies that,

$$\ln\left[\sum_{i}\prod_{j}\prod_{k}\left(1-c_{ijk}\right)^{x_{ijk}}\right]\leq\sum_{i}\ln\left[\prod_{j}\prod_{k}\left(1-c_{ijk}\right)^{x_{ijk}}\right].$$

Thus, $Min\sum_{i}\ln\left[\prod_{j}\prod_{k}\left(1-c_{ijk}\right)^{x_{ijk}}\right]$ constitutes an upper bound for the minimization problem. This would yield a lower bound for SAP3.2-P. We include this lower bounding scheme in order to present the idea. Actual solution procedure for SAP3.2 will be presented in the next chapter.

Upper Bounding Strategies

LP relaxation to SAP3.2-L produces loose upper bounds (see Table 6.1 and Table 6.2). It does not lead to an efficient solution procedure for SAP3.2-L.

Lagrangean Relaxation also failed to produce tight upper bounds. The upper bounds produced by Lagrangean Relaxation were no better than the upper bounds produced by the LP relaxation. We think that the failure to produce a tight upper bound is caused by the summation in equation (6.30); each term in the objective function of the Lagrangean subproblem controlled by one Lagrangean multiplier depends on some other term because of the summation. Thus, Lagrangean multipliers do not control and reduce the infeasibility independent of each other.

We add valid inequalities derived from the physical nature of the problem with the hope of getting tighter upper bounds. These valid inequalities are;

$$x_{ijk} \le y_{jk} \quad \text{for all } i, j, k \tag{6.37}$$

$$x_{ijk} \le \sum_{l \in K} y_{il} \quad \text{for all } i, j, k \tag{6.38}$$

$$\sum_{i} x_{ij,k=1} \le y_{j,k=1} \sum_{k} P_k \quad \text{for all } j \tag{6.39}$$

$$\sum_{i} x_{ij,k=2} \le y_{j,k=2} \quad \text{for all } j \tag{6.40}$$

$$\sum_{k} x_{ijk} \le 1 \quad \text{for all } i, j \tag{6.41}$$

$$\sum_{jk} x_{ijk} \le P_{k=1} \sum_{k} y_{ik} + y_{i,k=2} \quad \text{for all } i \tag{6.42}$$

Constraint sets (6.37) and (6.38) are the stronger version of constraint set (6.30). Surrogate constraint (6.39) limits the total number of linking variables emanating from a sector occupied by an AAD ship with the total number of ships in TG. Constraint (6.40) limits the number of linking variables to 0 or 1 depending on the presence of a SD ship in the sector. Constraint (6.41) limits the total number of linking variables between any pair of sectors. Constraint set (6.37) is stronger than constraints (6.40) and (6.41). Constraint (6.42) limits the total number of linking variables entering to each sector. If there is a ship of any type in sector i, there could be at most $P_{k=1}+1$ coverage links to that sector (i.e. the total number of AAD ships plus the self defense link).

Addition of constraints (6.37)-(6.42) to SAP3.2-L do not give promising results in terms of tightening the upper bound. We report the results in Table 6.3.

Table 6.3. Results of the Upper Bound Improvement Process.

Total Ships	Ship Types[1]	SAP3.2-P[2] (1)	SAP3.2-L[3] (2)	% Gap (1 vs. 2)	SAP3.2-L[3] w/ cuts (3)	% Gap (1 vs. 3)	Reduction in Upper Bound (%)
3	1,1,1	2.69	13.93	80.7	4.49	40.1	67.8
4	1,1,2	3.57	14.31	75.0	5.86	39.1	59.0
4	1,2,1	3.63	14.43	74.8	6.17	41.2	57.2
4	2,1,1	3.88	17.00	77.2	9.74	60.1	42.7
5	1,2,2	4.53	14.74	69.3	7.47	39.4	49.3
5	2,1,2	4.86	17.21	71.8	11.58	58.0	32.7
5	2,2,1	4.86	17.24	71.8	11.82	58.9	31.4
6	2,2,2	5.85	17.37	66.3	12.84	54.5	26.1
7	2,2,3	6.83	17.48	61.0	13.77	50.4	21.3
8	2,3,3	7.82	17.61	55.6	14.74	47.0	16.3
9	2,3,4	8.79	17.70	50.3	15.45	43.1	12.7
10	2,5,3	9.77	17.84	45.2	16.27	39.9	8.8
10	3,5,2	9.92	18.52	46.4	17.33	42.8	6.5

[1] Number of Ships of Type 1,2, and 3.
[2] SAP3.2-P Objective Calculated for SAP3.2-L Solution.
[3] Linear programming relaxation of SAP3.2-L

Test problems are generated for 19 sectors. When the number of ships is small compared to the number of sectors, the percent reduction in the upper bound and in the gap is impressive. A maximum of 67.8% reduction in upper bound is achieved when TG has 3 ships. However, the percent gap between the upper bound and the lower bound stayed between 39.1 and 60.1%. The percent reduction in upper bound decreases as total number of ships increases. One may consider increasing the number of sectors in order to decrease the ratio of total number of ships to total possible sectors, but this increases the number of variables and constraints immensely.

Solution approaches for SAP3.2-P and SAP3.2-L presented above and the continuation of the trials in the wake of preceding approaches produced unsatisfactory results for SAP3.2-L (Neither the Lagrangean relaxation nor the valid inequalities generated sufficiently tight upper bounds.). We present a new variant of SAP3.2, which maximizes the sum of coverage below. We call new SAP3.2 as SAP3.2-C. We introduce another model, SAP4 in the following sections before any discussion on the reasoning to use SAP3.2-C and SAP4 instead of SAP3.2-L. SAP4 also maximizes the sum of coverage.

SAP3.2-C model is the same as SAP3.2-P except the objective function. Here in SAP3.2-C, we maximize the total coverage provided to the ships of the TG.

$$Max \quad \sum_{i}\sum_{j}\sum_{k} c_{ijk} x_{ijk} \tag{6.43}$$

subject to

(6.28), (6.29), (6.30), (6.31), and (6.32).

6.4 SAP4 - SECTOR ALLOCATION MODEL-IV

Suppose that there are P AAW ships with identical air defense capabilities and R ND ships with no effective air defense capability, i.e. we restrict ourselves by two types of ships. Let m be the total number of sectors in which the warships composing the naval TG may be assigned, indexed $j \in M = \{1,...,m\}$. Let i be an alias for j.

c_{ij}: the coverage provided by the AAW ship at sector j to sector i.

$$y_j = \begin{cases} 1, & \text{if an AAW ship is allocated to sector } j \\ 0, & \text{otherwise.} \end{cases}$$

$$z_i = \begin{cases} 1, & \text{if we decide to defend sector } i \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if there is a ship at sector } j \text{ that can cover sector } i \text{ and we decide to defend sector } i \\ 0, & \text{otherwise.} \end{cases}$$

Then, SAP4 can be written as follows:

$$Max \quad \sum_i \sum_{j \neq i} c_{ij} x_{ij} \tag{6.44}$$

subject to

$$\sum_j y_j \leq P \tag{6.45}$$

$$x_{ij} \leq y_j \quad \text{for all } i, j, \quad i \neq j \tag{6.46}$$

$$\sum_i z_i \leq R \tag{6.47}$$

$$x_{ij} \leq z_i \quad \text{for all } i, j, \quad i \neq j \tag{6.48}$$

$$y_j + z_j \leq 1 \quad \text{for all } j \tag{6.49}$$

$$x_{ij} \geq 0 \quad \text{for all } i, j, \quad i \neq j \tag{6.50}$$

$$y_j \in \{0,1\} \quad \text{for all } j \tag{6.51}$$

$$z_i \in \{0,1\} \quad \text{for all } i \tag{6.52}$$

The objective function maximizes the sum of the coverage provided to the ships in TG. Constraints (6.45) and (6.47) enforce respectively at most P ships and R

ships to be allocated. Constraints (6.46) and (6.48) ensure that if there is no AAW ship allocated to sector j then there can be no coverage provided from sector j, and if there is no demand (ND ship) at sector i then there can be no coverage provided to sector i. Constraint (6.49) ensures that each sector can have at most one ship. Constraints (6.50), (6.51) and (6.52) enforce binary and non-negativity restrictions on the decision variables.

6.4.1 Discussion

In this section, we elaborate on the use of coverage instead of probability of no-leaker in the objective function.

In SAP, the objective is to determine a robust air defense formation for a naval TG with known ships and air defense capabilities. We still need to utilize the coverage parameter, which is the measure of how well one ship can defend herself or another ship against a perceived and aggregated threat. That is, we do not know the threat exactly, but we can predict the threat using information from different sources such as intelligence and surveillance. Alternatively, we can estimate the coverage parameter by aggregating the results from a number of likely scenarios. Although solving SAP using a probability of no-leaker objective function is desirable, it is unreasonable to accept the computational burden due to the probability of no-leaker objective function, considering the fact that the threat is defined vaguely.

We have also checked the quality of the solutions produced by a coverage model, here SAP4, through calculating the SAP3.2-L objective function value for

SAP4 solution and contrasting this with the genuine objective of SAP3.2-L. Table 6.4 depicts the % gap, which is calculated as follows:

$$\%Gap = \frac{Z_{SAP4} - Z_{SAP3.2-L}}{Z_{SAP3.2-L}} * 100$$

where Z_{SAP4} is the SAP3.2-L objective value calculated for the solution found with SAP4. Table 6.5 shows % gap between SAP3.2-L and SAP4 solutions in terms of the original SAP3.2-P objective function. Maximum % gap between two solutions for different combinations of the ships is less than two percent for both comparisons. These results enable us to state that the coverage objective is a good approximation for the probability of no-leaker objective. Therefore, we can try to solve SAP3.2-C instead of SAP3.2-P, reducing the computational burden substantially.

Table 6.4. % Gap Between SAP3.2-L and SAP4 Solutions in Terms of SAP3.2-L Objective Function.

Number of AAD Ships	Number of ND Ships						
	2	3	4	5	6	8	10
2	0.00	0.00	-0.31	-0.25	-0.03	0.00	0.00
3	-1.33	-1.12	-0.96	-0.84	-0.74	-0.46	-0.42
4	-0.73	-0.64	-0.56	-0.49	-0.44	-0.34	-0.28
5	-0.58	-0.23	-0.34	-0.17	-0.17	-0.02	-0.01
6	-0.40	-0.09	0.00	-0.07	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6.5. % Gap Between SAP3.2-L and SAP4 Solutions in Terms of SAP3.2-P Objective Function.

Number of AAD Ships	Number of ND Ships						
	2	3	4	5	6	8	10
2	0.55	0.69	1.19	1.18	1.37	1.23	1.06
3	0.41	0.65	0.82	0.94	1.04	1.29	1.29
4	1.10	1.22	0.15	-0.17	0.28	0.42	0.00
5	-0.33	-0.04	-0.01	-0.06	-0.04	0.02	0.01
6	-0.26	-0.12	-0.08	-0.10	0.26	0.02	0.01
8	-0.03	0.00	0.01	0.02	-0.02	-0.01	1.93

6.4.2 Solution Procedure

We have added the following valid inequalities derived from the physical nature of the problem to SAP4 model. These valid inequalities are;

$$\sum_j x_{ij} \leq P\, z_i \quad \text{for all } i \tag{6.53}$$

$$\sum_i x_{ij} \leq R\, y_j \quad \text{for all } j \tag{6.54}$$

Constraint (6.53) limits the number of linking variables by P, if there is a SD ship or ND ship in the sector. Otherwise, the number of linking variables is limited to zero. Constraint (6.54) limits the total number of linking variables by R, if there is an AAD ship in the sector. Otherwise, the number of linking variables is zero.

We can show the validity of the new inequalities, (6.53) and (6.54) using the following arguments:

$x_{ij} \leq z_i$ (constraint 6.48) then $\sum_i x_{ij} \leq \sum_i z_i \leq R$ (using constraint 6.47).

$x_{ij} \le y_j$ (constraint 6.46) then, if $y_j = 0$, $x_{ij} = 0 \;\; \forall i$ and if $y_j = 1$, $x_{ij} \le 1 \;\; \forall i$. Thus, $\sum_i x_{ij} \le R\, y_j$ for all j (constraint 6.54). The same reasoning is valid for constraint (6.53).

Linear programming (LP) relaxation of SAP4 produced integer results after adding valid inequalities (6.53) and (6.54). However, our research on unimodularity proof of the LP relaxation's coefficient matrix revealed a negative result: the coefficient matrix is not totally unimodular. Four out of 42 experiments shown in Table 6.6 gave fractional solutions. We tried to develop additional cuts that warrant integer solution. Following is another valid inequality that improves the quality of LP relaxation.

$$x_{ij} + x_{ji} + x_{ik} + x_{ki} + x_{jk} + x_{kj} \le 2 \quad \text{for all } i, j, k \quad \text{and} \quad i \ne j \ne k \qquad (6.55)$$

Equation (6.55) restricts the number of links between any set of three sectors, i.e. if there are two AAD ships and one ND ship (or two ND ships and one AAD ship) in three sectors, there should be two links, otherwise there should be less than two links. Addition of equation (6.55) cut three out of four fractional solutions. Thus, we produced 41 integer solutions out of 42 problem instances using LP relaxation.

Table 6.6. Results of SAP4 Using LP Relaxation.

Number of AAD Ships		Number of ND Ships						
		2	3	4	5	6	8	10
2	Obj. Value	3.68	5.47	7.21	8.96	10.71	14.12	17.42
	*Time / **Solution	0/+	0/+	0/+	0/+	0/+	0/+	0/+
3	Obj. Value	5.49	8.16	10.76	13.35	15.93	20.82	25.58
	Time / Solution	0/+	0/+	0/+	0/+	0/+	1/+	1/+
4	Obj. Value	7.26	10.79	14.21	17.51	20.75	26.98	33.03
	Time / Solution	1/+	0/+	0/+	0/+	1/+	0/+	0/+
5	Obj. Value	8.96	13.18	17.42	21.48	25.44	32.97	40.31
	Time / Solution	0/+	0/+	0/+	0/+	1/+	1/-	0/-
6	Obj. Value	10.63	15.57	20.56	25.39	29.90	39.05	47.57
	Time / Solution	0/+	0/+	1/+	0/+	1/-	0/+	0/-
8	Obj. Value	13.74	20.07	26.45	32.80	38.97	50.36	60.80
	Time / Solution	0/+	1/+	0/+	0/+	0/+	0/+	1/+

* Time in CPU Second.

** + shows that solution is integer, - shows that solution is fractional.

6.5 SAP5 - SECTOR ALLOCATION MODEL-V

Here, the objective is to maximize the minimum coverage of the ships in TG. A comprehensive presentation of the model is as follows:

Suppose that there are P AAD ships with identical air defense capabilities and R ships with no effective air defense capability. Let m be the total number of sectors in which the warships composing the naval TG may be assigned, indexed $j \in M = \{1,...,m\}$. Let i be an alias for j.

c_{ij} : the coverage provided by the AAW ship at sector j to sector i.

α : decision variable.

ϕ : a very large number.

$$y_j = \begin{cases} 1, & \text{if an AAW ship is allocated to sector } j \\ 0, & \text{otherwise.} \end{cases}$$

$$z_i = \begin{cases} 1, & \text{if we decide to defend sector } i \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if there is a ship at sector } j \text{ that can cover sector } i \text{ and we decide to defend sector } i \\ 0, & \text{otherwise.} \end{cases}$$

Then, SAP5 can be written as follows:

$$Max \quad \alpha \tag{6.56}$$

subject to

$$\sum_j y_j = P \tag{6.57}$$

$$x_{ij} \le y_j \quad \text{for all } i, j \tag{6.58}$$

$$\sum_i z_i = R \tag{6.59}$$

$$x_{ij} \le z_i \quad \text{for all } i, j \tag{6.60}$$

$$y_j + z_j \le 1 \quad \text{for all } j \tag{6.61}$$

$$\alpha \le \sum_j c_{ij} x_{ij} + \phi(1 - z_i) \quad \text{for all } i \tag{6.62}$$

$$x_{ij} \ge 0 \quad \text{for all } i, j, i \ne j \tag{6.63}$$

$$y_j \in \{0,1\} \quad \text{for all } j \tag{6.64}$$

$$z_i \in \{0,1\} \quad \text{for all } i \tag{6.65}$$

$$\alpha \ge 0 \tag{6.66}$$

Equations (6.56) and (6.62) enforce the maximization of the minimum coverage. The rest of the constraints are common in both SAP4 and SAP5 formulations.

Because of constraint (6.62), SAP5 has to be solved using MIP formulation. LP relaxation of SAP5 does not produce integer solutions. All experiments we carried out produced fractional solutions. Thus, the results are not reasonable in a maximin context. We investigate the trade-off between using SAP4 and using SAP5 in the next section.

6.5.1 Discussion

In this subsection, we elaborate on the use of sum of coverage and maximin coverage in the objective function. A tactical commander at sea may be better informed if he or she knows the minimum coverage of the ships in TG instead of the sum of coverage. (Conceptually, maximizing the minimum coverage is also closer to our original objective of maximizing the probability of no-leaker, which we had to give up due to nonlinearity. Both objectives take a conservative approach and try to minimize the risk.) Since we maximize the sum of coverage in SAP4, the model may produce unbalanced protection for the ships in TG. We may have heavily defended some ships and poorly defended others in the optimal solution. SAP5 should produce more balanced coverage for all ships in TG. However, SAP5 does not let us use the LP relaxation scheme and nice features of SAP4. Thus, comparison of SAP4 and SAP5 in terms of maximizing the minimum coverage versus computation time may be helpful to determine the relative merit of SAP5 formulation. Table 6.7 shows such a comparison of instances with SAP4 solutions calculated in terms of SAP5 objective, i.e. we report the minimum coverage obtained in SAP4 solution.

There are two different SAP5 solutions in Table 6.7. We solved SAP5 with relative termination criteria equals 0.1 (OPTCR=0.1) using GAMS/Cplex MIP solver. In many cases OPTCR=0.1 produced inferior solutions. We reduced relative termination criteria to 0.01 for those cases and reported the results in the same Table. SAP4 solves faster than SAP5 generally. Table 6.8 depicts percent gap between SAP4 solution (in terms of SAP5 objective function) and the best SAP5 solution. Note that SAP5 still have inferior solutions (3 AAD ships and 2 or 10 ND ships). Percent gap is zero for those cases in true optimality of SAP5 solutions.

Table 6.8 shows that SAP4 and SAP5 solutions are comparable. It is plausible to say that SAP4 does not produce solutions that are highly unbalanced. The maximum gap between SAP4 and SAP5 solutions in terms of maximin coverage is 7.0 percent. Before a more detailed discussion on the gap between SAP4 and SAP5 solutions, we go back to the discussion on the integrality of the solution for LP relaxed SAP4 formulation. Thirty-eight out of 42 experiments shown in Table 6.7 gave non-fractional solutions. We solved those instances that produce fractional solutions using MIP solver. The results reported in Table 6.9 show that the LP relaxation of SAP4 produce tight upper bounds even for the cases that produce fractional solutions. Note that, problem instances depicted in Table 6.9 produce fractional solutions when solved with LP relaxed formulation.

Table 6.7. Comparison of SAP4 and SAP5 Solutions. SAP4 Solution Objective is Calculated in Terms of SAP5 Objective Function.

# of AAD Ships		# of ND Ships						
		2	3	4	5	6	8	10
2	SAP5 Obj.(OPTCR=0.1)	1.80	1.71	1.63	1.7	1.61	1.70	1.532
	Time (sec.)	0	0	0	0	1	6	3
	SAP5 Obj.(OPTCR=0.01)		1.79	1.77	1.75	1.75		1.63
	Time (sec.)		0	1	1	1		5
	SAP4 Soln.Obj.	1.80	1.79	1.77	1.75	1.75	1.70	1.63
	Time (sec.)/Solution	0/+	0/+	0/+	0/+	0/+	0/+	0/+
3	SAP5 Obj.(OPTCR=0.1)	2.67	2.58	2.55	2.59	2.58	2.35	2.23
	Time (sec.)	1	1	1	1	1	3	5
	SAP5 Obj.(OPTCR=0.01)	2.67	2.67	2.60			2.43	2.35
	Time (sec.)	1	1	1			3	7
	SAP4 Soln.Obj.	2.69	2.67	2.60	2.59	2.580	2.41	2.36
	Time (sec.)/Solution	0/+	0/+	0/+	0/+	0/+	0/+	1/+
4	SAP5 Obj.(OPTCR=0.1)	3.56	3.42	3.42	3.30	3.25	3.01	2.97
	Time (sec.)	1	1	1	3	2	4	3
	SAP5 Obj.(OPTCR=0.01)		3.53			3.30	3.25	3.05
	Time (sec.)		1			2	5	4
	SAP4 Soln.Obj.	3.56	3.53	3.42	3.30	3.24	3.10	3.00
	Time (sec.)/Solution	1/+	0/+	0/+	0/+	0/+	0/+	0/+
5	SAP5 Obj.(OPTCR=0.1)	4.01	4.08	4.05	3.94	4.01	3.70	3.59
	Time (sec.)	1	2	1	2	2	3	3
	SAP5 Obj.(OPTCR=0.01)	4.41	4.32	4.24	4.14		4.00	3.66
	Time (sec.)	1	2	1	2		7	3
	SAP4 Soln.Obj.	4.41	4.25	4.24	4.06	3.96	3.86	3.66
	Time (sec.)/Solution	0/+	0/+	0/+	0/+	1/+	1/-	0/-
6	SAP5 Obj.(OPTCR=0.1)	5.21	5.07	4.66	4.73	4.75	4.73	4.13
	Time (sec.)	1	1	1	1	2	3	4
	SAP5 Obj.(OPTCR=0.01)			4.99	4.87			4.38
	Time (sec.)			1	2			7
	SAP4 Soln.Obj.	5.21	5.04	4.99	4.83	4.48	4.73	4.25
	Time (sec.)/Solution	0/+	0/+	0/+	0/+	1/-	0/+	0/-
8	SAP5 Obj.(OPTCR=0.1)	6.315	6.35	6.03	6.35	6.115	5.52	5.38
	Time (sec.)	1	2	1	1	4	4	2
	SAP5 Obj.(OPTCR=0.01)	6.72	6.55	6.39		6.22		
	Time (sec.)	1	2	1		5		
	SAP4 Soln.Obj.	6.71	6.54	6.39	6.35	6.165	5.51	5.03
	Time (sec.)/Solution	0/+	1/+	0/+	0/+	0/+	0/+	0/+

+ shows that solution is integer, - shows that solution is fractional

Table 6.8. % Gap Between SAP4 and SAP5 Solutions Calculated in Terms of SAP5 Objective Function.

# of AAD Ships	# of ND Ships						
	2	3	4	5	6	8	10
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	-0.7	0.0	0.0	0.0	0.0	0.8	-0.4
4	0.0	0.0	0.0	0.0	1.9	4.8	1.7
5	0.0	1.6	0.0	2.0	1.3	3.6	0.0
6	0.0	0.6	0.0	0.8	6.0	0.0	3.1
8	0.1	0.2	0.0	0.0	0.9	0.2	7.0

Table 6.9. Comparison of Optimal Objective Values of SAP4 for LP Relaxation and MIP Formulations.

# of AAD Ships		# of ND Ships		
		6	8	10
5	Z (LP Relax.)	-	33.00	40.38
	Z (MIP)	-	32.95	40.31
	% Gap	-	0.15	0.17
6	Z (LP Relax.)	29.94	-	47.57
	Z (MIP)	29.87	-	47.57
	% Gap	0.25	-	0.00

6.5.2 SAP4.5 - Sector Allocation Model-4.5

As to the discussion on balanced coverage of the ships in TG, tilting the objective function of SAP4 by using an objective function weight towards SAP5 objective is another course of action that we investigated. We developed an intermediate model between SAP4 and SAP5. Thus we refer to this model as SAP4.5, which is formulated below:

ε : objective function weight.

$$Max \quad \sum_{i}\sum_{j \neq i} c_{ij} x_{ij} + \varepsilon \alpha \tag{6.64}$$

subject to

(6.54), (6.55), (6.56), (6.57), (6.58), (6.59), (6.60), (6.61), (6.62), and (6.63).

We investigated for intermediate solutions in terms of maximin objective of SAP5 by using SAP4.5 with different objective function weights. The objective function of SAP4.5 includes two terms, namely a maxisum term and a maximin term. We expected to increase the quality of solution in terms of maximizing the minimum coverage by incrementally increasing the objective function weight of maximin term while keeping maxisum term as the more influential part in the objective function. We implemented this idea for several cases such as 6 AAD – 6 ND ships, 8 AAD – 10 ND ships, 4 AAD – 8 ND ships, 4 AAD – 6 ND ships, and 5 AAD – 6 ND ships. We failed to produce intermediate solutions for all those cases.

6.6 DISCUSSION

In this chapter, we have presented five different SAP models, SAP1 through SAP5. SAP1 maximizes the sum of expected coverage provided to the ships in TG. This model identifies each individual ship as a distinct entity. While SAP1 takes an indirect approach to maximization of the probability of no-leaker for the TG, SAP2 directly maximizes the probability of no-leaker for the TG using the same set of constraints. SAP3 has a major difference in handling the ships. SAP3 uses the idea of sister ships that have identical weaponry. Thus, we could categorize ships in several types and reduce the computational burden without sacrificing any fidelity. While SAP3.1 uses only one type of ship, SAP3.2 uses multiple types. Since we developed SAP3.2 further, we call the original SAP3.2 as SAP3.2-P that has a nonlinear objective function. Although we prove that the objective function of SAP3.2-P is concave, solving a nonlinear 0-1 integer programming problem is considered to be out of the scope of this research. Instead, we developed a linearization procedure for SAP3.2-P and named this model as SAP3.2-L. SAP3.2-C model is the same as SAP3.2-P except the objective function. In SAP3.2-C, we maximize the total coverage provided to the ships of the TG. SAP4 model maximizes the sum of the coverage provided to the ships in TG as in SAP3.2-C. However SAP4 has only two types of ships (i.e. AAD ships and ND ships). Cuts generated for SAP4 enable LP relaxation of SAP4 to produce integer solutions most of the time. Thus SAP4 can be used to solve SAP3.2-C in a branch and bound scheme. We focus on solving SAP using a branch and bound approach in the next chapter. In SAP5, the objective is to maximize the minimum coverage of the ships in TG. We investigate finding intermediate solutions between SAP4 and SAP5 by using a variation of SAP5, called SAP4.5. Summary of features and drawbacks of the models are given in Table 6.10.

Considering the models and the results presented in this chapter, we conclude that SAP3.2-C is the most suitable model for SAP. We implemented a branch and bound solution procedure for SAP3.2-C model, which is explained in the following chapter.

Table 6.10. Summary of SAP Models.

Model	Objective	Nonlinearity in Obj. Func.	Treatment of Ship Types	Solution Difficulty
SAP1	Maximization of the total expected coverage	Yes (resembles QAP)	Individual	Hard to solve
SAP2	Maximization of the probability of no-leaker	Yes	Individual	Hard to solve
SAP3.1-P	Maximization of the probability of no-leaker	Yes	Single ship type	Hard to solve
SAP3.2-P	Maximization of the probability of no-leaker	Yes	Multiple ship types	Hard to solve
SAP3.1-C	Maximization of the total coverage	No	Single ship type	Easy with LP relaxation.
SAP3.2-C	Maximization of the total coverage	No	Multiple ship types	Moderate
SAP4	Maximization of the total coverage	No	Two ship types	Easy with LP relaxation
SAP5	Maximization of the minimum coverage	No	Two ship types	Hard (LP relaxation is not suitable)
SAP4.5	Maximization of the weighted sum of total and minimum coverages	No	Two ship types	Hard (LP relaxation is not suitable)

CHAPTER VII

SOLUTION OF THE SECTOR ALLOCATION PROBLEM (SAP)

In this chapter, we develop the solution procedure for SAP using model SAP3.2-C. Our argument for the computational complexity of SAP3.2-L in preceding chapter is also valid for SAP3.2-C. Without a formal proof, we can say that SAP3.2-C is very hard to solve in terms of computational complexity. SAP3.2-C has additional constraints besides the constraints similar to those of p-median, which is an NP-Hard problem even when the network has a simple structure (Kariv and Hakimi, 1979). Thus, development of tight upper and lower bounds is very important for solving the problem successfully. We discuss our lower and upper bounding strategies in Section 7.1 and 7.2 respectively. We then present our branching strategies in the following section. We conclude this chapter with computational results, after presenting the branch and bound algorithms in Section 7.4.

7.1 LOWER BOUNDING STRATEGIES

We use two lower bounding strategies in the solution procedure. One of those strategies is the randomized heuristic, which was introduced in previous chapter. We used the randomized heuristic (taking the best of a large number of randomly generated solutions) to establish a simple lower bound for SAP3.2-C. This is taken as a first step towards developing tighter lower bounds. We generate test problems with 19 sectors, 3 AAD ships and different combinations of SD and ND ships. We use the same set of problems throughout this chapter. We report the results of SAP3.2-C and Randomized Heuristic in Table 7.1. The results reported for Randomized Heuristic are the best solution chosen out of 100,000 trials. Randomized heuristic performed well for the small test problems that we could solve to optimality, producing tight lower bounds for these problems.

Table 7.1. Comparison of SAP3.2-C and Randomized Heuristic Results.

Number of ND Ships		Number of SD Ships					
		0	2	3	4	5	6
1	SAP3.2C	9.49	15.40	18.40	21.39	24.37	27.25
	Random	9.49	15.29	18.24	21.14	24.19	26.69
	% Gap	0.00	0.71	0.87	1.17	0.74	2.06
2	SAP3.2C	12.08	18.00	20.99	23.97	26.85	29.66
	Random	12.08	17.84	20.74	23.79	26.29	29.22
	% Gap	0.00	0.89	1.19	0.75	2.09	1.48
3	SAP3.2C	14.60	20.59	23.57	26.45	29.26	32.06
	Random	14.49	20.34	23.39	25.89	28.82	31.60
	% Gap	0.75	1.21	0.76	2.12	1.50	1.43
4	SAP3.2C	17.20	23.17	26.05	28.86	31.66	34.42
	Random	17.04	22.99	25.49	28.42	31.20	33.99
	% Gap	0.93	0.78	2.15	1.52	1.45	1.24
6	SAP3.2C	22.37	28.06	30.86	33.62	36.32	38.95
	Random	22.19	27.62	30.40	33.19	35.90	38.59
	% Gap	0.80	1.57	1.48	1.27	1.15	0.92
8	SAP3.2C	27.26	32.82	35.52	38.15	40.78	43.40
	Random	26.82	32.39	35.10	37.79	40.59	43.15
	% Gap	1.61	1.30	1.18	0.94	0.47	0.57

The second lower bounding strategy is using the linear programming relaxation of SAP4. SAP4 is formulated for only two types of ships, hence we need to group the ships into AAD ships and ND ships. We combine SD and ND ships within the group of ND ships. Having an all integer solution for the relaxed SAP4 is another condition to be satisfied. Otherwise, we cannot use this procedure for developing a lower bound. If we have a fractional solution for the relaxed SAP4, the fractional solution constitutes an upper bound for SAP4 and we cannot guarantee that this solution will be a lower bound for SAP3.2-C. If the relaxed SAP4's solution is integer, we calculate the objective function value of SAP3.2-C using the solution generated by the relaxed SAP4 and add the coverage of the SD ships to produce a lower bound for SAP3.2-C. Table 7.2 depicts the results of the lower bounding strategy using the relaxation of SAP4. The second lower bounding strategy produced highly satisfactory results. We attained the optimal solution of SAP3.2-C in 34 out of 36 cases by using the solution of the linear programming relaxation of SAP4 with added cuts. The maximum percent gap between SAP3.2-C objective and the lower bound was 2.63. The relaxed SAP4 produced integer solutions for all the cases investigated. However, we have shown that the relaxed SAP4 could produce non-integer solutions as reported in the preceding chapter.

Table 7.2. Comparison of SAP3.2-C and Relaxed SAP4 Results.

Number of ND Ships		Number of SD Ships					
		0	2	3	4	5	6
1	SAP3.2C	9.49	15.40	18.40	21.39	24.37	27.25
	SAP4-LP*	9.24	15.40	18.40	21.39	24.37	27.25
	% Gap	2.63	0.00	0.00	0.00	0.00	0.00
2	SAP3.2C	12.08	18.00	20.99	23.97	26.85	29.66
	SAP4-LP	11.93	18.00	20.99	23.97	26.85	29.66
	% Gap	1.24	0.00	0.00	0.00	0.00	0.00
3	SAP3.2C	14.60	20.59	23.57	26.45	29.26	32.06
	SAP4-LP	14.60	20.59	23.57	26.45	29.26	32.06
	% Gap	0.00	0.00	0.00	0.00	0.00	0.00
4	SAP3.2C	17.20	23.17	26.05	28.86	31.66	34.42
	SAP4-LP	17.20	23.17	26.05	28.86	31.66	34.42
	% Gap	0.00	0.00	0.00	0.00	0.00	0.00
6	SAP3.2C	22.37	28.06	30.86	33.62	36.32	38.95
	SAP4-LP	22.37	28.06	30.86	33.62	36.32	38.95
	% Gap	0.00	0.00	0.00	0.00	0.00	0.00
8	SAP3.2C	27.26	32.82	35.52	38.15	40.78	43.40
	SAP4-LP	27.26	32.82	35.52	38.15	40.78	43.40
	% Gap	0.00	0.00	0.00	0.00	0.00	0.00

* Lower bound produced by using the solution of the linear programming relaxation of SAP4 with cut constraints.

7.2 UPPER BOUNDING STRATEGIES

We develop an upper bounding scheme for SAP3.2-C by using SAP4 and SAP3.1 models. Figure 7.1 shows a graphical representation of an instance of SAP3.2-C, where arrows indicate the defense support that can be provided. In SAP3.2-C model, AAD ships are assumed to be the global supporters of the TG. Self defense (SD) ships can defend themselves in addition to the support provided by AAD ships. Ships with no air defense capability (ND) must receive air defense support from AAD ships.

SAP4 model accounts for the interaction between the AAD ships and the other ships of the TG assuming they are all ND ships. However, we may add self defense contribution of each individual ship exogenously. This is depicted in Figure 7.2. SAP4 can only accommodate two types of ships. However, we can identify SD ships exogenously as a third ship type. SAP4 does not capture the interaction between the AAD ships. Solving the interaction among the AAD ships separately and adding its objective function value to SAP4 objective function value constitutes an upper bound for SAP3.2-C model. Interaction between a number of identical ships is captured in SAP3.1 model. Note that we assume infinite supply of rounds of SAM systems on board of AAD ships. An instance of SAP3.1 is shown in Figure 7.3. Here SAP3.1 is used with the objective function that maximizes the total coverage the identical ships provided to each other.

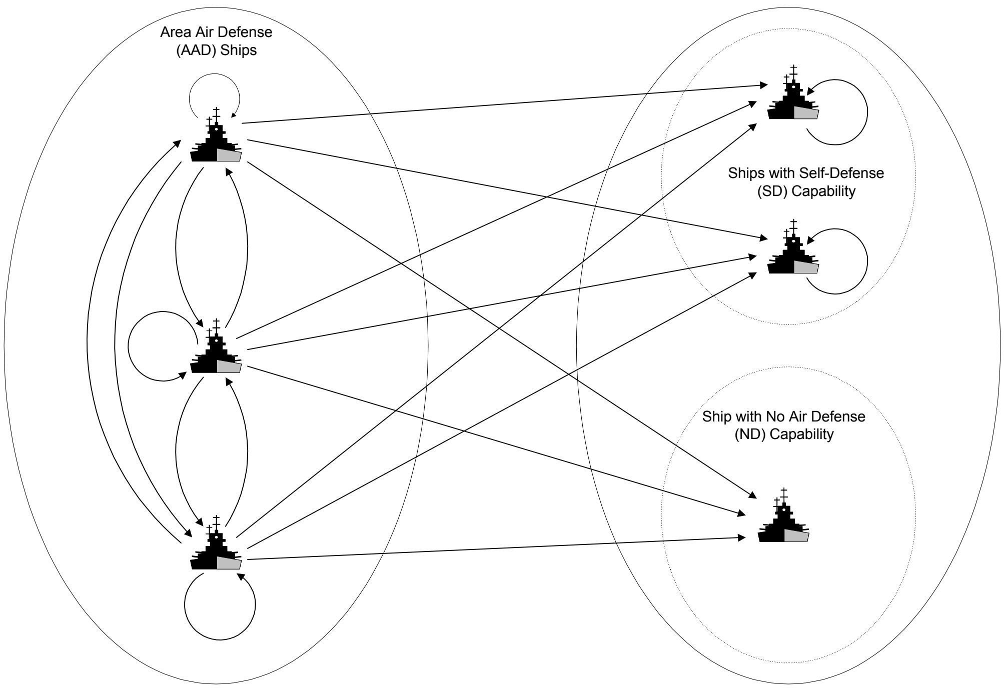

Figure 7.1. Representation of an Instance of SAP3.2-C Model with Three Different Ship Types (i.e. 3 AAD Ships, 2 SD Ships and 1 ND Ship).

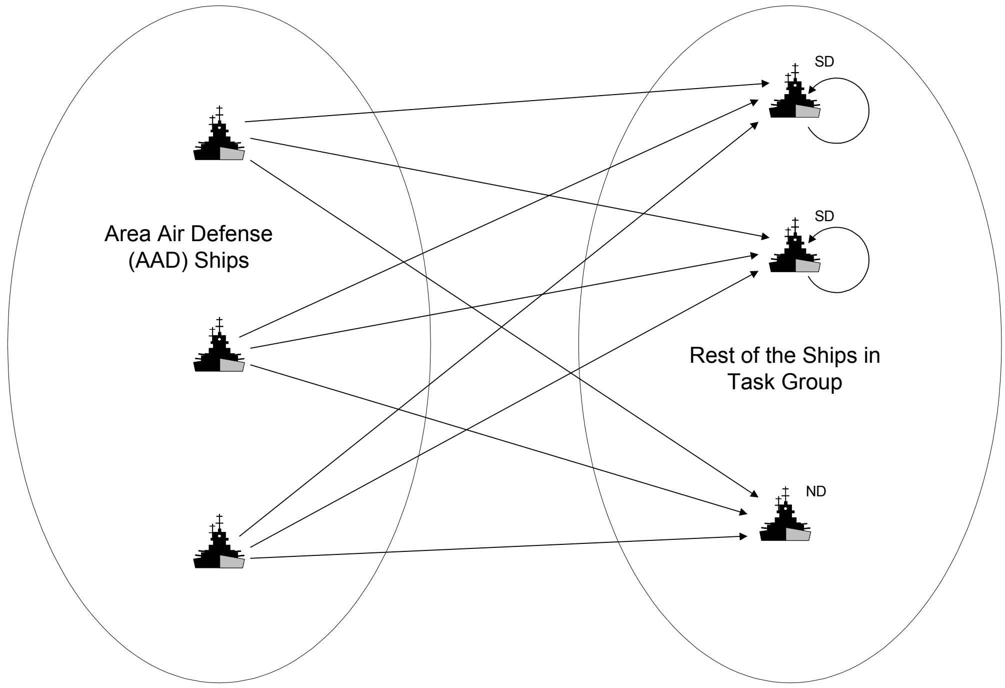

Figure 7.2. Representation of an Instance of SAP4 Model.

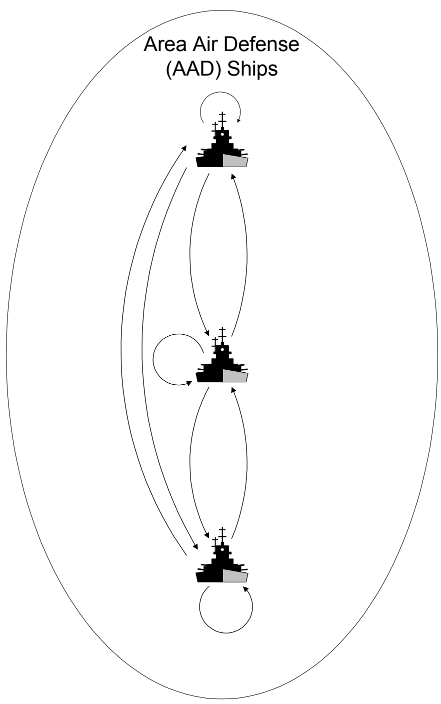

Figure 7.3. Representation of an Instance of SAP3.1 Model with Three Identical Ships of AAD Type.

We have checked the quality of upper bounds produced by the above upper bounding scheme. We used the same test problems with 19 sectors, 3 AAD ships and different combinations of SD and ND ships. The results are given in Table 7.3. The % gap between upper bound and SAP3.2-C objective value is less than 3 % with one exception, and less than 2 % in 29 of 36 cases. The 3.31 % gap marks the largest deviation. This is a promising upper bounding scheme provided that we can easily solve SAP3.1 and SAP4 models.

We presented the linear programming relaxation of SAP4 for an efficient solution procedure in the preceding chapter. However, we did not specifically address the solution procedure of SAP3.1-C before. SAP3.1-C is an easier problem than SAP3.2-C. We may directly solve SAP3.1-C by either using a MIP solver or enumerating all the solutions when the problem size is small and it warrants using any of those approaches in terms of computational time. This approach may be reasonable considering the fact that we need to solve SAP3.1-C only once. However, when the number of sectors is large, solving SAP3.1-C even for a small number of AAD ships may require extensive computational resources. That reasoning leads us to develop an efficient solution procedure for SAP3.1-C. We develop valid inequalities for SAP3.1-C that enable the linear programming relaxation to produce efficient upper bounds for SAP3.1-C below.

Table 7.3. Upper Bounds for SAP3.2-C Objective Function Using SAP4 and SAP3.1 Models.

Number of ND Ships		Number of SD Ships					
		0	2	3	4	5	6
1	SAP3.2C	9.49	15.40	18.40	21.39	24.37	27.25
	SAP4+SAP3.1	9.65	15.81	18.81	21.80	24.78	27.66
	% Gap	1.69	2.66	2.23	1.92	1.68	1.50
2	SAP3.2C	12.08	18.00	20.99	23.97	26.85	29.66
	SAP4+SAP3.1	12.34	18.41	21.40	24.38	27.09	30.07
	% Gap	2.15	2.28	1.95	1.71	0.89	1.38
3	SAP3.2C	14.60	20.59	23.57	26.45	29.26	32.06
	SAP4+SAP3.1	15.01	21.00	23.98	26.86	29.67	32.47
	% Gap	2.81	1.99	1.74	1.55	1.40	1.28
4	SAP3.2C	17.20	23.17	26.05	28.86	31.66	34.42
	SAP4+SAP3.1	17.61	23.58	26.46	29.27	32.07	34.83
	% Gap	2.38	1.77	1.57	1.42	1.30	1.19
6	SAP3.2C	22.37	28.06	30.86	33.62	36.32	38.95
	SAP4+SAP3.1	22.78	28.47	31.27	34.03	36.73	39.36
	% Gap	1.83	1.46	1.33	1.22	1.13	1.05
8	SAP3.2C	27.26	32.82	35.52	38.15	39.87	43.40
	SAP4+SAP3.1	27.67	33.23	35.93	38.56	41.19	43.81
	% Gap	1.50	1.25	1.15	1.07	3.31	0.94

The valid inequalities for SAP3.1-C are;

$$y_i + y_j \le 1 + x_{ij} \quad \text{for all } i, j \tag{7.1}$$

$$x_{ij} = x_{ji} \quad \text{for all } i, j \tag{7.2}$$

$$\sum_j x_{ij} \le P y_i \quad \text{for all } i \tag{7.3}$$

Constraint (7.1) limits the total number of AAD ships in two sectors by one plus the value of linking variable between the two sectors. If there is no link between two sectors, there could be at most one sector with an AAD ship. Constraint (7.2) equalizes the value of linking variable from sector i to sector j to the value of linking variable from sector j to sector i. Constraint (7.3) limits the number of linking variables by P, if there is an AAD ship in the sector. Otherwise, the number of linking variables is limited to zero.

Table 7.4. Comparison of Solving SAP3.1-C Using CPLEX MIP Solver and LP Relaxation of SAP3.1-C with Cuts.

Number of AAD Ships	CPLEX MIP SOLVER		LP Relax. of SAP3.1-C	
	Obj. Value	Time (sec)	Obj. Value	Time (sec)
2	3.12	5.0	3.12	0.0
3	6.85	7.0	6.87	0.0
4	12.02	12.0	12.02	0.0
5	18.44	17.0	18.44	0.0
6	26.08	20.0	26.08	0.5
7	34.82	25.0	34.82	0.0
8	44.80	21.0	44.80	0.5
9	55.92	21.0	55.92	0.0
10	67.82	16.0	67.82	0.5

* Runs carried out on a personal computer with 2.1 GHz CPU and 256 MB RAM

The quality of upper bounds produced by the linear programming relaxation of SAP3.1-C after adding the valid inequalities above depicted in Table 7.4. The second and third columns of Table 7.4 show the optimal value and the elapsed time of SAP3.1-C solution using MIP solver of CPLEX. Upper bounding scheme produces the optimal objective function value of SAP3.1-C for 8 out of 9 test problems. Percent gap between the upper bound and the optimal solution for the problem with 3 AAD ships is 0.35 percent. Thus, the quality of the upper bound is satisfactory. We then can use as an upper bound for SAP3.2-C the sum of the objective function values of relaxed SAP4 and relaxed SAP3.1-C with added cuts.

7.3 BRANCHING STRATEGIES

We consider six different branching strategies when we branch at a node in the branch and bound tree. We explain each one of those strategies below.

Branching Strategy 1 (BS1): In this strategy, we first branch on AAD ships. After AAD ships, we branch on SD ships and ND ships respectively. At each node we consider, we first check for non-integer optimal solution values for AAD ships. If there is any, we branch on that variable. If there is no non-integer variable for AAD ships, we check for non-integer variables for SD ships. If there is any, we branch on that variable, otherwise we branch on a non-integer variable for ND ships, if there is any.

Branching Strategy 2 (BS2): This strategy is very similar to BS1. In this strategy we just change the order of the precedence of AAD ships. We first branch on SD ships, then AAD ships and finally ND ships.

Branching Strategy 3 (BS3): In this strategy we first branch on SD ships, then ND ships and finally AAD ships.

In the next three strategies, we try to branch on the variables corresponding to different ship types in a cyclic manner. We first branch on one ship of each ship type then the second ship of each type and so on. At each node, we first check the last variable that was fixed and the corresponding ship type. We branch on a variable corresponding to the next ship type of the branching strategy, if there is any non-integer variable corresponding to that ship type. Otherwise, we continue with the variables corresponding to the next ship type in order of the branching strategy.

Branching Strategy 4 (BS4): In BS4, we consider the order of AAD, SD, and ND ship types. Assume that we last branched on a variable corresponding to an AAD ship. Then, we try to find a non-integer variable corresponding to SD ships. If there is any, we branch on that variable. If there is no non-integer variable for SD ships, we check for non-integer variables for ND ships. If there is any, we branch on that variable, otherwise we branch on a non-integer variable for AAD ships.

Branching Strategy 5 (BS5): This strategy is very similar to BS4. In this strategy we branch first according to SD ship, and then AAD ship, and finally ND ship. If the last variable that was fixed corresponds to a ND ship, we try to branch on a variable corresponding to a SD ship, thus start the precedence order from the beginning again.

Branching Strategy 6 (BS6): In this strategy we branch according to SD ship, and then ND ship, and finally AAD ship order. If the last variable that was fixed corresponds to a AAD ship, we try to branch on a variable corresponding to a SD ship, starting the precedence order from the beginning again.

7.4 BRANCH AND BOUND (B&B) ALGORITHM

In this section, we present two different versions of a branch and bound algorithm for SAP3.2-C. In one version, we use *depth first search (DFS)* branch selection strategy. *Best node first search (BNFS)* branch selection strategy is used in the second version.

7.4.1 Depth First Search Branch Selection Strategy

Let G be an ordered set of (partial) integer programs $\{IP^i\}$, each of which is of the form $Z_{IP}^i = \max\{cx : x \in S^i\}$ where $S^i \subseteq S$ and S is the polytope defined by the constraints of problem SAP3.2-C. Associated with each problem in G there is an upper bound $\overline{Z}^i \geq Z_{IP}^i$.

Step 1 // Initialization //

$G = \{IP\}$, $S^0 = S$.

Find a lower bound, $\underline{Z}_{IP}$ for the original problem;

- Find lower bound, LB_1 by random heuristic
- Solve SAP4 and calculate LB_2 from that solution by using SAP3.2-C objective function
- Set $\underline{Z}_{IP} = \max\{LB_1, LB_2\}$

Step 2 // Termination test //

If $G = \emptyset$, then the solution x^0 that yielded $\underline{Z}_{IP} = cx^0$ is optimal, i.e. if there is no sub-problem to be solved then the best solution found is the optimal solution.

Step 3 // Branch selection and solution //

Select and delete the first sub-problem IP^i from G. Solve its linear relaxation, RP^i. Let Z_R^i be the optimal objective function value and let x_R^i be an optimal solution to RP^i if one exists.

Step 4 // Pruning //

a. If RP^i is infeasible then prune that node and go to Step 2.

b. If $Z_R^i \le \underline{Z}_{IP}$ then prune that node and go to Step 2.

c. If $x_R^i \notin S^i$, i.e. the solution is fractional, and $Z_R^i > \underline{Z}_{IP}$ then find upper bound, $\overline{Z}^i$. If $\overline{Z}^i \le \underline{Z}_{IP}$ then prune that node and go to Step 2, otherwise go to Step 5.

d. If $x_R^i \in S^i$, i.e. the solution is integer, and $cx_R^i > \underline{Z}_{IP}$, let $\underline{Z}_{IP} = cx_R^i$. Delete from G all sub-problems with $\overline{Z}^i \le \underline{Z}_{IP}$ and $Z_R^i \le \underline{Z}_{IP}$. Prune that node and go to Step 2.

Step 5 // Branching //

Select a fractional variable, α to branch on. Add those two new sub-problems where $\alpha = 0$ and $\alpha = 1$ into the front of set G and go to Step 3.

7.4.2 Best Node First Search Branch Selection Strategy

Let G be a collection of (partial) integer programs $\{IP^i\}$, each of which is of the form $Z_{IP}^i = \max\{cx : x \in S^i\}$ where $S^i \subseteq S$ and S is the polytope defined by the constraints of problem SAP3.2-C and B^n is the set of n-dimensional binary vectors. Associated with each problem in G there is an upper bound $\overline{Z}^i \geq Z_{IP}^i$.

Step 1 // Initialization //

Find a lower bound, $\underline{Z}_{IP}$ for the original problem;

- Find lower bound, LB_1 by randomized heuristic
- Solve SAP4 and calculate LB_2 from that solution by using SAP3.2-C objective function
- Set $\underline{Z}_{IP} = \max\{LB_1, LB_2\}$

Solve SAP3.2-C using LP relaxation. If $x_R \notin \mathrm{B}^n$, i.e. the solution is fractional, add the original problem to G, $G = \{IP\}$, $S^0 = S$. Otherwise stop, i.e. LP relaxation produced integer optimal solution.

Step 2 // Termination test //

If $G = \varnothing$, then the solution x^0 that yielded $\underline{Z}_{IP} = cx^0$ is optimal, i.e. if there is no sub-problem to be solved then the best solution found is the optimal solution.

Step 3 // Branch selection//

Select and delete the best sub-problem (i.e. the LP relaxed sub-problem that has the largest objective function value) from G. If $Z_R^i \leq \underline{Z}_{IP}$ then, restart Step 3 to select another sub-problem from G.

Step 4 // Branching //

Select a fractional variable, α to branch on. Create two new sub-problems with $\alpha = 0$ and $\alpha = 1$ respectively.

For each new sub-problem do Step 5 and Step 6:

Step 5 // Solution //

Solve linear relaxation of the sub-problem i, RP^i. Let Z_R^i be the optimal objective function value and let x_R^i be an optimal solution to RP^i if one exists.

Step 6 // Pruning //

If RP^i is infeasible then

prune that node,

else if $Z_R^i \leq \underline{Z}_{IP}$ then

prune that node,

else if $x_R^i \notin \mathrm{B}^n$, i.e. the solution is fractional, and $Z_R^i > \underline{Z}_{IP}$ then

find upper bound, $\overline{Z}^i$ and if $\overline{Z}^i \leq \underline{Z}_{IP}$ then prune that node, otherwise add the sub-problem to G,

else if $x_R^i \in S^i$, i.e. the solution is integer, and $cx_R^i > \underline{Z}_{IP}$ then,

set $\underline{Z}_{IP} = cx_R^i$ and prune that node.

Step 7 // Continue//

Go back to Step 2.

7.5 COMPUTATIONAL RESULTS

In this section, we present the computational results for the solution procedure of SAP3.2-C. To test the solution procedure proposed in this chapter, we implemented the B&B algorithm in C. We solved the linear programming sub-problems by calling GAMS (General Algebraic Modelling Language) with CPLEX LP solver from C. We used the same set of test problems presented in preceding sections.

In order to make a decision on the branching strategy, we made 36 runs for three different problems. Each problem was solved for two different branch selection and six different branching strategies. The results of tests for this part are summarized in Table 7.5. Detailed results including the changes in percent of nodes pruned by lower and upper bounding schemes as the iterations continue are given in Appendix G.

As shown in Table 7.5, BS1 dominates the other branching strategies in terms of elapsed time end efficiency. In reaching the optimal solution, BS1 explores a fraction of nodes compared to the other branching strategies. Thus, we decide to use BS1 for further computational experiments. DFS branch selection strategy performs better than BNFS branch selection strategy when we use BS1. However, BNFS performs better than DFS for some other branch selection strategies. We continue using both of the branch selection strategies in the following experiments in order to explore the performance of those strategies better.

Table 7.5. Computational Results for Branching and Branch Selection Strategies.

	Branching Strategy	3 AAD, 2 SD, 2 ND Ships				3 AAD, 2 SD, 4 ND Ships				3 AAD, 3 SD, 3 ND Ships			
		% Pruned [1]LB	% Pruned [2]UB	[3]Nodes	Time (sec.)	% Pruned LB	% Pruned UB	Nodes	Time (sec.)	% Pruned LB	% Pruned UB	Nodes	Time (sec.)
Depth First Search	BS1	30.97	19.47	113	9.3	34.15	14.63	41	3.3	34.15	14.63	41	3.2
	BS2	25.38	24.70	591	47.9	19.82	29.03	217	19.5	21.11	28.36	469	43.5
	BS3	24.50	25.50	6313	435.5	20.10	29.85	4657	391.2	19.76	30.19	5905	498.8
	BS4	26.06	23.94	8713	707.9	30.15	19.83	2909	224.6	28.08	21.85	4975	387.7
	BS5	25.08	24.93	9075	621.2	25.16	24.78	6133	494.0	25.88	24.07	7739	618.4
	BS6	24.77	25.20	9305	765.6	26.36	23.59	5481	432.9	23.53	26.44	7229	627.7
Best Node First Search	BS1	30.97	19.47	113	10.4	21.05	28.07	57	5.3	21.05	28.07	57	5.3
	BS2	25.38	24.70	591	51.6	18.40	31.20	375	34.2	20.12	29.82	815	77.5
	BS3	24.50	25.50	6313	539.9	17.25	30.91	12577	1068.1	16.97	30.94	17365	1476.1
	BS4	32.32	17.69	5489	400.2	19.61	30.38	5105	435.8	20.85	29.13	7909	700.9
	BS5	29.20	20.80	6773	512.0	20.10	29.87	7255	657.6	20.46	29.51	9181	781.7
	BS6	29.01	20.99	6597	504.1	16.65	33.35	5473	474.5	22.72	26.46	9925	806.3

[1] LB: Lower Bound

[2] UB: Upper Bound

[3] Total number of nodes explored to find the optimal solution.

The test problems with 19 sectors, 3 AAD ships and different combinations of SD and ND ships are solved using GAMS/CPLEX MIP solver and proposed B&B algorithm, using DFS and BNFS with BS1. We used CPLEX with default settings. CPLEX uses best-bound search for branch selection, which chooses the unprocessed node with the best objective function of the associated LP relaxation. CPLEX automatically selects the branching strategy in default setting. That branching strategy allows CPLEX to select the best rule based on the problem and its progress.

Table 7.6. Computational Results of the Solution Procedures for SAP3.2-C.

Number of ND Ships			Number of SD Ships					
			0	2	3	4	5	6
1		Obj.[1]	9.49	15.40	18.40	21.39	24.37	27.25
	CPLEX[2]	Nodes[3]	6132	68813	1128490	90086	484814	222429
		Time[4]	20.00	150.77	2035.00	158.00	823.00	417.00
	DFS[5]	Nodes	251	161	113	67	25	25
		Time	21.42	13.05	9.47	5.69	2.10	2.60
	BNFS[6]	Nodes	391	161	113	67	57	25
		Time	32.13	12.89	10.21	5.34	5.28	2.59
2		Obj.	12.08	18.00	20.99	23.97	26.85	29.66
	CPLEX	Nodes	41124	48083	516661	15070	383980	14886
		Time	70.00	86.00	938.00	234.00	874.00	34.00
	DFS	Nodes	237	113	67	41	25	15
		Time	17.08	9.28	5.96	3.66	2.08	2.57
	BNFS	Nodes	293	113	67	57	25	15
		Time	24.18	10.52	5.70	5.26	2.24	1.38
3		Obj.	14.60	20.59	23.57	26.45	29.26	32.06
	CPLEX	Nodes	382827	56684	649829	1194060	1052971	94087
		Time	615.00	104.00	1248.00	2029.00	2359.00	158.00
	DFS	Nodes	161	67	41	25	15	11
		Time	11.94	5.64	3.20	3.31	1.58	2.11
	BNFS	Nodes	161	67	57	25	15	11
		Time	12.95	5.72	5.26	2.17	1.68	1.47
4		Obj.	17.20	23.17	26.05	28.86	31.66	34.42
	CPLEX	Nodes	230535	212275	769433	2632150	103972	43882
		Time	388.00	389.00	1286.00	5058.00	173.00	75.00
	DFS	Nodes	113	41	25	15	11	11
		Time	8.71	3.28	2.03	1.13	1.26	0.99
	BNFS	Nodes	113	57	25	15	11	11
		Time	10.46	5.29	2.13	1.48	0.99	1.07
6		Obj.	22.37	28.06	30.86	33.62	36.32	38.95
	CPLEX	Nodes	845652	547233	308749	181122	27460	4599
		Time	1322.00	993.00	493.00	312.00	47.00	9.00
	DFS	Nodes	61	15	11	11	11	11
		Time	4.39	1.28	1.21	1.10	0.99	0.86
	BNFS	Nodes	57	15	11	11	11	11
		Time	4.97	1.45	1.10	1.05	0.86	1.01
8		Obj.	27.26	32.82	35.52	38.15	40.78	43.40
	CPLEX	Nodes	1477829	354538	6296	3124	220	24
		Time	2909.00	578.00	13.00	7.00	2.00	2.00
	DFS	Nodes	15	11	11	11	11	11
		Time	1.22	1.03	1.01	0.91	0.93	0.95
	BNFS	Nodes	15	11	11	11	17	29
		Time	1.77	1.84	0.77	0.95	1.26	2.94

[1] Objective function value.
[2] GAMS/CPLEX solver with default settings.
[3] Total number of nodes explored to find the optimal solution.
[4] Time in second for a personal computer with 2 GHz CPU and 256 MB of RAM.
[5] Depth First Search.
[6] Best Node First Search.

Table 7.6 depicts the computational results of the proposed solution procedures for SAP3.2-C. B&B solution procedures using DFS and BNFS performed better than CPLEX in term of elapsed time and number of nodes explored. Although our implementation for solving the LP relaxed sub-problems in the B&B tree is not efficient in terms of time, we still perform better than CPLEX, since we need to explore only a very small fraction of nodes compared to CPLEX. Our solution procedure would solve faster if we could embed an LP solver for sub-problems within the procedure.

Tight lower and upper bounding schemes, and the efficient branching strategy enabled us to produce highly satisfactory results for the solution procedures.

CHAPTER VIII

INTEGRATED SOLUTION APPROACH FOR ROBUST SECTOR ALLOCATION

In this chapter, we present an integrated solution approach to attain a robust sector allocation for a naval TG by using MAP results within SAP.

As we discussed before, SAP requires sector-to-sector coverage values provided by an AAD ship. Thus, we need to feed SAP with this information. Sector-to-sector coverage values for a specified scenario can be generated using MAP. Then we can find the best formation against the specified threat using SAP. We can find a robust formation against two or more scenarios by aggregating the sector-to-sector coverage values of MAP solutions, and then by solving SAP using the aggregated sector-to-sector coverage values. In Section 2.3, we introduced two different interaction models between MAP and SAP. Integrated solution approach proposed above uses the Interaction Model-1.

In Interaction Model-1, we solve MAP for each sector pair and a number of representative attack scenarios and using aggregated results as input, we solve SAP. Information on the enemy inventory of warships and their weapon systems and the intelligence coming from different sources may be used to generate representative scenarios. Pairwise coverage values from MAP are input to SAP. Coverage values are calculated in a restrictive scenario having only one AAD and one ND ship.

In Interaction Model-2, we assume that we determine the formation of TG using sector allocation model and operate at sea. Then, in the presence of an immediate ASM threat, we solve MAP to optimize our air defense against the threat. Thus, we do not need to use SAP and MAP on-line.

We present our integrated solution approach on a sample problem. Assume that we need to allocate three AAD ships, two SD ships, and two ND ships to 19

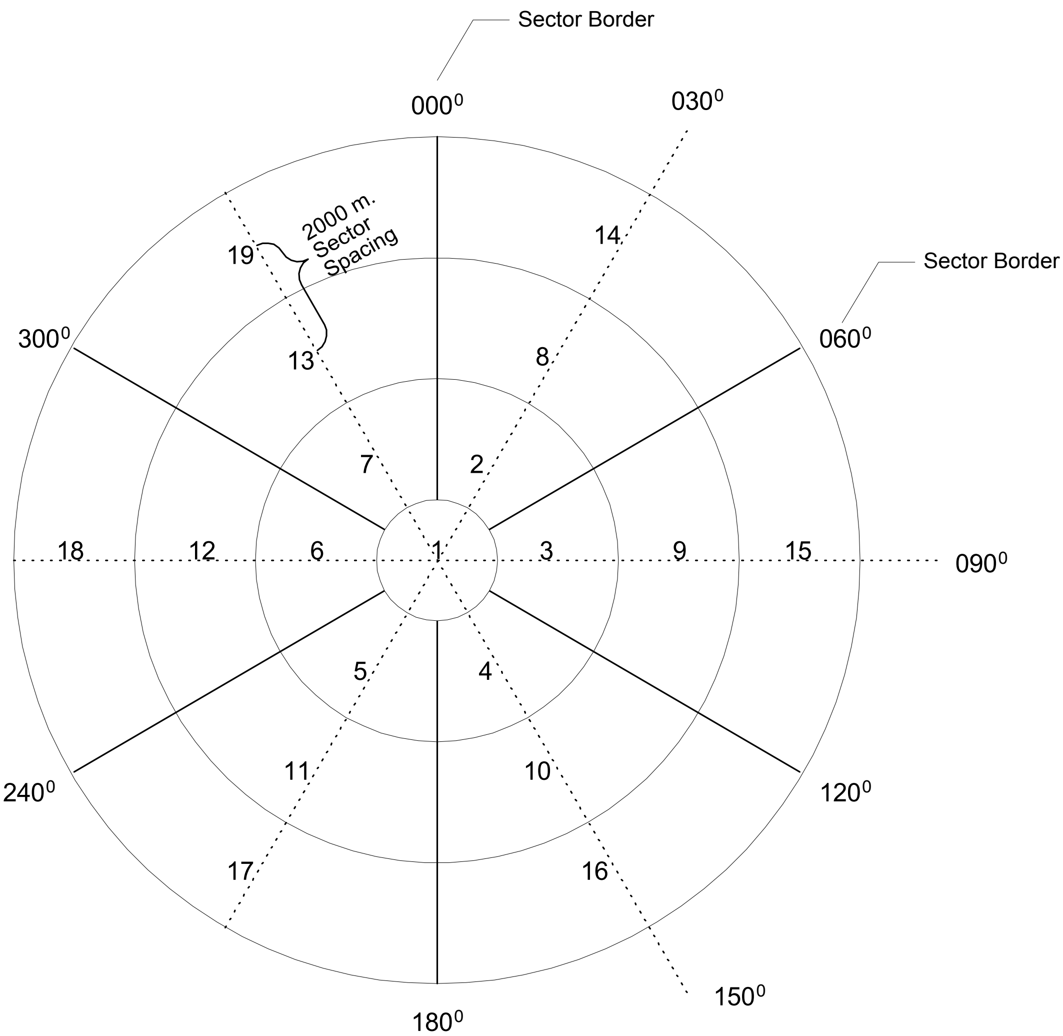

Figure 8.1. Geometry of a Sample SAP.

sectors with 2000 m. sector spacing and 60 degrees of bearing difference. Figure 8.1 depicts the geometry of the sample problem. Ships are assumed to be stationed at the center of the assigned sectors. Numbers at the center of the sectors represent the sector numbers.

We generated 6 different attack scenarios coming between 000 and 090 bearings (between North and East directions). For each attack scenario, we calculated the coverage provided by an AAD ship to a ND ship for every sector pair using MAP. Thus, we solved MAP using one AAD and one ND ships for a total of 19*19=361 times. Note that we allow ships to be stationed at the same sector in order to be able to calculate the self-defense capability of AAD ships. We take self-defense capability of the SD ships as half of the defense capability of AAD ships. Calculated coverage parameters for each scenario are reported in Appendix H. We then solved SAP with the coverage parameters for each scenario under consideration.

In Scenario 1, the air threat is one MM-38 Exocet ASM, which is coming from true North (000 degrees) and is initially detected at 21213.2 m. distance by the AAD ship. We use constant initial detection distance from the AAD ship for each run of MAP with a different sector pair. Figure 8.2 shows the result of SAP using MAP input for Scenario 1. Note that none of the centers of our sectors do lie on the flight path of the attacking ASM. Thus, SAP chooses the closest sectors to the attacker's line of flight for allocating the AAD ships. The resulting sector allocation is reasonable from a tactical point of view.

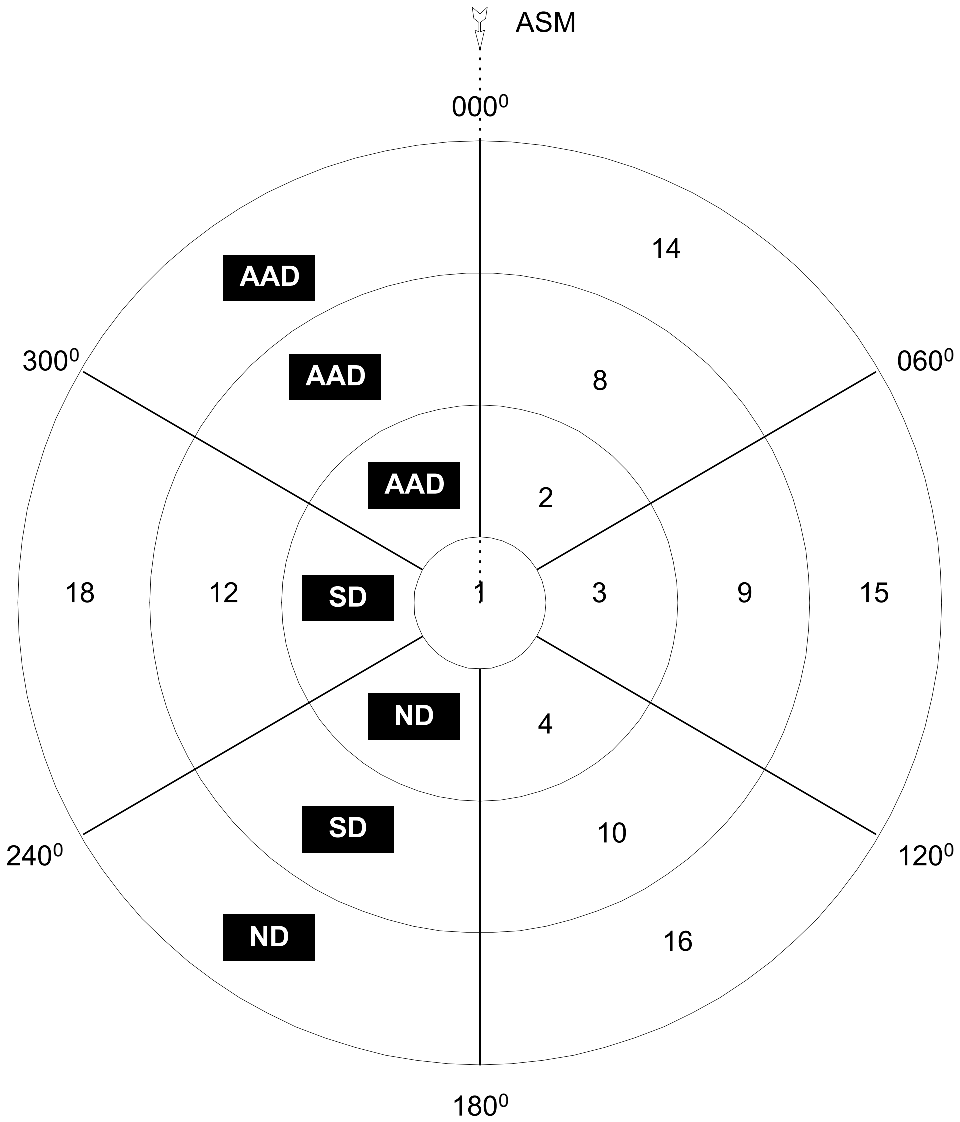

Figure 8.2. Sector Allocation for Scenario 1. AAD, SD, and ND Represent the Sectors of the Corresponding Ships in the Figure.

In Scenario 2, the air threat is again one MM-38 Exocet ASM, which is initially detected at 21213.2 m. distance by the AAD ship. However, the ASM is coming from 045 bearing in this case. Figure 8.3 shows the result of SAP using MAP input for Scenario 2. Similar to the result of Scenario 1, SAP chooses the closest sectors to the attackers line of flight for allocating the AAD ships.

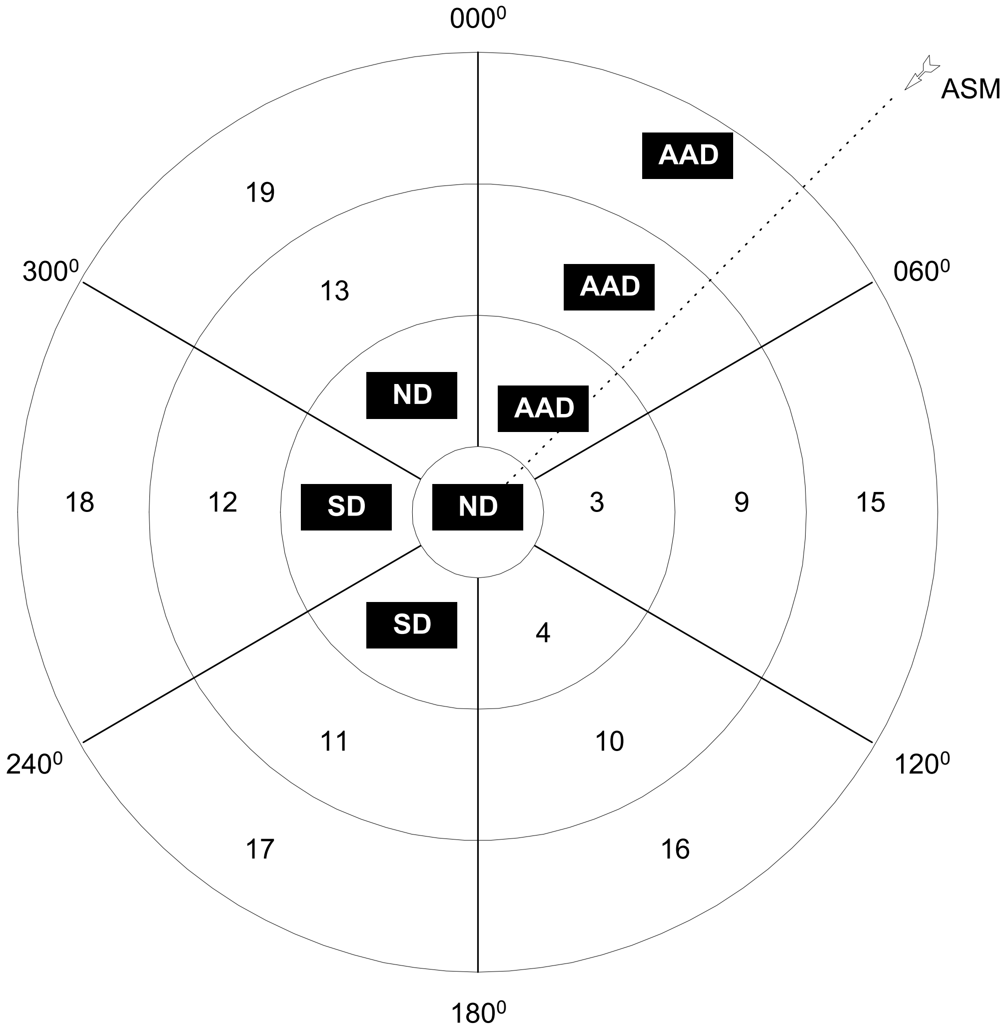

Figure 8.3. Sector Allocation for Scenario 2.

In Scenario 3, the Exocet ASM is coming from 090 (East) bearing. Figure 8.4 shows the result of SAP using MAP input for Scenario 3. Similar to the result of Scenario 1 and 2, SAP chooses the closest sectors to the attackers line of flight for allocating the AAD ships. However, AAD ships are allocated to the sectors with centers directly on the line of flight of the attacking ASM in this case. This condition enables the AAD ships in Scenario 3 to provide stronger coverage than the AAD ships in Scenario 1 and 2 (e.g. the objective function value in Scenario 3 is 12.338 as opposed to 9.767 in Scenario 1 and 9.336 in Scenario 2).

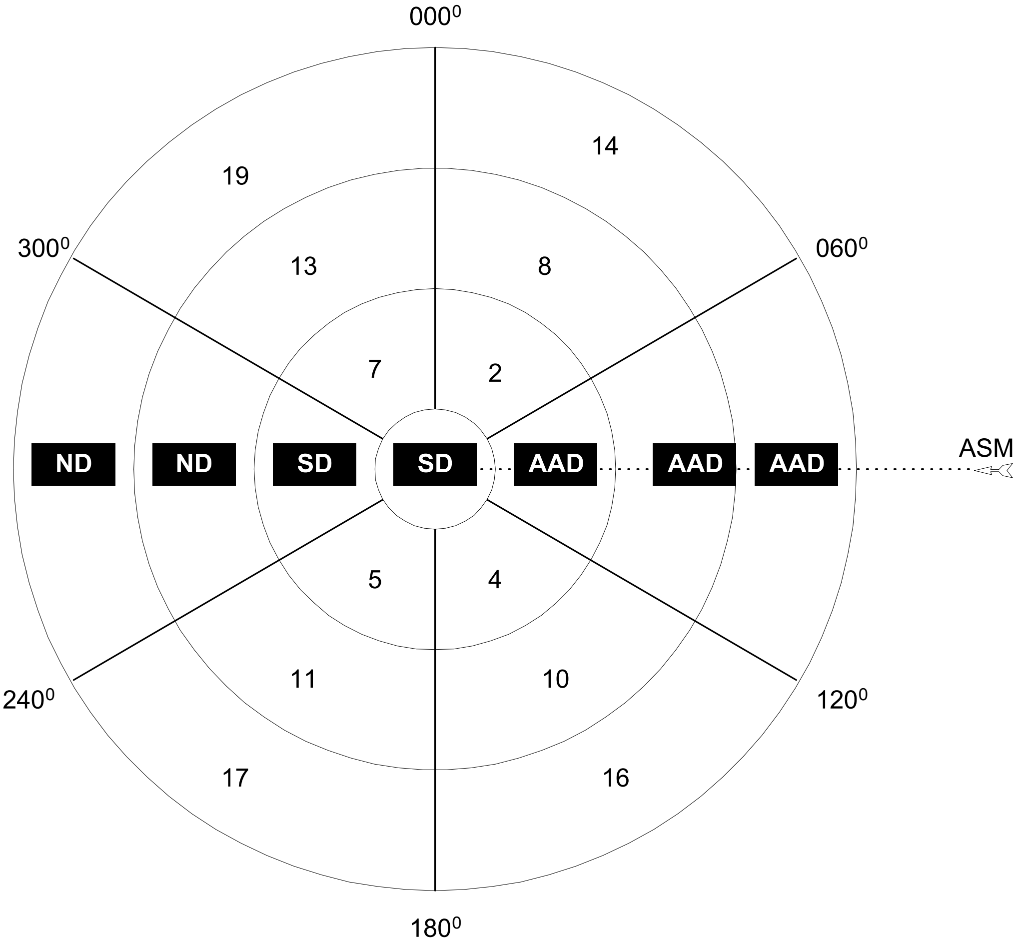

Figure 8.4. Sector Allocation for Scenario 3.

Assume that the first three scenarios are the representative scenarios for the expected threat. That is, we expect one ASM from bearing 000-090, but we do not know the exact bearing. Now, we need to aggregate the results in order to produce a robust formation that is reasonably strong against all the scenarios but not necessarily the best one against any of the scenarios.

Taking average of the coverage values for each sector pair across the scenarios is a candidate aggregation procedure. The resulting SAP solution for that approach is depicted in Figure 8.5. Sector allocation in Figure 8.5 may not overlap with what is expected. As we mention before, AAD ships provide stronger coverage in Scenario 3 compared to the Scenarios 1 and 2. Thus, the aggregated sector allocation is heavily affected by the results of Scenario 3. This result is reasonable considering the fact that we use small number of sectors and three scenarios.

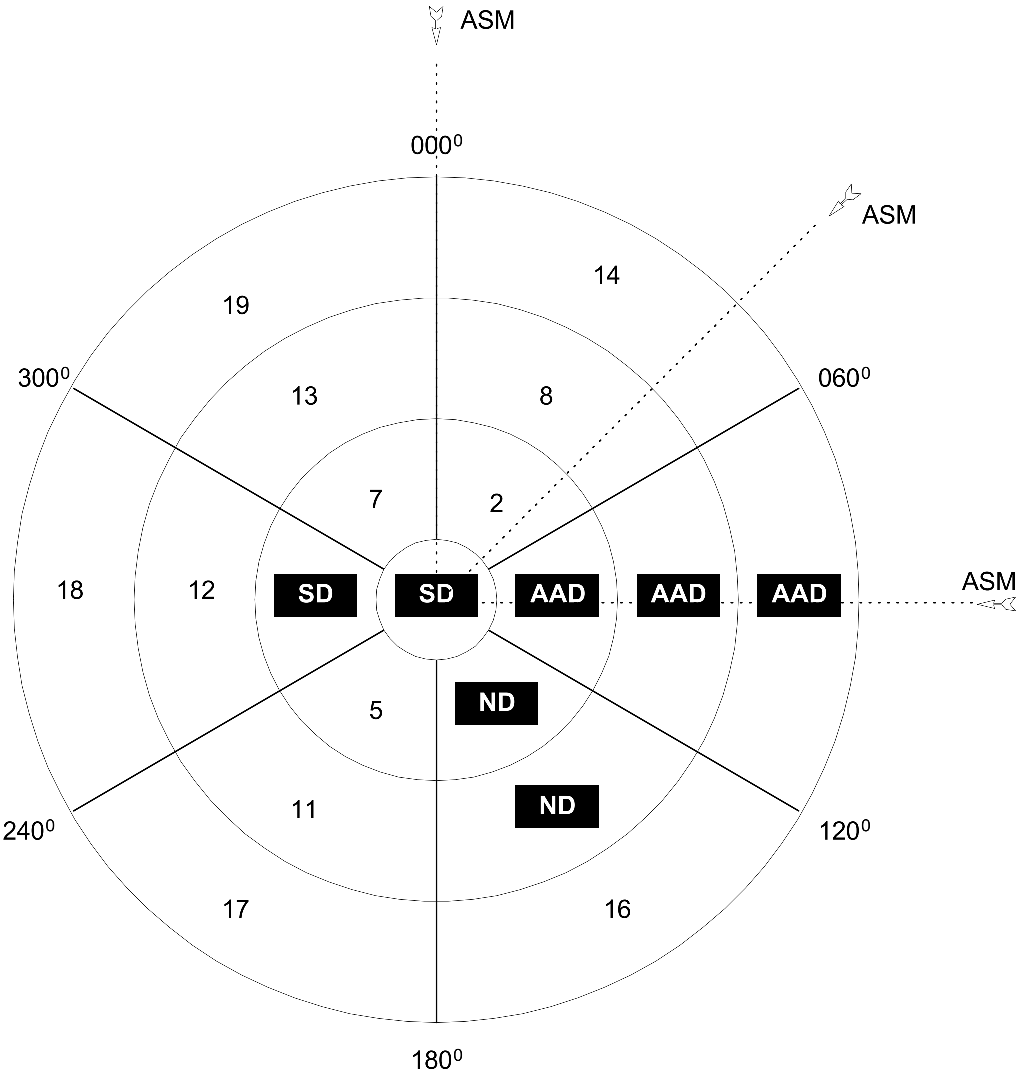

Figure 8.5. Aggregated Sector Allocation for Scenarios 1, 2, and 3 by Averaging the Coverage Values for Each Sector Pair.

Sectors, especially the outer ones, cover large areas. Thus the resolution of locations in sample problem is very low. In order to overcome this drawback, we need to use more sectors with small areas of control, and fine-tuned and relatively large number of representative scenarios.

Taking the necessary precautions by assuming the worst case may be an attractive approach to handling risks from a military perspective. This idea leads us to another aggregation procedure. For each sector pair, we can use the minimum sector-to-sector coverage across all the scenarios. Then, we maximize the minimum coverage in SAP. Thus we employ a risk averse approach in aggregating MAP results in order to produce a robust formation for TG. We present the results of this approach in Figure 8.6. The risk averse aggregation procedure produces more reasonable solutions than the averaging procedure. AAD ships are allocated to the sectors roughly in the middle of the representative attack scenarios.

We show the effect of formations on the solution of MAP in Table 8.1. For each formation generated above, we solve MAP using the attack Scenarios 1, 2 and 3. Note that the fist three formations are optimized for the corresponding attack scenarios. Thus, MAP attains the highest objective function values, when the disposition of the TG is optimized for the specific attack scenario. Robust 1-3 formation in Table 8.1 represents the formation produced by the risk averse aggregation scheme. Since the Robust 1-3 formation and the formation for Scenario 2 are very similar to each other, they produce the same results for MAP. Robust formation does not perform much better than those based on specific attack scenario.

However, the robust formation provides insurance against doing very poorly against a range of attack scenarios.

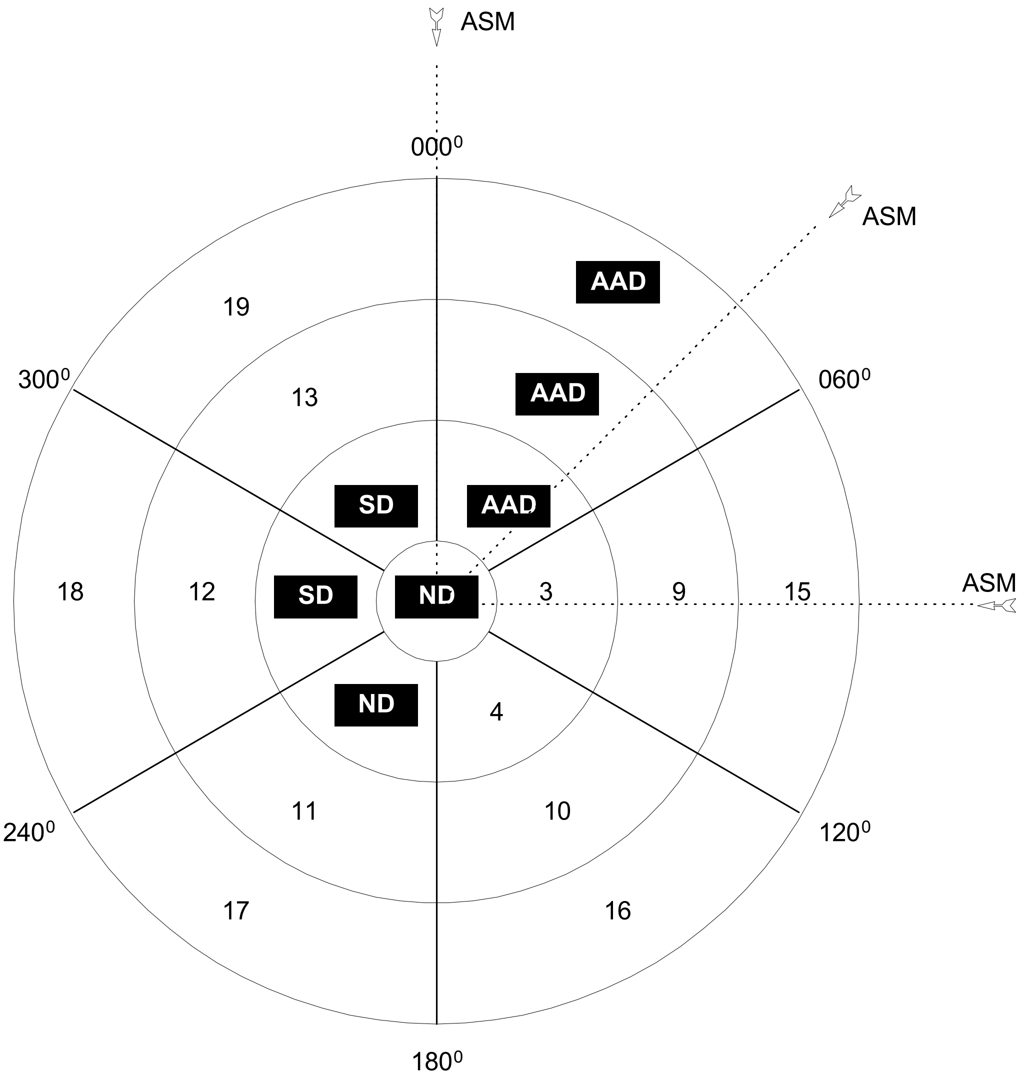

Figure 8.6. Aggregated Sector Allocation for Scenarios 1, 2, and 3 by Taking the Minimum Coverage Value for Each Sector Pair.

Table 8.1. Objective Function Values of MAP for Attack Scenarios and Formations.

Formation	Attack		
	Scenario 1	Scenario 2	Scenario 3
Scenario 1	0.487	0.265	0.185
Scenario 2	0.337	0.431	0.265
Scenario 3	0.185	0.302	0.693
Robust 1-3	0.337	0.431	0.265

In the next three scenarios, we use two different attacking ASMs coming from North, North-East, and East directions with approximately 10 degrees of bearing difference. We have one Harpoon ASM in addition to one MM-38 Exocet ASM. Harpoon is another widely used ASM in navies. Its parameters such as speed and probability of being shot down are similar to that of an Exocet. The initial detection distances of ASMs from AAD ship are approximately the same.

In Scenario 4, one Exocet and one Harpoon ASMs are coming from 000 (North) and 010 bearings respectively. Figure 8.7 shows the result of SAP using MAP input for Scenario 4. Similar to the result of Scenario 1, SAP chooses the closest sectors to the attackers line of flight for allocating the AAD ships.

In Scenario 5, one Exocet and one Harpoon ASMs are coming from 045 (North-East) and 055 bearings respectively. Figure 8.8 shows the result of SAP using MAP input for Scenario 5. Similar to the result of Scenario 2, SAP chooses the closest sectors to the attackers line of flight for allocating the AAD ships. However, the sectors of SD and ND ships are different from those in Scenario 2.

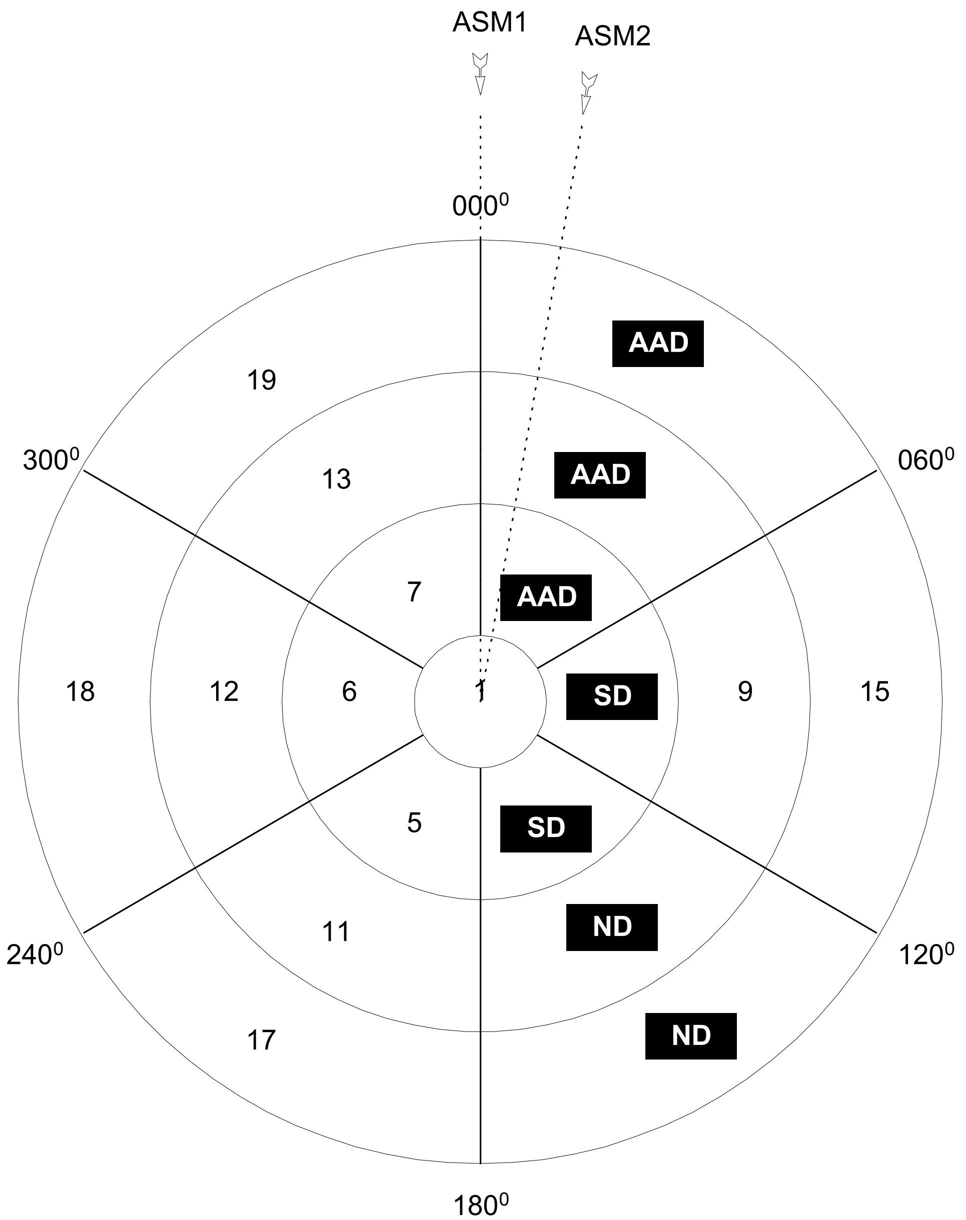

Figure 8.7. Sector Allocation for Scenario 4.

In Scenario 6, one Exocet and one Harpoon ASMs are coming from 090 (East) and 080 bearings respectively. Figure 8.9 shows the result of SAP using MAP input for Scenario 6. The sectors of the ships are almost the same as in Scenario 2.

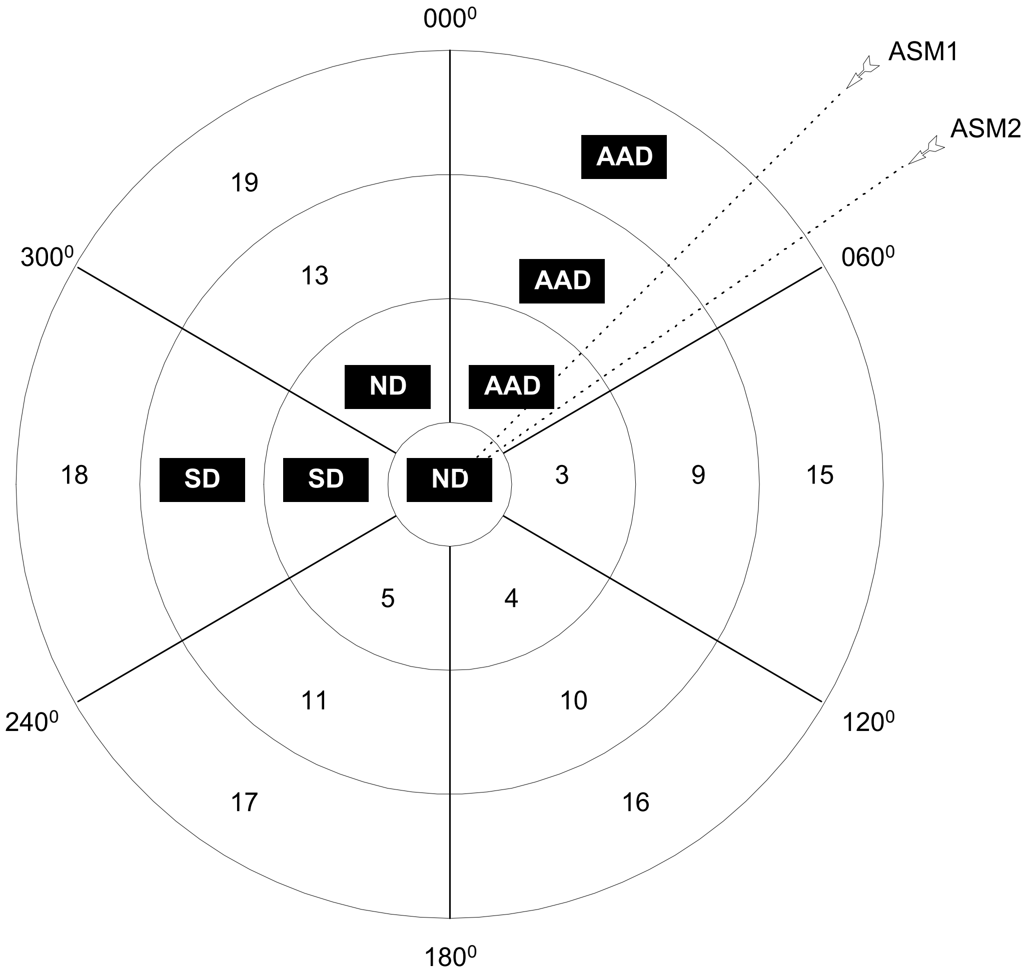

Figure 8.8. Sector Allocation for Scenario 5.

Risk averse aggregation of the coverage results of Scenarios 4, 5, and 6 produces similar output to the aggregation of Scenarios 1, 2, and 3, according to Figure 8.10. AAD ships are allocated to the sectors roughly in the middle of the representative attack scenarios.

Table 8.2 depicts the results of MAP for attack scenarios 4, 5, and 6 and the corresponding formations. The results are very similar to the ones in Table 8.1.

Robust 4-6 formation produces reasonable results. The robust formation enables the TG to increase its worst-case performance against a variety of attack scenarios.

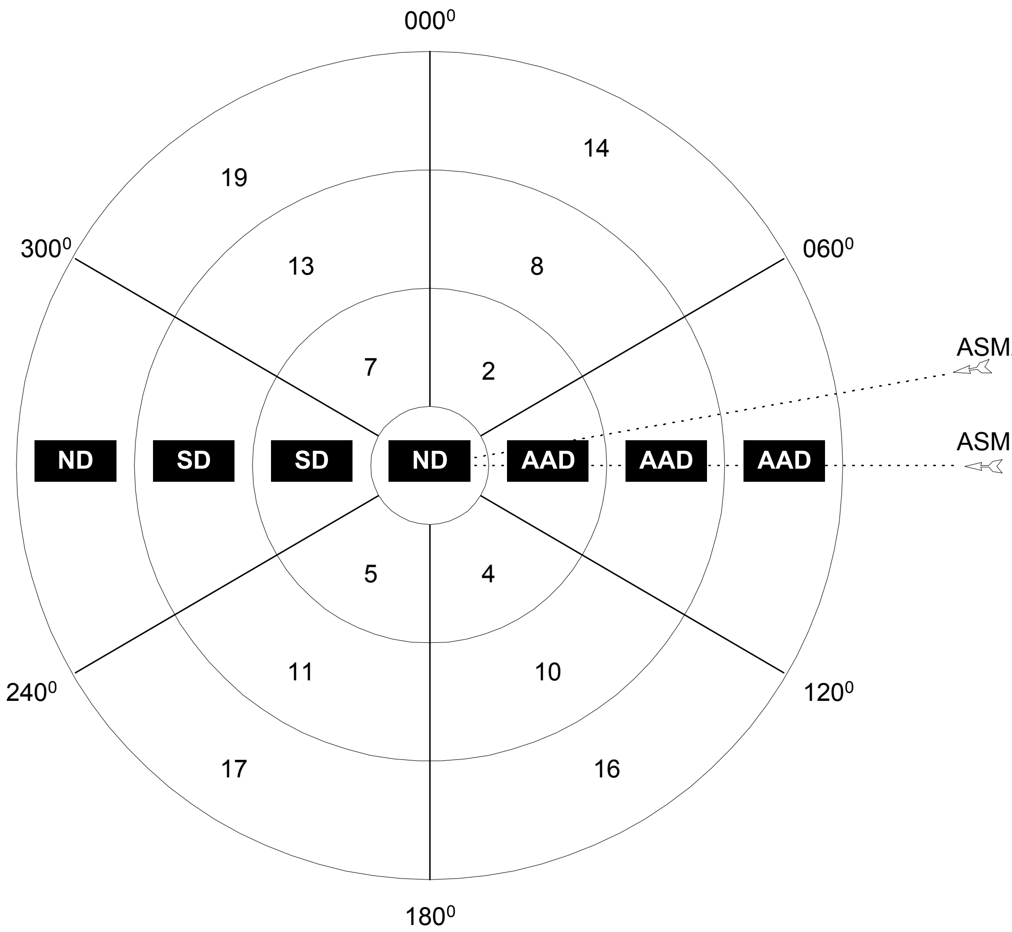

Figure 8.9. Sector Allocation for Scenario 6.

In this chapter, we have shown an integrated solution approach for MAP and SAP on sample scenarios. Two different coverage aggregation procedures in the development of the robust formation were presented. Aggregation schemes produced reasonable formations. We have shown the effect of robust formations on MAP solutions. We can define an integrated solution process for robust sector allocation as follows:

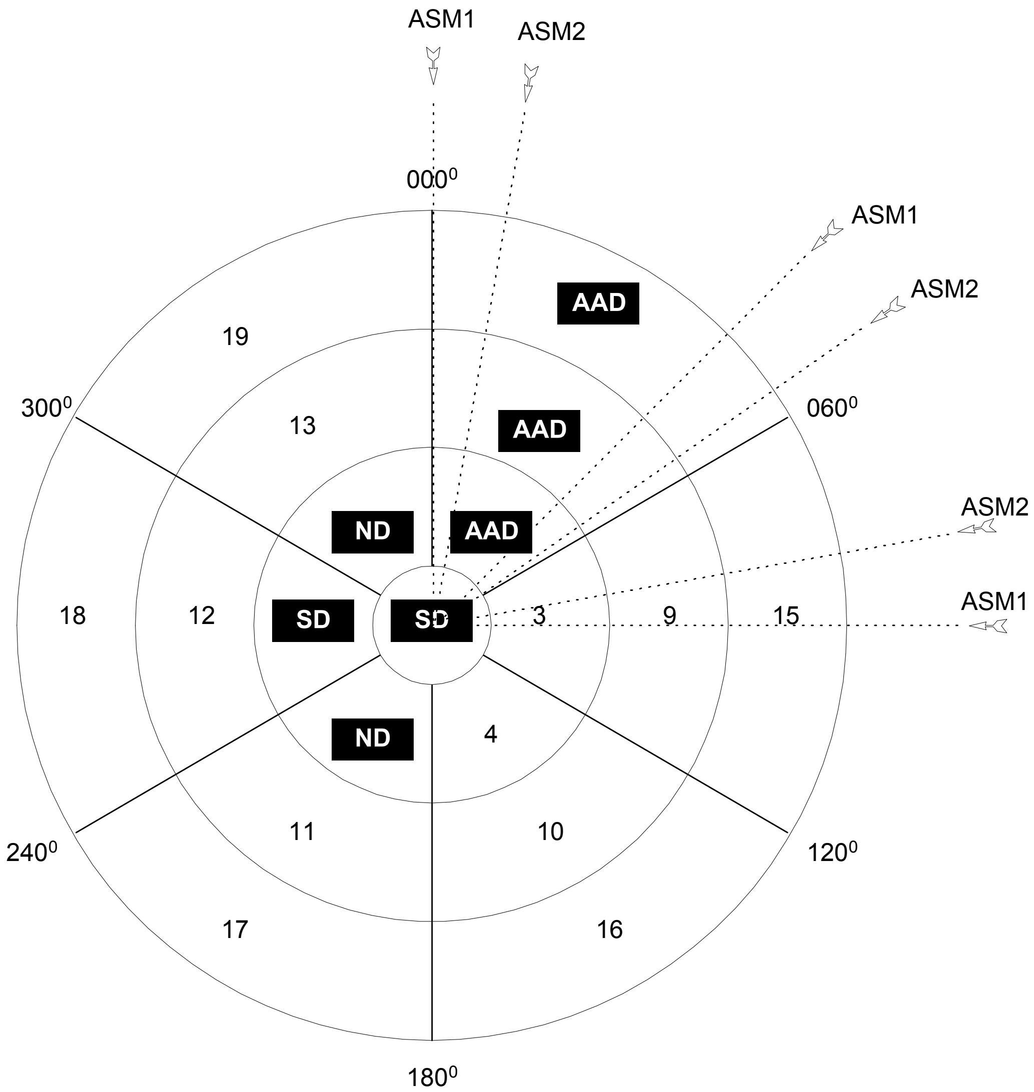

Figure 8.10. Aggregated Sector Allocation for Scenarios 4, 5, and 6 by Taking the Minimum Coverage Value for Each Sector Pair.

Table 8.2. Objective Values of MAP for Attack Scenarios and Formations.

Formation	Attack		
	Scenario 4	Scenario 5	Scenario 6
Scenario 4	0.391	0.245	0.141
Scenario 5	0.280	0.346	0.180
Scenario 6	0.103	0.219	0.394
Robust 4-6	0.280	0.346	0.180

Step 1: Define the representative attack scenarios using the intelligence information on general direction of threat, threat size, and enemy inventory of ASMs.

Step 2: For each scenario, find the sector-to-sector coverage values by solving MAP.

Step 3: Aggregate the sector-to-sector coverage values using an aggregation procedure.

Step 4: Solve SAP for the TG using aggregated coverage values.

CHAPTER IX

CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

In this dissertation, we developed solution methods for the air defense problem of a naval TG. We considered two interdependent problems, MAP and SAP. MAP can be defined as the optimal allocation of a set of defensive missile systems of a naval TG to a set of attacking air targets. SAP on the other hand, determines the air defense formation for a naval TG by locating ships in predefined sectors on the surface. We discussed special properties, assumptions, and environments of the problems. We also characterized the interaction between MAP and SAP.

We formulated three different missile allocation models and several variations. The first was the missile allocation model with no time dimension (MAP1). We treated MAP with a discretized time dimension (MAP2) in the second model. In the last model, we used continuous time dimension (MAP3). Theoretical development of those models and proposed solution approaches were given.

However, the mathematical programming models that were developed have not explicitly been used to solve MAP. Although mathematical programming models do guarantee an optimal solution (without loss of generality), they usually take much more than a few seconds in which we have to find solution for real time application of MAP.

MAP requires real time solution and very fast implementation without even sacrificing a single second. Thus, any solution procedure has to produce reasonable and high quality solutions in no more than several seconds. This is a must have feature of any solution algorithm that is eligible to be used in TEWA module of a warship.

Solving MAP for a large number of cases is a prerequisite for successfully solving SAP. Since this process requires running MAP many times for a single SAP solution, non-real time use of MAP also requires fast and high quality solutions.

Our solution approach for MAP uses construction and improvement heuristics. We developed two greedy construction algorithms for MAP. First of those algorithms, BEC heuristic, allocates SAM systems to incoming anti-ship missiles according to a measure called engagement potential. In QUC algorithm, we aim to engage each threat ASM at least once. Thus, we give precedence to the ASM that has the lowest number of SAM systems that can engage to it.

We developed two improvement heuristics, OC and 2OX. Our purpose in OC algorithm is to find the engagements that would increase the objective function value by changing the target ASM of an engagement under consideration and simultaneously considering the enhancement of the effectiveness of defense by increasing the total number of SAM missiles launched against target ASMs. Changing the target ASM means that an ASM will get one less shot while another

ASM will get one more shot. The ASM that gets one less shot after change is considered for another shot observing the SLS tactic.

Our purpose in 2OX algorithm is to find the engagement pairs that would increase the objective function value by exchanging the target ASMs of the SAMs in the engagements. With each exchange, we also try to increase the number of engagements done against the ASMs under consideration.

We tested our solution approach for 125 sample problems. Solution procedure gave highly successful results. We attained 121 optimal solutions out of 125 test problems. We generated 12 large test problems in order to be able to test the performance of improvement heuristics (OC, 2OX, OC+2OX, and 2OX+OC) in terms of computation time. The largest run time recorded was 1.17 seconds. Run times of the improvement heuristics for most of the other problems (44 out of 48 problems) were less than half a second.

We developed five different sector allocation models and several variations for SAP. We also investigated the validity of different objective functions. We identified the most suitable model for SAP. We developed cuts for linear programming relaxation of the models and proposed branch and bound solution approaches. Branch-and-bound solution approaches employ various branching and branch selection strategies along with the methods for deriving tight lower and upper bounds on the problem, in order to compose a viable solution strategy. The approach has been tested on some randomly generated problems. Our solution procedure performed better than CPLEX in term of computation time and number of nodes explored. Although our implementation for solving the LP relaxed sub-problems in the branch and bound tree is not efficient in terms of time, we still perform better than CPLEX, since we need to explore only a very small fraction of nodes compared to CPLEX. We owe this to tightness of our bounds and our problem specific branching strategy. Our solution procedure could solve even faster by using an embedded LP solver for sub-problems.

We have investigated SAP under two different assumptions: no information about the exact attack direction, and information coming from intelligence and surveillance sources about the direction of the attack. Our solution approach can solve SAP using any of those assumptions. Note that we need to define representative scenarios for the first assumption. In that case, we expect the possible attack from any direction between 000 and 360 degrees. Thus, solving SAP with the first assumption enables us to solve SAP with the second assumption and vice versa.

We integrated MAP and SAP problems together in order to come up with a robust sector allocation for a naval TG by using MAP results within SAP. Two different coverage aggregation procedures in the development of the robust formation were presented. Aggregation schemes produced reasonable formations. We have shown the effect of robust formations on MAP solutions.

Missile allocation model may be used in several areas such as in TEWA module of an AAW commander ship (on-line) as well as in decision-making process of the procurement of new air defense ships and in evaluating the capabilities of ships in inventory and the effectiveness of present tactics (off-line). Sector allocation model may be used to develop new formations and tactics to counter the perceived air threat likewise. Since these models are intended for use by the military planners, we addressed the ways of capturing the reality at the maximum extent.

The proposed solution approach for the TG air defense problem can be enhanced along several directions:

Comparison of the proposed methodology with the existing air defense policy and procedures may reveal more insight on the utility of the approach.

In addition to computational time requirement, solution quality of MAP for large problems needs to be investigated provided that an exact solution procedure for large size problems is developed.

We considered SLS firing policy in the solution procedures for MAP. Solution procedures may be developed to include other firing policies such as shoot-shoot-look-shoot-shoot (SSLSS) and shoot-shoot-look-shoot (SSLS) policies.

SLS firing policy has an implicit cost consideration. We only refire, if we do not shot down the threat ASM. Thus, we do not consume SAM rounds, if it is not necessary. However, we may consider cost component explicitly in a bicriteria optimization setting. This approach allows scrutinized investigation of alternative solutions for MAP in terms of both probability of no-leaker and cost.

Increasing the number of sectors by decreasing the bearing and sector spacing increases the resolution of sector allocations. Experimentation with increased sector resolution may help to understand the effect of relatively small changes in sector allocations.

Analyzing the sector allocations using the tools developed in this research and creating template formations for assumed attack sizes and different TG compositions would be beneficial to the commander of TG at sea. Large number of representative scenarios and different number of ship combinations with different air defense capabilities can be used in such an off-line study. Thus, we can create libraries of solutions for possible attack and defense scenarios. This might also provide improved guidelines on prescribing sector allocations.

We solve MAP in a static environment, assuming simultaneous attack. However, both simultaneous and sequential attack waves can occur in the dynamic environment of a real combat situation. Also attack size degrades as we shoot down some of the incoming ASMs, and this leaves some defensive capacity free to allocate against the surviving ASM threat. One way of allocation of the free capacity is solving MAP again with remaining ASMs and SAM rounds. High resolution simulation models can be developed to investigate the best use of MAP solutions in such a dynamic environment and thus, improve the solutions for SAP. Besides the single shot kill probabilities simulation model may include other stochastic elements such as acquisition distance of the threat by search radar, system reliability by each component, weather conditions, sea state. In order to treat both dynamism and stochasticity involved, an alternate approach based on simulation optimization may be used.

Finally, the integrated use of MAP and SAP in off-line analysis of various potential threats may produce results usable in developing a cost effective weapon and ammunition planning methodology.

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APPENDIX A

OPT-CHANGE (OC) ALGORITHM

Step 0: Select an initial feasible engagement list:

$$E = \{(i_1, j_1), (i_2, j_2), ..., (i_l, j_l) \mid k \in L = \{1, ..., l\}, (i_k, j_k) \in V, \\ t_{(i_k, j_k)} < t_{(i_{k'}, j_{k'})} \ \forall\, k \in L \text{ and } \forall\, k' \in L \Leftrightarrow i_k = i_{k'} \text{ and } |k| < |k'|\}$$

where $i \in N$, $j \in M$, $t_{(i_k, j_k)}$ is the time of the engagement (i_k, j_k) and V is the set of valid combinations of ASM and SAM systems, i.e. $(i, j) \in V$ if SAM system j can engage ASM i. Let the corresponding objective function value be $Z(E)$.

Step 1: Set $k = 0$, $E^* = E$, where E^* is the best engagement schedule that has been found so far. Set the logical variables “add1” and “add2” to “false”.

Step 2: Set $k = k + 1$. i.e. take the next engagement in the engagement list of E.

Step 3: Check the possibility of the change of target ASM for the engagement (i_k, j_k) in the engagement list for all possible targets except i_k, i.e. set $F = N \setminus \{i_k\} = \{f_1, f_2, ..., f_{n-1}\}$. Let $h \in H = \{1, 2, ..., n-1\}$ and set $h = 1$.

Step 4: If $h > 1$, then set $h = h + 1$. If $(f_h, j_k) \in V$ then go to Step 5 to find a SAM missile for ASM i_k to enhance the defense against it, otherwise go to Step 11.

Step 5: If there is at least one SAM system that can engage ASM i_k and has missiles left, then check the possibility of enhancement using all possible SAMs, i.e. set $G = M \setminus \{j_k\} = \{g_1, g_2, ..., g_{m-1}\}$. Let $t \in T = \{1,2,...,m-1\}$ and set $t = 1$. Set the logical variable "change" to "false".

Step 6: If $t > 1$, then set $t = t + 1$.

Step 7: If $d_{g_t} > 0$ and $(i_k, g_t) \in V$ then go to Step 8, otherwise go to Step 9. Note that d_{g_t} is the number of available rounds on SAM system g_t.

Step 8: Define a new engagement list, $\overline{E} = \{E \setminus \{(i_k, j_k)\}\} \cup \{(f_h, j_k), (i_k, g_t)\}$. Note that, (i_k, g_t) will be the last engagement of the engagement list. Check the feasibility of new engagement list $\overline{E}$ and calculate the objective function value, $Z(\overline{E})$. If $\overline{E}$ is feasible and $Z(\overline{E}) > Z(E^*)$ then change the engagement list, $E^* = \overline{E}$, update the objective function value $Z(E^*) = Z(\overline{E})$. Set $g^- = g_t$, the variables "add1" and "change" to "true", and "add2" to "false".

Step 9: If $t = m - 1$ then go to Step 10.

Else, go to Step 6.

Step 10: If the variable "change" has value "false" then, define a new engagement list, $\overline{E} = \{E \setminus \{(i_k, j_k)\}\} \cup \{(f_h, j_k)\}$. Check the feasibility of new engagement list $\overline{E}$

and calculate the objective function value, $Z(\overline{E})$. If $\overline{E}$ is feasible and $Z(\overline{E}) > Z(E^*)$ then change the engagement list, $E^* = \overline{E}$, update the objective function value $Z(E^*) = Z(\overline{E})$. Set the variable "add1" and "add2" to "false".

Step 11: If $h = n - 1$ then go to Step 12.

Else, go to Step 4.

Step 12: Consider changing the defending SAM for the engagement (i_k, j_k). Set $t = 1$.

Step 13: If $t > 1$, then set $t = t + 1$.

Step 14: If $d_{g_t} > 0$ and $(i_k, g_t) \in V$ then go to Step 15, otherwise go to Step 16.

Step 15: Define a new engagement list, $\overline{E} = \{E \setminus \{(i_k, j_k)\}\} \cup \{(i_k, g_t)\}$. Note that, we change (i_k, j_k) to (i_k, g_t) in the engagement list E. Check the feasibility of new engagement list $\overline{E}$ and calculate the objective function value, $Z(\overline{E})$. If $\overline{E}$ is feasible and $Z(\overline{E}) > Z(E^*)$ then change the engagement list, $E^* = \overline{E}$, update the objective function value $Z(E^*) = Z(\overline{E})$. Set $g^- = g_t$, $g^+ = j_k$, the variables "add1" to "false" and "add2" to "true".

Step 16: If $t = m - 1$ then go to Step 17.

Else, go to Step 13.

Step 17: If $k = l$ then go to Step 18.

Else, go to Step 2.

Step 18: If $Z(E) = Z(E^*)$ then stop.

Otherwise, set $E = E^*$, $Z(E) = Z(E^*)$, if variable "add1" has value "true", then set $l = l + 1$, $d_{g^-} = d_{g^-} - 1$, if variable "add2" has value "true", then set $d_{g^-} = d_{g^-} - 1$, $d_{g^+} = d_{g^+} + 1$ and go to Step 1.

$l[(n-1)m + (m-1)]$ different cases are considered for change and enhancement in each iteration of the algorithm. The computational complexity for OC algorithm is $O(lmn)$ per iteration.

APPENDIX B

2-OPT-EXCHANGE (2OX) ALGORITHM

Step 0: Select an initial feasible engagement list:

$$E = \{(i_1, j_1), (i_2, j_2), \ldots, (i_l, j_l) \mid k \in L = \{1, \ldots, l\}, (i_k, j_k) \in V,$$
$$t_{(i_k, j_k)} < t_{(i_{k'}, j_{k'})} \ \forall\, k \in L \text{ and } \forall\, k' \in L \Leftrightarrow i_k = i_{k'} \text{ and } |k| < |k'|\}$$

where $i \in N$, $j \in M$, $t_{(i_k, j_k)}$ is the time of the engagement (i_k, j_k) and V is the set of valid combinations of ASM and SAM systems, i.e. $(i, j) \in V$ if SAM system j can engage ASM i. Let the corresponding objective function value of the engagement list E be $Z(E)$. Set $E^* = E$, where E^* is the best engagement schedule that has been found so far.

Step 1: Set $k = 1$ and $h = 1$. Set the logical variables "add1" and "add2" to "false". Those logical variables are used to control whether the best engagement schedule that may be found has additional launhes against ASMs exchanged or not.

Step 2: Check the possibility of exchange of SAM allocation of the engagements k and $k+h$ in the engagement list: If $\{(i_k, j_{k+h}) \notin V$ or $(i_{k+h}, j_k) \notin V\}$ go to Step 18.

Step 3: Define a new engagement list, $\overline{E} = \{..., (i_k, j_{k+h}), ..., (i_{k+h}, j_k), ...\}$. Check the feasibility of new engagement list $\overline{E}$ and calculate the objective function value, $Z(\overline{E})$. If $\overline{E}$ is infeasible, then go to Step 18.

Step 4: If $Z(\overline{E}) > Z(E^*)$ then reset the best engagement list, $E^* = \overline{E}$, update the objective function value and set variables "add1" and "add2" to "false".

Step 5: Check for additional assignment against ASM i_k, i.e. set $G = M = \{g_1, g_2, ..., g_m\}$. Let $t \in T = \{1,2,...,m\}$ and set $t = 1$. Note that, we do not exclude SAM j_k from consideration, since change in a previous engagement may enable us to launch the same engagement (i_k, j_k) as the last engagement against ASM i_k. Set the logical variable "change" to "false". The variable "change" is used to control whether ASM i_k has additional launches against itself.

Step 6: If $t > 1$, then set $t = t + 1$.

Step 7: If $d_{g_t} > 0$ and $(i_k, g_t) \in V$ then go to Step 8, otherwise go to Step 10. Note that d_{g_t} is the number of available rounds on SAM system g_t.

Step 8: Define a new engagement list, $\overline{\overline{E}} = \overline{E} \cup \{(i_k, g_t)\}$. Note that, (i_k, g_t) will be the last engagement of the engagement list. Check the feasibility of new engagement list $\overline{\overline{E}}$ and calculate the objective function value, $Z(\overline{\overline{E}})$. If $\overline{\overline{E}}$ is feasible then set the variable "change" to "true", $g_{change} = g_t$ and go to Step 9, otherwise go to Step 10.

Step 9: If $Z\left(\overline{\overline{E}}\right) > Z\left(E^*\right)$ then reset the best engagement list, $E^* = \overline{\overline{E}}$, update the objective function value $Z\left(E^*\right) = Z\left(\overline{\overline{E}}\right)$. Set $g1^* = g_t$, the variable "add1" to "true" and "add2" to "false".

Step 10: If $t = m$ then go to Step 11.

Else, go to Step 6.

Step 11: Check for additional assignment against ASM i_{k+h}. Set $t = 1$.

Step 12: If $t > 1$, then set $t = t + 1$.

Step 13: If the variable "change" has value "true" go to Step 14, otherwise go to Step 16.

Step 14: If $g_t = g_{change}$ and $d_{g_t} > 1$ then go to Step 15,

else if $g_t \neq g_{change}$ and $d_{g_t} > 0$ then go to Step 15,

otherwise go to Step 17.

Step 15: If $\left(i_{k+h}, g_t\right) \in V$ then define a new engagement list, $\overline{\overline{\overline{E}}} = \overline{\overline{E}} \cup \left\{\left(i_{k+h}, g_t\right)\right\}$. Check the feasibility of new engagement list $\overline{\overline{\overline{E}}}$ and calculate the objective function value, $Z\left(\overline{\overline{\overline{E}}}\right)$. If $\overline{\overline{\overline{E}}}$ is feasible and $Z\left(\overline{\overline{\overline{E}}}\right) > Z\left(E^*\right)$ then change the engagement list, $E^* = \overline{\overline{\overline{E}}}$, update the objective function value $Z\left(E^*\right) = Z\left(\overline{\overline{\overline{E}}}\right)$, set $g2^* = g_t$, the variables "add2" "true", otherwise go to Step 17.

Step 16: If $d_{g_t} > 0$ then define a new engagement list, $\overline{\overline{E}} = \overline{E} \cup \{(i_{k+h}, g_t)\}$. Check the feasibility of new engagement list $\overline{\overline{E}}$ and calculate the objective function value, $Z(\overline{\overline{E}})$. If $\overline{\overline{E}}$ is feasible and $Z(\overline{\overline{E}}) > Z(E^*)$ then change the engagement list, $E^* = \overline{\overline{E}}$, update the objective function value $Z(E^*) = Z(\overline{\overline{E}})$. Set $g2^* = g_t$, the variable "add2" to "true" and "add1" to "false".

Step 17: If $t = m$ then go to Step 18.

Else, go to Step 12.

Step 18: If $k + 1 = l$, then go to Step 19.

Else,

if $k + h = l$ then set $t = 1$, $k = k + 1$ and go to Step 2.

if $k + h < l$ then set $h = h + 1$ and go to Step 2.

Step 19: If $Z(E) = Z(E^*)$ then go to step 20.

Otherwise, set $E = E^*$, $Z(E) = Z(E^*)$, if variable "add1" has value "true", then set $l = l + 1$, $d_{g1^*} = d_{g1^*} - 1$, if variable "add2" has value "true", then set $l = l + 1$, $d_{g2^*} = d_{g2^*} - 1$, and go back to Step 1.

Step 20: For each possible ASM pair, try changing all the engagements of those ASMs. If there is an improvement, update the engagement list E and go back to Step1, otherwise stop.

$\frac{l(l-1)}{2}(2m+1)$ different neighboring engagement lists are checked for exchange for each iteration of the algorithm. If an exchange is made, then the algorithm starts over again. Algorithm stops when no exchange is possible. Note that an undesirable exchange may be desirable after a change in the engagement list. Thus, we continue until no desirable exchange is left for the engagement list. The computational complexity for 2OX algorithm is $O\left(l^2 m\right)$ per iteration.

APPENDIX C

DATA FOR SAMPLE MAP GENERATION

Table C.1. Parameters of the Sample SAM Systems Used for Problem Generation.

Name	Speed (m/sec)	Minimum Range (km)	Maximum Range (km)	Type
SeaSparrow	850	1.5	16	Self-Defense
ESSM	1224	1.5	18	Self-Defense
Aster-15	986	1.5	30	Self-Defense
Barak	680	1.5	12	Self-Defense
SM-1	680	5.0	38	Area Air Defense
SM-2	850	5.0	170	Area Air Defense
Aster-30	1394	3.0	100	Area Air Defense

Table C.2. Parameters of the Sample ASMs Used for Problem Generation.

Name	Speed (m/sec)	Maximum Range (km)
Harpoon	289	130
MM-38 Exocet	306	41
Polyphem	221	61
Gabriel	238	19
Penguin	238	18
SS-N-26	1190	290
Maveric	850	25

Steps of the sample MAP generation are as follows:

1. Choose required number of SAM systems from sample list. Generate at least one area air defense SAM system for the problems having two or more SAM systems.

2. Determine the initial number of missiles of each SAM system in the launchers (no more than 9 missiles for each SAM system).

3. Choose an ASM from the sample list.

4. Determine the target ship of the threat ASMs.

5. Determine the initial detection range of the threat ASM from its target ship ranging from 5 to 40 km.

Note that we randomly generate all the information above using different random number streams.

APPENDIX D

RESULTS OF CONSTRUCTION HEURISTICS

Table D.1. Comparison of Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (First Set)

ASM		SAM 1	2	3	4	5
1	IE Obj	0.640	0.874	0.874	0.874	0.927
	BH Obj	0.640	0.874	0.874	0.874	0.927
	IE Sched.*	11 / 11	211 / 111	211 / 111	233 / 111	553 / 111
	BH Sched.*	11 / 11	211 / 111	212 / 111	233 / 111	253 / 111
	IE Time**	0.00	0.00	0.00	0.00	0.00
	BH Time**	0.00	0.00	0.00	0.00	0.00
2	IE Obj	0.160	0.416	0.559	0.416	0.602
	BH Obj	0.160	0.416	0.559	0.416	0.602
	IE Sched.	11 / 11	211 / 122	33211 / 22111	332 / 112	5553 / 1121
	BH Sched.	11 / 21	211 / 122	23311 / 12211	233 / 211	2553 / 2111
	IE Time	0.00	0.00	0.63	0.62	1.75
	BH Time	0.00	0.00	0.00	0.00	0.00
3	IE Obj	0.164	0.120	0.307	0.166	0.452
	BH Obj	0.164	0.120	0.307	0.166	0.452
	IE Sched.	11111 / 11223	211 / 312	33211 / 22311	3321 / 1123	55532 / 11213
	BH Sched.	11111 / 32121	211 / 321	23131 / 32121	2313 / 2131	25553 / 32111
	IE Time	0	0	0.422	0.422	9.812
	BH Time	0	0	0	0	0
4	IE Obj	0.051	0.339	0.096	0.118	0.383
	BH Obj	0.051	0.284	0.096	0.065	0.383
	IE Sched.	11111 / 11234	2211111 / 3411122	33211 / 24311	443321 / 221143	555332 / 123114
	BH Sched.	11111 / 32412	2211111 / 3241241	23311 / 32411	24433 / 32411	255533 / 324111
	IE Time	0.50	41.90	0.00	1.99	1294.61
	BH Time	0.00	0.00	0.00	0.00	0.00
5	IE Obj	0.016	0.138	0.159	0.037	0.225
	BH Obj	0.016	0.089	0.143	0.021	0.173
	IE Sched.	11111 / 12345	2211111 / 3411225	32211111 / 23411155	443321 / 221543	555332 / 234115
	BH Sched.	11111 / 32415	2211111 / 3241524	22311111 / 3241515	241433 / 123451	255533 / 324151
	IE Time	0.22	228.77	7008.60	4.96	8503.29
	BH Time	0.00	0.00	0.00	0.00	0.00

* IE or BH Sched: SAM Engagement Order / Target ASM Order

** Elapsed time in seconds.

Table D.2. Comparison of Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Second Set)

ASM		SAM				
		1	2	3	4	5
1	IE Obj	0.640	0.940	0.940	0.940	0.900
	BH Obj	0.640	0.940	0.940	0.940	0.900
	IE Sched.*	11 / 11	21/11	21/11	21/11	2/1
	BH Sched.*	11 / 11	21/11	21/11	21/11	2/1
	IE Time**	0.00	0.00	0.00	0.00	0.00
	BH Time**	0.00	0.00	0.00	0.00	0.00
2	IE Obj	0.080	0.705	0.705	0.705	0.675
	BH Obj	0.080	0.705	0.705	0.705	0.675
	IE Sched.	11/12	221/121	221/121	221/121	22/12
	BH Sched.	11/12	221/121	221/211	221/211	22/12
	IE Time	0.00	0.00	0.63	0.00	0.00
	BH Time	0.00	0.00	0.00	0.00	0.00
3	IE Obj	0.041	0.645	0.645	0.203	0.504
	BH Obj	0.038	0.645	0.645	0.203	0.474
	IE Sched.	1111/1233	222211/123313	222211/123313	221/123	44222/33123
	BH Sched.	1111/2131	222121/123133	222121/213133	221/213	2224/1233
	IE Time	0	2.76	2.82	0	0.11
	BH Time	0	0.6	0	0	0
4	IE Obj	0.007	0.420	0.419	0.198	0.281
	BH Obj	0.007	0.378	0.399	0.184	0.281
	IE Sched.	1111/1234	222211/123434	222211/123433	3333221/2444133	44222/44123
	BH Sched.	1111/1234	222211/123414	222211/241313	2213333/1234444	22244/12344
	IE Time	0.60	11.49	10.45	216.30	0.33
	BH Time	0.00	0.00	0.00	0.00	0.00
5	IE Obj	0.008	0.232	0.232	0.106	0.070
	BH Obj	0.007	0.143	0.143	0.106	0.070
	IE Sched.	1111111/1233445	222211/124533	222211/124533	3333221/4455123	44222/45123
	BH Sched.	1111111/1234515	222211/215431	222211/245131	2213333/1234554	22244/12345
	IE Time	1755.26	46.97	38.60	1127.91	2.15
	BH Time	0.00	0.00	0.00	0.00	0.00

* IE or BH Sched: SAM Engagement Order / Target ASM Order

** Elapsed time in seconds.

Table D.3. % Gap Between Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Second Set)

ASM	**SAM**				
	1	**2**	**3**	**4**	**5**
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	5.9	0.0	0.0	0.0	6.0
4	0.0	10.1	4.8	7.2	0.0
5	11.4	38.6	38.6	0.0	0.0

Table D.4. Comparison of Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Third Set)

ASM		SAM				
		1	2	3	4	5
1	IE Obj	0.360	0.520	0.520	0.400	0.400
	BH Obj	0.360	0.360	0.360	0.400	0.400
	IE Sched.*	11/11	21/11	21/11	4/1	4/1
	BH Sched.*	11/11	11/11	11/11	4/1	4/1
	IE Time**	0.00	0.00	0.00	0.00	0.00
	BH Time**	0.00	0.00	0.00	0.00	0.00
2	IE Obj	0.072	0.320	0.370	0.348	0.160
	BH Obj	0.072	0.256	0.282	0.348	0.160
	IE Sched.	111/112	21112/11222	21122/11222	44444/12222	44/12
	BH Sched.	111/121	12121/12122	12122/12122	44444/12222	44/12
	IE Time	0.00	0.60	3.52	0.16	0.00
	BH Time	0.00	0.00	0.00	0.00	0.00
3	IE Obj	0.018	0.156	0.216	0.251	0.072
	BH Obj	0.018	0.150	0.184	0.246	0.072
	IE Sched.	111/123	22111/13222	22211/12312	44444/12223	441/123
	BH Sched.	111/123	12211/12312	12212/12312	44444/12323	441/123
	IE Time	0	0.11	40.7	2.92	0.11
	BH Time	0	0.6	0	0	0
4	IE Obj	0.030	0.039	0.069	0.123	0.000
	BH Obj	0.030	0.035	0.046	0.108	0.000
	IE Sched.	1111111/1122344	22111/14223	22211/13422	44444/12234	-
	BH Sched.	1111111/1234142	12211/12341	12212/12344	44444/12344	441/123
	IE Time	413.99	0.44	269.35	10.11	0.33
	BH Time	0.00	0.00	0.00	0.00	0.00
5	IE Obj	0.006	0.009	0.052	0.069	0.000
	BH Obj	0.006	0.007	0.035	0.054	0.000
	IE Sched.	1111111/1122345	22111/15234	3322211/5513422	444443/123455	55441/55123
	BH Sched.	1111111/1234515	12211/12345	1221332/1234554	444434/123454	44155/12355
	IE Time	2190.99	2.82	1245.94	37.91	1.10
	BH Time	0.00	0.00	0.00	0.00	0.00

* IE or BH Sched: SAM Engagement Order / Target ASM Order

** Elapsed time in seconds.

Table D.5. % Gap Between Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Third Set)

ASM	**SAM**				
	1	**2**	**3**	**4**	**5**
1	0.0	30.8	30.8	0.0	0.0
2	0.0	20.0	23.8	0.0	0.0
3	0.0	4.1	14.8	2.0	0.0
4	0.0	11.1	33.3	12.5	0.0
5	8.3	22.2	33.3	22.2	0.0

Table D.6. Comparison of Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Fourth Set)

ASM		SAM				
		1	2	3	4	5
1	IE Obj	0.784	0.400	0.880	0.450	1.000
	BH Obj	0.784	0.400	0.880	0.450	1.000
	IE Sched.*	111 / 111	2/1	32/11	4/1	55555/11111
	BH Sched.*	111/111	2/1	32/11	4/1	55555/11111
	IE Time**	0.00	0.00	0.00	0.00	0.00
	BH Time**	0.00	0.00	0.00	0.00	0.00
2	IE Obj	0.204	0.263	0.449	0.135	0.899
	BH Obj	0.204	0.263	0.449	0.135	0.899
	IE Sched.	111/122	2111/1222	3211/1122	41/12	55555/11222
	BH Sched.	111/122	2111/1222	3211/1122	41/12	55555/12212
	IE Time	0.00	0.00	0.00	0.00	0.27
	BH Time	0.00	0.00	0.00	0.00	0.00
3	IE Obj	0.062	0.033	0.062	0.171	0.168
	BH Obj	0.062	0.033	0.062	0.171	0.168
	IE Sched.	111111/122333	22111/13222	211111/122333	433111/133222	55221/12332
	BH Sched.	111111/123323	22111/13222	211111/123323	33111/31222	55221/12332
	IE Time	1.21	0.6	0.17	4.45	0.00
	BH Time	0	0	0	0	0
4	IE Obj	0.037	0.013	0.021	0.098	0.065
	BH Obj	0.037	0.013	0.021	0.067	0.000
	IE Sched.	1111111/1122334	22111/22134	211111/211334	331111/231144	55221/34221
	BH Sched.	1111111/1234231	22111/22134	211111/211334	331111/231431	55122/12322
	IE Time	57.18	0.50	1.10	37.86	0.60
	BH Time	0.00	0.00	0.00	0.00	0.00
5	IE Obj	0.007	0.002	0.222	0.021	0.012
	BH Obj	0.007	0.002	0.222	0.020	0.010
	IE Sched.	1111111/1122345	22111/45123	3333332/1123451	331111/152234	55221/45132
	BH Sched.	1111111/1234514	22111/45123	3333332/2345111	331111/514234	55122/41235
	IE Time	512.35	0.11	229.59	166.42	0.11
	BH Time	0.00	0.00	0.00	0.00	0.00

* IE or BH Sched: SAM Engagement Order / Target ASM Order

** Elapsed time in seconds.

Table D.7. % Gap Between Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Fourth Set)

ASM	SAM				
	1	2	3	4	5
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	31.3	100.0
5	5.9	0.0	0.0	5.9	23.1

Table D.8. Comparison of Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Fifth Set)

ASM		SAM				
		1	2	3	4	5
1	IE Obj	0.760	0.852	0.760	0.698	0.098
	BH Obj	0.760	0.852	0.760	0.698	0.098
	IE Sched.*	1111/1111	2211/1111	1111/1111	22/11	55/11
	BH Sched.*	1111/1111	2211/1111	1111/1111	22/11	55/11
	IE Time**	0.00	0.00	0.00	0.00	0.00
	BH Time**	0.00	0.00	0.00	0.00	0.00
2	IE Obj	0.410	0.690	0.515	0.667	0.492
	BH Obj	0.410	0.690	0.515	0.595	0.456
	IE Sched.	1111/1122	22111/12112	333111/111222	444422/111122	5441154/2112211
	BH Sched.	1111/1221	22111/12212	311313/122121	224444/211111	411454/122121
	IE Time	0.00	0.00	1.28	0.00	0.28
	BH Time	0	0.00	0.00	0.00	0.00
3	IE Obj	0.082	0.320	0.125	0.288	0.111
	BH Obj	0.077	0.302	0.125	0.288	0.111
	IE Sched.	1111/1233	22111/12333	333111/333112	444422/333312	5444115/1333121
	BH Sched.	1111/1231	22111/12313	113133/123133	224444/123333	1145454/1231313
	IE Time	0.00	0.5	5.62	0.17	4.6
	BH Time	0	0	0	0	0
4	IE Obj	0.027	0.143	0.044	0.277	0.062
	BH Obj	0.027	0.118	0.044	0.277	0.062
	IE Sched.	1111/1234	22111/13224	333111/222134	322224/312342	5544411/3322214
	BH Sched.	1111/1234	22111/12343	131133/123422	222234/142332	5141544/3124322
	IE Time	0.00	0.50	31.20	1.26	26.76
	BH Time	0.00	0.00	0.00	0.00	0.00
5	IE Obj	0.014	0.037	0.014	0.058	0.014
	BH Obj	0.014	0.034	0.014	0.058	0.013
	IE Sched.	11111/12345	22111/24135	3333111/1155234	432222/521234	5544411/2211534
	BH Sched.	11111/12345	22111/12345	3111333/1234551	222243/341252	5411445/2134552
	IE Time	0.22	1.28	159.79	6.26	151.83
	BH Time	0.00	0.00	0.00	0.00	0.00

* IE or BH Sched: SAM Engagement Order / Target ASM Order

** Elapsed time in seconds.

Table D.9. % Gap Between Implicit Enumeration (IE) and the Best of Construction Heuristics (BH). (Fifth Set)

ASM	SAM				
	1	**2**	**3**	**4**	**5**
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	10.7	7.3
3	5.9	5.7	0.0	0.0	0.0
4	0.0	17.9	0.0	0.0	0.0
5	0.0	7.7	0.0	0.0	5.6

APPENDIX E

RESULTS OF IMPROVEMENT HEURISTICS

In this appendix, we report the detailed run results for the improvement algorithms. We generated 125 problems in order to measure the performance of the construction algorithms. Construction algorithms produced optimal solutions for 87 test problems. Using improvement algorithms, we solved 38 test problems for which the construction algorithms produced non-optimal solutions. For each test problem and solution algorithm pair, we report objective function value, engagement schedule, and the elapsed CPU time for the solution algorithm.

Table E.1. Results of Construction Heuristics and the Best Construction Heuristic (BH). (Problems I.4.2 – II.3.5)

*Problem #		Optimal	BH	OC	2OX	OC+2OX	2OX+OC
	Obj.Func.	0.3387	0.2844	0.2867	0.2918	0.3387	0.3387
I.4.2	**Schedule	2211111 / 3411122	2211111 / 3241241	2211111/3241141	2211111/3421241	2211111/3421121	2211111/3421211
	***Time	41.90	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.1175	0.0648	0.1161	0.0648	0.1175	0.1161
I.4.4	Schedule	443321 / 221143	24433 / 32411	244331/244113	24433/32411	244331/422113	244331/244113
	Time	1.99	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.1382	0.0889	0.1170	0.1382	0.1382	0.1382
I.5.2	Schedule	2211111 / 3411225	2211111 / 3241524	2211111/3241514	2211111/3412521	2211111/3421512	2211111/3412521
	Time	228.77	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.1588	0.1433	0.1433	0.1588	0.1588	0.1588
I.5.3	Schedule	32211111 / 23411155	22311111 / 3241515	22311111/32415151	22311111/34215151	22311111/34215151	22311111/34215151
	Time	7008.60	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.0367	0.0213	0.0363	0.0276	0.0367	0.0276
I.5.4	Schedule	443321 / 221543	241433 / 123451	241433/243451	241433/523411	241433/423251	241433/523411
	Time	4.96	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.2246	0.1733	0.2139	0.2246	0.2139	0.2246
I.5.5	Schedule	555332 / 234115	255533 / 324151	255533/241355	255533/245311	255533/241355	255533/245311
	Time	8503.29	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.0408	0.0384	0.0408	0.0384	0.0408	0.0408
II.3.1	Schedule	1111/1233	1111/2131	1111/3231	1111/1231	1111/3231	1111/3231
	Time	0.00	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.5043	0.4742	0.4742	0.4742	0.4742	0.4742
II.3.5	Schedule	44222/33123	2224/1233	2224/1233	2224/1233	2224/1233	2224/1233
	Time	0.11	0.00	0.00	0.00	0.00	0.00

* Problem #: Roman numeral shows the number of problem set, 2nd numeral shows the ASM number and 3rd numeral shows SAM system number

** Schedule: SAM engagement order / target ASM order

*** Time: Seconds for personal computer with AMD Athlon XP2000+ CPU

Table E.2. Results of Construction Heuristics and the Best Construction Heuristic (BH). (Problems II.4.2 – III.2.3)

*Problem #		Optimal	BH	OC	2OX	OC+2OX	2OX+OC
	Obj.Func.	0.4204	0.3781	0.4204	0.3781	0.4204	0.4204
II.4.2	**Schedule	222211/123434	222211/123414	222211/123434	222211/123414	222211/123434	222211/123434
	***Time	11.49	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.4194	0.3992	0.4194	0.3992	0.4194	0.4194
II.4.3	Schedule	222211/123433	222211/241313	222211/241333	222211/241313	222211/241333	222211/241333
	Time	10.45	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.1983	0.1840	0.1840	0.1840	0.1840	0.1840
II.4.4	Schedule	3333221/2444133	2213333/1234444	2213333/1234444	2213333/1234444	2213333/1234444	2213333/1234444
	Time	216.30	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.0083	0.0074	0.0083	0.0074	0.0083	0.0083
II.5.1	Schedule	1111111/1233445	1111111/1234515	1111111/3234415	1111111/1234515	1111111/3234415	1111111/3234415
	Time	1755.26	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.2324	0.1428	0.2324	0.2106	0.2324	0.2106
II.5.2	Schedule	222211/124533	222211/215431	222211/215433	222211/235411	222211/215433	222211/235411
	Time	46.97	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.2324	0.1428	0.2324	0.2106	0.2324	0.2106
II.5.3	Schedule	222211/124533	222211/245131	222211/245133	222211/245311	222211/245133	222211/245311
	Time	38.60	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.3203	0.2563	0.2563	0.3203	0.3203	0.3203
III.2.2	Schedule	21112/11222	12121/12122	12121/12122	12121/21221	12121/21221	12121/21221
	Time	0.60	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.3702	0.2822	0.2822	0.3702	0.3702	0.3702
III.2.3	Schedule	21122/11222	12122/12122	12122/12122	12122/21122	12122/21122	12122/21122
	Time	3.52	0.00	0.00	0.00	0.00	0.00

* Problem #: Roman numeral shows the number of problem set, 2nd numeral shows the ASM number and 3rd numeral shows SAM system number

** Schedule: SAM engagement order / target ASM order

*** Time: Seconds for personal computer with AMD Athlon XP2000+ CPU

Table E.3. Results of Construction Heuristics and the Best Construction Heuristic (BH). (Problems III.3.2 – III.5.2)

*Problem #		Optimal	BH	OC	2OX	OC+2OX	2OX+OC
	Obj.Func.	0.1562	0.1498	0.1498	0.1498	0.1498	0.1498
III.3.2	**Schedule	22111/13222	12211/12312	12211/12312	12211/22311	12211/22311	12211/22311
	***Time	0.11	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.2163	0.1843	0.1843	0.2163	0.2163	0.2163
III.3.3	Schedule	22211/12312	12212/12312	12212/12312	12212/21312	12212/21312	12212/21312
	Time	40.70	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.2509	0.2458	0.2509	0.2458	0.2509	0.2509
III.3.4	Schedule	44444/12223	44444/12323	44444/12223	44444/12323	44444/12223	44444/12223
	Time	2.92	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.0389	0.0346	0.0346	0.0389	0.0389	0.0389
III.4.2	Schedule	22111/14223	12211/12341	12211/12341	12211/12431	12211/12431	12211/12431
	Time	0.44	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.0691	0.0461	0.0691	0.0653	0.0691	0.0653
III.4.3	Schedule	22211/13422	12212/12344	12212/12314	12212/42341	12212/12314	12212/42341
	Time	269.35	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.1229	0.1075	0.1229	0.1075	0.1229	0.1229
III.4.4	Schedule	44444/12234	44444/12344	44444/12324	44444/12344	44444/12324	44444/12324
	Time	10.11	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.0061	0.0056	0.0061	0.0056	0.0061	0.0061
III.5.1	Schedule	1111111/1122345	1111111/1234515	1111111/1234215	1111111/1234515	1111111/1234215	1111111/1234215
	Time	2190.99	0.00	0.00	0.00	0.00	0.00
	Obj.Func.	0.0086	0.0067	0.0067	0.0086	0.0086	0.0086
III.5.2	Schedule	22111/15234	12211/12345	12211/12345	12211/12543	12211/12543	12211/12543
	Time	2.82	0.00	0.00	0.00	0.00	0.00

* Problem #: Roman numeral shows the number of problem set, 2nd numeral shows the ASM number and 3rd numeral shows SAM system number
** Schedule: SAM engagement order / target ASM order
*** Time: Seconds for personal computer with AMD Athlon XP2000+ CPU

Table E.4. Results of Construction Heuristics and the Best Construction Heuristic (BH). (Problems III.5.3 – IV.2.4)

*Problem #		Optimal	BH	OC	2OX	OC+2OX	2OX+OC
III.5.3	Obj.Func.	0.0518	0.0346	0.0518	0.0490	0.0518	0.0490
	**Schedule	3322211/5513422	1221332/1234554	1221332/1231554	1221332/4234551	1221332/1231554	1221332/4234551
	***Time	1245.94	0.00	0.00	0.00	0.00	0.00
III.5.4	Obj.Func.	0.0691	0.0538	0.0691	0.0538	0.0691	0.0691
	Schedule	444443/123455	444434/123454	444434/123554	444434/123454	444434/123554	444434/123554
	Time	37.91	0.00	0.00	0.00	0.00	0.00
IV.4.4	Obj.Func.	0.0979	0.0673	0.0979	0.0979	0.0979	0.0979
	Schedule	331111/231144	331111/231431	331111/231441	331111/231441	331111/231441	331111/231441
	Time	37.86	0.00	0.00	0.00	0.00	0.00
IV.4.5	Obj.Func.	0.0648	0.0000	0.0518	0.0000	0.0648	0.0518
	Schedule	55221/34221	55122/12322	55122/14322	55122/12322	55122/34122	55122/14322
	Time	0.60	0.00	0.00	0.00	0.00	0.00
IV.5.1	Obj.Func.	0.0073	0.0069	0.0073	0.0069	0.0073	0.0073
	Schedule	1111111/1122345	1111111/1234514	1111111/1232514	1111111/1234514	1111111/1232514	1111111/1232514
	Time	512.35	0.00	0.00	0.00	0.00	0.00
IV.5.4	Obj.Func.	0.0212	0.0200	0.0212	0.0200	0.0212	0.0212
	Schedule	331111/152234	331111/514234	331111/512234	331111/514234	331111/512234	331111/512234
	Time	166.42	0.00	0.00	0.00	0.00	0.00
IV.5.5	Obj.Func.	0.0125	0.0096	0.0096	0.0125	0.0125	0.0125
	Schedule	55221/45132	55122/41235	55122/41235	55122/45231	55122/45231	55122/45231
	Time	0.11	0.00	0.00	0.00	0.00	0.00
V.2.4	Obj.Func.	0.6668	0.5954	0.6668	0.5954	0.6668	0.6668
	Schedule	444422/111122	224444/211111	224444/221111	224444/211111	224444/221111	224444/221111
	Time	0.00	0.00	0.00	0.00	0.00	0.00

* Problem #: Roman numeral shows the number of problem set, 2nd numeral shows the ASM number and 3rd numeral shows SAM system number

** Schedule: SAM engagement order / target ASM order

*** Time: Seconds for personal computer with AMD Athlon XP2000+ CPU

Table E.5. Results of Construction Heuristics and the Best Construction Heuristic (BH). (Problems V.2.5 – V.5.5)

*Problem #		Optimal	BH	OC	2OX	OC+2OX	2OX+OC
V.2.5	Obj.Func.	0.4917	0.4560	0.4560	0.4560	0.4560	0.4560
	**Schedule	5441154/2112211	411454/122121	411454/122121	411454/122121	411454/122121	411454/122121
	***Time	0.28	0.00	0.00	0.00	0.00	0.00
V.3.1	Obj.Func.	0.0816	0.0768	0.0816	0.0768	0.0816	0.0816
	Schedule	1111/1233	1111/1231	1111/3231	1111/1231	1111/3231	1111/3231
	Time	0.00	0.00	0.00	0.00	0.00	0.00
V.3.2	Obj.Func.	0.3203	0.3022	0.3203	0.3022	0.3203	0.3203
	Schedule	22111/12333	22111/12313	22111/12333	22111/12313	22111/12333	22111/12333
	Time	0.50	0.00	0.00	0.00	0.00	0.00
V.4.2	Obj.Func.	0.1434	0.1177	0.1177	0.1434	0.1434	0.1434
	Schedule	22111/13224	22111/12343	22111/12343	22111/13242	22111/13242	22111/13242
	Time	0.50	0.00	0.00	0.00	0.00	0.00
V.5.2	Obj.Func.	0.0366	0.0338	0.0338	0.0366	0.0366	0.0366
	Schedule	22111/24135	22111/12345	22111/12345	22111/42315	22111/42315	22111/42315
	Time	1.28	0.00	0.00	0.00	0.00	0.00
V.5.5	Obj.Func.	0.0138	0.0131	0.0138	0.0131	0.0138	0.0138
	Schedule	5544411/2211534	5411445/2134552	5411445/2134152	5411445/2134552	5411445/2134152	5411445/2134152
	Time	151.83	0.00	0.00	0.00	0.00	0.00

* Problem #: Roman numeral shows the number of problem set, 2nd numeral shows the ASM number and 3rd numeral shows SAM system number

** Schedule: SAM engagement order / target ASM order

*** Time: Seconds for personal computer with AMD Athlon XP2000+ CPU

APPENDIX F

STATISTICAL COMPARISON OF IMPROVEMENT HEURISTICS

WILCOXON SIGNED RANK TEST

Table F.1. Summary of Calculations Required by Wilcoxon Test.

*Problem Number	Optimal Solution	OC+2OX		2OX+OC		Best	
		Solution	Ratio	Solution	Ratio	Solution	Ratio
I.4.2	0.3387	0.3387	1.000	0.3387	1.000	0.3387	1.000
I.4.4	0.1175	0.1175	1.000	0.1161	0.988	0.1175	1.000
I.5.2	0.1382	0.1382	1.000	0.1382	1.000	0.1382	1.000
I.5.3	0.1588	0.1588	1.000	0.1588	1.000	0.1588	1.000
I.5.4	0.0367	0.0367	1.000	0.0276	0.753	0.0367	1.000
I.5.5	0.2246	0.2139	0.952	0.2246	1.000	0.2246	1.000
II.3.1	0.0408	0.0408	1.000	0.0408	1.000	0.0408	1.000
II.3.5	0.5043	0.4742	0.940	0.4742	0.940	0.4742	0.940
II.4.2	0.4204	0.4204	1.000	0.4204	1.000	0.4204	1.000
II.4.3	0.4194	0.4194	1.000	0.4194	1.000	0.4194	1.000
II.4.4	0.1983	0.1840	0.928	0.1840	0.928	0.1840	0.928
II.5.1	0.0083	0.0083	1.000	0.0083	1.000	0.0083	1.000
II.5.2	0.2324	0.2324	1.000	0.2106	0.906	0.2324	1.000
II.5.3	0.2324	0.2324	1.000	0.2106	0.906	0.2324	1.000
III.2.2	0.3203	0.3203	1.000	0.3203	1.000	0.3203	1.000
III.2.3	0.3702	0.3702	1.000	0.3702	1.000	0.3702	1.000
III.3.2	0.1562	0.1498	0.959	0.1498	0.959	0.1498	0.959
III.3.3	0.2163	0.2163	1.000	0.2163	1.000	0.2163	1.000
III.3.4	0.2509	0.2509	1.000	0.2509	1.000	0.2509	1.000
III.4.2	0.0389	0.0389	1.000	0.0389	1.000	0.0389	1.000
III.4.3	0.0691	0.0691	1.000	0.0653	0.944	0.0691	1.000
III.4.4	0.1229	0.1229	1.000	0.1229	1.000	0.1229	1.000
III.5.1	0.0061	0.0061	1.000	0.0061	1.000	0.0061	1.000
III.5.2	0.0086	0.0086	1.000	0.0086	1.000	0.0086	1.000
III.5.3	0.0518	0.0518	1.000	0.0490	0.944	0.0518	1.000
III.5.4	0.0691	0.0691	1.000	0.0691	1.000	0.0691	1.000
IV.4.4	0.0979	0.0979	1.000	0.0979	1.000	0.0979	1.000
IV.4.5	0.0648	0.0648	1.000	0.0518	0.800	0.0648	1.000
IV.5.1	0.0073	0.0073	1.000	0.0073	1.000	0.0073	1.000
IV.5.4	0.0212	0.0212	1.000	0.0212	1.000	0.0212	1.000
IV.5.5	0.0125	0.0125	1.000	0.0125	1.000	0.0125	1.000
V.2.4	0.6668	0.6668	1.000	0.6668	1.000	0.6668	1.000
V.2.5	0.4917	0.4560	0.927	0.4560	0.927	0.4560	0.927
V.3.1	0.0816	0.0816	1.000	0.0816	1.000	0.0816	1.000
V.3.2	0.3203	0.3203	1.000	0.3203	1.000	0.3203	1.000
V.4.2	0.1434	0.1434	1.000	0.1434	1.000	0.1434	1.000
V.5.2	0.0366	0.0366	1.000	0.0366	1.000	0.0366	1.000
V.5.5	0.0138	0.0138	1.000	0.0138	1.000	0.0138	1.000

* Problem Number: Roman numeral shows the number of problem set, 2nd numeral shows the ASM number and 3rd numeral shows SAM system number

Table F.2. Test of OC+2OX Against 2OX+OC.

Problem Number	OC+2OX x_i	2OX+OC y_i	x_i-y_i	Signed Rank of $\lvert x_i - y_i \rvert$
4.2	0.00	0.00	0.00	
4.4	0.00	1.23	-1.23	-1
5.2	0.00	0.00	0.00	
5.3	0.00	0.00	0.00	
5.4	0.00	24.71	-24.71	-8
5.5	4.78	0.00	4.78	2
.3.1	0.00	0.00	0.00	
.3.5	5.97	5.97	0.00	
.4.2	0.00	0.00	0.00	
.4.3	0.00	0.00	0.00	
.4.4	7.21	7.21	0.00	
.5.1	0.00	0.00	0.00	
.5.2	0.00	9.37	-9.37	-5.5
.5.3	0.00	9.37	-9.37	-5.5
I.2.2	0.00	0.00	0.00	
I.2.3	0.00	0.00	0.00	
I.3.2	4.10	4.10	0.00	
I.3.3	0.00	0.00	0.00	
I.3.4	0.00	0.00	0.00	
I.4.2	0.00	0.00	0.00	
I.4.3	0.00	5.56	-5.56	-3.5
I.4.4	0.00	0.00	0.00	
I.5.1	0.00	0.00	0.00	
I.5.2	0.00	0.00	0.00	
I.5.3	0.00	5.56	-5.56	-3.5
I.5.4	0.00	0.00	0.00	
/.4.4	0.00	0.00	0.00	
/.4.5	0.00	20.00	-20.00	-7
/.5.1	0.00	0.00	0.00	
/.5.4	0.00	0.00	0.00	
/.5.5	0.00	0.00	0.00	
.2.4	0.00	0.00	0.00	
.2.5	7.26	7.26	0.00	
.3.1	0.00	0.00	0.00	
.3.2	0.00	0.00	0.00	
.4.2	0.00	0.00	0.00	
.5.2	0.00	0.00	0.00	
.5.5	0.00	0.00	0.00	
			W=	-32

$H_0: \quad E(x_i) = E(y_i) \qquad H_a: \quad E(x_i) < E(y_i)$

$W_{0.05} = -26$

Since $W < W_\alpha$, we reject the null hypothesis.

Table F.3. Test of "Best" Against 2OX+OC.

Problem Number	Best x_i	2OX+OC y_i	x_i-y_i	Signed Rank of $\|x_i$-$y_i\|$
I.4.2	0.00	0.00	0.00	
I.4.4	0.00	1.23	-1.23	-1
I.5.2	0.00	0.00	0.00	
I.5.3	0.00	0.00	0.00	
I.5.4	0.00	24.71	-24.71	-7
I.5.5	0.00	0.00	0.00	
II.3.1	0.00	0.00	0.00	
II.3.5	5.97	5.97	0.00	
II.4.2	0.00	0.00	0.00	
II.4.3	0.00	0.00	0.00	
II.4.4	7.21	7.21	0.00	
II.5.1	0.00	0.00	0.00	
II.5.2	0.00	9.37	-9.37	-4.5
II.5.3	0.00	9.37	-9.37	-4.5
III.2.2	0.00	0.00	0.00	
III.2.3	0.00	0.00	0.00	
III.3.2	4.10	4.10	0.00	
III.3.3	0.00	0.00	0.00	
III.3.4	0.00	0.00	0.00	
III.4.2	0.00	0.00	0.00	
III.4.3	0.00	5.56	-5.56	-2.5
III.4.4	0.00	0.00	0.00	
III.5.1	0.00	0.00	0.00	
III.5.2	0.00	0.00	0.00	
III.5.3	0.00	5.56	-5.56	-2.5
III.5.4	0.00	0.00	0.00	
IV.4.4	0.00	0.00	0.00	
IV.4.5	0.00	20.00	-20.00	-6
IV.5.1	0.00	0.00	0.00	
IV.5.4	0.00	0.00	0.00	
IV.5.5	0.00	0.00	0.00	
V.2.4	0.00	0.00	0.00	
V.2.5	7.26	7.26	0.00	
V.3.1	0.00	0.00	0.00	
V.3.2	0.00	0.00	0.00	
V.4.2	0.00	0.00	0.00	
V.5.2	0.00	0.00	0.00	
V.5.5	0.00	0.00	0.00	
			W=	-28

$H_0: \quad E(x_i) = E(y_i) \qquad H_a: \quad E(x_i) < E(y_i)$

$W_{0.05} = -22$

Since $W < W_\alpha$, we reject the null hypothesis.

APPENDIX G

COMPUTATIONAL RESULTS FOR BRANCHING AND BRANCH SELECTION STRATEGIES

Table G.1. Branching and Branch Selection Strategy Performances for the Problem with 3 AAD, 2 SD, 2 ND Ships.

	Branching Strategy	% for the Number of Nodes Explored*									Total Nodes Explored	Time (sec.)
		1000	2000	3000	4000	5000	6000	7000	8000	Total		
Depth First Search	BS1	**								31.0	113	9.31
										19.5		
	BS2									25.4	591	47.88
										24.7		
	BS3	26.0	25.4	26.6	26.4	25.4	24.7			24.5	6313	435.45
		23.9	24.5	23.3	23.6	24.5	25.3			25.5		
	BS4	27.1	29.1	28.3	27.5	27.2	26.9	26.6	26.3	26.1	8713	707.86
		22.6	20.8	21.6	22.4	22.8	23.1	23.3	23.7	23.9		
	BS5	24.9	26.5	26.7	26.3	25.9	26.0	25.9	25.6	25.1	9075	621.21
		24.8	23.4	23.2	23.7	24.0	24.0	24.0	24.4	24.9		
	BS6	26.2	26.8	26.3	26.4	26.5	26.1	25.7	25.3	24.8	9305	765.57
		23.5	23.1	23.7	23.6	23.5	23.9	24.2	24.7	25.2		
Best Node First Search	BS1									31.0	113	10.37
										19.5		
	BS2									25.4	591	51.62
										24.7		
	BS3	0.0	0.0	0.0	3.3	12.7	20.6			24.5	6313	539.94
		30.1	37.2	41.4	40.3	32.2	26.8			25.5		
	BS4	0.0	5.8	12.2	18.2	25.7				32.3	5489	400.23
		36.1	31.8	27.3	23.9	19.4				17.7		
	BS5	0.0	0.0	1.0	8.0	15.0	20.8			29.2	6773	511.96
		32.7	37.1	39.5	33.5	28.1	23.5			20.8		
	BS6	0.0	0.0	1.5	8.9	15.9	22.1			29.0	6597	504.14
		31.8	37.7	39.1	33.1	27.6	23.1			21.0		

* Numbers show the % of nodes pruned by lower bound and the upper bound respectively.

** Empty cells show that the optimal solution is found before the corresponding number of nodes explored.

Table G.2. Branching and Branch Selection Strategy Performances for the Problem with 3 AAD, 3 SD, 3 ND Ships.

	Branching Strategy	% for the Number of Nodes Explored*									Total Nodes Explored	Time (sec.)
		1000	2000	3000	4000	5000	6000	7000	8000	Total		
Depth First Search	BS1	**								34.2	41	3.20
										14.6		
	BS2									21.1	469	43.45
										28.4		
	BS3	20.0	21.0	21.1	21.2	20.7				19.8	5905	498.77
		29.6	28.8	28.8	28.7	29.1				30.2		
	BS4	30.0	29.9	29.4	28.8					28.1	4975	387.73
		19.5	19.7	20.4	21.0					21.9		
	BS5	28.8	29.6	29.1	28.3	27.8	27.5	26.4		25.9	7739	618.39
		20.7	20.2	20.7	21.5	22.1	22.4	23.5		24.1		
	BS6	25.2	23.9	23.7	24.1	24.8	24.3	23.8		23.5	7229	627.68
		24.5	25.9	26.1	25.8	25.1	25.6	26.1		26.4		
Best Node First Search	BS1									21.1	57	5.26
										28.1		
	BS2									20.1	815	77.48
										29.8		
	BS3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.0	17365	1476.15
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	30.9		
	BS4	0.0	0.0	0.7	3.1	5.5	8.4	13.7		20.9	7909	700.94
		30.3	30.0	35.1	32.4	32.9	34.0	32.4		29.1		
	BS5	0.0	0.0	0.0	0.5	2.6	4.5	7.5	12.5	20.5	9181	781.71
		30.8	31.9	33.8	35.7	33.9	31.7	33.2	32.8	29.5		
	BS6	0.0	0.0	0.0	0.6	2.9	4.9	7.6	12.0	22.7	9925	806.31
		30.7	32.5	34.4	35.9	33.6	33.3	33.8	31.6	26.5		

* Numbers show the % of nodes pruned by lower bound and the upper bound respectively.

** Empty cells show that the optimal solution is found before the corresponding number of nodes explored.

Table G.3. Branching and Branch Selection Strategy Performances for the Problem with 3 AAD, 2 SD, 4 ND Ships.

	Branching Strategy	% for the Number of Nodes Explored* 1000	2000	3000	4000	5000	6000	7000	8000	Total	Total Nodes Explored	Time (sec.)
Depth First Search	BS1	**								34.2	41	3.28
										14.6		
	BS2									19.8	217	19.52
										29.0		
	BS3	19.4	21.3	21.3	20.9					20.1	4657	391.16
		30.1	28.4	28.5	28.9					29.9		
	BS4	32.3	31.5							30.2	2909	224.60
		17.2	18.4							19.8		
	BS5	29.0	28.7	28.1	27.4	26.3	25.4			25.2	6133	494.00
		20.4	21.0	21.7	22.5	23.5	24.5			24.8		
	BS6	26.3	26.5	26.5	27.2	26.9				26.4	5481	432.90
		23.2	23.3	23.3	22.7	22.9				23.6		
Best Node First Search	BS1									21.1	57	5.29
										28.1		
	BS2									18.4	375	34.15
										31.2		
	BS3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.3	12577	1068.10
		26.5	29.6	32.2	35.3	37.8	39.7	41.1	42.2	30.9		
	BS4	0.0	0.0	3.7	8.4	18.0				19.6	5105	435.79
		31.9	35.6	36.1	36.1	31.0				30.4		
	BS5	0.0	0.0	0.6	3.3	6.3	10.9	17.2		20.1	7255	657.64
		30.2	33.5	36.0	33.4	32.2	33.5	31.0		29.9		
	BS6	0.0	0.0	0.5	3.2	10.5				16.7	5473	474.54
		32.7	33.9	36.6	40.0	36.5				33.4		

* Numbers show the % of nodes pruned by lower bound and the upper bound respectively.

** Empty cells show that the optimal solution is found before the corresponding number of nodes explored.

APPENDIX H

CALCULATED COVERAGE VALUES FOR SAMPLE SCENARIOS

In this appendix, we report the coverage values calculated by using MAP for six different sample scenarios. Sample problem has 19 sectors. We solve MAP for each coverage value (i.e. coverage provided by an AAD ship from sector *j* to sector *i*.). A total of 361 instances of MAP are to be solved for every scenario. We have one AAD and one ND ship in each MAP. Detail parameters of the scenarios are given below the corresponding tables.

Table H.1. Coverage Values Calculated for Scenario 1.

Sector i	Sector j																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.337	0.431	0.265	0.226	0.226	0.265	0.431	0.370	0.226	0.185	0.185	0.226	0.370	0.337	0.185	0.143	0.143	0.185	0.337
2	0.226	0.337	0.226	0.185	0.185	0.185	0.265	0.431	0.185	0.143	0.143	0.185	0.302	0.370	0.185	0.098	0.098	0.143	0.302
3	0.265	0.431	0.337	0.226	0.185	0.226	0.302	0.693	0.265	0.185	0.143	0.185	0.302	0.487	0.226	0.143	0.098	0.143	0.265
4	0.431	0.693	0.431	0.337	0.265	0.302	0.370	0.487	0.302	0.226	0.185	0.226	0.337	0.431	0.226	0.185	0.143	0.185	0.337
5	0.431	0.370	0.302	0.265	0.337	0.431	0.693	0.337	0.226	0.185	0.226	0.302	0.487	0.337	0.185	0.143	0.185	0.226	0.431
6	0.265	0.302	0.226	0.185	0.226	0.337	0.431	0.302	0.185	0.143	0.185	0.265	0.693	0.265	0.143	0.098	0.143	0.226	0.487
7	0.226	0.265	0.185	0.185	0.185	0.226	0.337	0.302	0.185	0.143	0.143	0.185	0.431	0.302	0.143	0.098	0.098	0.185	0.370
8	0.185	0.226	0.185	0.143	0.143	0.143	0.185	0.337	0.185	0.098	0.098	0.143	0.226	0.431	0.143	0.098	0.050	0.098	0.226
9	0.226	0.302	0.265	0.185	0.185	0.185	0.226	0.370	0.337	0.185	0.143	0.143	0.226	0.487	0.265	0.143	0.098	0.143	0.226
10	0.370	0.487	0.693	0.431	0.302	0.302	0.337	0.693	0.370	0.337	0.226	0.226	0.337	0.512	0.302	0.226	0.185	0.185	0.302
11	0.370	0.337	0.302	0.302	0.431	0.693	0.487	0.337	0.226	0.226	0.337	0.370	0.693	0.302	0.185	0.185	0.226	0.302	0.512
12	0.226	0.226	0.185	0.185	0.185	0.265	0.302	0.226	0.143	0.143	0.185	0.337	0.370	0.226	0.143	0.098	0.143	0.265	0.487
13	0.185	0.185	0.143	0.143	0.143	0.185	0.226	0.226	0.143	0.098	0.098	0.185	0.337	0.226	0.098	0.050	0.098	0.143	0.431
14	0.143	0.185	0.143	0.098	0.098	0.098	0.143	0.226	0.143	0.098	0.050	0.098	0.185	0.337	0.143	0.050	0.050	0.098	0.185
15	0.185	0.226	0.226	0.185	0.143	0.143	0.185	0.302	0.265	0.143	0.098	0.143	0.185	0.337	0.337	0.143	0.098	0.098	0.185
16	0.337	0.431	0.487	0.370	0.302	0.265	0.337	0.512	0.487	0.431	0.226	0.226	0.302	0.693	0.337	0.337	0.185	0.185	0.302
17	0.337	0.337	0.265	0.302	0.370	0.487	0.431	0.302	0.226	0.226	0.431	0.487	0.512	0.302	0.185	0.185	0.337	0.337	0.693
18	0.185	0.185	0.143	0.143	0.185	0.226	0.226	0.185	0.143	0.098	0.143	0.265	0.302	0.185	0.098	0.098	0.143	0.337	0.337
19	0.143	0.143	0.098	0.098	0.098	0.143	0.185	0.185	0.098	0.050	0.098	0.143	0.226	0.185	0.098	0.050	0.050	0.143	0.337

Scenario: MM-38 Exocet ASM, SM-1 Area Defense SAM System, Detection Distance of ASM From AAD Ship 21213.2 m., ASM's Bearing From ND Ship 000.

Table H.2. Coverage Values Calculated for Scenario 2.

Sector i	Sector j																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.337	0.512	0.370	0.265	0.226	0.226	0.302	0.460	0.337	0.185	0.143	0.185	0.265	0.431	0.302	0.143	0.143	0.143	0.226
2	0.226	0.337	0.265	0.185	0.143	0.185	0.226	0.512	0.265	0.143	0.143	0.143	0.226	0.460	0.265	0.143	0.098	0.098	0.185
3	0.226	0.302	0.337	0.226	0.185	0.185	0.226	0.337	0.370	0.185	0.143	0.143	0.185	0.337	0.337	0.143	0.098	0.098	0.185
4	0.302	0.337	0.512	0.337	0.226	0.226	0.265	0.337	0.487	0.265	0.185	0.143	0.226	0.337	0.370	0.185	0.143	0.143	0.185
5	0.512	0.460	0.487	0.370	0.337	0.302	0.337	0.431	0.370	0.265	0.226	0.226	0.265	0.431	0.337	0.185	0.143	0.143	0.226
6	0.370	0.487	0.337	0.265	0.265	0.337	0.512	0.582	0.302	0.185	0.185	0.226	0.337	0.676	0.265	0.185	0.143	0.185	0.265
7	0.265	0.370	0.265	0.185	0.185	0.226	0.337	0.487	0.265	0.143	0.143	0.185	0.302	0.582	0.226	0.143	0.098	0.143	0.265
8	0.143	0.226	0.185	0.143	0.143	0.143	0.185	0.337	0.185	0.098	0.098	0.098	0.185	0.512	0.185	0.098	0.050	0.098	0.143
9	0.185	0.226	0.226	0.185	0.143	0.143	0.143	0.265	0.337	0.143	0.098	0.098	0.143	0.265	0.370	0.143	0.098	0.098	0.143
10	0.265	0.265	0.337	0.302	0.226	0.185	0.226	0.265	0.460	0.337	0.185	0.143	0.185	0.265	0.582	0.265	0.143	0.098	0.143
11	0.460	0.431	0.582	0.487	0.512	0.337	0.337	0.431	0.431	0.337	0.337	0.265	0.265	0.401	0.370	0.265	0.226	0.185	0.226
12	0.337	0.370	0.302	0.265	0.265	0.370	0.487	0.431	0.265	0.185	0.185	0.337	0.460	0.487	0.226	0.143	0.143	0.226	0.337
13	0.185	0.265	0.185	0.143	0.143	0.185	0.265	0.337	0.185	0.143	0.098	0.143	0.337	0.370	0.185	0.098	0.098	0.143	0.302
14	0.143	0.143	0.143	0.098	0.098	0.098	0.143	0.226	0.143	0.098	0.050	0.098	0.143	0.337	0.143	0.050	0.050	0.050	0.143
15	0.143	0.143	0.185	0.143	0.098	0.098	0.143	0.185	0.226	0.143	0.098	0.098	0.098	0.226	0.337	0.143	0.050	0.050	0.098
16	0.226	0.226	0.265	0.265	0.185	0.185	0.185	0.226	0.337	0.302	0.143	0.143	0.143	0.226	0.431	0.337	0.143	0.098	0.143
17	0.431	0.431	0.676	0.582	0.460	0.337	0.337	0.401	0.487	0.370	0.512	0.265	0.265	0.401	0.401	0.302	0.337	0.226	0.226
18	0.302	0.337	0.265	0.226	0.265	0.337	0.370	0.370	0.226	0.185	0.185	0.370	0.582	0.401	0.226	0.143	0.143	0.337	0.431
19	0.143	0.185	0.185	0.143	0.143	0.143	0.185	0.265	0.143	0.098	0.098	0.143	0.265	0.302	0.143	0.098	0.050	0.143	0.337

Scenario: MM-38 Exocet ASM, SM-1 Area Defense SAM System, Detection Distance of ASM From AAD Ship 21213.2 m., ASM's Bearing From ND Ship 045.

Table H.3. Coverage Values Calculated for Scenario 3.

Sector i	Sector j																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.337	0.337	0.693	0.337	0.226	0.226	0.226	0.265	0.693	0.265	0.185	0.143	0.185	0.265	0.693	0.265	0.143	0.098	0.143
2	0.226	0.337	0.337	0.226	0.185	0.185	0.226	0.337	0.401	0.226	0.143	0.143	0.185	0.265	0.431	0.185	0.098	0.098	0.143
3	0.226	0.226	0.337	0.226	0.185	0.143	0.185	0.226	0.693	0.226	0.143	0.098	0.143	0.226	0.693	0.226	0.098	0.098	0.098
4	0.226	0.226	0.337	0.337	0.226	0.185	0.185	0.226	0.401	0.337	0.185	0.143	0.143	0.185	0.431	0.265	0.143	0.098	0.098
5	0.337	0.265	0.401	0.693	0.337	0.226	0.226	0.265	0.431	0.401	0.226	0.185	0.185	0.226	0.431	0.302	0.185	0.143	0.143
6	0.693	0.401	0.693	0.401	0.337	0.337	0.337	0.302	0.693	0.302	0.226	0.226	0.226	0.265	0.693	0.265	0.185	0.143	0.185
7	0.337	0.693	0.401	0.265	0.226	0.226	0.337	0.401	0.431	0.265	0.185	0.185	0.226	0.302	0.431	0.226	0.143	0.143	0.185
8	0.185	0.226	0.226	0.185	0.143	0.143	0.185	0.337	0.265	0.185	0.098	0.098	0.143	0.337	0.302	0.143	0.098	0.098	0.143
9	0.143	0.185	0.226	0.185	0.143	0.098	0.143	0.185	0.337	0.185	0.098	0.098	0.098	0.185	0.693	0.185	0.098	0.050	0.098
10	0.185	0.185	0.226	0.226	0.185	0.143	0.143	0.185	0.265	0.337	0.143	0.098	0.098	0.143	0.302	0.337	0.143	0.098	0.098
11	0.265	0.265	0.302	0.401	0.337	0.226	0.226	0.226	0.337	0.693	0.337	0.185	0.185	0.185	0.370	0.431	0.226	0.143	0.143
12	0.693	0.431	0.693	0.431	0.401	0.693	0.401	0.337	0.693	0.337	0.265	0.337	0.265	0.302	0.693	0.302	0.226	0.226	0.226
13	0.265	0.401	0.302	0.265	0.226	0.226	0.337	0.693	0.337	0.226	0.185	0.185	0.337	0.431	0.370	0.185	0.143	0.143	0.226
14	0.143	0.185	0.185	0.143	0.098	0.098	0.143	0.226	0.226	0.143	0.098	0.098	0.143	0.337	0.265	0.143	0.050	0.050	0.098
15	0.098	0.143	0.143	0.143	0.098	0.098	0.098	0.143	0.226	0.143	0.098	0.050	0.098	0.143	0.337	0.143	0.050	0.050	0.050
16	0.143	0.143	0.185	0.185	0.143	0.098	0.098	0.143	0.226	0.226	0.143	0.098	0.098	0.143	0.265	0.337	0.098	0.050	0.050
17	0.265	0.226	0.265	0.302	0.265	0.226	0.185	0.185	0.302	0.431	0.337	0.185	0.143	0.185	0.302	0.693	0.337	0.143	0.143
18	0.693	0.431	0.693	0.431	0.431	0.693	0.431	0.370	0.693	0.370	0.302	0.693	0.302	0.302	0.693	0.302	0.265	0.337	0.265
19	0.265	0.302	0.265	0.226	0.185	0.226	0.265	0.431	0.302	0.185	0.143	0.185	0.337	0.693	0.302	0.185	0.143	0.143	0.337

Scenario: MM-38 Exocet ASM, SM-1 Area Defense SAM System, Detection Distance of ASM From AAD Ship 21213.2 m., ASM's Bearing From ND Ship 090.

Table H.4. Coverage Values Calculated for Scenario 4.

Sector i	Sector j 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.245	0.370	0.180	0.126	0.126	0.165	0.346	0.317	0.141	0.089	0.089	0.126	0.284	0.280	0.103	0.055	0.055	0.089	0.245
2	0.126	0.245	0.126	0.089	0.089	0.089	0.165	0.370	0.103	0.055	0.055	0.089	0.188	0.317	0.089	0.027	0.027	0.055	0.188
3	0.165	0.346	0.245	0.126	0.089	0.126	0.188	0.394	0.180	0.089	0.055	0.089	0.188	0.391	0.141	0.055	0.027	0.055	0.165
4	0.346	0.394	0.370	0.245	0.165	0.188	0.284	0.391	0.219	0.126	0.089	0.126	0.245	0.370	0.141	0.089	0.055	0.089	0.229
5	0.370	0.317	0.219	0.180	0.245	0.346	0.394	0.280	0.141	0.103	0.126	0.188	0.391	0.270	0.103	0.068	0.089	0.126	0.331
6	0.180	0.219	0.141	0.103	0.126	0.245	0.370	0.219	0.103	0.068	0.089	0.165	0.394	0.193	0.068	0.038	0.055	0.126	0.391
7	0.126	0.180	0.103	0.089	0.089	0.126	0.245	0.219	0.089	0.055	0.055	0.089	0.346	0.219	0.068	0.027	0.027	0.089	0.284
8	0.089	0.126	0.089	0.055	0.055	0.055	0.089	0.245	0.089	0.027	0.027	0.055	0.126	0.370	0.068	0.027	0.008	0.027	0.126
9	0.126	0.188	0.165	0.089	0.089	0.089	0.126	0.284	0.245	0.089	0.055	0.055	0.126	0.391	0.180	0.055	0.027	0.040	0.126
10	0.284	0.391	0.394	0.346	0.188	0.188	0.245	0.394	0.317	0.245	0.126	0.126	0.229	0.394	0.219	0.126	0.089	0.089	0.188
11	0.317	0.280	0.219	0.219	0.370	0.394	0.391	0.270	0.141	0.141	0.245	0.284	0.394	0.242	0.103	0.089	0.126	0.188	0.394
12	0.141	0.141	0.103	0.089	0.103	0.180	0.219	0.141	0.068	0.055	0.089	0.245	0.317	0.141	0.055	0.027	0.055	0.165	0.391
13	0.089	0.103	0.068	0.055	0.055	0.089	0.126	0.141	0.055	0.027	0.027	0.089	0.245	0.141	0.038	0.014	0.027	0.055	0.346
14	0.055	0.089	0.055	0.027	0.027	0.027	0.055	0.126	0.055	0.027	0.008	0.027	0.089	0.245	0.055	0.008	0.008	0.027	0.089
15	0.089	0.126	0.126	0.089	0.055	0.055	0.089	0.188	0.165	0.055	0.027	0.040	0.089	0.245	0.245	0.055	0.027	0.027	0.089
16	0.245	0.331	0.391	0.284	0.188	0.165	0.229	0.394	0.391	0.346	0.126	0.126	0.188	0.394	0.280	0.245	0.089	0.089	0.188
17	0.280	0.270	0.193	0.219	0.317	0.391	0.370	0.242	0.141	0.141	0.370	0.391	0.394	0.232	0.103	0.103	0.245	0.245	0.394
18	0.103	0.103	0.068	0.068	0.089	0.141	0.141	0.103	0.055	0.038	0.068	0.180	0.219	0.103	0.038	0.027	0.055	0.245	0.280
19	0.055	0.068	0.038	0.027	0.027	0.055	0.089	0.089	0.027	0.014	0.027	0.055	0.126	0.103	0.027	0.008	0.008	0.055	0.245

Scenario: MM-38 Exocet and Harpoon ASM, SM-1 Area Defense SAM System, Detection Distance of ASMs From AAD Ship 21213.2 m., ASMs' Bearing From ND Ship 000 and 010.

Table H.5. Coverage Values Calculated for Scenario 5.

Sector i	Sector j																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.245	0.394	0.297	0.165	0.126	0.126	0.205	0.383	0.259	0.103	0.068	0.089	0.147	0.346	0.219	0.068	0.055	0.055	0.126
2	0.126	0.245	0.165	0.089	0.068	0.089	0.126	0.394	0.180	0.068	0.055	0.055	0.126	0.383	0.180	0.055	0.027	0.027	0.089
3	0.126	0.205	0.245	0.126	0.089	0.089	0.126	0.245	0.297	0.089	0.055	0.055	0.089	0.245	0.259	0.068	0.027	0.027	0.072
4	0.205	0.245	0.394	0.245	0.126	0.126	0.147	0.245	0.391	0.165	0.089	0.068	0.126	0.229	0.317	0.103	0.055	0.055	0.089
5	0.394	0.383	0.391	0.297	0.245	0.205	0.245	0.346	0.317	0.180	0.126	0.126	0.165	0.331	0.270	0.103	0.068	0.068	0.126
6	0.297	0.391	0.259	0.180	0.165	0.245	0.394	0.394	0.219	0.103	0.089	0.126	0.245	0.394	0.193	0.089	0.055	0.089	0.165
7	0.165	0.297	0.180	0.103	0.089	0.126	0.245	0.391	0.180	0.068	0.055	0.089	0.205	0.394	0.141	0.055	0.027	0.055	0.147
8	0.068	0.126	0.089	0.055	0.055	0.055	0.089	0.245	0.103	0.038	0.027	0.027	0.089	0.394	0.103	0.027	0.008	0.027	0.068
9	0.089	0.126	0.126	0.089	0.055	0.055	0.068	0.147	0.245	0.068	0.027	0.027	0.055	0.165	0.297	0.055	0.027	0.027	0.055
10	0.147	0.165	0.245	0.205	0.126	0.089	0.126	0.165	0.383	0.245	0.089	0.055	0.089	0.165	0.394	0.165	0.055	0.027	0.055
11	0.383	0.346	0.394	0.391	0.394	0.245	0.245	0.331	0.370	0.259	0.245	0.147	0.165	0.308	0.317	0.180	0.126	0.089	0.126
12	0.259	0.317	0.219	0.180	0.180	0.297	0.391	0.370	0.193	0.103	0.103	0.245	0.383	0.391	0.154	0.068	0.068	0.126	0.245
13	0.103	0.180	0.103	0.068	0.068	0.089	0.165	0.259	0.103	0.055	0.038	0.068	0.245	0.317	0.103	0.038	0.027	0.055	0.205
14	0.055	0.068	0.055	0.027	0.027	0.027	0.055	0.126	0.068	0.027	0.008	0.027	0.055	0.245	0.068	0.014	0.008	0.008	0.055
15	0.055	0.068	0.089	0.055	0.027	0.027	0.055	0.089	0.126	0.055	0.027	0.027	0.027	0.126	0.245	0.055	0.008	0.008	0.027
16	0.126	0.126	0.165	0.147	0.089	0.072	0.089	0.126	0.245	0.205	0.068	0.055	0.055	0.126	0.346	0.245	0.055	0.027	0.055
17	0.346	0.331	0.394	0.394	0.383	0.245	0.229	0.308	0.391	0.317	0.394	0.165	0.165	0.292	0.353	0.219	0.245	0.126	0.126
18	0.219	0.270	0.193	0.141	0.180	0.259	0.317	0.317	0.154	0.103	0.103	0.297	0.394	0.353	0.141	0.068	0.068	0.245	0.346
19	0.068	0.103	0.089	0.055	0.055	0.068	0.103	0.180	0.068	0.038	0.027	0.055	0.165	0.219	0.068	0.027	0.014	0.055	0.245

Scenario: MM-38 Exocet and Harpoon ASM, SM-1 Area Defense SAM System, Detection Distance of ASMs From AAD Ship 21213.2 m., ASMs' Bearing From ND Ship 045 and 055.

Table H.6. Coverage Values Calculated for Scenario 6.

Sector i	Sector j 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.245	0.259	0.394	0.245	0.126	0.126	0.141	0.193	0.394	0.165	0.089	0.068	0.103	0.180	0.394	0.147	0.055	0.038	0.068
2	0.126	0.245	0.245	0.126	0.089	0.089	0.126	0.259	0.308	0.126	0.055	0.055	0.089	0.193	0.331	0.089	0.027	0.027	0.055
3	0.126	0.141	0.245	0.126	0.089	0.068	0.089	0.141	0.394	0.126	0.055	0.038	0.055	0.141	0.394	0.126	0.027	0.027	0.038
4	0.141	0.141	0.259	0.245	0.126	0.089	0.103	0.141	0.344	0.245	0.089	0.055	0.068	0.103	0.370	0.165	0.055	0.027	0.038
5	0.259	0.193	0.344	0.394	0.245	0.141	0.141	0.180	0.370	0.308	0.126	0.089	0.103	0.141	0.370	0.205	0.089	0.055	0.068
6	0.394	0.344	0.394	0.308	0.245	0.245	0.259	0.232	0.394	0.205	0.126	0.126	0.141	0.193	0.394	0.165	0.089	0.068	0.103
7	0.245	0.394	0.308	0.165	0.126	0.126	0.245	0.344	0.331	0.147	0.089	0.089	0.141	0.232	0.331	0.108	0.055	0.055	0.103
8	0.089	0.126	0.126	0.089	0.055	0.055	0.089	0.245	0.165	0.072	0.027	0.027	0.068	0.259	0.205	0.055	0.027	0.027	0.055
9	0.068	0.089	0.126	0.089	0.055	0.038	0.055	0.103	0.245	0.089	0.027	0.027	0.038	0.103	0.394	0.089	0.027	0.008	0.027
10	0.103	0.103	0.141	0.141	0.089	0.055	0.068	0.089	0.193	0.245	0.068	0.038	0.038	0.068	0.232	0.245	0.055	0.027	0.027
11	0.193	0.180	0.232	0.344	0.259	0.141	0.141	0.141	0.280	0.394	0.245	0.103	0.089	0.103	0.317	0.331	0.126	0.055	0.055
12	0.394	0.370	0.394	0.331	0.308	0.394	0.344	0.280	0.394	0.229	0.165	0.245	0.193	0.232	0.394	0.188	0.126	0.126	0.141
13	0.165	0.308	0.205	0.147	0.126	0.126	0.245	0.394	0.229	0.108	0.072	0.089	0.245	0.370	0.251	0.089	0.055	0.055	0.141
14	0.055	0.089	0.089	0.055	0.027	0.027	0.055	0.126	0.126	0.055	0.027	0.027	0.055	0.245	0.147	0.040	0.008	0.008	0.038
15	0.038	0.055	0.068	0.055	0.027	0.027	0.027	0.055	0.126	0.055	0.027	0.008	0.027	0.068	0.245	0.055	0.008	0.008	0.008
16	0.068	0.068	0.103	0.103	0.055	0.038	0.038	0.055	0.141	0.141	0.055	0.027	0.027	0.055	0.180	0.245	0.038	0.008	0.014
17	0.180	0.141	0.193	0.232	0.193	0.141	0.103	0.103	0.232	0.370	0.259	0.103	0.068	0.103	0.232	0.394	0.245	0.068	0.055
18	0.394	0.370	0.394	0.331	0.331	0.394	0.370	0.317	0.394	0.251	0.205	0.394	0.232	0.232	0.394	0.188	0.147	0.245	0.180
19	0.147	0.205	0.165	0.108	0.089	0.126	0.165	0.331	0.188	0.089	0.055	0.089	0.245	0.394	0.188	0.072	0.040	0.055	0.245

Scenario: MM-38 Exocet and Harpoon ASM, SM-1 Area Defense SAM System, Detection Distance of ASMs From AAD Ship 21213.2 m., ASMs' Bearing From ND Ship 080 and 090.

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B.S. in Operations Analysis, Naval Academy, İstanbul, Turkey, 1991

Employment

Force Structure Analyst, Operations Research Division, Turkish Navy HQs, Ankara, August 2001 – Present.

Defence Research Fellow, Operational Research Division, Maritime OR Team, Canadian Department of National Defence HQs, Ottawa, September 2000 - August 2001.

Operations Research Project Officer, R&D Dept., Ops. Research Branch, Turkish Navy HQs, Ankara, November 1997 – August 2000.

Force Structure Analysis Officer, Plans and Policy Dept. Force Planning Branch, Turkish Navy HQs, Ankara, April 1997 – November 1997.

Operations Officer to the Commodore of the 1st FPBG Squadron, Turkish Navy FPB Command, Umuryeri, İstanbul, July 1994 – December 1994.

Communications Officer, TCG Turgutreis, 1st Frigate Squadron Command, Gölcük, Kocaeli, August 1991 – July 1994.

Journal Article

Karasakal O. and E. Karasakal, "A Maximal Covering Location Model in the Presence of Partial Coverage", *Computers and Operations Research* (forthcoming).

Articles in Proceedings

Karasakal O., “An Investigation on Defense Acquisition Systems”, *Defense R&D Seminar Proceedings*, Ankara, Turkey, 13 November 1999 (in Turkish).

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Karasakal O., “Status and Requirements for Modeling and Simulation in the Turkish Navy”, *Turkish Armed Forces Modeling and Simulation Seminar Proceedings*, Ankara, Turkey, 1-3 April 1998 (in Turkish).

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Kaplan A.C. and O. Karasakal, “A Model for Allocating Communication Budget and Time Series Analysis for the Year 2003”, Dz.K.K.lığı BİLKARDES D.Bşk.lığı, *Project Report* PR-2002/06, December 2002 (in Turkish).

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Eliiyi D. Türsel, E. Erdem and O. Karasakal, “Integrated Locational Planning of Schools in Turkey”, *Technical Report* No:02-05, Industrial Engineering Department, Middle East Technical University, May 2002.

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Karasakal O., “Analysis of Officer Billets Authorized for Promotion Using System Dynamics”, Dz.K.K.lığı BİLKARDES D.Bşk.lığı, *Project Report* No: DZBİLKARDES:TR-04, December 2001 (in Turkish).

Karasakal E. and O. Karasakal, “Multicriteria Decision Making Methods”, *Maritimes Regional Advisory Process Working Paper* 2001/46, Department of Fisheries and Oceans, Bedford Institute of Oceanography, Dartmouth, Nova Scotia, Canada, June 2001.

Karasakal O. and E. Karasakal, “Locating the Search and Rescue Bases in the Presence of Partial Coverage”, *DOR (MLA) Research Note* RN-2001/03, Operational Research Division, Department of National Defence, Ottawa, July 2001.

Karasakal O., “A Literature Review of Operational Research Methods for Modeling the Air Defence Problem”, *DOR (MLA) Research Note* RN-2001/02,

Operational Research Division, Department of National Defence, Ottawa, May 2001.

Karasakal O. and S. Akgün, "System and Equipment Selection Using the Analytic Hierarchy Process and Criteria Template for Selected Systems and Equipment", *Technical Report* No: DZBİLKARDES:TR-03, R&D Department, Turkish Navy HQs, June 2000 (in Turkish).

Karasakal O., **"**Optimization of Loading in Amphibious Operations: The HAMULE Model", *Technical Report* No: DZBİLKARDES: TR-02, R&D Department, Turkish Navy HQs, December 1999 (in Turkish)

Presentations at Conferences

Karasakal O. and E. Karasakal, "A Maximal Covering Location Model in the Presence of Partial Coverage", 23rd Operations Research and Industrial Engineering National Conferece, 3-5 July 2002, İstanbul.

Karasakal O. and E. Karasakal, "Locating the Search and Rescue Bases in the Presence of Partial Coverage", CORS 2001, Quebec City, Canada, 6-9 May 2001.

Karasakal O., "An Investigation on Defense Acquisition Systems", Defense R&D Seminar, Ankara, Turkey, 13 November 1999 (in Turkish)

Karasakal O. and E. Sayın, "An Application Oriented BPR Approach for Public Sector", 20th National Operations Research and Industrial Engineering Conference, Ankara, Turkey, 8-9 June 1999 (in Turkish).

Karasakal O., "Optimizing the Acquisition Process for Aircraft and Air Dropped Munitions", Turkish Armed Forces Modeling and Simulation Seminar, Ankara, Turkey, 1-3 April 1998 (in Turkish).

Karasakal O., "Status and Requirements for Modeling and Simulation in the Turkish Navy", Turkish Armed Forces Modeling and Simulation Seminar, Ankara, Turkey, 1-3 April 1998 (in Turkish).

Seminar

Karasakal O., "Optimal Air Defense Strategies for a Naval Task Group", Department of Industrial Engineering, METU, Ankara, 5 April 2002.

Workshop and Conferences Participated

Yeditepe University- IIASA-DAS Conference on Multicriteria Decision Making, Istanbul, Turkey, 31 August-5 September 1998.

23rd Operations Research and Industrial Engineering National Conference, Yeditepe University, İstanbul, 3-5 July 2002.

2001 CORS (Canadian Operational Research Society) Conference, Quebec City, Canada, 6-9 May 2001.

1999 Command and Control Research and Technology Symposium, United States Naval War College, Newport, RI, 29 June-1 July 1999.

1999 International Symposium on Modeling and Analysis of Command and Control, Paris, France, 12-14 January 1999.

Activities

Member of NATO RTO Simulation, Analysis and Studies (SAS) Panel Technical Team on Helicopter Mission Planning (SAS-045), September 2001 – Present.

Member of NATO RTO Simulation, Analysis and Studies (SAS) Panel Technical Team on Small Scale Contingencies (SAS-027), February 2000 – August 2000.

Member of NATO RTO Simulation, Analysis and Studies (SAS) Panel Technical Team on Long Term Defence Planning (SAS-025), June 1999 – August 2000.

Member of Institute for Operations Research and the Management Sciences (INFORMS), March 1996- Present.

Member of Decision Analysis Society of INFORMS, November 1998 - December 2002.

Member of Simulation Society of INFORMS, March 1996 - December 2002.

Member of Military Applications Society of INFORMS, March 1996 – Present.

Member of Optimization Society of INFORMS, March 1996 – December 1998.

Awards and Honours

Canadian Defense Research Fellow, September 2000-August 2001.

Ranking the second out of 196 students in the graduating class of Naval Academy, August 1991.

US Navy–Turkish Navy Exchange Cadet for on-job training on board USS Ponce at the US Navy 6^{th} Fleet, June-July 1991.

Naval Academy High Honour Student, Fall 1987, Spring 1988, Fall 1988, Spring 1989, Fall 1989, Spring 1990, Fall 1990, Spring 1991.