INVESTIGATION OF ESTIMATION ABILITIY OF HIGH SCHOOL STUDENTS

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ABSTRACT

INVESTIGATION OF THE COMPUTATIONAL ESTIMATION ABILITIY OF HIGH SCHOOL STUDENTS

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The purpose of this study was to investigate the high school students' ability on estimation and computational estimation. The study was conducted in Denizli with 153 ninth grade students who enrolled to general, Anatolian and foreign language high schools. The Estimation Ability Test was utilized. The three formats which are numbers format, answer format and problem format of the test were analyzed by with respect to school types and gender. The design of the present research was one of the experimental studies one group pretest- posttest design. The hypotheses of the study were tested by using analysis of covariance at the significance level 0,05. The results of the study indicated that: 1.There were statistically significant differences among the mean scores of students enrolled to different kinds of high schools with respect to estimation ability and computational estimation in the favor of Anatolian High School students. 2. There were statistically significant mean differences of students enrolled to different kinds of high schools with respect to sub-categories of the estimation ability test in the favor of Anatolian High School students. 3. There was no statistically significant mean difference between boys and girls on estimation ability. 4. There was statistically significant mean difference in some sub-categories of the estimations test in the favor of boys.

Keywords: Estimation, Computational Estimation, Gender, High School Students

ÖZ

LİSE ÖĞRENCİLERİNİN TAHMİNSEL HESAPLAMA BECERİSİNİN İNCELENMESİ

BOZ, Burçak
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Bu çalışmanın amacı, lise öğrencilerinin tahmin ve tahminsel hesaplama becerilerini araştırmaktır. Araştırma, Denizli'deki genel, Anadolu ve yabancı dil ağırlıklı liselerine kayıtlı 153 dokuzuncu sınıf öğrencisiyle yürütülmüştür. Bu araştırma için Tahmin Beceri Testi kullanılmıştır. Sayılar biçimi, cevap biçimi, soru biçimi olmak üzere üç biçimde okul çeşitleri ve cinsiyet değişkenleri analiz edildi. Araştırmanın deseni deneysel çalışmalardan biri olan tek grup öntest-sontest desenidir. Bu araştırmanın hipotezleri 0,05 anlamlılık düzeyinde kovaryans analizi kullanılarak test edilmiştir. Bu çalışmanın sonuçları göstermiştir: 1. Tahmin ve tahmin becerisi açısından farklı liselerde okuyan öğrencilerin ortalamaları arasında Anadolu Lisesi öğrencileri lehine anlamlı bir fark bulunmaktadır. 2. Tahmin Beceri Testinin alt kategorileri açısından farklı liselerde okuyan öğrencilerin ortalamaları arasında Anadolu Lisesi öğrencileri lehine istatistiksel olarak anlamlı bir fark bulunmaktadır. 3. Tahmin becerisi açısından kız ve erkek öğrenciler arasında istatistiksel olarak anlamlı bir fark bulunmamaktadır. 4. Tahmin beceri testinin bazı alt kategorileri açısından kız ve erkek öğrenciler arasında erkek öğrenciler lehine istatistiksel olarak anlamlı bir fark vardır.

Anahtar Kelimeler: Tahmin, Tahminsel Hesaplama, Cinsiyet, Lise Öğrencileri

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LIST OF ABBREVIATIONS

EAT..: Estimation Ability Test

AHS..: Anatolian High School

FHLS..: Foreign Language High School

HS...: High School

ANOVA...: Analysis of the Variance

ANCOVA..: Analysis of the Covariance

wrt...: with respect to

CHAPTER I

INTRODUCTION

With the developing technology, accurate and fast computation has gained great importance. While the computer is the perfect device in our age, human being faces with his own computation power time to time. During these times, instead of exact answers, rough and estimated solutions are so practical to use. Those times close answers or approximate solutions are more appropriate for the situations. Because of this, estimation becomes an important skill. The words such as "near, close to, about, almost…" are recognized for answering the question in the daily life while using mathematics. The ability to estimate the answer to an arithmetic problem rather than compute an answer with paper and pencil is an essential part of mathematics education (NCTM, 1989).

Estimation is defined as the process of producing an answer that is sufficiently close allows decisions to be made (Reys, 1986). Thompson (1979) called the estimation as educated guess. Smart (1982) defined estimation as forming an approximate opinion. As seen there are different definitions of the estimation. In the present study estimation ability can be defined as developing an idea as quickly and reasonable as possible about the quantity or sizes of something without actually counting or measuring. The study of estimation can be a means to help students developed an understanding of concepts and procedures, flexibility in working with numbers, and an awareness of the reasonableness of results (NCTM, 1989).

Estimation is important for three reasons: (1) it is used for more often than paper-pencil skills in everyday life (Reys & Reys, 1998); (2) it is particularly important as both adults and children do more work with calculators and computers (Glasgow,1998); and (3) Ways to check the reasonableness of results are vital (Suydam, 1985). In recent years there has been considerable discussion of the emphasis estimation, estimation skills and estimation strategies in the mathematics curriculum. Sowder (1992) stated that there is a great need for research studies on determining of the appropriate grade-level to teach/learn estimation and how instruction and grade levels and instruction influence one another.

A goal of mathematics education must have students to use mathematics in their everyday life, and estimate such things as total cost of items, unit price, and measurement of distance are certainly relevant uses of mathematics. In daily life many types of questions and situations make us use the estimation: Do I have enough cash to pay for these pencils? How much paint do I need for paint my room? How much time will take between school and home? How many watchers in the stadium? Answers of these questions involve estimating results of computations, estimating measures, and estimating numerosity. Estimation is generally divided into three categories: numerosity, measurement, and computational estimation (Sowder, 1992).

The first task is to estimate numerosity, or the number of objects, usually dots in an array. Baroody and Gatzke (1991) studied on numerosity estimation and got an excellent history about it.

The second task is to estimating measures in everyday situations (e.g., the weight of typical car or the length of time required for going to school). Such tasks have been used in the Third International Mathematics and Science Study (TIMSS) in which Measurement, Estimation and Number sense is a major category of items.

A third task is to estimate answers to numerical computations. It has been the most studied of the three tasks, although even it has not been extensively studied (Sowder, 1992). Computational estimation is one topic receiving increased attention among the other estimation types in mathematics education (NCTM, 1989). There are several definitions of computational estimation. For example, Dowker (1992) defined computational estimation as making reasonable guesses as to the approximate answer to arithmetic problems without or before actually doing the calculation. Similarly Berry (1998) defined it as making a reasonable approximation to the answer of a computation problem without the use of external tools. In the present study we used definition of LeFevre, Greenham, Waheed (1993) who stated that computational estimation is the process of simplifying an arithmetic problem using same set of rules or procedures to produce an approximation but satisfactory answer through mental computation**.**

Computational estimation and mental computation require number sense (Greeno 1991; McIntosh, Reys and Reys 1992; Sowder and Schappelle 1989). In the literature there are several definitions of number sense. For example, Greeno (1991) refers to number sense as several important but indefinable capabilities, including flexible mental computation numerical estimation and quantitative judgement. Berry (1998) defines the number sense as understanding numbers and their multiple relationships and recognizing the relative magnitude of numbers.

According to Reys (1986) mental computation is defined as without using an external device producing an exact answer. Both mental computation and estimation in any kinds can be done mentally. However the process of estimation produces a response that is close to the exact answer, mental computation must be give the exact result of the process. In the present study number sense is defined as used as an intuition about numbers and mental computation defined as the process of finding exact solution without using any external computing device.

The most common type of computational estimation problem requires estimating the results of computation by performing some mental computation on approximations of the original numbers. To be correct the answer must fall within a certain interval, as determined by the problem itself or some outside source, such as a teacher.

Mental computation and computational estimation are different but there is an interaction between them. So, computational estimation is an interaction and combination of mental computation, number concepts, technical arithmetic skill including rounding, place value and less straight forward processes such as mental compensation that rapidly and consistently result in answers that are reasonable close to a correctly computed results. This process is done internally, without the external use of a calculating or recording to.

The topic of computational estimation is taught to elementary school students beginning in about grade-3 in the USA and European countries (Reys, Rybolt, Bestgen & Wyatt, 1982). It is taught from grade 3 to grade eight (included). Unfortunately, many students could not master estimation skills even though they receive formal instruction on estimation for at least a short period of time during 6 years. There are some studies to examine estimation skills and strategies used by students in the primary and secondary schools, and why they did not master the taught subject estimation (Sowder, 2001; Crites, 1992; Heinrich, 1998; Berry, 1998; Damarin, Dziak, Stull & Whiteman, 1988). All the study point out that the score getting from the estimation tests was low relatively. It is not easy to explain the reasons of that situation. One of the reasons can be that teachers could not have enough competencies on estimation this can be concluded from the results of study conducted on pre-service teachers because in some study it was found that preservice teachers got low scores on the estimation test (Gliner, 1991; Goodman, 1991; Bestgen, Rybolt, Reys & Wyatt, 1980, Smith, 1993). It is a paradox, since when teachers' estimation ability is low, their students' ability also low, and those students grown up and being an adult with lack of estimation ability performance.

Estimation strategies are not universal. The researchers divided the estimation strategies into different sub groups. Several researchers have identified three general ways in which people estimate answers to computational problems: reformulation, translation, and compensation (e.g., Heinrich, 1998). Reformulation is rethinking the numbers in a more manageable manner. This procedure can be done by the help of rounding and truncating the numbers. A number can be truncated by the making number smaller, that the converse implication of the rounding. Translation is the converse the operations into more applicable situations. The last strategy of the estimation is compensation which is the making the estimation more close the actual answer. This can be done either intermediate of the estimating procedure or end of the procedure. In the present study these strategies were used.

In elementary mathematics curriculum of Turkey it is stated the importance of estimation ability however estimation strategies except rounding are not taught (MEB; 2000). The Turkish Ministry of Education stated that measurement and mental computation must be improved. In elementary school students must be encouraged to develop their estimation ability. Whether separate topic or in other subjects estimation and their strategies must taught deeply. In Turkey there are a few studies on estimation (Yazgan, Bintaş & Altun, 2002; Boz &Bulut, 2002). In the first study it was found that however a short of period time treatment estimation and mental computation ability of 5th grade students were improved. Boz and Bulut (2002) stated that the preservice elementary mathematics and science teachers and childhood teachers have some problems basic concepts of the numbers and number sense. The most challenge point was fractions of the number format in the Estimation Ability Test.

Consequently, the purpose of this study is to investigate the 9th grade students' overall computational estimation ability and estimation ability on number format, answer format and problem format with respect to type of school and gender after the instruction on estimation.

The mathematics must help the students in their daily life. The researches showed that estimation and mental computation are used in most part of the life. In recent time the mathematics researchers conceded importance of the estimation and estimation strategies in teaching mathematics. Although lots of educators agree with that, in our country except Yazgan et al. (2002); and Boz and Bulut (2002), there are not any researches on estimation and mental computation. This research contributes abundance knowledge in the Turkish mathematics education literature.

CHAPTER II

REVIEW OF THE LITERATURE

This chapter focuses on literature about the types of estimation; mental computation and computational estimation; the topic of number sense; the computational estimation strategies; and teaching estimation and strategies, process used by good computational estimators; why children fail to master computational estimation.

2.1 Types of Estimation

Researchers have defined "estimation" in different ways. Reys (1986) defined estimation as the "process of producing an answer that is sufficiently close allows decisions to be made". Micklo (1999) states that estimation is nothing more than to know quickly and quantity or size of something without actually counting or measuring it.

In the research literature estimation is not paid as much attention as other mathematical topics (Sowder, 1992; Reys, Reys & Penafiel, 1991).

Thompson (1979) calls an estimate an educated guess, usually made in context of the number of objects in a collection, the results of a numerical computation or the measure of an object.

As the estimation is used in many areas, mathematics educators divide it into three categories: numerosity estimation, measurement estimation and computational estimation (Munakata, 2002; Hanson & Hogan, 2000; Sowder, 1992).

Besides these three kinds of estimation types also there are other forms of estimation. Smart (1982) described estimating trigonometric functions, estimating numerical values of the derivative for a graph of a function, and estimating with a calculator (for example $6,159^{2,317}$) as the other examples. Probability and statistics are other areas where estimation skills will be useful and can lead to better understanding. An estimate of a probability is often all that we need and in some cases all that we can find in our daily life. For example estimating the probability of such an event that whether school will be closed because of snowy weather within the next month may or may not need calculations. It often depends on some information such as location and time of the year. If statistics is considered, the statisticians can often estimate the mean or the standard deviation of a set of the data depending more on the previous experiences rather than computing the exact results. Development of the ability to make such estimates has not been investigated (Sowder, 1992).

As estimation is mentioned in the literature in the word, only those three kinds of estimation types; numerosity, measurement estimation and computational estimation will be reviewed in this part of the study. After reporting some information about numerosity and measurement estimation briefly, computational estimation will be deeply concerned.

Numerosity is defined as estimating the number of objects, usually dots in an array (Hanson & Hogan, 2000; Sowder, 1992). "How many" is asked in order to find the number of the items in a set; that is the question of numerosity. In many cases an estimate is sufficient and perhaps all that is even possible. It is sometimes tried the estimate the number of people in a theater, the number of cars in a parking lot, the number of books on a library shelf.

The typical procedure used in numerosity estimation is to take a count of a sample, and then multiply it by the number of such samples estimated to be present. It is known as benchmark strategies and that is the most important strategy in the numerosity. So to estimate the number of people at a football game, we might count or estimate the people in a small section, estimate the number of such sections, and use the product as our estimate of the total.

The most important research study of all conducted up to now is Baroody and Gatzke's study (1991). Since qualitative study provides further understanding, they investigated the ability and used strategies of the gifted students by interviewing them. It was provided an excellent history research on numerosity estimation by these researchers. In the research, they interviewed 18 potentially gifted preschool-kindergarten children about their ability to perform three tasks:

(a) estimation tasks, where children were to estimate the number of dots in a set (b) number-referent task, where children decided whether a set of dots was larger or smaller than given reference numbers (c) order-of-magnitude task, where children decided where a set of dots fit in relation to two reference numbers. As a result of this study a majority of the children were successful on the number-referent task, but performance varied on the order of magnitude task.

Another search conducted by Crites in 1992 was also related with the numerosity. He developed 24-item test to identify skilled and less skilled estimators, 3^{rd}, 5^{th} and 7^{th} grades 36 students from small, rural, midwestern community. In addition to this test, an interview consisting of 20 questions was prepared to search the strategies for discrete quantities (numerosity) estimation that is used by these students. Crites firstly separated the students into two groups according to their skilled levels by using estimation test score. The results of both the estimation test and interview questions suggested that students' ability to make estimates of discrete quantity is generally poor. The fifth and the seventh grade students performed better than the third graders. This conclusion was supported in many research studies (Sowder, 1992). In the interview part of the study strategies were identified. Skilled estimators tended to use the higher-order strategies (benchmark, multiple benchmark, benchmark comparison, and decomposition/recomposition) whereas less skilled estimators were more likely to say "I don't know" to use strategies that relied on guessing or to use "false" strategies.

According to results of the study Crites concluded that from his research results students' weakness in this area could be due to a variety of factors: Poor number sense, inability to comprehend large-number quantities, undeveloped computational estimation or mental computation skills.

One of the studies on discrete quantity (numerosity) was done by Montague and van Garderen (2003). They investigated the different ability groups in fourth, sixth and eighth grade students' estimation ability and strategies with discrete quantities and the relationships among the mathematics achievement, estimation skills and academic self-perceptions. Despite the differences among the ability groups, it is evident that all students did quite poorly on the estimation test. When compared with the other ability groups, the intellectually gifted students significantly performed better on estimation measures. However they still did not perform well when their overall percentage correct was calculated.

The other type of estimation which is very similar to numerosity in terms of their strategies (benchmark is the most important one for both of the estimation types) is measurement estimation. This type of estimation contains everyday situations such as the weight of a typical car, the length of the time for a normal adult to walk a kilometer.

As stated before, Montague et al. (2003) and Crites (1992) searched the strategies of numerosity in different ability levels. Similarly, Mottram (1995) studied with different ability levels in measurement estimation strategies. All of the researchers agreed on the fact that the variety of the strategies expands and sophisticates as the ability level increases.

Moreover, Siegel, Goldsmith and Madson (1982) studied on both numerosity and measurement estimation with respect to the strategies of 2^{nd} through 8^{th} grade students. They wanted to assess developmental differences in estimation strategies of the children. In contradiction to Montague et al. (2003), Crites (1992) and Mottram (1995); Siegel et al. (1982) stated that there was a weak relationship between accuracy in estimation and used strategies. They also found age differences for measurement estimates that the more grade-level the more different sophisticated estimation strategies.

Mottram (1995) found significant correlations among students' ability to estimate and (1) students' perceptions of mathematics ability and to estimate; (2) teachers' ratings of students' mathematical ability and (3) ability to estimate measurement. No gender differences were found for estimation ability or for perceptions of mathematics ability. Males rated themselves significantly higher than females in ability to estimate.

In order to results of the Third International Mathematics and Science Study (TIMMS) Taylor, Simms, Kim and Reys (2001) investigated why the American third- and fourth-grade students scored lower than the international average in the measurement, estimation and number sense. The researchers conducted a survey on the 3rd and 4th grades. The surveys were distributed to 110 students to inquire about the use of metric measurement in the classroom. Most of those who responded stated that they used metric units mainly for measuring length, only a few used metric units for measuring capacity or weight. Others responded that metric units received greater emphasis in science lessons than in mathematics. While Taylor et al. (2001) surveyed measurement estimation on students, Forrester and Pike (1998) dealt with the same topic in a conversation-analytic approach for classroom observation to identify children and teachers' acts. They concluded that the significance of rough measurement in regard to estimation was clearly evidenced in the children's activities although they didn't find explicit instructions or using a nonstandard measuring tool in any teachers' talk. Also Sowder (1992) agreed that school-age children were weak in estimating measure. Comparing with adults' estimations, children were worse than adults though the strategies they used were fundamentally the same as those children used (Sowder, 1992).

In order to results of these studies researchers (Forrester&Pike,1998; Taylor et al.,2001; Sowder,1992) agreed that for students to acquire skill in estimation, they must have practical experiences in making estimates so that they can develop their own individual frames of reference for estimating the quantity of various types of measurement (numerousness, time, length, etc.). Crites (1992) stated that to improve the estimation ability of students, several suggestions have been made; these are:

- Students should be given opportunities to develop their own benchmarks
- It is important for students to observe their teachers' use of benchmarks and the benchmark and decomposition-recomposition strategies to estimate discrete quantities.
- Students can develop their own estimation skills by frequently making estimates in practical-application situations.

Among the three types estimation, computational estimation has been the most frequently studied while the literature on estimating numerosity and measurement remain sparse (Sowder, 1992; Munakata, 2002). Although a universal definition exists for the computational estimation, every researcher defines the concept in their own style. Dowker (1992) defined computational estimation as making reasonable guesses as to the approximate answers to arithmetic problems, without or before actually doing the calculations. The computational estimation has been defined by Reys, Rybolt, Bestgen and Wyatt:

> *"The interaction of mental computation, number concepts, arithmetic skills including rounding, place value and mental compensation that rapidly and consistently result in answers that are reasonably close to a correctly computed result. This process is done internally without the external use of a calculating or recording tool (p.307)."*

In his study Heinrich (1998) explained that the computational estimation is a multistep process, performed mentally, which requires that a number be rounded off and then used to calculate an answer using one of the four basic mathematical applications of addition, subtraction, multiplication or division. Sowder (1992) added the explanation of the computational estimation as problem requires estimating the results of a computation by performing some mental computation on approximations of the original numbers. Also she gave extra information to be correctness of the results as the answer must be fall within a certain interval, as determined by the problem itself or some outside source.

2.2 Relations of Mental Computation and Computational Estimation

Many researches agreed that computational estimation is so related with the mental computation. Also they said that mental computation and computational estimation are to be accomplished without the use of paper and pencil or other tools (Reys, Rybolt, Bestgen, Wyatt, 1982; Dowker, 1992).

According to Reys (1984) mental computation is important for estimation since it provides the cornerstone necessary for the diverse numeric processes used in the computational estimation. According to him, mental computation had two distinct characteristics. First, it produces an exact answer and second, it is performed mentally without the aid of external devices such as pencil and paper. He found that a person can be competent at mental computation but very poor at computational estimation simultaneously. However, converse is not true, that is, people who can good at computational estimation are not also good at mental computation.

Computation can be accomplished by various methods; mental, written, approximate and calculator, each appropriate given a particular problem context. In general if it is possible to solve the problem mentally, then mental computation would be the natural tool to choice. Often the mental strategy is an invented one and is based on conceptual understanding. However, if the numbers are too complex for mental computation but estimation provides a solution that addresses the problem context then computational estimation is an appropriate tool. Again, the estimation strategy employed is generally based on conceptual understanding although some standard techniques for estimating are also practical. If however, the result from estimation are inconclusive or if more precise results are needed then exact computation is needed. Either a calculator or a standard written technique is a natural tool of choice for tedious computation requiring an exact answer (Reys & Reys, 1998).

Similarly the meaning of estimation, the meaning of the mental computation is discussed with researches and giving so many explanations of it. Some of them are presented here.

According to Reys (1986) mental computation definition is the process of producing an exact answer to a computational problem without any external computational aid. While both mental computation and estimation can be done mentally the process of estimation produces a response that is close to the exact answer, which would be the result of the process of mental computation

In this study the mental computation is defined as finding an exact answer without using any electronic or paper-pencil techniques. Mental computation enhances a student's understanding of numbers, number properties, and operations on those numbers that is; it improved the number sense of the students. It also promotes flexible thinking and problem solving (Gay, 1997).

Some authors do not give importance on making clear distinction between the terms estimation and approximation. Smart (1982), for example, defines estimation as forming an approximate opinion of size, amount, or number that is sufficiently exact for a specified purpose. On the other hand, Hall (1984) claims that estimation and approximation are not synonymous and writes that whereas estimation is usually a mental exercise, approximating usually requires a tool of some kind. Approximating is attempting to close in a target value. It is often possible to get as one desires without ever reaching the exact value. This particular distinction between estimation and approximation is not universal. Siegel, Goldsmith and Madson (1982) consider estimation to be a process leading to a solution to a problem of counting or measurement; what we refer to as estimation they refer to as approximation.

The researchers emphasized the role of mental computation in bringing about a better understanding of the number system and estimation. Mental computation is also useful in its own right. In order to Hope (1986) in everyday world of the consumer and worker there is more need for exact or a reasonably accurate mental calculation than for a pencil-and-paper calculation. Like computational estimation, skill in mental computation is also associated with understanding the structure of the number system. Individuals skilled at mental computation use this understanding to their advantage while those poor at mental calculation tend to try to use mental analogues of paper-pencil algorithm. Because of that, these students could not cover the usefulness of mental computation (Sowder, 2001).

Mental computation and estimation play a valuable role in everyday life. Reys and Reys (1986) stated that surveys show mental computation and estimation are used in more than 80% of all real-world problem solving situations outside the classroom. In daily life people sometimes do not have calculator, paper-pencil or any other devices to make computation that's why they need their brains as stated by Maier (1977)- "Other computation tools may not always be available, but people always carry their brains with them" (p.47). On the other hand, Reys and Bestgen (1981) stated that it has often been found that students are more successful when computing an exact answer with paper and pencil than when estimating an answer.

Computational estimation and mental computation are frequently combined together as one topic in the research studies (Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991; Reys, Reys. and Penafiel, 1991; Bestgen et al., 1980; Sowder, 2001; Markovits & Sowder, 1994; Munakata, 2002; LeFevre A., Greenham S. L. and Waheed N., 1993). Estimation requires competence in mental computation.

Hanson and Hogan (2000) studied on level of the computational estimation ability and the number of computational estimation strategies with respect to different type of numbers on 45 college students. They prepared the three phases study to identify the students' ability. In first phase the students tested by 20-item estimation test on the overhead projector. In second phase students were tested individually to estimate their answers and to think aloud as they arrived their answers. The last phase of the study was containing again a testing with the sufficient time to compute the answers. The researchers concluded that the subjects did fairly well on the integer part of the test but the fraction and decimal part did relatively worse. It was categorized 23 "think aloud" estimation strategies used by participants in individual follow-up sessions.

Reys ,Reys, Nohda, Ishida, Yoshikawa and Shimizu (1991) and Reys, Reys and Penafiel (1991) were investigated students' computational estimation abilities in Japanese and Mexico. These were replicated study of Reys's study in 1982 in United States. The studies consisted of testing and interview parts for identifying the strategies of estimation. R.E. Reys et al. (1991) compared the Japanese students and American students' computational estimation ability. Although the questions of the test were not exactly the same the comparison of the students in both countries can be done by the helping some of the same items. The researchers concluded that the Japanese students performed better than the American students. In addition, the Japanese students were more reluctant to accept error.

Reys, Reys and Penafiel (1991) provided a framework about how good estimators produce estimates. Although Mexican students were not taught the estimation and their strategies, they performed as well as the other countries' students. They developed and improved the ability out-of-school activities, especially with consumer-type setting. In the study the researchers found that computational estimation has been recognized as an important mathematics topic, emphasized by professional organizations, and identified in the recent curricular recommendations in a number of countries including the United States, England and Japan.

In a short summary, computational estimation has been the focus of a considerable amount of research in recent years. It can be concluded that good estimators are flexible in their thinking, use a variety of estimation strategies, and demonstrate a deep understanding of numbers and operations. Poor estimators are bound to applying algorithms, find it difficult to think of a problem as having more than one right answer or solution procedure, do not value estimation, and often equate estimation with guessing.

Goodman (1986) discriminated high ability level and low ability level groups on computational estimation with the subjects of preservice elementary school teachers. The treatment score was distinguished the subjects in the ability levels. At the end of the study Goodman reached the same conclusion like Montague et al. (2003), Crites (1992) and Mottram (1995). He stated that the low ability group struggled with the estimation strategies.

He also gave an additionally results that the low ability groups had difficulties in all format of the estimation test.

Carpenter and his colleagues (1976), after analyzing The National Assessment of Educational Progress (NAEP) data on estimation, concluded that before students can estimate well, they had to develop a quantitative intuition, a feel for quantities represented by numbers. In more recent years, this quantitative intuition occurred to be referred to as number sense. The authors of the National Council of Teachers of Mathematics Standards proposed that children with number sense understand numbers and their multiple relationships recognized relative magnitude of numbers and the effect of operating on numbers and developed referents for quantities and measures. According to Carpenter et al., instruction on estimation and mental computation could provide a possibility for developing number sense, or quantitative intuition. Students who were good at estimation and mental computation were easily able to link symbols to concepts. They finally stated that estimation and mental computation were not only useful tools in everyday life but they could also lead to better number sense. NCTM (1989) also stated that mental computation and computational estimation require number sense.

2.3 Relations of Number Sense and Computational Estimation

The National Council of Teachers of Mathematics (NCTM, 1989) defines number sense as an intuition about numbers that is drawn from all the varied meaning of numbers.

Hatano (1988) describe two types of number sense experts: Routine and adaptive. Routine experts are able to solve familiar problems quickly and accurately but are not able to invent new procedures because they lack the rich conceptual knowledge of an adaptive expert. Adaptive experts can discover rules, invent algorithms and develop flexible uses of numbers.

Number sense is not broader domain than either estimation or mental computation (Greeno 1991; McIntosh, Reys & Reys 1992; Sowder & Schappelle 1989). On the other hand, it includes both mental computation and computational estimation which require number sense. Both computational estimation and mental computation are closely related to number sense. Number sense refers to an intuitive feeling for numbers and their various uses and interpretations; the ability to detect arithmetical errors and a common-sense approach to using numbers (Sowder and Schappelle, 1994). Although there is currently a great deal of interest in number sense has not been a focus in instruction. Sowder (1992) stated that it is difficult to define and asses number sense like higher-order thinking.

Assessing number sense, mental computational and all three kinds of estimation presents many difficulties. For example, with estimation there is the problem of multiple correct answers. With mental computation there is the difficulty of determining whether or not the computation was indeed carried out mentally, particularly in group settings where students have pencil and paper to write down answer. With number sense there is the lack of an operational definition on which to base assessment items. New types of assessment are needed to measure success in these areas. If the learning of these topics is not evaluated, they are unlikely to find their way into school curricula.

According to Sowder's (2001) study 26 middle school students representing a variety of backgrounds and achievement levels, were individually asked to estimate answers to 12 computational problems and explain how they obtained their answers. They were allowed to use writing materials. Results indicated that estimation skills are highly dependent on a student's "number sense".

The researchers interested in studying number sense, computational estimation and mental computation agree on the importance of these topics, but did not necessary agree on which were the most important research issues to pursue, how research should proceed, or how these topics should be incorporated into the curriculum. Sowder (1992) found a reason it was because of the lack of agreement that is primarily due to the different epistemological viewpoints of the investigators. All do agree, however, that number sense should permeate the curriculum and that computational estimation and mental computational should be incorporated into all instruction on computation.

Markovits and Sowder (1994) examined the effect of an intervention in the instruction of 12 seventh grade students for the purpose of developing number sense. The students were taught experimental units on number magnitude, mental computation and computational estimation. From the interviews and written measures, it was discovered that the students reorganized and used existing strategies rather than acquiring new knowledge structure. Markovits and Sowder stated that a brief instructional unit appeared to bring about positive changes in understanding most of aspects of number sense. Sowder (1995) also stated the same conclusion with developmental capacity of the number sense. She connected estimation and number sense in her research that instruction on estimation and mental computation can provide an avenue for developing number sense. Students who are good at estimation and mental computation can easily link symbols to concepts that contribute to development of number sense. According to Markovits and Sowder (1994) if students understand the relationship between number sense and mental computation, they can develop effective strategies to solve and estimate problems mentally.

Boz and Bulut (2002) searched the preservice mathematics, science and childhood teachers' estimation abilities. The Estimation Ability Test was conducted to participant to understand their performance on computational estimation ability.   The researchers concluded that these teachers' estimation abilities were moderately low. The preservice teachers struggled with mostly on fraction number questions. The other number related categories of the estimation test were also difficult for the participant. As a result of the test researchers concluded the preservice teachers' quantitative intuition was very poor.

2.4 Estimation Strategies

In the mathematics education literature there are different types of estimation strategies. It is mainly focused on the computational estimation strategies.

The researches done the other countries showed and also in our country while estimating a solution or given approximating results presented that the rounding is used as the only strategy. However researchers agreed that it was one of the strategies among lots of them. They also investigated which strategies were used by different grade levels of students, preservice teachers, or different age groups adults.

One of the researchers that the studied on strategies was Dowker (1992). He interviewed 44 pure mathematicians to learn the computation estimation strategies used by mathematicians. They were accurate estimators and they used great variety of strategies. Dowker conclude that people often develop their own non-school based techniques for computational estimation. Furthermore, Reys (1986) there were 5 types self-developed strategies by students; front-end, clustering, rounding, compatible numbers, special numbers. She stated that like the problem-solving techniques, estimation strategies are developed through instruction.

Berry (1998) investigated to 8^{th} grade students' computational estimation ability and the strategies they used. The researcher interviewed ten students using the interview format of the Accessing Computational Estimation (ACE) Test (Reys, Rybolt, Bestgen and Wyatt, 1982). Instead of using the entire test, only the interview format from the ACE test was used. The interviews were divided 4 segments: In the computation segment the subjects were presented with 5 problems and were asked to "think out loud" as they estimated solution to the problems. In the application segment, the subjects were presented with 10 problems and then asked to answer the interviewer's probes. In the calculators segment calculators were programmed to make systematic errors and the subjects were tested to see if they questioned the calculators' output. The last segment was the attitude/concepts segment. The questions in this segment were designed to learn about the subject's concept of estimation and to find out what factors, such as home, school, community activities and jobs, appeared to contribute to the development of estimation strategies. In the study also, the research reviewed the locally used school mathematics textbooks to find out what computational estimation skills were presented in textbooks and how these skills were taught. A front-end strategy such as rounding, compatible numbers and truncation was observed in many forms and in different situations in the interviews. Berry (1998) concluded that rounding was the most frequently observed strategy; however in some cases compatible numbers and truncation were used. On the other hand, Smith (1993) also investigated the preservice elementary teachers' conceptual understanding of computational estimation strategies. In her study, the results of the dialogues indicated that rounding was the only strategy that many of the preservice elementary teachers. The compatible numbers strategy refers to using set of numbers that when estimated can easily be manipulated mentally. Front-end estimation focuses on the left-most digit of a number to provide an initial estimate followed by mental adjustment to determine a better estimate. Averaging or clustering can be used when numbers cluster about a particular value.

Like Berry (1998), Heinrich (1998) conducted a research on middle school students about the computational estimation skills that the students had the performance of them. The main independent variable in this research is the instructional treatment or four topics of unit in lessons being taught the students in grade 6, 7 and 8 in this study. All students received instruction from the same teacher over a period of 4 weeks in order to eliminate any effects due to different teaching styles. After each concept of computational estimation was taught, each group of the students was given a unit test to measure the degree of learning that had taken place. Based on the pretest, two students at each grade level were interviewed to determine which strategies they used to estimate and how they learned to adapt strategies they knew or developed new ones. Students considered good estimators were interviewed. The posttest which was the same form of the pretest was given to all students participating in the study 4 weeks after the last lesson was taught to determine whether any difference between the pretest and posttest scores were due to competency on the concept of computational estimation. Like Crites (1992) and Siegel et al., (1982); Heinrich concluded that the superior calculation ability developed from additional experience and maturity. The students in grades 6, 7 and 8 demonstrated that they were capable of learning to perform computational estimation tasks in a short period of time. All three grades, sixth, seventh and eighth, found that the easiest strategy was translation and the most difficult one was the compensation. He concluded that the major problem experienced by students not estimation ability skills that were lack of computational skills.

The choice and use of these strategies developed flexibility in thinking about and using numbers that fit a particular situation. Students generally did poorly estimating percents, square roots and product of mixed numbers. Similarly with Berry (1998) and Heinrich (1998), Sowder (1984) found that errors on estimation problems could be attributed to a lack of understanding of number size which led students to make poor approximations.

Levine (1982) investigated the strategies college students used to estimate products and quotients of whole numbers and decimal numbers. Her strategy classifications scheme included strategies which involved fractional relationships, powers of ten, exponents, and rounding. Like Berry (1998), Levine (1982) concluded that the most frequently used strategy was rounding both numbers in the problem. The other frequently used strategy was proceeding algorithmically, where a form of a standard algorithm was used to calculate, estimate and then combine partial products or quotients. Students of lower quantities ability used an algorithmic procedure for estimation more likely to use a variety of different estimation strategies. The compatible numbers strategy is especially useful in working percent problems. Levine stated that an understanding of place value is essential to being able to estimate decimals. She listed strategy classification scheme, consisting of nine types of estimation strategies; proceeding algorithmically, rounding both number, rounding one number, fractions, known numbers, powers of ten, incomplete partial products, exponents and establishing bounds. Levine (1982) concluded that students should recognize that the leading nonzero digit is the important one in determining the relative size of a decimal number. The leading-digit estimate is one strategy to use of common fraction equivalents and compatible numbers.

Crites's (1992) study about the discrete quantities also relied on the possession of spatial visualization, measurement, mental computation and number sense skills. In his study he identified two main strategies multiple benchmark and decomposition/recomposition which were more sophisticated from the other strategies. Benchmark is the application of a known standard to the to-be-estimated item. The comparison is made by regular decomposition-recomposition where the-to-be-estimated item is grouped into terms small enough to compare with a benchmark. This strategy involves dividing the item to be estimated into smaller parts until a benchmark can be applied and then recombining the parts based on a comparison with a known benchmark. There are also irregular decomposition-recomposition strategies. If the item cannot be easily divided into parts or the parts are of different sizes, then irregular decomposition occurs.

According to Reys et al. (1991); Reys, Reys and Penafiel (1991); and Heinrich (1998) the three general estimation process namely reformulation, translation and compensation were observed. Reformulation is a changing the numerical data into more mentally manageable form. Of these three key processes reformulation was observed most frequently during the interviews, followed by compensation and translation. Translation is changing the equation or mathematical structure of the problem to a more mentally manageable form. Compensation is adjustments made into the initial, intermediate or final estimate to reflect and awareness of the relationship of the estimate to the exact answer. Translation is more sophisticated technique than reformulation. Several different dimensions of this research suggest that computational estimation has not been emphasized in Japanese elementary and junior high schools. From textbook, they found that written computation was emphasized and driven by algorithmic procedures. They also found that three different types of rounding are taught in fourth grade. However they found little evidence of this rounding being incorporated in later grades to foster computational estimation skill. From students they found confusion often surrounded the concept of estimation. Many students did not understand what they were supposed to do when they were asked to estimate. Although all of the good estimators mentioned rounding as a result skill, few of them recalled being taught to estimate in school. Most of them attributed their success in estimation to skills and techniques they had developed on their own.

Brame (1986) investigated the computational estimation strategies used by high-school students of limited computational estimation ability. The Assessing Computational Estimation (ACE) test was administered 460 students, and 40 of them were selected for interviews. Each students interviewed was asked to estimate the answers to 14 computation and application problems. A comparison of the interview results and ACE Test results showed that removing the time pressure did improve performance. Students used wide variety of estimation strategies; however sometimes they had no strategy for estimation and attempted to use exact calculation. All but one of the students used some form of the front-number strategies rounding and truncation in making mental estimates. Truncation was replaced by the use of rounding and compensation by the better estimators of the study. Although many of the estimators seemed to want to use compensation they were many times not successful in its use. Estimators of limited ability used rounding but not always consistently or according to the standard rounding rules. Other commonly used strategies were averaging, using compatible or easier numbers and using the largest number to eliminate choices. The students in the study were most successful on percent problems when they thought of percents as part of one hundred or in terms of an easier percent. They performed better than expected on division problems. Possibly this is because of the use of estimation in the traditional algorithm. A major difficulty encountered by the estimators of limited ability was the large-number syndrome. Connected to this problem was the power of ten error. A student made a power-of-ten error when his answer would have been acceptable if it had been multiplied by an appropriate power of ten. Mental calculation and development of number sense could be taught to aid in the development of estimation strategies (Berry, 1998).

2.5 Teaching Estimation and the Strategies

The necessity for developing computational estimation procedures has been well established. However, the teaching of this skill has been neglected in schools.

One of the researches was Bobis's (1991) study that investigated the effect of instruction on the development of computational estimation strategies of fifth graders. She conducted an experimental study with a control group. Her test contained the whole number, decimal numbers and fraction. After the quantitative part there was an interview part to identify to strategies they used. At the end of the study she supported her claims with data. All experimental groups have great improvement. In deeply the students were showed a little improvement to whole number addition, addition and multiplication involving decimal currency, whole number subtraction, division and multiplication items showed moderate improvement and fractions show a much greater level of improvement.

Bestgen et al. (1980) found the instruction on various strategies of computational estimation helped preservice teachers developed useful strategies for estimating. They used tree treatment groups to assess the impact of instruction: the control group, to which no instruction or practice on estimation was given; the first experimental group, to which estimation practice was given without instruction; and the second experimental group, to which both practice and instruction were provided. Pretest and posttest assessing estimation performance on computational estimation skills and general attitudes towards mathematics were administered to all groups. Though there was no significant difference in gains on estimation performance between the group given only practice and the group given both practice and instruction, there was a significant difference between the control group and the group given only practice. This result indicates that exposure to computational estimation problems, even in the absence of formal instruction, improves performance. The difference between the group that received instruction and the two groups receiving no instruction were related to attitudes toward mathematics. Instruction had a positive effect of instruction on subjects' general attitudes toward mathematics.

In Turkey, Yazgan, Bintaş and Altun (2002) investigated to improve the fifth grade students' mental computation and computational estimation. 26 participants were tested for understanding the students' mental computation and computation estimation ability. After eight weeks treatment posttest was conducted to identify the progress of these abilities. Yazgan et al. (2002) had the same results with Berry (1998) and Heinrich (1998). The researchers concluded that students improved their ability and used more sophisticated strategies.

In order to Crites (1992) for students to acquire skill in estimation they must have practical experiences in making estimates so that they can develop their own individual frames of reference for estimating the quantity of various types of measurement. Moreover, Trafton (1986) stressed the important role that teachers play in developing estimation ability in children. He made four points that can serve as the foundation for incorporating the important aspect of mathematics instruction into the curriculum, and he also offered several teaching suggestions to address each point. These points include helping students to (a) understand the legitimacy and usefulness of estimation, (b) develop flexible thinking and decision-making ability, (c) adjust initial estimates based on understanding the relationships between an estimate and the exact answer, and (d) build the recognition of sensible answers.

Gossard (1986), investigated to determine the kinds and amounts of computational estimation that were taught to middle school students then compare what they were taught with what they actually learned and what they actually used in real world mathematical problem solving. Three types of estimation were considered: reformulation, translation and compensation. When looked the results; all subjects learned the reformulation of the rounding whole numbers and decimal to a designated with a stronger understanding of rounding whole numbers. Translation estimation was not taught at all and compensation estimation was only taught with division of whole numbers. Only a small number of rounding estimates were made in the problem-solving sessions. Neither translation nor compensation was used in the problem-solving sessions at all. The subjects in this study did not receive adequate instruction for the types of estimation needed in the real world problem solving. Rounding of whole numbers and decimals was the only type taught, learned, and sparsely used in the problem-solving by the average-ability eight graders studied. The critical role of computational estimation in applied and nonapplied mathematical decision making warrants more time for instruction.

According to Chien (1990), it was used the self-study material for improving the computational estimation ability and he found that some strategies were more used than some others. Two other instructional studies used computer-based instructional units. Whiteman (1989) used computer to develop the strategies of computational estimation on middle school children. He concluded that the estimation activities had a greater influence on the development of estimation strategies for poor and fair estimators. Additional opportunities to practice generating estimates were more beneficial to good estimators than was instruction on specific strategies. Damarin, Dziak, Stull and Whiteman (1988) searched that teaching estimation with using a sequence of computer based activities to high school mathematics students. They asserted that the computer can be programmed to change the range of acceptable responses and thus adapt to the student's increasing level of skill. Computers can also be used to control the amount of time allotted for making estimates, thus discouraging attempts at exact calculations or providing students motivation for increasing their speed of estimation. They concluded the study with the result that the treatment was appropriate for students at each grade level and each class improved their estimation scores significantly; the amount of improvement varied across class.

Also Bright (1985) designed a search to determine the effectiveness of two microcomputer instructional games in teaching estimation of length and angle measurements to preservice elementary teachers. He conducted an experimental study with a control group and contradictory to Whiteman and Damarin et al. he found that the microcomputer games were not very effective at teaching estimation.

Developmental differences on computational estimation tasks have been documented. For example, in their survey of estimation skills in students of four countries Reys et al. (1999) reported that 8^{th} grade students in each target country outperformed 6^{th} grade students on problems involving the understanding of the meaning and effects of operations. In a separate study, Reys et al. (1991) found marked developmental differences between 5^{th} and 8^{th} grade students in Mexico; the 8^{th} grade students scored significantly higher on computational estimation tests.

Sowder (1984) stated that analyses with respect to grade level showed that while the younger students' preferred strategy for computational estimation was rounding, the reverse technique, computing then rounding, was more accepted by older students. In other words, older students, more than younger ones, showed a similarity for finding exact answers first, then rounding to make the solutions look like estimates. This statistics was explained by way of reference to the developmental differences in tolerance for error: older students seem to have a lower tolerance for error, a more pronounced need to find the exact answer, than younger students. Estimation performance improved with age, adults produce more accurate estimates than children (LeFevre, Greenham and Waheed, 1993; Crites, 1992; Heinrich, 1991). This is perhaps a result of the emphasis on the one correct answer in mathematics classes, especially at higher grade levels.

CHAPTER III

METHOD OF THE STUDY

This chapter explains the method used in this study of computational estimation. It also describes research design, main and sub problems of the study, hypotheses, variables, subjects of the study, the materials used, the procedures that were undertaken, assumptions and limitations, internal and external validity.

3.1 Research Design

In this study it was used the experimental study research method with one-group pre-post test design. In this design single group is measured or observed not only after being exposed to a treatment of some sort, but also before. Although this design is not the best experimental design, it is better than the one-shot case study since researcher at least knows whether any changes occurred (Fraenkel and Wallen, 1996). In the table 3.1 the one group pretest-posttest design is presented.

Table 3.1 *One Group Pretest-Posttest Design*

Pretest	Treatment	Posttest
Estimation Ability Test completed	Four weeks of estimation lessons	Ability Test completed

3.2 Main and Sub-Problems of the Study and Associated Hypotheses

In this section main and sub problems and hypotheses are stated.

3.2.1 Main and Sub-problems of the Study

Main problem of the study is stated as: “What is the 9th grade students’ estimation ability after the instruction on estimation?”

Based on the main problem, the following sub-problems are explored:

SP1. Are there any statistically significant differences among students enrolled to different type of schools with respect to estimation ability?

SP2. Are there any statistically significant mean difference among students enrolled to different type of schools with respect to estimation ability on Number Format, Answer Format and Question Format?

SP3. Is there any statistically significant mean difference between girls and boys enrolled to different type of schools with respect to estimation ability?

SP4. Is there any statistically significant mean difference between girls and boys with respect to estimation ability on Number Format, Answer Format and Question Format?

3.2.2 Hypotheses of the Study

The following null hypotheses are stated in order to investigate the main problems of the study. They were tested at a significance level of 0.05.

To examine the first sub-problem, the following one hypothesis is stated:

H1.1. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability.

To examine the second sub-problem nine hypotheses are stated as:

H.2.1. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Whole number.

H2.2. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Fraction.

H2.3. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Percent.

H2.4.There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Decimal.

H2.5.There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Numbers Only Category.

H2.6. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Application Category.

H2.7. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Open-ended Category.

H2.8. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Reference Number Category.

H2.9. There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Order of Magnitude Category.

To examine the third sub-problem one hypothesis is stated as:

H3.1. There is no statistically significant mean difference between girls and boys with respect to estimation ability.

To examine the fourth problem nine hypotheses are stated as:

H4.1 There is no statistically significant mean differences between girls and boys with respect to estimation ability on Whole number.

H4.2 There is no statistically significant mean differences between girls and boys with respect to estimation ability on Fraction.

H4.3 There is no statistically significant mean difference between girls and boys with respect to estimation ability on Percent.

H4.4 There is no statistically significant mean difference between girls and boys with respect to estimation ability on Decimal.

H4.5 There is no statistically significant mean difference between girls and boys with respect to estimation ability on Numbers Only Category.

H.4.6 There is no statistically significant mean difference between girls and boys with respect to estimation ability on Application Category.

H4.7 There is no statistically significant mean difference between girls and boys with respect to estimation ability on Open-ended Category.

H4.8 There is no statistically significant mean difference between girls and boys with respect to estimation ability on Reference Number Category.

H4.9 There is no statistically significant mean difference between girls and boys with respect to estimation ability on Order of Magnitude Category.

To test the interaction between school type and gender the following 10 hypotheses are stated:

1. There is no statistically significant interaction between school type and gender with respect to estimation ability.

2. There is no statistically significant interaction between school type and gender with respect to estimation ability Whole Number.
3. There is no statistically significant interaction between school type and gender with respect to estimation ability Fraction.
4. There is no statistically significant interaction between school type and gender with respect to estimation ability Decimal.
5. There is no statistically significant interaction between school type and gender with respect to estimation ability Percent.
6. There is no statistically significant interaction between school type and gender with respect to estimation ability Open-ended Category.
7. There is no statistically significant interaction between school type and gender with respect to estimation ability Reference Number Category.
8. There is no statistically significant interaction between school type and gender with respect to estimation ability Order of Magnitude.
9. There is no statistically significant interaction between school type and gender with respect to estimation ability Numbers Only Category.
10. There is no statistically significant interaction between school type and gender with respect to estimation ability Application Category.

3.3 Definition of the Terms

Estimation refers to process of producing an answer that is sufficiently close allows decisions to be made.

Computational estimation refers to the process of simplifying an arithmetic problem using same set of rules or procedures to produce an approximation but satisfactory answer through mental calculation.

Mental computation refers to the ability to calculate exact numerical answers without the aid of calculating devices or recording devices.

Number Sense refers to the number sense as understanding numbers and their multiple relationships and recognizing the relative magnitude of numbers.

Estimation strategies refer to reformulation, translation and compensation. In the reformulation means that the process of altering numeric data to produce more mentally manageable form. This strategy is divided into two methods that are rounding and truncation that is shorting of number to make it easier to manipulate. Translation refers to process of changing the mentally mathematical structure of the problem to a more mentally manageable form. The compensation refers to process of altering numeric data to produce a more mentally manageable form. This strategy is also divided into two sub methods; these are final compensation and intermediate compensation. The first one is an adjusting an initial estimate to more closely convey the user's knowledge of the error introduced by the strategy employed. The lasted is an adjusting numerical values prior to their being operated on to systematically correct for error.

3.4 Subjects of the Study

The subjects of the study are 153 students on the 9^{th} grade of three types of schools; the High School, Foreign Language High School and Anatolian High School of Denizli.

The students in the study sample were 59 boys and 94 girls. The students' ages are 15-16. All the students are in the same region of the Denizli, so that their social economic status is almost the same. Although, the students of Anatolian High School enrolled to after the High School Entrance Examination and the Foreign Language High School students are enrolled according to their primary school cumulative grade; these students are all living in that part of the region. Because of this, generally the students that participated in study all had same demographic characteristics to each other.

In order to that it can easily say the sample is representative of that region. Since the researcher is a teacher in Denizli, the sample of this study was selected by her for their conveniences for the researcher. Because of this convenience sampling was accomplished in this study. The distribution of the subjects wrt different high schools was given in Table 3.2.

Table 3.2 *The Distribution of the Subjects wrt Different High Schools*

Gender	High School	Foreign Language High School	Anatolian High School	Total
Boys	18	16	25	59
Girls	27	43	24	94
Total	45	59	49	153

3.5 Procedure

The procedure of the study consisted of following steps:

- Selection of the subjects
- The pilot study
- Development of the materials
- Pretest application
- The four weeks treatment and
- Posttest application

The subjects accounting in the study were selected by convenience sampling procedure. The students who were enrolled to normal High School and Foreign Language High School were in the researcher's own classes. On the other hand the Anatolian High School students were not the researcher's students. Some pilot studies were conducted on the selected subjects for testing the estimation ability test and the treatment period. According to pilot test results estimation ability test was redesigned.

The main study was conducted in Spring Semester of 2004. The study was lasted over a 6 weeks-period.

After determination of the subjects researcher developed the material that were used in the estimation lessons. The lessons plans and exercises on estimation strategies were presented in the Appendix A.

Before the treatment researcher conducted a pretest was given to each of the three participating schools for determining the students' level of estimation ability. Test items were presented using overhead projector; each item was shown for fifteen to twenty seconds with approximately three to five seconds between items. The students recorded their answers on answer sheets provided for them. Since the estimation ability is not a separate subject of Turkish Education Curriculum, the first measurement conducted to identify their estimation ability which they already have by their own experiences. Students were told that this was an estimation test. Because there is no direct Turkish translation of the word estimate, the word "*tahmin etmek"* meaning rough calculation, was used in the directions. Students were told that each problem would be timed and that they would have between 10 and 15 seconds to make and record their "rough calculations". The students were also told "not to copy the problem but to do the work in your head". Prior to starting the test, two sample problems were provided to explain students with the format and the time restriction of each item. This permitted students to adjust their seats to see the screen clearly and ask for further classification of the task, if necessary.

After pretest subjects exposed the training part. The researcher gave a treatment to the three types of the high schools students. She developed lessons plans and exercises sheets for enhancing the students' estimation ability. The treatment part will be explained with detailed in following parts.

After the four weeks treatment the posttest was administered in order to measure whether any gains were result of having just been taught the material or from having mastered the skill of computational estimation. The estimation ability test was used as a post and pre tests. Also post test was conducted as the same way of the pretest. In other word the questions were presented on the overhead projector and students had fifteen to twenty minutes for doing their estimation. Since the time period between pretest and posttest was long enough students didn't have any familiarity among them. After testing procedure was ended the data were analyzed.

Along the treatment when any of the students missed any lessons, these students explained and gave the same procedure as the others did. However, when some students missed the assessment on pre or post test, this time they cleaned out of the subjects' data.

3.6 Development of Materials

In the study Estimation Ability Test was used as pretest and posttest. The used test was adapted from Goodman (1991) 72-item test. The formats and categories of the test will be explained with examples in the next part the measuring instrument of the study. The questions of the test were adapted for Turkish culture and students. For example the questions that contained the dollar and miles arrangements were removed the test. These terms replaced with some other decimal content questions. In other words questions main computational concepts were protected but the story part of the problem was changed.

In the treatment part lessons plans, exercises and homework sheets were used. These materials were developed according to Heinrich's (1998) study. Although in Heinrich study the lesson plans were prepared as eight separate lessons, we combined two lessons and used as a one lesson so they became four lessons in the present study. One reason to doing that was, Heinrich study was on eighth grade students who were the primary students of our country; on the other hand we used same lessons plans and exercises sheets' questions for the ninth graders in this study.

3.7 Measuring Instrument

In the study Goodman's (1991) Estimation Ability Test (AET) was used. The original test contains 72 items but we eliminated one of them so we applied 71-item-test. The test which was used as pretest and posttest in the study adapted for Turkish students. In some items dollar applications with decimal numbers converted some similar mathematical concepts without the money meaning with decimal numbers applications. Goodman's test was designed to measure performance on three categories of estimation: Numbers (whole, fractions, decimal, per cent), problem format (numbers only or application), and answer format (open-ended, reference number, order of magnitude).

In the following every category and the examples were given in more detailed.

In the numbers category, it was divided into four separate parts; first one was whole numbers category consisted of 18 items.

- Estimating the following; 3235÷5=?

The second category of the numbers format was the fraction category that contained 17 items

- Estimating the following; 13/16 ÷ 1 7/8 =?

The third category was the decimal category consisted of 18 items

- True or False? 359,25÷19, 6<17

The last category of the number format was per cent category that involved in 18 items

- True or False? 35 per cent of 37, 50 is about 13

When the test investigated the problem format category, it divides the two separates parts.

The first one was numbers only category consisted on 36 items.

- Choose the best estimate; 967 is what percent of 214?

(a) 500

(b) 50

(c) 5

(d) None of these

The second category of the problem format was an application category which involved 35 items

- The population of a city is 19 700. This is a 9 per cent increase from last year. What was the population last year?

(a) 18 000

(b) 1800

(c) 180

(d) None of these

When the test items investigated in order to answer format, the test was separated into three different parts.

The first category was open-ended category with 27 items

- If a postman delivers 96 letters per day, about how many letters will he deliver in a year? - He works everyday in a year

It is the most common style of “real life” estimation. In this test’s for open-ended items the researcher established “acceptable interval” for each of the 27 items. I considered an estimate accurate if it was in the interval that was taken as within 50% of the exact answer. Besides

Gatze (1989) took the acceptable estimates within 25% of the actual answer we took 50% of interval. A wide margin of 50% of the exact answer with the large error was chosen for the reason that the previous researches for example Siegel, Goldsmith and Madson (1982), Baroody and Gatzke (1991) and also Crites (1992) took as the same interval in their studies.

The second category of the EAT according to answer format was the reference number category with 21 items

- 347 036 – 256 987=? Will this be about 90 000?

This type of estimation asks the students to decide the whether the answer to an estimation item is over or under a given reference number (Rubenstein, 1986).

The answer format's last category was the order of magnitude category. In the test there were 23 items on order of magnitude category.

- 0,59+93,703+8,071+29,2+267,15=?
  (a) 4000
  (b) 40
  (c) 400
  (d) None of these

In order to Rubenstein (1986) order of magnitude type is the sub style of the multiple choice estimation type. In this type students were focused on the order of the magnitudes of the choices. Although some researchers (Reys and Bestgen, 1981) stated that these types of items were not useful to evaluating the estimation performance (Rubenstein, 1986). Students' performance varied significantly depending on which format was used in tests (Rubenstein, 1986). The open-ended formats are more difficult than the order of magnitude or reference number type estimation for the students.

Table 3.3 *The Number Format Categories of the Estimation Ability Test*

Format	Categories	Number of Items
	Whole	18
Numbers Format	Fraction	17
	Percent	18
	Decimal	18

Table 3.4 *The Problem Format Categories of the Estimation Ability Test*

Format	Categories	Number of Items
Problem Format	Numbers Only	36
	Application	35

Table 3.5 *The Answer Format Categories of the Estimation Ability Test*

Format	Categories	Number of Items
Answer Format	Open-Ended	27
	Reference Number	21
	Order of Magnitude	23

3.8 Treatment

The researcher conducted four weeks treatment to the students. She applied estimation lessons. At the beginning of the lessons she introduced estimation and related concepts to the students. Firstly she asked some questions that need to estimate. For example she asked “how much hair they have got?”. The students firstly couldn’t say anything on the other hand when the researcher said they could say an approximate answer everyone said their own estimated result. So the researcher warmed up students by helping this type estimate required questions.
However students still didn’t want to estimate. In order to using estimation they resisted on using the mental computation abilities. When the teacher explained them the importance of the estimation then they started to use the estimation strategies. After students recognized estimation type questions researcher moved on the strategies of estimation.

The estimation lessons were taught by the researcher one hour of every week in every type of the high schools. Approximately four weeks researcher gave teaching seasons about estimation and their strategies. In the lessons first the researcher told the estimation and one of the related strategies (reformulation, translation, and compensation) then makes some exercises about the strategy. In the reformulation strategies students learned to redesigning the number to more manageable form. For example 243 can be rounded to 250.
Students could use this strategy by helping of rounding and truncation methods. 243 was rounded to 250. On the other hand in some computation 243 could be truncated to 240 so that the computation might be simpler than before.

The second strategy was translation. It was used to change the process of the structure on the problem to more mentally manageable form. For example in the computation (12x430) / 7, students could first apply the reformulation to 430 to 420 then they could divide 420 to 7 and found 60. Then the computation 12 x 60 could be estimated as 10 x 60 so the estimated result was 600.

The last strategy was compensation. It was used for smoothing the estimated answer and getting closer one. In other words compensation was used for making more reasonable answer that close to the exact answer by redesigning the estimated result. For example 600 that found in the before strategy was a small answer since 12 was truncated. Because of that 2x60 =120 could be add to 600, so 720 was more close answer to the exact answer.

Each estimation lessons contain exercises sheet consisted with strategies, and these sheets done by every student in the classes. At the end of every lesson, homework sheets are given to students for practicing the concept at home. These homework sheets formats have same format of sheets that studied in the lessons.

3.9 Variables of the Study

Independent variables in the present study are kinds of high schools (general high school, Anatolian high school and foreign language high school) and gender. Dependent variables are the posttest sores on Estimation Ability Test's three kinds of formats.

3.10 Data Analysis

Data analyses of the study were conducted by the following statistical techniques:

- Data of the present study were analyzed by using the SPSS package program.
- Reliability analysis was used to test the reliability of Estimation Ability Test scale administered in the present study.
- Descriptive statistics were used to get the mean and standard deviations of the responses of each item on Estimation Ability Test.
- ANCOVA and one-way ANOVA were used by the following reasons:
- To determine whether there are significance mean differences among groups with respect to their estimation ability levels where the pretest is taken as a covariate variable
    - To test for interactions as well as for main effects to variables.
    - To examine the differences among school types of the students.
    - To examine the differences between gender of the students.

3.11 Assumptions and Limitations

In this section, assumptions and limitations of the present study are discussed.

3.11.1 Assumptions

In the study no outside event occurred during the treatment to affect the students' estimation ability levels. The administrations of the scales were completed under standard conditions. Additionally, all the subjects of the pilot and experimental studies answered the measuring instruments accurately and sincerely.

3.11.2 Limitations

This study is limited to subjects enrolled at the high schools in rural areas of Denizli during 2003-2004 spring semester. The selection of the sample for the study did not conducted random sampling. Therefore the sample may not be fully representative of the population and generalizability is limited. Self-reported techniques, which require the subject to respond truthfully and willing, were used. Additionally, some of the classes consisted in the study were researchers own teaching classes. This may be produced some biased results.

3.12 Validity of the Study

Internal validity of a study means that observed differences on the dependent variable are directly related to the independent variable, but not due to some other unintended variable. There are three threats to internal validity that might also explain the results on the posttest. One of them is maturation that is over the time passing, very young students in particular will change in many ways because of simply aging and experience. On the other hand the study was taken a semester that the students maturation almost not influence on the experiment. The other one is data collector characteristics that are the characteristics of the data gatherers can also affect the results. However, all groups' data collectors are the same, so this threat was controlled. The other threat is testing that means a pretest sometimes can make students more alert to or aware of what may be going to take place, making them more sensitive to a responsive toward the treatment that subsequently occurred. This threat is under controlled by the statistical methods with ANCOVA analysis (Frankel & Wallen, 1996).

External validity is to which the results of a study can be generalized (Frankel & Wallen, 1996). One of the external validity is population validity. In this study convenience sampling was utilized. Because of this, generalizations of the findings of the study were limited. However, generalizations can be done on subjects having the same characteristics mentioned in the "Subjects of the Study" section. The other external validity is ecological validity. It is the degree to which results of a study can be extended to other setting or conditions. The measuring the instruments were used in regular classroom settings. Since the study is on ninth grade students, the results of the present study can be generalized to similar setting to this study.

The reliability of the EAT which was computed by using Croanbach Alpha was 0, 84 as it was expected.

CHAPTER IV

RESULTS AND CONCLUSIONS

This chapter includes the results of analyses of pre-treatment and post-treatment measures with respect to type of schools and gender. Conclusions are also presented.

4.1. The Results of Pre-treatment Measures with respect to Type of Schools and Gender

Before the treatment estimation ability test was administered to the subjects. The results of one-way ANOVA of the pre-treatment measures scores with respect to type of schools were given in Table4.1. In the analyses degrees of freedom was 2,152.

Table 4.1 *The Results of ANOVA of Pre-treatment Measures Scores wrt Type of Schools*

Variables	HS		FLHS		AHS		F
	Mean	SD	Mean	SD	Mean	SD	
Estimation Ability	22.600	5.154	29.288	7.624	32.469	6.961	25.90*
Whole Number	5.111	2.187	8.101	2.868	8.632	2.514	25.66*
Fraction	5.000	1.821	6.664	1.963	5.959	1.914	9.49*
Decimal	6.288	2.272	7.406	2.755	8.551	2.821	8.58*
Percent	6.200	2.312	7.135	2.562	9.326	2.779	18.75*
Numbers Only	12.177	3.024	15.745	3.857	17.040	3.469	24.14*
Application	10.422	3.180	13.542	4.523	15.428	4.420	17.43*
Open-Ended	5.000	3.240	9.525	4.387	11.530	4.368	31.44*
Order Of Magnitude	11.000	2.022	11.610	2.579	12.020	2.096	2.37
Reference Number	7.133	2.051	8.644	2.530	9.530	2.777	11.10*

* $P<0.05$

There were statistically significant mean differences between girls and boys with respect to all prior measures except Fraction. The differences were in the favor of boys.

Table 4.2 *The Results of t-Test Analyses of Pre-treatment Measures Scores wrt Gender*

Variables	Boys		Girls		t-value
	Mean	SD	Mean	SD	
Estimation Ability	32.305	7.684	25.851	6.802	5.43*
Whole Number	8.033	2.846	6.989	2.971	2.15*
Fraction	6.322	2.137	5.702	1.899	1.87
Decimal	8.508	2.873	6.776	2.502	3.93*
Percent	9.440	2.699	6.383	2.253	7.56*
Numbers Only	16.678	4.133	14.127	3.607	4.02*
Application	15.627	4.201	11.723	4.135	5.64*
Open-Ended	11.271	4.630	7.308	4.310	5.37*
Order Of Magnitude	12.339	2.233	11.074	2.210	3.43*
Reference Number	9.322	3.014	7.954	2.247	3.19*

* $P<0.05$

There were statistically significant mean differences among student who enrolled to different type of schools with respect to all pre-treatment measures except order of magnitude.
The results of t-test Analyses of pre-treatment measures with respect to gender were given in Table4.2. In the analyses degrees of freedom was 151.

4.2. The Results of Post-treatment Measures with respect to Type of Schools and Gender

As a result of analyses of pre-treatment measures, covariate variables were determined to test the hypotheses of post-treatment measures. In other words, prior measures of post measures were taken as covariate variables. For example, prior estimation ability was taken as a covariate variable to test the hypothesis of post estimation ability. Hypotheses of post-treatment measures were tested by using ANCOVA at the level of significance 0.05.

4.2.1The Results of the Analysis of Estimation Ability Test with respect to Type of School

The first hypothesis of the first subproblem (H1.1) was “There were no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability." The results were given in Table 4.3.

Table 4.3 *The Results of ANCOVA of Estimation Ability Test Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	10539.342	6	1756.557	68.494	0.00*
Intercept	1393.536	1	1393.536	54.338	0.00*
Prior Estimation Ability	2185.780	1	2185.780	85.231	0.00*
Type of School	2690.144	2	1345.072	52.449	0.00*
Gender	23.713	1	23.713	0.925	0.34
Type of School*Gender	11.928	2	5.964	0.233	0.79
Error	3744.240	146	25.645		
Total	180373.000	153			
Corrected Total	14283.582	152			

* p<0.05

As seen in Table 4.3, prior estimation ability score was statistically significant covariate (p <0.05). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability

($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS.
The students in AHS had moderately higher mean score than students in both HS and FLHS (M_{AHS} =42.306, SD_{AHS} =6,384; M_{HS} =24.666, SD_{HS} =5.900; M_{FLHS} =31.491, SD_{FLHS}=7.534). The students in FLHS were also better on estimation ability than the students in HS.

4.2.1.1 The Results of the Analysis of Numbers Format of Estimation Ability Test with respect to Type of School

The Numbers Format consists of whole number, decimal number; fractional number and percent number categories. The analyses of each category are shown below. The first hypothesis of the second subproblem (H2.1) was "There are no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on whole number. The results were given in Table 4.4.

Table 4.4 *The Results of ANCOVA of Whole Number Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1108.309	6	184.718	39.103	0.000*
Intercept	361.080	1	361.080	76.437	0.000*
Prior Whole Number	246.937	1	246.937	52.274	0.000*
Type of School	301.552	2	150.776	31.918	0.000*
Gender	9.056	1	9.056	1.917	0.168
Type of School*Gender	8.445	2	4.222	0.894	0.411
Error	689.691	146	4.724		
Total	13290.000	153			
Corrected Total	1798.000	152			

* p<0.05

As seen in Table4.4, prior estimation ability on Whole Number was statistically significant covariate ($p < 0.05$). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on whole number ($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS.

The students in Anatolian High School scored quite better than both the students in High School and those in Foreign Language High School (M_{AHS} =11.387, SD_{AHS} =2.352; M_{HS} =5.488, SD_{HS} =2.242; M_{FLHS} =8.830, SD_{FLHS} =2.913). The students in Foreign Language High School were also better scored at the Estimation Ability Test's Whole Number Category score than those in High School.

The second hypothesis of the second sub-problem (H2.2) was "There were no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Fraction". The results were given in Table 4.5.

Table4.5 *Results of ANCOVA of Fraction Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	602.882	6	100.480	22.748	0.000*
Intercept	484.463	1	484.463	109.679	0.000*
Prior Fraction	32.585	1	32.585	7.377	0.007*
Type of School	441.363	2	220.681	49.961	0.000*
Gender	37.906	1	37.906	8.582	0.004*
Type of School*Gender	4.356	2	2.178	0.493	0.612
Error	644.896	146	4.417		
Total	9982.000	153			
Corrected Total	1247.778	152			

$p<0.05$

As seen in Table4.5, prior estimation ability on Fraction was statistically significant covariate ($p < 0.05$). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Fraction ($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. On the other hand there was no statistically significant mean difference between those in FLHS and those in HS.

The students in Anatolian High School had moderately higher mean score than both the students in High School and Foreign Language High School ($M_{AHS} = 10.142$, $SD_{AHS} = 2.449$; $M_{HS} = 5.733$, $SD_{HS} = 1.911$; $M_{FLHS} = 6.796$, $SD_{FLHS} = 2.226$). Moreover the mean difference of the students in Foreign Language High School and in High School wasn't statistically significant to each other with respect to estimation ability on Fraction.

The third hypothesis of the second subproblem (H2.3) was "There were no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Percent". The results were given in Table 4.6.

As seen in Table4.6, prior estimation ability on Percent score was statistically significant covariate ($p < 0.05$). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability ($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS.

Table4.6 Results of ANCOVA of Percent Category Scores

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	451.148	6	75.191	16.844	0.000*
Intercept	302.820	1	302.820	67.838	0.000*
Prior Percent	77.588	1	77.588	17.381	0.000*
Type of School	102.655	2	51.328	11.498	0.000*
Gender	6.127	1	6.127	1.373	0.243
Type of School*Gender	10.426	2	5.213	1.168	0.314
Error	651.728	146	4.464		
Total	10452.000	153			
Corrected Total	1102.876	152			

* $p<0.05$

The students in Anatolian High School scored quite better than both the students in High School and those in Foreign Language High School (M_{AHS} =9.693, SD_{AHS} =2.338; M_{HS} =6.311, SD_{HS} =1.998; M_{FLHS} =7.406, SD_{FLHS} =2.560). The students in Foreign Language High School were also better scored at the estimation ability on Percent than the students in High School.

The fourth hypothesis of the second subproblem (H2.4) was “There were no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Decimal”. The results were given in Table4.7.

Table4.7 *Results of ANCOVA of Decimal Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	491.123	6	81.854	19.103	0.000*
Intercept	775.578	1	775.578	181.002	0.000*
Prior Decimal	47.216	1	47.216	11.019	0.001*
Type of School	234.550	2	117.275	27.369	0.000*
Gender	19.740	1	19.740	4.607	0.033*
Type of School*Gender	18.023	2	9.011	2.103	0.126
Error	625.596	146	4.285		
Total	13259.000	153			
Corrected Total	1116.719	152			

* $p<0.05$

As seen in Table4.7, prior estimation ability on Decimal score was statistically significant covariate ($p <0.05$). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability ($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS.

The students in Anatolian High School scored quite better than both the students in High School and those in Foreign Language High School ($M_{AHS}=11.081$, $SD_{AHS}=1.800$; $M_{HS}=7.133$, $SD_{HS}=2.117$; $M_{FLHS}=8.457$, $SD_{FLHS}=2.555$). The students in Foreign Language High School were also better scored at the estimation ability on Decimal measurement than those in High School.

4.2.1.2 The Results of the Analysis of Problem Format of Estimation Ability Test with respect to Type of School

In this section the Problem Format's two categories; Numbers Only and Application were analyzed in order to types of school.

The fifth hypothesis of the second subproblem (H2.5) was "There were no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Numbers Only Category". The results were given in Table 4.8.

Table4.8 *Results of ANCOVA of Numbers Only Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2296.186	6	382.698	33.682	0.000*
Intercept	516.225	1	516.225	45.434	0.000*
Prior Numbers Only	526.917	1	526.917	46.375	0.000*
Type of School	674.083	2	337.041	29.664	0.000*
Gender	5.928	1	5.928	0.522	0.471
Type of School*Gender	85.595	2	42.798	3.767	0.025*
Error	1658.873	146	11.362		
Total	51285.000	153			
Corrected Total	3955.059	152			

* p<0.05

As seen in Table4.8, prior estimation ability on Numbers Only score was statistically significant covariate ($p < 0.05$). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability ($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. On the other hand there was no statistically significant mean difference between those in FLHS and those in HS.

The students in Anatolian High School (M_{AHS} ==22.020, SD_{AHS} =3.556) had moderately higher mean score than both the students in High School (M_{HS} ==13.911, SD_{HS} =4.027) and Foreign Language High School (M_{FLHS} ==16.711, SD_{FLHS} =4.189). However, the students in Foreign Language High School and the students in High School didn't cause any statistically significant mean difference to each other.

The sixth hypothesis of the second subproblem is "There is no statistically significant mean difference among the students who enrolled to different types of high school with respect to estimation ability on Application Category". The results were given in Table4.9.

Table4.9 *Results of ANCOVA Application Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2927.249	6	487.875	53.396	0.000*
Intercept	977.217	1	977.217	106.953	0.000*
Prior Application	383.495	1	383.495	41.972	0.000*
Type of School	1044.331	2	522.166	57.149	0.000*
Gender	55.217	1	55.217	6.043	0.015*
Type of School*Gender	40.803	2	20.401	2.233	0.111
Error	1333.980	146	9.137		
Total	40356.000	153			
Corrected Total	4261.229	152			

* $p<0.05$

As seen in Table4.9, prior estimation ability on Application score was statistically significant covariate ($p < 0.05$). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability ($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS.

The students in Anatolian High School scored quite better than both the students in High School and Foreign Language High School (M_{AHS} =20.285, SD_{AHS} =3.769; M_{HS} =10.755, SD_{HS} =2.901; M_{FLHS} =14.779, SD_{FLHS} =4.247). The students in Foreign Language High School were also better scored at the estimation ability on Application than the students in High School.

4.21.3 The Results of the Analysis of Answer Format of Estimation Ability Test with respect to Type of School

In this part of the study the Answer Format of Estimation Ability Test's three categories are tested. These are Open-ended Category, Reference Number Category and Order of Magnitude Category.

The seventh hypothesis of the second subproblem is "There is no statistically significant mean difference among the students who enrolled to different types of high school with respect to estimation ability on Open-ended Category". The results were given in Table4.10.

Table 4.10 *Results of ANCOVA of Open-Ended Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4280.443	6	713.407	76.160	0.000*
Intercept	933.747	1	933.747	99.683	0.000*
Prior Open-Ended	843.603	1	843.603	90.059	0.000*
Type of School	1194.497	2	597.249	63.760	0.000*
Gender	4.172	1	4.172	0.445	0.506
Type of School*Gender	5.188	2	2.594	0.277	0.759
Error	1367.610	146	9.367		
Total	29692.000	153			
Corrected Total	5648.052	152			

* $p<0.05$

As seen in Table4.10, prior estimation ability on Open-ended score was statistically significant covariate ($p <0.05$). It was also found that there were statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability ($p<0.05$). In order to determine which schools caused that difference the Bonferroni test was conducted. . There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. On the other hand there was no statistically significant mean difference between those in FLHS and those in HS.

The students in Anatolian High School scored quite better than both the students in High School and Foreign Language High School (M_{AHS} ==18.816, SD_{AHS} =3.638; M_{HS} =7.644, SD_{HS} =3.772; M_{FLHS} ==11.050, SD_{FLHS} =4.636). However the students in Foreign Language High School and the students in High School didn't cause any statistically significant mean difference to each others.

The eighth hypothesis of the second subproblem was "There were no statistically significant mean differences among the students who enrolled to different types of high school with respect to estimation ability on Reference Number Category.” The results were given in Table4.11.

Table 4.11 *Results of ANCOVA of Reference Number Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	134.017	6	22.336	4.184	0.001*
Intercept	493.286	1	493.286	92.408	0.000*
Prior Reference Number	9.819	1	9.819	1.839	0.177
Gender	.342	1	0.342	0.064	0.801
Type of school	99.653	2	49.826	9.334	0.000*
Gender * Type of school	1.144	2	0.572	0.107	0.898
Error	779.368	146	5.338		
Total	20884.000	153			
Corrected Total	913.386	152			

* $p<0.05$

In the ANCOVA analysis; as seen in the Table 4.11 the prior Reference Number was not a statistically significant covariate variable. On the other hand the in order to table there was a statistically significant mean difference in the estimation ability on Reference Number with respect to school type. To investigate which schools caused the difference the Bonferroni Test was conducted. . There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS.

The students in Anatolian High School scored quite better than both the students in High School and Foreign Language High School (M_{HS} =10.222, SD_{HS} =2.494; M_{AHS} =12.489, SD_{AHS} =2.283; M_{FLHS} =11.457, SD_{FLHS} =2.152). The students in Foreign Language High School were also better scored at the estimation ability on Reference Number than the students in High School.

The last (ninth) hypothesis of the second subproblem is "There is no statistically significant mean difference among the students who enrolled to different types of high school with respect to estimation ability on Order of Magnitude Category". The results were given in Table4.12.

Table 4.12 *Results ANCOVA of Order of Magnitude Category Scores*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	669.818	6	111.636	16.694	0.000*
Intercept	555.759	1	555.759	83.106	0.000*
Prior Order of Magnitude	73.463	1	73.463	10.985	0.001*
Type of School	285.416	2	142.708	21.340	0.000*
Gender	56.422	1	56.422	8.437	0.004*
Type of School*Gender	8.256	2	4.128	0.617	0.541
Error	976.352	146	6.687		
Total	15521.000	153			
Corrected Total	1646.170	152			

* $p<0.05$

As seen in the Table 4.12, the prior Order of Magnitude was a statistically significant covariate variable. According to the results, there was a statistically significant difference in estimation ability on Order of Magnitude with respect to school type. To investigate which schools caused the difference Bonferroni Test was conducted. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS.

The students in Anatolian High School scored quite better than both the students in High School and Foreign Language High School (M_{HS} =10.222, SD_{HS} =2.494; M_{AHS} =12.489, SD_{AHS} =2.283; M_{FLHS} =11.457, SD_{FLHS} =2.152). The students in Foreign Language High

School were also better scored at the estimation ability on Order of Magnitude than the students in High School.

4.2.2 The Results of the Analysis of the Estimation Ability Test with respect to Gender

In this section all categories of the Estimation Ability Test's analysis with respect to gender are presented.

The hypothesis H3.1 of the third subproblem is: "There is no statistically significant mean difference between girls and boys with respect to estimation ability".

As seen in Table 4.3 there wasn't statistically significant difference the mean scores of boys and girls with respect to estimation ability ($p > 0.05$). Boys' mean score was higher and girls' mean score (M_{boy}=36.881, SD_{boy}=9.581; M_{girl}=30.478, SD_{girl}=8.964).

4.2.2.1 The Results of the Analysis of Numbers Format of Estimation Ability Test with respect to Gender

In this section Numbers Format of the test that contains Whole Number, Decimal Number, Fractional Number and Percent Number Categories is analyzed with respect to gender.

The first hypothesis of the fourth subproblem is "There is no statistically significant mean difference between girls and boys with respect to estimation ability on Whole number".

As seen in Table4.4, there wasn't statistically significant mean difference between boys and girls with respect to estimation ability on Whole Number ($p > 0.05$). The boys' mean score was higher than the girls' mean score (M_{boy}=9.508, SD_{boy}=3.158) M_{girl}=8.138, SD_{girl}=3.518).

The second hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Fraction”.

As seen in Table4.5, there was statistically significant mean difference between boys and girls with respect to estimation ability on Fraction ($p < 0.05$). The boys’ mean score was moderately higher than the girls’ mean score (M_{boy}=8.644, SD_{boy}=2.958; M_{girl}=6.872, SD_{girl}=2.595).

The third hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Percent”.

As seen in Table 4.6 there wasn’t statistically significant difference the mean scores of boys and girls with respect to estimation ability on Percent ($p > 0.05$). Boys’ mean score was higher and girls’ mean score (M_{boy}=8.898, SD_{boy}=2.814; M_{girl}=7.138, SD_{girl}=2.389).

The fourth hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Decimal”.

As seen in Table4.7, there was statistically significant mean difference between boys and girls with respect to estimation ability on Decimal ($p < 0.05$). The boys’ mean score was moderately higher than the girls’ mean score (M_{boy}=9.830, SD_{boy}=2.436; M_{girl}=8.329, SD_{girl}=2.725).

4.2.2.2 The Results of the Analysis of Problem Format of Estimation Ability Test with respect to Gender

In this section of the present study the Numbers Only and Application categories are analyzed with respect to gender groups.

The fifth hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Numbers Only Category”.

As seen in Table 4.8 there wasn’t statistically significant difference the mean scores of boys and girls with respect to estimation ability on Numbers Only ($p > 0.05$). Boys’ mean score was higher and girls’ mean score (M_{boy}=19.169, SD_{boy}=4.835; M_{girl}=16.595, SD_{girl}=5.036).

The sixth hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Application Category”.

As seen in Table 4.9 there wasn’t statistically significant difference the mean scores of boys and girls with respect to estimation ability on Application ($p > 0.05$). Boys’ mean score was higher and girls’ mean score (M_{boy}=17.711, SD_{boy}=5.690; M_{girl}=13.883 SD_{girl}=4.462).

4.2.2.3 The Results of the Analysis of Answer Format of Estimation Ability Test with respect to Gender

In this section of the present study the analysis the Answer Format’s categories that Open-ended Categories, Reference Number Categories and Order of Magnitude Category are analyzed with respect to gender.

The seventh hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Open-ended Category”.

As seen in Table 4.10 there wasn’t statistically significant difference the mean scores of boys and girls with respect to estimation ability on Open-ended ($p > 0.05$). Boys’ mean score was higher and girls’ mean score (M_{boy}=15.016, SD_{boy}=5.811; M_{girl}=10.978, SD_{girl}=5.771)

The eighth hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Reference Number Category”.

As seen in Table 4.11 there wasn’t statistically significant difference the mean scores of boys and girls with respect to estimation ability on Reference Number ($p > 0.05$). Boys’ mean score was higher and girls’ mean score (M_{boy}=15.016, SD_{boy}=5.811; M_{girl}=10.978, SD_{girl}=5.771).

The last (ninth) hypothesis of the fourth subproblem is “There is no statistically significant mean difference between girls and boys with respect to estimation ability on Order of Magnitude Category”.

As seen in Table4.12, there was statistically significant mean difference between boys and girls with respect to estimation ability on Order of Magnitude ($p < 0.05$). The boys’ mean score was moderately higher than the girls’ mean score (M_{boy}==10.728, SD_{boy} =3.473; M_{girl}=8.8, SD_{girl} =2.9).

We tested the hypotheses related to interaction between school types and gender with respect to all categories. There were no statistically mean differences all categories of the estimation test except numbers only category.

4.3 Conclusion

The conclusions of the present study can be stated in a summary. There were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. In addition there was statistically significant mean difference between those in FLHS and those in HS in the favor of FLHS with respect to estimation ability, whole number, percent, decimal, application, reference number and order of magnitude categories.

With respect to fraction, numbers only and open-ended categories, there were statistically significant mean differences between students in AHS and those in FLHS in the favor of AHS. There was also statistically significant difference between students in AHS and those in HS in the favor of AHS. On the other hand there was no statistically significant mean difference between those in FLHS and those in HS.

There wasn't statistically significant difference the mean scores of boys and girls with respect to estimation ability, whole number, percent, numbers only, application, open-ended., reference number categories. Additionally there was statistically significant mean difference between boys and girls with respect to estimation ability on fraction, decimal, and order of magnitude categories.

CHAPTER V

DISCUSSION AND RECOMMENDATIONS

In this chapter the results were discussed and some recommendations were given.

5.1 Discussion

The purpose of the study was to explore the computational estimation ability of the ninth grade students on the three formats of the Estimation Ability Test. This was achieved by analyses of the all formats and categories of the test. In the followings findings were discussed.

When the test was totally concerned the subjects performed rather poorly on the test (M_{total}= 32.9, SD_{total}=9.7). Maximum score that has been taken was 56 out of 71. It is consisted with the previous researches in with students generally performed poorly on computational estimation. The ninth grade students' performance on estimation tasks differed within the school types and gender differences. Additionally in the defined format groups the students served relatively different performance.

In Number format of the test the categories; Whole, Fraction, Decimal and Percent, it was founded that there were statistically significant differences among the mean score of the students who enrolled the different kinds of the schools. Thus, it can be stated that students at different high schools had different estimation ability. Anatolian High School students were more successful than the other two types of schools. This results might occur because the different mathematics and Turkish achievement of the students in different schools.

According to the data analysis the types of the schools are highly correlated with the students' mathematics and Turkish achievement. Because of this, the high mathematics and Turkish achievement could implied that could easily apply mathematics to their daily life and to improve easily own ways to answer for the practical estimation questions. The higher Turkish achievement might be effect the students estimation ability while the understanding of the questions on the overhead projector in a short time. The mathematics achievement seemed to be related to estimation ability according to literature that the results of the studies which differ in age-groups and settings served same results (Hanson & Hogan, 2000; Sowder, 1992). Through these results Foreign Language High School students' might have be higher estimation abilities than High School students' estimation ability. The results of this study proved these expectations. The Foreign Language high school students were more successful than the high school students with respect to estimation on numbers format. Like Levine (1982) Hanson and Hogan (2000) concluded that in their analyses the estimation ability was correlated with the SAT mathematics achievement but not significant correlation between the verbal score and the estimation ability score. In the problem format's two categories which were the numbers only and application, it can be seen the school type's differences were also occurred. In both categories Anatolian High School students performed better than Foreign Language high school students. This result might be considered again in mathematics achievement aspects because of the correlation between type of school and mathematics achievements of the students. In order to this, higher mathematics achievement could imply (cause) higher estimation ability. In addition, in the literature, Bestgen, Reys, Rybolt and Wyatt (1980) found that the students' self-perception of success in mathematics was highly related to success in estimation. Additionally, Levine (1982) concluded that college students' score on test of quantitative ability was positively correlated with score on a test of estimation skills. Similarly, in this study mathematics achievement or the school type of the students were found so related with the estimation score.

In the answer format of the test was concluded the same results like in the number format. The students in Anatolian high school were better than the other two types of the school types. The reasons mentioned in the number format might be similar for the other two formats. In other words, the mathematics and Turkish achievements of the students were moderately higher than the students in foreign language high school and high school.

According to the results of the present study the decimal and fractional categories were very difficult for the participants its reason could be that decimal was format of the interpretations of a fraction like stated by Gay (1997). The result of low ability on fraction and decimal numbers was supported by Goodman (1991), Hanson and Hogan (2000), Boz and Bulut (2002). They agreed that fraction was the most difficult concept of the entire numbers format. Although the subjects in the studies conducted by Goodman (1991), Hanson and Hogan (2000), and Boz and Bulut (2002) were all the higher age groups than the present study's subjects, the same results were gathered from the present study' participants. Fraction was the most difficult number category. Rubenstein (1986) similarly found that eight-grade students had more difficulties with decimals than with whole numbers. When the students who answer items on fraction typically tried to find a common denominator, they realized that they could not answer them.

When we take into consideration the results related to problem format's categories which were numbers only and application, there were statistically significant mean differences of the students with respect to two categories. It was found that the students were more successful in application category than in numbers only category. According to Gliner (1991), the number of correct answers in the application format was statistically significantly greater than the number of subjects answering problems correctly in the computation format. Similarly, Goodman (1991) stated the numbers only category was more difficult than the application category. In contradiction to Rubenstein' (1986) findings it was found that students performance in the application category are better than the performance on numbers only category of problem format. These findings also were consistent with findings of Goodman (1991), Bestgen et al. (1980) and .Reys, Reys and Penafiel (1991). The higher performance of the students on application might be some reasons. In numbers only category students tended to try to mentally compute, not estimate, and this may have contributed to more error. From the appearance of questions in the numbers only category, students could think these questions required mental computation. Because of this, the students might make more mistakes and got low score from these questions. In the literature students were better in numbers only questions rather than application question which did not require estimation. On the other hand, in the estimation ability test students were more successful in the application questions than in numbers only category questions. Additionally, estimation was appeared to be a natural process when problems were presented in an applied format. Even though students were asked to estimate rather than mentally compute answers, they appeared to have often chosen to try to make mental computation when problems were presented in the format of arithmetic computation problems. They were not as successful as those subjects who worked the problems when they were presented in an applied format. These results were contradictory to the usual success rates for exact answers using paper and pencil (Gliner, 1991). Applied problems in paper and pencil situation usually produced lower success rates than those for computational arithmetic problems.

The other important result of the findings of the present study was occurred in answer format. Among the three categories of the answer format which were open-ended, reference number and order of magnitude, surprisingly the mean score of the students were better mostly on open-ended category. In the study students scored high open-ended, reference number and order of magnitude respectively. This result was interesting since Rubenstein (1986) and Gatzke (1986) were stated different conclusions. While Rubenstein (1986) stated that open-ended type of estimation was the most difficult one in order to multiple choice and reference number estimation, Gatzke (1989) found that open-ended estimation which offered no reference point was definitely the most difficult type of estimation task with success rates sharply declining as the number of questions in a test increased. However, Result of the present study was contradict with the findings of Goodman (1991), Rubenstein (1986) and Gatzke (1989) who agreed that students had more difficulties with open-ended tasks but order of magnitude tasks were the easiest one. This could be explained by findings of researcher's informal conversation with a few students. At the end researcher concluded that the order of magnitude items were not understood very well by the responders. The students wanted to sign the choice which was the exact or more close the exact answer however in the choices there were orders of magnitudes of the answers like in the example:

The population of a city is 19 700. This is a 9 per cent increase from last year. What was the population last year?

(a) 18 000

(b) 1800

(c) 180

(d) None of these

From these results, it can be concluded that the students were not still familiar to the estimation and the different kinds of estimation questions. Because of this, in the study the reference number category of the answer format did not work as it was expected in the literature.

On the other hand, why the reference number category still less scored than open-ended category has not been answered yet. One reasonable explanation of these unexpected finding was that the students try to compute the problems for obtaining the exact results. Because of this, the open-ended category is the most successfully one. Open-ended responses have been shown to provide a more valid measure of computational estimation (Reys et al., 1980) because they provided no clues about the answer and because their nature guaranteed that a range of estimates would be produced. Although in the reference number category's questions there were two options (yes-no) significantly lots of students did not try to estimate, they only made up an answer and passed the other question. Since the researcher said that at the beginning of the test, time was very short and they had 15-20 seconds to answer each questions, everyone tried to estimate as possible as in a short time. Time was restricted because prior research studies had demonstrated that valid measures of computational estimation in a group setting were very difficult to obtain without controlling the time (Reys et al., 1991; Reys, Reys and Penafiel, 1991)**.** The subjects had not been taken an estimation test on the overhead projector like that before. Unfortunately they were not used to this procedure. Some students seemed confused, and others did not seem to understand what they were expected to do when asked to estimate. No students reported having taken an estimation test previously. Because of these reasons Answer Format's categories scores contradicted with the literature findings. The results of the researcher observation during the testing period students were not using estimation skills. Even they were trained in four weeks, most of the students resisted not to use the estimation strategies. Similarly, Sowder and Schappelle (1989) found that as students progressed in school they seemed to resist rounding and then computing, but when asked to estimate an answer they preferred to compute mentally and then round their answer. These findings were also pointed out in the present study. Especially some high mathematics achievement students resisted to estimate the answer, since they could compute mentally and gave an exact answer in a short time. When these students were warned they continued to these behaviors except a difference, they rounded the answer after finding the exact solution. These findings were consistent with findings of Reys et al. (1991) and Reys, Reys and Penafiel (1991), Hanson and Hogan's (2000). Reys et al. (1991) and Reys, Reys and Penafiel (1991) stated that many students in both Japan and Mexico were more comfortable solving computational problems exactly than estimating solutions. These researchers also concluded that students resisted giving estimates because they either did not understand the meaning of estimation or were reluctant accept error. In Hanson and Hogan's (2000) study students who were reluctant to estimate because they appeared confused about why anyone would want an estimate instead of an exact answer and simply wanted to show that they knew how to solve the problem exactly. Like these results, in the present study especially hard working and with high mathematics ability students didn't want to estimate the answer. They also wanted to compute the exact answers.

The reasons why students have so much trouble with estimation might be that mathematics is commonly associated with certainty and being able to get the right answer quickly. The other reason might be the students' poor number sense, inability to comprehend large-number quantities, undeveloped computational estimation or mental computation skills. Teachers tell students whether their answers are right or wrong, but rarely do they encourage students to explore the assumptions which led them to their answers. As a result children learn that there is only one correct answer and become afraid to offer alternative ones. Good estimators are comfortable with inexact answers and look for different ways to solve problems. According to Sowder (1992) current practice indicates that the majority of mathematics instructional time in elementary school, with estimates ranging up to 80 percent, is spent introducing, developing, practicing and establishing proficiency with written algorithms. This conclusion was also reached by the researcher of this study. Students were more depending on paper-pencil computation in the classes. Reys and Reys (1998) also suggested that teachers should give less importance to the written computation algorithm; besides this they must help the children for improving the mental computation and computational estimation. Moreover, the results of the present study showed that a short period of treatment was improved the ability on estimation. Estimation was crucial to becoming a good problem solver and experience and practice were critical to becoming a good estimator.

According the results of the present study although there was a significant difference between boys and girls in the favor of boys Reys, Reys and Penafiel (1991) studied the performance on estimation ability, there was no statistically significant difference in performance between boys and girls. Moreover, Reys et al. (1991) found in the fifth grade students' gender difference occurred in the favor of boys but in eight grade students gender groups there was not a difference. The results in the literature about gender differences on computational estimation skills had been mixed. In addition, the number format of the present study according to the whole number and percent categories there were no statistically significant mean differences in terms of gender. However, the other categories fraction and decimal there statistically significant mean differences in the favor of boys. Furthermore, in the numbers only category there were a gender differences in the favor of boys, on the other hand in application format there were no statistically significant mean differences with respect to gender. The differences were occurred always in favor of boys. Although estimation ability with respect to gender was not a significant factor as in the literature, in the numbers only category of the present study most students tended to perform exact computations. Because of this, boys performance appeared quite high than girls. Since as in the literature boys' computational ability is moderately higher than girls. Similarly the findings in the present study, Reys and Yang (1998) found no gender difference in performance on measure of number sense or computational skills, but on the other hand Rubenstein (1985) found that boys performed slightly better than girls on written computational estimation tasks, especially when problems required the selection of the appropriate order of magnitude (Munakata, 2002). According to Munakata (2002), the gender difference also was underlined in the favor of boys. Since boys excel on problems requiring the application of mathematics to real-life situations, whereas girls perform relatively better on textbook problems (Munakata, 2002). On the other hand, Mottram (1995) stated that no gender-related differences were found for estimation ability or for perceptions of mathematics ability. In his study males rated themselves significantly higher than females in ability to estimate. Additionally, Hanson and Hogan (2000) found that with higher mathematics achievement students had higher level of ability on computational estimation.

5.2. Recommendations

In this section recommendations stated for teachers, students, curriculum makers, teachers' educators and researchers in order to understand what estimation is and why estimation is important. Currently, computational estimation is taught very briefly to elementary school students in our country. It is usually presented only as front-end rounding. This limited exposure to computational estimation deprives students of the opportunity to learn methods which they consider "an easy way to do math." Computational estimation is considered by students to be both an important topic in mathematics and an easy way to solve problems. Students have a preference for the strategy used to solve computational estimation problems based, in part, on their own ability to rapidly add, subtract, multiply, and divide numbers. Some students found front-end rounding to be the easiest strategy to use while other students showed a preference for either reformulation or translation. Students should be taught all of the strategies and be given repeated opportunities to practice them. More time should be spent on teaching the concept of compensation in order to facilitate mastery of this concept. Teachers need to make aware of the importance attached to the area of computational estimation both by experts in the field of mathematics and by students themselves. Teachers should teach all four computational estimation strategies and provide sufficient time and opportunity for students to master each of the four strategies.

We can identify what the different groups can do about the estimation followings:

Teachers should:

- give emphasis on the number sense-mental computation-estimation in lessons;
- deal with weak students since in a short time these estimation abilities can be improved;
- warm up to the mathematics by help of implications of estimation in daily life especially in primary and middle school stage;
- introduce estimation with examples where rounded or estimated numbers are used;
- emphasize on and use real-world examples where only estimates are required;
- not require too much precision when they estimate, but they should utilize the language of estimation;
- should help students develop a respect for approximate answer and recognize that in computational estimation there is not just one right answer;
- teach mathematical estimation to students who may not have been successful with paper-pencil mathematical task may find that they are successful at estimating answers because this success might give them more confidence to succeed in mathematics; and by presenting the process of estimation in everyday applied settings prior to or concurrent with instruction on computational methods, students' chance of success may be increased.

Students should:

- talk together to discuss their answers to realize that a variety of answers are possible.
- improve their abilities whether school type methods or in their own methods.
- have more awareness while daily life computations whether mental one or estimation one.
- control the results of computer or calculators since their computational power

- be able to carry out rapid approximate calculations by first rounding off numbers.
- acquire some simple techniques for estimating quantity, length, weight, etc.
- decide when a particular result it precise enough for the purpose at hand.

Curriculum developers should :

- syntheses the estimation concepts in every units of mathematics in every level of the education.
- make people include estimation in the textbooks by providing students with opportunities to make mental calculations, develop hypotheses, reason mathematically and draw conclusion.
- design curriculum that reflect a balanced approach to type of computation- exact or approximate- and the methods used to compute –mental, written or calculators- multiple tools are all diverse in instruction and curriculum.
- give no allowance to teach estimation as a separate unit; rather that it is integrated systematically to provide regular estimation experiences prior to instructions on written algorithms. Because of this, estimation should become an integral part of the mathematics curriculum.
- Computation, estimation or methods for solving proportions should not be considered or taught as ends in themselves. In grade 5-8, computation and estimation should be integrated with the study of the concepts underlying fractions, decimals, integers and rational numbers as well as with the counting study of whole number concept.

Teachers' educators should:

- make the preservice teachers more awareness about the estimation and estimation abilities.
- teach the preservice teachers how these abilities can be taught.

Researchers should:

- give more attention on the number sense and related abilities to make clear the basic rules of the mathematics
- conduct more researchers on the numerosity, measurement estimation and computational estimation.
- conduct computational estimation on different levels to identified the differences and identified the teaching and learning tips
- conduct also qualitative researchers to identify the strategies that the students, preservice teachers and adults used.
- give special emphasis on the task of searching the strategies of estimation and additional time spent on teaching methods.
- introduce the estimation types by doing different researches to the mathematics curriculum.
- continue to investigate estimation ability in students, including computational mental estimation, a critical element of mathematical problem solving.

The sooner it is exposed the students to estimation and related skills, the more they will value estimation and understand when and how to apply it effectively.

REFERENCES

Baroody, A.J., & Gatzke, M.R. (1991). *The Estimation of Set Size by Potentially Gifted Kindergarten-age Children.*, Journal for Research in Mathematics Education, 22, 59-68.

Berry R. Q., (1998), *Computational Estimation Skills of Eighth Grade Students*, Unpublished doctoral dissertations, Newport University

Bestgen, B.J., Reys, R.E., Rybolt, J.F., & Wyatt, J.W. (1980), *Effectiveness of Systematic Instruction on Attitudes and Computational Estimation Skills of Preservice Elementary Teachers*, Journal for Research in Mathematics Education, march 1980, 124-136

Bobis, J. (1991), *The Effect of Instruction on the Development of Computational Estimation Strategies,* Mathematics Education Research Journal, Vol.3, No.1, 1991

Boz, B., Bulut, B. (2002) *İlköğretim Matematik, Fen Bilgisi ve Okul Öncesi Öğretmen Adaylarının Tahmin Becerilerinin İncelenmesi,* 5. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (ODTÜ Kültür Kongre Merkezi, Ankara)

Brame, O. H. (1986), *Computational Estimation Strategies Used by High School Students of Limited Computational Estimation Ability*, DAI-A 47/07, p. 1228

Bright, George W.(1985) *"Estimation as part of learning to measure" In Measurement in School Mathematics*, 1976 Yearbook of the National Council of Teachers of Mathematics, edited by Doyal Nelson, pp.87-104. Reston. Va.: The Council, 1976

Carpenter, T.P., Coburn, T.G., Reys, R.E., & Wilson, J.W. (1976) In Sowder, J. (1992), *Estimation and Number Sense*. In D.A. Grouws (Ed.), Handbook of research in mathematics teaching and learning (pp.371-389). New York : Macmillan

Chien, Y. C., (1990), *The Use of Self-study Material for Improving the Computational Estimation Skills of Preservice Teachers*, Edd Columbia University Teachers College 1990, DAI-A 51/03, p774.

Crites, T., (1992), *Skilled and Less-Skilled Estimators' Strategies for Estimating Discrete Quantities*, The Elementary School Journal, 92, 601-615.

Damarin, S.K., Dziak N.J., Stull, L., Whiteman, F., (1988), *Computer Instruction in Estimation: Improvement in High School Mathematics Students*, School Science and Mathematics, volume 88(6) October 1988

Dowker, A. (1992), *Computational Estimation Strategies of Professional Mathematicians*, Journal for Research in Mathematics Education, 23, pp. 45-55.

Fraenkel, J.R., Wallen, N.E; (1996), *How to Design and Evaluate Reseach in Education, Third Edition,* McGraw-Hill, INC. pp.268-269

Forrester, M. A., & Pike, C. D. (1998), *Learning to Estimate in the Mathematics Classroom: A conversation-Analytic Approach,*. Journal for Research in Mathematics Education, 29, 334-356.

Gatzke, M.R. (1989), *Kindergarten Children's Estimates of Numerosity*, DA-A 51/02, p.406

Gay, A.S. (1997), *A study of Middle School Student's Understanding of Number Sense Related to Percent*, Digital dissertations

Glasgow, B. (1998), *The Authority of the Calculator in the Minds of College Students*, School Science and Mathematics, Nov98, Vol.98, Issue 7, p 383, 6 charts, 1 diagram Item Number 1326713

Gliner, G.S. (1991), *Factors Contributing to Success in Mathematical Estimation in Preservice Teachers: Types of Problems and Previous Mathematical Experience*, Educational Studies in Mathematics, 22, 595-606

Goodman, T. (1991), *Computational Estimation Skills of Preservice elementary Teachers*, Int. J. Math. Educ. Sci. Technol., vol.22, no.2, 259-272

Gossard, P. N.(1986), *Computational Estimation in Applied Nonroutine Problem Solving (reformulation, translation, compensation, middle grade student*s), DAI-A46/09, p. 2606, Greeno,J.(1991), *Number sense as a situated knowing in a computational domain*, Journal for Research in Mathematical Education, 22(3), 170-218

Hall, L.T., Jr., (1984), *Estimation and Approximation- not synonyms*. In Sowder, J. (1992), Estimation and Number Sense. In D.A. Grouws (Ed.), Handbook of research in mathematics teaching and learning (pp.371-389). New York: Macmillan

Hatano, G. (1988), *Social and motivational bases for Mathematical Understanding*. In Sowder, J. (1992), Estimation and Number Sense. In D.A. Grouws (Ed.), Handbook of research in mathematics teaching and learning (pp.371-389). New York : Macmillan

Hanson, S.A., Hogan P.T., (2000), *Computational Estimation Skill of College Students*, Journal for Research in Mathematics Education, vol.31, Issue4, p483, 17p, 12 charts 1diagram.

Heinrich, E.J. (1998), *Characteristics and skills exhibited by middle school students in performing the tasks of computational estimation*, Unpublished doctoral dissertations, Fordham University, New York.

Hope, J.A. (1986), *Estimation and mental computation-Developing estimation skills in the primary grades*. In H. L. Schoen & M. J. Zweng (Eds.), Estimation and mental computation: 1986 yearbook (pp. 45-54). Reston, VA: National Council of Teachers of Mathematics.

LeFevre A., Greenham S. L. and Waheed N. (1993), *The Development of Procedural and Conceptual Knowledge in Computational estimation*, Cognition and Instruction, 1993, 11(2), 95-132

Levine,D.J., 1982,*Strategy, Use, and Estimation Ability of College Students*, Journal for Research in Mathematics Education 13, 350-359

National Council of Teachers of Mathematics (NCTM), *Curriculum and Evaluation Standards for School Mathematics* Reston, Va NCTM 1989

Maier, E. (1977), *Folk Math.* In Hope, J.A.,(1986) Estimation and mental computation, Developing estimation skills in the primary grades. In H. L. Schoen & M. J. Zweng

(Eds.), Estimation and mental computation: 1986 yearbook (pp. 45-54). Reston, VA: National Council of Teachers of Mathematics.

McIntosh, A., Reys, B.J., & Reys, R.E., (1992), *A proposed frame work for examining basic number sense*, For Learning of Mathematics, 12, 2-8

Micklo, S. J. (1999), *Estimation; Its More Than a Guess* ,Childhood Education, vol.5, no:3, 142-145

Milli Eğitim İlköğretim Okulu Ders Programları, *Matematik Progaramı 6-7 -8. Sınıflar*, Milli Eğitim Basımevi-İstanbul 2000

Montague, M., Van Garderen, D. (2003), *A Cross-Sectional Study of Mathematics Achievement, Estimation Skills, and Academic Self-Perception in Students of Varying Ability*; Journal of Learning Disabilities, Sep/Oct2003, vol.36, issue 5, p437, 12p

Mottram, R.D. (1995), *A Comparative Study of Computational Estimation Ability and Strategies Used in Estimation Problems*, DAI-A 57/02, p. 614

Munakata, M. (2002), *Relationships Among Estimation Ability, Attitudes Toward Estimation, Category Width And Gender in Students of Grades 5-11*, Unpublished doctoral dissertations, Columbia University

Reys, R.E. (1984), *Mental Computation and Estimation: Past, present and future*, The Elementary School Journal, 84(5), 547-557.

Reys B.J., (1986), Teaching *Computational Estimation: Concepts and Strategies*. In H. L. Schoen & M. J. Zweng (Eds.), Estimation and mental computation: 1986 yearbook (pp. 31-45). Reston, VA: National Council of Teachers of Mathematics.

Reys, R.E., & Bestgen, B.J., (1981), *Teaching and Assessing Computational Estimation Skills*. In Gliner, G.S. (1991), Factors Contributing to Success in Mathematical Estimation in Preservice Teachers: Types of Problems and Previous Mathematical Experience, Educational Studies in Mathematics, 22, 595-606

Reys, B. J., Reys, R. E. (1998); *Computation in the Elementary Curriculum: Shifting Emphasis.; Teaching Children Mathematics*; Vol.5 Issue 4, p. 236, 6p, 1 diagram

Reys,B.J., Reys,R.E., Penafiel,A.F. (1991),. *Estimation Performance and Strategy Use of Mexican 5th and 8th Grade Student Sample*, Educational Studies in Mathematics, 22,353-375

Reys,R.E., Reys, B.J., Nohda, N., Ishida, J., Yoshikawa, S., Shimizu, K. (1991), *Computational Estimation Performance and Strategies Used by 5th and 8th Grade Japanese Students*, Journal For Research in Mathematics Education, 22, 39-58

Reys, R.E. and Yang, D. (1998), *Relationships between the computational performance and number sense among sixth and eighth grades in Taiwan*, Journal for Research in Mathematics Education, 29(2), 225-237

Reys, R.E., Rybolt, J.F., Bestgen, B.J., & Wyatt, J.W. (1982), *Process Used by Good Computational Estimators*, Journal for Research in Mathematics Education, 13, 183-201

Rubentein, Rheta N., (1986), *Varieties of Estimation*, In H. L. Schoen & M. J. Zweng (Eds.), Estimation and mental computation: 1986 yearbook (pp. 16-30). Reston, VA: National Council of Teachers of Mathematics.

Siegel, A. W., Goldsmith, L. T., & Madson, C. R. (1982), *Skill in Estimation Problems of Extent and Numerosit,*. Journal for Research in Mathematics Education, 13, 211232.

Smart, J.R., (1982), *Estimation Skills in Mathematics*, School Science and Mathematics, vol.82 (8).

Smith, M. L., (1993), *Preservice Elementary Teachers' Conceptual Understanding of Computational Estimation Strategies within the Operations of Addition and Subtraction*. DAI-A 54/06, p.2084

Sowder, J. (1992), *Estimation and Number Sense*, In D.A. Grouws (Ed.), Handbook of research in mathematics teaching and learning (pp.371-389). New York : Macmillan

Sowder, T.J.(1984), *Computational Estimation Procedures of School Children*, Journal of Educational Research, July/August, 77, pp. 42-53.

Sowder.T. J & Schappelle B.P., (Eds.) (1989), *Establishing Foundation for Research on Number Sense and Related Topic: Report of a Conference*. In Sowder, J. (1992), Estimation and Number Sense. In D.A. Grouws (Ed.), Handbook of research in mathematics teaching and learning (pp.371-389). New York: Macmillan

Sowder J.T. (2001), *Computational Estimation Procedure of School Childr*en, Journal of Educational Research, July/August 1984, vol77 no.6

Suydam, M.N. (1985), *Recent Research on Mathematics Instruction*, eric/smeac mathematics education digest no: 2

Taylor, P.M., Simms, K., Kim, O., Reys, R.E., (2001), *Do your Students Measure Up?* [Online] (science.msfc nasa gov/new/home/headlines/ast30sep99_2 htm)

Thompson, A.G. (1979), *Estimating and Approximating*. In Sowder, J. (1992), Estimation and Number Sense. In D.A. Grouws (Ed.), Handbook of research in mathematics teaching and learning (pp.371-389). New York: Macmillan

Trafton, P. (1986), *Teaching computational estimation: Establishing an estimation mindset*. In H. L. Schoen & M. J. Zweng (Eds.), Estimation and mental computation: 1986 yearbook (pp. 16-30). Reston, VA: National Council of Teachers of Mathematics.

Yazgan, Y., Bintaş, J., Altun, M. (2002), *İlköğretim 5. Sınıf Öğrencilerinin Zihinden Hesap ve Tahmin Becerilerinin Geliştirilmesi*, 5. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (ODTÜ Kültür Kongre Merkezi, Ankara)

Whiteman, F. C. (1989), *The Role of Computer-Based Instruction in the Development of Strategies for Computational Estimation with Middle School Children*, DAI-A 49/09, p.2629

APPENDIX A

LESSON PLANS

Ders Planı- 1

Amaç: Tahmin konusunun tanıtılması ve genel stratejilere giriş

Süre: 45 dakika

İşleniş:

- Öğrencilere günlük hayatta zihinden hesaplamalar yaptıkları yerlere dair sorular sorulacak.
- Zihinden hesaplama anında en çok kullanılan işlemler olan dört işlemin önemi vurgulanacak.
- Zihinden hesaplamalar yaparken net cevap yerine yaklaşık cevapların da geçerli olduğu durumlar tartışılacak.
- Yaklaşık cevaplar gerektirecek sorular öğrencilere yöneltilecek. Örneğin; "Sınıftaki karatahtanın ağırlını tahmin ediniz., Kaç tane saç telimiz olabilir?,İçinde bulunduğumuz odanın yüksekliği kaç metre olabilir?, Marketten alacağınız 1 216 733TL ve 3 425 879TL olan iki ürüne cebinizdeki 5 000 000TL yeterli olacak mıdır?, Okul bahçesinde bulunan herhangi bir çam ağacının iğne yaprak sayısı kaç olabilir?, v.b"
- Üç farklı tahmin konusunu; yığın tahmini yapmak (numerosity), uzunluk, ağırlık gibi ölçmeye dayalı tahminler yapmak (measurement), ve hesaba dayalı , hesap yaparak tahminler yapmak(computational estimation) yüzeysel olarak anlatılacak.
- Hesaplamalı tahmin konusuna değinerek detaylara girilecek. Tahtaya hesaplamalı tahmin stratejilerinin şeması çizilip hepsi hakkında kısa açıklamalar yapılacak. "Sayıların yeniden düzenlenmesi (reformulation), işlemlerin yeniden düzenlenmesi (translation) ve düzenleme-düzeltme (compensation)".

- Stratejilere detaylı girmeden önce ilköğretimde öğrenilen ondalıklı sayıları yuvarlamaya dair hatırlatmalar yapılacak.
- Sayıların yeniden yapılandırılması stratejisinin iki alt metotları; ileri ve geri yuvarlama (Rounding & Truncation) hakkında bilgi verilip ve örnekler yapılacak.

Alıştırmalar:

- Fotokopi halinde çoğaltılmış olan Alıştırmalar-1 yaprak-uygulama kağıdı öğrencilere dağıtılacak.
- İlk soru örnek olması için öğretmen tarafından yapılacak sonrası öğrencilerle yüksek sesle yapılacak.
- Farklı cevaplarla geldiğinde bunların her ikisinin de doğru olduğu, yani tahmin ederek işlem yapıldığında hiçbir zaman net ve tek bir çözüm olmayacağının altı çizilecek.
- Öğretmen tarafından abartılı cevaplar söylenerek öğrencilerin tepkileri ölçülecek.
- Öğrencilerin cevapları incelenirken ileri-geri yuvarlamalardan hangisini kullandığını ve neden ileri, neden geri yuvarlama yaptığı tartışılacak.

Ödev: Evde konu tekrarı yapmaları ve pratiklik kazanmaları için ÖDEV-1 yaprak uygulama kağıdı öğrencilere dağıtılacak.

ALIŞTIRMALAR-1

1. Her bir sayıyı en yakın onlar basamağına yuvarlayınız.

47 53 76 84 27 91

2. Her bir sayıyı en yakın yüzler basamağına (ileri) yuvarlayınız.

258 342 571 839 447 763

3. Her bir sayıyı en yakın binler basamağına (ileri) yuvarlayınız.

3456 2567 4278 5892 6136 7498

4. Geri yuvarlamayı kullanarak sayıları en yakın yüzler basamağına yuvarlayınız.

765 833 956 635 793 619

5. Geri yuvarlamayı kullanarak sayıları en yakın binler basamağına yuvarlayınız.

2675 4383 5960 6035 7309 9569

6. Her bir sayı grubunu en yakın onlar basamağına ileri yuvarlayarak işlemleri yapınız.

76+29=? 43-22=? 89+76=? 64-38=? 58+93=?

7. Her bir sayı grubunu en yakın yüzler basamağına yuvarlayarak işlemleri yapınız.

345+489=? 571-323=? 657+338=?

722-567=?

489+817=? 3567+4376=? 4892-2459=? 7653+3738=?

5257-2943=? 7498-3841=? 8198+1926=?

8. Geri yuvarlamayı kullanarak her bir işlemi yapınız.

492-286=? 527+653=? 753-428=? 639+391=?

861-549=? 5376+7364=? 8429-4592=?

6537+7383=?

7525+9234=? 9847-8314=? 1988+6291=?

ÖDEV-1

1. Aşağıdaki sayı gruplarını ileri ve geri yuvarlayarak cevabı tahmin ediniz.

a) 4,75+5,29+7,36=? 678+334+458=?

346+527+769=? 9,26+6,38+4,67=?

12,58+14,79+16,27=? 653+148+320=?

b) 15,76-12,48=? 768-433=? 967-536=?

9,38-6,67=? 27,16-14,58=? 418-257=?

2. Aşağıdaki soruyu ileri ve geri yuvarlama kullanarak çözünüz?

“Eğer 20 kg şekerden aşağıdaki miktarlarda kullanırsam geriye kaçar kg şeker kalır?”

7,89 kg 6,38 kg 3,79 kg 5,47 kg 8,92 kg 4,18 kg

Ders Planı- 2

Amaç: Sayıların yeniden düzenlenmesi stratejisinin pekiştirilmesi

Süre: 45 dakika

İşleniş:

- Bir önceki derste yapılanlar kısa bir girişle tekrarlanacak.
- Ödev olarak verilenler üzerinden hızlıca geçilerek kontrol edilecek.
- Öğrencilere yeni öğrendikleri stratejinin gerekçelerini anlatabilmek için neden sayıları yeniden düzenlemek gerektiğini ve yararları sorulacak.
- Sayılarla daha kolay baş edebilmek ve onları yapıyor olunan işlem içinde uygun bir şekilde kullanabilmek için bu stratejinin uygulandığını anlamalarını sağlamak.

Alıştırmalar:

- Fotokopi halinde çoğaltılmış olan Alıştırmalar-2 yaprak-uygulama kağıdı öğrencilere dağıtılacak.
- İlk soru örnek olması için öğretmen tarafından yapılacak sonrası öğrencilerle yüksek sesle yapılacak.

Ödev: Evde konu tekrarı yapmaları ve pratiklik kazanmaları için ÖDEV-2 yaprak uygulama kağıdı öğrencilere dağıtılacak.

ALIŞTIRMALAR-2

Sayıları yeniden yapılandırarak işlemleri yapınız.

1. 47x8=? 69x4=? 73x22=? 93x12=?
2. 51x19=? 28x41=? 85x17=? 34x29=?
3. 357x21=? 522x14=? 973x17=? 139x16=?
4. 76÷4=? 57÷5=? 98÷9=? 42÷5=?
5. 378÷8=? 823÷9=? 1247÷4=? 1896÷5=?
6. 2612÷193=? 3158÷984=? 5532÷1096=? 7347÷2368=?

ÖDEV-2

Sayıları yeniden yapılandırarak işlemleri yapınız.

1. 129x8=? 347x19=? 721x9=? 458x17=?

2. 327x9=? 764x12=? 1236x23=? 1476x33=?

3. 2927x89=? 3651x13=? 1456x24=? 1731x28=?

4. 391÷9=? 768÷7=? 246÷5=? 841÷16=?

5. 468÷8=? 927÷18=? 276÷67=? 639÷15=?

6. 4851÷18=? 2211÷143=? 3723÷19=? 6362÷77=?

Ders Planı- 3

Amaç: İşlemlerin yeniden düzenlenmesiı stratejisinin tanıtımı

Süre: 45 dakika

İşleniş:

- Bir önceki ders konusu tekrar edilecek ve hesaplamalı tahmin yaparken sadece sayıların değil işlemlerin de üzerinde değişiklik yapılabildiğine dikkat çekilecek.
- Verilen işlemi daha kolay sonuca gidecek şekilde yeniden şekillendirmeye dayalı olan işlemlerin yeniden düzenlenmesi stratejisi tanıtılacak.
- Bu stratejiye dair birkaç örnek verilecek; örneğin, 227+179=? sorusunu 200x2=400 şeklinde toplama işlemini çarpma işlemine dönüştürerek cevaplanabileceği gösterilecek.
- Bu strateji aynı zamanda referans (benchmark) stratejisini de kullanmaya uygundur. Referans stratejisi ile 1/3 yerine %30 ya da ¼ yerine %25 kullanılabilmektedir. Bu da öğrencilere sayıyı sırasıyla 3 ya da 4 e bölebilme şansı tanır. Bu değişikliği yapabilmek işlemi öğrenci açısından rahatlatacaktır. Bu özellikleri anlatılarak öğrencilerin sorularda işlemleri yeniden yapılandırma stratejisini kullanmaları sağlanacak.
- Öğrencilerin sayıların yeniden düzenlenmesi ve işlemleri yeniden düzenlenmesi arasındaki farkı anlamalarını sağlamak.

Alıştırmalar:

- Fotokopi halinde çoğaltılmış olan Alıştırmalar-3 yaprak-uygulama kağıdı öğrencilere dağıtılacak.
- İlk soru örnek olması için öğretmen tarafından yapılacak sonrası öğrencilerle yüksek sesle yapılacak.

Ödev: Evde konu tekrarı yapmaları ve pratiklik kazanmaları için ÖDEV-3 yaprak uygulama kağıdı öğrencilere dağıtılacak.

ALIŞTIRMALAR-3

İşlemleri yeniden yapılandırarak aşağıdaki işlemlerin sonucunu tahmin ediniz.

1. 34+26+37=? 97+121+89=? 83+75+77=?

47+29+43=? 46+42+39=? 73+74+69=?

2. 278+339+321=? 656+747+712=? 471+522+482=?

537+483+514=? 529+495+751=? 814+827+783=?

3. 1736+1671+1896=? 4581+4788+8372=?

2834+3108+2743=? 2355+1965+2429=? 7288+6841+7416=?

4434+3722+4186=?

4. 23,436+29,322+31,428+36,243=? 33,298+36,492+39,435+31,875=?

41,679+38,143+40,673+42,513=? 54,142+47,361+53,219+46,392=?

63,766+68,229+66,328+62,437=? 82,451+86,327+88,523+83,683=?

ÖDEV-3

İşlemlerin yeniden yapılandırılması stratejisini kullanarak işlemleri yapınız.

1. (4,5+3,7+2,9)x38=?   3 3/4 +4 2/3 –2 3/5=?
   13 2/9 + 27 2/8 -19 5/6=?

2. 32,7x41,8=?   29,3x67,4=?   57,24x94,6=?

3. 357x13,4=?   428x22,5=?   678x18,7=?

4. 412x43,7=?   628,3x27,7=?   851x16,3=?

5. 958÷15,7=?   273÷23,7=?   412÷9,7=?

6. 563÷77,2=?   471÷87,35=?   722÷347,4=?

7. 256÷46,2=?   493÷64,8=?   532÷57=?

8. 374÷24,7=?   841÷13,2=?   387÷126=?

Ders Planı- 4

Amaç: Düzenleme-düzeltme stratejisini tanıtımı

Süre: 45 dakika

İşleniş:

- Sayıların ve işlemlerin yeniden yapılandırılması stratejileri tekrar edilecek.
- Öğrencilere, daha yakın tahminler yapabilmek için neler yapılabileceği sorulup düşünmeleri sağlanacak.
- Düzenleme ve düzeltme stratejisi tanıtılacak. Tahmin ederek elde edilen sonuçların gerçek cevaba daha da yakın olmasını bu strateji ile sağlarız. Sonuç üzerinde oynamalar yapılabildiği gibi işlemin ortasında da düzenleme ve düzeltme metodu kullanılabilmektedir. Sonda kullanılan düzenleme-düzeltme

stratejisine final-düzenleme düzeltme denirken, işlemin ortasında yapılana da (prior) başlangıç düzenleme düzeltme denir. Örneğin; 87 429+92 878+94 336 toplamı tüm sayıları en yakın on binlere yuvarladığımızda 90 000x3=270 000 şeklinde cevaplanabilmektedir. Düzenleme düzeltme stratejisi ile daha net bir cevap bulabiliriz. İlk sayının 90 000'e gelebilmesi için yaklaşık 3 000 daha olması gerekirken, diğer iki sayıdan toplam yaklaşık 6 000 kadar sayı eksilmesi gerekiyor. Bu iki düzenlemeyi birleştirirsek toplama 3 000 kadar daha ilave etmemiz gerekir (273 000). Bu da gerçek cevap olan 274 643 sayısına oldukça yakın bir cevap demektir.

- Öğrenciler alıştırmalardaki soruları sayıların yeniden düzenlenmesi, ileri-geri yuvarlamaları ve işlemlerin yeniden düzenlenmesi stratejilerinden her hangi birini ya da hepsini kullanarak cevabı tahmin ederler ve düzenleme düzeltme stratejisinin gerektirirliğini tartışırlar.

Alıştırmalar:

- Fotokopi halinde çoğaltılmış olan Alıştırmalar-4 yaprak-uygulama kağıdı öğrencilere dağıtılacak.
- İlk soru örnek olması için öğretmen tarafından yapılacak sonrası öğrencilerle yüksek sesle yapılacak.

Ödev: Evde konu tekrarı yapmaları ve pratiklik kazanmaları için ÖDEV-4 yaprak uygulama kağıdı öğrencilere dağıtılacak.

ALIŞTIRMALAR-4

Aşağıdaki soruları sayıların yeniden düzenlenmesi (ileri-geri yuvarlama), işlemlerin yeniden düzenlenmesi ve en son da düzenleme ve düzeltme stratejilerini kullanarak asıl cevaba en yakın sonucu tahmin ediniz.

1. 47x18=? 38x24=? 93x56=? 76x52=?
2. 247x36=? 372x17=? 612x47=? 558x73=?
3. 728x32=? 959x7,8=? 336x26,5=? 287x37,4=?

4. 58÷7,8=?   38÷11=?   76÷18,7=?   85,3÷6,2=?

5. 267÷92=?   463÷24=?   527÷87=?   344÷23=?

6. 632÷24=?   787÷132=?   351÷8,9=?   428÷13,2=?

ÖDEV-4

Aşağıdaki soruları sayıların yeniden düzenlenmesi (ileri-geri yuvarlama), işlemlerin yeniden düzenlenmesi ve en son da düzenleme ve düzeltme stratejilerini kullanarak asıl cevaba en yakın sonucu tahmin ediniz.

1. 37x29=?   46x33=?   94x28=?   72x47=?
2. 241x33=?   156x21=?   315x12=?   419x18=?
3. 1453x8=?   2319x12=?   3638x18=?   4127x23=?
4. 87÷13=?   356÷27=?   429÷13=?   5,47÷0,9=?
5. 1235÷43=?   4213÷57=?   8572÷46=?   1956÷26=?
6. 92,06÷17=?   4786÷47=?   186,2÷63=?   2246÷83=?

APPENDIX B

SAMPLE QUESTIONS FOR ESTIMATION ABILITY TEST

A) Sayılar   Formatı (Numbers Format)

1. Pozitif Tamsayılar (Whole Number)

“Bir postacı haftanın hergünü çalışmaktadır. Günde 96 mektup dağıyor ise bir yılda kaç mektup dağıtır?”

2. Kesirler (Fraction)

“14 ¾ m uzunluğundaki tabaka kartonlardan kaç tane 5/8 m lik parçalar kesilebilir?

3. Ondalıklı Kesirler (Decimal)

“Aşağıdaki ifade doğru mu yanlış mı?

359, 25÷19, 6 < 17”

4. Yüzdeler (Percent)

“967 sayısı 214 sayısının yüzde kaçıdır?

A) 500

B) 50

C) 5

D) Hiçbiri ”

B) Problem Formatı (Problem Format)

1. Sadece Sayılar Kategorisi (Numbers Only Category)

“Aşağıdaki işlemin sonucunu tahmin ediniz:

1 347 628 + 2 675 026 + 827 000 + 3 472 100=?”

2. Uygulama Kategorisi (Application Category)

"Bir sınavda Mustafa 75 sorudan 49 unu doğru yapmıştır. Mustafa soruların % 70 ini mi doğru cevaplamış olur?

( ) EVET ( ) HAYIR"

C) Cevap Formatı (Answer Format)

1. Açık Uçlu Kategori (Open-Ended Category)

"Aşağıdaki işlemin sonucunu tahmin ediniz:

23 x 95 =?"

2. Referans Kategorisi (Reference Number Category)

"Bir havuza bir çeşmeden saatte 3,35 litre su akamaktadır. 20 saat çeşme açık kalırsa havuzda 70 litreden fazla mı su olur?

( ) EVET ( ) HAYIR"

3. Büyüklük Sıralaması Kategorisi(Order of Magnitude Category)

"95 kg üzüm kurutulunca 65 kg oluyor. Buna gore ağırlıkta yüzde kaç azalma olur?

A) 3

B) 0,03

C) 0,3

D) Hiçbiri"