THE EFFECTS OF INSPECTION ERROR AND REWORK ON QUALITY LOSS FOR A NOMINAL-THE-BEST TYPE QUALITY CHARACTERISTIC

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ABSTRACT

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Taguchi defines quality loss as the loss imposed to the consumer for each unit of deviation from the target consumer requirements. In this thesis, the effects of inspection error and rework on quality loss are studied for a nominal-the-best type quality characteristic. The distribution of the quality characteristic in a production environment where there are inspection error and a separate rework facility is investigated. 100 % inspection policy is considered. After deriving the mean and variance of the resulting distribution of the quality characteristic, the true and simulated quality loss values for a number of scenarios are calculated. Furthermore, effects of deviation of the process mean from the target and variance of the rework are studied besides inspection error and process capability through a full factor factorial experimental design. Results are discussed for possible uses as quality improvement project selection criteria.

Keywords: quality loss, inspection error, rework, 100% inspection

ÖZ

NOMİNAL-EN-İYİ TÜRÜ BİR KALİTE KARAKTERİSTİĞİ İÇİN MUAYENE HATASI VE YENİDEN İŞLEMENİN KALİTE KAYBI ÜZERİNE ETKİLERİ

TAŞELİ, AYSUN

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Taguchi, kalite kaybını, hedef tüketici taleplerinden uzaklaşılan her birim karşılığında tüketiciye dayatılan kayıp olarak tanımlamaktadır. Bu tezde, nominal-en-iyi türü bir kalite karakteristiği için muayene hatası ve yeniden işlemenin kalite kaybı üzerindeki etkileri çalışılmıştır. Muayene hatalarının ve ayrı bir yeniden işleme biriminin bulunduğu bir üretim ortamında, kalite karakteristiğinin dağılımı incelenmiştir. Kalite karakteristiğinin nihai dağılımının ortalama ve standart sapması türetildikten sonra, farklı senaryolar için gerçek ve benzetim kalite kaybı değerleri elde edilmiştir. Ayrıca, muayene hatası ve süreç yeteneğinin yanında süreç ortalamasının hedeften sapmasının ve yeniden işlemenin varyansının etkileri de tam faktör faktöryel deney tasarımı ile çalışılmıştır. Sonuçlar, kalite iyileştirme projesini seçme kriterleri gibi olası kullanımlar için tartışılmıştır.

Anahtar Kelimeler: kalite kaybı, muayene hatası, yeniden işleme, %100 muayene

To my family

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CHAPTER 1

INTRODUCTION

The understanding of quality has changed especially during the last five decades. While quality is used to be evaluated by the number or percentage of defective items, it now is taken as a more sophisticated concept regarding a number of parameters. First of all, as the quality guru E. Deming states, the new philosophy of quality envisions continuity. Secondly, the consumer has become the true evaluator of product quality [6]. The customer requirements are now diagnostics of quality and qualifications of a product. Product or process requirements are set on the basis of customer demands. By translating the customer demands into a more technical language, product or process targets or specifications are set by the manufacturers [23]. Despite this increasing focus on consumer demands by producer, still the quality concept of Taguchi proposes an alternative view to the specifications and the conformance-nonconformance duality. He relates the quality to the loss to the society [32] (cited in [23]). And today, a manufacturer's aim is to decrease this quality loss while increasing profitability.

Unfortunately, it is not easy to achieve this goal. Many factors cause deviation of a quality characteristic from the target and the dispersion of the characteristic. Quality loss is typically proportional to such deviations and dispersion. Reworking is a factor that may have effects on the average deviation and dispersion of the quality characteristic values of the accepted items. Additionally, errors in measuring the true quality characteristic values, that is inspection errors, adversely affect quality loss calculations. Hence, knowing what kind of effects inspection error and rework have on quality loss will have contributions in quality improvement studies; such as the decision of the project that has the priority to be improved. The studies in the literature do not consider such effects. The previous studies have not considered effects of reworking items on the quality loss, either. Instead, the studies in the literature that consider inspection error and rework usually focus on determination of economically optimum specification limits or target values.

In [13], Fernell and Chhoker investigate the differences of acceptance sampling and 100 % inspection each with or without inspection error to decide which plan minimizes the expected loss (consumer's loss+producer's loss). In [10], Dhavale studies the distribution and effects of inspection in order to determine the distribution of number of defective items in a lot, while Agnihorti and Kenett [1] focus on the effects of the pattern that defective items follow on the performance for a production process with 100 % perfect inspection followed with rework. Another study considering inspection error is made by Greenberg and Strokes [16]. They try to determine the optimum number of test repetitions by maximizing the expected benefit model regarding the inspection error.

Phillips and Cho [25] study the distribution of accepted items to develop a model to determine the optimum specification region in a manufacturing environment where there are 100 % inspection, no inspection error, and items that are reworked return to the system at the target. Chen and Chung [5] also study the distribution of a quality characteristic that goes under 100 % inspection but with inspection error and do not consider rework. On the other hand, Irianto [20], [19] compares the production environments with or without a separate rework unit in terms of production, inspection and rework costs.

In this thesis, we study the effects of inspection error on the quality loss in different production environments. The probability distribution of a quality characteristic assuming 100 % inspection is investigated with or without inspection error and with or without rework conditions, separately. The significance of factors other than rework and inspection error are investigated with a designed experiment.

The thesis consists of four chapters. The first part of the next chapter gives some theoretical information about the basic concepts used in this study. The second part briefly reports relevant literature. In the third chapter, the resulting distributions and parameters of a quality characteristic are explained for four different production environments. Validation of the results for the production environment with rework and inspection error is presented in the last section of the third chapter. The valuation and discussion of experimental results are carried out in the fourth chapter. And the last chapter is a conclusion about the study including possible future research topics.

CHAPTER 2

LITERATURE REVIEW

2.1 Background

2.1.1 Quality Loss

The quality of a product is measured in terms of its characteristics which describe their performance [29]. According to the traditional understanding of quality, the quality characteristic values are not different as long as they are within some specification limits. But today, manufacturing a product that conforms to quality specifications is not sufficient to satisfy customers and keep the competitive position in the market.

Especially, after the World War II, a new concept of quality started to arise following the Continuous Improvement (or Never-Ending Improvement) of Deming. Previously, the quality control was related to the control of defective products, i.e. controlling the fraction defective [26]. It is used to think that after a certain percent of defective items or specifications are defined and those criteria were satisfied, no further improvement was necessary [11]. The items whose quality characteristics lie between the specified limits or the so-called 'customer tolerance' are accepted to have a good quality. This approach to quality is named as 'goal post' syndrome by Ross [30]. In the goal post model no loss is considered unless the quality characteristics of the product is out of specifications. An item which is very close to a limit but within specifications and another one which is at or close to the target are treated in the same way. But, an item which is again very close to a limit but out of specifications is accepted as non-conforming.

However, it is recognized that this approach does not make much sense since meeting the specifications of the producer is not enough to meet customer specifications for most of the cases. In fact, the effect of using a product which slightly satisfies and which slightly misses the specifications does not very much differ for a customer. Hence, a product that meets the tolerances may also negatively affect the customer satisfaction and position of the producer in the market. The new understanding of quality takes this important reality into consideration.

An important example illustrating for this difference is the Sony television customer preference study [26]. The research shows the fallacy of using number of defective (or fraction defective or percent defective) as a quality measure. It is recognized in the late 1970s that the customers prefer the television sets produced by Sony-Japan rather than those made by Sony-USA with a reasoning of the difference in their color density quality. Although both factories use identical designs and specification limits, Sony-Japan is preferred to Sony-USA. The distribution of color density of television sets produced by Sony- USA had a uniform distribution between the specification limits although the color density of television sets produced by Sony-USA had a normal distribution with a mean at the target and a standard deviation of 5/3. Although all most all the sets produced by Sony-USA are within specification limits, and about 0.3 % of the sets produced by Sony-Japan are outside the tolerance limits, the customers use their preferences in favor of the sets produced by Sony-Japan. In this case, the policy used by Sony-USA corresponds to the goal post syndrome of Ross. On the other hand, Sony-Japan factory regards the new philosophy rather than the percent defective.

According to the new philosophy, developed by Genichi Taguchi, every product produced imposes a loss to the consumer, even if its quality performance is within the specified limits. This loss can be generally defined as the loss in the product function or properties through its life cycle. The better the quality of a product is, the less it will loose its functionality and properties during its life cycle. Hence, if a product does not perform as it is expected, the consumer senses some loss. So, a quality loss function that can measure the loss of products even when they meet the tolerances should be developed [26].

Taguchi's quality loss functions express quality as a loss phenomena. Taguchi assures that a customer is fully satisfied only when the quality characteristic of the product is at the target (nominal) level. The loss, (thus the dissatisfaction) of the consumer increases as the quality characteristic deviates from the target. He emphasizes the importance of a quality performance that aims to reach the target value on the average with the minimum deviation from this average value. This can be named as performance consistency [31], [11].

That is why, he focuses more on the process rather than product, regards controlling the location and dispersion of the distribution as well as meeting the specifications and develops a quality measure which is a function of deviation of the process from the target value and the variation in the process.

In Figure 2.1, part (a)the step function represents the loss function for goal post syndrome. In part (b), the quadratic loss function of Taguchi is plotted. LSL is the lower specification limit and USL is the upper specification limit.

There are many types of quality loss function. Each type depends on a quality characteristic with a different nature. The loss functions expressing the relationship between quality and variability of the process from Taguchi's point of view are:

- Nominal-the-best Type
- Smaller-the-better Type
- Larger-the-better Type
- Asymmetric Type

Nominal-the-best Type: For this type of quality characteristic, target value is the nominal value. The quality characteristic may take values less than or more than the target value. When the value of the characteristic deviates from the target in either direction, the quality loss increases.

The quadratic loss function of Taguchi can be obtained as follows: Let L be the loss function of a product. Then, expanding L around the target value using Tay-

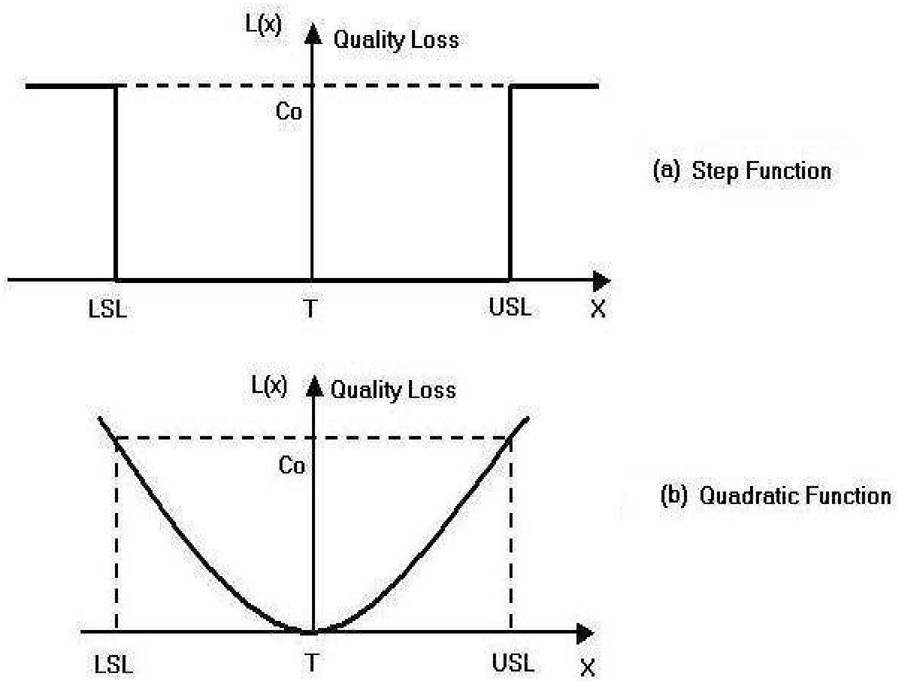

Figure 2.1: Step and Quadratic Loss Functions (Source: Ross [30])

lor expansion [34], and eliminating the higher order terms, the loss function can be approximated by

$$L(x) = c \cdot (x - T)^2$$

where x is the true quality characteristic, c is the quality loss coefficient and T is the target.

It is often trivial to determine c. To compute c, first the specification limits should be determined. Then, the loss at these limits should be computed (C_0). This loss should include all the losses such as the cost of repair, transportation, replacement or loss (dissatisfaction) of the customer due to the malfunction and lack of the product, etc. [26].

Then the coefficient is computed as

$$c = \frac{C_0}{{\Delta_0}^2}, \qquad \Delta_0 = USL - T$$

Smaller-the-better Type: Sometimes, a nonnegative characteristic has the ideal value of zero and the loss of an item increases as the quality characteristic value increases. Such characteristics are called smaller-the-better type quality characteristics. The quality loss function of such characteristics is

$$L(x) = c \cdot x^2$$

The quality loss coefficient is computed in the same way as it is computed for nominal-the-best type characteristics.

Larger-the-better Type: In some situations, for a nonnegative characteristic, the worst value is zero and the performance of the process gets better and better while the quality characteristic value increases (ideal value is infinity). Those characteristics are called larger-the-better type characteristics and the corresponding loss function is

$$L(x) = \frac{c}{x^2}$$

In this case, c is computed as $c = C_0 \cdot {\Delta_0}^2$.

Asymmetric Loss Function: There are also some cases where the loss of the product is not the same for equal amount of deviations from the target in opposite directions. Then, different quality loss coefficients can be computed. The quality loss function is

$$L(x) = \begin{cases} c_1 \cdot (x-T)^2, & x > T \\ c_2 \cdot (x-T)^2, & x \leq T \end{cases}$$

The types of quadratic loss functions are plotted in Figure 2.2.

The quality characteristic value x is different for each product. Hence, for a sample of observation or for a distribution, it is possible to talk about average unit loss [23]. The average quality loss of a nominal-the best-type quality characteristic is computed in [26]. The average quality loss functions for all types of quality characteristics are given in Table 2.1.

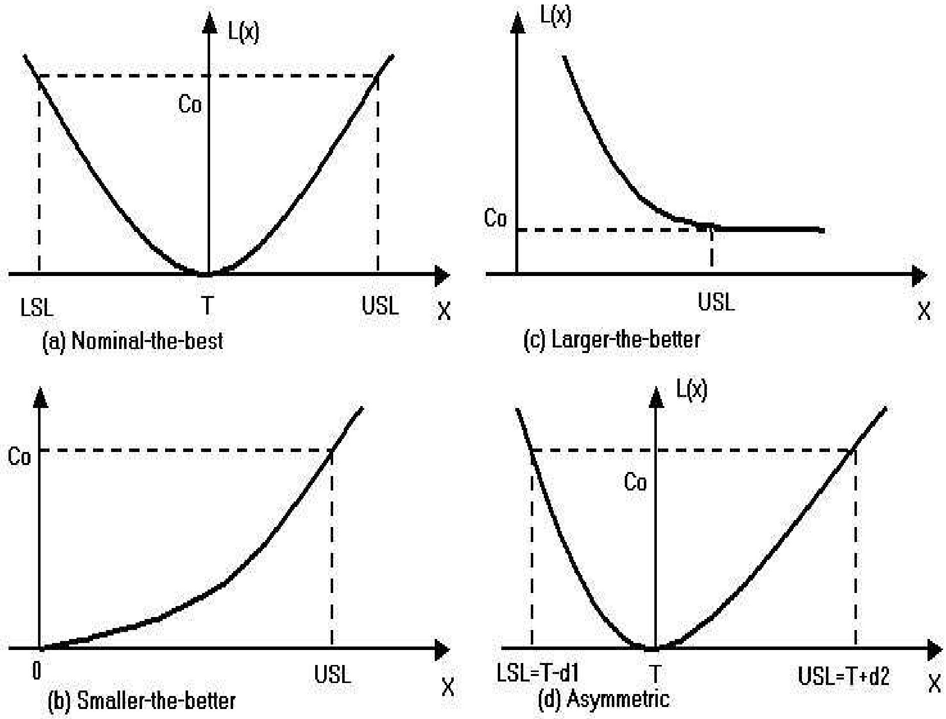

Figure 2.2: Types of Quadratic Loss Function(Source: Ross [30])

Table 2.1: Expected Quality Loss Functions

Type	Average Loss Function
Nominal-the-best	$E[L] = c \cdot [(\mu - T)^2 + \sigma^2]$
Smaller-the-better	$E[L] = c \cdot [\mu^2 + \sigma^2]$
Larger-the-better	$E[L] = c \cdot [1/\mu^2] \cdot [1 + 3\sigma^2/\mu^2]$
Asymmetric	$E[L] = \begin{cases} c_1 \cdot (\mu - T)^2 + \sigma^2, & x > T \\ c_2 \cdot (\mu - T)^2 + \sigma^2, & x \leq T \end{cases}$

where μ is the mean and σ is the standard deviation of the distribution of quality characteristic X. If μ and σ are unknown, they are estimated by the sample mean and standard deviation of X.

Kapur and Cho [22] consider a process with n correlated quality characteristics and develop a multivariate quality loss function based on the idea of Taguchi. If $x_1, x_2, x_3, ..., x_n$ are n nominal-the best type quality characteristics of a product, and $T_1, T_2, T_3, ..., T_n$ are the target values of $x_1, x_2, x_3, ..., x_n$ respectively, similar to the univariate case, using Taylor expansion and ignoring the higher order terms, the multivariate quality loss function can be expressed as

$$L(x_1, x_2, x_3, ..., x_n) = \sum_{i=1}^{n} c_{ij}(x_i - T_i)(x_j - T_j)$$

where $c_{ij} = c_{ji}$ is the proportionality constant depending on the losses at the specification limits. c_{ij} can be determined by using a regression method [8], [24](cited in [6]).

Then, the expected multivariate quality loss is

$$\begin{aligned} E[L(x_1, x_2, x_3, ..., x_n)] = & \textstyle\sum_{i=1}^{n} c_{ii}[(\mu_i - T_i)^2 + {\sigma_i}^2] \\ + & \sum_{i=2}^{n}\sum_{j=1}^{i-1} c_{ij}[\sigma_{ij} + (\mu_i - T_i)(\mu_j - T_j)], \end{aligned}$$

where σ_{ij} is the covariance of x_i and x_j.

The loss functions of Taguchi, however, do not consider the abasement of quality due to the usage of the product over time. On the other hand, Teran et al. [35](cited in [6]) state that the deviation of a quality characteristic of a product may change over time as a result of its use. Thus, the quality of the product will also be subject to a change during time. Reexpressing the expected quality loss function of Taguchi as 'an expected continuous cash flow stream that occurs during a time period (0,M)', they define the present worth of expected quality loss with the following equation:

$$\int_0^M E[L]e^{-rt}dt$$

where r is the consumer's discount rate and t represents the time.

Integrating the multivariate quality loss and present worth of expected quality loss, Chou and Chen [6], develop the present worth for expected multivariate quality loss (PWEMQL). They define PWEMQL as an additive function of three components:

1) present worth of expected multivariate quality loss due to variances,

2) present worth of expected multivariate quality loss due to means,

3) present worth of expected multivariate quality loss due to covariance.

According to Kano's approach to quality, Taguchi's functions are suitable for only one type of quality characteristics which are of Performance Quality. In order to obtain a more accurate approximation of consumer's quality loss, Teeravaraprug [33] integrates the quality model of Kano. Kano's model, cited in [9], can be defined as a set of ideas on quality that are based on clarifying and classifying the customer requirements and hence quality characteristics [33].

According to Kano, one type of quality requirements is Performance Quality. The loss of these characteristics can be expressed on a continuous scale. If these types of characteristics are at the customer requirement levels, then customer will be satisfied and the loss to the customer will be zero. If the quality characteristic is below the customer requirements (deviates from the required level), the loss to the customer increases. And lastly, if the quality characteristic is above the customer requirements, then the customer is pleased and the quality loss may be negative. Another type of quality requirements defined by Kano is Basic Quality. These requirements are expected to be existing. If such requirements exist, then the customer will be neutral and there will be no loss incurred to the consumer. But if they do not exist then there will be a constant amount of loss to the consumer. The third type of quality requirements is Excitement Quality type. This kind of requirements are the ones that the customer does not expect. That's why, if these requirements are not present, the customer will incur no loss. However, if they are satisfied, consumer's loss will be negative [33].

2.1.2 Mixture (Mixed) Distributions

Mixtures of distributions are often met in various biological, psychological and physical applications [2], where a process can be modelled by a number of simpler processes which arise in a hierarchical structure [4].

Berger and Casella [4] express a finite mixture of distributions as the distribution of a random variable X that depends on a quantity which also has a distribution. Suppose the realization of quantity E is the first step of the hierarchy and the realization of X depending on the realization of E is the second step of the hierarchy.

If E takes the value e_1 with probability p, the value e_2 with probability q=1-p, then the probability distribution of E can be defined as

$$E = \begin{cases} e_1, & \text{with probability } \ p \\ e_2, & \text{with probability } \ q = 1 - p \end{cases}$$

If X is distributed as $f_X(x)$ when e_1 occurs and as $g_X(x)$ when e_2 occurs, then the conditional probability distribution of X when E is given can be defined as

$$m_{(X|E)}(x|e) = \begin{cases} f_X(x), & \text{with probability} \;\; P(e_1) \\ g_X(x), & \text{with probability} \;\; P(e_2) \end{cases}$$

Then the probability distribution of X can be found as

$$\begin{aligned} h_X(x) &= P(e_1) \cdot f_X(x) + P(e_2) \cdot g_X(x) \\ &= p \cdot f_X(x) + q \cdot g_X(x) \end{aligned}$$

In other words, if E has the distribution $l_E(e)$ and given E, X has the distribution $m_{(X|E)}(x|e)$, then the distribution of X is found as

$$h_X(x) = l_E(e) \cdot m_{(X|E)}(x|e)$$

Of course, the stages of the hierarchy may be more than two, one can define as many steps (backwards) as possible to simplify the process. Berger & Casella [4] additionally give examples of many two and three-stage mixture models. They also study the mean and variance of mixed distributions.

Mixture or mixed distributions are also called heterogeneous distributions and are defined to be a combination of two or more populations in given proportions [17].

Behboodian shows how to find the distributions of some statistics like sample mean, sample variance and order statistics coming from populations with mixture distributions in [3]. He additionally studies the structure and also some statistics of finite mixture distributions [2].

2.1.3 Truncated Normal Distribution

One may talk about a truncated distribution in two situations. First situation is that, sampling not from the whole population but only a part of it is possible. One example is to sort the elements of a population and sampling among the ones which are under or below a certain value. The other situation arises when the individual values of observations below or above a certain value are not specified [17].

Consider a random variable X having a normal distribution with parameters μ and σ^2. Then, the probability density function $f(x)$ of X is specified as:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}, \qquad -\infty \leq x \leq \infty$$

If the values of X above some value x_R can not be observed - due to censoring or truncation- then, the resulting distribution is a right-truncated normal distribution with probability density function $f^{RT}(x)$ given by

$$f^{RT}(x) = \begin{cases} \frac{f(x)}{\int_{-\infty}^{x_R} f(x)}, & -\infty \leq x \leq x_R \\ 0, & x_R \leq x \leq \infty \end{cases}$$

where $f(x)$ is the normal probability density function [21].

If the values of X below some value x_L can not be observed due to similar reasons as above then, the resulting distribution is a left-truncated normal distribution with probability density function $f^{LT}(x)$ [21]given by

$$f^{LT}(x) = \begin{cases} 0, & -\infty \leq x \leq x_L \\ \frac{f(x)}{\int_{x_L}^{\infty} f(x)}, & x_L \leq x \leq \infty \end{cases}$$

But if the values of X which are below x_L and above x_R can not be observed, then the resulting distribution is said to be a doubly truncated normal distribution truncated at limits x_L and x_R [21]. The probability density function of this truncated normal distribution is

$$f^{DT}(x) = \begin{cases} \frac{f(x)}{\int_{x_L}^{x_R} f(x)}, & x_L \leq x \leq x_R \\ 0, & \text{otherwise} \end{cases}$$

The left, right and doubly truncated normal distributions are shown in Figure 2.3.

2.1.4 Process Capability

Process capability indices are used to express the relationship between technical specifications and production abilities on the production line. This relationship is important to both suppliers and purchasers. For this reason, process capability measures are widely used in industry to measure the producer's own ability to meet quality specifications [23].

Among the process capability indices the two widely used are process capability ratio C_p or PCR and C_{pk}.

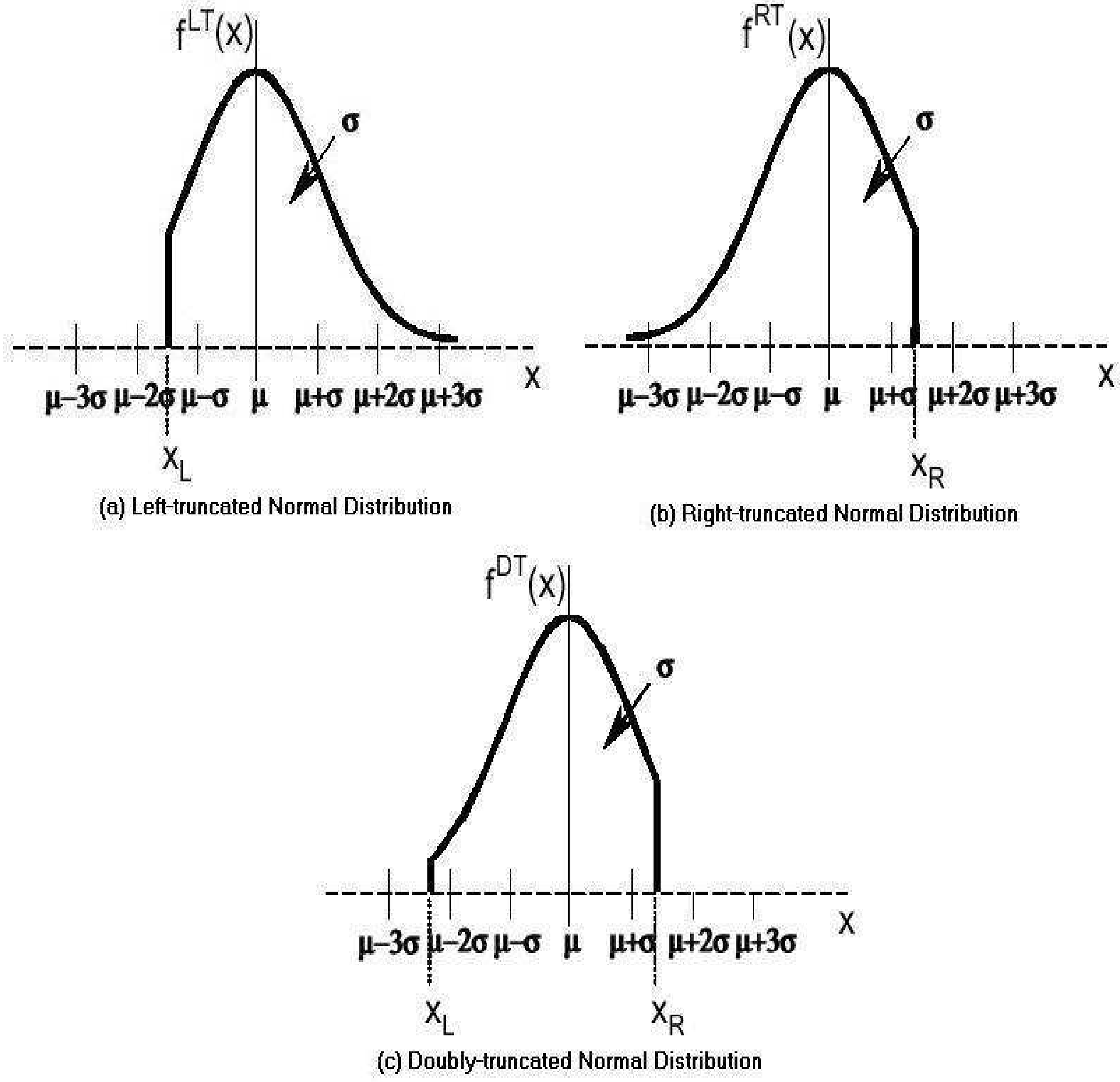

Figure 2.3: Left, Right and Doubly Truncated Normal Distributions (Source: Johnson [21])

C_p measures the potential capability of the production process to manufacture products that meet the specifications. Process capability ratio is defined as

$$C_p = \frac{USL - LSL}{6\sigma}$$

where USL and LSL are upper and lower specification limits, respectively.

C_p measures the potential capability provided that the process can be fitted to the target. Hence, it is suitable only for nominal-the-best type quality characteristics [23]. But it still does not take into account where the process mean is located relative to the specifications [36].

However, there may be situations where the process does not produce at the target even if it produces within the specifications. At that time, it is relevant to use another measure that considers shifts from the target. That measure is C_{pk} and is defined as

$$C_{pk} = min\{\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\}$$

When specifications are one sided, the following two measures can be used to measure process capability:

$$C_{pL} = \frac{\mu - LSL}{3\sigma}, \qquad \text{for processes that have only a lower limit}$$

and

$$C_{pU} = \frac{USL - \mu}{3\sigma}, \qquad \text{for processes that have only an upper limit}$$

Another process capability index C_{pm} is independently developed by Hsiang and Taguchi [18] and Chan et al. [7](cited in[29]). This new index has the advantage of applicability to a process where the target does not stand at the middle of the specifications and also presentability of the deviation between the process mean and the target value. C_{pm} is defined as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\mu - T)^2 + \sigma^2}}$$

A more advanced process capability index, C_{pmk}, which combines the properties of both C_{pk} and C_{pm} is proposed by Pearn et al. [27] as

$$C_{pmk} = min\{\frac{USL - \mu}{3\sqrt{(\mu - T)^2 + \sigma^2}}, \frac{\mu - LSL}{3\sqrt{(\mu - T)^2 + \sigma^2}}\},$$

or by Kolarik [23]as

$$C_{pmk} = \frac{C_{pk}}{\sqrt{1 + (\frac{\mu - T}{\sigma})^2}}$$

C_{pmk} index is proposed to be useful for two sided processes and provide more assurance than the quality indices C_{pk} and C_{pm} since $C_{pmk} \leq C_{pk}$ and $C_{pmk} \leq C_{pm}$ [28].

2.1.5 Measurement System Analysis

Any time result of a process is measured, a variability is observed among the values recorded. This variability is due to two factors: the first one is the variability of the items produced, the second is the imperfectness of the measurement. So, the variability in the measured values can be defined as:

$$\sigma^2_{measured\ values} = \sigma^2_{product} + \sigma^2_{measurement}$$

Hence, achieving an adequate gauge (gage, measurement or inspection) system capability is one of the aspects that need to be considered in process control and quality improvement studies [36]. There are two fundamental points to be concerned about the measurement system:

1) Accuracy

2) Precision

Accuracy: is about the bias between the measured value and the actual value of a quality characteristic. The main idea is that in a measurement system, when an item is measured repeatedly, each observed value will show a difference from the other. However, the average of the measured values should approach to the actual value of the quality characteristic. Hence, accuracy is about the location of the measurement values.

When there is an inaccurate measurement system; that is, the measurements are biased, one of the ways to get rid of the bias is calibrating the device used. Instrument calibration is a way to minimize the bias although it is not eliminated totally. And the bias can be ignored if its magnitude is small enough relative to the magnitude of the measurement values [23].

Precision: is about the variation of the measurements. This variation has two components:

1) Gauge Repeatability: expresses the variation observed when the measurement device fails to exactly repeat the measurement for the same item.

2) Operator Reproducibility: expresses the variation observed when different operators used in the measurement system fail to exactly reproduce the same measurement for the same parts using the same device.

Precision of measurement which is sometimes called the measurement error [14] (cited in [23]) is defined as:

$$\sigma^2_{measurement\ \ error} = \sigma^2_{gage\ \ repeatability} + \sigma^2_{operator\ \ reproducibility}$$

The ratio of $6\sigma_{measurement\ \ error}$ to the difference between specification limits is called the precision-to-tolerance (P/T) ratio and is sometimes used to evaluate gage capability [23].

$$P/T = \frac{6\sigma_{measurement\ \ error}}{USL - LSL}$$

The processes are generally accepted to have a good measurement systems if their "precision to tolerance values" are less than or equal to 10% [36].

2.2 Related Work

There are a number of studies on the effects of inspection error. These effects are usually studied in order to determine the optimum target or specification regions or cost. The production environments with or without rework where 100% inspection is applied are within the scope of our study.

It is possible to observe two types of inspection errors: Type I and Type II errors. Type I error is the error of rejecting a conforming item, and Type II error is the error of accepting a non-conforming item.

We can consider four cases:

1. no inspection error is observed
2. only type I error is observed
3. only type II error is observed
4. both types of inspection error are observed.

While studying the inspection error and its effects, it is trivial to determine what kind of an occurrence pattern inspection error follows (the probability distribution of inspection error).

In [13], a study to determine the economically optimal sampling plan is performed. Mathematical models are developed to design four different sampling plans. These plans are 100 % perfect inspection, 100 % inspection with inspection error, single sampling with perfect inspection, and single sampling with inspection error. Both types of inspection error are considered and no rework exists in the system.

Accepting Taguchi's argument that every product produced exposes a loss, which is producer's loss plus consumer's loss, to the society, the optimal sampling plan that minimizes this loss is sought. Taguchi's nominal-the-best type continuous loss function for both the consumer and the producer are integrated into the economic models and the producer's tolerance minimizing the expected loss is derived.

A similar study is made by Chen and Chung in [5], to determine the economically optimal target value for a production process. The effect of inspection error (type I and II) on the net expected income is investigated in production system models with one-sided and two-sided specification limits, separately.

In each of the above four cases, for a production environment where there is 100 % inspection, no rework and where the products that are out of specification limits are sold at a lower price, the most profitable target value is investigated.

In [10], the distribution of number of defective items left in a lot as a result of imperfect inspection is obtained using mixture distributions. The errors are accepted to be due to human error. The rates of the inspectors (Θ_i) are accepted to have a gamma distribution referring to the previous studies. The number of defective items in a lot after inspection given that the lot is inspected by inspector i $(X_i|\Theta_i)$ is assumed to have a poisson distribution with parameter Θ_i. So, the unconditional distribution of undetected defective items in a lot is found to be hypergeometric distribution.

In [20] and [19], two production systems where no item is scrapped are considered. In one of these systems rework is done in process unit and the other has a separate rework station. These two systems are compared with respect to their production, inspection and rework costs. With the assumption that the item coming from rework may have better quality characteristic, the loss ('cost of quality') to the consumer of the accepted items is included to the total cost function while determining which system is better. The optimum tolerance that minimizes the total cost is determined and the better system is selected according to optimum tolerance and minimum cost.

In order to find the quality loss, Irianto [20] drives the distribution of the output of a production process where there is rework regarding the inspection error. In the study, the true value of the quality characteristic (X) and the error (ε) are assumed to be independent. Hence the joint density of X and ε is expressed as

$h(x, \varepsilon) = f(x) * l(\varepsilon)$

By making the change of variable $X' = X - \varepsilon$, the joint distribution of X and X' is found as

$g(x, x_m) = h(x, x - x') * |J|$ where —J— is the jacobian matrix of transformation.

The distribution function of the output is derived by making the necessary truncation on this joint probability density function with respect to the measured characteristic value and mixing with the distribution of the output of rework station.

Phillips and Cho [25] develop an optimization model to determine the optimum specification regions . A production system where rework occurs when an item falls above the upper specification limit and a scrap occurs when an item falls below the lower specification limit. The reworked items are accepted to be at the target and there is no inspection error in the system. Then outgoing distribution of the quality characteristic is defined by a truncated normal distribution. Expected inspection, rework, scrap and quality cost (loss) are included in the total cost function. The specification limits that minimize the expected cost are determined.

Another study on inspection error is performed by Greenberg and Stokes [16]. The authors handle a problem where only one type of inspection error (Type I Error) which is rejecting a conforming item occurs. The items rejected do not go under any rework but are retested to avoid scrapping a conforming device. The authors formulate a maximization problem to find out whether this retesting procedure is beneficial or not. And if so, this formulation is also used to determine the number of optimum number of repeated inspections. The expected benefit is a function of probability of a defective item and the probability of imperfect inspection is constructed and maximized.

CHAPTER 3

DERIVATION OF DISTRIBUTION CHARACTERISTICS OF ACCEPTED ITEMS

3.1 Problem Definition

In this chapter, components of expected quality loss, namely the mean and variance for a quality characteristic subject to inspection error and rework are studied.

Production Environment: The general picture of the production environment is given in Figure 3.1.

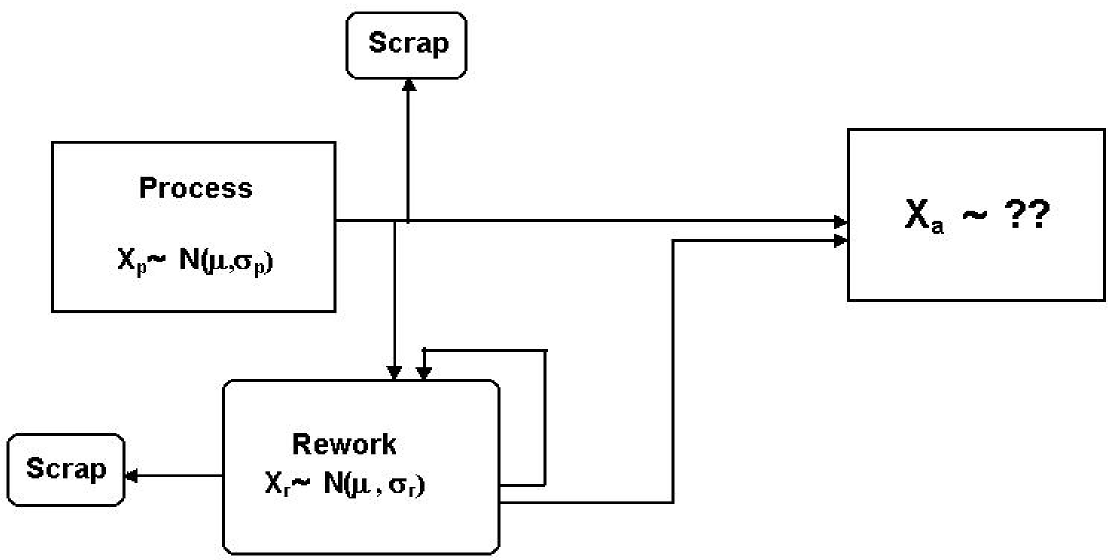

Figure 3.1: General Picture of the Production Environment

It is assumed that only one quality characteristic is produced at one work center or station (and the rework center), and the distribution of a quality characteristic is not affected by the operations performed at the succeeding stations. The quality characteristic X_p of items that are produced in a processing unit are assumed to have a normal distribution with mean μ_p and variance σ_p^2.

For the cases where rework is possible, a separate station is assigned for rework. Similar to the process, the quality characteristic X_r of items that are reworked is also assumed to have a normal distribution with parameters μ_r and σ_r^2.

Additionally, in this study, it is assumed that 100% inspection instead of acceptance sampling takes place after both process and rework. That is, every item is assumed to be inspected one by one after the production. The system is assumed to be producing items at the target and to have symmetric specification and scrap limits. The measurement tool is assumed to be calibrated, so that the measurement system produces accurate results. In this thesis, effects of the precision of the measurement system on quality loss is investigated. At the end of inspection, the items that are within the specification limits are accepted, those that are out of the specification limits but within the scrap limits are sent to rework. The items that are out of scrap limits are scrapped. These inspection rules are valid for the items that come from either process or rework.

Table 3.1 summarizes the assumptions.

Table 3.1: Assumptions

$X_p \sim N(\mu_p,\sigma_p^2)$
$X_r \sim N(\mu_r,\sigma_r^2)$
100% inspection.
Measurement system is accurate but not precise.
Two-sided symmetric scrap and specification limits, LSL=lower specification limit, USL= upper specification limit, LLs=lower scrap limit, ULs=upper scrap limit, where LLs <LSL, ULs> USL
Rework is performed at a separate unit.
Quality of a part or product characteristic is influenced by only one work center and rework center.
The processes are under statistical control.

Taguchi's nominal-the-best type quality loss function is chosen to be the most appropriate type for the examined production environment. Unit loss for a quality characteristic value x_a of an accepted item is

$$L = c(x_a - T)^2$$

Expected Quality Loss of the accepted items is

$$E(L) = c(\mu_a - T)^2 + {\sigma_a}^2$$

where μ_a is the mean and ${\sigma_a}^2$ is the variance of X_a.

To be able to measure the effects of inspection error on quality loss, the mean and variance of the quality characteristics of the accepted items are needed for both cases where there is no inspection error and there is inspection error.

Hence, at first, we need to determine the distribution of the quality characteristic X_a of accepted items for each case. The parameters (μ_a and ${\sigma_a}^2$) of the probability distribution are computed afterwards. Using these parameters, the expected quality loss can easily be computed.

3.2 Resulting Probability Distributions and Parameters

Once the distribution of the quality characteristic of the accepted items is determined, mean and variance can be derived by using the first and second moments.

$$E[X_a^t] \quad = \quad \int_{-\infty}^{\infty} x^t \, h_{X_a}(x) dx \qquad ,$$

where $h_{X_a}(x)$ is the probability distribution of the quality characteristic of the accepted items. The first and second moments of X_a provide the mean and the variance, respectively as follows:

$$\mu_a = E[X_a] \qquad \text{and} \qquad \sigma_a^2 = E[X_a^2] - E[X_a]^2$$

3.2.1 No Rework and No Inspection Error

In a production environment where there is neither rework nor inspection error, the items that are within the specification limits are accepted and the ones that are out of specification limits are scrapped. That is why, after inspection at the process unit, the distribution of the accepted items will be a distribution truncated at the specification limits [20],[19],[25].

$$h_{X_a} = f^T_{X_p}(x) = \frac{f_{X_p}(x)}{\int_{LSL}^{USL} f_{X_p}(x)dx} \qquad , \qquad LSL < x < USL$$

The moments of the accepted items (X_a) will be the truncated moments:

$$E[X_a^t] \;=\; \int_{-\infty}^{\infty} x^t \, h_{X_a}(x)dx = \int_{-\infty}^{\infty} x^t \, f^T_{X_p}(x)dx$$

The truncated moments of this distribution are derived by Phillips and Cho [25]. The moments are

$$E[X_a] \;=\; \mu_p + \frac{\sigma_p}{[F(\frac{USL-\mu_p}{\sigma_p}) - F(\frac{LSL-\mu_p}{\sigma_p})]} \cdot [\phi(\frac{LSL-\mu_p}{\sigma_p}) - \phi(\frac{USL-\mu_p}{\sigma_p})]$$

and

$$\begin{aligned} E[X_a^2] \;&=\; \mu_p^2 + \sigma_p^2 + \frac{2\mu_p\sigma_p}{F(\frac{USL-\mu_p}{\sigma_p}) - F(\frac{LSL-\mu_p}{\sigma_p})} \cdot [\phi(\frac{USL-\mu_p}{\sigma_p} - \phi(\frac{LSL-\mu_p}{\sigma_p})] \\ &+\; \frac{\sigma_p^2}{F(\frac{USL-\mu_p}{\sigma_p}) - F(\frac{LSL-\mu_p}{\sigma_p})} \cdot [(\frac{LSL-\mu_p}{\sigma_p})\phi(\frac{LSL-\mu_p}{\sigma_p}) \\ &-\; (\frac{USL-\mu_p}{\sigma_p})\phi(\frac{USL-\mu_p}{\sigma_p})] \end{aligned}$$

where $F(\cdot)$ is the cumulative distribution function and $\phi(\cdot)$ is the probability distribution function of a standard normal random variable.

3.2.2 Rework and No Inspection Error

As the second case, we consider the rework when inspection error is still of no concern. We obtained the final distribution and moments of the accepted items coming from both process and rework units are obtained in the following way:

The distribution of quality characteristic of the reworked items also will be truncated at the specification limits.

$$g^T_{X_r}(x) = \frac{g_{X_r}(x)}{\int_{LSL}^{USL} g_{X_r}(x)dx} \qquad , \qquad LSL < x < USL$$

Hence, the resulting distribution of the quality characteristic X_a of all items coming from both the process and rework will be a mixture of these two truncated distributions. We find the proportions at which the truncated distributions coming from process and rework are mixed as

$$p = \frac{\int_{LSL}^{USL} f_{X_p}(x)dx}{\int_{LL_s}^{UL_s} f_{X_p}(x)dx} \qquad \text{and} \qquad q = 1 - p$$

where UL_s and LL_s are the upper and lower scrap limits, respectively. It can also be understood from the above formulations that p is the proportion of the probability that the quality characteristic values are within specification limits to the probability that the quality characteristic values are within scrap limits. Consequently, q is the proportion of the probability that the quality characteristic values are outside the specification limits but within the scrap limits (that is, in the rework area) to the probability that the quality characteristic values are within scrap limits. So, p is the proportion of the accepted quality characteristics coming from the process and q is the proportion of the items that go to rework.

Then the mixture distribution turns out to be

$$\begin{aligned}
h_{X_a}(x) &= p \cdot f^T_{X_p}(x) + q \cdot g^T_{X_r}(x) \\
&= \frac{\int_{LSL}^{USL} f_{X_p}(x)dx}{\int_{LL_s}^{UL_s} f_{X_p}(x)dx} \cdot \frac{f_{X_p}(x)}{\int_{LSL}^{USL} f_{X_p}(x)dx} + (1 - \frac{\int_{LSL}^{USL} f_{X_p}(x)dx}{\int_{LL_s}^{UL_s} f_{X_p}(x)dx}) \cdot \frac{g_{X_r}(x)}{\int_{LSL}^{USL} g_{X_r}(x)dx} \\
&= \frac{f_{X_p}(x)}{\int_{LLs}^{ULs} f_{X_p}(x)dx} + q \cdot \frac{g_{X_r}(x)}{\int_{LSL}^{USL} g_{X_r}(x)dx} \\
&= \frac{1}{A} \cdot f_{X_p}(x) + \frac{q}{B} \cdot g_{X_r}(x) \qquad , \qquad LSL < x < USL
\end{aligned}$$

where

$$A = \int_{LLs}^{ULs} f_{X_p}(x)dx \qquad \text{and} B = \int_{LSL}^{USL} g_{X_r}(x)dx$$

Using the definition of a truncated moment of a random variable [25], the moments of X_a can be found as:

$$
\begin{aligned}
E[X_a^t] &= \int_{-\infty}^{\infty} x^t \, h_{X_a}(x)dx \\
&= \frac{1}{A} \cdot \int_{-\infty}^{\infty} x^t \, f_{X_p}(x)dx + \frac{q}{B} \cdot \int_{-\infty}^{\infty} x^t \, g_{X_r}(x)dx
\end{aligned}
$$

Then, the first and the second moments are:

$$
\begin{aligned}
E[X_a] &= \frac{1}{A} \cdot \mu_p [F(\frac{USL-\mu_p}{\sigma_p}) - F(\frac{LSL-\mu_p}{\sigma_p})] + \sigma_p [\phi(\frac{LSL-\mu_p}{\sigma_p}) - \phi(\frac{USL-\mu_p}{\sigma_p})] \\
&+ \frac{q}{B} \cdot \mu_r [F(\frac{USL-\mu_r}{\sigma_r}) - F(\frac{LSL-\mu_r}{\sigma_r})] + \sigma_r [\phi(\frac{LSL-\mu_r}{\sigma_r}) - \phi(\frac{USL-\mu_r}{\sigma_r})]
\end{aligned}
$$

and

$$
\begin{aligned}
E[X_a]^2 &= \frac{1}{A} \cdot \{(\mu_p^2 + \sigma_p^2)[F(\frac{USL-\mu_p}{\sigma_p}) - F(\frac{LSL-\mu_p}{\sigma_p})] \\
&- 2\mu_p\sigma_p[\phi(\frac{USL-\mu_p}{\sigma_p}) - \phi(\frac{LSL-\mu_p}{\sigma_p})] \\
&+ \sigma_p^2[(\frac{LSL-\mu_p}{\sigma_p})\phi(\frac{LSL-\mu_p}{\sigma_p}) - (\frac{USL-\mu_p}{\sigma_p})\phi(\frac{USL-\mu_p}{\sigma_p})]\} \\
&+ \frac{q}{B} \cdot \{(\mu_r^2 + \sigma_r^2)[F(\frac{USL-\mu_r}{\sigma_r}) - F(\frac{LSL-\mu_r}{\sigma_r})] \\
&- 2\mu_r\sigma_r[\phi(\frac{USL-\mu_r}{\sigma_r}) - \phi(\frac{LSL-\mu_r}{\sigma_r})] \\
&+ \sigma_r^2[(\frac{LSL-\mu_r}{\sigma_r})\phi(\frac{LSL-\mu_r}{\sigma_r}) - (\frac{USL-\mu_r}{\sigma_r})\phi(\frac{USL-\mu_r}{\sigma_r})]\} \quad ,
\end{aligned}
$$

3.2.3 No Rework and Inspection Error

It is assumed that both types (Type I and Type II) of inspection error are possible for the quality characteristic.

If there is inspection error in the process, when the real value of the quality characteristic is X_p, during inspection it is observed as Y_p with an E_p amount of deviation from the real value. That is,

$$Y_p = X_p + E_p$$

Here we can assume that $E_p \sim N(0, \epsilon_p^2)$. Then, the conditional distribution of the observed value given the actual quality characteristic value is Normal with parameters x_p and ϵ_p^2 ($Y_p|X_p \sim N(x, \epsilon_p^2)$) [5].

And the joint distribution of the actual and observed quality characteristic values can be defined as $l_{X_p,Y_p}(x,y) = g_{Y_p|X_p}(y|x).f_{X_p}(x)$ [5].

After the inspection, the joint distribution will be truncated over the observed quality characteristic y at the specification limits [20]. So, the resulting distribution of the quality characteristic of the accepted items is:

$$h_{X_a}(x) = \frac{\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dy}{\int_{-\infty}^{\infty}\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dydx} = \frac{\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dy}{\int_{LSL}^{USL} m_{Y_p}(y)dy} = \frac{1}{M}\cdot\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dy$$

where

$$M = \int_{LSL}^{USL} m_{Y_p}(y)dy$$

and $m(y)$ is the marginal distribution of Y.

We find the moments of X_a in the following way:

$$\begin{aligned} E[X_a^t] &= \int_{-\infty}^{\infty} x^t\, h_{X_a}(x)dx \\ &= \frac{1}{M}\cdot\int_{-\infty}^{\infty}\int_{LSL}^{USL} x^t\, l_{X_p,Y_p}(x,y)dydx \end{aligned}$$

Then,

$$\begin{aligned} E[X_a] &= \frac{1}{M}\cdot\{\mu_p[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\ &+ \frac{\sigma_p^2}{\sqrt{\sigma_p^2+\epsilon_p^2}}[\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - \phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})]\} \end{aligned}$$

and

$$\begin{aligned} E[X_a^2] &= \frac{1}{M}\cdot\{(\mu_p^2+\sigma_p^2)[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\ &- \frac{2\mu_p\sigma_p^2}{\sqrt{\sigma_p^2+\epsilon_p^2}}[\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - \phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\ &+ \frac{\sigma_p^4}{\sigma_p^2+\epsilon_p^2}[(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - (\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})]\} \end{aligned}$$

Details of the derivation of the moments, are provided in the Appendices A.1 and A.2.

3.2.4 Rework and Inspection Error

As the last case, we study the production environment where rework and two types of inspection error exist in both the process and the rework. We determine the distribution of the accepted items as a mixture distribution of truncated distribution of the items coming from the rework and truncated distribution of the items coming from the process. We also derive the moments of the resulting distribution.

Similar to the case of original processing unit, the conditional distribution of the observed quality characteristic Y_r of a reworked item is a normal distribution with mean x_r and a standard deviation of ϵ_r ($Y_r|X_r \sim N(x_r, \epsilon_r^2)$).

Then the joint distribution of the actual and observed quality characteristics at the rework can be defined as $h_{X_r,Y_r}(x,y) = g_{Y_r|X_r}(y|x) f_{X_r}(x)$.

After the inspection, the distribution of the reworked items will also be truncated at the specification limits with respect to the observed quality characteristic value as:

$$\frac{\int_{LSL}^{USL} h_{X_r,Y_r}(x,y)dy}{\int_{-\infty}^{\infty}\int_{LSL}^{USL} h_{X_r,Y_r}(x,y)dydx}$$

The resulting distribution of the quality characteristic X_a of all accepted items coming from both the process and rework will be a mixture of these two truncated distributions mixed at the proportions p and q, where p is the proportion of the probability that the observed quality characteristic values are within specification limits to the probability that the observed quality characteristic values are within scrap limits and q=1-p. p is calculated in the following way:

$$p = \frac{\int_{-\infty}^{\infty}\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dydx}{\int_{-\infty}^{\infty}\int_{LL_s}^{UL_s} l_{X_p,Y_p}(x,y)dydx} = \frac{\int_{LSL}^{USL} m_{Y_p}(y)dy}{\int_{LL_s}^{UL_s} m_{Y_p}(y)dy},$$

The mixture distribution of the accepted quality characteristic value X_a is

$$
\begin{aligned}
h_{X_a}(x) &= p \cdot \frac{\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dy}{\int_{-\infty}^{\infty}\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dydx} + q \cdot \frac{\int_{LSL}^{USL} h_{X_r,Y_r}(x,y)dy}{\int_{-\infty}^{\infty}\int_{LSL}^{USL} h_{X_r,Y_r}(x,y)dydx} \\
&= \frac{\int_{LSL}^{USL} m_{Y_p}(y)dy}{\int_{LL_s}^{UL_s} m_{Y_p}(y)dy} \cdot \frac{\int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dy}{\int_{LSL}^{USL} m_{Y_p}(y)dy} + q \cdot \frac{\int_{LSL}^{USL} h_{X_r,Y_r}(x,y)dy}{\int_{LSL}^{USL} n_{Y_r}(y)dy} \\
&= \frac{1}{M'} \cdot \int_{LSL}^{USL} l_{X_p,Y_p}(x,y)dy + \frac{q}{M''} \cdot \int_{LSL}^{USL} h_{X_r,Y_r}(x,y)dy
\end{aligned}
$$

where

$$
M' = \int_{LL_s}^{UL_s} m_{Y_p}(y)dy \text{and} M'' = \int_{LSL}^{USL} n_{Y_r}(y)dy
$$

The first and second moments of this distribution are:

$$
\begin{aligned}
E[X_a] &= \frac{1}{M'} \cdot \{\mu_p[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\
&+ \frac{\sigma_p^2}{\sqrt{\sigma_p^2+\epsilon_p^2}}[\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - \phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})]\} \\
&+ \frac{q}{M''} \cdot \{\mu_r[F(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}) - F(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})] \\
&+ \frac{\sigma_r^2}{\sqrt{\sigma_r^2+\epsilon_r^2}}[\phi(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}) - \phi(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})]\},
\end{aligned}
$$

and

$$
\begin{aligned}
E[X_a^2] &= \frac{1}{M'} \cdot \{(\mu_p^2+\sigma_p^2)[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\
&- \frac{2\mu_p\sigma_p^2}{\sqrt{\sigma_p^2+\epsilon_p^2}}[\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - \phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\
&+ \frac{\sigma_p^4}{\sigma_p^2+\epsilon_p^2}[(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - (\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})]\} \\
&+ \frac{q}{M''} \cdot \{(\mu_r^2+\sigma_r^2)[F(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}) - F(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})] \\
&- \frac{2\mu_r\sigma_r^2}{\sqrt{\sigma_r^2+\epsilon_r^2}}[\phi(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}) - \phi(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})] \\
&+ \frac{\sigma_r^4}{\sigma_r^2+\epsilon_r^2}[(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})\phi(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}) - (\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})\phi(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}))]\},
\end{aligned}
$$

3.3 Validation of the Formulas

The distribution and the moments of a quality characteristic in the production environment without inspection error and without rework is derived by [25]. The distribution when there is inspection error but no rework is determined by [5]. We added rework in both cases and derived the first two moments of the distributions. One way to validate the new moments is to check whether the corresponding moments of no inspection error cases can be obtained when ϵ_p and ϵ_r are assigned the value zero. When they are set to zero, it is observed that both moment formulas are the same.

Another method used to validate the formulas is to compute and compare the real and simulated mean, standard deviation and expected quality loss values. The real and simulated expected quality loss values are computed for different specification limits and different inspection error levels for both production environments with and without rework.

Real values are computed using the formulas derived. Simulation is done by using MATLAB language. The program file can be seen in Appendix A. Each run corresponds to a different production environment where 10000 items are produced at the process.

Simulation of the production environment without inspection error:
At the processing unit, quality characteristic values, X_p, of items are produced according to a normal distribution with mean 0 and standard deviation 1. Then, quality characteristic value of each item is checked; the items with quality characteristic values that are within specification limits are accepted, the ones with quality characteristic values that are out of scrap limits are scrapped and the items with quality characteristic values that are out of specification but within scrap limits are sent to rework. At rework unit, each item gains a new quality characteristic value, X_r according to a normal distribution with mean 0 and standard deviation 0.75. Then the items go under the same inspection procedure as in the processing unit.

Simulation of the production environment with inspection error:
After producing the real quality characteristic value, X_p, of an item according to a normal distribution with mean 0 and standard deviation 1 at the processing unit, the observed quality characteristic value, Y_p, of the same item is produced according to a normal distribution with mean x_p and standard deviation ϵ_p. Then, the item goes under inspection with respect to its observed quality characteristic. If the observed quality characteristic of the item is within specification limits, the item is accepted. If, the observed quality characteristic value of the item is out of scrap limits, the item is scrapped no matter what its real quality characteristic is. Lastly, if the observed quality characteristic value of the item is out of specification and within scrap limits the item is sent to rework. Since it is assumed that the reworked items will go under a more sophisticated operation, it is assumed that the standard deviation of the quality characteristics of the items reworked is less than the quality characteristics of the items produced at the process($\sigma_r < \sigma_p$). Additionally, it is observed that when $\sigma_r < \sigma_p$, the system works as if there is no rework option. That's why, to make rework meaningful and to see the effects of rework with better operation than that of the process, the above assumption is made.

When an item is reworked, it again gains a new real quality characteristic value, X_r, according to a normal distribution with mean 0 and standard deviation 0.75 just as the no inspection error case. In some situations, an item that should be scrapped may become conforming by going under relevant rework operations. That is why, we assume that even the items which are sent to rework due to inspection error instead of being scrapped can gain a new quality characteristic with respect to the distribution N(0, 0.75). Consequently, an item which is sent to rework also has a new observed quality characteristic, Y_r, produced according to a normal distribution with mean x_r and standard deviation ϵ_r. Then, the item goes under inspection, the same procedure as at the end of the processing unit is applied and the item is accepted, reworked or scrapped accordingly.

At the end of each run, the mean, standard deviation and expected loss for the true quality characteristic values of the accepted items are calculated. Target is accepted as 0 (T=0), and the loss coefficient is taken as 2 (c=2) while calculating the expected quality loss.

Simulation results are summarized by calculating the averages of 30 replicates of each run. The program is ran for 41 different levels of specification limits and 11 different levels of standard deviation of inspection error. Hence, there are totally 451 cases. Table 3.2 summarizes the set of parameters used to define the production environments.

Table 3.2: Set of Parameters Used in the Validation of the Formulas

Distributions	$X_p \sim N(0,1)$ $X_r \sim N(0,0.75)$
USL	1 to 5
LSL	-5 to -1
ULs	2 to 6
LLs	-6 to -2
ϵ_p	0 to 1
ϵ_r	0 to 0.75

First, the production environment with a separate rework station is considered. The distribution of the quality characteristic of the items produced by the process is assumed to be $N(0,1)$ and the distribution of the quality characteristic of the items coming from rework is assumed to be $N(0,0.75)$. The variety of scenarios for which the parameters and expected loss values are calculated are described in the following:

- Upper specification limit (USL) changes between 1 and 5 by 0.1 units.
- Lower specification limit (LSL) for each case is equal to -USL.
- Upper scrap limit (ULs) for each case is equal to USL+1.
- Lower scrap limit (LLs) for each case is equal to -LSL-1.
- Standard deviation of the inspection error at process (ϵ_p) changes between 0 and 1 by 0.1 units. The case $\epsilon_p = 0$ represents 'no inspection error'.
- Standard deviation of the inspection error at rework (ϵ_r) is equal to ϵ_p until $\epsilon_p = 0.7$. After ϵ_p exceeds this value, ϵ_r is taken to be equal to 0.75, the standard deviation of the quality characteristics of item at rework.

Similar steps are taken for a no rework production environment. The conditions are:

- Upper specification limit (USL) changes between 1 and 5 by 0.1 units.
- Lower specification limit (LSL) for each case is equal to -USL.
- Standard deviation of inspection error at the process (ϵ_p) changes between 0 and 1 by 0.1 units. The case $\epsilon_p = 0$ represents 'no inspection error'.

When Table 3.3 is checked, the closeness of the simulated and true expected quality loss, mean and standard deviation of the items can be noticed. The small values of maximum and average % relative deviations validate the accuracy of the formulas derived.

Table 3.3: Differences between the true and simulated quality loss, standard deviation and mean of accepted items for different production environments

		Production Environment			
		Rework ϵ_p=ϵ_r=0	Rework $0 < \epsilon_p < 1$ $0 < \epsilon_p < 0.75$	No Rework ϵ_p=0	No Rework $0 < \epsilon_p < 1$
Loss	Maximum Deviation	0.0117	0.0145	0.0128	0.0159
	Average % Relative Deviation	2.1054	0.1929	2.1285	0.1972
Std. Dev.	Maximum Deviation	0.0028	0.0036	0.0102	0.0587
	Average % Relative Deviation	1.0429	0.0956	9.3766	0.8940
Mean	Maximum Deviation	0.0051	0.0058	0.0102	0.0056
	Average % Relative Deviation	0.0153	0.0014	0.0147	0.0014

CHAPTER 4

EXPERIMENTAL RESULTS AND DISCUSSION

In this chapter the effects of inspection error and rework on expected quality loss are examined.

One may expect an increase in the loss function as the inspection error increases. However, this expectation is realized only for a limited number of specification limits. As specification limits get wider and wider, the expected loss values show a decrease with increasing inspection error. This is true for both production environments with and without rework. For the production environment without rework, after C_p exceeds 0.56 and for the production environment with rework, after C_p exceeds 0.5, we observe descending expected quality loss values corresponding to ascending standard error of inspection error. The expected quality loss values computed using the moments derived in Chapter 3 for a selected number of specification and scrap limits are presented in Figure 4.1 and Figure 4.2. The expected quality loss values are given in Appendices C.1 and C.2 (The figures present the results for only selected specification and scrap limits. Data are given through Appendices C-O).

As a reason for this decrease, it is supposed that as specification limits get wider and wider relative to the process standard deviation, that is, as the process capability increases, the system produces more items that are within specification limits and accepts less nonconforming items due to inspection error. We suppose that, although the number of non-conforming items that are accepted also increases with increasing inspection error, this number is always less than the growth in the number of conforming items that are scrapped. The quality loss corresponding to those scrapped items (which is not included in the expected quality loss) is bigger than the loss cor-

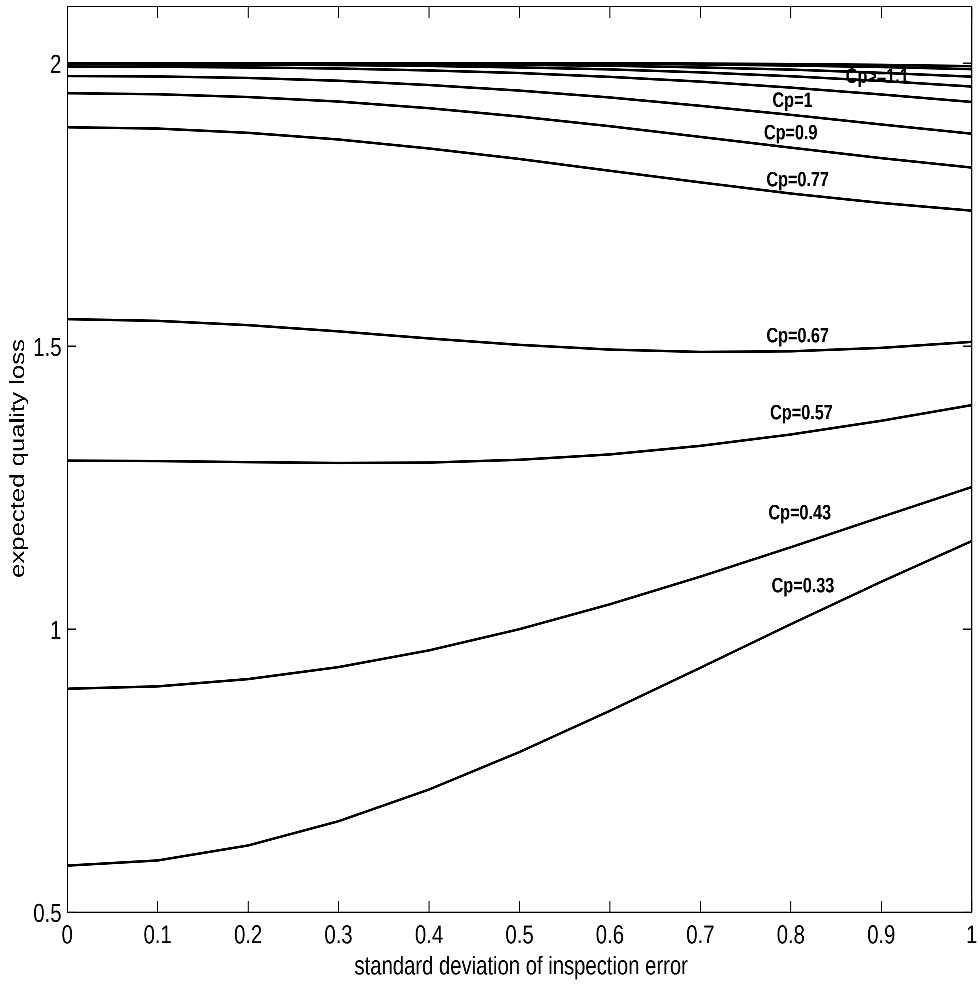

Figure 4.1: Expected Quality Loss Values in a Production Environment Without Rework ($X_p \sim N(0, 1)$, $0 \leq \epsilon_p \leq 1$, T=0, c=2

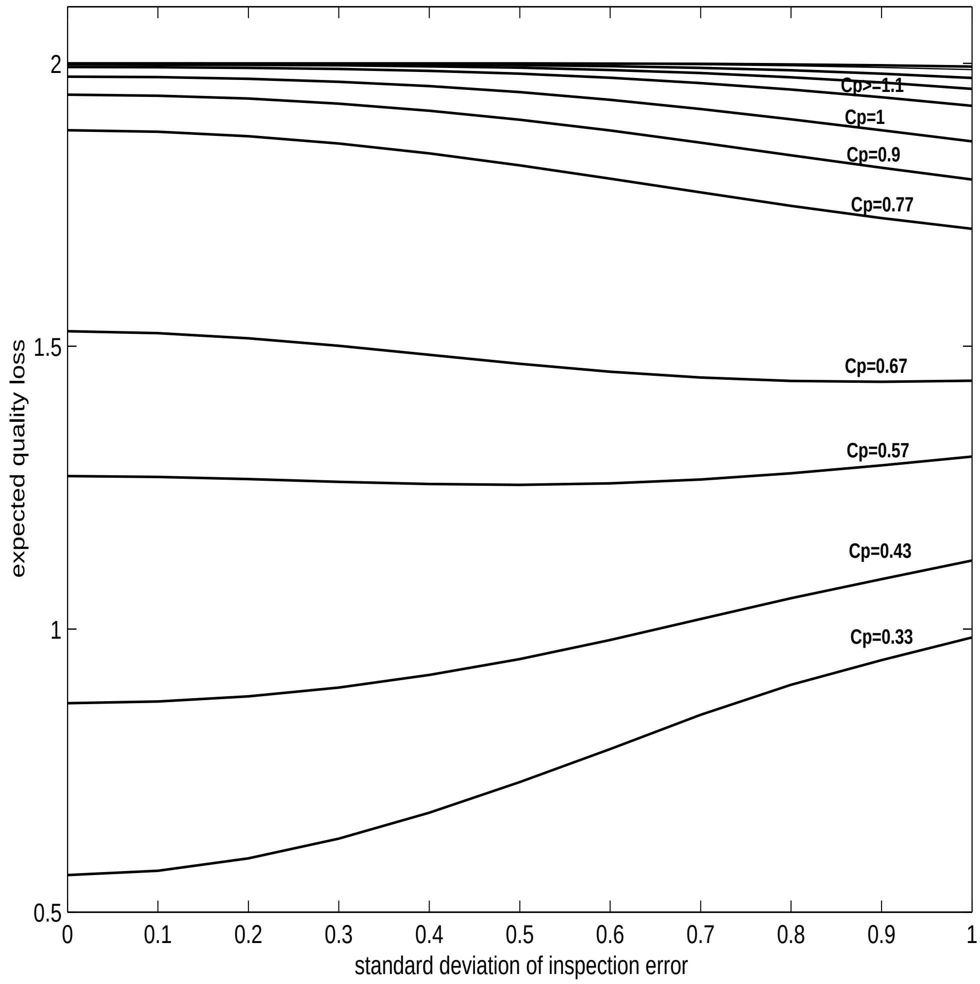

Figure 4.2: Expected Quality Loss Values in a Production Environment With Rework($X_p \sim N(0,1)$, $X_r \sim N(0,0.75)$, $0 \leq \epsilon_p \leq 1$, $0 \leq \epsilon_r \leq 0.75$, T=0, c=2)

responding to the accepted nonconforming items, hence the loss decreases.

We additionally think that although they both increase as inspection error increases, the number of conforming items that are sent to rework erroneously is always bigger than the number of items that are accepted due to inspection error instead of being reworked. And, the items, conforming or non-conforming, which are sent to rework return with better quality characteristics than that of items coming from process.

4.1 Simulation Results

4.1.1 Production Environment Without Rework

We first simulated the production environment without rework to obtain detailed information about the system. The details of the simulation environment is given in Section 3.3.

In a production environment without rework, all the items which are determined to be out of specification limits are scrapped. So, in this case, inspection error causes only erroneous scrap of conforming items or erroneous acceptance of non-conforming items.

We can define the relationship between the total loss of accepted items when there is no inspection error and the loss of the ones when there is inspection error as follows:

$$\sum L_0 = \sum L_1 - \sum L_{as} + \sum L_{sc}$$

where

$\sum L_o$ = total loss with no inspection error

$\sum L_1$ = total loss with inspection error

$\sum L_{sc}$ = total loss of conforming items which are scrapped due to inspection error

$\sum L_{as}$ = total loss of non-conforming items which are accepted due to inspection error

When we look at the simulation results, we see that, for all cases, the number of scraps and acceptances due to inspection error both increase as inspection error increases. However, as the system becomes better (as C_p increases), the proportion of number of items scrapped due to inspection error to the number of items accepted due to inspection error also increases (see Appendix D).

Hence, for a system with bad process capability, although the number of scrapped conforming items increases, the system also accepts a large amount of non-conforming items. Since the quality loss corresponding to the nonconforming items which are accepted is greater than the quality loss corresponding to the conforming items which are scrapped ($\sum L_{as} > \sum L_{sc}$), the expected quality loss increases as inspection error increases (see Appendices E.1 and E.2).

However, as C_p increases, the number of erroneous acceptances become relatively much smaller than the wrong scraps (see Appendices D, F.1 and F.2). This time, the quality loss corresponding to the conforming items which are scrapped becomes greater than the quality loss corresponding to the nonconforming items which are accepted ($\sum L_{sc} > \sum L_{as}$)(see Appendices E.1 and E.2). The quality loss corresponding to those scrapped conforming items ($\sum L_{sc}$) is not included in the calculation of the expected quality loss. Although there is the extra loss of the non-conforming items which are accepted ($\sum L_{as}$), they are not as much as the wrong scraps. As a result, the total quality loss of the accepted items show a decline as the inspection error increases for better systems.

The number erroneous acceptances and scraps for selected specification limits are visualized in Figure 4.3 and Figure 4.4 (refer to Appendices F.1 and F.2 to see all

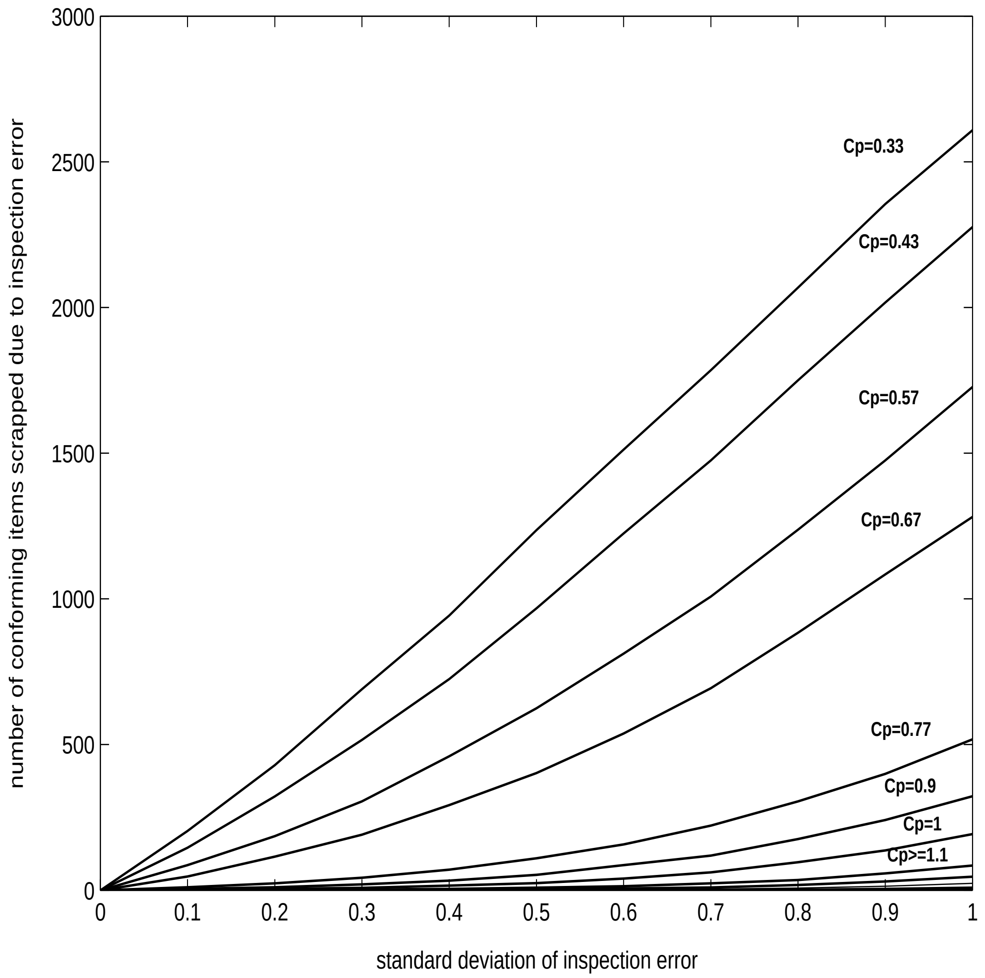

Figure 4.3: Number of Conforming Items Scrapped Due To Inspection Error in a Production Environment Without Rework ($X_p \sim N(0, 1)$, $0 \leq \epsilon_p \leq 1$)

cases).

Another way to investigate the decrease in the expected quality loss as inspection error increases is to check the change in the distribution of the accepted items. We figure out the change in the distribution of the accepted items by drawing the histograms of the distributions. In Figure 4.5, histograms of the true and observed quality characteristics of the accepted items for a variety of inspection error levels are presented. In this case, USL, LSL, ULs and LLs are assumed to be 2, -2, 3 and -3, respectively, and $X_p \sim N(0, 1)$.

It is observed that as inspection error increases, the distribution of the true qual-

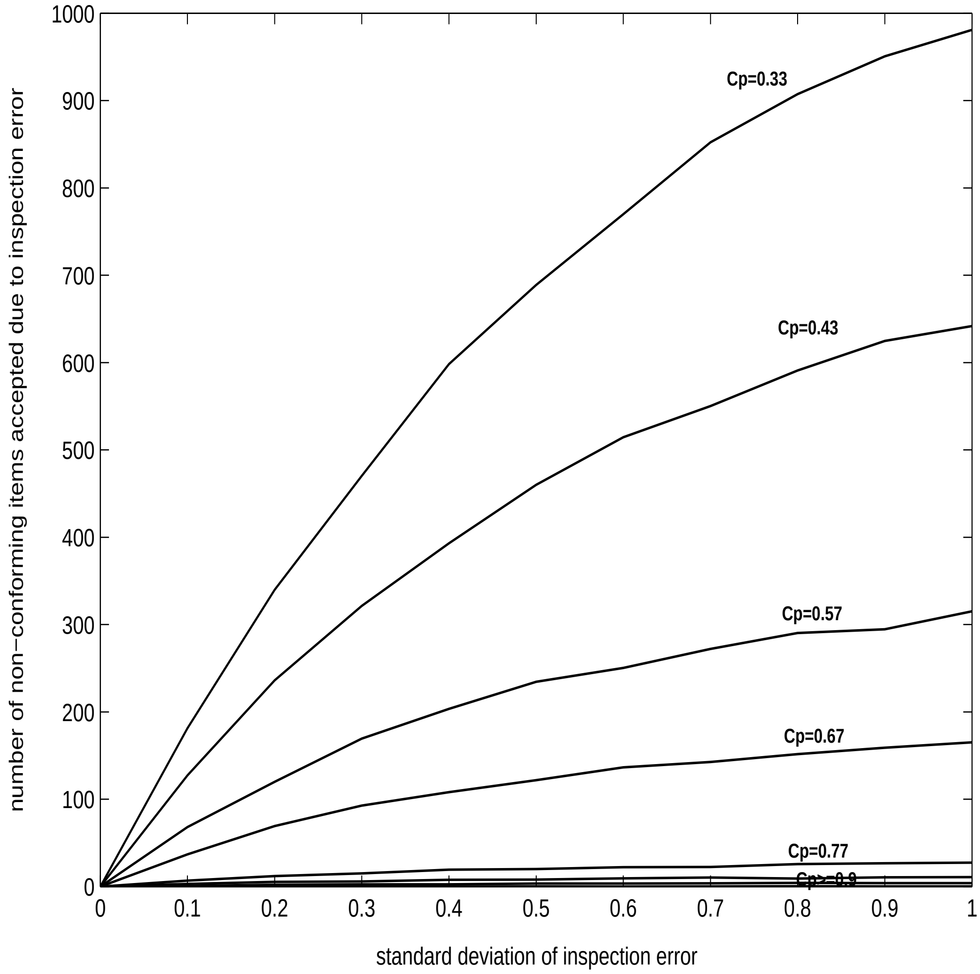

Figure 4.4: Number of Non-Conforming Items Accepted Due To Inspection Error Instead of Being Scrapped in a Production Environment Without Rework ($X_p \sim N(0,1)$, $0 \leq \epsilon_p \leq 1$)

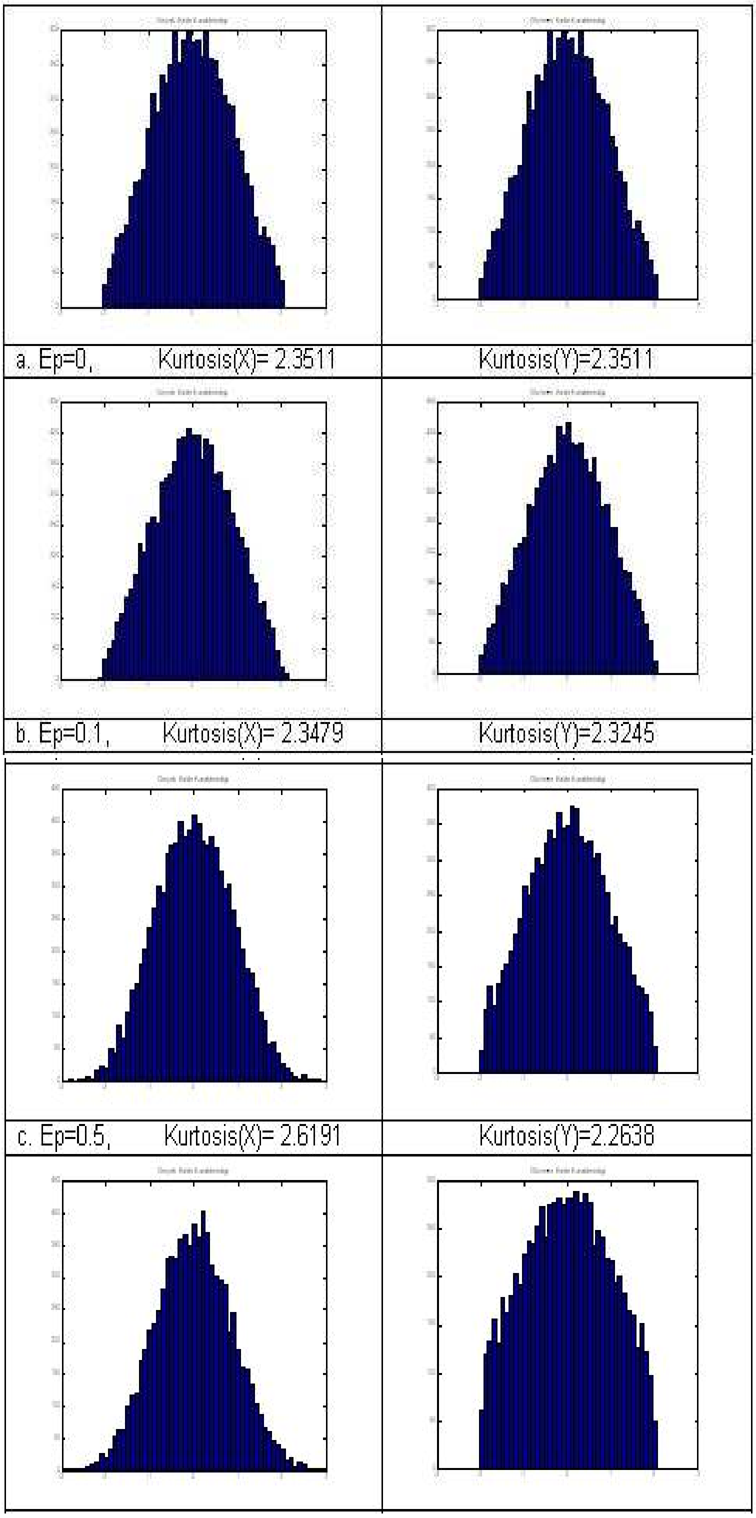

Figure 4.5: Histogram of True and Observed Data for a Production Environment Without Rework ($X_p \sim N(0,1), LSL = -2, USL = 2$)

ity characteristic values of the accepted items become more and more concentrated around the mean, hence target, and seem to get closer to a normal distribution. On the other hand, the distribution of the observed quality characteristic values become more like a uniform distribution as inspection error increases. Additionally, when the kurtosis values of the distributions are compared, it is recognized that the kurtosis of the true characteristic values is always greater than that of the observed quality characteristic values and is much closer to 3 which is the kurtosis of a normal distribution.

To strengthen the above claim, the standard deviations of the true and observed quality characteristic values of the accepted items are also recorded (refer to Appendices H.1 and H.2). It is recognized that, for higher values of specification limits, the standard deviation of the true quality characteristic values seem to decrease for increasing values of inspection error. But the standard deviation of the observed quality characteristic values increases more rapidly as inspection error increases for wider specifications (Figure 4.6 and Figure 4.7).

Excess Cost:

The decrease in the quality loss does not mean that inspection error is advantageous for the producer. It is observed that the number of conforming items scrapped due to inspection error always increases as inspection error increases for any case. And each time a conforming item is scrapped, the manufacturer faces an excess cost.

We accept this cost of scrapping a conforming item as the manufacturing cost of that item up to that processing step.

In the literature, manufacturing cost is generally divided into three parts which are:

1. Direct materials
2. Direct labor
3. Manufacturing overhead

Direct materials include the materials that enter into and become a part of the finished product [12]. They include all the materials and parts purchased from a

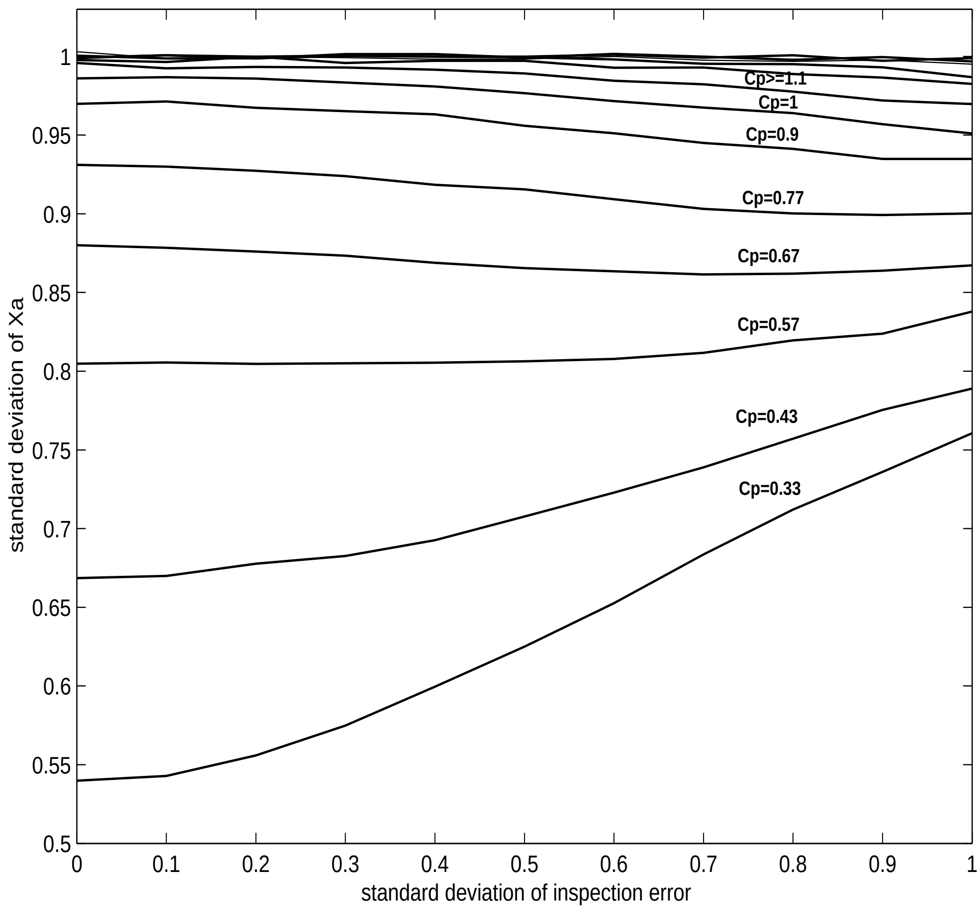

Figure 4.6: Standard Deviation of the True Quality Characteristics of the Accepted Items in a Production Environment Without Rework ($X_p \sim N(0,1)$, $0 \leq \epsilon_p \leq 1$)

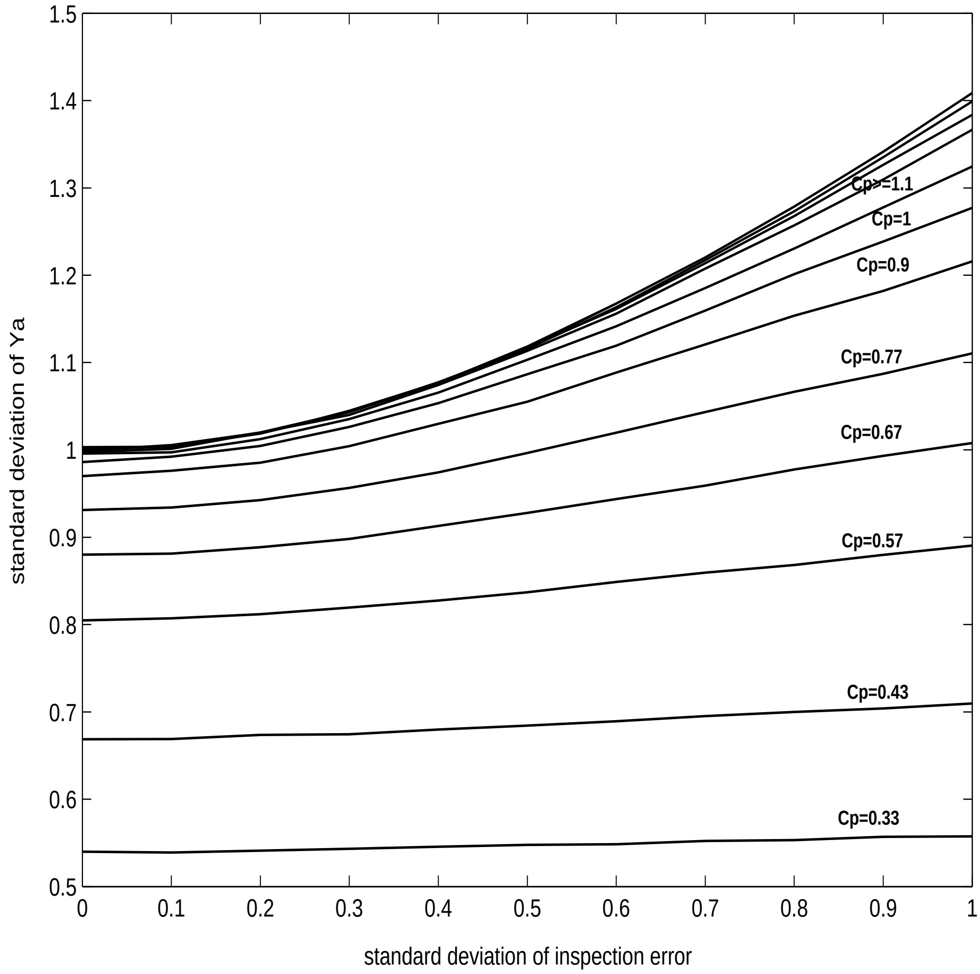

Figure 4.7: Standard Deviation of the Observed Quality Characteristics of the Accepted Items in a Production Environment Without Rework ($X_p \sim N(0,1)$, $Y_p \sim N(x, \epsilon_p)$, $0 \leq \epsilon_p \leq 1$)

supplier and integrated into the product as well as the materials that are used and operated by the manufacturer [15].

Direct labor includes the cost of the employees who work on the product in person. The laborer who do not directly work in the production phase but contribute to production indirectly such as by supervising or engineering are accepted as indirect labor and their cost is included in manufacturing overhead.

Manufacturing overhead includes all the costs of manufacturing except direct materials and direct labor such as maintenance and repairs of product equipment, heat and light, indirect materials and indirect labor, taxes, depreciation and insurance on manufacturing facilities, etc. [15].

We accept the manufacturing cost of a scrapped conforming item as all the expenses made for that item up to the current work station.

It is clearly observed that as standard deviation of inspection error increases, the scrapping cost (excess manufacturing cost) per item increases (Figure 4.8 and Appendix I).

4.1.2 Production Environment With Rework

We secondly simulated the production environment with rework. The details of the simulation are given in Section 3.3.

The relationship between the total quality loss with no inspection error and the total quality loss with inspection error in an environment with rework can be defined as follows:

$$\sum L_0 = \sum L_1 + \sum L_{sc} + \sum L_{sr} - \sum L_{as} - \sum L_{ar} + \sum L_{er}$$

where

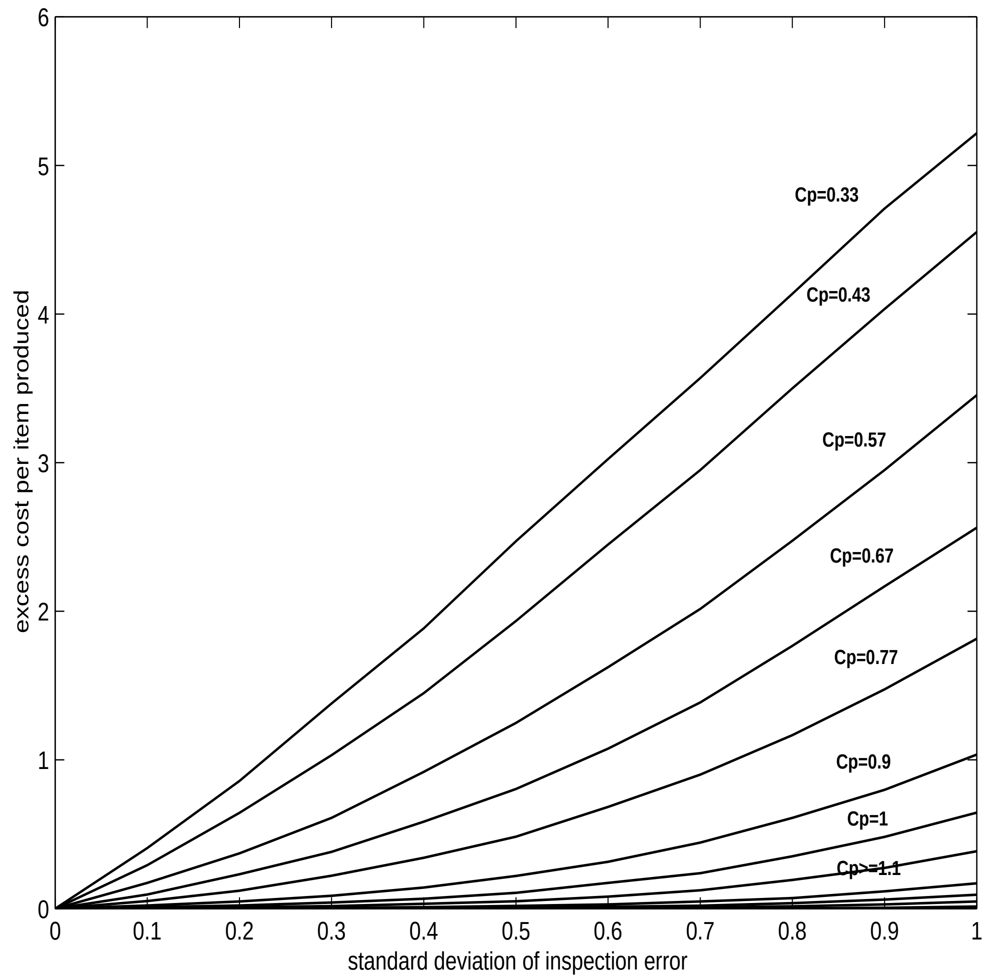

Figure 4.8: Excess Cost per Item Produced in a Production Environment Without Rework ($X_p \sim N(0,1)$, $Y_p \sim N(x, \epsilon_p)$, $0 \leq \epsilon_p \leq 1$)

$\sum L_0$= total loss with no inspection error

$\sum L_1$= total loss with inspection error

$\sum L_{sc}$= total loss of conforming items which are scrapped due to inspection error

$\sum L_{sr}$= total expected loss of items which are scrapped due to inspection error instead of being reworked

$\sum L_{as}$= total loss of items accepted instead of being scrapped due to inspection error

$\sum L_{ar}$ = total loss of items accepted instead of being reworked due to inspection error

$\sum L_{er}$= total loss due to erroneously reworked items

and

$$\sum L_{er} = \sum L_{rc} - \sum L_{nrc} - \sum L_{nrs}$$

where

$\sum L_{er}$ = total loss due to erroneously reworked items

$\sum L_{rc}$ = total loss of conforming items which are reworked instead of being accepted due to inspection error

$\sum L_{nrc}$ = sum of the new quality loss of erroneously reworked conforming items

$\sum L_{nrs}$ =sum of the new quality loss of items which are erroneously reworked instead of being scrapped

The flow of items and the corresponding losses are shown in Figure 4.9.

Similar to the case where there is no rework, the proportion of items that are scrapped erroneously to the number of items which are accepted instead of being scrapped show a generally increasing behavior for better process capabilities (see Appendix J.1). Additionally, in a production environment with rework, the same is true for the proportion of number of conforming items sent to rework due to inspection error and the number of items which are accepted instead of being sent to rework (see Appendix J.2). Consequently, the loss gained due to erroneously accepted or reworked items ($\sum L_{as} + \sum L_{ar} + \sum L_{nrc} + \sum L_{nrs}$) is less than the actual released loss due to the items which are erroneously scrapped or reworked instead of being accepted ($\sum L_{sc} + \sum L_{sr} + \sum L_{rc}$). This leads to the decrease in the expected quality loss with increasing standard deviation of inspection error as system gets better.

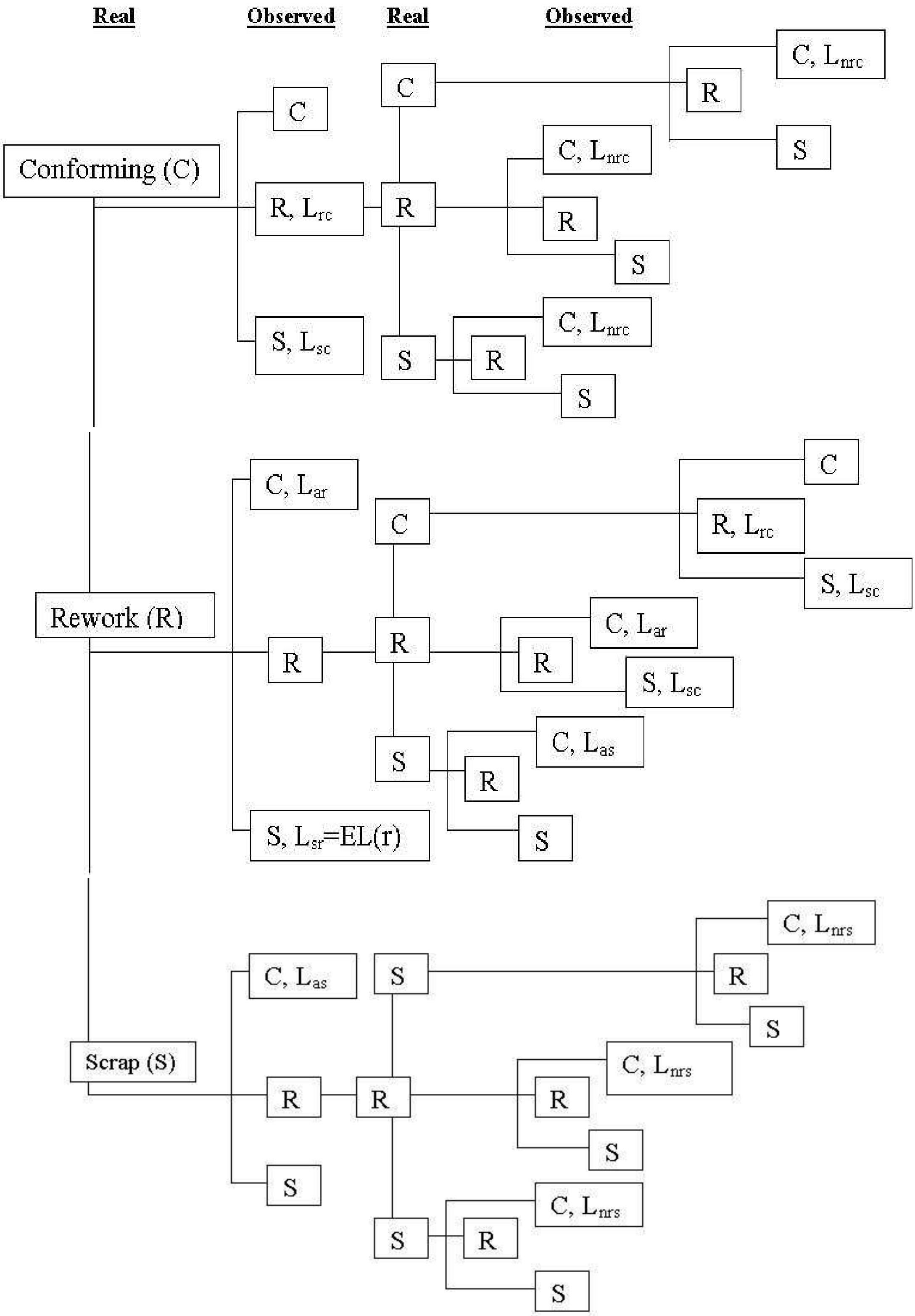

Figure 4.9: Tree Diagram of Flow of Items Produced

When we refer to Appendix K, it is seen that the increasing behavior of the difference of the total loss of items that is excluded due to inspection error ($\sum L_{sc} + \sum L_{sr} + \sum L_{rc}$) and the total loss of items that is included due to inspection error ($\sum L_{as} + \sum L_{ar} + \sum L_{nrc} + \sum L_{nrs}$) supports our claim.

We assume that the items which are sent to rework go under a more sophisticated operation, that is why we think that either conforming or non-conforming, the items which are sent to rework return with better quality characteristics, thus lower quality losses. It is observed that this claim is true (see Appendices L.1 and L.2). This may also be a reason of the decrease in the quality loss.

Another fact is that, reworking the nonconforming items with a better process than the usual process ($\sigma_p > \sigma_r$) also decreases the expected quality loss of the system. In the Appendices C.1 and C.2, it is clearly seen that the expected quality loss value for particular specification limits and standard deviation of inspection error in a production environment with rework is always smaller than that in a production environment without rework.

It is also observed that, for a given inspection error, the numbers of erroneous scrap or rework decrease as specification limits get wider and wider. The increase in the loss for a given inspection error as C_p increases is because the loss coefficient is kept constant.

Again the histograms and the kurtosis values (Figure 4.10) of the true and observed quality characteristics endorse the statements claimed by the authors about the decrease in the loss function corresponding to the increase in standard deviation of inspection error.

The descending behavior of the standard deviation of the true quality characteristic values and the opposite ascending behavior of the observed quality characteristic values can be examined in Figure 4.11 and Figure 4.12, respectively (refer to Appendices M.1 and M.2).

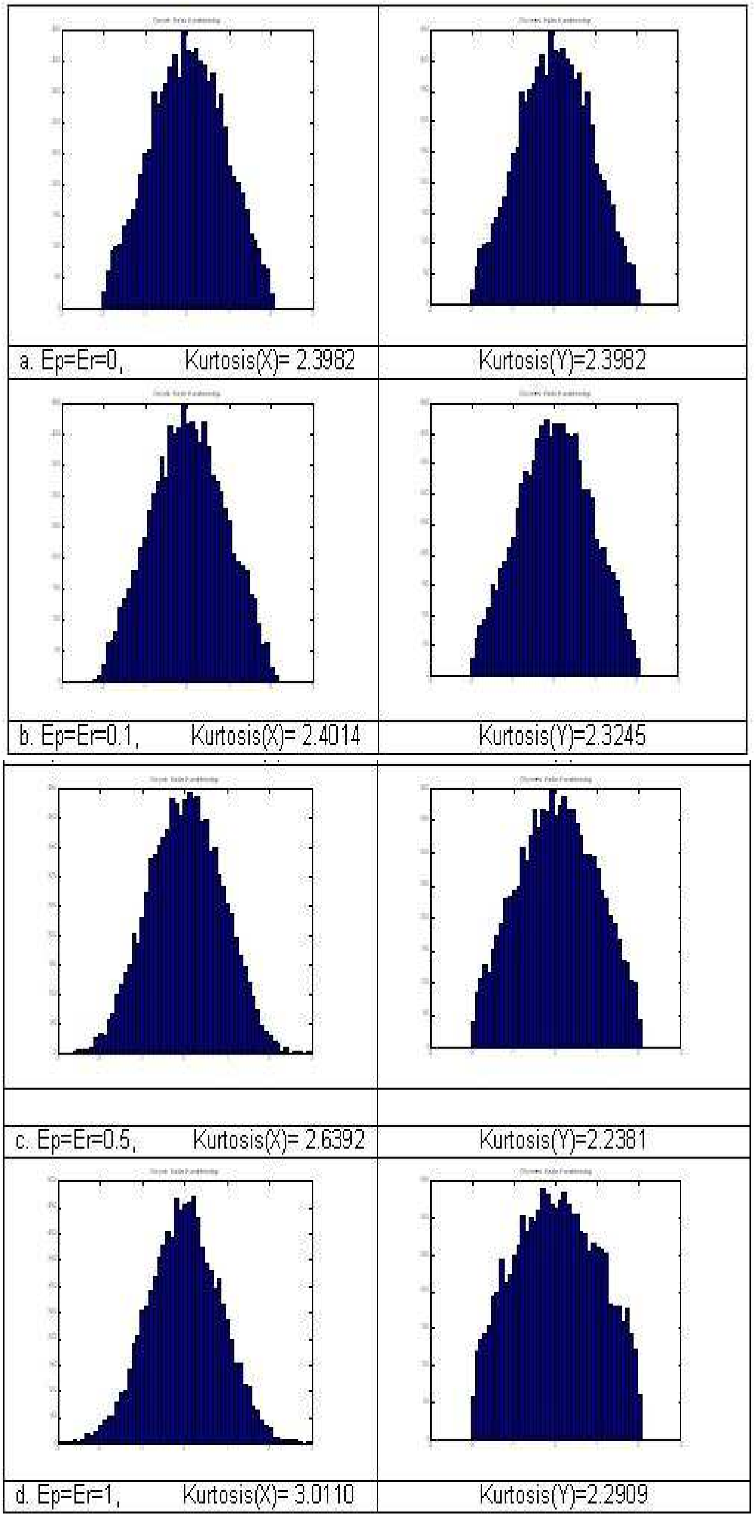

Figure 4.10: Histogram of True and Observed Data for a Production Environment With Rework ($X_p \sim N(0,1)$, $X_r \sim N(0,0.75)$, LSL=-2, USL=2, LLs=-3, ULs=3)

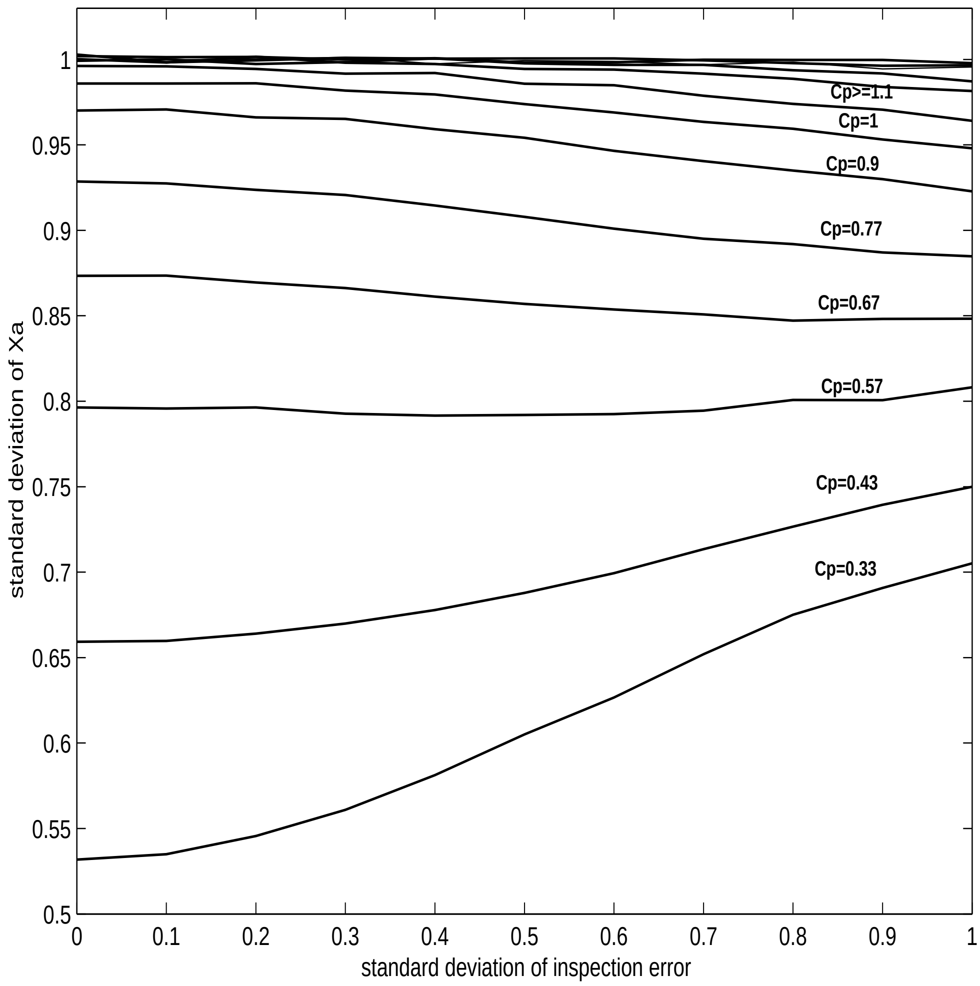

Figure 4.11: Standard Deviation of the True Quality Characteristics of the Accepted Items in a Production Environment with Rework ($X_p \sim N(0,1)$, $X_r \sim N(0,0.75)$, $0 \leq \epsilon_p \leq 1$, $0 \leq \epsilon_r \leq 0.75$)

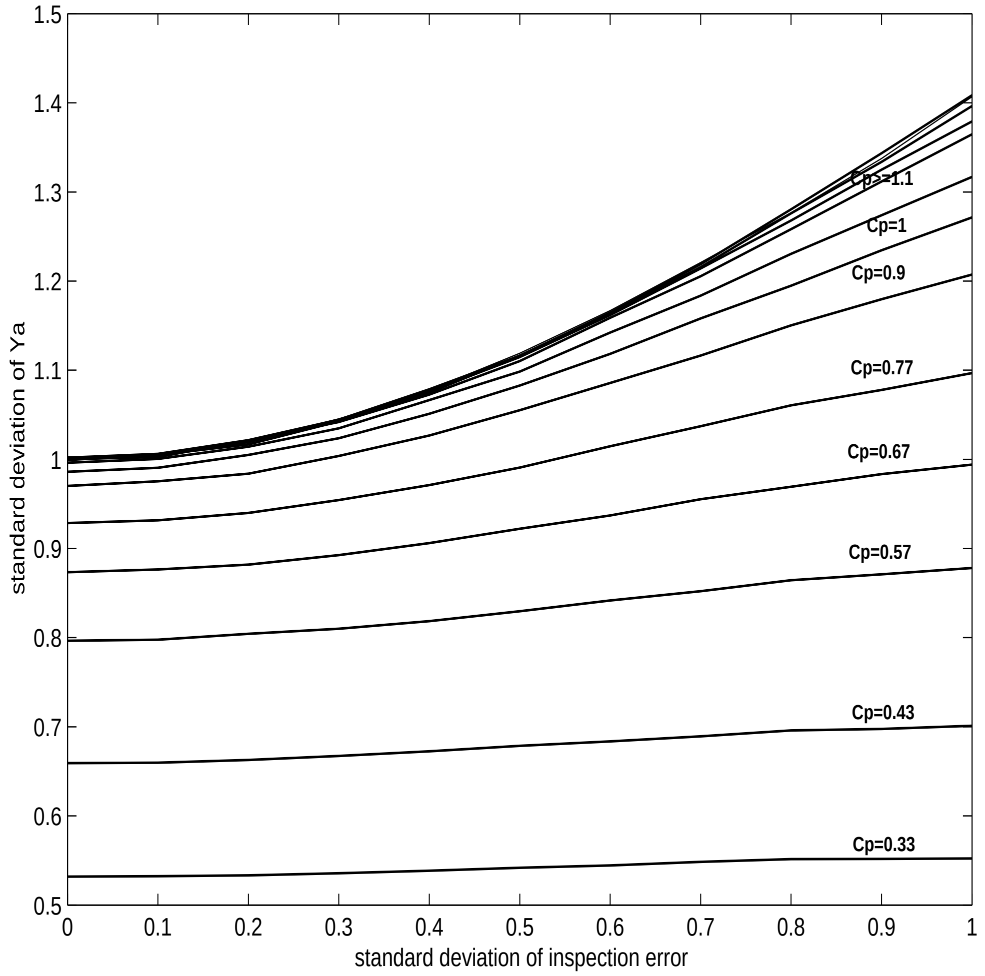

Figure 4.12: Standard Deviation of the Observed Quality Characteristics of the Accepted Items in a Production Environment with Rework ($X_p \sim N(0,1)$, $X_r \sim N(0,0.75)$, $Y_p \sim N(x_p,\epsilon_p)$, $Y_r \sim N(x_r,\epsilon_r)$, $0 \leq \epsilon_p \leq 1$, $0 \leq \epsilon_r \leq 0.75$)

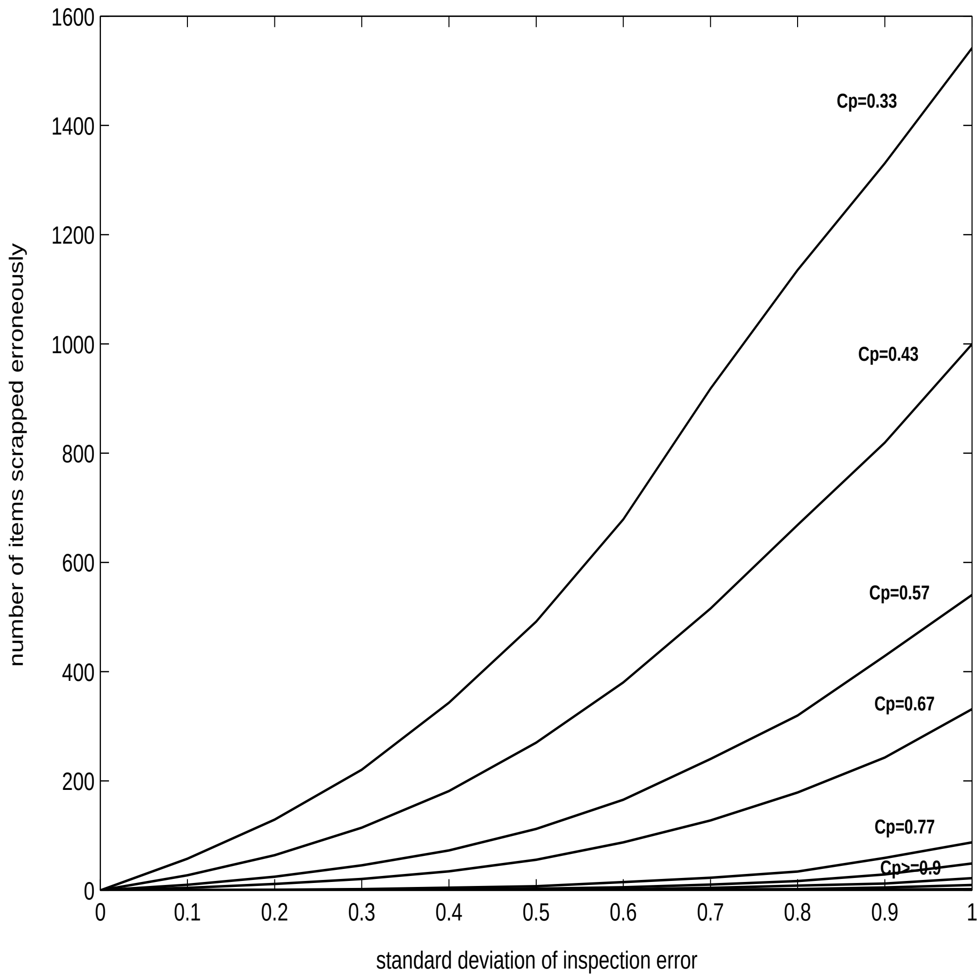

Figure 4.13: Number of Conforming Items Scrapped Due To Inspection Error in a Production Environment With Rework ($X_p \sim N(0,1)$, $X_r \sim N(0,0.75)$, $0 \leq \epsilon_p \leq 1$, $0 \leq \epsilon_r \leq 0.75$)

The decrease in the expected quality loss as inspection error gets bigger and bigger does not indicate that increasing inspection error is good for a system.

In spite of the fact that the expected quality loss values show a decline as inspection error increases for most of the cases, an obvious increment is observed in the number of items which are scrapped as inspection error increases for the same cases (Appendix N.1). A similar growth is also true for the number of conforming items which are sent to rework (Appendix N.2). These increases are clearly seen in Figure 4.13 and Figure 4.14.

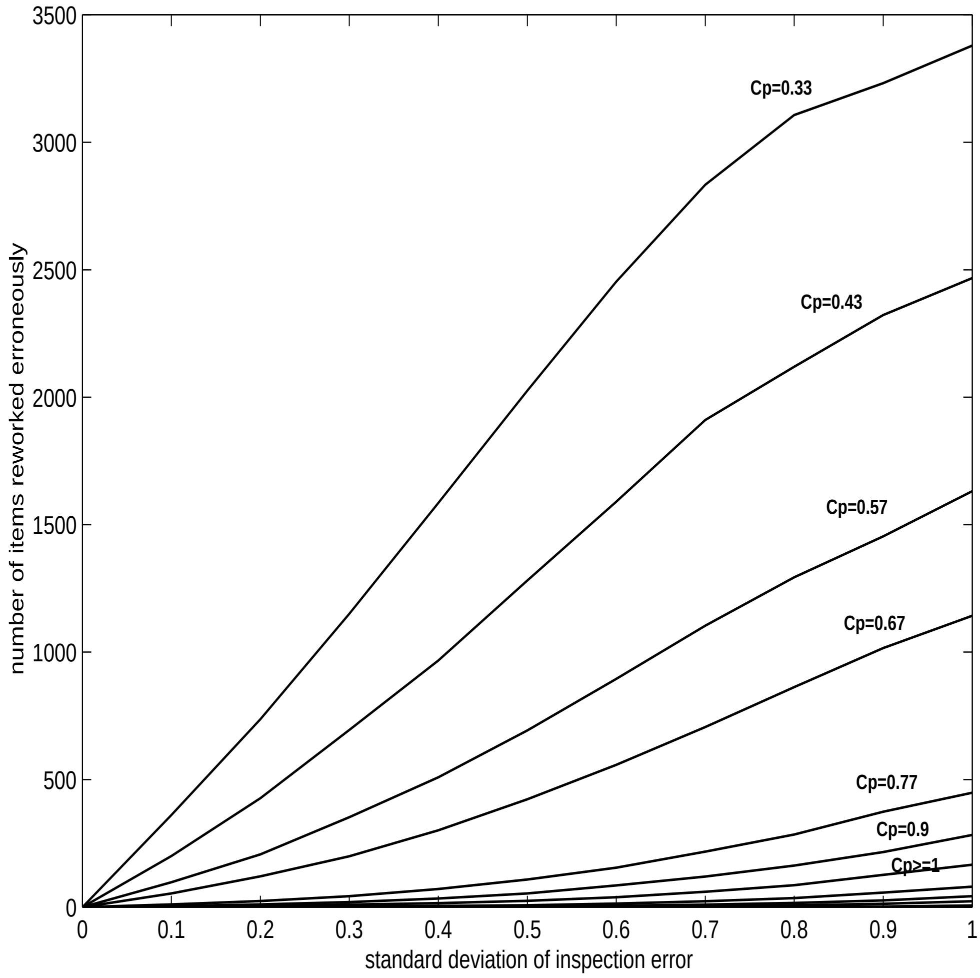

Figure 4.14: Number of Conforming Items Reworked Due To Inspection Error in a Production Environment With Rework ($X_p \sim N(0,1)$, $X_r \sim N(0,0.75)$, $0 \leq \epsilon_p \leq 1$, $0 \leq \epsilon_r \leq 0.75$)

That is why, for any specification limit, as inspection error increases, the number of erroneous scraps and reworks always increase which adds extra manufacturing, rework and inspection costs.

Excess Cost

In a production environment with rework and which is subject to inspection error, there are three types of wrong decisions which lead to some excess cost to the producer:

1. scrapping a conforming item : $C_{scrap} = C_{manuf}$
2. scrapping an item that should be reworked : $C_{scrap} = C_{manuf} - EC(r)$
3. reworking a conforming item or an item that should be scrapped : C_{rework} = C_{rework} + $C_{inspection}$

We assume that when a conforming item is scrapped, the producer has a n extra manufacturing cost as in the non-rework production environment. When an item that should be reworked is scrapped due to inspection error, an excess cost which is the difference between the cost of manufacturing that item up to that station and the expected cost when the item is reworked. That is;

$$C_{scrap} = C_{manuf} - EC(r)$$

When an item that is conforming or that should be scrapped is reworked due to inspection error, the manufacturer meets excess rework and inspection cost each time the item is reworked either correctly or due to inspection error. To show that although the expected quality loss decreases as inspection error increases, the excess scrapping rework and inspection costs with increasing inspection error, these costs are recorded separately.

Furthermore, the excess cost per item is also calculated.

Figure 4.15 indicates that inspection error has an adverse affect on these costs. It is observed that the average excess cost per item produced increases as standard deviation of inspection error increases for any system.

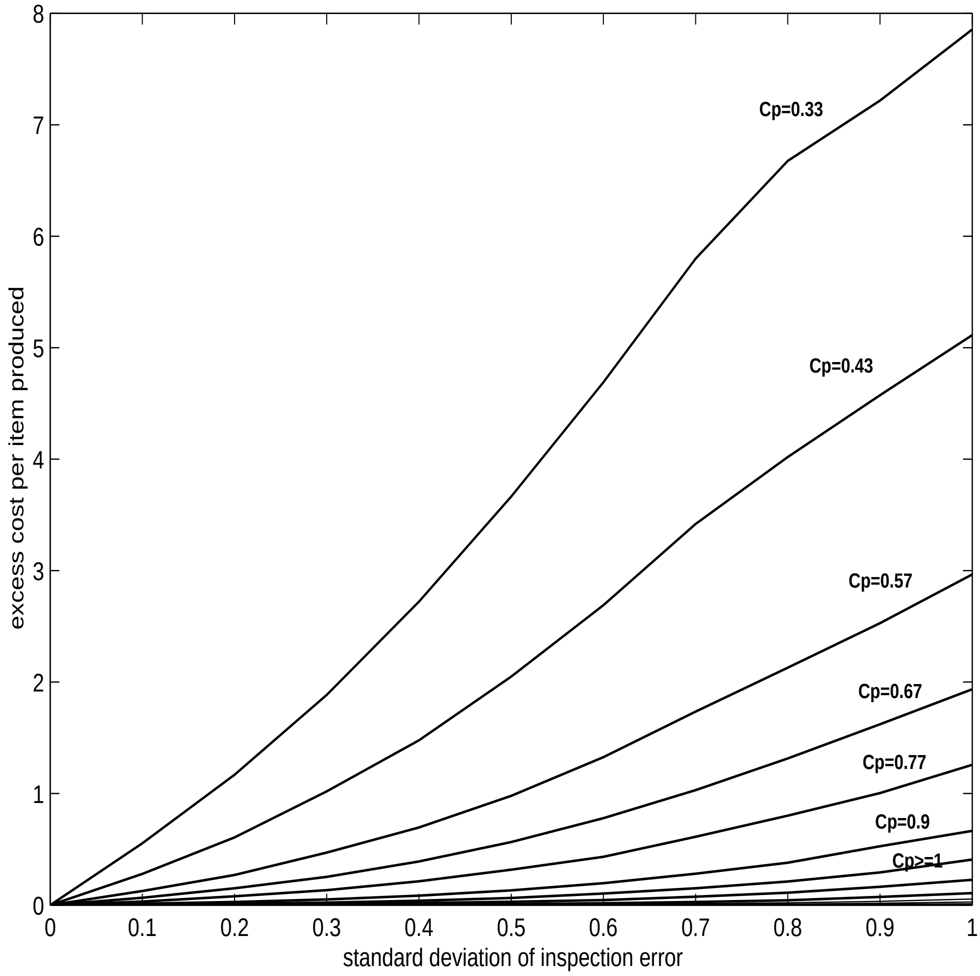

Figure 4.15: Average Excess Cost per Item Produced in a Production Environment With Rework ($X_p \sim N(0,1)$, $Y_p \sim N(x, \epsilon_p)$, $0 \leq \epsilon_p \leq 1$)

4.2 Discussion

In this study, we have studied the effects of inspection error and rework on quality loss. The anticipated result is that, the increasing standard deviation of inspection error also causes an increase in the quality loss. But, as the process capability gets better, a decrease is observed in the expected quality loss while inspection error increases. When we seek for the possible reasons of this result, in a production environment without rework, we recognize that the number of items that are scrapped due to inspection error appear to be more than the number of items which are erroneously accepted. Hence, the total released loss corresponding to the items which are erroneously scrapped becomes much more than the gain due to the total loss corresponding to the items which are erroneously accepted.

Similarly, for a production environment with rework, we find that as process becomes better, the number of items that are scrapped or reworked due to inspection error always appear to be more than the number of items accepted by mistake. The conforming items which are scrapped due to inspection error never return to the system again. And it is observed that the items that are reworked return to the system with better quality characteristics. That is why, the total quality loss (which is excluded from the average loss of the accepted items) corresponding to the items reworked or scrapped by mistake becomes bigger than the total quality loss (which is included in the average quality loss of accepted items) corresponding to the nonconforming items that are accepted due to inspection error with increasing standard deviation of inspection error as the system gets better and better.

The above findings explain the decrease in the expected quality loss of accepted items as standard deviation of inspection error increases for systems with better and better process capabilities in both production environments with and without rework.

Another observation about the accepted items as inspection error increases for a moderate process capability is that, the distribution of the quality characteristics of the accepted items becomes more and more peak and concentrated around the target value, which looks like normal distribution while inspection error increases. This also explains the decrease in the expected quality loss of accepted items as the standard deviation of inspection error increases with better process capability.

Even though these results seem to indicate that increasing inspection error is even good for production processes, growing number of scrap and rework of the conforming items proves the opposite. No matter what the process capability of a system is, increasing measurement error always brings about increasing number of erroneous scraps and reworks. These mean excess manufacturing, reworking and inspection costs. We computed the average excess cost per item produced and observed that this average excess cost always increases as gage system becomes worse, even though the expected quality loss seems to decrease.

Figure 4.16 and Figure 4.17 show some indicators for production systems with bad process capabilities (Cp=0.5) with and without rework, respectively. It is clearly seen that for systems with bad process capabilities, even a small standard deviation value of inspection error points out a significant badness in the measurement system. The figures clearly show the increase in the average quality loss with increasing standard deviation of inspection error. Besides the increasing quality loss, a sharp growth in the amount of scraps and reworks due to inspection error are observed. For a production system with bad capacity, the increase in the wrong scrap and wrong rework starts at even very small levels of measurement error (P/T ratio). These erroneous reworks and scraps mean excess cost for the manufacturer. Hence, the increase in the average excess cost in Figure 4.16 and Figure 4.17 are noticeable.

When the system gets better, the number of scraps and reworks due to inspection error seem to decrease relative to a worse production process. In Figure 4.18, average quality loss values, number of erroneous reworks and scraps are seen. Although the average loss declines as inspection error increases, the amount of wrong reworks reaches the top where expected loss is the minimum. The ascending view of reworks and scraps due to inspection error still must be taken into account. Nevertheless, since the increase in the number of scraps become more sharper after standard deviation of inspection error (and also P/T ratio) is 0.8, if manufacturing cost

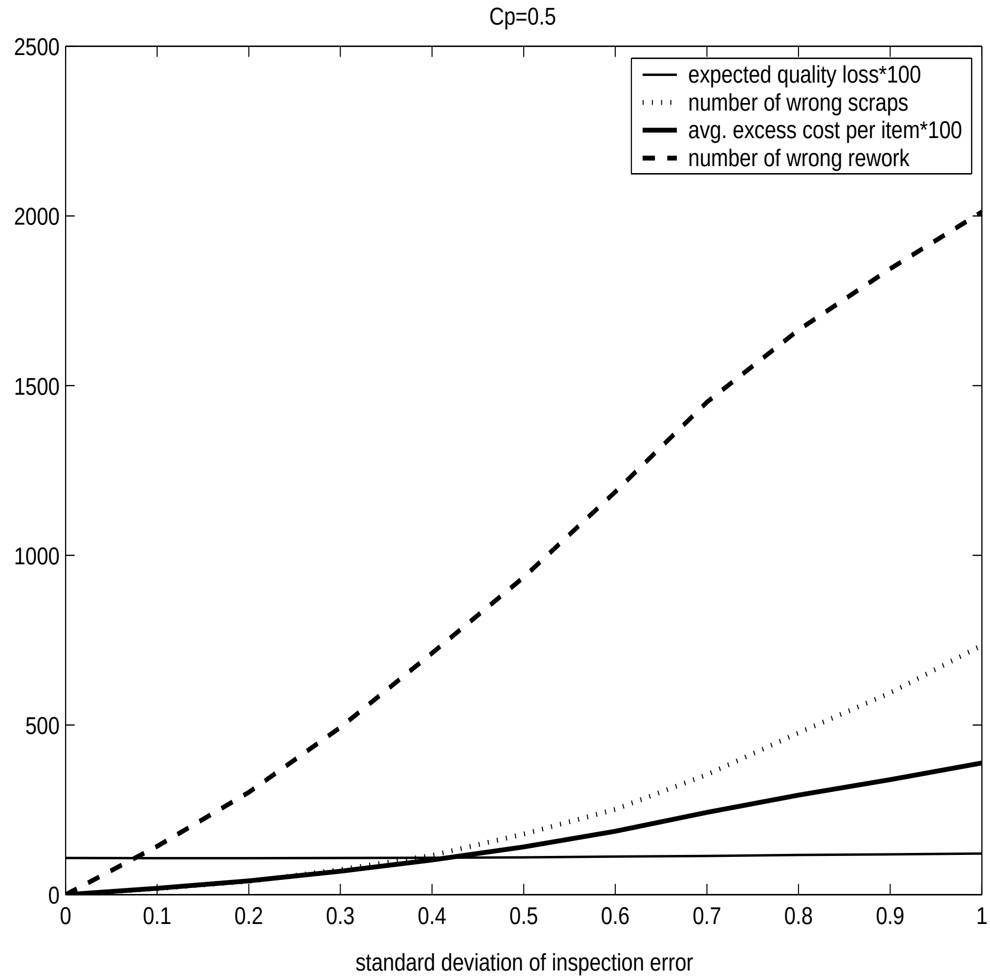

Figure 4.16: General View of The Findings in The Case of a Bad Process Capability for a Production Process with Rework, P/T Ratio:0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2, respectively

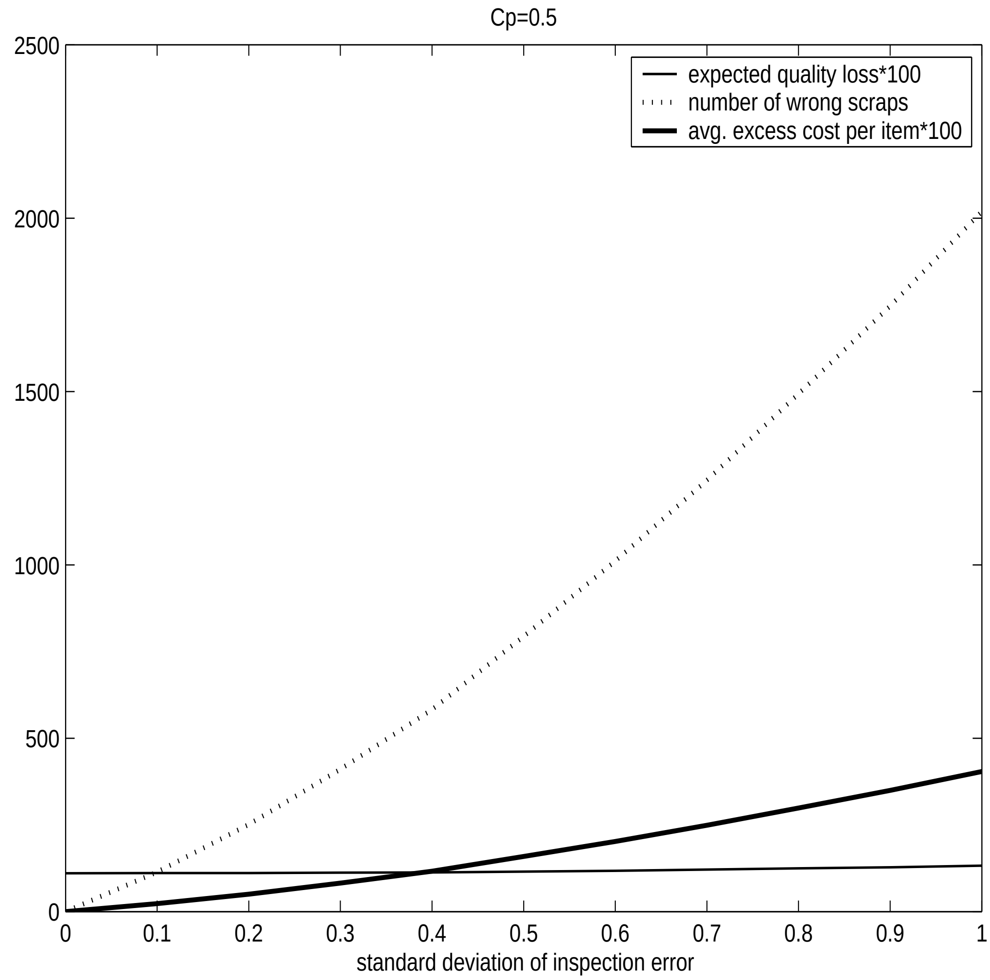

Figure 4.17: General View of The Findings in The Case of a Bad Process Capability for a Production Process without Rework, P/T Ratio:0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2, respectively

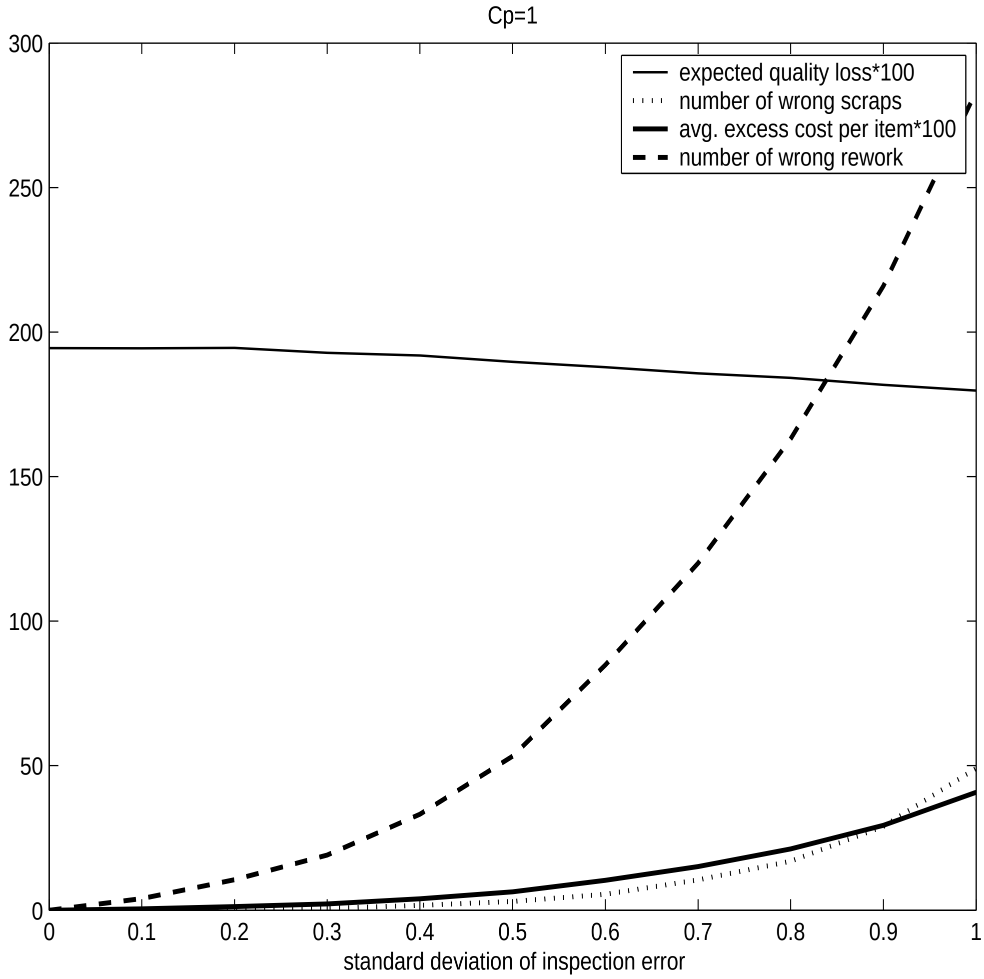

Figure 4.18: General View of The Findings in The Case of a Moderate Process Capability for a Production Process with Rework, P/T Ratio=standard deviation of inspection error

is more crucial rather than rework and re-inspection costs for the manufacturer, the measurement error may not critically affect the production even up to this level. In Figure 4.19, the results for a production process where there is no rework is proposed. A decrease in the average loss as the inspection error increases is seen. However, an increase in the average excess cost per item is noticeable. When the figure is closely examined, it can be observed that the increase in the number of wrong scraps becomes sharper after measurement error exceeds 0.5.

When process capability increases, (see Figure 4.20 and Figure 4.21) the number of scraps and reworks due to inspection error significantly decrease. However, it ad-

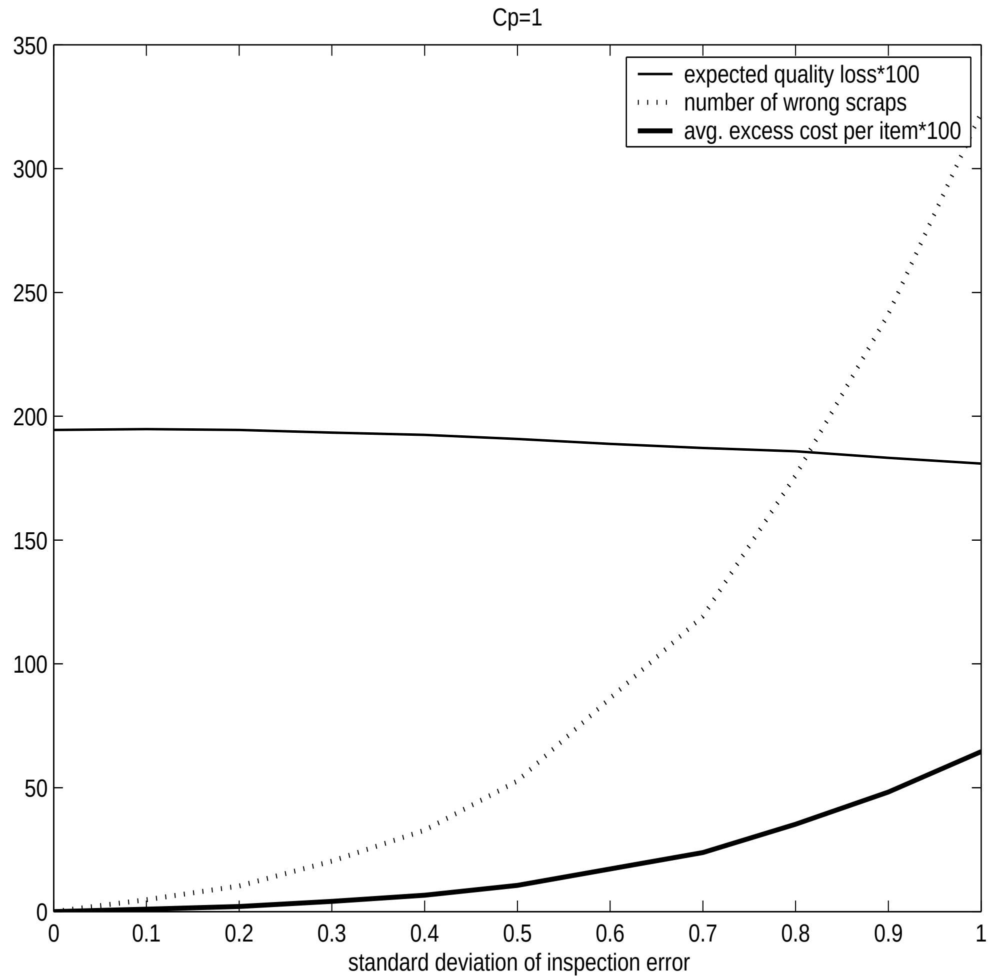

Figure 4.19: General View of The Findings in The Case of a Moderate Process Capability for a Production Process without Rework, P/T Ratio=standard deviation of inspection error

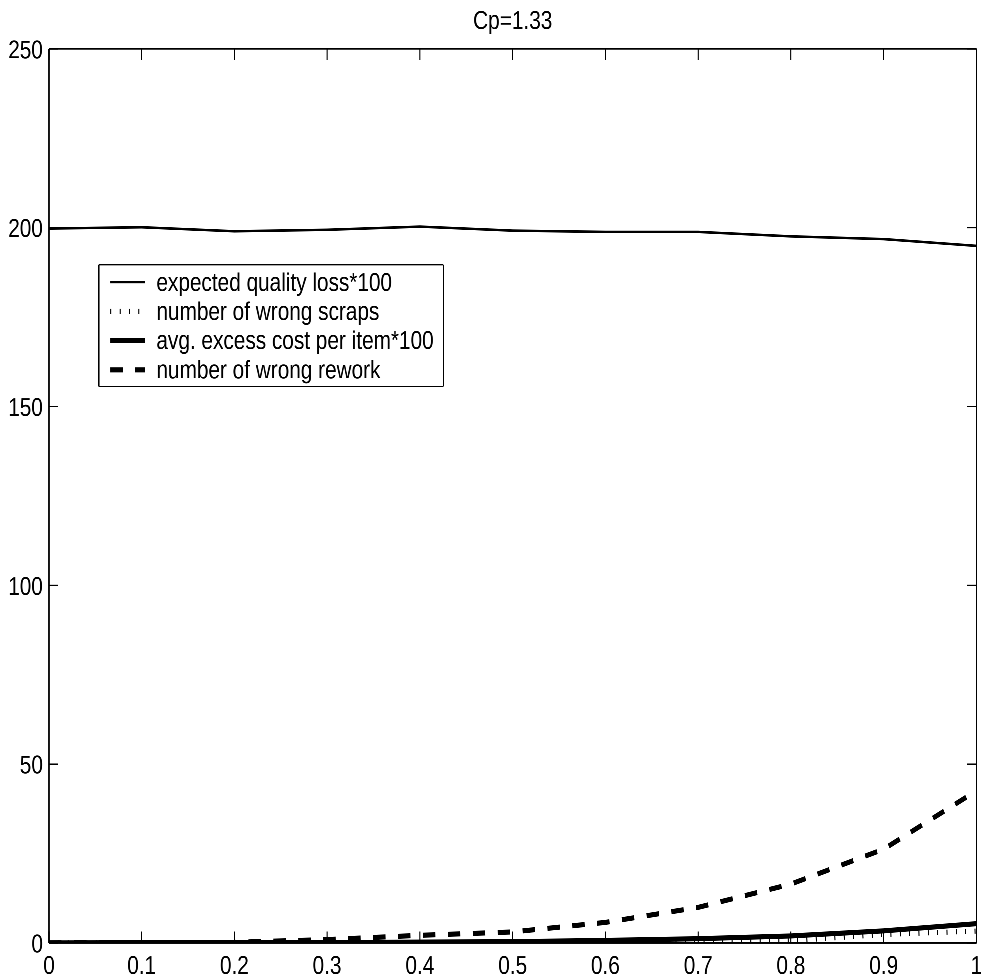

Figure 4.20: General View of The Findings in The Case of a Good Process Capability for a Production Process with Rework, P/T Ratio:0, 0.075, 0.15, 0.225, 0.3, 0.375, 0.45, 0.525, 0.6, 0.675, 0.75, respectively

ditionally is observed that no matter how small they are relative to worse systems, we observe an increase in the number of erroneous reworks and scraps done by mistake, hence the excess cost per item. It is also observed that the erroneous scraps and reworks show a sharp increase after measurement error exceeds approximately 0.42.

The picture is even better for production processes with very good process capabilities. For a process capability of 1.67 as an example (Figure 4.22 and Figure 4.23), the measurement error tolerated by the system may reach up to 60% till wrong scraps and reworks begin. Additionally, the scraps and reworks due to inspection error are observed to be in small amounts relative to the mass of production (10.000 items for

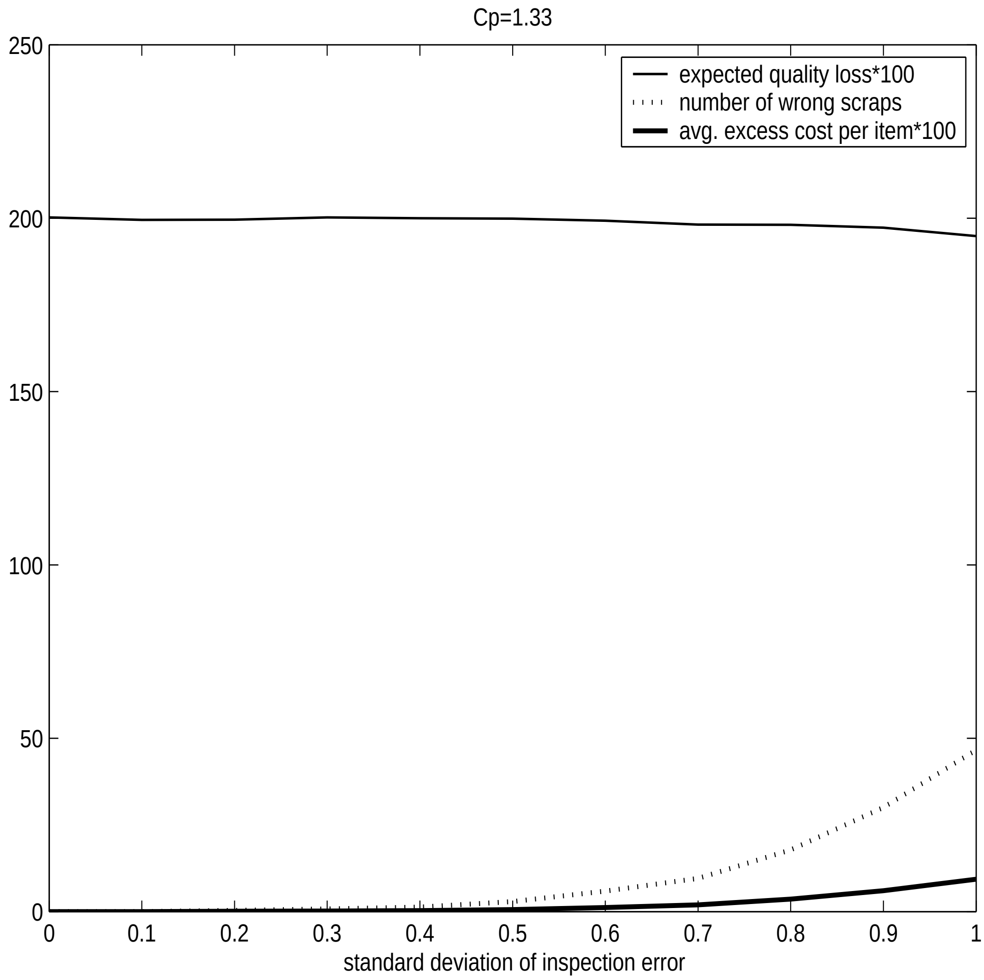

Figure 4.21: General View of The Findings in The Case of a Good Process Capability for a Production Process without Rework, P/T Ratio: 0, 0.075, 0.15, 0.225, 0.3, 0.375, 0.45, 0.525, 0.6, 0.675, 0.75, respectively

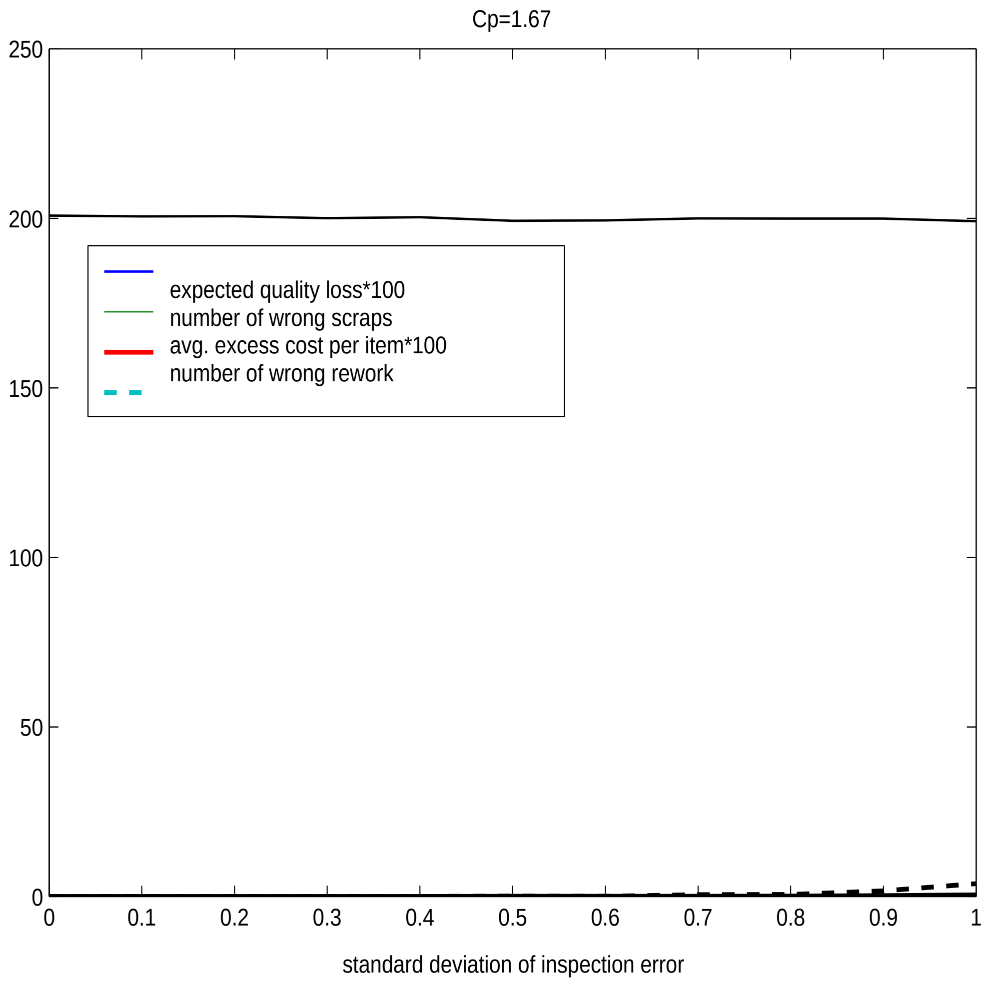

Figure 4.22: General View of The Findings in The Case of a Very Good Process Capability for a Production Process with Rework, P/T Ratio:0.06, 0.12, 0.18, 0.24, 0.3, 0.36, 0.42, 0.48, 0.54, 0.6, respectively

each simulation run), even for high levels of measurement error.

It is generally observed that, as process capability increases, the system becomes more and more tolerable against inspection error. What is trivial about this finding is that, if there is measurement error in the system, this also affects the calculation of process capability. In [28], Pearn et al. conduct some sensitivity analysis to measure the true process capability (C_p) based on C_{pmk} with measurement error. They obtain lower confidence bounds and critical values for hypothesis testing for true process capability when there is measurement error in the system. Thus, it should be strongly emphasized that we make the above comments considering the true pro-

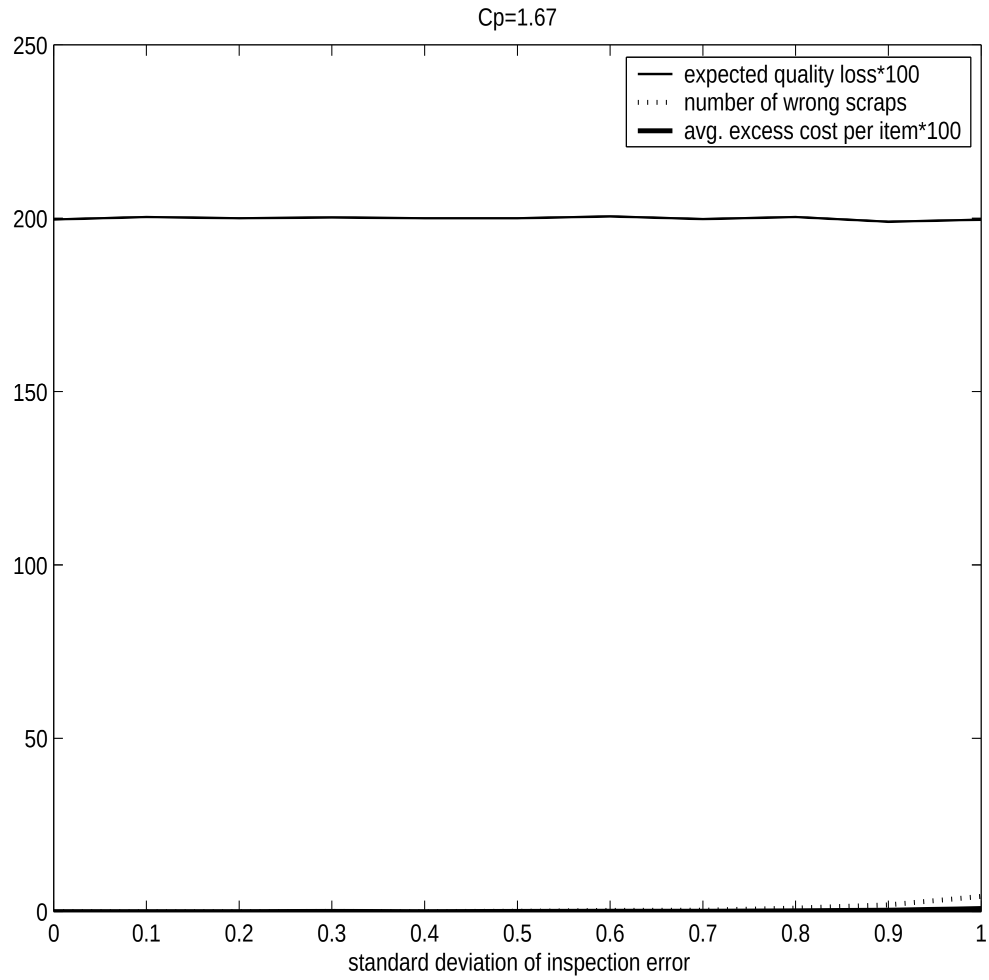

Figure 4.23: General View of The Findings in The Case of a Very Good Process Capability for a Production Process without Rework, P/T Ratio: 0.06, 0.12, 0.18, 0.24, 0.3, 0.36, 0.42, 0.48, 0.54, 0.6, respectively

cess capability values, not the observed process capability values which are subject to measurement error.

For production processes with good process capability, the measurement error does not cause serious amounts of erroneous reworks, scraps and hence excess cost till P/T ratio reaches up to 60%, even more for very good process capabilities. However, it is once more recognized that the measurement error should better not exceed 30% if a production processes has bad or the minimum required process capability. The negative effect of inspection error on expected quality loss is obviously observed in processes with bad process capacity. Although the inspection error seem to have positive effect on the loss to the customer as the process capability increases, processes may still need improvement so as to decrease extra manufacturing, rework and inspection costs.

It is also found from this study that, a rework facility where the items go under a more sophisticated operation has a decreasing effect on expected quality loss of the accepted items.

4.3 Design of Experiments

All the information collected on the effects of inspection error and specification limits, thus the gage and capacity measures, on the quality loss function are based on the simulation results and some descriptive statistics of the simulation data. To further study the effects of factors, deviation from the target and standard deviation of the quality characteristic produced in rework as well as inspection error and specification limits, a design of experiments is constructed.

The factors and level of factors are presented in Table 4.1.

The specification limit levels are selected according to the process capacity levels whose effects are sought. It is known that the minimum Process Capacity Ratio (C_p) required for an existing system is 1.33 [36]. The corresponding upper and lower specification limits are 4 and -4, respectively for $X_p \sim N(0,1)$. Another C_p value, 1.67 is selected to represent a better system. For this C_p value, the upper and lower specification limits are 5 and -5, respectively. And to analyze the systems that have

Table 4.1: Factors

	Factor Number								
	1	2	3	4	5	6			
Levels	$\mu_p - T$	$\mu_r - T$	σ_r	ϵ_p	ϵ_r	USL	LSL	ULs	LLs
1	0	0	$0.5\sigma_p$	$0.066\sigma_p$	$0.066\sigma_r$	$2\sigma_p$	$-2\sigma_p$	$3\sigma_p$	$-3\sigma_p$
2	$0.5\sigma_p$	$0.5\sigma_r$	σ_p	$0.1\sigma_p$	$0.1\sigma_r$	$3\sigma_p$	$-3\sigma_p$	$4\sigma_p$	$-4\sigma_p$
3	σ_p	σ_r		$0.2\sigma_p$	$0.2\sigma_r$	$4\sigma_p$	$-4\sigma_p$	$5\sigma_p$	$-5\sigma_p$
4	$1.5\sigma_p$	$1.5\sigma_r$		$0.3\sigma_p$	$0.3\sigma_r$	$5\sigma_p$	$-5\sigma_p$	$6\sigma_p$	$-6\sigma_p$
5				$0.4\sigma_p$	$0.4\sigma_r$				
6				$0.5\sigma_p$	$0.5\sigma_r$				

worse process capability level, the C_p values 0.67 and 1 are chosen. The corresponding specification limits are calculated and included as the 3^{rd} and 4^{th} levels in the design.

It is known that small shifts (up to, say, 1.5σ) from the process mean may not be detected using traditional Shewart control charts [36]. For this reason, the effects of deviation of the process mean from the target till it is more easily detected are investigated. The levels of deviation from the target for both the process and rework are selected as 0 (the process is at the target), 0.5σ, σ and finally, 1.5σ.

One of the levels of σ_r is exactly σ_p. The other level is $0.5\sigma_p$, the standard deviation of rework being smaller than the standard deviation of process.

The inspection error levels are determined regarding the gage capability of the production or rework processes. The processes are accepted to have a good measurement system if their "precision to tolerance values" are less than or equal to 10% [36]. And between a 10% and 30% P/T value, the processes are accepted to have reasonable measurement systems. So, for the three levels of P/T ratio which are 10%, 20% and 30%, and specification limit levels 2, 3, 4 and 5σ, the corresponding inspection error levels are calculated. It is observed that the standard deviation of the inspection error level changes between 0.066σ and 0.5σ. And 6 main values are included as the levels of inspection error both at process and rework in the design.

A full factorial experimental design is constructed, main effects and two-level interactions of the factors are studied. There are 6 factors with the levels presented in Table 4.1, so there are totally 4608 observations gathered on the response variable expected quality loss.

4.4 Analysis of Variance of Experimental Data

ANOVA is used to investigate the model and the relationship between the response variable, which is expected quality loss and the independent variables and two-way interactions of the factors. The model is :

$$Y = \mu + \sum \tau_i + \sum \sum \tau_i \tau_j + \epsilon$$

where

$$\begin{aligned} \mu &= \text{overall mean} \\ \tau_i &= \text{main effect of factor i} \\ \tau_i \tau_j &= \text{interaction effect of factors i and j} \\ \epsilon &= \text{error term} \\ Y &= \text{data} \end{aligned}$$

The model is constructed for the response variable 'Expected Quality Loss'.

As mentioned before, there are 6 factors with different number of levels. The factors and their levels are are given in Table 4.1.

ANOVA is applied to determine the factors and two-way interactions that have significant effects on expected quality loss. The following hypotheses on the significance of the main effects of the factors and two-way factor interactions:

H_o: There is no main effect of factor i ($\tau_i = 0$)
H_1: Factor i affects the model ($\tau_i \neq 0$)

$H_o^{'}$: There is no interaction effect of factors i and j ($\tau_i \tau_j = 0$)
$H_1^{'}$: The interaction of factor i and factor j affects the model ($\tau_i \tau_j \neq 0$)

Before interpreting the results of ANOVA, there are two assumptions that must be satisfied:

1. Error terms are distributed normally with mean 0 and constant variance.

2. All pairs of error terms are uncorrelated.

Different graphical tools are used to check the validity of these assumptions (see Appendix P.2). When we check the histogram of the residuals, they seem to have a normal distribution with mean zero. However, the normal probability plot shows a slight departure from normality. Plot of residuals versus fitted values show that the residuals do not seem to have constant variance. Since we think that the residuals seem to be lithesome far from satisfying the assumptions, Box-Cox transformation is applied to the response variable. However, a significant change in the structure of the residuals could not be obtained (the results of Box-Cox transformation and the residuals of the model constructed for the transformed expected quality loss are given in Appendix Q). Thus, the results of ANOVA are accepted to be the approximate results that give us an idea about which factors and two-way factor interaction significantly affect the expected quality loss.

Further study on the model is not performed since the main aim of this study is to derive the resulting distribution of the quality characteristic under given circumstances and to investigate the effects of inspection error on expected quality loss. We try to get an approximate idea about the factors affecting the behavior of the expected quality loss using Analysis of Variance, not to accurately fit a model that explains the behavior of expected quality loss. And the approximate results are consistent with our expectations.

According to Table 4.2, the corresponding 'p' values of the factors specification limits (limits, p=0.000), standard deviation of inspection error in the rework unit (E_r, p=0.000), standard deviation of inspection error in the processing unit (E_r, p=0.000) and deviation of process mean from the target (D_p, p=0.000) point out that these factors are significantly affecting expected quality loss. The table also indicates that, the standard deviation of items in the rework unit (S_r, p=0.864) and the deviation of rework mean from the target (D_r, p=0.144) do not affect the model significantly.

Since there are 6 factors, C(6,2)= 15 different two-way interactions exist in the model.

Table 4.2: Part of MINITAB Output of ANOVA Table, Main Effects

Source	DF	Seq SS	Adj SS	Adj MS	F	P
limits	3	7232.02	7232.02	2410.67	1.6E+06	0.000
E_r	5	5.77	5.77	1.15	771.73	0.000
E_p	5	0.23	0.23	0.05	31.35	0.000
D_r	3	0.01	0.01	0.00	1.80	0.144
S_r	1	0.00	0.00	0.00	0.03	0.864
D_p	3	2184.16	2184.16	728.05	4.9E+05	0.000

The related 'p' values of the interactions in Table 4.3 indicate that the two-way interactions of specification limits with inspection error in the rework unit (limitsxE_r, p=0.000), inspection error in the processing unit (limitsxE_p, p=0.000), and deviation of process mean from the target (limitsxD_p, p=0.000), the two-way interaction of inspection error in the rework unit with deviation of process mean from the target (E_rxD_p, p=0.000) significantly affect the expected quality loss. Additionally, it is found that the two-way interaction of inspection error in the processing unit with deviation of process mean from the target (E_pxD_p, p=0.000) and with inspection error in the rework unit (E_pxE_r, p=0.000) have significant effects on expected quality loss.

Table 4.3: Part of MINITAB Output of ANOVA Table, Two-Way Interactions

Source	DF	Seq SS	Adj SS	Adj MS	F	P
$limitsxE_r$	15	3.14	3.14	0.21	140.14	0.000
$limitsxE_p$	15	0.13	0.13	0.01	5.74	0.000
$limitsxD_r$	9	0.00	0.00	0.00	0.20	0.994
$limitsxS_r$	3	0.00	0.00	0.00	0.00	1.000
$limitsxD_p$	9	1241.27	1241.27	137.92	9.2E+04	0.000
E_rxE_p	25	0.67	0.67	0.03	18.05	0.000
E_rxD_r	15	0.02	0.02	0.00	0.90	0.559
E_rxS_r	5	0.00	0.00	0.00	0.04	0.999
E_rxD_p	15	8.45	8.45	0.56	376.74	0.000
E_pxD_r	15	0.05	0.05	0.00	2.17	0.005
E_pxS_r	5	0.00	0.00	0.00	0.00	1.000
E_pxD_p	15	0.40	0.40	0.03	17.85	0.000
D_rxS_r	3	0.00	0.00	0.00	0.00	1.000
D_rxD_p	9	0.02	0.02	0.00	1.47	0.152
S_rxD_p	3	0.00	0.00	0.00	0.04	0.989

These approximate results of ANOVA show consistence with the previous experimental results.

A main effects plot is a plot of the means at each level of a factor. An interactions plot is a plot of means for each level of a factor with the level of a second factor held constant. The complete ANOVA table, main effects and interactions plots can be seen from the Appendices P.1 and P.3.

CHAPTER 5

CONCLUSION AND FUTURE STUDY

In this thesis, we studied the effects of inspection error and rework on expected quality loss for a nominal-the-best type of quality characteristic. We assume a production environment where there is a separate facility for rework. We consider 100 % inspection (perfect inspection). We assume that only one quality characteristic is produced in each station on the production line and the processing units are independent of each other. Hence, the quality characteristic that is produced at one processing unit is not affected from the operations in the succeeding units.

The distribution of the quality characteristic of items is assumed to be normal with mean μ_p and standard deviation σ_p. The quality characteristic of items produced at rework also have a normal distribution with mean μ_r and σ_r. We assume that the items which are sent to rework go under more careful and detailed operation. That is the reason why, the standard deviation of the quality characteristic values of the items produced at rework is taken to be smaller than the standard deviation of the quality characteristic values of the items produced at process ($\sigma_r < \sigma_p$). We also assume that the mean of the quality characteristic values of the items produced at process and rework are equal to each other ($\mu_p = \mu_r$).

One other assumption that we make is that the inspection utility is calibrated and the measurement system is accurate. The standard deviation of the inspection error is assumed to be smaller than or equal to the standard deviation of the process. Lastly, the processes are assumed to be under statistical control.

To be able to calculate the expected quality loss of the accepted items, the mean and the standard deviation of the accepted items are needed. Hence, first of all, the distribution of the accepted items should be determined. We assume that the quality characteristic of items produced at process have a normal distribution with mean μ_p and variance σ_p^2. Then, the distribution of the accepted items in a production environment where there is no rework, no inspection error and two-sided symmetric specification limits is a normal distribution truncated at specification limits [25]. The parameters mean and variance of this truncated normal distribution are derived by Phillips and Cho [25].

We added the rework to the above no inspection error situation. Then, the resulting distribution of the accepted items coming from the process and rework turn out to be the mixture of the two truncated normal distributions. The parameters of this resulting distribution are also determined.

The distribution of the accepted items when there is inspection error in a production environment without rework is provided by Chen and Chung [5]. We derived the first and second moments of this distribution in order to find the mean and variance. Then, we added the rework and determined the parameters for this new case.

The moments for all above cases are given in Chapter 3.

Validation of the formulas we derived are done in two ways. First of all, by setting the inspection error to zero, we have found that the formulas are the same with the formulas for no inspection error case. Secondly, the validation is provided by comparison of the real and simulated mean and standard deviation and expected quality loss values.

By using the formulas derived, we computed the expected quality loss values for different specification limits and standard deviations of inspection error. While calculating the expected quality values, the quality characteristic values X_p of the items produced at process are accepted to have N(0,1) and the quality characteristic values X_r of the items produced at rework are accepted to have N(0,0.75). The standard deviation of the inspection error at processing unit is changed between 0 and 1 by

0.1 unit. The standard deviation of the inspection error at process changes between 0 and 1 by 0.1 unit. The standard deviation of the inspection error at reworking unit is taken as equal to the standard deviation of the inspection error till it is equal to the standard deviation of quality characteristic of items at rework. Afterwards, it is accepted to be 0.75. The upper specification and scrap limits are symmetric. The absolute value of the specification limits change between 1 to 5, the absolute value of the scrap limits change between 2 to 6 by 0.1 unit.

The maximum relative differences between the real and simulated mean, standard deviations and the loss values (given in Table 3.3 in Section 3.3) indicate that the results are quite close to each other and we can accept that the formulas we have derived are correct.

In both production environments with or without rework, as the contrary of our expectation, a decrease in the expected quality loss is observed with increasing standard deviation of inspection error as system becomes better (C_p increases). The system is simulated not only for the validation of the formulas, but also to further investigate the decrease in the expected quality loss with increasing inspection error and effects of inspection error and rework on quality loss.

It is recognized that as specification limits get wider and wider relative to standard deviation, the total loss of conforming items which are scrapped or reworked, due to inspection error, which is not included in the total loss of the accepted items, becomes more than the total loss of nonconforming items which are accepted erroneously, that is, the loss which exists in the total loss of the accepted items as standard deviation of the inspection error increases. That is because, as system gets better the number of items which are erroneously scrapped or reworked become much more than the ones which are erroneously accepted with increasing standard deviation of the inspection error relative to worse systems. As a result, the expected quality loss decreases as standard deviation of inspection error increases after C_p exceeds approximately 0.5.

However, this decline in the expected quality loss with increasing standard deviation of measurement error (P/T ratio) does not mean that when measurement error is high, the system is advantageous. An important fact is that, the number of scraps and reworks due to inspection error always increases with increasing standard deviation of increasing error. It is observed that this brings excess manufacturing, reworking and inspection cost to the manufacturer.

As a result, although the expected quality loss seems to decrease as measurement error increases, this decrease is not because the system produces better products when the measurement system gets worse. This decrease in the expected quality loss is just because of the items erroneously scrapped instead of being accepted or reworked. Hence, we reach the conclusion that when the measurement system of a process gets worse, this certainly has some penalties to the manufacturer. First of all, in a production environment without rework, the number of erroneous scraps caused by the measurement error increases and the excess cost that the manufacturer encounters definitely increases. In a production environment with rework, besides the erroneous scraps, the number of reworks due to measurement error also increases, and this causes excess rework and inspection cost to the consumer. From the findings of the study, it may also be concluded that the tolerance of the system against the measurement error increases as the system gets better(C_p increases). However, this may also be misleading since when the measurement system is not precise, the process capability values computed using that measurement system will probably be inaccurate. Hence, the results of this study, once more emphasize that a process with bad measurement system needs prior improvement especially when measurement error exceeds 30%.

Our study additionally shows that a rework unit which operates better than the processing unit leads to products with better quality characteristics, thus a decrease in the expected quality loss. That's why, one may prefer having a rework unit when he has to work with expensive raw materials and reworking is not as costly as scrapping a non-conforming item.

This study helps one to analyze a production process in different aspects. First of all, it is possible to observe the effects of measurement error on the expected quality loss for given deviation of the process mean from the nominal value and process capability. Furthermore, with the simulation of the system, one can get an idea about the effects of inspection error on the excess scrapping, rework and inspection costs (or generally, average excess cost per item produced). By just entering the parameters of a process, the resulting distribution of a certain quality characteristic of the accepted items can easily be obtained provided that the assumptions are valid. All these information can be used in deciding, for example, priorities of quality improvement projects. In such a case, one needs to compute the expected quality loss. Similarly, these results can also be used in the measurement system improvement studies.

Changing each assumption we make will open a new area of research. A future study may be deriving the distribution of quality characteristic of accepted items when the items produced have a non-normal quality characteristic. The effects of inspection error having a distribution other than normal may also be investigated. It may also be worthwhile to study the dependency of the working units and model the effects of other operations performed in other work stations on the distribution of quality characteristics of accepted items, hence on the quality loss. A similar study can be performed for the cases when the measurement system is not accurate. Effects of other types of inspection error, for example, when there is only one type of error can also be studied. Another possible research area can be to study the effects of measurement error for other types of (larger-the-better, smaller-the-better or asymmetric) quality characteristics. A combined rework facility could also constitute a future study case. Our next study will be investigating the factors effecting expected quality loss with more robust statistical analysis methods as robust regression or non-parametric regression.

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APPENDIX A

MATLAB PROGRAM USED TO SIMULATE THE SYSTEM

```
USL = 2;      LSL = -2;
ULs = 3;      LLs = -3;
m = 0;
tend=10000;
E1=1;     E2=0.75;     s = 0.75;
for i = 1:tend
x1 = normrnd(0,1,1,1);
y1=normrnd(x1,E1,1,1);
if LSL < y1 & y1 < USL
j=j+1;
Accx_p(j) = x1;
Accy_p(j) = y1;
elseif (LLs < y1 & y1 < LSL) | (USL < y1 & y1 < ULs)
x2 = normrnd(0,s,1,1);
if (LLs < x2 & x2 < LSL) | (USL < x2 & x2 < ULs)
while (LLs < x2 & x2 < LSL)|(USL < x2 & x2 < ULs)
x2 = normrnd(0,s,1,1);
if x2 < LLs | x2 > ULs | (LSL < x2 & x2 < USL)
break
end
end
end
y2 = normrnd(x2,E2,1,1);
while (LLs < y2 & y2 < LSL) | (USL < y2 & y2 < ULs)
```

x2=normrnd(0,s,1,1);

y2 = normrnd(x2,E2,1,1);

if $y2 < LLs \mid y2 > ULs \mid (LSL < y2 \,\&\, y2 < USL)$

break

end

end

if $(LSL < y2 \,\&\, y2 < USL)$

m=m+1;

Accx_r(m) = x2;

Accy_r(m) = y2;

end

end

end

if $m > 0$

Accx_r;

Accy_r;

Acc_x = [Accx_p Accx_r];

Acc_y = [Accy_p Accy_r];

else Acc_x = Accx_p;

Acc_y = Accy_p;

end

Mx = mean(Acc_x);

Sx = std(Acc_x);

k=2; T=0;

$L = k * [(Mx - T)^2 + Sx^2]$

APPENDIX B

DERIVATION OF MOMENTS OF THE RESULTING DISTRIBUTION OF ACCEPTED ITEMS IN A PRODUCTION ENVIRONMENT WITH REWORK AND INSPECTION ERROR

$$
\begin{aligned}
E[X_a] &= \int_{-\infty}^{\infty} x\, h_{X_a}(x)dx \\
&= \frac{1}{M'} \cdot \underbrace{\int_{-\infty}^{\infty} \int_{LSL}^{USL} x\, l_{X_p,Y_p}(x,y)dydx}_{\text{I}} + \frac{q}{M''} \cdot \int_{-\infty}^{\infty} \int_{LSL}^{USL} x\, h_{X_r,Y_r}(x,y)dydx
\end{aligned}
$$

$$
\begin{aligned}
I &= \int_{-\infty}^{\infty} \int_{LSL}^{USL} x\, g_{Y_p|X_p}(y|x)\, f_{X_p}(x)dydx \\
&= \int_{-\infty}^{\infty} \int_{LSL}^{USL} x\, \frac{1}{\sqrt{2\pi}\, \frac{1}{\epsilon_p}\, \exp -\frac{1}{2}(\frac{y-x}{\epsilon_p})^2\, \frac{1}{\sqrt{2\pi}}\, \frac{1}{\sigma_p}\, \exp -\frac{1}{2}(\frac{x-\mu}{\sigma_p})^2\, dydx}
\end{aligned}
$$

with the substitution u=1$\frac{}{\sigma^2}$ and $v = \frac{1}{\epsilon^2}$,

$$
I = \int_{LSL}^{USL} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{u+v}{uv}}} \exp -\frac{1}{2}\frac{uv}{u+v}(y-\mu)^2 \underbrace{\int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{1}{u+v}}} \exp -\frac{1}{2}\frac{(x-\frac{um+vy}{u+v})^2}{\frac{1}{u+v}}}_{\frac{um+vy}{u+v}}\, dxdy
$$

$$
\begin{aligned}
I &= \int_{LSL}^{USL} \frac{um+vy}{u+v} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{u+v}{uv}}} \exp -\frac{1}{2}\frac{u+v}{uv}(y-x)^2\, dy \\
&= (\frac{u\mu}{u+v} + \frac{v\mu}{u+v})[F(b) - F(a)] + \frac{v}{u+v}\sqrt{\frac{u+v}{uv}} \frac{1}{\sqrt{2\pi}} \int_a^b t\, \exp -\frac{1}{2}t^2 dt
\end{aligned}
$$

where $\quad t = \frac{y-\mu}{\sqrt{\frac{u+v}{uv}}}$, $a = \frac{LSL-\mu}{\sqrt{\frac{u+v}{uv}}}$ and $b = \frac{USL-\mu}{\sqrt{\frac{u+v}{uv}}}$

Then, by making the reverse substitutions of u and v

$$\begin{aligned} I &= \mu_p\,[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\ &+ \frac{\sigma_p^2}{\sqrt{\sigma_p^2+\epsilon_p^2}}\,[\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - \phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \end{aligned}$$

and

$$\begin{aligned} E[X_a] &= \frac{1}{M'} \cdot \{\mu_p[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\ &+ \frac{\sigma_p^2}{\sqrt{\sigma_p^2+\epsilon_p^2}}[\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - \phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})]\} \\ &+ \frac{q}{M''} \cdot \{\mu_r[F(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}) - F(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})] \\ &+ \frac{\sigma_r^2}{\sqrt{\sigma_r^2+\epsilon_r^2}}[\phi(\frac{LSL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}}) - \phi(\frac{USL-\mu_r}{\sqrt{\sigma_r^2+\epsilon_r^2}})]\} \end{aligned}$$

$$\begin{aligned} E[X_a^2] &= \int_{-\infty}^{\infty} x^2\, h_{X_a}(x)dx \\ &= \frac{1}{M'} \cdot \underbrace{\int_{-\infty}^{\infty}\int_{LSL}^{USL} x^2\, l_{X_p,Y_p}(x,y)dydx}_{\text{II}} + \frac{q}{M''} \cdot \int_{-\infty}^{\infty}\int_{LSL}^{USL} x^2\, h_{X_r,Y_r}(x,y)dydx \end{aligned}$$

$$\begin{aligned} II &= \int_{-\infty}^{\infty}\int_{LSL}^{USL} x^2\, g_{Y_p|X_p}(y|x)\, f_{X_p}(x)dydx \\ &= \int_{-\infty}^{\infty}\int_{LSL}^{USL} x^2\, \frac{1}{\sqrt{2\pi}}\,\frac{1}{\epsilon_p}\, \exp -\frac{1}{2}(\frac{y-x}{\epsilon_p})^2\, \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sigma_p}\, \exp -\frac{1}{2}(\frac{x-\mu}{\sigma_p})^2\, dydx \end{aligned}$$

with the substitution $\quad u = \frac{1}{\sigma^2} \quad$ and $\quad v = \frac{1}{\epsilon^2}$,

$$II = \int_{LSL}^{USL} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{u+v}{uv}}} \exp -\frac{1}{2}\frac{uv}{u+v}(y-\mu)^2 \underbrace{\int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{1}{u+v}}} \exp -\frac{1}{2}\frac{(x-\frac{um+vy}{u+v})^2}{\frac{1}{u+v}}}_{(\frac{um+vy}{u+v})^2+\frac{1}{u+v}} dx\,dy$$

$$\begin{aligned} II &= \frac{1}{(u+v)^2}\,[u^2\mu^2(F(b)-F(a)) + 2uv\mu \int_{LSL}^{USL} y\,\frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{u+v}{uv}}} \exp -\frac{1}{2}\frac{uv}{u+v}(y-\mu)^2 dy \\ &+ v^2 \int_{LSL}^{USL} y^2\,\frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{u+v}{uv}}} \exp -\frac{1}{2}\frac{uv}{u+v}(y-\mu)^2 dy] + \frac{1}{u+v}(F(b)-F(a)) \end{aligned}$$

Since

$$\int_{LSL}^{USL} y\,\frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{u+v}{uv}}} \exp -\frac{1}{2}\frac{uv}{u+v}(y-\mu)^2 dy = \mu\,[F(b)-F(a)] - \sqrt{\tfrac{u+v}{uv}}\,[\phi(b)-\phi(a)]$$

and

$$\int_{LSL}^{USL} y^2\,\frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{u+v}{uv}}} \exp -\frac{1}{2}\frac{uv}{u+v}(y-\mu)^2 dy$$

$$= (\mu^2 + \frac{u+v}{uv})\,[F(b)-F(a)] - 2\mu\,\sqrt{\tfrac{u+v}{uv}}\,[\phi(b)-\phi(a)]$$

$$+\frac{u+v}{uv}\,[-b\,\phi(b) + a\,\phi(a)]$$

where $a = \frac{LSL-\mu}{\sqrt{\frac{u+v}{uv}}}$ and $b = \frac{USL-\mu}{\sqrt{\frac{u+v}{uv}}}$

by making the inverse substitutions of u and v

$$\begin{aligned} II &= \{(\mu_p^2+\sigma_p^2)[F(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - F(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\ &- \frac{2\mu_p\sigma_p^2}{\sqrt{\sigma_p^2+\epsilon_p^2}}[\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - \phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})] \\ &+ \frac{\sigma_p^4}{\sigma_p^2+\epsilon_p^2}[(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})\phi(\frac{LSL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}}) - (\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})\phi(\frac{USL-\mu_p}{\sqrt{\sigma_p^2+\epsilon_p^2}})]\} \end{aligned}$$

and

$$
\begin{aligned}
E[X_a^2] &= \frac{1}{M'} \cdot \{(\mu_p^2 + \sigma_p^2)[F(\frac{USL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}}) - F(\frac{LSL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}})] \\
&- \frac{2\mu_p \sigma_p^2}{\sqrt{\sigma_p^2 + \epsilon_p^2}}[\phi(\frac{USL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}}) - \phi(\frac{LSL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}})] \\
&+ \frac{\sigma_p^4}{\sigma_p^2 + \epsilon_p^2}[(\frac{LSL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}})\phi(\frac{LSL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}}) - (\frac{USL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}})\phi(\frac{USL - \mu_p}{\sqrt{\sigma_p^2 + \epsilon_p^2}})]\} \\
&+ \frac{q}{M''} \cdot \{(\mu_r^2 + \sigma_r^2)[F(\frac{USL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}}) - F(\frac{LSL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}})] \\
&- \frac{2\mu_r \sigma_r^2}{\sqrt{\sigma_r^2 + \epsilon_r^2}}[\phi(\frac{USL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}}) - \phi(\frac{LSL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}})] \\
&+ \frac{\sigma_r^4}{\sigma_r^2 + \epsilon_r^2}[(\frac{LSL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}})\phi(\frac{LSL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}}) - (\frac{USL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}})\phi(\frac{USL - \mu_r}{\sqrt{\sigma_r^2 + \epsilon_r^2}}))]\},
\end{aligned}
$$

APPENDIX C.1

EXPECTED QUALITY LOSS OF ACCEPTED ITEMS IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5823	0.5914	0.6181	0.6608	0.7168	0.783	0.8558	0.932	1.0088	1.0839	1.1558
0.43	1.3	0.8949	0.8991	0.9118	0.933	0.9627	1.0001	1.044	1.0929	1.1448	1.1981	1.2513
0.57	1.7	1.2979	1.2971	1.2952	1.2937	1.2944	1.2991	1.3087	1.3238	1.3439	1.3682	1.3957
0.67	2	1.5475	1.5448	1.5371	1.526	1.5137	1.5023	1.4939	1.4899	1.4909	1.4969	1.5074
0.77	2.3	1.7337	1.7305	1.7212	1.707	1.6894	1.6704	1.6522	1.6364	1.6244	1.617	1.6141
0.9	2.7	1.8667	1.8842	1.8769	1.8651	1.8492	1.8304	1.8099	1.7891	1.7697	1.7528	1.7391
1	3	1.9467	1.9451	1.9402	1.932	1.9204	1.9057	1.8885	1.8697	1.8505	1.9321	1.8153
1.1	3.3	1.9772	1.9763	1.9735	1.9686	1.9613	1.9515	1.9391	1.9246	1.9084	1.8917	1.8752
1.23	3.7	1.9937	1.9934	1.9923	1.9903	1.9871	1.9823	1.9757	1.9671	1.9566	1.9444	1.9313
1.33	4	1.9979	1.9977	1.9973	1.9964	1.9948	1.9924	1.9888	1.9836	1.9768	1.9684	1.9585
1.43	4.3	1.9993	1.9993	1.9991	1.9988	1.9981	1.997	1.9952	1.9924	1.9884	1.9829	1.9761
1.56	4.7	1.9999	1.9999	1.9998	1.9997	1.9996	1.9992	1.9986	1.9975	1.9958	1.9931	1.9894
1.67	5	2	2	2	1.9999	1.9999	1.9997	1.9995	1.999	1.9981	1.9967	1.9946

APPENDIX C.2

EXPECTED QUALITY LOSS OF ACCEPTED ITEMS IN A PRODUCTION ENVIRONMENT WITH REWORK

$$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5654	0.5729	0.5948	0.6298	0.6757	0.7295	0.7882	0.8486	0.9016	0.9453	0.9855
0.43	1.3	0.869	0.872	0.8811	0.8967	0.9189	0.9472	0.9806	1.0178	1.0545	1.0884	1.1211
0.57	1.7	1.2704	1.2689	1.2651	1.2603	1.2564	1.2551	1.2576	1.2645	1.2754	1.2893	1.3051
0.67	2	1.5264	1.5232	1.5142	1.5007	1.4849	1.469	1.455	1.4445	1.4385	1.437	1.4391
0.77	2.3	1.7207	1.7171	1.7069	1.6908	1.6706	1.6482	1.6256	1.6045	1.5869	1.5734	1.5639
0.9	2.7	1.8815	1.8789	1.871	1.8581	1.8407	1.8196	1.7962	1.7718	1.7481	1.7265	1.7076
1	3	1.9445	1.9428	1.9376	1.9288	1.9163	1.9002	1.8811	1.8598	1.8375	1.8153	1.7944
1.1	3.3	1.9764	1.9755	1.9725	1.9673	1.9596	1.949	1.9355	1.9194	1.9011	1.8817	1.862
1.23	3.7	1.9935	1.9932	1.992	1.9899	1.9866	1.9815	1.9745	1.9651	1.9535	1.9399	1.9248
1.33	4	1.9978	1.9977	1.9972	1.9962	1.9947	1.9921	1.9883	1.9828	1.9754	1.966	1.9549
1.43	4.3	1.9993	1.9993	1.9991	1.9987	1.998	1.9969	1.995	1.992	1.9877	1.9818	1.9742
1.56	4.7	1.9999	1.9999	1.9998	1.9997	1.9995	1.9992	1.9985	1.9974	1.9955	1.9927	1.9887
1.67	5	2	2	2	1.9999	1.9999	1.9997	1.9995	1.999	1.9981	1.9965	1.9942

APPENDIX D

PROPORTION OF CONFORMING ITEMS SCRAPPED DUE TO INSPECTION ERROR TO THE NUMBER OF NON-CONFORMING ITEMS ACCEPTED DUE TO INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	1.1216	1.2639	1.4662	1.5759	1.7945	1.9638	2.0941	2.2789	2.4771	2.6593
0.43	1.3	0	1.145	1.3641	1.605	1.8436	2.1036	2.3799	2.6818	2.9621	3.2294	3.5463
0.57	1.7	0	1.2684	1.5505	1.8031	2.2597	2.6649	3.2439	3.7073	4.2596	5.0075	5.4813
0.67	2	0	1.2777	1.6615	2.0565	2.7038	3.3023	3.9416	4.8662	5.8289	6.8178	7.7619
0.77	2.3	0	1.2819	1.8006	2.3369	3.2177	3.8331	5.0827	6.385	7.813	9.6827	10.892
0.9	2.7	0	1.5073	1.9833	2.8304	3.6557	5.4551	7.1483	9.9671	11.8818	15.0063	18.9451
1	3	0	1.5714	1.9375	3.4407	4.2137	6.5602	9.2914	11.6515	19.563	23.1214	29.8765
1.1	3.3	0	2.15	2.2414	3.24	6.5342	7.2157	11.625	16.3274	23.072	34.437	49.8793
1.23	3.7	0	1.7143	1.5	7.3636	6.5714	11.9565	19.4762	30.913	31.1765	64.2593	97.5385
1.33	4	0	0	1.6667	2.4444	9.75	17.2	22	47.6667	66.75	180.2	139.8
1.43	4.3	0	0	0	4	0	12.6667	0	0	76.6667	101.75	362
1.56	4.7	0	0	0	0	0	0	0	0	0	0	267
1.67	5	0	0	0	0	0	0	0	0	0	0	0

APPENDIX E.1

TOTAL QUALITY LOSS OF CONFORMING ITEMS SCRAPPED DUE TO INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

1.0e+003*

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.3563	0.6519	0.9082	1.073	1.2303	1.3378	1.436	1.5338	1.6462	1.7581
0.43	1.3	0	0.4437	0.8768	1.2467	1.5437	1.8181	2.0445	2.2002	2.3723	2.5343	2.675
0.57	1.7	0	0.4605	0.8992	1.3394	1.8121	2.205	2.5426	2.8264	3.1365	3.388	3.6345
0.67	2	0	0.3516	0.7929	1.1944	1.6647	2.0392	2.4243	2.8146	3.1945	3.5395	3.792
0.77	2.3	0	0.2525	0.5553	0.9345	1.3171	1.6807	2.1272	2.4825	2.8571	3.2434	3.5613
0.9	2.7	0	0.1423	0.3073	0.5156	0.7713	1.0738	1.3755	1.7333	2.0893	2.3932	2.7874
1	3	0	0.0821	0.1685	0.3029	0.4457	0.6445	0.9534	1.1523	1.4843	1.7787	2.0795
1.1	3.3	0	0.0301	0.0846	0.1478	0.2672	0.3661	0.5328	0.7109	0.9711	1.2021	1.4799
1.23	3.7	0	0.0105	0.0274	0.0626	0.0954	0.172	0.2258	0.3417	0.4378	0.6288	0.7804
1.33	4	0	0	0.0099	0.02	0.032	0.0627	0.114	0.159	0.263	0.3573	0.497
1.43	4.3	0	0.0012	0.0035	0.0085	0.018	0.0323	0.0464	0.0829	0.1227	0.1923	0.2836
1.56	4.7	0	0	0	0	0.0029	0.0031	0.0103	0.0224	0.0404	0.0777	0.1265
1.67	5	0	0	0	0.0016	0	0.0017	0.0102	0.0088	0.0187	0.0399	0.0666

APPENDIX E.2

TOTAL QUALITY LOSS OF ITEMS ACCEPTED DUE TO INSPECTION ERROR INSTEAD OF BEING SCRAPPED IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

1.0e+003*

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.4074	0.8505	1.2861	1.7584	2.1627	2.5515	2.9631	3.29	3.5469	3.786
0.43	1.3	0	0.4699	0.9452	1.3754	1.7711	2.1666	2.5165	2.7803	3.0767	3.3365	3.489
0.57	1.7	0	0.4207	0.7827	1.1638	1.4572	1.7234	1.8932	2.0982	2.2786	2.3405	2.5572
0.67	2	0	0.3108	0.6122	0.8533	1.0196	1.1852	1.3609	1.4333	1.5583	1.6485	1.7204
0.77	2.3	0	0.2207	0.3845	0.5636	0.6487	0.7884	0.8541	0.8989	0.9719	1.0004	1.0998
0.9	2.7	0	0.1039	0.1877	0.2442	0.3169	0.3336	0.3734	0.3796	0.4378	0.4548	0.4729
1	3	0	0.0567	0.1023	0.1147	0.1553	0.1617	0.1898	0.21	0.1853	0.2157	0.2235
1.1	3.3	0	0.0149	0.0447	0.0588	0.0571	0.0815	0.0838	0.0918	0.1018	0.0968	0.0944
1.23	3.7	0	0.0066	0.0213	0.0109	0.0216	0.0226	0.0211	0.023	0.0349	0.0271	0.0272
1.33	4	0	0.0022	0.0066	0.0104	0.0047	0.0057	0.0096	0.0071	0.0098	0.0057	0.0119
1.43	4.3	0	0.0012	0	0.0025	0	0.0038	0	0	0.0038	0.0055	0.003
1.56	4.7	0	0	0	0	0	0	0	0	0	0	0.0015
1.67	5	0	0	0	0.0017	0	0	0	0	0	0	0

APPENDIX F.1

NUMBER OF CONFORMING ITEMS SCRAPPED DUE TO INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.2032	0.4294	0.6895	0.9429	1.2361	1.5118	1.7845	2.0677	2.3547	2.6085
0.43	1.3	0	0.1456	0.3221	0.5158	0.7248	0.9676	1.2242	1.4753	1.75	2.0172	2.2759
0.57	1.7	0	0.0862	0.1858	0.3052	0.4599	0.6248	0.8117	1.0083	1.237	1.475	1.727
0.67	2	0	0.0469	0.115	0.1904	0.2924	0.4023	0.5379	0.6936	0.8835	1.0838	1.281
0.77	2.3	0	0.0255	0.0599	0.1103	0.171	0.2419	0.3421	0.4506	0.5836	0.7375	0.9077
0.9	2.7	0	0.0103	0.0237	0.0428	0.0704	0.1095	0.1575	0.2219	0.305	0.3992	0.5178
1	3	0	0.0048	0.0103	0.0203	0.0329	0.0527	0.0861	0.1192	0.1761	0.2412	0.3227
1.1	3.3	0	0.0014	0.0043	0.0081	0.0159	0.0245	0.0403	0.0615	0.0961	0.1366	0.1929
1.23	3.7	0	0.0004	0.0011	0.0027	0.0046	0.0092	0.0136	0.0237	0.0353	0.0578	0.0845
1.33	4	0	0	0.0003	0.0007	0.0013	0.0029	0.0059	0.0095	0.0178	0.03	0.0466
1.43	4.3	0	0	0.0001	0.003	0.006	0.0013	0.0021	0.0041	0.0077	0.0136	0.0241
1.56	4.7	0	0	0	0	0.0001	0.0001	0.0004	0.001	0.0019	0.005	0.0089
1.67	5	0	0	0	0	0	0.0001	0.0003	0.0004	0.001	0.0019	0.0043

APPENDIX F.2

NUMBER OF ITEMS ACCEPTED DUE TO INSPECTION ERROR INSTEAD OF BEING SCRAPPED IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	181.2	339.73	470.26	598.33	688.83	769.83	852.16	907.33	950.6	980.9
0.43	1.3	0	127.16	236.1	321.4	393.13	460	514.36	550.1	590.8	624.63	641.76
0.57	1.7	0	67.93	119.83	169.26	203.53	234.46	250.23	271.96	290.4	294.56	315.06
0.67	2	0	36.73	69.23	92.56	108.13	121.83	136.46	142.53	151.56	158.96	165.03
0.77	2.3	0	19.86	33.26	47.2	53.13	63.1	67.3	70.56	74.7	76.16	83.33
0.9	2.7	0	6.83	11.96	15.13	19.26	20.06	22.03	22.26	25.66	26.6	27.33
1	3	0	3.03	5.33	5.9	7.8	8.03	9.26	10.23	9	10.43	10.8
1.1	3.3	0	0.66	1.93	2.5	2.43	3.4	3.46	3.76	4.16	3.96	3.86
1.23	3.7	0	0.23	0.73	0.36	0.7	0.76	0.7	0.76	1.13	0.9	0.86
1.33	4	0	0.06	0.2	0.3	0.13	0.16	0.26	0.2	0.26	0.16	0.33
1.43	4.3	0	0.03	0	0.06	0	0.1	0	0	0.1	0.13	0.06
1.56	4.7	0	0	0	0	0	0	0	0	0	0	0.03
1.67	5	0	0	0	0.03	0	0	0	0	0	0	0

APPENDIX G

DIFFERENCE BETWEEN THE SUM OF QUALITY LOSS WITHOUT INSPECTION ERROR AND THE SUM OF QUALITY LOSS WITH INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$$\sum L_o - \sum L_1 = \sum L_{sc} - \sum L_{as}$$

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

1.0e+003*

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	-0.0511	-0.1986	-0.3779	-0.6854	-0.9325	-1.2137	-1.527	-1.7562	-1.9007	-2.0279
0.43	1.3	0	-0.0262	-0.0683	-0.1287	-0.2274	-0.3485	-0.472	-0.5801	-0.7044	-0.8022	-0.814
0.57	1.7		0 0.0398	0.1165	0.1756	0.3549	0.4816	0.6495	0.7282	0.8579	1.0475	1.0773
0.67	2	0	0.0407	0.1807	0.341	0.6451	0.854	1.0634	1.3813	1.6361	1.8909	2.0716
0.77	2.3	0	0.0318	0.1708	0.371	0.6684	0.8923	1.2731	1.5836	1.8852	2.243	2.4615
0.9	2.7	0	0.0385	0.1196	0.2714	0.4544	0.7402	1.0021	1.3537	1.6515	1.9384	2.3145
1	3	0	0.0254	0.0663	0.1882	0.2904	0.4828	0.7636	0.9424	1.2991	1.5631	1.856
1.1	3.3	0	0.0152	0.0399	0.089	0.2101	0.2845	0.449	0.6192	0.8694	1.1053	1.3855
1.23	3.7	0	0.0039	0.0062	0.0517	0.0738	0.1494	0.2046	0.3186	0.4029	0.6017	0.7531
1.33	4	0	-0.0022	0.0033	0.0096	0.0273	0.057	0.1044	0.1518	0.2532	0.3517	0.485
1.43	4.3	0	0	0.0035	0.006	0.018	0.0285	0.0464	0.0829	0.1189	0.1868	0.2806
1.56	4.7	0	0	0	0	0.0029	0.0031	0.0103	0.0224	0.0404	0.0777	0.125
1.67	5	0	0	0	-0.0001	0	0.0017	0.0102	0.0088	0.0187	0.0399	0.0666

APPENDIX H.1

STANDARD DEVIATION OF TRUE QUALITY CHARACTERISTIC VALUES OF ITEMS ACCEPTED IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p / USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5398	0.5428	0.5559	0.5749	0.5995	0.625	0.6527	0.6836	0.7121	0.736	0.7606
0.43	1.3	0.6686	0.6699	0.6777	0.6826	0.6926	0.7077	0.7228	0.7389	0.7571	0.7754	0.7889
0.57	1.7	0.8084	0.8055	0.8047	0.805	0.8054	0.8062	0.8077	0.8116	0.8196	0.8239	0.8379
0.67	2	0.88	0.8784	0.876	0.8734	0.8688	0.8655	0.8635	0.8615	0.8619	0.8638	0.8673
0.77	2.3	0.9311	0.93	0.9273	0.9239	0.9184	0.9156	0.9093	0.9032	0.9003	0.8992	0.9002
0.9	2.7	0.9699	0.9714	0.9674	0.9652	0.9632	0.9559	0.9511	0.9451	0.9413	0.9348	0.9349
1	3	0.986	0.9868	0.9859	0.9834	0.9809	0.9766	0.9715	0.9674	0.964	0.9569	0.951
1.1	3.3	0.9958	0.9925	0.9932	0.9929	0.9915	0.9891	0.9846	0.9823	0.9777	0.972	0.9698
1.23	3.7	0.9977	0.9965	0.9998	0.9958	0.9972	0.9972	0.9929	0.9929	0.9889	0.9865	0.9826
1.33	4	1.0005	0.9987	0.9988	1.0004	0.9999	0.9995	0.9981	0.9953	0.9951	0.993	0.9868
1.43	4.3	1.003	0.9987	0.9996	0.9991	0.9982	0.9983	1	0.9976	0.9968	0.9976	0.9951
1.56	4.7	0.9994	1.0007	0.9987	1.0015	1.0014	0.9992	1.0016	0.9998	0.9979	0.9996	0.9968
1.67	5	0.999	1.0007	0.9998	1.0005	0.9999	0.9998	1.0011	0.9992	1.0007	0.9972	0.9989

APPENDIX H.2

STANDARD DEVIATION OF OBSERVED QUALITY CHARACTERISTIC VALUES OF ITEMS ACCEPTED IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5398	0.5389	0.5412	0.5432	0.5456	0.5478	0.5485	0.5523	0.5532	0.5569	0.5574
0.43	1.3	0.6686	0.6689	0.6737	0.6744	0.6798	0.6843	0.6892	0.6952	0.6999	0.704	0.7097
0.57	1.7	0.8048	0.8071	0.8118	0.8194	0.8276	0.837	0.8487	0.8593	0.8681	0.8798	0.8905
0.67	2	0.88	0.8812	0.8886	0.898	0.9128	0.9277	0.9436	0.959	0.9776	0.9931	1.0078
0.77	2.3	0.9311	0.9339	0.9426	0.9564	0.9743	0.9965	1.0198	1.0432	1.0665	1.0871	1.1106
0.9	2.7	0.9699	0.976	0.9854	1.0042	1.0299	1.0551	1.0886	1.1208	1.1535	1.1819	1.2158
1	3	0.986	0.9923	1.0046	1.0262	1.0536	1.0865	1.1193	1.1596	1.2012	1.2386	1.2771
1.1	3.3	0.9958	0.9972	1.0123	1.0353	1.0655	1.1032	1.1416	1.1852	1.2304	1.2777	1.3246
1.23	3.7	0.9977	1.0012	1.0198	1.0401	1.0744	1.1134	1.156	1.2075	1.2569	1.3097	1.3667
1.33	4	1.0005	1.0037	1.0188	1.0433	1.0769	1.1171	1.1614	1.2138	1.2678	1.3265	1.3837
1.43	4.3	1.003	1.0034	1.0199	1.0434	1.0773	1.1157	1.1632	1.2172	1.2732	1.3352	1.3992
1.56	4.7	0.9994	1.0057	1.0187	1.0454	1.0777	1.1163	1.1669	1.2206	1.2778	1.3422	1.408
1.67	5	0.999	1.0054	1.0196	1.0436	1.0766	1.1184	1.1675	1.2204	1.2785	1.3414	1.4087

APPENDIX I

EXCESS COST PER ITEM PRODUCED IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, \mu_p = 0, \sigma_p = 1$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.4065	0.8588	1.379	1.8858	2.4723	3.0236	3.5691	4.1354	4.7095	5.2169
0.43	1.3	0	0.2912	0.6441	1.0317	1.4496	1.9353	2.4483	2.9505	3.5001	4.0343	4.5517
0.57	1.7	0	0.1723	0.3716	0.6104	0.9199	1.2497	1.6235	2.0165	2.474	2.9501	3.4539
0.67	2	0	0.0939	0.2301	0.3807	0.5847	0.8047	1.0758	1.3872	1.7669	2.1676	2.5619
0.77	2.3	0	0.0509	0.1198	0.2206	0.3419	0.4837	0.6841	0.9011	1.1673	1.475	1.8153
0.9	2.7	0	0.0206	0.0475	0.0857	0.1409	0.2189	0.315	0.4439	0.6099	0.7983	1.0357
1	3	0	0.0095	0.0207	0.0406	0.0657	0.1054	0.1722	0.2385	0.3521	0.4825	0.6453
1.1	3.3	0	0.0029	0.0087	0.0162	0.0318	0.0491	0.0806	0.123	0.1923	0.2732	0.3857
1.23	3.7	0	0.0008	0.0022	0.0054	0.0092	0.0183	0.0273	0.0474	0.0707	0.1157	0.1691
1.33	4	0	0	0.0007	0.0015	0.0026	0.0057	0.0117	0.0191	0.0356	0.0601	0.0932
1.43	4.3	0	0.0001	0.0002	0.0005	0.0013	0.0025	0.0042	0.0083	0.0153	0.0271	0.0483
1.56	4.7	0	0	0	0	0.0002	0.0002	0.0007	0.002	0.0038	0.0099	0.0178
1.67	5	0	0	0	0.0001	0	0.0001	0.0007	0.0008	0.0019	0.0038	0.0087

APPENDIX J.1

PROPORTION OF CONFORMING ITEMS SCRAPPED DUE TO INSPECTION ERROR TO THE NUMBER OF NON-CONFORMING ITEMS ACCEPTED DUE TO INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITH REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	1.5406	1.6989	1.9036	2.1126	2.2804	2.4818	2.6189	2.6932	2.7057	2.7929
0.43	1.3	0	1.4403	1.6381	1.9076	2.1887	2.5124	2.8299	3.0781	3.2157	3.3636	3.4607
0.57	1.7	0	1.3575	1.5874	2.1117	2.4644	2.9231	3.4538	3.8794	4.3448	4.7254	5.0026
0.67	2	0	1.2879	1.7225	2.2085	2.702	3.322	4.0103	4.6832	5.6933	6.1962	6.824
0.77	2.3	0	1.3682	1.9702	2.274	3.3338	3.8013	4.935	6.3604	7.4646	8.4156	10.0063
0.9	2.7	0	1.4952	1.9859	2.5117	3.6851	5.1688	6.8726	9.2507	11.2031	14.6719	17.3928
1	3	0	1.2604	1.8011	2.6528	4.6981	5.9004	8.9894	13.1905	17.9816	20.3711	26.6426
1.1	3.3	0	1.4186	2.1724	3.9315	6.5652	6.8257	11.9687	16.4312	26.5876	34.4818	42.4872
1.23	3.7	0	2.1429	4	9.375	5.7273	5.175	19.8571	28.875	36.6552	70.75	85.4643
1.33	4	0	1.3333	4	4	21	30.6667	19.1111	33.1111	82.1667	157	141.6667
1.43	4.3	0	0.3333	2	2	0.5667	16	29.5	40.3333	6.9667	95.5	676
1.56	4.7	0	0	1	0.0333	0.0667	0.1667	0.7	1.4333	77	4.2667	8.7333
1.67	5	0	0	0	0	0	0.0667	0.0333	0.5	0.5667	1.6333	3.7667

APPENDIX J.2

PROPORTION OF CONFORMING ITEMS REWORKED DUE TO INSPECTION ERROR TO THE NUMBER OF NON-CONFORMING ITEMS ACCEPTED DUE TO INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITHOUT REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.0576	0.1293	3.3105	0.3961	0.1419	0.0793	0.0544	0.0417	0.0368	0.0342
0.43	1.3	0	0.0276	0.0642	1.718	0.3408	0.1589	0.0913	0.0629	0.0521	0.0458	0.0453
0.57	1.7	0	0.0101	0.0249	0.0457	0.4364	0.153	0.1035	0.0838	0.0681	0.0707	0.0718
0.67	2	0	0.0045	0.0115	0.0207	0.5175	0.21	0.1143	0.116	0.1117	0.0985	0.1058
0.77	2.3	0	0.0017	0.0038	0.0083	0.0146	0.1947	0.1958	0.2724	0.1531	0.1584	0.1706
0.9	2.7	0	0.0005	0.0011	0.002	0.069	0.0075	0.227	0.342	0.129	0.1486	0.6567
1	3	0	0.0001	0.0005	0.0006	0.0017	0.003	0.166	0.315	0.0169	0.2177	0.736
1.1	3.3	0	0.0001	0.0001	0.0002	0.0007	0.0012	0.0016	0.0045	0.0084	0.0124	0.667
1.23	3.7	0	0	0	0.0001	0.0001	0.0002	0.0006	0.0012	0.002	0.0051	0.0098
1.33	4	0	0	0	0	0	0	0.0001	0.0004	0.0006	0.0023	0.0033
1.43	4.3	0	0	0	0	0	0	0.0001	0.0001	0.0004	0.0007	0.0016
1.56	4.7	0	0	0	0	0	0	0.0001	0.0001	0	0.0002	0.0005
1.67	5	0	0	0	0	0	0	0	0	0	0.0002	0.0001

APPENDIX K

DIFFERENCE BETWEEN THE SUM OF QUALITY LOSS WITHOUT INSPECTION ERROR AND THE SUM OF QUALITY LOSS WITH INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITH REWORK

$$\sum L_o - \sum L_1 = \sum L_{sc} + \sum L_{sr} + \sum L_{er} - \sum L_{as} - \sum L_{ar}$$

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

1.0e+003*

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	-0.1722	-0.4573	-0.8169	-1.2565	-1.7639	-2.2251	-2.6915	-3.1381	-3.4571	-3.6908
0.43	1.3	0	-0.1324	-0.3247	-0.5464	-0.8177	-1.091	-1.3944	-1.7663	-2.0904	-2.4044	-2.5951
0.57	1.7	0	-0.0665	-0.1239	-0.0367	-0.0723	-0.1076	-0.119	-0.209	-0.3703	-0.4557	-0.6346
0.67	2	0	-0.0335	0.0417	0.1578	0.2879	0.411	0.5003	0.5819	0.6897	0.6108	0.5618
0.77	2.3	0	0.009	0.1323	0.2121	0.4862	0.6267	0.8327	1.0869	1.1764	1.2429	1.3462
0.9	2.7	0	0.0226	0.0888	0.1822	0.3641	0.593	0.7909	1.0438	1.2084	1.4715	1.5708
1	3	0	0.0029	0.045	0.118	0.2688	0.41	0.6478	0.8485	1.0507	1.154	1.4162
1.1	3.3	0	0.0058	0.0344	0.1041	0.1781	0.2528	0.3847	0.5536	0.7197	0.9124	1.0442
1.23	3.7	0	0.0062	0.0209	0.0489	0.0668	0.0901	0.1974	0.2939	0.3853	0.5133	0.6611
1.33	4	0	0.0008	0.0025	0.0171	0.048	0.0638	0.0984	0.1482	0.2142	0.2973	0.4006
1.43	4.3	0	-0.0027	0.0017	0.0013	0.0162	0.0226	0.0408	0.0655	0.1098	0.1669	0.2501
1.56	4.7	0	0	-0.0003	0.0012	0.0021	0.0043	0.0197	0.0324	0.0454	0.0647	0.1115
1.67	5	0	0	0	0	0	0.0021	0.0014	0.013	0.0127	0.0262	0.0542

APPENDIX L.1

TOTAL QUALITY LOSS OF ITEMS COMING FROM PROCESS IN A PRODUCTION ENVIRONMENT WITH REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5816	0.5913	0.6185	0.6587	0.7157	0.7842	0.8511	0.9295	1.0134	1.0862	1.1571
0.43	1.3	0.8956	0.8982	0.9119	0.9323	0.9637	0.9982	1.0401	1.0926	1.1454	1.2005	1.2531
0.57	1.7	1.2961	1.2959	1.2975	1.2906	1.2907	1.2979	1.3062	1.3222	1.3494	1.3591	1.3949
0.67	2	1.5455	1.5481	1.5358	1.5268	1.5132	1.5029	1.4968	1.4945	1.4888	1.4993	1.5076
0.77	2.3	1.7373	1.7353	1.7198	1.7111	1.6921	1.6712	1.6493	1.635	1.629	1.6169	1.6165
0.9	2.7	1.8875	1.8899	1.8723	1.8699	1.8486	1.8319	1.8051	1.7856	1.7701	1.7568	1.7344
1	3	1.9463	1.9463	1.9475	1.9305	1.9232	1.9022	1.8855	1.867	1.855	1.8334	1.8184
1.1	3.3	1.9858	1.9848	1.9791	1.9679	1.9703	1.9461	1.9436	1.9207	1.9048	1.8941	1.8719
1.23	3.7	2.0012	1.993	2.0067	1.9926	1.99	1.9788	1.9778	1.9687	1.9577	1.9408	1.9332
1.33	4	1.9973	2.0007	1.9893	1.994	2.0025	1.9914	1.988	1.9883	1.9767	1.9695	1.9527
1.43	4.3	2	1.9992	2.0012	2.0052	1.9897	1.9973	1.9956	1.9881	1.9942	1.9789	1.9839
1.56	4.7	2.0114	1.9936	1.9981	2.0039	2.0017	2.0024	2.0023	1.998	1.9918	1.9859	1.987
1.67	5	2.008	2.0057	2.0061	2.0002	2.0029	1.9927	1.9933	1.9996	1.9989	1.9989	1.9918

APPENDIX L.2

TOTAL QUALITY LOSS OF ITEMS COMING FROM REWORK IN A PRODUCTION ENVIRONMENT WITH REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5255	0.5249	0.5378	0.5588	0.5843	0.6173	0.6464	0.6877	0.7141	0.7103	0.7093
0.43	1.3	0.7453	0.7439	0.7457	0.7514	0.7375	0.754	0.7638	0.7782	0.7844	0.7911	0.7858
0.57	1.7	0.9596	0.9502	0.9639	0.9359	0.9268	0.9161	0.9118	0.8948	0.9128	0.8996	0.9096
0.67	2	1.0736	1.046	1.033	1.0224	1.0038	0.9949	0.9878	0.966	0.9591	0.9708	0.9694
0.77	2.3	1.105	1.0438	1.1348	1.0899	1.0658	1.0594	1.0547	1.028	1.0308	1.04	1.0266
0.9	2.7	1.0692	1.1628	1.136	1.1401	1.119	1.0823	1.1003	1.1169	1.0777	1.0811	1.0754
1	3	1.0906	1.0748	1.0901	1.2101	1.0799	1.1203	1.1374	1.0826	1.0636	1.1175	1.1019
1.1	3.3	1.0825	1.1715	1.0948	1.2088	1.0306	1.1388	1.1435	1.1598	1.0622	1.1341	1.1118
1.23	3.7	1.071	0.7612	1.1827	0.9964	0.9505	1.0326	1.0132	1.172	1.1545	1.1103	1.1753
1.33	4	0.3713	0.3009	1.2561	1.0516	1.0716	1.0847	1.0798	1.1848	1.0483	1.1358	1.0663
1.43	4.3	0.0329	0.2005	0.5926	0.1653	0.8156	1.356	0.8084	1.1865	1.1612	1.05	1.2682
1.56	4.7	0	0	0.0085	0.0384	0.0958	0.0408	0.5699	0.6908	0.9321	1.1393	1.2135
1.67	5	0	0.007	0	0	0	0.0069	0.0339	0.3238	0.5517	1.0401	1.055

APPENDIX M.1

STANDARD DEVIATION OF TRUE QUALITY CHARACTERISTIC VALUES OF ACCEPTED ITEMS IN A PRODUCTION ENVIRONMENT WITH REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5319	0.5349	0.5455	0.5609	0.5813	0.605	0.6267	0.6519	0.675	0.6908	0.7051
0.43	1.3	0.6593	0.6598	0.664	0.67	0.6778	0.6879	0.6994	0.7135	0.7266	0.7394	0.75
0.57	1.7	0.7964	0.7958	0.7964	0.7927	0.7915	0.792	0.7924	0.7945	0.8007	0.8007	0.8081
0.67	2	0.8733	0.8735	0.8695	0.8663	0.8612	0.857	0.8537	0.8508	0.8471	0.8481	0.8483
0.77	2.3	0.9285	0.9275	0.9237	0.9206	0.9146	0.9079	0.901	0.8951	0.892	0.8871	0.8847
0.9	2.7	0.9701	0.9707	0.9661	0.9652	0.9592	0.9541	0.9465	0.9404	0.935	0.93	0.9228
1	3	0.9859	0.9859	0.9861	0.9818	0.9795	0.9739	0.969	0.9635	0.9595	0.9532	0.948
1.1	3.3	0.9962	0.996	0.9945	0.9916	0.992	0.9858	0.985	0.9787	0.9739	0.9706	0.9641
1.23	3.7	1.0002	0.9982	1.0016	0.998	0.9973	0.9945	0.9941	0.9917	0.9886	0.9839	0.9816
1.33	4	0.9993	1.0001	0.9973	0.9984	1.0005	0.9977	0.9968	0.9968	0.9937	0.9918	0.9871
1.43	4.3	1	0.9998	1.0003	1.0012	0.9974	0.9993	0.9988	0.9969	0.9984	0.9944	0.9956
1.56	4.7	1.0028	0.9984	0.9995	1.0009	1.0004	1.0005	1.0006	0.9994	0.9979	0.9964	0.9966
1.67	5	1.002	1.0014	1.0015	1	1.0006	0.9981	0.9983	0.9999	0.9997	0.9997	0.9978

APPENDIX M.2

STANDARD DEVIATION OF OBSERVED QUALITY CHARACTERISTIC VALUES OF ACCEPTED ITEMS IN A PRODUCTION ENVIRONMENT WITH REWORK

$$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0.5319	0.5322	0.5332	0.5357	0.5384	0.5419	0.5443	0.5483	0.5514	0.5518	0.5522
0.43	1.3	0.6593	0.6597	0.6628	0.6673	0.6725	0.6786	0.6835	0.6892	0.6959	0.6975	0.7009
0.57	1.7	0.7964	0.7977	0.8042	0.8099	0.8185	0.8295	0.8417	0.852	0.8645	0.8711	0.8781
0.67	2	0.8733	0.8765	0.8818	0.8925	0.906	0.9222	0.937	0.9553	0.9693	0.9835	0.994
0.77	2.3	0.9285	0.9316	0.9399	0.9542	0.9712	0.9908	1.0147	1.0371	1.0607	1.078	1.0967
0.9	2.7	0.9701	0.9753	0.9839	1.0037	1.0267	1.0552	1.0857	1.1163	1.1504	1.1795	1.2072
1	3	0.9859	0.9906	1.005	1.0237	1.051	1.0828	1.1183	1.1581	1.1948	1.2345	1.2714
1.1	3.3	0.9962	1.0004	1.0142	1.0349	1.0665	1.0985	1.1423	1.1837	1.2305	1.2738	1.3168
1.23	3.7	1.0002	1.0027	1.0213	1.042	1.0726	1.1104	1.1587	1.2054	1.2581	1.3118	1.3647
1.33	4	0.9993	1.005	1.0169	1.0426	1.0763	1.1149	1.1619	1.2142	1.268	1.3251	1.3792
1.43	4.3	1	1.0046	1.0195	1.0447	1.0749	1.1165	1.1637	1.2156	1.2759	1.3338	1.3965
1.56	4.7	1.0028	1.0033	1.0197	1.0446	1.0776	1.119	1.167	1.2209	1.2767	1.3376	1.4069
1.67	5	1.002	1.0063	1.0214	1.0445	1.0786	1.1162	1.1648	1.219	1.2805	1.3435	1.4085

APPENDIX N.1

NUMBER OF ITEMS SCRAPPED DUE TO INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITH REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

1.0e+003*

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.0576	0.1293	0.2207	0.3433	0.4919	0.679	0.9183	1.1353	1.3307	1.5413
0.43	1.3	0	0.0276	0.0642	0.1145	0.1817	0.2702	0.3804	0.5158	0.6686	0.8195	0.9996
0.57	1.7	0	0.0101	0.0249	0.0457	0.0727	0.1122	0.1656	0.2403	0.3199	0.4289	0.5408
0.67	2	0	0.0045	0.0115	0.0207	0.0345	0.056	0.0876	0.1276	0.1788	0.2429	0.3315
0.77	2.3	0	0.0017	0.0038	0.0083	0.0146	0.026	0.0392	0.0636	0.097	0.132	0.1933
0.9	2.7	0	0.0005	0.0011	0.002	0.0046	0.0075	0.0151	0.0228	0.0344	0.0594	0.0876
1	3	0	0.0001	0.0005	0.0006	0.0017	0.003	0.0055	0.0105	0.0169	0.029	0.0491
1.1	3.3	0	0.0001	0.0001	0.0002	0.0007	0.0012	0.0016	0.0045	0.0084	0.0124	0.0222
1.23	3.7	0	0	0	0.0001	0.0001	0.0002	0.0006	0.0012	0.002	0.0051	0.0098
1.33	4	0	0	0	0	0	0	0.0001	0.0004	0.0006	0.0023	0.0033
1.43	4.3	0	0	0	0	0	0	0.0001	0.0001	0.0004	0.0007	0.0016
1.56	4.7	0	0	0	0	0	0	0.0001	0.0001	0	0.0002	0.0005
1.67	5	0	0	0	0	0	0	0	0	0	0.0002	0.0001

APPENDIX N.2

NUMBER OF ITEMS REWORKED DUE TO INSPECTION ERROR IN A PRODUCTION ENVIRONMENT WITH REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

1.0e+003*

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.3605	0.7371	1.1506	1.5848	2.0259	2.4536	2.8339	3.107	3.2316	3.3799
0.43	1.3	0	0.2001	0.4269	0.6944	0.9671	1.2808	1.5903	1.9108	2.1198	2.3222	2.4673
0.57	1.7	0	0.0973	0.2067	0.3523	0.5089	0.6927	0.8955	1.1034	1.2939	1.4546	1.6317
0.67	2	0	0.0532	0.1202	0.1991	0.3007	0.423	0.5578	0.7061	0.8625	1.016	1.1428
0.77	2.3	0	0.0266	0.0662	0.109	0.1721	0.2481	0.3288	0.4459	0.5554	0.6741	0.7952
0.9	2.7	0	0.0104	0.0234	0.043	0.071	0.1082	0.1546	0.2177	0.2849	0.3741	0.4487
1	3	0	0.004	0.0106	0.0191	0.0332	0.0533	0.0848	0.12	0.163	0.2159	0.2833
1.1	3.3	0	0.002	0.0042	0.0096	0.0151	0.0248	0.0383	0.0597	0.086	0.1264	0.1657
1.23	3.7	0	0.0005	0.0013	0.0025	0.0042	0.0069	0.0139	0.0231	0.0354	0.0566	0.0798
1.33	4	0	0.0001	0.0001	0.0009	0.0021	0.0031	0.0057	0.0099	0.0164	0.0262	0.0425
1.43	4.3	0	0	0.0001	0.0001	0.0006	0.0011	0.002	0.004	0.007	0.0127	0.0225
1.56	4.7	0	0	0.0001	0	0.0001	0.0002	0.0007	0.0014	0.0026	0.0043	0.0087
1.67	5	0	0	0	0	0	0.0001	0	0.0005	0.0006	0.0016	0.0038

APPENDIX O

AVERAGE EXCESS COST PER ITEM PRODUCED IN A PRODUCTION ENVIRONMENT WITH REWORK

$ULs = USL + 1, 0 < \epsilon_p < 1, 0 < \epsilon_r < 0.75, \mu_p = 0, \sigma_p = 1, \mu_r = 0, \sigma_r = 0.75$

	P/T Ratio	0	0.06-0.3	0.6-0.12	0.9-0.18	0.24-1.2	0.3-1.5	0.36-1.8	0.42-2.4	0.48-2.4	0.54-2.7	0.6-3
C_p	ϵ_p USL	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.33	1	0	0.5547	1.1687	1.8852	2.7224	3.6647	4.6901	5.798	6.6753	7.2166	7.8542
0.43	1.3	0	0.28	0.6074	1.0194	1.4779	2.0498	2.6894	3.4183	4.019	4.5735	5.1128
0.57	1.7	0	0.1256	0.2707	0.4697	0.6958	0.9805	1.3273	1.735	2.1286	2.5282	2.9662
0.67	2	0	0.0655	0.1508	0.2529	0.3902	0.565	0.7783	1.0316	1.3157	1.6211	1.9366
0.77	2.3	0	0.032	0.0788	0.133	0.2126	0.317	0.4325	0.6118	0.8021	1.0048	1.2586
0.9	2.7	0	0.0122	0.0274	0.0503	0.0852	0.1313	0.1961	0.2819	0.3794	0.5278	0.6653
1	3	0	0.0045	0.0123	0.0219	0.0391	0.0635	0.1032	0.1511	0.2119	0.2934	0.408
1.1	3.3	0	0.0024	0.0047	0.0108	0.0176	0.0292	0.0448	0.0741	0.1103	0.1635	0.2262
1.23	3.7	0	0.0006	0.0015	0.0028	0.0048	0.008	0.0163	0.0276	0.0429	0.0723	0.107
1.33	4	0	0.0001	0.0001	0.001	0.0024	0.0034	0.0066	0.0116	0.0192	0.0334	0.0533
1.43	4.3	0	0	0.0001	0.0001	0.0006	0.0012	0.0023	0.0046	0.0085	0.0155	0.028
1.56	4.7	0	0	0.0001	0	0.0001	0.0002	0.0009	0.017	0.0029	0.0052	0.0106
1.67	5	0	0	0	0	0	0.0001	0.0001	0.0005	0.0006	0.0021	0.0043

APPENDIX P.1

MINITAB ANOVA RESULTS

General Linear Model: Loss versus limits; E_r; E_p; D_r; S_r; D_p

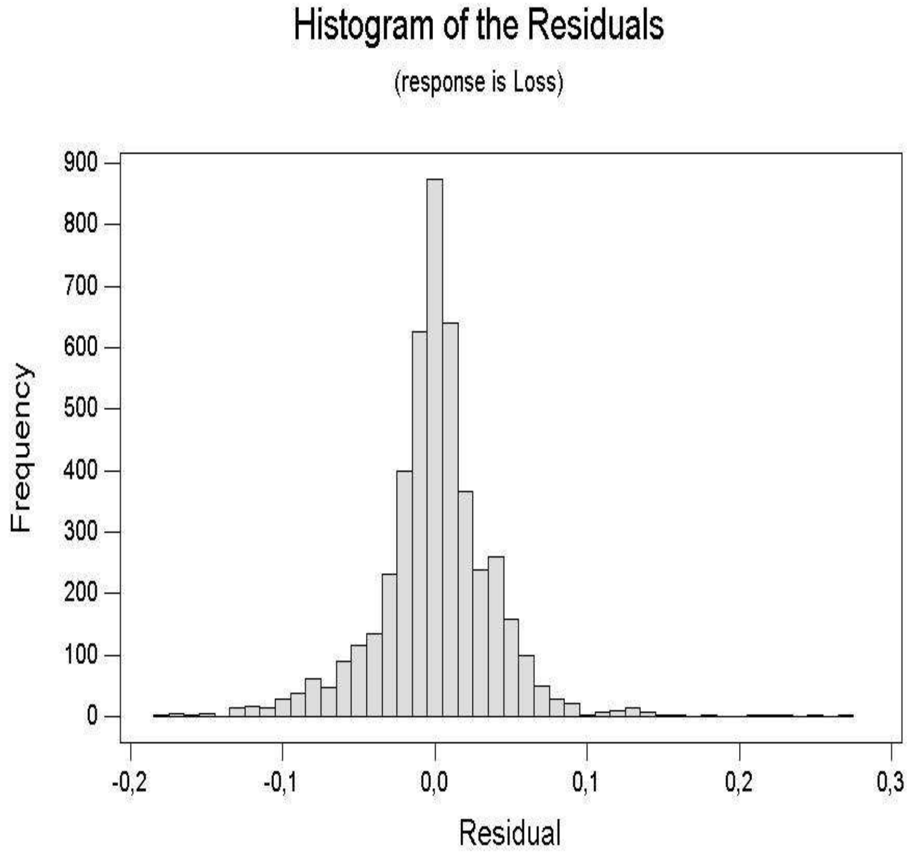
Factor	Type	Levels	Values
limits	fixed	4	1 2 3 4
E_r	fixed	6	1 2 3 4 5 6
E_p	fixed	6	1 2 3 4 5 6
D_r	fixed	4	1 2 3 4
S_r	fixed	2	1 2
D_p	fixed	4	1 2 3 4

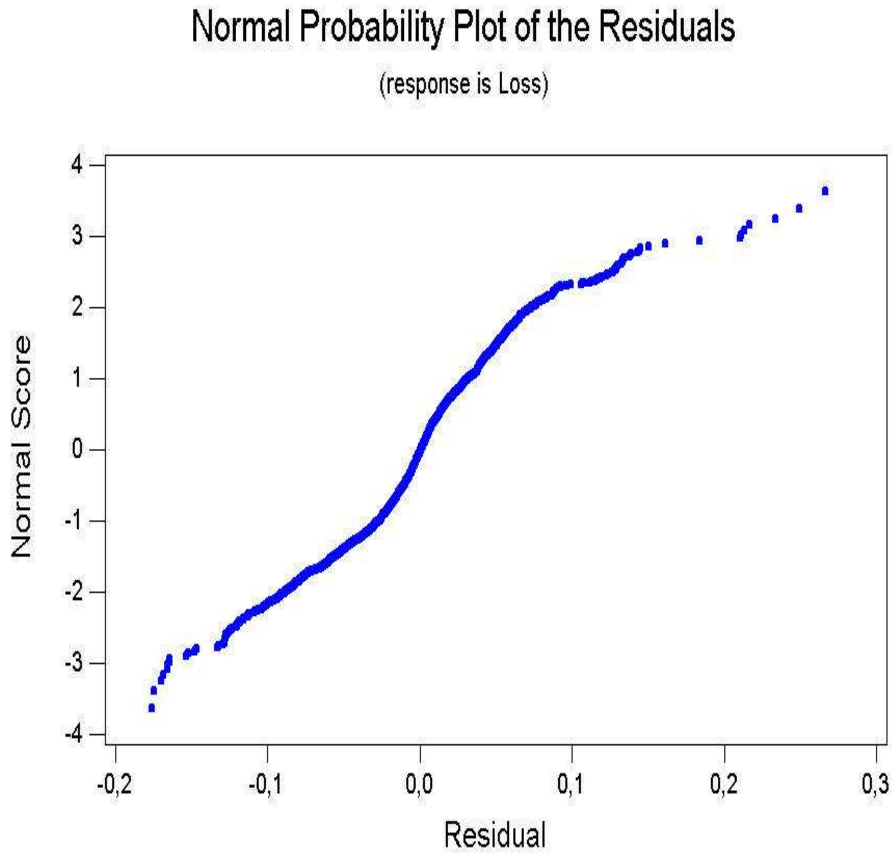
Analysis of Variance for Loss, using Adjusted SS for Tests

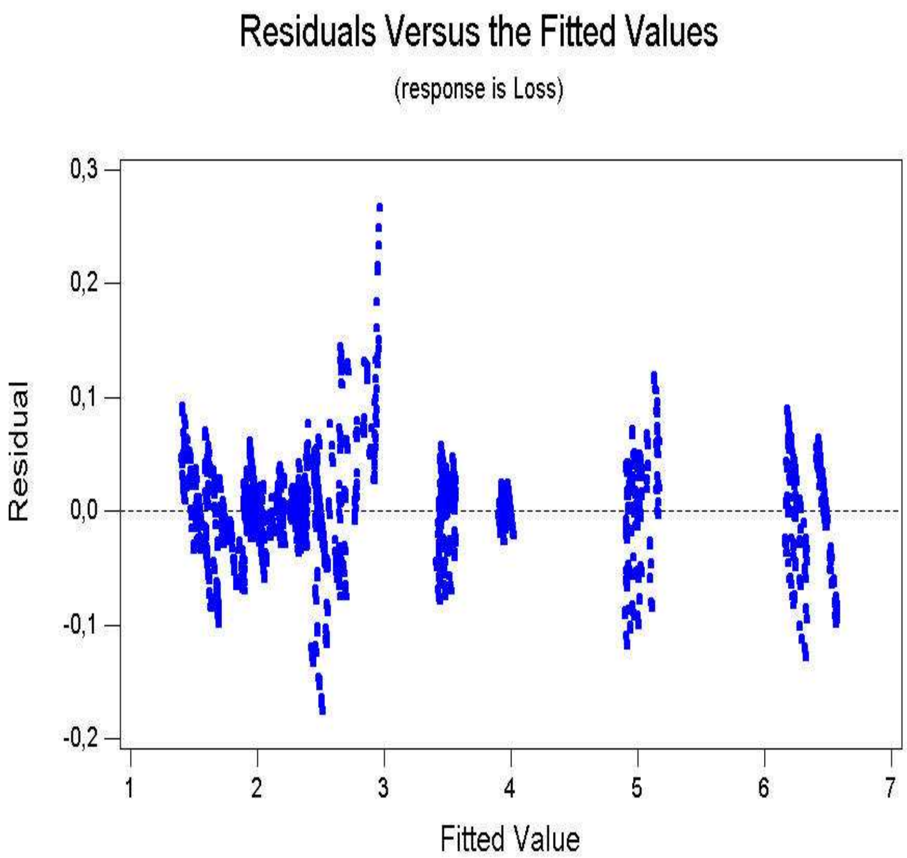
Source	DF	Seq SS	Adj SS	Adj MS	F	P
limits	3	7232,02	7232,02	2410,67	1,6E+06	0,000
E_r	5	5,77	5,77	1,15	771,73	0,000
E_p	5	0,23	0,23	0,05	31,35	0,000
D_r	3	0,01	0,01	0,00	1,80	0,144
S_r	1	0,00	0,00	0,00	0,03	0,864
D_p	3	2184,16	2184,16	728,05	4,9E+05	0,000
limits*E_r	15	3,14	3,14	0,21	140,14	0,000
limits*E_p	15	0,13	0,13	0,01	5,74	0,000
limits*D_r	9	0,00	0,00	0,00	0,20	0,994
limits*S_r	3	0,00	0,00	0,00	0,00	1,000
limits*D_p	9	1241,27	1241,27	137,92	9,2E+04	0,000
E_r*E_p	25	0,67	0,67	0,03	18,05	0,000
E_r*D_r	15	0,02	0,02	0,00	0,90	0,559
E_r*S_r	5	0,00	0,00	0,00	0,04	0,999
E_r*D_p	15	8,45	8,45	0,56	376,74	0,000
E_p*D_r	15	0,05	0,05	0,00	2,17	0,005
E_p*S_r	5	0,00	0,00	0,00	0,00	1,000
E_p*D_p	15	0,40	0,40	0,03	17,85	0,000
D_r*S_r	3	0,00	0,00	0,00	0,00	1,000
D_r*D_p	9	0,02	0,02	0,00	1,47	0,152
S_r*D_p	3	0,00	0,00	0,00	0,04	0,989
Error	4426	6,62	6,62	0,00		
Total	4607	10682,97				

APPENDIX P.2

MINITAB RESIDUAL ANALYSIS

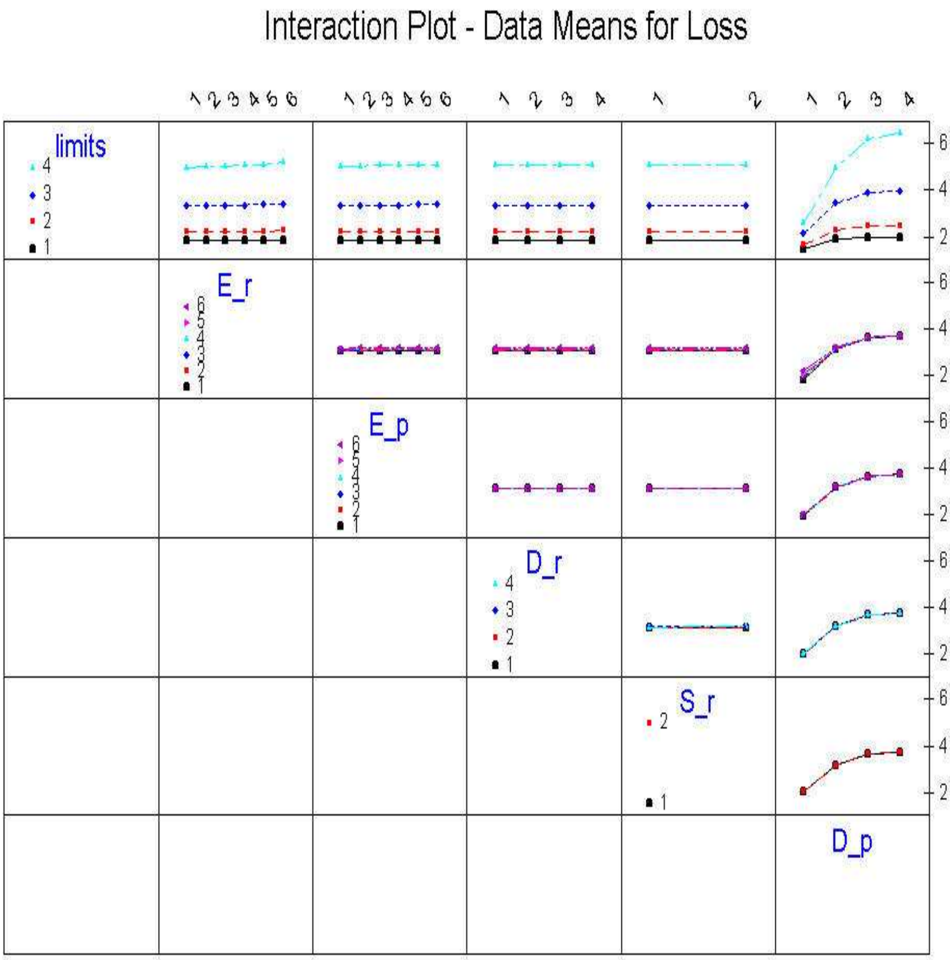



Normal Probability Plot of the Residuals
(response is Loss)
Normal Score
4
3
2
1
0
-1
-2
-3
-4
-0,2
-0,1
0,0
0,1
0,2
0,3
Residual


Residuals Versus the Fitted Values
(response is Loss)
Residual
0,3
0,2
0,1
0,0
-0,1
-0,2
1
2
3
4
5
6
7
Fitted Value

APPENDIX P.3

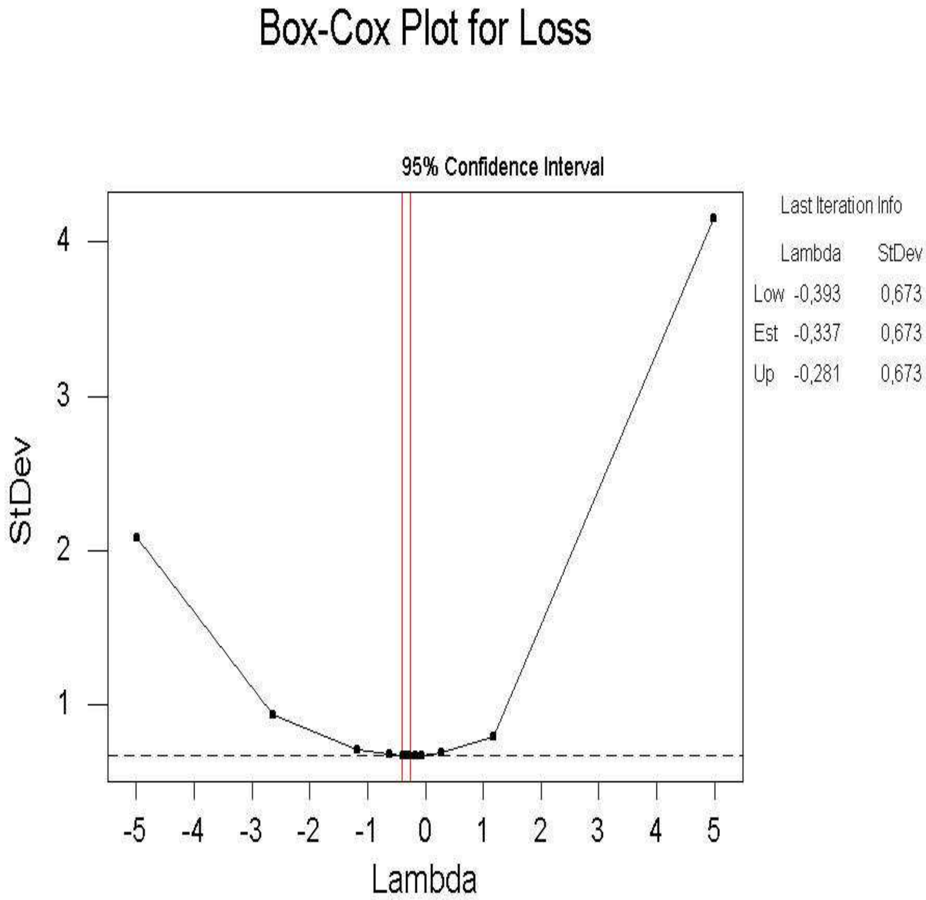
MAIN EFFECTS AND TWO-WAY INTERACTIONS PLOTS



Interaction Plot - Data Means for Loss
limits
4
3
2
1
E_r
6
5
4
3
2
1
E_p
6
5
4
3
2
1
D_r
4
3
2
1
S_r
2
1
D_p
6
4
2

APPENDIX Q

MINITAB BOX-COX TRANSFORMATION RESULTS

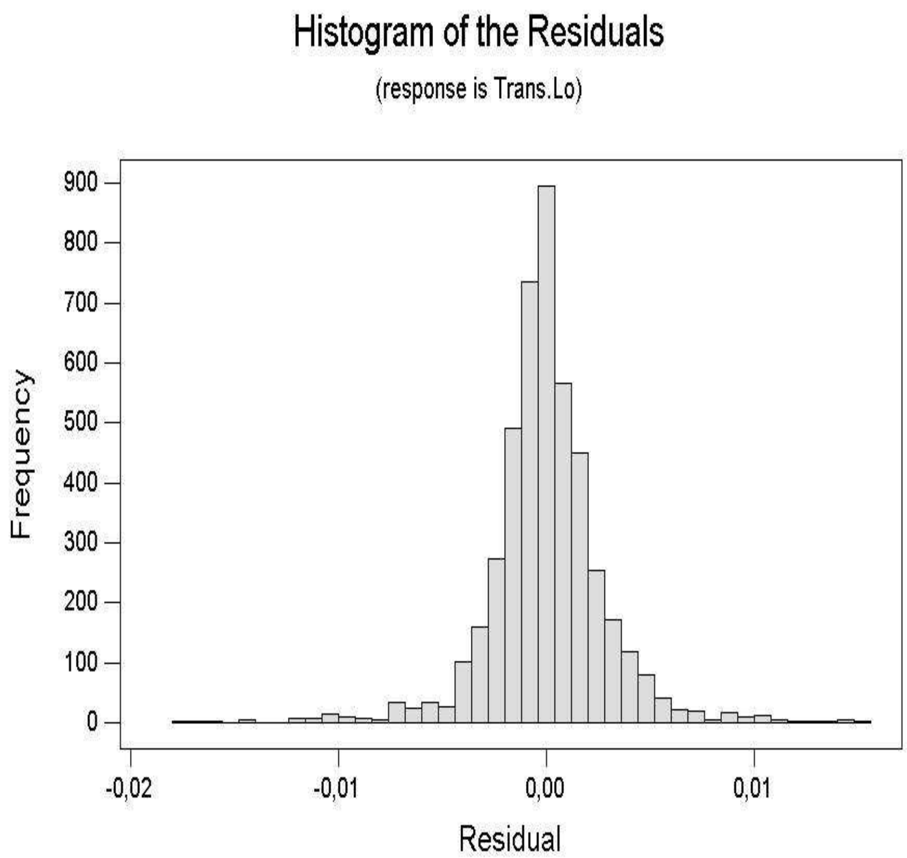


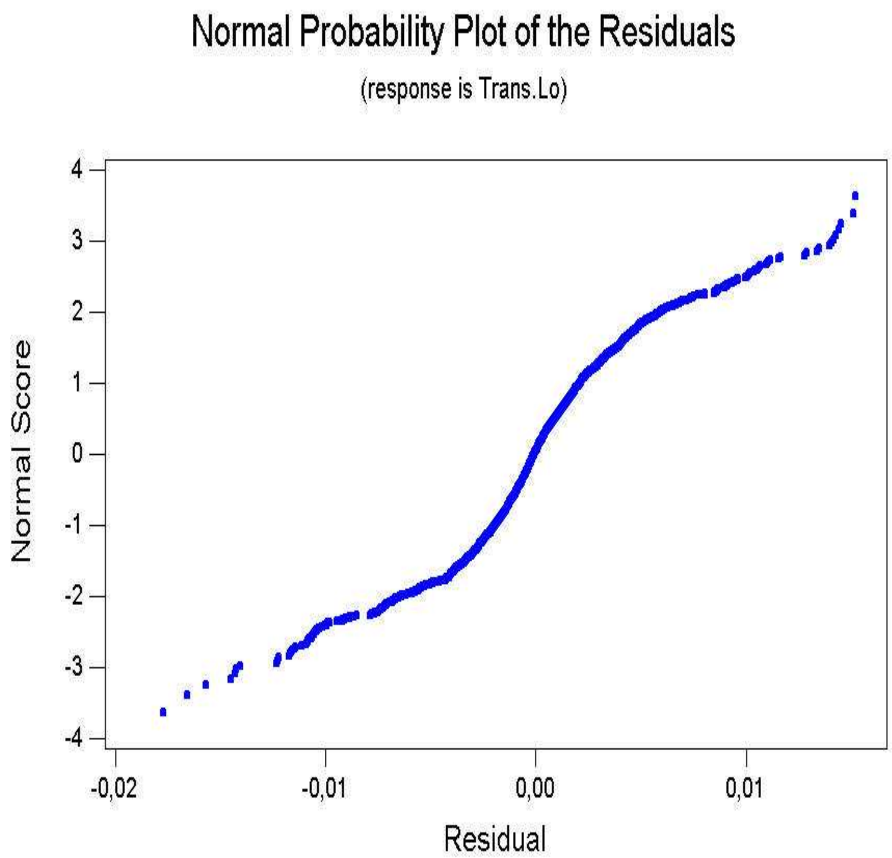
General Linear Model: Trans.Loss versus limits; E_r; E_p; D_r; S_r; D_p

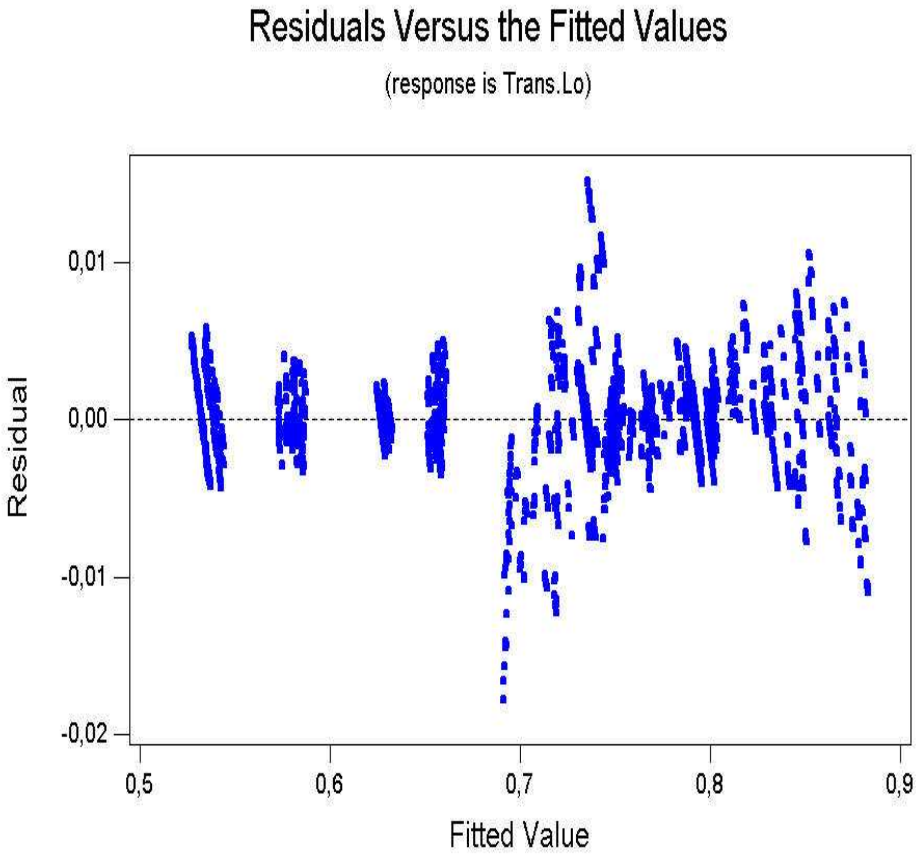
Factor	Type	Levels	Values
limits	fixed	4	1 2 3 4
E_r	fixed	6	1 2 3 4 5 6
E_p	fixed	6	1 2 3 4 5 6
D_r	fixed	4	1 2 3 4
S_r	fixed	2	1 2
D_p	fixed	4	1 2 3 4

Analysis of Variance for Trans.Lo, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
limits	3	32,84614	32,84614	10,94871	1,3E+06	0,000
E_r	5	0,05705	0,05705	0,01141	1405,43	0,000
E_p	5	0,00237	0,00237	0,00047	58,27	0,000
D_r	3	0,00007	0,00007	0,00002	2,67	0,046
S_r	1	0,00000	0,00000	0,00000	0,06	0,808
D_p	3	12,22169	12,22169	4,07390	5,0E+05	0,000
limits*E_r	15	0,00960	0,00960	0,00064	78,84	0,000
limits*E_p	15	0,00049	0,00049	0,00003	4,03	0,000
limits*D_r	9	0,00003	0,00003	0,00000	0,41	0,932
limits*S_r	3	0,00000	0,00000	0,00000	0,00	1,000
limits*D_p	9	1,35556	1,35556	0,15062	1,9E+04	0,000
E_r*E_p	25	0,00399	0,00399	0,00016	19,67	0,000
E_r*D_r	15	0,00012	0,00012	0,00001	1,00	0,446
E_r*S_r	5	0,00000	0,00000	0,00000	0,09	0,994
E_r*D_p	15	0,12021	0,12021	0,00801	987,09	0,000
E_p*D_r	15	0,00028	0,00028	0,00002	2,26	0,004
E_p*S_r	5	0,00000	0,00000	0,00000	0,01	1,000
E_p*D_p	15	0,00541	0,00541	0,00036	44,44	0,000
D_r*S_r	3	0,00000	0,00000	0,00000	0,00	1,000
D_r*D_p	9	0,00017	0,00017	0,00002	2,36	0,012
S_r*D_p	3	0,00000	0,00000	0,00000	0,07	0,975
Error	4426	0,03593	0,03593	0,00001		
Total	4607	46,65913				


Histogram of the Residuals
(response is Trans.Lo)
Frequency
900
800
700
600
500
400
300
200
100
0
-0,02
-0,01
0,00
0,01
Residual


Normal Probability Plot of the Residuals
(response is Trans.Lo)
Normal Score
4
3
2
1
0
-1
-2
-3
-4
-0,02
-0,01
0,00
0,01
Residual


Residuals Versus the Fitted Values
(response is Trans.Lo)
Residual
0,01
0,00
-0,01
-0,02
0,5
0,6
0,7
0,8
0,9
Fitted Value