INVESTIGATION OF SECONDARY SCHOOL STUDENTS' PERFORMANCE ON PROOF AND ATTITUDE TOWARDS PROOF IN GEOMETRY

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ABSTRACT

INVESTIGATION OF SECONDARY SCHOOL STUDENTS' PERFORMANCE ON PROOF AND ATTITUDE TOWARDS PROOF IN GEOMETRY

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The purpose of the study was to investigate secondary school students' performance on proof and attitude towards proof in geometry.

The research was conducted on 367 10th grade students. The numbers of subjects were 94, 96, 90 and 87 from General High Schools (GHS), Anatolian High Schools (AHS), Science High Schools (SHS) and Private High Schools (PHS) respectively. The number of girls and boys were 142 and 225 respectively.

To obtain the data of this study, the following measuring instruments were utilized: 1.Proof Performance in Geometry Test (PPGT); 2.Proof Attitude Scale in Geometry (PASG). They were developed by researchers.

The results indicated that: 1.There were statistically significant differences among the mean scores of students enrolled in different school types with respect to performance on proof in geometry; 2.There was no statistically significant difference between the mean scores of boys and girls with respect to performance on proof and attitude towards proof in geometry; 3.There were no statistically significant differences among the mean scores of

students enrolled in different school types with respect to attitude towards proof in geometry; 4.There was statistically significant correlation between secondary school students' performances on proof and attitude towards proof in geometry; 5.While students in SHS got the highest scores from each question, students in GHS got the lowest scores; 6. While most students in SHS perceived themselves as successful in geometry, most students in GHS perceived themselves unsuccessful.

Keywords: Performance on Proof in Geometry, Attitude Towards Proof in Geometry, Gender, High School.

ÖZ

ORTAÖĞRETİM ÖĞRENCİLERİNİN GEOMETRİDE İSPAT PERFORMANSLARININ VE GEOMETRİDE İSPATA YÖNELİK TUTUMLARININ İNCELENMESİ

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Bu çalışmanın amacı ortaöğretim öğrencilerinin geometride ispat performanslarının ve geometride ispata yönelik tutumlarının incelenmesidir.

Araştırma 367 10. sınıf öğrencisi ile yürütülmüştür. Genel Liselerden (GL) 94, Anadolu Liselerinden (AL) 96, Fen Liselerinden (FL) 90 ve Özel Liselerden (ÖL) 87 öğrenci seçilmiştir. Öğrencilerden 142'si kız ve 225'i erkektir.

Gerekli verileri toplamak için çalışmada şu ölçme araçları kullanılmıştır 1.Geometride İspat Performans Testi (GİPT), 2.Geometride İspata Yönelik Tutum Ölçeği (GİYTÖ); Bu testler araştırmacılar tarafından geliştirilmiştir.

Çalışmanın sonuçları şunları göstermiştir. 1.Geometride ispat performanslarına göre farklı okul çeşitlerinden öğrencilerin ortalama puanları arasında istatistiksel olarak anlamlı farklılıklar vardır; 2.Geometride ispat performanslarına ve ispata yönelik tutuma göre erkek öğrenciler ve kız öğrencilerin ortalama puanları arasında istatistiksel olarak anlamlı bir fark yoktur; 3.Geometride ispata yönelik tutumlara göre farklı okul çeşitlerinden öğrencilerin ortalama puanları arasında istatistiksel olarak anlamlı bir fark

yoktur; 4.Geometride öğrencilerin ispattaki performansı ile ispata yönelik tutumları arasında anlamlı bir ilişki vardır; 5.GİPT'indeki her bir sorudan en yüksek puanı Fen Lisesi öğrencileri alırken, en düşük puanı da Genel Lise öğrencileri almıştır; 6. Fen Lisesi öğrencilerinin büyük kısmı geometride kendini yeterli buluyorken, Genel Lise öğrencilerinin büyük kısmı kendini yetersiz bulmaktadır.

Anahtar Kelimeler: Geometride İspat Performansı, Geometride İspata Yönelik Tutum, Cinsiyet, Lise.

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LIST OF ABBREVIATIONS

PIF	: Personal Information Form
PASG	: Proof Attitude Scale in Geometry
PPGT	: Proof Performance in Geometry Test
PIF	: Personal Information Form
GHS	: General High School
PHS	: Private High School
AHS	: Anatolian High School
SHS	: Science High School

CHAPTER I

INTRODUCTION

Main purpose of education, especially courses given in the school, is to prepare students to the real life. When being adults in the society, students should be able to deal with problems faced in their job and daily life. They should be able to apply new ideas to those problems like solving problems in the school. One of effective guns to get rid of those problems easily is their reasoning ability obtained in the school. As Fitzgerald (1996) stated "One of the primary functions of education is the development of the ability to reason." (p.35). Therefore, schools have an important role to develop the reasoning ability of students.

Teachers try to teach students how to use their knowledge effectively when faced with various problems and deriving conclusion from various situations. In addition, Fitzgerald (1996) also stated that, "A review of school curriculum documents indicates that the development of reasoning ability is critically important for the schools at any level. Further confirmation may be gleaned from an examination of the self-study documents prepared by secondary schools for the accreditation previews. These documents would show that the school curriculum places a heavy emphasis on developing the reasoning capability of the child." (p.35). Moreover, Espojo, Good and Westmeyer (1975), Cohen (1980) expressed the view that one of the important aims of science education is to develop students' formal reasoning or thinking abilities. Furthermore, the authors of the National Council of Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards (1989) cited reasoning as one of four major and common goals of mathematics education across all grade levels. Therefore, it is easily understood that one of aims of main courses is to develop students' reasoning ability.

As mentioned above, developing reasoning ability and critical thinking are among undeniable aims of all courses in schools. Especially mathematics has an important role to contribute to the development of students' reasoning ability. Because, students can succeed in mathematics if and only if not memorizing some rules and theorems but applying existence knowledge to the different situations. In other words, students' success depends on their capability of reasoning and critical thinking. Szombathelyi and Szarvas (1998) stated that "Students in kindergarten through twelfth grade are expected to be able to observe patterns and relationships, to use them to make conjectures, and to construct arguments by drawing logical conclusions. They are encouraged to use deductive and inductive reasoning, formulate counterexamples and indirect proofs, and use mathematical induction-just as in the typical Hungarian K-12 curriculum. We should start promoting the use of reasoning, introduce critical thinking at an early age, and re-iterate its importance at each grade level."(p.677)

Importance of mathematical reasoning is not a new issue, it was mentioned in various reports in previous years such as the Reorganization of Mathematics in Secondary School Report (Mathematical Association of America, 1923), Progressive Education Report (NCTM,1940) and College Entrance Examination Report of 1958 (CEEB,1958).

Thus, we should investigate ways of developing reasoning ability, teaching analytical and critical thinking. In this point of view, proof is one of ways to reach our purpose in mathematics. As Knuth and Elliott (1998) stated that "The classroom practice aligned with this conception of proof emphasizes developing mathematical reasoning through the social interactions occurring within the classroom community."(p.714). Because, while students are proving something or listening to proof of something, they "make conjectures and present solutions; explore examples and counter examples to investigate a conjecture; try to convince themselves and one another of the validity of particular presentations, solutions, conjectures, and answers; [and] rely on mathematical evidence and argument to determine validity" (NCTM, 1991)

How does the proof relate reasoning ability? In order to answer this question, we should explain what a proof is. Proof has a multiplicity of meanings depending on which subject area it is used. For instance, as Tall (1989) proof can imply “beyond reasonable doubt” to a jury, “occurring with a certain probability”(p.28) to a statistician, and the result of empirical investigation to a scientist. On the other hand, Hoyles (1997) stated “within mathematical community, proof has a distinctive role. Proof lies at the heart of mathematics. It has traditionally separated mathematics from the empirical sciences as in dubitable method of testing knowledge which contrasts with natural induction from empirical pursuits.”(p.1) According to Wu (1997) Deductive mathematical proof offers human beings the purest way to distinguish right from wrong.

In mathematics education, empirical research on this topic has tended to focus on describing and analyzing students' responses to questions requiring proof. According to Bloom’s Taxonomy, proving is the fifth level of cognitive development. Thus, proof is difficult for many students. Hoyles (1997) stated that difficultness of proof for many students might be partly because of ambiguous meaning but also because proof requires the coordination of a range of competences – identifying, assumptions, organizing logical arguments – each of which, individually, is by no means trivial. A large body of evidence indicates that most students have difficulties in following or constructing formally presented deductive arguments, in understanding how they differ from empirical evidence, and in using them to derive further results (Balacheff, 1988; Chazan, 1993; Fischbein, 1982; Harel & Sowder, 1998; Porteous, 1994; Schoenfeld, 1989). Therefore, students should have an advanced reasoning ability to prove something.

Since undeniable contribution of proof to the students’ reasoning ability, it should be taught in the school. Yet, in recent years teachers go away from teaching proof because proof is considered too time-consuming for a class period (Szombathelyi & Szarvas, 1998). Thus, many students have a limited awareness of what proof is about (Hoyles,1997). Besides, according a

study conducted by Healy and Hoyles about proof (2000), the most successful students presented proofs in everyday language, not using algebra.

The NCTM's Curriculum and Evaluation Standards for School Mathematics (1989) cautions against the tendency to completely abandon proofs and focus only on the end results and formulas. In addition, as Polya (1957) stated that, removing all proofs from calculus may reduce the calculus to the level of the cookbook. The cookbook gives a detailed description or reasons for its recipes. There are some recommendations to get rid of these problems in the world. NCTM's Professional Standards for teaching mathematics (1991) recommended that teachers should consider mathematics as "a process involving problem solving, reasoning and communication". Hanna and Janhnje (1993) suggested that "A mathematics curriculum which aims to reflect the real role of rigorous proof in mathematics must present it as an indispensable tool in mathematics rather than as the very core of that science"(p.4). Geometric proofs are very important for analytical and critical thinking. Fawcett (1938) understood that the purpose of studying geometric proofs was to "cultivate critical and reflective thought". However, students are not good at proof in geometry because curriculum developers and mathematics teachers ignore it. McGivney and DeFranco (1995) stated that geometry, particularly geometric proof writing can be a vehicle to attaining the goal about NCTM's recommendation.

A study involving approximately 2700 High School students enrolled in ninety-nine geometry classes in thirteen General High Schools across five states showed that only 30 percent were able to complete geometry proofs successfully (Senk, 1985). According to Usiskin (1982), 50 percent of High School graduates complete a year of geometry, yet fewer than 15 percent of High School graduates master proof writing. It is seen that proof is very hard for students.

Advantages of studying geometric proofs are not bounded with achievement in geometry. Christofferson (1933) summarized the ideal

geometry classroom culture as one that would help “develop an attitude of mind which always tends to analyze situations, to understand their interrelationships, to question hasty conclusions, to express clearly, precisely and accurately non-geometric as well as geometric ideals” (cited in Fawcett 1938,p28) Geometry, especially proof writing, has to be viewed as a problem-solving activity. Through discourse, teachers can pose questions that engage and challenge their students while encouraging them to reason, clarify, justify, and communicate their ideas (NCTM, 1991). As mentioned above, the student-teacher and student-student dialogues create a classroom environment in which students support, argue and defend their thoughts throughout the solution process. The process of proving is undeniably complex, involving a range of student competencies--identifying assumptions, isolating given properties and structures, and organizing logical arguments--each of which is by no means trivial (Healy & Hoyles, 2000). Thus, students learn mathematical thinking in this away.

As a result, geometric proofs are very important for geometry achievement because it prevents memorizing facts and theorems. Instead, it provides students to derive basic facts and theorems themselves. Moreover, studying geometric proofs is very important for the development of mathematical reasoning ability of students. Unfortunately, we could not reach research study in geometric proofs.

In the light of the above considerations, it was decided to carry out a research study on secondary school students’ performance on proof and their attitude towards proof in geometry with respect to school type and gender.

CHAPTER II

LITERATURE REVIEW

In this chapter, the literature, related to the study, is reviewed and discussed. It is classified as importance of reasoning ability, proof in mathematics education, proof and geometry, gender differences in mathematics education and attitude towards proof.

2.1 Importance of Reasoning Ability

Reasoning ability is one of the distinguishing characteristics of human beings, which separate them from other living things. Development in science changes our daily lives and bring new challenges that increase the importance of reasoning ability. In relation to this, expectations from education also change and increase. Thus, the fundamental task of all core subject-matter disciplines, whether English, social studies, science or mathematics in school is the development of reasoning. (Fitzgerald, 1996).

To answer the question why reasoning is very important in education, we should look at the definition of reasoning. Reasoning has various meanings depending on subject area it is used. In Webster's (1986) dictionary, definition of proof is given as

> " The process of drawing conclusions, judgments, or inferences forms facts or premises …. The reasons, arguments, proof etc. resulting from this process…. To form conclusions, judgments or inferences from facts or premises ….. To convince, persuade by reasoning"

In addition the dictionary definition, According to Leighton (2004), if we wanted to give an example for reasoning, it would take on the character of a middleman in a company or enterprise. As a middleman, reasoning works

behind the scenes, coordinating ideas, premises, or beliefs in the pursuit of conclusions. These conclusions may sometimes find their way to the surface in the form of observable behavior as when someone exclaims "I have an idea!" or argues "I think your idea is not going to work because " Other times, the conclusions do not find their way to the surface but, rather, stay beneath the surface and function internally as antecedent conditions that feed into chains of productions for problem solving" (Simon, 1999, p676-677).

Moreover, Nicherson (2004) indicated that reasoning is defined as the process of drawing deductive inferences and as an aspect of thinking that is involved not only in drawing inferences but in making decisions and solving problems as well. Thus, as students develop reasoning power, they progress toward high levels of Bloom's taxonomy and encounter the need to synthesize or prove that what they reasoned will always be true (Fitzgerald, 1996). Besides, *Standard 3*, Mathematics as Reasoning, of the NCTM's Curriculum and Evaluation Standards, which are constructed as the result of various researches, for School Mathematics (1989) recognizes the importance of students' reasoning. The standard states that we cannot continue to settle for results like many that we obtain from the traditional curriculum. Hence, if the Standards document's call for student justifications in all areas of mathematics, rather than just in geometry, results in students' learning that reasoning in mathematics is as important as the facts of mathematics, then we will have made progress. If the Standards document's support for activities in which students generate conjectures leads naturally to questions of justification, then we have set a stage for mathematical reasoning.

In 1997 NCTM got Ross, formerly President of Mathematical Association of America, make research on mathematical reasoning. In this research, Ross (1997) stated that teachers should have recognized the theoretical nature of mathematics, which idealizes every situation, as well as the utilitarian interpretations of the abstract concepts and if reasoning ability was not developed in the students, then mathematics simply becomes a matter

of following a set of procedures and mimicking examples without thought as to why they made sense.

Although the reasoning objective has been of utmost importance in the minds of mathematics educators over time, perhaps we mathematics teachers have been more content oriented than goal oriented. We have usually left the reasoning goal to be achieved passively by transfer-of-training and not by direct instruction. (Fitzgerald, 1996)

2.2 Proof in Mathematics Education

Proof is not only important in mathematics education, but also important in daily life. Since questioning approach toward situations in the life is the key point of improvement and development of the society, we do not want to make our students be people who accept everything without questioning. Proof has an important role in students' questioning, because "not to prove is to accept any statement in life without questioning its appropriateness." (Fitzgerald, 1996,p35)

This study mainly concerns proof in geometry. Thus, we should discuss what a proof is. In Webster's (1986) dictionary definitions of proof are given as:

> "The cogency of evidence that compels acceptance by the mind of a truth or a fact. The process or an instance of establishing the validity of a statement by derivation from other statements in accordance with accepted or stipulated principles of reasoning. An act, effort or operation designed to establish or discover a fact or truth."

The last statement is not appropriate for mathematical proof. Because, Borowski and Borwein (1991) defined proof in his book "Dictionary of Mathematics" as "a proof is a sequence of statements, each of which is either validly derived form those preceding it or is an axiom or assumption."

However, the first and second statements together have a closer meaning for the proof and imply a procedure for reasoning or for constructing proof.

According to De Villiers (1987) function of proof in mathematics, has been defined almost exclusively as being to verify the correctness of mathematical statements. That is, proof is used mainly to remove either personal doubt or the doubt of skeptics

As Thomson (1996) emphasized that proof is the soul of mathematics as individuals generate conjectures and try to convince people (including themselves) about the truth and falsity of their conjecture. In fact, proving is one of the aspect of mathematical behavior that distinguishes mathematical behavior from scientific behavior in other disciplines (Dreyfus et al, 1990) Thus, reasoning and proof are unavoidable part of mathematical language. They offer people effective ways of communicating mathematical ideas and insights. These components, especially the ability to reason, are integral parts of understanding mathematics. They help the student to note and express patterns, structure, and similarities in mathematics and real-world situations as well.

As mentioned above, NCTM (1989) recommends activities which develop reasoning ability of students. Thus, mathematical proof, the last stage of justification, should be thought to develop students' mathematical reasoning. Authors of the NCTM also recommend that formal reasoning in the forms of all type of proofs, deductive proof; mathematical induction and indirect proof, should be thought to students. To emphasize the importance of proof, NCTM put following standards into NCTM's Curriculum and Evaluation Standards.

According to NCTM's Curriculum and Evaluation Standards (1989), instructional programs from prekindergarten through grade 12 should enable all students to

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Through the first premise, the NCTM hopes that students will become conscious that statements or propositions should be supported or rebutted with evidence by using systematic reasoning and this is the integral part of mathematics. Second premise's aim is to encourage students observe pattern and relationship, to guess about the particular case of a problem and to test their guesses (Sozmbathelyi & Szarvas, 1998). The third premise can begin at early ages. Firstly, students can support statements with basic examples; they should recognize that this is not yet a proof except for disproving a statement with only one counterexample. That is, students should learn that proving a specific case is not equivalent to proving in general case. Besides, they should learn to use on previous mathematical knowledge in order to support their conjectures. The fourth premise is the last stage. Students should identify the statement and decide which method of proof is appropriate to prove the statement. Some of these methods are proof by mathematical induction, direct proof or indirect proof (by contradiction).

As seen proof have a great importance in mathematics education, it is the time to mention and how students can succeed in proof.

Firstly, students' success in proof is greatly deal with specialized language associated with reasoning. Such words as axiom, postulate, conjecture, proof and theorem should be understood by students before they engage in the proof-making task (Thomson,1996) In order to measure grade 11 students' understanding of the terms axiom and theorem, National Assessment of Educational Progress (NAEP) gave a test in the 1985-86

academic year. Silver and Carpenter (1989) reported that "the majority of the [grade 11] students responded that a theorem was essentially a demonstration or an assumption, and fewer than one-fourth of the students were able to identify correctly the meaning of an axiom." Of course, it can easily be stated that students, who cannot distinguish between a theorem and its proof or between an assumption and a proof, are likely to have difficulty judging an argument's validity (Thomson, 1996)

After students understand basic concepts of proof, they should know how to write proof. Bell (1996) expected students to be aware of recognizing that a proof must do more than restate the result and they should also connect results to existing knowledge. Hence, students should translate their ideas and discovered relations to mathematical language. Thomson (1989) claimed that this translation is difficult for students because they should modify a statement in which concepts are intuitively based on experience to one in which the concepts are specified by formal definition and their properties are reconstructed through logical definitions.

Finally, student should decide which technique is appropriate to prove the assertion or proposition. Even students know above prerequisites of proof, it is not enough. As Galbraith (1995) declared that definitions were not sufficient for students using prior knowledge and preconceived notions about the content. Because, proof needs more advanced mathematical thinking. This is sometimes too difficult for students. Thus, they should choose correct technique, which are commonly direct proof, indirect proof (proof by contradiction), proof by construction and mathematical induction. Consequently, students should carry out technique by using definitions and hypothesis to prove statements. Of course, they should prove the statement step by step without omitting to show relations among steps.

Mentioned techniques for proof are explained.

2.2.1 Direct Proof

Gross (2002) stated a direct proof is a mathematical argument that uses rules of inference to derive the conclusion from the premises directly. In other words, it is to show that a given statement is true by combination of definitions, axioms and earlier theorems with mathematical manipulations. In direct proof, we use the implication, $p \Rightarrow q$. Here, p is known statement. That is, p is in their own right and may be obtained from theorems, corollaries, lemmas or definitions. In other words, it is the given information to make statement q be true. Gross (2002) gave the following example for direct proof.

Example:

Theorem: The sum of two even numbers x and y is even.

Proof:

(1) There exist numbers m and n such that

$x = 2m$ *and* $y = 2n$	*(by definition of ("even").*
(2) Then, $x + y = 2m + 2n$	*(by substitution).*
$= 2(m + n)$	*(by left distribution)*

which is even, by the definition of evenness

2.2.2 Indirect Proof

Gross (2002) stated an indirect proof uses rules of inference on the negation of the conclusion and on some of the premises to derive the negation of a premise. This result is called a contradiction. That is, after starting with given conditions, we assume the conclusion to be false and arrive at a contradiction (Szombathelyi & Szarvas, 1998). Gross (2002) gave the following example for indirect proof.

Example:

Theorem: If x^2 is odd, then so is x..

Proof:

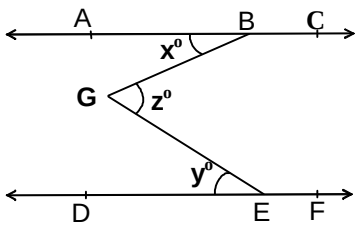
Assume that x is even	*(negation of conclusion).*
Say $x = 2n$	*(by definition of even).*
Then $x^2 = (2n)^2$	*(by substitution)*
$= 2n \cdot 2n$	*(definition of exponentiation)*
$= 2 \cdot 2n^2$	*(commutativity of multiplication)*
which is an even number	*(definition of even)*

which contradicts the premise that x^2 is odd.

2.2.3 Proof by construction

A constructive proof is a method of proof that shows the correctness of the statement with certain properties by creating or providing a method. That is constructing a concrete example with a property to show that something having that property exists. This method is generally used in geometric proofs.

Example:



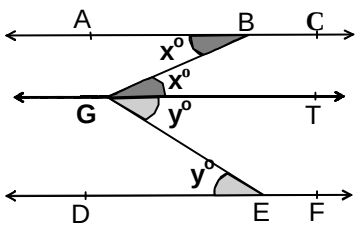
GİVEN:

- *AC//DF*
- $m(\widehat{ABG})=x$ $m(\widehat{BGE})=z^o$ $m(\widehat{GED})=y^o$

PROVE:

- $z=x+y$

To prove, we should construct a line passing through G and parallel to AC and DF



$m(\widehat{ABG})=m(\widehat{BGT})=x^o$ *(by interior reverse angles)*

$m(\widehat{DEG})=m(\widehat{EGT})=y^o$ *(by interior reverse angles)*

$m(\widehat{BGE})=x^o+y^o$

2.2.4 Mathematical Induction

According to Gauss's method if the Statement to be proved is logically correct for a first case, typically n = 1, and that if the statement can be found for any natural number more than one, n = k + 1, given that it happens for the previous one, n = k, then The statement is true for all natural numbers n. Franklin (1996) made an analogy about mathematical induction. This analogy was that imagine dominos each domino was placed so that the preceding one would push over the next one of the sequence when one push the first domino. Following example makes the hat mathematical induction is.

Example:

Theorem: If n is a natural number which is greater than 4 then $2^n > n^2$.

Proof:

Firstly, is it true for the first n, i.e. i=5

$2^5 > 5^2 = 32 > 25$ *(by substitution) so it is true for first n.*

Secondly, assume that statement is true for n=k. i.e. $2^k > k^2$

Finally, we will show that statement is true for n=k+1. i.e. $2^{k+1} > (k+1)^2$

$2^k > k^2$ *(by assumption)*

$2^k > 2k+1$ *(because* $2^k > k^2 > 2k+1$ *for* $k \geq 2$*)*

Summation of both sides gives $2^{k+1} > (k+1)^2$.

Although students know the mathematical language and techniques of proof, they have difficulties in proof. Students often believe that non-deductive arguments constitute a proof. According to Weber (2003), student's common beliefs about what constitutes a mathematical proof have an important role in those difficulties. Some of beliefs are below.

Firstly, an argument is a proof if and only if it is in accordance with a specific mathematical tradition. For instance, many pre-service teachers

believe a geometry argument must be in a two-column format to be a proof (Martin & Harel, 1989). Secondly, an argument is a proof if it is presented by or approved by an established authority, such as a teacher or a famous mathematician. Thirdly, checking that a general statement holds for one example, or perhaps several examples, is sufficient to demonstrate its veracity. The above faulty proof is an example. Finally, by way of an appropriate diagram, one can visually demonstrate that a certain property holds (Harel & Sowder, 1998). For example, to prove that the median of an isosceles triangle's base is the angle bisector of vertex angle at same time, some students will draw an isosceles triangle and median and says that median looks like dividing vertex angle into two equal parts.

Consequently, it is not easy to teach proofs for teachers. They should give all students rich opportunities and experiences with mathematical proof (Knuth & Ellilott,1998). In this way, students' understanding of mathematical proof can be discovered and we give a chance to students to utilize mentioned benefits of proof.

2.3 Proof and Geometry

After above considerations, it is unavoidable to agree with NCTM's Professional Standards for Teaching Mathematics (1991), that teachers should consider mathematics as "a process involving problem solving, reasoning, and communication.". McDivney and DeFranco (1995) declared that geometry, especially geometric proof writing, can be a way to reach mentioned goal. Silver and Carpenter (1989) stated that students who had completed a geometry course generally performed better on proof than did those students who had not studied geometry according to mentioned NAEP's test scores. Moreover, Geometric proofs provide success in geometry because they help students to obtain the knowledge and ability of how to use axioms and theorems instead of memorizing them (Senk,1989)

In addition, Szombathelyi and Szarvas (1998) emphasized the importance of geometric constructions, which are generally used proofs in geometry. They claimed that geometric construction not only funny for students but also helped students develop mathematical thinking and understanding and offered excellent opportunities for exploration, recognizing patterns, making and testing conjectures. In general, geometric proofs have an important place in our mathematics instruction. "The study of the elements of plane geometry yields still the best opportunity to acquire the idea of rigorous proof. If general education intends to bestow on the student the ideas of intuitive evidence and logical reasoning, it must reserve a place for geometric proof" (Pólya, 1957).

The Royal Society's Report (2001) about teaching and learning geometry emphasized that geometry was more suitable area of school mathematics in which to start dealing with proof. Reasons of why geometry was particularly suitable for mathematical thinking are presented by The Royal Society's Report. Briefly, they were followings:

i. While students dealt with proofs, they worked with concrete objects, which were already familiar (angles, parallel lines, triangles, circles). In other areas where students met proof, they often had to cope with additional difficulties such as the meaning of symbols, abstract statements and quantifiers.

ii. In geometry properties and relationships had to do with pre-existing mental objects, rather than objects which were based on abstract definitions.

iii. The statements were easily understandable. For example, the statement in geometry, "the sum of angles of a triangle is equal to 180°", was more intelligible the statement in calculus, "if $b^2<4ac$, then the equation $ax^2+bx+c=0$ has no real roots".

iv. The logical methods involved in basic plane geometry were likely to be less subtle than those in other introductory parts of mathematics; for example, they involved fewer quantifiers and statements were generally less complex.

v. In view of the points above, it was easily seen that pupils were able to prove statements in geometry earlier than in other parts of mathematics. In this way, they could use and develop their skills in logical thinking as soon as they emerged, rather than waited till a later stage in their mathematical education.

vi. Although most deduction in school mathematics took the form of a linear sequence with each conclusion following from the previous one, basic plane geometry involved contemplating several statements at the same time, reorganizing them and drawing conclusions from the collection as a whole.

vii. Geometry was extremely suitable for developing the skills of recognizing and aiming for fruitful intermediate objectives, providing an ideal visual and verbal scenario for considering where you were, where you want to go, and how you might get there. For example, marking all the angles and sides that you could establish to be equal for congruent triangles reveals the solution. So a little bit playing with the problem enabled students to solve.

viii. It was possible to do serious mathematical learning in geometry without having a perfect understanding of what axiom systems were and what the rules were for working with them.

ix. One of the unique features of mathematics was that it was the only subject where you did not have to take things on trust. On the other hand, geometry was a such a branch of mathematics demonstrating mathematics not address to only authority (whether that was a person, a book or a computer) but also to students.

x. In geometry, there was the situation where a small number of plausible assumptions leaded to a large number of surprising and appealing results.

Thus, it is easily seen that geometry is a good starting point to prove statements. However, there is still a problem how to guide students to prove. McGivney and DeFranco (1995) saw geometric proof as "problems to prove" because of their similar structure. They typically include given information; such operations as axioms, postulates, and theorems; and a specified goal. As with other problems that are in this category, the objective in geometry proof problems is to move from the given to the goal as efficiently as possible. Then, of course, problem solving strategies can be used to prove statements. These are:

- working backward and determining sub goals,
- identify alternative solution paths,
- eliminate irrelevant paths,
- check incorrect paths,
- discover a correct solution path in an efficient way.

In order to teach prove, it should be understood that what is the process of learning and making proof. There are some different descriptions about how students develop the ability to prove ideas formally. Batista and Clements (1995) stressed that Piaget described how this development happens without considering curricula and Van Hiele described it with curricula.

According to Piaget, developing the ability to construct a proof passed through some stages. (Batista & Clements,1992)

Stage 1. The child's thinking is not reflective, systematic, and logical. They cannot combine and discover relations among various pieces of data

collected or examples examined. Exploration proceeds randomly without a plan and conclusions may be contradictory.

Stage 2. Students make predictions by using empirical results and they try to justify their predictions. They expect results in their searches for information and think logically only about premises that they believe in.

Stage 3. Students progress beyond a belief. To conclude something is necessarily true, its premises should be true. They are capable of formal deductive reasoning based on any assumptions and so they can operate explicitly with a mathematical system.

Although Piaget's theory describe how thinking in general progresses from being non-reflective and unsystematic, to empirical, and finally to logical-deductive without curricula, Van Hiele’s theory deals specifically with geometric thought as it developed through several levels of sophistication under the influence of a school curriculum (Clements & Battista, 1992) According to Walle (2001), characteristics of these levels are following:

- The Van Hiele levels of geometric reasoning are sequential. i.e. Students must pass through level by level.
- In Contrast Piaget’s description of development, these levels are not age-dependent in the way
- Geometric experiences have the greatest influence on advancement through the levels.
- Instruction and language at a level higher than student’s level may reduce learning.

Van Hiele (1999) identified five levels through which children move sequentially on their way to geometric thinking.

Level 1. Visualization: Students can recognize and name shapes by their appearance without attention to parts, attributes, or properties. Although they may be able to recognize characteristics, they do not use them for recognition and sorting.

Level 2. Analysis: Students can identify properties of shapes and learn to use appropriate vocabulary related to properties, but cannot understand ordered relationships among shapes and their properties. Moreover, they cannot discern between necessary and sufficient properties

Level 3. Informal Deduction: Students are able to recognize relationships between and among properties of shapes. They are able to follow logical arguments using such properties.

Level 4. Deduction: Students can go beyond just identifying characteristics of shapes and are able to construct proofs using postulates or axioms and definitions. A typical High School geometry course should be taught at this level.

Level 5. Rigor: This is the highest level of thought in the Van Hiele hierarchy. Students at this level can work in different geometric or axiomatic systems, they can use techniques of proof such as indirect proof and proof by contra positive. This level would most likely be enrolled in a college level course in geometry.

Van Dormolen (1977) stated that at the first level, single cases are justified; conclusions are limited to the specific example. At the second level, justifications and conclusions may be made for specific cases but refer to collections of similar objects, such as a class of rectangles. Since at following level 3 students start to follow logical arguments, they can justify statements by forming arguments that is, they start to prove. Besides, De Villiers (1987) recommended that deductive reasoning in geometry first occurs at level 3. He says that because students at levels 1 or 2 cannot recognize logical

relationships and test the validity of their empirical observations. They cannot understand formal proof and they see it as justifying obviously.

Research by Senk (1989) recommended that a proof-oriented geometry course should be given at least at level 3 in the Van Hiele hierarchy. She found that less than 22 percent of students below level 3, however 57 percent, 85 percent, and 100 percent at levels 3, 4, 5, respectively. Unfortunately, over 70 percent of students begin High School geometry at level 1 or below, and only those students who enter at level 2 or higher become competent with proof by the end of the course (Senk 1989,: Shaughnessy & Burger 1985).

As a result, both Piaget's and van Hiele's theories suggested that students must pass through lower levels of geometric thought before the can attain higher levels and that this passage takes a considerable amount of time (Clements & Battista, 1992)

There are also researches about how students' performance on proof and geometric proof. For example, Fawcett (1938), Smith (1940), Reynolds (1967), Tall(1979), Senk (1985).

Fawcett's (1938) study reported on an experimental geometry course in which transfer of logical reasoning into contexts other than geometry was incorporated into the subject matter. Student comments interpreting business regulation documents or pieces of news were shown to argue for the existence of disciplinary value in demonstrative geometry provided that it was taught having transfer in mind. The kind of argumentation practices that Fawcett and his students were able to achieve is to be envied;

According to Smith (1940) students made marked improvement in understanding this construct if they made constructions to verify the validity of geometric statements. He also stated that improvement in pupils' understanding of "if-then" statements by letting them first make constructions to evaluate geometric statements. In his research he found that it enabled pupils to learn to clearly distinguish between the "given condition(s)" and the

"conclusion(s)", and laid the conceptual groundwork for an improved understanding the eventual deductive proof.

In his study, Reynolds (1967) concerned with students' ability of using indirect proof in geometry, especially context was lines and alternate interior angles. 14 percent of students wrote valid proofs, those students in the first year of secondary school and by about 70 percent of those in the sixth year of secondary school who were still taking mathematics. From 12 percent to 18 percent of the students at all grade levels arrived at a contradiction but did not know how to proceed from that point.

Tall (1979) showed that students in the first year of university expressed a strong preference for the generic proof over the standard proof by contradiction. He also remarked that this did not mean that the generic proof was preferable in the long term. Proof by contradiction was an essential element in formal mathematics and needed to be addressed, even though it involved significant cognitive difficulties. These difficulties were subtler than is often assumed.

A study involving approximately 2700 High School students enrolled in ninety-nine geometry classes in thirteen General High Schools across five states showed that only 30 percent were able to complete geometry proofs successfully. Yet fewer than 15 percent of High School graduates master proof writing. It is seen that proof is very hard for students. (Senk, 1985)

Unfortunately, we could not find research about student's performance on proof in geometry in Turkey. Thus, it is a good subject to deal with.

2.4 Gender Differences in Mathematics and Geometry

Some of authors claimed that there was still a gender gap in mathematics between boys and girls. For example, Li (1999) conducted a research about gender differences in mathematics and he favored males. Also, Sadker (1999) supported the existence of a gender gap in mathematics scores.

Odell and Schumacher stressed that "no one disputes that females consistently score lower, on average, than males on standardized tests”(p.37). Casey (1997) claimed that since boys were more confident and had better mental rotation skills, their scores were better in mathematics. Here, Casey dealt with high ability students. Sadker (1999) also stressed that more girls were in language arts programs while more boys were in math and science programs. Another point of view, Li (1999) stated that it was believed that male teachers were more effective at teaching mathematics and science, which may attribute to males doing better at math. Their male teachers serve as role models encouraging them to enjoy and get involved in mathematics.

There are still others who believe extinction of difference between boys and girls. Casey (1997) claims that "there are no gender differences in mathematics when all abilities and ages are compared; girls get better math grades while boys tend to do better on standardized tests.” "(p.37).Besides, “on college mathematics placement tests....females tend to score as well as males" (Odell & Schumacher, 1998,p.39). Walker and Plata (2000) stressed that in their study, which was about success of students in remedial mathematics courses, there were no significant differences in pass-fail frequencies between males and females.

There are also authors that found that females are better than males at mathematics. Noldon and Sedlacek (1998) found that males were more likely to earn lower grades in mathematics than predicted by standardized test scores". In addition Odell and Schumacher (1998) found that average female grades in college mathematics courses tended to be as good as or better than males.

In geometry, there are a few studies on gender differences. However, some studies dealt with it as a sub problem. For instance, Amy and Andrew (2004) stated that “results were also analyzed to determine if students of a certain gender perform at a higher level when taught using dynamic geometry software. Prior to implementation average test scores of males and females

showed no significant difference." Senk and Usiskin (1983) stated that there was no statistically significant mean difference between boys and girls in senior high school with respect to geometry achievement. However, Hanna (1986) found that there was a statistically significant mean difference in the favor of boys with respect to geometry achievement. Unfortunately, we could not reach any study about gender difference on performance on proof and attitude towards proof in geometry.

2.5 Attitude Towards Proof

Unfortunately, we could not reach research studies on students' attitude towards proof. However, to mention about students ideas about proof might be useful. For instance, Sowder and Harel (1998) stated that when they asked students their experience about proof. Students responded like; "What the words 'mathematical proof' means to me is a necessary evil" or "I remember hating those proofs in High School geometry. I remember thinking, 'Why do I need to prove this? It is so obvious' "or "For what reason? I guess I really felt that they were that important, since I was willing to trust whatever brilliant mathematician thought up the theorem."

Chazan (1993), whose research was about High School students' justification for their views of empirical evidence and mathematical proof, found that of the nine urban site interviews, only four did not think that measuring proves for all members of a set with infinite number of elements. Moreover, of the eight suburban site interviews, only five did not think that measuring proves for all members of a set with infinite number of elements. In addition, Williams (1979) also held such views. On one item of his test, 68% of the sample believed that an empirical argument was sufficient for proof; only 6.4% felt the need for a deductive proof. On another item, whose content was more familiar to students, 54% accepted an empirical argument; only 14% wanted a deductive proof. Thus, we can say that students generally do not know what a valid evidence for a proof is.

As it can be seen lots of students do not know what a proof is, how much it is important and how to construct a proof. Thus they do not like proofs.

CHAPTER III

METHOD OF THE STUDY

This chapter includes explanations of the problem and hypotheses, research design, subjects, definitions of terms, variables, and development of measuring tools, procedures, methods used to analyze data, assumptions and limitations.

3.1 Problem of the Present Study and Associated Hypotheses

In this section the main problem, related sub-problems and hypotheses related to these problems stated.

The main problem of the present study is following:

- MP: What is the secondary school students' performance on proof and attitude towards proof in geometry with respect to school type and gender?

Based on the main problem, the following sub problems are explored:

- SP1:What is the secondary school students' performance on proof in geometry with respect to school type and gender?
- SP2:What is the secondary school students' attitude towards proof in geometry with respect to school type and gender?

To examine the sub-problems, the following hypotheses are stated:

- $H_0$1: There are no statistically significant differences among the mean scores of students enrolled in different school types with respect to performance on proof in geometry.
- $H_0$2: There is no statistically significant difference between the

mean scores of boys and girls with respect to performance on proof in geometry.

- $H_0$3: There is no statistically significant interaction between school type and gender with respect to performance on proof in geometry.
- $H_0$4: There are no statistically significant differences among the mean scores of students enrolled in different school types with respect to attitude towards proof in geometry.
- $H_0$5: There is no statistically significant difference among the mean scores of boys and girls with respect to attitude towards proof in geometry.
- $H_0$6: There is no statistically significant interaction between school type and gender with respect to attitude towards proof in geometry.
- $H_0$7: There is no statistically significant correlation between secondary school students' performances on proof in geometry and attitude towards proof in geometry.

As seen, the hypotheses above are defined in the null form. They will be tested at significance level of 0.05.

3.2 Research Design

The purpose of this study was to investigate secondary school students' performance and attitude towards proof in geometry with respect to school type and gender. We utilized casual-comparative design and correlational study (Fraenkel & Wallen, 1996) as outlined Table 3.1.

Table 3.1 Research Design of The Study

Independent Variables		Dependent Variables
School Types	Gender	
General High Schools	Girls	Attitude towards proof in geometry
Anatolian High Schools	Boys	Performance on proof in geometry
Science High Schools		
Private High Schools		

3.3 Subjects of the Study

The subjects of the study consisted of 367 10th grade students from General High School (one school from İstanbul, two schools from Ankara), Anatolian High Schools (two schools from İstanbul, one school from Ankara), Science High School (one school from İstanbul, one school from Ankara) and Private High School (two schools from İstanbul, one school from Ankara). This study was carried out in 2004. The distribution of the subjects is given in the Table 3.2.

Table 3.2 Distributions of Subjects

			SCHOOL TYPE				TOTAL
			General	Anatolian	Science	Private	
GENDER	Girls	n	36	35	21	50	142
		%	*38*	*37*	*23*	*43*	*39*
	Boys	n	58	61	69	37	225
		%	*62*	*63*	*77*	*57*	*61*
	TOTAL		94	96	90	87	367

3.4 Definitions of Terms

The definitions of terms used in this study are given below to clarify and to avoid possible misunderstandings.

- Performance on proof in geometry refers to subject's score on the "Proof Performance in Geometry Test".
- Attitude towards proof in geometry refers to subject's score on the "Proof Attitude Scale in Geometry".
- School Type refers to General High School, Anatolian High School, Science High School and Private High School.

3.5 Variables

There are four variables in the main problem "What is the secondary school students' performance on proof and attitude towards proof in geometry with respect to school type and gender?" Performance on proof and attitude towards proof in geometry are dependent variables. On the other hand, school type and gender are independent variables.

3.6 Measuring Instruments

In the present study, the following instruments were used:

1. Proof Performance in Geometry Test (PPGT)
2. Proof Attitude Scale in Geometry (PASG)

Development process of each measuring instrument is explained below and their samples are given in Appendices.

3.6.1 Proof Performance in Geometry Test (PPGT)

PPGT was developed to assess subjects' performance on proof in geometry. The procedure followed in the development of PPGT is outlined below.

The item bank for PPGT derived from:

- Lecture notes, which were used by experienced teachers used.
- Suggestions of experienced teacher in giving geometry courses.
- Books, which were appropriate to National Education Ministry curriculum

After the curriculum was investigated, 12 questions were selected by taking the expert opinion. All items were written in Turkish because it would be applied to different types of school and education language was Turkish in some kind of school such as Science High Schools and General High Schools.

This performance test was administered to 136 11th grade students from four different types of school at the beginning of fall semester. The purposes of this administration were to determine how these questions were appropriate to assess performance on proof in geometry and to test whether or not there were misunderstandings and ambiguous. This performance test was assessed by using focused holistic scoring techniques presented in Table 3.3.

Table 3.3 Rubric Criteria for assessment of PPGT

What is done in solution?	Points
There is nothing done or irrelevant things are written.	0
Attempt to solve and beginning of the proof is done.	1
Most of proof is done but it is not complete.	3
Proof is done but some explanations are missing.	4
Complete proof is done.	6

After experts examined the results, PPGT was revised by changing inappropriate items with different items in the item bank. Table of content and a copy of revised version of PPGT were given in appendix A and appendix C respectively. This test was administered to 367 10th grade students from four different school types. For the rater reliability of PPGT two experts assessed the test. The relationship between these scores was tested by Pearson Correlation. Correlation coefficient was found 0.99.

3.6.2 Proof Attitude Scale in Geometry (PASG)

PASG was developed to assess subjects' attitude towards proof. The procedure followed in the development of PASG is outlined below. A copy of PASG was given in appendix D.

The item bank for PASG derived from:

- Students' thoughts, which were obtained by interviewing with secondary school students and taking their journals on proof.
- Scales of attitude towards probability, statistics, mathematics and technology used in previous years.

By taking expert opinion, we selected 47 items from this bank which consist of 23 negative and 24 positive items. The scale was written in Turkish because it would be applied to different types of school.

This attitude scale was administered to 136 11th grade students from four different types of school to test whether or not there were misunderstanding or ambiguous or unclear items. It was analyzed by using the "Statistical Packages for Social Sciences" (SPSS). They scaled on a five-point Liker Type scale: Strongly Agree, Agree, and Undecided, Disagree and Strongly Disagree. Positive items were scored starting form strongly agree as 5 to strongly disagree as 1 and negative items were scored starting form strongly agree as 1 to strongly disagree as 5.

To test the construct validity of the PASG, factor analysis was performed. According to the initial principal factor solution with iterations, the first four eigenvalues were 15.508, 5.940, 2.069 and 1.810. Factor loadings of the PASG in the first factor ranged between 0.826 and 0.398. Factor loadings of the PASG in the second factor ranged between 0.725 and 0.680. The first and second factors accounted for 33.0% and 12.6% of the total variation in the PASG scores respectively. The factor loadings with the value at 0.346 or above presented in Table 3.4.

Table 3.4 Factor Loadings for PASG with 47 items

ITEM NO	FACTORS 1	2	3	4	ITEMS
19	.826				İspat yapmayı öğrenmek gereksizdir.
24	.819				İspat yapmayı severim.
42	.816				İspat yapma konusunda daha çok bilgi edinmek isterim.
16	.794				İspat yapmak sıkıcıdır.
45	.794				İspat yapmayı öğrenmek bana çekici gelir.
32	.789				İspat yapma yeteneğimi geliştirmek isterim.
15	.783				İspat yapmak zihinsel gelişimime olumlu katkıda bulunur.
36	.765				İspat yapmayı öğrenmekten hoşlanmam.
47	.764				İspatlara çalışmak için zaman ayırmak istemem.
11	.745				İspat yapmak konunun iyice öğrenilmesini sağlar.
4	.726				İspat ile ilgili ileri düzeyde bilgi edinmek istemem.
28	.725				İspat yapmak ilginç değildir.
23	.717				İspatlanmış teoremlerin ispatını öğrenmenin bir faydası yoktur.
26	.700				İspat sorusu ile karşılaştığımda kendim ispatlamak yerine bana başkasının yapmasını tercih ederim.
35	.693				İspat yapmak bende ders çalışma isteği uyandırır.
27	.693				İspat yaparken bir probleme farklı açılardan bakmayı öğrenmemi sağlar.
41	.683				Derste ispat yapmak derse olan ilgimi azaltıyor.
5	.679				İspat yapmaya çalışmak beni huzursuz eder.

Table 3.4 (Continued)

ITEM NO	FACTORS 1	2	3	4	ITEMS
25	.669				İspat yapınca kendime güvenim artıyor.
30	.665				İspat yapmak öğretilmelidir.
8	.663				Ders çalışırken teoremlerin ispatlarını incelemekten hoşlanırım.
3	.658				Derste ispat yapılınca ders daha eğlenceli oluyor.
31	.648				İspat yapmayı öğrenmemin gerçek yaşamıma olumlu bir katkıda bulunmaz.
38	.643				Gelecekteki hayatımda ispat yapma yeteneğine ihtiyacım olmayacaktır.
7	.635				İspat düşünme yeteneğimi geliştirmez.
20	.615				İspat yapabilmek beni gururlandırır.
43	.596				Sınavda ispat sorulması gereklidir.
14	.580				İspat ile ilgili soruyu cevaplayamadığım zaman çözmek için çaba harcamam.
12	.534				Kendimi mutlu hissetmem için ispat yapmada başarılı olmam gerekir.
46	.398	.388			İspat yaparken aklım karışır.
40		.725			Kimseden yardım almadan ispat yapmayı başarabilirim.
1		.708			İspat yapabilme konusunda hiç endişe duymam.
21		.706			İspat yapmaktan korkmam.
13		.680			İspat ile ilgili sorularını cevaplarken rahatımdır.
2		.673			İspat yapmakta zorlanırım.
44		.641			İspat yapmakta yeteneksizimdir.
37		.603			İspat yapmakta kendime güvenirim.
9		.583			Sınavda ispat sorularından korkmam.
39		.533			İspat yapmaktan çekinirim.
18		.527			İspat yapmasını kolayca öğrenirim.
33	.303	.456			İspat ile ilgili tartışmalarda görüşümü destekleyen güçlü kanıtlar bulamam.
22	,315	,383			İspat yaparken değişik yaklaşımlar kullanabilirim.
34		,342		,318	Bir ispatı hiç kimse anlatmadan sadece kitaptan çalışarak anlayabilirim.
6			,524		Bir ispat tekniğini öğrendikten sonra benzer ispatları yapabilirim.

Table 3.4 (Continued)

ITEM NO	FACTORS 1	2	3	4	ITEMS
10			,519		Yapılan ispatı açıklayamam.
29			,463		İspat yaparken kendimi çaresiz hissederim.
17		,346		,357	İspat yaparken aklıma bir şey gelmez.

When items accumulated in each factor in Table 3.4 were examined, it could be easily seen that items generally loaded first and second factors. After items were read in each factor, while first factor consisted of items related to enjoyment and usefulness, the second factor consisted of items related to anxiety and self concept.

5th, 14th, 26th and 46th items were expected to be in second factor but they were in the first factor. In addition, 6th, 10th, 29th and 34th items were expected to be in first factor, they also accumulated second and third factors. Thus, we eliminated them to provide construct validity. On the other hand, 33rd, 22nd and 17th items were expected to be in second factor, they were accumulated to other factors, too. It was expected that we had also eliminated these items, but existence of these items was important for the validity of PASG. Therefore, we kept these items.

After eliminating items, factor analysis was performed again with left 39 items. According to the initial principal factor solution with iterations, the first four eigenvalues were 14.292, 5.264, 1.762 and 1.434. Factor loadings of the PASG in the first factor ranged between 0.837 and 0.614. Factor loadings of the PASG in the second factor ranged between 0.731 and 0.314. The first and second factors accounted for 36.7% and 13.5% of the total variation in the PASG scores respectively. The factor loadings with the value at 0.314 or above presented in Table 3.5.

Table 3.5 Factor Loadings for PASG with 39 items

ITEM NO	FACTORS 1	2	3	4	ITEMS
42	.837				İspat yapma konusunda daha çok bilgi edinmek isterim.
19	.833				İspat yapmayı öğrenmek gereksizdir.
24	.819				İspat yapmayı severim.
32	.811				İspat yapma yeteneğimi geliştirmek isterim.
16	.803				İspat yapmak sıkıcıdır.
15	.787				İspat yapmak zihinsel gelişimime olumlu katkıda bulunur.
36	.779				İspat yapmayı öğrenmekten hoşlanmam.
47	.770				İspatlara çalışmak için zaman ayırmak istemem.
11	.756				İspat yapmak konunun iyice öğrenilmesini sağlar.
28	.749				İspat yapmak ilginç değildir.
45	.742				İspat yapmayı öğrenmek bana çekici gelir.
23	.737				İspatlanmış teoremlerin ispatını öğrenmenin bir faydası yoktur.
4	.729				İspat ile ilgili ileri düzeyde bilgi edinmek istemem.
27	.721				İspat yaparken bir probleme farklı açılardan bakmayı öğrenmemi sağlar.
35	.717				İspat yapmak bende ders çalışma isteği uyandırır.
25	.705				İspat yapınca kendime güvenim artıyor.
30	.701				İspat yapmak öğretilmelidir.
41	.692				Derste ispat yapmak derse olan ilgimi azaltıyor.
8	.690				Ders çalışırken teoremlerin ispatlarını incelemekten hoşlanırım.
20	.658				İspat yapabilmek beni gururlandırır.
38	.654				Gelecekteki hayatımda ispat yapma yeteneğine ihtiyacım olmayacaktır.
7	.634				İspat düşünme yeteneğimi geliştirmez.
31	.614				İspat yapmayı öğrenmemin gerçek yaşamıma olumlu bir katkıda bulunmaz.
43	.606				Sınavda ispat sorulması gereklidir.
3	.599				Derste ispat yapılınca ders daha eğlenceli oluyor.
12	.575				Kendimi mutlu hissetmem için ispat yapmada başarılı olmam gerekir.

Table 3.5 (Continued)

ITEM NO	FACTORS 1	2	3	4	ITEMS
13		.731			İspat ile ilgili sorularını cevaplarken rahatımdır.
1		.720			İspat yapabilme konusunda hiç endişe duymam.
2		.707			İspat yapmakta zorlanırım.
21		.698			İspat yapmaktan korkmam.
40		-.671			Kimseden yardım almadan ispat yapmayı başarabilirim.
37		.634			İspat yapmakta kendime güvenirim.
44		.625			İspat yapmakta yeteneksizimdir.
9		.623			Sınavda ispat sorularından korkmam.
18		.480			İspat yapmasını kolayca öğrenirim.
39	.358	.478		-.344	İspat yapmaktan çekinirim.
33		-.461			İspat ile ilgili tartışmalarda görüşümü destekleyen güçlü kanıtlar bulamam.
22		.418			İspat yaparken değişik yaklaşımlar kullanabilirim.
17		.314		-.427	İspat yaparken aklıma bir şey gelmez.

Although items loaded on two factors in the study with 136 subjects, we could not give the names to sub-dimensions. We applied the scale to 367 subjects. Factor Loadings for PASG with 39 items with 367 subjects were given in Table 3.6.

Table 3.6 Factor Loadings for PASG with 39 items with 367 subjects

ITEM NO	FACTORS 1	2	3	4	ITEMS
38	.753				İspat yapmayı öğrenmek bana çekici gelir.
21	.750				İspat yapınca kendime güvenim artıyor.
35	.747				İspat yapma konusunda daha çok bilgi edinmek isterim.
15	.746				İspat yapmayı öğrenmek gereksizdir.
22	.743				İspat yaparken bir probleme farklı açılardan bakmayı öğrenmemi sağlar.
34	.736				Derste ispat yapmak derse olan ilgimi azaltıyor.
29	.723				İspat yapmayı öğrenmekten hoşlanmam.

Table 3.6. (Continued)

ITEM NO	FACTORS 1	2	3	4	ITEMS
26	.696				İspat yapma yeteneğimi geliştirmek isterim.
12	.679				İspat yapmak sıkıcıdır.
28	.674				İspat yapmak bende ders çalışma isteği uyandırır.
25	.671				İspat yapmayı öğrenmemin gerçek yaşamıma olumlu bir katkıda bulunmaz.
24	.663				İspat yapmak öğretilmelidir.
8	.661				İspat yapmak konunun iyice öğretilmesini sağlar.
19	.656				İspatlanmış teoremlerin ispatını öğrenmenin bir faydası yoktur.
11	.645				İspat yapmak zihinsel gelişimime olumlu katkıda bulunur.
3	.641				Derste ispat yapılınca ders daha eğlenceli oluyor.
23	.641				İspat yapmak ilginç değildir.
30	.640				İspat yapmakta kendime güvenirim.
6	.629				Ders çalışırken teoremlerin ispatlarını incelemekten hoşlanırım.
16	.622				İspat yapabilmek beni gururlandırır.
39	.601				İspatlara çalışmak için zaman ayırmak isterim.
31	.597				Gelecekteki hayatımda ispat yapma yeteneğine ihtiyacım olmayacaktır.
32	.582				İspat yapmaktan çekinirim.
9	.579				Kendimi mutlu hissetmem için ispat yapmakta başarılı olmam gerekir.
18	.564				İspat yaparken değişik yaklaşımlar kullanabilirim.
36	.546				Sınavda ispat sorulması gereklidir.
33	.510				Kimseden yardım almadan ispat yapmayı başarabilirim.
5	.465				İspat düşünme yeteneğini geliştirmez
4	.463				İspatla ilgili ileri düzeyde bilgi edinmek isterim.
27	.445				İspat ile ilgili tartışmalarda görüşümü destekleyen güçlü kanıtlar bulamam.
13	.428				İspat yaparken aklıma bir şey gelmez.
17	.224				İspat yapmaktan korkmam.
1	.388	.643			İspat yapabilme konusunda hiç endişe duymam.
2	.447	.628			İspat yapmakta zorlanırım.

Table 3.6. (Continued)

ITEM NO	FACTORS 1	2	3	4	ITEMS
7	.463	.569			Sınavda ispat sorularından korkmam.
10	.566	.566			İspat ile ilgili soruları cevaplarken rahatımdır.
37	.501	.513			İspat yapmakta yeteneksizimdir.
14	.405	.496			İspat yapmasını kolayca öğrenirim
20	.034				İspat yapmayı severim.

After the analysis, we obtained one dimension titled as general attitude towards proof in geometry as seen in Table 3.6. The alpha reliability coefficient of the PASG with 39 items was found 0.94 with SPSS package program. The total score was between 39 and 195.

3.7 Procedure

The present study was conducted to 10th grade students from four types of school at the end of semester in 2004. To all classes, before PPGT and PASG were administered, the purpose of the present study was explained. After PPGT was performed in one lecture hour (40 minutes), PASG was performed in 15 minutes.

For the present study four types of school in Turkey were chosen. i.e. General High School, Anatolian High Schools, Science High School and Private High School.

3.8 Analysis of Data

Data of the present study were analyzed by the following statistical techniques:

- Reliability analysis was used to test the reliability of the scale administered in the present study.

- Factor analysis was used to test the validity of the scale administered in the present study and to determine the dimensions of the scale.

- Univariate Analysis of Variance was used to test the secondary school students' performance on proof and attitude towards proof in geometry with respect to school type and gender.

- Pearson Correlation was used to test relationship between performance on proof and attitude towards proof in geometry.

- Cross-Tab was used to find out the parentages of students' performance on PPGT in each question with respect to school types and gender.

3.9 Assumptions and Limitations

As in other studies there are several assumptions and limitations of the present study. The main assumptions of the present study are the following:

- The administration of PPGT and PASG were completed under standard conditions.

- All subjects of the pilot and experimental studies answered the measuring instruments accurately and sincerely.

The limitations of the present study are as listed below:

- This study was limited 10th grade students in four school types at the end of semester in 2004.

- This study was limited to only chosen schools from Ankara and İstanbul.

CHAPTER IV

RESULTS

In the previous chapters, the theoretical background of the study, the review of the previous studies related to this study and method of the study were stated. This chapter contains two sections. The first section presents results of testing hypotheses and the second section presents the overall pictures of students' responses about PASG, PPGT and PIF.

4.1 Results of Inferential Statistics

The main purpose of the present study was to investigate secondary school students' performance on proof and attitude towards proof in geometry with respect to school type and gender.

4.1.1 Results of the Testing Hypotheses

The problem of the study is: "What are the secondary school students' performance on proof and attitude towards proof in geometry with respect to school type and gender?"

The hypotheses of the problem are:

- $H_0$1: There are no statistically significant differences among the mean scores of students enrolled in different school types with respect to performance on proof in geometry.
- $H_0$2: There is no statistically significant difference between the mean scores of boys and girls with respect to performance on proof in geometry.
- $H_0$3: There is no statistically significant interaction between school type and gender with respect to performance on proof in geometry.

- $H_0$4: There are no statistically significant differences among the mean scores of students enrolled in different school types with respect to attitude towards proof in geometry.

- $H_0$5: There is no statistically significant difference among the mean scores of boys and girls with respect to attitude towards proof in geometry.

- $H_0$6: There is no statistically significant interaction between school type and gender with respect to attitude towards proof in geometry.

- $H_0$7: There is no statistically significant correlation between secondary school students' performances on proof in geometry and attitude towards proof in geometry.

To examine the problem of the study $H_0$1, $H_0$2, $H_0$3, $H_0$4, $H_0$5 and $H_0$6 were tested by Univariate Analysis of Variance. Results were presented in Table 4.1. $H_0$7 was tested by Pearson Correlation.

4.1.1.1 Results of Testing $H_0$1

As seen in Table 4.1 the Univariate Analysis of Variance indicated that there were statistically significant differences among the PPGT mean scores of students who enrolled in different types of schools ($p<0.05$).

Table 4.1 Results of "Univariate Analysis of Variance"

Source	Dependent Variable	Type III Sum of Squares	Df	Mean Square	F	Sig.
School Type	PPGT	81152.15	3	27050.72	150.23	0.00*
	PASG	3125.82	3	1041.94	1.05	0.37
Gender	PPGT	0.20	1	0.20	0.00	0.97
	PASG	299.44	1	299.44	0.30	0.58
School Type	PPGT	745.01	3	248.34	1.38	0.25
* Gender	PASG	1312.97	3	437.66	0.44	0.72

Table 4.1 (Continued)

Source	Dependent Variable	Type III Sum of Squares	Df	Mean Square	F	Sig.
Error	PPGT	64641.11	359	180.06		
	PASG	355521.90	359	990.31		
Total	PPGT	415054	367			
	PASG	6362639	367			

*p<0.05

To determine which groups caused the significant difference in PPGT scores, Bonferroni Test was employed. The results of the Bonferroni Test analysis related to the student's performance on proof in geometry at different types of school were showed in Table 4.2.

Table 4.2 The results of the Bonferroni Test analysis related to the students' performance on proof in geometry with respect to different school types.

Dependent Variable	(I) School Type (High School)	(J) School Type (High School)	Mean Difference (I-J)	Sig.
PPGT	General	Anatolian	-23.60*	0.00
		Science	-44.05*	0.00
		Private	-8.60*	0.00
	Anatolian	General	23.60*	0.00
		Science	-20.45*	0.00
		Private	15.01*	0.00
	Science	General	44.05*	0.00
		Anatolian	20.45*	0.00
		Private	35.46*	0.00
	Private	General	8.60*	0.00
		Anatolian	-15.01*	0.00
		Science	-35.46*	0.00

*p<0.05

According to Bonferroni Test results, it was seen that there were statistically significant mean differences among each pair of school type ($p<0.05$).

Table 4.3 Means and Standard Deviations of the PPGT Scores with respect to school types

School Type	Mean	SD	n
Science	51.02	14.14	90
Anatolian	30.57	15.12	96
Private	25.98	21.38	87
General	6.97	7.49	94
TOTAL	25.98	21.38	367

As seen Table 4.3, while students in Science High Schools had the highest, students in General High Schools had the lowest PPGT scores ($M_{Science}=51.02$, $SD_{Science}=14.14$; $M_{Anatolian}=30.57$, $SD_{Anatolian}=15.12$; $M_{Private}=25.98$, $SD_{Private}=21.38$, $M_{General}=6.97$, $SD_{General}=7.49$).

4.1.1.2 Results of Testing $H_0 2$

As seen Table 4.1, there was no statistically significant mean difference between PPGT scores of boys and girls ($p>0.05$).

Table 4.4 Means and Standard Deviations of the PPGT Scores with respect to gender.

Gender	Mean	SD	n
Boys	28.2	21.86	225
Girls	22.48	20.17	142
TOTAL	25.98	21.38	367

As seen in Table 4.4, PPGT scores of boys was slightly higher than PPGT scores of girls (M_{Boys}=28.2, SD_{Boys}=21.86, M_{Girls}=22.48, SD_{Girls}=20.17).

4.1.1.3 Results of Testing $H_0$3

As seen in Table 4.1, there was no statistically significant interaction between school types and gender with respect to performance on proof in geometry ($p>0.05$). Mean scores and standard deviations of boys and girls with respect to school type are given in Table 4.5.

Table 4.5 Mean scores and standard deviations of boys and girls with respect to school type

School Type	Gender	Mean	Std. Deviation	n
General	Girls	6.75	7.89	36
	Boys	7.1	7.29	58
	Total	6.97	7.49	94
Anatolian	Girls	33.77	15.33	35
	Boys	28.74	14.8	61
	Total	30.57	15.12	96
Science	Girls	49.14	15.06	21
	Boys	51.59	13.92	69
	Total	51.02	14.14	90
Private	Girls	14.7	15.19	50
	Boys	16.73	16.05	37
	Total	15.56	15.5	87
Total	Girls	22.48	20.17	142
	Boys	28.2	21.86	225
	Total	25.98	21.38	367

4.1.1.4 Results of Testing $H_0 4$

As seen table 4.1, it was found that there was no statistically significant differences among the PASG means of scores of students who enrolled different types schools ($p>0.05$)

Table 4.6 Means and Standard Deviations of the PASG Scores with respect to types of schools.

School Type	Mean	SD	n
Science	131.66	46.07	90
Anatolian	131.57	22.29	96
Private	123.57	27.52	87
General	129.11	24.5	94
TOTAL	127.89	31.37	367

As seen in Table 4.6, Science and Anatolian High Schools' students PASG scores slightly higher than Private and General High Schools ($M_{Science}$=131.66, $SD_{Science}$=46.07, $M_{Anatolian}$=131.57, $SD_{Anatolian}$=22.29, $M_{Private}$=123.57, $SD_{Private}$=27.52, $M_{General}$=129.11, $SD_{General}$=24.5)

4.1.1.5 Results of Testing $H_0 5$

As seen Table 4.1, there was no statistically significant difference among the mean of PASG scores of boys and girls ($p>0.05$).

Table 4.7 Means and Standard Deviations of the PASG Scores with respect to gender.

Gender	Mean	SD	n
Boys	127.77	35.48	225
Girls	128.08	23.55	142
TOTAL	127.89	31.37	367

As seen in Table 4.7. Science and Anatolian High schools' students PASG scores were slightly higher than PASG scores of Private and General High Schools (M_{Boys}=127.77, SD_{Boys}=35.48, M_{Girls}=128.08, SD_{Girls}=23.55)

4.1.1.6 Results of Testing $H_0$6

As seen Table 4.1, there was no statistically significant interaction between school types and gender with respect to performance on proof in geometry. ($p>0.05$)

4.1.1.7 Results of Testing $H_0$7

There was a statistically significant correlation between PPGT and PASG scores. Although this Pearson Correlation Coefficient is too small ($r=0.15$, $p<0.05$).

4.2 Results of Descriptive Statistics

In this section, students' responses to questions in PPGT and PIF are presented.

4.2.1 Statistics About PPGT Questions

PPGT was assessed by using focused holistic scoring techniques presented in Table 3.6, besides Means and Standard Deviations of students' PPGT scores out of 100 with respect to school type and gender were given in Table 4.8 and Table 4.9 respectively. Moreover, for each question, assessments of students' responses were presented with respect to school type and gender.

Table 4.8 Means and Standard Deviations of the PPGT Scores out of 100 with respect to school

School Type	Mean	SD	n
Science	70.86	19.64	90
Anatolian	42.46	20.99	96

Table 4.8. (Continued)

School Type	Mean	SD	n
Private	21.62	21.53	87
General	9.68	10.40	94
TOTAL	36.09	29.69	367

Table 4.9 Means and Standard Deviations of the PPGT Scores out of 100 with respect to gender.

Gander	Mean	SD	n
Boys	39.16	30.37	225
Girls	31.22	28.01	142
TOTAL	36.09	29.69	367

Question 1 was given in Figure 4.1. The frequencies and percentages of students' responses with respect to school type were given in Table 4.10.

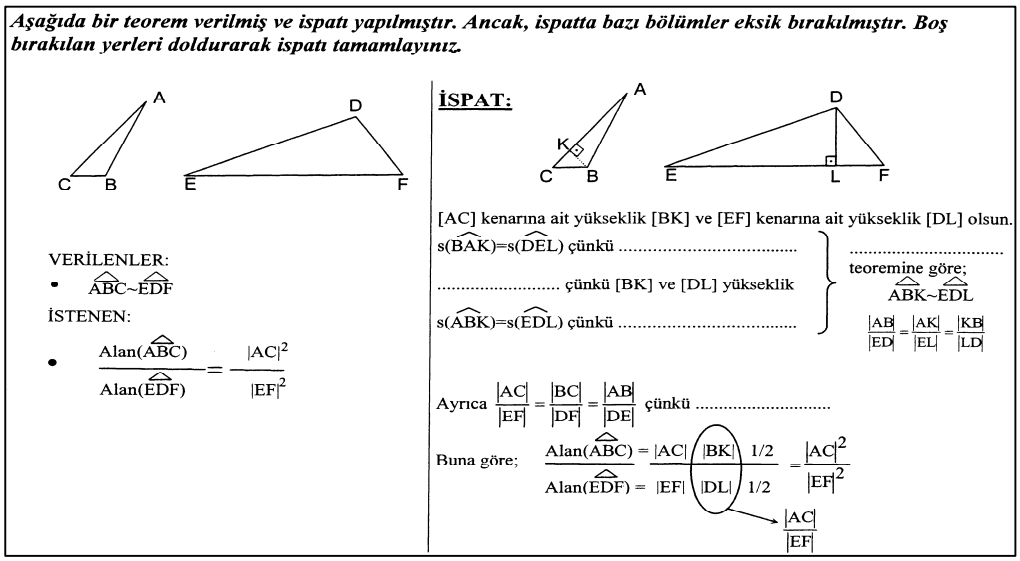


Figure 4.1 Question 1 in PPGT

Table 4.10 The frequencies and percentages of students' responses to question 1 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	**n**	87	55	20	74	236
	% within S.Type	92.60	57.30	22.20	85.10	64.30%
	% of Total	23.70	15.00	5.40	20.20	64.30%
1	**n**	7	27	18	7	59
	% within S.Type	7.40	28.10	20.00	8.00	16.10%
	% of Total	1.90	7.40	4.90	1.90	16.10%
3	**n**		7	27	1	35
	% within S.Type		7.30	30.00	1.10	9.50%
	% of Total		1.90	7.40	0.30	9.50%
4	**n**		5	22	4	31
	% within S.Type		5.20	24.40	4.60	8.40%
	% of Total		1.40	6.00	1.10	8.40%
6	**n**		2	3	1	6
	% within S.Type		2.10	3.30	1.10	1.60%
	% of Total		0.50	0.80	0.30	1.60%

As seen table 4.10, 64.30 % of students did not do anything or wrote irrelevant things to prove. While most of those students were in GHS (23.7%), AHS (15.00%) and PHS (20.20%), only 8.5% of them were in SHS. In general scores of students of GHS, AHS, and PHS were accumulated 0 and 1 point. On the other hand, most of the students' scores in SHS had 0, 1, 2, 3, and 4, that is, their scores were distributed evenly.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.11.

Table 4.11 The frequencies and percentages of students' responses to question 1 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	96	140	236
	% within Gender	67.60	62.20	64.30
	% of Total	26.20	38.10	64.30
1	**n**	21	38	59
	% within Gender	14.80	16.90	16.10
	% of Total	5.70	10.40	16.10
3	**n**	13	22	35
	% within Gender	9.20	9.80	9.50
	% of Total	3.50	6.00	9.50
4	**n**	11	20	31
	% within Gender	7.70	8.90	8.40
	% of Total	3.00	5.40	8.40
6	**n**	1	5	6
	% within Gender	0.70	2.20	1.60
	% of Total	0.30	1.40	1.60

Distribution of scores of girls and boys was similar. Most of them (i.e. 64.30 % of girls and 62.20 of boys) did not do anything or wrote irrelevant things to prove. On the other hand, boys' performance was a little bit better then girls' performance.

Question 2 was given in Figure 4.2. The frequencies and percentages of students' responses were given in Table4.12.

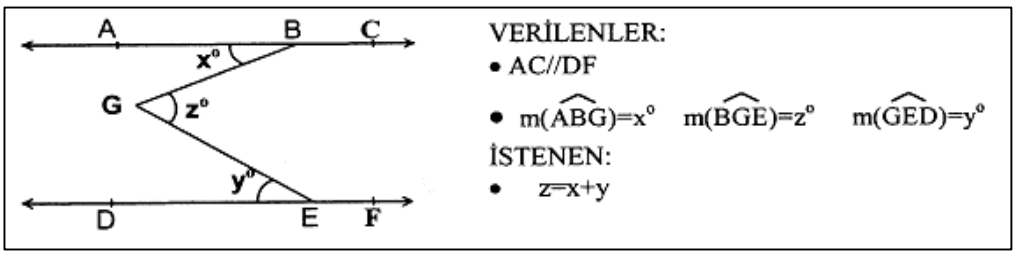


Figure 4.2 Question 2 in PPGT

Table 4.12 The frequencies and percentages of students' responses to question 2 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	**n**	25			14	39
	% within S.Type	26.60			16.10	10.60
	% of Total	6.80			3.80	10.60
1	**n**	4				4
	% within S.Type	4.30				1.10
	% of Total	1.10				1.10
3	**n**	6	2	1	6	15
	% within S.Type	6.40	2.10	1.10	6.90	4.10
	% of Total	1.60	0.50	0.30	1.60	4.10
4	**n**	25	27	10	25	87
	% within S.Type	26.60	28.10	11.10	28.70	23.70
	% of Total	6.80	7.40	2.70	6.80	23.70
6	**n**	34	67	79	42	222
	% within S.Type	36.20	69.80	87.80	48.30	60.50
	% of Total	9.30	18.30	21.50	11.40	60.50

As seen in the Table 4.12, while 23.7 % of students did complete proof, only 10.60 % of them did not do anything or wrote irrelevant things to prove. Students having 0 point were in GHS (6.30%) and in PHS (3.80%). On the other hand, scores of students were accumulated on 4 and 6 points in general.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.13.

Table 4.13 The frequencies and percentages of students' responses to question 2 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	n	20	19	39
	% within Gender	14.10	8.40	10.60
	% of Total	5.40	5.20	10.60
1	n	1	3	4
	% within Gender	0.70	1.30	1.10
	% of Total	0.30	0.80	1.10
3	n	7	8	15
	% within Gender	4.90	3.60	4.10
	% of Total	1.90	2.20	4.10
4	n	27	60	87
	% within Gender	19.00	26.70	23.70
	% of Total	7.40	16.30	23.70
6	n	87	135	222
	% within Gender	61.30	60.00	60.50
	% of Total	23.70	36.80	60.50

As shown in Table 4.13, distribution of scores of girls and boys was similar. Generally, boys and girls were successful in proving question 2. Most of them, (i.e. 61.30 % of girls and 60.00 of boys) proved question 2 completely.

Question 3 was given in Figure 4.3. The frequencies and percentages of students' responses were given in Table4.14.

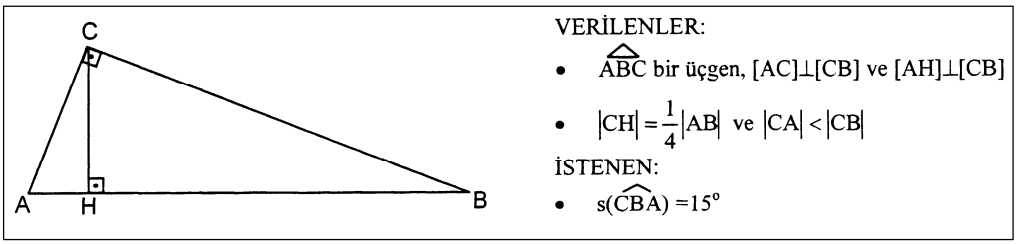


Figure 4.3 Question 3 in PPGT

Table 4.14 The frequencies and percentages of students' responses to question 3 with respect to school types.

Score	Frequencies and Percentages	School Type: General	Anatolian	Science	Private	Total
0	**n**	87	49	20	66	222
	% within S.Type	92.60	51.00	22.20	75.90	60.50
	% of Total	23.70	13.40	5.40	18.00	60.50
1	**n**	3	20	10	7	40
	% within S.Type	3.20	20.80	11.10	8.00	10.90
	% of Total	0.80	5.40	2.70	1.90	10.90
3	**n**	3	3	2	1	9
	% within S.Type	3.20	3.10	2.20	1.10	2.50
	% of Total	0.80	0.80	0.50	0.30	2.50
4	**n**		3	1		4
	% within S.Type		3.10	1.10		1.10
	% of Total		0.80	0.30		1.10
6	**n**	1	21	57	13	92
	% within S.Type	1.10	21.90	63.30	14.90	25.10
	% of Total	0.30	5.70	15.50	3.50	25.10

As seen in Table 4.14, while 60.50 % of students did not do anything or wrote irrelevant things to prove and 10.90% of them attempted to solve by doing the beginning of the proof, 25.10% of them did the complete proof. That is, students' points accumulated to 0 and 6 points. On the other hand, 92.60% of students in GHS had 0 points. On the other hand 63.30% of students in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.15.

Table 4.15 The frequencies and percentages of students' responses to question 3 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	88	134	222
	% within Gender	62.00	59.60	60.50
	% of Total	24.00	36.50	60.50
1	**n**	17	23	40
	% within Gender	12.00	10.20	10.90
	% of Total	4.60	6.30	10.90
3	**n**	5	4	9
	% within Gender	3.50	1.80	2.50
	% of Total	1.40	1.10	2.50
4	**n**		4	4
	% within Gender		1.80	1.10
	% of Total		1.10	1.10
6	**n**	32	60	92
	% within Gender	22.50	26.70	25.10
	% of Total	8.70	16.30	25.10

As seen in Table 4.15, girls and boys were not good at proving question such as 62.00 % of girls and 59.60% of boys did not do anything or

wrote irrelevant things to prove but 22.50% of girls and 26.70% of boys did the complete proof. Number of students who got 3 or 4 points from this question was 13. Generally, both girls and boys either did not do anything or did complete proof.

Question 4 was given in Figure 4.4. The frequencies and percentages of students' responses were given in Table 4.16.

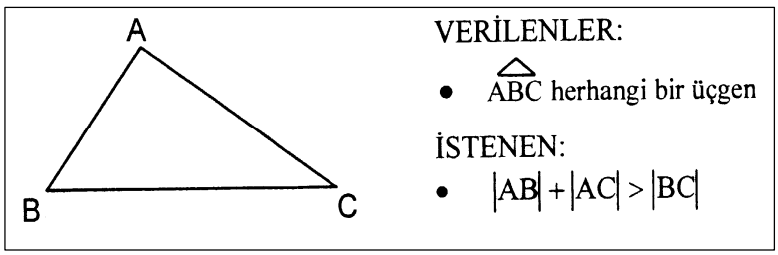


Figure 4.4 Question 4 in PPGT

Table 4.16 The frequencies and percentages of students' responses to question 4 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	**n**	88	42	25	72	227
	% within S.Type	93.60	43.80	27.80	82.80	61.90
	% of Total	24.00	11.40	6.80	19.60	61.90
1	**n**	3	29	26	5	63
	% within S.Type	3.20	30.20	28.90	5.70	17.20
	% of Total	0.80	7.90	7.10	1.40	17.20
3	**n**	1	4	8	3	16
	% within S.Type	1.10	4.20	8.90	3.40	4.40
	% of Total	0.30	1.10	2.20	0.80	4.40

Table 4.16 (Continued)

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
4	**n**	1	4	3	2	10
	% within S.Type	1.10	4.20	3.30	2.30	2.70
	% of Total	0.30	1.10	0.80	0.50	2.70
6	**n**	1	17	28	5	51
	% within S.Type	1.10	17.70	31.10	5.70	13.90
	% of Total	0.30	4.60	7.60	1.40	13.90

As seen in the Table 4.16, 61.90 % of students did not do anything or wrote irrelevant things to prove. While most of those students who got 0 point were in GHS (24.00%), AHS (11.40%) and PHS (19.60%), only 6.80% of them were in SHS. In general, scores of students in GHS, AHS and PHS were accumulated 0 and 1 point. While 93.60% of students in GHS and 82.80% students in PHS had 0 point, only 27.80% of students in SHS had 0. Most of students (31.10%) in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.17.

Table 4.17 The frequencies and percentages of students' responses to question 4 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	96	131	227
	% within Gender	67.60	58.20	61.90
	% of Total	26.20	35.70	61.90

Table 4.17 (Continued)

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
1	**n**	19	44	63
	% within Gender	13.40	19.60	17.20
	% of Total	5.20	12.00	17.20
3	**n**	7	9	16
	% within Gender	4.90	4.00	4.40
	% of Total	1.90	2.50	4.40
4	**n**	3	7	10
	% within Gender	2.10	3.10	2.70
	% of Total	0.80	1.90	2.70
6	**n**	17	34	51
	% within Gender	12.00	15.10	13.90
	% of Total	4.60	9.30	13.90

As seen in Table 4.17, girls and boys were not good at proving question such as 67.60% of girls and 58.20% of boys did not do anything or wrote irrelevant things to prove besides only 12% of girls and 15.10% of boys proved the statement completely. As seen, boys were slightly more successful than girls.

Question 5 was given in Figure 4.5. The frequencies and percentages of students' responses were given in Table4.18.

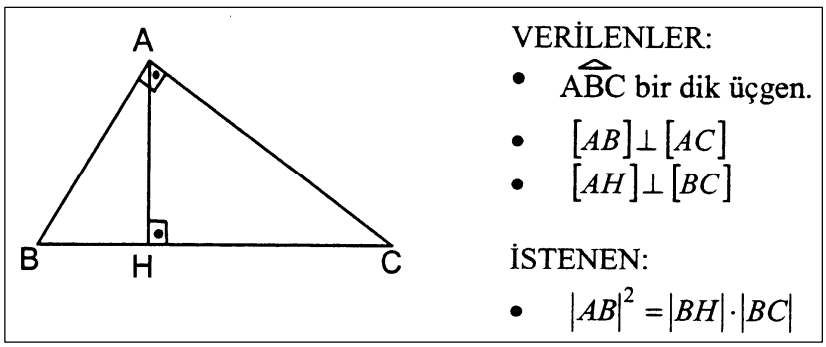


Figure 4.5 Question 5 in PPGT

Table 4.18 The frequencies and percentages of students' responses to question 5 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	**n**	66	23	3	41	133
	% within S.Type	70.20	24.00	3.30	47.10	36.20
	% of Total	18.00	6.30	0.80	11.20	36.20
1	**n**	11	9	6	18	44
	% within S.Type	11.70	9.40	6.70	20.70	12.00
	% of Total	3.00	2.50	1.60	4.90	12.00
3	**n**	7	8	8	10	33
	% within S.Type	7.40	8.30	8.90	11.50	9.00
	% of Total	1.90	2.20	2.20	2.70	9.00
4	**n**	3	12	11		26
	% within S.Type	3.20	12.50	12.20		7.10
	% of Total	0.80	3.30	3.00		7.10
6	**n**	7	44	62	18	131
	% within S.Type	7.40	45.80	68.89	20.70	35.69
	% of Total	1.90	12.00	16.89	4.90	35.69

As seen in the Table 4.18, while 36.20% of students did not do anything or wrote irrelevant things to prove, 35.69% of them did the complete proof. That is, students' scores accumulated 0 and 6 points. Whenever most of students started to prove, they achieved to complete proof. On the other hand, 70.20% of students in GHS and 47.10% students in PHS had 0 point; only 3.30% of students in SHS had 0. Most of students (68.89%) in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.19.

Table 4.19 The frequencies and percentages of students' responses to question 5 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	58	75	133
	% within Gender	40.80	33.30	36.20
	% of Total	15.80	20.40	36.20
1	**n**	22	22	44
	% within Gender	15.50	9.80	12.00
	% of Total	6.00	6.00	12.00
3	**n**	11	22	33
	% within Gender	7.70	9.80	9.00
	% of Total	3.00	6.00	9.00
4	**n**	7	19	26
	% within Gender	4.90	8.40	7.10
	% of Total	1.90	5.20	7.10
6	**n**	44	87	131
	% within Gender	31.00	38.20	35.40
	% of Total	12.00	23.40	35.40

As seen Table 4.19, 40.80 % of girls and 33.30% of boys did not do anything or wrote irrelevant things to prove but 31.00% of girls and 38.20% of boys did the complete proof. Besides, distribution of girls and boys' scores was similar. For instance, their scores accumulated on 0 and 6 points.

Question 6 was given Figure 4.6. The frequencies and percentages of students' responses were given in Table4.20.

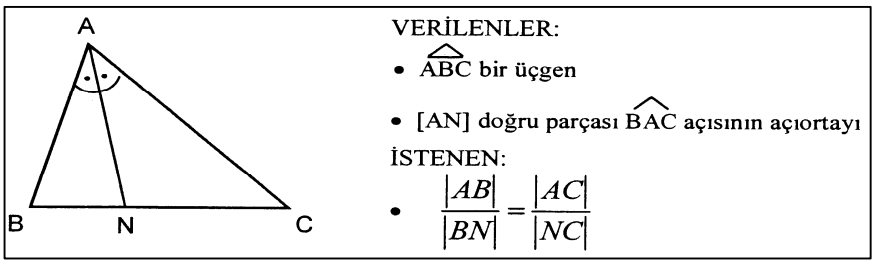


Figure 4.6 Question 6 in PPGT

Table 4.20 The frequencies and percentages of students' responses to question 6 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	n	88	63	15	75	241
	% within S.Type	93.60	65.60	16.70	86.20	65.70
	% of Total	24.00	17.20	4.10	20.40	65.70
1	n	4	12	10	3	29
	% within S.Type	4.30	12.50	11.10	3.40	7.90
	% of Total	1.10	3.30	2.70	0.80	7.90
3	n		3	3		6
	% within S.Type		3.10	3.30		1.60
	% of Total		0.80	0.80		1.60
4	n		2	10	1	13
	% within S.Type		2.10	11.10	1.10	3.50
	% of Total		0.50	2.70	0.30	3.50
6	n	2	16	52	9	79
	% within S.Type	2.10	16.70	57.80	10.34	21.53
	% of Total	0.50	4.40	14.20	2.45	21.53

As seen in the Table 4.20, while 65.70% of students did not do anything or wrote irrelevant things to prove, 21.53% of them proved the statement completely. That is, students' scores accumulated 0 and 6 points. While 93.60% of students in GHS, %86.20 students in PHS and 65.60% students in AHS had 0 point, only 16.70 % students in SHS had 0. Besides, most of students (57.80%) in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.21.

Table 4.21 The frequencies and percentages of students' responses to question 6 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	105	136	241
	% within Gender	73.90	60.40	65.70
	% of Total	28.60	37.10	65.70
1	**n**	11	18	29
	% within Gender	7.70	8.00	7.90
	% of Total	3.00	4.90	7.90
3	**n**	1	5	6
	% within Gender	0.70	2.20	1.60
	% of Total	0.30	1.40	1.60
4	**n**	3	10	13
	% within Gender	2.10	4.40	3.50
	% of Total	0.80	2.70	3.50
6	**n**	22	56	78
	% within Gender	14.80	24.90	21.00
	% of Total	5.70	15.30	21.00

As seen in Table 4.21, girls and boys were not good at proving question such as 73.90 % of girls and 60.40% of boys did not do anything or

wrote irrelevant things to prove but only 14.80% of girls and 24.90% of boys did the complete proof. Number of students who got 3 or 4 points from this question was 22. As a result, Most of girls and boys either did not do anything or did complete proofs generally.

Question 7 was given in Figure 4.7. The frequencies and percentages of students' responses were given in Table 4.22.

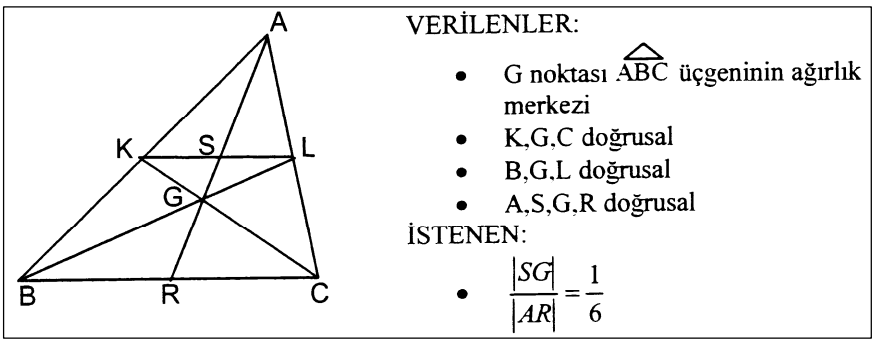


Figure 4.7 Question 7 in PPGT

Table 4.22 The frequencies and percentages of students' responses to question 7 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	n	69	26	13	58	166
	% within S.Type	73.40	27.10	14.40	66.70	45.20
	% of Total	18.80	7.10	3.50	15.80	45.20
1	n	9	31	22	24	86
	% within S.Type	9.60	32.30	24.40	27.60	23.40
	% of Total	2.50	8.40	6.00	6.50	23.40
3	n	14	14	23	4	55
	% within S.Type	14.90	14.60	25.60	4.60	15.00
	% of Total	3.80	3.80	6.30	1.10	15.00

Table 4.22 (Continued)

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
4	**n**	2	13	9		24
	% within S.Type	2.10	13.50	10.00		6.50
	% of Total	0.50	3.50	2.50		6.50
6	**n**		12	23	1	36
	% within S.Type		12.50	25.60	1.10	9.80
	% of Total		3.30	6.30	0.30	9.80

As shown in Table 4.22, most of students' scores distributed in 0, 1, 3 and students who did proof completely or with missing parts were only 16.3 % of all students. While students who had 0, 1 and 3 were in GHS (25.10%), PHS (23.40%), AHS (19.30%) and SHS (15.80%), students who had 4 and 6 were in GHS (0.50%), PHS (0.30%), AHS (6.80%) and SHS (8.80%).

The frequencies and percentages of students' responses with respect to gender were given in Table 4.23.

Table 4.23 The frequencies and percentages of students' responses to question 7 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	71	95	166
	% within Gender	50.00	42.20	45.20
	% of Total	19.30	25.90	45.20
1	**n**	35	51	86
	% within Gender	24.60	22.70	23.40
	% of Total	9.50	13.90	23.40

Table 4.23 (Continued)

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
3	**n**	22	33	55
	% within Gender	15.50	14.70	15.00
	% of Total	6.00	9.00	15.00
4	**n**	7	17	24
	% within Gender	4.90	7.60	6.50
	% of Total	1.90	4.60	6.50
6	**n**	7	29	36
	% within Gender	4.90	12.90	9.80
	% of Total	1.90	7.90	9.80

As seen in Table 4.23, distribution of scores of girls and boys was similar such as both girls and boys' scores were accumulated 0, 1 and 2 points. 50% of girls and 42.20% of boys got 0 point, 24.60% of girls and 22.70% of boys got 1 point, besides 15.50% of girls and 14.70% of boys got 2 points.

Question 8 was in Figure 4.8. The frequencies and percentages of students' responses were given in Table 4.24.

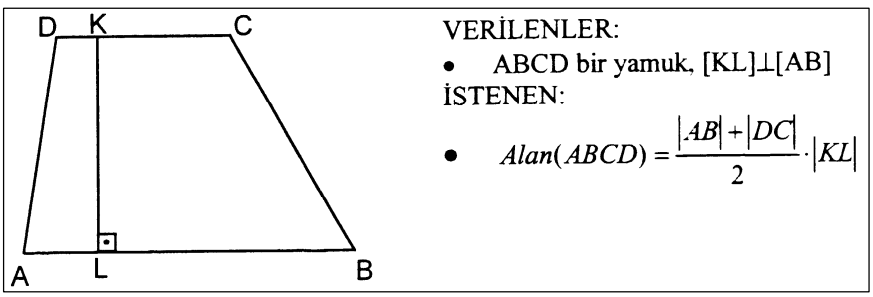


Figure 4.8 Question 8 in PPGT

Table 4.24 The frequencies and percentages of students' responses to question 8 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	**n**	76	33	6	62	177
	% within S.Type	80.90	34.40	6.70	71.30	48.20
	% of Total	20.70	9.00	1.60	16.90	48.20
1	**n**	11	4	1	4	20
	% within S.Type	11.70	4.20	1.10	4.60	5.40
	% of Total	3.00	1.10	0.30	1.10	5.40
3	**n**	2	3		1	6
	% within S.Type	2.10	3.10		1.10	1.60
	% of Total	0.50	0.80		0.30	1.60
4	**n**	1	3		2	6
	% within S.Type	1.10	3.10		2.30	1.60
	% of Total	0.30	0.80		0.50	1.60
6	**n**	4	53	83	18	158
	% within S.Type	4.30	55.20	92.20	20.70	43.10
	% of Total	1.10	14.40	22.60	4.90	43.10

As seen in the Table 4.24, while 48.20% of students did not do anything or wrote irrelevant things to prove, 43.10% of them did the complete proof. That is, students' scores accumulated 0 and 6 points. Whenever most of students started to prove, they achieved to complete proof. On the other hand, 80.90% of students in GHS, %71.30 of students in PHS and %34.40 of students in AHS had 0 point; only 6.70% of students in SHS had 0. Most of students (92.20%) in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.25.

Table 4.25 The frequencies and percentages of students' responses to question 8 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	75	102	177
	% within Gender	52.80	45.30	48.20
	% of Total	20.40	27.80	48.20
1	**n**	5	15	20
	% within Gender	3.50	6.70	5.40
	% of Total	1.40	4.10	5.40
3	**n**	3	3	6
	% within Gender	2.10	1.30	1.60
	% of Total	0.80	0.80	1.60
4	**n**	1	5	6
	% within Gender	0.70	2.20	1.60
	% of Total	0.30	1.40	1.60
6	**n**	58	100	158
	% within Gender	40.80	44.40	43.10
	% of Total	15.80	27.20	43.10

As seen in Table 4.25, 52.80 % of girls and 45.30% of boys did not do anything or wrote irrelevant things to prove but 40.80% of girls and 44.40% of boys did the complete proof. 8.60% of students got 1, 3 or 4 points from this question. Therefore, it could be stated that both girls and boys either did not do anything or did complete proofs generally.

Question 9 was given in Figure 4.9. The frequencies and percentages of students' responses were given in Table4.26.

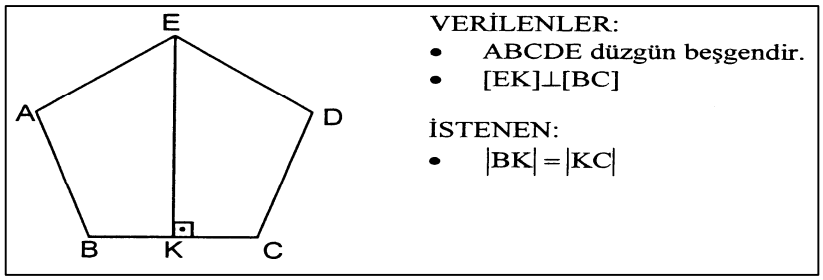


Figure 4.9 Question 9 in PPGT

Table 4.26 The frequencies and percentages of students' responses to question 9 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	**n**	80	37	10	51	178
	% within S.Type	85.10	38.50	11.10	58.60	48.50
	% of Total	21.80	10.10	2.70	13.90	48.50
1	**n**	7	11	7	12	37
	% within S.Type	7.40	11.50	7.80	13.80	10.10
	% of Total	1.90	3.00	1.90	3.30	10.10
3	**n**	3	12	4	2	21
	% within S.Type	3.20	12.50	4.40	2.30	5.70
	% of Total	0.80	3.30	1.10	0.50	5.70
4	**n**	4	23	12	8	47
	% within S.Type	4.30	24.00	13.30	9.20	12.80
	% of Total	1.10	6.30	3.30	2.20	12.80
6	**n**		13	57	14	84
	% within S.Type		13.50	63.30	16.10	22.90
	% of Total		3.50	15.50	3.80	22.90

As seen in the Table 4.26, although most of students' scores (71.40%) accumulated 0 and 6 points, accumulation on 1 and 4 (22.90%) were not neglectable. 48.50% of students did not do anything or wrote irrelevant things to prove, 22.90% of them did the complete proof. While 85.10% of students in GHS, %58.60 of students in PHS and %38.50 of students in SHS had 0 point, only 11.10 % of students in SHS had 0 point. and most of students (63.30%) in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.27.

Table 4.27 The frequencies and percentages of students' responses to question 9 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	78	100	178
	% within Gender	54.90	44.40	48.50
	% of Total	21.30	27.20	48.50
1	**n**	13	24	37
	% within Gender	9.20	10.70	10.10
	% of Total	3.50	6.50	10.10
3	**n**	5	16	21
	% within Gender	3.50	7.10	5.70
	% of Total	1.40	4.40	5.70
4	**n**	17	30	47
	% within Gender	12.00	13.30	12.80
	% of Total	4.60	8.20	12.80
6	**n**	29	55	84
	% within Gender	20.40	24.40	22.90
	% of Total	7.90	15.00	22.90

As seen in Table 4.27, 54.90 % of girls and 44.40% of boys did not do anything or wrote irrelevant things to prove on the other hand 20.40% of girls and 24.40% of boys did the complete proof. Although distribution of girls and boys' scores was similar, boys were a little bit more successful than girls.

Question 10 was given in Figure 4.10. The frequencies and percentages of students' responses were given in Table 4.28.

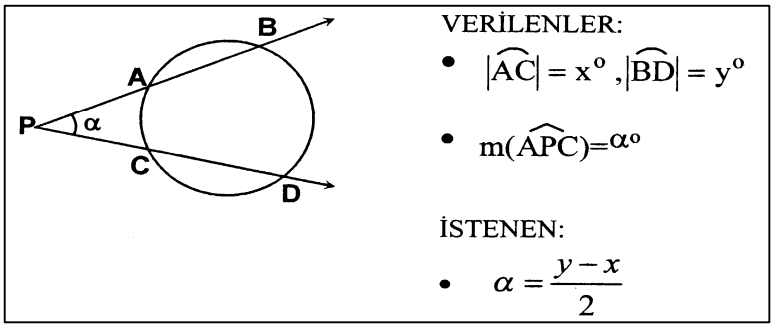


Figure 4.10 Question 10 in PPGT

Table 4.28 The frequencies and percentages of students' responses to question 10 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	**n**	93	63	11	77	244
	% within S.Type	98.90	65.60	12.20	88.50	66.50
	% of Total	25.30	17.20	3.00	21.00	66.50
1	**n**	1	10	4		15
	% within S.Type	1.10	10.40	4.40		4.10
	% of Total	0.30	2.70	1.10		4.10
3	**n**		4	3		7
	% within S.Type		4.20	3.30		1.90
	% of Total		1.10	0.80		1.90

Table 4.28 (Continued)

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
6	**n**		19	72	10	101
	% within S.Type		19.80	80.00	11.50	27.50
	% of Total		5.20	19.60	2.70	27.50

As seen in the Table 4.28, while 66.50% of students did not do anything or wrote irrelevant things to prove, 27.50% of them did the complete proof. There was no student who had 4 points. In other words, whenever students proved the statement, they did not leave any missing. On the other hand, 98.90% of students in GHS, %88.50 of students in PHS and %65.60 of students in AHS had 0 point; only 12.20% of students in SHS had 0. Most of students (80.00%) in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.30.

Table 4.29 The frequencies and percentages of students' responses to question 10 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	102	142	244
	% within Gender	71.80	63.10	66.50
	% of Total	27.80	38.70	66.50
1	**n**	8	7	15
	% within Gender	5.60	3.10	4.10
	% of Total	2.20	1.90	4.10

Table 4.29 (Continued)

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
3	**n**		7	7
	% within Gender		3.10	1.90
	% of Total		1.90	1.90
6	**n**	32	69	101
	% within Gender	22.50	30.70	27.50
	% of Total	8.70	18.80	27.50

As seen in Table 4.29, girls and boys were not good at proving question such as 71.80 % of girls and 63.10% of boys did not do anything or wrote irrelevant things to prove but 22.50% of girls and 30.70% of boys did the complete proof. There was no student who got 4 points. Both girls and boys' scores accumulated on 0 and 6 points.

Question 11 was given in Figure 4.11. The frequencies and percentages of students' responses were given in Table4.30.

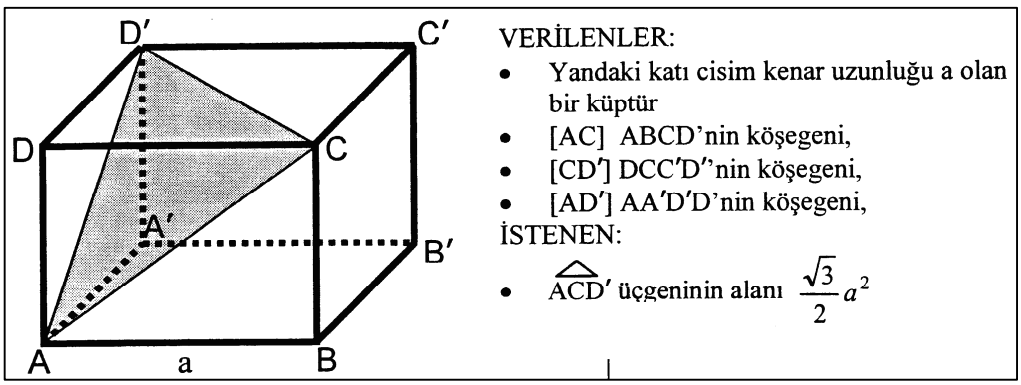


Figure 4.11 Question 11 in PPGT

Table 4.30 The frequencies and percentages of students' responses to question 11 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	n	88	37	15	63	203
	% within S.Type	93.60	38.50	16.70	72.40	55.30
	% of Total	24.00	10.10	4.10	17.20	55.30
1	n	1	2		2	5
	% within S.Type	1.10	2.10		2.30	1.40
	% of Total	0.30	0.50		0.50	1.40
3	n		5		1	6
	% within S.Type		5.20		1.10	1.60
	% of Total		1.40		0.30	1.60
4	n		2	3	3	8
	% within S.Type		2.10	3.30	3.40	2.20
	% of Total		0.50	0.80	0.80	2.20
6	n	5	50	72	18	145
	% within S.Type	5.30	52.10	80.00	20.70	39.50
	% of Total	1.40	13.60	19.60	4.90	39.50

As seen in the Table 4.30, while 55.30% of students did not do anything or wrote irrelevant things to prove, 39.50% of them did the complete proof. That is, students' scores accumulated 0 and 6 points. While 93.60% of students in GHS, %72.40 of students in PHS and %38.50 of students in AHS had 0 point, only 16.70 % of students in SHS had 0 and most of students (39.50%) in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender were given in Table 4.31.

Table 4.31 The frequencies and percentages of students' responses to question 11 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	91	112	203
	% within Gender	64.10	49.80	55.30
	% of Total	24.80	30.50	55.30
1	**n**	2	3	5
	% within Gender	1.40	1.30	1.40
	% of Total	0.50	0.80	1.40
3	**n**	4	2	6
	% within Gender	2.80	0.90	1.60
	% of Total	1.10	0.50	1.60
4	**n**	2	6	8
	% within Gender	1.40	2.70	2.20
	% of Total	0.50	1.60	2.20
6	**n**	43	102	145
	% within Gender	30.30	45.30	39.50
	% of Total	11.70	27.80	39.50

As seen Table 4.31, 64.10% of girls and 49.80% of boys did not do anything or wrote irrelevant things to prove on the other hand 30.30% of girls and 45.30% of boys did the complete proof. Although distribution of girls and boys' scores was similar, boys were more successful than girls.

Question 12 was given in Figure 4.12. The frequencies and percentages of students' responses were given in Table4.32.

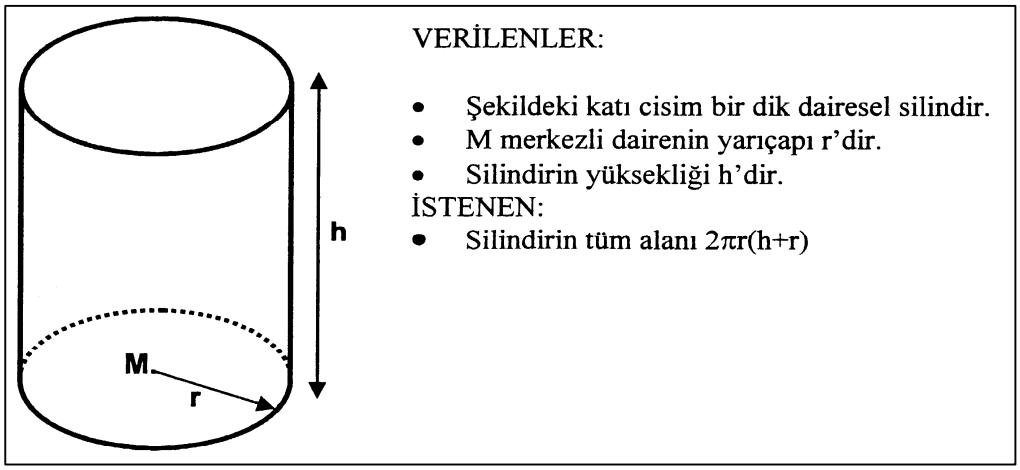


Figure 4.12 Question 12 in PPGT

Table 4.32 The frequencies and percentages of students' responses to question 12 with respect to school types.

Score	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
0	n	89	39	20	62	210
	% within S.Type	94.70	40.60	22.20	71.30	57.20
	% of Total	24.30	10.60	5.40	16.90	57.20
1	n	1	2	1	6	10
	% within S.Type	1.10	2.10	1.10	6.90	2.70
	% of Total	0.30	0.50	0.30	1.60	2.70
3	n	1	1	1		3
	% within S.Type	1.10	1.00	1.10		0.80
	% of Total	0.30	0.30	0.30		0.80
4	n	2	2	6	1	11
	% within S.Type	2.10	2.10	6.70	1.10	3.00
	% of Total	0.50	0.50	1.60	0.30	3.00
6	n	1	52	62	18	133
	% within S.Type	1.10	54.20	68.90	20.70	36.20
	% of Total	0.30	14.20	16.90	4.90	36.20

As seen in Table 4.32, while 57.20 % of students did not do anything or wrote irrelevant things to prove and 36.20% of them did complete proof. That is, students' points accumulated to 0 and 6 points except for students' points in GHS. %94.70 of students in GHS had 0 points. On the other hand %68.90 of students in SHS had 6 points.

The frequencies and percentages of students' responses with respect to gender are given in Table 4.33.

Table 4.33 The frequencies and percentages of students' responses to question 12 with respect to gender.

Score	Frequencies and Percentages	Gender		Total
		Girls	Boys	
0	**n**	95	115	210
	% within Gender	66.90	51.10	57.20
	% of Total	25.90	31.30	57.20
1	**n**	2	8	10
	% within Gender	1.40	3.60	2.70
	% of Total	0.50	2.20	2.70
3	**n**	1	2	3
	% within Gender	0.70	0.90	0.80
	% of Total	0.30	0.50	0.80
4	**n**	4	7	11
	% within Gender	2.80	3.10	3.00
	% of Total	1.10	1.90	3.00
6	**n**	40	93	133
	% within Gender	28.20	41.30	36.20
	% of Total	10.90	25.30	36.20

As seen in Table 4.33, 66.90% of girls and 51.10% of boys did not do anything or wrote irrelevant things to prove on the other hand 28.20% of girls and 41.30% of boys did the complete proof. Although distribution of girls and boys' scores was similar, boys were much more successful than girls.

4.2.2 Statistics About PIF Questions

The frequencies and percentages of students' responses to "How often does your teacher prove statements in geometry? were given in Table 4.35.

Table 4.34 The frequencies and percentages of students' responses to "How often does your teacher prove statements in geometry?

Answer	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
Never	n	14	2	0	6	22
	% within S. Type	15.05	2.27	0.00	7.23	6.21
	% of Total	3.95	0.56	0.00	1.69	6.21
Rare	n	32	15	6	13	66
	% within S. Type	34.41	17.05	6.67	15.66	18.60
	% of Total	9.04	4.24	1.69	3.67	18.60
Sometimes	n	31	34	23	23	111
	% within S. Type	33.33	38.64	25.56	27.71	31.40
	% of Total	8.76	9.60	6.50	6.50	31.40
Often	n	13	31	49	28	121
	% within S. Type	13.98	35.23	54.44	33.73	34.20
	% of Total	3.67	8.76	13.84	7.91	34.20
Always	n	3	6	12	13	34
	% within S. Type	3.23	6.82	13.33	15.66	9.60
	% of Total	0.85	1.69	3.39	3.67	9.60

As seen in Table 4.35, teachers in Science and Anatolian High Schools proved statements in lecture more frequently than teachers in General and Private High schools. For instance, 49.46% of students in GHS, 22.98% of

students in PHS, 19.32% of students in AHS and 6.67% of students in SHS stated their teachers never or rarely prove statements in the lecture

The frequencies and percentages of students' responses to "Is there any questions related to proof in geometry exams?" were given in Table 4.35.

Table 4.35 The frequencies and percentages of students' responses to "Is there any questions related to proof in geometry exams?"

Answer	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
Yes	n	8	51	88	4	151
	% within S. Type	8.70	56.70	97.80	4.80	42.40
	% of Total	2.20	14.30	24.70	1.10	42.40
No	n	84	39	2	80	205
	% within S. Type	91.30	43.33	2.22	95.24	57.58
	% of Total	23.60	10.96	0.56	22.47	57.58

As seen in Table 4.36, teachers in Science High Schools asked students prove the statements in exams. On the other, hand teachers in General and Private High Schools asked almost no question about proving statements in geometry exams. 8.70% of students in GHS and %4.80 of students in PHS stated their teacher did not ask any question related to proof in geometry exams. However, 56.70% of students in AHS and %97.80 of students in SHS stated there were questions related to proof in geometry exam.

The frequencies and percentages of students' perceptions about their success in geometry were given in Table 4.36.

Table 4.36 The frequencies and percentages of students' perceptions about their success in geometry.

Answer	Frequencies and Percentages	School Type				Total
		General	Anatolian	Science	Private	
Poor	n	17	6	2	11	36
	% within Geo. Ach. Level	47.2	16.7	5.6	30.6	100.0
	% within S. Type	18.1	6.3	2.2	12.9	9.9
	% of Total	4.7	1.6	.5	3.0	9.9
Fair	n	43	30	11	35	119
	% within Geo. Ach. Level	36.1	25.2	9.2	29.4	100.0
	% within S. Type	45.7	31.3	12.2	41.2	32.6
	% of Total	11.8	8.2	3.0	9.6	32.6
Good	n	28	44	48	28	148
	% within Geo. Ach. Level	18.9	29.7	32.4	18.9	100.0
	% within S. Type	29.8	45.8	53.3	32.9	40.5
	% of Total	7.7	12.1	13.2	7.7	40.5
Well	n	6	16	29	11	62
	% within Geo. Ach. Level	9.7	25.8	46.8	17.7	100.0
	% within S. Type	6.4	16.7	32.2	12.9	17.0
	% of Total	1.6	4.4	7.9	3.0	17.0

As seen in Table 4.36, most of students in SHS perceived themselves as successful in geometry in contrast to students in GHS. 85.50% of students in SHS perceived their success as good or well but 92.90 % of students of GHS perceived their success as poor or fair.

CHAPTER V

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

This chapter presents discussions and conclusions related to the results in the previous chapter, and recommendations are stated.

5.1 Discussion

Proof in geometry has been studied by several researchers and educators to investigate the students' competency on proof. In present study, it was investigated that students' performance on proof and their attitude towards proof in geometry. In order to reach this purpose, students' scores obtained from PPGT and PASG were analyzed with respect to school type and gender, hypothesis tested by Univariate Analysis of Variance and Pearson Correlation Coefficient. In addition to that descriptive analyses were utilized. In this section, results of study will be discussed.

The results showed that there were statistically significant mean differences among school type with respect to performance on proof in geometry. The factors which might have caused difference could be students' perceptions about their success in geometry, geometry instruction, classroom environment in these different types of schools.

The first factor could be students' perceptions about their success in geometry. It could be easily stated that students' perceptions varied through schools. For instance, while 47.2% of students in General High School perceived themselves as poor, 5.6% of students in Science High School perceived themselves as poor. Thus, there was a relationship between students' perceptions and performance on proof in geometry. This was not surprising because students were enrolled in Science High Schools, Anatolian High School and Private High School after taking enough scores form High

School Entrance Examination. In addition to this, education in the Science High Schools is based on mathematics and science fields. Therefore, this might cause the difference in PPGT scores of students.

The second factor which might have caused the differences in means scores of PPGT between General High Schools and others could be classroom environment. In the study, I observed that there was a big difference between class size of General High Schools and class size of other types of schools. For instance, all General High Schools in the study had over 45 students per classroom. However, there were less than 25 students in classrooms of other types of school. In addition to this, as a result of informal conversation with teachers, I learnt that General High Schools had less educational opportunities than others' educational opportunities. For instance, Private High Schools might use technology more than the other types of schools such as overhead projectors, multimedia rooms etc. Therefore, they could be reasons of the difference in means of PPGT scores of students in General High Schools be the lowest. According to attitude towards proof in geometry, there was no significant difference among school types. That is, students in General High Schools wanted to learn to prove as much as students in Science High Schools. Thus, classroom environment which is an external factor might cause the mean differences among school types.

The last factor which might have caused the difference among PPGT scores with respect to school type could be geometry instruction. As students' perceptions about their success on geometry and classroom environments change, teaching strategies in geometry also change. For instance, in this study some students in General High School wrote comments as 'We do not learn proof, why are you asking proofs?', 'It is obvious, there is nothing to prove' etc instead of answers. Moreover, in Personal Information Form, students were asked 'How often do your teacher prove statements in geometry.' 15.05 % of those students stated 'Never', 34.41% stated 'Rare' and 33.33% stated 'Sometimes'. Thus, it could be concluded that teachers did not prove statements in General High School frequently. Unlikely, 33.73% and 15.66%

of the students in Science High Schools answered the same question as 'Often' and 'Always', respectively. In addition to that, 97.80% of the students in Science High Schools stated 'Proofs are asked in the schools exams' but 91.30% of students in General High Schools stated 'Proofs are not asked in the school exams'. Therefore, it was easily seen that geometry instruction varied with respect to the type of the school. This might be a factor that caused a difference in PPGT scores with respect to school type.

Another important finding from the results is that, there was a statistically significant correlation between students' performance in geometry and their attitudes toward proof in geometry. It was not surprising that students with positive attitude towards proof in geometry got higher scores in PPGT.

In the present study, it was also found that there were no statistically significant mean differences between boys and girls with respect to performance on proof and attitude towards proof in geometry.

Unfortunately, we could not reach any study on gender differences with respect to performance on proof and attitudes toward proof in geometry. That's why we could not compare our findings with other studies. However, we compared the results of the study with the studies on geometry achievement conducted by the different researchers.

We could reach a few research studies about gender differences in geometry achievement. This finding was consistent with these research. For example, Senk and Usiskin (1983), Amy and Andrew (2004) found that boys and girls did not show statistically significant difference in geometry. On the other hand, the findings of the present study was not consistent with finding of Hanna (1986) who found that boys had better than girls at grade 8 in geometry. In this study, there was no gender difference in performance in proof and attitude toward proof in geometry. Their reasons could be that girls

and boys might have common teachers, took the same mathematics and geometry courses and none of them have privilege

In this study, there was no statistically significant differences among PASG mean scores of students with respect to school type. Unfortunately, we could not find any research related to attitude towards proof in geometry. Thus, this result could not be compared with other researches. Reason of this might be Turkish Education System. For example, success of a student generally was evaluated High School Entrance Examination and University Entrance Examination so teachers and students' primary aim might be to pass these exams. In addition to that, they might not be aware of their attitudes.

5.2 Conclusions

The problem of the study is: "What are the secondary school students' performance on proof in geometry and attitude towards proof in geometry with respect to school type and gender?"

In this section, in the light of findings, the following conclusions can be stated:

1. There were statistically significant differences among the mean scores of students enrolled to different school types with respect to performance on proof in geometry.

2. There were not statistically significant differences between the mean scores of boys and girls with respect to performance on proof in geometry.

3. There were not statistically significant interaction between school type and gender with respect to performance on proof in geometry.

4. There were not statistically significant differences among the mean scores of students enrolled to different school types with respect to attitude towards proof in geometry.

5. There were not statistically significant differences among the mean

scores of boys and girls attitude towards proof in geometry.

6. There were not statistically significant interaction between school type and gender with respect to attitude towards proof in geometry.

7. There was statistically significant correlation between secondary school students' performances on proof in geometry and attitude towards proof in geometry.

8. While students in SHS got the highest scores from each question, students in GHS got the lowest scores.

9. While most of students in SHS perceived themselves as successful in geometry, most of students in GHS perceived themselves unsuccessful.

5.3 Recommendations

In the present study many students were not successful in proving statements in geometry. According to the results of the research studies and experience of the researchers, there is a relationship between students' reasoning ability and competency on proof. Thus, students' reasoning ability must be improved. To reach this goal there are different ways. One of them can be that teachers should give importance on proof in their courses. They also have them be aware of necessity of proof for their success in school and real life. Students should learn not to accept everything directly; and they should ask 'Why?' and 'How?' when they do not understand something. To achieve this, teachers should spend more time to prove propositions and theorems in mathematics and ask proofs in exams. Teachers should have their students learn by inquiry rather than just memorizing facts.

Unfortunately, in several schools teachers could not spend time for proof and ask questions in the exams. Their reasons can be loaded curriculum, qualification of teachers on proof, negative attitudes toward proof of teachers

and students. To handle these problems curriculum should be revised in such a way that it gives chance teachers to emphasize on proofs. In addition, in-service teacher training program should be organized. We should revise programs in the Divisions of Elementary and Secondary Mathematics Education. Prospective and in service mathematics teachers should also be an aware of the importance of proof, competent on how to teach proofs and on overcoming students' difficulties in reasoning. These recommendations are not valid only for mathematics teachers but also for all teachers.

Several recommendations for the further studies can be stated: The sample size should be increased. Students at different grade levels should be included. For a deep investigation qualitative methods should be utilized.

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APPENDIX A

Table A.1 Table of content of PPGT

OBJECTIVES / TOPICS	To be able to prove measure of angles formed with parallel lines by drawing additional lines.	To be able to prove properties of special triangles.	To be able to prove properties of similar triangles.	To be able to prove basic properties of triangles.	To be able to prove basic properties of right triangles.	To be able to prove basic properties of quadrilaterals.	To be able to prove basic properties of polygons.	To be able to prove basic properties of circles	To be able to prove basic properties of solids.
Angels	8%								
Triangles		8%	1%	25%	8%				
Quadrilaterals						8%			
Polygones							8%		
Circles								8%	
Space Geometry and Solids									17%

APPENDIX B

Sevgili Öğrenciler:

Bu çalışma üç bölümden oluşmaktadır:

1. Kişisel Bilgiler;
2. Geometride İspat Yapabilme Becerisini Ölçen Başarı Testi;

Bu test sizin geometri dersindeki ispatlarda ne kadar başarılı olduğunuzu tespit etmek için hazırlanmıştır. Bu testi ne kadar ***ciddiyetle*** *çözerseniz elde edilecek sonuçlar o kadar güvenilir olacaktır.*

3. İspat Yapmaya Yönelik Tutum Ölçeği;

Yaptığımız çalışma sizlerin geometride ispat yapmaya yönelik tutumlarınızı belirlemek için yapılmaktadır. Ne kadar ***içten*** *cevap verecek olursanız çalışmadan elde edilecek sonuçlar o kadar güvenilir olacaktır.*

Bu test ve anket sadece araştırma amacıyla kullanılacaktır ve verdiğiniz cevaplar **kesinlikle gizli tutulacaktır**. Yardımlarınız için çok teşekkürler

1. KİŞİSEL BİLGİLER

Adınız Soyadınız : .....

Okulunuzun İsmi : .....

Sınıfınız : .....

Geçen Yıla Ait Matematik Notunuz: .....

Cinsiyetiniz : ( ) Kız ( ) Erkek

Geometri Başarınızı Derecelendiriniz :

( ) Çok İyi ( ) İyi ( ) Orta ( ) Zayıf

Geometri ile İlgili Yeteneğinizi Derecelendiriniz :

( ) Çok İyi ( ) İyi ( ) Orta ( ) Zayıf

Geometri anlatılan derslerde ne kadar sıklıkla ispat yapılmaktadır:

( ) Her zaman ( ) Sık Sık ( ) Bazen ( ) Nadiren ( ) Hiçbir Zaman

Geometri sınavında ispat sorusu soruluyor mu? ( ) Evet ( ) Hayır

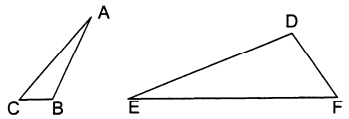
Figure B.1 Personal Information Form

APPENDIX C

2. GEOMETRİDE İSPAT BAŞARI TESTİ

Aşağıda bir teorem verilmiş ve ispatı yapılmıştır. Ancak, ispatta bazı bölümler eksik bırakılmıştır. Boş bırakılan yerleri doldurarak ispatı tamamlayınız.

1.



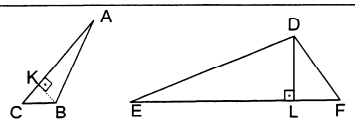
VERİLENLER:

- $\triangle ABC \sim \triangle EDF$

İSTENEN:

- $\dfrac{\text{Alan}(\triangle ABC)}{\text{Alan}(\triangle EDF)} = \dfrac{|AC|^2}{|EF|^2}$

İSPAT:



[AC] kenarına ait yükseklik [BK] ve [EF] kenarına ait yükseklik [DL] olsun.

$s(\widehat{BAK})=s(\widehat{DEL})$ çünkü .....

..... çünkü [BK] ve [DL] yükseklik

$s(\widehat{ABK})=s(\widehat{EDL})$ çünkü .....

..... teoremine göre; $\triangle ABK \sim \triangle EDL$

$\dfrac{|AB|}{|ED|} = \dfrac{|AK|}{|EL|} = \dfrac{|KB|}{|LD|}$

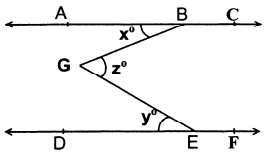
Ayrıca $\dfrac{|AC|}{|EF|} = \dfrac{|BC|}{|DF|} = \dfrac{|AB|}{|DE|}$ çünkü .....

Buna göre; $\dfrac{\text{Alan}(\triangle ABC)}{\text{Alan}(\triangle EDF)} = \dfrac{|AC|\,|BK|\,1/2}{|EF|\,|DL|\,1/2} = \dfrac{|AC|^2}{|EF|^2}$

$\dfrac{|AC|}{|EF|}$

Aşağıdaki ispatları verilenler ışığında detaylı şekilde yapınız. Kullandığınız her teoremi ve kurduğunuz sebep sonuç ilişkilerini mutlaka belirtiniz.

2.



VERİLENLER:

- AC//DF
- $m(\widehat{ABG})=x^\circ$ $m(\widehat{BGE})=z^\circ$ $m(\widehat{GED})=y^\circ$

İSTENEN:

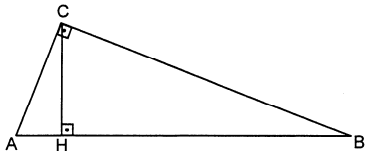
- $z=x+y$

İSPAT:

DİĞER SAYFAYA GEÇİNİZ

Figure C.1 Performance Test on Proof in Geometry Page 1

3.



VERİLENLER:

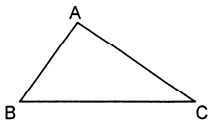
- $\widehat{ABC}$ bir üçgen, $[AC]\perp[CB]$ ve $[AH]\perp[CB]$
- $|CH| = \frac{1}{4}|AB|$ ve $|CA| < |CB|$

İSTENEN:

- $s(\widehat{CBA}) = 15^{\circ}$

İSPAT:

4.



VERİLENLER:

- $\widehat{ABC}$ herhangi bir üçgen

İSTENEN:

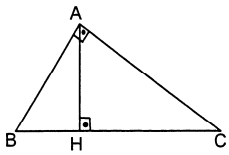
- $|AB| + |AC| > |BC|$

İSPAT:

DİĞER SAYFAYA GEÇİNİZ

Figure C.2 Performance Test on Proof in Geometry Page 2

5.



VERİLENLER:

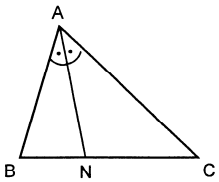
- $\widehat{ABC}$ bir dik üçgen.
- $[AB] \perp [AC]$
- $[AH] \perp [BC]$

İSTENEN:

- $|AB|^2 = |BH| \cdot |BC|$

İSPAT:

6.



VERİLENLER:

- $\widehat{ABC}$ bir üçgen
- [AN] doğru parçası $\widehat{BAC}$ açısının açıortayı

İSTENEN:

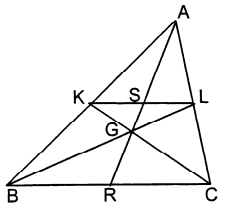
- $\dfrac{|AB|}{|BN|} = \dfrac{|AC|}{|NC|}$

İSPAT:

DİĞER SAYFAYA GEÇİNİZ

Figure C.3 Performance Test on Proof in Geometry Page 3

7.



VERİLENLER:

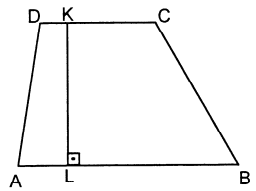
- G noktası $\widehat{ABC}$ üçgeninin ağırlık merkezi
- K,G,C doğrusal
- B,G,L doğrusal
- A,S,G,R doğrusal

İSTENEN:

- $\frac{|SG|}{|AR|} = \frac{1}{6}$

İSPAT:

8.



VERİLENLER:

- ABCD bir yamuk, [KL]⊥[AB]

İSTENEN:

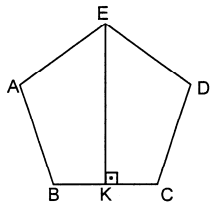
- $Alan(ABCD) = \frac{|AB| + |DC|}{2} \cdot |KL|$

İSPAT:

DİĞER SAYFAYA GEÇİNİZ

Figure C.4 Performance Test on Proof in Geometry Page 4

9.



VERİLENLER:

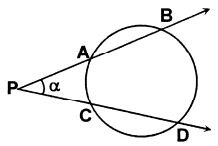
- ABCDE düzgün beşgendir.
- $[EK] \perp [BC]$

İSTENEN:

- $|BK| = |KC|$

İSPAT:

10.



VERİLENLER:

- $|\widehat{AC}| = x^o$, $|\widehat{BD}| = y^o$
- $m(\widehat{APC}) = \alpha^o$

İSTENEN:

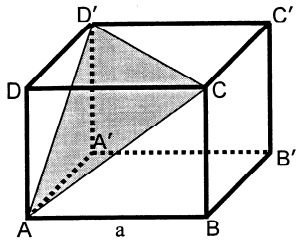
- $\alpha = \dfrac{y-x}{2}$

İSPAT:

DİĞER SAYFAYA GEÇİNİZ.

Figure C.5 Performance Test on Proof in Geometry Page 5

11.



VERİLENLER:

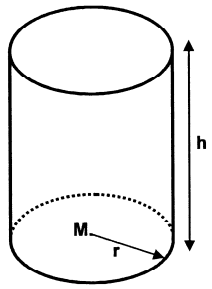
- Yandaki katı cisim kenar uzunluğu a olan bir küptür
- [AC] ABCD'nin köşegeni,
- [CD′] DCC′D′'nin köşegeni,
- [AD′] AA′D′D'nin köşegeni,

İSTENEN:

- $\overset{\triangle}{ACD'}$ üçgeninin alanı $\frac{\sqrt{3}}{2}a^2$

İSPAT:

12



VERİLENLER:

- Şekildeki katı cisim bir dik dairesel silindir.
- M merkezli dairenin yarıçapı r'dir.
- Silindirin yüksekliği h'dir.

İSTENEN:

- Silindirin tüm alanı $2\pi r(h+r)$

İSPAT:

TEST BİTTİ

Figure C.6 Performance Test on Proof in Geometry Page 6

APPENDIX D

3. İSPAT YAPMAYA YÖNELİK TUTUM ÖLÇEĞİ

Adınız Soyadınız : .....

Okulunuzun İsmi : .....

Sınıfınız : .....

YÖNERGE: Bu ölçek **geometri anlatılan derslerde** ispat yapma ile ilgili tutum cümleleri içermektedir. Bu cümlelerin doğru yada yanlış cevapları bulunmamaktadır. Yalnızca sizin doğru bulduğunuz cevaplar doğru kabul edilmektedir. Mümkün olduğunca, yaşadıklarınızı düşünerek karar veriniz. Bu cümleler, ifade edilen düşüncelere sizin ne derece katılıp katılmadığınızı belirtmeniz için "Kesinlikle Katılıyorum", "Katılıyorum", "Kararsızım", "Katılmıyorum" ve "Hiç Katılmıyorum" seçenekleri vardır. Örneğin, eğer cümlede belirtilen düşünceye katılmıyorsanız, ne derece katılmadığınızı göstermek için; düşünceye sadece katılmamanız durumunda "katılmıyorum" seçeneğini, kesinlikle karşı olmanız durumunda "Hiç katılmıyorum"seçeneğini işaretleyiniz. Fakat, ifade edilen düşünce hakkında olumlu ya da olumsuz bir görüş belirtmiyorsanız, yani, kararsızsanız "kararsızım" seçeneğini işaretleyiniz. Hiçbir cümlede fazla zaman kaybetmeden hızlı fakat dikkatli okuyarak hiç bir cümleyi boş bırakmadan samimi olarak kendi durumunuza uygun olan yuvarlağı işaretleyiniz.

Bu bilgiler gizli tutulacaktır. Yalnız siz ve araştırmacı tarafından bilinecektir. Bu araştırma dışında başka bir amaçla kullanılmayacaktır.

Yardımlarınız için çok teşekkürler.

	Kesinlikle Katılıyorum	Katılıyorum	Kararsızım	Katılmıyorum	Hiç Katılmıyorum
1. İspat yapabilme konusunda hiç endişe duymam.	O	O	O	O	O
2. İspat yapmakta zorlanırım.	O	O	O	O	O
3. Derste ispat yapılınca ders daha eğlenceli oluyor.	O	O	O	O	O
4. İspat ile ilgili ileri düzeyde bilgi edinmek istemem.	O	O	O	O	O
5. İspat düşünme yeteneğimi geliştirmez.	O	O	O	O	O
6. Ders çalışırken teoremlerin ispatlarını incelemekten hoşlanırım.	O	O	O	O	O
7. Sınavda ispat sorularından korkmam.	O	O	O	O	O
8. İspat yapmak konunun iyice öğrenilmesini sağlar.	O	O	O	O	O
9. Kendimi mutlu hissetmem için ispat yapmada başarılı olmam gerekir.	O	O	O	O	O
10. İspat ile ilgili sorularını cevaplarken rahatımdır.	O	O	O	O	O
11. İspat yapmak zihinsel gelişimime olumlu katkıda bulunur.	O	O	O	O	O
12. İspat yapmak sıkıcıdır.	O	O	O	O	O
13. İspat yaparken aklıma birşey gelmez.	O	O	O	O	O
14. İspat yapmasını kolayca öğrenirim.	O	O	O	O	O

ARKA SAYFAYA GEÇİNİZ.

Figure D.1 Proof Attitude Scale in Geometry Page 1.

	Kesinlikle Katılıyorum	Katılıyorum	Kararsızım	Katılmıyorum	Hiç Katılmıyorum
15. İspat yapmayı öğrenmek gereksizdir.	O	O	O	O	O
16. İspat yapabilmek beni gururlandırır.	O	O	O	O	O
17. İspat yapmaktan korkmam.	O	O	O	O	O
18. İspat yaparken değişik yaklaşımlar kullanabilirim.	O	O	O	O	O
19. İspatlanmış teoremlerin ispatını öğrenmenin bir faydası yoktur.	O	O	O	O	O
20. İspat yapmayı severim.	O	O	O	O	O
21. İspat yapınca kendime güvenim artıyor.	O	O	O	O	O
22. İspat yaparken bir probleme farklı açılardan bakmayı öğrenmemi sağlar.	O	O	O	O	O
23. İspat yapmak ilginç değildir.	O	O	O	O	O
24. İspat yapmak öğretilmelidir.	O	O	O	O	O
25. İspat yapmayı öğrenmemin gerçek yaşamıma olumlu bir katkıda bulunmaz.	O	O	O	O	O
26. İspat yapma yeteneğimi geliştirmek isterim.	O	O	O	O	O
27. İspat ile ilgili tartışmalarda görüşümü destekleyen güçlü kanıtlar bulamam.	O	O	O	O	O
28. İspat yapmak bende ders çalışma isteği uyandırır.	O	O	O	O	O
29. İspat yapmayı öğrenmekten hoşlanmam.	O	O	O	O	O
30. İspat yapmakta kendime güvenirim.	O	O	O	O	O
31. Gelecekteki hayatımda ispat yapma yeteneğine ihtiyacım olmayacaktır.	O	O	O	O	O
32. İspat yapmaktan çekinirim.	O	O	O	O	O
33. Kimseden yardım almadan ispat yapmayı başarabilirim.	O	O	O	O	O
34. Derste ispat yapmak derse olan ilgimi azaltıyor.	O	O	O	O	O
35. İspat yapma konusunda daha çok bilgi edinmek isterim.	O	O	O	O	O
36. Sınavda ispat sorulması gereklidir.	O	O	O	O	O
37. İspat yapmakta yeteneksizimdir.	O	O	O	O	O
38. İspat yapmayı öğrenmek bana çekici gelir.	O	O	O	O	O
39. İspatlara çalışmak için zaman ayırmak istemem.	O	O	O	O	O

Figure D.2 Proof Attitude Scale in Geometry Page 2.