

PROFIT ORIENTED DISASSEMBLY LINE BALANCING

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Approval of the Graduate School of Natural and Applied Sciences

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## **ABSTRACT**

### **PROFIT ORIENTED DISASSEMBLY LINE BALANCING**

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In this study, we deal with the profit oriented partial disassembly line balancing problem which seeks a feasible assignment of selected disassembly tasks to stations such that the precedence relations among the tasks are satisfied and the profit is maximized. We consider two versions of this problem. In the *profit maximization per cycle problem* (PC), we maximize the profit for a single disassembly cycle given the task times and costs, part revenues and demands and station costs. We propose a heuristic solution approach for PC based on the liner programming relaxation of our mixed integer programming formulation. In the *profit maximization over the planning horizon problem* (PH), the planning horizon is divided into time zones each of which may have a different disassembly rate and a different line balance. We also incorporate other issues such as finite supply of discarded product, subassembly and released part inventories availability, and smoothing of the number of stations across the zones. PH is decomposed into a number of successive per cycle problems, which are solved by a similar heuristic approach. Computational analysis is conducted for both problems and results are reported.

Keywords: Profit Oriented Partial Disassembly, Line Balancing, Remanufacturing.

## ÖZ

### KAR AMAÇLI DEMONTAJ HATLARININ DENGELENMESİ

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Bu çalışmada kar amaçlı kısmi demontaj hatlarının dengelenmesi problemi incelenmektedir. Bu problemde seçilen demontaj operasyonları sıralı bir istasyon dizisine atanırken, operasyonların öncelik sırası sağlanıp kar enbüyüklenmektedir. Bu amaçla problemin iki versiyonu incelenmektedir. *Çevrim başına karın enbüyüklenmesi probleminde (PC)*, verilen demontaj operasyon süreleri ve maliyetleri, parça talepleri ve gelirleri, ve istasyon maliyetleri doğrultusunda, tek bir demontaj çevrimi için kar enbüyüklenmektedir. PC problemi için karmaşık tam sayılı programlama formülasyonumuzu baz alan sezgisel bir çözüm yöntemi önerilmektedir. *Tüm planlama dönemindeki karın enbüyüklenmesi probleminde (PH)*, planlama dönemi her biri kendi hat dengesi ve çevrim süresi olan ardışık zaman aralıklarına bölünmektedir. Demontaj sistemlerinin kendine has doğasını yansıtan sökülecek ürün sayısının kısıtlı olması, sistemde kısmi olarak sökülmüş ürünlerin varlığı ve farklı zaman aralıklarda kullanılan istasyon sayılarının dengelenmesi gibi konular da problem tanımında kapsamaktadır. PH problemi birbirini takip eden çevrim başına karı enbüyükleme problemlerine ayrıştırılmakta ve benzer sezgisel yöntemler kullanılarak çözülmektedir. Her iki problem için de deneysel analizler yapılmakta ve sonuçları raporlanmaktadır.

Anahtar Kelimeler: Kar Amaçlı Kısmi Demontaj, Hat Dengelenmesi, Yeniden Üretim.

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1. MOTIVATION**

Environmental problems related with waste management and disposal of discarded products have led many countries to take legislative action to improve reuse, recycling and other forms of recovery. Recently, the European Parliament and the Council of the European Union have published the Waste Electric and Electronic Equipment (WEEE) Directive, which has come into effect on February 13, 2003. The Directive aims at the prevention of WEEE, facilitation of reuse, recycling and other forms of recovery so as to reduce the disposal of waste, and improve environmental performances of producers, distributors, consumers and especially of the operators who are directly involved in the treatment of WEEE (European Parliament and Council of European Union, 2003). The recovery of materials and products is not only ecologically necessary and driven by legislation but also economically challenging. De Brito and Dekker (2004) claim that direct economic gains can be achieved by reducing the use of “virgin” raw materials, decreasing disposal cost and adding value through recovery. For example, in Europe the disposal costs account to 12.5% of direct production costs for refrigerators and freezers (Ayres *et al.*, 1997 based on the study of Steinhilper, 1995). Only in Europe, the market for industrial recycling of the electronic products had a volume of \$144



million in 1995, and the volume was expected to increase to \$419 million by the year 2002 (Wiendahl and Brückner, 1999). Recently, in the United States the total value of the returned products was projected to have a yearly volume of \$100 billion (Stock *et al.*, 2004). There are also indirect economic gains involved with recovery of materials and products. These include market protection, green image and improved customer/supplier relations (de Brito and Dekker, 2004).

The WEEE Directive applies to electrical and electronic equipment clustered in ten categories. A non-exhaustive list of the products that are covered by the Directive includes refrigerators, washing machines, coffee machines, computers, printers, copiers, telephones, TV sets, electric car racing sets and heating regulators. The obligations forced by the Directive apply to products and producers irrespective of the selling technique including online sales. The “producers” who are responsible for WEEE are broadly defined as all companies that sell products within the scope of the WEEE Directive in the European Union. The producers are required to ensure that sufficient collection facilities are in place, treatment and recovery facilities are available, recovery and recycling targets are met, information on reuse and treatment of their products is made available. They are also pressured to implement design changes to enable reuse and recycling and to cease using hazardous materials such as lead, mercury and cadmium. The Directive defines the set of activities to be performed in treatment and recovery facilities as “depollution, disassembly, shredding, recovery, or preparation for disposal and any other operation carried out for the recovery and/or the disposal of the WEEE” (European Parliament and Council of European Union, 2003).

Disassembly is a major activity performed in treatment and recovery facilities. Disassembly is defined as a systematic method of sorting out a product into its constituent parts and

subassemblies (Gupta and Taleb, 1994). Disassembly can be *complete* if the product is fully disassembled or *partial* if only some parts and subassemblies are removed (Güngör and Gupta, 1999). Technical constraints such as irreversible connections and economic constraints such as the revenues obtained from recovered parts being lower than the disassembly costs (Lambert, 2002; Chen *et al.*, 1993) can hinder the realization of complete disassembly. Hence, *selective disassembly* that aims at unraveling of valuable and/or hazardous materials has been introduced as a partial disassembly process (Lambert, 1999 and 2002).

Due to its critical role in recovery of products and materials, disassembly has recently become an active research area. In the disassembly literature, most of the studies mention the differences in physical and operational characteristics of disassembly and assembly, “even though approaching disassembly as the reverse of assembly may sound reasonable” (Homem de Mello and Sanderson, 1990; Gupta and Taleb, 1994; Brennan *et al.*, 1994; Kochan, 1995; Penev, 1996; Tani and Güner, 1997; Wiendahl and Brückner, 1999; Güngör and Gupta, 1999, 2001a and 2001b; Lambert, 1999, 2002 and 2003; Dini *et al.*, 2001). Güngör and Gupta (1999) note that due to such differences it might not be appropriate to use the techniques and methodologies derived for assembly planning “as is” and hence they state the need for “new techniques and methodologies to specifically address disassembly planning”. Consequently, disassembly process planning and design of disassembly systems have become two frequently studied topics in the disassembly literature (Cui and Foressberg, 2003). Although there are many studies that explore disassembly systems, Güngör and Gupta (2002) speak out the growing need for designing and improving disassembly systems that optimize use of resources such as time, labor and money.

The majority of today's disassembly systems consist of a single workstation (Das and Caudill, 1999) where selective disassembly is carried out manually to retrieve highest value components or bulk parts that are easily recycled (Das *et al.*, 2000). Based on a 1999 study, it is reported that (1) more than 58% reuse and recycling rates are achieved for products such as washing machines, computers, telephones, kettles and refrigerators, (2) these activities are profitable, and (3) increasing the current reuse and recycling rates to the target levels set by the WEEE Directive is expected to increase costs (Commission of the European Communities, 2000).

Referring to the study by Michalkowski (1997), Wiendahl *et al.* (1998) report the possibility of improving the efficiency of a disassembly system that consists of single workstations up to 70% via interlinking of workstations. Das and Caudill (1999) foresee several benefits of high volume disassembly lines. These include achieving economies of scale and division of labor, which might lead to lower labor costs, greater degree of disassembly, and retrieval of a wider range of parts and materials. Thus, academicians propose disassembly lines as one of the settings in which disassembly can be performed efficiently despite their low flexibility, because they can yield high productivity rates and are suitable for automation (Wiendahl *et al.*, 1998; Das and Caudill, 1999; Güngör and Gupta, 2001b and 2002).

Penev (1996) points out the fact that the products that are disassembled today may have been produced 15 or 20 years ago. At that time, future use or disassembly of the products were probably not a concern. As the recyclability of discarded products increases, the automation of disassembly operations is expected to become feasible, which will lead to design and use of more disassembly lines. Based on a study by Boks and Tempelman (1998), Cui and Forsberg (2003) report that a breakthrough on the technical feasibility of full automation for electronic

products might be expected by 2010, but fully automated disassembly of brown and white goods might not be economically feasible until 2020.

Although, the majority of today's disassembly facilities consist of a single workstation, there are various companies that use disassembly lines. Zerlegezentrum Gruenborich (Germany) has different lines for disassembly of refrigerators, recovery of oil from radiators, recycling of TV tubes, recycling of asbestos containing apparatus, and processing of electronic waste (de Ron and Penev, 1995). Mirec BV (the Netherlands), which is a subsidiary of Philips, has a dismantling line for TV sets (de Ron and Penev, 1995). AVR and Prozon (the Netherlands) have built dismantling lines for refrigerators and freezers (de Ron and Penev, 1995). BMW (Germany) has a disassembly line where the disassembly operations are performed in the reverse order of assembly operations. Kansai Recycling Systems (Japan), which is a joint venture of Sharp Corporation and Mitsubishi Materials Corporation along with five other electronics companies, operates four disassembly lines for air conditioners, TV sets, refrigerators and washing machines.

## **1.2. STATEMENT OF THE PROBLEM**

In its simplest form, the disassembly process consists of a finite set of disassembly tasks some of which result in removal of parts that may have an associated demand. Disassembly tasks are often characterized by task costs, which may have a fixed component and a component that varies depending on the task time. Released parts have associated revenue figures that can be attained only if there is demand for them. While all tasks must be completed to produce the end product in an assembly line, the disassembly process does not have to be complete. In the presence of task costs and part revenues, discarded products should be disassembled to the extent it is profitable to do so. That is, disassembly is profit oriented and hence can be selective

or partial. Lambert and Gupta (2002) state that discarded products are to be disassembled in a cost effective manner where the target is to recover parts and materials, and there are no limitations on the demand satisfaction other than the market prices of the resulting parts and materials.

Performing a disassembly task may require certain equipment or machines and/or certain operator skills. Technological and physical conditions define precedence relations among the disassembly tasks. While assembly tasks are typically characterized by AND precedence relations, various types of precedence relations are involved in disassembly, such as *AND precedence*, *OR precedence* and *OR successor* relations. The additional precedence relations are mostly due to physical restrictions or processing alternatives.

Similar to an assembly line, a disassembly line is made up of an ordered sequence of stations often connected by some mechanical material handling equipment. Discarded products enter the disassembly line and move to downstream stations. Disassembly lines can be paced or unpaced. In a *paced disassembly line*, a set of disassembly tasks is performed at each station within the cycle time common to all stations. As the variability in task times is high, unpaced disassembly lines can be designed. In an *unpaced disassembly line*, all stations are allowed to operate at their own pace, and therefore subassemblies may wait to step in the downstream station and stations may become idle as they wait the next subassembly from the upstream station. Buffers are placed between stations to partially overcome arising difficulties (Becker and Scholl, 2003). Güngör and Gupta (2002) remark upon the advantages of paced lines over unpaced lines as incorporating less work in process, requiring less space, and having less chances of causing bottlenecks if properly designed. They also note that in order “to take advantage of the positive aspects of paced lines, its speed can be dynamically modified

throughout the entire disassembly process to minimize the negative effects of variability (including variability in demand)".

The basic *disassembly line balancing problem* (DLBP) can be stated as the assignment of disassembly tasks to an ordered sequence of stations such that various forms of precedence relations are satisfied and some measure of effectiveness is optimized. Due to long term effect of the balancing decisions, the objective has to be chosen carefully considering the strategic goals of the enterprise (Becker and Scholl, 2003). Commonly studied objectives include minimizing number of stations given cycle time, maximizing production rate (equivalently minimizing cycle time) given number of stations, maximizing the line efficiency (directly depends on the number of stations and cycle time, cost minimization and profit maximization. Profit seeking nature of disassembly systems should be taken into consideration in choosing the objective for DLBP.

In assembly line balancing problem (ALBP) the demanded entity is the end product whose demand must be fulfilled and therefore determines the production rate and hence the cycle time. In DLBP, however, several parts and subassemblies can be demanded in different quantities, implying different disassembly rates. Furthermore, when profitability is of concern, not all demand has to be met and not all tasks need to be performed. Therefore, determination of disassembly rate and the cycle time is not straightforward, and a new basis is needed to determine them.

Based on the characteristics of DLBP, we define the profit oriented partial DLBP as simultaneously answering the following questions.

§ Which tasks should we perform and which parts should we release?

- § How many stations should we open and how should we assign selected tasks to these stations such that various forms of precedence relations (such as AND precedence, OR precedence, and OR successor relations) are satisfied?
- § How should we decide on the cycle time?

Güngör and Gupta (2001b) provide a comparison of assembly and disassembly lines in terms of both technical and operational features. Based on their comparison, they conclude that special techniques need to be developed for the improvement of disassembly lines. They also note that the vast body of knowledge and experience developed in the ALBP literature may provide useful guidelines. However, differences in technical and operational features of assembly and disassembly lines imply that DLBP deserves special attention.

### **1.3. PURPOSE AND CONTRIBUTION OF THE DISSERTATION**

The main purpose of this study is to formulate and solve the DLBP with profit maximization objective under partial disassembly. We assume a single product is to be disassembled on a paced disassembly line. Although unpaced disassembly lines are more suitable when task time variability is high as is in disassembly, we consider paced disassembly lines for simplicity. The nature of disassembly requires line designs with flexibility in mind, allowing restructuring on a continuous basis as the input flow of discarded products and the demand for released parts and materials change. In such an environment, the problem is not to make a single-shot decision for a complex problem, but to make it repeatedly. Therefore, we aim at finding high quality solutions to our problems through fast solution procedures.

We characterize and formulate various forms of precedence relations specific to disassembly and provide a mixed integer programming formulation of the problem. In our formulation both

the number of stations and the cycle time are decision variables. Developing solution procedures for line balancing problems, which simultaneously minimize number of stations and cycle time, may require generation of all feasible combinations. This by itself is computationally intractable even when the set of tasks to be performed is known. The addition of task selection under partial disassembly further increases the complexity of the problem.

We consider two versions of DLBP: the *profit maximization per cycle problem* (PC) and the *profit maximization over the planning horizon problem* (PH). In PC, we maximize the profit for a single disassembly cycle in which a single discarded product is disassembled to a certain extent. In partial DLBP, revenue is obtained as long as there is demand for the released parts. When demand is consumed for some parts, performing some tasks may no longer be profitable. Hence, using the same PC solution throughout the planning horizon does not necessarily maximize the profit. Therefore, we define the second version of the problem. In PH, the planning horizon is divided into time zones each of which may have a different disassembly rate (cycle time) and a different line balance. We also incorporate other issues so as to reflect more realistic features of disassembly systems such as incurring inventory holding cost for parts released in excess of their demand, considering finite supply of discarded products, allowing accumulation of subassemblies in work-in-process (WIP) or use of previously accumulated WIP for further disassembly, and smoothing out the number of station used across different time zones.

The contributions of this study can be summarized as follows.

§ Disassembly precedence relations (other than AND precedence) are characterized and their mathematical formulations are provided.



- § Our PC formulation determines the tasks to be performed and the parts to be released under the objective of profit maximization. It also simultaneously decides on the number of stations required and the cycle time while balancing the partial disassembly line. To the best of our knowledge this is the first partial DLBP formulation in the literature.
- § An upper bounding scheme and a heuristic solution procedure are developed for PC.
- § We also define and formulate the PH problem for the first time by extending the PC formulation. PH takes the time dimension into consideration by dividing the planning horizon into time zones. It also incorporates finite supply of discarded products, subassembly and released part inventories.
- § Heuristic solution procedures for PH are developed. Besides allowing the number of stations to vary in different time zones, the effect of fixing the number of stations across the zones is also explored.

As a final summary we present a comparison of the current DLBP research and our research agenda in Table 1.1.

**Table 1.1** Comparison of current research and our research

	<b>Current Research</b>	<b>Our Research</b>
DLBP considers	PC	PC and PH
Objective	Min. number of stations Min. flow time	Max. profit
Demand	All demand is met	Profitable demand is met
Disassembly	Complete	Partial
Supply	Infinite	Infinite or finite
Subassembly Inventory	N/A	Included
Released Part Inventory	N/A	Included
Cycle Time	Given	Decision variable
Precedence Relations Basis	DPM	DPM and AND/OR graph
Mathematical formulation	N/A	Provided
Experimentation	N/A	Conducted

The next chapter starts with a brief description of the role of disassembly in recovery of products and materials. After providing a comparison of assembly and disassembly systems, a comparison of settings under which disassembly operations are performed is given. The DLBP problem is defined and the relevant literature is reviewed.

In Chapter 3, different precedence relation representations used in the disassembly literature are evaluated. A representation scheme that embraces them and unifies them into a simple representation suitable for DLBP is proposed. Examples taken from the literature are used to illustrate the proposed representation scheme. Mathematical representations of the precedence constraints are also given.

In Chapter 4, the PC problem environment is defined. A mixed integer mathematical programming formulation of PC is presented. The optimal solution of an instance is analyzed to explore the nature of the problem and state our motivation for developing a heuristic solution procedure. A mathematical programming based solution procedure and an upper bounding scheme are described. The results of a computational analysis are reported.

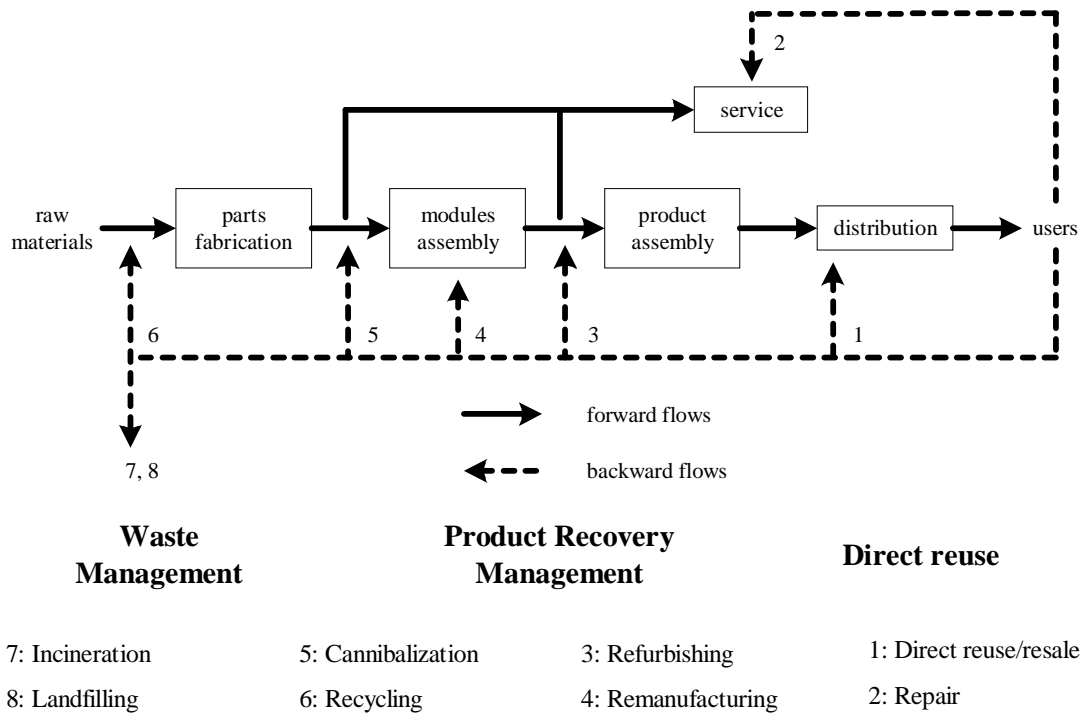
In Chapter 5, the PH problem is defined. Inventory valuation in disassembly environments is discussed and an inventory valuation scheme based on value added is proposed. The assumptions concerning the PH environment are described. A mathematical programming formulation is provided. Heuristic solution procedures are presented. The results of a computational analysis are reported.

In Chapter 6, we discuss our conclusions and present further research directions.

## CHAPTER 2

### DISASSEMBLY IN RECOVERY OF MATERIALS AND PRODUCTS

*Recovery* is the organization and execution of all activities associated with the reuse of discarded products and materials. *Product recovery management* covers administration of all returned and discarded products, parts, subassemblies and materials “that fall under the responsibility of a manufacturing company”. The objective is to recover the economic and ecological values as much as possible, thereby minimizing the amount of waste landfilled (Thierry *et al.*, 1995) and leading to energy savings (Guide *et al.* 2000). Consequently, the aim may be restated as closing the use-of-materials cycle in the supply chain (Guide *et al.* 2000). Figure 2.1 illustrates a closed-loop supply chain with product recovery and waste management activities. Recently van Nunen and Zuidwijk (2004) note that the high variety of closed-loop supply chains may include returns of new unused products due to commercial reasons (within up to 90 days of sale). Blackburn *et al.* (2004) analyze closed supply chains for commercial returns and remark upon the necessity of recognizing returned products as perishable assets. Perishability of the returns implies that their value is lost over time. This time concern demands for increased speed of recovery of products and materials.



**Figure 2.1** Closed-loop supply chain (Thierry et al., 1995)

Fleischmann *et al.* (1997) categorize types of products that are recovered in closed-loop supply chains into three, depending on when and why the discarded products are returned.

§ *Consumer goods* involve items such as copiers, refrigerators, computers and cars. Consumer goods are mostly returned at the end of their life cycles. This can be rather long and might imply outdated of the product. Here, another possibility is the return of items after the expiry of a leasing contract. In this case, timing of the return is known in advance.

§ *Rotable spare parts* include items such as machine parts and TV tubes. Rotable spares are returned upon failure of the item or for preventive maintenance; hence they are returned after a longer time and may have some defects.

§ *Packages* involve items such as pallets and bottles. They are returned rather quickly since they are no longer required once their content has been delivered.

## **2.1. RECOVERY OF PRODUCTS AND MATERIALS**

Material and product recovery is carried out mainly due to three reasons:

1. hidden economic value of solid waste,
2. market requirements, and
3. governmental regulations on environmental issues.

Many authors have adapted the categorization of different forms of recovery given in Figure 2.1 (Thierry *et al.*, 1995). In all of these recovery options, discarded products are collected, reprocessed and redistributed. Thierry *et al.* (1995) claim that the primary difference among these options is due to reprocessing. In the cannibalization option, only “a limited set of reusable parts” is recovered. In the repair, refurbishing and remanufacturing options the discarded products are “upgraded in terms of their quality and/or technology”. The degree of the upgrading is the largest in remanufacturing and the least in repair.

Güngör and Gupta (1999) categorize the recovery process into two: *material recovery (recycling)* and *product recovery (remanufacturing)*. In material recovery the discarded product is disassembled to separate and process materials (e.g. perform necessary chemical operations). In product recovery the discarded product is disassembled to clean, sort, replace or repair defective components, recondition, test, reassemble and inspect them.

### **2.1.1. Material Recovery (Recycling)**

Recycling is used to retrieve the material content of the discarded products. The identity and functionality of recycled products and components is lost in recycling. The retrieved materials

are used in producing original parts if the quality of materials is high, or in producing other parts if the quality is poor due to impurities.

Recycling is mainly driven by economic, environmental and regulatory factors. Several recovery facilities have been established to retrieve the economic value of used products. For example, virtually all metals in discarded cars (on average 75% of the weight of a car) are recycled in automobile recycling facilities in developed countries like Germany, the UK, and the United States. Consumer electronics industry is another example for economically driven material recovery process. A typical computer contains precious materials such as gold, silver, palladium and platinum. The recovery of these materials from consumer electronic products requires proper equipment and is generally completed in mass. Besides the recovery of high valued materials, environmental concerns demand the recovery of other materials such as plastics. Regulatory tire recycling is practiced by members of the European Union since 2003. The legislation forces the tire manufacturers to recycle a discarded tire for every new tire they sell (Guide and van Wassenhove, 2002). Currently regulatory electronic recycling is conducted in developed countries such as Germany, the Netherlands, Japan and the United States. In the near future an increase in electronic recycling is expected due to the WEEE Directive.

### **2.1.2. Product Recovery (Remanufacturing)**

Fleischmann *et al.* (1997) define remanufacturing as a process of bringing used products to “as new” condition through some necessary operations such as disassembly, overhaul and replacement. The identity of the used products and their components is retained as much as possible in remanufacturing where the quality standards employed are as rigorous as those for the new products. For example, BMW remanufactures high-value components such as engines, starter motors and alternators. The remanufactured parts are tested according to strict quality

standards to become a BMW Exchange Part. These parts are sold with the same warranty and quality at a price 30-50% cheaper than new parts.

Remanufacturing of existing components has the following added benefits (Bras and McIntosh, 1999):

- § It reduces the company's expenditure on acquiring and producing new components.
- § It reduces energy and matter consumption during manufacturing, besides reducing the material waste and amount landfilled.

In remanufacturing systems, similar to conventional production systems, there are operational, manufacturing, inventory, distribution and marketing related decisions to be made. Highly flexible structures characterize remanufacturing environments. Flexibility is required in order to handle uncertainties that are likely to arise due to several reasons such as condition of used products, arrival time and quantity of used products. In general, the existing methods for conventional production systems cannot be used for remanufacturing systems. For details the reader may refer to the surveys by Guide (2000) and Güngör and Gupta (1999).

### **2.1.3. Common Issues in Recovery of Materials and Products**

All the recovery options involve collection of discarded products and components, reprocessing and redistribution. Hence the common issues can be grouped under three main topics presented below:

- § *Collection issues:* In a product recovery environment, one of the major issues is to collect the discarded items and/or their packages. The collection issues involve transshipment of retired products that originate from multiple sources to a single destination. The high level of difficulty is due to the uncertainties involved in the process such as the quantity

of products to be collected, delivery logistics and placement of collection centers. As a consequence of the difficulty of the problem, most of the studies model the collection process independent of the distribution system of the manufactured products. Based on the current literature, Güngör and Gupta (1999) in their survey conclude that “the collection process is yet to be fully understood” and models that simultaneously incorporate collection and distribution systems need to be developed.

§ *Disassembly*: Lately, disassembly is one of the most actively researched areas in the context of material and product recovery. Various practical and theoretical techniques are being developed for manual and automatic disassembly processes, some of which will be discussed in the next section.

§ *Inventory control and production planning*: Many studies in the area of inventory control, production planning and scheduling in recovery of materials and products utilize the well-known Operations Research (OR) techniques. For several OR applications in environmental management, the reader may refer to the articles by Bloemhof-Ruwaard *et al.* (1995) and Daniel *et al.* (1997). The reader may also refer to Dekker *et al.* (2004) for quantitative models that address production and inventory management in closed-loop supply chains.

## **2.2. DISASSEMBLY**

Different characteristics of the product recovery options defined by Thierry *et al.* (1995) are summarized in Table 2.1. Although the purpose, quality requirements involved and the resulting product vary from one option to another, disassembly is performed in all options though at different levels. Disassembly, due to its crucial role in recovery of materials and products, has recently received a lot of attention in the literature.



**Table 2.1** Comparison of product recovery options (Thierry *et al.*, 1995)

	<b>Purpose</b>	<b>Level of Disassembly</b>	<b>Quality Requirements</b>	<b>Resulting Product</b>
Repair	Return used product to “working order”	To product level	Restore product to “working order”	Some parts fixed or replaced by spares
Refurbishing	Bring products to specified quality	To module level	Inspect all critical modules and upgrade to specified quality level	Some modules repaired/replaced
Remanufacturing	Bring products up to quality standards that are as rigorous as those for new ones	To part level	Inspect all modules and parts and upgrade to “as new” quality	Used and new modules/parts combined into new product
Cannibalization	Recover a limited set of usable parts from used products or components	Selective retrieval of parts	Depends on process in which parts are reused	Some parts reused; remaining product recycled/disposed
Recycling	Reuse materials from used products or components	To material level	High for production of original parts, less for other parts	Materials reused to produce new parts

### 2.2.1. Disassembly Planning and Scheduling

Gupta and Taleb (1994) define disassembly as a systematic method of separating a product into its constituent parts, components, subassemblies or other groupings. Contrary to assembly that has to be fulfilled completely, disassembly usually cannot be performed to full extent (Lambert, 2002). There are economic and technical constraints that necessitate the introduction of *partial disassembly* concept. Irreversible connections impose technical constraints. Disassembly costs’ being disproportional to revenues obtained from recovered parts and materials impose economic constraints (Lambert, 2002). Hence, depending on the product recovery option executed and the technical and economic constraints of the product under consideration, disassembly may be *partial* (product is not fully disassembled) or *complete* (product is fully disassembled). Chen *et al.* (1993), by analyzing the disassembly of an automobile dashboard, demonstrate that complete disassembly is not profitable due to current material recycling

technology and high costs that outweigh the revenues realized from recovered parts and materials.

Lambert (1999) defines *selective disassembly* as nondestructive, reversible disassembly of complex products into less complex subassemblies or single parts. He claims that selective disassembly can be conducted in order to:

- § repair and perform maintenance operations
- § make subassemblies available as service parts
- § remove parts prior to set free other, desired parts
- § make parts available that will be used for recycling
- § increase purity of materials by removing contaminants
- § comply regulations that prescribe removal of definite parts, materials or substances (mainly for environmental and safety reasons)
- § reduce the amount and harmfulness of the residual waste

The disassembly process is strongly affected by the type and quantity of the demand for specific parts and materials (Lambert, 2003). The disassembly process can be *supply* or *demand driven*. In *supply driven disassembly*, the discarded products are to be disassembled in a cost effective manner. The purpose is to recover parts and materials and there are no limitations on the demand satisfaction other than the market prices of the resulting parts and materials. In *demand driven disassembly*, the discarded products are disassembled to fulfill the demand regardless of whether or not it is cost efficient to do so (Lambert and Gupta, 2002).

In general, there are several issues studied in the disassembly literature, such as disassembly planning and scheduling, shop floor scheduling and control, capacity planning, forecasting and

facility layout. Among these, we focus on disassembly planning and scheduling and briefly review the relevant literature here.

The work published on disassembly planning and scheduling has been recently reviewed by Güngör and Gupta (1999), Lee *et al.* (2001) and Lambert (2003). Lee *et al.* (2001) suggest further research directions as well, including the integration of disassembly planning and scheduling.

Lee *et al.* (2001) define *disassembly planning* as a collection of issues including product representation, disassembly sequencing with disassembly level and end-of-life options. Lambert (2003) further classifies the issues studied under this topic into two levels, namely to sequence level (based on product structure) and to detailed level (based on component geometry). Lee *et al.* (2001) define *disassembly scheduling* as the problem of determining the number of used products to disassemble to fulfill the demand for parts and subassemblies.

Güngör and Gupta (1999) further group the research in the *disassembly planning* field in two major areas:

1. *Disassembly leveling*: Here the disassembly leveling problem is defined as “achieving a disassembly level to which the product of interest is disassembled to keep profitability and environmental features of the process at a desired level”. Increasing the disassembly level increases the purity of secondary materials (and the price) but results in longer disassembly times and higher costs (de Ron and Penev, 1995). Therefore, it is crucial to find a balance between the resources invested in the disassembly process and their return. In the literature, most studies aim to find the optimum balance between the resource requirement and the benefit of the disassembly process via cost analysis.

2. *Disassembly process planning*: A disassembly process plan (DPP) is a sequence of disassembly tasks, which begins with a product to be disassembled and terminates in a state where all the parts of interest are extracted (thus it could be either for partial or complete disassembly). The objective of disassembly process planning is to find optimal or near-optimal DPPs, which minimize the cost of disassembly (assuming that a certain level of disassembly is required) or obtain the best cost/benefit ratio for disassembly. The number of alternative DPPs grows exponentially as the number of the components increases in a product (Homem de Mello and Sanderson, 1990). Identifying the ‘best’ disassembly sequence requires a systematic approach operating under a given set of objectives and constraints. This problem is one of the most challenging problems in the field of disassembly planning.

*Disassembly scheduling* is alternatively known as reversed material requirement planning (MRP) in the literature, simply because the problem is a reversed form of the regular MRP (Lee *et al.*, 2001). The disassembly scheduling is the problem of determining the number of used products to disassemble together with the delivery deadline of the used products to fulfill the demand of individual parts and subassemblies with certain objective functions. The objective usually is to minimize the number of root products disassembled. The studies in this field include single product structures as well as multiple product structures with part and material commonality.

Due to the nature of the disassembly planning problem the associated studies consider supply driven disassembly while the disassembly scheduling research covers demand driven disassembly.

In the disassembly literature most of the studies remark why the operational characteristics of disassembly and assembly are different, even though “approaching disassembly as the reverse of assembly may sound reasonable”. As will be discussed in the following section, they claim that, both operational and physical differences between assembly and disassembly imply that the assembly planning knowledge may not be used ‘as is’ for the disassembly planning issues. Thus, they point out the need for new techniques and methodologies to specifically address disassembly planning and scheduling issues.

### **2.2.2. Assembly versus Disassembly**

In this section, we provide a summary of various remarks made in the literature on the differences between assembly and disassembly systems. We summarize these remarks in chronological order.

Contrary to the most recent studies, Homem de Mello and Sanderson (1990) claim that the problem of determining how to assemble a product can be transformed into an equivalent problem of determining how the same product can be disassembled. However, they also point out that not necessarily all assembly operations are reversible. Hence they define each disassembly operation as the logical reverse of the corresponding assembly operation, regardless of whether the disassembly operation is itself feasible or not. Consequently they use this definition to achieve the equivalence of the two problems. They further assume that disassembly operations under consideration refer to reverse of feasible assembly operations. Since assembly and disassembly operations are not necessarily reversible, they claim that there may be two different graphs representing the precedence relations for the same product, one corresponding to assembly and the other to disassembly operations.

Brennan *et al.* (1994) highlight general operational characteristics of disassembly and assembly lines, which are summarized in Table 2.2.

**Table 2.2** Comparison of operational and technical considerations of assembly and disassembly lines (Brennan *et al.*, 1994)

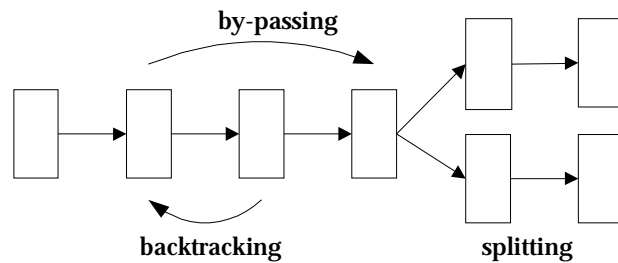
<b>Line Considerations</b>	<b>Assembly Line</b>	<b>Disassembly Line</b>
Demand	Dependent	Dependent
Demand sources	Single	Multiple
Demanded entity	End product	Individual parts/subassemblies
Precedence relations	Yes	Yes
Complexity related to precedence relations	High (includes physical and functional precedence constraints)	Moderate (mostly physical constraints)
Uncertainty related to quality of parts	Low	High
Uncertainty related to quantity of parts	Low	High
Uncertainty related to stations and material handling	Low to moderate	High
Reliability of the stations and material handling	High	Low
Multiple products	Yes	Yes
Flow process	Convergent	Divergent
Line flexibility	Low to moderate	High
Layout alternatives	Multiple	Multiple
Complexity of performance measures	Moderate	High
Known performance measures	Numerous	N/A
Complexity of “between stations inventory”	Moderate	High
Known techniques for optimization	Numerous	None
Problem complexity	NP-Hard	NP-Hard

They state that similarities between assembly and disassembly include dependent demand concepts, which relate to scheduling in discrete parts production system and general assumptions for an assembly system, and these remain unchanged for a disassembly environment. For instance there is a whole class of constraints, which is shared by assembly and disassembly problems, such as order due dates, setup time/cost of changeovers, and existence of precedence relations in routing. They point out that in a disassembly system, there

is a serious inventory planning and control problem, a much more complicated flow process, high degree of uncertainty in the structure and quality of products, and uncertainty factors associated with the reliability of workstations. Among the operational characteristics they compare, we find the following two issues as critical:

§ Although a product may have a specified number of tasks to be performed in both systems, in assembly there is a deterministic routing since all tasks have to be performed. However, disassembly can be partial or complete, or there might be failures due to several reasons. Such failures may cause the used product to backtrack to one of the upstream workstations or bypass downstream workstation(s). Figure 2.2 illustrates the stochastic routing in disassembly.

§ Operation time variability in assembly systems is in a low to moderate range. However it is very high in disassembly. Guide (2000) reports that even though similar units are disassembled, there are very large variances with a coefficient of variation as high as five.



**Figure 2.2** Sequence of steps in disassembly (Wiendahl *et al.*, 1998)

Kochan (1995) describes the following three factors that differentiate assembly from disassembly:

- § Disassembly facilities have to be highly flexible because the products originate from numerous manufacturers and they may have undergone different treatment by their users during their life cycle.
- § Lack of information about the product, its shape, material structure and position of joints means that there is a need for sensors as well as new methods for process planning and control.
- § All machinery must be designed and built with a high degree of robustness because of the presence of fluids and dirt that are likely to be encountered during the dismantling process.

Penev (1996) argues that assembly and disassembly processes complement each other. He proposes to clearly distinguish and investigate their common and distinct features. He believes this will reduce the effort exerted for the development of an “advanced disassembly process” by following the procedures that have been already introduced in assembly and can readily be applied to both processes. He further believes that by using the knowledge and experience gained in assembly, distinct aspects of disassembly can be investigated and its importance can be clarified. He claims that such an approach would shorten the time required to develop possible “advanced disassembly processes”. He considers the particular nature of input flow as the most obvious difference between disassembly and assembly. Since the discarded products contain various components that have been joined together by means of different assembly techniques, the disassembly process becomes very complicated as the input flow of the discarded products cannot be changed. However in assembly, the input flow can be deliberately chosen to simplify the assembly process during the design phase.



Tani and Güner (1997) compare assembly and disassembly to describe the identifiers of the disassembly process. From the experiments on manual disassembly of electric appliances such as washing machines and refrigerators, they observe that typically a human operator can disassemble them without referring to any instructions, manuals, or drawings. Hence they claim that:

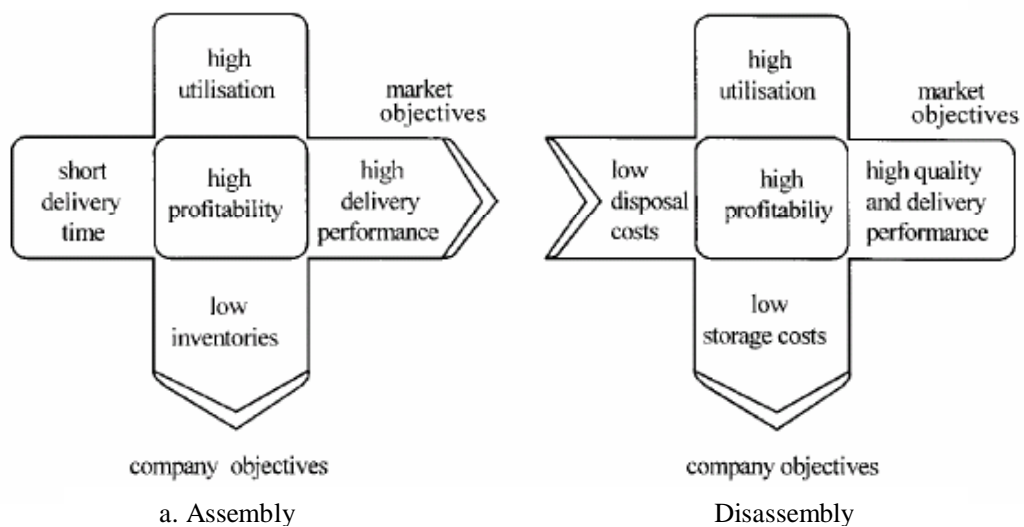
- § Disassembly of a product can be performed by finding a natural and easier way whereas in assembly, the process needs to be highly optimized and the sequence of assembling parts to form a product must be clearly defined.
- § Although the actual mechanism of disassembly is simpler than that of assembly, the operational scope of disassembly is much more complex than that of assembly.

Lambert (1999) claims that disassembly process is less precise especially when recovery of materials is targeted. He also considers “disassembly is not equivalent to inverse assembly” since disassembly might be conducted partially due technical and economic constraints (Lambert, 2002).

Wiendahl and Brückner (1999) believe that assembly and disassembly differ in their objectives as well as in their degree of unpredictability. Their demonstration of objective systems of assembly and disassembly as market and company objectives is presented in Figure 2.3. They make the following remarks regarding the two objective systems:

- § In assembly, customers ask for a fast flow of their products through the company and they demand on time deliveries. High utilization and low inventories are major yet conflicting objectives for the company.
- § In disassembly, delivery time and punctuality play less important roles for the customers. Moreover the disassembly system has two markets, the first one being the market where

the used products are obtained and the second one being the market where the disassembled components and materials are sold. The first market asks for cheap recycling while the second asks for good quality products. In disassembly, high utilization and low inventories are the major set of conflicting company objectives. The main difference from assembly is that, the capital is tied up in the used products as well as in the storage of large volumes of disassembled parts and materials with low value.



**Figure 2.3** Objective systems in planning and control for assembly and disassembly  
(Wiendahl and Brückner, 1999)

Güngör and Gupta (2001a) point out the following distinct features of disassembly:

- § Some joining release mechanisms may not be accessible for disassembly in the order in which they were assembled.
- § Joining techniques used during assembly may be irreversible.
- § Disassembly sequence planning should incorporate the unique characteristic of disassembly, due to uncertainty in structure and quality of returned products.

§ Reversing assembly plans may be inefficient and unpractical for the purpose of disassembly.

Dini *et al.* (2001) focus mainly on profitability of disassembly systems and make the following additional remarks on why reversing an assembly sequence plan may not be feasible for disassembly:

§ Due to damaged parts and connections, a discarded product may vary significantly from a new one.

§ During the disassembly process, destructive operations can be profitably performed (flame cutting, drilling of screw heads, etc.).

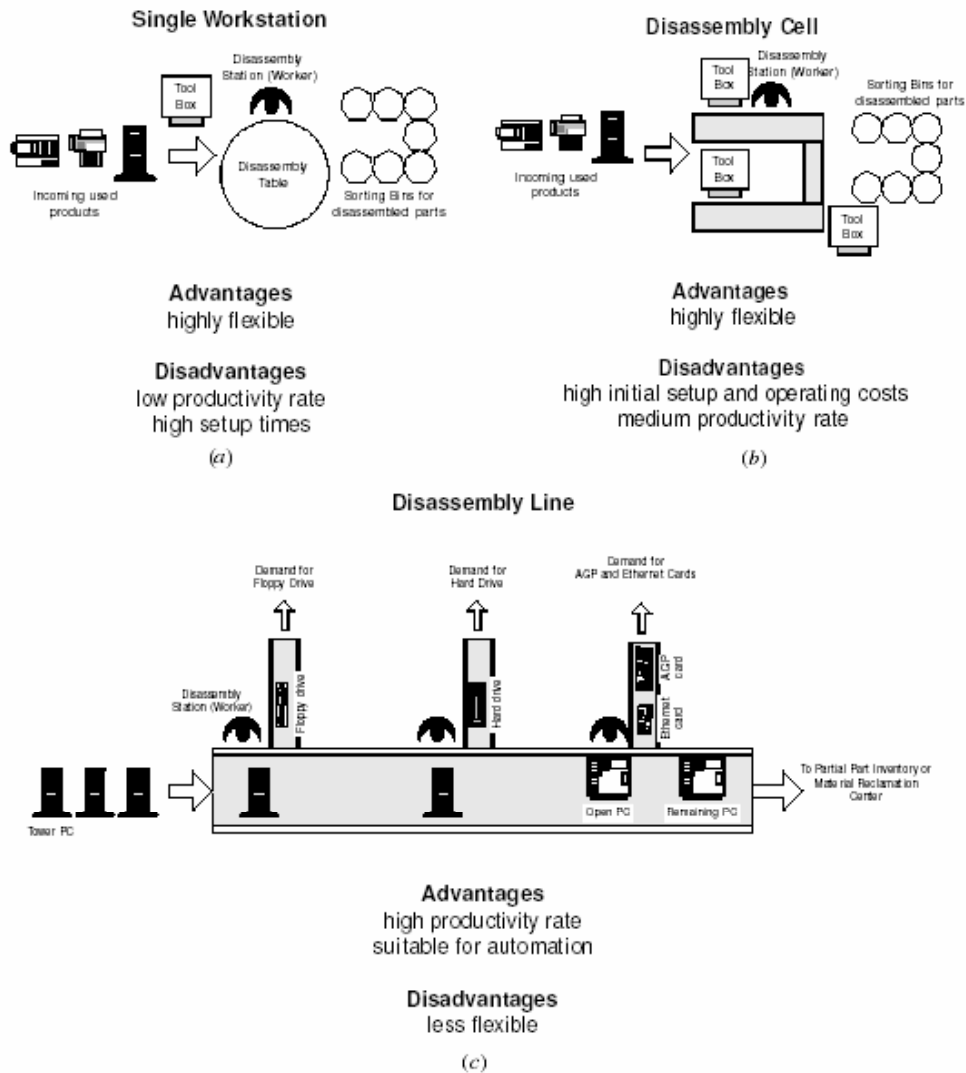
§ Reversing assembly plans may not be economically profitable.

Contrary to all the other summarized studies, Ketzenberg *et al.* (2004) assume a disassembly sequence can be obtained by reversing the assembly sequence. They verify their assumption by citing the study of Nasr *et al.* (1998) and reporting that more than fifty percent of the disassembly sequences are reversed assembly sequences.

### **2.2.3. Disassembly Systems**

Because disassembly is an unpredictable process, it requires general and flexible architectures that are capable of managing unknown processing times, probabilities of task failures, changing routing of a product and multiple products with a lot size of one. To fulfill such requirements, Wiendahl *et al.* (1998) propose three types of architectures that can be borrowed from the assembly process: the single workstation, the disassembly cell, and the disassembly line. As can be seen from Figure 2.4, a single workstation or a disassembly cell provides the highest

flexibility in sorting the parts according to their quantity and quality, however a disassembly line provides the highest productivity.



**Figure 2.4** Alternative architectures of disassembly systems (Güngör and Gupta, 1999)

It is not possible to argue in general which of these types is the most suitable for disassembly since each type has its special characteristics, strengths and weaknesses. Hence for each specific problem, the most appropriate type must be selected. Wiendahl *et al.* (1998) propose

the following criteria for such a selection procedure: flexibility, space requirement, productivity, costs, possibility of automation, amount of different product types to be disassembled, and amount of products to be disassembled per year.

The majority of today's disassembly plants consist of single workstations. Based on a 1999 study, it is reported that (1) more than 58% reuse and recycling rates are achieved for products such as washing machines, computers, telephones, kettles and refrigerators, (2) these activities are profitable, and (3) increasing the current reuse and recycling rates to the target levels set by the WEEE Directive is expected to increase costs (Commission of the European Communities, 2000).

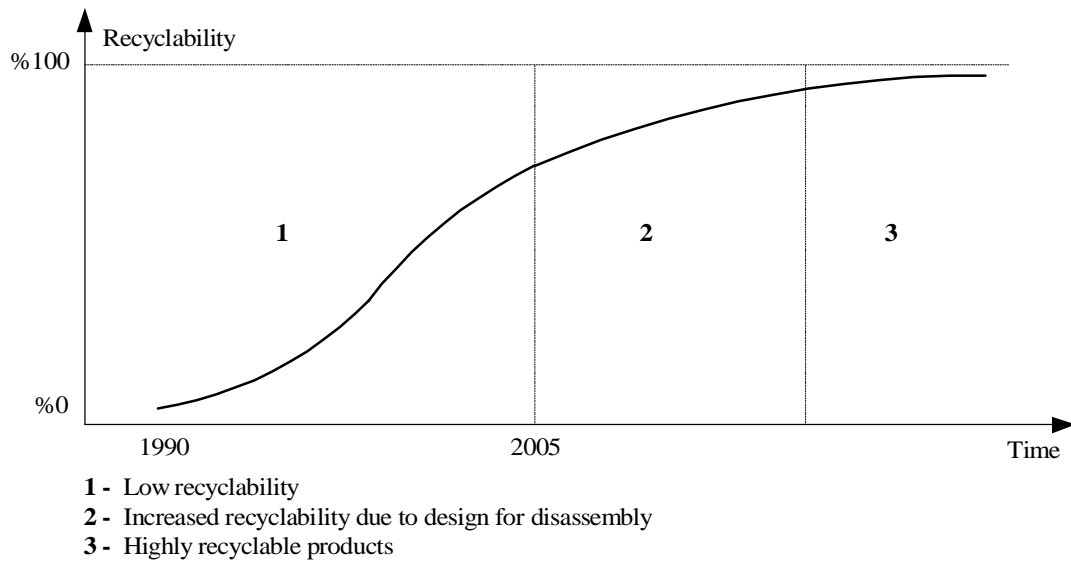
Referring to the study by Michalkowski (1997), Wiendahl *et al.* (1998) report the possibility of improving the system's efficiency up to 70% via interlinking of workstations. Hence, the disassembly line setting seems promising for disassembly of large products or small products in large quantities with few different product types to be disassembled.

Hence Das and Caudill (1999) propose high volume disassembly lines since they foresee the following benefits:

- § Economies of scale can be achieved.
- § Greater degree of disassembly (which leads to smaller carcass) can be attained.
- § Assembly technologies can be used more readily.
- § Division of labor, which leads to lower labor costs and better accessibility, is possible.
- § Analytical understanding of the entire disassembly operation becomes possible.
- § A wider range of parts and materials can potentially be reclaimed.

Furthermore, Tani and Güner (1997) claim that the disassembly line is the best choice for automated disassembly process, a feature that will be essential in the future disassembly systems.

Penev (1996) points out the fact that the products, which are available for recycling at the moment, may have been produced 15 to 20 years ago. At that time, future use and disassembly were probably not a concern. Today, the absence of such concerns causes enormous environmental problems as well as the loss of value added to products and materials that can be reused for different purposes. Figure 2.5 presents the product recovery phases of used products in terms of recyclability. As recyclability of the discarded products increases, automation of disassembly operations is possible, and hence will facilitate design and use of disassembly lines.



**Figure 2.5** Product recovery phases of used products (Penev, 1996)

Based on a study by Boks and Tempelman (1998), Cui and Forsberg (2003) report that a breakthrough on the technical feasibility of full automation for electronic products might be

expected by 2010. However, they also note that fully automated disassembly of brown and white goods might not be economically feasible until 2020.

Even though the products that have been disassembled during the last decade are not highly recyclable, there are still various companies that use disassembly lines to perform disassembly operations. Some examples taken from the literature include,

- § Zerlegezentrum Gruenbroich (Germany) has different lines for disassembly of products such as refrigerators, recovery of oil from radiators, recycling of TV tubes, recycling of asbestos containing apparatus, and processing of electronic waste (de Ron and Penev, 1995). (Dismantling is a destructive process while disassembly is nondestructive.)
- § Mirec BV (the Netherlands) is a subsidiary of Philips that treats electronic consumer products from Philips and other companies. It has a dismantling line for TV sets (de Ron and Penev, 1995).
- § AVR and Prozon (the Netherlands) have built dismantling lines for refrigerators and freezers (de Ron and Penev, 1995).
- § BMW (Germany) has a disassembly line where the operations are conducted in the reverse order of assembly operations (Thierry *et al.*, 1995).
- § Kansai Recycling Systems (Japan), which is a joint venture of Sharp Corporation and Mitsubishi Materials Corporation along with five other electronics companies, operates four disassembly lines for air conditioners, TV sets, refrigerators and washing machines.

Through our analysis of the literature and the requirements that are being set by legislations such as the WEEE directive, we also conclude that disassembly lines are an appropriate setting for disassembly. We believe their benefits such as achieving high productivity, higher recovery rates, economies of scale, and possibility of automation in near future are noteworthy.

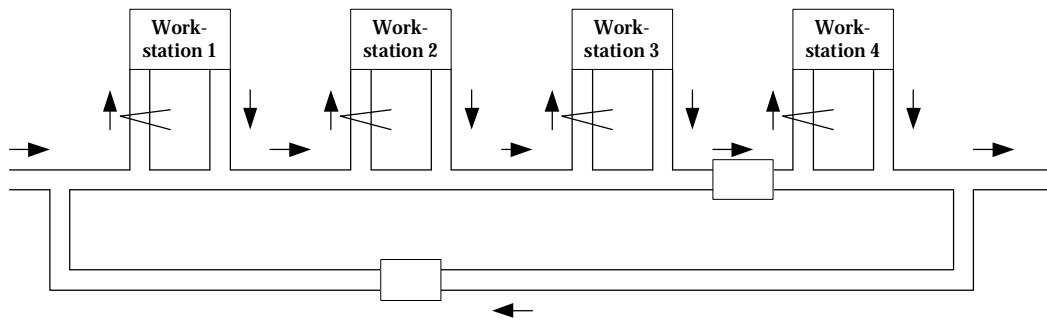
Moreover the legislations that require recovery of large volumes of similar products in near future justify suitability of disassembly lines.

#### **2.2.4. Disassembly Lines**

De Ron and Penev (1994) describe a research project conducted at a company disassembling refrigerators and processing them into environmentally friendly pieces and easy to reuse materials. Their project designs and develops a new disassembly line to reduce the total disassembly cost. For this purpose they analyze the product structure and present disassembling process, and they detect the main problems. Based upon their analysis they propose a new process, which is to determine the minimum number of operations required to fulfill the “environmental and economical demands”. Equipment capacity and cycle time are also considered. Their new design reduces the number of workers down to 6 from 15. The investment in the equipment they propose is compensated by the reduction in labor cost.

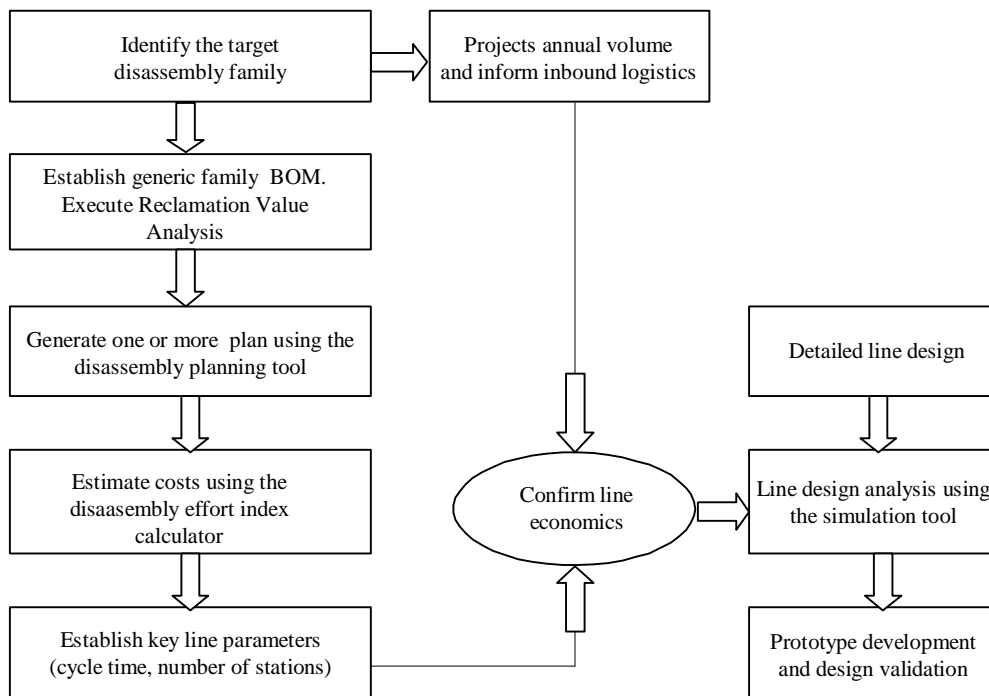
Wiendahl *et al.* (1998) argue that, due to the fuzzy and unpredictable nature of the disassembly processes, the disassembly cannot be performed as planned without a suitable layout structure. Hence they propose a ring or circle topology, which provides efficiency and flexibility required by the disassembly (see Figure 2.6). The proposed ring structure makes planning and control easier. Data capturing and learning turn out to be essential concepts in such a system.





**Figure 2.6** Layout of a ring structure (Wiendahl *et al.*, 1998)

Das and Caudill (1999) after analyzing current disassembly facilities and foreseeing benefits of disassembly lines, develop tools to support design of high volume disassembly lines. The process they propose is demonstrated in Figure 2.7.



**Figure 2.7** Proposed disassembly line design process (Das and Caudill, 1999)

Two examples are presented in their study. The first one is the HP DeskJet example, which is reported to have a total disassembly time of 72 seconds. The second one is from the automobile industry with the following properties:

- § Disassembly line consists of 12 workstations in total (main line consists of 9 workstations and there are 3 branch workstations).
- § The target end product is a steel only chassis.
- § Disassembly cycle time is 17 minutes, with several stations having a slack time of 2 to 3 minutes. 28 vehicles will be disassembled during an eight-hour shift.
- § Direct manpower for the line is 16 workers.
- § The conveying mechanism is a towline conveyor that pulls the custom designed pallets on a rail track.
- § The direct disassembly labor time per vehicle is 4.0 hours (Note that in the existing facilities partial disassembly requires between 5.0 and 8.0 hours of labor time).

Tang *et al.* (2001) propose an algorithm to facilitate the disassembly line design and optimization, which dynamically configures a large system into many disassembly lines, based on system status and demand sources. They claim that their algorithm guarantees the line balance and efficiency.

Güngör and Gupta (2002) present a study with the primary objective of illustrating the importance of disassembly lines in product recovery. They argue that various complications involved with disassembly processes require proper considerations. They summarize the issues that must be taken into account in a disassembly line setting under the following main headings:

- § *Product considerations:* The disassembly line under consideration may deal with only one type of product or it may disassemble a product family whose original configurations are slightly different from each other.
- § *Line considerations:* Layout and line speed are the two issues to be considered. Inspired from the assembly line layouts, they propose serial, circular, U-shaped, cellular and two-sided line layouts. Disassembly lines can be paced or unpaced.
- § *Part considerations:* The incoming used products may include parts with different quality due to proper or improper usage and different quantity mainly due to upgrading or downgrading during usage.
- § *Operational considerations:* Disassembly task times may vary depending on several factors that are related to the condition of the product and state of the disassembly workstation (or worker). Moreover, presence of defective parts in the incoming products may lead to failure of related disassembly tasks. These complications may cause a product to do one of the following: leave the current station early, skip next workstation(s), revisit (backtrack to) a preceding workstation or be taken off the line.
- § *Demand considerations:* In disassembly there may be demand for multiple parts (partial disassembly), or demand for all parts (complete disassembly). There are three types of demand in terms of the desired quality. First type may accept parts “as is”. Second type may not accept parts with any type of defect. Third type demand source may accept certain defective parts depending on the seriousness of the defect.
- § *Other considerations:* Additional uncertainty factors associated with the reliability of the workstations exist. Some parts may cause pollution or nuisance due to the nature of their contents (e.g. oil, gas etc.), which increases the chance of breakdowns or downtimes of workstations.

Ketzenberg *et al.* (2004) study design of mixed assembly-disassembly lines where disassembly and remanufacturing operations are performed to supply components of a product. They assume disassembly sequence is the reverse of the assembly sequence. In the mixed assembly-disassembly configuration, assembly and disassembly operations of a specific task are performed on the same station. For comparative purposes they also study a parallel configuration where there exist two separate lines, one for assembly and the other for disassembly. They use GI/G/c networks and simulation to analyze the problem. Finally they conclude that when variability of disassembly times and arrival times of discarded products is significantly higher than the variability of the corresponding assembly times, parallel configuration performs better than the mixed configuration.

### **2.3. DISASSEMBLY LINE BALANCING PROBLEM**

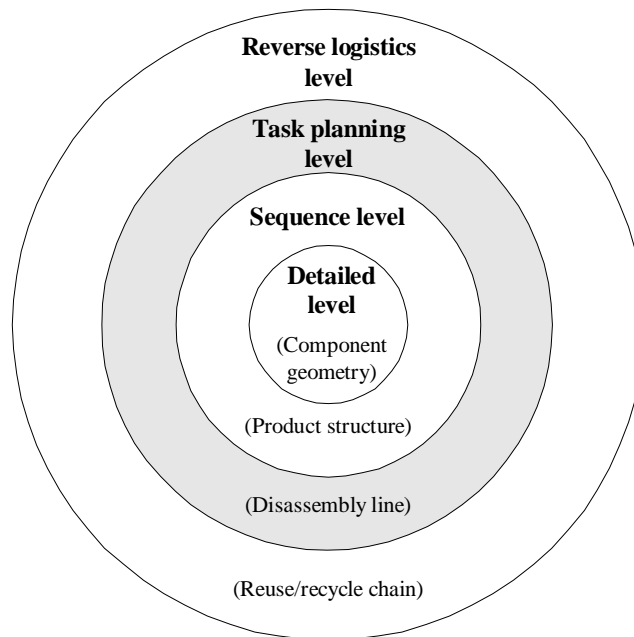
In its basic form, a disassembly line is characterized by

- § a finite set of disassembly tasks (some of which result in removal of parts), each having a task time,
- § a demand associated with each released part or subassembly, and
- § a set of precedence relations which specify permissible orderings of the disassembly tasks.

#### **2.3.1. Definition of Disassembly Line Balancing Problem**

The fundamental line balancing problem is defined as “assignment of tasks to an ordered sequence of stations, such that the precedence relations are satisfied and some measure of effectiveness is optimized” (Ghosh and Gagnon, 1989). This definition is also valid for the disassembly line balancing problem (DLBP).

Lambert (2003) calls DLBP's detail level as task level planning. In Figure 2.8 the onion model he proposes is depicted. This onion model gives an insight in positioning the DLBP within related fields of interest in the disassembly literature.



**Figure 2.8** Positioning of DLBP within related fields of interest (Lambert, 2003)

Güngör and Gupta (2002) study the DLBP and show “how some important factors in disassembly can be accommodated to balance a paced disassembly line by modifying the existing concepts of assembly line balancing”. They assume:

- § Disassembly of a single product. Used products have identical configuration, i.e. the exact quantity of the parts in each received product is known.
- § Infinite supply of used product.
- § Disassembly tasks times are deterministic and known.
- § Demand parameters are deterministic and known.

§ Products are disassembled completely.

§ The parts disassembled are accepted by the demand source “as is”.

They define objective of the DLBP as efficient utilization of the resources while meeting the demand. By efficient utilization of resources they mean “finding the minimum number of disassembly workstations required, optimally assigning the disassembly tasks to the workstations and improving the layout and material handling features of the disassembly line”. They consider demand-driven disassembly since they aim at meeting all the demand.

As Güngör and Gupta (2002) propose to minimize the number of workstations for a paced disassembly line, they have the cycle time ( $CT$ ) constraint. They assume the cycle time is longer than the longest disassembly task time and determine it as follows:

$$CT = \frac{\text{Duration of the planning period}}{\text{Number of products that need to be disassembled to meet demand}} \quad (2.1)$$

The number of products that need to be disassembled is determined by the part with the highest *demand level*, which is found by dividing the demand of the part by quantity of that part in the used product.

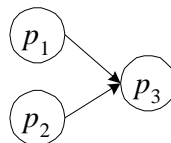
### 2.3.2. Precedence Relations in Disassembly

As in the assembly line balancing problem (ALBP), the precedence relations among the tasks must be satisfied when solving the DLBP. In ALBP, in order to perform a task, all of its predecessors must be complete. In the disassembly literature such precedence relations are called *AND precedence relations*. In ALBP these relations are developed considering the physical and functional constraints, since the objective of the assembly process is to create a

stable and functional end product. In the DLBP, the parts are removed from the used product without any concern of their interrelated functionality; only the physical constraints are important. Therefore, some precedence relations are relaxed by excluding the functionality constraints. This relaxation, however, does not make the disassembly problem any simpler because additional complications arise due to the nature of disassembly. These complications lead to definition of other types of precedence relations, which require distinct representation of the precedence relations (Güngör and Gupta, 2002). Moore *et al.* (2001) introduce the *OR precedence relation* and the *complex AND/OR precedence relation* for the disassembly case.

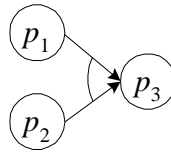
To explain different precedence relations types, let  $p_i$  represent part  $i$  in a product to be disassembled (or task  $i$  to be performed). The set of precedence relations defined in disassembly can be summarized as follows (Moore *et al.*, 2001) :

- § An *AND precedence* relation exists between  $p_1$  and  $p_2$  in relation to  $p_3$ , if both  $p_1$  and  $p_2$  must be removed before  $p_3$  can be removed (see Figure 2.9).



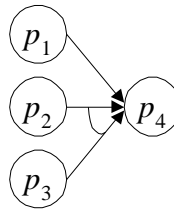
**Figure 2.9** An AND precedence relation example

- § An *OR precedence* relation exists between  $p_1$  or  $p_2$  in relation to  $p_3$ , if at least one of  $p_1$  or  $p_2$  must be removed before  $p_3$  (see Figure 2.10). We use an arc between two predecessor parts (or tasks) to denote the OR precedence.



**Figure 2.10** An OR precedence relation example

§ A complex AND/OR relation exists between  $p_1$ ,  $p_2$  and  $p_3$ , in relation to  $p_4$  if  $p_1$  along with at least one of  $p_2$  or  $p_3$  must be removed before  $p_4$  (see Figure 2.11).



**Figure 2.11** A complex AND/OR relation example

In ALBP, AND precedence relations among the tasks are usually represented by an acyclic graph in which each node represents an assembly task, and a unidirected edge connecting two nodes represents the relation between the two tasks. A binary matrix form of the precedence graph is also used to express these relations (Güngör and Gupta, 2002). However in DLBP, to represent the precedence relations, Güngör and Gupta (2002) use a disassembly precedence matrix (DPM), which strictly represents the geometrical relations among the parts. To generate the DPM, they use the algorithm developed by Güngör and Gupta (2001a). This algorithm evaluates the CAD (computer-aided design) drawing of the end product using a one-part-at-a-time approach and generates the DPM. In addition to the binary elements, the DPM contains another element ( $d$ ) that facilitates representation of OR and complex AND/OR relations among the parts (or tasks). Here  $d$  denotes the disassembly movement in  $x$ - $y$ - $z$  directions;  $d \in$



$\{x, -x, y, -y, z, -z\}$ . Thus the DPM is more complex compared to the binary matrix that represents the precedence relations in the ALBP. They mathematically represent DPM by  $R = [r_{ij}]$   $i, j = 1, \dots, n$  where  $n$  is the number of parts (or tasks) in the product and

$$r_{ij} = \begin{cases} 1, & \text{part } i \text{ AND precedes part } j \\ d, & \text{part } i \text{ OR precedes part } j \\ 0, & \text{otherwise} \end{cases} \quad (2.2)$$

The DPM representation seems to be somewhat restrictive. First of all, since the purpose is full disassembly, the precedence relations are based on only end items, i.e. parts. Secondly, this representation may not be appropriate for a disassembly line that performs selective disassembly, i.e. only certain subassemblies or modules are demanded. Thirdly, in this representation each task disassembles a part from the product. However, in disassembly process there might be some tasks that do not yield any part. Therefore the DPM generation procedure might be modified or other alternative representations of the precedence relations need to be explored.

Due to the restrictions involved with the DPM representation, we have investigated how precedence relations are represented in related fields of interest. The disassembly planning problem, which involves decisions such as how far to disassemble and how to sequence the disassembly tasks, also uses graphical representation of assembly drawings. Connection diagrams and logical expressions are used to represent precedence relations. Lambert (2003) categorizes the studies under the following two approaches.

1. *Hierarchical tree approach* is similar to the bill of materials which is commonly used in materials resource planning. This approach is mainly implemented for electrical and electronic equipment.
2. *Mechanical approach* originates from the assembly study. This approach is mainly used for the analysis of mechanical assemblies.

For the analysis of precedence relations we further focus on the studies following the mechanical approach.

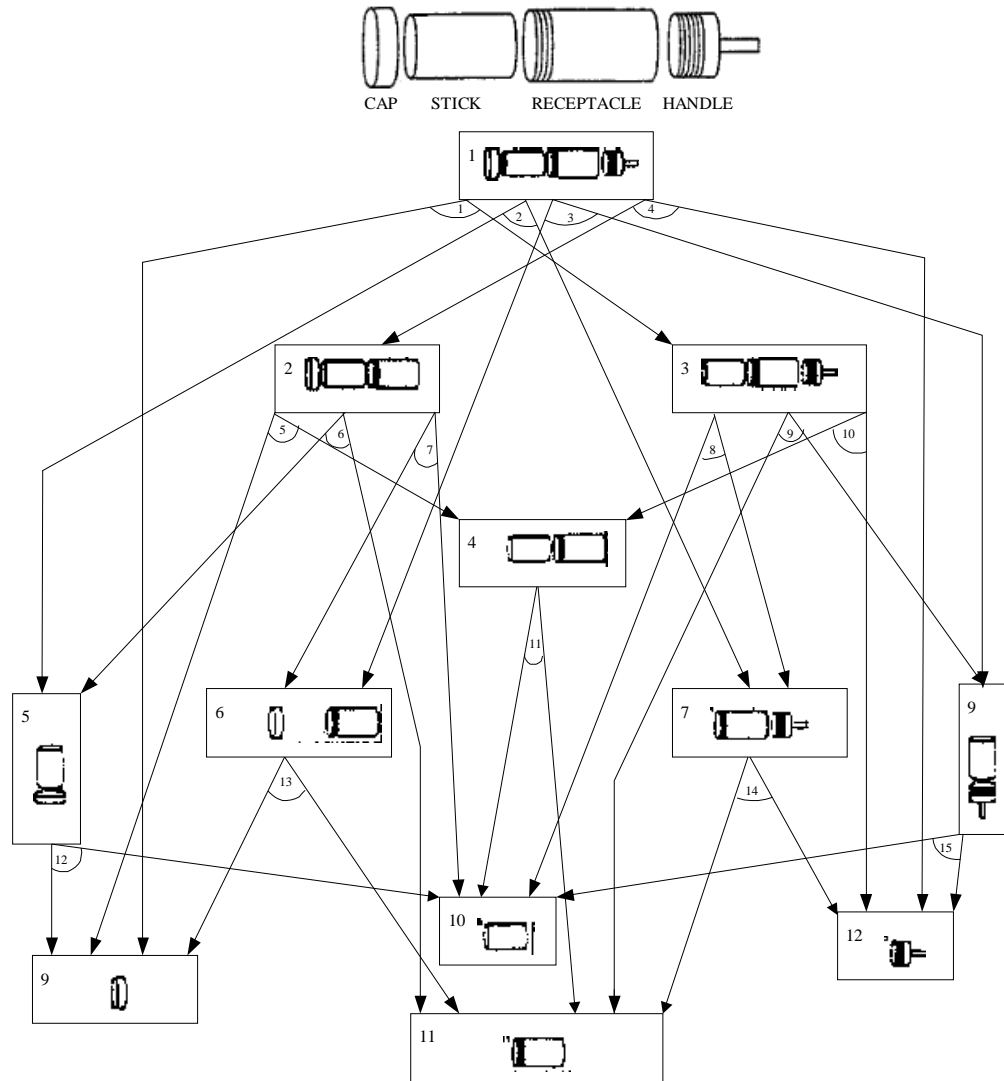
Homem de Mello and Sanderson (1990) state that available precedence relation representations have some restrictions like having the set of tasks fixed (only their order can change) and serial processing of tasks (one at a time). The *AND/OR graph* representation they propose does not have these two restrictions. Regarding the latter restriction, the AND/OR graph explicitly shows when operations can be executed in parallel and do not have any time dependence.

Penev and de Ron (1996) report that, compared to other graphs, they find the AND/OR graph more appropriate since it fulfills all the requirements for the representation of disassembly sequences.

Lambert (1997) introduces the concept of *disassembly graph*, which is based on AND/OR graphs. Here we briefly summarize the AND/OR graphs and disassembly graphs because we believe they can be used in representing the precedence relations in disassembly lines, though with some modifications which will be discussed in Chapter 3.

Figure 2.12 shows a sample product consisting of four parts and its *AND/OR graph*. Each node in the graph corresponds to a subassembly composed of a subset of parts. The root node corresponds to the whole assembly while the nodes at the very bottom level correspond to individual parts. The “hyperarcs” are the physically feasible decompositions of subassemblies into smaller subassemblies. Both the nodes and the hyperarcs have identification numbers. There are four hyperarcs leaving the root node. Each of these four hyperarcs corresponds to one way of disassembling the whole assembly. Each hyperarc points to the two nodes that are

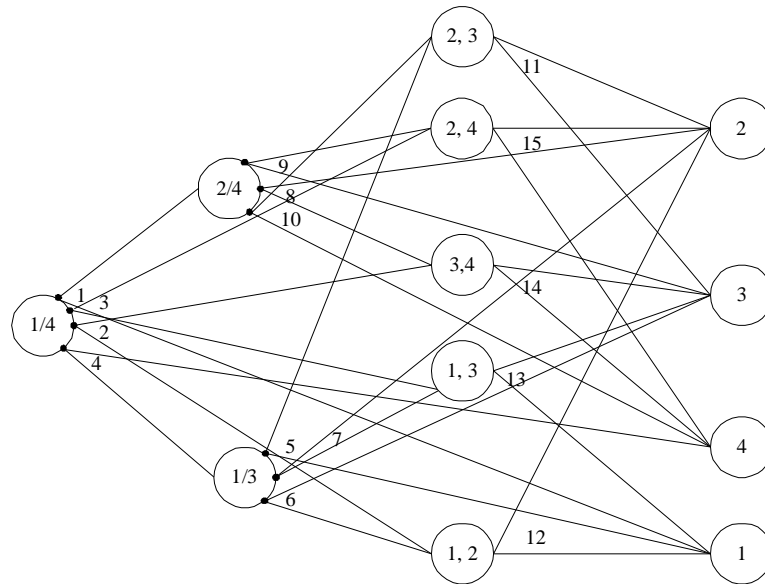
associated with the set of parts that describe the resulting subassemblies. Similarly the other nodes in the graph have one or more outgoing hyperarcs indicating all possible ways in which their corresponding subassembly can be disassembled.



**Figure 2.12** AND/OR graph example (Homem de Mello and Sanderson, 1990)

Any connected subassembly appears only once in the AND/OR graph, even though it may be the result of alternative disassembly operations. The subassembly of node 4, for example, may result from two different operations, represented by hyperarcs 5 and 10 coming from two distinct nodes (2 and 3, respectively). Hence the AND/OR graph encompasses all possible partial orderings of operations with a reduced number of nodes. Homem de Mello and Sanderson (1990) state this feature of the AND/OR graph representation makes it useful for the assembly and disassembly planning problem.

The *disassembly graph* (Lambert, 1997) displays all the subassemblies that may appear during the disassembly of a product in a proper way. An example disassembly graph is given in Figure 2.13 for the four-part product of Figure 2.12. The plane is divided into columns of nodes that are ordered according to the number of parts found in the present subassembly. The numbers labeling the nodes represent the parts that are present in a subassembly. Let part numbers 1, 2, 3 and 4 represent the parts cap, stick, receptacle and handle, respectively. Here **1/3** means parts “1 through 3” and **1, 3** means only parts “1 and 3”. The leftmost node denotes the complete assembly (1/4), while individual parts are displayed at the right hand side. Arc numbered 1 through 10 emanate from filled dots on the right hand sides of nodes. A filled dot is used when a choice can be made regarding how to further disassemble (a forward exclusive OR situation). Pairs of arcs represent actions regarding how a subassembly can be separated into two subassemblies and they have identification numbers. At each action two lines run to the right, each arriving at a new subassembly with fewer parts. Hence this approach has the restriction that at each step the subassembly under consideration is divided into exactly *two* new subassemblies.



**Figure 2.13** Disassembly graph of the four part product

Lambert (1999) presents the mathematical form of the disassembly graph as a *transition matrix*. The columns of the transition matrix correspond to actions while the rows correspond to subassemblies (including the whole product and individual parts). In the columns of the transition matrix, a “-1” entry represents the destruction of a subassembly while a “1” represents the subassemblies created by the corresponding action. All other elements of the matrix are zeros. Figure 2.14 illustrates the transition matrix of the disassembly graph presented in Figure 2.13.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1/4	1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0
1/3	0	0	0	0	1	-1	-1	-1	0	0	0	0	0	0	0	0
2/4	0	1	0	0	0	0	0	0	-1	-1	-1	0	0	0	0	0
1,2	0	0	1	0	0	0	1	0	0	0	0	0	-1	0	0	0
1,3	0	0	0	1	0	0	0	1	0	0	0	0	0	-1	0	0
2,3	0	0	0	0	0	1	0	0	0	0	1	-1	0	0	0	0
2,4	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	-1
3,4	0	0	1	0	0	0	0	0	1	0	0	0	0	0	-1	0
1	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	0
2	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	1
3	0	0	0	0	0	0	1	0	0	1	0	1	0	1	1	0
4	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1

**Figure 2.14** Transition matrix example

### 2.3.3. Solution Procedures for Disassembly Line Balancing Problem

There are very few studies that attempt to solve DLBP, since the problem has been defined very recently. In this section we summarize them briefly.

Güngör and Gupta (2002) study DLBP under complete disassembly. Their objective is to minimize the number of stations while meeting the demand of the recovered parts. Hence their version of the problem is demand-driven. They propose a priority rule based heuristic solution procedure for DLBP. Their procedure is station-oriented. In their approach, unassigned tasks become candidate tasks for the current workstation if their precedence relations are satisfied (including the OR precedence) and their task time is less than the unused time of the current workstation. Once the candidate tasks are identified, they use a priority function that determines which candidate task(s) will be assigned to the current workstation. The complexity of their heuristic is  $O(n^2 \log n)$ , where  $n$  represents the number of disassembly tasks. Although many factors can be incorporated into the priority function, they limit themselves to the following:

- § *Idle times of workstations*: Tasks are assigned to workstations such that the utilization of the line is maximized (within this component of the priority function the tasks having the smallest task time have higher priority).
- § *Disassembly of highly demanded parts*: Disassembly of highly demanded parts should take place at the earliest possible workstation(s) since the longer they stay on the line, the higher the chance of damaging them during the disassembly process.
- § *Disassembly of easily accessible parts, which precede the largest number of parts*: Parts that are easily accessible and precede many other parts should be removed as early as possible. Assigning these tasks to early workstations will increase the number of

candidate parts in the later stages of the disassembly line, which may lead to a better line balance.

- § *Disassembly of parts with hazardous content:* Parts with hazardous material content should be removed from the product as early as possible to reduce the possibility of hazardous material spill, as this could lead to breakdown of a workstation and/or material-handling system, and contamination of the demanded parts.
- § *Minimizing changes in disassembly direction:* The objective is to reduce the number of times the workpiece is re-oriented on the disassembly line. This provides smoothness of the overall disassembly procedure. For example, in an automated disassembly situation, the reduction in the number of re-orientations of the workpiece would require less complicated (and hence less expensive) machinery.

Güngör and Gupta (2001b) analyze DLBP in the presence of task failures. This study (although published earlier) incorporates the concept of failing tasks into the DLBP procedure presented in Güngör and Gupta (2002). In this version of the problem, if a task cannot be performed because of some defect, some or all of the remaining tasks may be disabled due to the precedence relations. This may result in various complications in the flow of used products on the disassembly line, e.g. early leaving, self-skipping, skipping, disappearing and revisiting. The problem is to assign tasks to workstations such that the effect of the defective parts is minimized. In their approach, they first generate an incomplete state network (ISN) representing all feasible states and their partial relations, where a state is a set of tasks already assigned to stations. For ISN generation they modify models developed for assembly line balancing by Gutjahr and Nemhauser (1964) and Erel and Gökçen (1999). This modification enables the incorporation of AND/OR precedence relations. The second step is to develop all possible relations among the states of the ISN via application of the cycle time constraint. The

resulting network is called the state network (SN). As a third step, they calculate the idle times of task assignments for each edge of SN where edges are relations between states of SN. They form the weighted state network (WSN) by using these idle times as weights of the edges. The fourth step is to find the shortest directed path from the first to the final node of WSN. At this step, they use Dijkstra's shortest path algorithm. This approach may yield multiple alternative balances with the same number of stations. Since task assignments by each alternative may cause different complications on the disassembly line, they determine the probabilities of several complications according to the failure probabilities of the tasks. In the fifth step, they use these probabilities to determine cost of the complications for each alternative via a complications cost function. Given costs of complications, the balance with minimum complication cost is selected.

McGovern and Gupta (2003) present a greedy heuristic procedure for multi objective DLBP under complete disassembly. They propose a two-phased solution where the first step constructs a feasible solution with minimum or near-minimum number of work stations using a "first-fit decreasing algorithm". The second step aims at balancing the workload of the stations and uses a 2-opt exchange local search algorithm to improve the initial solution. The sum of the square of the idle times for all workstations is used to smooth the workload across the stations

Ranky *et al.* (2003) propose a dynamic disassembly line balancing algorithm that aims at smoothing the load of the shopfloor. They present an extended version of the COMSOAL (Arcus, 1966) algorithm used in the assembly line balancing literature. The proposed Ranky-COMSOAL algorithm yields a balanced schedule in which all precedence relations are met and the flowtime of the discarded product on the disassembly line is minimized.



Despite the differences between DLBP and the generalized ALBP, some studies include common features with our study. Common issues include incorporating cost minimization, profit maximization, balancing the cycle time versus the number of stations, and solving the assembly line design problem with processing alternatives. Thus, we briefly summarize our findings on these issues in the next section.

#### **2.3.4. Relevant Work from Generalized ALBP Literature**

Becker and Scholl (2003) state that the establishment of assembly lines requires long term capital investment while operating the line leads to short-term operating costs such as wages, material, set-up, inventory and implementation costs. They report that Zapfel (1975) points out the necessity for comparing these costs with profit attained from the line especially in cases with “non-fixed production rate and varying levels of production quality”.

Deckro (1989) states that the installation and operating costs as well as the profits obtained from a line mainly depend on the number of stations and cycle time. He provides a zero-one integer programming formulation based on the models of Patterson and Albracht (1975) and Thangavelu and Shetty (1971). In order to balance the cycle time and the number of stations he proposes a weighted objective of these two elements. He uses a different constant weight associated with each workstation and another constant weight associated with the cycle time, which should all be in line with the decision makers’ preferences. He presents an example illustrating his formulation. He remarks that analyzing cost tradeoffs between cycle time and the number of stations might be useful at the aggregate planning stage and to achieve better overall line balances.

Becker and Scholl (2003) articulate that in order to solve cost-oriented models that mainly depend on cycle time and the number of stations, type-I (minimize the number of stations for a given cycle time) and type-II (minimize cycle time given the number of stations) problems of ALBP could be solved iteratively similar to solving the type-E (maximize line efficiency equivalently minimize the multiplication of cycle time and the number of stations) problem. However, they also mention that models that explicitly incorporate costs and/or profits are necessary when the balancing problem is related with the decision problem of selecting processing and/or equipment alternatives.

Pinto *et al.* (1983) present a method that simultaneously handles choice of processing alternatives and the assignment of tasks to stations in order to minimize the total labor and fixed costs over the expected life of the production line. They assume a “base set of tasks”, which are sufficient to operate the line and have lowest fixed costs, are identified. They further assume the existence of a set of independent processing alternatives (each of which can replace one or more of the base tasks) with an incremental fixed cost. They provide two integer programming formulations where one uses a fixed cycle time and the other treats cycle time as a decision variable. They provide upper and lower bounds on the total cost only for the fixed cycle time case, which they have embed in a branch and bound procedure. Although they describe an iterative approach for the cases where cycle time is a decision variable, they do not solve the problem. They illustrate their approach with fixed cycle time on two examples. The nature of this problem and our problem are similar in terms of some of the components in the objective function. However, the processing alternatives that are externally provided in this problem do not exist in our problem. Due to the various precedence relation types in our DLBP formulations, there might be alternative ways of releasing parts. Because of the complex nature of the precedence relations, detecting such alternatives externally may not be easy on the

precedence network. These alternatives might not be disjoint and their number may grow exponentially, which makes external treatment impractical. One of their findings related to our study is the importance of jointly considering the processing alternatives and task assignment decisions. They further explain their claim by considering a processing alternative that may reduce the total work content of the set of tasks it replaces. They note that the “assumed savings” cannot be achieved if the resulting line balance does not reduce the number of stations. In such a case, selection of the corresponding processing alternative leads only to an increase in the total idle time of the line.

The other studies we reviewed include cost-oriented ALBP models (Amen, 2000a, 2000b, 2001; and Buckin and Tzur, 2000) and profit-oriented ALBP models (Rosenblatt and Carlson, 1985; and Martin, 1994). As a consequence of the review, the following observations are noteworthy for our study. Amen (2000b) shows that balancing stations maximally might lead to missing a cost-oriented optimal solution. Similarly, Rosenblatt and Carlson (1985) illustrate that a solution that maximizes efficiency does not necessarily maximize the profit.

## **2.4. DISCUSSION**

DLBP is a recently defined problem. As the recyclability of the products increases and disassembly tasks are automated, disassembly lines will become more widespread. Analytical models that may be developed in the mean time can facilitate the efficient design and operation of disassembly lines. We also believe that the knowledge from ALBP can be used in analyzing DLBP. In order to achieve this, the similarities and differences between these two problems must be clearly identified. We believe the main differences concerning DLBP include the following:

1. The disassembly precedence diagram and assembly precedence diagram may be different since some operations may not be reversible (e.g. soldering). Still, the disassembly precedence diagram might be a symmetrical version of the assembly precedence diagram where irreversible tasks and their successors are administered to reflect corresponding changes, or it can be obtained by modifying the assembly precedence diagram to incorporate precedence relation types of disassembly.
2. Reversing of assembly operations for disassembly (i.e. item segregation) may not be optimal due to the differences in task times and profitability of the corresponding disassembly operations.
3. Alternative ways of disassembling products and subassemblies lead to incorporation of other precedence relations (i.e. OR precedence). As a consequence of these new precedence relations higher line flexibility can be attained. Hence disassembly line balancing should take into consideration these additional relations.
4. Unlike assembly, disassembly can be partial. It is necessary to decide which tasks should be performed and which parts should be released. In doing this, the objective should be profit maximization or cost minimization.
5. The uncertainties in quantity, quality and arrival times of the used products create an erratic (uncontrollable) input flow to the disassembly system.
6. Backtracking, bypassing or skipping stations due to failures of disassembly tasks lead to a variable flow on the line which may result in imbalances in the workloads of the stations.
7. Disassembly task times in general have higher variability compared to assembly tasks.

In this study, we concentrate on the first four issues.

There are very few studies that attempt to solve DLBP, since the problem has been defined very recently. To the best of our knowledge, a mathematical programming formulation of the problem, optimal solution seeking procedures, and a test bed of problems for experimentation are yet unavailable. The current studies are specific for the environments they have been defined for and may be improved to incorporate other issues. We identify the following improvement directions regarding the reviewed DLBP literature:

- § *Problem definition:* In Güngör and Gupta (2002), the cycle time is determined as a function of maximum demand level. Demand levels of other parts are ignored and the line is balanced as if all parts have the same demand. No remark is made regarding the excess quantities released. (Are they placed in inventory or scrapped or sold in some other market?) Due to recent developments in closed-loop supply chains that incorporate commercial returns, a time value component is added to the recovery process. This time value component requires such returned products to be recovered quickly and returned to the market before the product becomes obsolete. Within such a framework the disassembly rate should be maximized. In order to achieve this the cycle time should be incorporated as a decision variable in the problem. Hence the relationships among different part demand levels, cycle time, and line balancing should be explored in DLBP.
- § *Disassembly Precedence Matrix (DPM) representation:* Unless minimization of the number of orientation changes is aimed as in Güngör and Gupta (2002), it is not necessary to use the geometric direction information within the DPM. Hence, a simpler representation of precedence relations that includes 1, 0 and -1 entries may be defined. If orientation is really an important issue, the studies of Arcus (1966) and Schofield (1979) from ALBP with orientation literature can be examined as a starting point.
- § All of the precedence diagrams in the DLBP literature assume each task releases exactly one part which is not necessarily the case. In addition, the disassembly process planning

literature shows that precedence relation representations other than AND relations should also be considered in DLBP.

§ *Solution approach:* All the reviewed studies propose heuristics. First of all, a mathematical formulation of the problem considering all aspects in a general framework should be provided to get insight about the nature of the problem. Only then solution procedures that use it as a stepping stone could be developed.

§ *Objective function:* The objective functions studied minimize the number of workstations opened or minimize the flowtime. However, due to the previous discussion on disassembly, we believe minimization of disassembly costs or maximization of profit might be more suitable as DLBP objectives under partial disassembly.

## CHAPTER 3

### PRECEDENCE RELATIONS IN DLBP

In this chapter, we first introduce a representation scheme for the precedence relations of disassembly tasks suitable for line balancing decisions. Our purpose is to come up with a concise but flexible model for a variety of such relations, by combining alternative representations found in the literature.

We aim at developing an analytical model for DLBP that assigns disassembly tasks to a sequence of stations without violating the precedence relations among the tasks. The precedence constraints play a major role in the formulation of the problem. Therefore identification of different relation types and a proper representation of these relations is essential. As discussed in Chapter 2, the Disassembly Precedence Matrix (DPM) (Moore *et al.*, 2001; Güngör and Gupta, 2001b), AND/OR graphs (Homem de Mello and Sanderson, 1989) and disassembly graphs (Lambert, 1997) are commonly used schemes in the related literature.

We start by identifying different types of precedence relations in disassembly. We then describe how the representation schemes proposed in literature can be unified into a simpler form to include all these relation types. Two examples taken from the literature are used to

illustrate the proposed representation scheme. We also provide mathematical formulation of the precedence constraints together with their representation scheme. The proposed representation facilitates the development of mathematical programming formulations for DLBP, which will be discussed in subsequent chapters.

Gottipolu and Gosh (1997) state that there exist a vast number of alternative ways of assembling a product. As the number of such alternative feasible assembly sequences increases exponentially with increased the number of parts, representing all sequences becomes intractable. Thus, Gottipolu and Gosh (1997) remark on the need for “systematic and efficient ways to represent and evaluate all the available alternatives, and choose the best one satisfying the available resources”. They review the representation schemes proposed in the assembly process planning literature and conclude that graphical representations provide more compact and useful schemes that embrace all feasible sequences besides ordered lists of tasks, assembly states and feasible connections. Their list of most commonly used diagrammatic representation schemes include precedence diagrams (Prenting and Battaglin, 1964), state transition diagrams (Warrets, *et al.*, 1992), liaison graphs (De Fazio and Whitney, 1987) and AND/OR graphs (Homem de Mello and Sanderson, 1990). Based on our literature review and analysis, we conclude that these schemes can be used to represent feasible disassembly sequences as well.

Among these alternative representation schemes, the ALBP and DLBP literature traditionally stick to precedence diagrams in which operations are represented by nodes connected by unidirected arcs depicting the binary precedence relations. However, precedence diagrams are limited in the sense that they consider only the AND precedence relations. As a result, the set of tasks are predefined and fixed, and they are to be performed serially (Homem de Mello and Sanderson, 1990).



Our representation scheme for precedence relations is also based on the precedence diagram approach. However, to represent additional relation types, we incorporate AND/OR graph representation of Homem de Mello and Sanderson (1990) and disassembly graph approach of Lambert (1997). In doing this, we transform a product structure based representation to an operation based precedence diagram. We use the precedence relation types defined by Güngör and Gupta (2001b) and introduce a new precedence relation type. Although we focus on precedence relations in disassembly, our approach is applicable to assembly systems as well.

The AND, OR and complex AND/OR precedence relations are the three types introduced by Güngör and Gupta (2001a). Moreover, when AND and OR precedence relations are combined, two other types emerge. AND precedence relations might exist within OR precedence relations and vice versa. We also define a new relation type, namely the OR successor, which is observed in AND/OR graphs and disassembly graphs. Moreover when an OR predecessor of a task is an OR successor of another task, the OR successors within OR predecessors relation comes up. With the addition of these precedence relations we believe all combinations that may arise in disassembly systems are accounted for.

Our unified representation includes AND precedence, OR precedence and OR successor as the three main types. More complex precedence relation types (complex AND/OR, AND within OR, OR within AND, and OR precedence with OR successors) can be represented using the main types via the insertion of dummy tasks. We also transform our precedence diagram with  $n$  disassembly tasks to an  $n \times n$  matrix  $M=[m_{il}]$  called the disassembly precedence matrix (DPM).

Next, we start with introducing the three main precedence relation types. For each main relation type, we also provide an incident vector representation (a column or row of DPM) and

corresponding constraint in a typical DLBP formulation. In the following sections, we define more complex precedence relation types and show how they are simplified.

In formulating the precedence relations, we use the following notation and variables.

$i$  Disassembly task index,  $i = 0, 1, 2, \dots, n, A1, A2, \dots, D$

$k$  Station index,  $k = 1, 2, \dots, K$

PAND( $i$ ) Index set of AND predecessors of task  $i$

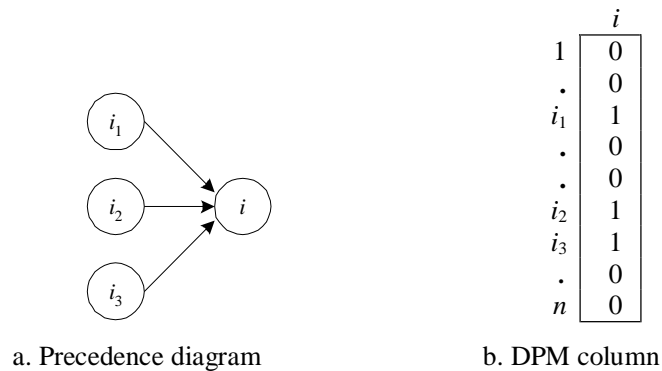
POR( $i$ ) Index set of OR predecessors of task  $i$

SOR( $i$ ) Index set of OR successors of task  $i$

$$x_{i,k} = \begin{cases} 1 & \text{if task } i \text{ is assigned to station } k \\ 0 & \text{otherwise} \end{cases}$$

### 3.1. AND PREDECESSORS

Tasks  $i_1$  through  $i_m$  are AND predecessors of task  $i$ , if all of them must be finished before starting task  $i$  (see Figure 3.1a for  $m=3$ ). The  $i^{\text{th}}$  column of our DPM contains ones in rows corresponding to tasks  $i_1$  through  $i_m$  and the entry is zero if the corresponding task is not an AND predecessor (Figure 3.1b).



**Figure 3.1** AND predecessor example

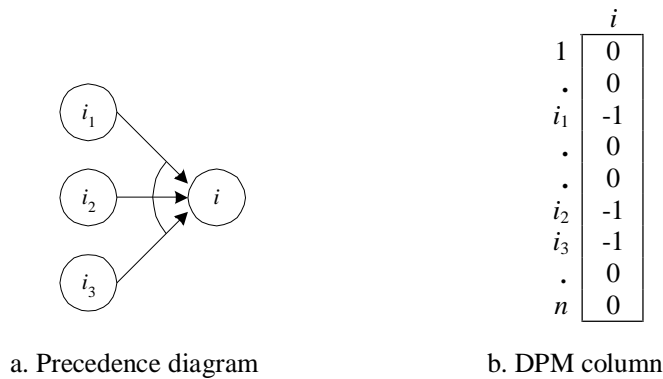
Note that we simply use the AND precedence representation of Güngör and Gupta (2001a) which is also very common in the ALBP literature.

We use the classical precedence constraint formulation of Bowman (1960) and White (1961) from the ALBP literature, which is given in constraint set (3.1). This constraint set allows the assignment of task  $i$  to station  $k$  only if all of its AND predecessors are already assigned to stations 1 through  $k$ .

$$x_{i,k} \leq \sum_{h=1}^k x_{l,h} \quad \forall k, i \text{ and } l \in \text{PAND}(i) \quad (3.1)$$

### 3.2. OR PREDECESSORS

Tasks  $i_1$  through  $i_m$  are OR predecessors of task  $i$ , if at least one of them must be finished before starting task  $i$  (see Figure 3.2a for  $m=3$ ).



**Figure 3.2** OR predecessor example

In Güngör and Gupta's (2001a) representation, the DPM column corresponding to task  $i$  contains one or more different  $d$  values that represent geometric directions ( $d \in \{x, -x, y, -y, z, -z\}$ ).

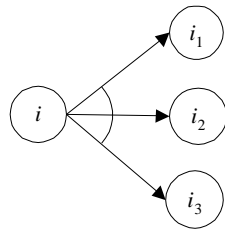
We represent OR precedence by “-1”s instead of direction codes since we are not interested in geometrical orientation of the tasks. We illustrate our DPM column corresponding to task  $i$  in Figure3.2b.

The OR precedence constraint set (3.2) states that task  $i$  cannot be assigned to station  $k$  unless at least one of its OR predecessors is assigned to one of the stations 1 through  $k$ .

$$x_{i,k} \leq \sum_{h=1}^k \sum_{l \in \text{POR}(i)} x_{l,h} \quad \forall k \text{ and } i \quad (3.2)$$

### 3.3. OR SUCCESSORS

Tasks  $i_1$  through  $i_m$  are OR successors of task  $i$ , if at most one of them can start after task  $i$  is finished (see Figure3.3a for  $m=3$ ).



a. Precedence diagram

	1	.	$i_1$	$i_2$	$i_3$	.	$n$
$i$	0	0	-1	-1	-1	0	0

b. DPM row

**Figure 3.3** OR successor example

This relationship is found in AND/OR graphs and disassembly graphs and has no corresponding representation in Güngör and Gupta’s (2001a) DPM. This precedence relation arises from the fact that, although there might be multiple ways of dividing a parent subassembly into child subassemblies, only one of them can be realized on the line. OR

successors of task  $i$  are denoted in the row corresponding to task  $i$  by “-1”s in columns of tasks  $i_1$  through  $i_m$  (see Figure 3.3b).

Two OR successor constraints facilitate the assignment of at most one of the OR successors of task  $i$  to station  $k$ , if task  $i$  is assigned to one of the stations 1 through  $k$ . The first constraint set (3.3) assures assignment of at most one of the OR successors of task  $i$  (to some station) if task  $i$  is performed. The second constraint set (3.4) allows assignment of the OR successors of task  $i$  to station  $k$ , if task  $i$  is assigned to one of the stations 1 through  $k$ .

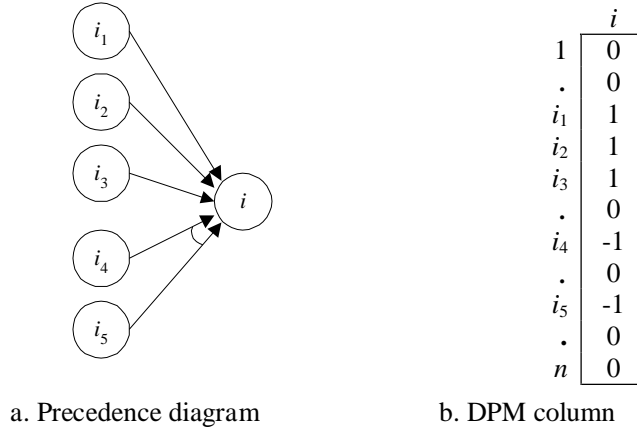
$$\sum_{k=1}^K \sum_{l \in \text{SOR}(i)} x_{l,k} \leq \sum_{k=1}^K x_{i,k} \quad \forall i \quad (3.3)$$

$$\sum_{l \in \text{SOR}(i)} x_{l,k} \leq \sum_{h=1}^k x_{i,h} \quad \forall k \text{ and } i \quad (3.4)$$

### 3.4. COMPLEX AND/OR PREDECESSORS

The complex AND/OR precedence arises when all of the tasks  $i_1$  through  $i_m$  must be finished along with at least one of the tasks  $i_{m+1}$  through  $i_{m+r}$  before task  $i$  can start (see Figure 3.4a for  $m=3$  and  $r=2$ ). In Güngör and Gupta (2001a)’s representation, the DPM column of task  $i$  contains direction codes and “1”s. In our representation, we use “1”s for AND predecessors and “-1”s for OR predecessors in the  $i^{\text{th}}$  column of DPM (see Figure 3.4b). Hence we obtain a representation with  $\text{PAND}(i)=\{i_1, \dots, i_m\}$  and  $\text{POR}(i)=\{i_{m+1}, \dots, i_{m+r}\}$ .

Constraint sets (3.1) and (3.2) are used together to represent the complex AND/OR precedence constraints over the sets  $\text{PAND}(i)$  and  $\text{POR}(i)$ .

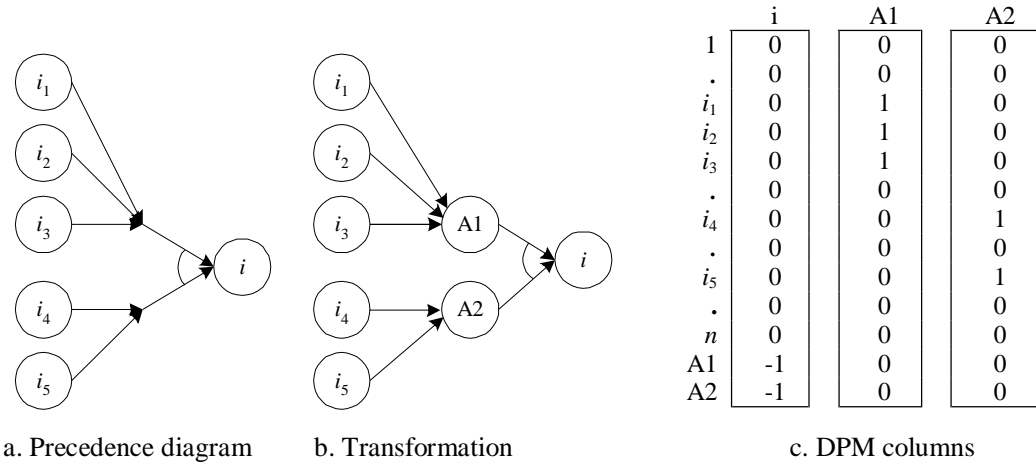


**Figure 3.4** Complex AND/OR predecessor example

### 3.5. AND WITHIN OR PREDECESSORS

Task sets  $i_1$  through  $i_m$  and  $i_{m+1}$  through  $i_{m+r}$  are AND within OR predecessors, if at least one of these two sets must be finished completely before task  $i$  can start (see Figure 3.5a for  $m=3$  and  $r=2$ ). In Güngör and Gupta's (2001a) representation, the DPM column corresponding to task  $i$  contains  $d$  values where at least two direction codes are equal. Before task  $i$ , all tasks with the same  $d$  (for instance tasks  $i_1$  through  $i_m$ ) must be finished to satisfy a single OR condition. In our transformation, two dummy tasks (A1 and A2) are introduced (see Figure 3.5b). Tasks  $i_1$  through  $i_{m+1}$  become AND predecessors of A1, and tasks  $i_{m+1}$  through  $i_{m+r}$  become AND predecessors of A2. Thus A1 and A2 become OR predecessors of task  $i$ . Corresponding DPM columns of tasks  $i$ , A1 and A2 are presented in Figure 3.5c. After the transformation, we obtain a simpler representation with  $\text{PAND}(A1)=\{i_1, \dots, i_m\}$ ,  $\text{PAND}(A2)=\{i_{m+1}, \dots, i_{m+r}\}$  and  $\text{POR}(i)=\{A1, A2\}$ .

Note that again, constraint sets (3.1) over  $\text{PAND}(A1)$  and  $\text{PAND}(A2)$ , and (3.2) over  $\text{POR}(i)$  are used to represent the AND within OR precedence constraint.

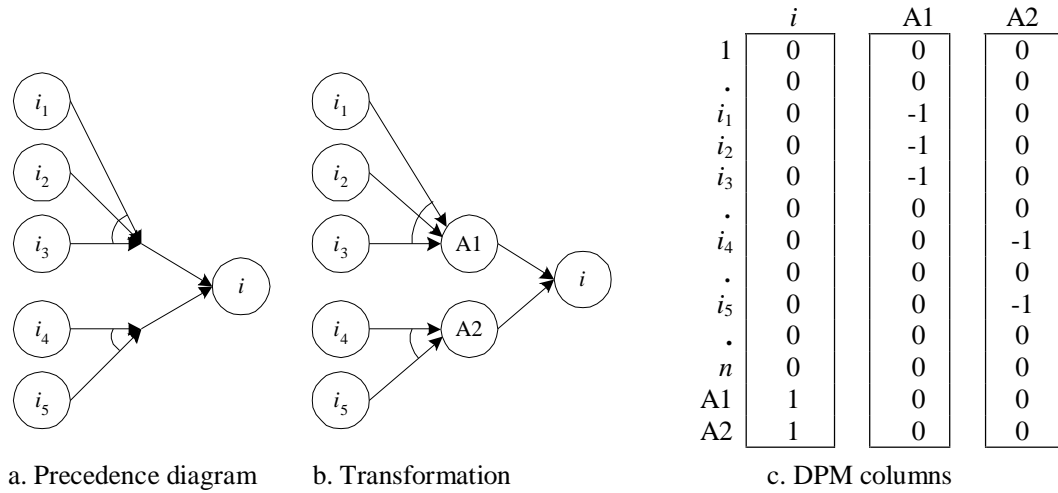


**Figure 3.5** AND within OR predecessors example

The advantage of this transformation is that AND within OR predecessors can be represented in DPM using the standard 0, 1, -1 codes of AND and OR precedence relation types. Also, the regular AND precedence and OR precedence constraints are applicable. The disadvantage, however, is that dummy tasks and new variables are added, increasing the problem size.

### 3.6. OR WITHIN AND PREDECESSORS

Tasks  $i_1$  through  $i_m$  and  $i_{m+1}$  through  $i_{m+r}$  are OR within AND predecessors, if at least one task in each of the two OR predecessor groups must be finished before starting task  $i$  (see Figure 3.6a for  $m=3$  and  $r=2$ ). In our transformation, we again define two dummy tasks, A1 and A2 (see Figure 3.6b). Tasks  $i_1$  through  $i_m$  become OR predecessors of A1 while tasks  $i_{m+1}$  through  $i_{m+r}$  become OR predecessors of A2. Thus A1 and A2 become AND predecessors of task  $i$ . Corresponding DPM columns of tasks  $i$ , A1 and A2 are given in Figure 3.6c. After the transformation, we again obtain a representation with  $\text{POR}(A1)=\{i_1, \dots, i_m\}$ ,  $\text{POR}(A2)=\{i_{m+1}, \dots, i_{m+r}\}$  and  $\text{PAND}(i)=\{A1, A2\}$ .



**Figure 3.6** OR within AND predecessor example

Here similarly, constraint sets (3.1) and (3.2) are used to represent the OR within AND precedence constraint over the sets  $POR(A1)$ ,  $POR(A2)$  and  $PAND(i)$ .

Note that a mathematical representation of this relation type can also be provided without inserting dummy tasks and using constraint set (3.2) as many times as the number of OR predecessor groups. This representation would require a different coding scheme in the DPM so that different OR precedence groups within the AND precedence can be discriminated. For instance constraints (3.5) and (3.6) would represent the corresponding relation depicted in Figure 3.6a. However, we stick to the representation where dummy tasks are inserted as we write the constraints over the sets  $POR(i)$  and  $PAND(i)$ , which are determined using the DPM. Hence we demand its entries to consist of only 1, 0 and  $-1$ .

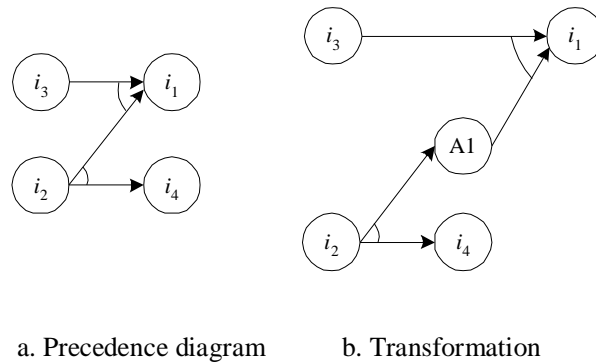
$$x_{i,k} \leq \sum_{h=1}^k x_{i_1,h} + x_{i_2,h} + x_{i_3,h} \quad \forall k \quad (3.5)$$

$$x_{i,k} \leq \sum_{h=1}^k x_{i_4,h} + x_{i_5,h} \quad \forall k \quad (3.6)$$



### 3.7. OR SUCCESSORS WITHIN OR PREDECESSORS

If one of the OR predecessors of task  $i_1$  is also one of the OR successors of task  $i_2$ , we have OR successors within OR predecessors type of precedence relation (see Figure 3.7a). In our transformation, we introduce dummy task A1 (see Figure 3.7b). After the transformation, we again obtain a simpler representation with  $SOR(i_2)=\{i_4, A1\}$ , and  $POR(i_1)=\{i_3, A1\}$ .



**Figure 3.7** OR successors within OR predecessor example

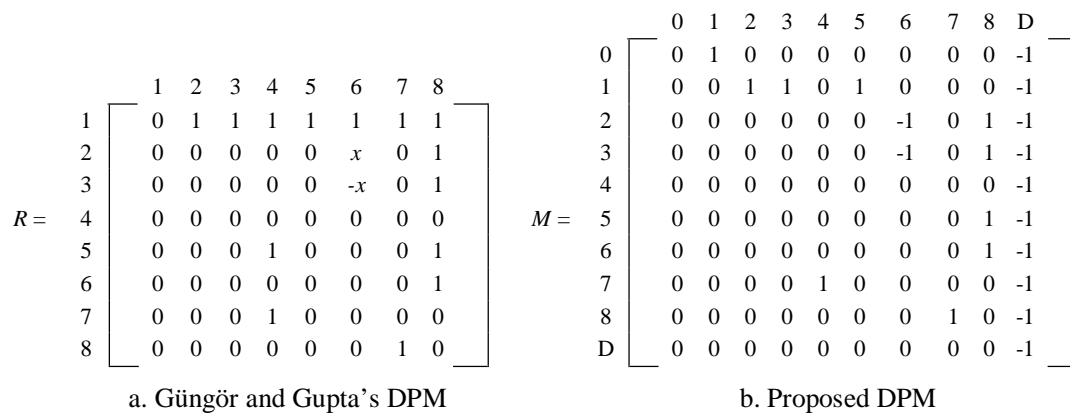
### 3.8. EXAMPLES

Our first example is taken from the DLBP literature. Güngör and Gupta (2002) illustrate their approach on an eight-part personal computer (PC) example. The information regarding each task that disassembles a PC part is provided in Figure 3.8. Their DPM representation and our disassembly precedence representation are illustrated in Figure 3.9a and Figure 3.9b, respectively. Unless minimization of the number of orientation changes is aimed as in Güngör and Gupta (2002), it is not necessary to use the geometric direction information within the DPM. Hence, a simpler and coherent representation of precedence relations that includes only 1, 0 and -1 entries may be defined.

Task (i)	Definition	$t_i$ (s)	Hazardous content	$d^*$
1	Removal of the top cover of the PC (TC)	14	No	-x
2	Removal of the floppy drive (FD)	10	No	x
3	Removal of the hard drive (HD)	12	No	-x
4	Removal of the back plane (BP)	18	No	x, -x, y, or -y
5	Removal of the PCI cards (PCI)	23	No	y
6	Removal of the two RAM modules (RAM)	16	No	z
7	Removal of the power unit (PU)	20	Yes	-x, x, or y
8	Removal of the mother board (MB)	36	No	z

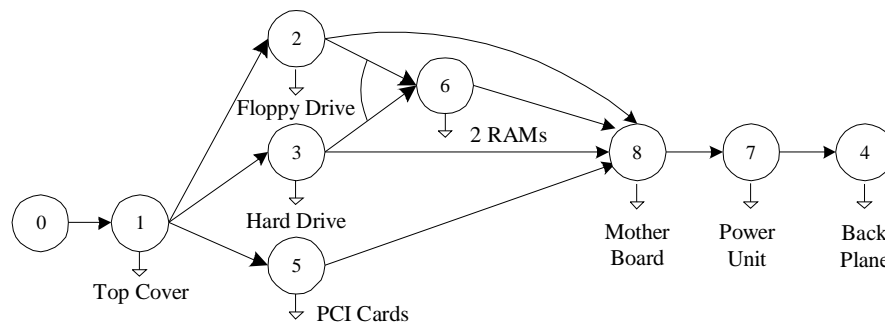
\* Identified during the analysis of the product to generate  $R$ .

**Figure 3.8** “Knowledge base” of PC example (Güngör and Gupta, 2002)



**Figure 3.9** DPM representations of PC example (Güngör and Gupta, 2002)

Figure 3.10 demonstrates the transformed precedence diagram of the PC example. Parts released by each task are also depicted with white ended arrows beneath each task. Note that OR predecessors of dummy task D are omitted in the transformed precedence diagram.



**Figure 3.10** Precedence diagram of PC example

Our second example, which is a 10-part ball-point pen (Lambert, 1997) is illustrated in Figure 3.11. The disassembly graph given in Figure 3.12 displays the 20 actual tasks that disassemble the ball-point pen and the 14 main subassemblies (including the whole ball-point pen) that arise during different stages of the disassembly. In order to transform the disassembly graph to our precedence diagram, three dummy tasks (A1, A2 and A3) are introduced. The resulting precedence diagram is depicted in Figure 3.13.

We use these precedence relations in formulating DLBP in the following chapters

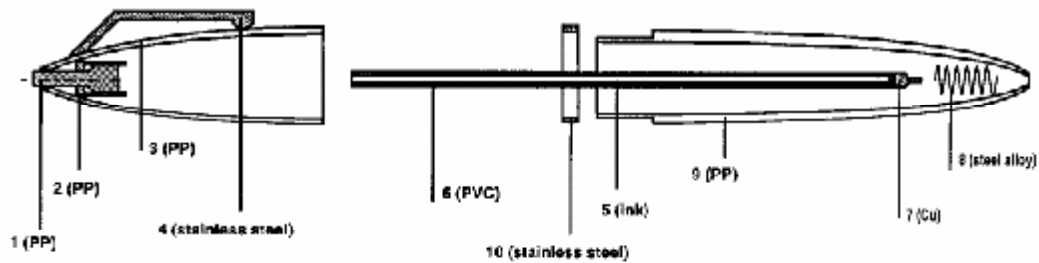


Figure 3.11 10-part ball-point pen example (Lambert, 1997)

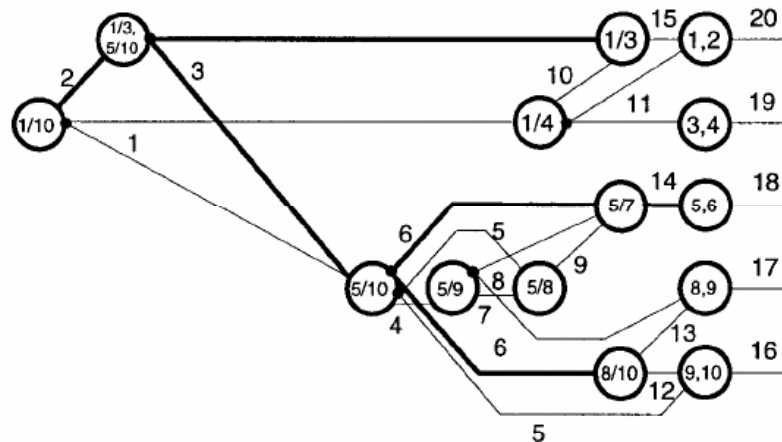
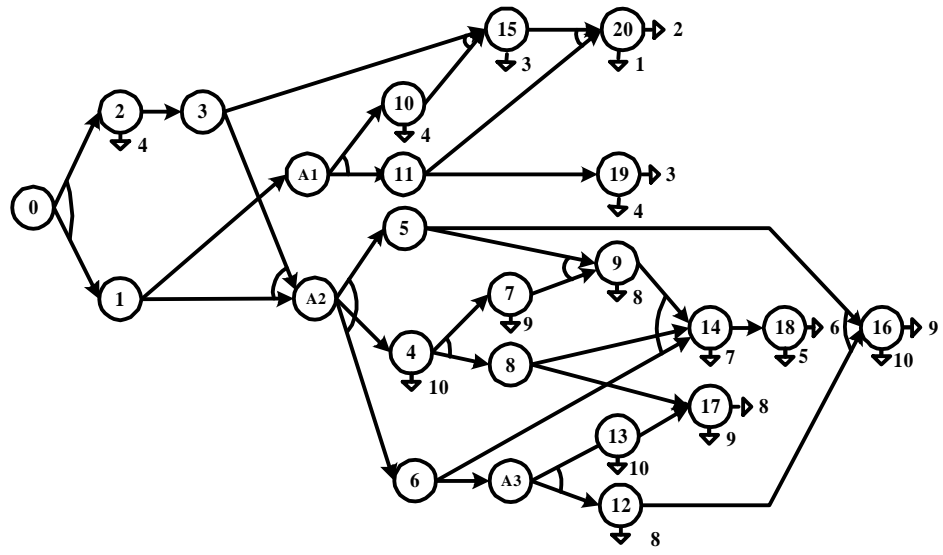


Figure 3.12 Disassembly graph of the 10-part ball-point pen example (Lambert, 1997)



**Figure 3.13** Transformed precedence diagram of the 10-part ball-point pen example

## CHAPTER 4

### PROFIT MAXIMIZATION PER CYCLE PROBLEM (PC)

A disassembly line is made up of an ordered sequence of *stations* often connected by some mechanical material handling equipment. The discarded products enter the line and proceed to downstream stations. There is a cost associated with opening and operating a station per unit time, called the *station cost*. This station cost might be associated with the wages of the operators working at the stations as well as time-variant costs associated with the tasks. The disassembly process consists of a set of *disassembly tasks* some of which result in removal of parts or materials that might have an associated demand. Performing a disassembly task may require certain equipment and/or machines and certain operator skills, resulting in a *task cost* that is incurred every time the task is performed. The time required to perform a disassembly task is called the *task time*. Technological and physical conditions define *precedence relations* among disassembly tasks. At each station, the assigned set of disassembly tasks is performed within the *cycle time* which is common to all stations if the line is paced. Although unpaced disassembly lines are more suitable when task time variability is high, as it is in disassembly, we consider paced disassembly lines for simplicity. *Revenue* is obtained for each unit of part removed from the discarded product if there is demand for it. Revenue can be positive or negative. A negative revenue implies that there is an associated *disposal cost*. In the presence

of revenues and costs, discarded products should be disassembled partially. That is, tasks should be performed and parts should be released as long as it is profitable to do so. Hence the *profit oriented DLBP* can be stated as the assignment of disassembly tasks to an ordered sequence of stations such that the precedence relations are satisfied and the profit is maximized. Our profit oriented DLBP treats the number of stations and the cycle time as decision variables.

There are several reasons that lead us to treat the cycle time as a decision variable rather than a parameter in our study. The first consideration is that in DLBP parts and subassemblies can be demanded in different quantities. The second reason arises due to partial disassembly and meeting demand as long as it is profitable to do so. In ALBP, the demanded entity is the end product whose demand must be fulfilled and therefore determines the production rate and the cycle time. In DLBP, new bases need to be defined so that disassembly rate and the corresponding cycle time can be properly determined. The third reason is that returned products, which include discarded products and commercial returns, are perishable assets. Since the value of these products decreases over time, the disassembly cycle time needs to be minimized to speed up recovery of products and materials. For these reasons we let the cycle time be a decision variable and include it in the station cost component of our profit maximizing objective function.

We consider two versions of the problem. In the *profit maximization per cycle problem (PC)*, we maximize the profit for a single disassembly cycle. In partial DLBP, revenue is obtained as long as there is demand for the released parts. When demand is satisfied for some parts, performing some tasks may not be profitable any longer. Hence, using the same PC solution in every cycle does not necessarily maximize the profit. Therefore, we define *profit maximization over the planning horizon problem (PH)*. In this version, the planning horizon consists of

multiple cycles each of which may have a different cycle length and a different line balance. We discuss formulation and solution of PC in this chapter and PH in the next chapter.

We summarize our problem environment in Section 4.1. We present a mixed integer mathematical programming formulation for PC in Section 4.2. In Section 4.3, we briefly analyze the optimal solution of an instance of PC problem to understand the nature of the problem and state our motivation for using heuristics. We present our solution procedure and upper bounding scheme in Section 4.4. We conclude this chapter with experimental settings and computational results.

#### **4.1. PROBLEM ENVIRONMENT**

We study the deterministic version of DLBP and adopt some assumptions of Güngör and Gupta (2001b and 2002). The assumptions of our DLBP formulation are as follows.

- A1. A single discarded product is to be partially disassembled on a paced disassembly line. All discarded products have identical configuration, i.e. each discarded product contains the same parts. The number of units of a certain part available in a discarded product can be one or more.
- A2. The disassembly precedence relations are given in the form of an AND/OR graph, a disassembly graph, a disassembly precedence matrix (DPM) or any other equivalent representation.
- A3. A disassembly task may result in removal of parts or subassemblies. Those that do release parts or subassemblies destroy a (parent) subassembly and create one or more (child) parts or subassemblies. However after the completion of each task, exactly one main subassembly remains on the line to be further disassembled. Note that this

assumption is not restrictive in the sense that the main subassembly may in fact consist of a number of detached subassemblies. For simplicity, we refer to the released parts or subassemblies as just “parts”, and the main subassembly as “the subassembly”.

- A4. The task times and costs are deterministic and known. They are independent of the station at which the task is performed. These parameters are set to zero for the dummy tasks.
- A5. A disassembly task cannot be split among two or more workstations.
- A6. The demand quantities and revenues (which can be positive or negative) for parts are deterministic and known. Every released part with a nonnegative revenue value generates the corresponding revenue upon demand satisfaction. A negative revenue value implies that there is a disposal cost associated with the released part and there is no demand for it.
- A7. A fixed station cost is incurred for opening and operating a station per unit time.
- A8. An upper limit on the cycle time is provided by the decision maker in relation with the desired disassembly rate.
- A9. The supply of discarded products is infinite.

Note that although in A6 and the subsequent stages of this study we consider and model parts with disposal costs. Another way of representing this could be by simply adding the disposal cost to the associated task cost. In such a representation, additional constraints ensuring the assignment of tasks that have to incur the disposal cost must be incorporated.

Assumption A9 implies that the supply of discarded products is more than the number of units that can be assembled according to the upper limit in cycle time defined in assumption A8.



## 4.2. PROBLEM FORMULATION

Before proceeding with the formulation we present our notation.

$n$	Number of actual tasks on the disassembly precedence matrix.
DMY	Index set of all dummy tasks, $DMY = \{0, A1, A2, A3, \dots, D\}$ , where task 0 is the first dummy task which precedes all tasks with no predecessors, task D is the last dummy task such that all tasks are its OR predecessors, and task $A_y$ is the $y^{\text{th}}$ dummy task inserted to simplify the precedence relations $y = 1, 2, \dots$
$i$	Disassembly task index, $i = 0, 1, 2, \dots, n, A1, A2, \dots, D$
$j$	Part index, $j = 1, 2, \dots, J$
$k$	Station index, $k = 1, 2, \dots, K$ ( $K = n$ when no upper bound is used)
$t_i$	Task time of disassembly task $i$
$c_i$	Cost of disassembly task $i$
PAND( $i$ )	Index set of AND predecessors of task $i$
POR( $i$ )	Index set of OR predecessors of task $i$
SOR( $i$ )	Index set of OR successors of task $i$
$K_{UB}$	Upper bound on number of stations
$m_{i,j}$	Number of units of part $j$ released by task $i$ . $m_{i,j} \geq 1$ if task $i$ releases part $j$ . $m_{i,j} = -1$ if part $j$ is in fact that a subassembly which might be further disassembled by task $i$
$d_j$	Demand for part $j$
$r_j$	Revenue realized for fulfilling per unit demand of part $j$
$R^+$	Index set of parts with nonnegative revenue
$R^-$	Index set of parts with negative revenue

- $CT_U$       Upper limit on cycle time
- $S$             Station cost for keeping a station open per unit time

Our decision variables are as follows.

- $q_j$           Number of units of revenue generating part  $j$  released
- $x_{i,k} = \begin{cases} 1 & \text{if task } i \text{ is assigned to station } k \text{ } i=0, 1, \dots, D \text{ and } k = 1, 2, \dots, K \\ 0 & \text{otherwise} \end{cases}$
- $CT$           Cycle time
- $u_k = \begin{cases} 0 & \text{when } x_{D,k} = 0 \\ CT & \text{when } x_{D,k} = 1 \end{cases}$

The proposed mathematical formulation for profit maximization per cycle problem is as follows.

**PC:**

$$\max \quad \sum_{j=1}^J r_j q_j - \sum_{i=0}^D \sum_{k=1}^K c_i x_{i,k} - S \sum_{k=1}^K k u_k \quad (4.1)$$

$$\text{s. t.} \quad x_{i,k} \leq \sum_{h=1}^k x_{l,h} \quad \forall k, i \text{ and } l \in \text{PAND}(i) \quad (4.2)$$

$$x_{i,k} \leq \sum_{h=1}^k \sum_{l \in \text{POR}(i)} x_{l,h} \quad \forall k \text{ and } i \quad (4.3)$$

$$\sum_{k=1}^K \sum_{l \in \text{SOR}(i)} x_{l,k} \leq \sum_{k=1}^K x_{i,k} \quad \forall i \quad (4.4)$$

$$\sum_{l \in \text{SOR}(i)} x_{l,k} \leq \sum_{h=1}^k x_{i,h} \quad \forall k \text{ and } i \quad (4.5)$$

$$\sum_{k=1}^K x_{i,k} \leq 1 \quad \forall i \quad (4.6)$$

$$\sum_{i=0}^D t_i x_{i,k} \leq CT \quad \forall k \quad (4.7)$$

$$q_j \leq d_j \quad \forall j \in R^+ \quad (4.8)$$

$$q_j \leq \sum_{i=0}^D \sum_{k=0}^K m_{i,j} x_{i,k} \quad \forall j \in R^+ \quad (4.9)$$

$$q_j \geq \sum_{i=0}^D \sum_{k=0}^K m_{i,j} x_{i,k} \quad \forall j \in R^- \quad (4.10)$$

$$\sum_{k=1}^K k x_{i,k} \leq \sum_{k=1}^K k x_{D,k} \quad \forall i \quad (4.11)$$

$$x_{i,k} \leq \sum_{i=0}^D t_i x_{i,k} \quad \forall k, i \in \text{DMY} \quad (4.12)$$

$$u_k \leq \left( \sum_{i=0}^D t_i \right) x_{D,k} \quad \forall k \quad (4.13)$$

$$\sum_{k=1}^K u_k \geq CT \quad (4.14)$$

$$CT \leq CT_U \quad (4.15)$$

$$CT \geq 0 \quad (4.16)$$

$$u_k \geq 0 \quad \forall k \quad (4.17)$$

$$q_j \geq 0 \quad \forall j \quad (4.18)$$

$$x_{i,k} \in \{0, 1\} \quad \forall i \text{ and } k \quad (4.19)$$

The objective function (4.1) maximizes profit per cycle and consists of three terms. The first term is the total revenue realized by the partial disassembly of a discarded product. The second term is the total cost of performing the assigned disassembly tasks. The third term is related with the total fixed cost of opening and operating stations throughout the cycle. Note that this cost term involves the product of the number of stations opened and the cycle time, both of which are decision variables. The arising nonlinearity is resolved by introducing decision variable  $u_k$  and corresponding constraints given in (4.13) and (4.14).

Note that, when the station cost is set to zero, the problem reduces to disassembly process planning (task and part selection) problem where  $K = 1$ .

The first four constraint sets given by (4.2) through (4.5) are related with the precedence relations. The first constraint set enables the assignment of task  $i$  to station  $k$  only if all its AND predecessors are already assigned to stations 1 through  $k$ . The second set prevents assignment of task  $i$  to station  $k$  unless at least one of its OR predecessors is assigned to one of the stations 1 through  $k$ . The third and fourth constraint sets are related with OR successor relations. The former set given in (4.4) guarantees assignment of at most one of the OR successors of task  $i$  (to some station) if task  $i$  is to be performed. The latter set represented by (4.5) assures assignment of an OR successor of task  $i$  to station  $k$ , if task  $i$  is previously assigned to stations 1 through  $k$ .

Constraint set (4.6) indicates that a task might be assigned to at most one station. Tasks may not be assigned to stations due to partial disassembly and restrictions that might be imposed by the precedence relations. The binary nature of the task assignment variables  $x_{i,k}$  are reflected in (4.19). Note that if some disassembly tasks must be performed, this can be assured by using equality version of (4.6) for those tasks.

The cycle time constraint presented in (4.7) enforces the work content of each station to remain within the cycle time.

Constraint sets (4.8) through (4.10) are related with determining the number of revenue generating units of part  $j$  released,  $q_j$ . Constraint set (4.8) ensures the total number released to be no more than the demand of part  $j$ , if the revenue of part  $j$  is nonnegative. Next two sets are

used to correctly determine the number of revenue generating units of part  $j$  released. The  $q_j$  value is determined as the total number released by assigned tasks with positive  $m_{i,j}$ . According to (4.9), if the revenue of part  $j$  is nonnegative,  $q_j$  cannot exceed the total number of parts  $j$  that are released by the assigned tasks. For example, consider the case where task 1 releases part 1 (which in fact is subassembly AB), hence  $m_{1,1} = 1$ . Suppose task 2 disassembles subassembly AB to release parts A and B (let 2 and 3 be part indices of A and B, respectively), hence  $m_{2,1} = -1$ ,  $m_{2,2} = 1$  and  $m_{2,3} = 1$ . With these parameter values, if task 1 is assigned but task 2 is not, then part 1 is released and can be used to satisfy the demand, if any. If both tasks 1 and 2 are performed, no part 1 is released ( $m_{1,1} \sum_k x_{1,k} + m_{2,1} \sum_k x_{2,k} = 0$ ) to satisfy the demand since it is further disassembled to release parts A and B. If revenue of part  $j$  is negative, (4.10) forces  $q_j$  to take the correct total number released under maximization objective. The model may release parts with negative revenue, when positive revenue gained from parts released later along the same branch of the precedence graph justifies this. Moreover,  $q_j$  is defined to be a continuous decision variable by constraint (4.18).

Constraint set (4.11) assures that no task is assigned to the stations following the station to which dummy task D is assigned. Constraint set (4.12) assures assignment of dummy tasks to stations to which some actual tasks are assigned. These two constraint sets can be perceived as technical constraints.

Constraints (4.13) and (4.14) are used to determine the total available time on the line, which is normally found by multiplying the number of stations opened and the cycle time. This is required to find the total station cost and results in nonlinearity. Decision variable  $u_k$  is used

for linearization. By definition,  $u_k$  takes a value of zero if the dummy task D is not assigned to station  $k$  and a value equal to the cycle time if it is assigned to station  $k$ . This relationship is conveyed using these two constraints. In (4.13),  $u_k$  is defined to be less than or equal to  $x_{D,k}$  times the total processing time of all tasks (which is used instead of big  $M$  since it is the largest value that can be attained when all tasks are performed). Since task D is assigned to the last of the  $k$  stations opened, only one of the  $u_k$  variables will take a positive value. In order to guarantee the corresponding  $u_k$  variable to take the value of the cycle time we impose (4.14). Note that  $\sum_{k=1}^K k u_k$  represents the total available time on the line. Moreover,  $u_k$  is defined as a continuous decision variable by constraint (4.17).

A constraint to assure that the cycle time decision variable is less than or equal to its upper limit determined by the decision maker and another one that defines  $CT$  as a continuous decision variable are added as (4.15) and (4.16), respectively.

### 4.3. AN EXAMPLE

Consider Güngör and Gupta's (2002) eight-part personal computer example whose precedence diagram is depicted in Figure 3.10. Figure 4.1 shows the number of parts released by each task. Part revenues ( $r_j$ ), part demands ( $d_j$ ), task times ( $t_i$ ) and task costs ( $c_i$ ) are summarized in Table 4.1. Assume that the upper limit on the cycle time is given as 40 time units.

$$m_{i,j} = \begin{matrix} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} \\ \mathbf{1} & \left[ \begin{array}{cccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right. \end{matrix}$$

**Figure 4.1** Number of parts released by tasks information for PC example

**Table 4.1** Part revenue and demand; task time and cost information

$j$	$r_j$	$d_j$	$i$	$t_i$	$c_i$
Top Cover (TC)	27	26	1	15	7
Floppy Drive (FD)	16	79	2	7	16
Hard Drive (HD)	22	99	3	17	19
Back Plane (BP)	21	34	4	15	10
PCI Cards (PCI)	24	73	5	9	6
RAMs (RAM)	14	94	6	12	4
Power Unit (PU)	30	48	7	7	18
Mother Board (MB)	31	45	8	4	16

The optimal solutions found under seven different station costs are summarized in Table 4.2. Each optimal solution is described with its profit ( $\pi^*$ ), number of stations ( $K^*$ ), cycle time ( $CT^*$ ), assignment of tasks to stations (i.e. the station number each task is assigned to) and the quantities of parts released. Note that the CPU time required to solve each instance using commercially available optimization software such as CPLEX 9.0 took less than 1 second.

**Table 4.2** Summary of optimal solutions

$S$	$\pi^*$	$K^*$	$CT^*$	Task assignments								Quantity of parts released							
				1	2	3	4	5	6	7	8	TC	FD	HD	BP	PCI	RAM	PU	MB
0.25	81	4	22	1	1	3	4	2	2	4	4	1	1	1	1	1	2	1	1
0.50	59	4	22	1	1	3	4	2	2	4	4	1	1	1	1	1	2	1	1
0.75	38	3	24	1	2	2		1	3	3	3	1	1	1		1	2	1	1
1.00	20	3	24	1	2	2		1	3	3	3	1	1	1		1	2	1	1
1.25	8	1	24	1				1				1				1			
1.50	2	1	24	1				1				1				1			
1.75	0	0	0																

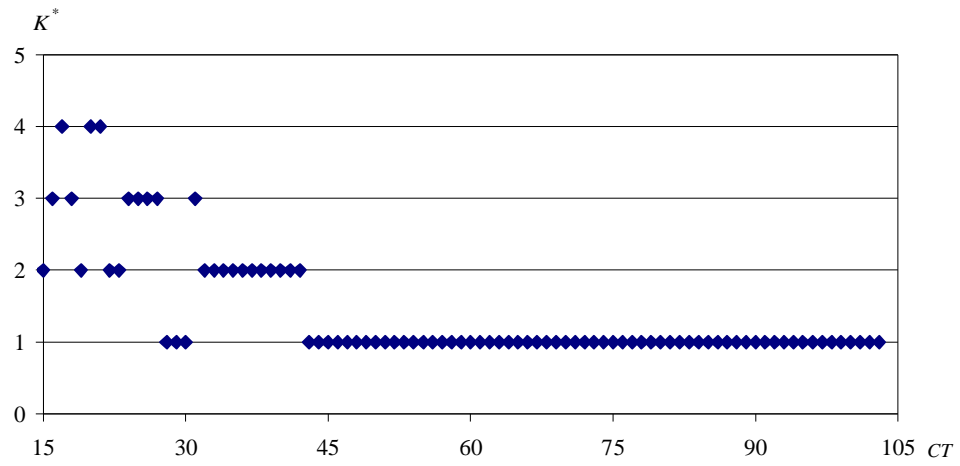
When the optimal solutions of the seven station costs are compared, a decrease in the profit and the number of stations opened is seen as the station cost increases. The set of selected tasks and released parts also change with the increasing station cost. In this instance this change is in the form of eliminating some tasks and parts. However in other instances, which especially involve AND/OR graph based precedence diagrams, the set of selected tasks and released parts change drastically.

Some further analysis is required to understand the relationship between the cycle time and the profit per cycle. For this purpose we fix the station cost at 1.00 and solve PC with all possible cycle time values to explore how profit changes as a function of cycle time.

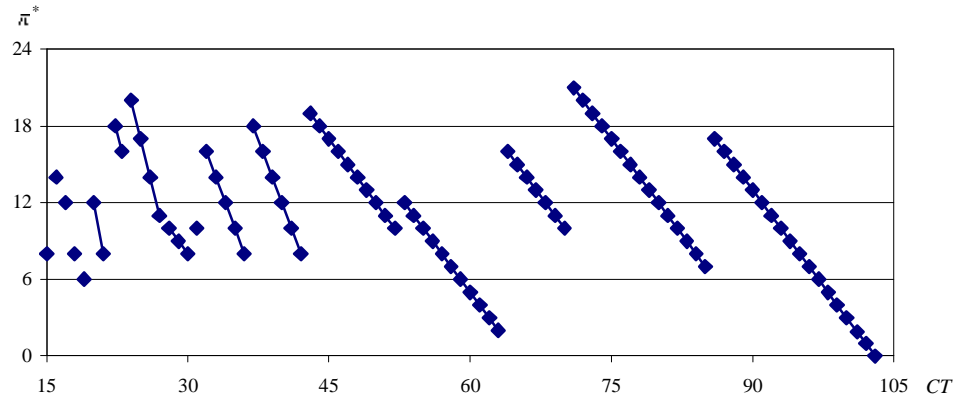
We start by setting the cycle time to the minimum feasible task time. We take the minimum instead of the maximum because the cycle time can be less than the maximum task time in partial disassembly. However, since the time of the first task (which precedes all other tasks) is 15, a cycle time shorter than 15 makes the problem infeasible. We then increase the cycle time until the profit becomes zero due to the increase in station cost. Hence we solve PC with every integer cycle time between 15 and 103. The profit attained ( $\pi^*$ ), the number of stations opened ( $K^*$ ), time available on the line ( $K^* CT$ ), the maximum work content ( $WC_{max}$ ), assignment of tasks to stations, and parts released are given in Appendix A for each cycle time value.

Figure 4.2 illustrates the change in the number of stations as cycle time increases. Figure 4.3 depicts how profit changes as a function of cycle time in this instance. Figure 4.4 shows the behavior of the profit as a function of the available time on the line ( $K^* CT$ ).

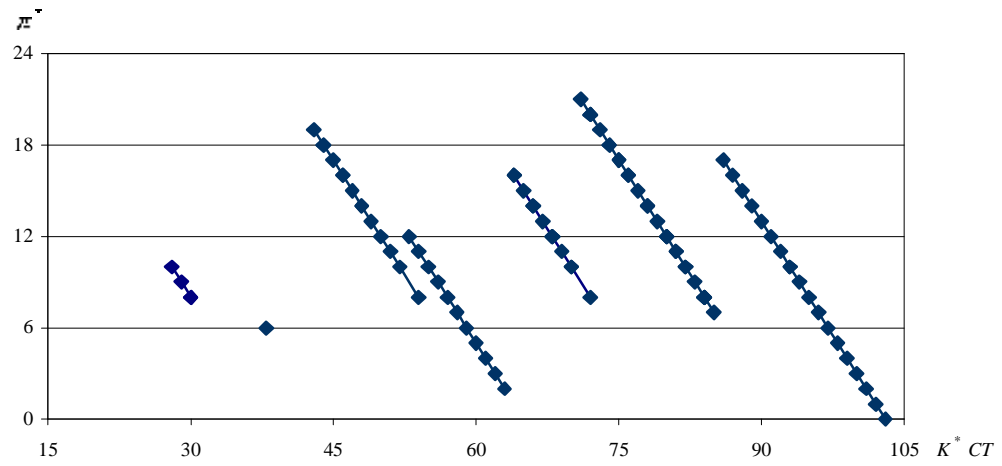




**Figure 4.2** Number of stations opened versus cycle time



**Figure 4.3** Profit versus cycle time



**Figure 4.4** Profit versus available time on the line

Our observations on these three figures include the following.

- § According to Figure 4.3, the profit as a function of the cycle time is piecewise linear. Each line segment corresponds to line balances where the same part set is released via the same set of tasks on the same number of stations. Within each line segment, further increase in the cycle time (beyond the maximum work content of the stations) results in idle time in all stations and hence the profit decreases linearly with the slope being equal to the increase in the available time on the line. This observation can easily be verified by Appendix A as the maximum work content remains unchanged for each such line segment.
- § The jumps between two profit line segments in Figure 4.3 correspond to a change in the number of stations and/or the selected task set and the released part set.
- § In Figure 4.4 each connected line segment represents solutions with the same set of selected tasks and released parts. However, the number of stations used given the cycle times may vary from one solution to another.
- § Among the solutions in which the same tasks are performed in different number of stations, the highest profit is attained when the available time on the line ( $K^*CT$ ) is minimum. For example in Appendix A, for cycle times in the [24, 27] and [37, 42] intervals, the same tasks are assigned to 3 and 2 stations, respectively. A maximum profit of 20 is attained when the available time on the line has a minimum value of 72 with 3 stations. This is an important observation since it pinpoints a basic aspect that can be utilized in developing solution procedures for PC. It clearly shows that it might be misleading to minimize the cycle time for a given number of stations. Such an approach may result in getting stuck at a local optimum. Hence, treating the cycle time as a decision variable prevents the model from getting stuck at local optima. This is achieved

by the third term in the objective function, which is a linearized form of the total time available on the line ( $K^* CT$ ).

- § Increasing the cycle time may ultimately reduce the number of stations and the idle time, and this increases the profit. However, this is highly interrelated with the selection of tasks, released parts and the resulting line balance. Therefore, although it is relatively easy to detect such relations in a given solution, predicting them prior to solving the problem would be impossible. This indicates that the task and part selection decisions must be made jointly with the task assignment decisions.
- § Another remark is on the upper limit on cycle time, which we assume to be given. Setting a tight upper limit to achieve a high disassembly rate might lead to a suboptimal solution as the truly optimal cycle time might have been excluded. This means that the determination of such an upper limit on the cycle time deserves special treatment and we acknowledge this fact by conducting a sensitivity analysis on this parameter.

Developing solution procedures for line balancing problems, which simultaneously minimize the number of stations and the cycle time, may require generation of all feasible combinations. This is by itself computationally intractable even when the set of tasks to be performed is known as in assembly or complete disassembly. The addition of task selection under partial disassembly further increases the complexity of our problem. If one had to develop a branch and bound scheme for the problem, choosing an effective branching strategy would be a challenge. It would be very difficult to decide whether one should start branching with the cycle time, part selection, task selection, or task assignment. Such a traditional procedure would also be very time consuming.

As a result of our analysis we conclude that it would be wiser to confine our solution procedure to a heuristic approach. The nature of disassembly requires line designs with flexibility in mind, allowing restructuring on a continuous basis as the input flow of discarded products and the demand for released parts or materials change. In such an environment, the problem is not to make a single-shot decision, but to make it repeatedly. Therefore, we aim at finding near optimum solutions through fast solution procedures.

#### **4.4. BOUNDING SCHEMES**

To jointly consider the task and part selection aspects with the line balancing aspect of the problem, we base our heuristic solution procedure on our mathematical formulation. The main idea is to solve a strengthened version of the linear programming (LP) relaxation of the PC formulation, to amend it to build up a feasible solution, and then to perform neighborhood search to improve it. Since our objective is to maximize profit, the optimal profit of the strengthened linear programming relaxation is an upper bound, and the amended and improved solution's profit provides a lower bound on profit. Below, we discuss bounding schemes in detail.

##### **4.4.1. Upper Bound on Profit**

Solving the linear programming relaxation of problem PC (PC-LP) yields an upper bound on the profit. The binary decision variables in the PC formulation indicate the assignment of tasks to stations if they are to be performed. Moreover, the binary nature of the assignment variables is fully utilized in precedence constraints and in constraints (4.13) and (4.14) that linearize the multiplication of cycle time and number of stations opened. Solving the linear programming relaxation may affect all these constraints.

Unfortunately, the PC-LP produces a weak relaxation. Our analysis of a number of preliminary solutions yields the following observations. (Using the previous example with a station cost of 1.0 these observations are illustrated in Appendix B.)

- O1. All  $K$  ( $K = n$ ) stations have tasks assigned to them, that is all stations are opened and none of them is empty.
- O2. Tasks are assigned to more than one station.
- O3. The total assignment percentage of a task ( $\sum_{k=1}^K x_{i,k} * 100$ ) may be less than 100%.
- O4. Hence the total idle time and balance delay of the resulting line balance are zero.
- O5. The cycle time constraint ensures that the work content of each of the  $K$  stations is less than or equal to the cycle time. Fractional assignment of tasks to all  $K$  stations and the precedence relations are arranged such that the minimum  $CT$  is attained.
- O6. Similar to actual tasks, the last dummy task D is also assigned to more than one station. Hence any of stations to which task D is assigned can have a positive value for the decision variable  $u_k = x_{D,k} CT$ . Among them the objective function tends to select the station with the smallest index to minimize the station cost component of the objective function. This implies that the true cost of opening stations is greater than the value accounted for in the objective function. Also, the total time available on the line, determined by the sum of ( $k u_k$ ) variables over all stations, is less than or equal to the true total time used on the line.
- O7. For any station  $k > 1$ , the total fractional assignment of some tasks to stations 1 through  $k$  is greater than total fractional assignment of their respective AND and OR predecessors. This turns out to be the case especially when the predecessors are dummy tasks or tasks that do not release profitable parts. Note that a similar behavior is not observed in linear programming relaxation of OR successor constraint (4.4). This constraint guarantees that

the total assignment of an OR successor does not exceed the total assignment of its predecessor.

If we eliminate observations O6 and O7, a tighter upper bound on profit can be obtained. For this purpose we propose “*logical inequalities*” in PC-LP to assure the following:

1. Total time available on the line is correctly determined and reflected in the station cost term.
2. Total assignment of a task to stations 1 through  $K$  cannot exceed
  - § total assignment of each of its AND predecessors to stations 1 through  $K$ ,
  - § total assignment of all of its OR predecessors to stations 1 through  $K$ .

Note that the term “logical inequality” is taken from the study of Süral and Bookbinder (2003) on the single vehicle routing problem with unrestricted backhauls. These backhauls are optional and revenue generating. They provide mixed integer programming formulations of the problem and aim at solving their linear programming relaxations. Although the formulation they propose has several benefits such as incorporating subtour elimination constraints apriori and being easily adaptable to different types of constrained vehicle routing problems, it yields a weak linear programming relaxation. To strengthen it they use methods such as constraint disaggregation and coefficient improvement, lifting the subtour elimination constraints and inserting logical inequalities. As the solution of the linear programming relaxation does not prevent profitable backhauls to be visited more than once, they insert a new constraint that does not lead to such infeasible solutions and call them “logical inequalities”.

### Logical Inequality – 1

A logical inequality is added to the LP relaxation to ensure that the total time used over all stations does not exceed the total time available on the line.

$$\sum_{k=1}^K \sum_{i=0}^D t_i x_{i,k} \leq \sum_{k=1}^K k u_k \quad (4.20)$$

This new constraint can be rewritten by using the definition of  $u_k$  variable ( $u_k = x_{D,k} CT$ ) as follows:

$$\sum_{k=1}^K \sum_{i=0}^D t_i x_{i,k} \leq CT \sum_{k=1}^K k x_{D,k} \quad (4.20')$$

Note that constraint set (4.7) in PC guarantees that the total time used in each station does not exceed the cycle time. If constraints in this set are summed up over all  $K$  stations, the following surrogate constraint is obtained:

$$\sum_{k=1}^K \sum_{i=0}^D t_i x_{i,k} \leq \sum_{k=1}^K CT = CT K \quad (4.21)$$

The expressions on the left hand side of (4.20') and (4.21) are the same. However, the right hand side of (4.20') is less than or equal to the right hand side of (4.21), since the number of stations opened is always less than or equal to  $K$ , i.e.  $\sum_{k=1}^K k x_{D,k} \leq K$ . This shows that the valid inequality (4.20') is a stronger version of surrogate constraint (4.21).

## Logical Inequality - 2

Two forms of the second logical inequality are proposed, one for AND precedence relations and another one for OR precedence relations.

Constraint (4.22) guarantees that the total fractional assignment of task  $i$  does not exceed the total fractional assignment of each of its AND predecessors.

$$\sum_{k=1}^K x_{i,k} \leq \sum_{k=1}^K x_{l,k} \quad \forall i \text{ and } l \in \text{PAND}(i) \quad (4.22)$$

Note that the AND precedence constraint set (4.2) in PC allows assignment of task  $i$  to station  $k$  if all AND predecessors are assigned to one of the stations 1 through  $k$ . If constraints in this set are summed up over all  $K$  stations the following surrogate constraint is obtained:

$$\sum_{k=1}^K x_{i,k} \leq \sum_{k=1}^K \sum_{h=1}^k x_{l,h} \quad \forall i \text{ and } l \in \text{PAND}(i) \quad (4.23)$$

Constraints (4.22) and (4.23) have the same term on their left hand sides. However, the right hand side of (4.22) is less than or equal to the right hand side of (4.23). When task  $l$  is not assigned, both right hand sides take the value of zero. Otherwise, if task  $l$  is assigned to one of the stations 1 through  $K$  (to station  $k$  for instance), the right hand side of (4.22) becomes one and is less than or equal to the right hand side of (4.23) which takes a value of  $K-k+1$ . This depicts that constraint (4.22) is a stronger version of the surrogate AND precedence constraint (4.23).



Similarly, constraint (4.24) assures that the total fractional assignment of task  $i$  is less than or equal to the total fractional assignment of all of its OR predecessors.

$$\sum_{k=1}^K x_{i,k} \leq \sum_{k=1}^K \sum_{l \in \text{POR}(i)} x_{l,k} \quad \forall i \quad (4.24)$$

Again, we can sum up the OR precedence constraints of PC given in set (4.3) over all  $K$  stations to obtain the following surrogate constraint (4.25).

$$\sum_{k=1}^K x_{i,k} \leq \sum_{k=1}^K \sum_{h=1}^k \sum_{l \in \text{POR}(i)} x_{l,h} \quad \forall i \quad (4.25)$$

As in the previous cases constraint (4.24) is stronger than the surrogate constraint and therefore is a cut for the linear programming relaxation.

In the computational analysis section, we illustrate that the upper bound on profit found by solving the strengthened linear programming relaxation formulation (PC-SLP) is promising in terms of both solution quality and computation time.

#### 4.4.2. Lower Bound on Profit

In our problem, which tasks to perform (equivalently which parts to release), how many stations to open and what cycle time to use are issues that need to be decided, as well as how to balance the disassembly line to maximize the profit. All of these issues are highly interrelated.

Our lower bounding scheme starts with a construction heuristic that takes the total fractional assignment information from the upper bound solution (PC-SLP) and converts this solution to a feasible (integer) one. The construction heuristic uses an “assignment procedure” that assigns the selected tasks to stations so as to maximize the profit in finding the line balance. Then, a two step improvement heuristic reevaluates the task selection and their assignments imposed by the construction heuristic. In the subsequent sections we briefly summarize our construction and improvement heuristics.

#### **4.4.2.1. Construction Heuristic**

The construction heuristic starts by assuming all tasks with a positive fractional assignment in the upper bound solution are “selected” and proceeds with a station oriented assignment procedure.

The cycle time is a decision variable in our problem. It is bounded from below by the minimum task time of the selected tasks and from above by a prespecified upper limit. All possible integral values of the cycle time within these bounds are enumerated in ascending order. For each of these values, the assignment of selected tasks is repeated using the station oriented assignment procedure.

#### **Assignment Procedure**

Given the set of selected tasks and a cycle time, the assignment procedure uses priority values computed for the tasks based on the task times, released part revenues and the precedence relations to assign the selected tasks to stations. These priority values are also called numerical scores in the ALBP literature. The five numerical scores used are provided in Table 4.3 with

their definitions and time complexities. Before proceeding with the details of the numerical scores we introduce the following notation.

- $N$  Total number of tasks (including dummy tasks) in the precedence diagram
- $DEL(i)$  Index set of tasks that cannot be assigned to any station if task  $i$  is assigned since they are in the same OR successor relationship with task  $i$
- $F(i)$  Index set of immediate successors of task  $i$
- $F^*(i)$  Index set of all successors of task  $i$

**Table 4.3** Definition of numerical scores used

<b>Numerical Score</b>	<b>Definition</b>	<b>Complexity</b>
$NX_1(i)$	The sum of task times for task $i$ and all tasks in $F^*(i)$ , i.e. positional weight (Helgeson and Birnie, 1961)	$O(N^2)$
$NX_2(i)$	The number of tasks in $F(i)$ , i.e. number of immediate successors (Mastor, 1970)	$O(N)$
$NX_3(i)$	The number of tasks in $F^*(i)$ , i.e. number of all successors (Tonge, 1965)	$O(N^2)$
$NX_4(i)$	The sum of revenues generated by parts released by task $i$ and all tasks in $F^*(i)$	$O(N^2)$
$NX_5(i)$	The sum of task times over all tasks - $NX_1(i)$	$O(N^2)$

The first three numerical scores are borrowed from the ALBP literature. The fourth numerical score is incorporated since for each task it relates the precedence relation structure and the revenue component of the profit maximization objective. The fifth numerical score is included as a result of our preliminary computational analysis. We observed that the first three numerical scores did not generate a sufficiently wide range of line balances as they create similar patterns at different stages of the assignment. A fifth numerical score was introduced to allow more variety. This new numerical score, considering the total task time, is a kind of complement of the first numerical score.

Once the numerical scores are computed we use a “station oriented” assignment procedure. Note that the assignment procedure is applied independently for each numerical score. In ALBP literature, this procedure, when used with the first numerical score, is called the *ranked positional weight* (RPW) technique by Helgeson and Birnie (1961). We start with the first station. The subsequent stations are considered consecutively. In each iteration, the task with the highest numerical score which is assignable to the current station is selected and assigned. When none of the assignable tasks fit in the current station the current station is closed and a new station is opened.

To implement this assignment procedure we keep two updated lists. The first one, feasible list (FL), stores the set of tasks that are feasible for assignment in non-increasing order of the corresponding numerical score. The second list stores the set of tasks that have been put in the task ordering list (TOL), again in non-increasing order of the corresponding numerical score.

The first step starts with putting task 0 in FL, if task 0 has been selected. While FL is not empty, the first task in FL, call it task  $i$ , is removed and put in TOL. The set of tasks that are OR successors together with task  $i$ , i.e. elements of  $DEL(i)$ , are deleted from FL since at most one OR successor can be performed. Immediate successors of task  $i$  are put in FL, if they were selected and are precedence feasible (that is all of their AND predecessors and at least one of their OR predecessors are elements of TOL). Once task ordering is complete, the second step in RPW heuristic is to assign tasks in TOL to the first feasible station (Askin and Standridge, 1993) such that the cycle time is not exceeded.

The above task assignment procedure is repeated for each combination of the cycle time and numerical score. The resulting line balance is evaluated to compute the profit. Among all these

solutions generated the one yielding the highest profit is selected. If the highest profit is negative, then the best solution is taken as no disassembly with a profit of zero.

The time complexity of the assignment procedure for a numerical score and cycle time combination is  $O(N \log N)$ . Hence, the time complexity of the construction heuristic for a single cycle time value is  $O(N^3 \log N)$ , when numerical score computations are included. Thus, the overall complexity of the heuristic is pseudopolynomial.

The differences between our assignment procedure and station oriented assignment procedures of ALBP are due to handling of the precedence relations specific to disassembly and include the following:

- § Precedence feasibility check incorporates OR precedence.
- § Due to OR successor relation, which allows assignment of at most one of the OR successors of a task, after placement of each task  $i$  in TOL, the FL list is examined and tasks that are elements of  $DEL(i)$  are removed from the FL list.

#### **4.4.2.2. Improvement Heuristic**

After an initial solution is obtained using the construction heuristic, a two step improvement heuristic is applied. If the initial set of tasks selected by the PC-SLP solution contains tasks with fractional total assignments (their total assignments are less than 1.0), a task deletion heuristic is used in the first step. Otherwise this step is omitted. The task deletion heuristic facilitates elimination of tasks that were fractionally assigned in the upper bound solution and become redundant after other fractional tasks are fully assigned by the construction heuristic. Typically, redundant OR predecessors can be eliminated by task deletion. In the second step a task insertion heuristic is executed. In our lower bounding scheme, task selection and

assignment decisions are not made simultaneously and the idle time cost does not have an effect on the task selection. The task insertion heuristic has a potential of increasing the profit by decreasing station idle times.

### **Step 1. Task Deletion**

The set of tasks with fractional total assignments in the PC-SLP solution are added to a deletion list (DL), in a forward breadth first search order starting with task 0. The solution of the construction heuristic is taken as the incumbent.

While the DL is not empty, the first candidate task in DL is removed from the selected tasks set and the assignment procedure given in Section 4.4.2.1 is executed. If the profit found at the end of the assignment procedure is greater than the profit of the incumbent solution, then the candidate task is deleted, otherwise the candidate task is labeled as selected and put back in the selected tasks list.

In a second trial, the set of tasks with fractional total assignments in the upper bound solution are added to DL, but this time in a backward breadth first search order starting with task D. The initial incumbent solution is again the solution of the construction heuristic. Similar to the forward case, while the DL is not empty, the first candidate task in DL is removed from the selected tasks set and the assignment procedure is executed. At each iteration a task is either removed from or left in the selected tasks lists permanently and the incumbent solution is updated accordingly.

The profits of the best forward run and backward run are compared and the one with the highest profit is recorded as the solution of the task deletion heuristic.

## **Step 2. Task Insertion**

The set of tasks that are not in the set of selected tasks are put in an insertion list (IL) first in a forward breadth first search order that starts with task 0, and then a backward breadth first search order that starts with task D.

While the IL list is not empty, the first candidate task in IL is removed and added to the selected tasks set and the assignment procedure is applied. If the profit evaluated at the end of the assignment procedure is greater than the profit of the incumbent solution, the candidate task is inserted in the selected tasks set. Note that tasks that are candidates for insertion but are not precedence feasible since some of their predecessors are not in the selected tasks list, are not assigned in the assignment procedure. Hence they do not lead to an improvement in the profit and are marked as not selected.

The same steps are repeated in a backward manner. The profits of the forward run and the backward run are compared and the one with the highest profit is recorded as the solution of the task improvement heuristic.

### **4.4.3. A Heuristic Upper Bound on Number of Stations ( $K_{UB}$ )**

In solving PC and its strengthened linear programming relaxation PC-SLP, the upper bound on the number of stations that can be opened ( $K$ ) is ordinarily set to the number of actual tasks ( $n$ ). This is a very loose bound, and for large problem instances, an upper bound on the number of stations is needed to reduce the computation time. The idea here is to allow assignment of as many tasks as possible that lead to release of revenue generating parts. To facilitate this we reduce the total cost in the objective function component by multiplying it with a small value  $\epsilon$

(we still need this term to prevent assignment of multiple OR predecessors unnecessarily). We also set  $K = 1$ . The resulting objective function is presented in (4.26).

**PK<sub>UB</sub>:**

$$\max \quad \sum_{j=1}^J r_j q_j - \varepsilon \sum_{i=0}^D (c_i + S t_i) x_{i,1} \quad (4.26)$$

s. t. (4.2) - (4.6), (4.8) - (4.12), (4.18), (4.19)

A set of tasks is selected by solving the linear programming relaxation of PK<sub>UB</sub>. Then, all tasks with positive fractional assignments are assigned to stations using the assignment procedure given in Section 4.4.2.1. In applying the assignment procedure, the cycle time is set to the maximum task time of the selected tasks to ensure that the assignment yields the maximum number of stations. The maximum number of stations found by different numerical scores is recorded as an estimate of the upper bound on the number of stations. Note that  $K_{UB}$  found in this manner may be smaller than the true upper bound. We resort this heuristic upper bound only for large problem instances

#### 4.4.4. Example Continued

Recall the personal computer example of Section 4.2 example with eight parts and eight tasks, taken from the DLBP literature (Güngör and Gupta, 2002).

The upper bound on profit obtained by solving PC-SLP with  $CT_U = 40$  and  $K = n = 8$  under seven different station costs, and the total fractional assignments of tasks and released parts are given in Table 4.4.



The numerical scores calculated for our example are presented in Table 4.5. Table 4.6 depicts the lower bound solutions.

**Table 4.4** PC-SLP solution for the example

$S$	$\pi^*$	$K^*$	$CT^*$	Selected tasks								Quantity of parts released								
				1	2	3	4	5	6	7	8	TC	FD	HD	BP	PCI	RAM	PU	MB	
0.25	81.50	8	11.4	+	+	+	+	+	+	+	+	+	1	1	1	1	1	2	1	1
0.50	60.00	8	10.8	+	+	+	+	+	+	+	+	+	1	1	1	1	1	2	1	1
0.75	38.75	8	8.9	+	+	+		+	+	+	+	+	1	1	1		1	2	1	1
1.00	23.50	8	6.7	+	+	+		+	+	+	+	+	1	0.5	0.5		1	2	0.5	0.5
1.25	10.13	8	6.7	+	+	+		+	+	+	+	+	1	0.5	0.5		1	2	0.5	0.5
1.50	2.00	8	3.4	+				+					1				1			
1.75	0.00	8	0.0																	

**Table 4.5** Numerical scores for the example

	Task number									
	0	1	2	3	4	5	6	7	8	D
$NX_1$	86	86	45	55	15	35	38	22	26	0
$NX_2$	2	4	3	3	1	2	2	2	2	0
$NX_3$	9	8	5	5	1	4	4	2	3	0
$NX_4$	199	199	126	132	21	106	110	51	82	0
$NX_5$	0	0	41	31	71	51	48	64	60	86

**Table 4.6** Lower bound solutions for the example

$S$	$\pi^*$	$K^*$	$CT^*$	Task assignments								Quantity of parts released							
				1	2	3	4	5	6	7	8	TC	FD	HD	BP	PCI	RAM	PU	MB
0.25	81	4	22	1	1	3	4	2	2	4	4	1	1	1	1	1	2	1	1
0.50	59	4	22	1	1	3	4	2	2	4	4	1	1	1	1	1	2	1	1
0.75	37	4	22	1	1	3	4	2	2	4	4	1	1	1	1	1	2	1	1
1.00	18	2	22	1	1			2	2			1	1			1	2		
1.25	8	1	24	1				1				1				1			
1.50	2	1	24	1				1				1				1			
1.75	0	0	0																

When the station cost is increased to 0.75 task 4 is not selected in the PC-SLP solution. The revenue yielded by task 4 is 21 while its task cost and the station cost associated with its task

time adds up to 21.25. Since the total cost of task 4 is greater than the arising revenue, task 4 is not profitable and hence is not selected in the upper bound solution. However, it is assigned to station 4 in the lower bound solution simply because it reduces the total idle time on the line. (The work content of station 4 is 11 due to the assignment of selected tasks 8 and 7 and there is an idle capacity of 10 time units.) This instance once again clearly illustrates that task selection and balancing decisions must be made jointly (Pinto *et al.*, 1983). As they are not made jointly in our construction heuristic, we utilize the task insertion heuristic to provide a chance for recovery.

For station cost 1.00, one can see how the task deletion step of our improvement heuristics eliminates selected tasks 3, 7 and 8.

Finally in Table 4.7 we illustrate the results of heuristic upper bound on the number of stations. We give the number of stations found in the optimal PC solution ( $K_{PC}^*$ ) and the lower bound solution ( $K_{LB}$ ).

**Table 4.7** Heuristic upper bound on the number of stations

$S$	$K_{UB}$	$K_{PC}^*$	$K_{LB}$
0.25	6	4	4
0.50	6	4	4
0.75	6	2	4
1.00	6	2	2
1.25	6	1	1
1.50	6	1	1
1.75	6	0	0
2.00	6	0	0

#### 4.5. COMPUTATIONAL ANALYSIS

The computational analysis conducted is based on ten problems. Five of these problems are originally created during the conduct of this study. The sixth problem uses the precedence relation and part release structure of the personal computer example (Güngör and Gupta, 2002). Precedence relations of the seventh and ninth problems are taken from the disassembly process planning literature (Lambert, 1999) and are transformed to equivalent task based precedence diagrams. They are originally given by disassembly graphs. Two versions of the seventh and ninth problems are used in our experimentation. In the first version (seventh and ninth problems) it is assumed that there is demand only for end parts. In the second version (eighth and tenth problems) it is assumed that there is also demand for all feasible subassemblies released at different stages of the disassembly. These subassemblies may not be further disassembled if they are used in satisfying their demand. The number of actual tasks in these ten problems varies between 8 and 30 while the number of parts released is between 4 and 29. Each problem is represented by a code that denotes the author(s) who defined the problem. This code also includes information on the number of actual tasks and the number of parts (or subassemblies) that have demand so as to give an insight on the size of the problem. For instance, GUN8T8 represents Güngör and Gupta's (2002) personal computer example with 8 actual tasks and 8 parts while LAM20T10 represents Lambert's (1999) ball-point pen example with 20 actual tasks and 10 parts. In Table 4.8 we present the basic features of these ten problems and in Appendix C we portray their precedence relations.

Each of these ten problems is conceived as a problem category. Keeping the number of tasks, the number of parts/subassemblies released/used by each task and precedence relations fixed, ten instances are created for each problem category. Parameters such as task times and costs, part demands and revenues vary from one instance to another. The base station cost per unit

time is set to unity. However, eight different levels of the station cost that range from a quarter of the base level to twice of the base level are used in solving each instance.

**Table 4.8** Basic features of the problems

Problem	Actual Tasks	Total Tasks	Total Parts	AND Prec.	OR Prec.	OR Succ.
GUN8T8	8	8	8	10	2	0
AKO8T6	8	11	6	9	3	0
AKO20T4-A	20	20	4	15	6	1
AKO20T4-B	20	20	4	15	5	1
AKO20T4-C	20	20	4	15	6	1
LAM20T10	20	25	10	5	8	5
LAM20T24	20	25	24	5	8	5
LAM30T10	30	49	10	16	11	10
LAM30T29	30	49	29	16	11	10
AKO30T12	30	39	12	22	9	5

In generating the task times and costs, we assume that the sum of task times is equal to the expected sum of task costs. The main reason of this scheme is to create a tradeoff between task costs and the base station cost due to task times. The task times for an instance are generated from discrete uniform distribution between one and twenty. After the task times of an instance are generated their mean is calculated. The task costs are generated using discrete uniform distribution where the lower limit is one and the upper limit is found using the mean of the generated task times. That is, the upper limit is equal to twice the mean of the task times minus the lower limit which is one.

Our generation scheme further assumes that the total revenue over all parts is equal to the total costs (including task costs and station costs associated with total task times). After Hence, in generating the part revenues, the lower and upper limits of discrete uniform distribution are determined by the average and variance of the total cost figures. Moreover, during the revenue generation, we allow a 5% probability of having a negative revenue. Finally, we generate

demand for parts from discrete uniform distribution between zero and ten. We call this set the *low demand variability data set*. Since such demand levels might be too low to justify the feasibility of a disassembly line, we create another set where only the demand parameters change and name it as the *high demand variability data set*. In this case we assume the probability of having no demand is 5% and the positive demand figures are generated using discrete uniform distribution between ten and one hundred.

We need an upper limit on the cycle time which we assume is provided by the decision maker. To determine this value in our computational analysis we stick to Güngör and Gupta's (2001b) cycle time determination approach which assumes all demand is to be met within the planning horizon (see equation (2.1)). In implementing their approach we assume the planning horizon is given as 400 time units in the low demand variability data set and 4000 time units in the high demand variability data set. However for sensitivity analysis purposes, after solving each instance of the low demand variability data set we recalculate the upper limit on cycle time considering the demand of only those parts that are released in the optimal solution. If this limit is different from the original one, we solve PC to optimality and execute our upper and lower bounding procedures once more to report on differences in solutions obtained. In calculating the heuristic upper bound on the number of stations we let  $\epsilon$  to take a value of 0.0001. All of our routines are coded in Visual Studio C 6.0. Callable libraries of ILOG CPLEX 9.0 are invoked to solve PC, PC-LP and PC-SLP throughout the computational analysis.

#### **4.5.1. Evaluation of the Logical Inequalities**

We solve each of the ten instances of a problem category with each of the eight levels of station cost and with low and high demand variability. We compare PC-LP, PC-LP with logical inequality-1 (LI-1), PC-LP with logical inequality-2 (LI-2), and PC-SLP which incorporates

both logical inequalities (LI-1 + LI-2). The profits obtained with each of these formulations for each problem instance at two demand variability levels are presented in Table D.2 of Appendix D. Also, percentage reduction in profit achieved over PC-LP by using logical inequalities are summarized for each problem category and station cost combination in Table D.1.

The average and minimum reductions in profit realized for each problem category are further summarized in Table 4.9 for the two demand variability levels. Over all 800 cases (10 problem categories x 10 instances x 8 station cost levels) LI-1 alone results in a 35% reduction over PC-LP for both demand variability levels. The same figure is 28% when LI-2 alone is used. When both of them are used simultaneously a 66% average reduction is achieved over the PC-LP solution.

**Table 4.9** Percentage reduction in profit with logical inequalities  
(average over 10 problem instances and 8 station costs)

Problem	Low Demand Variability Set						High Demand Variability Set					
	LI-1		LI-2		LI-1 + LI-2		LI-1		LI-2		LI-1 + LI-2	
	AVG.	MIN.	AVG.	MIN.	AVG.	MIN.	AVG.	MIN.	AVG.	MIN.	AVG.	MIN.
GUN8T8	63.89	12.25	18.35	0.19	76.54	18.91	62.69	10.81	16.78	0.00	75.34	18.17
AKO8T6	49.67	7.58	11.86	0.00	64.33	14.89	50.20	8.46	7.76	0.00	61.64	11.79
AKO20T4-A	12.81	1.19	20.47	8.48	55.63	13.97	13.05	1.36	20.09	8.48	55.24	13.97
AKO20T4-B	13.52	2.04	44.77	23.05	77.58	34.62	13.78	1.72	40.18	21.93	73.52	32.35
AKO20T4-C	14.45	1.84	24.90	12.00	60.26	16.71	15.09	2.19	25.59	12.00	61.78	16.71
LAM20T10	26.08	4.13	44.77	39.36	61.53	43.65	24.07	3.99	46.77	41.11	61.76	44.19
LAM20T24	61.60	11.25	1.55	0.00	62.11	12.44	61.26	10.10	1.62	0.00	61.81	10.71
LAM30T10	20.62	3.74	61.41	54.21	69.64	56.21	21.87	4.14	66.06	57.57	72.79	60.02
LAM30T29	67.21	11.80	0.00	0.00	67.21	11.80	65.20	13.54	0.00	0.00	65.20	13.54
AKO30T12	21.87	4.12	51.28	35.35	74.27	40.54	21.90	4.37	52.35	44.58	71.59	48.14
OVERALL	35.17	1.19	27.94	0.00	66.91	11.80	34.91	1.36	27.72	0.00	66.07	10.71

When Table 4.9 is examined it is seen that the smallest improvements by LI-2 and the largest improvements by LI-1 are observed for the problems where the number of parts and the

number of tasks are close to each other. A reverse relationship is observed for problems where the number of parts is small compared to the number of tasks.

The performances of PC-LP and the upper bound found by solving PC-SLP are further discussed in the subsequent sections.

#### **4.5.2. Evaluation of the Numerical Scores**

In Appendix E, we present the cycle time, the number of stations opened, and the profit found by our lower bounding scheme for both demand variability levels. The numerical scores that found the best solution are also indicated to evaluate their performances.

Tables 4.10 and 4.11 summarize the total number of times the best solution is found by each numerical score and the number of times the best solution is found uniquely by each numerical score over all 80 instances of a problem category. Note that the maximum number of instances a numerical score can find the best solution is bounded from above by the total number of solutions with positive profit.

When the number of times the best solution found is considered as the performance measure, the first numerical score yields the best in more than 79% of the cases in both data sets. The first three numerical scores that are borrowed from the ALBP literature that rely on task times and the precedence relations show similar performances. This is expected since tasks with a large number of successors might also have a large sum of task times for its successors. The fifth numerical score, which is a kind of complement of the first numerical score, is different from the precedence relation dependent numerical scores. When we consider the number of times the best solution is uniquely found as the performance measure, we see that the variety

introduced by  $NX_5$  is effective in finding the best solution when the other numerical scores fail to do so.

**Table 4.10** Comparison of numerical scores for low demand variability data set

Problem	Num of times LB > 0	Number of times best solution found					Number of times best solution is uniquely found				
		$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$
GUN8T8	63	58	56	56	53	57	5				5
AKO8T6	72	64	64	64	60	62					8
AKO20T4-A	75	44	59	41	41	41		17			8
AKO20T4-B	62	55	36	55	52	17		7			
AKO20T4-C	69	42	55	42	42	47		13			13
LAM20T10	80	54	57	56	52	58					21
LAM20T24	64	58	61	59	58	59		1			3
LAM30T10	80	71	71	71	71	40					9
LAM30T29	65	59	59	59	59	50					4
AKO30T12	80	56	33	36	32	24	10	5		2	15
TOTAL	710	561	551	539	520	455	15	45	0	2	86

**Table 4.11** Comparison of numerical scores for high demand variability data set

Problem	Num of times LB > 0	Number of times best solution found					Number of times best solution is uniquely found				
		$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$
GUN8T8	63	60	58	58	56	55	2				3
AKO8T6	72	66	61	61	61	52	5				6
AKO20T4-A	59	59	46	56	43	40					8
AKO20T4-B	67	48	49	48	46	38		4			7
AKO20T4-C	67	39	57	39	36	32		18			8
LAM20T10	80	56	56	56	53	55		1			19
LAM20T24	65	65	63	63	63	53	2				9
LAM30T10	80	71	71	71	71	40					19
LAM30T29	71	67	67	67	67	67					3
AKO30T12	80	47	33	26	33	32	10	1			24
TOTAL	721	578	561	545	529	464	19	24	0	0	105



As a result of this analysis it can be concluded that  $NX_3$  (number of all successors) numerical score can be excluded since all the best solutions found by it are readily found by at least one of the other numerical scores.

#### **4.5.3. Evaluation of the Bounds on Profit**

We present the results concerning our bounding schemes in Appendix F. Table F.1 is provided to summarize the average and maximum deviations from the optimal of the lower bound, upper bound (found solving the strengthened linear programming relaxation with logical inequalities) and linear programming relaxation solutions for each problem category and station cost level. In Table F.2 for each problem instance we present the upper bound on the number of stations and the upper limit on cycle time. We also include the profit, the cycle time, and the number of stations found by our heuristic solution procedure and by solving PC to optimality. The profits of the upper bound found by PC-SLP and the linear programming relaxation PC-LP are also presented for each instance.

In solving PC to optimality we first set  $K$  to the number of actual tasks ( $n$ ) and allow MIP solver of ILOG CPLEX 9.0 to search for the optimum for six hours. (In Table 4.12 we provide the number of continuous variables, binary variables and constraints for the PC formulation of each problem category when  $K = n$ .) If the optimum is not found within this time limit, we let  $K$  take the value of the estimated upper bound on the number of stations ( $K_{UB}$ ) and permit MIP solver to seek for the optimum for another six hours. However, in problem categories LAM30T10 and AKO30T12 we directly set  $K$  to  $K_{UB}$  after seeing that in more than ten instances no optimum is found within the given time limit. These problems are difficult to solve for a number of reasons. First of all, as the number of actual tasks increase, the number of variables and constraints increase drastically. Replacing  $n$  with  $K_{UB}$  in setting  $K$  allows us to

find the optimal solutions within the prespecified time limits. Secondly these two problems are more difficult compared to the other 30 task problem (LAM30T29) as the ratio of the number of parts released to the number of actual tasks decreases the problem becomes more difficult to solve.

**Table 4.12** Number of variables and constraints in PC formulation of each problem (with  $K = n$ )

Problem	Continuous Variables	Binary Variables	Constraints
GUN8T8	17	80	249
AKO8T6	15	88	254
AKO20T4-A	25	400	956
AKO20T4-B	25	400	936
AKO20T4-C	25	400	956
LAM20T10	31	500	1013
LAM20T24	45	500	1055
LAM30T10	41	1470	2830
LAM30T29	60	1470	2887
AKO30T12	43	1170	2471

The average and maximum deviations from the optimal of the upper bound, lower bound and linear programming relaxation solutions are summarized for each problem category in Tables 4.13 and 4.14. In these tables, each row reflects the measures derived over 10 problem instances and 8 station cost levels.

The proposed solution procedure's performance deteriorates as the number of stations in the solution increases and the ratio of the number of parts to the number of tasks decreases. For instance the number of stations opened in the solutions of problem AKO30T12 varies between 1 and 8. Thus the detriment of not balancing the line optimally emerges more significantly as higher station costs are incurred. On the other hand, in problems AKO20T4-A through AKO20T4-C a low ratio of the number of parts to the number of tasks leads to worse upper bound solutions which in return directly affect the quality of the lower bound solution to deteriorate.

**Table 4.13** Comparison of percentage deviations from the optimal for low demand variability data set (average of 10 problem instances and 8 station costs)

PROBLEM	LB			UB			PC-LP		
	AVG <sup>+</sup>	MAX <sup>+</sup>	OLB <sup>*</sup>	AVG <sup>+</sup>	MAX <sup>+</sup>	OUB <sup>*</sup>	AVG <sup>+</sup>	MAX <sup>+</sup>	OLP <sup>*</sup>
GUN8T8	0.08	5.00	79	2.54	42.86	63	724.72	6053.46	0
AKO8T6	0.19	5.49	76	3.75	56.82	53	454.45	4635.01	0
AKO20T4-A	0.43	3.13	55	15.46	500.00	9	455.34	22493.31	0
AKO20T4-B	3.14	71.05	69	24.51	205.26	9	454.35	11791.52	0
AKO20T4-C	3.21	100.00	50	15.55	56.44	14	202.02	626.82	0
LAM20T10	0.19	2.94	67	1.19	16.44	50	184.67	547.25	0
LAM20T24	0.07	4.48	79	8.55	341.62	53	346.45	3707.86	0
LAM30T10	0.07	1.04	70	1.04	4.04	16	247.22	457.85	0
LAM30T29	4.34	100.00	68	15.84	446.40	29	620.55	9066.00	0
AKO30T12	5.36	54.67	14	4.39	50.00	13	597.70	6473.26	0
OVERALL	1.70	100.00	627	8.86	500.00	309	421.85	22493.31	0

\* OLB, OUB and OLP: Number of optimal solutions found by LB, UB and LP relaxation out of 80 cases.

+ AVG and MAX: Average and maximum percentage deviations from the optimal over cases with nonzero optimal objective function values.

**Table 4.14** Comparison of percentage deviations from the optimal for high demand variability data set (average of 10 problem instances and 8 station costs)

PROBLEM	LB			UB			PC-LP		
	AVG <sup>+</sup>	MAX <sup>+</sup>	OLB <sup>*</sup>	AVG <sup>+</sup>	MAX <sup>+</sup>	OUB <sup>*</sup>	AVG <sup>+</sup>	MAX <sup>+</sup>	OLP <sup>*</sup>
GUN8T8	1.22	30.77	74	3.50	75.00	59	761.18	7775.00	0
AKO8T6	1.57	37.06	73	5.55	183.33	44	440.93	6254.69	0
AKO20T4-A	2.63	100.00	49	26.09	750.00	4	919.95	31146.52	0
AKO20T4-B	3.06	100.00	70	20.14	205.26	14	442.86	9149.06	0
AKO20T4-C	5.33	100.00	49	28.85	108.23	13	203.25	1247.11	0
LAM20T10	0.10	2.80	71	1.59	27.91	39	188.09	776.76	0
LAM20T24	0.50	11.11	75	9.50	341.62	46	356.36	5323.00	0
LAM30T10	0.07	1.04	70	0.98	4.04	16	288.21	641.91	0
LAM30T29	2.17	100.00	69	7.62	160.00	28	441.47	6012.78	0
AKO30T12	3.16	22.86	10	2.82	17.14	18	361.98	2062.36	0
OVERALL	1.95	100.00	610	10.37	750.00	281	436.17	31146.52	0

\* OLB, OUB and OLP: Number of optimal solutions found by LB, UB and LP relaxation out of 80 cases.

+ AVG and MAX: Average and maximum percentage deviations from the optimal over cases with nonzero optimal objective function values.

Tables 4.15 and 4.16 give the average and standard deviation of the CPU times (in seconds) to find the solutions over 80 instances of each problem category. These computations have been

made on a Toshiba Satellite Notebook with a Celeron 2800 MHz processor and 728 MB RAM memory.

**Table 4.15** Comparison of CPU solution time (in seconds) for low demand variability data set

PROBLEM	LB		PC		UB		PC-LP	
	AVG	STD DEV	AVG	STD DEV	AVG	STD DEV	AVG	STD DEV
GUN8T8* (80)	0.04	0.02	0.80	0.19	0.02	0.01	0.03	0.01
AKO8T6* (80)	0.05	0.02	1.00	0.27	0.03	0.01	0.03	0.01
AKO20T4-A* (74)	0.32	0.07	3173.26	7071.69	0.22	0.13	0.27	0.06
AKO20T4-B* (68)	0.30	0.08	2820.72	5223.73	0.22	0.21	0.24	0.04
AKO20T4-C* (72)	0.31	0.10	2714.32	6276.39	0.18	0.06	0.29	0.13
LAM20T10* (80)	0.42	0.06	1076.56	3699.63	0.29	0.22	0.40	0.07
LAM20T24* (80)	0.42	0.11	37.91	32.98	0.24	0.08	0.27	0.08
LAM30T10**	3.06	0.80	115.10	144.97	2.30	0.69	3.62	1.25
LAM30T29* (75)	2.90	1.50	5240.09	9058.62	2.33	1.56	2.53	1.08
AKO30T12**	1.85	0.42	2387.22	3769.91	1.31	0.36	3.01	1.04

\* Optimum is first sought for six hours using  $K = n$  and then with  $K = K_{UB}$ . The number in parenthesis indicate the number of instances out of 80 solved with  $K = n$

\*\* Optimum is directly sought using  $K = K_{UB}$ .

**Table 4.16** Comparison of CPU solution time (in seconds) for high demand variability data set

PROBLEM	LB		PC		UB		PC-LP	
	AVG	STD DEV	AVG	STD DEV	AVG	STD DEV	AVG	STD DEV
GUN8T8* (80)	0.04	0.01	0.76	0.20	0.02	0.01	0.03	0.01
AKO8T6* (80)	0.05	0.01	0.96	0.25	0.03	0.01	0.03	0.01
AKO20T4-A* (78)	0.31	0.06	994.95	3692.26	0.21	0.12	0.27	0.06
AKO20T4-B* (76)	0.31	0.09	1044.88	3368.02	0.20	0.05	0.25	0.04
AKO20T4-C* (74)	0.30	0.09	1648.75	4035.45	0.18	0.06	0.29	0.12
LAM20T10* (80)	0.42	0.06	560.29	3212.07	0.26	0.06	0.42	0.10
LAM20T24* (80)	0.40	0.10	432.19	3218.63	0.26	0.12	0.29	0.17
LAM30T10**	2.96	0.72	67.82	77.30	2.32	0.67	3.69	1.39
LAM30T29* (68)	2.59	1.28	5935.30	9472.69	2.00	1.30	2.18	0.75
AKO30T12**	1.74	0.39	2053.52	3727.58	1.39	0.37	3.02	1.08

\* Optimum is first sought for six hours using  $K = n$  and then with  $K = K_{UB}$ . The number in parenthesis indicate the number of instances out of 80 solved with  $K = n$

\*\* Optimum is directly sought using  $K = K_{UB}$ .

The upper and lower bounding schemes find the optimal in 649 of 800 instances (81%) in the low demand variability data set and in 637 of 800 instances (80%) in the high demand variability data set. When the overall performance of the lower bounding scheme is analyzed, it is seen that it produces fairly satisfactory results in very short time. It finds the optimal solution in 627 instances (78%) for low demand variability and 610 instances (76%) for high demand variability. The time it takes to find the heuristic solution is always within four seconds. The average deviation from the optimal is less than 2% for the two demand variability levels, when the optimal profit is positive. Note that when the 90 cases with zero optimal objective function values are included, the average deviation for the low demand variability data set becomes 1.5%. Similarly, if the 79 instances with zero objective function are incorporated, the average deviation for the high demand variability data set turns out to be 1.76%. Out of the 1600 instances the construction heuristic found the lower bound solution in 1423 of the instances. Among the remaining 177 instances, the task deletion step yielded the lower bound solution in 168 of the instances and the task insertion step of our improvement heuristics terminated with success only in 9 of them. The construction heuristic on the average deviates 12% from the lower bound, and the task deletion heuristic 0.3%.

The upper bound finds the optimal profit in more than 35% of the instances in both data sets. Its average deviation from the optimal for the low and high demand variability data sets is 9% and 10%, respectively. PC-SLP finds these bounds in less than three second. The maximum deviation from the optimal is 500% and 75% for low and high demand variability data sets respectively. This observed maximum deviation corresponds to an instance where the upper bound value is equal to 6 while the associated optimal objective function value is 1 in the lower demand variability data set and to an instance where the upper bound value is equal to 8.5 while the associated optimal objective function value is 1 in the high demand variability data

set. The PC-LP's average percent deviation is higher than 400% in both data sets. However, the percent deviation of the upper bound found by PC-SLP is within 10% of the optimal in both data sets. This empirically illustrates that the two logical inequalities introduced have strengthened the linear programming relaxation considerably.

The performance of the lower bound indicates that the proposed heuristic procedure yields promising results for small to medium sized partial DLBP with profit maximization objective. Next we attempt to find the largest problem size that can be solved within the proposed solution procedure.

#### **4.5.4. Large Size Problems**

Due to the unavailability of large size test problems in DLBP we started with merging three of our test problems, namely LAM30T29, AKO20T4-B, and AKO30T12 from the low demand variability data set, to obtain a larger problem with 80 actual tasks. A precedence diagram that connects these three problems in parallel is obtained. Similarly, we generate a problem with 160 actual tasks. Time, cost, revenue and demand parameter values of the selected problems are used for the large size problems. Finally, the size of the problem is doubled once more to obtain a 320 actual task problem. We stop increasing the problem size at this point since this problem is larger than the largest common ALBP test problem with 297 tasks (Scholl, 1999). Note that we let  $K = K_{UB}$  and  $CT_U = 40$  in solving PC-SLP formulation for the upper bound. In Table 4.17 we summarize the results for eight different station costs. Although the average percentage gap between the upper and lower bound solution is less than 4%, we observe a monotonic increase as the station cost increases. The reason for this is that the performance of the upper bound deteriorates as the station cost increases, because it does not allow idle time in stations and is not sensitive to the station cost. In Table 4.17 we also provide the CPU times in

seconds for the lower and upper bound procedures. (These computations have been also made on a Toshiba Satellite Notebook with a Celeron 2800 MHz processor and 728 MB RAM memory.) As the upper bound procedure is part of the lower bound procedure, lower bound CPU times also include the upper bound times. Based on very limited instances, we might say that the total CPU time of the lower bound increases approximately 15 times if we quadruple the problem size from 80 to 320.

**Table 4.17** Results for large problems

INSTANCE	$S$	$K_{UB}$	$\pi_{LB}$	$CT$	$K$	$\pi_{UB}$	LB and UB % Gap	LB CPU	UB CPU
Large Problem 1 $n = 80$ $N = 112$	0.25	19	586.75	37	9	590.25	0.59	21.27	20.66
	0.50	19	507.00	40	8	510.50	0.69	50.50	47.91
	0.75	19	427.00	40	8	432.25	1.21	9.75	9.02
	1.00	19	347.00	40	8	354.00	1.98	2.06	1.45
	1.25	19	266.00	37	8	276.00	3.62	20.41	19.75
	1.50	19	192.00	37	8	204.00	5.88	1.77	1.17
	1.75	18	116.00	36	7	140.50	17.44	1.56	1.05
	2.00	17	66.00	37	4	87.00	24.14	3.86	2.11
Large Problem 2 $n = 160$ $N = 222$	0.25	39	1318.25	37	19	1322.00	0.28	11.45	8.78
	0.50	39	1142.50	37	19	1150.00	0.65	14.75	8.94
	0.75	39	964.50	39	18	979.50	1.53	12.75	8.95
	1.00	38	795.00	36	17	816.00	2.57	13.06	6.86
	1.25	38	644.75	39	15	670.75	3.88	17.88	7.47
	1.50	38	502.00	38	15	538.17	6.72	18.84	6.36
	1.75	37	391.00	40	10	416.81	6.19	20.81	6.47
	2.00	36	291.00	40	10	321.00	9.35	14.80	6.09
Large Problem 3 $n = 320$ $N = 442$	0.25	80	2573.50	30	49	2585.00	0.44	145.91	111.78
	0.50	78	2214.00	31	46	2233.00	0.85	133.91	109.08
	0.75	79	1865.50	37	34	1889.88	1.29	245.02	198.24
	1.00	78	1567.00	35	32	1591.00	1.51	131.00	95.99
	1.25	76	1286.75	35	31	1320.25	2.54	586.39	540.22
	1.50	78	1010.00	40	24	1066.17	5.27	159.11	85.44
	1.75	75	776.50	39	22	833.31	6.82	161.25	76.47
	2.00	75	563.00	40	19	637.00	11.62	131.83	70.86

#### 4.5.5. Sensitivity Analysis on the Upper Limit of the Cycle Time

We conclude this chapter with a final remark on the sensitivity of our approach to determine the upper limit on the cycle time. Out of the 800 cases of the low demand variability data set, 100 cases resulted with a solution that required recalculation of the upper limit on the cycle time since the part with highest demand was not selected. This implies that the maximum number of products that need to be disassembled to satisfy all profitable demand for selected parts ( $D_2$ ) is smaller than the original figure ( $D_1$ ). This allows the upper limit on the cycle time ( $CT_{U-2}$ ) to be larger than the initially calculated value ( $CT_{U-1}$ ).

However, when these 100 instances are solved once again with the new upper limit on the cycle time it is observed that the optimal objective function value changes only in 12 of these cases. This means that in 788 (98.5%) cases the upper limit on the cycle time turns out to be not binding. In the remaining 12 cases, the objective function value improves since the enlarged upper limit tends to be not restrictive thereafter allowing the number of stations opened to be one, yielding zero idle time and hence lower station costs and higher profit. In Table 4.18 we report these instances and provide for each upper limit on cycle time, the optimal solution via the profit ( $\pi$ ), cycle time ( $CT$ ) and number of stations opened ( $K$ ) and finally the percentage improvements in the optimal objective function due to the new cycle time upper limit.



**Table 4.18** Sensitivity analysis results on the cycle time upper limit for 12 instances

INSTANCE	S	$K_{UB}$	BEFORE					AFTER					PERCENT IMPROV.
			$D_1$	$CT_{U-1}$	$\bar{\pi}_1$	$CT_1$	$K_1$	$D_2$	$CT_{U-2}$	$\bar{\pi}_2$	$CT_2$	$K_2$	
GUN8T8-6	0.50	6	8	50	29.00	28	2	7	57	29.50	55	1	1.72
GUN8T8-6	0.75	6	8	50	15.00	28	2	7	57	15.75	55	1	5.00
AKO20T4-B-7	0.25	5	6	66	138.50	41	2	5	80	141.00	72	1	1.81
AKO20T4-B-7	0.50	5	6	66	118.00	41	2	5	80	123.00	72	1	4.24
AKO20T4-B-7	0.75	5	6	66	97.50	41	2	5	80	105.00	72	1	7.69
AKO20T4-B-7	1.00	5	6	66	77.00	41	2	5	80	87.00	72	1	12.99
AKO20T4-B-7	1.25	5	6	66	56.50	41	2	5	80	69.00	72	1	22.12
AKO20T4-B-7	1.50	5	6	66	36.00	41	2	5	80	51.00	72	1	41.67
AKO20T4-B-7	1.75	5	6	66	28.75	27	1	1	400	33.00	72	1	14.78
LAM20T24-0	0.25	3	9	44	72.50	23	2	7	57	72.75	45	1	0.34
LAM20T24-8	0.75	5	9	44	21.50	25	2	8	50	22.25	49	1	3.49
LAM20T24-8	1.00	5	9	44	9.00	25	2	8	50	10.00	49	1	11.11

## CHAPTER 5

### PROFIT MAXIMIZATION OVER PLANNING HORIZON (PH)

In this chapter, we introduce the *profit maximization over planning horizon problem* (PH). As the demands of the parts selected in the PC solution are different, implementing this solution throughout the planning horizon is not reasonable if we want to maximize the profit over the planning horizon. We propose to divide the planning horizon into a number of *time zones*, each of which may contain a number of identical disassembly cycles. The aim of PH is to find a (different) balance for each zone (say in days) such that the profit realized over the entire planning horizon (say a month) is maximized. The zones may have different lengths in terms of the number of cycles run and the cycle time used.

We assume that the supply of discarded product and part demands are known at the beginning of the planning horizon. These figures may come from a master production schedule, say on a monthly basis. When these figures change within the month, a new instance of the PH problem must be solved. Essentially this fact adds a time dimension and requires a dynamic DLBP definition that involves a rolling planning horizon. Our goal is to provide solutions for real life cases where the demand, discarded product availability and even revenue figures may change continuously in a dynamic environment. We treat the inherent dynamic nature of the DLBP by decomposing it to a series of static problems. Therefore, in the rest of this study we confine

ourselves to a static disassembly environment where we assume parameters are known at the beginning of the planning horizon. Whenever a parameter value is changed, a new instance of the PH problem needs to be solved.

We incorporate other issues so as to reflect more realistic features of disassembly systems. First of all, inventory holding cost is incurred for parts released in excess of their demand. Secondly, we allow the supply of discarded products to be finite. Thirdly, since we are dealing with partial disassembly, we allow accumulation of subassemblies in work-in-process (WIP) inventories or use of previously accumulated WIP for further disassembly. Finally, although we allow different line balances in different zones of the planning horizon with no restriction on the number of stations, we analyze the effects of smoothing out the number of stations used across different zones.

The emphasis of this chapter is on defining the PH problem, formulating it, and analyzing the solutions in order to gain insight regarding the effects of partial disassembly, finite or infinite supply, different cycle times, accumulation and use of WIP inventories and invariant number of stations throughout the planning horizon. Although we propose a heuristic solution approach, solution quality is of secondary concern. We first discuss the inventory valuation concepts in disassembly environments in Section 5.1. We state the assumptions concerning the problem environment in Section 5.2 and give the formulation of the PH problem in Section 5.3. We describe our heuristic solution procedures in Section 5.4. Finally we present the results of our computational analysis in Section 5.5.

## **5.1. INVENTORY VALUATION**

The values of the main subassembly WIPs and released part inventories need to be determined in solving DLBP with profit maximization over planning horizon objective. In this section, after a literature review, our valuation method is presented.

### **5.1.1. Literature Review**

None of the studies in the disassembly process planning and DLBP literature explicitly consider valuation of inventories as far as we know. There are a number of studies discussing inventory valuation in remanufacturing environments with a focus on inventory control decisions. Most of these studies argue that an inventory valuation scheme based on traditional value added approach is not suitable for remanufacturing environments. With one exception, none of these studies consider disassembly and hence they are out of our scope.

Teunter (2001) is the only study that discusses disassembly and inventory valuation in reverse logistics. Given a fixed partial disassembly scheme (parts to be released are prespecified) and recovery operations for released parts and subassemblies, they calculate costs and net profits of the subassemblies and parts using the bill of materials of the discarded product. They compare the common valuation method (where the values of parts and subassemblies are based on their production costs) and reverse logistics valuation method (where the values of reusable parts and subassemblies are based on the net profit that could have been attained if they were sold instead of being kept in the inventory). They conclude that when inventory control decisions are in focus, the reverse logistics valuation method used together with average costing models provides results that are closer to those obtained with the net present value methods.

We cannot implement their inventory valuation approach since in our problem the partial disassembly scheme is not given and net profits are unknown. In our case, the problem is to determine this scheme (parts to be released) while maximizing the profit that includes a holding cost component related with the values of the inventories. Moreover, using the bill of materials to define values of parts or main subassemblies may not be appropriate in our case because, when OR precedence and OR successor relations are present, some subassemblies and parts can be released by more than one task each resulting in a different value added.

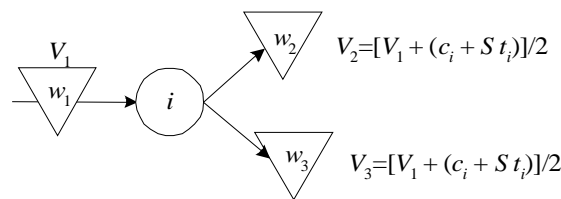
Motivated by the studies that use traditional value added approach in remanufacturing environments (Bayındır, 2002; Bayındır, Erkip and Güllü, 2003), we adopt an approach details of which are discussed below. As long as means of using the value added approach over a precedence diagram with various precedence types can be devised, this seems to be a promising inventory valuation method for PH.

### **5.1.2. Value Added Approach**

Our valuation scheme for main subassembly WIPs and released part (RP) inventories is based on opportunity (or value added) costs. The unit holding costs are determined in the traditional way by multiplying the inventory carrying charge with the total value added to subassembly or part. In the remanufacturing literature, the value of the discarded product is assumed to be zero since the collection cost is assumed to be negligible (Teunter, 2001). We follow this assumption.

The value added by a task is defined by its cost ( $c_i$ ) and the station cost associated with its task time ( $S t_i$ ). Similar to the common valuation method described by Teunter (2001), the subassemblies and parts yielded by a specific task, share equally the total value added in

previous stages (e.g.  $V_1$  in Figure 5.1) and the current stage. Moreover, if a subassembly or a part can be released by different tasks, its total value added can be determined by taking the average of values added over all these releasing tasks. As a result, the values of the WIP and RP inventories towards the end of the precedence graph are in general higher than those at the beginning.

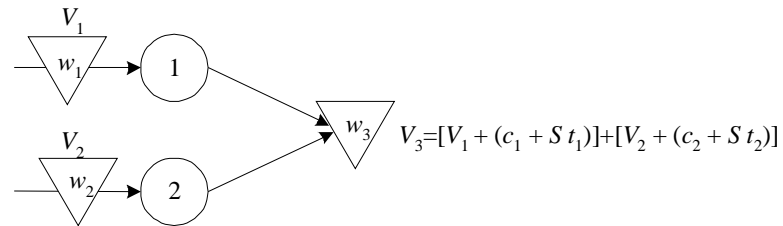


**Figure 5.1** Value added when two subassemblies are released by a task

The total value added to a main subassembly or released part is determined considering all tasks that are to be performed to obtain that main subassembly or released part. The AND and OR precedence relations, and OR successor relations affect the determination of this value. Below we briefly summarize how total values added are determined with respect to different precedence types.

### AND Precedence

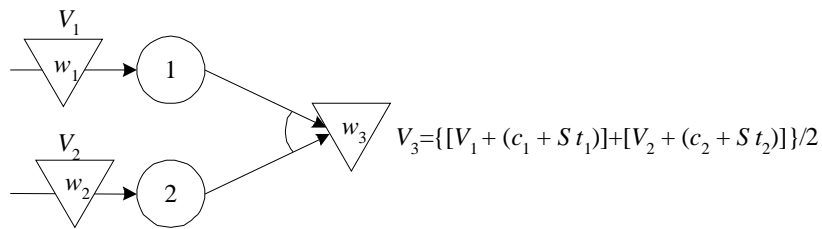
Consider the example given in Figure 5.2 where tasks 1 and 2 are AND predecessors of task  $i$ . Both tasks 1 and 2 must be completed to accumulate WIP inventory  $w_3$ . In this case the value added for WIP inventory  $w_3$  is the sum of values added by tasks 1 and 2 plus the total previous value added.



**Figure 5.2** Value added in AND precedence relation

### OR Precedence

Consider the example given in Figure 5.3 where tasks 1 and 2 are OR predecessors of task  $i$ . At least one of the tasks 1 and 2 must be completed to accumulate WIP inventory  $w_3$ . In this case the value added to WIP inventory  $w_3$  is the average of the values added by tasks 1 and 2 and the previous total values.

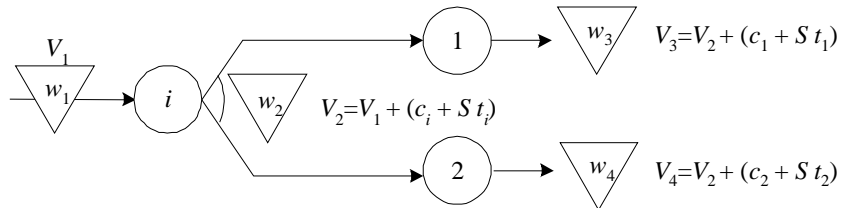


**Figure 5.3** Value added in OR precedence relation

### OR Successor

Consider the example given in Figure 5.4 where task 1 and 2 are OR successors of task  $i$ . If task  $i$  is not completed, at most one of the tasks 1 and 2 can be performed by using WIP inventory  $w_2$ . In this case the value added to WIP inventory  $w_3$  is the sum of the value added by

task 1 and the total previous value added  $V_2$ . Similarly, the value added to WIP inventory  $w_4$  is the sum of the value added by task 2 plus  $V_2$ .



**Figure 5.4** Value added in OR successor relation

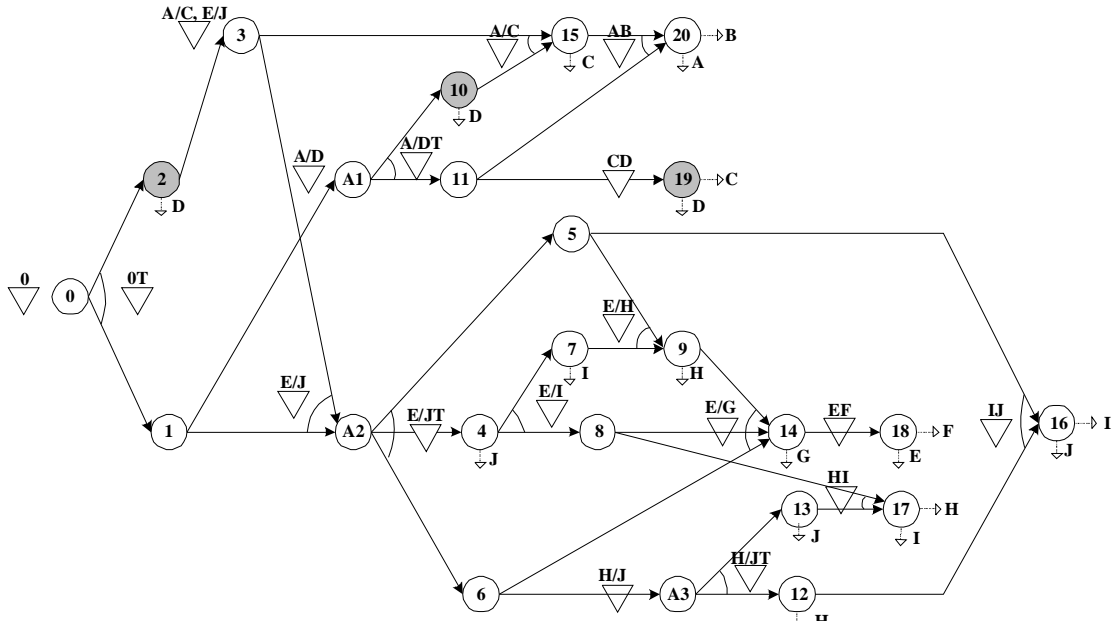
### 5.1.3. An Example

Consider the 10-part ball-point pen example's (Lambert, 1997) precedence diagram with WIP inventories given in Figure 5.5 and task time and cost data given in Table 5.1. The WIP inventories are labeled by the contents of the subassemblies they store. The subassembly label A/C represents the main subassembly ABC while the label AB represents the AB main subassembly. Tasks that release part D (tasks 2, 10 and 19) are illustrated with light gray.

**Table 5.1** Task time and cost data for 10-part ball-point pen example

$i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$t_i$	16	1	19	19	1	11	15	11	4	19	20	1	20	17	12	20	19	6	8	20
$c_i$	6	15	12	20	8	9	17	9	12	18	11	7	18	13	18	13	14	3	14	3





**Figure 5.5** Precedence diagram of 10-part ball-point pen example with WIP inventories

The values of WIP inventories 0, 0T, CD, AB and A/C, E/J, and values of RP inventories A, B and D are determined below.

$$\begin{aligned}
 V_0 &= 0 \\
 V_{0T} &= V_0 = 0 \\
 V_{A/C, E/J} &= (c_2 + S t_2 + V_{0T})/2 \\
 V_{CD} &= (c_{11} + S t_{11} + V_{A/DT})/2 \\
 V_{AB} &= \frac{(c_{11} + S t_{11} + V_{A/DT})/2 + (c_{15} + S t_{15} + V_{A/C})/2}{2} \\
 v_A &= (c_{20} + S t_{20} + V_{AB})/2 \\
 v_B &= (c_{20} + S t_{20} + V_{AB})/2 \\
 v_D &= \frac{(c_2 + S t_2 + V_{0T})/2 + (c_{10} + S t_{10} + V_{A/DT})/2 + (c_{19} + S t_{19} + V_{CD})/2}{3}
 \end{aligned}$$

Applying the proposed approach on the example problem we have calculated the values of the WIP and RP inventories with eight different station costs (see Tables 5.2 and 5.3). Main

subassembly E/I has the highest value under all the station costs. Because of a cascading effect, its value is conveyed to main subassemblies and parts that are succeeding it.

**Table 5.2** WIP inventory values for 10-part ball-point pen example

$w$	$S$							
	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A/C,E/J	7.63	7.75	7.88	8.00	8.13	8.25	8.38	8.50
E/J	8.59	10.81	13.03	15.25	17.47	19.69	21.91	24.13
A/D	5.00	7.00	9.00	11.00	13.00	15.00	17.00	19.00
A/C	13.03	15.94	18.84	21.75	24.66	27.56	30.47	33.38
AB	13.76	16.98	20.21	23.44	26.66	29.89	33.12	36.34
CD	10.50	14.00	17.50	21.00	24.50	28.00	31.50	35.00
E/I	16.67	20.16	23.64	27.13	30.61	34.09	37.58	41.06
H/J	10.17	12.66	15.14	17.63	20.11	22.59	25.08	27.56
E/H	13.57	15.99	18.42	20.84	23.27	25.70	28.12	30.55
E/G	12.56	14.99	17.43	19.87	22.31	24.75	27.18	29.62
HI	15.40	18.83	22.26	25.69	29.12	32.55	35.98	39.41
EF	14.90	18.25	21.59	24.93	28.28	31.62	34.97	38.31
IJ	8.57	9.87	11.17	12.47	13.77	15.07	16.37	17.67
OT	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E/JT	8.59	10.81	13.03	15.25	17.47	19.69	21.91	24.13
A/DT	5.00	7.00	9.00	11.00	13.00	15.00	17.00	19.00
H/JT	10.17	12.66	15.14	17.63	20.11	22.59	25.08	27.56

**Table 5.3** RP inventory values for 10-part ball-point pen example

$j$	$S$							
	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
A	10.88	14.99	19.11	23.22	27.33	31.45	35.56	39.67
B	10.88	14.99	19.11	23.22	27.33	31.45	35.56	39.67
C	15.13	17.98	20.84	23.69	26.54	29.39	32.24	35.09
D	11.58	13.67	15.75	17.83	19.92	22.00	24.08	26.17
E	9.70	12.12	14.55	16.97	19.39	21.81	24.23	26.66
F	9.70	12.12	14.55	16.97	19.39	21.81	24.23	26.66
G	14.90	18.25	21.59	24.93	28.28	31.62	34.97	38.31
H	13.02	15.41	17.80	20.19	22.58	24.97	27.36	29.75
I	16.36	19.98	23.59	27.21	30.83	34.45	38.07	41.69
J	15.51	18.97	22.43	25.89	29.35	32.81	36.27	39.73

Note that released part I, has the highest value since two out of three of its releasing tasks succeed WIP inventory of subassembly E/I.

#### **5.1.4. Discussion**

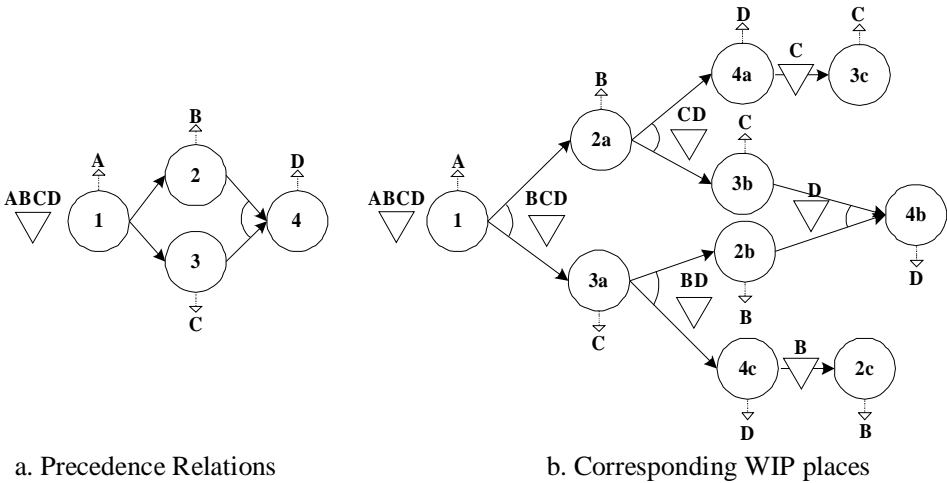
Determining WIP and RP inventory values based on the total value added approach has its advantages and disadvantages. The advantages include (1) once the formulas for each WIP and RP inventory are extracted using the precedence diagram of a problem category, the corresponding values can be calculated quickly for each instance, (2) the traditional value added approach used here reflects the differences among different WIP inventories (or different RP inventories) in a consistent way incorporating task costs and station costs. This leads to intuitive PH solutions. The solutions seek shorter cycle times due to high inventory holding costs, compared to cases with no or low inventory holding costs. The disadvantages, on the other hand, are mainly related with the assumptions made, such as sharing the total value added in a stage equally among released parts and taking averages for cases where a subassembly or part could be released by a number of different tasks (taking the minimum or maximum are other alternatives which can be evaluated in an experimentation).

## **5.2. PROBLEM ENVIRONMENT**

For DLBP with profit maximization over planning horizon objective we adopt assumptions A1-A8 from DLBP with profit maximization per cycle objective. Assumption A9, which is related with the supply of discarded products, is modified and described below together with additional assumptions.

- A9. The discarded product has finite supply, which is deterministic and known.
- A10. Partially disassembled main subassemblies can be stored in WIP inventories at different stages if they are not further disassembled. Alternatively, tasks whose predecessors are not finished can use preceding WIP inventories as input. A WIP inventory that stores main subassemblies with identical configuration is placed before task with AND or OR

predecessors, and after each task with OR successors. However, no WIP inventories are placed before tasks that are capable of performing the same disassembly operation on more than one configuration of the main subassembly. Note that this in fact is not restrictive. If the precedence relations are given in an AND/OR graph format such that each task disassembles parts from a specific main subassembly, WIP inventories can be placed before each task. For example, in the precedence diagram given in Figure 5.6a, a WIP inventory can be placed only before task 1 according to this assumption. For instance, the removal of task B via task 2 can be performed on main subassemblies BCD (meaning right after task 1), BD (meaning after tasks 1 and 3) or B (meaning after tasks 1, 3 and 4). However, if the same precedence relations are represented as in Figure 5.6b, a WIP inventory can be placed before each task.



**Figure 5.6** Deployment of WIP inventories

A11. All demands are satisfied within the planning horizon provided that it is profitable to do so. Unsatisfied part demands become lost sales.

- A12. The individual parts produced in excess of demand are stored in RP inventories. They can be used to satisfy the demand in future periods, but inventory holding cost is incurred.
- A13. Valuation of WIP and released parts inventories is based on their opportunity (or value added) costs. We assume that subassemblies or parts yielded by a certain task share the total value added equally. Moreover if a subassembly or a part can be released by a number of different tasks, its total value added is determined by the average of values added over all these tasks.
- A14. The cost of holding one unit of a WIP or RP inventory per unit time is calculated in the traditional way, by multiplying the inventory carrying charge with the total value added.

### **5.3. PROBLEM FORMULATION**

The DLBP with profit maximization over planning horizon formulation is presented in this section. We do not intend to solve this formulation within the scope of this study. When the number of zones is set to the number of products to be disassembled to meet total demand, and the number of cycles in a zone is set to one, the size of the formulation increases drastically. Moreover, since the cycle time of each zone, number of cycles in each zone and ending inventory levels are also decision variables, the objective function and some constraints become nonlinear. To the best of our efforts we could not resolve the nonlinearity in this formulation.

Before proceeding with the formulation we present our additional notation:

- $h$           Inventory carrying charge (\$/\$/unit time)
- $W$           Index set of WIP inventories

$w$	Index of subassembly kept in respective WIP inventory
$BWIP_w$	Beginning inventory level for WIP inventory of subassembly $w$
$BRP_j$	Beginning inventory level for released part (RP) inventory of part $j$
$V_w$	Total value added to unit WIP inventory of subassembly $w$
$v_j$	Total value added to one unit RP inventory of part $j$
$P$	Set of tasks with AND or OR predecessors and WIP inventories available before them
$Q$	Set of tasks with OR successors and WIP inventories available after them
$z$	Zone index, $z = 1, 2, \dots, Z$
$T$	Planning horizon

Our decision variables can be summarized as follows.

$x_{i,k,z} =$	$\begin{cases} 1 & \text{if task } i \text{ is assigned to station } k \text{ in zone } z \\ 0 & \text{otherwise} \end{cases}$
$q_{j,z}$	Number of revenue generating part $j$ released in zone $z$
$CT_z$	Cycle time of zone $z$
$\alpha_z$	Number of cycles in zone $z$
$u_{k,z} =$	$\begin{cases} 0 & \text{when } x_{D,k,z} = 0 \\ CT_z & \text{when } x_{D,k,z} = 1 \end{cases}$
$UW_{w,z} =$	$\begin{cases} 1 & \text{if subassemblies from WIP inventory } w \text{ are used in zone } z \\ 0 & \text{otherwise} \end{cases}$
$AW_{w,z} =$	$\begin{cases} 1 & \text{if subassemblies of WIP inventory } w \text{ are accumulated in zone } z \\ 0 & \text{otherwise} \end{cases}$
$EWIP_{w,z}$	Ending WIP inventory level of subassembly $w$ at the end of zone $z$
$ERP_{w,z}$	Ending released part inventory level of part $j$ at the end of zone $z$

Given an upper limit on the number of zones  $Z$ , the proposed mathematical formulation for PH is as follows.

**PH:**

$$\max \sum_{z=1}^Z \sum_{j=1}^J r_j q_{j,z} - \sum_{z=1}^Z \alpha_z \sum_{i=0}^D \sum_{k=1}^K c_i x_{i,k,z} - S \sum_{z=1}^Z \alpha_z \sum_{k=1}^K k u_{k,z} - \quad (5.1)$$

$$h \sum_{z=1}^Z \alpha_z CT_z \left[ \sum_{w \in W} V_w (EWIP_{w,z-1} + EWIP_{w,z})/2 + \sum_{j=1}^J v_j (ERP_{j,z-1} + ERP_{j,z})/2 \right]$$

$$\text{s. t. } x_{i,k,z} \leq \sum_{h=1}^k x_{i,h,z} \quad \forall z, k, i \notin P \text{ and } l \in \text{PAND}(i) \quad (5.2)$$

$$x_{i,k,z} \leq \sum_{h=1}^k \sum_{l \in \text{POR}(i)} x_{l,h,z} \quad \forall z, k \text{ and } i \notin P \quad (5.3)$$

$$\sum_{k=1}^K \sum_{l \in \text{SOR}(i)} x_{l,k,z} \leq \sum_{k=1}^K x_{i,k,z} \quad \forall z, i \notin Q \quad (5.4)$$

$$\sum_{l \in \text{SOR}(i)} x_{l,k,z} \leq \sum_{h=1}^k x_{i,h,z} \quad \forall z, k \text{ and } i \notin Q \quad (5.5)$$

$$\sum_{k=1}^K x_{i,k,z} \leq 1 \quad \forall z, i \quad (5.6)$$

$$\sum_{i=0}^D t_i x_{i,k,z} \leq CT_z \quad \forall z, k \quad (5.7)$$

$$\sum_{z=1}^Z q_{j,z} \leq d_j \quad \forall j \in R^+ \quad (5.8)$$

$$q_{j,z} \leq \alpha_z \sum_{i=0}^D \sum_{k=1}^K m_{i,j} x_{i,k,z} \quad \forall z, j \in R^+ \quad (5.9)$$

$$q_{j,z} \geq \alpha_z \sum_{i=0}^D \sum_{k=1}^K m_{i,j} x_{i,k,z} \quad \forall z, j \in R^- \quad (5.10)$$

$$\sum_{k=1}^K k x_{i,k,z} \leq \sum_{k=1}^K k x_{D,k,z} \quad \forall z, i, \quad (5.11)$$

$$x_{i,k,z} \leq \sum_{i=0}^D t_i x_{i,k,z} \quad \forall z, k \text{ and } i \in \text{DMY} \quad (5.12)$$

$$u_{k,z} \leq \left( \sum_{i=0}^D t_i \right) x_{D,k,z} \quad \forall z, k \quad (5.13)$$

$$\sum_{k=1}^K u_{k,z} \geq CT_z \quad \forall z, k \quad (5.14)$$

$$\sum_{z=1}^Z \alpha_z CT_z \leq T \quad (5.15)$$

$$AW_{w,z} + \sum_{k=1}^K x_{i,k,z} \leq 1 \quad \forall z, w \in W \setminus \{0\}, i \in P \text{ and } i \text{ succeeds } w \quad (5.16)$$

$$UW_{w,z} \leq \sum_{k=1}^K x_{i,k,z} \quad \forall z, w \in W \setminus \{0\}, i \in P \text{ and } i \text{ succeeds } w \quad (5.17)$$

$$x_{i,k,z} \leq UW_{w,z} + \sum_{h=1}^k x_{l,h,z} \quad \forall z, w \in W \setminus \{0\}, k, i \in P, i \text{ succeeds } w \text{ and } l \in \text{PAND}(i) \quad (5.18)$$

$$x_{i,k,z} \leq UW_{w,z} + \sum_{h=1}^k \sum_{l \in \text{POR}(i)} x_{l,h,z} \quad \forall z, w \in W \setminus \{0\}, k, i \in P \text{ and } i \text{ succeeds } w \quad (5.19)$$

$$UW_{w,z} + \sum_{l \in \text{PAND}(i)} \sum_{k=1}^K x_{l,k,z} \leq |\text{PAND}(i)| \quad \forall z, w \in W \setminus \{0\}, i \in P \text{ and } i \text{ succeeds } w \quad (5.20)$$

$$UW_{w,z} + \sum_{k=1}^K x_{l,k,z} \leq 1 \quad \forall z, w \in W \setminus \{0\}, i \in P \text{ and } i \text{ succeeds } w, \text{ and } l \in \text{POR}(i) \quad (5.21)$$

$$AW_{w,z} \leq \sum_{k=1}^K x_{l,k,z} \quad \forall z, w \in W \setminus \{0\}, i \in P \text{ and } i \text{ succeeds } w, l \in \text{PAND}(i) \quad (5.22)$$

$$\frac{1}{|\text{PAND}(i)|} \sum_{k=1}^K \sum_{l \in \text{PAND}(i)} x_{l,k,z} - \frac{|\text{PAND}(i)|-1}{|\text{PAND}(i)|} \leq \sum_{k=1}^K x_{i,k,z} + AW_{w,z} \quad \forall z, w \in W \setminus \{0\},$$

$i \in P \text{ and } i \text{ succeeds } w$

$$AW_{w,z} \leq \sum_{l \in \text{POR}(i)} \sum_{k=1}^K x_{l,k,z} \quad \forall z, w \in W \setminus \{0\}, i \in P \text{ and } i \text{ succeeds } w, \text{ POR}(i) \neq \emptyset \quad (5.24)$$

$$\sum_{k=1}^K x_{l,k,z} - AW_{w,z} \leq \sum_{k=1}^K x_{i,k,z} \quad \forall z, w \in W \setminus \{0\}, i \in P \text{ and } i \text{ succeeds } w, l \in \text{POR}(i) \quad (5.25)$$

$$UW_{w,z} + \sum_{k=1}^K x_{i,k,z} \leq 1 \quad \forall z, w \in W \setminus \{0\}, i \in Q \text{ and } i \text{ precedes } w \quad (5.26)$$

$$UW_{w,z} \leq \sum_{l \in \text{SOR}(i)} \sum_{k=1}^K x_{l,k,z} \quad \forall z, w \in W \setminus \{0\}, i \in Q \text{ and } i \text{ precedes } w \quad (5.27)$$

$$\sum_{l \in \text{SOR}(i)} x_{l,k,z} \leq \sum_{h=1}^k x_{i,h,z} + UW_{w,z} \quad \forall z, k, w \in W \setminus \{0\}, i \in Q \text{ and } i \text{ precedes } w \quad (5.28)$$

$$\sum_{l \in \text{SOR}(i)} \sum_{k=1}^K x_{l,k,z} \leq UW_{w,z} + \sum_{k=1}^K x_{i,k,z} \quad \forall z, w \in W \setminus \{0\}, i \in Q \text{ and } i \text{ precedes } w \quad (5.29)$$

$$\sum_{l \in \text{SOR}(i)} \sum_{k=1}^K x_{l,k,z} + AW_{w,z} \leq 1 \quad \forall z, w \in W \setminus \{0\}, i \in Q \text{ and } i \text{ precedes } w \quad (5.30)$$

$$AW_{w,z} \leq \sum_{k=1}^K x_{i,k,z} \quad \forall z, w \in W \setminus \{0\}, i \in Q \text{ and } i \text{ precedes } w \quad (5.31)$$



$$\sum_{k=1}^K x_{i,k,z} - AW_{w,z} \leq \sum_{l \in \text{SOR}(i)} \sum_{k=1}^K x_{l,k,z} \quad \forall z, k, w \in W \setminus \{0\}, i \in Q \text{ and } i \text{ precedes } w \quad (5.32)$$

$$EWIP_{0,z} = EWIP_{0,z-1} - \alpha_z \sum_{k=0}^K x_{0,k,z} \quad \forall z \quad (5.33)$$

$$EWIP_{w,z} = EWIP_{w,z-1} + \alpha_z (AW_{w,z} - UW_{w,z}) \quad \forall z, w \in W \setminus \{0\} \quad (5.34)$$

$$ERP_{j,z} = ERP_{j,z-1} + \alpha_z \sum_{i=0}^D m_{i,j} \sum_{k=1}^K x_{i,k,z} - q_{j,z} \quad \forall z, j \quad (5.35)$$

$$EWIP_{w,0} = BWIP_w \quad \forall w \quad (5.36)$$

$$ERP_{j,0} = BRP_j \quad \forall j \quad (5.37)$$

$$CT_z \geq 0 \quad \forall z \quad (5.38)$$

$$u_{k,z} \geq 0 \quad \forall k, z \quad (5.39)$$

$$q_{j,z} \geq 0 \quad \forall j, z \quad (5.40)$$

$$x_{i,k,z} \in \{0, 1\} \quad \forall z, k \text{ and } i \quad (5.41)$$

$$ERP_{j,z} \geq 0 \quad \forall z, j \quad (5.42)$$

$$EWIP_{w,z} \geq 0 \quad \forall z, w \in W \quad (5.43)$$

$$AW_{w,z}, UW_{w,z} \in \{0, 1\} \quad \forall z, w \in W \quad (5.44)$$

The objective function (5.1) maximizes profit over planning horizon and consists of four terms. The first term represents the total revenue earned from released parts over the planning horizon. The second term is the total cost of performing the assigned disassembly tasks in the planning horizon. The third term is related with the total fixed station cost in the planning horizon. Note that this cost term for each zone involves the product of the number of stations opened and the cycle time, both of which are decision variables. The arising nonlinearity is resolved by introducing decision variable  $u_{k,z}$  and corresponding constraints given in (5.13) and (5.14). The fourth term of the objective function represents the total holding cost associated with the

average WIP and RP inventory levels summed up over all zones. Note that this term involves the product of the number of cycles in a zone, cycle time and ending inventory levels which are all decision variables. Hence our objective function is still nonlinear.

The first four constraint sets given by (5.2) through (5.5) are related with the precedence relations written for all zones when there are no WIP inventories available (indicated by  $i \notin P$  or  $i \notin Q$ ) due to precedence diagram as discussed in assumption A10. The first constraint set (5.2) enables the assignment of a task to station  $k$  in zone  $z$  only if all its AND predecessors are already assigned to stations 1 through  $k$  in zone  $z$ . The second set (5.3) prevents assignment of a task to station  $k$  unless at least one of its OR predecessors is already assigned to one of the stations 1 through  $k$  in zone  $z$ . The third and fourth constraint sets are related with OR successor relations. The former set (5.4) guarantees assignment of at most one of the OR successors of task  $i$  in zone  $z$  if task  $i$  is to be performed in zone  $z$ . The latter set represented by (5.5) assures assignment of an OR successor station  $k$ , if task  $i$  is already assigned.

Constraint set (5.6) indicates that a task might be assigned to at most one station in each zone. Tasks may not be assigned to stations due to partial disassembly or restrictions that are imposed by the precedence relations. The binary nature of the task assignment variables  $x_{i,k,z}$  are reflected in (5.41). The cycle time constraint presented in (5.7) enforces the work content of each station in each zone to remain within the cycle time of that zone.

Constraint sets (5.8) through (5.10) are related with determining the number of revenue generating units released in zone  $z$ . The first constraint set (5.8) ensures the total number of revenue generating parts in all zones does not exceed their demand, if the revenue is nonnegative. Next two sets are used to correctly determine the number of revenue generating

units released. The  $q_{j,z}$  value is determined as the total number released in zone  $z$  by assigned tasks with positive  $m_{i,j}$ . According to set (5.9), if the revenue of part  $j$  is nonnegative,  $q_{j,z}$  cannot exceed the total number of parts  $j$  that are released by the assigned tasks in zone  $z$ . If revenue of part  $j$  is negative, set (5.10) forces  $q_{j,z}$  to take the correct total number released in each zone under maximization objective. Note that constraints (5.9) and (5.10) are nonlinear since they involve the product of the number of cycles in a zone and the task assignment decision variables. Moreover,  $q_{j,z}$  is defined to be a continuous decision variable by constraint (5.40).

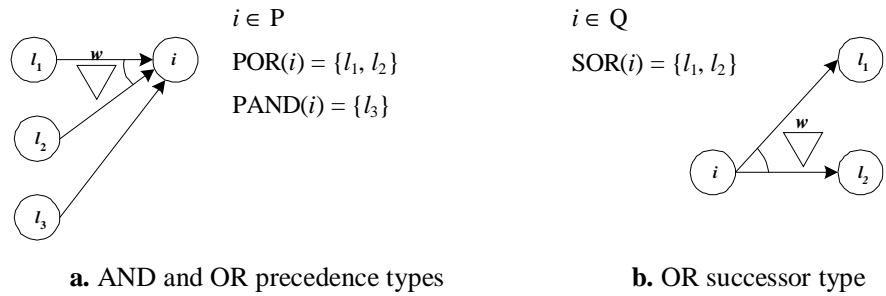
Constraint set (5.11) assures that in each zone no task is assigned to the stations following the station to which dummy task D is assigned. Constraint set (5.12) assures assignment of dummy tasks to stations to which some actual tasks are assigned for each zone. These two constraint sets can be perceived as technical constraints.

Constraints (5.13) and (5.14) are used to determine the total available time on the line for each cycle of each zone, which is calculated by multiplying the number of stations opened by the cycle time of the zone. Note that this is required to find the total station opening cost. By definition, decision variable  $u_{k,z}$  takes a value of zero if the dummy task D is not assigned to station  $k$  in zone  $z$  and a value equal to the cycle time of zone  $z$  if it is assigned to station  $k$ . This relationship is conveyed using these two equations. In (5.13),  $u_{k,z}$  is defined to be less than or equal to  $x_{D,k,z}$  times the total processing time of all tasks in each zone  $z$  (which is used instead of big  $M$  since it is the largest value that can be attained when all tasks are performed). Since task D is assigned to only one of the  $k$  stations opened in zone  $z$ , only one of the  $u_{k,z}$  variables will take a positive value in each zone. In order to guarantee the corresponding  $u_{k,z}$

variable to take the value of the cycle time of zone  $z$  we use in (5.14). Moreover,  $u_{k,z}$  is defined as a continuous decision variable by constraint (5.39).

Constraint (5.15) assures that the sum of zone lengths over all zones (i.e. the makespan) is less than the duration of the planning horizon.  $CT_z$  are defined as continuous decision variables in (5.38).

Constraints (5.16) through (5.32) are related with the accumulation or usage of main subassemblies in WIP inventories (including the discarded product denoted with the subassembly index 0). Sets (5.16) through (5.25) relate task  $i$  to its AND or OR predecessors and WIP inventory  $w$  preceding it (see Figure 5.7a). Sets (5.26) through (5.32) relate task  $i$  with respect to its OR successors and WIP inventory  $w$  succeeding it (see Figure 5.7b).



**Figure 5.7** Illustration of WIP inventory places

Constraints (5.16) through (5.32) are valid for each cycle of each zone. For the sake of simplicity and without loss of generality they are described below without mentioning the zone dimension.

The two constraint sets provided in (5.16) and (5.17) are related with proper utilization of a WIP that precedes a task. The former ensures that the main subassembly is either accumulated in WIP or further disassembled by the downstream task. The latter guarantees that if subassembly is used from WIP inventory, it should be further disassembled by the downstream task.

When WIP accumulation is allowed before a task, it can be performed either if its precedence relations are satisfied or if there is WIP inventory. This new option of using available WIP inventories leads to a change in precedence relations for the tasks succeeding a WIP inventory ( $i \in P$ ). The constraints (5.18) and (5.19) reflect these changes with respect to AND and OR precedence types. The former prevents assignment of such a task to station  $k$  unless all its AND predecessors are assigned to stations 1 through  $k$ , or unless available subassembly from the preceding WIP inventory is used. Similarly, the latter blocks assignment of such a task to station  $k$  unless at least one of its OR predecessors is assigned to stations 1 through  $k$ , or unless available subassembly is used from the WIP inventory.

Constraints (5.20) and (5.21) regulate actual flow of the main subassembly through the disassembly line. Task  $i$  is performed either on a subassembly from the preceding WIP inventory or on a subassembly released from the preceding tasks, but not on both. The first constraint set controls material flow over the AND predecessors and the second set over the OR predecessors.

The mechanism of accumulating WIP inventories is represented by constraints (5.22) through (5.25). These constraints indicate that WIP inventory  $w$  cannot be accumulated before task  $i$  if precedence relations of task  $i$  are violated. Note that predecessors of task  $i$  are also predecessors

of WIP inventory  $w$ . This again involves two sets with respect to predecessor types. Constraint set (5.22) prevents WIP accumulation unless all AND predecessors are done. Constraint (5.23), on the other hand, assures that WIP inventory is accumulated if all AND predecessors are completed but the succeeding task is not started. Constraint (5.24) similarly does not allow WIP accumulation unless at least one OR predecessor is done. Moreover, the accumulation of WIP inventory is guaranteed by constraint (5.25) if at least one OR predecessor is performed while the succeeding task is not performed.

Constraint sets (5.26) through (5.32) are related with the proper use of WIP when an upstream task is followed by both a WIP inventory and OR successors. Constraint (5.26) prohibits simultaneous use of WIP for further disassembly and production of the same subassembly via the upstream task. Set (5.27) guarantees that if WIP is used, subassembly is further disassembled by one of the OR successors. Constraints (5.28) and (5.9) prevent further disassembly unless there is inventory usage or upstream task is done to produce the subassembly. While (5.28) restricts further disassembly at a specific station, (5.29) controls it over the line. Set (5.30) indicates that both inventory accumulation and further disassembly by an OR successor is impossible. It also assures that at most one OR successor can be done. (5.31) makes inventory accumulation impossible unless the upstream task is performed, whereas (5.32) forces inventory accumulation when the upstream task produces a subassembly which is not disassembled further.

The nonlinear constraints (5.33) through (5.35) are end-of-zone balance equations for raw material (discarded product), WIP and RP inventories. Constraints (5.36) and (5.37) are used to define the beginning inventory levels of the first zone as initial conditions. Finally (5.42) and

(5.43) depict that the variables representing the ending inventory levels are continuous and (5.44) states that WIP inventory accumulation and usage decision variables are binary.

In Appendix G, an approach we followed in checking the redundancy of the several constraints in PH related with accumulation and usage of WIP subassemblies and the associated precedence relations of the tasks.

#### **5.4. PH SOLUTION SCHEMES**

We propose the following heuristic solution for PH. The number of zones ( $Z$ ), and the number of cycles in each zone ( $\alpha_z$ ) are determined by decomposing PH into a number of successive per cycle problems. We name this problem as PC-E. PC-E is basically a version of PH where the zone subscript is dropped. A different PC-E instance needs to be solved for each zone. For each zone, the PC-E solution yields a cycle time ( $CT_z$ ), the set of selected tasks to be assigned, their assignments to stations, parts to be disassembled to satisfy demand using finite supply of discarded product and subassemblies stored in WIP inventories. The number of cycles in a zone,  $\alpha_z$ , will be determined as the earliest cycle at the end of which one of the following conditions occurs:

- § Among the parts produced, the demand is satisfied for the part having the lowest demand.
- § WIP inventory of a subassembly is depleted.
- § Supply of discarded product is consumed.
- § The end of the planning horizon is reached, which means this is the final zone. In such a case the remaining portion of the planning horizon divided is by the cycle time of this zone and rounded down to the next integer to determine the number of cycles in the final zone.

After the unsatisfied demand and the ending WIP and RP inventory levels are updated using the current zone's solution and the number of cycles in that zone, PC-E is initialized and solved for the next zone. We refer to this procedure as the decomposed solution approach (PHD). The procedure terminates when one of the following circumstances arises:

- § All demand is satisfied.
- § All available WIP subassemblies and the supply of discarded product are used up.
- § PC-E solution of the current zone has nonpositive profit. In such a case this zone is expelled from the decomposed solution.
- § The end of the planning horizon is reached.

We apply the PHD procedure twice. The first one allows the number of stations to vary from one zone to the next and is called PHD with variable number of stations (PHD-VK). The second one fixes the number of stations to smooth it across the zones, and is named as PHD with fixed number of stations (PHD-FK). After executing PHD-VK, an average of number of stations ( $K_{avg}$ ) is found by weighing the number of stations in each zone with the number of cycles in that zone. PHD-FK can then be solved by fixing the number of stations as  $\lfloor K_{avg} \rfloor$  and  $\lceil K_{avg} \rceil$ . We compare the two PHD-FK solutions (PHD-FK<sup>-</sup> and PHD-FK<sup>+</sup>) and propose the one with the highest profit (PHD-FK<sup>\*</sup>) as our heuristic solution to PH.

If the original PH could be solved to optimality with variable number of stations and with fixed number of stations, then the profit of the former solution would be greater than or equal to the profit of the latter solution. The difference between them would yield the profit we lose by smoothing out the number of stations across the zones.



As an alternative solution procedure, one can borrow  $Z$ ,  $\alpha_z$  for  $z=1, 2, \dots, Z$ ,  $\lfloor K_{avg} \rfloor$  and  $\lceil K_{avg} \rceil$  from the respective PHD-FK<sup>-</sup> and PHD-FK<sup>+</sup> solutions, and solve the original PH formulation twice with these parameters. We refer to this alternative as the semi-direct solution approach. PHS-FK<sup>\*</sup> represents the best of the two solutions of this approach obtained with  $\lfloor K_{avg} \rfloor$  and  $\lceil K_{avg} \rceil$ . Since the nonlinearity involved with the holding cost component of the PH objective function is not resolved, we replace the cycle time decision variable of this component with the minimum possible cycle time and propose this approach as an alternative heuristic solution procedure. Although the resultant PH formulation is solved to optimality, this solution may not be optimal for the original PH because the parameters  $Z$ ,  $\alpha_z$ , and number of stations are imposed. Also, the profit value needs to be reevaluated using the optimal cycle time to correctly determine the holding cost associated with the inventories. The aim of using this alternative procedure is to provide a means of comparison for the proposed decomposed solution scheme. If the nonlinearity in the objective function could have been handled, then the profit of PHS-FK<sup>\*</sup> would have been greater than or equal to the profit of PHD-FK<sup>\*</sup>. Then, the difference between PHS-FK<sup>\*</sup> and PHD-FK<sup>\*</sup> would provide a measure regarding profit that is lost due to using the decomposed solution procedure.

The decomposed and semi-direct solution approaches are illustrated on the 10-part ball-point pen example whose precedence diagram is given in Figure 5.5. The results are given in Appendix H and are summarized in Table 5.4. The best profit value obtained by fixing the number of stations (PHD-FK<sup>\*</sup>) is 1.2% lower compared to PHD-VK profit. The semi-direct approach (PHS-FK<sup>\*</sup>) on the other hand, generates 11.6% higher profit than PHD-FK<sup>\*</sup>, because the former is less myopic and optimally solves the line balancing component of the problem. The relative difference between the profits of these two solution procedures is 62.437. The total revenue generated and demand satisfaction percentages of these two solutions are the same

(847 and 81.5% respectively). However, the PHD-FK\* solution has higher total task cost, total station cost, and inventory holding cost. Approximately 83% of the relative difference in the two profit values is due to the difference in total task cost, 15% is due to difference in station cost (since number of stations is fixed at three, PHD-FK\* solutions have longer cycle times), and remaining 2% is due to the difference in the holding cost.

**Table 5.4** Results of the PH solution procedures for the 10-part ball-point pen example

	PHD-VK	PHD-FK* = PHD-FK <sup>-</sup>	PHD-FK <sup>+</sup>	PHD-FK* = PHS-FK <sup>-</sup>	PHS-FK <sup>+</sup>
Profit	484.848	479.141	478.745	541.578	536.965
Holding cost	8.152	9.109	7.255	8.422	6.035
Makespan	101	113	87	100	82
Demand satisfaction %	81.48	81.48	81.48	81.48	81.48
Z	3	3	3	3	3
$\alpha_{avg}$	1	1	1	1	1
$K_{avg}$	3.33	-	-	-	-
K given	-	3	4	3	4

Our decomposed solution approach solves the extended per cycle problem PC-E for each zone. Next we define PC-E and describe our solution procedure for PC-E.

#### 5.4.1. Definition of PC-E

Letting the number of zones be one ( $Z = 1$ ) and the number of cycles in that zone be one ( $r_{z=1}$ ) in PH, we obtain the formulation of PC-E, the extended version of the PC problem. Moreover, we replace the planning horizon constraint (5.15) with  $CT_z \leq CT_U$ . In contrast with the PC version of the problem, we let the upper limit of the cycle time take the value of the sum of all task times. The same upper limit is used in our heuristic procedure for PC-E. Reasons for using this upper limit include the following. First of all when the number of stations is fixed, using an

upper limit that is based on complete satisfaction of unfilled demand may lead to suboptimal solutions. Secondly, due to the inventory holding cost that incurs over the whole zone length, the heuristic procedure keeps the cycle time as small as possible. Moreover, in cases with fixed number of stations, in order to minimize the station cost the PC-E model has to target the cycle time and select it as small as possible. Finally due to partial disassembly and having no limitations on demand satisfaction, makespan in most of the solutions turns out to be shorter than the planning horizon, allowing single station solutions to be implemented if it is profitable to do so.

In PC-E, which tasks to perform, equivalently which parts to release, which WIP inventories to accumulate or to consume, what number of stations and cycle time to use are issues that need to be decided. Unfortunately all of these issues are highly interrelated. Our solution procedure for PC-E starts with a construction heuristic that is based on the mathematical formulation of PC-E. It first solves a strengthened version of the linear programming relaxation of PC-E (PC-E-SLP). It borrows the total fractional task assignments, WIP inventory usage and accumulation information from this solution and tries to amend this solution in order to make it feasible while maximizing the profit. The assignment procedure of PC is modified to incorporate WIP usage and accumulation during the assignment of selected tasks to stations and evaluation of the resulting profit. Then, a two step improvement heuristic that evaluates the task selection, WIP usage and accumulation imposed by the construction solution is utilized. In Section 5.4.2 we describe the PC-E-SLP used in the construction heuristic. We briefly summarize our construction and improvement heuristics in Section 5.4.3.

### 5.4.2. PC-E-SLP Formulation

As in our solution procedure for PC, we intend to find an initial solution of PC-E by solving its linear programming relaxation. In this relaxation, we heuristically tackle the nonlinearity in the objective function. Then we relax the integrality constraints of the binary variables. Note that the nonlinearities in constraints (5.9), (5.10) and (5.33) - (5.35) are removed by setting  $\alpha_z=1$  in PC-E. Finally, we introduce two additional sets of logical inequalities that strengthen the formulation.

The nonlinearity in the total inventory holding cost term of the objective function (5.45) is removed by letting  $CT$  take the minimum possible value  $CT_L$ , which the minimum task time. This ensures that the profit found by solving the strengthened version of the linear programming relaxation of PC-E (PC-E-SLP) yields an upper bound on the profit. Note that  $CT$  is still left as a decision variable in the model except for the holding cost term.

$$\begin{aligned} \max \quad & \sum_{z=1}^Z \sum_{j=1}^J r_j q_{j,z} - \sum_{z=1}^Z \alpha_z \sum_{i=0}^D \sum_{k=1}^K c_i x_{i,k,z} - S \sum_{z=1}^Z \alpha_z \sum_{k=1}^K k u_{k,z} - \\ & h \sum_{z=1}^Z \alpha_z CT_L \left[ \sum_{w \in W} V_w (EWIP_{w,z-1} + EWIP_{w,z})/2 + \sum_{j=1}^J v_j (ERP_{j,z-1} + ERP_{j,z})/2 \right] \end{aligned} \quad (5.45)$$

In addition to LI-1 and LI-2 that we have used in PC-SLP, we introduce two additional logical inequalities to obtain PC-E-SLP.

#### Logical Inequality - 3

Two versions of third logical inequality are proposed. The first one is valid for AND precedence relations and the second one for OR precedence relations.

The first set (5.46) guarantees that the total fractional assignment of task  $i$  does not go beyond the total fractional assignment of each of its AND predecessors plus the fraction of upstream WIP used.

$$\sum_{k=1}^K x_{i,k} \leq UW_w + \sum_{k=1}^K x_{l,k} \quad \forall w \in W \setminus \{0\}, k, i \in P, i \text{ succeeds } w \text{ and } l \in \text{PAND}(i) \quad (5.46)$$

Similarly, the second set (5.47) assures that the total fractional assignment of task  $i$  is less than or equal to the total fractional assignment of all of its OR predecessors plus the fraction of upstream WIP used.

$$\sum_{k=1}^K x_{i,k} \leq UW_w + \sum_{k=1}^K \sum_{l \in \text{POR}(i)} x_{l,k} \quad \forall w \in W \setminus \{0\}, k, i \in P \text{ and } i \text{ succeeds } w \quad (5.47)$$

#### Logical Inequality – 4

The last logical inequality set given by (5.48) ensures that the amount of WIP  $w$  used must be less than or equal to the beginning inventory level of subassembly  $w$ .

$$UW_w \leq BWIP_w \quad \forall w \in W \setminus \{0\} \quad (5.48)$$

#### 5.4.3. Solution Procedure of PC-E

We briefly summarize our construction and improvement heuristics below.

### **Construction Heuristic**

The lower bounding scheme starts with a construction heuristic which assumes that all tasks with a positive fractional assignment in the PC-E-SLP solution and all tasks succeeding WIPs with positive fractional usage are selected. The assignment procedure described in Section 4.4.2.1 is modified only for precedence feasibility check. If there are no WIP inventories before a task, the precedence feasibility check remains unchanged. That is, a task is precedence feasible if all of its AND predecessors and at least one of its OR predecessors are elements of the task ordering list (TOL). However, if there are WIP inventories before a task, it can be added to the feasible list even if its predecessors are unassigned.

The assignment procedure uses the same numerical scores and enumerates the integer  $CT$  values. When the assignment procedure is completed, the ending balances of WIP and RP inventories are determined using the task assignments (which also provide information on the released parts), beginning inventory levels, WIP inventory accumulation or usage, and demand information. Then the calculated average levels of the WIP and RP inventories are multiplied by the cycle time found and the inventory carrying charge. Finally they are summed up to find the total inventory holding cost associated with the assignment, and the true objective function value is calculated.

### **Improvement Heuristic**

After an initial solution is obtained using the construction heuristic, a two step improvement heuristic is applied.

In the first step of the heuristic, a backward task deletion strategy is implemented. In each iteration, deletion of a task corresponds to WIP accumulation before that task. All tasks in the construction heuristic solution are added to a deletion list, DL, in a backward breadth first search order starting from task D.

While DL is not empty, the first candidate task in DL is removed from the selected tasks set and the assignment procedure is executed. If the profit evaluated at the end of the assignment procedure is higher than the profit of the incumbent solution, the candidate task is deleted and WIP inventory is accumulated before that task. Otherwise the candidate task is labeled as selected and returned to the selected tasks list.

In the second step of the heuristic, a forward task deletion strategy is implemented. In each iteration, deletion of a task corresponds to usage of WIP succeeding that task. All tasks that were selected at the end of the backward task deletion step and that have subassemblies available in WIP inventories are added to a deletion list, DL, in a forward breadth first search order starting from WIP 0.

While the DL list is not empty, the first candidate task in DL is removed from the selected tasks set and the assignment procedure is executed. If the profit evaluated at the end of the assignment procedure is higher than the profit of the incumbent solution, the candidate task is deleted and the WIP inventory available after the task is used. Otherwise the candidate task is labeled as selected and returned to the selected tasks list.

## 5.5. COMPUTATIONAL ANALYSIS

The aim of the computational analysis conducted is twofold. Firstly, we want to evaluate the performance of PHD against that of PHS, and we want to assess the effect of balancing the number of stations across the zones. Due to the large number of variables and constraints in PHS formulation, we confine ourselves to two small problem categories. We summarize the corresponding analysis in Section 5.5.1. Secondly, we want to evaluate PHD by assessing the effects of aiming partial disassembly, having finite or infinite supply, including cycle time as a decision variable, allowing accumulation and use of WIP inventories. The observations made over four problems with varying sizes are discussed in Section 5.5.2.

### 5.5.1. Comparison of PHD and PHS

The evaluation of the two proposed solution approaches is based on only the problem categories GUN8T8 and AKO8T6 under low and high demand variability levels. We limit ourselves with these two problem categories since the majority of the PHS instances of only these problems can be solved within the given time limit of 6 hours.

For each problem 20 instances (10 instances x 2 demand variability levels) are solved under the following settings.

1. *WIP inventory availability (WIP)*: We consider two levels.
  - N: No subassemblies are available in WIP inventories at the beginning of the planning horizon.
  - W: Subassemblies are available at the beginning of the planning horizon. With a probability of 0.5, the number of subassemblies in each WIP inventory is generated from discrete uniform distribution between 1 and 2. With the remaining probability, the WIP is empty.



Note that the beginning RP inventory levels are all set to zero.

2. *Supply of discarded product (SU)*: We again consider two levels.
  - I: Discarded product has finite supply. In this case, we set the number of discarded products to the maximum demand level (10 and 100 for low and high demand variability).
  - F: Discarded product has finite supply. The number of discarded products available is set to the average demand (rounded up to the nearest integer) in each problem instance. This means that slightly more than 50% of the demand can be met if it is profitable to do so.
3. *Station cost per unit time (S)*: 0.25 and 0.75 are used as low and moderate cost levels.
4. *Inventory carrying charge (h)*: Three levels are considered: 0, 0.001 and 0.005.

In this computational analysis, the station cost, inventory carrying charge and beginning WIP levels are intentionally set to relatively lower levels. Even in the PC experiment, there were several instances yielding no disassembly solutions with moderate station costs, because disassembly was not profitable. In PH, we increase the total cost by including WIP and RP inventory holding cost components. Parameter settings leading to high cost would result in no disassembly solution or disassembly in a single zone. We want to be able to observe the performances of PHD and PHS over a number of zones. We set the cost related parameters at low levels to ensure that disassembly is profitable and continues for a number of zones.

As the proposed PHD approach is myopic and the solution that maximizes the profit per cycle is selected in each zone, unfavorable beginning conditions are inherited by subsequent zones in terms of ending RP and WIP inventories. However, given the number of stations, number of zones and number of cycles in each zone, the PHS approach has an overall view and is less

myopic. We want to investigate if the myopic behavior of PHD becomes more apparent as the number of zones increases.

The proposed solution approaches are coded in Visual Studio C 6.0. Callable libraries of ILOG CPLEX 8.1 are invoked to solve PC-E-SLP and PHS. (Recall that PHS consists of the PH formulation where the objective function is replaced with (5.45) to resolve the nonlinearity involved with the inventory holding cost term.)

We present the results for problems GUN8T8 and AKO8T6 in Tables I.1 through I.4 of Appendix I. For each parameter combination, we report the average over 10 problem instances of the makespan (MS), the percentage of demand satisfied (Sat %), the total profit ( $\pi$ ), the total holding cost (H cost), the number of zones (Z), the number of cycles in a zone ( $\alpha_{avg}$ ), and the number of stations ( $K_{avg}$ ) for the PHD-VK approach. For the PHD-FK\* and PHS-FK\* approaches we provide the same measures except  $K_{avg}$ . The performance of the PHD approach is assessed by calculating the percentage gap in profit (SD) between PHS-FK\* and PHD-FK\*. Note that PHD-FK\* and PHS-FK\* represent the best of the two solutions found using  $\lfloor K_{avg} \rfloor$  and  $\lceil K_{avg} \rceil$ . However in some cases PHS-FK\* does not yield a solution within the given time limit of 6 hours for either PHS-FK<sup>+</sup> or PHS-FK<sup>-</sup>. For such cases we let PHD-FK\* take the value of PHD-FK<sup>+</sup> if PHS-FK<sup>+</sup> has generated the solution recorded in PHS-FK\* and vice versa. Finally, the effect of fixing the number of stations is appraised by computing the percentage gap in profit (VF) between PHD-VK and PHD-FK\*.

Table 5.5 summarizes the average percentage SD and VF gaps for each problem category, demand variability level and factor combination. Our observations follow.

**Table 5.5** Percentage SD and VF gaps (average of 10 problem instances)

Factor Combination				GUN8T8				AKO8T6			
				Low Demand V.		High Demand V.		Low Demand V.		High Demand V.	
<i>WIP</i>	<i>SU</i>	<i>S</i>	<i>h</i>	SD	VF	SD	VF	SD	VF	SD	VF
N	I	0.25	0.000	0.07	-3.84	0.26	-6.15	2.22	-0.68	1.13	-3.88
N	I	0.25	0.001	-0.72	0.05	-1.09	-2.16	2.31	-0.16	-0.21	-3.05
N	I	0.25	0.005	-11.12	0.39	-6.29	-3.28	1.32	0.59	-6.47	-8.88
N	I	0.75	0.000	0.00	0.00	0.27	0.06	0.28	0.01	0.19	0.04
N	I	0.75	0.001	-0.15	0.00	-10.79	-5.33	0.87	0.28	-0.34	-2.88
N	I	0.75	0.005	0.00	0.00	4.90	-0.62	0.57	0.23	-3.34	0.22
N	F	0.25	0.000	0.00	-2.49	0.22	-0.47	1.97	-1.11	1.28	0.41
N	F	0.25	0.001	0.00	-0.07	-1.01	2.39	2.16	-0.49	0.23	0.50
N	F	0.25	0.005	-2.74	1.08	-0.02	2.00	1.74	0.69	-2.65	-5.77
N	F	0.75	0.000	0.00	0.00	0.27	0.06	0.00	0.00	0.19	0.04
N	F	0.75	0.001	-0.15	0.00	-10.79	-5.33	0.46	0.31	0.10	-2.87
N	F	0.75	0.005	0.00	0.00	4.90	-0.62	1.17	0.23	-5.29	0.22
W	F	0.25	0.000	1.40	-2.33	0.86	0.68	1.06	-0.38	1.40	0.39
W	F	0.25	0.001	0.70	1.46	-0.47	3.73	2.12	0.52	0.78	0.15
W	F	0.25	0.005	1.77	5.65	-6.08	5.49	3.23	1.75	-2.76	2.41
W	F	0.75	0.000	0.00	0.00	1.13	0.11	0.60	0.00	2.00	0.07
W	F	0.75	0.001	-2.34	1.02	0.86	4.30	0.96	0.71	2.35	6.88
W	F	0.75	0.005	-1.49	-10.39	4.96	8.89	3.40	5.21	-7.92	-15.06
W	I	0.25	0.000	1.22	-4.02	0.71	-6.88	1.21	-0.55	1.45	0.48
W	I	0.25	0.001	0.65	1.30	-0.48	4.27	2.56	0.91	-1.08	0.18
W	I	0.25	0.005	-7.12	5.81	0.26	-6.15	4.48	2.98	-9.90	-0.97
W	I	0.75	0.000	0.00	0.00	-1.09	-2.16	0.59	0.00	2.44	0.07
W	I	0.75	0.001	-0.13	0.93	-6.29	-3.28	1.11	0.64	2.32	4.10
W	I	0.75	0.005	2.03	-10.24	0.27	0.06	0.05	2.17	-7.60	-15.40

**PHD-FK\* vs PHS-FK\***

In 51% (492 out of 960) of the instances both procedures yield solutions with the same profit figures. In 34% (327) of them a positive SD gap is observed. This represents the profit lost as a consequence of using the myopic decomposed approach (see the example provided in Appendix H). As far as these instances are concerned, PHD-FK\* is on the average within 1.5% of PHS-FK\*. When the instances with  $h = 0$  are considered, the PHS-FK\* approach provides better solutions as expected. The optimal solution for the PHS formulation is possible only under this condition as the nonlinearity of the inventory holding cost is resolved. The SD gap is within 2%. The differences mainly arise since PHS-FK\* has an overall view of the planning

horizon leading to better selection of tasks and parts, line balances with less idle time and more profit.

In the remaining 15% (141) of the instances a negative SD gap is detected. This implies that the lower limit on cycle time used in the PHS objective function and the cycle time of the PHS-FK\* solution vary drastically. This leads to a higher true holding cost figure than that is calculated in the PHS objective function. This is a natural consequence since PHS-FK\* attempts to find an optimal solution which in fact is a heuristic solution due to objective function (5.45). Recall that in solving PHD-FK\* all possible cycle times are enumerated. In the instances where the same set of tasks and parts are selected by both approaches, PHD-FK\* uses more stations than those used in the PHS-FK\* solutions. Hence it may incur higher station costs but lower inventory holding costs. The tradeoff between these two components determines whether the SD gap is positive or negative.

#### **PHD-VK vs PHD-FK\***

In 37.5% of the cases, PHD-VK and PHD-FK\* approaches yield the same solutions. These mainly consist of cases where  $K_{avg}$  has an integer value that is also explored in finding PHD-FK\*. 27% of the instances yield a positive VF gap meaning that PHD-VK has higher profit. This is because PHD-VK is unrestricted in number of stations. However, the performance of PHD-FK\* in these instances is on the average within 2% of PHD-VK.

Finally, 35.5% of the cases show a negative VF gap meaning PHD-FK\* has higher profit. There are two factors affecting this. The first one is due to solving PC-E heuristically. PC-E-SLP calculates the inventory holding cost using the lower limit on the cycle time, and it may change with the emerging cycle time of the zone. Also, while the PC-E-SLP solution is amended to

end up with a feasible solution and neighborhood searched is performed, the improvements achieved might be limited since task deletion and WIP insertion decisions are evaluated one after another. (Thus, simultaneous changes in the tasks performed and WIP usage decisions are omitted.) The second reason is due to the myopic nature of the PHD-VK approach. In each zone the selected solution has the maximum profit per cycle. The costs of ending WIP and RP inventories are covered profitably within the current zone. However, in the subsequent zones they may become burdens that are carried throughout the zone. Thus, in some of these cases the PHD-VK solution tends to have higher inventory holding cost and lower profit by the end of the planning horizon. The difference tends to increase as the number of cycles in the zones increases as it is observed in the high demand variability cases. The losses incurred by PHD-VK in these cases are on the average within 15% of PHD-FK.

### **Overall Remarks**

As far as this analysis is concerned, no generalization can be made as to whether rounding  $K_{avg}$  up or down to the nearest integer yields the best PHD-FK\* and PHS-FK\* solutions. In majority of the cases the one that is closest to  $K_{avg}$  gives the best solution. However, due to the fact that PC-E-SLP solutions might vary from those of PHD-VK (even for the very first zone), counter observations are made. Similarly the demand satisfaction percentage may increase or decrease in the best PHD-FK\* and PHS-FK\* solutions compared to PHD-VK's.

With these limited problem categories, we can conclude that PHD-FK\* solution quality is acceptable compared to PHS-FK\*. However its performance may deteriorate as the problem size increases or parameter settings are changed.

In Table 5.6 we provide the average and standard deviation of CPU times (in seconds) elapsed in finding the PHD-FK\* and PHS-FK\* solutions on an OEM Pentium IV personal computer with 2.4 GHz processor and 496 MB RAM memory. The computational times are acceptable when the difficulty of the PH problem is considered.

**Table 5.6** Comparison of CPU solution time (in seconds)  
(average and standard deviation of 240 instances)

		GUN8T8		AKO8T6	
		Low Demand V.	High Demand V.	Low Demand V.	High Demand V.
PHD-FK*	AVG	18.4	40.5	24.6	263.2
	STD DEV	125.8	156.2	55.5	909.8
PHS-FK*	AVG	550.9	742.3	538.9	2397.5
	STD DEV	2884.0	3940.8	3166.3	6370.6

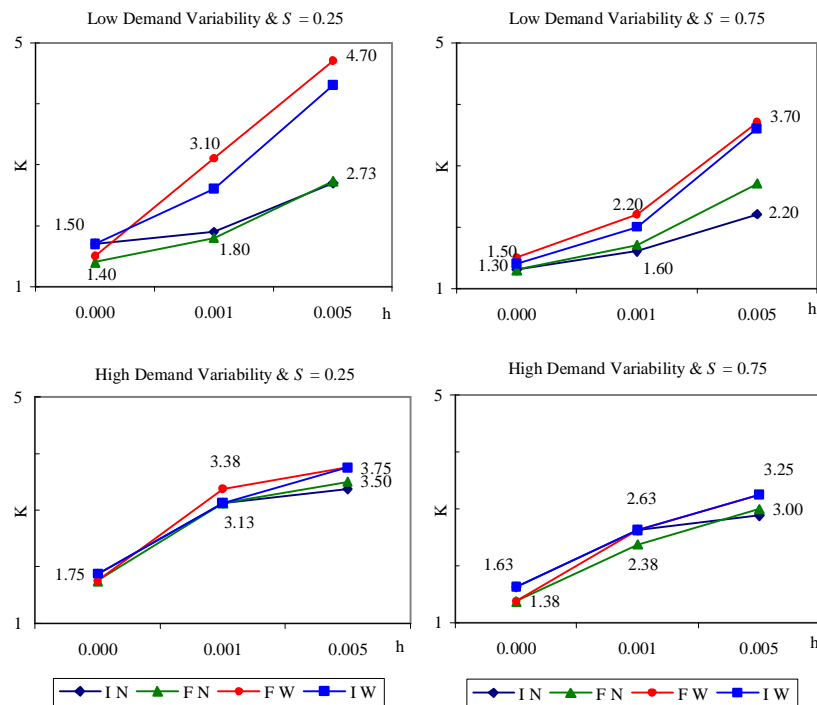
### 5.5.2. The Effects of Problem Parameters on PHD

We want to analyze the effects of having finite (F) versus infinite (I) supply of discarded product, having partially disassembled subassemblies available at the beginning of the planning horizon (W) versus having none (N). We consider the combinations of these two factors as our cases (IN, FN, FW, IW). Each case is studied under six settings composing of two levels of station costs,  $S = 0.25, 0.75$ , and three levels of inventory carrying charge,  $h = 0, 0.001, 0.005$ . The PHD-FK\* solution procedure is utilized to solve each case under the low and high demand variability levels of problems GUN8T8, AKO8T6, AKO20T4-C and LAM20T10. The precedence diagrams of these problems after deployment of the WIP inventories are given in Appendix J. The results are given in Appendix K. Tables K.1 through K.4 summarize the solutions of four problem categories. The solutions are represented using the average number of stations opened ( $K_{avg}$ ), the number of zones ( $Z$ ), the average number of cycles in a zone

( $\alpha_{avg}$ ), makespan (MS), the percentage of demand satisfied (Sat %), the total holding cost (H cost), the total profit ( $\bar{\pi}$ ), and the percentage of holding cost relative to profit (H/P).

### Average Number of Stations Opened ( $K_{avg}$ )

In all four problems and under both demand variability levels,  $K_{avg}$  increases as  $h$  increases. Figure 5.8 depicts  $K_{avg}$  for the 10-part ball-point pen example presented in Section 5.1.3. whereas Figures K.1 through K.3 provide the corresponding figures for the other three problems. Each corresponding figure depicts separately the four cases (IN, FN, FW, IW) under two demand variability levels and two station cost levels. As the high and low demand variability level problems yield entirely different PH solutions, they are not comparable.



**Figure 5.8** Average number of stations opened in LAM20T10 problem

Generally in FW and IW cases where initial WIPs are available, more stations are used compared to the contrary cases (FN and IN). This becomes more evident in the low demand variability instances.

As the beginning WIP levels are given, the only means of reducing the inventory holding cost and increasing the profit is through minimizing the cycle time and increasing the number of stations used. Given a demand variability level, the number of stations opened decreases as  $S$  increases as expected. Since the value added by a task incorporates the station cost associated with its task time, values of inventories and inventory holding cost increase, resulting in a decrease in the number of disassembly operations performed and percentage of demand satisfied.

#### **Average Number of Zones ( $Z$ ) and Average Number of Cycles in a Zone ( $\alpha_{avg}$ )**

Generally for a given demand variability level and a station cost,  $Z$  and  $\alpha_{avg}$  remain fairly constant as  $h$  increases (see Figures 5.9, 5.10, and K.4 through K.9). This is evident in the sense that each zone is terminated by demand depletion, consumption of WIP or supply of discarded products. Thus,  $Z$  and  $\alpha_{avg}$  are more sensitive to demand, supply and initial WIP than they are to  $h$ .

Generally speaking, subassembly availability (W) leads to higher  $Z$  figures relative to no WIP cases (N). In the high demand variability level, smaller values for  $Z$  are observed in IN and FN cases as compared to IW and FW cases due to the initial WIP conditions. However this is not apparent in the case in the low demand variability level, since the initial WIP levels and demand figures are close to each other.



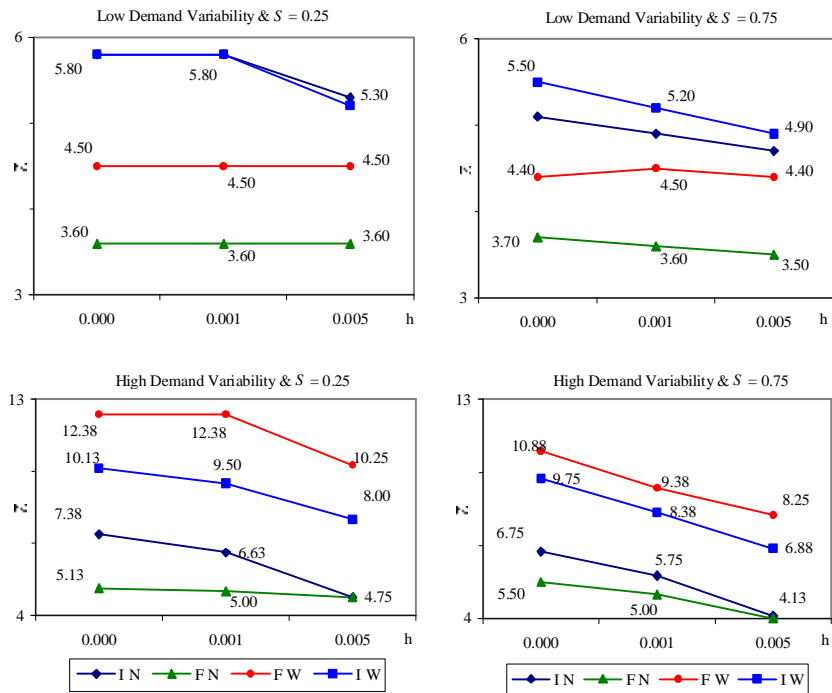


Figure 5.9 Number of zones in LAM20T10 problem

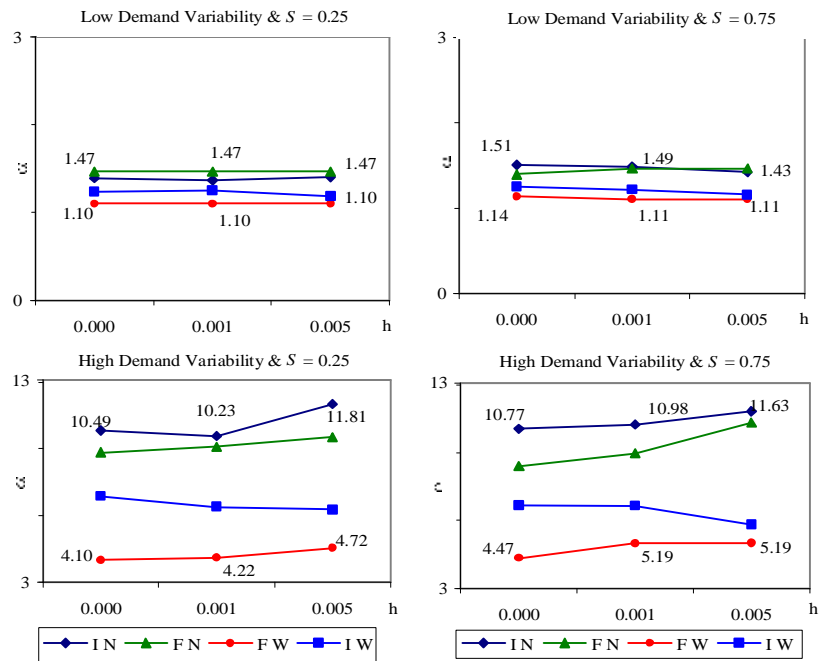


Figure 5.10 Average number of cycles in a zone in LAM20T10 problem

Higher  $\alpha_{\text{avg}}$  values are observed in IN and FN cases compared to IW and FW cases, as the beginning WIP levels (1 or 2) limit the number of cycles in a zone where WIPs are used for disassembly. For a given WIP setting, naturally the infinite supply cases (I) end up with higher  $\alpha_{\text{avg}}$  figures compared to finite supply cases (F). In the case of high demand variability, this becomes more apparent.

### **Makespan (MS) and Percentage of Demand Satisfied (Sat %)**

Makespan is usually shorter than the planning horizon and the corresponding demand satisfaction percentage is less than 100% (see Figures 5.11, 5.12, and K.10 through K.15). This is expected for the FW and FN cases. However for the cases with infinite supply (IW and IN) the same observation is still valid. Firstly, this is a direct implication of using partial disassembly and stopping the disassembly whenever it ceases to be profitable. Secondly, this is due to our problem environment which assumes that released parts are instantaneously used in fulfilling demand. A counter behavior could have been observed, if the released parts were stored in a finished goods inventory and if all demand were satisfied at the end of the planning horizon. We encounter exceptions where disassembly is conducted until the end of the planning horizon (in 198 out of 1920 cases) or 100% demand satisfaction (in 170 out of 1920 cases) is realized.

Generally makespan and demand satisfaction percentage decrease with increasing  $h$  or  $S$ . These results are intuitive in the sense that partial disassembly stops at the point it ceases to be profitable due to higher inventory holding or station costs.

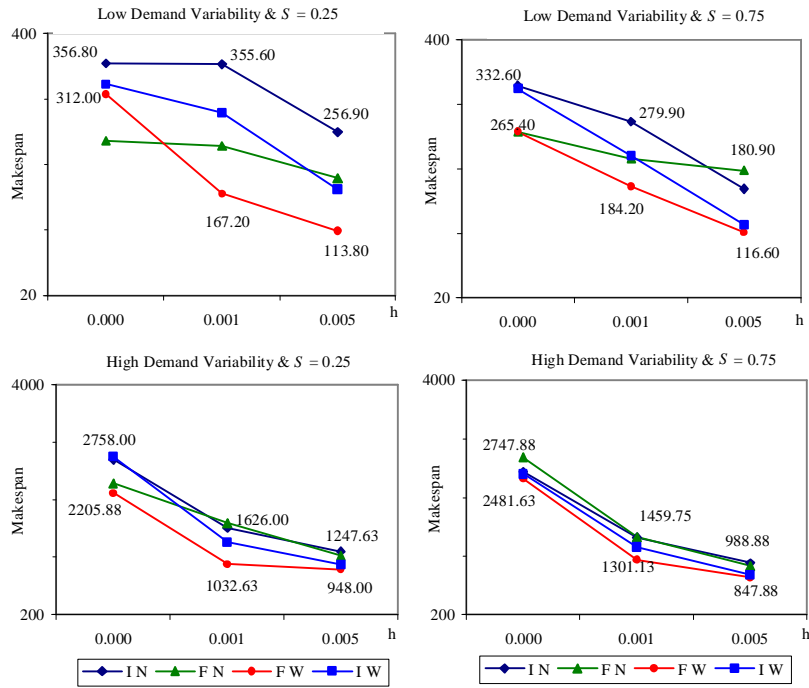


Figure 5.11 Makespan of LAM20T10 problem

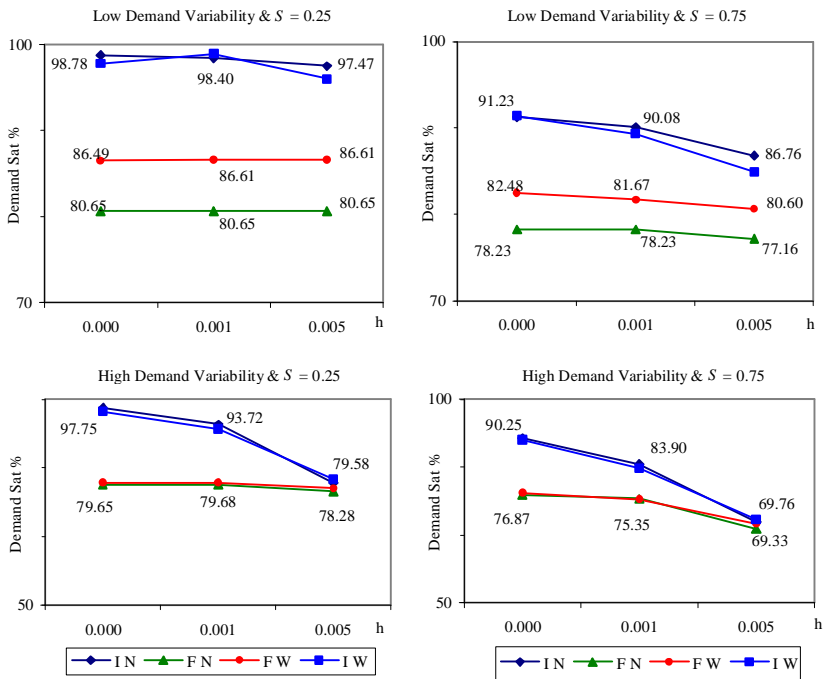


Figure 5.12 Percentage of demand satisfied in LAM20T10 problem

In the IW and FW cases the highest makespan decrease rates are observed as  $h$  increases. Depletion of initially given WIPs take some time during which inventory holding costs incur. Hence disassembly operations' profitability can be used up by the inventory holding cost leading to smaller makespan and demand satisfaction percentage.

### **Profit ( $\pi$ ) and Holding Cost to Profit Ratio (H/P)**

Generally speaking, the profit decreases and the H/P ratio increases with increasing  $h$  or  $S$  as expected (see Figures 5.13, 5.14, and K.16 through K.21). Although the profit values may change as  $h$  changes, the underlying solutions may not change in terms of selected parts and tasks. In 260 out of 640 instances (4 problems x 10 instances x 2 demand variability x 2 supply x 2 subassembly availability x 2 station cost levels) the demand satisfaction percentage, profit and holding cost figures did not change indicating no change in the solution. A fairly constant profit pattern with increasing  $h$  under FN and IN cases implies that the disassembly operations are conducted such that ending WIP or RP inventories are empty. Both of these measures are also sensitive to the number and location of WIPs and the ratio of the number of parts to the number of tasks, creating differences in the patterns observed over the four problems.

In the problems where the ratio of the number of parts to the number of tasks is high and the WIPs are located towards the end of the precedence diagram (problems GUN8T8 and AKO8T6), the H/P ratio under positive  $h$  is significantly higher in FW and IW cases than in FN and IN cases. For these problems the decrease in profit with increasing  $h$  is more evident.

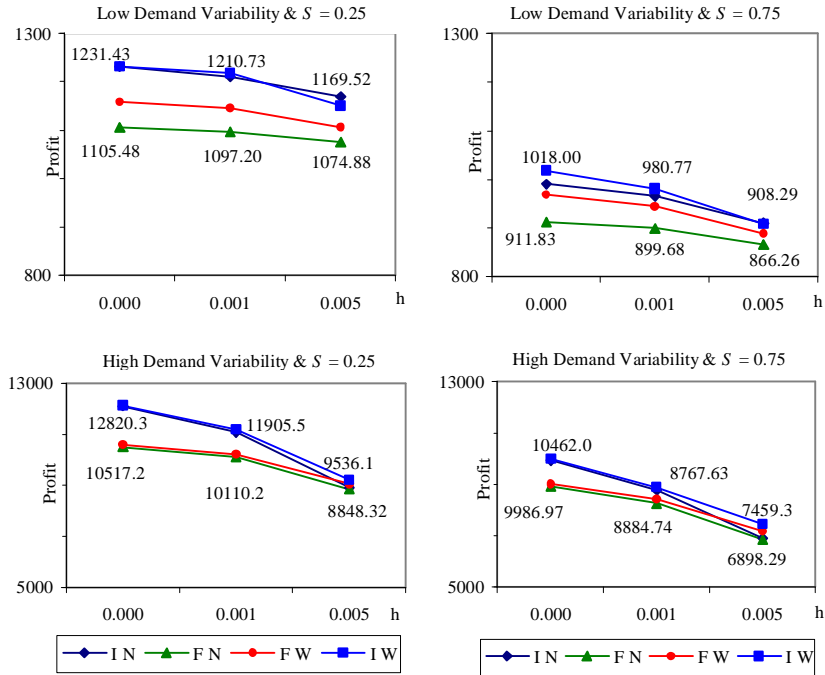


Figure 5.13 Profit in LAM20T10 problem

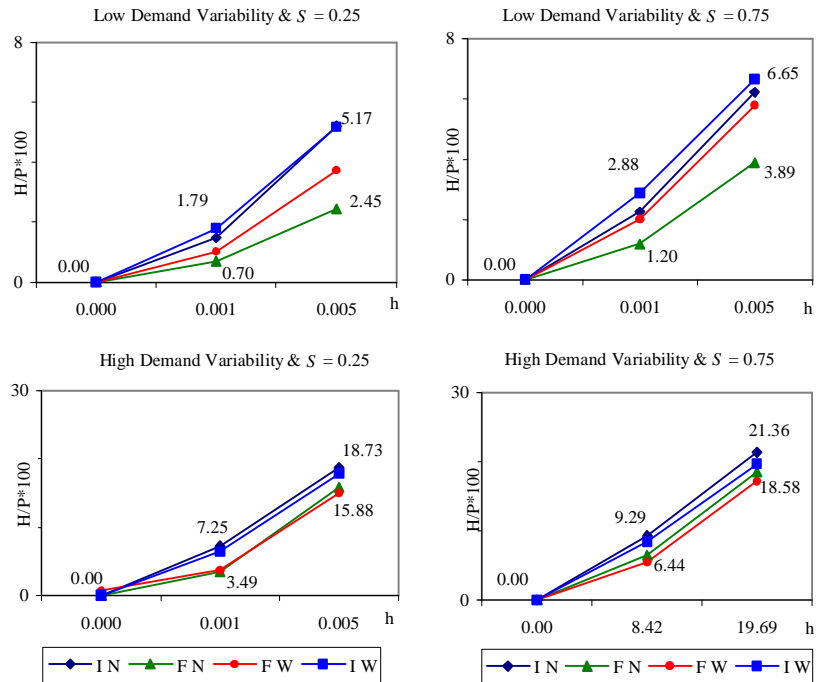


Figure 5.14 Holding cost to profit ratio in LAM20T10 problem

Higher profit values are attainable when the supply of discarded products is infinite compared to finite supply cases. When  $h$  is zero, WIP availability leads to higher profit in both cases as expected. No other generalization can be made as the profit and holding cost figures are also quite dependent on the ending RP and WIP levels.

## **CHAPTER 6**

### **CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH**

As environmental regulations come into effect and producers are obliged to collect their end-of-life products and recover the parts and materials, the problem of disassembling them in large volumes arises. Disassembly lines are proposed as one of the settings on which disassembly can be performed efficiently. With the use of disassembly lines, however, the need to balance the disassembly line emerges. In the presence of revenues from recovered parts and materials and costs of performing disassembly and associated with disposal, discarded products should be disassembled partially to maximize profit. In this dissertation, we defined, formulated and developed solution procedures for partial DLBP with profit maximization objective.

Following the analysis of DLBP environment, we considered two versions of the problem, PC and PH. PC is defined as the assignment of disassembly tasks to an ordered sequence of stations such that the precedence relations are satisfied and the profit per disassembly cycle is maximized. PH, on the other hand, defines the problem for the entire planning horizon by dividing into time zones and finds a different line balance for each zone. Moreover, PH considers other aspects of disassembly systems such as finite supply of discarded products, availability of subassembly and released part inventories for which holding costs are incurred.

Besides considering the two problems with different time spans, we explored how precedence relations are represented in disassembly literature and proposed a simple and unifying representation scheme suitable for DLBP.

We provided a mixed integer programming formulation of PC and used it in our computational analysis to evaluate the performance of the proposed upper bounding scheme and heuristic solution procedure. Since the nature of disassembly requires restructuring on a continuous basis as the flow of discarded products and demand for released parts and materials change, PC must be solved repeatedly. In addition, since we treated both the number of stations and cycle time as decision variables we confined ourselves to a heuristic solution procedure. Since partial disassembly was aimed tasks should only be performed and parts should only be released as long as it is profitable to do so. In order to maximize the profit the task selection and line balancing decisions must be made jointly. Thus we proposed a heuristic solution procedure that is based on the formulation of PC. We also strengthened the LP relaxation of this formulation to obtain an upper bound on profit. An initial feasible solution is constructed using this upper bound solution and then it is improved with neighborhood search heuristics.

We tested our solution approach on 10 problem categories having 8 to 30 disassembly tasks and 4 to 29 parts. Our proposed approach found the optimum in 77% of the 1600 instances solved. The average percentage deviation from the optimum is less than 2% and these solutions are obtained within 1 second of CPU time on the average. We also tried to explore the size of the largest problem that can be solved within reasonable time. A DLBP problem instance with 320 disassembly tasks was solved in 10 minutes. Our upper bounding scheme alone found the optimum in 35% of the 1600 instances. The average percentage deviation from the optimum is less than 10%.



We also provided a mathematical programming model for PH. Due to nonlinearities in the objective function as well as in some constraints we did not attempt to solve it. Our solution procedure (PHD) for PH involved decomposes the overall problem into a number of successive per cycle problems, determining the number of cycles according to the solution of the per cycle problems, each corresponding to a time zone. We determine the number of identical cycles in a time zone according to the solution of the per cycle problem. In the first pass (PHD-VK), the number of stations is allowed to vary from one zone to the next. In the second pass, the average number of stations found in the first pass is rounded up and down and the decomposed heuristic is rerun twice with imposed number of stations. Among the two solutions generated the one yielding the highest profit is declared as the solution to the problem with fixed number of stations (PHD-FK<sup>\*</sup>). To evaluate the performance of the PHD-FK<sup>\*</sup> heuristic, we attempt to solve a restricted version of PH, PHS, where the number of zones, the number of cycles in each zone and the cycle time in the holding cost component of the objective function are given. The results indicated that, with these limited problem categories, the solution quality and times of PHD-FK<sup>\*</sup> are acceptable when the difficulty of the PH problem is considered.

Possible further research directions related with the proposed solution procedures include the following.

- § Determining an upper limit on the cycle time: Uncertainties involved with the quality of the discarded products and the released parts can be considered here. Given probabilities for defective supply and parts, one can determine the number of products that must be disassembled. Then this figure can be used in determining the upper limit of the cycle time. However, the effect on profit of overestimating or underestimating this limit needs to be analyzed.

- § Objective function: In our experimental analysis the upper limit on cycle time turned out to be very constraining. The station cost term in the objective function might be replaced with  $S CT_U \sum_{k=1}^K k x_{D,k} + \epsilon CT$  which might result with satisfactory approximation.
- § Upper bounding scheme: To obtain tighter upper bounds other relaxations can be investigated. For instance partial linear programming relaxations can be used and their performances can be evaluated.
- § Numerical scores used in solution procedure: Numerical scores that fully reflect the nature of disassembly precedence relations (e.g. OR successors) need to be devised and incorporated in the proposed solution procedure.
- § Developing optimum seeking solution procedures for PC: Although choosing an effective branching strategy would be a challenge for approaches such as branch and bound, branch and cut and beam search approaches, the tight upper and lower bounds we propose could be employed in such approaches. It would be difficult to decide on how one should sequence the branching decisions which involve cycle time, part selection, task selection and task assignment. Furthermore, attention must be paid to the fact that unprofitable parts might be released by tasks that are on the path of highly profitable parts. Conditions for task selection must consider globally defined criteria which involve the precedence relations and the parameters along the branches into and out of the tasks. Such a traditional procedure might also be very time consuming.
- § Enhancing the PHD-VK, PHD-FK\* and PHS-FK\* solution procedures: We use PH-E-SLP in solving the decomposed problem. As the cycle time in the inventory holding cost component of this formulation is replaced by the lower limit on the cycle time, constructing a feasible solution that considers the true cycle time in the holding cost and improving this solution is difficult. This adversely affects the solution quality of PHD-VK at the moment. As the average number of stations is calculated using this solution, it

significantly affects the solutions of PHD-FK\* and PHS-FK\*. Thus, finding a way of linearizing the inventory holding cost component is another further research issue. If this issue can be resolved, PC-E-SLP could be solved to optimality, and one can measure exactly the performances of PHD-FK\* and PHS-FK\* in terms of the profit lost by fixing the number of stations. Solving PC-E-SLP iteratively until the lower limit of cycle time used in the holding cost component converges to the cycle time yielded by the solution could be another alternative.

- § Recording the best feasible solution found when PHS-FK<sup>+</sup> or PHS-FK<sup>-</sup> cannot be solved optimally within the given time limit: Thus the SD gap results change at the risk of penalizing the PHS-FK\* heuristic.
- § Decomposing PH into a part mix problem and a line balancing problem: An alternative solution procedure can be developed by first deciding the parts to be released (and the tasks to be performed). Then, the selected tasks can be assigned to stations as in the case of complete DLBP.

It is possible to extend DLBP with profit maximization objective to cover additional issues.

- § Mixed model DLBP with profit maximization can be defined. The input flow of discarded products could be composed of a family of products. Developing solution procedures for this problem is another research direction. The idea of combining the precedence diagrams of the models from ALBP literature could be applied and then our PC solution procedure can be used to find the solution for the mixed model problem.
- § Stochastic version of the DLBP with profit maximization can be considered as the disassembly tasks are known to have task durations with very high coefficient of variation. Formulating and developing solution procedures for this problem is another further research issue one may address. The uncertainties related with the quality of the

incoming products and the flow of the parts along the line due to failures in disassembly tasks can also be incorporated.

- § DLBP with profit maximization objective on unpaced disassembly lines can be formulated and solution procedures can be developed. As the variability involved in task times is high in disassembly systems, the research on unpaced lines is important. In unpaced lines additional decisions need to be given on the places and the sizes of the buffers that are utilized in regulating the flow of the subassemblies on the line.
- § In PH, instead of instantly fulfilling individual part demands, all demand can be met at the end of the planning horizon. In this periodic review case, all RP inventories need to be accumulated and holding cost has to be incurred.
- § Rolling horizon treatment of dynamic PH can be investigated. A starting point can be to compare via simulation the rolling horizon version of PH with our static approach based on decomposition.
- § A test problem library for DLBP needs to be developed including large size real life problems.

Regarding the proposed inventory valuation scheme the following can be explored.

- § In calculating the value added over OR predecessors, instead of taking the average of the values of all predecessors, one can use the minimum or maximum of these values.
- § A reverse logistic valuation scheme can be proposed if it can be adapted to allow partial disassembly.

It is possible to use the proposed precedence diagram representation and PC solution procedure in ALBP.

- § First of all, independent of the line balancing issues, the cons and pros of using precedence diagrams instead of other diagram based representations in assembly must be investigated.
- § If the representation of precedence relations were to incorporate AND/OR graphs or liaison graphs, the proposed representation scheme can be used to transform the given relations to precedence diagrams.
- § In such a case the profit oriented, cost oriented, Type-I, Type-II, and Type-E ALBP problems can be formulated and solved by adapting the proposed PC formulation and solution procedure.
- § Our PC approach adapted for the assembly line balancing with processing alternatives problem can be compared with the approaches proposed in the ALBP literature.

Finally, systems that integrate assembly and disassembly can be studied. One form of such integration is maintenance or renovation projects of (tanks, aircrafts) by first disassembling and then assembling on the same line. This can be done in sequel (assemble after disassembly is finished). Alternatively assembly and disassembly can be conducted simultaneously as in hybrid manufacturing/remanufacturing systems.

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## APPENDIX A

### PROFIT VS CYCLE TIME ANALYSIS

CT	$\bar{r}^*$	$K^*$	$K^* CT$	$WC_{max}$	Task assignments								Quantity of parts released							
					1	2	3	4	5	6	7	8	TC	FD	HD	BP	PCI	RAM	PU	MB
14	0	0	0	0																
15	8	2	30	15	1				2				1					1		
16	14	3	48	16	1	2			2	3			1	1				1	2	
17	12	4	68	17	1	3	2		3	4		4	1	1	1			1	2	1
18	8	3	54	18	1	2			2	3			1	1				1	2	
19	6	2	38	19	1	2			2				1	1					2	
20	12	4	80	20	1	2	3		4	2	4	4	1	1	1			1	2	1 1
21	8	4	84	20	1	2	3		4	2	4	4	1	1	1			1	2	1 1
22	18	2	44	22	1	1			2	2			1	1				1	2	
23	16	2	46	22	1	1			2	2			1	1				1	2	
24	20	3	72	24	1	2	2		1	3	3	3	1	1	1			1	2	1 1
25	17	3	75	24	1	2	2		1	3	3	3	1	1	1			1	2	1 1
26	14	3	78	24	1	2	2		1	3	3	3	1	1	1			1	2	1 1
27	11	3	81	24	1	2	2		1	3	3	3	1	1	1			1	2	1 1
28	10	1	28	28	1				1				1					1		
29	9	1	29	28	1				1				1					1		
30	8	1	30	28	1				1				1					1		
31	10	3	93	31	1	1	2	3	1	2	3	3	1	1	1	1		1	2	1 1
32	16	2	64	32	1	2	1		2	2	2	2	1	1	1			1	2	1
33	14	2	66	32	1	2	1		2	2	2	2	1	1	1			1	2	1
34	12	2	68	32	1	2	1		2	2	2	2	1	1	1			1	2	1
35	10	2	70	32	1	2	1		2	2	2	2	1	1	1			1	2	1
36	8	2	72	32	1	2	1		2	2	2	2	1	1	1			1	2	1
37	18	2	74	37	1	1	2		2	1	2	2	1	1	1			1	2	1 1
38	16	2	76	37	1	1	2		2	1	2	2	1	1	1			1	2	1 1
39	14	2	78	37	1	1	2		2	1	2	2	1	1	1			1	2	1 1
40	12	2	80	37	1	1	2		2	1	2	2	1	1	1			1	2	1 1
41	10	2	82	37	1	1	2		2	1	2	2	1	1	1			1	2	1 1
42	8	2	84	37	1	1	2		2	1	2	2	1	1	1			1	2	1 1
43	19	1	43	43	1	1			1	1			1	1				1	2	
44	18	1	44	43	1	1			1	1			1	1				1	2	
45	17	1	45	43	1	1			1	1			1	1				1	2	
46	16	1	46	43	1	1			1	1			1	1				1	2	
47	15	1	47	43	1	1			1	1			1	1				1	2	
48	14	1	48	43	1	1			1	1			1	1				1	2	

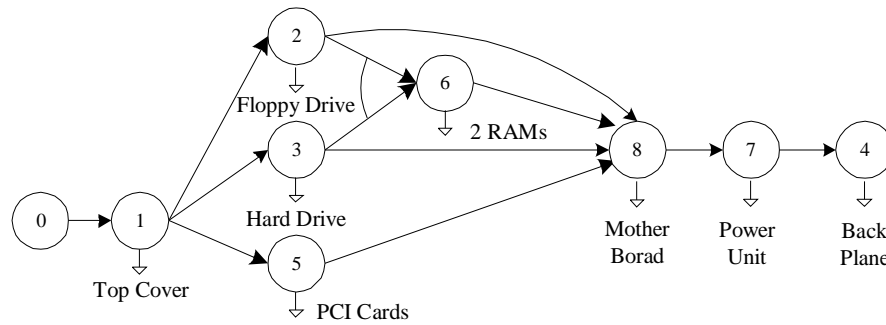
CT	F*	K*	K* CT	WC <sub>max</sub>	Task assignments								Quantity of parts released							
					1	2	3	4	5	6	7	8	TC	FD	HD	BP	PCI	RAM	PU	MB
49	13	1	49	43	1	1				1	1				1	2				
50	12	1	50	43	1	1				1	1				1	2				
51	11	1	51	43	1	1				1	1				1	2				
52	10	1	52	43	1	1				1	1				1	2				
53	12	1	53	53	1		1			1	1			1	2					
54	11	1	54	53	1		1			1	1			1	2					
55	10	1	55	53	1		1			1	1			1	2					
56	9	1	56	53	1		1			1	1			1	2					
57	8	1	57	53	1		1			1	1			1	2					
58	7	1	58	53	1		1			1	1			1	2					
59	6	1	59	53	1		1			1	1			1	2					
60	5	1	60	53	1		1			1	1			1	2					
61	4	1	61	53	1		1			1	1			1	2					
62	3	1	62	53	1		1			1	1			1	2					
63	2	1	63	53	1		1			1	1			1	2					
64	16	1	64	64	1	1	1			1	1	1		1	2		1			
65	15	1	65	64	1	1	1			1	1	1	1		2		1			
66	14	1	66	64	1	1	1			1	1	1	1		2		1			
67	13	1	67	64	1	1	1			1	1	1	1		2		1			
68	12	1	68	64	1	1	1			1	1	1	1		2		1			
69	11	1	69	64	1	1	1			1	1	1	1		2		1			
70	10	1	70	64	1	1	1			1	1	1	1		2		1			
71	21	1	71	71	1	1	1			1	1	1	1	1	2	1	1			
72	20	1	72	71	1	1	1			1	1	1	1	1	2	1	1			
73	19	1	73	71	1	1	1			1	1	1	1	1	2	1	1			
74	18	1	74	71	1	1	1			1	1	1	1	1	2	1	1			
75	17	1	75	71	1	1	1			1	1	1	1	1	2	1	1			
76	16	1	76	71	1	1	1			1	1	1	1	1	2	1	1			
77	15	1	77	71	1	1	1			1	1	1	1	1	2	1	1			
78	14	1	78	71	1	1	1			1	1	1	1	1	2	1	1			
79	13	1	79	71	1	1	1			1	1	1	1	1	2	1	1			
80	12	1	80	71	1	1	1			1	1	1	1	1	2	1	1			
81	11	1	81	71	1	1	1			1	1	1	1	1	2	1	1			
82	10	1	82	71	1	1	1			1	1	1	1	1	2	1	1			
83	9	1	83	71	1	1	1			1	1	1	1	1	2	1	1			
84	8	1	84	71	1	1	1			1	1	1	1	1	2	1	1			
85	7	1	85	71	1	1	1			1	1	1	1	1	2	1	1			
86	17	1	86	86	1	1	1	1		1	1	1	1	1	2	1	1			
87	16	1	87	86	1	1	1	1	1		1	1	1	1	2	1	1			
88	15	1	88	86	1	1	1	1	1	1		1	1	1	2	1	1			
89	14	1	89	86	1	1	1	1	1	1	1		1	1	2	1	1			
90	13	1	90	86	1	1	1	1	1	1	1	1		1	2	1	1			
91	12	1	91	86	1	1	1	1	1	1	1	1	1		2	1	1			
92	11	1	92	86	1	1	1	1	1	1	1	1	1		2	1	1			
93	10	1	93	86	1	1	1	1	1	1	1	1	1		2	1	1			
94	9	1	94	86	1	1	1	1	1	1	1	1	1		2	1	1			
95	8	1	95	86	1	1	1	1	1	1	1	1	1		2	1	1			
96	7	1	96	86	1	1	1	1	1	1	1	1	1		2	1	1			
97	6	1	97	86	1	1	1	1	1	1	1	1	1		2	1	1			
98	5	1	98	86	1	1	1	1	1	1	1	1	1		2	1	1			
99	4	1	99	86	1	1	1	1	1	1	1	1	1		2	1	1			
100	3	1	100	86	1	1	1	1	1	1	1	1	1		2	1	1			
101	2	1	101	86	1	1	1	1	1	1	1	1	1		2	1	1			
102	1	1	102	86	1	1	1	1	1	1	1	1	1		2	1	1			
103	0	1	103	86	1	1	1	1	1	1	1	1	1		2	1	1			



## APPENDIX B

### LOGICAL INEQUALITIES GENERATION EXAMPLE

Consider the PC example given in Figure B.1 and Table B.1 with a station cost of 1.00.



**Figure B.1** Precedence diagram of PC example

**Table B.1** Part revenue, demand and availability; task time and cost information

$j$	$r_j$	$d_j$	$a_j$	$i$	$t_i$	$c_i$
Top Cover (TC)	27	26	1	1	15	7
Floppy Drive (FD)	16	79	1	2	7	16
Hard Drive (HD)	22	99	1	3	17	19
Back Plane (BP)	21	34	1	4	15	10
PCI Cards (PCI)	24	73	1	5	9	6
RAMs (RAM)	14	94	2	6	12	4
Power Unit (PU)	30	48	1	7	7	18
Mother Board (MB)	31	45	1	8	4	16

The solution of the LP relaxation of the PC formulation is as follows.

$$\begin{aligned}
 K^* &= 1 \text{ (although } K = 8 \text{ stations are used)} \\
 u_1^* &= 9.9844 \\
 CT^* &= 9.9844 \\
 q_{TC}^* &= q_{HD}^* = q_{BP}^* = q_{PCI}^* = q_{PU}^* = q_{MB}^* = 1 \quad q_{FD}^* = 0.125 \quad q_{RAM}^* = 2 \\
 \sum_{j=TC}^{MB} r_j q_j^* &= 185 \\
 \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* &= 82 \\
 \sum_{i=0}^D t_i \sum_{k=1}^8 x_{i,k}^* &= 79.88 \text{ (represents total time used on the opened stations)} \\
 \sum_{k=1}^8 k u_k^* &= 9.9844 \text{ (represents total available time on the opened stations)} \\
 \mathcal{R}^* &= \sum_{j=TC}^{MB} r_j q_j^* - \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* - S \sum_{k=1}^8 k u_k^* = 185 - 82 - 1 * 9.9844 = 93.0156
 \end{aligned}$$

The percentage assignment of tasks to stations is depicted in Figure B.2.

$k$	WC	0	1	2	3	4	5	6	7	8	D
1	9.98	12.50	12.50	12.50	12.50	7.40	12.50	12.50	12.50	12.50	11.61
2	9.98	18.52	10.66		3.20	12.50	23.16	28.20		12.50	
3	9.98		31.02				53.68			12.50	
4	9.98		14.80			23.27		25.15	10.77	12.50	
5	9.98				5.95			34.15	62.50	12.50	
6	9.98		31.02		22.56				14.23	12.50	
7	9.98					56.83	10.66			12.50	88.39
8	9.98				55.79					12.50	
		31.02	100.00	12.50	100.00	100.00	100.00	100.00	100.00	100.00	100.00

**Figure B.2** Percentage assignment of tasks to stations in the PC-LP solution

The optimal solution of PC-LP with logical inequality - 1 is summarized below.

$$\begin{aligned}
 K^* &= 8 \\
 u_1^* &= 2 \text{ and } u_8^* = 6 \\
 CT^* &= u_1^* + u_8^* = 8 \\
 q_{TC}^* &= q_{PCI}^* = q_{PU}^* = q_{MB}^* = 1 \quad q_{FD}^* = q_{HD}^* = 0.125 \quad q_{RAM}^* = 2 \\
 \sum_{j=TC}^{MB} r_j q_j^* &= 144.75 \\
 \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* &= 55.38 \\
 \sum_{i=0}^D t_i \sum_{k=1}^8 x_{i,k}^* &= 50.00 \text{ (represents total time used on the opened stations)} \\
 \sum_{k=1}^8 k u_k^* &= 50 \text{ (represents total available time on the opened stations)} \\
 \bar{\pi}^* &= \sum_{j=TC}^{PB} r_j q_j^* - \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* - S \sum_{k=1}^8 k u_k^* = 144.75 - 55.38 - 1 * 50 = 39.38
 \end{aligned}$$

The percentage assignment of tasks to stations is depicted in Figure B.3.

k	WC	0	1	2	3	4	5	6	7	8	D
1	8.88	12.50	12.50	12.50	12.50		12.50	12.50		12.50	2.33
2	4.62						12.50	25.00		12.50	
3	6.12		12.50				25.00	12.50		12.50	
4	3.5							25.00		12.50	
5	7.87	9.38						25.00	62.50	12.50	
6	8.00		21.88				46.88			12.50	85.61
7	4.76	4.69	26.56				3.12			12.50	
8	7.09		26.56						37.25	12.50	12.06
		26.57	100.00	12.50	12.50	0.00	100.00	100.00	100.00	100.00	100.00

**Figure B.3** Assignment of tasks to stations in PC-LP with logical inequality-1 solution

The optimal solution of PC-LP with logical inequality-2 is summarized below.

$$\begin{aligned}
 K^* &= 8 \\
 u_1^* &= 10.75 \\
 CT^* &= 10.75 \\
 q_{TC}^* &= q_{FD}^* = q_{HD}^* = q_{PCI}^* = q_{PU}^* = q_{MB}^* = 1 \quad q_{RAM}^* = 2 \\
 \sum_{j=TC}^{PB} r_j q_j^* &= 199 \\
 \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* &= 96 \\
 \sum_{i=0}^D t_i \sum_{k=1}^8 x_{i,k}^* &= 86 \text{ (represents total time used on the opened stations)} \\
 \sum_{k=1}^8 k u_k^* &= 10.75 \text{ (represents total available time on the opened stations)} \\
 \bar{\pi}^* &= \sum_{j=TC}^{PB} r_j q_j^* - \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* - S \sum_{k=1}^8 k u_k^* = 199 - 96 - 1 * 10.75 = 92.25
 \end{aligned}$$

The percentage assignment of tasks to stations is depicted in Figure B.4.

<i>k</i>	WC	0	1	2	3	4	5	6	7	8	D
1	10.75	12.95	12.95	12.95	12.95		12.95	25.90	12.95	12.95	12.50
2	10.75		12.95	12.95	12.95		21.59		38.86	25.90	19.62
3	10.75	6.88				65.35			13.54		
4	10.75					34.65		46.27			
5	10.75		19.83		45.74						
6	10.75		19.83		11.08		65.46				
7	10.75			65.57	7.58				34.65	61.15	
8	10.75	80.17	34.4	8.53	9.70			27.83			67.88
		100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

**Figure B.4** Assignment of tasks to stations in PC-LP with logical inequality-2 solution

The optimal solution of PC-SLP (i.e. PC-LP with logical inequalities 1 and 2) is summarized below.

$$\begin{aligned}
 K^* &= 8 \\
 u_8^* &= 6.6875 \\
 CT^* &= 6.6875 \\
 q_{TC}^* &= q_{PCI}^* = 1 \quad q_{RAM}^* = 2 \quad q_{FD}^* = q_{HD}^* = q_{PU}^* = q_{MB}^* = 0.5 \\
 \sum_{j=TC}^{PB} r_j q_j^* &= 128.5 \\
 \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* &= 51.5 \\
 \sum_{i=0}^D t_i \sum_{k=1}^8 x_{i,k}^* &= 53.5 \text{ (represents total time used on the opened stations)} \\
 \sum_{k=1}^8 k u_k^* &= 53.5 \text{ (represents total available time on the opened stations)} \\
 \pi^* &= \sum_{j=TC}^{PB} r_j q_j^* - \sum_{i=0}^D c_i \sum_{k=1}^8 x_{i,k}^* - S \sum_{k=1}^8 k u_k^* = 128 - 47 - 1 * 53.5 = 23.5
 \end{aligned}$$

The percentage assignment of tasks to stations is depicted in Figure B.5.

k	WC	0	1	2	3	4	5	6	7	8	D
1	10.75		14.25	14.25	13.35		14.25				12.50
2	10.75			14.25	14.25		14.25	16.53			19.62
3	10.75			14.25			14.25	36.72			
4	10.75		14.25	7.25	22.40			1.96			
5	10.75	85.75	44.58								
6	10.75		26.92				7.25			50.00	
7	10.75						50.00	18.23			
8	10.75	14.25						26.56	50.00		67.88
		100.00	100.00	50.00	50.00	0.00	100.00	100.00	50.00	50.00	100.00

**Figure B.5** Assignment of tasks to stations in PC-SLP solution

## APPENDIX C

### PRECEDENCE RELATIONS OF PROBLEM CATEGORIES

#### C.1 GUN8T8

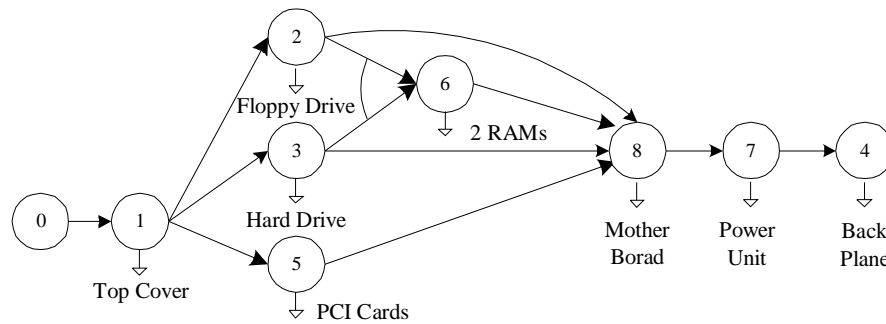


Figure C.1 Precedence diagram of Gungör and Gupta's (2002) 8 task 8 part PC problem

#### C.2 AKO8T6

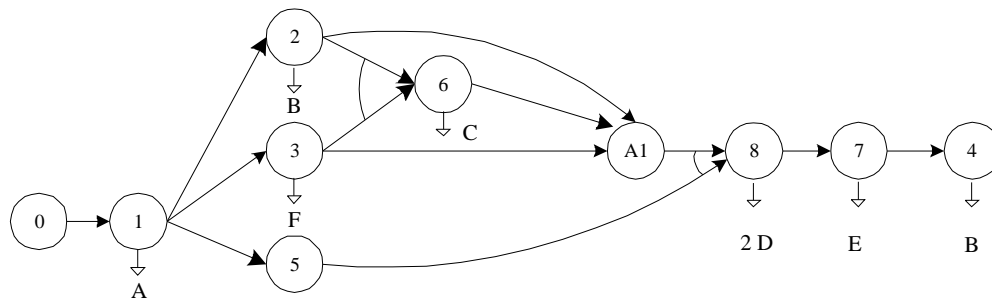
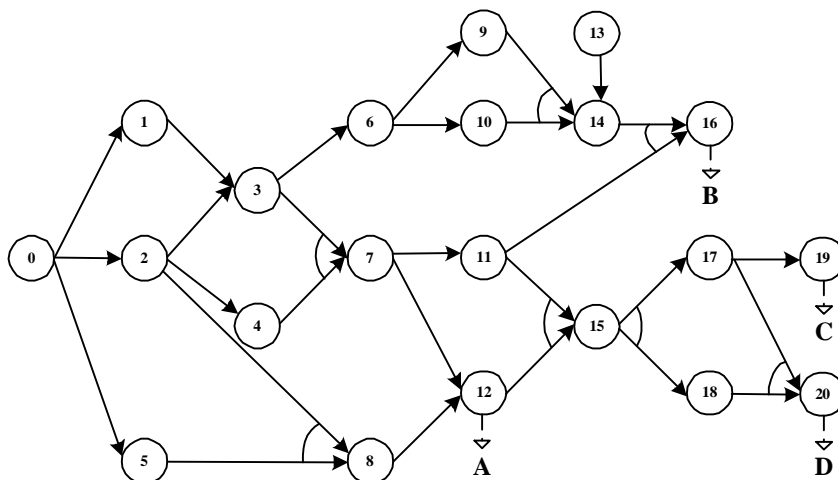


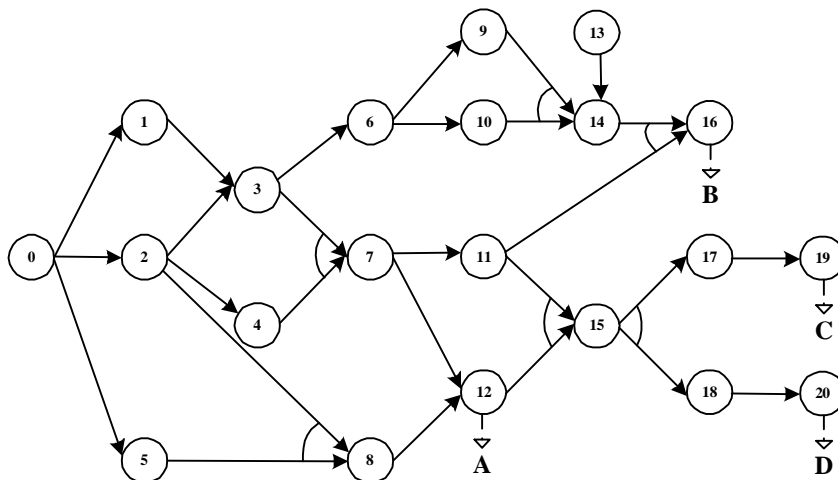
Figure C.2 Precedence diagram of 8 task 6 part problem

**C.3 AKO20T4-A**



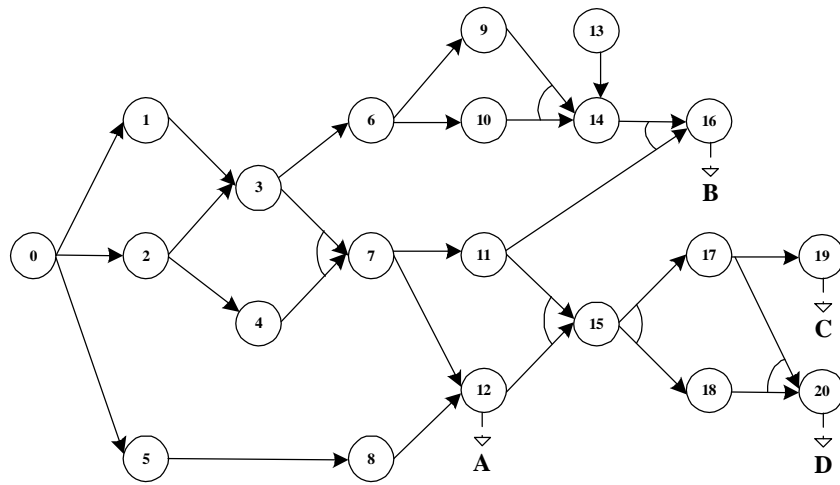
**Figure C.3** Precedence diagram of 20 task 4 part problem - A

**C.4 AKO20T4-B**



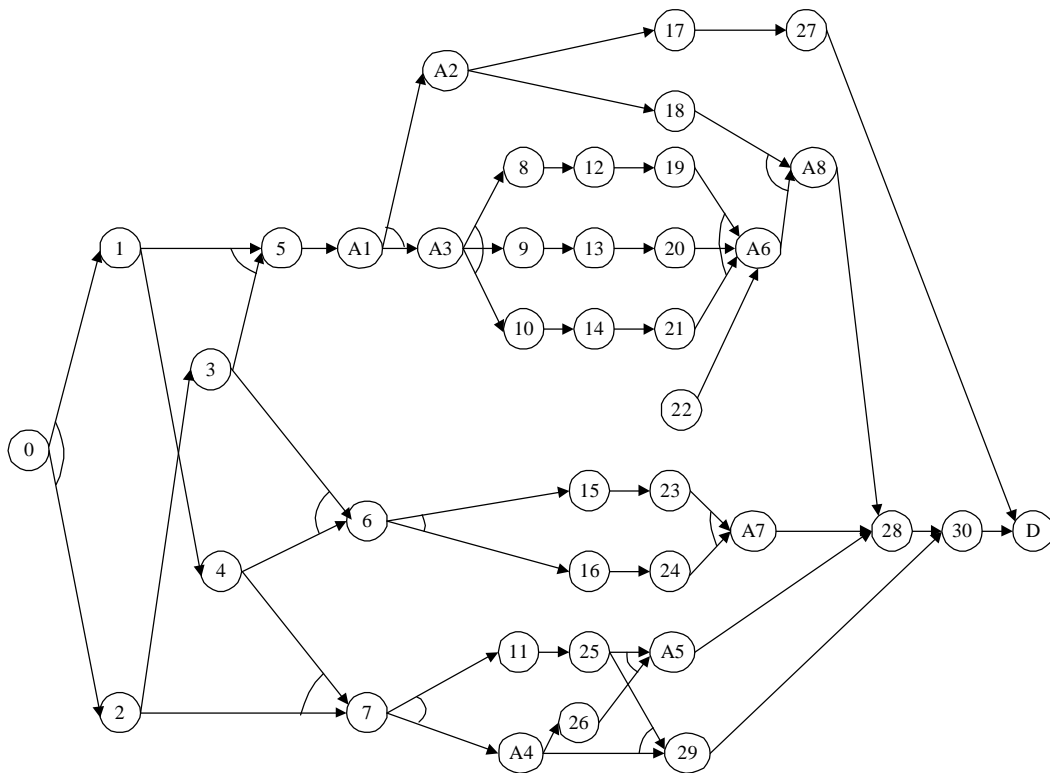
**Figure C.4** Precedence diagram of 20 task 4 part problem - B

**C.5 AKO20T4-C**



**Figure C.5** Precedence diagram of 20 task 4 part problem - C

**C.6 AKO30T12**



**Figure C.6** Precedence diagram of 30 tasks 12 part problem



C.7 LAM20T10 and C.8 LAM20T24

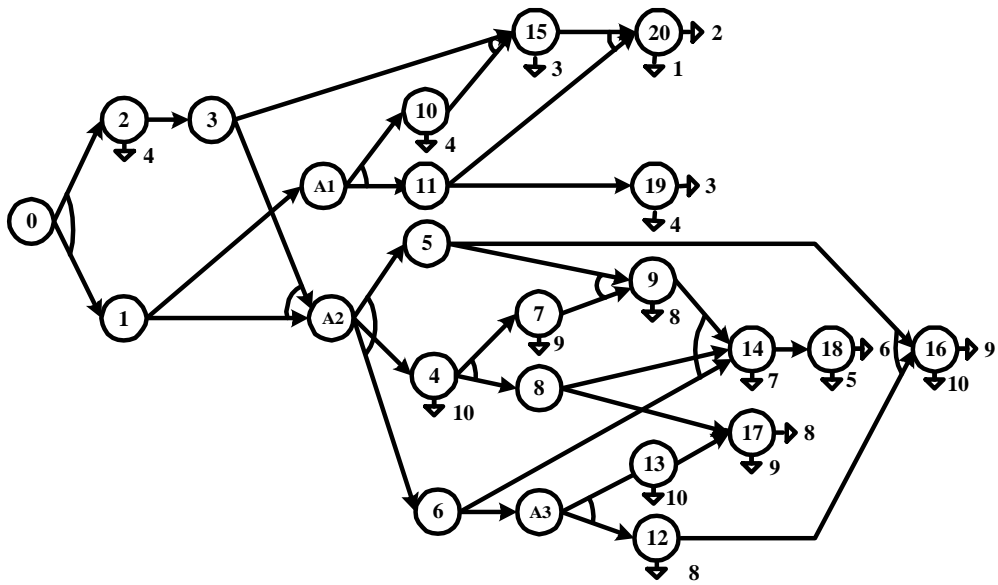


Figure C.7 Precedence diagram of Lambert's (1997) 20 tasks 10-part ball-point pen problem

C.9 LAM30T10 and C.10 LAM30T29

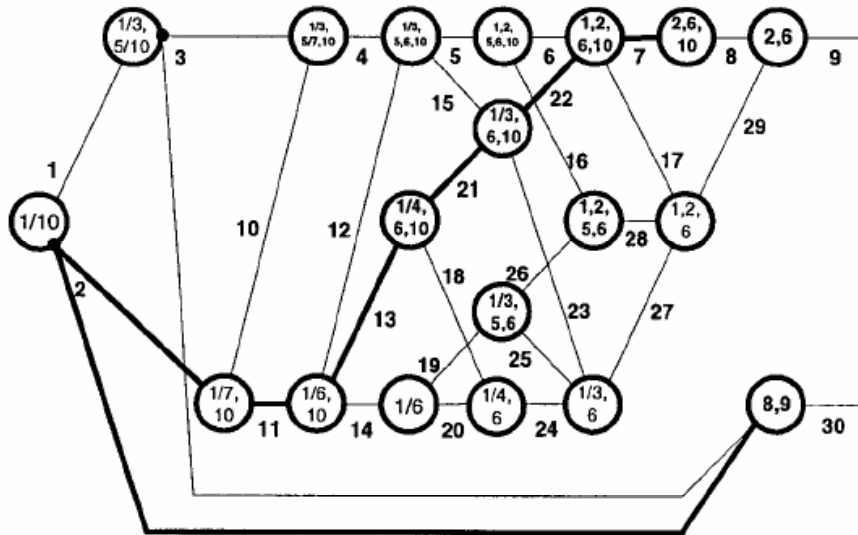


Figure C.8 Disassembly graph of Lambert's (1999) 30 tasks 10-part radio problem

## APPENDIX D

### EVALUATION OF THE LOGICAL INEQUALITIES

In Table D.1 the percentage reductions in profit achieved over the linear programming relaxation of PC formulation via the introduced inequalities are summarized by problem category. In Table D.2 profit obtained by linear programming relaxation of PC (PC-LP), PC-LP with logical inequality 1 (LI-1), PC-LP with logical inequality 2 (LI-2), PC-LP strengthened by both logical inequalities (LI-1+LI-2) are given for each instance solved.

**Table D.1** Percentage reduction in profit with logical inequalities  
(average of 10 problem instances)

PROBLEM	S	LOW DEMAND VARIABILITY DATA SET						HIGH DEMAND VARIABILITY DATA SET					
		IL-1		IL-2		IL-1+IL-2		IL-1		IL-2		IL-1+IL-2	
		AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN	AVG	MIN
GUN8T8	0.25	21.21	12.25	15.08	0.19	38.76	18.91	20.09	10.81	14.00	0.00	35.97	18.17
	0.50	40.23	21.92	15.84	0.39	58.21	38.78	39.09	21.92	14.67	0.00	55.00	31.72
	0.75	55.40	28.87	16.67	0.60	71.74	48.69	53.95	28.87	15.40	0.00	68.85	45.71
	1.00	66.40	36.08	17.59	0.82	80.39	58.66	64.87	36.08	16.20	0.00	79.89	58.62
	1.25	74.25	43.56	18.59	1.06	86.07	69.08	72.99	43.56	17.06	0.00	86.27	69.23
	1.50	80.16	51.32	19.70	1.49	89.99	74.27	78.89	51.32	18.02	0.00	89.94	71.99
	1.75	85.04	57.15	20.97	2.29	92.68	73.96	83.93	57.15	19.09	0.00	92.49	71.74
	2.00	88.41	63.22	22.35	3.12	94.52	73.63	87.68	63.22	19.83	0.00	94.33	71.46
AKO8T6	0.25	12.25	7.58	10.56	0.00	23.88	14.89	12.51	8.46	6.89	0.00	20.14	11.79
	0.50	24.65	15.33	10.91	0.00	37.71	26.95	24.86	17.16	7.12	0.00	33.83	21.43
	0.75	37.21	23.25	11.28	0.00	51.65	37.28	37.42	26.08	7.36	0.00	47.46	31.32
	1.00	48.52	31.35	11.66	0.00	64.27	46.40	48.86	33.23	7.60	0.00	60.03	41.46
	1.25	58.53	39.63	12.05	0.00	74.79	55.73	59.32	38.30	7.86	0.00	71.94	51.87
	1.50	65.82	43.84	12.43	0.00	82.63	64.45	66.73	43.33	8.12	0.00	81.83	59.11
	1.75	72.41	47.61	12.78	0.00	87.75	69.29	73.27	48.50	8.40	0.00	87.02	65.80
	2.00	77.97	51.46	13.17	0.00	91.97	74.27	78.61	53.80	8.71	0.00	90.91	72.67
AKO20T4-A	0.25	2.82	1.19	19.37	8.48	27.60	13.97	2.87	1.36	19.04	8.48	27.11	13.97
	0.50	5.66	2.41	19.68	8.68	36.14	19.67	5.76	2.73	19.34	8.68	35.48	19.67
	0.75	8.50	3.63	20.00	8.88	44.64	25.37	8.66	4.11	19.64	8.88	43.82	25.37
	1.00	11.35	4.85	20.31	9.08	53.05	31.09	11.57	5.49	19.94	9.08	52.05	31.09
	1.25	14.22	6.06	20.63	9.28	61.22	36.82	14.48	6.86	20.24	9.28	60.01	36.82
	1.50	17.08	7.27	20.95	9.48	68.15	42.56	17.41	8.24	20.55	9.48	67.76	42.56
	1.75	19.96	8.48	21.26	9.69	74.46	48.31	20.34	9.61	20.85	9.69	75.15	48.31
	2.00	22.85	9.70	21.58	9.89	79.77	54.08	23.28	11.00	21.16	9.89	80.52	54.08

**Table D.1** Percentage reduction in profit with logical inequalities (Continued)

		LOW DEMAND VARIABILITY DATA SET						HIGH DEMAND VARIABILITY DATA SET					
		IL-1		IL-2		IL-1+IL-2		IL-1		IL-2		IL-1+IL-2	
S													
AKO20T4-B	0.25	2.97	2.04	43.64	23.05	51.44	34.62	3.03	1.72	39.07	21.93	46.92	32.35
	0.50	5.97	4.09	43.96	23.52	59.57	46.69	6.08	3.46	39.38	22.40	55.11	42.31
	0.75	8.98	6.15	44.28	23.98	67.47	58.80	9.15	5.18	39.70	22.87	63.11	52.30
	1.00	11.99	8.21	44.61	24.45	75.16	70.53	12.21	6.91	40.02	23.35	71.01	59.64
	1.25	15.01	10.28	44.93	24.92	82.74	76.80	15.29	8.64	40.34	23.82	78.84	64.79
	1.50	18.05	12.35	45.26	25.39	89.99	82.83	18.38	10.38	40.66	24.30	85.97	69.96
	1.75	21.09	14.43	45.58	25.86	95.87	88.87	21.48	12.12	40.98	24.78	91.83	75.14
	2.00	24.14	16.51	45.91	26.34	98.41	92.57	24.59	13.86	41.30	25.26	95.35	78.59
AKO20T4-C	0.25	3.18	1.84	23.74	12.00	32.42	16.71	3.32	2.19	24.41	12.00	33.33	16.71
	0.50	6.38	3.71	24.07	12.12	41.38	21.51	6.66	4.40	24.74	12.12	42.54	21.51
	0.75	9.59	5.58	24.40	12.24	50.19	26.16	10.01	6.61	25.08	12.24	51.61	26.16
	1.00	12.81	7.45	24.73	12.36	58.58	30.83	13.37	8.83	25.42	12.36	60.25	30.83
	1.25	16.04	9.31	25.06	12.48	66.12	35.51	16.75	11.06	25.76	12.48	67.86	35.51
	1.50	19.28	11.19	25.40	12.61	72.62	40.20	20.13	13.29	26.10	12.61	74.37	40.20
	1.75	22.53	13.06	25.73	12.73	78.20	44.91	23.52	15.52	26.45	12.73	79.96	44.91
	2.00	25.79	14.94	26.07	12.86	82.56	49.63	26.93	17.76	26.79	12.86	84.33	49.63
LAM20T10	0.25	6.08	4.13	44.58	39.36	48.74	43.65	5.64	3.99	46.59	41.11	50.34	44.19
	0.50	12.18	8.12	44.64	39.41	52.72	45.89	11.27	7.81	46.64	41.19	53.92	46.58
	0.75	18.14	12.14	44.69	39.45	56.43	48.13	16.73	11.65	46.69	41.27	57.23	48.97
	1.00	23.95	16.17	44.75	39.50	60.02	50.39	22.05	15.49	46.75	41.35	60.41	51.37
	1.25	29.61	20.22	44.80	39.55	63.52	52.66	27.30	19.36	46.80	41.44	63.54	53.78
	1.50	34.92	24.29	44.86	39.60	66.94	54.94	32.21	23.24	46.86	41.51	66.58	56.21
	1.75	39.70	28.38	44.90	39.65	70.32	57.23	36.64	27.14	46.90	41.53	69.57	58.64
	2.00	44.03	32.48	44.94	39.70	73.55	59.53	40.75	31.05	46.94	41.55	72.48	61.09
LAM20T24	0.25	18.60	11.25	1.47	0.00	20.20	12.44	18.65	10.10	1.53	0.00	20.36	10.71
	0.50	35.49	21.07	1.49	0.00	36.66	21.07	35.21	17.63	1.55	0.00	36.47	17.63
	0.75	49.86	29.85	1.51	0.00	50.59	29.85	49.08	24.65	1.58	0.00	49.87	24.65
	1.00	62.69	36.16	1.54	0.00	63.04	36.16	62.02	29.77	1.61	0.00	62.41	31.77
	1.25	72.61	41.66	1.56	0.00	72.79	42.25	73.10	33.82	1.64	0.00	73.31	35.96
	1.50	80.36	45.22	1.59	0.00	80.41	45.77	80.40	37.90	1.66	0.00	80.47	38.52
	1.75	85.40	48.02	1.61	0.00	85.40	48.02	84.71	41.11	1.69	0.00	84.71	41.11
	2.00	87.83	50.29	1.64	0.00	87.83	50.29	86.92	43.72	1.72	0.00	86.92	43.72
LAM30T10	0.25	4.75	3.74	61.38	54.25	63.43	56.21	4.88	4.14	66.06	57.57	67.74	60.02
	0.50	9.40	7.22	61.39	54.24	65.34	58.05	9.75	8.29	66.06	57.58	69.28	62.49
	0.75	13.99	10.61	61.40	54.24	67.14	59.90	14.60	12.46	66.06	57.58	70.75	64.85
	1.00	18.53	14.02	61.40	54.23	68.85	61.75	19.45	16.63	66.06	57.59	72.14	66.72
	1.25	22.98	17.43	61.41	54.23	70.55	63.39	24.29	20.82	66.07	57.60	73.53	68.61
	1.50	27.41	20.85	61.42	54.22	72.24	64.52	29.16	25.02	66.07	57.60	74.91	70.50
	1.75	31.81	24.28	61.43	54.21	73.93	65.66	34.02	29.23	66.07	57.61	76.29	71.93
	2.00	36.09	27.72	61.44	54.21	75.63	66.81	38.77	33.45	66.07	57.62	77.68	72.86
LAM30T29	0.25	21.00	11.80	0.00	0.00	21.00	11.80	19.91	13.54	0.00	0.00	19.91	13.54
	0.50	39.41	22.99	0.00	0.00	39.41	22.99	37.49	24.67	0.00	0.00	37.49	24.67
	0.75	56.21	33.22	0.00	0.00	56.21	33.22	53.74	34.57	0.00	0.00	53.74	34.57
	1.00	70.59	41.41	0.00	0.00	70.59	41.41	67.40	41.94	0.00	0.00	67.40	41.94
	1.25	81.30	49.68	0.00	0.00	81.30	49.68	78.01	49.37	0.00	0.00	78.01	49.37
	1.50	87.53	58.01	0.00	0.00	87.53	58.01	85.68	56.89	0.00	0.00	85.68	56.89
	1.75	90.02	66.41	0.00	0.00	90.02	66.41	88.69	64.48	0.00	0.00	88.69	64.48
	2.00	91.63	74.88	0.00	0.00	91.63	74.88	90.68	72.16	0.00	0.00	90.68	72.16
AKO30T12	0.25	5.25	4.12	50.94	35.35	56.28	40.54	5.10	4.37	52.13	44.58	56.41	48.14
	0.50	10.46	8.26	51.04	35.44	61.74	45.84	10.20	8.76	52.19	44.61	60.78	51.74
	0.75	15.55	12.41	51.13	35.52	67.16	51.12	15.32	13.16	52.25	44.63	65.14	55.33
	1.00	20.33	16.57	51.23	35.61	72.32	56.11	20.23	17.57	52.32	44.66	69.45	58.73
	1.25	24.91	20.72	51.32	35.69	77.45	61.11	24.91	21.78	52.38	44.69	73.77	62.14
	1.50	29.04	24.08	51.42	35.78	82.13	66.13	29.26	24.99	52.44	44.71	78.08	65.56
	1.75	32.95	27.45	51.52	35.87	86.63	71.17	33.32	27.67	52.51	44.74	82.38	68.99
	2.00	36.45	29.71	51.62	35.96	90.41	76.22	36.89	29.72	52.57	44.77	86.69	72.43

**Table D.2** Profit values obtained with logical inequalities (by problem instance)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
GUN8T8-0	0.25	70.89	56.13	70.34	51.75	71.50	58.81	59.25	40.00
	0.50	68.78	39.25	67.69	30.50	69.68	44.25	56.50	18.00
	0.75	66.67	26.31	65.03	12.75	67.86	32.31	53.75	6.00
	1.00	64.56	16.50	62.38	7.00	66.03	23.13	51.00	5.00
	1.25	62.45	9.81	59.72	4.00	64.21	15.31	48.25	4.00
	1.50	60.34	6.00	57.06	3.00	62.38	9.66	45.50	3.00
	1.75	58.23	3.31	54.41	2.00	60.56	6.14	42.75	2.00
	2.00	56.12	1.00	51.75	1.00	58.74	3.25	40.00	1.00
GUN8T8-1	0.25	71.28	53.09	62.94	41.50	64.76	49.06	42.94	23.50
	0.50	68.68	33.63	59.88	18.00	62.52	31.13	39.88	18.00
	0.75	66.08	22.43	56.81	16.50	60.27	21.88	36.81	16.50
	1.00	63.48	17.81	53.75	16.00	58.03	17.25	33.75	16.00
	1.25	60.89	15.50	50.69	15.50	55.79	15.50	30.69	15.50
	1.50	58.29	15.00	47.63	15.00	53.55	15.00	27.63	15.00
	1.75	55.69	14.50	44.56	14.50	51.30	14.50	24.56	14.50
	2.00	53.09	14.00	41.50	14.00	49.06	14.00	23.50	14.00
GUN8T8-2	0.25	95.45	74.22	53.13	40.00	95.45	74.22	53.13	40.00
	0.50	92.03	52.78	51.25	25.00	92.03	52.78	51.25	25.00
	0.75	88.72	37.52	49.38	13.25	88.72	37.52	49.38	13.25
	1.00	85.82	27.07	47.50	3.00	85.82	27.07	47.50	3.00
	1.25	82.92	16.78	45.63	0.00	82.92	16.78	45.63	0.00
	1.50	80.02	7.58	43.75	0.00	80.02	7.58	43.75	0.00
	1.75	77.12	1.36	41.88	0.00	77.12	1.36	41.88	0.00
	2.00	74.22	0.00	40.00	0.00	74.22	0.00	40.00	0.00
GUN8T8-3	0.25	53.60	37.81	45.03	24.25	74.25	55.00	74.03	53.25
	0.50	51.35	24.71	42.06	18.00	71.50	36.21	71.06	29.50
	0.75	49.09	16.15	39.09	14.00	68.75	22.86	68.09	20.50
	1.00	46.83	10.42	36.13	10.00	66.00	12.76	65.13	12.00
	1.25	44.57	6.88	33.16	6.00	63.25	6.88	62.16	6.00
	1.50	42.31	5.50	30.19	2.00	60.50	5.50	59.19	2.00
	1.75	40.05	4.13	27.22	0.00	57.75	4.13	56.22	0.00
	2.00	37.79	2.75	24.25	0.00	55.00	2.75	53.25	0.00
GUN8T8-4	0.25	48.89	35.94	46.66	30.25	48.89	35.94	46.66	30.25
	0.50	47.04	24.63	44.31	20.50	47.04	24.63	44.31	20.50
	0.75	45.18	15.50	41.97	14.25	45.18	15.50	41.97	14.25
	1.00	43.33	11.00	39.63	11.00	43.33	11.00	39.63	11.00
	1.25	41.47	9.25	37.28	9.25	41.47	9.25	37.28	9.25
	1.50	39.62	7.50	34.94	7.50	39.62	7.50	34.94	7.50
	1.75	37.77	5.75	32.59	5.75	37.77	5.75	32.59	5.75
	2.00	35.91	4.00	30.25	4.00	35.91	4.00	30.25	4.00
GUN8T8-5	0.25	65.49	57.47	43.00	36.00	65.49	57.47	43.00	36.00
	0.50	64.34	50.24	42.00	30.00	64.34	50.24	42.00	30.00
	0.75	63.20	44.95	41.00	26.00	63.20	44.95	41.00	26.00
	1.00	62.05	39.67	40.00	22.00	62.05	39.67	40.00	22.00
	1.25	60.91	34.38	39.00	18.00	60.91	34.38	39.00	18.00
	1.50	59.76	29.09	38.00	14.00	59.76	29.09	38.00	14.00
	1.75	58.62	25.12	37.00	10.00	58.62	25.12	37.00	10.00
	2.00	57.47	21.14	36.00	6.00	57.47	21.14	36.00	6.00
GUN8T8-6	0.25	72.05	55.34	68.03	47.25	99.03	78.69	99.03	78.25
	0.50	69.60	38.07	65.06	29.50	96.06	57.25	96.06	55.50
	0.75	67.15	22.83	62.09	15.75	93.09	37.81	93.09	34.75
	1.00	64.70	9.47	59.13	4.00	90.13	20.13	90.13	14.00
	1.25	62.25	3.42	56.16	0.00	87.16	9.22	87.16	0.00
	1.50	59.90	1.26	53.19	0.00	84.30	6.24	84.19	0.00
	1.75	57.62	0.00	50.22	0.00	81.49	4.14	81.22	0.00
	2.00	55.33	0.00	47.25	0.00	78.69	2.06	78.25	0.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
GUN8T8-7	0.25	100.50	84.13	100.31	81.50	100.50	84.13	100.31	81.50
	0.50	98.01	67.88	97.63	60.00	98.01	67.88	97.63	60.00
	0.75	95.51	51.88	94.94	38.75	95.51	51.88	94.94	38.75
	1.00	93.02	39.38	92.25	23.50	93.02	39.38	92.25	23.50
	1.25	90.52	26.88	89.56	10.13	90.52	26.88	89.56	10.13
	1.50	88.19	16.56	86.88	2.00	88.19	16.56	86.88	2.00
	1.75	86.16	7.59	84.19	0.00	86.16	7.59	84.19	0.00
GUN8T8-8	0.25	67.77	53.25	59.38	41.00	87.94	66.50	87.94	66.50
	0.50	65.69	37.50	56.75	20.00	84.88	42.88	84.88	42.00
	0.75	63.61	25.25	54.13	4.50	81.81	28.00	81.81	21.00
	1.00	61.53	16.50	51.50	1.00	78.75	19.25	78.75	1.00
	1.25	59.46	9.94	48.88	0.00	75.69	12.69	75.69	0.00
	1.50	57.38	4.25	46.25	0.00	72.63	7.00	72.63	0.00
	1.75	55.30	0.32	43.63	0.00	69.56	1.31	69.56	0.00
GUN8T8-9	0.25	69.63	59.63	55.09	48.75	68.43	61.03	64.75	56.00
	0.50	68.19	50.94	54.19	41.50	67.37	52.56	63.50	46.00
	0.75	66.76	42.72	53.28	34.25	66.31	44.09	62.25	36.00
	1.00	65.32	36.25	52.38	27.00	65.25	37.38	61.00	27.00
	1.25	63.88	31.97	51.47	19.75	64.20	33.09	59.75	19.75
	1.50	62.44	27.69	50.56	12.50	63.14	28.81	58.50	12.50
	1.75	61.00	23.41	49.66	7.00	62.08	24.53	57.25	7.00
AKO8T6-0	0.25	112.88	94.69	95.13	75.00	107.59	91.72	102.13	82.00
	0.50	110.28	73.88	92.25	52.00	105.33	73.58	99.25	62.00
	0.75	107.68	53.94	89.38	29.00	103.06	56.42	96.38	43.00
	1.00	105.08	37.64	86.50	13.00	100.79	43.14	93.50	28.00
	1.25	102.48	27.65	83.63	6.25	98.52	32.32	90.63	13.00
	1.50	99.87	20.35	80.75	0.00	96.26	25.10	87.75	0.00
	1.75	97.27	13.28	77.88	0.00	93.99	18.04	84.88	0.00
AKO8T6-1	0.25	149.52	128.49	127.41	102.25	149.52	128.49	127.41	102.25
	0.50	146.52	104.46	123.81	73.50	146.52	104.46	123.81	73.50
	0.75	143.51	80.42	120.22	44.75	143.51	80.42	120.22	44.75
	1.00	140.51	58.14	116.63	19.00	140.51	58.14	116.63	19.00
	1.25	137.51	38.93	113.03	7.00	137.51	38.93	113.03	7.00
	1.50	134.50	27.13	109.44	0.00	134.50	27.13	109.44	0.00
	1.75	131.50	18.17	105.84	0.00	131.50	18.17	105.84	0.00
AKO8T6-2	0.25	128.49	10.93	102.25	0.00	128.49	10.93	102.25	0.00
	0.25	106.84	92.68	90.56	75.00	144.16	124.25	144.16	124.25
	0.50	104.82	76.49	88.13	60.50	141.31	101.50	141.31	101.50
	0.75	102.79	61.51	85.69	47.00	138.47	79.94	138.47	78.75
	1.00	100.77	50.46	83.25	34.50	135.63	62.34	135.63	56.00
	1.25	98.75	39.45	80.81	22.00	132.78	46.94	132.78	33.25
	1.50	96.72	28.49	78.63	9.50	129.94	33.47	129.94	12.00
AKO8T6-3	1.75	94.70	20.02	76.81	2.00	127.09	22.44	127.09	2.00
	2.00	92.68	14.78	75.00	0.00	124.25	14.78	124.25	0.00
	0.25	102.81	87.50	102.81	87.50	91.70	78.73	75.81	60.50
	0.50	100.63	70.00	100.63	70.00	89.85	63.90	73.63	43.00
	0.75	98.44	52.50	98.44	52.50	88.00	49.08	71.44	29.00
	1.00	96.25	37.92	96.25	35.00	86.14	37.17	69.25	22.00
	1.25	94.06	27.22	94.06	17.50	84.29	26.47	67.06	16.25
1.50	91.88	23.42	91.88	14.50	82.44	22.90	64.88	14.50	
1.75	89.69	20.59	89.69	12.75	80.58	20.36	62.69	12.75	
2.00	87.50	18.13	87.50	11.00	78.73	17.88	60.50	11.00	

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO8T6-4	0.25	81.19	73.86	64.00	57.00	91.12	83.41	72.94	65.50
	0.50	80.14	65.48	63.00	49.00	90.01	74.56	71.88	57.00
	0.75	79.10	57.10	62.00	41.00	88.91	65.72	70.81	48.50
	1.00	78.05	50.62	61.00	35.00	87.80	58.63	69.75	40.00
	1.25	77.00	46.05	60.00	31.00	86.70	53.49	68.69	31.50
	1.50	75.95	41.48	59.00	27.00	85.59	48.50	67.63	27.00
	2.00	73.86	32.37	57.00	19.00	83.38	38.52	65.50	19.00
AKO8T6-5	0.25	115.53	98.25	115.53	98.25	115.53	98.25	115.53	98.25
	0.50	113.06	78.50	113.06	78.50	113.06	78.50	113.06	78.50
	0.75	110.59	59.75	110.59	59.75	110.59	59.75	110.59	59.75
	1.00	108.13	43.00	108.13	43.00	108.13	43.00	108.13	43.00
	1.25	105.66	26.49	105.66	26.25	105.66	26.49	105.66	26.25
	1.50	103.19	17.71	103.19	9.50	103.19	17.71	103.19	9.50
	2.00	98.25	6.13	98.25	0.00	98.25	6.13	98.25	0.00
AKO8T6-6	0.25	122.65	107.61	117.06	103.50	142.59	125.99	142.59	125.75
	0.50	120.47	94.18	115.13	88.00	140.19	110.88	140.19	106.50
	0.75	118.29	81.01	113.19	72.50	137.78	95.76	137.78	87.25
	1.00	116.11	69.64	111.25	57.00	135.38	80.64	135.38	68.00
	1.25	113.93	58.27	109.31	41.50	132.97	65.52	132.97	48.75
	1.50	111.75	46.90	107.38	33.00	130.56	50.40	130.56	33.00
	2.00	107.61	24.17	103.50	16.00	125.99	24.17	125.75	16.00
AKO8T6-7	0.25	135.58	122.56	116.56	99.50	140.53	123.25	140.53	123.25
	0.50	133.72	107.67	114.13	80.00	138.06	108.36	138.06	103.50
	0.75	131.86	92.78	111.69	60.50	135.59	93.47	135.59	83.75
	1.00	130.00	77.89	109.25	41.00	133.13	78.58	133.13	64.00
	1.25	128.14	65.71	106.81	27.00	130.66	66.40	130.66	44.25
	1.50	126.28	58.33	104.38	19.00	128.19	59.03	128.19	27.50
	2.00	122.56	45.49	99.50	3.00	123.25	46.18	123.25	8.50
AKO8T6-8	0.25	86.78	80.20	76.97	69.75	108.27	98.78	105.56	95.50
	0.50	85.84	72.68	75.94	61.50	106.91	87.94	104.13	84.00
	0.75	84.90	65.16	74.91	53.25	105.56	77.09	102.69	72.50
	1.00	83.96	57.64	73.88	45.00	104.20	66.25	101.25	61.00
	1.25	83.02	50.12	72.84	36.75	102.85	55.41	99.81	49.50
	1.50	82.08	46.10	71.81	28.50	101.49	48.56	98.38	41.50
	2.00	80.20	38.93	69.75	12.00	98.78	41.25	95.50	27.00
AKO8T6-9	0.25	92.36	82.44	84.22	71.75	125.92	110.59	114.22	101.75
	0.50	90.94	71.11	82.44	57.50	123.73	93.06	112.44	87.50
	0.75	89.53	59.78	80.66	45.25	121.54	75.53	110.66	73.25
	1.00	88.11	48.44	78.88	37.00	119.35	59.00	108.88	59.00
	1.25	86.69	37.11	77.09	28.75	117.16	44.75	107.09	44.75
	1.50	85.28	25.78	75.31	20.50	114.97	30.50	105.31	30.50
	2.00	82.44	8.00	71.75	8.00	110.58	8.00	101.75	8.00
AKO20T4-A-0	0.25	355.86	348.38	279.99	260.75	355.86	348.38	279.99	260.75
	0.50	355.43	340.48	278.98	240.50	355.43	340.48	278.98	240.50
	0.75	355.00	332.60	277.96	222.00	355.00	332.60	277.96	222.00
	1.00	354.58	324.73	276.95	204.50	354.58	324.73	276.95	204.50
	1.25	354.16	316.89	275.94	187.00	354.16	316.89	275.94	187.00
	1.50	353.75	309.10	274.93	169.50	353.75	309.10	274.93	169.50
	2.00	352.93	293.55	272.90	134.50	352.93	293.55	272.90	134.50

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO20T4-A-1	0.25	389.27	378.84	319.89	298.75	389.27	378.84	319.89	298.75
	0.50	388.71	367.86	318.78	276.50	388.71	367.86	318.78	276.50
	0.75	388.16	356.87	317.66	254.25	388.16	356.87	317.66	254.25
	1.00	387.60	345.88	316.55	232.00	387.60	345.88	316.55	232.00
	1.25	387.05	334.89	315.44	209.75	387.05	334.89	315.44	209.75
	1.50	386.50	323.90	314.33	187.50	386.50	323.90	314.33	187.50
	1.75	385.95	312.91	313.21	165.25	385.95	312.91	313.21	165.25
AKO20T4-A-2	2.00	385.40	301.92	312.10	143.00	385.40	301.92	312.10	143.00
	0.25	154.32	146.49	114.90	94.00	231.06	218.49	191.65	166.00
	0.50	153.89	138.22	113.80	72.00	230.37	205.22	190.30	139.00
	0.75	153.47	129.94	112.70	50.00	229.68	191.94	188.95	112.00
	1.00	153.04	121.67	111.60	28.00	229.00	178.67	187.60	85.00
	1.25	152.62	113.40	110.50	6.00	228.32	165.40	186.25	58.00
	1.50	152.20	105.12	109.40	0.00	227.63	152.12	184.90	31.00
AKO20T4-A-3	1.75	151.77	96.85	108.30	0.00	226.95	138.85	183.55	4.00
	2.00	151.35	88.58	107.20	0.00	226.27	125.58	182.20	0.00
	0.25	334.13	330.14	281.74	257.75	246.21	242.87	194.95	175.00
	0.50	333.90	325.84	280.48	232.50	245.99	239.28	193.90	154.00
	0.75	333.67	321.55	279.21	207.25	245.78	235.68	192.85	133.00
	1.00	333.45	317.28	277.95	183.00	245.58	232.10	191.80	113.00
	1.25	333.23	313.03	276.69	162.00	245.39	228.56	190.75	96.25
AKO20T4-A-4	1.50	333.01	308.79	275.43	141.00	245.21	225.02	189.70	80.00
	1.75	332.78	304.55	274.16	120.00	245.03	221.47	188.65	67.25
	2.00	332.56	300.31	272.90	99.00	244.85	217.93	187.60	54.50
	0.25	228.17	222.12	167.91	147.25	228.17	222.12	167.91	147.25
	0.50	227.85	215.75	166.83	125.50	227.85	215.75	166.83	125.50
	0.75	227.53	209.38	165.74	103.75	227.53	209.38	165.74	103.75
	1.00	227.21	203.01	164.65	82.00	227.21	203.01	164.65	82.00
AKO20T4-A-5	1.25	226.89	196.63	163.56	60.25	226.89	196.63	163.56	60.25
	1.50	226.57	190.26	162.48	38.50	226.57	190.26	162.48	38.50
	1.75	226.25	183.89	161.39	19.63	226.25	183.89	161.39	19.63
	2.00	225.93	177.52	160.30	6.00	225.93	177.52	160.30	6.00
	0.25	316.55	305.64	261.31	229.25	316.55	305.64	261.31	229.25
	0.50	315.96	294.20	259.63	196.25	315.96	294.20	259.63	196.25
	0.75	315.37	282.76	257.94	163.75	315.37	282.76	257.94	163.75
AKO20T4-A-6	1.00	314.78	271.33	256.25	132.00	314.78	271.33	256.25	132.00
	1.25	314.20	259.90	254.56	100.25	314.20	259.90	254.56	100.25
	1.50	313.62	248.47	252.88	68.50	313.62	248.47	252.88	68.50
	1.75	313.04	237.05	251.19	36.75	313.04	237.05	251.19	36.75
	2.00	312.47	225.62	249.50	8.50	312.47	225.62	249.50	8.50
	0.25	510.87	501.40	467.53	439.50	510.87	501.40	467.53	439.50
	0.50	510.37	491.43	466.05	410.00	510.37	491.43	466.05	410.00
AKO20T4-A-7	0.75	509.87	481.48	464.58	380.50	509.87	481.48	464.58	380.50
	1.00	509.36	471.53	463.10	351.00	509.36	471.53	463.10	351.00
	1.25	508.87	461.59	461.63	321.50	508.87	461.59	461.63	321.50
	1.50	508.37	451.64	460.15	292.00	508.37	451.64	460.15	292.00
	1.75	507.87	441.70	458.68	262.50	507.87	441.70	458.68	262.50
	2.00	507.37	431.75	457.20	233.00	507.37	431.75	457.20	233.00
	0.25	251.31	242.50	179.63	153.50	251.31	242.50	179.63	153.50
AKO20T4-A-7	0.50	250.83	233.18	178.25	126.00	250.83	233.18	178.25	126.00
	0.75	250.35	223.86	176.88	98.50	250.35	223.86	176.88	98.50
	1.00	249.89	214.53	175.50	72.00	249.89	214.53	175.50	72.00
	1.25	249.42	205.21	174.13	49.88	249.42	205.21	174.13	49.88
	1.50	248.95	195.88	172.75	27.75	248.95	195.88	172.75	27.75
	1.75	248.48	186.56	171.38	5.63	248.48	186.56	171.38	5.63
	2.00	248.02	177.24	170.00	0.00	248.02	177.24	170.00	0.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO20T4-A-8	0.25	380.18	368.74	321.46	292.25	380.18	368.74	321.46	292.25
	0.50	379.56	356.69	319.93	261.50	379.56	356.69	319.93	261.50
	0.75	378.94	344.64	318.39	230.75	378.94	344.64	318.39	230.75
	1.00	378.33	332.59	316.85	200.00	378.33	332.59	316.85	200.00
	1.25	377.71	320.54	315.31	169.25	377.71	320.54	315.31	169.25
	1.50	377.10	308.54	313.78	138.50	377.10	308.54	313.78	138.50
	1.75	376.49	296.60	312.24	107.75	376.49	296.60	312.24	107.75
AKO20T4-A-9	0.25	338.26	329.11	280.59	253.75	338.26	329.11	280.59	253.75
	0.50	337.75	319.44	279.18	225.50	337.75	319.44	279.18	225.50
	0.75	337.25	309.77	277.76	197.25	337.25	309.77	277.76	197.25
	1.00	336.76	300.11	276.35	169.00	336.76	300.11	276.35	169.00
	1.25	336.27	290.45	274.94	140.75	336.27	290.45	274.94	140.75
	1.50	335.78	280.79	273.53	119.50	335.78	280.79	273.53	119.50
	1.75	335.30	271.13	272.11	102.25	335.30	271.13	272.11	102.25
AKO20T4-B-0	0.25	344.39	333.75	193.80	171.00	344.39	333.75	193.80	171.00
	0.50	343.82	322.48	192.60	147.00	343.82	322.48	192.60	147.00
	0.75	343.26	311.23	191.40	123.00	343.26	311.23	191.40	123.00
	1.00	342.69	300.00	190.20	99.00	342.69	300.00	190.20	99.00
	1.25	342.13	288.80	189.00	75.00	342.13	288.80	189.00	75.00
	1.50	341.56	277.61	187.80	51.00	341.56	277.61	187.80	51.00
	1.75	341.00	266.42	186.60	27.00	341.00	266.42	186.60	27.00
AKO20T4-B-1	0.25	340.44	255.24	185.40	9.00	340.44	255.24	185.40	9.00
	0.25	232.09	223.78	178.59	151.75	302.28	290.49	178.59	151.75
	0.50	231.65	215.03	177.18	123.50	301.65	278.02	177.18	123.50
	0.75	231.21	206.29	175.76	95.25	301.02	265.56	175.76	95.25
	1.00	230.77	197.55	174.35	68.00	300.40	253.11	174.35	68.00
	1.25	230.34	188.82	172.94	43.75	299.77	240.67	172.94	43.75
	1.50	229.90	180.09	171.53	24.17	299.15	228.23	171.53	24.17
AKO20T4-B-2	1.75	229.46	171.37	170.11	4.81	298.52	215.78	170.11	4.81
	2.00	229.02	162.64	168.70	0.00	297.90	203.34	168.70	0.00
	0.25	348.99	341.48	207.65	182.00	269.04	262.51	207.65	182.00
	0.50	348.58	333.46	206.30	155.00	268.68	255.58	206.30	155.00
	0.75	348.18	325.44	204.95	128.00	268.34	248.65	204.95	128.00
	1.00	347.77	317.43	203.60	101.00	267.99	241.73	203.60	101.00
	1.25	347.37	309.42	202.25	74.00	267.64	234.80	202.25	74.00
AKO20T4-B-3	1.50	346.97	301.40	200.90	47.00	267.29	227.87	200.90	47.00
	1.75	346.56	293.39	199.55	29.00	266.95	220.95	199.55	29.00
	2.00	346.16	285.37	198.20	13.50	266.60	214.02	198.20	13.50
	0.25	548.53	532.40	313.05	276.00	548.53	532.40	313.05	276.00
	0.50	547.67	515.32	311.10	237.00	547.67	515.32	311.10	237.00
	0.75	546.81	498.23	309.15	198.00	546.81	498.23	309.15	198.00
	1.00	545.96	481.14	307.20	159.00	545.96	481.14	307.20	159.00
AKO20T4-B-4	1.25	545.10	464.06	305.25	120.00	545.10	464.06	305.25	120.00
	1.50	544.24	446.97	303.30	81.00	544.24	446.97	303.30	81.00
	1.75	543.39	429.89	301.35	42.00	543.39	429.89	301.35	42.00
	2.00	542.53	412.80	299.40	10.50	542.53	412.80	299.40	10.50
	0.25	310.91	301.87	141.35	120.00	403.27	391.13	219.64	193.75
	0.50	310.40	292.31	140.20	98.00	402.62	378.32	218.28	166.50
	0.75	309.89	282.75	139.05	76.00	401.97	365.52	216.91	139.25
AKO20T4-B-4	1.00	309.38	273.19	137.90	54.00	401.32	352.72	215.55	112.00
	1.25	308.88	263.63	136.75	32.00	400.67	339.93	214.19	84.75
	1.50	308.37	254.07	135.60	11.00	400.02	327.13	212.83	57.50
	1.75	307.86	244.51	134.45	0.00	399.37	314.34	211.46	39.88
	2.00	307.36	234.96	133.30	0.00	398.73	301.54	210.10	23.00



**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO20T4-B-5	0.25	317.11	305.45	168.34	136.75	317.11	305.45	168.34	136.75
	0.50	316.48	293.15	166.68	103.50	316.48	293.15	166.68	103.50
	0.75	315.86	280.88	165.01	77.50	315.86	280.88	165.01	77.50
	1.00	315.24	268.60	163.35	52.00	315.24	268.60	163.35	52.00
	1.25	314.62	256.32	161.69	26.50	314.62	256.32	161.69	26.50
	1.50	314.01	244.05	160.03	4.67	314.01	244.05	160.03	4.67
	1.75	313.39	231.77	158.36	0.00	313.39	231.77	158.36	0.00
AKO20T4-B-6	2.00	312.77	219.49	156.70	0.00	312.77	219.49	156.70	0.00
	0.25	252.76	247.61	103.28	89.50	332.33	326.61	175.10	158.00
	0.50	252.48	242.15	102.55	75.00	332.01	320.54	174.20	140.00
	0.75	252.20	236.69	101.83	60.50	331.70	314.51	173.30	122.00
	1.00	251.92	231.24	101.10	46.00	331.39	308.49	172.40	104.00
	1.25	251.64	225.78	100.38	31.50	331.09	302.47	171.50	86.00
	1.50	251.36	220.32	99.65	17.00	330.78	296.45	170.60	68.00
AKO20T4-B-7	1.75	251.08	214.86	98.93	2.50	330.47	290.43	169.70	50.00
	2.00	250.80	209.40	98.20	0.00	330.16	284.41	168.80	32.00
	0.25	299.38	290.61	158.10	141.00	405.61	395.16	245.95	226.00
	0.50	298.90	281.30	157.20	123.00	405.04	384.23	244.90	205.00
	0.75	298.42	272.00	156.30	105.00	404.46	373.36	243.85	184.00
	1.00	297.94	262.69	155.40	87.00	403.90	362.50	242.80	163.00
	1.25	297.45	253.40	154.50	69.00	403.34	351.64	241.75	142.00
AKO20T4-B-8	1.50	296.97	244.10	153.60	51.00	402.78	340.78	240.70	121.00
	1.75	296.49	234.81	152.70	33.00	402.23	329.91	239.65	100.00
	2.00	296.01	225.51	151.80	22.00	401.67	319.05	238.60	86.00
	0.25	300.13	291.30	182.56	155.25	233.86	225.66	182.56	155.25
	0.50	299.65	281.87	181.13	126.50	233.42	216.93	181.13	126.50
	0.75	299.18	272.45	179.69	98.50	232.98	208.19	179.69	97.75
	1.00	298.71	263.03	178.25	74.00	232.54	199.46	178.25	69.00
AKO20T4-B-9	1.25	298.23	253.62	176.81	49.50	232.10	190.72	176.81	40.25
	1.50	297.76	244.20	175.38	25.00	231.67	181.99	175.38	22.00
	1.75	297.29	234.79	173.94	6.00	231.23	173.26	173.94	6.00
	2.00	296.81	225.40	172.50	0.00	230.79	164.56	172.50	0.00
	0.25	336.46	324.80	204.46	175.50	336.46	324.80	204.46	175.50
	0.50	335.84	312.48	202.92	145.00	335.84	312.48	202.92	145.00
	0.75	335.22	300.17	201.38	114.50	335.22	300.17	201.38	114.50
AKO20T4-C-0	1.00	334.61	287.85	199.84	86.00	334.61	287.85	199.84	86.00
	1.25	333.99	275.53	198.30	57.75	333.99	275.53	198.30	57.75
	1.50	333.37	263.22	196.76	29.50	333.37	263.22	196.76	29.50
	1.75	332.75	250.91	195.22	3.50	332.75	250.91	195.22	3.50
	2.00	332.14	238.60	193.69	0.00	332.14	238.60	193.69	0.00
	0.25	217.01	209.42	127.07	99.88	217.01	209.42	127.07	99.88
	0.50	216.58	201.32	125.64	71.25	216.58	201.32	125.64	71.25
AKO20T4-C-1	0.75	216.17	193.23	124.21	42.69	216.17	193.23	124.21	42.69
	1.00	215.76	185.14	122.78	18.50	215.76	185.14	122.78	18.50
	1.25	215.35	177.05	121.34	0.00	215.35	177.05	121.34	0.00
	1.50	214.94	168.98	119.91	0.00	214.94	168.98	119.91	0.00
	1.75	214.53	160.91	118.48	0.00	214.53	160.91	118.48	0.00
	2.00	214.12	152.83	117.05	0.00	214.12	152.83	117.05	0.00
	0.25	531.43	517.51	467.68	442.63	531.43	517.51	467.68	442.63
AKO20T4-C-1	0.50	530.65	502.82	466.36	416.50	530.65	502.82	466.36	416.50
	0.75	529.89	488.13	465.04	391.25	529.89	488.13	465.04	391.25
	1.00	529.12	473.45	463.73	366.00	529.12	473.45	463.73	366.00
	1.25	528.36	458.76	462.41	340.75	528.36	458.76	462.41	340.75
	1.50	527.60	444.08	461.09	315.50	527.60	444.08	461.09	315.50
	1.75	526.84	429.40	459.77	290.25	526.84	429.40	459.77	290.25
	2.00	526.09	414.71	458.45	265.00	526.09	414.71	458.45	265.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO20T4-C-2	0.25	306.92	290.87	205.09	168.75	306.92	290.87	205.09	168.75
	0.50	306.05	273.91	203.18	130.50	306.05	273.91	203.18	130.50
	0.75	305.19	256.94	201.26	92.25	305.19	256.94	201.26	92.25
	1.00	304.34	240.01	199.35	58.75	304.34	240.01	199.35	58.75
	1.25	303.48	223.08	197.44	31.94	303.48	223.08	197.44	31.94
	1.50	302.63	206.16	195.53	5.13	302.63	206.16	195.53	5.13
	1.75	301.78	189.23	193.61	0.00	301.78	189.23	193.61	0.00
AKO20T4-C-3	2.00	300.93	172.30	191.70	0.00	300.93	172.30	191.70	0.00
	0.25	246.73	238.48	166.01	137.75	246.73	238.48	166.01	137.75
	0.50	246.27	229.76	164.53	109.00	246.27	229.76	164.53	109.00
	0.75	245.83	221.05	163.04	84.25	245.83	221.05	163.04	84.25
	1.00	245.38	212.34	161.55	62.00	245.38	212.34	161.55	62.00
	1.25	244.94	203.64	160.06	42.63	244.94	203.64	160.06	42.63
	1.50	244.50	194.94	158.58	24.75	244.50	194.94	158.58	24.75
AKO20T4-C-4	1.75	244.06	186.24	157.09	6.88	244.06	186.24	157.09	6.88
	2.00	243.62	177.53	155.60	0.00	243.62	177.53	155.60	0.00
	0.25	310.86	304.06	273.04	254.75	310.86	304.06	273.04	254.75
	0.50	310.48	296.84	272.08	235.50	310.48	296.84	272.08	235.50
	0.75	310.11	289.62	271.11	216.25	310.11	289.62	271.11	216.25
	1.00	309.75	282.40	270.15	197.00	309.75	282.40	270.15	197.00
	1.25	309.39	275.18	269.19	177.75	309.39	275.18	269.19	177.75
AKO20T4-C-5	1.50	309.03	267.97	268.23	158.50	309.03	267.97	268.23	158.50
	1.75	308.66	260.75	267.26	139.25	308.66	260.75	267.26	139.25
	2.00	308.30	253.54	266.30	120.50	308.30	253.54	266.30	120.50
	0.25	225.18	217.14	180.79	157.75	225.18	217.14	180.79	157.75
	0.50	224.72	208.65	179.58	133.50	224.72	208.65	179.58	133.50
	0.75	224.27	200.17	178.36	109.25	224.27	200.17	178.36	109.25
	1.00	223.83	191.69	177.15	85.00	223.83	191.69	177.15	85.00
AKO20T4-C-6	1.25	223.39	183.21	175.94	60.75	223.39	183.21	175.94	60.75
	1.50	222.95	174.73	174.73	36.50	222.95	174.73	174.73	36.50
	1.75	222.52	166.25	173.51	12.25	222.52	166.25	173.51	12.25
	2.00	222.08	157.78	172.30	0.00	222.08	157.78	172.30	0.00
	0.25	406.98	395.09	319.22	285.38	406.98	395.09	319.22	285.38
	0.50	406.34	382.57	317.44	249.75	406.34	382.57	317.44	249.75
	0.75	405.70	370.04	315.66	214.13	405.70	370.04	315.66	214.13
AKO20T4-C-7	1.00	405.07	357.52	313.88	178.50	405.07	357.52	313.88	178.50
	1.25	404.44	345.00	312.09	149.88	404.44	345.00	312.09	149.88
	1.50	403.81	332.47	310.31	121.50	403.81	332.47	310.31	121.50
	1.75	403.18	319.95	308.53	93.13	403.18	319.95	308.53	93.13
	2.00	402.55	307.43	306.75	64.75	402.55	307.43	306.75	64.75
	0.25	353.99	339.90	273.71	249.50	353.99	339.90	273.71	249.50
	0.50	353.21	325.00	272.41	225.00	353.21	325.00	272.41	225.00
AKO20T4-C-8	0.75	352.42	310.13	271.12	200.50	352.42	310.13	271.12	200.50
	1.00	351.65	295.41	269.83	176.00	351.65	295.41	269.83	176.00
	1.25	350.87	280.74	268.53	151.50	350.87	280.74	268.53	151.50
	1.50	350.10	266.08	267.24	129.50	350.10	266.08	267.24	129.50
	1.75	349.33	251.41	265.94	112.75	349.33	251.41	265.94	112.75
	2.00	348.57	236.74	264.65	96.00	348.57	236.74	264.65	96.00
	0.25	400.71	390.29	292.92	262.88	400.71	390.29	292.92	262.88
AKO20T4-C-8	0.50	400.14	379.29	291.34	231.25	400.14	379.29	291.34	231.25
	0.75	399.57	368.31	289.76	199.63	399.57	368.31	289.76	199.63
	1.00	399.00	357.35	288.18	168.00	399.00	357.35	288.18	168.00
	1.25	398.44	346.44	286.59	139.88	398.44	346.44	286.59	139.88
	1.50	397.87	335.62	285.01	113.25	397.87	335.62	285.01	113.25
	1.75	397.31	324.79	283.43	89.81	397.31	324.79	283.43	89.81
	2.00	396.75	313.97	281.85	68.75	396.75	313.97	281.85	68.75

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO20T4-C-9	0.25	257.81	253.06	218.99	199.75	252.53	244.33	197.69	172.75
	0.50	257.55	247.98	217.98	179.50	252.08	235.67	196.38	146.50
	0.75	257.29	242.94	216.96	159.25	251.64	227.01	195.06	120.25
	1.00	257.04	237.90	215.95	139.00	251.20	218.36	193.75	94.00
	1.25	256.78	232.86	214.94	119.25	250.75	209.70	192.44	72.75
	1.50	256.52	227.83	213.93	100.00	250.31	201.06	191.13	53.75
	1.75	256.27	222.79	212.91	80.75	249.87	192.41	189.81	34.75
	2.00	256.01	217.76	211.90	61.50	249.43	183.76	188.50	15.75
LAM20T10-0	0.25	340.80	317.12	190.08	172.50	502.89	477.09	244.91	224.25
	0.50	339.55	292.19	189.15	156.00	501.53	449.93	243.83	204.50
	0.75	338.31	267.26	188.23	142.50	500.18	422.76	242.74	190.00
	1.00	337.06	242.33	187.30	129.00	498.82	395.60	241.65	176.00
	1.25	335.81	217.40	186.38	115.50	497.46	368.44	240.56	162.00
	1.50	334.57	192.47	185.45	102.00	496.10	341.28	239.48	148.00
	1.75	333.32	168.74	184.53	88.75	494.74	315.29	238.39	134.00
	2.00	332.07	154.21	183.60	80.00	493.39	297.40	237.30	125.00
LAM20T10-1	0.25	614.15	588.81	337.96	322.50	500.85	480.86	294.96	279.50
	0.50	612.81	563.03	336.93	310.00	499.78	460.73	293.93	267.00
	0.75	611.46	537.24	335.89	297.50	498.72	440.63	292.89	254.50
	1.00	610.12	511.45	334.85	285.00	497.65	420.54	291.85	242.00
	1.25	608.77	485.66	333.81	272.50	496.59	400.45	290.81	229.50
	1.50	607.43	459.88	332.78	260.00	495.52	380.37	289.78	217.00
	1.75	606.08	434.09	331.74	247.50	494.46	360.28	288.74	204.50
	2.00	604.74	408.30	330.70	235.00	493.39	340.20	287.70	192.00
LAM20T10-2	0.25	390.10	361.63	234.69	209.75	389.82	355.13	204.69	179.75
	0.50	388.60	331.65	233.38	183.50	387.99	318.80	203.38	153.50
	0.75	387.10	306.18	232.06	161.75	386.16	286.98	202.06	131.75
	1.00	385.61	280.88	230.75	144.00	384.33	255.15	200.75	114.00
	1.25	384.11	255.67	229.44	126.50	382.50	224.22	199.44	96.50
	1.50	382.61	233.47	228.13	112.00	380.67	199.49	198.13	82.00
	1.75	381.11	213.92	226.81	98.50	378.84	180.16	196.81	68.50
	2.00	379.61	196.75	225.50	85.00	377.01	163.72	195.50	55.00
LAM20T10-3	0.25	401.71	370.45	239.84	217.75	558.07	523.20	326.84	304.75
	0.50	400.06	337.55	238.68	194.50	556.23	486.50	325.68	281.50
	0.75	398.42	304.65	237.51	171.25	554.40	450.25	324.51	258.25
	1.00	396.77	271.75	236.35	150.00	552.56	415.80	323.35	237.00
	1.25	395.13	240.82	235.19	131.75	550.73	381.35	322.19	218.75
	1.50	393.48	216.96	234.03	113.50	548.89	350.80	321.03	200.50
	1.75	391.84	197.85	232.86	95.25	547.06	324.74	319.86	182.25
	2.00	390.19	182.70	231.70	78.00	545.22	301.77	318.70	164.00
LAM20T10-4	0.25	591.19	553.71	358.51	330.25	725.73	687.35	410.51	382.25
	0.50	589.21	514.30	357.03	300.50	723.70	646.95	409.03	352.50
	0.75	587.23	475.12	355.54	272.00	721.68	606.55	407.54	324.00
	1.00	585.25	436.00	354.05	244.00	719.66	566.15	406.05	296.00
	1.25	583.26	396.89	352.56	217.00	717.63	525.75	404.56	269.00
	1.50	581.28	359.86	351.08	192.00	715.61	487.35	403.08	244.00
	1.75	579.30	327.61	349.59	167.00	713.58	453.02	401.59	219.00
	2.00	577.32	296.97	348.10	142.00	711.56	421.21	400.10	194.00
LAM20T10-5	0.25	440.00	416.19	199.95	187.00	440.00	416.19	199.95	187.00
	0.50	438.74	391.13	198.90	176.00	438.74	391.13	198.90	176.00
	0.75	437.49	369.03	197.85	165.00	437.49	369.03	197.85	165.00
	1.00	436.24	351.25	196.80	154.00	436.24	351.25	196.80	154.00
	1.25	434.98	334.37	195.75	143.00	434.98	334.37	195.75	143.00
	1.50	433.73	317.50	194.70	132.00	433.73	317.50	194.70	132.00
	1.75	432.48	300.63	194.15	121.00	432.48	300.63	194.15	121.00
	2.00	431.23	283.76	193.60	110.00	431.23	283.76	193.60	110.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
LAM20T10-6	0.25	592.04	558.21	314.88	293.50	595.94	564.35	283.88	262.50
	0.50	590.26	522.59	313.75	274.00	594.28	531.10	282.75	243.00
	0.75	588.48	486.98	312.63	254.50	592.61	497.85	281.63	223.50
	1.00	586.70	451.36	311.50	235.00	590.95	464.60	280.50	204.00
	1.25	584.92	418.45	310.38	215.50	589.29	434.05	279.38	184.50
	1.50	583.14	386.04	309.25	196.00	587.63	404.00	278.25	165.00
	1.75	581.36	360.22	308.13	176.50	585.96	377.70	277.13	145.50
LAM20T10-7	2.00	579.58	334.41	307.00	157.00	584.30	351.40	276.00	126.00
	0.25	783.46	749.55	459.90	441.50	815.76	779.94	459.90	441.50
	0.50	781.68	713.86	458.80	423.00	813.88	742.23	458.80	423.00
	0.75	779.89	678.17	457.70	404.50	811.99	704.51	457.70	404.50
	1.00	778.11	642.47	456.60	386.00	810.11	666.80	456.60	386.00
	1.25	776.32	606.78	455.50	367.50	808.22	629.09	455.50	367.50
	1.50	774.54	571.09	454.45	349.00	806.34	591.38	454.45	349.00
LAM20T10-8	1.75	772.75	535.39	453.33	330.50	804.45	553.66	453.33	330.50
	2.00	770.97	499.86	452.60	312.00	802.57	515.95	452.60	312.00
	0.25	418.08	390.10	214.95	195.00	525.70	496.30	290.95	271.00
	0.50	416.61	360.65	213.90	174.00	524.16	466.70	289.90	250.00
	0.75	415.13	331.20	212.85	155.50	522.61	437.33	288.85	231.50
	1.00	413.66	303.14	211.80	137.00	521.06	407.95	287.80	213.00
	1.25	412.19	277.37	210.75	118.50	519.51	378.58	286.75	194.50
LAM20T10-9	1.50	410.72	254.64	209.70	100.00	517.97	353.99	285.70	176.00
	1.75	409.24	233.96	208.65	81.50	516.42	331.55	284.65	157.50
	2.00	407.77	213.28	207.60	63.00	514.87	309.22	283.60	139.00
	0.25	569.16	534.13	307.95	288.00	569.16	534.13	307.95	288.00
	0.50	567.31	497.25	306.90	267.00	567.31	497.25	306.90	267.00
	0.75	565.47	460.38	305.85	246.00	565.47	460.38	305.85	246.00
	1.00	563.63	426.35	304.80	225.00	563.63	426.35	304.80	225.00
LAM20T24-0	1.25	561.78	393.04	303.75	204.00	561.78	393.04	303.75	204.00
	1.50	559.94	360.17	302.70	183.00	559.94	360.17	302.70	183.00
	1.75	558.09	332.77	301.65	163.75	558.09	332.77	301.65	163.75
	2.00	556.25	305.90	300.60	145.00	556.25	305.90	300.60	145.00
	0.25	90.26	80.11	86.25	72.75	79.38	71.36	75.36	64.00
	0.50	89.73	69.70	85.50	64.00	78.95	63.20	74.73	57.50
	0.75	89.19	61.54	84.75	57.50	78.53	58.29	74.09	54.25
LAM20T24-1	1.00	88.66	54.85	84.00	52.00	78.11	54.85	73.45	52.00
	1.25	88.12	51.41	83.25	49.75	77.68	51.41	72.81	49.75
	1.50	87.58	47.98	82.50	47.50	77.26	47.98	72.18	47.50
	1.75	87.05	45.25	81.75	45.25	76.83	45.25	71.54	45.25
	2.00	86.51	43.00	81.00	43.00	76.41	43.00	70.90	43.00
	0.25	87.33	74.50	87.33	74.50	87.33	74.50	87.33	74.50
	0.50	86.65	61.00	86.65	61.00	86.65	61.00	86.65	61.00
LAM20T24-2	0.75	85.98	47.50	85.98	47.50	85.98	47.50	85.98	47.50
	1.00	85.30	34.00	85.30	34.00	85.30	34.00	85.30	34.00
	1.25	84.63	20.50	84.63	20.50	84.63	20.50	84.63	20.50
	1.50	83.95	7.00	83.95	7.00	83.95	7.00	83.95	7.00
	1.75	83.28	0.00	83.28	0.00	83.28	0.00	83.28	0.00
	2.00	82.60	0.00	82.60	0.00	82.60	0.00	82.60	0.00
	0.25	71.10	54.00	71.10	54.00	71.10	54.25	71.10	54.25
LAM20T24-2	0.50	70.20	36.00	70.20	36.00	70.20	38.50	70.20	38.50
	0.75	69.30	18.00	69.30	18.00	69.30	22.75	69.30	22.75
	1.00	68.40	2.00	68.40	2.00	68.40	7.00	68.40	7.00
	1.25	67.50	0.00	67.50	0.00	67.50	0.00	67.50	0.00
	1.50	66.60	0.00	66.60	0.00	66.60	0.00	66.60	0.00
	1.75	65.70	0.00	65.70	0.00	65.70	0.00	65.70	0.00
	2.00	64.80	0.00	64.80	0.00	64.80	0.00	64.80	0.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	LOW DEMAND VARIABILITY DATA SET					HIGH DEMAND VARIABILITY DATA SET			
	S	PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
LAM20T24-3	0.25	74.66	49.25	74.66	49.25	74.66	49.25	74.66	49.25
	0.50	73.33	33.50	73.33	33.50	73.33	33.50	73.33	33.50
	0.75	71.99	24.75	71.99	24.75	71.99	24.75	71.99	24.75
	1.00	70.65	19.00	70.65	19.00	70.65	19.00	70.65	19.00
	1.25	69.31	13.25	69.31	13.25	69.31	13.25	69.31	13.25
	1.50	67.98	7.50	67.98	7.50	67.98	7.50	67.98	7.50
	1.75	66.64	1.75	66.64	1.75	66.64	1.75	66.64	1.75
	2.00	65.30	0.00	65.30	0.00	65.30	0.00	65.30	0.00
LAM20T24-4	0.25	58.38	46.50	58.38	46.50	73.11	56.25	73.11	56.25
	0.50	57.75	34.00	57.75	34.00	72.23	38.50	72.23	38.50
	0.75	57.13	26.75	57.13	26.75	71.34	27.25	71.34	27.25
	1.00	56.50	20.00	56.50	20.00	70.45	20.00	70.45	20.00
	1.25	55.88	13.25	55.88	13.25	69.56	13.25	69.56	13.25
	1.50	55.25	6.50	55.25	6.50	68.68	6.50	68.68	6.50
	1.75	54.63	0.00	54.63	0.00	67.79	1.25	67.79	1.25
	2.00	54.00	0.00	54.00	0.00	66.90	0.00	66.90	0.00
LAM20T24-5	0.25	75.38	66.00	75.38	66.00	76.44	68.25	76.44	68.25
	0.50	74.75	59.00	74.75	59.00	75.88	62.50	75.88	62.50
	0.75	74.13	52.00	74.13	52.00	75.31	56.75	75.31	56.75
	1.00	73.50	45.00	73.50	45.00	74.75	51.00	74.75	51.00
	1.25	72.88	38.00	72.88	38.00	74.19	45.25	74.19	45.25
	1.50	72.25	31.00	72.25	31.00	73.63	39.50	73.63	39.50
	1.75	71.63	24.00	71.63	24.00	73.06	33.75	73.06	33.75
	2.00	71.00	17.00	71.00	17.00	72.50	28.00	72.50	28.00
LAM20T24-6	0.25	102.31	89.25	102.31	89.25	113.00	94.00	113.00	94.00
	0.50	101.63	77.50	101.63	77.50	112.00	76.00	112.00	76.00
	0.75	100.94	70.50	100.94	70.50	111.00	62.75	111.00	62.75
	1.00	100.25	64.00	100.25	64.00	110.00	50.00	110.00	50.00
	1.25	99.56	57.50	99.56	57.50	109.00	40.75	109.00	40.75
	1.50	98.88	51.00	98.88	51.00	108.00	35.50	108.00	35.50
	1.75	98.19	47.50	98.19	47.50	107.00	33.25	107.00	33.25
	2.00	97.50	44.00	97.50	44.00	106.00	31.00	106.00	31.00
LAM20T24-7	0.25	71.46	61.25	71.46	61.25	127.70	106.75	127.70	106.75
	0.50	70.93	50.50	70.93	50.50	126.40	86.00	126.40	86.00
	0.75	70.39	39.75	70.39	39.75	125.10	69.50	125.10	69.50
	1.00	69.85	29.00	69.85	29.00	123.80	53.00	123.80	53.00
	1.25	69.31	18.25	69.31	18.25	122.69	36.50	122.69	36.50
	1.50	68.78	11.00	68.78	11.00	121.63	22.50	121.63	22.50
	1.75	68.24	6.50	68.24	6.50	120.56	14.75	120.56	14.75
	2.00	67.70	2.00	67.70	2.00	119.50	8.00	119.50	8.00
LAM20T24-8	0.25	81.95	62.00	81.95	62.00	103.95	84.00	103.95	84.00
	0.50	80.90	41.00	80.90	41.00	102.90	63.00	102.90	63.00
	0.75	79.85	22.25	79.85	22.25	101.85	42.00	101.85	42.00
	1.00	78.80	10.00	78.80	10.00	100.80	21.00	100.80	21.00
	1.25	77.75	0.00	77.75	0.00	99.75	0.00	99.75	0.00
	1.50	76.70	0.00	76.70	0.00	98.70	0.00	98.70	0.00
	1.75	75.65	0.00	75.65	0.00	97.65	0.00	97.65	0.00
	2.00	74.60	0.00	74.60	0.00	96.60	0.00	96.60	0.00
LAM20T24-9	0.25	92.73	76.25	83.25	69.00	92.73	76.25	83.25	69.00
	0.50	91.87	58.90	82.50	54.00	91.87	58.90	82.50	54.00
	0.75	91.00	41.55	81.75	39.00	91.00	41.55	81.75	39.00
	1.00	90.13	24.20	81.00	24.00	90.13	24.20	81.00	24.00
	1.25	89.26	11.00	80.25	11.00	89.26	11.00	80.25	11.00
	1.50	88.40	0.00	79.50	0.00	88.40	0.00	79.50	0.00
	1.75	87.53	0.00	78.75	0.00	87.53	0.00	78.75	0.00
	2.00	86.66	0.00	78.00	0.00	86.66	0.00	78.00	0.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
LAM30T10-0	0.25	871.68	830.49	287.29	268.25	987.26	940.43	342.29	323.25
	0.50	870.19	792.26	286.58	251.50	985.57	894.92	341.58	306.50
	0.75	868.70	754.04	285.88	235.75	983.89	849.41	340.88	290.75
	1.00	867.20	716.28	285.17	222.00	982.20	803.90	340.17	277.00
	1.25	865.71	682.08	284.46	208.25	980.52	758.39	339.46	263.25
	1.50	864.22	647.94	283.75	194.50	978.83	712.88	338.75	249.50
	1.75	862.73	613.81	283.04	180.75	977.15	667.37	338.04	235.75
LAM30T10-1	2.00	861.25	579.67	282.33	167.00	975.46	621.86	337.33	222.00
	0.25	565.55	533.64	241.30	221.00	827.97	783.87	351.30	331.00
	0.50	564.45	501.12	240.60	200.00	826.45	738.76	350.60	310.00
	0.75	563.35	471.25	239.90	180.00	824.93	696.40	349.90	290.00
	1.00	562.25	442.38	239.20	164.00	823.41	655.05	349.20	274.00
	1.25	561.15	415.51	238.50	148.00	821.89	615.71	348.50	258.00
	1.50	560.05	388.65	237.80	132.00	820.37	576.37	347.80	242.00
LAM30T10-2	1.75	558.95	361.78	237.10	116.00	818.85	537.03	347.10	226.00
	2.00	557.85	334.91	236.40	100.00	817.33	497.69	346.40	210.00
	0.25	864.32	823.98	395.42	378.50	1088.01	1033.70	395.42	378.50
	0.50	862.93	783.07	394.83	362.00	1086.14	977.51	394.83	362.00
	0.75	861.53	744.01	394.25	345.50	1084.26	921.32	394.25	345.50
	1.00	860.14	705.06	393.67	329.00	1082.39	865.13	393.67	329.00
	1.25	858.75	666.13	393.08	312.50	1080.52	808.94	393.08	312.50
LAM30T10-3	1.50	857.36	627.21	392.50	296.00	1078.65	752.76	392.50	296.00
	1.75	855.97	588.29	391.92	279.50	1076.77	696.57	391.92	279.50
	2.00	854.58	551.10	391.33	263.00	1074.90	642.74	391.33	263.00
	0.25	1389.85	1325.68	442.20	421.00	1434.85	1370.37	442.20	421.00
	0.50	1387.63	1259.30	441.40	404.00	1432.63	1303.67	441.40	404.00
	0.75	1385.42	1192.92	440.60	387.00	1430.40	1236.96	440.60	387.00
	1.00	1383.20	1129.78	439.80	371.00	1428.18	1170.26	439.80	371.00
LAM30T10-4	1.25	1380.98	1066.95	439.00	355.50	1425.95	1103.56	439.00	355.50
	1.50	1378.77	1004.12	438.20	340.00	1423.73	1036.85	438.20	340.00
	1.75	1376.55	941.29	437.40	324.50	1421.51	970.15	437.40	324.50
	2.00	1374.34	878.46	436.60	309.00	1419.28	903.45	436.60	309.00
	0.25	960.96	918.33	331.24	309.25	1352.00	1283.38	331.24	309.25
	0.50	959.47	874.30	330.48	289.00	1349.63	1212.39	330.48	289.00
	0.75	957.97	830.31	329.73	270.50	1347.26	1141.40	329.73	270.50
LAM30T10-5	1.00	956.47	786.33	328.97	252.00	1344.90	1070.41	328.97	252.00
	1.25	954.98	742.35	328.21	235.00	1342.53	999.43	328.21	235.00
	1.50	953.48	700.36	327.45	219.00	1340.17	928.44	327.45	219.00
	1.75	951.99	658.97	326.69	203.00	1337.80	857.45	326.69	203.00
	2.00	950.49	617.58	325.93	187.00	1335.43	786.46	325.93	187.00
	0.25	990.35	943.82	432.35	413.50	1212.62	1149.84	432.35	413.50
	0.50	988.74	895.70	431.70	395.50	1210.45	1084.89	431.70	395.50
LAM30T10-6	0.75	987.14	847.59	431.05	383.75	1208.29	1019.94	431.05	383.75
	1.00	985.54	799.59	430.40	372.00	1206.12	955.00	430.40	372.00
	1.25	983.93	753.64	429.75	360.25	1203.95	890.05	429.75	360.25
	1.50	982.33	709.38	429.10	348.50	1201.78	825.10	429.10	348.50
	1.75	980.72	668.06	428.45	336.75	1199.62	761.66	428.45	336.75
	2.00	979.12	629.75	427.80	325.00	1197.45	701.21	427.80	325.00
	0.25	1455.54	1389.50	492.24	471.75	1498.00	1429.86	492.24	471.75
LAM30T10-6	0.50	1453.25	1321.17	491.48	452.50	1495.62	1359.35	491.48	452.50
	0.75	1450.95	1252.84	490.73	433.25	1493.25	1288.84	490.73	433.25
	1.00	1448.66	1184.50	489.97	414.00	1490.88	1218.34	489.97	414.00
	1.25	1446.36	1116.17	489.21	394.75	1488.50	1147.83	489.21	394.75
	1.50	1444.07	1047.84	488.45	375.50	1486.13	1077.32	488.45	375.50
	1.75	1441.77	979.51	487.69	356.25	1483.75	1006.81	487.69	356.25
	2.00	1439.48	911.17	486.93	337.00	1481.38	936.31	486.93	337.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
LAM30T10-7	0.25	1575.02	1504.71	633.03	607.75	1883.69	1801.90	633.03	607.75
	0.50	1572.59	1431.98	632.07	583.50	1880.87	1717.29	632.07	583.50
	0.75	1570.17	1359.24	631.10	559.25	1878.05	1632.67	631.10	559.25
	1.00	1567.75	1286.67	630.13	535.00	1875.23	1548.06	630.13	535.00
	1.25	1565.32	1214.10	629.17	510.75	1872.41	1463.45	629.17	510.75
	1.50	1562.90	1141.53	628.20	486.50	1869.59	1378.84	628.20	486.50
	1.75	1560.47	1069.41	627.23	462.25	1866.77	1294.23	627.23	462.25
LAM30T10-8	2.00	1558.05	998.31	626.27	439.00	1863.95	1209.62	626.27	439.00
	0.25	882.63	830.00	384.28	363.50	941.49	884.93	384.28	363.50
	0.50	880.78	776.84	383.57	343.00	939.54	826.42	383.57	343.00
	0.75	878.93	724.10	382.85	322.50	937.59	767.90	382.85	322.50
	1.00	877.08	672.22	382.13	302.00	935.64	709.40	382.13	302.00
	1.25	875.23	620.56	381.42	281.50	933.69	650.89	381.42	281.50
	1.50	873.38	568.91	380.70	261.00	931.74	592.38	380.70	261.00
LAM30T10-9	1.75	871.53	517.27	379.98	240.50	929.79	533.87	379.98	240.50
	2.00	869.68	473.51	379.27	220.00	927.84	483.89	379.27	220.00
	0.25	858.07	825.94	320.49	305.75	1156.55	1108.67	320.49	305.75
	0.50	856.96	795.12	319.98	290.50	1154.90	1059.14	319.98	290.50
	0.75	855.85	765.03	319.48	275.25	1153.25	1009.61	319.48	275.25
	1.00	854.74	734.94	318.97	260.00	1151.60	960.08	318.97	260.00
	1.25	853.63	704.84	318.46	244.75	1149.95	910.55	318.46	244.75
LAM30T29-0	1.50	852.52	674.75	317.95	229.50	1148.30	861.02	317.95	229.50
	1.75	851.41	644.66	317.44	214.25	1146.65	811.48	317.44	214.25
	2.00	850.30	614.57	316.93	199.00	1145.00	761.95	316.93	199.00
	0.25	119.23	98.25	119.23	98.25	119.23	98.25	119.23	98.25
	0.50	118.45	79.50	118.45	79.50	118.45	79.50	118.45	79.50
	0.75	117.68	60.75	117.68	60.75	117.68	60.75	117.68	60.75
	1.00	116.90	42.00	116.90	42.00	116.90	42.00	116.90	42.00
LAM30T29-1	1.25	116.13	23.50	116.13	23.50	116.13	23.50	116.13	23.50
	1.50	115.35	11.00	115.35	11.00	115.35	13.00	115.35	13.00
	1.75	114.58	6.83	114.58	6.83	114.58	6.96	114.58	6.96
	2.00	113.80	6.58	113.80	6.58	113.80	6.71	113.80	6.71
	0.25	83.30	63.00	83.30	63.00	83.30	63.00	83.30	63.00
	0.50	82.60	42.00	82.60	42.00	82.60	42.00	82.60	42.00
	0.75	81.90	21.00	81.90	21.00	81.90	21.00	81.90	21.00
LAM30T29-2	1.00	81.20	5.93	81.20	5.93	81.20	5.93	81.20	5.93
	1.25	80.50	5.68	80.50	5.68	80.50	5.68	80.50	5.68
	1.50	79.80	5.43	79.80	5.43	79.80	5.43	79.80	5.43
	1.75	79.10	5.18	79.10	5.18	79.10	5.18	79.10	5.18
	2.00	78.40	4.93	78.40	4.93	78.40	4.93	78.40	4.93
	0.25	143.42	126.50	143.42	126.50	115.42	99.75	115.42	99.75
	0.50	142.83	110.00	142.83	110.00	114.83	86.50	114.83	86.50
LAM30T29-3	0.75	142.25	95.00	142.25	95.00	114.25	74.75	114.25	74.75
	1.00	141.67	83.00	141.67	83.00	113.67	66.00	113.67	66.00
	1.25	141.08	71.00	141.08	71.00	113.08	57.25	113.08	57.25
	1.50	140.50	59.00	140.50	59.00	112.50	48.50	112.50	48.50
	1.75	139.92	47.00	139.92	47.00	111.92	39.75	111.92	39.75
	2.00	139.33	35.00	139.33	35.00	111.33	31.00	111.33	31.00
	0.25	89.42	72.50	89.42	72.50	98.11	75.25	98.11	75.25
LAM30T29-3	0.50	88.83	60.50	88.83	60.50	97.22	58.50	97.22	58.50
	0.75	88.25	50.25	88.25	50.25	96.33	42.00	96.33	42.00
	1.00	87.67	40.00	87.67	40.00	95.43	31.00	95.43	31.00
	1.25	87.08	29.75	87.08	29.75	94.54	23.00	94.54	23.00
	1.50	86.50	19.50	86.50	19.50	93.65	16.00	93.65	16.00
	1.75	85.92	14.50	85.92	14.50	92.76	14.50	92.76	14.50
	2.00	85.33	13.00	85.33	13.00	91.87	13.00	91.87	13.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
LAM30T29-4	0.25	120.16	95.75	120.16	95.75	120.16	95.75	120.16	95.75
	0.50	119.32	74.00	119.32	74.00	119.32	74.00	119.32	74.00
	0.75	118.48	58.00	118.48	58.00	118.48	58.00	118.48	58.00
	1.00	117.63	42.00	117.63	42.00	117.63	42.00	117.63	42.00
	1.25	116.79	26.00	116.79	26.00	116.79	26.00	116.79	26.00
	1.50	115.95	14.50	115.95	14.50	115.95	14.50	115.95	14.50
	1.75	115.11	14.25	115.11	14.25	115.11	14.25	115.11	14.25
	2.00	114.27	14.00	114.27	14.00	114.27	14.00	114.27	14.00
LAM30T29-5	0.25	78.29	57.75	78.29	57.75	87.29	66.75	87.29	66.75
	0.50	77.58	40.50	77.58	40.50	86.58	49.50	86.58	49.50
	0.75	76.88	28.25	76.88	28.25	85.88	37.25	85.88	37.25
	1.00	76.17	16.00	76.17	16.00	85.17	25.00	85.17	25.00
	1.25	75.46	6.50	75.46	6.50	84.46	12.75	84.46	12.75
	1.50	74.75	5.00	74.75	5.00	83.75	5.00	83.75	5.00
	1.75	74.04	3.50	74.04	3.50	83.04	3.50	83.04	3.50
	2.00	73.33	2.00	73.33	2.00	82.33	2.00	82.33	2.00
LAM30T29-6	0.25	76.24	55.75	76.24	55.75	96.24	75.75	96.24	75.75
	0.50	75.48	36.50	75.48	36.50	95.48	56.50	95.48	56.50
	0.75	74.73	17.25	74.73	17.25	94.73	37.25	94.73	37.25
	1.00	73.97	7.00	73.97	7.00	93.97	26.00	93.97	26.00
	1.25	73.21	1.92	73.21	1.92	93.21	15.50	93.21	15.50
	1.50	72.45	1.67	72.45	1.67	92.45	6.00	92.45	6.00
	1.75	71.69	1.42	71.69	1.42	91.69	1.92	91.69	1.92
	2.00	70.93	1.17	70.93	1.17	90.93	1.67	90.93	1.67
LAM30T29-7	0.25	98.03	72.75	98.03	72.75	126.03	100.75	126.03	100.75
	0.50	97.07	50.50	97.07	50.50	125.07	78.50	125.07	78.50
	0.75	96.10	32.75	96.10	32.75	124.10	60.75	124.10	60.75
	1.00	95.13	20.00	95.13	20.00	123.13	48.00	123.13	48.00
	1.25	94.17	7.50	94.17	7.50	122.17	35.50	122.17	35.50
	1.50	93.20	0.89	93.20	0.89	121.20	24.00	121.20	24.00
	1.75	92.23	0.64	92.23	0.64	120.23	17.50	120.23	17.50
	2.00	91.27	0.39	91.27	0.39	119.27	11.00	119.27	11.00
LAM30T29-8	0.25	86.36	67.75	86.36	67.75	86.36	67.75	86.36	67.75
	0.50	85.72	49.50	85.72	49.50	85.72	49.50	85.72	49.50
	0.75	85.08	31.25	85.08	31.25	85.08	31.25	85.08	31.25
	1.00	84.43	13.00	84.43	13.00	84.43	13.00	84.43	13.00
	1.25	83.79	2.25	83.79	2.25	83.79	3.00	83.79	3.00
	1.50	83.15	0.86	83.15	0.86	83.15	0.86	83.15	0.86
	1.75	82.51	0.61	82.51	0.61	82.51	0.61	82.51	0.61
	2.00	81.87	0.36	81.87	0.36	81.87	0.36	81.87	0.36
LAM30T29-9	0.25	106.36	88.75	106.36	88.75	112.48	97.25	112.48	97.25
	0.50	105.72	74.50	105.72	74.50	111.95	81.50	111.95	81.50
	0.75	105.08	60.25	105.08	60.25	111.43	65.75	111.43	65.75
	1.00	104.43	46.00	104.43	46.00	110.90	50.00	110.90	50.00
	1.25	103.79	32.25	103.79	32.25	110.38	34.25	110.38	34.25
	1.50	103.15	21.00	103.15	21.00	109.85	21.00	109.85	21.00
	1.75	102.51	16.75	102.51	16.75	109.33	16.75	109.33	16.75
	2.00	101.87	14.00	101.87	14.00	108.80	14.00	108.80	14.00
AKO30T12-0	0.25	668.03	629.14	356.77	321.00	879.97	834.60	451.77	416.00
	0.50	666.69	588.91	355.53	284.00	878.38	787.65	450.53	379.00
	0.75	665.34	548.96	354.30	248.50	876.80	740.76	449.30	343.50
	1.00	664.00	514.98	353.07	213.00	875.22	699.52	448.07	308.00
	1.25	662.66	483.00	351.83	177.50	873.65	660.28	446.83	272.50
	1.50	661.32	457.13	350.60	142.00	872.08	627.14	445.60	237.00
	1.75	659.98	431.28	349.37	106.50	870.50	594.03	444.37	201.50
	2.00	658.63	405.43	348.13	71.00	868.93	560.92	443.13	166.00



**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO30T12-1	0.25	944.67	901.92	463.57	422.00	998.47	952.03	463.57	422.00
	0.50	943.17	857.70	462.13	379.00	996.84	904.00	462.13	379.00
	0.75	941.67	813.51	460.70	336.00	995.20	855.99	460.70	336.00
	1.00	940.17	769.32	459.27	293.00	993.57	807.97	459.27	293.00
	1.25	938.67	726.64	457.83	250.00	991.94	761.45	457.83	250.00
	1.50	937.17	691.44	456.40	209.00	990.31	722.40	456.40	209.00
	1.75	935.67	660.16	454.97	170.50	988.68	687.29	454.97	170.50
AKO30T12-2	2.00	934.18	630.79	453.53	133.00	987.05	654.07	453.53	133.00
	0.25	775.21	730.40	457.62	417.50	845.11	793.27	457.62	417.50
	0.50	773.65	684.69	456.23	376.00	843.30	739.62	456.23	376.00
	0.75	772.09	640.04	454.85	334.50	841.49	685.98	454.85	334.50
	1.00	770.54	596.38	453.47	293.00	839.68	633.30	453.47	293.00
	1.25	768.98	557.56	452.08	251.50	837.87	585.45	452.08	251.50
	1.50	767.43	518.75	450.70	210.00	836.06	537.61	450.70	210.00
AKO30T12-3	1.75	765.88	482.67	449.32	168.50	834.25	492.49	449.32	168.50
	2.00	764.33	454.85	447.93	127.00	832.45	456.77	447.93	127.00
	0.25	828.34	794.18	535.52	492.50	1208.05	1148.20	669.52	626.50
	0.50	827.13	758.82	534.03	448.00	1205.96	1088.57	668.03	582.00
	0.75	825.93	723.46	532.55	403.75	1203.86	1029.19	666.55	537.75
	1.00	824.73	688.10	531.07	362.00	1201.76	969.89	665.07	496.00
	1.25	823.54	652.89	529.58	320.25	1199.67	910.59	663.58	454.25
AKO30T12-4	1.50	822.36	624.33	528.10	278.50	1197.57	851.29	662.10	412.50
	1.75	821.17	595.77	526.62	236.75	1195.48	793.07	660.62	370.75
	2.00	819.98	567.21	525.13	195.00	1193.38	737.15	659.13	329.00
	0.25	606.71	575.09	230.05	202.50	656.71	625.09	280.05	252.50
	0.50	605.61	542.36	229.10	174.00	655.61	592.36	279.10	224.00
	0.75	604.51	509.64	228.15	146.75	654.51	559.64	278.15	195.50
	1.00	603.42	483.44	227.20	121.00	653.42	533.44	277.20	167.00
AKO30T12-5	1.25	602.32	457.47	226.25	95.25	652.32	507.47	276.25	138.50
	1.50	601.22	431.51	225.30	69.50	651.22	481.51	275.30	110.00
	1.75	600.12	405.55	224.35	49.00	650.12	455.55	274.35	81.50
	2.00	599.02	385.04	223.40	34.00	649.02	435.04	273.40	53.00
	0.25	495.81	472.72	228.13	203.00	654.34	625.73	278.13	253.00
	0.50	495.01	448.84	227.27	177.00	653.35	596.12	277.27	227.00
	0.75	494.22	424.97	226.40	151.00	652.37	566.52	276.40	201.00
AKO30T12-6	1.00	493.42	401.10	225.53	125.00	651.38	536.92	275.53	175.00
	1.25	492.62	378.69	224.67	99.00	650.39	508.72	274.67	149.00
	1.50	491.83	364.39	223.80	73.00	649.41	487.11	273.80	123.00
	1.75	491.03	352.43	222.93	47.00	648.42	469.00	272.93	97.00
	2.00	490.24	344.59	222.07	23.00	647.43	455.02	272.07	71.00
	0.25	742.31	701.77	347.53	305.00	791.55	750.92	389.53	347.00
	0.50	740.91	659.84	346.07	261.00	790.14	708.86	388.07	303.00
AKO30T12-7	0.75	739.51	618.14	344.60	217.00	788.73	667.03	386.60	259.00
	1.00	738.11	582.31	343.13	176.00	787.33	626.73	385.13	218.00
	1.25	736.71	549.54	341.67	136.25	785.93	592.89	383.67	178.25
	1.50	735.31	516.78	340.20	96.50	784.52	559.92	382.20	138.50
	1.75	733.91	484.06	338.73	56.75	783.12	526.95	380.73	98.75
	2.00	732.51	451.34	337.27	46.00	781.72	494.00	379.27	59.00
	0.25	621.69	590.54	306.85	273.50	743.37	704.18	356.85	323.50
AKO30T12-7	0.50	620.60	560.77	305.70	239.00	741.98	663.63	355.70	289.00
	0.75	619.51	533.87	304.55	204.50	740.59	623.09	354.55	254.50
	1.00	618.42	506.99	303.40	170.00	739.22	584.44	353.40	220.00
	1.25	617.33	481.04	302.25	135.50	737.84	549.38	352.25	185.50
	1.50	616.24	456.85	301.10	101.00	736.47	516.07	351.10	151.00
	1.75	615.15	432.68	299.95	66.50	735.09	486.38	349.95	116.50
	2.00	614.05	412.50	298.80	44.00	733.72	466.17	348.80	82.00

**Table D.2** Profit values obtained with logical inequalities (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET				HIGH DEMAND VARIABILITY DATA SET			
		PC-LP	LI-1	LI-2	LI-1+LI-2	PC-LP	LI-1	LI-2	LI-1+LI-2
AKO30T12-8	0.25	558.37	523.65	241.76	205.75	767.51	723.72	337.76	301.75
	0.50	557.17	487.75	240.52	168.50	765.98	678.41	336.52	264.50
	0.75	555.96	452.32	239.28	132.00	764.46	633.09	335.28	227.25
	1.00	554.75	419.08	238.03	106.00	762.93	587.78	334.03	190.00
	1.25	553.55	386.31	236.79	81.38	761.41	542.47	332.79	152.75
	1.50	552.34	354.93	235.55	59.75	759.88	501.49	331.55	115.50
	1.75	551.14	329.98	234.31	41.63	758.35	470.08	330.31	78.25
AKO30T12-9	2.00	549.93	307.52	233.07	24.00	756.83	441.16	329.07	41.00
	0.25	399.94	377.02	164.15	139.50	660.30	628.40	297.15	272.50
	0.50	399.15	354.04	163.30	114.00	659.15	595.36	296.30	247.00
	0.75	398.36	333.46	162.45	88.50	658.02	562.32	295.45	221.50
	1.00	397.56	312.87	161.60	63.00	656.90	529.28	294.60	196.00
	1.25	396.77	292.29	160.75	37.50	655.78	496.24	293.75	170.50
	1.50	395.98	271.71	159.90	27.75	654.66	463.21	292.90	145.00
1.75	395.19	251.13	159.05	18.38	653.54	430.18	292.05	119.50	
2.00	394.40	230.54	158.20	9.00	652.42	399.72	291.20	94.00	

## APPENDIX E

### RESULTS OF THE PROPOSED PC SOLUTION PROCEDURE

In Table E.1 we summarize the results of the heuristic solution procedure. The definitions of the columns can be summarized as follows.

§	$K_{UB}$ :	The estimate of the upper bound on the number of stations
§	$CT_U$ :	The upper limit on cycle time found using equation (2.1)
§	$\pi$ :	The profit of the heuristics solution
§	$CT$ :	The cycle time of the heuristic solution
§	$K$ :	Number of stations of the heuristic solution
§	$NX_1$ :	Positional weight
§	$NX_2$ :	Number of immediate successors
§	$NX_3$ :	Number of all successors
§	$NX_4$ :	The sum of revenue generated by parts released by the task and all of its successors
§	$NX_5$ :	The sum of task times over all tasks - $NX_1$
§	$NX_p =$	$\begin{cases} 1 & \text{if the best solution is found by that numerical score} \\ 0 & \text{otherwise} \end{cases}$ where $p = 1, 2, 3, 4, 5$
§	$IMP =$	$\begin{cases} 1 & \text{if the task deletion insertion step improved the solution} \\ 2 & \text{if the task insertion step improved the solution} \\ 0 & \text{if the solution is not improved} \end{cases}$

**Table E.1 Results of the heuristic solution procedure**

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET								HIGH DEMAND VARIABILITY DATA SET												
		K <sub>UB</sub>	CT <sub>U</sub>	r	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP	K <sub>UB</sub>	CT <sub>U</sub>	r	CT	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP
GUN8T8-0	0.25	5	40	50.50	3	1	0	0	0	0	0	5	45	39.50	30	3	1	0	0	0	0	0
	0.50	5	40	28.00	3	1	0	0	0	0	0	5	45	17.00	30	3	1	0	0	0	0	0
	0.75	5	40	12.75	1	1	1	1	1	1	0	5	45	6.00	4	1	1	1	1	1	1	0
	1.00	5	40	7.00	1	1	1	1	1	1	0	5	45	5.00	4	1	1	1	1	1	1	0
	1.25	5	40	4.00	1	1	1	1	1	1	0	5	45	4.00	4	1	1	1	1	1	1	0
	1.50	5	40	3.00	1	1	1	1	1	1	0	5	45	3.00	4	1	1	1	1	1	1	0
	1.75	5	40	2.00	1	1	1	1	1	1	0	5	45	2.00	4	1	1	1	1	1	1	0
	2.00	5	40	1.00	1	1	1	1	1	1	0	5	45	1.00	4	1	1	1	1	1	1	0
GUN8T8-1	0.25	6	50	40.50	3	1	1	1	1	0	6	41	23.50	22	1	1	1	1	1	1	0	
	0.50	6	50	18.00	1	1	1	1	1	0	6	41	18.00	22	1	1	1	1	1	1	0	
	0.75	6	50	16.50	1	1	1	1	1	0	6	41	16.50	2	1	1	1	1	1	1	0	
	1.00	6	50	16.00	1	1	1	1	1	0	6	41	16.00	2	1	1	1	1	1	1	0	
	1.25	6	50	15.50	1	1	1	1	1	0	6	41	15.50	2	1	1	1	1	1	1	0	
	1.50	6	50	15.00	1	1	1	1	1	0	6	41	15.00	2	1	1	1	1	1	1	0	
	1.75	6	50	14.50	1	1	1	1	1	0	6	41	14.50	2	1	1	1	1	1	1	0	
	2.00	6	50	14.00	1	1	1	1	1	0	6	41	14.00	2	1	1	1	1	1	1	0	
GUN8T8-2	0.25	7	50	39.25	3	1	1	1	1	0	7	41	39.25	21	3	1	1	1	1	1	0	
	0.50	7	50	23.50	3	1	1	1	1	0	7	41	23.50	21	3	1	1	1	1	1	0	
	0.75	7	50	13.25	1	1	1	1	1	0	7	41	13.25	41	1	1	1	1	1	1	0	
	1.00	7	50	3.00	1	1	1	1	1	0	7	41	3.00	41	1	1	1	1	1	1	0	
	1.25	7	50	0.00	0					0	0	7	41	0.00	0	0					0	0
	1.50	7	50	0.00	0					0	0	7	41	0.00	0	0					0	0
	1.75	7	50	0.00	0					0	0	7	41	0.00	0	0					0	0
	2.00	7	50	0.00	0					0	0	7	41	0.00	0	0					0	0
GUN8T8-3	0.25	6	50	24.00	2	1	1	1	1	0	6	42	50.00	36	3	1	1	1	1	1	0	
	0.50	6	50	18.00	1	1	1	1	1	0	6	42	23.00	36	3	1	1	1	1	1	0	
	0.75	6	50	14.00	1	1	1	1	1	0	6	42	20.50	34	1	1	1	1	1	1	0	
	1.00	6	50	10.00	1	1	1	1	1	0	6	42	12.00	34	1	1	1	1	1	1	0	
	1.25	6	50	6.00	1	1	1	1	1	0	6	42	6.00	16	1	1	1	1	1	1	0	
	1.50	6	50	2.00	1	1	1	1	1	0	6	42	2.00	16	1	1	1	1	1	1	0	
	1.75	6	50	0.00	0					0	0	6	42	0.00	0	0					0	0
	2.00	6	50	0.00	0					0	0	6	42	0.00	0	0					0	0
GUN8T8-4	0.25	5	44	29.50	3	1	1	1	1	0	5	45	29.50	26	3	1	1	1	1	1	0	
	0.50	5	44	20.50	1	1	1	1	1	0	5	45	20.50	25	1	1	1	1	1	1	0	
	0.75	5	44	14.25	1	1	1	1	1	0	5	45	14.25	25	1	1	1	1	1	1	0	
	1.00	5	44	11.00	1	1	1	1	1	0	5	45	11.00	7	1	1	1	1	1	1	0	
	1.25	5	44	9.25	1	1	1	1	1	0	5	45	9.25	7	1	1	1	1	1	1	0	
	1.50	5	44	7.50	1	1	1	1	1	0	5	45	7.50	7	1	1	1	1	1	1	0	
	1.75	5	44	5.75	1	1	1	1	1	0	5	45	5.75	7	1	1	1	1	1	1	0	
	2.00	5	44	4.00	1	1	1	1	1	0	5	45	4.00	7	1	1	1	1	1	1	0	
GUN8T8-5	0.25	3	44	36.00	2	1	1	1	1	0	3	49	36.00	16	2	1	1	1	1	1	0	
	0.50	3	44	30.00	1	1	1	1	1	0	3	49	30.00	16	1	1	1	1	1	1	0	
	0.75	3	44	26.00	1	1	1	1	1	0	3	49	26.00	16	1	1	1	1	1	1	0	
	1.00	3	44	22.00	1	1	1	1	1	0	3	49	22.00	16	1	1	1	1	1	1	0	
	1.25	3	44	18.00	1	1	1	1	1	0	3	49	18.00	16	1	1	1	1	1	1	0	
	1.50	3	44	14.00	1	1	1	1	1	0	3	49	14.00	16	1	1	1	1	1	1	0	
	1.75	3	44	10.00	1	1	1	1	1	0	3	49	10.00	16	1	1	1	1	1	1	0	
	2.00	3	44	6.00	1	1	1	1	1	0	3	49	6.00	16	1	1	1	1	1	1	0	
GUN8T8-6	0.25	6	50	46.00	2	1	1	1	0	0	6	40	76.50	34	3	1	1	1	1	1	0	
	0.50	6	50	29.00	2					1	0	6	40	53.00	22	4	1	1	1	1	0	0
	0.75	6	50	15.00	2					1	0	6	40	31.00	22	4	1	1	1	1	0	0
	1.00	6	50	4.00	1	1	1	1	1	0	6	40	9.00	22	4	1	1	1	1	1	0	0
	1.25	6	50	0.00	0					0	0	6	40	0.00	0	0					0	0
	1.50	6	50	0.00	0					0	0	6	40	0.00	0	0					0	0
	1.75	6	50	0.00	0					0	0	6	40	0.00	0	0					0	0
	2.00	6	50	0.00	0					0	0	6	40	0.00	0	0					0	0
GUN8T8-7	0.25	6	50	81.50	2					1	0	6	40	81.00	22	4					1	0
	0.50	6	50	60.00	2					1	0	6	40	59.00	22	4					1	0
	0.75	6	50	38.50	2					1	2	6	40	37.00	22	4					1	2
	1.00	6	50	19.00	1	1	1	1	1	1	6	40	18.00	22	2	1	1	1	1	1	1	1
	1.25	6	50	8.25	1	1	1	1	1	1	6	40	8.00	24	1	1	1	1	1	1	1	0
	1.50	6	50	2.00	1	1	1	1	1	1	0	6	40	2.00	24	1	1	1	1	1	1	0
	1.75	6	50	0.00	0					0	0	6	40	0.00	0	0					0	0
	2.00	6	50	0.00	0					0	0	6	40	0.00	0	0					0	0
GUN8T8-8	0.25	5	50	38.00	3	1	1	1	0	0	6	42	61.75	39	3	1	1	1	1	0	0	0
	0.50	5	50	14.00	3	1	1	1	0	0	6	42	36.00	32	3	1	1	1	0	0	0	0
	0.75	5	50	4.50	1	1	1	1	1	0	6	42	12.00	32	3	1	1	1	0	0	0	0
	1.00	5	50	1.00	1	1	1	1	1	0	6	42	1.00	11	1	1	1	1	1	1	0	0
	1.25	5	50	0.00	0					0	0	6	42	0.00	0	0					0	0
	1.50	5	50	0.00	0					0	0	6	42	0.00	0	0					0	0
	1.75	5	50	0.00	0					0	0	6	42	0.00	0	0					0	0
	2.00	5	50	0.00	0					0	0	6	42	0.00	0	0					0	0

**Table E.1 Results of the heuristic solution procedure (Continued)**

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET									HIGH DEMAND VARIABILITY DATA SET													
		K <sub>UB</sub>	CT <sub>U</sub>	r	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP	K <sub>UB</sub>	CT <sub>U</sub>	r	CT	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP		
GUN8T8-9	0.25	3	80	48.75	1	1	1	1	1	1	0	3	43	56.00	40	1	1	1	1	1	1	0		
	0.50	3	80	41.50	1	1	1	1	1	1	0	3	43	46.00	40	1	1	1	1	1	1	0		
	0.75	3	80	34.25	1	1	1	1	1	1	0	3	43	36.00	40	1	1	1	1	1	1	0		
	1.00	3	80	27.00	1	1	1	1	1	1	0	3	43	27.00	29	1	1	1	1	1	1	0		
	1.25	3	80	19.75	1	1	1	1	1	1	0	3	43	19.75	29	1	1	1	1	1	1	0		
	1.50	3	80	12.50	1	1	1	1	1	1	0	3	43	12.50	29	1	1	1	1	1	1	0		
	1.75	3	80	5.25	1	1	1	1	1	1	1	3	43	5.25	29	1	1	1	1	1	1	1		
	2.00	3	80	0.00	0						0	0	3	43	0.00	0	0					0	0	
AKO8T6-0	0.25	6	50	74.50	2	1	1	1	1	1	0	5	93	82.00	92	1	1	1	1	1	1	1		
	0.50	6	50	51.00	2	1	1	1	1	1	0	5	93	62.00	80	1	1	1	1	1	1	0		
	0.75	6	50	27.50	2	1	1	1	1	1	0	5	93	43.00	60	1	1	1	1	1	1	0		
	1.00	6	50	13.00	1	1	1	1	1	1	0	5	93	28.00	60	1	1	1	1	1	1	0		
	1.25	6	50	6.25	1	1	1	1	1	1	0	5	93	13.00	60	1	1	1	1	1	1	0		
	1.50	6	50	0.00	0						0	0	5	93	0.00	0	0					0	0	
	1.75	6	50	0.00	0						0	0	5	93	0.00	0	0					0	0	
	2.00	6	50	0.00	0						0	0	5	93	0.00	0	0					0	0	
AKO8T6-1	0.25	7	50	100.25	3						1	0	7	83	101.50	59	2					1	0	
	0.50	7	50	69.50	3						1	0	7	83	72.00	59	2						1	0
	0.75	7	50	38.75	3						1	0	7	83	42.50	59	2						1	0
	1.00	7	50	19.00	1	1	1	1	1	1	0	7	83	19.00	48	1	1	1	1	1	1	1	0	
	1.25	7	50	7.00	1	1	1	1	1	1	0	7	83	7.00	48	1	1	1	1	1	1	1	0	
	1.50	7	50	0.00	0						0	0	7	83	0.00	0	0						0	0
	1.75	7	50	0.00	0						0	0	7	83	0.00	0	0						0	0
	2.00	7	50	0.00	0						0	0	7	83	0.00	0	0						0	0
AKO8T6-2	0.25	5	66	73.00	2	1	1	1	1	1	0	0	6	71	123.50	47	2	1	1	1	1	0	0	
	0.50	5	66	53.00	2	1	1	1	1	1	0	0	6	71	100.00	47	2	1	1	1	1	0	0	
	0.75	5	66	35.50	2						1	0	6	71	76.50	47	2	1	1	1	1	0	0	
	1.00	5	66	22.00	1	1	1	1	1	1	0	6	71	53.00	47	2	1	1	1	1	1	0	0	
	1.25	5	66	14.50	1	1	1	1	1	1	0	6	71	29.50	47	2	1	1	1	1	1	0	0	
	1.50	5	66	7.00	1	1	1	1	1	1	0	6	71	12.00	35	2							1	0
	1.75	5	66	2.00	1	1	1	1	1	1	0	6	71	2.00	12	1	1	1	1	1	1	1	0	
	2.00	5	66	0.00	0						0	0	6	71	0.00	0	0						0	0
AKO8T6-3	0.25	5	44	87.00	2	1	1	1	1	1	0	0	6	42	60.00	36	2	1	1	1	1	0	0	
	0.50	5	44	69.00	2	1	1	1	1	1	0	0	5	42	42.00	36	2	1	1	1	1	0	0	
	0.75	5	44	51.00	2	1	1	1	1	1	0	0	5	42	29.00	40	1	1	1	1	1	1	0	
	1.00	5	44	33.00	2	1	1	1	1	1	0	0	5	42	22.00	25	1	1	1	1	1	1	0	
	1.25	5	44	16.25	1	1	1	1	1	1	0	5	42	16.25	7	1	1	1	1	1	1	1	0	
	1.50	5	44	14.50	1	1	1	1	1	1	0	5	42	14.50	7	1	1	1	1	1	1	1	0	
	1.75	5	44	12.75	1	1	1	1	1	1	0	5	42	12.75	7	1	1	1	1	1	1	1	0	
	2.00	5	44	11.00	1	1	1	1	1	1	0	5	42	11.00	7	1	1	1	1	1	1	1	0	
AKO8T6-4	0.25	3	44	57.00	2	1	1	1	1	1	0	3	60	65.50	34	1	1	1	1	1	1	1	0	
	0.50	3	44	49.00	2	1	1	1	1	1	0	3	60	57.00	34	1	1	1	1	1	1	1	0	
	0.75	3	44	41.00	2	1	1	1	1	1	0	3	60	48.50	34	1	1	1	1	1	1	1	0	
	1.00	3	44	35.00	1	1	1	1	1	1	0	3	60	40.00	34	1	1	1	1	1	1	1	0	
	1.25	3	44	31.00	1	1	1	1	1	1	0	3	60	31.50	34	1	1	1	1	1	1	1	0	
	1.50	3	44	27.00	1	1	1	1	1	1	0	3	60	27.00	16	1	1	1	1	1	1	1	0	
	1.75	3	44	23.00	1	1	1	1	1	1	0	3	60	23.00	16	1	1	1	1	1	1	1	0	
	2.00	3	44	19.00	1	1	1	1	1	1	0	3	60	19.00	16	1	1	1	1	1	1	1	0	
AKO8T6-5	0.25	5	80	98.25	1	1	1	1	1	1	0	5	54	96.00	44	2						1	0	
	0.50	5	80	78.50	1	1	1	1	1	1	0	5	54	74.00	44	2							1	0
	0.75	5	80	59.75	1	1	1	1	1	1	0	5	54	58.25	23	3	1	1	1	1	1	0	0	
	1.00	5	80	43.00	1	1	1	1	1	1	0	5	54	41.00	23	3	1	1	1	1	1	0	0	
	1.25	5	80	26.25	1	1	1	1	1	1	0	5	54	23.75	23	3	1	1	1	1	1	0	0	
	1.50	5	80	9.50	1	1	1	1	1	1	0	5	54	6.50	23	3	1	1	1	1	1	0	0	
	1.75	5	80	3.50	1	1	1	1	1	1	0	5	54	3.50	22	1	1	1	1	1	1	1	0	
	2.00	5	80	0.00	0						0	0	5	54	0.00	0	0						0	0
AKO8T6-6	0.25	6	44	103.00	2	1	1	1	0	0	0	6	62	125.50	39	2	1	1	1	1	1	0	0	
	0.50	5	44	87.00	2	1	1	1	0	0	0	6	62	106.00	39	2	1	1	1	1	1	0	0	
	0.75	5	44	71.00	2	1	1	1	0	0	0	6	62	86.50	39	2	1	1	1	1	1	0	0	
	1.00	5	44	55.00	2	1	1	1	0	0	0	6	62	67.00	39	2	1	1	1	1	1	0	0	
	1.25	5	44	41.50	1	1	1	1	1	1	0	6	62	47.50	35	2	1	1	1	1	1	1	1	
	1.50	5	44	33.00	1	1	1	1	1	1	0	6	62	33.00	34	1	1	1	1	1	1	1	1	0
	1.75	5	44	24.50	1	1	1	1	1	1	0	6	62	24.50	34	1	1	1	1	1	1	1	1	0
	2.00	5	44	16.00	1	1	1	1	1	1	0	6	62	16.00	34	1	1	1	1	1	1	1	1	0
AKO8T6-7	0.25	5	66	98.00	2						1	0	5	41	119.75	31	3	1	1	1	1	1	0	
	0.50	5	66	77.00	2						1	0	5	41	96.50	31	3	1	1	1	1	1	0	
	0.75	5	66	56.00	2						1	2	5	41	73.25	31	3	1	1	1	1	1	0	
	1.00	5	66	35.00	2						1	1	5	41	50.00	31	3	1	1	1	1	1	1	0
	1.25	5	66	27.00	1	1	1	1	1	1	1	5	41	26.75	31	3	1	1	1	1	1	1	0	
	1.50	5	66	19.00	1	1	1	1	1	1	0	5	41	26.00	33	2	1	1	1	1	1	1	0	
	1.75	5	66	11.00	1	1	1	1	1	1	0	5	41	11.00	32	1	1	1	1	1	1	1	1	
	2.00	5	66	3.00	1	1	1	1	1	1	0	5	41	3.00	32	1	1	1	1	1	1	1	1	

**Table E.1 Results of the heuristic solution procedure (Continued)**

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET									HIGH DEMAND VARIABILITY DATA SET															
		$K_{UB}$	$CT_U$	$r$	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	CT	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP				
AKO8T6-8	0.25	3	44	69.75	1	1	1	1	1	1	0	3	40	95.50	23	2	1	0	0	0	0	0				
	0.50	2	44	61.50	1	1	1	1	1	1	0	3	40	84.00	23	2	1	0	0	0	0	0				
	0.75	2	44	53.25	1	1	1	1	1	1	0	3	40	72.50	23	2	1	0	0	0	0	0				
	1.00	2	44	45.00	1	1	1	1	1	1	0	3	40	61.00	23	2	1	0	0	0	0	0				
	1.25	2	44	36.75	1	1	1	1	1	1	0	3	40	49.50	23	2	1	0	0	0	0	0				
	1.50	2	44	28.50	1	1	1	1	1	1	0	3	40	41.50	29	1	1	1	1	1	1	1	0			
	1.75	2	44	20.25	1	1	1	1	1	1	0	3	40	34.25	29	1	1	1	1	1	1	1	0			
	2.00	2	44	12.00	1	1	1	1	1	1	0	3	40	27.00	29	1	1	1	1	1	1	1	0			
AKO8T6-9	0.25	5	44	71.00	5	1	1	1	1	1	0	6	44	101.00	12	5	1	1	1	1	1	0				
	0.50	5	44	56.00	5	1	1	1	1	1	0	6	44	86.00	12	5	1	1	1	1	1	0				
	0.75	5	44	45.25	1	1	1	1	1	1	0	6	44	71.00	12	5	1	1	1	1	1	0				
	1.00	5	44	37.00	1	1	1	1	1	1	0	6	44	56.00	12	5	1	1	1	1	1	0				
	1.25	5	44	28.75	1	1	1	1	1	1	0	6	44	41.00	12	5	1	1	1	1	1	0				
	1.50	5	44	20.50	1	1	1	1	1	1	0	6	44	26.00	12	5	1	1	1	1	1	0				
	1.75	5	44	13.25	1	1	1	1	1	1	1	6	44	11.00	12	5	1	1	1	1	1	0				
	2.00	5	44	8.00	1	1	1	1	1	1	0	6	44	8.00	21	1	1	1	1	1	1	0				
AKO20T4-A-0	0.25	5	57	260.50	2	1	1	0	0	1	0	5	47	260.50	41	2	1	1	0	0	1	0				
	0.50	5	57	240.00	2	1	1	0	0	1	0	5	47	240.00	41	2	1	1	0	0	1	0				
	0.75	5	57	219.50	2	1	1	0	0	1	1	5	47	219.50	41	2	1	1	0	0	1	1				
	1.00	5	57	200.00	2	1	1	1	1	1	1	5	47	200.00	38	2	1	1	1	1	1	1				
	1.25	5	57	181.00	2	1	1	1	1	1	1	5	47	181.00	38	2	1	1	1	1	1	1				
	1.50	5	57	162.00	2	1	1	1	1	1	1	5	47	162.00	38	2	1	1	1	1	1	1				
	1.75	5	57	143.00	2	1	1	1	1	1	1	5	47	143.00	38	2	1	1	1	1	1	1				
	2.00	5	57	124.00	2	1	1	1	1	1	1	5	47	124.00	38	2	1	1	1	1	1	1				
AKO20T4-A-1	0.25	8	66	298.00	2					1	0	8	44	297.00	24	4						1	0			
	0.50	7	66	275.00	2					1	0	7	44	273.00	24	4							1	0		
	0.75	7	66	252.00	2					1	0	7	44	249.00	24	4							1	0		
	1.00	7	66	229.00	2					1	0	7	44	225.00	24	4								1	0	
	1.25	7	66	206.00	2					1	0	7	44	201.00	24	4									1	0
	1.50	7	66	183.00	2					1	0	7	44	177.00	24	4									1	0
	1.75	7	66	160.00	2					1	0	7	44	153.00	24	4									1	0
	2.00	7	66	137.00	2					1	0	7	44	129.00	24	4										1
AKO20T4-A-2	0.25	7	100	94.00	1	1	1	1	1	1	0	8	43	163.00	40	3	1	1	1	1	1	1	0			
	0.50	6	100	72.00	1	1	1	1	1	1	0	7	43	133.00	40	3	1	1	1	1	1	1	0			
	0.75	6	100	50.00	1	1	1	1	1	1	0	7	43	103.00	40	3	1	1	1	1	1	1	0			
	1.00	6	100	28.00	1	1	1	1	1	1	0	7	43	73.00	40	3	1	1	1	1	1	1	0			
	1.25	6	100	6.00	1	1	1	1	1	1	0	7	43	43.00	40	3	1	1	1	1	1	1	0			
	1.50	6	100	0.00	0						0	0	7	43	13.00	40	3	1	1	1	1	1	1	0		
	1.75	6	100	0.00	0						0	0	7	43	0.00	0	0							0	0	
	2.00	6	100	0.00	0						0	0	7	43	0.00	0	0								0	0
AKO20T4-A-3	0.25	6	66	257.50	2		1	0	0	1	0	6	72	172.00	32	3	1	1	1	1	1	1	0			
	0.50	6	66	232.00	2		1	0	0	1	0	5	72	148.00	32	3	1	1	1	1	1	1	0			
	0.75	6	66	206.50	2		1	0	0	1	0	5	72	124.00	32	3	1	1	1	1	1	1	0			
	1.00	6	66	177.00	3		1	0	0	0	0	5	72	113.00	67	1	1	1	1	1	1	1	0			
	1.25	6	66	154.50	3		1	0	0	0	0	5	72	96.25	67	1	1	1	1	1	1	1	0			
	1.50	6	66	132.00	3		1	0	0	0	0	5	72	79.50	67	1	1	1	1	1	1	1	1			
	1.75	6	66	109.50	3		1	0	0	0	0	5	72	62.75	67	1	1	1	1	1	1	1	1			
	2.00	6	66	87.00	3		1	0	0	0	0	5	72	46.00	67	1	1	1	1	1	1	1	1			
AKO20T4-A-4	0.25	7	50	146.50	2	1	1	1	0	0	0	7	47	146.50	45	2	1	1	1	0	0	0				
	0.50	7	50	124.00	2	1	1	1	0	0	0	7	47	124.00	45	2	1	1	1	0	0	0				
	0.75	7	50	101.50	2	1	1	1	0	0	0	7	47	101.50	45	2	1	1	1	0	0	0				
	1.00	7	50	79.00	2	1	1	1	0	0	0	7	47	79.00	45	2	1	1	1	0	0	0				
	1.25	7	50	56.50	2	1	1	1	0	0	0	7	47	56.50	45	2	1	1	1	0	0	0				
	1.50	7	50	34.00	2	1	1	1	0	0	0	7	47	34.00	45	2	1	1	1	0	0	0				
	1.75	7	50	14.50	2		1	0	0	1	1	7	47	14.50	27	2		1	0	0	1	1				
	2.00	7	50	1.00	2		1	0	0	1	1	7	47	1.00	27	2		1	0	0	1	1				
AKO20T4-A-5	0.25	9	66	228.50	3	1	1	1	1	1	0	9	47	228.50	46	3	1	1	1	1	1	1	0			
	0.50	10	66	195.00	2		1	0	0	0	1	10	47	194.00	46	3	1	1	1	1	1	1	0			
	0.75	10	66	163.00	2		1	0	0	0	0	10	47	157.75	45	3	1	0	1	0	0	0				
	1.00	8	66	131.00	2		1	0	0	0	0	8	47	124.00	45	3	1	0	1	0	0	0				
	1.25	8	66	99.00	2		1	0	0	0	0	8	47	90.25	45	3	1	0	1	0	0	0				
	1.50	8	66	67.00	2		1	0	0	0	0	8	47	56.50	45	3	1	0	1	0	0	0				
	1.75	10	66	35.00	2		1	0	0	0	0	10	47	22.75	45	3	1	0	1	0	0	0				
	2.00	8	66	3.00	2		1	0	0	0	1	8	47	0.00	0	0							0	0		
AKO20T4-A-6	0.25	8	50	438.25	3	1	0	1	1	0	0	8	59	438.25	41	3	1	0	1	1	0	0				
	0.50	8	50	407.50	3	1	0	1	1	0	0	8	59	407.50	41	3	1	0	1	1	0	0				
	0.75	8	50	376.75	3	1	0	1	1	0	0	8	59	376.75	41	3	1	0	1	1	0	0				
	1.00	8	50	346.00	3	1	0	1	1	0	0	8	59	346.00	41	3	1	0	1	1	0	0				
	1.25	8	50	315.25	3	1	0	1	1	0	0	8	59	315.25	41	3	1	0	1	1	0	0				
	1.50	8	50	284.50	3	1	0	1	1	0	0	8	59	284.50	41	3	1	0	1	1	0	0				
	1.75	8	50	253.75	3	1	0	1	1	0	0	8	59	253.75	41	3	1	0	1	1	0	0				
	2.00	8	50	223.00	3	1	0	1	1	0	0	8	59	223.00	41	3	1	0	1	1	0	0				

**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET													
		$K_{UB}$	$CT_U$	$r$	$K$	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	$CT$	$K$	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP			
AKO20T4-A-7	0.25	6	40	153.25	3							6	68	153.25	37	3									
	0.50	6	40	125.50	3		1	0	1	1	0	6	68	125.50	37	3		1	0	1	1	1	0		
	0.75	6	40	97.75	3		1	0	1	1	0	6	68	97.75	37	3		1	0	1	1	1	0		
	1.00	6	40	70.00	3		1	0	1	1	1	6	68	70.00	37	3		1	0	1	1	1	1		
	1.25	6	40	42.25	3		1	0	1	1	1	6	68	42.25	37	3		1	0	1	1	1	1		
	1.50	6	40	14.50	3		1	0	1	1	1	6	68	14.50	37	3		1	0	1	1	1	1		
	1.75	6	40	0.00	0							0	0	0.00	0	0							0	0	
	2.00	6	40	0.00	0							0	0	0.00	0	0							0	0	
AKO20T4-A-8	0.25	8	57	290.75	3	1	1	1	1	1	0	8	42	290.00	22	6	1	0	1	0	0	0	0		
	0.50	8	57	258.50	3	1	1	1	1	1	0	8	42	257.00	22	6	1	0	1	0	0	0	0		
	0.75	8	57	226.25	3	1	1	1	1	1	0	8	42	224.00	22	6	1	0	1	0	0	0	0		
	1.00	8	57	194.00	3	1	1	1	1	1	0	8	42	191.00	22	6	1	0	1	0	0	0	0		
	1.25	8	57	161.75	3	1	1	1	1	1	0	8	42	158.00	22	6	1	0	1	0	0	0	0		
	1.50	8	57	129.50	3	1	1	1	1	1	0	8	42	125.00	22	6	1	0	1	0	0	0	0		
	1.75	8	57	97.25	3	1	1	1	1	1	0	8	42	92.00	22	6	1	0	1	0	0	0	0		
	2.00	8	57	65.00	3	1	1	1	1	1	1	8	42	59.00	22	6	1	0	1	0	0	0	1		
AKO20T4-A-9	0.25	6	50	252.00	6	1	1	1	1	0	0	6	51	252.00	20	6	1	1	1	1	0	0	0		
	0.50	6	50	222.00	6	1	1	1	1	0	0	6	51	222.00	20	6	1	1	1	1	1	0	0		
	0.75	6	50	192.00	6	1	1	1	1	0	0	6	51	192.00	20	6	1	1	1	1	1	0	0		
	1.00	6	50	162.00	6	1	1	1	1	0	0	6	51	162.00	20	6	1	1	1	1	1	0	0		
	1.25	6	50	132.00	6	1	1	1	1	0	0	6	51	132.00	20	6	1	1	1	1	1	0	0		
	1.50	6	50	102.00	6	1	1	1	1	0	1	6	51	102.00	20	6	1	1	1	1	1	0	1		
	1.75	6	50	72.00	6	1	1	1	1	0	1	6	51	72.00	20	6	1	1	1	1	1	0	1		
	2.00	6	50	42.00	6	1	1	1	1	0	1	6	51	42.00	20	6	1	1	1	1	1	0	1		
AKO20T4-B-0	0.25	7	40	169.50	3	1	1	1	1	0	0	7	42	169.50	34	3	1	1	1	1	0	0	0		
	0.50	7	40	144.00	3	1	1	1	1	0	0	7	42	144.00	34	3	1	1	1	1	1	0	0	0	
	0.75	7	40	118.50	3	1	1	1	1	0	0	7	42	118.50	34	3	1	1	1	1	1	0	0	0	
	1.00	7	40	93.00	3	1	1	1	1	0	0	7	42	93.00	34	3	1	1	1	1	1	0	0	0	
	1.25	7	40	67.50	3	1	1	1	1	0	0	7	42	67.50	34	3	1	1	1	1	1	0	0	0	
	1.50	7	40	42.00	3	1	1	1	1	0	0	7	42	42.00	34	3	1	1	1	1	1	0	0	0	
	1.75	7	40	16.50	3	1	1	1	1	0	0	7	42	16.50	34	3	1	1	1	1	1	0	0	0	
	2.00	7	40	0.00	0						0	0	7	42	0.00	0	0						0	0	
AKO20T4-B-1	0.25	9	66	151.50	3	1	0	1	1	0	0	9	41	151.50	38	3	1	0	1	1	0	0	0		
	0.50	7	66	123.00	3	1	0	1	1	0	0	7	41	123.00	38	3	1	0	1	1	0	0	0		
	0.75	8	66	94.50	3	1	0	1	1	0	0	8	41	94.50	38	3	1	0	1	1	0	0	0		
	1.00	8	66	66.00	3	1	0	1	1	0	1	8	41	66.00	38	3	1	0	1	1	0	1	0	1	
	1.25	8	66	37.50	3	1	0	1	1	0	1	8	41	37.50	38	3	1	0	1	1	0	1	0	1	
	1.50	8	66	9.00	3	1	0	1	1	0	1	8	41	9.00	38	3	1	0	1	1	0	1	0	1	
	1.75	8	66	0.00	0						0	0	8	41	0.00	0	0						0	0	
	2.00	8	66	0.00	0						0	0	8	41	0.00	0	0						0	0	
AKO20T4-B-2	0.25	8	40	180.50	3	1	1	1	1	1	0	6	43	180.50	38	3	1	1	1	1	1	1	0		
	0.50	8	40	152.00	3	1	1	1	1	1	0	6	43	152.00	38	3	1	1	1	1	1	1	0	0	
	0.75	8	40	123.50	3	1	1	1	1	1	0	6	43	123.50	38	3	1	1	1	1	1	1	0	0	
	1.00	8	40	95.00	3	1	1	1	1	1	0	6	43	95.00	38	3	1	1	1	1	1	1	1	0	
	1.25	8	40	66.50	3	1	1	1	1	1	0	6	43	66.50	38	3	1	1	1	1	1	1	1	0	
	1.50	8	40	38.00	3	1	1	1	1	1	0	6	43	38.00	38	3	1	1	1	1	1	1	1	0	
	1.75	8	40	9.50	3	1	1	1	1	1	1	6	43	9.50	38	3	1	1	1	1	1	1	1	1	
	2.00	8	40	0.00	0						0	0	6	43	0.00	0	0						0	0	
AKO20T4-B-3	0.25	10	40	268.75	5	1	0	1	1	0	0	10	50	272.00	43	4			1	0	0	1	0	0	
	0.50	10	40	222.50	5	1	0	1	1	0	0	10	50	229.00	43	4			1	0	0	1	0	0	
	0.75	10	40	176.25	5	1	0	1	1	0	0	10	50	186.00	43	4			1	0	0	1	0	0	
	1.00	10	40	130.00	5	1	0	1	1	0	0	10	50	143.00	43	4			1	0	0	1	0	0	
	1.25	10	40	83.75	5	1	0	1	1	0	0	10	50	100.00	43	4			1	0	0	1	0	0	
	1.50	10	40	37.50	5	1	0	1	1	0	0	10	50	57.00	43	4			1	0	0	1	0	0	
	1.75	10	40	0.00	0						0	0	10	50	14.00	43	4			1	0	0	1	0	0
	2.00	10	40	0.00	0						0	0	10	50	0.00	0	0						0	0	0
AKO20T4-B-4	0.25	8	50	117.00	5	1	1	1	1	0	0	8	41	192.50	38	3	1	1	1	1	1	0	0	0	
	0.50	8	50	92.00	5	1	1	1	1	0	0	8	41	164.00	38	3	1	1	1	1	1	0	0	0	
	0.75	7	50	67.00	5	1	1	1	1	0	0	7	41	135.50	38	3	1	1	1	1	1	0	0	0	
	1.00	7	50	42.00	5	1	1	1	1	0	0	7	41	107.00	38	3	1	1	1	1	1	1	0	0	
	1.25	7	50	17.00	5	1	1	1	1	0	0	7	41	78.50	38	3	1	1	1	1	1	1	0	0	
	1.50	12	50	0.00	0						0	0	12	41	50.00	38	3	1	1	1	1	1	0	0	0
	1.75	7	50	0.00	0						0	0	7	41	21.50	38	3	1	1	1	1	1	0	0	1
	2.00	7	50	0.00	0						0	0	7	41	0.00	0	0						0	0	0
AKO20T4-B-5	0.25	9	40	133.00	4		1	0	0	0	0	9	41	133.00	37	4			1	0	0	0	0	0	
	0.50	9	40	96.00	4		1	0	0	0	0	9	41	96.00	37	4			1	0	0	0	0	0	
	0.75	9	40	59.00	4		1	0	0	0	1	9	41	59.00	37	4			1	0	0	0	0	1	
	1.00	9	40	22.00	4		1	0	0	0	1	9	41	22.00	37	4			1	0	0	0	0	1	
	1.25	9	40	0.00	0						0	0	9	41	0.00	0	0						0	0	0
	1.50	9	40	0.00	0						0	0	9	41	0.00	0	0						0	0	0
	1.75	9	40	0.00	0						0	0	9	41	0.00	0	0						0	0	0
	2.00	9	40	0.00	0						0	0	9	41	0.00	0	0						0	0	0

**Table E.1 Results of the heuristic solution procedure (Continued)**

INSTANCE	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET												
	S	$K_{UB}$	$CT_U$	$r$	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	CT	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	
AKO20T4-B-6	0.25	8	57	88.25	3	1	0	1	1	0	0	9	53	158.00	36	2	1	1	1	1	1	0	
	0.50	5	57	72.50	3	1	0	1	1	0	0	6	53	140.00	36	2	1	1	1	1	1	0	
	0.75	5	57	56.75	3	1	0	1	1	0	0	6	53	122.00	36	2	1	1	1	1	1	0	
	1.00	5	57	41.00	3	1	0	1	1	0	0	6	53	104.00	36	2	1	1	1	1	1	0	
	1.25	5	57	25.25	3	1	0	1	1	0	0	6	53	86.00	36	2	1	1	1	1	1	0	
	1.50	5	57	9.50	3	1	0	1	1	0	0	6	53	68.00	36	2	1	1	1	1	1	0	
	1.75	5	57	0.00	0						0	0	6	53	50.00	36	2	1	1	1	1	0	
	2.00	5	57	0.00	0						0	0	6	53	32.00	36	2	1	1	1	1	1	1
AKO20T4-B-7	0.25	5	66	138.50	2	1	1	1	1	1	0	5	67	225.50	43	2						1	0
	0.50	5	66	118.00	2	1	1	1	1	1	0	5	67	204.00	43	2						1	0
	0.75	5	66	97.50	2	1	1	1	1	1	0	5	67	182.50	43	2						1	0
	1.00	5	66	77.00	2	1	1	1	1	1	0	5	67	161.00	43	2						1	0
	1.25	5	66	56.50	2	1	1	1	1	1	0	5	67	139.50	43	2						1	0
	1.50	5	66	36.00	2	1	1	1	1	1	0	5	67	118.00	43	2						1	0
	1.75	5	66	15.50	2	1	1	1	1	1	0	5	67	96.50	43	2						1	0
	2.00	5	66	22.00	1	1	1	1	1	1	0	5	67	86.00	39	1	1	1	1	1	1	1	0
AKO20T4-B-8	0.25	7	57	154.00	3	1	1	1	1	1	0	7	43	154.00	40	3	1	1	1	1	1	1	0
	0.50	7	57	124.00	3	1	1	1	1	1	0	7	43	124.00	40	3	1	1	1	1	1	1	0
	0.75	7	57	95.50	3	1	0	1	1	0	0	7	43	94.00	40	3	1	1	1	1	1	1	0
	1.00	7	57	70.00	3	1	0	1	1	0	0	7	43	64.00	40	3	1	1	1	1	1	1	0
	1.25	7	57	44.50	3	1	0	1	1	0	0	7	43	34.00	40	3	1	1	1	1	1	1	0
	1.50	7	57	19.00	3	1	0	1	1	0	0	7	43	19.00	33	2	1	0	1	1	1	0	0
	1.75	7	57	2.50	2	1	0	1	1	0	0	7	43	2.50	33	2	1	0	1	1	1	0	0
	2.00	7	57	0.00	0						0	0	7	43	0.00	0	0					0	0
AKO20T4-B-9	0.25	8	57	171.50	3	1	0	1	0	0	0	8	41	171.00	20	7	1	1	1	1	1	1	0
	0.50	7	57	137.00	3	1	0	1	0	0	0	7	41	136.00	20	7	1	1	1	1	1	1	0
	0.75	7	57	102.50	3	1	0	1	0	0	0	7	41	101.00	20	7	1	1	1	1	1	1	0
	1.00	7	57	70.00	3		1	0	0	0	0	7	41	64.00	27	5	1	0	1	0	0	0	0
	1.25	7	57	37.75	3		1	0	0	0	0	7	41	30.25	27	5	1	0	1	0	0	0	0
	1.50	7	57	5.50	3		1	0	0	0	0	7	41	0.00	0	0						0	0
	1.75	7	57	0.00	0						0	0	7	41	0.00	0	0					0	0
	2.00	7	57	0.00	0						0	0	7	41	0.00	0	0					0	0
AKO20T4-C-0	0.25	9	133	93.25	1	1	1	1	1	1	1	9	41	89.00	31	4	1	1	1	0	0	1	1
	0.50	9	133	66.50	1	1	1	1	1	1	1	9	41	58.00	31	4	1	1	1	0	0	1	1
	0.75	9	133	39.75	1	1	1	1	1	1	1	9	41	27.00	31	4	1	1	1	0	0	1	1
	1.00	9	133	0.00	0						0	0	9	41	0.00	0	0					0	0
	1.25	7	133	0.00	0						0	0	7	41	0.00	0	0					0	0
	1.50	9	133	0.00	0						0	0	9	41	0.00	0	0					0	0
	1.75	7	133	0.00	0						0	0	7	41	0.00	0	0					0	0
	2.00	7	133	0.00	0						0	0	7	41	0.00	0	0					0	0
AKO20T4-C-1	0.25	6	40	440.75	5		1	0	0	0	1	6	43	440.75	21	5		1	0	0	0	1	1
	0.50	6	40	414.50	5		1	0	0	0	0	6	43	414.50	21	5		1	0	0	0	0	1
	0.75	7	40	388.25	5		1	0	0	0	0	7	43	388.25	21	5		1	0	0	0	0	1
	1.00	7	40	362.00	5		1	0	0	0	0	7	43	362.00	21	5		1	0	0	0	0	1
	1.25	7	40	335.75	5		1	0	0	0	0	7	43	335.75	21	5		1	0	0	0	0	1
	1.50	7	40	309.50	5		1	0	0	0	0	7	43	309.50	21	5		1	0	0	0	0	1
	1.75	6	40	283.25	5		1	0	0	0	0	6	43	283.25	21	5		1	0	0	0	0	1
	2.00	6	40	257.00	5		1	0	0	0	0	6	43	257.00	21	5		1	0	0	0	0	1
AKO20T4-C-2	0.25	9	57	166.50	3						1	0	9	43	165.00	42	4		1	0	0	0	0
	0.50	9	57	126.00	3						1	0	9	43	123.00	42	4		1	0	0	0	0
	0.75	9	57	85.50	3						1	0	9	43	81.00	42	4		1	0	0	0	0
	1.00	9	57	45.00	3						1	1	9	43	39.00	42	4		1	0	0	0	1
	1.25	9	57	4.50	3						1	1	9	43	0.00	0	0					0	0
	1.50	9	57	0.00	0						0	0	9	43	0.00	0	0					0	0
	1.75	9	57	0.00	0						0	0	9	43	0.00	0	0					0	0
	2.00	9	57	0.00	0						0	0	9	43	0.00	0	0					0	0
AKO20T4-C-3	0.25	9	80	137.00	2		1	0	0	0	1	9	42	135.75	39	3		1	0	0	0	1	1
	0.50	9	80	109.00	2		1	0	0	0	0	9	42	106.50	39	3		1	0	0	0	0	1
	0.75	9	80	81.00	2		1	0	0	0	1	9	42	77.25	39	3		1	0	0	0	1	1
	1.00	7	80	53.00	2		1	0	0	0	1	7	42	48.00	39	3		1	0	0	0	1	1
	1.25	7	80	25.00	2		1	0	0	0	1	7	42	18.75	39	3		1	0	0	0	1	1
	1.50	7	80	0.00	0						0	0	7	42	0.00	0	0					0	0
	1.75	7	80	0.00	0						0	0	7	42	0.00	0	0					0	0
	2.00	7	80	0.00	0						0	0	7	42	0.00	0	0					0	0
AKO20T4-C-4	0.25	5	50	254.50	2	1	1	1	1	1	0	5	53	254.50	39	2	1	1	1	1	1	1	0
	0.50	5	50	235.00	2	1	1	1	1	1	0	5	53	235.00	39	2	1	1	1	1	1	1	0
	0.75	5	50	215.50	2	1	1	1	1	1	0	5	53	215.50	39	2	1	1	1	1	1	1	0
	1.00	5	50	196.00	2	1	1	1	1	1	0	5	53	196.00	39	2	1	1	1	1	1	1	0
	1.25	5	50	176.50	2	1	1	1	1	1	0	5	53	176.50	39	2	1	1	1	1	1	1	0
	1.50	5	50	157.00	2	1	1	1	1	1	0	5	53	157.00	39	2	1	1	1	1	1	1	0
	1.75	5	50	137.50	2	1	1	1	1	1	1	5	53	137.50	39	2	1	1	1	1	1	1	1
	2.00	5	50	118.00	2	1	1	1	1	1	1	5	53	118.00	39	2	1	1	1	1	1	1	1



**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET												
		$K_{UB}$	$CT_U$	$r$	$K$	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	$CT$	$K$	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP		
AKO20T4-C-5	0.25	7	57	157.50	2	1	1	1	1	1	0	7	51	157.50	49	2	1	1	1	1	1	0		
	0.50	7	57	133.00	2	1	1	1	1	1	0	7	51	133.00	49	2	1	1	1	1	1	0		
	0.75	6	57	108.50	2	1	1	1	1	1	0	6	51	108.50	49	2	1	1	1	1	1	0		
	1.00	6	57	84.00	2	1	1	1	1	1	0	6	51	84.00	49	2	1	1	1	1	1	0		
	1.25	7	57	59.50	2	1	1	1	1	1	0	7	51	59.50	49	2	1	1	1	1	1	0		
	1.50	7	57	35.00	2	1	1	1	1	1	0	7	51	35.00	49	2	1	1	1	1	1	0		
	1.75	7	57	10.50	2	1	1	1	1	1	0	7	51	10.50	49	2	1	1	1	1	1	0		
	2.00	6	57	0.00	0						0	0	6	51	0.00	0	0					0	0	
AKO20T4-C-6	0.25	11	57	278.25	3	1	1	1	1	0	1	11	54	278.25	49	3	1	1	1	1	0	1		
	0.50	11	57	241.50	3	1	1	1	1	0	1	11	54	241.50	49	3	1	1	1	1	0	1		
	0.75	11	57	204.75	3	1	1	1	1	0	1	11	54	204.75	49	3	1	1	1	1	0	1		
	1.00	11	57	168.00	3	1	1	1	1	0	1	11	54	168.00	49	3	1	1	1	1	0	1		
	1.25	11	57	131.25	3	1	1	1	1	0	1	11	54	131.25	49	3	1	1	1	1	0	1		
	1.50	11	57	94.50	3	1	1	1	1	0	1	11	54	94.50	49	3	1	1	1	1	0	1		
	1.75	11	57	57.75	3	1	1	1	1	0	1	11	54	57.75	49	3	1	1	1	1	0	1		
	2.00	9	57	21.00	3	1	1	1	1	0	1	9	54	21.00	49	3	1	1	1	1	0	1		
AKO20T4-C-7	0.25	7	50	249.00	2	1	1	1	1	1	0	7	93	249.00	50	2	1	1	1	1	1	0		
	0.50	6	50	224.00	2	1	1	1	1	1	0	6	93	224.00	50	2	1	1	1	1	1	0		
	0.75	6	50	199.00	2	1	1	1	1	1	0	6	93	199.00	50	2	1	1	1	1	1	0		
	1.00	6	50	174.00	2	1	1	1	1	1	0	6	93	174.00	50	2	1	1	1	1	1	0		
	1.25	6	50	149.00	2	1	1	1	1	1	0	6	93	149.00	50	2	1	1	1	1	1	0		
	1.50	6	50	124.00	2	1	1	1	1	1	1	6	93	124.00	50	2	1	1	1	1	1	1		
	1.75	6	50	99.00	2	1	1	1	1	1	1	6	93	99.25	65	1	1	1	1	1	1	1		
	2.00	6	50	74.00	2	1	1	1	1	1	1	6	93	83.00	65	1	1	1	1	1	1	1		
AKO20T4-C-8	0.25	10	50	258.00	3					1	1	10	63	258.00	44	3					1	1		
	0.50	10	50	225.00	3					1	1	10	63	225.00	44	3					1	1		
	0.75	10	50	192.00	3					1	1	10	63	192.00	44	3					1	1		
	1.00	10	50	159.00	3					1	1	10	63	159.00	44	3					1	1		
	1.25	10	50	126.00	3					1	1	10	63	126.00	44	3					1	1		
	1.50	10	50	93.00	3					1	1	10	63	93.00	44	3					1	1		
	1.75	10	50	60.00	3					1	1	10	63	60.00	44	3					1	1		
	2.00	10	50	27.00	3					1	1	10	63	27.00	44	3					1	1		
AKO20T4-C-9	0.25	6	40	199.00	3	1	1	1	1	1	0	7	53	171.25	37	3	1	1	1	1	0	0		
	0.50	7	40	178.00	3	1	1	1	1	1	0	7	53	143.50	37	3	1	1	1	1	0	0		
	0.75	7	40	157.00	3	1	1	1	1	1	0	7	53	115.75	37	3	1	1	1	1	0	0		
	1.00	7	40	136.00	3	1	1	1	1	1	0	8	53	88.00	37	3	1	1	1	1	0	0		
	1.25	7	40	115.00	3	1	1	1	1	1	1	8	53	45.50	38	3		1	0	0	0	1		
	1.50	7	40	94.00	3	1	1	1	1	1	1	8	53	18.00	39	2	1	1	1	1	1	1		
	1.75	7	40	73.00	3	1	1	1	1	1	1	8	53	0.00	0	0						0	0	
	2.00	7	40	52.00	3	1	1	1	1	1	1	7	53	0.00	0	0						0	0	
LAM20T10-0	0.25	4	44	172.50	2	1	1	1	1	0	0	4	46	223.00	46	2	1	1	1	1	1	0		
	0.50	3	44	155.00	2	1	1	1	1	0	0	3	46	204.00	34	2	1	0	1	0	0	0		
	0.75	3	44	141.00	2	1	1	1	1	0	0	3	46	190.00	28	2	1	1	1	1	0	0		
	1.00	3	44	127.00	2	1	1	1	1	0	0	3	46	176.00	28	2	1	1	1	1	0	0		
	1.25	4	44	113.00	2	1	1	1	1	0	0	4	46	162.00	28	2	1	1	1	1	0	0		
	1.50	3	44	99.00	2	1	1	1	1	0	0	3	46	148.00	28	2	1	1	1	1	0	0		
	1.75	3	44	88.75	1	1	1	1	1	1	0	3	46	134.00	28	2	1	1	1	1	0	0		
	2.00	4	44	80.00	1	1	1	1	1	1	0	4	46	125.00	35	1	1	1	1	1	1	0		
LAM20T10-1	0.25	5	44	322.50	2	1	1	1	1	1	0	5	46	279.50	25	2	1	1	1	1	1	0		
	0.50	5	44	310.00	2	1	1	1	1	1	0	5	46	267.00	25	2	1	1	1	1	1	0		
	0.75	5	44	297.50	2	1	1	1	1	1	0	5	46	254.50	25	2	1	1	1	1	1	0		
	1.00	5	44	285.00	2	1	1	1	1	1	0	5	46	242.00	25	2	1	1	1	1	1	0		
	1.25	5	44	272.50	2	1	1	1	1	1	0	5	46	229.50	25	2	1	1	1	1	1	0		
	1.50	5	44	260.00	2	1	1	1	1	1	0	5	46	217.00	25	2	1	1	1	1	1	0		
	1.75	5	44	247.50	2	1	1	1	1	1	0	5	46	204.50	25	2	1	1	1	1	1	0		
	2.00	5	44	235.00	2	1	1	1	1	1	0	5	46	192.00	25	2	1	1	1	1	1	0		
LAM20T10-2	0.25	7	44	206.00	4	1	1	1	1	1	0	7	44	176.00	30	4	1	1	1	1	1	0		
	0.50	7	44	183.00	2					1	0	7	44	153.00	44	2						1	0	
	0.75	6	44	161.00	2					1	0	6	44	131.00	44	2						1	0	
	1.00	6	44	138.00	2	1	0	0	0	1	0	6	44	108.00	38	2	1	0	0	0	0	1	0	
	1.25	6	44	119.00	2	1	0	0	0	1	0	6	44	89.00	38	2	1	0	0	0	0	1	0	
	1.50	6	44	103.00	2	1	1	1	1	1	0	6	44	73.00	30	2	1	1	1	1	1	1	0	
	1.75	6	44	88.00	2	1	1	1	1	1	0	6	44	58.00	30	2	1	1	1	1	1	1	0	
	2.00	6	44	73.00	2	1	1	1	1	1	0	6	44	43.00	30	2	1	1	1	1	1	1	0	
LAM20T10-3	0.25	6	66	217.50	2					1	0	6	54	304.50	47	2						1	0	
	0.50	6	66	194.00	2					1	0	6	54	281.00	47	2							1	0
	0.75	6	66	170.50	2					1	0	6	54	257.50	47	2							1	0
	1.00	5	66	149.00	2	1	1	1	1	0	0	5	54	236.00	37	2	1	1	1	1	1	0	0	
	1.25	6	66	130.50	2	1	1	1	1	0	0	6	54	217.50	37	2	1	1	1	1	1	0	0	
	1.50	5	66	112.00	2	1	1	1	1	0	0	5	54	199.00	37	2	1	1	1	1	1	0	0	
	1.75	5	66	93.50	2	1	1	1	1	0	0	5	54	180.50	37	2	1	1	1	1	1	0	0	
	2.00	5	66	78.00	1	1	1	1	1	1	0	5	54	162.00	37	2	1	1	1	1	1	0	0	

**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET									HIGH DEMAND VARIABILITY DATA SET												
		$K_{UB}$	$CT_U$	$r$	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP	$K_{UB}$	$CT_U$	$r$	CT	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP	
LAM20T10-4	0.25	7	44	327.75	3					1	0	7	40	378.25	27	5	1	0	0	1	0	0	
	0.50	7	44	295.50	3					1	0	7	40	344.50	27	5	1	0	0	1	0	0	
	0.75	7	44	266.00	3	1	1	1	1	0	0	7	40	318.00	40	3	1	1	1	1	0	0	
	1.00	7	44	236.00	3	1	1	1	1	0	0	7	40	288.00	40	3	1	1	1	1	0	0	
	1.25	7	44	210.75	5		1	1	0	1	0	7	40	262.75	21	5		1	1	0	1	0	
	1.50	6	44	184.50	5		1	1	0	1	0	6	40	236.50	21	5		1	1	0	1	0	
	1.75	6	44	158.25	5		1	1	0	1	0	6	40	210.25	21	5		1	1	0	1	0	
	2.00	6	44	132.00	5		1	1	0	1	0	6	40	184.00	21	5		1	1	0	1	0	
LAM20T10-5	0.25	4	44	187.00	1	1	1	1	1	1	0	4	40	186.50	23	2	1	1	1	1	1	0	
	0.50	4	44	176.00	1	1	1	1	1	1	0	4	40	175.00	23	2	1	1	1	1	1	0	
	0.75	4	44	165.00	1	1	1	1	1	1	0	4	40	163.50	23	2	1	1	1	1	1	0	
	1.00	4	44	154.00	1	1	1	1	1	1	0	4	40	152.00	23	2	1	1	1	1	1	0	
	1.25	4	44	143.00	1	1	1	1	1	1	0	4	40	140.50	23	2	1	1	1	1	1	0	
	1.50	4	44	132.00	1	1	1	1	1	1	0	4	40	129.00	23	2	1	1	1	1	1	0	
	1.75	4	44	121.00	1	1	1	1	1	1	0	4	40	117.50	23	2	1	1	1	1	1	0	
	2.00	4	44	110.00	1	1	1	1	1	1	0	4	40	106.00	23	2	1	1	1	1	1	0	
LAM20T10-6	0.25	6	44	293.50	2	1	1	1	1	1	0	4	42	262.50	39	2	1	1	1	1	1	1	
	0.50	6	44	274.00	2	1	1	1	1	1	0	5	42	243.00	39	2	1	1	1	1	1	0	
	0.75	5	44	254.50	2	1	1	1	1	1	0	4	42	223.50	39	2	1	1	1	1	1	0	
	1.00	5	44	235.00	2	1	1	1	1	1	0	5	42	204.00	39	2	1	1	1	1	1	0	
	1.25	5	44	215.50	2	1	1	1	1	1	0	5	42	184.50	39	2	1	1	1	1	1	0	
	1.50	4	44	196.00	2	1	1	1	1	1	0	5	42	165.00	39	2	1	1	1	1	1	0	
	1.75	5	44	176.50	2	1	1	1	1	1	0	5	42	145.50	39	2	1	1	1	1	1	0	
	2.00	5	44	157.00	2	1	1	1	1	1	0	5	42	126.00	39	2	1	1	1	1	1	0	
LAM20T10-7	0.25	6	44	440.50	2					1	0	6	40	440.50	39	2					1	0	
	0.50	6	44	421.00	2					1	0	6	40	421.00	39	2					1	0	
	0.75	6	44	401.50	2					1	0	6	40	401.50	39	2					1	0	
	1.00	6	44	382.00	2					1	0	6	40	382.00	39	2					1	0	
	1.25	6	44	362.50	2					1	0	6	40	362.50	39	2					1	0	
	1.50	6	44	343.00	2					1	0	6	40	343.00	39	2					1	0	
	1.75	6	44	323.50	2					1	0	6	40	323.50	39	2					1	0	
	2.00	6	44	304.00	2					1	0	6	40	304.00	39	2					1	0	
LAM20T10-8	0.25	6	57	195.00	2		1	0	0	0	0	6	45	271.00	42	2		1	0	0	0	0	
	0.50	6	57	174.00	2	1	1	1	1	0	1	6	45	250.00	37	2	1	1	1	1	0	0	
	0.75	5	57	155.50	2	1	1	1	1	0	0	5	45	231.50	37	2	1	1	1	1	0	0	
	1.00	6	57	137.00	2	1	1	1	1	0	0	6	45	213.00	37	2	1	1	1	1	0	0	
	1.25	6	57	118.50	2	1	1	1	1	0	0	6	45	194.50	37	2	1	1	1	1	0	0	
	1.50	6	57	100.00	2	1	1	1	1	0	0	6	45	176.00	37	2	1	1	1	1	0	0	
	1.75	6	57	81.50	2	1	1	1	1	0	0	6	45	157.50	37	2	1	1	1	1	0	0	
	2.00	6	57	63.00	2	1	1	1	1	0	0	6	45	139.00	37	2	1	1	1	1	0	0	
LAM20T10-9	0.25	6	44	288.00	2					1	0	6	43	288.00	42	2					1	0	
	0.50	6	44	267.00	2					1	0	6	43	267.00	42	2					1	0	
	0.75	6	44	246.00	2					1	0	6	43	246.00	42	2					1	0	
	1.00	6	44	225.00	2					1	0	6	43	225.00	42	2					1	0	
	1.25	6	44	204.00	2					1	0	6	43	204.00	42	2					1	0	
	1.50	6	44	183.00	2					1	0	6	43	183.00	42	2					1	0	
	1.75	6	44	158.50	3	1	1	1	1	0	0	6	43	158.50	26	3	1	1	1	1	0	0	
	2.00	6	44	139.00	3	1	1	1	1	0	0	6	43	139.00	26	3	1	1	1	1	0	0	
LAM20T24-0	0.25	4	44	72.50	2		1	1	0	0	0	4	40	64.00	36	1	1	1	1	1	1	0	
	0.50	4	44	64.00	1	1	1	1	1	1	0	3	40	57.50	17	1	1	1	1	1	1	0	
	0.75	3	44	57.50	1	1	1	1	1	1	0	3	40	54.25	9	1	1	1	1	1	1	0	
	1.00	4	44	52.00	1	1	1	1	1	1	0	4	40	52.00	9	1	1	1	1	1	1	0	
	1.25	4	44	49.75	1	1	1	1	1	1	0	4	40	49.75	9	1	1	1	1	1	1	0	
	1.50	4	44	47.50	1	1	1	1	1	1	0	4	40	47.50	9	1	1	1	1	1	1	0	
	1.75	3	44	45.25	1	1	1	1	1	1	0	3	40	45.25	9	1	1	1	1	1	1	0	
	2.00	4	44	43.00	1	1	1	1	1	1	0	4	40	43.00	9	1	1	1	1	1	1	0	
LAM20T24-1	0.25	5	44	74.50	2	1	1	1	1	1	0	5	40	74.50	27	2	1	1	1	1	1	0	
	0.50	5	44	61.00	2	1	1	1	1	1	0	5	40	61.00	27	2	1	1	1	1	1	0	
	0.75	5	44	47.50	2	1	1	1	1	1	0	5	40	47.50	27	2	1	1	1	1	1	0	
	1.00	5	44	34.00	2	1	1	1	1	1	0	5	40	34.00	27	2	1	1	1	1	1	0	
	1.25	5	44	20.50	2	1	1	1	1	1	0	5	40	20.50	27	2	1	1	1	1	1	0	
	1.50	5	44	7.00	2	1	1	1	1	1	0	5	40	7.00	27	2	1	1	1	1	1	0	
	1.75	5	44	0.00	0						0	0	5	40	0.00	0	0					0	0
	2.00	5	44	0.00	0						0	0	5	40	0.00	0	0					0	0
LAM20T24-2	0.25	6	44	54.00	4	1	1	1	1	1	0	6	40	52.00	18	4	1	1	1	1	1	0	
	0.50	5	44	36.00	4	1	1	1	1	1	0	5	40	34.00	18	4	1	1	1	1	1	0	
	0.75	5	44	18.00	4	1	1	1	1	1	0	5	40	16.00	18	4	1	1	1	1	1	0	
	1.00	5	44	2.00	1	1	1	1	1	1	0	5	40	7.00	28	1	1	1	1	1	1	0	
	1.25	5	44	0.00	0						0	0	5	40	0.00	0	0					0	0
	1.50	5	44	0.00	0						0	0	5	40	0.00	0	0					0	0
	1.75	5	44	0.00	0						0	0	5	40	0.00	0	0					0	0
	2.00	5	44	0.00	0						0	0	5	40	0.00	0	0					0	0

**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET								HIGH DEMAND VARIABILITY DATA SET														
		$K_{UB}$	$CT_U$	$r$	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	CT	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP		
LAM20T24-3	0.25	6	44	46.75	3	1	1	1	1	0	0	6	40	46.75	39	3	1	1	1	1	1	0	0	
	0.50	6	44	33.50	1	1	1	1	1	1	0	6	40	33.50	39	1	1	1	1	1	1	1	0	
	0.75	6	44	24.75	1	1	1	1	1	1	0	6	40	24.75	23	1	1	1	1	1	1	1	0	
	1.00	4	44	19.00	1	1	1	1	1	1	0	6	40	19.00	23	1	1	1	1	1	1	1	0	
	1.25	6	44	13.25	1	1	1	1	1	1	0	6	40	13.25	23	1	1	1	1	1	1	1	0	
	1.50	6	44	7.50	1	1	1	1	1	1	0	6	40	7.50	23	1	1	1	1	1	1	1	0	
	1.75	4	44	1.75	1	1	1	1	1	1	1	4	40	1.75	19	1	1	1	1	1	1	1	1	
	2.00	4	44	0.00	0						0	0	4	40	0.00	0	0						0	0
LAM20T24-4	0.25	6	44	45.50	2	1	1	1	1	1	0	6	41	53.75	27	3	1	1	1	1	1	1	0	
	0.50	6	44	32.00	2	1	1	1	1	1	0	7	41	33.50	27	3	1	1	1	1	1	1	0	
	0.75	6	44	26.75	1	1	1	1	1	1	0	6	41	27.25	41	1	1	1	1	1	1	1	0	
	1.00	6	44	20.00	1	1	1	1	1	1	0	7	41	20.00	27	1	1	1	1	1	1	1	0	
	1.25	6	44	13.25	1	1	1	1	1	1	0	6	41	13.25	27	1	1	1	1	1	1	1	0	
	1.50	6	44	6.50	1	1	1	1	1	1	0	7	41	6.50	21	1	1	1	1	1	1	1	1	
	1.75	5	44	0.00	0						0	0	6	41	1.25	21	1	1	1	1	1	1	1	0
	2.00	5	44	0.00	0						0	0	6	41	0.00	0	0							0
LAM20T24-5	0.25	4	44	66.00	2	1	1	1	1	1	0	4	41	68.25	23	1	1	1	1	1	1	1	1	0
	0.50	4	44	59.00	2	1	1	1	1	1	0	4	41	62.50	23	1	1	1	1	1	1	1	1	0
	0.75	4	44	52.00	2	1	1	1	1	1	0	5	41	56.75	23	1	1	1	1	1	1	1	1	0
	1.00	4	44	45.00	2	1	1	1	1	1	0	4	41	51.00	23	1	1	1	1	1	1	1	1	0
	1.25	4	44	38.00	2	1	1	1	1	1	0	4	41	45.25	23	1	1	1	1	1	1	1	1	0
	1.50	4	44	31.00	2	1	1	1	1	1	0	4	41	39.50	23	1	1	1	1	1	1	1	1	0
	1.75	4	44	24.00	2	1	1	1	1	1	0	4	41	33.75	23	1	1	1	1	1	1	1	1	0
	2.00	4	44	17.00	1	1	1	1	1	1	1	4	41	28.00	23	1	1	1	1	1	1	1	1	0
LAM20T24-6	0.25	6	44	88.50	2	1	1	1	1	0	0	6	40	94.00	40	2	1	0	0	0	0	0	0	
	0.50	5	44	77.50	1	1	1	1	1	1	0	5	40	75.00	35	2	1	0	0	0	0	0	0	
	0.75	5	44	70.50	2						1	0	5	40	62.00	26	2	1	1	1	1	1	1	0
	1.00	5	44	64.00	2						1	0	5	40	49.00	26	2	1	1	1	1	1	1	0
	1.25	5	44	57.50	2						1	0	5	40	40.75	21	1	1	1	1	1	1	1	0
	1.50	5	44	51.00	1	1	1	1	1	1	0	5	40	35.50	21	1	1	1	1	1	1	1	1	0
	1.75	5	44	47.50	1	1	1	1	1	1	0	5	40	33.25	9	1	1	1	1	1	1	1	1	0
	2.00	5	44	44.00	1	1	1	1	1	1	0	5	40	31.00	9	1	1	1	1	1	1	1	1	0
LAM20T24-7	0.25	6	50	61.25	1	1	1	1	1	1	0	6	40	106.25	29	3	1	1	1	1	1	0	0	
	0.50	6	50	50.50	1	1	1	1	1	1	0	6	40	85.00	17	4	1	1	1	1	1	0	0	
	0.75	6	50	39.75	1	1	1	1	1	1	0	5	40	68.00	17	4	1	1	1	1	1	0	0	
	1.00	6	50	29.00	1	1	1	1	1	1	0	6	40	51.00	17	4	1	1	1	1	1	0	0	
	1.25	6	50	18.25	1	1	1	1	1	1	0	6	40	34.00	17	4	1	1	1	1	1	0	0	
	1.50	6	50	11.00	2	1	1	1	1	1	0	6	40	22.50	31	1	1	1	1	1	1	1	1	0
	1.75	6	50	6.50	2	1	1	1	1	1	0	6	40	14.75	31	1	1	1	1	1	1	1	1	0
	2.00	6	50	2.00	2	1	1	1	1	1	0	6	40	8.00	26	1	1	1	1	1	1	1	1	0
LAM20T24-8	0.25	5	44	62.00	2		1	0	0	0	0	5	40	82.50	30	3	1	1	1	1	1	0	0	
	0.50	5	44	41.00	2		1	0	0	0	0	5	40	60.00	30	3	1	1	1	1	1	0	0	
	0.75	5	44	21.50	2	1	1	1	1	1	0	5	40	37.50	30	3	1	1	1	1	1	0	0	
	1.00	5	44	9.00	2	1	1	1	1	1	0	5	40	15.00	30	3	1	1	1	1	1	0	0	
	1.25	5	44	0.00	0						0	0	5	40	0.00	0	0						0	
	1.50	6	44	0.00	0						0	0	6	40	0.00	0	0						0	
	1.75	5	44	0.00	0						0	0	5	40	0.00	0	0						0	
	2.00	6	44	0.00	0						0	0	6	40	0.00	0	0						0	
LAM20T24-9	0.25	4	44	69.00	2	1	1	1	1	1	0	4	42	69.00	30	2	1	1	1	1	1	1	0	
	0.50	4	44	54.00	2	1	1	1	1	1	0	4	42	54.00	30	2	1	1	1	1	1	1	0	
	0.75	4	44	39.00	2	1	1	1	1	1	0	4	42	39.00	30	2	1	1	1	1	1	1	0	
	1.00	4	44	24.00	2	1	1	1	1	1	0	4	42	24.00	30	2	1	1	1	1	1	1	0	
	1.25	4	44	11.00	1	1	1	1	1	1	0	4	42	9.75	15	3	1	1	1	1	1	1	1	0
	1.50	4	44	0.00	0						0	0	5	42	0.00	0	0						0	
	1.75	4	44	0.00	0						0	0	4	42	0.00	0	0						0	
	2.00	4	44	0.00	0						0	0	4	42	0.00	0	0						0	
LAM30T10-0	0.25	5	50	267.50	2	1	1	1	1	1	0	5	44	322.50	35	2	1	1	1	1	1	1	0	
	0.50	5	50	250.00	2	1	1	1	1	1	0	5	44	305.00	35	2	1	1	1	1	1	1	0	
	0.75	5	50	233.50	2	1	1	1	1	0	0	5	44	288.50	29	2	1	1	1	1	1	0	0	
	1.00	5	50	219.00	2	1	1	1	1	0	0	6	44	274.00	29	2	1	1	1	1	1	0	0	
	1.25	5	50	204.50	2	1	1	1	1	0	0	5	44	259.50	29	2	1	1	1	1	1	0	0	
	1.50	5	50	190.00	2	1	1	1	1	0	0	6	44	245.00	29	2	1	1	1	1	1	0	0	
	1.75	5	50	175.50	2	1	1	1	1	0	0	5	44	230.50	29	2	1	1	1	1	1	0	0	
	2.00	5	50	161.00	2	1	1	1	1	0	0	5	44	216.00	29	2	1	1	1	1	1	0	0	
LAM30T10-1	0.25	5	50	219.00	2	1	1	1	1	0	0	5	62	329.00	46	2	1	1	1	1	1	0	0	
	0.50	5	50	196.00	2	1	1	1	1	0	0	5	62											

**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET											
		$K_{UB}$	$CT_U$	$r$	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	CT	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	
LAM30T10-2	0.25	7	44	378.50	2	1	1	1	1	0	1	7	41	378.50	35	2	1	1	1	1	0	1	
	0.50	7	44	361.00	2	1	1	1	1	1	0	7	41	361.00	34	2	1	1	1	1	1	0	0
	0.75	7	44	344.00	2	1	1	1	1	1	0	7	41	344.00	34	2	1	1	1	1	1	1	0
	1.00	6	44	327.00	2	1	1	1	1	1	0	6	41	327.00	34	2	1	1	1	1	1	1	0
	1.25	6	44	310.00	2	1	1	1	1	1	0	6	41	310.00	34	2	1	1	1	1	1	1	0
	1.50	6	44	293.00	2	1	1	1	1	1	0	6	41	293.00	34	2	1	1	1	1	1	1	0
	1.75	6	44	276.00	2	1	1	1	1	1	0	6	41	276.00	34	2	1	1	1	1	1	1	0
	2.00	6	44	259.00	2	1	1	1	1	1	0	6	41	259.00	34	2	1	1	1	1	1	1	0
LAM30T10-3	0.25	7	44	418.50	3	1	1	1	1	1	0	6	40	418.50	26	3	1	1	1	1	1	0	
	0.50	5	44	399.00	3	1	1	1	1	1	0	4	40	399.00	26	3	1	1	1	1	1	1	0
	0.75	4	44	379.50	3	1	1	1	1	1	0	4	40	379.50	26	3	1	1	1	1	1	1	0
	1.00	4	44	365.00	2	1	1	1	1	1	0	4	40	365.00	34	2	1	1	1	1	1	1	0
	1.25	4	44	348.00	2	1	1	1	1	1	0	4	40	348.00	34	2	1	1	1	1	1	1	0
	1.50	4	44	331.00	2	1	1	1	1	1	0	4	40	331.00	34	2	1	1	1	1	1	1	0
	1.75	5	44	314.00	2	1	1	1	1	1	0	5	40	314.00	34	2	1	1	1	1	1	1	0
	2.00	4	44	297.00	2	1	1	1	1	1	0	4	40	297.00	34	2	1	1	1	1	1	1	0
LAM30T10-4	0.25	6	44	304.50	5	1	1	1	1	1	0	6	41	304.50	22	5	1	1	1	1	1	1	0
	0.50	6	44	286.00	2	1	1	1	1	1	0	6	41	286.00	40	2	1	1	1	1	1	1	0
	0.75	5	44	266.00	2	1	1	1	1	1	0	5	41	266.00	40	2	1	1	1	1	1	1	0
	1.00	5	44	246.00	2	1	1	1	1	1	0	5	41	246.00	40	2	1	1	1	1	1	1	0
	1.25	5	44	230.00	2	1	1	1	1	1	0	5	41	230.00	34	2	1	1	1	1	1	1	0
	1.50	6	44	213.00	2	1	1	1	1	1	0	5	41	213.00	34	2	1	1	1	1	1	1	0
	1.75	5	44	196.00	2	1	1	1	1	1	0	5	41	196.00	34	2	1	1	1	1	1	1	0
	2.00	6	44	179.00	2	1	1	1	1	1	0	5	41	179.00	34	2	1	1	1	1	1	1	0
LAM30T10-5	0.25	6	44	412.50	2					1	0	6	41	412.50	41	2						1	0
	0.50	6	44	395.50	3	1	1	1	1	0	1	6	41	395.50	19	3	1	1	1	1	0	0	1
	0.75	6	44	381.50	2	1	1	1	1	1	0	5	41	381.50	25	2	1	1	1	1	1	1	0
	1.00	5	44	369.00	2	1	1	1	1	1	0	5	41	369.00	25	2	1	1	1	1	1	1	0
	1.25	5	44	356.50	2	1	1	1	1	1	0	5	41	356.50	25	2	1	1	1	1	1	1	0
	1.50	4	44	344.00	2	1	1	1	1	1	0	4	41	344.00	25	2	1	1	1	1	1	1	0
	1.75	4	44	331.50	2	1	1	1	1	1	0	4	41	331.50	25	2	1	1	1	1	1	1	0
	2.00	4	44	319.00	2	1	1	1	1	1	0	4	41	319.00	25	2	1	1	1	1	1	1	0
LAM30T10-6	0.25	5	44	471.00	2	1	1	1	1	0	0	5	50	471.00	40	2	1	1	1	1	0	0	0
	0.50	5	44	451.00	2	1	1	1	1	0	0	5	50	451.00	40	2	1	1	1	1	1	0	0
	0.75	5	44	431.00	2	1	1	1	1	0	0	5	50	431.00	40	2	1	1	1	1	1	0	0
	1.00	5	44	411.00	2	1	1	1	1	0	0	5	50	411.00	40	2	1	1	1	1	1	0	0
	1.25	5	44	391.00	2	1	1	1	1	0	0	5	50	391.00	40	2	1	1	1	1	1	0	0
	1.50	5	44	371.00	2	1	1	1	1	0	0	5	50	371.00	40	2	1	1	1	1	1	0	0
	1.75	5	44	351.00	2	1	1	1	1	0	0	5	50	351.00	40	2	1	1	1	1	1	0	0
	2.00	5	44	331.00	2	1	1	1	1	0	0	5	50	331.00	40	2	1	1	1	1	1	0	0
LAM30T10-7	0.25	7	44	606.00	4					1	1	7	45	606.00	26	4						1	1
	0.50	6	44	580.00	4					1	1	6	45	580.00	26	4						1	1
	0.75	6	44	554.00	4					1	1	6	45	554.00	26	4						1	1
	1.00	6	44	528.00	4					1	1	6	45	528.00	26	4						1	1
	1.25	6	44	502.00	4					1	1	6	45	502.00	26	4						1	1
	1.50	6	44	476.00	4					1	1	6	45	476.00	26	4						1	1
	1.75	6	44	457.00	3					1	1	6	45	457.00	32	3						1	1
	2.00	6	44	433.00	3					1	0	6	45	433.00	32	3						1	0
LAM30T10-8	0.25	5	44	363.50	2	1	1	1	1	0	1	5	48	363.50	41	2	1	1	1	1	1	0	1
	0.50	5	44	343.00	2	1	1	1	1	0	0	5	48	343.00	41	2	1	1	1	1	1	0	0
	0.75	5	44	322.50	2	1	1	1	1	0	0	5	48	322.50	41	2	1	1	1	1	1	0	0
	1.00	5	44	302.00	2	1	1	1	1	0	0	5	48	302.00	41	2	1	1	1	1	1	0	0
	1.25	5	44	281.50	2	1	1	1	1	0	0	5	48	281.50	41	2	1	1	1	1	1	0	0
	1.50	5	44	261.00	2	1	1	1	1	0	0	5	48	261.00	41	2	1	1	1	1	1	0	0
	1.75	5	44	240.50	2	1	1	1	1	0	0	5	48	240.50	41	2	1	1	1	1	1	0	0
	2.00	5	44	220.00	2	1	1	1	1	0	0	5	48	220.00	41	2	1	1	1	1	1	0	0
LAM30T10-9	0.25	4	50	305.50	2	1	1	1	1	0	0	6	42	305.50	31	2	1	1	1	1	1	0	0
	0.50	4	50	290.00	2	1	1	1	1	0	0	5	42	290.00	31	2	1	1	1	1	1	0	0
	0.75	4	50	274.50	2	1	1	1	1	0	0	5	42	274.50	31	2	1	1	1	1	1	0	0
	1.00	4	50	259.00	2	1	1	1	1	0	0	5	42	259.00	31	2	1	1	1	1	1	0	0
	1.25	4	50	243.50	2	1	1	1	1	0	0	5	42	243.50	31	2	1	1	1	1	1	0	0
	1.50	4	50	228.00	2	1	1	1	1	0	0	5	42	228.00	31	2	1	1	1	1	1	0	0
	1.75	4	50	212.50	2	1	1	1	1	0	0	5	42	212.50	31	2	1	1	1	1	1	0	0
	2.00	4	50	197.00	2	1	1	1	1	0	0	5	42	197.00	31	2	1	1	1	1	1	0	0
LAM30T29-0	0.25	6	44	96.75	3	1	1	1	1	1	0	5	40	96.75	27	3	1	1	1	1	1	1	0
	0.50	5	44	76.50	3	1	1	1	1	1	0	5	40	76.50	27	3	1	1	1	1	1	1	0
	0.75	5	44	56.25	3	1	1	1	1	1	0	5	40	56.25	27	3	1	1	1	1	1	1	0
	1.00	5	44	36.00	3	1	1	1	1	1	0	5	40	36.00	27	3	1	1	1	1	1	1	0
	1.25	5	44	21.00	2	1	1	1	1	1	0	5	40	21.00	26	2	1	1	1	1	1	1	0
	1.50	5	44	8.00	2	1	1	1	1	1	0	5	40	13.00	28	1	1	1	1	1	1	1	0
	1.75	4	44	0.00	0					0	0	4	40	6.00	28	1	1	1	1	1	1	1	1
	2.00	4	44	0.00	0					0	0	4	40	0.00	0	0						0	0

**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET																
	S	K <sub>UB</sub>	CT <sub>U</sub>	r	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP	K <sub>UB</sub>	CT <sub>U</sub>	r	CT	K	NX <sub>1</sub>	NX <sub>2</sub>	NX <sub>3</sub>	NX <sub>4</sub>	NX <sub>5</sub>	IMP					
LAM30T29-1	0.25	5	44	59.25	3	1	1	1	1	1	0	5	40	59.25	33	3	1	1	1	1	1	1	0				
	0.50	5	44	34.50	3	1	1	1	1	1	0	5	40	34.50	33	3	1	1	1	1	1	1	0				
	0.75	5	44	9.75	3	1	1	1	1	1	0	5	40	9.75	33	3	1	1	1	1	1	1	0				
	1.00	5	44	0.00	0						0	0	5	40	0.00	0	0						0	0			
	1.25	5	44	0.00	0						0	0	5	40	0.00	0	0							0	0		
	1.50	7	44	0.00	0						0	0	5	40	0.00	0	0								0	0	
	1.75	5	44	0.00	0						0	0	5	40	0.00	0	0									0	0
	2.00	7	44	0.00	0						0	0	5	40	0.00	0	0										0
LAM30T29-2	0.25	6	44	126.50	2	1	1	1	1	0	1	7	43	98.50	31	2	1	1	1	1	1	1	1	0			
	0.50	6	44	109.00	2	1	1	1	1	1	0	7	43	84.00	29	2	1	1	1	1	1	1	1	0			
	0.75	6	44	95.00	2	1	1	1	1	0	0	6	43	74.75	35	1	1	1	1	1	1	1	1	0			
	1.00	6	44	83.00	2	1	1	1	1	0	0	6	43	66.00	35	1	1	1	1	1	1	1	1	0			
	1.25	6	44	71.00	2	1	1	1	1	0	0	6	43	57.25	35	1	1	1	1	1	1	1	1	0			
	1.50	6	44	59.00	2	1	1	1	1	0	0	6	43	48.50	35	1	1	1	1	1	1	1	1	0			
	1.75	6	44	47.00	2	1	1	1	1	0	0	6	43	39.75	35	1	1	1	1	1	1	1	1	0			
	2.00	6	44	35.00	2	1	1	1	1	0	0	6	43	31.00	35	1	1	1	1	1	1	1	1	0			
LAM30T29-3	0.25	6	44	71.00	4	1	1	1	1	1	0	6	42	74.50	35	2	1	1	1	1	1	1	1	0			
	0.50	5	44	60.50	1	1	1	1	1	1	0	5	42	57.00	35	2	1	1	1	1	1	1	1	0			
	0.75	5	44	50.25	1	1	1	1	1	1	0	5	42	41.25	19	3	1	1	1	1	1	1	1	0			
	1.00	5	44	40.00	1	1	1	1	1	1	0	5	42	31.00	32	1	1	1	1	1	1	1	1	0			
	1.25	5	44	29.75	1	1	1	1	1	1	0	5	42	23.00	32	1	1	1	1	1	1	1	1	0			
	1.50	5	44	19.50	1	1	1	1	1	1	0	5	42	16.00	6	1	1	1	1	1	1	1	1	0			
	1.75	5	44	14.50	1	1	1	1	1	1	0	5	42	14.50	6	1	1	1	1	1	1	1	1	0			
	2.00	5	44	13.00	1	1	1	1	1	1	0	5	42	13.00	6	1	1	1	1	1	1	1	1	0			
LAM30T29-4	0.25	5	44	93.25	3	1	1	1	1	1	0	5	41	93.25	37	3	1	1	1	1	1	1	1	0			
	0.50	5	44	73.00	2					1	2	6	41	73.00	41	2								1	2		
	0.75	5	44	55.00	2	1	1	1	1	1	0	5	41	55.00	34	2	1	1	1	1	1	1	1	0			
	1.00	5	44	38.00	2	1	1	1	1	1	0	6	41	38.00	34	2	1	1	1	1	1	1	1	0			
	1.25	5	44	21.00	2	1	1	1	1	1	0	5	41	21.00	34	2	1	1	1	1	1	1	1	0			
	1.50	5	44	14.50	1	1	1	1	1	1	0	5	41	14.50	1	1	1	1	1	1	1	1	1	0			
	1.75	5	44	14.25	1	1	1	1	1	1	0	5	41	14.25	1	1	1	1	1	1	1	1	1	0			
	2.00	5	44	14.00	1	1	1	1	1	1	0	5	41	14.00	1	1	1	1	1	1	1	1	1	0			
LAM30T29-5	0.25	6	50	57.00	2					1	0	6	42	63.25	33	3	1	1	1	1	1	1	0	0			
	0.50	6	50	40.50	1	1	1	1	1	1	0	6	42	49.00	25	2	1	1	1	1	1	1	1	0			
	0.75	6	50	28.25	1	1	1	1	1	1	0	7	42	36.50	25	2	1	1	1	1	1	1	1	0			
	1.00	5	50	16.00	1	1	1	1	1	1	0	5	42	24.00	25	2	1	1	1	1	1	1	1	0			
	1.25	5	50	6.50	1	1	1	1	1	1	0	5	42	11.50	25	2	1	1	1	1	1	1	1	0			
	1.50	5	50	5.00	1	1	1	1	1	1	0	5	42	5.00	6	1	1	1	1	1	1	1	1	0			
	1.75	5	50	3.50	1	1	1	1	1	1	0	5	42	3.50	6	1	1	1	1	1	1	1	1	0			
	2.00	5	50	2.00	1	1	1	1	1	1	0	5	42	2.00	6	1	1	1	1	1	1	1	1	0			
LAM30T29-6	0.25	5	44	55.00	2	1	1	1	1	0	0	5	42	75.00	40	2	1	1	1	1	1	1	0	0			
	0.50	6	44	35.00	2	1	1	1	1	0	0	5	42	55.00	40	2	1	1	1	1	1	1	0	0			
	0.75	5	44	16.50	1	1	1	1	1	1	2	5	42	36.50	42	1	1	1	1	1	1	1	1	2			
	1.00	5	44	7.00	1	1	1	1	1	1	0	5	42	26.00	42	1	1	1	1	1	1	1	1	0			
	1.25	5	44	0.00	0						0	0	5	42	15.50	42	1	1	1	1	1	1	1	0			
	1.50	5	44	0.00	0						0	0	5	42	6.00	30	1	1	1	1	1	1	1	0			
	1.75	5	44	0.00	0						0	0	5	42	1.50	6	1	1	1	1	1	1	1	1			
	2.00	5	44	0.00	0						0	0	5	42	0.00	0	0								0	0	
LAM30T29-7	0.25	7	44	71.00	4					1	1	7	43	99.00	26	4							1	1			
	0.50	6	44	50.00	3					1	0	6	43	78.00	26	3							1	1			
	0.75	6	44	32.00	2	1	1	1	1	1	0	6	43	60.00	26	2	1	1	1	1	1	1	1	0			
	1.00	6	44	19.00	2	1	1	1	1	1	0	6	43	47.00	26	2	1	1	1	1	1	1	1	0			
	1.25	6	44	3.25	1	1	1	1	1	1	2	6	43	28.00	26	2	1	1	1	1	1	1	1	0			
	1.50	6	44	0.00	0						0	0	6	43	24.00	26	1	1	1	1	1	1	1	0			
	1.75	6	44	0.00	0						0	0	6	43	17.50	26	1	1	1	1	1	1	1	0			
	2.00	6	44	0.00	0						0	0	6	43	11.00	26	1	1	1	1	1	1	1	1	0		
LAM30T29-8	0.25	5	44	66.50	2	1	1	1	1	1	1	5	40	65.75	27	3	1	1	1	1	1	1	1	1			
	0.50	5	44	45.50	3	1	1	1	1	1	0	5	40	45.50	27	3	1	1	1	1	1	1	1	0			
	0.75	5	44	25.25	3	1	1	1	1	1	0	5	40	25.25	27	3	1	1	1	1	1	1	1	0			
	1.00	5	44	5.00	3	1	1	1	1	1	0	5	40	5.00	27	3	1	1	1	1	1	1	1	0			
	1.25	5	44	2.25	1	1	1	1	1	1	0	5	40	3.00	32	1	1	1	1	1	1	1	1	0			
	1.50	5	44	0.00	0						0	0	5	40	0.00	0	0								0	0	
	1.75	5	44	0.00	0						0	0	5	40	0.00	0	0									0	0
	2.00	5	44	0.00	0						0	0	5	40	0.00	0	0										0
LAM30T29-9	0.25	4	44	88.00	2	1	1	1	1	0	0	4	40	96.50	33	2	1	1	1	1	1	1	1	0			
	0.50	4	44	73.00	2	1	1	1	1	0	0	4	40	80.00	33	2	1	1	1	1	1	1	1	0			
	0.75	4	44	58.00	2	1	1	1	1	0	0	4	40	63.50	33	2	1	1	1	1	1	1	1	0			
	1.00	4	44	43.00	2	1	1	1	1	0	0	4	40	47.00	33	2	1	1	1	1	1	1	1	0			
	1.25	4	44	28.75	1	1	1	1	1	1	2	4	40	30.50	33	2	1	1	1	1	1	1	1	0			
	1.50	4	44	21.00	1	1	1	1	1	1	0	4	40	21.00	30	1	1	1	1	1	1	1	1	0			
	1.75	4	44	16.75	1	1	1	1	1	1	0	4	40	16.75	11	1	1	1	1	1	1	1	1	0			
	2.00	4	44	14.00	1	1	1	1	1	1	0	4	40	14.00	11	1	1	1	1	1	1	1	1	0			

**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET								HIGH DEMAND VARIABILITY DATA SET															
		$K_{UB}$	$CT_U$	$r$	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	CT	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP			
AKO30T12-0	0.25	10	44	320.00	4	1	0	0	0	0	0	10	41	415.00	38	4	1	0	0	0	0	0	0		
	0.50	10	44	282.00	4	1	0	0	0	0	1	10	41	377.00	38	4	1	0	0	0	0	0	1		
	0.75	10	44	244.00	4	1	0	0	1	0	0	10	41	339.00	37	4	1	0	0	1	0	0	0		
	1.00	10	44	207.00	4	1	0	0	1	0	0	10	41	302.00	37	4	1	0	0	0	1	0	0		
	1.25	10	44	170.00	4	1	0	0	1	0	0	10	41	265.00	37	4	1	0	0	1	0	0	0		
	1.50	10	44	133.00	4	1	0	0	1	0	0	10	41	228.00	37	4	1	0	0	1	0	0	0		
	1.75	10	44	96.00	4	1	0	0	1	0	0	10	41	191.00	37	4	1	0	0	1	0	0	0		
	2.00	10	44	59.00	4	1	0	0	1	0	0	10	41	154.00	37	4	1	0	0	1	0	0	0		
AKO30T12-1	0.25	12	44	420.00	6	1	0	1	0	0	0	12	40	420.00	30	6	1	0	1	0	0	0	0		
	0.50	12	44	375.00	6	1	0	1	0	0	0	12	40	375.00	30	6	1	0	1	0	0	0	0		
	0.75	12	44	330.00	6	1	0	1	0	0	0	12	40	330.00	30	6	1	0	1	0	0	0	0		
	1.00	12	44	285.00	6	1	0	1	0	0	0	12	40	285.00	30	6	1	0	1	0	0	0	0		
	1.25	11	44	240.00	6	1	0	1	0	0	0	11	40	240.00	30	6	1	0	1	0	0	0	0		
	1.50	11	44	206.00	4		1	0	0	0	0	12	40	206.00	40	4		1	0	0	0	0	0		
	1.75	11	44	161.75	5	1	0	1	0	0	0	11	40	161.75	31	5	1	0	1	0	0	0	0		
	2.00	10	44	123.00	5	1	0	1	0	0	0	10	40	123.00	31	5	1	0	1	0	0	0	0		
AKO30T12-2	0.25	9	44	415.00	8	1	0	0	0	0	0	9	41	415.00	22	8	1	0	0	0	0	0	0		
	0.50	9	44	371.00	8	1	0	0	0	0	0	9	41	371.00	22	8	1	0	0	0	0	0	0		
	0.75	9	44	327.00	8	1	0	0	0	0	0	9	41	327.00	22	8	1	0	0	0	0	0	0		
	1.00	9	44	283.00	8	1	0	0	0	0	0	9	41	283.00	22	8	1	0	0	0	0	0	0		
	1.25	9	44	239.00	8	1	0	0	0	0	0	9	41	239.00	22	8	1	0	0	0	0	0	0		
	1.50	9	44	195.00	8	1	0	0	0	0	0	9	41	195.00	22	8	1	0	0	0	0	0	0		
	1.75	9	44	151.00	8	1	0	0	0	0	0	9	41	151.00	22	8	1	0	0	0	0	0	0		
	2.00	9	44	107.00	8	1	0	0	0	0	0	9	41	107.00	22	8	1	0	0	0	0	0	0		
AKO30T12-3	0.25	11	50	491.00	4				1	0	0	10	42	624.75	37	5							1	0	
	0.50	10	50	445.00	4				1	0	0	10	42	578.50	37	5								1	0
	0.75	10	50	397.00	4	1	0	1	0	0	0	10	42	528.00	36	5								1	0
	1.00	10	50	353.00	4	1	0	1	0	0	0	10	42	483.00	36	5								1	0
	1.25	10	50	309.00	4	1	0	1	0	0	0	10	42	438.00	36	5								1	0
	1.50	10	50	265.00	4	1	0	1	0	0	0	10	42	393.00	36	5								1	0
	1.75	10	50	221.00	4	1	0	1	0	0	0	10	42	348.00	36	5								1	0
	2.00	10	50	177.00	4	1	0	1	0	0	0	10	42	303.00	36	5								1	0
AKO30T12-4	0.25	7	44	201.75	3	1	1	1	1	0	0	7	40	251.75	39	3	1	1	1	1	0	0	0	0	0
	0.50	7	44	172.50	3	1	1	1	1	0	0	7	40	222.50	39	3	1	1	1	1	0	0	0	0	0
	0.75	7	44	143.00	4					1	0	7	40	193.25	39	3	1	1	1	1	0	0	0	0	0
	1.00	7	44	116.00	4					1	0	7	40	164.00	39	3	1	1	1	1	0	0	0	0	0
	1.25	7	44	89.00	4					1	0	7	40	134.75	39	3	1	1	1	1	0	0	0	0	0
	1.50	8	44	62.00	4					1	0	7	40	105.50	39	3	1	1	1	1	0	0	0	0	0
	1.75	7	44	45.50	2	1	1	1	1	0	0	7	40	76.25	39	3	1	1	1	1	0	0	0	0	0
	2.00	7	44	34.00	1	1	1	1	1	1	0	7	40	47.00	39	3	1	1	1	1	0	0	0	0	0
AKO30T12-5	0.25	7	44	200.50	3	1	1	1	1	0	0	7	40	250.50	38	3	1	1	1	1	0	0	0	0	0
	0.50	7	44	172.00	3	1	1	1	1	0	0	7	40	222.00	38	3	1	1	1	1	0	0	0	0	0
	0.75	7	44	143.50	3	1	1	1	1	0	0	7	40	193.50	38	3	1	1	1	1	0	0	0	0	0
	1.00	7	44	115.00	3	1	1	1	1	0	0	7	40	165.00	38	3	1	1	1	1	0	0	0	0	0
	1.25	7	44	86.50	3	1	1	1	1	0	0	7	40	136.50	38	3	1	1	1	1	0	0	0	0	0
	1.50	7	44	58.00	3	1	1	1	1	0	0	7	40	108.00	38	3	1	1	1	1	0	0	0	0	0
	1.75	7	44	29.50	3	1	1	1	1	0	0	7	40	79.50	38	3	1	1	1	1	0	0	0	0	0
	2.00	7	44	23.00	1	1	1	1	1	1	0	7	40	51.00	38	3	1	1	1	1	0	0	0	0	0
AKO30T12-6	0.25	12	50	304.00	4		1	0	0	0	0	12	40	343.00	32	6	1	1	1	1	0	0	0	0	0
	0.50	12	50	259.00	4		1	0	0	0	0	12	40	295.00	32	6	1	1	1	1	0	0	0	0	0
	0.75	12	50	214.00	4		1	0	0	0	0	12	40	247.00	32	6	1	1	1	1	0	0	0	0	0
	1.00	10	50	170.00	5	1	1	0	0	0	0	10	40	212.00	33	5	1	1	0	0	0	0	0	0	0
	1.25	10	50	128.75	5	1	1	0	0	0	0	10	40	170.75	33	5	1	1	0	0	0	0	0	0	0
	1.50	10	50	87.50	5	1	1	0	0	0	0	10	40	129.50	33	5	1	1	0	0	0	0	0	0	0
	1.75	10	50	46.25	5	1	1	0	0	0	0	9	40	88.25	33	5	1	1	0	0	0	0	0	0	
	2.00	10	50	46.00	1	1	1	1	1	1	0	9	40	47.00	33	5	1	1	0	0	0	0	0	0	
AKO30T12-7	0.25	8	44	270.50	5					1	0	8	43	320.50	30	5								1	0
	0.50	8	44	233.00	5					1	0	8	43	283.00	30	5								1	0
	0.75	8	44	195.50	5					1	0	8	43	245.50	30	5								1	0
	1.00	8	44	158.00	5					1	0	8	43	208.00	30	5								1	0
	1.25	8	44	120.50	5					1	0	8	43	170.50	30	5								1	0
	1.50	8	44	83.00	5					1	0	8	43	133.00	30	5								1	0
	1.75	8	44	45.50	5					1	0	8	43	95.50	30	5								1	0

**Table E.1** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET										
		$K_{UB}$	$CT_U$	$r$	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP	$K_{UB}$	$CT_U$	$r$	CT	K	$NX_1$	$NX_2$	$NX_3$	$NX_4$	$NX_5$	IMP
AKO30T12-9	0.25	6	50	139.00	4					1	0	7	42	272.00	26	4					1	0
	0.50	6	50	113.00	4					1	0	7	42	246.00	26	4					1	0
	0.75	7	50	87.00	4					1	0	7	42	220.00	26	4					1	0
	1.00	6	50	61.00	4					1	0	7	42	194.00	26	4					1	0
	1.25	7	50	17.00	3	1	0	1	0	0	0	7	42	168.00	26	4					1	0
	1.50	7	50	16.50	1	1	1	1	1	1	0	7	42	142.00	26	4					1	0
	1.75	7	50	10.25	1	1	1	1	1	1	0	7	42	116.00	26	4					1	0
	2.00	7	50	4.00	1	1	1	1	1	1	0	7	42	90.00	26	4					1	0

## APPENDIX F

### EVALUATION OF THE BOUNDS ON PROFIT

In Tables F.1 and F.2 the average and maximum percentage deviations from the optimal are reported for the heuristic solution or the lower bound (LB), the upper bound (UB) and the linear programming relaxation of PC (LP). In calculating the percentage deviation from the optimal, instances with zero optimal objective function value are excluded. The OLB, OUB and OLP columns depict the number of times the optimal solution is found by the respective solution procedure. In counting the number of times the optimal solution is found, the instances with zero optimal objective function value are taken into account. In Table F.3 all corresponding solutions are listed separately for the two data sets.

**Table F.1** Percentage deviations from the optimal for low demand variability data set (average of 10 problem instances)

PROBLEM	<i>s</i>	LB			UB			PC-LP		
		AVG	MAX	OLB	AVG	MAX	OUB	AVG	MAX	OLP
GUN8T8	0.25	0.00	0.00	10	2.11	7.89	3	73.17	143.19	0
	0.50	0.00	0.00	10	5.99	42.86	6	178.49	369.24	0
	0.75	0.00	0.00	10	0.56	5.00	8	380.80	1313.65	0
	1.00	0.50	5.00	9	1.75	17.50	9	1280.18	6053.46	0
	1.25	0.00	0.00	10	3.25	22.73	9	600.61	1461.22	0
	1.50	0.00	0.00	10	0.00	0.00	10	1382.78	4309.38	0
	1.75	0.00	0.00	10	6.67	33.33	9	1040.09	2811.43	0
	2.00	0.00	0.00	10	0.00	0.00	9	1861.68	5511.83	0



**Table F.1** Percentage deviations from the optimal for low demand variability data set (average of 10 problem instances) (Continued)

PROBLEM	S	LB			UB			PC-LP		
		AVG	MAX	OLB	AVG	MAX	OUB	AVG	MAX	OLP
AKO8T6	0.25	0.07	0.74	9	0.83	2.74	3	33.61	51.52	0
	0.50	0.21	2.11	9	2.88	14.15	3	68.79	116.24	0
	0.75	0.55	5.49	9	6.01	32.39	4	136.15	291.56	0
	1.00	0.54	5.41	9	7.73	56.82	6	275.92	708.29	0
	1.25	0.00	0.00	10	5.94	51.72	8	579.13	1864.37	0
	1.50	0.00	0.00	10	4.46	35.71	9	536.26	1281.76	0
	1.75	0.00	0.00	10	0.00	0.00	10	1306.71	4635.01	0
2.00	0.00	0.00	10	0.00	0.00	10	1173.48	3985.19	0	
AKO20T4-A	0.25	0.06	0.22	7	0.24	0.69	1	40.03	64.17	0
	0.50	0.07	0.37	8	0.58	1.58	1	61.60	113.74	0
	0.75	0.16	0.59	7	0.99	2.73	1	93.41	206.93	0
	1.00	0.49	2.21	6	1.82	4.32	1	150.79	446.58	0
	1.25	0.44	1.36	6	4.47	18.05	1	409.37	2443.64	0
	1.50	0.55	1.82	7	15.82	91.38	1	380.24	1616.90	0
	1.75	0.82	2.39	7	13.62	42.01	1	435.60	1460.36	0
2.00	1.07	3.13	7	103.37	500.00	2	4342.14	22493.31	0	
AKO20T4-B	0.25	0.16	0.87	8	1.47	2.70	0	114.82	186.42	0
	0.50	0.42	2.14	8	3.64	6.52	0	164.60	248.25	0
	0.75	1.08	5.96	8	8.15	25.00	0	246.98	409.46	0
	1.00	3.81	26.67	8	17.98	73.33	0	414.33	950.81	0
	1.25	2.55	22.96	9	27.39	88.24	0	676.23	1716.91	0
	1.50	8.88	71.05	9	67.14	168.52	0	1465.58	2545.92	0
	1.75	11.52	46.09	9	105.92	205.26	2	4559.37	11791.52	0
2.00	0.00	0.00	10	0.00	0.00	7	1245.51	1245.51	0	
AKO20T4-C	0.25	0.11	0.45	6	1.36	7.10	1	55.44	132.72	0
	0.50	0.26	1.18	6	1.66	7.14	2	86.70	225.69	0
	0.75	0.50	2.56	6	2.66	7.39	1	142.52	443.82	0
	1.00	11.00	100.00	5	9.59	42.31	1	320.28	1559.68	0
	1.25	5.67	45.45	6	42.95	287.12	2	615.04	3578.57	0
	1.50	1.19	4.55	7	8.37	22.73	1	240.88	537.01	0
	1.75	1.94	8.33	7	19.52	49.69	2	545.76	2019.20	0
2.00	4.94	22.22	7	56.44	154.63	4	626.82	1390.94	0	
LAM20T10	0.25	0.03	0.23	8	0.26	1.82	7	96.87	135.29	0
	0.50	0.14	0.64	7	0.19	1.18	7	113.41	149.29	0
	0.75	0.15	1.05	8	0.35	2.26	7	132.42	166.97	0
	1.00	0.20	1.55	8	0.90	4.35	6	155.71	201.94	0
	1.25	0.22	2.16	9	1.16	6.30	6	182.95	247.84	0
	1.50	0.29	2.94	9	1.59	8.74	6	216.81	310.72	0
	1.75	0.22	2.16	9	2.26	11.93	5	261.10	402.14	0
2.00	0.28	2.80	9	2.80	16.44	6	318.11	547.25	0	
LAM20T24	0.25	0.00	0.00	10	0.87	5.35	6	27.44	59.71	0
	0.50	0.45	4.48	9	0.15	1.49	9	63.42	118.88	0
	0.75	0.00	0.00	10	0.35	3.49	9	129.30	285.00	0
	1.00	0.00	0.00	10	15.77	146.61	8	530.76	3320.00	0
	1.25	0.00	0.00	10	0.00	0.00	8	286.37	711.48	0
	1.50	0.00	0.00	10	4.92	28.04	5	498.88	1099.29	0
	1.75	0.00	0.00	10	68.32	341.62	4	1011.04	3707.86	0
2.00	0.00	0.00	10	0.00	0.00	4	956.36	3285.00	0	
LAM30T10	0.25	0.05	0.49	9	0.36	1.06	2	179.13	232.10	0
	0.50	0.00	0.00	10	0.63	2.04	2	195.40	248.08	0
	0.75	0.07	0.65	9	0.67	1.69	2	212.14	272.03	0
	1.00	0.10	0.81	8	0.83	1.64	2	230.28	295.98	0

**Table F.1** Percentage deviations from the optimal for low demand variability data set (average of 10 problem instances) (Continued)

PROBLEM	$s$	LB			UB			PC-LP		
		AVG	MAX	OLB	AVG	MAX	OUB	AVG	MAX	OLP
LAM30T10	1.25	0.00	0.00	10	0.00	0.00	8	286.37	711.48	0
	1.50	0.00	0.00	10	4.92	28.04	5	498.88	1099.29	0
	1.75	0.00	0.00	10	68.32	341.62	4	1011.04	3707.86	0
	2.00	0.00	0.00	10	0.00	0.00	4	956.36	3285.00	0
LAM30T10	0.25	0.05	0.49	9	0.36	1.06	2	179.13	232.10	0
	0.50	0.00	0.00	10	0.63	2.04	2	195.40	248.08	0
	0.75	0.07	0.65	9	0.67	1.69	2	212.14	272.03	0
	1.00	0.10	0.81	8	0.83	1.64	2	230.28	295.98	0
	1.25	0.10	0.59	8	1.02	2.16	2	250.71	323.33	0
	1.50	0.15	1.04	8	1.28	2.72	2	274.21	354.85	0
	1.75	0.05	0.51	9	1.57	3.34	2	301.63	391.59	0
	2.00	0.06	0.56	9	1.93	4.04	2	334.21	457.85	0
LAM30T29	0.25	0.03	0.27	9	2.03	6.33	1	29.64	40.59	0
	0.50	0.11	1.09	9	4.29	21.74	2	76.81	139.42	0
	0.75	1.21	9.30	7	14.02	95.35	3	204.81	661.86	0
	1.00	10.26	100.00	8	29.42	160.00	4	668.78	2606.67	0
	1.25	7.57	45.83	7	6.98	25.00	4	944.25	3624.08	0
	1.50	0.98	5.88	9	4.90	29.41	5	704.10	1395.00	0
	1.75	16.67	100.00	9	74.40	446.40	5	2165.24	9066.00	0
	2.00	0.00	0.00	10	0.00	0.00	5	1153.00	3566.67	0
AKO30T12	0.25	0.30	0.74	3	0.30	0.55	2	136.51	199.61	0
	0.50	0.73	1.71	2	0.64	1.27	2	174.28	250.13	0
	0.75	1.63	3.43	1	1.05	2.25	1	227.03	350.12	0
	1.00	2.53	5.71	0	1.95	3.66	1	302.99	531.05	0
	1.25	9.41	54.67	1	3.37	7.02	1	434.64	958.06	0
	1.50	8.90	37.74	1	6.34	12.10	0	632.52	1394.26	0
	1.75	12.75	36.92	1	9.49	22.43	1	1006.13	2331.92	0
	2.00	6.61	33.33	5	11.96	50.00	5	1867.55	6473.26	0
OVERALL	$\forall s$	1.70	100.00	627	8.86	500.00	309	421.85	22493.31	0

**Table F.2** Percentage deviations from the optimal for high demand variability data set (average of 10 problem instances)

PROBLEM	$s$	LB			UB			PC-LP		
		AVG	MAX	OLB	AVG	MAX	OUB	AVG	MAX	OLP
GUN8T8	0.25	0.00	0.00	10	2.28	7.69	3	71.41	175.57	0
	0.50	2.43	20.69	8	3.33	16.67	4	156.23	309.88	0
	0.75	1.15	8.82	8	7.92	75.00	7	345.24	1030.94	0
	1.00	4.08	30.77	8	2.52	17.50	8	1404.50	7775.00	0
	1.25	0.00	0.00	10	3.79	26.56	9	651.80	1505.20	0
	1.50	0.00	0.00	10	0.00	0.00	10	1518.72	4309.38	0
	1.75	0.00	0.00	10	6.67	33.33	9	1061.44	2927.99	0
	2.00	0.00	0.00	10	0.00	0.00	9	1919.92	5773.56	0
AKO8T6	0.25	0.20	2.04	9	0.63	2.92	3	27.41	52.84	0
	0.50	0.51	5.13	9	1.61	7.25	3	57.80	113.93	0
	0.75	0.30	2.98	9	2.58	10.93	4	109.06	237.68	0
	1.00	1.53	15.25	9	2.59	8.47	5	204.18	639.53	0
	1.25	3.71	37.06	9	3.91	12.71	5	449.20	1864.37	0
	1.50	3.47	27.78	9	3.58	17.31	7	486.28	1046.53	0
	1.75	2.85	22.81	9	9.28	60.23	8	1522.68	6254.69	0
	2.00	0.00	0.00	10	30.56	183.33	9	1199.75	4008.33	0

**Table F.2** Percentage deviations from the optimal for high demand variability data set (average of 10 problem instances) (Continued)

PROBLEM	s	LB			UB			PC-LP		
		AVG	MAX	OLB	AVG	MAX	OUB	AVG	MAX	OLP
AKO20T4-A	0.25	0.06	0.25	7	0.64	1.84	0	39.19	63.98	0
	0.50	0.14	0.55	7	1.53	4.51	0	59.94	99.86	0
	0.75	0.91	4.43	5	2.32	8.74	0	88.19	156.12	0
	1.00	0.78	4.62	6	3.72	16.44	1	131.52	256.98	0
	1.25	1.20	7.67	6	8.10	34.88	1	213.46	490.34	0
	1.50	1.96	13.74	6	28.33	138.46	0	515.57	1651.04	0
	1.75	4.91	31.58	6	15.21	42.01	0	456.05	1460.36	0
	2.00	13.77	100.00	6	176.87	750.00	2	6973.64	31146.52	0
AKO20T4-B	0.25	0.07	0.75	9	0.97	2.63	1	93.84	136.65	0
	0.50	0.42	2.16	8	2.19	5.61	1	133.39	222.94	0
	0.75	1.22	7.34	8	5.23	25.00	1	197.45	409.46	0
	1.00	4.57	26.67	8	12.29	73.33	1	326.98	950.81	0
	1.25	4.72	38.27	8	11.12	20.00	1	430.42	699.39	0
	1.50	11.11	100.00	9	38.26	168.52	1	1055.22	3223.84	0
	1.75	0.00	0.00	10	99.71	205.26	2	2891.76	9149.06	0
	2.00	0.00	0.00	10	0.00	0.00	6	649.40	931.75	0
AKO20T4-C	0.25	0.18	1.20	6	2.03	12.22	1	58.54	143.83	0
	0.50	0.46	3.15	6	3.63	22.84	1	95.20	273.42	0
	0.75	0.93	6.90	6	8.57	58.10	1	175.32	700.63	0
	1.00	2.30	17.02	6	8.11	29.17	1	200.14	547.52	0
	1.25	14.45	100.00	5	58.84	356.25	2	745.75	4235.45	0
	1.50	7.57	44.62	6	16.80	65.38	1	311.92	670.19	0
	1.75	16.19	100.00	6	108.23	631.58	2	1247.11	5160.47	0
	2.00	4.90	22.22	8	62.60	154.63	4	668.33	1390.94	0
LAM20T10	0.25	0.03	0.22	8	0.40	2.13	5	104.04	135.92	0
	0.50	0.12	1.01	8	0.29	1.29	5	120.29	153.59	0
	0.75	0.03	0.29	9	0.41	1.89	6	138.62	194.78	0
	1.00	0.13	0.92	8	0.98	4.59	5	160.87	252.60	0
	1.25	0.00	0.00	10	1.45	8.43	5	187.40	329.77	0
	1.50	0.00	0.00	10	2.03	12.33	5	219.11	421.46	0
	1.75	0.22	2.16	9	2.95	18.10	4	259.85	553.17	0
	2.00	0.28	2.80	9	4.24	27.91	4	314.57	776.76	0
LAM20T24	0.25	0.37	3.70	9	1.28	5.35	5	28.14	59.71	0
	0.50	1.75	10.67	7	1.58	6.94	6	64.49	118.88	0
	0.75	1.11	11.11	9	4.18	26.39	6	126.40	285.00	0
	1.00	0.00	0.00	10	4.60	40.00	7	276.37	877.14	0
	1.25	0.00	0.00	10	2.52	12.82	6	315.61	815.51	0
	1.50	0.00	0.00	10	0.91	6.38	6	522.28	1099.29	0
	1.75	0.00	0.00	10	79.16	341.62	5	1692.72	5323.00	0
	2.00	0.00	0.00	10	0.00	0.00	5	468.08	1393.75	0
LAM30T10	0.25	0.05	0.49	9	0.33	1.06	2	219.04	341.83	0
	0.50	0.00	0.00	10	0.55	1.31	2	235.92	371.90	0
	0.75	0.07	0.65	9	0.65	1.69	2	253.47	406.49	0
	1.00	0.10	0.81	8	0.80	1.64	2	272.21	442.30	0
	1.25	0.10	0.59	8	0.98	2.16	2	292.90	481.18	0
	1.50	0.15	1.04	8	1.22	2.72	2	316.23	526.25	0
	1.75	0.05	0.51	9	1.50	3.34	2	342.75	579.09	0
	2.00	0.06	0.56	9	1.83	4.04	2	373.20	641.91	0
LAM30T29	0.25	0.30	2.69	8	2.18	6.33	0	27.90	40.59	0
	0.50	0.00	0.00	10	4.77	21.74	0	70.39	139.42	0
	0.75	1.19	9.30	7	14.06	95.35	1	178.16	661.86	0
	1.00	10.66	100.00	7	24.09	160.00	3	502.97	1930.00	0

**Table F.2** Percentage deviations from the optimal for high demand variability data set (average of 10 problem instances) (Continued)

PROBLEM	S	LB			UB			PC-LP		
		AVG	MAX	OLB	AVG	MAX	OUB	AVG	MAX	OLP
LAM30T29	1.25	3.61	17.65	7	5.12	18.18	4	567.09	2693.06	0
	1.50	0.00	0.00	10	0.00	0.00	8	563.42	1575.00	0
	1.75	0.00	0.00	10	5.48	27.78	6	834.84	6012.78	0
	2.00	0.00	0.00	10	0.00	0.00	6	1210.01	4016.67	0
AKO30T12	0.25	0.24	0.60	3	0.33	0.65	2	132.42	160.08	0
	0.50	0.58	1.33	2	0.72	1.51	2	159.90	192.68	0
	0.75	1.22	2.47	1	1.13	2.27	2	194.83	239.76	0
	1.00	1.53	3.51	1	1.65	3.17	2	240.50	307.98	0
	1.25	2.45	5.21	1	2.27	4.68	2	303.49	411.01	0
	1.50	3.36	7.69	1	3.22	6.95	2	397.54	584.58	0
	1.75	5.51	11.67	1	5.03	11.90	3	557.75	938.84	0
	2.00	10.38	22.86	0	8.25	17.14	3	909.44	2062.36	0
OVERALL	$\forall S$	1.95	100.00	610	10.37	750.00	281	436.17	31146.52	0

**Table F.3** Results of the heuristic solution procedure

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET													HIGH DEMAND VARIABILITY DATA SET									
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution		UB		PC-LP	
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\pi$	$\pi$	$\pi$			CT	K	$\pi$	CT	K	$\pi$	$\pi$		
GUN8T8-0	0.25	8	40	50.50	30	3	50.50	30	3	51.75	70.89	8	45	39.50	30	3	39.50	30	3	40.00	71.50			
	0.50	8	40	28.00	30	3	28.00	30	3	30.50	68.78	8	45	17.00	30	3	17.00	45	2	18.00	69.68			
	0.75	8	40	12.75	23	1	12.75	23	1	12.75	66.67	8	45	6.00	4	1	6.00	4	1	6.00	67.86			
	1.00	8	40	7.00	23	1	7.00	23	1	7.00	64.56	8	45	5.00	4	1	5.00	4	1	5.00	66.03			
	1.25	8	40	4.00	4	1	4.00	4	1	4.00	62.45	8	45	4.00	4	1	4.00	4	1	4.00	64.21			
	1.50	8	40	3.00	4	1	3.00	4	1	3.00	60.34	8	45	3.00	4	1	3.00	4	1	3.00	62.38			
	1.75	8	40	2.00	4	1	2.00	4	1	2.00	58.23	8	45	2.00	4	1	2.00	4	1	2.00	60.56			
	2.00	8	40	1.00	4	1	1.00	4	1	1.00	56.12	8	45	1.00	4	1	1.00	4	1	1.00	58.74			
GUN8T8-1	0.25	8	50	40.50	34	3	40.50	34	3	41.50	71.28	8	41	23.50	22	1	23.50	22	1	23.50	64.76			
	0.50	8	50	18.00	22	1	18.00	22	1	18.00	68.68	8	41	18.00	22	1	18.00	22	1	18.00	62.52			
	0.75	8	50	16.50	2	1	16.50	2	1	16.50	66.08	8	41	16.50	2	1	16.50	2	1	16.50	60.27			
	1.00	8	50	16.00	2	1	16.00	2	1	16.00	63.48	8	41	16.00	2	1	16.00	2	1	16.00	58.03			
	1.25	8	50	15.50	2	1	15.50	2	1	15.50	60.89	8	41	15.50	2	1	15.50	2	1	15.50	55.79			
	1.50	8	50	15.00	2	1	15.00	2	1	15.00	58.29	8	41	15.00	2	1	15.00	2	1	15.00	53.55			
	1.75	8	50	14.50	2	1	14.50	2	1	14.50	55.69	8	41	14.50	2	1	14.50	2	1	14.50	51.30			
	2.00	8	50	14.00	2	1	14.00	2	1	14.00	53.09	8	41	14.00	2	1	14.00	2	1	14.00	49.06			
GUN8T8-2	0.25	8	50	39.25	21	3	39.25	21	3	40.00	95.45	8	41	39.25	21	3	39.25	21	3	40.00	95.45			
	0.50	8	50	23.50	21	3	23.50	41	1	25.00	92.03	8	41	23.50	21	3	23.50	41	1	25.00	92.03			
	0.75	8	50	13.25	41	1	13.25	41	1	13.25	88.72	8	41	13.25	41	1	13.25	41	1	13.25	88.72			
	1.00	8	50	3.00	41	1	3.00	41	1	3.00	85.82	8	41	3.00	41	1	3.00	41	1	3.00	85.82			
	1.25	8	50	0.00	0	0	0.00	0	0	0.00	82.92	8	41	0.00	0	0	0.00	0	0	0.00	82.92			
	1.50	8	50	0.00	0	0	0.00	0	0	0.00	80.02	8	41	0.00	0	0	0.00	0	0	0.00	80.02			
	1.75	8	50	0.00	0	0	0.00	0	0	0.00	77.12	8	41	0.00	0	0	0.00	0	0	0.00	77.12			
	2.00	8	50	0.00	0	0	0.00	0	0	0.00	74.22	8	41	0.00	0	0	0.00	0	0	0.00	74.22			
GUN8T8-3	0.25	8	50	24.00	48	2	24.00	48	2	24.25	53.60	8	42	50.00	36	3	50.00	36	3	53.25	74.25			
	0.50	8	50	18.00	16	1	18.00	16	1	18.00	51.35	8	42	23.00	36	3	29.00	34	1	29.50	71.50			
	0.75	8	50	14.00	16	1	14.00	16	1	14.00	49.09	8	42	20.50	34	1	20.50	34	1	20.50	68.75			
	1.00	8	50	10.00	16	1	10.00	16	1	10.00	46.83	8	42	12.00	34	1	12.00	34	1	12.00	66.00			
	1.25	8	50	6.00	16	1	6.00	16	1	6.00	44.57	8	42	6.00	16	1	6.00	16	1	6.00	63.25			
	1.50	8	50	2.00	16	1	2.00	16	1	2.00	42.31	8	42	2.00	16	1	2.00	16	1	2.00	60.50			
	1.75	8	50	0.00	0	0	0.00	0	0	0.00	40.05	8	42	0.00	0	0	0.00	0	0	0.00	57.75			
	2.00	8	50	0.00	0	0	0.00	0	0	0.00	37.79	8	42	0.00	0	0	0.00	0	0	0.00	55.00			

**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET													
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP	
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\bar{\pi}$	$\pi$	
GUN8T8-4	0.25	8	44	29.50	26	3	29.50	39	2	30.25	48.89	8	45	29.50	26	3	29.50	26	3	30.25	48.89				
	0.50	8	44	20.50	25	1	20.50	25	1	20.50	47.04	8	45	20.50	25	1	20.50	25	1	20.50	47.04				
	0.75	8	44	14.25	25	1	14.25	25	1	14.25	45.18	8	45	14.25	25	1	14.25	25	1	14.25	45.18				
	1.00	8	44	11.00	7	1	11.00	7	1	11.00	43.33	8	45	11.00	7	1	11.00	7	1	11.00	43.33				
	1.25	8	44	9.25	7	1	9.25	7	1	9.25	41.47	8	45	9.25	7	1	9.25	7	1	9.25	41.47				
	1.50	8	44	7.50	7	1	7.50	7	1	7.50	39.62	8	45	7.50	7	1	7.50	7	1	7.50	39.62				
	1.75	8	44	5.75	7	1	5.75	7	1	5.75	37.77	8	45	5.75	7	1	5.75	7	1	5.75	37.77				
	2.00	8	44	4.00	7	1	4.00	7	1	4.00	35.91	8	45	4.00	7	1	4.00	7	1	4.00	35.91				
GUN8T8-5	0.25	8	44	36.00	16	2	36.00	32	1	36.00	65.49	8	49	36.00	16	2	36.00	16	2	36.00	65.49				
	0.50	8	44	30.00	16	1	30.00	16	1	30.00	64.34	8	49	30.00	16	1	30.00	16	1	30.00	64.34				
	0.75	8	44	26.00	16	1	26.00	16	1	26.00	63.20	8	49	26.00	16	1	26.00	16	1	26.00	63.20				
	1.00	8	44	22.00	16	1	22.00	16	1	22.00	62.05	8	49	22.00	16	1	22.00	16	1	22.00	62.05				
	1.25	8	44	18.00	16	1	18.00	16	1	18.00	60.91	8	49	18.00	16	1	18.00	16	1	18.00	60.91				
	1.50	8	44	14.00	16	1	14.00	16	1	14.00	59.76	8	49	14.00	16	1	14.00	16	1	14.00	59.76				
	1.75	8	44	10.00	16	1	10.00	16	1	10.00	58.62	8	49	10.00	16	1	10.00	16	1	10.00	58.62				
	2.00	8	44	6.00	16	1	6.00	9	1	6.00	57.47	8	49	6.00	16	1	6.00	9	1	6.00	57.47				
GUN8T8-6	0.25	8	50	46.00	50	2	46.00	50	2	47.25	72.05	8	40	76.50	34	3	76.50	34	3	78.25	99.03				
	0.50	8	50	29.00	28	2	29.00	28	2	29.50	69.60	8	40	53.00	22	4	55.00	28	3	55.50	96.06				
	0.75	8	50	15.00	28	2	15.00	28	2	15.75	67.15	8	40	31.00	22	4	34.00	28	3	34.75	93.09				
	1.00	8	50	4.00	28	1	4.00	28	1	4.00	64.70	8	40	9.00	22	4	13.00	28	3	14.00	90.13				
	1.25	8	50	0.00	0	0	0.00	0	0	0.00	62.25	8	40	0.00	0	0	0.00	0	0	0.00	87.16				
	1.50	8	50	0.00	0	0	0.00	0	0	0.00	59.90	8	40	0.00	0	0	0.00	0	0	0.00	84.30				
	1.75	8	50	0.00	0	0	0.00	0	0	0.00	57.62	8	40	0.00	0	0	0.00	0	0	0.00	81.49				
	2.00	8	50	0.00	0	0	0.00	0	0	0.00	55.33	8	40	0.00	0	0	0.00	0	0	0.00	78.69				
GUN8T8-7	0.25	8	50	81.50	43	2	81.50	43	2	81.50	100.50	8	40	81.00	22	4	81.00	22	4	81.50	100.50				
	0.50	8	50	60.00	43	2	60.00	43	2	60.00	98.01	8	40	59.00	22	4	59.00	22	4	60.00	98.01				
	0.75	8	50	38.50	43	2	38.50	43	2	38.75	95.51	8	40	37.00	22	4	38.00	24	3	38.75	95.51				
	1.00	8	50	19.00	43	1	20.00	24	3	23.50	93.02	8	40	18.00	22	2	20.00	24	3	23.50	93.02				
	1.25	8	50	8.25	43	1	8.25	43	1	10.13	90.52	8	40	8.00	24	1	8.00	24	1	10.13	90.52				
	1.50	8	50	2.00	24	1	2.00	24	1	2.00	88.19	8	40	2.00	24	1	2.00	24	1	2.00	88.19				
	1.75	8	50	0.00	0	0	0.00	0	0	0.00	86.16	8	40	0.00	0	0	0.00	0	0	0.00	86.16				
	2.00	8	50	0.00	0	0	0.00	0	0	0.00	84.13	8	40	0.00	0	0	0.00	0	0	0.00	84.13				
GUN8T8-8	0.25	8	50	38.00	32	3	38.00	32	3	41.00	67.77	8	42	61.75	39	3	61.75	39	3	66.50	87.94				
	0.50	8	50	14.00	32	3	14.00	32	3	20.00	65.69	8	42	36.00	32	3	36.00	32	3	42.00	84.88				
	0.75	8	50	4.50	30	1	4.50	30	1	4.50	63.61	8	42	12.00	32	3	12.00	32	3	21.00	81.81				
	1.00	8	50	1.00	11	1	1.00	11	1	1.00	61.53	8	42	1.00	11	1	1.00	11	1	1.00	78.75				
	1.25	8	50	0.00	0	0	0.00	0	0	0.00	59.46	8	42	0.00	0	0	0.00	0	0	0.00	75.69				
	1.50	8	50	0.00	0	0	0.00	0	0	0.00	57.38	8	42	0.00	0	0	0.00	0	0	0.00	72.63				
	1.75	8	50	0.00	0	0	0.00	0	0	0.00	55.30	8	42	0.00	0	0	0.00	0	0	0.00	69.56				
	2.00	8	50	0.00	0	0	0.00	0	0	0.00	53.22	8	42	0.00	0	0	0.00	0	0	0.00	66.50				
GUN8T8-9	0.25	8	80	48.75	29	1	48.75	29	1	48.75	69.63	8	43	56.00	40	1	56.00	40	1	56.00	68.43				
	0.50	8	80	41.50	29	1	41.50	29	1	41.50	68.19	8	43	46.00	40	1	46.00	40	1	46.00	67.37				
	0.75	8	80	34.25	29	1	34.25	29	1	34.25	66.76	8	43	36.00	40	1	36.00	40	1	36.00	66.31				
	1.00	8	80	27.00	29	1	27.00	29	1	27.00	65.32	8	43	27.00	29	1	27.00	29	1	27.00	65.25				
	1.25	8	80	19.75	29	1	19.75	29	1	19.75	63.88	8	43	19.75	29	1	19.75	29	1	19.75	64.20				
	1.50	8	80	12.50	29	1	12.50	29	1	12.50	62.44	8	43	12.50	29	1	12.50	29	1	12.50	63.14				
	1.75	8	80	5.25	29	1	5.25	29	1	7.00	61.00	8	43	5.25	29	1	5.25	29	1	7.00	62.08				
	2.00	8	80	0.00	0	0	0.00	0	0	3.00	59.57	8	43	0.00	0	0	0.00	0	0	3.00	61.02				
AKO8T6-0	0.25	8	50	74.50	47	2	74.50	47	2	75.00	112.88	8	93	82.00	92	1	82.00	80	1	82.00	107.59				
	0.50	8	50	51.00	47	2	51.00	47	2	52.00	110.28	8	93	62.00	80	1	62.00	80	1	62.00	105.33				
	0.75	8	50	27.50	47	2	27.50	47	2	29.00	107.68	8	93	43.00	60	1	43.00	60	1	43.00	103.06				
	1.00	8	50	13.00	27	1	13.00	27	1	13.00	105.08	8	93	28.00	60	1	28.00	60	1	28.00	100.79				
	1.25	8	50	6.25	27	1	6.25	27	1	6.25	102.48	8	93	13.00	60	1	13.00	60	1	13.00	98.52				
	1.50	8	50	0.00	0	0	0.00	0	0	0.00	99.87	8	93	0.00	0	0	0.00	0	0	0.00	96.26				
	1.75	8	50	0.00	0	0	0.00	0	0	0.00	97.27	8	93	0.00	0	0	0.00	0	0	0.00	93.99				
	2.00	8	50	0.00	0	0	0.00	0	0	0.00	94.67	8	93	0.00	0	0	0.00	0	0	0.00	91.72				

**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET																			
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP							
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\bar{\pi}$	$\pi$							
AKO8T6-1	0.25	8	50	100.25	41	3	101.00	40	3	102.25	149.52	8	83	101.50	59	2	101.50	59	2	102.25	149.52	8	83	101.50	59	2	101.50	59	2	102.25	149.52
	0.50	8	50	69.50	41	3	71.00	40	3	73.50	146.52	8	83	72.00	59	2	72.00	59	2	73.50	146.52	8	83	72.00	59	2	72.00	59	2	73.50	146.52
	0.75	8	50	38.75	41	3	41.00	40	3	44.75	143.51	8	83	42.50	59	2	42.50	59	2	44.75	143.51	8	83	42.50	59	2	42.50	59	2	44.75	143.51
	1.00	8	50	19.00	48	1	19.00	48	1	19.00	140.51	8	83	19.00	48	1	19.00	48	1	19.00	140.51	8	83	19.00	48	1	19.00	48	1	19.00	140.51
	1.25	8	50	7.00	48	1	7.00	48	1	7.00	137.51	8	83	7.00	48	1	7.00	48	1	7.00	137.51	8	83	7.00	48	1	7.00	48	1	7.00	137.51
	1.50	8	50	0.00	0	0	0.00	0	0	0.00	134.50	8	83	0.00	0	0	0.00	0	0	0.00	134.50	8	83	0.00	0	0	0.00	0	0	0.00	134.50
	1.75	8	50	0.00	0	0	0.00	0	0	0.00	131.50	8	83	0.00	0	0	0.00	0	0	0.00	131.50	8	83	0.00	0	0	0.00	0	0	0.00	131.50
	2.00	8	50	0.00	0	0	0.00	0	0	0.00	128.49	8	83	0.00	0	0	0.00	0	0	0.00	128.49	8	83	0.00	0	0	0.00	0	0	0.00	128.49
AKO8T6-2	0.25	8	66	73.00	40	2	73.00	40	2	75.00	106.84	8	71	123.50	47	2	123.50	47	2	124.25	144.16	8	71	123.50	47	2	123.50	47	2	124.25	144.16
	0.50	8	66	53.00	40	2	53.00	35	2	60.50	104.82	8	71	100.00	47	2	100.00	47	2	101.50	141.31	8	71	100.00	47	2	100.00	47	2	101.50	141.31
	0.75	8	66	35.50	35	2	35.50	35	2	47.00	102.79	8	71	76.50	47	2	76.50	47	2	78.75	138.47	8	71	76.50	47	2	76.50	47	2	78.75	138.47
	1.00	8	66	22.00	30	1	22.00	30	1	34.50	100.77	8	71	53.00	47	2	53.00	47	2	56.00	135.63	8	71	53.00	47	2	53.00	47	2	56.00	135.63
	1.25	8	66	14.50	30	1	14.50	30	1	22.00	98.75	8	71	29.50	47	2	29.50	70	1	33.25	132.78	8	71	29.50	47	2	29.50	70	1	33.25	132.78
	1.50	8	66	7.00	30	1	7.00	30	1	9.50	96.72	8	71	12.00	35	2	12.00	70	1	12.00	129.94	8	71	12.00	35	2	12.00	70	1	12.00	129.94
	1.75	8	66	2.00	12	1	2.00	12	1	2.00	94.70	8	71	2.00	12	1	2.00	12	1	2.00	127.09	8	71	2.00	12	1	2.00	12	1	2.00	127.09
	2.00	8	66	0.00	0	0	0.00	0	0	0.00	92.68	8	71	0.00	0	0	0.00	0	0	0.00	124.25	8	71	0.00	0	0	0.00	0	0	0.00	124.25
AKO8T6-3	0.25	8	44	87.00	36	2	87.00	36	2	87.50	102.81	8	42	60.00	36	2	60.00	36	2	60.50	91.70	8	42	60.00	36	2	60.00	36	2	60.50	91.70
	0.50	8	44	69.00	36	2	69.00	36	2	70.00	100.63	8	42	42.00	36	2	42.00	36	2	43.00	89.85	8	42	42.00	36	2	42.00	36	2	43.00	89.85
	0.75	8	44	51.00	36	2	51.00	36	2	52.50	98.44	8	42	29.00	40	1	29.00	40	1	29.00	88.00	8	42	29.00	40	1	29.00	40	1	29.00	88.00
	1.00	8	44	33.00	36	2	33.00	36	2	35.00	96.25	8	42	22.00	25	1	22.00	25	1	22.00	86.14	8	42	22.00	25	1	22.00	25	1	22.00	86.14
	1.25	8	44	16.25	7	1	16.25	7	1	17.50	94.06	8	42	16.25	7	1	16.25	7	1	16.25	84.29	8	42	16.25	7	1	16.25	7	1	16.25	84.29
	1.50	8	44	14.50	7	1	14.50	7	1	14.50	91.88	8	42	14.50	7	1	14.50	7	1	14.50	82.44	8	42	14.50	7	1	14.50	7	1	14.50	82.44
	1.75	8	44	12.75	7	1	12.75	7	1	12.75	89.69	8	42	12.75	7	1	12.75	7	1	12.75	80.58	8	42	12.75	7	1	12.75	7	1	12.75	80.58
	2.00	8	44	11.00	7	1	11.00	7	1	11.00	87.50	8	42	11.00	7	1	11.00	7	1	11.00	78.73	8	42	11.00	7	1	11.00	7	1	11.00	78.73
AKO8T6-4	0.25	8	44	57.00	16	2	57.00	32	1	57.00	81.19	8	60	65.50	34	1	65.50	34	1	65.50	91.12	8	60	65.50	34	1	65.50	34	1	65.50	91.12
	0.50	8	44	49.00	16	2	49.00	32	1	49.00	80.14	8	60	57.00	34	1	57.00	34	1	57.00	90.01	8	60	57.00	34	1	57.00	34	1	57.00	90.01
	0.75	8	44	41.00	16	2	41.00	32	1	41.00	79.10	8	60	48.50	34	1	48.50	34	1	48.50	88.91	8	60	48.50	34	1	48.50	34	1	48.50	88.91
	1.00	8	44	35.00	16	1	35.00	16	1	35.00	78.05	8	60	40.00	34	1	40.00	34	1	40.00	87.80	8	60	40.00	34	1	40.00	34	1	40.00	87.80
	1.25	8	44	31.00	16	1	31.00	16	1	31.00	77.00	8	60	31.50	34	1	31.50	34	1	31.50	86.70	8	60	31.50	34	1	31.50	34	1	31.50	86.70
	1.50	8	44	27.00	16	1	27.00	16	1	27.00	75.95	8	60	27.00	16	1	27.00	16	1	27.00	85.59	8	60	27.00	16	1	27.00	16	1	27.00	85.59
	1.75	8	44	23.00	16	1	23.00	16	1	23.00	74.90	8	60	23.00	16	1	23.00	16	1	23.00	84.48	8	60	23.00	16	1	23.00	16	1	23.00	84.48
	2.00	8	44	19.00	16	1	19.00	16	1	19.00	73.86	8	60	19.00	16	1	19.00	16	1	19.00	83.38	8	60	19.00	16	1	19.00	16	1	19.00	83.38
AKO8T6-5	0.25	8	80	98.25	79	1	98.25	79	1	98.25	115.53	8	54	96.00	44	2	98.00	40	2	98.25	115.53	8	54	96.00	44	2	98.00	40	2	98.25	115.53
	0.50	8	80	78.50	79	1	78.50	79	1	78.50	113.06	8	54	74.00	44	2	78.00	40	2	78.50	113.06	8	54	74.00	44	2	78.00	40	2	78.50	113.06
	0.75	8	80	59.75	67	1	59.75	67	1	59.75	110.59	8	54	58.25	23	3	58.25	23	3	59.75	110.59	8	54	58.25	23	3	58.25	23	3	59.75	110.59
	1.00	8	80	43.00	67	1	43.00	67	1	43.00	108.13	8	54	41.00	23	3	41.00	23	3	43.00	108.13	8	54	41.00	23	3	41.00	23	3	43.00	108.13
	1.25	8	80	26.25	67	1	26.25	67	1	26.25	105.66	8	54	23.75	23	3	23.75	23	3	26.25	105.66	8	54	23.75	23	3	23.75	23	3	26.25	105.66
	1.50	8	80	9.50	67	1	9.50	67	1	9.50	103.19	8	54	6.50	23	3	9.00	28	1	9.50	103.19	8	54	6.50	23	3	9.00	28	1	9.50	103.19
	1.75	8	80	3.50	22	1	3.50	22	1	3.50	100.72	8	54	3.50	22	1	3.50	22	1	3.50	100.72	8	54	3.50	22	1	3.50	22	1	3.50	100.72
	2.00	8	80	0.00	0	0	0.00	0	0	0.00	98.25	8	54	0.00	0	0	0.00	0	0	0.00	98.25	8	54	0.00	0	0	0.00	0	0	0.00	98.25
AKO8T6-6	0.25	8	44	103.00	32	2	103.00	32	2	103.50	122.65	8	62	125.50	39	2	125.50	39	2	125.75	142.59	8	62	125.50	39	2	125.50	39	2	125.75	142.59
	0.50	8	44	87.00	32	2	87.00	32	2	88.00	120.47	8	62	106.00	39	2	106.00	39	2	106.50	140.19	8	62	106.00	39	2	106.00	39	2	106.50	140.19
	0.75	8	44	71.00	32	2	71.00	32	2	72.50	118.29	8	62	86.50	39	2	86.50	39	2	87.25	137.78	8	62	86.50	39	2	86.50	39	2	87.25	137.78
	1.00	8	44	55.00	32	2	55.00	32	2	57.00	116.11	8	62	67.00	39	2	67.00	39	2	68.00	135.38	8	62	67.00	39	2	67.00	39	2	68.00	135.38
	1.25	8	44	41.50	34	1	41.50	34	1	41.50	113.93	8	62	47.50	35	2	47.50	35	2	48.75	132.97	8	62	47.50	35	2	47.50	35	2	48.75	132.97
	1.50	8	44	33.00	34	1	33.00	34	1	33.00	111.75	8	62	33.00	34	1	33.00	34	1	33.00	130.56	8	62	33.00	34	1	33.00	34	1	33.00	130.56
	1.75	8	44	24.50	34	1	24.50	34	1	24.50																					

**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET															
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP			
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\bar{\pi}$	$\pi$			
AKO8T6-8	0.25	8	44	69.75	33	1	69.75	33	1	69.75	33	1	69.75	86.78	8	40	95.50	23	2	95.50	23	2	95.50	23	2	95.50	108.27
	0.50	8	44	61.50	33	1	61.50	33	1	61.50	33	1	61.50	85.84	8	40	84.00	23	2	84.00	23	2	84.00	23	2	84.00	106.91
	0.75	8	44	53.25	33	1	53.25	33	1	53.25	33	1	53.25	84.90	8	40	72.50	23	2	72.50	23	2	72.50	23	2	72.50	105.56
	1.00	8	44	45.00	33	1	45.00	33	1	45.00	33	1	45.00	83.96	8	40	61.00	23	2	61.00	23	2	61.00	23	2	61.00	104.20
	1.25	8	44	36.75	33	1	36.75	33	1	36.75	33	1	36.75	83.02	8	40	49.50	23	2	49.50	23	2	49.50	23	2	49.50	102.85
	1.50	8	44	28.50	33	1	28.50	33	1	28.50	33	1	28.50	82.08	8	40	41.50	29	1	41.50	29	1	41.50	29	1	41.50	101.49
	1.75	8	44	20.25	33	1	20.25	33	1	20.25	33	1	20.25	81.14	8	40	34.25	29	1	34.25	29	1	34.25	29	1	34.25	100.14
	2.00	8	44	12.00	33	1	12.00	33	1	12.00	33	1	12.00	80.20	8	40	27.00	29	1	27.00	29	1	27.00	29	1	27.00	98.78
AKO8T6-9	0.25	8	44	71.00	12	5	71.00	12	5	71.75	92.36	8	44	101.00	12	5	101.00	12	5	101.00	12	5	101.75	125.92			
	0.50	8	44	56.00	12	5	56.00	12	5	57.50	90.94	8	44	86.00	12	5	86.00	12	5	86.00	12	5	87.50	123.73			
	0.75	8	44	45.25	33	1	45.25	33	1	45.25	89.53	8	44	71.00	12	5	71.00	12	5	71.00	12	5	73.25	121.54			
	1.00	8	44	37.00	33	1	37.00	33	1	37.00	88.11	8	44	56.00	12	5	56.00	12	5	56.00	12	5	59.00	119.35			
	1.25	8	44	28.75	33	1	28.75	33	1	28.75	86.69	8	44	41.00	12	5	41.00	12	5	41.00	12	5	44.75	117.16			
	1.50	8	44	20.50	33	1	20.50	33	1	20.50	85.28	8	44	26.00	12	5	26.00	12	5	26.00	12	5	30.50	114.97			
	1.75	8	44	13.25	21	1	13.25	21	1	13.25	83.86	8	44	11.00	12	5	14.25	33	1	16.25	33	1	16.25	112.77			
	2.00	8	44	8.00	21	1	8.00	21	1	8.00	82.44	8	44	8.00	21	1	8.00	21	1	8.00	21	1	8.00	110.58			
AKO20-A-0	0.25	20	57	260.50	41	2	260.50	41	2	260.75	355.86	20	47	260.50	41	2	260.50	41	2	260.75	355.86						
	0.50	20	57	240.00	41	2	240.00	41	2	240.50	355.43	20	47	240.00	41	2	240.00	41	2	240.50	355.43						
	0.75	20	57	219.50	41	2	220.50	37	2	222.00	355.00	20	47	219.50	41	2	220.50	37	2	222.00	355.00						
	1.00	20	57	200.00	38	2	202.00	37	2	204.50	354.58	20	47	200.00	38	2	202.00	37	2	204.50	354.58						
	1.25	20	57	181.00	38	2	183.50	37	2	187.00	354.16	20	47	181.00	38	2	183.50	37	2	187.00	354.16						
	1.50	20	57	162.00	38	2	165.00	37	2	169.50	353.75	20	47	162.00	38	2	165.00	37	2	169.50	353.75						
	1.75	20	57	143.00	38	2	146.50	37	2	152.00	353.34	20	47	143.00	38	2	146.50	37	2	152.00	353.34						
	2.00	20	57	124.00	38	2	128.00	37	2	134.50	352.93	20	47	124.00	38	2	128.00	37	2	134.50	352.93						
AKO20-A-1	0.25	20	66	298.00	46	2	298.50	45	2	298.75	389.27	20	44	297.00	24	4	297.75	31	3	298.75	389.27						
	0.50	20	66	275.00	46	2	276.00	45	2	276.50	388.71	20	44	273.00	24	4	274.50	31	3	276.50	388.71						
	0.75	20	66	252.00	46	2	253.50	45	2	254.25	388.16	20	44	249.00	24	4	251.25	31	3	254.25	388.16						
	1.00	20	66	229.00	46	2	231.00	45	2	232.00	387.60	20	44	225.00	24	4	228.00	31	3	232.00	387.60						
	1.25	20	66	206.00	46	2	208.50	45	2	209.75	387.05	20	44	201.00	24	4	204.75	31	3	209.75	387.05						
	1.50	20	66	183.00	46	2	186.00	45	2	187.50	386.50	20	44	177.00	24	4	181.50	31	3	187.50	386.50						
	1.75	20	66	160.00	46	2	163.50	45	2	165.25	385.95	20	44	153.00	24	4	158.25	31	3	165.25	385.95						
	2.00	20	66	137.00	46	2	141.00	45	2	143.00	385.40	20	44	129.00	24	4	135.00	31	3	143.00	385.40						
AKO20-A-2	0.25	20	100	94.00	88	1	94.00	88	1	94.00	154.32	20	43	163.00	40	3	163.00	40	3	166.00	231.06						
	0.50	20	100	72.00	88	1	72.00	88	1	72.00	153.89	20	43	133.00	40	3	133.00	40	3	139.00	230.37						
	0.75	20	100	50.00	88	1	50.00	88	1	50.00	153.47	20	43	103.00	40	3	103.00	40	3	112.00	229.68						
	1.00	20	100	28.00	88	1	28.00	88	1	28.00	153.04	20	43	73.00	40	3	73.00	40	3	85.00	229.00						
	1.25	20	100	6.00	88	1	6.00	88	1	6.00	152.62	20	43	43.00	40	3	43.00	40	3	58.00	228.32						
	1.50	20	100	0.00	0	0	0.00	0	0	0.00	152.20	20	43	13.00	40	3	13.00	40	3	31.00	227.63						
	1.75	20	100	0.00	0	0	0.00	0	0	0.00	151.77	20	43	0.00	0	0	0.00	0	0	4.00	226.95						
	2.00	20	100	0.00	0	0	0.00	0	0	0.00	151.35	20	43	0.00	0	0	0.00	0	0	0.00	226.27						
AKO20-A-3	0.25	20	66	257.50	51	2	257.50	51	2	257.75	334.13	20	72	172.00	32	3	172.00	48	2	175.00	246.21						
	0.50	20	66	232.00	51	2	232.00	51	2	232.50	333.90	20	72	148.00	32	3	148.00	48	2	154.00	245.99						
	0.75	20	66	206.50	51	2	206.50	51	2	207.25	333.67	20	72	124.00	32	3	129.75	67	1	133.00	245.78						
	1.00	20	66	177.00	30	3	181.00	51	2	183.00	333.45	20	72	113.00	67	1	113.00	67	1	113.00	245.58						
	1.25	20	66	154.50	30	3	155.50	51	2	162.00	333.23	20	72	96.25	67	1	96.25	67	1	96.25	245.39						
	1.50	20	66	132.00	30	3	132.00	45	2	141.00	333.01	20	72	79.50	67	1	79.50	67	1	80.00	245.21						
	1.75	20	66	109.50	30	3	109.50	30	3	120.00	332.78	20	72	62.75	67	1	62.75	67	1	67.25	245.03						
	2.00	20	66	87.00	30	3	87.00	45	2	99.00	332.56	20	72	46.00	67	1	46.00	67	1	54.50	244.85						
AKO20-A-4	0.25	20	50	146.50	45	2	146.50	45	2	147.25	228.17	20	47	146.50	45	2	146.50	45	2	147.25	228.17						
	0.50	20	50	124.00	45	2	124.00	45	2	125.50	227.85	20	47	124.00	45	2	124.00	45	2	125.50	227.85						
	0.75	20	50	101.50	45	2	101.50	45	2	103.75	227.53	20	47	101.50	45	2	101.50	45	2	103.75	227.53						
	1.00	20	50	79.00	45	2	79.00	45	2	82.00	227.21	20	47	79.00	45	2	79.00	45	2	82.00	227.21						
	1.25	20	50	56.50	45	2	56.50	45	2	60.25	226.89	20	47	56.50	45	2	56.50	45	2	60.25	226.89						
	1.50	20	50	34.00	45	2	34.00	45	2	38.50	226.57	20	47	34.00	45	2	34.00	45	2	38.50	226.57						
	1.75	20	50	14.50	27	2	14.50	27	2	19.63	226.25	20	47	14.50	27	2	14.50	27	2	19.63	226.25						
	2.00	20	50	1.00	27	2	1.00	27	2	6.00	225.93	20	47	1.00	27	2	1.00	27	2	6.00	225.93						

**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET																			
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP							
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\pi$	CT	K	$\pi$	CT	K			
AKO20-A-5	0.25	20	66	228.50	46	3	229.00	34	4	229.25	316.55	20	47	228.50	46	3	229.00	34	4	229.25	316.55	20	47	228.50	46	3	229.00	34	4	229.25	316.55
	0.50	20	66	195.00	64	2	195.00	64	2	196.25	315.96	20	47	194.00	46	3	195.00	34	4	196.25	315.96	20	47	194.00	46	3	195.00	34	4	196.25	315.96
	0.75	20	66	163.00	64	2	163.00	64	2	163.75	315.37	20	47	157.75	45	3	162.25	43	3	163.75	315.37	20	47	157.75	45	3	162.25	43	3	163.75	315.37
	1.00	20	66	131.00	64	2	131.00	64	2	132.00	314.78	20	47	124.00	45	3	130.00	43	3	132.00	314.78	20	47	124.00	45	3	130.00	43	3	132.00	314.78
	1.25	20	66	99.00	64	2	99.00	64	2	100.25	314.20	20	47	90.25	45	3	97.75	43	3	100.25	314.20	20	47	90.25	45	3	97.75	43	3	100.25	314.20
	1.50	20	66	67.00	64	2	67.00	64	2	68.50	313.62	20	47	56.50	45	3	65.50	43	3	68.50	313.62	20	47	56.50	45	3	65.50	43	3	68.50	313.62
	1.75	20	66	35.00	64	2	35.00	64	2	36.75	313.04	20	47	22.75	45	3	33.25	43	3	36.75	313.04	20	47	22.75	45	3	33.25	43	3	36.75	313.04
	2.00	20	66	3.00	64	2	3.00	64	2	8.50	312.47	20	47	0.00	0	0	1.00	43	3	8.50	312.47	20	47	0.00	0	0	1.00	43	3	8.50	312.47
AKO20-A-6	0.25	20	50	438.25	41	3	439.00	40	3	439.50	510.87	20	59	438.25	41	3	439.00	40	3	439.50	510.87	20	59	438.25	41	3	439.00	40	3	439.50	510.87
	0.50	20	50	407.50	41	3	409.00	40	3	410.00	510.37	20	59	407.50	41	3	409.00	40	3	410.00	510.37	20	59	407.50	41	3	409.00	40	3	410.00	510.37
	0.75	20	50	376.75	41	3	379.00	40	3	380.50	509.87	20	59	376.75	41	3	379.00	40	3	380.50	509.87	20	59	376.75	41	3	379.00	40	3	380.50	509.87
	1.00	20	50	346.00	41	3	349.00	40	3	351.00	509.36	20	59	346.00	41	3	349.00	40	3	351.00	509.36	20	59	346.00	41	3	349.00	40	3	351.00	509.36
	1.25	20	50	315.25	41	3	319.00	40	3	321.50	508.87	20	59	315.25	41	3	319.00	40	3	321.50	508.87	20	59	315.25	41	3	319.00	40	3	321.50	508.87
	1.50	20	50	284.50	41	3	289.00	40	3	292.00	508.37	20	59	284.50	41	3	289.00	40	3	292.00	508.37	20	59	284.50	41	3	289.00	40	3	292.00	508.37
	1.75	20	50	253.75	41	3	259.00	40	3	262.50	507.87	20	59	253.75	41	3	259.00	40	3	262.50	507.87	20	59	253.75	41	3	259.00	40	3	262.50	507.87
	2.00	20	50	223.00	41	3	229.00	40	3	233.00	507.37	20	59	223.00	41	3	229.00	40	3	233.00	507.37	20	59	223.00	41	3	229.00	40	3	233.00	507.37
AKO20-A-7	0.25	20	40	153.25	37	3	153.25	37	3	153.50	251.31	20	68	153.25	37	3	153.25	37	3	153.50	251.31	20	68	153.25	37	3	153.25	37	3	153.50	251.31
	0.50	20	40	125.50	37	3	125.50	37	3	126.00	250.83	20	68	125.50	37	3	125.50	37	3	126.00	250.83	20	68	125.50	37	3	125.50	37	3	126.00	250.83
	0.75	20	40	97.75	37	3	97.75	37	3	98.50	250.35	20	68	97.75	37	3	97.75	37	3	98.50	250.35	20	68	97.75	37	3	97.75	37	3	98.50	250.35
	1.00	20	40	70.00	37	3	70.00	37	3	72.00	249.89	20	68	70.00	37	3	70.00	37	3	72.00	249.89	20	68	70.00	37	3	70.00	37	3	72.00	249.89
	1.25	20	40	42.25	37	3	42.25	37	3	49.88	249.42	20	68	42.25	37	3	42.25	37	3	49.88	249.42	20	68	42.25	37	3	42.25	37	3	49.88	249.42
	1.50	20	40	14.50	37	3	14.50	37	3	27.75	248.95	20	68	14.50	37	3	14.50	37	3	27.75	248.95	20	68	14.50	37	3	14.50	37	3	27.75	248.95
	1.75	20	40	0.00	0	0	0.00	0	0	5.63	248.48	20	68	0.00	0	0	0.00	0	0	5.63	248.48	20	68	0.00	0	0	0.00	0	0	5.63	248.48
	2.00	20	40	0.00	0	0	0.00	0	0	0.00	248.02	20	68	0.00	0	0	0.00	0	0	0.00	248.02	20	68	0.00	0	0	0.00	0	0	0.00	248.02
AKO20-A-8	0.25	20	57	290.75	43	3	290.75	43	3	292.25	380.18	20	42	290.00	22	6	290.00	22	6	292.25	380.18	20	42	290.00	22	6	290.00	22	6	292.25	380.18
	0.50	20	57	258.50	43	3	258.50	43	3	261.50	379.56	20	42	257.00	22	6	257.00	33	4	261.50	379.56	20	42	257.00	22	6	257.00	33	4	261.50	379.56
	0.75	20	57	226.25	43	3	226.25	43	3	230.75	378.94	20	42	224.00	22	6	224.00	33	4	230.75	378.94	20	42	224.00	22	6	224.00	33	4	230.75	378.94
	1.00	20	57	194.00	43	3	194.00	43	3	200.00	378.33	20	42	191.00	22	6	191.00	33	4	200.00	378.33	20	42	191.00	22	6	191.00	33	4	200.00	378.33
	1.25	20	57	161.75	43	3	161.75	43	3	169.25	377.71	20	42	158.00	22	6	158.00	33	4	169.25	377.71	20	42	158.00	22	6	158.00	33	4	169.25	377.71
	1.50	20	57	129.50	43	3	129.50	43	3	138.50	377.10	20	42	125.00	22	6	125.00	22	6	138.50	377.10	20	42	125.00	22	6	125.00	22	6	138.50	377.10
	1.75	20	57	97.25	43	3	97.25	43	3	107.75	376.49	20	42	92.00	22	6	92.00	33	4	107.75	376.49	20	42	92.00	22	6	92.00	33	4	107.75	376.49
	2.00	20	57	65.00	43	3	65.00	43	3	77.50	375.88	20	42	59.00	22	6	59.00	33	4	77.50	375.88	20	42	59.00	22	6	59.00	33	4	77.50	375.88
AKO20-A-9	0.25	20	50	252.00	20	6	252.00	40	3	253.75	338.26	20	51	252.00	20	6	252.00	40	3	253.75	338.26	20	51	252.00	20	6	252.00	40	3	253.75	338.26
	0.50	20	50	222.00	20	6	222.00	40	3	225.50	337.75	20	51	222.00	20	6	222.00	20	6	225.50	337.75	20	51	222.00	20	6	222.00	20	6	225.50	337.75
	0.75	20	50	192.00	20	6	192.00	40	3	197.25	337.25	20	51	192.00	20	6	192.00	40	3	197.25	337.25	20	51	192.00	20	6	192.00	40	3	197.25	337.25
	1.00	20	50	162.00	20	6	162.00	40	3	169.00	336.76	20	51	162.00	20	6	162.00	40	3	169.00	336.76	20	51	162.00	20	6	162.00	40	3	169.00	336.76
	1.25	20	50	132.00	20	6	132.00	40	3	140.75	336.27	20	51	132.00	20	6	132.00	40	3	140.75	336.27	20	51	132.00	20	6	132.00	40	3	140.75	336.27
	1.50	20	50	102.00	20	6	102.00	40	3	119.50	335.78	20	51	102.00	20	6	102.00	40	3	119.50	335.78	20	51	102.00	20	6	102.00	40	3	119.50	335.78
	1.75	20	50	72.00	20	6	72.00	20	6	102.25	335.30	20	51	72.00	20	6	72.00	40	3	102.25	335.30	20	51	72.00	20	6	72.00	40	3	102.25	335.30
	2.00	20	50	42.00	20	6	42.00	40	3	85.00	334.82	20	51	42.00	20	6	42.00	40	3	85.00	334.82	20	51	42.00	20	6	42.00	40	3	85.00	334.82
AKO20-B-0	0.25	20	40	169.50	34	3	169.50	34	3	171.00	344.39	20	42	169.50	34	3	169.50	34	3	171.00	344.39	20	42	169.50	34	3	169.50	34	3	171.00	344.39
	0.50	20	40	144.00	34	3	144.00	34	3	147.00	343.82	20	42	144.00	34	3	144.00	34	3	147.00	343.82	20	42	144.00	34	3	144.00	34	3	147.00	343.82
	0.75	20	40	118.50	34	3	118.50	34	3	123.00	343.26	20	42	118.50	34	3	118.50	34	3	123.00	343.26	20	42	118.50	34	3	118.50	34	3	123.00	343.26
	1.00	20	40	93.00	34	3	93.00	34	3	99.00	342.69	20	42	93.00	34	3	93.00	34	3	99.00	342.69	20	42	93.00	34	3	93.00	34	3	99.00	342.69
	1.25	20	40	67.50	34	3	67.50																								







Table F.3 Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET													
		K <sub>U</sub>	CT <sub>U</sub>	LB Solution			PC Solution			UB		PC-LP		K <sub>U</sub>	CT <sub>U</sub>	LB Solution			PC Solution			UB		PC-LP	
				π	CT	K	π	CT	K	π	π̄	π	CT			K	π	CT	K	π	CT	K	π	CT	K
AKO20-C-6	0.25	20	57	278.25	49	3	279.00	48	3	285.38	406.98	20	54	278.25	49	3	279.00	48	3	285.38	406.98				
	0.50	20	57	241.50	49	3	243.00	48	3	249.75	406.34	20	54	241.50	49	3	243.00	48	3	249.75	406.34				
	0.75	20	57	204.75	49	3	207.00	48	3	214.13	405.70	20	54	204.75	49	3	207.00	48	3	214.13	405.70				
	1.00	20	57	168.00	49	3	171.00	48	3	178.50	405.07	20	54	168.00	49	3	171.00	48	3	178.50	405.07				
	1.25	20	57	131.25	49	3	135.00	48	3	149.88	404.44	20	54	131.25	49	3	135.00	48	3	149.88	404.44				
	1.50	20	57	94.50	49	3	99.00	48	3	121.50	403.81	20	54	94.50	49	3	99.00	48	3	121.50	403.81				
	1.75	20	57	57.75	49	3	63.00	36	4	93.13	403.18	20	54	57.75	49	3	63.00	48	3	93.13	403.18				
	2.00	20	57	21.00	49	3	27.00	48	3	64.75	402.55	20	54	21.00	49	3	27.00	48	3	64.75	402.55				
AKO20-C-7	0.25	20	50	249.00	50	2	249.50	49	2	249.50	353.99	20	93	249.00	50	2	249.50	49	2	249.50	353.99				
	0.50	20	50	224.00	50	2	225.00	49	2	225.00	353.21	20	93	224.00	50	2	225.00	49	2	225.00	353.21				
	0.75	20	50	199.00	50	2	200.50	49	2	200.50	352.42	20	93	199.00	50	2	200.50	49	2	200.50	352.42				
	1.00	20	50	174.00	50	2	176.00	49	2	176.00	351.65	20	93	174.00	50	2	176.00	49	2	176.00	351.65				
	1.25	20	50	149.00	50	2	151.50	49	2	151.50	350.87	20	93	149.00	50	2	151.50	49	2	151.50	350.87				
	1.50	20	50	124.00	50	2	127.00	49	2	129.50	350.10	20	93	124.00	50	2	127.00	49	2	129.50	350.10				
	1.75	20	50	99.00	50	2	102.50	49	2	112.75	349.33	20	93	99.25	65	1	102.50	49	2	112.75	349.33				
	2.00	20	50	74.00	50	2	78.00	49	2	96.00	348.57	20	93	83.00	65	1	83.00	65	1	96.00	348.57				
AKO20-C-8	0.25	20	50	258.00	44	3	258.00	44	3	262.88	400.71	20	63	258.00	44	3	258.00	44	3	262.88	400.71				
	0.50	20	50	225.00	44	3	225.00	44	3	231.25	400.14	20	63	225.00	44	3	225.00	44	3	231.25	400.14				
	0.75	20	50	192.00	44	3	192.00	44	3	199.63	399.57	20	63	192.00	44	3	192.00	44	3	199.63	399.57				
	1.00	20	50	159.00	44	3	159.00	44	3	168.00	399.00	20	63	159.00	44	3	159.00	44	3	168.00	399.00				
	1.25	20	50	126.00	44	3	126.00	44	3	139.88	398.44	20	63	126.00	44	3	126.00	44	3	139.88	398.44				
	1.50	20	50	93.00	44	3	93.00	44	3	113.25	397.87	20	63	93.00	44	3	93.00	44	3	113.25	397.87				
	1.75	20	50	60.00	44	3	60.00	44	3	89.81	397.31	20	63	60.00	44	3	60.00	44	3	89.81	397.31				
	2.00	20	50	27.00	44	3	27.00	44	3	68.75	396.75	20	63	27.00	44	3	27.00	44	3	68.75	396.75				
AKO20-C-9	0.25	20	40	199.00	28	3	199.00	28	3	199.75	257.81	20	53	171.25	37	3	171.25	37	3	172.75	252.53				
	0.50	20	40	178.00	28	3	178.00	28	3	179.50	257.55	20	53	143.50	37	3	143.50	37	3	146.50	252.08				
	0.75	20	40	157.00	28	3	157.00	28	3	159.25	257.29	20	53	115.75	37	3	115.75	37	3	120.25	251.64				
	1.00	20	40	136.00	28	3	136.00	28	3	139.00	257.04	20	53	88.00	37	3	88.00	37	3	94.00	251.20				
	1.25	20	40	115.00	28	3	115.00	28	3	119.25	256.78	20	53	45.50	38	3	60.25	37	3	72.75	250.75				
	1.50	20	40	94.00	28	3	94.00	28	3	100.00	256.52	20	53	18.00	39	2	32.50	37	3	53.75	250.31				
	1.75	20	40	73.00	28	3	73.00	28	3	80.75	256.27	20	53	0.00	0	0	4.75	37	3	34.75	249.87				
	2.00	20	40	52.00	28	3	52.00	28	3	61.50	256.01	20	53	0.00	0	0	0.00	0	0	15.75	249.43				
LA20T10-0	0.25	20	44	172.50	37	2	172.50	37	2	172.50	340.80	20	46	223.00	46	2	223.50	45	2	224.25	502.89				
	0.50	20	44	155.00	28	2	156.00	27	2	156.00	339.55	20	46	204.00	34	2	204.00	34	2	204.50	501.53				
	0.75	20	44	141.00	28	2	142.50	27	2	142.50	338.31	20	46	190.00	28	2	190.00	28	2	190.00	500.18				
	1.00	20	44	127.00	28	2	129.00	27	2	129.00	337.06	20	46	176.00	28	2	176.00	28	2	176.00	498.82				
	1.25	20	44	113.00	28	2	115.50	27	2	115.50	335.81	20	46	162.00	28	2	162.00	28	2	162.00	497.46				
	1.50	20	44	99.00	28	2	102.00	27	2	102.00	334.57	20	46	148.00	28	2	148.00	28	2	148.00	496.10				
	1.75	20	44	88.75	35	1	88.75	35	1	88.75	333.32	20	46	134.00	28	2	134.00	28	2	134.00	494.74				
	2.00	20	44	80.00	35	1	80.00	35	1	80.00	332.07	20	46	125.00	35	1	125.00	35	1	125.00	493.39				
LA20T10-1	0.25	20	44	322.50	25	2	322.50	25	2	322.50	614.15	20	46	279.50	25	2	279.50	25	2	279.50	500.85				
	0.50	20	44	310.00	25	2	310.00	25	2	310.00	612.81	20	46	267.00	25	2	267.00	25	2	267.00	499.78				
	0.75	20	44	297.50	25	2	297.50	25	2	297.50	611.46	20	46	254.50	25	2	254.50	25	2	254.50	498.72				
	1.00	20	44	285.00	25	2	285.00	25	2	285.00	610.12	20	46	242.00	25	2	242.00	25	2	242.00	497.65				
	1.25	20	44	272.50	25	2	272.50	25	2	272.50	608.77	20	46	229.50	25	2	229.50	25	2	229.50	496.59				
	1.50	20	44	260.00	25	2	260.00	25	2	260.00	607.43	20	46	217.00	25	2	217.00	25	2	217.00	495.52				
	1.75	20	44	247.50	25	2	247.50	25	2	247.50	606.08	20	46	204.50	25	2	204.50	25	2	204.50	494.46				
	2.00	20	44	235.00	25	2	235.00	25	2	235.00	604.74	20	46	192.00	25	2	192.00	25	2	192.00	493.39				
LA20T10-2	0.25	20	44	206.00	30	4	206.00	30	4	209.75	390.10	20	44	176.00	30	4	176.00	40	3	179.75	389.82				
	0.50	20	44	183.00	44	2	183.00	44	2	183.50	388.60	20	44	153.00	44	2	153.00	44	2	153.50	387.99				
	0.75	20	44	161.00	44	2	161.00	44	2	161.75	387.10	20	44	131.00	44	2	131.00	44	2	131.75	386.16				
	1.00	20	44	138.00	38	2	138.00	38	2	144.00	385.61	20	44	108.00	38	2	109.00	44	2	114.00	384.33				
	1.25	20	44	119.00	38	2	119.00	38	2	126.50	384.11	20	44	89.00	38	2	89.00	38	2	96.50	382.50				
	1.50	20	44	103.00	30	2	103.00	30	2	112.00	382.61	20	44	73.00	30	2	73.00	30	2	82.00	380.67				
	1.75	20	44	88.00	30	2	88.00	30	2	98.50	381.11	20	44	58.00	30	2	58.00	30	2	68.50	378.84				
	2.00	20	44	73.00	30	2	73.00	30	2	85.00	379.61	20	44	43.00	30	2	43.00	30	2	55.00	377.01				

**Table F.3** Results of the heuristic solution procedure (Continued)

		LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET													
INSTANCE	S	$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP	
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\bar{\pi}$	$\pi$	
LA20T10-3	0.25	20	66	217.50	47	2	217.75	31	3	217.75	401.71	20	54	304.50	47	2	304.75	31	3	304.75	558.07				
	0.50	20	66	194.00	47	2	194.50	31	3	194.50	400.06	20	54	281.00	47	2	281.50	31	3	281.50	556.23				
	0.75	20	66	170.50	47	2	171.25	31	3	171.25	398.42	20	54	257.50	47	2	258.25	31	3	258.25	554.40				
	1.00	20	66	149.00	37	2	149.00	37	2	150.00	396.77	20	54	236.00	37	2	236.00	37	2	237.00	552.56				
	1.25	20	66	130.50	37	2	130.50	37	2	131.75	395.13	20	54	217.50	37	2	217.50	37	2	218.75	550.73				
	1.50	20	66	112.00	37	2	112.00	37	2	113.50	393.48	20	54	199.00	37	2	199.00	37	2	200.50	548.89				
	1.75	20	66	93.50	37	2	93.50	37	2	95.25	391.84	20	54	180.50	37	2	180.50	37	2	182.25	547.06				
	2.00	20	66	78.00	57	1	78.00	57	1	78.00	390.19	20	54	162.00	37	2	162.00	37	2	164.00	545.22				
LA20T10-4	0.25	20	44	327.75	43	3	328.50	42	3	330.25	591.19	20	40	378.25	27	5	378.25	27	5	382.25	725.73				
	0.50	20	44	295.50	43	3	297.00	42	3	300.50	589.21	20	40	344.50	27	5	348.00	40	3	352.50	723.70				
	0.75	20	44	266.00	40	3	266.00	40	3	272.00	587.23	20	40	318.00	40	3	318.00	40	3	324.00	721.68				
	1.00	20	44	236.00	40	3	237.00	21	5	244.00	585.25	20	40	288.00	40	3	289.00	21	5	296.00	719.66				
	1.25	20	44	210.75	21	5	210.75	21	5	217.00	583.26	20	40	262.75	21	5	262.75	21	5	269.00	717.63				
	1.50	20	44	184.50	21	5	184.50	21	5	192.00	581.28	20	40	236.50	21	5	236.50	21	5	244.00	715.61				
	1.75	20	44	158.25	21	5	158.25	21	5	167.00	579.30	20	40	210.25	21	5	210.25	21	5	219.00	713.58				
	2.00	20	44	132.00	21	5	132.00	21	5	142.00	577.32	20	40	184.00	21	5	184.00	21	5	194.00	711.56				
LA20T10-5	0.25	20	44	187.00	44	1	187.00	44	1	187.00	440.00	20	40	186.50	23	2	186.50	23	2	187.00	440.00				
	0.50	20	44	176.00	44	1	176.00	44	1	176.00	438.74	20	40	175.00	23	2	175.00	23	2	176.00	438.74				
	0.75	20	44	165.00	44	1	165.00	44	1	165.00	437.49	20	40	163.50	23	2	163.50	23	2	165.00	437.49				
	1.00	20	44	154.00	44	1	154.00	44	1	154.00	436.24	20	40	152.00	23	2	152.00	23	2	154.00	436.24				
	1.25	20	44	143.00	44	1	143.00	44	1	143.00	434.98	20	40	140.50	23	2	140.50	23	2	143.00	434.98				
	1.50	20	44	132.00	44	1	132.00	44	1	132.00	433.73	20	40	129.00	23	2	129.00	23	2	132.00	433.73				
	1.75	20	44	121.00	44	1	121.00	44	1	121.00	432.48	20	40	117.50	23	2	117.50	23	2	121.00	432.48				
	2.00	20	44	110.00	44	1	110.00	44	1	110.00	431.23	20	40	106.00	23	2	106.00	23	2	110.00	431.23				
LA20T10-6	0.25	20	44	293.50	39	2	293.50	39	2	293.50	592.04	20	42	262.50	39	2	262.50	39	2	262.50	595.94				
	0.50	20	44	274.00	39	2	274.00	39	2	274.00	590.26	20	42	243.00	39	2	243.00	39	2	243.00	594.28				
	0.75	20	44	254.50	39	2	254.50	39	2	254.50	588.48	20	42	223.50	39	2	223.50	39	2	223.50	592.61				
	1.00	20	44	235.00	39	2	235.00	39	2	235.00	586.70	20	42	204.00	39	2	204.00	39	2	204.00	590.95				
	1.25	20	44	215.50	39	2	215.50	39	2	215.50	584.92	20	42	184.50	39	2	184.50	39	2	184.50	589.29				
	1.50	20	44	196.00	39	2	196.00	39	2	196.00	583.14	20	42	165.00	39	2	165.00	39	2	165.00	587.63				
	1.75	20	44	176.50	39	2	176.50	39	2	176.50	581.36	20	42	145.50	39	2	145.50	39	2	145.50	585.96				
	2.00	20	44	157.00	39	2	157.00	39	2	157.00	579.58	20	42	126.00	39	2	126.00	39	2	126.00	584.30				
LA20T10-7	0.25	20	44	440.50	39	2	440.50	39	2	441.50	783.46	20	40	440.50	39	2	440.50	39	2	441.50	815.76				
	0.50	20	44	421.00	39	2	421.00	39	2	423.00	781.68	20	40	421.00	39	2	421.00	39	2	423.00	813.88				
	0.75	20	44	401.50	39	2	401.50	39	2	404.50	779.89	20	40	401.50	39	2	401.50	39	2	404.50	811.99				
	1.00	20	44	382.00	39	2	382.00	39	2	386.00	778.11	20	40	382.00	39	2	382.00	39	2	386.00	810.11				
	1.25	20	44	362.50	39	2	362.50	39	2	367.50	776.32	20	40	362.50	39	2	362.50	39	2	367.50	808.22				
	1.50	20	44	343.00	39	2	343.00	39	2	349.00	774.54	20	40	343.00	39	2	343.00	39	2	349.00	806.34				
	1.75	20	44	323.50	39	2	323.50	39	2	330.50	772.75	20	40	323.50	39	2	323.50	39	2	330.50	804.45				
	2.00	20	44	304.00	39	2	304.00	39	2	312.00	770.97	20	40	304.00	39	2	304.00	39	2	312.00	802.57				
LA20T10-8	0.25	20	57	195.00	42	2	195.00	42	2	195.00	418.08	20	45	271.00	42	2	271.00	42	2	271.00	525.70				
	0.50	20	57	174.00	37	2	174.00	42	2	174.00	416.61	20	45	250.00	37	2	250.00	42	2	250.00	524.16				
	0.75	20	57	155.50	37	2	155.50	37	2	155.50	415.13	20	45	231.50	37	2	231.50	37	2	231.50	522.61				
	1.00	20	57	137.00	37	2	137.00	37	2	137.00	413.66	20	45	213.00	37	2	213.00	37	2	213.00	521.06				
	1.25	20	57	118.50	37	2	118.50	37	2	118.50	412.19	20	45	194.50	37	2	194.50	37	2	194.50	519.51				
	1.50	20	57	100.00	37	2	100.00	37	2	100.00	410.72	20	45	176.00	37	2	176.00	37	2	176.00	517.97				
	1.75	20	57	81.50	37	2	81.50	37	2	81.50	409.24	20	45	157.50	37	2	157.50	37	2	157.50	516.42				
	2.00	20	57	63.00	37	2	63.00	37	2	63.00	407.77	20	45	139.00	37	2	139.00	37	2	139.00	514.87				
LA20T10-9	0.25	20	44	288.00	42	2	288.00	42	2	288.00	569.16	20	43	288.00	42	2	288.00	42	2	288.00	569.16				
	0.50	20	44	267.00	42	2	267.00	42	2	267.00	567.31	20	43	267.00	42	2	267.00	42	2	267.00	567.31				
	0.75	20	44	246.00	42	2	246.00	42	2	246.00	565.47	20	43	246.00	42	2	246.00	42	2	246.00	565.47				
	1.00	20	44	225.00	42	2	225.00	42	2	225.00	563.63	20	43	225.00	42	2	225.00	42	2	225.00	563.63				
	1.25	20	44	204.00	42	2	204.00	42	2	204.00	561.78	20	43	204.00	42	2	204.00	42	2	204.00	561.78				
	1.50	20	44	183.00	42	2	183.00	42	2	183.00	559.94	20	43	183.00	42	2	183.00	42	2	183.00	559.94				
	1.75	20	44	158.50	26	3	162.00	38	2	163.75	558.09	20	43	158.50	26	3	162.00	42	2	163.75	558.09				
	2.00	20	44	139.00	26	3	143.00	38	2	145.00	556.25	20	43	139.00	26	3	143.00	38	2	145.00	556.25				



**Table F.3 Results of the heuristic solution procedure (Continued)**

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET													
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP	
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\bar{\pi}$	$\pi$	
LA20T24-7	0.25	20	50	61.25	43	1	61.25	43	1	61.25	71.46	20	40	106.25	29	3	106.25	29	3	106.75	127.70				
	0.50	20	50	50.50	43	1	50.50	43	1	50.50	70.93	20	40	85.00	17	4	85.00	34	2	86.00	126.40				
	0.75	20	50	39.75	43	1	39.75	43	1	39.75	70.39	20	40	68.00	17	4	68.00	34	2	69.50	125.10				
	1.00	20	50	29.00	43	1	29.00	43	1	29.00	69.85	20	40	51.00	17	4	51.00	17	4	53.00	123.80				
	1.25	20	50	18.25	35	1	18.25	43	1	18.25	69.31	20	40	34.00	17	4	34.00	34	2	36.50	122.69				
	1.50	20	50	11.00	9	2	11.00	18	1	11.00	68.78	20	40	22.50	31	1	22.50	31	1	22.50	121.63				
	1.75	20	50	6.50	9	2	6.50	18	1	6.50	68.24	20	40	14.75	31	1	14.75	31	1	14.75	120.56				
2.00	20	50	2.00	9	2	2.00	9	2	2.00	67.70	20	40	8.00	26	1	8.00	26	1	8.00	119.50					
LA20T24-8	0.25	20	44	62.00	42	2	62.00	42	2	62.00	81.95	20	40	82.50	30	3	82.50	30	3	84.00	103.95				
	0.50	20	44	41.00	42	2	41.00	42	2	41.00	80.90	20	40	60.00	30	3	60.00	30	3	63.00	102.90				
	0.75	20	44	21.50	25	2	21.50	25	2	22.25	79.85	20	40	37.50	30	3	37.50	30	3	42.00	101.85				
	1.00	20	44	9.00	25	2	9.00	25	2	10.00	78.80	20	40	15.00	30	3	15.00	30	3	21.00	100.80				
	1.25	20	44	0.00	0	0	0.00	0	0	4.85	77.75	20	40	0.00	0	0	0.00	0	0	4.94	99.75				
	1.50	20	44	0.00	0	0	0.00	0	0	4.60	76.70	20	40	0.00	0	0	0.00	0	0	4.69	98.70				
	1.75	20	44	0.00	0	0	0.00	0	0	4.35	75.65	20	40	0.00	0	0	0.00	0	0	4.44	97.65				
2.00	20	44	0.00	0	0	0.00	0	0	4.10	74.60	20	40	0.00	0	0	0.00	0	0	4.19	96.60					
LA20T24-9	0.25	20	44	69.00	30	2	69.00	30	2	69.00	92.73	20	42	69.00	30	2	69.00	30	2	69.00	92.73				
	0.50	20	44	54.00	30	2	54.00	30	2	54.00	91.87	20	42	54.00	30	2	54.00	30	2	54.00	91.87				
	0.75	20	44	39.00	30	2	39.00	30	2	39.00	91.00	20	42	39.00	30	2	39.00	30	2	39.00	91.00				
	1.00	20	44	24.00	30	2	24.00	30	2	24.00	90.13	20	42	24.00	30	2	24.00	30	2	24.00	90.13				
	1.25	20	44	11.00	44	1	11.00	44	1	11.00	89.26	20	42	9.75	15	3	9.75	15	3	11.00	89.26				
	1.50	20	44	0.00	0	0	0.00	0	0	6.84	88.40	20	42	0.00	0	0	0.00	0	0	6.84	88.40				
	1.75	20	44	0.00	0	0	0.00	0	0	6.59	87.53	20	42	0.00	0	0	0.00	0	0	6.59	87.53				
2.00	20	44	0.00	0	0	0.00	0	0	6.34	86.66	20	42	0.00	0	0	0.00	0	0	6.34	86.66					
LA30T10-0	0.25	30	50	267.50	35	2	267.50	35	2	268.25	871.68	30	44	322.50	35	2	322.50	35	2	323.25	987.26				
	0.50	30	50	250.00	35	2	250.00	35	2	251.50	870.19	30	44	305.00	35	2	305.00	35	2	306.50	985.57				
	0.75	30	50	233.50	29	2	233.50	29	2	235.75	868.70	30	44	288.50	29	2	288.50	29	2	290.75	983.89				
	1.00	30	50	219.00	29	2	219.00	29	2	222.00	867.20	30	44	274.00	29	2	274.00	29	2	277.00	982.20				
	1.25	30	50	204.50	29	2	204.50	29	2	208.25	865.71	30	44	259.50	29	2	259.50	29	2	263.25	980.52				
	1.50	30	50	190.00	29	2	190.00	29	2	194.50	864.22	30	44	245.00	29	2	245.00	29	2	249.50	978.83				
	1.75	30	50	175.50	29	2	175.50	29	2	180.75	862.73	30	44	230.50	29	2	230.50	29	2	235.75	977.15				
2.00	30	50	161.00	29	2	161.00	29	2	167.00	861.25	30	44	216.00	29	2	216.00	29	2	222.00	975.46					
LA30T10-1	0.25	30	50	219.00	46	2	219.00	46	2	221.00	565.55	30	62	329.00	46	2	329.00	46	2	331.00	827.97				
	0.50	30	50	196.00	46	2	196.00	32	2	200.00	564.45	30	62	306.00	46	2	306.00	32	2	310.00	826.45				
	0.75	30	50	180.00	32	2	180.00	32	2	180.00	563.35	30	62	290.00	32	2	290.00	32	2	290.00	824.93				
	1.00	30	50	164.00	32	2	164.00	32	2	164.00	562.25	30	62	274.00	32	2	274.00	32	2	274.00	823.41				
	1.25	30	50	148.00	32	2	148.00	32	2	148.00	561.15	30	62	258.00	32	2	258.00	32	2	258.00	821.89				
	1.50	30	50	132.00	32	2	132.00	32	2	132.00	560.05	30	62	242.00	32	2	242.00	32	2	242.00	820.37				
	1.75	30	50	116.00	32	2	116.00	32	2	116.00	558.95	30	62	226.00	32	2	226.00	32	2	226.00	818.85				
2.00	30	50	100.00	32	2	100.00	32	2	100.00	557.85	30	62	210.00	32	2	210.00	32	2	210.00	817.33					
LA30T10-2	0.25	30	44	378.50	35	2	378.50	35	2	378.50	864.32	30	41	378.50	35	2	378.50	35	2	378.50	1088.01				
	0.50	30	44	361.00	34	2	361.00	34	2	362.00	862.93	30	41	361.00	34	2	361.00	34	2	362.00	1086.14				
	0.75	30	44	344.00	34	2	344.00	34	2	345.50	861.53	30	41	344.00	34	2	344.00	34	2	345.50	1084.26				
	1.00	30	44	327.00	34	2	327.00	34	2	329.00	860.14	30	41	327.00	34	2	327.00	34	2	329.00	1082.39				
	1.25	30	44	310.00	34	2	310.00	34	2	312.50	858.75	30	41	310.00	34	2	310.00	34	2	312.50	1080.52				
	1.50	30	44	293.00	34	2	293.00	34	2	296.00	857.36	30	41	293.00	34	2	293.00	34	2	296.00	1078.65				
	1.75	30	44	276.00	34	2	276.00	34	2	279.50	855.97	30	41	276.00	34	2	276.00	34	2	279.50	1076.77				
2.00	30	44	259.00	34	2	259.00	34	2	263.00	854.58	30	41	259.00	34	2	259.00	34	2	263.00	1074.90					
LA30T10-3	0.25	30	44	418.50	26	3	418.50	26	3	421.00	1389.85	30	40	418.50	26	3	418.50	26	3	421.00	1434.85				
	0.50	30	44	399.00	26	3	399.00	34	2	404.00	1387.63	30	40	399.00	26	3	399.00	35	2	404.00	1432.63				
	0.75	30	44	379.50	26	3	382.00	34	2	387.00	1385.42	30	40	379.50	26	3	382.00	34	2	387.00	1430.40				
	1.00	30	44	365.00	34	2	365.00	34	2	371.00	1383.20	30	40	365.00	34	2	365.00	34	2	371.00	1428.18				
	1.25	30	44	348.00	34	2	348.00	34	2	355.50	1380.98	30	40	348.00	34	2	348.00	34	2	355.50	1425.95				
	1.50	30	44	331.00	34	2	331.00	34	2	340.00	1378.77	30	40	331.00	34	2	331.00	34	2	340.00	1423.73				
	1.75	30	44	314.00	34	2	314.00	34	2	324.50	1376.55	30	40	314.00	34	2	314.00	34	2	324.50	1421.51				
2.00	30	44	297.00	34	2	297.00	34	2	309.00	1374.34	30	40	297.00	34	2	297.00	34	2	309.00	1419.28					

**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET										HIGH DEMAND VARIABILITY DATA SET																			
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP							
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\bar{\pi}$	$\pi$							
LA30T10-4	0.25	30	44	304.50	22	5	306.00	40	2	309.25	960.96	30	41	304.50	22	5	306.00	40	2	309.25	1352.00	30	41	304.50	22	5	306.00	40	2	309.25	1352.00
	0.50	30	44	286.00	40	2	286.00	40	2	289.00	959.47	30	41	286.00	40	2	286.00	40	2	289.00	1349.63	30	41	286.00	40	2	286.00	40	2	289.00	1349.63
	0.75	30	44	266.00	40	2	266.00	40	2	270.50	957.97	30	41	266.00	40	2	266.00	40	2	270.50	1347.26	30	41	266.00	40	2	266.00	40	2	270.50	1347.26
	1.00	30	44	246.00	40	2	248.00	34	2	252.00	956.47	30	41	246.00	40	2	248.00	34	2	252.00	1344.90	30	41	246.00	40	2	248.00	34	2	252.00	1344.90
	1.25	30	44	230.00	34	2	231.00	34	2	235.00	954.98	30	41	230.00	34	2	231.00	34	2	235.00	1342.53	30	41	230.00	34	2	231.00	34	2	235.00	1342.53
	1.50	30	44	213.00	34	2	214.00	34	2	219.00	953.48	30	41	213.00	34	2	214.00	34	2	219.00	1340.17	30	41	213.00	34	2	214.00	34	2	219.00	1340.17
	1.75	30	44	196.00	34	2	197.00	34	2	203.00	951.99	30	41	196.00	34	2	197.00	34	2	203.00	1337.80	30	41	196.00	34	2	197.00	34	2	203.00	1337.80
	2.00	30	44	179.00	34	2	180.00	34	2	187.00	950.49	30	41	179.00	34	2	180.00	34	2	187.00	1335.43	30	41	179.00	34	2	180.00	34	2	187.00	1335.43
LA30T10-5	0.25	30	44	412.50	41	2	412.50	41	2	413.50	990.35	30	41	412.50	41	2	412.50	41	2	413.50	1212.62	30	41	412.50	41	2	412.50	41	2	413.50	1212.62
	0.50	30	44	395.50	19	3	395.50	19	3	395.50	988.74	30	41	395.50	19	3	395.50	19	3	395.50	1210.45	30	41	395.50	19	3	395.50	19	3	395.50	1210.45
	0.75	30	44	381.50	25	2	381.50	25	2	383.75	987.14	30	41	381.50	25	2	381.50	25	2	383.75	1208.29	30	41	381.50	25	2	381.50	25	2	383.75	1208.29
	1.00	30	44	369.00	25	2	369.00	25	2	372.00	985.54	30	41	369.00	25	2	369.00	25	2	372.00	1206.12	30	41	369.00	25	2	369.00	25	2	372.00	1206.12
	1.25	30	44	356.50	25	2	356.50	25	2	360.25	983.93	30	41	356.50	25	2	356.50	25	2	360.25	1203.95	30	41	356.50	25	2	356.50	25	2	360.25	1203.95
	1.50	30	44	344.00	25	2	344.00	25	2	348.50	982.33	30	41	344.00	25	2	344.00	25	2	348.50	1201.78	30	41	344.00	25	2	344.00	25	2	348.50	1201.78
	1.75	30	44	331.50	25	2	331.50	25	2	336.75	980.72	30	41	331.50	25	2	331.50	25	2	336.75	1199.62	30	41	331.50	25	2	331.50	25	2	336.75	1199.62
	2.00	30	44	319.00	25	2	319.00	25	2	325.00	979.12	30	41	319.00	25	2	319.00	25	2	325.00	1197.45	30	41	319.00	25	2	319.00	25	2	325.00	1197.45
LA30T10-6	0.25	30	44	471.00	40	2	471.00	40	2	471.75	1455.54	30	50	471.00	40	2	471.00	40	2	471.75	1498.00	30	50	471.00	40	2	471.00	40	2	471.75	1498.00
	0.50	30	44	451.00	40	2	451.00	40	2	452.50	1453.25	30	50	451.00	40	2	451.00	40	2	452.50	1495.62	30	50	451.00	40	2	451.00	40	2	452.50	1495.62
	0.75	30	44	431.00	40	2	431.00	40	2	433.25	1450.95	30	50	431.00	40	2	431.00	40	2	433.25	1493.25	30	50	431.00	40	2	431.00	40	2	433.25	1493.25
	1.00	30	44	411.00	40	2	411.00	40	2	414.00	1448.66	30	50	411.00	40	2	411.00	40	2	414.00	1490.88	30	50	411.00	40	2	411.00	40	2	414.00	1490.88
	1.25	30	44	391.00	40	2	391.00	40	2	394.75	1446.36	30	50	391.00	40	2	391.00	40	2	394.75	1488.50	30	50	391.00	40	2	391.00	40	2	394.75	1488.50
	1.50	30	44	371.00	40	2	371.00	40	2	375.50	1444.07	30	50	371.00	40	2	371.00	40	2	375.50	1486.13	30	50	371.00	40	2	371.00	40	2	375.50	1486.13
	1.75	30	44	351.00	40	2	351.00	40	2	356.25	1441.77	30	50	351.00	40	2	351.00	40	2	356.25	1483.75	30	50	351.00	40	2	351.00	40	2	356.25	1483.75
	2.00	30	44	331.00	40	2	331.00	40	2	337.00	1439.48	30	50	331.00	40	2	331.00	40	2	337.00	1481.38	30	50	331.00	40	2	331.00	40	2	337.00	1481.38
LA30T10-7	0.25	30	44	606.00	26	4	606.00	26	4	607.75	1575.02	30	45	606.00	26	4	606.00	26	4	607.75	1883.69	30	45	606.00	26	4	606.00	26	4	607.75	1883.69
	0.50	30	44	580.00	26	4	580.00	26	4	583.50	1572.59	30	45	580.00	26	4	580.00	26	4	583.50	1880.87	30	45	580.00	26	4	580.00	26	4	583.50	1880.87
	0.75	30	44	554.00	26	4	554.00	26	4	559.25	1570.17	30	45	554.00	26	4	554.00	26	4	559.25	1878.05	30	45	554.00	26	4	554.00	26	4	559.25	1878.05
	1.00	30	44	528.00	26	4	529.00	32	3	535.00	1567.75	30	45	528.00	26	4	529.00	32	3	535.00	1875.23	30	45	528.00	26	4	529.00	32	3	535.00	1875.23
	1.25	30	44	502.00	26	4	505.00	32	3	510.75	1565.32	30	45	502.00	26	4	505.00	32	3	510.75	1872.41	30	45	502.00	26	4	505.00	32	3	510.75	1872.41
	1.50	30	44	476.00	26	4	481.00	32	3	486.50	1562.90	30	45	476.00	26	4	481.00	32	3	486.50	1869.59	30	45	476.00	26	4	481.00	32	3	486.50	1869.59
	1.75	30	44	457.00	32	3	457.00	32	3	462.25	1560.47	30	45	457.00	32	3	457.00	32	3	462.25	1866.77	30	45	457.00	32	3	457.00	32	3	462.25	1866.77
	2.00	30	44	433.00	32	3	433.00	32	3	439.00	1558.05	30	45	433.00	32	3	433.00	32	3	439.00	1863.95	30	45	433.00	32	3	433.00	32	3	439.00	1863.95
LA30T10-8	0.25	30	44	363.50	41	2	363.50	41	2	363.50	882.63	30	48	363.50	41	2	363.50	41	2	363.50	941.49	30	48	363.50	41	2	363.50	41	2	363.50	941.49
	0.50	30	44	343.00	41	2	343.00	41	2	343.00	880.78	30	48	343.00	41	2	343.00	41	2	343.00	939.54	30	48	343.00	41	2	343.00	41	2	343.00	939.54
	0.75	30	44	322.50	41	2	322.50	41	2	322.50	878.93	30	48	322.50	41	2	322.50	41	2	322.50	937.59	30	48	322.50	41	2	322.50	41	2	322.50	937.59
	1.00	30	44	302.00	41	2	302.00	41	2	302.00	877.08	30	48	302.00	41	2	302.00	41	2	302.00	935.64	30	48	302.00	41	2	302.00	41	2	302.00	935.64
	1.25	30	44	281.50	41	2	281.50	41	2	281.50	875.23	30	48	281.50	41	2	281.50	41	2	281.50	933.69	30	48	281.50	41	2	281.50	41	2	281.50	933.69
	1.50	30	44	261.00	41	2	261.00	41	2	261.00	873.38	30	48	261.00	41	2	261.00	41	2	261.00	931.74	30	48	261.00	41	2	261.00	41	2	261.00	931.74
	1.75	30	44	240.50	41	2	240.50	41	2	240.50	871.53	30	48	240.50	41	2	240.50	41	2	240.50	929.79	30	48	240.50	41	2	240.50	41	2	240.50	929.79
	2.00	30	44	220.00	41	2	220.00	41	2	220.00	869.68	30	48	220.00	41	2	220.00	41	2	220.00	927.84	30	48	220.00	41	2	220.00	41	2	220.00	927.84
LA30T10-9	0.25	30	50	305.50	31	2	305.50	31	2	305.75	858.07	30	42	305.50	31	2	305.50	31	2	305.75	1156.55	30	42	305.50	31	2	305.50	31	2	305.75	1156.55
	0.50	30	50	290.00	31	2	290.00	31	2	290.50	856.96	30	42	290.00	31	2	290.00	31	2	290.50	1154.90	30	42	290.00	31	2	290.00	31	2	290.50	1154.90
	0.75	30	50	274.50	31	2	274.50	31	2	275.25	855.85	30	42	274.50	31	2	274.50	31	2	275.25	1153.25	30	42	274.50	31	2	274.50	31	2	275.25	1153.25
	1.00	30	50	259.00	31	2	259.00	31	2	260.00	854.74																				

**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET												HIGH DEMAND VARIABILITY DATA SET																	
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP							
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\pi$	CT	K	$\pi$	CT	K	$\pi$	CT	K
LA30T29-1	0.25	30	44	59.25	33	3	59.25	33	3	63.00	83.30	30	40	59.25	33	3	59.25	33	3	63.00	83.30	30	40	59.25	33	3	59.25	33	3	63.00	83.30
	0.50	30	44	34.50	33	3	34.50	33	3	42.00	82.60	30	40	34.50	33	3	34.50	33	3	42.00	82.60	30	40	34.50	33	3	34.50	33	3	42.00	82.60
	0.75	30	44	9.75	33	3	10.75	31	1	21.00	81.90	30	40	9.75	33	3	10.75	31	1	21.00	81.90	30	40	9.75	33	3	10.75	31	1	21.00	81.90
	1.00	30	44	0.00	0	0	3.00	31	1	5.93	81.20	30	40	0.00	0	0	4.00	23	1	5.93	81.20	30	40	0.00	0	0	4.00	23	1	5.93	81.20
	1.25	30	44	0.00	0	0	0.00	0	0	5.68	80.50	30	40	0.00	0	0	0.00	0	0	5.68	80.50	30	40	0.00	0	0	0.00	0	0	5.68	80.50
	1.50	30	44	0.00	0	0	0.00	0	0	5.43	79.80	30	40	0.00	0	0	0.00	0	0	5.43	79.80	30	40	0.00	0	0	0.00	0	0	5.43	79.80
	1.75	30	44	0.00	0	0	0.00	0	0	5.18	79.10	30	40	0.00	0	0	0.00	0	0	5.18	79.10	30	40	0.00	0	0	0.00	0	0	5.18	79.10
	2.00	30	44	0.00	0	0	0.00	0	0	4.93	78.40	30	40	0.00	0	0	0.00	0	0	4.93	78.40	30	40	0.00	0	0	0.00	0	0	4.93	78.40
LA30T29-2	0.25	30	44	126.50	35	2	126.50	35	2	126.50	143.42	30	43	98.50	31	2	98.50	29	2	99.75	115.42	30	43	98.50	31	2	98.50	29	2	99.75	115.42
	0.50	30	44	109.00	34	2	109.00	34	2	110.00	142.83	30	43	84.00	29	2	84.00	29	2	86.50	114.83	30	43	84.00	29	2	84.00	29	2	86.50	114.83
	0.75	30	44	95.00	24	2	95.00	24	2	95.00	142.25	30	43	74.75	35	1	74.75	35	1	74.75	114.25	30	43	74.75	35	1	74.75	35	1	74.75	114.25
	1.00	30	44	83.00	24	2	83.00	24	2	83.00	141.67	30	43	66.00	35	1	66.00	35	1	66.00	113.67	30	43	66.00	35	1	66.00	35	1	66.00	113.67
	1.25	30	44	71.00	24	2	71.00	24	2	71.00	141.08	30	43	57.25	35	1	57.25	35	1	57.25	113.08	30	43	57.25	35	1	57.25	35	1	57.25	113.08
	1.50	30	44	59.00	24	2	59.00	24	2	59.00	140.50	30	43	48.50	35	1	48.50	35	1	48.50	112.50	30	43	48.50	35	1	48.50	35	1	48.50	112.50
	1.75	30	44	47.00	24	2	47.00	24	2	47.00	139.92	30	43	39.75	35	1	39.75	35	1	39.75	111.92	30	43	39.75	35	1	39.75	35	1	39.75	111.92
	2.00	30	44	35.00	24	2	35.00	24	2	35.00	139.33	30	43	31.00	35	1	31.00	35	1	31.00	111.33	30	43	31.00	35	1	31.00	35	1	31.00	111.33
LA30T29-3	0.25	30	44	71.00	19	4	71.00	19	4	72.50	89.42	30	42	74.50	35	2	74.50	35	2	75.25	98.11	30	42	74.50	35	2	74.50	35	2	75.25	98.11
	0.50	30	44	60.50	41	1	60.50	41	1	60.50	88.83	30	42	57.00	35	2	57.00	35	2	58.50	97.22	30	42	57.00	35	2	57.00	35	2	58.50	97.22
	0.75	30	44	50.25	41	1	50.25	41	1	50.25	88.25	30	42	41.25	19	3	41.25	19	3	42.00	96.33	30	42	41.25	19	3	41.25	19	3	42.00	96.33
	1.00	30	44	40.00	41	1	40.00	41	1	40.00	87.67	30	42	31.00	32	1	31.00	32	1	31.00	95.43	30	42	31.00	32	1	31.00	32	1	31.00	95.43
	1.25	30	44	29.75	41	1	29.75	41	1	29.75	87.08	30	42	23.00	32	1	23.00	32	1	23.00	94.54	30	42	23.00	32	1	23.00	32	1	23.00	94.54
	1.50	30	44	19.50	41	1	19.50	41	1	19.50	86.50	30	42	16.00	6	1	16.00	6	1	16.00	93.65	30	42	16.00	6	1	16.00	6	1	16.00	93.65
	1.75	30	44	14.50	6	1	14.50	6	1	14.50	85.92	30	42	14.50	6	1	14.50	6	1	14.50	92.76	30	42	14.50	6	1	14.50	6	1	14.50	92.76
	2.00	30	44	13.00	6	1	13.00	6	1	13.00	85.33	30	42	13.00	6	1	13.00	6	1	13.00	91.87	30	42	13.00	6	1	13.00	6	1	13.00	91.87
LA30T29-4	0.25	30	44	93.25	37	3	93.50	41	2	95.75	120.16	30	41	93.25	37	3	93.50	41	2	95.75	120.16	30	41	93.25	37	3	93.50	41	2	95.75	120.16
	0.50	30	44	73.00	41	2	73.00	41	2	74.00	119.32	30	41	73.00	41	2	73.00	34	2	74.00	119.32	30	41	73.00	41	2	73.00	34	2	74.00	119.32
	0.75	30	44	55.00	34	2	56.00	34	2	58.00	118.48	30	41	55.00	34	2	56.00	34	2	58.00	118.48	30	41	55.00	34	2	56.00	34	2	58.00	118.48
	1.00	30	44	38.00	34	2	39.00	34	2	42.00	117.63	30	41	38.00	34	2	39.00	34	2	42.00	117.63	30	41	38.00	34	2	39.00	34	2	42.00	117.63
	1.25	30	44	21.00	34	2	22.00	34	2	26.00	116.79	30	41	21.00	34	2	22.00	34	2	26.00	116.79	30	41	21.00	34	2	22.00	34	2	26.00	116.79
	1.50	30	44	14.50	1	1	14.50	1	1	14.50	115.95	30	41	14.50	1	1	14.50	1	1	14.50	115.95	30	41	14.50	1	1	14.50	1	1	14.50	115.95
	1.75	30	44	14.25	1	1	14.25	1	1	14.25	115.11	30	41	14.25	1	1	14.25	1	1	14.25	115.11	30	41	14.25	1	1	14.25	1	1	14.25	115.11
	2.00	30	44	14.00	1	1	14.00	1	1	14.00	114.27	30	41	14.00	1	1	14.00	1	1	14.00	114.27	30	41	14.00	1	1	14.00	1	1	14.00	114.27
LA30T29-5	0.25	30	50	57.00	44	2	57.00	44	2	57.75	78.29	30	42	63.25	33	3	65.00	36	2	66.75	87.29	30	42	63.25	33	3	65.00	36	2	66.75	87.29
	0.50	30	50	40.50	49	1	40.50	49	1	40.50	77.58	30	42	49.00	25	2	49.00	25	2	49.50	86.58	30	42	49.00	25	2	49.00	25	2	49.50	86.58
	0.75	30	50	28.25	49	1	28.25	49	1	28.25	76.88	30	42	36.50	25	2	36.50	25	2	37.25	85.88	30	42	36.50	25	2	36.50	25	2	37.25	85.88
	1.00	30	50	16.00	49	1	16.00	49	1	16.00	76.17	30	42	24.00	25	2	24.00	25	2	25.00	85.17	30	42	24.00	25	2	24.00	25	2	25.00	85.17
	1.25	30	50	6.50	6	1	6.50	6	1	6.50	75.46	30	42	11.50	25	2	11.50	25	2	12.75	84.46	30	42	11.50	25	2	11.50	25	2	12.75	84.46
	1.50	30	50	5.00	6	1	5.00	6	1	5.00	74.75	30	42	5.00	6	1	5.00	6	1	5.00	83.75	30	42	5.00	6	1	5.00	6	1	5.00	83.75
	1.75	30	50	3.50	6	1	3.50	6	1	3.50	74.04	30	42	3.50	6	1	3.50	6	1	3.50	83.04	30	42	3.50	6	1	3.50	6	1	3.50	83.04
	2.00	30	50	2.00	6	1	2.00	6	1	2.00	73.33	30	42	2.00	6	1	2.00	6	1	2.00	82.33	30	42	2.00	6	1	2.00	6	1	2.00	82.33
LA30T29-6	0.25	30	44	55.00	40	2	55.00	40	2	55.75	76.24	30	42	75.00	40	2	75.00	40	2	75.75	96.24	30	42	75.00	40	2	75.00	40	2	75.75	96.24
	0.50	30	44	35.00	40	2	35.00	40	2																						



**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET											HIGH DEMAND VARIABILITY DATA SET																		
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP							
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\pi$	CT	K	$\pi$	CT	K			
LA30T29-8	0.25	30	44	66.50	41	2	66.50	41	2	67.75	86.36	30	40	65.75	27	3	65.75	27	3	67.75	86.36	30	40	65.75	27	3	65.75	27	3	67.75	86.36
	0.50	30	44	45.50	27	3	46.00	41	2	49.50	85.72	30	40	45.50	27	3	45.50	27	3	49.50	85.72	30	40	45.50	27	3	45.50	27	3	49.50	85.72
	0.75	30	44	25.25	27	3	25.50	41	2	31.25	85.08	30	40	25.25	27	3	25.25	27	3	31.25	85.08	30	40	25.25	27	3	25.25	27	3	31.25	85.08
	1.00	30	44	5.00	27	3	5.00	11	1	13.00	84.43	30	40	5.00	27	3	5.00	27	3	13.00	84.43	30	40	5.00	27	3	5.00	27	3	13.00	84.43
	1.25	30	44	2.25	11	1	2.25	11	1	2.25	83.79	30	40	3.00	32	1	3.00	32	1	3.00	83.79	30	40	3.00	32	1	3.00	32	1	3.00	83.79
	1.50	30	44	0.00	0	0	0.00	0	0	0.86	83.15	30	40	0.00	0	0	0.00	0	0	0.86	83.15	30	40	0.00	0	0	0.00	0	0	0.86	83.15
	1.75	30	44	0.00	0	0	0.00	0	0	0.61	82.51	30	40	0.00	0	0	0.00	0	0	0.61	82.51	30	40	0.00	0	0	0.00	0	0	0.61	82.51
	2.00	30	44	0.00	0	0	0.00	0	0	0.36	81.87	30	40	0.00	0	0	0.00	0	0	0.36	81.87	30	40	0.00	0	0	0.00	0	0	0.36	81.87
LA30T29-9	0.25	30	44	88.00	30	2	88.00	30	2	88.75	106.36	30	40	96.50	33	2	96.50	33	2	97.25	112.48	30	40	96.50	33	2	96.50	33	2	97.25	112.48
	0.50	30	44	73.00	30	2	73.00	30	2	74.50	105.72	30	40	80.00	33	2	80.00	33	2	81.50	111.95	30	40	80.00	33	2	80.00	33	2	81.50	111.95
	0.75	30	44	58.00	30	2	58.00	30	2	60.25	105.08	30	40	63.50	33	2	64.00	30	2	65.75	111.43	30	40	63.50	33	2	64.00	30	2	65.75	111.43
	1.00	30	44	43.00	30	2	43.00	44	1	46.00	104.43	30	40	47.00	33	2	49.00	30	2	50.00	110.90	30	40	47.00	33	2	49.00	30	2	50.00	110.90
	1.25	30	44	28.75	33	1	32.00	44	1	32.25	103.79	30	40	30.50	33	2	34.00	30	2	34.25	110.38	30	40	30.50	33	2	34.00	30	2	34.25	110.38
	1.50	30	44	21.00	30	1	21.00	30	1	21.00	103.15	30	40	21.00	30	1	21.00	30	1	21.00	109.85	30	40	21.00	30	1	21.00	30	1	21.00	109.85
	1.75	30	44	16.75	11	1	16.75	11	1	16.75	102.51	30	40	16.75	11	1	16.75	11	1	16.75	109.33	30	40	16.75	11	1	16.75	11	1	16.75	109.33
	2.00	30	44	14.00	11	1	14.00	11	1	14.00	101.87	30	40	14.00	11	1	14.00	11	1	14.00	108.80	30	40	14.00	11	1	14.00	11	1	14.00	108.80
AKO30T12-0	0.25	30	44	320.00	38	4	320.00	38	4	321.00	668.03	30	41	415.00	38	4	415.00	38	4	416.00	879.97	30	41	415.00	38	4	415.00	38	4	416.00	879.97
	0.50	30	44	282.00	38	4	283.00	36	4	284.00	666.69	30	41	377.00	38	4	378.00	36	4	379.00	878.38	30	41	377.00	38	4	378.00	36	4	379.00	878.38
	0.75	30	44	244.00	37	4	247.00	36	4	248.50	665.34	30	41	339.00	37	4	342.00	36	4	343.50	876.80	30	41	339.00	37	4	342.00	36	4	343.50	876.80
	1.00	30	44	207.00	37	4	211.00	36	4	213.00	664.00	30	41	302.00	37	4	306.00	36	4	308.00	875.22	30	41	302.00	37	4	306.00	36	4	308.00	875.22
	1.25	30	44	170.00	37	4	175.00	36	4	177.50	662.66	30	41	265.00	37	4	270.00	36	4	272.50	873.65	30	41	265.00	37	4	270.00	36	4	272.50	873.65
	1.50	30	44	133.00	37	4	139.00	36	4	142.00	661.32	30	41	228.00	37	4	234.00	36	4	237.00	872.08	30	41	228.00	37	4	234.00	36	4	237.00	872.08
	1.75	30	44	96.00	37	4	103.00	36	4	106.50	659.98	30	41	191.00	37	4	198.00	36	4	201.50	870.50	30	41	191.00	37	4	198.00	36	4	201.50	870.50
	2.00	30	44	59.00	37	4	67.00	36	4	71.00	658.63	30	41	154.00	37	4	162.00	36	4	166.00	868.93	30	41	154.00	37	4	162.00	36	4	166.00	868.93
AKO30T12-1	0.25	30	44	420.00	30	6	421.00	44	4	422.00	944.67	30	40	420.00	30	6	420.00	30	6	422.00	998.47	30	40	420.00	30	6	420.00	30	6	422.00	998.47
	0.50	30	44	375.00	30	6	377.00	44	4	379.00	943.17	30	40	375.00	30	6	375.00	30	6	379.00	996.84	30	40	375.00	30	6	375.00	30	6	379.00	996.84
	0.75	30	44	330.00	30	6	333.00	44	4	336.00	941.67	30	40	330.00	30	6	330.00	30	6	336.00	995.20	30	40	330.00	30	6	330.00	30	6	336.00	995.20
	1.00	30	44	285.00	30	6	289.00	44	4	293.00	940.17	30	40	285.00	30	6	286.00	40	4	293.00	993.57	30	40	285.00	30	6	286.00	40	4	293.00	993.57
	1.25	30	44	240.00	30	6	246.00	40	4	250.00	938.67	30	40	240.00	30	6	246.00	40	4	250.00	991.94	30	40	240.00	30	6	246.00	40	4	250.00	991.94
	1.50	30	44	206.00	40	4	208.00	30	5	209.00	937.17	30	40	206.00	40	4	208.00	30	5	209.00	990.31	30	40	206.00	40	4	208.00	30	5	209.00	990.31
	1.75	30	44	161.75	31	5	170.50	30	5	170.50	935.67	30	40	161.75	31	5	170.50	30	5	170.50	988.68	30	40	161.75	31	5	170.50	30	5	170.50	988.68
	2.00	30	44	123.00	31	5	133.00	30	5	133.00	934.18	30	40	123.00	31	5	133.00	30	5	133.00	987.05	30	40	123.00	31	5	133.00	30	5	133.00	987.05
AKO30T12-2	0.25	30	44	415.00	22	8	416.00	43	4	417.50	775.21	30	41	415.00	22	8	415.25	35	5	417.50	845.11	30	41	415.00	22	8	415.25	35	5	417.50	845.11
	0.50	30	44	371.00	22	8	373.00	43	4	376.00	773.65	30	41	371.00	22	8	371.50	35	5	376.00	843.30	30	41	371.00	22	8	371.50	35	5	376.00	843.30
	0.75	30	44	327.00	22	8	330.00	43	4	334.50	772.09	30	41	327.00	22	8	327.75	35	5	334.50	841.49	30	41	327.00	22	8	327.75	35	5	334.50	841.49
	1.00	30	44	283.00	22	8	287.00	43	4	293.00	770.54	30	41	283.00	22	8	284.00	35	5	293.00	839.68	30	41	283.00	22	8	284.00	35	5	293.00	839.68
	1.25	30	44	239.00	22	8	244.00	43	4	251.50	768.98	30	41	239.00	22	8	240.25	35	5	251.50	837.87	30	41	239.00	22	8	240.25	35	5	251.50	837.87
	1.50	30	44	195.00	22	8	201.00	43	4	210.00	767.43	30	41	195.00	22	8	196.50	35	5	210.00	836.06	30	41	195.00	22	8	196.50	35	5	210.00	836.06
	1.75	30	44	151.00	22	8	158.00	43	4	168.50	765.88	30	41	151.00	22	8	152.75	35	5	168.50	834.25	30	41	151.00	22	8	152.75	35	5	168.50	834.25
	2.00	30	44	107.00	22	8	115.00	43	4	127.00	764.33	30	41	107.00	22	8	110.00	41	4	127.00	832.45	30	41	107.00	22	8	110.00	41	4	127.00	832.45
AKO30T12-3	0.25	30	50	491.00	46	4	491.00	46	4	492.50	828.34	30	42	491.00	37	5	491.00	37	5	492.50	1208.05	30	42	491.00	37	5	491.00	37	5	492.50	1208.05
	0.50	30	50	445.00	46	4	445.00	46	4	448.00	827.13	30	42	445.00	37	5	445.00	37	5	448.00	1205.96	30	42	445.00	37	5	445.00	37	5	448.00	1205.96
	0.75	30	50	397.00	44	4	401.50	34	5	403.75	825.93	30	42	397.00	36	5	397.00	34	5	401.50	1203.86	30	42	397.00	36	5	397.00	34	5	401.50	1203.86
	1.00	30	50	353.00	44	4	359.00	34	5	362.00	824.73	30	42	353.00	36	5	353.00	34	5	362.00	1201.76	30	42	353.00	36	5	353.00	34	5	362.00	1201.76
	1.25	30	50	309.00	44	4	316.50	34	5	320.25	823.54	30																			

**Table F.3** Results of the heuristic solution procedure (Continued)

INSTANCE	S	LOW DEMAND VARIABILITY DATA SET											HIGH DEMAND VARIABILITY DATA SET												
		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP		$K_U$	$CT_U$	LB Solution			PC Solution			UB		PC-LP	
				$\pi$	CT	K	$\pi$	CT	K	$\pi$	$\bar{\pi}$	$\pi$	CT			K	$\pi$	CT	K	$\pi$	CT	K	$\bar{\pi}$	$\pi$	
AKO30T12-5	0.25	30	44	200.50	38	3	202.00	36	3	203.00	495.81	30	40	250.50	38	3	252.00	36	3	253.00	654.34				
	0.50	30	44	172.00	38	3	175.00	36	3	177.00	495.01	30	40	222.00	38	3	225.00	36	3	227.00	653.35				
	0.75	30	44	143.50	38	3	148.00	36	3	151.00	494.22	30	40	193.50	38	3	198.00	36	3	201.00	652.37				
	1.00	30	44	115.00	38	3	121.00	36	3	125.00	493.42	30	40	165.00	38	3	171.00	36	3	175.00	651.38				
	1.25	30	44	86.50	38	3	94.00	36	3	99.00	492.62	30	40	136.50	38	3	144.00	36	3	149.00	650.39				
	1.50	30	44	58.00	38	3	67.00	36	3	73.00	491.83	30	40	108.00	38	3	117.00	36	3	123.00	649.41				
	1.75	30	44	29.50	38	3	40.00	36	3	47.00	491.03	30	40	79.50	38	3	90.00	36	3	97.00	648.42				
	2.00	30	44	23.00	9	1	23.00	9	1	23.00	490.24	30	40	51.00	38	3	63.00	36	3	71.00	647.43				
AKO30T12-6	0.25	30	50	304.00	45	4	304.00	45	4	305.00	742.31	30	40	343.00	32	6	344.75	37	5	347.00	791.55				
	0.50	30	50	259.00	45	4	259.00	45	4	261.00	740.91	30	40	295.00	32	6	298.50	37	5	303.00	790.14				
	0.75	30	50	214.00	45	4	214.00	45	4	217.00	739.51	30	40	247.00	32	6	253.25	33	5	259.00	788.73				
	1.00	30	50	170.00	33	5	171.00	41	4	176.00	738.11	30	40	212.00	33	5	212.00	33	5	218.00	787.33				
	1.25	30	50	128.75	33	5	130.00	41	4	136.25	736.71	30	40	170.75	33	5	170.75	33	5	178.25	785.93				
	1.50	30	50	87.50	33	5	89.00	41	4	96.50	735.31	30	40	129.50	33	5	129.50	33	5	138.50	784.52				
	1.75	30	50	46.25	33	5	55.00	36	1	56.75	733.91	30	40	88.25	33	5	88.25	33	5	98.75	783.12				
	2.00	30	50	46.00	36	1	46.00	36	1	46.00	732.51	30	40	47.00	33	5	51.00	37	4	59.00	781.72				
AKO30T12-7	0.25	30	44	270.50	30	5	272.00	36	4	273.50	621.69	30	43	320.50	30	5	322.00	36	4	323.50	743.37				
	0.50	30	44	233.00	30	5	236.00	36	4	239.00	620.60	30	43	283.00	30	5	286.00	36	4	289.00	741.98				
	0.75	30	44	195.50	30	5	200.00	36	4	204.50	619.51	30	43	245.50	30	5	250.00	36	4	254.50	740.59				
	1.00	30	44	158.00	30	5	164.00	36	4	170.00	618.42	30	43	208.00	30	5	214.00	36	4	220.00	739.22				
	1.25	30	44	120.50	30	5	128.00	36	4	135.50	617.33	30	43	170.50	30	5	178.00	36	4	185.50	737.84				
	1.50	30	44	83.00	30	5	92.00	36	4	101.00	616.24	30	43	133.00	30	5	142.00	36	4	151.00	736.47				
	1.75	30	44	45.50	30	5	56.00	36	4	66.50	615.15	30	43	95.50	30	5	106.00	36	4	116.50	735.09				
	2.00	30	44	44.00	37	1	44.00	37	1	44.00	614.05	30	43	58.00	30	5	70.00	36	4	82.00	733.72				
AKO30T12-8	0.25	30	44	204.00	39	4	205.00	38	4	205.75	558.37	30	40	300.00	39	4	301.00	38	4	301.75	767.51				
	0.50	30	44	165.00	39	4	167.00	38	4	168.50	557.17	30	40	261.00	39	4	263.00	38	4	264.50	765.98				
	0.75	30	44	126.75	37	3	131.25	35	3	132.00	555.96	30	40	222.00	39	4	225.00	38	4	227.25	764.46				
	1.00	30	44	99.00	37	3	105.00	35	3	106.00	554.75	30	40	183.00	39	4	187.00	38	4	190.00	762.93				
	1.25	30	44	67.00	28	3	78.75	35	3	81.38	553.55	30	40	144.00	39	4	149.00	38	4	152.75	761.41				
	1.50	30	44	46.00	30	2	54.00	40	2	59.75	552.34	30	40	105.00	39	4	111.00	38	4	115.50	759.88				
	1.75	30	44	31.00	30	2	34.00	40	2	41.63	551.14	30	40	66.00	39	4	73.00	38	4	78.25	758.35				
	2.00	30	44	16.00	30	2	16.00	30	2	24.00	549.93	30	40	27.00	39	4	35.00	38	4	41.00	756.83				
AKO30T12-9	0.25	30	50	139.00	26	4	139.50	34	3	139.50	399.94	30	42	272.00	26	4	272.50	34	3	272.50	660.30				
	0.50	30	50	113.00	26	4	114.00	34	3	114.00	399.15	30	42	246.00	26	4	247.00	34	3	247.00	659.15				
	0.75	30	50	87.00	26	4	88.50	34	3	88.50	398.36	30	42	220.00	26	4	221.50	34	3	221.50	658.02				
	1.00	30	50	61.00	26	4	63.00	34	3	63.00	397.56	30	42	194.00	26	4	196.00	34	3	196.00	656.90				
	1.25	30	50	17.00	36	3	37.50	34	3	37.50	396.77	30	42	168.00	26	4	170.50	34	3	170.50	655.78				
	1.50	30	50	16.50	25	1	26.50	41	1	27.75	395.98	30	42	142.00	26	4	145.00	34	3	145.00	654.66				
	1.75	30	50	10.25	25	1	16.25	41	1	18.38	395.19	30	42	116.00	26	4	119.50	34	3	119.50	653.54				
	2.00	30	50	4.00	25	1	6.00	41	1	9.00	394.40	30	42	90.00	26	4	94.00	34	3	94.00	652.42				

## APPENDIX G

### REDUNDANCY CHECK FOR PH FORMULATION

Due to the numerous constraints involved with the PH formulation, an approach to identify redundant constraints involved with WIP accumulation and usage is proposed. The idea is implemented separately for each main precedence type.

For each constraint related with a main precedence relation type, we enumerate all possible decisions as to whether or not WIP is accumulated, WIP is used, the main disassembly is further disassembled and precedence relations are satisfied (see Table G.1). Each decision has a binary verdict, leading to  $2^4$  possible combinations. For each decision, a value of 1 represents that the verdict is affirmative, (i.e. Accumulate WIP=1 means WIP is accumulated) and 0 represents the opposite.

Without loss of generality, consider AND precedence relation type. In PH the corresponding constraints are (5.16) - (5.18), (5.20), (5.22) and (5.23). Consider constraint (5.16) which ensures that a subassembly is either accumulated in respective WIP or further disassembled by the downstream task. Thus it enforces us to do at most one of the following. Accumulate WIP (that is placed before the downstream task and do not disassemble further) or disassemble that subassembly further by performing the downstream task. This means that accumulate WIP and further disassemble decisions cannot be affirmative (hence their corresponding variables cannot take a value of 1) simultaneously. In Table G.1, among the 16 possibilities generated, the realization of the first four with simultaneous 1s are banned by constraint (5.16). We depict this by shading these combinations with gray under the column of constraint (5.16). Similarly

consider constraint (5.17) that guarantees further disassembly of a main subassembly if it is used from the WIP inventory. This constraint does not allow an affirmative verdict for the use WIP decision while the verdict of to disassemble further decision is zero. Thus it prohibits combinations numbered 5, 6, 13 and 14 and we illustrate this by shading the corresponding rows. When the other four constraints are analyzed with regard to the 16 possibilities, we obtain Table G.2.

**Table G.1** Enumerated decisions with respect to WIP accumulation and usage

Decision Number	Accumulate WIP	Disassemble further	Use WIP	All AND Pred.s are done	(5.16)	(5.17)
1	1	1	1	1		
2	1	1	1	0		
3	1	1	0	1		
4	1	1	0	0		
5	1	0	1	1		
6	1	0	1	0		
7	1	0	0	1		
8	1	0	0	0		
9	0	1	1	1		
10	0	1	1	0		
11	0	1	0	1		
12	0	1	0	0		
13	0	0	1	1		
14	0	0	1	0		
15	0	0	0	1		
16	0	0	0	0		

In a similar manner, we enumerate the 16 decisions for OR precedence and OR successor relations and present them in Tables G.3 and G.4.

By analyzing Tables G.2-G.4 we observe the following.

1. None of the constraints are redundant. The area shaded by one constraint never dominates the area shaded by another.
2. In all precedence relation types, among the 16 alternatives generated, the corresponding constraints allow only the 4 cases described below.

**Table G.2** Redundancy check for AND Precedence Constraints

Decision Number	Accumulate WIP	Disassemble further	Use WIP	All AND Pred.s are done	(5.16)	(5.17)	(5.18)	(5.20)	(5.22)	(5.23)
1	1	1	1	1						
2	1	1	1	0						
3	1	1	0	1						
4	1	1	0	0						
5	1	0	1	1						
6	1	0	1	0						
7	1	0	0	1						
8	1	0	0	0						
9	0	1	1	1						
10	0	1	1	0						
11	0	1	0	1						
12	0	1	0	0						
13	0	0	1	1						
14	0	0	1	0						
15	0	0	0	1						
16	0	0	0	0						

**Table G.3** Redundancy check for OR Precedence Constraints

Decision Number	Accumulate WIP	Disassemble further	Use WIP	At least one OR Pred. is done	(5.16)	(5.17)	(5.19)	(5.21)	(5.24)	(5.25)
1	1	1	1	1						
2	1	1	1	0						
3	1	1	0	1						
4	1	1	0	0						
5	1	0	1	1						
6	1	0	1	0						
7	1	0	0	1						
8	1	0	0	0						
9	0	1	1	1						
10	0	1	1	0						
11	0	1	0	1						
12	0	1	0	0						
13	0	0	1	1						
14	0	0	1	0						
15	0	0	0	1						
16	0	0	0	0						

**Table G.4** Redundancy check for OR Successor Constraints

Decision Number	Accumulate WIP	Disassemble further	Use WIP	Task with OR successors is done	(5.30)	(5.27)	(5.29)&(5.39)	(5.26)	(5.31)	(5.32)
1	1	1	1	1						
2	1	1	1	0						
3	1	1	0	1						
4	1	1	0	0						
5	1	0	1	1						
6	1	0	1	0						
7	1	0	0	1						
8	1	0	0	0						
9	0	1	1	1						
10	0	1	1	0						
11	0	1	0	1						
12	0	1	0	0						
13	0	0	1	1						
14	0	0	1	0						
15	0	0	0	1						
16	0	0	0	0						

**Case 1a** (for AND and OR precedence): If all AND predecessors of a task (at least one of the OR predecessors) are completed and further disassembly is not performed by that task, WIP is accumulated. This case is represented by decision number 7 in Tables G.2 and G.3.

**Case 2a** (for AND and OR precedence): If all AND predecessors of a task (at least one of the OR predecessors) are not completed, but subassemblies from the preceding WIP are used, then further disassembly is performed by that task. This case is represented by decision number 10 in Tables G.2 and G.3.

**Case 3a** (for AND and OR precedence): If all AND predecessors of a task (at least one of the OR predecessors) are finished, and WIP is not accumulated, then further disassembly must be conducted by that task. This case is represented by decision number 11 in Tables G.2 and G.3.

**Case 4a** (for AND and OR precedence): This case, which corresponds to do nothing, is represented by decision number 16 in Tables G.2 and G.3.

**Case 1b** (OR successor): If a task that has OR successors is performed, but none of its successors are performed (leading to no further disassembly decision), WIP is accumulated after that task. This case is represented by decision number 7 in Table G.4.

**Case 2b** (OR successor): If a task that has OR successors is not performed, but the WIP placed after it is used, then further disassembly is performed by one of the OR successors of task  $i$ . This case is represented by decision number 10 in Table G.4.

**Case 3b** (OR successor): If task  $i$  is performed, and WIP is not accumulated, then further disassembly is performed by one of the OR successors of that task. This case is represented by decision number 11 in Table G.4.

**Case 4b** (OR successor): This case, which corresponds to do nothing, is represented by decision number 16 in Table G.4.

## APPENDIX H

### DECOMPOSED AND SEMI-DIRECT SOLUTION PROCEDURE EXAMPLE

We solve the 10-part ball-point pen example under finite supply, subassembly WIP availability, a station cost of 0.25 and an inventory carrying charge of 0.001.

The solution provided by the decomposed solution procedure with variable number of stations (PHD-VK) is depicted in Figure H.1. The solution procedures of decomposed and semi-direct solution procedures with  $\lceil K_{\text{avg}} \rceil$  stations (PHD-FK<sup>+</sup> and PHS-FK<sup>+</sup>, respectively) are shown in Figures H.2 and H.3. Finally, the solution procedures of decomposed and semi-direct solution procedures with  $\lfloor K_{\text{avg}} \rfloor$  stations (PHD-FK<sup>-</sup> and PHS-FK<sup>-</sup>, respectively) are shown in Figures H.4 and H.5.

Zone		0	1	2	3		
-----							
$\alpha$		0	1	1	1		
$CT_L$		0	1	1	1		
$CT$		0	53	17	31		
$CT_U$		0	231	231	231		
$K$		0	3	4	3		
PlanHorUsed		0	53	70	101		
Profit	0.000	268.417	118.644	97.787			
PartRevenue	0.000	457.000	195.000	195.000			
TaskCost	0.000	146.000	58.000	70.000			
StationCost	0.000	39.750	17.000	23.250			
HoldingCost	0.000	2.833	1.356	3.963			
TotalProfit	0.000	268.417	387.061	484.848			
-----							
Task time	1- 6- 8	0	1	0	0	} Assignment of tasks to stations in each zone	
	2- 3- 3	0	0	3	1		
	3- 7- 2	0	0	0	1		
	4- 2- 8	0	0	1	1		
	5-17- 4	0	0	0	0		
	6-16-14	0	1	0	0		
Task cost	7-15- 2	0	1	1	2		
	8- 9-14	0	0	0	0		
	9-16-10	0	1	2	2		
	10-16-11	0	0	0	0		
	11-20- 7	0	2	0	0		
	12-19- 5	0	0	0	0		
	13- 9-15	0	3	0	0		
	14-14-11	0	2	3	3		
	15-19-10	0	2	0	1		
	16-18-15	0	3	0	0		
	17- 2-16	0	3	0	0		
	18-16-12	0	3	4	3		
	19- 6-14	0	3	0	0		
20- 1-12	0	3	2	3			
A1- 0- 0	0	1	0	0			
A2- 0- 0	0	1	1	1			
A3- 0- 0	0	1	0	0			
-----							
Part demand	A- 0- 1	0	0	0	0	} Number of units of parts released in each zone	
	B- 3- 1	0	1	1	1		
	C- 2- 1	0	2	0	0		
	D- 4- 1	0	1	1	1		
	E- 3- 1	0	1	1	1		
	F- 0- 1	0	0	0	0		
	G- 6- 1	0	1	1	1		
	H- 2- 1	0	2	0	0		
	I- 6- 1	0	3	1	1		
	J- 1- 1	0	1	0	0		
	-----						
Part revenue	A- 0 44	0	1	2	3	} Ending RP inventory levels	
	B- 0 40	0	0	0	0		
	C- 0 35	0	0	0	1		
	D- 0 36	0	0	0	0		
	E- 0 43	0	0	0	0		
	F- 0 43	0	1	2	3		
	G- 0 40	0	0	0	0		
	H- 0 39	0	0	1	2		
Beg. inventory levels	I- 0 36	0	0	0	0		
	J- 0 42	0	1	2	3		
	-----						
	Beg. WIP inventory levels	0- 3	3	2	1	0	} Ending WIP inventory levels
		OmD- 1	1	1	2	2	
		EnJ- 2	2	2	1	1	
		AnD- 0	0	0	0	0	
		ABC- 1	1	0	0	0	
AB- 1		1	1	0	0		
CD- 0		0	0	0	0		
EnI- 1		1	0	0	0		
HnJ- 0		0	0	0	0		
EnH- 1		1	1	1	1		
EFG- 0		0	0	0	0		
HI- 0		0	0	0	0		
EF- 0		0	0	0	0		
IJ- 1		1	0	0	0		
OT- 0		0	0	0	0		
EnJT- 0		0	0	0	0		
AnDT- 0		0	0	0	0		
HnJT- 0		0	0	0	0		

Figure H.1 PHD-VK Solution



Zone	0	1	2	3
$\alpha$	0	1	1	1
$CT_L$	0	1	1	1
$CT$	0	41	17	29
$CT_U$	0	231	231	231
$K$	4	4	4	4
PlHUsed	0	41	58	87
Profit	0.000	267.808	118.644	92.293
TPReven	0.000	457.000	195.000	195.000
TTCost	0.000	146.000	58.000	70.000
TSCost	0.000	41.000	17.000	29.000
THCost	0.000	2.192	1.356	3.707
TProfit	0.000	267.808	386.452	478.745
-----				
1- 6- 8	0	1	0	0
2- 3- 3	0	0	3	1
3- 7- 2	0	0	0	1
4- 2- 8	0	0	1	1
5-17- 4	0	0	0	0
6-16-14	0	1	0	0
7-15- 2	0	1	1	3
8- 9-14	0	0	0	0
9-16-10	0	2	2	1
10-16-11	0	0	0	0
11-20- 7	0	3	0	0
12-19- 5	0	0	0	0
13- 9-15	0	2	0	0
14-14-11	0	2	3	3
15-19-10	0	3	0	2
16-18-15	0	4	0	0
17- 2-16	0	2	0	0
18-16-12	0	4	4	4
19- 6-14	0	4	0	0
20- 1-12	0	1	2	2
A1- 0- 0	0	1	0	0
A2- 0- 0	0	1	1	1
A3- 0- 0	0	1	0	0
-----				
A- 0- 1	0	0	0	0
B- 3- 1	0	1	1	1
C- 2- 1	0	2	0	0
D- 4- 1	0	1	1	1
E- 3- 1	0	1	1	1
F- 0- 1	0	0	0	0
G- 6- 1	0	1	1	1
H- 2- 1	0	2	0	0
I- 6- 1	0	3	1	1
J- 1- 1	0	1	0	0
-----				
A- 0 44	0	1	2	3
B- 0 40	0	0	0	0
C- 0 35	0	0	0	1
D- 0 36	0	0	0	0
E- 0 43	0	0	0	0
F- 0 43	0	1	2	3
G- 0 40	0	0	0	0
H- 0 39	0	0	1	2
I- 0 36	0	0	0	0
J- 0 42	0	1	2	3
-----				
0- 3	3	2	1	0
0mD- 1	1	1	2	2
EnJ- 2	2	2	1	1
AnD- 0	0	0	0	0
ABC- 1	1	0	0	0
AB- 1	1	1	0	0
CD- 0	0	0	0	0
EnI- 1	1	0	0	0
HnJ- 0	0	0	0	0
EnH- 1	1	1	1	1
EFG- 0	0	0	0	0
HI- 0	0	0	0	0
EF- 0	0	0	0	0
IJ- 1	1	0	0	0
OT- 0	0	0	0	0
EnJT- 0	0	0	0	0
AnDT- 0	0	0	0	0
HnJT- 0	0	0	0	0

Figure H.2 PHD-FK<sup>+</sup> solution with  $[K_{avg}]$

Zone	0	1	2	3
$\alpha$	0	1	1	1
$CT_L$	0	34	31	17
$CT$	0	231	231	231
$CT_U$	0	231	231	231
$K$	4	4	4	4
PlHUsed	0	34	65	82
Profit	0.000	232.269	186.640	118.056
TPReven	0.000	347.000	305.000	195.000
TTCost	0.000	79.000	85.000	58.000
TSCost	0.000	34.000	31.000	17.000
THCost	0.000	1.731	2.360	1.944
TProfit	0.000	232.269	418.909	536.965
-----				
1- 6- 8	0	0	0	0
2- 3- 3	0	3	1	3
3- 7- 2	0	0	1	0
4- 2- 8	0	0	1	1
5-17- 4	0	1	0	0
6-16-14	0	0	0	0
7-15- 2	0	3	2	1
8- 9-14	0	0	0	0
9-16-10	0	1	2	2
10-16-11	0	0	0	0
11-20- 7	0	0	0	0
12-19- 5	0	0	0	0
13- 9-15	0	0	0	0
14-14-11	0	2	3	3
15-19-10	0	2	1	0
16-18-15	0	4	4	0
17- 2-16	0	0	0	0
18-16-12	0	4	3	4
19- 6-14	0	0	0	0
20- 1-12	0	3	4	4
A1- 0- 0	0	0	0	0
A2- 0- 0	0	1	1	1
A3- 0- 0	0	0	0	0
-----				
A- 0- 1	0	0	0	0
B- 3- 1	0	1	1	1
C- 2- 1	0	1	1	0
D- 4- 1	0	1	1	1
E- 3- 1	0	1	1	1
F- 0- 1	0	0	0	0
G- 6- 1	0	1	1	1
H- 2- 1	0	1	1	0
I- 6- 1	0	2	2	1
J- 1- 1	0	1	0	0
-----				
A- 0 44	0	1	2	3
B- 0 40	0	0	0	0
C- 0 35	0	0	0	0
D- 0 36	0	0	0	0
E- 0 43	0	0	0	0
F- 0 43	0	1	2	3
G- 0 40	0	0	0	0
H- 0 39	0	0	0	1
I- 0 36	0	0	0	0
J- 0 42	0	0	2	3
-----				
0- 3	3	2	1	0
0mD- 1	1	2	2	3
EnJ- 2	2	1	1	0
AnD- 0	0	0	0	0
ABC- 1	1	0	0	0
AB- 1	1	1	1	0
CD- 0	0	0	0	0
EnI- 1	1	0	0	0
HnJ- 0	0	0	0	0
EnH- 1	1	1	1	1
EFG- 0	0	0	0	0
HI- 0	0	0	0	0
EF- 0	0	0	0	0
IJ- 1	1	1	0	0
OT- 0	0	0	0	0
EnJT- 0	0	0	0	0
AnDT- 0	0	0	0	0
HnJT- 0	0	0	0	0

Figure H.3 PHS-FK<sup>+</sup> solution with  $[K_{avg}]$

Zone	0	1	2	3
$\alpha$	0	1	1	1
$CT_L$	0	1	1	1
$CT$	0	53	29	31
$CT_U$	0	231	231	231
$K$	3	3	3	3
PlHUsed	0	53	82	113
Profit	0.000	268.417	112.937	97.787
TPReven	0.000	457.000	195.000	195.000
TTCost	0.000	146.000	58.000	70.000
TSCost	0.000	39.750	21.750	23.250
THCost	0.000	2.833	2.313	3.963
TProfit	0.000	268.417	381.354	479.141
-----				
1- 6- 8	0	1	0	0
2- 3- 3	0	0	1	1
3- 7- 2	0	0	0	1
4- 2- 8	0	0	1	1
5-17- 4	0	0	0	0
6-16-14	0	1	0	0
7-15- 2	0	1	2	2
8- 9-14	0	0	0	0
9-16-10	0	1	1	2
10-16-11	0	0	0	0
11-20- 7	0	2	0	0
12-19- 5	0	0	0	0
13- 9-15	0	3	0	0
14-14-11	0	2	2	3
15-19-10	0	2	0	1
16-18-15	0	3	0	0
17- 2-16	0	3	0	0
18-16-12	0	3	3	3
19- 6-14	0	3	0	0
20- 1-12	0	3	1	3
A1- 0- 0	0	1	0	0
A2- 0- 0	0	1	1	1
A3- 0- 0	0	1	0	0
-----				
A- 0- 1	0	0	0	0
B- 3- 1	0	1	1	1
C- 2- 1	0	2	0	0
D- 4- 1	0	1	1	1
E- 3- 1	0	1	1	1
F- 0- 1	0	0	0	0
G- 6- 1	0	1	1	1
H- 2- 1	0	2	0	0
I- 6- 1	0	3	1	1
J- 1- 1	0	1	0	0
-----				
A- 0 44	0	1	2	3
B- 0 40	0	0	0	0
C- 0 35	0	0	0	1
D- 0 36	0	0	0	0
E- 0 43	0	0	0	0
F- 0 43	0	1	2	3
G- 0 40	0	0	0	0
H- 0 39	0	0	1	2
I- 0 36	0	0	0	0
J- 0 42	0	1	2	3
-----				
0- 3	3	2	1	0
0mD- 1	1	1	2	2
EnJ- 2	2	2	1	1
AnD- 0	0	0	0	0
ABC- 1	1	0	0	0
AB- 1	1	1	0	0
CD- 0	0	0	0	0
EnI- 1	1	0	0	0
HnJ- 0	0	0	0	0
EnH- 1	1	1	1	1
EFG- 0	0	0	0	0
HI- 0	0	0	0	0
EF- 0	0	0	0	0
IJ- 1	1	0	0	0
OT- 0	0	0	0	0
EnJT- 0	0	0	0	0
AnDT- 0	0	0	0	0
HnJT- 0	0	0	0	0

Figure H.4 PHD-FK solution with  $[K_{avg}]$

Zone	0	1	2	3
$\alpha$	0	1	1	1
$CT_L$	0	1	1	1
$CT$	0	31	34	35
$CT_U$	0	231	231	231
$K$	3	3	3	3
PlHUsed	0	31	65	100
Profit	0.000	211.948	176.773	152.858
TPReven	0.000	312.000	269.000	266.000
TTCost	0.000	75.000	64.000	83.000
TSCost	0.000	23.250	25.500	26.250
THCost	0.000	1.802	2.727	3.892
TProfit	0.000	211.948	388.720	541.578
-----				
1- 6- 8	0	0	0	0
2- 3- 3	0	1	2	3
3- 7- 2	0	1	0	0
4- 2- 8	0	1	0	1
5-17- 4	0	0	3	0
6-16-14	0	0	0	0
7-15- 2	0	2	1	1
8- 9-14	0	0	0	0
9-16-10	0	2	2	2
10-16-11	0	0	0	0
11-20- 7	0	0	0	0
12-19- 5	0	0	0	0
13- 9-15	0	0	0	0
14-14-11	0	3	2	3
15-19-10	0	0	1	2
16-18-15	0	1	0	1
17- 2-16	0	0	0	0
18-16-12	0	3	3	3
19- 6-14	0	0	0	0
20- 1-12	0	1	3	3
A1- 0- 0	0	0	0	0
A2- 0- 0	0	1	3	1
A3- 0- 0	0	0	0	0
-----				
A- 0- 1	0	0	0	0
B- 3- 1	0	1	1	1
C- 2- 1	0	0	1	1
D- 4- 1	0	1	1	1
E- 3- 1	0	1	1	1
F- 0- 1	0	0	0	0
G- 6- 1	0	1	1	1
H- 2- 1	0	1	1	0
I- 6- 1	0	2	1	2
J- 1- 1	0	1	0	0
-----				
A- 0 44	0	1	2	3
B- 0 40	0	0	0	0
C- 0 35	0	0	0	0
D- 0 36	0	0	0	0
E- 0 43	0	0	0	0
F- 0 43	0	1	2	3
G- 0 40	0	0	0	0
H- 0 39	0	0	0	1
I- 0 36	0	0	0	0
J- 0 42	0	1	1	3
-----				
0- 3	3	2	1	0
0mD- 1	1	1	2	3
EnJ- 2	2	2	1	0
AnD- 0	0	0	0	0
ABC- 1	1	2	1	0
AB- 1	1	0	0	0
CD- 0	0	0	0	0
EnI- 1	1	1	0	0
HnJ- 0	0	0	0	0
EnH- 1	1	1	1	1
EFG- 0	0	0	0	0
HI- 0	0	0	0	0
EF- 0	0	0	0	0
IJ- 1	1	0	1	0
OT- 0	0	0	0	0
EnJT- 0	0	0	0	0
AnDT- 0	0	0	0	0
HnJT- 0	0	0	0	0

Figure H.5 PHS-FK solution with  $[K_{avg}]$

## APPENDIX I

### COMPARISON OF PHD AND PHS

In this Appendix we present the results for problems GUN8T8 and AKO8T6 in Tables I.1 through I.4.

For each parameter combination, we report the average over 10 problem instances of the following for the PHD-VK approach.

- § Makespan (MS)
- § Percentage of demand satisfied (Sat %)
- § Profit ( $\pi$ )
- § Holding cost (H cost)
- § Number of zones ( $Z$ ),
- § Number of cycles in a zone ( $\alpha_{\text{avg}}$ )
- § Number of stations ( $K_{\text{avg}}$ ).

For the PHD-FK\* and PHS-FK\* approaches we provide the same measures except  $K_{\text{avg}}$ .

**Table I.1** PHD-VK, PHD-FK\* and PHS-FK\* solutions of problem GUN8T8 with low demand variability  
(average of 10 problem instances)

WIP	SU	S	h	PHD-VK						PHD-FK*						PHS-FK*						SD		
				MS	Sat %	$\pi$	H Cost	Z	$\alpha_{rs}$	$K_r$	$\alpha_r$	MS	Sat %	$\pi$	H Cost	Z	$\alpha_r$	MS	Sat %	$\pi$	H Cost	Z	$\alpha_r$	Gap
N	I	0.25	0.000	268.00	69.50	168.63	0.00	4.30	2.09	1.14	264.80	72.27	176.63	0.00	3.30	2.23	264.40	72.27	176.83	0.00	3.30	2.23	0.07	-3.84
N	I	0.25	0.001	213.00	70.98	169.29	5.81	4.30	2.27	1.54	214.50	66.79	168.96	5.12	3.20	2.29	230.30	72.27	167.53	8.30	3.20	2.29	-0.72	0.05
N	I	0.25	0.005	190.90	64.20	159.28	8.82	4.10	2.42	1.55	204.10	64.20	158.66	9.87	3.10	2.37	203.70	67.12	146.44	25.93	3.00	2.40	-1.12	0.39
N	I	0.75	0.000	109.00	34.19	73.65	0.00	2.90	3.43	1.00	90.30	29.52	70.20	0.00	1.70	3.17	93.30	29.52	70.20	0.00	1.70	3.17	0.00	0.00
N	I	0.75	0.001	106.20	34.45	73.56	0.06	2.90	3.43	1.06	90.30	29.78	70.11	0.06	1.70	3.17	88.80	29.52	70.04	0.16	1.70	3.17	-0.15	0.00
N	I	0.75	0.005	106.20	34.45	73.32	0.30	2.90	3.43	1.06	90.30	29.78	69.87	0.30	1.70	3.17	90.30	29.78	69.87	0.30	1.70	3.17	0.00	0.00
N	F	0.25	0.000	224.90	59.58	138.70	0.00	3.90	1.36	1.16	179.30	61.43	143.90	0.00	3.40	1.42	218.30	61.43	143.90	0.00	3.40	1.42	0.00	-2.49
N	F	0.25	0.001	166.70	60.32	137.88	4.45	3.80	1.48	1.61	161.10	56.14	137.85	4.20	3.20	1.56	172.60	58.87	137.80	5.15	3.20	1.56	0.00	-0.07
N	F	0.25	0.005	140.20	53.36	126.34	7.73	3.50	1.64	1.74	112.90	53.36	124.93	9.57	3.20	1.56	165.40	56.98	122.28	19.87	3.20	1.56	-2.74	1.08
N	F	0.75	0.000	90.50	27.75	56.73	0.00	2.60	2.17	1.00	97.00	25.08	54.93	0.00	2.10	1.60	83.00	25.08	54.93	0.00	2.10	1.60	0.00	0.00
N	F	0.75	0.001	87.70	28.01	56.70	0.00	2.60	2.17	1.06	97.60	25.34	54.90	0.00	2.10	1.60	78.50	25.08	54.82	0.11	2.10	1.60	-0.15	0.00
N	F	0.75	0.005	87.70	28.01	56.69	0.01	2.60	2.17	1.06	97.60	25.34	54.89	0.01	2.10	1.60	80.00	25.34	54.89	0.01	2.10	1.60	0.00	0.00
W	F	0.25	0.000	226.60	62.68	166.10	0.00	4.50	1.50	1.14	212.90	63.14	171.18	0.00	3.60	1.36	261.00	64.26	174.55	0.00	3.60	1.36	1.40	-2.33
W	F	0.25	0.001	129.20	64.53	159.20	9.32	4.50	1.70	2.05	165.70	62.06	156.94	10.78	3.60	1.36	133.80	63.72	159.14	11.36	3.60	1.36	0.70	1.46
W	F	0.25	0.005	88.80	56.61	132.62	25.10	4.50	1.82	2.86	113.20	55.93	124.86	28.69	3.60	1.33	91.60	63.74	128.42	30.90	3.70	1.29	1.77	5.65
W	F	0.75	0.000	110.90	36.05	77.13	0.00	4.10	1.87	1.09	119.00	36.05	77.13	0.00	3.10	1.38	119.00	36.05	77.13	0.00	3.10	1.38	0.00	0.00
W	F	0.75	0.001	79.00	33.62	67.94	7.28	4.20	1.94	1.32	89.20	34.42	67.29	8.23	3.20	1.36	83.30	36.25	66.43	8.77	3.20	1.36	-2.34	1.02
W	F	0.75	0.005	36.50	24.93	44.10	18.28	3.40	2.34	1.73	42.70	26.86	42.50	18.72	2.60	1.18	44.40	27.08	42.08	17.52	2.40	1.15	-1.49	-10.39
W	I	0.25	0.000	269.70	72.60	196.03	0.00	4.70	1.81	1.09	256.80	74.54	204.75	0.00	3.60	2.03	259.00	75.66	208.00	0.00	3.60	2.03	1.22	-4.02
W	I	0.25	0.001	177.70	75.74	189.95	12.85	4.70	2.10	1.87	206.10	73.27	187.44	14.71	3.60	2.03	190.90	76.01	189.85	14.90	3.60	2.03	0.65	1.30
W	I	0.25	0.005	128.00	65.18	160.13	28.45	4.20	2.42	2.42	147.20	65.24	150.22	38.36	3.40	2.18	146.70	71.87	142.51	53.69	3.20	2.35	-7.12	5.81
W	I	0.75	0.000	129.40	42.49	94.05	0.00	3.50	2.68	1.06	137.50	42.49	94.05	0.00	2.50	2.53	137.50	42.49	94.05	0.00	2.50	2.53	0.00	0.00
W	I	0.75	0.001	93.70	39.40	83.56	8.44	3.40	2.88	1.27	103.90	40.19	82.91	9.39	2.40	2.67	95.30	41.03	82.77	9.13	2.40	2.67	-0.13	0.93
W	I	0.75	0.005	46.80	29.37	56.02	21.63	2.70	3.14	1.65	55.80	31.30	53.57	22.78	1.70	2.75	60.10	31.89	54.41	23.19	1.70	2.75	2.03	-10.24

**Table I.2** PHD-VK, PHD-FK\* and PHS-FK\* solutions of problem GUN8T8 with high demand variability (average of 10 problem instances)

WIP	SU	S	h	PHD-VK						PHD-FK*						PHS-FK*						SD			
				MS	Sat%	$\pi$	H Cost	Z	$\alpha_{p^*}$	$K_{p^*}$	MS	Sat%	$\pi$	H Cost	Z	$\alpha_{p^*}$	MS	Sat%	$\pi$	H Cost	Z	$\alpha_{p^*}$	Gap	VF	
N	I	0.25	0.000	2912.80	61.50	1989.80	0.00	4.50	18.69	1.06	2431.70	66.17	1988.23	0.00	3.50	20.70	20.70	2386.40	66.17	1992.88	0.00	3.50	20.70	0.18	-4.94
N	I	0.25	0.001	2007.50	59.08	1830.73	86.37	4.20	20.86	1.69	2008.50	56.57	1785.21	101.51	2.90	20.08	20.08	1881.10	56.81	1778.73	109.27	2.90	20.08	-0.76	-2.34
N	I	0.25	0.005	1822.00	50.37	1722.97	109.65	3.70	22.26	1.72	1762.20	53.18	1721.78	75.35	2.90	20.08	20.08	1498.33	49.48	1515.52	174.28	2.56	19.08	-3.93	-2.61
N	I	0.75	0.000	1679.30	38.14	853.48	0.00	3.10	27.63	1.00	1628.40	36.58	770.65	0.00	1.90	27.50	27.50	1600.40	36.84	772.55	0.00	1.90	27.50	0.19	0.04
N	I	0.75	0.001	1352.20	31.14	768.30	0.15	2.80	28.89	1.00	1293.20	33.43	715.73	0.00	1.40	30.27	30.27	1130.30	32.93	685.65	23.58	1.40	30.27	-7.55	-3.73
N	I	0.75	0.005	1305.10	33.45	773.03	3.34	3.00	28.03	1.13	1061.50	33.49	694.65	6.28	1.50	29.37	29.37	1125.50	34.72	718.65	6.28	1.50	29.37	3.43	-0.43
N	F	0.25	0.000	2607.11	52.46	1752.28	31.11	4.33	11.80	1.06	1694.50	48.01	1336.15	0.00	2.70	10.51	10.51	1565.70	39.70	1022.88	0.00	2.10	9.46	0.18	-0.42
N	F	0.25	0.005	1468.10	44.20	1333.00	76.75	3.40	25.53	1.45	1333.40	40.87	1177.84	80.10	2.10	10.68	10.68	1471.30	41.13	1172.92	43.85	2.10	10.68	-0.89	2.09
N	F	0.75	0.000	1363.00	37.66	1222.73	109.84	3.10	26.66	1.52	1273.00	37.90	1112.16	74.18	2.10	10.68	10.68	934.20	28.86	843.83	49.80	1.50	7.78	-0.02	1.75
N	F	0.25	0.001	1377.90	28.54	528.63	0.00	2.80	28.31	0.80	1288.00	26.99	445.80	0.00	1.70	12.47	12.47	1305.00	27.24	447.70	0.00	1.70	12.47	0.23	0.05
N	F	0.75	0.001	1050.80	21.55	443.45	0.15	2.40	28.69	0.80	991.80	23.84	390.88	0.00	1.20	15.23	15.23	828.90	23.34	360.80	23.58	1.20	15.23	-9.44	-4.66
N	F	0.75	0.005	1003.70	23.85	448.18	3.34	2.60	27.83	0.93	760.10	23.90	369.80	6.28	1.30	14.33	14.33	824.10	25.12	393.80	6.28	1.30	14.33	4.29	-0.54
W	F	0.25	0.000	2334.40	45.91	1491.35	0.00	5.10	20.03	0.82	2152.60	47.94	1343.40	0.00	4.10	6.89	6.89	1505.40	33.10	888.15	0.00	2.40	5.64	0.72	0.60
W	F	0.25	0.001	1082.70	46.90	1430.69	114.31	4.70	22.80	2.07	1154.90	46.82	1285.14	107.26	3.80	6.23	6.23	881.30	38.85	911.95	87.53	3.00	5.24	-0.41	3.31
W	F	0.25	0.005	1019.40	41.27	1267.47	154.88	4.50	22.95	2.05	1012.00	40.00	1141.88	89.82	3.50	5.70	5.70	748.80	32.89	751.02	161.91	2.70	4.76	-5.21	5.08
W	F	0.75	0.000	1389.90	28.93	548.88	0.00	4.90	23.19	0.83	1302.70	28.53	460.28	0.00	3.70	7.13	7.13	1309.10	27.45	405.25	0.00	2.30	6.84	1.13	1.14
W	F	0.75	0.001	830.30	25.41	462.06	40.39	5.70	22.87	1.23	977.70	25.21	363.46	48.42	3.33	5.58	5.58	627.60	20.49	247.25	26.98	1.60	5.58	0.86	3.82
W	F	0.75	0.005	601.60	26.22	344.79	127.18	4.40	24.79	1.61	522.30	25.79	292.71	71.14	3.40	7.05	7.05	302.88	15.88	156.53	31.69	1.25	3.67	4.96	8.90
W	I	0.25	0.000	2433.10	45.48	1413.58	0.00	4.60	23.93	0.71	2153.90	52.23	1412.25	0.00	3.80	6.81	6.81	732.50	19.64	285.18	0.00	1.20	2.41	0.71	-6.88
W	I	0.25	0.001	1044.40	45.85	1336.28	123.80	4.20	26.39	2.00	1208.10	43.95	1184.57	85.54	2.40	4.61	4.61	771.10	35.97	810.94	86.68	2.40	4.61	-0.48	4.27
W	I	0.25	0.005	928.90	38.41	1168.86	153.46	4.20	26.39	1.93	817.70	37.13	1044.96	85.27	2.90	5.08	5.08	649.60	30.12	649.89	162.44	2.10	4.13	-6.48	5.51
W	I	0.75	0.000	1379.50	27.53	480.98	0.00	3.50	26.67	0.70	1331.30	27.13	398.08	0.00	2.30	6.84	6.84	1301.60	27.45	405.25	0.00	2.30	6.84	1.13	0.11
W	I	0.75	0.001	821.00	24.02	395.33	39.59	3.20	28.49	1.10	964.50	23.81	296.73	47.24	1.90	6.72	6.72	627.60	20.49	247.25	26.98	1.60	5.58	0.86	4.34
W	I	0.75	0.005	479.33	19.87	287.17	67.47	3.00	28.66	1.31	350.70	17.39	172.06	45.09	1.80	3.27	3.27	260.50	13.82	137.02	27.60	1.50	3.13	3.97	5.65

**Table I.3** PHD-VK, PHD-FK\* and PHS-FK\* solutions of problem AKO8T6 with low demand variability  
(average of 10 problem instances)

WIP	SU	S	h	PHD-VK						PHD-FK*						PHS-FK*						SD		
				MS	Sat %	F <sub>r</sub>	H Cost	Z	c <sub>h,4</sub>	K <sub>h,4</sub>	MS	Sat %	π	H Cost	Z	c <sub>h,4</sub>	MS	Sat %	π	H Cost	Z	c <sub>h,4</sub>	Gap	VF
N	I	0.25	0.000	251.60	85.57	289.20	0.00	4.30	1.97	1.16	213.30	86.34	288.95	0.00	3.50	2.08	264.30	86.99	294.73	0.00	3.60	2.01	2.22	-0.68
N	I	0.25	0.001	246.40	85.57	284.83	4.49	4.20	1.99	1.19	213.30	86.34	283.69	5.26	3.50	2.08	212.80	86.99	289.42	4.73	3.50	2.08	2.31	-0.16
N	I	0.25	0.005	202.50	83.65	274.76	14.44	4.10	2.19	1.56	195.60	83.15	272.63	16.10	3.30	2.23	214.90	83.56	276.25	17.45	3.40	2.20	1.32	0.59
N	I	0.75	0.000	198.80	70.21	167.43	0.00	4.00	2.18	1.09	208.90	69.97	167.40	0.00	3.10	2.16	214.60	69.97	168.25	0.00	3.10	2.16	0.28	0.01
N	I	0.75	0.001	191.10	68.98	163.48	3.57	3.90	2.15	1.12	209.50	69.28	162.95	3.45	3.00	2.09	198.60	68.98	165.00	2.15	3.00	2.09	0.87	0.28
N	I	0.75	0.005	187.90	67.56	156.32	9.30	3.60	2.22	1.17	202.30	67.56	156.17	9.61	2.70	2.23	195.60	67.73	157.07	9.51	2.70	2.23	0.57	0.23
N	F	0.25	0.000	211.80	70.48	233.90	0.00	4.10	1.58	1.15	159.80	71.48	235.50	0.00	3.40	1.36	234.30	71.89	239.53	0.00	3.50	1.29	1.97	-1.11
N	F	0.25	0.001	207.10	70.48	230.57	3.23	4.10	1.59	1.17	155.20	71.48	231.26	4.14	3.40	1.36	182.80	71.89	235.35	3.60	3.40	1.36	2.16	-0.49
N	F	0.25	0.005	167.70	67.78	223.61	9.21	3.90	1.73	1.55	128.00	67.28	222.25	11.20	3.30	1.37	174.10	70.01	225.20	11.05	3.30	1.37	1.74	0.69
N	F	0.75	0.000	159.00	56.69	134.73	0.00	3.50	1.83	1.12	151.20	56.69	134.73	0.00	2.90	1.45	174.10	56.46	134.73	0.00	2.90	1.45	0.00	0.00
N	F	0.75	0.001	155.40	56.23	132.09	2.33	3.50	1.84	1.14	151.70	56.52	131.56	2.21	2.80	1.42	161.10	55.99	132.16	1.51	2.80	1.42	0.46	0.31
N	F	0.75	0.005	153.70	54.57	127.56	5.19	3.30	1.92	1.18	189.70	54.57	127.41	5.49	2.60	1.53	159.90	54.51	128.76	4.87	2.60	1.53	1.17	0.23
W	F	0.25	0.000	177.60	71.69	273.65	0.00	4.80	1.44	1.18	204.40	72.19	274.50	0.00	3.80	1.16	200.70	72.42	277.53	0.00	3.80	1.16	1.06	-0.38
W	F	0.25	0.001	111.40	71.69	263.59	8.14	4.90	1.59	1.98	129.30	71.92	261.94	9.96	3.90	1.13	120.90	72.42	268.23	7.40	3.90	1.13	1.12	0.52
W	F	0.25	0.005	82.80	70.92	236.88	29.60	4.90	1.66	2.71	91.50	72.05	233.02	33.78	3.90	1.13	90.60	73.05	242.24	29.91	3.90	1.13	3.23	1.75
W	F	0.75	0.000	156.60	63.75	178.90	0.00	4.70	1.51	1.08	174.00	63.75	178.90	0.00	3.70	1.16	175.50	64.25	180.18	0.00	3.70	1.16	0.60	0.00
W	F	0.75	0.001	109.10	63.22	167.11	10.09	4.80	1.60	1.58	138.40	63.45	166.35	10.92	3.90	1.11	140.80	64.54	167.92	10.06	3.90	1.11	0.96	0.71
W	F	0.75	0.005	88.50	62.35	131.95	39.85	4.80	1.65	1.97	87.89	61.85	118.77	40.79	3.56	1.11	88.89	62.99	124.07	37.87	3.56	1.11	3.40	5.21
W	I	0.25	0.000	219.10	87.23	327.38	0.00	4.50	1.83	1.20	228.60	88.23	328.63	0.00	3.70	1.90	235.00	88.38	332.50	0.00	3.70	1.90	1.21	-0.55
W	I	0.25	0.001	149.70	87.23	314.05	11.23	4.80	1.77	1.83	172.00	87.46	310.95	13.75	3.90	1.77	143.40	88.29	318.93	10.50	4.00	1.75	2.56	0.91
W	I	0.25	0.005	117.60	86.46	276.13	43.12	4.70	1.94	2.53	125.50	87.19	268.36	44.59	3.90	1.64	132.20	89.27	282.22	44.11	4.00	1.70	4.48	2.98
W	I	0.75	0.000	193.80	76.21	210.60	0.00	4.30	1.91	1.08	207.80	76.21	210.60	0.00	3.50	1.95	214.70	76.71	211.88	0.00	3.50	1.95	0.59	0.00
W	I	0.75	0.001	144.50	75.68	195.70	12.97	4.60	1.87	1.51	171.70	75.91	194.94	14.41	3.90	1.73	184.30	76.77	196.76	13.14	3.90	1.73	1.11	0.64
W	I	0.75	0.005	121.80	74.28	150.50	52.12	4.50	2.05	1.88	150.40	74.21	149.39	54.63	3.60	1.83	124.80	75.33	148.56	53.09	3.60	1.73	0.05	2.17

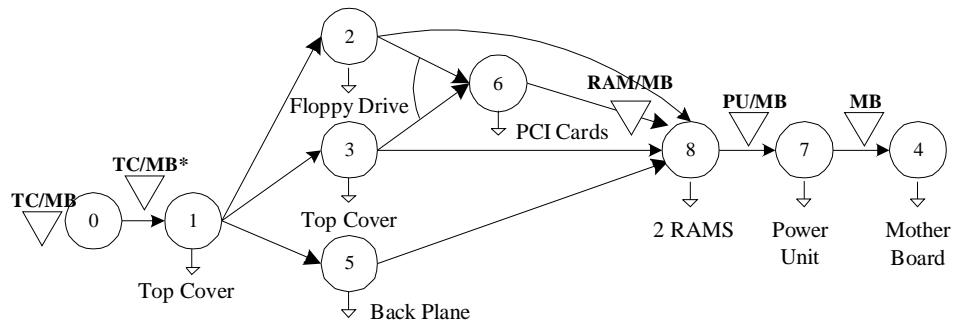
**Table I.4** PHD-VK, PHD-FK\* and PHS-FK\* solutions of problem AKO8T6 with high demand variability  
(average of 10 problem instances)

WIP	SU	S	h	PHD-VK						PHD-FK*						PHS-FK*						SD		
				MS	Sat %	$\pi$	H Cost	Z	$\alpha_{r^*}$	$K_{r^*}$	MS	Sat %	$\pi$	H Cost	Z	$\alpha_{r^*}$	MS	Sat %	$\pi$	H Cost	Z	$\alpha_{r^*}$	Gap	VF
N	I	0.25	0.000	3335.43	65.16	1998.14	0.00	4.71	14.00	1.08	2611.86	71.65	1977.82	0.00	3.71	14.50	2547.14	71.65	1984.46	0.00	3.71	14.50	0.26	-6.15
N	I	0.25	0.001	2047.29	61.94	1777.33	123.39	4.29	17.07	1.98	2027.86	58.39	1689.14	144.54	2.86	13.61	1845.86	58.73	1679.88	155.61	2.86	13.61	-1.09	-2.16
N	I	0.25	0.005	1815.71	50.00	1611.32	156.64	3.71	18.38	2.02	1676.00	53.54	1600.45	105.23	2.86	13.61	1265.83	48.06	1270.84	258.61	2.33	11.04	-6.29	-3.28
N	I	0.75	0.000	1957.00	38.96	660.89	0.00	3.29	20.25	1.00	1884.29	36.73	542.57	0.00	2.00	15.91	1844.29	37.10	545.29	0.00	2.00	15.91	0.27	0.06
N	I	0.75	0.001	1489.71	28.96	539.21	0.22	2.86	22.06	1.00	1405.43	32.23	464.11	0.00	1.29	19.86	1172.71	31.52	421.14	33.68	1.29	19.86	-10.79	-5.33
N	I	0.75	0.005	1422.43	32.25	545.98	4.77	3.14	20.82	1.19	1074.43	32.32	434.00	8.97	1.43	18.57	1165.86	34.07	468.28	8.97	1.43	18.57	4.90	-0.62
N	F	0.25	0.000	3028.29	61.65	1959.21	0.00	4.71	12.65	1.08	2261.43	64.94	1774.50	0.00	3.43	13.11	2111.00	53.07	1326.96	0.00	2.57	11.61	0.22	-0.47
N	F	0.25	0.005	1971.57	59.51	1770.00	109.64	4.14	16.28	1.92	1745.57	54.74	1548.34	114.43	2.57	13.35	1976.14	55.12	1541.32	62.65	2.57	13.35	-1.01	2.39
N	F	0.75	0.000	1821.43	50.16	1612.48	156.92	3.71	17.90	2.03	1660.00	50.50	1454.51	105.98	2.57	13.35	1208.86	37.59	1071.18	71.14	1.71	9.20	-0.02	2.00
N	F	0.25	0.001	1957.00	38.96	660.89	0.00	3.29	20.25	1.00	1828.57	36.73	542.57	0.00	2.00	15.91	1852.86	37.10	545.29	0.00	2.00	15.91	0.27	0.06
N	F	0.75	0.001	1489.71	28.96	539.21	0.22	2.71	20.80	1.00	1405.43	32.23	464.11	0.00	1.29	19.86	1172.71	31.52	421.14	33.68	1.29	19.86	-10.79	-5.33
N	F	0.75	0.005	1422.43	32.25	545.98	4.77	3.00	19.56	1.19	1074.43	32.32	434.00	8.97	1.43	18.57	1165.86	34.07	468.28	8.97	1.43	18.57	4.90	-0.62
W	F	0.25	0.000	3202.86	61.62	1986.79	0.00	6.29	9.19	1.01	2911.00	64.52	1775.43	0.00	5.14	8.70	2008.86	43.32	1125.07	0.00	2.71	6.91	0.86	0.68
W	F	0.25	0.001	1414.71	63.05	1900.73	162.70	5.71	13.15	2.80	1485.71	62.92	1693.36	152.07	4.71	7.76	1117.29	51.54	1160.22	123.89	3.57	6.35	-0.47	3.73
W	F	0.25	0.005	1331.14	55.00	1669.64	219.50	5.43	13.35	2.76	1295.57	53.18	1493.30	122.55	4.29	7.00	928.00	43.03	934.93	225.54	3.14	5.65	-6.08	5.49
W	F	0.75	0.000	1970.71	39.33	687.11	0.00	4.57	14.10	1.00	1846.14	38.76	568.68	0.00	3.29	9.77	1870.14	39.22	578.93	0.00	3.29	9.77	1.13	0.11
W	F	0.75	0.001	1171.29	34.29	564.35	56.44	5.71	13.64	1.57	1377.86	34.01	423.90	67.49	2.71	7.98	896.57	29.27	353.21	38.54	2.29	7.98	0.86	4.30
W	F	0.75	0.005	844.57	35.45	401.86	175.39	3.86	16.37	2.11	731.29	34.84	335.61	95.32	2.86	9.67	484.60	25.41	250.45	50.70	2.00	5.87	4.96	8.89
W	I	0.25	0.000	3475.86	64.97	2019.39	0.00	6.14	10.18	1.02	3077.00	74.62	2017.50	0.00	5.43	9.73	1046.43	28.06	407.39	0.00	1.71	3.44	0.71	-6.88
W	I	0.25	0.001	1492.00	65.50	1908.97	176.85	6.00	13.98	2.86	1725.86	62.78	1692.24	122.20	3.43	6.59	1101.57	51.38	1158.49	123.83	3.43	6.59	-0.48	4.27
W	I	0.25	0.005	1327.00	54.86	1669.81	219.23	5.57	13.70	2.76	1168.14	53.04	1492.80	121.81	4.14	7.26	928.00	43.03	928.41	232.05	3.00	5.89	-6.48	5.51
W	I	0.75	0.000	1970.71	39.33	687.11	0.00	4.57	14.10	1.00	1901.86	38.76	568.68	0.00	3.29	9.77	1859.43	39.22	578.93	0.00	3.29	9.77	1.13	0.11
W	I	0.75	0.001	1172.86	34.31	564.76	56.56	4.14	16.70	1.57	1377.86	34.01	423.90	67.49	2.71	9.59	896.57	29.27	353.21	38.54	2.29	7.98	0.86	4.34
W	I	0.75	0.005	719.00	29.80	430.75	101.21	4.00	14.99	1.97	501.00	24.84	245.80	64.41	2.57	4.67	372.14	19.74	195.75	39.43	2.14	4.48	3.97	5.65

## APPENDIX J

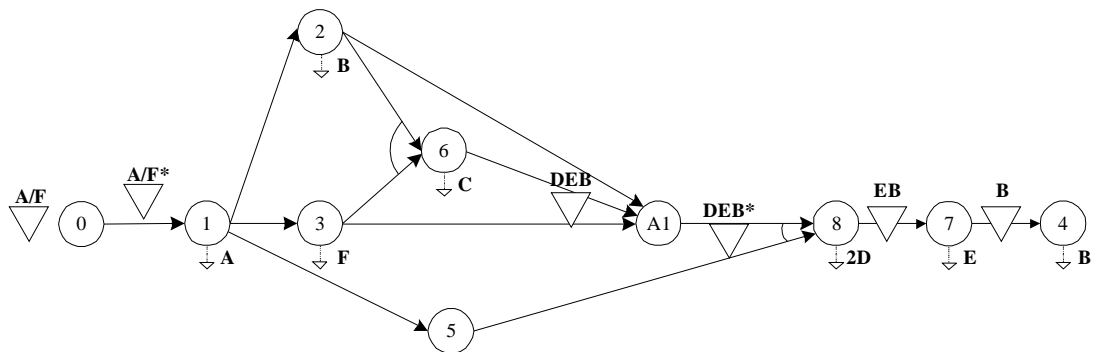
### PRECEDENCE RELATIONS OF PROBLEM CATEGORIES WITH WIP

#### J.1 GUN8T8



**Figure J.1** Precedence diagram of Gungör and Gupta's (2002) 8 task 8 part PC problem with WIP

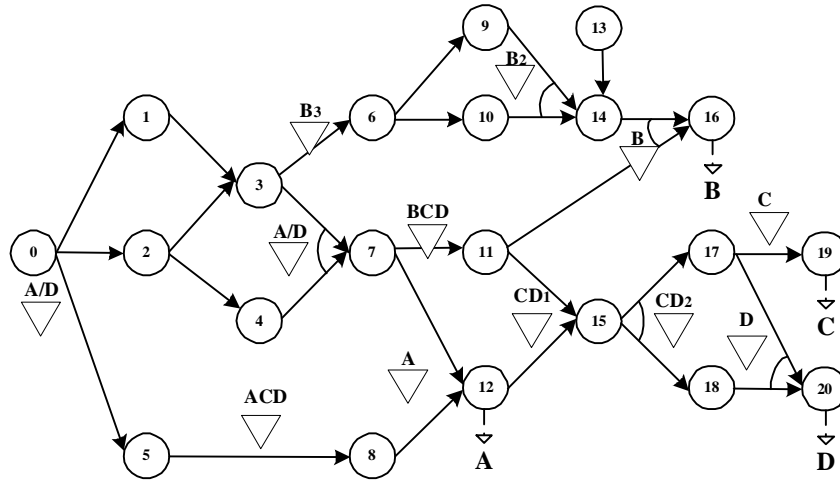
#### J.2 AKO8T6



**Figure J.2** Precedence diagram of 8 task 6 part problem with WIP

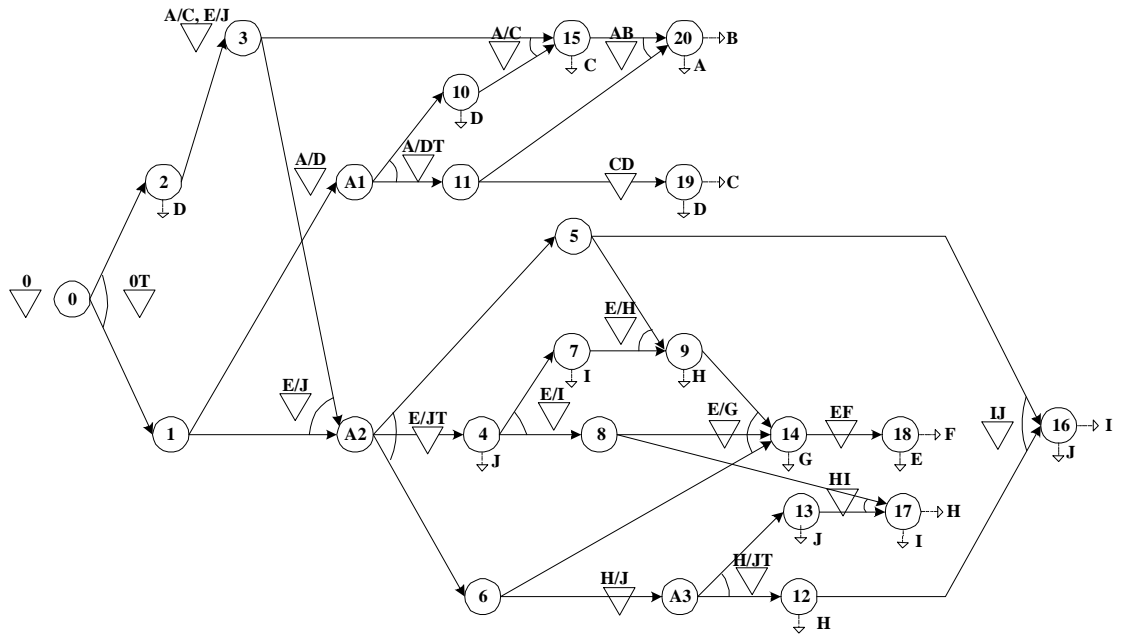


**J.3 AKO20T4-C**



**Figure J.3** Precedence diagram of 20 task 4 part problem with WIP

**J.4 LAM20T10**



**Figure J.4** Precedence diagram of Lambert's (1997) 20 tasks 10 part ball point pen problem with WIP

## APPENDIX K

### THE EFFECTS OF PROBLEM PARAMETERS ON PHD

#### K.1. PHD Results

**Table K.1** PHD results for GUN8T8 problem

WIP	SU	S	h	K <sub>avg</sub>	MS	Low Demand Variability						High Demand Variability							
						Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P	K <sub>avg</sub>	MS	Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P
N	I	0.25	0.000	1.20	264.80	72.27	3.30	2.23	176.63	0.00	0.00	1.38	2802.88	68.12	3.75	22.42	2101.72	0.00	0.00
N	I	0.25	0.001	1.40	214.50	66.79	3.20	2.29	168.96	5.12	3.03	1.75	2085.75	55.52	2.88	21.19	1824.38	141.37	7.75
N	I	0.25	0.005	1.50	204.10	64.20	3.10	2.37	158.66	9.87	6.22	1.88	1765.50	50.28	2.88	21.19	1753.75	91.84	5.24
N	I	0.75	0.000	0.90	90.30	29.52	1.70	3.17	70.20	0.00	0.00	1.00	1804.13	36.76	2.25	23.26	711.97	0.00	0.00
N	I	0.75	0.001	0.90	90.30	29.78	1.70	3.17	70.11	0.06	0.09	1.13	1372.75	31.83	1.63	26.72	641.34	0.00	0.00
N	I	0.75	0.005	0.90	90.30	29.78	1.70	3.17	69.87	0.30	0.43	1.25	1083.13	31.90	1.75	25.59	615.00	7.85	1.28
N	F	0.25	0.000	1.20	179.30	61.43	3.40	1.42	143.90	0.00	0.00	1.38	2469.63	62.61	4.25	13.20	1993.03	0.00	0.00
N	F	0.25	0.001	1.40	161.10	56.14	3.20	1.56	137.85	4.20	3.04	1.50	1803.13	53.65	3.38	13.58	1752.02	141.04	8.05
N	F	0.25	0.005	1.60	112.90	53.36	3.20	1.56	124.93	9.57	7.66	1.88	2084.88	48.42	3.38	13.58	1680.41	92.50	5.50
N	F	0.75	0.000	0.90	97.00	25.08	2.10	1.60	54.93	0.00	0.00	1.00	1749.13	35.76	2.50	16.80	660.41	0.00	0.00
N	F	0.75	0.001	0.90	97.60	25.34	2.10	1.60	54.90	0.00	0.00	1.13	1378.88	30.83	1.88	20.26	589.78	0.00	0.00
N	F	0.75	0.005	0.90	97.60	25.34	2.10	1.60	54.89	0.01	0.02	1.25	1089.25	30.91	2.00	19.14	563.43	7.85	1.39
W	F	0.25	0.000	1.10	212.90	63.14	3.60	1.36	171.18	0.00	0.00	1.13	2820.75	62.68	6.25	8.54	2030.00	0.00	0.00
W	F	0.25	0.001	1.80	165.70	62.06	3.60	1.36	156.94	10.78	6.87	2.38	1592.75	57.49	5.25	8.75	1772.87	171.01	9.65
W	F	0.25	0.005	2.80	113.20	55.93	3.60	1.33	124.86	28.69	22.98	2.50	1406.88	48.37	5.00	8.29	1595.56	159.85	10.02
W	F	0.75	0.000	1.00	119.00	36.05	3.10	1.38	77.13	0.00	0.00	1.13	1765.00	36.23	5.00	9.86	678.13	0.00	0.00
W	F	0.75	0.001	1.20	89.20	34.42	3.20	1.36	67.29	8.23	12.24	1.38	1248.88	31.39	4.79	7.62	570.84	48.88	8.56
W	F	0.75	0.005	1.70	42.70	26.86	2.60	1.18	42.50	18.72	44.05	2.13	761.38	31.70	4.50	9.75	437.33	116.58	26.66
W	I	0.25	0.000	1.20	256.80	74.54	3.60	2.03	204.75	0.00	0.00	1.29	3223.86	71.29	5.71	10.82	2210.07	0.00	0.00
W	I	0.25	0.001	1.60	206.10	73.27	3.60	2.03	187.44	14.71	7.85	2.57	1896.29	61.61	4.00	7.68	1882.46	164.41	8.73
W	I	0.25	0.005	2.10	147.20	65.24	3.40	2.18	150.22	38.36	25.53	2.71	1330.29	51.18	4.57	8.94	1685.04	176.18	10.46
W	I	0.75	0.000	1.00	137.50	42.49	2.50	2.53	94.05	0.00	0.00	1.00	2058.00	39.40	3.71	10.86	686.14	0.00	0.00
W	I	0.75	0.001	1.20	103.90	40.19	2.40	2.67	82.91	9.39	11.32	1.43	1408.43	33.87	3.14	10.68	557.06	54.19	9.73
W	I	0.75	0.005	1.60	55.80	31.30	1.70	2.75	53.57	22.78	42.52	2.00	729.17	28.27	3.33	6.69	382.02	112.02	29.32

Table K.2 PHD results for AKO8T6 problem

WIP	SU	s	h	Low Demand Variability								High Demand Variability							
				K <sub>avg</sub>	MS	Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P	K <sub>avg</sub>	MS	Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P
N	I	0.25	0.000	1.30	213.30	86.34	3.50	2.08	288.95	0.00	0.00	1.00	2006.57	68.32	3.86	11.42	2117.50	0.00	0.00
N	I	0.25	0.001	1.30	213.30	86.34	3.50	2.08	283.69	5.26	1.85	1.43	1357.43	64.94	3.57	11.70	1953.44	128.31	6.57
N	I	0.25	0.005	1.50	195.60	83.15	3.30	2.23	272.63	16.10	5.91	1.57	1147.71	58.18	3.29	12.66	1820.67	196.15	10.77
N	I	0.75	0.000	1.00	208.90	69.97	3.10	2.16	167.40	0.00	0.00	1.00	1661.14	57.81	3.29	11.97	1256.82	0.00	0.00
N	I	0.75	0.001	1.00	209.50	69.28	3.00	2.09	162.95	3.45	2.12	1.00	1390.29	52.93	3.14	11.78	1164.37	51.48	4.42
N	I	0.75	0.005	1.10	202.30	67.56	2.70	2.23	156.17	9.61	6.15	1.00	1349.57	48.82	2.71	13.32	1155.35	25.32	2.19
N	F	0.25	0.000	1.10	159.80	71.48	3.40	1.36	235.50	0.00	0.00	1.00	1570.43	62.91	3.71	9.61	1932.04	0.00	0.00
N	F	0.25	0.001	1.20	155.20	71.48	3.40	1.36	231.26	4.14	1.79	1.43	1163.29	59.82	3.57	9.58	1811.01	94.07	5.19
N	F	0.25	0.005	1.40	128.00	67.28	3.30	1.37	222.25	11.20	5.04	1.57	1419.57	53.06	3.29	10.32	1738.72	101.42	5.83
N	F	0.75	0.000	1.00	151.20	56.69	2.90	1.45	134.73	0.00	0.00	1.00	1529.00	54.02	3.57	9.42	1145.68	0.00	0.00
N	F	0.75	0.001	1.00	151.70	56.52	2.80	1.42	131.56	2.21	1.68	1.00	1349.43	49.59	3.14	10.30	1066.92	39.19	3.67
N	F	0.75	0.005	1.10	189.70	54.57	2.60	1.53	127.41	5.49	4.31	1.00	1264.57	45.54	2.57	12.43	1050.13	20.97	2.00
W	F	0.25	0.000	1.00	204.40	72.19	3.80	1.16	274.50	0.00	0.00	1.00	1711.86	62.48	6.43	5.59	1966.93	0.00	0.00
W	F	0.25	0.001	1.90	129.30	71.92	3.90	1.13	261.94	9.96	3.80	1.86	956.14	59.73	6.43	5.42	1822.05	116.27	6.38
W	F	0.25	0.005	2.50	91.50	72.05	3.90	1.13	233.02	33.78	14.50	3.29	729.71	56.79	6.00	5.17	1548.28	258.32	16.68
W	F	0.75	0.000	1.00	174.00	63.75	3.70	1.16	178.90	0.00	0.00	1.00	1404.71	54.23	5.71	5.96	1187.71	0.00	0.00
W	F	0.75	0.001	1.40	138.40	63.45	3.90	1.11	166.35	10.92	6.57	1.14	1027.71	50.42	5.43	5.85	1052.69	87.70	8.33
W	F	0.75	0.005	1.70	87.89	61.85	3.56	1.11	118.77	40.79	34.34	1.57	787.86	44.77	5.57	5.06	906.38	174.40	19.24
W	I	0.25	0.000	1.20	228.60	88.23	3.70	1.90	328.63	0.00	0.00	1.00	1972.57	67.84	6.00	7.33	2150.29	0.00	0.00
W	I	0.25	0.001	1.80	172.00	87.46	3.90	1.77	310.95	13.75	4.42	1.86	1126.14	64.80	5.86	7.10	1958.92	149.58	7.64
W	I	0.25	0.005	2.50	125.50	87.19	3.90	1.64	268.36	44.59	16.61	3.29	678.00	60.08	5.71	6.47	1639.32	297.86	18.17
W	I	0.75	0.000	1.00	207.80	76.21	3.50	1.95	210.60	0.00	0.00	1.00	1632.00	57.97	5.43	7.60	1297.39	0.00	0.00
W	I	0.75	0.001	1.30	171.70	75.91	3.90	1.73	194.94	14.41	7.39	1.14	1184.14	53.71	5.14	7.16	1146.31	102.37	8.93
W	I	0.75	0.005	1.60	150.40	74.21	3.60	1.83	149.39	54.63	36.57	1.57	838.00	47.99	4.86	6.82	996.86	192.03	19.26

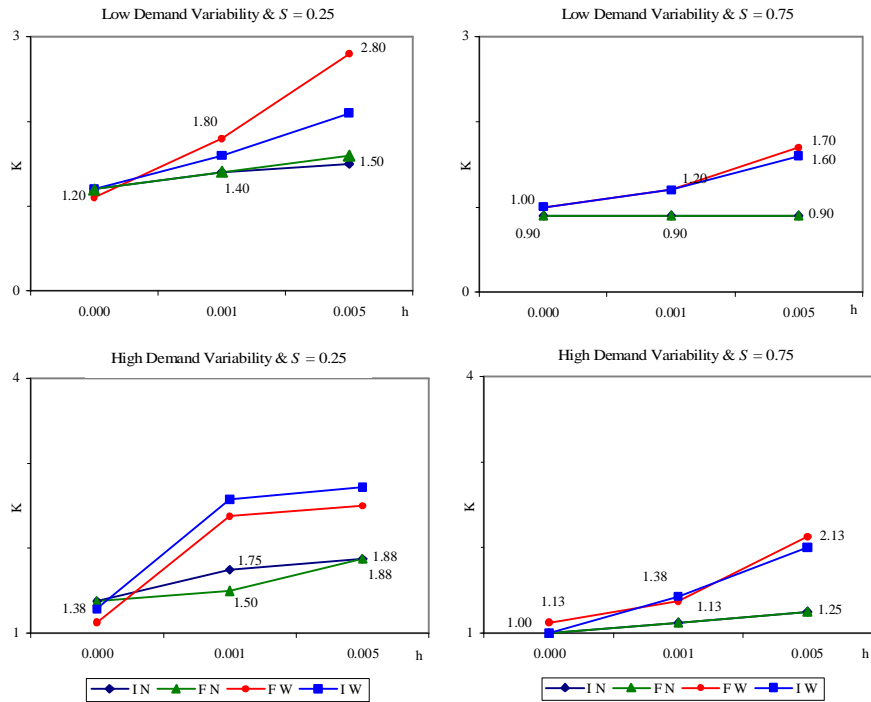
Table K.4 PHD results for LAM20T10 problem

WIP	SU	s	h	Low Demand Variability															
				K <sub>avg</sub>	MS	Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P	K <sub>avg</sub>	MS	Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P
N	I	0.25	0.000	1.70	356.80	98.78	5.80	1.39	1231.43	0.00	0.00	1.90	2920.70	96.05	7.30	10.57	12820.23	0.00	0.00
N	I	0.25	0.001	1.90	355.60	98.40	5.80	1.37	1210.73	17.85	1.47	3.20	1805.50	94.04	6.90	10.30	11905.52	976.13	8.20
N	I	0.25	0.005	2.70	256.90	97.47	5.30	1.40	1169.52	60.91	5.21	3.40	1382.70	80.37	5.00	11.85	9536.07	1967.46	20.63
N	I	0.75	0.000	1.30	332.60	91.23	5.10	1.51	990.10	0.00	0.00	1.70	2666.50	90.28	6.80	10.71	10462.08	0.00	0.00
N	I	0.75	0.001	1.60	279.90	90.08	4.90	1.49	966.14	21.73	2.25	2.70	1645.60	84.32	5.90	10.94	9204.34	946.66	10.28
N	I	0.75	0.005	2.20	180.90	86.76	4.70	1.43	910.10	56.80	6.24	2.90	1126.80	68.49	4.20	11.76	7106.70	1456.47	20.49
N	F	0.25	0.000	1.40	244.40	80.65	3.60	1.47	1105.48	0.00	0.00	1.80	2508.80	79.05	5.30	9.45	11330.40	0.00	0.00
N	F	0.25	0.001	1.80	237.00	80.65	3.60	1.47	1097.20	7.67	0.70	3.20	1765.20	79.23	5.20	9.73	10873.07	439.71	4.04
N	F	0.25	0.005	2.73	189.80	80.65	3.60	1.47	1074.88	26.32	2.45	3.50	1286.50	77.40	5.30	9.72	9454.64	1631.09	17.25
N	F	0.75	0.000	1.30	265.00	78.23	3.70	1.40	911.83	0.00	0.00	1.40	2955.50	77.03	6.10	8.55	9506.93	0.00	0.00
N	F	0.75	0.001	1.70	225.40	78.23	3.60	1.47	899.68	10.77	1.20	2.50	1599.60	75.86	5.30	9.46	8711.82	624.41	7.17
N	F	0.75	0.005	2.70	207.70	77.16	3.50	1.47	866.26	33.71	3.89	3.00	1104.30	67.43	4.20	11.15	7069.79	1355.56	19.17
W	F	0.25	0.000	1.50	312.00	86.49	4.50	1.10	1158.98	0.00	0.00	1.80	2381.30	79.60	11.60	4.63	11420.63	54.10	0.47
W	F	0.25	0.001	3.10	167.20	86.61	4.50	1.10	1146.69	11.58	1.01	3.40	1231.50	79.79	11.70	4.67	10971.24	467.65	4.26
W	F	0.25	0.005	4.70	113.80	86.61	4.50	1.10	1106.52	41.28	3.73	4.10	1015.50	78.69	11.70	4.44	9641.14	1595.46	16.55
W	F	0.75	0.000	1.50	265.40	82.48	4.40	1.14	968.53	0.00	0.00	1.30	2307.20	69.72	9.70	4.24	8776.65	0.00	0.00
W	F	0.75	0.001	2.20	184.20	81.67	4.50	1.11	944.02	19.01	2.01	2.30	1211.00	68.07	8.40	4.89	8097.01	526.14	6.50
W	F	0.75	0.005	3.70	116.60	80.60	4.40	1.11	888.83	51.47	5.79	2.90	855.70	62.70	7.30	5.00	6811.04	1327.24	19.49
W	I	0.25	0.000	1.70	326.60	97.76	5.80	1.24	1232.08	0.00	0.00	1.50	2247.90	77.52	8.10	5.81	9703.13	0.00	0.00
W	I	0.25	0.001	2.60	284.90	98.90	5.80	1.25	1218.61	21.79	1.79	2.50	1112.50	74.14	7.60	5.39	8962.99	572.03	6.38
W	I	0.25	0.005	4.30	174.10	96.01	5.20	1.19	1150.67	59.49	5.17	3.00	817.30	64.35	6.40	5.28	7366.43	1318.19	17.89
W	I	0.75	0.000	1.40	328.40	91.36	5.50	1.25	1018.00	0.00	0.00	1.30	1985.30	71.86	7.80	5.62	7989.58	0.00	0.00
W	I	0.75	0.001	2.00	229.60	89.30	5.20	1.22	980.77	28.20	2.88	2.10	1040.90	66.33	6.70	5.61	7107.79	598.41	8.42
W	I	0.75	0.005	3.60	127.90	84.93	4.90	1.16	908.29	60.41	6.65	2.60	678.30	56.28	5.50	4.88	5967.41	1174.74	19.69

**Table K.4** PHD results for AKO20T4-C problem

Low Demand Variability											High Demand Variability								
WIP	SU	S	h	K <sub>avg</sub>	MS	Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P	K <sub>avg</sub>	MS	Sat %	Z	α <sub>avg</sub>	Profit	H Cost	H/P
N	I	0.25	0.000	1.09	294.20	76.03	2.00	2.75	928.85	0.00	1.18	1.57	3296.71	83.08	3.14	17.90	10189.32	0.00	0.00
N	I	0.25	0.001	1.61	265.40	76.03	2.00	2.75	919.55	8.55	1.38	3.00	2497.00	79.57	2.86	18.11	9591.06	373.12	3.89
N	I	0.25	0.005	2.86	229.80	77.95	2.10	2.60	911.31	32.99	1.67	3.00	2056.43	76.11	2.43	22.39	8398.88	1400.91	16.68
N	I	0.75	0.000	0.94	238.60	58.01	1.60	2.60	665.18	0.00	1.21	1.57	3018.43	79.61	2.71	19.01	7682.79	0.00	0.00
N	I	0.75	0.001	1.08	238.60	58.01	1.60	2.60	653.66	11.51	1.29	2.43	2329.29	76.21	2.43	22.44	7032.92	512.33	7.28
N	I	0.75	0.005	1.94	195.40	57.99	1.50	2.72	628.71	31.76	1.63	1.57	2154.00	54.81	2.00	15.32	6041.16	353.42	5.85
N	F	0.25	0.000	1.11	289.40	72.29	2.70	1.80	870.95	0.00	1.18	1.43	2976.71	73.56	3.86	11.55	9545.79	0.00	0.00
N	F	0.25	0.001	1.80	262.80	73.45	2.90	1.77	876.10	7.72	1.39	2.57	2179.71	73.56	3.57	13.04	9226.10	276.54	3.00
N	F	0.25	0.005	2.80	248.90	73.45	3.00	1.75	851.65	28.95	1.44	2.86	2541.71	69.95	3.29	13.97	8385.37	843.92	10.06
N	F	0.75	0.000	0.95	224.40	54.66	2.00	1.90	623.08	0.00	1.18	1.43	2973.86	73.56	3.86	11.55	7412.36	0.00	0.00
N	F	0.75	0.001	1.17	226.60	55.81	2.20	1.87	618.89	10.46	1.30	2.29	2410.29	72.43	3.57	13.04	6903.25	415.29	6.02
N	F	0.75	0.005	1.79	204.40	54.64	2.20	1.87	590.02	30.45	1.34	1.57	2149.86	54.73	2.57	12.73	6055.40	288.53	4.76
W	F	0.25	0.000	1.15	259.60	76.11	4.40	1.12	1058.53	0.00	0.95	1.43	2551.29	65.48	13.14	4.62	8643.00	0.00	0.00
W	F	0.25	0.001	3.29	147.10	75.73	4.30	1.11	1030.14	14.88	1.09	4.00	1558.57	67.81	9.71	5.57	8539.91	437.96	5.13
W	F	0.25	0.005	4.52	125.70	74.86	4.30	1.11	954.95	59.28	1.09	4.86	1486.29	67.38	9.86	5.28	7474.01	1364.44	18.26
W	F	0.75	0.000	1.15	266.00	74.55	4.20	1.13	866.03	0.00	0.95	1.57	2364.00	67.95	12.71	4.46	7554.07	0.00	0.00
W	F	0.75	0.001	2.21	189.00	73.93	4.20	1.11	827.03	26.47	1.06	2.71	1978.86	67.22	10.57	4.81	6588.41	550.72	8.36
W	F	0.75	0.005	3.83	128.90	73.93	4.20	1.11	741.00	84.45	1.09	4.00	883.43	58.26	6.57	5.49	5916.36	674.32	11.40
W	I	0.25	0.000	1.11	319.80	84.55	5.00	1.29	1155.78	0.00	0.97	1.57	3308.00	83.60	7.43	9.17	10389.57	0.00	0.00
W	I	0.25	0.001	3.10	194.90	83.54	4.90	1.24	1123.97	20.53	1.19	4.14	1659.57	81.18	7.00	8.50	9430.69	634.28	6.73
W	I	0.25	0.005	4.25	144.80	80.44	4.40	1.24	1037.57	74.11	1.22	4.71	1156.43	71.88	5.71	8.64	7586.15	1858.28	24.50
W	I	0.75	0.000	1.01	298.00	81.86	4.70	1.28	944.33	0.00	1.00	1.43	3082.00	77.34	6.86	8.51	8056.18	0.00	0.00
W	I	0.75	0.001	1.99	224.00	79.22	4.40	1.26	902.99	34.07	1.18	3.29	1686.29	76.87	6.43	8.37	7001.26	648.99	9.27
W	I	0.75	0.005	3.34	130.30	79.22	4.50	1.22	780.36	111.27	1.18	3.86	852.86	56.31	5.71	5.66	5894.82	645.96	10.96

**K.2. Average Number of Stations Opened ( $K_{avg}$ )**



**Figure K.1** Average number of stations in GUN8T8 problem

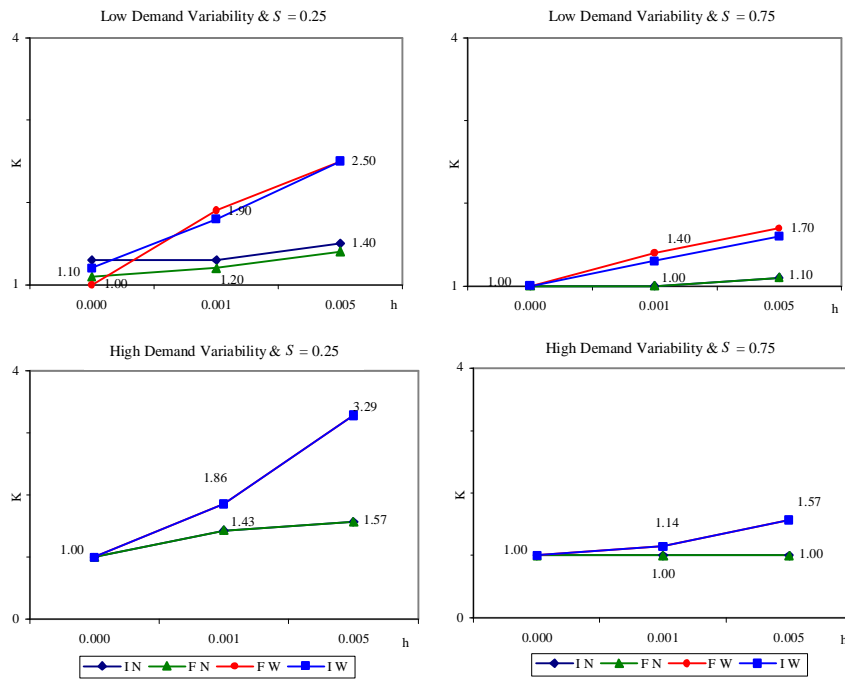


Figure K.2 Average number of stations in AKO8T6 problem

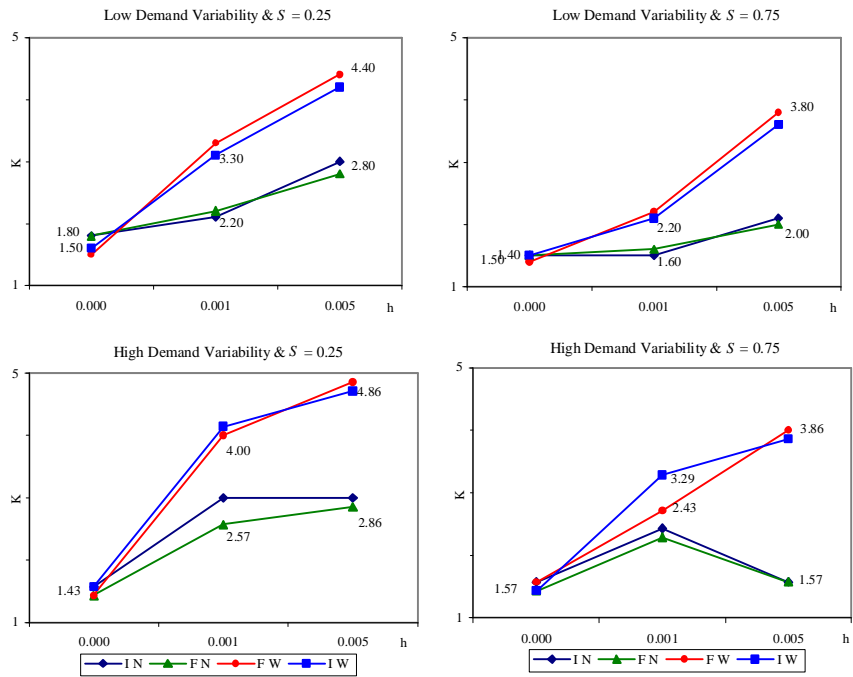


Figure K.3 Average number of stations in AKO20T4-C problem

### K.3. Average Number of Zones ( $Z$ ) and Average Number of Cycles in a Zone ( $\alpha_{avg}$ )

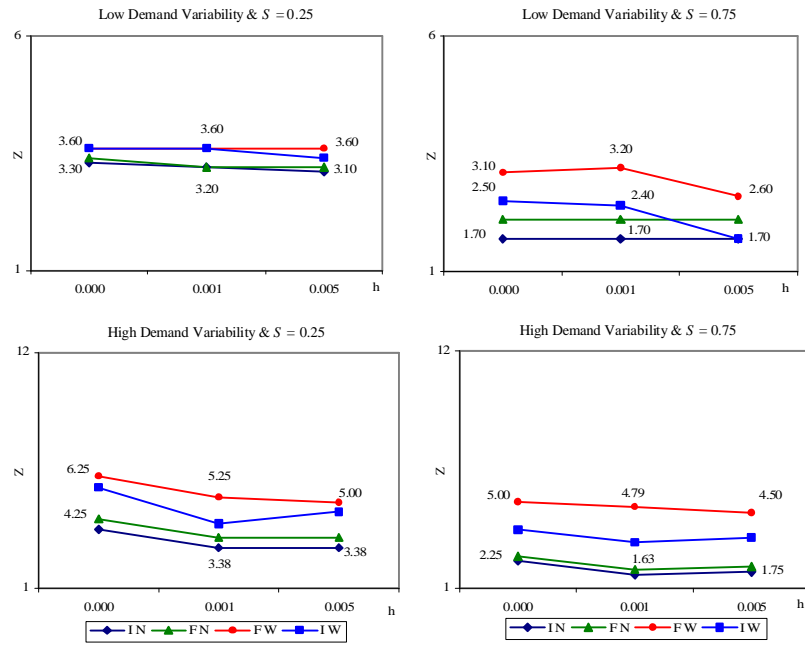


Figure K.4 Number of zones in GUN8T8 problem

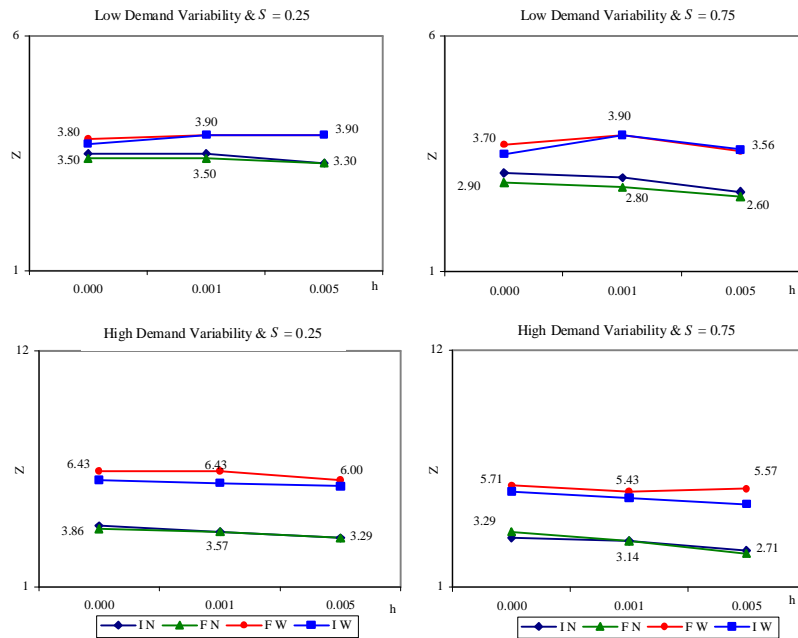


Figure K.5 Number of zones in AKO8T8 problem

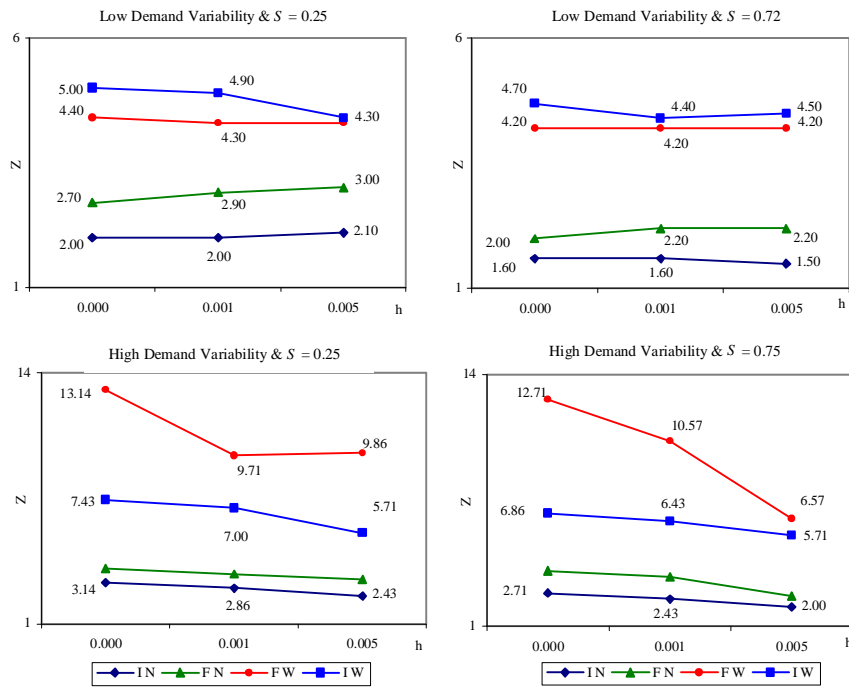


Figure K.6 Number of zones in AKPO20T4-C problem

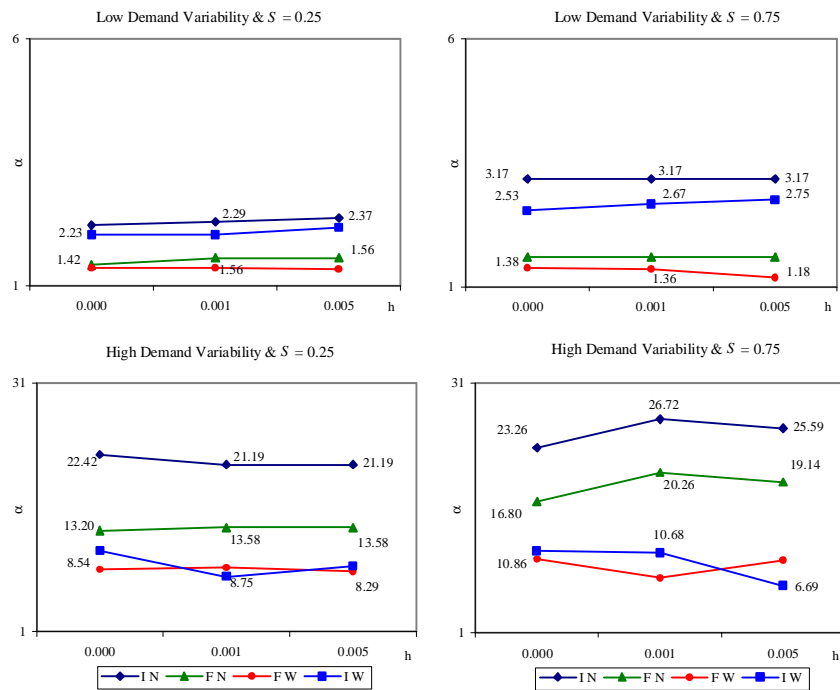


Figure K.7 Average number of cycles in a zone in GUN8T8 problem

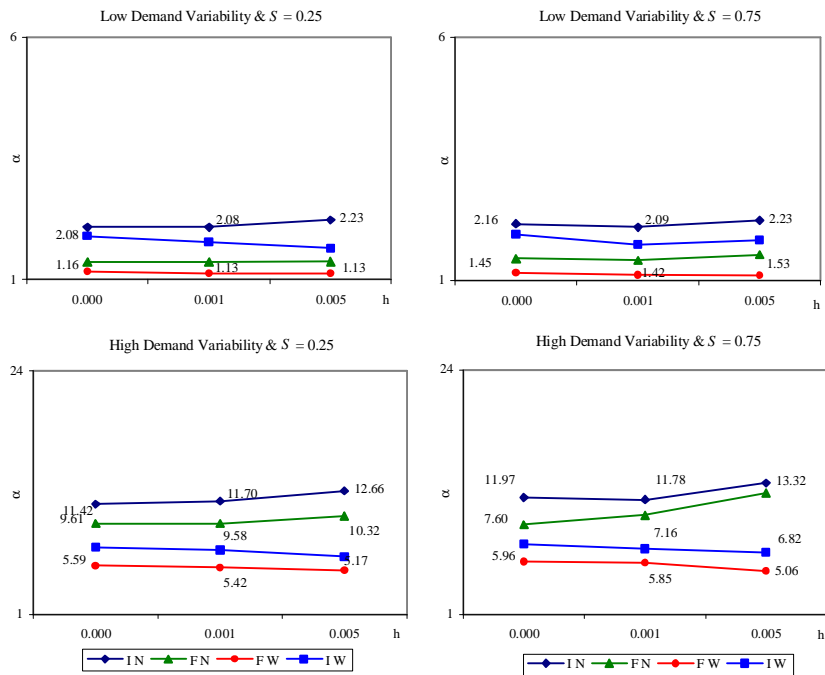


Figure K.8 Average number of cycles in a zone in AKO8T6 problem

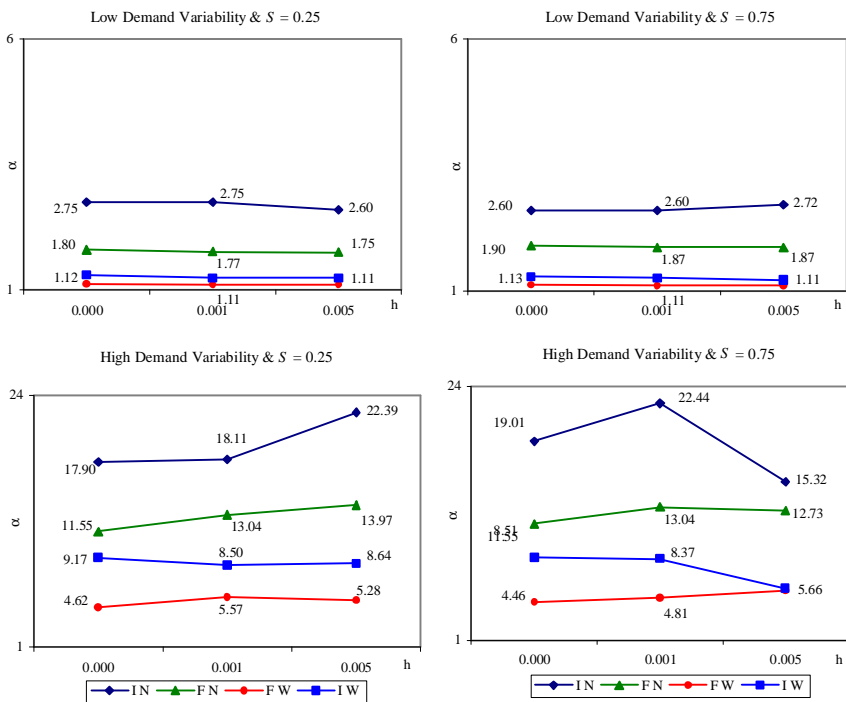


Figure K.9 Average number of cycles in a zone in AKO20T4-C problem



### K.4. Makespan (MS) and Percentage of Demand Satisfied (Sat %)

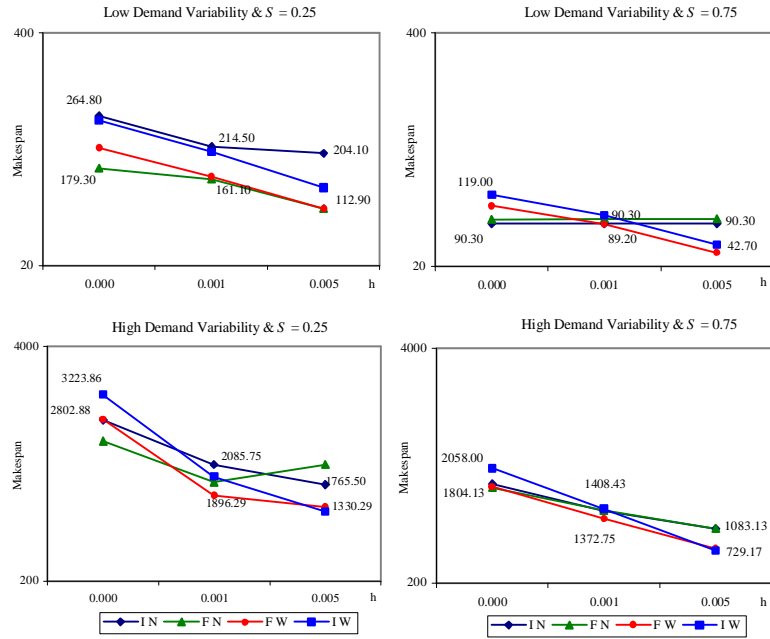


Figure K.10 Makespan of GUN8T8 problem

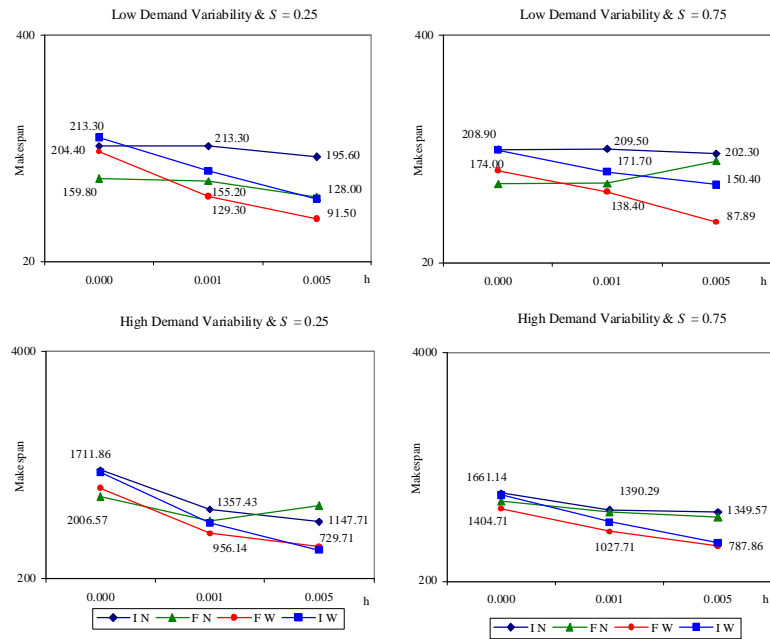


Figure K.11 Makespan of AKO8T6 problem

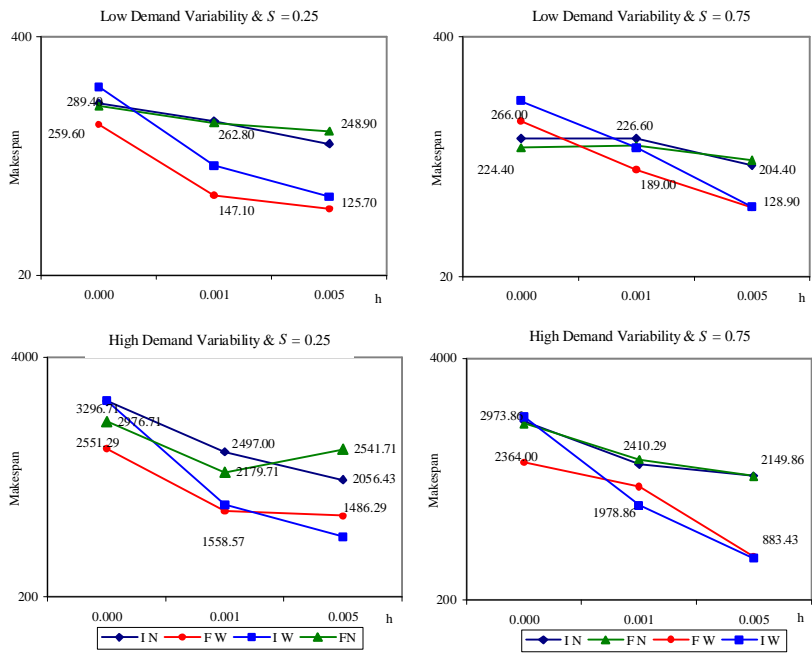


Figure K.12 Makespan of AKO20T4-C problem

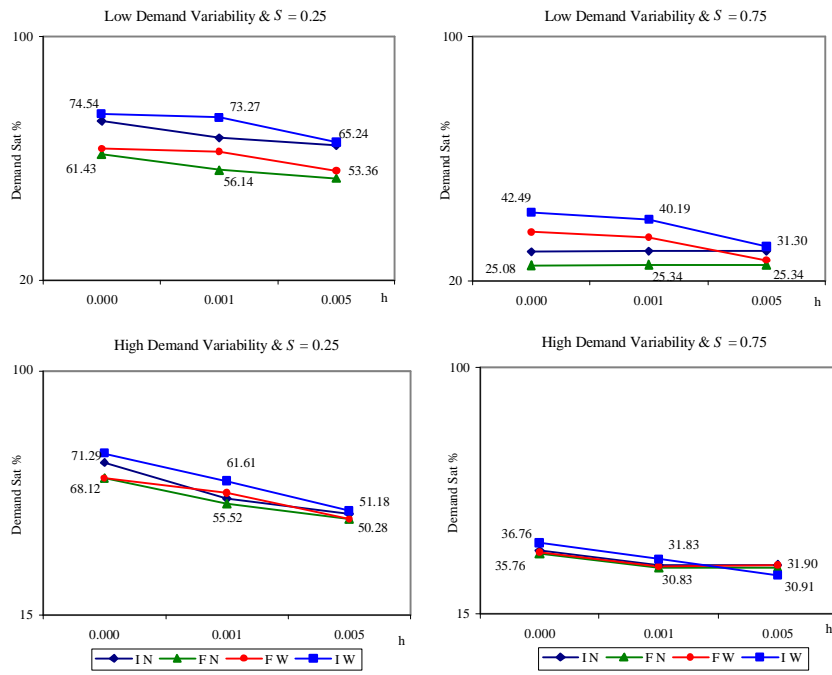


Figure K.13 Percentage of demand satisfied in GUN8T8 problem

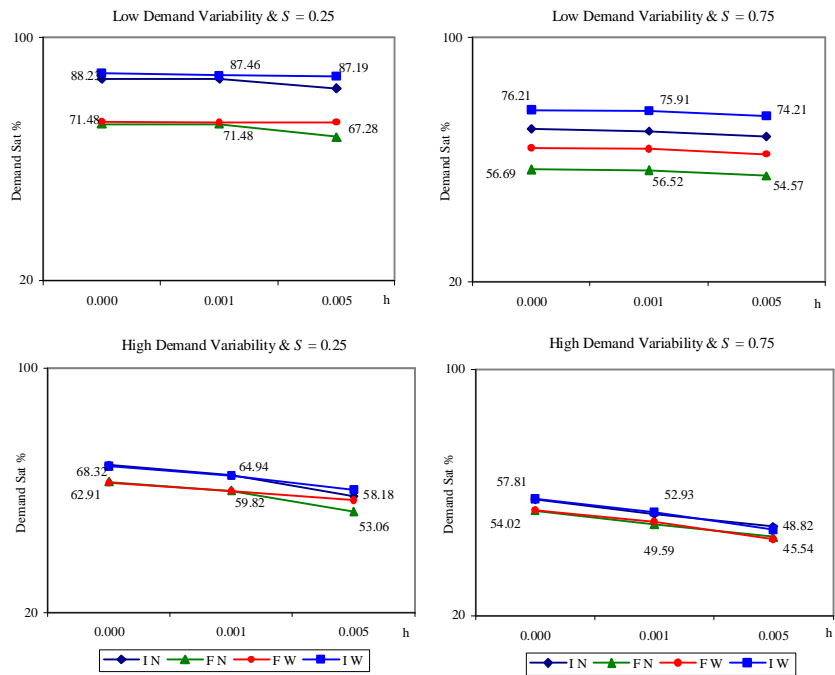


Figure K.14 Percentage of demand satisfied in AKO8T6 problem

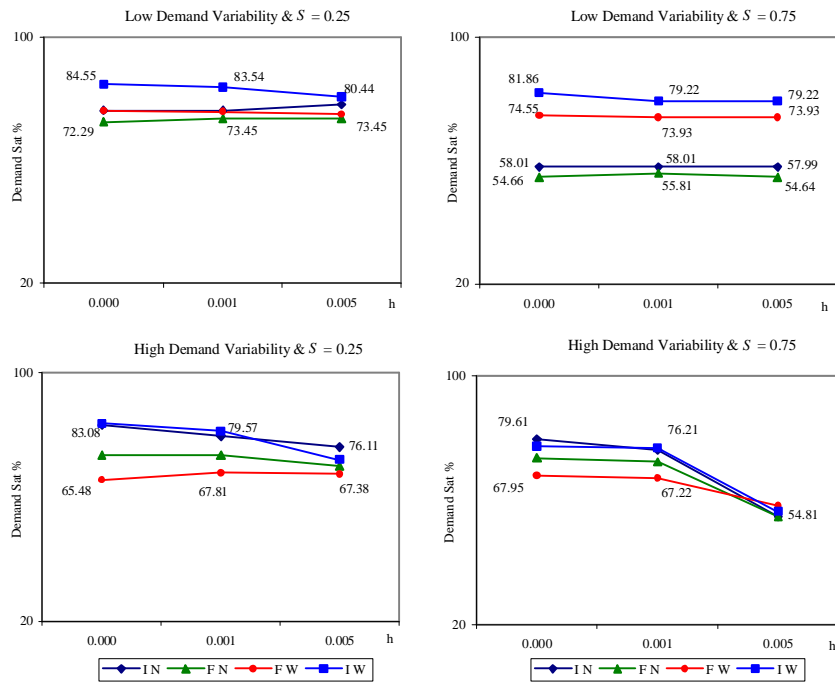


Figure K.15 Percentage of demand satisfied in AKO20T4-C problem

### K.5. Profit ( $\pi$ ) and Holding Cost to Profit Ratio (H/P)

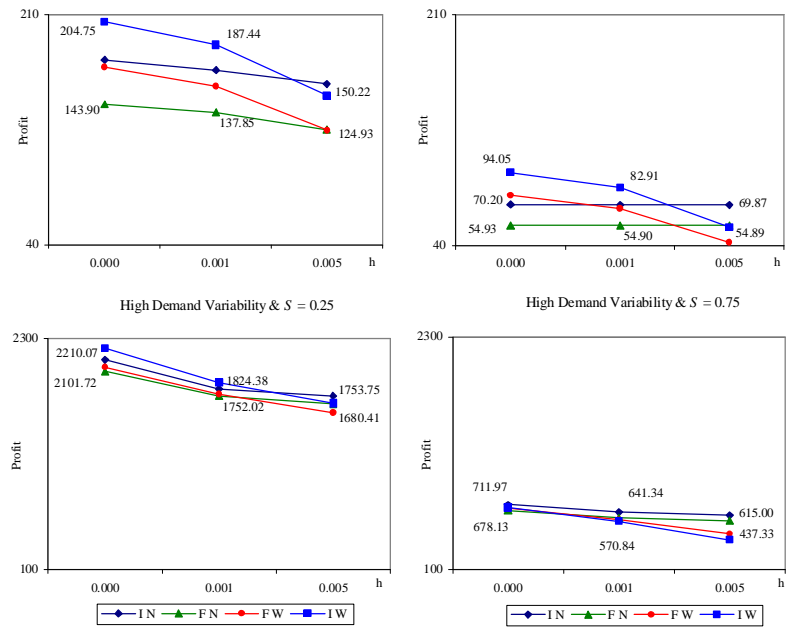


Figure K.16 Profit in GUN8T8 problem

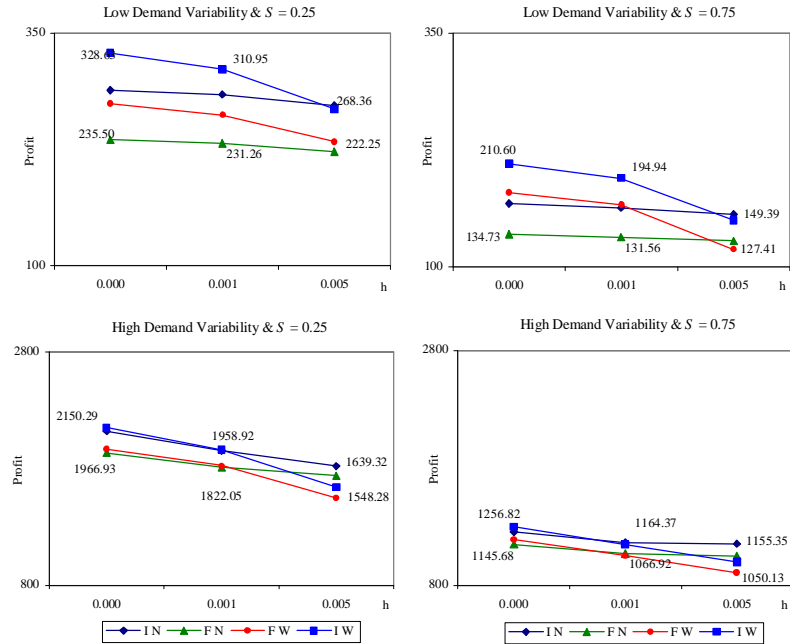


Figure K.17 Profit in AKO8T6 problem

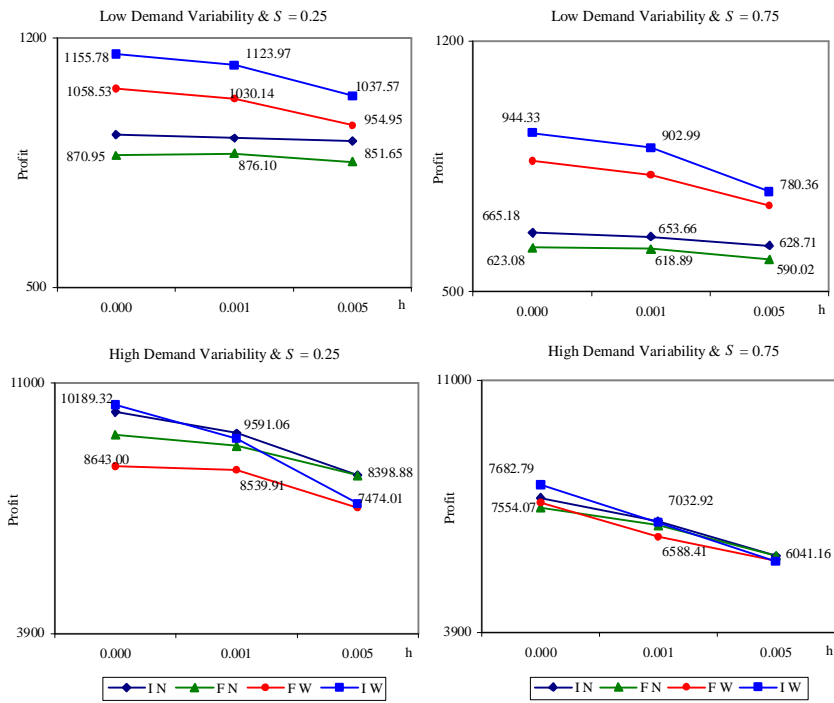


Figure K.18 Profit in AKO20T4-C problem

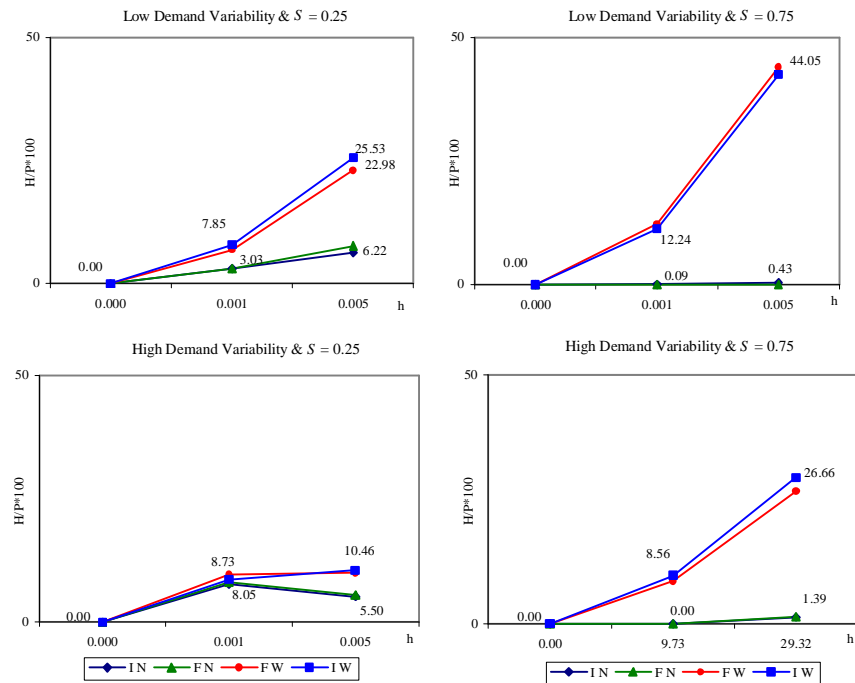


Figure K.19 Holding cost to profit ratio in GUN8T8 problem

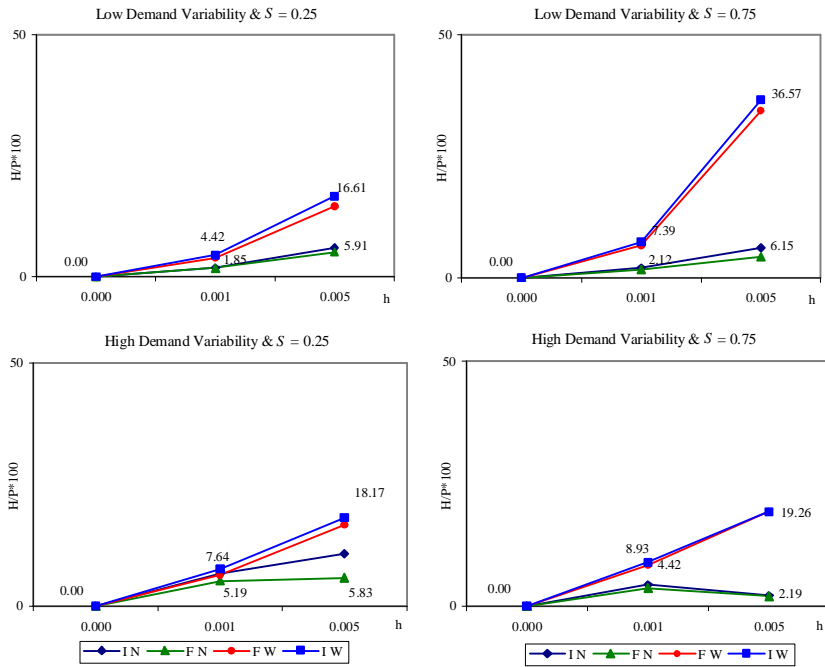


Figure K.20 Holding cost to profit ratio in AKO8T6 problem

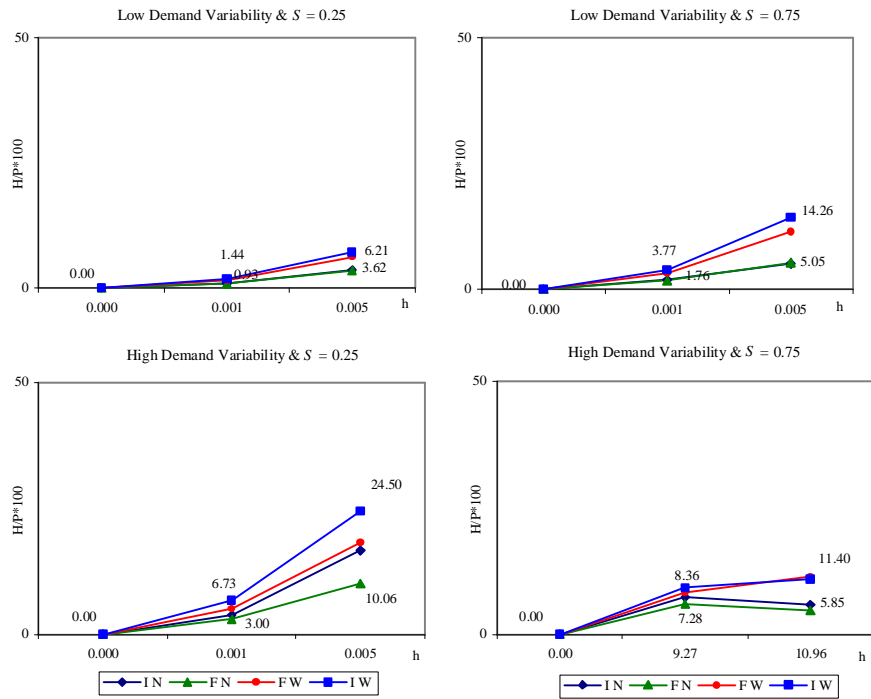


Figure K.21 Holding cost to profit ratio in AKO20T4-C problem

## VITA

Fatma Tevhide Altekin was born in Kütahya on March 29, 1974. She graduated in 1992 from İzmir Bornova Anadolu Lisesi. She received her B. S. degree in July 1996 and M.S. degree in December 1999 in Industrial Engineering from the Middle East Technical University. She has worked as a teaching and research assistant in the department during September 1996 and September 2004. She was enrolled as a part time instructor as of September 2004 in Informatics Institute of Middle East Technical University. Her main areas of interest are disassembly systems, design of assembly lines, mathematical programming, operations management and simulation.