THE EFFECTS OF MULTIPLE REPRESENTATIONS-BASED INSTRUCTION ON SEVENTH GRADE STUDENTS' ALGEBRA PERFORMANCE, ATTITUDE TOWARD MATHEMATICS, AND REPRESENTATION PREFERENCE

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Oylum Akkuş Çıkla

ABSTRACT

THE EFFECTS OF MULTIPLE REPRESENTATIONS-BASED INSTRUCTION ON SEVENTH GRADE STUDENTS' ALGEBRA PERFORMANCE, ATTITUDE TOWARD MATHEMATICS, AND REPRESENTATION PREFERENCE

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The purpose of this study was to investigate the effects of multiple representations-based instruction on seventh grade students' algebra performance, attitudes toward mathematics, and representation preference compared to the conventional teaching. Moreover, it was aimed to find out how students use multiple representations in algebraic situations and the reasons of preferring certain modes of representations.

The study was conducted in four seventh grade classes from two public schools in Ankara in the 2003-2004 academic year, lasting eight weeks.

For assessing algebra performance, three instruments called algebra achievement test, translations among representations skill test, and Chelsea diagnostic algebra test were used. To assess students' attitudes towards mathematics, mathematics attitude scale, to determine students' representation preferences before and after the treatment representation preference inventory were administered. Furthermore, as qualitative data, interview task protocol was prepared and interviews were carried out with the students from experimental and control classes.

The quantitative analyses were conducted by using multivariate covariance and chi square analyses. The results revealed that multiple representations-based instruction had a significant effect on students' algebra performance compared to the conventional teaching. There was no significant difference between the experimental and control groups in terms of their attitudes towards mathematics. The chi square analyses revealed that treatment made a significant contribution to the students' representation preferences.

The results of the interviews indicated that the experimental group students used variety of representations for algebra problems and were capable of using the most appropriate one for the given algebra problems.

Keywords: Mathematics education, multiple representations, representation preference, algebra, learning algebra.

ÖZ

ÇOKLU TEMSİL TEMELLİ ÖĞRETİMİN YEDİNCİ SINIF ÖĞRENCİLERİNİN CEBİR PERFORMANSINA, MATEMATİĞE KARŞI TUTUMUNA VE TEMSİL TERCİHLERİNE ETKİSİ

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Bu çalışma çoklu temsil temelli öğretimin, geleneksel öğretim yöntemiyle karşılaştırıldığında yedinci sınıf öğrencilerinin cebir performanslarına, matematiğe karşı tutumlarına ve temsil tercihlerine olan etkisini araştırmayı amaçlamıştır. Ayrıca; öğrencilerin cebirsel problemlerle karşılaştıkları zaman, çoklu temsilleri nasıl kullandıklarının ortaya çıkarılması ve onların temsil tercihlerinin nedenlerinin araştırılması da amaçlanmıştır.

Çalışma iki devlet okulundan alınan dört yedinci sınıf üzerinde 2003-2004 öğretim yılında gerçekleştirilmiş ve 8 hafta sürmüştür.

Veri toplama amacıyla, birçok ölçme aracı kullanılmıştır. Cebir performansını değerlendirme amacıyla; cebir başarı testi, temsil biçimleri arasında dönüştürme beceri testi ve Chelsea cebir tanı testi olmak üzere üç araç kullanılmıştır. Öğrencilerin matematiğe karşı tutumlarını belirleme amacıyla matematiğe karşı utum ölçeği ve öğrencilerin temsil tercihlerini tespit etmek için deneyden önce ve sonra, temsil biçimi tercih ölçeği uygulanmıştır. Bunların yanısıra; deney ve kontrol gruplarından öğrencilerle görüşmeler yapılmıştır.

Elde edilen niceliksel veriler, yapılan çoklu kovaryans analizi ve kaykare testi ile incelenmiştir. Analiz sonuçlarına göre; gruplar arasında cebir başarı testi, temsil biçimleri arasında dönüştürme beceri testi ve Chelsea cebir tanı testinden alınan puanlara göre, deney grubu lehine istatistiksel olarak manidar bir fark bulunmuştur, ancak gruplar arasında matematiğe karşı tutum ölçeği puanlara göre deney grubu lehine istatistiksel olarak manidar bir fark bulunamamıştır. Kaykare analizi sonuçlarına göre; deney, öğrencilerin temsil tercihlerini manidar olarak değiştirmiştir.

Öğrencilerle yapılan görüşmeler sonucunda, deney grubu öğrencilerinin verilen cebir problemleri için farklı temsil biçimlerini kullanabildikleri ve bunlardan verilen duruma en uygun olanını seçebildikleri ortaya çıkmıştır.

Anahtar Kelimeler: Matematik eğitimi, çoklu temsil, temsil tercihi, cebir, cebir öğrenimi.

Dedicated to the families of Karoğlu, Akkuş, and Çıkla

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TABLE OF CONTENTS

LIST OF TABLES

TABLE

LIST OF FIGURES

FIGURE

LIST OF ABBREVITIONS

ABBREVIATION	
EG:	Experimental group
CG:	Control group
MRI:	Multiple representations-based instruction
CI:	Conventional instruction
TRST:	Translations among representations skill test
OBACT:	Objective based achievement test
CDAT:	Chelsea diagnostic algebra test
ATMS:	Attitude towards mathematics scale
MGPS:	Mathematics grade of previous semester
PRECDAT:	Pretest of Chelsea diagnostic algebra test
PREATMS:	Pretest of attitude towards mathematics scale
POSTCDAT:	Posttest of Chelsea diagnostic algebra test
POSTATMS:	Posttest of attitude towards mathematics scale
LMRTM:	Lesh multiple representational translation model
JMRTM:	Janvier multiple representational translation model
MANCOVA:	Multivariate analysis of covariance
ANCOVA:	Univariate analysis of covariance
SES:	Socio economic status
Sig:	Significance
Df:	Degree of freedom
N:	Sample size
α:	Significance level

CHAPTER 1

INTRODUCTION

Mathematics instruction has always gained attention from mathematics educators who are trying to make conscious efforts to do what is best for the students. From the beginning of the 1950s mathematics educators have been taken precautions against drill and practice as a primary teaching technique since they felt that children would view mathematics as a body of unrelated facts and procedures rather than as a unified system of concepts and operations that state certain patterns and relationships that exist in the real world. (Larkin, 1991; Leitzel, 1991; Nair & Pool, 1991; Resnick & Ford, 1981). The nature of mathematics instruction is being altered nowadays (Pape, Bell, & Yetkin, 2003). To make students appreciate this kind of mathematics, lots of suggestions have been made by many mathematics educators (Resnick & Ford, 1981; Pape, Bell, & Yetkin, 2003). The concept of multiple representations is one of them.

Different meanings can be given to the concept of multiple representations in connection with the teaching and learning of mathematics. It was generally defined as providing the same information in more than one form of external mathematical representation by Goldin and Shteingold (2001). Before examining the concept of multiple representations deeply, it seems to be crucial to review the meaning of representation concept.

1.1 The Concept of Internal and External Representations

In the related literature, various definitions can be encountered related to the concept of "representation" (Abram, 2001; Ainsworth, Bibby, & Wood, 1997; 2002; Brenner, Herman, Ho, & Zimmer, 1999; diSessa & Sherin, 2000; Eisner, 1997; English, 1997; Goldin, 1998a; Hatfield, Edwards, & Bitter, 1993; Janvier, 1987b; Kaput, 1994; Lesh, 1987b; Schwartz, 1984;). Seeger, Voight, & Werschescio (1998) summarized some of those definitions in very general terms as follows:

"…representation is

- any kind of mental state with a specific content.
- a mental reproduction of a former mental state.
- a picture, symbol, or sign.
- symbolic tool one has to learn their language.
- a something "in place of" something else "

Hall (1996) clarified Dewey's view of representation like, "representation is a process of transforming a problematic situation through inquiry and the development of an experience during that activity".

Representations can be categorized into two classes, namely internal and external. Internal representations are defined as "individual cognitive configurations inferred from human behavior describing some aspects of the process of mathematical thinking and problem solving"; on the other hand, external representations can be described as "structured physical situations that can be seen as embodying mathematical ideas" (Goldin & Janvier, 1998, p. 3). According to a constructivist view, internal representations are inside the students' heads, and external representations are situated in the students' environments (Cobb, Yackel, and Wood, 1992). Goldin (1998a) investigated these two phenomenons further. He mentioned that any physical situation including mathematical objects can be defined as external representations. For instance; a number line, illustrated relationships among numbers or a computer-based environment in which mathematical construct can be manipulated as external representations. Internal representations, on the other hand, to the learner only. This means that what students conceptualize in their minds can be labeled as internal representation (Goldin, 1998a).

External representations are the main focus of this dissertation, and the researcher agrees with the definition of Lesh, Post, and Behr (1987a). External representations can be defined as physical embodiments of the ideas, concepts, and procedures, and by them, mathematical ideas can be manipulated by the learners.

Edward (1998) made a powerful and unique distinction between internal and external representations. He clarified that internal representations are something constructed by learners only. They can be figures, ways of solving problems, or schemas. In contrast to the internal representations, external representations are shared conventionally. They have some common language to be communicated within people. For example, in mathematics tables, graphics, tree diagrams can be illustrated as external representations. In addition to Edward's interpretation, Pape and Tchoshanov (2001) made a beneficial framework by which the interaction between internal and external representations is explained. This interaction can be seen in Figure 1. 1.

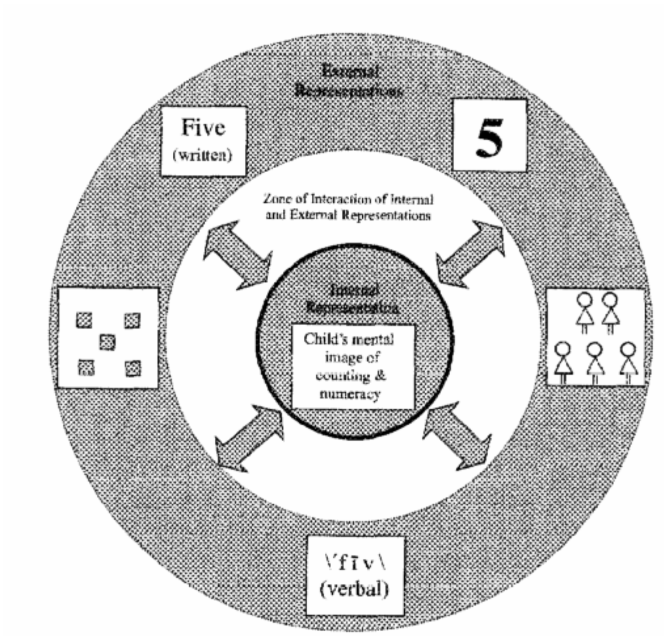


Figure 1.1 The interaction between internal and external representations for understanding numeracy (Pape & Tchoshanov, 2001).

They illustrated the external and internal representations in the context of numeracy. The five distinct external manifestations of the concept of "five" can be seen at the outer circle of the figure. Children's internal representations of "five" are situated at the inner circle, and the relationship between these two kinds of representations is described in the zone of interaction of internal and external representations. Here, as an external representations; drawing, manipulatives, written and verbal symbols can be identified. According to Pape and Tchoshanov (2001), “Mathematical concepts are learned through the gradual building of mental images for primary concepts such as the number of objects in a set or complex natural phenomenon.” (p. 119). They further explained that learners should remark the variety of external representations of the mathematical ideas to create internal representations and conceptualize the mathematical knowledge behind these representations (Pape & Tchoshanov, 2001), so that as Skemp (1986) implied learners can create their appropriate schema to understand abstract mathematical concept.

1.2 The Concept of Multiple Representations

So far the external and internal representations and the distinction between them were sketched, now it was aimed to turn briefly to the concept of multiple representations. The concept of multiple representations includes various representation types and in addition to these representation types, interconnectedness between these representations can also be taken into account. Within the theory of multiple representations understanding includes the following characteristics:

“a. Identifying a mathematical idea in a set of different representations,

b. Manipulating the idea within a variety of representations,

c. Translating the idea from one representation to another,

d. Constructing connections between internal representations in one’s network of representations,

e. Being able to decide the appropriate representation to use in a given problem,

f. Identifying the strengths and weaknesses, differences and similarities of various representation of a concept” (Owens & Clements, 1998, p. 203).

The above six statements would seem to be the crucial points of multiple representations. Even (1998) stated that to be able to identify and represent the same concept in different representational modes, being flexible in passing through the representations, being able to select the most suitable one among various representational modes, and realizing the advantages and disadvantages of representations are the crucial issues for conceptual understanding in mathematics. Hitt (1999) supported this view by stating, "A central goal of mathematics teaching is taken to be that the students be able to pass from one representation type to another without falling into contradictions." (p. 134). Besides, the usage of multiple representations of concepts yield deeper and flexible understanding (Keller & Hirsch, 1998).

Kaput (1991) claimed that to make students create their internal representations, introducing them to variety of external representations should become the aim of mathematics instruction. Also, Janvier (1987a) added that;

> Adults and children are daily confronted with a multiplicity of external representations in mathematical classrooms, school textbooks, and other teaching materials in mathematics in order to make them problem solver using their internal representations (p. 167).

The concept of multiple representations in mathematics education is emphasized by many researchers (Even, 1998; Hitt, 1998; Leinenbach & Raymond, 1996) and also supported by the mathematics education standards developed by the National Council of Teachers of Mathematics (NCTM) and National Council's Science Standard (NRC) in USA (NCTM, 2000; NRC, 1996). To these standards, representation which is accepted as a process standard is central to the study of mathematics. These documents call for students to be able to develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations (NCTM, 2000). The national standards in USA set three expectations for school mathematics for all grade levels from preschool to twelve grades.

1. Create and use representations to organize, record, and communicate mathematical ideas.
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena (NCTM, 2000, p.67).

According to these standards, students are not only encouraged to use multiple representations of mathematical concepts, but also, to create them, use them as a tools for mathematics learning, apply them to the mathematical situations, and also make translations among them (NCTM, 2000). The role of the teachers in order to encourage students to use multiple representations was also noticed by the standards (NCTM, 2000). The teachers should give opportunities to the students to create their own representations, and guide students to see the similarities and differences among representational modes for one particular mathematics object or situation (NCTM, 2000; Smith, 2004a).

1.3 The Concept of Multiple Representations in Algebra

A growing body of research in cognitive psychology, cognitive science, and mathematics education gave importance on the role of using multiple representations in mathematical learning (diSessa, Hammer, & Sherin, 1991; Kaput, 1994; Post, Behr, & Lesh, 1988; Tishman & Perkins, 1997). The need for students to use multiple representations is widely accepted. When the traditional mathematics instruction and curriculum are examined, they only focus on symbol manipulation skills and rote memorization (Brenner, Brar, Duran, Mayer, Moseley, Smith, & Webb, 1995; Moseley & Brenner, 1997), even the fundamental mathematical concepts are generally offered to the students in abstract forms (Pape & Tchoshanov, 2001). The same idea could also be reached when the conventional mathematics classrooms would be visited in Turkey. Particularly, in algebra lessons, in Turkish middle schools, the emphasis is only on the symbolic part of algebraic concepts (MEB, 2002). In addition, instruction focuses mostly on the procedural skills such as solving linear equations, or finding the solution set of a given inequality system. The utilization of multiple representations in algebra classes is avoided by many mathematics teachers. However, algebra has used various representation systems to express ideas and processes and it is one of the cornerstones of school mathematics (Herscovics, 1989; Lubinski & Otto, 2002). This branch deals with symbolizing general numerical relationships, mathematical structures and with operating on those structures (Kieran, 1989; Smith, 2004b). It does not begin in formal schooling years, that kind of thinking appears very early, expands through the years, and continues throughout the life. The importance of algebra was also advocated by NCTM Standards. According to NCTM;

> to think algebraically; one must be able to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts. Each of these components evolves as students grow and mature (NCTM, 2000, p. 64).

Algebraic reasoning involves representing, generalizing, and formulating patterns and regularity in all aspects of mathematics (van de Walle, 2001). That's why in all parts of mathematics the branch of algebra is crucial and students should deepen their algebraic thinking skills in order to be successful in mathematics and life itself. Since algebra is one of the subjects that seem to be the least concrete for pupils, they find algebra challenging in school mathematics. Due to its difficulty, it puts forward serious obstacles in the process of effective and meaningful learning in mathematics (NCTM, 2000). Students entering algebra classes often have difficulty understanding and working with variables and their notations (Kieran & Chalouh, 1992). However, conceptualizing variables and manipulating them are key features of algebra learning. One way of making the process of learning algebra meaningful and effective for middle grade students is to use multiple representations. The use of multiple representations, in other words, expressing algebraic concepts in different forms, such as, verbal, numerical, and graphical has an unavoidable contribution on meaningful algebra learning (Brenner, et al. 1995; Özgün-Koca, 2001). For instance, to understand an algebraic variable and fluently work with it, pupils should be engaged in using multiple representations, such as tabular and graphical representations of that variable, so that they can notice that two different representational modes indicate the same mathematical concept.

Much discussions and related suggestions have taken place about the instruction of algebra (Davies, 1988; Koedinger & Nathan, 2000; McGregor & Price, 1999; Wagner, 1983; Wagner & Kieran, 1999; Yerushalmy & Gilead, 1997). Most of the mathematics educators have been complaining that students view algebra just as the process of manipulating symbols and getting the correct result (Blanton & Kaput, 2003; Kaput, 1986; Moseley & Brenner, 1997; Pirie & Martin, 1997; van Dyke & Craine, 1997). They claimed that students in algebra classes tend to use only equation expression to represent algebra concept, they mostly ignore other type of representations. McCoy, Thomas, and Little (1996) acknowledged that;

> the traditional symbol-manipulation algebra courses in which students learned to simplify algebraic expressions and solve equations with little connection to real-world application are no longer sufficient. There is a need to foster students' algebraic models in real-world contexts using multiple representational tasks (p. 42).

Being aware of this problem in algebra classes, it is considered that there is a need for students to learn variety of representations and to make translations among them and for teachers to introduce their students with the concept of multiple representations.

After reviewing Lesh Multiple Representational Translations Model (LMRTM) and Janvier's Representational Translations Model (JRTM), the researcher combined these two multiple representational models to fit the research questions of this study, and to provide better mathematics instruction and conceptual algebra understanding in algebra classes. When Lesh model is deeply examined, it can be understood that his definition for representation had some similarities with the definition of Janvier (1987b). Lesh defined representation as external (and therefore observable) embodiments of students' conceptualizations for internal representations (Lesh, Post, & Behr, 1987b).

According to Lesh (1979) and Janvier (1987b), conceptual understanding relies on students' having experiences representing contents in each of representational modes. Cramer and Bezuk (1991) also explained Lesh' point of view as; understanding in mathematics could be defined as the ability to represent a mathematical idea in multiple ways and to make connections among different representational modes. Lesh (1979) suggested a model for multiple representations of mathematical concepts. This model has five modes of representations, which are: (1) real-world situations, (2) manipulatives, (3) pictures, (4) spoken symbols, (5) written symbols.

Although the meanings of and relationships between the above representational modes will be discussed in Chapter 2, it can be said that Lesh model emphasizes the transformations within a single mode and the translations among these modes of representation (Lesh, Post, & Behr, 1987a). From his point of view, using multiple representations in school algebra emerges as a beneficial vehicle that supports a conceptual shift to meaningful learning in schools (Lesh, 1979). Hence, the framework of this study is shaped by an alternative instructional approach, which involves multiple representations of algebra topics in middle school.

1.4 Purpose of the Study

The purpose of this study is to examine the effects of a treatment based on multiple representations on seventh grade students' performance in algebra, attitude towards mathematics, and representation preference. Furthermore, this study has sought to aim the followings:

- to reveal how students use multiple representations in algebraic situations, such as; algebra word problems.
- to investigate the representation preferences of the students before and after the unit of instruction and to examine the reasons of preferring certain kinds of representations.

1.5 Specific Research Questions of this Study

This study attempted to answer the following research questions:

1. What are the effects of the multiple representations-based instruction compared to conventional teaching method on seventh grade students' algebra performance, attitudes toward mathematics and representation preference when students' gender, mathematics grade of previous semester, age, prior algebra level, and attitudes towards mathematics are controlled?

2. What are the students' preferences of representations before and after the unit of instruction?

3. How do the students use multiple representations when they encounter an algebraic situation?

1.6 Hypotheses of the Study

The following hypotheses were tested to answer the research questions:

Null Hypothesis 1: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the collective dependent variables of the seventh grade students' posttest scores on the Algebra Achievement Test (AAT), translations among representations skill test (TRST), and Chelsea diagnostic algebra test (CDAT), and attitudes towards mathematics scale (ATMS) when students' gender, age, mathematics grade in previous semester, the scores on the pre-implementation of the CDAT (PRECDAT), and on the pre-implementation of the ATMS (PREATMS) are controlled.

Null Hypothesis 2: There will be no significant effects of two methods of teaching (multiple representation-based and conventional) on the population means of the seventh grade students' scores on the AAT, after controlling their age, MGPS, and the PRECDAT scores.

Null Hypothesis 3: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the seventh grade students' scores on TRST, after controlling their age, MGPS, and PRECDAT scores.

Null Hypothesis 4: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the seventh grade students' scores on the post implementation of CDAT, after controlling their age, MGPS, and PRECDAT scores.

Null Hypothesis 5: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the seventh grade students' scores on the post implementation of ATMS, after controlling their age, MGPS, and PRECDAT scores.

1.7 Definitions of the Important Terms

Representation: It is typically a sign or a configuration of signs, the numeral or objects. It stands for something other than itself (Seeger, Voight, & Werschescio, 1998).

Internal Representations: Mental configurations on students' minds (Goldin, 1990).

External representations: External embodiments for students (Kaput, 1994).

Multiple representations-based instruction: It is a kind of instruction that involves a multiple representations of a concept. This means that in order to explain a concept, to use variety of external representations such as; tables, graphs, pictures, etc. for comprehending the concept.

Conventional teaching method: It is a kind of teaching method in which only symbolic mode of representation was utilized to teach mathematical concepts by teachers (Monk, 2004; Smith, 2004a).

Algebra performance: Seventh grade student's performance on the instruments of the AAT, TRST, and CDAT.

1.8 Significance of the Study

The issues of what instructional approaches should be used in algebra classes have not been solved yet. No matter which instructional approach is used, the primary goal of mathematics instruction should be to help students in forming conceptual understanding. Janvier (1987b) mentioned that if the teachers enrich their algebra classrooms by placing multiple representations, the students can more efficiently make connections between the meaning of algebraic concepts and the way of representing them, therefore they simply "go for the meaning, beware of the syntax" which results in conceptual understanding (Janvier, 1987c).

In addition to Janvier's view, Lesh, Landau, and Hamilton (1983) stated that "If translations among representational modes abilities are such obvious components of mathematics understanding and problem solving, why are they so often omitted from instruction and testing? One reason is that many translation types are not easily "bookable", other reasons stem from the fact that so many research questions remain unresolved concerning the exact roles that translations play in the acquisition and use of mathematical ideas, and about the instructional outcomes that can be expected if they are taught effectively" (p. 264). Here they identified the general textbooks representation systems (mostly pictures, written language and symbols) as "bookable" representations. Since they are convenient, most of the time teachers prefer to use them rather than the variety of representational modes.

The improvement of mathematical understanding and representational thinking of students necessitate flexible use of multiple representations and the interaction of external and internal representations (Pape & Tchoshanov, 2001). Since making meaningful translations in representational modes plays a crucial role in acquisition of mathematical concepts and there is an unanswered question about the instructional outcome of using multiple representations, the research questions of this study are worth to be investigated.

Since this study focuses on the effects of multiple representation-based environments in mathematics classroom, its results should help mathematics educators who seek alternative pedagogical instructions in classroom settings. Furthermore, if a teacher is aware of his/her students understanding of the multiple representations and what kind of learning is supported by multiple representation-based environments, s/he can better choose and utilize an appropriate type of methods, manipulatives, or activities to meet the needs of students.

Moreover, providing students with a multiple representation-based algebra instruction would promote a conceptual shift to thinking algebraically. Therefore, receiving such kind of instruction makes students more competent in the area of algebra.

1.9 Assumptions of the Study

Four assumptions for this study were listed as follows:

1. The seventh grade students participated in the study represent typical middle school students.
2. Participants responded honestly on the measuring instruments.
3. All instruments were administered to the experimental and control groups under the same conditions.
4. The possible differences between teachers and the researcher have no influence on the results of the study.

1.10 Limitations of the Study

There are several identifiable factors that limit the generalizability of the research or that would have enhanced the effectiveness of the treatment.

A major limitation is the act of researcher as a mathematics teacher. Because of the unwillingness of the teachers and administrators in the selected schools, the researcher was obligated to implement the treatment in both experimental groups. As it was the case personal biases and enthusiasm may have influenced the results of the study. Therefore, there may be a bias favoring the implementation of multiple representation-based instructions in treatment groups.

This study is limited by the modes of multiple representations included only in the Lesh and Janvier model. There was no attempt to use any computer-based representations or graphing calculators.

Another drawback of instructing the experimental group by the researcher is the teacher difference for the experimental and control groups. The two experimental groups were instructed by the researcher, whereas the two control groups were given conventional teaching by their regular mathematics teacher. This case may have an impact on the different test scores of the students.

Another limitation of the study was using a conventional sampling. Due to the restrictions, two schools which were conventional for the researchers were determined as a sample.

In spite of the fact that the control groups' teachers' background has common features, the way in which each teacher implemented the conventional seventh grade algebra curriculum was an important consideration.

Limited experience with the sample lessons based on multiple representations can also be seen as a drawback. Before the treatment the students in the treatment groups were introduced with multiple representations throughout sample lesson plans about rational numbers only for 4 hours. The students were coming to a traditional classroom in which a traditional teacher taught the mathematics lessons and traditional textbooks were being used, introducing this new approach should be lasting at least one month.

Multiple representation based instruction was implemented in seventh grade classes for this study. However, only two seventh grade classes were taken as experimental groups. Therefore, until replication studies dealing with different student populations are conducted, the applicability of this instructional method to other groups such as to the sixth and eight graders remains an open question.

Besides, algebra was chosen as the content for this study, this narrow focus limits to generalize the results of this study to other contents in mathematics.

CHAPTER 2

CONCEPTUAL FRAMEWORK FOR THE STUDY

The following chapter describes the underlying theory that comprises the conceptual framework for this study. Ideas from several psychologists and mathematics educators formed the foundation for the research questions of this study. Even though no single unifying theory includes multiple representations, fundamentals of different theories provide a theoretical framework for conceptualizing multiple representations. Theories related with multiple representations advocated by the pioneers in this area are discussed in this chapter and this discussion is concluded with a multiple representations model which shapes this study.

2.1 Early Pioneers in the Area of Multiple Representations in Mathematics Education

The theory of multiple representations in understanding and manipulation of mathematical concepts has gained importance with Dienes' works. He devoted a career to design materials for teaching mathematics and conducting experiments to enlighten certain aspects of mathematical concept acquisition. He influenced by Piagetian theory and worked closely with Bruner on an experimental mathematics project (Resnick & Ford, 1981). In Dienes' works, the concept of multiple representations was named as "Perceptual Variability Principle" which means presenting the same conceptual structure in the form of as many perceptual equivalents as possible so that children could have the mathematical essence of an abstraction (Dienes, 1960). According to Dienes, the concepts must be presented in multiple embodiments; that is to say, children should work with several different kinds of materials, all of which embody the concept of interest (Dienes, 1960). Multiple embodiments are viewed in the book of Resnick and Ford (1981) as a variety of environment in which the children could see the structure from several different perspectives and build up a rich store of mental images belonging to each concept.

Dienes claimed that children are not accustomed to mathematical concepts in their daily life, and those concepts should be introduced to them within the realm of concrete experiences (Resnick & Ford, 1981). Due to this reason, he designed a set of mathematical materials called multibase arithmetic blocks or Dienes blocks. They are made up of wood showing different base systems. He also cautions that using only mathematical materials would create a handicap for conceptualization of mathematical activities. Symbolization should also be placed in children's minds. He believes that as symbols are applied, mathematical concepts could be free from their concrete referents and be the new tools for creating new symbols (Lesh, Post, & Behr, 1987a; Resnick & Ford, 1981).

In addition to the works of Dienes, Bruner made a significant contribution to the multiple representations theory. As cited in Resnick and Ford (1981), Bruner conducted studies in teaching cases with children. He examined the cognitive processes of children and how children represented the concepts mentally (Resnick & Ford, 1981). Bruner (1960) claims that mental development of children includes the construction of a model of the world in the child's mind, an internalized set of structures for representing the world around them (Bruner, 1960). These structures have definite features, and in the course of development, they and the features that rule them alter in certain systematic ways (Bruner, 1960). When teachers transmitting the structure to the students, they face with a problem of finding the language and ideas that the other person would be able to use if they were attempting to explain the same thing. In most cases the language that is used would not fit the child's schema. Bruner (1960) argued that how past experience is coded and processed in child's mind so that it may indeed be relevant and usable in the present when needed. Such a system of coding and processing is what he called as a representation. He (1966) also argued that the importance of multiple forms of representations by stating that:

> Any idea or problem or body of knowledge can be presented in a form simple enough so that any particular learner can understand it in a recognizable form. The ways in which humans mentally represented acts, objects, and ideas could be translated into ways of presenting concepts in classroom. And even though some students might be quite ready for a purely symbolic presentation, it seemed wise to present at least the iconic mode as well, so that learners would have mental images to fall back on in case their symbolic manipulations failed (Bruner, 1966; p. 44).

Like Dienes, Bruner suggested that development of concepts involves successive restructurings of facts and relations, which came from children's interactions with and active manipulation of their environment (Bruner, 1966). He describes three modes of representation; namely, enactive, iconic, and symbolic. Enactive representation is a mode of representation through appropriate motor response (Cramer & Karnowski, 1995). Resnick and Ford (1981) illustrated enactive representation of Bruner as:

> …what we are seeing in children who figure addition problems by tapping their fingers against chin in an obvious counting motion. Counting for these children may be represented as a motor act (p. 59).

The second representation mode of Bruner is iconic, that is visualizing an operation or manipulation as a way of not only remembering the act but also recreating it mentally if it is necessary (Resnick & Ford, 1981). For instance, a child learning numbers between 1 and 10 might use the pictures of numbers arranged from "1" as a smallest picture to "10" as a biggest picture. Therefore, he could understand the numbers with reference to pictures of those numbers.

The last mode of representation is also the most abstract mode which is symbolic representation. In this mode of representation a symbol - a word or a mark - stands for something but does not look like that thing (Cramer & Karnowski, 1995). For example, numerals do not resemble their wordings (Resnick & Ford, 1981). According to Bruner, these three representational modes are developmental. The development of each mode is depending on the previous mode and after a long term practice with one mode, one can make transition to the next mode (Bruner, 1966).

2.2 A Constructivist View on the Concept of Representation

The concept of representation has also been investigated by constructivists. Their naming "representational view of mind" argues that representation is an active construction (Seeger, Voight, & Werschescio, 1998; von Glasersfeld, 1987b). Vergnaud defined representation (1987) as an important element in the course of teaching and learning mathematics. This importance is appreciated for not only the use of symbolic systems is inevitable in mathematics, but also it is rich, varied, and universal. As a constructivist Goldin (2000) defined representational modes as a system including spoken and written symbols, static figural models and pictures, manipulative models, and real world situations. He claims further, representational system includes signs like letters, numerals and operations with signs like obtaining multidigit numerals from single digit numerals or composing a word using letters. According to him, a representational system has intrinsic (within itself) and extrinsic (with other systems of representation) structure. It is essential to provide a model consisting of interactions both within one representational mode and among the representational mode to enhance learning and problem solving in mathematics (Goldin, 1990).

Goldin (1998a) highlighted the close relationship between external and internal representations as in Figure 2.1;

Signifier (External Representations)

↕

Signified (Internal Representations)

Figure 2.1 The relationship between external and internal representations according to Goldin (1998a).

In this outline, it can be grasped that there is a close relationship between external and internal representations in terms of semiotic views. He identifies the external representational system as constructs to understand mathematics (Goldin, 1998a). They are easy to use, permit visualization, and universal. Internal representational system was defined by him as again constructs of mathematical behaviors (Goldin, 1998b). With the help of them how individual learn and conceptualize can be understood. He also added that the interaction between the two systems is not just essential, it is the whole point. In order to trace this interaction, the child's environment should be designed for obtaining all kinds of representations ranging from spoken language to mathematical symbols (Goldin, 1990).

2.3 Semiotic View on Multiple Representations

After the brief introduction of Goldin's semiotic and multiple representation resemblance, the influences on multiple representation theory from semiotic domain can be explained in detail. Semiotics deal with signs and actions of signs. In semiotic, there are three important terms, sign, interpretant, and object (Vile & Lerman, 1996). As cited in Klein (2003), Peirce indicated that; sign refers to something which stands to somebody for something. If this sign has a meaning in somebody's mind, it is called interpretant, and what this sign belongs to, is called object (Vile & Lerman, 1996).

Klein (2003) combined semiotic view with multiple representations. According to him, representations are signs from which students learn something. He suggested that in mathematics curriculum the "object" from Peircian view can be considered the topic that should be taught, the "sign" is the representations used for teaching a topic, and "interpretant" is also signs that would help to conceptualize the object. Therefore, he defines a representation as a sign or combination of signs like Janvier (1998). For instance, a manipulative of an equation can be seen as a sign, its object is an equation, and its interpretant is accompanying text to this manipulative for equation.

External and internal representational systems from semiotic point of view were also examined by Skemp (1986). He argued that "a concept is a purely mental object" (p. 64)". Since no one can have the ability of observing someone else's mind directly, means and indicators of the minds should be used for interpreting concepts. These are called external representations, according to Skemp. They are visible; and they should be mentally related with an idea. This idea is the meaning of that representation. He also claimed that an external representation is meaningless without its attached idea (Skemp, 1986).

Dufour-Janvier, Bednarz, and Belanger (1987) also clarified the internal and external representations by combining semiotic views. They claimed that the meaning of a signified refers to the internal representations which can be concerned more particularly "mental images corresponding to internal formulations which was a construction of reality" (Dufour-Janvier, et al. 1987, p. 110). External representations on the other hand refer to "all external symbolic organizations (symbol, schema, diagrams, etc.) that have as their objective to represent externally a certain mathematical reality" (Dufour-Janvier, et al. 1987, p. 110), which can be combined by the meaning of signifier.

2.4 Kaput's View on Multiple Representations

In addition to the above researchers mentioned above, Kaput (1989, 1991, 1994) also proposed an important theory of understanding within multiple representational particularly in technology context. He distinguished between internal and external representations by stating; the former is referred as "mental structures" and the latter as "notation systems" (Kaput, 1991). He defined those terms as follows; "Mental structures are means by which an individual organizes and manages the flow of experience, and notation systems are materially realizable cultural or linguistic artifacts shared by a cultural or language community" (Kaput, 1991, p. 55).

According to him, notation systems can be anything such as a mark on the paper or a sign on a computer screen, and they are used by the people to organize their mental structures (Kaput, 1991). He claimed that when someone is talking about notational system, its mental structure should also be considered. One can not learn something from notational systems when those systems are told separately from mental structures (Kaput, 1989). He also supports the view of von Glasersfeld (1987b) who argues that;

"A representation does not represent by itself, it needs interpreting and to be interpreted, it needs an interpreter" (p. 216).

Kaput (1987) further stated that his semiotic-oriented definition by saying any particular representation should include five entities:

1. the represented world,
2. the representing world,
3. the aspects of the world being represented
4. the aspects of the representing world doing the representing
5. the relation between these two worlds.

Kaput (1994) echoes the constructivist view of representations by claiming that the act of representation is involved in the relation between the representing thing and the represented thing. In mathematics, the correspondence between the represented and representing world; or in other words, the signified and the signifier should be established for achieving a permanent and meaningful learning (Kaput, 1993). The interaction in Figure 2.2 should be built in the early years of children's mathematical activities.

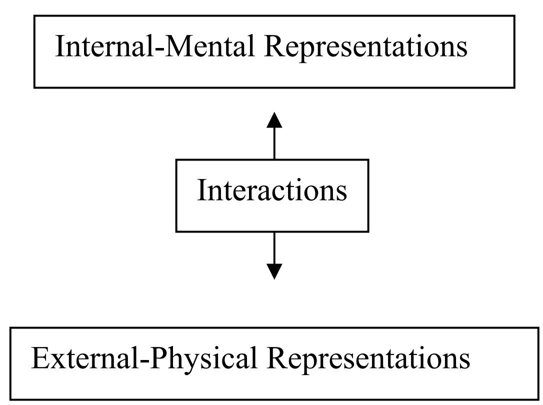


Figure 2.2 The interactions between internal and external representations

Kaput (1994) argued that internal representations are mental configurations that should be created and developed by the person himself. They are not observable, whereas external representations can be physical configurations and they can be observed, such as equations, pictures, or computer signs (Kaput, 1991). He gave an explanatory example including internal and external representations.

> …sometimes an individual externalizes in his or her internal structures in physical form , by writing, speaking, manipulating the elements of some concrete system, and so on. For example consider the graph drawn in Cartesian coordinates by a person to represent the equation y+3x-6 = 0. The particular graph is not an isolated drawing from its equational context, its table configuration, and its several meanings (Kaput, 1989, p. 169).

As it was stated before, Kaput is particularly interested in the representational role of technology. He believes that computer technology is a popular media to link the mathematical representations, such as graphs, tables, and formulas (Blanton & Kaput, 2003; Kaput, 1994). He states that usage of dynamic media makes the viewing of representations and performing the translations among representations more easier (Kaput, 1991). To him, computers are the most helpful carrier for children to externalize their internal representations in multiple ways. He stated that;

> …with more than one representation available at any given time, we can have our cake and eat it too, in the sense of being able to trade on the accessibility and strengths of different representations without being limited by the weakness of any particular one (Kaput, 1991, p.70).

As a result of his various research studies in using technology as a multiple representation supplier in mathematics education, he concluded that students need to express the connected link between the external representations and they should be forced to generate new representational modes (Kaput, 1994).

2.5 Janvier's View on Multiple Representations

The usage of multiple representations in mathematical learning was investigated in depth by Claude Janvier who edited a book about the problems of representation in mathematical learning. He defined "understanding" as a cumulative process mainly based upon the capacity of dealing with an "ever-enriching" set of representations (Janvier, 1987b, p. 67). He said that a representation "may be a combination of something written on paper, something existing in the form of physical objects and carefully constructed arrangement of idea in one's mind" (Janvier, 1987b, p. 68). A representation can also be identified as a combination of three components: written symbols, real objects, and mental images. There are two important key terms in a theory of representation that are; "to mean or to signify, as they are used to express the link existing between external representation (signifier) and internal representation (signified)" (Janvier, Girardon, & Morand, 1993, p. 81). External representations were defined as "acts stimuli on the senses or embodiments of ideas and concepts", whereas internal representations are regarded as "cognitive or mental models, schemas, concepts, conceptions, and mental objects" which are illusive and not directly observed (Janvier, Girardon, & Morand, 1993, p. 81). He emphasized the external representations by further definitions. They include "some material organization of symbols such as diagram, graph, schema, which refers to other entities or 'modelizes' various mental processes" (Janvier, 1987a, p. 147). He made a visual resemblance between a representation and a star in Figure 2.3.

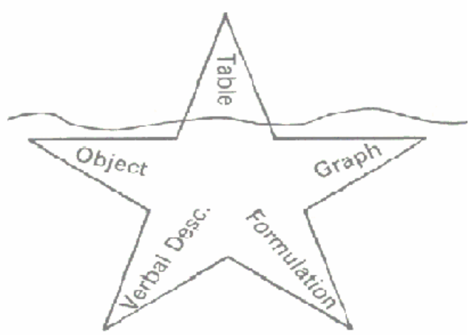


Figure 2.3 The visual resemblance between a representation and a star

To him, a representation would be a sort of star-like iceberg that would show one point at a time. A translation would occur while going from one point to another when dealing external representations which were also named as schematization by Janvier (1987c). By a translation it was meant that "the psychological processes involved in going from one mode of representation to another; for example, from an equation to a graph" (Janvier, 1987c, p. 27). This representational process is presented in Table 2.1 in detail.

Table 2.1 Janvier's Representational Translations Model (JRTM)

From-To	Situations, pictures and verbal descriptions	Tables	Graphs	Formulas
Situations, pictures, verbal descriptions	_	Measuring	Modeling	Analytical Modeling
Tables	Reading	_	Plotting	Fitting
Graphs	Interpretation	Reading Off	_	Curve Fitting
Formulas	Parameter Recognition	Computing	Sketching	_

Janvier named the diagonal cells in which the translations between two same representational modes occur, like from tables to tables as transposition. The names given to the cells can be changed according to the "context in which a particular translation is achieved" (Janvier, 1987c, p. 27). He further mentioned that "transitional representations are pedagogical devices in order to clarify concepts in mathematics, with strengths and limitations that were explored" (Janvier, 1987b, p. 69).

There can be two kinds of different translations in Janvier's view of representation: direct and indirect translations. Direct translations might be carried out from one representational mode to the other one without using any other kind of representational mode between this translation; for instance, from an equation to a representation of a table. On the other hand, a translation from an equation to a table can be conducted by making translations from an equation to a graph, and then to a table. In this case this kind of translation process is called as indirect translation (Janvier, 1998).

Another important point about the translation process is the source and target phenomenon. Any translation involves at least two modes of representations forms source and target. He claimed that "to achieve directly and correctly a given translation, one has to look at it from target point of view means which representation mode one would like to have after the translation and derive the results" (Janvier, 1987b, p. 68). The cognitive processes of students might be changed with respect to being source or target of one representational mode. Therefore, teachers should design their instruction considering each representation either as a source or as a target (Janvier, 1987c).

Dufour-Janvier, et al. (1987) investigated the multiple representation theory in terms of students' usage of external representations in classrooms and their drawbacks. They claimed that using conventional representations as mathematical tools, rejecting one representation to another in a given mathematical situation, making translations from one representation to another, are expected from the learner in traditional mathematics. All these expectations suppose that "the learner has grasped the multiple representations; that he knows the possibilities, the limits, and the effectiveness of each" (Dufour-Janvier, et al., 1987, p. 111). Moreover, the learner is supposed to choose the appropriate mode of representation depending on the mathematical task. For instance, given an algebra problem to solve, the equation and the graph might not be giving equal access to the same information and possibilities. Hence, to meet all these expectations, instructional strategies should be improved in a way that they include variety of representations and are flexible to use translation processes in representations (Dufour-Janvier, et al. 1987).

2.6 Lesh's Multiple Representations Model

Another approach to the theory of multiple representations which is called Lesh Multiple Representations Translations Model (LMRTM) has been suggested by Richard Lesh (1979). His theory draws the theoretical framework of this study since he improved a model involving translations among representational modes and transformation within one representational mode, and parts of this model were used in order to design the lesson plans used in this study.

According to Lesh, Post and Behr (1987b), representations are crucial for understanding mathematical concepts. They defined representation as "external (and therefore observable) embodiments of students' internal conceptualizations" (Lesh, et al. 1987b, p. 34). This model suggests that if a student understands a mathematical idea she or he should have the ability of making translations between and within modes of representations. He identified five distinct modes of representations that occur in mathematics learning and problem solving; they are "(1) real-world situations –in which knowledge is organized around "real world" events; (2) manipulatives –in which the "elements" in the system have little meaning but the "built in" relationships and operations fit many everyday situations; (3) pictures or diagrams –static figural models; (4) spoken symbols –it can be everyday language; (5) written symbols –in which specialized sentences and phrases take place" (Lesh, et al. 1987b, p. 38). Lesh model can also be described by using the diagram in Figure 2.4.

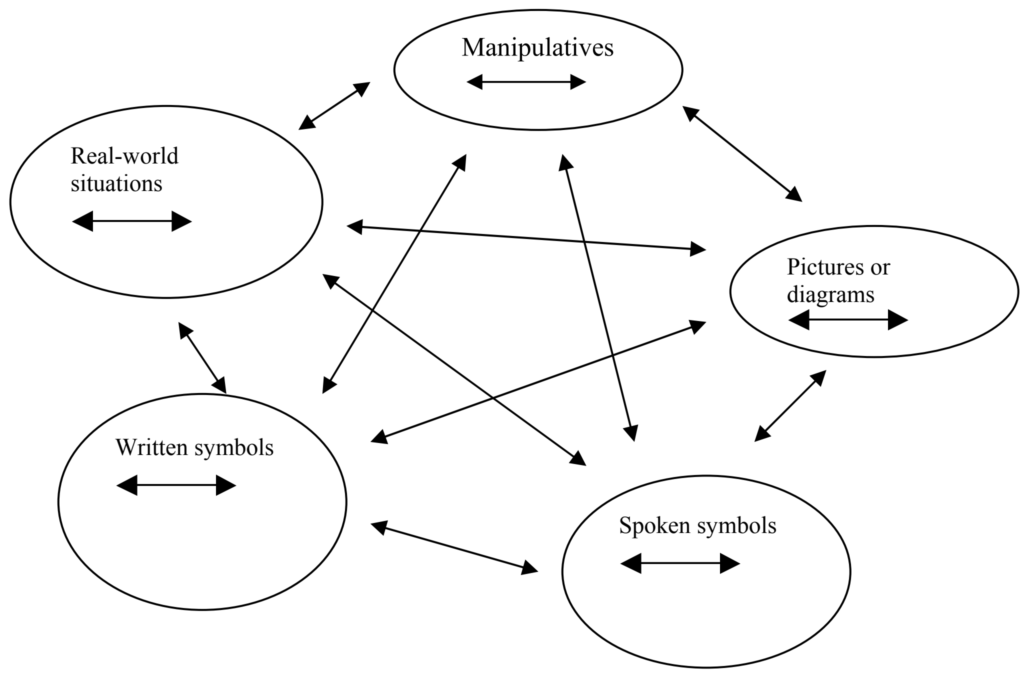


Figure 2.4 Lesh Multiple Representations Translations Model (LMRTM)

In the model given in Figure 2.4, not only the five distinct types of representational modes (or systems), but also the translations among them and transformations within them are important. The translations among representations aim to require students to establish a relationship (or mapping) from one representational system to another, preserving structural characteristics and meaning. Lesh (1979) and Lesh and Kelly (1997) indicated that as a students' concept of a given idea evolves, the related underlying translation networks become more complex, and to make a translation between representational modes, first a student should conceptualize the mathematical idea within given representational system. From this point of view, a good problem solver should be able to "sufficiently flexible" in using variety of representational systems.

He also added that most of the representations can be identified on a continuum between "transparent" and "opaque" (Lesh, et al. 1987b, p. 40). "A transparent representation would have no meaning of its own, apart from the situation or thing that it is modeling at any given moment; furthermore, all of its meaning would be endowed by the student. An opaque representation would have significant meaning in and of itself, quite apart from that imposed on it by a particular student. Clearly, many mathematical representations like Cartesian coordinate system have significant characteristics that are both transparent and opaque" (p. 40).

According to Lesh, for students to understand a mathematical concept like "1/3", they should be able to:

1. recognize "1/3" embedded in a variety of different representational systems,
2. flexibly manipulate "1/3" within given representational system,
3. accurately translate the idea from one representational system to another (Lesh, et al. 1987b, p. 33).

He claimed further, "As a student's concept of a given idea evolves, the related underlying transformation/translation networks become more complex; and teachers who are successful at teaching these ideas often do so by reversing this evolutionary process; that is, teachers simplify, concretize, particularize, illustrate, and paraphrase these ideas, and imbed them in familiar situations" (p. 36).

Cramer and Bezuk (1991) mentioned that Lesh model should not be interpreted as a hierarchical model for representation like Bruner's model. He emphasized particularly the relationship among the modes of representations and the transformation within one single mode of representation. Lesh indicated that "…translation requires establishing a relationship (or mapping) from one representational system to another preserving structural characteristics and meaning in much the same way as in translating from one written language to another. Translation (dis)abilities are significant factors influencing both mathematical learning and problem-solving performance, and that strengthening or remediating these abilities facilitate the acquisition and use of elementary mathematical ideas" (Lesh, et al. 1987b, p. 38)". He also added that;

> A child who has difficulty in translating from real situations to written symbols may find it helpful to begin by translating from real situations to spoken words and then translate from spoken words to written symbols; or it may be useful to practice the inverse of the troublesome translation, i.e., identifying familiar situations that fit given situations (Lesh, Landau, & Hamilton, 1983, p. 267).

According to Lesh, in some cases a translation within representational systems can be plural; this means that a student may begin to solve a problem by making a translation from one representational system to another, and then may map from this representational system to another system; therefore, s/he can combine more than two representational systems in one problem setting (Lesh, et al. 1987b, p. 40). He illustrated this view with the following example.

"Al has an after school job. He earns \$6 per hour if he works 15 hours per week. If he works more than 15 hours, he gets paid "time and a half" for overtime. How many hours must Al work to earn \$135 during one week?"

A student attempted to solve the above algebraic word problem she or he uses three modes of representations, which are:

1. from English sentences to an algebraic sentence
2. from an algebraic sentence to an arithmetic sentence
3. from arithmetic sentence back into the original problem situation (Lesh, et al. 1987b, p. 33).

Therefore, for achieving the correct result of the problem, more than one mode of representation and the translations among the representational modes should be utilized.

2.7 The Multiple Representations Model for this Research Study

As stated throughout this chapter, many researchers have tried to respond to the need for mathematical understanding within representational context by generating ideas and theories. However, they have not reached a unifying theory yet. There are many research ideas suggested by the mathematics educators about learning mathematics in multiple representational contexts. Within those ideas, the researcher is interested in investigating particularly students' ability to use the given representational model for solving problems, and to make translations among the representational modes. Therefore, it is impossible using only one point of view to design this research study. After synthesizing a number of theories about multiple representations, this study emphasizes a multiple representational translations model combined from the models belonging to Lesh and Janvier. The five distinct representational modes; namely, manipulatives, real-world situations, written symbols, spoken symbols, and pictures or diagrams in LMRTM were directly included to the model of this study. Some of those representational modes were named differently referring the JRTM. Instead of "written symbols" from LMRTM, wording of "formulas" from JRTM was decided to include in this study since in the algebra topics within the scope of this dissertation students dealt particularly with the formulas including the first degree equations. Besides in lieu of the combination of "situations, pictures, and verbal descriptions", the researcher decided to use those representational modes separately, so "manipulatives", "pictures or diagrams", and "spoken symbols" were taken from LMRTM instead of "situations, pictures, and verbal descriptions" form JRTM. In addition to those representational modes, "tables" and "graphs" were taken separately from JRTM. So the new combined model was finally formed. JRTM was revised in light of the Lesh (1979) ideas as appeared in Table 2.2

Table 2.2 The combined model of Lesh and Janvier for translations among representation modes

From \ To	Spoken Symbols	Tables	Graphs	Formulas (Equations)	Manipulatives	Real Life Situations	Pictures
Spoken Symbols	–	Measuring	Sketching	Abstracting	Acting out	Acting out	Drawing
Tables	Reading	_	Plotting	Fitting	Modeling	Modeling	Visualizing
Graphs	Interpretation	Reading Off	_	Fitting	Modeling		
Formulas (Equations)	Reading	Computing	Sketching	–	Concretizing	Exemplifying	Localizing
Manipulatives	Describing	Exemplifying	Concretizing	Symbolizing	–	Simplifying, Generalizing	Drawing
Real Life Situations	Describing	Exemplifying	Plotting	Modeling, Abstracting	Particularizing	–	Modeling
Pictures or Diagrams	Describing	Describing	Sketching	Abstracting	Constructing	Situationalizing	–

The present study investigated using multiple representations theory in seventh graders' algebra courses honoring the tenets of LMRTM and JRTM, and examined the effects of a treatment based on the combined model of multiple representations on students' algebra performance, attitudes towards mathematics, and representation preference. This chapter has taken into account the theories behind multiple representations. Various ways that theory of multiple representations has been involved in research studies will be presented next in Chapter 3.

CHAPTER 3

LITERATURE REVIEW

The aim of this literature review is to provide a coherent summary of the research studies about the usage of multiple representations in mathematics, particularly in algebra, and student's representation preferences. It is meant to obtain a basis and argument for this study. The review of literature is divided into six sections. The first section focuses on literature that provides an insight into the studies about algebra. The second section reviews the literature about multiple representations. The third section emphasizes a particular mathematics field, algebra, in connection with multiple representations, and the literature related to student's representation preferences is presented in the fourth section. Moreover, a coherent summary of the reviewed literature is drawn in the fifth section, and finally, the results and implications of the researches documented so far are assessed in terms of the need of the current study.

3.1 Research Studies Focusing on Algebra

As a well-known fact, algebra is a milestone for students to develop their mathematical adventure throughout life. It is a vehicle for logical reasoning and it improves abstract thinking (Stacey & McGregor, 1999). Due to the fact that algebra is a crucial branch for students, there exists an extensive range of research studies that looks teaching and learning of algebra, students' performance, difficulties, and misconceptions at various domains in algebra. Some of these studies in relation with the scope of this dissertation will be discussed here.

Austin and Thompson (1997) integrated literature and algebra for sixth and seventh graders. They presented algebraic activities integrated with stories in two classrooms. Students were asked to explore patterns in algebra through reading stories and after reading them they were responsible to explain stories in algebraic concepts, like patterns and functions. It was found that most of the students were lack in ability to express algebraic objects in an abstract manner; however, they could give verbal and written expressions of the patterns in stories. Furthermore, by integrating literature in algebra classrooms, Austin and Thompson mentioned that the students were more comfortable with studying algebra. As they were exploring algebraic relationships by using stories, they developed self-confidence in algebra. It was recommended to build algebra concepts by introducing them in real-world applications like stories before abstract symbolic forms.

Another research study about using writing in algebra was conducted by Miller and England (1989) who believed that writing in algebra is a kind of integration in two different disciplines and it increases students' interest and achievement in algebra. Their aim was to investigate how writing is used in algebra classrooms. For this purpose, they observed three high school algebra classrooms in which teachers use writing as an instructional approach for algebra. They also conducted interviews with students and teachers to take their perceptions about the effectiveness of writing in algebra. The teachers claimed that asking students to write about an algebraic concept, challenges them at first. They could perform essential procedures in algebra to solve questions; however, they could write nothing about an equation. Miller and England reported that, in these writings, the teachers noticed the errors and misconceptions of the students about algebra. Furthermore, the students found algebra meaningful as they were writing about it, since they considered the meaning of the equations; for example, when they were asked to write the meaning of it. Otherwise, they just solved equations and found the correct answer. It was also reported that students' willingness to attend the algebra class was increased.

According to Harel (1989), students might better abstract algebraic concepts and representations of those concepts if they are given in a form of physical embodiments, like concrete or semi-concrete. To present linear algebra in physical embodiment forms, Harel designed a study with 72 sophomores from linear algebra course. The participants divided in two groups. One group received formal linear algebra course whereas the other group had an linear algebra course including some geometric representations of algebraic concepts, such as; vector-space. At the end of the four weeks, the students were given the same test. The results showed that the students in treatment group performed better than the students in control group. The researcher mentioned that his claim about giving algebraic concepts in physical embodiments can increase algebra achievement was verified.

There exists a variety of instructional approaches to algebra. Pirie and Martin (1997)'s case study involved Pirie-Kieren model for successful growth of algebraic understanding as a teaching method. They took one low ability eighth grade classroom and observed the period of teaching linear equations content in this classroom. The teachers' way of teaching linear equations was to use real-life modeling and realistic metaphors to make meaningful algebraic concepts. For instance, she used fence or two pan balance for equality, and bank balances, rates of interest, or change in temperature for solving equations. He believed that equation is a dynamic thing and it should be perceived as a whole, rather than the combination of letters and signs. At the beginning of the class they introduced real-life situations and students tried to model it mathematically, and then they were presented activity sheets and worksheets to work on in small groups. It was reported that all his pupils built a sound concept image for the equations and solution of equations. According to the teacher, the reason of this was, he did not reduce algebra to meaningless symbol manipulation and rote memorization of facts and procedures.

Stacey and MacGregor (2000) claimed that students have difficulties in solving algebra word problems due to many reasons. They tried to investigate students' strategies in solving algebra word problems. First the researchers developed four test items about algebra word problems, and after administering the items in 12 secondary schools in Australia, they examined written solutions of 900 students, and they also conducted interviews with 30 students. It was reported that students used variety of strategies to solve algebraic word problems. Some of the students used no algebraic method to solve the problems. However, some of them preferred algebraic methods even it was the most time consuming method. Besides the researchers pointed out that, the students perceived an equation as a formula for finding an answer, an operation for reaching the result, and a description of necessary relationships. It was recommended that to make students appreciate the power of algebra as a problem solving tool, the teachers should present algebra problems in different contexts rather than routine ones.

There is a growing body of literature devoted to the new instructional approaches in algebra. One of them was Hofmann and Hunter (2003)'s research study about applying a new curriculum in beginning algebra course. Inspite of the traditional algebra curriculum which focuses on teaching procedural skills for solving problems, they implemented a curriculum in which just-in-time approach was placed on. Problems from daily life were chosen, real-life applications in algebra were designed, and also calculators for solving problems were recommended. As well as classroom activities they designed a new textbook which had three parts, class work, group work, homework and also a software align with the textbook for their new algebra approach. This curriculum was implemented in one of the eight grade classrooms for thirthy-four hours. When the performance of the students in algebra and attitudes towards mathematics were compared with the previous years, it was remarked that using new algebra curriculum had a significant effect on students' achievement and attitude. The researchers claimed that using real-life applications in algebra motivated students and they found algebra more relevant and meaningful. Besides, it was reported that students would no longer ask the common 'Why do I need this?' question.

Like Hofmann and Hunter (2003)' research study, Hollar and Norwood (1999) conducted a research about comparing a graphing-approach curriculum and conventional curriculum in algebra. The purpose of the researchers was to find out how the curriculum on graphing helped reification of the function concept. The sample consisted of 90 college students, 46 in the treatment class and 44 in the control class. In treatment class, TI-82s were used to enable students to use multirepresentational approach including graphical approach in algebra. The activities included exploration and discovery examples to make students to notice patterns and investigate discoveries in algebra. Students were required to use different forms of representations and to move flexibly among representations in activities with the help of graphing calculators. At the end of the treatment two groups received a function test and a mathematics attitude scale. The researchers reported a significant difference between the treatment and the control group in function test; however, no significant difference was detected between groups in terms of their attitudes towards mathematics.

Thompson and Senk (2001) investigated the effectiveness of advanced algebra-based curriculum and instruction on students' algebra achievement. The participants of this research study were 150 tenth, eleventh, and twelveth graders from four high schools in experimental groups, and 156 students from same grades and schools for comparison classes. The new algebra curriculum and instruction which emphasized skills, properties, uses, and representations in algebra were implemented in experimental classes for one year. "Skills" represent the symbolic part of second-year algebra classes, "properties" represent the reasoning expectations in algebra, "uses" deal with applications of algebra in real world situations, and "representations" includes pictures, graphs, and objects connecting to the symbolic part of algebra. For the instructional part, lecturing was not the primary method in algebra classes; class activities, small group discussions, writing in mathematics, graphing calculators, and project-based teaching were also used by the teachers. To assess the effectiveness of the curriculum and instruction, eight instruments were administered to the participants. Three of them were achievement tests, and the others were survey asking students' and teachers' opinion about this kind of curriculum.

According to Thompson and Senk (2001), it can be said that students in experimental classes did significantly better on all instruments when compared with the control classes. Especially the items regarding reasoning in algebra and using different types of representation in algebra, experimental group students performed higher than the others, and also on conservative test items involving traditional algebra items the experimental group students performed well. The researchers concluded that implementing reform-based algebra curriculum increased algorithmic and procedural skills valued by parents, teachers, and administrators as well. It was highly recommended to conduct longitudinal studies with new curriculum materials and instructional methods since the growth in reasoning abilities can be more clearly determined in studies that take long time.

3.2 Research Studies Focusing on Multiple Representations

Using multiple representations in mathematics and science has been examined in various research studies. Generally, all of the studies suggested that introducing mathematical and scientific concepts by using multiple representations makes students better comprehend the relationships between the concepts. Some of them are summarized here.

Project MRC which was a 3-year project funded by the National Science Foundation was one of the comprehensive projects about multiple representations. Sherin (2000) indicated the aim of this project, in very broad terms, was to investigate students' "Meta-Representational Competence (MRC)" which means students' knowledge about their external representations. MRC focuses on four essential competences, the ability to invent novel representations, the ability to critique existing representations, knowledge of the functions that representations could perform, and knowledge that facilitates the rapid learning of new representation. The specific goals of this project were to identify student's MRC, investigate the relationship between MRC and conceptual development in science and mathematics, and explore the instructional implications of MRC. The data of this project came from the students at the middle and high school level. As a data collection method, laboratory interviews if any experiment is given, clinical interviews, observations, written exams related to the tasks were given to the students.

As a result of this project, it was argued that by the help of Project MaRC, it is now possible to design a new mathematics and science curriculum which can prepare students for the real life better since introducing multiple representations to the students would enable them model real life situations mathematically. They further claimed that meta-representational skills are crucial targets for instruction of standard representations and concepts. If students were engaged in activities involving meta representational skills, they posses significant knowledge about the related concepts.

diSessa and Sherin (2000) carried out a research study as a part of Project MaRC. In this study, it was aimed to examine, in very broad terms, what students know about representations and what is possible for them to learn. They named the ability having full capacity about constructing, using, critiquing, modifying, and designing external representations as “Meta-Representational Competence (MRC)”.

The subjects were eighth, ninth, tenth, and eleventh graders. Those students were involved in classroom activities in which scenarios about using multiple representations were included. For instance, they were asked to explain a specified pattern of motion using several external representations. The different kinds of scenarios were applied several times until the students individually designed representations, and then they were taught to discuss and compare the works with the whole class. Interviews with students and classroom observations were used as a data collection method.

According to diSessa and Sherin (2000), the most important result of this research was that students have a “deep, rich, and generative understanding of representation”. They believed that engaging students in MRC activities, the students were better promoted to be succesful in science and mathematics. Furthermore, the students mentioned that understanding some mathematics concepts is easier when multiple representations of those concepts were presented to them. From the classroom observations, the authors concluded that students have a better chance to discuss their ideas and be creative in using and designing external representations when they were involved in multiple representations-based instruction in mathematics. After this research, diSessa and Sherin suggested that further studies investigating the children’s cultural experience and interaction with external representations should be done.

Thomas, Mulligan, and Goldin (2002) investigated children’s internal representations from their drawings and explanations of the numbers from 1 to 100. Their aim was to infer children’s internal representations from their external representations. The sample consisted of 172 children from the grades K to 6. They interviewed with all children to describe their understandings of numeration, and there were 89 different tasks in five categories: counting, grouping, place value, structure of numeration, and visualization. Among all tasks, the visual task was posed first, so that it was considered not to let children to be influenced the external representations before they create their internal representations. 264 interview scripts, together with the external pictorial and notational representations were collected from this sample.

The researchers concluded that cognitive representations of numeration can be developed over time. From the students' external representations, it can be said that students used a variety of representations of the numerals 1–100 than the researchers expected. There was an evidence that their representations were highly unconventional. The students coded formal number representations in a shape of spiral, or flashing cards. The researchers concluded that if children have more developed internal representations, their external representations would be more coherent and well-organised, and so as to their numerical understandings. It was suggested to conduct longitudinal studies about children's internal systems of representation by the help of task-based interviews.

Some calculus concepts were also investigated within multiple representation contexts. Girard's (2002) study was one of them. She investigated students' understanding of limit and derivative concepts in terms of multiple representational views. During the instruction, the use of graphing calculator was emphasized as well as the multiple representations of the concepts of limit and derivative. A multiple representation approach was used to develop the limit and derivative concepts from numerical, graphical, and algebraic views as an instruction. To illustrate, the limit concept was firstly introduced to the students from a numerical approach in which tabular mode of representation was used to calculate limits, and then a graphical mode of representation which was a visually complement of tables was introduced to the students, lastly a translation from these two modes of representations to algebraic mode of representation was made. Data was collected by an instrument in which participants were given tasks including items about conceptual understanding and representational methods of solution of limit and derivative.

Girard (2002) concluded that after having one semester multiple representation-based instructions, the calculus students in her sample demonstrated a multiple representational knowledge when solving limit and derivative problems. She mentioned that algebraic mode of representation was the most common representation mode among the students, however tabular and graphical modes of representations were also used to solve limit and derivative problems. It was suggested that designing multiple representation-based instruction might be beneficial to avoid excessive and inappropriate usage of algebraic mode of representation.

Noble, Nemirovsky, Wright, and Tierney (2001) conducted a research study which was a part of SimCalc and Mathematics of Change Projects. Their purpose was to examine students learning in multiple mathematical environments which included manipulative materials like number tables, fraction bars, or computer software to think mathematically. They preferred to conduct this study in one fifth grade classroom in Boston. The classroom activities were video taped, especially one focus group including two girls and two boys were examined. Moreover the students in this focus group were interviewed after the 4-week unit. In classroom sessions, the researchers particularly focused on how students work with number tables, physical materials, and computer simulations contributed to their understanding of the mathematics of change.

The classroom observations and individual interview results were presented with respect to the three mathematical environments; Cuisenaire rods-and-meterstick, table, and the trips software. The participants tried to conceptualize mathematical change in cuisenaire rods-and-meterstick by using the meters as manipulatives, one student took a step of 2 cm and the other one took a step of 4 cm. In table environment, the participants required to present the previous activity with meterstick in tabular form. They showed their steps in two column table. As a last mathematical environment, specific software was used. Two iconic boy and girl figures were generated to take the steps in meterstick. It was reported that in the first environment, all of the participants did not conceptualize the aim of the activity and the mathematics behind it, however; when they met with tabular mathematical environment they dealt with numbers in tables well. Afterwards, they used the software and they said that by using this software they established the connection among these three environments, and the concept made sense to them. The researchers concluded that the three mathematical environments were prepared in resemblance to make a connection between different representations regarding one mathematical concept, and by this way students reached the essential knowledge. Besides they mentioned that not only the multiple representations, but also the connections and subordinations among them are also important for students' learning in mathematics.

3.3 Research Studies Focusing on Multiple Representations in Algebra Context

In the related literature, one can encounter with several studies about using multiple representations in algebra. Here, some of them were reviewed and discussed.

Silva, Moses, Rivers, and Johnson (1990) conducted a research study which was a part of "The Algebra Project". This project lasted seven years about integrating algebra and multiple representations. The assumption of the researchers was that a student could gradually arrive the symbolic representation of events by passing through 5 steps which are; "physical events, picture or model of this event, intuitive language description of this event, a description of this event in English, and a symbolic representation of the event". At the beginning, only sixth graders were included in the study, but after a while seventh and eight grade students were also involved. It was planned to teach algebra in a kind of instruction called "Open Program". In this program students were taken to some trips in which they could explore algebraic concepts in a view of multiple representations. For example, one trip was to Boston's subway system (it is called as a physical event). This trip provided a context in which algebra concepts can be investigated. Students were asked some questions, like "Where are we going?", "How many stops will it take to get there?" After the trip, students were asked to write about the trip, sketch graphs for the trips they made, and collect data about them. As the students engaged in the trip activity, they had a chance to deal with external representations, like table and graph. The researchers implied that, all students had an efficacy to do algebra by the help of this project. This was the most beneficial gain of using multiple representations in algebra. Furthermore, the participants learned to represent algebraic and arithmetic concepts in different kinds of representations.

Brenner, Brar, Duran, Mayer, Moseley, Smith and Webb (1995) investigated the role of multiple representations in learning algebra, particularly in the concept of functions. Their aim was to propose an instructional unit regarding with the new reforms in mathematics education and opposing to the conventional mathematics instruction. There were four principles which were taken into consideration before preparing lesson plans; (1) instead of using only symbol manipulation of middle school pre-algebra concepts, using multiple representations (words, tables, graphs, and symbols) of those concepts were emphasized, (2) instead of presenting an isolated problems to the students, they used thematic situations for the problems, (3) they emphasized not only the product of problem solving, but also the process of it, (4) they preferred to teach by using guided-discovery approach in which students are allowed to use different representations and construct their own representations. A 20-day unit about functional relation between two variables was prepared by the researchers and a team of mathematics teachers. This unit involves cooperative group work activities in which students are encouraged to use representations, to make translations among representations, and to create their own representations in meaningful mathematical contexts. The sample consisted of three experimental and one control classes from seventh and eighth graders.

The results of this research study revealed that except for one test, the students in experimental groups performed better than the control group students on the instruments. The researchers also stated that the experimental groups' students were engaged in using many representational modes, and they made reasonable preferences among the given representational modes for solving problems. The researchers reported those higher gains in students' test results can be counted as an evidence for claiming that multiple representation based instruction did support students' conceptual understanding in algebra units. They argued that the textbooks and the context of conventional instruction involve also certain kinds of multiple representations like graphical representations of functions, but these representational modes are given isolated, the connections among them were not valued.

One of the research studies implementing a multiple representations-based curriculum was carried out by Yerushalmy (1997). The aim of this study was to make students "representation designers"; that is, students would participate in creating and using certain representation types. In this study, to introduce the students with the concept of multiple representations, an algebra curriculum of seventh graders was modified regarding to the three principles; "(1) describing phase: building the concepts of function, (2) solving phase: comparing functions, and (3) generalizing various aspects of functions." (p. 433). In every experiment, special software was used for making 38 seventh graders to conceptualize the concept of function. The translations from numerical mode of representation to the verbal mode of representation were used to reach the symbols and graphs of functions. The entire instruction which was held in small group work and in thematic mathematical problems had four levels. First, students designed a representation of dependency in two variables. In a second level, they explained what they did; thirdly, they solved the given problems using various strategies, and in a final step, they made discussions about the contributions of different strategies to the solutions. It was also pointed out that the whole experiment was about modeling a situation mathematically by using functions.

Yerushalmy (1997) reported that, although the students were familiar with the conventional representations of a function like f( ), they preferred to use their own representation when they were asked to write a representation for function, and therefore, their usage of representations affected their generalizations also. It was also noted that students in that study used the symbolic representations in modeling accurately and appropriately. Tabular representations including two-column, three-column, and multi-column tables were used as an "organizational framework" for the students. They began with tables to solve the problems, they generalized the information situated in that table; and finally, they represented in an equation of graphical modes. Moreover, the sample was in favor of using graphs. As they implied graphical representations are visual by which one can think and discuss, and they are convenient for reading results without computations. And they viewed tables as only bridge representations to build graphs.

He made his conclusions by indicating that multiple representations of functions would not be just tools for understanding mathematics for students; they are ends also, since they might be used for creating new mathematics. Furthermore, this kind of curriculum in which structured activities about modeling situations mathematically took place supports the sense making of algebra.

Pitts (2003) examined pre-service mathematics teachers' knowledge about function by focusing on the representational translation ability between algebraic and graphical representational modes in her dissertation. The specific aims of her study were determining the three sorts of "mental models" of pre-service mathematics teachers; that are: mental models that are used when they tried to solve the algebraic problems including the translations between algebraic and graphical representations, mental models that they had when they were examining students' answers to algebraic problems, conceptual models that were used when they were dealing with students' misconceptions. The sample of this study constituted of 59 seniors who were studying at the department of mathematics education in four different universities in USA. The researcher first administered a two part open-ended survey to the pre-service mathematics teachers; then 10 of them were interviewed. After the data collection process, data was analyzed qualitatively. Of the various effective results of this research, the results especially related to the aim of this literature review were presented. It was found that many teachers have the ability to make flexible translations between graphical and algebraic modes of representations for solving the problems. In addition to this, majority of the pre-service mathematics teachers failed to notice students' failures in answering problems. Those pre-service mathematics teachers rarely determined the misconceptions of the students about the process and object views of the functions either.

The results were presented as evidence by Pitts (2003) in order to claim that the pre-service teachers have dynamic understanding in problems requiring representational translations by means of the mathematics courses they had, but they had difficulties in analyzing the students' responses to the problems and determining the students' misconceptions. Therefore, she denoted that there is a lack in teacher education program in terms of "knowledge for teaching."

Swafford and Langrall (2000) studied on representations of students for describing algebraic problem solving situations. The specific goals of this study were to investigate whether students would give correct answers when the specific values of the variable were given in an algebraic problem, whether students would generalize the relationship that they found from the problem, whether they would be able to describe the problem situation verbally and symbolically, and whether they would be able to use the symbolization to solve the problems. To achieve these goals, the researchers interviewed 10 sixth graders who had not taught formal algebra before. In interviews, six verbal algebraic problems were presented to the students. The interviewees were required to solve the problems; afterwards, they were asked to describe the situation verbally and symbolically.

Swafford and Langrall (2000) highlighted that the participants performed well on the interview tasks generally. They solved the questions correctly, described the algebraic situations verbally and symbolically, and made accurate generalizations about functional relationships. Out of interview tasks 98% was solved correctly by the students. Besides, in 88% of all interview tasks the relationships were described verbally, and only in 50% of all interview tasks the relationships were described symbolically. In 20% of all interview tasks students used symbolization to solve algebra problems. It was reported that the most challenging procedure for students was to represent a situation symbolically and to write an equation. The students who wrote equations for problems, solved them also. The other problem was encountered in making tables. The students mentioned that they perceived tables as a sense maker of the problem so they wanted to use them mostly; however, they found the tables also confusing because they focused on the connections between consecutive values in the columns and ignored the relation between the dependent and independent variables. Therefore, they could not notice the algebraic relation in the whole table, they only remarked the patterns in numbers.

Remarkable suggestions related to the representational context of algebra were put by Swafford and Langrall (2000). The researchers paid attention to the considerable benefit in investigating the same problem from multiple representations' view since every representational mode has its own advantage. It was also suggested that the students should be forced to establish the link between the different kinds of representations in a problem context. For further studies, the researchers specified a need to examine best instructional method using representational modes in supporting meaningful learning.

In Cai (2004)'s paper two studies investigating students' early algebra learning and teachers' beliefs on U.S. and Chinese students' algebraic thinking were reported. The purpose of the first study was to examine the impact of teaching algebraic concepts on U.S. and Chinese students' preference of solution strategies and representations, whereas in the second study, U.S. and Chinese teachers' scoring of student responses was examined. The U.S. sample of the study involved 115 6th, 109 7th, and 110 8th grade students, and the Chinese sample consisted of 196 4th, 213 5th, and 200 6th grade students.

For this research study, as a teaching and learning material "national unified textbooks" was used in Chinese sample. This curriculum included the unit of equation solving where the algebraic concepts were taught by using letters to express formulas. In

U.S. sample, the algebra tiles was used to model additions of polynomials and equation solving. in order to assess students, the researcher administreed a test including four algebraic task relevant to algebraic word problems. The results of the study suggested that U.S. and Chinese students performed differently on the tasks and used different strategies and representations to solve algebra problems. Across the four tasks, Chinese students at each grade level rarely utilized visual representational modes, however; U.S. students used them much more frequently. Besides Chinese students had tendency to use symbolic representations for solving algebra word problems, whereas U.S. students used arithmetic approaches for algebraic situations. In general, Chinese students preferred to utilize symbolic representations and U.S. students preferred to use visual representations (algebra tiles, drawing, etc.). From the teachers' point of view it was observed that Chinese teachers paid attention to symbolic representations than visual ones in teaching algebra, therefore their students preferred to use symbolic representations. On the other hand, U.S. teachers more focused on the representing problems visually. This had an effect on students' representation preference. Hence it can be said that teachers' way of instruction directly influenced students' preferences in solving algebra problems. The researcher recommended that other factors which could have impact on students' representation preference and algebra learning should be investigated.

In Cifarelli's (1998) research, the constructions of mental representations in problem solving situations were examined. The sample was composed of 14 freshmen calculus course students chosen from one of the universities in USA. Interviews were carried out for realizing the purpose of the study. In interviews, there were nine algebra word problems in which the participants were introduced a problem situation. Data were gathered from the videotapes and written protocols of interview. Written summaries which brought light into their understanding of problems of participants were prepared and also macroscopic summaries of the student's performance were prepared. By preparing them it was aimed to get deeper information about how the participants constructed the required representations and at which level they possessed the abstraction of solution methods.

From the results of this study, it was inferred that four of the participants reflected their solution methods at a more abstract level, they barely used other kinds of representations except for symbols. Two of them recognized the necessary diagrammatic representations for problems, but they could not pass to the abstraction level for solving problems. And the other two participants reached the correct answer without carrying out the necessary solution activity. These two participants were accepted as they were at the level of "structural abstraction". Furthermore, he claimed that the representation types and the way of using them have an influence on the participants' solution activity. For instance, he said that one of the participants did not reach the correct solution only because she insisted on using the diagrammatic representation. This student was assigned to the "recognition level" since she arranged her cognitive actions in a way that enabled her to recognize the superiority of one type of representation. If she would have tried another mode of representation, she could have found the required answer. One point that deserves to be mentioned is, the participants had a tendency to use the same representation type for all problems. Generally, they started to solve the given task by applying the same solution methods and representations that were used in the previous problem. This was called the level of "re-presentation" that means after getting a correct response by using one kind of representation, fixing this representation type for every problem.

Cifarelli (1998) concluded that mathematical knowledge in problem solving situations can be inferred by examining the ways of constructing representations of problem solvers. According to him, there are bodies of research investigating the student's representations in problem solving activity; however, the literature lacks research studies for examining the problem solving situations in which the representations that were chosen by the students do not work well and need to be altered.

Baker, Corbett, and Koedinger (2001) presented a research particularly on graphical representations of data in pre-algebra context. They focused on examining the students' performance on "interpreting, generating, and selecting" external representations for data analysis. For this purpose, students were inquired to make interpretations on graphical representations, choose the suitable type of representation, and form different representations. Moreover, the researchers asked two teachers in order to make them to predict their students' performance on representations. The sample of this study involved 52 seventh and eighth graders from three pre-algebra courses in two schools in USA.

The participants were given 3-4 exercises in which they were asked to draw a histogram, scatterplot, or stem-and-leaf-plots accordingly the given data set in a table, to respond to the questions about those graphical representations or to select the most suitable type of graphical representations for a given exercise. According to the results of the study, the overall performance of students on interpretation of graphs was moderate. Significant differences were found in their performance in terms of the type of the graphs. 15 students preferring histograms showed remarkable performance than 12 student interpreting scatterplots and 13 students interpreting stem-and-leaf plots. However, the participants poorly performed on the selection and generation of graphs. Particularly histograms and scatterplots were generated by none of the students.

Baker, Corbett, and Koedinger (2001) noticed one salient finding that the students performed poorly on interpretation of stem-and-leaf plots, but this graphical representation was the only one which was generated by all of the students. This finding was interpreted as students have a tendency to generate representations just by considering the "surface features" of them, like they want to draw them or they like the way of appearing stem-and-leaf plots. When it was asked to interpret this graph, the students need to know the "deeper features" of this graph, so some of them failed.

Moseley and Brenner (1997) called for multiple representations-based curriculum that foster problem solving in middle school algebra. They suggested that the algebraic objects and processes should be given with different representational modes including words, tables and graphs. The crucial point in this curriculum was focusing on using variety of representations in small group works. The experimental group in which multiple representation-based curriculums took place had 15 students whereas the control group in which stressing on symbol manipulation of skills was involved had 12 students. Data was gathered for this study by pre and post clinical interviews. The findings of this study pointed out that the experimental group students were more capable of working with variables, expressions, and equations than the control group students. They took this finding as an evidence for saying that multiple representations-based curriculum would not only make students to acquire various representational modes and use them fluently but also it would promote a better conceptualization in algebraic learning. The researchers recommended that the persistence of algebraic knowledge gathered from multiple representations-based curriculum should be documented in further studies.

Herman (2002) presented a relationship between visual preference for using multiple representations and algebra in her dissertation. In general, the aim of her study was to investigate students' usage of multiple representations to solve algebra problems by the help of graphical calculators. The topic of polynomial, exponential and logarithmic functions were chosen in order to examine students' usage of algebraic, graphical and tabular modes of representations and the interactions among them. It was also sought to find a connection between students' visual preferences and the strategies they preferred to solve the algebra problems with graphing calculators.

In order to obtain data from 38 freshmen college students, pretest and posttest including algebra problems, mathematics processing instrument, and questionnaires were used as instruments. In addition to them, interviews were conducted with the participants to get information about their preferences for representational modes. The advance algebra course that lasted eight weeks was modified for the purpose of the study. Multiple ways of approaching the algebra problems by technology were emphasized. Every student was required to have a graphic calculator which enables the students use multiple representations in algebra.

The results of the study denoted that the most preferable representation modes were algebraic and graphical. It was reported that for solving problems in pre and posttests the students had choices to select one or more representational modes. Numerous students generally chose using only algebra, only a graph, or algebra and a graph. The tabular representation type was hardly used by few of the students. When the students were asked to explain why they had certain kind of preferences for using representations on questionnaires and interviews, they argued that the algebraic mode of representation was seemed to be more mathematical, more familiar than others, and more emphasized in all mathematics courses. They also mentioned that if they would not have a chance to use graphic calculators, they could not use graphs for solving problems at all because they believed that using graphs is only appropriate if one has a graphic calculator. Furthermore, the tabular mode of representation was rejected due to the fact that it is barely used among the algebra course instructors that the participants have had so far. From those comments of the participants, Herman (2002) purported that there are many factors affecting the use of representational modes. Some of them are beliefs of students and their instructors of “what is mathematically proper”, accuracy of the representations, and the nature of problem. Another finding of this study was, no significant relationship between students’ visual learning preferences and their preferences for representation types was found. It was also revealed that the likelihood of having correct answers to the problems inclined with respect to the number of representations participants used. They mentioned the same situation in the interviews by saying that the conceptual algebra understanding was deepened by multiple representations since they comprehended the logic behind the structure of algebra by manipulating its objects in various forms.

Numerous recommendations were documented to bring light into further studies. Two of them were according to the researcher, investigating the persistence of the idea that algebraic representation mode is more mathematical when compared with others would be fascinating, and it would seem to be necessary to investigate whether students’ use of one type of representational mode results in more accurate answers to problems than use of more than one representational mode.

Another dissertation investigating the role of multiple representations in students’ learning was completed by Özgün-Koca (2001). The aims of this dissertation were finding out the effects of computer software having multiple representational context on student learning and determining the students’ attitudes towards and preferences for multiple representations (tables, graphs, and equations) in algebra. There were two experimental and one control group having sample size of 10, 10, and 5 accordingly. VideoPoint linked multiple representational software packages which were about gathering distance and time data from QuickTime movies was introduced to the students in one experimental group. For the second experimental group, the semi linked version of this software was designed. In linked VideoPoint, the relations between the modes of representations were established in two ways; on the contrary, in semi-linked version of the software only the linkage between graphical and algebraic modes of representations was provided to the user. Data was collected from 25 freshmen Algebra I students from a public school in 10-week experiment. The sources of data were pre and post mathematics tests, students' works from computer lab sessions; computer based and the follow up interviews with students and teacher, survey, and observations.

The results concerning the theme of my dissertation were presented here. The students from two experimental groups did significantly better than the control group students on posttest in general, and they were also capable of giving more productive explanations for answering to the questions than control group students who clarified their answers by stressing on procedural steps. The researcher also found that there is a specific reason for students to make a choice in favor of one representation. These could be finding an exact answer with a certain type of representation or the visual advantage of one kind of representation. Especially, the algebraic mode of representation was popular among the participants. The reasons for preferring it, was explained like one can easily get the accurate answer with equations and it has "ready-made steps" to work on. Moreover, all of the students' attitudes towards using multiple representations in mathematics are similar. Although they mentioned that focusing on one mode of representation is easier for them, they appreciated all modes of representations. Another point is, students saw the relationships between different modes of representations, and superiorities of one representation to another by the help of VideoPoint software. The growing need in technology-oriented multiple representations environment and their usage in today's classrooms was pointed for future studies.

One dissertation was also devoted to using multiple representations in algebra by Selzer (2000). The aim of this dissertation was to apply Bruner's representation model to algebra teaching. He believed that using the sequence of enactive, iconic, and symbolic representations for learning algebra concepts was the most efficient way for students, especially lower ability students. 38 male seventh graders were the participants of this quasi-experimental research study. This experiment took only three days. First day, students were given treatment following the sequence of enactive, iconic, and symbolic, on the second day the sequence of iconic, symbolic, and enactive was followed; and finally, on the last day symbolic, enactive, and iconic sequence was followed. The activities were about polynamials operations, and performing the operations by numbers and variables (symbolic), by rods and blocks (enactive), and by diagrams of rods and blocks (iconic). At the end of the treatment an algebra achievement test covering polynomial operations symbolically was administered to the participants.

The results showed that none of the effect sizes between the instructions yielded significant results. However, the students taught by the enactive, iconic, and symbolic sequence got higher scores than the students taught by the other sequences. Besides, for the lower ability students this sequence was mentioned to be the best sequence. The researcher concluded that among all type of multiple representations, the enactive type was the most suitable for introducing algebraic concepts. Moreover, it was highligthed that there is a need in algebra classrooms for multiple representations-based instructions, particularly for hands-on activities. In addition to the instructional need, for the researcher the mathematics textbooks should also be re-designed in order to cover multiple representations for algebra.

According to Huntley, Rasmussen, Villarubi, and Fey (2000), an efficient algebra curriculum should be based on realistic context. They investigated the effectiveness of such curriculum on students' algebra achievement and algebraic thinking when compared with conventional algebra curriculum. They conducted a research study to examine the effects of NCTM Standards-based curriculum on algebra as a part of a research project called Core-Plus Mathematics Project (CPMP). They particularly focused on the quantitative problems in algebra. Their purpose was to examine the students' abilities in formulating those problems, doing algebraic calculations, and reasoning algebraically in authentic problem contexts. The subjects of this study were ninth graders from 36 high schools. Students were randomly assigned to the experimental groups. Those subjects were provided variety of instruments to assess the effectiveness of standards-based curriculum in algebra. Three quantitative instruments and one interview protocol for both teachers and students were prepared. Of quantitative ones, two of them were multiple-choice tests assessing students' algebraic thinking, and other one included open-ended items related to algebraic word problems. For data analysis, they used rubrics indicating minor and major errors as well.

The results showed that for the questions requiring specific algebraic skills and problem solving strategies, experimental group students outperformed than conventional class students. However, students in control classes performed better than the students in experimental classes on the items requiring pure algebraic symbol manipulation. Furthermore, experimental group students were better in items involving translational skills among representations than control group students. The most difficult task for control group students was the translation from tabular to symbolic mode. The authors claimed that an algebra curriculum that emphasizes multiple representations of algebraic ideas made students deal with mathematics problems involving representational fluency. Moreover the researchers suggested that traditional curriculums should take the advantage of the representational features of graphing calculators in algebra since it makes easier to use graphical mode of representation. One suggestion was made for future studies; the researchers paid attention the need for using qualitative data collection methods in determining students' reasoning abilities in algebra, and also they recommended to analyze students' work when they were dealing with different kinds of representations in algebra.

A great deal of literature has looked at how translations among representational model occur and what are the beneficial points of being able to make such translations in mathematics learning. Many researchers have emphasized that making translations among representational modes is a gatekeeper of competent mathematical thinking (Brenner, Herman, Ho, & Zimmer, 1999, Janvier, 1987b; Lesh, 1979). For instance, Brenner, et al. (1999) conducted a cross-national comparison about representational competence including 895 sixth graders from China (223), Taiwan (224), Japan (177), and United States (271). The aim of this study was to examine the performance of students, in judging the appropriateness of representational modes for variety of mathematics items related to fractions which were presented them beforehand. There were transformations between within the symbolic form of representations, and translations between visual and symbolic representations. The students were expected to judge whether the given solutions to the items included correct or incorrect representations.

According to the results, the Chinese students ranked first on all the items, the Taiwanese students ranked second, the Japanese students ranked third, and the U.S. sample ranked last. Overall, all the students had difficulty in choosing the correct representations for the items. They generally solved the problems; however they were not capable of representing the solutions in a correct way. There were two items requiring the transformation within symbolic mode of representation. Chinese students got significantly higher scores on these items whereas U. S. students got the lowest scores. For the items including the translations of fraction to decimal, fraction to proportion, simple number sentences to algebraic number sentences the Chinese and Taiwanese students showed the highest scores. Besides the only item type that American students showed the highest scores was the item including visual representation for solution.

The results of this study were discussed in detail, like the Asian students outscored the American students on almost all the test items. The Asian sample was more successful than American students for the items that required understanding of formal symbolism. Each sample had some difficulty on the items particularly about flexibility across different modes of representations. In the light of the results of this study, the researchers suggested that mathematics classroom activities should comprise of different representational modes since flexible integration of different mathematical representations is one of the ways for advanced mathematical thinking.

Shama and Layman (1997) did a research in a part of a project called “Maryland Collaborative for Teachers’ Preparation” (MCTP). The aim was to investigate the application of representations in introductory physics class of pre-service teachers. Students are encouraged to study with science situations, mathematical representations, and be able to make transformations among mathematical representational modes. 29 students were involved in this study. They were observed on the task and their conversations were recorded. During the data analysis, it was remarked that there are two kinds of mathematical representations that students used, graphical and algebraic and two types of scientific situations; a real event and a story. The responses of students were classified on the basis of which representation type they choose for which scientific situation. The researchers explained the real event and story situations in science and highlighted particular distinctions between those two. The students prefer graphical type of representation to a graph indicating a concrete view, and also they refer algebraic representation to a formula and expression.

The results of scripts showed that at first the three students preferred to make a graph after the experiment and then translate this graph to a referring algebraic form.

After they have dealt with many physics experiments, they had a tendency to write the algebraic expression, and then to sketch the graph. The students indicated that when an experiment is given to them they found easy to make a translation from a graph to an algebraic form, but when they meet with a science story they prefer to write an algebraic statement of it and then to translate it to the graph. Furthermore, all the students said that it is easier to make a translation from a graph to an algebraic sentence than from an algebraic sentence to the graph. In the former the scientific situation was first simplified to a graph and then was elaborated by the help of graph.

The factors that have effects on the translations among representations of learners were explored by Ainsworth and van Labeke (2002). In order to achieve their goal, they conducted three experiments including four groups of children who were five years old. One group received pictorial representations, the second group received only mathematical representations, the third group received mixed representations including pictorial and mathematical representations, and the fourth group had no intervention as a control group. The experiments were designed on a computer-based environment; the specific software called CENTS was used here. In the context of CENTS, students are dealing with external mathematical representations in order to improve their computational skills with numbers. An instrument called "Think and Solve Mental Math" was administered to the children as a pre and post test.

According to the results of this study, children did translations from pictorial representation to the mathematical representation and they failed to make translations in reverse. Furthermore, even children who did well on translations between pictorial and mathematical did not get the highest score on post test. However, the children who preferred mostly the mathematical representations got significantly higher scores on post test administration. This finding oriented the researchers to the idea that dealing with different representations and being able to make translations among them might not be a guarantee a good performance on mathematics tests.

At last, the researchers hypothesized that if two external representations support the same concept, the translations between those external representations are easy and beneficial for mathematics learning, whereas if the external representations are dissimilar, the students have difficulty in making translation between them. Moreover, they concluded that it is better to focus on one single suitable external representation to result in successful learning if the external representation preferred, includes all the necessary information.

Another study about representational translations was belonged to Hitt (1998). He conducted a research about translations among representational modes adopting the representation theory of Kaput. His sample was constituted of 30 secondary level mathematics teachers. He aimed to look at the errors in the concept of function of those mathematics teachers. He designed 14 questionnaires about the content, task, and different representations of functions and every week, the teachers took two questionnaires. In the questionnaires, the teachers were asked to make a translation between graphical type of representation and real-life context. They were asked to draw a container that fits the given graph that had height as an independent variable and volume as a dependent variable. Among those teachers, 24 of them got the correct answer for the first question, and only 13 of them had the correct translation for the second one. And they were also asked to draw a graph for given container type in two questions. For the first question out of 30 teachers, only two of them drew the correct graph and for the other question, the number of correct answers was increased to five. As a result, most of the teachers failed to make translations among graphical mode of representation and real life representation. He added that the main reason behind those failures was, the teachers did not consider the independent variable which was height of the container. Therefore, he concluded that teachers could not be able to make translations between systems of representations involving function concept coherently.

Even (1998) introduced his study with the sentence below. “The ability to identify and represent the same thing in different representations, and flexibility in moving from one representation to another, allow one to see rich relationships, develop a better conceptual understanding, broaden and deepen one’s understanding, and strengthen one’s ability to solve problems (Even, 1998, p. 105).” And he explained his aim as there are three different ways that affect linking representations of functions; “different ways of approaching functions, context of the representations, and underlying notations”. In this research study he explored these three ways deeply. The sample of this study included 162 pre-service secondary mathematics education seniors from eight universities in USA.

For data collection an open-ended questionnaire was used. There are non-standard mathematical problems including the conceptions of functions.

When the results of this study were examined, it can be said that students preferred different ways to approach functions. These ways can be classified as global approach and point-wise approach, but the discussion about these approaches is beyond the scope of this literature review. He also found that the context of the given function had an influence on student's representational modes. For instance, if they were given with a trigonometric function, they attempted to draw a graph to solve the questions about it. Furthermore, he stated that underlying notation is also remarkable for the seniors since they had some difficulties in translating representations of functions regarding the notations. When they were given with a partial function none of the students accomplished to translate it in a graphical type of representation. He completed his study by arguing that there is a very challenging process behind the translations between representations and this process should be investigated deeply.

Koedinger and Terao (2002) conducted a research study on translations among verbal, pictorial, and algebraic modes of representations. Their aim was to investigate the role of using pictorial representations in students' algebra learning. To accomplish this goal they used "Picture Algebra Strategy" which had two steps. At the first step, the students were asked to translate the given algebraic word problem (verbal mode of representation) to a picture mode of representation, and then the translation of this mode of representation to algebraic mode was required. At the second step they are asked to solve this equation. For gathering data, 35 sixth grade students' solutions to the three algebraic word problems were analyzed. The overall performance of sixth graders on three questions was outstanding. For each question the percentage of correct answers was approximately 68%. The authors divided diagrammatic representation strategies into five categories; size-preserving, incomplete, wrong, abstract, and no diagram. In size-preserving strategy, the students chose the box figure for representing the quantities with respect to the size of the quantity. Bigger sizes were represented with bigger boxes. If there were missing quantities or pictures, the authors named that strategy as incomplete. "wrong" pictures had boxes that were irrelevant with the sizes. "Abstract" pictures were named so since it is the reverse process, i.e representing the larger box with a small picture. According to the results of the study, although students performed well on the questions, the authors pointed out that “Picture Algebra” was not a cure for developing algebraic thinking in students since after they represented the algebraic context with diagrams; they still had obstacles to translate this representational mode to the algebraic one to solve the equation. The researchers also added that using pictures might facilitate students’ learning in algebra; however, it is not such kind of method reaching every student for supporting them in algebraic sense making.

Schwarz, Dreyfus, and Bruckheimer (1990) investigated problems and misconceptions about functions by using the triple representation model which integrated algebraic, graphical and tabular forms of representations. In this model, the researchers paid attention to the translations within and between the representational modes. This model included open-ended classroom activities. The participants learned by performing classroom activities in these three representations, and then they compared the three representational modes in different context. It was found that this model made student develop high thinking strategies in the concept of function.

Hines (2002) also examined students’ interpretation of linear functions in dynamic physical models which were mechanical tools for visualizing functions, and the way they used the tables, equations, and graphs to represent functions. This study was a case study with one eight-grade male student. He was a purposive sample because of his willingness to join this study. He was given the algebra, geometry and proportional reasoning tests of the Chelsea diagnostic tests as instruments. The researcher interviewed with the subject by using several tasks. The subject was required to explain the given problem in written words, to draw pictures, to create tables, equations, and graphs. The researcher gave the main focus on the subjects’ interpretations on functional relationship while he were experiencing them with the models, tables, equations, and graphs.

The researcher reported that the subject entered the interview procedure with little knowledge of functions. He could not interpret equations, and graphs as representations for functions. After his interaction with physical models and other representations of functions in the process of interview, he developed a generalized view of linear functions. The researcher mentioned that using different representations might help to build a deep knowledge of linear functions. This research showed that when students are given the opportunities to create and interpret representations, they can develop a better understanding for mathematics concepts by linking representations.

3.4 Research Studies Focusing on Student's Representation Preferences

When the relevant research literature is examined, it is noticed that the researchers emphasize not only using multiple representations in certain topics, but also students' representation preferences. For instance, Keller and Hirsch (1998) investigated whether or not students have representation preferences, how they are contextually related, and the extent to which representation preference was influenced by the occurrence of number of representation. The sample of this study comprised of 39 students from graphics calculator section and 40 from regular calculus section. After administrating the representation preference survey developed by the researchers to the both groups as a pre-test, 13 weeks of treatment was conducted. The post-test which had almost the same features with pre-test was administered followed by the treatment.

Their study indicated that students have preferences for various representations, and they can be affected by the contextualized and purely mathematical problems. Most students had an orientation towards the equational mode of representation in purely mathematical settings; in contrast, they preferred tabular form of representations in contextualized settings. Students in graphic calculator section have more likely graphical representations for both contextualized and purely mathematical settings. Moreover, they found several factors affecting student's preferences for representations. Some of them can be listed as; students' experiences with representations, students' perceptions of the usage of the representations, the level of the task, and the context. In this study, it was highly recommended that student's dependence on equational mode of representation should be shifted to the other modes of representations by providing them multiple representational environments mostly including technology.

Several studies have also examined representational preferences of students in mathematics classes. A study carried out by Özgün-Koca (1998) was about examining students' representation preferences when they were asked to solve a mathematics problem, and finding out the effects of students' thinking process when they were dealing with mathematical representations. As a part of her dissertation, she focused also on students' thoughts, beliefs, and attitudes towards representations in computer and non-computer settings. The subjects were 16 students from remedial mathematics class in which the primary goal was to make students believe that they can do mathematics. In this study several data collection methods were used. Interview was conducted with the classroom teacher in order to understand her use of multiple representations in class. Two observation sessions, one was in a regular classroom setting and the other one was in a computer class setting, were carried out by the researcher to see how multiple representations were using in a classroom. Lastly, a Likert type questionnaire was administered to the students to get students' attitudes towards mathematics and multiple representations, their strategies and preferences related to the use of representations, and effects of technology on their opinions and attitudes.

Although the results of this study can be discussed in terms of attitudes towards mathematics and students' problem solving strategies, the main focus is on the results related to multiple representations. Students (10 out of 14) generally agreed that mathematics problems can be solved using lots of representational modes, however they found easier to focus on one mode and deal with it. Moreover most of the students (10 out of 14) found equations and graphs more clear. They also mentioned that they were more comfortable with equations to solve a mathematical problem. The least appealing representational mode among students was graphs. From the computer related items the researcher got surprising results since the most popular representation type was graphs since the students said that technology made graphing easier. Besides, students have some specific reasons to choose the representation type. For instance, they preferred equation since they tought that this representation type is the only way to reach the exact answer. By using tables they said that they could see as many solutions as possible and this type of representation was also helpful to sketch the graphs. The students also added that if they need to think in a visual manner they would choose to sketch the graph. Summing up those significant results, the researcher suggested that as a teacher and researcher we can provide a rich learning environment including multiple representations instead of focusing only one type of representation, we can make students appreciate the different types of representations, experience with those representations, and choose the most meaningful one to them.

Knuth's (2000) research goal was to examine students' abilities to select, use and translate between algebraic and graphical representations of functions. For achieving this goal, the researcher preferred to investigate the translations between representations in the context of Cartesian Connection. There were five classes from college preparatory mathematics courses for this study. In each class, the students were given 10 minutes work as a warm-up activity. A problem including the algebraic and graphical solutions was given to them and they were expected to work on the problems in small groups. The responses to the problems were categorized as algebraic and graphical solutions. The researcher clarified that some students used equation in his or her solution, some of them preferred to use graph, and few of them used graphs to support the algebraic solution. So this kind of answers was categorized as algebraic solutions.

The results were a little bit surprising in terms of choosing representation types since more than three fourths of all students preferred the algebraic method as a primary solution method, and only less than a third of the students' first choice was graphical representation. Besides, as it was reported many students did not even notice that the graphical representation was a solution method. The author discussed the possible reasons of those results. The major reason was mentioned as the nature of the problems. When asked to find the solution of a problem, one can have a tendency to find the exact answer so it was normal to use equation at a first glance. In this sense, graphical representations might hide the necessary information, so students may not be aware that this type of representation is also a solution method. Finally, Knuth (2000) suggested that the goal of mathematics instruction should be not giving importance on procedural skills but emphasizing a more flexible understanding including variety of representations.

The goal of Boulton-Lewis (1998)' research was to investigate children's representations of symbols and operations with manipulative and strategies of problem solving. For this purpose, 29 children from first, second, and third grades were interviewed. The children were presented mathematical tasks which are about addition and subtraction of numbers. Besides, the classroom teachers were also interviewed in order to identify the representations and strategies that they used in the classroom activities. The results of this study denoted that children from all level preferred to use manipulative rather than carrying out the operations mentally. In fact, children from second and third grades had a more tendency to use the manipulative than the first graders who preferred especially fingers as manipulatives. The other children mixed variety of concrete materials (counters, arithmetic blocks) to solve the problems whenever it was necessary. The researcher suggested that if children is taught the relation between the concrete materials and the mental images regarding this material he or she should be familiar to this representation type and without increasing the load of representations, it is better to introduce one type of representation at one time so that the children could make a one-to-one mapping between the external representation and the internal one.

A tool for supporting multiple representations was introduced in Ainsworth and van Labeke's (2002) study. This tool, DEMIST, is a kind of software which allows the learner to use multiple representations for problem solving. This study aimed to evaluate the effectiveness of DEMIST at supporting for conceptualizing different representational modes. The second aim of the study was to investigate the decision making process of learners if they were given many representations. In this software, the learners were engaged in different modes of representations for particular topic rather than exposing the superiorities of only one type of representation. Therefore, it was possible to use concrete materials, tables, graphs to support the context. For carrying out the research study, several scenarios including learning units consisted of multiple representations modes like histograms, dynamic animations were prepared. When 18 undergraduate students pre and post test results were compared, it was revealed that there was no significant difference between the test results (The authors did not provide sufficient information about the instruction and data collection periods of the study).

Students' representation preferences were investigated in this study, also. It was reported that the majority of the students gave 40% of their time to work with three representational modes, 32% of their time to work with four representational modes. They tried to explore different kinds of representations. An average of 73 representations out of 80 were discovered by the participants, although not all of them were used efficiently. A kind of representation preference was specified by the researchers according to the percentage of time used:

X versus Time Graph> Terms > Value > Chart > XY Graph > Animation > Table > Equation > Pie Chart

The most frequently used representation type was X versus Time Graph to test the hypothesis of the learning units and to make translations between representational modes.

The intuitions of the students for algebra concepts given in three representational modes-table, graph, and diagram- were investigated by Dreyfus and Eisenberg in 1982. They took 443 students from sixth grade through ninth in Israel. It was noticed that the intuitions have altered according to the groups. High achievers preferred graphs for all algebraic concepts, whereas low achievers used tabular representation. It was recommended that algebraic concepts should be introduced in a graphical representation mode for high achievers, and in tabular representation form for low achievers.

3.5 A Coherent Summary of the Literature Review

Algebra, as a gatekeeper of mathematics, has been an important field of much discussion and research in mathematics education. It would be impossible to consider this field without representation systems. A good deal of important studies which showed progress in the multiple representations of algebra has been conducted. The most relevant studies were cited during the course of literature review chapter.

As the literature confirms, some sort of perspectives on multiple representations is being implemented in many algebra classrooms. This view reflects a kind of reform efforts along with NCTM Standards for school mathematics (NCTM, 2000). An overview indicates that much research have been conducted on the effectiveness of multiple representations on calculus and function concepts (Cifarelli, 1998; Even, 1998; Girard, 2002; Keller & Hirsch, 1998; Özgün-Koca, 2001; Pitts, 2003; Yerushalmy, 1997). Some of the studies have been focused on the success of using technology as an aid to multiple representations (Herman, 2002; Ainsworth & van Labeke, 2002). What is important, however, is that the studies provide insights and point to the method of multiple representation-based instructions as a growing issue for concern in the teaching of the algebra unit at any school level.

Although a notable amount of research has been done in a relatively short time periods, researches on the effect of using multiple representations in algebra contents indicated that a multiple representations-based instruction for teaching algebra can positively influence algebra achievement and it helps students to explore different ways of mathematical representations in many contexts.

The body of research discussed above indicated a certain need on implementation of multiple representations for many mathematical concepts for all grade levels since higher gain is possible in conceptual algebra understanding by the help of this kind of instruction (Brenner, et al., 1995; Girard, 2002). Additionally, in the literature there has been a growing interest in attempting to teach translational skills in representation modes in science and mathematics contexts (Brenner, et al. 1999; diSessa & Sherin, 2000; Swafford & Langrall, 2000), but there is no considerable evidence why these skills have not been performed by the students well (Even, 1998; Herman, 2002;).

The literature also suggests that persistence of knowledge gained from multiple representation-based instruction and influences on why students prefer one representational mode over another when solving algebra problems should be put into context of further investigations in mathematics education.

As an overall conclusion from the literature review, reform-based instructions and curriculum like multiple representations have the power to make better algebra teaching and learning process.

Need for this Study

In general, the major purpose of this study is to provide seventh graders with a multiple representations-based instruction that will help them to see conceptual frame of algebra which typically appears in lots of topics in their middle school, high school and even in college curricula. The positive effect of using this kind of instruction on students' algebra learning is obvious. There exists an extensive range of research looking at the effects of providing calculus and function concepts in multiple representations context;

most of them determined students` representation skills in mathematics. More research is needed in this area, due to the mixed results reported in a few studies. For instance, none of the studies reported in the literature examined the translations among all representational modes included in Janvier (1987b) and Lesh (1979) models. In these models, the students` understandings in translations from a network of representations are measured by the number and strength of the translations they carry out. Generally, the translation skills between two modes are investigated. This study integrated these two multiple representations model for providing a new teaching approach for algebra and illustrate the full range of algebra activities about pre-algebra concepts, linear equations, equation solving, and algebra word problems with understanding.

Besides, this non-technology oriented study presents the first step to investigate the differential effects of multiple representations-based instruction and conventional instruction in seventh graders algebra performance in general. Unlike Brenner, et al. (1999) study, this study involved two treatment and two control groups. By implementing multiple representations-based instruction in two seventh grade classrooms from two different schools, it was aimed to make reasonable interpretations about the treatment itself.

Significant changes in students` performance take time. In the literature, there were some applications including short time period. These applications will not be adequate since they do not provide students with a long term treatment. In contrast, this study also differs from most other studies in this respect since it included two-month treatment consisting of a whole algebra unit in seventh grade curriculum.

Furthermore, the literature is lacking in studies dealing with different contexts in multiple representations specifically. The current study will contribute to the body of knowledge by addressing this issue. As Knuth (2000) pointed out the nature of the problem is quite plausible for the students when choosing a certain representation type for solving algebra problems. To deal with this issue, variety of problem contexts was presented in algebra activities, so that students could see different problem types.

One weakness of the studies is that after implementing multiple representation-based instructions in one or two groups, the researchers generally preferred to administer an instrument involving the items about only multiple representations. They did not attempt to measure the students` gain in the given concept by using any other instrument. After having multiple representation-based instructions, it seems to be easier to demonstrate representation skills and good performance in tests including multiple representations. According to the researcher, performing well in the instruments not only related to multiple representations but also algebra is an indicator for having a good conception about using multiple representations in algebra. Therefore, this enabled the researcher to develop and use variety of instruments to measure students` gain in algebra and multiple representations. This study will contribute the literature by examining the effects of multiple representations-based approach to teaching algebra by collecting data from a variety of sources, such as `Translations among representations test`, `Chelsea diagnostic algebra test`, and `Algebra Achievement Test`.

From the research point of view, there is one further issue. There is growing evidence from a number of researches that students have tendencies for certain representational modes (Boulton-Lewis, 1998; Herman, 2002), and also symbolic type of representational mode was reported as the most popular mode particularly among college students (Ainsworth & van Labeke, 2002; Keller & Hirsch, 1998). The need for investigating the reasons behind this preference was confirmed in the literature. In order to address this need, in addition to quantitative methods for data collection and analyses, qualitative approaches were also used. Interviews were conducted with the students to explore their usage of multiple representations in algebra and reasons of preferences for representational modes in the present study.

To sum up, it can be said that, this study presents a different instructional method for algebra and it includes many applications of multiple representations concept.

CHAPTER 4

METHODOLOGY

The aim of this chapter is to present the intended procedures for the study. It includes descriptions of the overall research design, the population and the sample, the description of the variables, the data collection instruments, the design of the instruction, the pilot study of the instruction, the treatment procedure, the data analyses approach used to address each research question, power analysis, and limitations of the study. Chapter 4 concludes with the assumptions of this study.

4.1 Research Design

The first research question was examined through a quasi-experimental research design since this study do not include the use of random assignment of participants to both experimental and control groups (Fraenkel & Wallen, 1996). This research design can be visualized as in Figure 4.1.

School A	O_1	G_1	O_2
	O_3	G_2	O_4
School B	O_5	G_3	O_6
	O_7	G_4	O_8

Figure 4.1 The visual representation of research design for the first research question.

In Figure 4.1, the symbols O_1, O_3, O_5, O_7 represent scores on the instruments that were used as pretests. The two experimental groups are represented as G_1, G_3. The two control groups both received only traditional instructions that are represented as G_2, G_4. The symbols O_2, O_4, O_6, O_8, represent scores on the post administration of the instruments.

4.2 Population and Sample

The target population of this study consists of all seventh grade students from public schools in Çankaya district. There are 103 public schools in this region. However two schools from this district are determined as the accessible population of this study. These schools were named as School A and School B. There are 2 seventh grade classes in School A, and 7 seventh grade classes in School B. This is the population for which the results can be generalized. To protect each school's anonymity, the characteristics of them were given in pseudonyms. School A is located in a high socioeconomic status (SES) residential area. The location of this school can be considered as urban, and most of the students are coming from the neighboring areas of the school. The school had a student population of 377. It had a total seventh grade population of 57 students in two classes. Of the total seventh grade population, 26 of the students were male, and 31 of the students were female. School B was also a public school which was located in the campus of one of the large universities in Turkey. Most of the students in this school are children of the academic personnel of this university; therefore, the socioeconomic status (SES) of their families can be considered as high, comprising family income, social life standards, and educational level of parents. This school had a total population 1369 of students. There were 91 male seventh graders and 83 female seventh graders in 5 classes.

Due to the fact that getting permission for an experimental study and conducting an eight-week instruction in two classes are accepted only by few of the schools, the sample chosen from the accessible population was a convenient sample. All seventh grade classrooms, namely 7-A and 7-B, from the first school (School A) and two classes, called 7-C and 7-D, from the second school (School B) were included in the sample. Since the mathematics teachers of the groups 7-A and 7-D were also classroom teacher of them, they wanted to continue with their classes. Therefore, these classes were chosen as control groups, and 7-B and 7-C were identified as experimental groups. There are 15 girls and 13 boys in experimental group and 16 girls and 13 boys in control group taken from School A. On the other hand, the experimental group from School B consists of 17 girls and 21 boys and in the control group the number of girls and boys were equal, that is 18. The participants in this study ranged in age from 11 years to 14 years old. Table 4.1 presents a summary of the number of students in the experimental and control groups in terms of gender.

Table 4.1 The number of students in the experimental and control groups in terms of gender.

		Female	Male	Total
School A	Exp. (7-B)	15	13	28
	Cont. (7-A)	16	13	29
School B	Exp. (7-C)	17	21	38
	Cont. (7-D)	18	18	36

4.3 Variables of the Study

In this study there were 12 variables that can be classified as dependent and independent variables. Table 4.2 presents a list of those variables.

Table 4.2 Classification of the variables

Variable Type	Name	Value Type	Scale Type
Dependent	Algebra achievement posttest scores	Continuous	Interval
Dependent	Translations among	Continuous	Interval

	representations skill posttest scores		
Dependent	Attitudes towards mathematics scale posttest scores	Continuous	Interval
Dependent	Representation preference inventory posttest scores	Categorical	Nominal
Dependent	Chelsea diagnostic algebra test posttest scores	Continuous	Interval
Independent	Attitudes towards mathematics scale pretest scores	Continuous	Interval
Independent	Representation preference inventory pretest scores	Categorical	Nominal
Independent	Chelsea diagnostic algebra pretest scores	Continuous	Interval
Independent	Treatment	Categorical	Nominal
Independent	Gender	Categorical	Nominal
Independent	Age	Continuous	Interval
Independent	Mathematics grades in previous semester	Continuous	Interval

4.3.1 Dependent Variables

The four of the dependent variables are the student's raw scores on objective-based achievement, translations among representations skill, attitudes towards mathematics scale, and conceptual algebra level posttest. These raw scores were obtained from Algebra Achievement Test (AAT), Translations among Representations Skill Test (TRST), Chelsea Diagnostic Algebra Test (CDAT), and Attitudes towards Mathematics Scale (ATMS), respectively. These four dependent variables are continuous and measured on interval scale. The possible minimum and maximum scores range from 0 to

40 for the AAT, 0 to 45 for the TRST, 0 to 55 for the CDAT, and 0 to 80 for the ATMS, respectively.

There is one categorical dependent variable, namely representation preference inventory posttest scores. Representation Preference Inventory (RPI) was used to measure students' representation preference survey posttest scores. It has four categories as tabular, graphical, algebraic, and other representation types.

4.3.2 Independent Variables

The independent variables of this study can be classified in two groups as covariates and group membership.

The student's raw scores on the ATMS as a pretest, the pre-implementation of the CDAT, gender, age, and mathematics grades in previous semester (MGPS) were considered as covariates. Treatment (Multiple representations-based and conventional instructions) was considered as group membership so this was a categorical variable.

4.4 Instrumentation

Instruments used in this study were Algebra Achievement Test (AAT), Translations among Representations Skill Test (TRST), Chelsea Diagnostic Algebra Test (CDAT), Attitudes towards Mathematics Scale (ATMS), and Representation Preference Inventory (RPI). To assess students' performance in algebra; AAT, TRST, and CDAT were used. In addition to these instruments, interview task protocol was used to collect more information and insight about the participants' understanding of using multiple representations in algebra.

4.4.1 Algebra Achievement Test (AAT)

One of the instruments measuring students' algebra performance was the AAT. To analyze students' answers more deeply and to explore their computation and problem solving skills intensively, essay type questions were used in AAT. The test combines traditional school algebra test items including symbolic manipulations and computations in algebra. AAT consists of 10 items addressing key learning goals specified in the Mathematics Curriculum for Elementary Schools, published by Turkish Ministry of National Education (MEB, 2002). All the test items except two of them were adapted from the related literature, including one question from Third International Mathematics and Sciences Study (Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, & Smith, 1999). The fifth and tenth items were developed by the researcher. This instrument was presented in Appendix A.

Although the concept of symmetry was covered in the official mathematics curriculum, the AAT did not include items about symmetry topic since it would be taught in spring semester within the unit of geometry in seventh grade classes participated in the study.

A pilot study for this instrument was conducted with 102 seventh grade students chosen from two different public schools. The given time for completing the initial version of the AAT was 40 minutes in the pilot study. However, the students completed this test 15 minutes earlier. To score the students' responses to each question in AAT, four-point rubric was used. The highest point of 4 indicated a complete understanding of underlying mathematical concepts and procedures while the lowest point of 0 was given for irrelevant or no responses (Lane, 1993). The minimum and maximum possible scores from the test items are 0 and 40 points, respectively. Internal consistency reliability estimate for the AAT was measured by Cronbach alpha to be .85.

After its pilot study, the test had its final form. The required time for this instrument was reduced to be 30 minutes. For obtaining evidence about the face and content validity of this instrument, the AAT was checked by three experienced middle school mathematics teachers in terms of its format and content. They agreed on the appropriateness of the language, and the difficulty level of the items. The value of Cronbach alpha from the post implementation of the AAT was .90.

4.4.2 Translations among Representations Skill Test (TRST)

Another instrument for assesing students' algebra performance was the TRST. The purpose of this test was to obtain data about students' abilities in making translations among different representational modes. Duration of the test was 40 minutes. It was scored by using a three-point holistic scoring rubric. The highest point of 3 was awarded for responses showing that the problem was solved correctly and that the appropriate translations among representations were used. The lowest point of 0 indicated if the response is completely wrong or immaterial to the problem. The possible minimum score was 0, and the possible maximum score was 45.

TRST contains 15 open-ended items which were designed to measure skills of translation among representations, use of certain representations, and creating new representations. The item numbers and attributing translation skills was given in Table 4.3.

Table 4.3 The item numbers and attributing translation skills of TRST.

From \ To	Verbal	Algebraic	Tabular	Graphical	Real-World Situations
Verbal		1, 2, 5		11	
Algebraic			12		4
Tabular		6, 7		10	
Graphical			8, 9		
Real-World Situations		13			
Diagram		3			
All modes of representations			14, 15		

All modes of representations: Students can use all representational modes indicated in LMRM for solving the items 14 and 15.

Table 4.3 depicts the view that, except for the items 11 and 12 all the other items required a translation from one representation mode to another, such as from tabular representation to graphical one. For example, item 11 requires a translation from the verbal representation to the graphical one. The translations from the real-world situations to verbal, tabular, and graphical representations and the translations from verbal representations to real-world situations were ignored since in describing a real-world situation, a verbal representation is used anyhow and vise versa. In a similar vein, for the translation from the real-world situations to tabular and graphical representations, interview task protocol was developed.

Open-ended item type was considered as the most appropriate item type for this instrument since this kind of item type would reveal the cognitive process of students' translations among representations. The test was a compilation of items from numerous sources (Özgün-Koca, 2001; Mullis, et al., 1999; Lesh, Post, & Behr, 1987b; Frielander, & Tabach, 2001). The adapted version of the instrument was given in Appendix B.

To obtain the validity of the TRST, the items of this instrument were evaluated on three main criteria, namely; item content, item format, and examinee responses to the items. For obtaining valid results for the first two criteria, the instrument was submitted for review and recommendations to three mathematics teachers. They were asked to evaluate the items in terms of their accuracy, wording, difficulty, importance (the importance of the knowledge or skill to be measured), and bias (the possible favoring of some groups or unfamiliarity to specified groups). Taking into consideration the opinions of the experts, some improvements were made on the items. For instance, the wordings of some items were criticized because of their unfamiliarity to the students. Those items were revised for clarity. All the experts found the answer key correct; they agreed that the difficulty level of the items were suitable for the seventh graders. Besides, the importance of the knowledge and skills measured by the test items was appreciated and they came to an agreement that the items included no possible bias for members of identified groups. These experts' opinions could be considered as an evidence for the face and content validity of this instrument.

To acquire the examinee responses to the items and rework on the instrument, the pilot study of the TRST was conducted with 150 seventh grade students from two public schools. In the initial version of the instrument there were 17 open-ended items. After getting the responses from the students, two of them were not comprised in the last version of the instrument. It was concluded from the obtained data that those items were extremely difficult and complicated for the students. For instance, for one discarded item from the TRST, it can be said that, all of the students in pilot group did this item incorrectly. This item included a translation from the given diagram to the algebraic mode of representation by illustrating the Sierpinski triangles. None of the students found the correct formula for solving the question, because the necessary formula was 3^n Since the discarded two items were measuring the same translational skills (translation from the given diagram to the symbolic mode) along with the other test items, taking them out did not violate the construct of the instrument. The internal reliability estimate of the TRST was found to be .79 by calculating the Cronbach alpha coefficient. This value indicates a high reliability measure. The easy and ordinary types of items were put in the beginning, whereas the items that were comprehensive and requiring the knowledge of all representational modes were numbered as the last two. All the items were provided with the sufficient blank spaces for solution processes.

4.4.3 Chelsea Diagnostic Algebra Test (CDAT)

CDAT was developed by the Concepts in Secondary Mathematics and Science Team (CSMST) (Hart, Brown, Kerslake, Küchemann, & Ruddock, 1985; 1998) in order to determine 13-15 years old British children's algebraic thinking levels. The test was designed to measure the conceptual knowledge of elementary algebra. According to CSMST, there are six different categories of interpreting and using the "letter".

1. Letter evaluated: In this category, a letter is assigned to a numerical value from the outset. As an example; the item "What can you say about a if a+5=8?" can be given.

2. Letter not used: In this category, children ignore the letter or aware of it without giving it a meaning. The following item can be given as an example: "If a+b=43, find a+b+2=?".

3. Letter used as an object: The letter is seen as shorthand for an object or as an object in its own right. For instance; the item "2a+5a=?"

4. Letter used as a specific unknown: In this class, children treat a letter as a specific but unknown number, and can operate upon it directly, as in the item "Add 4 onto n+5".

5. Letter used as a generalized number: The letter is regarded as representing several values rather than just one. As an example; the item “What can you say about c if c+d=10 and c is less than d?” can be given.

6. Letter used as a variable: Children view the letter as a representative of a range of unspecified values, and a systematic relationship exist between two such sets of values. The foolowing item can be an example; “Which is larger, 2n or n+2? Explain”.

The first three categories indicate a low level of response. Children possessing these levels have a low level understanding of algebra, where they need to be able to cope with items that require the use of a letter as a specific unknown (Küchemann, 1998).

Apart from these six categories, four levels of algebra understanding were developed with respect to the children’s responses and the items themselves. The items in Level 1 are purely numerical and extremely easy. Level 2 items differ from Level 1 items in their increased complexity. The items in Level 2 include the letter as objects. Children at Level 3 can use letters as specific unknowns. In Level 4, children can deal with the items that require specific unknowns and which have a complex structure (Küchemann, 1998).

The item numbers and indicating levels were given in Table 4.4.

Table 4.4 The item numbers and indicating levels of the CDAT

Item Numbers	Levels
5a, 6a, 8, 7b, 9a, 13a,	1
7c, 9b, 9c, 11a, 11b, 13d, 15a	2
1, 2, 4c, 5c, 9d, 13b, 13h, 14, 15b, 16	3
3, 4e, 7d, 13e, 17a, 18b, 20, 21, 22, 23	4

The items were scored as 1 point if the response is correct, and 0 if the response is wrong. The published Cronbach reliability measure of this instrument was 0.70.

CDAT was translated and adapted into Turkish for this study. It was translated from English to Turkish by three research assistants from the department of mathematics education and one mathematics teacher. A backward translation was done by the researcher and two mathematics teachers to check whether the translation was equivalent to the English version. After the translation phase, the instrument was checked by a mathematician and two mathematics teachers in terms of its face and content validity. A mathematician was asked to solve all the questions and write down the correct answers. Two mathematics teachers were asked to critique the instrument in terms of its difficulty level, properness and familiarity of the items to the seventh graders, and any possible biases or favoring to some groups. Their agreements on this instrument can be taken as a valid measure of the CDAT. For gaining examinee responses to this test, a pilot study was conducted with 125 seventh graders from two public schools. The students answered the items in this test approximately in 60 minutes; therefore, this duration was set to be the completion time of the final version of the CDAT. The discrimination power of the items ranged from 0.20 to 0.60, and the item difficulties ranged from 0 to 0.94. Reliability measure as based on KR-20 coefficient was found to be 0.93. There were 55 items in the translated version of CDAT. The possible minimum and maximum scores were 0 and 55 respectively.

4.4.4 Attitudes towards Mathematics Scale (ATMS)

Mathematics attitude scale developed by Aşkar (1986) was used in order to determine students' attitudes toward mathematics. This scale was selected due to its appropriateness to the middle school students as Aşkar (1986) suggested. It consists of 20 items including 10 positive and 10 negative statements. This attitude scale uses a five-point Likert type of scale-response format with five possible responses: strongly agree, agree, undecided, disagree, and strongly disagree. Marking in the appropriate place located near the statements describing the possible choices, indicates the answer. In the scoring procedure, a choice of "strongly agree" was scored as 4 point, "strongly disagree" as 0 point for positive attitude items whereas a choice of "strongly agree" was scored as 0 point, "strongly disagree" as 4 point for negative attitude items. Possible scores for the ATMS ranged from 0 to 80 with higher scores denoting positive attitude towards mathematics and lower scores denoting negative attitude towards mathematics. This scale could be completed approximately in 10 minutes. The published reliability estimate of the scale was reported to be .96 by Cronbach alpha coefficient. For this study, Cronbach alpha reliability estimate from the post implementation of the ATMS was found as .95. The ATMS was presented in Appendix D.

4.4.5 Representation Preference Inventory (RPI)

The representation preference inventory was developed by the researcher for this particular study, which would address the second research question that is to identify the representation preferences of students. It consists of 15 questions in which students were given the problem and three different ways (tabular, graphical, and algebraic) of representing the problem. They were asked to choose one of the given representations to solve the given problem. The crucial point of this survey was that the students were not required to solve the problems since the solutions of the problems were already existed. They were only responsible to choose the favoring representation among three different kinds of representations. The items of the survey were selected and developed from the research articles and dissertations. The adapted version was given in Appendix E.

The pilot study of RPI was carried out with 120 seventh and eighth graders from three public schools. They completed the survey in approximately 40 minutes. Therefore, this duration was taken as a completion time for the final version of RPI. Each student was required to make a choice for presented representation types that were “table”, “graph”, “equation”, and “other”. Students’ representation preferences were classified with respect to their maximum frequency on a certain representation type. Students choosing the three representations with equal frequency were classified as not having a representation preference. The most preferred type of representation was algebraic, followed by tabular, while the least preferred types of representations were graphical and other ways (arithmetic, using proportions). In the light of this pilot study, some improvements were made on the survey. First of all, response type was changed. In the pilot version of the survey students were asked to order their preferences for representations from the most they like to the least by using the numbers 1, 2, 3, and 4. However, this format was difficult to understand for the examinees. Therefore, it was modified to make participants choose one of the presented types of representations. Another issue in the pilot version was that, students attempted to solve the given problem which was not required. Since solving the problem was wasting time for them, they were warned in the actual administration of the RPI.

For the face validity of the inventory, the RPI was submitted to two mathematics teachers. They examined the RPI in terms of its accuracy and format of the items. They found the given solution ways correct and agreed on the appropriateness of the item formats. These opinions can be accepted as a valid measure for this survey.

4.4.6 Interview Task Protocol (ITP)

Smith (2004a) states that students` representational abilities can be deduced by investigating their usage of representations in mathematical situations. Due to this reason, interviewing with students seemed to be the best method to understand the students` understanding in a multiple representation context. Semi-structured interviews were conducted to obtain data on how students used different representational modes when they were solving algebra problem and to obtain deeper understanding about the possible reasons of their representation preferences on representation preference inventory. In general, there were three types of questions, the aim of asking the first type of questions is to know about students' demographic information and algebra background. The following questions were posed to the participants.

Q1: Are you going to private school to support the school? If yes, when did you start?

Q2: Are you having private tutoring? If yes, when did you start?

Q3: When have you had the first experience about the concept of "algebra", such as; equations?

The second type of questions were aimed to obtain information about their use of multiple representations in algebra situations. Each question had one algebraic situation and needed generalization. The students were questioned on why they chose one type of representation over others in the third part. The second and third types of questions were given along with the analyses of these questions. In these questions, it was decided that, there is a possibility for students not to understand the term "algebra", therefore instead of this term, "equations", The interview process involved the purposeful sampling of 21 students from both of the schools' experimental classes and 4 students from control classes. Purposeful sampling, as used in qualitative research methods, selects information rich cases for in-depth studies (Maxwell, 1996; Patton, 1990).

Each interview lasted approximately 120 minutes. These interviews took place in the teacher's room in each school at times that suited to students' schedules and all the interviews were audio taped with the permission of the student. During the interviews, there were some rules that the researcher must obey and situations that the researcher should provide for the participating students. First of all, the researcher informed the interview participants about the purpose and the content of the interview, and then she asked each of the participant's permission to record all the interview session by audio recorder. For facilitating understanding of students' thoughts, it is crucial that the participants feel comfortable and willing to give honest answers to the questions. Hence, the subjects were told that they have the freedom to express any views or concerns they have about the treatment during the interviews. Besides, during each interview, the researcher paid careful attention to listen to the students. Before the interviews, the researcher provided students' pretests and posttests along with paper, and a pencil. Only equipment for the interviews was an audio recorder.

The think aloud interview protocol was tested with two seventh grade students who were chosen randomly from the low and high achiever groups. After explaining the aim of this pilot study to the pupils, they were given the questions separately. The students indicated that they were not familiar with this type of questions. Particularly for the questions requiring drawing, they mentioned that they found these questions very challenging, however; this challenge made them solve the questions, and they liked them. While solving the questions, one of the students was quite confident in her approaches to the interview questions and easily moved among representations. She transformed one representation into another by using combination of logic and prior knowledge. The other student however was much less confident in his translations among representational modes. His graphs were tentative. In the light of their answers the interview questions were modified.

4.5 Instructional Design

In general, the term instructional design indicates the systematic process of translating an instruction into plans for instructional materials, activities, and evaluation (Smith & Ragan, 1999; Wiggins & McTighe; 1998). For this study, the instructional design consisted of daily lesson plans in which several activities took place was given in Figure 4.2.

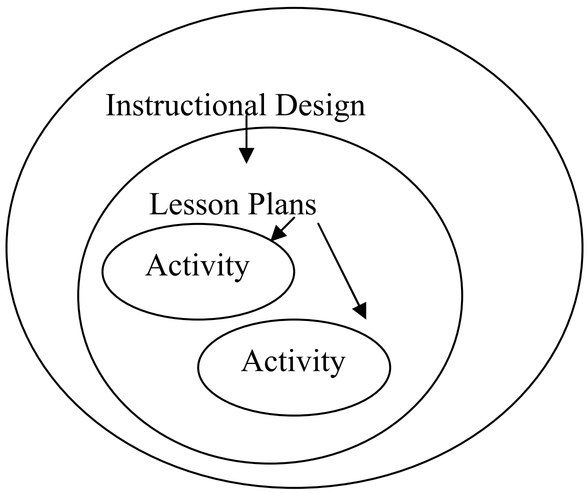


Figure 4.2 The schema of the instructional design of this study

The instructional design of the activities was based on the research and theoretical discussion presented in the part of conceptual framework and literature review. These instructional activities were based on Lesh and Janvier model for translations among representations. Lesh model was improved in a way that had some common features with Janvier model by the researcher. Table 4.5 presents the combined model for the activities that were used in the treatment.

Table 4.5 The combined model of Lesh and Janvier for translations among representation modes

From \ To	Spoken Language (Verbal Descriptions)	Tables	Graphs	Formulas (Equations)	Manipulatives	Real Life Situations	Pictures
Spoken Language (Verbal Descriptions)	–	Measuring	Sketching	Abstracting	Acting out	Acting out	Drawing
Tables	Reading	_	Plotting	Fitting	Modeling	Modeling	Visualizing
Graphs	Interpretation	Reading Off	_	Fitting	Modeling		
Formulas (Equations)	Reading	Computing	Sketching	–	Concretizing	Exemplifying	Localizing
Manipulatives	Describing	Exemplifying	Concretizing	Symbolizing	–	Simplifying, Generalizing	Drawing
Real Life Situations	Describing	Exemplifying	Plotting	Modeling, Abstracting	Particularizing	–	Modeling
Pictures	Describing	Describing	Sketching	Abstracting	Constructing	Situationalizing	_

82

There are 21 distinct activities which were involved in the lesson plans of the instructional unit in order to aid in teaching of a unit on Equations and Line Graphs. Equations and Line Graphs unit is the third unit of seventh grade mathematics curriculum. Ministry of National Education Mathematics Curriculum includes seven objectives and 62 behavioral objectives for this unit. In this curriculum, the duration for this unit was determined as 32 lesson hours indicating eight-weeks. This instructional unit does not comprise the objective of “Understanding the Symmetry” since the teachers of control groups mentioned that they planned to make students accomplish this objective in geometry unit with related goals. Therefore, they decided to give emphasis to the other objectives since this objective was not included in the lesson plans of control groups, the researcher decided not to include it in the lesson plans for experimental group, as well.

All 21 lesson plans which had distinct contexts and problem situations were developed in order to reflect the procedure of translations among representations, transformations within a specified representation, usage of any representational mode in dealing with algebraic situation. They were developed and selected based on the following principles.

- Instead of emphasizing symbol manipulation, representation skills were emphasized. In particular, students were required to learn constructing the multiple representations of algebraic situations, including expressing them in tables, graphs, and symbols.
- Instead of teaching these representation skills in isolation, it was anchored within meaningful thematic situations. Each activity was anchored within a mathematical context such as bouncing a tennis ball, or finding the money savings of children.
- Instead of direct instruction in how to construct and use mathematical representations in algebra, students were only guided in the activities to explore different representations and to develop their understanding of each one.

The instructional design of the study was given in Appendix H. The required representational translations within the activities in the instructional unit were presented in Table 4.6.

Table 4.6 The required representational translations within the activities

From / To	Verbal	Real-World Situations	Manipulative	Table	Graph	Algebraic	Drawing
Verbal	-	-	-	10	11, 21	1, 2, 3, 4, 6, 10, 16, 18, 19	-
Real-World Situations	-	-	-	13, 14	21	19	19
Manipulative	-	-	-	2, 5, 6, 17, 18	-	-	3
Table	2, 3, 4, 5, 6, 10, 18	-	-	-	10, 13, 14, 16	5, 10, 11, 12, 17, 20	-
Graph	14, 16, 21	21	-	19	-	10, 13	-
Algebraic	1, 5, 11, 19	19	-	10, 20	10, 12, 17, 18	-	-
Drawing	-	-	2, 5, 17, 18	3	-	10	-
All Representations				15			

In Table 4.6, different representational translations were mentioned. In a verbal mode of representation, the relation was expressed in words relating to a mathematical sentence. In a tabular mode of representation, the relation was expressed as two corresponding columns of numbers with each column representing a variable. In a graphical mode of representation, the relation was expressed as a line of equation with the x-axis representing one variable and y-axis representing the other, and in a symbolic mode of representation, the relation was expressed as an equation with unknown representing a variable.

For the instructional unit addressed within this dissertation most of the activities listed above were adapted from different sources or developed by the researcher. Those activities are known as multiple representation-based activities in the related literature. Adaptations were made on the activities including the appropriateness of the context for the participants. The instructional unit was controlled in order to assure the mathematical correctness of the activities and appropriateness to the multiple representation concepts by the advisor of the researcher. All the activities were designed on an activity sheet where the pupils were expected to conjecture and explain what was happening. All of the lesson plans were presented in Appendix K.

Prior to the application of lesson plans, a pilot implementation of them was conducted.

4.5.1 Piloting procedure of the instructional unit

Piloting is integral to the development process of the instruction since decisions regarding the development of instruction would be made based on the results of its pre-implementation. Pilot study of this instructional design was conducted as a part of ongoing work in preparation for this study. It was undertaken to collect information which would comment or relate to the instruction and the procedure, as well as suggesting criticisms and possible improvements. Piloting procedure that had two phases was completed with a sample of preservice elementary mathematics teachers and independently, with seventh grade students drawn from two public schools.

4.5.1.1 Procedure of the pilot study with preservice elementary mathematics teachers

The first phase of the pilot study of the instruction involving lesson plans was carried out with 53 senior preservice elementary mathematics teachers in the spring semester of 2002-2003 academic years. This piloting procedure took place in the course of analyzing textbooks of mathematics with the permission of the course instructor. Before piloting, there was a brief introductory session about the purpose of this pilot study and the researcher's dissertation. Students enrolled in this course were informed the concept of multiple representations and the objective of algebra unit in seventh grade mathematics curriculum. They were asked to participate in the activities as if they were seventh graders and then critise the activities in terms of

a. how well the activity sheets were likely to achieve the objectives,

b. how interesting and useful the tasks were from an instructional point of view,

c. if the concepts were within the competence of seventh grade students for whom it was intended.

More generally, preservice teachers were asked to note down any characteristics or features of the activity sheets they liked, and any they disliked or thought were physically and psychology harmful to the students. Furthermore, they were asked to examine the activities with respect to the mathematical language, the appropriateness to the level of students, and the difficulty level.

In the scope of this piloting, preservice teachers were involved 8 activities which were the activities of "Ancient Theatre", "x-y", "The Temperature", "12 Giant Man", "Walking Tour", "Geometric Figures", "Inequalities", "Stories in Graphs" in turn in order. The total time they spent on those activities was eight hours; hence, evaluating each activity lasted approximately one hour. First, the senior preservice teachers involved in the activities and then examined the activity sheets. Following this experience, all the preservice teachers were brought together for a small-group discussion based on the notes they had made. The reason for arranging such a discussion was to allow agreements and disagreements to be followed up and justified between participants so that reasons for comments would become more clear. Also, it was hoped that suggestions for improvement could act as a catalyst and stimulate further ideas for modifying the activities. There was a strong measure of agreement between all the participants on both the good points of the activity sheets, and with the criticisims that were made according to their comments in the small-group discussion and their notes about the activity sheets.

The main criticisms centered on the tasks, that had unclear directions that might be challenging for seventh graders. In the activity of "Ancient Theatre", they all agreed that the usefulness of the activity, but they found the way of writing this activity sheet extremely confusing. In the initial version of this activity sheet, there was one question requiring a translation from the tabular mode of representation to the symbolic mode by asking that; “Write a mathematical sentence satisfying this table”. One of the students said that a student would have no idea if the activity sheet is given in this way. Therefore the mathematical language of this activity was modified concerning the suggestions of the preservice teachers. Instead of the above sentence, “Construct a formula for satisfying this table” was written. The activities “x-y”, “The Temperature”, “12 Giant Man” were considered very engaging and easy for the specified pupils. So they suggested that the duration for those activities should be reduced to one hour. “Walking Tour”, “Geometric Figures” were the most popular activities among the preservice teachers who involved this pilot study. They liked the latter activity more than the former since they were allowed to use toothpicks for constructing the required geometric figures in the activity. They recommended that instead of using toothpick, less harmful things should be considered as a manipulative for this activity, like sticks cut from paper. Hence, cotton sticks were decided to use. The last two activities were also gained positive criticism from the preservice teachers. They found the tabular representation for solving inequalities very interesting and enjoyable for seventh graders and themselves also. All the positive and negative comments were sent to the last activity. There was a strong debate among preservice teachers about the activity of “Stories in Graph”. The purpose of this activity was to make students to translate real-life situations to the graphical representations and vice versa. The main idea of the activity were found to be difficult and open to discussion. One of the preservice teachers said that different stories could be made up for the same graph and this would create an inconsistency among the pupils whereas the other student argued that being open to different interpretations is the crucial idea of this activity. This makes students understand that one graph can be interpreted in many ways, it would not be confusing. After some time, they decided a new writing format on the activity and wanted to be informed what would happen in the actual implementation of this activity.

In the light of suggestions of the preservice teacher, the applied activities were improved.

4.5.1.2 Procedure of the pilot study with middle school students

The second phase of piloting was conducted with seventh grade children to test out the appropriateness of the activity sheets for students and, to determine the amount of pre-training time needed to complete the activity sheets. The initial versions of some activities were piloted in four groups of seventh graders at two state schools. The groups were made up of approximately 30 students having almost equal number of male and female students. The students covered the middle range of ability and had little information about elementary algebra. This experiment took 18 hours for all classes. 4 hours were spent in one school with two seventh grade classes, and 14 hours were spent in the other school.

This pilot study covered mostly the activities that had not been implemented in the first phase of piloting with the preservice teachers. Those activities were named as the followings; “The Pattern of Houses”, “Cutting a String”, “Cinema Hall”, “Folding a Paper”, “The Scale”, “A Journey to Planets”, “x-y”, “Bouncing a Ball”, “Saving Money”, “Making a Frame”. For implementing those activities, some manipulatives were prepared beforehand. A ball of wool, scissor, a model of a scale, the pattern blocks in the shape of square and triangle, the papers, 10 tennis balls, and thumbtacks.

Before the implementation of the activities, a short introductory talk was given about the researcher, aim of the study, and activity sheets. The subjects worked collaboratively as pairs, groups, and individually under the supervision of the researcher. The students were asked to discuss the tasks following the activity sheets freely with their partner. Answers to the questions were to be noted down, and students were also asked to comment on any features they found ambiguous.

During this pilot study, students were observed when they were on the given task. Whenever it was necessary, the researcher gave explanations about the given activities and guided the pupils. In general, the children were extremely enthusiastic, even excited, particularly in using the scale model for equations and construction of tables, and their progress through all the tasks was much quicker than expected. The first two activities (“The Pattern of Houses” and “Cutting a String”) could be considered as warm-up activities in which students were provided explanations and help. In those activities, students did not possess the skills to take an independent approach to the task. They were all requiring some help from the researcher about all the steps of the activity sheets. For the activities "Cinema Hall" and "Folding a Paper", there was no negative comments from the participants except the necessary equation was found to be hard by some of the students. The most remarkable activity was the one in which the scale model was used. Students were interested in this manipulative and they were able to provide satisfactory answers to this task. The activities of "A Journey to Planets", and "Bouncing a Ball" were the most popular activities as well since the context of the former was found appealing to the students, and bouncing a tennis ball for the second activity was enjoyable. The students got a little bit bored during the activity of "Saving Money" because of its long sentences for explaining the questions; however, the "Making a Frame" and "x-y" activities received an unexpected attention from the pupils.

The pilot study conducted in seventh grade classrooms revealed that the syntax of some activities was still confusing. Specifically, saying "write the mathematical expression of this table" was not clear for many students. They did not see the connection between the table appearance and the required algebraic expressions. The major difficulty was that students had difficulty in writing about mathematics, constructing of a mathematical sentence. This might be a result of being unaccustomed to general difficulty with writing or particularly writing in mathematics class. Moreover, the students had difficulties in translating directly from the tabular mode of representation to the graphical mode, and writing an equation from the table was also confusing to the students. They often wrote several equations satisfying the numbers in each row. In addition, students barely graphed the equations on a graph-paper.

Some corrections and adjustments about the activities were carried out based on this pilot study. The language of the activities was corrected in the light of the experiences from this implementation and the suggestions of the preservice teachers. To help alleviate the translations from the mode of representations to verbal statements, pupils do not have to be required to construct verbal statements in full sentences. It is more important for participants to have something in writing as a representation that constructing full sentences. For the difficulty of translating from a tabular representation to graphical mode, students were asked to write the necessary (x,y) pairs from the table and then plot them on coordinate axis. In a similar manner, it was decided that students will be warned that they should develop a formula including all the rows of the given table, not separate formulas for each row. It was seen from the pilot study that using graph-paper created a barrier for plotting a graph for participants; hence, instead of using extra graph-paper, the students would draw the graphs on their mathematics notebooks. Lastly, the specified durations for each activity were also tested. While the time for some activities was reduced, for others it was increased.

In summary, the two phases of piloting were clearly beneficial, gave strong encouragement to the rationale and design of the instruction, and provided some comments for the modification of the activity sheets with suggestions for supplementary tasks in the classroom.

4.6 Procedure

The timeline of this study is explained in Table 4.7.

Table 4.7 Timeline of the study

Tasks	Time
Reviewing related literature	Since 2001
Preparation of instructional unit	January-June 2003
Development of the instruments	February-June 2003
Piloting of instruments and instructional unit	June-September 2003
Observation of the experimental classes before the treatment	September-October 2003
Administration of the pretests	November 2003
Implementation of the treatment	December-January 2003-2004
Administration of the posttests	January 2004
Conducting interviews	February-March 2004

In 2001-2002 academic year, this study started with a detailed literature review relating education from the primary and secondary sources. Having read all the obtained documents, the research problem was narrowed and specified.

Then the instructional design of the study was developed. This phase was a continuing process. The content of algebra using multiple representations was decided to be presented by preparing activity sheets for the students. Each activity has a theme and enables students to use different modes of representations. To prepare the activities, several books, articles, and dissertations were examined and most of the activities were adapted from these sources, and four of them were developed by the researcher. At first, 55 activities were obtained. Those activities were investigated by the academic advisor of the researcher with respect to the level of possessing multiple representations of algebra. Then the activity sheet format and required materials for the activities were prepared for the pilot study of the treatment.

The third phase of the study involved selection and development of the instruments. Seven distinct instruments were decided to be used in this study.

Table 4.8 Selection and development of the instruments

Instruments	Selection or development of them
AAT	Selected and adapted from different sources
TRST	Selected and adapted from different sources
CDAT	Adapted from the original source
ATMS	Directly used from the original sources
RPI	Developed by the researcher
ITP	Selected and adapted from different sources

As it can be understood from Table 4.8, the ATMS was directly used since it was developed in Turkish.

Pilot studies for the instruments and instruction took place in 2001-2002 spring and 2002-2003 fall academic semesters. Of seven instruments AAT, TRST, CDAT, RPI, and ITP were piloted in six schools. Like instruments, the pilot study of the instructional unit was also conducted in two distinct schools with seventh and eighth grade students, and with elementary mathematics preservice teachers. After piloting procedure, 55 activities were reduced to 21.

Approval for this study was requested from the Ministry of National Education in September 2003, and obtained one month later. This approval included the permission of conducting a two months study with seven graders in three different schools. Getting this approval enables the researcher to visit the schools, observe the seventh grade students in their mathematics classes, and conduct the study. The two classes in one of the schools in which the researcher had a permission to implement the study were observed during the month of September and one week in October. After this period, the mathematics teacher of those classes was not willing to accept the researcher for observing the students and also for implementing her study, therefore this school was eliminated before the implementation of the treatment. Therefore, the study was conducted with the remaining two schools. Four classes in those schools were observed from mid-October to December, four times a week. Observing the classes helped the researcher to know the students and the classroom setting better before the treatment. Names of the students were memorized, the communication among students was explored during this period. Moreover, the students got used to the researcher, and they create a friendly and warm environment between each other. Hence, when the treatment begun, the students had already known the researcher, and got on well with her. This atmosphere made the treatment and observations more natural and ordinary.

The general procedure of the actual implementation of the study was overviewed in Table 4.9.

Table 4.9 The general procedure of the actual implementation of the study

	RPI/ATMS/CDAT			Multiple rep. based Treatment	Conventional Teaching	AAT/TRST/RPI/ATMS/CDAT					ITP
EG1	X	X	X	X		X	X	X	X	X	X
CG1	X	X	X		X	X	X	X	X	X	X
EG2	X	X	X	X		X	X	X	X	X	X
CG2	X	X	X		X	X	X	X	X	X	X

In both groups, instruments as pretest were completed before the implementation of the treatment. Students took the instruments in a quiet classroom atmosphere. They were given the CDAT first for 40 minutes lesson hour and additional 10 minutes from the break. Then the ATMS was administered in 20 minutes. The last instrument was RPI which lasted one lesson hour after the administration of the ATMS. During the administration of the instruments the sudents were asked to read the problems or statements and try their best to solve the problem, and give their honest answer. Students were encouraged to show all of their work in the space available on the test paper. They were assisted only if they had difficulty reading the words or phrases on the test, and were also provided with sufficient time to complete the test. No feedback was given regarding the accuracy of their works during all instruments' administration.

After the administration of instruments, the treatment phase began. There were a total of four groups involved in this study. Two of the groups were designated as the experimental while the other two served as the control group. Experimental groups (EG) received their instruction four lesson hours in a week, with each session lasting 40 minutes. They received 32 sessions of instruction in which multiple representation-based instruction was performed. The control group (CG) also received 32 sessions of instruction. However, this group worked on traditional way of performing algebra.

A multiple representations-based instruction was introduced in experimental classes through sample lessons about fractions and verbal arithmetic problems. Before the treatment, it seemed to be necessary to make students familiar with multiple representations-based instruction so that they are ready for the actual implementation. By the help of sample lessons, the fundamentals of multiple representations were explained to the students, especially those features that were to be used frequently in the treatment (setting up tables, the idea of graph, etc.). It took 4 hours before the treatment. Thereafter, the students were ready for the actual implementation.

The experimental groups explored algebraic content through multiple representations approach. Throughout this instruction, emphasis was placed upon the multiple representations of algebraic concepts. The two experimental classes met two days each week for eighty minutes, one in the early morning, and the other one in the late morning. During the treatment it was noticed that all of the students were enthusiastic about the implementation and willing to participate in every activity. Even when it was the break time, they did not want to take a short break, instead they wanted to continue with the activities. Some opportunities arose in the experimental groups for the researcher to interact with the students by answering questions or talking informally during those breaks. Even though this involvement was not initially planned, it proved to be beneficial for the students in becoming more comfortable with the researcher. It also provided insight into student thinking and difficulties with multiple representations use.

The researcher periodically visited both control groups for eight weeks. The adherence of the control group teacher's traditional teaching was judged by the researcher's presence in the classrooms. Those observations were conducted to determine the degree to which the teachers were using traditional methods to teach algebra for seventh grade students. In the first control group, the researcher observed all of the lessons, and in the second control group the researcher was able to observe 24 out of 32 lessons.

Across both the experimental and control group' conditions, the researcher gave the same assignments from the worksheets that the teacher provided and from the textbook.

At the end of the treatment, students' understandings of algebra was elicited through posttests administered to both experimental and control groups. The way of administering all the instruments as posttests were in the same way those described as pretests.

After collecting and analyzing the pretest and posttest data, the researcher conducted the follow-up interviews with selected students in an effort to have an in-depth understanding of their utilization of multiple representations in algebraic situations. For this purpose semi-structured protocol for the interviews was used.

4.7 Treatment Procedure

In this part, treatments in experimental and control groups were explained, seperately.

4.7.1 Treatment in Experimental Group

The experimental groups explored algebra content of seventh grade curriculum by multiple representations approach. After introducing this approach, by the help of sample lessons, the treatment took in each treatment group. Two classes, namely 7-B in School A and 7-C in School B were taught by the researcher.

Treatment in this study was primarily given through activities based on multiple representations. This approach was used to present and develop concepts from verbal, algebraic, graphical, and tabular standpoints. To illustrate, the concept of equation was first introduced from a numerically intuitive approach in which tables were used to collect the data and refine them on activity sheet. Then a verbal representation was used to verbally complement what was the relationship among the numbers in the tabular type of representation. Finally, a transition and development was made to the algebraic representation. The usage of multiple representations varied for each activity presented in this treatment. For instance, for the topic of equations, first the tabular representation then the verbal representation were constructed; however, for conceptualizing the concept of graph, first, the algebraic representation, and then the other representations were used. The usage of multiple representations also varied for the activities. Even after the algebraic representations were introduced to the students and conceptualized, the tabular, verbal and graphical interpretations of these concepts were not ignored. Many times, students obtained answers in an algebraic form, they were asked to interpret them in different representational modes as well. For example, students were not only required to translate graphical representation to algebraic but also vice versa. It was aimed to make students to understand that the final achieving point is not the algebraic form; the translation from an algebraic type of representation to a graphical one was also appreciated.

Activities were given to the students and they were responsible to deal with them. They were daily or sometimes weekly activities by which students were provided opportunities to demonstrate how manipulatives, tables, graphs, verbal expressions, and abstract symbolic representations fit well in one context. Open-ended responses were needed within each activity, and the students were encouraged to show all their work.

While implementing the treatment, first of all, the class was organized with respect to the activity requirements of that particular day. Then, the researcher or one of the students distributed the activity sheet, and if applicable necessary manipulatives. The students were given some time to read and understand the activity. After that, the whole class discussed the activity and its requirements. Then the phase of dealing with the activity sheets was begun. When the students were on the given task, the researcher provided feedback to the students on their errors and questions. At last, students had a chance to demonstrate their approaches including multiple representations to deal with the activities. Their works were discussed with whole class. The errors, questions or unclear parts were taken into account by the researcher when she was making a conclusion for the students.

The actualization of treatment can be illustrated in one of the activities which is Activity 20, "Inequalities". In this activity, students were responsible to find out the main characteristics of inequalities using the tabular representation. At the beginning, the activity sheets were given to the students, and then they examined the activity. They filled the given table by necessary numbers. After they explored the concept of inequality, the part of translation from one representation to another came. For the translation, the daily life situations and the algebraic representational modes were selected by the researcher. She asked one daily life example to the inequality of "$x-3<7$". Students gave their daily life situations, like;

> There are x number of teachers in one school, then 3 of them are appointed to another school, and the number of the remaining teachers was less than 7" (Anıl, 14.01.2004).
>
> Let us say that the number of the desks in our class was x, we get rid of 3 of them, then there are less than 7 desks in our class" (Gülsemin, 14.01.2004)

After getting students translations among representations, all of them were discussed in class. It is compulsory for the students to keep the activity sheets in the folder that the researcher gave them, since they did all the works on those papers. They were also responsible to bring their folder to the class every mathematics lesson. Besides the activity sheets, the students used their mathematics notebooks in order to take notes from the blackboard.

Class periods were 40 minutes for both groups. The Table 4.10 presents the construct of lesson plans of experimental.

Table 4.10 The construct of lesson plans of experimental groups

Desired Learning Outcomes	Student Activity	Teacher Activity
Introduction: 5 minutes		
Become aware that lesson has begun	Listen to explanation of lesson	Open lesson
Become interested and curious about the lesson		Distribute today's activity sheet
Know what lesson will cover and what will happen during the lesson		Explain the main idea of today's activity
Body: 30 minutes	Take notes	Guide students when necessary
Understand the main concept of the lesson	Fill activity sheet	
Make all necessary translations among representations	Discuss the ideas with other students	
Conclusions: 5 minutes		
Recall and consolidates experiences	Recall and share the main concept of the activity	Review main points of lesson

98

Naturally there were times when the schedule in Table 4.10 was not followed exactly. This was in cases where students had some difficulties referring to the activities or worked through them faster than it was expected. The detail teaching scheme with respect to the dates can be seen for each treatment group in Appendix I.

At the end of each unit the homeworks were assigned from the textbook of "İlköğretim Matematik 7" (Yıldırım, 2001), and in addition to this book, the extra tests provided by their mathematics teacher were given to the students.

4.7.2 Treatment in Control Group

As the students in experimental groups explored algebra by using multiple representations approach, the teachers in control groups took a conventional approach during eigth weeks. In control groups, the researcher made observations to document the climate and characteristics of those classes. By the help of those observations, it can be said that the control group teachers adhered to the curriculum to teach the algebra topics. Typically, the teacher would provide instruction by giving an explanation of the strategy and a solving template of the problem. The class would watch and take notes (copying the explanations and the problems with the solutions) as the teacher solved the problem. Then, students would work individually on homework problems as the teacher monitored their efforts.

The mathematics teacher for the control group in School A graduated from one of the education institutes, and she has been teaching mathematics for 28 years, 20 of those teaching years for middle schools mathematics. The other mathematics teacher graduated from one of the education faculties in Turkey, and she has been teaching mathematics for 18 years, 10 of those teaching years for middle schools mathematics. Both of them used traditional method of organization and instruction for their students.

Out of seven units in seventh grade mathematics curriculum, one unit was covered during the fall semester of 2003-2004 academic year. This unit which is an introduction to the algebra, deals with equations, inequalities and graphs. Teachers' lesson plans for teaching this unit were typical lesson plans that can be encountered in many seventh grade classrooms. One problem with this conventional is that it ignores the fact that mathematical concepts and procedures were connected to each other; hence, they should not be taugth as seperate concepts (NCTM, 2000). A conventional algebra instruction can be characterized by its emphasis on procedural skills and manipulating symbolic expressions. In control groups the teacher usually began by providing the rules for operations for manipulating algebraic concepts. For example, shifting 3 to the right side of the equation with its opposite sign is the first step for solving the equation “4x+3=12”. After providing students with this rule, the teacher demonstrated several examples that incorporated the rule. The same process was then repeated for the other rules and procedures. Throughout the lesson presentation the teacher asked for questions from the students and asked them to help her to solve the equation. She called students to the blackboard to practice several exercises.

The students in control groups were responsible for listening to the teacher, taking notes from the blackboard, and solving the questions that the teacher asked to them. The typical characteristic of both control groups teachers was that they mentioned the necessary rule for the topic, solved examples by themselves, and then called students from the specified class list to solve more examples. Almost all of the mathematics lessons in control groups were held following the same order. In control groups, the same homeworks and extra worksheets were supplied to the students by their teachers throughout the term. In Table 4.11, a comparison of treatment and control groups can be understood.

Table 4.11 A comparison of treatment and control groups.

Treatment Groups	Control Groups
Multiple representations-based instruction took place	Conventional teaching was the main method of teaching
Students were actively involved in their learning process by dealing with activity sheeets	Students were responsible for listening to teacher, taking notes, and solving the questions
The researcher taught the algebra unit	The algebra unit was taught by the mathematics teachers
The groups were observed by two preservice mathematics teachers and their regular mathematics teacher during the treatment	The groups were observed by the researcher
The researcher acted as a facilitator to make students to develop algebra concepts	The mathematics teachers acted as an information giver

4.8 Treatment Verification

Treatment verification was carried out by the presence of two preservice elementary mathematics teachers and one mathematics teacher in both experimental groups. They observed the whole treatment process in two experimental groups. The classroom notes that the preservice elementary mathematics teachers took, were examined by the researcher. Besides, the responses of the experimental group students for the interview questions were also counted as treatment verification.

4.9 Analyses of Data

In order to uncover the role of multiple representations-based instruction on seventh grade students' algebra performance, both quantitative and qualitative analyses of data proposed by the research questions were used.

4.9.1 Quantitative Data Analyses

Quantitative data analyses were classified as descriptive and inferential statistics. All the statistical analyses were carried out by using both Excel and SPSS 9.0.

4.9.1.1 Descriptive Statistics

Data was initially examined to obtain descriptive statistics of the mean, median, mode, standard deviation, skewness, kurtosis, maximum and minimum values, and the describing graphs were presented in this part of statistics for experimental and controls groups involved in this study.

4.9.1.2 Inferential Statistics

To test the null hypothesis, the statistical technique of Multiple Analysis of Covariance (MANCOVA) was used for comparing the mean scores of control and experimental groups separately on the AAT, TRST, CDAT, and the ATMS. For the first section, since this study comprised multiple independent and dependent variables, the inferential statistical analysis was based on the multivariate general linear model, which is a generalization of the univariate general linear model, but includes more than one dependent variable (Cohen & Cohen, 1983). The statistical model variable entry order used for this analysis is summarized in Table 4.12.

Table 4.12 MANCOVA variable-set composition and statistical model entry order

Variable Set	Entry Order	Variable Name
A (Covariates)	1st	X1: Gender X2: Age X3: Mathematics Grade of Previous Semester X4: PREATMS X5: PRECDAT
B (Group Membership)	2nd	X6: Teaching Method
C (Covariates*Group Interaction)	3rd	X7: X1*X6 X8: X2*X6 X9: X3*X6 X10: X4*X6 X11: X5*X6
D (Dependent Variables)	4rt	Y1: POSTATMS Y2: POSTCDAT Y3: AAT Y4: TRST

In addition to MANCOVA, to answer the second research question, frequency and percentages were calculated and they were tested by using Chi square analysis.

4.9.2 Qualitative Data Analyses

The conceptual framework of the study guided the qualitative analyses of data obtained from the students' RPI and interviews. The responses from participants were transcribed and coded. The focus of the analyses was on how students use multiple representations in algebra, ways of the students' understandings of representational modes offered by the treatment and the reasons why the students made the choices that they did when they solving problems on pretests and posttests.

Results of the data analyses were presented in Chapter 5.

4.10 Power Analysis

Before the treatment the effect size was set to small (f^2= .20) since the effect of this treatment is unknown in the related literature, even a small effect size may have practical significance. During the analyses, the probability of rejecting true null hypothesis (making Type 1 error) was specified as .05 which is commonly used value in the educational studies. This study involved 131 middle school students and 11 variables. For those values power of the study was calculated as .97. Therefore, the probability of failing to reject the false null hypothesis (making Type 2 error) was calculated as .03.

CHAPTER 5

RESULTS

This chapter presents the results of the study. It begins with a review of the purpose of the study and hypotheses generated by the research questions that guided the development of the study. It was followed by results of the descriptive, inferential, and qualitative analyses. Impressions from the experimental and control groups were reported after the analyses, and this chapter ends with the summary of the results.

5.1 Purpose of the Study

The purpose of this study can be summarized as following:

1. To examine the effects of a treatment, based on multiple representations, on seventh grade students' performance in algebra, attitude towards mathematics, and representation preference.
2. To investigate the representation preferences of the students before and after the unit of instruction.
3. To reveal how students use multiple representations in algebra word problems.
4. To investigate the reasons of preferring certain kinds of representations.

5.2 Null Hypotheses

As presented in Chapter 1, the null hypotheses of this study were as follows;

Null Hypothesis 1: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the collective dependent variables of the seventh grade students' posttest scores on the Algebra Achievement Test (AAT), translations among representations skill test (TRST), and Chelsea diagnostic algebra test (CDAT), and attitudes towards mathematics scale (ATMS) when students' gender, age, mathematics grade in previous semester, the scores on the pre-implementation of the CDAT (PRECDAT), and on the pre-implementation of the ATMS (PREATMS) are controlled.

Null Hypothesis 2: There will be no significant effects of two methods of teaching (multiple representation-based and conventional) on the population means of the seventh grade students' scores on the AAT, after controlling their age, MGPS, and the PRECDAT scores.

Null Hypothesis 3: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the seventh grade students' scores on TRST, after controlling their age, MGPS, and PRECDAT scores.

Null Hypothesis 4: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the seventh grade students' scores on the post implementation of CDAT, after controlling their age, MGPS, and PRECDAT scores.

Null Hypothesis 5: There will be no significant effects of two methods of teaching (multiple representations-based versus conventional) on the population means of the seventh grade students' scores on the post implementation of ATMS, after controlling their age, MGPS, and PRECDAT scores.

Statistical analyses can be divided in two parts; quantitative and qualitative analyses. Quantitative data analyses has two dimensions; descriptive and inferential.

5.3 Descriptive Statistics

Descriptive statistics collected on the data to identify means, standard deviations, kurtosis, skewness, minimum and maximum scores for the four groups were summarized in Table 5.1.

Table 5.1 Descriptive statistics related to the scores from PRECDAT, PREATMS, POSTCDAT, POSTATMS, TRST, and AAT for experimental and control groups.

Groups	Variable	N	Mean	SD	Min.	Max.	Skewness	Kurtosis
EG	PRECDAT	66	25.85	8.29	10	42	.015	-.764
	PREATMS	66	56.61	13.47	12	77	-.717	.561
	POSTCDAT	66	34.92	8.20	17	48	-.355	-.671
	POSTATMS	66	56.64	14.99	18	80	-.746	.101
	TRST	66	29.73	9.38	6	42	-.831	.056
	AAT	66	27.85	8.39	5	40	-.905	.560
CG	PRECDAT	65	21.12	7.98	1	40	-.352	.014
	PREATMS	65	54.75	16.12	19	80	-.243	-.722
	POSTCDAT	65	24.05	9.4	7	46	.588	-.207
	POSTATMS	65	53.72	16.07	11	79	-.355	-.378
	TRST	65	21.32	10.45	2	47	-.214	-.289
	AAT	65	20.94	11.25	0	40	-.528	-.118

PRECDAT: Pretest of Chelsea Diagnostic Algebra Test
PREATMS: Pretest of Attitude towards Mathematics Scale
POSTCDAT: Posttest of Chelsea Diagnostic Algebra Test
POSTATMS: Posttest of Attitude towards Mathematics Scale
TRST: Translations among Representations Skill Test
AAT: Algebra Achievement Test

As it can be seen in Table 5.1, for all instruments mean scores of the EG were higher than the CG mean scores. Of all the instruments, the EG and CG had the highest mean score from the instruments of PREATMS.

When the mean scores from the pre administration of the instruments and post administrations of them were compared, for the instrument CDAT, the EG showed an increase from 25.85 to 34.92. An increase in mean scores was observed for the CG only from 21.12 to 24.05. Similarly, from PREATMS to POSTATMS, the EG showed a mean increase of just .03 points, whereas the CG showed a decrease of 1.03 points.

In addition to the numerical descriptive statistics, simple and clustered boxplots were also formed. From Figure 5.1 to Figure 5.4 displayed the clustered boxplots related to the instruments the PRECDAT and POSTCDAT, the PREATMS and POSTATMS, TRST, and AAT for two groups.

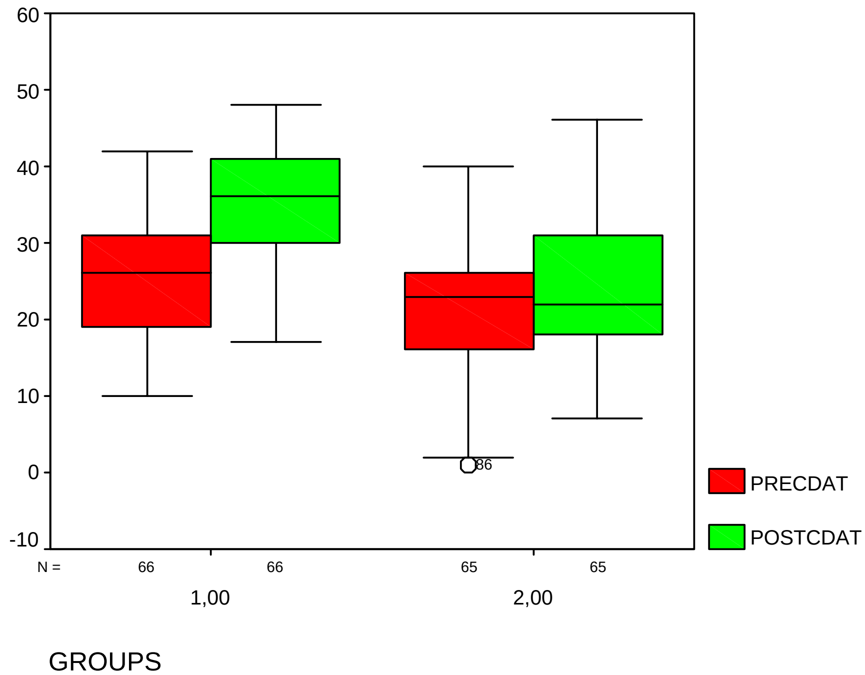


Figure 5.1 Clustered boxplot of the PRECDAT and POSTCDAT for the EG and CG

(1 indicates the EG and 2 indicates the CG)

In boxplot, the box includes mid 50% and each whisker represents upper and lower 25 % of the cases (Green, Salkind, & Akey, 2003) As Figure 5.1 indicated only one lower outlier was detected in the PRECDAT of the CG. The median of the EG was higher than the median of the CG for both PRECDAT and POSTCDAT. The range of the scores from PRECDAT and POSTCDAT for the EG was smaller than the scores from the CG. Besides, the EG students got the maximum score on POSTCDAT. Furthermore the scores of the CG students from POSTCDAT lied in the first quartile of the scores of the EG students from POSTCDAT.

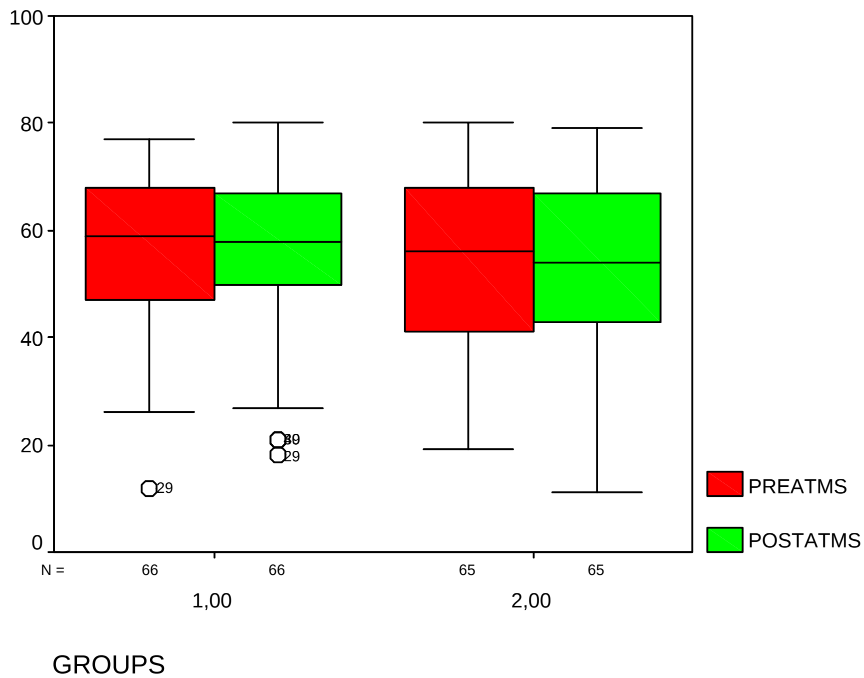


Figure 5.2 Clustered boxplot of the PREATMS and POSTATMS for the EG and CG

(1 indicates the EG and 2 indicates the CG)

Since the mean score of the EG had an increase .03 from pretest to posttest, the boxplot distribution of the PREATMS and POSTATMS were almost equivalent. There was one lower outlier for PREATMS of the EG, and two lower outliers for POSTATMS of the EG.

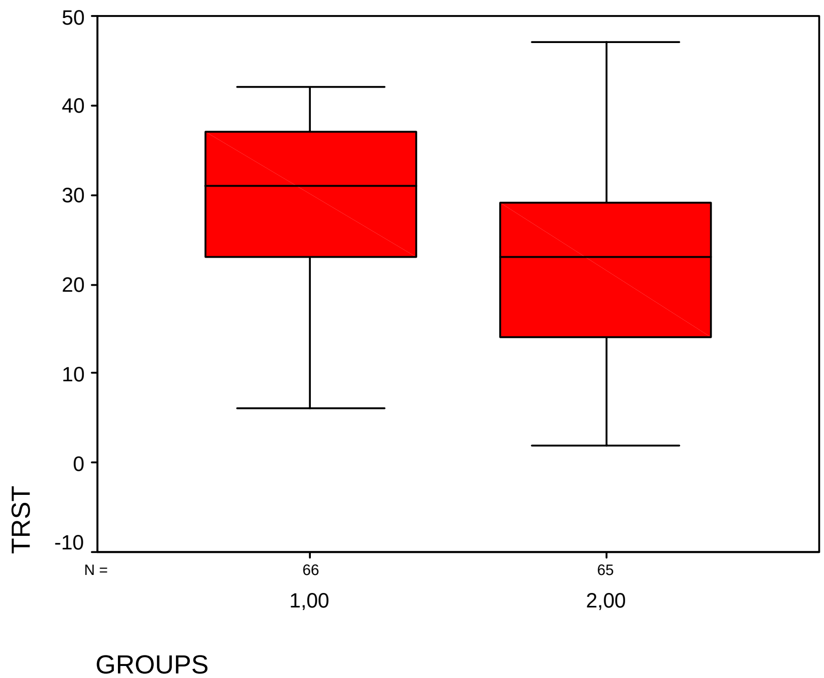


Figure 5.3 Simple boxplot of the TRST for the EG and CG
(1 indicates the EG and 2 indicates the CG)

As it can be seen in Figure 5.3, the median of TRST was higher for the EG than the CG. The CG students' scores lied in the first quartile of EG students' scores. The maximum and minimum scores were gained by the students from the CG.

In addition to the boxplot of the scores on TRST, the frequencies and percentages of the correct responses of the EG and the CG on each task of the TRST were calculated to show the distribution of students' responses for each task on the test items (see Appendix B). The calculations indicated that the EG students' correct response percentages were higher than the CG students' correct response percentages except for the items 6 and 13. On the items requiring a translation from verbal, diagram, or tabular to algebraic mode, the EG students performed better than the CG students. It can be argued that students from the EG carried out translations from different representational modes to the conventional algebraic mode well. Furthermore, the EG students made the translations among variety of representational modes except algebraic mode better than the CG students. For instance for the translation from verbal to graphical mode in item 11, the correct response percentage of the EG students was 66.7% whereas the correct response percentage of the CG students was only 27.7%. This can be perceived as an evidence for indicating that the EG students used the translations among representational modes better that the CG students. For the items 14 and 15 that were available to use all representational modes, again the EG students' correct responses (43.9%) were higher than the CG students (13.8% and 29.2%). This means that the EG students were not only capable of making translations from one representational mode to the other, but they were good at using different modes of representations for the given question also.

On the other hand, for the item 13, the correct response percentages of the EG and the CG were equal, and for the sixth item, the correct response percentages of the CG was slightly higher. In item 13, students were required to make a translation from real-world situations to algebraic mode of representation which was demonstrated on a figure of two pan balance scale. They were asked to write an algebraic equation regarding the situation shown by a scale. For the sixth item, a translation from the tabular mode of representation to algebraic mode was asked. The same translation was asked on the subsequent questions, as well. In these two questions, generally students were given a table in which some cells were in numbers and others were left in letters, they should use some algebraic knowledge to fill out this table. Although for the sixth item the correct response percentage of the EG students was lower than the CG students' response percentage, for the seventh item which had the same objective with the sixth one, the EG students achieved higher (53%) than the CG students (16.9%).

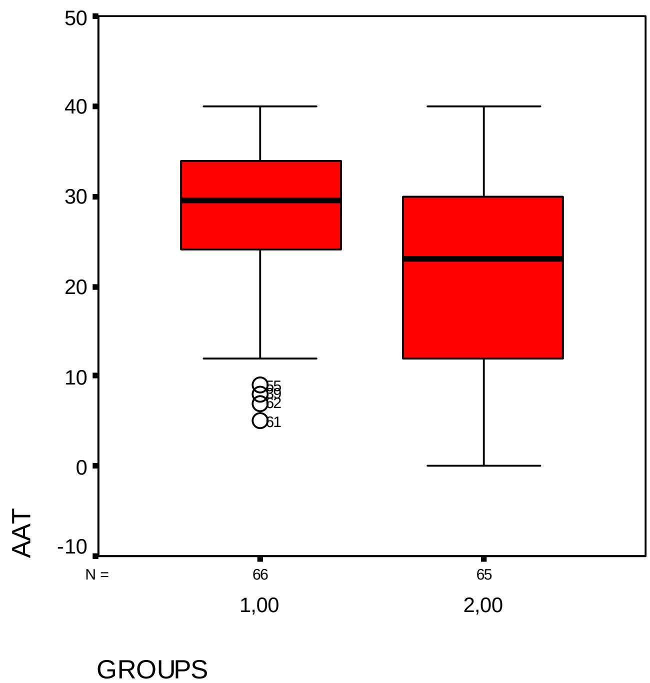


Figure 5.4 Simple boxplot of the AAT for the EG and CG
(1 indicates the EG and 2 indicates the CG)

From Figure 5.4, it can be said that there were four lower outlier, in the EG, whereas there was none in the CG. The median of the EG was higher than the CG. The distribution of the scores from AAT was more spread in the CG than the EG.

5.4 Inferential Statistics

This part covers the analysis of data that was collected during the study. Since the two experimental groups' characteristics and also two control groups' characteristics were found to be similar, the experimental groups were combined in one experimental group, and the control groups were combined in one control group for the inferential statistics. Firstly, determination of the covariates, the verifications of multivariate analysis of covariance (MANCOVA) assumptions, MANCOVA as a statistical model, the analyses of the hypotheses, and the follow-up analysis took place. Then it is followed by the analyses for RPI.

5.4.1 Missing Data Analyses

There were no missing data on the implementation of the pretests and posttests.

5.4.2 Determination of the Covariates

Before clarifying the assumptions of MANCOVA, five independent variables, namely; gender, age, mathematics grade of previous semester (MGPS), PRECDAT, and PREATMS scores of the participants were set as possible confounding variables of this study. Hence these five independent variables were taken as covariates in order to statistically equalize the differences between the EG and CG. The correlations between the predetermined independent variables and dependent variables were calculated and tested for their statistical significance to decide which independent variables should be selected as covariates in MANCOVA. The results of these correlations and significances were presented in Table 5.2.

Table 5.2 Pearson and point-biserial correlation coefficients between preset covariates and dependent variables and their significance test for the EG and the CG.

	Correlation Coefficients			
	POSTCDAT	POSTATMS	TRST	AAT
Gender**	-.152	.056	-.082	.124
Age	-.046	-.281*	-.159	-.193*
MGPS	.103	.480*	.435*	.530*
PRECDAT	.313*	.319*	.452*	.370*
PREATMS	.027	.717*	.180*	.312*
POSTCDAT	1.000	.017	.313*	.215*
POSTATMS	.017	1.00	.244*	.346*
TRST	.313*	.244*	1.000	.538*
AAT	.215*	.346*	.538*	1,.00

* Correlation is significant at the 0.05 level (2-tailed).
** Point biserial correlation coefficients were presented.

As presented in Table 5.2, all of the preset covariates have significant correlations with at least one of the dependent variables except gender of the student. Therefore the gender was discarded from the covariate set, and the other four independent variables determined in the covariate set for the inferential analyses for the EG and the CG.

5.4.3 Assumptions of MANCOVA

In analysis of MANCOVA there are five underlying assumptions that need to be verified

1. Normality
2. Multicollinearity
3. Homogeneity of regression
4. Equality of variances
5. Independency of observations

For normality assumption, skewness and kurtosis values were examined. These values of scores on POSTCDAT, POSTATMS, TRST, and AAT were in almost acceptable range (for skewness .015 - .905, and for kurtosis .014 - .764) for a normal distribution (Duatepe, 2004) as indicated in Table 5.1.

To test for multicollinearity, the correlations between covariates should be calculated for all groups (Green, et al., 2003). Table 5.3 indicates these correlations among covariates.

Table 5.3 Significance test of correlations among covariates for the EG and the CG

	Correlation Coefficients			
	Age	MGPS	PRECDAT	PREATMS
Age		-.232*	-.073	-.133
MGPS	-.232*		.473*	.370*
PRECDAT	-.073	.473*		.213*
PREATMS	-.133	.370*	.213*	

* Correlation is significant at the 0.05 level (2-tailed).

As implied in Table 5.3, the values of correlations among covariates were less than .80 (Green, et al. 2003), so it can be said that there was no multicollinearity among the four covariates for this study. Therefore, it can be said that there is no interaction effect of covariates on posttest scores; and so this assumption was validated.

Homogeneity of regression indicates that the slope of a regression of covariates on a dependent variable must be constant over different values of group membership (Crocker & Algina, 1986). To test this assumption, multiple regression analysis with enter method was conducted. Each dependent variable was regressed hierarchically on the independent variables using the entry order of covariate variables as Block 1, group membership as Block 2, and interaction between the covariates and group membership as Block 3. If the interactions were significant, then the homogeneity of the regression assumption would be violated and a valid MANCOVA would not be conducted. Table 5.4 displays the result of the multiple regression analysis.

Table 5.4 Results of the multiple regression analysis for homogeneity of regression assumption for both groups with respect to the post instruments.

	Change Statistics				
IV Block Added	R Square Change	F Change	df1	df2	Sig. F Change
		POSTCDAT			
Block 1	.103	3.603	4	126	.008
Block 2	.209	38.005	1	125	.000
Block 3 (Block1*Block2)	.048	2.258	4	121	.067
		POSTATMS			
Block 1	.595	46.242	4	126	.000
Block 2	.011	3.617	1	125	.059
Block 3 (Block1*Block2)	.001	.211	4	121	.810
		TRST			
Block 1	.272	11.798	4	126	.000
Block 2	.125	25,942	1	125	.000
Block 3 (Block1*Block2)	.043	2.325	4	121	.060
		AAT			
Block 1	.316	14.528	4	126	.000
Block 2	.165	39.835	1	125	.000
Block 3 (Block1*Block2)	.003	.361	4	121	.698

From Table 5.4, it can be understood that the interaction terms did not result in significance (F(4,121) = 2.258, p= .067, F(4,121) = .211, p= .810, F(4,121) = 2.325, p= .060, F(4,121) = .361, p= .698) for each dependent variable. Thus, there was no significant interaction between the covariates and the independent variables block, and the second assumption of MANCOVA model was therefore satisfied.

The fourth assumption of satisfying the equality of variances was controlled by Levene's Test of Equality. Table 5.5 shows the results of this test.

Table 5.5 Levene's test for equality of error variances for all posttest scores for the EG and the CG.

	F	df1	df2	Sig.
		EG and CG		
POSTCDAT	.864	1	129	.354
POSTATMS	.566	1	129	.453
TRST	4.396	1	129	.308
AAT	6.083	1	129	.105

As it appears in Table 5.5, all F values were found non-significant. This indicates the error variances of four dependent variables across groups were equal.

Lastly, independency of the observations assumption was checked. To validate this assumption the researcher observed the groups during the administration of all pre and posttest. From the observations it can be mentioned that the participants did all the tests by themselves.

5.4.4 Findings Related to Hypotheses Testing

Findings of the study related to the hypotheses were presented in this section. Hypotheses related to the first research question were:

Null hypothesis 1: There will be no significant effects of two methods of teaching (MRI and CI) on the population means of the collective dependent variables of the seventh grade students' scores on POSTCDAT, POSTATMS, TRST, and AAT after controlling their age, MGPS, PRECDAT scores, and PREATMS scores.

To test the first hypothesis the statistical technique of MANCOVA was used.

The covariates of this study were AGE, MGPS, PRECDAT, and PREATMS, and the dependent variables were POSTCDAT, POSTATMS, TRST, and AAT. Group membership was taken as a fixed factor of this study. The results of MANCOVA was showed in Table 5.6.

Table 5.6 Multivariate test results

Effect	Wilks' Lambda	F	Hypothesis df	Error df	Sig.	Eta Squared	Observed Power
Intercept	.929	2.318	4	122	.061	.071	.659
PRECDAT	.920	2.657	4	122	.036	.080	.727
PREATMS	.554	24.577	4	122	.000	.446	1.000
AGE	.947	1.720	4	122	.150	.53	.514
MGPS	.740	10.708	4	122	.000	.260	1.000
GROUP	.629	17.964	4	122	.000	.371	1.000

According to the results displayed in Table 5.6, 37% of the total variance of model for the collective dependent variables of the POSCDAT, POSTATMS, TRST, and AAT was explained by group membership of the participants. Using the Wilks' Lambda test, significant main effects were detected between the groups EG1 and CG1 (λ = .63, p = .000). This means that statistically significant differences were identified between the multiple representation based and conventional instruction on the collective dependent variables of the POSTCDAT, POSTATMS, TRST, and AAT. Therefore, the first null hypothesis was rejected. In other words, multiple representation-based instruction has an effect on the collective dependent variables of the POSTCDAT, POSTATMS, TRST, and AAT.

Null Hypothesis 2: There will be no significant effects of two methods of teaching (MRI and CI) on the population means of the seventh grade students' scores on AAT after controlling their age, MGPS, PRECDAT scores, and PREATMS scores.

Null Hypothesis 3: There will be no significant effects of two methods of teaching (MRI and CI) on the population means of the seventh grade students' scores on TRST after controlling their age, MGPS, PRECDAT scores, and PREATMS scores.

Null Hypothesis 4: There will be no significant effects of two methods of teaching (MRI and CI) on the population means of the seventh grade students' scores on POSTCDAT after controlling their age, MGPS, PRECDAT scores, and PREATMS scores.

Null Hypothesis 5: There will be no significant effects of two methods of teaching (MRI and CI) on the population means of the seventh grade students' scores on POSTATMS after controlling their age, MGPS, PRECDAT scores, and PREATMS scores.

The significant finding of a group effect from MANCOVA, allowed further statistical analysis to be done in order to determine the exact nature of significant differences found in main effect. To test the null hypotheses 2, 3, 4, and 5 separate univariate analyses of covariance (ANCOVA) were then carried out on each dependent variable in order to test the effect of the group membership. Table 5.7 presents the results of the ANCOVA.

Table 5.7 Tests of between-subjects effects

Source	Dependent Variable	Type III Sum of Squares	df	F	Sig.	Eta Squared	Observe Power
Corrected Model	POSTCDAT	4336.882	5	11.330	.000	.312	1.000
	POSTATT	18691.256(c)	5	36.731	.000	.595	1.000
	TRST	5993.116(d)	5	16.495	.000	.398	1.000
	AAT	5697.662(e)	5	16.870	.000	.403	1.000
Intercept	POSTCDAT	55.351	1	.723	.397	.006	135
	POSTATT	731.655	1	7.189	.008	.054	.758
	TRST	45.498	1	.626	.430	.005	.123
	AAT	31.440	1	.465	.496	.004	.104
PRECDAT	POSTCDAT	275.489	1	3.598	.060	.028	.469
	POSTATT	180.835	1	1.777	.185	.014	.263
	TRST	358,355	1	4.932	.028	.038	.596
	AAT	14,768	1	.219	.641	.002	.075
PREATMS	POSTCDAT	36.344	1	.475	.492	.004	.105
	POSTATT	10052.628	1	98.774	.000	.441	1.000
	TRST	2.888	1	.040	.842	.000	.054
	AAT	111.300	1	1.648	.202	.013	.247
AGE	POSTCDAT	2.251	1	.029	.864	.000	.053
	POSTATT	665.135	1	6.535	.012	.050	.718
	TRST	16.311	1	.224	.636	.002	.076
	AAT	22.424	1	.332	.566	.003	.088
MGPS	POSTCDAT	19.270	1	.252	.617	.002	.079
	POSTATT	629.801	1	6.188	.014	.047	.694
	TRST	1166.130	1	16.048	.000	.114	.978
	AAT	1996.254	1	29.552	.000	.191	1.000
GROUP	POSTCDAT	2909.618	1	38.005	.000	.233	1.000
	POSTATT	6.398	1	.063	.802	.001	.057
	TRST	1885.026	1	25.942	.000	.172	.999
	AAT	1234.192	1	18.271	.000	.128	.989
Error	POSTCDAT	9569.775	125				
	POSTATT	12721.706	125				
	TRST	9083.052	125				
	AAT	8443.696	125				
Corrected Total	POSTCDAT	13906.656	130				
	POSTATT	31412.962	130				
	TRST	15076.168	130				
	AAT	14141.359	130				

As it was shown in Table 5.7, it can be said that multiple representations-based instruction has a significant effect on the dependent variable posttest scores of the AAT ($F(1,125) = 18.271$, $p = .000$). So null hypothesis 2 was also rejected.

From Table 5.7, it can also be revealed that, multiple representations-based instruction has a significant effect on the dependent variable posttest scores of the TRST (F(1,125) = 25.942, p = .000). So null hypothesis 3 was rejected.

From Table 5.7, it can be stated that, multiple representations-based instruction has a significant effect on the dependent variable scores of POSTCDAT (F(1,125) = 38.005, p = .000). So null hypothesis 4 was rejected.

Null Hypothesis 5 was failed to reject since this instruction has no significant effect on the dependent variable posttest scores of POSTATMS (F(1,125) = .063, p = .802).

Hence, multiple representations-based instruction has a significant effect on the POSTCDAT, TRST, and AAT, and no significant effect on the POSTATMS. Although multiple representations-based instruction had no significant effect on the EG students, their posttest attitude scores were higher than the CG students.

Furthermore, for the observed treatment effects, it was obvious that the value of eta squared for the scores of the POSTCDAT was .233 which is almost equal to the medium effect size, for the scores of the TRST and AAT was .172, and .128, respectively which is almost equal to the medium effect size. This explains 23% of the variance in the POSTCDAT, 17% of the variance in the TRST, and 13% of the variance in the AAT were related to the treatment. Power for the scores of the POSTCDAT was found as 1.00, for the scores of the TRST and AAT was .92, and .78 respectively.

The adjusted means of the dependent variables for the EG and CG were computed by discarding the effects of the covariates from the dependent variables. Table 5.8 presents the adjusted means.

Table 5.8 Prior and adjusted means of the dependent variables

Dependent Variable	Group Membership	Prior Mean	Adjusted Mean
POSTCDAT	EG	35.89	35.89
	CG	19.14	19.14
POSTATMS	EG	60.71	60.71
	CG	58.38	58.38
TRST	EG	32.61	32.61
	CG	21.28	20.65
AAT	EG	32.21	32.21
	CG	24.48	24.37

Table 5.8 denoted that the prior and adjusted means for the EG was higher than the CG for all posttests. Besides for the CG there was a .63 point decrease on the TRST mean scores, and .11 point decrease on the AAT mean scores after

extracting the covariates from those posttests. The other adjusted means remain same as prior mean values.

5.4.5 Step-down analyses

Step-down analyses were carried out as significant MANCOVA follow up analysis. By the help of this analysis, the unique importance of dependent variables which were found as significant in the MANCOVA was investigated. Since there were three significant dependent variables namely, POSTCDAT, TRST, and AAT, three step-down analyses were conducted. By doing so, any possible variance overlap among the dependent variables was planned to be detected. Table 5.9 displays the result of this analysis with the POSTCDAT as the dependent variable of highest priority.

Table 5.9 Step-down ANCOVA of POSTCDAT

Source	df	Mean Square	F	Sig.
GROUP	1	2596.379	33.294	.000
TRST	1	103.937	1.333	.250
AAT	1	.175	.002	.962

As appears in Table 5.9, the effect of multiple representations-based instruction had still significant effect on POSTCDAT [F(1,131)= 33.294, p= .000]. This result showed that performance on POSTCDAT was significantly and uniquely influenced by multiple representations-based instruction after accounting its effect on the TRST and AAT scores.

The same analysis was performed by taking the TRST as the highest priority dependent variable. Table 5.10 shows the result of this analysis.

Table 5.10 Step-down ANCOVA of TRST

Source	df	Mean Square	F	Sig.
GROUP	1	541.272	7.188	.008
POSTCDAT	1	100.361	1.333	.250
AAT	1	2578.188	34.238	.000

As Table 5.10 depicts that the effect of multiple representations-based instruction had still significant effect on the TRST [F(1,131)= 7.188, p= .008]. This result indicated that performance on the TRST was significantly and uniquely influenced by multiple representations-based instruction after accounting its effect on the POSTCDAT and AAT scores.

By taking AAT as the highest priority dependent variable, step-down analysis was performed one more time. Table 5.11 presents the result of this analysis.

Table 5.11 Step-down ANCOVA of AAT

Source	df	Mean Square	F	Sig.
GROUP	1	493.345	7.231	.006
POSTCDAT	1	10.173	.321	.574
TRST	1	8.818	.278	.600

As Table 5.11 displays, the effect of multiple representations-based instruction had still significant effect on the AAT [F(1,131)= 7.231, p= .006]. This result suggested that performance on the AAT was significantly and uniquely influenced by the treatment after accounting its effect on the POSTCDAT and TRST scores. The unique contribution that AAT made in the presence of the covariates (TRST and POSTCDAT) was significant.

Representation Preferences of Students

For the second research question, the representation preferences of the students were assessed with respect to the frequency of preferring one of the representational modes for all the items in RPI. Students' representation preferences were categorized as 1 for tabular representation, 2 for graphical representation, 3 for symbolic algebraic representation, 4 for other type of representations, and 5 for no distinct representation. The frequency of preferring representational modes on the PRERPI and POSTRPI for the EG and CG was presented in Table 5.12.

Table 5.12 The frequency and percentages of preferring representational modes of the EG and CG students on PRERPI and POSTRPI.

EG	Table (1)	Graph (2)	Symbolic (3)	Other (4)	No Pref. (5)
PRERPI	13 (19.4%)	1 (1.5%)	47 (70.1%)	4 (6%)	1 (2%)
POSTRPI	24(36.3%)	8(11.9%)	34(50.7%)	-	-
CG	Table (1)	Graph (2)	Symbolic (3)	Other (4)	No Pref. (5)
PRERPI	17 (26.2%)	5(7.7%)	42(64.6%)	1 (1.5%)	-
POSTRPI	5 (8.5%)	4 (6.8%)	56 (87%)	-	-

As explained in Chapter 3, the responses of participants to each item on RPI were classified into the following five classes: table, graph, symbolic, other modes, and no preference of representations. From Table 5.12, it can be understood that, the symbolic representational mode was the most preferable type of representation for the EG and CG groups. Although the general tendency of the EG students seemed to be to use symbolic mode of representation, a variation in choosing representations can be observed in their representation preference after the treatment. Before the treatment, 70.1% of the EG students, preferred the symbolic mode of representation, few students (19.4%) preferred tables to solve questions, and only one student's preference was graphical mode of representation. However, on the POSTRPI a decrease in the percentages of preference for symbolic mode of representation can be detected. For this mode of representation, it can be revealed that there was a 19.4 point decrease for the preference for the EG students. On the other hand, there was an increase in the percentages of the EG students for their other representational modes. For tabular mode of representation, the percentage was increased to 36.3% from 19.4%, and for graphical mode, there was a 10.4 point increase in the EG students representation preferences. This means that after the treatment, the 19.4% of the EG students altered their preferences from excessive use of symbolic mode to the tabular or graphical modes for solving algebraic questions. Since the EG students learned other modes of representations and were capable of using them, they preferred other modes of representations on highest rate on POSTRPI.

A remarkable case was encountered for the other type of representational modes or no preference for any representation. On the implementation of the PRERPI, there were four students who preferred other type of representational modes (aritmetic, drawing, etc.) in the EG, however those students' preferences were classified as tabular, graphical, or symbolic mode after the treatment. There was no preference for other type of representations on the implementation of POSTRPI for the EG students.

In a similar way, 64.6% of the CG students decided to choose the symbolic mode, 7.7% of the CG students preferred the symbolic mode, and 26.2% of the CG students preferred to use graphical mode of representation on the PRERPI. It can be mentioned that the most preferred representational mode was again symbolic mode among the CG students on the PRERPI. On the POSTRPI, it can be observed that, the CG students' representation preferences were altered sharply since the percentage of preferring symbolic mode of representation was increased to 87% and the percentage of preferring tabular and graphical modes of representations was decreased to 8.5% and 6.8%, respectively. It was a surprising result because after the conventional teaching in the CG classes, 22.4% of the students changed their previous representational preferences, and preferred to use symbolic mode of representation. Similarly, on the PRERPI, 26.2% of the CG students preferred to use tables for solving the questions, however; on the POSTRPI, tables were appealed only by 8.5% of the CG students. Besides, on the PRERPI, there was one student who preferred other type of representational modes, after the treatment there was no such a preference. The influence of using symbolic mode of representation in conventional teaching dominantly by teachers on the students' representation preferences can be observed clearly by the help of these results.

From the frequencies and percentages of students' representation preferences, it can be claimed that, the EG students' utilization of representational modes varied after the multiple representation-based treatment. To test whether the representation preferences of the EG and the CG students on the POSTRPI were significantly different for each representational mode after the treatment, chi-square test was conducted. The results of this test were given in Table 5.13, 5.14 and 5.15, respectively.

Table 5.13 The results of chi-square test for the EG on POSTRPI

	Observed N	Expected N	Residual
Tabular	25	22.3	2.7
Graphical	8	22.3	-14.3
Algebraic	34	22.3	11.7
Total	67		

Table 5.14 The results of chi-square test for the CG on POSTRPI

	Observed N	Expected N	Residual
Tabular	17	21.3	-4.3
Graphical	5	21.3	-16.3
Algebraic	42	21.3	20.7
Total	64		

Table 5.15 Chi-square test statistics for the EG and CG on POSTRPI

	EG	CG
Chi-Square	15.612	33.406
df	2	2
Asymp. Sig.	.000	.000

The result of the Chi-square test was significant, x^2 (2, N=67) = 15.612, p=.000 for the EG. Therefore the results suggested that the students, in general, prefered different representational modes as a function of multiple representations-based instruction. Similarly, the result of the Chi-square test was also significant, x^2 (2, N=64) = 33.406, p=.000 for the CG. Hence it can be indicated that the CG students in general prefered the symbolic representational mode as a function of conventional teaching.

5.5 Qualitative Data Analyses

Qualitative data of this study were obtained by interview task protocol. Interviewing is a commonly used data collection method among mathematics educators. As Patton (1990) indicated, the main purpose of interviewing is to find out what is in someone else's mind. Since conducting interviews enables researchers to dig the students' thoughts deeper, this way of data collection was seemed to be compulsory for this study in order to understand how students use multiple representations in algebra. In general, participants were asked on how they use multiple representations in algebra, how they made translations among representations, and why they initially choose a given representation over others.

The specific aims of this interview can be organized as follows:

1. to understand how students use different representational modes when they solve problems related to algebra,
2. to address the reasons of their preferences on RPI,

The researcher used standardized open-ended semi-structured interviews (Maxwell, 1996). In the interview, pre-determined wording and sequence for each question was used for each respondent. After the questions, probes and follow-up questions were directed to deepen the interview responses.

The analysis focused on the transcripts of 21 students taken from two experimental groups for whom extensive data are obtainable in an effort to answer the interview questions. Semi-structured interviews which allow the interviewer the opportunity to probe further related to the questions were conducted by the researcher. The prompts were given to further explore students' thoughts. Those prompts varied according to the responses of the students.

For the interviews "information rich cases" (Bogdan & Biklen, 1992) were selected as participants in purposeful sampling. The students were chosen considering the fact that they would be good informants who could explain more about multiple representations. 21 students (11 male and 10 female) from the EG and 4 students (1 male and 3 female) from the CG were selected as participants.

After selecting the interview participants, each participant was interviewed in the same order by the researcher individually, going through the interview questions in order, asking further questions when it was essential and appropriate to clarify in more detail some of the responses of the interviewees.

The researcher as an interviewer was represented by "I". Letters for the names of the participants were assigned for purposes of protecting their identities.

The analyses of the interview data were conducted with respect to the interview questions which can be classified as demographic, knowledge and opinion type of questions (Patton, 1990; Zazkis & Hazzan, 1998) and the analyses of the interview data were presented here.

5.5.1 Demographic Information

To obtain data about the EG participants' demographic information and their past experiences about algebra, three questions which were given in Chapter 4 were posed to them. The results indicated that except for five students, all of the participants were going to the private schools to support the school. Besides of five students, 3 of them were taking private tutoring beyond the mathematics lessons in their schools. Hence, it can be argued that except for two participants, all of them were supported by private schools or tutoring as an additional aid to mathematics. Table 5.16 depicted the general view about the past algebra experiences of the participants.

Table 5.16 A summary of participants' first experience with algebra

First experience with algebra	Number of the participants
Third grade	2
Fifth grade	3
Sixth grade	12
Seventh grade	1
During the treatment	3

Nevertheless, the students have already met some sort of algebraic representations in some way before the treatment. Table 5.16 describes that; students' first experience with algebra (i.e. the use of letters to represent numbers) also varied. Majority of the participants' (12 of 21) first experience with algebraic expressions was in their sixth grade, either by the mathematics lessons or the help of the family members. Three of them experienced algebra when they were in fifth, and two of them have met algebraic expressions in their third grade. Only one participant was introduced algebra in seventh grade, and three of them were experienced algebra during the treatment. Although the algebra concepts were firstly mentioned in the unit of "Equations and Line Graphs" of seventh grade, according to the national curriculum (MEB, 2002), even in third grades, students started to be taught by the concept of equations in schools or informally outside the schools. The participants indicated that particularly in solving arithmetic word problems included in the third grade, their teachers have explained the method of solving equations algebraically for those arithmetic word problems. Therefore, almost all of the participants knew something about the content of the instructional unit before the treatment.

5.5.2 Participants' Use of Multiple Representations in Algebraic Situations

During the interview, there were questions that are aimed at exploring what factual information the participants had about the posing question (Patton, 1990; Zaskis & Hazzan, 1999). There were five such questions that included conceptually-oriented algebra tasks to be completed.

The frequency of correct and incorrect responses, and also items with no response with respect to each question, and the frequency of representation preferences of participants with respect to each question, Table 5.17 and 5.18 can be constructed.

Table 5.17 The frequency of response types with respect to the questions.

	Correct	Incorrect	No response
Q1	14	6	1
Q2	20	1	-
Q3	19	2	-
Q4	19	2	-

Note: Q1, Q2, Q3, and Q4denotes the five questions that posed to the participants during the interviews.

From the Table 5.17, the correct responses were higher than the incorrect responses. This indicated that the EG participants performed well on the interview questions.

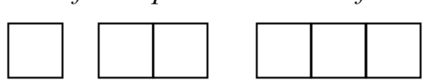
Table 5.18 The frequency of representation preferences of participants with respect to each question.

	Q1	Q2	Q3	Q4
Arithmetic	3	8	_	_
Table	1	5	2	_
Symbolic	4	10	2	1
Verbal	_	4	_	1
Graph	_	_	1	_
Diagram	_	_	3	_
Diagram to symbolic	6	_	1	_
Verbal to symbolic	1	1	1	5
Arithmetic to symbolic	2	_	_	_
Table to symbolic	3	1	12	14

In Table 5.18, the representation preferences of the participants were displayed. In general all representational modes and translations among these modes were used by them. From the table, it can be observed that the most popular representational mode was symbolic one. Tabular mode and arithmetic procedure were also appealing to the participants. The diagrammatic mode of representation had the least frequency. The translation from the tabular mode to the symbolic mode was commonly used among participants; however, making translations to solve the questions were less used compared to using only one mode of representation.

To interpret the interview results in a more detail way, it seemed to be necessary to give the question and make the discussion with respect to them. Firstly, the interview question was given, and then the number of students who responded this question correctly and incorrectly was presented, afterwards how the students used multiple representations for solving these questions was explained. The first question was given below.

Below you can see three figures made up of toothpicks. How could you find the number of toothpicks in order to form the nth figure?



This question was not a common question that the participants usually encountered in their mathematics lessons and textbooks. It was expected for the participants from the EG that they could connect this question with the activities of the treatment and use different types of representations to solve this question. However, it was experienced by most students as more difficult than the other four questions. Only 14 of the students could answer it correctly. This question required to make a generalization considering the growing figures of squares and open to use different representational modes.

When the transcripts of the participants were analyzed, they preferred all representational modes in order to solve this question. Since in this question, a specific figure was given; participants B and U tried to use this figure to find the answer. They added new squares to this figure without attempting to use any other representational mode. In a similar way, participants A and N had a tendency to replace *n* with any number when solving the problem, therefore to reach the *nth* figure; they tried to replace n with a number, like 4 or 7.

I: Why do you think that, n should be 4?

A: Because after the third figure, the fourth figure comes, so n should be four.

I: Why do you think that, n should be 7?

N: I counted the squares, there are 6 squares so n is the square seventh, I put 7 instead of n because we do not know n, it should be a number.

I: Can we just say n, without putting a number instead of n?

N: No, we can't. If we do not put a number instead of n, how can we solve this question?

The participants A and N insisted on saying that the *n*th figure means the fourth, or seventh figure. They did not recognize that *n* indicates an algebraic way of looking at the question, so they failed to reach the solution. Dealing with the problem arithmetically resulted in wrong answer since this question needs to be interpreted with the algebraic point of view including generalization to the *n*th figure. Those participants used no representations; they just followed the arithmetic rules on the given figure. However, participant C formed a table in order to see the pattern of the figure number and the number of the toothpicks used but she could not establish a relationship between this table and the necessary algorithm to find the toothpick number in the *n*th figure. In fact, she was unsuccessful to make a translation between the tabular representation and symbolic one. Participant P and T failed to find the correct answer as well. They chose symbolic mode of representation for this question. They tried to write an equation by counting the toothpicks in each figure.

Participant P first wrote y=2x, and he tried to put the number of the toothpick as x and the number of the figures as y to verify this equation. He used trial-and-error method to reach the solution; however after trying few equations, he gave up and could not get the correct answer.

Even with probing, participant J showed no awareness that she figured out the question.

The participants who solved the question correctly ended their approaches to Question 1 with symbolic representation. Respondents E, G, K, and V preferred constructing equation to solve the problem. They represented the number of figures by using several letters like n, ş, x and the number of toothpick used as k and y. They immediately wrote the equation which were y=3x+1 (participantV), k=3ş+1 (participant K), k=3n+1 (participant G), and y=3n+1 (participant E) without attempting to use any other representational mode as an aid to reach the equation. Participants D, F, O, R, and S used the given diagram to reach the symbolic representation. They made a translation from diagrammatic mode of representation to the symbolic mode without using tabular representation as a vehicle between these two modes. Participants D mentioned that he focused on the figure only and thought the numbers of the toothpicks in his mind, and then he formed an equation related to these numbers.

A translation from tabular mode of representation to symbolic mode was established by the respondent H and M. They approached the problem firstly by forming a table and tried to solve the problem in this form of representation, but they could not. Therefore, they constructed an equation by using the numbers located in the columns and rows of the table. Student M used table in a different way. She said that it is a kind of representation that made her attain the symbolic mode of representation. She further explained;

M: In tables, everything you need is there, it is a suitable vehicle to find out the relationship between numbers. I make use of table to find equation.

Only one student preferred to describe the problem verbally. He wrote what he understood from the given diagram in words and then translates the words into the symbols as in an equation. In summary, for the first question, the representational modes that were used were symbolic, tabular, diagrammatic, arithmetic, and verbal. It can be revealed that, students' use of representational modes can be altered with respect to the given question. For this question, some students tried to use the figure only, some of them preferred to construct a table related with the figure, and some of them wrote the necessary equation. It can also be argued that, tables are perceived as a mean to reach the symbolic mode of representation, not an end for this question. Moreover, there were no attempts for drawing a graph to solve the question.

The second question was:

In a shop there is a return fee for the beverages. For one bottle the return fee costs 5 million TL.

a. *For 6 and 12 bottles how much return fee should be paid respectively?*
b. *What kind of a relationship exists between the return bottles and their return fee?*
c. *For 100 bottles how much return fee should be paid respectively?*
d. *How many bottles does the shop owner have after paying 300 million TL for return fee?*

This one was a typical algebra word problem. There were two fundamentally different kinds of quantities, namely bottle and return fee in this question and participants were required to calculate return fees of the bottles while the number of the returned bottles was changing. Since the participants solved questions like that before, it was not a challenging question for them. Except for one student, all of them solved this question correctly.

The general tendency of the participants for the first and second part of the question was to use arithmetic representation. This means building a proportional relationship between the return fee for 1 bottle and for 6 and 12 bottles. Respondents B, E, F, G, N, P, T, and V established a cross multiplication algorithm between the given and asked quantities. The proportion was like;

If for 1 bottle the return fee is 5 million TL.
for 6 bottle x

They preferred to carry out the above procedure for solving this part of the question and for expressing a relationship between the two quantities. Participants D and U, however, chose to convey their ideas about the question verbally. They wrote the relationship between the return fee and the number of bottles in words like;

U: The return money should be 5 times of the bottles.

On the other hand, a tabular representation was preferred by the participants A and H. These students constructed a table including the numbers representing two quantities which were bottles and return fee, and answer all the questions by using this table.

Symbolic, verbal, and graphical representational modes were the respondents' choices for the parts of c and d. Respondent G was the only student who preferred graphical representation to find the return fee for 100 bottles and to find the number of bottles for 300 million TL. return fee. When she was asked about her representation, she said that;

G: I like constructing graphs. To solve the question drawing a graph seemed to be the best way since you can see the increase in the graph, and you can easily decide how much should the owner pay for 100 bottles.

I: Isn't it difficult to build up a graph including the points (100,500)

G: Actually no, because I did not start from 1 for x-axis, I started with 50 so 100 was easily pointed on this graph.

Unlike G, participants F, N, P, and T preferred verbal representation for the part of c and d of this question. Although they did not use this mode of representation for the first two parts, finding the return fee for 100 bottles and finding the number of bottles for 300 million TL. return fee by explaining in words was perceived much more easier than the other modes of representations for those participants.

Symbolic representation mode was found as appealing by respondents A, B, D, E, H, U, and V. They explained the reasoning behind their preferences as follows;

A: If we have an equation, the risk of failure reduced. We can just look at this equation, put the given numbers into this equation and find the solution. I think, using an equation is better, nice, and more amusing.

H: Firstly I constructed a table to see what I have clearly, and then I wrote an equation from this table, I would use a table as a means of starting point before moving to equation.

U: When I wrote the relationship in words, I read it then I figured out the equation from these words.

For participants I, J, K, M, and O the representation preference showed no alteration for all parts of the second question. They all preferred symbolic mode of representation for solving this interview question. They preferred to stick with one way to do each options of this problem. On the other hand, four participants C, L, R, and S preferred to use tabular representational mode for all parts of this question. Three of them did not show any hesitation to begin with constructing a table and continued to solve the question by using it. However, participant L first tried to write an equation for the options c and d, and then he could not continue with this equation. He added the following opinion:

L: First I write an equation, ….(he tried to write, but he could not make it). Let's forget about the equation I should make a table first, and then I discovered the equation. Table can show the way to the solution.

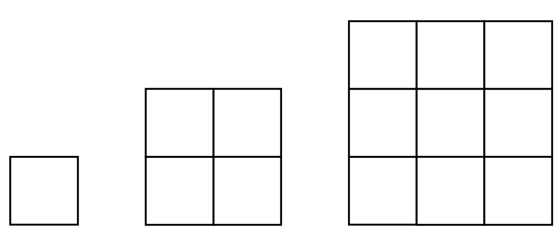
As a whole, it can be mentioned that, for the second question, representation preferences varied according to the students' perceptions of the parts of this question and representational modes. Two participants indicated that they used the easiest way to reach the solution, so they tried one mode of representation for the first and second part of the question, whereas they used other modes of representations for the third and fourth parts. In general, participants preferred to use arithmetic or tabular modes of representations since these parts included small numbers that can be easily multiplied or divided in a proportion and put in a table. With these numbers, performing arithmetic operations were not difficult. However; for the third and fourth parts of the question there were numbers, such as 100 and 300. Some of the participants noticed the need for constructing an equation or a table to reach the equation when they saw bigger numbers. Participant L also mentioned that;

L: For 100 and 300 I could not use proportions because they are too big. It is better to write an equation, so I can put any number and find the correct result.

Hence, it can be argued that for some participants, representation preference depends on the numbers in the given question and the way they perceive those numbers as large or small. Besides, the symbolic mode of representation was seen among some of the participants as a representational mode to be utilized only for the questions involving large numbers complicated situations. If small numbers were given in a question, they did not attempt to use the symbolic mode of representation. On the other hand, there are participants who used the same representational preference for all the parts of the question. They were not influenced by the given numbers in the question.

The third question was the following one:

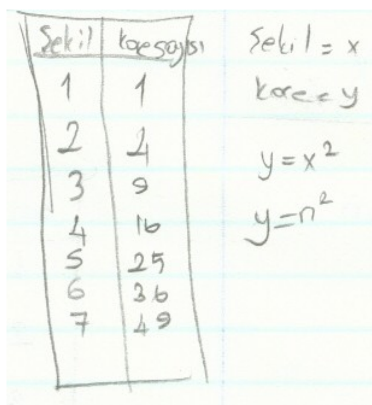
a. How many of the smallest squares will be in the next figure if this pattern continues?



b. How many of the smallest squares will be in the 7th and nth figures if this pattern continues?

In the third question participants were given with a pattern including growing squares, and were asked to find the number of smallest squares in seventh and *nth* figures. 19 participants reached the correct answer for this question, whereas only two of the participants failed to give the correct response.

The responses for the third knowledge question were surprising since half of the students preferred the same representation type which was establishing a translation from the tabular representation to the symbolic mode. These participants first constructed a table including the number of figure in one column and the number of squares for this figure in the other column. The following table can be given as an example of those participants.



As the sample table indicated, those participants put the number 7 in this column and find the regarding number for the squares as 49. When they were probed the reasons of preferring this method, they explained that by constructing a table they can summarize the essential numbers in one type of representation. After drawing a table, they remarked that an equation should be written to find the answer for the *nth* figure. So they wrote the necessary equation and carried out the procedures to find the number of the squares in the nth figure. Students I and V conveyed the ideas about their preferences like this;

I: First I think in my mind, and then I realize that I should organize the numbers, table is the best way of it. And also it helps to find the relationship. In table you can see clearly. Till the third figure I could not see the relationship, but in the fourth figure I figured it out and wrote the equation.

V: I tried to construct an equation here, but before the equation I should make a table then go with the equation. In table I searched the relationship between numbers, and then I use this relationship in the equation.

Participants H and K preferred also a translation from the tabular representation to the symbolic mode for solving this question however they constructed the table only to make the situation clear and then they wrote an equation. As a third step, they put 7 into an equation then find 49, and carried out the same procedures for *n*. they put *n* and find n^2 as a correct response. They did not use the tabular representation to find the number of smallest figure for the seventh figure, in order to find the answer they preferred a symbolic representation in the same way with the *nth* figure. When they were asked the reason behind their preference, participant H mentioned that;

H: For n I have to write a table, so I write this table at the beginning for using for the seventh figure and nth figure also.

Different translations among representational modes were also utilized by the interviewers. For instance F preferred to explain the mathematical situation verbally, like;

F: From the figures I understood that it is a growing figure and it includes squares of the numbers, like 1 is the square of 1, 4 is the square of 2, and 9 is the square of 3, so 49 should be the answer, because it is the square of 7. For the nth figure the results is n^2*, the square of n.*

This showed that, participant D used the given figure, and made a translation from this figure to verbal mode of representation; afterwards she translated this mode into an equation for the *nth* figure.

Participant U also preferred to go on with the given figures; he found an equation from the given figures. He did not draw a table or a graph; he directly wrote the equation by considering the relationship between numbers. He noticed that the figure gave a clue that there are numbers whose squares were taken, so he constructed the equation.

Participant A was the only student who preferred graphical and symbolic representations for the first and second parts of the question separately. He did not make a translation between two modes of representations; rather he used graphical representation of $y=x^2$ for the first part and symbolic representation for the second. Plotting this graph was very challenging for seventh grade students. Although respondent A said that he preferred the graphical representation, he did not draw this graph correctly. He just put the numbers in the axes of graph, like constructing a table. He could not plot the graph of $y=x^2$. An equation as symbolic representation was used by respondent A for the second part of the question.

Eight of the participants used the same representation type for two parts of this question. E, S, and T preferred to construct an equation by referring the number of figures as ş, and the number of squares as k, and then wrote the equation of k=2ş. They put 7 and n into this equation to find the number of the squares. E explained his idea like this;

E: When someone says "n" I understand that the figures go infinity, there is no end. If I use an equation to represent the situation, this is the most suitable way since in an equation you can put any number, so it is infinity. So if there is an equation, why do I waste time with using other kind of representations?

The explanation of the participant E was very interesting since the participant E saw no need for other kind of representations if she has an equation.

Participants B and C reached the wrong answer since their tabular representation was remained along with the second part of the question. They insisted on representing the mathematical situation in tabular form for seventh and *nth* figure. However, they normally could not find the answer for the *nth* figure in the table; therefore, they decided to replace n by a number 8, and fin the result as 64. C said that:

C: I considered n as 8. So the answer is 64.

I: Is this answer true for the nth figure?

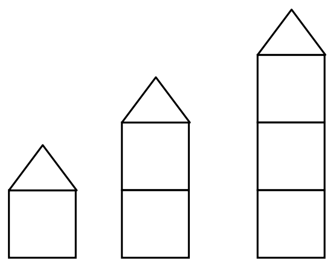
C: Actually, no. But in a table how can I find the number of squares for n? I should translate n to a number.

I: Why did you choose 8?

C: Because 7 was followed by 8.

As a result, the third knowledge question was appropriate for using several representations like the other two questions. The participants showed the examples of using different modes of representations for an algebra problem like this. Although, it can not be argued that symbolic mode is the most convenient representational mode to solve this question, some of the participants' efforts to construct a graph or to write sentences about the figures seemed not practical for this question. From the given figure the relationship between the number of the figures and the small squares can be seen for the seventh figure and $y=x^2$ can be constructed as an essential equation. Not being familiar with the second degree equations might be the reasons for not writing an equation for all participants. In this question, the same situation in the first question can be observed. Some of the participants treated *n* as an unknown, not a variable. They tried to replace *n* by certain numbers. These students can be classified as the students who think arithmetically, not algebraically to solve the problems. For them using the tabular mode of representation seemed to be the only way since to construct an equation, they should be persuaded that all the mathematical situations can not be represented by numbers.

The subsequent question was:



Dila made this pattern using sticks. What is the relationship between the number of toothpicks and the figures? Explain how you got your answer. How many toothpicks will be used in (n+2). figure?

This one had common features with the third question. Both of them included a figure, and asked questions about the consecutive figures. The responses for this interview question did not vary as much as in the third question, and the frequency of the correct answer was the same with the frequency in the third question.

For this question, among the participants, only one student followed the wrong procedures and concluded with an incorrect answer. She started with a verbal type of representation indicating the relationship between the number of toothpicks and the number of figures. However, she could not pass to the other type of representations. She thought that the only way to solve the question was to represent the mathematical situation verbally, so she replaced (n+2) by a number. She first tried 4 for *n*, and found 6 for (n+2). Then she decided that 6 was the correct answer. The researcher made her aware of something wrong with deeper probing; however, she did not recognize it.

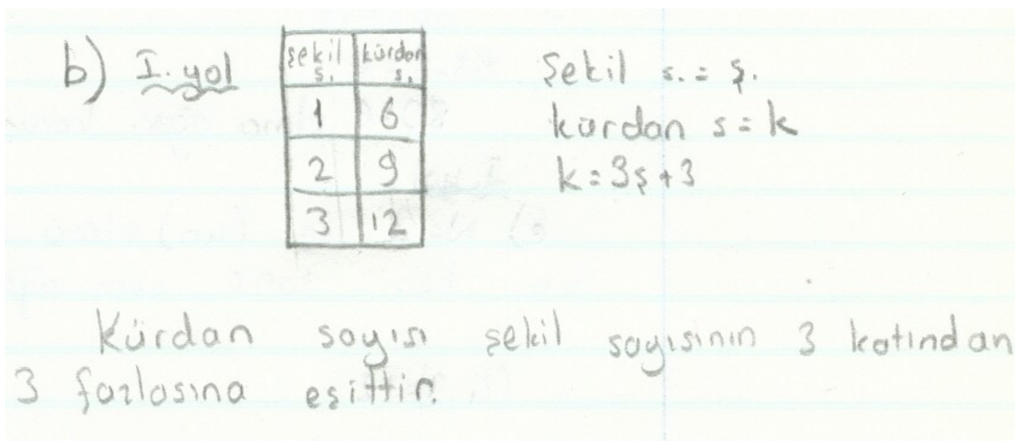
Except for the student G, the other students got the correct answers for the question four. Among them only D used a symbolic representation for expressing the relationship, and for finding the number of toothpicks in the (n+2). figure. He constructed an equation representing the number of figures by x, and the number of toothpicks by y, then wrote y=3x+3 as required relationship, and then he put (n+2) in place of x, carried out the necessary algorithms, and found y as (3n+9) which was the correct answer.

Setting up a translation between the verbal and symbolic representational modes was preferred favorably. Respondents B, C, E, L, and P described the mathematical context in this question in words. C mentioned that;

C: There is a relationship between the given things. The number of the figures is less than the toothpicks. And when I multiply the figure numbers by 3 and add another 3 to what I have, I found the number of toothpicks.

After explaining the relationship verbally, they translated this verbal statement into an equation which was the same as the participant G constructed.

The other 14 respondents preferred a tabular representation, and translated this tabular representation to a symbolic mode as illustrated in one of the sample table from the participant G.



The above sample table was constructed by 14 students who had almost the identical features. It has two columns, one represents the number of the figures, and the other one represents the number of toothpicks. Some of the students filled the table till the fourth figure, some of them continued with the table till they found the regarding equation. After the equation was found, the students put (n+2) into the equation then find the correct answer. Two of the students explained this process as follows;

M: When you said a relationship, I firstly think a path from a table to an equation. I created a table, then I looked at this table, I tried to interpret the numbers in the table. I searched the common things among the numbers, and I catch a *pattern. After this I translate this pattern to an equation, and finally I ended with an answer.*

R: When I am searching a relationship in a table I try several relationships like "multiply 4, and add 3", or "divide 2". I try until one of them fits. Then I write "table relationship" in symbols, equation I mean.

Of all representation modes, a translation from a tabular to symbolic mode of representation was appeared to be most preferred one by the students. The interesting thing can be that, the students' representation preferences varied according to the given figure for this question. In the previous question, the given figure in which growing squares were placed was simple. It involved only one type of geometric figure which was a square. However, in this question a figure was perceived a little bit complicated by most of the students. It included a house combining squares and one triangle, therefore the complex feature of the figure might be a reason for students to have a tendency for constructing a table to solve the question. In addition to the table, verbal explanations for the given algebraic situations were also used by the participants.

In addition to the students from experimental classes, three students from control classes were interviewed. The same knowledge questions were posed to those students within the same interview situation with the participants from experimental classes. The researcher chose the most hardworking students from control classes for the interview. It was observed that the participants from control classes had difficulties in solving the interview questions. Particularly the last two questions were found very hard among the participants. An important point was noticed that this group of participants tried none of the different representational modes except symbolic mode for solving interview questions. Even constructing a table was the easiest way to solve the given question, the participants insisted on trying to formulate equations. Besides they generally attempted to try the arithmetic procedure firstly. They did not recognize the algebraic pattern for the third question, for instance. When they could not reach the solution, they changed their strategy and attempted to use trial-and-error method for almost all questions. One of the participants from control classes mentioned that;

X: I could not understand these problems. Are they algebraic or arithmetic? I think arithmetic because no equation was given to us, so it is not an algebraic question, so I choose to use trial-and-error method.

It was remarkable that students from experimental groups used variety of representational modes whereas control group' students utilized only symbolic mode of representation for solving knowledge questions.

The participants were also responsible to explain the reasons for their representation preferences in the interviews and in the RPI by responding the question of;

Which type of representation you choose usually, and why?

This question sought to understand the reasons of their representation preferences, and the factors that have an influence on those preferences.

There were four groups of responses,

1. participants prefer equation for all question types,
2. participants prefer table for all question types,
3. participants prefer graph for all question types,
4. participants preferences depend on the question type.

Participants A, D, F, I, K, L, M, O, and P were included in the first group. They said that they always prefer equation for solving algebra questions. They explained their reasons, like;

It gives me a pleasure (participant A).

It is easy (participant D).

There is less risk of failure than other methods (participant F).

It is practical to use (participant I).

I like to construct and solve an equation (participant K).

It takes less time than the other methods (participant L).

It is challenging (participant s O).

It is certain, and more mathematical (participant M).

It makes sense to me (participant O).

It is mysterious with its all unknowns (participant L).

It gives me enthusiasm (participant K).

It arouses my curiosity (participant A).

It don't bother me, it is easy (participant D).

When the above responses were looked over, it is amazing that the reason of students' tendencies for symbolic mode of representation was mostly emotional. They like to deal with equations, and felt pleasure from them. They were also found using an equation more mathematical and certain.

There were four respondents contained in second group. Tabular representation was preferred by them for all types of questions. They preferred using a table for solving algebraic problems since it is a visual vehicle for them. All the numbers are given in a table, so it is easy to discover the relationship between them, and also in a table organized information was given.

The third group has only one member who was respondent U. He mentioned that he prefers graphical mode of representation to solve the problems that were similar to the problems in activity sheets. He explained his reason as:

U: I know drawing a graph was difficult and sometimes time consuming; however in a graph you can see the previous and further numbers. I mean that in a graph besides the answer, many numbers were located so you can interpret the situation more meaningfully.

Participants B, C, H, J, N, S, and V were from the fourth group. Their preferences changes accordingly with the type of question. They were open to use any representation mode if the question requires. Respondents J and V said that knowing and using many ways to solve questions are better than knowing only one way. Participant N and S claimed that, if they were given with a table they certainly use this table to reach the result, but if there were no table and they have to construct it for reaching a solution, they did not prefer to construct rather they continued with symbolic mode of representation. On the other hand participants B, C, and H preferences depend on the given numbers in the question. If the numbers were small, in fact less than 20, they were willing to make a table including the numbers. They further explained that if the numbers are getting larger, their preferences change in favor of symbolic mode of representation since they said that dealing with large numbers was easy in an equation.

Furthermore, participants S and N stated that they could prefer tabular, graphical and symbolic mode of representations depending on the situation. If they were dealing with questions including unknowns like x, y, and z they preferred symbolic mode of representations since an equation means unknowns for them. If finding a relationship between the numbers was asked in a question, they said that their representation preference would be tabular, because the sequence of numbers can be easily seen so the relationship can be directly found. And the graphical mode of representation was preferred by them if a question asks about any increase or decrease in a given mathematical situation. They indicated that sometimes they used one representational mode to verify other mode of representation. When they were having a trouble in symbolic mode of representation, they preferred to construct a table of the equation to verify the symbolic representational mode.

The results from the interviews were supported by the responses of students on the PRERPI and POSTRPI. Although the EG and CG students' representation preferences were changed after the treatment and conventional teaching, the reasons of using representations were generally similar, and they varied according to the question and the way of student's perceptions of representations. Most of the students wrote that if a table, a graph, and an equation were given as a way to solution in a question, they would use firstly the table since it was the easiest way. All the numbers can be seen in the table, and the correct response can be detected from those numbers. Few students marked graphical representational modes, according to them the most important reason was graphs are complicated representational modes, they were afraid of being failed while dealing with the graphs. For the questions involving letters and large numbers like 100 or 500, most of the students preferred to use symbolic mode of representation. As a reason, they wrote that to construct a table until the number 100 is impossible, so they should construct an equation, or they clarified that if there is a question involving letters, such as x and y, an equation should be constructed. According to her, tables and graphs only involve numbers, not letters. Majority of the CG students indicated that the symbolic mode of representation was the most mathematical one. They wrote that, the other modes of representations can not be used as an alternative for this mode since symbolic mode is certain, correct, and easy to use.

5.6 Impressions of Experimental Classes

During the treatment in both experimental groups, the researcher kept a daily journal and the two preservice mathematics teachers of the researcher took classroom notes. The two presevice teachers were informed about the role of multiple representations in mathematics education, the activities in treatment, the role of the researcher in this treatment, and their responsibility in the study, before the treatment. The preservice teachers helped in writing notes about what went on during the treatment. The researcher also kept a daily journal after class. Notes were based on what the students said and did during treatment.

From the notes of preservice teachers, it can be observed that they found the treatment very beneficial for the students. They have some common interpretations about the experimental classrooms` environment. First of all, they wrote that all activities were very appealing to the students, and all of them were interested in them.

According to the preservice teachers, the reasons for the students` interest were; the activities made them active participants in the mathematical thinking process, they generated and discussed the questions about the meaning of an equation or graph, they also found relationships as they were studying on the activity sheets and established connections among each other`s thoughts, and also between mathematical ideas and their outside experiences. He added that the activities were so interesting to the students that they were studying on the activity sheets and discussing the representation types even during the breaks, and he wrote that sometimes students did not want to have a break for their mathematics course since they wanted to complete the current activity and take the subsequent one.

One of the preservice teacher mentioned that the most surprising thing in the experimental classrooms was the high achievers did not get bored at all. He believed that this is due to the activities. In some activities, the students were allowed to pass another activity if they complete the previous one. Therefore, high and low achievers can work on the different activities at the same time and this was observed as a learning opportunity for students by this preservice teacher.

Both observers agreed that the most difficult task for students was making translations from an equation to a sentence. They said that some of the students asked for their help about this translation. One of the observers noticed that students were not accustomed to using nonconventional symbols in an equation. He illustrated that if an equation was given with x`s and y`s, solving of this equation was not an obstacle for the students; however if it included variables called as a`s and b`s or *`s and %`s they got confused since they have never used such letters and symbols in lieu of x and y. Moreover, one of the observers remarked that at the beginning of the treatment students stucked on one representaion mode to solve the problems. After sometime they began using different modes of representations, and for solving problems either at the blackboard or on activity sheets when they encountered any trouble with the representation mode thay preferred, they were able to use another representational mode to support the previous one. He wrote that by this way the students noticed the superiorities of some representational modes and used different modes of representations as a complementary to each other.

5.7 Impressions of Control Classes

The researcher who attended all the sessions of both control classes kept daily journals during the sessions. In control classess, in addition to giving a conventional instruction, some points deserve to be mentioned here. By a coincidence, the way of teaching algebra was almost same in these two control classes. The focus was given only on the procedural knowledge in teaching. Both teachers tried to teach the algebra concept by first introducing the exercise or problem, and then the whole unit was built on this exercise. For instance, for the topic of `First degree equations with one unknown` without discussing the meaning of an equation they immediately jumped on the solving the equations part since this part is the only important part to be taught for these teachers. And during the entire topic they only gave the solving equations part. They introduced highly challenging exercises and problems about equation solving to the students; however, as long as the researcher was observing there were no interpretation about the solving procedure of the equations. The students were forced to memorize the techniques that can be used with certain type of questions. Rote memorization in solving equations and drawing graphs were the main emphasis.

Furthermore, the teachers of the control classes did not try to use representational modes to make connections among algebraic concepts. They only used tabular representation for drawing graphs easily, and equational mode for presenting graphs. Any other kind of utilization of representations or any backward translations were not used in the control classes.

From the researcher point of view, a unique observation should be mentioned here. After one month from the treatment in experimental classes, the researcher administered another instrument which was about proportional reasoning to experimental and control classes in School A. Administering this instrument was not related to the purpose of this study, it was a part of another research, however; the surprising point was students in experimental class and control class have attempted to use totally different approaches for solving problems about ratio and proportion. Control class` students tried traditional methods, such as; setting cross multiplication algorithm between given numbers; however, experimental class` students used different representations for approaching given problems. They tried to apply what they learned in their algebra classes to ratio and proportion content. When their solving strategies were examined by the researcher, some of the students preferred to form a table and tried to find out a proportional relation between numbers within the table instead of using only cross multiplication algorithm. Besides, a few students tried to create their own representations and used these representations to solve the questions. Examining the papers of experimental class` students made the researcher notice that the utilization of multiple representations in other mathematics contexts was a consequence of the treatment.

5.8. Summary of the Results

On the foundation of the analyses of both quantitative and qualitative data, the following summary of results can be driven.

5.8.1 Summary of Descriptive Statistics

The descriptive statistics including sample size, mean, standard deviation, minimum and maximum scores, skewness and kurtosis reported the demographics of the sample. According to the results related to all of the instruments, the EG had the higher scores comparing the CG, and also from the instrument PREATMS both groups got the highest mean score.

5.8.2 Summary of the Inferential Statistics

Statistical analyses of quantitative data concerning performance in algebra, attitudes towards mathematics, and representation preferences were summarized in this part in relation to each of the hypothesis guiding the study.

- There was a significant effect of two methods of teaching on the population means of the collective dependent variables of seventh grade students' scores on the AAT, TRST, POSTCDAT, and POSTATMS after controlling their age, the MGPS, PRECDAT scores, and PREATMS scores.
- There was a significant effect of two methods of teaching on the population means of the seventh grade students' scores on the AAT, after controlling their age, the MGPS, the PRECDAT scores, and the PREATMS scores.
- There was a significant effect of two methods of teaching on the population means of the seventh grade students' scores on the TRST, after controlling their age, the MGPS, the PRECDAT scores, and the PREATMS scores.
- There was a significant effect of two methods of teaching on the population means of the seventh grade students' scores on the POSTCDAT, after controlling their age, the MGPS, the PRECDAT scores, and the PREATMS scores.
- There was no significant effect of two methods of teaching on the population means of the seventh grade students' scores on the POSTATMS, after controlling their age, the MGPS, the PRECDAT scores, and the PREATMS scores.
- The EG students had a tendency to use mostly symbolic mode of representations, however; their representation preferences were varied after the treatment.
- The CG students representation preferences were varied before the conventional teaching, however, they generally changed their representation preferences in favor of symbolic mode of representation after the conventional teaching.

5.8.3 Summary of the Qualitative Data

The interview was used to determine how students used different representational modes when they solve algebraic problems, and what are the reasons of their representation preferences. For the aim of the interview, four questions were directed to the students. Their responses shed light onto the participants' perceptions of multiple representations in algebra.

Summing up the interview results briefly, it can be argued that students understood from the treatment that there are various representation types to approach problems. Knowing different types of representations and relationships between them were very helpful for solving the problems for the students. They also realized that they understood better if they know how to do the problems with more than one method.

Some of the students preferred the same representation mode for all questions, whereas the representation preferences of some participants varied depending on the type of questions. Students' representation preferences vary with respect to the question type, their perception of the representational mode. For instance; if a question includes complicated figures, the students preferred to construct a table, and then to create an equation with respect to the table. For choosing symbolic mode of representation, the major reason was being certain. For tabular mode of representation, the reason can be written as being easy and available to see all numbers, and lastly for graphical mode of representation, the students argued that they can see an increase and decrease in the numbers if they use graphs. In addition to these reasons, students indicated that, they could use the representational mode they like to deal with, even if it is not the most convenient representational mode for the given question.

CHAPTER 6

DISCUSSION, CONCLUSIONS, AND IMPLICATIONS

In this chapter, the results of this study, in the light of the theoretical and empirical findings obtained from the research literature relating with the multiple representations in algebra were discussed. The implications of the findings of the present study, the internal and external validity of the present research design, and the suggestions for future research about multiple representations in algebra were given in Chapter 6. This chapter ties together research questions and literature presented in Chapter 1, 2 and 3, with the methods and results presented in Chapter 4 and 5.

6.1 Discussion

The purposes of this research study were to examine the effects of a treatment based on multiple representations on seventh grade students' performance in algebra, attitude towards mathematics, and representation preference. Furthermore, it was aimed to reveal how students use multiple representations in algebraic situations, and to investigate the representation preferences of the students before and after the unit of instruction and to examine the reasons of preferring certain kinds of representations. The discussion part was presented with respect to the each variable in this study.

6.1.1 Students' Performance in Algebra

The effect of the treatment based on multiple representations was investigated by the first research question which was a quantitative one. By posing this question, the focus was given on finding a significant difference between the experimental and control groups in terms of the scores on the algebra achievement test (AAT), translations among representational skills test (TRST), and Chelsea diagnostic algebra test (CDAT).

The results revealed that, 37% of the total variance of MANCOVA model for the collective dependent variables of the AAT, TRST, and the POSCDAT was explained by group membership of the participants. Therefore, the results of this study were of practical significance. Before the treatment, power of this study was calculated as .97, and after the treatment MANCOVA analysis was calculated this value as 1.00, which was higher than the preset value. According to the follow-up analyses, it was found that the performance of experimental group students on each instrument was significantly and uniquely influenced by multiple representations-based treatment after accounting the effect of each instrument on the other instruments.

This research study has documented that multiple representations-based instruction versus the conventional instruction did make a significant influence on the algebra performance of students. The results of this study are supported in the literature by numerous studies. One of them is Brenner`s (1995) and her colleagues study. In this study, they conducted only 20 days multiple representations unit including variables and algebraic problem solving. After their treatment they implemented four instruments related to algebra learning to the seventh and eight graders. Although the treatment took short period of time, significant difference was found between experimental and control group of students in favor of the students in experimental groups.

There might be various reasons to result in positive influences of multiple representations-based instruction on students' algebra performance. Visualization in algebraic objects, connections between algebraic ideas, the significance of translational abilities in algebra problem solving (Lesh, Post, & Behr, 1987b) can be counted as what multiple representations-based instruction provide for students. By the help of this instruction, students avoid memorization in algebra learning, and understand concepts meaningfully. As suggested in Swafford and Langrall`s (2000) study; multiple representations-based instruction promotes conceptual algebra understanding and makes students conceptualize algebraic objects. The findings of this study is also consistent with the findings of previous studies (Özgün-Koca, 2001; Pitts, 2003) that provided evidence for the effectiveness of multiple representations-based instruction in engaging students in meaningful algebra learning.

In the treatment, particularly the translations among representational modes were stressed and valued by the researcher. In conventional algebra teaching, translation among representations might occur only when the students are required to draw a graph. In this case, instead of constructing a table to represent the given equation, they only identified two points where the line passess through. Then, by the help of this information, a graph could be drawn. However, the multiple representatios-based instruction emphasizes the translations from variety of representational modes to the other modes. Therefore, students could have the opportunity to notice that one mathematical concept can be represented in several ways and these ways can be complementary to understand this concept. The same task of drawing the graph of a linear equation is taken in a way that, students analyze the equation through daily life situations, plain language, tables, and graphs. In that sense, drawing the graph of an equation is not an end but it is a means of interpreting the existing mathematical situation.

According to Smith (2004a), students should learn the conventional representational modes to improve their mathematical reasoning, however; idiosynractic representations which are specific to certain problems and belong to the individual, should also be valued in the mathematics classrooms in order to establish a conceptual link between these modes of representations and conventional ones. Additionally, the results from the literature review (Cifarelli, 1998; Herman, 2002; Martinez, 2001) in conjuction with the results of this study indicate that the use of multiple representations gives the opportunity of making translations among representational modes. Herman (2002) aimed to investigate students` use of representations to solve algebra problems with the help of graphic calculators. For this aim, the data was collected from 38 college algebra students. When the students` responses to the pre and post tests were examined, it was found that students' translations ability from different modes of representations to the conventional representations was improved after the multiple representations-based instruction. This result agrees with the findings of the current study.

From the classroom observations, it can be implied that after introducing multiple representations-based instruction, students were better able to establish connections between variety of representational modes. Particulary, the translations from the symbolic mode of representation to the tabular or graphical modes of representations were beneficial for students to see that the symbolic mode of representation is not the only end point of algebra tasks. Generally, students perceived that equations are the last achieving point, and all other representational modes can only be used to reach this representational mode. However, the experimental group students noticed the fact that equation is one of the most common representational modes, like graphs or tables.

Another argument from the treatment was that multiple representations-based instruction made students better problem solvers, since they were dealing with the thematic activities and every activity had a problem situation. To handle this problematic situations, students used multiple representations, such as they made a translation from daily life situation like cutting a string to construct a tabular representation. To further refute the argument, it was claimed in the literature that flexible use of multiple representations and making translations among multiple representations enable students better problem solvers (diSessa & Sherin, 2000; Girard, 2002; Lesh, Post, & Behr, 1987b; Yerushalmy & Gilead, 1997). Students in this treatment commented that when they were encountered different modes of representations they felt comfortable in using them. This research supports the view that to enhance mathematical understanding, connections within and between multiple representational modes should be established (Even, 1998; Lesh, Post, & Behr, 1987b). In the algebra courses, generally the translation from an equational mode to a graphical mode was overemphasized. However, in this treatment not only this translation emphasized, but also translations among other representational modes were supported.

Furthermore, the results of this study were in alignment with the theoretical views of multiple representation-based instruction on some articles (Brenner, et al. 1997; Elliot, 1996; Frielander & Tabach, 2001; Janvier, 1987b; Klein, 2003; Lapp, 1999; Laughbaum, 2003; Lesh, Post, & Behr, 1987b; NCTM, 2000; Pape & Tchoshanov, 2001; van Dyke & Craine, 1997; Vergnaud, 1998). Klein (2003) argued that multiple representations promote students` recall and understanding of the material. If a student is provided with a text and an accompanying graphic, he or she uses the graphic to organize the necessary information and text to learn the details. Eventually, to have and effective instruction, multiple representations should be used in complementary purposes, rather than contradictory. In general, multiple representations of mathematical concepts are perceived as vehicles to communicate powerfully, to learn meaningfully, and result in conceptual understanding in the literature (Kamii, Kirkland, & Lewis, 2001) and this view is supported in the present study also.

6.1.2 Students' Attitudes towards Mathematics

As for the attitudes of the experimental and control group students towards mathematics, no significant difference was found between groups. The mean score of the experimental group students' attitudes towards mathematics before the treatment was 56.61 and after the treatment this value was increased to only 56.64. For control group students, their mean score from the pre implementation of attitude towards mathematics scale was 54.75, and the mean score of the post implementation was 53.72. Since the mean scores of both group students were already high before the treatment, no significant difference was found between the experimental and control groups' mean scores.

As the related literature implied, the improvement of positive mathematics attitudes is in relation with the active involvement of the students in the multiple representations-based activities (Diezmann & English, 2001; Monk, 2004). The increase in the experimental group students' attitudes and the decrease in the control group students' attitudes were negligible. This was not a surprising finding, because the students' attitudes towards mathematics were already very high before the treatment. Therefore, having a sample of students who enjoyed mathematics very much revealed no significant effect of the treatment on attitudes towards mathematics.

6.1.3 Representation Preferences of the Students

Students' preferences for representations were examined by posing second research question and the reasons behind their representation preferences were investigated by the fourth qualitative question.

From the percentage data of the representation preferences, it can be mentioned that before the treatment, the equations were the most preferable mode of representation. 70.1% of the experimental groups and 64.6% of the control groups preferred to use equations to solve the problems on the representation preferences inventory. The tabular representation mode was also popular among students before the treatment. For the experimental group, the usage of tabular representation mode was 19.4%, for the control group it was 26.2%. 1.5% of the experimental group students and 7.7% of the control group students preferred to use graphical mode of representation before the treatment.

On the other hand, after the treatment, the percentages for preferring representational modes were changed. Numerous students from the experimental groups still preferred to use equation mode for solving problems (50.7%). For control groups` students, this percentage was found to be quite high (87%) compared to the experimental groups. Tabular representation mode was still preferable among students. The percentages of students preferring to use tabular approach were 36.3% and 8.5% for experimental and control groups respectively. Graphical representation mode was rarely selected as an algebra problem solving technique. For experimental group, it was 11.9%, and for control group it was only 6.8%. Hence, out of three representation modes, students preferred to use predominantly algebraic and tabular approaches after the treatment.

It can also be argued that, for experimental groups` students tendency to use symbolic mode of representation was decreased slightly after the treatment, whereas for them, using the other two modes of representation was also appealing. This can be an evidence for claiming that students` tendency to use representational modes was changed after the multiple representation-based treatment since they encountered situations in which other representation modes can be implemented and valued. Although, using symbolic mode of representation was one of the aims in mathematics (Kaput, 1994) and making students remark the necessity of using this mode of representation in mathematical reasoning and other disciplines (Smith, 2004a), this treatment made students notice the other mode of representations in addition to the symbolic mode, and use the most convenient mode of representation among all the representational modes.

From the interview results, it can be understood that almost all of the students in experimental groups met equations somehow, before the treatment. Therefore these students were familiar with the symbolic mode of representations, and that's why the percentages of preferring symbolic mode was higher than the percentages after the treatment. During the treatment, those students encountered several representational modes, in addition to the symbolic mode, and they solved algebra questions by using different representational modes rather than the symbolic mode, and this case made them alter their preferences. After having known all modes of representations, the experimental group students decided on their representation preferences not by thinking their familiarity of the representational mode, but the convenience of the representational mode for the specific question.

On the other hand, the case in control group students was totally different and confusing since before the conventional teaching the control group students were willing to use the graphical and tabular modes of representations in addition to the symbolic mode. After the conventional teaching, this case was changed. The students decided to prefer symbolic mode dominantly. According to the observations conducted in control groups, this situation was expected because of the conventional teaching method of the mathematics teachers of control groups. Both teachers were teaching the unit of equations and line graphs by using solely symbolic mode of representations. Two teachers had almost the same technique to teach this unit. In introducing the unit, they both divided the blackboard into several parts and wrote the questions that require the solutions of equations, proceeding from simple to difficult ones. They called students to the blackboard for solving these questions. If a student failed to solve the given question correctly, the teacher solved and explained it. In this manner, teachers covered a lot of questions about solving equations and drawing the graphs of equations. The symbolic mode was dominantly used in these classrooms, so it altered students tendencies towards symbolic mode of representation.

The reasons behind the preferences for representations of students were identified by interview questions and the representations preference inventory (RPI). During interviews and on the RPI, students were asked to provide reasons for their preference of representations. The results of the interview scripts revealed that, there were four group of students in terms of their use of representations. Specifically these groups were the students who preferred, (1) to use algebraic mode of representation for all questions, (2) to use table for all questions, (3) to use graph for all questions, and (4) the preferences for representations changed according to the type of question. The students who preferred to use mostly symbolic mode of representation explained their reasons. They thought that symbolic mode of representation is more mathematical than other modes; therefore knowing how to construct an equation and solve it should take the priority over tables and graphs. The reasons for preferring tabular mode of representation were explained generally like, it was visual, easy to discover the relationship between numbers, and also more organized than the other representation modes. There was only one student favoring the use of graphical mode of representation. He mentioned that he knew drawing a graph is difficult and time consuming but he could see the previous and further numbers in graphs, hence it seemed to him more sensible and meaningful to him. For the students in the last group, they pointed out that they were open to use any kinds of representations that a question requires. They said that some problems lend themselves to certain modes of representations. They noticed that some representation modes are more efficient than others in given situations.

Many students mentioned that being familiar with equational mode of representation and knowing the algorithmic process of this representation takes precedence over using tables and graphs. After the treatment, students became comfortable using other modes of representations; they preferred to use tables and graphs also. Some thought that a table was more accurate than an equation; others said that the table was unnecessary; it can only be used as a vehicle to reach the equation.

As a conclusion, the reason why one representation is preferred over another for solving algebra problems involves the perception of students about the representations and what is mathematically sound, the nature of a given problem, the belief about the level of accuracy that a certain mode of representation can produce a solution for the problem, and also whether or not they enjoy to use it.

The findings related to representation preferences of the students are in line with Keller and Hirch (1998) study. In their study they focused on students\` representation preferences and the reasons for choosing certain type of representations. They found that students\` representation preferences were influenced by the nature of the problem. In the regular calculus section students preferred mostly symbolic mode of representation to solve calculus problem, whereas in the calculus section in which graphing calculators are utilized the common representational mode was graphical type of representation. This result has resemblance with one of the result of this dissertation that says; after the treatment, experimental group students had more tendency to use different representational modes rather than the symbolic mode for solving algebra problems.

Additional examples of studies lending support for the findings in Chapter 5 that students prefer different modes of representations after they have introduced with the utilization of multiple representations (Knuth, 2000; Özgün-Koca, 1998).

However, Boulton-Lewis (1998) and Ainsworth and Peevers (2003) reported conflicting results. Boulton-Lewis' study, involves use of manipulatives and his aim was to examine childrens` representations of mathematical symbols and operations with manipulatives. He conducted interviews with the students from first, second, and third grades. In contrast to many of the studies in the literature, the results of this study revealed that for young children it is better to introduce only one type of representation at one time, after they internalize this external representation and the concept it refers, the young children can be given another type of representation. Ainsworth and Peevers (2003) found that students preferred to use single mode of representation, especially; the varbal mode for solving physics and mathematics problems. However; using only one mode of representation was associated with slower performance by the researchers. The conflicting evidence to the results found in the literature review and to the results presented in Chapter 5 stimulates the need for further research studies.

6.1.4 Students' Utilization of Multiple Representations in Algebra

Using of multiple representations in algebraic situations by the students was examined with the help of the third research question that required a qualitative investigation through semi-structured interviews. The aim of the interviews was to understand how students use multiple representations when they solve problems relaed to algebra.

21 students from two experimental classes and 4 students from one control classes were interviewed for this purpose. First, they were asked three questions about whether or not they are taking tutoring and they are attending any private courses besides the school, and about their first algebra experience. The results of these questions revealed that, most of the students had private support for mathematics lessons, and they experienced algebraic expressions before the treatment.

Afterwards, the researcher posed four questions including algebra word problems. Students were asked to solve all of the questions by using representations. According to the students` responses, they use different representations according to the algebraic questions. Their ways of using different representations varied in terms of the nature of the problems and their perception of the representations. For instance, for some the questions involving large numbers or complicated figures, tabular and symbolic mode of representations were the most popular modes of representations. However, students' level of involvement in certain mode of representation influenced the way of using it for solving algebra questions. If they are familiar with a graphical mode of representation, they try to use it, or if they enjoy using tabular mode of representation, they use it for every question.

Lastly, which type of representations the participants want to use and what kind of reasons they have for their representation preferences were asked to them.

Most of the students mentioned that in some of the problems using certain mode of representation is efficient than other modes. Therefore they make their preference according to this fact, since they know the superiorities and shortcomings of the representational modes.

The results of the qualitative part were aligning with the results of Hines (2002) who investigated one student's experiences with linear functions. He argued that students could use variety of representational modes in dealing with algebraic concepts. The results of Herman's (2002) study were also supportive for this dissertation. She argued that, students' representational preferences can be varied according to the content of the problem, students' perceptions of the representations, and being familiar with the certain type of representations.

Along with the above results, the qualitative part of this research contributes an additional component to the conceptual framework presented in Chapter 2. In the theory of multiple representations, several factors influencing students` representation preferences were listed. These factors were presented in the study of Özgün-Koca (1998) as following;

Table 6.1 Reasons for students` preferences for representations

Internal Effects	Personal preferences Previous experience Previous knowledge Beliefs about mathematics Rote learning
External Effects	Presentation of problem Problem itself Sequential mathematics curriculum Dominance of algebraic representation in teaching Technology and graphing utilities

These all stand out important factors determining the reasons of students` choices for certain representational modes. However according to numerous comments from interview subjects and students in experimental groups, one factor can be added to the above list. From the interview scripts and informal communication with the students during the treatment, an emotional factor can be contributed to the factors that affect students` representation preference. It was observed that if students like to practice one of representation modes, whether or not using this type of representation is suitable for this problem situation, they use it. In interviews, one of the students said that;

G: I know using table is time consuming here and I love to use tables, I like to put numbers in it, so I am going to use it.

This emotional factor might be coming from the fact that; as students practiced certain type of representation and recognized the benefits of it, they seem hardly willing to use another type of representation over the one they are familiar with. However, there were some cases in which a student insisted on using graphs although he performed not well on the items of TRST related with graphs. Therefore, it was recognized that students` appreciation and enjoying of the type of representation has an influence on choosing a representation mode for solving algebra problems.

6.2. Validity Issues

Possible threats to the internal and external validity of this study and ways of controlling and minimizing them were discussed in this part.

6.2.1 Internal Validity

Internal validity refers to how well the research was conducted and how confidently the researcher can make conclusions that the variance in the dependent variable was gathered only by the indenpendent variable (Campell & Stanley, 1966). The procedure for minimizing and controlling possible threats to the internal validity of the study was discussed here.

In this study, students were not randomly assigned to the experimental and control groups. This can cause the subject characteristics threat to the study. The characteristics -mathematics grade in previous semester, previous algebra achievement, previous mathematics attitude, gender, and age- that could potentially influence the outcomes of the study were specified as potential extraneous variables to posttests. To determine any relation between the extraneous variables and dependent variables, they were put as a covariate set in MANCOVA analysis to match the student on these variables. Possible subject characteristics were minimized and group equivalency was satisfied by this statistical remedy.

The treatment was conducted by the researcher in two experimental groups. This is another threat to the internal validity since the characteristics, teaching ability, motivation of the researcher and also biases toward the treatment of her might have an influence on the students' performance and attitude. For reducing implementation effect, the researcher tried to be unbiased during the treatment. In addition to the researcher, two elementary mathematics preservice teachers in one class and the classroom teacher in another class were present for observing the lessons and the behaviors of the researcher.

It was thought that outcomes of this study could be influenced by Hawthorne effect which was not under control in this study. However, since the duration of treatment was eight weeks, any Hawthorne effect that might be caused by the novel teaching method can be reduced.

For controlling data collector characteristics and data collector bias, during the administration of the instruments both classroom teachers and the researcher were present at the classrooms. Therefore, this threat was not viable.

Testing procedure in classes was relatively uniform; this might reduce history effect; however, classroom interactions before testing that might influence testing scores were unknown.

Location threat was reduced by satisfying similar situations in two schools. The location was four similar seventh grade mathematics classess. Furthermore, no outside events affecting participants' responses were notified during the administration of the instruments.

Since all subjects were pretested before the treatment, this might be another threat to internal validity. Pretesting subjects in a research study might make them to react more or sometimes less strongly to the treatment than they would have had they did not take the pretest (Isaac & Michael, 1971). For minimizing the effect of this threat, both groups were pretested beforehand. Besides, there was a sufficient time period for posttests for subjects to desensitize the pretest. To partial out the effect of pretesting statistically, the pretests were put in MANCOVA analysis as a covariate.

Mortality means dropping out of the research study (Crocker & Algina, 1986). There were no missing data in the pretests and posttests. All of the subjects attended the treatment regularly, in a few lessons, some of the subjects did not join the class due to the bad weather conditions.

Another possible threat can be considered as maturation which means natural changes in subjects as a result of the normal passage of the time period (Campell & Stanley, 1966). However this was not a threat for this study since the length of the treatment was eight week and also both groups had the same amount of time period. If any maturation was occurred in subjects during the treatment, it affected both groups.

To sum up, possible threats to internal validity were taken into consideration and the researcher tried to reduce the impact of those threats.

6.2.2 External validity

The accessible population of the study was the seventh grade students in Çankaya district in Ankara. The participants of this study were the seventh grade students of two schools from this district. The use of a nonrandom sample of convenience limits the generalizability of this research study for the population' external validity. However, the results presented in this study can be applied to a broader population of samples having similar characteristics with the sample of this study.

Treatment and testing were conducted in regular classroom settings during the regular lesson hours. Although the research study was conducted in four different classes from two different schools, the conditions of the schools were quite similar. Besides, the conditions in all of the classes were more or less the same, the size of the classes were around 30, the sitting arrangements and the lighting were equal in four classrooms; therefore, the threats to the ecological validity were also controlled.

6.3 Implications for Practice

Effective mathematics instruction needs more than lecturing. It reqiires active involvement of the students in mathematical learning process. Multiple representations-based instruction meets this need in mathematics classroom. Using multiple representations in mathematical contexts and giving opportunities including manipulating representations to the students can be accounted as reform-oriented attempts (Battista, 1999; Boaler, 1998; NCTM, 2000). However, it was noted by Monk (2004) that the aim should be to teach students to use multiple representations in a particular mathematical context and to use variety of representations at the same time, rather than to use only one representation for all situations.

In mathematics classrooms, teachers are responsible for designing constructivist situations and concrete connections for students so that scaffolding of knowledge can be achieved. Teachers should also encourage students to think about connections between multiple representations. Laughbaum (2003) claims that teachers should spend some time of the mathematics lesson on the relationships between manipulative and abstract symbols of algebra. According to Stylianou and Kaput (2002), the lack of mathematical understanding comes from not being able to make connections between different representational modes of mathematical concepts and processes.

In interviews students agreed the idea that they like to be engaged in all kinds of representations for solving algebra problems. Therefore, teachers should emphasize applications of multiple representations.

In a very simple way, establishing relationships between representational modes can be conducted in part of a daily lesson by making students to think about any situation that represents a mathematical object. It can be a daily-life situation, a table, or a poet. Afterwards, students can be provided opportunities to discuss the similarities and differences of variety of representations, as Herman (2002) suggested so that students can recognize relationships between different modes of representations and appreciate the superiorities and disadvantages of some kind of representational modes over others. As it can be understood, discussion about representations should be an inevitable part of mathematics lessons.

This study confirmed the need for considering other kinds of representations, such as; representations used in graphic calculator and computer programs or representations that students create and unique for them. As it was suggested by Özgün-Koca (2001), computer-based applications can be used to provide linked and semi-linked representations, and graphical form of representations. These applications can make students to abstract mathematical concepts from virtual world. Besides, allowing students to create their own representations for solving algebra problems makes them more creative and flexible in mathematics (Piez & Voxman, 1997). In this study it was observed that, students were mainly restricted by four types of representations which are tabular, graphical, algebraic, and verbal. This can be due to the activities or researcher`s emphasize on those representation types. However, students should be given an opportunity that they can use representations that they invent or create.

Moreover, using multiple representations in teaching of mathematics should be emphasized in preservice teacher education programs, as well as in in-service teacher education seminars. In preservice mathematics teachers' education programs, there are method courses in which the ways of instructing mathematics are introduced. These courses can be modified considering the need for multiple representations in mathematics in order to make preservice students familiar with the concept of multiple representations and remark the importance of this concept in mathematics teaching. By this way, the perservice teachers would be capable of preparing their lesson plans including multiple representations in mathematics. In addition to the method courses, the multiple representations of concepts should be emphasized in mathematics courses of preservice teachers as well. Particularly, the use of computer technology can provide promising opportunities for the different representations of mathematical concepts. In this sense, preservice teachers would better see the benefits of multiple representations-based instruction while they are learning mathematics.

One further implication can be suggested for the mathematics textbooks and other teaching materials. The mathematics textbooks for elementary students are lacking connections among representational modes of mathematical concepts. Generally, the only representational mode in the mathematics textbooks for the unit of equations and line graphs is symbolic mode (Yıldırım, 2001). It will be beneficial to include other representational modes in addition to the symbolic mode. Textbooks should not consider these representational modes as separated topics, but should give a clear attention to the translations and relationships among them. This attempt will help students to understand the mathematics by making connections between daily life situations and symbolic mathematical representations or verbal statements of the problem and tabular representation of it.

The usage of multiple representations should also be valued in the new mathematics curriculum due to its various advantages. Multiple representations-based instruction should be implemented in various topic in mathematics curriculum, such as fractions, geometry and equations. The involvement of multiple representations in mathematics curriculum will accordingly make teachers give more importance to multiple representations in instruction.

6.4 Theoretical Implications

This study confirmed Lesh and Janvier Multiple Representations Theory which were explained in detail in Chapter 2. The theory of multiple representations based on the premise that students learn the mathematical concepts and build new concepts making a meaningful relationship between the previous ones, only by dealing with variety of representational modes of the concepts, and communicating with these modes of representations. Though it was not the intentional to look for evidence to support the theories considering multiple representations, it is consistent with the learning experiences in the current study.

Janvier (1987a) suggests that conceptual learning occurs when a student makes a relationship her everyday situations, concrete and abstract representations of a mathematical concept. Lesh, Post, & Behr (1987b) describes how this learning can happen in a process of multiple representations in which students make different relationships among modes of representations. It was possible to observe this process throughout this research study, as experimental group students made translations within and between representational modes in algebra unit. The findings of this study confirm Lesh` theory that multiple representations of mathematical objects are crucial for learning process of students.

The current study does not agree with the idea that students might be confused when they are provided with more than one representation. If translations among representation modes are established, they can develop deeper understanding and more likely to use different representation modes for solving one problem, instead of being lost in variety of representations. This view is supported by the participants in Herman (2002) study. They said that recognizing multiple representations for solving problems made them better decision makers about which mode they were most comfortable, and they also claimed that they would change the representation mode when one is not working, and they can approach in many ways to solve problems.

6.5 Recommendations for Further Research

Generally, having known what students gathered from multiple representation-based instruction in this research study suggests some ideas for further research studies in algebra classrooms. In this manner, future research can focus on teachers and teaching strategies in algebra classrooms. All of the data for this study was collected from students. Future research could combine data from students and their teachers, because teachers have also impact on shaping students' representation preferences. What teaching strategies and representation types are used within algebra classrooms by teachers and how those representations are conceptualized by the students seems to be worthwhile to study. Some students during the interviews claimed that they prefer to use equational mode of representation to solve algebra problems because it seems to them more mathematical and they were taught with more emphasis on this kind of representation. Such study examining the reasons of that belief and the degree of teacher effects on that belief would be a deeper level of investigation after this study.

Multiple representation-based instruction can totally be replicated in small groups since as students were dealing with representational modes; they need to discuss their thought with others. Small group works would give them this opportunity. Besides, the replications of this study can be conducted with a random sample so that the results could be generalized over a wider population.

Another way of looking closer at the understanding in the theory of multiple representations would be qualitative case studies. As Goldin (2004) suggested examining only students` interaction with external representations can be insufficient to conceptualize the entire process of learning in multiple representations. There is a need to analyze the `inside` of the children's mind, their creative processes in representational contexts, and internal representations of students. In this study interviews were conducted in a qualitative data collection purpose; however, there is a need for a structured task based interviews to answer the question of how students create new representations, how they use the representational modes in problem solving, how they demonstrate these representations in a mathematical situations. In my opinion, it would be interesting to examine more closely students behaviors when they are dealing with multiple representations in mathematics.

Further studies could also be conducted beyond the algebra courses. Multiple representation-based approach can be implemented to every topic in mathematics. Algebra was chosen for this study, however it is strongly recommended to use this instruction method in geometry and fractions since these two topics are available to use hands-on materials and visualization.

This study was carried out in an eight week period. Further research can apply multiple representations-based approach for longer periods, and incorporating at least two units; such as; fractions and geometry in sixth grade or algebra and ratio and proportion in seventh grade. If the treatment would last longer and if it includes not only one topic, a better chance to gain evidence on students` mathematical learning in multiple representations environment could be catched.

Possible studies in this area could look more closely on gender issues in representation preferences. It was beyond the aim of this study to investigate the gender differences in students` preferences regarding to representational modes. However, it would be interesting to find out evidence like girls are more likely to prefer visual representations than boys.

Another recommendation for further research comes to the surface when the researcher was implementing the multiple representations-based approach in seventh graders. Giving importance on activities including multiple representations in primary schools would be beneficial to understand students` early representations for mathematical concepts before they learn any conventional mathematical representations. Therefore, research studies can be conducted in early grades about multiple representations.

In this study during the treatment, students were provided activity sheets by which they were in a way guided to use representational modes. For instance, they were presented a table in which a relationship among numbers was hidden. They were asked to translate this table into an equation, and then in a daily life situation. It might be worthy to investigate students` behavior without providing any guidance. It would also be beneficial to observe what kind of representations students invent and use when they encounter a table and are asked to represent this table in another form without giving directions or leading them to use certain representations.

To assess and describe the depth of understanding of algebraic concepts attained by the students, special attention should be paid to the instrument selection. In this study instruments including open-ended format was found more effective in assessing and characterizing the algebraic understanding of students than multiple-choice format. It might be recommended to enrich the format of the instruments by combining two or more item formats in one instrument.

Finally, from the treatment it can be claimed that multiple representations-based instruction could make a substantial differences in the ways that students understand algebraic concepts. However it remains an unanswered question that what long term advantages this type of instruction could provide. It is hoped that further research will be able to respond this question.

6.6 Final Thought

Several reform-oriented efforts in the field of mathematics education gave importance on students` conceptual understanding in mathematics and encouraging the utilization of many techniques to support this understanding (Greeno, 2004; Sfard, 2004). If it is mainly the area of learning in algebra, it can be seen that the reform attempts are presented to use manipulative materials, concrete embodiments, and computer modeling for encouraging conceptual understanding in algebra and multiple representations of algebraic concepts for establishing connection among these concepts (Shore, 1999; Smith, 1994). According to the researcher, mathematics educators ought to recognize to make connections between concepts for the mathematics instruction for all students. Nowadays, many attempts can be observed to improve mathematics instruction. Multiple representations-based instruction for conceptual algebra understanding is just the one that the researcher implemented and appreciated the benefits of using this method. Giving opportunity to new instructional methods like multiple representations-based instruction in mathematics classrooms makes students better mathematics learner. As Klein (2003) implied;

`Learning to create and interpret representations using specific media such as texts, graphics, and even videotapes are themselves curricular goals for many teachers and students` (p. 49). As a two-year experienced mathematics teacher before, the researcher could say that in traditional mathematics classroom, there is a need to encourage students to think more deeply on mathematical concepts, to intrinsically motivate for learning, to make students appreciate the nature of mathematics by getting rid of rote memorization, and to avoid overemphasizing mathematical rules and algorithms. In fact, new instructional methodologies like multiple representations-based instruction might address this need.

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APPENDIX A

ALGEBRA ACHIEVEMENT TEST

Adı Soyadı:

Sevgili öğrenci;
Bu test denklemler ve doğru grafikleri ünitesi ile ilgili 10 sorudan oluşmaktadır. Bu testten alacağınız puan, üçüncü yazılı notunuzun bir kısmını oluşturmaktadır. Lütfen tüm soruları cevaplamaya çalışınız. Sınav süresi 40 dakikadır.

Başarılar….

SORULAR

1) 12x = 4(x+5)   ise   x = ?

2) a = 3 ise $\frac{(5a+3)}{(4a-3)}$ = ?

3) $\frac{2(2x-1)}{3}+\frac{(x-2)}{2}=\frac{1}{6}$ denkleminin çözüm kümesini bulunuz?

4) Bir dikdörtgenin uzunluğu, genişliğinden 1 cm. daha fazladır. Dikdörtgenin çevresi 26 cm. olduğuna göre; uzunluğu ne kadardır?

5) Aklımdaki bir sayıdan 7 çıkarıp, sonucu dörde bölüp, 13 eklersem 20 elde ediyorum. Aklımdaki sayı kaçtır?

6) Aşağıdaki noktaların hepsini aynı koordinat düzleminde (x-y ekseninde) gösteriniz. Noktaları birleştirip ortaya çıkan geometrik şeklin adını yazınız.
A(0,3)   B(-6,3)   C(0,-6)   D(-6,-6)

7) x+2 = y doğrusunun üzerinde olan A(0,?) noktasının ordinatını bulunuz.

8) 3x+4y = 24 doğrusunun grafiğini çiziniz.

9) $x \in Z^{+}$ ise 12x-10 < 6x+32 denklemini sağlayan x değerlerinin toplamı nedir?

10) $-3u+\frac{2}{3}\geq 4(u-1)$ eşitsizliğinin çözüm kümesini bulunuz.

APPENDIX B

TRANSLATIONS AMONG REPRESENTATIONS SKILL TEST

Adı Soyadı:

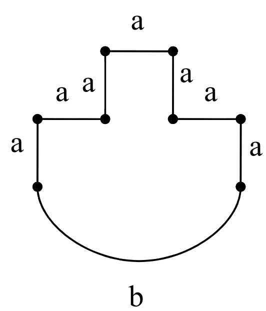
Sevgili öğrenci;
Bu test denklemler ve doğru grafikleri ünitesi ile ilgili 15 sorudan oluşmaktadır. Bazı sorular bir ya da birkaç alt soru içermektedir. Bu testten alacağınız puan, üçüncü yazılı notunuzun bir kısmını oluşturmaktadır. Lütfen tüm soruları cevaplamaya çalışınız.
Sınav süresi 60 dakikadır.
Başarılar….

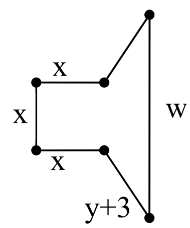
SORULAR

1) Aşağıdaki ifadeleri yanıtlayınız.
(a) Tufan, Umut'tan 12 cm. uzundur. Umut *h* cm. boyunda ise Tufan'ın boyu ....cm'dir.
(b) Mert'in ağırlığı Cem'den 5 kg. azdır. Cem y kilo ağırlığında ise, Mert'in kilosu....................'dır.
(c) Serpil'de Esra'nın 2 katı kadar pul vardır. Esra'nın n tane pulu varsa, Serpil'in;...........kadar pulu vardır.

2) s ve t birer sayı olmak üzere, s t'den 8 fazladır. s ile t arasındaki ilişkiyi gösteren bir denklem yazınız.

3) Aşağıdaki şekillerin çevrelerini yazınız.





Çevre = 　　　　　　　　　　　　　　　Çevre =

4) x+9 = 23 denklemini ifade edecek bir problem yazınız.

5) x = 3 iken 17'ye eşit olan bir denklem yazınız.

6) Aşağıda boyu eninden 4 cm. daha uzun olan dikdörtgenler ile ilgili bir tablo verilmiştir. Tablodaki eksik bilgileri doldurunuz.

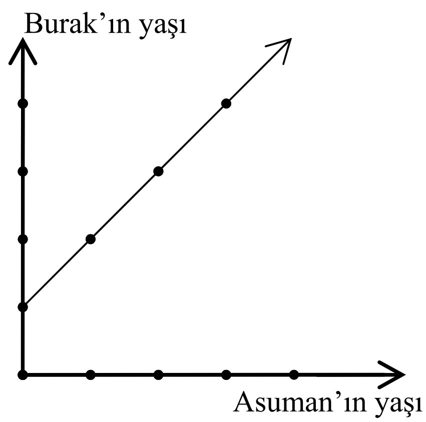
En (cm.)	**Boy (cm.)**	**Çevre (cm.)**
1		
5		
	12,4	
		56
a		
	2a	

7)

s	1	2	3	4	5
m	4	7	10	13	16

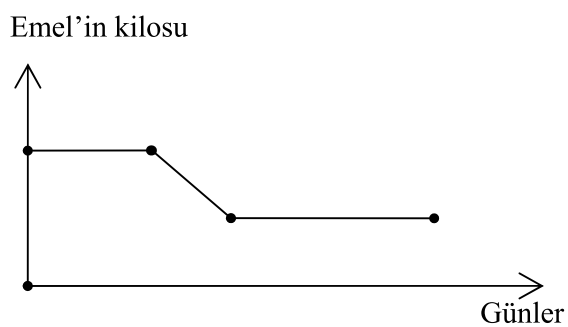
Yukarıdaki tabloda verilen sayıların tümünü düşünerek, m'yi s'den nasıl elde edebileceğimizin formülünü yazınız. 　　　　　m =

8) Aşağıda grafiği verilen ilişkinin tablosunu oluşturunuz.



Asuman	Burak

9) Aşağıdaki grafik Emel'in kilosunun 12 gün içinde nasıl değiştiğini göstermektedir. Bu grafiğe göre, Emel'in kilosunun zamana göre değişimini gösteren tabloyu doldurunuz.

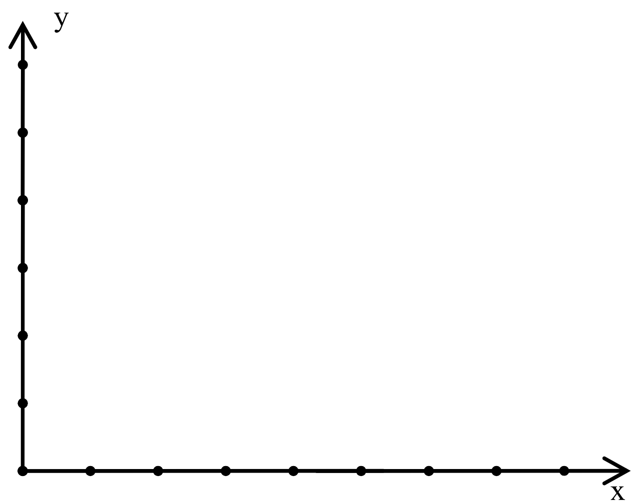


Günler	1	2	3	4	5	6	7	8	9	10	11	12
Emel'in Kilosu	133											130

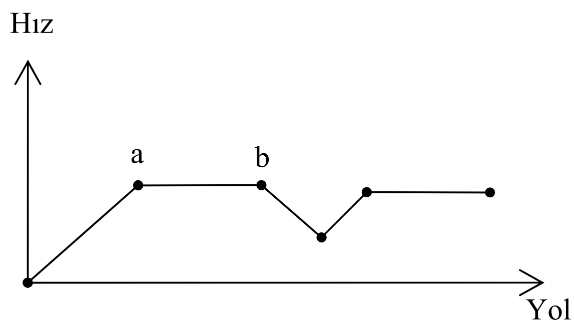
10)

p	0	1	2	3	4	5
r	2	5	8	11	14	17

a) Tablodaki (p,r) ikililerini koordinat ekseninde birer nokta olarak yazınız.
b) p, x-ekseninde r de y-ekseninde yer almak üzere tablodaki sayıların grafiğini çiziniz.



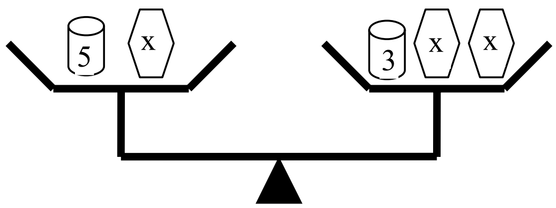
11) Aşağıdaki grafik Umut'un okuldan eve kadar süren yürüyüşünü göstermektedir. Buna göre; *a-b* aralığında Umut'un yürüyüşünün nasıl olabileceğini yazınız.



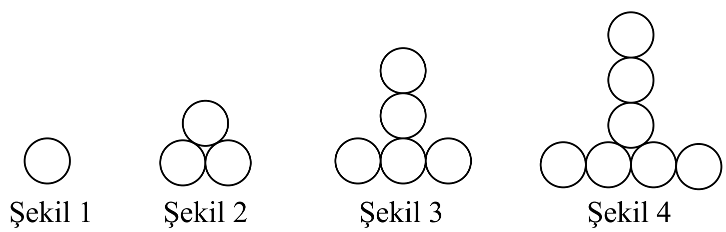
12) 3x-4 = y denklemini sağlayan x ve y sayılarından bazılarını aşağıdaki tabloya yazınız.

x					
y					

13) Aşağıda sol ve sağ kefesinde eşit ağırlık bulunduğundan dengede olan bir terazi modeli görülmektedir. Eğer bu terazi dengede ise, x nedir? (Denklem kurarak bulunuz.)



14)



Yukarıdaki şekilleri inceleyiniz. 100. şekilde kaç tane top olacaktır? (Yanıtı bulmak için yaptığınız tüm işlemleri aşağıda gösteriniz). (Sayarak bulmaya çalışmayın)

15) Gökalp ansiklopedi satarak haftada 16 milyon TL. kazanmakta ve haftalığına ek olarak sattığı her ansiklopedi için 5 milyon TL. daha almaktadır. Buna göre;

a) Bir haftada 3 tane ansiklopedi satan Gökalp ne kadar para kazanır?

b) Gökalp iki haftada n tane ansiklopedi satarsa kaç TL. kazanır? Açıklayınız.

APPENDIX C

CHELSEA DIAGNOSTIC ALGEBRA TEST

Adı Soyadı:

Sevgili öğrenci;
Bu test genel cebir konularını kapsayan 22 sorudan oluşmuştur. Bazı sorular bir ya da birkaç alt soru içermektedir. Bu testten alacağınız puan, üçüncü yazılı notunuzun bir kısmını oluşturmaktadır. Lütfen tüm soruları cevaplamaya çalışınız.
Sınav süresi 60 dakikadır.
Başarılar….

KAVRAMSAL CEBİR TESTİ

1) Belirtilenlere göre aşağıdaki boşlukları doldurunuz.

a) *x* → (*x+2*)

6 → …..
r → …..

b) *x* → (*4x*)

3 → ......

2) Aşağıdakilerden en küçük ve en büyük olanı yazınız

	en küçük	en büyük
n+1, *n+4*, *n-3*, *n*, *n-7*	....	....

3) Hangisi daha büyüktür, *2n* ya da *n+2* ?
Yanıtınızı açıklayınız:...

4) a) n'ye 4 eklendiğinde ***"n+4"*** olarak yazılır. Aşağıdaki ifadelerin her birine 4 ekleyiniz.

8	*n+5*	*3n*
....	....	....

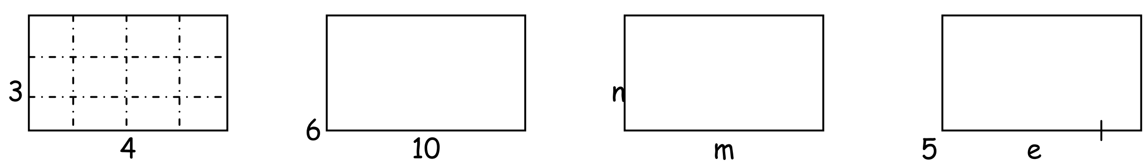
b) n 4 ile çarpıldığında ***"4n"*** olarak yazılır. Aşağıdaki ifadelerin her birini 4 ile çarpınız.

8	*n+5*	*3n*
....	....	....

5) a + b = 43 ise a + b + 2 = ...
n - 246 = 762 ise n - 247 = ...
e + f = g ise e + f + g = ...

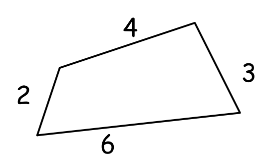
6) a +5 = 8 ise a nedir? ....
b + 2, 2b'ye eşit ise b nedir?.........................

7) Aşağıdaki şekillerin alanı nedir?

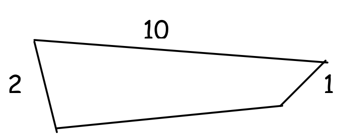


Alan = ... Alan = ... Alan = ... Alan= ....

8) Yandaki şeklin çevresi, 6+3+4+2 = 15'tir.

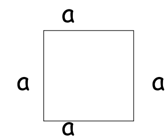


Buna göre, aşağıdaki şeklin çevresi nedir?



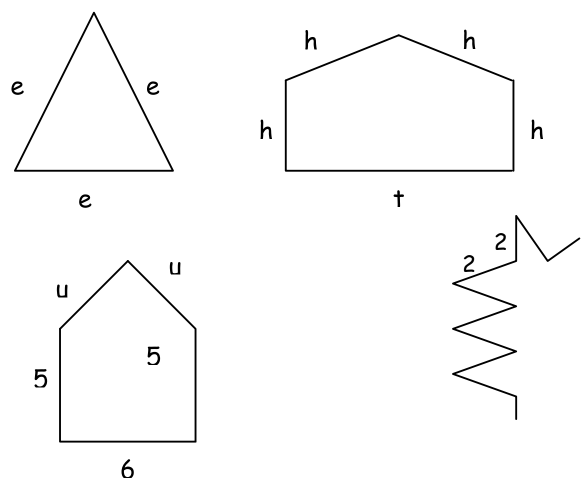
Çevre = ...

9) Yandaki karenin kenar uzunluğu g birimdir.
Bu karenin çevresi, Ç = 4a olarak gösterilir.



Buna göre, aşağıdaki şekillerin çevrelerini nasıl yazarız?

2



Bir kısmı çizilmeyen yandaki şeklin toplam n kenarı vardır ve herbir kenar uzunluğu 2cm'dir.

10) Kırtasiyede satılan bilgisayar dergilerinin tanesi 8, müzik dergilerinin tanesi 6 milyon liradır.
b harfi satın alınan bilgisayar dergilerinin sayısını,
m harfi de müzik dergilerinin sayısını gösteriyorsa;

8b+6m neyi göstermektedir? ....
Toplam kaç tane dergi alınmıştır?...

11) Eğer u = v+3 ve v = 1 ise, u = ? ....

Eğer m = 3n+1 ve n = 4 ise, m = ? ....

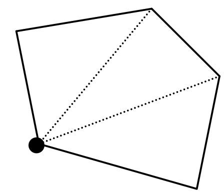
12) Eğer Özlem'in Ö, Atakan'ın da A kadar misketi varsa, ikisinin sahip olduğu toplam misket miktarını nasıl yazarsınız?...

13) a+3a ifadesi sade haliyle 4a olarak yazılır.
Buna göre; aşağıdaki ifadeleri yazılabiliyor ise sade halleriyle yazınız.

2a + 5a = ...	
2a + 5b = ...	3a - (b + a) = ...
(a + b) + a = ...	a + 4 + a - 4 = ...
2a + 5b + a = ...	3a – b + a = ...
(a - b) + b = ...	(a + b) + (a - b) = ...

14) Eğer r = s + t ve r + s + t = 30 ise r = ...

15) Yandaki gibi bir şekilde köşegen sayısı
kenar sayısından 3 çıkarılarak bulunabilir.
Buna göre; 5 kenarlı bir şeklin 2 köşegeni vardır.
57 kenarlı bir şeklin ....köşegeni vardır.
k kenarlı bir şeklin ....köşegeni vardır.



16) Eğer c + d = 10 ve c, d'den küçük ise c = ...

17) Ahmet'in haftalık kazancı 20 milyon liradır ve fazla mesai yaptığı her saat başına 2 milyon lira daha almaktadır.
Eğer s harfi yapılan fazla mesai saatini ve k harfi de Ahmet'in toplam kazancını gösteriyorsa; s ile k arasındaki ilişkiyi gösteren bir denklem yazınız:...
Eğer Ahmet 4 saat fazla mesai yaparsa, toplam kazancı ne olur?..

18) Aşağıdaki ifadeler ne zaman doğrudur? Her zaman, Asla, Bazen?
Doğru yanıtın altını çiziniz. Yanıtınız "Bazen" ise ne zaman olduğunu açıklayınız.

A+B+C = C+A+B Her zaman Asla Bazen, ...

L+M+N = L+P+N Her zaman Asla Bazen, ...

19) a = b + 3 iken b 2 artırıldığında a ne olur?...

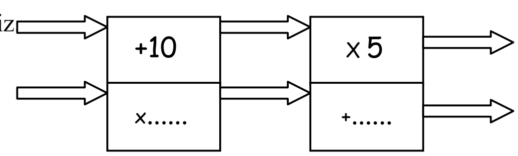
f= 3g + 1iken g 2 artırıldığında f ne olur?...

20) Isırgan büfede kekler k liraya, börekler b liraya satılmaktadır.
Eğer 4 kek ve 3 börek alırsam,
4k + 3b ifadesi ne anlama gelir? ....

21) Kırtasiyede satılan mavi kalemlerin her biri 5, kırmızı kalemlerin her biri 6 milyon liradır. Biraz mavi ve kırmızı kalem alırsam, toplam 90 milyon lira ödüyorum.
Eğer m alınan mavi kalem sayısını,
k alınan kırmızı kalem sayısını gösteriyorsa,
m ve k hakkında ne yazabilirsiniz?..

22)
Yandaki makineyi herhangi bir sayı ile besleyebilirsiniz

Aynı etkiye sahip başka bir makine bulabilir misiniz?



APPENDIX D

ATTITUDE TOWARD MATHEMATICS SCALE

Sevgili öğrenci, bu ölçek sizin matematik dersine yönelik düşüncelerinizi öğrenmek için hazırlanmıştır. Ölçekte belirtilen ifadelerden hiçbirinin kesin cevabı yoktur. Her ifadeyle ilgili görüş, kişiden kişiye değişebilir. Bunun için vereceğiniz yanıtlar kendi görüşünüzü yansıtmalıdır. Her ifadeyle ilgili düşüncenizi yazmadan önce, o ifadeyi dikkatlice okuyunuz, sonra ifadede belirtilen düşüncenin, sizin düşünce ve duygunuza ne derecede uygun olduğuna aşağıda belirtilen derecelendirmeyi düşünerek karar veriniz.

Hiç katılmıyorsanız, Hiç Uygun Değildir
Katılmıyorsanız, Uygun Değildir,
Kararsız iseniz, Kararsızım
Kısmen katılıyorsanız, Uygundur
Tamamen katılıyorsanız, Tamamen Uygundur seçeneğini
İşaretleyiniz.

Ad Soyad:______________________ Cinsiyet:________ Sınıf:____

	Tamamen Uygundur	Uygundur	Kararsızım	Uygun Değildir	Hiç uygun Değildir
1. Matematik sevdiğim bir derstir.					
2. Matematik dersine girerken büyük bir sıkıntı duyarım.					
3. Matematik dersi olmasa öğrencilik hayatı daha zevkli olurdu.					
4. Arkadaşlarımla matematik tartışmaktan zevk alırım.					
5. Matematiğe ayrılan ders saatlerinin fazla olmasını dilerim.					
6. Matematik dersi çalışırken canım sıkılır.					
7. Matematik dersi benim için angaryadır.					
8. Matematikten hoşlanırım.					
9. Matematik dersinde zaman geçmek bilmez.					
10. Matematik dersi sınavından çekinirim.					
11. Matematik benim için ilgi çekicidir.					
12. Matematik bütün dersler içinde en korktuğum derstir.					
13. Yıllarca matematik okusam bıkmam.					
14. Diğer derslere göre matematiği daha çok severek çalışırım.					
15. Matematik beni huzursuz eder.					
16. Matematik beni ürkütür.					
17. Matematik dersi eğlenceli bir derstir.					
18. Matematik dersinde neşe duyarım.					
19. Derslerin içinde en sevimsizi matematiktir.					
20. Çalışma zamanımın çoğunu matematiğe ayırmak isterim.					

APPENDIX E

REPRESENTATIONS PREFERENCE INVENTORY

Adı Soyadı:

Sevgili öğrenci;
Bu ölçek senin hangi tip gösterim biçimini tercih ettiğini belirleme amacı ile hazırlanmıştır. Ölçekte verilen soruları çözmeden, çözümü yapmak için hangi tip gösterim biçimini seçeceğini belirtmen ve bu seçiminin nedenini belirtmen gerekmektedir. Lütfen her soru için uygun olan nedeni mutlaka belirtin.

Teşekkür ederim..

Asuman bir şekerci dükkanında kasiyer olarak çalışmaktadır. Şu an 42 milyon lirası olan Asuman, çalıştığı her saat için 7 milyon lira daha kazanmaktadır. Bu durum tablo, grafik ve denklem kullanılarak aşağıda belirtilmiştir.

Gösterim Biçimleri

<table>
<tr><th>TABLO</th><th>GRAFİK</th><th>DENKLEM</th></tr>
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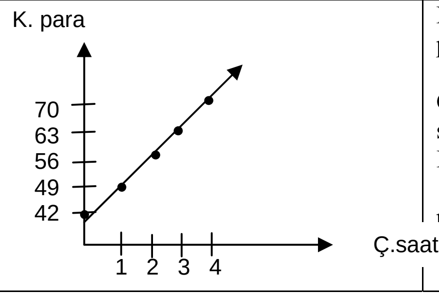

</td>
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Çalışma saati=s
İse;

p= 42+7s</td>
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1) "Asuman'ın 3 saat sonunda kazandığı para nedir?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

2) "Asuman'ın 6 saat sonunda kazandığı para nedir?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

Türker bilgisayar almak için para biriktirmektedir. Birinci gün kumbarasına 1,5 milyon TL. atan Türker, hergün aynı miktardaki parayı kumbarasına koymaktadır. Bu durum tablo, grafik ve denklem kullanılarak aşağıda belirtilmiştir.

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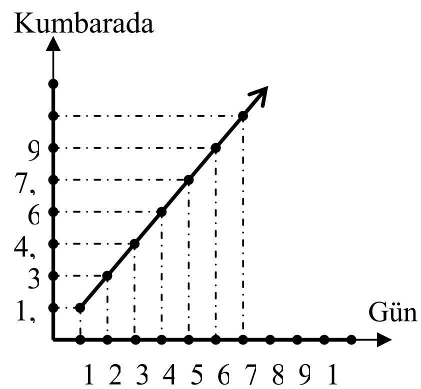

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Gün = g
İse;

P = 1,5.g</td>
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Gösterim Biçimleri

3) "Türker'in kumbarasında 5. günün sonunda kaç lirası olur?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

□ Tablo □ Grafik □ Denklem □ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

4) "Türker'in kumbarasında 30. günün sonunda kaç lirası vardır?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

□ Tablo □ Grafik □ Denklem □ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

Küçük bir kasabada saati 2 milyon liraya bisiklet kiralayan bir şirket vardır. Bu şirketin saat hesabı karşılığında kazandığı para miktarı; tablo, grafik ve denklem kullanılarak aşağıda belirtilmiştir.

Gösterim Biçimleri

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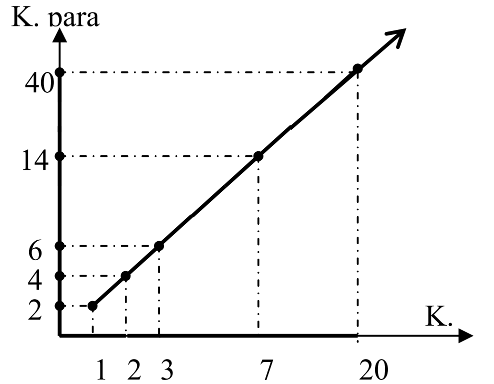

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Kiralama saati=s
İse;

p= 2s</td>
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5) “7 saat bisiklet kiralayan biri ne kadar ücret öder?” Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

6) “100 saat bisiklet kiralayan biri ne kadar ücret öder?“ Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

7) “x saat bisiklet kiralayan biri ne kadar ücret öder?“ Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

Ferhan 195 milyon lira ödeyerek bir ağaç budama makinesi satın alırlar. Her ağacı 5 milyon liraya budamaya karar veren Ferhan'ın kazandığı para miktarı; tablo, grafik ve denklem kullanılarak aşağıda belirtilmiştir.

Gösterim Biçimleri

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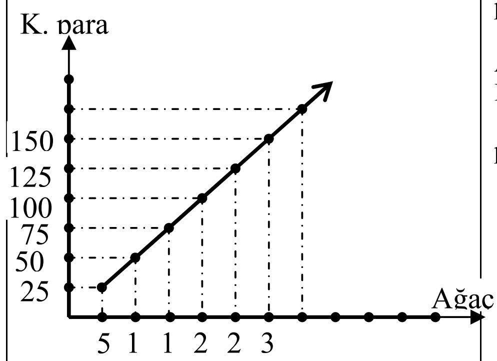

</td>
<td>Kazanılan para=p

Ağaç sayısı=a
İse;

p= 5a</td>
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8) "Ferhan'ın bu işi yaparak kara geçmesi için en az ne kadar ağaç budaması gerekir?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

9) "35 ağaç budandıktan sonra Ferhan'ın kazandığı para ne kadardır?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

10) "Ferhan'ın 500 milyon lira kazanabilmesi için kaç adet ağaç budaması gerekir?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

☐ Tablo ☐ Grafik ☐ Denklem ☐ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

Bir sinema sahibi film arasında sık sık mısır yiyen izleyiciler için iki farklı ödeme planı önermiştir. İlki, izleyicinin 3 milyon lira ödeyip her mısır kutusu başına 500 bin lira daha ödemesi, ikincisi ise izleyicinin 2 milyon lira ödeyip her mısır kutusu başına 750 bin lira daha ödemesidir.

11) "Birinci plana göre 3 kutu mısır alan bir kişinin ödediği para nedir?" Bu sorunun yanıtını bulmak için aşağıda belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz?

□ Tablo □ Grafik □ Denklem □ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

12) "İkinci plana göre 6 kutu mısır alan bir kişinin ödediği para nedir?" Bu sorunun yanıtını bulmak için aşağıda belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz?

□ Tablo □ Grafik □ Denklem □ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

13) "Kaçıncı kutu mısırda bu iki planın aynı paraya denk geldiğini bulunuz?" Bu sorunun yanıtını bulmak için aşağıda belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz?

□ Tablo □ Grafik □ Denklem □ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

İki farklı DVD kiralama şirketi müşteri kazanmak için fiatlarına yeni bir uygulama getirmişlerdir. İlk firma, yıllık 5 milyon lira DVD kiralama ücreti alıp, her DVD başına 1 milyon lira almaktadır. İkinci şirket yıllık kira ücreti almayıp, her DVD başına 2 milyon lira kira almaktadır.

14) "İkinci şirketten 11 DVD kiralayan bir kişinin ne kadar para öder?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

□ Tablo □ Grafik □ Denklem □ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

15) "Kiralanan kaçıncı DVD'de iki şirkete de aynı para ödenir?" Bu sorunun yanıtını bulmak için belirtilen gösterim biçimlerinden hangisini kullanmayı tercih edersiniz? Neden?

□ Tablo □ Grafik □ Denklem □ Diğer yollar (Hangi yolu tercih ettiğinizi mutlaka belirtiniz.)

APPENDIX F

THE FREQUENCY AND PERCENTAGE OF EACH REPRESENTATION TYPE FOR TRANSLATIONAL AMONG REPRESENTATIONS SKILL TEST

Questions	Representation types	EG (n=65)	CG (n=66)
1	Verbal to algebraic	53 (80,3)	40 (61,5)
2	Verbal to algebraic	46 (69,7)	35 (53,8)
3	Diagrammatic to algebraic	43 (65,2)	37 (56,9)
4	Algebraic to real-world situations	42 (63,69)	26 (40,0)
5	Verbal to algebraic	42 (63,6)	14 (21,5)
6	Tabular to algebraic	14 (21,2)	24 (36,9)
7	Tabular to algebraic	35 (53,0)	11 (16,9)
8	Graphical to tabular	53 (80,3)	32 (49,2)
9	Graphical to tabular	29 (43,9)	19 (29,2)
10	Tabular to graphical	40 (60,6)	13 (20,0)
11	Verbal to graphical	44 (66,7)	18 (27,7)
12	Algebraic to tabular	43 (65,2)	21 (32,3)
13	Real-world situations to algebraic	28 (42,4)	28 (43,1)
14	All representational modes	29 (43,9)	9 (13,8)
15	All representational modes	29 (43,9)	19 (29,2)

APPENDIX G

TURKISH EXCERPTS FROM INTERVIEW WITH STUDENTS

- Çünkü üçüncü şekilden sonra, dördüncü gelir, böylece n 4 olur (Participant A, p. 129)
- Ben kareleri saydım. 6 tane var, n de yedinci kare olur, n yerine 7 koydum, çünkü n'i bilmiyorum bir sayı olmalı (Participant N, p. 129)
- Hayır yapamayız. Eğer n yerine bir sayı koymazsak, soruyu nasıl çözeceğiz? (Participant N, p. 129)
- Tabloda ihtiyacın olan her şey veriliyor, sayılar arasında bir ilişki bulmak için uygun bir araç. Ben denklem bulmak için tabloyu kullanıyorum (Participant M, p. 130)
- Geri dönen para, şişe miktarının 5 katı olmalı (Participant U, p. 131)
- Grafik yapmayı seviyorum. Grafiğin içinde artışı görebildiğim için soru çözerken en iyi yol grafik çizmek gibi geliyor bana. Bu grafiğe bakarak ta 100 şişe için ne kadar ödeneceğine kolaylikla karar verebilirsin (Participant G, p. 132).
- Aslında, hayır, çünkü x eksenine yazmaya 1'den başlamadım, 50'den başladım böylece 100'ü hemen grafikte gösterebilirim yani (Participant G, p. 132).
- Eğer elimizde bir denklem varsa, hata yapma riskimiz azalır. Denkleme bakarız, verilen sayıları denkleme koyarız ve sonucu buluruz. Bence denklem kullanmak daha iyi, daha güzel ve çok eğlendirici (Participant A, p. 132).
- Öncelikle elimde ne var net olarak görmek için tablo oluştururum, sonra bu tablodan denklemi yazarım. Yani denkleme geçmeden önce bir başlangıç olarak tabloyu kullanırım (Participant H, p. 132).
- İlişkiyi sözcüklerle yazdığım zaman okuyorum ve bu sözlerden denklemi çıkarıyorum (Participant U, p. 132).
- Önce bir denklem yazdım….Denklemi unutalım, önce tablo yapmalıyım sonra denklemi bulurum. Tablo bana çözüm yolunu gösterir (Participant L, p. 133).
- 100 ve 300 için orantı kullanamam, çünkü çok büyükler. En iyisi denklem yazmak böylece herhangi bir sayıyı koyup, doğru sonucu bulurum (Participant L, p. 133).
- Önce şöyle bir aklımdan düşünüyorum, sonra sayıları organize etmem gerektiğini fark ediyorum, tablo bunun için en iyi yol. Ayrıca ilişki bulmaya da yarar. Tablo varken elinde açıkça görürsüm, 3. şekle kadar ben hiç göremedim ilişkiyi ama 4. şekilde anladım ve denklemi yazdım (Participant I, p. 134).
- Burada bir denklem kurmaya çalıştım ama önden önce bir tablo yapmalıyım ki denkleme geçiş yapabileyim. Tabloda sayılar arasındaki ilişkileri araştırabiliyordum ve bu ilişkiyi denklem için kullandım (Participant V, p. 135).
- N için benim bir table yazmam lazım. Yedinci ve n inci şekilde kullanabilmek içic, tabloyu başta yazıyorum (Participant H, p. 135).
- Şekillerden şunu anlıyorum, bu büyüyen bir şekil ve içinde sayıların kareleri var, örneğin 1'in karesi 1, 2'nin karesi 4, 3'ün karesi 9, cevap 49 olmalı çünkü bu 7'nin karesi. N. Şekil için de n kare olmalı (Participant F, p. 135).
- Biri "n" derse ben şekillerin sonsuza gittiğini anlıyorum, yani sonu yok. Eğer bu durumu ifade etmek için denklem kullanırsam bu en uygun yol olur çünkü bir denkleme sayıları koyarsın, bu da sonsuz yapar. Yani eğer denklem varsa neden diğer şeyleri kullanarak vaktimi harcayayım ki (Participant E, p. 136).
- n'i 8 diye düşündüm ve cevabı 64 buldum (Participant C, p. 136).
- Aslında değil. Ama tabloda n için kare sayısını nasıl bulabilirim ki? n'yi bir sayıya çevirmek zorundayım (Participant C, p. 136).
- Çünkü 7'den sonra 8 gelir, o yüzden (Participant C, p. 136).
- Verilen şeyler arasında bir ilişki var. Şekil sayısı, kürdan sayısından daha az ve şekil sayılarını 3 ile çarparsam ve bir tane 3 daha eklersem, kürdan sayısını bulurum (Participant C, p. 138).
- İlişki deyince siz, tablodan denkleme bir yol düşündüm. Tablo oluşturdum, sonra bu tabloya baktım ve tablodaki sayıları yorumlamaya çalıştım. Bu sayılar arasındaki ortak şeylere baktım ve bir ilişki buldum, tekrarlı ilişki yani. Bundan sonra da bu ilişkiyi denkleme çevirdim ve sonunda da yanıtı buldum ben (Participant M, p. 138).
- Ben tablodan ilişki bulmaya çalışırken birçok şeyi deniyorum. Mesela, "4 ile çarp, 3 ekle" ya da "2'ye böl" gibi. Bir tanesi tutuyor sonra bu tablo ilişkisini sembolle yani denklem halinde yazıyorum (Participant R, p. 139).

- Bu problemi anlamadım ben. Sayısal işlemlerle mi yoksa denklemlerle mi falan çözülecek. Bence sayısal işlemlerle çözeceğim çünkü sorularda denklem verilmemiş yani denklem konusuyla ilgili değil, ben deneyerek bulacağım (Participant X, p. 139).
- Eğlenceli (Participant A, p. 140).
- Kolay (Participant D, p. 140).
- Diğer metotlara göre yanlış yapma riski az (Participant F, p. 140).
- Kullanımı kolay (Participant I, p. 140).
- Bir denklemi kurmayı ve çözmeyi çok seviyorum (Participant K, p 140).
- Diğerlerine göre daha az zaman alıyor (Participant L, p. 140).
- Zor şeyleri severim, bana zor geliyor (Participant O, p 140).
- Kesin, denklemler (Participant M, p. 140).
- Daha anlamlı geliyor (Participant O, p. 140).
- Tüm bilinmeyenleriyle gizemli bir şey (Participant L, p. 140).
- Heyecan veriyor bana (Participant K, p. 140).
- Merakımı artırıyor (Participant A, p. 140).
- Beni hiç üzmez, çok kolay (Participant D, p. 140).
- Grafik çizmenin zor ve bazen zaman kaybına yol açtığını biliyorum ama grafikte önceki ve sonraki sayıları görebilirsin. Demeye çalıştığım şu; sayının yanın da grafikte birçok sayı var yerleştirilmiş olarak, böylece matematiksel durumu daha anlamlı yorumlayabilirsiniz (Participant U, p. 141).

APPENDIX H

DESCRIPTIVE STATISTICS RELATED TO THE SCORES FROM PRECDAT, PREATMS, POSTCDAT, POSTATMS, TRST, and AAT FOR EXPERIMENTAL GROUP FROM SCHOOL A (EG1) AND EXPERIMENTAL GROUP FROM SCHOOL B (EG2)

Groups	Variable	N	Mean	SD	Min.	Max.	Skewness	Kurtosis
EG1	PRECDAT	28	28.36	6.97	13	40	-.773	.318
	PREATMS	28	57.46	11.58	33	72	-.384	-.999
	POSTCDAT	28	35.89	7.29	22	47	-.342	-.917
	POSTATMS	28	60.71	12.27	35	78	-.766	-.333
	TRST	28	32.61	7.64	10	42	-1.00	1.318
	AAT	28	32.21	4.74	25	40	.054	-1.121
EG2	PRECDAT	38	23.74	8.89	10	42	-.352	-.467
	PREATMS	38	56.03	14.83	12	77	.584	.753
	POSTCDAT	38	34.21	8.83	17	48	-.783	-.709
	POSTATMS	38	53.58	16.21	18	80	-.574	-.122
	TRST	38	28.53	9.61	6	41	-.616	-.473
	AAT	38	24.63	9.07	5	40	-.516	-.370

PRECDAT: Pretest of Chelsea Diagnostic Algebra Test
PREATMS: Pretest of Attitude towards Mathematics Scale
POSTCDAT: Posttest of Chelsea Diagnostic Algebra Test
POSTATMS: Posttest of Attitude towards Mathematics Scale
TRST: Translations among Representations Skill Test
AAT: Algebra Achievement Test

APPENDIX I

THE NUMBER OF THE ITEMS FROM THE AAT AND RELATED SEVENTH GRADE MATHEMATICS LESSON OBJECTIVES

Items	Related Seventh Grade Mathematics Lesson Objectives
1	Understanding the propositions, open propositions, and equations
2	Understanding mathematical expressions
3	Solving the first degree equations with one unknown
4	Solving the first degree equations with one unknown
5	Solving the first degree equations with one unknown
6	Understanding the coordinates of a point on a plane.
7	Understanding the coordinates of a point on a plane.
8	To be able to draw graphs
9	Solving the first degree inequalities with one unknown
10	Solving the first degree inequalities with one unknown

APPENDIX J

THE INSTRUCTIONAL DESIGN OF THE STUDY

Activity Names	Related Seventh Grade Mathematics Lesson Objectives	Additional Objectives	Type of Representational Translations	Required Lesson Hours
Verbal Statements	1	Use table to classify the information	Verbal to Symbolic Symbolic to Verbal	2
The Pattern of Houses	1, 2	Use table to classify the information Understand patterns in algebra	Manipulative to Table Table to Verbal Verbal to Symbolic	2
Cutting a String	1, 2	Use table to classify the information Understand patterns in algebra	Manipulative to Table Table to Verbal Verbal to Symbolic	2
Cinema Hall	1, 2, 3	Use table to classify the information Understand patterns in algebra	Table to Verbal Verbal to Symbolic	1
Ancient Theatre	1, 2, 3	Use table to classify the information Understand patterns in algebra Understand the relationship btw. table and equation	Drawing to Manipulatives Manipulatives to Table Table to Symbolic	1
Folding a Paper	1, 2	Use table to classify the information Understand patterns in algebra	Manipulative to Table to Verbal to symbolic	1
The Scale	2, 3	Understand the connection btw. the real-world materials and algebra	Manipulative to drawing to symbolic to verbal	2
Algebra Tiles	1, 2, 3	Understand the connection btw. the manipulatives and algebra	Manipulative to symbolic	1
Coordinate System	6	Understand the connection btw. the manipulatives and algebra	Manipulative to graph	1
A Journey to Planets	2, 3	Understand the concept and meaning of equations	Drawing to algebraic Table to verbal Table to graph to symbolic Verbal to table Graph to algebraic Algebraic to table Algebraic to graph	2
x-y	1, 2, 3, 6, 7	Understand the relation btw. table, graph, and equation	Table to algebraic to verbal to graph	1

The Temperature	1, 2, 6, 7	Understand the relation btw. table, graph, and equation	Table to algebraic to graph	1
12 Giant Man	1, 2, 6, 7	Collect data Understand the relation btw. table, graph, and equation	Real world situations to table to graph to symbolic	1
Bouncing a Ball	1, 6, 7	Collect data Understand the relation btw. table, graph, and equation	Real world situations to table to graph to symbolic	1
Saving Money	1, 2, 3, 6	Understand the relation btw. table, graph, and equation	All translations among representations are used. Students could create and use their own representations	2
Walking Tour	1, 2, 3, 6, 7	Understand the relation btw. table, graph, and equation Use table for analyzing data Interpret different graphs	Table to graph to verbal to symbolic	2
Geometric Figures	1, 2, 6, 7	Understand the relation btw. table, graph, and equation Use table for analyzing data Interpret different graphs	Drawing to manipulatives to table to symbolic	1
Making a Frame	1, 2, 6, 7	Understand the relation btw. table, graph, and equation Use table for analyzing data	Manipulative to table to verbal to symbolic	1
Verbal Problems	1, 3	Understand the connection btw. the verbal and mathematical statements Model the real-world situations mathematically	Real world situations to drawing Real world situations to symbolic Verbal to symbolic Symbolic to verbal	2
Inequalities	3, 4	Use table for analyzing data	Table to symbolic Symbolic to table Symbolic to real life situation	2
Story in Graphs	6, 7	Understand the relation btw. the graphs and verbal statements Interpreting the graphs	real life situation to graph verbal to graph graph to real world situations graph to verbal	2

APPENDIX K

TEACHING SCHEMA FOR THE EXPERIMENTAL GROUP IN SCHOOL A

Week Beginning	Content	Activities
December 3rd, 2003	Being familiar with the verbal statements of the algebraic concepts. Making translations from manipulative to table, verbal, and algebraic modes.	1, 2, and 3
December 10th, 2003	Making translations from table to verbal and algebraic modes. Making translations from manipulative to table, verbal, and algebraic modes.	4, 5, and 6
December 17th, 2003	Making translations from algebraic to verbal modes. Solving first degree equations with one unknown. Making translations from manipulative to algebraic modes.	7, 8, and 9
December 24th, 2003	Making translations among all types of representations. Making translations from algebraic to graphical modes.	10, 11, and 12.
December 31st, 2003	Making translations from real life situations to the other representation modes.	13 and 14.
January 7th, 2004	Students create and use their own representations. Making translations from graphical and tabular modes to the algebraic modes.	15, 16, and 17.
January 14th, 2004	Understanding the connection between the verbal and mathematical statements. Applying the real-world situations to mathematics	18 and 19.
January 21st, 2004	Understanding the relation btw. the graphs and verbal statements Interpreting the graphs Being able to use table to conceptualize inequality.	20 and 21.

APPENDIX I

TEACHING SCHEMA FOR THE EXPERIMENTAL GROUP IN SCHOOL B

Week Beginning	Content	Activities
December 8th, 2003	Being familiar with the verbal statements of the algebraic concepts. Making translations from manipulative to table, verbal, and algebraic modes.	1, 2, and 3 (last part of Activity 3 was given as a homework)
December 15th, 2003	Making translations from table to verbal and algebraic modes. Making translations from manipulative to table, verbal, and algebraic modes.	4, 5, and 6
December 22nd, 2003	Making translations from algebraic to verbal modes. Solving first degree equations with one unknown. Making translations from manipulative to algebraic modes.	7, 8, and 9
December 29th, 2003	Making translations among all types of representations. Making translations from algebraic to graphical modes.	10, 11, and 12.
January 5th, 2004	Making translations from real life situations to the other representation modes.	13 and 14.
January 12th, 2004	Students create and use their own representations. Making translations from graphical and tabular modes to the algebraic modes.	15, 16, and 17 (Last part of Activity 3 was given as homework).
January 19th, 2004	Understanding the connection between the verbal and mathematical statements. Applying the real-world situations to mathematics. Understanding the relation btw. the graphs and verbal statements Interpreting the graphs Being able to use table to conceptualize inequality.	18, 19, 20, and 21 (The last two activities were completed in extra hours).

APPENDIX M

LESSON PLANS

1.ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
Ek Hedefler: Bilgiyi düzenlemek için tablo kullanabilme.
Konu: Matematiksel İfadeler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem
Materyal, Araç-Gereç: Kartonlar
Etkinlik Süresi: 80 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri: Bu etkinlikte sadece sözelden cebirsel ifadeye ya da cebirsel ifadeden sözel ifadeye geçiş vardır.

	Sözel	Cebirsel
Sözel		**x**
Cebirsel	**x**	

İşleniş:

1) Aşağıdaki tabloyu tahtaya çiziniz (Not: Tablonun büyüklüğü sınıftaki öğrenci sayısına göre düzenlenmelidir).

Sözel İfadeler	Sembolik ifadeler

2) Öğrencilere önceden hazırlayacağınız üzerinde aşağıdaki tabloda verilenlere benzer matematiksel ve sözel ifadeler olan uzun kartonları, her öğrencide bir adet olacak şekilde dağıtınız. Dağıtma işlemi sırasında öğrencilerin bu ifadelerle basitten zora doğru karşılaşmaları için, hangi öğrenciye en kolay, hangi öğrenciye en zor kartonu verdiğinizi belirleyiniz.
3) Bu işlemden sonra sınıftan en kolay sözel ifadenin bulunduğu katrona sahip öğrenciyi kaldırıp, o öğrencinin kartonunda yazan ifadeyi tabloya yerleştirmesini sağlayınız. Bu ifadenin matematiksel olarak aynısına sahip olan diğer öğrencinin tahtaya gelerek tablodaki uygun yere kartını yerleştirmesini isteyiniz.
4) Bu işleme tablonun tamamı doldurulana kadar devam ediniz. Tamamlanmış bir tablo aşağıdaki gibi olabilir.

Sözel İfadeler	Sembolik ifadeler
Üçün beş fazlası	3+5
Onbeşin sekiz eksiği	15-8
Kırsekizin dörtte üçü	48x3/4
Bir sayının dört fazlası	*+4
Bir sayının dört katının iki fazlası	4#+2
Bir sayının yedi fazlasının yarısı	(x+7)/2
Kenar uzunluğu a olan bir karenin çevresi	4a
Kenar uzunluğu b olan bir eşkenar üçgenin çevresi	3b
Pi sayısının 2 eksiğinin yarısı	(π-2)/2
Uzun kenarı x, kısa kenarı y olan bir dikdörtgenin çevresi	2(x+y)

Bu etkinlikte öğretmenin bilinmeyen sayı yerine çeşitli harfler kullanıp, en son x ifadesini kullanmaya dikkat etmesi gerekir.
5) Etkinlikle matematiksel ifadelerin tanıtılmasından sonra, öğretmen önerme, değişken, bilinmeyen, sabit ve denklem kavramlarını tanıtıp, örneklendirir.

2. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
2. Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
Ek Hedefler: Öğrencilerin,
. örüntü gelişim anlayışlarını,
. bilgiyi düzenlemek için tablo kullanma becerilerini,
. tahmin yapmak için gerekli yöntemi seçebilme becerilerini değerlendirme.
Konu: Matematiksel ifadeler, önermeler, denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem
Materyal, Araç-Gereç: Renkli örüntü blokları, etkinlik sayfası
Etkinlik Süresi: 80 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

	Materyal	Tablo	Sözel	Cebirsel
Materyal		**x**		
Tablo			**x**	
Sözel				**x**
Cebirsel				

İşleniş

Öğrenciler renkli desen bloklarını kullanarak, onlara dağıtılan etkinlik sayfasındaki sorulara cevap verirler.

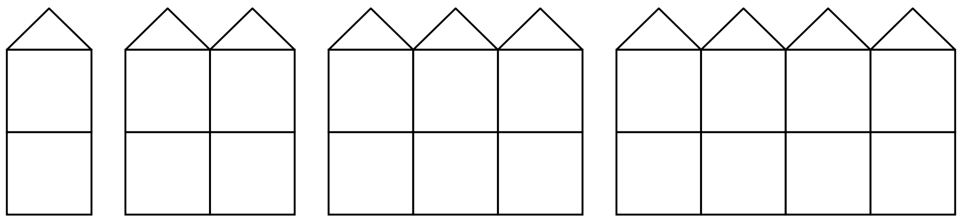
ETKİNLİK SAYFASI

Grup adı
Ad-Soyad:

Bu etkinlik sayfasında size ev şeklinde çeşitli desenler verilmiştir. Bu desenlerin yapısında belli bir matematiksel uyum vardır. Sizden istenen bu matematiksel uyumu fark etmeniz ve ortaya çıkarmanızdır.

Elinizdeki yeşil ve turuncu renkli desen bloklarını kullanarak, aşağıda gördüğünüz ev şeklindeki desenleri oluşturunuz.

1. desen 2. desen 3. desen 4. desen



Oluşturduğunuz şekli dikkate alarak, aşağıdaki tabloyu doldurunuz.

	Kare Sayısı	Üçgen Sayısı	Toplam Parça Sayısı
1. Desen	2		
2. Desen			
3. Desen		3	
4. Desen			12

Yukarıdaki tabloya göre;

1) Beşinci desenin nasıl olacağını aşağıya çiziniz.

2) Onbeşinci deseni oluşturmak için kaç parçaya ihtiyacınız vardır? Neden?

3) Bu seri içinde herhangi bir ev şeklinde desen oluşturmak için, toplam kaç parça gerekeceğini anlatan bir kural yazıp, bu kuralı yazarak olarak açıklayınız.

4) Yukarıda yazdığınız kuralı matematiksel olarak ifade ediniz.

3.ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
2. Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
Ek Hedefler: Öğrencilerin,
. örüntü gelişim anlayışlarını,
. bilgiyi düzenlemek için tablo kullanma becerilerini,
. tahmin yapmak için gerekli yöntemi seçebilme becerilerini değerlendirme.
Konu: Matematiksel ifadeler, önermeler, denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem
Materyal, Araç-Gereç: İp, makas, etkinlik sayfası.
Etkinlik Süresi: 40 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

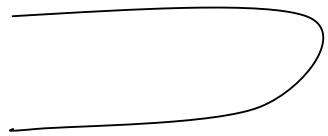
	Materyal	Tablo	Sözel	Cebirsel
Materyal		x		
Tablo			x	
Sözel				x
Cebirsel				

İşleniş:
Öğrenciler dağıtılan ip ve makasları kullanarak etkinlik sayfasındaki sorulara cevap verirler.

ETKİNLİK SAYFASI

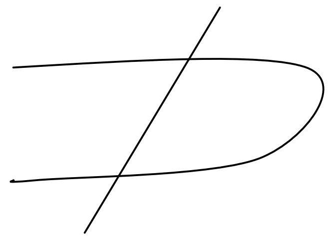
Grup Adı:
Grup Üyelerinin Adı:

Size verilen ipi aşağıdaki gibi kıvırınız.

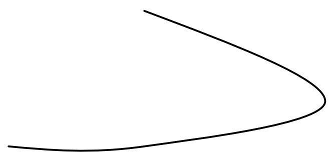


Şimdi de ipi belirtilen çizgiden makasla kesiniz.

1. Kesim

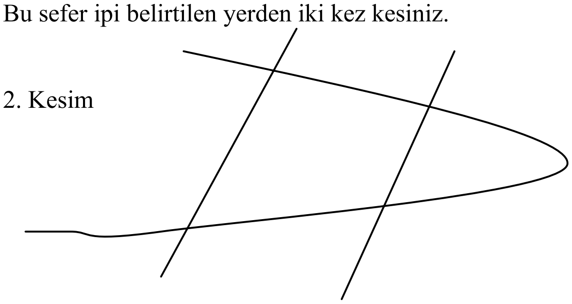


Başka bir ip alıp yine aynı şekilde kıvırınız.



Bu sefer ipi belirtilen yerden iki kez kesiniz.

2. Kesim



İp kesme işlemini bu şekilde sürdürdüğünüzde; ipi her kesişinizde belli miktada ip parçası oluştuğunu fark ettiniz mi? Bu gözleminize göre aşağıdaki tabloyu doldurunuz.

Kesim Sayısı	0	1	2	3	4	5
Parça Sayısı						

Yukarıdaki tabloya göre;

1) Tablodaki sayıların tümünü dikkate alarak kesim sayısı ve oluşan parça sayısı arasındaki genel ilişkiyi yazarak anlatınız.

2) İpi kesmeden, 6. 7. ve 8. kesimlerde kaç tane parça oluşacağını nasıl bulursunuz? Açıklayınız.

3) Görüldüğü gibi kesim sayısından yola çıkarak parça sayısını bulmak mümkün. 20. kesimde kaç parça oluşacağını yazılı olarak anlatınız.

4) 20. kesimde kaç parça oluşacağını denklem kullanarak ifade ediniz.

5) 21 parçaya sahip olabilmeniz için ipi kaç kez kesmeniz gereklidir? Nasıl bulduğunuzu açıklayarak yazınız.

4. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme
2. Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
3. Hedef: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
Konu: Matematiksel ifadeler, önermeler ve denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı
Materyal, Araç-Gereç: Etkinlik sayfası.
Etkinlik Süresi: 40 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

	Tablo	Sözel	Cebirsel
Tablo		x	
Sözel			x
Cebirsel			

İşleniş: Öğretmen etkinlik sayfalarını gruplara dağıtır ve onlardan sayfadaki durumu matematiksel olarak analiz etmelerini ve sorulan sorulara cevap vermelerini ister.

ETKİNLİK SAYFASI

Ad - Soyad:

Bir sinemaya gittiğinizi düşünelim. Bu sinemanın ilk sırasında 10 koltuk var. Her sırada, önündekinden 2 koltuk daha fazla var.
Buna göre;
1) 2. sırada kaç koltuk vardır? Açıklayınız.

2) 3. sırada kaç koltuk vardır? Açıklayınız.

3) 4. sırada kaç koltuk vardır? Açıklayınız.

4) 10. sırada kaç koltuk vardır? Açıklayınız.

5) Sıra ve koltuk sayısı arasındaki ilişkiyi tablolaştırınız.

6) Sıra sayısını s, sıralardaki koltuk sayısını k ile ifade etsek, k'yı s cinsinden nasıl yazabilirsiniz?

7) Biletleri kontrol eden kişi her sırada kaç koltuk olduğunu bilmek istiyor. Tek tek sayması zor olacağından bunun için kolay bir yol bulması gerekiyor. Bilet kontrolü yapan kişi eğer sıra sayısını biliyorsa, kaç tane koltuk olduğunu nasıl hesaplar? Yazarak açıklayınız.

8) Yazdığınız denkleme göre; 21. sırada kaç koltuk vardır?

9) Eğer son sırada 100 koltuk varsa, sinema salonunda kaç sıra vardır?

5. ETKİNLİK

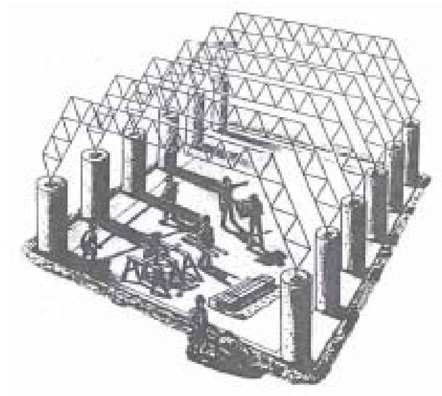
Etkinlikle örtüşen MEB Hedefleri:
Hedef 1: Matematiksel ifadeleri kavrayabilme.
Hedef 2: Önerme, açık önerme ve denklemleri kavrayabilme.
Hedef 3: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
Ek Hedefler:
Öğrencilerin,
. örüntü gelişim anlayışlarını,
. bilgiyi düzenlemek için tablo kullanma becerilerini,
. tablo ve denklem arasındaki ilişkiyi sezebilme,
becerilerini geliştirme.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı.
Materyal, Araç-Gereç: Etkinlik sayfası, pamuk çubuklar.
Etkinlik Süresi: 40 dakikalık bir ders saati
Kullanılan Gösterim Biçimleri:

	Çizim	Materyal	Tablo	Cebirsel	Sözel
Çizim		x			
Materyal			x		
Tablo				x	
Cebirsel					x
Sözel					

Somutlaştırma Veriyi düzenleme Sembolleştirme Yazma
Çizim ⟶ Materyal ⟶ Tablo ⟶ Cebirsel ⟶ Sözel

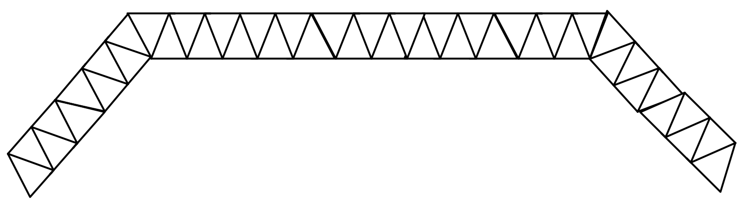
İşleniş: Öğretmen öğrencilerin sıra arkadaşları ile grup olmalarını sağlar ve onlara etkinlik sayfalarını ve pamuk çubukları dağıtır. Öğrencilerden etkinlik sayfasındaki problemi okumalarını ve anlaşılmayan noktaları sormalarını ister. Öğrencilerden gelen olası soruları yanıtladıktan sonra, öğretmen öğrencileri etkinlik ile uğraşmaları için serbest bırakır. Öğretmen gruplar arasında dolaşarak, öğrencilerin sorularına yanıt verir ve öğrencileri etkinlik üzerinde çalışırken gözler.

ETKİNLİK SAYFASI



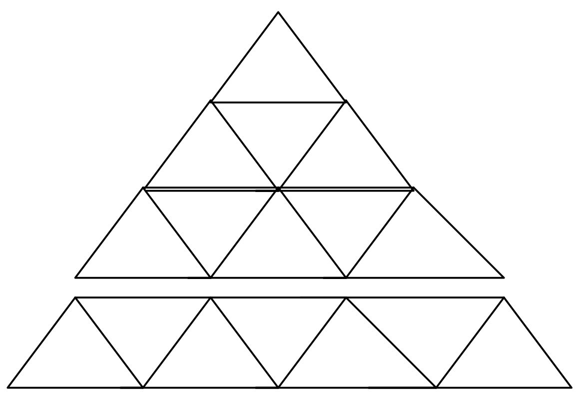
Ad- soyad:

Efes'te antik bir tiyatronun tekrar yapılması için belediye tarafından çalışmalara başlanmıştır. Temel atıldıktan sonra ilk iş olarak ustalar sütunları dikip, bu sütunları metal üçgenlerden oluşan kirişlerle birbirine bağlamaya başlamışlardır. Aşağıda kullanılan bir kiriş örneği görülmektedir.



Hep beraber bu antik tiyatroya gidelim ve çalışanlara biz de yardım edelim, ne dersiniz?

Size verilen pamuk çubukları kullanarak aşagıda görülen metal üçgenleri sıranızda oluşturunuz.



Oluşturduğunuz şekillere göre aşağıdaki tabloyu doldurunuz.

Şekil	Kiriş Uzunluğu	Kullanılan Çubuk Sayısı
1	1	3
2	2	
3	3	
4	4	

5	5	

2) Görüldüğü gibi kullanılan metal çubuk sayısı ile kiriş uzunluğu arasında bir ilişki vardır. Tablodaki sayıların tümünü kapsayan kuralı, ç çubuk sayısı ve k kiriş uzunluğu olacak şekilde matematiksel olarak yazınız.

3) Cebirsel olarak ifade ettiğiniz bu kuralı, sözcüklerle yazarak anlatınız.

4) Bulduğunuz kurala göre; uzunluğu 61 birim olan bir kiriş yapmak için kaç adet metal çubuk gereklidir?

5) Bulduğunuz kurala göre; 119 tane metal çubuk kullanarak kaç birim uzunluğunda bir kiriş yapılabilir?

Sizin gibi bu durum üzerinde uğraşan üç yedinci sınıf öğrencisi; Mercan, Tufan ve Masal aşağıdaki formülleri bulmuşlardır.

Mercan $ç = 3k+(k-1)$
Tufan $ç = k+(k-1)+2k$
Masal $ç = 3+(k-1)4$

6) Sizin yazdığınız formül, bu üç formülden birine benziyor mu?
7) Bu üç formül doğru mu? Neden, açıklayınız.
8) Yazdığınız formülü kullanarak, tüm sahneyi yapmak için gereken çubuk sayısını bulunuz.

6. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
2. Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
Ek Hedefler: Öğrencilerin,
. örüntü gelişim anlayışlarını,
. bilgiyi düzenlemek için tablo kullanma becerilerini,
değerlendirme.
Konu: Matematiksel ifadeler, önermeler ve denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı, üslü sayılar
Materyal, Araç-Gereç: A3 kağıtlar, etkinlik sayfası.
Etkinlik Süresi: 40 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

	Materyal	Tablo	Sözel	Cebirsel
Materyal		x		
Tablo			x	
Sözel				x
Cebirsel				

İşleniş: Öğretmen daha önceden hazırladığı A3 kağıtları ve etkinlik sayfalarını ikili gruplara dağıtır. Öğrencilerden bu kağıtları istedikleri sayıda ikiye katlayarak, etkinlik sayfasında belirtilen soruları yanıtlamalarını ister.

ETKİNLİK SAYFASI

Ad-Soyad:

Size dağıtılan kağıtları istediğiniz sayıda ortadan ikiye katlayabilirsiniz. Katlama sayısı ile katlama sonrası kağıtta kat yerinden sayıldığında oluşan bölge sayısı arasında bir ilişki vardır. Aşağıdaki tabloyu doldurmak için katlama işlemini yaparak ve verilen soruları yanıtlayarak bu ilişkiyi bulmaya çalışın.

Katlama Sayısı	Oluşan Bölge Sayısı
0	
1	
2	
3	
4	
5	

1) Yukarıdaki tabloda bulunan tüm sayıları düşünerek katlama sayısı ile oluşan bölge sayısı arasındaki ilişkiyi yazarak anlatınız.

2) Yazarak anlattığınız bu ilişkinin matematiksel formülünü yazınız.

3) Yukarıda yazdığınız formülü kullanarak 8 katlama sonucunda kaç bölge oluşacağını hesaplayınız.

4) 64 bölge oluşturmak için, kağıdınızı kaç kez katlamanız gerekir?

7. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
2. Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
3 Hedef: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
Ek Hedefler:
Öğrencilerin,
. gerçek yaşamda kullanılan materyaller ve denklem arasındaki ilişkiyi kurma, becerilerini geliştirme.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı, önermeler.
Materyal, Araç-Gereç:Terazi, misket, çeşitli ağırlıklar
Etkinlik Süresi: 80 dakikalık bir ders saati
Etkinlik Tipi: Toplu sınıf etkinliği

Kullanılan gösterim biçimleri:

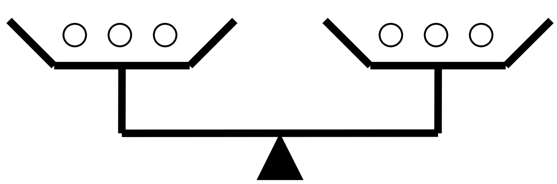
	Materyal	Çizim	Cebirsel	Sözel
Materyal		x		
Çizim			x	
Cebirsel				x
Sözel				

İşleniş: Öğretmen bu ders için sınıfa önceden hazırladığı terazi modelini götürür. Masanın üzerine bu teraziyi yerleştirerek, öğrencilere bir bilinmeyenli denklem çözümleri konusuna bu teraziyi inceleyerek başlayacaklarını duyurur ve terazinin tanıtımına başlar.

Denklem çözümüne geçmeden önce, eşitliğin ne olduğunu kavratmak için, öğretmen "denge" kavramından sözeder. Bunun için terazinin her iki kefesine eşit miktarda misket, tavla pulu, silgi, kalem yerleştirerek terazinin dengede kaldığını öğrencilere gösterir. Bu noktada öğretmen öğrencilerden birkaçının tahtaya gelmesini sağlayarak, onlara hazırlanan materyali keşfetmeleri için bir fırsat vermiş olur.

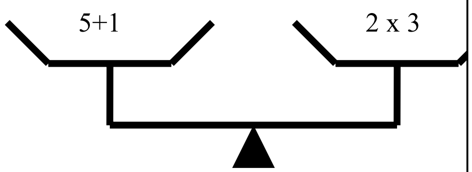
Daha sonra öğretmen öğrencilerden bu gösterimi çizime dökmelerini ister. Defterlerine birer terazi çizmelerini ve denge kavramını çizerek göstermelerini ister. Öğrencilerin çizimleri aşağıdaki gibi olabilir. Aynı çizim tahtaya da yapılabilir.

3 misket = 3 misket

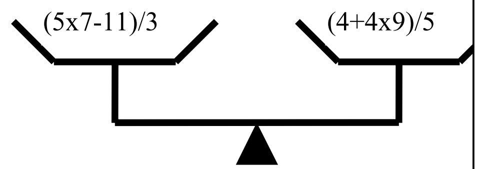


Terazinin nasıl dengede kaldığını sayılar yardımıyla göstermek isteyen öğretmen aşağıdaki modelleri tahtaya çizerek, onlara terazinin dengede olup olmadığını sorar.

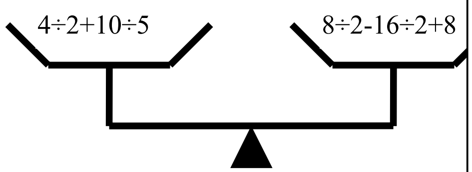
a)



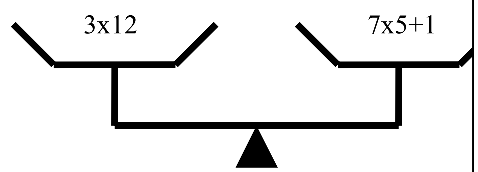
b)



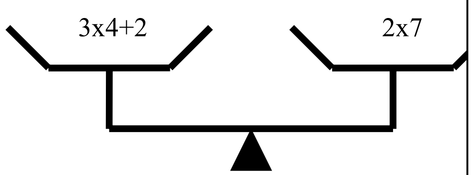
c)



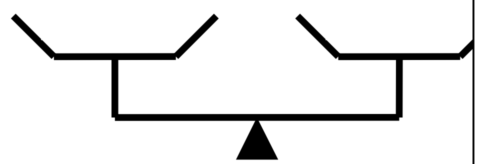
d)



e)



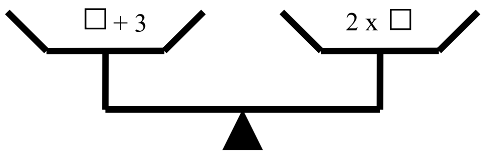
f)



(Bu terazinin kefelerine dengede olan bazı sayısal ifadeler yazalım)

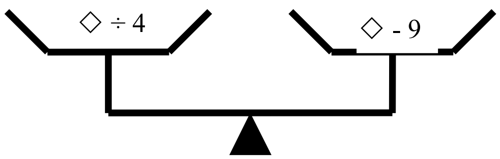
Öğrencilerden yanıtlar alındıktan sonra, yanıtların doğruluğu ve nasıl bulundukları üzerine tartışılır, ikinci aşamada öğretmen terazi modelindeki kefelere ağırlığı bilinen tavla
pulları ya da silgilerin yanısıra, ağırlığını bilmediği kutucuklar yerleştirip, bilinmeyen ağırlıkları bulmak için ne gibi bir yol izlenebileceğini öğrencilerle bulmaya çalışır. Bu model üzerinde yeteri kadar çalışıldıktan sonra, öğretmen modellenen matematiksel durumun öğrenciler tarafından çizimle ifade edilmesini ister. Ve öğrencilere aşağıdaki çizimlerde görülen bilinmeyen değerin ne olduğunu sorar.

1)



Yandaki terazinin dengede olması için yerine hangi sayı gelmelidir ? Bu ifadeyi denklem olarak yazın

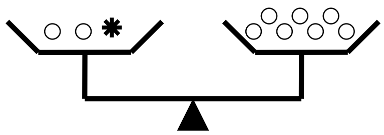
2)



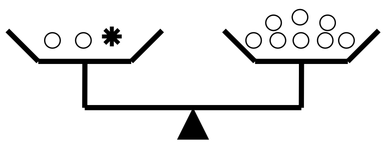
Yandaki terazinin dengede olması için ◇ yerine hangi sayı gelmelidir ? Bu ifadeyi denklem olarak yazın.

3) Aşağıda terazi ile gösterilen modelleri sembollerle ifade edip, yazılı olarak anlatınız.

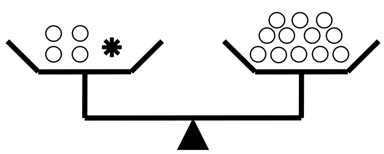
a)



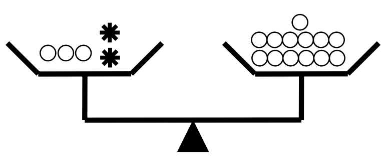
b)



c)

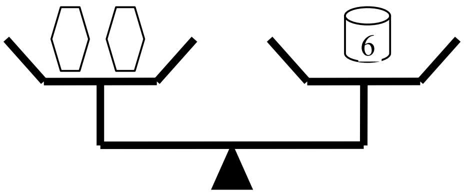


d)

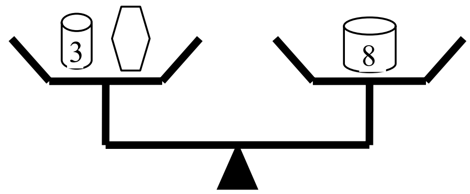


4) Teraziler dengede olduğuna göre, külçelerin değerini bulunuz, denklemi yazınız.

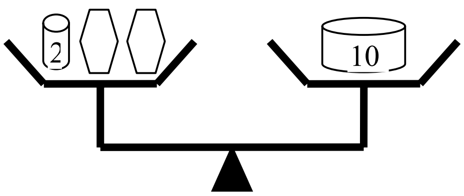
a)



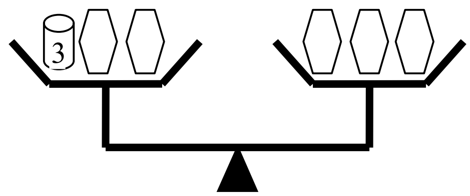
b)



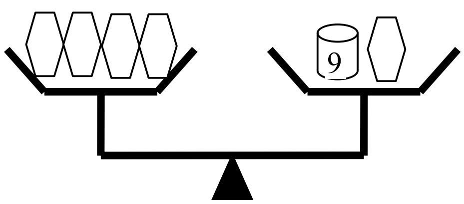
c)



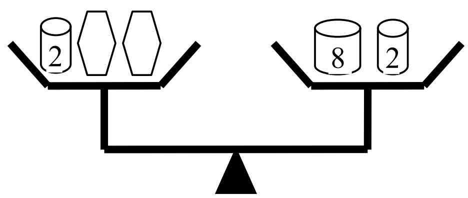
d)



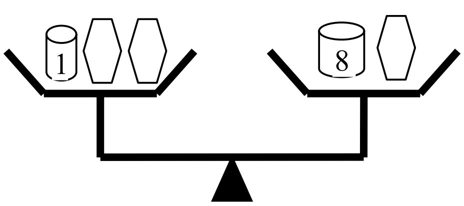
e)



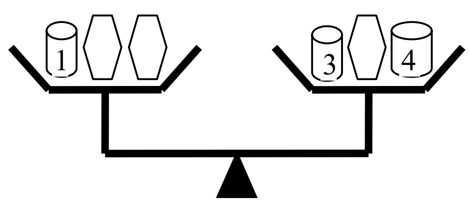
f)



g)

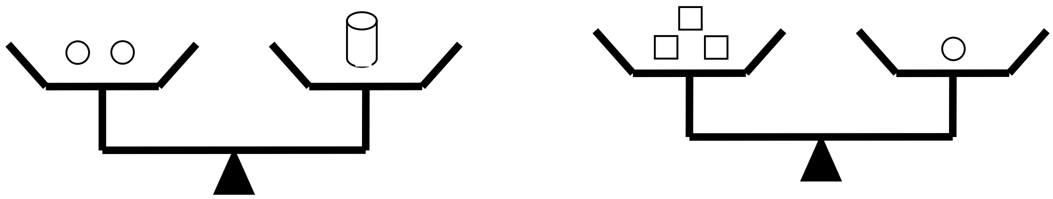


h)



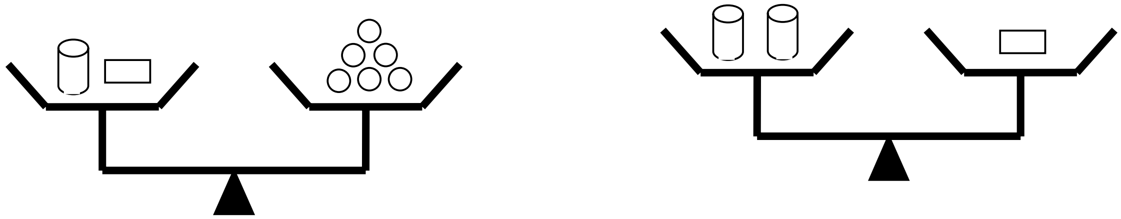
Üçüncü aşamada ise artık terazi modelinin kullanılmasına gerek kalmamıştır. Öğrencilerden gereken durumları terazi çizerek ifade etmeleri istenebilir. Onlardan aşağıdaki çizimleri sembolleştirmeleri istenir.

A)

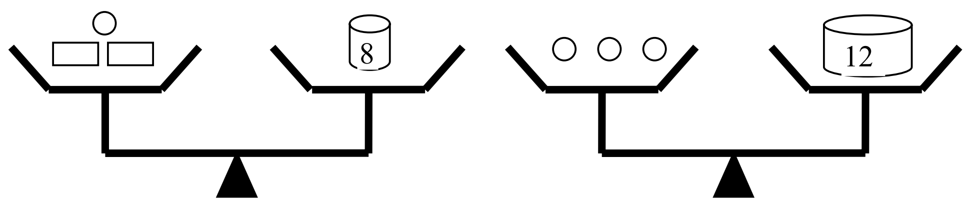


Hangi şekil daha ağırdır? Açıklayın.
Hangi şekil daha hafiftir? Açıklayın.

B)



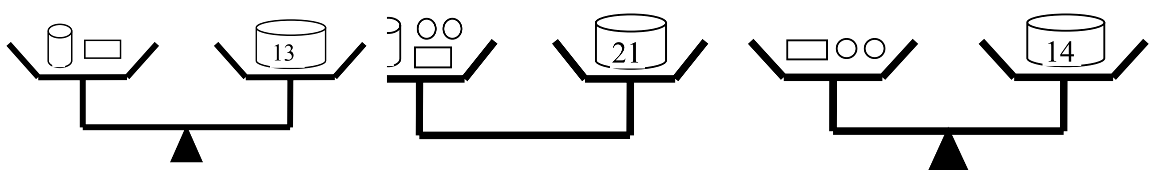
İki daireyi hangi şekil dengeler? Açıklayın.



C)

Herbir şeklin ağırlığı nedir ?Açıklayın.

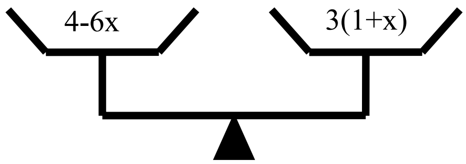
D)



Herbir şeklin ağırlığı nedir ?Açıklayın.

Bu işlemden sonra, denklem çözümleri ile ilgili sembolik işlemlere geçilip, öğrenci katılımı sağlanarak örnekler çözülür.

E)

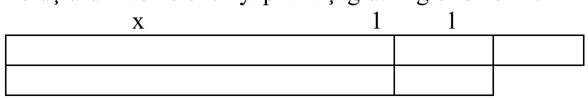


8. ETKİNLİK

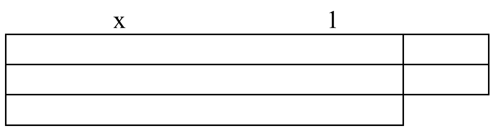
Etkinlikle örtüşen MEB Hedefleri:
Hedef 1: Matematiksel ifadeleri kavrayabilme.
Hedef 2: Önerme, açık önerme ve denklemleri kavrayabilme.
Hedef 3: Birinci dereceden bir bilinmeyenli denklem çözümleri.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı.
Materyal, Araç-Gereç:Etkinlik sayfası, çubuklar veya kartonlar (algebra tiles), cetvel, makas.
Etkinlik Süresi: 40 dakikalık bir ders saati
Etkinlik Tipi: 3 kişilik grup etkinliği
Kullanılan gösterim biçimleri:

	Materyal	Cebirsel
Materyal		x

İşleniş: Öğretmen öğrencilerin üçer kişilik gruplar olmalarını sağlayıp, önceden hazırladığı cebir çubuklarını öğrencilere dağıtır. Dağıtılan bu çubukları kullanarak öğrencilerden (2x+3) ve (3x+2) ifadelerini oluşturmalarını ister. Öğrencilerden oluşturulması istenen yapılar aşağıdaki gibi olabilir.



2x+3



3x+2

Gruplar bu yapıları oluşturduktan sonra öğretmen öğrencilerden (2x+3) + (3x+2) ifadesini cebir çubukları yardımıyla oluşturmalarını ister. Beklenen yapı aşağıdaki gibi olabilir.

Öğretmen öğrencilerden bu yapıyı cebirsel olarak ifade etmelerini ister.

Beklenen ifade; (2x+3) + (3x+2) = 5x +5 ifadesidir.

Ayı modellemeler, 2.(3x+1) = 6x+2, (4x+3)+3 = 4x+6, (x+1)+2(x+3) = 3x+7 ifadeleri için de yapılıp çözümleri bulunur.

9. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
Hedef 6: Düzlemde bir noktanın koordinatlarını kavrayabilme.
Ek Hedefler: Grafik yorumlama becerisi
Konu:
Düzlemde bir noktanın koordinatları
Ön Koşul Bilgileri: Denklemler.
Materyal, Araç-Gereç: Koordinat düzlemi materyali, Türkiye haritası
Etkinlik Süresi: 40 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

	Materyal	Çizim
Materyal		x

İşleniş: Öğretmen bu ders için sınıfa önceden hazırladığı koordinat düzlemi modelini götürür. Öğretmen bu modeli bir sinema salonu olarak tanıttıktan sonra, materyal üzerinde her bir sinema salonunu eksenin bir bölgesine benzeterek, nokta yerleştirme işleminin yapılabilmesi için, öğrencileri tahtaya çağırır ve onların materyali keşfetmelerine izin verir.

Daha sonra öğretmen öğrencilerden bu gösterimi çizime dökmelerini ister. Defterlerine gördükleri gibi bir sinema salonu çizmelerini ve bu salon üzerinde onlara verilen koltuk numaralarını belirtmelerini ister. kavramını Son aşamada sınıfça bu çizimleri koordinat eksenine dönüştürürler.

Öğretmen öğrencilerden sıra arkadaşları ile grup olmalarını ister ve her gruba x-y koordinat düzlemine yerleştirilmiş Türkiye haritası materyalini dağıtır. Bu materyal üzerinde öğrenciler onlardan istenen illerin hangi koordinatlar üzerinde olduğunu etkinlik sayfasına yazarlar. Daha sonra bu işlem tahtada kontrol edilir. Ders öğrencilerin defterlerine x-y koordinat düzlemi çizmesi ile son bulur.

10. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
Hedef 2: Önerme, açık önerme ve denklemleri kavrayabilme.
Hedef 3: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
Ek Hedefler: Öğrencilerin denklemleri kavramalarını sağlama.
Konu: Denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, matematiksel ifadeler, bilinmeyen kavramı.
Materyal, Araç-Gereç: Etkinlik sayfası
Etkinlik Süresi: 80 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

	Çizim	Tablo	Sözel	Grafik	Cebirsel
Çizim					x
Tablo			x	x	x
Sözel		x			x
Grafik					x
Cebirsel		x		x	

İşleniş:

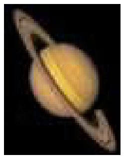
Öğretmen sınıfa bu ders denklem çözümlerini işleyeceklerini duyurur ve onlara denklem çözümü ile ilgili çeşitli yollar olduğunu söyleyip, etkinlik sayfasında bu farklı yolları bulabileceklerini duyurur. Daha sonra öğrencilere etkinlik sayfasını okumalarını ve anlamadıkları noktaları sormalarını ister. Olası soruları yanıtladıktan sonra, öğrencileri etkinliği yapmaları için serbest bırakır.

ETKİNLİK SAYFASI

Ad-Soyad:

Umut rüyasında galakside bulunan diğer gezegenlere bir yolculuk yapıp, o gezegenlerin matematik derslerine katıldığını görmüş. Rüya bu ya; matematik sadece dünyada değil diğer gezegenlerde de önemli bir dersmiş. Denklemler konusunun nasıl öğretildiğine dair bazı bilgiler toplayan Umut'un size sormak istediği sorular var. Hadi beraberce inceleyelim☺.

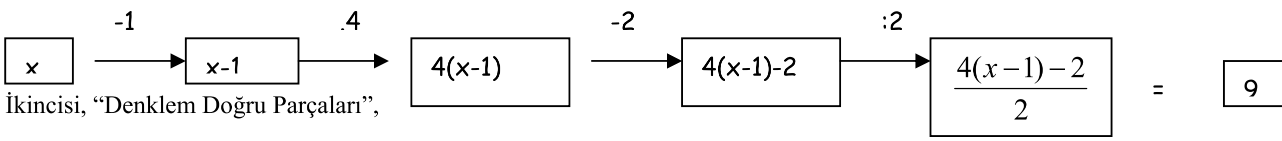
SATÜRN



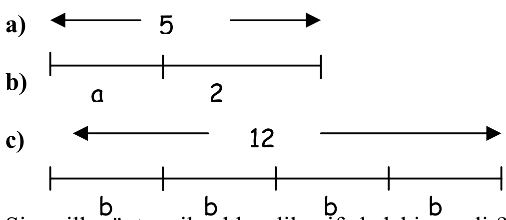
Satürn'de eskiden beri öykü yöntemi kullanılırmış. Yani bir denklemi çözmek için öncelikle o denklemin öyküsünü yazarlarmış. Aynen aşağıdaki gibi;

"x adında bir sayı varmış. Bu sayıdan 1 çıkarıp, sonucu 4 ile çarpıyoruz ve sonra 2 çıkarıyoruz ve bunu da 2'ye bölüyoruz. Sonuç 9 çıkıyor. x kimdir?"

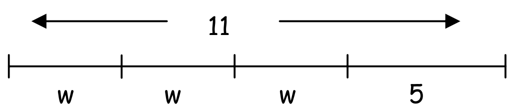
Bu tip denklemleri çözmek için Satürn'dekiler iki yöntemden yararlanırmış. İlki; yukarıdaki öykü için örnek verilen "Denklem Kutuları",



İkincisi, "Denklem Doğru Parçaları",



Sizce ilk yöntem ile elde edilen ifadedeki x nedir? Bulunuz.



İkinci yöntem ile elde edilen ifadeleri matematiksel denklem olarak yazsak nasıl yazarız?

a)

b)

c)

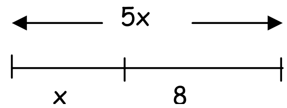
Diyelim ki; siz bundan sonra bu gezegende yaşayacaksınız, aşağıdaki denklem öykülerini nasıl çözersiniz?

1) a adında bir sayı varmış. Bu sayının 5 katından 8 çıkarınca 22 elde ediliyormuş. Bu sayı kimmiş? Çözmeden önce denklem kutusunu mutlaka çizin.

2) y'nin 2 katından 9 çıkarırsam ve sonucu 8'e bölersem 3 çıkıyor. y kimdir? Çözmeden önce denklem kutusunu mutlaka çizin.

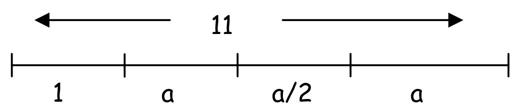
3) Aşağıdaki Denklem Doğru Parçaları'nın ifade ettiği denklemleri yanına yazın ve buradaki bilinmeyeni bulun.

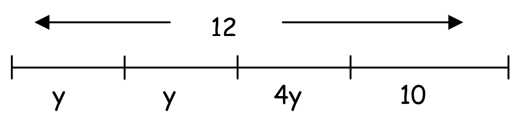
a)



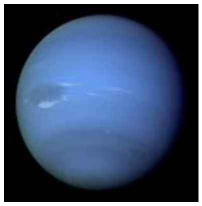
b)

c)

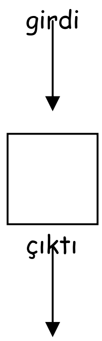




NEPTÜN



Umut'un ikinci durağı Neptün. Bu gezegen kendi icatları olan makineler ile ünlü imiş. Bu makinelerden birisi de "Dönüştürme Makinesi". Bu makine verilen sayılara belli işlemler uygulayıp yeni sayılar veriyor. Örneğin aşağıdaki gibi;



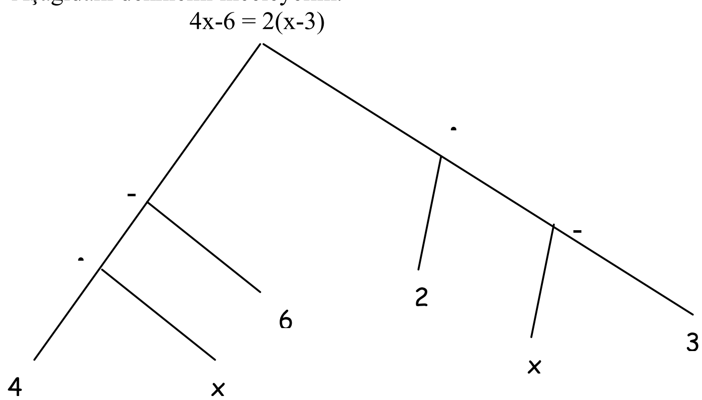
girdi	çıktı
2	4
4	8
5	10
10	12
17	34

Yukarıdaki tabloda bulunan tüm sayılar için, yapılan ortak işlemi yazarak anlatınız.

Yukarıda yazdığınızı ifade eden tek bir denklem kurunuz.

JUPİTER

Bu gezegende ise Umut denklemin sağı ve solundaki ifadeler için kullanılan ağaç dalları görmüş. Aşağıdaki denklemi inceleyelim.



Yukarıdaki örnekte görülen ağaç dallarını kullanarak aşağıdaki denklemleri ifade edip, x'i bulunuz..

a) 5x+8 = 10x

b) 3x-18 = x+6

c) x+14 = 7(5+3x)

MARS

Bu gezegende Umut eşit bölmeli çubuklarla denklemler konusunun işlendiğini görmüş. Örneğin aşağıdaki gibi iki eşit uzunlukta çubuk çizilmiş ve buradan bir denklem yazmanız ve a değernini bulmanız istense, nasıl yaparsınız?

3	3	2a

24

Eğer a değerini doğru olarak bulduysanız aşağıdaki çubuklarının denklemlerini yazıp, bilinmeyeni bulabilirsiniz demektir. Hadi bakalım☺

a)

2	3	a

14

b)

x+4	3	11

2x	17

AY

Umut artık Dünya'ya döncekken bize en yakın olan gezegene de uğramış. Ay'da geçerli bir denklem oyunu varmış, adı da; "Kuralımı Bul!". Bu oyunu oynamak için iki kişi gerekiyor, bunlar da sen ve sıra arkadaşın olabilir. Önce sen aklından bir kural yaratıyorsun, sonra arkadaşın bu kurala uyan sayıları tablolaştırıyor, sen bu tablonun grafiğini çizerken sıra arkadaşın da kuralı harfli ifade kullanarak yazıyor. İlk kuralı Umut söylüyor, siz bulun bakalım.

"Söylediğin her sayının 2 katını al ve 8 ekle."

Bu kurala uygun olarak yazılabilecek 6 sayıyı kullanarak bir tablo yapın.

Tablodaki sayıları (x,y) ikilileri olacak şekilde koordinat düzleminde gösterin.

Kuralı denklem kullanarak ifade edin.

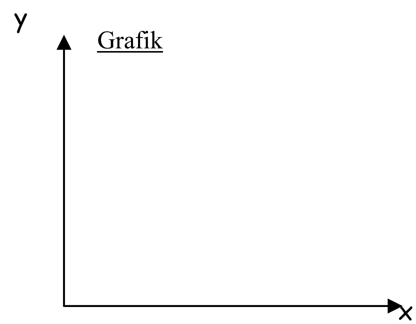
Aşağıda bu oyunu oynamış Ay gezegeni öğrencilerinin tamamlamadan bıraktığı bazı matematiksel durumlar var, bunlardan bazılarının tablosu, bir kısmının grafiği ya da denklemi eksik, öncelikle onları tamamlayın.

1) Tablo

x	y
0	8
1	10
2	12
3	14
4	16
.	.
.	.

Denklem

Grafik



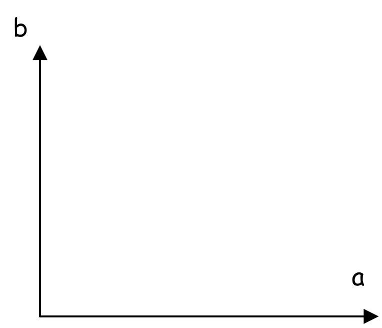
2) Denklem

(3 x a) + 1 = b

Tablo

a	b

Grafik

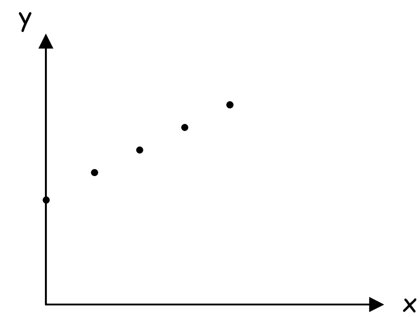


3) Denklem

Tablo

x	Y
0	3
1	4
2	
3	
4	
.	

Grafik



Kuralımı Bul oyunun sıra arkadaşınızla iki el oynayıp, sonuçları bu sayfaya yazınız.

11. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
2. Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
3. Hedef: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
6. Hedef: Düzlemde bir noktanın koordinatlarını kavrayabilme.
7. Hedef: Grafik çizebilme.
Ek Hedefler: Öğrencilerin,
. grafik çizebilme,
. verilen verideki sabit değişenin ne olduğunu bulma,
. . tablo, grafik ve sembolik gösterim arasındaki ilişkinin farkına varma becerilerini değerlendirme.

Konu: Birinci dereceden bir bilinmeyenli denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı, tablo kullanımı
Materyal, Araç-Gereç: Etkinlik sayfası, kareli kağıt
Etkinlik Süresi: 40 dakikalık bir ders saati
Kullanılan Gösterim Biçimleri:

	Tablo	Cebirsel	Sözel	Grafik
Tablo		x		
Cebirsel			x	
Sözel				x
Grafik				

İşleniş: Bu etkinlik aslında bir konuya ısınma etkinliği olarak adlandırılabilir. Bu etkinlikten sonra doğrusal ilişkiler ile grafik çizebilme konusu ile ilgili etkinlikler yapılacaktır. Öğretmen öğrencilere ekteki etkinlik sayfasını dağıtır ve onlardan bu etkinlik sayfasındaki soruları yapmalarını ister.

ETKİNLİK SAYFASI

Ad,Soyad:

Bu etkinlik sayfasında size tamamlanmayı bekleyen bir tablo verilmiştir. Sizden istenen x ve y arasındaki ilişkiyi kullanarak bu tabloyu tamamlamanız ve sorulan sorulara yanıt vermenizdir.

x	y
2	-
3	-
4	16
5	20
6	24
7	-
8	32

1) y yerine yazılacak değerleri bulup tabloyu doldurun.

2) Verilen x değerlerinden yola çıkarak y değerlerini bulmanızı sağlayacak genel kuralı ifade eden denklemi aşağıya yazın.

3) Bu kuralı yazarak açıklayınız.

4) Ekteki kareli kağıda bir koordinat ekseni çizip, bu eksen üzerinde tablodaki sayıları (x,y) ikilileri şeklinde gösteriniz ve bu ikililerin grafiğini çiziniz.

5) Bu kuralı kullanarak verilen bir x = -1 değeri için y değerini hesaplayın. Bu noktayı 4. soruda çizmiş olduğunuz grafik üzerinde renkli kalemle gösterin.

6) Bu kuralı kullanarak verilen bir x = -2 değeri için y değerini hesaplayın. Bu noktayı 4. soruda çizmiş olduğunuz grafik üzerinde renkli kalemle gösterin.

7) Bu kuralı kullanarak verilen bir y = 64 değeri için x değerini hesaplayın.

8) Bu kuralı kullanarak verilen bir y = 200 değeri için x değerini hesaplayın.

12. ETKİNLİK

Etkinliğin örtüştüğü MEB Hedefleri:
Hedef 1: Matematiksel ifadeleri kavrayabilme.
Hedef 2: Önerme, açık önerme ve denklemleri kavrayabilme.
Hedef 6: Düzlemde bir noktanın koordinatlarını kavrayabilme.
Hedef 7: Grafik çizebilme
Ek Hedefler:
Öğrencilerin,
. tablo okuma ve kullanma,
becerilerini geliştirme.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Düzlemde bir noktanın koordinatları
Doğru grafikleri.
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı.
Materyal, Araç-Gereç:Etkinlik sayfası.
Etkinlik Süresi: 40 dakikalık bir ders saati
Kullanılan gösterim biçimleri:

	Tablo	Cebirsel	Grafik
Tablo		x	
Cebirsel			x

İşleniş: Öğretmen öğrencilere önceden hazırladığı etkinlik sayfalarını dağıtır. Öğrencilerden etkinlik sayfasını dikkatlice okumalarını ve anlaşılmayan noktaları sormalarını isteyip, varsa sorulara yanıt verir. Etkinlik süresince, öğretmen gruplar arasında dolaşarak, öğrencilerin sorularına yanıt verir. Etkinlik tamamlandıktan sonra, öğretmen konuyla ilgili daha ayrıntılı örnekleri öğrencilerle beraber çözer.

ETKİNLİK SAYFASI

Ad – Soyad:

Trabzon ilindeki gece ve gündüz sıcaklıklarına ilişkin, Ocak ayının ilk haftası örnek olarak verilmiştir.

Tarih	Gece Sıcaklığı	Gündüz Sıcaklığı
1 Ocak	2	6
2 Ocak	1	5
3 Ocak	0	4
4 Ocak	-1	3
5 Ocak	-2	2
6 Ocak	-3	1
7 Ocak	-4	0

1) Gece ve gündüz sıcaklığını değişken olarak alıp, gece sıcaklığına x, gündüz sıcaklığına y diyerek; tablodaki veriyi ifade eden denklemi yazınız.

2) Yazdığınız denklemi, ekteki kareli kağıdı kullanarak x-y koordinat düzleminde gösteriniz.

Aşağıdaki tabloda bir odanın gece ve gündüz sıcaklık ortalaması °C cinsinden verilmiştir.

Gece Sıcaklığı	**Gündüz Sıcaklığı**
-3	7
0	4
2	2
10	-6
4	0
8	-4
-10	14

1) Gece ve gündüz sıcaklığını değişken olarak alıp, gece sıcaklığına x, gündüz sıcaklığına y diyerek; tablodaki veriyi ifade eden denklemi yazınız.

2) Yazdığınız denklemi, ekteki kareli kağıdı kullanarak x-y koordinat düzleminde gösteriniz.
3) Çizilen iki grafiği inceleyip, aradaki farkı yazınız.

13. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
2.Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
6. Hedef: Düzlemde bir noktanın koordinatlarını kavrayabilme.
7. Hedef: Grafik çizebilme.
Ek Hedefler: Öğrencilerin,
. veri toplama ve veriyi tablo haline getirme,
. grafik çizme ,
. değişkenin farkına varma becerilerini değerlendirme.
Konu: Grafikler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, tablo kullanma.
Materyal, Araç-Gereç: Etkinlik sayfası, kareli kağıt
Etkinlik Süresi: 40 dakikalık bir ders saati

Kullanılan gösterim biçimleri:

	Gerçek Hayat Durumu	Tablo	Grafik	Cebirsel
Gerçek Hayat Durumu		x		
Tablo			x	
Grafik				x
Cebirsel				

İşleniş: Öğretmen her öğrenciye bir etkinlik sayfası dağıtıp, onlardan bu etkinlik sayfasındaki durumu analiz etmelerini ister.

ETKİNLİK SAYFASI

Ad-Soyad:

12 Dev Adam son basketbol maçlarına otobüsle gitmeye karar vermişler ve 800 kilometrelik yolu saatte 100 kilometre ile almışlar.

1) Gidilen yolu x, zamanı t ile gösterecek şekilde, her bir saat sonunda katedilen yolun tablosunu yapın.

2) Tablodaki verinin grafiğini, x ekseni zamanı, y ekseni alınan yolu gösterecek şekilde ekteki kareli kağıda çiziniz.

3) Grafik üzerindeki noktaları tek bir doğru üzerinde birleştirmek doğru olur mu? Neden?

4) 2 saatte alınan yol kaç kilometredir? Tablodan mı yoksa grafikten mi bulmak daha kolay? Neden?

5) Tablodan zaman ve yola bağlı bir formül yazabilir misiniz?

6) $3\frac{3}{4}$ saatte alınan yolu yazdığınız formülü kullanarak bulabilir misiniz?

7) $3\frac{3}{4}$ saatte alınan yolu tablodan ve grafikten bulabilir misiniz? Nasıl?

8) Yazılan formüle göre, x saatte alınan yol kaç kilometredir?

9) Yazılan formüle göre, (2x+6) saatte alınan yol kaç kilometredir?

10) Yazılan formüle göre, n kilometrelik yol kaç saatte gidilir?

11) Yazılan formüle göre, $\frac{n}{9}$ kilometrelik yol kaç saatte gidilir?

14. ETKİNLİK

Etkinliğin Örtüştüğü MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
6. Hedef: Düzlemde bir noktanın koordinatlarını kavrayabilme.
7. Hedef: Grafik çizebilme.

Ek Hedefler: Öğrencilerin,
. veri toplama ve veriyi tablo haline getirme,
. grafik çizme ,
. değişkenin farkına varma becerilerini değerlendirme.

Konu: Grafikler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, tablo kullanma.
Materyal, Araç-Gereç: Top, etkinlik sayfası, saat, kareli kağıt
Etkinlik Süresi: 80 dakikalık iki ders saati
Etkinlik Tipi: 4 kişilik grup etkinliği

Kullanılan Gösterim Biçimleri:

	Gerçek Hayat Durumu	Tablo	Grafik	Sözel
Gerçek Hayat Durumu		x		
Tablo			x	
Grafik				x
Sözel				

İşleniş: Öğretmen öğrencilerden dörder kişilik gruplar oluşturmalarını ister ve her gruba birer top ve saat verir. Öğretmen bugün yapacakları etkinliğin amacını duyurur ve gruplara etkinlik sayfasını dağıtır.

ETKİNLİK SAYFASI

Ad-Soyad:

Bugün topları kullanarak bir matematik deneyi yapacağız. Bu deneyde amaç; 2 dakika içinde her bir grup üyesinin bir topu kaç kez saydırabileceğinin hesaplanması ve bu durumu matematiksel olarak ifade edilmesidir.

"Saydırma" ile kastedilen, tenis topunun belden aşağı bırakılıp tutulmasıdır. Bu çalışma dört kişilik bir ekip çalışmasıdır. Grup üyelerinden bir tanesi zamanı tutarken, diğeri topu saydıracak, başka bir üye bu zıplatma sayısını belirlemek için sayarken, sonuncu üye de bu sayma işlemini tabloya yazacaktır. Zamanı kontrol eden kişi her 10 saniyede haber vererek, tabloyu dolduran öğrencinin 2 dakikalık zaman aralığını 10 saniyelik zaman aralıklarına bölmesini sağlayacaktır. Elde edeceğiniz veriyi ekteki sayfalara tablo halinde belirtebilirsiniz.

Grubun her üyesi topu saydırıp, üyelerin rolleri değişene kadar etkinlik sürecektir. Böylece her grup üyesinin elinde aynı tablonun doldurulmuş hali olacaktır.

1. Kişi için;

Zaman (saniye)	Bu aralıktaki saydırma sayısı	Bu ana kadar olan toplam saydırma sayısı
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		
110		
120		

2. Kişi için;

Zaman (saniye)	Bu aralıktaki saydırma sayısı	Bu ana kadar olan toplam saydırma sayısı
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		
110		
120		

3. Kişi için;

Zaman (saniye)	Bu aralıktaki saydırma sayısı	Bu ana kadar olan toplam saydırma sayısı
0		
10		
20		
30		
40		
50		
60		
70		

80		
90		
100		
110		
120		

4. Kişi için;

Zaman (saniye)	Bu aralıktaki saydırma sayısı	Bu ana kadar olan toplam saydırma sayısı
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		
110		
120		

Bu tablolara göre;
1) Şimdi herkes kendi top saydırma grafiğini oluştursun. Bunun için x eksenini zaman, y eksenini aralıktaki saydırma sayısı olarak alıp, ekteki kareli kağıda grafiği çizebilirsiniz.

2) Toplam saydırma sayısı ile, zaman arasındaki ilişkiyi yazarak açıklayınız.

15. ETKİNLİK

Etkinliğin Örtüştüğü MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
2. Hedef: Önerme, açık önerme ve denklemleri kavrayabilme.
3. Hedef: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
6. Hedef: Düzelmde bir noktanın koordinatlarını kavrayabilme.
Ek Hedefler: Öğrencilerin,
. verilen verideki sabit değişenin ne olduğunu bulma,
. tablo, grafik ve sembolik gösterim arasındaki ilişkinin farkına varma becerilerini değerlendirme.

Konu: Birinci dereceden bir bilinmeyenli denklemler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı, tablo kullanımı
Materyal, Araç-Gereç: Etkinlik sayfası.
Etkinlik Süresi: 80 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri: Bu etkinlikte öğrenciler varolan representation biçimleri arasında dönüştürme yapıyorlar ve istedikleri representation biçimini kullanmakta özgürler.

İşleniş: Öğretmen öğrencilere ekteki etkinlik sayfasını dağıtır ve onlardan bu etkinlik sayfasındaki soruları yapmalarını ister.

ETKİNLİK SAYFASI

Ad-Soyad:

Fotoğrafta görülen Gökçe, Göksu, Deniz ve Mercan adındaki dört arkadaş geçen yıl boyunca harçlıklarını biriktirmişlerdir. Her birinin elindeki toplam para miktarı milyon TL cinsinden dört farklı şekilde aşağıda gösterilmiştir. Beraberce inceleyelim☺

Gökçe

Aşağıdaki tabloda Gökçe'nin her hafta sonunda kaç milyon TL. Biriktirdiği görülmektedir. Tablo yıl sonuna kadar bu şekilde devam etmektedir.

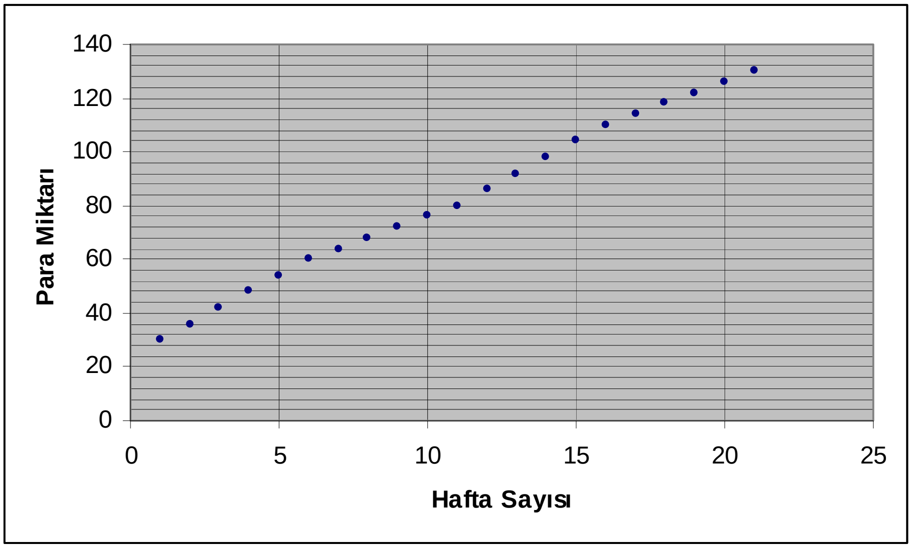
Hafta Sayısı	1	2	3	4	5	6	7	8	9	...
Biriken Para Miktarı	7 mil.	14 mil.	21 mil.	28 mil.	35 mil.	42 mil.	49 mil.	56 mil.	63 mil.	...

Göksu

Göksu bir yıl boyunca her ay eşit miktarda harçlık alarak toplam 300 milyon TL. biriktirmiştir.

Deniz

Aşağıdaki grafik Deniz'in ilk 20 hafta boyunca biriktirdiği parayı göstermektedir. Grafikte görülen para biriktirme oranı yıl sonuna kadar bu şekilde devam etmektedir.



Mercan
Hafta sayısını x ile gösterirsek, Mercan'ın bir yıl boyunca biriktirdiği para 300-5x kadardır.

Bu verilere göre;

1) Gökçe'nin 1 hafta sonundaki kazancı kaç liradır?

2) Göksu'nun 1 ay sonundaki kazancı kaç liradır?

3) Deniz'in 5 hafta sonundaki kazancı kaç liradır?

4) Mercan'ın 20 hafta sonundaki kazancı kaç liradır?

5) Mercan 100 milyon lirayı kaç haftada biriktirir?

6) Deniz 130 milyon lirayı kaç haftada biriktirir?

7) Her bir çocuğun biriktirdiği paranın yıl boyunca nasıl değiştiğini sözcüklerle anlatınız.

Gökçe:

Göksu:

Deniz:

Mercan:

8) Gökçe ve Göksu'nun yıl boyunca biriktirdikleri para miktarını gösteren grafikleri ekteki kareli kağıda çizin. Grafikleri çizerken x eksenini hafta sayısı, y eksenini biriken para miktarı olarak alabilirsiniz.

7) Bu dört çocuktan iki tanesini örnek olarak alıp, bu ikisinin biriktirdikleri para miktarını karşılatırın. Bu karşılaştırmayı yapabilmek için; tablo, grafik, sözel ve sembolik ifadeler kullanabilirsiniz.
7) Bu dört çocuktan iki tanesini örnek olarak alıp, bu ikisinin biriktirdikleri para miktarını karşılatırın. Bu karşılaştırmayı yapabilmek için; tablo, grafik, sözel ve sembolik ifadeler kullanabilirsiniz.
8)

Elvan Gökçe'nin 15 hafta sonunda ne kadar para biriktirdiğini bulmak istiyor. Bu nedenle, aşağıdaki tabloyu kullanıyor.

Hafta Sayısı	11	12	13	14	15
Biriken Para Miktarı	77	84	91	98	105

Elvan tabloya bakıp 15 hafta sonunda biriken para miktarının 105 milyon TL. olduğunu söylüyor. Ulaş bu hesaplama için başka bir yol daha olduğunu söylüyor. Gökçe'nin yılın başında hiç parası olmadığı ve her hafta biriktirdiği para 7 milyon TL arttığı için, hafta sayısının 7 katını alıyor ve 7x15=105 buluyor.

Sizce Elvan ve Ulaş'ın kullandıkları yöntemler doğru mu? Neden?

Siz hangisini tercih ederdiniz? Nedenini belirterek açıklayınız.

16. ETKİNLİK

Etkinlikle Örtüşen MEB Hedefleri:
Hedef 1: Matematiksel ifadeleri kavrayabilme.
Hedef 2: Önerme, açık önerme ve denklemleri kavrayabilme.
Hedef 3: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
Hedef 6: Düzlemde bir noktanın koordinatlarını kavrayabilme.
Hedef 7: Grafik çizebilme
Ek Hedefler:
Öğrencilerin,
. doğrusallık kavramı ile gerçek yaşam durumları arasındaki ilişkiyi kurma,
. veriyi düzenlemek için tablo kullanma,
. grafik çizebilme,
. tablo, grafik, denklem arasındaki ilişkiyi sezebilme,
becerilerini geliştirme.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Düzlemde bir noktanın koordinatları
Doğru grafikleri.
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı.
Materyal, Araç-Gereç:Etkinlik sayfası.
Etkinlik Süresi: 40 dakikalık bir ders saati
Kullanılan Gösterim Biçimleri:

	Tablo	Grafik	Sözel	Cebirsel
Tablo		x		
Grafik			x	
Sözel				x

İşleniş: Öğretmen öğrencilerin üçer kişilik grup olmalarını sağladıktan sonra, onlara önceden hazırladığı etkinlik sayfalarını dağıtır. Öğrencilerden etkinlik sayfasındaki problem durumunu okumalarını ve anlaşılmayan noktaları sormalarını ister. Etkinlik sayfasındaki öğrenci adları yerine, grup üyelerinin adlarını yazabileceklerini söyledikten sonra, öğrencileri etkinlik sayfasında belirtilen soruları yanıtlamaları için serbest bırakır. Etkinlik süresince, öğretmen gruplar arasında dolaşarak, öğrencilerin sorularına yanıt verir.

ETKİNLİK SAYFASI

Ad-Soyad:

7B sınıfındaki spor kolu öğrencileri, çevresi 6 kilometre olan okul sahasında bir yürüyüş düzenlemeye karar verirler. Bu mesafenin ortalama ne kadar zamanda yürüneceğini hesaplamak için, öğrenciler kendi aralarında yürüyüş denemeleri yapmaya başlıyorlar. Bu deney sonucunda elde edilen üç öğrencinin yürüme hızları aşağıda verilmiştir.

Ad	Yürüme Hızı
Umut	Saniyede 1 metre
Türker	Saniyede 1,5 metre
Ulaş	Saniyede 2 metre

Bu yürüme hızlarına dayanarak, herbir öğrenci için aynı sürelerde alınan farklı uzaklıklar aşağıdaki gibi tablolaştırılmıştır.

	Alınan Yol (m.)		
Zaman (Saniye)	Umut	Türker	Ulaş
0	0	0	0
1	1	1,5	2
2			
3			
4			

1) Belirtilen saniyelerde bu üç öğrencinin aldığı yolu tabloya yazın.

2) Aynı koordinat düzlemine, x ekseni zamanı y ekseni alınan yolu göstermek üzere bu üç öğrencinin zaman-yol grafiğini çiziniz (Grafik çizimi için ekteki kareli kağıdı kullanabilirsiniz.).

3) Çizilen grafiklerin farklı olmasının nedeni ne olabilir?

4) Her bir öğrenci için, zaman ve yürünen mesafe arasındaki ilişkiyi yazarak açıklayınız.

Umut:

Türker:

Ulaş:

5) 4. soruda yazdığınız herbir ilişkiyi denklem ile gösteriniz.

Umut:

Türker:

Ulaş:

6) Öğrencilerin yürüme hızları bu denklemi nasıl etkilemektedir?

7) Yazdığınız bu denklemleri kullanarak, her bir öğrencinin 1 dakikada yürüyebileceği mesafeyi hesaplayınız.

8) Bu denklemleri kullanarak, okul çevresindeki yürüyüşü herbir öğrencinin ne kadar zamanda tamamlayabileceğini hesaplayınız.

17. ETKİNLİK

Etkinlikle Örtüşen MEB Hedefleri:
Hedef 1: Matematiksel ifadeleri kavrayabilme.
Hedef 2: Önerme, açık önerme ve denklemleri kavrayabilme.
Hedef 6: Düzlemde bir noktanın koordinatlarını kavrayabilme.
Hedef 7: Grafik çizebilme
Ek Hedefler:
Öğrencilerin,
. doğrusallık kavramı ile gerçek yaşam durumları arasındaki ilişkiyi kurma,
. veriyi düzenlemek için tablo kullanma,
. grafik çizebilme,
. tablo, grafik, denklem arasındaki ilişkiyi sezebilme,
becerilerini geliştirme.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Düzlemde bir noktanın koordinatları
Doğru grafikleri.
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı.
Materyal, Araç-Gereç:Etkinlik sayfası, pamuk çubuklar.
Etkinlik Süresi: 40 dakikalık bir ders saati
Kullanılan gösterim Biçimleri:

	Çizim	Materyal	Tablo	Cebirsel	Grafik
Çizim		x			
Materyal			x		
Tablo				x	
Cebirsel					x

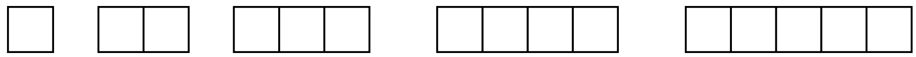
İşleniş: Öğretmen öğrencilerin sıra arkadaşları ile grup olmalarını sağlar ve onlara önceden hazırladığı etkinlik sayfalarını ve pamuk çubukları dağıtır. Öğrencilerden etkinlik sayfasındaki problemi okumalarını ve anlaşılmayan noktaları sormalarını ister. Öğrencilerden gelen olası soruları yanıtladıktan sonra, öğretmen öğrencileri etkinlik ile uğraşmaları için serbest bırakır. Öğretmen gruplar arasında dolaşarak, öğrencilerin sorularına yanıt verir.

ETKİNLİK SAYFASI

Ad-Soyad:

Aşağıdaki kare ve düzgün sekizgenleri pamuk çubukları kullanarak sıranın üzerinde oluşturun.

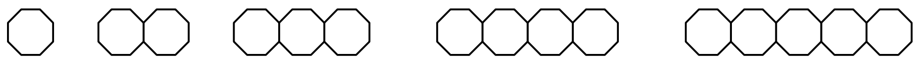
A)



Error!

Şekil 1 Şekil 2 Şekil 3 Şekil 4 Şekil 5

B)



Şekil 1 Şekil 2 Şekil 3 Şekil 4 Şekil 5

Kenar uzunlukları 1 birim olan bu geometrik şekillerin, şekil sayıları ile çevreleri arasında bir ilişki vardır. Bu ilişkiyi göstermek için;

1) Kareler ve düzgün sekizgenler için şekil sayısı ve çevre uzunluğunun yer aldığı bir tablo oluşturun.

Kareler için; Düzgün sekizgenler için;

2) Şekil sayısı ve çevre uzunluğu arasındaki ilişkiyi gösteren denklemleri her bir şekil için yazınız.
Kareler için;

Düzgün Sekizgenler için;

3) 25 karenin yanyana gelmesiyle oluşan şeklin çevresi kaç çubuktan oluşur?

4) 50 düzgün sekizgenin yanyana gelmesiyle oluşan şeklin çevresi kaç çubuktan oluşur?

5) n tane karenin yanyana gelmesiyle oluşan şeklin çevresi kaç çubuktan oluşur?

6) Yazdığınız denklemlerin grafiklerini, x ekseni şekil sayısını y ekseni çevre uzunluğunu gösterecek şekilde tek bir koordinat ekseninde çiziniz. (Çizim için ekteki kareli kağıdı kullanabilirsiniz.)

18. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
Hedef 1: Matematiksel ifadeleri kavrayabilme.
Hedef 2: Önerme, açık önerme ve denklemleri kavrayabilme.
Hedef 6: Düzlemde bir noktanın koordinatlarını kavrayabilme.
Hedef 7: Grafik çizebilme
Ek Hedefler:
Öğrencilerin,
. doğrusallık kavramını farkedebilme,
. veriyi düzenlemek için tablo kullanma,
. grafik çizebilme,
. tablo, grafik, denklem arasındaki ilişkiyi sezebilme,
becerilerini geliştirme.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Düzlemde bir noktanın koordinatları
Doğru grafikleri.
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, bilinmeyen kavramı.
Materyal, Araç-Gereç:Etkinlik sayfası, dama taşları.
Etkinlik Süresi: 80 dakikalık bir ders saati
Kullanılan Gösterim Biçimleri:

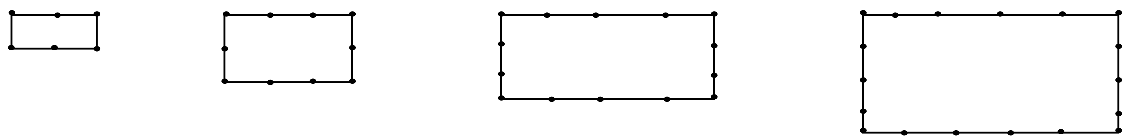
	Çizim	Materyal	Tablo	Sözel	Cebirsel	Grafik
Çizim		x				
Materyal			x			
Tablo				x		
Sözel					x	
Cebirsel						x

İşleniş: Öğretmen öğrencilerin sıra arkadaşları ile grup olmalarını sağlar ve onlara önceden hazırladığı etkinlik sayfalarını ve dama taşlarını dağıtır. Öğrencilerden etkinlik sayfasındaki problemi okumalarını ve anlaşılmayan noktaları sormalarını ister. Öğrencilerden gelen olası soruları yanıtladıktan sonra, öğretmen öğrencileri etkinlik ile uğraşmaları için serbest bırakır. Öğretmen gruplar arasında dolaşarak, öğrencilerin sorularına yanıt verir.

ETKİNLİK SAYFASI

Ad-Soyad:

Size dağıtılmış olan taşları kullanarak aşağıda verilen fotoğraf çerçevesi desenini oluşturun.



1. Şekil 2. Şekil 3. Şekil 4. Şekil

1) Şekil numarası ile fotoğraf çerçevesini oluşturmak için kullanılan taş sayısı arasındaki ilişkiye ait verileri tablo halinde aşağıya çiziniz.

2) Oluşturduğunuz fotoğraf çerçevelerinde, şeklin çerçevesinde bulunan taş sayısı ile şekil numarası arasında bir ilişki var mı? Varsa bu ilişkiyi yazarak anlatınız.

3) 4'ten fazla şeklin yapılmasında kaç adet dama taşı kullanılacağını hesaplamak için gereken denklemi yazınız.

4) Ekteki kareli kağıda tablodaki tüm sayıları içeren grafiği çiziniz. Grafikte x eksenini şekil numarası, y eksenini ise çerçeveyi oluşturmak için kullanılan taş sayısı olarak alabilirsiniz.

5) Bu denklemi kullanarak, 43. şekli oluşturmak için kaç adet dama taşı gerektiğini hesaplayınız.

6) Kaçıncı şekilde, fotoğraf çerçevesini oluşturmak için 66 dama taşı kullanmak gereklidir?

19. ETKİNLİK

Etkinlikle örtüşen MEB Hedefleri:
1. Hedef: Matematiksel ifadeleri kavrayabilme.
3. Hedef: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
Ek Hedefler: Öğrencilerin,
. verilen sözel bir ifadeyi cebirsel bir ifadeye dönüştürme,
. gerçek hayat durumlarını matematiksel durumlara uygulama,
becerilerini değerlendirme.
Konu: Denklemler
Ön Koşul Bilgileri: Rasyonel sayılarda dört işlem, birinci dereceden bir bilinmeyenli denklemler.
Materyal, Araç-Gereç:Etkinlik sayfası.
Etkinlik Süresi: 80 dakikalık bir ders saati

Kullanılan gösterim biçimleri:

	Gerçek Hayat Durumu	Sözel	Çizim	Cebirsel
Gerçek Hayat Durumu			x	x
Sözel				x
Çizim				
Cebirsel		x		

İşleniş: Öğretmen bugün yapacakları etkinliğin amacını duyurur ve sınıfla beraber etkinlik sayfasındaki problemleri tahtada çözerler.

ETKİNLİK SAYFASI

Bu etkinlik sayfası kapsamında öğrenciler hep beraber aşağıdaki gibi çeşitli problemleri tahtada çözerler.

1) 3 katının yarısı 37 olan sayı kaçtır?

2) Toplamları 12, farkları 27 olan sayıları bulalım.

3) Masal'ın yaşının 7 katı, babasının yaşına eşittir. Masal ile babasının yaşlarının toplamı 40 olduğuna gore, Masal'ın yaşı nedir?

4) Ardışık 3 doğal sayının toplamı 33 ise, bu sayıların çarpımını bulunuz.

5) Nurdan ile Gülden'in iki yıl önceki yaşlarının toplamı 46'dır. Nurdan'ın yaşının 2 katı Gülden'in yaşına eşit olduğuna gore, Gülden'in yaşı kaçtır?

6) "13x+30" 'u ifade edecek bir günlük hayat durumu söyleyelim.

7) Bir dikdörtgenin uzun kenarı kısa kenarından 4 fazladır. Bu dikdörtgenin çevresi 48 cm. ise uzun kenarı kaç cm dir? Çizerek gösteriniz.

20. ETKİNLİK

Etkinliğin Örtüştüğü MEB Hedefleri:
Hedef 3: Birinci dereceden bir bilinmeyenli denklemleri çözebilme.
Hedef 4: Birinci dereceden bir bilinmeyenli eşitsizlikleri çözebilme.
Ek Hedefler:
Öğrencilerin,
. veriyi düzenlemek için tablo kullanma,
becerilerini geliştirme.
Konu:
Birinci dereceden bir bilinmeyenli denklemler
Birinci dereceden bir bilinmeyenli eşitsizlikler
Ön Koşul Bilgileri: Doğal sayılarda dört işlem, tablo okuma, bilinmeyen kavramı.
Materyal, Araç-Gereç:Etkinlik sayfası.
Etkinlik Süresi: 40 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

	Tablo	Cebirsel
Tablo		x

İşleniş: Öğretmen bu etkinliğin bireysel olduğunu öğrencilere duyurup, önceden hazırladığı etkinlik sayfalarını dağıtır. Öğrencilerden etkinlik sayfasındaki tabloyu doldurmalarını ve verilen soruları bu tabloya göre yanıtlamalarını ister. Etkinlik süresince, öğretmen gruplar arasında dolaşarak, öğrencilerin sorularına yanıt verir. Etkinlikten sonra öğretmen tahtada eşitsizlik çözümleri ile ilgili kitaptan örnek çözecektir.

ETKİNLİK SAYFASI

Ad-Soyad:

Aşağıdaki tabloyu belirtilen a değerini esas alarak doldurunuz.

a	6-3a	3(2-a)	(a-4)/3	-6a	a+14
-5					
-4					
-3					
-2					
-1					
0					
1					
2					
3					
4					
5					

Tabloya göre;

1) a'nın hangi değeri için, -6a = a+14 olur.

2) a'nın hangi değeri için, a+14 = 6-3a olur.

3) a'nın hangi değeri için, -6a < a+14 olur.

4) a'nın hangi değeri için, 6-3a > 3(2-a) olur.

5) a'nın hangi değeri için, (a-4)/3 > -6a olur.

6) 3. soruda bulduğunuz a değerinin doğruluğunu belirtilen eşitsizliği çözerek kontrol ediniz.

21. ETKİNLİK

Etkinlikle Örtüşen MEB Hedefleri:
Hedef 6: Düzlemde bir noktanın koordinatlarını kavrayabilme.
Hedef 7: Grafik çizebilme
Ek Hedefler:
Öğrencilerin,
. grafiksel gösterimi sözel ifadeye çevirebilme,
. sözel ifadeyi grafiksel gösterime çevirebilme,
becerilerini geliştirme.
Konu:
Düzlemde bir noktanın koordinatları
Doğru grafikleri.
Ön Koşul Bilgileri:
Materyal, Araç-Gereç: Etkinlik sayfası
Etkinlik Süresi: 40 dakikalık bir ders saati

Kullanılan Gösterim Biçimleri:

	Gerçek Yaşam Durumu	Sözel	Grafik
Gerçek Yaşam Durumu			x
Sözel			x
Grafik	x	x	

İşleniş: Öğretmen öğrencilerin sıra arkadaşları ile grup olmalarını sağlar ve onlara önceden hazırladığı etkinlik sayfalarını dağıtır. Öğrencilerden etkinlik sayfasını okumalarını ve anlaşılmayan noktaları sormalarını ister. Öğrencilerden gelen olası soruları yanıtladıktan sonra, öğretmen öğrencileri etkinlik ile uğraşmaları için serbest bırakır. Öğretmen gruplar arasında dolaşarak, öğrencilerin sorularına yanıt verir.

ETKİNLİK SAYFASI

Ad-Soyad:

A) ÖYKÜLERDEN GRAFİKLERE

Bu kısımda sizden, verilen çeşitli durumları okuyarak, bunları ifade edecek grafikleri çizmeniz bekleniyor.

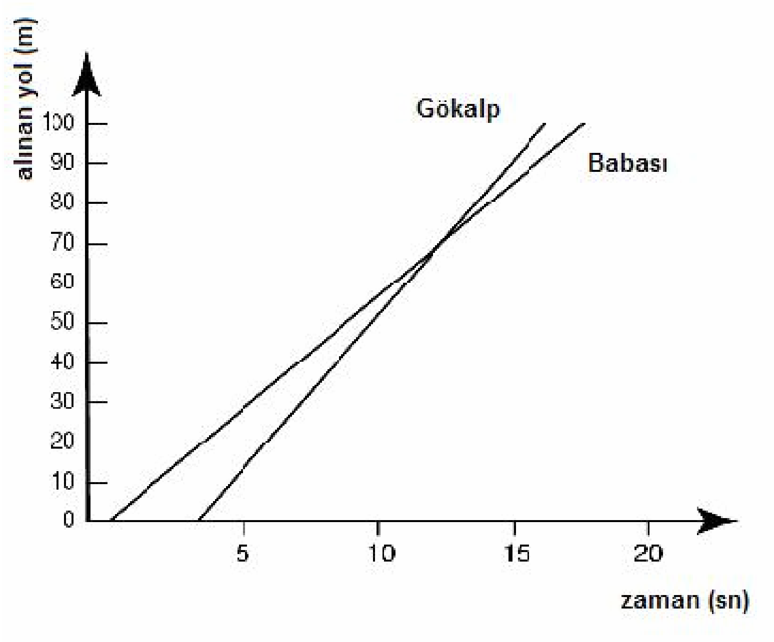
1) Okuldan eve bisikletle dönmeyi seven Eylül, toplam 6 kilometrelik yolun yarısını 30 dakikada alıp arkadaşının evinin önüne gelmiştir. Tam burada kaskı düşen Eylül, kaskını tekrar takmak için durup 5 dakika harcamış ve 30 dakika daha yola devam edip evine varmıştır.

2) Haftasonu tatili için büyükbabanızın evine gidip onun küçük bahçesindeki çimleri biçtiğinizi düşünelim. Siz çimleri biçtikçe bahçede kesilmesi gereken çim miktarı azalacaktır. Bahçedeki çimlerin yarısı bitene kadar hep aynı hızda çimleri biçtiğinizi düşünelim. Sonra yorulup kısa bir su molası veriyorsunuz ve aynı hızda devam ederek çimlerin hepsini biçiyorsunuz.

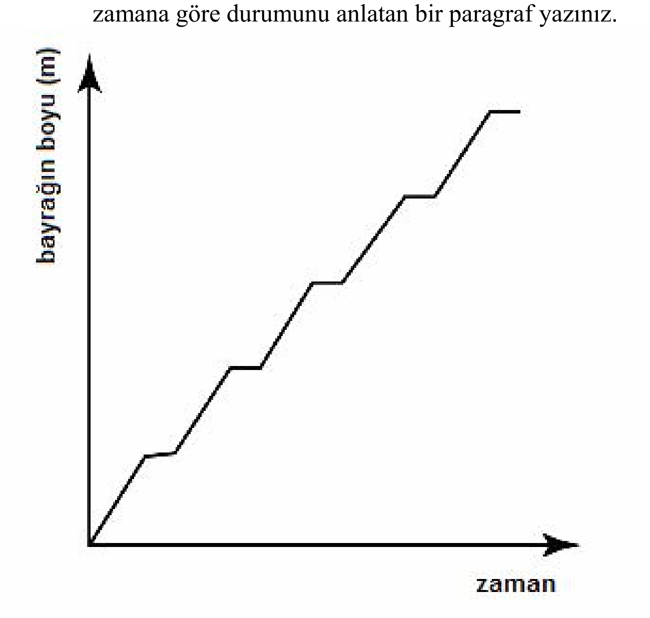
B) GRAFİKLERLERDEN ÖYKÜLERE

Bu kısımda sizden, verilen grafikleri okuyup yorumlamanız ve grafiğin ifade ettiği durumları sözcükler kullanarak yzmanız bekleniyor.

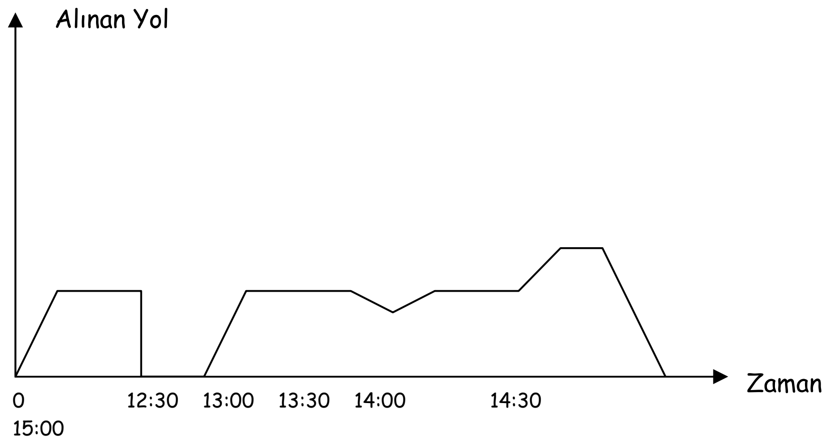
1) Gökalp ve babası 100 metrelik bir koşu yarışması yapmaya karar veriyorlar. Gökalp, babasından 3 saniye sonra koşmaya başlıyor. Aşağıdaki grafik Gökalp ve babasının bu yarışta ne kadar uzağa koştuğu hakkında bilgi vermektedir. Bu grafiğe bakarak, kimin yarışı kazanacağını söyleyebilir misiniz?



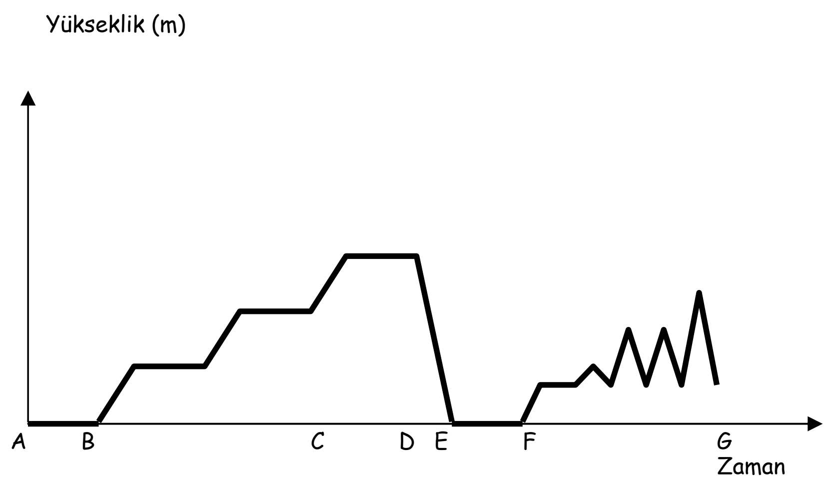
2) Aşağıdaki grafik bir bayrağın göndere çekilmesini anlatıyor. Bu grafikten yola çıkarak, bayrağın zamana göre durumunu anlatan bir paragraf yazınız.



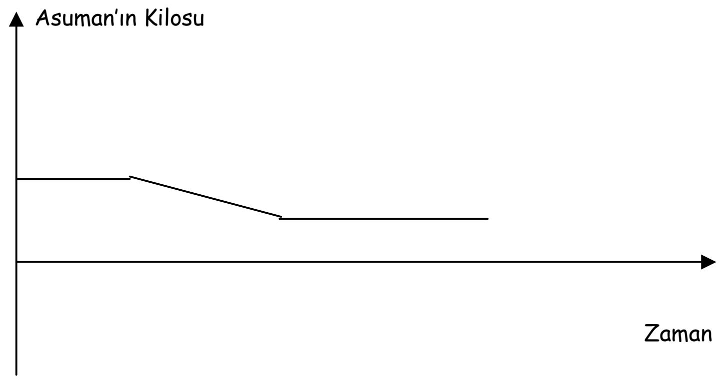
3) Cumartesi günü dedesine gitmek üzere yola çıkan Burak, yürümeye başlar. Dedesine vardığında saat 15:00'dır. Aşağıdaki grafik Burak'ın zamana göre hızını göstermektedir. Böyle bir grafik çizilebilmesi için, Burak'ın dedesine gelene kadar neler yapmış olması gereklidir?



4) Sude Pazar günü ailesi ile beraber çocuk parkına gider. Aşağıdaki grafikte Sude'nin oyun parkında bir oyuncak ile oynarken, yerden yüksekliğini belirtmektedir. Buna göre, grafiğin her aralığı için Sude'nin hangi oyuncakla oynamış olabileceğini yazıp, nedenini açıklayınız.



5) Aşağıdaki grafik Asuman'ın 12 gündeki kilosunun zamana göre değişimini göstermektedir. Buna göre; 12 günlük süre içinde Asuman'ın kilosundaki değişimi anlatınız.



APPENDIX N
RAW DATA OF THE STUDY

NO	GROUP	GENDER	AGE	MGPS	PRECDAT	PREATMS	POSTCDAT	POSTATMS	TRST	AAT
1	1	1	13	4	27	55	27	58	34	26
2	1	1	14	2	32	47	24	37	42	25
3	1	1	13	3	19	42	33	36	31	30
4	1	1	14	4	26	50	36	44	38	38
5	1	1	13	4	38	43	41	65	42	34
6	1	2	13	4	29	33	31	70	28	30
7	1	2	13	3	29	51	25	53	26	31
8	1	2	13	5	35	65	42	64	38	36
9	1	1	12	4	24	64	42	69	23	28
10	1	2	13	4	31	69	41	67	31	28
11	1	1	13	4	25	57	30	67	35	34
12	1	1	13	5	29	72	47	73	35	40
13	1	1	13	4	29	65	42	73	41	40
14	1	1	13	3	14	48	22	35	23	27
15	1	2	13	4	30	48	36	55	30	31
16	1	1	13	3	24	41	35	50	42	34
17	1	1	13	4	24	72	36	74	33	26
18	1	1	13	4	28	63	37	67	38	32
19	1	1	13	5	40	62	47	72	42	38
20	1	1	13	5	37	72	44	74	37	38
21	1	2	12	5	31	70	43	78	33	39
22	1	1	13	4	34	72	25	64	35	26
23	1	2	13	4	33	40	39	52	21	35
24	1	1	12	5	32	61	39	57	31	25
25	1	2	13	3	13	53	27	50	10	30
26	1	2	12	4	14	58	33	66	24	33
27	1	2	13	5	34	70	45	71	39	34
28	1	1	13	4	33	66	36	59	31	34
29	2	2	13	3	18	65	19	65	18	22
30	2	2	12	5	16	78	11	73	22	33
31	2	2	13	5	25	80	21	74	34	40
32	2	2	13	2	10	47	19	33	3	17
33	2	2	12	4	18	72	9	73	27	30
34	2	2	13	4	12	38	18	39	27	31
35	2	2	13	5	19	56	23	59	31	32
36	2	1	13	4	18	71	17	66	20	23
37	2	1	14	1	2	64	23	36	4	1
38	2	2	13	5	25	76	20	78	41	38
39	2	2	13	3	11	45	16	46	11	23
40	2	1	13	5	21	80	17	76	27	32
41	2	2	13	5	12	74	19	67	26	27
42	2	1	13	5	24	68	31	66	33	36
43	2	1	13	4	27	77	11	75	32	33
44	2	1	13	3	10	41	16	44	5	17
45	2	1	13	2	5	59	23	54	6	12
46	2	1	13	5	24	59	18	60	34	34
47	2	1	13	4	16	57	19	54	26	24
48	2	1	15	1	1	41	17	47	2	1
49	2	2	13	4	25	53	16	37	21	29

50	2	1	14	5	30	80	22	79	33	34
51	2	1	14	4	14	59	31	53	17	16
52	2	1	13	4	22	60	31	57	21	25
53	2	2	13	3	12	45	19	46	11	23
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55	2	2	13	4	16	58	17	61	23	28
56	2	1	13	1	31	49	23	44	3	3
57	2	1	14	2	30	64	7	61	9	6
58	1	2	13	3	25	12	30	18	30	12
59	1	1	13	5	30	59	19	57	34	31
60	1	1	13	2	14	44	31	41	41	20
61	1	2	13	3	12	53	30	53	31	30
62	1	2	13	4	38	62	24	58	40	40
63	1	2	13	3	18	59	34	59	19	21
64	1	2	13	3	16	47	38	39	16	15
65	1	2	13	3	18	45	26	54	19	7
66	1	1	13	3	23	61	28	55	21	23
67	1	1	13	4	35	52	31	21	39	22
68	1	1	13	5	41	74	33	73	36	36
69	1	2	13	5	42	72	32	80	40	40
70	1	1	13	5	26	66	45	61	35	34
71	1	1	13	2	26	60	40	60	28	30
72	1	2	13	3	29	51	17	58	29	31
73	1	1	13	5	29	53	41	54	28	27
74	1	2	13	2	17	51	42	53	23	9
75	1	1	14	4	25	68	46	54	38	34
76	1	2	13	4	26	51	46	50	31	25
77	1	2	13	5	13	72	40	66	20	31
78	1	1	13	5	40	61	46	58	37	24
79	1	2	13	5	30	75	48	78	23	30
80	1	1	13	4	11	62	26	56	36	22
81	1	1	13	5	31	67	40	68	29	27
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83	1	1	13	5	41	68	48	67	41	35
84	1	1	13	5	24	77	35	67	35	28
85	1	1	15	2	16	47	36	21	11	13
86	1	2	13	1	10	26	36	30	12	8
87	1	1	13	5	20	71	32	70	38	24
88	1	2	13	4	18	41	35	27	25	18
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91	1	2	13	2	19	33	41	30	27	25
92	1	1	13	5	25	48	25	58	31	29
93	1	1	13	2	17	64	17	52	6	5
94	1	1	13	4	19	73	19	75	21	24
95	1	2	13	5	25	44	25	46	40	29
96	2	1	13	3	25	59	29	27	12	23
97	2	2	13	4	7	53	22	50	47	35
98	2	2	14	5	33	45	11	42	29	22
99	2	2	13	3	13	19	21	15	30	24
100	2	2	13	5	17	35	43	26	34	22
101	2	2	13	4	24	38	7	40	12	7

102	2	2	13	4	26	38	23	36	16	20
103	2	1	13	2	13	38	46	37	22	7
104	2	2	13	3	29	75	42	11	27	31
105	2	1	13	3	20	62	39	43	25	12
106	2	1	14	5	34	38	41	45	26	24
107	2	2	13	4	26	76	26	79	18	21
108	2	1	13	5	25	56	17	56	28	3
109	2	1	13	5	28	70	36	74	25	7
110	2	1	13	4	16	21	37	45	26	7
111	2	1	13	5	31	51	22	54	29	6
112	2	1	13	4	18	36	19	42	36	29
113	2	2	13	4	22	60	31	55	10	36
114	2	1	13	5	19	72	31	67	4	35
115	2	1	14	5	25	28	33	35	5	33
116	2	1	13	3	31	49	31	72	7	22
117	2	1	13	2	23	48	30	68	29	5
118	2	1	13	5	29	80	23	75	19	8
119	2	2	13	5	25	56	25	49	19	13
120	2	1	13	5	28	71	26	72	22	4
121	2	1	13	5	28	32	43	43	26	29
122	2	1	13	2	14	47	43	38	20	9
123	2	2	13	4	34	70	40	68	15	14
124	2	1	13	2	23	56	30	48	32	3
125	2	2	13	4	19	56	16	56	5	18
126	2	2	13	4	18	68	22	70	5	21
127	2	1	13	5	28	36	17	58	24	24
128	2	1	13	4	40	21	26	35	14	29
129	2	2	13	3	23	49	19	55	24	27
130	2	2	12	4	20	38	23	67	24	16
131	2	2	13	2	23	35	18	46	23	5

APPENDIX O

RUBRIC TO EVALUATE AAT

MATHEMATICS SCORING RUBRIC: A GUIDE TO SCORING EXTENDED-RESPONSE ITEMS			
Score Level	MATHEMATICAL KNOWLEDGE Knowledge of mathematical principles and concepts which result in a correct solution to a problem.	STRATEGIC KNOWLEDGE Identification of important elements of the problem and the use of models, diagrams, symbols and /or algorithms to systematically represent and integrate concepts.	EXPLANATION Written explanation and rationales that translate into words the steps of the solution process and provide justification for each step. Though important, the length of response, grammar and syntax are not the critical elements of this dimension.
4	• shows complete understanding of the problem's mathematical concepts and principles • uses appropriate mathematical terminology & notations including labeling the answer if appropriate; that is, whether or not the unit is called for in the stem of the item • executes algorithms completely and correctly	• identifies all the important elements of the problem and shows complete understanding of the relationships among elements • reflects an appropriate and systematic strategy for solving the problem • gives clear evidence of a complete and systematic solution process	• gives a complete written explanation of the solution process employed; explanation addresses both what was done and why it was done • may include a diagram with a complete explanation of all its elements
3	• shows nearly complete understanding of the problem's mathematical concepts and principles • uses nearly correct mathematical terminology and notations • executes algorithms completely; computations are generally correct but may contain minor errors	• identifies most of the important elements of the problem and shows general understanding of the relationships among them • reflects an appropriate strategy for solving the problem • solution process is nearly complete	• gives a nearly complete written explanation of the solution process employed; clearly explains what was done and begins to address why it was done • may include a diagram with most of the elements explained

2	• shows some understanding of the problem's mathematical concepts and principles • may contain major computational errors	• identifies some important elements of the problem but shows only limited understanding of the relationships among them • appears to reflect an appropriate strategy but the application of strategy is unclear, or a related strategy is applied logically and consistently • gives some evidence of a solution process	• gives some written explanation of the solution process employed, either explains what was done or addresses why it was done; explanation is vague or difficult to interpret • may include a diagram with some of the elements explained
1	• shows limited to no understanding of the problem's mathematical concepts and principles • may misuse or fail to use mathematical terms • may contain major computational errors	• fails to identify important elements or places too much emphasis on unimportant elements • may reflect an inappropriate or inconsistent strategy for solving the problem • gives minimal evidence of a solution process; process may be difficult to identify • may attempt to use irrelevant outside information	• gives minimal written explanation of the solution process; may fail to explain what was done and why it was done • explanation does not match the presented solution process • may include minimal discussion of the elements in a diagram; explanation of significant elements is unclear
0	• no answer attempted	• no apparent strategy	• no written explanation of the solution process is provided

APPENDIX P

RUBRIC TO EVALUATE TRST

Score levels	Indicators
3 Points	A three-point response is complete and correct. This response • demonstrates a thorough understanding of the mathematical concepts and/or procedures embodied in the task. • the representations are correct. • indicates that the student has completed the task correctly, using mathematically sound procedures. • contains clear, complete explanations and/or adequate work when required.
2 Points	A two-point response is partially correct. This response • demonstrates partial understanding of the mathematical concepts and/or procedures embodied in the task. • the representations are essentially correct. • addresses most aspects of the task, using mathematically sound procedures. • may contain an incorrect solution but applies a mathematically appropriate process with valid reasoning and/or explanation. • may contain a correct solution but provides incomplete procedures, reasoning, and/or explanations. • may reflect some misunderstanding of the underlying mathematical concepts and/or procedures.
1 Point	A one-point response is incomplete and exhibits many flaws but is not completely incorrect. This response • demonstrates only a limited understanding of the mathematical concepts and/or procedures embodied in the task. • The representations are partially correct. • may address some elements of the task correctly but reaches an inadequate solution and/or provides reasoning that is faulty or incomplete. • exhibits multiple flaws related to a misunderstanding of important aspects of the task, misuse of mathematical procedures, or faulty mathematical

	reasoning. • reflects a lack of essential understanding of the underlying mathematical concepts. • may contain a correct numerical answer but required work is not provided.
0 Points	A zero-point response is completely incorrect, irrelevant or incoherent, or a correct response that was arrived at using an obviously incorrect procedure.

VITA

PERSONAL INFORMATION

Surname, Name: Çıkla-Akkuş, Oylum

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EDUCATION

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MS	Hacettepe University-Measurement and Evaluation in Education	2000
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WORK EXPERIENCE

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1998- Present	Hacettepe University, Elementary Mathematics Education Department	Research Assistant
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