RANKING UNITS BY TARGET-DIRECTION-SET VALUE EFFICIENCY ANALYSIS AND MIXED INTEGER PROGRAMMING

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

TAYYAR BÜYÜKBAŞARAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

APRIL 2005

Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science

Prof. Dr. Çağlar Güven
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science

Prof. Dr. Murat Köksalan
Supervisor

Examining Committee Members

Assist. Prof. Dr. Sedef Meral	(METU, IE)	
Prof. Dr. Murat Köksalan	(METU, IE)	
Prof. Dr. Jyrki Wallenius	(HSE, Int. MBA)	
Dr. Seçil Şavaşaneril	(METU, IE)	
Prof. Dr. Meral Azizoğlu	(METU, IE)	

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Name, Last name : Tayyar Büyükbaşaran

Signature :

ABSTRACT

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AND
MIXED INTEGER PROGRAMMING

Büyükbaşaran, Tayyar

M.Sc., Department of Industrial Engineering

Supervisor: Prof. Dr. Murat Köksalan

April 2005, 111 pages

In this thesis, two methods are proposed in order to rank units: Target-direction-set value efficiency analysis (TDSVEA) and mixed integer programming (MIP) technique. Besides its ranking ability based on preferences of a decision maker (DM), TDSVEA, which modifies the targeted projection approach of Value Efficiency Analysis (VEA) and Data Envelopment Analysis (DEA), provides important information to analyzer: targets and distances of units from these targets, proposed input allocations in order to project these targets, the lack of harmony between the DM and the manager of the unit etc. In MIP technique, units select weights of the criteria from a feasible weight space in order to outperform maximum number of other units. Units are then ranked according to their outperforming ability. Mixed integer programs in this technique are simplified by domination and weight-domination relations. This simplification procedure is further simplified using transitivity between relations. Both TDSVEA and MIP technique are applied to rank research universities and these rankings are compared to those of other ranking techniques.

Keywords: Ranking, Target-direction-set Value Efficiency Analysis (TDSVEA), Mixed Integer Programming (MIP) technique, University ranking

ÖZ

HEDEF-YÖN-TESPİTLİ FAYDASAL ETKİNLİK ANALİZİ
VE
KARMA TAM SAYILI PROGRAMLAMA
İLE
ÜNİTELERİN SIRALANMASI

Büyükbaşaran, Tayyar

Yüksek Lisans, Endüstri Mühendisliği Bölümü

Tez Yöneticisi: Prof. Dr. Murat Köksalan

Nisan 2005, 111 sayfa

Bu tez çalışmasında, ünitelerin sıralanması için iki metot önerilmiştir: hedef-yön-tespitli faydasal etkinlik analizi (TDSVEA) ve karma tam sayılı programlama (MIP) tekniği. Faydasal etkinlik analizi (VEA) ve Veri Zarflama Analizi'nin (DEA) hedef tespiti ve hedefe yönelme yaklaşımlarını geliştiren TDSVEA, üniteleri karar vericinin (DM) tercihlerine göre sıralamanın yanı sıra, bazı önemli ek bilgiler de sağlar: ünitelerin hedefleri ve bu hedefe olan uzaklıkları, bu hedefe ulaşmak için yönelecekleri girdi dağılımı, DM ve ünite yöneticisi arasındaki uyum eksikliği vb. Karma tam sayılı programlama tekniğinde, üniteler en fazla sayıda üniteden daha iyi olabilecek şekilde, kriterlere, olurlu ağırlık uzayından ağırlık seçerler. Üniteler daha iyi olma kabiliyetlerine göre sıralanırlar. Bu teknikte yer alan karma tam sayılı programlar, baskınlık ve olurlu ağırlıklar göz önünde bulundurulduğunda baskınlık ilişkileri sayesinde basitleştirilmektedir. Bu basitleştirme işlemi, ilişkiler arasında kopyalamalar sayesinde ayrıca basitleştirilmiştir. TDSVEA ve MIP tekniği, araştırma üniversitelerinin sıralanmasında uygulanmış ve bu sıralamalar diğer sıralama yöntemlerinin sıralamalarıyla karşılaştırılmıştır.

Anahtar Kelimeler: Sıralama, hedef-yön-tespitli faydasal etkinlik analizi (TDSVEA), karma tam sayılı programlama (MIP) tekniği, üniversitelerin sıralanması

To my wife, my parents and my sister...

ACKNOWLEDGMENTS

The author wishes to express his deepest gratitude to his supervisor Prof. Dr. Murat Köksalan for his guidance, advice, criticism, encouragements and insight throughout the research.

The author would like to show his gratitude to jury members, especially to Prof. Dr. Jyrki Wallenius from International MBA program of Helsinki School of Economics, for insightful criticisms and suggestions.

The author would like to thank to his parents, Musa Büyükbaşaran and Nermin Büyükbaşaran, and sister, Gül Büyükbaşaran, for their unlimited belief, love and support.

The author also wishes to present special thanks to his wife, Zeycan Esra Büyükbaşaran, for her peerless love and moral support.

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LIST OF ABREVIATIONS

D:	Domination Matrix
DEA:	Data Envelopment Analysis
DM:	Decision maker
DMU:	Decision making unit
GDM:	Global decision maker
MCDM:	Multi-Criteria Decision Making
MIP:	Mixed Integer Program(ming)
MOLP:	Multi-Objective Linear Program(ming)
MPS:	Most preferred solution
PVE:	Potential Value Efficiency
TDSVE(A):	Target-direction-set Value Efficiency (Analysis)
VE(A):	Value Efficiency (Analysis)

CHAPTER 1

INTRODUCTION

Ranking different alternatives according to some criteria is one of the frequently encountered real life problems. When there exists only one criterion and score of each alternative in that criterion is apparent and easily quantifiable, ranking is easy. However, when there are more than one criterion, the ranking problem is not be that easy. Different techniques are developed in the literature in order to rank alternatives when there exist multiple criteria; some of them are simple such as fixed weight technique while some of them are more sophisticated.

This study proposes two different ranking techniques: one is Target-direction-set Value Efficiency Analysis (TDSVEA), which is a modification of Data Envelopment Analysis (DEA) and of its extension - Value Efficiency Analysis (VEA), and the other gets its name by the tool it uses, i.e. mixed integer programming (MIP) technique.

Besides its ranking ability based on the preference information of a decision maker (DM), TDSVEA provides important information to analyzer depending on the context. It helps the DM in defining targets for the ranked units and shows how 'far' the units are from these targets. It may show how the input allocation should be in order to attain DM's preferred targets. It may also identify the possible disharmony between the DM and managers of ranked units.

In MIP technique, parallel to DEA methodology, the alternatives select their own weights for the criteria from a weight space in order to outperform maximum number of other alternatives. Mixed integer programs of this technique are simplified by domination and weight-domination relations. Moreover, obtaining these relations is simplified by copying relations from other relations based on transitivity.

After the introduction in this chapter, the literature is reviewed briefly on DEA, some of DEA’s extensions, and ranking techniques in Chapter 2.

In Chapter 3, MIP procedure, its simplification and simplification of its simplification are presented.

In Chapter 4, TDSVEA is introduced. Additional information provided by TDSVEA is explained.

In Chapter 5, for test purposes, TDSVEA and MIP procedure are used to rank the research universities, and the results compared with other techniques.

In Chapter 6, the study is concluded and further research areas are discussed.

CHAPTER 2

LITERATURE REVIEW

In this chapter, the literature is reviewed on Data Envelopment Analysis (DEA) and Multi Criteria Decision Making, on the efficiency measures of DEA, on some extension of DEA, and on some ranking methods.

2.1. Data Envelopment Analysis and Multiple Criteria Decision Making:

Data Envelopment Analysis (DEA), proposed by Charnes et al. (1978), assesses the efficiency of similar units (decision making units – DMUs) considering inputs which they consume and outputs which they produce. The scope of DMU types are very wide: can change from cities (Charnes et al. 1989) to the students (Korhonen et al. 2003), from the banks (Charnes et al. 1990) to the academic departments in a university (Sinuany-Stern et al. 1994 and Belton and Vickers 1992), from agriculture (Battese 1992) to production and service industry (Seiford 1997 and Tavares 2002). The aim of DEA "is not in general to … select one DMU; the intention is rather to identify which are not 'efficient' in some sense, and to assess where the efficiencies arise" (Stewart 1996, pp. 654). Ease of application and wideness of scope of DEA made it one of the most used techniques in productivity and efficiency analysis. Detailed bibliographies of DEA are Seiford (1997) and Tavares (2002).

In some Multiple Criteria Decision Making (MCDM) problems, it is assumed that there exist some alternatives from which a Decision Maker (DM) selects. These alternatives are evaluated according to their performance on each of the criteria. The aim of MCDM in this case is to provide support to the DM in the process of making the choice between alternatives (Stewart, 1996), and includes some types of selection: ranking and outranking of the alternatives (Keeney and Raiffa 1976, Roy 1991), finding the most preferred solution (MPS) (Zionts 1981, Köksalan et al. 1984, Köksalan and Rizi 2001), finding the most preferred subset of alternatives (Köksalan, 1989), or placing alternatives in preference classes (Köksalan and Ulu, 2003).

Although two of the three pioneers of DEA had also developed Goal Programming (Charnes and Cooper, 1961), which is an important development of Multiple Criteria Decision Making, researchers of DEA and MCDM did not cooperate much until the second half of 1990s (Joro et al., 1998). Before this time, Golany (1988) used an interactive approach of MCDM to identify the solutions on the efficient frontier in DEA. Kornbluth (1991) identified that DEA models can be stated as Multi–Objective Linear Fractional Programming models. Belton and Vickers (1992, 1993) used another visual interactive approach to DEA context. They also argued that "… there were [are] a number of ways in which practitioners of DEA and MCDA [MCDM] could [can] learn from each other" (Belton and Vickers 1993, pp. 883). Doyle and Green (1993) used DEA methodology and concepts in order to solve a bi-criteria problem with "aggressive formulation". They also called researchers on MCDM fields to take notice of the material the two fields have in common. Two parallel but distinct organized researches to compare and contrast MCDM and DEA come after mid 1990s. Stewart (1996) compares the aim, the model formulation and the efficiency concept of DEA and MCDM and identifies some linkages between the two fields. Joro et al. (1998) identifies that despite the differences in terminology, DEA and Multiple Objective Linear Programming (an important tool in MCDM) address very similar problem. They also show that these two models are significantly similar in structure in generating efficient solutions. They conclude, "DEA and MOLP should not be seen as substitutes, but rather as complements. We show that they cross-fertilize each other. MOLP provides interesting extensions to DEA and DEA provides new areas of application to MOLP." (Joro et al. 1998, pp. 962).

Bouyssou (1999) studies the proposals to use DEA as a tool for multiple criteria, identifies the problems with the 'folk technique' (Doyle and Green, 1993), super-efficiency approach (Andersen and Petersen, 1993) and cross-efficiency approach (Sexton et al., 1986). He discusses "The reluctance to introduce **properly modeled preference information (weights, trade-offs, utility functions**, etc.) leads to methods the normative properties of which can easily be questioned" (Bouyssou 1999, pp. 977) and concludes "…, DEA could benefit from some ideas that have received much attention in the MCDM community, e.g. the distinction between efficiency and convex efficiency, **the importance – and difficulty – of modeling preferences**, the necessity to consider normative properties in order to guide the creation of new aggregation methods." (Bouyssou 1999, pp. 978, bolds are not in the original but by the author).

Farinaccio and Ostanello (1999) indicate the analogy of the problem tackled by DEA and the ranking problem in MCDM. In spite of this analogy, they argue that "The multiplicity of 'criteria' (i.e. input and output factors) involved in DEA models **does not necessarily mean** that such kind of representation [considering DEA as an MCDM tool] might answer some questions which concerns the **preferences on a given candidate set**, as other MCDA/M tools can do validly (both conceptually and formally)" (Farinaccio and Ostanello 1999, pp. 1, bolds are not in the original but by the author). As Bouyssou (1999), they illustrate that the efficiency concept in DEA corresponds to the convex efficiency concept of MCDM and, therefore, the ranking according to Pareto preferences of DEA may be problematic when there exist a global evaluation especially in the strategic level of decisions. Consequently, they conclude, "The DM's problem [Global decision maker (GDM) or DM who makes strategic level decisions] tackled is a typical aggregation problem that cannot be solved by DEA,…" (Farinaccio and Ostanello 1999, pp. 7). They imply that the GDM's ranking or selection problem cannot be solved only by DEA, but it requires some MCDM techniques or some modifications and extension on DEA.

From the references above the general conclusion may be drawn as: DEA and MCDM have much in common, they may feed each other, however, while establishing this, the conceptual and methodical similarities and differences between them should be clearly understood and the linkage between them should be formed accordingly. In fact, *intentionally* or *naturally* linking one's aims with other's methodology (e.g., MCDM's selection aim with DEA methodology of selection of best possible weights by DMU or DEA's finding efficiencies aim with MCDM's interactive methodologies) has been occurring while the two camps of the researchers interact with and crossly refer to each other. Bouyssou (1999) and Farinaccio and Ostanello (1999) warn the researchers about the problems and difficulties in this linkage and particularly they suggest usage of preference information in the form of utility function, trade-offs methods, weight restriction methods or some types of aggregation methods of MCDM while using DEA for selection or ranking. Actually, although the ranking with DEA started with value free approaches, the general trend in the ranking with DEA is usage of the preference information somehow. This situation will be analyzed in the next section.

Based on the general conclusion drawn in the previous paragraph, two ranking procedures are developed in this study: first procedure extends the value efficiency analysis of DEA by incorporating additional preferential information; second one uses mixed integer programming and a weight selection method of DMUs similar to DEA. Before the explanation of these procedures, two important issues, efficiency measures in DEA and ranking efforts in DEA, will be surveyed in the next section.

2.2. A Brief Survey on Efficiency Measures and Ranking in DEA:

Simply, the idea in DEA is to measure the efficiency of a unit by looking at the ratio of its weighted outputs to weighted inputs (in input oriented case) and comparing this ratio with the ratios of other similar units. Those that have 'high' ratios compared to others are said to be efficient and those that have 'low' ratios are said to be inefficient. Traditionally, a measure of inefficiency is obtained by radially projecting the inefficient units toward the efficient frontier which is obtained from efficient units.

Radial measure, introduced by Debreu (1951) and Farrell (1957), is used in order to evaluate the efficiencies of these similar units. This measure is popular because it has an apparent economic meaning, namely, regardless of prices, it corresponds to proportion of cost decrease (revenue increase) while holding revenue (cost) constant in order to eliminate the technical inefficiency in input (output) oriented case. Nevertheless, considering the technical efficiency definition of Koopmans (1951) instead of the definition of Debreu and Farrell mentioned above, several non-radial measures are proposed such as Färe (1975), Färe and Lovell (1978), Färe et al. (1983), and Zieschang (1984). A survey of these non-radial measures can be found in De Borger et al. (1998). These radial and non-radial measures are generally defined as output based (expand outputs of DMU until it reaches the efficient frontier while holding inputs constant) or as input based (shrink inputs of DMU until it reaches the efficient frontier while holding outputs constant).

A newer generation of the efficiency measures, namely global efficiency measures (GEM), is proposed by Ali and Lerme (1991), Tone (1993), Lovell and Pastor (1995), and Lovell et al. (1995). These measures are global in the sense that they simultaneously increase the level of outputs and decrease the level of inputs in order to eliminate the technical efficiency. For a survey of this generation scheme, one can refer to Pastor (1995).

All measures discussed hitherto give the same efficiency values (e.g. 1 or 0) to the efficient units. Therefore, ranking of only inefficient units are possible (although this ranking are questioned by Bouyssou, 1999 and Farinaccio and Ostanello, 1999). In order to rank efficient units and increase the ranking power of DEA, different ranking methods such as super-efficiency method (Andersen and Petersen, 1993), evaluation of cross-efficiency matrix (Sexton et al., 1986), benchmarking (Torgersen et al., 1996) and utilization of multivariate statistical techniques (Sinuany-Stern et al., 1994) are proposed. Again, ranking by these methods are questioned and criticized by Bouyssou (1999). A detailed review of these ranking methods is in Adler et al. (2002).

Although all these methods increase the ranking power of the DEA, they do not incorporate the preference information of decision makers or decision making units (DMUs) in their analysis. Therefore, assigning efficiency points or ranking according to their goal attainment levels is not possible with these methods. Actually, efficiency measure of these techniques may measure the DMU's goal attainment level incorrectly (e.g. for inadequateness of radial measures in representing the preference structure see Korhonen et al., 2003).

The studies that incorporate preference of decision makers and/or DMU to DEA can be gathered in two separate sets of papers[1]: one set does this by restricting weights (e.g. Thompson et al. 1986, Dyson and Thannassolis 1988, Cook and Kress 1990, Charnes et al. 1990, Thompson et al. 1995 and Green and Doyle 1995), alternatively, the other set does this by selecting an ideal (and sometimes hypothetical) DMU or preferred input/output target (Golany 1988, Zhu 1996, Halme et al. 1999 and Joro et al. 2003). One may refer to Adler et al. (2002) for the discussion and the review of these two sets. The second set, in particular 'value efficiency analysis' concept of Halme et al. (1999) and Joro et al. (2003), will be discussed extensively in section 4.2.

The approach of Zhu (1996) may be particularly important for this study. Instead of the traditional radial projection, he suggests a non-radial projection to project an efficient and targeted input/output level, which is also suggested in this study at Target-Direction-Set Value Efficiency Analysis (TDSVEA). He identifies two possible cases justifying why to project non-radially instead of radially: management may prefer one input reduction (output expansion) to other input reduction (output expansion) or/and some input reductions (output expansion) may not be possible. By assigning preference weights (he proposes that these weights may come from price/cost levels of outputs/inputs; he also states that expert judgments with some techniques such as Delphi method or AHP is another possibility), he finds a scalar non-radial efficiency score, i.e. *Russell measure*. He argues that his approach improves that of Thanassoulis and Dyson (1992). Joro (1998) compares the models of identifying target units in DEA: weight-based general preference structure and target setting with ideal targets by Thanassoulis and Dyson (1992), weights-based preference structure by Zhu (1996) and reference point approach by Korhonen and Laakso (1986). She argues that reference point approach is superior when compared to others.

[1] These classifications are based on Adler et al. (2002).

2.3. Value Efficiency Analysis:

As stated before, Value Efficiency Analysis (VEA), introduced in Halme et al. (1999), incorporates the preference information of a DM. In this section only the literature and the basics of the VEA will be presented, technical details are given in section 4.2 and stated in references. Firstly, decision maker selects the most preferred solution (MPS), which is on the efficient frontier (pseudoconcave utility function assumption is made in VEA). Secondly, since it is hard to generate exactly, the contour of the utility function of the DM, which passes through the MPS, is approximated by tangent cones. Finally, each unit is ranked according to its proportional distance from the origin to these tangent cones (this distance is an optimistic estimate of the actual value efficiency score). Joro et al. 2003, extends this approach in two ways: first, when additional information from the DM is not obtained, a bound for actual value efficiency score is established and second, with an interactive approach actual value efficiency score of each unit is obtained. Pseudoconcavity assumption of VEA is criticized in Cherchye et al. (2001).

Halme and Korhonen (2000) extend study of Halme et al. (1999) by utilizing additional information of 'prices' of inputs and outputs through restricting the weights of VEA. This extension increases the discrimination power of VEA.

Some practical aspects, examples and extensions of VEA can be found in Korhonen et al. (2002). Particularly, they point out the issues associated with finding MPS.

Two applications of VEA, to academic research (Korhonen et al. 2001) and to parishes in Helsinki (Halme 2002), illustrate that VEA well fits to real life problems.

2.4. Some Ranking Methods:

Generally, in a ranking problem there exists a number of alternatives $A = \{a_1,\dots,a_n\}$. These alternatives will be ranked (partially or fully) according to their scores in a number of criteria $C=\{c_1,\dots,c_k\}$. Below is a non-comprehensive list of the techniques in ranking:

1) Fixed weights approach: This is the simplest ranking approach. Firstly, fixed weights are given to each criterion, secondly individual scores multiplied by these weights are added and finally these total values for units are compared. Its ease to application make it the most used approach especially in publicly announced rankings such as university rankings, sportsmen rankings etc. Predetermined weights are generally criticized, and no consensus can be achieved by all the stakeholders of the ranking. Generally, it is not used in academic research but one can search the World Wide Web for applications and criticism of this approach.

2) Statistical methods (such as principal components analysis): In statistical methods, the general aim is decreasing the number of criteria through finding hypothetical new criterion (or criteria) which represents initial criteria. These new criteria are linear combinations of old criteria, orthogonal to each other and selected so as to explain the maximum proportion of that explained by old criteria (more detailed statistical methods in Johnston and Wichern, 1982). Moreover, ranking methodology utilizes statistical techniques such as rank correlation coefficients in order to tackle some conceptual aspects such as Consensus Ranking Problem (see Emond and Mason, 2002).

3) Rankings based on voting: Using voting information, grouped alternative comparisons or pair-wise comparisons according to votes, these methods provide a final ranking of alternatives. Lansdowne (1996) provides some ordinal ranking techniques based on vote information: Condorcet order (ranking according to pair-wise comparison of the alternatives in the votes), Borda's method (ranking according to total number of voting points), Bernardo's method (ranking by maximizing the total agreement), Cook-Seiford's method (ranking by minimizing the total disagreement), Köhler's method (ranking by two sequential algorithms which finds the best among unranked alternatives based on outranking matrix) and Arrows-Raynaud method (ranking by two sequential algorithms which finds the worst among unranked alternatives based on outranking matrix).

4) MCDM methods: Methods such as multivariate utility approaches (Keeney and Raiffa, 1976), AHP (Saaty, 1980), PROMETHEE (Brans et al., 1984) and

ELECTRE (Roy, 1991) etc. are available in MCDM literature. Some of these methods make full ranking while some make partial ranking.

5) Method of '*TheCenter*': This method is applied by *TheCenter*[2], a research enterprise focused on the competitive national context for major American research universities. They rank each unit in each criterion. If for a criterion, unit is ranked between 1^{st} to m^{th}, then it gets bold-1. If for that criterion, unit is ranked between m^{th} to p^{th} ($p>m$) then it gets weak-1. Where bold-1 >> weak-1, units are ranked according to number of their bold-1 and weak-1 scores (e.g. unit that gets 1 bold-1 and 0 weak-1 has higher ranking than units that get 0 bold-1 and 23 weak-1s). The units that get same bold-1 and weak-1 scores have the same ranking. However, this method is subject to some practical difficulties: assume that in the stated example the unit ranked $m+1^{st}$ in each of 23 criteria gets 23 weak-1s, on the other hand, unit ranked m^{th} only in 1 criterion and the last in the other 22 criteria gets 1 bold-1 score. In this case, the resulted ranking may be questionable. Moreover, the selection of m and p affects the ranking.

6) DEA and its derivatives such as VEA: Efficiency scores of original DEA models (CCR or BCC) may be used to rank inefficient alternatives. As explained in section 2.2 the ranking power of the DEA is increased by numerous methods. For example, Value Efficiency Analysis (VEA) incorporates the preference information to Data Envelopment Analysis in order to obtain ranking of DMUs (Halme et al. 1998 and Joro et al. 2003). This approach with its applications and extensions will be discussed in section 4.2.

7) Target-Direction-Set Value Efficiency Analysis (TDSVEA): In this study, the method TDSVEA will be introduced and developed in Chapter 4. When there exists a global decision maker (GDM) who can provide the information needed by TDSVEA, the analysis results in a complete ranking of alternatives.

8) Mixed integer programming: In this study, a mixed integer programming based approach will be introduced and developed in Chapter 3. With this method, parallel to DEA, each unit selects its own weights of criteria so as to outperform maximum number of other units.

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[2] Information about this enterprise can be found in web site http://thecenter.ufl.edu .

CHAPTER 3

RANKING BY MIXED INTEGER PROGRAMMING

3.1. Mixed Integer Program for Ranking:

In practice, many ranking problems are solved by externally attaching weights to criteria. Score of a unit in a criterion is multiplied by weight attributed to that criterion in order to obtain weighted score values. The units are then ranked according to their final score, which is obtained by aggregating (generally directly summing) weighted score values. School rankings and sport rankings are two common examples. In this ranking methodology, since the weights of the criteria are fixed, objections about weight structure may arise. Especially, for the units, whose final score difference is small, only small differences in the fixed weights may change ranking significantly. Moreover, the methodology does not allow the flexibility of suggesting ordinal weights rather than cardinal weights. Many incompletely defined weight structures, like criterion 1 is more important than criterion 2 or criterion 3 is four to five times more important than criterion 4, are not applicable with this methodology. Rather, it is said that criterion 1 has a weight 0.35 but criterion 2 has a weight 0.34 or criterion 3 has a weight 0.28 whereas criterion 4 has a weight 0.07 (actually, for the specified example, criterion 3 should have a weight between 0.28 and 0.35 if criterion 4 has a weight 0.07, however, with fixed weight method such a weight setting is not appropriate, weight of criterion 3 is either 0.28, 0.34 or a number between them.)

Another ranking method is provided by a research center, "*TheCenter*", which ranks American research universities. *TheCenter* first ranks universities in their individual scores in each criterion. The university that is ranked in a criterion within the best m universities gets a point of bold-1 (**1**). The university that is ranked in a criterion between m+1th and pth get a point of weak-1 (*1*), and the university that is ranked worse than pth does not get a score in this criterion. In their scoring, bold-1 is much greater than weak-1 (i.e. **1**>>*1*). Therefore, a unit with 5 bold-1s and no weak-1s has a better rank than a unit with 4 bold-1s and 20 weak-1s. Finally, *TheCenter* ranks universities according to their bold-1 and weak-1 scores. If two universities get the same points they are ranked in equal standings. *TheCenter* specifies m as 25, p as 50 in its ranking. It uses 9 criteria and its standings are (9,0), (8,1), (8,0), (7,2), (7,1), (7,0), (6,3), (6,2), (6,1), (6,0),…, where first number in parenthesis shows bold-1 score and the second number shows weak-1 score (e.g. (6,2) means the university is ranked between 1st and 25th in 6 criteria, and between 26th and 50th in 2 criteria). This method eliminates some objections of fixed weight methods, however, it is apt to some practical difficulties. With this method, a unit, which is 1st in k-1 criteria but m+1th in only one criterion, is ranked in worse standing than a unit, which is mth in all k criteria. These types of extreme situations may question ranking ability of this method, especially when the criteria are not equally important.

In MIP technique provided in this study, regarding their scores in k criteria, n units will be ranked according to outperforming ability of each unit to other units by means of selecting best possible weights (attached to criteria) among a specified weight set. Each unit selects its own weight set from the specified feasible weight space, in order to outperform maximum number of other units. Therefore, each unit selects its own best possible weights and, consequently, weights attached to criteria probably become different for each unit. The other units are outperformed by the evaluated unit if their final scores are lower than the final score of the evaluated unit with the selected weights of the evaluated unit. This methodology can allow many weight structures, which the fixed weight method does not allow, such as ordinal weights, weight bounds, etc. In fact, the fixed weight method is a special case of this methodology where feasible weight space is a single point. Therefore, this methodology is more flexible. Moreover, this methodology is prone to fewer objections, since the weights are not fixed but produced from a larger weight space.

Since rankings are subject to change if the feasible weights differ, the selection of weight space is extremely important for the analysis. The feasible weight space should convey the known information about attainable weights.

The methodology starts with identifying the criteria that will be used for ranking. Then, the individual raw scores (X_{ij}) of each unit (i=1,…, n) in each criterion (j=1,…, k) are obtained. These raw scores are translated to z-scores of each unit in each criterion by the following formula:

$$Z_{ij} = \frac{X_{ij} - \overline{X}_j}{\sigma_j}$$

Where

Z_{ij} : z-score of unit i (i=1,…, n) for criterion j (j=1,…, k),

X_{ij} : actual raw score for unit i in criterion j,

$\overline{X}_j$: mean value of actual scores of all units for criterion j (i.e. $\frac{\sum_{i=1}^{n} X_{ij}}{n}$),

σ_j : standard deviation of actual scores of all units for criterion j.

Therefore, z-score measures how many standard deviations the actual raw score of a unit in a criterion deviates from the mean score for that criterion. Thus, the scaling problem of actual scores is solved.

Utilizing these z-scores, the methodology then uses the following mixed integer programming, which is solved for each unit in the analysis, in order to obtain rank of that unit:

Model (I)

$$\text{Minimize} \quad n - \sum_{i=1, i \neq DMU_0}^{n} Y(i)$$

s.t.

$$\left(\sum_{j=1}^{k} w_j * Z_{DMU_0 j} \right) \geq \left(\sum_{j=1}^{k} w_j * Z_{ij} \right) + M * Y(i) - M$$

$$\text{for } \forall \ i = 1,..., DMU_0 - 1, DMU_0 + 1,..., n \quad (1)$$

$$w \in W \quad (2)$$

$$Y(i) \in \{0,1\}$$

$$w_j \geq 0 \text{ for } \forall \ j = 1,..., k$$

where

DMU_0 is the unit whose rank is evaluated by this program, i.e. for each $DMU_0 = 1,..., n$, a different program will be solved (total of n different programs will be solved),

w_j are decision variables that correspond to weight of criterion $j \, (j = 1,..., k)$ for DMU_0,

$Y(i)$ are binary variables, if weighted z-score of DMU_0 can outperform that of unit i with the selected weights w_j, then $Y(i)$ takes 1, otherwise takes 0,

w is a column vector whose coefficients are w_j,

W determines the feasible weight space, also see Table 1,

M is a large number,

Z_{ij} is the z-score for unit i in criterion j.

Objective function determines rank of DMU_0, which is total units - number of outperformed units with selected weights that means lower ranks are better

Determination of the feasible weight space ($w \in W$) is important. This may be achieved by different linear equalities and/or inequalities. As a guide for different weight space constraints, one can see Table 1. Other constraints may be as well included provided that they are linear.

Table 1 – Some possible weight constraints

Meaning of Constraint	Constraint
Criterion j is more important than criterion m	$w_j – w_m \geq 0$
Criterion j is t times more important than criterion m	$w_j – t\ w_m = 0$
Criterion j is at least t times more important than criterion m	$w_j – t\ w_m \geq 0$
Criterion j is t_1 to t_2 ($t_1 < t_2$) times more important than criterion m	$w_j – t_1\ w_m \geq 0$ and $w_j + t_2\ w_m \leq 0$
Weights should be positive (No zero weights)	$w_j \geq \varepsilon$, where ε is non-Archimedean term
At least one weight should be positive	$\sum_{j=1}^{k} w_j \geq \varepsilon$, where ε is non-Archimedean term
Normalizing criterion weights	$\sum_{j=1}^{k} w_j = 1$
(If weights are normalized), weight for criterion j is at least p_1%	$w_j \geq p_1/100$
(If weights are normalized), weight for criterion j is at most p_2%	$w_j \leq p_2/100$

The objective function of the MIP shows the standing of the unit that is evaluated. If it gives value 1 then it means the unit evaluated outperforms all the other n-1 units with its selected weights. If the objective function value for a unit is k, then the unit is ranked at k^{th} standing, which means it outperforms n-k other units with its selected weights. The units that can be outperformed by the evaluated unit can be obtained from the optimal Y(i) values. If Y(i) value is 1 for the DMU_0, then DMU_0 outperforms unit i with its selected weights. Selected weights of DMU_0 are the w vector values (w_j's) at the optimal solution.

3.2. Simplification of MIP by D and WD matrices and simplification of obtaining these matrices:

Before explaining the simplification procedure, some concepts should be clarified. If a binary variable, say Y(p), is 1 in Model (I) for the evaluated unit DMU_0, this means that DMU_0 *can outperform* the unit p (or DMU_0 *outperforms* the unit p *with its selected weights*). In other words, it may or may not outperform the unit p with other weights (one cannot know this without further information). If it is known that a unit, DMU_0, *outperforms* the other unit, p, for *all feasible weight spaces*, then *weight-domination* exist between DMU_0 and unit p. If it is known that DMU_0 *outperforms* the other unit for *all non-negative weights, feasible or not,* then DMU_0 *dominates* p.

If it is known that a unit outperforms another unit for any non-negative weights (i.e. dominates it), which is feasible or not, or if it is known that a unit outperforms another unit for all feasible weights $w \in W$ (i.e. weight domination exists), then for the evaluation of dominating and dominated units (or for the evaluation of the units subject to weight-domination), assessing outperforming relation in MIPs of one of these units for the other unit with the feasible weight set is not needed. For example, if unit 2 dominates unit 5 (i.e. unit 2 has z-scores at least as large as unit 5 where at least in one score strict inequality exists), then for evaluating unit 2, the variable Y(5) is not needed (its value is 1 and known a priori) and also the constraint that includes Y(5) is redundant. Also, for evaluating unit 5, the variable Y(2) is not needed (its value is 0) and therefore the constraint that includes Y(2) is unnecessary. Consequently, the MIP that evaluates unit 2 (dominating unit) becomes (DMU_0=2):

$$
\begin{aligned}
&\text{Minimize } \mathrm{n} - 1 - \sum_{\mathrm{i}=1, \mathrm{i} \neq 5,2}^{n} \mathrm{Y(i)} \\
&s.t. \\
&\left(\sum_{\mathrm{j}=1}^{k} \mathrm{w_j} * \mathrm{Z_{2j}} \right) \geq \left(\sum_{\mathrm{j}=1}^{k} \mathrm{w_j} * \mathrm{Z_{ij}} \right) + \mathrm{M} * \mathrm{Y(i)} - \mathrm{M} \\
&\qquad\qquad\qquad\qquad \text{for } \forall\, \mathrm{i} = 1,3,4,6,7,\ldots, \mathrm{n} \quad (1) \\
&\mathrm{w} \in \mathrm{W} \qquad\qquad\qquad\qquad\qquad\qquad (2) \\
&\mathrm{Y(i)} \in \{0,1\} \\
&\mathrm{w_j} \geq 0 \text{ for } \forall\, \mathrm{j} = 1,\ldots, \mathrm{k}
\end{aligned}
$$

In this MIP, the objective function becomes [$\text{Minimize } \mathrm{n} - 1 - \sum_{\mathrm{i}=1, \mathrm{i} \neq 5,2}^{n} \mathrm{Y(i)}$] instead of [$\text{Minimize } \mathrm{n} - \sum_{\mathrm{i}=1, \mathrm{i} \neq 2}^{n} \mathrm{Y(i)}$] since Y(5) is known to be 1. Moreover, the constraint that includes Y(5) in constraints (1) is deleted.

The MIP that evaluates unit 5 (dominated unit) becomes (DMU_0=5):

$$
\begin{aligned}
&\text{Minimize } n - \sum_{i=1, i\neq 2,5}^{n} Y(i) \\
&s.t. \\
&\left(\sum_{j=1}^{k} w_j * Z_{5j}\right) \geq \left(\sum_{j=1}^{k} w_j * Z_{ij}\right) + M * Y(i) - M \\
&\qquad\qquad\qquad\qquad \text{for } \forall\ i = 1,3,4,6,7,....,n \quad (1) \\
&w \in W \qquad\qquad\qquad\qquad\qquad\qquad\qquad (2) \\
&Y(i) \in \{0,1\} \\
&w_j \geq 0 \text{ for } \forall\ j = 1,...,k
\end{aligned}
$$

In this MIP, the objective function becomes $[\text{Minimize } n - \sum_{i=1, i\neq 2,5}^{n} Y(i)]$ instead of $[\text{Minimize } n - \sum_{i=1, i\neq 5}^{n} Y(i)]$ since Y(2) is known to be 0. Furthermore, the constraint that includes Y(2) in constraints (1) is deleted.

To sum up, if unit m (weight-)dominates unit p, the approach in Table 2 will be applied:

Table 2 – Simplification of MIP if unit m (weight-)dominates unit p

Affected MIP	Variable Whose Value is Known	Simplification of MIP
For m (DMU_0=m)	Y(p)=1	Objective function: $\text{Minimize } n - 1 - \sum_{i=1, i\neq p,m}^{n} Y(i)$ instead of $\text{Minimize } n - \sum_{i=1 i\neq m}^{n} Y(i)$ Constraints (1): Delete constraint that corresponds to Y(p), which is $\left(\sum_{j=1}^{k} w_j * Z_{mj}\right) \geq \left(\sum_{j=1}^{k} w_j * Z_{pj}\right) + M * Y(p) - M$
For p (DMU_0=p)	Y(m)=0	Objective function: $\text{Minimize } n - \sum_{i=1, i\neq m,p}^{n} Y(i)$ instead of $\text{Minimize } n - \sum_{i=1 i\neq p}^{n} Y(i)$ Constraints (1): Delete constraint that corresponds to Y(m), which is $\left(\sum_{j=1}^{k} w_j * Z_{pj}\right) \geq \left(\sum_{j=1}^{k} w_j * Z_{mj}\right) + M * Y(m) - M$

The domination relations can be summarized in a matrix, *domination matrix D*. Rows and columns of matrix corresponds to units. When m dominates p, $D_{mp} = +1$ and $D_{pm} = -1$, i.e. when one sees +1, then the unit corresponding to row dominates the unit corresponding to column, which means the unit that corresponds to row has higher final score than the unit that corresponds to column for any non-negative weights (for -1, vice versa). If two units (r and s) cannot dominate each other, which means for some weight set a unit has higher final score and for some other set the other unit has higher final score, then entries corresponding those units will be equal to zero ($D_{rs} = 0$ and $D_{sr} = 0$). The entries in the main diagonal do not give any information and they will be shown by X. An example for this matrix is shown in Figure 1.

	1	…	m	…	r	…	p	…	s	…	n
1	X										
⋮											
m			X				+1				
⋮											
r					X				0		
⋮											
p			-1				X				
⋮											
s					0				X		
⋮											
n											X

Figure 1 – Domination matrix (D)

As can be seen from the figure, the information on the upper triangle (the lower triangle) of the matrix is sufficient to generate information on the lower triangle (upper triangle). If an entry D_{ij} is +1 (-1), then D_{ji} is –1 (+1). If D_{ij} is equal to zero then D_{ji} is also zero. Therefore, only the entries on the upper triangle (or the lower triangle) are needed (without main diagonal, a total of $\frac{n(n-1)}{2}$ entries are needed in order to generate D). Among these entries, those that are +1 and –1 simplify the MIP for the related units as in Table 2.

Lemma 1: If unit m dominates unit r and unit r dominates unit s, then m dominates s.

Proof: Follows by transitivity.

Implication of this lemma to the domination matrix is that: as in Figure 2, if in a row m an entry +1 exists in column r, then all the +1s in row r (dark shaded entries in row r, $D_{r\,r+1}$ to $D_{r\,n}$) can be copied to corresponding entries of row m (dark shaded entries in row m, $D_{m\,r+1}$ to D_{mn}), and if in a row p an entry -1 exists in column s, all the –1s in row s (light shaded entries in row s, $D_{s\,s+1}$ to D_{sn}) can be copied to corresponding entries of row p (light shaded entries in row p, $D_{p\,s+1}$ to D_{pn}); as in Figure 3, if in a column r an entry +1 exists in row m, all the -1s in column r (light shaded entries in column r, D_{1r} to $D_{m-1\,r}$) can be copied to corresponding entries of column m (light shaded entries in column m, D_{1m} to $D_{m-1\,m}$), and if in a column s an entry -1 exists in row p, all the +1s in column s (dark shaded entries in column s, D_{1s} to $D_{p-1\,s}$) can be copied to corresponding entries of column p (dark shaded entries in column p, D_{1p} to $D_{p-1\,p}$).

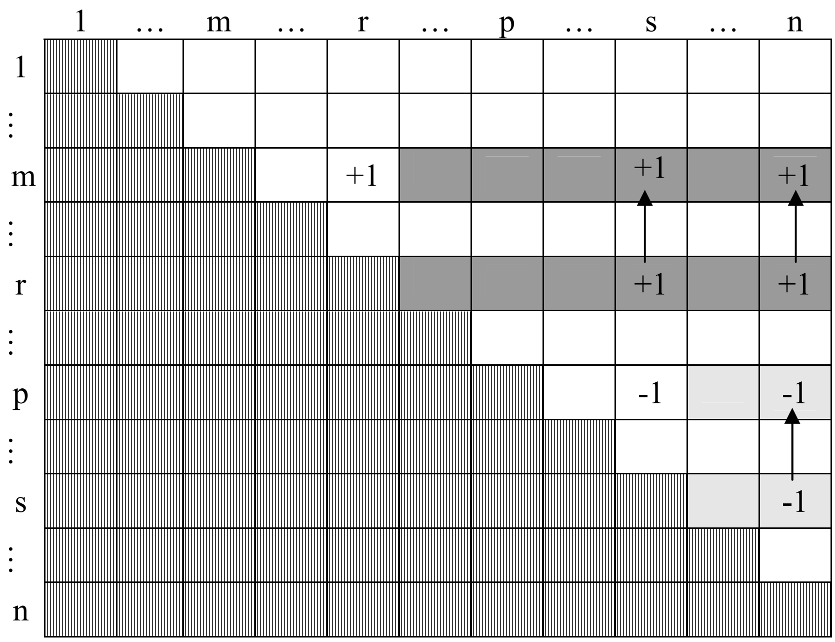


Figure 2 – Implication of Lemma 1 to Domination and Weighted Domination matrix rows

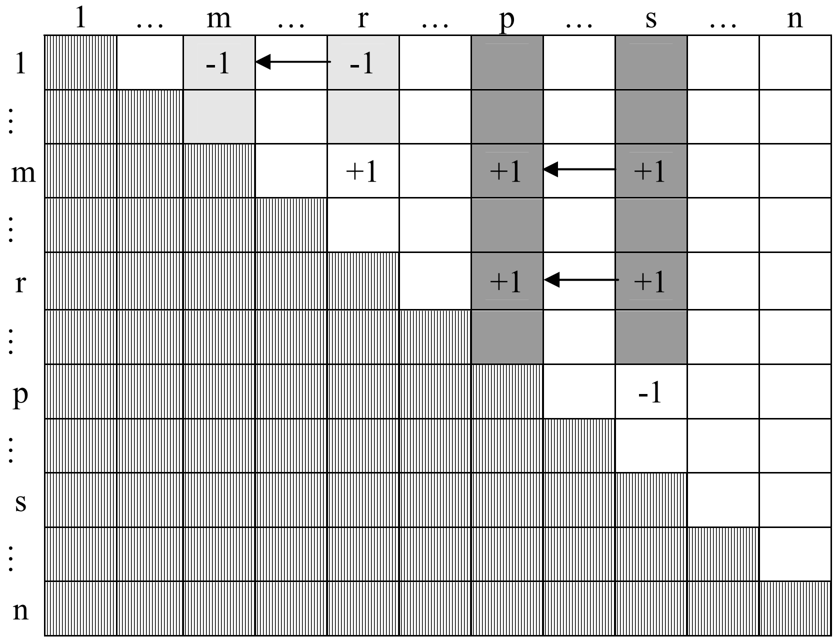


Figure 3 – Implication of Lemma 1 to Domination matrix and Weighted Domination matrix columns

Since copying entries from other rows and columns is possible, this implication may greatly simplify the filling of Domination matrix, if there exist many positive and negative domination relations. When copying entries are made, duplication of effort for identifying domination relation is eliminated. In the best possible case (where all $D_{i\ i+1}$ entries are +1 or all $D_{i\ i+1}$ entries are –1), looking for the domination relation for only n-1 times is enough to fill all entries in domination matrix. However, if any positive or negative domination relations do not exist, then for all $\frac{n(n-1)}{2}$ entries domination relation should be investigated.

If number of +1s or –1s in domination matrix is large, then the MIP for all the units can be greatly simplified. For example, MIP for unit 1 in domination matrix in Figure 4 is:

$$\text{Minimize} \quad n - 5 - \sum_{i=1, i \notin A}^{n} Y(i) \qquad \text{(since unit 1 dominates 5 other units)}$$

$s.t.$

$$\left(\sum_{j=1}^{k} w_j * Z_{1j} \right) \geq \left(\sum_{j=1}^{k} w_j * Z_{ij} \right) + M * Y(i) - M \text{ for } \forall\, i \notin A \qquad (1)$$

$$w \in W \qquad (2)$$

$$Y(i) \in \{0,1\} \; \forall\, i \notin A$$

$$w_j \geq 0 \text{ for } \forall\, j = 1,\dots,k$$

$$A = \{1,2,3,4,5,15,16,17\}$$

	1	2	3	4	5	…	15	16	17	…	n
1		+1	-1	+1	+1	0	+1	+1	-1	0	0
2											
3											
4											
5											
⋮											
15											
16											
17											
⋮											
n											

Figure 4 – Example for domination relation

As can be seen from the model and Figure 4 above, for each +1 or –1 in a row, in the MIP to evaluate unit that matches the row, the binary variable and the constraint in constraints (1) of MIP corresponding to the unit that matches the column is deleted.

Identification of an entry D_{mp} can be obtained by directly searching vectors of scores of units, m and p. From score of first criterion to score of the last criterion, this method searches whether m or p has higher score than the other in all criteria.

Domination matrix discussed hitherto does not employ feasible weight space information, but it looks at the domination relations. As discussed before, in the ranking approach discussed here, each unit selects its own best possible weight set from a feasible weight space. Therefore, the domination structure regarding *only feasible weight space* (*weight-domination structure*) provides another simplification for the MIP. For domination structure regarding only feasible weight set, *weighted domination matrix, WD*, will be used. It is very similar to domination matrix, D, however, this matrix, rather than using all positive quadrant, uses the weight space exactly in MIP (i.e. $w \in W$) in order to determine weight-domination relations. Nevertheless, some information in D is applicable to WD also. The following lemma will explain this.

Lemma 2: Domination relations, +1 and –1, in D are valid for WD as weight domination relations, but the converse may not be true.

Proof: For dominations relations in D, feasible weight space is determined as $W_1 = \bigcap_{j=1}^{k} (w_j \geq 0)$. For weight-domination relations in WD, weight space is determined as $W_2 = \left[\bigcap_{j=1}^{k} (w_j \geq 0) \right] \cap \left[w \in W \right]$. Therefore, $W_2 \subseteq W_1$. Thus, if relations, +1 and –1, are valid for D, then it should be valid for WD; but all +1 and –1 which are valid for WD may not be valid for D.

Implication of this lemma for weighted domination matrix is that: any entry, +1 and –1, in D is directly copied to corresponding entry in WD. However, the 0 entries in D can be 0, +1 or –1 entries in WD.

Hence, +1 or –1 entries in D shorten the approach in three ways: they simplify filling the domination matrix, they simplify filling the weighted domination matrix, and finally they simplify MIP for corresponding units.

The 0 entries in D can be 0, +1 or –1 entries in WD. Therefore, for the identification of domination structure of entries in WD, which are 0 in D (i.e. $D_{mp} = 0$), can be obtained from the following LP:

$$
\begin{aligned}
&\text{Minimize } \mathrm{d}^{m} + \mathrm{d}^{p} \\
&\left(\sum_{\mathrm{j=1}}^{k} \mathrm{w_j} * \mathrm{Z_{mj}}\right) - \left(\sum_{\mathrm{j=1}}^{k} \mathrm{w_j} * \mathrm{Z_{pj}}\right) - \mathrm{d}^{m} + \mathrm{d}^{p} = 0 \\
&\mathrm{w} \in \mathrm{W} \\
&\mathrm{w_j} \geq 0 \text{ for } \forall\ \mathrm{j} = 1,\dots,\mathrm{k} \\
&\mathrm{d}^{m}, \mathrm{d}^{p} \geq 0
\end{aligned}
$$

In this model, if objective function turns out to be 0, then neither unit m nor unit p outperforms the other ($WD_{mp} = 0$) for all feasible weights. If d^m is positive, then m outperforms p ($WD_{mp} = +1$) for all feasible weights, and if d^p is positive, then p outperforms m ($WD_{mp} = -1$) for all feasible weights. Both d^m and d^p cannot be positive at the same time because of the objective function and that their columns are linearly dependent.

In addition to simplification according to Lemma 2 (copying +1 and –1 entries in D to WD matrix), since Lemma 1 is valid for WD in terms of weight-domination relations (rather than domination relations), the discussion about simplification of filling D is applicable to filling WD also: as in Figure 2, if in a row m an entry +1 exists in column r, then all the +1s in row r (dark shaded entries in row r, $WD_{r\ r+1}$ to WD_{rn}) can be copied to corresponding entries of row m (dark shaded entries in row m, $WD_{m\ r+1}$ to WD_{mn}), and if in a row p an entry -1 exists in column s, all the –1s in row s (light shaded entries in row s, $WD_{s\ s+1}$ to WD_{sn}) can be copied to corresponding entries of row p (light shaded entries in row p, $WD_{p\ s+1}$ to WD_{pn}); as in Figure 3, if in a column r an entry +1 exists in row m, all the -1s in column r (light shaded entries in column r, WD_{1r} to $WD_{m-1\ r}$) can be copied to corresponding entries of column m (light shaded entries in column m, WD_{1m} to $WD_{m-1\ m}$), and if in a column s an entry -1 exists in row p, all the +1s in column s (dark shaded entries in column s, WD_{1s} to $WD_{p-1\ s}$) can be copied to corresponding entries of column p (dark shaded entries in column p, WD_{1p} to $WD_{p-1\ p}$).

As can be guessed, duplication of effort is possible for the 0 entries in D matrix if they are also 0 in WD matrix, since for them an LP is solved for WD matrix after comparing vectors for D matrix. However, for +1 and –1 entries in D matrix, only vector comparison for D matrix or only solving LP for WD matrix is sufficient.

Then, why one need D matrix if it provides possible duplication of efforts? First of all, determining relations for D matrix is easier (vector comparison is easier than solving the specified LP). Secondly, if the feasible weight space is subject to change or if it is modified frequently, unless the new weight space is a subspace or superspace of old weight space, all the information in WD matrix except that comes from D matrix is lost. However, when the weight space is changed, information in D matrix is not lost and will be valid for new situation. Therefore, a tradeoff exist in using D matrix: if the weight space is frequently changed, the D matrix provides reusable information, but if the weight space never changes, D matrix provides nothing additional to information of WD matrix but possible duplication of effort, especially when there exist many 0 entries in both D and WD matrices. Hence, the usage of D matrix in the simplification of MIP may be omitted in some cases.

Another point that should be paid attention in this simplification procedure is that only entries +1 and –1 provide usable information and simplification. 0 entries do not provide any simplification. Moreover, if all the entries are 0 in both D matrix and WD matrix, then no simplification is provided whereas vector comparison for D matrix is applied $\frac{n(n-1)}{2}$ times and the LP provided for WD matrix is solved for $\frac{n(n-1)}{2}$ times. Hence, the simplification approach by D matrix and/or WD matrix is used only if the possibility of finding positive or negative (weighted) domination relation is sufficiently high. In fact, this possibility cannot be known a priori, however some guesstimate may prevent loss of effort.

In fact, the matrix WD can be filled fully by only one LP:

$$\text{Minimize} \sum_{i=1}^{\text{n-1}} \sum_{r=i+1}^{n} \mathrm{d}^{-ir} + \mathrm{d}^{+ir}$$

$$\left(\sum_{\mathrm{j=1}}^{k} \mathrm{w}_{\mathrm{jir}} * \mathrm{Z}_{\mathrm{ij}} \right) - \left(\sum_{\mathrm{j=1}}^{k} \mathrm{w}_{\mathrm{jir}} * \mathrm{Z}_{\mathrm{mj}} \right) - \mathrm{d}^{-ir} + \mathrm{d}^{+ir} = 0 \text{ for i} = 1,\ldots,n-1 \text{ and}$$
$$\mathrm{r} = \mathrm{i}+1,\ldots,\mathrm{n}$$

$$w \in W$$

$$\mathrm{w}_{\mathrm{jir}} \geq 0 \text{ for } \mathrm{j} = 1,\ldots,\mathrm{k}; \;\; \mathrm{i} = 1,\ldots,n-1 \text{ and } \mathrm{r} = \mathrm{i}+1,\ldots,\mathrm{n}$$

$$\mathrm{d}^{-ir}, \mathrm{d}^{+ir} \geq 0 \text{ for i} = 1,\ldots,n-1 \text{ and r} = \mathrm{i}+1,\ldots,\mathrm{n}$$

In this LP for each i and r, at most one d term, either d^{-ir} or d^{+ir}, may be positive. Both cannot be positive because of the objective function. If d^{-ir} is positive then unit i outperforms r for all feasible weights (i.e. put +1 to WD_{ir}), and if d^{+ir} is positive then unit r outperforms i for all feasible weights (i.e. put -1 to WD_{ir}). If both are zero, neither unit outperforms the other (i.e. put 0 to WD_{ir}) for all feasible weights. With this LP all of the entries in WD can be obtained. However, with this method the simplification procedure, which is provided by Lemma 1, cannot be provided, whereas simplification of the LP of WD by Lemma 2 can still be valid. In other words, since all matrix is filled at once by only one LP, the simplification of the matrix filling through copying +1 and -1 entries from a row or a column to other row or column is not possible. Nevertheless, the +1 and -1 entries in D matrix can be still directly copied to WD. Therefore, if D matrix has been filled and +1 and –1 entries have been provided, the LP for WD can be simplified as:

$$\text{Minimize} \sum_{i=1}^{n\text{-}1} \sum_{r=i+1, if\ D_{ir}=0}^{n} d^{-ir} + d^{+ir}$$

$$\left(\sum_{j=1}^{k} w_{jir} * Z_{ij} \right) - \left(\sum_{j=1}^{k} w_{jir} * Z_{mj} \right) - d^{-ir} + d^{+ir} = 0 \text{ for } i = 1,\ldots, n-1 \text{ and}$$

$$r = i+1,\ldots,n \text{ if } D_{ir} = 0$$

$$w \in W$$

$$w_{jir} \geq 0 \text{ for } j = 1,\ldots,k;\ i = 1,\ldots,n-1 \text{ and } r = i+1,\ldots,n$$

$$d^{-ir}, d^{+ir} \geq 0 \text{ for } i = 1,\ldots,n-1 \text{ and } r = i+1,\ldots,n$$

The constraints and d variables for +1 and -1 entries in D have been deleted in this LP. Only those entries which are 0 in D have been included for the LP for WD.

As discussed, the simplification procedure, which directly copies +1 or –1 entries in a row or a column to other row and column is not possible if one LP for filling all the weighted domination matrix is applied. On the other hand, this method does not employ $\frac{n(n-1)}{2}$ small LPs for filling the matrix in the worst possible case but employs only one large LP. Nonetheless, simplification procedure through direct copies of +1 and –1 entries from D to WD is still possible with one LP approach. The linear program in one LP approach (filling all entries in the matrix by one large

LP) is much more complicated than linear programs in multiple LP approach (filling each entry in the matrix by one LP). The selection from these two possible approaches of filling WD, one LP without simplification by Lemma 1 or multiple LP with simplification by Lemma 1, is case dependent. For example, when simplification with only WD matrix is applied, in the best possible case of multiple LP approach (entries in WD are either all +1 or all –1), solving n - 1 simple linear programs is sufficient for filling WD matrix in multiple LP approach, whereas solving one large linear program is needed in one LP approach. On the other hand, in the worst possible case of multiple LP approach (all entries in WD are 0), $\frac{n(n-1)}{2}$ simple linear programs should be solved, whereas, solving one large linear program is sufficient. The selection between two alternative simplifications of MIP methods may depend on possibility of finding +1 or –1 entries in D and/or WD matrix (actually, these entries are the ultimate goal of the simplification of MIP method – see Table 2). If possibility of finding +1 or –1 entries in D and/or WD matrix is very high, then multiple LP approach may provide a better alternative. If n is large and possibility of finding +1 or –1 entries in WD matrix is small, although its linear program becomes more complicated, one LP approach may be selected since number of linear programs in the multiple LP approach may grow very large. If possibility of finding +1 or –1 entries in D and/or WD matrix is sufficiently small, then the analyzer may skip the simplification of MIP procedure fully, which means s/he selects neither of the alternatives. In this case the analyzer may use ranking by MIP without simplification. However, generally, possibility of finding +1 or –1 entries in D and/or WD matrix is not known a priori. Ranking procedure with partial/full simplification and without simplification is summarized in Figure 5.

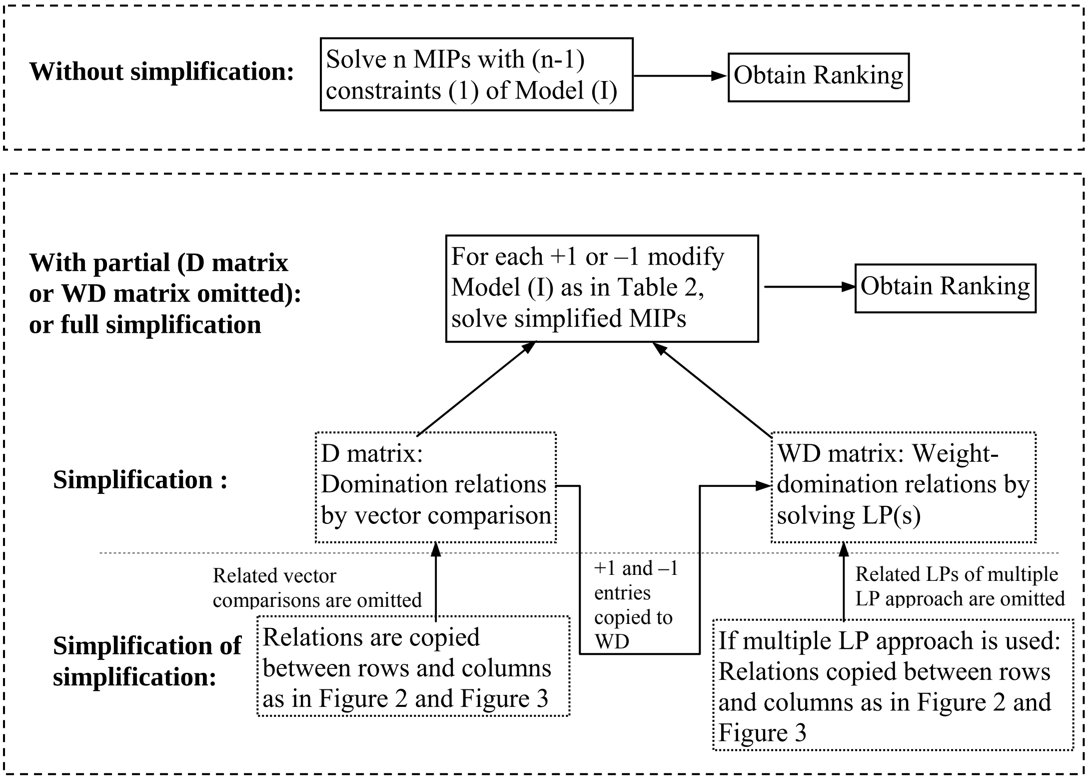


Figure 5 - Ranking procedure with partial/full simplification and without simplification

3.3. Ranking by MIP:

After the simplification procedure is applied, the simplified MIP discussed in previous sections is solved for each unit. As discussed, the objective function of the MIP for a unit provides its rank. If the objective function of MIP for a unit is k, then the unit is ranked at k^{th} standing, which means it outperforms other n-k units with its selected weights. There may exist more than one unit at k^{th} standing, therefore each standing provides a possible ranking group. Depending on the ranking case at hand, standings (ranking groups) may have gaps, i.e. although there exist one or more units at k^{th} standing, there may not exist any unit at $k\text{-}2^{th}$, $k\text{-}1^{th}$, $k{+}1^{th}$, $k{+}2^{th}$ standings etc. This nonconsecutive structure of the ranking by MIP will propose two new terms: reduced rank and discrepancy indexes. Consequently, the standing, the reduced rank for the standing and the discrepancy indexes of the standing will determine the final ranking of the MIP procedure.

After obtaining the standings by MIP, the standings and their corresponding units are sorted in ascending order. The other revealing information, which has been provided by MIP such as the weight set selected by each unit, outperformed and non-outperformed units by each unit with its selected weight set etc., may be presented in this arrangement. This arrangement may include gapped standings and reduced rank eliminates this gapped structure. Reduced rank for the standing is obtained by eliminating the gaps in successive rank values. For example, 1^{st} rank group, whose objective function of MIP for each unit in the group is 1, may be obtained. If in the ordered arrangement, MIP for each unit in the successive rank group in ordered arrangement has objective function of, let's say, 4, this is the 4^{th} rank group. But, there does not exist 2^{nd} or 3^{rd} rank groups. Reduced rank gives rank of 2 for the second group if such a case exists (i.e. 4^{th} rank group is 2^{nd} reduced rank group). Therefore, reduced rank maintains a consecutive ranking of groups. However, reduced rank disregards important information with such a procedure. Returning to example, each unit in the first group, which has 1^{st} standing and 1^{st} reduced rank, outperforms all the other units with its selected weights. Conversely, each unit in the second group, which has 4^{th} standing and 2^{nd} reduced rank, outperforms all the other but three units with its selected weights. Discrepancy indexes re-provide this information. Discrepancy of k^{th} reduced rank group from its superior group is the standing difference between k^{th} reduced rank group and $k\text{-}1^{st}$ reduced rank group. In the example, discrepancy of 2^{nd} reduced rank group from its superior group (1^{st} reduced rank group) is 4-1=3. Discrepancy of k^{th} reduced rank group from its inferior group is the standing difference between k^{th} reduced rank group and $k+1^{st}$ reduced rank group. In the example, let's say, next standing in the ordered form is 6^{th}, consequently this rank group is in 6^{th} standing, 3^{rd} reduced rank group. Then, discrepancy of 4^{th} rank and 2^{nd} reduced rank group from its inferior group is 6-4=2. Therefore, discrepancy indexes indicate 'how far' the jumps are in gapped standings. High discrepancy superior - inferior indexes show high discrepancy of group from previous – successive groups in the ordered form, respectively.

CHAPTER 4

TARGET-DIRECTION-SET VALUE EFFICIENCY ANALYSIS

4.1. Data Envelopment Analysis:

In Data Envelopment Analysis (DEA), there are n comparable Decision Making Units (DMUs) each consuming l inputs and producing k outputs. $X \in \Re_{+}^{lxn}$ and $Y \in \Re_{+}^{kxn}$ are the input and output matrices, whose entries are nonnegative. x_j and y_j are column vectors which correspond to inputs and outputs of j$^{\text{th}}$ DMU (DMU_j), respectively. x_{rj} (y_{pj}) denotes quantity of input r (output p) consumed (produced) by DMU_j.

Under constant returns to scale assumption, Charnes et al. (1978) proposed following linear fractional model:

$$\min \frac{\sum_{\mathrm{r}=1}^{l} \nu_{\mathrm{r}} x_{r0}}{\sum_{\mathrm{p}=1}^{k} \mu_{\mathrm{p}} y_{p0}}$$

$$s.t.$$

$$\frac{\sum_{\mathrm{r}=1}^{l} \nu_{\mathrm{r}} x_{rj}}{\sum_{\mathrm{p}=1}^{k} \mu_{\mathrm{p}} y_{pj}} \geq 1, \; j = 1,...,n,$$

$$\mu_{\mathrm{p}}, \nu_{\mathrm{r}} \geq 0, \; p = 1,...,k; \; r = 1,...,l.$$

Subscript ‘0’ refers to the unit under consideration. μ_p and ν_r are weights of output p and input r, respectively. Therefore, in DEA the measure of efficiency of a DMU_0 is defined as the ratio of weighted sum of its inputs to a weighted sum of its outputs subject to corresponding ratio for each DMU be greater than or equal to one. Charnes et al. (1979) recognized the problem of using non-negativity constraint of variables and replaced it by strict positivity constraint.

This linear fraction model is formulated as linear program by setting the denominator in the objective function equal to arbitrary constant (in general, 1) and minimizing the numerator as:

$$
\begin{aligned}
&\min \ \sum_{r=1}^{l} \nu_r x_{r0} \\
&s.t. \\
&\sum_{p=1}^{k} \mu_p y_{p0} = 1 \\
&\sum_{r=1}^{l} \nu_r x_{rj} - \sum_{p=1}^{k} \mu_p y_{p0} \geq 0, \ j = 1,\ldots,n, \\
&\mu_p, \nu_r \geq \varepsilon, \ p = 1,\ldots,k; \ r = 1,\ldots,l, \\
&\varepsilon > 0 \text{ (non - Archimedian).}
\end{aligned}
$$

Taking the dual of this model, referring the first letters of its introducers' surnames, so-called 'output oriented CCR – primal model' can be obtained:

$$
\begin{aligned}
&\max \ \theta + \varepsilon(1^T s^+ + 1^T s^-) \\
&s.t. \\
&Y\lambda - \theta y_0 - s^+ = y_0 \\
&X\lambda + s^- = x_0 \\
&\lambda, s^+, s^- \geq 0 \\
&\varepsilon > 0 \text{ (non - archimedian)}
\end{aligned}
$$

x_0 (y_0) is the current input (output) vector, which includes x_{r0} (y_{p0}) in its entries, for the unit evaluated. s^+ (s^-) is slack vector corresponding to outputs (inputs).

This model is modified so as to conform to varying returns to scale by Banker et al. (1984). The following is the output oriented BCC - primal model:

$$
\begin{aligned}
&\max\ \theta + \varepsilon(1^T s^+ + 1^T s^-) \\
&s.t. \\
&Y\lambda - \theta y_0 - s^+ = y_0 \\
&X\lambda + s^- = x_0 \\
&1^{\mathrm{T}}\lambda = 1 \text{ where } 1 \text{ is column vector of } 1\text{s} \\
&\lambda, s^+, s^- \geq 0 \\
&\varepsilon > 0 \text{ (non - archimedian)}
\end{aligned}
$$

After these models and their duals, several modified models of DEA have been proposed such as additive models, multiplicative models, assurance region models (Thompson et al. 1986), and cone-ratio models (Charnes et al., 1990). Joro et al. (1998) makes a structural comparison between multiple objective linear programming and DEA, and specifies a reference point model, which proposes most of the extended models of DEA as special cases. This model is:

$$
\begin{aligned}
&\max\ \sigma + \varepsilon(1^T s^+ + 1^T s^-) \\
&s.t. \\
&Y\lambda - \sigma w^y - s^+ = g^y \\
&X\lambda + \sigma w^x + s^- = g^x \\
&\mathrm{A}\lambda \leq b \\
&\lambda, s^+, s^- \geq 0 \\
&\varepsilon > 0 \text{ (non - archimedian)}
\end{aligned}
$$

where, vector g^y (g^x) is aspiration levels for outputs (inputs), and vector w^y >0 (w^x>0) is the weighting vectors for outputs (inputs). In Korhonen (1997), one can see the details of this model and the situations in which this model turns out to be the other extended models as with a discussion about the other models and the efficiency concept of DEA.

In general, the value of θ (σ in reference point model) and slacks s^+ and s^- determine efficiency of the unit. If the value of θ (σ in reference point model) is 1 and slacks are 0 then the unit is efficient. Otherwise, the unit is inefficient and value of θ shows the amount of inefficiency (i.e. for output oriented models, how much to increase outputs - all outputs should be increased by a proportion $1/\theta$ - in order to make unit (weakly) efficient).

4.2. Value Efficiency Analysis:

Value efficiency analysis (VEA), first introduced in Halme et al. (1999), is a means to incorporate preference information into DEA. Originally in DEA, preference information of a decision maker is not included but only the technical and price efficiencies of DMUs are evaluated. Here, only terminology and illustrative examples of VEA will be presented, all other technical details can be found in references.

In Halme et al. (1999), a decision maker (DM) identifies his/her most preferred input-output vector (the most preferred solution – MPS) among all points on the production possibility frontier using an interactive multi objective linear programming (MOLP) search procedure. Through using achievement scalarizing function (ASF) (Wierzbicki, 1980), this search procedure (Korhonen and Laakso, 1986) generates the points on the efficient frontier by projecting a vector to the efficient frontier. The DM then identifies his/her most preferred point among these points on the frontier. Since the search procedure only finds a local optima for DM's utility function, pseudoconcave utility function assumption should be made. Since local optima of a pseudoconcave function over a convex set is also global optima (Bazaraa et al., 1993), this point is assumed to be the most preferred solution over the whole production possibilities. However, if one is sure about globality of the optima, pseudoconcavity assumption is not needed.

Some practical aspects, illustrative examples and some extensions of VEA can be found in Korhonen et al. (2002). Particularly, practical aspects associated with finding MPS and its extension, such as approaches when more than one MPS identification by the DM, are stated. Since the MPS identification is a step in Target-direction-set value efficiency analysis (TDSVEA), one can refer to the reference related with practical approaches in this step.

After the MPS is identified, the value efficiency measure is found as the proportional radial distance of the DMU to the contour of the utility function that passes through the MPS. Since the contour of the utility function through the MPS is difficult to establish, it is approximated by possible tangent cones that cross the

MPS. Tangent cones are possible limits for the contour of pseudoconcave utility function and these limits are most probably closer to the origin than the actual contour. Therefore, the value efficiency score by this method gives the optimistic estimates of true ones.

In Figure 6, a hypothetical value efficiency analysis, which includes five DMUs (A to F) using the same amount (one unit) of one input and producing only two outputs[3], is shown. Assume that the DM identifies B as the MPS (note that the MPS should be on the efficient frontier under pseudoconcavity utility assumption). Then the tangent cones for this MPS are C_1B and BC_2. In the figure, value efficiency measures of E and F are illustrated. For F, optimistic estimate of VE measure (OF/OV^o_F) is same as standard radial measure of Debreu and Farrell. On the other hand, for E, it (OF/OV^o_E) is smaller than standard radial measure (OE/OR_E).

As discussed, this method gives only an optimistic estimate of VE scores. In Joro et al. (2003), this method is extended in two ways. First, without any additional information input by the DM, a bound is established for the true VE score. Second, by direct interaction with the DM, true VE score is obtained by a linear direction search. In Figure 6, these two extensions can be seen. For pseudoconcave utility functions, a contour can be only between the two cone sets: one is tangent cone set C_1B and BC_2, and the other is N_1B N_2, above which all points dominate the MPS. VE scores regarding the tangent cone set provides optimistic estimates (OF/OV^o_F) for true value efficiency scores, whereas VE scores regarding the other set provides pessimistic ones (OF/OV^n). Then, true value efficiency score is between these two values. Regarding the second extension, starting from OF, a linear search through OV^n is done. The DM stops the search when the vector OV^a gives him the same satisfaction with the MPS.

In Halme et al. (1999), Joro et al. (2003) and all the others papers that use VEA, the same type of procedure which employs the radial projection is applied. However, the radial projection itself is a value free type of projection. Without changing any

[3] In this situation the resultant efficient frontier may be considered as an estimate for the well-known production possibility frontier. If there exist one input and one output, the resultant efficient frontier may be considered as an estimate for production function under the assumption that the DMUs are very similar units.

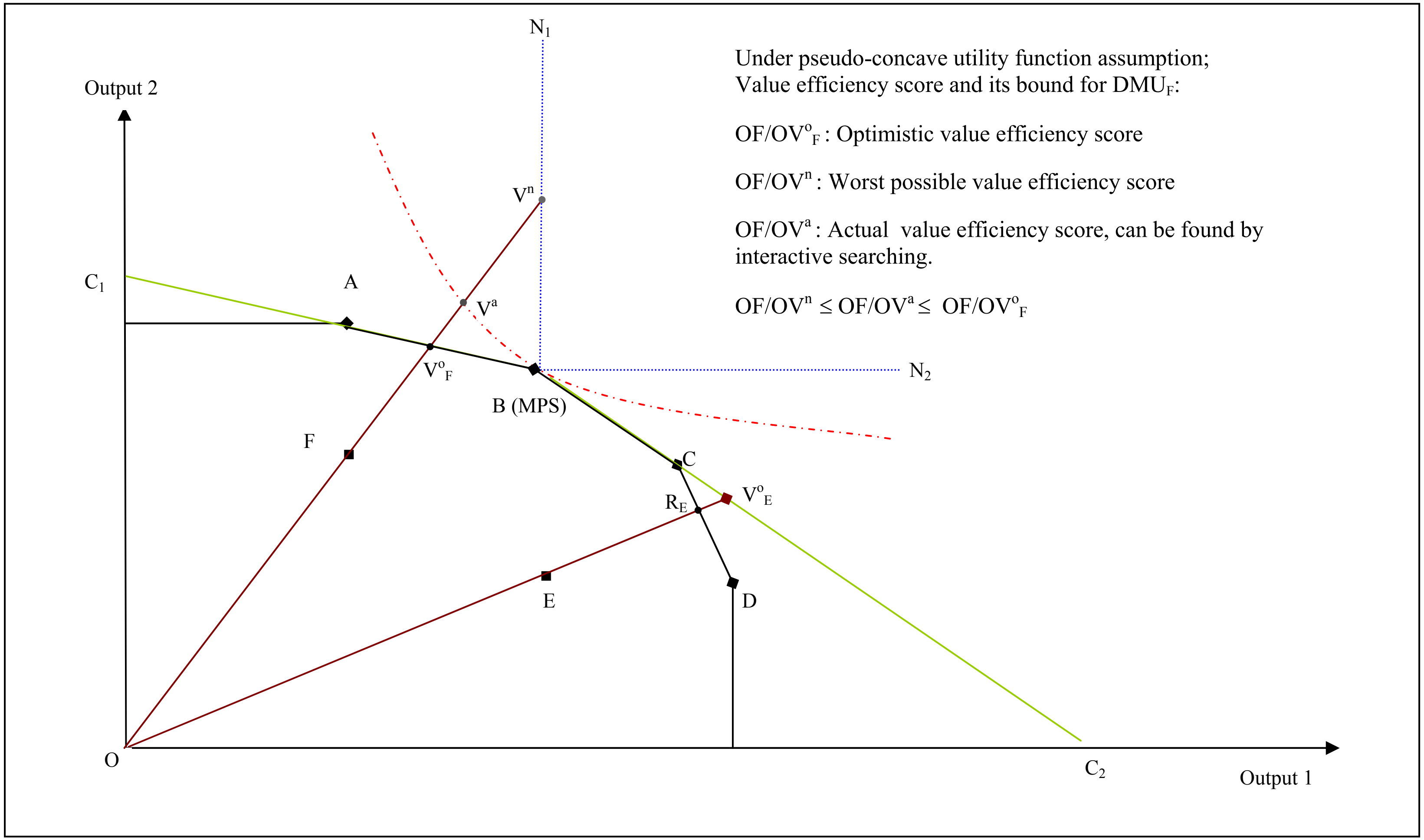


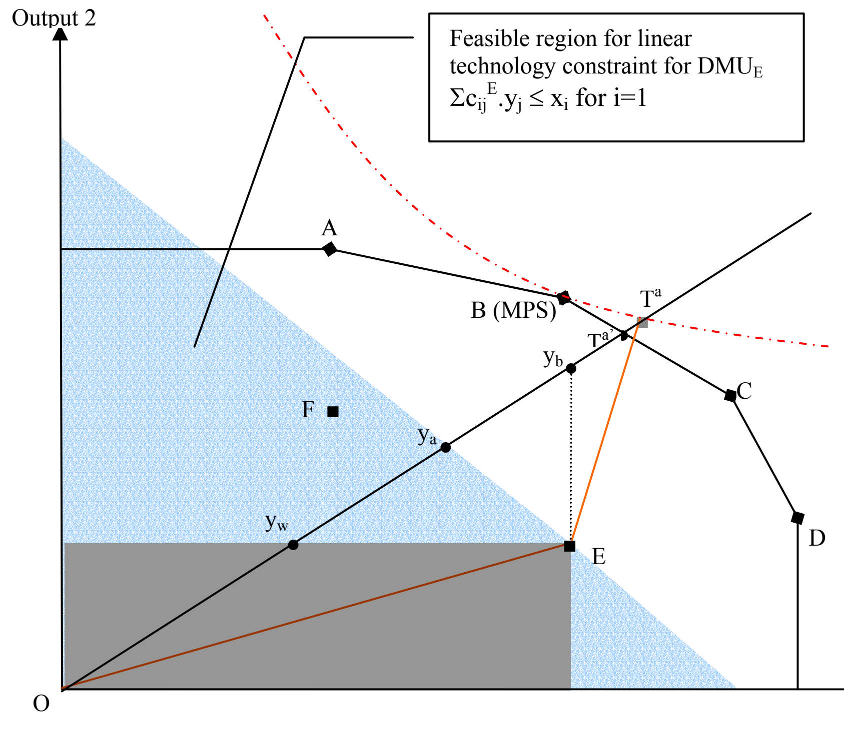
Figure 6 – Value efficiency analysis

inputs and without changing actual proportional values for outputs (in output based DEA), it provides how much to expand outputs of the DMU until it reaches the contour of utility function that passes through the MPS. It does not employ any preferences of the DM about the direction of improvement of the DMU. In real world, DMUs do not necessarily expand or plan to expand with the same and only same output proportion. If DMUs are any production or service organizations their managers select a new bundle of products or services in the following year or probably plan to expand their current production level with other output proportions. Therefore, they may select other target directions rather than the radial projection (see Appendix A for a partial analysis of DM's preferences for selecting the projection direction). If the objective is to identify the price and technical efficiency of the DMU, a value-neutral radial projection may be meaningful. On the other hand, if the objective is to set target according to preferences and to identify how much DMUs are far away from the preferred target, a value-neutral radial projection may not be meaningful. In fact, in the following sections it will be shown how radial projection may overestimate the value inefficiencies of the DMUs. It will also be shown that through simply rearranging input allocation among outputs and thus substituting outputs, a value inefficient DMU may become value efficient (and even value super-efficient). Consequently, even optimistic value efficiency estimates of radial projection may underestimate the value efficiency potential of DMUs. In the next section, a tool that provides the efficiency measures regarding to a target input/output set will be explained since it will be used in the target-direction-set value efficiency analysis provided in this study.

4.3. Target mix approach for measuring efficiency:

In Joro (2000), an approach to measure efficiency, when a DM provides a targeted input/output values (target mix), is developed. With assumption of nonnegative substitution between outputs (in output oriented case), this approach identifies a bound for the efficiency score regarding to the target mix. One can see the general outline of the approach here, but the technical details will be left to the reference.

In Figure 7, an MPS (DMU_B) is identified by the DM with a method explained in section 4.2 and assume that an output mix of T^a (how this target is found will be



Substitution equivalent, y_a, (if DMU changes input allocation, how much can it produce in direction of targeted projection) determination:
Assume that DMU_E has a linear technology constraint
$\Sigma c_{ij}^{E}.y_j \leq x_i$ for all i and $y_j \geq 0$
where y_j is production of j^{th} output, x_i is amount of i^{th} input, c_{ij} is amount of input i required to produce one unit of output j. This produces a convex region.
And assume that this technology constraint is known (managers or engineers can determine this technology constraint). x_i, y_j ,c_{ij}'s are parametric values and one tries to determine y_a. Then, it is the intersection point of the surface of convex region ($\Sigma c_{ij}^{0}.y_j \leq x_i$ for all i and $y_j \geq 0$) and radial projection through OT^a (or $OT^{a'}$). (It can be shown that there exists only one intersection point). This point is y_a and can be found with the following LP:

$$\text{Max} \sum_{j=1}^{k} y_j$$

s.t
linear technology constraint: $\Sigma c_{ij}^{0}.y_j \leq x_i$ for i=1,…,*l* (*l* is total number of inputs)

line OT^a: $\frac{y_j}{y_j^{T^a}} = \frac{y_{j+1}}{y_{j+1}^{T^a}}$ for j = 1,...,k-1 (k is total number of outputs)

$y_j \geq 0$

When T^a and y_a are known, the true target-direction-set value efficiency measure can be obtained.

Figure 7 – Target mix approach to measure efficiency in DEA and in Target-direction-set Value Efficiency Analysis

explained later) will be targeted for E. The shaded rectangle in the figure is free disposable hull (FDH) for E since any point in this rectangle can be obtained freely only by discarding some outputs (with implicit assumption that discarding outputs is costless). Therefore, output mix y_w can be achieved freely by E. In target mix approach, a non-positive marginal rate of substitution is assumed (without changing input levels but possibly only changing the allocation of inputs to outputs, if one type of output is decreased by one unit, how many units of other type of output can be increased). When this substitution is zero (which means no additional output can be produced if one output is decreased by one unit – corresponds to free disposable case) the output mix y_w can be achieved. This is the worst possible output mix that can be achieved on the radial ray OT^a by reallocation of inputs of E. When this substitution is minus infinite (which means unbounded additional output can be produced if one output is decreased by only a small perturbation – which is not possible in practice but gives upper bound, in fact) the output mix y_b can be achieved. This is the best possible output mix that can be achieved on the radial ray OT^a by reallocation of inputs of E. In Joro (2000), it is assumed that actual output mix that can be achieved by substitution, y_a (substitution equivalent of E) cannot be found but it should be between Oy_w and Oy_b. Therefore, actual target mix efficiency score Oy_a/OT^a should be between lower level Oy_w/OT^a and upper level, Oy_b/OT^a. In that paper, the possibility of finding y_a is not discussed. In this study, an approach to find y_a will be discussed.

4.4. A method to find substitution equivalent of DMUs in Target mix approach:

As explained, in Joro (2000), finding substitution equivalents (y_a) of DMUs is not discussed. In this section, under the assumption of local linear technology and no additional cost such as setup costs to substitute outputs, a method to find y_a will be discussed.

Assume that DMU_E has the following linear technology constraint around its current output mix (point E):

$$\sum_{j=1}^{k} c_{ij}^{E}.y_j \leq x_i \ \text{ for all } i = 1,\ldots, l.$$

$$y_j \geq 0$$

where y_j is current production of j^{th} output by E, x_i is current amount of i^{th} input usage by E, c_{ij} is amount of i^{th} input required to produce one unit of j^{th} output . Each DMU can have its own linear technology, these different technologies are represented in c_{ij}^{0} and its current input and output level where superscript 0 represents the DMU that is evaluated (it is E in the constraint presented above). With this constraint, since DMU's current level of inputs and outputs is on the boundary of constraint, it is implicitly assumed that DMU currently uses its own technology efficiently. Otherwise, its linear technology constraint cannot be determined without making further assumptions.

In this constraint y_j and x_i values are known since they are current output and input values for DMU_E. Nevertheless, there may exist situations in which c_{ij} values may not be known and they should be estimated by different techniques or expert opinions. However, even in these situations, some estimates about c_{ij} values can be obtained. Since technology is assumed to be linear only around the current input-output levels (locally linear assumption), it may produce satisfactory results even if the actual technology constraint of the unit is an unknown nonlinear constraint. With linear constraint, a good estimate for the actual substitution equivalent can be obtained provided that the substitution equivalent is not very 'far' from actual mix (if 'far', linearization may loss its effectiveness). Of course, closeness and distance are problem dependent issues and when the linearization of this nonlinear constraint is used, attention must be paid to any inconvenience of this assumption. With locality assumption also good estimates for substitution equivalent can be obtained when the actual technology constraint is linear around current output set with given set of parameters c_{ij} regardless of the form of the technology constraint and set of parameters in other regions. Nonetheless, same concerns as in the previous situation are valid.

When the target mix T^a for the considered DMU is known, y_a for that DMU could be found by the following linear program:

$$\text{Max} \sum_{j=1}^{k} y_j$$

s.t.

linear technology constraint: $\sum_{j=1}^{k} c_{ij}^{0}.y_j \le x_i$ for i=1,…, l where l is total number of inputs

line OT^a: $\frac{y_j}{y_j^{\,0}} = \frac{y_{j+1}}{y_{j+1}^{\,0}}$ for j = 1,...,k-1 where k is total number of outputs

$y_j \ge 0$

Or equivalently:

$$\text{Max} \sum_{j=1}^{k} y_j$$

s.t.

linear technology constraint: $\sum_{j=1}^{k} c_{ij}^{0}.y_j \le x_i$ for i=1,…,l where l is total number of inputs

line OT^a: $y_{j+1}^{\,0}\, y_j = y_j^{\,0}\, y_{j+1}$ for j = 1,...,k-1 where k is total number of outputs

$y_j \ge 0$

where y_j^0 is the j^{th} output value of target mix (T^a). Note that direction OT^a is same as direction $OT^{a'}$. Therefore, T^a and $T^{a'}$ can be used interchangeably in this LP.

As implied, this LP should be solved for each DMU whose substitution equivalent is aimed to be found. For each DMU, the direction $OT^{a'}$ will be different as well as c_{ij}^0, and different y_a values[4] will be found for different DMUs.

[4] After finding y_a, validity of locally linear technology assumption may be tested by one easy but incomplete way. The DM may be asked to if the DMU was in input-output mix that corresponds to y_a, what $c_{ij}^{y_a}$ values (technology parameter estimates for substitution equivalent, y_a) would be. With these estimates the analyzer may look at whether the current input-output mix (e.g. DMU_E) is feasible or not. If it is not feasible, the assumption or the consistency of the DM may be questioned, since if it is possible to substitute from current mix to y_a, it would also be possible to substitute from y_a to current mix.

Since the direction of $OT^{a'}$ is positive, the objective function will give the intersection of line $OT^{a'}$ and the boundary of convex region $\sum_{j=1}^{k} c_{ij}^{0}.y_j \leq x_i$ which is exactly the substitution equivalent of E, y_a. In fact, the objective function can be

$$\text{Max} \sum_{j=1}^{k} a_j y_j$$

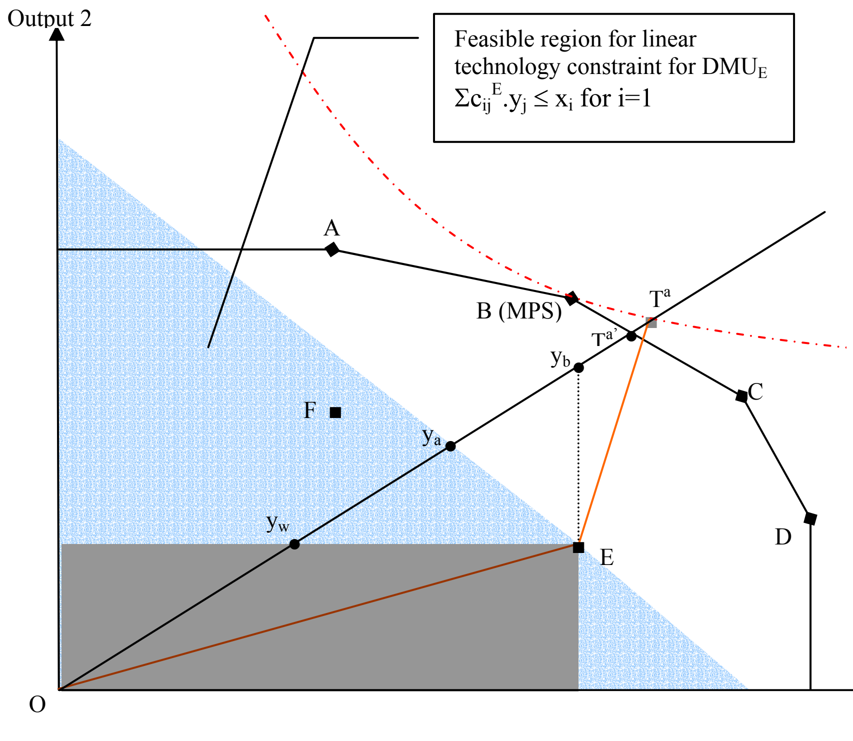
where a_j values are any positive numbers.

In Figure 8, finding substitution equivalent for DMU_E is illustrated. As discussed before, linearity assumption of technology constraint is not needed everywhere but only between point E and y_a. Even the linearity assumption of technology constraint is not needed in this region if E and y_a are sufficiently close. In this situation, linear constraints may provide good estimates for actual y_a whatever the form and parameters of the actual technology constraints are. One can consider these situations by trying different conditions for the constraint in Figure 8.

4.5. Target-direction-set Value Efficiency analysis:

As explained, radial projection is itself a value free projection in terms of DM's preferences. It does not incorporate the value judgments, but shows only how much to increase outputs with same proportions while holding inputs constant in order to be (value) efficient. However, the radial projection is only one of infinite number of alternative projection directions. When a DMU or the DM plans a direction for their future, it is only pure chance that this targeted direction be the radial one. In fact, in a different context inappropriateness of radial projection in target setting is identified (Korhonen et al., 2003).

Instead of projecting radially, concerning the current conditions (prices, condition of their competitors, taxes, its mission and vision etc.) the DMU or the DM may probably select a different direction. This direction is identified by current input-output level of the DMU and by the weight attached by the DMU or the DM to each output. Assuming that weight γ^0_j is attached to projection of j^{th} output of evaluated



Substitution equivalent, y_a, (if DMU changes input allocation, how much can it produce in direction of targeted projection) determination:
Assume that DMU_E has a linear technology constraint
$\Sigma c_{ij}^{E}.y_j \leq x_i$ for all i and $y_j \geq 0$
where y_j is production of j^{th} output, x_i is amount of i^{th} input, c_{ij} is amount of input i required to produce one unit of output j. This produces a convex region.
And assume that this technology constraint is known (managers or engineers can determine this technology constraint). x_i, y_j ,c_{ij}'s are parametric values and one tries to determine y_a. Then, it is the intersection points of the surface of convex region ($\Sigma c_{ij}^{0}.y_j \leq x_i$ for all i and $y_j \geq 0$) and radial projection through OT^a (or $OT^{a'}$). (It can be shown that there exists only one intersection point). This point is y_a and can be found with the following LP:

$$\text{Max} \sum_{j=1}^{k} y_j$$

s.t

linear technology constraint: $\Sigma c_{ij}^{0}.y_j \leq x_i$ for i=1,…,l (l is total number of inputs)

line OT^a: $\frac{y_j}{y_j^{T^a}} = \frac{y_{j+1}}{y_{j+1}^{T^a}}$ for j = 1,...,k-1 (k is total number of outputs)

$y_j \geq 0$

When T^a and y_a are known, true target-direction-set value efficiency measure can be obtained.

Figure 8– Finding substitution equivalent of DMU_E

unit DMU_0 by the DMU or the DM, the line,

$$\frac{y_j - y_j^0}{\gamma_j^0} = \frac{y_{j+1} - y_{j+1}^0}{\gamma_{j+1}^0} \text{ for } j = 1,...,k\text{-}1 \text{ where k is total number of outputs}$$

determines the target direction where y_j^0 defines current level of j^{th} output of DMU_0. Weights for the projection define the preference of the DM or the DMU for projection.[5]

In Figure 9, a target direction for DMU_E is illustrated. Since, the radial projection may be undesirable in the current conditions or arduous to achieve, the DM selects a new direction possibly more desirable in current conditions and easier to achieve (see the situations explained in Appendix A). More importantly, this new direction is more convenient to the DM's preferences. In this direction, bound points T^o or T^n can be found with a similar method to finding V^o and V^n described in Joro et al. (2003). Furthermore, requiring additional input from the DM, T^a can be found by a linear search like V^a is found in the same paper.

Since a new direction is selected for target direction, efficiency scores with standard radial projection of VEA cannot be used. However, radial efficiency measures after finding the substitution equivalent gives new value efficiency scores, target-direction-set value efficiency (TDSVE) scores or potential value efficiency (PVE) scores. This score provides, after the reallocation of inputs (reaching substitution equivalent of DMU), how much to increase outputs in order to be target-direction-set value efficient (i.e. in order to reach T^a, which gives the same satisfaction as the most preferred solution to the DM and is in the direction of the DM's targeted projection). In a sense these scores are potential scores since it determines the efficiency after substituting outputs. The PVE score comprises the original advantage of radial measure stated in section 2.2; it still defines in what proportion revenue should be increased to become value efficient with the same cost (terminology such as cost and revenue is problem dependent and can be changed according to context). Nevertheless, it differs from the original radial measures in that it uses the substitution equivalent rather than current output-input levels of the

[5] See Appendix A for a discussion about the projection direction selection of the DM.

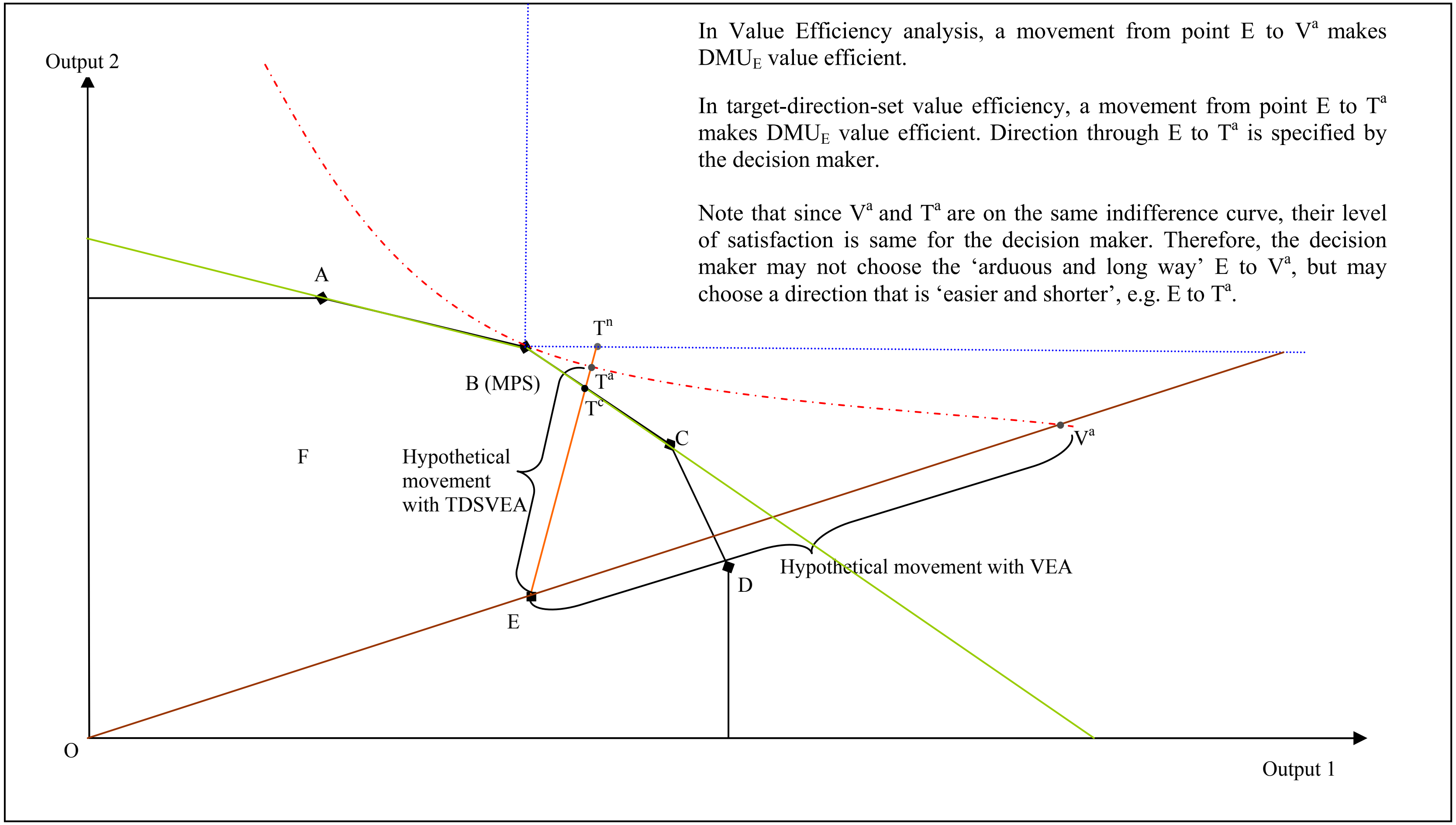


Figure 9– Target-direction-set Value Efficiency Analysis

DMU to project radially, i.e. it estimates the potential value efficiency in the sense that only if the DMU substitutes the outputs, its value efficiency would be the specified value. In different words, since the DMU and its substitution equivalent are technologically equal (i.e. produced by same input levels and therefore with same cost), radial projection of substitution equivalent resembles radial projection of DMU. Only difference between them is proportion of output: in target-direction-set approach this proportion conforms to preferences of the DM. However, as discussed before, linear technology assumption should not be taken for granted and it should not be forgotten that substitution equivalent is a hypothetical unit and testing its underlying assumption is important. Furthermore, if one is not sure about globality of the MPS on the production possibility frontier, pseudoconcave utility function assumption should be tested as in Joro et al. (2003). Then the following methodology can be applied to the DMU by the interaction of the DM in order to get potential value efficiency score (PVE) or target-direction-set value efficiency score (TDSVE score):

1) The decision maker (DM) determines the most preferred solution (MPS) in the production possibility frontier with achievement scalarizing function of Wierzbicki (1980) and search procedure of Korhonen and Laakso (1986). The illustration and explanation of the procedure can be found in Halme et al. (1999) and Korhonen et al. (2002). As discussed there other methods to find the MPS may also be used. This MPS may be represented by (x^*, y^*), where they are current level of inputs and outputs of the MPS, respectively.
2) In order to identify targeted projection, the DM gives weights $\gamma^0{}_j$ (superscript 0 means the DMU under evaluation) for outputs (j=1,…, k) of the DMU. The weights show compensation between outputs for the projection. These weights and the current position of the DMU determine the target-direction-set projection for the DMU (see Appendix A). For example direction ET^a in Figure 10 can be identified as:

$\frac{y_j - y_j^E}{\gamma_j^E} = \frac{y_{j+1} - y_{j+1}^E}{\gamma_{j+1}^E}$ for j = 1,..., k-1 where k is total number of outputs (k=2 in Figure 10) where y_j^E shows the current output levels for the DMU_E.

3) After direction is given, by an interactive linear search as in Joro et al. (2003), targeted hypothetical unit (T^a), which gives the same satisfaction as the MPS in the specified direction, is found as can be seen in Figure 10. In order to test consistency of the DM and/or pseudoconcavity of the utility function, following modified approach of Joro et al. (2003) can be used:

   a. For testing pseudoconcavity assumption and consistency of the DM, search should be started from point E (current output-input level of DMU_E) through the direction specified in step 2. For this test two projected points will be needed: one is the projection of the DMU with specified direction in step 2 to *the efficient frontier* T^e, the other is projection of the DMU with specified direction in step 2 *on tangent cones of the MPS*, T^c.

   b. T^e can be found by the following LP:

$$
\begin{aligned}
&\max = Z = \sigma + \varepsilon(1^T s^+ + 1^T s^-) \\
&s.t. \\
&Y\lambda - \gamma\sigma y_0 - s^+ = y_0 && (1) \\
&X\lambda + s^- = x_0 && (2) \\
&A\lambda + \mu = b && (3) \\
&s^+, s^- \geq 0 && (4) \\
&\lambda_j \geq 0, \\
&\mu_j \geq 0,
\end{aligned}
$$

   In this linear program, *n* is total number of DMUs, *k* is total number of outputs and *l* is total number of inputs. *Y* is *k x n* matrix, which denotes current output levels for DMUs, whereas *X* is *l x n* matrix, which denotes current input levels for them. y_0 and x_0 are output and input vectors of the DMU under evaluation, respectively. Constraint (3) may or may not exist in the model. If it exists then it represents the restriction set on DEA weights λ and several DEA models can be

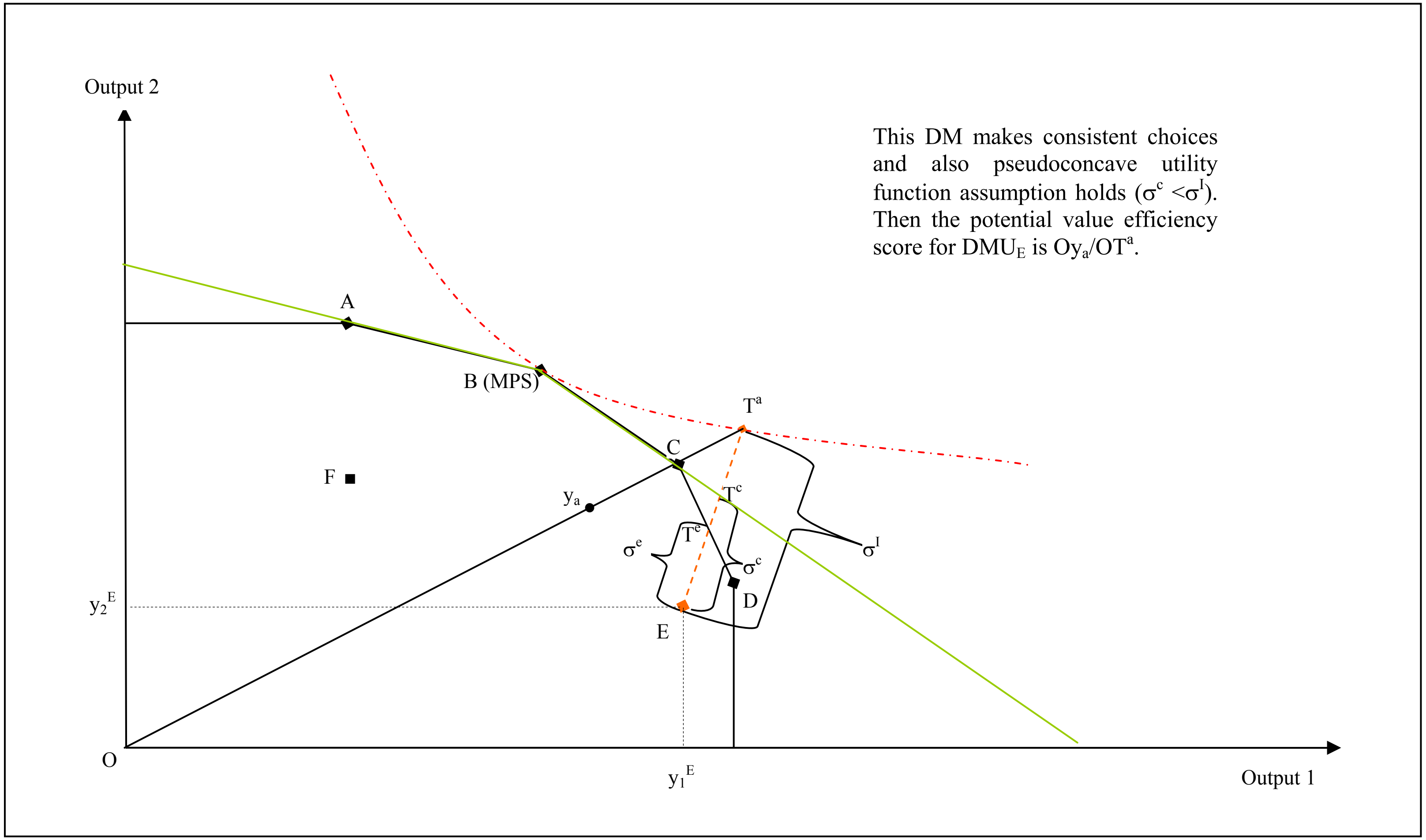


Figure 10 – An example for TDSVEA methodology

represented by this restriction (e.g. if constraint (3) is $1^T\lambda = 1$, then the model resembles BCC model) and through μ values, which are the slacks of the constraint (3). For the possible types of constraints, one may see Korhonen et al. (2003). s^+ and s^-, namely slack variable vectors, include slack variables in order to get rid of projection to weakly efficient points rather than strongly efficient points. γ is a kxk diagonal matrix which includes weights γ_j discussed in step 2 in its diagonal. σ is a scalar. Optimum value of the variable σ in this LP, which will be denoted as σ^e, is particularly important. The projected point calculations and consistency checking needs this value. Then the projected point becomes $T^e = y_0 + \gamma\, \sigma^e\, y_0$.

c. The other point, T^c (Figure 10), can be found with a modified version of the value efficiency linear program presented in Halme et al. (1999):

$$
\begin{aligned}
&\max = Z = \sigma + \varepsilon(1^T s^+ + 1^T s^-) \\
&s.t. \\
&Y\lambda - \gamma\sigma y_0 - s^+ = y_0 \qquad (1) \\
&X\lambda + s^- = x_0 \qquad (2) \\
&A\lambda + \mu = b \qquad (3) \\
&s^+, s^- \geq 0 \qquad (4) \\
&\lambda_j \geq 0, \text{if } \lambda_j^* = 0, \text{otherwise } \lambda_j \text{ is unrestricted, for } j = 1,2,...,n, \\
&\qquad \text{where n is total number of DMUs} \qquad (5) \\
&\mu_j \geq 0, \text{if } \mu_j^* = 0, \text{otherwise } \mu_j \text{ is unrestricted, for } j = 1,2,...,k, \\
&\qquad \text{where k is total number of outputs} \qquad (6) \\
&\text{where } \lambda^* \text{ and } \mu^* \text{ correspond to the MPS } (x^*, y^*) \qquad (7) \\
&y^* = Y\lambda^* \\
&x^* = X\lambda^*
\end{aligned}
$$

All the parameters, variables, constraints (1), (2), (3) and (4) are as in step 3.b. One important point in the formulation is, if the MPS is linear combinations of j^{th} DMU with other DMUs (condition (7) in the formulation), the nonnegativity constraint on DEA weight λ_j and μ_j is relaxed (λ_j and μ_j become unrestricted in sign). Then projection to tangent cone rather than to efficient frontier becomes possible. σ is a scalar. Optimum value of the variable σ in this LP, which will be denoted as σ^c, is particularly important. Then the projected point to tangent cone, T^c, becomes $y_0+\gamma\ \sigma^c\ y_0$.

d. If pseudoconcavity and consistency checking will be applied, interactive linear search should be started from the current input and output level (x_0, y_0). While x_0 is held constant, output level should be changed by $y_0+\gamma\ \sigma\ y_0$ where σ is a scalar and γ is the diagonal matrix in step 3.b. By changing σ from 0 to positive numbers, one can search for the T^a (preference equivalent with the MPS on the specified direction). Assume that the DM specifies σ as σ^I (the DM identifies that he is indifferent between the MPS and the hypothetical DMU $y_0+\gamma^T\sigma^I\ y_0$) in Figure 10. Then the pseudoconcavity and consistency checking as in Joro et al. (2003) can be done:

$$\text{If } \sigma^I \text{ is between } \begin{cases} \left[0, \sigma^e\right), \text{ go step3.d.i} \\ \left[\sigma^e, \sigma^c\right), \text{ go step3.d.ii} \\ \left[\sigma^c, \sigma^n\right), \text{ then finish step3.d. The targeted point } T^a \text{ can} \\ \qquad\qquad\qquad \text{be found as } y_0 + \gamma^T \sigma^I y_0 \end{cases}$$

i. No consistent choice situation: The DM has chosen $\sigma^I < \sigma^e$, which means that the $y_0+\gamma^T\sigma^I y_0$ is dominated by $y_0+\gamma^T\sigma^e y_0$. But this situation is inconsistent with strictly increasing value function. The DM has selected an MPS, which is less preferred to another efficient point. He/she should select either new σ^I or new MPS.

ii. Inconsistent choice situation: The DM has chosen $\sigma^e \leq \sigma^I < \sigma^c$. Either the pseudoconcavity assumption is true but the DM has made an inconsistent choice or the DM has made a consistent choice but the pseudoconcavity assumption is not true.

This test provides valuable but incomplete information since while $\sigma^I \in [\sigma^c, \sigma^n)$, the DM can still be inconsistent or pseudoconcavity assumption can still be invalid.

d'. If pseudoconcavity assumption and consistency of the DM will not be tested, step 3.b can be eliminated, whereas 3.c should still be applied. Search should be started from T^c in specified direction in step 2. Then, in interactive linear search, σ^0 is not started from zero but started from σ^c. If σ^I will be the result of this search, the target point T^a can be found as $y_0+\gamma^T\sigma^I y_0$.

4) A projection line through the origin to this target point is specified as $\frac{y_j}{y_j^{T^a}} = \frac{y_j}{y_{j+1}^{T^a}}$ for j = 1,..., k-1, where k is total number of outputs, where j^{th} output value for target points, T^a, will be denoted as $y_j^{T^a}$ for j=1,…,k.

5) For the DMU under evaluation, substitution equivalent, y_a, which denotes, after the reallocation of inputs, how many units of outputs will be produced in the radial direction through target point, is found with the LP specified in Section 4.4 which is:

Max $\sum_{j=1}^{k} y_j$

s.t.

linear technology constraint: $\sum_{j=1}^{k} c_{ij}^0 . y_j \le x_i$ for i=1,…,*l* where *l* is total number of inputs

line OT^a: $y_{j+1}^0 \, y_j = y_j^0 \, y_{j+1}$ for j = 1,...,k-1 where k is total number of outputs

$y_j \ge 0$

6) TDSVE score (or potential value efficiency score after substituting outputs) is Oy_a/OT^a.

Graphical representation of the methodology can be seen in Figure 10. The DMU under evaluation is E. The DM selects MPS as B, therefore $(x^*, y^*) = (x_B, y_B)$. Note that a hypothetical unit such as any point between B and C can also be selected. The

DM selects target preference weights, γ_j^E for j=1,..,k (k is total number of outputs). In the figure, these weights correspond to direction ET^a. The analyzer selects to test pseudoconcavity of value function and consistency of the DM. S/he, solving the LPs specified in step 3.b and step 3.c, finds σ^e, T^e and σ^c, T^c respectively. After these LPs, linear search is applied and the DM specifies σ^I. The DM passes consistency check and the target point T^a is found. With LP specified in step 5, substitution equivalent, y_a, is found. Potential efficiency score is Oy_a/OT^a.

If value efficiency scores of Halme et al. (1999) are in hand, then comparison of these scores and potential efficiency scores may provide important information. Harmony index is such information and will be explained in section 4.6.a. Another information is the difference between potential scores and value efficiency scores. If the DM knows the technology and capabilities of DMU and wants the DMU to project 'shortly and easily' to his/her target, then one should expect this difference be positive. The DM should specify a direction for the DMU be in a better position after substituting its outputs. If this difference is negative then the analyzer may suspect about consistency of the DM and should warn the DM about the consequences of this selection. The analyzer directly explains that if this target is applied to the DMU under consideration it may be a worse situation than the radial projection. Of course, this should be only a warning and the preference structure of the DM should not be affected by the analyzer.

Input levels and their allocation after reallocation (substitution equivalent, y_a) may also be usable information for the DM. Projection needs a kind of productivity or technology improvement. If the productivity or technology improvement is better or more possible in the radial direction, (that is projecting with the same output proportions) then the DM may determine that firstly immediate substitution of output (DMUs' output levels is immediately changed from its current level to its substitution equivalent, y_a) should occur. After substitution, radial productivity and technology improvement may direct DMU to its target point (e.g. for DMU_E in Figure 10, the path is first to substitute outputs by reallocating inputs from point E to y_a and then project from y_a to T^a), therefore, output proportions do not change in the projection. In this case, the input allocation of the DMU is provided by the LP in section 4.4 as $\sum_{j=1}^{k} c_{ij}^{0}.y_j^a = x_i$ for i=1,…, l where l is total number of inputs and y_j^a is optimum values of this LP and determines j^{th} output level of y_a, therefore i^{th} input assigned to production of j^{th} output is $c_{ij}^{0}.y_j^a$). As a second possibility, the DM decides to project from current input and output levels of DMUs to target points (e.g. for DMU_E in Figure 10, the path is from point E to directly point T^a). In this case output proportions always change while the projection occurs. TDSVEA provides these two options to the DM, and the DM selects between them. Obviously, if the DMU has a super-efficient potential value efficiency score (as DMU_E in Figure 12), the DM most probably selects the immediate substitution.

As discussed, the TDSVEA modifies the value free projection of VEA, by a DM's selected projection direction. Doing this it requires more information than VEA for establishing the target direction and for finding substitution equivalent. On the other hand, it provides valuable information such as more meaningful targets for units, (potential) distance of units from these targets, input allocations after substitution, a modified ranking of DMUs, lack of harmony between the DM and the manager of DMU (see section 4.6.a) etc. Target-direction-set value efficiency analysis can be applied in many situations with many contexts. Two specific situations will be illustrated in the next section in order to demonstrate the usefulness of the approach.

4.6. Two contexts where the TDSVEA is useful:

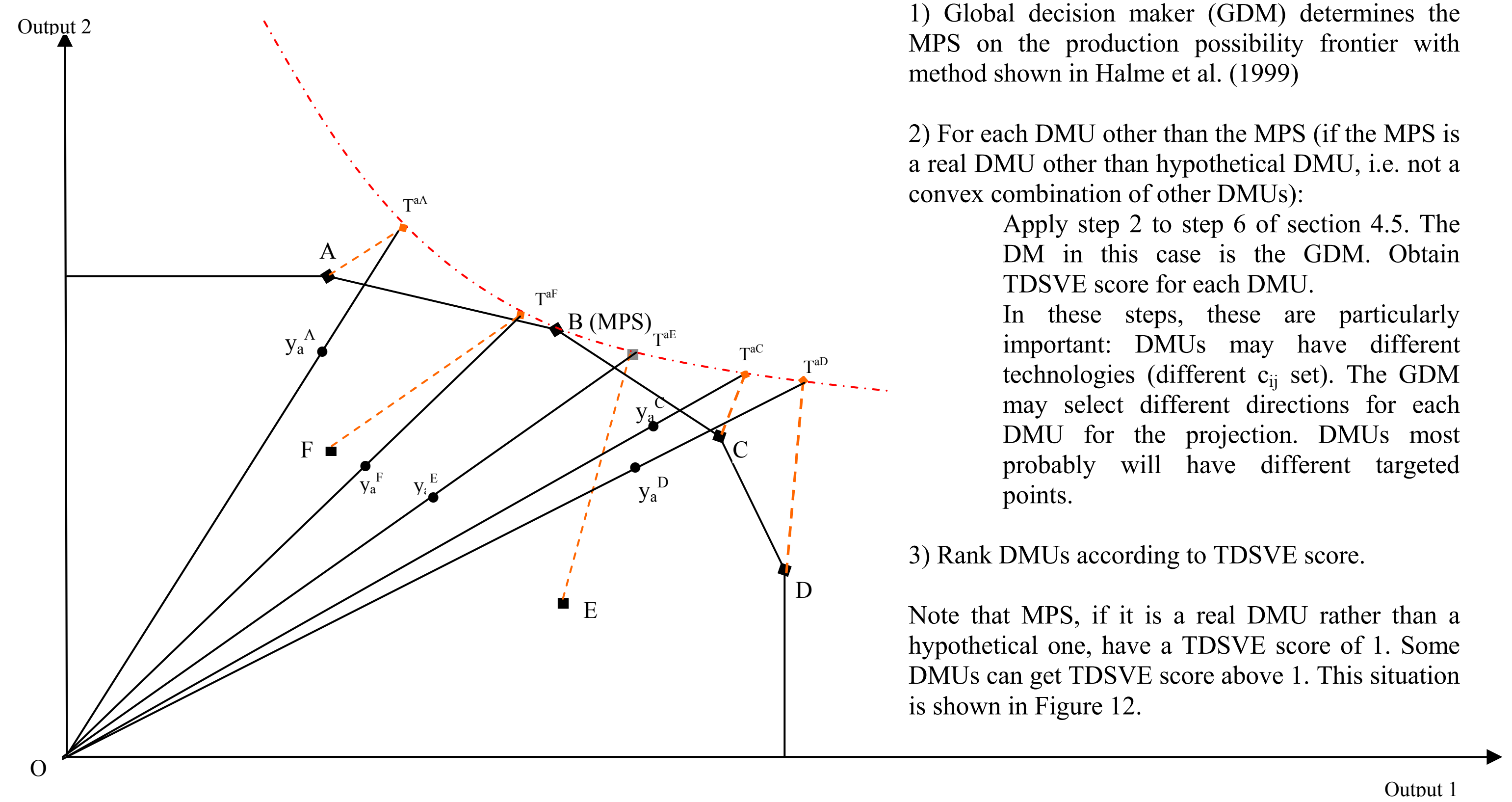
a) Complete ranking when a global decision maker exists:

There exist some situations that decision maker has the power to project all DMUs to a hypothetical and equivalent unit to his MPS, all the information about the technology of all DMUs, and the power to manage all DMUs. A CEO may evaluate comparable branches of his/her company, a municipality may appraise the grocery stores, markets etc., or a country may compare firms in a sector according to its national goals. The important thing that should be paid attention to is that DMUs should be comparable and similar units, which is the original assumption in DEA.

When the assumptions about global decision maker stated above partly or fully hold, a complete ranking of all DMUs through target-direction-set value efficiency analysis is possible. The methodology is very similar to those in section 4.5 but requires some modifications. New methodology and a graphical example are provided in Figure 11. In this methodology, steps in section 4.5 are applied to each DMU, to obtain PVE score. With this approach a complete ranking of DMUs, substitution equivalents of the DMUs and some useful information such as harmony index of DMUs can be obtained.

It is worth consideration that some DMUs can get TDSVE score above one, which means after reallocating its inputs, its substitution equivalent in the targeted direction is preferred even to the MPS. Therefore, it has a potential to be the actual MPS if it reallocates its inputs. An example for this situation is provided in Figure 12. Note that DMU_E in Figure 12 has a TDSVE score above 1 ($Oy_a/OT^a>1$). On the other hand, if the value efficiency score provided in the Joro et al. (2003) is applied to DMU_E, even the optimistic estimates (OE/OV^o) is lower than 1. Therefore, even the optimistic estimates of value efficiency score in Joro et al. (2003) may underestimate the DMUs' potential value efficiency scores.

Differences between direction of current output mix (OE or OV^a) and direction through substitution equivalent (Oy_a or OT^a) may provide useful information. Local manager of the DMU selects an output mix OE, but according to preferences of global decision maker, this selection should be Oy_a. If they are not equivalent then two possibilities may exist: the local manager conformed to previous preferences of global decision maker but preference structure of global decision maker has changed, or preference structure of global decision maker has not changed but the local manager did not conform to preferences of global decision maker. In the second possibility, a lack of harmony may exist between local manager and global manager (this problem may be considered parallel as the problem indicated by Farinaccio and Ostanello 1999, which is discussed in Chapter 2 of this thesis study: operational level decisions by the local managers of DMUs may produce different consequences than strategic level decisions by GDM). This lack of harmony may be measured by the angle between OE and Oy_a or if the actual value efficiency scores



Methodology for ranking:

1) Global decision maker (GDM) determines the MPS on the production possibility frontier with method shown in Halme et al. (1999)

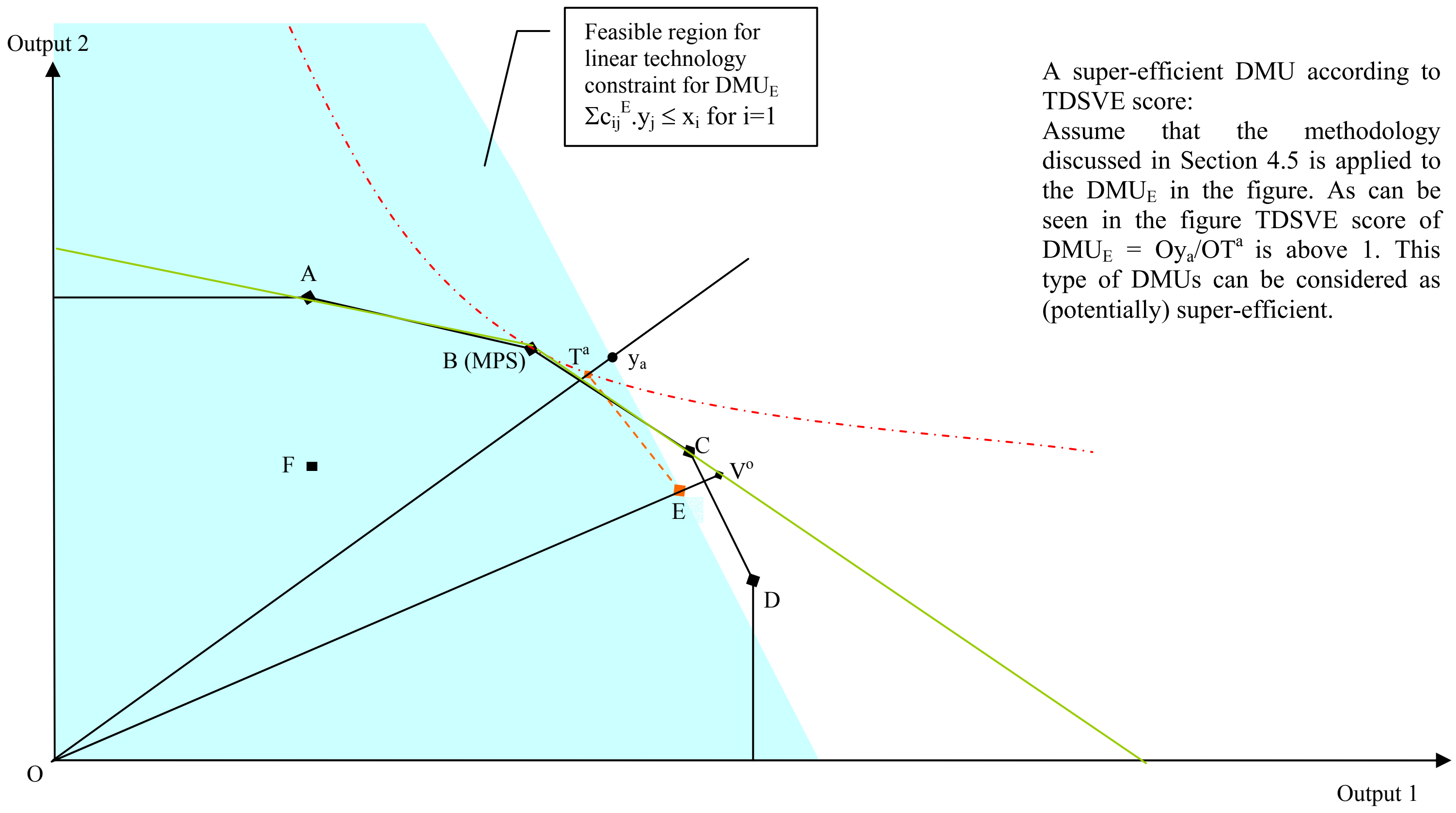
2) For each DMU other than the MPS (if the MPS is a real DMU other than hypothetical DMU, i.e. not a convex combination of other DMUs):

> Apply step 2 to step 6 of section 4.5. The DM in this case is the GDM. Obtain TDSVE score for each DMU.
>
> In these steps, these are particularly important: DMUs may have different technologies (different c_{ij} set). The GDM may select different directions for each DMU for the projection. DMUs most probably will have different targeted points.

3) Rank DMUs according to TDSVE score.

Note that MPS, if it is a real DMU rather than a hypothetical one, have a TDSVE score of 1. Some DMUs can get TDSVE score above 1. This situation is shown in Figure 12.

Figure 11 – Complete ranking when there is a global decision maker



A super-efficient DMU according to TDSVE score:
Assume that the methodology discussed in Section 4.5 is applied to the DMU_E in the figure. As can be seen in the figure TDSVE score of $DMU_E = Oy_a/OT^a$ is above 1. This type of DMUs can be considered as (potentially) super-efficient.

Figure 12 – Super efficient unit in TDSVEA

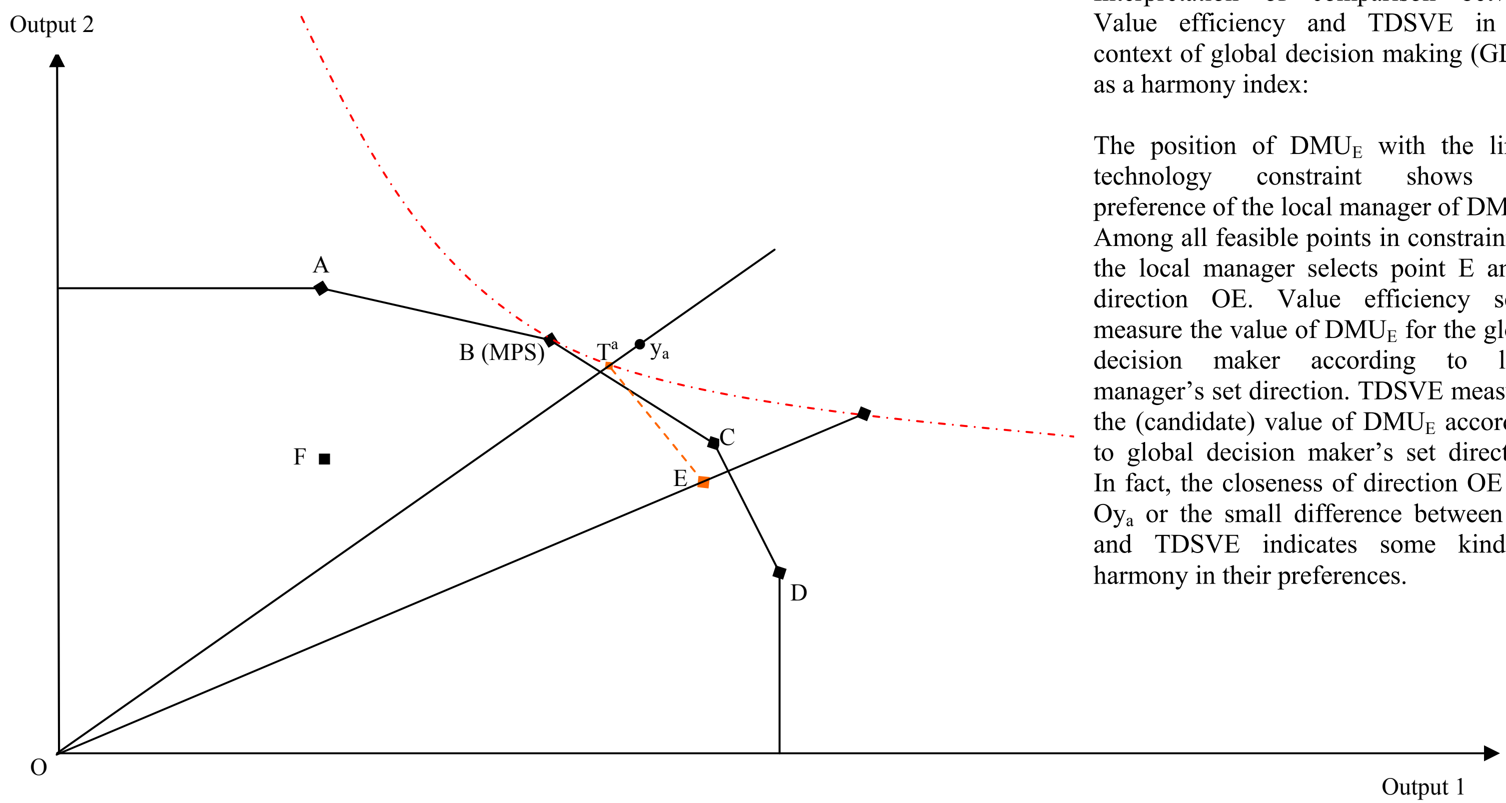
exist then it may be measured by the ratio VE score / TDSVE score. The local manager and global decision maker may need a compromise. However, first possibility should be always considered when harmony index is interpreted. If the analysis is repeated in a time frame, inferences about GDM preferences may be evaluated and differences in his/her preference structure may be taken into account. An illustration of harmony index is shown in Figure 13.

The ratio, as discussed before, VE score / TDSVE score may also identify the consistency of the GDM. Since GDM knows the capabilities and technologies of all DMUs, if s/he suggests easier and shorter way to project for the DMU than original radial projection, this ratio should be more than 1 (see Appendix A). If this ratio is less than 1, then the GDM should be warned about the consequences of his/her choice. But, as discussed before this should be only a warning; preference structure of the DM should not be affected.

Discussion about projection from y_a or from current input and output levels of DMU is valid in this situation. The GDM may select the projection from y_a for some DMUs, and the projection from current input and output levels of DMU for others. If the projection from y_a is selected, corresponding input allocation is obtained from LP in step 5 of the algorithm presented in section 4.5.

b) DMU identifies where to project:

There exist some situations that the decision maker has the power to attain the goal of only one DMU. This type of situation may usually occur when a DMU compares itself with other competitors. Then, finding TDSVE scores of other DMUs may be meaningless since projection of other DMUs by preferences of the DM is pointless. The DM identifies only his/her DMU's TDSVE score. Therefore, ranking according to TDSVE score becomes impossible. However, TDSVE score still has important information about where the DM wants to project and how far s/he is from this objective. Moreover, since the TDSVE score shows the potential for the DMU under evaluation, its knowledge is valuable. Discussion about the projection from y_a or from the current input and output levels of DMU is valid also in this situation. The difference of the potential value efficiency score and current value efficiency score may provide valuable information about the consistency of the DM. As in section 4.6.a, harmony index may show that either preference structure of the DM has changed or there exists lack of harmony between the DM and the less hierarchical units in the DMU.



Interpretation of comparison between Value efficiency and TDSVE in the context of global decision making (GDM) as a harmony index:

The position of DMU_E with the linear technology constraint shows the preference of the local manager of DMU_E. Among all feasible points in constraint set the local manager selects point E and a direction OE. Value efficiency score measure the value of DMU_E for the global decision maker according to local manager's set direction. TDSVE measures the (candidate) value of DMU_E according to global decision maker's set direction. In fact, the closeness of direction OE and Oy_a or the small difference between VE and TDSVE indicates some kind of harmony in their preferences.

Figure 13 – Illustration of harmony index

CHAPTER 5

APPLICATIONS OF TDSVEA AND MIP TECHNIQUE

5.1. Ranking American Research Universities by MIP:

In this section, ranking by mixed integer procedure with its all simplification methodology will be illustrated on a sample data. These data[6] are obtained from the 2003 report of *TheCenter*[7], which ranks American research universities with its own methodology (see section 2.4 and 3.1). All the information in the report is not used but it is filtered out to suit the ranking methodology at hand. Therefore, 146 American research universities are ranked according to 8 criteria. These criteria are shown in Table 3. The universities and their representative unit numbers are presented in Table 6.

Table 3 - 8 Criteria for ranking universities

1) 2001 Total Research in $1000
2) 2001 Federal Research in $1000
3) 2002 Endowment Assets in $1000
4) 2002 Annual Giving in $1000
5) 2002 National Academy Members
6) 2002 Faculty Awards
7) 2002 Doctorates Granted
8) 2001 Postdoctoral Appointees

The criterion, 'SAT scores' used by *TheCenter*, is not taken into account in this analysis, since the quantitative data for these is given as bounds. Although several data conversion approaches, taking median or average of bounds etc., may be used, these are not employed in this analysis in order not to deviate from original data. In

[6]The discussion about sensitivity of data to weights of the criteria can be seen in Appendix B.
[7] Detailed information about *TheCenter* and data used in this application can be found on the web site: http://thecenter.ufl.edu .

the report there are more than 146 universities, however, data for all criteria are not available for all universities.

In the ranking method presented in this study, a mixed integer program (MIP) for each university should be solved. This MIP, through finding evaluated university's optimum weights selected from the provided feasible weight space, finds maximum possible number of outperformed universities by the university under evaluation. These MIPs can be simplified through domination and/or weighted domination matrices. Entries of domination matrix, which represents positive, i.e. +1, (unit corresponding to the row dominates unit corresponding to the column), negative, i.e. –1, (unit corresponding to the row is dominated by the unit corresponding to the column) and neutral, i.e. 0, (neither unit dominates the other) domination relations, show whether a university dominates the other in the considered criteria. Establishing an entry of domination matrix requires pairwise comparison of criteria score vectors of units if simplification by Lemma 1 (copying entry from other rows and columns) is not possible for the related entry. In addition to the simplification of MIP, domination matrix provides first simplification of filling weighted domination matrix through implication of Lemma 2: positive and negative domination relations of domination matrix can directly be copied to weighted domination matrix. Moreover, if the feasible weight space is changed, the information in the weighted domination matrix becomes useless, whereas, the information in domination matrix is still usable. Thus, simplifying both MIP and the formation of weighted domination matrix with the robust information in its entries, domination matrix provides an important step of simplification procedure. Entries of weighted domination matrix give whether a university outperforms the other considering the determined feasible weight space. Filling of remaining entries (after copying from domination matrix) of weighted domination matrix can be obtained by two ways: one large linear program (one LP approach) or multiple simple linear programs (multiple LP approach). In one LP approach, the LP provides entries of all weighted domination matrix, but second simplification procedure of filling weighted domination matrix (by Lemma 1) is not possible. In multiple LP approach, second simplification procedure of filling weighted domination matrix (copying entries between the rows and columns of WD by Lemma 1) is possible but each LP provides only one entry of weighted domination matrix.

As discussed in Chapter 3, the simplification of MIP may or may not be employed. Moreover, if it is employed, domination matrix step or weighted domination matrix step may be omitted. In this section, full simplification procedure of MIP will be employed for two different feasible weight spaces. Nevertheless, for illustration purposes, the implications of omitting any step of simplification of MIP will be presented. Furthermore, implications of the simplification of filling domination matrix, and of first simplification and second simplification of filling weighted domination matrix will be illustrated.

The ranking of 146 American research universities by MIP with full simplification procedure is explained step by step. Firstly, individual raw score of each university in each criterion is turned to z-scores. After z-score conversion, upper triangle of 146*146 domination matrix, D, is formed through vector comparisons presented in section 3.2. In the particular example, $\frac{145*146}{2}=10585$ domination relations exist. As described in section 3.2, ordering of universities and the simplification ordering can change the results of the simplification procedure. For test purposes, for simplification by Lemma 1, no complex rules or heuristic are employed. Alphabetical order of universities and bottom to up approach (alphabetically order universities, then start the simplification from bottom row leftmost entry[8] of upper triangle and while copying possible entries continue with one upper row's leftmost entry to rightmost entry till uppermost row rightmost entry of the upper triangle of the D and WD matrix is reached) are used. Simplification by Lemma 1 (2208 entries are copied between the rows and consequently, 2208 pairwise comparisons are saved) and pairwise vector comparisons of units (10585-2208 = 8377 comparisons) produce a result that among these 10585 domination relations a sum of 4670 relations are positive (+1 for related entry in D) or negative (-1 for related entry in D), which in fact allows a remarkable simplification. The related MIPs of dominated and dominating units for each related positive or negative relations will

[8] In fact, there exists only 1 entry in bottom row of upper triangle

be simplified as in section 3.2. Moreover, these 4670 entries will be directly copied to weighted domination matrix. In fact, the amount of the +1 and -1 entries can vary for different ranking cases. In this particular example, a good amount of positive and negative domination relations (high amount of +1 and-1 entries) exists, which is useful for the simplification procedure.

After the domination relations are established, a weight space is suggested. If the criteria were approximately equally important, w_j takes a value around 0.125 for all criteria. Rather than using exact weights, the following ranges are used:

$0.12 \leq w_j \leq 0.13$ for $j=1,\ldots,8$.

A relatively small feasible weight space is intentionally selected in order to assess the potential of the simplification by weighted dominance matrix. Since a smaller feasible weight space leads to more positive or negative weight-domination relations for weighted domination matrix, simplification by the weighted dominance matrix is expected to be more. In extreme situation, where the weights are fixed quantities, which, in fact, corresponds to fixed weight ranking procedure, all weighted domination relations are positive or negative. Therefore, ranking by MIP is simplified so that solving MIP is not needed at all; solutions are trivial and can be calculated by hand. The feasible weight space is selected as large enough to prevent such trivial cases. At least some neutral relations (0 entries in weighted domination matrix, WD) should be obtained. Of course, this weight space is selected for only testing purposes. Normally, weight space should be selected by considerable efforts of experts and it represents the true relations between criteria (some possible relations are shown in Table 1). The specific weight structure used in the test gives the same bounds for all criteria and assumes that criteria are more or less equally important.

Through simplification by the implication of Lemma 2, 4670 entries, which are +1 or -1 in D, have been copied from the D matrix to WD matrix. Therefore, $\frac{145*146}{2}-4670=5915$ entries are evaluated for WD. Since probability of finding +1 and –1 is high, multiple LP simplification methodology is selected. This methodology depends on the order of universities in WD matrix since this changes the order of LPs solved. Arbitrarily, the alphabetical order of universities is used to sort universities in WD matrix as in the D matrix. Bottom to up approach (last row of WD is filled first, then in the previous row those entries, which cannot be copied from D matrix and which cannot be copied from following rows and columns by second simplification of formation of WD by Lemma 1, are filled and the procedure goes on like this) is used in order to fill the WD matrix as the D matrix.

First simplification of formation of WD (copying entries from D to WD) has produced significant results: 4670 of 10585 entries has been copied to WD. Since multiple LP approach is used, second simplification of formation of WD (copying entries between the rows and columns of WD), which is an implication of Lemma 1, is possible. Second simplification also provides significant results. Among the remaining 5915 (10585-4670) entries after first simplification, 5217 entries can be copied between the rows and columns of the WD according to Lemma 1. Therefore, through these two simplifications, a total of 9887 (4670+5217) entries are copied from other entries. Only 698 entries need solving simple LPs of multiple LP approaches. Therefore, multiple LP approach (5915-5217 = 698 simple LPs are needed) is better than single LP approach (1 large LP is needed and simplification by Lemma 1 cannot be used). Through these 698 simple LPs and two simplification procedures of formation of WD, all the entries in WD are obtained.

For the multiple LP alternative for filling WD, if simplification by the implication of only Lemma 1 (copying of +1 or –1 between WD matrix rows and columns) would be used without simplification of Lemma 2 (copying entries from D to WD) – i.e. simplification by D is omitted, 9267 (among possible of 10585) entries would be copied. Corresponding to theoretical maximum copying (10585-145=10440) according to Lemma 1, this would correspond to 9267/10440 = 88.76 % savings from needed simple LPs for filling WD. Through 1318 (10585-9267) simple LPs and second simplification procedure of formation of WD, all the entries in WD would be obtained. For the test case at hand, multiple LP alternative (10585-9267=1318 simple LPs are needed) would be better than single LP approach (1 large LP is needed and simplification by Lemma 1 cannot be used). Considering the two simplification procedures, first is the simplification made through both D and WD matrices and second is the simplification made through only the WD matrix, the difference between the total number of simple LPs necessary to obtain full weight-domination relations in WD matrix (698 if both D and WD is used versus 1318 if only WD is used) comes from those entries in WD matrix, which cannot be copied between rows and columns of WD matrix but can be copied from the D matrix to WD matrix.

Among the possible 10585 entries of WD, 10443 entries (4670 entries, which are −1 or +1, has been copied from D matrix; among remaining 5915 (10585-4670) entries, 5773 entries are found to be –1 or +1 by second simplification procedure and multiple LP approach) are –1 (university corresponding to row is weight-dominated by university corresponding to column in particular feasible weight space) or +1 (university corresponding to row weight-dominates university corresponding to column in particular feasible weight space), therefore, simplify the MIPs of weight-dominating and weight-dominated units. Only 142 relations are neutral, and cannot provide simplification of MIP.

Consequently, simplification procedure of MIP produces 10443 (4670+5773) simplifying entries (+1s or -1s) from possible of 10585 entries, which corresponds to 98.67% of all possible entries. From 146 MIPs for all units, 28 MIPs are fully simplified (no need to solve that MIP since all Y(i) values are known from WD matrix, i.e. the row corresponding to the unit in full WD matrix, in which all entries on both upper and lower triangle are filled, includes only –1 or +1 but not 0). If no simplification procedure is applied each of 146 MIPs should include 145 constraints (1) of Model (I). After simplification procedure, 41 MIPs include only 1 constraint (1) of Model (I), 35 MIPs include only 2 constraints (1) of Model (I), 18 MIPs include only 3 constraints (1) of Model (I), 9 MIPs includes only 4 constraints (1) of Model (I), 9 MIPs include only 5 constraints (1) of Model (I), 4 MIPs includes only 6 constraints (1) of Model (I) and 2 MIPs includes only 7 constraints (1) of Model (I). No MIP includes more than 7 constraints (1) of Model (I). Therefore, the simplification produces very significant results. Results of simplifications for this weight structure are summarized in Table 4.

In order to analyze the effect of weight space change, the methodology is also applied for the following feasible weight space:

$$0.05 \le w_j \le 0.20 \text{ for } j=1,\ldots,8$$

$$\sum_{j=1}^{8} w_j = 1 .$$

The weights for each criterion include $w_j = 0.125$ (If the weights were equally important the weight for each criterion would be 0.125) in this weight space also. However, the weight bound is wider for each criterion. On the other hand, normalization of the weights is included in this weight space. Therefore, neither of these two feasible weight spaces is a subspace of the other. Nevertheless, the current feasible weight space is larger than the first weight space. Consequently, more neutral relations in WD are expected. As mentioned before, the relations in D matrix have not been changed since they are valid for any feasible weight space. Therefore, information such as positive and negative relations in D matrix, the simplification of filling D matrix through the implication of Lemma 1, and simplification of filling WD matrix through copying entries from D matrix (by Lemma 2) are same for this feasible weight space also (compare Table 4 and Table5). On the other hand, the WD matrix and all the related information have changed. Ranking by MIP methodology parallel to first feasible weight space is applied for current feasible weight space. The results are summarized in Table 5.

As can be seen from Table 4 and Table 5, the information related to D matrix has not changed. Number of neutral relations in WD (column I) has been increased from 142 to 1887. Therefore, total number of constraints (1) of Model (I) in 146 MIPs has been increased from 384 (2*142) to 3774 (2*1887). Moreover, since number of positive and negative relations decrease, in the current feasible weight space, number of second simplifications of WD formation, when multiple LP approach is used, produces worse results than the first weight space. The numbers 5217 and 9267 for the first feasible weight space has decreased to 3609 and 6473 for the current feasible weight space, corresponding to cases if both simplification of MIP through D and WD is used, and if only simplification of MIP through WD is used, respectively. Final result of all simplification procedures get also worse, as expected, since the number of positive and negative relations decreases because of the larger feasible weight space. But still, the simplification procedure produces a significant final result: among the possible 10585 entries, 8698 entries are positive or negative which corresponds to 82.17% of all entries. Final results of the simplification procedures with first and second feasible spaces are shown in Table 7.

After the simplification procedure is applied, the required MIPs with required amount of constraints have been solved according to the particular weight spaces. Objective function of MIPs, reduced ranks, and discrimination indices are tabulated in Table 8 and Table 9.

As a last caution for simplification of MIP and simplification of this simplification, results of these procedures of the ranking methodology provided in this study is case dependent. The simplification procedure of the MIP with z-scores for universities and with the weight space provided here produces significant outcomes. Moreover, simplification of this simplification (i.e. the simplification of WD filling through copying entries from D to WD according to Lemma 2 and through copying +1 and –1 among rows and columns of D and WD according to Lemma 1) produces substantial results. However, these results may change if the case at hand changes. Despite that, except for some extreme situations, computational savings is expected through the use of the simplification procedure of MIP provided in this study.

With several weight spaces, the MIP technique without simplification is applied to rank these 146 universities. Results can be seen in Table 6. This table also includes the weight spaces that are used in the simplification procedure. It can be seen that when the weight space becomes wider, standings (objective function of MIPs) improve for all universities as expected since expansion of feasible space should at least not worsen objective of MIPs. Therefore, number of reduced ranks tends to decrease (the last row of Table 6). Inclusion of zero, one, near one or near zero values into weight space bounds with normalization may affect (and change) this tendency since it may mean that units that have smaller (higher) scores in one or several criteria can more easily get rid of (amplify) the effect of these scores with assigning zero or near zero (one or near one) weights. The last row of Table 6 supports this proposition. For example, while total number of reduced ranks of the feasible space ($w_j \in [0.05,0.20]$ and $\Sigma\ w_j = 1$) is 83, the feasible space ($w_j \in [0.000001,0.25]$ and $\Sigma\ w_j = 1$) has 90 reduced ranks, which is the one of the two cases that do not suit the tendency. Similar observation is valid for feasible space ($w_j \in [0.000001,0.5]$ and $\Sigma\ w_j = 1$) and ($w_j \in [0.000001,0.99999]$ and $\Sigma\ w_j = 1$).

The shaded cells in Table 6 indicate some specific cases where the method of *TheCenter* and fixed equal weight approach produce very different results than the ranking by MIP. For example, Unit 54 – Rice university, which is 77th with fixed equal weights and between 44th and 46th with the method by *TheCenter* (since all the units which have (2, 1) as does Rice University share the ranks between 44th and 46th), may become 14th (13th reduced rank) with a weight space $w_j \in [0.00001, 0.99999]$ and $\Sigma\ w_j = 1$). This means, if Rice University would have a chance to show its originality[9], it may have a higher ranking. With fixed weights in fixed equal weight method and fixed m and p in the method of *TheCenter*, it cannot show its originality and is ranked among the last units. Another example is Unit 145 – Yeshiva University. It is 48th with fixed equal weights and between 50th and 54th with the method by TheCenter. On the other hand, it may become 4th standing (4th reduced rank) with a weight space $w_j \in [0.00001, 0.99999]$ and $\Sigma\ w_j = 1$) if it would find a chance to show its originality[10]. Although these weights are too extreme, it is possible to show significant differences between rankings with less extreme weights.

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[9] With this specific feasible weight space, $w_j \in [0.00001, 0.99999]$ and $\Sigma\ w_j = 1$ for j=1,...8, a unit can select comparatively very high weights for the criteria in which it has higher scores compared to others - i.e. unit's comparative advantage, and can select comparatively very low scores for the criteria in which it has lower scores compared to others - i.e. unit's comparative disadvantage. In individual criterion ranking, Rice University is, 123rd, 107th, 14th, 78th, 40th, 82nd, 98th and 92nd in criterion 1-8, respectively. It is also between 1st and 25th in SAT scores which is the omitted criteria in this study. Its originality is in criteria 3- total endowment assets.

[10] In individual criterion ranking, Yeshiva University is, 75th, 55th, 44th, 71st, 64th, 95th, 96th and 4th in criterion 1-8, respectively. Its originality is in criteria 8- post doctoral appointees.

For the cases, when the ranking approach allows and accounts for the originalities of units and the differences between units, MIP may be a better option than fixed weights approach and fixed rule approach such as the method of *TheCenter*. Especially, for the cases, in which originality is important rather than standardization, ranking results are motivation for the future directions of the units and they affect the behavior of the units such as input allocation; ranking by MIP technique rather than the fixed weight and fixed rule techniques may be a better alternative.

MIP technique also allows for categorization as well as ranking. Two or more units may share the same standing (and reduced rank) and they constitute a category (a category of units that can outperform other X units). Method of *TheCenter* also allows categorization while fixed weight approaches do not, except for extremely rare ties. In fixed weight approaches, a 'keen' ranking occurs (probably number of ranks is equal to number of units), however, there do not exist layers that include two or more units (if same exact final weighted scores do not occur by chance). Especially, when the weight space becomes larger, MIP may provide more categorization since the standings of each unit probably increase (or at least do not get worse) which results in a denser ranking. For the cases, where the categorization rather than the sharp rankings are more appropriate, MIP technique may be an alternative.

To sum up, MIP technique provides a flexible ranking tool in accounting for originalities and allowing categorization. Because of all, the weight space is larger, it allows categorization rather than 'keen' ranking and all the units 'race' against each other in the same conditions; its ranking results may be less questionable and debatable at least than fixed weight techniques

Table 4 - Summary of simplification procedure of ranking by MIP with first feasible weight space

If both simplification of MIP through D and WD is applied

				Needed if multiple LP approach is used			Multiple or one LP approaches produces same results			Number of MIP that contains X constraints (1) of Model (I)									
Number of possible domination relation (A)	Number of positive or negative relations in D (Also first simplification of WD formation) (B)	Simpli-fication in D formation by Lemma 1	Number of possible domination relation in WD after first simplification of WD formation (C)=(A-B)	Second simpli-fication of WD formation by Lemma 1 (D)	Number of LPs needed in multiple LP approach of formation of WD (E) = (C-D)	Number of positive or negative relations copied from D (F) = (B) (D+E+F)=(A)	Number of positive or negative relations in WD found from multiple LP approach (G)	Total number of positive or negative relations in WD (H) = (F+G)	Number of neutral relations in WD which cannot simplify MIPs (I)= (A-H)	X=0 (J1)	X=1 (J2)	X=2 (J3)	X=3 (J4)	X=4 (J5)	X=5 (J6)	X=6 (J7)	X=7 (J8)	X=8 or more (J9)	Total cons. (J)=2* (I) = ∑X*Ji
10585	4670	2208	5915	5217	698	4670	5773	10443	142	28	41	35	18	9	9	4	2	0	**284**

If only simplification of MIP through WD is applied

				Needed if multiple LP approach is used			Multiple or one LP approaches produces same results			Number of MIP that contains X constraints (1) of Model (I)									
Number of possible domination relation (A)	Number of positive or negative relations in D (Also first simplification of WD formation) (B)	Number of possible domination relation in WD after first simplification of WD formation (C)=(A)		Second simplificat ion of WD formation by Lemma 1 (D)	Number of LPs needed in multiple LP approach of formation of WD (E) = (C-D) (D+E)=(A)	Number of positive or negative relations copied from D (F) = (B)	Number of positive or negative relations in WD found from multiple LP approaches (G)	Total number of positive or negative relations in WD (H) = (G)	Number of neutral relations in WD which cannot simplify MIPs (I)= (A-H)	X=0 (J1)	X=1 (J2)	X=2 (J3)	X=3 (J4)	X=4 (J5)	X=5 (J6)	X=6 (J7)	X=7 (J8)	X=8 or more (J9)	Total cons. (J)=2* (I) = ∑X*Ji
10585	NA	10585		9267	1318	NA	10443	10443	142	28	41	35	18	9	9	4	2	0	**284**

If only simplification of MIP through D is applied

				Numbers of constraints (1) of Model (I) (c) are very erratic between 22 c to 137 c. Therefore, number of MIP that contains X constraints (1) of Model (I) cannot be fully tabulated here															
Number of possible domination relation (A)	Number of positive or negative relations in D (B)	Simpli-fication in D formation by Lemma 1	Number of neutral relations in D which cannot simplify MIPs (C)= (A-B)	X=22 J(1)	X=25 (J2)	X=26 (J3)	…	…	Mode of # of c: X=95 (J25)	…	…	…	…	…	…	…	X=136 (J75)	X=137 (J76)	Total cons. (J)=2*(C) = ∑X* Ji
10585	4670	2208	5915	1	1	1			6								1	1	**11830**

If simplification procedure is fully omitted:

Number of MIP that contains 145 constraints (1) of Model (I):	146	Total constraints (1) (145*146):	**21170**

Table 5 – Summary of simplification procedure of ranking by MIP with second feasible weight space

If both simplification of MIP through D and WD is applied																			
				Needed if multiple LP approach is used			Multiple or one LP approaches produces same results			Numbers of constraints (1) of Model (I) (c) are very erratic between 1 c to 55 c. Therefore, number of MIP that contains X c cannot be fully tabulated here									
Number of possible domination relation (A)	Number of positive or negative relations in D (Also first simplification of WD formation) (B)	Simplification in D formation by Lemma 1	Number of possible domination relation in WD after first simplification of WD formation (C)=(A-B)	Second simplification of WD formation by Lemma 1 (D)	Number of LPs needed in multiple LP approach of formation of WD (E) = (C-D)	Number of positive or negative relations copied from D (F) = (B) (D+E+F)=(A)	Number of positive or negative relations in WD found from multiple LP approach (G)	Total number of positive or negative relations in WD (H)=(F+G)	Number of neutral relations in WD which cannot simplify MIPs (I)= (A-H)	X=1 (J1)	X=2 (J2)	…	1st Mode of # of c: X=19 (J18)	…	2nd Mode of # of c: X=26 (J25)	…	X=53 (J46)	X=55 (J47)	Total cons. (J)=2* (I) = ∑X*Ji
10585	4670	2208	5915	3609	2306	4670	4028	8698	1887	1	1		9		9		2	0	**3774**

If only simplification of MIP through WD is applied																			
				Needed if multiple LP approach is used			Multiple or one LP approaches produces same results			Numbers of constraints (1) of Model (I) (c) are very erratic between 1 c to 55 c. Therefore, number of MIP that contains X c cannot be fully tabulated here									
Number of possible domination relation (A)	Number of positive or negative relations in D (Also first simplification of WD formation) (B)	Number of possible domination relation in WD after first simplification of WD formation (C)=(A)		Second simplification of WD formation by Lemma 1 (D)	Number of LPs needed in multiple LP approach of formation of WD (E) = (C-D) (D+E)=(A)	Number of positive or negative relations copied from D (F) = (B)	Number of positive or negative relations in WD found from multiple LP approaches (G)	Total number of positive or negative relations in WD (H) = (G)	Number of neutral relations in WD which cannot simplify MIPs (I)= (A-H)	X=1 (J1)	X=2 (J2)	…	1st Mode of # of c: X=19 (J18)	…	2nd Mode of # of c: X=26 (J25)	…	X=53 (J46)	X=55 (J47)	Total cons. (J)=2* (I) = ∑X*Ji
10585	NA	10585		6473	4112	NA	8698	8698	1887	1	1		9		9		2	0	**3774**

If only simplification of MIP through D is applied																			
				Numbers of constraints (1) of Model (I) (c) are very erratic between 22 c to 137 c, therefore, cannot be fully tabulated here															
Number of possible domination relation (A)	Number of positive or negative relations in D (B)	Simplification in D formation by Lemma 1	Number of neutral relations in D which cannot simplify MIPs (C)= (A-B)	X=22 J(1)	X=25 (J2)	X=26 (J3)	…	…	Mode of # of c: X=95 (J25)	…	…	…	…	…	…	…	X=136 (J75)	X=137 (J76)	Total cons. (J)=2*(C) = ∑X* Ji
10585	4670	2208	5915	1	1	1			6								1	1	**11830**

If simplification procedure is fully omitted:			
Number of MIP that contains 145 constraints (1) of Model (I):	146	Total constraints (1) (145*146):	**21170**

Table 6 – Rank by *TheCenter*, by fixed equal weights and by MIP technique with several feasible weight spaces ($w_j \in$ [lowerlimit,upperlimit] and $\Sigma\, w_j = 1$)

Unit No	University	Rank of The Center[11]			If equal fixed weights	Rank with our model with weights between [0.124999, 0.125001]	Rank with our model with weights between [0.1225, 0.1275]	Rank with our model with weights between [0.12,0.13]	Rank with our model with weights between [0.10,0.15]	Rank with our model with weights between [0.05,0.20]	Rank with our model with weights between [0.000001, 0.25]	Rank with our model with weights between [0.000001, 0.50]	Rank with our model with weights between [0.000001, 0.99999]	Rank with our model with weights between [0,1]	Rank with our model with weights between [0.12,0.13] without normalization[12]
1	Baylor College of Medicine	34	3	1	44	44	43	43	38	30	21	18	17	17	43
2	Binghamton University				133	133	132	132	128	123	117	105	93	93	133
3	Boston College				113	113	113	113	108	95	84	58	39	39	113
4	Boston University	49	1	2	47	47	47	47	47	43	37	29	26	25	47
5	Brandeis University				97	97	97	96	94	79	69	65	60	60	97
6	Brown University	41	2	1	70	70	70	70	68	57	51	36	25	25	70
7	California Institute of Technology	25	3	5	34	34	33	33	28	21	16	11	6	6	33
8	Carnegie Mellon University	50	1	1	71	71	71	71	68	62	56	46	39	39	71
9	Case Western Reserve University				46	46	46	45	45	41	37	32	26	26	46
10	Catholic University of America				134	134	134	134	133	128	123	117	113	113	134
11	City University of NY – Grd. S. and University C.				123	123	120	120	119	97	80	56	44	44	120
12	Clark University				145	145	144	144	143	141	133	123	113	105	144
13	Clemson University				106	106	105	105	104	99	94	86	79	79	105
14	College of William and Mary				125	125	125	123	122	119	109	100	80	80	125
15	Colorado School of Mines				137	137	137	137	135	130	125	118	96	93	137
16	Colorado State University				87	87	86	85	78	69	64	58	57	57	86
17	Columbia University	4	8	1	12	12	12	12	10	6	4	2	2	2	12
18	Cornell University	5	8	1	13	13	12	12	11	7	4	4	3	3	12
19	Creighton University				139	139	138	138	137	130	120	112	107	107	138
20	Dartmouth College	42	2	1	66	66	65	64	60	50	40	26	18	18	65
21	Drexel University				120	120	120	120	120	117	108	103	94	93	120
22	Duke University	8	8	0	19	19	17	17	17	15	13	10	9	9	17
23	Emory University	28	3	5	31	31	30	30	26	19	17	8	6	6	30
24	Florida Atlantic University				136	136	135	135	133	128	122	118	112	105	136
25	Florida International University				135	135	134	134	133	131	127	124	122	121	134

[11] Second column shows bold-1 score while third one shows weak-1 score. First one is the order of universities. As stated before, if their bold-1 and weak-1 scores are same, *TheCenter* ranks universities at the same standing, but in this table orders in first column are increased by one in order to establish comparability.

[12] This feasible weight space do not include normalization equation, $\Sigma\, w_j = 1$.

Table 6 (continued)

Unit No	University	Rank of The Center[11]			If equal fixed weights	Rank with our model with weights between [0.124999, 0.125001]	Rank with our model with weights between [0.1225, 0.1275]	Rank with our model with weights between [0.12,0.13]	Rank with our model with weights between [0.10,0.15]	Rank with our model with weights between [0.05,0.20]	Rank with our model with weights between [0.000001, 0.25]	Rank with our model with weights between [0.000001, 0.50]	Rank with our model with weights between [0.000001, 0.99999]	Rank with our model with weights between [0,1]	Rank with our model with weights between [0.12,0.13] without normalization[12]
26	Florida State University				81	81	80	79	76	69	63	54	49	49	80
27	George Washington University				88	88	87	85	83	74	67	57	50	50	87
28	Georgetown University	51	1	1	72	72	71	71	68	57	52	45	39	38	71
29	Georgia Institute of Technology	44	1	6	43	43	43	43	41	33	29	25	22	21	43
30	Harvard University	1	9	0	1	1	1	1	1	1	1	1	1	1	1
31	Indiana University - Bloomington				63	63	62	61	58	49	44	37	31	31	62
32	Indiana University-Purdue University - Indianapolis				89	89	87	86	79	64	53	45	41	41	88
33	Iowa State University				64	64	63	63	62	57	54	49	48	48	63
34	Johns Hopkins University	6	8	1	3	3	3	3	2	1	1	1	1	1	3
35	Lehigh University				119	119	119	119	116	104	91	60	50	50	119
36	Louisiana State University - Baton Rouge				78	78	76	72	70	65	61	56	53	53	77
37	Massachusetts Institute of Technology	2	9	0	4	4	4	4	4	3	3	2	2	2	4
38	Medical College of Wisconsin				124	124	122	120	120	110	100	85	82	82	121
39	Medical University of South Carolina				118	118	117	115	109	97	79	70	61	61	116
40	Michigan State University	38	2	3	38	38	37	37	35	29	24	19	18	18	37
41	Michigan Technological University				141	141	141	140	140	136	132	130	115	105	141
42	New York University	36	2	4	35	35	34	34	31	25	20	15	11	11	34
43	North Carolina State University				52	52	51	48	46	43	37	32	30	30	49
44	Northwestern University	26	3	5	29	29	29	29	27	23	20	15	13	13	29
45	Ohio State University - Columbus	27	3	5	24	24	24	22	20	16	15	8	4	4	24
46	Oklahoma State University - Stillwater				110	110	105	103	103	95	88	76	66	66	106
47	Oregon Health & Science University				86	86	84	83	78	68	60	55	52	52	83
48	Oregon State University				90	90	90	89	88	77	73	69	67	67	90
49	Pennsylvania State University - University Park	23	4	2	23	23	23	21	20	17	14	12	12	12	23
50	Polytechnic University				143	143	143	143	143	141	131	112	90	84	143
51	Princeton University	19	5	1	18	18	17	16	15	13	5	3	3	3	17
52	Purdue University - West Lafayette	45	1	4	39	39	38	37	35	31	28	23	21	21	38
53	Rensselaer Polytechnic Institute				109	109	105	105	101	93	79	62	58	57	105
54	Rice University	43	2	1	77	77	71	71	69	53	44	24	14	14	72
55	Rockefeller University	46	1	4	55	55	54	52	47	38	28	24	22	22	53
56	Rush University				126	126	126	126	124	118	111	102	90	90	126
57	Rutgers the State University of NJ - New				53	53	53	50	48	43	40	31	28	28	53

Table 6 (continued)

Unit No	University	Rank of The Center[11]			If equal fixed weights	Rank with our model with weights between [0.124999, 0.125001]	Rank with our model with weights between [0.1225, 0.1275]	Rank with our model with weights between [0.12,0.13]	Rank with our model with weights between [0.10,0.15]	Rank with our model with weights between [0.05,0.20]	Rank with our model with weights between [0.000001, 0.25]	Rank with our model with weights between [0.000001, 0.50]	Rank with our model with weights between [0.000001, 0.99999]	Rank with our model with weights between [0,1]	Rank with our model with weights between [0.12,0.13] without normalization[12]
	Brunswick														
58	Saint Louis University - St. Louis				122	122	120	120	120	114	104	78	49	49	120
59	Stanford University	3	9	0	2	2	2	2	2	2	1	1	1	1	2
60	State Univ. of New York Health Science C. - Brooklyn				144	144	143	143	143	139	132	128	125	121	143
61	Stony Brook University				51	51	48	47	46	42	33	27	26	26	48
62	Syracuse University				115	115	114	114	112	101	93	79	53	53	114
63	Texas A&M University	29	3	4	27	27	27	26	25	17	14	8	8	8	27
64	Thomas Jefferson University				114	114	113	113	107	96	78	62	55	55	113
65	Tufts University				73	73	71	70	68	57	49	37	28	28	71
66	Tulane University				91	91	90	90	87	79	73	66	56	56	90
67	University at Buffalo				68	68	67	66	64	57	51	44	38	38	66
68	University of Akron - Akron				131	131	131	131	128	123	118	113	109	109	131
69	University of Alabama - Birmingham				57	57	56	55	53	48	39	30	26	26	57
70	University of Arizona	30	3	4	30	30	30	29	27	24	22	21	20	20	29
71	University of Arkansas - Fayetteville				111	111	110	106	104	97	90	65	59	59	109
72	University of California - Berkeley	9	8	0	5	5	5	5	5	3	2	1	1	1	5
73	University of California - Davis	39	2	3	36	36	35	33	32	25	20	14	14	14	36
74	University of California - Irvine				60	60	60	58	56	51	48	39	34	34	59
75	University of California - Los Angeles	11	7	1	11	11	11	10	9	4	3	2	2	2	10
76	University of California - Riverside				105	105	105	104	101	96	87	68	58	58	105
77	University of California - San Diego	22	4	3	16	16	16	15	15	11	7	5	4	4	15
78	University of California - San Francisco	24	4	1	25	25	25	24	19	16	12	9	7	7	24
79	University of California - Santa Barbara	52	1	1	65	65	65	63	61	52	48	33	24	23	65
80	University of California - Santa Cruz				116	116	115	115	113	102	90	69	66	65	115
81	University of Chicago	21	4	4	28	28	28	27	24	19	17	11	11	11	27
82	University of Cincinnati - Cincinnati				54	54	53	52	50	46	42	35	33	33	53
83	University of Colorado - Boulder	37	2	4	37	37	37	36	33	24	20	14	12	12	37
84	University of Colorado Health Sciences Center				79	79	79	75	69	60	54	45	44	44	77
85	University of Connecticut - Storrs				95	95	95	93	89	81	73	65	56	56	95
86	University of Delaware				93	93	93	91	89	79	73	55	42	42	93
87	University of Florida	33	3	3	26	26	25	25	24	17	14	9	6	6	25
88	University of Georgia	53	1	1	56	56	55	54	51	44	38	28	23	23	55

Table 6 (continued)

Unit No	University	Rank of The Center[11]			If equal fixed weights	Rank with our model with weights between [0.124999, 0.125001]	Rank with our model with weights between [0.1225, 0.1275]	Rank with our model with weights between [0.12,0.13]	Rank with our model with weights between [0.10,0.15]	Rank with our model with weights between [0.05,0.20]	Rank with our model with weights between [0.000001, 0.25]	Rank with our model with weights between [0.000001, 0.50]	Rank with our model with weights between [0.000001, 0.99999]	Rank with our model with weights between [0,1]	Rank with our model with weights between [0.12,0.13] without normalization[12]
89	University of Hawaii - Manoa				98	98	97	97	96	83	73	66	60	60	97
90	University of Houston - University Park				112	112	108	105	101	94	83	72	65	65	109
91	University of Illinois - Chicago				58	58	58	57	56	51	49	44	40	40	57
92	University of Illinois - Urbana-Champaign	20	5	1	20	20	20	20	17	15	13	8	5	5	20
93	University of Iowa				42	42	42	42	41	35	31	29	29	29	42
94	University of Kansas - Lawrence				75	75	72	71	70	66	59	52	49	49	72
95	University of Kentucky				62	62	62	62	61	57	54	46	46	46	62
96	University of Louisville				107	107	105	104	102	96	86	76	67	67	105
97	University of Maine - Orono				127	127	126	126	123	121	115	108	100	96	126
98	University of Maryland - Baltimore				85	85	83	79	75	62	55	48	39	39	83
99	University of Maryland - College Park	47	1	4	41	41	41	40	39	31	27	22	18	18	41
100	University of Massachusetts - Amherst				80	80	79	75	73	66	59	47	41	41	79
101	University of Medicine & Dentistry of New Jersey				96	96	96	95	92	79	72	66	64	64	96
102	University of Miami				74	74	73	71	69	63	55	53	48	48	72
103	University of Michigan - Ann Arbor	12	7	1	7	7	6	6	5	4	2	2	2	2	6
104	University of Minnesota - Twin Cities	13	7	1	15	15	15	15	15	12	11	7	7	7	15
105	University of Missouri - Columbia				61	61	61	61	58	54	50	46	45	45	61
106	University of Nebraska - Lincoln				83	83	83	82	79	70	62	60	55	55	83
107	University of Nebraska Medical Center				128	128	128	127	125	121	117	106	99	99	128
108	University of Nevada - Reno				121	121	120	120	120	117	109	99	94	87	120
109	University of New Mexico - Albuquerque				84	84	83	80	77	67	63	57	52	52	83
110	University of North Carolina - Chapel Hill	18	5	3	22	22	21	21	19	17	16	15	15	15	21
111	University of Notre Dame	48	1	3	67	67	65	62	57	47	37	24	17	17	65
112	University of Oklahoma - Norman				104	104	101	100	97	90	81	69	63	63	101
113	University of Oklahoma Health Sciences Center				132	132	132	131	130	123	119	116	109	109	132
114	University of Oregon				99	99	99	98	97	87	75	69	63	63	99
115	University of Pennsylvania	7	8	1	9	9	9	9	5	4	4	4	4	4	9
116	University of Pittsburgh - Pittsburgh	35	2	5	32	32	30	29	27	21	18	15	13	13	30
117	University of Rhode Island				130	130	128	128	126	121	119	109	104	104	128
118	University of Rochester				50	50	49	48	47	43	36	32	30	30	48
119	University of South Carolina - Columbia				76	76	72	71	70	65	60	53	47	47	74
120	University of South Florida				92	92	91	90	89	81	76	66	62	62	91
121	University of Southern California	17	6	2	14	14	14	13	11	4	3	1	1	1	14

Table 6 (continued)

Unit No	University	Rank of The Center[11]			If equal fixed weights	Rank with our model with weights between [0.124999, 0.125001]	Rank with our model with weights between [0.1225, 0.1275]	Rank with our model with weights between [0.12,0.13]	Rank with our model with weights between [0.10,0.15]	Rank with our model with weights between [0.05,0.20]	Rank with our model with weights between [0.000001, 0.25]	Rank with our model with weights between [0.000001, 0.50]	Rank with our model with weights between [0.000001, 0.99999]	Rank with our model with weights between [0,1]	Rank with our model with weights between [0.12,0.13] without normalization[12]
122	University of Tennessee - Knoxville				82	82	79	76	74	66	59	48	43	43	79
123	University of Texas - Arlington				142	142	140	140	140	139	131	119	114	114	141
124	University of Texas - Austin	31	3	4	21	21	21	21	20	16	14	7	3	3	21
125	University of Texas - Dallas				138	138	138	138	136	129	125	120	109	109	138
126	University of Texas Health Science C. - Houston				100	100	99	99	98	91	81	72	66	66	99
127	University of Texas Health Science Ctr - San Antonio				101	101	100	99	98	87	79	72	71	69	100
128	University of Texas Medical Branch - Galveston				108	108	105	105	102	90	77	62	55	55	105
129	University of Texas SW Medical Center - Dallas				49	49	48	47	47	42	35	28	26	26	47
130	University of the Pacific				146	146	146	146	145	144	138	130	120	120	146
131	University of Tulsa				140	140	140	140	139	130	123	80	50	50	140
132	University of Utah				45	45	45	45	44	41	37	31	27	27	45
133	University of Vermont				117	117	117	116	114	105	101	95	90	87	116
134	University of Virginia	32	3	4	33	33	33	32	31	22	19	15	12	12	33
135	University of Washington - Seattle	14	7	1	8	8	7	6	5	4	3	2	2	2	7
136	University of Wisconsin - Madison	15	7	1	6	6	6	5	5	3	1	1	1	1	6
137	University of Wyoming				129	129	128	127	126	123	122	114	111	105	128
138	Vanderbilt University	40	2	3	40	40	40	39	36	31	24	22	18	18	40
139	Virginia Commonwealth University				103	103	101	101	99	96	93	87	81	81	101
140	Virginia Polytechnic Institute and State University				59	59	57	56	55	49	45	40	35	35	57
141	Wake Forest University				102	102	101	100	97	87	72	62	46	46	101
142	Washington State University - Pullman				94	94	93	93	89	83	80	69	65	65	93
143	Washington University in St. Louis	16	7	1	17	17	17	17	15	14	10	8	7	7	17
144	Yale University	10	7	2	10	10	9	6	5	3	3	2	2	2	9
145	Yeshiva University	54	1	1	48	48	47	45	45	36	25	8	4	4	47
146	Arizona State University - Tempe				69	69	59	48	48	47	45	43	42	41	48
SUM	**Number of Reduced Ranks**				**146**	**146**	**95**	**92**	**89**	**83**	**90**	**83**	**84**	**78**	**90**

Table 7 – Final result of simplification procedure by first and second feasible weight space

Final Result of Simplification Procedure with First Feasible Space									
	# of constraint in constraints (1) of model (I)	**0 cons**	**1 cons**	**2 cons**	**3 cons**	**4 cons**	**5 cons**	**6 cons**	**7 cons**
	in # of MIP	28	41	34	19	9	9	4	2

Final Result of Simplification Procedure with Second Feasible Space																																																
	# of constraint in constraints (1) of model (I)	**1 cons**	**2 cons**	**3 cons**	**4 cons**	**6 cons**	**7 cons**	**8 cons**	**9 cons**	**10 cons**	**11 cons**	**12 cons**	**13 cons**	**14 cons**	**15 cons**	**16 cons**	**17 cons**	**18 cons**	**19 cons**	**20 cons**	**21 cons**	**22 cons**	**23 cons**	**24 cons**	**25 cons**	**26 cons**	**27 cons**	**28 cons**	**29 cons**	**30 cons**	**31 cons**	**32 cons**	**33 cons**	**34 cons**	**35 cons**	**36 cons**	**37 cons**	**38 cons**	**39 cons**	**41 cons**	**42 cons**	**43 cons**	**44 cons**	**45 cons**	**46 cons**	**48 cons**	**53 cons**	**55 cons**
	in # of MIP	1	1	1	1	1	1	2	2	4	2	4	3	2	3	5	2	1	9	3	7	5	2	2	2	9	5	5	5	4	3	6	4	5	7	4	4	4	1	1	1	5	1	1	1	1	2	1

Table 8 - Results of ranking of 146 American research universities by MIP with first feasible weight space

Unit Number	University	Standing (objective of MIP)	Reduced Rank	Discrepancy from superior group	Discrepancy from inferior group
30	Harvard University	1	1	NA	1
59	Stanford University	2	2	1	1
34	Johns Hopkins University	3	3	1	1
37	Massachusetts Institute of Technology	4	4	1	1
72	University of California - Berkeley	5	5	1	1
103	University of Michigan - Ann Arbor	6	6	1	1
136	University of Wisconsin - Madison	6	6	1	1
135	University of Washington - Seattle	7	7	1	2
115	University of Pennsylvania	9	8	2	1
144	Yale University	9	8	2	1
75	University of California - Los Angeles	10	9	1	2
17	Columbia University	12	10	2	2
18	Cornell University	12	10	2	2
121	University of Southern California	14	11	2	1
77	University of California - San Diego	15	12	1	2
104	University of Minnesota - Twin Cities	15	12	1	2
22	Duke University	17	13	2	3
51	Princeton University	17	13	2	3
143	Washington University in St. Louis	17	13	2	3
92	University of Illinois - Urbana-Champaign	20	14	3	1
110	University of North Carolina - Chapel Hill	21	15	1	2
124	University of Texas - Austin	21	15	1	2
49	Pennsylvania State University - University Park	23	16	2	1
45	Ohio State University - Columbus	24	17	1	1
78	University of California - San Francisco	24	17	1	1
87	University of Florida	25	18	1	2
63	Texas A&M University	27	19	2	2
81	University of Chicago	27	19	2	2
44	Northwestern University	29	20	2	1
70	University of Arizona	29	20	2	1
23	Emory University	30	21	1	3
116	University of Pittsburgh - Pittsburgh	30	21	1	3
7	California Institute of Technology	33	22	3	1
134	University of Virginia	33	22	3	1
42	New York University	34	23	1	2
73	University of California - Davis	36	24	2	1
40	Michigan State University	37	25	1	1
83	University of Colorado - Boulder	37	25	1	1
52	Purdue University - West Lafayette	38	26	1	2
138	Vanderbilt University	40	27	2	1
99	University of Maryland - College Park	41	28	1	1
93	University of Iowa	42	29	1	1
1	Baylor College of Medicine	43	30	1	2
29	Georgia Institute of Technology	43	30	1	2
132	University of Utah	45	31	2	1
9	Case Western Reserve University	46	32	1	1
4	Boston University	47	33	1	1
129	University of Texas SW Medical Center - Dallas	47	33	1	1
145	Yeshiva University	47	33	1	1
61	Stony Brook University	48	34	1	2

Table 8 (continued)

Unit Number	University	Standing (objective of MIP)	Reduced Rank	Discrepancy from superior group	Discrepancy from inferior group
118	University of Rochester	48	34	1	2
146	Arizona State University - Tempe	48	34	1	2
43	North Carolina State University	50	35	2	4
55	Rockefeller University	54	36	4	3
57	Rutgers the State University of NJ - New Brunswick	54	36	4	3
82	University of Cincinnati - Cincinnati	54	36	4	3
88	University of Georgia	57	37	3	1
69	University of Alabama - Birmingham	58	38	1	2
91	University of Illinois - Chicago	58	38	1	2
140	Virginia Polytechnic Institute and State University	58	38	1	2
74	University of California - Irvine	60	39	2	2
105	University of Missouri - Columbia	62	40	2	1
31	Indiana University - Bloomington	63	41	1	1
95	University of Kentucky	63	41	1	1
33	Iowa State University	64	42	1	2
20	Dartmouth College	66	43	2	1
79	University of California - Santa Barbara	66	43	2	1
111	University of Notre Dame	66	43	2	1
67	University at Buffalo	67	44	1	3
6	Brown University	70	45	3	1
8	Carnegie Mellon University	71	46	1	1
28	Georgetown University	71	46	1	1
65	Tufts University	71	46	1	1
54	Rice University	72	47	1	2
94	University of Kansas - Lawrence	72	47	1	2
102	University of Miami	72	47	1	2
119	University of South Carolina - Columbia	74	48	2	3
36	Louisiana State University - Baton Rouge	77	49	3	2
84	University of Colorado Health Sciences Center	77	49	3	2
100	University of Massachusetts - Amherst	79	50	2	1
122	University of Tennessee - Knoxville	79	50	2	1
26	Florida State University	80	51	1	3
47	Oregon Health & Science University	83	52	3	3
98	University of Maryland - Baltimore	83	52	3	3
106	University of Nebraska - Lincoln	83	52	3	3
109	University of New Mexico - Albuquerque	83	52	3	3
16	Colorado State University	86	53	3	1
27	George Washington University	87	54	1	1
32	Indiana University-Purdue University - Indianapolis	88	55	1	2
48	Oregon State University	90	56	2	1
66	Tulane University	90	56	2	1
120	University of South Florida	91	57	1	2
86	University of Delaware	93	58	2	2
142	Washington State University - Pullman	93	58	2	2
85	University of Connecticut - Storrs	95	59	2	1
101	University of Medicine & Dentistry of New Jersey	96	60	1	1
5	Brandeis University	97	61	1	2
89	University of Hawaii - Manoa	97	61	1	2
114	University of Oregon	99	62	2	1
126	University of Texas Health Science Center - Houston	99	62	2	1
127	University of Texas Health Science Ctr - San Antonio	100	63	1	1
112	University of Oklahoma - Norman	101	64	1	4

Table 8 (continued)

Unit Number	University	Standing (objective of MIP)	Reduced Rank	Discrepancy from superior group	Discrepancy from inferior group
139	Virginia Commonwealth University	101	64	1	4
141	Wake Forest University	101	64	1	4
13	Clemson University	105	65	4	1
53	Rensselaer Polytechnic Institute	105	65	4	1
76	University of California - Riverside	105	65	4	1
96	University of Louisville	105	65	4	1
128	University of Texas Medical Branch - Galveston	105	65	4	1
46	Oklahoma State University - Stillwater	106	66	1	3
71	University of Arkansas - Fayetteville	109	67	3	1
90	University of Houston - University Park	110	68	1	3
3	Boston College	113	69	3	1
64	Thomas Jefferson University	113	69	3	1
62	Syracuse University	114	70	1	1
80	University of California - Santa Cruz	115	71	1	1
39	Medical University of South Carolina	116	72	1	3
133	University of Vermont	116	72	1	3
35	Lehigh University	119	73	3	1
11	City University of NY - Graduate Sch and University Ctr	120	74	1	1
21	Drexel University	120	74	1	1
58	Saint Louis University - St. Louis	120	74	1	1
108	University of Nevada - Reno	120	74	1	1
38	Medical College of Wisconsin	121	75	1	4
14	College of William and Mary	125	76	4	1
56	Rush University	126	77	1	2
97	University of Maine - Orono	126	77	1	2
107	University of Nebraska Medical Center	128	78	2	3
117	University of Rhode Island	128	78	2	3
137	University of Wyoming	128	78	2	3
68	University of Akron - Akron	131	79	3	1
113	University of Oklahoma Health Sciences Center	132	80	1	1
2	Binghamton University	133	81	1	1
10	Catholic University of America	134	82	1	2
25	Florida International University	134	82	1	2
24	Florida Atlantic University	136	83	2	1
15	Colorado School of Mines	137	84	1	1
19	Creighton University	138	85	1	2
125	University of Texas - Dallas	138	85	1	2
131	University of Tulsa	140	86	2	1
41	Michigan Technological University	141	87	1	2
123	University of Texas - Arlington	141	87	1	2
50	Polytechnic University	143	88	2	1
60	State Univ. of New York Health Science Ctr - Brooklyn	143	88	2	1
12	Clark University	144	89	1	2
130	University of the Pacific	146	90	2	NA

Table 9 - Results of ranking of 146 American research universities by MIP with second feasible weight space

Unit Number	University	Standing (objective of MIP)	Reduced Rank	Discrepancy from superior group	Discrepancy from inferior group
30	Harvard University	1	1	NA	1
34	Johns Hopkins University	1	1	NA	1
59	Stanford University	2	2	1	1
37	Massachusetts Institute of Technology	3	3	1	1
72	University of California - Berkeley	3	3	1	1
136	University of Wisconsin - Madison	3	3	1	1
144	Yale University	3	3	1	1
75	University of California - Los Angeles	4	4	1	2
103	University of Michigan - Ann Arbor	4	4	1	2
115	University of Pennsylvania	4	4	1	2
121	University of Southern California	4	4	1	2
135	University of Washington - Seattle	4	4	1	2
17	Columbia University	6	5	2	1
18	Cornell University	7	6	1	4
77	University of California - San Diego	11	7	4	1
104	University of Minnesota - Twin Cities	12	8	1	1
51	Princeton University	13	9	1	1
143	Washington University in St. Louis	14	10	1	1
22	Duke University	15	11	1	1
92	University of Illinois - Urbana-Champaign	15	11	1	1
45	Ohio State University - Columbus	16	12	1	1
78	University of California - San Francisco	16	12	1	1
124	University of Texas - Austin	16	12	1	1
49	Pennsylvania State University - University Park	17	13	1	2
63	Texas A&M University	17	13	1	2
87	University of Florida	17	13	1	2
110	University of North Carolina - Chapel Hill	17	13	1	2
23	Emory University	19	14	2	2
81	University of Chicago	19	14	2	2
7	California Institute of Technology	21	15	2	1
116	University of Pittsburgh - Pittsburgh	21	15	2	1
134	University of Virginia	22	16	1	1
44	Northwestern University	23	17	1	1
70	University of Arizona	24	18	1	1
83	University of Colorado - Boulder	24	18	1	1
42	New York University	25	19	1	4
73	University of California - Davis	25	19	1	4
40	Michigan State University	29	20	4	1
1	Baylor College of Medicine	30	21	1	1
52	Purdue University - West Lafayette	31	22	1	2
99	University of Maryland - College Park	31	22	1	2

Table 9 (continued)

Unit Number	University	Standing (objective of MIP)	Reduced Rank	Discrepancy from superior group	Discrepancy from inferior group
138	Vanderbilt University	31	22	1	2
29	Georgia Institute of Technology	33	23	2	2
93	University of Iowa	35	24	2	1
145	Yeshiva University	36	25	1	2
55	Rockefeller University	38	26	2	3
9	Case Western Reserve University	41	27	3	1
132	University of Utah	41	27	3	1
61	Stony Brook University	42	28	1	1
129	University of Texas SW Medical Center - Dallas	42	28	1	1
4	Boston University	43	29	1	1
43	North Carolina State University	43	29	1	1
57	Rutgers the State University of NJ - New Brunswick	43	29	1	1
118	University of Rochester	43	29	1	1
88	University of Georgia	44	30	1	2
82	University of Cincinnati - Cincinnati	46	31	2	1
111	University of Notre Dame	47	32	1	1
146	Arizona State University - Tempe	47	32	1	1
69	University of Alabama - Birmingham	48	33	1	1
31	Indiana University - Bloomington	49	34	1	1
140	Virginia Polytechnic Institute and State University	49	34	1	1
20	Dartmouth College	50	35	1	1
74	University of California - Irvine	51	36	1	1
91	University of Illinois - Chicago	51	36	1	1
79	University of California - Santa Barbara	52	37	1	1
54	Rice University	53	38	1	1
105	University of Missouri - Columbia	54	39	1	3
6	Brown University	57	40	3	3
28	Georgetown University	57	40	3	3
33	Iowa State University	57	40	3	3
65	Tufts University	57	40	3	3
67	University at Buffalo	57	40	3	3
95	University of Kentucky	57	40	3	3
84	University of Colorado Health Sciences Center	60	41	3	2
8	Carnegie Mellon University	62	42	2	1
98	University of Maryland - Baltimore	62	42	2	1
102	University of Miami	63	43	1	1
32	Indiana University-Purdue University - Indianapolis	64	44	1	1
36	Louisiana State University - Baton Rouge	65	45	1	1
119	University of South Carolina - Columbia	65	45	1	1
94	University of Kansas - Lawrence	66	46	1	1
100	University of Massachusetts - Amherst	66	46	1	1
122	University of Tennessee - Knoxville	66	46	1	1
109	University of New Mexico - Albuquerque	67	47	1	1

Table 9 (continued)

Unit Number	University	Standing (objective of MIP)	Reduced Rank	Discrepancy from superior group	Discrepancy from inferior group
47	Oregon Health & Science University	68	48	1	1
16	Colorado State University	69	49	1	1
26	Florida State University	69	49	1	1
106	University of Nebraska - Lincoln	70	50	1	4
27	George Washington University	74	51	4	3
48	Oregon State University	77	52	3	2
5	Brandeis University	79	53	2	2
66	Tulane University	79	53	2	2
86	University of Delaware	79	53	2	2
101	University of Medicine & Dentistry of New Jersey	79	53	2	2
85	University of Connecticut - Storrs	81	54	2	2
120	University of South Florida	81	54	2	2
89	University of Hawaii - Manoa	83	55	2	4
142	Washington State University - Pullman	83	55	2	4
114	University of Oregon	87	56	4	3
127	University of Texas Health Science Ctr - San Antonio	87	56	4	3
141	Wake Forest University	87	56	4	3
112	University of Oklahoma - Norman	90	57	3	1
128	University of Texas Medical Branch - Galveston	90	57	3	1
126	University of Texas Health Science Center - Houston	91	58	1	2
53	Rensselaer Polytechnic Institute	93	59	2	1
90	University of Houston - University Park	94	60	1	1
3	Boston College	95	61	1	1
46	Oklahoma State University - Stillwater	95	61	1	1
64	Thomas Jefferson University	96	62	1	1
76	University of California - Riverside	96	62	1	1
96	University of Louisville	96	62	1	1
139	Virginia Commonwealth University	96	62	1	1
11	City University of NY - Graduate Sch and University Ctr	97	63	1	2
39	Medical University of South Carolina	97	63	1	2
71	University of Arkansas - Fayetteville	97	63	1	2
13	Clemson University	99	64	2	2
62	Syracuse University	101	65	2	1
80	University of California - Santa Cruz	102	66	1	2
35	Lehigh University	104	67	2	1
133	University of Vermont	105	68	1	5
38	Medical College of Wisconsin	110	69	5	4
58	Saint Louis University - St. Louis	114	70	4	3
21	Drexel University	117	71	3	1
108	University of Nevada - Reno	117	71	3	1
56	Rush University	118	72	1	1
14	College of William and Mary	119	73	1	2
97	University of Maine - Orono	121	74	2	2

Table 9 (continued)

Unit Number	University	Standing (objective of MIP)	Reduced Rank	Discrepancy from superior group	Discrepancy from inferior group
107	University of Nebraska Medical Center	121	74	2	2
117	University of Rhode Island	121	74	2	2
2	Binghamton University	123	75	2	5
68	University of Akron - Akron	123	75	2	5
113	University of Oklahoma Health Sciences Center	123	75	2	5
137	University of Wyoming	123	75	2	5
10	Catholic University of America	128	76	5	1
24	Florida Atlantic University	128	76	5	1
125	University of Texas - Dallas	129	77	1	1
15	Colorado School of Mines	130	78	1	1
19	Creighton University	130	78	1	1
131	University of Tulsa	130	78	1	1
25	Florida International University	131	79	1	5
41	Michigan Technological University	136	80	5	3
60	State Univ. of New York Health Science Ctr - Brooklyn	139	81	3	2
123	University of Texas - Arlington	139	81	3	2
12	Clark University	141	82	2	3
50	Polytechnic University	141	82	2	3
130	University of the Pacific	144	83	3	NA

5.2. Ranking by TDSVEA:

The same data in section 5.1 is used for ranking universities using target-direction-set value efficiency analysis. With the assumption that there exists a global decision maker (this may be a congress unit, a ministry etc. who tries to direct universities according to a national goal), universities will be ranked according to their target direction set value efficiency score (potential value efficiency score) in this section.

Outputs of the universities, which are represented by shifted z-scores, are as in Table 10. All universities are assumed to have same amount of input (this will be represented as one fictitious input with value of 1). Global Decision Maker (GDM) knows output levels (shifted z-scores of each university in previously defined 8 criteria) and input level (for test purposes it is assumed that each university uses only one unit of a hypothetical input) of all universities. GDM also knows the technological capabilities of all universities (c_{ij} values) and has the power to make each DMUs project to an input-output combination equivalent to his most preferred solution (MPS).

For test purposes, assume that the utility function of the global decision maker is known (actually in the analysis it is not known and asking questions to GDM is necessary), which is $U(z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8) = z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8$, where z_i is the shifted z-score of university in i^{th} output (shifted z-scores are obtained from z-score by adding same positive value to them in order to make output values positive). This function is linear; therefore, it conforms to pseudo-concave utility function assumption of the analysis. With this utility function GDM will select Harvard University (unit 30) as MPS with a utility 44.470. Therefore, the contour of utility function that passes through MPS is the hyper-plane $z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 = 44.470$.

Technology constraint (since only one input exists, only one constraint exists) of the universities is obtained by random generation of c_{ij} values for each university. These randomly generated c_{ij} values are assumed to be c_{ij} values provided by GDM.

In order to simulate the direction setting of the GDM, utility function gradient at the current shifted z-score set of university is used (in fact, for the specific utility function the gradient (1,1,1,1,1,1,1,1) is same for all shifted z-scores).

Consequently, all the decisions and information that should be provided by GDM is simulated. The MPS is found by the utility function, technology constraints are obtained from random generation of technology parameters (c_{ij} values), target directions are identified with the gradient of utility function at the current level of shifted z-scores of each university, which is same (1,1,1,1,1,1,1,1) for each university for the specified case.

As in Figure 7 in Chapter 4, a hypothetical unit T^a, which is at the targeted direction from the university and which gives the same satisfaction as the MPS to the GDM, for each university can be found as the intersection of the contour of the utility function that passes through MPS and the line that passes from the current shifted z-scores of university at the targeted direction (gradient of utility function for the current shifted z-scores of university, which is same (1,1,1,1,1,1,1,1) for each university for the specified case). After T^a is found for each university, substitution equivalent of each university, y_a, can be found as the intersection of the radial line OT^a and technology constraint of the university. After y_a is found, target-direction-set value efficiency score can be obtained as $[Oy_a]/[OT^a]$ for each university. Finally, the universities can be ranked according to their individual target-direction-set value efficiency score.

The procedure described above is applied here for each of 146 universities. In order to make a comparison, also value efficiency analysis is applied here to the same 146 universities with the same shifted z-score values and the same utility function. Current shifted z-score values, T^a, c_{ij} values, y_a, V^a (Figure 6 in Chapter 4), Target-direction-set value efficiency (TDSVE) scores, ranks according to TDSVE scores, value efficiency (VE) scores (Figure 6 in Chapter 4), rank according to VE scores, ranks with fixed equal weights and standings of universities by MIP with two weight spaces used in simplification are given in Table 10. Pearson rank correlation coefficients for different pairs of these ranking techniques are provided in Table 11.

In Table 10, where j stands for output j, current shifted z-score$_j$ /V^a_j for any outputs (j=1,…,8) will give the VE score of unit, whereas y_{aj}/T^a_j for any outputs will give the TDSVE score. Because of the selection of projection direction (1,1,1,1,1,1,1,1), the difference between T^a_j and current shifted z-score$_j$ is same for all outputs for a unit. Since there is only one input whose value is 1 for all units, $\sum_{j=1}^{8}$ (c_{ij}^0 . current shifted z-score$_j$) = 1 as well as $\sum_{j=1}^{8}$ (c_{ij}^0 . y_{aj}) = 1.

As can be seen from Table 10 and Table 11, TDSVEA scores give different rankings compared to other techniques (minimum rank correlations are related with TDSVEA scores). The reasons for this are twofold. Firstly, the scores in TDSVEA are potential scores, i.e. they are related with potential of the unit that is ranked. Since the technologies of the units (c_{ij}'s) are varying, their potentials are varying accordingly. Secondly, the ranking in TDSVEA is done according to preference information of the DM. Although, also in VEA, the ranking is done according to preference information of the DM, for the specific case it will provide the very similar ranking by fixed equal weights[13]. On the other hand, in TDSVEA, different from VEA, projection directions are also determined by the DM; therefore, scores of VEA and TDSVEA are expected to be different. As discussed in the footnote, the other ranking techniques than TDSVEA are expected to produce similar rankings. In TDSVEA, units whose potential is more appropriate to preferences of the DM have higher rankings than in VEA (e.g. light shaded cells in Table 10), while units whose potential is less appropriate to preferences of the DM have lower rankings than in VEA (e.g. dark shaded cells in Table 10).

[13] If ranking in VEA would be done by z-scores rather than shifted z-scores (if it would be possible), ranking by VEA and by fixed equal weights would provide the same ranking since assumed utility function is linear with equal weights. However, ranking by actual z-scores is not possible in VEA since VEA uses only nonnegative scores. But still, the rankings are very similar (Pearson rank correlation between VEA and fixed equal weight technique is 0.99995). Since in MIP, especially for the 1st weight space, the weight space is narrow and includes 0.125 (if fixed equal weight is considered with normalization, weight of each criterion becomes 0.125) the rankings are expected to be similar with ranking by fixed equal weights, therefore with VEA. The results conform to expectations (Table 11).

As can be seen from y_a (substitution equivalent) and T^a (target at selected direction) columns of Table 10, considering in-unit values (i.e. values in a row of Table 10), shifted z-scores of outputs of target and substitution equivalent for each unit approach each other, which conforms to the preferences of the DM[14], although some deviation occurs, which is caused by current shifted z-scores and technological capabilities for outputs of each unit. Moreover, considering between-units values, the targets and the substitution equivalents of units differ because current situations and capabilities of units vary.

Differences between TDSVE scores and VE scores may be considered as disharmony between GDM and the local managers of DMUs. If the local managers consider preferences of the GDM and technologies of the units, then they would use this information and try to make a production, which is similar to substitution equivalent level of unit, therefore the two scores would be similar.

If the selected utility function considers also the technological capabilities of units and the DM's preferences conform[15], the DM would select such projection directions which would make TDSVE scores ≥ VE score. However, in this example the selected directions do not consider the technological capabilities of units, therefore, substitution equivalent of the unit may need more proportional increase in outputs than current output values of the unit in order to achieve a satisfaction value which is equivalent to MPS (i.e. TDSVE score – VE score < 0). The results in Table 10 verify this proposition, while 84 units have a positive difference (TDSVE score – VE score), 62 units have negative difference. In extreme situation, when the DM tries to maximize TDSVE score, all units have non-negative (probably positive) differences[16].

To sum up, results of TDSVEA conform to the expectations: TDSVEA ranks units according to capabilities (potentials) of them to satisfy the preferences of the DM.

[14] Utility function $U = z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8$ assigns same weight to each output and selected direction (1,1,1,1,1,1,1,1) also assigns the same weight to each output, i.e. γ_j=1, for projection.
[15] See Appendix A.
[16] See Appendix A.

As VEA, it gets scores for units. VEA score shows how much radial projection is needed for a unit in order to achieve an input-output mix for it, which gives DM the same satisfaction level as the MPS. However TDSVEA, different from VEA, while doing this, uses a projection direction based on the preferences of the DM (see Appendix A for a partial analysis of preferences of the DM in selecting the projection direction) and a substitution equivalent of that unit, which is technologically equivalent to current unit. This would lead to reflecting the DM's preferences to the ranking better in many applications.

Table 10 - TDSVEA and its scores, VEA and its scores, rank of universities by TDSVEA, VEA, fixed equal weights technique and MIP with two weight spaces used in simplification procedure

	Current shifted z-scores								T^a								c_{ij}							
Unit #	z1	z2	z3	z4	z5	z6	z7	z8	z1	z2	z3	z4	z5	z6	z7	z8	c11	c21	c31	c41	c51	c61	c71	c81
1	2.39	2.28	1.12	1.05	0.99	1.22	0.21	1.84	6.57	6.47	5.31	5.24	5.17	5.40	4.39	6.02	0.027	0.029	0.023	0.027	0.027	0.021	3.431	0.022
2	0.18	0.29	0.70	0.32	0.69	0.48	0.62	0.50	5.28	5.39	5.80	5.42	5.79	5.58	5.72	5.60	0.028	0.030	0.028	0.023	0.021	0.029	1.476	0.027
3	0.25	0.41	1.16	0.82	0.71	0.48	0.67	0.60	5.18	5.34	6.10	5.75	5.64	5.41	5.60	5.53	0.021	0.021	0.024	0.023	0.022	0.025	1.344	0.024
4	1.11	1.55	0.97	1.17	0.97	1.97	1.40	0.73	5.45	5.89	5.31	5.50	5.31	6.30	5.74	5.07	0.024	0.029	0.023	0.024	0.029	0.022	0.566	0.021
5	0.38	0.50	0.88	0.89	0.92	0.94	0.56	0.74	5.23	5.34	5.72	5.74	5.77	5.79	5.41	5.59	0.027	0.021	0.026	0.026	0.024	0.021	1.552	0.022
6	0.62	0.74	1.38	1.11	1.11	0.94	0.83	0.78	5.25	5.37	6.01	5.74	5.74	5.57	5.46	5.41	0.027	0.021	0.023	0.025	0.024	0.028	1.007	0.025
7	1.37	1.93	1.26	1.40	2.89	1.41	0.78	1.90	5.33	5.88	5.21	5.35	6.85	5.36	4.73	5.85	0.027	0.022	0.027	0.026	0.024	0.024	0.894	0.027
8	0.95	1.09	1.02	0.62	1.18	0.57	0.99	0.87	5.61	5.75	5.68	5.28	5.84	5.23	5.65	5.53	0.021	0.022	0.025	0.026	0.022	0.027	0.855	0.027
9	1.27	1.62	1.35	1.26	1.13	1.41	0.99	1.24	5.56	5.91	5.64	5.55	5.42	5.69	5.27	5.53	0.028	0.029	0.020	0.023	0.022	0.028	0.772	0.028
10	0.17	0.35	0.74	0.46	0.69	0.38	0.40	0.50	5.28	5.46	5.85	5.57	5.80	5.50	5.51	5.61	0.030	0.025	0.028	0.029	0.028	0.022	2.306	0.027
11	0.08	0.25	0.70	0.29	0.69	0.38	1.54	0.66	5.08	5.24	5.69	5.29	5.68	5.38	6.54	5.66	0.022	0.021	0.023	0.027	0.022	0.027	0.602	0.022
12	0.08	0.26	0.76	0.41	0.71	0.20	0.12	0.53	5.27	5.44	5.95	5.60	5.90	5.39	5.31	5.71	0.027	0.024	0.025	0.021	0.029	0.028	7.503	0.029
13	0.82	0.66	0.79	0.66	0.69	0.48	0.64	0.60	5.72	5.56	5.69	5.56	5.59	5.38	5.55	5.50	0.026	0.022	0.023	0.026	0.022	0.025	1.374	0.027
14	0.28	0.37	0.86	0.54	0.69	0.57	0.32	0.56	5.33	5.41	5.91	5.59	5.73	5.62	5.37	5.61	0.022	0.025	0.022	0.029	0.022	0.028	2.829	0.024
15	0.21	0.34	0.74	0.48	0.76	0.38	0.28	0.52	5.31	5.45	5.85	5.59	5.86	5.49	5.39	5.63	0.025	0.029	0.024	0.028	0.024	0.029	3.256	0.028
16	1.05	1.12	0.74	0.48	0.80	0.57	0.83	1.08	5.78	5.86	5.47	5.22	5.54	5.31	5.57	5.81	0.023	0.020	0.027	0.027	0.021	0.021	1.047	0.020
17	2.22	3.01	2.76	2.99	2.63	4.20	2.59	1.33	5.08	5.87	5.61	5.84	5.49	7.05	5.45	4.19	0.021	0.026	0.028	0.027	0.027	0.025	0.195	0.027
18	2.77	2.34	2.09	3.91	2.49	3.45	2.36	2.32	5.62	5.19	4.95	6.77	5.35	6.31	5.21	5.17	0.023	0.027	0.026	0.025	0.027	0.020	0.220	0.029

	y_a								V^a								Scores and Ranks						
Unit #	z1	z2	z3	z4	z5	z6	z7	z8	z1	z2	z3	z4	z5	z6	z7	z8	TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
1	0.41	0.40	0.33	0.33	0.32	0.34	0.27	0.37	9.59	9.16	4.51	4.22	3.98	4.90	0.84	7.37	0.062	119	0.249	44	44	43	30
2	0.56	0.57	0.61	0.57	0.61	0.59	0.60	0.59	2.10	3.41	8.30	3.80	8.09	5.63	7.32	5.91	0.106	102	0.085	133	133	133	123
3	0.62	0.63	0.72	0.68	0.67	0.64	0.67	0.66	2.18	3.58	10.17	7.16	6.21	4.18	5.84	5.24	0.119	91	0.114	113	113	113	95
4	1.30	1.40	1.26	1.31	1.26	1.50	1.37	1.21	5.02	7.01	4.39	5.27	4.38	8.88	6.32	3.30	0.238	47	0.221	47	47	47	43
5	0.56	0.57	0.61	0.61	0.62	0.62	0.58	0.60	2.94	3.80	6.71	6.84	7.06	7.22	4.32	5.68	0.107	100	0.131	97	97	97	79
6	0.81	0.83	0.93	0.89	0.89	0.86	0.85	0.84	3.68	4.41	8.20	6.58	6.57	5.58	4.92	4.63	0.155	78	0.169	70	70	70	57
7	1.02	1.12	0.99	1.02	1.31	1.02	0.90	1.12	4.74	6.64	4.33	4.81	9.97	4.85	2.68	6.54	0.191	69	0.290	34	34	33	21
8	0.97	0.99	0.98	0.91	1.01	0.90	0.98	0.96	5.79	6.65	6.22	3.79	7.22	3.49	6.08	5.33	0.173	73	0.163	71	71	71	62
9	1.10	1.16	1.11	1.09	1.07	1.12	1.04	1.09	5.51	7.04	5.86	5.48	4.91	6.10	4.28	5.39	0.197	63	0.231	46	46	46	41
10	0.38	0.40	0.43	0.41	0.42	0.40	0.40	0.41	2.04	4.23	8.96	5.54	8.31	4.65	4.78	6.06	0.073	115	0.083	135	134	134	128
11	1.05	1.09	1.18	1.10	1.18	1.12	1.36	1.17	0.77	2.39	6.76	2.80	6.67	3.73	14.98	6.46	0.207	59	0.103	120	123	120	97
12	0.13	0.13	0.15	0.14	0.14	0.13	0.13	0.14	1.17	3.72	11.09	5.97	10.32	2.89	1.79	7.63	0.024	140	0.069	145	145	144	141
13	0.67	0.65	0.66	0.65	0.65	0.63	0.65	0.64	6.84	5.51	6.58	5.50	5.75	4.00	5.39	5.01	0.117	94	0.120	106	106	105	99
14	0.33	0.34	0.37	0.35	0.36	0.35	0.33	0.35	2.98	3.91	9.18	5.76	7.31	6.07	3.40	5.95	0.062	120	0.094	125	125	125	119
15	0.29	0.29	0.31	0.30	0.32	0.30	0.29	0.30	2.48	4.10	8.87	5.80	9.09	4.61	3.35	6.27	0.054	123	0.083	134	137	137	130
16	0.86	0.87	0.81	0.78	0.82	0.79	0.83	0.87	6.99	7.49	4.92	3.23	5.38	3.82	5.55	7.20	0.149	80	0.150	87	87	86	69
17	2.45	2.83	2.71	2.82	2.65	3.40	2.63	2.02	4.56	6.18	5.66	6.12	5.40	8.60	5.31	2.73	0.483	11	0.488	12	12	12	6
18	2.64	2.44	2.32	3.18	2.51	2.96	2.45	2.43	5.68	4.79	4.29	8.02	5.11	7.08	4.84	4.75	0.470	13	0.488	13	13	12	7

Table 10 (continued)

Unit #	Current shifted z-scores z1	z2	z3	z4	z5	z6	z7	z8	T^a z1	z2	z3	z4	z5	z6	z7	z8
19	0.17	0.30	0.78	0.45	0.69	0.20	0.42	0.52	5.30	5.43	5.91	5.58	5.82	5.33	5.56	5.65
20	0.73	0.84	1.76	1.12	0.92	1.59	0.20	0.76	5.31	5.42	6.34	5.70	5.50	6.17	4.78	5.34
21	0.23	0.41	0.79	0.49	0.76	0.57	0.47	0.64	5.26	5.44	5.82	5.52	5.78	5.60	5.50	5.66
22	2.35	2.14	2.13	2.92	1.72	2.06	1.40	2.18	5.81	5.60	5.59	6.38	5.18	5.52	4.86	5.64
23	1.51	1.72	2.93	2.37	0.95	1.50	1.00	1.60	5.38	5.60	6.80	6.25	4.82	5.37	4.88	5.47
24	0.21	0.36	0.73	0.54	0.71	0.38	0.19	0.52	5.32	5.47	5.85	5.66	5.83	5.50	5.31	5.64
25	0.33	0.44	0.71	0.36	0.69	0.29	0.27	0.58	5.44	5.55	5.82	5.47	5.80	5.40	5.38	5.69
26	0.76	0.73	0.85	0.87	0.83	0.66	1.41	0.81	5.46	5.44	5.55	5.58	5.53	5.37	6.12	5.51
27	0.51	0.69	1.01	0.74	0.76	0.94	1.27	0.65	5.26	5.44	5.76	5.49	5.51	5.69	6.02	5.40
28	0.67	1.05	0.99	1.05	0.78	1.59	0.44	0.70	5.33	5.71	5.65	5.71	5.44	6.26	5.10	5.36
29	1.93	1.49	1.22	1.05	1.27	2.06	1.46	0.67	6.11	5.67	5.39	5.22	5.45	6.24	5.64	4.85
30	2.33	2.86	9.15	5.07	6.74	5.31	3.12	9.99	2.33	2.86	9.15	5.07	6.74	5.31	3.12	9.99
31	0.70	0.64	0.93	0.90	0.87	1.31	1.98	0.88	5.24	5.18	5.48	5.44	5.42	5.86	6.53	5.42
32	1.01	0.85	0.90	1.44	0.83	0.29	0.14	1.06	5.77	5.60	5.66	6.19	5.58	5.05	4.90	5.82
33	1.16	0.78	0.85	0.87	0.87	1.31	1.36	0.98	5.70	5.33	5.40	5.42	5.42	5.86	5.91	5.53
34	6.16	7.93	1.52	3.47	2.14	3.36	2.13	3.56	7.95	9.71	3.31	5.25	3.93	5.15	3.92	5.35
35	0.24	0.36	1.02	0.58	0.90	0.57	0.44	0.60	5.22	5.34	6.00	5.56	5.88	5.55	5.42	5.58
36	1.19	0.80	0.79	0.87	0.71	0.76	1.26	0.72	5.88	5.49	5.47	5.56	5.39	5.44	5.94	5.41
37	2.72	2.89	3.33	2.48	6.11	3.27	2.88	2.68	5.00	5.17	5.61	4.75	8.38	5.54	5.15	4.96
38	0.57	0.79	0.71	0.43	0.69	0.20	0.07	0.82	5.61	5.83	5.75	5.46	5.72	5.23	5.11	5.86

Unit #	c_{ij} c11	c21	c31	c41	c51	c61	c71	c81	y_a z1	z2	z3	z4	z5	z6	z7	z8
19	0.023	0.026	0.024	0.021	0.025	0.021	2.176	0.029	0.41	0.42	0.45	0.43	0.45	0.41	0.43	0.43
20	0.023	0.029	0.026	0.022	0.023	0.028	3.921	0.028	0.27	0.27	0.32	0.29	0.28	0.31	0.24	0.27
21	0.023	0.023	0.021	0.023	0.020	0.028	1.937	0.022	0.46	0.47	0.50	0.48	0.50	0.49	0.48	0.49
22	0.020	0.030	0.026	0.027	0.028	0.023	0.440	0.021	1.86	1.79	1.79	2.04	1.66	1.77	1.55	1.80
23	0.025	0.027	0.021	0.023	0.029	0.024	0.697	0.023	1.23	1.28	1.56	1.43	1.10	1.23	1.12	1.25
24	0.023	0.026	0.026	0.024	0.025	0.024	4.747	0.025	0.20	0.21	0.22	0.22	0.22	0.21	0.20	0.22
25	0.028	0.024	0.025	0.025	0.022	0.022	3.341	0.029	0.29	0.29	0.31	0.29	0.31	0.28	0.28	0.30
26	0.021	0.026	0.021	0.023	0.022	0.027	0.618	0.023	1.17	1.16	1.19	1.19	1.18	1.15	1.31	1.18
27	0.027	0.028	0.023	0.027	0.024	0.025	0.686	0.023	1.03	1.07	1.13	1.08	1.08	1.12	1.18	1.06
28	0.028	0.028	0.021	0.026	0.030	0.026	1.855	0.026	0.51	0.54	0.54	0.54	0.52	0.59	0.49	0.51
29	0.021	0.021	0.022	0.026	0.027	0.024	0.530	0.022	1.57	1.45	1.38	1.34	1.40	1.60	1.45	1.24
30	0.026	0.025	0.023	0.027	0.022	0.022	0.010	0.022	2.33	2.86	9.15	5.07	6.74	5.31	3.12	9.99
31	0.028	0.025	0.020	0.020	0.020	0.026	0.428	0.029	1.41	1.40	1.47	1.47	1.46	1.58	1.76	1.46
32	0.021	0.026	0.024	0.030	0.028	0.024	5.962	0.024	0.19	0.19	0.19	0.21	0.18	0.17	0.16	0.19
33	0.023	0.021	0.030	0.028	0.028	0.029	0.604	0.025	1.24	1.16	1.18	1.18	1.18	1.28	1.29	1.20
34	0.027	0.022	0.028	0.024	0.021	0.027	0.150	0.022	5.05	6.18	2.11	3.34	2.50	3.27	2.49	3.40
35	0.024	0.025	0.026	0.026	0.026	0.026	2.016	0.024	0.44	0.45	0.50	0.47	0.49	0.47	0.45	0.47
36	0.025	0.028	0.024	0.023	0.028	0.028	0.673	0.028	1.17	1.09	1.09	1.11	1.08	1.09	1.19	1.08
37	0.029	0.024	0.022	0.021	0.025	0.026	0.151	0.021	2.90	3.00	3.25	2.76	4.86	3.21	2.99	2.88
38	0.028	0.027	0.024	0.024	0.020	0.025	11.71	0.021	0.09	0.09	0.09	0.08	0.09	0.08	0.08	0.09

Unit #	V^a z1	z2	z3	z4	z5	z6	z7	z8	Scores and Ranks: TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
19	2.15	3.76	9.83	5.67	8.69	2.51	5.37	6.58	0.077	112	0.079	139	139	138	130
20	4.09	4.74	9.91	6.29	5.18	8.95	1.15	4.27	0.051	125	0.178	66	66	66	50
21	2.36	4.18	8.11	5.04	7.73	5.82	4.80	6.52	0.087	108	0.098	121	120	120	117
22	6.20	5.65	5.62	7.71	4.54	5.43	3.69	5.74	0.320	32	0.379	19	19	17	15
23	4.95	5.65	9.62	7.79	3.10	4.92	3.30	5.24	0.229	49	0.305	31	31	30	19
24	2.54	4.36	8.97	6.62	8.68	4.70	2.35	6.35	0.038	133	0.082	137	136	136	128
25	4.02	5.36	8.60	4.38	8.31	3.53	3.31	7.06	0.053	124	0.083	136	135	134	131
26	4.87	4.72	5.46	5.61	5.33	4.28	9.09	5.20	0.214	57	0.155	81	81	80	69
27	3.48	4.66	6.84	5.04	5.15	6.41	8.60	4.39	0.196	64	0.147	88	88	87	74
28	4.09	6.45	6.04	6.43	4.79	9.76	2.71	4.30	0.095	104	0.163	72	72	71	57
29	7.72	5.96	4.86	4.18	5.09	8.23	5.84	2.68	0.257	45	0.250	43	43	43	33
30	2.33	2.86	9.15	5.07	6.74	5.31	3.12	9.99	1.000	1	1.000	1	1	1	1
31	3.77	3.48	5.05	4.88	4.74	7.12	10.75	4.77	0.269	42	0.185	63	63	63	49
32	6.93	5.79	6.15	9.84	5.66	1.99	0.96	7.25	0.033	135	0.146	89	89	88	64
33	6.30	4.23	4.65	4.72	4.77	7.17	7.41	5.32	0.218	54	0.183	64	64	64	57
34	9.07	11.67	2.24	5.10	3.15	4.95	3.14	5.24	0.636	3	0.679	3	3	3	1
35	2.26	3.37	9.66	5.49	8.50	5.40	4.18	5.69	0.084	109	0.106	119	119	119	104
36	7.49	5.05	4.93	5.47	4.46	4.74	7.90	4.53	0.199	61	0.159	78	78	77	65
37	4.60	4.89	5.63	4.19	10.33	5.52	4.87	4.54	0.580	5	0.591	4	4	4	3
38	5.96	8.25	7.42	4.46	7.14	2.07	0.74	8.53	0.015	142	0.096	124	124	121	110

Table 10 (continued)

Unit #	Current shifted z-scores z1	z2	z3	z4	z5	z6	z7	z8	T^a z1	z2	z3	z4	z5	z6	z7	z8
39	0.77	0.76	0.72	0.56	0.71	0.11	0.20	1.07	5.73	5.72	5.68	5.52	5.67	5.06	5.16	6.03
40	1.68	1.22	0.95	2.39	0.80	1.31	2.45	1.26	5.75	5.28	5.01	6.45	4.87	5.38	6.52	5.33
41	0.24	0.39	0.71	0.35	0.71	0.20	0.22	0.53	5.40	5.54	5.86	5.51	5.86	5.35	5.37	5.68
42	1.23	1.37	1.27	2.79	1.41	1.13	2.38	1.25	5.19	5.34	5.24	6.76	5.38	5.10	6.35	5.22
43	1.89	1.07	0.83	1.49	1.09	0.76	1.71	0.70	6.27	5.45	5.21	5.86	5.47	5.14	6.09	5.08
44	1.64	1.62	2.18	2.10	1.46	1.69	2.00	1.16	5.48	5.46	6.02	5.94	5.30	5.53	5.84	5.00
45	2.44	1.64	1.16	2.06	1.06	2.06	3.55	1.25	6.11	5.31	4.83	5.73	4.73	5.73	7.22	4.92
46	0.61	0.46	0.76	0.57	0.73	0.48	1.06	0.61	5.52	5.37	5.67	5.48	5.64	5.39	5.97	5.52
47	0.90	1.21	0.81	0.88	0.80	1.04	0.17	0.92	5.63	5.94	5.54	5.61	5.53	5.77	4.90	5.65
48	1.00	0.98	0.80	0.69	0.78	0.38	0.88	0.78	5.78	5.76	5.59	5.48	5.56	5.17	5.66	5.57
49	2.58	2.17	1.03	1.67	1.25	2.52	2.98	1.18	6.22	5.82	4.68	5.32	4.90	6.17	6.63	4.83
50	0.11	0.27	0.75	0.33	0.78	0.20	0.09	0.56	5.30	5.45	5.93	5.52	5.96	5.38	5.28	5.74
51	0.97	0.92	4.79	2.12	2.52	2.90	1.31	1.40	4.43	4.38	8.24	5.58	5.97	6.35	4.76	4.85
52	1.62	1.09	1.23	1.91	1.02	1.69	2.34	1.15	5.68	5.16	5.29	5.97	5.08	5.75	6.41	5.22
53	0.34	0.46	0.95	0.83	0.95	0.48	0.64	0.67	5.24	5.37	5.86	5.74	5.85	5.39	5.55	5.58
54	0.32	0.55	2.14	0.85	1.16	0.76	0.59	0.78	5.00	5.22	6.81	5.53	5.83	5.44	5.27	5.46
55	0.95	0.72	1.32	1.17	1.70	1.87	0.15	1.36	5.37	5.13	5.74	5.59	6.11	6.29	4.56	5.77
56	0.49	0.56	0.83	0.55	0.73	0.11	0.16	0.66	5.55	5.62	5.89	5.61	5.79	5.17	5.22	5.72
57	1.35	0.83	0.86	1.01	1.30	1.13	2.08	0.83	5.75	5.23	5.26	5.40	5.70	5.53	6.48	5.22
58	0.26	0.48	1.03	0.50	0.69	0.20	0.61	0.58	5.29	5.51	6.06	5.53	5.72	5.23	5.64	5.61

Unit #	c_{ij} c11	c21	c31	c41	c51	c61	c71	c81	y_a z1	z2	z3	z4	z5	z6	z7	z8
39	0.027	0.029	0.028	0.021	0.030	0.022	4.415	0.024	0.24	0.24	0.24	0.23	0.24	0.21	0.22	0.25
40	0.024	0.028	0.023	0.028	0.028	0.029	0.303	0.027	1.92	1.77	1.68	2.16	1.63	1.80	2.18	1.78
41	0.027	0.023	0.028	0.025	0.023	0.027	4.275	0.023	0.23	0.23	0.24	0.23	0.24	0.22	0.22	0.24
42	0.030	0.024	0.023	0.029	0.023	0.027	0.305	0.026	1.77	1.82	1.79	2.31	1.84	1.74	2.17	1.78
43	0.026	0.026	0.028	0.027	0.023	0.023	0.467	0.024	1.64	1.43	1.36	1.53	1.43	1.34	1.59	1.33
44	0.026	0.029	0.026	0.022	0.021	0.028	0.352	0.023	1.81	1.80	1.99	1.96	1.75	1.83	1.93	1.65
45	0.028	0.025	0.030	0.029	0.025	0.029	0.191	0.027	2.54	2.21	2.01	2.38	1.97	2.38	3.00	2.04
46	0.023	0.020	0.027	0.028	0.024	0.022	0.842	0.029	0.92	0.90	0.95	0.92	0.94	0.90	1.00	0.92
47	0.023	0.024	0.030	0.025	0.025	0.020	4.957	0.025	0.22	0.24	0.22	0.22	0.22	0.23	0.19	0.22
48	0.021	0.024	0.028	0.025	0.026	0.025	0.993	0.020	0.88	0.88	0.85	0.84	0.85	0.79	0.86	0.85
49	0.027	0.028	0.024	0.029	0.025	0.025	0.226	0.024	2.50	2.34	1.88	2.14	1.97	2.48	2.66	1.94
50	0.029	0.029	0.030	0.030	0.020	0.024	9.818	0.025	0.10	0.10	0.11	0.10	0.11	0.10	0.10	0.11
51	0.023	0.024	0.020	0.021	0.029	0.021	0.494	0.026	1.35	1.34	2.52	1.70	1.82	1.94	1.45	1.48
52	0.029	0.026	0.027	0.030	0.023	0.025	0.315	0.028	1.87	1.70	1.74	1.96	1.67	1.89	2.11	1.72
53	0.029	0.025	0.030	0.022	0.027	0.025	1.372	0.027	0.61	0.62	0.68	0.66	0.68	0.62	0.64	0.65
54	0.021	0.027	0.022	0.023	0.023	0.024	1.440	0.026	0.59	0.61	0.80	0.65	0.68	0.64	0.62	0.64
55	0.023	0.029	0.024	0.026	0.024	0.026	5.294	0.022	0.21	0.20	0.23	0.22	0.24	0.25	0.18	0.23
56	0.023	0.021	0.028	0.028	0.028	0.029	5.695	0.025	0.18	0.18	0.19	0.18	0.19	0.17	0.17	0.19
57	0.026	0.026	0.025	0.020	0.026	0.021	0.398	0.023	1.65	1.50	1.51	1.55	1.64	1.59	1.86	1.50
58	0.027	0.028	0.023	0.029	0.024	0.024	1.487	0.024	0.56	0.59	0.65	0.59	0.61	0.56	0.60	0.60

Unit #	V^a z1	z2	z3	z4	z5	z6	z7	z8	Scores and Ranks: TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
39	7.04	6.94	6.56	5.07	6.46	0.96	1.80	9.74	0.042	128	0.110	118	118	116	97
40	6.22	4.49	3.49	8.82	2.97	4.85	9.06	4.67	0.335	26	0.271	38	38	37	29
41	3.24	5.17	9.43	4.71	9.48	2.65	2.88	7.01	0.042	129	0.075	141	141	141	136
42	4.26	4.76	4.41	9.69	4.92	3.92	8.27	4.35	0.341	24	0.288	35	35	34	25
43	8.83	5.01	3.90	6.94	5.08	3.54	8.00	3.27	0.261	44	0.214	52	52	50	43
44	5.27	5.21	7.01	6.77	4.71	5.43	6.43	3.75	0.330	28	0.310	29	29	29	23
45	7.16	4.81	3.40	6.04	3.11	6.02	10.39	3.65	0.416	18	0.342	24	24	24	16
46	5.16	3.86	6.42	4.80	6.18	4.02	8.95	5.18	0.167	74	0.119	110	110	106	95
47	5.94	8.01	5.36	5.83	5.33	6.86	1.12	6.12	0.040	132	0.151	86	86	83	68
48	7.08	6.90	5.68	4.91	5.53	2.72	6.20	5.54	0.152	79	0.141	90	90	90	77
49	7.46	6.28	2.98	4.85	3.62	7.31	8.64	3.42	0.402	20	0.345	23	23	23	17
50	1.62	3.87	10.79	4.81	11.23	2.86	1.35	8.05	0.019	141	0.070	143	143	143	141
51	2.56	2.43	12.61	5.59	6.63	7.63	3.44	3.68	0.305	35	0.380	18	18	17	13
52	5.98	4.04	4.55	7.06	3.76	6.24	8.67	4.27	0.329	29	0.270	39	39	38	31
53	2.83	3.86	7.97	6.97	7.94	4.01	5.37	5.63	0.116	95	0.119	109	109	105	93
54	2.01	3.41	13.33	5.33	7.22	4.72	3.66	4.89	0.117	92	0.160	77	77	72	53
55	4.59	3.46	6.38	5.67	8.19	9.04	0.71	6.54	0.040	131	0.207	55	55	54	38
56	5.35	6.10	9.06	6.02	8.00	1.15	1.72	7.16	0.033	136	0.092	126	126	126	118
57	6.40	3.96	4.10	4.78	6.16	5.36	9.87	3.92	0.287	39	0.210	53	53	54	43
58	2.67	4.93	10.56	5.10	7.06	2.04	6.27	5.94	0.107	101	0.097	123	122	120	114

Table 10 (continued)

Unit #	Current shifted z-scores								T^a							
	z1	z2	z3	z4	z5	z6	z7	z8	z1	z2	z3	z4	z5	z6	z7	z8
59	3.01	3.60	4.44	4.84	6.39	2.62	3.15	3.69	4.61	5.20	6.04	6.44	7.99	4.22	4.75	5.30
60	0.25	0.43	0.70	0.28	0.69	0.20	0.03	0.50	5.44	5.62	5.89	5.46	5.87	5.38	5.22	5.69
61	1.09	1.05	0.71	0.44	0.97	1.78	2.07	1.54	5.46	5.42	5.07	4.81	5.33	6.15	6.43	5.91
62	0.32	0.50	1.01	0.64	0.69	0.57	0.72	0.57	5.26	5.44	5.95	5.59	5.63	5.51	5.66	5.52
63	2.54	1.54	2.41	1.45	1.06	1.31	2.89	1.11	6.33	5.32	6.19	5.23	4.84	5.09	6.67	4.89
64	0.60	0.80	0.78	0.58	0.78	0.38	0.00	1.12	5.55	5.74	5.72	5.52	5.72	5.33	4.94	6.06
65	0.71	0.86	1.01	1.08	0.85	0.57	0.53	1.63	5.37	5.53	5.67	5.75	5.52	5.24	5.20	6.30
66	0.67	0.72	0.98	0.79	0.73	0.94	0.76	0.68	5.46	5.51	5.77	5.58	5.52	5.73	5.55	5.46
67	1.20	1.08	0.88	0.51	0.78	0.66	1.31	1.34	5.80	5.68	5.48	5.11	5.38	5.26	5.91	5.94
68	0.20	0.31	0.75	0.48	0.69	0.38	0.44	0.64	5.28	5.39	5.84	5.56	5.77	5.47	5.52	5.73
69	1.48	1.94	0.81	0.77	0.87	0.94	0.70	1.32	5.94	6.40	5.28	5.24	5.34	5.41	5.16	5.79
70	2.30	1.98	0.83	1.46	1.32	2.06	2.12	1.61	6.16	5.84	4.69	5.32	5.18	5.92	5.98	5.47
71	0.54	0.44	0.97	0.90	0.71	0.48	0.59	0.65	5.45	5.35	5.88	5.81	5.62	5.39	5.50	5.56
72	2.78	2.05	1.56	2.50	5.40	2.62	4.64	2.86	5.30	4.57	4.08	5.02	7.92	5.13	7.16	5.38
73	2.70	1.59	0.88	1.08	1.30	1.04	1.98	2.04	6.69	5.58	4.88	5.07	5.29	5.03	5.97	6.04
74	1.16	1.12	0.75	0.65	1.23	1.50	0.99	1.28	5.65	5.61	5.23	5.13	5.71	5.99	5.47	5.77
75	4.29	2.97	1.29	3.10	2.07	2.52	3.41	2.73	7.07	5.74	4.06	5.87	4.84	5.30	6.18	5.51
76	0.64	0.46	0.72	0.56	0.76	0.48	0.64	1.09	5.54	5.36	5.62	5.46	5.66	5.38	5.55	5.99
77	3.46	3.24	0.81	1.27	2.80	1.69	1.58	3.00	6.80	6.57	4.15	4.61	6.14	5.03	4.92	6.34
78	3.26	2.66	1.08	2.34	2.47	1.04	0.46	1.64	6.97	6.36	4.78	6.05	6.17	4.74	4.16	5.34

Unit #	c_{ij}								y_a							
	c11	c21	c31	c41	c51	c61	c71	c81	z1	z2	z3	z4	z5	z6	z7	z8
59	0.029	0.026	0.023	0.030	0.027	0.022	0.081	0.024	3.25	3.66	4.26	4.54	5.63	2.97	3.35	3.73
60	0.021	0.025	0.024	0.029	0.021	0.021	12.69	0.029	0.03	0.03	0.04	0.03	0.04	0.03	0.03	0.04
61	0.025	0.026	0.023	0.020	0.026	0.030	0.388	0.025	1.57	1.56	1.46	1.39	1.54	1.77	1.86	1.70
62	0.024	0.023	0.025	0.030	0.027	0.029	1.228	0.030	0.66	0.68	0.74	0.70	0.70	0.69	0.71	0.69
63	0.026	0.022	0.023	0.021	0.027	0.025	0.253	0.021	2.46	2.07	2.41	2.03	1.88	1.98	2.59	1.90
64	0.020	0.025	0.023	0.024	0.023	0.025	25.32	0.027	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
65	0.026	0.027	0.029	0.021	0.023	0.023	1.577	0.025	0.59	0.60	0.62	0.63	0.60	0.57	0.57	0.69
66	0.026	0.026	0.024	0.020	0.028	0.023	1.138	0.023	0.75	0.76	0.79	0.77	0.76	0.79	0.76	0.75
67	0.027	0.030	0.029	0.025	0.026	0.022	0.631	0.027	1.22	1.20	1.15	1.08	1.13	1.11	1.24	1.25
68	0.030	0.025	0.027	0.021	0.026	0.021	2.092	0.028	0.42	0.43	0.47	0.44	0.46	0.44	0.44	0.46
69	0.028	0.020	0.023	0.022	0.025	0.024	1.165	0.021	0.86	0.92	0.76	0.76	0.77	0.78	0.75	0.84
70	0.029	0.022	0.021	0.029	0.022	0.020	0.339	0.026	2.08	1.97	1.59	1.80	1.75	2.00	2.02	1.85
71	0.027	0.026	0.024	0.026	0.026	0.029	1.499	0.024	0.59	0.58	0.64	0.63	0.61	0.58	0.59	0.60
72	0.024	0.021	0.023	0.023	0.030	0.025	0.108	0.024	3.12	2.69	2.40	2.96	4.66	3.02	4.21	3.17
73	0.029	0.026	0.025	0.025	0.027	0.025	0.361	0.027	2.11	1.76	1.54	1.60	1.67	1.58	1.88	1.90
74	0.022	0.029	0.023	0.024	0.024	0.026	0.818	0.026	1.04	1.03	0.96	0.94	1.05	1.10	1.00	1.06
75	0.026	0.023	0.023	0.025	0.025	0.029	0.150	0.027	3.70	3.00	2.13	3.07	2.53	2.77	3.24	2.88
76	0.028	0.027	0.028	0.026	0.023	0.025	1.363	0.025	0.65	0.63	0.66	0.64	0.66	0.63	0.65	0.70
77	0.024	0.025	0.028	0.027	0.020	0.027	0.377	0.027	2.38	2.31	1.46	1.62	2.15	1.76	1.73	2.22
78	0.023	0.026	0.022	0.020	0.026	0.023	1.426	0.026	1.01	0.92	0.69	0.88	0.90	0.69	0.60	0.77

Unit #	V^a								Scores and Ranks						
	z1	z2	z3	z4	z5	z6	z7	z8	TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
59	4.23	5.05	6.24	6.80	8.98	3.68	4.42	5.19	0.705	2	0.712	2	2	2	2
60	3.68	6.26	10.16	4.03	9.91	2.87	0.44	7.24	0.006	145	0.069	144	144	143	139
61	5.04	4.85	3.27	2.04	4.48	8.23	9.55	7.12	0.289	38	0.216	51	51	48	42
62	2.85	4.40	8.97	5.72	6.09	5.06	6.39	5.08	0.125	90	0.113	115	115	114	101
63	7.92	4.79	7.51	4.50	3.30	4.09	9.00	3.46	0.388	21	0.321	27	27	27	17
64	5.34	7.08	6.85	5.08	6.90	3.40	0.01	9.91	0.000	146	0.113	114	114	113	96
65	4.36	5.30	6.21	6.67	5.24	3.51	3.26	10.04	0.109	98	0.162	73	73	71	57
66	4.76	5.11	6.98	5.61	5.21	6.69	5.40	4.81	0.138	85	0.141	91	91	90	79
67	6.90	6.19	5.04	2.93	4.48	3.81	7.53	7.68	0.210	58	0.174	68	68	67	57
68	2.27	3.55	8.61	5.49	7.87	4.41	5.00	7.38	0.080	110	0.087	131	131	131	123
69	7.44	9.77	4.09	3.90	4.41	4.76	3.52	6.68	0.144	83	0.198	57	57	58	48
70	7.50	6.45	2.71	4.76	4.30	6.71	6.90	5.24	0.338	25	0.307	30	30	29	24
71	4.57	3.69	8.19	7.58	6.01	4.04	4.97	5.52	0.108	99	0.118	111	111	109	97
72	5.08	3.75	2.85	4.57	9.86	4.77	8.47	5.23	0.589	4	0.548	5	5	5	3
73	9.54	5.62	3.13	3.81	4.59	3.66	6.99	7.23	0.315	33	0.283	36	36	36	25
74	5.96	5.78	3.83	3.32	6.31	7.72	5.07	6.58	0.183	71	0.194	60	60	60	51
75	8.55	5.91	2.57	6.17	4.12	5.02	6.79	5.44	0.523	8	0.502	11	11	10	4
76	5.32	3.82	6.00	4.66	6.31	3.98	5.37	9.10	0.117	93	0.120	105	105	105	96
77	8.63	8.08	2.03	3.18	6.99	4.21	3.95	7.50	0.351	23	0.401	16	16	15	11
78	9.73	7.93	3.22	6.99	7.37	3.09	1.37	4.88	0.145	82	0.335	25	25	24	16

Table 10 (continued)

Unit #	Current shifted z-scores z1	z2	z3	z4	z5	z6	z7	z8	T^a z1	z2	z3	z4	z5	z6	z7	z8
79	0.77	0.91	0.73	0.52	1.56	1.41	1.13	0.97	5.35	5.48	5.30	5.10	6.13	5.98	5.70	5.54
80	0.45	0.49	0.72	0.39	0.87	0.57	0.51	0.96	5.40	5.44	5.67	5.34	5.83	5.52	5.46	5.91
81	1.25	1.59	2.29	2.11	1.88	1.69	1.90	1.45	5.05	5.39	6.09	5.91	5.68	5.49	5.70	5.25
82	1.24	1.35	1.13	1.58	0.78	1.04	1.21	0.96	5.65	5.76	5.54	5.99	5.19	5.45	5.62	5.37
83	1.29	1.74	0.78	0.87	1.27	2.52	1.47	2.29	5.33	5.79	4.83	4.91	5.31	6.56	5.51	6.33
84	1.04	1.39	0.74	0.57	0.87	0.94	0.21	1.22	5.74	6.09	5.44	5.27	5.57	5.64	4.91	5.92
85	0.62	0.50	0.74	0.60	0.69	0.85	1.25	0.82	5.44	5.31	5.55	5.42	5.50	5.66	6.07	5.63
86	0.53	0.60	1.12	0.69	0.87	0.76	0.77	0.85	5.33	5.40	5.91	5.49	5.67	5.56	5.57	5.65
87	2.25	1.46	0.97	2.06	1.04	1.59	3.49	1.85	5.99	5.19	4.71	5.79	4.77	5.33	7.22	5.58
88	1.72	0.82	0.90	0.84	0.83	0.66	2.25	0.99	6.17	5.26	5.34	5.28	5.27	5.11	6.70	5.44
89	1.02	1.09	0.76	0.44	0.78	0.38	0.61	0.68	5.87	5.94	5.61	5.29	5.63	5.24	5.46	5.53
90	0.38	0.42	0.83	0.58	0.87	0.48	1.03	0.66	5.29	5.34	5.75	5.49	5.79	5.39	5.95	5.58
91	1.48	1.33	0.74	1.01	0.80	1.22	1.00	1.20	5.96	5.80	5.21	5.48	5.28	5.69	5.47	5.67
92	2.45	1.94	0.99	1.56	1.86	2.52	3.46	1.19	6.02	5.52	4.56	5.13	5.44	6.10	7.04	4.76
93	1.62	1.59	1.01	1.11	1.09	1.69	1.83	1.48	5.76	5.74	5.16	5.26	5.23	5.83	5.97	5.62
94	0.63	0.62	0.98	0.83	0.80	1.31	1.15	0.87	5.31	5.29	5.65	5.50	5.47	5.98	5.83	5.54
95	1.35	0.99	0.88	0.87	0.73	1.04	1.22	1.16	5.89	5.53	5.42	5.41	5.27	5.58	5.76	5.70
96	0.51	0.45	0.92	0.64	0.69	0.85	0.49	0.77	5.41	5.36	5.83	5.54	5.59	5.76	5.40	5.67
97	0.45	0.44	0.75	0.36	0.69	0.57	0.20	0.61	5.52	5.51	5.81	5.42	5.75	5.63	5.26	5.67
98	1.52	1.10	0.75	0.68	0.92	0.38	0.44	0.98	6.24	5.82	5.48	5.40	5.65	5.11	5.17	5.70

Unit #	c_{ij} c11	c21	c31	c41	c51	c61	c71	c81	y_a z1	z2	z3	z4	z5	z6	z7	z8
79	0.020	0.027	0.025	0.030	0.028	0.022	0.738	0.020	1.03	1.06	1.03	0.99	1.19	1.16	1.10	1.07
80	0.027	0.026	0.023	0.021	0.026	0.030	1.738	0.023	0.52	0.52	0.54	0.51	0.56	0.53	0.52	0.56
81	0.021	0.029	0.026	0.028	0.028	0.023	0.360	0.023	1.66	1.77	2.00	1.94	1.87	1.80	1.87	1.73
82	0.029	0.029	0.025	0.021	0.022	0.026	0.656	0.023	1.21	1.23	1.19	1.28	1.11	1.17	1.20	1.15
83	0.025	0.030	0.022	0.022	0.022	0.029	0.488	0.027	1.45	1.57	1.31	1.33	1.44	1.78	1.49	1.72
84	0.027	0.023	0.029	0.027	0.027	0.027	3.929	0.024	0.28	0.30	0.27	0.26	0.27	0.28	0.24	0.29
85	0.020	0.023	0.029	0.029	0.020	0.024	0.706	0.022	1.05	1.02	1.07	1.04	1.06	1.09	1.17	1.08
86	0.026	0.030	0.026	0.027	0.025	0.023	1.122	0.026	0.73	0.74	0.81	0.76	0.78	0.77	0.77	0.78
87	0.025	0.024	0.023	0.022	0.021	0.027	0.207	0.029	2.48	2.15	1.95	2.40	1.98	2.21	3.00	2.32
88	0.030	0.026	0.024	0.022	0.027	0.024	0.366	0.027	1.80	1.54	1.56	1.54	1.54	1.49	1.96	1.59
89	0.020	0.021	0.026	0.020	0.022	0.024	1.447	0.029	0.67	0.67	0.64	0.60	0.64	0.59	0.62	0.63
90	0.026	0.023	0.022	0.027	0.024	0.027	0.870	0.020	0.87	0.87	0.94	0.90	0.95	0.88	0.97	0.91
91	0.024	0.021	0.026	0.023	0.027	0.023	0.810	0.028	1.10	1.07	0.97	1.01	0.98	1.05	1.01	1.05
92	0.026	0.027	0.026	0.030	0.026	0.020	0.196	0.027	2.56	2.34	1.94	2.18	2.31	2.59	2.99	2.02
93	0.027	0.022	0.028	0.030	0.025	0.021	0.418	0.022	1.67	1.66	1.49	1.52	1.51	1.69	1.73	1.63
94	0.021	0.022	0.021	0.021	0.023	0.030	0.740	0.026	1.02	1.01	1.08	1.05	1.05	1.15	1.12	1.06
95	0.026	0.027	0.024	0.025	0.027	0.025	0.669	0.026	1.21	1.14	1.12	1.11	1.09	1.15	1.19	1.17
96	0.020	0.021	0.025	0.029	0.028	0.027	1.779	0.023	0.51	0.51	0.55	0.52	0.53	0.54	0.51	0.54
97	0.029	0.030	0.028	0.026	0.023	0.029	4.523	0.023	0.22	0.22	0.23	0.22	0.23	0.23	0.21	0.23
98	0.026	0.029	0.022	0.021	0.027	0.028	1.888	0.028	0.58	0.54	0.51	0.50	0.52	0.47	0.48	0.53

Unit #	V^a z1	z2	z3	z4	z5	z6	z7	z8	Scores and Ranks: TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
79	4.31	5.06	4.06	2.92	8.68	7.86	6.29	5.39	0.194	65	0.179	65	65	66	52
80	4.07	4.39	6.47	3.50	7.85	5.12	4.59	8.57	0.095	103	0.111	116	116	115	102
81	3.92	5.02	7.21	6.63	5.93	5.31	5.99	4.57	0.329	30	0.318	28	28	27	19
82	5.95	6.50	5.41	7.58	3.75	4.97	5.82	4.60	0.214	56	0.208	54	54	54	46
83	4.69	6.35	2.86	3.16	4.64	9.19	5.35	8.34	0.271	40	0.275	37	37	37	24
84	6.64	8.88	4.73	3.64	5.57	6.00	1.34	7.76	0.049	126	0.157	79	79	77	60
85	4.58	3.65	5.43	4.42	5.04	6.24	9.20	6.01	0.192	66	0.136	95	95	95	81
86	3.86	4.32	8.04	4.96	6.31	5.46	5.53	6.11	0.138	84	0.139	93	93	93	79
87	6.83	4.41	2.95	6.24	3.15	4.83	10.58	5.59	0.415	19	0.330	26	26	25	17
88	8.52	4.05	4.43	4.14	4.09	3.28	11.13	4.92	0.292	36	0.202	56	56	57	44
89	7.89	8.41	5.87	3.39	6.04	2.97	4.72	5.29	0.114	96	0.129	98	98	97	83
90	3.19	3.60	7.05	4.91	7.41	4.05	8.75	5.61	0.164	75	0.118	112	112	110	94
91	7.53	6.74	3.75	5.11	4.08	6.20	5.07	6.09	0.185	70	0.197	58	58	58	51
92	6.83	5.42	2.75	4.35	5.19	7.04	9.66	3.32	0.425	17	0.358	20	20	20	15
93	6.33	6.21	3.95	4.35	4.24	6.59	7.14	5.77	0.289	37	0.256	42	42	42	35
94	3.93	3.82	6.04	5.15	4.98	8.13	7.14	5.39	0.192	68	0.162	75	75	72	66
95	7.31	5.34	4.77	4.69	3.97	5.60	6.62	6.27	0.206	60	0.185	62	62	63	57
96	4.24	3.80	7.74	5.33	5.76	7.12	4.14	6.44	0.095	105	0.119	107	107	105	96
97	4.97	4.85	8.17	3.96	7.53	6.26	2.17	6.66	0.040	130	0.091	127	127	126	121
98	10.00	7.23	4.96	4.45	6.06	2.53	2.91	6.44	0.093	106	0.152	85	85	83	62

Table 10 (continued)

Unit #	Current shifted z-scores								T^a								c_{ij}							
	z1	z2	z3	z4	z5	z6	z7	z8	z1	z2	z3	z4	z5	z6	z7	z8	c11	c21	c31	c41	c51	c61	c71	c81
99	1.69	1.51	0.83	0.98	1.11	1.87	2.47	1.09	5.82	5.63	4.96	5.11	5.24	6.00	6.59	5.22	0.025	0.022	0.020	0.027	0.028	0.023	0.318	0.020
100	0.66	0.67	0.72	0.48	0.87	1.04	1.64	0.85	5.36	5.37	5.43	5.18	5.58	5.74	6.34	5.56	0.024	0.027	0.027	0.029	0.020	0.021	0.533	0.025
101	1.05	0.99	0.76	0.44	0.69	0.76	0.35	0.91	5.88	5.81	5.59	5.26	5.51	5.58	5.18	5.74	0.027	0.025	0.026	0.030	0.023	0.025	2.419	0.023
102	1.00	1.21	0.90	1.12	0.71	0.76	0.64	0.87	5.67	5.88	5.57	5.79	5.38	5.43	5.31	5.54	0.029	0.028	0.020	0.020	0.021	0.028	1.300	0.026
103	3.73	3.70	2.28	1.88	2.31	4.20	3.51	2.15	6.33	6.30	4.89	4.48	4.91	6.80	6.11	4.75	0.029	0.021	0.026	0.022	0.028	0.026	0.138	0.026
104	2.88	2.54	1.43	2.61	1.56	1.78	3.22	2.12	6.18	5.85	4.73	5.91	4.86	5.08	6.52	5.43	0.026	0.023	0.022	0.021	0.027	0.020	0.202	0.026
105	1.13	0.83	0.87	1.16	0.78	1.31	1.43	0.88	5.65	5.35	5.40	5.69	5.30	5.84	5.95	5.40	0.023	0.024	0.026	0.024	0.029	0.027	0.576	0.024
106	1.02	0.62	0.97	0.90	0.71	0.66	1.21	0.70	5.74	5.34	5.69	5.62	5.43	5.39	5.93	5.42	0.026	0.027	0.028	0.023	0.026	0.023	0.707	0.030
107	0.43	0.43	0.76	0.62	0.69	0.29	0.13	0.67	5.50	5.50	5.83	5.69	5.76	5.36	5.20	5.74	0.026	0.025	0.029	0.022	0.025	0.028	6.992	0.025
108	0.42	0.51	0.75	0.40	0.71	0.66	0.39	0.50	5.45	5.54	5.77	5.43	5.74	5.69	5.42	5.53	0.027	0.025	0.021	0.026	0.021	0.025	2.325	0.024
109	1.02	1.19	0.77	0.61	0.73	0.76	1.07	0.64	5.74	5.91	5.49	5.33	5.46	5.48	5.79	5.36	0.025	0.026	0.024	0.028	0.024	0.022	0.804	0.026
110	1.91	2.17	1.21	2.07	1.51	2.34	2.23	2.07	5.54	5.80	4.85	5.71	5.14	5.97	5.86	5.70	0.029	0.028	0.028	0.021	0.027	0.027	0.293	0.022
111	0.34	0.49	1.95	1.53	0.71	1.41	0.64	0.82	4.93	5.08	6.53	6.12	5.29	5.99	5.23	5.40	0.029	0.029	0.028	0.028	0.025	0.029	1.249	0.021
112	0.62	0.53	0.88	0.93	0.71	0.29	0.89	0.63	5.50	5.42	5.77	5.82	5.60	5.18	5.77	5.51	0.021	0.021	0.023	0.021	0.028	0.030	1.006	0.020
113	0.42	0.46	0.77	0.49	0.69	0.29	0.07	0.66	5.51	5.55	5.86	5.58	5.78	5.38	5.16	5.75	0.024	0.022	0.021	0.022	0.029	0.026	#####	0.021
114	0.29	0.52	0.80	0.93	0.83	0.85	0.77	0.67	5.15	5.38	5.67	5.80	5.69	5.72	5.63	5.53	0.023	0.030	0.024	0.025	0.024	0.023	1.145	0.027
115	2.93	3.31	2.36	3.48	2.80	2.99	2.18	3.01	5.62	6.00	5.05	6.17	5.49	5.68	4.87	5.70	0.023	0.024	0.029	0.021	0.023	0.030	0.223	0.024
116	2.19	2.58	1.26	1.01	1.16	1.50	1.92	1.96	6.06	6.46	5.13	4.89	5.03	5.37	5.79	5.84	0.027	0.030	0.025	0.020	0.023	0.024	0.368	0.023
117	0.37	0.61	0.71	0.41	0.69	0.20	0.42	0.58	5.45	5.68	5.79	5.48	5.76	5.27	5.49	5.65	0.025	0.027	0.022	0.025	0.021	0.026	2.188	0.021
118	1.49	1.69	1.25	0.83	1.20	0.94	1.04	1.19	5.86	6.06	5.61	5.20	5.57	5.31	5.41	5.56	0.029	0.024	0.028	0.023	0.021	0.029	0.742	0.029

Unit #	y_a								V^a								Scores and Ranks						
	z1	z2	z3	z4	z5	z6	z7	z8	z1	z2	z3	z4	z5	z6	z7	z8	TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
99	1.94	1.88	1.65	1.70	1.75	2.00	2.20	1.74	6.53	5.81	3.20	3.79	4.28	7.22	9.51	4.21	0.334	27	0.259	41	41	41	31
100	1.24	1.24	1.25	1.20	1.29	1.33	1.47	1.29	4.24	4.29	4.65	3.07	5.63	6.66	10.53	5.50	0.231	48	0.155	80	80	79	66
101	0.43	0.43	0.41	0.39	0.41	0.41	0.38	0.42	7.89	7.39	5.73	3.26	5.15	5.67	2.66	6.82	0.074	113	0.133	96	96	96	79
102	0.72	0.75	0.71	0.74	0.68	0.69	0.68	0.70	6.18	7.48	5.54	6.93	4.39	4.67	3.98	5.39	0.127	89	0.162	74	74	72	63
103	3.47	3.46	2.68	2.46	2.69	3.73	3.35	2.61	6.99	6.94	4.29	3.53	4.33	7.88	6.59	4.03	0.549	7	0.533	7	7	6	4
104	2.80	2.65	2.14	2.67	2.20	2.30	2.95	2.45	7.08	6.25	3.51	6.40	3.82	4.37	7.91	5.22	0.452	15	0.407	15	15	15	12
105	1.28	1.22	1.23	1.29	1.21	1.33	1.35	1.23	5.98	4.41	4.64	6.18	4.14	6.97	7.60	4.65	0.227	50	0.189	61	61	62	54
106	1.10	1.03	1.09	1.08	1.04	1.03	1.14	1.04	6.71	4.05	6.37	5.92	4.66	4.35	7.91	4.60	0.192	67	0.153	83	83	83	70
107	0.15	0.15	0.16	0.15	0.15	0.14	0.14	0.15	4.80	4.78	8.40	6.86	7.64	3.24	1.43	7.42	0.027	138	0.090	128	128	128	121
108	0.40	0.41	0.43	0.40	0.42	0.42	0.40	0.41	4.34	5.21	7.66	4.11	7.29	6.81	4.00	5.15	0.074	114	0.097	122	121	120	117
109	1.02	1.05	0.98	0.95	0.97	0.98	1.03	0.95	6.68	7.82	5.07	4.01	4.82	4.97	7.02	4.18	0.178	72	0.152	84	84	83	67
110	2.03	2.13	1.78	2.09	1.88	2.19	2.15	2.09	5.50	6.24	3.49	5.96	4.33	6.71	6.41	5.94	0.366	22	0.348	22	22	21	17
111	0.65	0.67	0.86	0.81	0.70	0.79	0.69	0.71	1.94	2.79	10.98	8.65	4.01	7.95	3.64	4.62	0.132	88	0.177	67	67	66	47
112	0.82	0.81	0.86	0.87	0.83	0.77	0.86	0.82	5.00	4.34	7.18	7.60	5.77	2.37	7.22	5.10	0.149	81	0.123	104	104	101	90
113	0.08	0.08	0.09	0.08	0.09	0.08	0.08	0.09	4.81	5.33	8.92	5.70	7.95	3.37	0.82	7.66	0.015	143	0.086	132	132	132	123
114	0.69	0.72	0.76	0.78	0.77	0.77	0.76	0.74	2.27	4.07	6.31	7.35	6.54	6.71	6.05	5.27	0.135	87	0.127	99	99	99	87
115	2.72	2.90	2.44	2.98	2.65	2.75	2.35	2.75	5.66	6.40	4.56	6.72	5.41	5.78	4.21	5.81	0.483	10	0.517	9	9	9	4
116	1.96	2.09	1.66	1.58	1.63	1.74	1.88	1.89	7.19	8.47	4.12	3.32	3.80	4.92	6.30	6.44	0.324	31	0.305	32	32	30	21
117	0.42	0.44	0.45	0.42	0.44	0.41	0.42	0.44	4.16	6.83	7.98	4.54	7.68	2.22	4.68	6.46	0.077	111	0.089	130	130	128	121
118	1.16	1.20	1.12	1.03	1.11	1.05	1.07	1.10	6.88	7.82	5.77	3.85	5.56	4.35	4.82	5.52	0.199	62	0.217	50	50	48	43

Table 10 (continued)

Unit #	Current shifted z-scores								T^a								c_{ij}							
	z1	z2	z3	z4	z5	z6	z7	z8	z1	z2	z3	z4	z5	z6	z7	z8	c11	c21	c31	c41	c51	c61	c71	c81
119	0.73	0.69	0.83	0.90	0.69	1.04	1.44	0.85	5.41	5.36	5.51	5.58	5.36	5.71	6.11	5.53	0.029	0.021	0.027	0.027	0.025	0.027	0.594	0.022
120	1.11	0.75	0.81	0.67	0.73	0.57	0.93	0.69	5.90	5.54	5.60	5.45	5.52	5.36	5.72	5.48	0.027	0.021	0.024	0.030	0.025	0.020	0.934	0.027
121	2.14	2.39	1.74	6.15	1.48	2.24	2.85	1.95	5.09	5.34	4.69	9.11	4.44	5.20	5.80	4.90	0.026	0.025	0.029	0.027	0.022	0.029	0.186	0.021
122	0.74	0.66	0.91	0.96	0.69	0.57	1.57	0.82	5.45	5.37	5.61	5.66	5.39	5.28	6.28	5.52	0.021	0.028	0.020	0.027	0.022	0.025	0.554	0.026
123	0.17	0.32	0.70	0.31	0.69	0.20	0.39	0.55	5.32	5.47	5.86	5.46	5.84	5.35	5.55	5.71	0.029	0.025	0.029	0.025	0.028	0.024	2.361	0.027
124	1.86	1.94	1.35	1.82	1.91	1.97	3.68	1.05	5.49	5.56	4.98	5.44	5.53	5.59	7.30	4.67	0.024	0.030	0.030	0.021	0.020	0.023	0.193	0.028
125	0.16	0.30	0.77	0.30	0.69	0.38	0.35	0.58	5.29	5.43	5.90	5.43	5.82	5.51	5.48	5.71	0.020	0.023	0.025	0.021	0.025	0.025	2.636	0.029
126	0.83	1.01	0.73	0.61	0.78	0.57	0.45	0.65	5.70	5.88	5.60	5.47	5.65	5.44	5.32	5.52	0.025	0.024	0.021	0.023	0.023	0.027	1.951	0.030
127	0.76	0.86	0.80	0.55	0.69	0.94	0.17	0.80	5.64	5.73	5.67	5.43	5.56	5.82	5.04	5.67	0.024	0.026	0.023	0.029	0.026	0.027	5.061	0.030
128	0.69	0.80	0.83	0.67	0.71	0.29	0.20	1.12	5.60	5.71	5.74	5.57	5.62	5.20	5.11	6.03	0.028	0.024	0.021	0.028	0.029	0.027	4.261	0.025
129	1.42	1.39	1.04	1.44	1.25	1.22	0.25	1.72	5.77	5.74	5.40	5.79	5.60	5.58	4.60	6.08	0.024	0.022	0.029	0.029	0.023	0.026	3.064	0.020
130	0.07	0.24	0.75	0.43	0.69	0.11	0.16	0.50	5.27	5.44	5.95	5.63	5.89	5.31	5.36	5.71	0.023	0.028	0.030	0.027	0.021	0.027	5.883	0.026
131	0.14	0.30	1.02	0.42	0.69	0.20	0.07	0.54	5.29	5.45	6.17	5.56	5.84	5.35	5.22	5.69	0.022	0.030	0.022	0.020	0.020	0.021	12.07	0.026
132	1.27	1.35	0.85	1.54	1.09	1.87	1.24	1.19	5.54	5.62	5.12	5.81	5.36	6.15	5.51	5.46	0.020	0.029	0.026	0.029	0.024	0.022	0.622	0.028
133	0.52	0.66	0.78	0.53	0.73	0.66	0.29	0.74	5.48	5.62	5.74	5.49	5.69	5.62	5.24	5.70	0.020	0.026	0.025	0.030	0.028	0.021	3.100	0.024
134	0.97	1.31	1.52	2.82	1.13	1.97	1.83	1.47	4.92	5.25	5.46	6.77	5.08	5.91	5.78	5.41	0.028	0.023	0.025	0.030	0.024	0.023	0.390	0.023
135	3.66	4.04	1.24	2.59	2.52	4.01	2.59	2.97	6.28	6.66	3.85	5.21	5.14	6.63	5.21	5.59	0.029	0.026	0.023	0.023	0.030	0.028	0.169	0.025
136	3.75	2.89	1.18	3.35	2.28	4.94	3.74	1.73	6.34	5.48	3.77	5.94	4.87	7.53	6.33	4.32	0.025	0.026	0.028	0.025	0.025	0.021	0.136	0.025
137	0.32	0.41	0.75	0.43	0.71	0.48	0.29	0.61	5.39	5.48	5.82	5.50	5.78	5.55	5.36	5.69	0.026	0.028	0.029	0.024	0.027	0.026	3.088	0.029
138	1.20	1.51	1.68	2.25	1.02	1.50	1.07	1.58	5.29	5.61	5.78	6.35	5.11	5.59	5.17	5.67	0.020	0.022	0.026	0.026	0.024	0.020	0.697	0.023

Unit #	y_a								V^a								Scores and Ranks						
	z1	z2	z3	z4	z5	z6	z7	z8	z1	z2	z3	z4	z5	z6	z7	z8	TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
119	1.17	1.16	1.20	1.21	1.16	1.24	1.33	1.20	4.56	4.28	5.17	5.60	4.27	6.44	8.95	5.30	0.217	55	0.161	76	76	74	65
120	0.94	0.88	0.89	0.87	0.88	0.85	0.91	0.87	7.90	5.33	5.76	4.74	5.23	4.07	6.62	4.91	0.159	76	0.140	92	92	91	81
121	2.45	2.57	2.26	4.38	2.13	2.50	2.79	2.36	4.55	5.08	3.70	13.09	3.16	4.78	6.06	4.15	0.481	12	0.470	14	14	14	4
122	1.24	1.22	1.28	1.29	1.23	1.20	1.43	1.26	4.79	4.25	5.85	6.16	4.43	3.68	10.14	5.27	0.227	51	0.155	82	82	79	66
123	0.38	0.39	0.41	0.39	0.41	0.38	0.39	0.40	2.26	4.24	9.40	4.14	9.21	2.66	5.22	7.43	0.071	116	0.075	142	142	141	139
124	2.35	2.39	2.14	2.34	2.37	2.40	3.13	2.00	5.33	5.55	3.87	5.21	5.46	5.63	10.52	3.00	0.429	16	0.349	21	21	21	16
125	0.34	0.35	0.38	0.35	0.38	0.36	0.36	0.37	1.99	3.73	9.74	3.82	8.67	4.85	4.41	7.36	0.065	118	0.079	138	138	138	129
126	0.50	0.52	0.49	0.48	0.50	0.48	0.47	0.49	6.56	7.99	5.79	4.80	6.19	4.52	3.55	5.19	0.088	107	0.126	100	100	99	91
127	0.21	0.22	0.21	0.20	0.21	0.22	0.19	0.21	6.12	6.85	6.40	4.40	5.50	7.54	1.36	6.40	0.038	134	0.125	101	101	100	87
128	0.25	0.25	0.25	0.24	0.25	0.23	0.22	0.26	5.77	6.70	6.98	5.60	5.96	2.45	1.71	9.40	0.044	127	0.119	108	108	105	90
129	0.38	0.38	0.36	0.38	0.37	0.37	0.30	0.40	6.49	6.35	4.78	6.59	5.72	5.59	1.15	7.90	0.066	117	0.218	49	49	47	42
130	0.16	0.17	0.18	0.17	0.18	0.16	0.16	0.18	1.06	3.61	11.38	6.46	10.42	1.60	2.39	7.65	0.031	137	0.066	146	146	146	144
131	0.08	0.08	0.09	0.08	0.08	0.08	0.08	0.08	1.83	3.94	13.51	5.49	9.08	2.63	0.94	7.15	0.014	144	0.076	140	140	140	130
132	1.25	1.27	1.16	1.32	1.21	1.39	1.25	1.24	5.44	5.78	3.64	6.62	4.66	8.04	5.31	5.10	0.226	52	0.233	45	45	45	41
133	0.32	0.33	0.33	0.32	0.33	0.33	0.30	0.33	4.74	5.98	7.07	4.82	6.64	6.01	2.58	6.73	0.058	121	0.110	117	117	116	105
134	1.52	1.63	1.69	2.10	1.57	1.83	1.79	1.67	3.33	4.48	5.20	9.67	3.88	6.73	6.27	5.02	0.310	34	0.292	33	33	33	22
135	3.26	3.46	2.00	2.71	2.67	3.45	2.71	2.91	6.90	7.62	2.33	4.89	4.75	7.57	4.89	5.61	0.520	9	0.530	8	8	7	4
136	3.51	3.04	2.09	3.29	2.70	4.17	3.51	2.39	7.00	5.40	2.20	6.26	4.26	9.23	6.99	3.24	0.554	6	0.536	6	6	6	3
137	0.31	0.31	0.33	0.31	0.33	0.31	0.30	0.32	3.52	4.56	8.37	4.78	7.92	5.32	3.24	6.85	0.057	122	0.090	129	129	128	123
138	1.17	1.24	1.28	1.41	1.13	1.24	1.14	1.26	4.53	5.70	6.35	8.51	3.83	5.66	4.05	5.95	0.221	53	0.265	40	40	40	31

Table 10 (continued)

Unit #	Current shifted z-scores z1	z2	z3	z4	z5	z6	z7	z8
139	0.67	0.73	0.78	0.55	0.71	0.66	0.60	0.83
140	1.38	0.91	0.85	0.99	0.95	1.04	1.86	0.79
141	0.66	0.92	1.05	0.85	0.71	0.48	0.11	0.77
142	0.67	0.62	0.92	0.65	0.80	0.66	0.91	0.93
143	2.54	2.72	2.42	1.64	1.53	2.99	0.98	2.19
144	2.02	2.43	5.87	2.84	3.20	2.71	1.77	1.95
145	0.97	1.18	1.11	0.87	0.90	0.66	0.60	3.45
146	0.79	0.73	0.79	0.86	0.76	1.31	1.79	0.72

Unit #	T^a z1	z2	z3	z4	z5	z6	z7	z8
139	5.55	5.61	5.66	5.43	5.59	5.54	5.48	5.71
140	5.86	5.39	5.32	5.46	5.42	5.51	6.34	5.27
141	5.54	5.79	5.93	5.72	5.59	5.36	4.99	5.65
142	5.47	5.42	5.72	5.46	5.61	5.46	5.71	5.73
143	5.99	6.17	5.87	5.08	4.98	6.43	4.42	5.63
144	4.75	5.15	8.60	5.56	5.92	5.43	4.49	4.68
145	5.32	5.53	5.47	5.23	5.25	5.02	4.96	7.80
146	5.39	5.33	5.39	5.47	5.36	5.92	6.39	5.32

Unit #	c_{ij} c11	c21	c31	c41	c51	c61	c71	c81
139	0.024	0.021	0.029	0.021	0.024	0.026	1.464	0.029
140	0.020	0.026	0.022	0.030	0.027	0.025	0.444	0.026
141	0.028	0.025	0.023	0.025	0.021	0.025	7.780	0.027
142	0.022	0.028	0.030	0.024	0.027	0.025	0.954	0.025
143	0.024	0.021	0.029	0.028	0.024	0.028	0.605	0.026
144	0.026	0.024	0.029	0.028	0.024	0.029	0.251	0.021
145	0.029	0.029	0.024	0.021	0.027	0.024	1.285	0.022
146	0.025	0.020	0.028	0.027	0.024	0.029	0.474	0.023

Unit #	y_a z1	z2	z3	z4	z5	z6	z7	z8
139	0.62	0.62	0.63	0.60	0.62	0.62	0.61	0.64
140	1.55	1.42	1.41	1.44	1.43	1.46	1.68	1.39
141	0.14	0.15	0.15	0.14	0.14	0.13	0.13	0.14
142	0.85	0.84	0.89	0.85	0.87	0.85	0.89	0.89
143	1.62	1.67	1.58	1.37	1.34	1.74	1.19	1.52
144	2.18	2.37	3.95	2.56	2.72	2.50	2.07	2.15
145	0.72	0.75	0.74	0.71	0.71	0.68	0.67	1.06
146	1.35	1.33	1.35	1.37	1.34	1.48	1.60	1.33

Unit #	V^a z1	z2	z3	z4	z5	z6	z7	z8
139	5.37	5.91	6.26	4.45	5.72	5.34	4.81	6.71
140	7.03	4.63	4.32	5.01	4.81	5.27	9.47	4.04
141	5.32	7.36	8.43	6.80	5.71	3.84	0.89	6.21
142	4.83	4.47	6.66	4.73	5.82	4.80	6.55	6.70
143	6.66	7.14	6.34	4.29	4.01	7.83	2.56	5.73
144	3.96	4.74	11.49	5.55	6.25	5.30	3.46	3.82
145	4.42	5.38	5.09	4.00	4.11	3.04	2.76	15.77
146	4.53	4.20	4.54	4.97	4.36	7.57	10.29	4.12

Unit #	Scores and Ranks: TDSVE Score	Rank (TDSVEA)	VE Score	Rank (VEA)	Fixed equal weights	Standing in MIP (1st W)	Standing in MIP (2nd W)
139	0.111	97	0.124	103	103	101	96
140	0.264	43	0.197	59	59	58	49
141	0.025	139	0.124	102	102	101	87
142	0.155	77	0.138	94	94	93	83
143	0.270	41	0.382	17	17	17	14
144	0.460	14	0.511	10	10	9	3
145	0.136	86	0.219	48	48	47	36
146	0.250	46	0.174	49	48	48	47

Table 11 – Pearson rank correlation coefficients for different pairs of ranking techniques

PEARSON (r)- Rank correlations between rankings	TDSVEA	VEA	Fixed Equal Weights	MIP (1^{st} W)	MIP (2^{nd} W)
TDSVEA	1	0,87293	0,87238	0,87129	0,85305
VEA	0,87293	1	0,99995	0,99967	0,99273
Fixed Equal Weights	0,87238	0,99995	1	0,9997	0,99249
MIP (1^{st} W)	0,87129	0,99967	0,9997	1	0,99339
MIP (2^{nd} W)	0,85305	0,99273	0,99249	0,99339	1

CHAPTER 6

CONCLUSION AND FURTHER RESEARCH AREAS

In this study, ranking of units subject to multiple criteria is addressed. Two ranking methods, TDSVEA and MIP technique, are proposed. TDSVEA takes information from DM to obtain a target direction, to attain a target for the unit, and to make output substitutions. The method ranks units considering after making (possible) output substitutions how much the unit should expand its outputs without changing the levels of inputs to achieve a level which gives a satisfaction to the DM equivalent to that of the most preferred solution. While ranking units, TDSVEA provides the analyzer with important information such as input allocation after substituting outputs and lack of harmony index between the DM and manager of the unit. The MIP technique uses mixed integer programs (MIPs) in order to rank units. In these programs, the evaluated unit selects the best weights from a feasible weight space in order to outperform maximum number of other units. MIPs can be simplified by domination and weight-domination relations. Furthermore, copying entries between relations further simplifies the simplification procedure.

These ranking techniques are applied to rank research universities of the United States of America. For TDSVEA, all the information that comes from the DM is simulated and a final ranking is obtained. This ranking shows the potential of each unit to satisfy the decision maker after making output substitution. For MIP technique, different feasible weight spaces are tried to obtain final rankings. It is shown that MIP technique allows categorization and accounts for the originalities of units. Simplification procedure of MIP and simplification of its simplification are applied to two different weight spaces. For these weights spaces, simplification produces significant computational savings.

For testing purposes, in the simplification of simplification procedure of MIP, no complex rules and/or heuristics are used in order to distinguish the power of the simplification procedure. However, some rules and procedures may increase the efficiency of the simplification procedure, e.g., ordering of units in the descending (or ascending) order of a criterion in which the scores have a high standard deviation. A more sophisticated rule may be, after making principal component analysis and obtaining first principal component, ordering units in the descending or ascending order of this principal component. Other rules may also be tried, especially if the problem size is so large that it could present computational difficulties.

In this study, a real DM does not exist, but the information that should come from him/her, such as feasible weight space in MIP and projection directions in TDSVEA, is simulated. Some assumptions such as pseudoconcave utility and locally linear technology assumptions are made for these techniques. Applications with real DMs and checking the validity of these two procedures with all their assumptions may be considered.

TDSVEA provides some important information. Moreover, some additional information can be obtained, especially including time frame to the analysis, such as evolution of units and the disharmony of the DM and the local manager of the DMU in consecutive evaluation periods.

APPENDIX A

PARTIAL ANALYSIS OF PREFERENCES OF DM

In this appendix, a partial analysis of preferences of DM is made, in order to examine the behavior of him/her and consequently understand what TDSVEA tries to do. This analysis is partial in that it will analyze only some cases on some assumptions. These cases and assumptions will neither include all possible preference structure of the DM nor represent all real life perfectly, therefore this analysis is only a partial one but not complete. Starting from some possible instances, they will only enlighten some aspects in TDSVEA and what it may achieve. Firstly from the situation with looser assumptions, three situations are analyzed.

The first situation is in which the DM knows the technological ability of DMU and considers direction for the projection of the DMU, which is exactly same situation in TDSVEA. For illustration purposes it is assumed that technological ability of the units (c_{ij} values) can be represented by only one parameter (t). When selecting the projection direction, main assumption is (global) decision maker ((G)DM) selects the direction according to his/her utility function's gradient, which can change with the current output levels and current technology levels of the DMUs. Therefore, there exist some additional dimension(s) when selecting the projection dimension. This is shown in Figure 14. Two dimensions represent conventional output dimensions of output-oriented Data Envelopment Analysis with single input and two outputs. The other dimension (z-dimension) shows the technological ability of the DMU (this is represented by t). The meaning of including technology parameter in the analysis is that the DM considers the technology of the unit and selects a direction, which is suitable to its ability.

In Figure 14, considering the technological ability (utility function is defined in three-dimensional space), the DM selects a non-radial projection (which is the gradient of utility function contour on the technology plane $t=t^E$) rather than a radial projection, because the unit's ability is more appropriate to this direction (or the DM believes this).

In fact, since the utility function of the global decision maker is difficult to establish explicitly, direction of the projection is determined by directly asking the GDM or attaching a weight, γ^0_j , to j^{th} output of DMU_0, under consideration. The line,

$$\frac{y_j - y_j^0}{\gamma_j^0} = \frac{y_{j+1} - y_{j+1}^0}{\gamma_{j+1}^0} \quad \text{for } j = 1,\ldots,k\text{-}1 \text{ where k is total number of outputs}$$

determines the target direction where y_j^0 defines current level of j^{th} output of DMU_0. These weights, γ^0_j, are assumed to be determined by the gradient of the utility function that passes through the current output levels and current technological ability of the DMU in Figure 14.

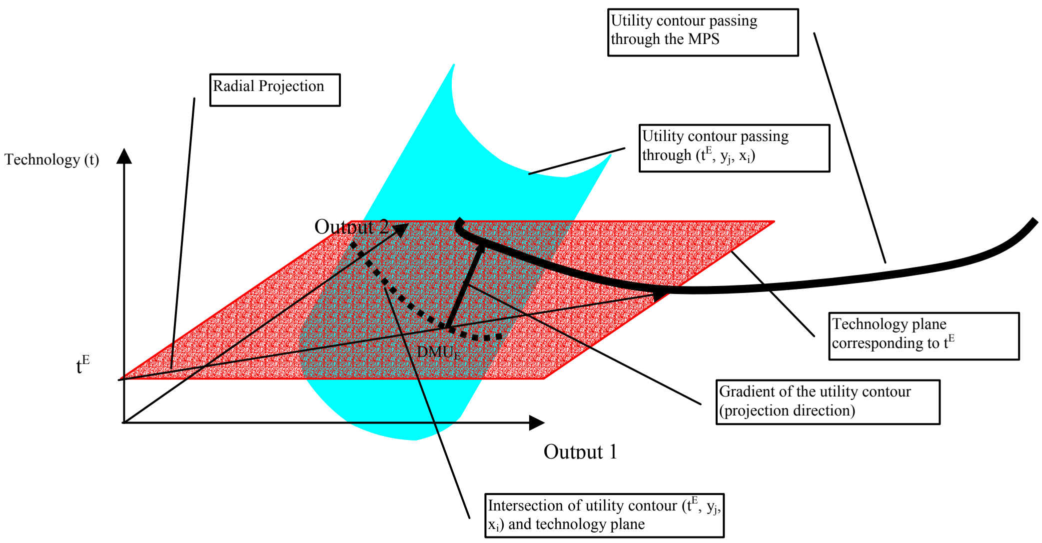

Figure 14 – Selection of projection direction considering the technological ability of the DMU

In this situation the DM selects projection direction considering the technological ability and preferences about outputs, which is more appropriate and therefore more meaningful for DMU. Assumption that DM selects a projection direction considering the DMUs ability is not an inappropriate one in the real life since if the

DM knows the abilities of DMU for projection s/he will select a projection appropriate to it.

In the second situation, the DM selects a projection direction, which makes the DMU's TDSVEA score maximum. In Figure 15, there exist two outputs and one input. The MPS selection is previously done. The DM selects a projection direction for the unit, which makes its TDSVEA score maximum, i.e. considering possible substitution equivalents of the DMUE on line [FS, FF], s/he selects a direction (i.e. [DMU_E, T^{aE}), which makes target of unit T^{aE} and substitution equivalent of unit y_a^E such that [Oy_a^E]/[OT_a^E] is maximum. In Figure 15, s/he obtains a super-efficient unit. Assumption that the DM selects a projection direction considering the DMUs potential is not an inappropriate one in the real life since if the DM knows the potential of the DMUs, s/he will select a projection direction such that after making output substitutions the DMU have input-output mix, which is the most satisfactory (in some situations as in Figure 15, even more satisfactory than the MPS).

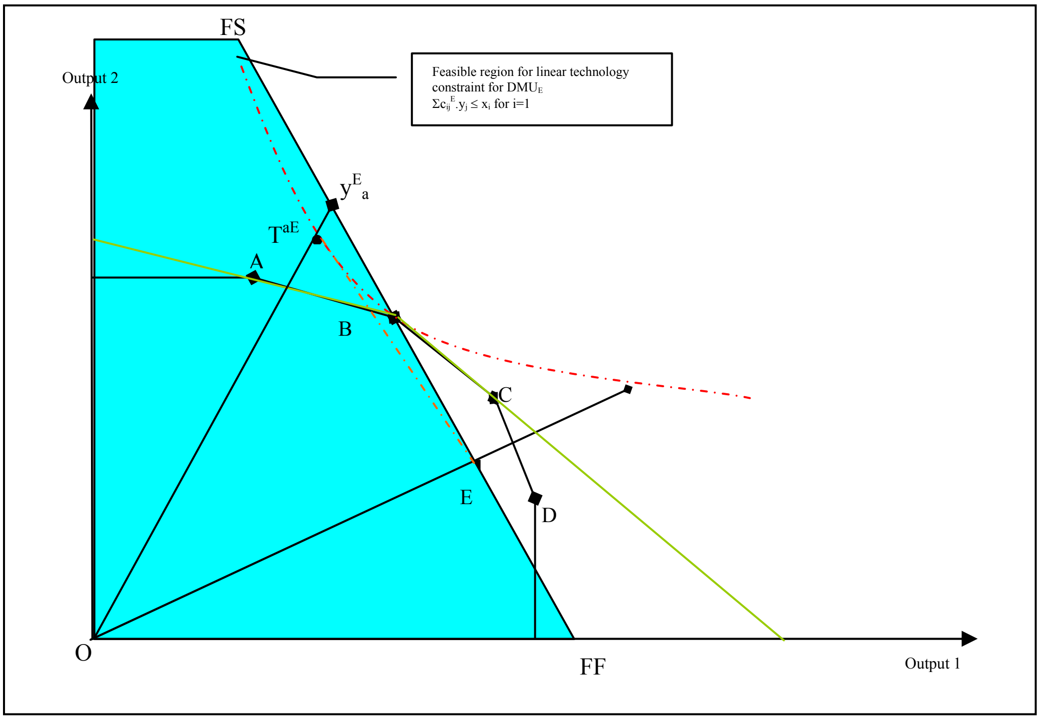

Figure 15– Selection of projection direction such that TDSVE score of the unit is maximum.

In third situation, the DMU selects a direction such that it is perpendicular to utility function contour that passes through the MPS. In this situation, projecting in any direction is assumed to cost the same for the DMU[17]. Therefore, the projection direction should be such that it gives the shortest distance from the current input-output mix to the contour of the utility curve that passes through the MPS in Euclidian space. Then, projecting to an input-output mix, which gives the DM the same satisfactory level with the MPS, will be made with the least possible cost or effort. This situation is shown in Figure 16.

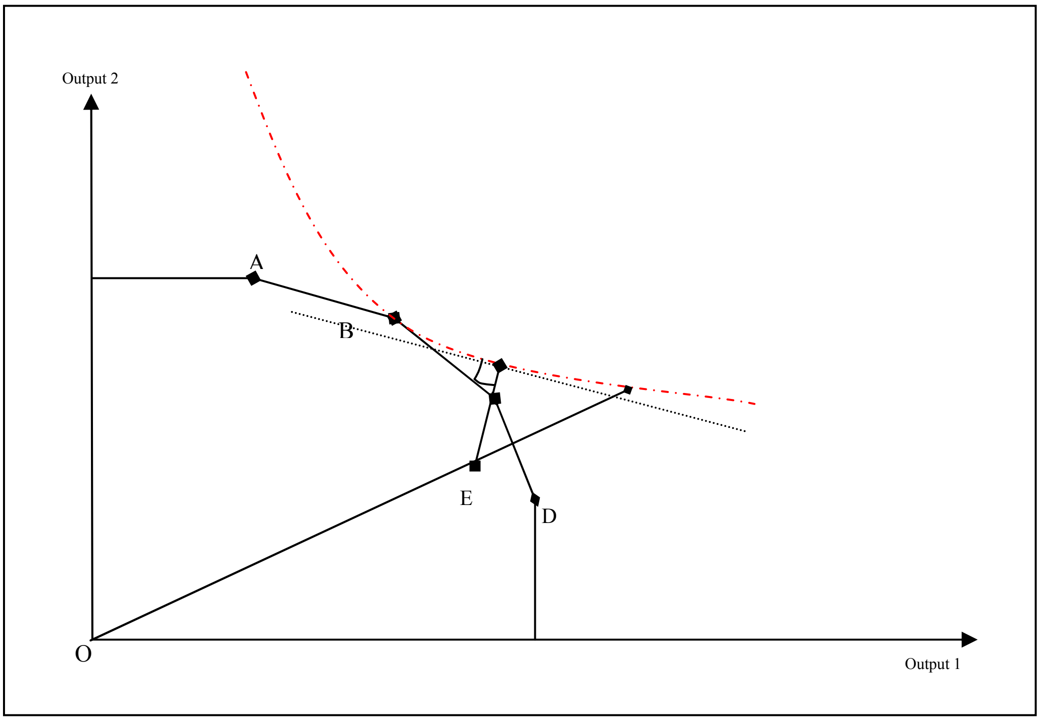

Figure 16 – Selection of projection direction perpendicular to the contour that pass through MPS

[17] Actually, projecting 1 unit in any direction will cost different for different projection abilities of DMUs. Here, it is assumed that unit has such a projection ability, projection 1 unit in any direction cost same for the DMU.

APPENDIX B

SENSITIVITY OF DATA USED IN THE APPLICATIONS TO WEIGHTS OF THE CRITERIA

In this appendix, the data is tested to see whether it shows sensitivity to change in weights of criteria. Some of the universities that are ranked by *TheCenter* (54 of 146 universities) are ranked by fixed weight methods with different weight sets of 8 criteria. These weight sets are produced from the base weights 0.10 with 10% (0.11), 25% (0.125), and 50% (0.15) increase. Different combinations of each of these increases are made and 3*126 different weight sets are obtained[18]. For each increase percentage, Pearson rank correlation coefficients are found for each pair (total number of pairs is 126*125 = 15750 for each increase percentage). Among these pairs the one(s) with minimum Pearson rank correlations for each increase percentage is (are) tabulated in Table 12.

Table 12 – Summary table for sensitivity analysis

	10% increase			**25% increase**					**50% increase**		
	Weight set no, Rankings and Differences			**Weight set no, Rankings and Differences**					**Weight set no, Rankings and Differences**		
University	**30**	**109**	**Dif**	**28**	**16**	**18**	**Dif1**	**Dif2**	**16**	**28**	**Dif**
Harvard University	1	1	0	1	1	1	0	0	1	1	0

[18] Of these 126 weights sets, first C(8,4) = 70 are generated so that weight of 4 criteria are increased (e.g. weight set 0.11,0.11,0.11,0.11,0.1,0.1,0.1,0.1). In 70-98 (C(8,2) = 28), 2 out of 8 weights are increased (e.g. weight set 0.11,0.11,0.1,0.1,0.1,0.1,0.1,0.1), and in 98-126 (C(8,6) = 28), 6 out of 8 weights (e.g. weight set 0.11,0.11,0.11,0.11,0.11,0.11,0.1,0.1) are increased by the specified amount. Actually, since normalization is not applied, increase of weights of k criteria with specified amount (e.g. 10%) corresponds to decrease of weights of 8-k criteria by an amount $\left(1-\frac{1}{1+increase}\right)$ (e.g. 1- (1/1.1) ≅ 0.09). Therefore, these weight sets correspond to both increase and decrease of the weights of criteria.

Table 12(continued)

University	10% increase			25% increase					50% increase		
	Weight set no, Rankings and Differences			Weight set no, Rankings and Differences					Weight set no, Rankings and Differences		
	30	**109**	**Dif**	**28**	**16**	**18**	**Dif1**	**Dif2**	**16**	**28**	**Dif**
Massachusetts Institute of Technology	4	4	0	4	4	4	0	0	4	4	0
Stanford University	3	2	1	3	2	3	1	1	2	3	1
Columbia University	12	12	0	13	12	12	1	0	10	13	3
Cornell University	13	11	2	11	13	11	2	2	13	11	2
Johns Hopkins University	2	3	1	2	3	2	1	1	3	2	1
University of Pennsylvania	9	9	0	9	9	10	0	1	9	9	0
Duke University	18	17	1	18	19	18	1	1	17	18	1
University of California - Berkeley	8	6	2	5	7	7	2	0	7	5	2
Yale University	11	13	2	12	11	13	1	2	11	12	1
University of California - Los Angeles	10	10	0	10	10	9	0	1	12	10	2
University of Michigan - Ann Arbor	6	7	1	8	6	6	2	0	6	7	1
University of Minnesota - Twin Cities	15	15	0	16	15	15	1	0	15	16	1
University of Washington - Seattle	7	8	1	7	8	8	1	0	8	6	2
University of Wisconsin - Madison	5	5	0	6	5	5	1	0	5	8	3
Washington University in St. Louis	17	18	1	17	18	17	1	1	18	17	1
University of Southern California	14	14	0	14	14	14	0	0	14	14	0
University of North Carolina - Chapel Hill	23	24	1	22	24	23	2	1	24	22	2
Princeton University	19	20	1	20	17	24	3	7	16	24	8
University of Illinois - Urbana-Champaign	20	19	1	19	20	19	1	1	19	19	0
University of Chicago	28	28	0	30	28	30	2	2	27	30	3
University of California - San Diego	16	16	0	15	16	16	1	0	20	15	5
Pennsylvania State University - University Park	21	22	1	21	22	20	1	2	23	21	2
University of California - San Francisco	26	26	0	24	27	26	3	1	30	20	10
California Institute of Technology	35	37	2	34	39	36	5	3	40	33	7
Northwestern University	30	30	0	31	29	31	2	2	28	31	3
Ohio State University - Columbus	24	23	1	25	23	22	2	1	22	25	3
Emory University	32	32	0	32	31	33	1	2	32	34	2
Texas A&M University	27	27	0	27	26	27	1	1	26	27	1
University of Arizona	29	29	0	28	30	28	2	2	29	29	0
University of Texas - Austin	22	21	1	23	21	21	2	0	21	23	2
University of Virginia	33	33	0	35	32	34	3	2	31	36	5
University of Florida	25	25	0	26	25	25	1	0	25	26	1
Baylor College of Medicine	43	43	0	42	43	43	1	0	44	39	5
University of Pittsburgh - Pittsburgh	31	31	0	29	33	29	4	4	34	28	6
New York University	34	34	0	36	34	35	2	1	33	37	4
University of Colorado - Boulder	37	39	2	37	38	38	1	0	37	35	2
Michigan State University	38	36	2	38	35	37	3	2	35	38	3
University of California - Davis	36	35	1	33	37	32	4	5	38	32	6
Vanderbilt University	40	41	1	41	41	41	0	0	41	42	1
Brown University	51	51	0	51	51	52	0	1	51	51	0
Dartmouth College	48	50	2	49	49	49	0	0	49	49	0

Table 12(continued)

University	10% increase			25% increase					50% increase		
	Weight set no, Rankings and Differences			Weight set no, Rankings and Differences					Weight set no, Rankings and Differences		
	30	**109**	**Dif**	**28**	**16**	**18**	**Dif1**	**Dif2**	**16**	**28**	**Dif**
Rice University	54	54	0	54	54	54	0	0	53	54	1
Georgia Institute of Technology	42	42	0	43	42	42	1	0	42	43	1
Purdue University - West Lafayette	39	38	1	39	36	39	3	3	36	40	4
Rockefeller University	46	47	1	47	46	47	1	1	46	47	1
University of Maryland - College Park	41	40	1	40	40	40	0	0	39	41	2
University of Notre Dame	50	49	1	50	48	50	2	2	48	53	5
Boston University	44	44	0	44	44	44	0	0	43	45	2
Carnegie Mellon University	53	53	0	52	53	51	1	2	54	50	4
Georgetown University	52	52	0	53	52	53	1	1	52	52	0
University of California - Santa Barbara	49	48	1	48	50	48	2	2	50	48	2
University of Georgia	47	45	2	46	45	46	1	1	45	46	1
Yeshiva University	45	46	1	45	47	45	2	2	47	44	3
WEIGHT SET in %	**30**	**109**		**28**	**16**	**18**			**16**	**28**	
2001Total Research	10	10		12.5	10	12.5			10	15	
2001Federal Research	11	10		12.5	10	12.5			10	15	
2002Endowment Assets	11	10		10	12.5	10			15	10	
2002 Annual Giving	11	11		10	12.5	10			15	10	
2002National Academy Members	10	10		12.5	10	10			10	15	
2002Faculty Awards	11	10		10	12.5	10			15	10	
2002Doctorates Granted	10	11		10	12.5	12.5			15	10	
2001Postdoctoral Appointees	10	10		12.5	10	12.5			10	15	
Pearson rank correlation coefficient	0.99802			0.99329					0.97873		

In Table 13, for example for 10% increase case, weight set 30 (0.1, 0.11, 0.11, 0.11, 0.1, 0.11, 0.1, 0,1) and weight set 109 (0.1, 0.1, 0.1, 0.11, 0.1, 0.1, 0.11, 0.1) give the minimum Pearson rank correlation coefficient (0.99802) among all 126*125 pairs. In this table, column 'Dif' shows the absolute value of differences of rankings of each unit by these two weight sets. As can be seen from this table, the rankings of the universities change. Therefore, it can be said that data is somewhat sensitive to weights of criteria.

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