ANALYSIS OF SERIAL INVENTORY SYSTEMS UNDER NONSTATIONARY DEMAND

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

FEVZI BASKAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

INDUSTRIAL ENGINEERING

DECEMBER 2004

Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Çağlar Güven
Head of the Program

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Prof. Dr. Nesim Erkip
Co – Supervisor

Prof. Dr. Refik Güllü
Supervisor

Examining Committee Members

Prof. Dr. Ömer Kırca	(METU, IE)	
Prof. Dr. Refik Güllü	(METU, IE)	
Prof. Dr. Nesim Erkip	(METU, IE)	
Prof. Dr. Nur Evin Özdemirel	(METU, IE)	
Assist. Prof. Dr. Osman Alp	(BİLKENT, IE)	

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name:

Signature:

ABSTRACT

ANALYSIS OF SERIAL INVENTORY SYSTEMS UNDER NONSTATIONARY DEMAND

Baskan, Fevzi
M.S., Department of Industrial Engineering
Supervisor: Prof. Dr. Refik Güllü
Co-Supervisor: Prof. Dr. Nesim Erkip

December 2004, 123 pages

In this study we consider a two-echelon supply chain with a nonstationary demand process. The retailer batches the customer demand for a predetermined number of periods before placing an order to the supplier. We show that the demand process for the supplier is more variable than that for the retailer. It is observed that the supplier can reduce the variability of orders by tracking the exogenous demand occurring at the retailer's side.

Keywords: Supply Chain Management, Order Batching, Value of Information

ÖZ

DEĞİŞKEN TALEPLERLE ÇALIŞAN SERİ ENVANTER SİSTEMLERİNİN İNCELENMESİ

Baskan, Fevzi

Yüksek Lisans, Endüstri Mühendisliği Bölümü

Tez Yöneticisi: Prof. Dr. Refik Güllü

Ortak Tez Yöneticisi: Prof. Dr. Nesim Erkip

Aralık 2004, 123 sayfa

Bu çalışmada stokastik ve değişken taleplerle çalışan iki seviyeli bir tedarik zinciri ele alınmıştır. Satıcı periyodik aralıklarda tedarikçiye sipariş verir. Buna göre tedarikçinin gördüğü talebin, satıcının gözlemlediği talebe göre daha değişken olduğu gösterilmiştir. Tedarikçinin satıcının gözlemlemiş olduğu talep verileri hakında bilgi sahibi olmasının tedarikçinin verdiği siparişin varyansını düşürdüğü de gözlenmiştir.

Anahtar Kelimeler: Tedarik Zinciri Yönetimi, Toplu Sipariş, Bilginin Değeri

To My Family (Specially to My Uncle Mustafa)

ACKNOWLEDGMENTS

First I would like to thank my supervisors Prof. Dr. Nesim Erkip and Prof. Dr. Refik Güllü for their continuous support and guidance throughout the study. I have learned a lot from their superior vision.

I owe thanks to my friends, Talip Kerem Koçkesen, Ulaş Özen, İsmail Serdar Bakal, for their existence and friendship.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Supply chain is a network of production facilities and distribution centers. The important functions performed in a supply chain involve the procurement of materials, transformation of materials into intermediate and finished products, and distribution of these products to customers. Managing supply chain is the integration of these functions. Coordination among the members of the chain is the key driving factor in the management of supply chain.

Analytical studies of supply chains need to reflect essential characteristics of supply networks observed in practice. One of the most important characteristics of a supply network is the underlying demand process. In most real environments demand is uncertain, possibly nonstationary, and hence hard to forecast. Among several other methods, time series analysis is used to model and forecast demand processes. On the research side, the impact of non-stationarity and variance parameters of the time series model on the performance of the chain is an important issue. On the other hand, the practitioners try to come up with a good representation of their data by fitting a time series model and by making predictions of future demand using the model.

In this study, we analyze a two-echelon supply chain with a retailer and a supplier. The demand observed by the retailer is nonstationary and the retailer uses an order-up-to level policy to determine its order amount from the supplier.

The supplier, in turn, places order from an outside source. There are lead-times associated with the retailer and supplier orders. Of course, the supplier can fulfill the order of the retailer as long as it has sufficient on-hand inventory.

In supply chains, each member orders with an upstream stage, but they do not necessarily place an order every time they experience a demand. A retailer, for instance, may batch or accumulate demands. There are many common reasons for an inventory system to work with order-batching: The supplier may not be able to handle frequent order processing because the time and cost of processing an order can be substantial. Some manufacturers place orders when they run their material requirements planning systems. Savings that can be obtained by ordering in truck-loads, or by consolidating orders for several items would force a manufacturer or retailer to order less frequently.

In this study our main aim is to model and understand the effect of order-batching on the system performance and optimal safety stock levels. To achieve this objective, we let the retailer in the system to follow a particular batching policy. Specifically, it batches the customer demand for a fixed, predetermined number of periods, and places an order with the supplier for the batched amount. The supplier, on the other hand, follows a standard order-up-to level policy by placing an order at the outside source every time an order is placed by the retailer.

An important observation in supply chain management, known as the bullwhip effect, suggests that the demand variability increases as one moves up a supply chain. Order-batching is cited as one of the sources of bullwhip effect. Please note that the supplier observes lumped orders at regular intervals, and does not observe potentially important periodic customer demand information. Another objective of this study is to understand the degree of the bullwhip effect under the prescribed order-batching policy.

In an attempt to streamline their supply chains, companies have engaged in information sharing practices. Information sharing may enable companies to overcome the bullwhip effect. In order to observe the possible benefits of information sharing, we designed a study that compares two retailer-supplier chains. In one of the chains, the retailer not only conveys the batched order to the supplier, but also passed the detailed demand information. In the second chain, the supplier only receives the order information. In both of the chains, the retailer and the supplier are assumed to know the relevant parameters of the demand process (the mean, the standard deviation and, the non-stationarity parameter).

This study uses two different tools; analytical derivations and simulation to observe the designed system. These methods work in parallel: analytically derived equations are used in simulation. In the simulation part of the study we aim to analyze how the system performs when the retailer and the supplier works in coordination.

1.1 General Description of the System

This study concentrates on a single item inventory system. There are two parties involved: the supplier, or the upstream stage and the retailer, or the downstream stage. The retailer observes a nonstationary demand process. For practical purposes, for this kind of demand processes, systems generally depend on forecasts based on time series of prior demand, e.g. moving averages. And these forecasts are used on the belief that the most recent demand observations are the best predictors of the future demand. Exponential-weighted moving average forecasting model is an example to this kind of forecasting. Our demand process behaves like a random walk , that is its evolution changes in directions and rates of growth or decline. And for the type of demand processes, to which the retailer in our work is exposed, the exponential-weighted moving average forecasting provides the minimum mean square forecast, Muth (1960), Box et al. (1994).

We build the system using the demand process and the forecasting model described above, with deterministic replenishment lead-times for the supplier and the retailer. We also assume that the retailer observes the customer demand for a fixed, predetermined number of periods and then places order at the supplier in batches. From the analysis of the model, we determine the safety stock levels that should be maintained by the supplier and the retailer. We also investigate the managerial implications of order-batching and information sharing in supplier-

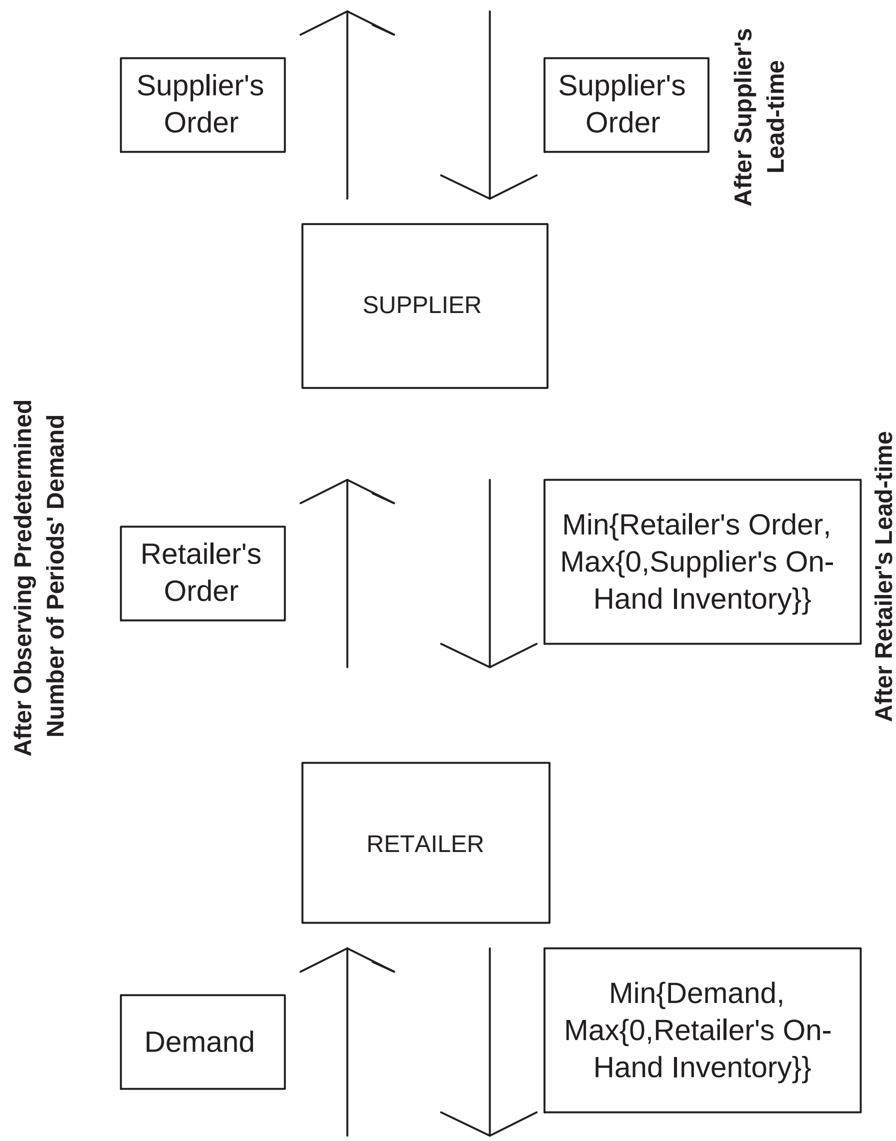

Figure 1.1: System Structure

retailer chains.

System structure can be seen in Figure 1.1. In this system, retailer gives order in every R periods of time and receives this amount after L periods (lead-time for the retailer to receive the order) if the supplier has enough amount to fulfill the order, otherwise the supplier sends the amount of positive inventory on-hand to the retailer and batches the remaining part of the order. In general; in a period the retailer first observes that period's demand, determines that period's order quantity, receives the order from L periods ago, and then fills the demand from inventory. Any demand that can not be met from the shelf is backordered. In this context we derive inventory and order equations for the retailer and observe the retiler's performance under different environmental conditions.

The supplier also has deterministic lead-time to receive its order, this value is denoted by K, an integer value, multiplied by R. In each R period time, supplier first observes the order given by the retailer, determines that period's order, receives the order from KR periods ago, and then fills the demand from inventory. Again for the upstream stage the demand that can not be met from inventory is backordered. By using an order-up-to level policy and above described structure we derive required equations for the supplier and observe its performance throughout the study.

We follow the following order in the remaining part of our study; in the next chapter we mention about the related literature under main groupings and state the relations of them with our work.

In chapter 3; we define the system analytically and study on the derivations of the system variables. We mention about the performance of the members of the system and give some insights about the initial results of the study.

Chapter 4 is devoted to the simulation model. After the model is described, steps taken for verification and performance measures are presented. Case where actual demand information becomes available for the supplier is compared with the no information case. Finally, results of the computational experience are discussed.

Conclusion comes with chapter 5. We state all the results reached throughout the work.

CHAPTER 2

RELATED LITERATURE

2.1 Literature Review

We study a single item inventory system of a two-echelon model. The end item demand is nonstationary, integrated moving average process of (0,1,1), for which exponential-weighted moving average forecasting provides the minimum mean square forecast. We introduce batch ordering into retailer's process, and try to see its implications. Both parts of the system employs order-up-to policies to determine their order amounts. Our main purpose is to see the effect of batching on the systems defined in many different context in the literature. What is its effect on the general truths of such systems. In the literature there are many studies considering inventory systems for echelon models and they reach some general results as: bullwhip effect and main causes of this, importance of information sharing under some scenarios and how this can improve the performance of considered systems. It is our concern to observe the realizability of these results under batch ordering for the system having nonstationary exogenous demand. In studying with these we used two method; while analytically deriving the formulas for the variables of the system we observed the performance of them with simulation tools. During our literature search we were not so lucky to find enough batching applications, although this increases our motivation to study on the subject it makes the study a bit more difficult. This was not the case about other topics, there are huge amounts of study on the literature related with the topics we mentioned previously in this paragraph. After some study we tried to group the related literature under three main headings: Group 1 consists of the materials discussing the information structure and bullwhip effect, materials in group 2 concentrates on general processes and descriptions of two-echelon systems, and the last group is about nonstationary demand processes, forecasting models that are suitable for these processes, analytical derivations of different equations and computational studies with simulation.

2.1.1 Bullwhip Effect and Importance of Information Sharing

Under supply chain modeling headings, many studies are on the topics of bullwhip effect, the increase in the variances of the demand observed by the members as we move up in a supply chain, and the importance of demand information sharing between the players of the system. These are really important and popular topics nowadays. Bullwhip effect is important because the companies that have no concern on this topic can face one or more of the following problems: They have excessive inventory piling up, their forecast for product is poor, they may have insufficient or excessive capacities, their customer service becomes poor due to unavailable products or long backlogs, they can not have exact production planning, and they have to pay high costs for corrections, such as for expedited shipments and overtime. This is a big problem and can be observed in most of the supply chain. Even when consumer sales do not seem to vary much, there is pronounced variability in the retailers' order to the suppliers. Our next topic in this grouping, information sharing, has been viewed as a major strategy to encounter this problem. In most context, by letting the supplier have information of demand observed by the retailer, the harmful effect of demand distortion can be ameliorated. Indeed, demand information sharing by a downstream operator to its supplier is the cornerstones of initiatives such as Quick Response (QR) and Efficient Consumer Response (ECR). Generally information sharing is embedded in programs like Vendor-Managed-Inventory (VMI) or Continuous Replenishment

Programs (CRP).

Lee et al. (1997a) study on definition, main causes and methods of counteraction of the bullwhip effect. They state that the ordering patterns share a common, recurring theme: the variabilities of an upstream site are always greater than those of downstream site, and discusses main causes for this statement as:

- Demand Forecast Updating: Sterman's experiment the "beer game" showed that players do not make decision rationally and human behavior, such as misconceptions about inventory and demand information, may cause the bullwhip effect. An important factor here is each player's thought process in projecting the demand pattern based on what he or she observes. When a downstream operation places an order, the upstream manager processes that pice of information as a signal for future product demand. By considering this signal, upstream stage readjusts its demand forecast and updates its order to its supplier. This contributes to bullwhip effect.

- Order Batching: Companies prefer to batch or accumulate their demands before issuing an order. They have many reasons to work in this manner, such as they may have a supplier that can not handle frequent order processing because the time and cost of processing an order can be substantial. Many companies place an order when they run their material requirement planning systems, and a company with slow moving items may prefer to order on a regular cyclical basis. When a company, having one of the reasons above to order in batches is considered, it is observed that its supplier faces erratic stream of orders. There may be a spike in demand at one time during any cycle followed by no demands for the rest of the cycle. This variability is of course higher than the variability of demand observed by the company itself. Therefore periodic ordering amplifies variability and contributes to bullwhip effect.

- Price Fluctuation: Estimates shows that almost 80 percent of the transactions between the manufacturers and retailers in the grocery industry were made in a "forward buy" arrangement in which items were bought in advance of requirements, generally because of manufacturers' attractive price offer, price discounts, quantity discounts, coupons, rebates, and so on. This promotions could be costly to the supply chain. When the price of the product is low customer buys more than needed amount. When the price turns to its normal value, customer stops buying until it has finished its inventory. As a result, the customer's buying pattern does not show its consumption pattern, and the variations of the buying quantities is much higher than those of the consumption rate.

- Rationing and Shortage Gaming: When the product demand exceeds the supply, manufacturers often rations its product to customers. One of the method in doing this is to allocate the amount in proportion to the amount ordered. Customers know these rationing behavior of the manufacturer and they exaggerate their real needs when they order. When demand cools orders will suddenly disappear. This gaming strategy gives little information about the real demand of the product to the manufacturer, especially in the early stage of the product.

After defining the main causes of the bullwhip effect they concentrate on the strategies to mitigate it. They examine how companies tackle each of the four causes and categorize the various initiatives and other possible remedies based on the underlying coordination mechanism, namely, information sharing, channel alignment, and operational efficiency.

- Avoid Multiple Demand Forecast Updates: Bullwhip effects are created when every members of the supply chain use their immediate downstream input for forecasting. One remedy to repetitive processing of consumption data in a supply chain is to make demand data available to the upstream stage. By this way they can update their forecast with the same raw data. One way for information sharing that can be used by the supply chain partners is Electronic Data Interchange (EDI).

- Break Order Batches: Companies need to devise strategies that leads to smaller batches or more frequent resupply. One main reason for batching was the cost of ordering this may be reduced by the implementation of EDI technologies. One other reason was the high transportation costs. Distributor tries to give a full truck load of order to reduce the transporting cost. Nowadays some manufacturers induce their distributors to order assortments of different products, to tackle with this problem. The use of third-party logistics company also helps making small batch replenishment economical. By consolidating loads from multiple suppliers located near each other, a company can realize full truck load economies without the batches coming from the same supplier.

- Stabilize Prices: The manufacturer can reduce the incentives for retail forward buying by establishing a uniform wholesale pricing policy. In the grocery industry major manufactures have moved to an everyday low price (EDLP) or value pricing strategy.

- Eliminate Gaming in Shortage Situations: In the case of shortage, instead of allocating products based on orders, allocating in proportion to past sales records make customers have no incentive to exaggerate their orders. The sharing of capacity and inventory information of the supplier helps to alleviate customer's anxiety and, consequently, lessen their need to engage in gaming.

In the same year with an other study, Lee et al. (1997b) develop simple mathematical models of supply chains that capture essential aspects of the institutional structure and optimizing behaviors of members. They study with the formulation of the causes and cures that they explained in their previous paper. They demonstrate through the models that the bullwhip effect is an outcome of the strategic interactions among rational supply chain members. They employ mathematical models to explain the outcome of rational decision making and their results suggest that companies attempting to gain control of the bullwhip effect are better served by attacking the institutional and inter-organizational infrastructure and related processes.

In an inventory management experimental context, Sterman (1989) reports evidence of the bullwhip effect in the "Beer Distribution Game". His experiment involves supply chain consisting of four members and making decisions individually, by only considering the input from their immediate downstream stages. Under the linear cost structure, the experiment proves that the variances of orders amplify as one moves up in the supply chain, confirming the bullwhip effect. He states that this phenomenon is a consequence of players' systematic irrational behavior, or "misperceptions of feedback".

In their study Chen et al. (2000a) quantify the bullwhip effect for simple two-stage supply chains consisting of a single downstream stage and a single upstream stage. Their focus is on determining the impact of demand forecasting on the bullwhip effect. They do not assume that the retailer knows exact form of the customer demand process. Instead, the retailer uses a standard demand forecasting technique to estimate certain parameters of the demand process. They study not only to show the existence of the bullwhip effect but also to quantify it. They also consider a multistage supply chain and the impact of centralized demand information on the bullwhip effect. Their results show that the smoother the demand forecast, the smaller the increase in variability, with longer lead times, the retailer must use more demand data in order to reduce the bullwhip effect, and centralizing customer demand information can significantly reduce the bullwhip effect but not totally eliminate it.

Chen et al. (2000b) demonstrate use of exponential smoothing forecast by the retailer can cause the bullwhip effect and contrast these results with the increase in variability due to use of a moving average forecast. They quantify the bullwhip effect for simple two-stage supply chains. They consider two types of demand processes, a correlated demand process and a demand process with a linear trend. Their results show that the magnitude of the increase in variability depends on both the nature of the demand process and on the forecasting technique used by the retailer. They demonstrate that, for a certain demand process, the variance of order placed by the retailer using a moving average forecast will be less than the variance of the order placed by the same retailer using an exponential smoothing forecast.

Lee, So, and Tang (2000) try to quantify the benefits of information sharing between the partners of supply chains and to identify the drivers of the magnitudes of these benefits. They study with a simple two level supply chain having nonstationary end demands by using analytical models. They show that information sharing alone could provide significant inventory reduction and cost savings to the manufacturer. They also suggest that the underlying demand process and the lead times have significant impact on the magnitudes of cost savings and inventory reductions associated with information sharing. Their specific results state that the manufacturer would experience greater savings from the sharing of information when: the demand correlation over time is high, the demand variance within each time period is high or the lead times are long. These stated conditions fit the profile of most high-tech products, therefore they say that the information sharing would be especially useful for improving the efficiency of the supply chains in this industry.

As an extension to study of Lee, So, and Tang (2000), Raghunathan (2001) studies with the same structure by making the manufacturer reduce the variance of its forecast farther by using the entire order history to which it has access. This study shows, analytically and using simulation, that even in Lee, So, and Tang (2000)'s model, sharing of demand information is of limited value when the parameters of the AR(1) process are known to both parties. This is so because the manufacturer can forecast the demand information shared by the retailer with a high degree of accuracy using retailer order history; the accuracy increases monotonically with each subsequent time period, and the value of information shared by the retailer decreases in the same manner with each time period, converging to zero in the limit.

Cachon and Fisher (2000) study the value of sharing demand and inventory data in a model with one supplier, N identical retailers, and stationary stochastic consumer demand. They compare a traditional information policy that does not utilize information sharing with the new models of sharing information between the partners of the supply chains. They study on not only the variance reduction benefit of the information sharing but also faster and cheaper order processing benefits of it, which leads to shorter lead-times and smaller batch sizes. Their results show that utilizing information technology to accelerate and smooth the physical flow of goods through a supply chain is more valuable than using it to expand the flow of information.

2.1.2 Two-Echelon Systems and Batch Ordering

In the literature there are huge amounts of study concentrating on inventory systems defined for two-echelon, retailer and supplier, supply chains. Their concentration are on the system random variables and coordinated performances of the parties involved. In some other studies they introduce order batching for one or more members of the chain. While we were searching the available history for batch ordering we reached studies generally working with fixed batch sizes, for periodic review inventory replenishment policies. These follows in this subsection. Our two-echelon model differs here in that we make the retailer define its order in every ordering period by updating its forecast and considering its order-up-to level, does not order a fixed amount or a multiple of a fixed amount. Supplier does not batch its order, it orders every period by using its own order-up-to policy.

Caplin (1985) considers the impact of batch ordering on the bullwhip effect. In his study retailer follows a continuous review (s, S) inventory policy. For such an inventory policy the order amount is fixed and has the value of $S - s$. Therefore, the variability of the orders placed by the retailer is due only to the variability in the time between orders. He proves that if the demands faced by the retailer are i.i.d., then the variance of the orders placed by the retailer is greater than the variance of the demand faced by the retailer, and that the variance of the orders increases linearly in the size of orders.

In his study Cachon (2001) considers a two-echelon supply chain with stochastic and discrete consumer demand. He studies on the topics of batch ordering, periodic inventory review, and deterministic transportation times. The system consists of one central warehouse and N identical retailers, which review their inventory periodically, implement reorder point policies and order quantities equal integer multiple of a fixed batch size, and have deterministic transportation times to receive their orders. He defines safety stock exactly for the lower echelon and details a good approximation for the upper echelon. His results show that system costs generally increase substantially if the upper echelon is restricted to carry no inventory, or if the upper echelon is required to provide a high fill rate. Also it is stated that although in many cases it is optimal to set upper echelon's reorder point to yield near zero safety stock, in some cases this simple heuristic can significantly increase supply chain operating costs. As a final observation he states that the policies selected under the assumption of continuous inventory review can perform poorly if implemented in an environment with periodic review.

Cachon (1999) studies supply chain variability in a model with one supplier and N retailers that face stochastic demand. Retailers order some multiple of a fixed batch size in scheduled, fixed intervals. He studies on exact determination of the system costs. It is stated that the supplier's demand variance is maximized when the retailers' orders are synchronized, i.e., all N retailers order in the same periods. And it is minimized when the retailers' orders are balanced, i.e., the same number of retailers order at each period. His results show that when retailers order in balance, the supplier's demand variance is reduced when the retailer order intervals are lengthened or when retailer's batch size is reduced.

Shapiro and Byrnes (1992) empirically studies on demand variance in medical supply industry. They observe that final demand shows little variance but the demand from the hospital shows great fluctuations. As a remedy they implemented standing order policies with the hospitals. A fixed amount of materials is shipped at fixed intervals unless an order is placed by the hospital. As a consequence of this implementation, hospitals need less storage and supplier's production efficiency is increased. Their results shows that reducing the variance of the demand observed by the supplier benefits a supply chain.

In her field study Rosenbaum (1981) deals with the application of a heuristic model which was developed to aid in determining safety-stock placement in the Eastman Kodak Comapany's two-level finished goods inventory systems. Company's inventory control system determined such that every location set its service level individually. She developed a heuristic model to minimize total company safety-stock inventory while guaranteeing a specified percentage of customer demand will be filled from on-hand inventory. Her field tests show that better customer service could be provided with the same amount of inventory, or perhaps even less, if the interaction of the service levels in the two echelons was considered.

Veinott (1965) introduces the idea of a myopic solution in which the single variable dynamic program can be solved with only information from the current period. He considers a dynamic nonstationary multi-product inventory model in which the demand in each of a sequence of periods of equal length are random vectors. He shows that when demand is independent over time and replenishment lead-time is constant the base-stock policy is optimal.

Cheung and Hausman (2000) study a two-echelon supply chain model with continuous review policy. There is a supplier serving to N retailers. They concentrate on the performance of the supplier serving N many locations, each of which uses decentralized (Q, R) policy based on installation stocks. The order quantity at each location is not identical and they show that under this circumstances the steady-state distribution of inventory position is often uniformly distributed. They also performs test to evaluate classical poisson approximation, and they reach a conclusion stating that although this approximation is an efficient estimator of the exact performance, its use in optimization may lead to moderate to significant loss in the resulting cost.

2.1.3 Nonstationary Demand and Related Forecasting Models

In this subsection we concentrate on the studies that design their system such that the end item demand observed by the retailer is nonstationary and they introduce a forecasting model to predict the future demand values. They use some methods to observe the performance of the system they construct; some use analytical tools such as derivations, some use simulation techniques and some use both simultaneously for their purposes. In this grouping our study fits to the last one. Following papers are about the effect of demand process and forecasting model on the general structure and performance of the system. Some of them give general structures for integrated moving average processes and some deals with which forecasting technique is more beneficial to these systems.

In his study Graves (1999) considers an adaptive base-stock policy for a single item inventory system, where the demand faced by the retailer is nonstationary, specifically integrated moving average process of order (0,1,1). It is stated in his study that for this kind of process the exponential-weighted moving average provides minimum mean square forecast. Under such a scenario he builds a single item inventory system assuming a deterministic lead time for the parts of the model. He analytically derives the equations for the system random variables, and finds the safety stocks for the supply chain partners. He observes that required inventory behaves much differently for the case of nonstationary demand compared with stationary demand. His results show that the upstream stage's demand process is not only nonstationary but also more variable than that of the downstream stage. One other interesting result of his study, special to his environment, is that there is no benefit for the system in letting the upstream stage see the exogenous demand. Last result for his study is that for when $\alpha > 0$, for nonstationary demand, retailer's lead-time impacts the safety stock requirements for both the retailer and the supplier of a two-stage supply chain. This is considered as a clue for the improvement seekers for such systems, it is more beneficial to concentrate on the reduction possibilities of retailer's lead-time than supplier's.

Zhao et al. (2002) presents a study on the effect of forecasting model selection on the value of information sharing on a supply chain, consisting of a capacitated supplier and multiple retailers. They use a computer simulation model to examine demand forecasting and inventory replenishment decisions of the retailers, and production decisions of the supplier under different demand patterns and capacity constraints. Analysis of outputs of comprehensive simulation studies bring following results:

- Information sharing can significantly increase the performance of the supply chain. Especially sharing future order information is more beneficial than only sharing demand information.

- The value of the information sharing is effected by the demand pattern, the forecasting technique used, and the capacity tightness.

- Benefits to different parties through information sharing could be quite different under different conditions.

In her general study based on Autoregressive Integrated Moving Average (ARIMA) time series model, Gilbert (2002) constructs a model that gives the ARIMA models of orders and inventories for any given ARIMA model of consumer demand and the lead time. The significance of her findings lies under the fact that they apply to any ARIMA (p,d,q) demand and any number of stages in a supply chain under the assumption that the order of a given stage becomes the demand of an immediate upstream stage. Therefore they provide a general frame-work for understanding the bullwhip effect and the importance of information sharing.

Aviv (2001) constructs two models for a two-stage supply chain consisting of a retailer and a supplier. In his first model each member forecasts locally and integrates the adjusted forecasts into its replenishment process. These forecast performed individually by the partners of the system can be correlated. The system is decentralized therefore the day to day forecasting is available for the stages locally. In his second model he forces the partners jointly maintain and update a single forecasting process, collaborative forecasting. His results show that both of the initiatives can provide substantial benefits to the supply chain, but the magnitude of these benefits significantly depends on the specific setting. Local forecasting is beneficial when forecasting strength increases, whereas the benefits of collaborative forecasting increase when both or either of the initiatives, quick response and advanced demand information, are implemented.

Aviv (2002) develops a stylized framework to describe a two-level supply chain, consisting of a retailer and a supplier, that faces an auto regressive demand process. He extends the literature by modeling the ability of the parts to observe early market signals, hence increase the performance of their forecasting methods. He examines three types of supply chain configurations: in the first configuration policy parameters are coordinated between the parts to minimize system wide costs without sharing demand signal information, for the next the supplier takes the role of managing the whole system and no demand signal information is transferred to it, and the third setting is the environment where inventory is managed centrally and all demand related information is shared. As a result he demonstrates that the consideration of Vendor Managed Inventory and Collaborative Forecasting And Replenishment programs are significantly important as the demand process is more correlated across periods, and as the companies need to explain larger portion of the demand uncertainty through the use of early demand information. He argues that the determination of the best policies for the systems depends on the understanding of the interaction between the explanatory power of the supply chain members.

Lovejoy (1990) shows that a simple order-up-to policy with an order-up-to level specified by a critical fractile can be optimal or near optimal for a more general class of demand distributions. In his study some parameter of the demand distribution is not known with certainty, and estimates of the parameter are updated in a static fashion as demand is observed through time, with either exponentially smoothed or Bayesian updating. His analysis derives from a two-stage reduction of a dynamic programming formulation of the problem. The first stage begins with a two dimensional state space , and the reduction of this to revised dynamic program with a single state variable. The second stage of analysis involves reducing the dynamic program with a single state variable to one with a "zero dimensional" state space, i.e., a static optimization problem. This is the definition of the "myopic" solution to the inventory problem, and the optimal policy can easily be derived by calculating a critical fractile.

Reddy and Rajendran (2004) deal with a supply chain with nonstationary customer demand and different levels of information sharing among the partners of the system. Their purpose is to minimize the sum of inventory, shortage and transport costs. They derived mathematical equations to determine dynamic order-up-to levels as a function of forecasted demand, replenishment lead-time and safety factor. Simulation techniques are used to observe the analytically derived dynamic order-up-to policies based on various heuristic settings. Their major conclusions can be stated as follows:

- Irrespective of the basis for forecasting at every divisions, the use of forecasting technique at any divisions for determining replenishment lead-times seems to be effective in reducing the holding and shortage costs.

- The use of customer demand information sharing across the partners of the chain appears to be very effective in reducing holding and shortage costs for every partner.

- In the case where there is limited information sharing across the members in the supply chain (they share the demand data only with their immediate upstream stages) coupled with the use of forecasting technique for determining the replenishment lead-time, substantial reduction in total cost is observed as compared to the case where no demand data is shared.

- The use of forecasted lead-time with heuristic settings of safety factor appears quite effective for different levels of information sharing.

2.2 Relation Between Our Study and Listed Literature

In his inventory management experiment Sterman (1989) reports evidence of the bullwhip effect. His study concentrates on a chain consisting of four members and many other studies deal with other types of chains. Lee et al. presents two studies in the same year 1997 on this subject, in the first one they try to define the reasons, why bullwhip effect is observed, and the possible remedies for each of the case. They define order batching as a triggering factor for bullwhip effect and we observe this in our study as well. In their second study they study on the quantification of their results in the previous study. Chen et al. (2000a) again study on quantifying the bullwhip effect for simple two-stage supply chains consisting of a single downstream stage and a single upstream stage and they focus on determining the impact of demand forecasting on the bullwhip effect. In their other study Chen et al. (2000b) try to see the effect of exponential smoothing forecast to their observations for different demand processes , a correlated demand process and a demand process with a linear trend. We use exponential-weighted forecast and nonstationary demand with batch ordering and as in the studies listed above we try to observe and quantify the bullwhip effect throughout our study.

There are many studies in the literature studying on the remedies of bullwhip effect, and some of them are concentrated on the information sharing topic. It is generally observed that in most of the structures having the upstream stage observe the real demand data of the downstream stage increases the performance of the upstream stage. And for the sake of the chain this must be promoted by the upstream stages by providing some incentives to the downstream stages. Lee, So, and Tang (2000) quantify the benefits of information sharing between the members of supply chains and identify the drivers of the magnitudes of these benefits. We study both analytically and by modeling on this subject and observe the importance of this topic. Raghunathan (2001) extends the study performed by Lee, So, and Tang (2000) by making the supplier use the whole demand history. He shows how this effects the value of information sharing. Cachon and Fisher (2000) study the topic by changing the model, one supplier, N identical retailers, and stationary stochastic consumer demand.

There are not many studies in the literature about order batching, especially when the order amount is not fixed. Caplin (1985) considers the impact of batch ordering on the bullwhip effect for a continuous review (s, S) inventory policy. The order amount for his system is fixed and has the value of $S - s$. On the other hand, Cachon (2001) deals with a two-echelon supply chain with stochastic and discrete consumer demand with batch ordering, periodic inventory review, and deterministic transportation times. Reorder point policies and order quantities equal integer multiple of a fixed batch size is implemented. We deal with batch ordering but since the retailer considers the demand accumulated during batching cycle and orders the amount by taking into account the observed demand and some forecast changes during its lead time, the order amount is not fixed.

Rosenbaum (1981) deals with a heuristic model which was developed to aid in determining safety-stock placement in the Eastman Kodak Comapany's two-level finished goods inventory systems. And she shows that the interaction between the service levels of the members of the chain increases the performance of the supply chain. We determine safety-stock values for the members of our model and by using these set service levels for them. Especially in our simulation study we try different combinations of service levels and observe the changes in the general performance of the chain.

In a general study based on Autoregressive Integrated Moving Average (ARIMA) time series model, Gilbert (2002) constructs a model that gives the ARIMA models of orders and inventories for any given ARIMA model of consumer demand and the lead time. We study with a class of ARIMA demand process and derive order equations and inventory variables similarly.

As we have mentioned previously our inspiration to study on this subject is somehow comes from the study of Graves (1999). We introduce batch ordering to a similar system to his defined system. We used same demand process and forecasting model for the downstream stage and derive analytical equations for the random variables. We extend our study to simulation modelling of the processes defined, and we observe the performance of the members of the supply chain. One interesting result of Graves's study was as we stated in the previous section of this chapter that there is no benefit for the system letting the supplier see the exogenous demand. We analyzed this under our systems dynamics and it is shown that it is not applicable for our model. In his study by some coincidence some magical cancellation of the equations for the supplier takes place and same methodology for the retailer can be applied to the supplier as well. We see that this is not the case for our study due to the complexity derived by the order batching.

We talked about the complexity we face due to batch ordering in the previous paragraph. We can not apply same policies directly to the supplier so we need to find an other method to work with the supplier. We constructed a similar order-up-to policy for the supplier to the one used in the study of Lee, So, and Tang (2000). The member uses the mean and variance of the demand accumulate during its lead time to derive the equation for its order-up-to level and then uses this to find its order quantity. They study to quantify the value of information sharing as we do in our study. Also in one extension to their study Raghunathan (2001) shows that when the supplier utilize whole of its demand history there seems no benefit from the demand data sharing. We again use similar methodology with these but we do not employ the whole history.

Lovejoy (1990) shows the optimality of critical-fractile inventory policy for more general class of demand distributions and Veinott (1965) establishes the optimality of base stock policy when demand is independent over time and there is a constant replenishment lead time. We use both these results in our study but we differ from this prior study in that we take these as given do not try to establish the optimality of these.

Reddy and Rajendran (2004) deals with supply chain having nonstationary demand and making observations with the help of simulation tools. We follow the same structure throughout our study.

CHAPTER 3

ANALYSIS OF A TWO-ECHELON SUPPLY CHAIN WITH NONSTATIONARY DEMAND AND BATCH ORDERING

In this part of the study, we analyze the system analytically first starting from the explanations of the processes involved; the demand process and forecast model for the retailer. The chapter continues with the inventory control and order-up-to policies of the retailer. Inventory equations for three different periods, that have certain characteristics, in a batching cycle are derived for the retailer for both of the cases when lead-time value of the retailer is smaller than the value of batching number and when bigger than it. The supplier's equations are derived analytically. Safety stock determination process for the supplier and the retailer is explained. In the last part of the chapter we show the effect of the information sharing on the performance of the supplier.

3.1 Notation and Descriptions

The notation used in the problem formulation is provided below:

R : number of periods the retailer batches its order

L : lead time for the retailer to receive its order from upstream

L_s : lead time for the supplier to receive its order, $L_s = KR$ and K is an

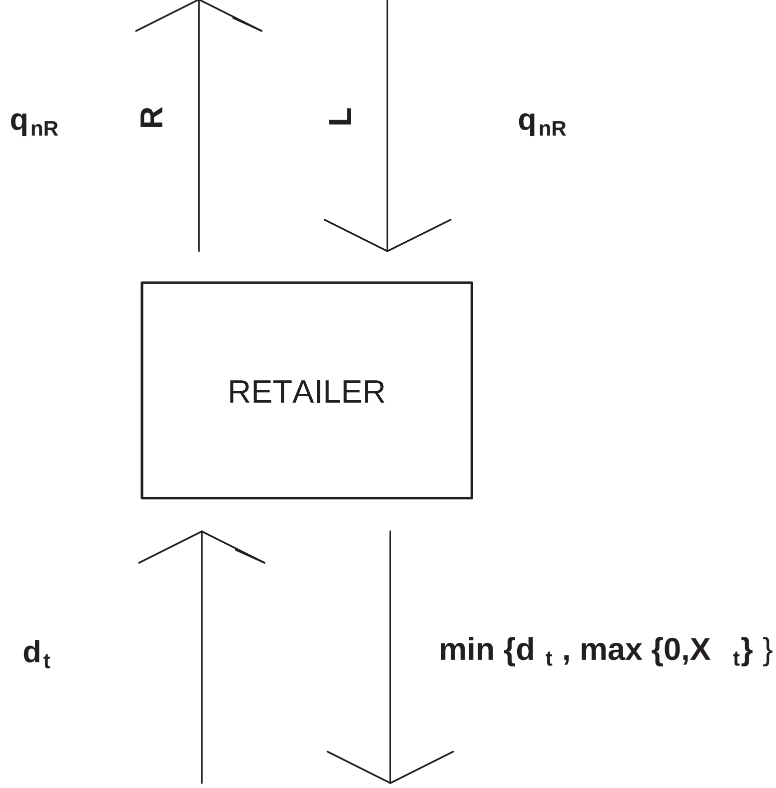

Figure 3.1: Retailer's System Description

integer, $K = 1, 2, 3...$

X_0 : starting inventory level for the retailer

XS_0 : starting inventory level for the supplier

d_t : demand realized by the retailer in period t

F_t : forecast made after observing the demand in period $t-1$, for demand in period t

X_{nR} : on hand inventory(or backorders) for the retailer at the end of period nR, $n =$ 0,1,2...

XS_{nR} : on hand inventory(or backorders) for the supplier at the end of period nR, $n =$ 0,1,2...

q_{nR} : retailer's order quantity at period nR

p_{nR} : supplier's order quantity at period nR

I_{nR} : inventory on hand for the supplier at period nR

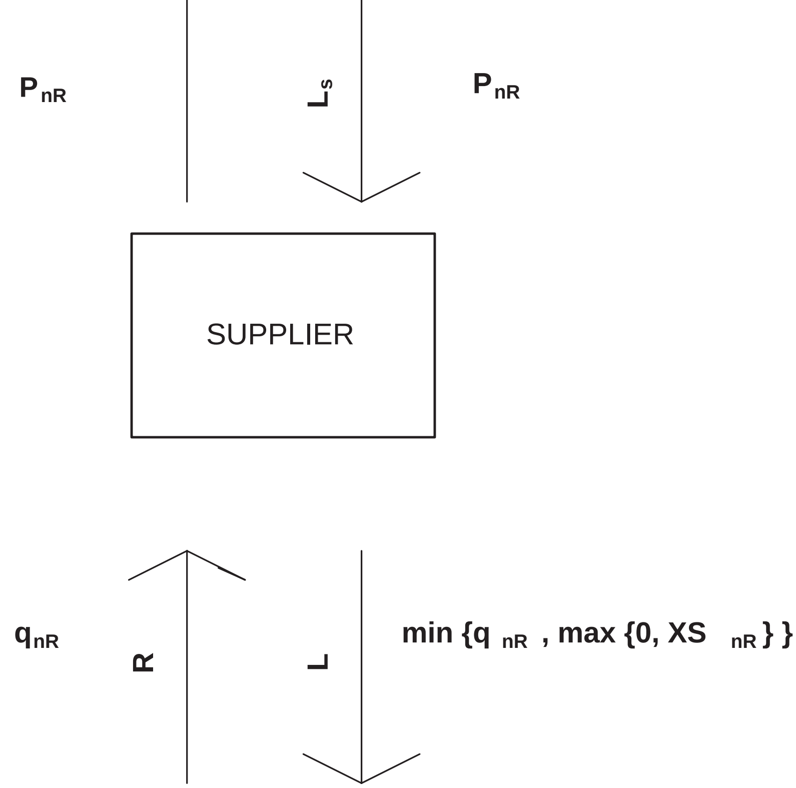

Figure 3.2: Supplier's System Description

3.2 The System

We study a two-level supply chain consisting of a retailer and a supplier. The end customer demand is a nonstationary stochastic process. The replenishment lead-times for the supplier and the retailer are fixed and they use different order-up-to level policies to determine their order amounts.

The general process that is applied in the retailer side can be seen in Figure 3.1. In general, in a period the retailer faces customer demand, d_t, determines that period's order quantity, q_{nR}, receives the order from lead-time, L, periods ago, and then fills the demand from inventory. Any demand that can not be met from inventory is backordered. This general structure is not applicable for all the periods since the retailer in this system batches its demand for a predetermined number of periods. Parts of this general structure is observed in different periods in a batching cycle: in every period customer demand is filled, in the ordering period order is placed to the supplier, and in the receiving period the previous ordered amount is received from the supplier.

The structure for the supplier is seen in Figure 3.2. In a period the supplier first observes its demand, q_{nR}, determines that period's order, p_{nR}, receives the order from L_s periods ago (p_{nR-L_s}), and then fills the retailer's order from inventory. It backorders the amount that can not be met from inventory.

In this chapter we study on the retailer and the supplier separately. Analytical equations for the retailer will be derived by assuming that an uncapacitated supplier serves to it.

3.3 Retailer's Process

3.3.1 Demand Process

Notation for the demand process:

μ : expected demand and forecast value for any period

α : parameter determining the weight of the most recent demand in determining the next period's demand

ϵ_i : random noise term in demand, normally distributed with mean 0 and variance σ^2, i = 0,1,2...

σ : standard deviation of random noise term in demand equation

Demand is defined as in an autoregressive integrated moving average (ARIMA) process:

$$\begin{aligned} d_1 &= \mu + \epsilon_1 \\ d_t &= d_{t-1} - (1-\alpha)\epsilon_{t-1} + \epsilon_t \quad \text{for } t = 2,3,...., \end{aligned} \tag{3.1}$$

where d_t is the observed demand in period t, α and μ are known parameters, and (ϵ_t) is a time series of independent and identically distributed random variables. It is assumed that $0 \leq \alpha \leq 1$. This is known as the integrated moving average (IMA) process of order $(0, 1, 1)$.

The model can be expanded by backward substitution as follows:

$$d_t = \epsilon_t + \alpha\epsilon_{t-1} + \alpha\epsilon_{t-2} + ... + \alpha\epsilon_1 + \mu \tag{3.2}$$

Demand process can be expressed as a function of the time series of random noise or independent shocks with the help of above representation. Each period there is a shift in the mean of the demand process and this is proportional to the size of the shock.

Here in this process, α plays an important role; as it grows d_t depends more on recent data and it can also be viewed as the measure of inertia in the process; the larger it is, the less inertia there is in the process. Larger α values result in less stable or more transitory environment. When α value is equal to 1; the demand process is a random walk on a continuous state space, we can easily observe from the model for the demand process that new demand is just only noise term added previous demand.

When we derive the variance and expected value of the demand equation, Equation 3.2, we have the followings:

$$
\begin{aligned}
d_t &= \epsilon_t + \mu + \alpha \sum_{i=1}^{t-1} \epsilon_i \\
E[d_t] &= \mu \\
V[d_t] &= \sigma^2 + \alpha^2 (t-1)\sigma^2 \\
&= \sigma^2(1 + \alpha^2(t-1))
\end{aligned}
\tag{3.3}
$$

It is seen from the variance equation of demand, (3.3), that the variance of demand depends on time and increases with the increasing value of it.

To observe the values of demand changing with time we simulated the system with different α values and draw the following graphs. The variance increase of the demand process with time can be observed from these figures.

In Figure 3.3, the value of α is 0.0 and this makes the demand normally distributed with mean μ and variance σ^2. Figure 3.4 shows the demand value changing with time for $\alpha = 0.5$ and Figure 3.5 for $\alpha = 1.0$. In these example figures value of σ and μ are 1.0 and 10 respectively.

3.3.2 Forecast Model

We employed the model which is stated in Graves (1999) as the one providing minimum mean square forecast for these kinds of demand processes; first-order

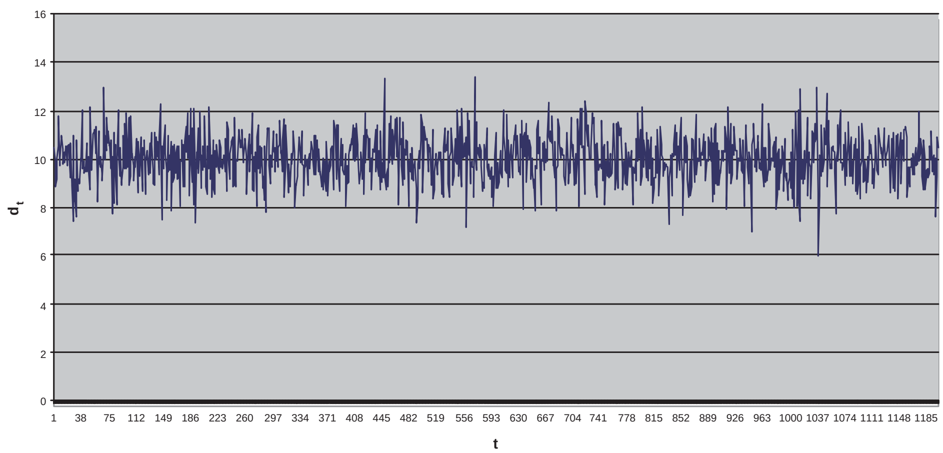

Figure 3.3: Demand, d_t, Values Observed by the Retailer ($\alpha = 0.0$)

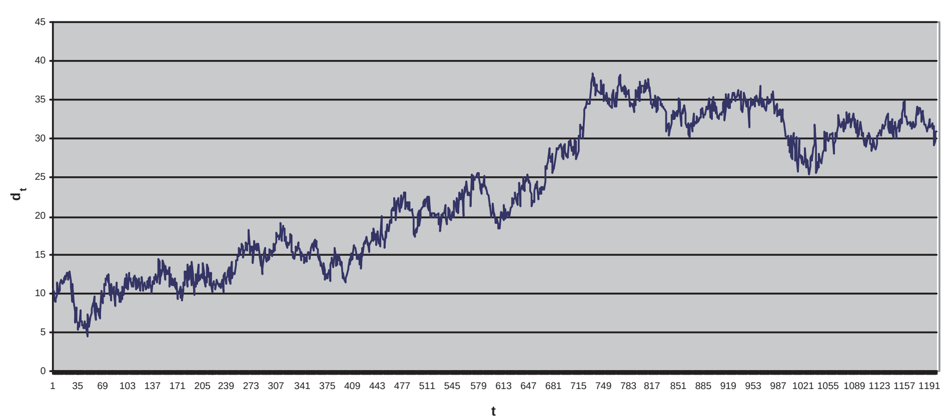

Figure 3.4: Demand, d_t, Values Observed by the Retailer ($\alpha = 0.5$)

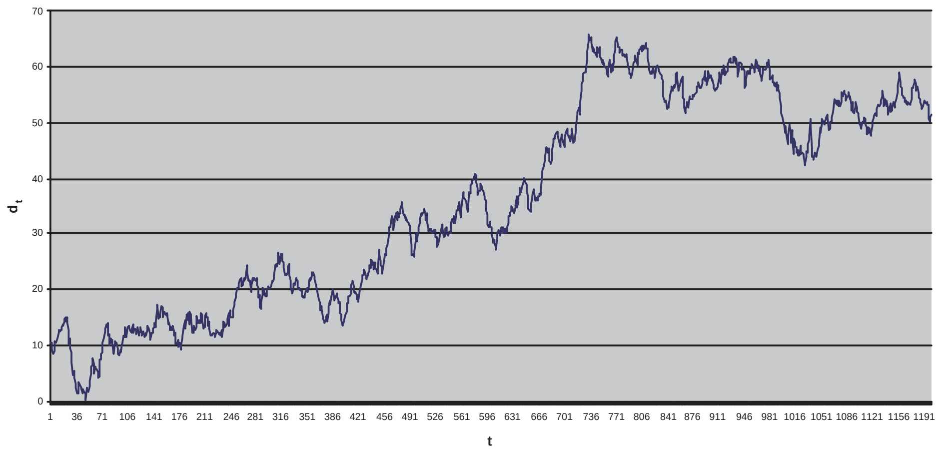

Figure 3.5: Demand, d_t, Values Observed by the Retailer ($\alpha = 1.0$)

exponential-weighted moving average with parameter α and initial forecast μ. F_{t+1} is defined as the forecast, made after observing demand in time period t, for demand in period $t+1$:

$$\begin{aligned} F_1 &= \mu \text{ and} \\ F_{t+1} &= \alpha d_t + (1-\alpha)F_t \text{ for } t = 1, 2, ... \end{aligned} \tag{3.4}$$

It can be shown by induction after subtracting equation (3.4) from (3.1) that the forecast error is:

$$d_t - F_t = \epsilon_t \text{ for } t = 1, 2... \tag{3.5}$$

Since we know that $E(\epsilon_t) = 0$ for any value of t, the exponential -weighted moving average is an unbiased forecast and the forecast error is random noise term for time period t.

The forecast can be expressed in terms of random noise terms by using equations (3.4) and (3.5):

$$F_{t+1} = F_t + \alpha\epsilon_t$$

$$= \alpha\epsilon_t + \alpha\epsilon_{t-1} + ... + \alpha\epsilon_1 + \mu \quad (3.6)$$

As a general rule of this model of forecast, at time t the forecast of demand in any future period equals the F_{t+1}. This is a general fact defined as τ period ahead forecast is equal to one period ahead forecast for these kinds of forecast models.

3.4 Order Mechanism

We considere a single item inventory system for two-echelon supply chain and introduce batch ordering for the retailer into this system. Let q_{nR} be the order placed in period nR for delivery in period $nR + L$. We assumed that in each order period, $nR; n = 0, 1, 2...$, we first observe d_{nR}, determine that period's order quantity (q_{nR}), and fill the demand form inventory. This process changes for the receiving periods, $nR + L; n = 0, 1, 2...$, where we first observe d_{nR+L}, receive the order, q_{nR}, from L periods ago, and then fill the demand from inventory. We have two different inventory balance equations in different time periods. These are:

$$X_{nR} = X_{nR-1} - d_{nR} \text{ for ordering periods} \quad (3.7)$$

$$X_{nR+L} = X_{nR+L-1} - d_{nR+L} + q_{nR} \text{ for receiving periods} \quad (3.8)$$

for other periods the equation is the same as the one for X_{nR}.

X_t stands for the on-hand inventory (or backorders) at the end of period t, and we set an initial inventory level X_0 and $q_t = R\mu$ for $t \leq 0$.

The system works with base stock policy and adjusts the base stock as the demand forecast changes. We used order-up-to policy to derive an equation for q_t taking into consideration this change.

$$\begin{aligned} Y_{nR} &= \sum_{i=1}^{L+R-1} F_{nR,nR+i} \\ &= (L + R - 1)F_{nR+1} \quad (3.9) \\ I_{nR} &= Y_{(n-1)R} - \sum_{i=1}^{R} d_{(n-1)R+i} \\ Y_{nR} &= I_{nR} + q_{nR} \\ q_{nR} &= Y_{nR} - I_{nR} \\ &= Y_{nR} - \left\{ Y_{(n-1)R} - \sum_{i=1}^{R} d_{(n-1)R+i} \right\} \quad (3.10) \end{aligned}$$

where I_{nR} is the inventory position at the beginning of period nR and Y_{nR} is the order up to level.

When we modify Equation 3.9 for $Y_{(n-1)R}$ and put it into Equation 3.10 our order equation can be found as follows:

$$q_{nR} = (L + R - 1)\left\{F_{nR+1} - F_{(n-1)R+1}\right\} + \sum_{i=1}^{R} d_{(n-1)R+i} \tag{3.11}$$

In these derivations our main equation for the order up to point calculation is taken as the total forecasted demand for $L + R - 1$ periods since we have to consider the time until the next order can be received. After giving order in period nR we give next order in $(n + 1)R$ and receive it at $(n + 1)R + L$ so we have to take into consideration $L + R - 1$ periods.

3.5 Order Amplification

The downstream faces the demand d_t and orders q_t from the upstream so q_t becomes the demand that upstream faces. In this section we compared the two demand processes and saw the increase in the variance of demand as we move up in the chain, the bullwhip effect for this process.

We employed (3.1), (3.6), and (3.11) to find following, derivation of this is provided in the Appendix A.1.

$$q_{nR} = \sum_{i=0}^{R-1} \{(L + R - 1 + i)\alpha + 1\} \epsilon_{nR-i} + R\alpha \sum_{i=1}^{(n-1)R} \epsilon_i + R\mu \tag{3.12}$$

To write $q_{(n+1)R}$ with respect to q_{nR} and noise terms we modified 3.12 and then subtracted 3.12 from 3.13:

$$\begin{aligned} q_{(n+1)R} &= \sum_{i=0}^{R-1} \{(L + R - 1 + i)\alpha + 1\} \epsilon_{(n+1)R-i} \\ &+ R\alpha \sum_{i=1}^{nR} \epsilon_i + R\mu \qquad (3.13) \\ q_{(n+1)R} - q_{nR} &= \sum_{i=0}^{R-1} \{(L + R - 1 + i)\alpha + 1\} \left(\epsilon_{(n+1)R-i} - \epsilon_{nR-i}\right) \\ &+ R\alpha \sum_{i=(n-1)R+1}^{nR} \epsilon_i \qquad (3.14) \end{aligned}$$

where

$$R\alpha \sum_{i=(n-1)R+1}^{nR} \epsilon_i = R\alpha \sum_{i=0}^{R-1} \epsilon_{nR-i} \tag{3.15}$$

and when we substitute it;

$$q_{(n+1)R} - q_{nR} = \sum_{i=0}^{R-1} \left(\{(L+R-1+i)\alpha+1\} \left(\epsilon_{(n+1)R-i} - \epsilon_{nR-i}\right) + R\alpha\epsilon_{nR-i} \right) \tag{3.16}$$

Let

$$\begin{aligned}
\gamma_{nR-i} &= \{(L+R-1+i)\alpha+1\} \epsilon_{nR-i} \\
\gamma_{(n+1)R-i} &= \{(L+R-1+i)\alpha+1\} \epsilon_{(n+1)R-i} \\
\beta_i &= \frac{R\alpha}{(L+R-1+i)\alpha+1}
\end{aligned}$$

then Equation 3.16 becomes

$$\begin{aligned}
q_{(n+1)R} - q_{nR} &= -\sum_{i=0}^{R-1} (1-\beta_i)\gamma_{nR-i} + \sum_{i=0}^{R-1} \gamma_{(n+1)R-i} \\
q_{(n+1)R} &= q_{nR} - \sum_{i=0}^{R-1} (1-\beta_i)\gamma_{nR-i} + \sum_{i=0}^{R-1} \gamma_{(n+1)R-i} \qquad (3.17)
\end{aligned}$$

Since above equation has noise terms related to previous periods multiplied with coefficients bigger than one and summed for R periods our q_{nR} is more variable than both the demand process and the q_t in Graves (1999).

In Graves's work, the derived equation for the retailer's order is similar to the demand process seen by it and with some modifications the same forecasting technique used to forecast the demand of retailer can be used to estimate the demand observed by the supplier. As we compare the derivation above with the demand equation observed by the retailer we see that this is not the case for our model due to the complications with summations in Equation 3.17.

3.5.1 Comparison of the Variance of q_{nR} with that of Demand for R Periods

In this part of our study we worked on the variances of demand process observed by the retailer and its order quantity, that is the demand for the supplier. Retailer batches its orders for R periods and this order replenishes the demand observed in R many previous periods and adds some correction factor for the change in the forecast. So here it is wise to compare the variance of demand accumulated during last R periods with the variance of the order given by the retailer. The order given by the retailer is found as follows in the previous calculations:

$$q_{nR} = \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\} \epsilon_{nR-i} + R\alpha \sum_{i=1}^{(n-1)R} \epsilon_i + R\mu$$

and the expected value and variance of this is:

$$\begin{aligned} E[q_{nR}] &= R\mu \\ V[q_{nR}] &= \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}^2 \sigma^2 + R^2\alpha^2(n-1)R\sigma^2 \end{aligned} \quad (3.18)$$

The equation for total demand over R periods:

$$\begin{aligned} \sum_{i=1}^{R} d_{(n-1)R+i} &= R\left\{\sum_{i=1}^{(n-1)R} \alpha\epsilon_i + \mu\right\} + \sum_{i=0}^{R-1}(1+i\alpha)\epsilon_{nR-i} \\ &= R\alpha \sum_{i=1}^{(n-1)R} \epsilon_i + R\mu + \sum_{i=0}^{R-1}(1+i\alpha)\epsilon_{nR-i} \end{aligned}$$

When we take expected value and variance of above equation we have:

$$\begin{aligned} E[\sum_{i=1}^{R} d_{(n-1)R+i}] &= R\mu \\ V[\sum_{i=1}^{R} d_{(n-1)R+i}] &= R^2\alpha^2(n-1)R\sigma^2 + \sum_{i=0}^{R-1}(1+i\alpha)^2\sigma^2 \end{aligned} \quad (3.19)$$

As we compare the equation for the variance of order, Equation 3.18 with the equation for variance of R periods accumulated demand, Equation 3.19 it can easily be seen that the variance of order is bigger than that of the demand. Therefore there is amplification of the exogenous demand process as we move from downstream stage to the upstream stage. This amplification can be calculated as a measure of difference between the different terms in the equations above:

$$Amplification = \sum_{i=0}^{R-1} \left\{ \{(L+R-1+i)\alpha+1\}^2 - (1+i\alpha)^2 \right\} \sigma^2$$

This is the popular phenomenon called "bullwhip effect" in the literature. Most of the works that deal with supply chain modelling consider this effect as a major or complementary topic of their work.

This above result is similar to that mentioned in Lee et al.(1997) as demand signal processing and batching among the causes of bullwhip effect.

In his article, Sterman(1989) works on this phenomenon and tries to make it more realizable for the beginners using "beer distribution game" as an experimental context.

For the system observing stationary demand Chen et al. (1996) show how the moving-average forecast can introduce bullwhip effect to this serial system. They work on quantifying the value of this effect and use quantifying variance amplification for this purpose. In a successive paper they extend to exponential forecast model and to the case where the demand has linear trend.

Cachon (1998) works on the system having many retailers and one supplier supplying to all. Retailers have stationary demand, he works on how the variability of the orders placed by each retailer changes by their structure and parameters of the order.

3.6 Characterization of Inventory for the Retailer When $L < R$

We tried to find inventory balance equation with respect to noise terms and initial value, X_0. Since we have two different equations for different time periods we worked on both separately to derive the inventory random variables specific to the characteristics of the periods.

Our equations for different time periods are as follows:

$$X_i = X_{i-1} - d_i \text{ for } i = 1, 2, .., L-1$$

$$
\begin{aligned}
X_L &= X_{L-1} - d_L + q_0 \\
X_{L+1} &= X_L - d_{L+1} \\
X_i &= X_{i-1} - d_i \quad \text{for } i = L+2, L+3, .., R-1 \\
X_R &= X_{R-1} - d_R \\
X_{R+1} &= X_R - d_{R+1} \\
X_i &= X_{i-1} - d_i \quad \text{for } i = R+2, R+3, .., R+L-1 \\
X_{R+L} &= X_{R+L-1} - d_{R+L} + q_R \\
X_{R+L+1} &= X_{R+L} - d_{R+L+1} \\
X_i &= X_{i-1} - d_i \quad \text{for } i = R+L+2, R+L+3, .., 2R-1 \\
X_{2R} &= X_{2R-1} - d_{2R} \\
X_i &= X_{i-1} - d_i \quad \text{for } i = 2R+1, 2R+2, .., 2R+L-1 \\
X_{2R+L} &= X_{2R+L-1} - d_{2R+L} + q_{2R} \\
X_{nR} &= X_{nR-1} - d_{nR} \qquad (3.20) \\
X_i &= X_{i-1} - d_i \quad \text{for } i = nR+1, nR+2, .., nR+L-1 \\
X_{nR+L} &= X_{nR+L-1} - d_{nR+L} + q_{nR} \qquad (3.21)
\end{aligned}
$$

As we work in this fashion, we realize that there is a form of equation for the receiving periods and an other form of equation for the remaining periods. In the later form of equation the retailer only fills the demand from inventory whereas in the former one it receives its previous order and then fills the demand. In the coming parts of our work we will specifically deal with these characteristics of the periods.

The form of the equation for receiving period is:

$$X_{nR+L} = X_{nR+L-1} - d_{nR+L} + q_{nR}$$

The form of the equations for the periods other than the receiving period is:

$$X_{nR} = X_{nR-1} - d_{nR}$$

3.6.1 Inventory Equation for Ordering Periods

As we have defined previously retailer gives order in every R periods, we tried to find inventory equations represented by noise terms and initial inventory variable by backward substitution of inventory equations for different periods.

Starting from Equation 3.20, backward substitution and replacing $q_{nR}, n = 0, 1, 2, ...$ with the Equation 3.11 we found the inventory equation as follows, the derivation can be found in Appendix A.2.

$$X_{nR} = X_0 - \sum_{i=0}^{R-1}(1+i\alpha)\epsilon_{nR-i} + (L-1)\sum_{i=1}^{(n-1)R}\alpha\epsilon_i \qquad (3.22)$$

From Equation 3.22 and the assumption that ϵ_t is a time series of normally-distributed i.i.d random variables with mean and variance, zero and σ respectively, we find the variance and mean of X_{nR}.

$$\begin{aligned} V(X_{nR}) &= \sigma^2\sum_{i=0}^{R-1}(1+i\alpha)^2 + (L-1)^2\alpha^2(n-1)R\sigma^2 \qquad (3.23) \\ StdDev(X_{nR}) &= \sigma\sqrt{\sum_{i=0}^{R-1}(1+i\alpha)^2 + (L-1)^2\alpha^2(n-1)R} \\ E(X_{nR}) &= X_0 \end{aligned}$$

As can be seen from the above results, variance of the inventory equation for the order periods X_{nR}'s depends on n, the time, and increases with increasing value of it. This was not an expected result, since this was not the case for a similar process in Graves (1999). In his work this variance term is independent of time; the time passes after giving order to the next order coincides with the time necessary to receive the order, so some calculations become applicable that makes the process independent of time. Since we changed the process by making the retailer batch its orders complexity of the structure is increased and unfortunately we do not have the same situation. It makes the process hard to think on intuitively and make some logical interpretation. To ease our job we explore some methods to make the system work more understandable, and we realized that working with small alpha values would decrease the complexity and be meaningful to work especially in practical observations. .

3.6.2 Inventory Equation for Receiving Periods

Downstream receives order in every $nR + L$ periods. In our calculations for the inventory random variable for these periods again we used backward substitution with Equation 3.21 and found the following equation, the derivation can be found in Appendix A.3.

$$X_{nR+L} = X_0 - \sum_{i=0}^{L-1}(1+i\alpha)\epsilon_{nR+L-i} + (R-1)\sum_{i=1}^{nR}\alpha\epsilon_i + (R-L)\mu \quad (3.24)$$

From Equation 3.24 and the assumption that ϵ_t is a time series of normally-distributed i.i.d random variables with mean zero and variance σ^2, we find the variance and mean of X_{nR+L}.

$$V(X_{nR+L}) = \sigma^2\sum_{i=0}^{L-1}(1+i\alpha)^2 + (R-1)^2\alpha^2 nR\sigma^2$$

$$Std(X_{nR+L}) = \sigma\sqrt{\sum_{i=0}^{L-1}(1+i\alpha)^2 + (R-1)^2\alpha^2 nR}$$

$$E(X_{nR+L}) = X_0 + (R-L)\mu$$

This variance also depends on time and increases with increasing value of it, so we have the same difficulty to interpret inventory equations for receiving periods

3.6.3 Inventory Equation for one Period Prior to Receiving Period

We also study the period prior to receiving due to the similarity of the process in this period with that of Graves (1999). No order, given between time nR and $(n+1)R + L$, is received for $(R + L - 1)$ periods of time except the one given at time nR. This order is received at the end of the interval defined above and when we do not consider the inventory processes in other periods this process is very similar to the one in Graves (1999). Since in his process, when the retailer orders, it has to wait for lead time many periods to receive the amount ordered, it does not have any chance to give a new order and receive it before the previous order is received. We tried same methods applied previously to find the inventory equation represented by noise terms for these periods. Inventory random variable for X_{nR+L-1}'s has the same structure with the periods other than receiving period, the retailer fills the demand from inventory without receiving any previous order. The derived inventory equation for this period is as follows, the derivation can be found in Appendix A.4.

$$X_{nR+L-1} = X_0 - \sum_{i=0}^{R+L-2} \{(1+i\alpha)\epsilon_{nR+L-1-i}\} - (L-1)\mu \tag{3.25}$$

From Equation 3.25 and the assumption that ϵ_t is a time series of normally-distributed i.i.d random variables with mean and variance, zero and σ^2 respectively, we find the variance and mean of X_{nR+L-1}.

$$V(X_{nR+L-1}) = \sigma^2 \sum_{i=0}^{R+L-2} (1+i\alpha)^2$$

$$Std(X_{nR+L-1}) = \sigma \sqrt{\sum_{i=0}^{R+L-2} (1+i\alpha)^2} \tag{3.26}$$

$$E(X_{nR+L-1}) = X_0 - (L-1)\mu \tag{3.27}$$

As we expect this variance does not depend on n, the time, and results are similar to the ones in Graves (1999) in that: limits for summation term was 0 and $(L-1)$ in Graves's work and these values are 0 and $(L+R-1)$ for our case. Since we use $(L+R-1)$ instead of L in our calculations $(L+R-2)$ is like $(L-1)$ in Graves (1999).

3.7 Characterization of Inventory for the Retailer When $L > R$ $(L = R + k, k < R)$

In this section, we examine the inventory levels for the case of $L > R; L = R+k$. In doing this we want to see whether having $L > R$ or $L < R$ makes any difference on the inventory random variables and the performance of the system. For the remaining part of this section we work on the similar equations as in the previous section.

3.7.1 Inventory Equation for Order Periods

Starting from Equation 3.20, backward substitution and replacing $q_{nR}, n = 0, 1, 2, ...$ with the Equation 3.11 we derived the inventory equation for this special period similar to the one when $L < R$, the derivation is provided in Appendix A.5.

$$X_{nR} = X_0 - \sum_{i=0}^{2R-1} (1 + i\alpha)\epsilon_{nR-i} + (k-1) \sum_{i=1}^{(n-2)R} \alpha\epsilon_i - R\mu \quad (3.28)$$

From Equation 3.28 and the assumption that ϵ_t is a time series of normally-distributed i.i.d random variables with mean and variance, zero and σ respectively, we find the variance and mean of X_{nR}.

$$\begin{aligned} V(X_{nR}) &= \sigma^2 \sum_{i=0}^{2R-1} (1 + i\alpha)^2 + (k-1)^2\alpha^2(n-2)R\sigma^2 \\ Std(X_{nR}) &= \sigma\sqrt{\sum_{i=0}^{2R-1} (1 + i\alpha)^2 + (k-1)^2\alpha^2(n-2)R} \\ E(X_{nR}) &= X_0 - R\mu \end{aligned}$$

This variance term depends on time and increases with increasing value of it.

3.7.2 Inventory Equation for Receiving Periods

We worked in the same manner as in the previous derivations and reached the inventory equation for this period, the derivation can be found in Appendix A.6.

$$X_{nR+L} = X_0 - \sum_{i=0}^{L-1} (1 + i\alpha)\epsilon_{nR+k-i} + (R-1) \sum_{i=1}^{(n-1)R} \alpha\epsilon_i + (R-L)\mu \quad (3.29)$$

From Equation 3.29 and the assumption that ϵ_t is a time series of normally-distributed i.i.d random variables with mean and variance, zero and σ^2 respectively, we find the variance and mean of X_{nR+k}

$$\begin{aligned} V(X_{nR+k}) &= \sigma^2 \sum_{i=0}^{L-1} (1 + i\alpha)^2 + (R-1)^2\alpha^2(n-1)R\sigma^2 \\ Std(X_{nR+L}) &= \sigma\sqrt{\sum_{i=0}^{L-1} (1 + i\alpha)^2 + (R-1)^2\alpha^2(n-1)R} \\ E(X_{nR+L}) &= X_0 + (R-L)\mu \end{aligned}$$

Variance term depends on time. It can be seen from above equation that taking $L < R$ or $L > R$ does not change this since we reach the same result in both of the cases.

3.7.3 Inventory Equation for one Period Prior to Receiving Periods

The inventory equation for this period is derived as follows and derivation is provided in Appendix A.7.

$$X_{nR+k-1} = X_0 - \sum_{i=0}^{R+L-2} \{(1+i\alpha)\epsilon_{nR+k-1-i}\} - (L-1)\mu \tag{3.30}$$

From Equation 3.30 and the assumption that ϵ_t is a time series of normally-distributed i.i.d random variables with mean and variance, zero and σ^2 respectively, we find the variance and mean of X_{nR+L-1}.

$$\begin{aligned} V(X_{nR+k-1}) &= \sigma^2 \sum_{i=0}^{R+L-2} (1+i\alpha)^2 \\ Std(X_{nR+k-1}) &= \sigma \sqrt{\sum_{i=0}^{R+L-2} (1+i\alpha)^2} \\ E(X_{nR+k-1}) &= X_0 - (L-1)\mu \end{aligned}$$

For this period the variance term does not depend on time. This is due to the coincidence mentioned previously in the text, in determining inventory equation for the same period when $L < R$.

Here also we can easily say that does not make any difference to our calculations to take $L < R$ or $L > R$. And we arbitrarily chose $L < R$ case to work on in our following studies.

3.7.4 Safety Stock Policies

For the inventory control policy defined as in Equation 3.11, the inventory at every constant variance period is normally distributed with the same mean and standard deviation. As previously mentioned these periods are the special ones which have important characteristics. These are the only periods that have variances independent of time. From Equation 3.26 the standard deviation is a function of parameters for the demand process, the lead time, L and batching number, R. X_0 is the initial inventory level, and it is used to determine different service levels in the computational studies. It works as a control variable. In the system construction these constant variance periods have the smallest inventory levels in any cycle, and the expected value of inventory level, Equation 3.27, for these periods can also be thought as the safety stock for the system, which is defined at the beginning of the process. We use probability of not stocking out occasions as the service level determining factor. In doing this we set the expected value of inventory level at this constant variance period to a multiple of the standard deviation of it. This multiplying factor is defined from the cumulative distribution function for the standard normal random variable.

It can be shown as:

$$\begin{aligned} E[X_{nR+L-1}] &= Std[X_{nR+L-1}] \\ X_0 - (L-1)\mu &= z\sigma\sqrt{\sum_{i=0}^{R+L-2}(1+i\alpha)^2} \end{aligned} \tag{3.31}$$

Where z is the variable generated from cumulative distribution function of standard normal. In the computation study we define different service levels and use suitable z values for them to calculate necessary safety stock value for the retailer to serve for that service level.

3.7.5 Comparison of the Findings With Those of Graves (1999)

The retailer batches its orders for R periods, and gives order in every Rth period. This differentiates the order mechanism totally from the order process used for the retailer in Graves (1999). In his study the supplier has almost the same demand pattern as the one for retailer and the supplier can easily estimate its demand by employing exponential-weighted moving average, the same forecasting model for the retailer provided that it knows the demand parameters of the retailer. However, this can not be the case for our structure due to complications generated by order-batching. Equation for the order-batching,Equation 3.17 does not have the same structure with the demand model, so same method is not applicable for the supplier to forecast its own demand, it has more complicated demand process than the supplier of Graves (1999).

We construct the analytical structure having R, the batching number bigger than 1, and we have an operation in between 0 and R time period. So we can not take R equal to 1 directly in our analytical results to reduce the system for the retailer to the system defined for the retailer in Graves (1999). This is due to the fact that we aim to study with batching and see its effect, and R values bigger than 1 is used.

3.8 Supplier's Process

The retailer orders after observing demand for R periods. This accumulated demand is reached to the supplier in every R period of time, at the time of order for the retailer and if the on-hand inventory is enough to fill this quantity, the quantity is immediately sent to the retailer, otherwise the amount on-hand is sent and the remaining part is backordered. There is L time lag for the retailer to receive its order after the order is sent by the supplier. The supplier has a lead time defined as integer multiple of R. This is not a constraint but it eases the calculations. It can be understood that it does not change the processes to allow the supplier's lead time take values not integer multiple of R, since we mostly concentrate on the process between supplier and retailer not on the supplier's inventory process. The supplier sends the quantity ordered by the retailer if it has the quantity on the shelf at the time when it observes the order, so it does not matter whether the supplier make the quantity ready some time smaller than R, earlier than the time it observes the order, or it receives it at time of the order. So we can consider supplier working in R periods intervals.

3.8.1 Order-up-to Policy Used and Order Mechanism

The supplier observes the demand, q_{nR} in each R periods of time, determines that period's order quantity, p_{nR}, receives the order form KR periods ago, p_{nR-KR}, and then fills the demand from inventory. Any demand that can not be met from inventory is backordered.

In determining the supplier's order quantity for each period order-up-to policy is employed. This policy is used in Lee et al (2000). This is the one minimizing the total expected holding and shortage costs in the lead time, in period $(nR, nR + KR)$.

Order-up-to level for the supplier:

$$Y_{nR} = m_{nR} + k\sqrt{V_{nR}}$$

where

$$k = \phi^{-1}[p/(p+h)]$$

m_{nR} and V_{nR} are the conditional expectation and variance of the total demand over the lead time, respectively, where

$$\begin{aligned} m_{nR} &= E(\sum_{i=1}^{K} q_{(n+i)R} | q_{nR}) \\ V_{nR} &= \sigma^2 V^2 \\ &= Var(\sum_{i=1}^{K} q_{(n+i)R} | q_{nR}) \end{aligned}$$

By using this order-up-to level we can deduce the order quantity for the supplier as follows:

$$I_{nR} = Y_{(n-1)R} - q_{nR} \qquad (3.32)$$

$$p_{nR} = Y_{nR} - I_{nR} \qquad (3.33)$$

If we replace inventory variable in 3.33 with Equation 3.32 we have the following order equation.

$$p_{nR} = Y_{nR} - Y_{(n-1)R} + q_{nR}$$

To derive the equation for the order quantity of the supplier we have to first of all study on the value of order-up-to level, Y_{nR}.

Let B_{nR} be the total shipment quantity over the suppler's lead time, and analytically it can be shown as;

$$B_{nR} = \sum_{i=1}^{K} q_{(n+i)R}$$

We study this to define it in terms of the currently observed demand and process parameters only. In doing this we employed previously derived equation for q_{nR}. Especially the one in Equation 3.14.

$$\begin{aligned} q_{nR} &= \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\,\epsilon_{nR-i} + R\alpha \sum_{i=1}^{(n-1)R} \epsilon_i + R\mu \\ q_{(n+1)R} &= q_{nR} + \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+1)R-i} - \epsilon_{nR-i}\right) + R\alpha \sum_{i=(n-1)R+1}^{nR} \epsilon_i \end{aligned}$$

by using this above formula we wrote $q_{(n+i)R}$ with respect to q_{nR} and ϵ_i's, the derivation can be found in Appendix A.8.

$$q_{(n+i)R} = q_{nR} + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\}\left(\epsilon_{(n+i)R-j} - \epsilon_{nR-j}\right) + R\alpha \sum_{j=(n-1)R+1}^{(n+i-1)R} \epsilon_j$$

As we reach the compact equation for $q_{(n+i)R}$, the next step is to find a compact representation for B_{nR}, total shipment quantity over the supplier's lead time. We derived it as following and the derivation is provided in Appendix A.9.

$$\begin{aligned} B_{nR} &= Kq_{nR} + K\sum_{j=0}^{R-1} \{-(L-1+j)\alpha-1\}\,\epsilon_{nR-j} \\ &+ \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\}\,\epsilon_{(n+K)R-j} \\ &+ \sum_{j=0}^{R-1} [\{(L-1+j)\alpha+1+RK\alpha)\}\,\epsilon_{(n+1)R-j} \\ &+ \{(L-1+j)\alpha+1+R\alpha(K-1)\}\,\epsilon_{(n+2)R-j} \\ &+ \{(L-1+j)\alpha+1+R\alpha(K-2)\}\,\epsilon_{(n+3)R-j} \\ &+ \sum_{t=3}^{K-2} \{(L-1+j)\alpha+1+R\alpha(K-t)\}\,\epsilon_{(n+t+1)R-j} \end{aligned} \qquad (3.34)$$

We reached the compact form of the demand, observed by the supplier, during lead time. As previously defined, expected value and variance term of this compact term are to be used in the order-up-to level. These values are computed;

$$\begin{aligned}
m_{nR} &= E(\sum_{i=1}^{K} q_{(n+i)R}|q_{nR}) \quad \text{(since } E(\epsilon_i)\text{'s are zero this expected value is;)} \\
m_{nR} &= Kq_{nR} \\
V_{nR} &= Var(\sum_{i=1}^{K} q_{(n+i)R}|q_{nR}) \\
&= K^2 \sum_{j=0}^{R-1} \{R\alpha - (L+R-1+j)\alpha - 1\}^2 \sigma^2 \\
&+ \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}^2 \sigma^2 \\
&+ \sum_{j=0}^{R-1} [\{(L-1+j)\alpha + 1 + RK\alpha)\}^2 \\
&+ \sum_{j=0}^{R-1} \sum_{t=1}^{K-2} \{(L-1+j)\alpha + 1 + R\alpha(K-t)\}^2
\end{aligned}$$

When we take it to σ^2 parenthesis;

$$\begin{aligned}
V_{nR} &= \sigma^2 \sum_{j=0}^{R-1} [K^2 \{-(L-1+j)\alpha - 1\}^2 + \{(L+R-1+j)\alpha + 1\}^2 \\
&+ \sum_{t=0}^{K-2} \{(L-1+j)\alpha + 1 + R\alpha(K-t)\}^2] \qquad (3.35)
\end{aligned}$$

This variance term does not depend on time, so it is easier to study as compared to the case for the retailer. After finding the necessary terms from the demand observed during lead time, we studied with the order quantity derivation of the supplier. In order to do this, we first need to study with the order-up-to level then the formula that gives the order equation for the supplier.

$$p_{nR} = Y_{nR} - Y_{(n-1)R} + q_{nR} \qquad (3.36)$$

$$Y_{nR} = m_{nR} + kV_{nR} \qquad (3.37)$$

we place (3.37) into (3.38);

$$p_{nR} = (m_{nR} + kV_{nR}) - (m_{(n-1)R} + kV_{(n-1)R}) + q_{nR}$$

Here the terms containing plus and minus variance drop since the variance does not depend on time.

$$p_{nR} = m_{nR} - m_{(n-1)R} + q_{nR}$$

where $m_{nR} = Kq_{nR}$ and $m_{(n-1)R} = Kq_{(n-1)R}$

$$\begin{aligned} p_{nR} &= Kq_{nR} - Kq_{(n-1)R} + q_{nR} \\ &= K(q_{nR} - q_{(n-1)R}) + q_{nR} \end{aligned} \tag{3.38}$$

This equation can be interpreted as follows. The supplier sees the current demand and orders by adjusting this value with the difference between this new value and the previous one multiplied with lead time. This adjustment helps the supplier follow the changes in the demand it observes and modify its order quantity accordingly. Here the supplier works much more similar to the retailer in Graves (1999). They both use their current observed demand and adjust it. The only difference occurs in determining the adjustment policy.

3.8.2 Safety Stock Determination

In determining the safety stock for the supplier the variance of the demand observed by the supplier during lead time is taken as a base. After that same method, which is applied for the retailer, is applied. The quantity in the order-up-to level equation, kV_{nR}, is the safety stock value. And the starting inventory for the supplier in the computational studies is the expected order-up-to quantity.

We use standard normal distribution in our service level determination as in the case for the retailer. In doing this we take different probability of not stockout occasion values theoretically and try to observe the performance of the system in these conditions accordingly. The logic behind using the variance of the demand observed during lead time is more general. The variance of the demand during lead time is the cause that makes the performer run out of stock, so modifications of this variance value is the best way to determine the safety stock.

3.8.3 Inventory Variable for the Supplier

In this part we study on the inventory requirements for the upstream stage. P_{nR} is the order placed in period nR by the upstream to its supplier. The lead-time for replenishment to the upstream sage is KR: An order placed in period nR is for delivery in period $(n+K)R$. As it is mentioned previously; in each period nR, the upstream first observes q_{nR}, determines this period's order (p_{nR}), receives the order from KR periods ago ($p_{(n-K)R}$), and then fills the downstream order from inventory. Any demand that can not be met from inventory is backordered. The inventory equation mentioned is

$$XS_{nR} = XS_{(n-1)R} - q_{nR} + p_{(n-K)R} \tag{3.39}$$

where XS_{nR} denotes the on-hand inventory (or backorders) at the end of period nR. We assume an initial inventory level XS_0, and that $p_{nR} = R\mu$ for $nR <= 0$.

When we put order equation, Equation 3.38, into its place in the Equation 3.39, and continue on backward substitution we reach the following formula for the inventory equation of the supplier. The derivation of the equation can be seen in Appendix A.10.

$$\begin{aligned} XS_{nR} &= XS_0 + K[\sum_{j=0}^{R-1} \{(L-1+j)\alpha+1\}\,\epsilon_{(n-K)R-j}] \\ &- \sum_{j=0}^{R-1}[\{(L+KR-1+j)\alpha+1\}\,\epsilon_{(n-K+1)R-j} \\ &+ \{(L+(K-1)R-1+j)\alpha+1\}\,\epsilon_{(n-K+2)R-j} \\ &+ \sum_{t=2}^{K-1} \{(L+(K-t)R-1+j)\alpha+1\}\,\epsilon_{(n-K+t+1)R-j} \end{aligned}$$

We found the inventory random variable for the supplier in terms of initial inventory and the random noise terms. Now we are ready to take the expected value and variance of it. $E[\epsilon_{nR}] = 0$ and $Var[\epsilon_{nR}] = \sigma^2$.

$$\begin{aligned} E[XS_{nR}] &= XS_0 \\ Var[XS_{nR}] &= K^2 \sum_{j=0}^{R-1} \sum_{t=0}^{K-1} \{(L+(K-t)R-1+j)\alpha+1\}^2 \end{aligned}$$

We require more upstream stage safety stock as compared to classical textbook case of stationary demand ($\alpha = 0$). Also in the nonstationary demand environment the relationship between the upstream lead-time and the upstream safety stock becomes convex. This is so due to increasing rate of increase in safety stock with lead-time after some point on.

Again for the case of having nonstationary demand ($\alpha > 0$), the standard deviation of the upstream stage does not only depends on the upstream lead-time and the α value of the exogenous demand but also on the downstream lead-time. Therefore in the nonstationary environment the downstream lead-time has effect on both determining the safety stock requirement of downstream and that of supplier. This can be thought as a clue in improvement studies of these kinds of systems, there is more improvement possibilities in working to reduce the lead-time of the downstream than paying same attention on the upstream.

And also special to our case the variance depends on the value of the batching number, and it increases with the increasing value of it. This is stated in the literature as one of the main causes of the bullwhip effect.

3.8.4 Information Sharing

The previous calculations related with the supplier's process consider the case where observed demand information is not shared between the downstream and the upstream. The upstream stage only knows the demand process and the parameters of it. Now we extend the study to the case where the downstream provides the upstream with the last demand data additionally. So the upstream knows; the order quantity of the downstream, the parameters of the demand observed by the downstream, and last R periods' noise terms(through the sharing of demand values for last batching periods) while determining the order-up-to level at the end of period nR. In doing this the upstream uses the demand observed during lead time and takes the expected value and variance of it accordingly. The compact equation for this demand was found in the Equation 3.34 as:

$$B_{nR} = Kq_{nR} + K \sum_{j=0}^{R-1} \{-(L-1+j)\alpha - 1\} \epsilon_{nR-j}$$

$$
\begin{aligned}
&+ \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\} \epsilon_{(n+K)R-j} \\
&+ \sum_{j=0}^{R-1} \sum_{t=0}^{K-2} \{(L-1+j)\alpha+1+R\alpha(K-t)\} \epsilon_{(n+t+1)R-j}
\end{aligned}
$$

When the upstream has the information of the last demand data, the upstram knows the value of $\sum_{j=0}^{R-1} \epsilon_{nR-j}$. So the expected value of this term in B_{nR} is no more zero and the variance of it is zero now. By considering these we study the expected value and variance equations for the case of information sharing.

$$
\begin{aligned}
m_{nR}^* &= Kq_{nR} + K \sum_{j=0}^{R-1} \{-(L-1+j)\alpha-1\} \epsilon_{nR-j} \\
V_{nR}^* &= \sigma^2 (\sum_{j=0}^{R-1} [\{(L+R-1+j)\alpha+1\}^2 \\
&+ \sum_{t=0}^{K-2} \{(L+R(K-t)-1+j)\alpha+1\}^2
\end{aligned}
$$

m_{nR}^* and V_{nR}^* are the conditional expectation and variance of the total demand over the lead time when information is shared, respectively.

These values are compared with the previous values of them, when information is not shared. And after making necessarry modifications, the equations of conditional expectation and variance of the total demand over the lead time when information shared can be represented by the equations of them when information is not shared as follows:

$$
\begin{aligned}
m_{nR}^* &= m_{nR} + K \sum_{j=0}^{R-1} \{-(L-1+j)\alpha-1\} \epsilon_{nR-j} \\
V_{nR}^* &= V_{nR} - \sigma^2 \sum_{j=0}^{R-1} K^2 \{-(L-1+j)\alpha-1\}^2
\end{aligned}
$$

It can be observed from above equations that the variance does not depend on time. In addition, note from above equation that $V_{nR}^* < V_{nR}$. Thus, information sharing would reduce the variance of the total shipment quantity over the upstream stage's lead-time, KR. In this case, the upstream stage's order-up-to level is given by:

$$
Y_{nR}^* = m_{nR}^* + kV_{nR}^*
$$

and its order quantity:

$$p^*_{nR} = Y^*_{nR} - Y^*_{(n-1)R} + q_{nR}$$

We put the new values into their places to see the order quantity of the upstream stage when it receives additional demand information from the downstream stage.

$$p^*_{nR} = (m^*_{nR} + kV^*_{nR}) - (m^*_{(n-1)R} + kV^*_{(n-1)R}) + q_{nR}$$

since variance does not depend on time the variance terms cancel out

$$p^*_{nR} = m^*_{nR} - m^*_{(n-1)R} + q_{nR}$$

replace m^*_{nR} values with their compact forms

$$\begin{aligned} p^*_{nR} &= (Kq_{nR} + K\sum_{j=0}^{R-1}\{-(L-1+j)\alpha - 1\}\epsilon_{nR-j}) \\ &- (Kq_{(n-1)R} + K\sum_{j=0}^{R-1}\{-(L-1+j)\alpha - 1\}\epsilon_{(n-1)R-j}) \\ &+ q_{nR} \\ &= Kq_{nR} - Kq_{(n-1)R} + q_{nR} \\ &+ K\sum_{j=0}^{R-1}\{-(L-1+j)\alpha - 1\}(\epsilon_{nR-j} - \epsilon_{(n-1)R-j}) \end{aligned}$$

First line of the this order equation is the order equation of the order quantity for the upstream when demand information is not shared, Equation 3.38. The remaining part of the above equation is therefore added due to the sharing of information.

$$p^*_{nR} = p_{nR} + K\sum_{j=0}^{R-1}\{-(L-1+j)\alpha - 1\}(\epsilon_{nR-j} - \epsilon_{(n-1)R-j}) \qquad (3.40)$$

One important observation here is the case when $\alpha = 0$:

$$\begin{aligned} p^*_{nR} &= p_{nR} + K\sum_{j=0}^{R-1}\{-1\}(\epsilon_{nR-j} - \epsilon_{(n-1)R-j}) \\ &= Kq_{nR} - Kq_{(n-1)R} + q_{nR} - K\sum_{j=0}^{R-1}(\epsilon_{nR-j} - \epsilon_{(n-1)R-j}) \qquad (3.41) \end{aligned}$$

compact form for q_{nR} from Equation3.12 is:

$$q_{nR} = \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\epsilon_{nR-i} + R\alpha \sum_{i=1}^{(n-1)R} \epsilon_i + R\mu$$

when $\alpha = 0$ it becomes:

$$q_{nR} = \sum_{i=0}^{R-1} \epsilon_{nR-i} + R\mu$$

and the summation in the above equation is equal to

$$\sum_{i=0}^{R-1} \epsilon_{nR-i} = q_{nR} - R\mu$$

let's put this value into its places in Equation 3.41

$$\begin{aligned} p^*_{nR} &= K(q_{nR} - q_{(n-1)R}) + q_{nR} - K[(q_{nR} - R\mu) - (q_{(n-1)R} - R\mu)] \\ &= K(q_{nR} - q_{(n-1)R}) + q_{nR} - K(q_{nR} - q_{(n-1)R}) \\ p^*_{nR} &= q_{nR} \end{aligned}$$

When the process is stationary ($\alpha = 0$), and the demand data is shared between the two players of the system, the upstream orders the same quantity that it observes as demand from the downstream. It orders the same quantity which is demanded from it.

In this point our system differentiates from Graves (1999), in that one of the important observations of his study is: 'There is not any benefit from providing the upstream stage with additional information about the exogenous demand or about the order process of the downstream stage'. Upstream stage needs to know the parameters of exponential-weighted moving average (or equivalently the IMA(0,1,1) demand process), and needs to observe its demand process, q_{nR}. This observation was reached by seeing the result of the order process of the upstream stage, it was very similar to that of the downstream stage and the downstream forecast was the same as the upstream forecast, and this was an unbiased forecast both for d_t, and q_{nR}. In addition the demand process seen by the upstream was also an IMA(0,1,1) process.

For our case information sharing increases the performance of the system, especially the performance of the supplier. So it is important for the upstream stage to observe extra demand information of the downstream stage. At this point the supplier must provide some incentives for the retailer to motivate it to share this extra data.

CHAPTER 4

SIMULATION STUDIES

Throughout Chapter 3 we analytically study the performance of the retailer and the supplier separately. We derive equations for the retailer assuming that a supplier, which has infinite capacity, supplies to the retailer and the supplier orders from an outside source. We need to use simulation to observe the retailer and the supplier working in coordination.

In this chapter we simulate the system to observe the performance of the retailer and the supplier in coordination as shown in Figure 4.1. In the figure, the retailer faces the end customer demand, d_t, places an order, q_{nR}, to the capacitated supplier and the supplier orders, P_{nR}, from an outside source. We use the derived equations in the previous chapter and make modifications to connect the retailer to the supplier: The retailer can get what it orders only if the supplier has enough inventory to fulfill the amount ordered. The supplier checks the on-hand inventory and sends the ordered amount if it is possible, if not the supplier sends the positive amount of inventory on-hand, and backorders the remaining amount.

We observe the system performance in practice by using simulation model. We use performance measures, such as time to satisfy backorder, backorder occasions, average inventories, etc., to interpret the general function of the system. The long-run performance of the system is also observed by increasing the replication

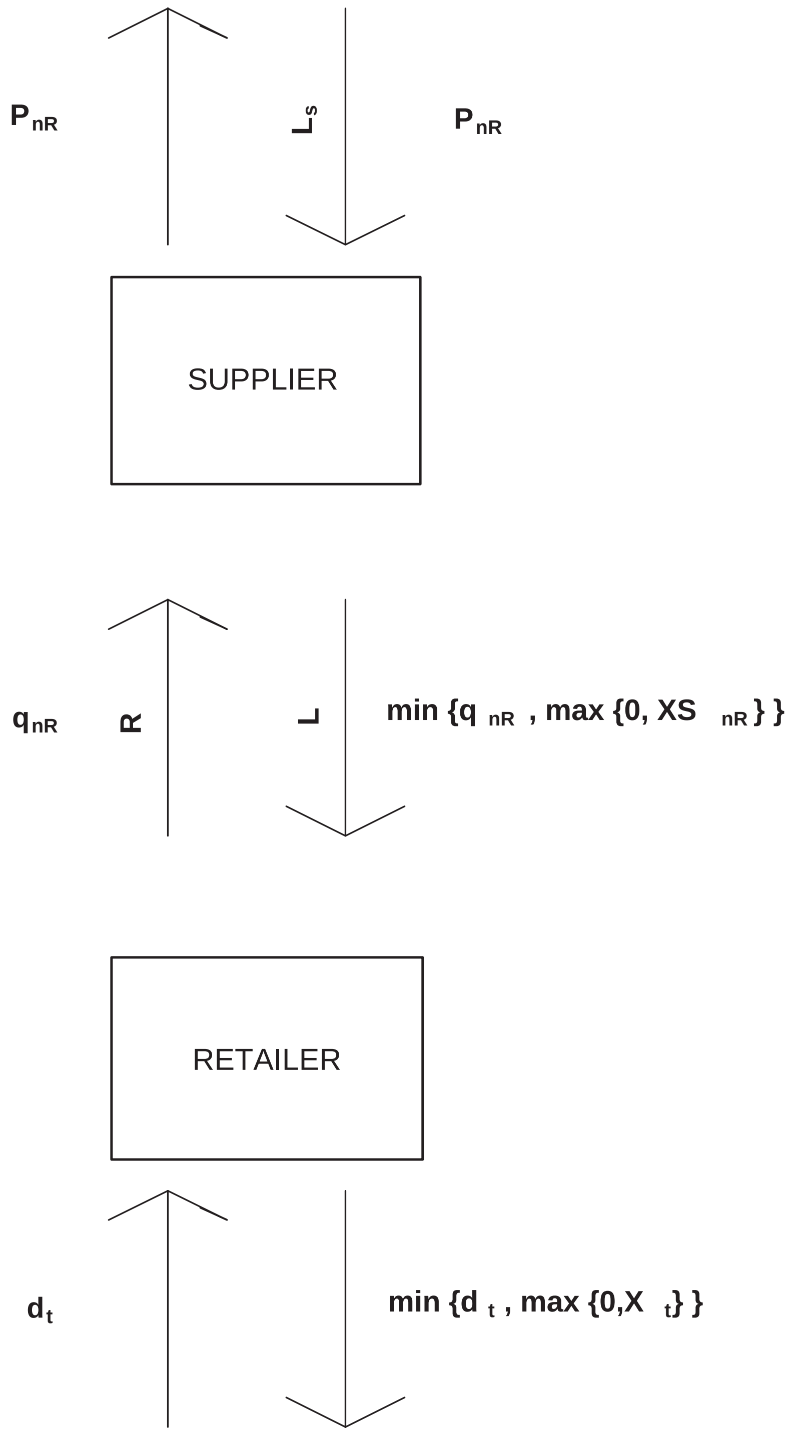

Figure 4.1: Coordinated System

lengths for the runs. This observation is used to determine a suitable rolling horizon for the designed system. As mentioned previously variance of the retailer's inventory equation, e.g. Equation 3.23, for the periods other than the constant variance period is time dependent and when we run the model for longer time periods, this dependence results in higher variances, and increases the difficulty in interpretation of the results. In addition, we aim to observe effect of order-batching and information sharing on the system when the retailer and the supplier works in coordination.

In safety-stock determination for the retailer, the variance of the inventory equation for the constant variance period is used. This safety stock value is used in determination of the retailer's service level.

We design mainly three different models; the retailer works with uncapacitated supply, the retailer and the supplier work in coordination, and the retailer and supplier both work in coordination and the retailer shares recent demand information with the supplier. The first model is used to have insights about the time effect on the analytically derived equations for the downstream stage. It is difficult to interpret the behavior of the equations derived analytically due to the complications resulting from nonstationarity. We analyze outputs of this model and equations are drawn with respect to time to analyze their structure. We study with different α values and run lengths to see their effects.

The second model is directly related with the coordination of the supplier and the retailer. We modified the model by introducing the equations derived for the supplier; its inventory equation, and order equation derived from an order-up-to policy. In this model the retailer is not free to receive any amount it orders since there is an other process that runs in parallel for the supplier and the retailer can only receive from the supplier if it has inventory on the self, otherwise the retailer needs to wait to receive the order till the supplier replenishes its inventory on-hand. In this model the supplier observes only retailer's order.

Last modification of the model adds processing of shared information to the second model. In this model the supplier is also provided with the last demand data of the retailer. The supplier uses this extra demand information in the inventory control policy. The change that is brought by the usage of extra demand data, can be seen from the Equation 3.40, where order equation for the supplier, when information is shared, is equal to the order equation for the supplier when information is not shared plus an other term. Analytically we see the effect of information sharing and we also aim to observe it with the simulation modelling.

In the remaining part of the chapter, we mention about the simulation model, including verification efforts, description of the performance measures observed, and issues of output analysis. We will conclude the chapter with the presentation of the results for each performance measure.

4.1 Modeling

The model is written in Visual C++. We use LCG random number generator, prime modulus multiplicative linear congruential generator based on Marse and Roberts' portable FORTRAN random-number generator UNIRAN. Then we use a function, provided in Law and Kelton (2000), to obtain normally distributed random variables by transforming the generated random numbers.

In this part of the chapter we will demonstrate the model; order of events, flow of the program, the modules of the model, and different parameters employed.

4.1.1 Order of Events

- Demand Generation
  - LCG random number generator generates random numbers
  - These random numbers are transformed to normally distributed noise terms, ϵ_i's
  - Demand is generated by using Equation 3.2 and noise terms
  - Negatively generated demand values are equated to zero without disturbing the general generation process of the model: The generation process uses demand equation, (3.2), that use ϵ_i's to generate the demand values.

- Forecast Generation
  - Demand values are imported from the demand generation module
  - These demand values are put into Equation 3.4 to generate forecast values.
- The Retailer's Order Generation

  Generated demand and forecast values are put into Equation 3.11 to generate the order amount.
- The Supplier's Order Generation

  The retailer's order amount is processed in Equation 3.38 to get the values for the supplier's order amount.
- The Supplier's Inventory Generation
  - The supplier starts the run with inventory on-hand value of order-up-to level quantity
  - At time zero the supplier fills the first demand of the retailer
  - The supplier's order amount and the demand that is observed by the supplier are imported into Equation 3.39 to get the values for the inventory random variable of the supplier.
- The Retailer's Inventory Generation
  - The supplier's inventory on hand and order values are checked to see whether it can fill the retailer's demand or not
  - Then the general equation, (3.8), is employed to get the values of inventory random variable of the retailer.

4.1.2 Parameters of the Model

In this section of the chapter, we will talk about the parameters used in the model. One example value for each parameter will also be given in the table after definition part. In the following sections of the chapter different values of these parameters will be used to observe the system performance under different combinations. The used parameters are:

μ: the mean of demand process and forecast model.

σ: the standard deviation of the noise terms, ϵ_i's.

α: the parameter for the demand process and the forecast model.

R: the batching number, a fixed number of periods that the downstream stage accumulates its observed demand before placing an order to the upstream stage.

L: the lead-time for the downstream stage. The time passes between placing of an order and receiving that order for the downstream stage.

β_r: the service level for the retailer; probability of no stockout occasions for the retailer.

Xo: the initial inventory value of the downstream stage. It is determined according to the desired service level in the model.

K: the number that determines the lead-time for the upstream stage while multiplied with R.

β_s: the service level for the supplier; probability of no stockout occasions for the supplier.

XSo: the initial inventory for the upstream stage

Example values for these parameters are provided in the Table 4.1 to give insights about their usage.

4.1.3 Settings for the Parameters

In this section we will define settings for the parameters defined in the previous section. Some of these parameters are free to be chosen by the programmer and some are to be determined through other parameters. We define the system by giving suitable values to the independent parameters to have the expected performance of the system. Characteristics of the parameters for the supplier and the retailer can be explained as follows:

- Parameters for the retailer are; R, L, X_0, β_r. In this set R, L and β_r are

Table 4.1: Example Values for the Parameters

parameter	value		
μ	10		
σ	1		
$replen$	20		
$precision$	0.1		
R	2	3	4
L	0	2	4
α	0	0.1	0.15
K	1	2	3

free to be chosen and choices of these parameters affect the values of the dependent parameters.

X_0 is determined through other parameters. The parameters that involve in determining the value of X_0 are: lead-time for the retailer, L, mean demand value, μ, batching number, R, inertia parameter of demand process, α, variance of ϵ_i's, σ^2, and service level for the retailer, β_r. In setting the value of X_0 we use the constant variance period's calculations. The expected value of the inventory equation in the constant variance period is defined as the safety stock value, since it is the minimum value on-hand throughout the cycle, just before replenishing the inventory. It is found with Equation 3.27 that the value of this expectation is $X_0 - (L-1)\mu$ and we equate this value to the safety stock value. The safety stock is defined using this period's variance and the standard normal distribution function, $z\sigma\sqrt{\sum_{i=0}^{R+L-2}(1+i\alpha)^2}$ where $\Phi(z)$ is the cumulative distribution function for the standard normal random variable. Under these definitions X_0 is found by using the Equation 3.31 as follows:

$$X_0 = (L-1)\mu + z\sigma\sqrt{\sum_{i=0}^{R+L-2}(1+i\alpha)^2}$$

- Parameters for the supplier are; K, XS_0, β_s. In this set K and β_s are free to be chosen. Choices for these parameters affect the values of dependent parameters for the supplier.

As in the case for determining the value of X_0, we use independent parameters to reach the value of supplier's initial inventory, XS_0. The prameters used for this purpose are: lead-time value for the supplier, KR, lead-time value for the retailer, L, batching number, R, mean demand value, μ, demand parameters, α and σ, and the service value for the supplier, β_s. In determining the value of initial inventory for the upstream stage we use an order-up-to level policy and the variance term of the demand observed during lead-time of the upstream stage. The supplier starts the system with the inventory value of expected order-up-to level plus a safety. Safety stock is determined through the use of the variance of the demand, Equation 3.35, observed during the lead-time of the supplier. We add this safety to expected order-up-to level for the supplier, $KR\mu$, and have the initial inventory on-hand value for the supplier as;

$$XS_0 = KR\mu + zs\sqrt{V_{nR}} \tag{4.1}$$

where $\Phi(zs)$, is the cumulative distribution for the standard normal random variable.

4.2 Model Verification

This section is devoted to the verification studies of the model. We will talk about the tests we perform in the following subsections; debugging and runs with deterministic input.

4.2.1 Debugging

We debug each module of the program in order to test if they run properly. We import the values of all the variables generated by the model into an excel sheet, and check one by one all the periods for all of the replications. Example tests of the models are provided in Tables B.1, B.2, and B.3 of Appendix B.

In debugging, we first take the normally generated ϵ_i's in the first column of an excel sheet. And these ϵ_i values are tested with graphical and statistical methods to see whether or not they are truly generated from normal distribution with desired mean and variance. Then the values of each variable generated as output to the model are imported into the same excel sheet. Known analytical equations for the demand, forecast, inventory random variable for the retailer, order equation for the retailer, inventory random variable for the supplier and order equation for the supplier are formulated with the help of excel functions and the results generated by excel are compared with the original model output. The results are positive, the program is successful to pass these tests.

4.2.2 Runs with Deterministic Input

We take $\alpha = 0$, to have a stationary i.i.d. demand process with mean μ, and variance σ^2. We are more familiar with this process to interpret and see whether the process is working as intended or not than the original nonstationary demand process used throuhgout the study. In this case we take different combinations of other parameters, and observe the performance of the model.

In another trial σ value is taken to be zero. This means that the system works with deterministic customer demand process, having the mean value μ as its constant value. This process removes the complications resulting from the demand process and it becomes easy to observe how other variables of the system perform. We observe the function of the model by giving different values to the other parameters in the parameter set.

These tests with deterministic input show that the model runs as designed. Example results for this subsection can be seen in Appendix Tables B.4 - B.7.

4.3 Performance Measures

After verifying that the model runs accuratly in the previous section we design performance measures to interpret how the system functions. We define different parameter combinations and watch the effects of a change in any parameter, by observing the change it brings over the performance measures.

- Time to satisfy backorder: This is defined as the average number of periods that pass between the period that the inventory on-hand value of the retailer or the supplier becomes negative and the period that this value becomes positive. The program counts the number of times that inventory level hits below zero with one variable, and the time passes during these occurrences for the inventory variable to recover with an other variable. Then by using these two variables the performance measure is calculated.

  This performance measure shows the ability of policies employed by the retailer and supplier for backorder recovery.

- Backorder occasions: The model counts the number of times inventory variable of the retailer and the supplier drops below zero. We observe these values and try to reach a conclusion about the performance of the retailer and the supplier.

- Difference between set and observed service levels: We define different service levels for the retailer and the supplier. The parameters are arranged according to theoretic values and run results are compared with the theoretically expected ones. We see how the coordination of the supplier and the retailer effects their system performance by comparing the separetely set sevice levels for each with the service levels resulting from the coordinated work of the supplier and the retailer.

- Average on-hand inventory and backorder: The program accumulates the positive inventory values both for the upstream stage and for the downstream stage. Then we divide these values by replication length to have the desired performance measure for average on-hand inventory. This performance measure is very important for managerial purposes since inventory is cost and it must be kept under control. By observing values of it under different conditions desired system can be constructed.

The program also accumulates the negative quantities of the inventory random variable for the retailer and the supplier and divide this to replication length to find average backorder. Firms pay high attention to this. They may lose their loyal customers, and may pay high penalty costs.

4.4 Analysis of System Output

4.4.1 Precision

We aim to achieve 90 percent relative precision on the value of the retailer's average inventory in the constant variance period. In order to approximately satisfy the 90 percent precision, we use the following procedure:

- Make an initial n replications and find the estimate for mean and variance of the constant variance period's inventory. Assume these values remain constant as n increases.
- Let $0 < \gamma < 1$ be desired level of relative precision.
- Let $n_r^*(\gamma)$ be the total number of replications to obtain desired precision, γ.
- Then the required number of replications is found as minimum i found from the equation;

$$n_r^*(\gamma) = min\left\{i >= n; (t_{i-1,1-\alpha/2}\sqrt{S^2(n)/i})/|\bar{X}(n)| <= \gamma\right\}$$

Where the estimated mean and variance of the constant variance period's inventory are as follows:

$$\begin{aligned} S^2(n) &= [1/(n-1)]\sum_{j=1}^{n}(\bar{X}_j - \bar{X}(n))^2 \\ \bar{X}(n) &= [1/n]\sum_{j=1}^{n}\bar{X}_j \end{aligned}$$

$\bar{X}_j$ is the average inventory value in the constant variance period for the replication j. And $\bar{X}(n)$ is the average of replication averages, $\bar{X}_j$'s.

After finding the required number of replications the program makes that many replications to reach the desired results for the observed values.

4.4.2 Confidence Interval

A confidence interval is constructed for the mean of the inventory random variable value of the constant variance period, X_{nR+L-1}. The theoretical expected value for the constant variance period's inventory is checked with the bounds of the intervals that are constructed through the program run.

After constructing confidence intervals for a defined number of runs, we construct a confidence interval for the true coverage of the confidence intervals constructed. The following procedure is applied:

- Make l many simulation runs and construct a confidence interval for each run.

- Let yl be the number of confidence intervals that cover the theoretical mean for the inventory value of the constant variance period.

- This yl is binomially distributed with probability of success p (a confidence interval contains the observed mean with probability p).

- Expected value of yl is $l * p$ and variance of it is $l * p * (1 - p)$

- Let $\hat{p} = yl/l$ then expected value of $\hat{p}$ is $lp/l = p$

Then the confidence interval for the true coverage, p, is;

$$\hat{p} + (-)Z_{1-\alpha/2}\sqrt{\hat{p}(1-\hat{p})/l}$$

Results for different parameter sets can be found in Appendix Tables B.8, and B.9.

4.4.3 Long Run Performance of the System

Nonstationary customer demand and the order-batching policy applied by the retailer increase the complexity of the system. Analytical derivations in Chapter 3 shows that inventory variables for the peiods other than constant variance peiod have time dependent variances. The time dependency of this variances increase the difficulty of system planning for long rolling horizons. The variances increase with increasing value of time and explanation of the performance of the variables get difficult, if the system is observed far away in the future.

In this part of the study we aim to determine suitable values for the replication length and demand parameter α in order to have meaningful run results that can be explained. The model is run with different α values and replication lengths to decide on the values of them. Figure 4.2 shows the value of retailer's inventory with respect to time for number of cycles in a replication,$n = 1500$, batching number, $R = 4$, and demand parameter, $\alpha = 0.5$ in a specific replication. In Figure 4.3 the value of α in the previous figure is changed to 0.15 and the value of inventory random variable variates less. We also decrease the replication length, $n * R$, in Figure 4.4 to 4000. After observing the performance of the inventory variable with different parameter sets by drawing the value of the variable with time, we decide on the suitable values of the parameters as $\alpha =< 0.15$ and $n <= 1000$.

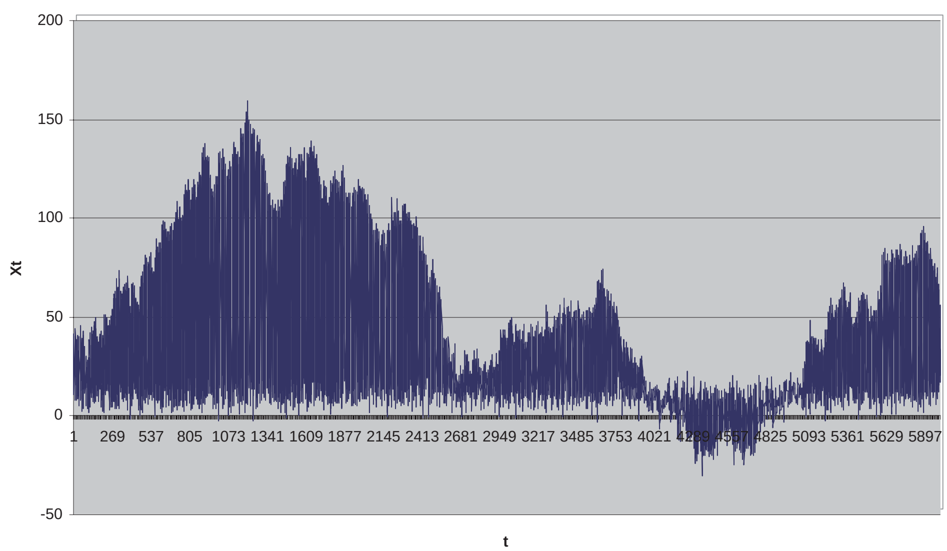

Figure 4.2: X_t for $\alpha = 0.5$ and $n = 1500$

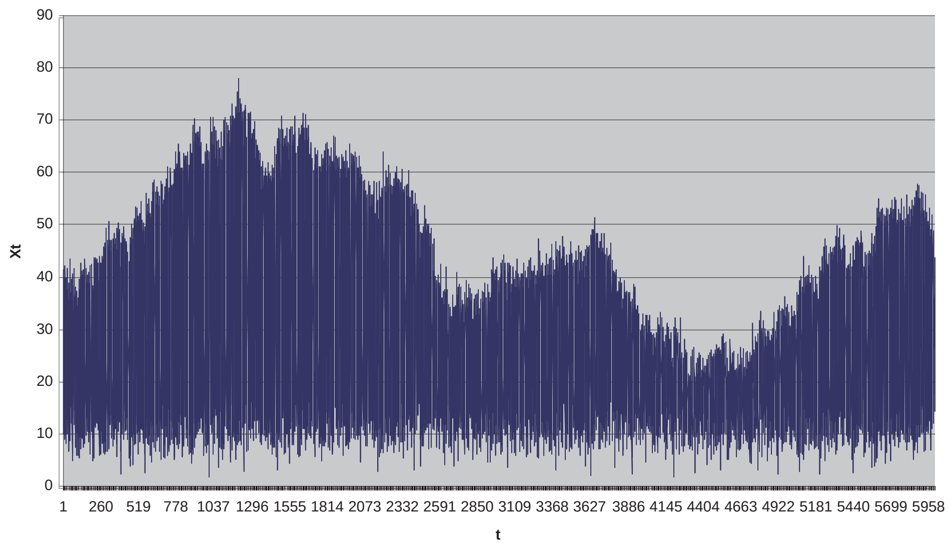

Figure 4.3: X_t for $\alpha = 0.15$ and $n = 1500$

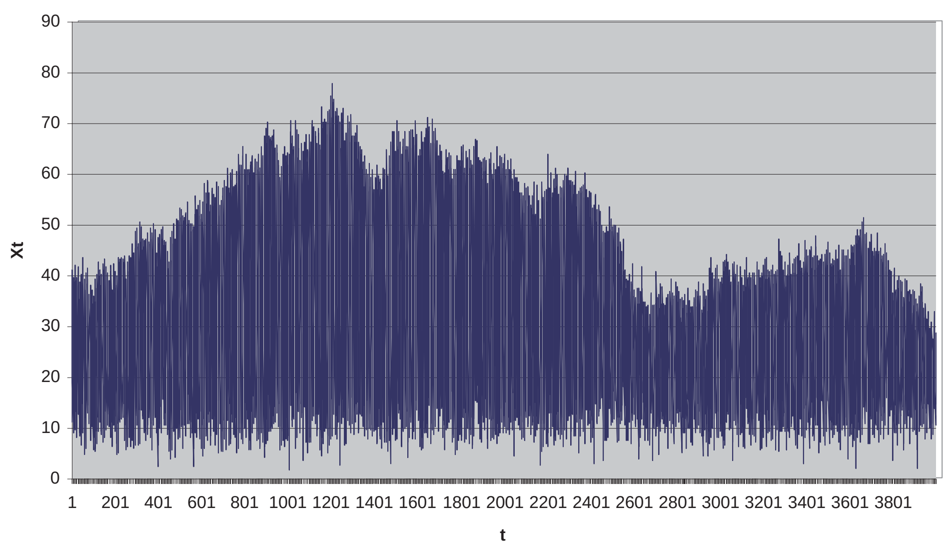

Figure 4.4: X_t for $\alpha = 0.15$ and $n = 1000$

For the remaining part of model runs we take replication length as 1000 and the value of demand parameter, α smaller than or equal to 0.15.

4.5 Results

This section is devoted to the presentation of the simulation study. We mainly concentrate on the models that we have defined; in the first model the supplier and the retailer work in coordination and the supplier observes only the order placed by the retailer. In the second model the supplier observes the last customer demand data in addition to the order placed by the retailer.

The paramater set is defined as in Table 4.1 and runs are made by changing the values of the parameters in this set (L, probability of nonstockout occasions for the supplier, probability of nonstockout occasions for the retailer). The effects of the changes in the values of the parameters on the results of the model are monitored by observing the performance meaasures defined in this chapter.

In the remaining part of this section we will demostrate the results for each performance measure separately. We choose different values for the parameters in the parameter set and provide the values of performance measures in tabulated forms. Coding of the values for the parameter set is as follows: L has values $0, 2, 4$ and probability of nonstockout occasions have values .85,.90,.95,.98. We run the rogram by fixing the following values of the other parameters; $\alpha = 0.1$, $\sigma = 1$, $n = 1000$, $R = 4$, $K = 2R$.

4.5.1 Time to Satisfy Backorder

This performance measure gives insight about how fast the system recovers, or how fast the retailer or the supplier responds to a backorder occasion on the average. Table 4.2 provides observed values of this performance measure both for the retailer and the supplier. The parameter set has different values as defined in the beginning of this section.

Values in the Table 4.2 show that keeping all the other variables fixed if L, lead-time value for the retailer is increased, time to satisfy backorder value increases as well. Bigger lead-time values for the retailer increase the time to recover negative inventory, since it can not replenish its inventory fast.

Another observation from the table is that; increase in the supplier's theoretically set service level decreases the time to satisfy backorder values both for the supplier and the retailer. When the supplier is designed to have less probability of stockout occasions, recovery period becomes shorter as well.

4.5.2 Backorder Occasions

In setting safety stock values for the retailer and the supplier we use service level based on probability of no stockout occasions. We derive analytical equations assuming that the retailer and the supplier work separately. An infinite capacity source supplies both to the retailer and the supplier. These theoretical values are used as input to the model where the supplier and the retailer work in coordination. The changes in the observed values of the service level due to different combinations of the parameters are analyzed in Table 4.3.

It can be seen from Table 4.3 that when the theoretical service level for the supplier is increased the observed service level for the retailer also increases although its theoretically set value for the service level is fixed. This is due to the fact that we set the theoretical value for the retailer assuming an uncapacitated supplier for it. It is observed that when the supplier serves with high service level, close to zero stockout probability, the observed value of the service level for the retailer gets closer to its theoretical value. An example from the table to explain this is: for the parameter values [0,.85,.95] observed values for service levels are .85233 and .88544, respectively. The retailer's observed value is smaller than its theoretical value since the supplier works with probability of nonstockout occasions of .85233. When the supplier works with high service level as in the [0,.98,.95] entry of the table we see that observed values for service levels become .98100 and .94640, respectively. The retailer's observed service level is closer to the theoretically set value.

Table 4.2: Time to Satisfy Backorder

Parameter Set ($[L, \beta_s, \beta_r]$)	**Supplier's Time to Satisfy Bacorder**	**Retailer's Time to Satisfy Bacorder**
[0,.85,.95]	1.11989	1.01178
[0,.90,.95]	1.07250	1.00629
[0,.95,.95]	1.02104	1.00485
[0,.98,.95]	1.01964	1.00000
[2,.85,.95]	1.10992	1.02064
[2,.90,.95]	1.06077	1.01278
[2,.95,.95]	1.01646	1.00635
[2,.98,.95]	1.01250	1.00813
[4,.85,.95]	1.10682	1.03371
[4,.90,.95]	1.04914	1.01425
[4,.95,.95]	1.01440	1.00625
[4,.98,.95]	0.91667	1.00333
[0,.85,.98]	1.11989	1.01061
[0,.90,.98]	1.06764	1.01294
[0,.95,.98]	1.02156	1.01056
[0,.98,.98]	1.01964	1.00000
[2,.85,.98]	1.10136	1.03557
[2,.90,.98]	1.05826	1.00681
[2,.95,.98]	1.01646	1.01548
[2,.98,.98]	1.01250	1.00833
[4,.85,.98]	1.09917	1.03226
[4,.90,.98]	1.04326	1.01274
[4,.95,.98]	1.01440	1.00455
[4,.98,.98]	0.91667	1.00455

Table 4.3: Backorder Occasions

Parameter Set ($[L, \beta_s, \beta_r]$)	**Supplier's Probability of No stockout**	**Retailer's Probability of No stockout**
[0,.85,.95]	0.85233	0.88544
[0,.90,.95]	0.90382	0.91206
[0,.95,.95]	0.94887	0.93461
[0,.98,.95]	0.98100	0.94640
[2,.85,.95]	0.85935	0.89745
[2,.90,.95]	0.90927	0.91955
[2,.95,.95]	0.95220	0.93820
[2,.98,.95]	0.98160	0.94680
[4,.85,.95]	0.86724	0.90533
[4,.90,.95]	0.91456	0.92489
[4,.95,.95]	0.95500	0.94320
[4,.98,.95]	0.98260	0.94840
[0,.85,.98]	0.85233	0.92600
[0,.90,.98]	0.90387	0.95032
[0,.95,.98]	0.94940	0.96840
[0,.98,.98]	0.98100	0.97840
[2,.85,.98]	0.86757	0.94270
[2,.90,.98]	0.90877	0.95985
[2,.95,.98]	0.95220	0.97180
[2,.98,.98]	0.98160	0.97960
[4,.85,.98]	0.86985	0.94820
[4,.90,.98]	0.91060	0.96480
[4,.95,.98]	0.95500	0.97380
[4,.98,.98]	0.98260	0.97860

4.5.3 Difference Between Set and Observed Service Levels

The probability of nonstockout occasions, that is defined for the supplier and the retailer analytically, is inspected by constructing confidence intervals over the observed probability of nonstockout occasions.

Table 4.4 provides confidence intervals constructed over observed probability of nonstockout occasions for the retailer and the supplier. These confindence intervals are used to check whether the theoretically set probability of nonstockout values are in the interval or not. It is seen from the table that for every combination of theoretically set values supplier's theoretical probability of nonstockout value is always in between the bounds of confidence intervals constructed. However it is not the case for the retailer, since it has a capacitated supplier as opposed to the assumption in analytical derivation part. We see from Table 4.4 that when the supplier has high probability of nonstockout occasions, the retailer's theoretical probability of nonstockout occassion value is in between the bounds of the confidence interval. Wheras, if the supplier works with more than .05 stockout probability, the retailer's theoretical probability of nonstockout value is bigger than the upper bound of the confidence interval.

4.5.4 Average On-hand Inventory

Inventory holding is among the main decision variables in designing general systems. We aim to observe the effects of the paramaters in the defined set on the average positive inventory values of the retailer and the supplier.

In Table 4.5, we see that as the replenishment lead-time for the retailer increases fixing the other parameters, both the retailer's and the supplier's average on-hand inventory values increase. This means that reducing the replenishment lead-time, L, could benefit both the retailer and the supplier.

In addition we can observe from the Table 4.5 that; increasing the value of probability of nonstockout occasions, increases the time weighted on-hand inventory value both for the supplier and the retailer. They need to keep more inventory to serve with high service levels.

Table 4.4: Comparison of Set and Observed Values

Supplier's Theoretical P (no stockout)	Retailer's Theoretical P (no stockout)	Observed CI for Supplier's P (no stockout)	Observed CI for Retailer's P (no stockout)
0.98	0.85	(0.978219,0.984981)	(0.838787,0.862413)
0.98	0.90	(0.978219,0.984981)	(0.889776,0.907024)
0.98	0.95	(0.978219,0.984981)	(0.940353,0.953247)
0.98	0.98	(0.978219,0.984981)	(0.975985,0.983215)
0.95	0.85	(0.945863,0.958537)	(0.828045,0.855955)
0.95	0.90	(0.945863,0.958537)	(0.879413,0.900187)
0.95	0.95	(0.945863,0.958537)	(0.931330,0.945070)
0.95	0.98	(0.945863,0.958537)	(0.967528,0.976072)
0.90	0.85	(0.894879,0.915521)	(0.806254,0.838546)
0.90	0.90	(0.894879,0.915521)	(0.857792,0.884208)
0.90	0.95	(0.894879,0.915521)	(0.913136,0.934064)
0.90	0.98	(0.894879,0.915521)	(0.953459,0.966941)

4.5.5 Average Negative Inventory

Certain companies may loose their loyal customers or be punished with big penalties because of backordering. We observe the effects of the paramaters in the defined set on the average negative inventory values of the retailer and the supplier.

In Table 4.6, we see that as the replenishment lead-time for the retailer increases fixing the other parameters, both the retailer's and the supplier's average negative inventory values increase. This again shows the importance of reduction in the replenishment lead-time, L, for the system as a whole.

It can be observed from the Table 4.5 that; increasing the value of probability of nonstockout occasions, decreases the time weighted negative inventory value both for the supplier and the retailer. Keeping more inventory to serve with high service levels, decreases the values of average negative inventory.

Table 4.5: Time Weighted On-hand Inventory

Parameter Set ($[L, \beta_s, \beta_r]$)	**Supplier's On-hand Inventory**	**Retailer's On-hand Inventory**
[0,.85,.95]	7.22957	17.92239
[0,.90,.95]	8.66201	18.01082
[0,.95,.95]	11.04102	17.60981
[0,.98,.95]	13.63264	17.38250
[2,.85,.95]	8.65113	19.09509
[2,.90,.95]	10.46458	18.61853
[2,.95,.95]	13.17196	18.62790
[2,.98,.95]	16.05434	18.71438
[4,.85,.95]	10.55297	20.26978
[4,.90,.95]	12.66991	20.04303
[4,.95,.95]	15.72705	19.93664
[4,.98,.95]	18.99449	20.03039
[0,.85,.98]	7.22957	18.68546
[0,.90,.98]	8.78760	18.17699
[0,.95,.98]	11.12999	18.07792
[0,.98,.98]	13.63264	18.15651
[2,.85,.98]	9.07147	19.57230
[2,.90,.98]	10.66732	19.86835
[2,.95,.98]	13.17196	19.72043
[2,.98,.98]	16.05434	19.80915
[4,.85,.98]	10.68133	21.22656
[4,.90,.98]	12.92733	21.16409
[4,.95,.98]	15.72705	21.34547
[4,.98,.98]	18.99449	21.44100

Table 4.6: Time Weighted Negative Inventory

Parameter Set ($[L, \beta_s, \beta_r]$)	**Supplier's Negative Inventory**	**Retailer's Negative Inventory**
[0,.85,.95]	-0.49075	-0.24630
[0,.90,.95]	-0.28984	-0.15300
[0,.95,.95]	-0.12803	-0.08354
[0,.98,.95]	-0.04361	-0.04977
[2,.85,.95]	-0.53609	-0.24941
[2,.90,.95]	-0.32090	-0.16145
[2,.95,.95]	-0.14391	-0.09862
[2,.98,.95]	-0.04949	-0.06746
[4,.85,.95]	-0.56852	-0.25810
[4,.90,.95]	-0.34247	-0.17314
[4,.95,.95]	-0.15512	-0.10970
[4,.98,.95]	-0.05371	-0.08106
[0,.85,.98]	-0.49075	-0.17381
[0,.90,.98]	-0.29520	-0.10112
[0,.95,.98]	-0.12832	-0.04551
[0,.98,.98]	-0.04361	-0.02104
[2,.85,.98]	-0.51141	-0.15148
[2,.90,.98]	-0.31918	-0.09301
[2,.95,.98]	-0.14391	-0.04971
[2,.98,.98]	-0.04949	-0.02749
[4,.85,.98]	-0.56251	-0.14997
[4,.90,.98]	-0.34861	-0.09366
[4,.95,.98]	-0.15512	-0.05330
[4,.98,.98]	-0.05371	-0.03180

4.5.6 Comparison of the Models with and without Information Sharing

In analytical part of the study we realize that the supplier benefits from observing the last customer demand information. We aim to observe this also with simulation tools. It is not directly observable with the outputs of the sample runs, since we need to run the system with smaller α values. Taking α smaller decreases the effect of information sharing and increases the difficulty of detecting the benefit with the model output. Due to complications coming from order-batching, results of some replications deviate and these replications dominate the observed average. It is difficult to observe the benefit with the presence of these outlier replications. We use box ploting to find which replications are outliers. The procedure for the box plot is as follows:

- The program counts the number of stockout occasions for the supplier in each replication.
- Replications are ordered in the order of increasing number of nonstockout occasions
- The replications, that are in the order of 75th percent and 25th percent, are found.
- The difference between these two replication's nonstockout occasions is calculated.
- Then multiplication of this difference value with 1.5 is added to the value of 75th percent replication to find the upper bound of the box plot and subtracted from the value of 25th percent replication to find the lower bound of the box plot.
- The replications that have nonstockout values not in this interval are thrown out.

An example of box plot output is provided in Appendix Tables B.10 and B.11.

After throwing out the outlier replications t-testing is used to compare the output of the models with information sharing and without information sharing. The values compared are the number of stockout occasions for the supplier. An example application, with theoretical probability of nonstockout value .98 both for the supplier and the retailer, of this test can be found in Appendix Tables B.12 - B.15. The result of this example is provided in the Table 4.7.

Table 4.7: t-Test: Paired Two Sample for Means

	Variable 1	**Variable 2**
Mean	4.235294118	0.941176471
Variance	3.441176471	2.183823529
Observations	17	17
Pearson Correlation	0.187757325	
Hypothesized Mean Difference	0	
df	16	
t Stat	6.335676759	
P(T¡=t) one-tail	4.94835E-06	
t Critical one-tail	1.745884219	
P(T¡=t) two-tail	9.89671E-06	
t Critical two-tail	2.119904821	

We see from the example table, (4.7), that the p value is smaller than 0.05. This means that the two variables tested are different from each other and mean value for the case of information sharing is smaller. Since the t-test is performed on the number of stockout occasions, the supplier with customer demand information benefits.

Table 4.8 provides t-test comparison on the value of stockout occasion for different values of the parameters in the parameter set. It is seen from the table that almost all of the p values for the t-test performed on the sample runs are less than 0.05. This shows that the tested means are different and the mean for the supplier who observes extra demand information performs better.

Table 4.8: Model Comparison

Parameter Set ($[L, \beta_s, \beta_r]$)	**t-table Value**	**t-test Critic**	**p-value**
[0,.85,.95]	1.65833	4.80272	2.39971E-06
[0,.90,.95]	1.67022	1.98034	2.60903E-02
[0,.95,.95]	1.73406	6.19531	3.78664E-06
[0,.98,.95]	1.75305	7.37865	1.14835E-06
[2,.85,.95]	1.66600	1.83531	3.52666E-02
[2,.90,.95]	1.68288	0.92760	1.79522E-01
[2,.95,.95]	1.74588	4.63261	1.38287E-04
[2,.98,.95]	1.74588	6.33568	4.94835E-06
[4,.85,.95]	1.67203	1.33454	9.36673E-02
[4,.90,.95]	1.69236	0.50419	3.08738E-01
[4,.95,.95]	1.75305	4.78351	1.20788E-04
[4,.98,.95]	1.74588	5.41036	2.88857E-05
[0,.85,.98]	1.65833	4.80272	2.39971E-06
[0,.90,.98]	1.70113	1.00370	1.62059E-01
[0,.95,.98]	1.74588	5.44513	2.69779E-05
[0,.98,.98]	1.75305	7.37865	1.14835E-06
[2,.85,.98]	1.68023	-0.69733	2.44632E-01
[2,.90,.98]	1.71387	1.29929	1.03359E-01
[2,.95,.98]	1.74588	4.63261	1.38287E-04
[2,.98,.98]	1.74588	6.33568	4.94835E-06
[4,.85,.98]	1.68488	-0.30752	3.80043E-01
[4,.90,.98]	1.74588	2.76214	6.94160E-03
[4,.95,.98]	1.75305	4.78351	1.20788E-04
[4,.98,.98]	1.74588	5.41036	2.88857E-05

CHAPTER 5

CONCLUSION

In this study, a two-echelon supply chain with a retailer and a supplier is analyzed. The end customer demand observed by the retailer is nonstationary. The retailer batches the customer demand for a fixed, predetermined number of periods, and uses an order-up-to policy to determine its order amount from the supplier. Every time an order is placed by the retailer, the supplier places an order at the outside source by using a standard order-up-to policy. There are lead-times for the retailer and the supplier to receive given orders. They can only fulfill the order of the lower echelon as long as they have sufficient on-hand inventory.

In the first part of Chapter 3, we first concentrate on the retailer's process. We present a model for a single-item inventory system with deterministic lead-time but subject to stochastic, nonstationary demand process. We analytically derive order and inventory equations for the supplier assuming it is supplied by an infinite capacity supplier. In the second part of the chapter we examine how the model extends to a supply chain context. We find that the upstream demand is a complicated process due to order-batching used by the retailer. Inventory and order equations are derived for the retailer. In the last part of the chapter we analytically compare the supplier's performance for the models with demand information sharing and without demand information sharing.

The fourth chapter is devoted to simulation. We study on the retailer's and the supplier's processes analytically in Chapter 3 separately. In Chapter 4 we aim to observe their performance while working in coordination. We define four different performance measures; time to satisfy backorder, backorder occasions, difference between set and observed values and time weighted on-hand and negative inventory. We observe these values under different combinations of the parameters and understand the system performance. In addition, in order to observe the effect of information sharing we code a new model. The results of the model with information sharing is compared with the results of the model without information sharing.

We have the following observations from this study:

- As it is mentioned previously basic structure of our model is similar to that of Graves (1999). One of its results is interesting and differs from the general results in the literature. It says that "there is no benefit from letting the upstream stage have exogenous demand information". We study on the implications of this in our system and we realize that it is not true for our case. The supplier can benefit from using the extra demand information to reduce the variance of forecast for the demand observed during its lead-time. And this reduction in the variance increases the performance of the supplier.

- We compare the demand observed by the retailer and its order to have some insights about the demand processes observed by the retailer and the supplier. We take R period accumulated demand observed by the retailer and its order in this comparison, since the retailer's order is for the accumulated demand. We realize that the variance of demand observed by the supplier is bigger than that of the retailer. This is one of the well-known issues in the literature about supply chains, the bullwhip effect.

- We observe that it is not wise to work with batch ordering while the demand parameter or the process inertia parameter α has big values. Variances of the retailer's inventory equations in the periods other than the constant variance period in a cycle depends on time and increase with increasing value of it. These variances also depend on the value of α. It is difficult to work for long rolling horizon in this system. If α is bigger planning for the long rolling horizon becomes more complicated.

Further studies can be conducted on finding a better forecasting model to forecast the demand observed by the supplier and consistently constructing a new order-up-to policy for it. Lead-times or/and ordering period can be stochastic and the supplier can also batch its demand.

REFERENCES

[1] Aviv, Y., 2001, 'The Effect of Collaborative Forecasting on Supply Chain Performance', Management Science, Vol. 47, No. 10, 1326-1343.

[2] Aviv, Y., 2002, 'Gaining Benefits from Joint Forecasting and Replenishment Processes: The Case of Auto-Correlated Demand', Manufacturing and Service Operations Management, Vol. 4, No. 1, 55-74.

[3] Box, G.E.P., Jenkins, G.M. and Reinsel, G.C., 1994, 'Time Series Analysis Forecasting and Control', 3rd Ed. Holden-Day, San Francisco, CA., 110-114.

[4] Cachon, G.P., 1999, 'Managing Supply Chain Demand Variability with Scheduled Ordering Policies', Management Science, Vol. 45, No. 6, 843-856.

[5] Cachon, G.P. and Fisher, M., 2000, 'Supply Chain Inventory Management and the Value of Shared Information', Management Science, Vol. 46, No. 8, 1032-1048.

[6] Cachon, G.P., 2001, 'Exact Evaluation of Batch-Ordering Inventory Policies in Two-Echelon Supply Chains with Periodic Review', Operations Research, Vol. 49, No. 1, 79-98.

[7] Caplin, A.S., 1985, 'The Variability of Aggregate Demand with (S,s) Inventory Policies', Econometrica, Vol. 53, 1396-1409.

[8] Chen, F., Drezner, Z., Ryan, J.K. and Simchi-Levi, D., 2000a, 'Quantifying the Bullwhip Effect in a Simple Supply Chain: The Impact of Forecasting, Lead Times, and Information', Management Science, Vol. 46, No. 3, 436-443.

[9] Chen, F., Drezner, Z. and Simchi-Levi, D., 2000b, 'The Impact of Exponential Smoothing Forecasts on the Bullwhip Effect', Naval Research Logistics, Vol. 47, 269-286.

[10] Cheung, K., Hausman, W.H., 2000, 'An Exact Performance Evaluation for the Supplier in a Two-Echelon Inventory System', Operations Research, Vol. 48, No. 4, 646-653.

[11] Gilbert, K., 2002, 'An Autoregressive Integrated Moving Average Supply Chain Model', Working Paper, University of Tenessee, Knoxville, TN 37848.

[12] Graves, S.C., 1999, 'A Single-Item Inventory Model for a Nonstationary Demand Process', Manufacturing and Service Operations Management, Vol. 1, No. 1, 50-61.

[13] Lee, H.L., Padmanabhan, V. and Whang, S., 1997a, 'The Bullwhip Effect in Supply Chains', Sloan Management Review, Spring????, 93-102.

[14] Lee, H.L., Padmanabhan, V. and Whang, S., 1997b, 'Information Distortion in a Supply Chain: The Bullwhip Effect', Management Science, Vol. 43, No. 4, 546-558.

[15] Lee, H.L., So, K.C. and Thang, C.S., 2000, 'The Value of Information Sharing in a Two-Level Supply Chain', Management Science, Vol. 46, No. 5, 626-643.

[16] Lovejoy, W.S., 1990, 'Myopic Policies for Some Inventory Models with Uncertain Demand Distributions', Management Science, Vol. 47, No. 6, 724-738.

[17] Muth, J.F., 1960, 'Optimal Properties of Exponential Weighted Forecasts', American Statistical Association Journal, Vol. 55, 299-306.

[18] Raghunathan, S., 2001, 'Information Sharing in a Supply Chain: A Note on its Value when Demand Is Nonstationary', Management Science, Vol. 47, No. 4, 605-610.

[19] Reddy, A.M., Rajendran, C., 2004, 'A Simulation Study of Dynamic Order-up-to Policies in a Supply Chain with Non-Statioanary Customer Demand and Information Sharing', Advanced Manufacturing Technology.

[20] Rosenbaum, B.A., 1981, 'Inventory Placement in a Two-Echelon Inventory System: An Application', Management Science, Vol. 16, 195-207.

[21] Shapiro, R. and Brynes, J., 1992, 'Intercompany Operating Ties: Unlocking the Value in Channel Restructuring', Working Paper, Harward Business School, Cambridge, MA.

[22] Sterman, J.D., 1989, 'Modeling Managerial Behavior Misperceptions of Feedback in a Dynamic Decision Making Experiment', Management Science, Vol. 35, No. 3, 321-339.

[23] Veinott, A., 1965, 'Optimal Policy for a Multi-Product, Dynamic, Nonstationary Inventory Problem', Management Science, Vol. 12, No. 3, 206-222.

[24] Zhao, X., Xie, J. and Leung, J., 2002, 'The Impact of Forecasting Model on the Value of Information Sharing in a Supply Chain', European Journal of Operational Research, Vol. 142, 321-344.

APPENDIX A

ANALYTICAL DERIVATIONS

A.1.Derivation of Order for the Retailer:

We employed 3.1, 3.6, and 3.11 to find following:

$$
\begin{aligned}
q_{nR} &= (L+R-1)\left\{F_{nR+1} - F_{(n-1)R+1}\right\} + \sum_{i=1}^{R} d_{(n-1)R+i} \\
&= (L+R-1)(\alpha\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_1 + \mu) \\
&- (L+R-1)(\alpha\epsilon_{(n-1)R} + \alpha\epsilon_{(n-1)R-1} + ... + \alpha\epsilon_1 + \mu) \\
&+ \left\{d_{(n-1)R+1} + d_{(n-1)R+2} + ... + d_{nR}\right\} \\
&= (L+R-1)\left\{(\alpha\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_{(n-1)R+1}\right\} \\
&+ \sum_{j=1}^{R}(\mu + \sum_{i=1}^{(n-1)R+j} \epsilon_i) \\
&= (L+R-1)\left\{(\alpha\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_{(n-1)R+1}\right\} \\
&+ R(\alpha\epsilon_{(n-1)R} + \alpha\epsilon_{(n-1)R-1} + ... + \alpha\epsilon_1 + \mu) \\
&+ \{1 + (R-1)\alpha\}\,\epsilon_{(n-1)R+1} + \{1 + (R-2)\alpha\}\,\epsilon_{(n-1)R+2} + ... + \{1 + (R-R)\alpha\}\,\epsilon_{nR} \\
&= (L+R-1)\left\{(\alpha\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_{(n-1)R+1}\right\} \\
&+ R(\sum_{i=1}^{(n-1)R} \alpha\epsilon_i + \mu) + \sum_{i=0}^{R-1}(1 + i\alpha)\epsilon_{t-i} \\
&= (L+R-1)\alpha\sum_{i=0}^{R-1}\epsilon_{nR-i} + R(\sum_{i=1}^{(n-1)R}\alpha\epsilon_i + \mu) + \sum_{i=0}^{R-1}(1 + i\alpha)\epsilon_{nR-i} \\
q_{nR} &= \sum_{i=0}^{R-1}\{(L+R-1+i)\alpha + 1\}\,\epsilon_{nR-i} + R\alpha\sum_{i=1}^{(n-1)R}\epsilon_i + R\mu
\end{aligned}
$$

A.2.Derivation of iInventory Equation for the Retailer in Ordering Periods

Starting from 3.20, backward substitution and replacing $q_{nR}, n = 0, 1, 2, ...$ with the equation 3.11 we found:

$$\begin{aligned}
X_{nR} &= X_{nR-1} - d_{nR} \\
&= X_{nR-2} - d_{nR-1} - d_{nR} \\
&= X_{nR-3} - d_{nR-2} - d_{nR-1} - d_{nR} \\
X_{nR} &= X_{nR-i} - \sum_{j=0}^{i-1} d_{nR-j} \quad \text{for } i = 4, 5, .., R - L - 2 \\
&= X_{(n-1)R+L+1} - d_{(n-1)R+L+2} - d_{(n-1)R+L+3} - ... - d_{nR} \\
&= X_{(n-1)R+L} - d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - ... - d_{nR} \\
&= X_{(n-1)R+L-1} - d_{(n-1)R+L} + q_{(n-1)R} \\
&- d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - ... - d_{nR}
\end{aligned}$$

where

$$q_{(n-1)R} = (L + R - 1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}$$

substitute this and continue on backward substitution

$$\begin{aligned}
X_{nR} &= X_{(n-1)R+L-1} - d_{(n-1)R+L} + ((L + R - 1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-2)R+i}) - d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - ... - d_{nR} \\
&= X_{(n-1)R} - d_{(n-1)R+1} + ((L + R - 1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-2)R+i}) - d_{(n-1)R+2} - d_{(n-1)R+3} - ... - d_{nR} \\
&= X_{(n-2)R+L} - d_{(n-2)R+L+1} - d_{(n-2)R+L+2} - ... - d_{nR} \\
&+ ((L + R - 1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&= X_{(n-2)R+L-1} - d_{(n-2)R+L} + ((L + R - 1)\left\{F_{(n-2)R+1} - F_{(n-3)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-3)R+i}) - d_{(n-2)R+L+1} - d_{(n-2)R+L+2} - ... - d_{(n-1)R+L} \\
&+ ((L + R - 1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&- d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - ... - d_{nR}
\end{aligned}$$

$$
\begin{aligned}
&= X_{(n-3)R+L-1} - d_{(n-3)R+L} + ((L+R-1)\left\{F_{(n-3)R+1} - F_{(n-4)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-4)R+i}) - d_{(n-3)R+L+1} - d_{(n-3)R+L+2} - ... - d_{(n-2)R+L} \\
&+ ((L+R-1)\left\{F_{(n-2)R+1} - F_{(n-3)R+1}\right\} + \sum_{i=1}^{R} d_{(n-3)R+i}) \\
&- d_{(n-2)R+L+1} - d_{(n-2)R+L+2} - ... - d_{(n-1)R+L} \\
&+ ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&- d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - ... - d_{nR}
\end{aligned}
$$

since plus and minus forecast terms in q_{nR}, $n = 1, 2, 3...$ cancel each other we have:

$$
\begin{aligned}
X_{nR} &= X_{2R+L-1} - d_{2R+L} + ((L+R-1)\{F_{2R+1} - F_{R+1}\} + \sum_{i=1}^{R} d_{R+i}) \\
&+ (L+R-1)\left\{F_{(n-1)R+1} - F_{2R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{2R+i} + \sum_{i=1}^{R} d_{3R+i} + ... + \sum_{i=1}^{R} d_{(n-2)R+i} \\
&- d_{2R+L+1} - d_{2R+L+2} - ... - d_{nR} \\
&= X_{R+L-1} - d_{R+L} + (L+R-1)\{F_{R+1} - F_1\} + \sum_{i=1}^{R} d_i \\
&+ (L+R-1)\left\{F_{(n-1)R+1} - F_{R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{R+i} + \sum_{i=1}^{R} d_{2R+i} + ... + \sum_{i=1}^{R} d_{(n-2)R+i} \\
&- d_{R+L+1} - d_{R+L+2} - ... - d_{nR} \\
&= X_{L-1} - d_L + q_0 + (L+R-1)\left\{F_{(n-1)R+1} - F_1\right\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-2)R+i} \\
&- d_{L+1} - d_{L+2} - ... - d_{nR} \\
&= X_0 - d_1 - d_2 - ... - d_L + q_0 + (L+R-1)\left\{F_{(n-1)R+1} - F_1\right\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-2)R+i} \\
&- d_{L+1} - d_{L+2} - ... - d_{nR}
\end{aligned}
$$

substituting initial values; $q_0 = R\mu$ and $F_1 = \mu$ so:

$$
X_{nR} = X_0 + R\mu + (L+R-1)\left\{F_{(n-1)R+1} - \mu\right\} + \sum_{i=1}^{(n-1)R} d_i
$$

$$
\begin{aligned}
&- && d_1 - d_2 - ... - d_{nR} \\
&= && X_0 + R\mu - (L+R-1)\mu + (L+R-1)F_{(n-1)R+1} \\
&+ && d_1 + d_2 + ... + d_{(n-1)R} \\
&- && d_1 - d_2 - ... - d_{nR} \\
&= && X_0 - (L-1)\mu + (L+R-1)F_{(n-1)R+1} \\
&- && d_{(n-1)R+1} - d_{(n-1)R+2} - ... - d_{nR} \\
&= && X_0 - (L-1)\mu + (L-1)F_{(n-1)R+1} \\
&- && d_{(n-1)R+1} - d_{(n-1)R+2} - ... - d_{nR} + (R)F_{(n-1)R+1} \\
&= && X_0 - (L-1)\mu + (L-1)F_{(n-1)R+1} \\
&- && (d_{nR} - F_{(n-1)R+1}) - (d_{nR-1} - F_{(n-1)R+1}) - ... - (d_{(n-1)R+1} - F_{(n-1)R+1}) \\
&= && X_0 - (L-1)\mu + (L-1)F_{(n-1)R+1} \\
&- && (\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_1 + \mu - \alpha\epsilon_{(n-1)R} - \alpha\epsilon_{(n-1)R-1} - ... - \alpha\epsilon_1 - \mu) \\
&- && (\epsilon_{nR-1} + \alpha\epsilon_{nR-2} + ... + \alpha\epsilon_1 + \mu - \alpha\epsilon_{(n-1)R} - \alpha\epsilon_{(n-1)R-1} - ... - \alpha\epsilon_1 - \mu) \\
&- && \sum_{i=2}^{R-2}(\epsilon_{nR-i} + \alpha\epsilon_{nR-i-1} + ... + \alpha\epsilon_1 + \mu - F_{(n-1)R+1}) \\
&- && (\epsilon_{(n-1)R+1} + \alpha\epsilon_{(n-1)R} + ... + \alpha\epsilon_1 + \mu - \alpha\epsilon_{(n-1)R} - \alpha\epsilon_{(n-1)R-1} - ... - \alpha\epsilon_1 - \mu) \\
&= && X_0 - (L-1)\mu + (L-1)F_{(n-1)R+1} \\
&- && (\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- && (\epsilon_{nR-1} + \alpha\epsilon_{nR-2} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- && (\epsilon_{nR-2} + \alpha\epsilon_{nR-3} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- && \sum_{i=3}^{R-2}(\epsilon_{nR-i} + \alpha\epsilon_{nR-i-1} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- && (\epsilon_{(n-1)R+1}) \\
&= && X_0 - (L-1)\mu + (L-1)F_{(n-1)R+1} - \sum_{i=0}^{R-1}(1+i\alpha)\epsilon_{nR-i} \\
&= && X_0 - (L-1)\mu - \sum_{i=0}^{R-1}(1+i\alpha)\epsilon_{nR-i} \\
&+ && (L-1)\left\{\alpha\epsilon_{(n-1)R} + \alpha\epsilon_{(n-1)R-1} + ... + \alpha\epsilon_1 + \mu\right\} \\
&= && X_0 - \sum_{i=0}^{R-1}(1+i\alpha)\epsilon_{nR-i} - (L-1)\mu
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad (L-1)\sum_{i=1}^{(n-1)R}\alpha\epsilon_i + (L-1)\mu \\
X_{nR} \quad &= \quad X_0 - \sum_{i=0}^{R-1}(1+i\alpha)\epsilon_{nR-i} + (L-1)\sum_{i=1}^{(n-1)R}\alpha\epsilon_i
\end{aligned}
$$

$$
\begin{aligned}
X_{nR+L} &= X_{nR+L-1} - d_{nR+L} + q_{nR} \\
&= X_{nR+L-2} - d_{nR+L-1} - d_{nR+L} + (L+R-1)\left\{F_{nR+1} - F_{(n-1)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-1)R+i}) \\
&= (X_{nR+L-i} - d_{nR+L} - d_{nR+L-1} - \dots - d_{nR+L-i+1} + (L+R-1)\left\{F_{nR+1} - F_{(n-1)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-1)R+i}) \ \text{ for } i = 3,4,..,R-1 \\
&= X_{(n-1)R+L} - d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - \dots - d_{nR+L} \\
&+ ((L+R-1)\left\{F_{nR+1} - F_{(n-1)R+1}\right\} + \sum_{i=1}^{R} d_{(n-1)R+i}) \\
&= X_{(n-1)R+L-1} - d_{(n-1)R+L} + ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&- d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - \dots - d_{nR+L} \\
&+ ((L+R-1)\left\{F_{nR+1} - F_{(n-1)R+1}\right\} + \sum_{i=1}^{R} d_{(n-1)R+i})
\end{aligned}
$$

plus and minus forecast terms in $q_{nR}, n = 1,2,3...$ cancel each other

$$
\begin{aligned}
X_{nR+L} &= X_{R+L-1} - d_{R+L} + (L+R-1)\{F_{R+1} - F_1\} + \sum_{i=1}^{R} d_i \\
&- d_{R+L+1} - d_{R+L+2} - \dots - d_{nR+L} \\
&+ (L+R-1)\{F_{nR+1} - F_{R+1}\} \\
&+ \sum_{i=1}^{R} d_{R+i} + \sum_{i=1}^{R} d_{2R+i} + \dots + \sum_{i=1}^{R} d_{(n-1)R+i} \\
&= X_{L-1} - d_L + q_0 - d_{L+1} - d_{L+2} - \dots - d_{nR+L} \\
&+ (L+R-1)\{F_{nR+1} - F_1\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + \dots + \sum_{i=1}^{R} d_{(n-1)R+i} \\
&= X_0 + R\mu - d_1 - d_2 - \dots - d_{nR+L} \\
&+ (L+R-1)\{F_{nR+1} - F_1\} \\
&+ d_1 + d_2 + \dots + d_{nR} \\
&= X_0 + R\mu - d_{nR+1} - d_{nR+2} - \dots - d_{nR+L} \\
&+ (L+R-1)\{F_{nR+1} - \mu\}
\end{aligned}
$$

$$
\begin{aligned}
&= X_0 + R\mu - d_{nR+1} - d_{nR+2} - ... - d_{nR+L} + LF_{nR+1} \\
&- (L + R - 1)\mu + (R - 1)F_{nR+1} \\
&= X_0 - (d_{nR+L} - F_{nR+1}) - (d_{nR+L-1} - F_{nR+1}) - ... - (d_{nR+1} - F_{nR+1}) \\
&+ (R - 1)\{\alpha\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_1 + \mu\} - (L - 1)\mu \\
&= X_0 - (\epsilon_{nR+L} + \alpha\epsilon_{nR+L-1} + ... + \alpha\epsilon_{nR+1}) \\
&- (\epsilon_{nR+L-1} + \alpha\epsilon_{nR+L-2} + ... + \alpha\epsilon_{nR+1}) \\
&. \\
&. \\
&- (\epsilon_{nR+1}) \\
&+ (R - 1)\sum_{i=1}^{nR} \alpha\epsilon_i + (R - 1)\mu - (L - 1)\mu \\
X_{nR+L} &= X_0 - \sum_{i=0}^{L-1}(1 + i\alpha)\epsilon_{nR+L-i} + (R - 1)\sum_{i=1}^{nR} \alpha\epsilon_i + (R - L)\mu
\end{aligned}
$$

A.4. Derivation of Inventory Equation for the Retailer in the Period Prior to Receiving

$$
\begin{aligned}
X_{nR+L-1} &= X_{nR+L-2} - d_{nR+L-1} \\
&= X_{nR+L-3} - d_{nR+L-2} - d_{nR+L-1} \\
&= X_{nR+L-i} - d_{nR+L-1} - d_{nR+L-2} - ... - d_{nR+L-i+1} \quad \text{for } i = 4, 5, .., L-1 \\
&= X_{nR} - d_{nR+1} - d_{nR+2} - ... - d_{nR+L-1} \\
&= X_{(n-1)R+L} - d_{(n-1)R+L+1} - d_{(n-1)R+L+2} - ... - d_{nR+L-1} \\
&= X_{(n-1)R+L-1} - d_{(n-1)R+L} - d_{(n-1)R+L+1} - ... - d_{nR+L-1} \\
&+ ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&= X_{(n-2)R+L-1} - d_{(n-2)R+L} - d_{(n-2)R+L+1} - ... - d_{nR+L-1} \\
&+ ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&+ ((L+R-1)\left\{F_{(n-2)R+1} - F_{(n-3)R+1}\right\} + \sum_{i=1}^{R} d_{(n-3)R+i})
\end{aligned}
$$

plus and minus forecast terms in $q_{nR}, n = 1, 2, 3...$ cancel each other

$$
\begin{aligned}
X_{nR+L-1} &= X_{L-1} - d_L + q_0 - d_{L+1} - d_{L+2} - ... - d_{nR+L-1} \\
&+ (L+R-1)\left\{F_{(n-1)R+1} - F_1\right\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-2)R+i} \\
&= X_0 + R\mu - d_1 - d_2 - ... - d_{nR+L-1} \\
&+ (L+R-1)\left\{F_{(n-1)R+1} - \mu\right\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-2)R+i} \\
&= X_0 + R\mu - d_{nR+L-1} - d_{nR+L-2} - ... - d_{(n-1)R+1} \\
&+ (L+R-1)F_{(n-1)R+1} - (L+R-1)\mu \\
&= X_0 - (d_{nR+L-1} - F_{(n-1)R+1}) - (d_{nR+L-2} - F_{(n-1)R+1}) - ... \\
&- (d_{(n-1)R+1} - F_{(n-1)R+1}) - (L-1)\mu \\
&= X_0 - (L-1)\mu - (\epsilon_{nR+L-1} + \alpha\epsilon_{nR+L-2} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- (\epsilon_{nR+L-2} + \alpha\epsilon_{nR+L-3} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- (\epsilon_{nR+L-3} + \alpha\epsilon_{nR+L-4} + ... + \alpha\epsilon_{(n-1)R+1})
\end{aligned}
$$

$$
\begin{aligned}
& - \sum_{i=4}^{R+L-2} (\epsilon_{nR+L-i} + \alpha\epsilon_{nR+L-i-1} + ... + \alpha\epsilon_{(n-1)R+1}) \\
& - (\epsilon_{(n-1)R+1}) \\
X_{nR+L-1} &= X_0 - \sum_{i=0}^{R+L-2} \{(1+i\alpha)\epsilon_{nR+L-1-i}\} - (L-1)\mu
\end{aligned}
$$

A.5. Derivation of Inventory for the Retailer in Ordering Periods (when L ¿ R)

Starting from 3.20, backward substitution and replacing $q_{nR}, n = 0, 1, 2, ...$ with the equation 3.11 we found:

$$\begin{aligned}
X_{nR} &= X_{nR-1} - d_{nR} \\
&= X_{nR-2} - d_{nR-1} - d_{nR} \\
&= X_{nR-3} - d_{nR-2} - d_{nR-1} - d_{nR} \\
&= X_{nR-4} - d_{nR-3} - d_{nR-2} - d_{nR-1} - d_{nR} \\
&= X_{nR-i} - d_{nR} - d_{nR-1} - ... - d_{nR-i+1} \quad \text{for } i = 5, 6, .., R + k - 2 \\
&= X_{(n-1)R+k+1} - d_{(n-1)R+k+2} - d_{(n-1)R+k+3} - ... - d_{nR} \\
&= X_{(n-1)R+k} - d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR} \\
&= X_{(n-1)R+k-1} - d_{(n-1)R+k} + q_{(n-2)R} \\
&- d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR}
\end{aligned}$$

where

$$q_{(n-2)R} = (L + R - 1) \left\{ F_{(n-2)R+1} - F_{(n-3)R+1} \right\} + \sum_{i=1}^{R} d_{(n-3)R+i}$$

then we substitute this and continue on backward substitution

$$\begin{aligned}
X_{nR} &= X_{(n-1)R+k-1} - d_{(n-1)R+k} + ((L + R - 1) \left\{ F_{(n-2)R+1} - F_{(n-3)R+1} \right\} \\
&+ \sum_{i=1}^{R} d_{(n-3)R+i}) - d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR} \\
&= X_{(n-1)R} - d_{(n-1)R+1} + ((L + R - 1) \left\{ F_{(n-2)R+1} - F_{(n-3)R+1} \right\} \\
&+ \sum_{i=1}^{R} d_{(n-3)R+i}) - d_{(n-1)R+2} - d_{(n-1)R+3} - ... - d_{nR} \\
&= X_{(n-2)R+k} - d_{(n-2)R+k+1} - d_{(n-2)R+k+2} - ... - d_{nR} \\
&+ ((L + R - 1) \left\{ F_{(n-2)R+1} - F_{(n-3)R+1} \right\} + \sum_{i=1}^{R} d_{(n-3)R+i}) \\
&= X_{(n-2)R+k-1} - d_{(n-2)R+k} + ((L + R - 1) \left\{ F_{(n-3)R+1} - F_{(n-4)R+1} \right\} \\
&+ \sum_{i=1}^{R} d_{(n-4)R+i}) - d_{(n-2)R+k+1} - d_{(n-2)R+k+2} - ... - d_{(n-1)R+k} \\
&+ ((L + R - 1) \left\{ F_{(n-2)R+1} - F_{(n-3)R+1} \right\} + \sum_{i=1}^{R} d_{(n-3)R+i})
\end{aligned}$$

$$
\begin{aligned}
&- \quad d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR} \\
&= \quad X_{(n-3)R+k-1} - d_{(n-3)R+k} + ((L+R-1)\left\{F_{(n-4)R+1} - F_{(n-5)R+1}\right\} \\
&+ \quad \sum_{i=1}^{R} d_{(n-5)R+i}) - d_{(n-3)R+k+1} - d_{(n-3)R+k+2} - ... - d_{(n-2)R+k} \\
&+ \quad ((L+R-1)\left\{F_{(n-3)R+1} - F_{(n-4)R+1}\right\} + \sum_{i=1}^{R} d_{(n-4)R+i}) \\
&- \quad d_{(n-2)R+k+1} - d_{(n-2)R+k+2} - ... - d_{(n-1)R+k} \\
&+ \quad ((L+R-1)\left\{F_{(n-2)R+1} - F_{(n-3)R+1}\right\} + \sum_{i=1}^{R} d_{(n-3)R+i}) \\
&- \quad d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR}
\end{aligned}
$$

plus and minus forecast terms in $q_n R, n = 1, 2, 3...$ cancel each other

$$
\begin{aligned}
X_{nR} &= X_{2R+k-1} - d_{2R+k} + ((L+R-1)\{F_{R+1} - F_1\} + \sum_{i=1}^{R} d_i) \\
&+ (L+R-1)\left\{F_{(n-2)R+1} - F_{R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{R+i} + \sum_{i=1}^{R} d_{2R+i} + ... + \sum_{i=1}^{R} d_{(n-3)R+i} \\
&- d_{2R+k+1} - d_{2R+k+2} - ... - d_{nR} \\
&= X_{R+k-1} - d_{R+k} + q_0 + (L+R-1)\left\{F_{(n-2)R+1} - F_1\right\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-3)R+i} \\
&- d_{R+k+1} - d_{R+k+2} - ... - d_{nR} \\
&= X_0 - d_1 - d_2 - ... - d_{nR} + q_0 + (L+R-1)\left\{F_{(n-2)R+1} - F_1\right\} \\
&+ d_1 + d_2 + ... + d_{(n-2)R}
\end{aligned}
$$

We have; $q_0 = R\mu$, $F_1 = \mu$, and $L = R + k$ so after making necessary cancellation on plus and minus demand terms:

$$
\begin{aligned}
X_{nR} &= X_0 - d_{nR} - d_{nR-1} - ... - d_{(n-2)R+1} \\
&+ R\mu + (R+k+R-1)\left\{F_{(n-2)R+1} - \mu\right\} \\
&= X_0 - d_{nR} - d_{nR-1} - ... - d_{(n-2)R+1} + (2R)F_{(n-2)R+1} \\
&- (R+k-1)\mu + (k-1)F_{(n-2)R+1} \\
&= X_0 - (d_{nR} - F_{(n-2)R+1}) - (d_{nR} - F_{(n-2)R+1}) - ...
\end{aligned}
$$

$$
\begin{aligned}
& - \quad (d_{(n-2)R+1} - F_{(n-2)R+1}) \\
& - \quad (R+k-1)\mu + (k-1)\left\{\sum_{i=1}^{(n-2)R} \alpha\epsilon_i + \mu\right\} \\
& = \quad X_0 - (\epsilon_{nR} + \alpha\epsilon_{nR-1} + ... + \alpha\epsilon_{(n-2)R+1}) \\
& - \quad (\epsilon_{nR-1} + \alpha\epsilon_{nR-2} + ... + \alpha\epsilon_{(n-2)R+1}) \\
& - \quad (\epsilon_{nR-2} + \alpha\epsilon_{nR-3} + ... + \alpha\epsilon_{(n-2)R+1}) \\
& - \quad \sum_{i=3}^{2R-2} (\epsilon_{nR-i} + \alpha\epsilon_{nR-i-1} + ... + \alpha\epsilon_{(n-2)R+1}) \\
& - \quad (\epsilon_{(n-2)R+1}) \\
& - \quad R\mu + (k-1)\sum_{i=1}^{(n-2)R} \alpha\epsilon_i \\
X_{nR} & = \quad X_0 - \sum_{i=0}^{2R-1} (1+i\alpha)\epsilon_{nR-i} + (k-1)\sum_{i=1}^{(n-2)R} \alpha\epsilon_i - R\mu
\end{aligned}
$$

A.6. Derivation of Inventory for the Retailer in Receiving Period (When L ¿ R)

$$
\begin{aligned}
X_{nR+k} &= X_{nR+k-1} - d_{nR+k} + q_{(n-1)R} \\
&= X_{nR+k-2} - d_{nR+k-1} - d_{nR+k} + ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&= (X_{nR+k-i} - d_{nR+k} - d_{nR+k-1} - ... - d_{nR+k-i+1} + ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} \\
&+ \sum_{i=1}^{R} d_{(n-2)R+i})) \quad \text{for } i = 3, 4, .., R-1 \\
&= X_{(n-1)R+k} - d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR+k} \\
&+ ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i}) \\
&= X_{(n-1)R+k-1} - d_{(n-1)R+k} + ((L+R-1)\left\{F_{(n-2)R+1} - F_{(n-3)R+1}\right\} + \sum_{i=1}^{R} d_{(n-3)R+i}) \\
&- d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR+k} \\
&+ ((L+R-1)\left\{F_{(n-1)R+1} - F_{(n-2)R+1}\right\} + \sum_{i=1}^{R} d_{(n-2)R+i})
\end{aligned}
$$

plus and minus forecast terms in $q_{nR}, n = 1, 2, 3...$ cancel each other

$$
\begin{aligned}
X_{nR+k} &= X_{R+k-1} - d_{R+k} + q_0 \\
&+ (L+R-1)\left\{F_{(n-1)R+1} - F_1\right\} \\
&- d_{R+k+1} - d_{R+k+2} - ... - d_{nR+k} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-2)R+i} \\
&= X_0 + R\mu - d_1 - d_2 - ... - d_{nR+k} \\
&+ (L+R-1)\left\{F_{(n-1)R+1} - F_1\right\} \\
&+ d_1 + d_2 + ... + d_{(n-1)R} \\
&= X_0 + R\mu - d_{(n-1)R+1} - d_{(n-1)R+2} - ... - d_{nR+k} \\
&+ (R+k+R-1)\left\{F_{(n-1)R+1} - \mu\right\} \\
&= X_0 + R\mu - d_{(n-1)R+1} - d_{(n-1)R+2} - ... - d_{nR+k} + (R+k)F_{nR+1} \\
&- (R+k+R-1)\mu + (R-1)F_{(n-1)R+1}
\end{aligned}
$$

$$
\begin{aligned}
&= X_0 - (d_{nR+k} - F_{(n-1)R+1}) \\
&- (d_{nR+k-1} - F_{(n-1)R+1}) - ... - (d_{(n-1)R+1} - F_{(n-1)R+1}) \\
&+ (R-1)\left\{\alpha\epsilon_{(n-1)R} + \alpha\epsilon_{(n-1)R-1} + ... + \alpha\epsilon_1 + \mu\right\} - (R+k-1)\mu \\
&= X_0 - (\epsilon_{nR+k} + \alpha\epsilon_{nR+k-1} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- (\epsilon_{nR+k-1} + \alpha\epsilon_{nR+k-2} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- \sum_{i=2}^{R+k-2} (\epsilon_{nR+k-i} + \alpha\epsilon_{nR+k-i-1} + ... + \alpha\epsilon_{(n-1)R+1}) \\
&- (\epsilon_{(n-1)R+1}) \\
&+ (R-1)\sum_{i=1}^{(n-1)R} \alpha\epsilon_i + (R-1)\mu - (R+k-1)\mu \\
X_{nR+L} &= X_0 - \sum_{i=0}^{L-1}(1+i\alpha)\epsilon_{nR+k-i} + (R-1)\sum_{i=1}^{(n-1)R} \alpha\epsilon_i + (R-L)\mu
\end{aligned}
$$

A.7. Derivation of Inventory Equation for the Retailer in the Period Prior to Receiving (When L ¿ R)

$$
\begin{aligned}
X_{nR+k-1} &= X_{nR+k-2} - d_{nR+k-1} \\
&= X_{nR+k-3} - d_{nR+k-2} - d_{nR+k-1} \\
&= X_{nR+k-i} - d_{nR+k-1} - d_{nR+k-2} - ... - d_{nR+k-i+1} \quad \text{for } i = 4, 5, .., k-1 \\
&= X_{nR} - d_{nR+1} - d_{nR+2} - ... - d_{nR+k-1} \\
&= X_{nR-i} - d_{nR+k-1} - d_{nR+k-2} - ... - d_{nR-i+1} \quad \text{for } i = 1, 2, .., R-k-1 \\
&= X_{(n-1)R+k} - d_{(n-1)R+k+1} - d_{(n-1)R+k+2} - ... - d_{nR+k-1} \\
&= X_{(n-1)R+k-1} - d_{(n-1)R+k} - d_{(n-1)R+k+1} - ... - d_{nR+k-1} \\
&+ ((L+R-1)\left\{F_{(n-2)R+1} - F_{(n-3)R+1}\right\} + \sum_{i=1}^{R} d_{(n-3)R+i}) \\
&= X_{(n-2)R+k-1} - d_{(n-2)R+k} - d_{(n-2)R+k+1} - ... - d_{nR+k-1} \\
&+ ((L+R-1)\left\{F_{(n-2)R+1} - F_{(n-3)R+1}\right\} + \sum_{i=1}^{R} d_{(n-3)R+i}) \\
&+ ((L+R-1)\left\{F_{(n-3)R+1} - F_{(n-4)R+1}\right\} + \sum_{i=1}^{R} d_{(n-4)R+i})
\end{aligned}
$$

plus and minus forecast terms in $q_{nR}, n = 1, 2, 3...$ cancel each other

$$
\begin{aligned}
X_{nR+k-1} &= X_{R+k-1} - d_{R+k} + q_0 - d_{R+k+1} - d_{R+k+2} - ... - d_{nR+k-1} \\
&+ (L+R-1)\left\{F_{(n-2)R+1} - F_1\right\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-3)R+i} \\
&= X_0 + R\mu - d_1 - d_2 - ... - d_{nR+k-1} \\
&+ (L+R-1)\left\{F_{(n-2)R+1} - \mu\right\} \\
&+ \sum_{i=1}^{R} d_i + \sum_{i=1}^{R} d_{R+i} + ... + \sum_{i=1}^{R} d_{(n-3)R+i} \\
&= X_0 + R\mu - d_{nR+k-1} - d_{nR+k-2} - ... - d_{(n-2)R+1} \\
&+ (L+R-1)F_{(n-2)R+1} - (L+R-1)\mu \\
&= X_0 - (d_{nR+k-1} - F_{(n-2)R+1}) - (d_{nR+k-2} - F_{(n-2)R+1}) - ... \\
&- (d_{(n-2)R+1} - F_{(n-2)R+1}) - (L-1)\mu \\
&= X_0 - (L-1)\mu - (\epsilon_{nR+k-1} + \alpha\epsilon_{nR+k-2} + ... + \alpha\epsilon_{(n-2)R+1})
\end{aligned}
$$

$$
\begin{aligned}
&- \quad \left(\epsilon_{nR+k-2} + \alpha\epsilon_{nR+k-3} + ... + \alpha\epsilon_{(n-2)R+1}\right) \\
&- \quad \left(\epsilon_{nR+k-3} + \alpha\epsilon_{nR+k-4} + ... + \alpha\epsilon_{(n-2)R+1}\right) \\
&- \quad \sum_{i=4}^{2R+k-2} \left(\epsilon_{nR+k-i} + \alpha\epsilon_{nR+k-i-1} + ... + \alpha\epsilon_{(n-2)R+1}\right) \\
&- \quad \left(\epsilon_{(n-2)R+1}\right)
\end{aligned} \tag{A.1}
$$

A.8. Derivation to Represent $q_{(n+i)R}$ with respect to qnR and ϵ_i's

$$q_{(n+2)R} = q_{(n+1)R} + \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+2)R-i} - \epsilon_{(n+1)R-i}\right) + R\alpha \sum_{i=nR+1}^{(n+1)R} \epsilon_i$$

place $q_{(n+1)R}$ with its value;

$$\begin{aligned} q_{(n+2)R} &= q_{nR} + \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+1)R-i} - \epsilon_{nR-i}\right) + R\alpha \sum_{i=(n-1)R+1}^{nR} \epsilon_i \\ &+ \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+2)R-i} - \epsilon_{(n+1)R-i}\right) + R\alpha \sum_{i=nR+1}^{(n+1)R} \epsilon_i \\ q_{(n+3)R} &= q_{(n+2)R} + \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+3)R-i} - \epsilon_{(n+2)R-i}\right) \\ &+ R\alpha \sum_{i=(n+1)R+1}^{(n+2)R} \epsilon_i \end{aligned}$$

place $q_{(n+2)R}$ with its value;

$$\begin{aligned} q_{(n+3)R} &= q_{nR} + \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+1)R-i} - \epsilon_{nR-i}\right) + R\alpha \sum_{i=(n-1)R+1}^{nR} \epsilon_i \\ &+ \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+2)R-i} - \epsilon_{(n+1)R-i}\right) + R\alpha \sum_{i=nR+1}^{(n+1)R} \epsilon_i \\ &+ \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+3)R-i} - \epsilon_{(n+2)R-i}\right) + R\alpha \sum_{i=(n+1)R+1}^{(n+2)R} \epsilon_i \\ q_{(n+4)R} &= q_{(n+3)R} + \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+4)R-i} - \epsilon_{(n+3)R-i}\right) \\ &+ R\alpha \sum_{i=(n+2)R+1}^{(n+3)R} \epsilon_i \end{aligned}$$

place $q_{(n+3)R}$ with its value;

$$\begin{aligned} q_{(n+4)R} &= q_{nR} + \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+1)R-i} - \epsilon_{nR-i}\right) + R\alpha \sum_{i=(n-1)R+1}^{nR} \epsilon_i \\ &+ \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+2)R-i} - \epsilon_{(n+1)R-i}\right) + R\alpha \sum_{i=nR+1}^{(n+1)R} \epsilon_i \\ &+ \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\}\left(\epsilon_{(n+3)R-i} - \epsilon_{(n+2)R-i}\right) + R\alpha \sum_{i=(n+1)R+1}^{(n+2)R} \epsilon_i \end{aligned}$$

$$+ \sum_{i=0}^{R-1} \{(L+R-1+i)\alpha+1\} \left(\epsilon_{(n+4)R-i} - \epsilon_{(n+3)R-i}\right) + R\alpha \sum_{i=(n+2)R+1}^{(n+3)R} \epsilon_i$$

.

.

.

As we continue writing the equation for the coming $q_{(n+i)R}$, we saw that plus and minus ϵ terms have same coefficients. We can easily drop these terms and reach the general term for $q_{(n+i)R}$.

$$q_{(n+i)R} = q_{nR} + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\} \left(\epsilon_{(n+i)R-j} - \epsilon_{nR-j}\right) + R\alpha \sum_{j=(n-1)R+1}^{(n+i-1)R} \epsilon_j$$

A.9. Derivation of B_{nR}, Total Shipment Qunatity for the Supplier Over Its Lead Time

$$
\begin{aligned}
B_{nR} &= \sum_{i=1}^{K} q_{(n+i)R} \\
&= q_{(n+1)R} + q_{(n+2)R} + q_{(n+3)R} + ... + q_{(n+K)R} \\
&= q_{nR} + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \left(\epsilon_{(n+1)R-j} - \epsilon_{nR-j}\right) + R\alpha \sum_{j=(n-1)R+1}^{nR} \epsilon_j \\
&+ q_{nR} + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \left(\epsilon_{(n+2)R-j} - \epsilon_{nR-j}\right) + R\alpha \sum_{j=(n-1)R+1}^{(n+1)R} \epsilon_j \\
&+ q_{nR} + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \left(\epsilon_{(n+3)R-j} - \epsilon_{nR-j}\right) + R\alpha \sum_{j=(n-1)R+1}^{(n+2)R} \epsilon_j \\
&+ \sum_{t=4}^{K} (q_{nR} + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \left(\epsilon_{(n+t)R-j} - \epsilon_{nR-j}\right) + R\alpha \sum_{j=(n-1)R+1}^{(n+t-1)R} \epsilon_j)
\end{aligned}
$$

we brought together the similar terms;

$$
\begin{aligned}
B_{nR} &= Kq_{nR} - K \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \epsilon_{nR-j} \\
&+ \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \left(\epsilon_{(n+1)R-j} + \epsilon_{(n+2)R-j} + ... + \epsilon_{(n+K)R-j}\right) \\
&+ R\alpha \sum_{j=(n-1)R+1}^{nR} \epsilon_j + (K-1) \sum_{j=nR+1}^{(n+1)R} \epsilon_j \\
&+ (K-2) \sum_{j=(n+1)R+1}^{(n+2)R} \epsilon_j + ... + \sum_{j=(n+K-2)R+1}^{(n+K-1)R} \epsilon_j) \\
B_{nR} &= Kq_{nR} + R\alpha K \sum_{j=(n-1)R+1}^{nR} \epsilon_j - K \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \epsilon_{nR-j} \\
&+ \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \epsilon_{(n+K)R-j} \\
&+ \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\} \left(\epsilon_{(n+1)R-j} + \epsilon_{(n+2)R-j} + ... + \epsilon_{(n+K-1)R-j}\right) \\
&+ R\alpha \left\{ (K-1) \sum_{j=0}^{R-1} \epsilon_{(n+1)R-j} + (K-2) \sum_{j=0}^{R-1} \epsilon_{(n+2)R-j} + ... + \sum_{j=0}^{R-1} \epsilon_{(n+K-1)R-j} \right\}
\end{aligned}
$$

combine last two lines above, that have same terms multiplied with different coefficients;

$$\begin{aligned}
B_{nR} &= Kq_{nR} + K\sum_{j=0}^{R-1}\left\{R\alpha - (L+R-1+j)\alpha - 1\right\}\epsilon_{nR-j} \\
&+ \sum_{j=0}^{R-1}\left\{(L+R-1+j)\alpha + 1\right\}\epsilon_{(n+K)R-j} \\
&+ \sum_{j=0}^{R-1}[\left\{(L+R-1+j)\alpha + 1 + R\alpha(K-1)\right\}\epsilon_{(n+1)R-j} \\
&+ \left\{(L+R-1+j)\alpha + 1 + R\alpha(K-2)\right\}\epsilon_{(n+2)R-j} \\
&+ \left\{(L+R-1+j)\alpha + 1 + R\alpha(K-3)\right\}\epsilon_{(n+3)R-j} \\
&+ \sum_{t=4}^{K-1}\left\{(L+R-1+j)\alpha + 1 + R\alpha(K-t)\right\}\epsilon_{(n+t)R-j}
\end{aligned}$$

and after last cancellation, it turns out to have the following equation:

$$\begin{aligned}
B_{nR} &= Kq_{nR} + K\sum_{j=0}^{R-1}\left\{-(L-1+j)\alpha - 1\right\}\epsilon_{nR-j} \\
&+ \sum_{j=0}^{R-1}\left\{(L+R-1+j)\alpha + 1\right\}\epsilon_{(n+K)R-j} \\
&+ \sum_{j=0}^{R-1}[\left\{(L-1+j)\alpha + 1 + RK\alpha)\right\}\epsilon_{(n+1)R-j} \\
&+ \left\{(L-1+j)\alpha + 1 + R\alpha(K-1)\right\}\epsilon_{(n+2)R-j} \\
&+ \left\{(L-1+j)\alpha + 1 + R\alpha(K-2)\right\}\epsilon_{(n+3)R-j} \\
&+ \sum_{t=3}^{K-2}\left\{(L-1+j)\alpha + 1 + R\alpha(K-t)\right\}\epsilon_{(n+t+1)R-j}
\end{aligned}$$

$$
\begin{aligned}
XS_{nR} &= XS_{(n-1)R} - q_{nR} + K(q_{(n-K)R} - q_{(n-K-1)R}) + q_{(n-K)R} \\
&= XS_{(n-2)R} - q_{(n-1)R} + K(q_{(n-K-1)R} - q_{(n-K-2)R}) + q_{(n-K-1)R} \\
&- \quad q_{nR} + K(q_{(n-K)R} - q_{(n-K-1)R}) + q_{(n-K)R} \\
&= XS_{(n-3)R} - q_{(n-2)R} + K(q_{(n-K-2)R} - q_{(n-K-3)R}) + q_{(n-K-2)R} \\
&- \quad q_{(n-1)R} + K(q_{(n-K-1)R} - q_{(n-K-2)R}) + q_{(n-K-1)R} \\
&- \quad q_{nR} + K(q_{(n-K)R} - q_{(n-K-1)R}) + q_{(n-K)R} \\
&= XS_{(n-4)R} - q_{(n-3)R} + K(q_{(n-K-3)R} - q_{(n-K-4)R}) + q_{(n-K-3)R} \\
&- \quad q_{(n-2)R} + K(q_{(n-K-2)R} - q_{(n-K-3)R}) + q_{(n-K-2)R} \\
&- \quad q_{(n-1)R} + K(q_{(n-K-1)R} - q_{(n-K-2)R}) + q_{(n-K-1)R} \\
&- \quad q_{nR} + K(q_{(n-K)R} - q_{(n-K-1)R}) + q_{(n-K)R} \\
XS_{nR} &= XS_{2R} - q_{3R} + K(q_{(3-K)R} - q_{(2-K)R}) + q_{(3-K)R} \\
&- \quad q_{4R} + K(q_{(4-K)R} - q_{(3-K)R}) + q_{(4-K)R} \\
&- \quad \sum_{t=5}^{n} q_{(t)R} + K(q_{(t-K)R} - q_{(t-1-K)R}) + q_{(t-K)R} \\
XS_{nR} &= XS_{R} - q_{2R} + K(q_{(2-K)R} - q_{(1-K)R}) + q_{(2-K)R} \\
&- \quad q_{3R} + K(q_{(3-K)R} - q_{(2-K)R}) + q_{(3-K)R} \\
&- \quad \sum_{t=4}^{n} q_{(t)R} + K(q_{(t-K)R} - q_{(t-1-K)R}) + q_{(t-K)R} \\
XS_{nR} &= XS_{0} - q_{R} + K(q_{(1-K)R} - q_{(-K)R}) + q_{(1-K)R} \\
&- \quad q_{2R} + K(q_{(2-K)R} - q_{(1-K)R}) + q_{(2-K)R} \\
&- \quad \sum_{t=3}^{n} q_{(t)R} + K(q_{(t-K)R} - q_{(t-1-K)R}) + q_{(t-K)R}
\end{aligned}
\tag{A.2}
$$

plus and minus q_{nR} terms cancels each other

$$XS_{nR} = XS_0 - q_{nR} - q_{(n-1)R} - ... - -q_{(n-K+1)R} + K - q_{(n-K)R}$$

we can write above equation as;

$$XS_{nR} = XS_0 - (q_{nR} - q_{(n-K)R}) - (q_{(n-1)R} - q_{(n-K)R})$$

$$- \quad ... - (q_{(n-K+1)R} - q_{(n-K)R})$$

Here we use noise term representation of q_{nR} values to define the inventory variable in terms of the random noise terms. In doing this we first of all need to make some modifications on the equation found for $q_{(n+i)R}$, to have $q_{(n-K+i)R}$.

$$\begin{aligned}
q_{(n+i)R} &= q_{nR} + R\alpha \sum_{j=(n-1)R+1}^{(n+i-1)R} \epsilon_j + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n+i)R-j} \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{nR-j} \\
q_{(n-K+i)R} &= q_{(n-K)R} + R\alpha \sum_{j=(n-K-1)R+1}^{(n-K+i-1)R} \epsilon_j + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n-K+i)R-j} \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n-K)R-j}
\end{aligned}$$

let's use this last equation in the equation **??**

$$\begin{aligned}
XS_{nR} &= XS_0 - (q_{(n-K+(K))R} - q_{(n-K)R}) - (q_{(n-K+(K-1))R} - q_{(n-K)R}) \\
&- \quad ... - (q_{(n-K+(1))R} - q_{(n-K)R}) \\
&= XS_0 - (R\alpha \sum_{j=(n-K-1)R+1}^{(n-1)R} \epsilon_j + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{nR-j} \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n-K)R-j}) \\
&- (R\alpha \sum_{j=(n-K-1)R+1}^{(n-2)R} \epsilon_j + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n-1)R-j} \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n-K)R-j}) \\
&- \sum_{t=3}^{K} (R\alpha \sum_{j=(n-K-1)R+1}^{(n-t)R} \epsilon_j + \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n-t+1)R-j} \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha + 1\}\, \epsilon_{(n-K)R-j})
\end{aligned} \tag{A.3}$$

then combine similar terms

$$
\begin{aligned}
XS_{nR} &= XS_0 + K[\sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\}\, \epsilon_{(n-K)R-j}] \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\} \sum_{i=0}^{K-1} \epsilon_{(n-i)R-j} \\
&- R\alpha \left\{ K \sum_{j=(n-K-1)R+1}^{(n-K)R} \epsilon_j + (K-1) \sum_{j=(n-K)R+1}^{(n-K+1)R} \epsilon_j + ... + \sum_{j=(n-2)R+1}^{(n-1)R} \epsilon_j \right\} \\
XS_{nR} &= XS_0 + K[\sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\}\, \epsilon_{(n-K)R-j}] \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\} (\epsilon_{nR-j} + \epsilon_{(n-1)R-j} + ... + \epsilon_{(n-K+1)R-j}) \\
&- R\alpha \left\{ K \sum_{j=0}^{R-1} \epsilon_{(n-K)R-j} + (K-1) \sum_{j=0}^{R-1} \epsilon_{(n-K+1)R-j} + ... + \sum_{j=0}^{R-1} \epsilon_{(n-1)R-j} \right\}
\end{aligned}
$$

again combine the similar terms in the last two rows of the above equation:

$$
\begin{aligned}
XS_{nR} &= XS_0 + K[\sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\}\, \epsilon_{(n-K)R-j}] \\
&- \sum_{j=0}^{R-1} [\{(L+R-1+j)\alpha+1+R\alpha(K-1)\}\, \epsilon_{(n-K+1)R-j} \\
&+ \{(L+R-1+j)\alpha+1+R\alpha(K-2)\}\, \epsilon_{(n-K+2)R-j} \\
&+ \sum_{t=3}^{K-1} \{(L+R-1+j)\alpha+1+R\alpha(K-t)\}\, \epsilon_{(n-K+t)R-j} \\
&- R\alpha K \sum_{j=0}^{R-1} \epsilon_{(n-K)R-j} - \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\}\, \epsilon_{nR-j} \\
XS_{nR} &= XS_0 + K[\sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1-R\alpha\}\, \epsilon_{(n-K)R-j}] \\
&- \sum_{j=0}^{R-1} [\{(L+R-1+j)\alpha+1+R\alpha(K-1)\}\, \epsilon_{(n-K+1)R-j} \\
&+ \{(L+R-1+j)\alpha+1+R\alpha(K-2)\}\, \epsilon_{(n-K+2)R-j} \\
&+ \sum_{t=3}^{K-1} \{(L+R-1+j)\alpha+1+R\alpha(K-t)\}\, \epsilon_{(n-K+t)R-j} \\
&- \sum_{j=0}^{R-1} \{(L+R-1+j)\alpha+1\}\, \epsilon_{nR-j} \\
XS_{nR} &= XS_0 + K[\sum_{j=0}^{R-1} \{(L-1+j)\alpha+1\}\, \epsilon_{(n-K)R-j}]
\end{aligned}
$$

$$
\begin{aligned}
- & \sum_{j=0}^{R-1} [\{(L + KR - 1 + j)\alpha + 1\} \epsilon_{(n-K+1)R-j} \\
+ & \{(L + (K-1)R - 1 + j)\alpha + 1\} \epsilon_{(n-K+2)R-j} \\
+ & \sum_{t=2}^{K-1} \{(L + (K-t)R - 1 + j)\alpha + 1\} \epsilon_{(n-K+t+1)R-j}
\end{aligned}
\tag{A.4}
$$

APPENDIX B

COMPUTATIONS

Table B.1: An Example of Expected Result Observations, α=0, β_s=0.99, β_r=0.99, replication-9 (Information is Shared)

t	ϵ_t	d_t	F_t	x_t
0	0.0000	0.0000	0.0000	15.2019
1	-2.8276	7.1724	10.0000	8.0294
2	0.7288	10.7288	10.0000	37.3006
3	-2.9307	7.0693	10.0000	30.2314
4	-1.5067	8.4933	10.0000	21.7380
5	0.6094	10.6094	10.0000	11.1286
6	0.0199	10.0199	10.0000	34.5725
7	0.6977	10.6977	10.0000	23.8749
8	0.1961	10.1961	10.0000	13.6788
9	1.7060	11.7060	10.0000	1.9728
10	-0.7350	9.2650	10.0000	34.2309
11	0.2479	10.2479	10.0000	23.9830
12	0.6095	10.6095	10.0000	13.3734
13	-0.2640	9.7360	10.0000	3.6374
14	2.9111	12.9111	10.0000	27.5276
15	-0.5295	9.4705	10.0000	18.0571
16	0.3498	10.3498	10.0000	7.7073
17	0.4364	10.4364	10.0000	-2.7291
18	-1.1670	8.8330	10.0000	29.9610
19	0.4870	10.4870	10.0000	19.4740
20	-0.6990	9.3010	10.0000	10.1730
21	0.2060	10.2060	10.0000	-0.0330
22	-1.0633	8.9367	10.0000	32.8587
23	-0.4819	9.5181	10.0000	23.3406
24	-0.0427	9.9573	10.0000	13.3833

Table B.2: An Example of Expected Result Observations, α=0, β_s=0.99, β_r=0.99, replication-9 (Information is Shared)(Continued)

t	x_t	q_{nr}	ixs_{nr}	ip_{nr}	b/o
1	15.2019	40.0000	51.3967	40.0000	0.0000
2	8.0294				
3	37.3006				
4	30.2314				
5	21.7380	33.4638	17.9329	20.3915	0.0000
6	11.1286				
7	34.5725				
8	23.8749				
9	13.6788	41.5231	16.4098	41.5231	0.0000
10	1.9728				
11	34.2309				
12	23.9830				
13	13.3734	41.8284	-5.0271	41.8284	5.0271
14	3.6374				
15	27.5276				
16	18.0571				
17	7.7073	42.4674	-5.9715	42.4674	5.9715
18	-2.7291				
19	29.9610				
20	19.4740				
21	10.1730	39.0574	-3.2005	39.0574	3.2005
22	-0.0330				
23	32.8587				
24	23.3406				

Table B.3: An Example of Expected Result Observations, α=0, β_s=0.99, β_r=0.99, replication-9 (Information is not Shared)

t	x_t	q_{nr}	xs_{nr}	p_{nr}	b/o
1	15.2019	40.0000	51.3967	40.0000	0.0000
2	8.0294				
3	37.3006				
4	30.2314				
5	21.7380	33.4638	17.9329	20.3915	0.0000
6	11.1286				
7	34.5725				
8	23.8749				
9	13.6788	41.5231	16.4098	57.6415	0.0000
10	1.9728				
11	34.2309				
12	23.9830				
13	13.3734	41.8284	-5.0271	42.4391	5.0271
14	3.6374				
15	27.5276				
16	18.0571				
17	7.7073	42.4674	10.1470	43.7454	0.0000
18	-2.7291				
19	35.9325				
20	25.4455				
21	16.1445	39.0574	13.5287	32.2374	0.0000
22	5.9385				
23	36.0592				
24	26.5411				

Table B.4: Example of $\alpha = 0.0$ Deterministic Input Trial ($\sigma = 0.0, r = 4, l = 2, X_0 = 10, XS_0 = 80$)

t	X_t	d_t	q_{nr}	xs_{nr}	p_{nr}	b/o
0	10	0	40	40	40	0
1	0	10				
2	30	10				
3	20	10				
4	10	10	40	0	40	0
5	0	10				
6	30	10				
7	20	10				
8	10	10	40	0	40	0
9	0	10				
10	30	10				
11	20	10				
12	10	10	40	0	40	0
13	0	10				
14	30	10				
15	20	10				
16	10	10	40	0	40	0
17	0	10				
18	30	10				
19	20	10				
20	10	10	40	0	40	0
21	0	10				
22	30	10				
23	20	10				
24	10	10	40	0	40	0
25	0	10				
26	30	10				
27	20	10				

Table B.5: Example of $\alpha = 0.0$ Deterministic Input Trial ($\sigma = 0.0, r = 4, l = 0.0, X_0 = 30, XS_0 = 80$)

t	X_t	d_t	q_{nr}	xs_{nr}	p_{nr}	b/o
0	30	0	40	40	40	0
1	20	10				
2	10	10				
3	0	10				
4	30	10	40	0	40	0
5	20	10				
6	10	10				
7	0	10				
8	30	10	40	0	40	0
9	20	10				
10	10	10				
11	0	10				
12	30	10	40	0	40	0
13	20	10				
14	10	10				
15	0	10				
16	30	10	40	0	40	0
17	20	10				
18	10	10				
19	0	10				
20	30	10	40	0	40	0
21	20	10				
22	10	10				
23	0	10				
24	30	10	40	0	40	0
25	20	10				
26	10	10				
27	0	10				

Table B.6: Example of $\alpha = 0.0$ Deterministic Input Trial ($\sigma = 1.0, r = 3, l = 2, X_0 = 10, XS_0 = 60$)

t	X_t	d_t	q_{nr}	xs_{nr}	p_{nr}	b/o
0	10	0	30	30	30	0
1	0	10				
2	20	10				
3	10	10	30	0	30	0
4	0	10				
5	20	10				
6	10	10	30	0	30	0
7	0	10				
8	20	10				
9	10	10	30	0	30	0
10	0	10				
11	20	10				
12	10	10	30	0	30	0
13	0	10				
14	20	10				
15	10	10	30	0	30	0
16	0	10				
17	20	10				
18	10	10	30	0	30	0
19	0	10				
20	20	10				
21	10	10	30	0	30	0
22	0	10				
23	20	10				
24	10	10	30	0	30	0
25	0	10				
26	20	10				
27	10	10	30	0	30	0

Table B.7: Example of $\alpha = 0.0$ Deterministic Input Trial ($\sigma = 1.0, r = 3, l = 2, X_0 = 12.5631, XS_0 = 60$)

t	**Xt**	**demand**	**qnr**	**xsnr**
0	12.5631	0	30	30
1	3.873624	8.689476		
2	23.68256	10.19107		
3	12.54668	11.13587	30.01642	-0.01642
4	2.241272	10.30541		
5	22.33521	9.906063		
6	11.86653	10.46868	30.68015	-0.69656
7	2.005192	9.861342		
8	21.97152	10.03367		
9	11.30117	10.67035	30.56536	-1.22909
10	0.618231	10.68294		
11	20.39258	10.25849		
12	11.08984	9.302738	30.24417	-0.12938
13	2.306384	8.783456		
14	23.77085	9.879414		
15	13.89728	9.873568	28.53644	1.78476
16	5.038736	8.858543		
17	23.48072	10.22384		
18	13.18941	10.29131	29.37369	2.334044
19	3.50967	9.67974		
20	22.52371	10.35964		
21	11.5727	10.95102	30.9904	-1.82765
22	2.751118	8.82158		
23	22.13036	9.783511		
24	11.77091	10.35945	28.96454	-0.58125
25	1.687899	10.08301		
26	24.01903	7.879806		

Table B.8: 90 Percent CI for $\alpha = 0.2$ and 100 Simulations

For the precision level of 0.05 optimal replication number is found to be 10 so we continued to work on these many replications. (Absolute precision of a CI is its H.L) To show how well the confidence interval will perform in terms of coverage in practice we decided to make several simulation runs and construct a confidence interval for the actual coverage of individual simulation's CI's. Average of CI (half length/mean) is also calculated as a measure of the precisions of the CI's. 90Alpha=0.2 with negative demand, 100 simulation

sim-no	**CI**	**mean**	**stdev**
1	(4.018457,4.160785)	4.08962	0.01507
2	(3.992180,4.111309)	4.05174	0.01056
3	(4.142827,4.252752)	4.19779	0.00899
4	(4.002726,4.091907)	4.04732	0.00592
5	(4.028461,4.177829)	4.10315	0.01660
6	(4.074551,4.184065)	4.12931	0.00892
7	(4.057841,4.144867)	4.10135	0.00564
8	(4.022164,4.102485)	4.06233	0.00480
9	(4.020836,4.076856)	4.04885	0.00234
10	(4.051611,4.170432)	4.11102	0.01051

90% CI for the true coverage

Number of intervals that cover the desired mean is 89

Proportion of 100 CIs that cover desired mean is 0.890000

CI for coverage is, 0.890000+(-) 0.051470 that is: (0.838530,0.941470)

Average of CI (half length/mean) for 100 CIs is 0.012830

Table B.9: 90 Percent CI for $\alpha = 0.2$ and 500 Simulations

sim-no	**CI**	**mean**	**stdev**
1	(4.018457,4.160785)	4.08962	0.01507
2	(3.992180,4.111309)	4.05174	0.01056
3	(4.142827,4.252752)	4.19779	0.00899
4	(4.002726,4.091907)	4.04732	0.00592
5	(4.028461,4.177829)	4.10315	0.01660
6	(4.074551,4.184065)	4.12931	0.00892
7	(4.057841,4.144867)	4.10135	0.00564
8	(4.022164,4.102485)	4.06233	0.00480
9	(4.020836,4.076856)	4.04885	0.00234
10	(4.051611,4.170432)	4.11102	0.01051

90% CI for the true coverage

Number of intervals that cover the desired mean is 456

Proportion of 500 CIs that cover desired mean is 0.912000

CI for coverage is, 0.912000+(-) 0.020841 that is: (0.891159,0.932841)

Average of CI (half length/mean) for 500 CIs is 0.013316

Table B.10: Box Ploting for Outlier Determination

Original Output		**Sorted Output**	
nonstockout value	**replication no**	**nonstockout value**	**replication no**
0	1	0	1
42	2	0	3
0	3	0	5
1	4	0	13
0	5	0	14
15	6	0	15
2	7	0	16
5	8	0	17
4	9	0	20
1	10	1	4
11	11	1	10
1	12	1	12
0	13	1	18
0	14	1	19
0	15	2	7
0	16	4	9
0	17	5	8
1	18	11	11
1	19	15	6
0	20	42	2

Table B.11: Box Ploting for Outlier Determination (continued)

In 25th Percent: Replication 14 has value 0

In 75th Percent: Replication 7 has value 2

Difference for Box Plot found as 2 (2-0)

Upper bound of interval= 2+2*1.5=5

Lower bound of interval= 0-2*1.5=-3

Replications that are thrown out are:

nonstockout value	**replication no**
11	11
15	6
42	2

Table B.12: t-test Studies-Number of Stockout Occassions In Each Replication (Probability of not stockout for retailer=.98 and for supplier=.98)

Outlier replications are not removed		
repno	**no-info**	**info**
1	5	0
2	9	42
3	7	0
4	7	1
5	6	0
6	3	15
7	5	2
8	7	5
9	3	4
10	5	1
11	8	11
12	1	1
13	4	0
14	4	0
15	4	0
16	3	0
17	4	0
18	3	1
19	3	1
20	1	0

Table B.13: t-test Studies(continued)-Test Results for the Previos Table

t-Test: Paired Two Sample for Means		
	Variable 1	**Variable 2**
Mean	4.6	4.2
Variance	4.778947368	95.11578947
Observations	20	20
Pearson Correlation	0.495204857	
Hypothesized Mean Difference	0	
df	19	
t Stat	0.201544013	
P(T¡=t) one-tail	0.42120765	
t Critical one-tail	1.729131327	
P(T¡=t) two-tail	0.8424153	
t Critical two-tail	2.093024705	

Table B.14: t-test Studies(continued)-Number of Stockout Occassions In Remaining Replications After Removing the Outlier Ones(Probability of not stockout for retailer=.98 and for supplier=.98)

Outlier replications are removed		
repno	**no-info**	**info**
1	5	0
3	7	0
4	7	1
5	6	0
7	5	2
8	7	5
9	3	4
10	5	1
12	1	1
13	4	0
14	4	0
15	4	0
16	3	0
17	4	0
18	3	1
19	3	1
20	1	0

Table B.15: t-test Studies(continued)-Test Results for the Case of Outliers Are Removed

t-Test: Paired Two Sample for Means		
	Variable 1	**Variable 2**
Mean	4.235294118	0.941176471
Variance	3.441176471	2.183823529
Observations	17	17
Pearson Correlation	0.187757325	
Hypothesized Mean Difference	0	
df	16	
t Stat	6.335676759	
P(T¡=t) one-tail	4.94835E-06	
t Critical one-tail	1.745884219	
P(T¡=t) two-tail	9.89671E-06	
t Critical two-tail	2.119904821	