RISK ANALYSIS OF THE GOVERNMENT DOMESTIC DEBT STOCK IN TURKEY: COST-AT-RISK APPROACH

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ABSTRACT

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In this study, stochastic simulation based risk analysis is applied to the government domestic debt stock in Turkey with the motivation to identify the cost and risk characteristics of alternative debt financing strategies. Future path of interest rates is simulated by using the yield curve forecasting framework in Diebold and Li (2002), which is founded on the Nelson-Siegel yield curve model. Yield curve simulation is based on the estimated term structure of interest rates for the period June 2001-July 2004. Simulated yield curves are generally upward sloped and concave. Contrary to the common observation, long-term yields are more volatile compared to short-term yields. Under each financing strategy, debt is rolled over on top of simulated term structure of interest rates. Alternative financing strategies are compared with respect to absolute Cost-at-Risk, relative Cost-at-Risk and relative risk measures computed from the simulated cost distributions. Results of the risk analysis are influenced by the characteristics of the simulated term structure of interest rates and the additional yield imposed on the coupon bonds, which is assumed to reflect risk perception of investors for increased maturity.

Keywords: Government Debt, Stochastic Simulation, Cost-at-Risk

ÖZ

KAMU İÇ BORÇ STOKUNUN RİSK ANALİZİ: RİSKE MARUZ MALİYET YAKLAŞIMI

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Yüksek Lisans, Ekonomi Bölümü

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Bu çalışmada, rassal benzetişim yöntemine dayanan risk analizi alternatif kamu borçlanma stratejilerinin maliyet ve risk unsurlarını belirlemek amacıyla Türkiye'de kamu iç borç stokuna uygulanmıştır. Borçlanma maliyetini etkileyen faiz oranlarının benzetişiminde Diebold ve Li (2002) tarafından geliştirilen ve Nelson-Siegel modeline dayanan, getiri eğrisi tahmin yaklaşımı kullanılmıştır. Getiri eğrisi benzetişimi Haziran 2001-Temmuz 2004 dönemi için tahmin edilen getiri eğrisi modeline dayanmaktadır. Benzetişim sonucu elde edilen getiri eğrileri pozitif eğimli ve dışbükeydir. Genel gözlemin aksine, uzun vadeli getirilerdeki dalgalanmalar, kısa vadeli getirilere göre daha yüksek olmaktadır. Belirlenen borçlanma stratejileri çerçevesinde, kamu borc stoku benzetişim sonucu elde edilen faizler kullanılarak çevrilmiştir. Farklı finansman stratejileri Mutlak Riske Maruz Maliyet, Göreli Riske Maruz Maliyet ile Göreli Risk ölçütleri çerçevesinde karşılaştırılmıştır. Risk analizinin sonuçları, simulasyon sonucu elde edilen getiri eğrisinin özellikleri ile kuponlu tahviller üzerine konan ve yatırımcıların uzayan vade karşısındaki risk algılamasını yansıtan ek getiri varsayımlarından etkilenmektedir.

Anahtar Kelimeler: Kamu Borç Stoku, Rassal Benzetişim, Riske Maruz Maliyet

To My Family,

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CHAPTER 1

INTRODUCTION

Optimal design of public debt in terms of denomination, maturity structure and indexation features has been an issue of both academic and practitioner research in the field of public debt management. However, these lines of work consider the issue from different perspectives. In the theoretical literature, under the optimal taxation approach, the objective of the government is to minimize welfare losses resulting from distortionary taxation. Hence the government is motivated to smooth tax rates over time. Under this framework risk is the budgetary risk, more specifically it is the risk of having to change taxes in response to the shocks hitting the government budget. On the other hand, under the public debt management practice objective of the government is to minimize the financial cost of servicing debt with due regard to risk, which is the potential variation in the financial cost. The scope of risk concept under debt management practice is limited to the debt servicing cost. However in recent years, there is a tendency towards measuring the risk under a budgetary framework, acknowledging that macroeconomic shocks not only affect debt costs but also other components of the government's budget.

In our study we approach the optimal debt design issue from the practitioner's perspective, where the objective of government is to finance maturing debt and fiscal deficit at lowest possible cost with due regard to risk. In recent years there is increased focus of countries on managing financial and operational risk inherent in the government debt portfolio (IMF and World Bank, 2002). This tendency in the field renders identification of cost and risk characteristics associated with alternative financing strategies and corresponding debt structures an important component of public debt management. Methods that are used to serve this purpose differ among countries. Most use deterministic models and few use stochastic models (IMF and World Bank, 2002). Cost and risk measures in the stochastic models are computed from a simulated cost distribution. Cost is the mean (or median) of the distribution whereas risk is measured as deviation from the mean with a given probability. These distribution-based measures are adapted to the sovereign debt management framework from financial and corporate sector.

In Turkey, Undersecretariat of Treasury is the responsible institution for establishing and executing the sovereign debt management strategy. Basic principles of debt and risk management are stated as the maintenance of sustainable, transparent and accountable borrowing policy consistent with the monetary and fiscal policies on account of the macroeconomic balances and the fulfillment of financing requirements at the lowest possible cost in the medium and long term with regard to the determined levels of risk. Within this framework debt management strategy of the Treasury comprises a cost and risk analysis where risk measure is based on a stochastic model.

In this thesis, market risk of the government domestic debt stock, associated with the changes in interest rates, is measured under a set of borrowing strategies, which comprise discounted securities and coupon bonds denominated in domestic currency. Risk measures are computed from simulated cost distributions obtained from a stochastic simulation model. The model consists of two parts. First part comprises the stochastic model, using which random elements that affect the debt cost are modeled and simulated. Second part contains strategy simulation, in which for each borrowing strategy debt is rolled over on top of the simulated economic environment. The study is structured as follows: Chapter 2 gives a brief overview of public debt management, empirical studies in the field and a summary of risk measures used in public debt management practice. Chapter 3 refers to the recent advances in the field of public debt management in Turkey and analyzes the present structure of the public debt stock. Chapter 4 and Chapter 5 describe the stochastic model and the strategy simulation components of the stochastic simulation analysis. Chapter 6 contains market risk analysis of the domestic currency denominated portion of the government debt stock in Turkey. Chapter 7 concludes.

CHAPTER 2

LITERATURE REVIEW ON SOVEREIGN DEBT MANAGEMENT WITH A RISK MANAGEMENT FOCUS

2.1 Introduction

Well-structured and sustainable public debt stock along with sound fiscal and monetary policies is the premises of a stable economy. High public debt stock besides crowding-out private sector activity constitutes a source of instability in the economy. Adverse developments relating to the public debt poses substantial risk not only to the government's balance sheet but also balance sheet of the other sectors, since government debt instruments are held as assets by the rest of the economy. High and poorly structured debt raises concerns related to the sustainability of the debt stock. Coupled with international capital mobility damages financial stability. Poorly structured debt in terms of maturity mix and currency denomination has been important factors in inducing and propagating economic crises (IMF and World Bank, 2002). Short maturity, floating rate and foreign currency denominated debt are considered to be risky. Short maturity and floating rate debt expose government budget to changing financial market conditions when this debt has to be rolled over. Refinancing problem faced by the government is exacerbated when the debt to be refinanced is in foreign currency. Inability of the government to borrow in terms of foreign currency either stemming from non-adequate foreign exchange in the economy or reluctance of investor to lend in foreign currency can exert upward pressure on the exchange rate. Moreover, this situation faced by the government consequently results in depletion of the central bank reserves. Sound debt structures alleviate the risk perceptions related to the sustainability of the debt stock by reducing the exposure of the debt stock to the interest rate and the exchange rate movements. In that respect, sound debt management along with policies for managing contingent liabilities[1] can reduce vulnerability of a country to financial risk by reducing the probability that government's debt portfolio will cause instability in the economy. Sound debt management is not a just the concern of the highly indebted countries with debt sustainability problems. Increase in the cost of debt results in welfare losses as tax rates are adjusted to finance the gap in government budget. Welfare loss is aggravated further if cost of the foreign debt stock is increased since resources of the economy are transferred abroad. Moreover, governments are debtors in international financial markets. Therefore governments are expected to have transparent and accountable debt management practices including a risk management policy. From the investors perspective sovereign debt is a financial asset with high credit quality and high degree of liquidity. In that respect, to maintain the reputation of the government in international financial markets, governments are expected to have risk management culture (Storkey, 2001).

Public debt management has gone through significant developments in the past 10-15 years on account of the risks associated with high and poorly structured debt stock and lack of well-specified government objectives for public debt management. In 2001, IMF and the World Bank had prepared guidelines to assists debt managers in their efforts to reduce financial vulnerability, with the contribution of national debt management experts. Guidelines were extended with an accompanying document containing the country experiences relating to

[1]Contingent liabilities are not government debt and do not appear in government's cash accounts but become government debt and give rise to governments payment obligations in case a certain risk occurs. Contingent liabilities are classified into two as open and hidden. Open contingent liabilities could arise due to provided guarantees, whereas hidden contingent liabilities occur as a result of default by public institutions and private institutions on the fulfillment of their obligations, which are not under guarantee of the government. The government although not obliged by Law, undertakes the obligations by virtue of being a State. (Republic of Turkey Undersecretariat of Treasury, April, 2003)

the development of public debt management. Countries included in the survey were at different stages in terms of public debt management and the level of economic and financial development. In the guidelines sovereign debt management is defined as

> a process of establishing and executing a strategy for managing the government's debt in order to raise required amount of funding, achieve its risk and cost objective, and to meet any other sovereign debt management goals the government may have set, such as developing and maintaining an efficient market for government securities (IMF and World Bank Guidelines, pp. 1)

Primary objective of the government may change depending on the stage of financial development of the economy. However, in most cases the main concern of the government is to raise necessary funds at lowest possible cost with due regard to risk. Other objectives of the government, such as developing and maintaining efficient domestic financial market is a means of reducing dependence on short term and foreign currency linked debt and thus in the long-run serves the objective of reducing cost and risk associated with debt servicing. From the country surveys three points had emerged relating to the direction of practice in the field (IMF and World bank Guidelines, 2002):

1. The objectives for managing debt and institutional framework for meeting the objectives are becoming more formalized.
2. There is a convergence in approaches taken by the countries to promote well functioning domestic financial markets.
3. There is high level of awareness of the importance of risk management of public debt and growing consensus for the appropriate techniques for managing risk.

> Prudent debt management practices include: (i) recognition of the benefits for clear objectives for debt management; (ii) weighing risk against cost considerations; (iii) separation and coordination of debt and monetary management objectives and accountabilities; (iv) limit on debt expansion; (v) the need to carefully manage refinancing and market risk and the interest cost of debt; (vi) the necessity of developing a sound institutional structure and policies for reducing operational risk, including clear delegation of responsibilities and associated among agencies involved in debt management; and (vii) the need to carefully identify and manage risks associate with contingent liabilities (IMF and World Bank, 2002, pp. 13).

As listed above, management of market and refinancing risk is one of the practices of prudent sovereign debt management. Market risk refers to the risk of changes in the cost of debt associated with the movements in market prices, such as interest rates, exchange rates, and commodity prices. Refinancing (rollover) risk is the risk that debt will have to be rolled over at an unusually high cost or in extreme cases cannot be rolled over at all. Refinancing risk and market risk overlap when refinancing risk is limited to the risk that debt might have to be rolled over at high interest rates.

In this general framework along with cost and risk objectives, government exercises debt management practices to enhance liquidity in the secondary markets and to broaden the investor base with the objective of reducing borrowing costs in the long term. Broadening investor base is implemented by diversifying the stock of debt among different maturities, thorough a range of different market instruments or by issuing securities targeted at specific investors. This practice reduces the risk that the pricing of government securities could be affected by the actions of small number of market participants. Another debt management practice is the issuance of benchmark securities at key maturities to enhance market liquidity, thereby reduce liquidity premium on yield of the government securities and lower government borrowing costs. Benchmark securities are constructed by issuing the same security over several auctions and repurchasing the older issues prior to maturity that are no longer actively traded in the market. However this strategy motivated to reduce debt service cost is a factor increasing the rollover risk (IMF and World Bank, 2002).

Increased transparency in public debt management activities goes along with the institutional advances in the field. Intention behind increased transparency is the view that risk premiums on government securities and thereby borrowing costs are minimized as uncertainty regarding the objectives and conduct of debt management and the state of government finances are reduced

Governments are not always capable of pursuing the public debt management objectives. Sovereign debt managers are faced with constraints. Prevailing macroeconomic conditions in the economy, investors' reluctance to lend to the government and the risk perception of the market hinder active debt management. In these instances governments in order to raise necessary amount of financing are forced to borrow in short-term or in foreign currency denominated debt. Moreover, in countries with less developed financial markets government face more severe cost and risk tradeoffs compared to countries with strong economies and developed financial markets. For example in countries with less developed financial markets governments pursue the objective of extending the maturity by issuing securities indexed to inflation or exchange rate.

One of the practices of prudent sovereign debt management is the management of refinancing and market risk and the interest cost of debt. This issue is the focus of our study. In the remaining of this chapter, we initially introduce risk measures associated with refinancing and market risk. The following section gives an overview of the risk management practices and the empirical work in the field.

2.2 Sovereign Debt Management with a Risk Management Focus

2.2.1 Risk Measures in Sovereign Debt Management

In this part risk measures computed under stochastic simulation framework are introduced. Prior to that, other risk concepts used by debt managers to assess the rollover and the market risks of the debt portfolio are briefly explained. We refer to them as conventional risk measures.

Conventional risk measures

Average term to maturity (ATM): The average term to maturity is the average remaining time to maturity of debt instruments that make up the debt. A longer ATM indicates that debt instruments are rolled over less frequently and therefore there is lower refinancing risk and less uncertainty regarding future debt cost

Duration: Duration is also the weighted average of the remaining maturity of the debt stock. However duration considers not only the principal payment but the present value of all expected cash flows through the lifespan of a debt instrument. Longer duration lowers the rollover risk. Most commonly used duration measure is Macaulay’s duration. Macaulay duration of a bond is defined as,

$$D = \sum_{t=1}^{N} t * \frac{CP_t}{(1+i)^t} \Big/ \sum_{t=1}^{N} \frac{CP_t}{(1+i)^t} ,$$

where, t = time until cash payment is made

CP_t = cash payment at time t

I = interest rate

N = time to maturity of the bond

(Mishkin, 1997)

Duration by itself is not a sufficient risk measure. Same duration can be obtained by a composition of debt instruments issued at various maturities. Thus, two debt portfolios with same duration may respond differently to an increase in the interest rates due to different maturity structures. Analysis in Riskgaldskatoret (2002) concludes that in short-term perspective primary factor affecting the risk of debt portfolio is the maturity profile.

Maturity profile: Maturity profile is the amount of debt that matures in any given year. Well-distributed maturity profile reduces the refinancing risk, which is the risk that relatively large proportion of the debt will have to be rolled over in a period of high interest rates. Smooth maturity profile limits the amount of debt that will be refinanced in a specified period of time (e.g. within a year). In this respect, related risk measure is the proportion of debt that will be refinanced within a year. A similar risk concept is the interest rate refixing risk, which is the risk that the interest rate on large part of the debt will be refixed when interest rates are unfavorable. This risk concept extends the coverage of the refinancing risk by including the debt whose interest will be reset within a specified period. Thus interest rate refixing risk provides a more comprehensive assessment of the exposure of debt stock to interest rate movements. Risk measure defined within this context is the fixed-rate share. It is computed as the proportion of debt that does not mature or need to be re-priced within a year relative to the total interest rate bearing debt stock (Canada Department of Finance, 2003).

Risk Measures Based on Stochastic Simulation

Risk measures obtained through stochastic simulation models complement the existing risk management tools. These measures are calculated from a simulated cost distribution. Simulation based risk models are described in detail in section 5.4. In this section we briefly introduce the concept by illustrating cost-at-risk (CaR), which is one of the main tools used in within this class of risk measures.

Simulation based risk models used in sovereign debt management practice are risk measures taken from the financial and the corporate sector and adopted to the sovereign risk management framework. Value-at-Risk (VaR) and Cash–Flow-at-Risk (CFaR) are two of these measures. Value at Risk measure is used in financial community to measure the risk associated with the market value of a portfolio. Value at Risk (VaR) expresses the maximum decline, with a given probability, in the market value of portfolio over a given period. In majority of the debt management practices debt cost is not computed from the market value of the debt portfolio. Instead, only realized cost, which is the costs when payment is made, is taken into consideration. Unrealized mark-to market[2] costs are not considered. Mark-to market costs are the changes in the market value of the debt stock resulting from the movements in the market prices. In other words value of the sovereign debt is not adjusted as market prices change.[3] Since bulk of the debt is left outstanding until maturity, direct application of VaR approach to debt management is regarded as being irrelevant. [4] Thus, instead of VaR, a similar measure used in sovereign debt management is Cost-at-Risk. Cost-at-Risk

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[2] Realized mark-to-market costs are cost arising from debt buybacks and swaps.

[3] An exception to this practice is New Zealand Debt Management Office(NZDMO). NZDMO manages market risk associated with tactical trading through the use of VaR measure (IMF and World Bank Guidelines, 2002).

[4]In Garcia (2002) Value-at-Risk of the debt stock is calculated as the variation in the present value of the debt portfolio acknowledging it as an irrelevant measure from the viewpoint of the government.

measure is based on debt costs rather than market value. Once the statistical distribution of the debt costs is obtained, CaR is the maximum cost that could occur with some probability in a particular time period. For example with 95 percent probability, CaR is the 95th quartile value of the cost distribution (Figure 2.1).

Cash–flow-at-risk is another risk concept used in the corporate literature. CFaR measure takes account of the impact of risk factors on the firm's cash flows (Jorion, 2001). When this methodology is adapted to the government sector, besides the public debt, the impact of fluctuations in market prices on other components of the fiscal accounts is taken into consideration (Garcia, 2002). Given this characteristic of the CFaR measure it can be classified as a risk measure under Asset and Liability Management Framework.[5]

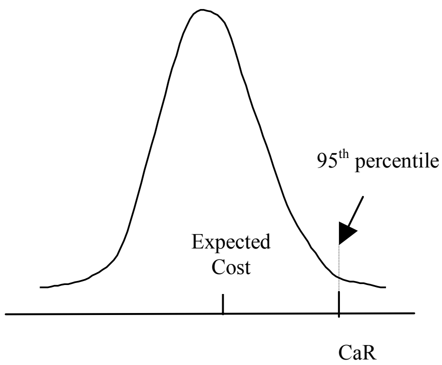


Figure 2.1 Illustration of Cost-at-Risk

2.2.2. Risk Management Practices for Sovereign Debt and Empirical Studies

Debt management strategy involves a choice on the structure of the debt stock and features of the borrowing instruments. Borrowing instruments can vary in terms of maturity structure, currency denomination and the terms of

[5]Asset Liability Management Framework is explained in section 2.2.2.

indexation. Each debt structure resulting from the debt management strategy entails different cost and risk characteristics. Risk management is motivated with the objective of identifying and controlling the risk involved in the debt stock and to the extent possible reducing the risk by modifying the debt structure. Focus of our analysis is on the risks affecting the cost of government debt stock, which refers to the financial cost of servicing debt over medium to long run. Therefore among various risk encountered in public debt management we concentrate on the market risk and the refinancing risk. These are the risks associated with the financial cost of the debt stock.[6]

Changes in interest rates affect debt-servicing cost for both domestic and foreign debt on the new issues when the maturing debt is refinanced and on floating rate debt at the time when interest rates on these securities are reset (e.g. coupon payments of floating rate debt). Hence, short term or floating rate debt is usually considered to be more risky than long-term debt. There are tradeoffs associated with different debt structures. One of these tradeoffs is related to the maturity structure. Short-term debt is less costly compared to long-term debt however it is more risky. This tradeoff has two premises. First one is the frequent refinancing associated with short-term borrowing. Second one is the presumption that on the average yield curves are positively sloped and short-term rates are more volatile with respect to long-term rates (Bolder, 2003; Diebold and Li, 2002). Foreign currency debt also entails a tradeoff. Foreign currency denominated/ indexed debt reduces the cost of borrowing however increases the exposure of debt stock to depreciation of the exchange rate.

Under the debt management framework, financial risk considerations that are mentioned above are limited to the debt stock, where risk is considered as the potential increase in the debt service cost. An alternative to the debt management framework is to consider risk under broader Asset and Liability Management (ALM) framework. Under this approach, risk is envisioned as the potential destabilizing impact on the budget of the financial shocks to debt service (Valendia, 2002). In response to an unexpected increase in debt service cost, governments can either raise taxes or cut spending. Both alternatives entail social

[6] Full coverage of the risks encountered in sovereign debt management can be found in IMF and World Bank (2001).

and economic costs. The objective of applying ALM approach is to smooth out the impact of budgetary shocks via examining various characteristics of assets and selecting liabilities with matching characteristics. Under this approach, correlation of financial shocks with government revenues and spending is taken into consideration. ALM approach to risk assessment of sovereign debt portfolio coincides with the theoretical perspective on the optimal structure of the public debt provided with the optimal taxation approach. The objective of the government under optimal taxation approach[7] is to minimize the loss from distortionary taxation. Governments look for the debt structure that minimizes the variation in tax rates. In order to serve this objective optimal debt structure should be such that debt returns are positively related to the output (tax base) and negatively related to the government spending. In that respect, structure of the public debt is considered as an insurance against shocks affecting the government budget.

The objective of selecting the optimal borrowing structure requires a framework for measuring cost and risk related to alternative borrowing strategies. Methods used to measure cost and market risk of sovereign debt portfolio differ across countries. Most use deterministic scenarios and few use stochastic models. Countries that use stochastic simulation models comprise Brazil, Colombia, Denmark, New Zealand, Canada, Italy, Portugal and Sweden. Recently Turkish Treasury, authority of managing sovereign debt, has also adopted a risk measure based on a stochastic simulation model. Debt management offices that have initially applied this method are New Zealand, Denmark and Sweden. Models developed by debt management offices have introduced a framework for measuring the risk of sovereign debt portfolio using stochastic simulation method. In the following debt management practices of these countries are briefly explained. New Zealand Debt Management Office (NZDMO) manages the debt portfolio at both strategic and tactical level. Strategic management refers to the management of the overall parameters of the portfolio in terms of currency mix and interest rate sensitivity. Strategic management is exercised under asset-liability management framework. Accordingly, NZDMO has reduced net foreign

[7] Overview of public debt management theory and optimal taxation approach can be found in Missale(1997) and Leong(1999).

currency debt to zero. This step was taken under the consideration that the value of government's assets is sensitive to the movements in domestic interest rates but not to the movement in the exchange rate. Insensitive nature of the assets to the exchange rate implies that when interest expenditure of the government changes assets do not change in a manner that would reduce volatility in the government budget. The other consideration was the vulnerability against the movements in the exchange rate, which could not be hedged effectively given the magnitude of the external portfolio and the capacity of the New Zealand foreign exchange market. Tactical management refers to the discretionary management of the net portfolio debt within the established limits around the strategic portfolio. VaR is used by NZDMO for managing market risk associated with tactical trading. Consistent with its commitment to transparency and predictability policy in debt management, NZDMO doesn't engage in tactical trading with respect to the domestic debt portfolio. Sweden Debt Management Office (SNDO) interprets the risk concept in terms of how debt costs affect the overall stability of the government finances. In this respect, SNDO has adopted an ALM approach as the starting point for debt portfolio analysis. SNDO uses stochastic simulation based risk measure to evaluate risk exposure of debt stock to the movements in the interest rate, exchange rate and inflation (Bergström and Holmlund, 2000; Bergström et al, 2002). Sweden also engages in tactical trading to benefit from the movements in exchange rate and interest rates. Tactical trading is limited to the foreign currency debt. In Denmark, Danmarks National Bank (DNB) undertakes the administrative functions related to the government debt management. The objective of debt management focuses on reducing the risk of negative spillover effects from government debt to the economy. In that regard, interest rate and exchange rate risk are considered to be the most important risk factors. In order to limit exchange rate risk borrowing takes place only in Euro and Danish kroner. Management of the interest rate risk is based on a duration target and, smoothening of the redemption profile. CaR model is used to support the decision making to select the borrowing strategy and duration target. Using CaR model different strategies in terms of issuance strategy, amount of buybacks and duration target are analyzed (IMF and World Bank Guidelines, 2002).

Among debt management offices there is a tendency towards moving to a risk analysis under ALM framework. Measuring cost with respect to Gross Domestic Product (GDP) is considered as an attempt in this direction. In the stochastic model developed by SNDO cost is measured with respect to GDP, where GDP is seen as a measure of business cycle related influences on budget.[8] Under the presumption that budget balance and GDP are positively correlated, debt portfolio with relatively stable cost to GDP ratio is regarded as less risky. Some countries explicitly incorporate specific assets and liabilities such as foreign exchange reserves and contingent liabilities in an overall risk management structure.[9] Under this framework coordinating the maturity and the currency composition of the foreign currency debt with that of foreign exchange reserves held by the government or central bank is found to be useful so as to hedge the governments exposure to interest rate and exchange rate risk (IMF and World Bank, 2002). Valendia(2002) proposes alternative methods for measuring risk under ALM approach.

Empirical studies of both the debt management offices and other researchers in this field are motivated to compare cost and risk characteristics of various debt management strategies with the perspective of selecting the optimal debt strategy amongst the alternatives. Identification of the optimal strategy can either be accomplished at one step or using a two-step procedure. Stochastic simulation is the first stage of the latter method. It provides set of borrowing strategies each with cost and risk characteristics. Further stage is the selection of the optimal strategy amongst them. This task can be accomplished under judgmental or mathematical optimization framework on account of the other objectives of the government along with cost and risk. Hahm and Kim (2002) provide means of choosing the optimal portfolio in two stages. Initially stochastic simulation method is used to compute cost and risk of alternative borrowing strategies. Each borrowing strategy corresponds to a debt structure. Afterwards efficient portfolio set is obtained by selecting the portfolio that yields lowest

[8] In the model developed by SNDO, GDP is generated within a model. Hence it is possible to capture correlations between interest rates, exchange rates and GDP in a consistent manner (Bergström and Holmlund, 2000; Bergström et al., 2002).

[9] Brazil, Demark, New Zealand and United Kingdom

expected cost at each level of standard deviation (risk). Given the efficient portfolio set and exogenous risk targets (duration, CaR, etc.), optimal portfolio is selected by minimizing CaR and deviations from other exogenous risk targets. Alternatively, the problem faced by the government can be formulated as stochastic dynamic optimization problem where given the outstanding debt stock and the simulated paths for the variables effecting it, the role of debt manager is to find the borrowing policy (control variable) that minimizes the objective function that is the cost of debt. There are studies that have approached the problem from this perspective, in which case optimal portfolio is selected at one step. Grill and Östberg (2003) approach the government's problem as a financial optimization problem. The practice in that case is different from setting alternative strategies and then evaluating their risk and cost characteristics. Instead a goal is set, such as minimizing cost and risk of the debt, the strategy is then chosen via optimization. Italian debt management office has adopted a similar perspective. Once a scenario for the evolution of the random variables that affect the cost of debt stock is set-up, portfolio optimization is formulated as a finite dimensional Linear Programming Problem (Cannata et al., 2004). In Bolder (2003) governments borrowing decision is also conceptualized as an optimal control problem in a stochastic setting. The government is trying to optimally select the composition of its debt portfolio to minimize expected debt cost subject to risk and liquidity constraints. However due to practical complexities regarding the use of dynamic programming technique analysis rely on simulation of alternative debt management strategies.

2.3 Conclusion

Main objective of the government is to raise necessary funds at lowest possible cost with due regard to risk. In that respect, prudent risk management practice includes management of the refinancing and the market risk and the interest cost of debt. In this chapter initially distribution based market risk measures along with conventional risk measures associated with refinancing and market risk were briefly introduced. After that an overview of the risk management practices and the empirical work on sovereign risk analysis were provided. The following chapter covers recent advances in the public debt management practice and the structure of public debt stock in Turkey.

CHAPTER 3

PUBLIC DEBT MANAGEMENT AND THE STRUCTURE OF SOVEREIGN DEBT STOCK IN TURKEY

In the recent years substantial progress has taken place in the field of public debt and risk management in Turkey. Law on the Regulation of Public Financing and Debt Management was put into effect in March 28, 2002. Moreover, "Regulation on the Principles and Procedures for the Coordination and Execution of Debt and Risk Management" went in to effect in September 1, 2002. Another innovation is the primary dealership system, which was reintroduced in 2002. This chapter covers these advances in public debt management and analysis of the structure of sovereign debt stock.

3.1 Recent Advances in Public Debt Management

Law on the organization of public financing and debt management is one of the basic laws of public financial administration.[10] It has introduced arrangements to ensure fiscal discipline and accountability and measures to increase transparency in the public debt management. The law sets out the legal foundations for fiscal risk management and the general principles and strategies related to the management of debt and receivables. Within this context, it contains provisions on the formation of the infrastructure related to the

[10] Legislation related to debt management can be found in Republic of Turkey Undersecretariat of Treasury (April, 2003)

management of the risks faced by the Treasury's debt portfolio and its portfolio of guarantees and claims. Accordingly, Risk Management unit and Debt Management Committee were established within the Undersecretariat of the Treasury. Risk Management Unit is responsible for defining and measuring the existing measurable and controllable risk, establishing tolerance limits, tracking and reporting these risks and keeping risk structure under control. Fiscal risks include foreign exchange risk, cross-rate risk, interest rate risk, rollover risk, liquidity risk, credit risk, operational risk and political macroeconomic risk.[11] Fiscal risk management encompasses Treasury's domestic and foreign debt portfolio, and its portfolio of guarantees and claims. Thus contingent liabilities are taken into account (Republic of Turkey Undersecretariat of Treasury (RTUT), , April 2003). Debt Management Committee on the other hand is responsible for ensuring coordination and efficiency in debt management.

Additionally, the regulation set the principles for public debt and risk management, duties of the Debt Management Committee and the Risk Management Unit. Accordingly the principles of the public debt and risk management were defined as:

- Maintenance of sustainable, transparent and accountable borrowing policy in consistency with monetary and fiscal policies taking account of macroeconomic balances,
- And, the fulfillment of financing requirements at the lowest possible cost in the medium and long term in accordance with the levels of risk determined in consideration of domestic and external market conditions and cost factors (RTUT, April 2003, pp. 1).

Another development in the field of public debt management is the Primary Dealership System. Primary Dealership System was first introduced in 2000, halted due to 2000 and 2001 crises and was reintroduced in September 2002. Primary dealership was introduced to increase efficiency and liquidity of the primary and secondary markets. Two main obligations are imposed on the banks participating in the system. Under this system each primary dealer (PD) bank is obliged to purchase certain amount of the debt instrument issued in the

[11] Risk definitions are given in technical terms appendix.

primary market. This obligation reduces rollover risk since certain amount of debt can always be raised. Other obligation of the PD banks is to continuously buy and sell benchmark government securities, set by the Treasury, in the Istanbul Stock Exchange Bond and Bill Market, in order to ensure liquidity in the secondary market. Presence of banks willing to buy and sell on the domestic borrowing instruments is intended to increase general demand for domestic borrowing instruments and broaden investor base. Currently there are 12 PD banks two of which are foreign banks.

Progress in the direction of enhanced fiscal discipline and transparency significantly contribute to the decline in the borrowing costs of the government. Transparency enables market participants to anticipate borrowing policies of the coming periods. Importance of transparency is underlined in the RTUT(April, 2003). As communication among debt managers and market participants increase, and as the uncertainty relating to the borrowing policies diminish so will the risk premium on government securities. This will help government reduce borrowing costs. Having briefly emphasized the legal and organizational developments in the field of public debt management, in the remaining part of the chapter structure of public sector debt will be analyzed.

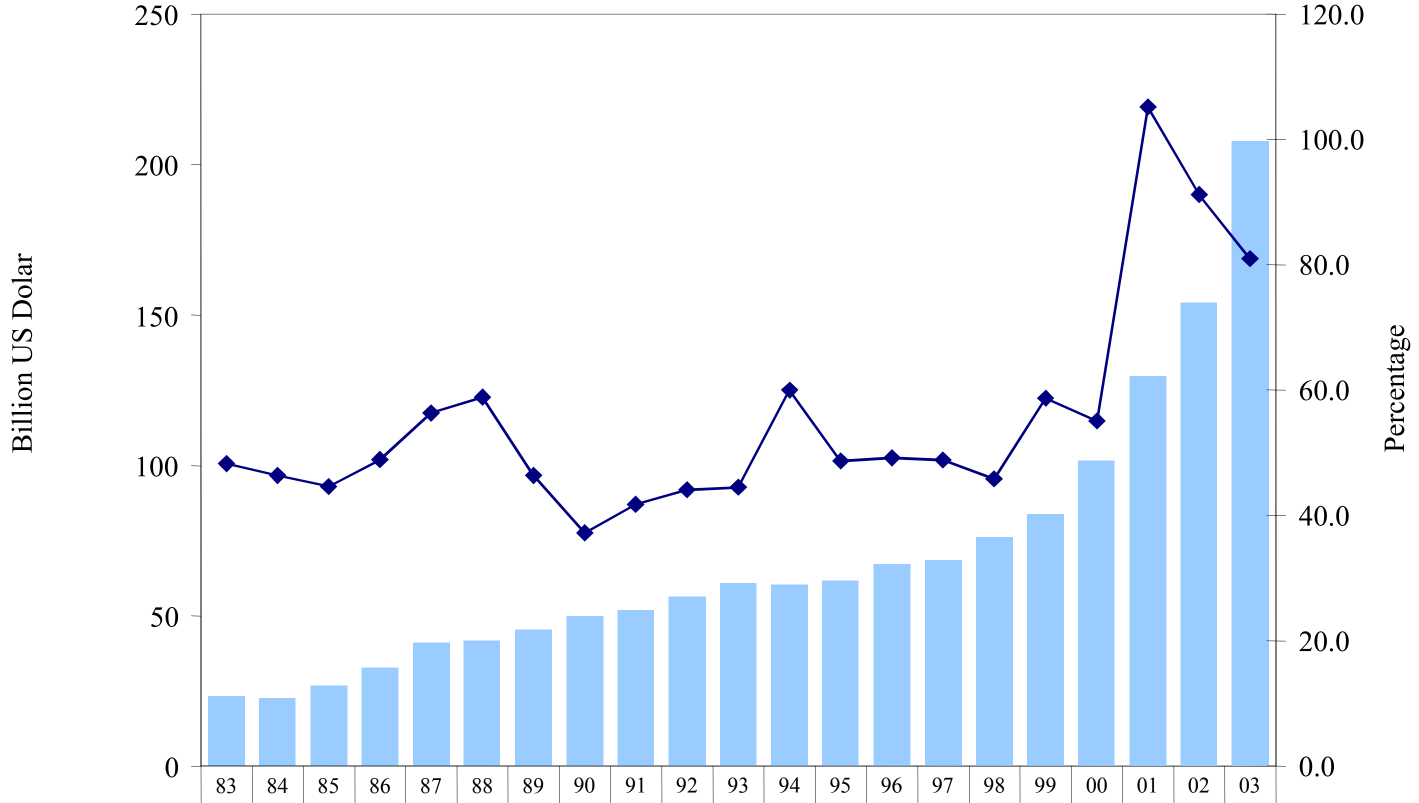
3.2 Debt Stock of the Public Sector

Gross debt stock of the public sector in Turkey increased from 48.3 percent of GDP in 1983 to 81 percent in 2003 (Figure 3.1). Decomposition of the change in the debt stock displays several important points (Table 3.1, Figure 3.2). Interest expenditure during the period has risen sharply due to increased borrowing from the markets and ensuing high level of interest rates. Interest expenditure, which constituted 2 percent of GDP in 1983 increased to 17.1 percent of GDP in 2002. Another important observation is the impact of movements in the exchange rate on the evolution of the debt stock during the period 1983-2002. Excluding the years from 1997 to 2004, increase in the debt stock resulting from the adjustments in the value of foreign debt expressed in domestic currency has been greater than the interest expenditure.

In the period 1983-1993, public sector borrowing requirement is positive indicating lack of fiscal discipline. Although primary budget was in surplus position following the 1992 until 1997, the magnitude of the primary surplus remained low compared to the heightening interest expenditures. Under a transparent, fiscal framework increase in the debt stock should be explained by revaluation of the foreign currency denominated portion of the debt stock and the budget balance. In the Turkish case for several years we observe considerable amount of residual increases in the debt stock that cannot be explained by the exchange rate movements or the budget deficits. In major part, source of these increases is the securities issued to cover the off budget expenditure of the central government or the other public institutions. Substantial amount of residual increase in the debt stock was observed in the year 2001. Prior to the year 2001, central government total debt stock to GDP ratio was around 50 percent. Hike in the debt stock mainly resulted from the increase in the non-cash debt[12], which was issued within the context of banking sector operation. In May 2001, specially designed, non-cash debt instruments were issued to the state banks and to the private banks under Saving Depository Insurance Fund (SDIF) to strengthen their capital structure. From mid 1980's and onwards, off-budget expenditures were carried out by the use of state banks, which resulted in accumulation of duty losses. Consequently debt instruments were issued to the state banks to cover the accrued losses. Substantial increase in the debt stock in 2001, for the aforementioned reasons has underlined the importance of fiscal discipline and the need to establish a control over contingent liabilities.

Despite the declining trend in the debt stock to GDP ratio, following the substantial increase in the year 2001, public debt sustainability is still a concern in Turkey. Along with its high level, the structure of the debt stock in terms of maturity and currency composition raise concerns.

[12] Non-cash securities are debt instruments against which no cash is received.



	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	00	01	02	03
Total Debt Stock (Billion Dolar)	23.5	22.8	27.0	32.9	41.2	41.9	45.5	49.9	51.8	56.3	60.9	60.3	61.7	67.2	68.5	76.2	83.8	101.	129.	154.	207.
Debt stock/GDP (Right axis)	48.3	46.4	44.7	48.9	56.4	58.9	46.4	37.2	41.8	44.1	44.5	60.0	48.7	49.2	48.9	45.8	58.7	55.1	105.	91.2	81.0

Figure 3.1. Gross Debt of the Public sector (1983-2003)

Source: State Planning Organization(SPO), Treasury.

Table 3.1 Decomposition of the Change in the Gross Total Public Sector Debt Stock (Share in GDP)

	Change in Total Debt Stock	Interest Expenditures	Foreign Debt Exchange Rate Effect	Primary Surplus	Other
1983	15.9	2.0	9.1	3.4	1.3
1984	15.6	1.9	5.7	1.7	6.3
1985	18.3	2.6	6.7	1.1	7.9
1986	22.9	3.0	8.6	3.1	8.2
1987	26.3	3.9	16.1	1.0	5.4
1988	12.9	3.6	6.0	1.8	1.5
1989	10.4	5.1	4.8	2.4	-1.8
1990	18.6	6.1	11.1	4.1	-2.8
1991	20.0	6.2	10.7	4.5	-1.4
1992	20.2	8.3	10.8	3.8	-2.7
1993	37.2	10.9	24.5	-2.9	4.7
1994	18.8	9.6	11.6	-4.5	2.1
1995	23.6	11.7	12.0	-2.9	2.8
1996	23.7	9.1	12.9	-1.2	3.0
1997	18.8	13.0	8.1	-3.3	1.1
1998	27.8	15.5	12.3	0.2	-0.1
1999	18.6	17.6	5.1	-5.7	1.6
2000	66.7	24.2	19.6	-8.0	30.9
2001	23.6	19.5	4.4	-6.9	6.6
2002	10.6	17.1	-4.6	-8.4	6.5

Source: SPO, Treasury.

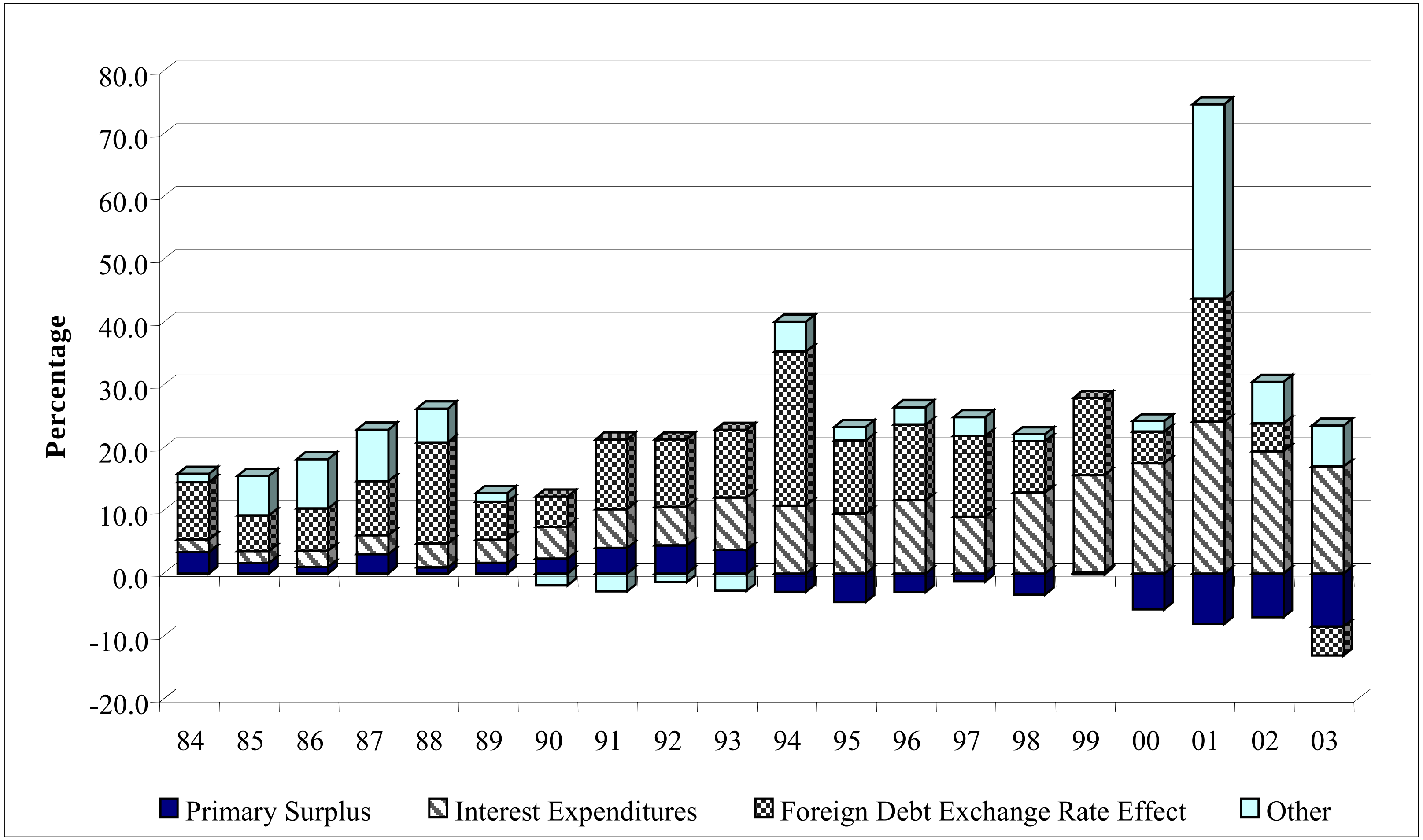


Figure 3.2. Decomposition of the Change in the Gross Total Public Debt Stock (Share in GDP)

Source: SPO, Treasury

Structure of the Central Government Debt Stock

Structure of the government debt stock is analyzed on the basis of the central government debt stock. Major segment of the public sector debt stock is held by the central government, thus sensitivity of the debt stock with respect to the movements in the interest rate and the exchange rate can be captured by the evaluation of the central government debt stock.

External debt stock of the central government, which constitutes 30 percent of the total central government debt stock, mainly consists of program credits from international organizations and Eurobond issues. Domestic debt stock comprises securities denominated in foreign currency, fixed rate securities and securities indexed to the interest rate, the inflation and the exchange rate. Together with the external debt, 44 percent of the total debt stock is linked to exchange rate and 49 percent of the total debt stock is composed of floating rate debt (Table 3.2). Given the high share of foreign exchange linked debt, the impact of movements in the exchange rate on the evolution of the debt stock has been considerable (Figure 3.2).

Table 3.2: Structure of the Central Government Debt Stock

June 2004	Quadr. TL	Million US dollar	Share (%)
Domestic Debt Stock	209.0	140.7	100.0
TL denominated	170.9	115.1	81.8
Fixed	83.2	56.0	39.8
Floating	87.7	59.0	42.0
Fx denominated/FX indexed	38.1	25.7	18.2
Fixed	16.7	11.3	8.0
Floating	21.4	14.4	10.2
TL denominated	170.9	115.1	81.8
Fx denominated/FX indexed	38.1	25.7	18.2
Fixed	99.9	67.3	47.8
Floating	109.1	73.4	52.2
Foreign Debt Stock	88.5	63.4	100.0
Fixed	52.9	37.9	59.8
Floating	35.6	25.5	40.2
Total Debt Stock	297.5	204.1	100.0
TL denominated	170.9	115.1	56.4
Fx denominated/FX indexed	126.6	89.1	43.6
Fixed	152.8	105.2	51.4
Floating	144.7	98.9	48.6

Source: Treasury.

Short average time to maturity is a major weakness of the domestic debt stock. As of June 2004, average remaining maturity of the debt stock is 20.2 months. Duration of the TL denominated domestic debt stock is within 5.0-5.5 months band and duration of the foreign debt stock is around 4.2 years (RTUT, October 2004). In year 2001, maturity of the overall debt stock has increased as the non-cash securities with relatively high maturity were issued within the framework of the aforementioned banking sector operation. Non-cash borrowing takes place under special circumstances. Regular means of financing debt service is via cash borrowing from the markets. Hence, in the period after 2001, overall maturity of the domestic debt stock declined approaching the maturity of cash debt stock, as the share and remaining maturity of non-cash debt stock decreased. This trend will continue in the coming years, average term to maturity of overall debt stock will approach the average term to maturity of the cash debt stock. Maturity of cash borrowing will determine the maturity of the debt stock. In this context, increasing the maturity of borrowing is a major issue of the debt management strategy.

Table 3.3: Average Remaining Maturity of Cash Domestic Borrowing and Domestic Debt Stock (Months)

	2000	2001	2002	2003	2004 June
Avg. Remaining Maturity of Total Debt Stock (1)	15.5	38.5	32.1	25.1	20.2
Avg. Remaining Maturity of Cash Debt Stock (2)	9.4	19.2	12.8	12.4	11.6
Maturity of Cash Borrowing	5.1	6.2	10	15.4	15.1
(1)-(2)	6.2	19.3	19.3	12.8	8.6

Source: Treasury

Recently stochastic simulation based risk analysis has been adapted to compare cost and risk characteristics of alternative borrowing policies and to develop a strategic benchmark policy. Risk is measured as Cost-at-Risk. Treasury does not announce borrowing benchmarks, that is risk limits, in terms of duration targets or fixed rate share for the debt stock. However, within the framework of strategic benchmark practice, which is determined according to the cost and risk analysis, announces borrowing objectives. Components of this strategy for the year 2004 were:

- To raise funds mainly in TL
- To use fixed rate TL instruments as a major source of domestic borrowing.
- To increase average maturity of domestic borrowing, including FX-denominated and indexed securities, over a year taking into account market conditions.
- To keep certain level of cash reserves throughout the year to reduce the liquidity risk associated with cash and debt management.

In year 2004, Treasury implemented a borrowing policy within the objectives set by the strategic benchmarks practice. For the January-June period domestic borrowing was done in major part through the issue of fixed rate securities. And domestic borrowing was dominated by securities denominated in domestic currency (Table 3.4).

Table 3.4. Structure of Domestic Borrowing

January-June 2004	Share (%)
Domestic Borrowing	
Fixed	81.0
Floating	19.0
TL denominated	96.0
Fx denominated/FX indexed	4.0

Source: RTUT, August 2004.

CHAPTER 4

STOCHASTIC MODEL: INTEREST RATE MODELING

4.1 Introduction

Stochastic model is part of the stochastic simulation analysis, in which random variables that affect the debt cost are modeled. This is the part where source of randomness is introduced into the stochastic simulation framework. Random variables to be modeled are determined by the structure of the debt stock and the scope of the analysis. In our study, set of borrowing strategies are compared with respect to the interest rate risk, which is the market risk associated with the movements in the interest rates. Hence our focus is on modeling the interest rates of different maturities. One way of doing this is the term structure or yield curve modeling. Term structure model to be used in the risk management analysis need to fit to cross-sectional set of observations and at the same time capture the inter- temporal dynamics of the term structure of interest rates. In order to serve this purpose, for modeling and simulating the yield curve, we use the dynamic framework proposed by Diebold and Li (2002). In this framework three factor Nelson-Siegel (1987) yield curve model is used to fit the yield curve in each period. After that, each factor of the yield curve is estimated as an autoregressive model, and the yield curve is forecasted by forecasting the factors.

In the remaining of this chapter, we give a brief overview of the interest rate modeling methods used under the stochastic simulation models developed in the empirical literature that have influenced our study. Afterwards basic yield concepts are defined, then the yield curve modeling framework that is used in our analysis is introduced.

4.2 Interest Rate Modeling in Stochastic Model Based Sovereign Risk Analysis

Yield curve or term structure of interest rates is the set of interest rates for different maturities. Yield curve models have been used under the market risk analysis framework developed in Bolder (2002) and Danish National Bank (1998, 2001). In these aforementioned empirical studies, Cox, Ingersoll and Ross (CIR) model is used to fit the term structure of interest rates.[13] CIR Model belongs to the family of affine term structure models, which along with CIR includes Vasicek (1977) and Duffie and Kan (1996).[14] Models in this class are formulated under the assumption that dynamics of the term structure of interest rates depend on the evolution of some observed and unobserved factor, also named state variable. Affine models assign a stochastic process to the state variable. Thereafter, dynamics of the entire term structure is derived from the state variable with the use of no-arbitrage[15] assumption in the underlying financial market. Affine term structure models were criticized in Duffe (2000) for their poor forecast performance. These models belong to the category of equilibrium models classified under dynamic yield curve models.[16] Nelson-Siegel(1987) model that is utilized in our risk analysis framework does not belong to the class of dynamic yield curve models, infact it is a static model. However under Diebold and Li(2002) approach the model is structured in a dynamic framework. In the early version of the sovereign risk model developed by Danish National Bank, Nelson Siegel yield curve model was utilized.

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[13] Danish National Bank (2001) one factor CIR is used, whereas Bolder(2002) models term structure of interest rates with a two factor CIR Model.

[14] Overview of the affine term structure models can be found in James and Webber (2000), Bolder(2001).

[15] Arbitrage is defined in the technical terms appendix.

[16] Overview of the dynamic yield curve models can be found in Yan (2001).

However model was not used in a dynamic framework as in Diebold and Li (2002). Initially, historical yield curves were estimated for each time increment using Nelson-Siegel model. Afterwards, the curves that will be used in the simulation were randomly chosen from the estimated yield curves (Danish National Bank, 1998).

Methods other than term structure modeling were used in the models developed by the World Bank and the Swedish National Debt Office (SNDO). In the World Bank Model, interest rate along with other financial variables is modeled as a simple stochastic process (Valendia, 2002). Whereas in the SNDO model short term and long term interest rates are modeled under a parsimonious macroeconomic framework, in which short term rate with three month maturity is determined on the basis of a monetary policy rule that central bank assumes to follow, the Taylor rule and After that, to obtain the long-term rate, in the first version of the model, spread between the three-month rate and the ten-year rate is modeled as a regime switching autoregressive process (Bergström and Holmlund, 2000). Whereas in the extended model nominal long-term yield is modeled on the basis of real return requirement, which depend on its lagged values and the capacity utilization in the economy (Bergström et al., 2002). The nominal yields for the maturities in between the three-month rate and the ten-year rate are obtained through interpolation.

4.3. Yield Curve

In this section we define the concepts of simple and continuously compounded yield to maturity within the context of discounted (or zero-coupon) bond. Discounted bond is a bond that is bought at a price below its face value, and the face value is repaid at maturity, there are no interim payments.

Yield to maturity is the interest rate that equates the present value of a payments received from a debt instrument to its value today (Mishkin, 1997). Simple yield to maturity is obtained from the following bond price function.[17]

[17] In our analysis time to maturity is express in terms of days. Therefore, in the formulation of the bond price function, (1+Z(t,T)) is raised to the power (T-t)/365 instead of T-t. Bond price function is formulated in this way in Bolder (2001) and Bayazıt (2004).

$$P(t,T) = \frac{1}{\left(1 + Z(t,T)\right)^{\left(\frac{(T-t)}{365}\right)}}$$

Where P (t, T) be the price of a discounted bond at time t, that pays a value 1 at maturity T. Z (t, T) is the yield implied by the return of a bond with remaining time to maturity of T-t days.

In yield curve modeling the concept of continuously compounded yield is used. Simple yield corresponds to discretely compounded return of a bond. Continuously compounded yield is obtained by making continuous the compounding frequency. Price equation for the continuously compounded yield curve is given as[18],

$$P(t,T) = e^{-\frac{T-t}{365} * Z(t,T)}$$

And the continuously compounded rate is,

$$Z(t,T) = -\ln P(t,T) * \frac{365}{(T-t)} \quad , \qquad t < T$$

(James and Webber, 2000)

4.4. Nelson-Siegel Yield Curve Model

Nelson-Siegel (1987) proposed a model to represent term structure of interest rates, which is capable of producing humped, monotonic and S-shaped yield curves using four parameters.[19] In this approach, initially forward rate function is drawn as a solution to a second order differential or a difference

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[18] Formulation of James and Webber (2000) is $Z(t,T) = -\ln P(t,T) * \frac{1}{(T-t)}$.

[19] Nelson-Siegel model is capable of generating curves with one hump. Svensson (1994) model is the extended version of the Nelson-Siegel model and it is capable of fitting yield curve shapes with two humps or u-shapes.

equation of the form represented in equation (1), where r (m) is the instantaneous forward rate at maturity m. [20]

$$r(m) = \alpha_1 r(m-1) + \alpha_2 r(m-2) + \alpha_0 \quad (1)$$

Solution to the difference equation for the case of equal roots is:

$$r(m) = \beta_0 + \beta_1 \exp(-m/\tau) + \beta_2 \left[(m/\tau)\exp(-m/\tau)\right] \quad (2)$$

Yield to maturity on a bill, denoted R(m) is the average of the forward rates r (m),

$$R(m) = 1/m \int_0^m r(x)dx$$

$$R(m) = \beta_0 + (\beta_1 + \beta_2)\left[1 - \exp(-m/\tau)\right]/(m/\tau) - \beta_2 \exp(-m/\tau) \quad (3)$$

$\{\beta_0, \beta_1, \beta_2, \tau\}$ are the parameters of the model and m stands for maturity. In these functions, τ is a positive constant determined by α_1 and α_2; β_0, β_1, β_2 are constants determined by the initial conditions on the forward rate. The parameter τ determine the rate at which the terms exp (-m/ τ) decay to zero. Limiting value of R(m) and r(m) as m approach to infinity is β_0 and as m approach to zero is (β_0 + β_1).

Forward rate curve (equation 2) is composed of long term, short term and medium term components. Long-term component, weighted with β_0, is a constant and does not decay to zero in the limit. Short-term component, weighted with β_1, starts at value one and decays to zero and the medium-term component, weighted with β_2, starts out at zero and decays to zero (Figure 4.1). At the point where medium component is maximized maturity is equal to the value of τ. Hence, τ specifies the position of the hump on the curve. Weight of the medium term component, β_2, determines the magnitude and direction of the hump. If β_2 is positive (negative), hump (u-shape) will occur at τ (Bolder and Streliski, 1999). Given these characteristics of the yield curve components, with the appropriate

[20] Instantaneous rate is the rate of interest with an extremely short maturity. In reality it does not exist, it is a theoretical construct used to facilitate the modeling process (Bolder, 2001).

choice of parameters, which are weights for the components, Nelson-Siegel model is capable of generating shapes including humps, S shapes, and monotonic curves.

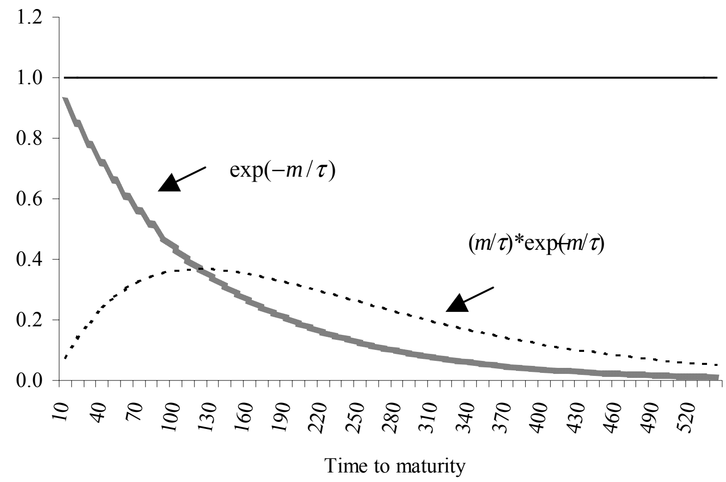

Figure 4.1. Components of the Forward Curve

Note: Paths of the forward rate components are computed using τ value equal to 121, which is the average τ value for our sample period.

Diebold and Li (2002) propose different interpretation to the parameters of the slightly modified specification of the Nelson-Siegel model. Re-specified model is given below.

$$R(m) = \beta_0 + \beta_1 \left[\frac{1 - exp(-m/\tau)}{(m/\tau)} \right] + \beta_2 \left[\frac{1 - exp(-m/\tau)}{(m/\tau)} - exp(-m/\tau) \right] \tag{4}$$

Model parameters β_0, β_1, $\beta_{2,}$ which were defined as weights for the long term, short term and medium term components of the yield curve in Nelson-Siegel (1987) are now interpreted as level, slope, curvature factors. In equation (4) values multiplied by parameters β_0, β_1, β_2 are defined as loadings. In that respect loading of β_0 is constant, 1, in the limit it doesn't approach to zero. Therefore, β_0 is the long-term factor and it is interpreted as the level of the yield curve. Loading of β_1, denominated as $\left(\frac{1-exp(-m/\tau)}{(m/\tau)}\right)$ starts at value one and approaches to zero in the limit. Therefore β_1 is evaluated as the short-term factor. Loading of β_2, $\left(\frac{1-exp(-m/\tau)}{(m/\tau)}-exp(-m/\tau)\right)$, starts at zero, increases with maturity and after a point starts to decrease again. Hence, β_2 is interpreted as the medium term factor. Increase in β_0, increases all yields equally, hence β_0 control the level of the yield curve. Short-term factor controls the slope of the yield curve. Increase in β_1 increases short yields relatively more compared to long yields, consequently changing the slope of the yield curve. And medium term factor is related to the curvature, given that change in β_2 will have little effect on the very short and very long yields, but will affect medium term yields. Thus change in β_2 will alter the curvature of the yield curve. These interpretations were supported by the calculations of Diebold and Li (2002) using their database, where they have demonstrated that level, slope and the curvature of the yield curve were affected in major part by the long-term, short-term and the medium term components respectively. Therefore β_0, β_1, and β_2 were treated as level, slope and curvature of the yield curve.

Yield curve simulation framework: Diebold and Li (2002) Approach

Nelson-Siegel method provides a static curve fitting tool. Diebold and Li approach has given a dynamic framework to the model by interpreting yield curve factors as time varying variables. As a result yield curve equation (4) turns into a dynamic equation .

$$R(m)=L_t+S_t\left(\frac{1-exp(-m/\tau)}{(m/\tau)}\right)+C_t\left(\frac{1-exp(-m/\tau)}{(m/\tau)}-exp(m/\tau)\right) \quad (5)$$

L_t, S_t and C_t are the time varying counterparts of β_0, β_1, and β_2 parameters respectively. In order to forecast yield curves, Nelson-Siegel factors are modeled and forecasted as univariate autoregressive and vector autoregressive processes.

4.5 Conclusion

Yield curve models have been used for the purpose of modeling and simulating interest within the framework of stochastic simulation based sovereign debt risk analysis. These yield curve models however belong to the class of affine term structure models. Models in this class are formulated under the assumption that dynamics of the term structure of interest rates depend on the evolution of some observed and unobserved factor, also named state variable. Hence models in this class have a dynamic nature, which enables them to be used under a simulation framework. Diebold and Li (2002) approach provides an alternative framework for simulating interest rates by transforming static Nelson-Siegel yield curve model into a dynamic model.

CHAPTER 5

STRATEGY SIMULATION MODEL AND ASSOCIATED RISK MEASURES

5.1 Introduction

Strategy simulation is part of the stochastic simulation analysis where for each borrowing strategy, debt is rolled over using the simulated paths of the random variables generated via the stochastic model. Strategy simulation is the core of the stochastic simulation analysis. In each step, role of the strategy simulation is to determine the amounts that will be borrowed at different maturities. Moreover, strategy simulation also keeps track of the debt cost and the debt service (principal and interest payments) of each period. There are some elements in the strategy simulation that are under control of the practitioner. These include the initial portfolio choice and the financing strategy. In this chapter, aforementioned key elements are discussed, next strategy simulation framework is described and finally cost and risk measures are defined. Strategy simulation framework applied in this thesis is explained in Chapter 6.

5.2 Key issues in strategy simulation

5.2.1 Initial portfolio choice

There are two possible initial portfolio alternatives. One of them is to use the actual portfolio and its maturity structure. Drawback of this approach is that starting with the actual portfolio can make the results from different debt management strategies less definite, since starting from a common portfolio will influence the results. Drawback arising from this option will become a less of a concern as the maturity of the actual portfolio shortens and the period of the analysis extends. The other alternative is to work with steady state portfolio. Initial portfolio is in steady state if the proportions of debt instruments in the overall portfolio are identical to the weights of the borrowing instruments in the financing strategy vector. Steady state portfolio is preferred to actual portfolio when the motivation of the simulation analysis is comparison of the long-term cost and risk characteristics of different debt portfolios rather than moving from one portfolio to the other (Bolder, 2003, Bergström et al, 2000). However use of steady state portfolio disregards the cost and time required to transform actual portfolio to the steady state portfolio.

5.2.2 Financing Strategy

Financing strategy indicates how much of the borrowing requirement to allocate among borrowing instruments. Financing strategies can be formulated in two ways. First is to work with strategies stating how the borrowing requirement is to be financed at each period as in Bolder (2003) and Hahm and Kim (2003). These strategies could either be static or dynamic. When the strategy is static, portion of each borrowing instrument in total borrowing is fixed and predetermined On the contrary, in dynamic strategies weights associated with each borrowing instrument is not fixed instead its conditional upon future realizations of some variables, such as the interest rates. The second way of formulating the strategies is to define strategies in terms of duration target and as a target for allocation of the debt stock between different types of debt as in Bergström and Holmlund (2000) and Bergström et al. (2000, 2002).

No matter in which of way the financing strategy is defined, common practice is to work with predetermined strategies. In other words, decision on which borrowing instrument to issue or the duration of the debt stock is not conditional on the realization of the random variables affecting the cost of the debt stock. Public debt management practice is assumed to be exercised within a transparent and predictable manner. In that respect, government adheres to the predetermined borrowing policy, despite fluctuations in the random variables

5.3 Simulation Framework

Strategy simulation is the core of the stochastic simulation analysis. In this part debt is rolled over each period under a predetermined financing strategy. There is no common framework for strategy simulation. Methods applied by the practitioners are shaped by the debt management practice and objectives. A simple simulation framework is to assume exogenous or zero government budget balance and roll over the maturing debt under a static financing strategy defined in terms of vector of fixed weights as in Hahm and Kim (2003). The level of complexity of the simulation framework could be extended in various ways. Basic, simple framework covers debt issues. One extension is to include debt buybacks as in the simulation framework developed by Swedish National Debt Office and Danmarks National Bank. Buybacks are an important component of the debt management practice in Denmark. Bolder(2003) contains a comprehensive strategy simulation framework, which is based on a stochastic model where the evolution of the term structure of interest rates, macroeconomic business cycles and government's financial position are jointly modeled. Distinguished feature of the model is that it takes into account the effect of financing strategy on government's financial position and interest rates.

5.4 Cost Measures

In this part we identify two aspects of measuring the sovereign debt cost. These issues comprise the coverage of the cost and the means of measuring it. In broad terms cost associated with the debt stock could be defined as a total of three components (Grill and Östberg, 2003). These include coupon cost, maturity cost and mark-to-market cost. Coupon cost is simply the coupon payment. Maturity cost is the difference between the amount received when the bond is issued and the amount paid at maturity. This occur when price at the time of issue is different from the face value of a bond and for the inflation linked bonds or bonds denominated in foreign currency as their principal payments are adjusted at maturity on account of the changes in inflation and the exchange rate. Interest paid for zero coupon bonds fall under this category. Mark-to-Market cost is the difference between the amount previously received and price paid when the security is bought back before maturity.

Cost can be measured either in cash flow basis or mark-to-market basis. In cash flow basis costs only occur when money is paid out, under this framework mark-to-market effects stemming from varying interest rates or exchange rates are not included. Under cash flow basis only realized mark-to-market cost arising from debt buybacks or swaps are recorded. When cost is measured under mark-to-market basis value of the debt portfolio is adjusted as interest rates and exchange rates change. Common practice is measuring the cost in cash flow basis, because these costs are the costs that actually affect the government's budget. An exception is the practice of New Zealand Debt Management Office (NZDMO). NZDMO actively manages debt portfolio to benefit from movements in the exchange rate and interest rates. Therefore they compute market value of the debt portfolio and measure cost as the increase in this value.

Another issue when costs are measured in cash flow basis is related to the period cost is recorded. Cost can be recorded in period it is paid or in period it belongs. When costs are reflected in the period they occur, then they are defined in accrual terms.[21] ESA95 criterion[22] for calculating government debt cost is on accrual terms. For each bond total cost is distributed over its existence period. Thus, cost over a given period is measured by the cost of bonds only for the days that fall within the period considered. Measuring cost in accrual terms provides a better comparison of alternative strategies in a specified period since costs are reflected in the period they occur. To give an example suppose costs are recorded in the period payment is made in a risk analysis with one-year simulation horizon. In this case bonds with maturity higher than a year will have no interest cost for that year. Under this framework, as a result of the recording practice short-term costs will appear more costly.

[21] Turkish Treasury computes costs on an accrual basis.

[22]ESA95 Manual on General Government deficit and debt
http://www.imf.org/external/bopage/pdf/99-35.pdf

5.5 Risk Measures Computed from Strategy Simulation Model Results

Under stochastic simulation framework two dimensions of risk can be specified (Bergström et al., 2002). These are the scenario risk and the time series risk. Under scenario risk, alternative strategies are compared on the basis of their average long-term cost and risk features. It is the risk that overall debt cost will exceed a certain amount in a specified period. Time series risk defines another risk dimension, which is the variability of costs between the years.

Scenario risk is computed from the average cost distribution of the analysis period. Average cost distribution is calculated by taking the average of the annual costs for each simulation run. Figure 5.1 displays the simulation results of the first strategy. 1000 Simulated paths are summarized into a single distribution representing average cost. Once the average cost distribution is obtained scenario risk can be calculated using various risk measures. These measures comprise absolute Cost-at-Risk, relative Cost-at-Risk, conditional tail Cost-at-Risk, relative conditional tail Cost-at-Risk (Bolder, 2003) and relative risk (Bergström and Holmlund, 2000) and (Bergström et al., 2002). In the following, we describe these measures one by one.

Absolute Cost-at-Risk (CaR) is the largest amount of government debt cost over a given time horizon that is not exceeded with probability 1-p, using statistical terminology $P(X \leq \text{CaR}) = 1 - \text{p}$. When p is set to .05, absolute CaR is the 95th percentile of the debt cost distribution, implying that absolute CaR is not exceeded by 95 percent of the debt cost observations. Relative CaR is the distance between absolute CaR (95th percentile) and the mean (or median) of the distribution. When absolute CaR and relative CaR is used to compare alternative financing strategies, expensive financing strategies also appear more risky compared to financing strategies dominated by less costly short-term borrowing.

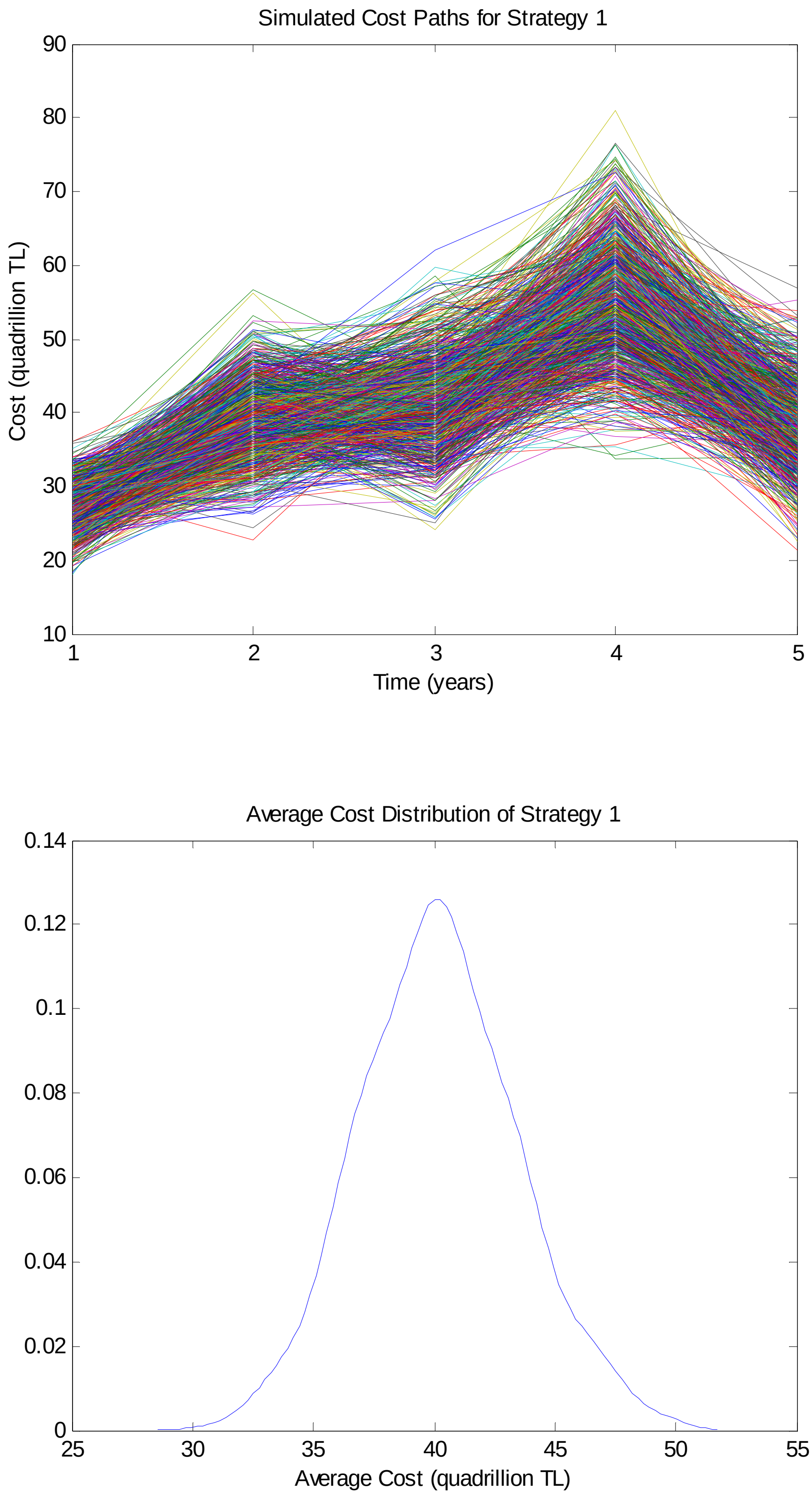

Figure 5.1.Illustration of the Simulation Output

Conditional tail CaR is the expected debt cost for a given period conditional on being on the tail of a distribution. Tail of the distribution is defined as the observations beyond the CaR, i.e 95th percentile of the distribution. Conditional tail CaR is the average of the debt cost greater CaR. Relative Conditional tail CaR is the distance between Conditional tail CaR and the mean of the distribution. Risk identified with conditional tail CaR describes a much worse case scenario compared to CaR.

Relative risk measure proposed in Bergström et al. (2002) is similar to relative Cost-at-risk. Instead of the absolute deviation, relative deviation is used with the reason that percentile distance in absolute terms tends to be bigger the higher the expected cost is. As a result with absolute measures, high cost strategies also tend to look riskier. This risk measure is computed as the relative distance between the 95th and the 50th percentiles of the simulated distribution:

$$\mathrm{Re}\,lative\ risk = \frac{P_{95}(Cost)}{P_{50}(Cost)} - 1$$

Scenario risk handles one dimension of the risk involved in sovereign debt management, that is the risk that overall debt cost will exceed a certain amount in a specified period. The other dimension is the time series risk, which identifies the variability of costs over the years.

Time series risk in Bergström et al. (2002) is computed as follows. A straight line is fitted to the cost path obtained from each simulation run. Then absolute deviations from the cost path are calculated. Thus, for each simulation run a distribution of absolute deviations is obtained. This is repeated for total number of simulations (e.g. 1000 times). Consequently, time series risk of a financing strategy is computed as the average of the relative distance between 95th and 50th percentile of the absolute deviation distribution of each simulation run. Higher the time series risk, higher is the variation in costs between the years.

Bolder (2003) proposes an alternative risk measure that incorporates time dimension into the risk analysis. Proposed risk measure is calculated from a conditional debt cost distribution. In this method debt cost is estimated as an autoregressive time series model. A forecast error from the model that is captured by conditional volatility provides a notion of risk. This measure provides a measure of uncertainty relating to the debt charges of the subsequent period given debt charge of the current period.

5.6 Conclusion

In this chapter strategy simulation framework, key issues in strategy simulation and methods for measuring cost and risk were introduced. Strategy simulation is the part of the stochastic simulation analysis, in which debt is rolled over, and borrowing requirements of the government are met under the random environment generated via the stochastic model. There are no common rules for strategy simulation. Framework developed by practitioners is country specific, shaped by debt management objectives and practices. Strategy simulation framework used in our study is explained in the following chapter.

CHAPTER 6

RISK ANALYSIS OF THE PUBLIC DEBT STOCK IN TURKEY USING STOCHASTIC SIMULATION MODEL

6.1 Introduction

Risk analysis based on stochastic simulation is a new concept in the field of public debt management. As pointed out in Chapter 2, it is only used in few countries. Even in countries that are considered to be advance on account of their public debt management backgrounds (e.g. Denmark, Sweden), stochastic simulation based risk analysis is a work in progress. Recently, risk management unit within Turkish Treasury has also adopted a new framework for risk analysis where market risk is measured based on a stochastic model. Treasury has disclosed basic features of their model along with some very brief results. However details of the model has not been published. Analysis carried out in this thesis is among the first studies attempting to measure market risk associated with the government debt in Turkey, under a stochastic simulation framework. However, analysis is partial, in the sense that among the market risk that debt stock is exposed to, only the interest rate risk associated with domestic currency (Turkish lira) denominated portion of the debt stock is evaluated. Interest rates are modeled via term structure modeling. This enables us to simulate interest rates of various maturities by modeling only few, in Nelson-Siegel case three, factors of the yield curve.

Our stochastic simulation model has a five-year horizon, which covers 2005-2009 period. Simulation is carried out for five years in monthly steps that is 60 periods. In this chapter the framework and the results of the stochastic simulation analysis are presented. In the first section, the empirics of the yield curve estimation are explained. In the second section strategy simulation framework is described and its results are evaluated.

6.2 Yield Curve Estimation for Turkey

Among the yield curve models, we used the Nelson-Siegel method for fitting the yield curves. Nelson-Siegel model is capable of fitting various yield shapes and it is easy to interpret. We have estimated three factor Nelson-Siegel model for each month of the period June 2001-July 2004 using Turkish secondary market data for government securities.23 Afterwards the extracted level, slope and curvature components of the yield curve are modeled as autoregressive processes. Then, we proceed to the simulation of these factors and thereby the yield curves for the 60-month period into the future. Data is introduced in the next section.

6.2.1 Data

Data used in the estimations are monthly continuously compounded yields and their corresponding maturities. Yields are calculated from end-of-month prices of the government securities from the Turkish Secondary Government Securities Market, from June 2001 through July 2004. Data is obtained from daily bulletin of the Istanbul Stock Exchange (ISE). Only discounted securities are used in the estimations which for the 1992-2004 period comprise on average 95 percent of the secondary government securities market in Turkey (Alper et al., 2004b). Securities with a maturity of less than a month are excluded from the estimations, hence minimum maturity is 30 days.[24]

[23] Yoldaş(2002) and Alper et al.(2004a) have estimated yield curves in Turkey using Nelson-Siegel method with secondary market government securities data.

[24] Data filtering is done in a similar manner in Yoldaş(2002) and Alper et al.(2004a). In Yoldaş(2002) floating rate bonds, coupon bonds, inflation linked bonds and T-bills with time to maturity of less than

6.2.2 Fitting the Yield Curves

We fit the yield curve using Nelson-Siegel Model (1987) as in equation (4). Nonlinear least squares and ordinary least squares (OLS) estimation methods are used in a complementary framework for estimating the yield curve parameters $\{\beta 0, \beta 1, \beta 2, \tau\}$. [25] Estimations are done using Eviews 5 econometrics software.

Initially the yield curve is estimated using non-linear least squares for various initial τ values in a range of 30 to 100. Given the initial values, two different results are identified; i) Non-linear estimation converges to multiple results, more specifically low (high) initial τ values converge to results in which estimated τ value is low (high).26 ii) Non-linear estimation converges to a single solution. Estimation procedure is finalized if a statistically and economically significant single result is obtained. If the parameters of the single solution are not significant OLS estimation is carried out over a grid of values for τ, and the value of τ that provides the best fit among the significant parameter estimates is chosen. In cases where there is multiple results, initially we chose the estimated solution with smaller sum of squared residuals (SSR). Then, if the non-linear estimation results are of the high (low) τ value we proceed by decreasing (increasing) τ by increments of 10 and estimating the yield curve trough OLS until significant results are obtained.27 Estimation results are presented in

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a month were excluded from the sample. In Alper et al.(2004), data sample includes discounted securities with time to maturity greater than ten days.

[25] Different methods have been used to fit the yield curves using Nelson-Siegel Model. Yield curve equation (Equation 1) becomes a linear model when the value of τ is given. Thus for a given value of τ, remaining parameters of the model can be estimated using ordinary least squares (OLS). Hence, in Nelson-Siegel (1987), best-fitting values of the yield curve parameters is found by repeating OLS estimation over a grid of values for τ. Diebold and Li (2002) uses nonlinear least squares estimation method. In Bolder and Streliski (1999) Nelson-Siegel curves are not determined through statistical estimation but in pure optimization framework.

[26] We initially estimated yield curves by excluding securities with remaining maturity of less than 10 days. In that case we observed a higher tendency for small τ values to converge to solutions with small values of τ.

[27]. Existence of heteroscedasticity in the error terms necessitates use of heteroscedasticity consistent error terms. These statistical issues were taken into account when deciding upon the statistical soundness of the estimated curves.

Appendix III. Estimated parameters are all statistically significant for each month of the Jun 2001-July 2004 period. In the estimations we control for the correlation, heteroscedasticity and normality in the residuals. Diagnostics of the yield curve equations reveal that assumptions related to the disturbances hold except for few dates where the heteroscedasticity and/or the normality assumptions are not satisfied. For the estimated parameters t-statistics are computed using White-Heteroscedasticity consistent covariances.

Along with statistical significance we impose constrains on the values that β_0 and β_1 can take on account of the economic interpretation attached to these parameters. When maturity approaches to infinity and zero, spot yield is given by β_0 and $\beta_0+\beta_1$ respectively. Thus constraints $R(0) \geq 0$ and $R(\infty) \geq 0$ which correspond to $\beta_0+\beta_1 \geq 0$ and $\beta_0 \geq 0$ apply. In the previous work by Yoldaş(2002) and Alper et al (2004a) constrained non linear estimation method was used.[28] In our estimations we impose an additional constraint on β_1, which is the negative of the slope of the curve when $R(0)$ and $R(\infty)$ are the shortest and the longest yields respectively. We imposed β_1 to be negative (positive) when raw curve had upward (downward) slope. When the raw curve is upward sloped, this constraint implies that as maturity increases yield will not fall below the shortest yield.

In the yield curve estimation, value of τ affects the fit of the curve, thus it is an important choice. Value of τ determines the location of the hump in the forward curve (and thereby the yield curve).29 Small values of τ correspond to rapid decay in the regressors and therefore will be able to fit curvature at low maturities well while being unable to fit excessive curvature over longer maturity ranges. Likewise, large values of τ result in slow decay in the regressors that can fit curvature over longer maturity ranges but they will be unable to follow extreme curvature at short maturities (Nelson-Siegel, 1987). Average τ value for our sample is 121. Given that our sample is in the range of 30-523 days, average value corresponds to the 23 percent of the upper limit of the maturity range. Estimated τ values are lower in Alper et al. (2004a) compared to our estimation

[28] Alper et al. (2004a) impose only the constraint $R(0) \geq 0$.

[29] At the point where medium term component of the forward curve is maximized value of τ is equal to the maturity (Figure 4.1).

results. In our opinion difference stems from the maturity range used in the estimations. Alper et al (2004a) includes maturities within the range of 10 to 30 days. We had initially estimated the yield curves by including securities with remaining maturity of greater than 10 days. In this case, when short maturities are included, nonlinear estimation converges to solutions with low τ values. The implication is that, at low τ values, estimated yield curve fits the curvature located in the short end of the curve. This leads to a bias, since curvature at the short end of the yield curve does not reflect the overall curvature of the curve.

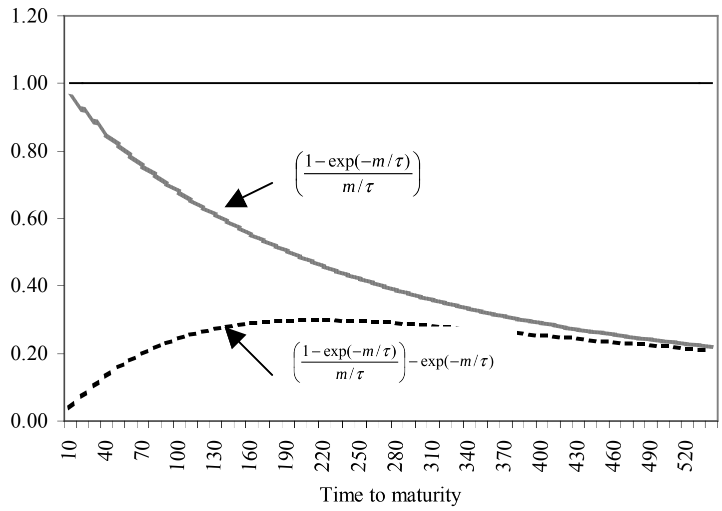

Figure 6.1. Yield Curve Loadings

In figure 6.1 yield curve loadings, other words regressors, are plotted for τ=121, where time to maturity ranges from 10 to 540 days covering the maturity range in our yield curve estimations. Thus, figure 6.2 displays the loadings that prevail in our analysis for the average τ value of the sample. Even though loadings of β_0 and β_1 enable interpretation of these factors as level and slope, path displayed by the loading of β_2, given the short maturity range, necessitates modification on the interpretation of the medium term factor. It is observed that the loading of β_2 after reaching a maximum decays back to zero at a slow pace. Change in β_2 does not only effect the medium term but also long term yields.

Our concern is not static curve fitting that is to estimate the best fitting curve at one point in time but to obtain consistent values for the model parameters so as to capture the dynamics of the yield curve and to obtain plausible forecasts for the future periods. Value of τ that provide the best fit could vary considerably and taking different values for τ each period could lead to fluctuating parameters and little gain in precision. In order to overcome fluctuations in estimated parameters Nelson-Siegel (1987) and Diebold and Li (2002) have used same τ value for the whole sample period. Nelson-Siegel has pointed out the value of τ is best chosen by fitting across data sets rather than by selecting the value for each individual data set. Employing the same approach in our study resulted in higher volatility in parameters. Therefore we proceeded using the estimated parameters obtained from the solutions where value of τ provided best fit.

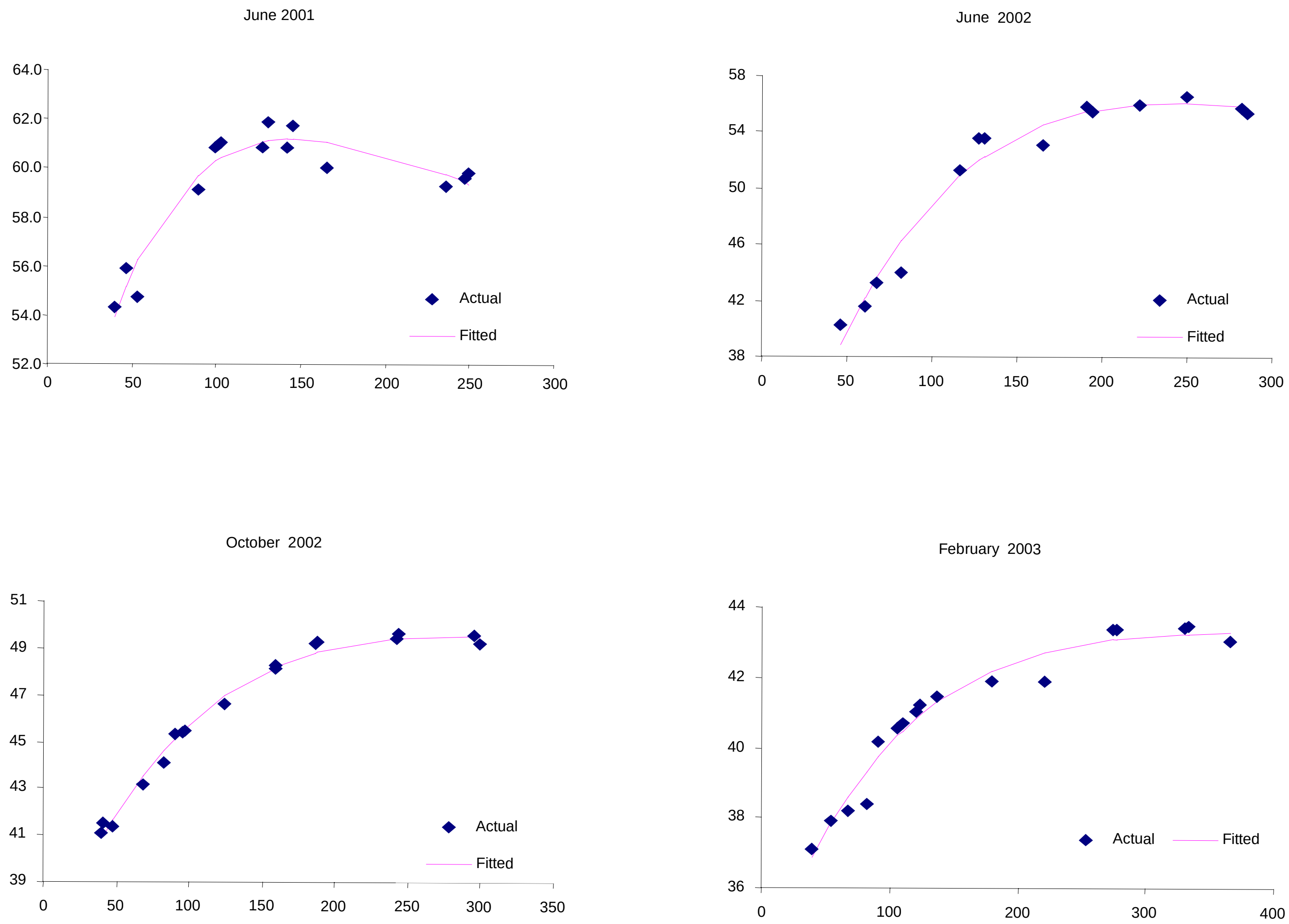

Figure 6.2. Examples of Fitted Yield Curves

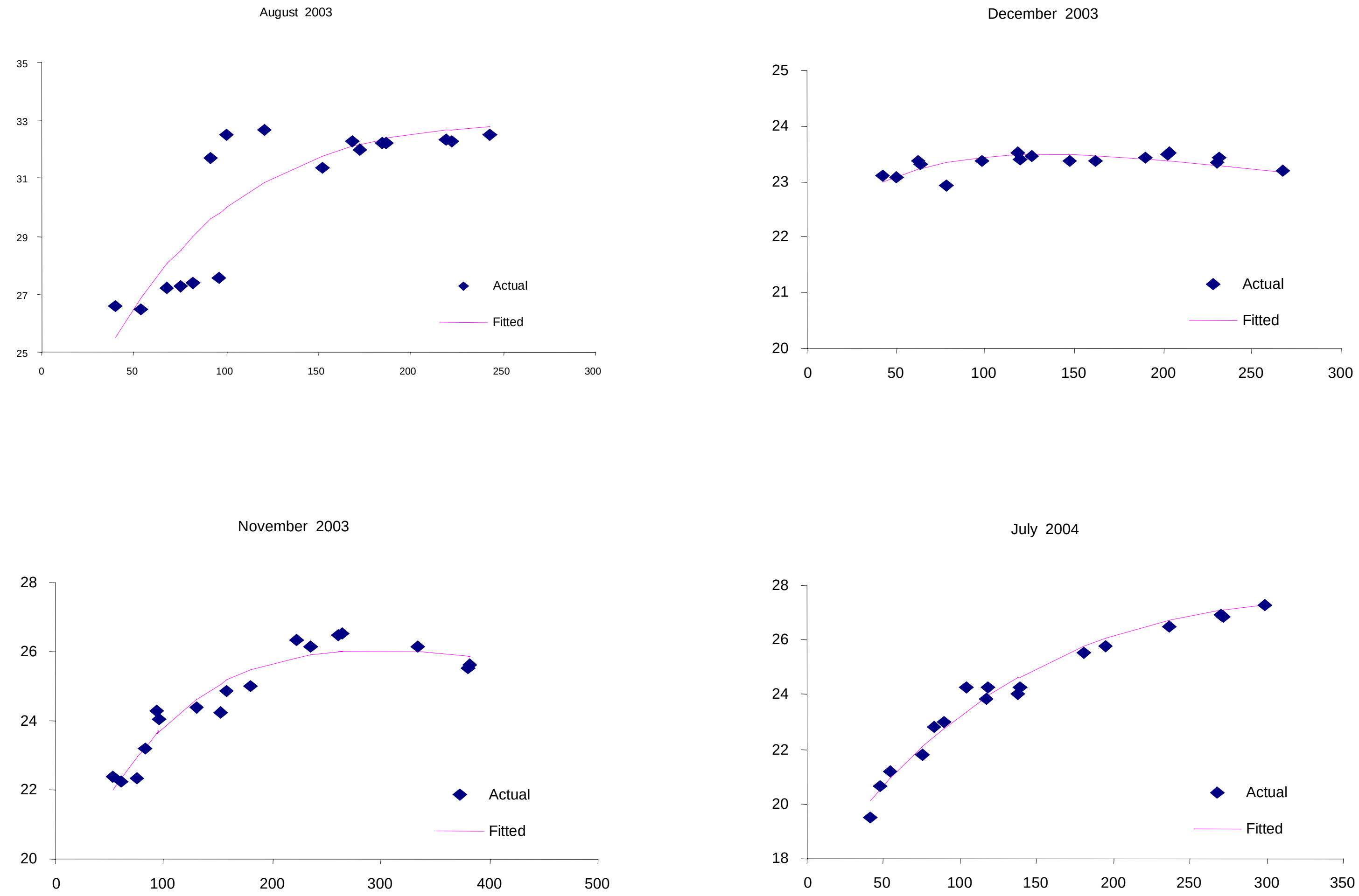

Figure 6.2. Examples of Fitted Yield Curves (cont’d)

In Figure 6.2, we plot raw yields with fitted curves for some selected dates. For the period under consideration, estimated yield curves are mostly upward sloped and humped or concave. Downward sloping curve is observed only in December 2003. Three-factor model is capable of replicating monotonic, humped and concave shapes. However it is not capable of replicating the raw yield when there is more than one interim optimum, that is more than one hump or u-shape. In general Nelson Siegel model provided good fit for the raw yield curves in our sample period. An exception to this is August 2003. However, the reason behind poor fit is the problematic nature of the raw yield curve, not the inadequacy of the model.

Graphical representation and descriptive statistics of the estimated yield curves are presented in Table 6.1 and Figure 6.3. Average estimated yield curve for the June 2001-July 2004 period is upward sloping and concave. Examples of fitted yield curves also confirm this average characteristic (Figure 6.2). In terms of volatility of the interest rates our estimation results depart from the typical observation that short end of the term structure is more volatile than the long end of the yield curve (Bolder, 2003; Diebold and Li, 2002). Our estimation results validate the contrary; volatility in the long end is higher compared to the short end. This results from the high and negative correlation among β_0 and β_1, such that variance of β1 is less than the covariance of β0 and β1 in absolute terms (Table 6.2).[30] Correlation of β_2 with the other two factors are low, whereas correlation between β_0 and β_1 is high implying that when there is a shock to the level component yield curve not only shifts but also its slope changes. Estimated yields and the estimated level factor, β_0, are persistent and highly variable relative to their mean. Compared to the level coefficient β_1 and β_2 are less persistent.

[30] Variance of the long-term yield is given by $VAR(R(\infty))=VAR(\beta_0)$ and variance of the short rate is given by $VAR(\beta_0+ \beta_1) = VAR(\beta_0)+ VAR(\beta_1)+2COV(\beta_0, \beta_1)$.Then $VAR(R(\infty))>VAR(R(0))$ when $VAR(\beta_1)+ COV(\beta_0, \beta_1) <0$ that is $VAR(\beta_1)< -COV(\beta_0, \beta_1)$.

Table 6.1. Descriptive Statistics of the Estimated Yield Curve

Maturity (Days)	Mean	St. dev.	Min.	Max.	ρ (1)	ρ (12)
30	31.4	10.4	16.9	47.8	0.9	0.72
90	32.8	11.3	17.1	51.7	0.89	0.69
180	34.1	12	17.3	56.2	0.87	0.67
270	34.9	12.4	17.4	58	0.87	0.66
360	35.4	12.6	17.5	58.9	0.86	0.65
450	35.8	12.7	17.6	59.5	0.86	0.64
540	36	12.8	17.6	59.8	0.86	0.64
β_0	37.5	13.4	17.8	61.7	0.84	0.59
β_1	-7.3	6.2	-25.2	0.8	0.4	0.25
β_2	29.4	18.5	-10.1	85.4	0.48	0.41
τ	121	47.5	40	190	0.43	-0.13

Note: ρ(1) and ρ(12) are the 1st and the 12th order autocorrelations.

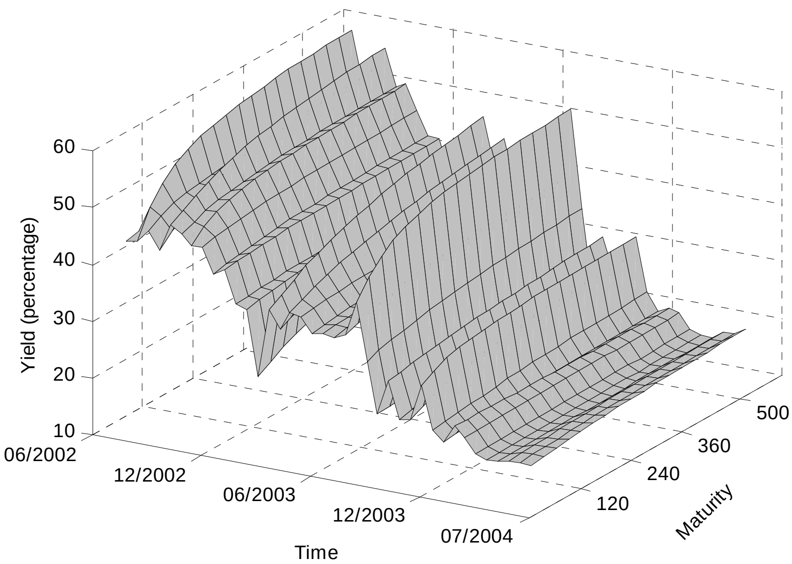

Figure 6.3. Estimated yield curves for the period June 2001 – July 2004

Table 6.2 Covariance/Correlation Matrix of the Estimated Parameters

		Correlation		
		B_0	β_1	β_2
Variance-Covariance	β_0	178.9	-0.7	0.2
	β_1	-59.4	38.3	-0.3
	β_2	50.5	-28.8	343.3

6.2.3 Modeling Yield Curve Factors

The model for the estimated yield curve factors will be used in simulating future paths for the term structure of interest rates. There are three factors to model. Time series plots of the yield curve factors are displayed in figures 6.4-6.6. Level factor of the yield curve, β_0, which in the limit corresponds to the long-term interest rate, is plotted together with the primary market average compounded rate for discounted securities. Level coefficient displays a declining trend during the July 2001- July 2004 period, as interest rates are falling from considerably high levels that prevailed in the previous years due to the favorable economic environment established within the framework of the stabilization program that initiated following the 2000 and 2001 crises.

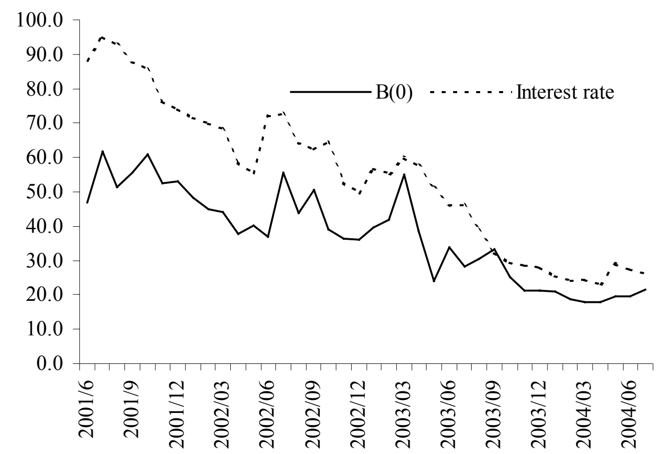

Figure 6.4 Interest Rate* and Level Factor of the Yield Curve

* Average compounded primary market rate for discounted securities.

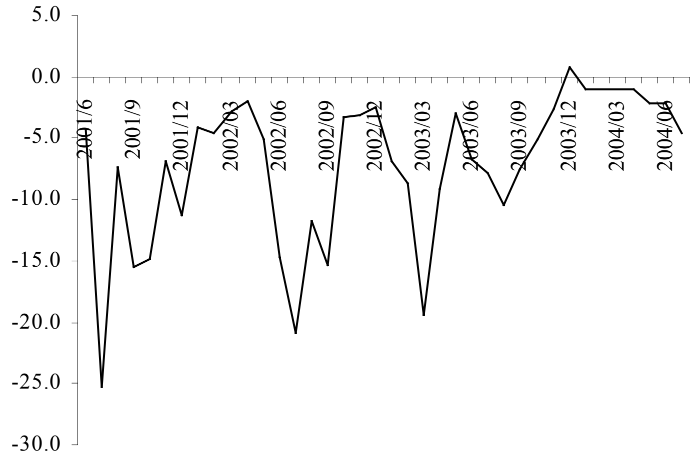

Figure 6.5 Slope Factor of the Yield Curve

Figure 6.6 Curvature Factor of the Yield Curve

Declining path of the factor β_0 reveals non-stationary nature of the series for the sample period. ADF test suggest that for the given sample period β_1 and

β_2 are stationary, whereas β_0 is non-stationary for the sample under consideration (Table 6.3).[31]

Table 6.3 ADF Test Results for Levels of the Estimated Yield Curve Factors

	β_0 ADF(1)	$R\beta_0$ ADF(0)	β_1 ADF(0)	β_2 ADF(0)
Test statistic	-1.31	-4.1	-3.88	-3.67
5% Critical value	-2.95	-2.94	-2.94	-2.94

Estimated factors are modeled as autoregressive processes (Table 6.4). However due to the non-stationary nature of the level coefficient, it is deflated with the expected inflation and the resulting deflated series is modeled, which we refer to as real β_0 ($R\beta_0$). Real β_0 is stationary for the sample period (Table 6.3). Level coefficient refers to the long-term interest rate. For this reason, when deflating the level coefficient, we imitated the Fisher equation, where reel interest rate is obtained by deducting inflation expectations from the nominal interest rate (Dornbush and Fisher, 1998). Real interest rate and the real level coefficient are plotted together in Figure 6.7.

[31] Alper et al (2004b) finds all the Nelson-Siegel yield curve factors to be stationary for the period 1992-2004.

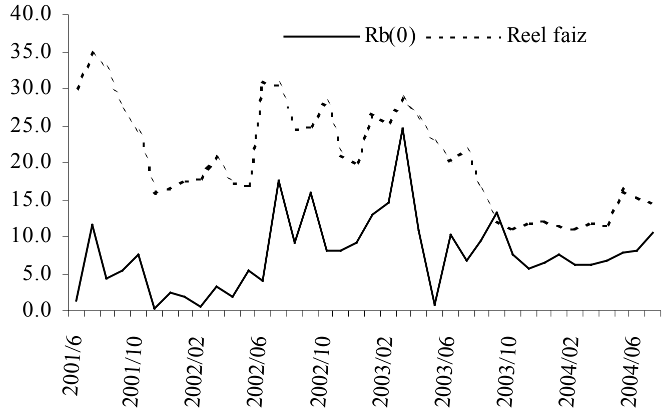

Figure 6.7 Real Interest Rate and the Real Level Factor

Table 6.4 Estimation Results of the Yield Curve Factors

	Equations*		
	$R\beta_0$	β_1	β_2
C	4.89 (4.37)	-2.73 (-3.11)	18.33 (5.01)
$R\beta_0(-1)$	0.3 (2.44)	-	-
$\beta_1(-1)$	-	0.38 (4.22)	-
$\beta_2(-1)$	-	-	0.37 (3.56)
D0107	-	-20.87 (-6.19)	-
D0109	-	-9.91 (-3.98)	-
D0206	-	-10.02 (-3.69)	52.48 (4.49)
D0207	11.52 (3.07)	-12.65 (-2.98)	-
D0303	15.41 (4.02)	-13.39 (-2.95)	-33.19 (-2.82)
D0309	-	-	-36.38 (-3.11)
R-squared	0.52	0.77	0.65
Std. dev. of residuals	3.52	3.02	10.83
	Diagnostics**		
Breusch-Godfrey Serial Correlation LM Test	2.51 (0.29)	0.30 (0.86)	0.81 (0.67)
White Heteroskedasticity Test	1.58 (0.81)	8.59 (0.28)	3.51 (0.62)
Jarque-Bera Normality Test	0.51 (0.77)	3.62 (0.16)	1.15 (0.56)

* In parenthesis are the t-statistics.

**In parenthesis are the probabilities.

Note: D0107, D0109, D0206, D0207, D0303, D0309 are the dummy variables. First two digits after the letter D stand for the year, following two digits stand for the month.

6.2.4 Yield Curve Simulation Framework and the Simulation Results[32]

Simulation horizon covers the period from year 2005 to 2009. Specification for simulation is

$$R(m) = \beta_{0t} + \beta_{1t}\left(\frac{1-\exp(-m/\tau)}{(m/\tau)}\right) + \beta_{2t}\left(\frac{1-\exp(-m/\tau)}{(m/\tau)} - \exp(m/\tau)\right) \text{, t=1, ..,60}$$

$$\begin{aligned} R\beta_{0t} &= \delta_0 + \delta_1 R\beta_{1t-1} + \varepsilon_t^{R\beta_0} \\ \beta_{1t} &= \eta_0 + \eta_1\beta_{1t-1} + \varepsilon_t^{\beta_1} \\ \beta_{2t} &= \gamma_0 + \gamma_1\beta_{2t-1} + \varepsilon_t^{\beta_2} \\ \beta_{0t} &= \left[\left(1 + R\beta_{0t}/100\right)\left(1 + \pi_t^e/100\right) - 1\right] * 100 \end{aligned} \quad \text{, t=1,...,60.}$$

where, π_t^e is the expected inflation.

Residuals of the above equations have the following specification:

$$\begin{bmatrix} \varepsilon_t^{R\beta_0} \\ \varepsilon_t^{\beta_1} \\ \varepsilon_t^{\beta_2} \end{bmatrix} = \begin{bmatrix} \sigma_{R\beta_0} \\ -\sigma_{\beta_1} \\ \sigma_{\beta_2} \end{bmatrix} * v_t \text{, t=1, ..., 60.}$$

where, $v_t \sim N(0,1)$

Random variable v_t is the source of randomness in the simulation model. Residual of the slope equation is taken as the negative of the random shock v_t on account of the negative correlation of the slope coefficient with the level and the curvature (Table 6.2). During simulation, to obtain level coefficient from the simulated real level series we make assumptions regarding the inflation expectations that will prevail for the simulation horizon. On account of the

[32] For yield curve simulation, MATLAB Software is used.

declining inflation, expected annual inflation is assumed to be 8 percent for the end of year 2005 and 5 percent for the remaining period. Initial values for $R\beta_0$, β_1, β_2 are 10.4, -4.61 and 26.8 respectively. Value of τ is fixed at 180, the value which prevailed for the yield curve equation estimated for July 2004. Simulation is carried out for 1000 times over 5 years in monthly steps, that is 60 periods. In each simulation run yields for 3, 6, 9, 12, 15, 18 months maturities are calculated.

In Figure 6.8 average estimated yield curve for the sample period and the average simulated yield curve[33] are plotted together. Average simulated yield curve in major part possesses the characteristics of the average estimated curve. Simulated curve is upward sloped and concave, as the average estimated curve. However unlike the estimated curve, simulated curve has a steeper short end and the long end of the simulated curve slopes downwards.

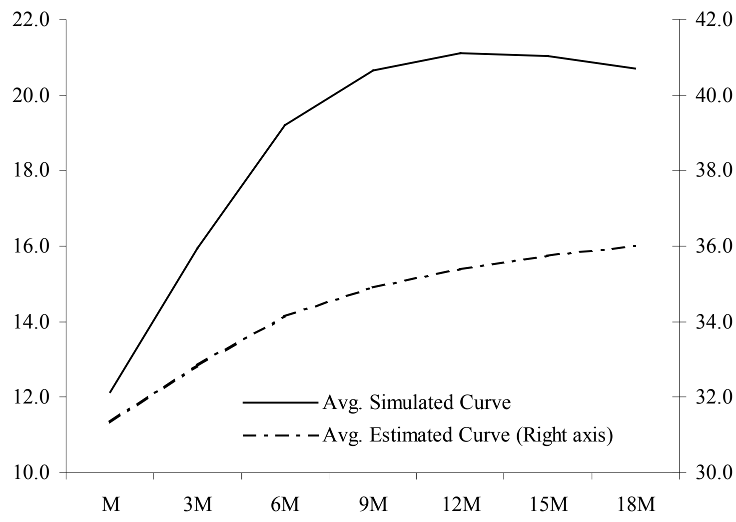

Figure 6.8. Average Estimated and the Average Simulated Yield Curve

Average of simulated yield curves for each period are plotted in figure 6.9. Yields display declining trend till the 24th period, afterwards follow a stable

[33] Average simulated yield is calculated by taking average of simulated yields over 1000 simulation runs and 60 months.

path. This results from the restriction imposed on the yield curve simulation. After year 2006 expected inflation is fixed at 5 percent. Volatility of the simulated yields is lower compared to the estimated period, however volatility in the long end is still higher than the volatility in the short end of the curve. Moreover, persistency of the simulated yields has declined compared to the estimated yields for the June 2001-July 2004 period. Another prominent feature of the simulated yields is that the yields with a maturity of 12 months and greater have similar characteristics in terms of volatility and persistency (Table 6.4). Thus characteristics of the yield curves that will be used in the debt simulation exercise can be summarized as follows;

- Yield curve is on the average upward sloping and concave. Long-term yields are higher than short term yields. However after maturity of 12 months, yields slightly decline.
- Long term yields are more volatile compared to short term yields
- Yields decline for two-year period until the year 2007 and thereafter follow a stable path.

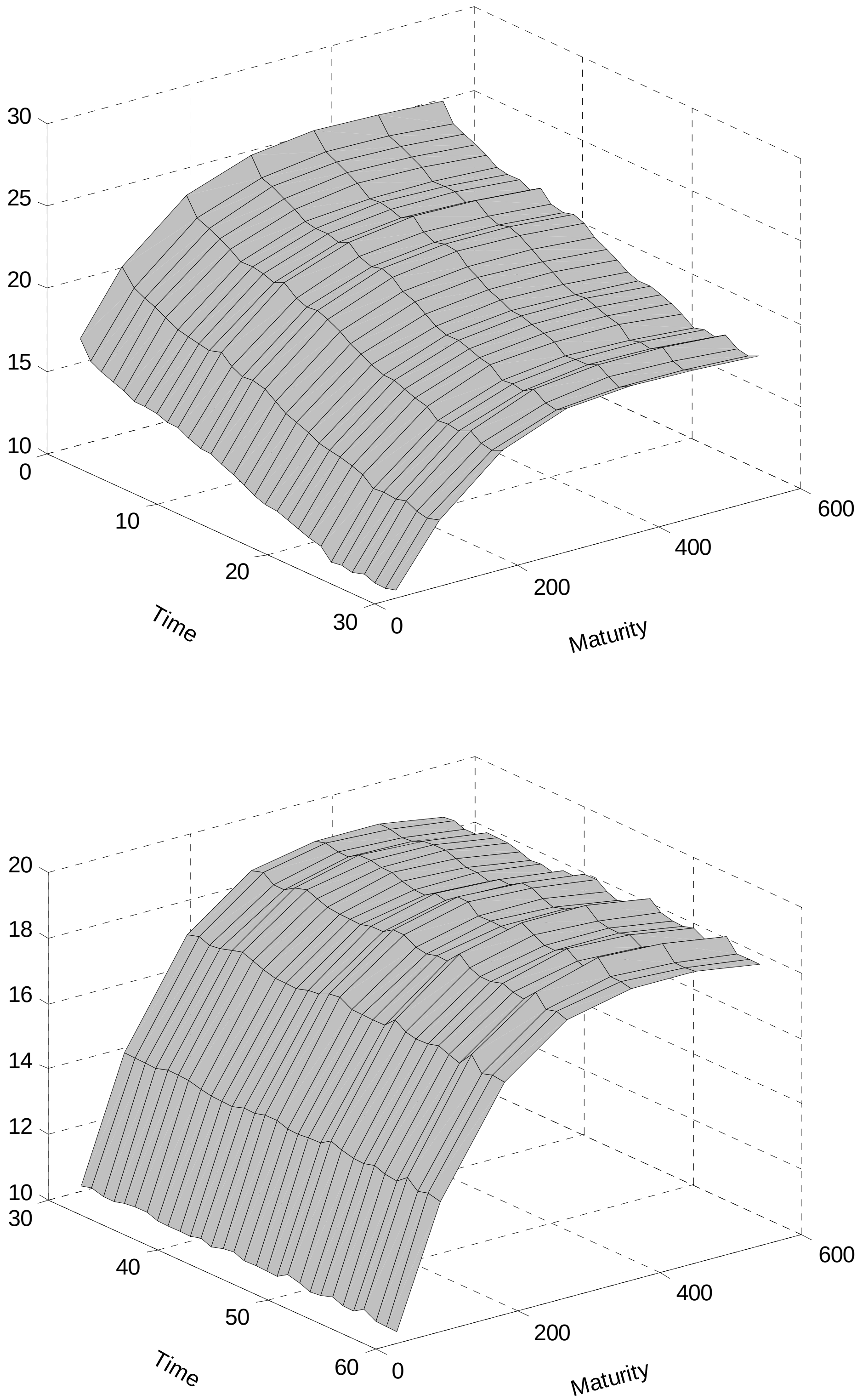

Figure 6.9. Average Simulated Yield Curves for the 2005-2009 Period

Table 6.5. Descriptive Statistics of the Simulated Yield Curve

Maturity (Days)	Average	St. dev.	Min.	Max.	ρ(1)	ρ(12)
30	11.2	2.4	6.2	18.3	0.52	0.19
90	14.9	3.8	5.8	24.8	0.38	0.05
180	18.0	5.1	5.3	31.0	0.35	0.02
270	19.4	5.8	5.0	34.0	0.35	0.01
360	19.8	6.1	4.7	35.1	0.35	0.00
450	19.8	6.2	4.4	35.3	0.35	0.00
540	19.5	6.1	4.2	34.8	0.35	0.00

Note: ρ(1) and ρ(12) are the 1st and the 12th order autocorrelations

6.3. Strategy Simulation Model for the Public Domestic Debt Stock in Turkey[34]

In this section we introduce the strategy simulation part of the stochastic simulation model, in which debt is rolled over under each strategy, on top of simulated term structure of interest rates. Every strategy is run for 1000 simulated interest rate paths and each simulation is run for 5 years in monthly steps, that is 60 periods. At each step government finances net borrowing requirement in line with the predetermined financing strategy. Net borrowing requirement of the government consists of maturing debt, interest payment and the primary (non-interest) deficit.

The Initial Portfolio Choice and the Financing Strategy

The aim of our analysis is to compare cost and risk characteristics of alternative financing strategies defined as vector of weights assigned to borrowing instruments. Financing strategies are static, in other words weights assigned to each borrowing instrument remains unchanged over the simulation horizon. There are 18 borrowing strategies formulated using 14 different borrowing instruments (Table 6.7). Borrowing instruments comprise 6

[34] MATLAB software is used for strategy simulation.

discounted securities with maturity of 3, 6, 9, 12, 15 and 18 months and 8 coupon bonds. Coupon bonds are identified with three features. These include maturity (2 year, 3 year), coupon type (flexible, fixed) and coupon period (quarterly, semiannually). Coupon bonds are formulated using different combinations of these features. For example, there are four types of three-year bonds. These include three-year bonds with quarterly and fixed coupon payments, three-year bonds with quarterly and flexible coupon payments, three-year bonds with semiannually and fixed coupon payments and three-year bonds with semiannually and flexible coupon payments. Interest rate to be applied to the discounted bonds is computed from the present value implied by the simulated yield of the respective maturity. Computations are based on the price equation of continuously compounded yield described in Chapter 4. Hence interest rate of the discounted bond is obtained directly from the simulated curve. However the same practice is not applied for the coupon bonds. Yield for the coupon bonds are computed using the information provided from the yield curve with some additional assumptions. Ordinary method would be to use the yield for the maturity of the bond obtained directly from the simulated yield curve and compute corresponding coupon rates. This method is not applicable in our case due to the limitations posed by the estimated yield curve. Yield curve estimations were carried out using discounted bonds with a maturity range of 30-523 days. On account of the short maturity range, yields greater than 18 months are not forecasted from the estimated yield curve. Therefore the yields for the coupon bonds are not computed directly from the simulated yield curve. Instead, it is computed over the simulated 12-month yield with an additional yield that is assumed to reflect investors risk perception for increased maturity. Assumptions relating to the investors behavior rely on two premises. Firstly, it is assumed that investors will demand additional yield over the 12-month yield because of the increased maturity and risk perceptions. Additional yield increases with maturity of the bond and the coupon period. Secondly, additional yield demanded for floating rate bonds is assumed to be lower than the additional yield on fixed rate bonds. This inference is based on the presumption that under macroeconomic framework of declining interest rates, return from investing in fixed rate bonds is higher and hence investors would demand lower additional yield for fixed coupon bonds. Consequently, coupon payments are determined by the interest rate of the discounted bond computed from the simulated yield curve that has the same maturity as the coupon period, that is the coupon rate, and the additional return over this rate. Additional return is computed on account of the additional yield on the bond. Under discrete time framework present value of the coupon bond is expressed as in the following equation.

$$P = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C+100}{(1+r)^n}$$

where P and C stand for the present value and the coupon payments respectively. Yield to maturity is the interest rate, r, that equates the present value of the payments received from a debt instrument to its value today. Therefore given the yield for the coupon bond and the coupon rate, additional return is the difference between the interest rate implied by the yield of the coupon bond and the coupon rate. In the simulation, average of the additional return for the period is added to the coupon rate of each month (Table 6.6).

Table 6.6 Interest Rates Applied to the Coupon Bonds

	Fixed Coupon Bonds		Flexible Coupon Bonds	
	Additional return over the coupon rate	Additional yield over 12-month discounted bond	Additional return over the coupon rate	Additional yield over 12-month discounted bond
Two-Year bonds with quarterly coupon payments	1.95	0.0	2.21	1.0
Two-Year bonds with semiannual coupon payments	2.96	1.0	3.53	2.0
Three-Year bonds with quarterly coupon payments	2.47	2.0	2.74	3.0
Three-Year bonds with semiannual coupon payments	4.08	3.0	4.65	4.0

In a financing strategy that involves a coupon bond, 15 percent of the total borrowing is restricted to be of the discounted bond with maturity equal to the coupon period. For coupon bonds, depending on the coupon period, interest rate that will be used in computing the coupon payment is set three or six months before the time of the coupon payment. This is inline with the current practice in debt management in Turkey.

In the strategy simulation the actual initial domestic currency denominated debt portfolio and the implied maturity structure is utilized. Since maturity of the debt stock is short, drawback arising from the use of actual portfolio is not expected influence the results in a significant magnitude. Size of the initial debt portfolio used in the analysis is 162 quadrillion Turkish liras.

Table 6.7. Definitions of the Financing Strategies

Strategy 1	100 % 9 Month discounted securities
Strategy 2	100 % 12 Month discounted securities
Strategy 3	100% 15 Month discounted securities
Strategy 4	100 % 18 Month discounted securities
Strategy 5	15% 3 month discounted securities 85% 2 Year, Floating rate; coupon payments 3 months
Strategy 6	15% 3 month discounted securities 85% 2 Year , Fixed rate; coupon payments 3 months
Strategy 7	15% 3 month discounted securities 85% 3 Year , Floating rate; coupon period 3 months
Strategy 8	15% 3 month discounted securities 85% 3 Year , Fixed rate; coupon period 3 months
Strategy 9	15% 6 month discounted securities 85% 2 Year , Floating rate; coupon period 6 months
Strategy 10	15% 6 month discounted securities 85% 2 Year , Fixed rate; coupon period 6 months
Strategy 11	15% 6 month discounted securities 85% 3 Year , Floating rate; coupon period 6 months
Strategy 12	15% 6 month discounted securities 85% 3 Year , Fixed rate; coupon period 6 months
Strategy 13	Short-term discounted securities 50% 9 Month, 50% 12 Month securities
Strategy 14	Long-term discounted securities 50% 15 Month, 18 Month
Strategy 15	2 year securities, coupon period 3 month 42.5 % Fixed rate 42.5 % Floating rate 15% 3 month discounted securities
Strategy 16	2 year securities, coupon period 6 month 42.5 % Fixed rate 42.5 % Floating rate 15% 6 month discounted securities
Strategy 17	3 year securities, coupon period 3 month 42.5 % Fixed rate 42.5 % Floating rate 15% 3 month discounted securities
Strategy 18	3 year securities, coupon period 6 month 42.5 % Fixed rate 42.5 % Floating rate 15% 6 month discounted securities

Strategy Simulation

Strategy simulation starts with an initial debt portfolio represented in terms of a maturity matrix of dimension 60 by 2. The information embedded in the maturity matrix is the debt service of the subsequent 60 months. Each row of the matrix corresponds to the time increments, i.e. months. Principal payments and the interest payments are located at the first and the second column respectively.

For each financing strategy simulation is repeated for 1000 times. In each simulation run debt portfolio is rolled over for 5 years in monthly steps. In each step following tasks are performed;

1. Borrowing requirement for the current month is calculated as the total debt service (interest + principal) minus the primary budget surplus of the government.[35] Debt service is the sum of the first row of the maturity matrix.
2. Once the debt service of the current month is calculated, first row of the maturity matrix is eliminated and a new row of zeros is added to the end of the matrix. Thereby, size of the maturity matrix is kept unchanged and it is prepared for the subsequent step. After these arrangements first row of the matrix now represents debt service of the next period
3. Total borrowing requirement is distributed among borrowing instruments in accordance with associated weights.
4. Nominal cost is computed.
5. Next, accrued interest and principal payments are distributed over the related cells in the maturity matrix.

Steps of strategy simulation along with the MATLAB codes are provided in the Appendix II.

[35] Primary budget surplus is exogenous to the model. It is calculated as 5 percent of the GNP of the year. Our assumption is that 50 percent of the primary surplus will be used to finance debt service arising from the TL denominated portion of the debt stock.

6.4 Cost and Risk Measures for Public Domestic Debt in Turkey

In this section results of the risk analysis are presented. Cost measure covers the coupon cost and the maturity cost of discounted securities. It is measured under cash flow basis and it is recorded under accrual terms as defined by the ESA95 criteria. Given that the simulation horizon is not very long, by using accrual based recording practice we overcome the bias associated with recording costs to the payment date. As explained in chapter 5, when costs are recorded in the payment date , cost associated with short-term borrowing appear more costly. This is because, cost of long-term borrowing falls outside the simulation period. However recording costs in accrual terms as defined by the ESA95 criteria complicates strategy simulation framework. Therefore initially, instead of distributing the cost over the existence period of the bond we record cost to the period in which the borrowing takes place. Under each financing strategy, simulation provides an output in the form of nominal cost distributions for each month. Below we illustrate how nominal cost is calculated at each step of the simulation run. Suppose bond is issued in the first month of year 1. First coupon payment of the bond will fall in year 1, third and fourth coupon payments will fall in year 2, and the last coupon payment will fall in year 3. Nominal cost of the bond is computed as follows:

$$Cost(1,1) = C(7,1) + C(1,2) + C(7,2) + C(1,3)$$

where,

$Cost(i, j)$ *is the cost in month* i, *year* j

$C(i, j)$ *is the coupon payment in month* i, *year* j

Once the nominal cost is computed, for each strategy costs are adjusted by deducting the interest payments that fall outside the simulation horizon. Under this approach, total cost of a strategy, which is the sum of all the adjusted monthly costs, and the average cost computed from the total cost are accrual based as defined by the ESA95 criteria. However, even after the adjustment, the annual costs still reflect the cost of borrowing, not the cost that fall in that year.

Simulation output is monthly. However, in the risk analysis, average costs of the simulation horizon are used to compare alternative strategies. Average cost of a strategy is calculated by dividing the total cost of a strategy, which is obtained by summing over monthly costs adjusted for off-period interest payments, by the number of years. At the end of 1000 simulation runs, for each strategy we have 1000 average costs (i.e. a cost distribution) for the period 2005-2009. The expected cost of the strategy is then defined as the median (50^{th} percentile) of the average cost distribution.

In the following part of this section results of the strategy simulation are presented. Financing strategies are compared with respect to their average cost and scenario risk. Scenario risk for each strategy is computed using three different risk measures: absolute CaR, relative CaR and relative risk. For every risk measure each strategy is expressed in terms of a single cost and a risk value. Every strategy is then located as a point on the cost and risk surface. Results are displayed in Table 6.8 and Figures 6.9-6.11. Each of the aforementioned risk measures provides different information. Relative risk, measured as the percentage deviation from the expected cost, enables one to compare alternative strategies on the basis of volatility of the expected cost. On the other hand, relative CaR and absolute CaR, measured in terms of domestic currency, provide a measure to assess the amount of maximum excess and total cost that government would have to undertake at a specified probability.

In figure 6.9 relative risk measure is plotted against nominal cost. Strategies are clustered in to five groups. These groups comprise strategies involving discounted securities; 2 year coupon bonds with quarterly coupon payments; 2 year coupon bonds with semiannual coupon payment; 3 year coupon bonds with quarterly coupon payments and 3 year coupon bonds with semiannual coupon payments (Figure 6.9). It is observed that, with respect to the relative risk, data points corresponding to each strategy are in general clustered into three risk levels. First level with minimum risk comprises two and three year bonds with quarterly coupon payments. Relative risk associated with strategies falling under this group is around 10 percent. Implying that at 5 percent significance level, average nominal cost for the period 2005-2009 can be over 10 percent of the expected cost. Second level with higher risk includes strategies with two and three year bonds with semiannual coupon payments. Risk associated with this group is in the range of 12-14 percent of the expected cost. Third level with highest risk comprises discounted securities, where the relative risk range is 14-16 percent. Relative risk, thus volatility of the strategies including discounted securities is higher compared to the strategies including coupon bonds. Furthermore, within the latter group of strategies coupon period determines the risk level. Strategy groups involving bonds with quarterly coupon periods are less volatile compared to strategy groups involving bonds with semiannual coupon periods. There appears to be a tradeoff among borrowing strategies containing discounted securities and borrowing strategies comprising coupon bonds. Shifting from former to the latter increases the cost but reduces the volatility. There is no significant tradeoff within the strategies containing coupon bonds with identical coupon periods. Within these strategies, at the similar levels of risk, the cost increases with maturity. Relative risk enables one to assess the volatility of the expected cost.

In terms of Relative CaR measure, strategies characterized with high cost also appear to be more risky. Relative CaR is determined by the volatility and the size of the expected cost. Discounted securities and bonds with semiannual coupon payments constitute the highest risk groups (Figure 6.10). For discounted securities expected cost is low, however its volatility is high. For the coupon bonds on the other hand size of the expected cost is high however its volatility is low. Final risk measure is the absolute CaR, which indicates the total maximum amount of cost that the government would undertake at the specified probability level. Like the relative CaR measure, when absolute CaR is used strategies with high cost tend to have higher risk. Effect of the size of the expected cost on the risk level is more apparent under the absolute CaR measure. In terms of absolute CaR, strategies comprising discounted securities are characterized with lowest risk. Absolute CaR and expected cost increase with maturity and the coupon period (Figure 6.11).

One prominent observation is that strategies with semiannual coupon payments are more risky compared to bonds with quarterly payments (Figure 6.9, 6.10, 6.11). This results from the higher volatility of six-month interest rate with respect to the three-month rate. Furthermore, within each group of strategies fixed rate bonds are more risky compared to floating rate bonds. The underlying reason behind this unexpected outcome may possibly be explained by the moderate change in the volatility of the 3 and 6-month rate during the 5-year simulation horizon (Figure 6.12). Coupon bonds with same coupon period but different coupon features would have different risk characteristics if volatility of the interest rates affecting cost, highly fluctuated over time.

In terms of relative risk and relative CaR discounted securities are characterized with the highest risk. Main reason is the higher volatility of the long-end of the yield curve along with the frequent financing need associated with discounted securities compared to coupon bonds. Costs associated with coupon bonds are exposed to volatility in either three or six month yield. On the other hand, discounted bonds, which are issued in the maturity range of 9 to 18 months, are exposed to higher volatility. Among the discounted securities there is no cost and risk tradeoff. As maturity increases from 9-months to 12-months, risk increases along with cost. Moreover, strategies containing securities with a maturity of 12-months and higher have the same cost characteristic. However, after 12-months risk increases with the maturity. Even though discounted securities possess the highest risk in terms of relative risk and relative CaR, since they are the least costly strategy group they are characterized with lowest Absolute CaR.

Table 6.8. Computed Nominal Cost and Risk Measures (Quadrillion TL)

	Expected Cost-50th percentile (1)	Absolute CaR-95th percentile (2)	Relative CaR (2)-(1)	Relative Risk ((2)-(1))/(1)*100 (Percentage)
Strategy 1	40.16	45.86	6.27	14.12
Strategy 2	42.08	48.45	7.20	15.32
Strategy 3	42.45	49.16	7.36	15.16
Strategy 4	42.12	48.54	8.51	16.11
Strategy 5	49.95	53.46	3.12	9.10
Strategy 6	49.74	53.41	3.68	9.98
Strategy 7	57.10	60.92	3.32	8.44
Strategy 8	58.77	63.46	4.65	10.57
Strategy 9	55.06	60.71	5.67	12.37
Strategy 10	53.31	59.02	6.36	13.15
Strategy 11	61.27	67.23	6.36	11.97
Strategy 12	62.46	69.45	8.05	13.84
Strategy 13	41.22	47.14	6.70	14.42
Strategy 14	42.30	48.90	7.89	15.43
Strategy 15	49.86	53.51	3.17	8.90
Strategy 16	53.66	59.26	5.73	12.16
Strategy 17	57.94	61.91	3.93	9.45
Strategy 18	62.66	68.84	7.12	12.81

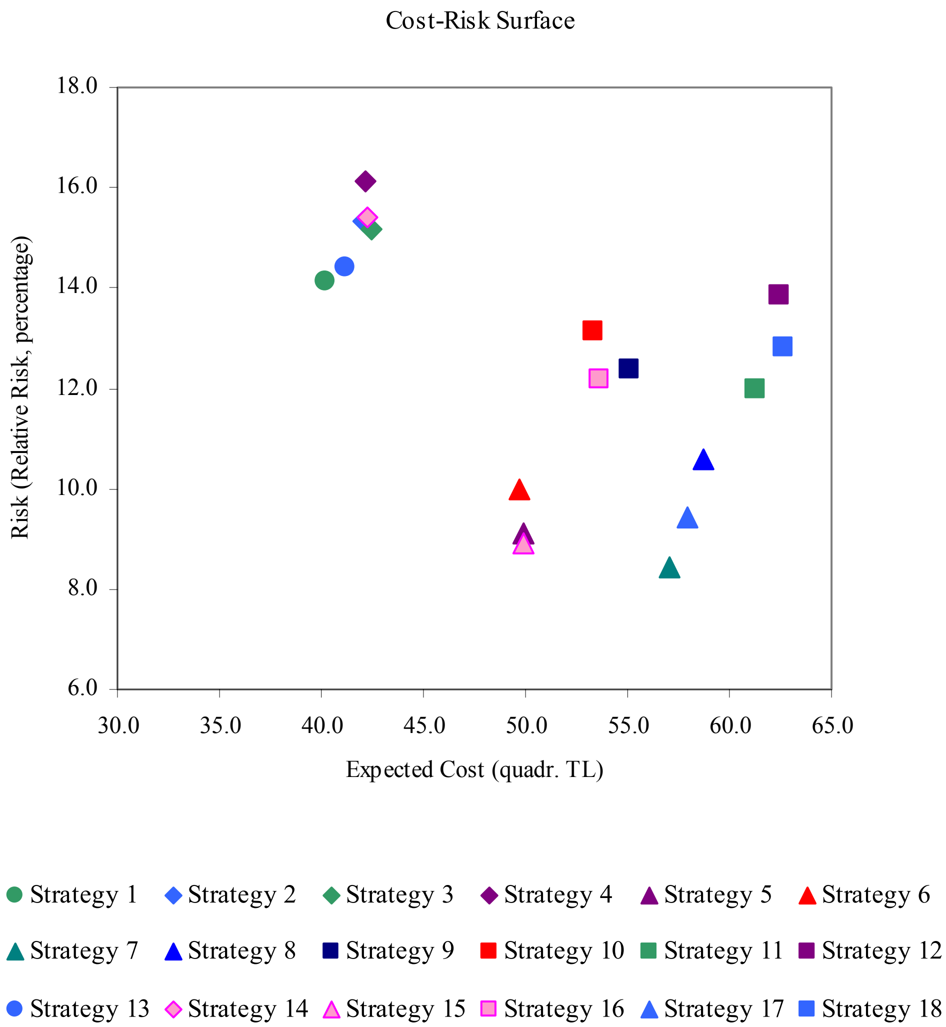

Figure 6.9. Scenario Risk Calculated by Relative Risk Measure

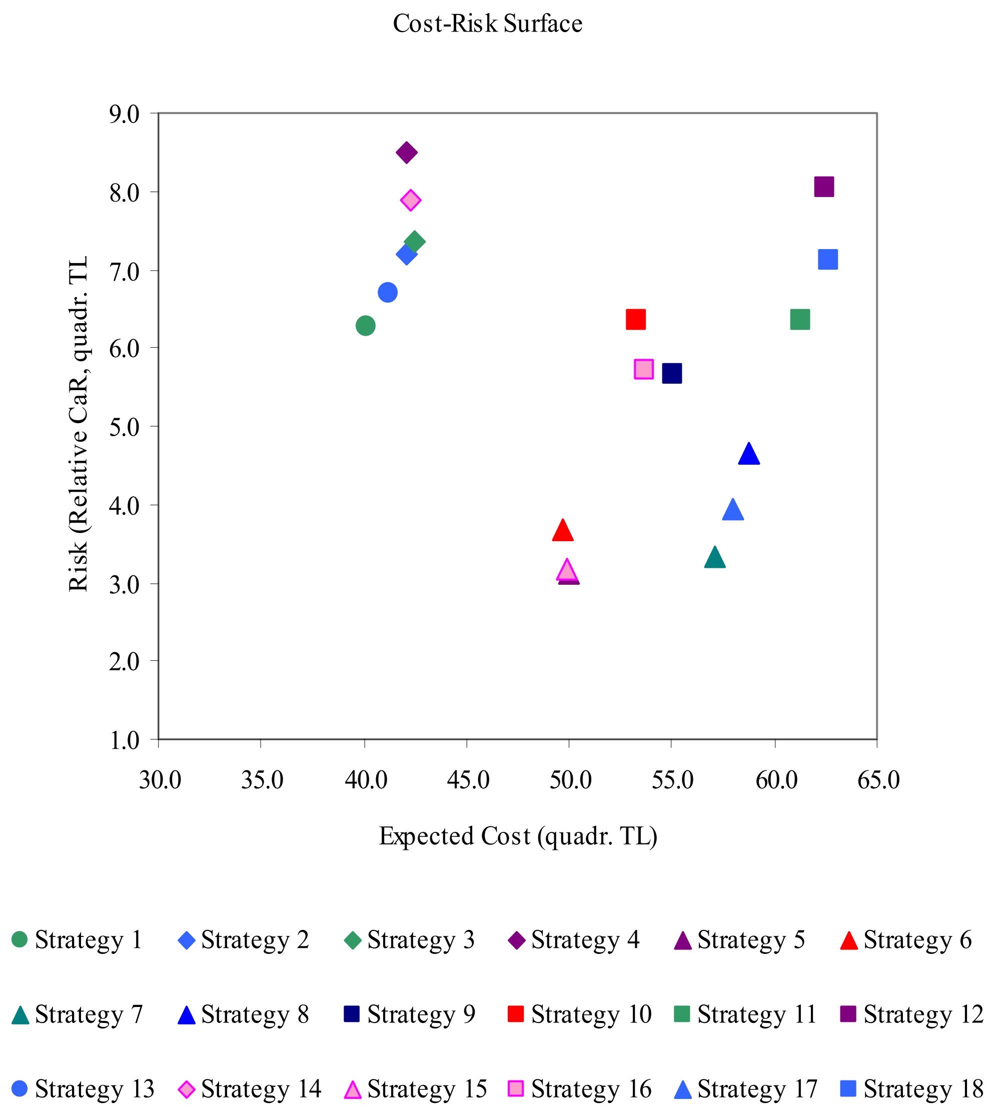

Figure 6.10 Scenario Risk Calculated by Relative Cost-at-Risk Measure

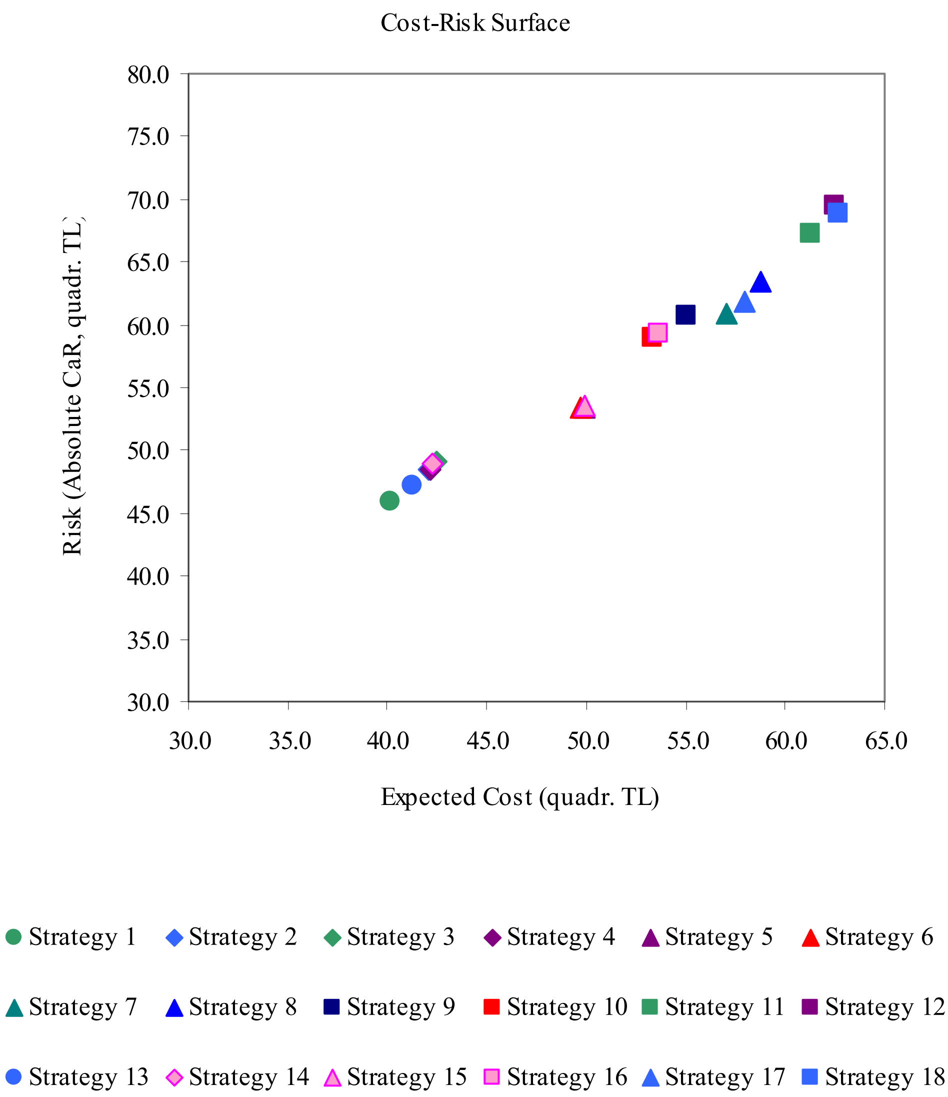

Figure 6.11 Scenario Risk Calculated by Absolute Cost-at-Risk Measure

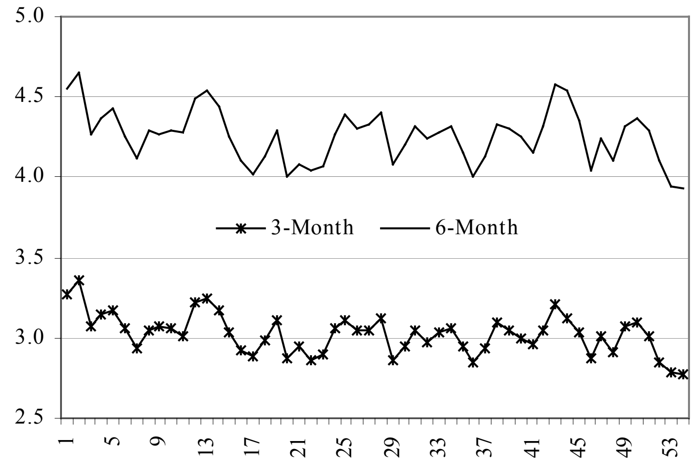

Figure 6.12: Volatility of theSimulated 3-Month and 6-Month Yield

Maturity, coupon period and the coupon characteristics are the factors affecting the cost of a strategy. In the following, as strategies are compared, interplay between cost and the aforementioned factors are discussed. Other features of the bond kept unchanged, higher coupon period is a factor increasing cost, since 6 month rates are higher compared to 3 month rates. Under the framework of declining interest rates, fixing the coupon rate at the time when bond is issued is a factor increasing the cost of a strategy. On the other hand additional yield imposed on the flexible coupon bonds are higher compared to fixed rate bonds. This factor increases the cost of floating rate bonds over fixed rate bonds. In our analysis, except for the strategy group comprising two-year bonds with quarterly coupon payments, within each group, nominal cost of strategies with fixed coupon payments are higher compared to strategies with flexible coupon payments. Another factor that affects the cost is maturity of the bonds. Yield of a bond increases with maturity hence three year bonds are more costly compared to two year bonds.

6.5 Implications of the Results for Government Debt Risk Management in Turkey

As pointed out in Chapter 2, market risk is only one of the debt management objectives. Hence results obtained under the risk analysis framework in this thesis are not the final words on the design of the financing strategy. Instead it provides a tool for debt management to evaluate cost and risk characteristics of alternative debt structures. Moreover it is limited to the domestic currency denominated portion of the debt stock and considers only the risk associated with the movements in the interest rate. Thus, results obtained from the stochastic simulation analysis should be evaluated under this framework.

Expected average cost associated with financing strategies range from TL 40.1 quadrillion, associated with 9 month discounted securities (strategy 1), to TL 62.7 quadrillion, associated with 3 year bond with semiannual coupon payments. This cost range indicates that considerable cost savings can be accomplished by the appropriate choice of a strategy. Relative risk and relative CaR point out to the tradeoff in the debt management practice. Shifting from the borrowing strategies involving discounted securities to the borrowing strategies comprising coupon bonds increases the cost but reduces the volatility. Risk and cost characteristics of the strategies are highly dependent on the characteristics of the yield curve and the assumptions regarding the additional yield imposed on the model for coupon bonds. For the simulation horizon yield curves are upward sloped and concave. Long-term yields are more volatile than the short-term yields and finally yields have a declining pattern till 2007 and thereafter follow a stable path. Volatility of the yields is highly influential on the relative risk characteristics of the strategies. Accordingly coupon bonds indexed to three month and six month interest rates are less volatile compared to the discounted securities. Moreover bonds with semiannual coupon payments are more volatile compared to bonds with quarterly payments. Cost difference between the coupon bonds and discounted securities stem from the additional yield imposed over the 12 month yield. The gap between these expected costs would reduce in size as the uncertainties, risk perceptions and consequently additional yield demanded by the investors decline. Under this framework if the government is not faced with any other risk such as refinancing risk and there is no cost burden on the government arising from debt sustainability concerns then the government would be inclined to borrow in terms of discounted securities. Since these securities possess lowest cost and the lowest risk features in terms of absolute CaR, which is the maximum total amount of cost that government would have to undertake. However given the short maturity of the government domestic debt stock in Turkey and high risk premium embedded in the interest rates, improvement of the debt structure necessitates extension of the borrowing maturity. In order to accomplish this structural change government has to issue long-term coupon bonds. Therefore we can evaluate the results of our analysis by constraining available strategies to the coupon bonds. Among strategies containing coupon bonds, strategy group with lowest cost and risk are the two-year bond with quarterly coupon payments. If the government has the objective of increasing the maturity further. Then given the lower volatility of the three-month rate compared to six month rate our analysis suggest the issuance of bonds with quarterly coupon payments. Cost and risk measures of a strategy are more sensitive to the choice of coupon period rather than the coupon feature whether it is flexible or fixed. If the confidence among the investors that interest rates will decrease in the forthcoming period is constituted, additional yields imposed on the coupon bonds would decline. Consequently the government would be able to increase the maturity by issuing bonds over two years without incurring such high cost as implied by the results of our analysis.

Results of the risk analysis are highly depended on the characteristics of the simulated term structure of interest rates and the additional yields imposed on the coupon bonds. Optimal strategy would depend on the extent of the decline in the additional yields. However whatever the additional yields are under the presumption that interest rates with this characteristics will prevail in the economy issuing bonds with quarterly coupon payments would reduce the cost and limit the exposure of the debt stock to interest rate variability. Moreover depending on the magnitude of the additional yield that prevails on the flexible rate over fixed rate bond, flexible rate bonds could be issued to benefit from declining interest rates.

6.6 Conclusion

In this chapter results of the stochastic model and the strategy simulation were presented. The analysis points out that results of the strategy simulation are dependent on the characteristics of the simulated term structure of interest rates. Hence it is necessary to be able to adequately model the nature of term structure of interest rates, in order to perform risk management analysis. Yield curve modeling approach that we have adopted displayed a good performance in terms of fitting the data for the 2001 July-2004 June period and producing simulated curves having similar characteristics with the estimated yield curves. The deficiency of the model is the short estimation period, which covers three years. If the period was extended to cover the period 1992-2004, for which data is available, yield curve estimation results, hence the characteristics of the simulated curves could be different. Extending the period would provide better assessment of the interest rate risk since more information regarding the evolution of the interest rates would be used. However shortness of the period does not degrade the outcome of the strategy simulation, since results are evaluated on account of the random environment generated from the stochastic model acknowledging that different random environment would lead to different results.

Strategy simulation framework adopted in this thesis is very simple and could be extended in various aspects. One extension would be to cover greater portion of the public debt stock by including debt instruments denominated in terms of foreign currency. Another improvement could be obtained by modifying the cost measure. Measuring the cost in accrual terms as specified by the ESA95 criteria was an adequate choice. Thereby we have overcome the bias resulting from recording costs to the payment date or to the period borrowing takes place. However the way cost is measured could be improved by measuring cost in real terms. When costs are measured in real terms relative weight of the cost that accrues in a shorter period of time is increased. Thus, real cost provides more adequate assessment of the debt burden on the government's budget. Another extension would be to extend the simulation horizon. Extending the simulation horizon would not only improve the scenario risk analysis but would also provide a framework to evaluate the trend in the cost paths associated with the financing strategies.

CHAPTER 7

CONCLUSION

In this study stochastic simulation based risk analysis method is applied to the domestic currency denominated portion of the Turkish sovereign debt stock. Simulation horizon is five years and covers the 2005-2009 period. Cost and risk of alternative borrowing strategies consisting of discounted and coupon bonds were computed using three different risk measures; absolute CaR, relative CaR and relative risk. Relative risk, measured as the percentage deviation from the expected cost, enables one to compare alternative strategies on the basis of volatility of the expected cost. On the other hand, relative CaR and absolute CaR, measured in terms of domestic currency, provide a measure to assess the amount of maximum excess cost and total cost that government would have to undertake at the specified probability. Term structure of interest rates, only source of uncertainty in the model, was estimated and simulated using Diebold and Li (2002) approach, in which dynamic framework for forecasting yield curves is established by utilizing the static Nelson-Siegel(1987) three factor model. The estimation period for the yield curve covers July 2001-June 2004. Following the estimation yield curve factors are modeled as an autoregressive processes. Afterwards yield curves for the 2005-2009 period are simulated by simulating these factors. Simulated yield curves in general reflect the characteristics of the estimated curves. In that regard, simulated yield curves are; in major part upward sloped and concave; yields with a maturity greater then 12 months slightly decrease compared to the 12 month yield; long term yields are more volatile than short-term yields; and yields decline till 2007 then follow a stable path in the 2007-2009 period.

Strategy simulation, in which debt is rolled over under each strategy, is applied on top of simulated term structure of interest rates. Formulated strategies comprise discounted securities and coupon bonds. Yield of the discounted securities are computed directly from the yield curve. Whereas yield of the coupon bonds is computed by imposing an additional yield over the simulated 12-month yield that is assumed to reflect investors risk perception for increased maturity. Results of the risk analysis are highly depended on the characteristics of the simulated term structure of interest rates and the additional yields imposed on the coupon bonds. Relative risk and relative CaR point out to the tradeoff in the strategies. Shifting from the borrowing strategies containing discounted securities to the borrowing strategies comprising coupon bonds increases cost but reduces volatility, i.e. relative risk. However in terms of absolute risk there are no tradeoffs. Risk and cost increase with maturity as strategies with high costs also appear to be more risky. Absolute CaR of a strategy is determined by the volatility and the expected cost. Given that absolute CaR is the maximum cost that will be undertaken by the government, among other risk measures it is the most adequate one to assess the market risk. Strategies including discounted securities are characterized with highest volatility however with lowest expected cost. Consequently these strategies are characterized with lowest risk in terms of absolute CaR. Under this framework if the government is not faced with any other risk such as refinancing risk and there is no cost burden on the government arising from debt sustainability concerns then the government would be inclined to borrow in terms of discounted securities. However given the short maturity of the government domestic debt stock in Turkey and high risk premiums embedded in the interest rates, debt management practice is motivated to increase the maturity in order to improve the structure of the debt stock. Then government, motivated to increase maturity of the debt stock, will issue long-term coupon bonds. Among the coupon bonds, strategy group with the lowest cost and risk are the two-year bond with quarterly coupon payments. If the government has the objective of increasing the maturity further. Then given the lower volatility of the three-month rate compared to six month rate our analysis suggest the issuance of bonds with quarterly coupon payments. This practice would reduce the cost and limit the exposure of the debt stock to interest rate variability. Cost and risk measures of a strategy are more sensitive to the choice of coupon period rather than the coupon feature whether it is flexible or fixed. Furthermore, the gap between the expected cost associated with discounted bonds and coupon bonds results from the additional yield demanded by investors on account of the uncertainties regarding future state of the economy. If the confidence among the investors that interest rates will decrease in the forthcoming period is constituted additional yields imposed on the coupon bonds would decline. Consequently the government would be able to increase the maturity by issuing bonds over two years without incurring such high cost as implied by the results of our analysis.

REFERENCES

Alper, C. Emre, Aras Akdemir and Kazim Kazimov (2004a), "Estimating the Term Structure of Government Securities in Turkey," Bogazici University Economics Working Paper No. ISS/EC-2004-03.

_________(2004b), "Estimating Yield Curves In Turkey: Factor Analysis Approach," Bogazici University Working Paper No. ISS/EC-2004-04.

Bayazıt, Derviş (2004), "Yield Curve Estimation and Prediction with Vasicek Model," Master Thesis Submitted to the Graduate School of Applied Mathematics. METU.

Bergström, Pal and Anders Holmlund (2000), "A Simulation Model Framework for Government Debt Analysis," Technical report, Riksgaldskontoret (Swedish National Debt Office).

Bergström, Pal, Anders Holmlund and Sara Lindberg (2002), "The SNDO's Simulation Model for Government Debt Analysis," Technical report, Riksgaldskontoret (Swedish National Debt Office).

Bolder, D. J. and Streliski D. (1999), "Yield Curve Modeling at the Bank of Canada," Technical Report No. 84, Bank of Canada.

Bolder, D. J. (2001), "Affine Term Structure Models: Theory and Implementation," Working Paper 2001-15, Bank of Canada.

Bolder, D. J. (2002), "Towards a more Complete Debt Strategy Framework," Working Paper, Bank of Canada.

Bolder, D. J. (2003), "A Stochastic Simulation Framework for the Government of Canada Debt Strategy," Working Paper 2003-10, Bank of Canada.

Canada Department of Finance (2003), Debt Management Report – 2002-2003.

Cannata M., Davide Iacovoni, Stefano Scalera and Manuel Turco (2003), "Broadening the Approach for Italian Debt Management,"paper presented at the Banca d'Italia Sixth Workshop on Public Finance, Perugia, Italy.

Cox, J. C., Ingersoll, J. E. and Ross, S. A. (1985), "A Theory of the Term Structure of Interest Rates," Econometrica, 53(2), 385-407

Danish National Bank (1998), "The Interest and Refinancing Risk on Domestic Government Debt – Cost-at-Risk," Discussion Paper.

Danish Government Borrowing and Debt 1999, Chapter 9: Scenario Analyses of the Government Debt.

Danish Government Borrowing and Debt 2001, Chapter 9: Interest Models for CaR.

Diebold, Francis X. and Canlin Li (2002), "Forecasting the Term Structure of Government Bond Yields," PIER Working Paper No. 02-026. http://ssrn.com/abstract=325300

Garcia, M. (2002), "Public Debt Management, Monetary Policy and Financial Institutions," Puc-Rio Working Paper.

Grill, T. and Hakan Östberg (2003), "A Financial Optimization approach to Quantitative Analysis of long Term Government Debt Management in Sweden," Master's Thesis at the Division of Optimization, Department of Mathematics, LinkÖpin Institute of Technology.

Hahm, Joo-Ho and Jinho Kim (2003), "Cost-at-Risk and Benchmark Government Debt Portfolio in Korea," International Economic Journal, 17(2).

James J. and Webber N. (2000), Interest Rate Modeling, New York: John Wiley & Sons.

Jorion P. (2001), *Value at Risk*, McGraw-Hill, New York. 2nd edition.

Nelson C. R. and Siegel A. F. (1987), “Parsimonious Modeling of the Yield Curves,” Journal of Business, 60(4).

IMF and World Bank (2001), *Guidelines for Public Debt Management.*

IMF and World Bank (2002), Guidelines for Public Debt Management: Accompanying Document.

Leong D. (1999), “Debt Management Theory and Practice,” Treasury Occasional Paper No.10

Missale A. (1997), “Managing the Public Debt: The Optimal Taxation Approach,” Journal of Economic Surveys, 11(3).

Riksgaldskontoret (Swedish National Debt Office) (2002), Duration, Maturity Profile and the Risk of Increased Costs for Central Government Debt, Report.

Republic of Turkey Undersecretariat of Treasury (April, 2003), Public Debt Management Report.

Republic of Turkey Undersecretariat of Treasury (October, 2004), Public Debt Management Report.

Storkey, Ian (May 2001), “Sovereign Debt Management: A Risk Management Focus,” The Finance and Treasury Professional.

Türkiye Cumhuriyeti Başbakanlık Hazine Müsteşarlığı (Ağustos 2004), Kamu Borç Yönetimi Raporu.

The New Palgrave Dictionary of Money and Finance (1992). Edited by Peter Newman, Murray Miltage and john Eatwell . The Stockton Press, New york. The Macmillan Press Limited, London.

Velandia-Rubiano, Antonio (2002), “A Risk Quantification Model for Public Debt Management.”

Yan, Hong (2001), “Dynamic Models of Term Structure,” Financial Analyst Journal, 57(4).

APPENDICES

APPENDIX I

TECHNICAL TERMS

Arbitrage: An arbitrage opportunity is an investment strategy that guarantees a positive payoff in some contingency with no possibility of negative payoff and with no net investment. Presence of arbitrage with the existence of optimal portfolio strategy for any competitive agent who prefers more to less. Absence of arbitrage follows from individual rationality of a single agent (The New Palgrave Dictionary of Money and Finance, 1992)

Credit risk or counter-party risk: the risk that stems from failure of the counter party to fulfill its obligations.

Cross-rate risk: Variation in the relative values of debt stock denominated in various foreign currencies in the debt portfolio.

Forward rates: forward rates are the rates contracted today for borrowing and lending in the future. Their value can be derived from pure discount bond prices under no-arbitrage condition.

Let $r_t(T_1)$ be the continuously compounded spot rate of a discounted bond at time t that pays 1 unit at the maturity date T1 and $f_t(T_1, T_2)$ be the continuously compounded forward rate available at time t for borrowing at time T_1 and repaying at time T_2. For no-arbitrage to hold, following condition must be satisfied:

$$e^{r_t(T_2)(T_2 - t)} = e^{r_t(T_1)(T_1 - t)} e^{f(T_1, T_2)(T_2 - T_1)}$$

so that,

$$f_t(T_1, T_2) = \frac{1}{(T_{2-T_1})}\left[r_t(T_2)(T_{2-t}) - r_t(T_1)(T_1 - t)\right]$$

(James J. and Webber N, 2000)

Foreign exchange risk: Change in the value of debt stock denominated in foreign currency resulting from variations in exchange rates.

Liquidity risk: There are two forms of liquidity risk. Asset liquidity risk arises when a transaction can not be conducted at prevailing market prices due to the size of the asset relative to the transaction volume. Funding liquidity risk, also known as the cash flow risk refers to the inability to meet payment obligations Jorion (2001).

Operational risk: Risk arising from possible staff errors, system failures.

APPENDIX II

YIELD CURVE ESTIMATION RESULTS

Table I.1 Yield Curve Estimation Results

		Parameters				Diagnostics*				Maturity
						Ser. Corr.			Number of	range
		B0	B1	B2	Tau	LM Test	Heteroscedasticity	Jarque-Bera	variables	(Days)
2001	Jun	46.9	-4.2	54.2	74	1.30 (0.52)	2.01 (0.36)	1.57 (0.46)	14	40-250
	Jul	61.7	-25.2	40.9	40	0.51 (0.77)	1.34 (0.51)	2.74 (0.25)	14	36-218
	Aug	51.5	-7.5	42	42	2.75 (0.25)	2.40 (0.66)	1.33 (0.51)	15	42-189
	Sep	55.6	-15.5	50.9	70	0.38 (0.82)	2.21 (0.32)	0.58 (0.75)	12	40-159
	Oct	60.8	-14.9	27.5	110	0.10 (0.94)	2.62 (0.26)	2.86 (0.24)	13	42-233
	Nov	52.4	-6.9	25.5	80	3.98 (0.13)	1.60 (0.44)	0.78 (0.68)	12	37-208
	Dec	52.9	-11.3	12.9	40	4.53 (0.10)	3.96 (0.13)	2.06 (0.36)	11	44-191
2002	Jan	48.4	-4.1	21.7	80	1.15 (0.56)	0.26 (0.87)	0.77 (0.68)	14	34-217
	Feb	44.8	-4.6	37.1	110	3.57 (0.16)	7.27 (0.02)	0.14 (0.93)	19	33-342
	Mar	44.1	-2.7	19.3	110	1.22 (0.54)	2.34 (0.30)	1.55 (0.46)	11	40-313
	Apr	37.8	-2	20.5	90	3.63 (0.16)	0.26 (0.87)	0.69 (0.71)	15	50-344
	May	40.1	-5.1	39.6	140	2.29 (0.31)	1.99 (0.57)	1.43 (0.49)	18	37-313
	Jun	36.9	-14.7	85.4	110	1.28 (0.52)	3.22 (0.19)	0.48 (0.79)	14	47-285
	Jul	55.5	-21	44.2	180	4.14 (0.12)	5.30 (0.07)	0.6 (0.74)	17	35-252
	Aug	43.8	-11.8	47.6	160	1.28 (0.52)	0.78 (0.67)	1.47 (0.48)	13	55-251
	Sep	50.6	-15.3	45.8	180	0.73 (0.69)	4.95 (0.17)	1.45 (0.48)	17	37-275
	Oct	39	-3.4	40.3	140	1.70 (0.42)	2.86 (0.23)	0.6 (0.74)	18	40-300
	Nov	36.4	-3.2	21.4	130	2.07 (0.35)	2.20 (0.69)	0.24 (0.89)	15	40-369
	Dec	36.1	-2.6	32.5	130	1.94 (0.37)	1.29 (0.52)	3.75 (0.15)	15	36-337

*In paranthesis are the probabilities. Test statistics are compared with chi-square critical values.

Table I.1 Yield Curve Estimation Results (Cont'd)

		Parameters				Diagnostics*			Number of variables	Maturity range (Days)
		$\beta 0$	$\beta 1$	$\beta 2$	τ	Ser. Corr. LM Test	Heteroscedasticity	Jarque -Bera		
2003	Jan	39.8	-6.9	29.9	190	0.44 (0.80)	1.55 (0.45)	0.89 (0.64)	23	30-362
	Feb	41.9	-8.6	15.1	111	1.97 (0.37)	1.17 (0.55)	4.78 (0.09)	18	40-369
	Mar	54.9	-19.4	-9.3	50	1.85 (0.39)	7.22 (0.30)	1.82 (0.40)	23	49-364
	Apr	38.7	-9.1	27.6	170	0.37 (0.82)	2.49 (0.28)	2.07 (0.35)	23	49-364
	May	24.1	-2.9	56.9	160	0.61 (0.73)	11.5 (0.04)	11.77 (0.00)	26	33-404
	Jun	33.9	-6.8	38	190	2.26 (0.32)	8.98 (0.10)	1.19 (0.55)	22	43-373
	Jul	28.3	-7.8	48	170	1.85 (0.39)	2.49 (0.28)	2.55 (0.28)	22	48-384
	Aug	30.4	-10.5	21.6	90	1.78 (0.41)	7.76 (0.02)	2.16 (0.34)	25	40-390
	Sep	33.3	-7.6	-10.1	40	2.72 (0.25)	10.6 (0.00)	1.16 (0.56)	25	22-390
	Oct	25.1	-5.1	15.6	190	2.99 (0.22)	5.93 (0.05)	1.1 (0.59)	23	33-362
	Nov	21.2	-2.7	20	140	1.58 (0.45)	9.20 (0.10)	0.81 (0.67)	18	54-383
	Dec	21.2	0.8	6.1	90	3.49 (0.17)	3.06 (0.38)	1.03 (0.60)	24	43-393
2004	Jan	21	-1.0	10.2	90	4.17 (0.12)	1.91 (0.38)	1.2 (0.55)	22	33-418
	Feb	18.6	-1.0	13.1	130	1.86 (0.39)	3.72 (0.44)	0.61 (0.74)	23	40-425
	Mar	17.8	-1.0	12.1	100	1.72 (0.42)	4.87 (0.30)	2.72 (0.26)	22	35-511
	Apr	17.9	-1.0	16.5	160	0.10 (0.94)	0.94 (0.62)	0.75 (0.69)	25	40-523
	May	19.6	-2.2	36.1	170	0.24 (0.88)	4.13 (0.38)	0.5 (0.78)	26	37-492
	Jun	19.6	-2.1	33.3	160	1.42 (0.49)	3.68 (0.59)	2.46 (0.29)	22	42-462
	Jul	21.6	-4.6	26.8	180	0.65 (0.72)	11.6 (0.00)	0.02 (0.99)	27	41-496

*In paranthesis are the probabilities. Test statistics are compared with chi-squared critical values

APPENDIX III

STRATEGY SIMULATION

Control variable

$W = [w_1 \cdots w_{14}]$ $\rightarrow$ Borrowing strategy vector

w_i, $i = 1, \cdots, 14$ $\rightarrow$ weight associated with each borrowing instrument.

Exogeneous variables

$GNPN_{1\times 8} \rightarrow$ GNP vector. Vector elements are the annual GNP for the period 2005-2013.

$F\ell_{60\times 1000} \rightarrow$ Interest rate matrix. Where $\ell = 90, 180, 270, 360, 450, 540$.
Elements of the matrix are the 60 period interest rate simulations of the corresponding maturity.

Each column stands for a simulation run. Matrices are the output of yield curve simulation.

$FZ\ell_{60\times 1} \rightarrow$ Interest rate vector. Where $\ell = 90, 180, 270, 360, 450, 540$.

$M_{72\times 2} \rightarrow$ Initial maturity matrix

Simulation

1) Initially GNP is adjusted for each year

```
for k=1:12
   GNP(:,:,k)=GNPN;
end
                 ⋮
for k=49:60
   GNP(:,1:4,k)=GNPN(1,5:8);
end
```

2) Debt Simulation

for j=1:1000

$FZ\ell = F\ell(1:60, j);$

for k=1:n

Step 1:Total financing need for the k^{th} period of the j^{th} simulation run is computed:

$N = sum(M(1,:)) - (GNP(1,1,k) * 0.05/24);$

Step 2: Interest and the principal payment of the k^{th} period of the j^{th} simulation run is computed:

F(j,k)=M(1,2);
AN(j,k)=M(1,1);

Step 3: Maturity matrix is adjusted. First row is eliminated and row of zeros is added to the end of the matrix

M(1,:)=[ ];
M(72,1)=0;

Step 4: Financing need is distributed according to the financing strategy.

$Addi2, \quad i = 3,6,9,...,36$ are the total interest payments of the i^{th} subsequent month.

M(3,1)=M(3,1)+N*w1;

Add32= N*w1*FZ90(1,1)+N*w7*FZ90 (1,1)+ N*w8*FZ90(1,1) + N*w9*FZ90(1,1) +N*w10*FZ90(1,1);

M(3,2)=M(3,2)+Add32;

M(6,1)=M(6,1)+N*w2;

Add62=N*w2*FZ180(1,1)+ N*w7*FZ90(3,1) +N*w8*FZ90(1,1) +N*w9*FZ90(3,1) +N*w10*FZ90(1,1) + N*w11*FZ180(1,1)+N*w12*FZ180(1,1) +N*w13*FZ180(1,1) + N*w14*FZ180(1,1);

M(6,2)=M(6,2)+Add62;

•
•
•

```
    Add362=N*w9*FZ90(33,1) +N*w10*FZ90(1,1)
+N*w13*FZ180(30,1)
           +N*w14*FZ180(1,1);
    M(36,2)=M(36,2)+Add362;
    M(36,1)=N*w9+N*w10+N*w13+N*w14;
```

Step 5: Nominal cost is computed. CSTN is the nominal cost for the k^{th} period of the j^{th} simulation

```
CSTN=Add32+Add62+Add92+Add122+Add152+Add182+Ad
d212+Add242+Add272+Add302+Add332+Add362;
```

Step 6: Interest rates are adjusted for the next period

$$FZ\ell(1,:) = [\ ];$$
$$FZ\ell(60,1) = 0;$$

```
  end
end
```