A PHYSICAL MODEL FOR DIMENSIONAL REDUCTION AND ITS EFFECTS ON THE OBSERVABLE PARAMETERS OF THE UNIVERSE

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ABSTRACT

A PHYSICAL MODEL FOR DIMENSIONAL REDUCTION AND ITS EFFECTS ON THE OBSERVABLE PARAMETERS OF THE UNIVERSE

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In this thesis, assuming that higher spatial dimensions existed only during the inflationary prematter phases of the universe, we construct a $(1+D)$-dimensional $(D>3)$, nonsingular, homogeneous and isotropic Friedmann model for dimensional reduction. In this model, dimensional reduction occurs in the form of a phase transition that follows from a purely thermodynamical consideration that the universe heats up during the inflationary prematter phases. When the temperature reaches its Planck value $T_{pl,D}$, which is taken as the maximum attainable physical temperature, the phase of the universe changes from one prematter era with D space dimensions to another prematter era with $(D-1)$ space dimensions where $T_{pl,D}$ is higher. In this way, inflation gets another chance to continue in the lower dimension and the reduction process stops when we reach $D=3$ ordinary space dimensions. As a specific model, we investigate the evolution of a $(1+4)$-dimensional universe and see that dimensional reduction occurs when a critical length parameter $l_{4,3}$ reaches the Planck length of the lower dimension. Although the predictions of our model for the cosmological parameters are beyond the ranges accepted by recent measurements for closed geometry, for a broad range of initial conditions they are within the acceptable ranges for open geometry.

Keywords: Dimensional reduction, inflation, phase transition.

ÖZ

BOYUTSAL İNDİRGENME VE EVRENİN GÖZLEMSEL PARAMETRELERİNE ETKİSİ İÇİN FİZİKSEL BİR MODEL

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Bu tezde yüksek boyutlu uzayların sadece evrenin şiştiği madde-öncesi adı verilen evrelerinde varolduğunu kabul ederek boyutsal indirgenme için $(1+D)$-boyutlu $(D > 3)$, tekil olmayan, homojen ve izotropik bir Friedmann evren modeli kuruyoruz. Bu modelde boyutsal indirgenme ısının evrenin şiştiği madde-öncesi evreler boyunca arttığı düşüncesinden kaynaklanan bir faz dönüşümü şeklinde meydana geliyor. Sıcaklık erişilebilecek en yüksek sıcaklık olan Planck sıcaklığına $(T_{pl,D})$ ulaştığında evren D boyuttan $(D-1)$ boyutlu ve aynı zamanda Planck sıcaklığının da daha yüksek olduğu baska bir madde-öncesi evreye geçiş yapıyor. Bu yolla şişme düşük boyutta da devam edebilme olanağını elde ediyor ve indirgenme süreci $D = 3$ olan olağan uzay boyutlarına ulaşıldığında sona eriyor. Özel bir model olarak $(1 + 4)$-boyutlu bir evrenin gelişimini inceliyoruz ve boyutsal indirgenmenin uzunluk parametresinin $(l_{4,3})$ düşük boyutun Planck uzunluğuna yaklaştığında meydana geldigini görüyoruz. Kapalı geometri için modelimizin öngörüleri son gözlemsel veriler ışığında kabul gören aralıkların dışında olmasına karşın, geniş bir ilk başlangıç durumu aralığında açık bir geometri için kabul

edilebilir aralıklardadır.

Anahtar Kelimeler: Boyutsal indirgenme, şişme, faz dönüşümü.

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CHAPTER 1

INTRODUCTION

Although the standard model of hot big bang cosmology has been successful in explaining the homogeneous expansion of the universe and the 2.73 K cosmic microwave background radiation, it has certain difficulties stemming from the puzzling initial conditions. These difficulties include the well known singularity, flatness and horizon problems (see e.g., Wald 1984, pp. 91-117 and Kolb & Turner 1990, pp. 261-269). In the standard model, flatness problem arises from the extreme fine tuning of the initial values of the energy density ρ and Hubble parameter H so that ρ is close to the critical value ρ_{cr} ($\equiv 3H^2c^2/8\pi G$) to an accuracy of one part in 10^{-55}. This kind of fine tuning is necessary to produce a universe that survives $\sim 10^{10}$ years which is an age prediction that complies with observational results. The horizon problem originates from the homogeneity assumption of the early universe, which consists of at least $\sim 10^{83}$ separate regions which are causally disconnected. On the other hand, the problem of initial singularity weakens the model in the sense that it causes infinities in the initial values of the physical quantities such as density, pressure and temperature.

Inflation mechanism was first suggested as a rescue from the horizon and the flatness problems by incorporating into the standard model an initial phase of accelerated expansion called the "inflationary epoch" (Guth 1981, Linde 1982 and Albrecht & Steinhardt 1982). During this epoch, inflation is provided by the vacuum dominance of the universe which is filled by an unusual form of matter having a positive energy density and negative pressure. In this context, the idea of starting the evolution of the universe from the Planck density ρ_{pl} ($= 5.1581 \cdot 10^{93}$ gr/cm^3) (Gliner 1970, Markov 1982 and Rosen 1985) was considered together with the idea of inflation. During the past two decades, several cosmological models aiming the singularity-free description of the universe were proposed (Israelit & Rosen 1989, Blome & Priester 1991, Starkovich & Cooperstock 1992, Bayın et al. 1994, Rebhan 2000 and Karaca & Bayın 2002).

In the paper by Israelit & Rosen (1989), the universe was modelled as a closed Friedmann-Robertson-Walker spacetime which starts with inflation from a cold nonsingular Planck-state characterized by a density equal to ρ_{pl} and a vacuum equation of state $P = -\rho$. Since the matter filling the universe during the inflationary phase would be under extreme conditions, and thus it would behave very differently from the ordinary matter, it is called "prematter" to distinguish it from the ordinary matter. After the period of inflation, there is a transition period into a radiation dominated era. Once the universe enters into the radiation era, it behaves as predicted by the standard cosmological models. Furthermore, all different eras and transition periods between these eras are governed by suitably chosen general equations of state which reduce under appropriate conditions to that of the desired era.

Few years later, Blome & Priester (1991) constructed a closed model according to which the universe starts contracting from an infinite size towards a minimum radius, which is larger than Planck length l_{pl} ($= 1.6160 \cdot 10^{-33} cm$), and then re-expands. According to this bouncing model, during the contraction phase the universe is in a quantum vacuum state and after attaining a minimum size, it experiences a phase transition into the radiation dominated period where its evolution is a Friedmann-Lemaitre evolution as in the big bang models of the universe.

A field theoretical description of the evolution of a singularity-free cosmological model was given by Starkovich & Cooperstock (1992) who proposed a cosmological field theory to describe the evolution of the universe by means of a single scalar field which is minimally coupled to the Ricci curvature, and which describes all the phases that the universe undergoes. The cosmological model built upon the consideration of this field theory describes an oscillating singularity-free closed universe in which transitions between different eras (prematter, radiation and matter) result from thermodynamically imposed boundary conditions rather than a finely tuned mechanism. In this oscillating singularity-free closed universe model, the universe initially is in a "vacuum-like" state described by an equation of state of the form $P = (\gamma - 1)\rho$, where $\gamma \approx 10^{-3}$ and inflation arises due to this "vacuum-like" characteristic of the equation of state. During the inflationary phase, due to the unusual characteristics of the equation of state, temperature rises although the universe expands. Inflation ends when the temperature reaches the Planck temperature T_{pl} $(= 1.4170 \cdot 10^{32}\ K)$ which is taken as the maximum attainable temperature.

In a paper by Bayın et al (1994), a closed singularity-free cosmological model was built upon the ideas presented in the paper by Starkovich & Cooperstock (1992). Furthermore, in this work inflation is driven by a scalar field which is conformally coupled to the Ricci curvature and the scalar field potential, which is responsible for the vacuum energy, was obtained from the equation of state instead of being assumed a priori. The approach developed by Bayın et al. (1994) was applied to open geometry by Karaca & Bayın (2002) and it was shown that a singularity-free cosmological model is realistically possible. This model starts with an initial expansion rate, i.e., $\dot{a} \neq 0$, because the initially static condition $(\dot{a} = 0)$ used in the closed universe models can no longer be used in the open universe case. This gives a two-parameter universe model in which one of the parameters determines the strength of the initial vacuum dominance of the universe, while the other corresponds to its initial expansion rate.

Recently, a "soft bang" model of the universe instead of "big bang" was proposed by Rebhan (2000). According to this model, the universe is initially in a static state with a closed geometry and devoid of any singularity. Its expansion is triggered by an instability and is initially quite slow but later reaches a rate of a typical inflationary universe model that starts from a big bang. Then the universe is carried to the usual Friedmann-Lemaitre evolution by a phase transition.

Higher dimensions played an important role in the search for a unified description of the known fundamental interactions of nature (strong, weak, electromagnetic and gravitational interactions) and a big industry has been generated for several decades by the works aiming a unified description of nature which requires theories with additional space dimensions such as Kaluza-Klein, eleven-dimensional supergravity and ten-dimensional superstring theories. We do not know whether we, human beings, possess the capacity of having the access to higher dimensions. But, if theories with additional space dimensions exist, then our universe would probably be in their low-energy limit. Thus, the above mentioned theories are expected to make predictions which comply with observations. Unfortunately, so far none of these theories have been successful in this sense. Although an acceptable physical mechanism necessary to explain how one could go from these theories to our familiar low-energy spacetime cosmology is still lacking, it is generally believed that such processes could take place in the form of compactification as in Kaluza-Klein theories (see e.g. Kolb & Turner 1990, Overduin & Wesson 1997), where the evolution of the extra dimensions is considered different from that of today's observed ones. In this scenario, extra dimensions somehow contract to a size at the order of l_{pl}, while the others continue expanding.

From the fact that we perceive the world to have three spatial dimensions, the discussions about the existence of higher dimensions become meaningful at sufficiently small length scales and the existence of higher dimensions is generally correlated with the very early phases of the universe where we reach very small length scales ($\sim l_{pl}$) at which it is believed that higher spatial dimensions become resolvable and come onto the same footing as the standard ones. As mentioned earlier, inflation was originally proposed to solve the long standing problems of modern cosmology such as flatness and horizon. But, what forces us to consider inflation in the same context with higher dimensions is the scale (Planck scale) at which it starts.

In this thesis, we explore the cosmological consequences of the possibility that the universe once had more than three space dimensions during the prematter phase and study how it may have evolved into the present three dimensional state. Reduction of dimensions could take place as in Kaluza-Klein theories (Kolb & Turner 1990), where the extra dimensions contract to Planck size homogeneously and isotropically, while the observed three dimensions expand.

The remainder of this thesis is organized as follows. In Chapter II, we discuss the laws of physics in higher dimensions and write the corresponding Planck scale. In Chapter III, we present our model together with its dynamical equations and solve them analytically under the initial conditions inspired by Gliner-Markov ideas as in the previous papers (Starkovich & Cooperstock 1992, Bayın et al. 1994, Karaca & Bayın 2002). In Chapter IV, we present a specific dimensional reduction scenario from $D = 4$ to $D = 3$ space dimensions for closed and open geometries, respectively. Finally, Chapter V contains a summary and discussion of our main results.

CHAPTER 2

PHYSICS IN HIGHER DIMENSIONS

2.1 Laws of Nature and Fundamental Constants

Since we have no direct experience of a higher dimensional world, we can only guess how the laws of nature might have looked like in higher dimensions by expressing the current laws of nature in D dimensions. In this way, we could gain some insight about how a D-dimensional universe might have looked like, and if it existed, how it may have evolved into its present state. In this context, we are primarily interested in expressing the Einstein's gravitational field equations in D dimensions. Effects of other interactions are naturally included in the equations of state that we use.

In the study of a D-dimensional universe even if we could extend the current laws of nature to D dimensions, it is still not certain that the current constants of nature, namely the speed of light c, Newton's constant of gravitation G, Planck constant (divided by 2π) $\hbar$ and Boltzmann's constant k, will remain the same. Hence, our next task is to decide upon the numerical values of the natural constants to be used in D dimensions.

In quantum mechanics, $\hbar$ represents our inability of measuring the momentum p and position x of a particle simultaneously with complete precision. In D dimensions, assuming that quantum properties of matter are independent of direction, uncertainty for each dimension is given as

$$\Delta x_i \Delta p_i \geq \frac{\hbar_D}{2}. \tag{2.1}$$

Uncertainty for D dimensions can then be written as

$$\Delta x_1 \Delta x_2 ..... \Delta x_D \Delta p_1 \Delta p_2 ..... \Delta p_D \geq \left(\hbar_D/2\right)^D. \tag{2.2}$$

We now isolate the contribution of the "disappearing" dimension and write Eq. (2.2) as

$$\Delta x_2 ..... \Delta x_D \Delta p_2 ..... \Delta p_D \geq \frac{\left(\hbar_D/2\right)^D}{\left(\Delta x_1 \Delta p_1\right)}. \tag{2.3}$$

Before dimensional reduction, one could say that the existence of dimensions greater than D are excluded a priori. After dimensional reduction we expect the reduced dimensions to be evacuated, however they may continue to exist at the Planck scale. Hence, we could take

$$\Delta x_1 \Delta p_1 \approx \hbar_D, \tag{2.4}$$

which converts Eq. (2.3) into

$$\Delta x_2 ..... \Delta x_D \Delta p_2 ..... \Delta p_D \geq \left(\hbar_D/2\right)^{D-1} \tag{2.5}$$

as the uncertainty relation for the remaining $(D-1)$-dimensional space. In the light of the foregoing discussion, we argue that dimensional reduction does not affect the value of $\hbar_D$ and take it as a constant which is independent of D and set $\hbar_D = \hbar = 1.0546 \times 10^{-27}\ \mathrm{cm}^2 gr/\sec$.

In $(1+3)$-dimensional space-time, c represents the speed of light in vacuum and its significance lies in the fact that in special theory of relativity it defines an upper limit to the speed of propagation of causal effects. In $(1+D)$-dimensional Minkowski space-time, line element is given as

$$ds^2 = c_D^2 dt^2 - d\vec{\sigma}_D^2, \tag{2.6}$$

where $d\vec{\sigma}_D$ is the proper distance in D dimensions. Speed of light could in principle be measured by bouncing light between two points in space with a proper distance $\Delta\vec{\sigma}_D$, and by measuring the transit time Δt. Then the speed of light c_D is obtained from the ratio $(c_D =)\, d\vec{\sigma}_D/dt$. From this we conclude that as long as the proper distance between the selected points remains the same and the speed of light is isotropic, we could take c_D as a constant independent of D and use $c_D = c = 2.9979 \times 10^{10} cm/\sec$ for the speed of light in vacuum for all D.

In Newton's theory of gravitation, the strength of the gravitational force between two masses m_1 and m_2 is proportional to the Newton's constant of gravitation G, whose numerical value is equal to $6.6720 \times 10^{-8}\ cm^3/gr\sec^2$. Assuming that Newton's gravitational field equation holds also in D dimensions, we write

$$\vec{\nabla}_D \cdot \vec{g}_D = -A_D G_D \rho_D. \tag{2.7}$$

Here, $\vec{g}_D$ is the gravitational field vector representing the gravitational force per unit mass, ρ_D is the mass density, $\vec{\nabla}_D$ is the divergence operator in D dimensions and A_D is the solid angle subtended by a sphere in a D-dimensional space. Integrating both sides of Eq. (2.7) in a D-dimensional space and applying Gauss's divergence theorem, we write the force law between two masses m_1 and m_2 as

$$\vec{F}_D = -G_D \frac{m_1 m_2}{r_D^{D-1}} \hat{e}_r. \tag{2.8}$$

As can be seen from Eq. (2.8), the force law goes as $1/r^{D-1}$. For example, if we set $D = 3$, which corresponds to the current number of space dimensions, the force law goes as $1/r^2$ as it should be. We can now relate G_D to the acceleration of a test mass m_1 and its separation from another mass m_2 as

$$G_D = \left| \frac{d^2 \vec{r}_D}{dt^2} \right| \frac{r_D^{D-1}}{m_1}, \tag{2.9}$$

where $\vec{r}_D$ represents the radius vector connecting m_1 to m_2 in D dimensions. We immediately notice that G_D has a dimension that depends on D, i.e., $[G_D] = cm^D/gr\,\sec^2$. Assuming that the equivalence principle is also valid in D dimensions, we argue that the acceleration of the test masses are independent of their masses. If we take the ratio of Eq. (2.9) for two different test masses with the same distance r, we see that G_D should be treated as a constant in a given dimension. However, neither Eq. (2.7) nor Eq. (2.9) can guarantee that its value will remain the same in a universe with a different D. Later, we will consider different numerical values for G_D in our calculations and by comparing our numerical results with observational data we will argue in favor of an expression that relates G_D in different dimensions.

Finally, in classical statistical mechanics, equipartition theorem asserts that the mean value of each independent quadratic term in the Hamiltonian is equal to $\frac{1}{2}kT$. Provided that the laws of thermodynamics do not change with D, we argue that each space dimension will bring a new quadratic degree of freedom to the Hamiltonian and take $k = 1.3807 \times 10^{-16} erg/K$ as the Boltzmann constant for all D.

The foregoing discussion makes it clear that the value of G as a fundamental constant of nature is not as firm as that of c, $\hbar$ and k. Now, we are ready to write the Planck scale applicable to $(1 + D)$-dimensional space-time.

2.2 Planck Quantities in D dimensions

The traditionally admitted view among cosmologists is to start the evolution of the universe from the Planck scale which is formed by the fundamental constants of nature in ordinary four dimensional space-time and which represents the scale of extreme physical quantities such as length, time, density and temperature. In ordinary $(1 + 3)$-dimensional space-time, Planck scale quantities are given as

$$\begin{aligned}
&\text{Planck length} &&: l_{pl} = \sqrt{\frac{G\hbar}{c^3}} = 1.6160 \cdot 10^{-33} cm, \\
&\text{Planck time} &&: t_{pl} = \sqrt{\frac{G\hbar}{c^5}} = 5.3907 \cdot 10^{-44} sec, \\
&\text{Planck mass} &&: m_{pl} = \sqrt{\frac{\hbar c}{G}} = 2.1768 \cdot 10^{-5} gr, \\
&\text{Planck density} &&: \rho_{pl} = \frac{c^5}{\hbar G^2} = 5.1584 \cdot 10^{93} gr/cm^3, \\
&\text{Planck temperature} &&: T_{pl} = \sqrt{\frac{\hbar c^5}{G}} \frac{1}{k} = 1.4170 \cdot 10^{32} K.
\end{aligned}$$

The fact that the force law for gravitational field depends on D, affects the definitions of various Planck quantities which can now be given as

$$\begin{aligned}
&\text{Planck length} &&: l_{pl,D} = \left(\frac{G_D \hbar}{c^3}\right)^{\frac{1}{D-1}}, \\
&\text{Planck time} &&: t_{pl,D} = \left(\frac{c^{D+2}}{G_D \hbar}\right)^{\frac{1}{1-D}}, \\
&\text{Planck mass} &&: m_{pl,D} = \left(\frac{\hbar^{D-2} c^{4-D}}{G_D}\right)^{\frac{1}{D-1}}, \\
&\text{Planck density} &&: \rho_{pl,D} = \left(\frac{c^{4+2D}}{\hbar^2 G_D^{D+1}}\right)^{\frac{1}{D-1}}, \\
&\text{Planck temperature} &&: T_{pl,D} = \left(\frac{\hbar^{D-2} c^{D+2}}{G_D}\right)^{\frac{1}{D-1}} \frac{1}{k}.
\end{aligned}$$

Naturally, numerical values of these quantities will depend on the model adapted for the D dependence of G_D and the D itself. In order to get an idea about how the various Planck quantities change with D, we take the numerical value of G_D to be the same as that of G_3 and present our results in Table 2.1. From this table it is striking to note that as the number of dimensions increases Planck length and Planck time increase while the other Planck quantities decrease.

Table 2.1: Various Planck Quantities for $G_D = 6.6720 \times 10^{-8}\ cm^D/gr\,\sec^2$.

D	3	4	5
$l_{pl,D}(cm)$	$1.6160 \cdot 10^{-33}$	$1.4170 \cdot 10^{-22}$	$4.0200 \cdot 10^{-17}$
$m_{pl,D}(gr)$	$2.1768 \cdot 10^{-5}$	$2.5545 \cdot 10^{-16}$	$8.7508 \cdot 10^{-22}$
$t_{pl,D}(\sec)$	$5.3905 \cdot 10^{-44}$	$4.5935 \cdot 10^{-33}$	$1.3409 \cdot 10^{-27}$
$\rho_{pl,D}(gr/cm^D)$	$5.1581 \cdot 10^{93}$	$7.1031 \cdot 10^{71}$	$8.3355 \cdot 10^{60}$
$T_{pl,D}(K)$	$1.4170 \cdot 10^{32}$	$1.6628 \cdot 10^{21}$	$5.6962 \cdot 10^{15}$

2.3 Adiabatic and Isothermal Processes in D Dimensions

In a D dimensional universe we write the first law of thermodynamics as

$$dQ_D = dU_D + P_D dV_D, \tag{2.10}$$

where U_D is the internal energy of the system, and dQ_D and $P_D dV_D$ represent the heat exchanged and the work done by the system on the environment, respectively. For the cosmological models we use the so-called "gamma-law" equation of state

$$P_D = (\gamma_D - 1)\,\rho_D, \tag{2.11}$$

which covers the entire range of basic equations of state corresponding to different eras in the history of the universe. Here, γ_D is a dimensionless constant parameter, and P_D and ρ_D represent the pressure and the energy density in the universe, respectively. For homogeneous and isotropic models we can now write the first law as

$$dQ = d(\rho_D V_D) + (\gamma_D - 1)\,\rho_D dV_D. \tag{2.12}$$

Integrating Eq. (2.12) for adiabatic processes where $dQ = 0$, we get

$$\rho_D V_D^{\gamma_D} = c_{1,D}, \;\; P_D V_D^{\gamma_D} = c_{2,D}, \tag{2.13}$$

where $c_{1,D}$ and $c_{2,D}$ are integration constants which have units that depend on the space dimensions of the universe. We now combine the first and second laws of thermodynamics to give

$$T dS_D = dU_D + P_D dV_D. \tag{2.14}$$

Using

$$dS_D(V_D, T) = \frac{1}{T}[d(\rho_D V_D) + P_D dV_D], \tag{2.15}$$

for homogeneous and isotropic models, we get from Eq. (2.15)

$$\frac{\partial S_D(V_D, T)}{\partial V_D} = \frac{1}{T}\left[\rho_D(T) + P_D(T)\right], \tag{2.16}$$

$$\frac{\partial S_D(V_D, T)}{\partial T} = \frac{V_D}{T}\frac{\partial \rho_D(T)}{\partial T}. \tag{2.17}$$

Using the integrability condition of Eq. (2.15):

$$\frac{\partial}{\partial T_D}\left(\frac{\partial S_D}{\partial V_D}\right) = \frac{\partial}{\partial V_D}\left(\frac{\partial S_D}{\partial T_D}\right), \tag{2.18}$$

we obtain

$$\frac{dP_D}{dT} = \frac{\rho_D + P_D}{T}. \tag{2.19}$$

Substituting the equation of state given in Eq. (2.11) into Eq. (2.19), we get

$$P_D T^{\frac{\gamma_D}{1-\gamma_D}} = c_{3,D}, \tag{2.20}$$

which is also true for adiabatic processes. Here, $c_{3,D}$ stands for a constant of integration. Using Eq. (2.15) and the integrability condition given in Eq. (2.19), we could now obtain, aside from an additive constant, a general expression for the entropy of the universe as

$$S_D(V_D, T) = \frac{V_D}{T}\left(P_D + \rho_D\right). \tag{2.21}$$

This expression is valid for both isothermal and isentropic processes. Using the equation of state given in Eq. (2.11), it is now possible to write the entropy as

$$S(V_D, T) = \frac{V_D \gamma_D}{T}\rho_D, \tag{2.22}$$

or by using Eqs. (2.11) and (2.20) in the following form:

$$S_D = V_D \frac{\gamma_D}{\gamma_{D-1}} T^{\frac{1}{\gamma_{D-1}}} c_{3,D}. \tag{2.23}$$

As in the standard model of the universe, we assume that the expansion of the universe is adiabatic in all eras. Then, the conservation of energy can be expressed as

$$\frac{d}{dt}\left(\rho_D V_D\right) = -P_D \frac{dV_D}{dt}. \tag{2.24}$$

With the help of Eqs. (2.19) and (2.24), it is possible to express the conservation of entropy as

$$\frac{d}{dt} S_D = 0, \tag{2.25}$$

where S_D is given in Eq. (2.21). For isentropic processes in D dimensions, $S = constant$ and from Eq. (2.23) we get

$$a^D T^{\frac{1}{\gamma_D - 1}} = constant, \tag{2.26}$$

where a represents the scale factor of the universe. For instance, for $D = 3$ and isotropic radiation, $\gamma_D = \gamma_3 = 4/3$, we get

$$a^3 T^3 = constant. \tag{2.27}$$

Thus, as the universe expands adiabatically temperature of the radiation decreases. This is in contrast to what happens during the prematter era, where the universe heats up due to the fact that $(\gamma_D - 1)$ is a negative number.

2.4 Blackbody Formula for the Massless Scalar Field

Finally, for later use we derive the general form of the energy density of homogeneous massless scalar field with a thermal spectrum in D dimensions. This energy can be written as

$$u_D = \frac{U_D}{V_D} = \int_0^\infty \frac{\hbar \omega g(\omega)}{(e^{\frac{\hbar\omega}{kT}} - 1)} d\omega, \tag{2.28}$$

where

$$g(\omega) = \frac{A_D \omega^{D-1}}{(2\pi c)^D} \tag{2.29}$$

represents the total number of effectively massless degrees of freedom. Integrating Eq. (2.28), we obtain

$$u_D = \frac{A_D}{(2\pi c\hbar)^D} \Gamma(D+1)\zeta(D+1)(kT)^{D+1}, \tag{2.30}$$

where $\Gamma(D+1)$ and $\zeta(D+1)$ are gamma and Riemann zeta functions, respectively.

CHAPTER 3

DESCRIPTION OF THE COSMOLOGICAL MODEL

In our model, the universe starts its journey as a D (> 3) dimensional Friedmann model (D denotes the number of space dimensions) with a density equal to the Planck density of the corresponding dimension. Existence of higher dimensions and their reduction to ordinary dimensions is assumed to take place during the inflationary prematter phases of the universe. Evolution of the universe starts with a period of exponentially rapid expansion called inflationary prematter phase, which is characterized by a "vacuum-like" equation of state $P_D \approx -\rho_D$. Here, we exclude the case $P_D = -\rho_D$ because it gives rise to eternal inflation. It is to be noted from Eq.(2.26) that due to the unusual characteristics of the equation of state that we adopt (Lima & Maia 1995), temperature increases while the universe expands isentropically. This expansion continues until the maximum allowed temperature, i.e., the Planck temperature ($T_{pl,D}$) of that dimension is reached. At this point, we postulate that the universe undergoes a phase transition to a lower dimension where the Planck temperature is higher. In this way, the universe finds more room for further inflation. This process continues until we reach $D = 3$, which is the current number of space dimensions. At this point, either the last reduction may carry the universe directly into the standard radiation era where $\gamma_r = 4/3$ or the prematter era may continue one more time before the radiation era begins. After the radiation era, the universe is eventually carried to the era of matter dominance where $\gamma_3 = 1$. In what follows, we consider the second alternative in which the prematter era continues also in ordinary dimensions.

3.1 Field Equations

We consider homogeneous and isotropic $(1+D)$-dimensional space-times where the metric is given as

$$ds^2 = c^2 dt^2 - a_D^2(t) d\sigma_k^2. \tag{3.1}$$

Here, $a_D(t)$ represents the scale factor of the universe, t is the comoving time and k is the curvature parameter, while $d\sigma_k^2$ is given as

$$d\sigma_k^2 = d\chi^2 + F_k^2(\chi)[d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + ... + \sin^2\theta_1 \sin^2\theta_2 ..... \sin^2\theta_{D-2} d\theta_{D-1}^2], \tag{3.2}$$

where

$$F_k(\chi) = \begin{cases} \sin\chi \;\; if \;\; k = +1, \;\; 0 \leq \chi \leq \pi, \\ \chi \;\; if \;\; k = 0, \;\; 0 \leq \chi \leq \infty, \\ \sinh\chi \; if \;\; k = -1, \;\; 0 \leq \chi \leq \infty, \end{cases} \tag{3.3}$$

and the independent variables are in the following ranges: $0 \leq \theta_n \leq \pi$, $0 \leq \theta_{D-1} \leq 2\pi$, $n = 1, 2, ..., D-2$.

We also consider that the universe is filled with a perfect fluid whose energy-momentum tensor is given as

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu - P g_{\mu\nu}, \tag{3.4}$$

where u_μ is the four-velocity of comoving observers and it has the following non-vanishing components:

$$T^0{}_0 = \rho_D, \quad T^1{}_1 = T^2{}_2 = ... = T^D{}_D = -P_D. \tag{3.5}$$

Using these in Einstein's gravitational field equations written in $(1+D)$-dimensional space-time:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -2 \frac{A_D G_D}{c^4} T_{\mu\nu}, \tag{3.6}$$

we get

$$\frac{D(D-1)}{2a_D^2} \left[\left(\frac{\dot{a}_D}{c} \right)^2 + k \right] = 2 \frac{A_D G_D}{c_D^4} \rho_D, \tag{3.7}$$

$$(1-D) \left(\frac{\ddot{a}_D}{c_D^2 a_D} \right) - \frac{(D-1)(D-2)}{2a_D^2} \left[\left(\frac{\dot{a}_D}{c_D} \right)^2 + k \right] = 2 \frac{A_D G_D}{c_D^4} P_D, \tag{3.8}$$

where the subscript D denotes the value of that quantity in D dimensions, A_D is the solid angle subtended by a sphere in a D dimensional space ($A_3 = 4\pi$, $A_4 = 2\pi^2$, $A_5 = \frac{8\pi^2}{3}, ...$) and a dot denotes differentiation with respect to the cosmic time t. Using the equation of state in the form $P_D = (\gamma_D - 1)\,\rho_D$ and combining Eqs. (3.7) and (3.8) we find an equation involving only the scale factor $a_D(t)$:

$$\frac{\ddot{a}_D}{a_D} + c^2\left(\frac{D\gamma_D}{2} - 1\right)\left(\frac{\dot{a}_D^2 + k}{a_D^2}\right) = 0. \tag{3.9}$$

3.2 Differential Equations for the Scale Factor

The evolution of the scale factor during different eras of the universe could be obtained by solving Eq. (3.9) for a given γ_D. In order to solve this equation for any γ_D, we introduce conformal time η defined through the following relation:

$$cdt = a_D(\eta)d\eta. \tag{3.10}$$

Under this transformation, Eq. (3.9) takes the following form:

$$\frac{a_D''}{a_D} + \left(\frac{D\gamma_D}{2} - 2\right)\left(\frac{a_D'}{a_D}\right)^2 + \left(\frac{D\gamma_D}{2} - 1\right)k = 0. \tag{3.11}$$

Here, a prime denotes differentiation with respect to the conformal time. If we define a new dependent variable given as

$$u_D \equiv \frac{a_D'}{a_D} = \frac{d\ln a_D}{d\eta},$$

Eq. (3.11) becomes the Riccati equation of the following form:

$$u_D' + c_D u_D^2 + kc_D = 0, \tag{3.12}$$

where

$$c_D = \frac{D}{2}\gamma_D - 1. \tag{3.13}$$

During the prematter eras, c_D is a negative number and its closeness to -1 represents the strength of the inflation that the universe undergoes. We will later see that c_D will be treated as one of the parameters of our cosmological model. In what follows, we will consider equations of state such that $c_D \neq 0$. Eq. (3.9) can be solved by defining a new variable w_D through

$$u_D \equiv \frac{1}{c_D}\frac{w'_D}{w_D} = \left[\ln\left(w_D^{1/c_D}\right)\right]'. \tag{3.14}$$

In terms of this new variable, Eq. (3.12) can be written as

$$w''_D + kc_D^2 w_D = 0. \tag{3.15}$$

In order to find the evolution of the scale factor in different geometries, this equation has to be solved for different values of k.

3.3 Solutions for the Scale Factor

From Eq. (3.15), we now obtain the solutions for the scale factor for different geometries as follows:

$$a_D(\eta) = \begin{cases} a_{0,D}[\cos(c_D\eta + \delta_D)]^{\frac{1}{c_D}}, & if\ k = +1, \\ a_{0,D}(\eta + \delta_D)^{\frac{1}{c_D}}, & if\ k = 0, \\ a_{0,D}[\sinh(c_D\eta + \delta_D)]^{\frac{1}{c_D}}, & if\ k = -1. \end{cases} \tag{3.16}$$

Here, we have introduced the subscript D to identify the era to which it applies, and a_{0D} and δ_D are integration constants to be determined from the initial and boundary conditions.

We now choose closed geometry, i.e., $k = 1$ to demonstrate how our model works. However, it is to be noted that our results are easily adaptable to other geometries, i.e., $k = 0$ and $k = -1$. If the initial number of space dimensions of the universe is D, then the total number of prematter eras in our model will be $(D - 2)$. We now write the solutions for the scale factor and the corresponding equations of state for $k = 1$ as

$$1^{st} \text{ prematter era:} \begin{cases} a_D(\eta) = a_{0,D}^{(p)}[\cos(c_D\eta + \delta_D]^{\frac{1}{c_D}}, \\ 0 \le \eta \le \eta_D, \\ P_D = \left(\frac{2c_D+2-D}{D}\right)\rho_D. \end{cases}$$

$$2^{nd} \text{ prematter era:} \begin{cases} a_{D-1}(\eta) = a_{0,D-1}^{(p)}[\cos(c_{D-1}\eta + \delta_{D-1}]^{\frac{1}{c_{D-1}}}, \\ \eta_D \le \eta \le \eta_{D-1}, \\ P_{D-1} = \left(\frac{2c_{D-1}+3-D}{D-1}\right)\rho_{D-1}. \end{cases}$$

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$$(D-2)^{th}\ \text{prematter era:} \begin{cases} a(\eta) = a_0^{(p)}[\cos(c_3\eta + \delta_{D-1}]^{\frac{1}{c_3-1}}, \\ \eta_4 \le \eta \le \eta_r, \\ P = \left(\frac{2c_3-1}{3}\right)\rho. \end{cases}$$

$$\text{radiation era:} \begin{cases} a(\eta) = a_0^{(r)}[\cos(\eta + \delta_r], \\ \eta_r \le \eta \le \eta_m, \\ P = \frac{1}{3}\rho. \end{cases}$$

$$\text{matter era:} \begin{cases} a(\eta) = a_0^{(m)}[\cos(\frac{\eta}{2} + \delta_m]^2, \\ \eta_m \le \eta, \\ P = 0. \end{cases}$$

Here, we have used p, r and m as subscripts (or superscripts) to denote the prematter, radiation and matter eras, respectively, and subscript D identifies the prematter era in higher dimensions. In this thesis, physical quantities which do not carry superscripts or subscripts are understood to correspond to ordinary four-dimensional space-time.

3.4 Initial and Boundary Conditions

We start the evolution of the universe with a prematter era from D (> 3) dimensions, and take the initial density as the Planck density of D, i.e., $\rho_{pl,D}$. We now write Eq. (3.7) in terms of conformal time

$$\frac{D(D-1)}{2}\left[\left(\frac{a'_D}{a_D^2}\right)^2+\frac{1}{a_D^2}\right]=\frac{2A_D G_D}{c^4}\rho_D. \tag{3.17}$$

Considering Eq. (3.17) at $\eta = 0$, we get the following quadratic equation in $a_D^2(0)$:

$$d_D a_D^4(0)-a_D^2(0)-v^2=0, \tag{3.18}$$

where v is the initial expansion rate of the universe, i.e.,

$$v=a'_D(0), \tag{3.19}$$

and

$$d_D=\frac{4A_D G_D}{D(D-1)c^4}\rho_{pl,D}. \tag{3.20}$$

Eq. (3.18) is quadratic in $a_D^2(0)$ and has the following physical solution for the initial size of the universe:

$$a_D(0)=\sqrt{\frac{\sqrt{1+4d_D v^2}+1}{2d_D}}, \tag{3.21}$$

which reflects the singularity-free character of our cosmological model.

According to our consideration, dimensional reduction takes place by a first order phase transition whose duration can be taken as negligibly short compared with the durations of the constant D eras in the history of the universe. Denoting the time of the transition as η_D, we may write Eq. (3.7) just before the dimensional reduction at $\eta = \eta_D - \mid \epsilon \mid$ as

$$\frac{D(D-1)}{2}\left[\left(\frac{a'_D}{a_D^2}\right)^2 + \frac{1}{a_D^2}\right] = \frac{2A_D G_D}{c^4}\rho_D, \tag{3.22}$$

where ϵ is a small number. After the dimensional reduction at η_D, the number of space dimensions becomes $D-1$ and Eq. (3.7) takes the following form:

$$\frac{(D-1)(D-2)}{2}\left[\left(\frac{a'_{D-1}}{a_{D-1}^2}\right)^2 + \frac{1}{a_{D-1}^2}\right] = \frac{2A_{D-1} G_{D-1}}{c^4}\rho_{D-1}, \tag{3.23}$$

at $\eta = \eta_D + \mid \epsilon \mid$. Since the dimensional reduction takes place in a time interval whose duration is expected to be significantly smaller than those of the constant D eras and the evolution of the universe during the constant D eras is inflationary, we could argue that during the transition from D to $D-1$, the change in $a_D(\eta)$ is negligible. Hence, as one of our junction conditions for dimensional reduction we take the scale factor of the universe to be continuous at the transition point. Moreover, from the field equations in Eqs. (3.7) and (3.8) we see that the kinematics of the evolution of the universe is driven by $G_D\rho_D$ and $G_D P_D$ combinations. Thus, in the limit as $\epsilon \rightarrow 0$ a discontinuity in $a'_D(\eta)$ would imply an infinite jump in either $G_D\rho_D$ or $G_D P_D$. Indeed, we should expect that during dimensional reduction there would be a discontinuity in ρ_D due to the fact that the available volume is reduced. However, since the change in volume is finite, a discontinuity in ρ_D should also be expected to be finite. Thus, in this case an infinite jump in $G_D\rho_D$ would also imply an infinite jump in G_D. Since all the physically relevant quantities change by finite amounts, it seems difficult to justify an infinite jump in G_D. Hence, for the time being we argue that taking $a'_D(\eta)$ as continuous during dimensional reduction is also a good working assumption.

Similarly, the phase transitions between prematter and radiation eras as well as between radiation and matter eras are also expected to be significantly smaller than those individual eras. We can thus use the same junction conditions for the evolution of the scale factor at the boundaries between different eras. Furthermore, since Hubble parameter is defined as the ratio of scale factor and its derivative, this assumption would also make the Hubble parameter continuous at the transition points.

Of course, strictly speaking neither the scale factor nor its derivative is continuous during the course of dimensional reduction. We should also expect that the universe deviates from both homogeneity and isotropy assumptions during the transition. What is worse is that due to the decomposition of higher dimensions, our understanding of the concept of dimension would be significantly different from the current one. In this case, it would be hard to talk about the applicability of the Friedmann models. In this thesis, we are not interested in the details of dimensional reduction. So, the study of what happens during the course of dimensional reduction is beyond the scope of the present work.

In the light of the foregoing assumptions we consider Eqs. (3.22) and (3.23) in the limit as $\epsilon \to 0$ and take their ratios to obtain

$$\frac{G_D}{G_{D-1}} = \frac{D}{D-2}\frac{A_{D-1}}{A_D}\left[\frac{\rho_{D-1}}{\rho_D}\right]_{\eta=\eta_D}. \tag{3.24}$$

With the help of the equation (3.24), we have thus related the change in G_D to that in ρ_D. Thus, in order to find how G_D changes as D changes, we have to know the discontinuity in density when the universe undergoes dimensional reduction. We may express this discontinuity by defining a "critical length" parameter:

$$l_{D,D-1} = \left[\frac{\rho_{D-1}}{\rho_D}\right]_{\eta=\eta_D}, \tag{3.25}$$

with $[l_{D,D-1}] = cm$. We then get the following expression for the change in G_D:

$$\frac{G_D}{G_{D-1}} = \frac{D}{D-2}\frac{A_{D-1}}{A_D} l_{D,D-1}. \tag{3.26}$$

Here, $l_{D,D-1}$ defines a characteristic length scale which marks the point at which dimensional reduction takes place. In the next section, when we discuss a specific dimensional reduction scenario from $D = 4$ to $D = 3$, we will see that the critical length parameter will play the role of one of the parameters of our cosmological model.

Furthermore, since the expansion of the universe is assumed to be adiabatic during the constant D eras of the universe, total entropy is constant. However, during dimensional reduction from D to $(D-1)$, which we interpret as a phase transition, entropy is expected to change; either in the form of an increase or a decrease. As can be seen from Eq. (2.22), entropy is defined up to an additive constant. Since we do not know whether this constant would differ in different dimensions, the zero level of entropy in different dimensions cannot be determined. Thus, we cannot make any decision about the nature of the change in entropy in this model.

Finally, the boundary conditions that the scale factor and its derivative are continous at points η_D, $\eta_{D-1}, ..., \eta_4, \eta_r$ and η_m allow us to determine the integration constants as

$$a_{0,D}^{(p)} = \left[\frac{d_D}{d_D - H_D^2(0)}\right]^{\frac{c_D+1}{2c_D}} d_D^{-1/2}, \tag{3.27}$$

$$a_{0,D-1}^{(p)} = a_{0,D}^{(p)} \left[\cos\left(c_D \eta_D + \delta_D\right)\right]^{\frac{1}{c_D} - \frac{1}{c_{D-1}}}, \tag{3.28}$$

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$$a_0^{(p)} = a_{0,4}^{(p)} \left[\cos\left(c_4 \eta_4 + \delta_4\right)\right]^{\frac{1}{c_4} - \frac{1}{c_3}}, \tag{3.29}$$

$$a_0^{(r)} = a_0^{(p)} \left[\cos\left(c_3 \eta_r + \delta_3\right)\right]^{\frac{1}{c_3} - 1}, \tag{3.30}$$

$$a_0^{(m)} = \frac{a_0^{(r)}}{\cos(\eta_m + \delta_r)}, \tag{3.31}$$

$$\delta_D = -\arctan\left(\frac{H_D(0)}{\sqrt{d_D - H_D^2(0)}}\right), \tag{3.32}$$

$$\delta_{D-1} = \delta_D + (c_D - c_{D-1})\,\eta_D, \tag{3.33}$$

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$$\delta_3 = \delta_4 + (c_4 - c_3)\,\eta_4, \tag{3.34}$$

$$\delta_r = \delta_3 + (c_3 - 1)\,\eta_r, \tag{3.35}$$

$$\delta_m = \delta_r + \frac{\eta_m}{2}, \tag{3.36}$$

where $H_D(0)$ denotes the initial value of the Hubble parameter in D dimensions.

3.5 Transition Times in the Model

During the prematter era, the universe is assumed to be comprised of a material substance which expands adiabatically and which is described by an equation of state similar to that of the vacuum, i.e., $P_D \simeq -\rho_D$. Using Eq. (2.23) we write the corresponding entropy as

$$S_D = V_D \frac{\gamma_D}{\gamma_{D-1}} T^{\frac{1}{\gamma_{D-1}}} c_{3,D}. \tag{3.37}$$

From the form of the equation of state, it is clear that $\gamma_D < 1$. In this case, the fluid filling the universe will be under tension rather than pressure. So, the properties of this fluid will be very different than the ones that we are acquainted.

As can be seen from Eq. (3.37), unlike the ordinary matter, temperature of the prematter with $\gamma_D < 1$ increases in the course of an adiabatic expansion. Thus, the universe heats up while it inflates during the prematter era, and inflation continues until the maximum allowed temperature $T_{pl,D}$ is reached. When the temperature reaches the Planck temperature of that dimension, we postulate that the universe undergoes a phase transition to a lower dimension where the Planck temperature is higher. Thus, inflation gets another chance to continue in the lower dimension. Of course, whether this is necessary or not will be seen when we compare the final results with observational data.

We also assume that the fluid filling the universe attained thermal equilibrium at the at the end of the prematter era and has a thermal spectrum. At this point, we take the density of the universe equal to that of a massless scalar field with a thermal spectrum at $T_{pl,D}$ as

$$\rho\left(\eta_D\right) = \frac{A_D}{\left(2\pi c\hbar\right)^D}\Gamma\left(D+1\right)\zeta\left(D+1\right)\left(kT_{pl,D}\right)^{D+1}. \qquad (3.38)$$

Using this expression for energy density in Einstein's field Eq. (3.17), we find the conformal time at which dimensional reduction occurs as follows

$$\begin{aligned} \eta_D &= \frac{1}{c_D}\left\{\arccos\left[\left(\frac{\left(2\pi c\hbar\right)^D \rho_{pl,D}}{A_D\Gamma(D+1)\zeta(D+1)\left(kT_{pl,D}\right)^{D+1}}\right)^{\frac{c_D}{2(c_D+1)}}\sqrt{\frac{d_D - H_D^2(0)}{d_D}}\right]\right. \\ &+ \left.\arctan\left(\frac{H_D(0)}{\sqrt{d_D - H_D^2(0)}}\right)\right\}, \qquad (3.39) \end{aligned}$$

which also represents the duration of the first prematter era.

In order to find conformal times corresponding to transitions between different eras we return back to Eqs. (2.21) and (2.25), and express the conservation of entropy in terms of the scale factor as

$$\frac{d}{dt}\left(\frac{a_D^D \rho_D}{T}\right) = 0, \tag{3.40}$$

where the general form of the equation of state in Eq. (2.11) was used. Carrying out differentiation in Eq. (3.40), we obtain

$$D\frac{\dot{a}_D}{a_D} - \frac{\dot{T}}{T} + \frac{\dot{\rho}_D}{\rho_D} = 0. \tag{3.41}$$

From the Einstein's field Eqs. (3.7) and (3.8), one may write

$$\dot{\rho}_D + D\gamma_D \rho_D \frac{\dot{a}_D}{a_D} = 0, \tag{3.42}$$

which describes the time evolution of the energy density. Using Eq. (3.41) in Eq. (3.42) and, as usual, defining the Hubble parameter H as

$$H_D \equiv \frac{\dot{a}_D}{a_D}, \tag{3.43}$$

we could write Eq. (3.42) in the form

$$\frac{\dot{T}}{T} + DH_D\left(\gamma_D - 1\right) = 0. \tag{3.44}$$

In terms of the conformal time η, this gives us the equation describing the time evolution of the temperature as follows:

$$\frac{T'}{T} + D\frac{a'_D}{a_D}\left(\gamma_D - 1\right) = 0. \tag{3.45}$$

Upon integration in conformal time, Eq. (3.45) gives

$$\frac{a_D(\eta_f)}{a_D(\eta i)} = \left[\frac{T(\eta_f)}{T(\eta i)}\right]^{\frac{1}{D(1-\gamma_D)}}, \tag{3.46}$$

where subscripts i and f denote the initial and final instants of conformal time in a given era, respectively. Similarly, we can obtain the time evolution of the density by returning back to Eq. (3.42) as

$$\frac{\rho_D(\eta_f)}{\rho_D(\eta i)} = \left[\frac{a_D(\eta_i)}{a_D(\eta_f)}\right]^{D\gamma_D}. \tag{3.47}$$

Like first order phase transitions, we assume that during the stage of dimensional reduction, temperature remains constant. Then, Eqs. (3.46) and (3.47) are useful in finding transition times between different eras in the history of the universe as well as densities corresponding to phase transitions and present.

CHAPTER 4

A SPECIFIC DIMENSIONAL REDUCTION SCENARIO

In this chapter, in the light of the foregoing chapter we present a numerical model for both closed and open geometries, respectively. According to this model, the universe starts from $D = 4$ prematter and evolves into $D = 3$ prematter. The second prematter evolves into radiation, and finally into an era of matter dominance.

4.1 Closed Universe Case

For closed geometry, i.e., $k = 1$, the scale factor in different eras with the corresponding equations of states are as follows:

$$a(\eta) = \begin{cases} 1^{st} \text{ premater era:} \begin{cases} a_{0,4}^{(p)}[\cos(c_4\eta_4 + \delta_4)]^{\frac{1}{c_4}}, \\ 0 \le \eta \le \eta_4, \\ P_4 = \left(\frac{2c_4 - 1}{2}\right)\rho_4. \end{cases} \\ 2^{nd} \text{ prematter era:} \begin{cases} a_0^{(p)}[\cos(c_3\eta_4 + \delta_4)]^{\frac{1}{c_4}}, \\ \eta_4 \le \eta \le \eta_r, \\ P = \left(\frac{2c_3 - 1}{3}\right)\rho. \end{cases} \\ \text{radiation era:} \begin{cases} a_0^{(r)}[\cos(\eta + \delta_r)], \\ \eta_r \le \eta \le \eta_m, \\ P = \frac{1}{3}\rho. \end{cases} \\ \text{matter era:} \begin{cases} a_0^{(m)}[\cos(\frac{\eta}{2} + \delta_m)]^2, \\ \eta_m \le \eta, \\ P = 0. \end{cases} \end{cases} \tag{4.1}$$

Here, the integrations constants can be found by setting $D = 4$ in Eqs. (3.27)-(3.36). The results are as follows

$$a_{0,4}^{(p)} = \left[\frac{d_4}{d_D - H_D^2(0)}\right]^{\frac{c_4+1}{2c_4}} d_4^{-1/2}, \tag{4.2}$$

$$a_0^{(p)} = a_{0,4}^{(p)} \left[\cos\left(c_4\eta_4 + \delta_4\right)\right]^{\frac{1}{c_4} - \frac{1}{c_3}}, \tag{4.3}$$

$$a_0^{(r)} = a_0^{(p)} \left[\cos\left(c_3\eta_r + \delta_3\right)\right]^{\frac{1}{c_3} - 1}, \tag{4.4}$$

$$a_0^{(m)} = \frac{a_0^{(r)}}{\cos(\eta_m + \delta_r)}, \tag{4.5}$$

$$d_4 = \frac{A_4 G_4}{3c^4} \rho_{pl,4}, \tag{4.6}$$

$$\delta_4 = -\arctan\left(\frac{H_4(0)}{\sqrt{d_4 - H_4^2(0)}}\right), \tag{4.7}$$

$$\delta_3 = \delta_4 + (c_4 - c_3)\,\eta_4, \tag{4.8}$$

$$\delta_r = \delta_3 + (c_3 - 1)\,\eta_r, \tag{4.9}$$

$$\delta_m = \delta_r + \frac{\eta_m}{2}. \tag{4.10}$$

At the point of dimensional reduction, we take the density equal to that of a massless scalar field having a thermal spectrum at $T_{pl,4}$. And, making use of Eq. (3.38) and setting $D = 4$, we obtain the energy density at the end of the first prematter era as

$$\rho_4(\eta_4) = \frac{A_4 \Gamma(5) \zeta(5) (k T_{pl,4})^5}{(2\pi\hbar c)^4}. \tag{4.11}$$

Then, solving Eq. (3.17) for η_4, which marks the conformal time at which dimensional reduction takes place, we get the duration of the first prematter era as

$$\eta_4 = \frac{1}{c_4}[\arccos(argm1) - \delta_4], \tag{4.12}$$

where

$$argm1 = (3.1727)^{\frac{c_4}{2(c_4+1)}}[\frac{d_4 - H_4^2(0)}{d_4}]. \tag{4.13}$$

According to our previous consideration, the universe starts its journey from $\rho_{pl,4}$ and while it expands, its temperature rises due to the unusual form of the equation of state describing the universe during the first prematter era, i.e., $P_4 \approx -\rho_4$. However, the temperature cannot go beyond $T_{pl,4}$. At this point, we postulate that when the temperature reaches $T_{pl,4}$, the universe undergoes a phase transition to the $D = 3$ universe where the Planck temperature is higher and the equation of state takes the form $P \approx -\rho$. And, inflation continues in this second prematter era until the temperature reaches $T_{pl,3}$ and a phase transition that links the second prematter era to the radiation era occurs. The rest is like the standard model of the universe, where the radiation era ($P = \rho/3$) is linked by a first order phase transition to the era of matter dominance ($P = 0$). We also assume that after the transition between radiation and matter eras, radiation is decoupled from matter, and behaves like a perfect fluid with $\gamma_3 = 4/3$.

Eq. (3.46) would allow us to determine the durations of different eras once we know the temperatures at which different phase transitions take place. For the second prematter era, initial and final temperatures are given as $T_i = T_{pl,4}$ and $T_f = T_{pl,3}$, respectively. Upon this, one can get from Eq. (3.46)

$$\frac{a(\eta_f)}{a(\eta_i)} = \left(\frac{T_{pl,3}}{T_{pl,4}}\right)^{\frac{1}{1-2c_3}}, \tag{4.14}$$

which gives the duration of the second prematter era in conformal time as

$$\eta_3 = \frac{1}{c_3}[\arccos(argm2) - \delta_3], \tag{4.15}$$

where

$$argm2 = \left(\frac{T_{pl,3}}{T_{pl,4}}\right)^{\frac{c_3}{1-2c_3}} argm1. \tag{4.16}$$

At this point, for our later purposes it would be useful to write the gravitational constant G_4 and the remaining Planck quantities in $D = 4$. They are as follows:

$$G_4 = \frac{4l_{4,3}}{\pi} G_3, \tag{4.17}$$

$$\rho_{pl,4} = \frac{4.7496 \cdot 10^{71}}{l_{4,3}^{5/3}} \ gr/cm^4, \tag{4.18}$$

$$T_{pl,4} = \frac{1.5356 \cdot 10^{21}}{l_{4,3}^{1/3}} \ K, \tag{4.19}$$

$$l_{pl,4} = 1.4924 \cdot 10^{-22} \ l_{4,3}^{1/3} \ cm, \tag{4.20}$$

where Eq. (3.24) has been used to relate G_4 to G_3. Then one may write

$$argm2 = argm1 \left(\frac{\pi l_{pl}}{4l_{4,3}} \right)^{\frac{c_3}{3(2c_3-1)}} . \tag{4.21}$$

Similarly, we can find the conformal time at which radiation era ends, i.e., η_m. From Eq. (3.46), one may write

$$\frac{a(\eta_f)}{a(\eta_i)} = \frac{T_{pl}}{T_m}, \tag{4.22}$$

where $T_i = T_{pl}$ and $T_f = T_m$ have been used for the radiation era. Here, T_m represents the temperature at the last phase transition which is also known as the "recombination temperature." Solving Eq. (4.22), we end up with

$$\eta_m = \eta_r + \arccos(argm3) - \arccos(argm2), \tag{4.23}$$

where

$$argm3 = \frac{T_{pl}}{T_m} argm2. \tag{4.24}$$

Making use of Eq. (3.46), we write

$$\frac{a(\eta_f)}{a(\eta_i)} = \frac{T_m}{T_{now}}, \tag{4.25}$$

where T_{now} is the present day temperature of the cosmic microwave background radiation (CMBR). Solving this equation, we obtain the age of the universe in conformal time as

$$\eta_{now} = 2[\arccos(argm4) - \delta_m], \tag{4.26}$$

where

$$argm4 = \sqrt{\frac{T_m}{T_{now}}} argm3. \tag{4.27}$$

4.1.1 Observable Parameters in the Model

We can now obtain expressions for physical quantities such as Hubble parameter, age and density. We can find comoving times corresponding to the conformal ones by going back to the definition given in Eq. (3.10). Assuming that $t = 0$ at $\eta = 0$, we get

$$t = a_{0,D} \int_0^{\eta} \left[\cos\left(c_D \eta' + \delta_D\right)\right]^{1/c_D} d\eta'. \tag{4.28}$$

This integral depends upon the values of c_D and has to be computed numerically in the prematter eras. Whereas, for the radiation ($c_r = 1$) and matter ($c_m = 1/2$) eras, integration yields analytical expressions. The expressions corresponding to the comoving times corresponding to η_4, η_r, η_m and η_{now} can be given as

$$t_4 = a_{0,4}^{(p)} \int_0^{\eta_4} \left[\cos\left(c_4\eta + \delta_4\right)\right]^{1/c_4} d\eta, \tag{4.29}$$

$$t_r = t_4 + a_0^{(p)} \int_{\eta_4}^{\eta_3} \left[\cos\left(c_3\eta + \delta_3\right)\right]^{1/c_3} d\eta, \tag{4.30}$$

$$t_m = t_r + a_0^{(r)} \left[\sin(\eta_m + \delta_r) - \sin(\eta_r + \delta_r)\right], \tag{4.31}$$

$$t_{now} = t_m + \frac{a_0^{(m)}}{2} \left[\sin(\eta_{now} + 2\delta_m) - \sin(\eta_m + 2\delta_m) + (\eta_{now} - \eta_m)\right]. \tag{4.32}$$

Hubble parameters at η_4, η_r, η_m and η_{now} can now be obtained by using

$$H(\eta) = \frac{a'(\eta)}{a^2(\eta)}, \tag{4.33}$$

as

$$H(\eta_4) = -\frac{\sin\left(c_4\eta_4 + \delta_4\right)}{a_{0,4}^{(p)} \left[\cos\left(c_4\eta_4 + \delta_4\right)\right]^{\frac{1}{c_4}+1}} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}, \tag{4.34}$$

$$H(\eta_r) = -\frac{\sin\left(c_3\eta_r + \delta_3\right)}{a_0^{(p)} \left[\cos\left(c_3\eta_r + \delta_3\right)\right]^{\frac{1}{c_3}+1}} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}, \tag{4.35}$$

$$H(\eta_m) = -\frac{\sin\left(\eta_m + \delta_r\right)}{a_0^{(r)} \cos^2\left(\eta_m + \delta_r\right)} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}, \tag{4.36}$$

$$H(\eta_{now}) = -\frac{\sin\left(\eta_{now}/2 + \delta_m\right)}{a_0^{(m)} \cos^3\left(\eta_{now}/2 + \delta_m\right)} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}. \tag{4.37}$$

Finally, to find densities at the transition points and present we make use of Eq. (3.47). First, we choose $(\eta_i, \eta_f) = (0, \eta_4)$ and get the density at the point of dimensional reduction as

$$\rho_4(\eta_4) = \left[\frac{\left[\frac{d_4 - H_4^2(0)}{d_4}\right]}{(argm1)^2}\right]^{\frac{c_4+1}{c_4}} \rho_{pl,4}. \tag{4.38}$$

Here, we should notice the fact that during the dimensional reduction from $D = 4 \longrightarrow 3$, the density has a discontinuity given by Eq. (3.25) and at the beginning of the second prematter era the universe has the following density:

$$\rho_3(\eta_4) = l_{4,3}\rho_4(\eta_4). \tag{4.39}$$

By choosing $(\eta_i, \eta_f) = (\eta_4, \eta_r)\,,\,(\eta_r, \eta_m)\,,\,(\eta_m, \eta_{now})$, we obtain, respectively

$$\rho(\eta_r) = \left(\frac{argm1}{argm2}\right)^{\frac{2(c_3+1)}{c_3}} \rho_3(\eta_4), \tag{4.40}$$

$$\rho(\eta_m) = \left(\frac{T_m}{T_{pl}}\right)^4 \rho(\eta_r), \tag{4.41}$$

$$\rho(\eta_{now}) = \left(\frac{T_{now}}{T_m}\right)^3 \rho(\eta_m). \tag{4.42}$$

4.1.2 Comparison with Observations

Even though we have listed all the physical parameters in the previous section, different regimes have to be investigated to construct numerical models. To this end, we will consider different cases corresponding to G_4. Let us start with the special case in which the numerical value of the gravitational constant in $D = 4$ does not differ from the one currently used. In this case, from Eq. (4.17) it follows that

$$\frac{4l_{4,3}}{\pi} = 1 \ cm. \tag{4.43}$$

From Eqs. (4.16) and (4.42), we may write the present value of the density as

$$\rho(\eta_{now}) = 6.2269 \cdot 10^{-35} T_m \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{2(2-c_3)}{3(2c_3-1)}} \left(1-\kappa^2\right)^{-\frac{1+c_4}{c_4}} \ gr/cm^3, \tag{4.44}$$

where we have set

$$\kappa = \frac{H_4(0)}{\sqrt{d_4}}. \tag{4.45}$$

From Eq. (3.7), we notice that for closed geometry $0 \leq \kappa < 1$. And, since the evolution of the universe is inflationary during the prematter eras, we have

$$c_4 \ and \ c_3 \in (-1, 0) \, . \tag{4.46}$$

Then, one may write the inequality:

$$0 < \left(1-\kappa^2\right)^{-\frac{1+c_4}{c_4}} \leq 1. \tag{4.47}$$

Furthermore, the power of the term $(4l_{4,3}/\pi l_{pl})$ in Eq. (4.44) is in the following range:

$$-\frac{4}{3} < \frac{2(2-c_3)}{3(2c_3-1)} < -\frac{2}{3}. \tag{4.48}$$

Thus, using Eqs. (4.43), (4.44), (4.47) and (4.48) we end up with

$$\rho(\eta_{now}) < 8.5750 \cdot 10^{-57} \ T_m. \tag{4.49}$$

At this point, following Kolb & Turner (1990, p.77) we may write

$$T_m = (1 + z_{dec}) T_{now}, \tag{4.50}$$

where z_{dec} represents the red shift at recombination and $T_{now} = 2.73\ K$. Using the numerical value $z_{dec} = 1090$ which is in good agreement with the recent measurements (see e.g., Spergel et al. 2003 and Bennett et al. 2003) one gets

$$T_m = 2978\ K. \tag{4.51}$$

Upon this numerical value, from Eq. (132) we end up with

$$\rho(\eta_{now}) < 2.5536 \cdot 10^{-53}\ gr/cm^3. \tag{4.52}$$

The present value of the total density of the universe ρ_0 is given through the present value of the density parameter $\Omega_0 \equiv \rho_0/\rho_{cr}$, where $\rho_{cr} = 3H_0^2 c^2/8\pi G$ and H_0 represents the present value of the Hubble parameter. The recent Wilkinson Microwave Anisotropy Probe (WMAP) data based on CMBR anisotropy (see e.g., Spergel et al. 2003 and Bennett et al. 2003) strongly suggests that we live in a universe with a nearly flat geometry ($\Omega_0 \approx 1$) and which is composed of $\sim 1/3$ of matter (baryonic+dark with $\Omega_m \approx 0.3$) and $\sim 2/3$ of an exotic form of matter usually called dark energy or quintessence ($\Omega_q \approx 0.7$). Upon this, one may write $\rho_0 \approx 10^{-29} h^2\ gr/cm^3$ where $h \equiv H_0/(100\ km\ s^{-1} Mpc^{-1})$. In the light of recent measurements (see e.g., Jimenez et al. 2003, Koopmans et al. 2003, Altavilla et al. 2004), h could be considered to be restricted to the interval [0.6, 0.8]. This implies that ρ_0 is at the order of $10^{-30} gr/cm^3$. Hence, for the special case in which the numerical value of G_4 is equal to that of G_3, the prediction of our model for the present value of the total density of the universe is too far away from the values suggested by recent measurements.

We have investigated the special case where the numerical value of G_4 is equal to that of G_3. We have seen that this does not produce meaningful numerical results. We now consider the possibility that the gravitational constant in $D = 4$ is different from the one in $D = 3$. To this end, we consider a general expression for G_4 as in the following:

$$G_4 = \frac{4l_{4,3}}{\pi} G_3, \tag{4.53}$$

and look for values that will produce observationally meaningful results. It is possible to see that the arguments in Eqs. (4.13), (4.16), (4.24) and (4.27) satisfy the following relations:

$$argm1 > argm2, \tag{4.54}$$

$$argm4 > argm3 > argm2, \tag{4.55}$$

which guarantee the correct time ordering of the various eras in the history of the universe, i.e., $0 < \eta_4 < \eta_r < \eta_m < \eta_{now}$. From Eqs. (4.21) and (4.54), we also see that $(4l_{4,3}/\pi l_{pl})$ which is a relation that we will use when we discuss the numerical results of our model.

Furthermore, $argm1$, $argm2$, $argm3$ and $argm4$ should be numbers smaller than or equal to unity. From Eqs. (4.13), (4.16) , it is easy to see that $argm1$ and $argm2$ satisfy this requirement. From Eqs. (4.24) and (4.27), we have the following conditions:

$$\frac{T_{pl}}{T_m} argm2 \leq 1, \tag{4.56}$$

$$\frac{T_{pl}}{\sqrt{T_m T_{now}}} argm2 \leq 1, \tag{4.57}$$

where we will consider only the second condition because it is more restrictive than the other one. Making use of Eq. (4.13) and re-arranging, we get

$$c_4 \leq -\frac{\left[127.22 - 0.87\left[\ln(T_m) + \frac{c_3}{c_3-1}\ln(\frac{4l_{4,3}}{\pi l_{pl}})\right] + 1.73\ln|1-\kappa^2|\right]}{\left[128.22 - 0.87\left[\ln(T_m) + \frac{c_3}{c_3-1}\ln(\frac{4l_{4,3}}{\pi l_{pl}})\right] + 1.73\ln|1-\kappa^2|\right]}, \tag{4.58}$$

which determines an upper limit for c_4. On the other hand, the lower bound is already known to be -1 from the inflationary character of the equation of state describing the universe in the first prematter era.

Then, using Eqs. (4.44), (4.47) and (4.48) we write

$$\rho(\eta_{now}) \leq 1.8544 \cdot 10^{-31} \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{-\frac{2}{3}} gr/cm^3. \tag{4.59}$$

Here, we conclude that $l_{4,3} \simeq l_{pl}$ to produce the results compatible with the observed figures of the density. In this case, from Eq. (4.53) we see that G_4 is at the order of $10^{-40} cm^4/gr\,\sec^2$. In addition to this, since $0 < \frac{c_3}{c_3-1} < \frac{1}{2}$, the inequality will not be affected too much by the choice of c_3. Thus, Eq. (4.58) reduces to

$$c_4 \lesssim -\frac{127.22 - 0.87\ln(T_m) + 1.73\ln[1-\kappa^2]}{128.22 - 0.87\ln(T_m) + 1.73\ln[1-\kappa^2]}, \tag{4.60}$$

and using the above mentioned numerical value for T_m, we get

$$c_4 \lesssim -\frac{120.10 + 1.73\ln[1-\kappa^2]}{121.10 + 1.73\ln[1-\kappa^2]}, \tag{4.61}$$

as the upper bound for c_4.

4.1.3 Numerical Results for Observable Parameters

In this model, in order to produce numerical results for the cosmological parameters of the universe one has to assign numerical values to the characteristic parameters of the model, i.e., c_3, c_4, $l_{4,3}$ and the initial value of the Hubble constant $H(0)$. Since these parameters are interconnected, some experimentation with numbers is necessary to produce meaningful results. In determining the critical parameters of the model we first consider the present value of the density predicted by our model, i.e.,

$$\rho(\eta_{now}) = 1.8544 \cdot T_m \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{2(2-c_3)}{3(2c_3-1)}} gr/cm^3. \tag{4.62}$$

It is evident that as the numerical value of the term $(4l_{4,3}/\pi l_{pl})$ gets far from unity, the present value of the density predicted in Eq. (14) falls rapidly below $10^{-30} gr/cm^3$, which represents the order of the present value of the total density. We also notice that within an interval very close to unity, the predictions of the model for the cosmological parameters of the universe do not change appreciably. We accordingly choose $(4l_{4,3}/\pi l_{pl}) = 1.1$, which gives $G_4 = 1.1862 \cdot 10^{-40}\ cm^4/gr\,\sec^2$.

Considering different values of κ, which is a measure of the initial expansion rate of the universe, we determine upper bounds for c_4 from Eq. (4.61). The lower bound is already known to be -1. Within these limiting values, in Tables 4.1-4.4 we give the ranges for c_4 in which we obtain observationaly meaningful results. On the other hand, we note that numerical results do not depend significantly on c_3. In order to see the dependence of the numerical results on the parameter c_4, we also present numerical results corresponding to the current values of Hubble parameter H_0, age t_0, density ρ_0. Although the results we obtain for density are in compliance with the recent measurements, the results for Hubble parameter and age are beyond the observed ranges, i.e., Hubble constant H_0 $(60\, km\ s^{-1}Mpc^{-1} - 80\, km\ s^{-1}Mpc^{-1})$, age t_0 $(1.2 \cdot 10^{10}\, yr - 1.6 \cdot 10^{10}\, yr)$ and density ρ_0 $(\sim 10^{-30}\ gr/cm^3)$.

Table 4.1: Results for $\kappa = 0$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$, $T_m = 2978\ K$.

$-0.9929 \leq c_4 \leq -0.9918$
$9.94\ km\ s^{-1} Mpc^{-1} \leq H_0 \leq 13.60\ km\ s^{-1} Mpc^{-1}$
$4.89 \times 10^{10}\ yr \leq t_0 \leq 5.70 \times 10^{10}\ yr$
$\rho_0 = 3.48 \times 10^{-31}\ gr\ cm^{-3}$

Table 4.2: Results for $\kappa = 0.3$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$, $T_m = 2978\ K$.

$-0.9929 \leq c_4 \leq -0.9918$
$10.66\ km\ s^{-1} Mpc^{-1} \leq H_0 \leq 13.60\ km\ s^{-1} Mpc^{-1}$
$4.32 \times 10^{10}\ yr \leq t_0 \leq 5.50 \times 10^{10}\ yr$
$\rho_0 = 3.48 \times 10^{-31}\ gr\ cm^{-3}$

Table 4.3: Results for $\kappa = 0.6$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$, $T_m = 2978\ K$.

$-0.9929 \leq c_4 \leq -0.9918$
$12.20\ km\ s^{-1} Mpc^{-1} \leq H_0 \leq 13.56\ km\ s^{-1} Mpc^{-1}$
$5.10 \times 10^{10}\ yr \leq t_0 \leq 6.22 \times 10^{10}\ yr$
$\rho_0 = 3.47 \times 10^{-31}\ gr\ cm^{-3}$

Table 4.4: Results for $\kappa = 0.9$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$, $T_m = 2978\ K$.

$-0.9928 \leq c_4 \leq -0.9916$
$9.67\ km\ s^{-1} Mpc^{-1} \leq H_0 \leq 13.45\ km\ s^{-1} Mpc^{-1}$
$4.04 \times 10^{10}\ yr \leq t_0 \leq 5.78 \times 10^{10}\ yr$
$\rho_0 = 3.44 \times 10^{-31}\ gr\ cm^{-3}$

4.2 Open Universe Case

In the previous section, we have seen that the numerical results that we obtained for today's value of Hubble parameter and age of the universe are beyond the ranges accepted by observations. We now consider the solutions of the field equations for the open geometry. We already know that the solution for the scale factor for open geometry is

$$a_D\left(\eta\right) = a_{0,D}\left[\sinh\left(c_D\eta + \delta_D\right)\right]^{1/c_D}. \tag{4.63}$$

Then, the scale factor in different eras will take the following forms:

$$a = \begin{cases} 1^{st} \text{ premater era:} \begin{cases} a_{0,4}^{(p)}[\sinh(c_4\eta_4 + \delta_4)]^{\frac{1}{c_4}}, \\ 0 \leq \eta \leq \eta_4, \\ P_4 = \left(\frac{2c_4-1}{2}\right)\rho_4. \end{cases} \\ 2^{nd} \text{ prematter era:} \begin{cases} a_0^{(p)}[\sinh(c_3\eta_4 + \delta_4)]^{\frac{1}{c_4}}; \\ \eta_4 \leq \eta \leq \eta_r, \\ P = \left(\frac{2c_3-1}{3}\right)\rho. \end{cases} \\ \text{radiation era:} \begin{cases} a_0^{(r)}[\sinh(\eta + \delta_r)], \\ \eta_r \leq \eta \leq \eta_m, \\ P = \frac{1}{3}\rho. \end{cases} \\ \text{matter era:} \begin{cases} a_0^{(m)}[\sinh(\frac{\eta}{2} + \delta_m)], \\ \eta_m \leq \eta, \\ P = 0. \end{cases} \end{cases} \tag{4.64}$$

The boundary condition that the scale factor and its derivative are continous at points η_4, η_r and η_m, allows us to determine the integration constants as

$$a_{0,4}^{(p)} = \left[\frac{d_4}{H_4^2(0) - d_4}\right]^{\frac{c4+1}{2c_4}} d_4^{-1/2}, \tag{4.65}$$

$$a_0^{(p)} = a_{0,4}^{(p)}[\sinh(c_4\eta_4 + \delta_4)\ ]^{\frac{1}{c_4} - \frac{1}{c_3}}, \tag{4.66}$$

$$a_0^{(r)} = a_0^{(p)}[\sinh(c_3\eta_r + \delta_3)\ ]^{\frac{1}{c_3} - 1}, \tag{4.67}$$

$$a_0^{(m)} = \frac{a_0^{(r)}}{\sinh(\eta_m + \delta_r)}, \tag{4.68}$$

$$d_4 = \frac{A_4 G_4}{3c^4}\rho_{pl,4}, \tag{4.69}$$

$$\delta_4 = \ln\sqrt{\frac{H_4(0) + \sqrt{H_4^2(0) - d_4}}{H_4(0) - \sqrt{H_4^2(0) - d_4}}}, \tag{4.70}$$

$$\delta_3 = \delta_4 + (c_4 - c_3)\,\eta_4, \tag{4.71}$$

$$\delta r = \delta_3 + (c_3 - 1)\,\eta_r, \tag{4.72}$$

$$\delta_m = \delta_r + \frac{\eta_m}{2}. \tag{4.73}$$

For open geometry, using conformal time we may write Eq. (2.3) as in the following form:

$$\frac{D(D-1)}{2}\left[\left(\frac{a'_D}{a_D^2}\right)^2 - 1\right] = \frac{2A_D G_D}{c^4}\rho_D, \tag{4.74}$$

which, when considered at $\eta = 0$ with $D = 4$, gives the following quadratic equation in $a_4^2(0)$:

$$d_4 a_4^2(0) + a_4^2(0) - v^2 = 0, \tag{4.75}$$

which has the following physical solution:

$$a_4(0) = \sqrt{\frac{\sqrt{1+4d_4 v^2}-1}{2d_4}}, \tag{4.76}$$

where v again represents the initial expansion rate of the universe, i.e.,

$$v = a_4'(0) > 0. \tag{4.77}$$

We may solve Eq. (4.74) to obtain the conformal time η_4 that gives the duration of the prematter era

$$\eta_4 = \ln\left[\frac{(3.1727)^{\frac{c_4}{2(1+c_4)}} + \sqrt{(3.1727)^{\frac{c_4}{2(1+c_4)}} + \frac{d_4}{H_4^2(0)-d_4}}}{\frac{H_4(0)+\sqrt{H_4^2(0)-d_4}}{\sqrt{H_4^2(0)-d_4}}}\right]^{\frac{1}{c_4}}. \tag{4.78}$$

This expression indicates also the conformal time at which dimensional reduction from $D=4$ to $D=3$ occurs. The conformal time corresponding the duration of the radiation era can be found from Eq. (4.14) as

$$\begin{aligned}\eta_r &= \frac{1}{c_3}\ln\left[\sqrt{1+\left(\frac{T_{pl}}{T_{pl,4}}\right)^{\frac{2c_3}{1-2c_3}}\sinh^2(c_3\eta_4+\delta_3)}\right. \\ &+ \left.\left(\frac{T_{pl}}{T_{pl,4}}\right)^{\frac{c_3}{1-2c_3}}\sinh(c_3\eta_4+\delta_3)\right] - \delta_3. \end{aligned} \tag{4.79}$$

Similarly, the duration of the matter dominated era and the age of the universe in conformal time can be obtained from Eqs. (4.22) and (4.95) respectively as

$$\eta_m = \left\{\ln\left[\frac{T_{pl}}{T_m}\sinh(\eta_3+\delta_r) + \sqrt{1+\left(\frac{T_{pl}}{T_m}\sinh(\eta_3+\delta_r)\right)^2}\right] - \delta_r\right\}, \tag{4.80}$$

$$\eta_{now} = 2\left\{\ln\left[\sqrt{\frac{T_m}{T_{now}}}\sinh(\eta_r+\delta_r) + \sqrt{\frac{T_m}{T_{now}}\sinh^2(\eta_r+\delta_r)+1}\right] - \delta_m\right\}. \tag{4.81}$$

4.2.1 Observable Parameters in the Model

Using Eqs. (3.10) and 4.63), one gets real time corresponding to conformal time as

$$t = a_{0,D} \int_0^{\eta} \left[\sinh\left(c_D \eta' + \delta_D\right)\right]^{1/c_D} d\eta', \tag{4.82}$$

where again we assumed that $t = 0$ at $\eta = 0$. As in the closed case, the integral in Eq. (4.82) depends upon the values of c_D and has to be computed numerically in the first and second prematter eras,respectively

$$t_4 = a_{0,4}^{(p)} \int_0^{\eta_4} \left[\sinh\left(c_4 \eta + \delta_4\right)\right]^{1/c_4} d\eta, \tag{4.83}$$

$$t_r = t_4 + a_0^{(p)} \int_{\eta_4}^{\eta_3} \left[\sinh\left(c_3 \eta + \delta_3\right)\right]^{1/c_3} d\eta. \tag{4.84}$$

Since $c_r = 1$ and $c_m = 1/2$ for radiation and matter eras, respectively, Eq. (4.82) can be integrated to yield analytical expressions as follows:

$$t_m = t_r + a_0^{(r)} \left[\cosh\left(\eta_m + \delta_r\right) - \cosh\left(\eta_r + \delta_r\right)\right], \tag{4.85}$$

$$t_{now} = t_m + \frac{a_0^{(m)}}{2} \left[\sinh\left(\eta_{now} + 2\delta_m\right) - \sinh\left(\eta_m + 2\delta_m\right) + \left(\eta_m - \eta_{now}\right)\right]. \tag{4.86}$$

Using Eq. (4.33), we get Hubble parameters at η_4, η_r, η_m and η_{now}, respectively

$$H(\eta_4) = \frac{\cosh\left(c_4 \eta_4 + \delta_4\right)}{a_{0,4}^{(p)} \left[\sinh\left(c_4 \eta_4 + \delta_4\right)\right]^{\frac{1}{c_4}+1}} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}, \tag{4.87}$$

$$H(\eta_r) = \frac{\cosh\left(c_3 \eta_r + \delta_3\right)}{a_0^{(p)} \left[\sinh\left(c_3 \eta_r + \delta_3\right)\right]^{\frac{1}{c_3}+1}} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}, \tag{4.88}$$

$$H(\eta_r) = \frac{\cosh(\eta_{now}/2 + \delta_r)}{a_0^{(r)} \sinh^2(\eta_m + \delta_r)} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}\ , \tag{4.89}$$

$$H(\eta_{now}) = \frac{\cosh(\eta_{now}/2 + \delta_m)}{a_0^{(m)} \cosh^3(\eta_{now}/2 + \delta_m)} \times 9.2503 \cdot 10^{29}\ km\ s^{-1} Mpc^{-1}\ . \tag{4.90}$$

As in the closed case, during dimensional reduction density has a discontinuity given by Eq. (4.39) and the expressions corresponding to densities at phase transitions can be expressed as

$$\rho_4(\eta_4) = \frac{\rho_{pl,4}}{3.1727}, \tag{4.91}$$

$$\rho(\eta_r) = \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{2(c_3+1)}{3(2c_3-1)}} \rho_3(\eta_4), \tag{4.92}$$

$$\rho(\eta_m) = \left(\frac{T_m}{T_{pl}}\right)^4 \rho(\eta_r), \tag{4.93}$$

$$\rho(\eta_{now}) = \left(\frac{T_{now}}{T_m}\right)^3 \rho(\eta_m). \tag{4.94}$$

4.2.2 Comparison with Observations

Firstly, we again consider the special case in which the numerical values of gravitational constants in different space dimensions are the same. For this case, from Eq. (4.94) the present value of the density can be obtained as

$$\rho(\eta_{now}) = 6.2269 \cdot 10^{-35} T_m \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{2(2-c_3)}{3(2c_3-1)}} gr/cm^3, \tag{4.95}$$

which is the same expression that we obtained in closed universe case. From the previous section, we know that this expression cannot produce observationally meaningful results for density. We now allow the numerical value of G_4 to differ from that of G_3 and write the relation between them as in the following:

$$G_4 = \frac{4l_{4,3}}{\pi} G_3. \tag{4.96}$$

For the recombination temperature previously specified as 2978 K, we end up with the same expression for density as in Eq. (4.59). Thus, we conclude that the requirement that $l_{4,3} \simeq l_{pl}$ to produce the results compatible with the observed figures of the density is also valid for the open geometry. Then, making use of Eq. (4.68) we may write the expression corresponding to the present value of the Hubble parameter in Eq. (4.90) as

$$\begin{aligned} H(\eta_{now}) = & \sqrt{\frac{d_4}{3.1727}\left[1 + \lambda^2 \frac{T_{pl}^2}{T_m T_{now}} \left(\frac{4l}{\pi l_{4,3}}\right)^{\frac{2c_3}{3(1-2c_3)}}\right]} \\ & \times \frac{\sqrt{T_m T_{now}^3}}{T_{pl}^2} \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{(c_3+1)}{3(2c_3-1)}} \times 9.2503 \cdot 10^{29} \ km\ s^{-1} Mpc^{-1}, \end{aligned} \tag{4.97}$$

where

$$\lambda = \frac{(3.1727)^{\frac{c_4}{2(1+c_4)}}}{\sqrt{\frac{d_4}{H_4^2(0) - d_4}}}. \tag{4.98}$$

Referring to Eq. (4.46), one may write the following inequalities:

$$\left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{-\frac{2}{9}} < \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{2c_3}{3(1-2c_3)}} < 1, \tag{4.99}$$

$$\left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{-\frac{1}{3}} < \left(\frac{4l_{4,3}}{\pi l_{pl}}\right)^{\frac{1+c_3}{3(2c_3-1)}} < 1, \tag{4.100}$$

and since $l_{4,3} \simeq l_{pl}$,we obtain

$$H(\eta_{now}) \simeq 10.73\sqrt{1 + 2.06 \cdot 10^{60} \lambda^2} \ km\ s^{-1} Mpc^{-1}. \tag{4.101}$$

Recent observations indicate that Hubble parameter is in the following range:

$$60\ km\ s^{-1}Mpc^{-1} \leq H(\eta_{now}) \leq 80\ km\ s^{-1}Mpc^{-1}. \tag{4.102}$$

Then, combining Eq. (4.101) and (4.102), we get

$$5.59 \lesssim \sqrt{1 + 2.06 \cdot 10^{60}\lambda^2} \lesssim 7.46. \tag{4.103}$$

Then, using Eqs. (4.45) and (4.98) we end up with two cases depending on the initial value of the expansion rate. These are:

$$\sqrt{1 + 2.06 \cdot 10^{60}\lambda^2} \lesssim 7.46, \tag{4.104}$$

which determines an upper bound for c_4 as

$$c_4 \lesssim -\frac{116.80 + 0.87\ln[\kappa^2 - 1]}{117.80 + 0.87\ln[\kappa^2 - 1]}, \tag{4.105}$$

and

$$\sqrt{1 + 2.06 \cdot 10^{60}\lambda^2} \gtrsim 5.59, \tag{4.106}$$

which sets a lower bound for c_4 as

$$c_4 \gtrsim -\frac{117.30 + 0.87\ln[\kappa^2 - 1]}{118.30 + 0.87\ln[\kappa^2 - 1]}. \tag{4.107}$$

4.2.3 Numerical Results for the Observable Parameters

Since the present value of the density is bounded by the same expression for both geometries, as we do in the closed universe case may choose the critical length parameter again as $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$. We again note that our model is insensitive to the choice of c_3, whereas it is sensitive to the value of c_4. In order to see the nature of this dependence, we choose as a particular case $\kappa = 1.1$. Upon this, we make use of Eqs. (4.105) and (4.107) which together set a range for c_4 in the light of recent Hubble constant measurements. For $\kappa = 1.1$, we find the corresponding range as $[-0.99145, -0.99141]$. The numerical results for the cosmological parameters (Hubble parameter, age, density and scale factor) for $c_4 = -0.99145$, -0.99144, -0.99143, -0.99142, -0.99141 are listed in Tables 4.5-4.9. In these tables, the first line correspond to the beginning of the universe. The other lines correspond to the transitions between different eras in the history of the universe. The discontinuity in density during dimensional reduction from $D = 4$ to $D = 3$ is also indicated in the third line. We see that the predictions of our cosmological model for this range are within the observed ranges for the cosmological parameters.

At this point, we note that the above range for the parameter c_4 is not the only range that produces numerical results in accord with the observational data. For different κ values corresponding to different initial expansion rates of the universe, from Eqs. (4.105) and (4.107) we obtain different ranges for c_4. For example, if we consider $\kappa = 10.0$, the range for c_4 that yields numerical results compatible with observations is $[-0.99182, -0.99179]$. Finally, for $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$ we give some Planck quantities in our model in Table 4.10.

Table 4.5: Results for $c_4 = -0.99145$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}$ cm, $T_m = 2978$ K.

$t\,(yr)$	$a\,(cm)$	$\rho\left(\frac{gr}{cm^3}\right)$	$H\left(\frac{km}{s \cdot mpc}\right)$
0	$1.42 \cdot 10^{-33}$	$2.72 \cdot 10^{126}$	$1.57 \cdot 10^{63}$
very small #	$2.99 \cdot 10^{-4}$	$8.58 \cdot 10^{125}$	$7.99 \cdot 10^{62}$
very small #	$3.07 \cdot 10^{-4}$	$1.20 \cdot 10^{93}$ $1.14 \cdot 10^{93}$	$7.44 \cdot 10^{62}$
$4.49 \cdot 10^{5}$	$1.22 \cdot 10^{25}$	$4.63 \cdot 10^{-22}$	$5.02 \cdot 10^{5}$
$1.58 \cdot 10^{10}$	$1.59 \cdot 10^{28}$	$2.06 \cdot 10^{-31}$	59.03

Table 4.6: Results for $c_4 = -0.99144$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}$ cm, $T_m = 2978$ K.

$t\,(yr)$	$a\,(cm)$	$\rho\left(\frac{gr}{cm^3}\right)$	$H\left(\frac{km}{s \cdot mpc}\right)$
0	$1.42 \cdot 10^{-33}$	$2.72 \cdot 10^{126}$	$1.57 \cdot 10^{63}$
very small #	$2.76 \cdot 10^{-4}$	$8.58 \cdot 10^{125}$	$7.99 \cdot 10^{62}$
very small #	$2.83 \cdot 10^{-4}$	$1.20 \cdot 10^{93}$ $1.14 \cdot 10^{93}$	$7.44 \cdot 10^{62}$
$4.48 \cdot 10^{5}$	$1.12 \cdot 10^{25}$	$4.63 \cdot 10^{-22}$	$5.03 \cdot 10^{5}$
$1.47 \cdot 10^{10}$	$1.47 \cdot 10^{28}$	$2.06 \cdot 10^{-31}$	63.73

Table 4.7: Results for $c_4 = -0.99143$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}$ cm, $T_m = 2978$ K.

$t\,(yr)$	$a\,(cm)$	$\rho\left(\frac{gr}{cm^3}\right)$	$H\left(\frac{km}{s \cdot mpc}\right)$
0	$1.42 \cdot 10^{-33}$	$2.72 \cdot 10^{126}$	$1.57 \cdot 10^{63}$
very small #	$2.55 \cdot 10^{-4}$	$8.58 \cdot 10^{125}$	$7.99 \cdot 10^{62}$
very small #	$2.62 \cdot 10^{-4}$	$1.20 \cdot 10^{93}$ $1.14 \cdot 10^{93}$	$7.44 \cdot 10^{62}$
$4.48 \cdot 10^{5}$	$1.04 \cdot 10^{25}$	$4.63 \cdot 10^{-22}$	$5.04 \cdot 10^{5}$
$1.37 \cdot 10^{10}$	$1.36 \cdot 10^{28}$	$2.06 \cdot 10^{-31}$	68.81

Table 4.8: Results for $c_4 = -0.99142$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$, $T_m = 2978\ K$.

$t\,(yr)$	$a\,(cm)$	$\rho\left(\frac{gr}{cm^3}\right)$	$H\left(\frac{km}{s \cdot mpc}\right)$
0	$1.42 \cdot 10^{-33}$	$2.72 \cdot 10^{126}$	$1.57 \cdot 10^{63}$
very small #	$2.36 \cdot 10^{-4}$	$8.58 \cdot 10^{125}$	$7.99 \cdot 10^{62}$
very small #	$2.42 \cdot 10^{-4}$	$1.20 \cdot 10^{93}$ $1.14 \cdot 10^{93}$	$7.44 \cdot 10^{62}$
$4.47 \cdot 10^{5}$	$9.60 \cdot 10^{24}$	$4.63 \cdot 10^{-22}$	$5.06 \cdot 10^{5}$
$1.27 \cdot 10^{10}$	$1.26 \cdot 10^{28}$	$2.06 \cdot 10^{-31}$	74.31

Table 4.9: Results for $c_4 = -0.99141$, $c_3 = -0.1$, $l_{4,3} = 1.3961 \cdot 10^{-33}\ cm$, $T_m = 2978\ K$.

$t\,(yr)$	$a\,(cm)$	$\rho\left(\frac{gr}{cm^3}\right)$	$H\left(\frac{km}{s \cdot mpc}\right)$
0	$1.42 \cdot 10^{-33}$	$2.72 \cdot 10^{126}$	$1.57 \cdot 10^{63}$
very small #	$2.18 \cdot 10^{-4}$	$8.58 \cdot 10^{125}$	$7.99 \cdot 10^{62}$
very small #	$2.24 \cdot 10^{-4}$	$1.20 \cdot 10^{93}$ $1.14 \cdot 10^{93}$	$7.44 \cdot 10^{62}$
$4.47 \cdot 10^{5}$	$8.88 \cdot 10^{24}$	$4.63 \cdot 10^{-22}$	$5.07 \cdot 10^{5}$
$1.18 \cdot 10^{10}$	$1.16 \cdot 10^{28}$	$2.06 \cdot 10^{-31}$	80.25

Table 4.10: Some Planck quantities and gravitational constant in our model for $D = 3$ and $D = 4$.

D	3	4
$G_D\ (cm^D/gr\,\mathrm{sec}^2)$	$6.6720 \cdot 10^{-8}$	$1.1862 \cdot 10^{-40}$
$l_{pl,D}(cm)$	$1.6160 \cdot 10^{-33}$	$1.6680 \cdot 10^{-33}$
$t_{pl,D}(\mathrm{sec})$	$5.3905 \cdot 10^{-44}$	$5.5648 \cdot 10^{-33}$
$\rho_{pl,D}(gr/cm^D)$	$5.1581 \cdot 10^{93}$	$2.7243 \cdot 10^{126}$
$T_{pl,D}(K)$	$1.4170 \cdot 10^{32}$	$1.3740 \cdot 10^{32}$

CHAPTER 5

DISCUSSION AND SUMMARY

We have constructed a nonsingular cosmological model for dimensional reduction with the assumption that the universe once had more than three space dimensions during the inflationary prematter phase and investigated the possibility that it underwent a dimensional reduction to lower and eventually to three spatial dimensions. The key ingredient to obtain a singularity-free model was the assumption that physical quantities are limited by their Planck values. In this model, inflation arises due to the form of the equation of state used to describe the perfect fluid filling the universe. And, heating is compatible with inflation in the sense that inflation stops in a given era as a result of a purely thermodynamical requirement which is the attainment of a maximum physical temperature; namely the Planck temperature corresponding to the number of space dimensions in a given era. In this way, different prematter eras in the history of the universe are linked to each other by first order phase transitions during which the universe is assumed to lower the number of space dimensions. The discontinuities across the boundaries manifested themselves in the forms of the equations of state used to describe consecutive eras.

Dimensional reduction may occur as generally believed in the form of compactification by some unknown effects which are probably of quantum-gravity character. In this work, we are not interested in the details of what happens during dimensional reduction; but in a way independent of the details of the mechanism governing dimensional reduction, we impose the boundary conditions that the scale factor and its first derivative are continuous accross the boundaries between D and $(D-1)$ dimensional prematter eras. The legitimacy of boundary conditions follows from the fact that we use inflationary "vacuum-like" equations of state ($P_D \approx -\rho_D$) to describe the constant D prematter eras and argue that the transition period is sufficiently smaller than the durations of the prematter eras. From the field Eqs. (3.7) and (3.8), it is obvious that as long as the scale factor and its derivative remain continuous during transition periods, the products $G_D\rho_D$ and G_DP_D, which drives the kinematics of the dimensional reduction, are continuous physical quantities accross the boundaries and neither G nor ρ (or P) is assumed to be a priori continuous physical quantity at the transitions.

In order to demonstrate the basic features of our cosmological model for both closed and open FRW geometries, we have considered a specific model which starts with a $D = 4$ prematter era and evolves into a $D = 3$ prematter era. In this numerical model, the magnitude of the discontinuities in G, ρ and P are represented by a critical length parameter $l_{4,3}$ which signals the dimensional reduction and which is shown to be one of the parameters of our cosmological model. Indeed, from the definition given in Eq. (3.25) alone, the critical length parameter can also be interpreted as the size or volume in which the extra dimension continues to exist. It is of considerable importance to conclude that the critical length parameter has to approach the Planck length of the lower dimension ($D = 3$) if the later epochs are to yield the present features of the universe. The possibility that the universe evolves with a gravitational constant that remains numerically the same in all D is ruled out by showing that in such a case the critical length parameter takes a value at the order of 1 cm and the model predicts a density that is far beyond the recent density measurements.

Although we neither propose nor give preference to any mechanism to explain the details of how the extra dimensions became so small that they are currently unobservable, one may think of the critical length parameter as the Kaluza-Klein radius which represents the size of the compact extra dimension in the five-dimensional Kaluza-Klein models with inflation (see e.g. Abbott, Barr & Ellis 1984 and references therein). In this work, we derive the size of the extra dimension from purely observational constraints, instead of assuming a priori. In this way, it is shown that the current size of extra dimension, which is the most important relic of a dimensional reduction in the past of the universe, has a crucial observational effect on its future evolution.

Moreover, the specific model we have presented here has the property of being parametric in the sense that the numerical results are heavily dependent on the values of the parameters c_4 and κ which determine the vacuum dominance of the first prematter era and the initial expansion rate of the universe, respectively. We should expect that higher dimensions present a flatness problem like the one which is present in traditional big bang models of the universe. In our model, the flatness problem is solved by carrying the universe to a nearly flat geometry with the help of a vacuum like equation of state used in the first prematter era. Since the geometry of the universe is already made flat at the beginning of the second prematter era, the existence of the second prematter era should not have a significant effect on the observable parameters of the universe. As expected, the numerical results about the present properties of the universe reflect this fact. They are hardly dependent on the value of c_3 which represents the vacuum dominance of the second prematter era. It should be noted that for a closed geometry the predictions of our model for the present properties of the universe are not in agreement with the currently accepted range for Hubble parameter and age. On the other hand, for an open geometry the predictions of our model for the present values of the cosmological parameters comply with observations for a broad range of initial conditions. We believe that this feature of the model could shed light on discussions centering on the question concerning the topological character of space by favoring an open geometry if our universe once experienced a dimensional reduction in its past.

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