OFDM PAPR REDUCTION WITH LINEAR CODING AND CODEWORD MODIFICATION

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ABSTRACT

OFDM PAPR REDUCTION WITH LINEAR CODING AND CODEWORD MODIFICATION

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In this thesis, reduction of the Peak-to-Average Power Ratio (PAPR) of Orthogonal Frequency Division Multiplexing (OFDM) is studied. A new PAPR reduction method is proposed that is based on block coding the input data and modifying the codeword until the PAPR is reduced below a certain threshold. The method makes use of the error correction capability of the block code employed.

The performance of the algorithm has been investigated through theoretical models and computer simulations. For performance evaluation, a new gain parameter is defined. The gain parameter considers the SNR loss caused by modification of the codeword together with the PAPR reduction achieved. The gain parameter is used to compare a plain OFDM system with the system employing the PAPR reduction algorithm. The algorithm performance is examined through computer simulations and it is found that power reductions around 2-3 dB are obtained especially for low to moderate number of channels and relatively strong codes.

Keywords: OFDM, PAPR, block code, SNR

ÖZ

DOĞRUSAL KODLAMA VE KODLANMIŞ VERİ DEĞİŞTİRİLMESİ YÖNTEMİYLE OFDM İÇİN TEPE GÜÇ/ORTALAMA GÜÇ ORANI AZALTIMI

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Bu tezde, Dikgen Sıklık Bölümlü Çoğullama (OFDM) yönteminde Tepe Güç/Ortalama Güç Oranı (PAPR) düşürülmesi üzerine çalışılmıştır. Giriş verisinin blok kodlanması ve kodlanmış verinin PAPR'ı belirli bir eşik seviyesinin altına düşene kadar değiştirilmesi temeline dayanan yeni bir PAPR azaltım metodu önerilmiştir. Metod, uygulanan blok kodun hata düzeltme yeteneğinden faydalanır.

Algoritmanın performansı teorik modeller ve bilgisayar benzetimleri aracılığıyla incelenmiştir. Performans değerlendirmesi için yeni bir kazanç parametresi tanımlanmıştır. Kazanç parametresi, kodlanmış verinin modifikasyonundan kaynaklanan SNR kaybı ile birlikte başarılan PAPR azaltımını da dikkate alır. Kazanç parametresi, basit OFDM sistemi ile PAPR düşürme algoritmasını kullanan sistemi karşılaştırmak için kullanılmıştır. Algoritmanın performansı bilgisayar benzetimleri ile incelenmiştir ve ortalama kanal sayıları ile nispeten güçlü kodlar için 2-3 dB civarında güç azaltımının elde edildiği bulunmuştur.

Anahtar Kelimeler: Dikgen Sıklık Bölümlü Çoğullama (OFDM), Tepe Güç/Ortalama Güç Oranı (PAPR), blok kod, Sinyal-Gürültü Oranı (SNR)

To Evrim

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LIST OF ABBREVIATIONS

MCM	Multicarrier Modulation
OFDM	Orthogonal Frequency Division Multiplexing
SNR	Signal-to-Noise Ratio
ICI	Intercarrier Interference
ISI	Intersymbol Interference
FFT	Fast Fourier Transform
IFFT	Inverse Fast Fourier Transform
DFT	Discrete Fourier Transform
IDFT	Inverse Discrete Fourier Transform
PAPR	Peak-to-Average Power Ratio
DAB	Digital Audio Broadcasting
DVB	Digital Video Broadcasting
EHF	Extremely High Frequency
BER	Bit Error Rate
SLM	Selected Mapping
PTS	Partial Transmit Sequences
pdf	Probability Density Function

CHAPTER 1

INTRODUCTION

Multicarrier Modulation (MCM) is the principle of transmitting data by dividing the input data stream into several lower rate parallel bit streams and using these substreams to modulate several carriers. The first systems using MCM were military HF radio links in 1960s. In a classical MCM system, the total signal frequency band is divided into N nonoverlapping frequency subchannels. Each subchannel is modulated with a separate symbol and then the N subchannels are frequency multiplexed. Spectral overlap of channels is avoided to eliminate interchannel interference. However, this leads to inefficient use of the available spectrum. To cope with the inefficiency, the ideas proposed from the 1960s were to use MCM and Frequency Division Multiplexing (FDM) with overlapping subchannels. Orthogonal Frequency Division Multiplexing (OFDM) is a special case of multicarrier transmission. The word orthogonal indicates that there is a mathematical relationship between the frequencies of carriers in the system. In a normal MCM system, many carriers are spaced apart in such a way that the signals can be received using conventional filters and demodulators. In such receivers, guard bands are introduced between different carriers and in frequency domain which results in reduction of spectrum efficiency. In OFDM, the carriers are arranged such that the frequency spectrum of the individual carriers overlap and the signals are still received without adjacent carrier interference. In order to achieve this, the carriers are chosen to be mathematically orthogonal. The receiver acts as a bank of demodulators, translating each carrier down to DC and integrating the resulting signal over a symbol period to obtain raw data. In 1971, Weinstein and Ebert [3] showed that OFDM waveforms can be generated using a Discrete Fourier Transform (DFT) at the transmitter and receiver for the modulation and demodulation.

For a long time, usage of OFDM in practical systems was limited. Main reasons for this limitation were the complexity of real time Fourier Transform and the linearity required in RF power amplifiers. However since 1990s, OFDM is used for wideband data communications over mobile radio FM channels, High-bit-rate Digital Subscriber Lines (HDSL, 1.6Mbps), Asymmetric Digital Subscriber Lines (ADSL, up to 6Mbps), Very-high-speed Digital Subscriber Lines (VDSL, 100Mbps), Digital Audio Broadcasting (DAB), and High Definition Television (HDTV) terrestrial broadcasting.

OFDM has many advantages over single carrier systems. The implementation complexity of OFDM is significantly lower than that of a single carrier system with equalizer. When the transmission bandwidth exceeds coherence bandwidth of the channel, resultant distortion may cause intersymbol interference (ISI). Single carrier systems solve this problem by using a linear or nonlinear equalization. The problem with this approach is the complexity of effective equalization algorithms. OFDM systems divide available channel bandwidth into a number of subchannels. By selecting the subchannel bandwidth smaller than the coherence bandwidth of the frequency selective channel, the channel appears to be nearly flat and no equalization is needed. Also by inserting a guard time at the beginning of OFDM symbol during which the symbol is cyclically extended, intersymbol interference (ISI) and intercarrier interference (ICI) can be completely eliminated, if the duration of guard period is properly chosen. This property of OFDM makes the single frequency networks possible. In single frequency networks, transmitters simultaneously broadcast at the same frequency, which causes intersymbol interference. Additionally, in relatively slow time varying channels, it is possible to significantly enhance the capacity by adapting the data rate per subcarrier according to the signal-to-noise ratio (SNR) of that particular subcarrier. Another advantage of OFDM over single carrier systems is its robustness against narrowband interference because such interference affects only a small percentage of the subcarriers.

Beyond all these advantages, OFDM has some drawbacks compared to single carrier systems. Two of the problems with OFDM are the carrier phase noise and frequency offset. Carrier phase noise is caused by imperfections in the transmitter and receiver oscillators. Frequency offsets are created by differences between oscillators in transmitter and receiver, Doppler shifts, or phase noise introduced by nonlinear channels. There are two destructive effects caused by a carrier frequency offset in an OFDM system. One is the reduction of signal amplitude since sinc (.) functions are shifted and no longer sampled at the peak, and the other is the introduction of ICI from the other carriers. The latter is caused by the loss of orthogonality between the subchannels. Sensitivity to phase noise and frequency offsets increases with the number of subcarriers and with the constellation size used for subcarrier modulation. For single carrier systems, phase noise and frequency offsets only give degradation in the receiver SNR, rather than introducing ICI. That is why the sensitivity to frequency offsets and phase noise are mentioned as disadvantages of OFDM relative to single carrier systems.

The most important disadvantage of OFDM systems is that highly linear RF amplifiers are needed. An OFDM signal consists of a number of independently modulated subcarriers, which can give a large Peak-to-Average Power Ratio (PAPR) when added up coherently. When N signals are added with the same phase, they produce a peak power that is N times the average power. In order to avoid nonlinear distortion, highly linear amplifiers are required which cause a severe reduction in power efficiency. Several methods are explained in the literature in order to solve this problem.

Most widely used methods are *clipping* and *peak windowing* the OFDM signal when a high PAPR is encountered. However these methods distort the original OFDM signal resulting in an increase in the bit error probability. There are other methods that do not distort the signal. Two of these methods can be listed as *Partial Transmit Sequences* [5,6] and *Selected Mapping* [7,8]. The principle behind these methods is to transmit the OFDM signal with the lowest PAPR value among

a number of candidates all of which represent the same information. *Coding* is another commonly used method. In this case the information bits are coded in a way that no high peaks are generated. Mainly Golay codes, Reed Muller codes and linear block codes are used [9,10,11,12].

In this thesis a method based on coding is proposed to reduce the PAPR value of an OFDM signal. The method uses linear block coding and the error correction capability of the code. In the literature there are a number of similar methods proposed to reduce the PAPR value. Their performance is evaluated according to the final PAPR value achieved only. However, some of the methods result in an increase in Bit Error Rate (BER) and some of the methods require transmission of side information on a secure channel for the correct reception of the transmitted data. These effects need to be considered for accurate performance calculations. Hence, there is a deficiency in the performance evaluation criterion. This thesis provides a means of performance evaluation of a PAPR reducing algorithm.

This thesis is organized as follows. Chapter 2 is a rather detailed overview of OFDM. Main equations are derived and main techniques of OFDM such as windowing, guard time & cyclic prefixing, synchronization, bit loading and PAPR are explained. Afterwards, applications of OFDM are discussed.

Chapter 3 provides a deep insight into the PAPR problem by discussing the methods used in the literature. Block diagram of each method is provided; their advantages and disadvantages are discussed.

In Chapter 4, the PAPR reduction method suggested in the thesis is explained. Detailed block diagrams and some MATLAB outputs are given to clarify how the algorithm works.

Chapter 5 explains the second objective of the thesis: We suggest a method to compare the algorithm implementing transmission system with a plain OFDM

modulating system under a very low clipping rate condition. The SNR loss caused by the algorithm is taken into account and the systems are compared for the case of equal information bit error probabilities. Derivations of the performance comparison equations are provided in this chapter.

Chapter 6 contains the graphs explaining system behavior. Additionally, this chapter provides the numerical results obtained by algorithm implementation and system comparison.

Finally, Chapter 7 includes some concluding remarks.

CHAPTER 2

OVERVIEW OF OFDM

In this chapter, first the general structure of an OFDM system will be mentioned. The second part will be about main techniques of OFDM. Finally, applications of OFDM will be examined.

2.1. General Structure of OFDM

The basic principle of OFDM is to split a high rate input data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the transmission rate is slower in parallel subcarriers, a frequency selective channel appears to be flat to each subcarrier. ISI is eliminated almost completely by adding a guard interval at the beginning of each OFDM symbol. However, instead of using an empty guard time, this interval is filled with a cyclically extended version of the OFDM symbol. This method is used to avoid ICI.

OFDM is a special case of Multicarrier Modulation (MCM). In MCM, input data stream is divided into lower rate substreams, and these substreams are used to modulate several subcarriers. In general, the spacing between these subcarriers is large enough such that individual spectrum of subcarriers do not overlap. Therefore the receiver uses a bandpass filter tuned to that subcarrier frequency in order to demodulate the signal. In OFDM, subcarrier spacing is kept at minimum, while still preserving the time domain orthogonality between subcarriers, even though the individual frequency spectrum may overlap. The minimum subcarrier spacing should equal to 1/T, where T is the symbol period.

An OFDM symbol in baseband is defined as:

$$x(t) = \sum_{i=-N/2}^{N/2-1} a_{i+N/2} \exp(j\frac{2\pi it}{T}) s(t) \qquad 0 \le t \le T, \tag{2.1}$$

where, $a_{i+N/2}$ denotes the complex symbol modulating the i-th carrier, s(t) is the time window function defined in the interval [0,T], N is the number of subcarriers, and T is the OFDM symbol period. Subcarriers are spaced $\Delta f=1/T$ apart. The correlation coefficient between the subcarriers may be defined as:

$$\rho_{kn} = \frac{1}{T}\int_0^T \exp(j\frac{2\pi kt}{T}) \exp(-j\frac{2\pi nt}{T}) dt \tag{2.2}$$

As can be seen from (2.2),

$$\rho_{kn} = \begin{cases} 1, & n = k \\ 0, & n \ne k \end{cases} \tag{2.3}$$

Therefore, OFDM signal of the form (2.1) satisfies the condition of mutual orthogonality between subcarriers in the symbol interval. In order to obtain the data modulating the k-th subcarrier OFDM symbol should be downconverted with a frequency of k/T, and then integrated over the symbol period [1]. We can assume that s(t) is a rectangular window defined in [0,T] for simplicity. Then above operation may be shown as:

$$\text{Resultant signal} = \frac{1}{T}\int_0^T \exp(-j\frac{2\pi kt}{T}) \sum_{i=-N/2}^{N/2-1} a_{i+N/2} \exp(j\frac{2\pi it}{T}) dt$$

$$= \sum_{i=-N/2}^{N/2-1} a_{i+N/2} \frac{1}{T}\int_0^T \exp(-j\frac{2\pi kt}{T}) \exp(j\frac{2\pi it}{T}) dt\,. \tag{2.4}$$

Using (2.3) and (2.4) together,

Resultant signal= $a_{k+N/2}$.

Considering the sequential transmission of symbols, the baseband signal at the OFDM modulator output can be expressed as:

$$x(t) = \sum_{n} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,n} \exp(j\frac{2\pi i(t-nT)}{T}) s(t-nT), \tag{2.5}$$

where $a_{i+N/2,n}$ represents the data modulating the i-th carrier of the n-th OFDM symbol. The n-th OFDM symbol is transmitted in the time interval [nT, nT+T]. Assuming that the windowing function s(t) is nonzero only in [0,T] interval, if N samples are taken from x(t) at time instants {nT+kT/N, k=0…N-1}, the result will be [1]:

y[k]=x(nT+kT/N)

$$= \sum_{n} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,n} \exp(j\frac{2\pi i(nT+kT/N-nT)}{T}) s(nT+kT/N-nT)$$

$$= \sum_{i=-N/2}^{N/2-1} a_{i+N/2,n} \exp(j\frac{2\pi ik}{N}) s(kT/N), \quad 0 \le \text{k} \le \text{N-1}. \tag{2.6}$$

It can be seen from (2.6) that, N samples taken from n-th OFDM symbol at a rate of N/T can be obtained by taking N-point inverse Discrete Fourier Transform (IDFT) of the input data $a_{k,n}$, k=0…N-1, weighted by the window function s(t) at the sampling time instants.

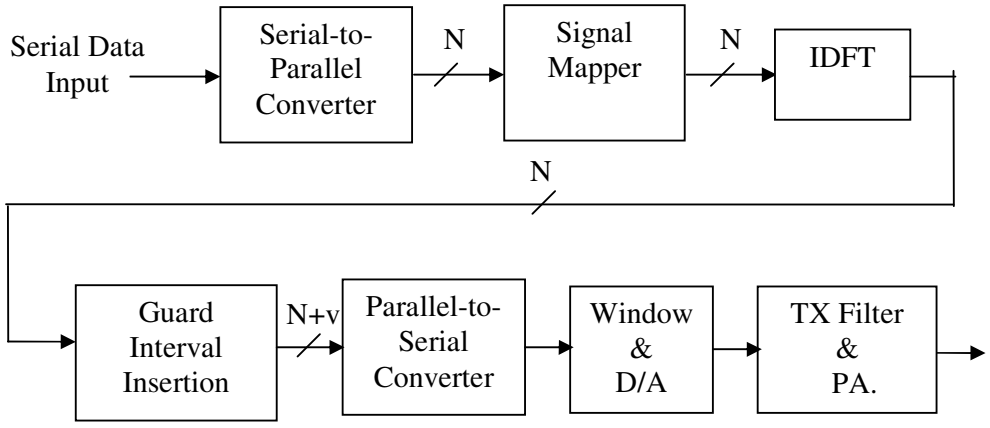

Figure 2.1 Block Diagram of an OFDM Transmitter

Figure 2.1 shows a basic OFDM transmitter structure. The serial input data stream is divided into frames of N_f bits. These N_f bits are arranged into N groups, where N is the number of subcarriers. The number of bits in each of the N groups determines the constellation size for that particular subcarrier. For example, if all the subcarriers are modulated by QPSK then each of the groups consists of 2 bits, if 16-QAM modulation is used each group contains 4 bits. This scheme is called as *fixed loading*. However, this is not the only way of distributing input bits among the subchannels. N_f bits could be divided among subcarriers according to the channel states. Therefore, one of the subcarriers can be modulated with 16-QAM whereas another one can be modulated with 32-QAM, etc. In this case, the former subcarrier consists of 4 bits and the latter subcarrier consists of 5 bits. This scheme is named as *adaptive loading*. Hence, OFDM can be considered as N independent QAM channels, each having a different QAM constellation but each operating at the same symbol rate 1/T. After signal mapping, N complex points are obtained. These complex points are passed through an IDFT block. Cyclic prefix of length v is added to the IDFT output in order to combat with ICI and ISI. After Parallel–to-Serial conversion, windowing function is applied. The output is fed into a Digital-

to-Analog converter operating at a frequency of N/T. Finally transmit filter is applied in order to provide necessary spectrum shaping before power amplification.

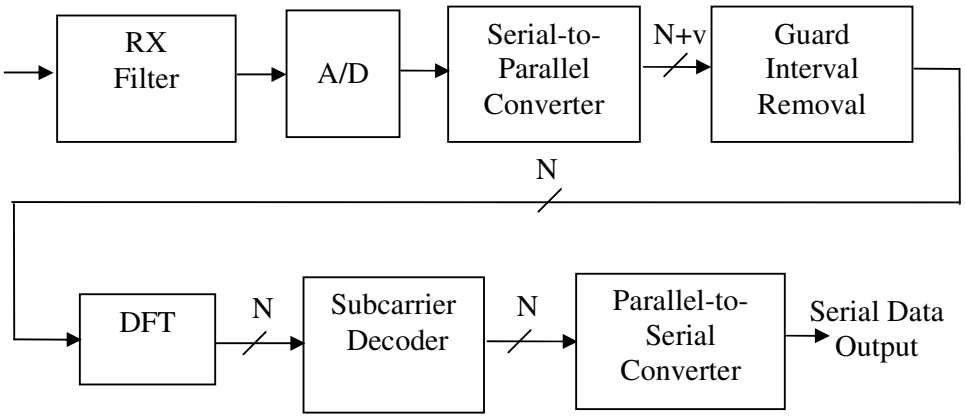

Figure 2.2 Block Diagram of an OFDM Receiver

Figure 2.2 gives the block diagram of an OFDM receiver. The receiver implements inverse operations of the transmitter. Received signal is passed through a receive filter at passband and an Analog-to-Digital converter operating at a frequency of N/T. After these downconverting and sampling operations, cyclic prefix is removed from the signal and a DFT operation is performed on the resultant complex points in order to demodulate the subcarriers. Subcarrier decoder converts obtained complex points to the corresponding bit stream.

2.2. Basic Techniques in OFDM

2.2.1. Windowing

An OFDM signal consists of a number of unfiltered QAM subcarriers. As a result, the out-of-band spectrum decreases slowly. Therefore, the windowing function

used at the transmitter should be selected in order to avoid the intercarrier interference (ICI) among subcarriers. The spectrum of the baseband OFDM signal given in (2.1) can be written as:

$$X(f) = \sum_{i=-N/2}^{N/2-1} a_{i+N/2} \int_{-\infty}^{\infty} \exp(-j2\pi ft) \exp(j\frac{2\pi it}{T}) s(t) dt$$

$$= \sum_{i=-N/2}^{N/2-1} a_{i+N/2} \int_{-\infty}^{\infty} \exp(-j2\pi (f - \frac{i}{T})t) s(t) dt$$

$$= \sum_{i=-N/2}^{N/2-1} a_{i+N/2} S(f - \frac{i}{T}), \tag{2.7}$$

where S(f) is the Fourier transform of s(t). As calculated in (2.1), a similar correlation coefficient can be written and it can be stated that the frequency domain function S(f) should satisfy

$$S(\frac{k-n}{T}) = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases} . \tag{2.8}$$

S(f) could be chosen as sinc(fT) which satisfies the above condition. Then, s(t) is a rectangular window. The problem about using a rectangular time window is the high out-of-band radiation since the tails of S(f) decreases only with 1/f. An alternative should be a raised cosine time window to reduce out-of-band radiation. In this case, the time window becomes

$$s(t) = \begin{cases} \frac{1}{2}\left[1 + \cos(\pi + \frac{\pi t}{\beta T})\right], 0 \le t \le \beta T \\ 1, \beta T \le t \le T \\ \frac{1}{2}\left[1 + \cos(\frac{\pi (t-T)}{\beta T})\right], T \le t \le (1+\beta)T \end{cases} \tag{2.9}$$

where $0 \le \beta \le 1$ is the roll-off factor and T is the symbol interval, which is shorter than the total OFDM symbol duration since we allow adjacent symbols to partially overlap in the roll-off region. The spectrum, S(f), can be expressed as

$$S(f) = \sin c(fT)\frac{\cos(\pi\beta fT)}{1-4\beta^2 f^2 T^2} \tag{2.10}$$

which satisfies the no ICI condition stated in (2.8). The advantage of raised cosine window over rectangular window is the faster decay rate of the tails of spectrum. Tails decay with $1/f^3$, which considerably reduces the out-of-band radiation. However, as mentioned before, there is an overlap region between adjacent OFDM frames, which increases the multipath delay sensitivity.

2.2.2. Guard Time and Cyclic Prefix

When the OFDM signal is passed through a multipath fading channel, the channel may introduce intersymbol interference (ISI) and intercarrier interference (ICI) to the signal. These effects should be removed from the signal in order to receive correct data at the demodulator. To examine the effects of multipath delay, assume that the channel impulse response is given by

$$h(t) = \sum_{l=0}^{L} h_l \delta(t - \frac{lT}{N}) \tag{2.11}$$

where L is the number of taps, h_l's are the mutually uncorrelated zero-mean complex Gaussian tap weight coefficients. Equation (2.11) is called the Tapped Delay Line (TDL) model of a frequency selective slowly fading channel [13]. Tap spacing is T/N, which is the reciprocal of the two sided bandwidth of the OFDM signal. For simplicity, assume that the channel remains constant during consecutive OFDM symbols and maximum delay LT/N is less than OFDM symbol period T, which guarantees that the intersymbol interference (ISI) on a symbol is only

limited by the previous symbol. The signal at the receiver's correlator input, assuming no noise, can be shown as

$$r(t) = \sum_{l=0}^{L} h_l x(t - \frac{lT}{N}), \tag{2.12}$$

where the transmitted OFDM symbol $x(t)$ can be represented as

$$x(t) = \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m} \exp(j \frac{2\pi i(t-mT)}{T}) s(t-mT). \tag{2.13}$$

At the receiver side, received signal is correlated with the complex exponential at the k^{th} subcarrier frequency over the interval [nT, nT+T] in order to demodulate the k^{th} subcarrier of the n^{th} OFDM symbol. Therefore, the correlator output can be represented as [1]

$$I_{k,n} = \frac{1}{T} \int_{nT}^{nT+T} r(t) \exp(-j \frac{2\pi k(t-mT)}{T}) dt. \tag{2.14}$$

Using (2.12) and (2.13), $I_{k,n}$ can be rewritten as

$$I_{k,n} = \sum_{m} \sum_{l=0}^{L} h_l \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m}$$
$$\frac{1}{T} \int_{nT}^{nT+T} \exp(j \frac{2\pi i(t-mT-lT/N)}{T}) \exp(-j \frac{2\pi k(t-mT)}{T}) s(t-mT-lT/N) dt$$

Changing the integration limits to [0,T] by replacing t with t+NT and rearranging the above equation

$$I_{k,n} = \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m} \sum_{l=0}^{L} h_l e^{-j2\pi il/N}$$
$$\frac{1}{T}\int_0^T \exp(j\frac{2\pi i(t+(n-m)T)}{T}) \exp(-j\frac{2\pi k(t+(n-m)T)}{T}) s(t+(n-m)T-lT/N)dt$$

(2.15)

where s(t) is the rectangular window defined on interval [0,T]. It can be seen from (2.15) that, both m=n and m=n-1 indexes affect the summation over m. After some manipulations, (2.15) can be written in the form

$$I_{k,n} = (\sum_{l=0}^{L} h_l(1-l/N)e^{-j2\pi kl/N}) a_{k+N/2,n}$$
$$+ \sum_{i,i\neq k} a_{i+N/2,n} \sum_{l=0}^{L} h_l e^{-j2\pi il/N} \frac{1}{T} \int_{lT/N}^{T} \exp(j\frac{2\pi(i-k)t}{T})dt$$
$$+ \sum_{i} a_{i+N/2,n-1} \sum_{l=0}^{L} h_l e^{-j2\pi il/N} \frac{1}{T} \int_{T-lT/N}^{T} \exp(j\frac{2\pi(i-k)t}{T})dt \quad . \qquad (2.16)$$

In (2.16), the first term is the desired data, scaled by a complex number depending on the channel taps. The second term represents the intercarrier interference (ICI) caused by the other subcarriers belonging to the current OFDM symbol. Finally, the third term represents the intersymbol interference (ISI) caused by the subcarriers of previous OFDM symbol.

ISI can be avoided by insertion of a guard time at the beginning of each OFDM symbol. Guard time should be longer than the maximum possible delay spread of the channel. Let T_g be the length of guard interval, insertion of the guard interval may be seen in Figure 2.3.

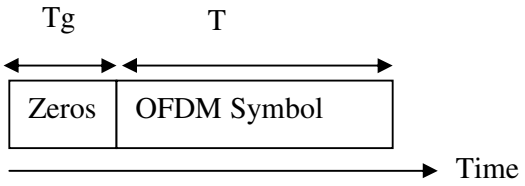

Figure 2.3 Insertion of Guard Interval

In this case $T_s = T_g + T$ becomes the new OFDM symbol time. The m[th] OFDM symbol is defined in $[mT_s, (m+1)T_s]$ interval and the symbol information is contained in $\lfloor mT_s + T_g, (m+1)T_s \rfloor$ interval. Therefore OFDM signal is defined as [1]

$$x(t) = \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m} \exp(j\frac{2\pi i(t - mT_s - T_g)}{T}) s(t - mT_s - T_g) \qquad (2.17)$$

where s(t) is the rectangular window defined over interval [0,T]. Hence n[th] OFDM symbol can be represented as

$$x_m(t) = \begin{cases} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m} \exp(j\frac{2\pi i(t - mT_s - T_g)}{T}), mT_s + T_g \le t \le (m+1)T_s \\ 0, mT_s \le t \le mT_s + T_g \end{cases} . \quad (2.18)$$

The correlator output for the k[th] subcarrier of the n[th] OFDM symbol is given by

$$I_{k,n} = \sum_{m} \sum_{l=0}^{L} h_l \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m}$$
$$\frac{1}{T} \int_{nT_s+T_g}^{(n+1)T_s} \{\exp(j\frac{2\pi i(t - mT_s - T_g - lT/N)}{T}) \exp(-j\frac{2\pi k(t - mT_s - T_g)}{T})$$
$$s(t - mT_s - T_g - lT/N)dt\}$$

$$= \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m} \sum_{l=0}^{L} h_l e^{-j2\pi il/N} \frac{1}{T} \int_0^T \{\exp(j\frac{2\pi i(t+(n-m)T_s)}{T})$$
$$\exp(-j\frac{2\pi k(t+(n-m)T_s)}{T}) s(t+(n-m)T_s - lT/N)dt\} \quad . \tag{2.19}$$

In this case there is no ISI and only the index m=n involves the sum. When s(t) is a rectangular window defined over [0,T], we obtain

$$I_{k,n} = \left(\sum_{l=0}^{L} h_l (1-l/N) e^{-j2\pi kl/N} \right) a_{k+N/2,n} +$$
$$\sum_{i,i\neq k} a_{i+N/2,n} \sum_{l=0}^{L} h_l e^{-j2\pi il/N} \frac{1}{T} \int_{lT/N}^{T} \exp(j2\pi(i-k)/T)dt \tag{2.20}$$

The first term in (2.20) represents the desired symbol scaled by a complex number depending on the channel tap coefficients. The second term is the intercarrier interference (ICI) caused by other subcarriers of the same OFDM symbol. Therefore, it can be stated that, inserting a guard time between OFDM symbols, during which no signal is transmitted, only the ISI can be avoided, ICI is still present.

In order to avoid ICI, the last part of the OFDM symbol can be added to the beginning of the symbol. This part is called the *cyclic prefix*. Insertion of a cyclic prefix to the OFDM symbol can be represented as in Figure 2.4.

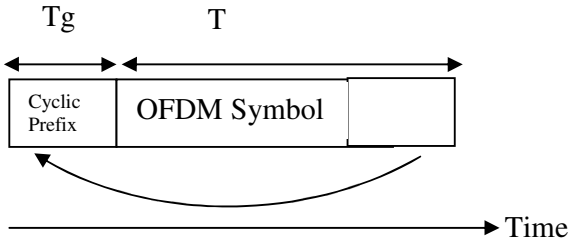

Figure 2.4 Insertion of Cyclic Prefix

In this case, transmitted OFDM signal can be expressed as

$$x(t) = \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m} \exp(j\frac{2\pi i(t - mT_s - T_g)}{T}) s(t - mT_s) \tag{2.21}$$

where s(t) is the rectangular window defined over $[0,T_s]$. Hence the n^{th} OFDM symbol is given by

$$x_n(t) = \sum_{i=-N/2}^{N/2-1} a_{i+N/2,n} \exp(j\frac{2\pi i(t - nT_s - T_g)}{T}), nT_s \le t \le (n+1)T_s \,. \tag{2.22}$$

Then the correlator output for the k^{th} subcarrier of the n^{th} OFDM symbol is

$$\begin{aligned} I_{k,n} = \sum_{m} \sum_{i=-N/2}^{N/2-1} a_{i+N/2,m} \sum_{l=0}^{L} h_l e^{-j2\pi il/N} \frac{1}{T} \int_0^T \{\exp(j\frac{2\pi i(t + (n-m)T_s)}{T}) \\ \exp(-j\frac{2\pi k(t + (n-m)T_s)}{T}) s(t + (n-m)T_s + T_g - lT/N) dt\} \end{aligned} \tag{2.23}$$

Since the rectangular window is defined over $[0,T_s]$, the correlator output can be expressed as

$$\begin{aligned} I_{k,n} &= \sum_{l=0}^{L} h_l e^{-j2\pi kl/N} a_{k+N/2,n} + \\ &\sum_{i,i\neq k} a_{i+N/2,n} \sum_{l=0}^{L} h_l e^{-j2\pi il/N} \frac{1}{T} \int_0^T \exp(j2\pi(i-k)/T) dt \end{aligned}$$

$$= \left(\sum_{l=0}^{L} h_l e^{-j2\pi kl/N} \right) a_{k+N/2,n} \tag{2.24}$$

which contains only the desired symbol, free from intercarrier interference (ICI) and intersymbol interference (ISI). Therefore, by inserting a guard time longer than the maximum delay spread of the channel and by cyclically extending the OFDM

symbol over the guard time eliminates both the ICI and ISI completely and the channel appears to be flat fading for each subcarrier.

If we take $N+\nu$ samples from $x_n(t)$ given by (2.22) at a sampling rate of $(N+\nu)/T_s$ where $\nu = NT_g/T$ (T_g is selected properly in order to obtain an integer value of ν), then we have

$$y_n[k] = \sum_{i=-N/2}^{N/2-1} a_{i+N/2,n} \exp(-j2\pi i \frac{k-v}{N}), \quad 0 \le k \le N+v-1 \ . \tag{2.25}$$

If we focus on (2.25), it can be observed that, $N+\nu$ samples of OFDM symbol can be generated by taking N-point IDFT of sequence $\{a_i\}$ and then adding last ν samples to the beginning of the IDFT output.

Let $u_n[k]$ represent the N-point IDFT of sequence $\{a_{i,n}\}$ given by

$$u_n[k] = \sum_{i=-N/2}^{N/2-1} a_{i+N/2,n} \exp(-j2\pi i \frac{k}{N}), \quad 0 \le k \le N-1. \tag{2.26}$$

Comparing (2.25) and (2.26), it can be seen that

$$y_n[k] = u_n[k-\nu], \quad \nu \le k \le N+\nu-1. \tag{2.27}$$

Also for $k < \nu$,

$$y_n[k] = u_n[N-(\nu-k)], \quad 0 \le k < \nu. \tag{2.28}$$

Equations (2.27) and (2.28) demonstrate that, OFDM symbol can be reconstructed from the samples of IDFT output, together with the addition of the cyclic prefix.

2.2.3. Synchronization

An OFDM receiver needs to perform synchronization operation prior to subcarrier demodulation. Two types of synchronization are needed. First, symbol boundaries are determined and then proper sampling instants are found out in order to minimize ICI and ISI effects. Second step of synchronization is used to eliminate the carrier frequency offset on the received signal. In an OFDM system, the orthogonality assumption is valid only if the transmitter and the receiver use exactly the same frequency. Any frequency offset results in intercarrier interference (ICI). Additionally, a practical oscillator does not produce a carrier at exactly one frequency, instead it produces a carrier that is phase modulated by random phase jitter. Hence, frequency is never perfectly constant resulting in some ICI in OFDM receiver.

In this section, sensitivity of OFDM systems to phase noise, frequency offset and timing errors should be examined. Then two synchronization techniques used in OFDM systems will be introduced.

2.2.3.1. Sensitivity to Phase Noise

In [14], power spectrum density of an oscillator signal with phase noise is modeled by a Lorentzian spectrum, which is equal to the squared magnitude of a first order lowpass filter transfer function. The single sided spectrum is given by

$$S(f) = \frac{1/\pi f_1}{1+|f-f_c|^2/f_1^2} \ . \qquad (2.29)$$

In (2.30), f_1 is the 3 dB linewidth of the oscillator signal and f_c is the carrier frequency.

Phase noise has two main effects. First, the phase noise results in attenuation and rotation of the received signal. Second and more important effect is the ICI, because phase noise changes the 1/T separation between subcarriers in the frequency domain. In [14], the degradation in SNR caused by phase noise is given as

$$SNR_{loss} \approx \frac{11}{6\ln 10} 4\pi\beta T \frac{E_s}{N_0} \tag{2.30}$$

where β is the 3 dB one sided bandwidth of the power spectrum density of the carrier, T is the symbol period and E_s/ N_0 is the symbol-to-noise energy ratio. From (2.30), it is seen that phase noise degradation is proportional with the ratio of subcarrier bandwidth (β) and the subcarrier spacing (1/T). It can be seen from [14] that, for a negligible SNR degradation of less than 0.1 dB, the 3 dB phase noise bandwidth has to be about 0.1 to 0.01 percent of the subcarrier spacing, depending on the modulation. For example, if 64 QAM modulation is used in an OFDM system with a subcarrier spacing of 300 kHz, the 3 dB bandwidth should be 30 Hz at most.

2.2.3.2. Sensitivity to Frequency Offset

As stated in [14], the degradation in SNR caused by a frequency offset which is small relative to subcarrier spacing can be approximated as

$$SNR_{loss} \approx \frac{10}{3\ln 10} (\pi \Delta f T)^2 \frac{E_s}{N_0} \ . \tag{2.31}$$

As shown in [14], for a negligible degradation of about 0.1 dB, maximum tolerable frequency offset is less than 1% of subcarrier frequency. For example, for an OFDM system at a carrier frequency of 5 GHz and a subcarrier spacing of 300 kHz, the oscillator accuracy needs to be 3 kHz or 0.6 ppm. Initial frequency error

of a low cost oscillator will not meet this requirement. Therefore, a frequency synchronization technique has to be applied before the FFT block of the receiver.

2.2.3.3. Sensitivity to Timing Errors

OFDM is more robust to timing errors relative to phase noise and frequency offsets. The symbol timing offset may vary over an interval equal to the guard time without causing ICI and ISI. Interferences occur only when the FFT interval extends over a symbol boundary or extends over the roll-off region of a system. Therefore, we can state that OFDM demodulation is quite insensitive to timing errors. However, in order to achieve the best possible multipath robustness, there is an optimal timing instant. Deviation from this timing instant increases the sensitivity to delay spread, so the system can handle less delay spread than it is designed for. To minimize this sensitivity, the system should be designed such that the timing error is small compared to the guard interval.

2.2.3.4. Synchronization Techniques

There are several OFDM synchronization techniques used in the literature. This section describes the synchronization methods in two categories: *Synchronization Using Cyclic Extension* and *Synchronization Using Special Training Symbols.*

2.2.3.4.1. Synchronization Using Cyclic Extension

Because of cyclic prefix, the first T_g seconds of an ODFM symbol is the same as the last T_g seconds of the symbol. This property can be used for both timing and frequency synchronization by using a system given in Figure 2.5. This system correlates a T_g long part of the OFDM signal with a part that is T seconds delayed [15].

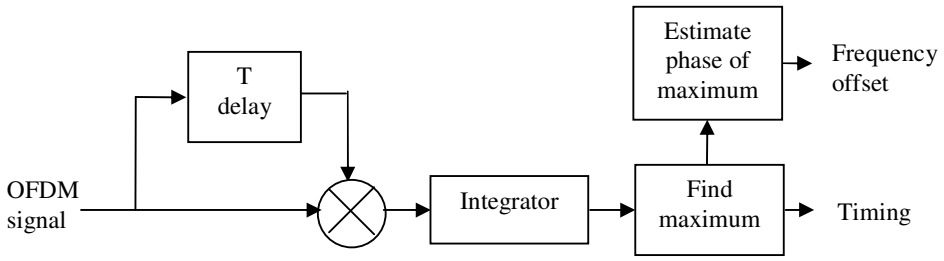

Figure 2.5 Synchronization Using Cyclic Prefix

Synchronization using cyclic extension is only effective when a large number of subcarriers are used, preferably more than 100. Once the symbol timing is known, the cyclic extension correlator output can be used to estimate the frequency offset. The phase of the correlator output equals the phase drift between samples that are T seconds apart. Hence, the frequency offset can be calculated by simply dividing the correlation phase by $2\pi T$. It should be noted that, this method is useful up to a maximum frequency offset of half the subcarrier spacing.

Synchronization technique using cyclic prefix is appropriate for tracking or blind synchronization, where no training signals are available.

2.2.3.4.2. Synchronization Using Special Training Symbols

When using cyclic prefix for synchronization purpose, it may take 10 or more OFDM symbols to have a reasonable performance [2]. For high data rate packet transmission, the synchronization time needs to be as short as possible, preferably a few OFDM symbols only. To achieve this, special OFDM training symbols, for which the receiver knows the data content, can be used [16-18]. This method uses the entire received training signal to achieve synchronization while the cyclic extension method uses only a fraction of each symbol.

Figure 2.6 shows the block diagram of a matched filter that can be used to correlate the input signal with the known training signal. In the figure, T is the sampling interval and c_i are the matched filter coefficients. Both symbol timing and frequency offset can be estimated from correlation peaks in the matched filter output. It should be noted that matched filtering is applied on the OFDM signal before FFT operation is applied.

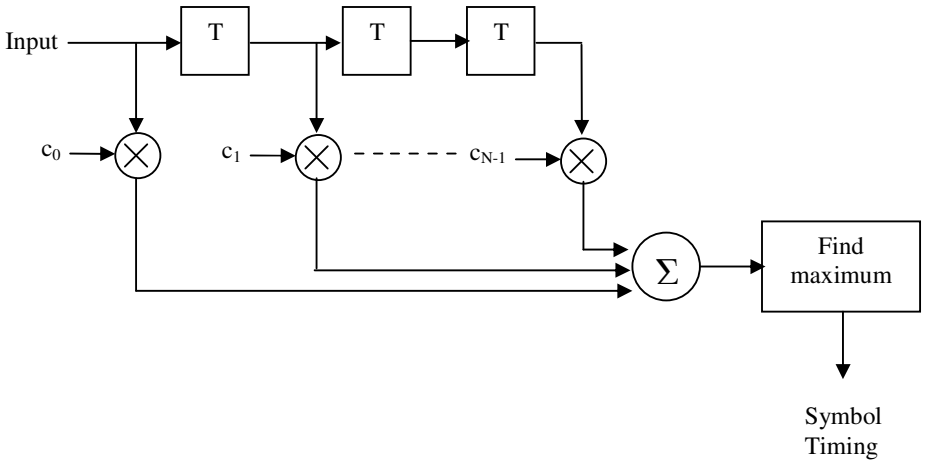

Figure 2.6 Synchronization Using Special Training Symbols

2.2.4. Bit loading

Another important concept about OFDM is the bit loading. If the available transmit power and input data is equally divided among the subcarriers, the technique is named as *fixed* or *uniform loading*. The subcarriers at which the channel attenuation is maximum, mostly determine the performance of a fixed loading OFDM system under Rayleigh fading. Under frequency selective fading, such systems may perform poorly. An efficient method to cope with this problem is to employ *adaptive loading* of the available transmit power and input bits over the subcarriers according to channel state. By this way, subcarriers experiencing severe

fading are loaded with a small number of bits, or may not be used at all. The advantage it provides in increasing the capacity can be computed for fading channels as described in [26].

2.2.5. Peak-to-Average Power Ratio (PAPR)

Amplitude of the OFDM signal has an almost Rayleigh distribution and exhibits strong fluctuations. Therefore, the resultant Peak-to-Average Power Ratio (PAPR) can be rather high. In the worst case, PAPR of an OFDM system with N subchannels may reach up to a value of N. High PAPR value has some disadvantages for practical implementations and needs to be reduced to an acceptable level. In the literature various methods are proposed for the purpose of PAPR reduction. Next chapter deals with PAPR problem and the proposed solutions in more detail.

2.3. Applications of OFDM

OFDM is used as the transmission technique in digital audio and television broadcasting applications, and in wireless LAN applications. This section describes the main applications of OFDM.

2.3.1. Digital Audio Broadcasting (DAB)

DAB is the successor of current analog audio broadcasting based on AM and FM and offers improved sound quality, comparable to that of CD quality, new data services and a higher spectrum efficiency. DAB was standardized in 1995 by the European Telecommunications Standards Institute (ETSI) as the first standard to use OFDM [32, 33]. The basis for this standard was the specification developed by the European Eureka 147 DAB project, which started in 1988.

DAB has four transmission modes using different sets of OFDM parameters, which are listed in Table 2.1. The parameters for modes I to III are optimized for use in specific frequency bands, while mode IV was introduced to provide a better coverage range at the cost of an increased vulnerability to Doppler shifts. DAB can be operated above 30 MHz and the upper limits of the carrier frequency are given in Table 2.1.

Table 2.1 DAB OFDM Parameters

	Mode I	Mode II	Mode III	Mode IV
Number of subcarriers	1536	384	192	768
Subcarrier spacing	1 kHz	4 kHz	8 kHz	2 kHz
Symbol time	1.246 ms	0.3115 ms	0.1558 ms	0.623 ms
Guard time	0.246 ms	0.0615 ms	0.0308 ms	0.123 ms
Carrier frequency	< 375 MHz	< 1.5 GHz	< 3 GHz	< 1.5 GHz
Transmitter separation	< 96 km	< 24 km	< 12 km	< 48 km

One important reason to use OFDM for DAB is the possibility to use a single frequency network, which greatly enhances the spectrum efficiency. As long as the delay between two signals is smaller than the guard time, no ISI and ICI occur due to the simultaneous transmission.

The transmitted data consists of a number of audio signals, sampled at 48 kHz with an input resolution up to 22 bits. The digital audio signal is compressed to a rate in the range of 32 to 384 kbps, depending on the desired quality. Transmission is done on 24 ms long frames. The start of a frame is indicated by a null symbol, which is a silence period that is slightly larger than the duration of an OFDM symbol. Then a reference OFDM symbol is sent which serves as the starting point for the differential decoding of the QPSK modulated subcarriers.

The digital input data is encoded by a rate 1/4 convolutional code with a constraint length of 7. The coding rate can be increased up to 8/9 by puncturing. This yields a total maximum data rate of approximately 2.2 Mbps. The coded data are then interleaved in frequency domain by distributing the input data over subcarriers, which avoids large burst errors in the case of a deep fade affecting a group of subcarriers.

2.3.2. Terrestrial Digital Video Broadcasting

Research on a digital system for TV broadcasting has been carried out since the late 1980s. In 1993, a pan-broadcasting industry group started the Digital Video Broadcasting (DVB) project. Within this project, a set of specifications was developed for the delivery of digital television over satellites, cable and through terrestrial transmitters. The terrestrial DVB system was standardized in 1997 [31, 34].

Terrestrial DVB uses OFDM with two possible modes, using 1705 and 6817 subcarriers, respectively. These modes are referred to us 2k and 8k modes, which correspond to the sizes of IFFT and FFT blocks used in multicarrier modulation and demodulation. The symbol period of 8k system is about 896 μs, while the guard time may have four different values between 28 to 224 μs. The corresponding values for the 2k system are four times smaller.

At the DVB-T transmitter, the input data is divided into groups of 188 bytes, which are scrambled and coded by an outer shortened (204,188) Reed-Solomon (RS) code over GF(256). This code can correct up to 8 byte errors in a frame of 204 bytes. The coded bits are then interleaved and then coded again by a rate 1/2 convolutional code, with a constraint length of 7. The coding rate can be increased for higher throughput by puncturing to 2/3, 3/4, 5/6 or 7/8. The encoded bits are then interleaved by an inner interleaver and then mapped onto QPSK, 16 QAM or 64 QAM symbols.

The receiver side employs coherent detection of QAM signals which requires the estimation of channel effects. This is accomplished by using pilot subcarriers. For the 8k mode, in each symbol there are 768 pilots, so 6048 subcarriers are allocated to data traffic. The 2k mode has 192 pilots and 1512 data subcarriers. The position of pilots varies from symbol to symbol with a pattern that repeats after 4 OFDM symbols. The pilots allow the receiver to estimate the channel both in frequency and time.

2.3.3. Magic WAND

The Magic WAND (Wireless ATM Network Demonstrator) project was a part of European ACTS (Advanced Communications Technology and Servers) program [25]. The Magic WAND consortium members implemented a prototype wireless ATM network based on OFDM. This prototype had a large impact on standardization activities in the 5 GHz band. The wireless ATM application based on OFDM forms the basis for the standardization of the HYPERLAN type II physical layer.

The main parameters of the WAND physical layer are listed in Table 2.2. The number of subcarriers is 16, with a guard time of 400 ns. With this guard time, rms delay spreads up to 100 ns are completely tolerated without employing equalization. While this is sufficient for most of the office buildings, a realistic product would require more delay spread robustness in order to cover large office buildings and factory halls.

As can be seen from Table 2.2, the subcarrier modulation is 8 PSK with a symbol rate of 13.3 Msymbols/s, yielding a raw data rate of 40 Mbps. The rate 1/2 complementary coding reduces the net rate to 20 Mbps. The subcarrier spacing is 1.25 MHz, which gives a total bandwidth of 20 MHz. The packet preamble is 8.4 μs in duration and consists of one OFDM symbol, repeated 7 times. This preamble

is used for packet detection, automatic gain control, frequency offset estimation, symbol timing and channel estimation. Transmission is done on half-slots, which consists of 9 OFDM symbols, carrying 27 bytes. An ATM cell is mapped on two consecutive half-slots, which form a full-slot. Of the 54 bytes carried on one full-slot, 53 bytes are allocated to ATM cell traffic and remaining 1 byte is reserved for physical layer control signaling.

Table 2.2 Main Parameters of the WAND OFDM Modem

Number of subcarriers	16
Modulation	8-PSK
Coding	Rate 1/2, length 8 complementary coding
Net bit rate	20 Mbps
Symbol time	1.2 μs
Guard time	0.4 μs
Windowing	Raised cosine, with 0.2 roll-off
Subcarrier spacing	1.25 MHz
Training length	7 symbols
Carrier frequency	5.2 GHz
Peak output power	1 W

The WAND modem uses complementary codes for both forward-error correction and PAP reduction. Length 8 complementary codes are used, with 4 8-PSK symbols input, generating eight complex code outputs. By taking the IFFT of this complex coded output, an OFDM signal with a low PAP ratio is obtained. To encode 16 subcarriers, two length 8 codes are interleaved, which maximizes the benefits of frequency diversity. The minimum distance of the length 8 code is 4 symbols, and hence three arbitrary subcarriers can be erased without causing errors, provided that the data loaded onto the remaining subcarriers are not in error.

The performance of Magic WAND modem under Rayleigh fading was reported in [2]. According to the results provided in [2] , for a delay spread of 50 ns, an average SNR of approximately 30 dB is required to reach 10^{-6} BER, without antenna diversity. At these average SNR, the packet error rate is about $5x10^{-5}$.

2.3.4. IEEE 802.11, HYPERLAN/2 and MMAC Wireless LAN Standards

Since the beginning of 90's, wireless local area networks (WLAN) for the 900 MHz, 2.4GHz and 5 GHz ISM (Industrial Scientific and Medical) bands have been available, based on proprietary techniques. In June 1997, the Institute of Electrical and Electronics Engineers (IEEE) approved an international interoperability standard [35]. The standard specifies medium access control (MAC) procedures and three different physical layers (PHY). Two of the specified PHYs are radio based using 2.4 GHz band and the third uses infrared light. All PHYs support a data rate of 1 Mbps and optionally 2 Mbps.

User demand for higher bit rates and the international availability of the 2.4 GHz band has encouraged the development of a higher speed extension to the 802.11 standard. In July 1998, a proposal was selected for standardization, which describes a PHY providing a basic rate of 11 Mbps and a fall back rate of 5.5 Mbps. A second IEEE 802.11 working group has moved on to standardize yet another PHY option, which offers higher bit rates in the 5.2 GHz band. This development was motivated by the adoption, in January 1997, by the US Federal Communications Commission (FCC), of a modification to FCC part 15 rules. The modification makes available 300 MHz spectrum in the 5.2 GHz band, intended for use by a new category of unlicensed equipment called "Unlicensed National Information Infrastructure (UNII)" devices.

Like the IEEE 802.11 standard, the European ETSI HYPERLAN type I [36] specifies both MAC and PHY layers. Unlike 802.11, however, there is no HYPERLAN type I compliant products available in the market. A newly formed

ETSI working group called “Broadband Radio Access Networks (BRAN)” has started to work on extensions to HYPERLAN standard which is referred as HYPERLAN type II [37].

In Japan, equipment manufacturers and service providers are cooperating in the Multimedia Mobile Access Communication (MMAC) project to define new wireless standards similar to those of IEEE 802.11 and ETSI BRAN. Additionally, MMAC is also looking into possibility for ultra-high speed wireless indoor LANs supporting large volume data transmission at speeds up to 156 Mbps using EHF band (30-300 GHz).

In July 1988, the IEEE 802.11 standardization group decided to select OFDM as the basis for their new 5 GHz standard, called IEEE 802.11a, targeting a range of data rates from 6 up to 54 Mbps [38]. This new standard is the first one which uses OFDM in packet-based high speed data communications. Following the IEEE 802.11 decisions, ETSI BRAN and MMAC also adopted OFDM for their physical layer standards. The three bodies have worked in close cooperation since then to make sure that the differences among the various standards are kept at a minimum level, thereby enabling world wide equipment manufacturing.

Table 2.3 lists the main parameters of OFDM physical layer adopted in all these standards with some minor changes. A key parameter that largely determines the choice of the other parameters is the guard interval of 800 ns. This provides robustness to rms delay spreads up to several hundreds of nanoseconds. In practice, this means that any indoor environment including large factory buildings can be covered.

To limit the relative amount of power and time spent on the guard time, the symbol duration is chosen to be 4 μs. This determines the subcarrier spacing at 312.5 kHz. By using 48 data subcarriers, uncoded data rates of 12 to 72 Mbps can be achieved by using variable modulation types from BPSK to 64 QAM. In addition to 48 data

subcarriers, each OFDM symbol contains an additional 4 pilot subcarriers, which can be used to track the residual carrier frequency offset that remains after an initial frequency correction during the training phase of the packet. The training phase of the packet consists of three parts, a 8 μs long first part consisting of 10 short symbols to be used in AGC and coarse frequency offset estimation, followed by another 8 μs part consisting of two OFDM symbols used for fine frequency offset estimation, symbol timing and channel estimation purposes and a 4 μs final part consisting of one OFDM symbol used primarily for physical layer signaling (indicating coding rate, modulation type, packet length etc. that are used in the rest of the packet carrying the user data).

To correct the errors encountered in deep fades, forward-error correction across the subcarriers is used with variable coding rates, yielding net data rates from 6 up to 54 Mbps. The main code is a rate 1/2, constraint length 7 convolutional code and higher coding rates can be obtained by puncturing the mother code to 2/3 and 3/4. The rate 2/3 code is used together with 64 QAM to obtain a net data rate of 48 Mbps. The rate 1/2 code is used in conjunction with BPSK, QPSK and 16 QAM, yielding net data rates of 6,12 and 24 Mbps, respectively. Finally, the rate 3/4 code is used in conjunction with BPSK, QPSK,16 QAM and 64 QAM, yielding net data rates of 9,18,36 and 54 Mbps, respectively. As an example of the error rate performance of the receiver under Rayleigh fading, it was reported in [2] that, for the 24 Mbps mode, in order to obtain a packet error rate of 10^{-4}, an average SNR of about 22dB is required for an rms delay spread of 100 ns without antenna diversity.

Table 2.3 Main Parameters of the OFDM physical layer standards (IEEE 802.11a, ETSI HYPERLAN type II and MMAC)

Data Rate	6, 9, 12, 18, 24, 36, 48, 54 Mbps
Modulation	BPSK, QPSK, 16 QAM, 64 QAM
Coding Rate	1/2, 2/3, ¾
Number of subcarriers	52
Number of pilots	4
OFDM symbol duration	4 µs
Guard interval	800 ns
Subcarrier spacing	312.5 kHz
Total bandwidth	16.56 MHz
Channel spacing	20 MHz
Maximum transmit Power	1 W (802.11 at 2.4-2.4835 and 5.725-5.825 GHz Bands) 250 mW (802.11 at 5.25-5.35 GHz) 50 mW (802.11 at 5.15-5.25 GHz) 100 mW (HYPERLAN at 2.4-2.4835 GHz) 10 mW (MMAC at 2.471-2.497 GHz)

CHAPTER 3

PAPR REDUCTION TECHNIQUES

3.1. Introduction

OFDM signal is essentially the sum of many independently modulated sine waves and its amplitude has an almost Rayleigh distribution. Amplitude of OFDM signal exhibits strong fluctuations and the resultant Peak-to-Average Power Ratio (PAPR) can be rather high. In the worst case, N signals with the same phase are added up resulting in a peak power that is N times the average power; i.e. PAPR may reach a value of N. For an OFDM system with 256 subcarriers, PAPR may reach to 24 dB.

High value of PAPR brings disadvantages like an increased complexity of A/D and D/A converters and a reduced efficiency of RF power amplifiers. In a practical system, before transmission, OFDM signal is passed through a power amplifier that is always peak power limited. If the squared magnitude of the OFDM signal is larger than the saturation point of the power amplifier at any time instant, then the signal will be clipped. Clipping destroys the orthogonality between subcarriers resulting in an increase in the BER when compared with the nonclipped case [19]. PAPR is important for OFDM since it is a measure of the clipping probability.

In the literature various methods are proposed for the purpose of PAPR reduction. In this chapter, some PAPR reducing methods will be discussed. First section represents the signal distorting methods such as *clipping, peak windowing* and *peak cancellation*. Later, scrambling techniques will be mentioned and the most frequently used two scrambling methods, *selected mapping* and *partial transmit sequences,* will be described in detail. Last section should consider *coding* techniques used to decrease PAPR value of the OFDM signal.

3.2. Clipping and Peak Windowing

The simplest way to reduce PAPR is to clip the signal such that the peak amplitude becomes limited to a desired level [20]. Figure 3.1 shows the clipping operation. Clipping is the simplest solution in terms of algorithm complexity; however there are two basic problems associated this technique. First problem is the introduction of self interference which degrades the Bit Error Rate (BER) performance. Second, nonlinear distortion of OFDM signal increases the out-of-band radiation. This effect can be clarified by considering clipping operation as a multiplication of the OFDM signal by a rectangular window function. Amplitude of the rectangular function equals 1 if the OFDM amplitude is below the threshold and smaller than 1 if the amplitude exceeds the threshold. The spectrum of the clipped OFDM signal is the spectrum of the input OFDM signal convolved with the spectrum of the windowing function. Therefore, the out-of-band characteristics are mainly determined by the wider spectrum of the two, which is the spectrum of the rectangular window function. The spectrum of a rectangular function has a very slow roll of that is proportional to the reciprocal of the frequency.

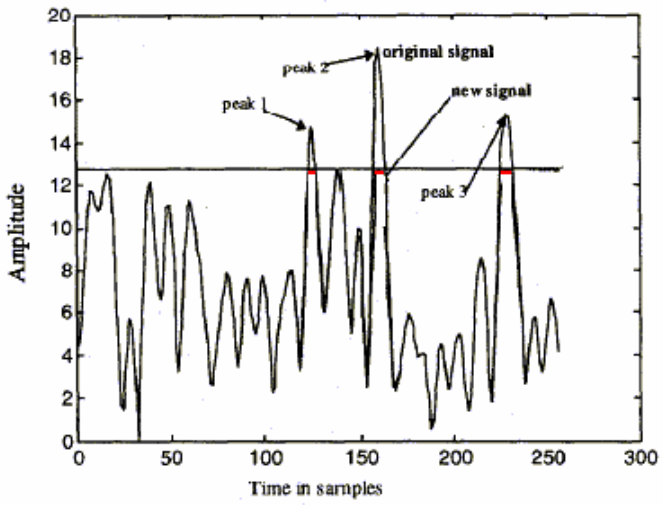

Figure 3.1 Clipping of OFDM Signal

To reduce the effect of out-of-band radiation problem of the clipping, another method is to multiply large signal peaks with a nonrectangular window [20]. Figure 3.2 demonstrates how windowing is applied to an OFDM signal. Any window can be used provided that it satisfies good spectral properties. To minimize out-of-band radiation, ideally the window should be as narrowband as possible. On the other hand, the window should not be too long in the time domain, because that implies many samples are affected, which increases the BER. Examples of suitable window functions are cosine, Kaiser and Hamming window.

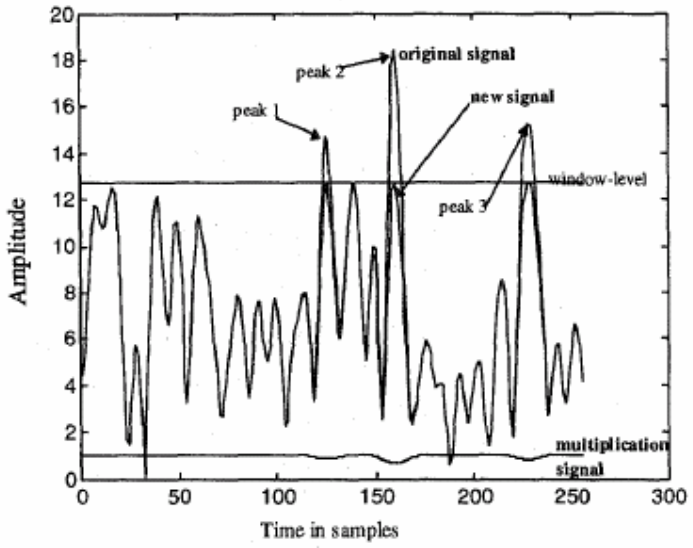

Figure 3.2 Peak Windowing of OFDM Signal

3.3. Peak Cancellation

All signal distortion techniques reduce the amplitude of OFDM samples whose power exceeds a certain threshold. When using clipping and peak windowing techniques, this reduction is achieved by means of a nonlinear distortion resulting in a certain amount of out-of-band radiation. This undesirable effect can be avoided by using linear peak cancellation method, where a time shifted and scaled

reference function is subtracted from the signal, such that each subtracted reference function reduces the peak power of at least one signal sample. Furthermore, selecting a reference function with approximately the same bandwidth as the transmitted signal can assure that the peak power reduction does not cause an increase in the out-of-band radiation. An example of a suitable reference function is the multiplication of a sinc function and a raised cosine function. If the windowing function is same as the function used for OFDM symbol windowing, then the reference function has the same bandwidth as of the OFDM signal. Hence, peak cancellation will not degrade the out-of-band spectrum properties. By making the reference signal window narrower, a tradeoff can be made between the complexity of the peak cancellation calculations and an increase in out-of-band power. The peak cancellation technique was described in [21].

Peak cancellation can be done digitally after generation of the digital OFDM symbols. It involves a peak power detector, a comparator to see if the peak power exceeds the threshold and a scaling of the peak and surrounding samples. Figure 3.3 shows the block diagram of an OFDM transmitter with peak cancellation [2]. Incoming data is first coded and converted from a serial bit stream to blocks of N complex signal samples. An IFFT operation is performed on each of these blocks. Then a cyclic prefix is added, extending the symbol size from N to $N+N_c$ samples. After parallel-to-serial conversion, the peak cancellation procedure is applied to reduce PAPR. Except for the peak cancellation block, there is no difference with a standard OFDM transmitter. There is no difference for the receiver, so any standard OFDM receiver can be used.

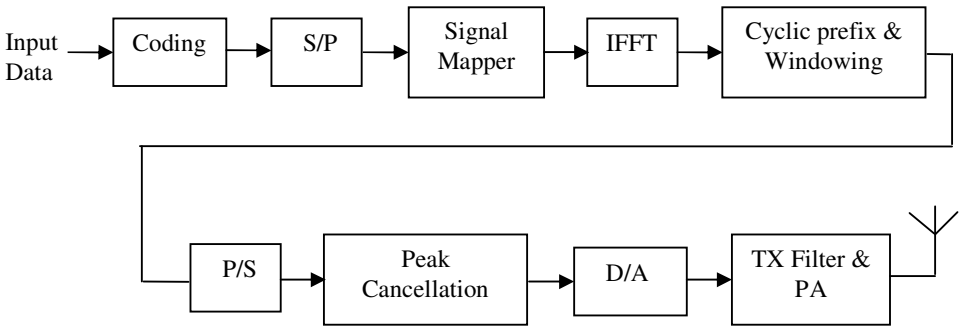

Figure 3.3 OFDM Transmitter with Peak Cancellation

In Figure 3.3, the peak cancellation was done after parallel-to-serial conversion stage. It is possible to implement peak cancellation immediately after the IFFT stage, as given in Figure 3.4 [2]. In this case, the cancellation is done on a symbol-by-symbol basis. An efficient way to generate the cancellation signal without using a stored reference function is to use a lowpass filter in the frequency domain. In Figure 3.4, for each OFDM symbol, it is detected which samples exceed a certain amplitude. Then, for each signal peak, an impulse is generated whose phase equals the peak phase and whose amplitude equals the peak amplitude minus the desired maximum amplitude. The impulses are then lowpass filtered on a symbol-by-symbol basis. Lowpass filtering is achieved in the frequency domain by taking the FFT, setting all outputs to zero whose frequencies exceed the frequency of the highest subcarrier, and then transforming the signal back by an IFFT.

Peak cancellation method seems to be a fundamentally different method than clipping and peak windowing. However, in [2] it is shown that peak cancellation is identical to clipping followed by filtering. This method was proposed as a PAPR reduction technique in [22].

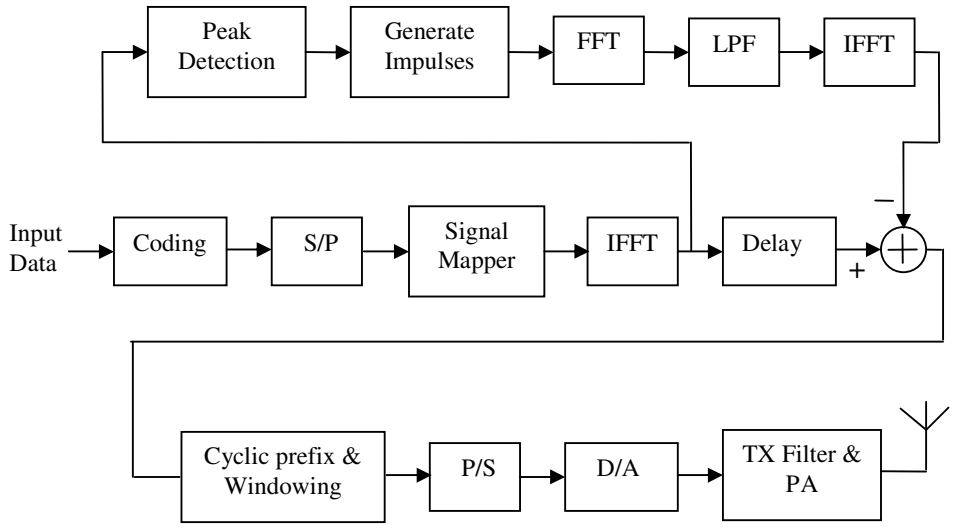

Figure 3.4 Peak Cancellation Using FFT/IFFT to Generate Cancellation Signal

3.4. Scrambling Techniques

The basic idea of symbol scrambling is that, for each OFDM symbol, the input sequence is scrambled by a certain number of scrambling sequences. The output signal with the smallest PAPR is transmitted. For uncorrelated scrambling sequences, the resulting OFDM signals and corresponding PAPR values will be uncorrelated, so if the PAPR for one OFDM symbol has a probability p of exceeding a certain level without scrambling, the probability is decreased to p^k by using k scrambling codes. Hence, symbol scrambling does not guarantee a PAPR below some low level; rather, it decreases the probability that high PAPR values occur. Initially proposed scrambling techniques were *Selected Mapping (SLM)* and *Partial Transmit Sequences (PTS)* which will be described in detail in the following sections. SLM and PTS improve the PAPR statistics by introducing little redundancy and also solve the problem signal distortion techniques which may cause in-band distortion and out-of-band noise.

3.4.1. Selected Mapping

Assume that an OFDM symbol is scrambled by M different scrambling sequences. Then M statistically independent OFDM symbols represent the same information. If the symbol with the lowest PAPR is selected for transmission, the probability that the PAPR value of the selected symbol ($PAPR_{low}$) exceeds a certain threshold z is given by

$$P\{PAPR_{low} > z\} = (P\{PAPR_{init} > z\})^{M} \quad (3.1)$$

where $PAPR_{init}$ is the PAPR value of the original symbol. The key idea of *Selected Mapping* (SLM) is to choose one particular signal that exhibits the lowest PAPR value among M candidates, all representing the same information. The block diagram of SLM is shown in Figure 3.5.

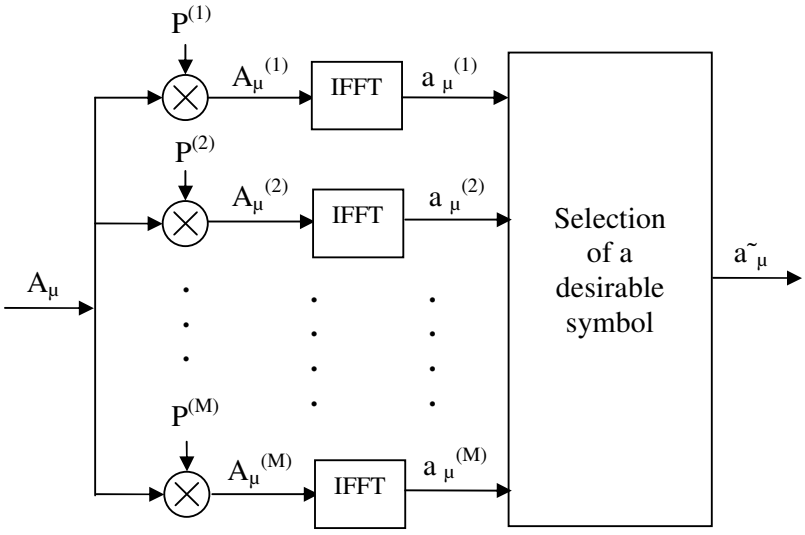

Figure 3.5 Block Diagram of Selected Mapping Method

In Figure 3.5, the M independent OFDM symbols $a_\mu^{(1)}$.... $a_\mu^{(M)}$ represent the same information and the symbol $\tilde{a}_\mu$ with the lowest PAPR is selected for transmission. The way of choosing $P^{(k)}$ vectors is as follows

$P^{(k)}=[P_1^{(k)}, ..., P_N^{(k)}]$,

with $P_v^{(k)} = e^{j\theta_v^{(k)}}$ where $\theta_v^{(k)} \in [0, 2\pi)$, $1\leq v \leq N$, $1\leq k <M$. After mapping the information to the subcarrier amplitude $A_{\mu,v}$, each OFDM symbol is multiplied with the M vector $P^{(k)}$, resulting in a set of M different OFDM symbols $A_\mu^{(k)}$ with components represented by [7]

$$A_{\mu,v}^{(k)} = A_{\mu,v}. P_v^{(k)}, \quad 1\leq v \leq N, 1\leq k \leq M \tag{3.2}$$

All new M OFDM symbols are transformed into time domain defined by

$$a_\mu^{(k)} = \text{IFFT} \{ A_\mu^{(k)}\}. \tag{3.3}$$

Finally, the symbol with the lowest PAPR is selected, which is represented as $\tilde{a}_\mu$ in Figure 3.5. Notice that, increasing the number distinct vectors used for scrambling (i.e. M), the PAPR reduction amount is increased.

In order to implement SLM effectively, the components of $P_v^{(k)}$ should consist of $\{\pm 1, \pm j\}$ as multiplication with these components can be implemented simply by interchanging, adding and subtracting real and imaginary parts. For the receiver to demodulate the data, it should know which vector is used at the transmitter to scramble the data. The most important disadvantage of SLM appears at this stage, the transmitter has to transfer side information to distinguish the vector used for scrambling. The number of required bits to be transmitted as side information is $\lceil \log_2 M \rceil$. Side information should be protected carefully; hence it should be sent from a secure channel, which increases system cost.

In [8], a method is suggested in order to avoid transmission of side information in a SLM system. In this technique, to generate M different transmit sequences $a^{(k)}$, $1 \leq k \leq M$ representing the same information word q, labels $b^{(k)}$ are inserted as a prefix to q. The labels are M different binary vectors of length $\lceil \log_2 M \rceil$. The concatenated word vector of the label and the information word is fed into a scrambler. The labels are hence used to drive the scrambler into one of the M different states before scrambling the information word q itself. Finally, the specific transmit sequence number $\tilde{u}$, which possesses the lowest peak power, is selected for transmission. This method removes the need for side information, however as classical SLM approach, it reduces the probability of the OFDM symbol PAPR to exceed a certain threshold, does not guarantee a certain reduction value.

3.4.2. Partial Transmit Sequences

The main principle of *Partial Transmit Sequences* (PTS) method is to partition an OFDM symbol A_μ into V disjoint subblocks $A_\mu^{(k)}$ (k=1, …, V). The simple and effective method is to divide an OFDM symbol into subblocks of the same size as shown in Figure 3.6. All subcarrier positions in $A_\mu^{(k)}$, which are represented in another subblock, are set to zero. Therefore, A_μ equals to the sum of all subblocks as given by

$$A_\mu = \sum_{k=1}^{V} A_\mu^{(k)} \tag{3.4}$$

Figure 3.7 gives the block diagram of a PTS system. After partitioning the OFDM symbol, each $A_\mu^{(k)}$ is multiplied by a complex-valued phase rotation factor $b_\mu^{(k)}$, where $b_\mu^{(k)} = e^{j\theta_\mu^{(k)}}$ where $\theta_\mu^{(k)} \in [0, 2\pi)$, $1 \leq k \leq V$. A phase rotation of all subcarriers included in the subblock k by the same angle $\theta_\mu^{(k)} = \arg(b_\mu^{(k)})$ is performed. To calculate time domain signal $\tilde{a}_\mu$, the subblocks are transformed by V separate and parallel IFFTs as following [6]

$$\tilde{a}_{\mu} = \sum_{k=1}^{V} b_{\mu}^{(k)} IFFT\{A_{\mu}^{(k)}\} = \sum_{k=1}^{V} b_{\mu}^{(k)} a_{\mu}^{(k)} . \quad (3.5)$$

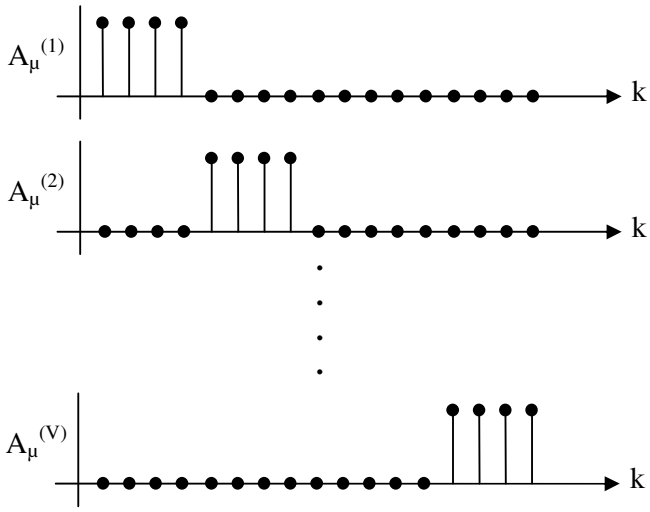

Figure 3.6 Subblock Configuration for Partial Transmit Sequences Method

From Figure 3.7, peak value of OFDM symbol can be reduced by suitably selecting the phase rotation parameter $b_{\mu}^{(k)}$ in the peak value optimization block. To implement PTS effectively, the components of $b_{\mu}^{(k)}$ should consist of $\{\pm 1, \pm j\}$. In order to demodulate the received data, the receiver has to know information about $b_{\mu}^{(k)}$, transmitter therefore has to transfer the side information as the case with SLM technique.

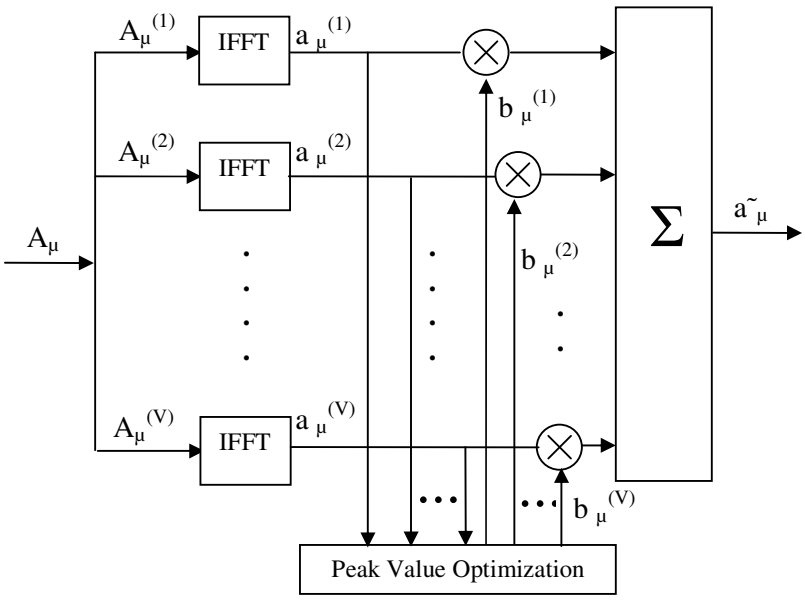

Figure 3.7 Block Diagram of Partial Transmit Sequences Method

3.5. Coding

Only a small fraction of all possible OFDM symbols have a high PAPR value. This statement suggests another solution to the high PAPR problem based on coding. The PAPR value can be reduced by using a code that only produces OFDM symbols for which the PAPR value is below some threshold.

In the literature, different coding techniques are used for this purpose. One of the most frequently used coding methods is the block coding scheme. In [9], a table is given indicating PEP (Peak Envelope Power) values for all possible 4 bit binary sequences normalizing the power of individual subcarriers to 1 W. Then codewords with the highest PEP values are determined. The paper offers the peak-to-mean envelope power ratio (PMEPR) reduction of the multicarrier signal by block coding the data such that a 3-bit data word is mapped onto a 4-bit code word such that set of permissible words does not contain those that result in excessive PEP values. [9] also considers an 8 carrier OFDM system and gives some

numerical results such that, if no block coding is used, PMEPR equals 9.03 dB, if half of the possible codewords are permissible, PMEPR equals 4.45 dB and if quarter of the possible codewords are permissible PMEPR equals 3.01 dB. These results are based on an exhaustive search through all possible codewords. The paper shows significant PEP reductions are possible; however it does not describe how to perform encoding and decoding operation. Additionally, as described in [9], the potential of this technique is limited because as the number of carriers and the number of modulation levels increases, the ratio of the number of permissible codewords to all codewords must be reduced to achieve a desired PMEPR. On the other hand, [9] has an important result: Inspection of PEP (Peak Envelope Power) values for eight carriers with BPSK modulation confirmed that many but not all of the codewords that resulted in low PEP values were Golay complementary sequences. However, the few codewords that yielded the minimum PEP were not Golay complementary sequences. Consequently, using the Golay complementary sequences would result in a PMEPR greater than the optimum, for a given code rate. Nevertheless, it would give a code that is open to mathematical encoding and decoding and may have error detection/correction capability.

It was already mentioned in [23] that the correlation properties of Golay complementary sequences translate into a relatively small PAPR of 3 dB when the codes are used to modulate an OFDM signal. For OFDM with a large number of channels, it may not be feasible to generate a sufficient number of complementary codes with a length equal to the number of channels. To avoid this problem, [24] suggests to split the total number of channels and applying a complementary code to each group of subchannels increasing the code rate at the cost of error correction capability and PAPR reduction. It may be concluded that, complementary codes have good properties to serve as both PAPR reduction codes and as forward error correction codes in OFDM systems. Reasonable coding rates can be achieved up to length 16 [24]. Unfortunately, the achievable code rate decreases for increasing code length. In practice, Golay codes were actually implemented in a prototype OFDM modem operating at 20 Mbps for the European Magic WAND project [25].

In [12], the idea is to use the standard array of a linear block code in order to reduce the PAPR value of OFDM, not for error correction. The algorithm suggested in [12] is as follows. As a first step, a binary information sequence is divided into blocks of k bits. Each information block is encoded into a codeword c by the encoder of an $[n,k]$ linear block code and then U distinct vectors are constructed as $c+e_1$, $c+e_{2,\ldots\ldots,}c+e_U$, where $e_1=0$ and $e_{2,\ldots\ldots,}e_U$ are properly selected so as to represent correctable erroneous patterns. Each scrambled vector is mapped to a sequence of N modulation symbols by a signal mapper and then transformed to an OFDM signal by IDFT. Finally, a signal with minimum PAPR is chosen from U distinct signals as the transmit signal corresponding to an information block of k bits. The code used at the transmit stage is a single error correcting Hamming code since the code is not used for error correction purposes. If needed, a channel encoder may be used after the scrambling stage. At the receiver, the N-point DFT and the signal demapper convert the received signal in a received binary vector r. When convolutional codes, turbo codes, or low-density parity-check (LDPC) codes are employed as a channel code for error correction, the signal demapper may be combined with their soft decision decoder. The syndrome calculated from r is used for estimating the codeword c. Finally; the codeword is converted into an information block of k bits. This scheme is a modified version of Selected Mapping (SLM), in which the transmit signal is selected as a signal with minimum PAPR from differently scrambled signals of the information by a number of random sequences. Compared with conventional SLM, above described scheme requires only the syndrome decoding of the received signal and therefore no side information on the choice of the transmit signal needs to be transmitted. This method seems to be open to further investigation, because it describes an easy-to-implement method without the disadvantage of transmitting side information; however it lacks the numeric performance results.

Additionally, there are other works in the literature which implements a combination of above described techniques, to reduce the PAPR value of OFDM.

CHAPTER 4

PAPR REDUCTION WITH LINEAR CODING AND CODEWORD MODIFICATION

4.1. Introduction

In this chapter, the algorithm proposed in this thesis work to reduce PAPR value of an OFDM modulating system will be explained. As described in Chapter 3, various methods are used in the literature for this purpose. However, all the proposed methods have some drawbacks, such as out-of-band radiation for clipping method, side information transmission for Selected Mapping and Partial Transmit Sequences methods and low coding rates for coding implementing methods. The aim of this work is to offer a new method which does not distort the signal that increases out-of-band radiation, instead uses block coding of the input data and then modifies the coded data iteratively until the PAPR reduces below a certain level. The method makes use of the advantage of implemented coding scheme's error correction capability. Additionally, the suggested method does not necessitate transmission of side information. The method uses block coding, however, a single or double error correcting code will be enough.

First, a block diagram will be given to provide a general overview of the system. Then, the details of the algorithm will be discussed and a flow chart will be supplied to better understand what algorithm does. Next, basic parameters of the algorithm will be described and some simulation outputs will be given. Finally, PAPR reduction will be demonstrated using simulation results.

4.2. Description of the Method

Figure 4.1 gives the block diagram of the system. In proposed method, the input data is distributed to N - Encoder& Mapper Blocks. Details of these blocks and their operation will be described later. Outputs of Encoder & Mapper Blocks are fed into an IFFT block. This operations form regular OFDM symbols. The Peak-to-Average Power Ratio (PAPR) values of OFDM symbols are calculated and compared with a certain threshold β. If the PAPR value is below the threshold, no operation is performed on the symbol and it is directly passed to the output. However, if PAPR exceeds the threshold, the algorithm is applied to that symbol until the PAPR is reduced below β.

Determination of the threshold is critical, since it determines the PAPR reduction achieved, but choosing a very low value will increase the number of calculations needed, which results in computational inefficiency. Hence, there is a trade off between PAPR reduction value and implementation complexity of the system. A criterion to choose a proper threshold value will be suggested in the next chapter.

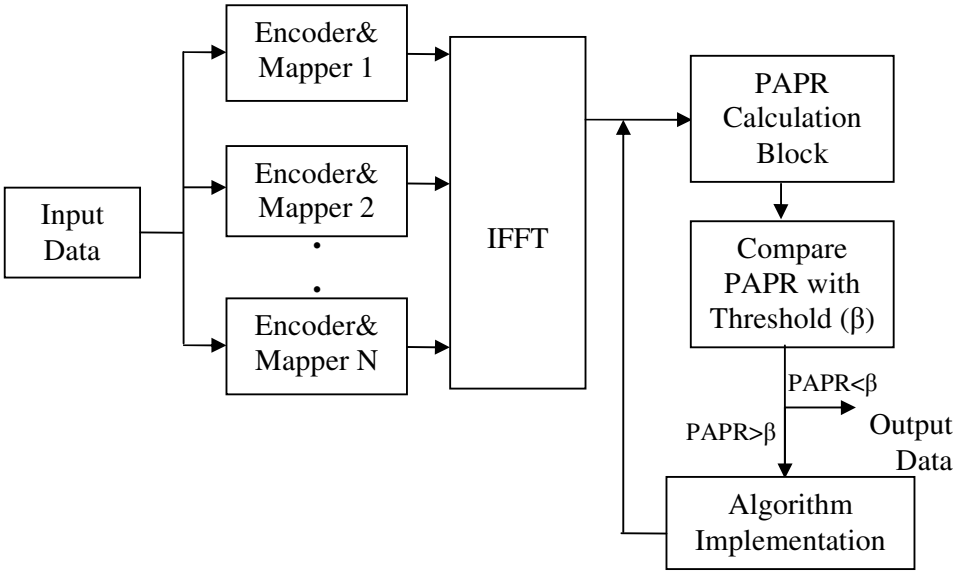

Figure 4.1 System Block Diagram

The operations performed in an Encoder & Mapper block is shown in Figure 4.2. Input data is distributed to N of these blocks and each of the block waits until k-bits are collected. Then k-bits are encoded into n- bits using an (n,k) linear block code. The error correction capability of the selected code is important for the performance calculation of the system. This issue will also be discussed in detail in the next chapter. Later, a parity value is calculated and added to the end of n-bits in order to obtain an even number of bits before modulation. Finally, (n+1) bits are passed through a QPSK modulator and (n+1)/2 QPSK modulated symbols are obtained as Encoder & Mapper block output. Note that any modulation can be used at this stage. However, QPSK modulation is chosen for simplicity.

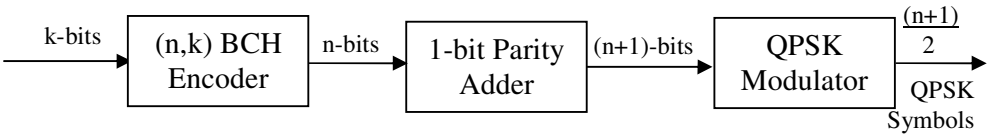

Figure 4.2 Encoder & Mapper Block Diagram

Assume that the OFDM system used has 4 channels, for illustration. The system uses (n,k) code and 1-bit parity, then the Encoder & Mapper outputs are as shown in Figure 4.3.

In Figure 4.3, (n+1)/2 QPSK symbols are generated on each Encoder & Mapper block for k information bits.

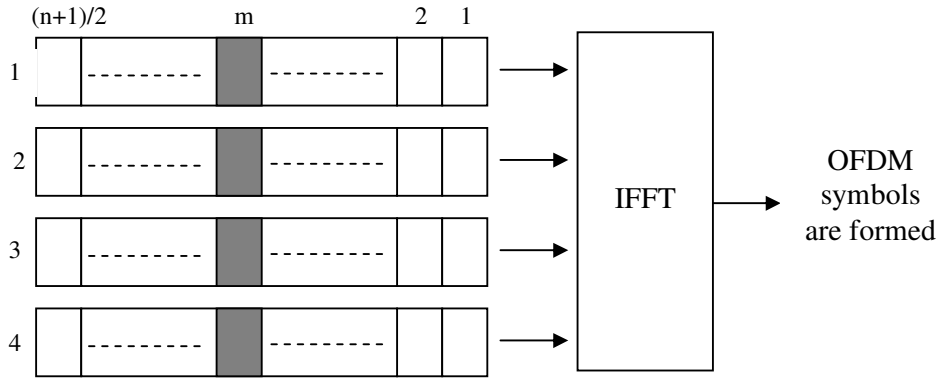

Figure 4.3 Encoder & Mapper Output for Single Coded Block

Gray Encoding is employed for QPSK modulation. Bit assignments of the QPSK modulation is shown in Figure 4.4.

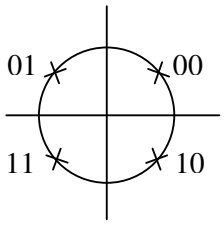

Figure 4.4 QPSK Bit Assignments

Assume that the m^{th} OFDM symbol, which corresponds to the m^{th} QPSK symbol from each Encoder & Mapper, produces a PAPR value that exceeds the threshold. m^{th} symbol consists of 4 complex numbers that are IFFT outputs. Assume that the complex number at index-1 of the m^{th} symbol, i.e. y[1], has the highest magnitude, hence causes a peak.

The operations performed to reduce this peak value are shown in Figure 4.5.

As given previously, OFDM modulation can be represented as

$$y[n] = \sum_{i=0}^{N-1} a_i \exp(j\frac{2\pi in}{N}), \qquad 0 \le n \le N-1 \tag{4.1}$$

where a_i's are QPSK modulator outputs. Using (4.1), y[1] can be written explicitly as

$$y[1] = a_0 + a_1 \exp(j\frac{\pi}{2}) + a_2 \exp(j\pi) + a_3 \exp(j\frac{3\pi}{2}). \tag{4.2}$$

Assume that a_i's are the solid phasors as shown in Figure 4.5 on the *Before Modulation* column. Then y[1] is the sum of 4 components shown with solid arrows in *After Modulation* column which are just the shifted versions of a_i's appropriate to Equation (4.2).

Examining the *After Modulation* column, it can be seen that rotating the second component in the indicated direction results in a decrease in y[1] amplitude. The rotation of 90 degrees in the clockwise direction can be reflected prior to the modulation side as shown in *Before Modulation* column of Figure 4.5 with dashed arrow. Using QPSK Bit Assignments it is observed that the given rotation can be achieved by changing second Encoder & Mapper output from '10' to '11' which results in 1-bit modification of the input data.

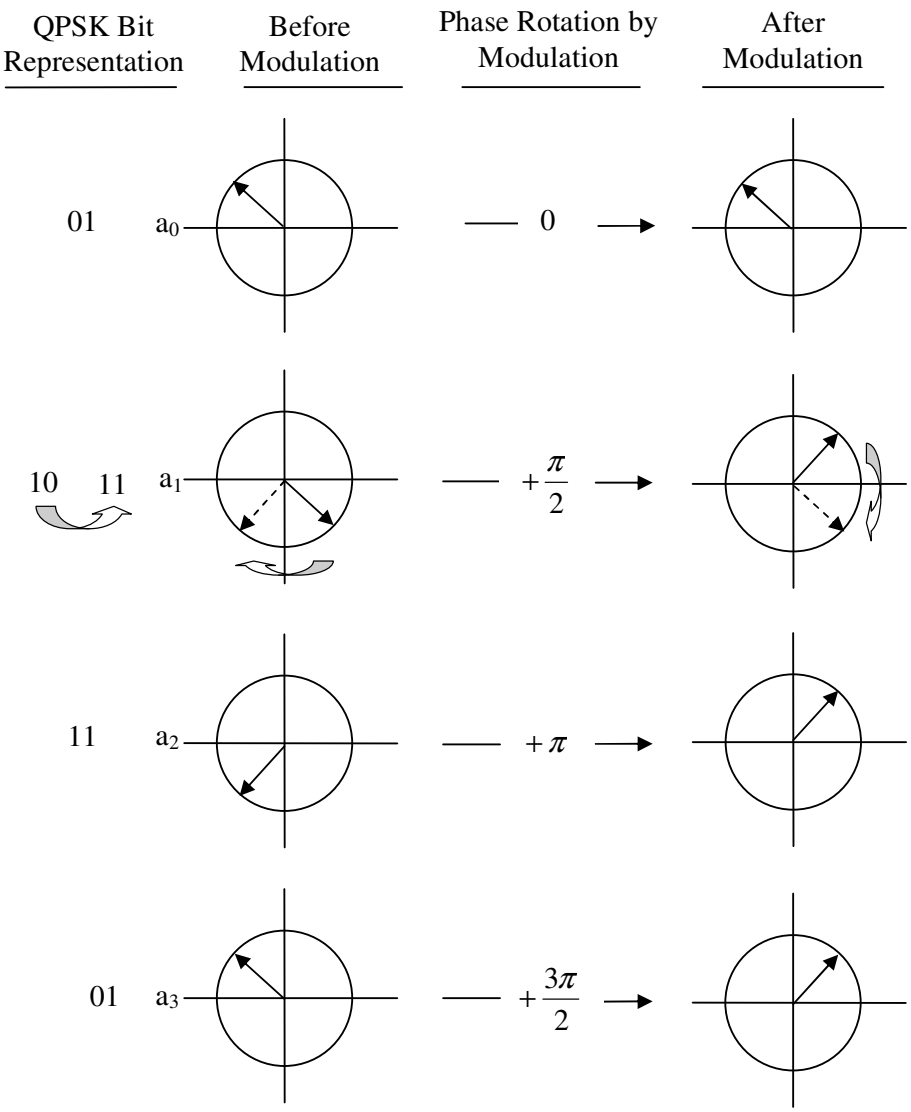

Figure 4.5 QPSK Modification for PAPR Reduction

After modifications of Figure 4.5 are applied on the m^{th} OFDM symbol, the amplitude of the y[1], which caused the high PAPR value, is changed as shown in Figure 4.6.

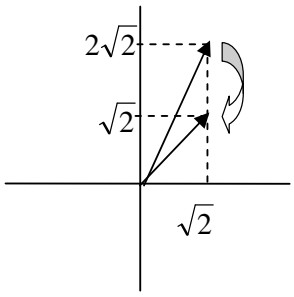

Figure 4.6 Amplitude Reduction of y[1]

As described in previous pages, the main principle of the proposed algorithm is to modify QPSK modulator outputs in a way to reduce the PAPR value of the OFDM symbol. Modification of modulator outputs results in a change of information to be transmitted. Encoders are used to eliminate this effect. Error correcting codes make the system resistant to bit errors. Therefore, if the number of bits changed at the transmitter per coded block is below the error correcting capability of the decoder, information should be received correctly at the receiver. However, it should be noted that the main objective of the error correcting code employed is to reduce the PAPR value without loss of information. Therefore, if errors occurred during transmission over the channel are to be compensated, an outer code should be used together with an interleaver or a scrambler. In simulations, only 1 bit modification per coded block is allowed since more bit modifications result in unacceptable SNR degradation.

In order to explain the method better, two flow charts are provided. Figure 4.7 shows a general flow chart of the method. Details of the modification block are given in Figure 4.8. The proposed algorithm is implemented in the modification block.

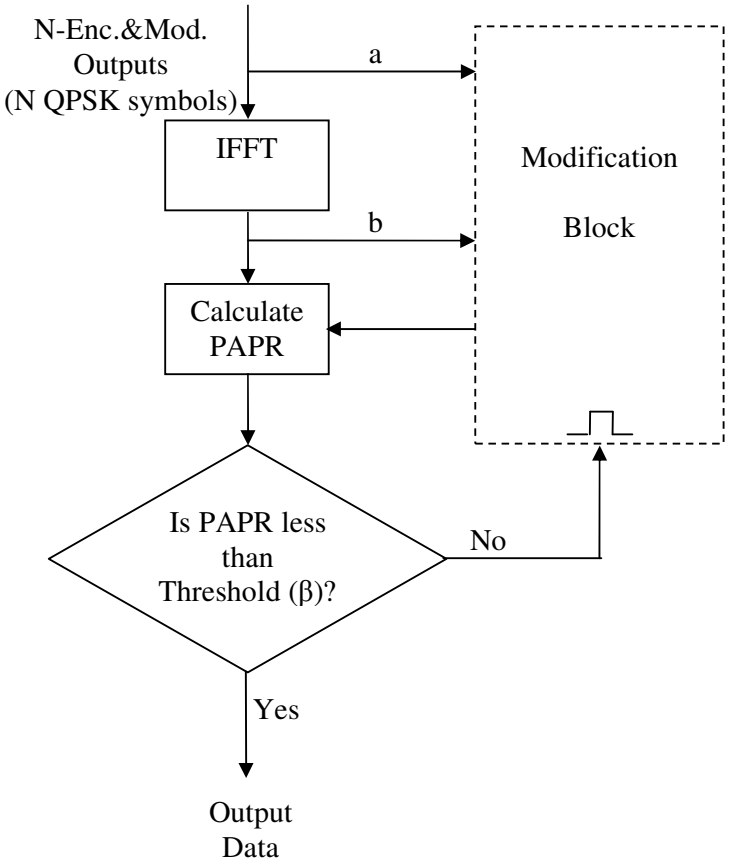

Figure 4.7 Algorithm Flow Chart 1/2

As seen in Figure 4.7, the algorithm block is enabled when the OFDM symbol PAPR value exceeds the threshold value. The block uses OFDM symbol with high PAPR value and its components before the IFFT block. The algorithm is implemented in an iterative manner until the PAPR value is reduced below the desired value.

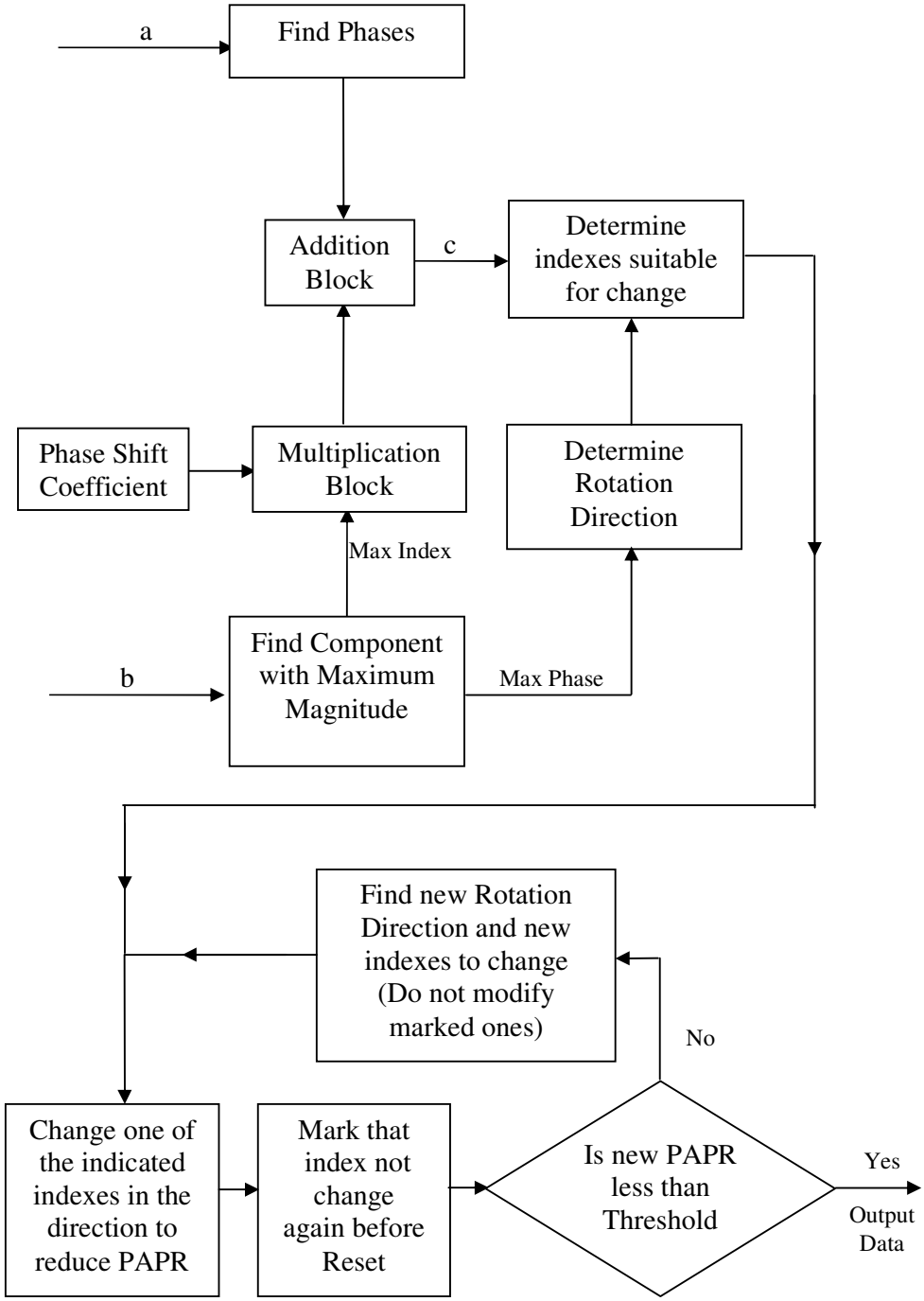

Figure 4.8 Algorithm Flow Chart 2/2

The modification block is described in detail in Figure 4.8. When a symbol PAPR exceeds the threshold value, the index of OFDM symbol component; i.e. IFFT output with maximum amplitude is determined which corresponds to parameter *n* of Equation (4.1). The phase of this component is calculated to find out its direction on the x-y plane. This information is used to determine which rotation of QPSK symbols should be most beneficial for the reduction of PAPR value. Another input to the modification block is the Encoder & Mapper outputs prior to IFFT block that are a_i's of Equation (4.1) given previously as

$$y[n] = \sum_{i=0}^{N-1} a_i \exp(j\frac{2\pi in}{N}),\ 0 \le n \le N-1.$$

Using the index of the high PAPR creating component as parameter *n* of Equation (4.1), phase rotations on a_i's caused by IFFT are determined. Another set of parameters labeled as c_i's ($0 \le i \le N-1$) are found out at this stage and shown in Figure 4.8. These parameters represent the phases of the phasors, summation of which gives the IFFT output resulting in high PAPR. Since the most beneficial rotation was determined previously, c_i's that can be rotated in the desired direction are marked. Then, the marked c_i's are rotated one by one until the PAPR value is reduced below the threshold. The beneficial rotation and suitable c_i's for rotation are recalculated after each iteration, because each rotation may change the direction of the maximum amplitude component. Additionally, rotation of c_i's are achieved by rotation of a_i's; that is, QPSK modulator outputs. Another critical point is the number of rotations per Encoder & Mapper block outputs. In the simulated system, only one bit modification per coded block is allowed, therefore when a QPSK symbol is rotated, its Encoder & Mapper is marked as 'outputs can not be changed until the next coded block'. This marking process guarantees that no more than 1 bit will be changed for an encoded block of n bits.

4.3. Fundamental Simulation Parameters

4.3.1. Index-to-Change

As described previously an OFDM signal is the IFFT of N modulated symbols. In this thesis, QPSK modulation is used in simulations. The suggested algorithm is based on the rotation of QPSK symbols in order to reduce the PAPR of the OFDM signal using the error correction capability of a linear block code. The first step is determination of the peak causing phasor when a high PAPR value is encountered. Then direction of this component is calculated (angle and quadrant of the complex number). The direction information is used to determine which of the QPSK symbols may be changed to reduce PAPR value of the OFDM symbol. The parameter named as *Index-to-change* is the indexes (in the range 1 to N) of QPSK symbols selected for modification. However, there is a restriction for the modification of QPSK symbols. Only 90° rotations are allowed. Since Gray encoding is employed a 90° rotation results in 1 bit change. In other words, the algorithm allows only 1 bit change per coded block. This restriction is necessary to limit SNR increase due to codeword modification. After a QPSK symbol is rotated, its Encoder & Mapper block is marked in order not to rotate any other QPSK symbol during that codeword time to satisfy 1 bit modification per coded block limitation. When a new codeword arrives at Encoder & Mapper blocks, all the marks are removed and all new QPSK symbols are ready to change if selected by the algorithm.

4.3.2. Rotation Direction

Another important parameter used in the algorithm is the *Rotation Direction.* This parameter indicates the quadrant and angle of the phasor that results in high PAPR As described in the previous sections, the phasor resulting high PAPR is the sum of phase shifted versions of N QPSK symbols. In order to reduce the amplitude of this component, some of the N vectors contributing to the sum, will be moved in

the opposite direction. Therefore, it is necessary to determine the direction of the phasor that causes the problem. In the algorithm, when a symbol with high PAPR is met, the direction of the high PAPR causing phasor is determined. Then the *Rotation Direction* parameter is found out, which is only the opposite of the peak causing phasor direction. Then one of the suitable QPSK symbols is rotated in the desired direction. After this step, *Rotation Direction* parameter is recalculated if the PAPR value is still higher than the threshold. Calculation is necessary at each step, since a rotation of one of the QPSK symbols may take the PAPR causing component from one quadrant to the other still with a high PAPR value.

Figure 4.10 shows the same case as in Figure 4.9. Again the same OFDM symbol with a PAPR value higher than the threshold is determined. During OFDM symbol time, *Rotation Direction* is recalculated at each step. The *Rotation Direction* parameter used in Figure 4.10 is coded as follows:

1: Rotate in +x direction (Transition from 2^{nd} quadrant to 1^{st} or 3^{rd} quadrant to 4^{th})
2: Rotate in -x direction (Transition from 1^{st} quadrant to 2^{nd} or 4^{th} quadrant to 3^{rd})
3: Rotate in +y direction (Transition from 4^{th} quadrant to 1^{st} or 3^{rd} quadrant to 2^{nd})
4: Rotate in -y direction (Transition from 1^{st} quadrant to 4^{th} or 2^{nd} quadrant to 3^{rd})

In order to clarify this point, an example is given: If the rotation direction is calculated as 2, a rotation in –x direction is necessary. Therefore, QPSK symbols in the 1^{st} and 4^{th} quadrant may be moved to 2^{nd} and 3^{rd} quadrant respectively, to reduce PAPR. For this case, all of the 1^{st} and 4^{th} quadrant components are marked as *suitable-to-change* and one of these components, that is, an element of one of the non-marked Encoder & Mapper block, is rotated in the desired direction.

4.3.3. PAPR

The most important performance parameter is the Peak-to-Average Power Ratio (PAPR) in this thesis' context. In the literature the PAPR of the baseband OFDM signal is defined as [12], [28], [29]

$$PAPR = \max_{0 \le t < T} \frac{|\, s(t) \,|^2}{E\left\langle |\, s(t) \,|^2 \right\rangle} \tag{4.3}$$

where s(t) is the baseband OFDM signal and T is the OFDM symbol duration.

As mentioned in [12], the PAPR of s(t) can be estimated from its LN samples where L is called the oversampling factor. These samples may be obtained from the LN-point IFFT of the QPSK modulated symbols with (L-1)N zero padding. In [30], it is shown that if no oversampling is used, then, the difference between the continuous time PAPR and the discrete time PAPR of an OFDM signal can be as high as 1 dB. However, when an oversampling factor of 4 is used, the difference is negligible. Therefore, it may be concluded that an oversampling factor of 4 is enough to estimate the continuous time PAPR given in (4.3).

Using the above defined relations, the PAPR of an OFDM symbol is calculated as follows

$$PAPR = \max_{0 \le k \le 4N} \frac{|\, A_k \,|^2}{\sum_{i=1}^{4N} |\, A_i \,|^2 \,/\, 4N} \quad . \tag{4.4}$$

System PAPR is the maximum of the calculated symbol PAPR's during the operation. As can be seen from (4.4), L=4 oversampling factor is used in simulations and A_k's are the 4N-point IFFT samples of the Encoder & Mapper block outputs, that is IFFT of the QPSK modulated symbols.

Figure 4.9 through Figure 4.15 show some simulation results. The simulations are carried out in MATLAB SIMULINK environment. Two simulation models are created, first one is the model of a 32-channel OFDM system and the second is the model of a 128-channel system. The *Index-to-change* and the *Rotation Direction* parameters are recalculated after each QPSK symbol modification. In addition, system variables are reset at the beginning of a new coded block.

In the SIMULINK models, blocks from Communications Blockset and DSP Blockset are used besides general SIMULINK blocks. The random binary generation, BCH encoding, QPSK modulation and IFFT operations are performed using SIMULINK libraries. In order perform some special functions, look-up tables and S-functions are created. Additionally, C-MEX S-functions are written to handle functions having complex inputs and outputs. Moreover, Stateflow components are used to generate some special trigger signals and control logic necessary for the system.

Figure 4.9 shows PAPR of a 32-channel OFDM system without the algorithm implementation. PAPR is given in decibels. In the simulations, the OFDM symbol duration is 0.1 seconds. Therefore, there are 15 000 OFDM symbols in the below figure. The solid line shows the threshold value that will be used during algorithm implementation. As described previously, the PAPR of an OFDM system with N channels can reach up to a value of N ($10\log_{10}$N dB). Thus, for a 32-channel system, the theoretical maximum value of PAPR is 32 (15.05 dB), however, the probability of observing such high PAPR values is very small. (This issue will be investigated in detail in the next chapter.) The highest PAPR value seen in Figure 4.9 is 10.83 dB, which is the system PAPR value.

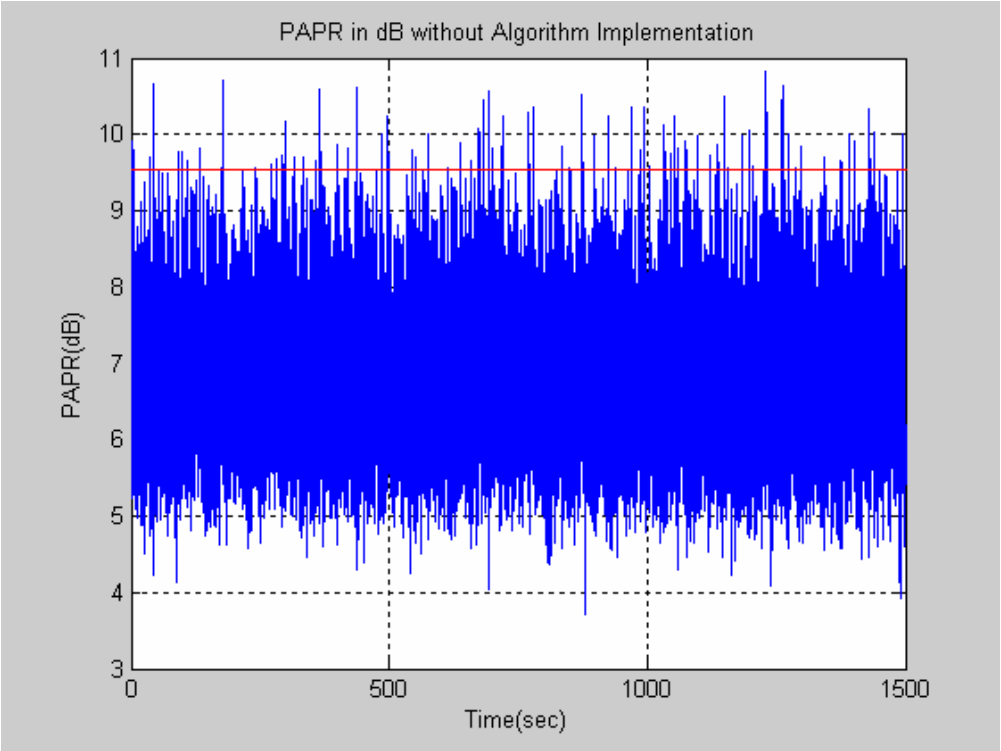

Figure 4.9 PAPR (in dB) of an OFDM System without Algorithm Implementation

Simulations are repeated for the same information data sequence while implementing the algorithm. This case is shown in Figure 4.10. Again there are 32 channels in the system and OFDM symbol duration is 0.1 seconds.

In Figure 4.10, the threshold is set to 9 (9.54 dB). From the figure, it is seen that the algorithm succeeded to lower the PAPR value below the threshold and the system PAPR value is reduced to 9.54 dB. (A method will be suggested in the next chapter in order to choose a suitable threshold value.) The threshold value used in the simulations is lower than the suggested value. However, it is chosen to provide easier visualization of the operation of the algorithm.

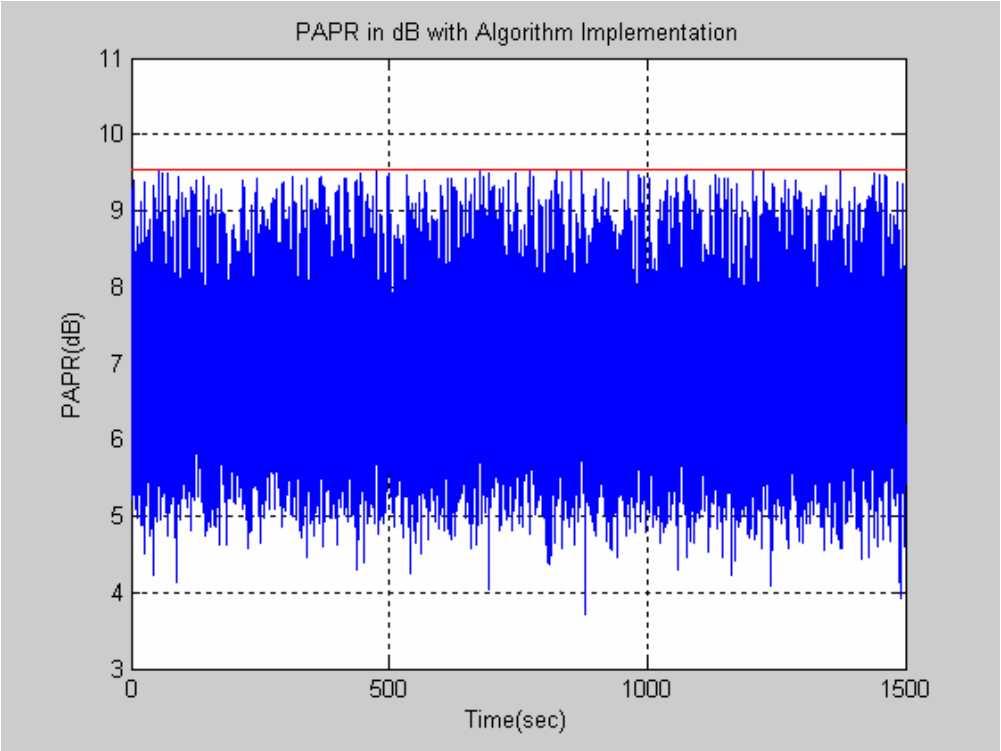

Figure 4.10 PAPR (in dB) of an OFDM System with Algorithm Implementation

It will be helpful to demonstrate how the simulation parameters change simultaneously while the PAPR reduction algorithm works on a symbol. The PAPR value of an OFDM symbol is reduced step-by-step and at each step the *Index-to-Change* and *Rotation Direction* parameters are recalculated. The PAPR values obtained by the ordinary OFDM modulation were much smaller than the theoretical maximum value of the PAPR because of the probabilistic behavior of this parameter. Consequently, a new simulation model was built. The aim of the model is to generate an extreme condition where the OFDM symbols with higher PAPR values are produced and the algorithm is operated on these symbols. For this purpose, some of the Encoder & Mapper outputs are modified by adding a certain bit pattern at the end of randomly generated input data. This operation made it possible to observe a PAPR value of 28.12 periodically. Figure 4.11 shows the intermediate steps of the algorithm while working on one of these symbols. For the sample system, at most 25 iterations are allowed during a symbol period. In the

simulated case, the initial PAPR value is 28.12 and at each iteration PAPR is reduced step-by-step. After 14 steps of implementation the PAPR value is reduced below the threshold value of 10 used in these simulations. After the PAPR is reduced below the threshold value, the obtained PAPR value is held by the system.

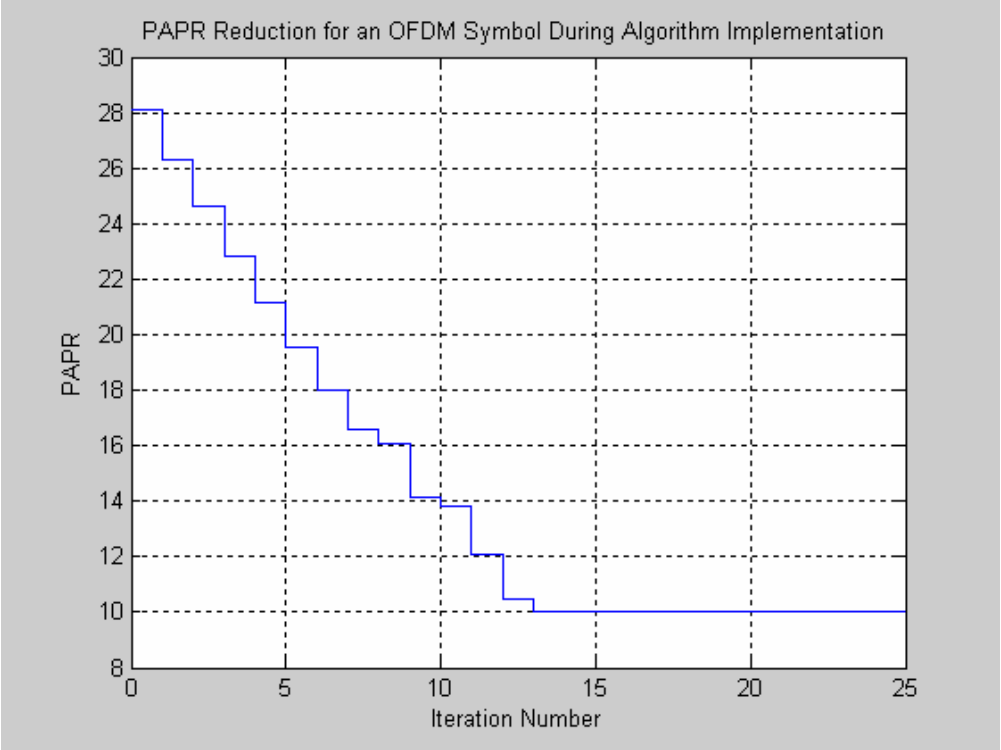

Figure 4.11 Step-by-Step PAPR Reduction During Algorithm Implementation

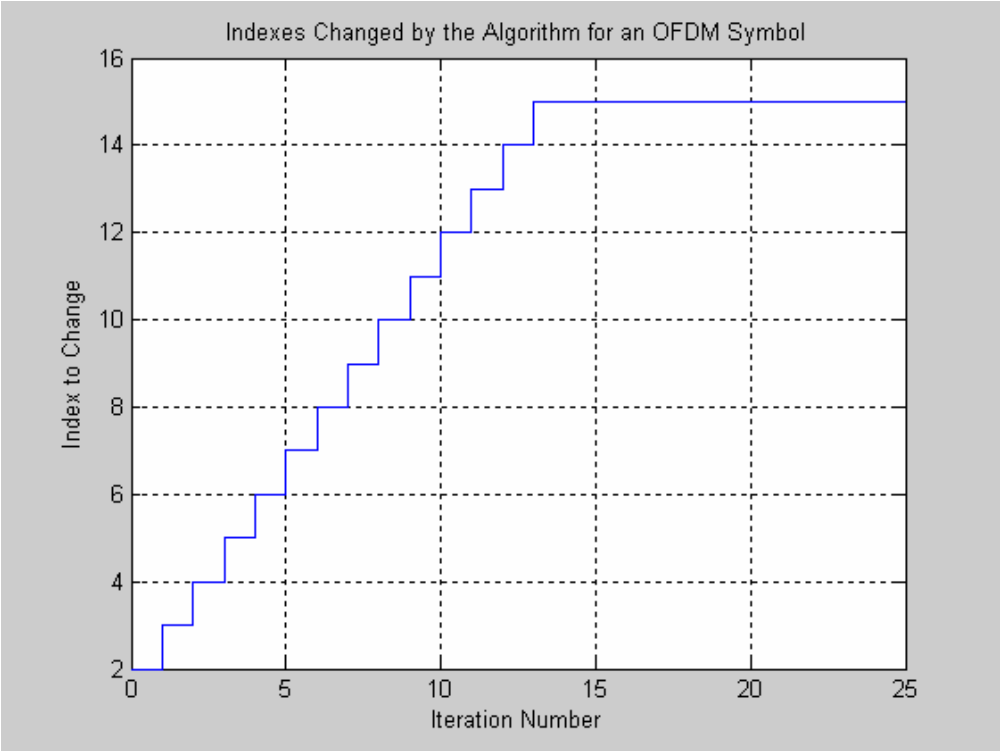

Figure 4.12 *Index-to-Change* Parameter for the Sample OFDM Symbol

Figure 4.12 shows the sample case for *Index-to-Change* parameter determination. In Figure 4.12, the algorithm determines a symbol with a PAPR value higher than the threshold and modifies QPSK symbols belonging to that symbol during symbol duration. After each iteration, an index is marked as *suitable-to-change*. In the above case, initially 2nd QPSK symbol is rotated, and then 3rd, 4th etc. indexed QPSK symbols are selected. After 14 modifications out-of-32 candidates, the PAPR is reduced below the threshold and the algorithm stops working.

Figure 4.13 shows how the *Rotation Direction* changes as the algorithm proceeds. *Rotation Direction* parameter is initially calculated as 1, which indicates a +x rotation is needed to reduce PAPR. Then, this parameter is recalculated as 3 after 2 rotations and so on.

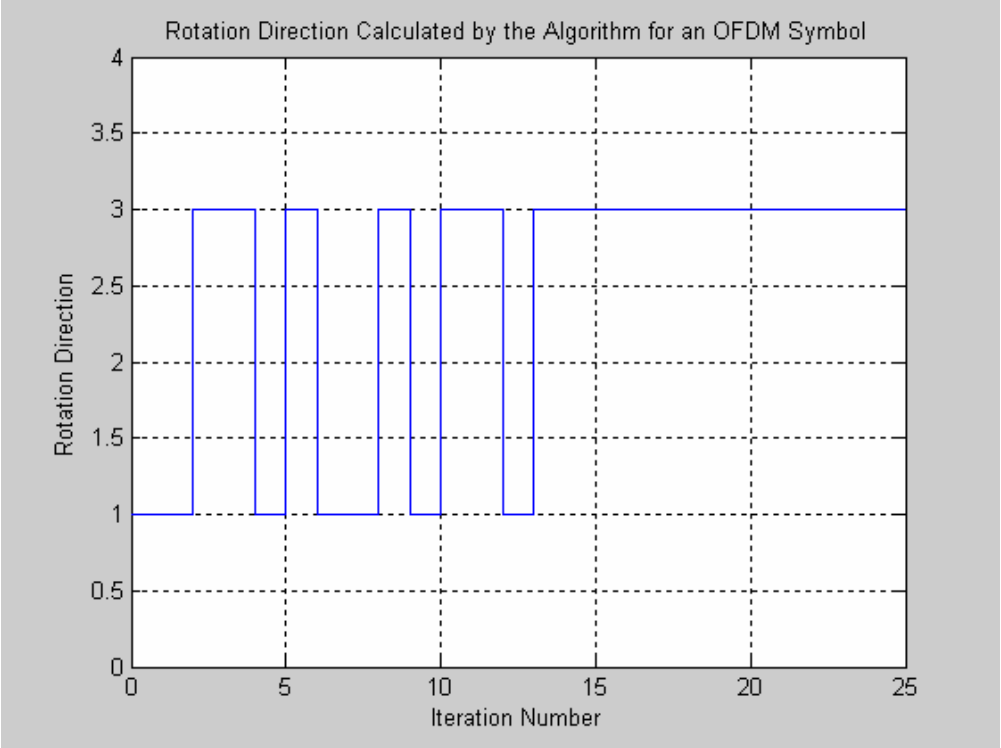

Figure 4.13 *Rotation Direction* Parameter for a Sample OFDM Symbol

4.4. Simulation Results of PAPR Reduction

Figure 4.9 and Figure 4.10 show a case of PAPR reduction, however, the PAPR reduction is low since it could not be possible to observe a high PAPR value. As described in the previous section, in order to give a real indication of the algorithm performance, high PAPR value producing OFDM symbols are injected periodically into the simulated system and the algorithm behavior is observed. Figure 4.14 shows the case for a 32-channel system. In the simulated case, a PAPR value of 28.12 (14.49 dB) is obtained which is close to the theoretical maximum of a 32-channel system and the simulation results show that this value is reduced to a value of 10.7 (10.29 dB) after applying the algorithm.

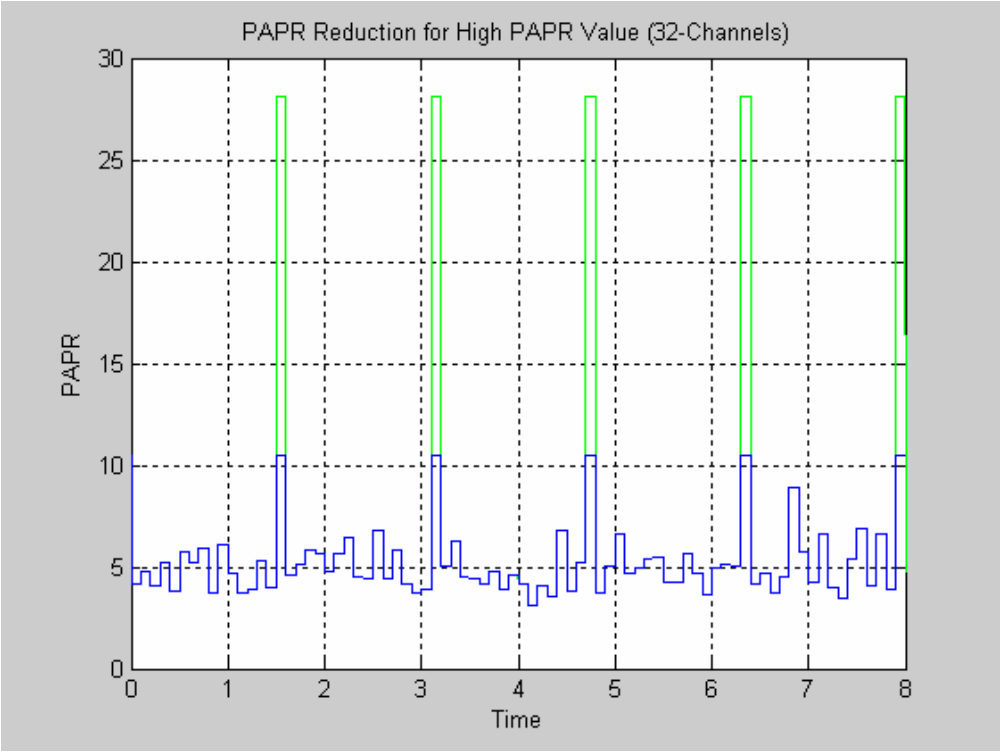

Figure 4.14 PAPR Reduction for 32-Channel OFDM System

The second model used in simulations is a 128-channel OFDM system employing a (127,106) 2-error correcting BCH code. Again, high PAPR generating OFDM symbols are generated and algorithm performance is observed. The simulation results are given in Figure 4.15. As can be seen from the figure, a high PAPR value of 112.5 (20.51 dB) is reached and the algorithm reduced this PAPR value to a level of 18 (12.55 dB) achieving a PAPR reduction of approximately 8 dB.

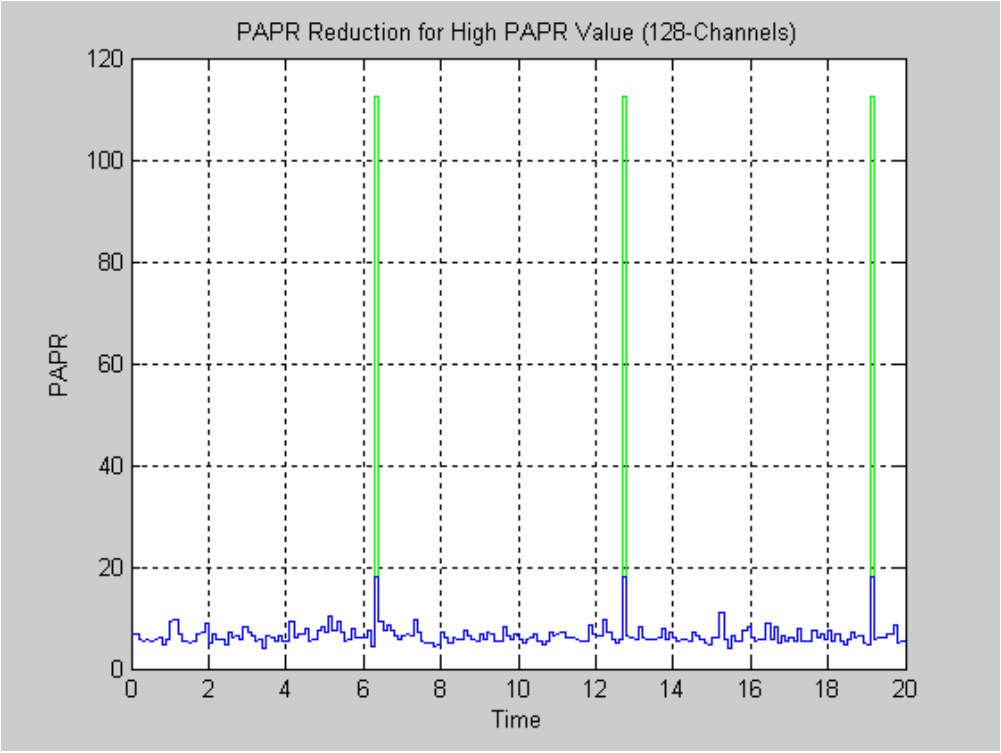

Figure 4.15 PAPR Reduction for 128-Channel OFDM System

CHAPTER 5

PERFORMANCE OF THE CODING BASED PAPR REDUCTION ALGORITHM

The algorithm suggested in Chapter 4 reduces the PAPR of an OFDM modulating system. However, the algorithm necessitates the modification of codewords which results in an increase of the input Signal-to-Noise Ratio (SNR) in order to satisfy the same information bit error probability as the simple OFDM system. Therefore, the performance of the system implementing the PAPR reduction algorithm depends not only on the PAPR reduction but also the SNR increase of the system to maintain a certain BER which degrades the performance.

While examining the work done in the literature, it has been observed that there is a lack of suitable performance measure criteria. The main criterion used by the suggested methods is the achieved PAPR reduction, however, each of the methods has some drawbacks and these effects should be considered to evaluate the real performance of the methods. In this chapter, we propose a performance evaluation criterion for PAPR reduction systems that takes the SNR loss into account.

In this chapter, two different systems will be considered; first system is the plain OFDM system and the second system uses the algorithm described in Chapter 4. First, general comparison criteria will be introduced. Then, coded OFDM operation will be explained and the Probability Density Function (pdf) of OFDM signal will be derived. Next, the bit error probability, coded block error probability and Signal-to-Noise Ratio relations will be derived for the two systems. Finally, the gain parameter that gives the overall system performance will be derived, and some gain parameter calculations for different system parameters will be given.

5.1. General Comparison Criteria

The parameters used for system evaluation are the information bit rate R_b, receiver noise level N_0 and the transmit power. Two systems are compared for equal information bit error probabilities. For the two systems with equal R_b, N_0 and information bit error probabilities, the system with lower transmit power will be evaluated as the system with better performance. A gain parameter will be derived using these parameters and two systems will be compared based on the ratio of their gain parameters.

Assume that there are two systems, first is the plain OFDM modulating system (Sys1) and the second system (Sys2) implements the suggested algorithm. In order to compare two systems on a fair basis, a very low clipping rate will be assumed for both Sys1 and Sys2. Since the OFDM technique is used mainly for video and audio transmission, having a sufficiently low rate clipping will not cause a serious problem. The calculations in this chapter are performed for 1 clipping-per-day assumption. However, it is possible to repeat calculations for different clipping rates.

5.2. Coded OFDM Operation

Assume that the OFDM system has N channels. In the sample OFDM transmitter used during simulations, there are N Encoder & Mapper blocks. Each Encoder & Mapper uses (n,k) coding; i.e. each k bits are coded into n bits. Then a parity bit is produced and added at the end of n bits. Next, these (n+1) bits are used to modulate the subcarriers (QPSK, QAM etc.) and finally N-point IFFT is computed in order to form OFDM symbols. The block diagram of the sample transmitter is given in Figure 5.1.

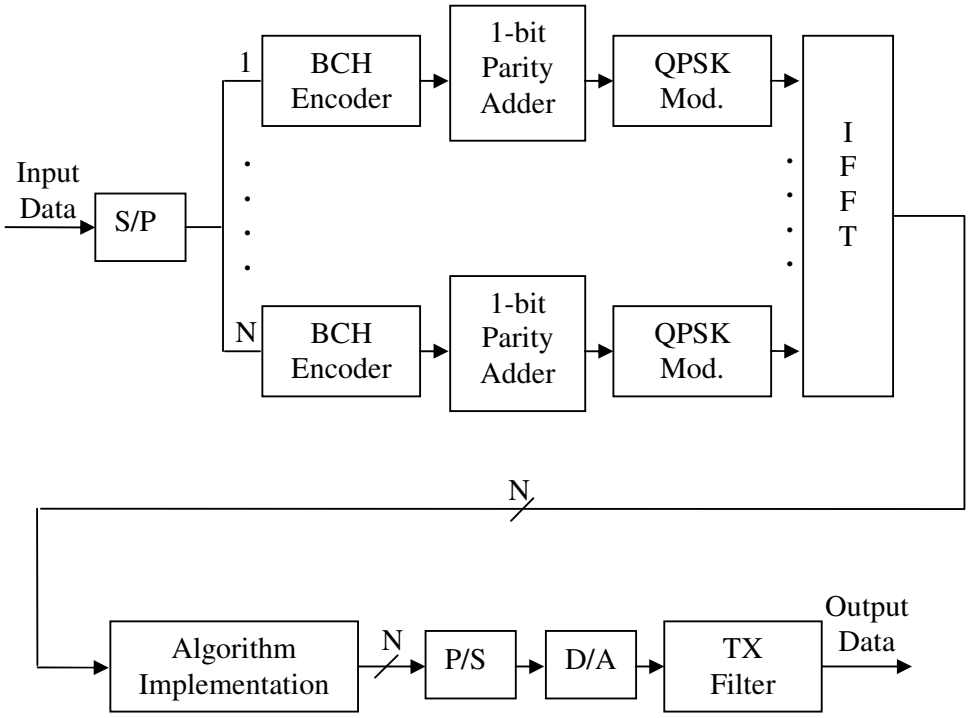

Figure 5.1 Block Diagram of the Sample OFDM Transmitter

In order to derive the relationship between the OFDM symbol period and the information bit rate, timing diagrams of the sample system will be provided. Figure 5.2 shows the input data to the transmitter that will be distributed among Encoder & Mapper blocks.

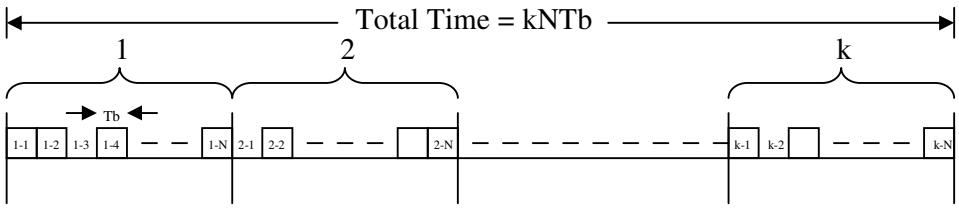

Figure 5.2 OFDM Transmitter Input Block

In Figure 5.2, N denotes the number of channels, T_b denotes the information bit period and k denotes the uncoded block width to be coded into n-bits by the encoder. After kN bits are collected at the input, these bits will be distributed among N-Encoder & Mapper blocks which will form N-*PreCode Blocks*. The procedure of bit distribution is given in Figure 5.3.

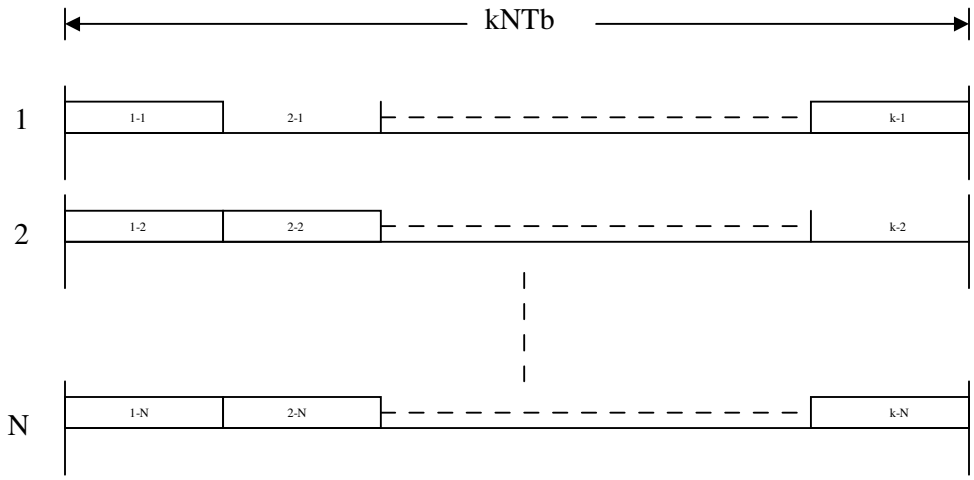

Figure 5.3 Construction of N *PreCode Blocks*

After coding, adding parity and modulating (QPSK, QAM etc.) each of the k-bit blocks produce (n+1) bit blocks with one parity bit added. Finally, (n+1)/L modulated symbols are produced from each of the k-bit blocks where L is the modulation width (e.g. L=2 for QPSK). Each of these (n+1)/L modulated symbols at each of the Encoder & Mapper blocks will be called as a *Coded Block* during rest of this chapter. Since a modulated symbol from each of the N Encoder & Mapper block produces an OFDM symbol after IFFT stage, (n+1)/L OFDM symbols are generated from kN input bits. This procedure is shown in Figure 5.4.

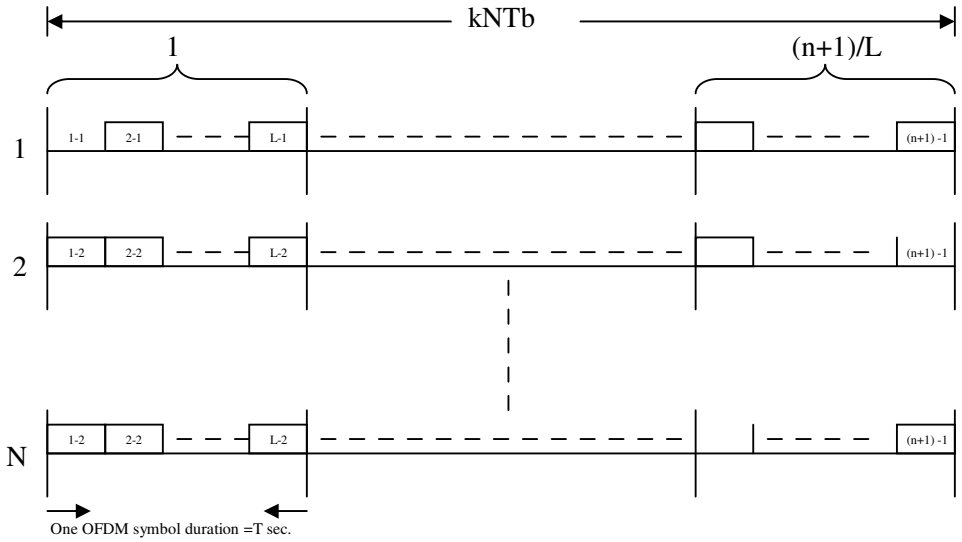

Figure 5.4 Construction of N Coded Blocks

The block diagram of the receiver for the proposed system is given in Figure 5.5.

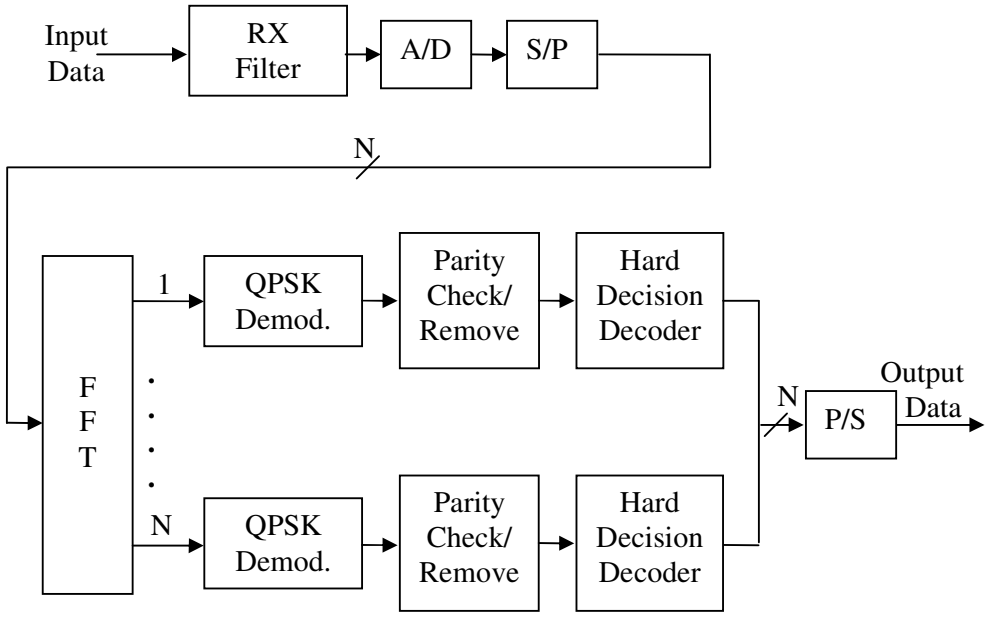

Figure 5.5 Block Diagram of the Sample OFDM Receiver

5.3. PDF (Probability Density Function) of Instantaneous Power of OFDM Signal

Let $z=x_R + j\ x_I$ be one of the samples of an OFDM symbol. Assume, x_R and x_I are statistically independent and identically distributed Gaussian random variables with zero mean and variance σ^2. Then, the instantaneous power of this sample can be defined as a random variable y as

$$y=|z^2| = x_R^2 + x_I^2 . \quad (5.1)$$

The expected value of y is:

$$E<y>=E< x_R^2 + x_I^2> = E< x_R^2> + E< x_I^2> . \quad (5.2)$$

Variances of x_R and x_I are by definition:

$$Var\ (x_R) = E < x_R^2> - [E< x_R >]^2, \quad (5.3)$$

$$Var\ (x_I) = E < x_I^2> - [E< x_I >]^2. \quad (5.4)$$

Then;

$$Var\ (x_R) = E < x_R^2> = \sigma^2, \quad (5.5)$$

$$Var\ (x_I) = E < x_I^2> = \sigma^2. \quad (5.6)$$

Therefore;

$$E<y> =2\sigma^2 = P_{mean} \quad (5.7)$$

Due to the characteristics of x_I and x_R that were assumed before, the random variable "y" has a central chi-square distribution [13]:

$$P_y(y) = \frac{1}{\sigma^n 2^{n/2}\Gamma(n/2)} y^{(n/2)-1} e^{-y/2\sigma^2} \quad y \geq 0; \qquad \text{for} \qquad y = \sum_{i=1}^{n} x_i^2 \tag{5.8}$$

Since n=2 in our case;

$$P_y(y) = \frac{1}{2\sigma^2\Gamma(1)} y^0 e^{-y/2\sigma^2}. \tag{5.9}$$

$\Gamma(p)$ is the gamma function defined as [13]:

$$\Gamma(p) = (p-1)! \quad , p \text{ an integer } >0. \tag{5.10}$$

Then the pdf of OFDM instantaneous power may be expressed as

$$P_y(y) = \frac{1}{2\sigma^2} \; e^{-y/2\sigma^2} = \frac{e^{-y/P_{mean}}}{P_{mean}}. \tag{5.11}$$

It follows that the probability that the instantaneous power is greater than β times its mean power turns out to be

$$P(y \geq \beta.P_{mean}) = \int_{\beta.P_{mean}}^{\infty} P_y(y)dy = \int_{\beta.P_{mean}}^{\infty} \frac{e^{-y/P_{mean}}}{P_{mean}} = e^{-\beta}. \tag{5.12}$$

Therefore;

P(A sample PAPR exceeds certain threshold β)= $e^{-\beta}$. (5.13)

Since an OFDM symbol consists of N samples;

P(PAPR not exceeding the threshold for one OFDM symbol) $= (1-e^{-\beta})^N$ (5.14)

where N is also the number of subcarriers.

Equation (5.14) is valid under the assumption that the samples are mutually uncorrelated. However, as described in Chapter 4, in order to calculate discrete time PAPR more accurately, at least a 4-times oversampling is necessary. And samples become correlated when oversampling is applied. In [2] it is explained that the distribution for N subcarriers and oversampling can be approximated by the distribution for αN subcarriers without oversampling, with α larger than one. Hence, the effect of oversampling is approximated by adding a certain number of independent samples. The parameter α is determined to be 2.8 with computer simulations in [2]. This result lacks theoretical justification, however; it is widely used in the literature and will also be used in this thesis. Accepting this argument, (5.14) may be rewritten as:

$$\text{P(PAPR not exceeding } \beta \text{ in one OFDM symbol)} = \left(1 - e^{-\beta}\right)^{2.8N} \qquad (5.15)$$

Then;

$$\text{P(PAPR exceeding threshold in one OFDM symbol)} = 1 - \left(1 - e^{-\beta}\right)^{2.8N} \qquad (5.16)$$

5.4. Bit Error Probability, Block Error Probability and SNR Loss Relations for Sys1

In the previous chapter, the simulations for a 32-Channel OFDM system were given and the coding scheme was (31,21) 2-error correcting BCH code. The next step is to examine the bit error probability, the block error probability and the SNR loss relations. The simulated system parameters will be used for the derivation of the mentioned relations.

Assume that Sys1 is the plain OFDM modulating system that uses the (31,21) 2-error correcting BCH code. In this section, the relation between bit error probabilities and block error probabilities of plain OFDM system will be derived.

The coded bit error probability for Sys1 can be expressed as [13]:

$$Pb_1 = Q\left(\sqrt{2\frac{E_c}{N_{01}}}\right) \quad , \tag{5.17}$$

where $\frac{E_c}{N_{01}}$ denote the Signal-to-Noise Ratio (SNR) per coded bit.

Since there are *(n+1)* bits per codeword after parity generation and each codeword conveys *k* bits of information, the SNR per coded bit can be given in terms of SNR per information bit as [13]:

$$\frac{E_c}{N_{01}} = \frac{k}{(n+1)}\frac{E_b}{N_{01}}. \tag{5.18}$$

(5.17) can be rewritten as:

$$Pb_1 = Q\left(\sqrt{2\frac{k}{(n+1)}\frac{E_b}{N_{01}}}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{k}{(n+1)}\frac{E_b}{N_{01}}}\right). \tag{5.19}$$

For the case of (31,21) BCH code:

$$Pb_1 = \frac{1}{2}erfc\left(\sqrt{\frac{21}{32}\frac{E_b}{N_{01}}}\right). \tag{5.20}$$

The probability of receiving a codeword in error will be defined as the *block error probability*. In the sample system, the generated parity bit is not used at the receiver. The only purpose of adding a parity bit is to obtain an even number of bits before QPSK modulation. Another solution to this problem may be employing BPSK modulation for the last remaining bit of the encoder output. However, we decided on using a parity bit and employing QPSK modulation for all encoder

output bits. Therefore, for a t-error correcting (n,k) code (we consider a hard-decision detection at the receiver):

$P_{e,block}$=P(number of bit errors ≥ (t+1) in n bits)

=1-P(t bit error + (t-1) bit error + …….+ 1 bit error + no error in n bits).

Then;

$$P_{e,block} = 1 - \left[\begin{matrix} \binom{n}{t} Pb_1^{\,t} (1-Pb_1)^{n-t} + \binom{n}{t-1} Pb_1^{\,t-1} (1-Pb_1)^{n-t+1} + \ldots \\ \ldots + nPb_1 (1-Pb_1)^{n-1} + (1-Pb_1)^n \end{matrix} \right]. \qquad (5.21)$$

For the sample case of (31,21) 2-error correcting code

$P_{e,block}$=P(number of bit errors ≥ 3 in 31 bits)

=1-P(2 bit error + 1 bit error + no error in 31 bits).

Then;

$$P_{e,block} = 1 - \left[\binom{31}{2} Pb_1^{\,2} (1-Pb_1)^{29} + 31Pb_1 (1-Pb_1)^{30} + (1-Pb_1)^{31} \right] \quad . \qquad (5.22)$$

Using polynomial approximations:

$$P_{e,block} \cong 1 - \left[\begin{matrix} \frac{31.30}{2} Pb_1^{\,2} \left(1 - 29Pb_1 + \frac{29.28}{2} Pb_1^{\,2} - \ldots \right) \\ + 31Pb_1 \left(1 - 30Pb_1 + \frac{30.29}{2} Pb_1^{\,2} - \frac{30.29.28}{3!} Pb_1^{\,3} + \ldots \right) \\ + \left(1 - 31Pb_1 + \frac{31.30}{2} Pb_1^{\,2} - \frac{31.30.29}{3!} Pb_1^{\,3} + \frac{31.30.29.28}{4!} Pb_1^{\,4} - \ldots \right) \end{matrix} \right]$$

From the above equation we find out that:

$$P_{e,block} \cong 4{,}495x10^{3} Pb_1^{\ 3} - 9{,}44x10^{4} Pb_1^{\ 4} \tag{5.23}$$

for the sample case of (31,21) 2-error correcting code employed at Encoder & Mapper blocks.

5.5. Choosing Transmitter Threshold for Sys1

As described previously, in order to compare the two systems, a clipping rate of 1 per day will be used for calculations. Therefore, it is necessary to set the PAPR threshold value appropriately using this constraint. Timing diagram of Figure 5.4 will be used for this purpose. As seen from the figure, the relation between information bit duration (Tb) and OFDM symbol duration (T) is as follows:

$$kNT_b = \frac{n+1}{L} T \, . \tag{5.24}$$

Thus OFDM symbol duration turns out to be

$$T = \frac{kNT_b L}{n+1} \quad . \tag{5.25}$$

The sampling period T_s is

$$T_s = \frac{T}{N} . \tag{5.26}$$

Then T_s is found as

$$T_s = \frac{kT_b L}{n+1} = \frac{kL}{(n+1)R_b} \, . \tag{5.27}$$

Define a new parameter N_{24} as the number of samples per day in an OFDM system. Since a day has 86400 seconds N_{24} may be expressed as

$$N_{24} = \frac{86400}{T_s} = \frac{86400}{\frac{kL}{(n+1)R_b}} = \frac{86400}{L}\frac{(n+1)}{k}R_b . \tag{5.28}$$

Let P_1 be the probability that a single sample exceeds the threshold. Since we expect that only one OFDM sample exceeds a certain threshold per day, P_1 may be roughly approximated as

$$P_1 \cong \frac{1}{N_{24}} . \tag{5.29}$$

As found from the probability density function of OFDM that:

$$\text{P(PAPR of an OFDM sample exceeds the threshold } \beta) = e^{-\beta} \tag{5.30}$$

Using equations (5.28), (5.29) and (5.30) together, a relationship is obtained between the code rate, information bit rate and the minimum transmitter threshold value to satisfy one clip per day constraint

$$e^{-\beta_{\min}} = \frac{kL}{86400(n+1)R_b} . \tag{5.31}$$

Therefore, the desired threshold value is obtained as

$$\beta_{\min} = \ln\left(\frac{86400(n+1)R_b}{kL}\right). \tag{5.32}$$

Note that the threshold value does not depend on the number of channels. This result may be explained in the way that for the same information bit rate as the

number of channels increase, less number of OFDM symbols are produced per unit time, at the same time the probability of having a high PAPR value increases because of the increase in number of channels. These two factors compensate each other. Additionally, (5.32) gives a minimum value of the PAPR threshold that will be used for calculations. It is possible to select a larger threshold value which reduces the clipping probability; despite degrades the advantage of applying the algorithm.

5.6. Bit Error Probability, Block Error Probability and SNR Loss Relations for Sys2

Sys2 is the sample system applying the PAPR reduction algorithm. Sys1 and Sys2 will be compared for equal information bit error probabilities. It can be assumed that receiving an erroneous block means receiving half of the bits in error. Therefore, information bit error probability approximately equals half of the block error probability. Since the block error probability of Sys1 was written in terms of its coded bit error probability in (5.21), we need to write the block error probability ($P_{e,block}$) in terms of coded bit error probability (Pb_2) for Sys2 and we can equate block error probabilities of the two systems.

The second system implements the PAPR reduction algorithm. Therefore, the block error probability depends on the probability of observing a PAPR value higher than the threshold during that block time. Assume that Sys2 employs a t-error correcting (n,k) code. Then, the term *block* denotes the coded block of *n* bits.

$P_{e,block}$=P(No high PAPR in the block) P(number of bit errors $\geq$ (t+1))
+P(1 high PAPR in the block) P(number of bit errors $\geq$ t)+…

Using the fact that the probability of more than 1 high PAPR in a block is approximately equal to zero, the above equation may be approximated as

$P_{e,block} \approx$ P(No high PAPR in the block) P(number of bit errors $\geq$ (t+1))
+P(1 high PAPR in the block) P(number of bit errors $\geq$ t). (5.33)

(It is assumed that modifying an OFDM block means modifying output data from each Encoder & Mapper block; i.e. worst case assumption)

Again using the fact that the probability of 2 or more samples with high PAPR values in a block approximately equals zero, it may be concluded that

P(No high PAPR in the block) + P(1 high PAPR in the block) $\approx$ 1. (5.34)

Additionally, if a coded block is modified due to a high PAPR, only 1 bit per coded block is modified which is the assumption used in Chapter 4.

(5.33) can be simplified, using some short form representations of the above probabilities and (5.34). Therefore, (5.33) can be expressed as

$$P_{e,block} = P_{NoPAPR} P2 + (1 - P_{NoPAPR}) P1 \quad (5.35)$$

where
P2 = P(number of bit errors $\geq$ (t+1) in n bits)
P1 = P(number of bit errors $\geq$ t in n bits)

The probability P2 is in the same form as in (5.21), only the coded bit error probability of Sys1 (i.e. Pb_1) should be replaced with the coded bit error probability of Sys2 (i.e. Pb_2). Hence, P2 is in the form

$$P2 = 1 - \left[\begin{matrix} \binom{n}{t} Pb_2{}^{t} (1 - Pb_2)^{n-t} + \binom{n}{t-1} Pb_2{}^{t-1} (1 - Pb_2)^{n-t+1} + \ldots \\ \ldots + nPb_2 (1 - Pb_2)^{n-1} + (1 - Pb_2)^{n} \end{matrix} \right] \quad (5.36)$$

P1 can be written as

P1 = P(number of bit errors $\geq$ t in n bits)

=1-P((t-1) bit error + (t-2) bit error + ….. + 1 bit error + no error in n bits).

Then;

$$P1 = 1 - \left[\begin{matrix} \binom{n}{t-1} Pb_2{}^{t-1} (1-Pb_2)^{n-t+1} + \binom{n}{t-2} Pb_2{}^{t-2} (1-Pb_2)^{n-t+2} \ldots \\ \ldots + nPb_2 (1-Pb_2)^{n-1} + (1-Pb_2)^n \end{matrix} \right] \tag{5.37}$$

In order to find the block error probability of the system using the PAPR reduction algorithm, the last step is to derive $P_{NoPAPR.}$:

P_{NoPAPR} = P(a block length of M OFDM symbols have low PAPR)

Using (5.15) in this context:

P(PAPR not exceeding β in one OFDM symbol) $= \left(1-e^{-\beta}\right)^{2.8N}$.

It may be easily seen that:

$$P_{NoPAPR} = \left(1-e^{-\beta}\right)^{2.8NM} \tag{5.38}$$

where β is the PAPR threshold of the system, N is the number of channels and M=(n+1)/L for codeword length of *n* and modulation width of L (e.g. for QPSK modulation L=2).

For the sample case of a (31,21) 2-error correcting BCH code, the probability P2 can be written in the form of (5.23) as

$$P2 \cong 4{,}495x10^3 Pb_2{}^3 - 9{,}44x10^4 Pb_2{}^4 \ . \tag{5.39}$$

Using equation (5.37), P1 turns out to be

$$P1 = 1 - \left[31Pb_2(1-Pb_2)^{30} + (1-Pb_2)^{31}\right] \tag{5.40}$$

Again, using the polynomial approximations:

$$P1 \cong 1 - \left[\begin{array}{l} 31Pb_2(1-30Pb_2 + \frac{30.29}{2}Pb_2{}^2 - \frac{30.29.28}{6}Pb_2{}^3 + ...) + \\ (1-31Pb_2 + \frac{31.30}{2}Pb_2{}^2 - \frac{31.30.29}{6}Pb_2{}^3 + \frac{31.30.29.28}{24}Pb_2{}^4 - ...) \end{array}\right]$$

$$P1 \cong 465Pb_2{}^2 - 8{,}99x10^3\,Pb_2{}^3 + 94{,}365x10^3\,Pb_2{}^4 . \tag{5.41}$$

Using equations (5.35), (5.38), (5.39), (5.41), the block error probability of Sys2 employing a (31,21) 2-error correcting code for the sample case may be rewritten as:

$$\begin{aligned} P_{e,block} &= \left(1-e^{-\beta_2}\right)^{2.8NM}\left(4{,}495x10^3\,Pb_2{}^3 - 9{,}44x10^4\,Pb_2{}^4\right) + \\ &\left(1-\left(1-e^{-\beta_2}\right)^{2.8NM}\right)\left(465Pb_2{}^2 - 8{,}99x10^3\,Pb_2{}^3 + 94{,}365x10^3\,Pb_2{}^4\right). \end{aligned} \tag{5.42}$$

As seen from (5.42) for Sys2, the block error probability depends on the coded bit error probability, the PAPR threshold, the number of channels, the codeword length and the modulation.

Finally, for the (31,21) BCH code, the coded bit error probability for Sys2 is given by:

$$Pb_2 = Q\left(\sqrt{\frac{21}{32}\frac{Eb}{N_{02}}.2}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{21}{32}\frac{Eb}{N_{02}}}\right) \tag{5.43}$$

5.7. Choosing Transmitter Threshold for Sys2

In order to be fair when comparing the two systems, the next step is to choose a proper threshold value for Sys2 employing the same constraint as the plain OFDM system. For a plain OFDM system, the system was allowed to clip the input data once a day; this was a constraint to determine its amplifier threshold value. The same criterion can be applied to Sys2. As discussed previously, it is possible to have to change almost all of the Encoder & Mapper outputs in order to reduce a high PAPR value. This is a bottleneck for the algorithm. To make this point more clear it may be argued that the algorithm may fail if there are two or more high PAPR samples during one coded block time. This restriction may be used to determine a proper threshold value for the amplifier of Sys2. For this purpose Sys2 will be allowed to clip the input signal when there are more than one high PAPR values in a single coded block. Therefore, the threshold will be chosen to make the probability of committing more than one high PAPR's in a coded block one-per-day.

Assume that the information bit rate is R_b bits/sec. If the system has N channels, each Encoder & Mapper block has R_b/N bits/sec. When using (n,k) encoders at Encoder & Mapper blocks, each k-bit of the information data is used to form a n-bit codeword. After k-bits are collected at Encoder & Mapper blocks, each are coded into n-bits and one bit parity is added at the end of each codeword. This structure of (n+1) bit codewords at N-Encoder & Mapper blocks that are formed simultaneously will be named as a *coded block* for the rest of this section. This means that, one codeword at each Encoder & Mapper block formed simultaneously, constitutes a *coded block* and results in (n+1)/L OFDM symbols where L is the modulation width, e.g. L=2 for QPSK modulation. Since an OFDM symbol consists of N samples, a *coded block* has N(n+1)/L samples. Additionally, ignoring coding latency, since each of the Encoder & Mapper blocks should have k-bits of information to form a *coded block*, T_{block}, which is the *coded block* duration, may be given as

$$T_{block} = \frac{kN}{R_b} \text{ sec.} \tag{5.44}$$

Define a new parameter M_{24} as the number of *coded blocks* per day, in the system implementing PAPR reduction algorithm. Since a day has 86400 seconds, M_{24} becomes

$$M_{24} = \frac{86400}{T_{block}} = \frac{86400R_b}{kN} \text{ blocks.} \tag{5.45}$$

Since we expect that for only one coded block, two or more samples exceed a certain threshold per day, we can use the same assumption as (5.29) replacing the variable N_{24} with M_{24} and we may be conclude that

$$P_1 \cong \frac{1}{M_{24}}. \tag{5.46}$$

On the other hand, P_1 may be written as:

P_1= 1-[P(1 sample PAPR>β in a coded block) + P(No sample PAPR> β in a coded block)]

Since a *coded block* consists of (n+1)/L OFDM symbols, a new parameter M may be defined as the number of OFDM symbols per coded block.

$$M = \frac{n+1}{L} \tag{5.47}$$

Using the equation derived as (5.13) which states that

P(A sample PAPR exceeds certain threshold β)= $e^{-\beta}$

P_1 may be expressed as

$$P_1 = 1 - \left[MNe^{-\beta}(1-e^{-\beta})^{MN-1} + (1-e^{-\beta})^{MN} \right]. \tag{5.48}$$

Using polynomial approximations and the assumption that $e^{-3\beta} << e^{-2\beta}$, (5.48) may be rewritten as

$$P_1 \cong 1 - \left[MNe^{-\beta}\left(1-(MN-1)e^{-\beta}\right) + \left(1 - MNe^{-\beta} + \binom{MN}{2} e^{-2\beta}\right)\right]$$

$$\cong 1 - \left[MNe^{-\beta} - MN(MN-1)e^{-2\beta} + 1 - MNe^{-\beta} + \frac{MN(MN-1)}{2} e^{-2\beta} \right]$$

$$P_1 \cong \frac{MN(MN-1)}{2} e^{-2\beta}. \tag{5.49}$$

Using equations (5.45), (5.46) and (5.49) together, a relationship between the selected code, information bit rate, clipping rate, number of channels and the transmitter threshold value may be obtained as

$$\frac{MN(MN-1)}{2} e^{-2\beta} \cong \frac{kN}{86400 R_b}. \tag{5.50}$$

Finally, the desired threshold value for the algorithm implementing system may be written approximately as

$$\beta \cong -\frac{1}{2} \ln\left(\frac{kN}{86400 R_b} \frac{2}{MN(MN-1)} \right). \tag{5.51}$$

When the information bit rate, modulation type used, clipping rate (changes the value 86400 which denotes the number of seconds per clipping period) and the code is determined for the system, (5.51) depends only on the PAPR threshold value and the number of channels used in the OFDM system.

5.8. Gain Definition Used for Performance Comparison

As mentioned previously, the lowpass equivalent of an OFDM signal can be written as:

$$s(t) = \sum_{k=1}^{N} a_k e^{jw_k t}$$

where $a_k = \pm 1, \pm j$ for QPSK modulation and N channels.

The average power of s(t) is

$$P_{av} = E\left\{ |s(t)|^2 \right\} = \sum_{k=1}^{N} \sum_{n=1}^{N} E\left\{ a_k a_n^{*} \right\} e^{jw_k t} e^{-jw_n t} = N . \quad (5.52)$$

Assuming that the output signal is clipped at level $\pm\lambda$, the peak output power can not exceed λ^2. That is:

$$P_{Peakout} \le \lambda^2 . \quad (5.53)$$

Using (5.52) and (5.53):

$$PAPR_{out} = \frac{P_{peakout}}{P_{av}} \cong \frac{\lambda^2}{N} \quad (5.54)$$

Or:

$$P_{av} = P_{peakout}\left(\frac{N}{\lambda^2} \right) \quad (5.55)$$

The system SNR can be written using (5.55) as:

$$\frac{E_b}{N_0} = \frac{P_{peakout} T_b}{N_0} \left(\frac{N}{\lambda^2} \right) \tag{5.56}$$

Equation (5.56) will be used to compare the two systems. First one does not use the PAPR reduction algorithm but it clips the signal above a certain level. The second system uses our algorithm and clipping at the same rate as the simple OFDM modulating system. In order to compare their performances, equation (5.56) will be written for both systems:

Sys 1: Plain OFDM with clipping

$$\left(\frac{E_b}{N_{01}} \right) = \left(\frac{P_{peakout1}}{N_{01}.R_b} \right) \left(\frac{N}{\lambda_1^{\,2}} \right) = \rho_1 \left(\frac{N}{\lambda_1^{\,2}} \right) \tag{5.57}$$

Sys 2: PAPR reduction algorithm and clipping

$$\left(\frac{E_b}{N_{02}} \right) = \left(\frac{P_{peakout2}}{N_{02}.R_b} \right) \left(\frac{N}{\lambda_2^{\,2}} \right) = \rho_2 \left(\frac{N}{\lambda_2^{\,2}} \right) \tag{5.58}$$

Using (5.57) and (5.58), ρ_1 and ρ_2 may be written explicitly as:

$$\rho_1 = \left(\frac{P_{peakout1}}{N_{01}.R_b} \right) = \left(\frac{E_b}{N_{01}} \right) \left(\frac{\lambda_1^{\,2}}{N} \right) \tag{5.59}$$

$$\rho_2 = \left(\frac{P_{peakout2}}{N_{02}.R_b} \right) = \left(\frac{E_b}{N_{02}} \right) \left(\frac{\lambda_2^{\,2}}{N} \right) \tag{5.60}$$

From the first part of equations (5.59) and (5.60) it is observed that for a system with constant N_0 and R_b values, ρ value is an indication of the transmitter power. Therefore, a system with a lower ρ value is preferred as the transmitter power is reduced. Another comparison may be the case of constant N_0 and transmitter power in which case the system with a lower ρ value can transmit at a higher information bit rate.

Additionally, as can be seen from (5.59) and (5.60), the parameters ρ_1 and ρ_2 are indication of the real performance of two systems since these parameters give an indication of SNR loss caused by codeword modification and also the PAPR reduction gained by the algorithm when two systems are compared. The ratio of ρ_1/ρ_2 will be used to compare two systems. At this point it is clear that as the ratio of ρ_1/ρ_2 increases, the algorithm gain is increased.

As used before, two 32-channel, OFDM systems implementing (31,21) 2-error correcting BCH codes will be compared. The parameters ρ_1 and ρ_2 will be calculated for a sample case to clarify how the comparison is done. For the sample case assume that the information bit error probability is chosen as:

$$P_{b,\inf o} = (7.043/2)x10^{-9},$$

It may be assumed that receiving an erroneous block (block means 31 bits) results in receiving half of the bits in error. Then we have:

$$P_{b,\inf o} \approx \frac{P_{e,block}}{2} \quad .$$

Therefore:

$$P_{e,block} \approx 7.043x10^{-9}.$$

Using the block error probability expression (5.23) derived previously for (31,21) BCH code:

$$P_{e,block} \cong 4{,}495x10^{3}\,Pb_1^{\,3} - 9{,}44x10^{4}\,Pb_1^{\,4}$$

(P_{b1} in this equation corresponds to coded bit error probability of system 1.)

And also using equation (5.20):

$$Pb_1 = \frac{1}{2}erfc\left(\sqrt{\frac{21}{32}\frac{E_b}{N_{01}}}\right)$$

Solving this equation SNR for the first system is found out to be $(E_b/N0)_1$=10.

Then, 1clip/day constraint should be taken into account. Equation (5.32) was derived for this purpose as

$$\beta = \ln\left(\frac{86400(n+1)R_b}{kL}\right).$$

When the previously used parameters are inserted into the equation as a (31,21) code, 32-channel system, L equals 2 and R_b is 8Mbits/sec, β can be found as 27.

Using equations (5.57) and (5.59), ρ_1 is found as 270.

In what follows, similar calculations will be done for the second system and ρ_2 value will be determined. For this case the block error probability expression of System 2 that implements the algorithm will be used. This equation was previously found in (5.42) as:

$$P_{e,block} = \left(1-e^{-\beta_2}\right)^{2.8NM}\left(4{,}495x10^{3}\,Pb_2{}^{3} - 9{,}44x10^{4}\,Pb_2{}^{4}\right)+$$
$$\left(1-\left(1-e^{-\beta_2}\right)^{2.8NM}\right)\left(465Pb_2{}^{2} - 8{,}99x10^{3}\,Pb_2{}^{3} + 94{,}365x10^{3}\,Pb_2{}^{4}\right)$$

where Pb_2 corresponds to coded bit error probability of system 2.

If the two systems are assumed to result in the same information bit error probability then it may be assumed that their block error probabilities are also equal.

Since some of the parameters were determined for the first system, they will also be used for the second system. Using previous assumptions, N is 32, M is 16 and the block error probability is $7.043x10^{-9}$. PAPR threshold value of second system can be calculated using (5.51) as 16.29. Then, (5.42) should be solved in order to find the coded bit error probability of the system. System parameters and the calculated PAPR threshold result in a coded bit error probability of 0.00011. Then, the SNR of the second system, i.e. $(E_b/N_0)_2$, will be calculated using the equation:

$$\left(\frac{E_b}{N_{02}}\right) = \frac{32}{21} erfcinv^2\left(2Pb_2\right). \tag{5.61}$$

From (5.61), $(E_b/N_0)_2$ is found out to be 10.40 which results in ρ_2 value of 169.4 using equation (5.60).

5.9. Summary

Assume that the information bit error probability, number of channels, employed code and the information bit rate are given. In order to compare the straightforward clipping system (Sys1) with the system implementing PAPR reduction algorithm (Sys2), the following steps should be applied:

1) Block error probability equation of Sys1 is calculated for the given code in the form of (5.21). (For the (31,21) 2-error correcting code, the equation is found out to be (5.23).)

2) Pb_1 is calculated using the equation found in Step 1.

3) SNR of Sys1 is determined applying (5.19) for the given code.

4) From the provided system parameters, PAPR threshold of Sys1 is calculated using (5.32).

5) Gain parameter is calculated from (5.59) using the SNR and the PAPR threshold values determined in Step 3 and Step4.

6) For Sys2, block error probability equation is written in the form of (5.35) using the equations (5.36), (5.37) and (5.38). (For the (31,21) 2-error correcting code, this equation is calculated as in (5.42).)

7) The PAPR threshold value of Sys2 is calculated using the previously determined parameters and equation (5.51).

8) PAPR threshold value is inserted into the block error probability equation of Sys2 and the coded bit error probability, i.e. Pb_2, is calculated.

9) Pb_2 value is used to determine the SNR of Sys2 using the equation (5.19) for the given code.

10) Gain parameter of Sys2 is calculated employing (5.60).

The two systems can be compared using gain parameters and their ratio. In the next chapter, numerical results will be provided using this procedure.

CHAPTER 6

NUMERICAL RESULTS

This chapter is devoted to the graphical and tabular results of the derivations of Chapter 5. First part will be the graphs of the previously derived equations and second part will be tables of gain parameters obtained for different conditions.

6.1. Basic System Behavior

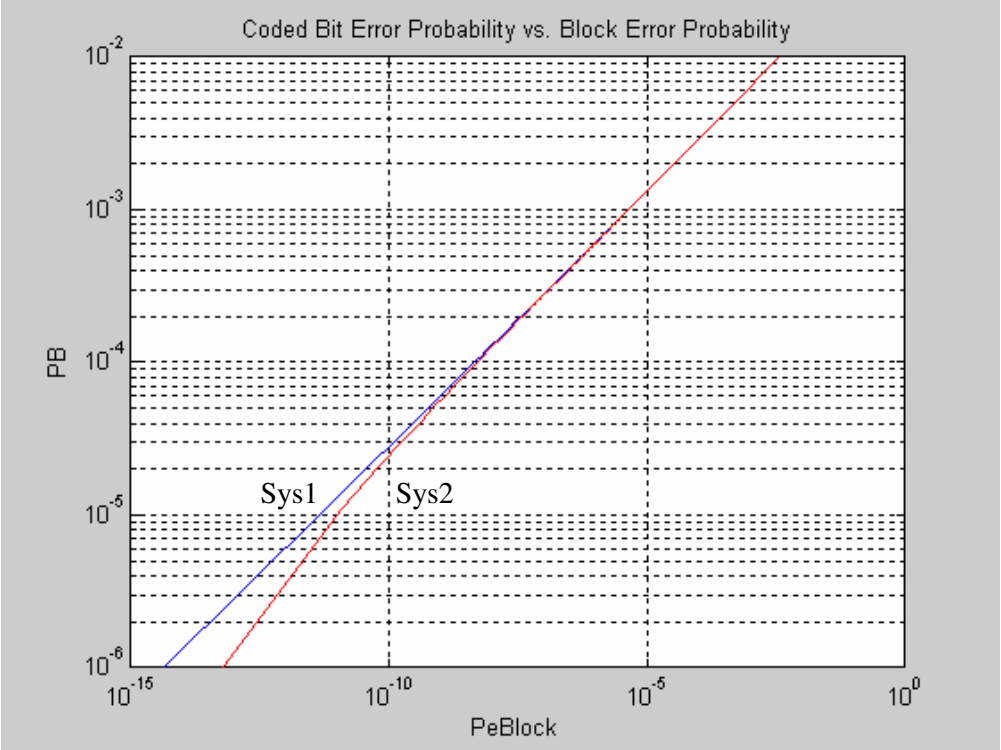

Figure 6.1 Coded Bit Error Probability versus Block Error Probability

Figure 6.1 shows the block error probability and coded bit error probability relations for the two sample systems of (31,21) BCH code employing systems with 32 channels having an information bit rate of 8 Mbits/sec and using a clipping rate of 1-per-day. Equations (5.23) and (5.42) are employed and the PAPR threshold of Sys2 is calculated from the sample system parameters using (5.51). As discussed before, information bit error probability may be assumed to be equal to the half of the block error probability. Therefore, block error probability is a direct indication of the information bit error probability. As seen from Figure 6.1, if low information bit error probabilities are desired, the system using PAPR reduction algorithm is to have a much lower coded bit error probability than the simple OFDM modulating system in order to have the same information bit error probability.

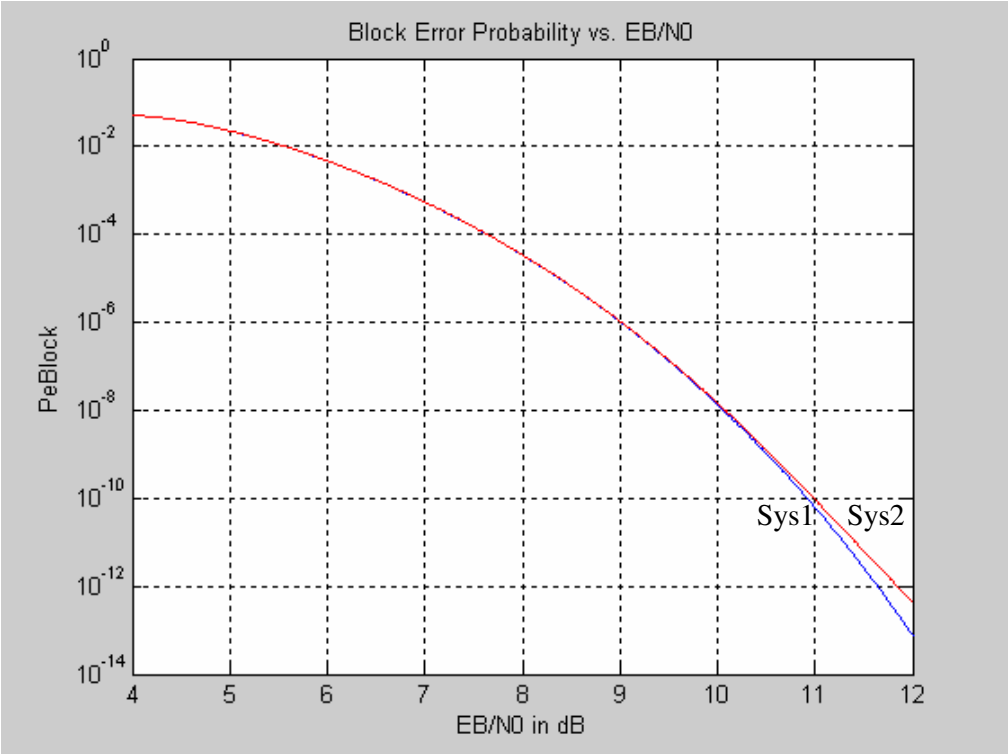

Figure 6.2 Block Error Probability versus Signal-to-Noise Ratio

Figure 6.2 shows Eb/N_0 versus block error probability for the two sample systems as used in Figure 6.1. In order to reflect the effect of information bit error probability to SNR, equations (5.20) and (5.43) are used. As seen from the figure, as the desired information bit error probability level is decreased, Sys2 should operate at a somewhat higher SNR than the plain OFDM system.

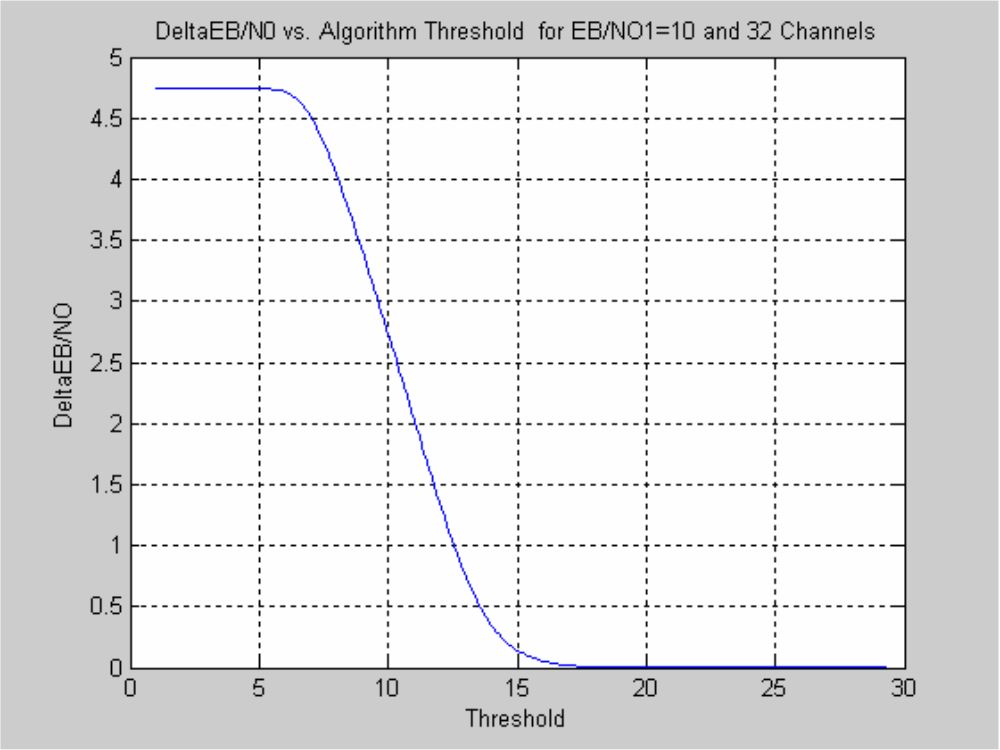

Figure 6.3 Signal-to-Noise Ratio Loss versus Algorithm Threshold

Figure 6.3 shows the relation between the PAPR threshold of Sys2 and the SNR loss caused by the algorithm. This figure is drawn for the sample system; i.e., the system has 32 channels and employs (31,21) BCH code. Also, plain OFDM system's SNR is set to 10 dB which means setting the coded bit error probability for the system (seen from equation (5.20)) and therefore setting the information bit error probability (seen from equation (5.23)) since information bit error probability

is one of the initially set parameters for a system design. From Figure 6.3 it may be observed that when the PAPR threshold of Sys2 is set to approximately 16 or 17, the SNR loss caused by the algorithm nearly approaches to zero, as the probability of exceeding the threshold is very low for higher threshold values. However, this mode of operation does not cure the disadvantages caused by the high PAPR.

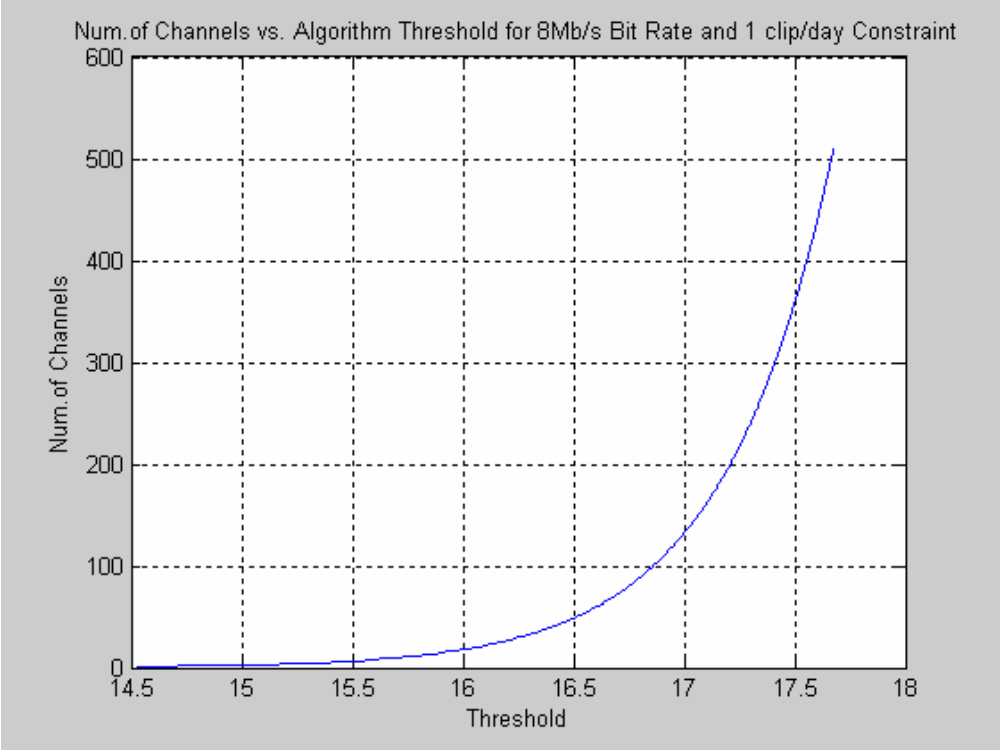

Figure 6.4 Number of Channels versus Algorithm Threshold

Figure 6.4 is plotted to show how the PAPR threshold value of Sys2 changes as the number of channels changes. As seen from Figure 6.4, the suitable transmitter threshold value increases much less slowly than the number of channels, this is due to very small probabilities of committing a high PAPR value. Additionally, the PAPR threshold value for a 32-channel system is found out to be 16.29, which results in the clipping probability of one-per-day.

Figure 6.5 gives a plot of the transmitter threshold values of the system implementing PAPR reduction algorithm versus the ρ_2 parameter. For this plot the SNR value of the first system is found out to be 10 dB and also the system PAPR threshold, i.e. β_1 is found out to be 27 to satisfy the one clipping-per-day constraint since the assumptions of 32-channels and (31,21) BCH code are used. Therefore the parameter ρ_1 is constant and equal to 270 with these system parameters.

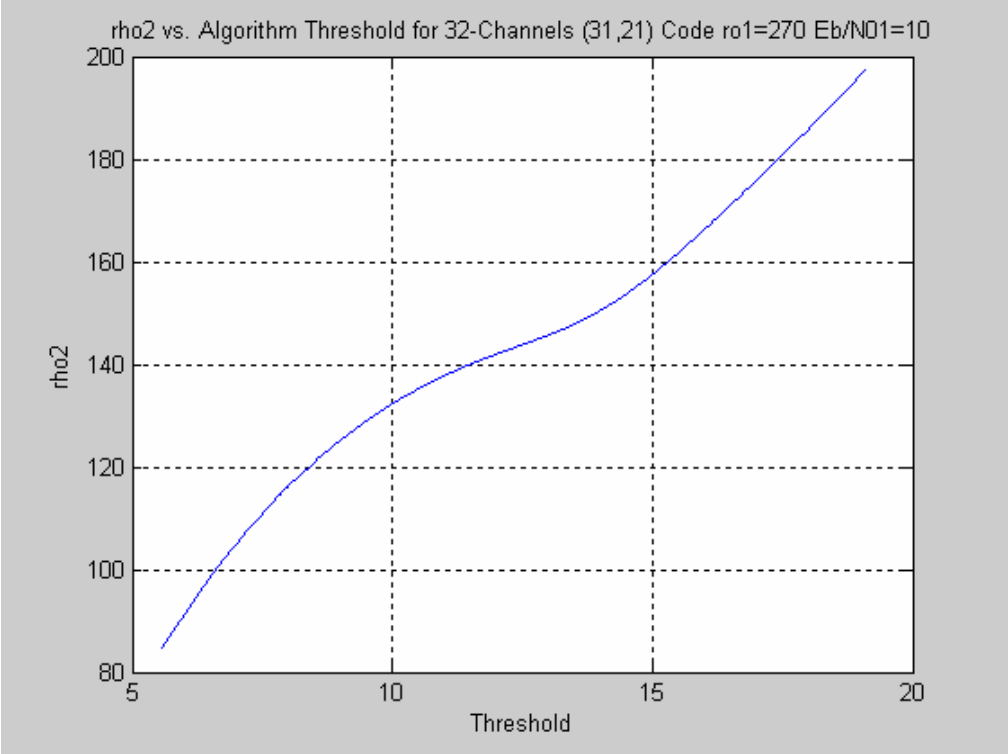

Figure 6.5 ρ_2 versus Algorithm Threshold

Additionally, from Figure 6.5 it is observed that as the PAPR threshold of the system implementing the PAPR reduction algorithm is increased, the ρ_2 value is also increased. This result stems from the fact that, when the system transmits the OFDM symbols with higher PAPR values, it necessitates a higher transmitter power.

As explained before ρ_1/ρ_2 is an indication of the gain obtained by the algorithm. Figure 6.6 is the graph of Gain Parameter Ratio versus PAPR threshold of Sys2 for the sample case. From figure it can be seen that, as the PAPR threshold of Sys2 is reduced, the algorithm gain is increased at the cost of increasing the number of calculations since a lower threshold value necessitates the modification of more OFDM samples than a higher threshold case. For a constant information bit rate, code and clipping rate, the PAPR threshold of the simple OFDM system is constant as given in (5.32). Therefore, an increase in the PAPR threshold of Sys2 results in a decrease in the algorithm gain, i.e. ρ_1 / ρ_2, since the increase of the threshold indicates an increase of the transmitter power. Additionally, the minimum PAPR threshold should be determined according to the clipping rate constraint. Therefore, the gain parameter ratios of Figure 6.6 should be examined for the threshold values greater than the minimum threshold value determined by the clipping rate.

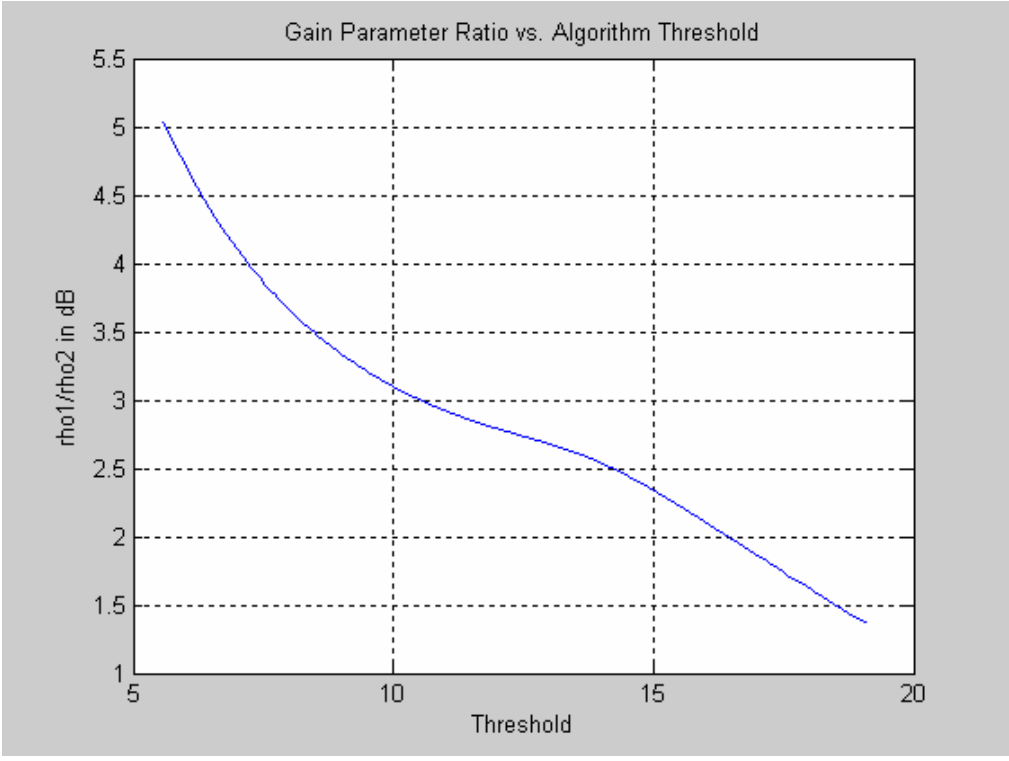

Figure 6.6 Gain Parameter Ratio versus Algorithm Threshold

Figure 6.7 is the graph of the algorithm gain over plain OFDM system versus the number of subchannels. The graph is plotted again for the sample system. The threshold value of Sys1 is constant as the code, clipping rate and information bit rate are kept as constant. For Sys2 the threshold values are calculated for the changing number of channels using (5.51). Then, (5.42) is used to determine the coded bit error probability of the second system which will be used to determine the SNR loss caused by the algorithm as given in (5.61).

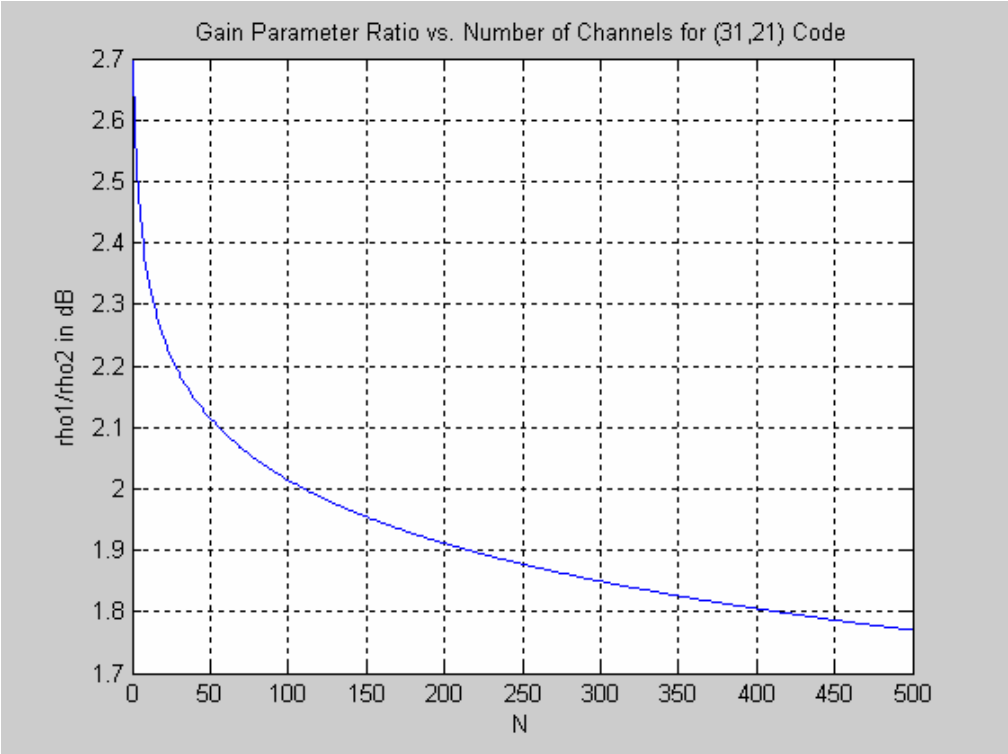

Figure 6.7 Gain Parameter Ratio versus Number of Channels

6.2. Tables of Gain Parameters

In order to better understand the effect of system parameters on the algorithm performance, six tables are provided (Table 6.1 – Table 6.6). The tables are used to compare a plain OFDM system; i.e.Sys1, with a system implementing PAPR reduction algorithm; i.e. Sys2. The Beta1 and Beta2 columns correspond to the PAPR threshold values of Sys1 and Sys2, respectively. These values are calculated using equations (5.32) and (5.51). Additionally, ρ_1 and ρ_2 are the gain parameters of Sys1 and Sys2. Gain parameters are calculated employing (5.59) and (5.60) for varying system conditions.

The information bit error probability is set to a value of 10^{-6} and kept constant for all of the tables. Therefore, the tables can be compared based on the equal information bit error probability condition. The tables are obtained for the cases of 32 and 128 system subchannels, 1-per-day and 1-per-month clipping rates and 8 Mbits/sec and 100 Mbits/sec information bit rates.

In order to obtain the values in the tables, initially block error probability equations are written. Equations (5.23) and (5.42) are derived for the special case of (31,21) 2-error correcting BCH code. (5.21) and (5.34) should be modified for the selected code. Coded bit error probabilities, PB_1 and PB_2 are calculated using the previously derived block error probability equations and used for the calculation of SNR values of Sys1 and Sys2. After determining Beta1 and Beta2, gain parameters ρ_1 and ρ_2 are calculated.

Each of the six tables includes four small tables. First three tables can be used to compare codes of different lengths with the same error correction capability. The fourth table indicates the case of approximately equal rate codes with various lengths and error correction capabilities. The generator polynomials of the codes used in the tables are given in Appendix A.

The effect of system parameters will be analyzed in the following sections. The tables will be compared to observe the effect of the number of channels, clipping rate, information bit rate and different coding schemes on the system performance. These effects will be considered separately.

6.2.1. Number of Channels

The effect of number of channels can be observed by comparison of Table 6.1 to Table 6.3 and alternatively, Table 6.4 to Table 6.6. The comparison is made with all other system parameters, i.e. clipping rate and information bit rate, are kept constant. Also, equivalent rows of the tables are compared which corresponds to keeping the employed code constant also.

As can be seen from the tables, threshold Beta1 does not change with the number of channel variations. This result is due to (5.32) which shows that Beta1 is independent of the number of channels. SNR of Sys1 is equal for corresponding rows of all tables, i.e. when the same code is applied to the system. This can be explained in the way that, the coefficients of the block error probability equation of Sys1 in the form of (5.21) depends only on the applied code. Since the information bit error probability is constant for all cases, the block error probability is also constant for equal code case. Therefore, the coded bit error probabilities are also equal resulting in equal SNR values. As a result, the gain parameter of Sys1 is constant when the code, the clipping rate and information bit rate are constant and the number of channels is varying. In other words, the transmit power is constant for Sys1 when system parameters other than number of channels are kept constant.

For Sys2, it is observed from Figure 6.4 that, Beta2 is increased as the number of channels increases. Also the SNR of Sys2 shows a slight increase for increasing number of channels. Therefore, the gain parameter of Sys2 is also increased as the number of channels is increased which results in an increase of transmit power.

The gain ratio, expressed as ρ_1/ρ_2, is reduced for increasing number of channels which can be observed from Figure 6.7.

6.2.2. Clipping Rate

The effect of clipping rate can be observed by the comparison of Table 6.1 to Table 6.4 and Table 6.2 to Table 6.5 and alternatively, Table 6.3 to Table 6.6. When the clipping rate is reduced from 1 clip-per-day to 1 clip-per-month, the Beta1 parameter of Sys1 is increased which can be seen from (5.32). Also the SNR of Sys1 is constant when the rows of tables with the same code are compared. This issue was described in section 6.2.1. in detail. Therefore, the gain parameter of Sys1 is increased when the clipping rate is reduced.

For Sys2, Beta2 is increased for reduced clipping rate as can be observed form (5.51). The gain parameter is also increased which results in an increase in the transmit power of Sys2 for the case of reduced clipping rate.

The gain parameters of the two systems are both increased when the clipping rate is reduced. However, as can be seen form the tables, since ρ_1 increases more rapidly than ρ_2, the gain ratio is also increased. This implies that, as the clipping rate is reduced, the transmit power of both systems are increased, yet the advantage of applying the PAPR reduction algorithm is increased as a result of the increase of gain ratio.

6.2.3. Information Bit Rate

The effect of information bit rate can be observed by the comparison of Table 6.1 to Table 6.2 and Table 6.4 to Table 6.5. As the information bit rate is increased, the same discussion applies as the clipping rate reduction case. In summary, when the information bit rate is increased, Beta1 and ρ_1 of Sys1 are increased. Sys2 also behaves in the same way, i.e. Beta2 and ρ_2 are increased. In other words, both

systems require more transmit power, as the information bit rate is increased. Moreover, the gain ratio is also increased which shows the enhanced advantage of applying the PAPR reduction algorithm instead of just clipping.

6.2.4. Code Rate

In order to observe the effect of the code rate, the codes having the same error correction capability in the same table will be compared with each other. For example, single error correcting codes of Table 6.1 can be compared with each other. From this comparison, it may be stated that Beta1 is slightly decreased for increasing code rate of the same error correction capacity. This result can be observed from (5.32). Beta1 is related approximately to the reciprocal of the code rate. Therefore, an increase in the code rate, results in a decrease in Beta1. However, there is not a direct relation between the code rate and ρ_1. Therefore, between the codes of equal error correction capability, the one with the least transmit power can be chosen as the ideal case. For example, when Table 6.1 is examined, if the plain OFDM system is to be used, the (31,26) code seems to be the most advantageous among four single error correcting codes.

For the case of Sys2, as the code rate is increased, Beta2 is also increased. It can be seen from (5.51) that inside the logarithm expression, there is a factor of approximately k/n^2. An increasing code rate results in a decrease of this factor, therefore, Beta2 is increased. For the gain parameter of Sys2, the same discussion applies as the gain parameter of Sys1. An optimum code with the lowest ρ_1 value can be selected among the alternative codes of equal error correction capability.

When the gain ratio is considered, it can de observed that the ratio is decreased as the higher rate code is selected from a set of codes with equal error correction capability.

6.2.5. Code Length

In this section, the effect of code length will be examined. The codes with equal rates will be considered. The fourth part of the tables shows the case of two equal rate codes. First is a (255,247) single error correcting code and the second one is the (1023,993) 3-error correcting code with approximately equal rate of 0.97.

Since the code rates are equal, Beta1 does not change for different codes. When ρ_1 is considered, a longer code of equal rate has more error correction capability than a shorter code. Therefore, when the system employs a longer code of the same rate as a shorter code, then a lower SNR value will be enough because of the higher error correction capacity. As the Beta1 value is constant, if the longer code is applied a lower transmit power will be enough. Hence, ρ_1 is reduced for longer codes of the same rate.

For Sys2, as described before, Beta2 is approximately related to the ratio of k/n^2. This factor can be written as *rate/n.* Since the rate is constant for both codes, *rate/n* factor is smaller for the longer code. Hence, Beta2 is increased for the longer code of equal rate. As for Sys1, the SNR value of Sys2 is much lower for the longer code. The reduction of SNR is higher than the increase in Beta2. Therefore, ρ_2 is lower for the longer code. As for Sys1, Sys2 also requires a lower transmit power for the longer code of same rate.

When the gain ratio is examined, it is observed that the ratio is reduced for increasing code length.

Table 6.1 Gain Parameter for 32-Channels, Clipping Rate of One-per-Day and Information Bit Rate of 100 Mbits/sec

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ρ2(dB)	CodeRate
(15,11) 1-err	29.5	17.2	283.9	165.6	2.34	0.73
(31,26) 1-err	29.3	17.4	264.1	157.4	2.25	0.84
(255,247) 1-err	29.2	18.4	282.0	178.9	1.98	0.97
(1023,1013) 1-err	29.2	19.1	313.9	207.7	1.79	0.99

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(15,7) 2-err	30.0	17.4	295.8	171.9	2.36	0.47
(31,21) 2-err	29.6	17.6	226.1	134.2	2.26	0.68
(255,239) 2-err	29.2	18.4	219.2	138.3	2.00	0.94
(1023,1003) 2-err	29.2	19.1	247.8	162.4	1.84	0.98

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,231) 3-err	29.2	18.4	187.6	118.3	2.00	0.90
(1023,993) 3-err	29.2	19.1	213.3	139.7	1.84	0.97

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,247) 1-err	29.2	18.4	282.0	178.9	1.98	0.97
(1023,993) 3-err	29.2	19.1	213.3	139.7	1.84	0.97

Table 6.2 Gain Parameter for 32-Channels, Clipping Rate of One-per-Day and Information Bit Rate of 8Mbits/sec

Code (n,k)	Beta1	Beta2	P1	ρ2	ρ1/ρ2(dB)	CodeRate
(15,11) 1-err	27.0	16.0	259.6	154.1	2.26	0.73
(31,26) 1-err	26.8	16.2	241.4	146.9	2.16	0.84
(255,247) 1-err	26.7	17.1	257.5	169.1	1.83	0.97
(1023,1013) 1-err	26.6	17.8	286.7	198.9	1.59	0.99

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(15,7) 2-err	27.4	16.1	270.9	159.5	2.30	0.47
(31,21) 2-err	27.0	16.3	206.7	124.7	2.20	0.68
(255,239) 2-err	26.7	17.2	200.3	129.1	1.90	0.94
(1023,1003) 2-err	26.6	17.8	226.3	152.1	1.73	0.98

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(31,16) 3-err	27.3	16.4	208.4	125.4	2.20	0.52
(255,231) 3-err	26.7	17.2	171.4	110.2	1.92	0.90
(1023,993) 3-err	26.6	17.8	194.9	130.5	1.74	0.97

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,247) 1-err	26.7	17.1	257.5	169.1	1.83	0.97
(1023,993) 3-err	26.6	17.8	194.9	130.5	1.74	0.97

Table 6.3 Gain Parameter for 128-Channels, Clipping Rate of One-per-Day and Information Bit Rate of 100 Mbits/sec

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ρ2(dB)	CodeRate
(15,11) 1-err	29.5	17.9	283.9	172.6	2.16	0.73
(31,26) 1-err	29.3	18.1	264.1	164.0	2.07	0.84
(255,247) 1-err	29.2	19.1	282.0	186.7	1.79	0.97
(1023,1013) 1-err	29.2	19.8	313.9	217.4	1.59	0.99

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(15,7) 2-err	30.0	18.1	295.8	178.8	2.19	0.47
(31,21) 2-err	29.6	18.2	226.1	139.6	2.09	0.68
(255,239) 2-err	29.2	19.1	219.2	143.6	1.84	0.94
(1023,1003) 2-err	29.2	19.8	247.8	168.4	1.68	0.98

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(31,16) 3-err	29.8	18.4	227.7	140.3	2.10	0.52
(255,231) 3-err	29.2	19.1	187.6	122.7	1.84	0.90
(1023,993) 3-err	29.2	19.8	213.3	144.8	1.68	0.97

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,247) 1-err	29.2	19.1	282.0	186.7	1.79	0.97
(1023,993) 3-err	29.2	19.8	213.3	144.8	1.68	0.97

Table 6.4 Gain Parameter for 32-Channels, Clipping Rate of One-per-Month and Information Bit Rate of 100 Mbits/sec

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ρ2(dB)	CodeRate
(15,11) 1-err	32.0	18.9	316.6	181.7	2.41	0.73
(31,26) 1-err	32.0	19.1	294.7	172.4	2.33	0.84
(255,247) 1-err	32.0	20.1	314.8	194.5	2.09	0.97
(1023,1013) 1-err	32.0	20.8	350.6	224.2	1.94	0.99

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(31,21) 2-err	32.0	19.2	252.0	147.2	2.33	0.68
(255,239) 2-err	32.0	20.1	244.8	151.0	2.10	0.94
(1023,1003) 2-err	32.0	20.8	276.7	176.7	1.95	0.98

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,231) 3-err	32.0	20.1	209.4	129.1	2.10	0.90
(1023,993) 3-err	32.0	20.8	238.2	152.1	1.95	0.97

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,247) 1-err	32.0	20.1	314.8	194.5	2.09	0.97
(1023,993) 3-err	32.0	20.8	238.2	152.1	1.95	0.97

Table 6.5 Gain Parameter for 32-Channels, Clipping Rate of One-per-Month and Information Bit Rate of 8 Mbits/sec

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ρ2(dB)	CodeRate
(15,11) 1-err	30.4	17.6	292.3	169.7	2.36	0.73
(31,26) 1-err	30.2	17.9	272.0	161.2	2.27	0.84
(255,247) 1-err	30.1	18.8	290.4	182.7	2.01	0.97
(1023,1013) 1-err	30.0	19.5	323.4	211.6	1.84	0.99

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(15,7) 2-err	30.8	17.8	304.5	176.2	2.38	0.47
(31,21) 2-err	30.4	18.0	232.7	137.6	2.28	0.68
(255,239) 2-err	30.1	18.8	225.8	141.6	2.03	0.94
(1023,1003) 2-err	30.0	19.5	255.2	166.0	1.87	0.98

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,231) 3-err	30.1	18.9	193.2	121.1	2.03	0.90
(1023,993) 3-err	30.0	19.5	219.7	142.8	1.87	0.97

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,247) 1-err	30.1	18.8	290.4	182.7	2.01	0.97
(1023,993) 3-err	30.0	19.5	219.7	142.8	1.87	0.97

Table 6.6 Gain Parameter for 128-Channels, Clipping Rate of One-per-Month and Information Bit Rate of 100 Mbits/sec

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ρ2(dB)	CodeRate
(15,11) 1-err	32.0	19.6	316.6	188.4	2.25	0.73
(31,26) 1-err	32.0	19.8	294.7	178.7	2.17	0.84
(255,247) 1-err	32.0	20.8	314.8	201.4	1.94	0.97
(1023,1013) 1-err	32.0	21.5	350.6	232.2	1.79	0.99

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(15,7) 2-err (Note1)	31.1	18.6	306.7	184.2	2.22	0.47
(31,21) 2-err	32.0	19.9	252.0	152.5	2.18	0.68
(255,239) 2-err	32.0	20.8	244.8	156.2	1.95	0.94
(1023,1003) 2-err	32.0	20.5	276.7	182.6	1.81	0.98

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,231) 3-err	32.0	20.8	209.4	133.6	1.95	0.90
(1023,993) 3-err	32.0	20.5	238.2	157.1	1.81	0.97

Code (n,k)	Beta1	Beta2	ρ1	ρ2	ρ1/ ρ2(dB)	CodeRate
(255,247) 1-err	32.0	20.8	314.8	201.4	1.94	0.97
(1023,993) 3-err	32.0	20.5	238.2	157.1	1.81	0.97

Note1:10Mbit/sec

Finally, it is observed from the tables that applying the PAPR reduction algorithm reduces the transmit power of the system when compared to the plain OFDM modulating system. The tables demonstrate that when all the other system parameters are kept constant, the transmit power of the system applying the PAPR reduction algorithm is reduced for the cases of

- Less number of channels,
- Higher clipping rate,
- Lower information bit rate,
- Longer codes (more error correcting codes) among equal rate alternatives.

Additionally, the system applying the PAPR reduction algorithm becomes more advantageous compared to the plain OFDM system for the cases of

- Less number of channels,
- Lower clipping rate,
- Higher information bit rate,
- Shorter codes among equal error correcting alternatives,
- Shorter codes among equal rate alternatives.

CHAPTER 7

SUMMARY AND CONCLUSIONS

In this thesis, an algorithm to reduce PAPR of an OFDM signal is proposed. In the literature, the proposed methods are evaluated only by means of the PAPR reduction achieved by the technique. PAPR reduction is not a fair criterion to compare the performance of different systems. The performance of the suggested algorithm is discussed by means of a new performance measure criterion that takes the basic system parameters like the peak transmit power, bit rate and noise level into account.

There are many different methods proposed in the literature for the purpose of reducing the PAPR of an OFDM signal. The methods can be grouped under three main categories as clipping, scrambling and coding. In this thesis, a new method is offered for the PAPR reduction. The method is based on block coding the input data and introducing deliberate errors prior to transmission iteratively, until the PAPR goes below a previously determined threshold value. The modification of the input data does not result in a signal distortion since the employed block code has a sufficient error correction capability.

Each of the methods proposed in the literature has a drawback and these effects need to be considered for a real performance measurement. In this thesis, the performance is measured by the comparison of an OFDM system applying the PAPR reduction algorithm, with a simple clipping OFDM system. Two systems are compared under the assumption that equal information bit error probabilities should be satisfied for both. Additionally, a very low probability of clipping the OFDM signal is allowed for both of the systems, since a very low clipping rate can be acceptable for OFDM services. If clipping is allowed for the plain system, the

system implementing the PAPR reduction algorithm will be allowed to fail at the same rate. The algorithm fails when there are more than one high PAPR values in a single coded block. Using the clipping rate as a constraint, two different PAPR threshold values are determined for the systems. The algorithm tries to reduce the PAPR value below the selected threshold. Finally, a gain parameter is defined and used for both systems. The gain parameter is simply the ratio of the transmit power to the product of receiver N_0 and the information bit rate (R_b). When two systems are considered separately, the gain parameter is a measure of the transmit power required by the systems under equal N_0 and R_b conditions. Another significance of the gain parameter is that, for the system implementing the PAPR reduction algorithm, the gain parameter considers the PAPR reduction together with the SNR loss caused by the modification of the codeword. Consequently, the ratio of the gain parameters of the two systems is an indication of the PAPR reduction and SNR loss caused by the algorithm compared to a plain system; alternatively, it is a measure of the reduction of transmit power obtained by the algorithm implementation.

The simulation results indicate that, application of the algorithm results in significant reduction in the PAPR values. In order to examine the performance potential of the algorithm, extreme conditions were generated by injecting high PAPR producing symbols in two simulation cases. Through this method it was observed that a PAPR reduction of approximately 4 dB was achieved for a 32-channel system whereas the 128-channel system reached up to a value of approximately 8 dB. The provided values indicate merely the achieved PAPR reduction. However, when the clipping probability is taken into consideration, the gain parameters of the two systems compared and their ratios are calculated and tabulated for different system conditions. These results show that a gain ratio in the range of 1.59 dB to 2.41 dB can be obtained for different cases. The calculations indicate that the transmit power of the system using the PAPR reduction algorithm is reduced when the number of channels is not very high, clipping rate is increased,

information bit rate is reduced and the code length is increased for constant code rate.

As a future work, the QPSK modulation used in simulations may be replaced with other modulations such as QAM and the effect of modulation type can be examined. Moreover, it is possible to implement different coding schemes such as convolutional codes at various rates and error correction capabilities. From Figure (4.12) it is observed that, some of the iterations result in an insufficient PAPR reduction. Consequently, a better implementation may be to skip those modifications which do not contribute much to PAPR reduction. That method could result in a decrease in the SNR of the system implementing the PAPR reduction algorithm. Therefore, the transmitter power could be reduced. Furthermore, another improvement may be realized when two bit modifications per coded block are allowed using a code with more error correction capability than the one used for the sample system. This flexibility lets the algorithm to reduce the PAPR values of the two OFDM symbols per coded block which indicates that the transmitter power of the system implementing the PAPR reduction algorithm could be further reduced.

REFERENCES

[1] B. Canpolat, *Performance Analysis of Adaptive Loading OFDM Under Rayleigh Fading With Diversity Combining,* Ph.D. Thesis, Middle East Technical University, Ankara, September 2001.

[2] R.V. Nee, R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House Publishers: Boston-London, 2000

[3] S.B. Weinstein, P.M. Ebert, *Data Transmission by Frequency Division Multiplexing Using the Discrete Fourier Transform*, IEEE Trans. On Commun. Tech., Vol.COM-19, pp.628-634, October 1971

[4] O. Edfords, M. Sandell, J.-J. V. de Beek, D. Landström, F. Sjöberg, *An Introduction to Orthogonal Frequency-Division Multiplexing*, September 1996

[5] L.J. Cimini, N.R. Sollenberger, *Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequences*, IEEE Communications Letters, Vol.4, No.3, March 2000

[6] S.H. Müller, J.B. Huber, *OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences*, Electronics Letters, Vol.33, No.5, February 1997

[7] R.W. Bauml, R.F.H. Fischer, J.B. Huber, *Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping*, Electronics Letters, Vol.32, No.22, October 1996

[8] M. Breiling, S.H. Müller-Weinfurtner, J.B. Huber, *SLM Peak-Power Reduction Without Explicit Side Information*, IEEE Communications Letters, Vol.5, No.6, June 2001

[9] T.A. Wilkinson, A.E. Jones, *Minimisation of the Peak to Mean Envelope Power Ratio of Multicarrier Transmission Schemes by Block Coding*, Vehicular Technology Conference, Vol.2, pp.825-829, July 1995

[10] P. Fan, X.-G. Xia, *Block Coded Modulation for the Reduction of the Peak to Average Power Ratio in OFDM Systems*, IEEE Transactions on Consumer Electronics, Vol.45, No.4, November 1999

[11] J.A. Davis, J. Jedwab, *Peak-to-Mean Power Control in OFDM, Golay Complementary Sequences, and Reed-Muller Codes*, IEEE Transactions on Information Theory, Vol.45, No.7, November 1999

[12] K. Yang, S. Chang, *Peak-to-Average Power Control in OFDM Using Standard Arrays of Linear Block Codes*, IEEE Communications Letters, Vol.7, No.4, April 2003

[13] J.G. Proakis, *Digital Communications, 3rd Edition*, McGraw-Hill, Inc., 1995

[14] T. Pollet, M.V.Bladel, M.Moeneclaey, *BER Sensitivity of OFDM Systems to Carrier Frequency Offset and Wiener Phase Noise,* IEEE Transactions on Communications, Vol.43,No.2/3/4, February-April 1995

[15] M. Sandell, J.J. Van de Beek, P.O. Borjesson, *Timing and Frequency Synchronization in OFDM Systems Using the Cyclic Prefix,* Proc. of Int. Symp. On Synchronization, pp.16-19, Germany, December 1995

[16] T.M. Schmidl, D.C. Cox, *Robust Frequency and Timing Synchronization for OFDM,* IEEE Transactions on Communications, Vol.45, pp.1613-1621, December 1997

[17] P.H. Moose, *A Technique for Orthogonal Frequency Division Multiplexing Frequency Offset Correction,* IEEE Transactions on Communications, Vol.42, pp.2908-2914, October 1994

[18] W.D. Warner, C. Leung, *OFDM/FM Frame Synchronization for Mobile Radio Data Communication,* IEEE Transactions on Vehicular Technology, Vol.42, pp. 302-313, August 1993

[19] D. Wulich, N. Dinur, A. Glinowiecki, *Level Clipped High-Order OFDM,* IEEE Transactions on Communications, Vol.48, No.6, June 2000

[20] R.D. Van Nee, A.D. Wild, *Reducing the Peak-to-Average Power Ratio of OFDM,* Vehicular Technology Conference, Vol.3, pp.2072-2076, May 1998

[21] T. May, H. Rohling, *Reducing the Peak-to-Average Power Ratio in OFDM Radio Transmission Systems,* Proc. of VTC 1998, pp.2774-2778, Canada, May 1998

[22] X. Li, L.J. Cimini, *Effects of Clipping and Filtering on the Performance of OFDM*, IEEE Communications Letters, Vol.2, No.5, May 1998

[23] B.M. Popovic, *Synthesis of Power Efficient Multitone Signals with Flat Amplitude Spectrum,* IEEE Transactions on Communications,Vol.39, No.7, July 1991

[24] R.D. Van Nee, *OFDM Codes for Peak-to-Average Power Reduction and Error Correction*, Proc. Globecom 96, pp.740-744, London, November 1996

[25] J. Mikkonen, P. Aldis, G.A. Awater, A. Lunn, D. Hutchinson, *The Magic WAND-Functional Overview*, IEEE Journal on Sel. Areas in Communications, Vol.16, No.6, August 1998

[26] B.Canpolat, Y.Tanik, *Performance Analysis of Adaptive Loading OFDM Under Rayleigh Fading,* IEEE Transactions on Vehicular Technology, Vol.53, pp. 1105-1115, July 2004

[27] N. Carson, T.A. Gulliver, *PAPR Reduction of OFDM Using Selected Mapping, Modified RA Codes and Clipping,* Vehicular Technology Conference, Vol.2, pp.1070-1073, September 2002

[28] K. Sathanantan, C. Tellambura, *Coding to Reduce Both PAR and PICR of an OFDM Signal,* IEEE Communications Letters, Vol.6, No.8, August 2002

[29] H. Ochiai, *On the Distribution of the Peak-to-Average Power Ratio in OFDM Signals,* IEEE Transactions on Communications, Vol.49, No.2, February 2001

[30] C. Tellambura, *Computation of the Continuous-Time PAR of an OFDM Signal with BPSK Subcarriers,* IEEE Communications Letters, Vol.5, No.5, May 2001

[31] U. Reimers, *DVB-T: The COFDM-Based System for Terrestrial Television,* Electronics and Communication Engineering Journal, Vol.16, pp.953-972, August 1998

[32] W.H.W. Tuttlebee, D.A. Hawkins, *Consumer Digital Radio: From Concept to Reality,* Electronics and Communication Engineering Journal, Vol.10, pp.263-276, December 1998

[33] ETSI, *Radio Broadcasting Systems: Digital Audio Broadcasting to Mobile, Portable and Fixed Receivers,* ETS300-401, February 1995

[34] ETSI, *Digital Video Broadcasting: Framing Structure, Channel Coding and Modulation for Digital, Terrestrial Television,* EN300-744, August 1997

[35] IEEE, *Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications,* IEEE 802.11, November 1997

[36] ETSI, *Radio Equipment and Systems HIgh PErformance Radio Local Area Network (HIPERLAN) Type I*, ETS300-652, October 1996

[37] ETSI, *Broadband Radio Access Network (BRAN); HIPERLAN Type II Technical Specification Part I-Physical Layer,* DTS/BRAN030003-1, October 1999

[38] IEEE, *Supplement to Standard for Telecommunications and Information Exchange Between Systems-LAN/MAN Specific Requirements-Part II: Wireless MAC and PHY Specifications: High Speed Physical Layer in the 5 GHz Band,* P802.11a/D7.0, July 1999

APPENDIX A

GENERATOR POLYNOMIALS OF THE BCH CODES

Generator Polynomial of the Single Error Correcting BCH Codes

i) Generator Polynomial for (15,11) BCH code:

$$g(x) = x^4 + x + 1. \tag{A.1}$$

ii) Generator Polynomial for (31,26) BCH code:

$$g(x) = x^5 + x^2 + 1. \tag{A.2}$$

iii) Generator Polynomial for (255,247) BCH code:

$$g(x) = x^8 + x^4 + x^3 + x^2 + 1. \tag{A.3}$$

iv) Generator Polynomial for (1023,1013) BCH code:

$$g(x) = x^{10} + x^3 + 1. \tag{A.4}$$

Generator Polynomial of the Double Error Correcting BCH Codes

i) Generator Polynomial for (15,7) BCH code:

$$g(x) = x^8 + x^7 + x^6 + x^4 + 1. \tag{A.5}$$

ii) Generator Polynomial for (31,21) BCH code:

$$g(x) = x^{10} + x^9 + x^8 + x^6 + x^5 + x^3 + 1. \tag{A.6}$$

iii) Generator Polynomial for (255,239) BCH code:

$$g(x) = x^{16} + x^{14} + x^{13} + x^{11} + x^{10} + x^9 + x^8 + x^6 + x^5 + x + 1. \tag{A.7}$$

iv) Generator Polynomial for (1023,1003) BCH code:

$$g(x) = x^{20} + x^{12} + x^{11} + x^6 + x^5 + x^4 + x^2 + x + 1. \quad \text{(A.8)}$$

Generator Polynomial of the Triple Error Correcting BCH Codes

i) Generator Polynomial for (31,16) BCH code:

$$g(x) = x^{15} + x^{11} + x^{10} + x^9 + x^8 + x^7 + x^5 + x^3 + x^2 + x + 1. \quad \text{(A.9)}$$

ii) Generator Polynomial for (255,231) BCH code:

$$g(x) = x^{24} + x^{23} + x^{21} + x^{20} + x^{19} + x^{17} + x^{16} + x^{15} + x^{13} + x^8 + x^7 + x^5 + x^4 + x^2 + 1. \quad \text{(A.10)}$$

iii) Generator Polynomial for (1023,993) BCH code:

$$g(x) = x^{30} + x^{28} + x^{23} + x^{21} + x^{19} + x^{16} + x^{12} + x^8 + x^4 + x + 1. \quad \text{(A.11)}$$