DESIGN OF A MOBILE ROBOT TO MOVE ON ROUGH TERRAIN

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ABSTRACT

DESIGN OF A MOBILE ROBOT TO MOVE ON ROUGH TERRAIN

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In this thesis work, a mobile robot is designed to be used in search and rescue operations to help the human rescue workers. The difficult physical conditions in the ruins obstruct the movement. Therefore, it is aimed to design a search and rescue robot which can move easily on rough terrain and climb over the obstacles. The designed robot is made up of three modules. A connecting unit is designed that is situated between each module. This connecting unit which is composed of two universal and one revolute joint gives 5 DOF relative motions to the modules. On the other hand, the wheel's continuous contact with the ground is important while moving on rough terrain. In order to increase the adaptation of the robot to the rough terrain the rear axle is connected to the body with a revolute joint. Besides, skid steering system is used in the design of the robot to attain a compact and light solution which requires few parts.

In the study, kinematic equations and dynamic equations of the robot are obtained to be used by the control program. The dynamic equations are obtained by using the Newton – Euler formulation. The forces, which are transmitted by the connecting unit to the modules, and the reaction forces formed between the wheels and the ground are derived by using these equations.

"Follow-the-Leader approach" is used as a control strategy to make the modules move in formation and to reduce the tracking problem. In this approach, the first module is the leader and the second and third modules follow it. A Matlab program is written to control the robot by using the constructed mathematical model of the robot. The reaction forces between the wheels and the ground are calculated through using the Matlab program written. Moreover to make the simulations of the robot for some cases, a model is constructed in ADAMS program.

Key Words: Wheeled Mobile Robots, Search and Rescue Robots, Dynamic Modeling, Control.

ÖZ

ENGEBELİ ARAZİDE HAREKET EDEN MOBİL ROBOT TASARIMI

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Bu tez çalışmasında, arama-kurtarma operasyonlarında kurtarma görevlilerine yardım etmek için kullanılacak bir mobil robot tasarlanmıştır. Enkazlardaki zorlu fiziksel koşullar hareketi kısıtlamaktadır. Bu nedenle, engebeli arazide kolaylıka hareket edebilecek ve engelleri aşabilecek bir arama-kurtarma robotyu tasarlamayı amaçladık. Tasarlanan robot, üç modülden oluşmaktadır. Her modülün arasına yerleştirilecek bir bağlantı birimi tasarlandı. İki universal ve bir döner mafsaldan oluşan bu bağlanti birimi modüllerin 5 serbestlik dereceli bağıl hareket yapmalarını sağlar. Öte yandan, engebeli arazide hareket ederken tekerleklerin zeminle sürekli teması önemlidir. Robotun engebeli araziye adaptasyonunu arttırmak için arka aks gövdeye döner bir mafsal ile bağlanmıştır. Bunun yanısıra, az sayıda parça gerektiren kompakt ve hafif bir çözüm elde etmek amacı ile robotun tasarımında skid steering yöntemi kullanılmıştır.

Bu çalışmada, kontrol programlarında kullanılamak üzere robotun kinematik ve dinamik denklemleri elde edilmiştir. Dinamik denklemler Newton – Euler formülasyonu kullanılarak elde edilmiştir. Modüllere bağlantı birimleri vasıtası ile iletilen kuvvetler ve tekerlekler ile zemin arasında oluşan reaksiyon kuvvetleri bu denklemler kullanılarak elde edilmiştir.

Modüllerin bir düzende hareket etmesi ve takip sorununu azaltmak için "lideri takip et" yöntemi bir control stratejisi olarak kullanılmıştır. Bu yöntemde, birinci modül liderdir ve ikinci ve üçüncü modüller onu takip eder. Robotu control edebilmek için, robotun matematiksel modeli kullanılarak bir Matlab programı yazılmıştır. Tekerlekler ile zemin arasında oluşan reaksiyon kuvvetleri bu yazılan Matlab programı kullanılarak hesaplanmıştır. Ayrıca, robotun bazı durumlardaki simulasyonunu yapmak için ADAMS programında bir modeli oluşturulmuştur.

Anahtar Sözcükler: Tekerlekli Mobil Robotlar, Arama-Kurtarma Robotları, Dinamik Modelleme, Kontrol.

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$F_0\left\{O;\vec{u}_1^{(0)},\vec{u}_2^{(0)},\vec{u}_3^{(0)}\right\}$	: Inertial frame fixed to the earth
$F_1\left\{G_1;\vec{u}_1^{(1)},\vec{u}_2^{(1)},\vec{u}_3^{(1)}\right\}$	: Frame fixed to the body of the first module
$F_7\left\{G_7;\vec{u}_1^{(7)},\vec{u}_2^{(7)},\vec{u}_3^{(7)}\right\}$	: Frame fixed to the body of the second module
$F_{13}\left\{G_{13};\vec{u}_1^{(13)},\vec{u}_2^{(13)},\vec{u}_3^{(13)}\right\}$	: Frame fixed to the body of the third module
β_1	: Heading angle of the first module
β_7	: Heading angle of the second module
β_{13}	: Heading angle of the third module
$\dot{\theta}_2$	: Angular velocity of the right front wheel of the first module relative to the body of the first module
$\dot{\theta}_3$	: Angular velocity of the left front wheel of the first module relative to the body of the first module
$\dot{\theta}_8$	: Angular velocity of the right front wheel of the second module relative to the body of the second module
$\dot{\theta}_9$	: Angular velocity of the left front wheel of the second module relative to the body of the second module
$\dot{\theta}_{14}$	: Angular velocity of the right front wheel of the third module relative to the body of the third module
$\dot{\theta}_{15}$	: Angular velocity of the left front wheel of the third module relative to the body of the third module
γ_5	: angular displacement of the caster wheel of the first module relative to the body of the first module
γ_{11}	: angular displacement of the caster wheel of the second module relative to the body of the second module

γ_{17}	: angular displacement of the caster wheel of the third module relative to the body of the third module
$\vec{\omega}_1, \vec{\omega}_7, \vec{\omega}_{13}$	: Angular velocity of first, second, third modules with respect to the fixed reference frame
$\vec{\alpha}_1, \vec{\alpha}_7, \vec{\alpha}_{13}$	: Angular acceleration of first, second, third modules with respect to the fixed reference frame
m_1	: the mass of the body of the first module
m_2	: the mass of the right front wheel
m_3	: the mass of the left front wheel
m_4	: the mass of the body 4
m_5	: the mass of the rear wheel
$\vec{F}_{21}$	: the internal force between the right front wheel and the body of the first module
$\vec{F}_{31}$	: the internal force between the left front wheel and the body of the first module
$\vec{F}_{41}$	: the internal force between the body 4 and the body of the first module
$\vec{F}_{54}$	: the internal force between the rear wheel and body 4
$\vec{F}_{02}$	: the external force applied from the ground to the right front wheel
$\vec{F}_{03}$	: the external force applied from the ground to the left front wheel
$\vec{F}_{05}$	: the external force applied from the ground to the rear wheel
$\vec{F}_{61}$	: the external force applied from the joint frame to the body of the first module
$\breve{J}_1$	: the inertia dyadic of the body of the first module

$\breve{J}_2$	: the inertia dyadic of the right front wheel
$\breve{J}_3$	: the inertia dyadic of the left front wheel
$\breve{J}_4$	: the inertia dyadic of the body 4
$\breve{J}_5$	: the inertia dyadic of the rear wheel
$\vec{M}_{12}$	: the internal moment between the right front wheel and the body of the first module
$\vec{M}_{13}$	: the internal moment between the left front wheel and the body of the first module
$\vec{M}_{14}$	: the internal moment between the body 4 and the body of the first module
$\vec{M}_{45}$	: the internal moment between the rear wheel and body 4
T_2	: the motor torques supplied from right motor
T_3	: the motor torques supplied from left motor
μ	: the friction coefficient

CHAPTER 1

INTRODUCTION

1.1 History of the Search and Rescue Robots

Natural disasters such as earthquakes, storms, floods, etc., occur with great frequency and cause enormous hazards, in all over the world. According to the World Disaster Report 2000, which shows the disaster damage in 1990-1999, the damage by the earthquakes is more serious than the other disasters. Unfortunately earthquakes are frequently happenings and are very dangerous in Turkey. In the history of Turkey there are many large-scale earthquakes such as İzmit earthquake on August 17, 1999 and Düzce earthquake on November 12, 1999, which have occurred with great frequency and resulted in enormous damages. In Table 1.1, the large-scale earthquakes in the 20th century are represented with their regions, magnitudes and the number of deaths.

Table 1.1 Human damage by large-scale earthquake disasters in the 20th century [1]

Year	Country/Region	Magnitude	Number of Death
1908	Italy	M 7.0	110000
1915	Italy	M 6.9	32600
1920	China	M 8.6	220000
1923	Japan	M 7.9	142800
1927	China	M 7.9	80000
1935	Pakistan	M 7.6	60000
1939	Turkey/Erzurum	M 7.8	32700
1970	Peru	M 7.6	66800
1976	China	M 7.8	242700
1990	Iran	M 7.3	41000
1999	Turkey/Izmit	M 7.4	17127

One major reason for the high number of deaths is that, in almost every large-scale earthquake many people are buried under collapsed buildings and it is very difficult to rescue these people. The most important tasks in rescue operations are to evaluate the situation, to locate the victims and to establish a first contact with them [2]. However doing all these tasks is both difficult and dangerous for the rescue workers. There is also a risky situation under the collapsed structure because of its instability. Besides, it is difficult to access the people who are buried under the ruins, from the small openings.

Time is very important in the rescue operations. The rubble is often structurally unsafe, so the rescue workers operate slowly and carefully. The number of the rescued people in Kobe earthquake is shown in the Figure 1.1. As it is seen from the figure, when the time passes, the survival rate reduces. After the first 72 hours, which is called "golden 72 hours", the survival rate reduces drastically [3]. It is important to rescue the people, who are buried under the collapsed structures, as soon as possible.

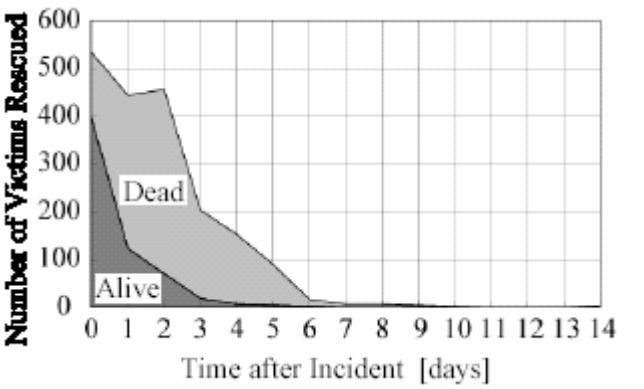


Figure 1.1 Number of victims rescued by Kobe Fire Department in Hanshin Awaji (Kobe) Earthquake [3]

When the search and rescue activities of the previous large-scale earthquake disasters were investigated, it was seen that the disasters were so large that the number of the rescue workers was completely insufficient. Non-professionals saved many victims, who were buried under the collapsed buildings. Search and rescue robots are good solutions for this problem. They can be used to assist the human rescue workers in rescue operations and minimize the risk of the rescue workers. Search and rescue robots can enter the collapsed structure, find the victims and generate human readable maps of the environment to help human rescuers determine the location of the victims. This reduces the rescue operation time and enables to search more collapsed structures.

The World Trade Center (WTC) disaster on September 11, 2001 is the first place where the search and rescue robots are used. The robots were used for; searching for victims, structural inspection of the rubble and detection of hazardous materials and found more than 10 people under the rubble pile [4]. WTC disaster created a training arena for the researchers because they worked with the true domain experts. Besides, WTC disaster is very important for the search and rescue robotics because the robots performed all their tasks well and were accepted by the rescue community.

1.2 Functions of Search and Rescue Robots

Robots are being used in many areas of daily life, military activities and medical operations [5]. As a consequence of studies on using robots in search and rescue activities will result in the extensive use of robots in risky situations such as fire fighting, mining, military inspections etc. In case of a disaster, the rescue workers should pay attention to reach the victims immediately despite the possible fires, and additional collapses or chemical obstacles. Moreover, it is hard to find the victim in the ruins and assist the rescue operation for a rescue worker in a bad structured area. However, the robots used minimizes the risk of human life and maximizes the performance of rescue operation by exploring the environment,

searching the victims buried under the ruins and mapping the most efficient way to reach the victims.

A mobile robot used for such kind of search and rescue activities can be any kind of vehicle, which can search and map the environment, move on unstructured ground with the help of on-board sensors and controllers. There are three locomotion types for robots: wheeled type, crawler type and legged type locomotion. All of them have advantages and disadvantages on rough terrain.

Taking into consideration the studies made on mobile robots used for search and rescue operations, it may be asserted that the robot designed should display high efficiency in its road ability on rough terrain, detect the place of the victim and find the best route to reach the victim. Additionally, it should be easy to control the robot via a responsive interface provided by the control mechanisms such as cameras microphones or 3D maps.

In order to encourage the works on search and rescue robots (SAR) competitions are organized which give the researchers to test their robots designs, software and control mechanisms in a constructed or natural environment. The rescue robot league competitions are very important for the search and rescue robotics. The first competition was held in Texas/USA, in July 2000. In these competitions the capabilities of the robots are tested in The National Institute of Standards and Technology (NIST) test arenas, where the challenges of a collapsed building are simulated. The arena consisted of three sections; Yellow, Orange and Red arenas. The three arenas differ from each other with their physical complexities. The yellow arena is the easiest one and consists of a planar maze. It is reconfigurable to test the mapping and planning capabilities of the robots. The orange arena is of average difficulty. It is a semi-collapsed structure and includes two levels. The robots may have to climb stairs or ramp to reach the second level where the hazards such as holes exist. The red arena is a fully collapsed structure. It is very difficult to traverse. The robots may have to use passageways under the rubble or through pipes to reach certain arenas.

In these competitions the robots try to find the maximum number of victims, and locate the hazards in minimum time without disruption of the test arena. The robots receive points for [6]:

- number of victims located
- number of hazards detected
- mapping of victim and hazard locations
- staying within the time limits
- dropping off a package to victims representing first aid, a radio, or food and water
- quality of communications with humans
- tolerance of communication dropout

They lose points for:

- causing damage to the environment, victims or themselves
- failing to exit within time limits.

1.3 Examples of Search and Rescue Robot

MOIRA and SOURYU-I are the two striking examples of robot studies carried out by university teams.

MOIRA is a snake-like mobile inspection robot developed for rescue activities by Kyoto University. It has multi links and these links have crawlers on their four directions. The distinctive characteristic of MOIRA is that it can go through the ruins despite the restrictions on any directions (up-side, down-side, right-side, left-side) [7].

Fig.1.2 MOIRA [7]

The main goals in the design of MOIRA are to overcome the problems of directional restrictions, lack of control when the robot rolls on ruins and lack of roadability on irregular ground when the robot goes into narrow debris [7]. Having a 4-sided crawler defeats the restrictions on four sides [7]. Thus it can go deep into the ruins and overcomes the first problem. Also having a 4-sided crawler helps to control it, when it rolls no recovery actions are needed. Besides, by introducing connected crawler type and passive joints it becomes easier to adapt the irregular ground conditions.

SOURYU-I is a connected crawler vehicle for inspection of narrow and winding space, developed by Tokyo Institute of Technology and represented in aimed to search people stuck in the ruins of a collapsed building. SOURYU-I is a three segmented robot having crawlers on both sides of front, center and rear segments. All the crawlers are driven simultaneously by the motor, located in the center segment. The front and the rear segments are connected to the center segment by active joint mechanisms. By active joint mechanisms, it is easier to move on irregular terrain.

The main goal in the design of SOURYU-I is to attain the high mobility while having independent driving ability and remote control of the system. Another important point in the design of SOURYU-I is to control the pitch and yaw angles.

By controlling the joint angles, yaw and pitch angles of the segments are modified and turning and climbing actions of the robot are done. Besides it is aimed to create a lightweight structure having hollow spaces for the rescue equipments [8].

Fig.1.3 SOURYU-1 [8]

One of the competitions, which stimulate these studies, is the RoboCup Rescue Robot Competition held periodically in different regions. Some of the robots represented in this periodical competition are TOIN PELICAN, CEDRA, ROBHAZ-DT3 and KURT 3D.

TOIN PELICAN is a mobile rescue robot developed by Toin University of Yokohama and represented by the Japanese team in "Urban Search and Rescue Robots Competition" held in Lisbon between June 27th –July 5th, 2004. The robot introduced by the team is unique with its 6 adaptive crawlers. Since the crawlers can change their forms, the robot is capable of moving both on flat surface and rough surface. With the sensors carried by the 5-degrees of freedom, it gathers information from many angles [9].

TOIN PELICAN robot, with its dimensions of 854 mm length, 482 mm width, 110 mm height and has the ability to adapt to the different ground conditions by changing the position of its crawlers [9]. When it moves on a flat ground the front and back parts are raised which results in little friction. On the other hand, when it moves on a rough ground it flattens the front and back parts so that it reduces the effects of the ground conditions [9]. Besides, the TOIN PELICAN can move over the steps by adapting the crawlers to the ground conditions. While moving over the step, the body of the robot is made to stand up by folding the back crawler inside [9].

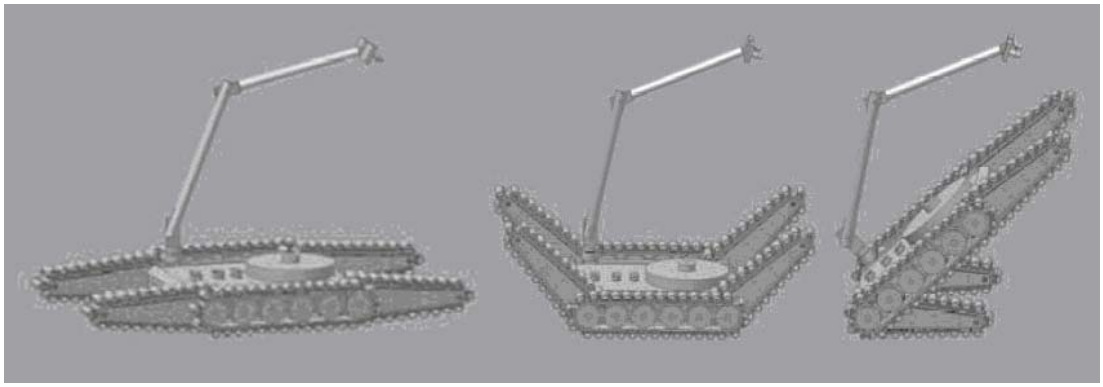


Fig.1.4 TOIN PELICAN moving on flat ground, in debris and over the step [9].

Shrimp Rescue Robot at Center of Excellence in Design Robotics and Automation (CEDRA), and represented by the Iranian team in "Urban Search and Rescue Robots Competition" held in Padova, in July 2003. Shrimp rescue robot is designed for search activities in bad conditioned environments. The Shrimp Rescue Robot has six wheels; front wheel placed on a front-fork mechanism, back wheel, and four side wheels assembled through parallel bogies system. The design of the robot makes it possible for the robot to adjust rough arenas and obstacles such that all six wheels touch the ground simultaneously. It adapts itself to the changing conditions and maintains it stability in rough terrain [10].

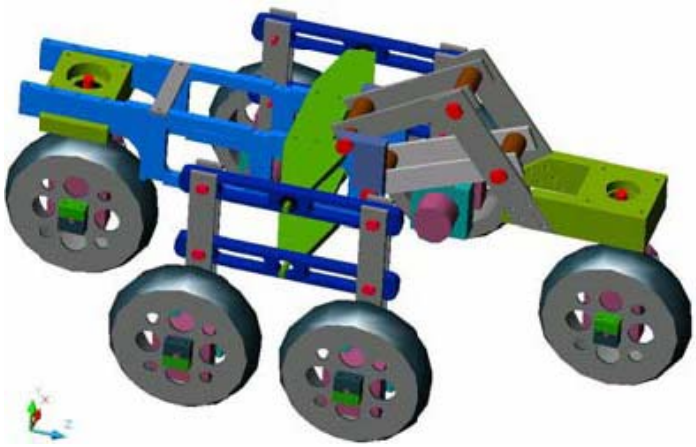


Fig.1.5 CAD model of the modified SHRIMP Rescue Robot [10].

The front fork enables the robot to stabilize itself on the ground and keeps its contact with the ground at all times. Also the four bar mechanism design in the front wheel causes the wheel to move up accordingly. Such use of system enables the robot to move on obstacles and climb the level differences. Another advantage of this robot is that, it maneuvers easily even in limited areas. Six wheels have separate drivers, the front and back wheels have angle adjusting and controlling system. Speed difference in side wheels and adjust the angle of front and back wheels causes the steering. This steering strategy increase the accuracy of robot maneuvers and the robot can turn in its place with minimum slip. Moreover, the design strategy of the robot considers energy consumption by storing the energy in the spring when the robot moves in horizontal direction [10]. Shrimp Rescue Robot can be remotely controlled with the data provided by the microphones and cameras placed on the robot. The map of the environment is prepared with the video images obtained from the cameras and the exact places of the victims can be identified.

ROBHAZ-DT3 is a mobile rescue robot represented by the Korean team in "Urban Search and Rescue Robots Competition" held in New Orleans, between April 24th –April 27th, 2004. ROBHAZ-DT3 is a newly developed mobile robot system with double tracks. The ROBHAZ-DT3 consists of three parts: the front body with

track, rear body with track, and travel limit mechanism of passive joint. The front-left and rear-left tracks have the common driving shaft, also the front-right and rear-left track too. Thus, the two tracks at each side rotate in the same direction. The rotational passive adaptation mechanism equipped between the front and rear body enables the ROBHAZ-DT3 to have good adaptability to uneven terrain including stairways. Use of passive adaptation mechanism simplifies the system and reduces energy consumption despite the use of track mechanism. The main reason to work on a double track mechanism is to overcome the problem of fixed mass center, faced when a single-track system is used. With the chained mechanism used, the mass center varies according to the ground conditions and the passivity is acquired [11]. The passivity can improve adaptability to a rough terrain and the motion occurs successively during stairway climbing.

Fig. 1.6 ROBHAZ-DT3 [11].

KURT3D is a mobile rescue robot developed by Fraunhofer Institute and represented by the German team in “Urban Search and Rescue Robots Competition” held in Lisbon between June 27th –July 5th, 2004. KURT3D has two types; one for the outdoor use and the other for indoor use. What differs in these two types is their

number of wheels and maximum velocities. The one used for indoor search activities has 6 wheels and maximum velocity of 5.2 m/s [12]. However, the one used for outdoor search activities has 4 wheels and a maximum velocity of 1.5 m/s [12].

KURT3D is unique with its autonomous mapping capacity of the environment. The concept is based on the remote control of the robot according to the map scanned through the sensors placed on the robot [12]. Via these sensors the robot can search through the ruins and determine the location of victims. The data gathered through laser scanners, video cameras and microphones are sent through wireless LAN to the operator [12]. Thus, the operator is able to control KURT3D remotely. Manual intervention of the operator is also possible since he can watch the images collected by the robot.

Figure 1.7 KURT3D outdoor edition (left) and indoor edition (right) [12].

1.4 The Objective and Scope of the Thesis

This study intends to design a robot that will be used in search and rescue operations. Therefore, the conceptual design of the search and rescue robot is made which has three modules. In order to control these three modules, “Follow-the-Leader approach” is used, which leads to the movement of the multiple modules in

formation. Besides, it is aimed to simulate the robot's behaviors for different cases through the use of computer programs. Therefore, a computer simulation is developed.

Chapter 1 includes a literature survey about the search and rescue robots in order to attain a general opinion about the previous researches. The functions and advantages of search and rescue robots are discussed referring to the examples demonstrated.

In Chapter 2, the search and rescue robot that we designed is introduced. The structure of the robot, design parameters and the systems selected are described in this chapter.

Chapter 3 covers the detailed mathematical models of the system, which are obtained through using Newton-Euler formulation. The mathematical models obtained are used to construct the model of the robot in Matlab. Therefore, all the kinematic and dynamic equations used in Matlab program are also declared in this chapter.

Control strategies are stated in Chapter 4. Several simulations are carried out in this chapter for different conditions where the path is longitudinal or circular. The results of the simulations are presented for several cases performed.

Finally, in Chapter 5 of the thesis study, the results obtained are discussed and future recommendations are stated.

CHAPTER 2

DESIGN PARAMETERS and SELECTED SYSTEMS

2.1 Design Overview and Selected Systems

A search and rescue mission is extremely challenging and dangerous. The search and rescue area is a highly unstructured and dynamic environment, where the mission is 0time critical. Very little information about the environment may exist. Figure 2.1 and Figure 2.2 illustrate the type of environment. There is totally unstructured rubble, which may be unstable and contain many hazards.

Figure 2.1 Partially Collapsed Building from İzmit Earthquake

Figure 2.2 A View of World Trade Center Disaster

As can be seen from Figure 2.1 and Figure 2.2, the mobility requirements for search and rescue robots are challenging. They must be traverse over piles of rubble, up and down stairs and steep ramps, through small openings. The surface that they must traverse may be composed of a variety of materials, including carpeting, concrete blocks, wood and other construction materials. There may be gaps, holes and discontinuities in the surface that the robot traverses. Moreover, the surface may be unstable and the robot may destabilize the area if it is too heavy. Therefore, the search and rescue robot should have the following functions to move inside the collapsed structures:

- High mobility over debris
- Space to carry equipment used to search for victims
- Independent driving
- Resistance against water, dust, high and low temperature
- Light weight

In the design of the mobile robot, the locomotion type used by robot must be selected carefully. The selection of the type of the locomotion is crucial to the performance of the robot. There are three basic types of locomotion for mobile robots, which are based on wheels, crawlers and legs [13].

Legged mobile robots are well adapted to unstructured environment because they can insure their stability in a wide range of situations, but they are mechanically complex and require a lot of control resources. They demonstrate low speed motion and high power consumption with the other solutions [14].

Crawlers demonstrate good off-road abilities because of their stability and good friction coefficient during motion. Its advantages are robustness and high speed in driving, but friction looses between the surface and the crawlers when the robot is turning are high. It consumes more energy than the others [14].

Wheeled mobile robots are the optimal solutions for well-structured environments. But off-road, their mobility is often very limited and highly depends on the type of environment and the typical size of encountered obstacles. Although they are limited in their terrain adaptability, the mechanisms can be simple and lightweight [14].

Search and rescue robot must be small in order to go into gaps and it is very difficult to build a small, legged type mobile robot. The crawler type robot consumes more energy. Because of simplicity, lightweight and less energy consumption, wheels are selected for the locomotion type of the designed robot robot.

Search and rescue robot can go into narrow spaces, thus it must be in small size. However, it must carry equipment to search for victims and need large volume, therefore size of the robot is a problem. This problem is solved by making the robot in connected body type form. The connected body mobile robot is a mobile robot composed of several connected body segments linked linearly. Their low height bodies are proper to go into narrow spaces and the robot can pass through anywhere the first segment can pass through. Moreover, their connected structures are proper to climb over obstacles.

Connected body type mobile robots can be classified into active joint and passive joint type. The joints are actuated in the active joint types. Active joint types can go over wide gaps and climb over obstacles. However, active joint mechanism is weak against the shock of high-speed movement or of fall on rough terrain because the joints are activated with gear-head motors. Although passive joint types cannot go over wide gaps, they move at high speed on rough terrain and their adaptability to rough terrain is high. Connected body type is selected for the designed robot because of its high-durability.

The steering system determines the mobility of the robot and affects the positioning and obstacle avoidance of the robot. The types of the steering systems are; independent steering, Ackerman steering and skid steering.

Independent steering explicitly articulates each of the wheels to the desired heading. Apart from the issues of actuation complexity and accuracy of coordination control, this scheme provides advantages to the maneuverability of mobile robots, especially those operating in unprepared terrains. Advantages of explicit steering include more aggressive steering with better dead reckoning (due to less slip of the wheels) and lower power consumption. The downside of explicit steering is a higher actuator count, part count, and the necessary swept volume [15]. Ackerman steering is another steering method used almost exclusively in the automotive industry. In this steering system, inner front wheel is rotated a slightly sharper angle than the outside wheel when turning. Skid steering is achieved by creating a differential thrust between the left and right sides of the vehicle thus causing a change in heading. This is an effective and easy solution to steering the robot. However, it is not as accurate as other steering methods; certain characteristics including friction, wheel slippage and other unpredictable attributes can cause problems.

Skid steering can be compact, light, require few parts, and exhibit agility from point turning to line driving using only the motions, components, and swept volume needed for straight driving. For these reasons skid steering is suitable system for the designed robot.

2.2 The Structure of The Robot

Connected body type mobile robots have complex structures and they need complex control systems. By making the segments identical to each other easy maintenance and easy control is accomplished. Easy maintenance is important because when one unit is broken down, by only exchanging the broken unit, the robot continues to work. The designed robot consists of three identical segments.

These segments are connected by passive joints. Each segment consists of one body and four wheels where the front wheels are driven and the rear wheels are towed. A view of the model of robot is shown in Figure 2.3.

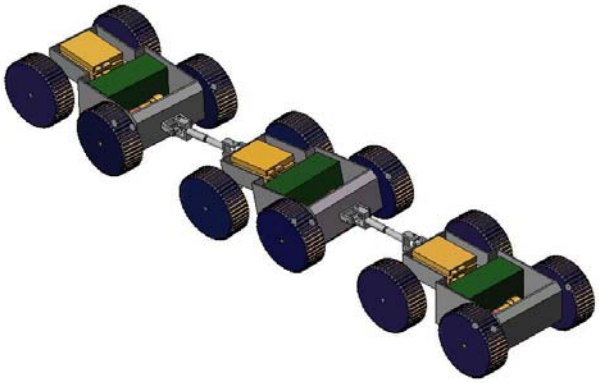


Figure 2.3 CAD model of the Robot

It is important for the robot that all the wheels, especially the driven wheels, have always contact with the ground. The control of the robot is obtained by the driven front wheels. When four wheels are connected the body rigidly, in rough terrain, one or more wheels may lose contact with the ground. To avoid this, the rear axle is connected to the body with a revolute joint as shown in Figure 2.4. This increases the adaptation of the robot to the rough terrain. Therefore, the wheels connected the body from three points and all the wheels always have contact with the ground.

Figure 2.4 Rotating Rear Axle

In addition, the rear axle is connected the body with a part, which is connected the body with a revolute joint as shown in Figure 2.5. This creates a caster effect and while the robot is turning, the rear axle rotates with respect to the body of the robot and relatively low steering power is needed.

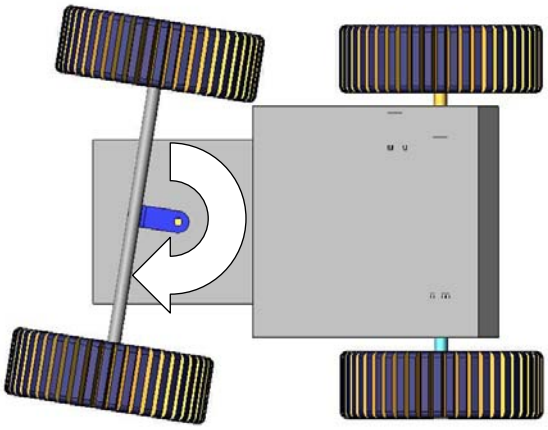


Figure 2.5 Rear Axle is Connected to a Caster Part

The segments of the robot are connected each other by two connecting units, which are composed of two universal and one revolute joint. These connecting units give 5 DOF relative motion to the segments. Connecting unit is shown in Figure 2.6. All the joints are passive joints, not actuated. The passive joints increase the adaptation of the robot to complex environments. The revolute joint can rotate freely on the roll axes and this improves the fitting of the robot on the ground surface.

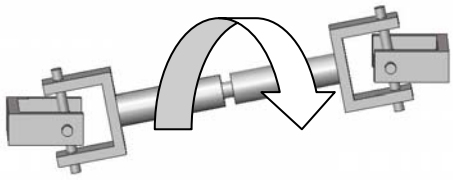


Figure 2.6 Connecting Unit

Each segment has its own battery, control cards and motors. The body of the segment is considered to house the battery, motors, control cards, sensors, etc. The size of the segments determined according to the equipments which they carried. Some equipments of the designed robot have already been decided and by using these equipments, the sizes of the segments are determined. The size of the segment is shown in Figure 2.7. The diameter of the wheel is higher than the body of the segment and the front wheels go forward to the front end of the body of the segment. By this property, the wheels interferes the obstacle and the robot may not stall because of the obstacle.

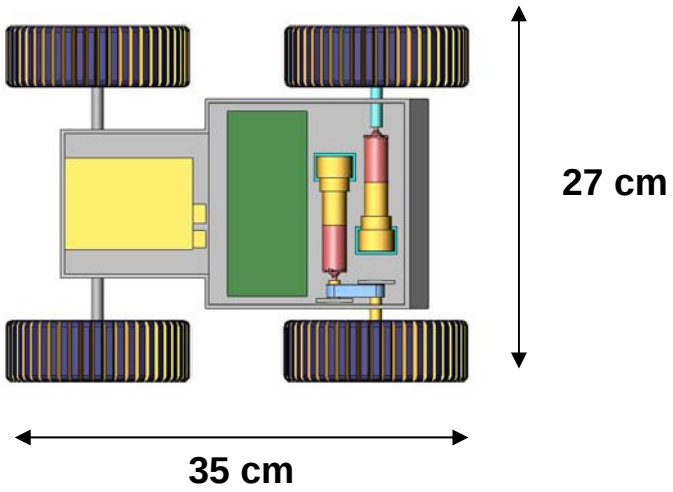


Figure 2.7 Sizes of the Segments

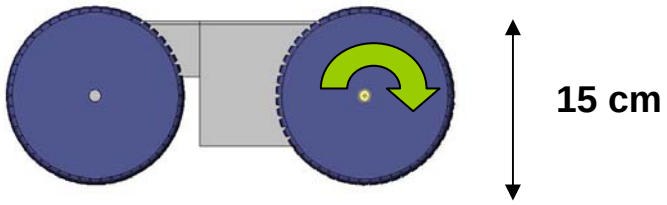


Figure 2.8 Sizes of the Segments

CHAPTER 3

MATHEMATICAL MODEL OF THE SYSTEM

3.1 Definition of the System

The system is composed of three modules and two connecting units, which connect the modules to each other. Each module is composed of two driven wheels on the front and two passive wheels on the rear. The front wheels of each module are independently driven by two motors. The rear wheels are connected to the body with a rotating axle. The connecting unit allows a 5 DOF relative motion of one module with respect to the other module. It is composed of two universal joints and a revolute joint.

The motion of the robot is analyzed on a horizontal surface. The forces, which are transmitted by the connecting unit to the modules, and the reaction forces formed between the wheels and the ground are analyzed by using the mathematical model.

While analyzing the dynamics of the system some simplifying assumptions are made. They are:

- The wheels and the surface are assumed to be rigid.
- There is no slip at the front wheels
- Rear wheels of each module are replaced by a single caster wheel
- The friction force of the rear caster wheel is neglected in its longitudinal direction

3.2 Kinematic Equations

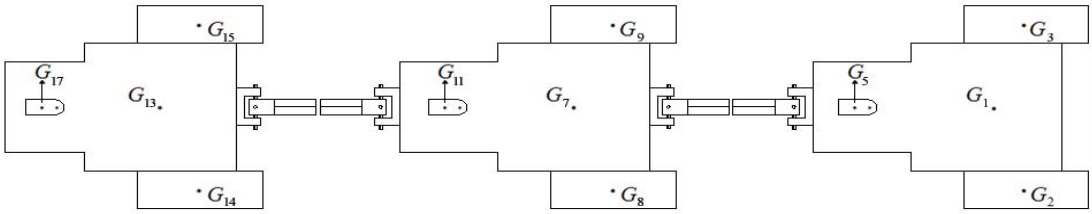


Figure 3.1 Center of Masses of the Rigid Bodies

The robot driven on a horizontal surface is shown in Figure 3.2. $F_0\left\{O;\vec{u}_1^{(0)},\vec{u}_2^{(0)},\vec{u}_3^{(0)}\right\}$ is defined as the inertial frame fixed to the earth, $F_1\left\{G_1;\vec{u}_1^{(1)},\vec{u}_2^{(1)},\vec{u}_3^{(1)}\right\}$ as the frame fixed to the body of the first module, $F_7\left\{G_7;\vec{u}_1^{(7)},\vec{u}_2^{(7)},\vec{u}_3^{(7)}\right\}$ as the frame fixed to the body of the second module and $F_{13}\left\{G_{13};\vec{u}_1^{(13)},\vec{u}_2^{(13)},\vec{u}_3^{(13)}\right\}$ as the frame fixed to the body of the third module. $\vec{u}_1^{(1)}$, $\vec{u}_1^{(7)}$ and $\vec{u}_1^{(13)}$ are defining the direction in which each module travels along the way.

The rotation of the frames $F_1\left\{G_1;\vec{u}_1^{(1)},\vec{u}_2^{(1)},\vec{u}_3^{(1)}\right\}$, $F_7\left\{G_7;\vec{u}_1^{(7)},\vec{u}_2^{(7)},\vec{u}_3^{(7)}\right\}$ and $F_{13}\left\{G_{13};\vec{u}_1^{(13)},\vec{u}_2^{(13)},\vec{u}_3^{(13)}\right\}$ with respect to frame $F_0\left\{O;\vec{u}_1^{(0)},\vec{u}_2^{(0)},\vec{u}_3^{(0)}\right\}$ are designated by the heading angles β_1, β_7 and β_{13}.

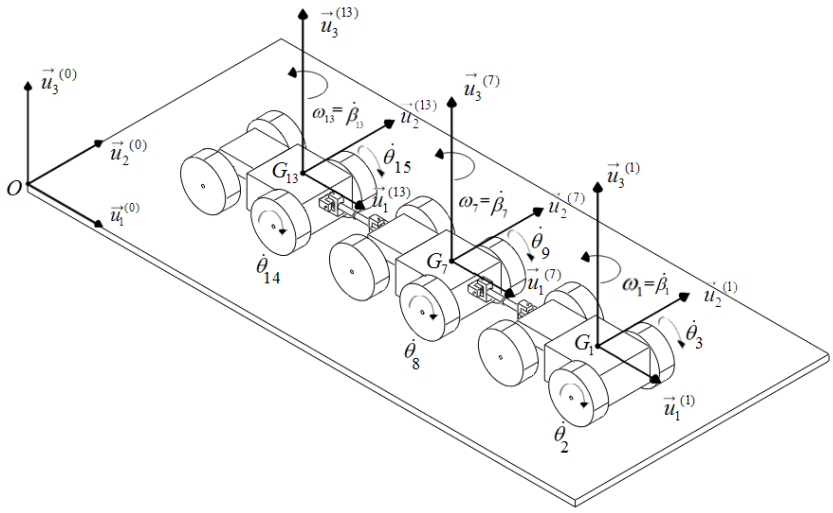


Figure 3.2 Robot on Horizontal Surface

In Figure 3.2; the parameters are defined as:

β_1, β_7, β_{13}	: heading angle of the first, second and third modules
$\dot{\theta}_2$	: angular velocity of the right wheel of the first module relative to the body of the first module
$\dot{\theta}_3$	: angular velocity of the left wheel of the first module relative to the body of the first module
$\dot{\theta}_8$	: angular velocity of the right wheel of the second module relative to the body of the second module
$\dot{\theta}_9$	: angular velocity of the left wheel of the second module relative to the body of the second module
$\dot{\theta}_{14}$	: angular velocity of the right wheel of the third module relative to the body of the third module
$\dot{\theta}_{15}$	: angular velocity of the left wheel of the third module relative to the body of the third module

γ_5 :angular displacement of the caster wheel of the first module relative to the body of the first module

γ_{11} :angular displacement of the caster wheel of the second module relative to the body of the second module

γ_{17} :angular displacement of the caster wheel of the third module relative to the body of the third module

Successive rotations observed in the fixed reference frame $F_0\left\{O;\vec{u}_1^{(0)},\vec{u}_2^{(0)},\vec{u}_3^{(0)}\right\}$ can be designated as:

$$F_0 \overset{\vec{u}_3^{(0)},\beta_1}{\rightarrow} F_1 \tag{3.1}$$

$$F_0 \overset{\vec{u}_3^{(0)},\beta_7}{\rightarrow} F_7 \tag{3.2}$$

$$F_0 \overset{\vec{u}_3^{(0)},\beta_{13}}{\rightarrow} F_{13} \tag{3.3}$$

The transformation matrices between the frames are as follows:

$$\hat{C}^{(0,1)} = e^{\tilde{u}_3\beta_1} \tag{3.4}$$

$$\hat{C}^{(0,7)} = e^{\tilde{u}_3\beta_7} \tag{3.5}$$

$$\hat{C}^{(0,13)} = e^{\tilde{u}_3\beta_{13}} \tag{3.6}$$

The angular velocity of each module with respect to the fixed reference frame F_0 can be written as:

$$\vec{\omega}_{1/0} = \vec{\omega}_1 = \dot{\beta}_1\vec{u}_3^{(0)} = \dot{\beta}_1\vec{u}_3^{(1)} \tag{3.7}$$

$$\vec{\omega}_{7/0} = \vec{\omega}_7 = \dot{\beta}_7\vec{u}_3^{(0)} = \dot{\beta}_7\vec{u}_3^{(7)} \tag{3.8}$$

$$\vec{\omega}_{13/0} = \vec{\omega}_{13} = \dot{\beta}_{13}\vec{u}_3^{(0)} = \dot{\beta}_{13}\vec{u}_3^{(13)} \tag{3.9}$$

The angular accelerations of each module are derived by differentiating the above expressions in the fixed reference frame F_0.

$$\vec{\alpha}_{1/0} = \vec{\alpha}_1 = D_0\vec{\omega}_{1/0} = \ddot{\beta}_1\vec{u}_3^{(0)} \tag{3.10}$$

$$\vec{\alpha}_{7/0} = \vec{\alpha}_7 = D_0\vec{\omega}_{7/0} = \ddot{\beta}_7\vec{u}_3^{(0)} \tag{3.11}$$

$$\vec{\alpha}_{13/0} = \vec{\alpha}_{13} = D_0\vec{\omega}_{13/0} = \ddot{\beta}_{13}\vec{u}_3^{(0)} \tag{3.12}$$

Top and side views of one module can be seen in Figure 3.3 and Figure 3.4.

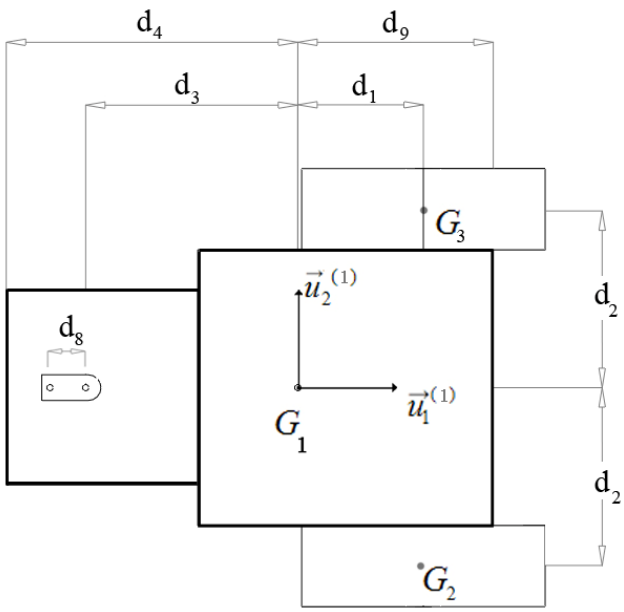


Figure 3.3 Top view of the First Module

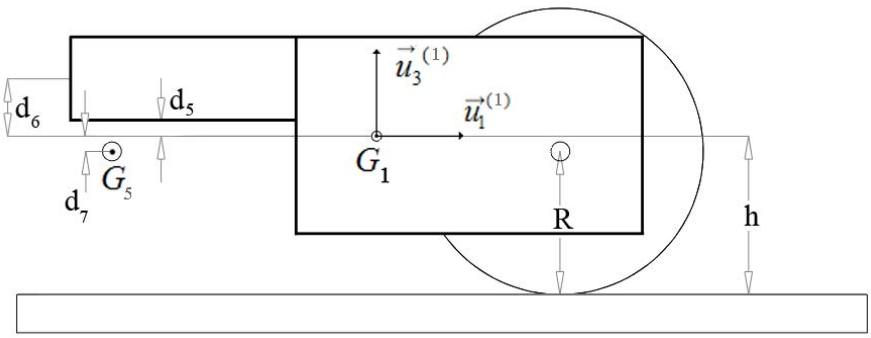


Figure 3.4 Side view of the First Module

In these Figures;

G_1 : center of mass of the body of the module

G_2 : center of mass of the right front wheel

G_3 : center of mass of the left front wheel

G_5 : center of mass of the rear wheel

R : radius of the wheels

h : height of the center of mass of the body of the module

d_1 : the distance between G_1 and the front axle in $\vec{u}_1^{(1)}$ direction

d_2 : the distance between G_1 and G_2, G_3 in $\vec{u}_2^{(1)}$ direction

d_3 : the distance between G_1 and point A in $\vec{u}_1^{(1)}$ direction

d_4 : the distance between G_1 and rear connecting unit in $\vec{u}_1^{(1)}$ direction

d_5 : the distance between G_1 and point A in $\vec{u}_3^{(1)}$ direction

d_6 : the distance between G_1 and rear connecting unit in $\vec{u}_3^{(1)}$ direction

d_7 : the distance between G_5 and point A in $\vec{u}_3^{(1)}$ direction

d_8 : the distance between point A and point B in $\vec{u}_1^{(1)}$ direction

d_9 : the distance between G_1 and front connecting unit in $\vec{u}_1^{(1)}$ direction

The position vectors of points G_1, G_2, G_3 and G_5 on the first module with respect to the origin O of the fixed reference frame $F_0\left\{O;\vec{u}_1^{(0)},\vec{u}_2^{(0)},\vec{u}_3^{(0)}\right\}$ can be written by Equations 3.13 to 3.16, where x_1 and y_1 are the displacements of G_1, in $\vec{u}_1^{(0)}$ and $\vec{u}_2^{(0)}$ directions.

$$\vec{r}_{G_1/O} = \vec{r}_{G_1} = x_1\vec{u}_1^{(0)} + y_1\vec{u}_2^{(0)} + h\vec{u}_3^{(0)} \tag{3.13}$$

$$\vec{r}_{G_2/O} = \vec{r}_{G_2} = x_1\vec{u}_1^{(0)} + y_1\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} + d_1\vec{u}_1^{(1)} - d_2\vec{u}_2^{(1)} \tag{3.14}$$

$$\vec{r}_{G_3/O} = \vec{r}_{G_3} = x_1\vec{u}_1^{(0)} + y_1\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} + d_1\vec{u}_1^{(1)} + d_2\vec{u}_2^{(1)} \tag{3.15}$$

$$\vec{r}_{G_5/O} = \vec{r}_{G_5} = x_1\vec{u}_1^{(0)} + y_1\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} - \left(d_8\cos(\gamma_5) + d_3\right)\vec{u}_1^{(1)} - d_8\sin(\gamma_5)\vec{u}_2^{(1)} \tag{3.16}$$

Similarly the position vectors of points G_7, G_8, G_9 and G_{11} on the second module with respect to the origin O of the fixed reference frame $F_0\left\{O;\vec{u}_1^{(0)},\vec{u}_2^{(0)},\vec{u}_3^{(0)}\right\}$ can be written by Equations 3.17 to 3.20, where x_2 and y_2 are the displacements of G_7, in $\vec{u}_1^{(0)}$ and $\vec{u}_2^{(0)}$ directions.

$$\vec{r}_{G_7/O} = \vec{r}_{G_7} = x_2\vec{u}_1^{(0)} + y_2\vec{u}_2^{(0)} + h\vec{u}_3^{(0)} \tag{3.17}$$

$$\vec{r}_{G_8/O} = \vec{r}_{G_8} = x_2\vec{u}_1^{(0)} + y_2\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} + d_1\vec{u}_1^{(7)} - d_2\vec{u}_2^{(7)} \tag{3.18}$$

$$\vec{r}_{G_9/O} = \vec{r}_{G_9} = x_2\vec{u}_1^{(0)} + y_2\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} + d_1\vec{u}_1^{(7)} + d_2\vec{u}_2^{(7)} \tag{3.19}$$

$$\vec{r}_{G_{11}/O} = \vec{r}_{G_{11}} = x_2\vec{u}_1^{(0)} + y_2\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} - \left(d_8\cos(\gamma_{11}) + d_3\right)\vec{u}_1^{(7)} - d_8\sin(\gamma_{11})\vec{u}_2^{(7)} \tag{3.20}$$

Similarly the position vectors of points G_{13}, G_{14}, G_{15} and G_{17} on the third module with respect to the origin O of the fixed reference frame $F_0\left\{O;\vec{u}_1^{(0)},\vec{u}_2^{(0)},\vec{u}_3^{(0)}\right\}$ can be written by Equations 3.21 to 3.24, where x_3 and y_3 are the displacements of G_{13}, in $\vec{u}_1^{(0)}$ and $\vec{u}_2^{(0)}$ directions.

$$\vec{r}_{G_{13}/O} = \vec{r}_{G_{13}} = x_3\vec{u}_1^{(0)} + y_3\vec{u}_2^{(0)} + h\vec{u}_3^{(13)} \tag{3.21}$$

$$\vec{r}_{G_{14}/O} = \vec{r}_{G_{14}} = x_3\vec{u}_1^{(0)} + y_3\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} + d_1\vec{u}_1^{(13)} - d_2\vec{u}_2^{(13)} \tag{3.22}$$

$$\vec{r}_{G_{15}/O} = \vec{r}_{G_{15}} = x_3\vec{u}_1^{(0)} + y_3\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} + d_1\vec{u}_1^{(13)} + d_2\vec{u}_2^{(13)} \tag{3.23}$$

$$\begin{aligned}\vec{r}_{G_{17}/O} = \vec{r}_{G_{17}} = {} & x_3\vec{u}_1^{(0)} + y_3\vec{u}_2^{(0)} + R\vec{u}_3^{(0)} - \left(d_8\cos(\gamma_{17}) + d_3\right)\vec{u}_1^{(13)} \\ & -d_8\sin(\gamma_{17})\vec{u}_2^{(13)}\end{aligned} \tag{3.24}$$

The velocity of points G_1, G_2, G_3 and G_5 can be obtained by differentiating the position vectors $\vec{r}_{G_1}, \vec{r}_{G_2}, \vec{r}_{G_3}$ and $\vec{r}_{G_5}$ in the fixed reference frame F_0, as follows:

$$\vec{V}_{G_1} = D_0\vec{r}_{G_1} = \dot{x}_1\vec{u}_1^{(0)} + \dot{y}_1\vec{u}_2^{(0)} \tag{3.25}$$

$$\vec{V}_{G_2} = D_0\vec{r}_{G_2} = \dot{x}_1\vec{u}_1^{(0)} + \dot{y}_1\vec{u}_2^{(0)} + d_1 D_0\vec{u}_1^{(1)} - d_2 D_0\vec{u}_2^{(1)} \tag{3.26}$$

$$\vec{V}_{G_3} = D_0\vec{r}_{G_3} = \dot{x}_1\vec{u}_1^{(0)} + \dot{y}_1\vec{u}_2^{(0)} + d_1 D_0\vec{u}_1^{(1)} + d_2 D_0\vec{u}_2^{(1)} \tag{3.27}$$

$$\begin{aligned}\vec{V}_{G_5} = D_0\vec{r}_{G_5} &= \dot{x}_1\vec{u}_1^{(0)} + \dot{y}_1\vec{u}_2^{(0)} + d_8\dot{\gamma}_1 \sin(\gamma_1)\vec{u}_1^{(1)} \\ &-\left(d_8\cos(\gamma_1) + d_3\right) D_0\vec{u}_1^{(1)} - d_8\dot{\gamma}_1\cos(\gamma_1)\vec{u}_2^{(1)} - d_8\sin(\gamma_1) D_0\vec{u}_2^{(1)}\end{aligned} \tag{3.28}$$

where

$$D_0\vec{u}_1^{(1)} = D_1\vec{u}_1^{(1)} + \vec{\omega}_{1/0} \times \vec{u}_1^{(1)} \tag{3.29}$$

and

$$D_0\vec{u}_2^{(1)} = D_1\vec{u}_2^{(1)} + \vec{\omega}_{1/0} \times \vec{u}_2^{(1)} \tag{3.30}$$

Differentiation of a unit vector in its own frame gives zero. Therefore:

$$D_1\vec{u}_1^{(1)} = D_1\vec{u}_2^{(1)} = 0 \tag{3.31}$$

By substituting Equation (3.31) into Equations (3.29) and (3.30) one can get the following equations:

$$D_0\vec{u}_1^{(1)} = \vec{\omega}_{1/0} \times \vec{u}_1^{(1)} = \dot{\beta}_1\vec{u}_3^{(1)} \times \vec{u}_1^{(1)} = \dot{\beta}_1\vec{u}_2^{(1)} \tag{3.32}$$

$$D_0\vec{u}_2^{(1)} = \vec{\omega}_{1/0} \times \vec{u}_2^{(1)} = \dot{\beta}_1\vec{u}_3^{(1)} \times \vec{u}_2^{(1)} = -\dot{\beta}_1\vec{u}_1^{(1)} \tag{3.33}$$

By substituting Equation (3.32) and (3.33) into Equation (3.26), (3.27) and (3.28), the velocities of the points G_2, G_3 and G_5 can be written as:

$$\vec{V}_{G_2} = \dot{x}_1\vec{u}_1^{(0)} + \dot{y}_1\vec{u}_2^{(0)} + d_1\dot{\beta}_1\vec{u}_2^{(1)} + d_2\dot{\beta}_1\vec{u}_1^{(1)} \tag{3.34}$$

$$\vec{V}_{G_3} = \dot{x}_1\vec{u}_1^{(0)} + \dot{y}_1\vec{u}_2^{(0)} + d_1\dot{\beta}_1\vec{u}_2^{(1)} - d_2\dot{\beta}_1\vec{u}_1^{(1)} \tag{3.35}$$

$$\begin{aligned}\vec{V}_{G_5} &= \dot{x}_1\vec{u}_1^{(0)} + \dot{y}_1\vec{u}_2^{(0)} + d_8\dot{\gamma}_1\sin(\gamma_1)\vec{u}_1^{(1)} - \left(d_8\cos(\gamma_1) + d_3\right)\dot{\beta}_1\vec{u}_2^{(1)} \\ &\quad -d_8\dot{\gamma}_1\cos(\gamma_1)\vec{u}_2^{(1)} + d_8\sin(\gamma_1)\dot{\beta}_1\vec{u}_1^{(1)}\end{aligned} \tag{3.36}$$

Similarly the velocity of the points G_7, G_8, G_9 and G_{11} on the second module and G_{13}, G_{14}, G_{15} and G_{17} on the third module are obtained same as with the points G_1, G_2, G_3 and G_5 on the first module.

$$\vec{V}_{G_7} = D_0\vec{r}_{G_7} = \dot{x}_2\vec{u}_1^{(0)} + \dot{y}_2\vec{u}_2^{(0)} \tag{3.37}$$

$$\vec{V}_{G_8} = D_0\vec{r}_{G_8} = \dot{x}_2\vec{u}_1^{(0)} + \dot{y}_2\vec{u}_2^{(0)} + d_1\dot{\beta}_7\vec{u}_2^{(7)} + d_2\dot{\beta}_7\vec{u}_1^{(7)} \tag{3.38}$$

$$\vec{V}_{G_9} = D_0\vec{r}_{G_9} = \dot{x}_2\vec{u}_1^{(0)} + \dot{y}_2\vec{u}_2^{(0)} + d_1\dot{\beta}_7\vec{u}_2^{(7)} - d_2\dot{\beta}_7\vec{u}_1^{(7)} \tag{3.39}$$

$$\begin{aligned}\vec{V}_{G_{11}} &= D_0\vec{r}_{G_{11}} = \dot{x}_2\vec{u}_1^{(0)} + \dot{y}_2\vec{u}_2^{(0)} + d_8\dot{\gamma}_{11}\sin(\gamma_{11})\vec{u}_1^{(7)} \\ &\quad -\left(d_8\cos(\theta_{D_2}) + d_3\right)\dot{\beta}_7\vec{u}_2^{(7)} - d_8\dot{\gamma}_{11}\cos(\gamma_{11})\vec{u}_2^{(7)} + d_8\sin(\gamma_{11})\dot{\beta}_7\vec{u}_1^{(7)}\end{aligned} \tag{3.40}$$

$$\vec{V}_{G_{13}} = D_0\vec{r}_{G_{13}} = \dot{x}_3\vec{u}_1^{(0)} + \dot{y}_3\vec{u}_2^{(0)} \tag{3.41}$$

$$\vec{V}_{G_{14}} = D_0\vec{r}_{G_{14}} = \dot{x}_3\vec{u}_1^{(0)} + \dot{y}_3\vec{u}_2^{(0)} + d_1\dot{\beta}_{13}\vec{u}_2^{(13)} + d_2\dot{\beta}_{13}\vec{u}_1^{(13)} \tag{3.42}$$

$$\vec{V}_{G_{15}} = D_0\vec{r}_{G_{15}} = \dot{x}_3\vec{u}_1^{(0)} + \dot{y}_3\vec{u}_2^{(0)} + d_1\dot{\beta}_{13}\vec{u}_2^{(13)} - d_2\dot{\beta}_{13}\vec{u}_1^{(13)} \tag{3.43}$$

$$\begin{aligned}\vec{V}_{G_{17}} &= D_0\vec{r}_{G_{17}} = \dot{x}_3\vec{u}_1^{(0)} + \dot{y}_3\vec{u}_2^{(0)} + d_8\dot{\gamma}_{17}\sin(\gamma_{17})\vec{u}_1^{(13)} \\ &\quad -\left(d_8\cos(\gamma_{17}) + d_3\right)\dot{\beta}_{13}\vec{u}_2^{(13)} - d_8\dot{\gamma}_{17}\cos(\gamma_{17})\vec{u}_2^{(13)} + d_8\sin(\gamma_{17})\dot{\beta}_{13}\vec{u}_1^{(13)}\end{aligned} \tag{3.44}$$

By differentiating the velocities of points G_1, G_2, G_3 and G_5 in F_0 frame accelerations of points can be derived. The following equations represent these accelerations.

$$\vec{a}_{G_1} = \ddot{x}_1 \vec{u}_1^{(0)} + \ddot{y}_1 \vec{u}_2^{(0)} \tag{3.45}$$

$$\vec{a}_{G_2} = \ddot{x}_1 \vec{u}_1^{(0)} + \ddot{y}_1 \vec{u}_2^{(0)} + \ddot{\beta}_1 \left(d_1 \vec{u}_2^{(1)} + d_2 \vec{u}_1^{(1)} \right) + \dot{\beta}_1^2 \left(-d_1 \vec{u}_1^{(1)} + d_2 \vec{u}_2^{(1)} \right) \tag{3.46}$$

$$\vec{a}_{G_3} = \ddot{x}_1 \vec{u}_1^{(0)} + \ddot{y}_1 \vec{u}_2^{(0)} + \ddot{\beta}_1 \left(d_1 \vec{u}_2^{(1)} - d_2 \vec{u}_1^{(1)} \right) + \dot{\beta}_1^2 \left(-d_1 \vec{u}_1^{(1)} - d_2 \vec{u}_2^{(1)} \right) \tag{3.47}$$

$$\begin{aligned} \vec{a}_{G_5} &= \ddot{x}_1 \vec{u}_1^{(0)} + \ddot{y}_1 \vec{u}_2^{(0)} \\ &+ \begin{pmatrix} d_8 \ddot{\gamma}_5 \sin(\gamma_5) + d_8 \dot{\gamma}_5^2 \cos(\gamma_5) + 2 d_8 \dot{\gamma}_5 \dot{\beta}_1 \cos(\gamma_5) \\ + d_8 \ddot{\beta}_1 \sin(\gamma_5) + \left(d_8 \cos(\gamma_5) + d_3 \right) \dot{\beta}_1^2 \end{pmatrix} \vec{u}_1^{(1)} \\ &+ \begin{pmatrix} 2 d_8 \dot{\gamma}_5 \dot{\beta}_1 \sin(\gamma_5) + d_8 \sin(\gamma_5) \dot{\beta}_1^2 - \left(d_8 \cos(\gamma_5) + d_3 \right) \ddot{\beta}_1 \\ - d_8 \ddot{\gamma}_5 \cos(\gamma_5) + d_8 \dot{\gamma}_5^2 \sin(\gamma_5) \end{pmatrix} \vec{u}_2^{(1)} \end{aligned} \tag{3.48}$$

Similarly accelerations of the points G_7, G_8, G_9 and G_{11} on the second module and the points G_{13}, G_{14}, G_{15} and G_{17} on the third module can be derived by differentiating the velocities in F_0 frame.

$$\vec{a}_{G_7} = \ddot{x}_2 \vec{u}_1^{(0)} + \ddot{y}_2 \vec{u}_2^{(0)} \tag{3.49}$$

$$\vec{a}_{G_8} = \ddot{x}_2 \vec{u}_1^{(0)} + \ddot{y}_2 \vec{u}_2^{(0)} + \ddot{\beta}_7 \left(d_1 \vec{u}_2^{(7)} + d_2 \vec{u}_1^{(7)} \right) + \dot{\beta}_7^2 \left(-d_1 \vec{u}_1^{(7)} + d_2 \vec{u}_2^{(7)} \right) \tag{3.50}$$

$$\vec{a}_{G_9} = \ddot{x}_2 \vec{u}_1^{(0)} + \ddot{y}_2 \vec{u}_2^{(0)} + \ddot{\beta}_7 \left(d_1 \vec{u}_2^{(7)} - d_2 \vec{u}_1^{(7)} \right) + \dot{\beta}_7^2 \left(-d_1 \vec{u}_1^{(7)} - d_2 \vec{u}_2^{(7)} \right) \tag{3.51}$$

$$\begin{aligned} \vec{a}_{G_{11}} &= \ddot{x}_2 \vec{u}_1^{(0)} + \ddot{y}_2 \vec{u}_2^{(0)} \\ &+ \begin{pmatrix} d_8 \ddot{\gamma}_{11} \sin(\gamma_{11}) + d_8 \dot{\gamma}_{11}^2 \cos(\gamma_{11}) + 2 d_8 \dot{\gamma}_{11} \dot{\beta}_7 \cos(\gamma_{11}) \\ + d_8 \ddot{\beta}_7 \sin(\gamma_{11}) + \left(d_8 \cos(\gamma_{11}) + d_3 \right) \dot{\beta}_7^2 \end{pmatrix} \vec{u}_1^{(7)} \\ &+ \begin{pmatrix} 2 d_8 \dot{\gamma}_{11} \dot{\beta}_7 \sin(\gamma_{11}) + d_8 \sin(\gamma_{11}) \dot{\beta}_7^2 - \left(d_8 \cos(\gamma_{11}) + d_3 \right) \ddot{\beta}_7 \\ - d_8 \ddot{\gamma}_{11} \cos(\gamma_{11}) + d_8 \dot{\gamma}_{11}^2 \sin(\gamma_{11}) \end{pmatrix} \vec{u}_2^{(7)} \end{aligned} \tag{3.52}$$

$$\vec{a}_{G_{13}} = \ddot{x}_3 \vec{u}_1^{(0)} + \ddot{y}_3 \vec{u}_2^{(0)} \tag{3.53}$$

$$\vec{a}_{G_{14}} = \ddot{x}_3\vec{u}_1^{(0)} + \ddot{y}_3\vec{u}_2^{(0)} + \ddot{\beta}_{13}\left(d_1\vec{u}_2^{(13)} + d_2\vec{u}_1^{(13)}\right) + \dot{\beta}_{13}^{\,2}\left(-d_1\vec{u}_1^{(13)} + d_2\vec{u}_2^{(13)}\right) \tag{3.54}$$

$$\vec{a}_{G_{15}} = \ddot{x}_3\vec{u}_1^{(0)} + \ddot{y}_3\vec{u}_2^{(0)} + \ddot{\beta}_{13}\left(d_1\vec{u}_2^{(13)} - d_2\vec{u}_1^{(13)}\right) + \dot{\beta}_{13}^{\,2}\left(-d_1\vec{u}_1^{(13)} - d_2\vec{u}_2^{(13)}\right) \tag{3.55}$$

$$\begin{aligned}\vec{a}_{G_{17}} &= \ddot{x}_3\vec{u}_1^{(0)} + \ddot{y}_3\vec{u}_2^{(0)} \\ &+ \begin{pmatrix} d_8\ddot{\gamma}_{17}\sin(\gamma_{17}) + d_8\dot{\gamma}_{17}^2\cos(\gamma_{17}) + 2d_8\dot{\gamma}_{17}\dot{\beta}_{13}\cos(\gamma_{17}) \\ +d_8\ddot{\beta}_{13}\sin(\gamma_{17}) + \left(d_8\cos(\gamma_{17}) + d_3\right)\dot{\beta}_{13}^{\,2} \end{pmatrix}\vec{u}_1^{(13)} \\ &+ \begin{pmatrix} 2d_8\dot{\gamma}_{17}\dot{\beta}_{13}\sin(\gamma_{17}) + d_8\sin(\gamma_{17})\dot{\beta}_{13}^{\,2} - \left(d_8\cos(\gamma_{17}) + d_3\right)\ddot{\beta}_{13} \\ -d_8\ddot{\gamma}_{17}\cos(\gamma_{17}) + d_8\dot{\gamma}_{17}^2\sin(\gamma_{17}) \end{pmatrix}\vec{u}_2^{(13)}\end{aligned} \tag{3.56}$$

The angular velocities of the wheels of the first module with respect to the fixed reference frame F_0 can be written as:

$$\vec{\omega}_{2/0} = \vec{\omega}_{2/1} + \vec{\omega}_{1/0} \tag{3.57}$$

$$\vec{\omega}_{3/0} = \vec{\omega}_{3/1} + \vec{\omega}_{1/0} \tag{3.58}$$

$$\vec{\omega}_{5/0} = \vec{\omega}_{5/1} + \vec{\omega}_{1/0} \tag{3.59}$$

where

$$\vec{\omega}_{2/1} = \dot{\theta}_2\vec{u}_2^{(1)} \tag{3.60}$$

$$\vec{\omega}_{3/1} = \dot{\theta}_3\vec{u}_2^{(1)} \tag{3.61}$$

$$\vec{\omega}_{5/1} = \dot{\gamma}_5\vec{u}_3^{(1)} \tag{3.62}$$

The terms $\dot{\theta}_2$ and $\dot{\theta}_3$ denote the angular velocities of the right and left front wheels of the first module.

The angular velocities of the wheels of the second module with respect to the fixed reference frame F_0 can be written as:

$$\vec{\omega}_{8/0} = \vec{\omega}_{8/7} + \vec{\omega}_{7/0} \tag{3.63}$$

$$\vec{\omega}_{9/0} = \vec{\omega}_{9/7} + \vec{\omega}_{7/0} \tag{3.64}$$

$$\vec{\omega}_{11/0} = \vec{\omega}_{11/7} + \vec{\omega}_{7/0} \tag{3.65}$$

where

$$\vec{\omega}_{8/7} = \dot{\theta}_8 \vec{u}_2^{\,(7)} \tag{3.66}$$

$$\vec{\omega}_{9/7} = \dot{\theta}_9 \vec{u}_2^{\,(7)} \tag{3.67}$$

$$\vec{\omega}_{11/7} = \dot{\gamma}_{11} \vec{u}_3^{\,(7)} \tag{3.68}$$

The terms $\dot{\theta}_8$ and $\dot{\theta}_9$ denote the angular velocities of the right and left front wheels of the second module.

The angular velocities of the wheels of the third module with respect to the fixed reference frame F_0 can be written as:

$$\vec{\omega}_{14/0} = \vec{\omega}_{14/13} + \vec{\omega}_{13/0} \tag{3.69}$$

$$\vec{\omega}_{15/0} = \vec{\omega}_{15/13} + \vec{\omega}_{13/0} \tag{3.70}$$

$$\vec{\omega}_{17/0} = \vec{\omega}_{17/13} + \vec{\omega}_{13/0} \tag{3.71}$$

where

$$\vec{\omega}_{14/13} = \dot{\theta}_{14} \vec{u}_2^{\,(13)} \tag{3.72}$$

$$\vec{\omega}_{15/13} = \dot{\theta}_{15} \vec{u}_2^{\,(13)} \tag{3.73}$$

$$\vec{\omega}_{17/13} = \dot{\gamma}_{17} \vec{u}_3^{\,(13)} \tag{3.74}$$

The terms $\dot{\theta}_{14}$ and $\dot{\theta}_{15}$ denote the angular velocities of the right and left front wheels of the third module.

The angular accelerations of the wheels of the first module are obtained by differentiating the Equations (3.57), (3.58) and (3.59) in the fixed reference frame F_0.

$$\vec{\alpha}_{2/0} = D_0\vec{\omega}_{1/0} + D_0\vec{\omega}_{2/1} \tag{3.75}$$

$$\vec{\alpha}_{3/0} = D_0\vec{\omega}_{1/0} + D_0\vec{\omega}_{3/1} \tag{3.76}$$

$$\vec{\alpha}_{5/0} = D_0\vec{\omega}_{1/0} + D_0\vec{\omega}_{5/1} \tag{3.77}$$

where

$$D_0\vec{\omega}_{1/0} = \vec{\alpha}_{1/0} = \ddot{\beta}_1\vec{u}_3^{(1)} \tag{3.78}$$

$$D_0\vec{\omega}_{2/1} = D_0\left(\dot{\theta}_2\vec{u}_2^{(1)}\right) = D_1\left(\dot{\theta}_2\vec{u}_2^{(1)}\right) + \vec{\omega}_{1/0} \times \dot{\theta}_2\vec{u}_2^{(1)} \tag{3.79}$$

$$D_0\vec{\omega}_{3/1} = D_0\left(\dot{\theta}_3\vec{u}_2^{(1)}\right) = D_1\left(\dot{\theta}_3\vec{u}_2^{(1)}\right) + \vec{\omega}_{1/0} \times \dot{\theta}_3\vec{u}_2^{(1)} \tag{3.80}$$

$$D_0\vec{\omega}_{5/1} = D_0\left(\dot{\gamma}_5\vec{u}_3^{(1)}\right) = D_1\left(\dot{\gamma}_5\vec{u}_3^{(1)}\right) + \vec{\omega}_{1/0} \times \dot{\gamma}_5\vec{u}_3^{(1)} \tag{3.81}$$

Rearranging the terms in Equations (3.79), (3.80) and (3.81) gives;

$$D_0\left(\dot{\theta}_2\vec{u}_2^{(1)}\right) = \ddot{\theta}_2\vec{u}_2^{(1)} + \dot{\beta}_1\vec{u}_3^{(1)} \times \dot{\theta}_2\vec{u}_2^{(1)} \tag{3.82}$$

$$D_0\left(\dot{\theta}_3\vec{u}_2^{(1)}\right) = \ddot{\theta}_3\vec{u}_2^{(1)} + \dot{\beta}_1\vec{u}_3^{(1)} \times \dot{\theta}_3\vec{u}_2^{(1)} \tag{3.83}$$

$$D_0\left(\dot{\theta}_{D_1}\vec{u}_3^{(1)}\right) = \ddot{\gamma}_5\vec{u}_3^{(1)} \tag{3.84}$$

Substituting Equations (3.78) and (3.82) into Equation (3.75), Equations (3.78) and (3.83) into Equation (3.76) and Equations (3.78) and (3.84) into Equation (3.77) the angular accelerations of the wheels of the first module can be derived as in the following equations.

$$\vec{\alpha}_{2/0} = \ddot{\beta}_1 \vec{u}_3^{(1)} + \ddot{\theta}_2 \vec{u}_2^{(1)} - \dot{\beta}_1 \dot{\theta}_2 \vec{u}_1^{(1)} \tag{3.85}$$

$$\vec{\alpha}_{3/0} = \ddot{\beta}_1 \vec{u}_3^{(1)} + \ddot{\theta}_3 \vec{u}_2^{(1)} - \dot{\beta}_1 \dot{\theta}_3 \vec{u}_1^{(1)} \tag{3.86}$$

$$\vec{\alpha}_{5/0} = \left(\ddot{\beta}_1 + \ddot{\gamma}_5 \right) \vec{u}_3^{(1)} \tag{3.87}$$

The angular accelerations of the wheels of the second module are obtained by differentiating the Equations (3.63), (3.64) and (3.65) in the fixed reference frame F_0.

$$\vec{\alpha}_{8/0} = D_0 \vec{\omega}_{7/0} + D_0 \vec{\omega}_{8/7} \tag{3.88}$$

$$\vec{\alpha}_{9/0} = D_0 \vec{\omega}_{7/0} + D_0 \vec{\omega}_{9/7} \tag{3.89}$$

$$\vec{\alpha}_{11/0} = D_0 \vec{\omega}_{7/0} + D_0 \vec{\omega}_{11/7} \tag{3.90}$$

Similarly the angular accelerations of the wheels on the second module are obtained same as with the angular accelerations of the wheels on the first module.

$$\vec{\alpha}_{8/0} = \ddot{\beta}_7 \vec{u}_3^{(7)} + \ddot{\theta}_8 \vec{u}_2^{(7)} - \dot{\beta}_7 \dot{\theta}_8 \vec{u}_1^{(7)} \tag{3.91}$$

$$\vec{\alpha}_{9/0} = \ddot{\beta}_7 \vec{u}_3^{(7)} + \ddot{\theta}_9 \vec{u}_2^{(7)} - \dot{\beta}_7 \dot{\theta}_9 \vec{u}_1^{(7)} \tag{3.92}$$

$$\vec{\alpha}_{11/0} = \left(\ddot{\beta}_7 + \ddot{\gamma}_{11} \right) \vec{u}_3^{(7)} \tag{3.93}$$

The angular accelerations of the wheels of the third module are obtained by differentiating the Equations (3.69), (3.70) and (3.71) in the fixed reference frame F_0.

$$\vec{\alpha}_{14/0} = D_0 \vec{\omega}_{13/0} + D_0 \vec{\omega}_{14/13} \tag{3.94}$$

$$\vec{\alpha}_{15/0} = D_0 \vec{\omega}_{13/0} + D_0 \vec{\omega}_{15/13} \tag{3.95}$$

$$\vec{\alpha}_{17/0} = D_0 \vec{\omega}_{13/0} + D_0 \vec{\omega}_{17/13} \tag{3.96}$$

Similarly the angular accelerations of the wheels on the third module are obtained same as with the angular accelerations of the wheels on the first module.

$$\vec{\alpha}_{14/0} = \ddot{\beta}_{13}\vec{u}_3^{(13)} + \ddot{\theta}_{14}\vec{u}_2^{(13)} - \dot{\beta}_{13}\dot{\theta}_{14}\vec{u}_1^{(13)} \tag{3.97}$$

$$\vec{\alpha}_{15/0} = \ddot{\beta}_{13}\vec{u}_3^{(13)} + \ddot{\theta}_{15}\vec{u}_2^{(13)} - \dot{\beta}_{13}\dot{\theta}_{15}\vec{u}_1^{(13)} \tag{3.98}$$

$$\vec{\alpha}_{17/0} = \left(\ddot{\beta}_{13} + \ddot{\gamma}_{17}\right)\vec{u}_3^{(13)} \tag{3.99}$$

Front wheels are assumed to roll without slipping so one can rewrite the velocities of the points G_2 and G_3, which are the centers of mass of the front wheels of the first module as follows:

$$\vec{V}_{G_2} = \left(\dot{\theta}_2\vec{u}_2^{(1)}\right)\times\left(R\vec{u}_3^{(1)}\right) = R\dot{\theta}_2\vec{u}_1^{(1)} \tag{3.100}$$

$$\vec{V}_{G_3} = \left(\dot{\theta}_3\vec{u}_2^{(1)}\right)\times\left(R\vec{u}_3^{(1)}\right) = R\dot{\theta}_3\vec{u}_1^{(1)} \tag{3.101}$$

Note that;

$$\vec{V}_{G_2}\cdot\vec{u}_1^{(1)} = R\dot{\theta}_2 \tag{3.102}$$

$$\vec{V}_{G_3}\cdot\vec{u}_1^{(1)} = R\dot{\theta}_3 \tag{3.103}$$

and

$$\vec{V}_{G_2}\cdot\vec{u}_2^{(1)} = 0 \tag{3.104}$$

$$\vec{V}_{G_3}\cdot\vec{u}_2^{(1)} = 0 \tag{3.105}$$

In order to use the above equations one should resolve the velocities of the front wheels of the first module derived in Equations (3.34) and (3.35) in frame $F_1\left\{G_1;\vec{u}_1^{(1)},\vec{u}_2^{(1)},\vec{u}_3^{(1)}\right\}$. The Equations (3.34) and (3.35) can be rewritten in matrix notation as follows:

$$\bar{V}_{G_2}^{(1)} = \dot{x}_1\bar{u}_1^{(0/1)} + \dot{y}_1\bar{u}_2^{(0/1)} + d_1\dot{\beta}_1\bar{u}_2^{(1/1)} + d_2\dot{\beta}_1\bar{u}_1^{(1/1)} \tag{3.106}$$

$$\bar{V}_{G_3}^{(1)} = \dot{x}_1\bar{u}_1^{(0/1)} + \dot{y}_1\bar{u}_2^{(0/1)} + d_1\dot{\beta}_1\bar{u}_2^{(1/1)} - d_2\dot{\beta}_1\bar{u}_1^{(1/1)} \tag{3.107}$$

Equation (3.106) and (3.107) can be rewritten as:

$$\bar{V}_{G_2}^{(1)} = \dot{x}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_1 + \dot{y}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_2 + d_1\dot{\beta}_1\bar{u}_2 + d_2\dot{\beta}_1\bar{u}_1 \tag{3.108}$$

$$\bar{V}_{G_3}^{(1)} = \dot{x}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_1 + \dot{y}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_2 + d_1\dot{\beta}_1\bar{u}_2 - d_2\dot{\beta}_1\bar{u}_1 \tag{3.109}$$

Rearranging the terms, one can get the following equations:

$$\begin{aligned}\bar{V}_{G_2}^{(1)} &= \dot{x}_1\left(\bar{u}_1\cos(\beta_1) - \bar{u}_2\sin(\beta_1)\right) + \dot{y}_1\left(\bar{u}_2\cos(\beta_1) + \bar{u}_1\sin(\beta_1)\right)\\ &\quad + d_1\dot{\beta}_1\bar{u}_2 + d_2\dot{\beta}_1\bar{u}_1\end{aligned} \tag{3.110}$$

$$\begin{aligned}\bar{V}_{G_3}^{(1)} &= \dot{x}_1\left(\bar{u}_1\cos(\beta_1) - \bar{u}_2\sin(\beta_1)\right) + \dot{y}_1\left(\bar{u}_2\cos(\beta_1) + \bar{u}_1\sin(\beta_1)\right)\\ &\quad + d_1\dot{\beta}_1\bar{u}_2 - d_2\dot{\beta}_1\bar{u}_1\end{aligned} \tag{3.111}$$

After some manipulations, velocity of point G_2 and G_3 resolved in first module fixed frame, F_1, can be rewritten in vector notation as follows:

$$\begin{aligned}\vec{V}_{G_2} &= \left(\dot{x}_1\cos(\beta_1) + \dot{y}_1\sin(\beta_1) + d_2\dot{\beta}_1\right)\vec{u}_1^{(1)}\\ &\quad + \left(-\dot{x}_1\sin(\beta_1) + \dot{y}_1\cos(\beta_1) + d_1\dot{\beta}_1\right)\vec{u}_2^{(1)}\end{aligned} \tag{3.112}$$

$$\begin{aligned}\vec{V}_{G_3} &= \left(\dot{x}_1\cos(\beta_1) + \dot{y}_1\sin(\beta_1) - d_2\dot{\beta}_1\right)\vec{u}_1^{(1)}\\ &\quad + \left(-\dot{x}_1\sin(\beta_1) + \dot{y}_1\cos(\beta_1) + d_1\dot{\beta}_1\right)\vec{u}_2^{(1)}\end{aligned} \tag{3.113}$$

When Equation (3.112) and (3.113) are substituted into Equation (3.102) and (3.103), the following two equations are obtained for the independent variables $\dot{\theta}_2$ and $\dot{\theta}_3$.

$$\dot{\theta}_2 = \frac{\dot{x}_1 \cos(\beta_1) + \dot{y}_1 \sin(\beta_1) + d_2 \dot{\beta}_1}{R} \tag{3.114}$$

$$\dot{\theta}_3 = \frac{\dot{x}_1 \cos(\beta_1) + \dot{y}_1 \sin(\beta_1) - d_2 \dot{\beta}_1}{R} \tag{3.115}$$

Also when Equation (3.112) and (3.113) are substituted into Equation (3.104) and (3.105), the following equations are obtained:

$$\dot{\beta}_1 = \frac{-\dot{x}_1 \sin(\beta_1) + \dot{y}_1 \cos(\beta_1)}{-d_1} \tag{3.116}$$

Similarly the velocities of the points G_8 and G_9, which are the center of mass of the front wheels of the second module as follows:

$$\vec{V}_{G_8} = \left(\dot{\theta}_8 \vec{u}_2^{(7)}\right) \times \left(R\vec{u}_3^{(7)}\right) = R\dot{\theta}_8 \vec{u}_1^{(7)} \tag{3.117}$$

$$\vec{V}_{G_9} = \left(\dot{\theta}_9 \vec{u}_2^{(7)}\right) \times \left(R\vec{u}_3^{(7)}\right) = R\dot{\theta}_9 \vec{u}_1^{(7)} \tag{3.118}$$

Note that;

$$\vec{V}_{G_8} \cdot \vec{u}_1^{(7)} = R\dot{\theta}_8 \tag{3.119}$$

$$\vec{V}_{G_9} \cdot \vec{u}_1^{(7)} = R\dot{\theta}_9 \tag{3.120}$$

and

$$\vec{V}_{G_8} \cdot \vec{u}_2^{(7)} = 0 \tag{3.121}$$

$$\vec{V}_{G_9} \cdot \vec{u}_2^{(7)} = 0 \tag{3.122}$$

In order to use the above equations one should resolve the velocities of the front wheels of the second module derived in Equations (3.38) and (3.39) in frame $F_7\left\{G_7;\vec{u}_1^{(7)},\vec{u}_2^{(7)},\vec{u}_3^{(7)}\right\}$. The Equations (3.38) and (3.39) can be rewritten in matrix notation as follows:

$$\bar{V}_{G_8}^{(7)} = \dot{x}_2\bar{u}_1^{(0/7)} + \dot{y}_2\bar{u}_2^{(0/7)} + d_1\dot{\beta}_7\bar{u}_2^{(7/7)} + d_2\dot{\beta}_7\bar{u}_1^{(7/7)} \tag{3.123}$$

$$\bar{V}_{G_9}^{(7)} = \dot{x}_2\bar{u}_1^{(0/7)} + \dot{y}_2\bar{u}_2^{(0/7)} + d_1\dot{\beta}_7\bar{u}_2^{(7/7)} - d_2\dot{\beta}_7\bar{u}_1^{(7/7)} \tag{3.124}$$

Equation (3.123) and (3.124) can be rewritten as:

$$\bar{V}_{G_8}^{(7)} = \dot{x}_2 e^{-\tilde{u}_3\beta_7}\bar{u}_1 + \dot{y}_2 e^{-\tilde{u}_3\beta_7}\bar{u}_2 + d_1\dot{\beta}_7\bar{u}_2 + d_2\dot{\beta}_7\bar{u}_1 \tag{3.125}$$

$$\bar{V}_{G_9}^{(7)} = \dot{x}_2 e^{-\tilde{u}_3\beta_7}\bar{u}_1 + \dot{y}_2 e^{-\tilde{u}_3\beta_7}\bar{u}_2 + d_1\dot{\beta}_7\bar{u}_2 - d_2\dot{\beta}_7\bar{u}_1 \tag{3.126}$$

Rearranging the terms, one can get the following equations:

$$\begin{aligned}\bar{V}_{G_8}^{(7)} &= \dot{x}_2\left(\bar{u}_1\cos(\beta_7) - \bar{u}_2\sin(\beta_7)\right) + \dot{y}_2\left(\bar{u}_2\cos(\beta_7) + \bar{u}_1\sin(\beta_7)\right)\\ &\quad + d_1\dot{\beta}_7\bar{u}_2 + d_2\dot{\beta}_7\bar{u}_1\end{aligned} \tag{3.127}$$

$$\begin{aligned}\bar{V}_{G_9}^{(7)} &= \dot{x}_2\left(\bar{u}_1\cos(\beta_7) - \bar{u}_2\sin(\beta_7)\right) + \dot{y}_2\left(\bar{u}_2\cos(\beta_7) + \bar{u}_1\sin(\beta_7)\right)\\ &\quad + d_1\dot{\beta}_7\bar{u}_2 - d_2\dot{\beta}_7\bar{u}_1\end{aligned} \tag{3.128}$$

After some manipulations, velocity of point G_8 and G_9 resolved in second module fixed frame, F_7, can be rewritten in vector notation as follows:

$$\begin{aligned}\vec{V}_{G_8} &= \left(\dot{x}_2\cos(\beta_7) + \dot{y}_2\sin(\beta_7) + d_2\dot{\beta}_7\right)\vec{u}_1^{(7)}\\ &\quad + \left(-\dot{x}_2\sin(\beta_7) + \dot{y}_2\cos(\beta_7) + d_1\dot{\beta}_7\right)\vec{u}_2^{(7)}\end{aligned} \tag{3.129}$$

$$\begin{aligned}\vec{V}_{G_9} &= \left(\dot{x}_2\cos(\beta_7) + \dot{y}_2\sin(\beta_7) - d_2\dot{\beta}_7\right)\vec{u}_1^{(7)}\\ &\quad + \left(-\dot{x}_2\sin(\beta_7) + \dot{y}_2\cos(\beta_7) + d_1\dot{\beta}_7\right)\vec{u}_2^{(7)}\end{aligned} \tag{3.130}$$

When Equation (3.129) and (3.130) are substituted into Equation (3.119) and (3.120), the following two equations are obtained for the independent variables $\dot{\theta}_8$ and $\dot{\theta}_9$:

$$\dot{\theta}_8 = \frac{\dot{x}_2 \cos(\beta_7) + \dot{y}_2 \sin(\beta_7) + d_2 \dot{\beta}_7}{R} \tag{3.131}$$

$$\dot{\theta}_9 = \frac{\dot{x}_2 \cos(\beta_7) + \dot{y}_2 \sin(\beta_7) - d_2 \dot{\beta}_7}{R} \tag{3.132}$$

Also when Equation (3.129) and (3.130) are substituted into Equation (3.121) and (3.122), the following equations are obtained:

$$\dot{\beta}_7 = \frac{-\dot{x}_2 \sin(\beta_7) + \dot{y}_2 \cos(\beta_7)}{-d_1} \tag{3.133}$$

Similarly the velocities of the points G_{14} and G_{15}, which are the center of mass of the front wheels of the third module as follows:

$$\vec{V}_{G_{14}} = \left(\dot{\theta}_{14} \vec{u}_2^{(13)}\right) \times \left(R\vec{u}_3^{(13)}\right) = R\dot{\theta}_{14} \vec{u}_1^{(13)} \tag{3.134}$$

$$\vec{V}_{G_{15}} = \left(\dot{\theta}_{15} \vec{u}_2^{(13)}\right) \times \left(R\vec{u}_3^{(13)}\right) = R\dot{\theta}_{15} \vec{u}_1^{(13)} \tag{3.135}$$

Note that;

$$\vec{V}_{G_{14}} \cdot \vec{u}_1^{(13)} = R\dot{\theta}_{14} \tag{3.136}$$

$$\vec{V}_{G_{15}} \cdot \vec{u}_1^{(13)} = R\dot{\theta}_{15} \tag{3.137}$$

and

$$\vec{V}_{G_{14}} \cdot \vec{u}_2^{(13)} = 0 \tag{3.138}$$

$$\vec{V}_{G_{15}} \cdot \vec{u}_2^{(13)} = 0 \tag{3.139}$$

In order to use the above equations one should resolve the velocities of the front wheels of the third module derived in Equations (3.42) and (3.43) in frame $F_{13}\left\{G_{13};\vec{u}_1^{(13)},\vec{u}_2^{(13)},\vec{u}_3^{(13)}\right\}$. The Equations (3.42) and (3.43) can be rewritten in matrix notation as follows:

$$\bar{V}_{G_{14}}^{(13)} = \dot{x}_3\bar{u}_1^{(0/13)} + \dot{y}_3\bar{u}_2^{(0/13)} + d_1\dot{\beta}_{13}\bar{u}_2^{(13/13)} + d_2\dot{\beta}_{13}\bar{u}_1^{(13/13)} \tag{3.140}$$

$$\bar{V}_{G_{15}}^{(13)} = \dot{x}_3\bar{u}_1^{(0/13)} + \dot{y}_3\bar{u}_2^{(0/13)} + d_1\dot{\beta}_{13}\bar{u}_2^{(13/13)} - d_2\dot{\beta}_{13}\bar{u}_1^{(13/13)} \tag{3.141}$$

Equation (3.140) and (3.141) can be rewritten as:

$$\bar{V}_{G_{14}}^{(13)} = \dot{x}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_1 + \dot{y}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_2 + d_1\dot{\beta}_{13}\bar{u}_2 + d_2\dot{\beta}_{13}\bar{u}_1 \tag{3.142}$$

$$\bar{V}_{G_{15}}^{(13)} = \dot{x}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_1 + \dot{y}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_2 + d_1\dot{\beta}_{13}\bar{u}_2 - d_2\dot{\beta}_{13}\bar{u}_1 \tag{3.143}$$

Rearranging the terms, one can get the following equations:

$$\begin{aligned}\bar{V}_{G_{14}}^{(13)} &= \dot{x}_3\left(\bar{u}_1\cos(\beta_{13}) - \bar{u}_2\sin(\beta_{13})\right) \\ &\quad + \dot{y}_3\left(\bar{u}_2\cos(\beta_{13}) + \bar{u}_1\sin(\beta_{13})\right) + d_1\dot{\beta}_{13}\bar{u}_2 + d_2\dot{\beta}_{13}\bar{u}_1\end{aligned} \tag{3.144}$$

$$\begin{aligned}\bar{V}_{G_{15}}^{(13)} &= \dot{x}_3\left(\bar{u}_1\cos(\beta_{13}) - \bar{u}_2\sin(\beta_{13})\right) \\ &\quad + \dot{y}_3\left(\bar{u}_2\cos(\beta_{13}) + \bar{u}_1\sin(\beta_{13})\right) + d_1\dot{\beta}_{13}\bar{u}_2 - d_2\dot{\beta}_{13}\bar{u}_1\end{aligned} \tag{3.145}$$

After some manipulations, velocity of point G_{14} and G_{15} resolved in third module fixed frame, F_{13}, can be rewritten in vector notation as follows:

$$\begin{aligned}\vec{V}_{G_{14}} &= \left(\dot{x}_3\cos(\beta_{13}) + \dot{y}_3\sin(\beta_{13}) + d_2\dot{\beta}_{13}\right)\vec{u}_1^{(13)} \\ &\quad + \left(-\dot{x}_3\sin(\beta_{13}) + \dot{y}_3\cos(\beta_{13}) + d_1\dot{\beta}_{13}\right)\vec{u}_2^{(13)}\end{aligned} \tag{3.146}$$

$$\begin{aligned}\vec{V}_{G_{14}} &= \left(\dot{x}_3\cos(\beta_{13}) + \dot{y}_3\sin(\beta_{13}) - d_2\dot{\beta}_{13}\right)\vec{u}_1^{(13)} \\ &\quad + \left(-\dot{x}_3\sin(\beta_{13}) + \dot{y}_3\cos(\beta_{13}) + d_1\dot{\beta}_{13}\right)\vec{u}_2^{(13)}\end{aligned} \tag{3.147}$$

When Equation (3.146) and (3.147) are substituted into Equation (3.136) and (3.137), the following two equations are obtained for the independent variables $\dot{\theta}_{14}$ and $\dot{\theta}_{15}$:

$$\dot{\theta}_{14} = \frac{\dot{x}_3\cos(\beta_{13}) + \dot{y}_3\sin(\beta_{13}) + d_2\dot{\beta}_{13}}{R} \tag{3.148}$$

$$\dot{\theta}_{15} = \frac{\dot{x}_3\cos(\beta_{13}) + \dot{y}_3\sin(\beta_{13}) - d_2\dot{\beta}_{13}}{R} \tag{3.149}$$

Also when Equation (3.146) and (3.147) are substituted into Equation (3.138) and (3.139), the following equations are obtained:

$$\dot{\beta}_{13} = \frac{-\dot{x}_3\sin(\beta_{13}) + \dot{y}_3\cos(\beta_{13})}{-d_1} \tag{3.150}$$

Equation (3.151) can be obtained by multiplying Equations (3.114) and (3.115) by R and then adding these equations:

$$R\left(\dot{\theta}_2 + \dot{\theta}_3\right) = 2\left(\dot{x}_1\cos(\beta_1) + \dot{y}_1\sin(\beta_1)\right) \tag{3.151}$$

Equation (3.152) can be obtained by multiplying Equations (3.114) and (3.115) by R and then subtracting these equations:

$$R\left(\dot{\theta}_2 - \dot{\theta}_3\right) = 2d_2\dot{\beta}_1 \tag{3.152}$$

The following equation can be obtained by inserting Equation (3.116) into Equation (3.152):

$$R\left(\dot{\theta}_2 - \dot{\theta}_3\right) = \frac{2d_2\left(-\dot{x}_1\sin(\beta_1) + \dot{y}_1\cos(\beta_1)\right)}{-d_1} \tag{3.153}$$

Equations (3.151) and (3.153) can be written in the matrix representation form as follows:

$$\begin{bmatrix} 2\cos(\beta_1) & 2\sin(\beta_1) \\ \dfrac{2d_2\sin(\beta_1)}{d_1} & \dfrac{-2d_2\cos(\beta_1)}{d_1} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} R\left(\dot{\theta}_2 + \dot{\theta}_3\right) \\ R\left(\dot{\theta}_2 - \dot{\theta}_3\right) \end{bmatrix} \tag{3.154}$$

After some manipulations, the expressions for $\dot{x}_1$ and $\dot{y}_1$ are obtained in terms of $\dot{\theta}_2$ and $\dot{\theta}_3$ as shown in the following equations:

$$\dot{x}_1 = \frac{R}{2}\cos(\beta_1)\left(\dot{\theta}_2 + \dot{\theta}_3\right) + \frac{Rd_1}{2d_2}\sin(\beta_1)\left(\dot{\theta}_2 - \dot{\theta}_3\right) \tag{3.155}$$

$$\dot{y}_1 = \frac{R}{2}\sin(\beta_1)\left(\dot{\theta}_2 + \dot{\theta}_3\right) - \frac{Rd_1}{2d_2}\cos(\beta_1)\left(\dot{\theta}_2 - \dot{\theta}_3\right) \tag{3.156}$$

The expression for $\dot{\beta}_1$ is obtained in terms of $\dot{\theta}_2$ and $\dot{\theta}_3$ from Equation (3.152) as follows:

$$\dot{\beta}_1 = \frac{R}{2d_2}\left(\dot{\theta}_2 - \dot{\theta}_3\right) \tag{3.157}$$

Equation (3.158), (3.159) and (3.160) are obtained by direct differentiation of the Equation (3.155), (3.156) and (3.157).

$$\begin{aligned}\ddot{x}_1 = {} & \frac{R}{2}\cos(\beta_1)\left(\ddot{\theta}_2 + \ddot{\theta}_3\right) - \frac{R}{2}\dot{\beta}_1 \sin(\beta_1)\left(\dot{\theta}_2 + \dot{\theta}_3\right) \\ & + \frac{Rd_1}{2d_2}\sin(\beta_1)\left(\ddot{\theta}_2 - \ddot{\theta}_3\right) + \frac{Rd_1}{2d_2}\dot{\beta}_1 \cos(\beta_1)\left(\dot{\theta}_2 - \dot{\theta}_3\right)\end{aligned} \tag{3.158}$$

$$\begin{aligned}\ddot{y}_1 = {} & \frac{R}{2}\sin(\beta_1)\left(\ddot{\theta}_2 + \ddot{\theta}_3\right) + \frac{R}{2}\dot{\beta}_1 \cos(\beta_1)\left(\dot{\theta}_2 + \dot{\theta}_3\right) \\ & - \frac{Rd_1}{2d_2}\cos(\beta_1)\left(\ddot{\theta}_2 - \ddot{\theta}_3\right) + \frac{Rd_1}{2d_2}\dot{\beta}_1 \sin(\beta_1)\left(\dot{\theta}_2 - \dot{\theta}_3\right)\end{aligned} \tag{3.159}$$

$$\ddot{\beta}_1 = \frac{R}{2d_2}\left(\ddot{\theta}_2 - \ddot{\theta}_3\right) \tag{3.160}$$

Equation (3.161) can be obtained by multiplying Equations (3.131) and (3.132) by the term ‘R’ and then adding these equations.

$$R\left(\dot{\theta}_8 + \dot{\theta}_9\right) = 2\left(\dot{x}_2 \cos(\beta_7) + \dot{y}_2 \sin(\beta_7)\right) \tag{3.161}$$

Equation (3.162) can be obtained by multiplying Equations (3.131) and (3.132) by the term ‘R’ and then subtracting these equations.

$$R\left(\dot{\theta}_8 - \dot{\theta}_9\right) = 2d_2\dot{\beta}_7 \tag{3.162}$$

The following equation can be obtained by inserting Equation (3.133) into Equation (3.162).

$$R\left(\dot{\theta}_8 - \dot{\theta}_9\right) = \frac{2d_2\left(-\dot{x}_2 \sin(\beta_7) + \dot{y}_2 \cos(\beta_7)\right)}{-d_1} \tag{3.163}$$

Equations (3.161) and (3.163) can be written in the matrix representation form as follows:

$$\begin{bmatrix} 2\cos(\beta_7) & 2\sin(\beta_7) \\ \dfrac{2d_2 \sin(\beta_7)}{d_1} & \dfrac{-2d_2 \cos(\beta_7)}{d_1} \end{bmatrix} \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} R\left(\dot{\theta}_8 + \dot{\theta}_9\right) \\ R\left(\dot{\theta}_8 - \dot{\theta}_9\right) \end{bmatrix} \tag{3.164}$$

After some manipulations, the expressions for $\dot{x}_2$ and $\dot{y}_2$ are obtained in terms of $\dot{\theta}_8$ and $\dot{\theta}_9$ as shown in the following equations:

$$\dot{x}_2 = \frac{R}{2}\cos(\beta_7)\left(\dot{\theta}_8 + \dot{\theta}_9\right) + \frac{Rd_1}{2d_2}\sin(\beta_7)\left(\dot{\theta}_8 - \dot{\theta}_9\right) \tag{3.165}$$

$$\dot{y}_2 = \frac{R}{2}\sin(\beta_7)\left(\dot{\theta}_8 + \dot{\theta}_9\right) - \frac{Rd_1}{2d_2}\cos(\beta_7)\left(\dot{\theta}_8 - \dot{\theta}_9\right) \tag{3.166}$$

The expression for $\dot{\beta}_7$ is obtained in terms of $\dot{\theta}_8$ and $\dot{\theta}_9$ from Equation (3.162) as follows:

$$\dot{\beta}_7 = \frac{R}{2d_2}\left(\dot{\theta}_8 - \dot{\theta}_9\right) \tag{3.167}$$

Equation (3.168), (3.169) and (3.170) are obtained by direct differentiation of the Equation (3.165), (3.166) and (3.167).

$$\begin{aligned}\ddot{x}_2 &= \frac{R}{2}\cos(\beta_7)\left(\ddot{\theta}_8+\ddot{\theta}_9\right)-\frac{R}{2}\dot{\beta}_7\sin(\beta_7)\left(\dot{\theta}_8+\dot{\theta}_9\right)\\ &+\frac{Rd_1}{2d_2}\sin(\beta_7)\left(\ddot{\theta}_8-\ddot{\theta}_9\right)+\frac{Rd_1}{2d_2}\dot{\beta}_7\cos(\beta_7)\left(\dot{\theta}_8-\dot{\theta}_9\right)\end{aligned} \tag{3.168}$$

$$\begin{aligned}\ddot{y}_2 &= \frac{R}{2}\sin(\beta_7)\left(\ddot{\theta}_8+\ddot{\theta}_9\right)+\frac{R}{2}\dot{\beta}_7\cos(\beta_7)\left(\dot{\theta}_8+\dot{\theta}_9\right)\\ &-\frac{Rd_1}{2d_2}\cos(\beta_7)\left(\ddot{\theta}_8-\ddot{\theta}_9\right)+\frac{Rd_1}{2d_2}\dot{\beta}_7\sin(\beta_7)\left(\dot{\theta}_8-\dot{\theta}_9\right)\end{aligned} \tag{3.169}$$

$$\ddot{\beta}_7 = \frac{R}{2d_2}\left(\ddot{\theta}_8-\ddot{\theta}_9\right) \tag{3.170}$$

Equation (3.171) can be obtained by multiplying Equations (3.148) and (3.149) by R and then adding these equations.

$$R\left(\dot{\theta}_{14}+\dot{\theta}_{15}\right) = 2\left(\dot{x}_3\cos(\beta_{13})+\dot{y}_3\sin(\beta_{13})\right) \tag{3.171}$$

Equation (3.172) can be obtained by multiplying Equations (3.148) and (3.149) by R and then subtracting these equations.

$$R\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) = 2d_2\dot{\beta}_{13} \tag{3.172}$$

The following equation can be obtained by inserting Equation (3.150) into Equation (3.172):

$$R\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) = \frac{2d_2\left(-\dot{x}_3\sin(\beta_{13})+\dot{y}_3\cos(\beta_{13})\right)}{-d_1} \tag{3.173}$$

Equations (3.171) and (3.172) can be written in the matrix representation form as follows:

$$\begin{bmatrix} 2\cos(\beta_{13}) & 2\sin(\beta_{13}) \\ \dfrac{2d_2\sin(\beta_{13})}{d_1} & \dfrac{-2d_2\cos(\beta_{13})}{d_1} \end{bmatrix} \begin{bmatrix} \dot{x}_3 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} R\left(\dot{\theta}_{14}+\dot{\theta}_{15}\right) \\ R\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) \end{bmatrix} \tag{3.174}$$

After some manipulations, the expressions for $\dot{x}_3$ and $\dot{y}_3$ are obtained in terms of $\dot{\theta}_{14}$ and $\dot{\theta}_{15}$ as shown in the following equations:

$$\dot{x}_3 = \frac{R}{2}\cos(\beta_{13})\left(\dot{\theta}_{14}+\dot{\theta}_{15}\right) + \frac{Rd_1}{2d_2}\sin(\beta_{13})\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) \tag{3.175}$$

$$\dot{y}_3 = \frac{R}{2}\sin(\beta_{13})\left(\dot{\theta}_{14}+\dot{\theta}_{15}\right) - \frac{Rd_1}{2d_2}\cos(\beta_{13})\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) \tag{3.176}$$

The expression for $\dot{\beta}_{13}$ is obtained in terms of $\dot{\theta}_{14}$ and $\dot{\theta}_{15}$ from Equation (3.172) as follows:

$$\dot{\beta}_{13} = \frac{R}{2d_2}\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) \tag{3.177}$$

Equation (3.178), (3.179) and (3.180) are obtained by direct differentiation of the Equation (3.175), (3.176) and (3.177).

$$\begin{aligned} \ddot{x}_3 = {} & \frac{R}{2}\cos(\beta_{13})\left(\ddot{\theta}_{14}+\ddot{\theta}_{15}\right) - \frac{R}{2}\dot{\beta}_{13}\sin(\beta_{13})\left(\dot{\theta}_{14}+\dot{\theta}_{15}\right) \\ & + \frac{Rd_1}{2d_2}\sin(\beta_{13})\left(\ddot{\theta}_{14}-\ddot{\theta}_{15}\right) + \frac{Rd_1}{2d_2}\dot{\beta}_{13}\cos(\beta_{13})\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) \end{aligned} \tag{3.178}$$

$$\begin{aligned} \ddot{y}_3 = {} & \frac{R}{2}\sin(\beta_{13})\left(\ddot{\theta}_{14}+\ddot{\theta}_{15}\right) + \frac{R}{2}\dot{\beta}_{13}\cos(\beta_{13})\left(\dot{\theta}_{14}+\dot{\theta}_{15}\right) \\ & - \frac{Rd_1}{2d_2}\cos(\beta_{13})\left(\ddot{\theta}_{14}-\ddot{\theta}_{15}\right) + \frac{Rd_1}{2d_2}\dot{\beta}_{13}\sin(\beta_{13})\left(\dot{\theta}_{14}-\dot{\theta}_{15}\right) \end{aligned} \tag{3.179}$$

$$\ddot{\beta}_{13} = \frac{R}{2d_2}\left(\ddot{\theta}_{14} - \ddot{\theta}_{15}\right) \tag{3.180}$$

When one resolve the velocities of the center of mass of the body of the first module, point G_1, and center of mass of rear wheel on the first module, point G_5, derived previously derived in Equations (3.25) and (3.36) in frame $F_1\left\{G_1;\vec{u}_1^{(1)},\vec{u}_2^{(1)},\vec{u}_3^{(1)}\right\}$, the following equations are obtained in matrix notation:

$$\bar{V}_{G_1}^{(1)} = \dot{x}_1\bar{u}_1^{(0/1)} + \dot{y}_1\bar{u}_2^{(0/1)} \tag{3.181}$$

$$\begin{aligned}\bar{V}_{G_5}^{(1)} &= \dot{x}_1\bar{u}_1^{(0/1)} + \dot{y}_1\bar{u}_2^{(0/1)} + d_8\dot{\gamma}_5\sin(\gamma_5)\bar{u}_1^{(1/1)} - \left(d_8\cos(\gamma_5) + d_3\right)\dot{\beta}_1\bar{u}_2^{(1/1)} \\ &\quad -d_8\dot{\gamma}_5\cos(\gamma_5)\bar{u}_2^{(1/1)} + d_8\sin(\gamma_5)\dot{\beta}_1\bar{u}_1^{(1/1)}\end{aligned} \tag{3.182}$$

Equation (3.181) and (3.182) can be rewritten as:

$$\bar{V}_{G_1}^{(1)} = \dot{x}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_1 + \dot{y}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_2 \tag{3.183}$$

$$\begin{aligned}\bar{V}_{G_5}^{(1)} &= \dot{x}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_1 + \dot{y}_1 e^{-\tilde{u}_3\beta_1}\bar{u}_2 + d_8\dot{\gamma}_5\sin(\gamma_5)\bar{u}_1 - \left(d_8\cos(\gamma_5) + d_3\right)\dot{\beta}_1\bar{u}_2 \\ &\quad -d_8\dot{\gamma}_5\cos(\gamma_5)\bar{u}_2 + d_8\sin(\gamma_5)\dot{\beta}_1\bar{u}_1\end{aligned} \tag{3.184}$$

Rearranging the terms, one can get the following equations:

$$\bar{V}_{G_1}^{(1)} = \dot{x}_1\left(\bar{u}_1\cos(\beta_1) - \bar{u}_2\sin(\beta_1)\right) + \dot{y}_1\left(\bar{u}_2\cos(\beta_1) + \bar{u}_1\sin(\beta_1)\right) \tag{3.185}$$

$$\begin{aligned}\bar{V}_{G_5}^{(1)} &= \dot{x}_1\left(\bar{u}_1\cos(\beta_1) - \bar{u}_2\sin(\beta_1)\right) + \dot{y}_1\left(\bar{u}_2\cos(\beta_1) + \bar{u}_1\sin(\beta_1)\right) \\ &\quad +d_8\dot{\gamma}_5\sin(\gamma_5)\bar{u}_1 - \left(d_8\cos(\gamma_5) + d_3\right)\dot{\beta}_1\bar{u}_2 - d_8\dot{\gamma}_5\cos(\gamma_5)\bar{u}_2 \\ &\quad +d_8\sin(\gamma_5)\dot{\beta}_1\bar{u}_1\end{aligned} \tag{3.186}$$

Equation (3.185) and (3.186) can be written in vector notation as follows:

$$\vec{V}_{G_1} = \left(\dot{x}_1 \cos(\beta_1) + \dot{y}_1 \sin(\beta_1)\right)\vec{u}_1^{(1)} + \left(-\dot{x}_1 \sin(\beta_1) + \dot{y}_1 \cos(\beta_1)\right)\vec{u}_2^{(1)} \quad (3.187)$$

$$\vec{V}_{G_5} = \left(\dot{x}_1 \cos(\beta_1) + \dot{y}_1 \sin(\beta_1) + d_8\dot{\gamma}_5 \sin(\gamma_5) + d_8\dot{\beta}_1 \sin(\gamma_5)\right)\vec{u}_1^{(1)}$$
$$+\left(-\dot{x}_1 \sin(\beta_1) + \dot{y}_1 \cos(\beta_1) - \left(d_8 \cos(\gamma_5) + d_3\right)\dot{\beta}_1 - d_8\dot{\gamma}_5 \cos(\gamma_5)\right)\vec{u}_2^{(1)}$$

(3.188)

When one resolve the velocities of center of mass of the body of the second module, point G_7, and center of mass of rear wheel on the second module, point G_{11}, derived previously derived in Equations (3.37) and (3.40) in frame $F_7\left\{G_7;\vec{u}_1^{(7)},\vec{u}_2^{(7)},\vec{u}_3^{(7)}\right\}$, the following equations are obtained in matrix notation:

$$\overline{V}_{G_7}^{(7)} = \dot{x}_2\overline{u}_1^{(0/7)} + \dot{y}_2\overline{u}_2^{(0/7)} \quad (3.189)$$

$$\overline{V}_{G_{11}}^{(7)} = \dot{x}_2\overline{u}_1^{(0/7)} + \dot{y}_2\overline{u}_2^{(0/7)} + d_8\dot{\gamma}_{11} \sin(\gamma_{11})\overline{u}_1^{(7/7)} - \left(d_8 \cos(\gamma_{11}) + d_3\right)\dot{\beta}_7\overline{u}_2^{(7/7)}$$
$$-d_8\dot{\gamma}_{11} \cos(\gamma_{11})\overline{u}_2^{(7/7)} + d_8 \sin(\gamma_{11})\dot{\beta}_7\overline{u}_1^{(7/7)}$$

(3.190)

Equation (3.189) and (3.190) can be rewritten as:

$$\overline{V}_{G_7}^{(7)} = \dot{x}_2 e^{-\tilde{u}_3\beta_7}\overline{u}_1 + \dot{y}_2 e^{-\tilde{u}_3\beta_7}\overline{u}_2 \quad (3.191)$$

$$\overline{V}_{G_{11}}^{(7)} = \dot{x}_2 e^{-\tilde{u}_3\beta_7}\overline{u}_1 + \dot{y}_2 e^{-\tilde{u}_3\beta_7}\overline{u}_2 + d_8\dot{\gamma}_{11} \sin(\gamma_{11})\overline{u}_1 - \left(d_8 \cos(\gamma_{11}) + d_3\right)\dot{\beta}_7\overline{u}_2$$
$$-d_8\dot{\gamma}_{11} \cos(\gamma_{11})\overline{u}_2 + d_8 \sin(\gamma_{11})\dot{\beta}_7\overline{u}_1$$

(3.192)

Rearranging the terms, one can get the following equations:

$$\overline{V}_{G_7}^{(7)} = \dot{x}_2\left(\overline{u}_1 \cos(\beta_7) - \overline{u}_2 \sin(\beta_7)\right) + \dot{y}_2\left(\overline{u}_2 \cos(\beta_7) + \overline{u}_1 \sin(\beta_7)\right) \quad (3.193)$$

$$\bar{V}_{G_{11}}{}^{(7)} = \dot{x}_2\left(\bar{u}_1\cos(\beta_7) - \bar{u}_2\sin(\beta_7)\right) + \dot{y}_2\left(\bar{u}_2\cos(\beta_7) + \bar{u}_1\sin(\beta_7)\right)$$
$$+d_8\dot{\gamma}_{11}\sin(\gamma_{11})\bar{u}_1 - \left(d_8\cos(\gamma_{11}) + d_3\right)\dot{\beta}_7\bar{u}_2 - d_8\dot{\gamma}_{11}\cos(\gamma_{11})\bar{u}_2 \quad (3.194)$$
$$+d_8\sin(\gamma_{11})\dot{\beta}_7\bar{u}_1$$

Equation (3.193) and (3.194) can be written in vector notation as follows:

$$\vec{V}_{G_7} = \left(\dot{x}_2\cos(\beta_7) + \dot{y}_2\sin(\beta_7)\right)\vec{u}_1{}^{(7)} + \left(-\dot{x}_2\sin(\beta_7) + \dot{y}_2\cos(\beta_7)\right)\vec{u}_2{}^{(7)} \quad (3.195)$$

$$\vec{V}_{G_{11}} = \left(\dot{x}_2\cos(\beta_7) + \dot{y}_2\sin(\beta_7) + d_8\dot{\gamma}_{11}\sin(\gamma_{11}) + d_8\dot{\beta}_7\sin(\gamma_{11})\right)\vec{u}_1{}^{(7)}$$
$$+\left(-\dot{x}_2\sin(\beta_7) + \dot{y}_2\cos(\beta_7) - \left(d_8\cos(\gamma_{11}) + d_3\right)\dot{\beta}_7 - d_8\dot{\gamma}_{11}\cos(\gamma_{11})\right)\vec{u}_2{}^{(7)}$$
$$(3.196)$$

When one resolve the velocities of center of mass of the body of the third module, point G_{13}, and center of mass of rear wheel on the third module, point G_{17}, derived previously derived in Equations (3.41) and (3.44) in frame $F_{13}\left\{G_{13};\vec{u}_1{}^{(13)},\vec{u}_2{}^{(13)},\vec{u}_3{}^{(13)}\right\}$, the following equations are obtained in matrix notation:

$$\bar{V}_{G_{13}}{}^{(13)} = \dot{x}_3\bar{u}_1{}^{(0/13)} + \dot{y}_3\bar{u}_2{}^{(0/13)} \quad (3.197)$$

$$\bar{V}_{G_{17}}{}^{(13)} = \dot{x}_3\bar{u}_1{}^{(0/13)} + \dot{y}_3\bar{u}_2{}^{(0/13)} + d_8\dot{\gamma}_{17}\sin(\gamma_{17})\bar{u}_1{}^{(13/13)}$$
$$-\left(d_8\cos(\gamma_{17}) + d_3\right)\dot{\beta}_{13}\bar{u}_2{}^{(13/13)} - d_8\dot{\gamma}_{17}\cos(\gamma_{17})\bar{u}_2{}^{(13/13)} \quad (3.198)$$
$$+d_8\sin(\gamma_{17})\dot{\beta}_{13}\bar{u}_1{}^{(13/13)}$$

Equation (3.197) and (3.198) can be rewritten as:

$$\bar{V}_{G_{13}}{}^{(13)} = \dot{x}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_1 + \dot{y}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_2 \quad (3.199)$$

$$\bar{V}_{G_{17}}{}^{(13)} = \dot{x}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_1 + \dot{y}_3 e^{-\tilde{u}_3\beta_{13}}\bar{u}_2 + d_8\dot{\gamma}_{17}\sin(\gamma_{17})\bar{u}_1 - \left(d_8\cos(\gamma_{17}) + d_3\right)\dot{\beta}_{13}\bar{u}_2$$
$$-d_8\dot{\gamma}_{17}\cos(\gamma_{17})\bar{u}_2 + d_8\sin(\gamma_{17})\dot{\beta}_{13}\bar{u}_1$$
$$(3.200)$$

Rearranging the terms, one can get the following equations:

$$\bar{V}_{G_{13}}{}^{(13)} = \dot{x}_3\left(\bar{u}_1\cos(\beta_{13}) - \bar{u}_2\sin(\beta_{13})\right) + \dot{y}_3\left(\bar{u}_2\cos(\beta_{13}) + \bar{u}_1\sin(\beta_{13})\right) \quad (3.201)$$

$$\begin{aligned}\bar{V}_{G_{17}}{}^{(13)} &= \dot{x}_3\left(\bar{u}_1\cos(\beta_{13}) - \bar{u}_2\sin(\beta_{13})\right) + \dot{y}_3\left(\bar{u}_2\cos(\beta_{13}) + \bar{u}_1\sin(\beta_{13})\right) \\ &+ d_8\dot{\gamma}_{17}\sin(\gamma_{17})\bar{u}_1 - \left(d_8\cos(\gamma_{17}) + d_3\right)\dot{\beta}_{13}\bar{u}_2 - d_8\dot{\gamma}_{17}\cos(\gamma_{17})\bar{u}_2 \quad (3.202) \\ &+ d_8\sin(\gamma_{17})\dot{\beta}_{13}\bar{u}_1\end{aligned}$$

Equation (3.201) and (3.202) can be written in vector notation as follows:

$$\begin{aligned}\vec{V}_{G_{13}} &= \left(\dot{x}_3\cos(\beta_{13}) + \dot{y}_3\sin(\beta_{13})\right)\vec{u}_1{}^{(13)} \\ &+ \left(-\dot{x}_3\sin(\beta_{13}) + \dot{y}_3\cos(\beta_{13})\right)\vec{u}_2{}^{(13)}\end{aligned} \quad (3.203)$$

$$\begin{aligned}\vec{V}_{G_{17}} &= \left(\dot{x}_3\cos(\beta_{13}) + \dot{y}_3\sin(\beta_{13}) + d_8\dot{\gamma}_{17}\sin(\gamma_{17}) + d_8\dot{\beta}_{13}\sin(\gamma_{17})\right)\vec{u}_1{}^{(13)} \\ &+ \left(-\dot{x}_3\sin(\beta_{13}) + \dot{y}_3\cos(\beta_{13}) - \left(d_8\cos(\gamma_{17}) + d_3\right)\dot{\beta}_{13} - d_8\dot{\gamma}_{17}\cos(\gamma_{17})\right)\vec{u}_2{}^{(13)}\end{aligned} \quad (3.204)$$

Accelerations of points G_1, G_2, G_3 and G_5, resolved in frame $F_1\left\{G_1;\vec{u}_1{}^{(1)},\vec{u}_2{}^{(1)},\vec{u}_3{}^{(1)}\right\}$, can be obtained by differentiation of Equations (3.187), (3.112), (3.113) and (3.188) as follows:

$$\vec{a}_{G_1} = D_1\vec{V}_{G_1}{}^{(1)} + \vec{\omega}_{1/0}\times\vec{V}_{G_1}{}^{(1)} \quad (3.205)$$

$$\vec{a}_{G_2} = D_1\vec{V}_{G_2}{}^{(1)} + \vec{\omega}_{1/0}\times\vec{V}_{G_2}{}^{(1)} \quad (3.206)$$

$$\vec{a}_{G_3} = D_1\vec{V}_{G_3}{}^{(1)} + \vec{\omega}_{1/0}\times\vec{V}_{G_3}{}^{(1)} \quad (3.207)$$

$$\vec{a}_{G_5} = D_1\vec{V}_{G_5}{}^{(1)} + \vec{\omega}_{1/0}\times\vec{V}_{G_5}{}^{(1)} \quad (3.208)$$

After some manipulations the above equations become:

$$\vec{a}_{G_1} = \left(\ddot{x}_1 \cos(\beta_1) + \ddot{y}_1 \sin(\beta_1)\right)\vec{u}_1^{(1)} + \left(-\ddot{x}_1 \sin(\beta_1) + \ddot{y}_1 \cos(\beta_1)\right)\vec{u}_2^{(1)} \quad (3.209)$$

$$\begin{aligned}\vec{a}_{G_2} &= \left(\ddot{x}_1 \cos(\beta_1) + \ddot{y}_1 \sin(\beta_1) + d_2\ddot{\beta}_1 - d_1\dot{\beta}_1^2\right)\vec{u}_1^{(1)} \\ &+ \left(-\ddot{x}_1 \sin(\beta_1) + \ddot{y}_1 \cos(\beta_1) + d_1\ddot{\beta}_1 + d_2\dot{\beta}_1^2\right)\vec{u}_2^{(1)}\end{aligned} \quad (3.210)$$

$$\begin{aligned}\vec{a}_{G_3} &= \left(\ddot{x}_1 \cos(\beta_1) + \ddot{y}_1 \sin(\beta_1) - d_2\ddot{\beta}_1 - d_1\dot{\beta}_1^2\right)\vec{u}_1^{(1)} \\ &+ \left(-\ddot{x}_1 \sin(\beta_1) + \ddot{y}_1 \cos(\beta_1) + d_1\ddot{\beta}_1 - d_2\dot{\beta}_1^2\right)\vec{u}_2^{(1)}\end{aligned} \quad (3.211)$$

$$\begin{aligned}\vec{a}_{G_5} &= \begin{pmatrix} \ddot{x}_1 \cos(\beta_1) + \ddot{y}_1 \sin(\beta_1) + d_8\ddot{\gamma}_5 \sin(\gamma_5) + d_8\dot{\gamma}_5^2 \cos(\gamma_5) \\ +2d_8\dot{\gamma}_5\dot{\beta}_1 \cos(\gamma_5) + d_8\ddot{\beta}_1 \sin(\gamma_5) + \left(d_8 \cos(\gamma_5) + d_3\right)\dot{\beta}_1^2 \end{pmatrix}\vec{u}_1^{(1)} \\ &+ \begin{pmatrix} -\ddot{x}_1 \sin(\beta_1) + \ddot{y}_1 \cos(\beta_1) + 2d_8\dot{\gamma}_5\dot{\beta}_1 \sin(\gamma_5) + d_8 \sin(\gamma_5)\dot{\beta}_1^2 \\ -\left(d_8 \cos(\gamma_5) + d_3\right)\ddot{\beta}_1 - d_8\ddot{\gamma}_5 \cos(\gamma_5) + d_8\dot{\gamma}_5^2 \sin(\gamma_5) \end{pmatrix}\vec{u}_2^{(1)}\end{aligned} \quad (3.212)$$

Accelerations of points G_7, G_8, G_9 and G_{11}, resolved in frame $F_7\left\{G_7;\vec{u}_1^{(7)},\vec{u}_2^{(7)},\vec{u}_3^{(7)}\right\}$, can be obtained by differentiation of Equations (3.195), (3.129), (3.130) and (3.196) as follows:

$$\vec{a}_{G_7} = D_7\vec{V}_{G_7}^{(7)} + \vec{\omega}_{7/0} \times \vec{V}_{G_7}^{(7)} \quad (3.213)$$

$$\vec{a}_{G_8} = D_7\vec{V}_{G_8}^{(7)} + \vec{\omega}_{7/0} \times \vec{V}_{G_8}^{(7)} \quad (3.214)$$

$$\vec{a}_{G_9} = D_7\vec{V}_{G_9}^{(7)} + \vec{\omega}_{7/0} \times \vec{V}_{G_9}^{(7)} \quad (3.215)$$

$$\vec{a}_{G_{11}} = D_7\vec{V}_{G_{11}}^{(7)} + \vec{\omega}_{7/0} \times \vec{V}_{G_{11}}^{(7)} \quad (3.216)$$

After some manipulations the above equations become:

$$\begin{aligned}\vec{a}_{G_7} &= \left(\ddot{x}_2 \cos(\beta_7) + \ddot{y}_2 \sin(\beta_7)\right)\vec{u}_1^{(7)} \\ &+ \left(-\ddot{x}_2 \sin(\beta_7) + \ddot{y}_2 \cos(\beta_7)\right)\vec{u}_2^{(7)}\end{aligned} \quad (3.217)$$

$$\vec{a}_{G_8} = \left(\ddot{x}_2 \cos(\beta_7) + \ddot{y}_2 \sin(\beta_7) + d_2\ddot{\beta}_7 - d_1\dot{\beta}_7{}^2\right)\vec{u}_1{}^{(7)} + \left(-\ddot{x}_2 \sin(\beta_7) + \ddot{y}_2 \cos(\beta_7) + d_1\ddot{\beta}_7 + d_2\dot{\beta}_7{}^2\right)\vec{u}_2{}^{(7)} \tag{3.218}$$

$$\vec{a}_{G_9} = \left(\ddot{x}_2 \cos(\beta_7) + \ddot{y}_2 \sin(\beta_7) - d_2\ddot{\beta}_7 - d_1\dot{\beta}_7{}^2\right)\vec{u}_1{}^{(7)} + \left(-\ddot{x}_2 \sin(\beta_7) + \ddot{y}_2 \cos(\beta_7) + d_1\ddot{\beta}_7 - d_2\dot{\beta}_7{}^2\right)\vec{u}_2{}^{(7)} \tag{3.219}$$

$$\vec{a}_{G_{11}} = \begin{pmatrix} \ddot{x}_2 \cos(\beta_7) + \ddot{y}_2 \sin(\beta_7) + d_8\ddot{\gamma}_{11} \sin(\gamma_{11}) + d_8\dot{\gamma}_{11}^2 \cos(\gamma_{11}) \\ +2d_8\dot{\gamma}_{11}\dot{\beta}_7 \cos(\gamma_{11}) + d_8\ddot{\beta}_7 \sin(\gamma_{11}) + \left(d_8 \cos(\gamma_{11}) + d_3\right)\dot{\beta}_7{}^2 \end{pmatrix}\vec{u}_1{}^{(7)} + \begin{pmatrix} -\ddot{x}_2 \sin(\beta_7) + \ddot{y}_2 \cos(\beta_7) + 2d_8\dot{\gamma}_{11}\dot{\beta}_7 \sin(\gamma_{11}) + d_8 \sin(\gamma_{11})\dot{\beta}_7{}^2 \\ -\left(d_8 \cos(\gamma_{11}) + d_3\right)\ddot{\beta}_7 - d_8\ddot{\gamma}_{11} \cos(\gamma_{11}) + d_8\dot{\gamma}_{11}^2 \sin(\gamma_{11}) \end{pmatrix}\vec{u}_2{}^{(7)} \tag{3.220}$$

Accelerations of points G_{13}, G_{14}, G_{15} and G_{17}, resolved in frame $F_{13}\left\{G_{13}; \vec{u}_1{}^{(13)}, \vec{u}_2{}^{(13)}, \vec{u}_3{}^{(13)}\right\}$, can be obtained by differentiation of Equations (3.203), (3.146), (3.147) and (3.204) as follows:

$$\vec{a}_{G_{13}} = D_{13}\vec{V}_{G_{13}}{}^{(13)} + \vec{\omega}_{13/0} \times \vec{V}_{G_{13}}{}^{(13)} \tag{3.221}$$

$$\vec{a}_{G_{14}} = D_{13}\vec{V}_{G_{14}}{}^{(13)} + \vec{\omega}_{13/0} \times \vec{V}_{G_{14}}{}^{(13)} \tag{3.222}$$

$$\vec{a}_{G_{15}} = D_{13}\vec{V}_{G_{15}}{}^{(13)} + \vec{\omega}_{13/0} \times \vec{V}_{G_{15}}{}^{(13)} \tag{3.223}$$

$$\vec{a}_{G_{17}} = D_{13}\vec{V}_{G_{17}}{}^{(13)} + \vec{\omega}_{13/0} \times \vec{V}_{G_{17}}{}^{(13)} \tag{3.224}$$

After some manipulations the above equations become:

$$\vec{a}_{G_{13}} = \left(\ddot{x}_3 \cos(\beta_{13}) + \ddot{y}_3 \sin(\beta_{13})\right)\vec{u}_1{}^{(13)} + \left(-\ddot{x}_3 \sin(\beta_{13}) + \ddot{y}_3 \cos(\beta_{13})\right)\vec{u}_2{}^{(13)} \tag{3.225}$$

$$\vec{a}_{G_{14}} = \left(\ddot{x}_3 \cos(\beta_{13}) + \ddot{y}_3 \sin(\beta_{13}) + d_2\ddot{\beta}_{13} - d_1\dot{\beta}_{13}{}^2\right)\vec{u}_1{}^{(13)} + \left(-\ddot{x}_3 \sin(\beta_{13}) + \ddot{y}_3 \cos(\beta_{13}) + d_1\ddot{\beta}_{13} + d_2\dot{\beta}_{13}{}^2\right)\vec{u}_2{}^{(13)} \tag{3.226}$$

$$\vec{a}_{G_{15}} = \left(\ddot{x}_3 \cos(\beta_{13}) + \ddot{y}_3 \sin(\beta_{13}) - d_2\ddot{\beta}_{13} - d_1\dot{\beta}_{13}{}^2\right)\vec{u}_1{}^{(13)} + \left(-\ddot{x}_3 \sin(\beta_{13}) + \ddot{y}_3 \cos(\beta_{13}) + d_1\ddot{\beta}_{13} - d_2\dot{\beta}_{13}{}^2\right)\vec{u}_2{}^{(13)} \tag{3.227}$$

$$\vec{a}_{G_{17}} = \begin{pmatrix} \ddot{x}_3 \cos(\beta_{13}) + \ddot{y}_3 \sin(\beta_{13}) + d_8\ddot{\gamma}_{17} \sin(\gamma_{17}) + d_8\dot{\gamma}_{17}^2 \cos(\gamma_{17}) \\ +2d_8\dot{\gamma}_{17}\dot{\beta}_{13} \cos(\gamma_{17}) + d_8\ddot{\beta}_{13} \sin(\gamma_{17}) + \left(d_8 \cos(\gamma_{17}) + d_3\right)\dot{\beta}_{13}{}^2 \end{pmatrix}\vec{u}_1{}^{(13)} + \begin{pmatrix} -\ddot{x}_3 \sin(\beta_{13}) + \ddot{y}_3 \cos(\beta_{13}) + 2d_8\dot{\gamma}_{17}\dot{\beta}_{13} \sin(\gamma_{17}) + d_8 \sin(\gamma_{17})\dot{\beta}_{13}{}^2 \\ -\left(d_8 \cos(\gamma_{17}) + d_3\right)\ddot{\beta}_{13} - d_8\ddot{\gamma}_{17} \cos(\gamma_{17}) + d_8\dot{\gamma}_{17}^2 \sin(\gamma_{17}) \end{pmatrix}\vec{u}_2{}^{(13)} \tag{3.228}$$

3.3 Dynamic Equations

The robot system is composed of three modules and two connecting units which connect the modules to each other. Each module system is composed of 5 rigid bodies, which are the right front wheel, left front wheel, rear wheel, rear axle and the caster part, and the body of the module. The rear axle and the caster part are assumed to have negligible mass. Two connecting units are the other rigid bodies in the robot system.

3.3.1 Equation of Motion of the First Module

For the first module, body 1 is the body of module, body 2 is the right front wheel, body 3 is the left front wheel, body 4 is the rear axle and the caster part, body 5 is the rear wheel and body 6 is the connecting unit between the first and the second modules. Newton Equations of the five rigid bodies of the first module are, shown in the following equations:

$$m_1\vec{a}_{G_1} = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} + \vec{F}_{61} + m_1\vec{g} \tag{3.229}$$

$$m_2\vec{a}_{G_2} = -\vec{F}_{21} + \vec{F}_{02} + m_2\vec{g} \tag{3.230}$$

$$m_3\vec{a}_{G_3} = -\vec{F}_{31} + \vec{F}_{03} + m_3\vec{g} \tag{3.231}$$

$$m_4\vec{a}_{G_4} = -\vec{F}_{41} + \vec{F}_{54} + m_4\vec{g} \tag{3.232}$$

$$m_5\vec{a}_{G_5} = -\vec{F}_{54} + \vec{F}_{05} + m_5\vec{g} \tag{3.233}$$

where

m_1 : the mass of the body of the first module

m_2 : the mass of the right front wheel

m_3 : the mass of the left front wheel

m_4 : the mass of the body 4

m_5 : the mass of the rear wheel

$\vec{F}_{21}$: the internal force between the right front wheel and the body of the first module

$\vec{F}_{31}$: the internal force between the left front wheel and the body of the first module

$\vec{F}_{41}$: the internal force between the body 4 and the body of the first module

$\vec{F}_{54}$: the internal force between the rear wheel and body 4

$\vec{F}_{02}$: the external force applied from the ground to the right front wheel

$\vec{F}_{03}$: the external force applied from the ground to the left front wheel

$\vec{F}_{05}$: the external force applied from the ground to the rear wheel

$\vec{F}_{61}$: the external force applied from the joint frame to the body of the first module

Body 4 is assumed to have negligible mass, i.e.

$$m_4 = 0 \tag{3.234}$$

Then Equation (3.232) becomes

$$\vec{F}_{41} = \vec{F}_{54} \tag{3.235}$$

Euler Equations of the body, right front wheel, left front wheel, body 4 and rear wheel of the first module are shown in the following equations:

$$\begin{aligned} \breve{J}_1 \cdot \vec{\alpha}_1 + \vec{\omega}_1 \times \breve{J}_1 \cdot \vec{\omega}_1 = & -\vec{M}_{12} - \vec{M}_{13} - \vec{M}_{14} \\ & + \vec{r}_1 \times \vec{F}_{21} + \vec{r}_2 \times \vec{F}_{31} + \vec{r}_3 \times \vec{F}_{41} + \vec{r}_4 \times \vec{F}_{61} \end{aligned} \tag{3.236}$$

$$\breve{J}_2 \cdot \vec{\alpha}_2 + \vec{\omega}_2 \times \breve{J}_2 \cdot \vec{\omega}_2 = \vec{M}_{12} + \vec{r}_5 \times \vec{F}_{02} \tag{3.237}$$

$$\breve{J}_3 \cdot \vec{\alpha}_3 + \vec{\omega}_3 \times \breve{J}_3 \cdot \vec{\omega}_3 = \vec{M}_{13} + \vec{r}_6 \times \vec{F}_{03} \tag{3.238}$$

$$\breve{J}_4 \cdot \vec{\alpha}_4 + \vec{\omega}_4 \times \breve{J}_4 \cdot \vec{\omega}_4 = \vec{M}_{14} - \vec{M}_{45} + \vec{r}_7 \times (-\vec{F}_{41}) + \vec{r}_8 \times \vec{F}_{54} \tag{3.239}$$

$$\breve{J}_5 \cdot \vec{\alpha}_5 + \vec{\omega}_5 \times \breve{J}_5 \cdot \vec{\omega}_5 = \vec{M}_{45} + \vec{r}_9 \times \vec{F}_{05} \tag{3.240}$$

where

$\breve{J}_1$: the inertia dyadic of the body of the first module

$\breve{J}_2$: the inertia dyadic of the right front wheel

$\breve{J}_3$: the inertia dyadic of the left front wheel

$\breve{J}_4$: the inertia dyadic of the body 4

$\breve{J}_5$: the inertia dyadic of the rear wheel

$\vec{M}_{12}$: the internal moment between the right front wheel and the body of the first module

$\vec{M}_{13}$: the internal moment between the left front wheel and the body of the first module

$\vec{M}_{14}$: the internal moment between the body 4 and the body of the first module

$\vec{M}_{45}$: the internal moment between the rear wheel and body 4

Body 4 is assumed to have negligible mass, i.e.

$$\breve{J}_4 = 0 \tag{3.241}$$

Substituting Equations (3.241) and (3.235) into Equation (3.239), one can get the following equation:

$$\vec{M}_{45} = \vec{M}_{14} + \vec{r}_7 \times (-\vec{F}_{41}) + \vec{r}_8 \times \vec{F}_{41} \tag{3.242}$$

The reaction forces formed between the wheels and the ground and the force, which is transmitted by the connecting unit to the module, are needed. To eliminate the internal forces from the Newton Equations, first calculate the internal forces from the Equations (3.230), (3.231) and (2.233).

$$\vec{F}_{21} = \left(\vec{F}_{02} + m_2(\vec{g} - \vec{a}_{G_2})\right) \tag{3.243}$$

$$\vec{F}_{31} = \left(\vec{F}_{03} + m_3(\vec{g} - \vec{a}_{G_3})\right) \tag{3.244}$$

$$\vec{F}_{41} = \left(\vec{F}_{05} + m_5(\vec{g} - \vec{a}_{G_5})\right) \tag{3.245}$$

By substituting these forces into the Equation (3.229), the Equation (3.229) becomes

$$\begin{aligned} m_1\vec{a}_{G_1} &= \left(\vec{F}_{02} + m_2(\vec{g} - \vec{a}_{G_2})\right) + \left(\vec{F}_{03} + m_3(\vec{g} - \vec{a}_{G_3})\right) \\ &+ \left(\vec{F}_{05} + m_5(\vec{g} - \vec{a}_{G_5})\right) + \vec{F}_{61} + m_1\vec{g} \end{aligned} \tag{3.246}$$

Rearranging the terms in Equation (3.246), one can get the following equation:

$$m_1(\vec{a}_{G_1} - \vec{g}) + m_2(\vec{a}_{G_2} - \vec{g}) + m_3(\vec{a}_{G_3} - \vec{g}) + m_5(\vec{a}_{G_5} - \vec{g}) - \vec{F}_{61} = \vec{F}_{02} + \vec{F}_{03} + \vec{F}_{05} \tag{3.247}$$

Similarly to eliminate the internal moment $\vec{M}_{14}$ and the internal forces from the Euler Equations, first calculate the internal moment $\vec{M}_{14}$ from the Equations (3.240) and (3.242).

$$\vec{M}_{14} = \left(\breve{J}_5 \cdot \vec{\alpha}_5 + \vec{\omega}_5 \times \breve{J}_5 \cdot \vec{\omega}_5\right) - \vec{r}_9 \times \vec{F}_{05} - \vec{r}_7 \times (-\vec{F}_{41}) - \vec{r}_8 \times \vec{F}_{41} \tag{3.248}$$

Substitute this moment and the internal forces into the Equation (3.236). Then the Equation (3.236) becomes

$$\begin{aligned} \breve{J}_1 \cdot \vec{\alpha}_1 + \vec{\omega}_1 \times \breve{J}_1 \cdot \vec{\omega}_1 = &-\vec{M}_{12} - M_{13} - \left(\breve{J}_5 \cdot \vec{\alpha}_5 + \vec{\omega}_5 \times \breve{J}_5 \cdot \vec{\omega}_5\right) \\ &+\vec{r}_7 \times -\left(\vec{F}_{05} + m_5(\vec{g} - \vec{a}_{G_5})\right) + \vec{r}_8 \times \left(\vec{F}_{05} + m_5(\vec{g} - \vec{a}_{G_5})\right) \\ &+\vec{r}_1 \times \left(\vec{F}_{02} + m_2(\vec{g} - \vec{a}_{G_2})\right) + \vec{r}_2 \times \left(\vec{F}_{03} + m_3(\vec{g} - \vec{a}_{G_3})\right) \\ &+\vec{r}_3 \times \left(\vec{F}_{05} + m_5(\vec{g} - \vec{a}_{G_5})\right) + \vec{r}_4 \times \vec{F}_{61} + \vec{r}_9 \times \vec{F}_{05} \end{aligned} \tag{3.249}$$

Rearranging the terms in Equation (3.249), one can get the following equation:

$$\begin{aligned}&\left(\breve{J}_1\cdot\vec{\alpha}_1+\vec{\omega}_1\times\breve{J}_1\cdot\vec{\omega}_1\right)+\left(\breve{J}_5\cdot\vec{\alpha}_5+\vec{\omega}_5\times\breve{J}_5\cdot\vec{\omega}_5\right)-\vec{r}_1\times m_2(\vec{g}-\vec{a}_{G_2})\\&-\vec{r}_2\times m_3(\vec{g}-\vec{a}_{G_3})-\vec{r}_3\times m_5(\vec{g}-\vec{a}_{G_5})-\vec{r}_7\times -m_5(\vec{g}-\vec{a}_{G_5})\\&-\vec{r}_8\times m_5(\vec{g}-\vec{a}_{G_5})-\vec{r}_4\times\vec{F}_{61}=-\vec{M}_{12}-\vec{M}_{13}+\vec{r}_1\times\vec{F}_{02}+\vec{r}_2\times\vec{F}_{03}\\&+\vec{r}_3\times\vec{F}_{05}+\vec{r}_9\times\vec{F}_{05}+\vec{r}_7\times(-\vec{F}_{05})+\vec{r}_8\times\vec{F}_{05}\end{aligned}\tag{3.250}$$

The inertia dyadics of the body and the wheels can be defined by the following equations:

$$\breve{J}_1=J_{11}\vec{u}_1^{(1)}\vec{u}_1^{(1)}+J_{22}\vec{u}_2^{(1)}\vec{u}_2^{(1)}+J_{33}\vec{u}_3^{(1)}\vec{u}_3^{(1)}+J_{31}\vec{u}_3^{(1)}\vec{u}_1^{(1)}+J_{31}\vec{u}_1^{(1)}\vec{u}_3^{(1)}\tag{3.251}$$

$$\breve{J}_2=\breve{J}_3=J_n\vec{u}_1^{(1)}\vec{u}_1^{(1)}+J_s\vec{u}_2^{(1)}\vec{u}_2^{(1)}+J_n\vec{u}_3^{(1)}\vec{u}_3^{(1)}\tag{3.252}$$

$$\breve{J}_5=J_n\vec{u}_1^{(5)}\vec{u}_1^{(5)}+J_s\vec{u}_2^{(5)}\vec{u}_2^{(5)}+J_n\vec{u}_3^{(5)}\vec{u}_3^{(5)}\tag{3.253}$$

The location of the center of the mass of the module and inertia dyadic of the body and the wheels are calculated by the program, ADAMS, in which the model of the robot is created.

The position vectors $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, $\vec{r}_4$, $\vec{r}_5$, $\vec{r}_6$, $\vec{r}_7$, $\vec{r}_8$ and $\vec{r}_9$ can be written using Figure 3.3 and Figure 3.4.

$$\vec{r}_1=d_1\vec{u}_1^{(1)}-d_2\vec{u}_2^{(1)}-d_7\vec{u}_3^{(1)}\tag{3.254}$$

$$\vec{r}_2=d_1\vec{u}_1^{(1)}+d_2\vec{u}_2^{(1)}-d_7\vec{u}_3^{(1)}\tag{3.255}$$

$$\vec{r}_3=-d_3\vec{u}_1^{(1)}+d_5\vec{u}_3^{(1)}\tag{3.256}$$

$$\vec{r}_4=-d_4\vec{u}_1^{(1)}+d_6\vec{u}_3^{(1)}\tag{3.257}$$

$$\vec{r}_5=-R\vec{u}_3^{(1)}\tag{3.258}$$

$$\vec{r}_6 = -R\vec{u}_3^{(1)} \tag{3.259}$$

$$\vec{r}_7 = \frac{d_8}{2}\vec{u}_1^{(5)} = \frac{d_8}{2}\cos(\theta_{D_1})\vec{u}_1^{(1)} - \frac{d_8}{2}\sin(\theta_{D_1})\vec{u}_2^{(1)} \tag{3.260}$$

$$\vec{r}_8 = -\frac{d_8}{2}\vec{u}_1^{(5)} = -\frac{d_8}{2}\cos(\theta_{D_1})\vec{u}_1^{(1)} + \frac{d_8}{2}\sin(\theta_{D_1})\vec{u}_2^{(1)} \tag{3.261}$$

$$\vec{r}_9 = -R\vec{u}_3^{(1)} \tag{3.262}$$

The force and moment terms in Newton and Euler Equations can be written in expanded form as follows. Remember the resistance force of the rear wheel is neglected.

$$\vec{F}_{02} = F_{02_x}\vec{u}_1^{(1)} + F_{02_y}\vec{u}_2^{(1)} + F_{02_z}\vec{u}_3^{(1)} \tag{3.263}$$

$$\vec{F}_{03} = F_{03_x}\vec{u}_1^{(1)} + F_{03_y}\vec{u}_2^{(1)} + F_{03_z}\vec{u}_3^{(1)} \tag{3.264}$$

$$\vec{F}_{05} = F_{05_y}\vec{u}_2^{(1)} + F_{05_z}\vec{u}_3^{(1)} \tag{3.265}$$

$$\vec{F}_{61} = F_{61_x}\vec{u}_1^{(1)} + F_{61_y}\vec{u}_2^{(1)} \tag{3.266}$$

$$\vec{M}_{12} = M_{12_x}\vec{u}_1^{(1)} + T_2\vec{u}_2^{(1)} + M_{12_z}\vec{u}_3^{(1)} \tag{3.267}$$

$$\vec{M}_{13} = M_{13_x}\vec{u}_1^{(1)} + T_3\vec{u}_2^{(1)} + M_{13_z}\vec{u}_3^{(1)} \tag{3.268}$$

Where T_2 and T_3 are the motor torques supplied from right and left motors. The unknowns are $\ddot{x}_1$, $\ddot{y}_1$, F_{02_x}, F_{02_y}, F_{02_z}, F_{03_x}, F_{03_y}, F_{03_z}, F_{05_y}, F_{05_z} M_{12_x}, M_{12_z}, M_{13_x} and M_{13_z}. T_2 and T_3 are the inputs of the system. The connecting unit only translates compression and tension forces in its longitudinal direction to the modules. For simplification and for calculating the force transmitted by the connecting unit to the modules, the connecting unit is assumed as a spring. Therefore magnitude of $\vec{F}_{61}$ is calculated by considering the current deflection of the spring and direction of $\vec{F}_{61}$ is calculated from the current positions.

There are two cases for the motion of the module. Case 1; there is lateral slip at the rear wheel and Case 2; there is no slip at the rear wheel.

3.3.1.1 Case 1: Lateral Slip at Rear Wheel

For Case 1, $\vec{F}_{05_y}$ can be written as:

$$\vec{F}_{05_y} = \mu \vec{F}_{05_z} \tag{3.269}$$

There are twelve equations formed from Equations (3.247), (3.250), (3.237) and (3.238) and but thirteen unknowns. Therefore an assumption can be made without loss of generality such that;

$$\frac{F_{02_y}}{F_{02_z}} = \frac{F_{03_y}}{F_{03_z}} \tag{3.270}$$

The lateral components of the front wheels are proportional to their normal components. Then there exist twelve unknowns and twelve equations. Three scalar equations can be obtained by expending the Equation (3.247) in the first module fixed frame components where masses of the wheels are equal to each other and written as m_2.

$$m_1 a_{G_{1_x}} + m_2 a_{G_{2_x}} + m_2 a_{G_{3_x}} + m_2 a_{G_{5_x}} - F_{61_x} = F_{02_x} + F_{03_x} \tag{3.271}$$

$$m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_2 a_{G_{3_y}} + m_2 a_{G_{5_y}} - F_{61_y} = F_{02_y} + \frac{F_{03_z}}{F_{02_z}} F_{02_y} + \mu F_{05_z} \tag{3.272}$$

$$-m_1 g_z - 3m_2 g_z = F_{02_z} + F_{03_z} + F_{05_z} \tag{3.273}$$

After some manipulations three scalar equations can be obtained by expanding the Equation (3.250) in the first module fixed frame components.

$$
\begin{aligned}
&J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\
&= -M_{12_x} - M_{13_x} + R\mu F_{05_z} - d_2 F_{02_z} + d_7 F_{02_y} + d_2 F_{03_z} + d_7 \frac{F_{03_z}}{F_{02_z}} F_{02_y} \\
&-d_5 \mu F_{05_z} + d_8 \sin(\gamma_5) F_{05_z}
\end{aligned}
\tag{3.274}
$$

$$
\begin{aligned}
&\omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2x}} + 2 d_1 m_2 g_z - d_7 m_2 a_{G_{3x}} + d_5 m_2 a_{G_{5x}} - d_3 m_2 g_z \\
&-d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} = -T_2 - T_3 - d_7 F_{02_x} - d_1 F_{02_z} - d_7 F_{03_x} \\
&-d_1 F_{03_z} + d_3 F_{05_z} + d_8 \cos(\gamma_5) F_{05_z}
\end{aligned}
\tag{3.275}
$$

$$
\begin{aligned}
&J_{33}\alpha_{1_z} + J_n \alpha_{5_z} + d_1 m_2 a_{G_{2y}} + d_2 m_2 a_{G_{2x}} + d_1 m_2 a_{G_{3y}} - d_2 m_2 a_{G_{3x}} \\
&-d_3 m_2 a_{G_{5y}} - d_8 \cos(\gamma_5) m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 a_{G_{5y}} + d_4 F_{61_y} = \\
&-M_{12_z} - M_{13_z} + d_1 F_{02_y} + d_2 F_{02_x} + d_1 \frac{F_{03_z}}{F_{02_z}} F_{02_y} - d_2 F_{03_x} \\
&-d_3 \mu F_{05_z} - d_8 \cos(\gamma_5) \mu F_{05_z}
\end{aligned}
\tag{3.276}
$$

Six scalar equations can be obtained by expending the Equation (3.237) and (3.238) in the first module fixed frame components

$$J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} = M_{12_x} + R F_{02_y} \tag{3.277}$$

$$J_s \alpha_{2_y} = T_2 - R F_{02_x} \tag{3.278}$$

$$J_n \alpha_{2_z} = M_{12_z} \tag{3.279}$$

$$J_n \alpha_{3_x} + \omega_{3_y} J_n \omega_{3_z} - \omega_{3_z} J_s \omega_{3_y} = M_{13_x} + R \frac{F_{03_z}}{F_{02_z}} F_{02_y} \tag{3.280}$$

$$J_s \alpha_{3_y} = T_3 - R F_{03_x} \tag{3.281}$$

$$J_n \alpha_{3_z} = M_{13_z} \tag{3.282}$$

By substituting Equation (3.272), (3.277) and (3.280) into Equation (3.274) and Equation (3.271) into Equation (3.275), and after some manipulations, one can get the following equations:

$$\begin{aligned}
&J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\
&+ J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} + J_n \alpha_{3_x} + \omega_{3_y} J_n \omega_{3_z} - \omega_{3_z} J_s \omega_{3_y} \\
&- R\left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y} \right) = -d_2 F_{02_z} + d_2 F_{03_z} \\
&+ \left(-d_5 \mu + d_8 \sin(\gamma_5) - d_7 \mu \right) F_{05_z}
\end{aligned} \tag{3.283}$$

$$\begin{aligned}
&\omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2x}} + 2 d_1 m_2 g_z - d_7 m_2 a_{G_{3x}} + d_5 m_2 a_{G_{5x}} \\
&- d_3 m_2 g_z - d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\
&+ d_7 \left(m_1 a_{G_{1x}} + m_2 a_{G_{2x}} + m_2 a_{G_{3x}} + m_2 a_{G_{5x}} - F_{61_x} \right) = \\
&- d_1 F_{02_z} - d_1 F_{03_z} + \left(d_3 + d_8 \cos(\gamma_5) \right) F_{05_z}
\end{aligned} \tag{3.284}$$

Equations (3.273), (3.283) and (3.284) can be written in the matrix representation form as follows:

$$\begin{bmatrix} -m_1 g_z - 3m_2 g_z \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\ + J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} + J_n \alpha_{3_x} + \omega_{3_y} J_n \omega_{3_z} - \omega_{3_z} J_s \omega_{3_y} \\ -R\left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y} \right) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2_x}} + 2d_1 m_2 g_z - d_7 m_2 a_{G_{3_x}} + d_5 m_2 a_{G_{5_x}} \\ -d_3 m_2 g_z - d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\ +d_7 \left(m_1 a_{G_{1_x}} + m_2 a_{G_{2_x}} + m_2 a_{G_{3_x}} + m_2 a_{G_{5_x}} - F_{61_x} \right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ -d_2 & d_2 & -\left(d_7 \mu + d_5 \mu - d_8 \sin(\gamma_5) \right) \\ -d_1 & -d_1 & d_3 + d_8 \cos(\gamma_5) \end{bmatrix} \begin{bmatrix} F_{02_z} \\ F_{03_z} \\ F_{05_z} \end{bmatrix}$$

(3.285)

After some manipulations the expressions for F_{02_z}, F_{03_z} and F_{05_z} obtained as in the following equations:

$$F_{02_z} = -\frac{\begin{pmatrix}\left(-d_2d_3 - d_2d_8\cos(\gamma_5) + d_1d_5\mu + d_1d_7\mu - d_1d_8\sin(\gamma_5)\right)\\ \left(-m_1g_z - 3m_2g_z\right)\end{pmatrix}}{2d_2\left(d_3 + d_8\cos(\gamma_5) + d_1\right)}$$
$$-\frac{\begin{pmatrix}J_{31}\alpha_{1_z} + d_7m_2a_{G_{2y}} + d_7m_2a_{G_{3y}} - d_5m_2a_{G_{5y}} - d_8\sin(\gamma_5)m_2g_z + d_6F_{61_y}\\ +J_n\alpha_{2_x} + \omega_{2_y}J_n\omega_{2_z} - \omega_{2_z}J_s\omega_{2_y} + J_n\alpha_{3_x} + \omega_{3_y}J_n\omega_{3_z} - \omega_{3_z}J_s\omega_{3_y}\\ -R\left(m_1a_{G_{1_y}} + m_2a_{G_{2_y}} + m_2a_{G_{3_y}} + m_2a_{G_{5_y}} - F_{61_y}\right)\end{pmatrix}}{2d_2}$$
$$-\frac{\begin{pmatrix}\left(d_5\mu + d_7\mu - d_8\sin(\gamma_5) + d_2\right)\\ \begin{pmatrix}\omega_{1_z}{}^2J_{31} - d_7m_2a_{G_{2_x}} + 2d_1m_2g_z - d_7m_2a_{G_{3_x}} + d_5m_2a_{G_{5_x}}\\ -d_3m_2g_z - d_8\cos(\gamma_5)m_2g_z - d_6F_{61_x} + T_2 + T_3\\ +d_7\left(m_1a_{G_{1_x}} + m_2a_{G_{2_x}} + m_2a_{G_{3_x}} + m_2a_{G_{5_x}} - F_{61_x}\right)\end{pmatrix}\end{pmatrix}}{2d_2\left(d_3 + d_8\cos(\gamma_5) + d_1\right)} \tag{3.286}$$

$$F_{03_z} = \frac{\begin{pmatrix}\left(-d_2d_3 - d_2d_8\cos(\gamma_5) + d_1d_5\mu + d_1d_7\mu - d_1d_8\sin(\gamma_5)\right)\\ \left(-m_1g_z - 3m_2g_z\right)\end{pmatrix}}{2d_2\left(d_3 + d_8\cos(\gamma_5) + d_1\right)}$$
$$+\frac{\begin{pmatrix}J_{31}\alpha_{1_z} + d_7m_2a_{G_{2y}} + d_7m_2a_{G_{3y}} - d_5m_2a_{G_{5y}} - d_8\sin(\gamma_5)m_2g_z\\ +d_6F_{61_y} + J_n\alpha_{2_x} + \omega_{2_y}J_n\omega_{2_z} - \omega_{2_z}J_s\omega_{2_y} + J_n\alpha_{3_x} + \omega_{3_y}J_n\omega_{3_z}\\ -\omega_{3_z}J_s\omega_{3_y} - R\left(m_1a_{G_{1_y}} + m_2a_{G_{2_y}} + m_2a_{G_{3_y}} + m_2a_{G_{5_y}} - F_{61_y}\right)\end{pmatrix}}{2d_2}$$
$$+\frac{\begin{pmatrix}\left(d_5\mu + d_7\mu - d_8\sin(\gamma_5) + d_2\right)\\ \begin{pmatrix}\omega_{1_z}{}^2J_{31} - d_7m_2a_{G_{2_x}} + 2d_1m_2g_z - d_7m_2a_{G_{3_x}} + d_5m_2a_{G_{5_x}}\\ -d_3m_2g_z - d_8\cos(\gamma_5)m_2g_z - d_6F_{61_x} + T_2 + T_3\\ +d_7\left(m_1a_{G_{1_x}} + m_2a_{G_{2_x}} + m_2a_{G_{3_x}} + m_2a_{G_{5_x}} - F_{61_x}\right)\end{pmatrix}\end{pmatrix}}{2d_2\left(d_3 + d_8\cos(\gamma_5) + d_1\right)} \tag{3.287}$$

$$F_{05_z} = \frac{d_1\left(-m_1 g_z - 3m_2 g_z\right)}{\left(d_3 + d_8\cos(\gamma_5) + d_1\right)} + \frac{\begin{pmatrix} \omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2_x}} + 2d_1 m_2 g_z - d_7 m_2 a_{G_{3_x}} + d_5 m_2 a_{G_{5_x}} \\ -d_3 m_2 g_z - d_8\cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\ +d_7\left(m_1 a_{G_{1_x}} + m_2 a_{G_{2_x}} + m_2 a_{G_{3_x}} + m_2 a_{G_{5_x}} - F_{61_x}\right) \end{pmatrix}}{\left(d_3 + d_8\cos(\gamma_5) + d_1\right)} \tag{3.288}$$

From Equation (3.272), the expression for F_{02_y} is obtained as in the following equation:

$$F_{02_y} = \frac{\left(m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_3 a_{G_{3_y}} + m_5 a_{G_{5_y}} - F_{61_y}\right) - \mu F_{05_z}}{\left(1 + \frac{F_{03_z}}{F_{02_z}}\right)} \tag{3.289}$$

From Equation (3.278) and (3.281), F_{02_x} and F_{03_x} are obtained as in the following equations:

$$F_{02_x} = \frac{T_2}{R} - \frac{J_s \ddot{\theta}_2}{R} \tag{3.290}$$

$$F_{03_x} = \frac{T_3}{R} - \frac{J_s \ddot{\theta}_3}{R} \tag{3.291}$$

By substituting Equations (3.290) and (3.291) into (3.271), one can get the following equation:

$$m_1 a_{G_{1_x}} + m_2 a_{G_{2_x}} + m_2 a_{G_{3_x}} + m_2 a_{G_{5_x}} - F_{61_x} = \frac{T_2 + T_3}{R} - \frac{J_s\left(\ddot{\theta}_2 + \ddot{\theta}_3\right)}{R} \tag{3.292}$$

By substituting Equations (3.279), (3.282), (3.272) (3.290) and (3.291) into (3.276), one can get the following equation:

$$\begin{aligned}
&J_{33}\alpha_{1_z} + J_n\alpha_{5_z} + d_1 m_2 a_{G_{2_y}} + d_2 m_2 a_{G_{2_x}} + d_1 m_2 a_{G_{3_y}} - d_2 m_2 a_{G_{3_x}} \\
&-d_3 m_2 a_{G_{5_y}} - d_8 \cos(\gamma_5) m_2 a_{G_{5_y}} - d_8 \sin(\gamma_5) m_2 a_{G_{5_y}} + d_4 F_{61_y} \\
&-d_1\left(m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_2 a_{G_{3_y}} + m_2 a_{G_{5_y}} - F_{61_y}\right) = -J_n\ddot{\theta}_2 - J_n\ddot{\theta}_3 \\
&+d_2\frac{T_2}{R} - d_2\frac{J_s\ddot{\theta}_2}{R} - d_2\frac{T_3}{R} + d_2\frac{J_s\ddot{\theta}_3}{R} - \left(d_1\mu + d_3\mu + d_8\mu\cos(\gamma_5)\right)F_{05_z}
\end{aligned} \tag{3.293}$$

$\ddot{\theta}_2$ and $\ddot{\theta}_3$ are obtained from the Equations (3.292) and (3.293).

3.3.1.2 Case 2: No Slip at Rear Wheel

For Case 2, there is no slip at the wheels. While the robot goes on a defined path, for example, goes on a straight line or goes on a circular arc, when there are no slip at all the wheels, the robot system is one degree of freedom. The unknowns are $\ddot{x}_1$, F_{02_x}, F_{02_y}, F_{02_z}, F_{03_x}, F_{03_y}, F_{03_z}, F_{05_y}, F_{05_z} M_{12_x}, M_{12_z}, M_{13_x} and M_{13_z}. There are twelve equations formed from Equations (3.247), (3.250), (3.237) and (3.238) but thirteen unknowns. Therefore an assumption can be made without loss of generality such that;

$$\frac{F_{02_y}}{F_{02_z}} = \frac{F_{03_y}}{F_{03_z}} \tag{3.294}$$

The lateral components of the front wheels are proportional to their normal components. Then there exist twelve unknowns and twelve equations. Three scalar equations can be obtained by expending the Equation (3.247) in the first module fixed frame components where masses of the wheels are equal to each other and written as m_2.

$$m_1 a_{G_{1_x}} + m_2 a_{G_{2_x}} + m_2 a_{G_{3_x}} + m_2 a_{G_{5_x}} - F_{61_x} = F_{02_x} + F_{03_x} \tag{3.295}$$

$$m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_2 a_{G_{3_y}} + m_2 a_{G_{5_y}} - F_{61_y} = F_{02_y} + \frac{F_{03_z}}{F_{02_z}} F_{02_y} + F_{05_y} \tag{3.296}$$

$$-m_1 g_z - 3m_2 g_z = F_{02_z} + F_{03_z} + F_{05_z} \tag{3.297}$$

After some manipulations three scalar equations can be obtained by expanding the Equation (3.250) in the first module fixed frame components.

$$\begin{aligned} & J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\ & = -M_{12_x} - M_{13_x} + RF_{05_y} - d_2 F_{02_z} + d_7 F_{02_y} + d_2 F_{03_z} + d_7 \frac{F_{03_z}}{F_{02_z}} F_{02_y} \\ & -d_5 F_{05_y} + d_8 \sin(\gamma_5) F_{05_z} \end{aligned} \tag{3.298}$$

$$\begin{aligned} & \omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2_x}} + 2d_1 m_2 g_z - d_7 m_2 a_{G_{3_x}} + d_5 m_2 a_{G_{5_x}} - d_3 m_2 g_z \\ & -d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} = -T_2 - T_3 - d_7 F_{02_x} - d_1 F_{02_z} - d_7 F_{03_x} \\ & -d_1 F_{03_z} + d_3 F_{05_z} + d_8 \cos(\gamma_5) F_{05_z} \end{aligned} \tag{3.299}$$

$$\begin{aligned} & J_{33}\alpha_{1_z} + J_n \alpha_{5_z} + d_1 m_2 a_{G_{2_y}} + d_2 m_2 a_{G_{2_x}} + d_1 m_2 a_{G_{3_y}} - d_2 m_2 a_{G_{3_x}} \\ & -d_3 m_2 a_{G_{5_y}} - d_8 \cos(\gamma_5) m_2 a_{G_{5_y}} - d_8 \sin(\gamma_5) m_2 a_{G_{5_y}} + d_4 F_{61_y} = \\ & -M_{12_z} - M_{13_z} + d_1 F_{02_y} + d_2 F_{02_x} + d_1 \frac{F_{03_z}}{F_{02_z}} F_{02_y} - d_2 F_{03_x} \\ & -d_3 F_{05_y} - d_8 \cos(\gamma_5) F_{05_y} \end{aligned} \tag{3.300}$$

Six scalar equations can be obtained by expending the Equation (3.237) and (3.238) in the first module fixed frame components.

$$J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} = M_{12_x} + RF_{02_y} \tag{3.301}$$

$$J_s \alpha_{2_y} = T_2 - RF_{02_x} \tag{3.302}$$

$$J_n\alpha_{2_z} = M_{12_z} \tag{3.303}$$

$$J_n\alpha_{3_x} + \omega_{3_y} J_n\omega_{3_z} - \omega_{3_z} J_s\omega_{3_y} = M_{13_x} + R\frac{F_{03_z}}{F_{02_z}}F_{02_y} \tag{3.304}$$

$$J_s\alpha_{3_y} = T_3 - RF_{03_x} \tag{3.305}$$

$$J_n\alpha_{3_z} = M_{13_z} \tag{3.306}$$

From Equations (3.302) and (3.305), F_{02_x} and F_{03_x} are obtained as in the following equations:

$$F_{02_x} = \frac{T_2}{R} - \frac{J_s\alpha_{2_y}}{R} \tag{3.307}$$

$$F_{03_x} = \frac{T_3}{R} - \frac{J_s\alpha_{3_y}}{R} \tag{3.308}$$

By substituting Equations (3.303), (3.306), (3.307), (3.308) and (3.296) into Equation (3.300), the equation for F_{05_y} can be obtained as in the following form:

$$F_{05_y} = \frac{\begin{pmatrix} J_{33}\alpha_{1_z} + J_n\alpha_{5_z} + d_1 m_2 a_{G_{2_y}} + d_2 m_2 a_{G_{2_x}} + d_1 m_2 a_{G_{3_y}} - d_2 m_2 a_{G_{3_x}} \\ -d_3 m_2 a_{G_{5_y}} - d_8\cos(\gamma_5) m_2 a_{G_{5_y}} - d_8\sin(\gamma_5) m_2 a_{G_{5_y}} + d_4 F_{61_y} \\ -d_1\left(m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_2 a_{G_{3_y}} + m_2 a_{G_{5_y}} - F_{61_y}\right) + J_n\alpha_{2_z} + J_n\alpha_{3_z} \\ -d_2\left(\dfrac{T_2 - J_s\alpha_{2_y}}{R}\right) + d_2\left(\dfrac{T_3 - J_s\alpha_{3_y}}{R}\right) \end{pmatrix}}{-\left(d_1 + d_3 + d_8\cos(\gamma_5)\right)} \tag{3.309}$$

By substituting Equation (3.296), (3.301), (3.304) and (3.309) into Equation (3.298) and Equation (3.271) into Equation (3.275), and after some manipulations, one can get the following equations:

$$
\begin{aligned}
&J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\
&+ J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} + J_n \alpha_{3_x} + \omega_{3_y} J_n \omega_{3_z} - \omega_{3_z} J_s \omega_{3_y} \\
&-\left(R + d_7\right)\left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y}\right) \\
&+\left(d_5 + d_7\right)\frac{\left(\begin{array}{l} J_{33}\alpha_{1_z} + J_n \alpha_{5_z} + d_1 m_2 a_{G_{2y}} + d_2 m_2 a_{G_{2x}} + d_1 m_2 a_{G_{3y}} - d_2 m_2 a_{G_{3x}} \\ -d_3 m_2 a_{G_{5y}} - d_8 \cos(\gamma_5) m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 a_{G_{5y}} + d_4 F_{61_y} \\ -d_1\left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y}\right) + J_n \alpha_{2_z} + J_n \alpha_{3_z} \\ -d_2\left(\dfrac{T_2 - J_s \alpha_{2_y}}{R}\right) + d_2\left(\dfrac{T_3 - J_s \alpha_{3_y}}{R}\right) \end{array}\right)}{-\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)} \\
&= -d_2 F_{02_z} + d_2 F_{03_z} + d_8 \sin(\gamma_5) F_{05_z}
\end{aligned}
\tag{3.310}
$$

$$
\begin{aligned}
&\omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2x}} + 2 d_1 m_2 g_z - d_7 m_2 a_{G_{3x}} + d_5 m_2 a_{G_{5x}} \\
&-d_3 m_2 g_z - d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\
&+d_7\left(m_1 a_{G_{1x}} + m_2 a_{G_{2x}} + m_2 a_{G_{3x}} + m_2 a_{G_{5x}} - F_{61_x}\right) = \\
&-d_1 F_{02_z} - d_1 F_{03_z} + \left(d_3 + d_8 \cos(\gamma_5)\right) F_{05_z}
\end{aligned}
\tag{3.311}
$$

Equations (3.295), (3.310) and (3.311) can be written in the matrix representation as in the following form:

$$
\begin{bmatrix}
-m_1 g_z - 3m_2 g_z \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\begin{array}{l}
J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\
+J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} + J_n \alpha_{3_x} + \omega_{3_y} J_n \omega_{3_z} - \omega_{3_z} J_s \omega_{3_y} \\
-\left(R + d_7\right)\left(m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_2 a_{G_{3_y}} + m_2 a_{G_{5_y}} - F_{61_y}\right) \\
+\left(d_5 + d_7\right) \dfrac{\left(\begin{array}{l}
J_{33}\alpha_{1_z} + J_n \alpha_{5_z} + d_1 m_2 a_{G_{2_y}} + d_2 m_2 a_{G_{2_x}} + d_1 m_2 a_{G_{3_y}} - d_2 m_2 a_{G_{3_x}} \\
-d_3 m_2 a_{G_{5_y}} - d_8 \cos(\gamma_5) m_2 a_{G_{5_y}} - d_8 \sin(\gamma_5) m_2 a_{G_{5_y}} + d_4 F_{61_y} \\
-d_1\left(m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_2 a_{G_{3_y}} + m_2 a_{G_{5_y}} - F_{61_y}\right) + J_n \alpha_{2_z} + J_n \alpha_{3_z} \\
-d_2\left(\dfrac{T_2 - J_s \alpha_{2_y}}{R}\right) + d_2\left(\dfrac{T_3 - J_s \alpha_{3_y}}{R}\right)
\end{array}\right)}{-\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)}
\end{array} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\begin{array}{l}
\omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2_x}} + 2d_1 m_2 g_z - d_7 m_2 a_{G_{3_x}} + d_5 m_2 a_{G_{5_x}} \\
-d_3 m_2 g_z - d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\
+d_7\left(m_1 a_{G_{1_x}} + m_2 a_{G_{2_x}} + m_2 a_{G_{3_x}} + m_2 a_{G_{5_x}} - F_{61_x}\right)
\end{array}
\end{bmatrix}
$$

$$
= \begin{bmatrix} 1 & 1 & 1 \\ -d_2 & d_2 & \left(d_8 \sin(\gamma_5)\right) \\ -d_1 & -d_1 & d_3 + d_8 \cos(\gamma_5) \end{bmatrix} \begin{bmatrix} F_{02_z} \\ F_{03_z} \\ F_{05_z} \end{bmatrix}
$$

(3.312)

$$
\begin{aligned}
F_{02_z} = {} & \frac{\left(d_2 d_3 + d_2 d_8 \cos(\gamma_5) + d_8 \sin(\gamma_5) d_1\right)\left(-m_1 g_z - 3 m_2 g_z\right)}{2 d_2 \left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)} \\
& - \frac{\left(\begin{array}{l}
J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\
+ J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} + J_n \alpha_{3_x} + \omega_{3_y} J_n \omega_{3_z} - \omega_{3_z} J_s \omega_{3_y} \\
- \left(R + d_7\right)\left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y}\right) \\
+ \left(d_5 + d_7\right) \dfrac{\left(\begin{array}{l}
J_{33}\alpha_{1_z} + J_n \alpha_{5_z} + d_1 m_2 a_{G_{2y}} + d_2 m_2 a_{G_{2x}} + d_1 m_2 a_{G_{3y}} - d_2 m_2 a_{G_{3x}} \\
- d_3 m_2 a_{G_{5y}} - d_8 \cos(\gamma_5) m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 a_{G_{5y}} + d_4 F_{61_y} \\
- d_1 \left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y}\right) + J_n \alpha_{2_z} + J_n \alpha_{3_z} \\
- d_2 \left(\dfrac{T_2 - J_s \alpha_{2_y}}{R}\right) + d_2 \left(\dfrac{T_3 - J_s \alpha_{3_y}}{R}\right)
\end{array}\right)}{-\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)}
\end{array}\right)}{2 d_2 \left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)} \\
& + \frac{\left(d_8 \sin(\gamma_5) - d_2\right)\left(\begin{array}{l}
\omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2x}} + 2 d_1 m_2 g_z - d_7 m_2 a_{G_{3x}} + d_5 m_2 a_{G_{5x}} \\
- d_3 m_2 g_z - d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\
+ d_7 \left(m_1 a_{G_{1x}} + m_2 a_{G_{2x}} + m_2 a_{G_{3x}} + m_2 a_{G_{5x}} - F_{61_x}\right)
\end{array}\right)}{2 d_2 \left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)}
\end{aligned}
\tag{3.313}
$$

$$F_{03_z} = -\frac{\left(-d_2 d_3 - d_2 d_8 \cos(\gamma_5) + d_8 \sin(\gamma_5) d_1\right)\left(-m_1 g_z - 3m_2 g_z\right)}{2d_2\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)}$$

$$+\frac{\left(\begin{array}{l} J_{31}\alpha_{1_z} + d_7 m_2 a_{G_{2y}} + d_7 m_2 a_{G_{3y}} - d_5 m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 g_z + d_6 F_{61_y} \\ + J_n \alpha_{2_x} + \omega_{2_y} J_n \omega_{2_z} - \omega_{2_z} J_s \omega_{2_y} + J_n \alpha_{3_x} + \omega_{3_y} J_n \omega_{3_z} - \omega_{3_z} J_s \omega_{3_y} \\ -\left(R + d_7\right)\left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y}\right) \\ + \left(d_5 + d_7\right) \dfrac{\left(\begin{array}{l} J_{33}\alpha_{1_z} + J_n \alpha_{5_z} + d_1 m_2 a_{G_{2y}} + d_2 m_2 a_{G_{2x}} + d_1 m_2 a_{G_{3y}} - d_2 m_2 a_{G_{3x}} \\ -d_3 m_2 a_{G_{5y}} - d_8 \cos(\gamma_5) m_2 a_{G_{5y}} - d_8 \sin(\gamma_5) m_2 a_{G_{5y}} + d_4 F_{61_y} \\ -d_1\left(m_1 a_{G_{1y}} + m_2 a_{G_{2y}} + m_2 a_{G_{3y}} + m_2 a_{G_{5y}} - F_{61_y}\right) + J_n \alpha_{2_z} + J_n \alpha_{3_z} \\ -d_2\left(\dfrac{T_2 - J_s \alpha_{2_y}}{R}\right) + d_2\left(\dfrac{T_3 - J_s \alpha_{3_y}}{R}\right) \end{array}\right)}{-\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)} \end{array}\right)}{2d_2\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)}$$

$$-\frac{\left(d_8 \sin(\gamma_5) - d_2\right)\left(\begin{array}{l} \omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2x}} + 2d_1 m_2 g_z - d_7 m_2 a_{G_{3x}} + d_5 m_2 a_{G_{5x}} \\ -d_3 m_2 g_z - d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\ +d_7\left(m_1 a_{G_{1x}} + m_2 a_{G_{2x}} + m_2 a_{G_{3x}} + m_2 a_{G_{5x}} - F_{61_x}\right) \end{array}\right)}{2d_2\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)}$$

(3.314)

$$F_{05_z} = \frac{d_1\left(-m_1 g_z - 3m_2 g_z\right)}{\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)}$$

$$+\frac{\left(\begin{array}{l} \omega_{1_z}{}^2 J_{31} - d_7 m_2 a_{G_{2x}} + 2d_1 m_2 g_z - d_7 m_2 a_{G_{3x}} + d_5 m_2 a_{G_{5x}} \\ -d_3 m_2 g_z - d_8 \cos(\gamma_5) m_2 g_z - d_6 F_{61_x} + T_2 + T_3 \\ +d_7\left(m_1 a_{G_{1x}} + m_2 a_{G_{2x}} + m_2 a_{G_{3x}} + m_2 a_{G_{5x}} - F_{61_x}\right) \end{array}\right)}{\left(d_1 + d_3 + d_8 \cos(\gamma_5)\right)} \qquad (3.315)$$

From Equation (3.296), the expression for F_{02_y} is obtained as in the following equation:

$$F_{02_y} = \frac{\left(m_1 a_{G_{1_y}} + m_2 a_{G_{2_y}} + m_3 a_{G_{3_y}} + m_5 a_{G_{5_y}} - F_{61_y}\right) - F_{05_y}}{\left(1 + \frac{F_{03_z}}{F_{02_z}}\right)} \tag{3.316}$$

By substituting Equations (3.307) and (3.308) into (3.295), one can get the following equation:

$$m_1 a_{G_{1_x}} + m_2 a_{G_{2_x}} + m_2 a_{G_{3_x}} + m_2 a_{G_{5_x}} - F_{61_x} = \frac{T_2 + T_3}{R} - \frac{J_s\left(\ddot{\theta}_2 + \ddot{\theta}_3\right)}{R} \tag{3.317}$$

The system is one degree of freedom so;

$$\ddot{\theta}_2 = k_\theta \ddot{\theta}_3 \tag{3.318}$$

$\ddot{\theta}_2$ and $\ddot{\theta}_3$ are obtained from Equation (3.317) and (3.318).

3.3.2 Equation of Motion of the Motor

The motor itself also shows a dynamic behavior. While determining the equations of motion of the robot, the motor dynamic equations should also be derived too. The motor and load equivalent circuit is shown in Figure 3.5 where;

R_a : the armature winding resistance

L_a : the armature winding inductance

i_a : the armature winding current

T_m : torque generated by the motor

E_g : back electromotive force (back e.m.f.)

E_g is an internally generated voltage proportional to the motor velocity ω_m .

$$E_g = k_e \, \omega_m \tag{3.319}$$

where

k_e : motor voltage constant.

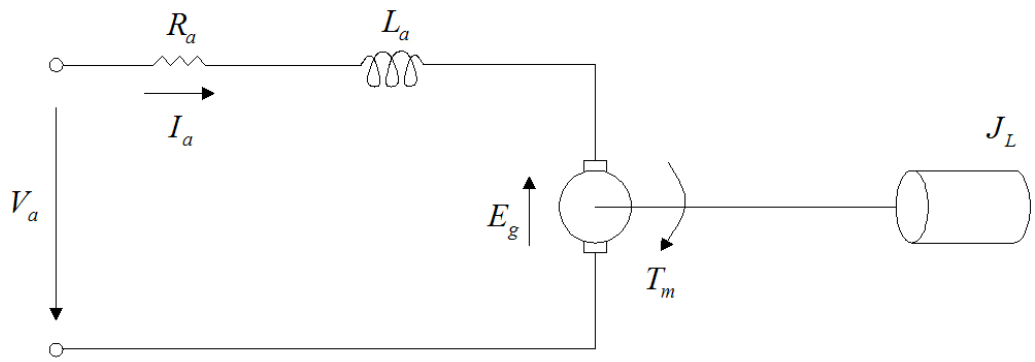


Figure 3.5 Permanent magnet DC motor and load

The electrical equation of the motor is

$$v_a = R_a i_a + k_e \omega_m + L_a \frac{di_a}{dt} \tag{3.320}$$

The transient response of the motor voltages changes in motor current is assumed to be negligible, i.e., $L_a \dfrac{di_a}{dt}$ is taken as zero [16].

The generated torque of the motor can be expressed as

$$T_m = k_m i_a \tag{3.321}$$

where

k_m : motor torque constant

The generated torque T_m is opposed by a variety of torques. Equating all these torques we get the mechanical equation of the motor.

$$J_m \dot{\omega}_m = T_m - b_m \omega_m - T_L \tag{3.322}$$

where

J_m : mass moment of inertia of the motor

b_m : rotational viscous damping constant for the motor

T_L : torque transmitted to the load

Coulomb friction is neglected in obtaining the dynamic model of motor.

Equation (3.323) can be rewritten as:

$$i_a = \frac{\left(v_a - k_e \omega_m\right)}{R_a} \tag{3.323}$$

By substituting Equation (3.321) and (3.323) into Equation (3.322) and after some manipulations, one can get the following equation:

$$T_L = \frac{\left(v_a - k_e \omega_m\right) k_m}{R_a} - J_m \dot{\omega}_m - b_m \omega_m \tag{3.324}$$

The load torque is the torque transmitted to wheels. By substituting this torque equation into the equation of motion of the robot, armature voltage v_a is becomes the input to the system.

CHAPTER 4

CONTROL STRUCTURE AND SIMULATION

4.1 Follow-the-Leader Approach

The follow-the-leader approach is used as a control strategy in the designed robot. This control strategy is used for moving multiple vehicles in formation. In follow-the-leader approach; one robot is designated as the leader and the remaining robots follow the leader's motion off-set by a distance. Leader-following essentially reduces to a tracking problem where stability of the tracking error is shown through standard control theoretic techniques [17].

The advantage of leader-following is that maneuvers can be specified in terms of the leader's motion. One disadvantage of the follow-the-leader approach is that the leader's motion is independent of the followers. Hence, if the following robots are unable to maintain a small tracking error, the leader does not slow down and formation is broken. In the designed robot, first module is the leader and second and third modules are following it. Between each module there is a connecting unit and this unit prevents the breaking off the formation. When the tracking error is increased, the connecting unit applies a force to each module and the leader slows down while the follower accelerates.

4.2 Control Structure

The block diagram of the robot system is shown in Figure 4.1. As seen from this figure the reference angular velocities of the left and right front wheels 'ω_{ref}' are inputs to the system. These two inputs are determined by the user. By defining these two inputs the path that the first module will follow is also defined. By giving the same value to these two inputs we define a longitudinal path for the first module. Besides, by giving these two inputs at a constant ratio a circular path is defined for the first module.

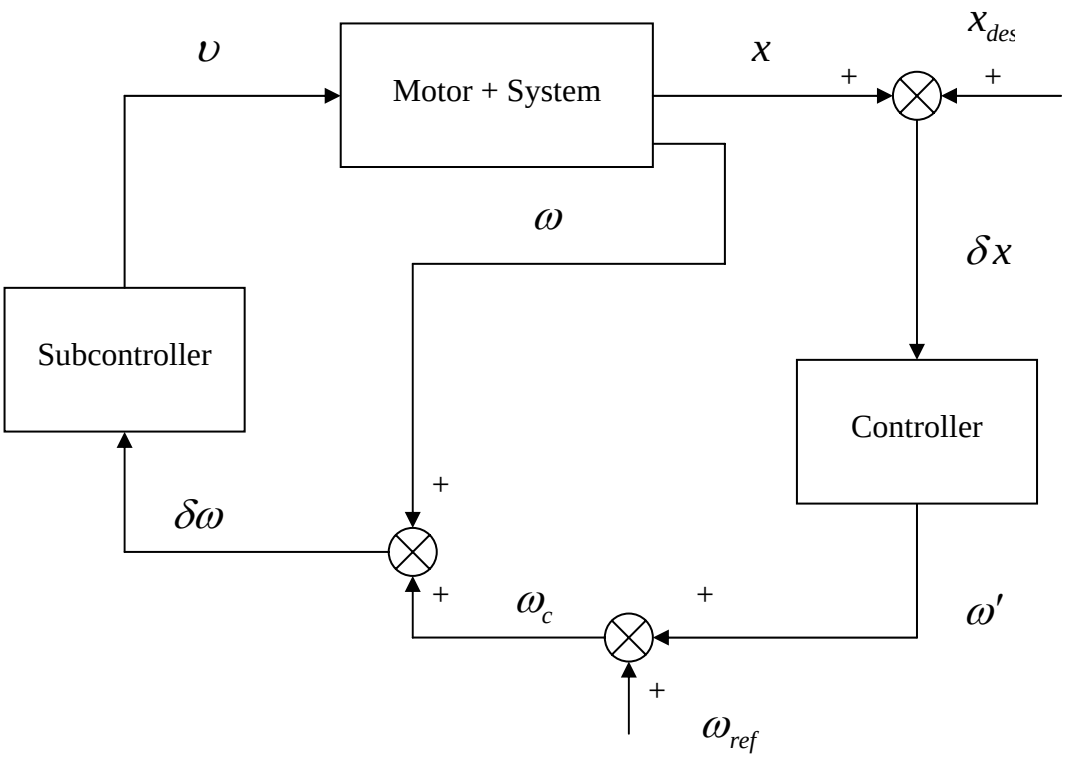


Figure 4.1 Block diagram of the system

By adding the reference angular velocities 'ω_{ref}' and the corrective angular velocities 'ω'', the control angular velocities 'ω_c' are obtained. Afterwards, the actual angular velocities of the left and right front wheels are compared with these

control angular velocities and the errors are minimized by the proportional controllers and the proper motor voltages 'υ' are produced. These motor voltages go to the right and left motor dynamics and the motor torques are produced. The angular accelerations of the left and right front wheels are calculated from the equation of motion of the robot system with these torques. Then the position of the center of mass of the module, heading angle, and the orientation of the connecting unit are calculated from the system equations.

Additionally the desired position of the center of mass of the module and the desired heading angle are calculated by using the reference angular velocities. Afterwards, these values are compared with the actual position of the center of mass of the module and actual heading angle and the errors are minimized by the PID controllers. As a consequence, the proper corrective angular velocities 'ω'' are produced.

The PID control technique provides a three controller with gains k_p, k_i and k_d where

k_p : proportional gain

k_i : integral gain

k_d : derivative gain

These gains act on the error that occurs between the desired response and the actual response to provide command information to the robot. The proportional gain is directly multiplied by the error while the derivative gain is multiplied by the derivative of the error and the integral gain is multiplied by the integral of the error. This process is carried out continuously, with the actual response moving towards the desired response.

Each of the three gains in the PID controller affects different aspects of the system's response. The proportional gain will shorten the system rise time, but will also increase the system overshoot or oscillation about the desired response. The derivative gain provides damping to the system, generally decreasing any overshoot or oscillation in the system. Finally, the integral gain is intended to primarily eliminate any steady-state error in the system [18].

There are many formal ways to select values for the k_p, k_i and k_d gains in PID controller. There is also the approach of trial and error. With a good system model and tool such as Matlab, trial and error is a reasonable approach so the values for the k_p, k_i and k_d gains are selected with this approach in this study.

4.3 Case Studies

This section covers the case studies of the robot traveling along a longitudinal and a circular path. These case studies are performed on the horizontal surface. In these studies only the first module's front wheel's reference angular velocities are given as input to the system. By giving the left and the right wheel's reference angular velocities at a constant ratio, a circular path is defined. On the other hand, by giving equal reference angular velocities, a longitudinal path is defined. An upper level controller will define the desired path that the robot will follow in the future works of this study. However, both a longitudinal and a circular path is defined in this study. The simulation is interrupted consciously when the first module completes the circular path defined. Therefore, in both cases, the simulation is defined to be 100 seconds.

4.3.1 Case 1: Robot Traveling on a Longitudinal Path

Figure 4.2 shows the reference angular velocities given. Equal values are defined to the left and the right wheels as reference angular velocities, in order to define a longitudinal path.

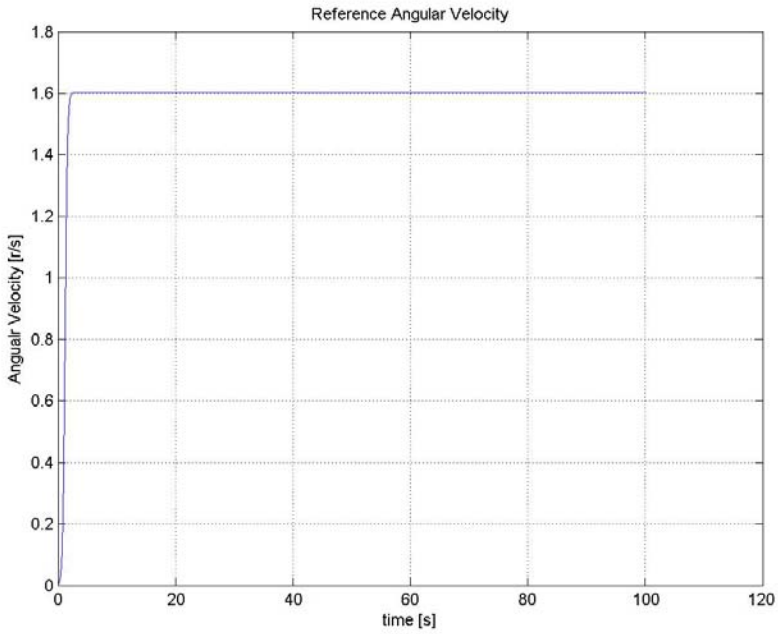


Figure 4.2 Reference Angular Velocities of Right and Left Wheel

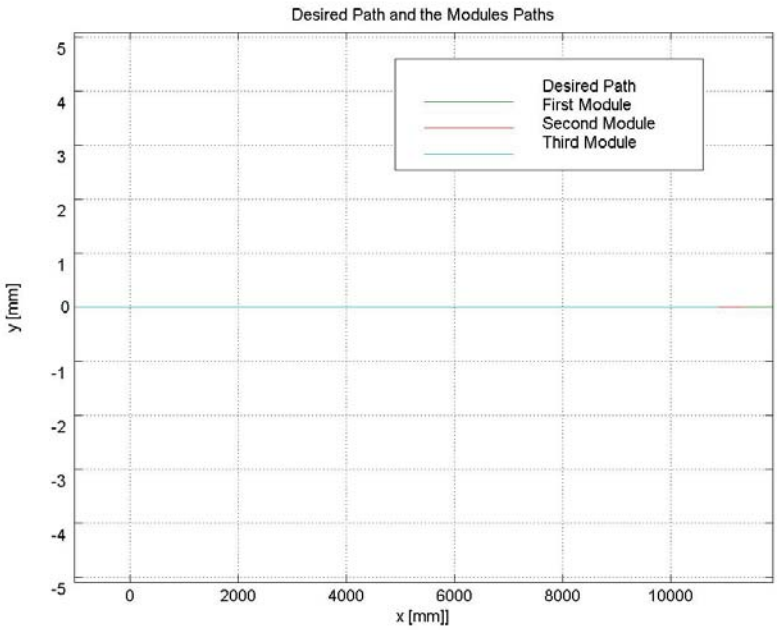


Figure 4.3 The Desired Longitudinal Path for the First Module and the Position of the Three Modules

The desired longitudinal path that the first module should follow, which is defined by the reference angular velocities and the movement of center of masses of the modules are shown in Figure 4.3. The distance between the modules is 510 mm at the beginning of their movement where they are lined in formation. It should also be noted that the connecting units between the bodies does not transmit force at the beginning.

The desired and the actual angular velocities for right and left wheels of the first module are shown in Figure 4.4. As it may be concluded from the figure the actual angular value reaches its maximum value within 3 seconds and the desired value within 15 seconds.

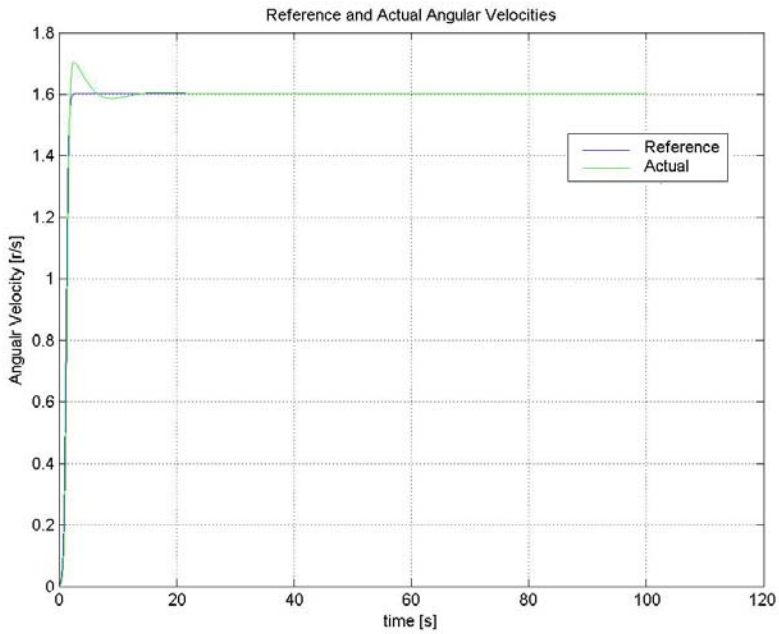


Figure 4.4 Reference and Actual Angular Velocities of the Right and Left Wheels of the First Module

Following the desired longitudinal path, the first module creates an error until it reaches the desired velocity, which is then used in the controller. The error between the desired and actual path and its change in time is shown in Figure 4.5.

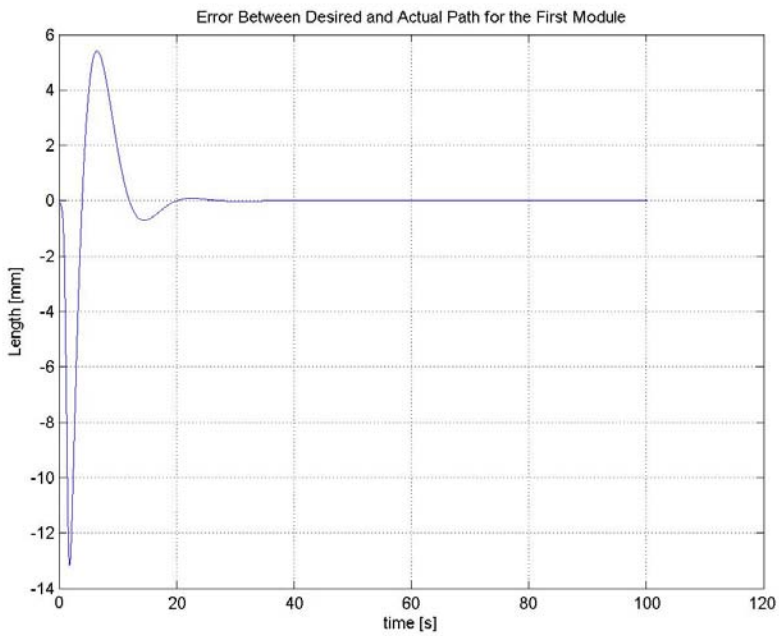


Figure 4.5 Error Between Desired and Actual Path for the First Module

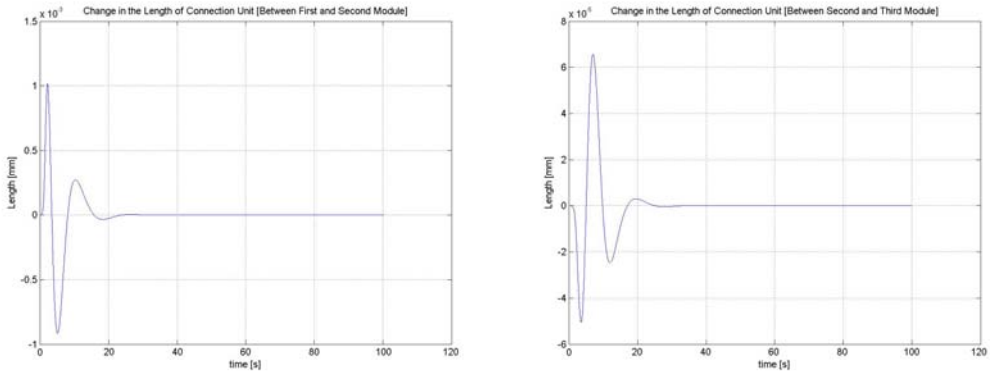


Figure 4.6 Changes in the Length of First and Second Connection Unit

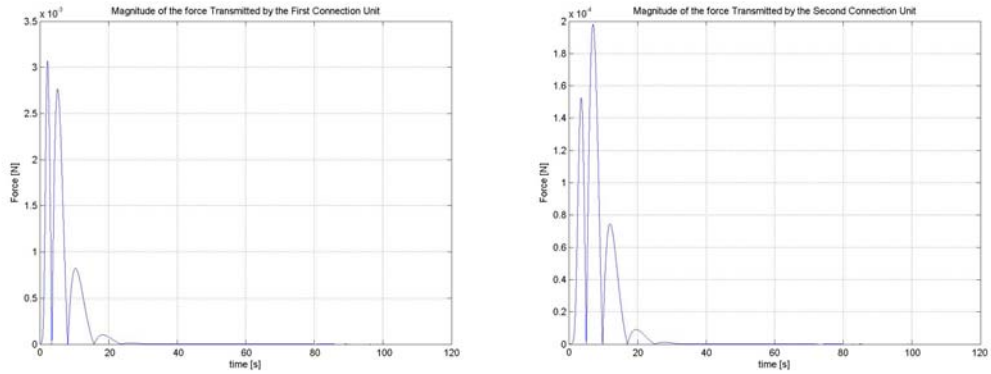


Figure 4.7 Force Transmitted by the First Connection Unit

The connecting unit is assumed as a spring and the changes in the length of the connection unit are shown in Figure 4.6. The desired path is straight line and a simple path therefore the changes in the length of the connection unit are so small and the force transmitted by the connection units is also small.

4.3.2 Case 2: Robot Traveling on a Circular Path

Figure 4.2 shows the reference angular velocities given. The ratio of left wheel's reference angular velocity over the right wheel's reference angular velocity is 1/1.1. As shown in the Figure 4.8 left and right wheels reference angular velocities increased from 0 r/s to their maximum values within 3 seconds and then remained constant.

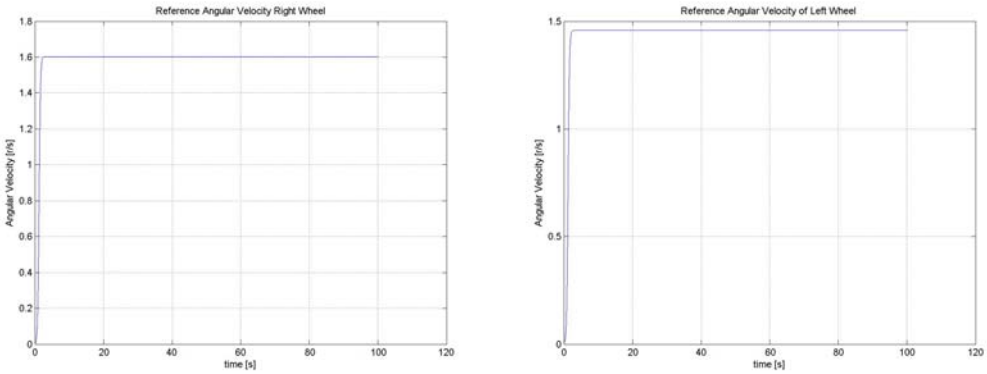


Figure 4.8 Reference Angular Velocities of Right and Left Wheel

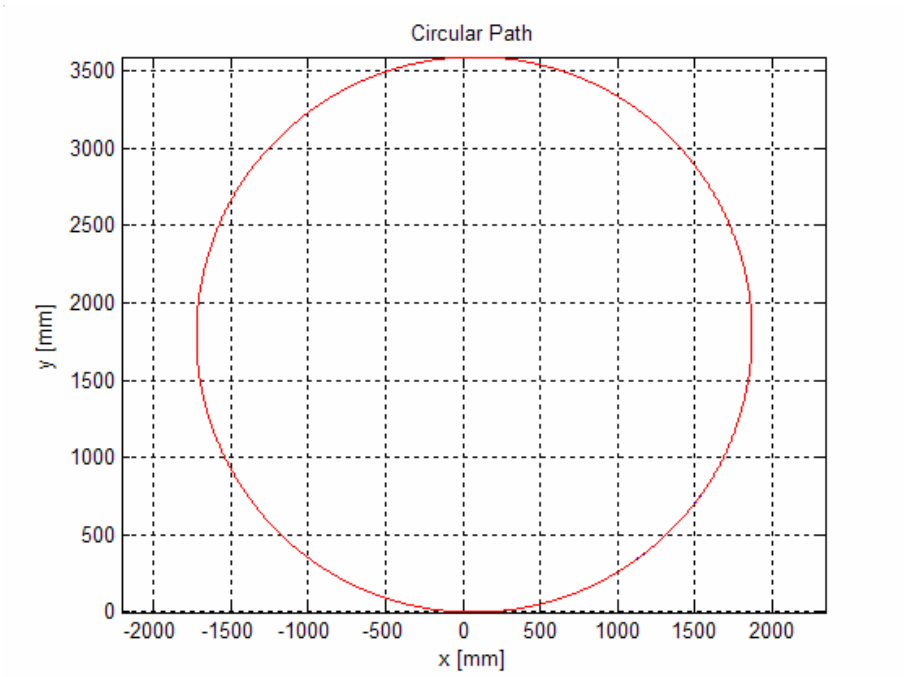


Figure 4.9 The Desired Circular Path for the First Module

Figure 4.9 shows the desired circular path that the first module should follow, which is defined by the reference angular velocities.

The path shown in Figure 4.10 demonstrates the movement of center of masses of the modules when the three modules move together. At the beginning, the three modules are in formation on a straight line where the distance between their centers of masses is 510 mm. Besides, at the beginning, there is no force transmission to the bodies through the connection units.

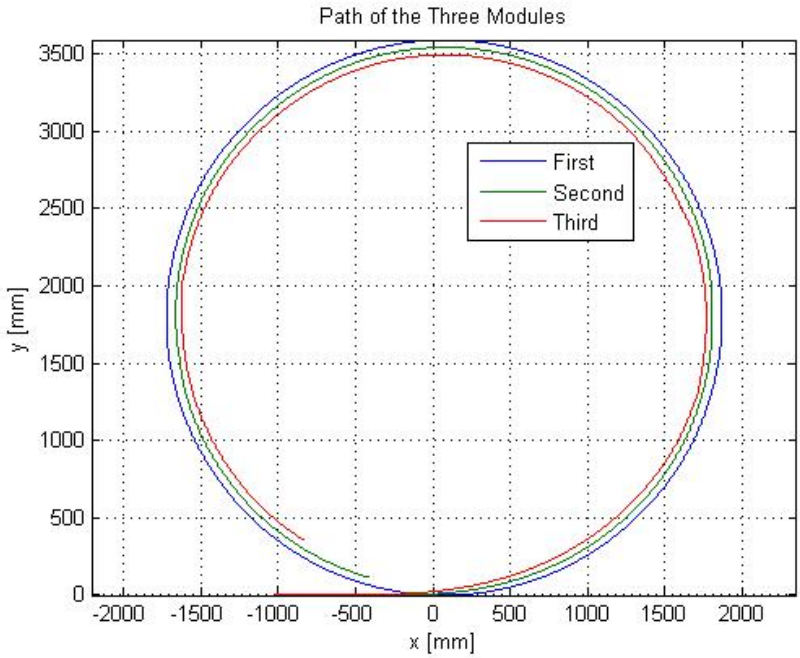


Figure 4.10 Positions of the Three Modules

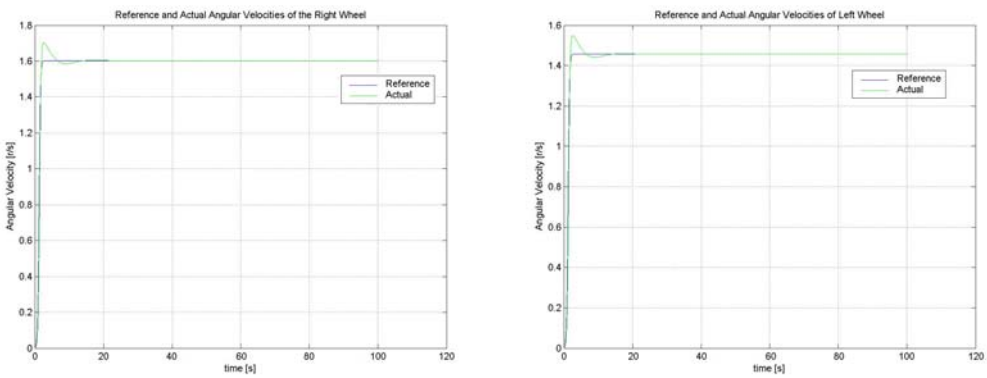


Figure 4.11 Reference and Actual Angular Velocities of the Right and Left Wheels of the First Module

In Figure 4.11 the desired and the actual angular velocities for right and left wheels of the first module are shown, where the actual angular value reaches the desired value within 15 seconds.

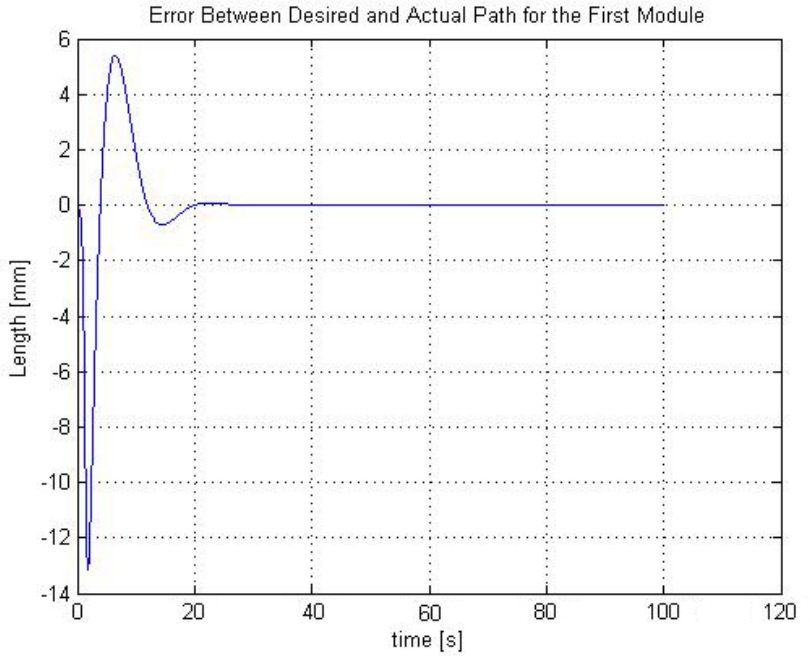


Figure 4.12 Error Between Desired and Actual Path for the First Module

While the first module follows the desired path, error occurs since it can not reach the desired velocity rapidly. Then, the error obtained is used in the controller. Figure 4.12 shows the amount of error between the desired and actual path.

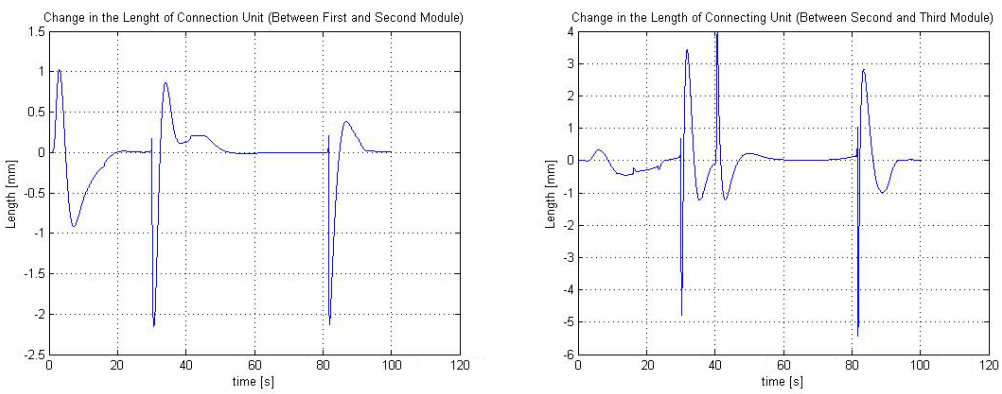


Figure 4.13 Change in the Length of First and Second Connection Unit

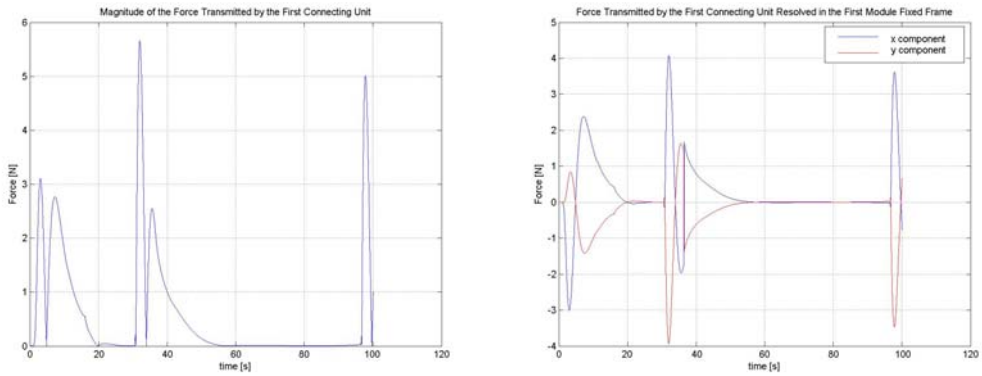


Figure 4.14 Force Transmitted by the First Connection Unit

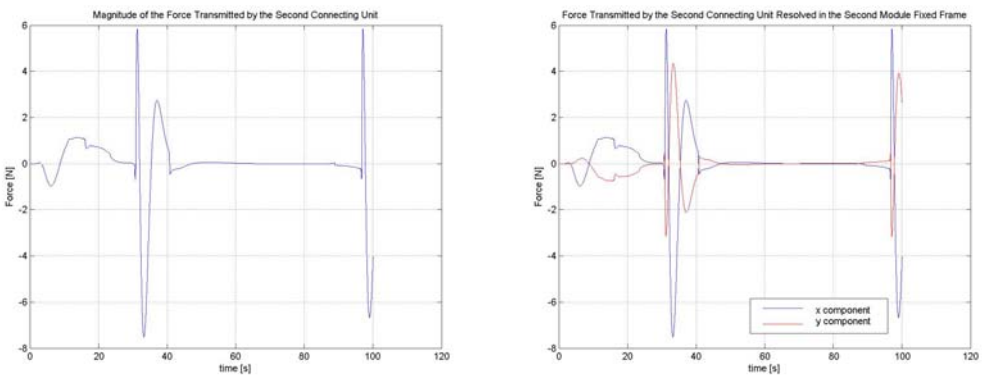


Figure 4.15 Force Transmitted by the Second Connection Unit

The connecting unit is assumed as a spring and the changes in the length of the connection unit are shown in Figure 4.13. The forces created by the connection unit according to these changes and the magnitude of the force it applied to the modules are also calculated. At the same time, the orientation of connecting unit in the reference frame is obtained from the positions of the modules. The direction of the force transmitted to the modules is found out, in relation to whether the connection unit is in compression or tension. These forces resolved in module fixed reference frames and the components of the forces in these frames are found. Figure 4.14 shows the magnitude of the force that transmitted by the first connecting unit where the Figure 4.15 shows the magnitude of the force that is transmitted by the second connecting unit.

4.4 Simulations

A model of the designed robot is performed in ADAMS. Simulations are carried out for two obstacles. Figure 4.16 represent the simulations screenshots for the two cases with different obstacles. These figures are the two different obstacles used in the simulations. In the first case, the distance between the two steps is shorter than the axle distance. However, in the second case, the distance between the two steps is longer than the axle distance.

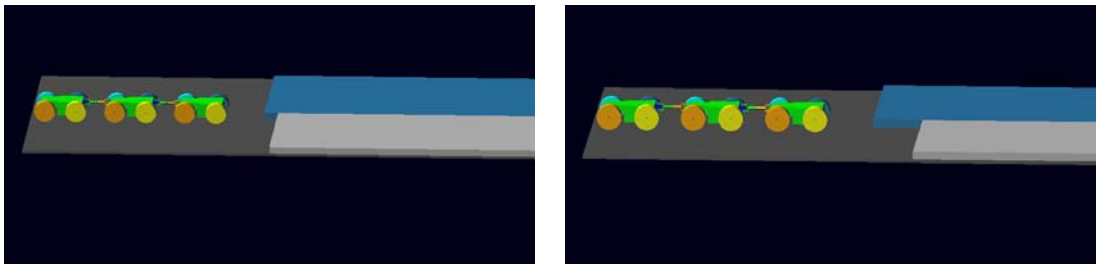


Figure 4.16 Two different obstacles used in simulations

In figures 4.17, 4.18, 4.19 and 4.20 the screenshots from the simulation are represented, where the robot is climbing the first obstacle. In this first case the distance between the two steps is shorter than the axle distance.

In figures 4.21, 4.22, 4.23 and 4.24 the screenshots from the simulation are represented, where the robot is climbing the second obstacle. In this second case the distance between the two steps is longer than the axle distance.

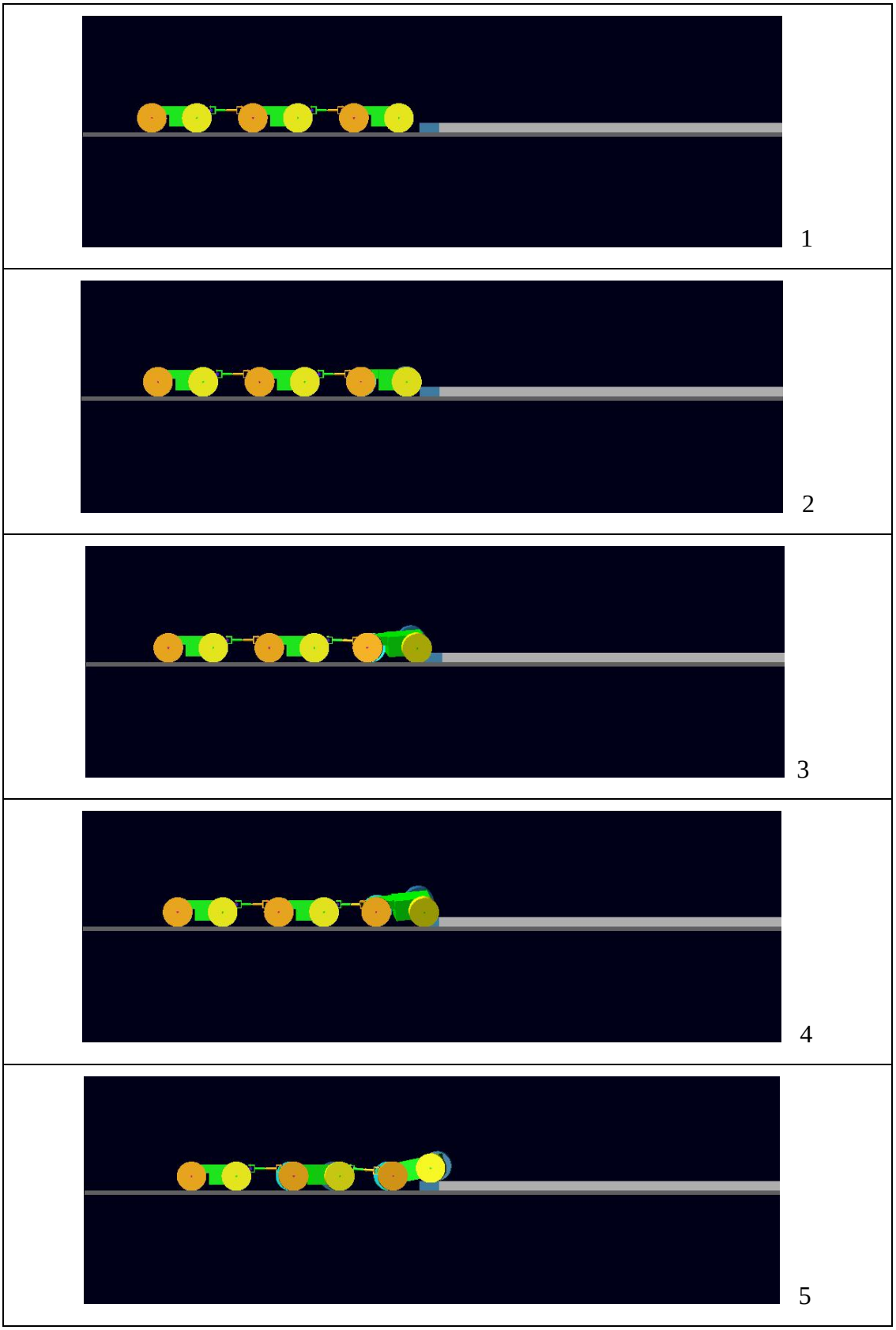


Figure 4.17 Simulation Screenshots of the First Case

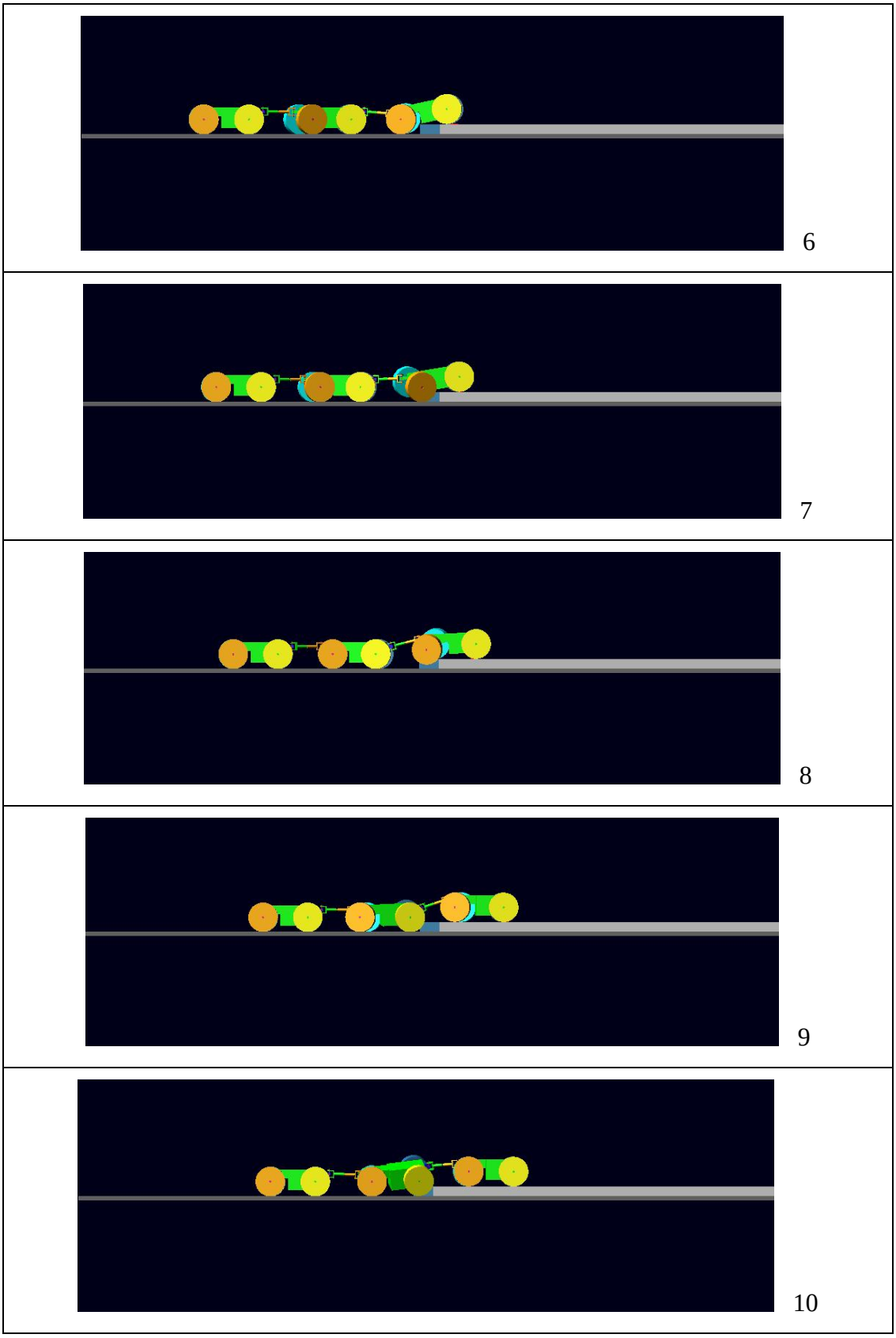
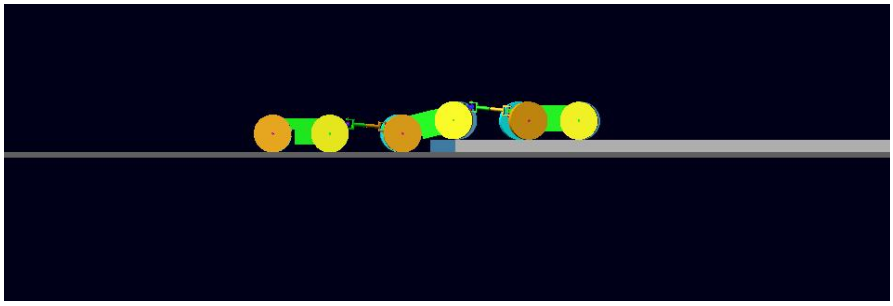


Figure 4.18 Simulation Screenshots of the First Case (Continued-1)



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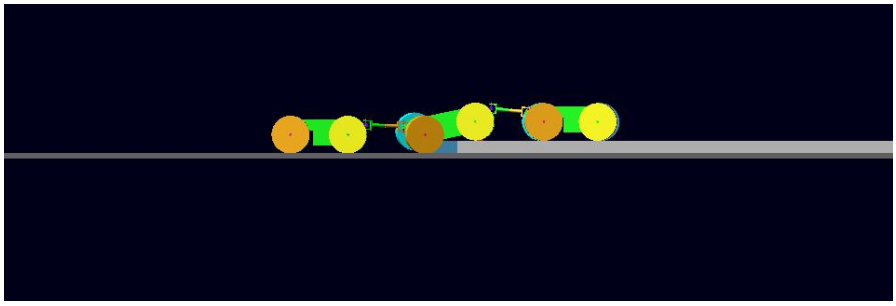

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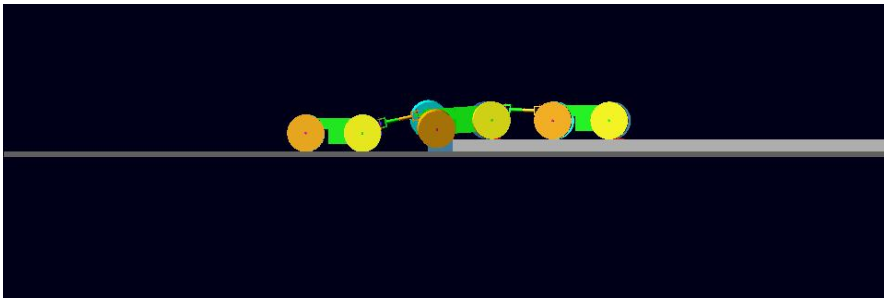

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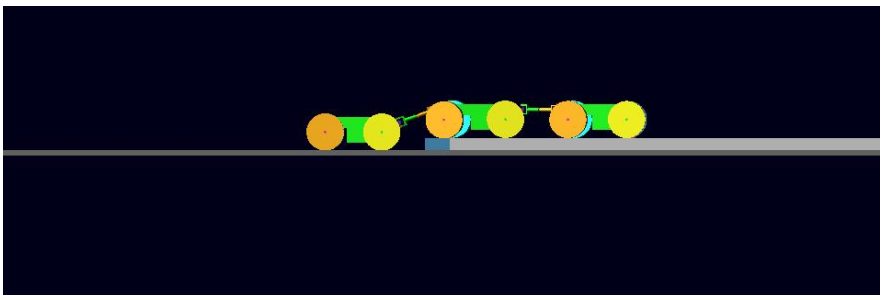

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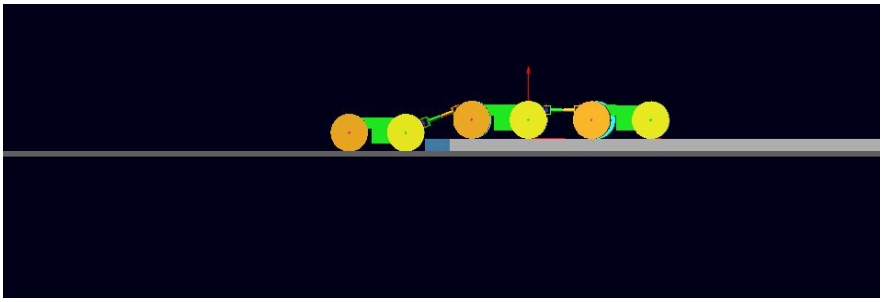

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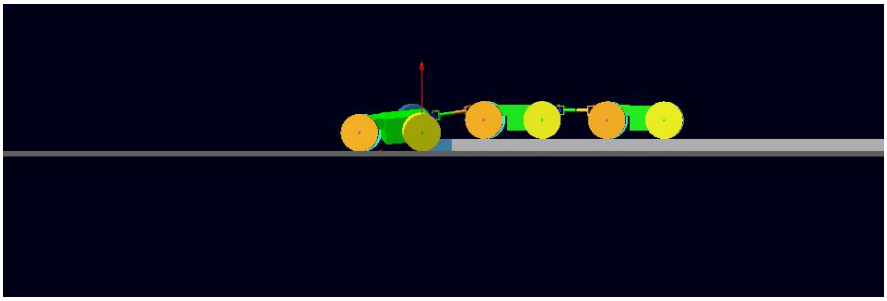
Figure 4.19 Simulation Screenshots of the First Case (Continued-2)



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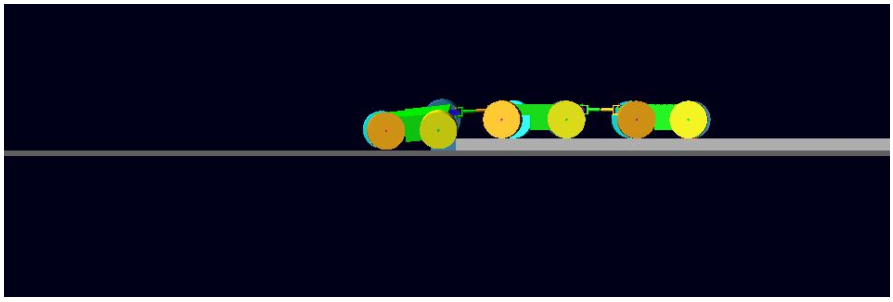

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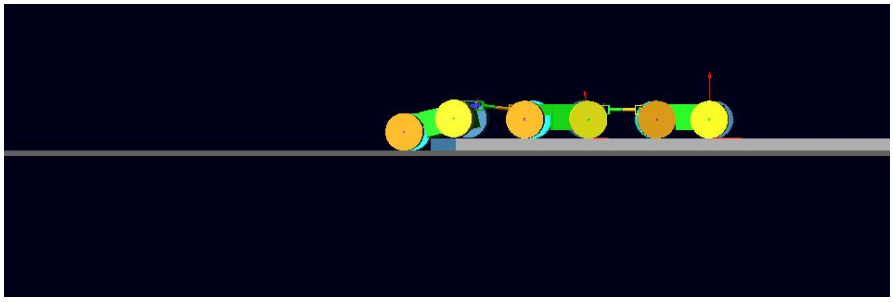

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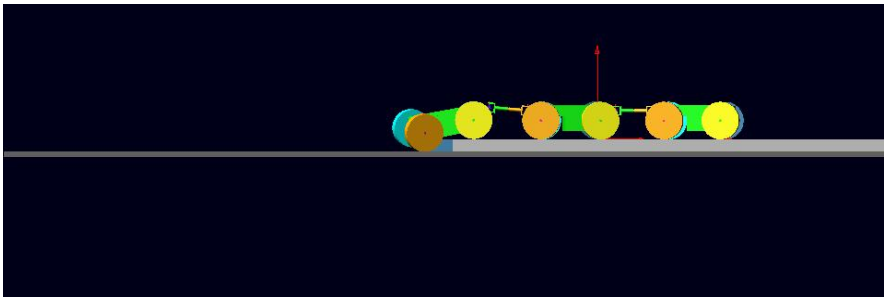

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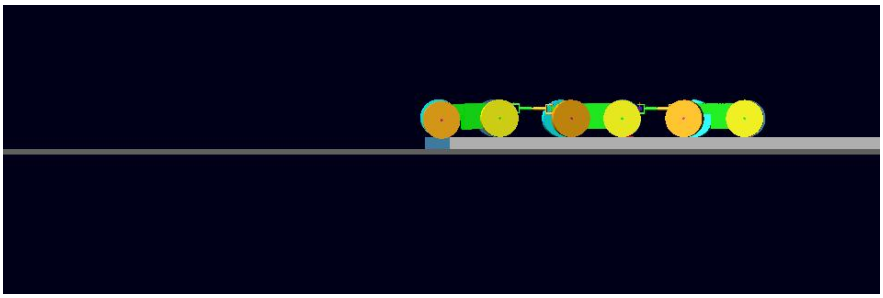

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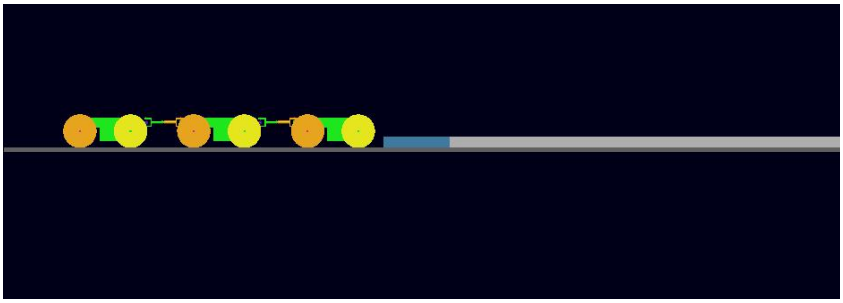
Figure 4.20 Simulation Screenshots of the First Case (Continued-3)



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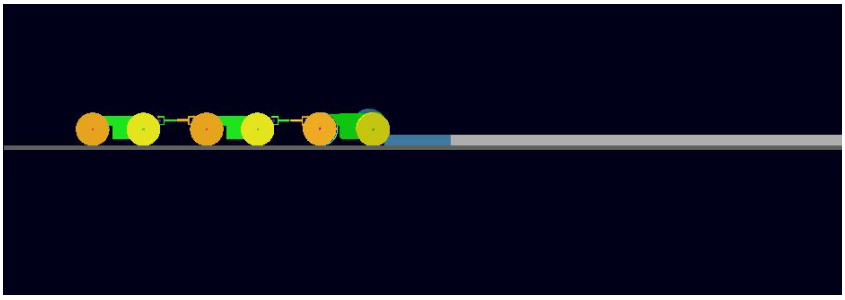

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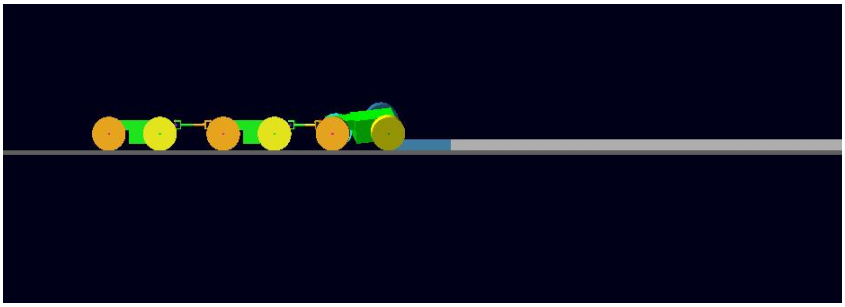

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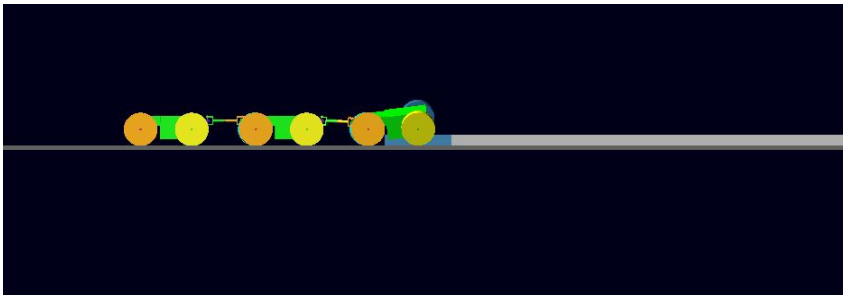

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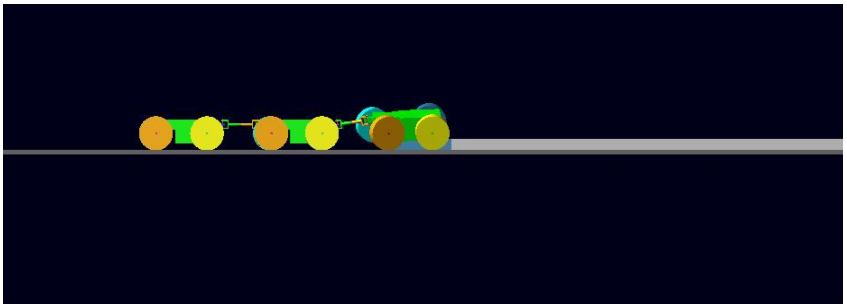

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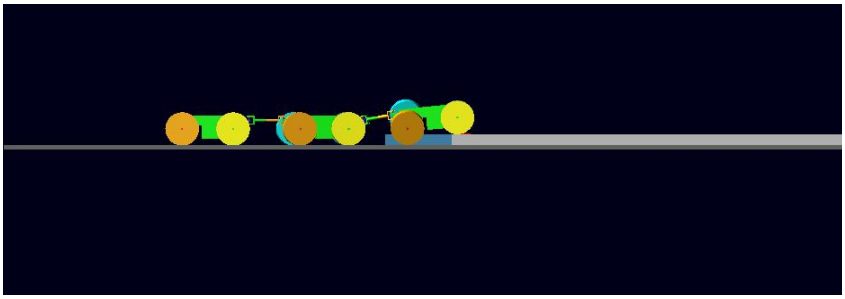
Figure 4.21 Simulation Screenshots of the Second Case



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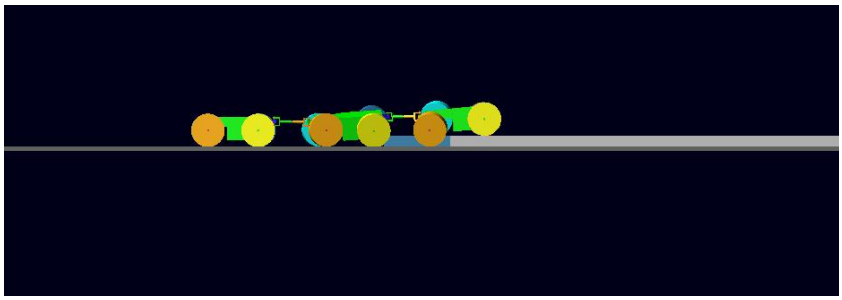

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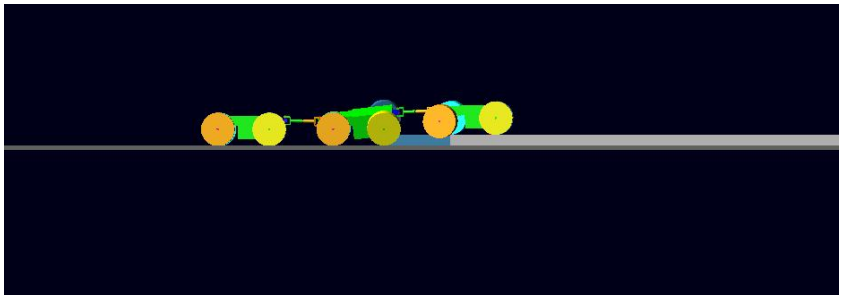

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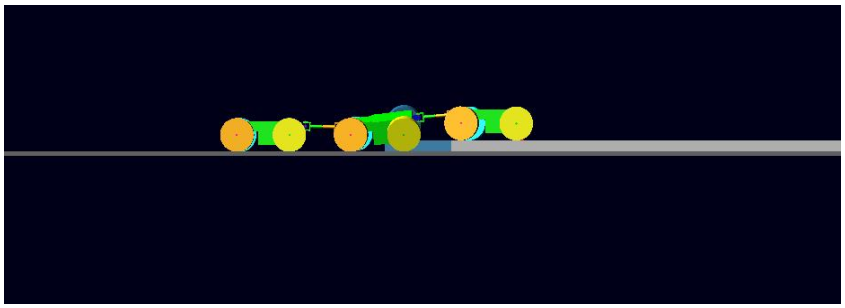

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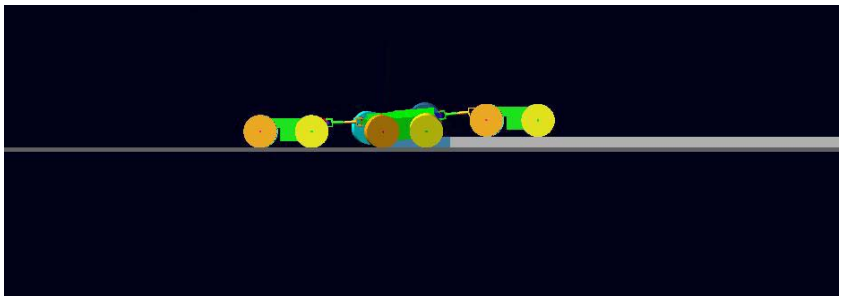

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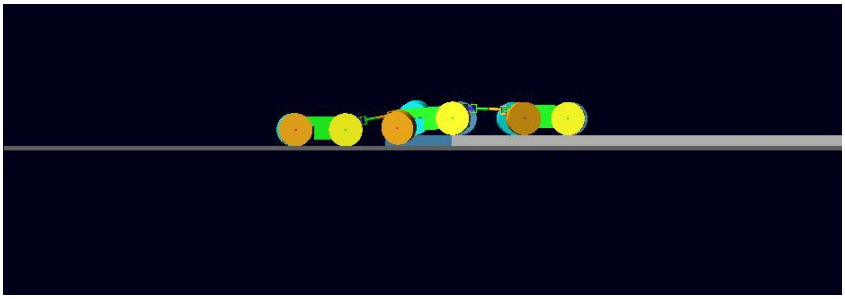
Figure 4.22 Simulation Screenshots of the Second Case (Continued-1)



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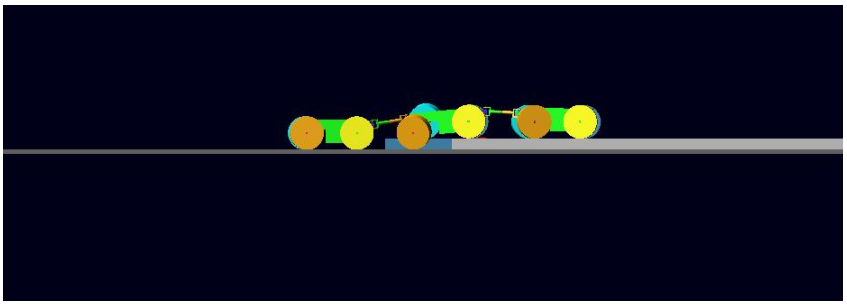

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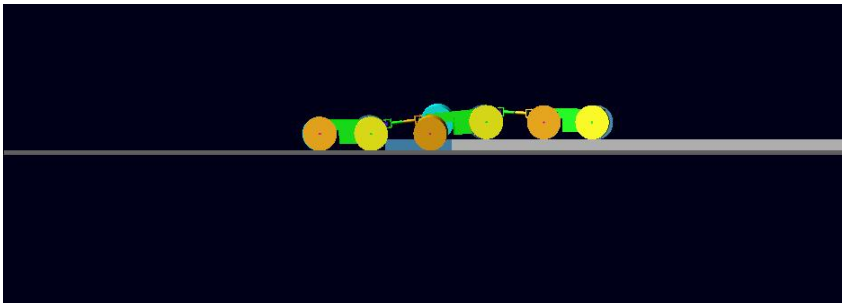

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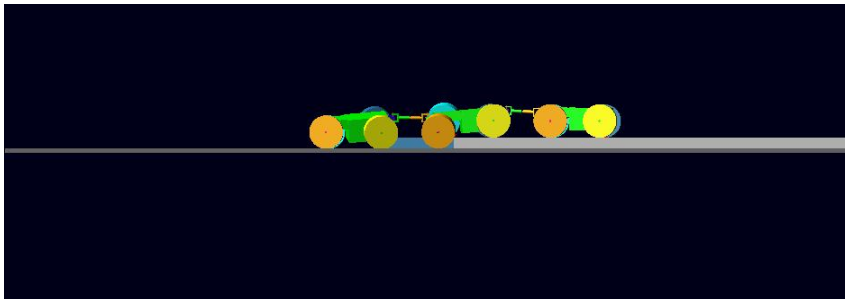

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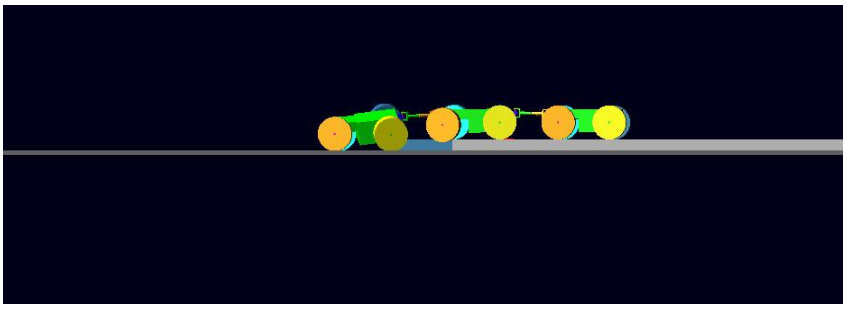

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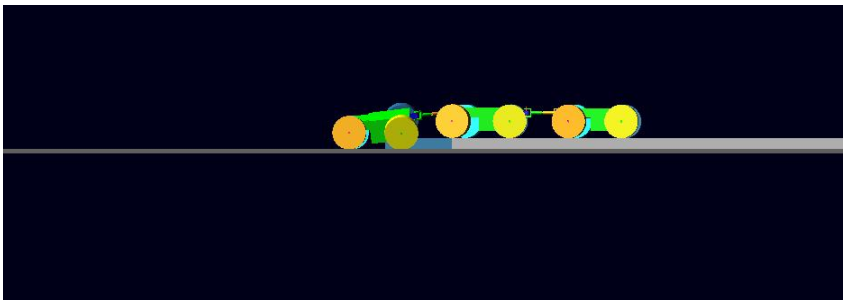
Figure 4.23 Simulation Screenshots of the Second Case (Continued-2)



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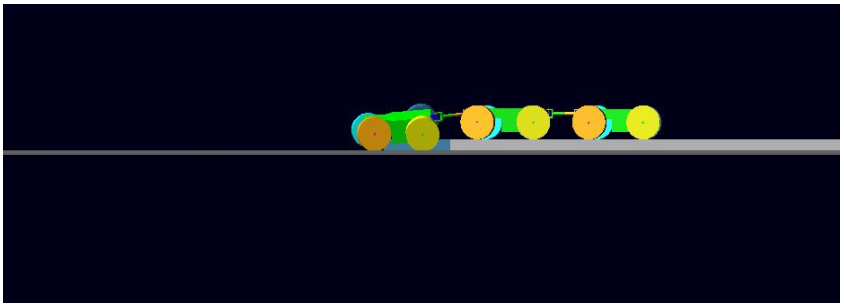

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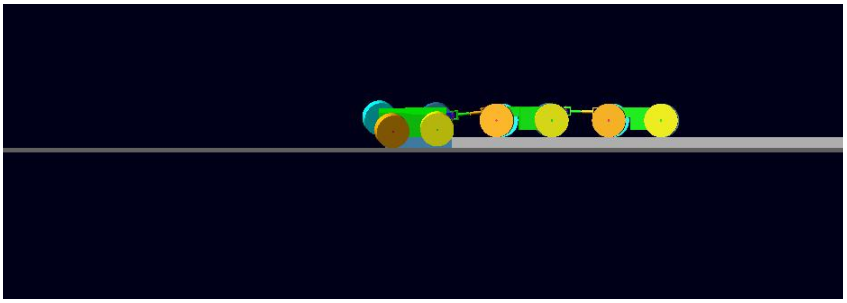

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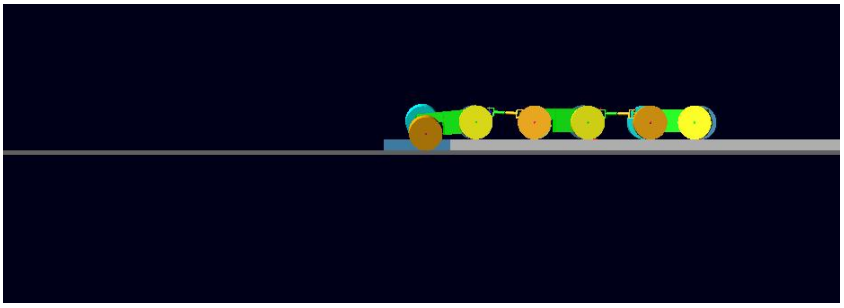

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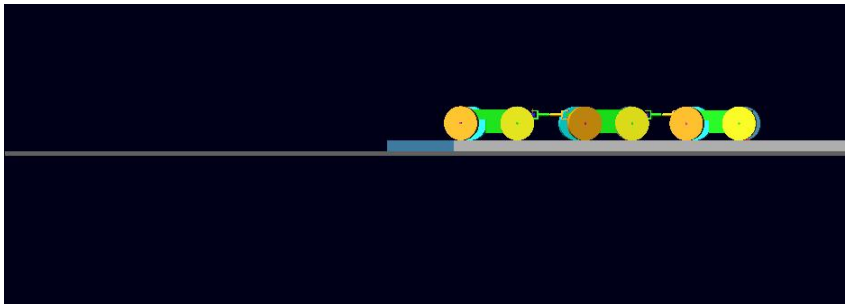

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Figure 4.24 Simulation Screenshots of the Second Case (Continued-3)

It can be observed from the both simulations that the wheels have always contact with the ground while climbing the obstacles. The rear axle connected to the body with two revolute joint enables this. It is important for cotrolling the robot that all the wheels, especially the front wheels always have contact with the ground since the control action is at the front wheels.

CHAPTER 5

SUMMARY AND CONCLUSION

Since Turkey has faced with many earthquake disasters througout the history and will probably face in the future also, new technologies on mobile robots should be used in search and rescue operations. Design and control of a robot that moves on rough terrain is considered within the scope of this thesis study. The main objective in the design of this robot is to use it in search and rescue operations, which decreases the risk for human rescue workers. As a consequence of using robots in rescue operations, the operation time reduces.

During the study, a robot consists of three modules which enables to pass through the small gaps was designed. Although they have considerably large volumes to carry equipments they are small in size as a result of their modular structure. Since the environment under the rubble is highly unstructured there is always the risk of sticking in ruins. Therefore a robot consisting of three modules is designed. If one module is stuck in the ruins or slips, the other two modules can rescue it. Owing to the modular system used, it is possible to add or remove modules from the system. This gives the user the opportunity of responding to the needs of different circumstances as well as easy maintenance.

The three modules of the robot are connected to each other through two connecting units, which are composed of two universal and one revolute joint. All the joints are passive joints in order to increase the adaptation of the robot to rough terrain.

After the design of the robot, detailed mathematical models of the system are obtained. However, during the studies on the mathematical model, it is assumed that there is no slip at the front wheels. Since the inputs are given to the modules from the front wheels, when there is slip at the front wheels it is impossible to control the robot. Moreover, the rear wheels of each module are assumed as being single caster wheel where the friction force of the rear caster wheel is neglected in its longitudinal direction.

In the study, the kinematic and the dynamic equations of the model were obtained. Newton – Euler formulation is used to derive the dynamic equations of the model. The reaction forces between the wheels and the ground are calculated by using these equations. The forces transmitted to the modules by the connecting units are also computed with these equations.

After the mathematical models, control structure of the system is constructed, where "Follow-the-Leader approach" is used as a control strategy. This approach enabled to move three modules in formation where the first module is designated as the leader and the second and third modules follow the first one's motion off-set by a distance. Therefore, inputs are only defined only for the first module instead of all modules.

Throughout the study, Matlab is used to write the control program. While writing the control program, PID controllers are used to minimize the differences between the actual and the desired positions of the center of mass of the modules as well as the differences between the heading angles. Through the use of PID control technique three gains are provided of which values are selected by using trial and error approach. Apart from that, two case studies are performed on the horizontal surface where the robot travels along a longitudinal and a circular path.

After the mathematical models and control structure of the system are constructed, simulations are carried out in ADAMS. The model of the robot constructed in ADAMS can perform as a base for the future researches and can be used to further enhance the design and control of the robot. This study can also be regarded as a basic reference for its manufacturing process. Another future research on the robot may be the upper level control, which can be applied on the existing mathematical models obtained in this thesis study. As a future work, a prototype of the designed robot can be produced and field-tests can be performed, which may aid to improve the proposed control algorithm and to re-evaluate the design criteria.

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