OPTIMUM DESIGN OF MULTISTEP SPUR GEARBOX

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

FATİH MEHMET ÖZTÜRK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

DECEMBER 2005

Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. Canan ÖZGEN
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Kemal İDER
Head of the Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Prof. Dr. M.A. Sahir ARIKAN
Supervisor

Examining Committee Members

Prof. Dr. Metin AKKÖK	(METU, ME)	
Prof. Dr. M.A. Sahir ARIKAN	(METU, ME)	
Prof. Dr. Levent PARNAS	(METU, ME)	
Asst. Prof. Dr. İlhan KONUKSEVEN	(METU, ME)	
Prof. Dr. Can ÇOĞUN	(Gazi Unv., ME)	

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Fatih Mehmet ÖZTÜRK

Signature:

ABSTRACT

OPTIMUM DESIGN OF MULTISTEP SPUR GEARBOX

Öztürk, Fatih Mehmet
M.Sc., Department of Mechanical Engineering
Supervisor: Prof. Dr. M.A. Sahir ARIKAN

December 2005, 143 pages

Optimum design of multistep gearbox, since many high-performance power transmission applications (e.g., automotive, space industry) require compact volume, has become an important interest area. This design application includes more complicated problems that are not taken into account while designing single stage gear drives. Design applications are generally made by trial and error methods depending on the experience and the intuition of the designer.

In this study, using Visual Basic 6.0, an interactive program is developed for designing multistep involute standard and nonstandard spur gearbox according to the American Gear Manufacturers Association (AGMA) Standards 218.01 and 2001-B88. All the equations for calculating the pitting resistance geometry factor I, and the bending strength geometry factor J, are valid for external spur gears that are generated by rack-type tools (rack cutters or hobs). The program is made for two-stage to six-stage gear drives, which are commonly used in the industry. Compactness of gear pairs and gearbox, and equality of factor of safety against bending failure is taken as the design objective. By considering the total required gear ratio, the number of reduction stages is input by the user. Gear ratios of every stage is distributed to the stages according to the total gear ratio that satisfies the required precision (from ±0.1 to ±0.00001 on overall gear ratio) depending on the user selected constraints (unequal gear ratio for every stage, noninteger gear ratio e.g.). Dimensional design is determined by considering bending stress, pitting stress, and involute interference constraints. These steps are carried out iteratively until a desirable solution is acquired. The necessary parameters for configuration design such as number of teeth, module, addendum modification coefficient, are selected from previously determined gear pairs that satisfies the constraints by user interaction considering the performance criterion from the developed program. The positions of gears and shafts are determined automatically in order to keep the volume of gearbox as minimum while satisfying the nonlinear spatial constraints (center distance constraint for proper meshing of gear pairs, face distance constraint for proper assembly of pinion and gear having same shaft, gear interference constraint for preventing interferences between gears, shaft interference constraint for preventing interferences between gears and shafts) by using DLL (Dynamic Link Library) technology of Lingo 8.0 optimization software together with Visual Basic 6.0. If shaft interference constraint is removed then cantilevered mounting of gear pairs would also be possible, otherwise the gears should be mounted between bearings. Visual output of assembly is made by using Autodesk Inventor 7.0, automatically by the program.

Keywords: Gears, Spur Gears, Multistep Gearbox, Dimensional Design, Configuration Design

ÖZ

ÇOK KADEMELİ DÜZ DİŞLİ KUTUSUNUN OPTİMUM TASARIMI

Öztürk, Fatih Mehmet
Yüksek Lisans, Makine Mühendisligi Bölümü
Tez Yöneticisi: Prof. Dr. M.A. Sahir Arıkan

Aralık 2005, 143 sayfa

Çok kademeli dişli kutusunun optimum tasarımı, yüksek performanslı güç iletim uygulamalarının(otomotiv, uzay endüstrisi vb.) kompakt hacim gereksiniminden dolayı, önemli bir ilgi alanı haline gelmiştir. Tek kademeli dişli çarkların tasarımında göz önüne alınmayan, daha karmaşık problemleri içermektedir. Tasarım uygulamaları genellikle tasarımcının deneyimlerine ve kabullenmelerine bağlı olarak deneme ve yanılma yöntemiyle yapılmaktadır.

Bu çalışmada, Visual Basic 6.0 programı kullanılarak, çok kademeli evolvent profilli standart ve tashihli düz dişlilerin Amerikan Dişli Üreticileri Birliği 218.01 ve 2001-B88 Standartlarına göre tasarımını yapan interaktif bir program geliştirilmiştir. Yüzey basınç geometri faktörü I ve eğilme mukavemeti faktörü J'nin hesaplanması için gerekli olan bütün denklemler, kremayer tipindeki takımlar (kremayer takım veya freze) tarafından oluşturulan dış düz dişli çarklar için geçerlidir. Program sanayide çoğunlukla kullanılan iki kademeden, altı kademeye kadar olan çok kademeli dişliler için yapılmıştır. Dişli çiftlerinin ve dişli kutusunun kompakt olması ve eğilme dayanım güvenlik katsayılarının eşit olması, tasarım amacı olarak alınmıştır. İstenilen toplam çevrim oranına göre, kademe sayısı kullanıcı tarafından belirlenmektedir. Her kademenin çevrim oranı, istenilen toplam çevrim oranını istenilen hassasiyeti (±0.1'den ±0.00001'e kadar) sağlayacağı şekilde, kullanıcının belirlediği sınırlamalara (her kademe için eşit olmayan çevrim oranı, tamsayı olmayan çevrim oranı vb.) bağlı olarak dağıtılmaktadır. Dişlilerin boyutları, eğilme mukavemeti, yüzey basıncı ve evolvent interferansı göz önüne alınarak belirlenmiştir. Bu basamaklar istenilen sonuç sağlanıncaya kadar iteratif bir şekilde yapılmıştır. Konfigürasyon tasarımı için gerekli olan diş sayıları, modül, modifikasyon katsayıları gibi parametreler, daha önce belirlenmiş gerekli şartları sağlayan dişli çiftlerinden, performans kriterlerinin değerlendirilmesiyle kullanıcı tarafından seçilmektedir. Dişlilerin ve şaftların konumları, lineer olmayan sınır şartlarını (dişli çiflerinin uygun eşleşmesi için merkezler arası uzaklık sınırlaması, aynı şafttaki pinyon ve dişlinin uygun montajı için yüzey uzaklık sınırlaması, dişliler arasındaki interferansı engellemek için dişli interferansı sınırlaması, dişlilerle şaftlar arasındaki interferansı engellemek için şaft interferansı sınırlaması) sağlayacak şekilde dişli kutusunun hacminin minimum olması için, Lingo 8.0 optimizasyon programının DLL (Dinamik Bağlantı Kütüphanesi) teknolojisinin Visual Basic 6.0 ile birlikte kullanılmasıyla, otomatik olarak yapılmaktadır. Eğer şaft interferansı sınırlaması kaldırılırsa, dişli çiftlerinin ankastre yataklanması da mümkün olabilmektedir, aksi takdirde dişlilerin rulmanlar arasında yataklanması gerekmektedir. Montaj resminin görüntüsel çıktısı Autodesk Inventor 7.0 kullanılarak program tarafından otomatik olarak yapılmaktadır.

Anahtar Kelimeler: Dişliler, Düz Dişliler, Çok Kademeli Dişli Kutusu, Boyutsal Tasarım, Konfigürasyon Tasarımı

To My Parents

ACKNOWLEDGMENTS

I would like to thank to my thesis supervisor Prof. Dr. M.A. Sahir Arıkan for his support, guidance and suggestions throughout my work.

I also would like to thank to Prof. Dr. Can Çoğun for his interest and valuable suggestions.

I would like to thank to Özlem Dağdelen, Ulvi Dağdelen and Demiralp Hatipoğlu for their suggestions and contributions.

Finally, I would like to thank to my parents for their continous support and encouragement throghout my M.Sc. study.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

NOMENCLATURE

a	Addendum
a^c	Rack cutter addendum
a_c	Chordal addendum
b	Dedendum
b^c	Rack cutter dedendum
BHN_p	Brinell hardness number of pinion
BHN_g	Brinell hardness number of gear
C	Standard Center distance
C_m	Modified center distance
C_{arack}	Rack cutter addendum factor
C_b	Rack cutter dedendum factor
C_f	Rack cutter tooth tip fillet radius factor
c	Bottom clearence
C_a	Application factor for pittting resistance
C_f	Surface condition factor
C_H	Hardness ratio factor
C_L	Life factor for pitting resistance
C_m	Load distriburion factor for pitting resistance
C_{ma}	Mesh alignment factor
C_{mc}	Lead correction factor
C_{mf}	Face load distribution factor
C_{mt}	Transverse load distribution factor
C_e	Mesh alignment correction factor
C_{pf}	Pinion proportion factor
C_{pm}	Pinion proportion modifier

C_p	Elastic coefficient
C_R	Reliability factor for pitting resistance
C_s	Size factor for pitting resistance
C_T	Temperature factor for pitting resistance
C_v	Dynamic factor for pitting resistance
D	Gear pitch diameter
D_{HPSTC}	Gear diameter at HPSTC
D_{LPSTC}	Gear diameter at LPSTC
D_b	Gear base diameter
D_o	Gear outside diameter
D_r	Gear root diameter
D_x	Diameter at which the load is acting to the gear tooth
d	Pinion pitch diameter
d_{HPSTC}	Pinion diameter at HPSTC
d_{LPSTC}	Pinion diameter at LPSTC
d_b	Pinion base diameter
d_o	Pinion outside diameter
d_r	Pinion root diameter
d_x	Diameter at which the load is acting to the pinion tooth
E_G	Modulus of elasticity of gear
E_P	Modulus of elasticity of pinion
e_t	Total lead mismatch between mating teeth
F	Face width
G	Tooth stiffness constant
H,L,M	Parameters used for calculation of K_f
h_f	Height of the Lewis parabola
HPSTC	Highest point of single-tooth contact
I	AGMA geometry factor for pitting resistance
J	AGMA geometry factor for bending strength
K_f	Stress correction factor
K_a	Application factor for bending strength
K_L	Life factor for bending strength

K_m	Load distriburion factor for bending strength
K_R	Reliability factor for bending strength
K_s	Size factor for bending strength
K_T	Temperature factor for bending strength
K_v	Dynamic factor for bending strength
LPSTC	Lowest point of single-tooth contact
m	Module
m_f	Slope of the tooth root fillet curve
m_p	Contact ratio, Slope of the Lewis parabola
m_g	Gear ratio
m_a	Aspect ratio
N	Number of load cycles
n_p	Pinion speed
n_G	Gear speed
N_p	Number of teeth for pinion
N_G	Number of teeth for gear
n_{bp}	Factor of safety for pinion against bending failure
n_{bG}	Factor of safety for gear against bending failure
n_{cp}	Factor of safety for pinion against surface failure
n_{cG}	Factor of safety for gear against surface failure
p_b	Base pitch
p^c	Rack cutter pitch
P	Transmitted power
Q_v	AGMA quality number
R_i	Radius of a point on involute gear tooth profile
r	Radius
r_L	Theoretical limit radius
r_f	Rack cutter tip fillet radius
r_i	Radius of a point on involute pinion tooth profile
r_o	Outside radius
r_{ps}	Standard pitch radius
r_{ti}	Radius at which involute tooth profile starts

s_f	Tooth thickness at critical section
S_c	Surface fatigue strength
S_t	Bending strength
S_{ac}	Allowable contact stress
S_{at}	Allowable bending stress
t	Circular tooth thickness at standard pitch radius
T	Transmitted torque
t_t	Circular tooth thickness at tooth tip
t^c	Rack cutter tooth thickness at pitch line
x	Addendum modification coefficient (rack shift) coefficient
x_f	x-coordinate on the trochoid fillet profile
x_i	x-coordinate on the involute fillet profile
x_t	x-coordinate on the point of tangency
V_t	Pitch line velocity
V_{tmax}	Pitch line velocity limit
W_t	Transmitted tangential load
Y	Tooth form factor
y_L	y-coordinate of the vertex of the Lewis parabola
y_f	y-coordinate on the trochoid fillet profile
y_i	y-coordinate on the involute fillet profile
y_t	y-coordinate on the point of tangency
Z	Length of line of action
Z_a	Length of pinion addendum (gear dedendum) portion of the line of action
Z_b	Length of pinion dedendum (gear addendum) portion of the line of action
Z_c	Distance between the pitch point and HPSTC of gear (LPSTC of pinion) measured along the line of action when the gear is not undercut
Z_e	Distance between the pitch point and HPSTC of pinion (LPSTC of gear) measured along the line of action when the pinion is not undercut

Δ	Half width of the hob-tooth land
ε	Distance between the pitch point and the contact point measured measured along the line of action
ε_x	x-component of ε
ε_y	x-component of ε
ρ	Radius of curvature
ρ_f	Minimum radius of curvature of the fillet curve
ϕ	Pressure angle
ϕ_L	Load angle
ϕ_t	Pressure angle at tooth tip
ϕ^c	Rack cutter pressure angle
β,γ,θ	Construction angles
s_b	Bending Stress
s_c	Contact Stress
μ_p	Poison's ratio for pinion
μ_G	Poison's ratio for gear

<u>Subscripts</u>

i	Number of stage
G	Gear
m	Modified
P	Pinion
s	Shaft

CHAPTER 1

INTRODUCTION

Gears are used to transmit torque and angular velocity in a wide variety of applications. The spur gear is designed to operate on parallel shafts and having teeth parallel to the shaft axis.

Gears are standardized with respect to the tooth shape and size. The American Gear Manufacturers Association (AGMA) publishes standards for their design. Most of the gears manufactured today have involute profiles in order to provide constant angular velocity ratio during meshing. The design process has become a challenge because of the complicated shape and geometry of gears.

The first formula for computing bending stress in gear teeth has been explained by Wilfred Lewis in 1892 and it still remains the basis for most gear design today. He derived an equation for determining the stress in a gear tooth by treating the tooth as a cantilever beam. This equation considers only the bending of the tooth and neglects the compression due to radial component of the transmitted load. At that time, stress concentration factors are not also included in the equation. The tooth root stress is calculated by using the transmitted tangential load, face width, circular pitch and Lewis Form Factor (Y). This formula for bending stress assumed no load sharing between teeth. It was calculated by assuming that the greatest force is exerted at the tip of the tooth. If the contact ratio is 1, the load will be applied at the tip of the tooth, creating the largest possible bending moment. In this case, one tooth is leaving contact just as the next is beginning contact, this is undesirable because slight errors in the tooth spacing will cause oscillations in the velocity, vibration, and noise which are not desired to achieve a quality gear set. At larger contact ratios than 1, examination of run-in teeth show that the tip loading is not the worst because another pair of teeth will be in contact when this condition occurs. These will occur toward the center of the mesh region where the load is applied at a lower position on the tooth, rather than its tip. This point is called the highest point of single-tooth contact or HPSTC.

Then American Gear Manufacturers Association (AGMA) introduced new equations to calculate both the bending strength and pitting resistance of gears including the radial component of the transmitted load and load sharing in stress calculations based on the studies of Earl Buckingham, Dolan and Broghamer. Dolan and Broghamer conducted a photoelastic investigation of stress concentration which is used for the calculation of K_f (stress correction factor by AGMA). Buckingham introduced an equation to calculate the pitting resistance of gears. Besides, the geometry factor J was established in which the stress concentration is also taken into consideration.

Based on the work of Lewis [1], AGMA [2] has established an improved model for the elastic behaviour of gear teeth, which uses the Lewis parabola to determine the critical section by the tangency point of the parabola and the tooth profile.

Mitchiner and Mabbie [3] developed a method valid for rack type, standard spur gears, and Tables for the AGMA J factor for usual sets of tooth proportions were given but neither hob offsets nor backlash were considered.

Based on the work of Errichello [4], AGMA [2] developed an augmented method considering spur and helical gears, rack-type or pinion-type cutters, tool protuberance, tool offsets, and backlash. This method is general and provides extremely accurate results, but involves an iterative procedure very tedious to implement into a computer program.

Since the calculation of geometry factor J require iterative procedures, Pedrero, Fuentes and Estrems [5], developed an approximate method for the determination of the bending strength factor for external spur and helical gear teeth by analytical methods. This method is also valid for any set of tooth proportions, rack shift coefficients, backlash etc. but as the name implies the method is approximate.

The design of a gear pair usually requires some iteration. Not enough information typically exists in the problem statement to directly solve for the unknowns. The design is made by assuming some initial values and a trial solution is done.

Usually, the overall gear ratio and either the power and speed, or the torque and speed of one shaft are defined in the problem. The parameters to be defined are the number of teeth for the pinion and gear, the module, the face width, the safety factors. Some design decisions regarding the mesh-accuracy required, the number of cycles, the pressure angle, the tooth form (standard or non-standard), the gear manufacturing method, operating temperature range, and desired reliability must be made.

The studies on the design of gear drives have focused on the dimensional design of single-stage gear drives so far. In recent years, the need to achieve things like high gear ratio, high power capacity, and high efficiency in a compact unit increases the studies focused on applications of multi-stage gear drives.

Most of the studies have used various optimization techniques to overcome this complicated problem since there are a lot of conflicting parameters exist.

Optimum design of multi-stage gear drive, introduces a number of challenges, particularly if the design involves optimization of both its kinematic structure and gear geometries. The resulting optimization problem involves design variables which can be both integer-valued (for pinion and gear teeth), discrete-valued (for module), and real valued (for face width). The feasibility of a design solution depends on the satisfaction of a number of equality and inequality constraints involving strength and gear meshing restrictions.

Chong et al. proposed an automation algorithm for the design of two- and three-stage cylindrical gear drives. Two types of dimensional design processes are presented. The first design process uses the total volume of gears to determine gear ratio and uses K factor, unit load, and aspect ratio to determine gear ratios, the second one makes use of Niemann's formula and center distance to calculate gear ratio and gear dimensions. Then configuration design is done to determine the position of gears to minimize the geometrical volume of a gearbox by using simulated annealing algorithm [6].

Chong and Bae automated the design process by integrating the dimensional design and configuration design process. Number of reduction stages is assigned by the designer, overall gear ratio is split to stages using random search method, the dimensional design is made by generate and test method, and configuration design is done to minimize the geometrical volume of a gearbox [7].

Bae et al. presented a design algorithm to automate the preliminary design of multi-stage gear drives including both the dimensional design of gears and the configuration design of the gear train [8].

Hsu constructed an optimization model to select the reduction ratios that minimize the angular backlash of a gear train, under constraints on total gear ratio and available space [9].

Deb demonstrated the use of a multi-objective evolutionary algorithm, which is capable of solving the multi-speed gearbox design problem involving mixed discrete and real-valued parameters and more than one objectives, and is capable of finding multiple nondominated solutions in a single simulation run [10].

Rosic presented an analytical and computer aided procedure for the multicriteria design optimization of gear train transmission by applying the optimization methods in the field of gear train transmission including Monte Carlo Method [11].

Thompson et al. made a generalized optimal design formulation with multiple objectives applicable to a gear train of arbitrary complexity. Optimal value of diametral pitch is determined at which tooth bending fatigue failure and surface fatigue failure are equally likely [12].

Chong et al. demonstrated the analysis of the relation between the geometrical volume and the vibration of a gear pair satisfying strength and geometric constraints. The addendum modification coefficients are determined by equalizing the ratios of bending strengths and geometry factors between pinion and gear for optimal design case [13].

Dolen et al. investigates the optimal design of a four-stage gear train using genetic algorithms with five different encoding schemes [14].

All the studies are based on optimization techniques and use some heuristic search methods (simulated annealing methods, genetic algorithms) by simplifying the complicated formulation of the problem. Except Kubo, the addendum modification is not considered since they are mainly focused on preliminary design.

In this study, an interactive program is developed for designing multistep involute standard and non-standard spur gearbox according to the AGMA Standards. The program is made for two-stage to six stage gear drives. Compactness of gear pairs and gearbox, and equality of factor of safety against bending failure is taken as the design objective. By considering the total required gear ratio, the number of reduction stages is determined by the user. Gear ratios of every stage is distributed to the stages according to the total gear ratio that satisfies the required precision depending on the user selected constraints. Depending on the designer all alternative gear pairs or the selected ones are used for dimensional design. Dimensional design is determined by considering bending stress, pitting stress, and involute interference constraints. The necessary parameters for configuration design such as number of teeth, module, addendum modification coefficient, are selected from previously determined gear pairs that satisfies the constraints by user interaction considering the minimum volume criteria and the performance criterion from the developed program. The positions of gears and shafts are determined in order to keep the volume of gearbox as minimum while satisfying the spatial constraints by using Lingo 8.0 optimization software. Visual output of assembly is made automatically by using Autodesk Inventor 7.0.

In this thesis:

- In Chapter 2, spur gear geometry is explained.
- In Chapter 3, AGMA rating equations and factors are explained.
- In Chapter 4, addendum modification on gears is explained.
- In Chapter 5, multi-stage spur gear drive design is given.
- In Chapter 6, sample application problem is solved by the developed program.
- In Chapter 7, results of this study are explained and future works on this subject is given.
- In Appendix, user guide for the developed program and configuration design constraints for six stage spur gear drive are given.

CHAPTER 2

SPUR GEAR GEOMETRY

2.1 Spur Gears

Spur gears are used to transmit torque and angular velocity in a wide variety of applications. They are designed to operate on parallel shafts and having teeth parallel to the shaft axis.

An infinite number of curves can be used for gear-teeth profiles, which will produce conjugate action. The most used tooth profile form is the involute shape. Small changes in center distance do not affect tooth action.

The most common pressure angles are 14.5°, 20°, and 25°. The lower pressure angle has the advantage of smoother and quieter tooth action because of the larger profile contact ratio. Also lower loads are imposed on the support bearings because of a decreased radial component. The problem of undercutting associated with small numbers of pinion teeth is more severe with the lower pressure angle. The larger pressure angles have the advantages of better load carrying capacity.

2.1.1 Spur Gear Nomenclature

Figure 2.1 shows a gear with the standard nomenclature and the terms explanations are as follows:

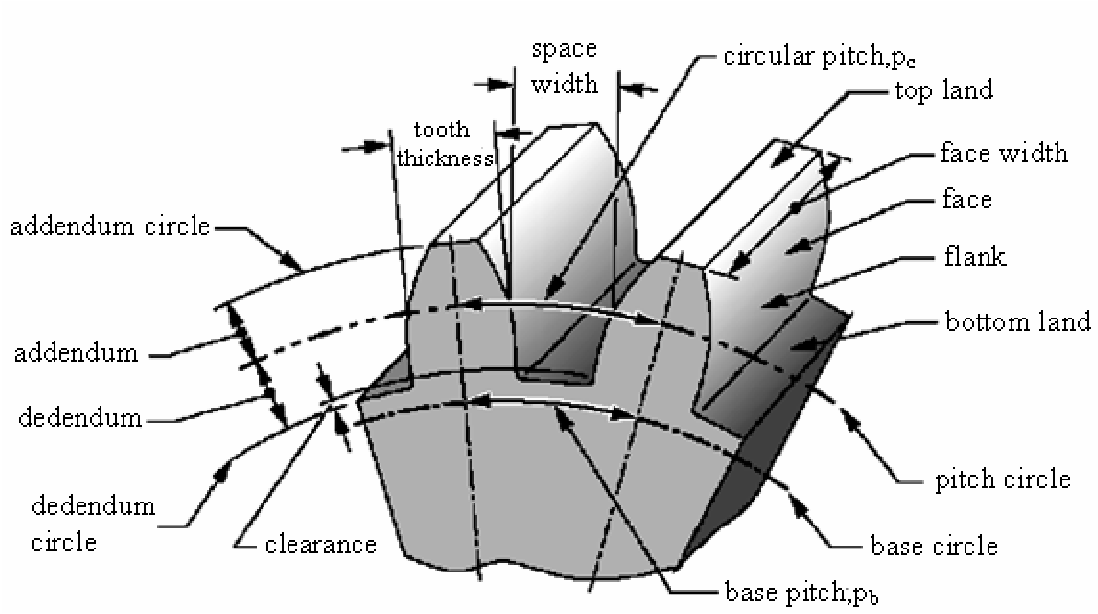


Figure 2.1 Gear Tooth Nomenclature [15]

Pinion and gear: The smaller of the two gears as the pinion and the other as the gear.

Pitch circle: It is an imaginary circle on a gear determined by dividing the number of teeth in the gear by the diametral pitch. Pitch circles of mating gears are tangent to each other.

Base circle: The circle from which the involute tooth profiles are generated.

Addendum circle: The circle passing from the tip of the tooth.

Dedendum circle: The imaginary circle tangent to the bottoms of the tooth spaces.

Base pitch: The distance along the line of action between successive involute tooth surfaces.

Circular pitch: The arc length along the pitch circle circumference measured from a point on one tooth to the same point on the next.

Face width: Length of the teeth in an axial plane.

Module: The ratio of the pitch diameter (mm) to the number of teeth.

Addendum: The radial distance between the outside diameter and the pitch circle.

Dedendum: The radial distance between the root diameter and the pitch circle.

Clearance: The amount between the gear dedendum and the addendum of the mating gear. The dedendum is slightly larger than the addendum to provide a small amount of clearance between the tip of one mating tooth and the bottom of the tooth space of the other.

Backlash: It is defined as the gap between mating teeth measured along the circumference of the pitch circle. Manufacturing tolerances precludes a zero backlash, as all teeth cannot be exactly the sane dimensions, and all must mesh without jamming.

Pressure angle: The angle between the tangent to the base circle and the line drawn normal to the line of centers at pitch point. It is shown in the Figure 2.3. The standard values are 14.5°, 20°, and 25°. The most commonly used is 20° and the 14.5° now being obsolete.

Involute: The involute of a circle is a curve that can be generated by unwrapping a taut string from a cylinder, as shown in the Figure 2.2. The string is always tangent to the base circle. The center of curvature of the involute is always normal to the string with the base circle. A tangent to the involute is always normal to the string, which is the instantaneous radius of curvature of the involute curve.

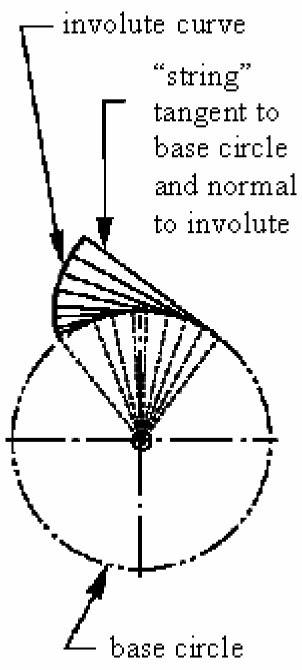


Figure 2.2 Development of the involute of a circle [15]

2.1.2 Standard Modules

The module is a basic parameter, by which a gear tooth is specified and is equal to the ratio d/N, where d is the pitch diameter in millimetre and N is the number of teeth.

Modules are given with respect to the ISO 54 standard [16]. The modules are divided into two groups as the preferred series and the second choice series. In the Table 2.1 the values in the brackets are valid for the second choice series. Mostly the preferred series are used, only for some special applications the second choice series are used. In this study user can select the preferred series or both the preferred and the second choice modules in order to increase the alternatives.

Table 2.1 Standard Modules

0.12	0.6	2	(5.5)	(22)
(0.14)	(0.65)	(2.25)	(5.75)	25
0.16	0.7	2.5	6	(27)
(0.18)	(0.75)	(2.75)	(6.5)	(28)
0.2	0.8	3	(7)	(30)
(0.22)	(0.85)	(3.25)	8	32
0.25	0.9	(3.5)	(9)	(36)
(0.28)	(0.95)	(3.75)	10	(39)
0.3	1	4	(11)	40
(0.35)	(1.125)	(4.25)	12	(42)
0.4	1.25	(4.5)	(14)	(45)
(0.45)	(1.375)	(4.75)	16	50
0.5	1.5	5	(18)	(55)
(0.55)	(1.75)	(5.25)	20	60

2.1.3 Mesh Geometry

Figure 2.3 shows a pair of involute pair of tooth forms in two positions, just beginning contact and about to leave contact.

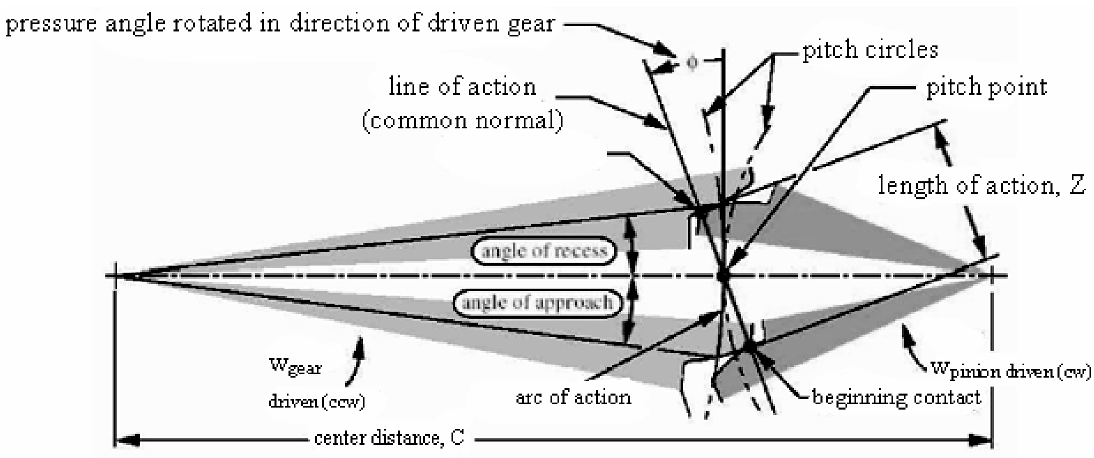


Figure 2.3 Mesh Geometry [15]

The points of beginning and leaving contact define the mesh of the pinion and gear. The distance along the line of action between these points within the mesh is called the length of action Z, defined by the intersections of the respective addendum circles with the line of action. The distance along the pitch circle within the mesh is the arc of action, and the angles subtended by these points and the lines of centers are the angle of approach and angle of recess. The arc of action on both pinion and gear pitch circles must be the same length for zero slip between the theoretical rolling cylinders. The length of action Z can be calculated from the gear and pinion geometry.

2.1.4 Undercutting

The involute tooth form is only defined outside of the base circle. In some cases, the dedendum will be large enough to extend below the base circle. If so, then the portion of tooth below the base circle will not be an involute and will interfere with the tip of the tooth on the mating gear, which is an involute. If the gear is cut with a standard gear shaper or a hob the cutting tool will also interfere with the portion of tooth below the base circle and will cut away the interfering material. This results in an undercut tooth.

Undercutting weakens the tooth by removing material at its root. The maximum moment and maximum shear from the tooth loaded as a cantilever beam both occur in this region. Severe undercutting will cause early tooth failure. Interference and its attendant undercutting can be prevented simply by avoiding gears with too few teeth.

2.1.5 Contact Ratio

It is the ratio of length of line of action divided by the base pitch and also contact ratio defines the average number of teeth in contact at any one time.

For contact ratios between 1 and 2, which are common for spur gears, there will still be times during the mesh when one pair of teeth will be taking the entire load.

However, these will occur toward the center of the mesh region where the load is applied at a lower position on the tooth, rather than its tip. This point is called the highest point of single tooth contact (HPSTC).

The minimum required contact ratio is 1.2 and the larger is better. Mostly the range is between 1.4 and 2.

2.1.6 Transmitted Tangential Load, W_t

In most gear applications, the torque is not constant. Therefore, the transmitted tangential load will vary. To obtain values of the operating tangential load, the designer should use the values of power and speed at which the driven device will perform. W_t represents the tooth load due to the driven apparatus.

If the rating is calculated on the basis of uniform load, the transmitted tangential load is calculated by the following equations:

$$W_t = \frac{1000P}{v_t} = \frac{2000T}{d} = \frac{1.91\times10^7 P}{n_p d} \tag{2.1}$$

$$v_t = \frac{p.n_p d}{60000} \tag{2.2}$$

where,

T : transmitted pinion torque, (Nm)
P : transmitted power, (kW)
V_t : pitch line velocity at operating pitch diameter, (m/s)
n_p : pinion speed, (rpm)

2.1.7 Criteria for Tooth Capacity

2.1.7.1 Relationship of Pitting Resistance and Bending Strength Ratings

There are two major differences between the pitting resistance and the bending strength ratings. Pitting is a function of the Hertzian contact (compressive) stresses between two cylinders and is proportional to the square root of the applied tooth load. Bending strength is measured in terms of the bending (tensile) stress in a cantilever plate and is directly proportional to this same load. The difference in nature of the stresses induced in the tooth surface areas and at the tooth root is reflected in a corresponding difference in allowable limits of contact and bending stress numbers for identical materials and load intensities.

The analysis of the load and stress modifying factors is similar in each case, so many of these factors have identical numerical values.

2.1.7.2 Pitting Resistance

The pitting of gear teeth is considered to be a fatigue phenomenon. In most industrial practice, corrective and non-progressive initial pitting is not deemed serious. Initial pitting is characterized by small pits which do not extend over the entire face width or profile height of the affected teeth. The definition of acceptable initial pitting varies widely with gear application. Initial pitting occurs in localized, overstressed areas. It tends to redistribute the load by progressively removing high contact spots. Generally, when the load has been reduced or redistributed, the pitting stops.

The aim of the pitting resistance formula is to determine a load rating at which destructive pitting of the teeth does not occur during their design life. The ratings for pitting resistance are based on the formulas developed by Hertz for contact pressure between two curved surfaces, modified for the effect of load sharing between adjacent teeth.

2.1.7.3 Bending Strength

The bending strength of gear teeth is a fatigue phenomenon related to the resistance to cracking at the tooth root fillet in external gears and at the critical section in internal gears.

The basic theory employed in this analysis assumes the gear tooth to be rigidly fixed at its base, thus the critical stress occurs in the fillet. If the rim supporting the gear tooth is thin relative to the size of the tooth and the gear pitch diameter, another critical stress may occur not at the fillet but in the root area. The strength ratings are based on plate theory modified to consider:

- The compressive stress at tooth roots caused by the radial component of tooth loading.
- Non-uniform moment distribution resulting from the inclined angle of the load lines on the teeth.
- Stress concentrations at the tooth root fillets.
- The load sharing between adjacent teeth in contact.

The intent of the AGMA strength rating formula is to determine the load which can be transmitted for the design life of the gear drive without causing cracking or failure.

Occasionally, wear, surface fatigue, or plastic flow may limit bending strength due to stress concentrations around large, sharp cornered pits or wear steps on the tooth surface.

2.1.8 Fundamental Rating Formulas

Two fundamental equations are used according to the AGMA standards as defined in [17], one for pitting resistance and the other for bending strength.

2.1.8.1 Pitting Resistance

The fundamental pitting resistance formula for gear teeth is:

$$s_C = C_P \sqrt{\frac{W_t C_a}{C_v} \frac{C_s}{dF} \frac{C_m C_f}{I}} \qquad (2.3)$$

where,

s_C : Contact stress, MPa

C_p : Elastic coefficient, $[MPa]^{0.5}$

W_t : Transmitted tangential load, N

C_a : Application factor for pitting resistance

C_s : Size factor for pitting resistance

C_m : Load distribution factor for pitting resistance

C_f : Surface condition factor for pitting resistance

C_v : Dynamic factor for pitting resistance

F : Net face width of narrowest member, mm

I : Geometry factor for pitting resistance

d : Operating pitch diameter of pinion, mm

2.1.8.2 Bending Stress

The fundamental bending stress formula for gear teeth is:

$$s_b = \frac{W_t K_a}{K_v} \frac{1.0}{Fm} \frac{K_s K_m}{J} \qquad (2.4)$$

where,

s_b : Bending stress, MPa

W_t : Transmitted tangential load, N

K_a : Application factor for bending strength

K_s : Size factor for bending strength

K_m : Load distribution factor for bending strength

K_v : Dynamic factor for bending strength

F : Net face width of narrowest member, mm

m : Metric module in plane of rotation, mm

J : Geometry factor for bending strength

2.1.8.3 Allowable Contact Stress Number

The relation of calculated contact stress number to allowable contact stress number is:

$$s_c \leq S_c \frac{C_L C_H}{C_T C_R} \tag{2.5}$$

where,

s_c : Contact stress, MPa

S_c : AGMA surface fatigue strength, MPa

C_L : Life factor for pitting resistance

C_H : Hardness ratio factor for pitting resistance

C_T : Temperature factor for pitting resistance

C_R : Reliability factor for pitting resistance

2.1.8.4 Allowable Bending Stress Number

The relation of calculated contact stress number to allowable contact stress number is:

$$s_b \leq S_t \frac{K_L}{K_T K_R} \tag{2.6}$$

where,

s_b : Bending stress, MPa

S_t : AGMA Bending Strength, MPa

K_L : Life factor for bending strength

K_T : Temperature factor for bending strength

K_R : Reliability factor for bending strength

The remaining equations and the factors will be explained in the following chapters including the developed program.

CHAPTER 3

AGMA RATING FACTORS

3.1 Dynamic Factors, C_v and K_v

Dynamic Factors, C_v and K_v, account for internally generated gear tooth loads which are included by non-conjugate meshing action of the gear teeth. Even if the input torque and speed are constant, significant vibration of the gear masses, therefore dynamic tooth forces can exist. These forces result from the relative displacements between the gears as they vibrate in response to an excitation known as "transmission error".

When transmission error is unavailable, it is reasonable to use pitch (spacing) and profile accuracy. Q_v is the transmission accuracy level number. Q_v can be same as the quality number for the lowest quality member in the mesh from AGMA 2000-A88 when manufacturing techniques ensure equivalent transmission accuracy, or when the pitch and profile accuracy are the same as AGMA 2000-A88 tolerances.

Due to the approximate nature of the empirical curves and the lack of measured tolerance values at the design stage of the job, the dynamic factor curve should be selected based on experience with the manufacturing methods and operating considerations of the design.

Where gearing is manufactured using process controls which provide tooth accuracy's which correspond to $Q_v > 12$ limits, or where the design and manufacturing techniques ensure a low transmission error which is equivalent to this accuracy, values of C_v and K_v between 0.90 and 0.98 may be used depending on the specifier's experience with similar applications and the degree of accuracy actually achieved. To use these values, the gearing must be maintained in accurate alignment and adequately lubricated so that its accuracy is maintained under the operating conditions. Spur gears should have properly designed profile modification.

The empirical curves of Figure 3.1 are generated by the following equations for values of Q_v, such that $6 < Q_v < 11$. The curves may be extrapolated beyond the end points shown in Figure 3.1 based on experience and careful consideration of the factors influencing dynamic load. For the purposes of computer calculations, Equation 3.4 defines the end points of the curves on Figure 3.1.

$$C_v = K_v = \left(\frac{A}{A + \sqrt{200 v_t}} \right)^B \qquad (3.1)$$

where,

$$A = 50 + 56(1.0 - B) \qquad (3.2)$$

$$B = \frac{(12 - Q_V)^{0.667}}{4} \qquad (3.3)$$

Q_v : transmission accuracy level number

$$v_{t\max} = \frac{[A + (Q_v - 3)]^2}{200} \qquad (3.4)$$

where,

V_{tmax}: pitch line velocity maximum at operating pitch diameter (end point of C_v and K_v curves on Figure 3.1), m/s

For Qv = 5

$$C_v = K_v = \frac{50}{50 + \sqrt{200 v_t}} \quad (3.5)$$

This equation should be used for gearing where the manufacturing process can be expected to give accuracies resulting in $Q_v < 6$. Equation 3.5 should not be used where V_t exceeds 13 m/s.

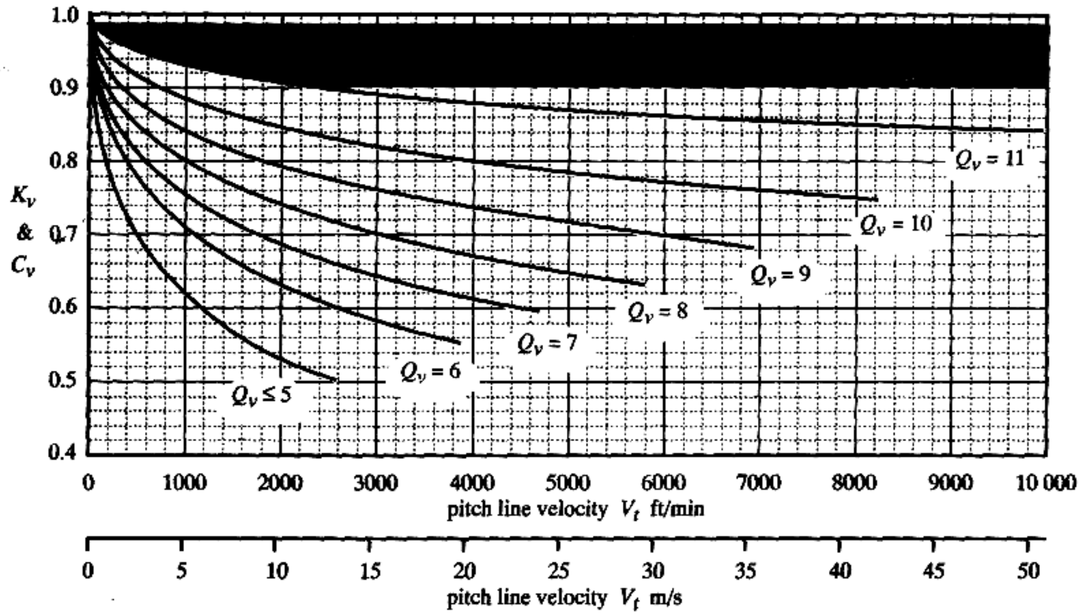


Figure 3.1 Dynamic Factors, C_v and K_v [18]

3.2 Application Factors, C_a and K_a

The loading model assumed that the transmitted load W_t, was uniform with time. The fluctuating moments on the teeth described in that section are due to the teeth coming into and out of mesh under a uniform or average load. If either the driving or driven machine has time-varying torques or forces, then these will increase the loading felt by the gear teeth above the average values.

In the absence of definitive information about the dynamic loads in the driving and driven machines, an application factor K_a can be applied to increase the tooth stress based on the shockiness of the machinery connected to the gear train. For example, if the gear train connects an electric motor to a centrifugal water pump (both of which are smooth-running devices) there is no need to increase the average loads and $K_a = 1$. But, if a single-cylinder, internal-combustion engine drives a rock crusher through a gear train, both the power source and the driven device deliver shock loads to the gear teeth and $K_a > 1$. Table 3.1 shows some AGMA-suggested values for K_a based on the assumed level of shock loading in driving and driven devices

Table 3.1 AGMA application factors C_a and K_a [15]

	DRIVEN MACHINE		
POWER SOURCE	Uniform	Moderate Shock	Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light shock (Multicylinder shock)	1.25	1.50	2 or higher
Medium shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

3.3 Elastic Coefficient, C_p

The elastic coefficient, C_p, is defined by the following equation:

$$C_p = \sqrt{\frac{10}{p\left[\left(\frac{1.0 - m_p^2}{E_p}\right) + \left(\frac{1.0 - m_G^2}{E_G}\right)\right]}} \qquad (3.6)$$

where,

C_P : Elastic coefficient, $[\text{Mpa}]^{0.5}$

μ_p and μ_G : Poisson's ratio for pinion and gear, respectively

E_p and E_G : Modulus of elasticity for pinion and gear, respectively, (Mpa)

The values of C_p for various combinations of gear and pinion materials are shown in Table 3.2.

Table 3.2 Elastic Coefficient, C_p [18]

Pinion Material	Modulus of Elasticity $E_p{}^2$ lb/in^2 (Mpa)	Gear Material and Combined Elastic Coefficient, C_p, [lb/in^2], ([Mpa] $^{0.5}$)					
		Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable Iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular Iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast Iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Al. Bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin Bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

3.4 Surface Condition Factor, C_f

The surface condition factor, C_f, used only in the pitting resistance formula, depends on:

- Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shot peening
- Residual stress
- Plasticity effects (work hardening)

Standard surface condition factors for gear teeth have not yet been established for cases where there is a detrimental surface finish effect.

In such cases, some surface finish factor greater than unity should be used.

3.5 Size Factors, C_s and K_s

The size factor reflects non uniformity of material properties. It depends primarily on:

- Tooth size
- Diameter of parts
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment of materials

Standard size factors for gear teeth have not yet been established for cases where there is a detrimental size effect. In such cases, some size factor greater than unity should be used.

The size factor may be taken as unity for most gears, provided a proper choice of steel is made for the size of the part and its heat treatment and hardening process.

3.6 Load Distribution Factors, C_m and K_m

Load distribution factor reflects the non-uniform distribution of the load along the lines of contact ($C_m = K_m$). It depends on:

- Gear tooth manufacturing accuracy
- Alignment of the axes of rotation of the pitch cylinders of the mating gear elements.

- Elastic deflections of gear elements, shafts, bearings, housing and foundations which support the gear elements.
- Bearing clearances
- Hertzian contact and bending deformations at the tooth surface.
- Thermal expansion and distortion due to operating temperature
- Centrifugal deflections due to operating speed
- Tooth crowning and end brief

Values for C_m and K_m can be obtained from:

$$C_m = K_m = C_{mf} C_{mt} \tag{3.7}$$

where,

C_{mf} : face load distribution factor
C_{mt} : transverse load distribution factor

Transverse load distribution factor accounts for the non-uniform distribution of load among the gear teeth, since standard procedures to evaluate the influence of C_{mt} have not been established, a value of unity may be used.

Face load distribution factor accounts for the non-uniform distribution of load across the gearing face width. There are two basic methods to evaluate it, empirical methods and analytical method.

Empirical method is used when:

- $F/d \leq 2$
- Gear elements are mounted between bearings (not overhung)
- Face width is up to 1016 millimetres
- Contact across full face width of narrowest member when loaded.

Following equation is used to calculate C_{mf} by this method:

$$C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) \tag{3.8}$$

where,

C_{mc} : lead correction factor

C_{pf} : pinion proportion factor

C_{ma} : mesh alignment factor

C_e : mesh alignment correction factor

Above mentioned factors are found as follows: C_{mc} modifies peak load intensity when crowning or lead modification is applied.

C_{mc} : 1.0 for gear unmodified leads.

C_{mc} : 0.8 for gear with leads properly modified crowning.

C_{mc} : 0.8 for gear with lead correction.

C_{pf} accounts for deflections due to load (Figure 3.7)

when,

$F \le 25.4$ mm

$$C_{pf} = \frac{F}{10.d} - 0.025 \tag{3.9}$$

$25.4 < F \le 431.8$ mm

$$C_{pf} = \frac{F}{10.d} - 0.00375 + 0.000492.F \tag{3.10}$$

$431.8 < F \le 1016$ mm

$$C_{pf} = \frac{F}{10.d} - 0.1109 + 0.000815F - 0.000000353F^2 \tag{3.11}$$

For values $\frac{F}{10d}$ less than 0.05, 0.05 is used for this value in the above equations.

The pinion proportion modifier, C_{pm}, alters C_{pf}, based on the location of the pinion relative to its bearing centerline.

C_{pm} : 1.0 for straddle mounted pinions with $(S_1/S) < 0.175$

C_{pm} : 1.1 for straddle mounted pinions with $(S_1/S) \geq 0.175$

where,

S_1 : the offset of the pinion, i.e., the distance from the bearing span centerline to the pinion mid face, mm as shown in the Figure 3.2.

S : the bearing span, i.e., the distance between the bearing centerlines, mm

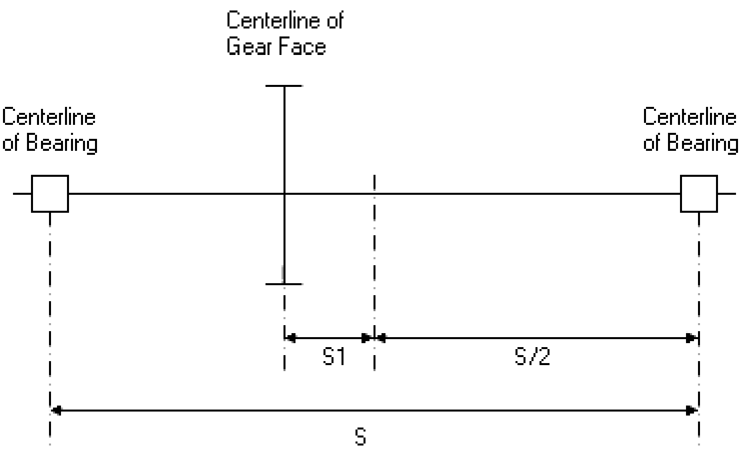


Figure 3.2 Evaluation of S and S_1

The mesh alignment factor accounts for the misalignment of the axes of rotation of the pitch cylinders of the mating gear elements from all causes other then elastic deformations. The mesh alignment factor can be obtained from Figure 3.3.

The four curves of Figure 3.3 provide representative values for C_{ma} based on the accuracy of gearing and misalignment effects which can be expected for the four classes gearing shown.

The values for the four curves of Figure 3.3 are defined as follows:

$$C_{ma} = A + B(F) + C(F)^2 \qquad (3.12)$$

where, empirical constants A, B, and C are given in Table 3.3

Table 3.3 Empirical Constants A, B, C

	A	B	C
Curve 1 Open Gearing	2.47×10^{-1}	0.657×10^{-3}	-1.186×10^{-7}
Curve 2 Commercial Enclosed Gear Units	1.27×10^{-1}	0.622×10^{-3}	-1.69×10^{-7}
Curve 3 Precision Enclosed Gear Units	0.675×10^{-1}	0.504×10^{-3}	-1.44×10^{-7}
Curve 4 Extra Precision Enclosed Gear Units	0.380×10^{-1}	0.402×10^{-3}	-1.27×10^{-7}

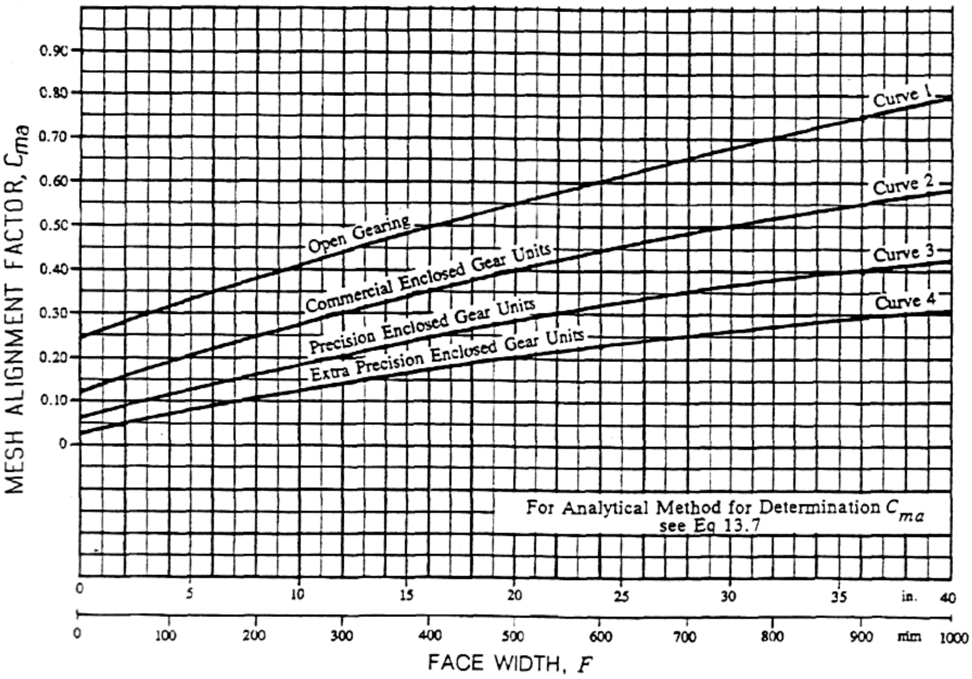


Figure 3.3 Mesh Alignment Factor, C_{ma} [18]

During the program the gearing type is asked from the user that is to be selected from the list given below and then C_{ma} is calculated:

- Open gearing
- Commercial enclosed gear units
- Precision enclosed gear units
- Extra precision enclosed gear units

C_e is used to modify the mesh alignment when the manufacturing or assembly techniques improve the effective mesh alignment.

C_e = 0.8 when gearing is adjusted at the assembly or when the compatibility of the gearing is improved by lapping.

C_e = 1.0 for all other conditions.

Analytical method is used when:

- F/d > 2
- Applications with overhung gear element
- Application with long shafts subjects to large deflections
- Applications where contact does not extend across full face or narrowest member when loaded.

Two conditions are given to the user input:

- Contact across the entire face width under normal operating load (the most commonly encountered condition), the face load distribution factor can be evaluated with the following equation:

$$C_{mf} = 1 + \frac{G.e_t.F}{2W_t} \tag{3.13}$$

- Total tooth contact length under normal operating load is less than the face width, the face load distribution factor can be evaluated with the following equation:

$$C_{mf} = \sqrt{\frac{2.G.e_t.F}{W_t}} \tag{3.14}$$

where,

G : tooth stiffness constant, MPa

The average mesh stiffness of a single pair of teeth in the normal direction. The usual range of this value that is compatible with this analysis, for steel gears is 1.0 to $1.4\text{x}10^4$ MPa. As advised by AGMA the most conservative value, the highest is taken for the developed program.

e_t : total lead mismatch between mating teeth, mm
p_b : base pitch

The amount of lead mismatch cannot be known at the design stage, then an average value is calculated as follows:

$$e_t = \frac{W_t}{1000.C_e} \tag{3.15}$$

where,

C_e : error coefficient

the usual range for C_e is between 0-15000, then 7500 is selected for evaluation of C_e

3.7 Allowable Stress Numbers, S_{ac} and S_{at}

The allowable stress numbers depend on:

- Material composition and cleanliness
- Mechanical properties
- Residual stress
- Hardness
- Type of heat treatment (surface or through hardened)

An allowable stress number for unity application factor, 10 million cycles of load application, 99 percent reliability and unidirectional loading, is determined or estimated from laboratory and field experience for each material and condition of that material. This stress number is designated S_{ac} or S_{at}. The allowable stress numbers are adjusted for design life load cycles by the use of life factors. The allowable stress numbers for gear materials vary with material composition, cleanliness, quality, heat treatment, and processing practices. For materials other than steel, a range is shown, and the lower values should be used for general design purposes. The allowable stress numbers are shown in Tables 3.4, 3.5 and Figures 3.4, 3.5.

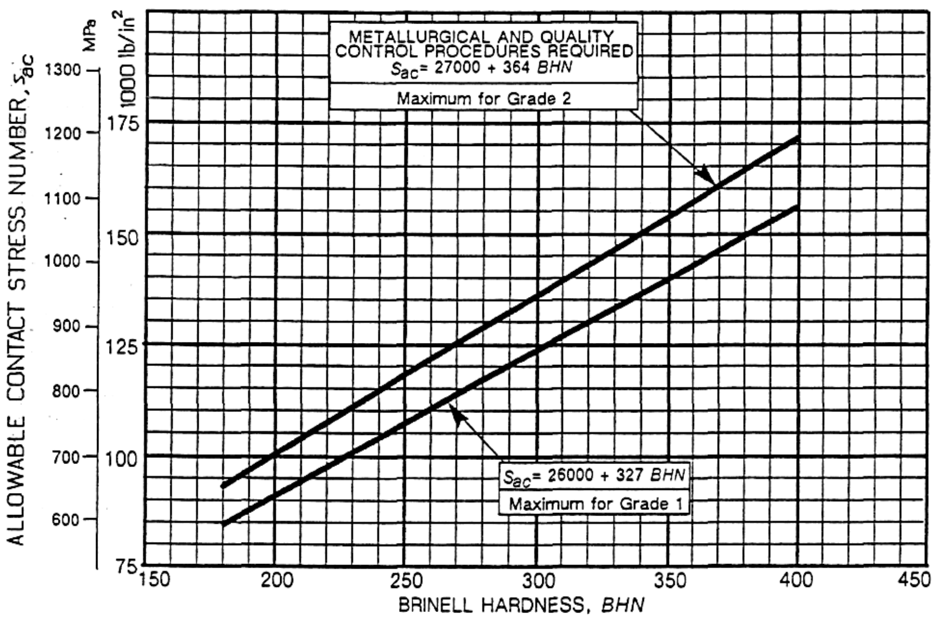


Figure 3.4 Allowable Contact Stress Number for Steel Gears, S_{ac} [18]

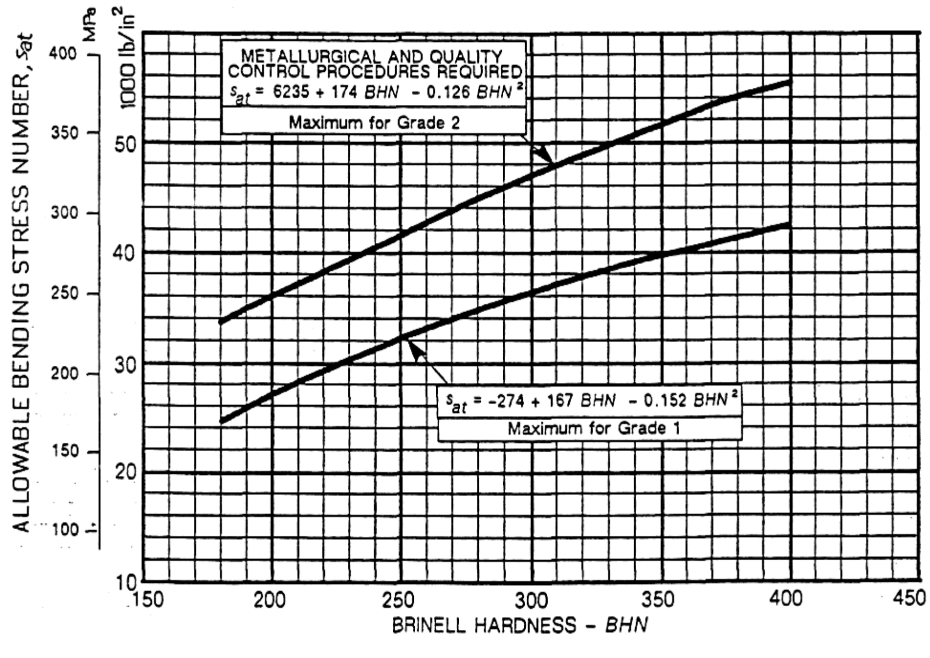


Figure 3.5 Allowable Bending Stress Number for Steel Gears, S_{at}[18]

Table 3.4 Allowable Contact Stress Number, S_{ac} [17]

Material	AGMA Class	Commercial Designation	Heat Treatment	Minimum Hardness at Surface	S_{ac}	
					Psi	MPa
Steel	A-1 Through A-5	-	Through-hardened and tempered	180 BHN & less 240 BHN 300 BHN 360 BHN 400 BHN	85-95000 105-115000 120-135000 145-160000 155-170000	(590-660) (720-790) (830-930) (1000-1100) (1100-1200)
			Flame- or induction-hardened	50 HRC 54 HRC	170-19000 175-195000	(1200-1300) (1200-1300)
			Carburised and case-hardened	55 HRC 60 HRC	180-200000 200-225000	(1250-1400) (1400-1550)
		AISI 4140 AISI 4340 Nitralloy 135 M 2 ½ % chrome	Nitrided Nitrided Nitrided Nitrided Nitrided	48 HRC 46 HRC 60 HRC 54 HRC 60 HRC	155-180000 150-175000 170-195000 155-172000 192-216000	(1100-1250) (1050-1200) (1170-1350) (1100-1200) (1300-1500)
Cast Iron	20 30 40		As cast As cast As cast	- 175 BHN 200 BHN	50-60000 65-75000 75-85000	(340-410) (450-520) (520-590)
Nodular (Ductile) Iron	A-7-a A-7-c A-7-d A-7-e	60-14-18 80-55-06 100-70-03 120-90-02	Annealed Quenched & Tempered	140 BHN 180 BHN 230 BHN 270 BHN	90-100% of S_c value of steel with same hardness	
Malleable Iron (Pearlitic)	A-8-c A-8-e A-8-f A-8-i	45007 50005 53007 80002	- - - -	165 BHN 180 BHN 195 BHN 240 BHN	72000 78000 83000 94000	(500) (540) (570) (650)
Bronze	Bronze 2	AGMA 2C	Sand-cast Sand-cast	Tensile strength minimum 40000 lb/in^2 275(MPa)	30000	(205)
	Al/Br 3	ASTM B-148-52 Alloy 9C	Heat-Treated	Tensile strength minimum 90000 lb/in^2 620(MPa)	65000	(450)

Table 3.5 Allowable Bending Stress Number, S_{at} [17]

Material	AGMA Class	Commercial Designation	Heat Treatment	Minimum Hardness at Surface	CORE	S_{at} Psi	MPa
Steel	A-1 Through A-5	-	Through-hardened and tempered	180 BHN & less 240 BHN 300 BHN 360 BHN 400 BHN	- - - - -	25-33000 31-41000 36-47000 40-52000 42-56000	(170-230) (210-280) (250-320) (280-360) (290-390)
			Flame- or induction-hardened with type B pattern	50-54 HRC	-	45-55000	(310-380)
			Flame- or induction-hardened with type B pattern		-	22000	(150)
			Carburised and case-hardened	55 HRC 60 HRC	-	55-65000 55-70000	(380-450) (380-480)
		AISI 4140 AISI 4340 Nitralloy 135 M 2 ½ % chrome	Nitrided Nitrided Nitrided Nitrided	48 HRC 46 HRC 60 HRC 54-60 HRC	300 BHN 300 BHN 300 BHN 300 BHN	34-45000 36-47000 38-48000 55-65000	(230-310) (250-325) (260-330) (380-450)
Cast Iron	20 30 40		As cast As cast As cast	- 175 BHN 200 BHN	- - -	5000 8500 13000	(35) (69) (90)
Nodular (Ductile) Iron	A-7-a A-7-c A-7-d A-7-e	60-14-18 80-55-06 100-70-03 120-90-02	Annealed Quenched & Tempered	140 BHN 180 BHN 230 BHN 270 BHN	- - - -	90-100% of S_t value of steel with same hardness	
Malleable Iron (Pearlitic)	A-8-c A-8-e A-8-f A-8-i	45007 50005 53007 80002	- - - -	165 BHN 180 BHN 195 BHN 240 BHN	- - - -	10000 13000 16000 21000	(70) (90) (110) (145)
Bronze	Bronze 2	AGMA 2C	Sand-cast Sand-cast	Tensile strength minimum 40000 lb/in^2 275(MPa)		5700	(40)
	Al/Br 3	ASTM B-148-52 Alloy 9C	Heat-Treated	Tensile strength minimum 90000 lb/in^2 620(MPa)		23600	(160)

3.8 Hardness Ratio Factor, C_H

The pinion generally has a smaller number of teeth than the gear and consequently is subjected, to more cycles of contact stress. If both the pinion and the gear are through-hardened, then a uniform surface strength can be obtained by making the pinion harder than the gear. A similar effect can be obtained when a surface-hardened pinion is mated with a through-hardened gear. The hardness-ratio factor C_H is used only for the gear. Its purpose is to adjust the surface strengths for this effect. The values of C_H are obtained from the equation give below:

$$C_H = 1.0 + A(m_G - 1.0) \qquad (3.16)$$

where,

$$A = 8.98.10^{-3}\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29.10^{-3} \qquad (3.17)$$

$$m_G = \frac{N_G}{N_P} \qquad (3.18)$$

The terms H_{BP} and H_{BG} are the Brinell hardnesses (10-mm ball at 3000-kg load) of the pinion and gear, respectively. The term m_G is the speed ratio and is valid only when $(H_{BP}/H_{BG}) \leq 1.70$.

3.9 Life Factors, C_L and K_L

The life factors, C_L and K_L, adjust the allowable stress numbers for the required number of cycles of operation. For the purpose of this Standard, N, the number of load cycles is defined as the number of mesh contacts, under load, of the gear tooth being analyzed. AGMA allowable stress numbers are established for 10^7 tooth load cycles at 99 percent reliability. The life factor adjusts the allowable stress numbers for design lives other than 10^7 cycles.

The life factor accounts for the S/N characteristics of the gear material as well as for the gradual increased tooth stress which may occur from tooth wear, resulting in increased dynamic effects and from shifting load distribution which may occur during the design life of the gearing.

A C_L or K_L value of unity (1) may be used, where justified by experience, beyond 10^7 cycles.

3.9.1 Life Factors for Steel Gears

At the present time there is insufficient data to provide accurate life curves for all types of gears and gear applications. Experience, however, suggests life curves for pitting and strength of steel gears as shown in Figure 3.6 and Figure 3.7. These figures do not include data for nitrided gears. The upper portion of the shaded zones on the figures may be used for general commercial applications. The lower portions of the shaded zones are typically used for critical service applications where little pitting and tooth wear is permissible and where smoothness of operation and low vibration levels are required.

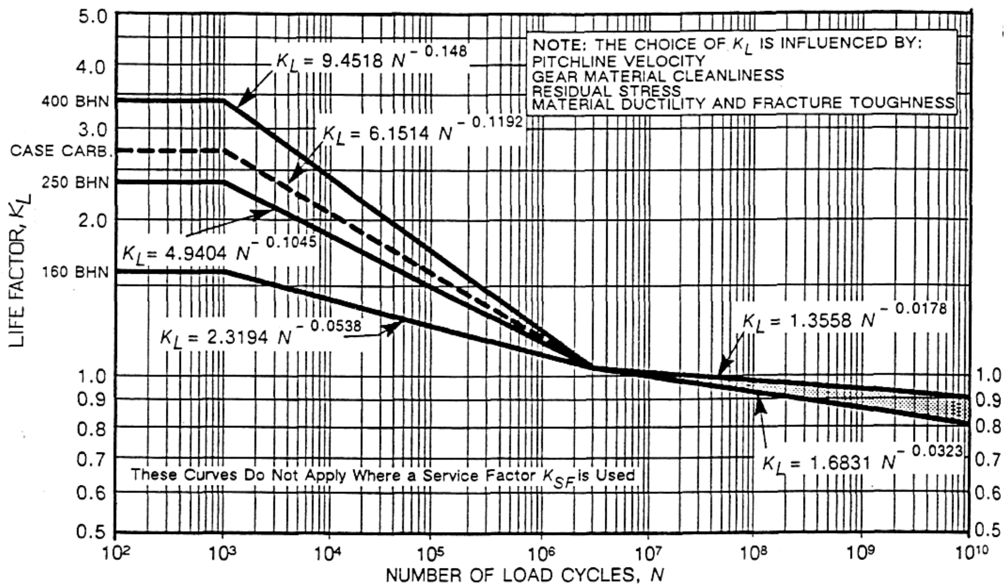


Figure 3.6 Bending Strength Life Factor, K_L [18]

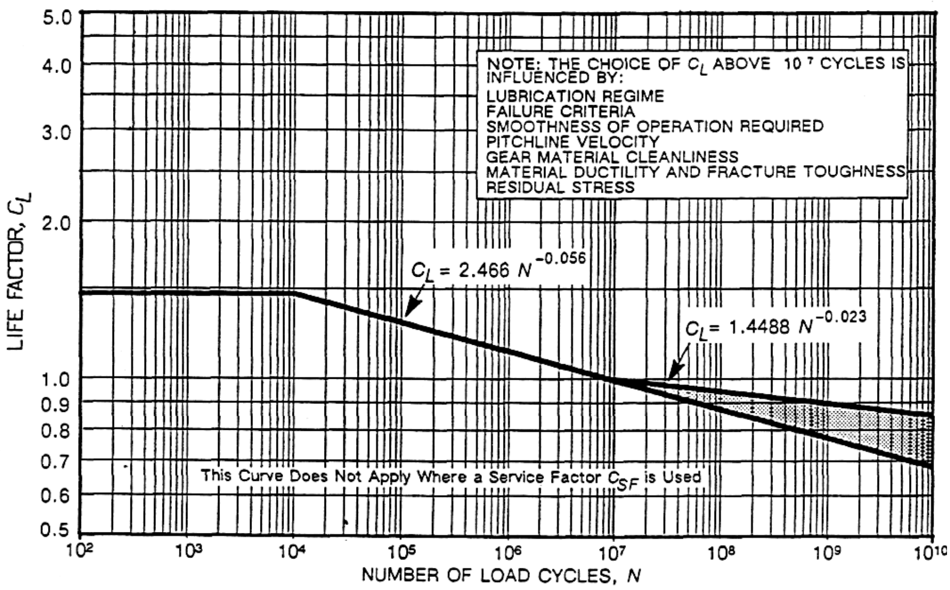


Figure 3.7 Pitting Resistance Life Factor, C_L[18]

3.10 Reliability Factors, C_R and K_R

The AGMA strengths presented are all based on a reliability R=0.99 corresponding to 10^7 cycles of life. Table 3.6 contains reliability factors which may be used to modify this allowable stresses to change that probability. These numbers are based on data developed for bending and pitting, failure by the U.S Navy. Other values may be used if specific data is available.

Table 3.6 Reliability Factors, C_R and K_R [15]

Requirements of application	C_R,K_R
Fewer than one failure in 10000	1.5
Fewer than one failure in 1000	1.25
Fewer than one failure in 100	1
Fewer than one failure in 10	0.85

And also with Equation 3.19 C_R and K_R is calculated as:

$$C_R = \begin{cases} 0.7 - 0.15 \log(1 - R) & 0.9 \leq R < 0.99 \\ 0.5 - 0.25 \log(1 - R) & 0.99 \leq R < 0.9999 \end{cases} \tag{3.19}$$

3.11 Temperature Factors, C_T and K_T

The lubricant temperature is a reasonable measure of gear temperature. For steel materials in oil temperatures up to 120°C (250°F), K_T can be set to 1. For higher temperatures, K_T can be estimated from:

$$K_T = \frac{460 + T_F}{620} \tag{3.20}$$

where T_F is the oil temperature in °F and is valid only for steel gears.

3.12 AGMA Geometry Factor for Pitting Resistance I

This factor takes into account the radii of curvature of the gear teeth and the pressure angle shown in the Figure 3.8. AGMA defines Equation 3.21 for I:

$$I = \frac{\cos(f^c)}{d\left(\frac{1}{r_p} + \frac{1}{r_g}\right)} \tag{3.21}$$

where,

ϕ^c : rack cutter pressure angle

ρ_p : radii of curvature of the pinion teeth

ρ_g : radii of curvature of the gear teeth

d : pitch diameter of pinion

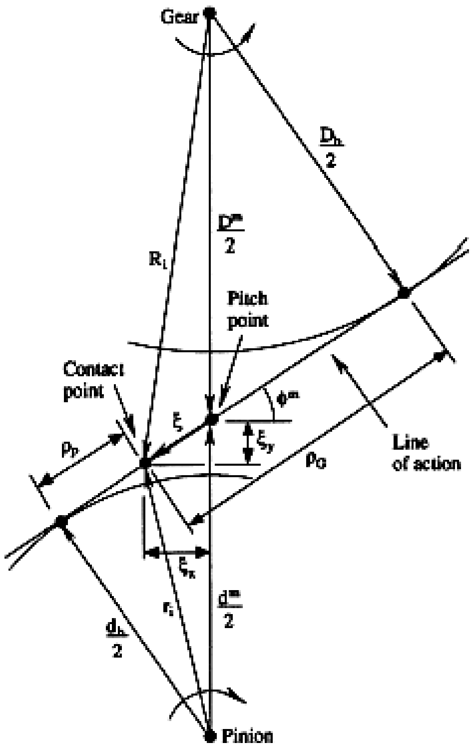


Figure 3.8 Contact Point and Radii of Curvature [19]

ρ_p and ρ_g for any contact position can be found from the Figure 3.8 by the following equations.

$$x = \frac{d}{2}\sin\left(f^c\right) - \left[\left(r_i\right)^2 - \frac{\left(d_b\right)^2}{4}\right]^{0.5} = \left[\left(R_i\right)^2 - \frac{\left(D_b\right)^2}{4}\right]^{0.5} - \frac{d}{2}\sin\left(f^c\right) \quad (3.22)$$

$$x_x = x\cos\left(f^c\right) \quad (3.23)$$

$$x_y = x\cos\left(f^c\right) \quad (3.24)$$

$$r_i = \left[\left(x_x\right)^2 + \left(\frac{d}{2} - x_y\right)^2\right]^{0.5} \quad (3.25)$$

$$R_i = \left[(x_x)^2 + \left(\frac{D}{2} - x_y \right)^2 \right]^{0.5} \qquad (3.26)$$

$$r_p = \left[(r_i)^2 + \frac{(d_b)^2}{4} \right]^{0.5} \qquad (3.27)$$

$$r_g = \left[(R_i)^2 + \frac{(D_b)^2}{4} \right]^{0.5} \qquad (3.28)$$

With respect to AGMA standard 218.01, LPSTC for the pinion is taken as the most critical point for calculation of AGMA geometry factor I, and all of the tables are prepared this rule.

"Although the rule is correct for most of the cases, there are cases for which it is not applicable. In x-zero gear pairs, a large positive pinion addendum modification coefficient results in a pinion having a very long addendum, and a very short dedendum; and a gear with a very long dedendum and a very short addendum. These tooth geometries shift the single tooth pair contact region of the pinion (region between HPSTC and LPSTC) above its pitch circle. A similar, but reverse shift is observed on the gear. Standard gears, or gears with small addendum modification coefficients have their pitch circles within this region; thus LPSTC is below, HPSTC is above the pitch circle. When the single tooth pair contact regions are shifted in this way, not LPSTC, but HPSTC becomes the most critical point" [20].

When running the developed program, the user is asked for radii of curvature calculation options, as defined by AGMA the critical, at LPSTC or both at LPSTC and at HPSTC.

3.13 Contact Ratio m_p

Contact ratio m_p defines the average number of teeth in contact at any one time. It is calculated from Equation 3.29.

$$m_p = \frac{Z_a + Z_b}{p_b} = \frac{Z}{p_b} \tag{3.29}$$

where,

Z_a: length of the pinion addendum (gear dedendum) portion of the line of action

Z_b: length of the pinion dedendum (gear addendum) portion of the line of action

Z_c: distance between the pitch point and HPSTC of gear (LPSTC of pinion) measured along the line of action when the gear is not undercut

Z_e: distance between the pitch point and HPSTC of pinion (LPSTC of pinion) measured along the line of action when the pinion is not undercut.

p_b: base pitch

When the gears are not undercut, pinion dedendum and addendum portions of the length of action can be calculated by using the intersection points of the outside circles with the line of action. Contact ratio, and pinion dedendum and addendum portions of the line of action can be calculated by using the following equations [20].

where,

$$Z_a = \frac{\left(d_0{}^2 - d_b{}^2\right)^{0.5} - d\left(\sin f^c\right)}{2} \qquad r_{tiG} \leq r_{LG} \tag{3.30}$$

$$Z_b = \frac{\left(D_0{}^2 - D_b{}^2\right)^{0.5} - D\left(\sin f^c\right)}{2} \qquad r_{tiP} \leq r_{LP} \tag{3.31}$$

$$Z_a = \frac{D \sin f^c - 2\left[r_{tiG} - \left(\frac{D_b}{2}\right)^2\right]^{\frac{1}{2}}}{2} \qquad r_{tiG} > r_{LG} \tag{3.32}$$

$$Z_b = \frac{d \sin f^c - 2\left[r^2_{tiP} - \left(\frac{d_b}{2} \right)^2 \right]^{\frac{1}{2}}}{2} \qquad r_{tiP} > r_{LP} \tag{3.33}$$

In the above equations, condition $r_{ti} > r_L$ indicates undercutting.

3.14 AGMA Geometry Factor for Bending Strength J

The geometry factor J is calculated from a complicated algorithm defined in AGMA standard 908-B89 which is based on methods given in [3]. The calculation of the geometry factor J is explained in the following sections after the definition of rack cutter geometry, spur gear geometry and their equations. The equation is simply as follows.

$$J = \frac{Y}{K_f} \tag{3.34}$$

3.14.1 Rack Cutter Geometry

In Figure 3.9 rack cutter geometry and in the Table 3.7 its dimensions with respect to the AGMA 908-B89 are given.

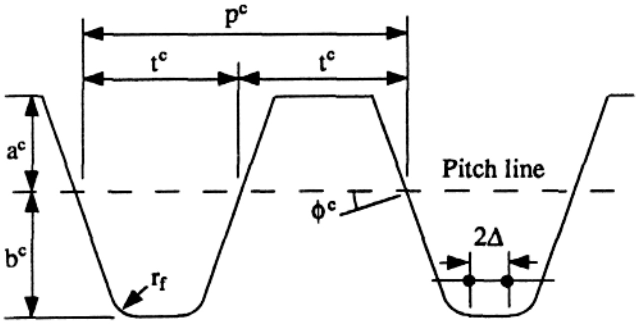


Figure 3.9 Rack Cutter Geometry [19]

Rack cutter dimension are found from the following equations.

$$a^c = C_{ac}\, m \qquad (3.35)$$

$$b^c = C_b\, m \qquad (3.36)$$

$$r_f = C_f\, m \qquad (3.37)$$

Table 3.7 Rack Cutter Dimensions

Standard	ϕ^c	a^c	b^c	r_f	t^c
AGMA 908-B89	14.5°	1.0 m	1.157 m	0.157 m	$\pi m/2$
AGMA 908-B89	20°	1.0 m	1.250 m	0.250 m	$\pi m/2$
AGMA 908-B89	25°	1.0 m	1.350 m	0.270 m	$\pi m/2$

Amount of rack cutter offset used to cut gears with non-standard proportions is expressed by using the addendum modification (rack shift) coefficient. Positive coefficients are used for advances. Amount of rack cutter shift is found by multiplying the addendum modification coefficient by the module.

Pinion and gear dimensions are found from the following equations whether they are modified or not. When there is no rack shift then x_p and x_G are zero and the equations are valid for standard proportions.

$$d = m\, N_p \qquad (3.38)$$

$$D = m\, N_G \qquad (3.39)$$

$$r_p = (m\, N_p) / 2 \qquad (3.40)$$

$$r_G = (m\, N_G) / 2 \qquad (3.41)$$

$$d_o = m\, N_p + 2\, a^c + 2\, m\, x_p \quad (3.42)$$

$$D_o = m\, N_G + 2\, a^c + 2\, m\, x_G \quad (3.43)$$

$$r_{op} = d_o / 2 \quad (3.44)$$

$$r_{oG} = D_o / 2 \quad (3.45)$$

$$d_r = m\, N_p - 2\, b^c + 2\, m\, x_p \quad (3.46)$$

$$D_r = m\, N_G - 2\, b^c + 2\, m\, x_G \quad (3.47)$$

$$d_b = m\, N_p \cos(\phi) \quad (3.48)$$

$$D_b = m\, N_G \cos(\phi) \quad (3.49)$$

$$p_b = \pi\, m \cos(\phi) = p^c \cos(\phi) \quad (3.50)$$

$$C = m \frac{N_p + N_G}{2} \quad (3.51)$$

$$c = C - \frac{D_o + D_r}{2} \quad (3.52)$$

3.14.2 Spur Gear Tooth Geometry

In order for the fundamental law of gearing to be true, the gear tooth contours on mating teeth must be conjugates of one another. There is infinite number of possible conjugate pairs that could be used, but only a few curves have been practical application as gear teeth. The cycloid is still used as a tooth form in some watches and clocks, but most gears use the involute of a circle for their working portion and the trochoid as the fillet portion.

The involute function is calculated from Equation 3.53.

$$\text{inv}(\phi) = \tan(\phi) - \phi \qquad (3.53)$$

Coordinates of involute (working portion) of tooth profile is calculated by the following equations. The coordinate system and tooth profile is given in Figure 3.10.

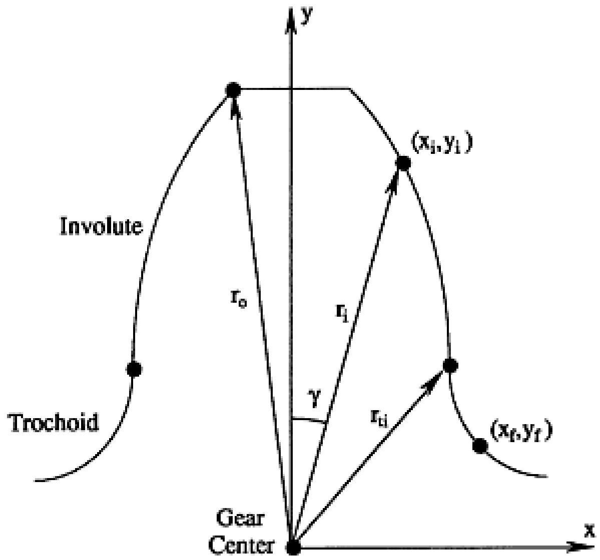


Figure 3.10 Coordinate system and tooth profile [19]

$$x_i = r_i \sin(\gamma) \qquad r_{ti} \le r_i \le r_o \qquad (3.54)$$

$$y_i = r_i \cos(\gamma) \qquad r_{ti} \le r_i \le r_o \qquad (3.55)$$

where,

$$\gamma = \frac{t}{2r} + inv(f^c) - inv(arc(\cos(\frac{r\cos(f^c)}{r_i}))) \qquad r_{ti} \le r_i \le r_o \qquad (3.56)$$

$$r_{ti} = (x_f^2 + y_f^2)^{0.5} \qquad \text{at } \theta = \theta_{max} \qquad (3.57)$$

Circular tooth thicknesses at pitch circles and tooth tips are calculated by the following equations:

$$t_p = m(\frac{p}{2} + 2x_P \tan(f^c) - backlash) \tag{3.58}$$

$$t_G = m(\frac{p}{2} + 2x_G \tan(f^c) - backlash) \tag{3.59}$$

$$t_{tp} = do(\frac{t_P}{d} + inv(f^c) - inv(f_{tP})) \tag{3.60}$$

$$t_{tG} = Do(\frac{t_G}{D} + inv(f^c) - inv(f_{tG})) \tag{3.61}$$

where,

$$\phi_{tP} = \mathrm{arc}(\cos(d_b/d_o)) \tag{3.62}$$

$$\phi_{tG} = \mathrm{arc}(\cos(D_b/D_o)) \tag{3.63}$$

According to the AGMA 908-B89, limiting value for the tooth thickness calculated at tooth tip is 0.3 module. During the developed program this is also taken as upper limit for modification. The limit of this approach occurs when the pinion tooth becomes pointed (excessively-thinned tooth tip).

Coordinates of trochoid (fillet portion) of tooth profile is calculated by the following equations [19].

$$x_f = \left(r - b^c + m\cdot\chi + rf\right)\cdot\sin(\beta + \theta) - r\cdot\theta\cdot\cos(\beta + \theta) \ldots \\ + \frac{-rf}{\left[\left(b^c - m\cdot\chi - r_f\right)^2 + r^2\cdot\theta^2\right]^{0.5}}\cdot\left[\left(b^c - m\cdot\chi - r_f\right)\cdot\sin(\beta + \theta) + r\cdot\theta\cdot\cos(\beta + \theta)\right] \tag{3.64}$$

$$y_f = \left(r - b^c + m\cdot\chi + rf\right)\cdot\cos(\beta + \theta) + r\cdot\theta\cdot\sin(\beta + \theta) \;...$$
$$+ \frac{-rf}{\left[\left(b^c - m\cdot\chi - r_f\right)^2 + r^2\cdot\theta^2\right]^{0.5}}\cdot\left[\left(b^c - m\cdot\chi - r_f\right)\cdot\cos(\beta + \theta) + r\cdot\theta\cdot\sin(\beta + \theta)\right] \quad (3.65)$$

For both of the above equations $\theta_{min} \le \theta \le \theta_{max}$.

where,

$$\beta = \frac{p}{N} - \frac{\Delta}{r} \quad (3.66)$$

$$\Delta = \frac{pm}{4} - (b^c - r_f)\tan(f^c) - \frac{r_f}{\cos(f_c)} \quad (3.67)$$

$$\theta_{min} = 0 \quad (3.68)$$

$$\theta_{max} = \frac{b^c - mx - r_f}{r\tan(f_c)} \quad (3.69)$$

3.15 Calculation of Geometry factor J

The AGMA factor J employs a modified value of the Lewis form factor. Y is the tooth form factor obtained from a generated layout of the tooth profile and K_f is the stress correction factor based on a formula deduced from a photoelastic investigation of stress concentration in gear teeth over 40 years ago.

$$J = \frac{Y}{K_f} \quad (3.70)$$

$$Y = \frac{1}{\frac{\cos(\phi_L)}{\cos(\phi^c)}\left(\frac{6h_f}{s_f^{\,2}} - \frac{\tan(\phi_L)}{s_f}\right.} \tag{3.71}$$

$$K_f = H + \left(\frac{s_f}{r_f}\right)^{L} + \left(\frac{s_f}{r_f}\right)^{M} \tag{3.72}$$

where,

$$H = 0.331 - 0.436\,\phi^c \tag{3.73}$$

$$L = 0.324 - 0.492\,\phi^c \tag{3.74}$$

$$M = 0.261 + 0.545\,\phi^c \tag{3.75}$$

While calculating the geometry factor J, the problem is to determine the critical section parameters (s_f, h_f, ρ_f) from the condition of tangency of the Lewis parabola and the root trochoid (fillet portion), and numerical techniques with iterative procedures are required. This procedure is done by the methods defined in [19].

Height of the Lewis parabola h_f, and corresponding tooth thickness at critical section s_f is calculated by finding the coordinates of tangency point of the Lewis parabola and the root.

It is seen from Figure 3.11 that y-coordinate of the vertex of the parabola y_L can be found by making use of the equation of the line whose slope is $\tan(\phi_L)$ and passing through a point on the involute tooth profile (x_i, y_i). Values of them are found from Equation 3.76.

$$y_L = y_i - x_i \tan(\phi_L) \tag{3.76}$$

The load angles are found from the following equations according to the diameters at which the load is acting (HPSTC or Tip loading). During the calculation of AGMA bending stress factor J, HPSTC is considered the most critical point and use HPSTC diameters at the following equations, when the tip loading is the case then outside diameters are used.

$$\phi_{LP} = \frac{(d_x^{\ 2} - d_b^{\ 2})^{0.5}}{d_b} - (\frac{t_P}{d} + inv(f^{\,c})) \tag{3.77}$$

$$\phi_{LG} = \frac{(D_x^{\ 2} - D_b^{\ 2})^{0.5}}{D_b} - (\frac{t_G}{D} + inv(f^{\,c})) \tag{3.78}$$

where,

when $r_{tiP} \leq r_{LP;}$

$$d_{HPSTC} = 2\left[(\frac{d_b}{2})^2 + (\frac{d\sin(f^{\,c})}{2} + Z_e)^2\right]^{0.5} \tag{3.79}$$

$$Z_e = p_b - \frac{1}{2}\left[D_o^{\ 2} - D_b^{\ 2}\right]^{0.5} - \left[D^2 - D_b^{\ 2}\right]^{0.5} \tag{3.80}$$

when $r_{tiG} \leq r_{LG}$;

$$D_{HPSTC} = 2\left[(\frac{D_b}{2})^2 + (\frac{D\sin(f^{\,c})}{2} + Z_c)^2\right]^{0.5} \tag{3.81}$$

$$Z_c = p_b - \frac{1}{2}\left[d_o^{\ 2} - d_b^{\ 2}\right]^{0.5} - \left[d^2 - d_b^{\ 2}\right]^{0.5} \tag{3.82}$$

when $r_{tiP} > r_{LP}$;

$$d_{HPSTC}=2\{\left[\frac{d_b}{2}\right]^2+\left[\left(r^2{}_{tiP}-\left(\frac{d_b}{2}\right)^2\right)^{\frac{1}{2}}+P_b\right]^2\}^{\frac{1}{2}} \quad (3.83)$$

when $r_{tiG} > r_{LG}$;

$$D_{HPSTC}=2\{(\frac{D_b}{2})^2+\left[\left(r_{tiG}^2-\left(\frac{D_b}{2}\right)^2\right)^{\frac{1}{2}}+P_b\right]^2\}^{\frac{1}{2}} \quad (3.84)$$

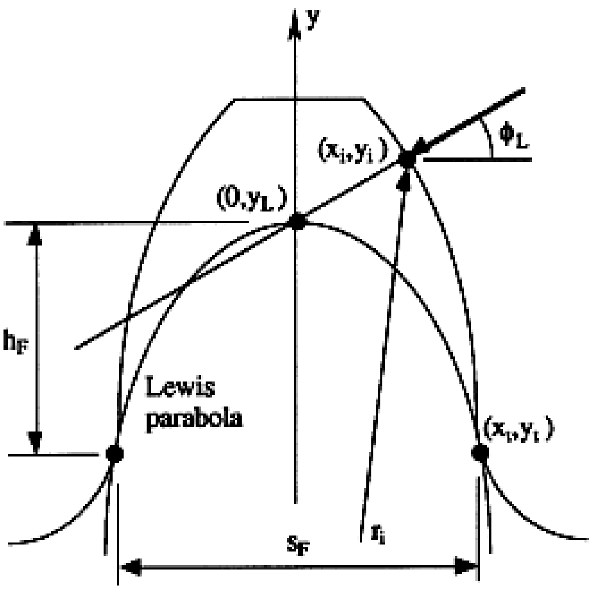


Figure 3.11 Lewis Parabola [19]

An opening down parabola, having its vertex at (y_L, 0) can be expressed by using Equation 3.85.

$$y = -cx^2 + y_L \quad (3.85)$$

By taking derivative with respect to the x then, the slope of the parabola is found as follows:

$$m_p = \frac{d_y}{d_x} = -2cx \tag{3.86}$$

Slope of the tooth root fillet curve, m_f is found from Equation 3.87:

$$m_f = \frac{\frac{d_{y_f}}{dq}}{\frac{d_{x_f}}{dq}} \tag{3.87}$$

By equating two slopes at the point of tangency (x_t,y_t), between the Lewis parabola and the fillet curve and also using the opening down parabola equation, θ, c and then critical section parameters are found by using Newton Raphson method [21].

$$y_f(\theta) = -(c\, x_f(\theta))^2 + y_L \tag{3.88}$$

and

$$-2\, c\, x_f(\theta) = \frac{\frac{d_{y_f}}{dq}}{\frac{d_{x_f}}{dq}} \tag{3.89}$$

are two simultaneous nonlinear equations with two unknowns, θ and c. They can be expressed in the form of

$$U(\theta,c) = y_f(\theta) + (c\, x_f(\theta))^2 - y_L = 0 \tag{3.90}$$

$$V(\theta,c) = -2\,c\,x_f(\theta) - \frac{\frac{d_{y_f}}{dq}}{\frac{d_{x_f}}{dq}} = 0 \qquad (3.91)$$

Thus the solution would be the values of c and θ that make the functions U(θ,c) and V(θ,c) equal to zero. Newton-Raphson method used the derivative (slope) of a function to estimate the root. This estimate was based on a first-order Taylor series expansion,

$$x_{i+1} = x_i - \frac{f(x_i)}{f^{'}(x_i)} \qquad (3.92)$$

which is the single-equation form for the Newton-Raphson method. A similar multiequation form is used for this study. The denominators of each of these equations are formally referred to as the determinant of the Jacobian of the system. It can be employed iteratively until an acceptable solution is obtained.

In this study initial guesses for c = 0 and θ = π/32,

$$\theta_{i+1} = \theta_i - \frac{(u(q,c)\frac{\partial}{\partial c}v(q,c)) - (v(q,c)\frac{\partial}{\partial c}u(q,c))}{(\frac{\partial}{\partial q}u(q,c)\frac{\partial}{\partial c}v(q,c)) - (\frac{\partial}{\partial c}u(q,c)\frac{\partial}{\partial q}v(q,c))} \qquad (3.93)$$

$$c_{i+1} = c_i - \frac{(v(q,c)\frac{\partial}{\partial c}u(q,c)) - (u(q,c)\frac{\partial}{\partial c}v(q,c))}{(\frac{\partial}{\partial q}u(q,c)\frac{\partial}{\partial c}v(q,c)) - (\frac{\partial}{\partial c}u(q,c)\frac{\partial}{\partial q}v(q,c))} \qquad (3.94)$$

After calculation of Equation 3.93 and 3.94, values are used for the Equation 3.90 and 3.91 and this process is done iteratively. Then c and θ are ready to be used for the critical section parameters.

$$h_f = y_L - y_t \tag{3.95}$$

$$s_f = 2\, x_t \tag{3.96}$$

$$\rho = \frac{\left(\left(\frac{\partial x_f}{\partial q}\right)^2 + \left(\frac{\partial y_f}{\partial q}\right)^2\right)^{3/2}}{\left| \frac{\partial x_f}{\partial q}\frac{\partial^2 y_f}{\partial q^2} - \frac{\partial y_f}{\partial q}\frac{\partial^2 x_f}{\partial q^2} \right|} \tag{3.97}$$

ρ_f is the minimum radius of curvature occurs at the tangency point of fillet curve and the root circle, so it is found by using $\theta = 0$ in the Equation 3.97.

CHAPTER 4

ADDENDUM MODIFICATION FOR GEARS

The most extensively used modification method is the addendum modification for gear manufacturing and it can be applied with standard tools and machines.

Modification of the addendum of the pinion, and in most cases the gear member, is recommended for gears serving the following applications:

- Meshes in which either or both members have small numbers of teeth.
- Meshes operating on non-standard center distances because of limitations on ratio or center distances.
- Meshes of speed-increasing drives.
- Meshes designed to carry maximum power for the given weight allowance. (This type of gearing is usually designed to achieve the best balance in strength, wear, specific sliding, pitting, or scoring.)
- Meshes in which an absolute minimum of energy loss through friction is to be achieved.

Only base circles of gears affect the kinematics of a gear pair which have gears with involute tooth profiles. Addendum modification is based on this property of involute gears. By changing the diameters of pitch circles of the pinion and the gear, they can be run at any center distance. If one of the gears is thought as a cutter with infinite number of teeth, i.e. the rack cutter, the mating gear can be run with this gear at different pitch diameters properly. Also gears manufactured by the same cutter can be run with each other. If the pitch diameter of the gear to be manufactured coincides with the datum line of this cutter, a standard gear is obtained which is called as O-Gear (Gear with no modification). If the cutter is withdrawn from the gear by (x/m), a gear which is called as plus V-Gear is obtained. If withdrawal is made in the opposite direction by (-x/m) i.e., the cutter is advanced towards the gear, a minus V-Gear is obtained. Here, x is called as the addendum modification coefficient. It is positive for withdrawal, negative for advance of cutter. Upper limit of addendum modification is formed by point tooth and lower limit of that is formed by cutter interference.

4.1 Types of Corrected Gear Mechanisms

Based on the correction aspect, gearing can be classified into the following types: O-Gearing, V-Gearing, and V-O-Gearing

- *O-Gearing:* In standard gearing, the gear teeth are generated with standard proportions in the normal manner without any profile correction or modification. Two 0-Gears are used. (Standard gear drive).
- *V-Gearing:* In V-Gearing the sum of the profile corrections of the two mating gears is not equal to zero. However, the sum is positive in almost all cases in order to take advantage of the beneficial effects of positive correction. Usually, the sum is so divided that the pinion gets the bigger share for the positive correction. Sometimes a V-Pinion may mate with a normal, uncorrected gear. It all depends on how the situation warrants it.
- *V-O-Gearing:* In V-O-Gearing, the mating pair of gears receive numerically equal but opposite values for correction factors. Normally, the smaller gear (pinion) is provided with positive correction and the larger gear with negative correction. Hence in such gears (x_p + x_G =0). In these type of gearing, one of the gears is a plus V-Gear and the other is a minus V-Gear with the same amount of addendum modification coefficient. This is also known as “Long and Short Addendum Gearing” in AGMA standards and “V-Null Gearing” in German technical literature.

4.1.1 O-Gearing

It is a standard gearing which consists of two O-Gears. These type of gearing is used when the number of teeth is equal to or greater than the minimum number of teeth not to have undercutting and when there are not any special requirements for the center distance and tooth strength. Generating pitch diameters and operating pitch diameters are the same for these gears.

The mechanism is defined as

$$x_P = x_G = 0 \tag{4.1}$$

$$x_T = x_P + x_G = 0 \tag{4.2}$$

$$\phi = \phi^c \tag{4.3}$$

Then the dimensions are as follows:

$$d = m\, N_p \tag{4.4}$$

$$D = m\, N_G \tag{4.5}$$

$$d_o = m\, N_p + 2\, a^c \tag{4.6}$$

$$D_o = m\, N_G + 2\, a^c \tag{4.7}$$

$$d_r = m\, N_p - 2\, b^c \tag{4.8}$$

$$D_r = m\, N_G - 2\, b^c \tag{4.9}$$

$$d_b = m\, N_p \cos(\phi) \tag{4.10}$$

$$D_b = m\, N_G \cos(\phi) \tag{4.11}$$

$$c = C - \frac{D_o + D_r}{2} \tag{4.12}$$

$$t_p = m(\frac{p}{2}) \tag{4.13}$$

$$t_G = m(\frac{p}{2}) \tag{4.14}$$

$$t_{tp} = do(\frac{t_P}{d} + inv(f) - inv(f_{tP})) \tag{4.15}$$

$$t_{tG} = Do(\frac{t_G}{D} + inv(f) - inv(f_{tG})) \tag{4.16}$$

$$\phi_{tP} = arc(\cos(d_b/d_o)) \tag{4.17}$$

$$\phi_{tG} = arc(\cos(D_b/D_o)) \tag{4.18}$$

$$C = m\frac{N_p + N_G}{2} \tag{4.19}$$

where,

a^c	:	Rack cutter addendum
b^c	:	Rack cutter dedendum
C	:	Operating Center distance
c	:	Bottom clearance
D	:	Gear pitch diameter
D_b	:	Gear base diameter
D_o	:	Gear outside diameter
D_r	:	Gear root diameter
d	:	Pinion pitch diameter
d_b	:	Pinion base diameter

d_o	:	Pinion outside diameter
d_r	:	Pinion root diameter
m	:	Module
N_p	:	Number of teeth for pinion
N_G	:	Number of teeth for gear
t	:	Circular tooth thickness at standard pitch radius
t_t	:	Circular tooth thickness at tooth tip
x	:	Addendum modification coefficient (rack shift) coefficient
ϕ	:	Pressure angle
ϕ_t	:	Pressure angle at tooth tip
ϕ^c	:	Rack cutter pressure angle

<u>Subscripts</u>

P	:	Pinion
G	:	Gear
m	:	Modified

4.1.2 V-O-Gearing

In V-O-Gearing, because of the fact that the amount of correction is equal and opposite, their effect is to nullify each other as far as certain dimensions are concerned, so that the two pitch circles contact each other at the point P and the working pressure angle remains as the standard pressure angle as in the case of uncorrected gear. Also, the centre distance remains unaltered and is equal to the sum of the pitch circle radii. However, unlike uncorrected gearing system, the reference line of the reference profile (the basic rack) does not pass through the normal pitch point P in this case. It is shifted away by an amount equal to the numerical value of xm (mm). In case of the pinion, which normally receives the positive correction, the cutter is moved away from the gear blank centre by an extra amount of xm (mm) while cutting, so that an enlarge pinion with its tip diameter increased by an amount of 2xm (mm) is produced. In case of the gear, the cutter is moved towards the gear centre by the same amount, so that its diameter is smaller by an amount of 2xm (mm) than in the case of uncorrected gears.

Since topping is not necessary for V-O-Gearing, the gear blanks are simply made bigger or smaller, as the case may be, by the amount indicated before feeding them to the gear-cutting machine.

The V-O-Gearing is normally meant where the reduction ratio is large. Thicker pinion teeth are ensured and the gear teeth also do not become significantly weak. However, V-O-Gearing is not recommended for small reduction ratios as it tends to weaken the teeth of the gear. The V-O-Gearing is also sometimes recommended where for certain specific reasons the normal tooth-thickness of the gear pair or the specific sliding velocities between the meshing teeth flanks are to be changed. Besides, since normally the pinion teeth are weaker than gear teeth when both are made of the same material, they are more vulnerable to breakage and wear. The, V-O-Gearing system tends to equalise the tooth strength and thereby reduces the susceptibility to such damage.

This type of gearing is recommended mainly for the following cases.

- If the number of teeth is smaller than the minimum number of teeth not to have under cutting, a plus V-Gearing is used in order to avoid interference. In contrast, if center distance is not required to be changed and a number of teeth of the gear is large enough, a minus V-Gearing is used. Thus, the sum of addendum modification coefficients is zero. V-O-Gearing have the same center distances as O-Gearing. Generating pitch diameters of the gears are tangent to each other but not coincident with the datum line of cutter.
- If pinion and gear strengths are required to be balanced this type of modification can also be applied.

The mechanism is defined as:

$$x = x_P = -x_G \tag{4.20}$$

$$x_T = x_P + x_G = 0 \quad (4.21)$$

$$\phi = \phi^c \quad (4.22)$$

Then dimensions are as follows:

$$d = m\ N_p \quad (4.23)$$

$$D = m\ N_G \quad (4.24)$$

$$d_o = m\ N_p + 2\ a^c + 2\ m\ x_p \quad (4.25)$$

$$D_o = m\ N_G + 2\ a^c + 2\ m\ x_G \quad (4.26)$$

$$d_r = m\ N_p - 2\ b^c + 2\ m\ x_p \quad (4.27)$$

$$D_r = m\ N_G - 2\ b^c + 2\ m\ x_G \quad (4.28)$$

$$d_b = m\ N_p \cos(\phi) \quad (4.29)$$

$$D_b = m\ N_G \cos(\phi) \quad (4.30)$$

$$c = C - \frac{D_o + D_r}{2} \quad (4.31)$$

$$t_p = m(\frac{p}{2} + 2x_P \tan(f)) \quad (4.32)$$

$$t_G = m(\frac{p}{2} + 2x_G \tan(f)) \quad (4.33)$$

$$t_{tp} = do(\frac{t_P}{d} + inv(f) - inv(f_{tP})) \quad (4.34)$$

$$t_{tG} = Do(\frac{t_G}{D} + inv(f) - inv(f_{tG})) \tag{4.35}$$

$$\phi_{tP} = arc(\cos(d_b/d_o)) \tag{4.36}$$

$$\phi_{tG} = arc(\cos(D_b/D_o)) \tag{4.37}$$

$$C = m\frac{N_p + N_G}{2} \tag{4.38}$$

4.1.3 V-Gearing

On this gearing, generating pitch diameters of the gears are not tangent to each other while running. So, center distances of these mechanisms are different form V-O-gearing and O-gearing. If a center distance other than the standard center distance is required, this type of modification can be used.

The mechanism is defined as:

$$x_P \neq -x_G \tag{4.39}$$

$$x_T = x_P + x_G \neq 0 \tag{4.40}$$

$$\phi \neq \phi^c \tag{4.41}$$

For this case the generating pitch diameters are not tangent to each other. A nonzero backlash term β should be used in the equations when it is desired to have a circumferential backlash B during meshing. The modified pressure angle (ϕ^m), backlash term (β), and total addendum modification coefficient (x_T) is found as follows

$$f^m = \arccos(\frac{C^s \cos(f^c)}{C^m} \tag{4.42}$$

$$\beta = \frac{B}{m(N_p + N_G)} \frac{\cos(f^c)}{\cos(f^m)} \tag{4.43}$$

$$x_T = (inv(f^m) - inv(f^c) - b) \frac{N_p + N_G}{2} \tag{4.44}$$

Allocation of the sum of the total addendum modification coefficients on pinion and gear teeth is will be made for the desired performance criteria and this case is beyond the scope of this study.

4.2 Applications of Addendum Modification

In this study, addendum modification is applied for three main reasons, namely to have compact gears, to have gears with small numbers of teeth without undercutting, and to have gears with better balance of strength between the pinion and the gear. Increasing that addendum modification of teeth makes the gear teeth stronger. Decrease in the addendum modification of teeth has the reverse on strength of teeth.

4.3 Interference and Undercutting

The involute tooth form is only defined outside of the base circle. In some cases the dedendum will be large enough to extend below the base circle. If so, then the portion of tooth below the base circle will not be an involute and will interfere with the tip of the tooth on the mating gear, which is an involute. If the gear is cut with a standard gear shaper or a hob the cutting tool will also interfere with the portion of tooth below base circle and will cut away the interfering material. This results in an undercut gear. Undercutting weakens the tooth by removing material at its root. The maximum moment and maximum shear from the tooth loaded as a cantilever beam both occur in this region. Severe undercutting will cause early tooth failure. To prevent undercutting, x_{gmin}, must be equal to or greater than the expression in Equation 4.45, derived from the Figure 4.1;

$$x_{g\,\min} = h_{ao} - r_{ao}\left(1 - \sin f\right) - \frac{n}{2}\sin^2 f \qquad (4.45)$$

where,

x_{gmin} : the generating rack shift coefficient

h_{ao} : nominal tool addendum

ρ_{ao} : tool tip radius

n : pinion or gear number of teeth

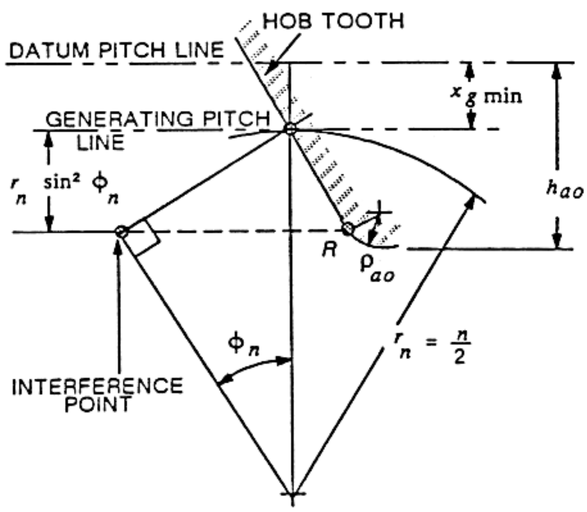


Figure 4.1 Undercutting Criteria [2]

Interference and its attendant undercutting can be prevented simply by avoiding gears with too few teeth. If a pinion has a large number of teeth, they will be small compared to its diameter. As the number of teeth is reduced for a fixed diameter pinion, the teeth must become larger. At some point, the dedendum will exceed the radial distance between the base circle and the pitch circle, and interference will occur.

If the theoretical limit radius of a gear, which is the radius at which involute profile on a gear should start in order to make use of the full length of the involute profile of the mating gear, is smaller than the radius at which involute tooth profile starts, involute interference occurs. As mentioned before when gear teeth are produced by a generating process using a rack cutter or hobbing, interference is automatically eliminated since the cutting tool removes the interfering portion. This results in a reduced contact ratio. Theoretical limit radii for the pinion and the gear, which are calculated by using gear kinematics, are given below [22].

$$r_{LP} = \frac{1}{2}\left(\left[d_b\right]^2 + \left[(d+D)\sin f - \left(D_o^2 - D_b^2\right)^{\frac{1}{2}}\right]^2\right\}^{\frac{1}{2}} \qquad (4.46)$$

$$r_{LG} = \frac{1}{2}\left(\left[D_b\right]^2 + \left[(d+D)\sin f - \left(d_o^2 - d_b^2\right)^{\frac{1}{2}}\right]^2\right\}^{\frac{1}{2}} \qquad (4.47)$$

CHAPTER 5

MULTISTEP GEAR BOX DESIGN

5.1 Overview

A gear train is any collection of two or more meshing gears. Single stage gear drive (a pair of gears) is the simplest form of gear train and limited to a ratio from 1:1 to 8:1. Ratios of 10:1 are also possible for ordinary spur gear design practices [23]. To get a train ratio greater than 10:1, it is necessary to compound the train.

Until now, the studies on the design of gear drives have been focused on single stage gear drives. Nowadays, with increasing the applications of the gear drives in high speed and small space, the need for designing multi-stage gear drives become an important interest subject. A number of complicated problems should be considered in the design of multi-stage gear drives which don't exist in the single-stage gear drives. These problems are summarized as follows:

- Determination of the number of reduction stages,
- Determination of the gear ratios of each stage,
- Determination of dimension of gears,
- Configuration design of the gear drive elements.

5.2 Determination of the Number of Reduction Stages

The number of reduction stage is input from the designer considering the overall gear ratio, available space, and other design specifications. Some simple recommendations are proposed the number of reduction stages. As explained before, it is recommended to handle gear ratios from 1:1 to 8:1 (max. 10:1) in a single reduction for spur gear design practices [23]. It is recommended by AGMA to add another stage if the gear ratio of stage is greater than 5:1 [24]. Then the number of reduction stages is determined by the intuition of the designer. Where larger ratios are needed, the choice of gear arrangement is determined by finding the arrangement with the fewest number of parts that will do the job adequately for the compactness case.

In this study the number of reduction stage is an input from the user. The chance is given to the user to change the range of gear ratios of each stage by changing the number of teeth of pinion and gear. Thus a given overall gear ratio will be allocated between diverse number of stages. And also the upper and lower limits of each stage will be defined by the user.

5.3 Determination of the Gear Ratios of Each Stage

5.3.1 Determination of Number of Teeth in Each Gears

The gear ratio of a gear train can be easily found if the number of teeth in each gear is known.

$$m_G = \frac{N_{G_1}}{N_{P_1}} \frac{N_{G_2}}{N_{P_2}} \frac{N_{G_3}}{N_{P_3}} \frac{N_{G_4}}{N_{P_4}} \ldots\ldots \quad (5.1)$$

where,

m_G : Overall gear ratio
N_P : Number of teeth of pinion
N_G : Number of teeth of gear

To explain the situation easily, a two-stage gear drive is shown in Figure 5.1. The overall gear ratio of the two-stage gear train is found from Equation 5.2 easily.

$$m_G = \frac{N_{G_1}}{N_{P_1}} \frac{N_{G_2}}{N_{P_2}} \qquad (5.2)$$

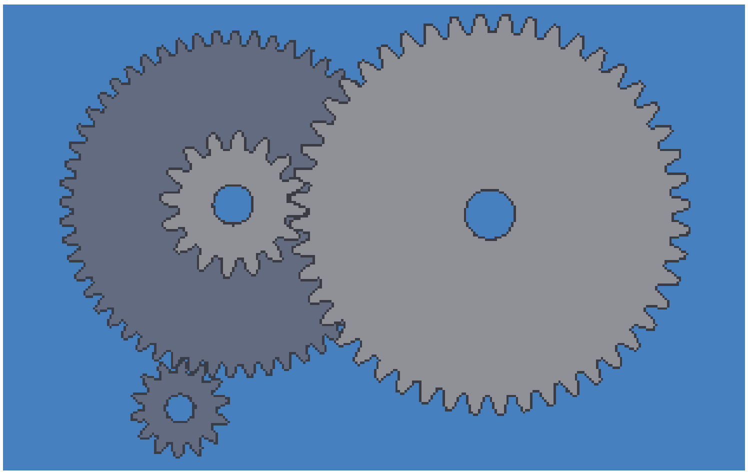


Figure 5.1 Two-stage Gear Train

The inverse problem that of finding the number of teeth of each gear that satisfies the overall gear ratio is much more difficult. This problem is defined in literature as an optimization problem [25]. The objective of the gear train design is to find the number of teeth in each of the gears to minimize the error between the obtained gear ratio and the required gear ratio. Since the number of teeth must be integers, all variables are strictly integers. The optimization problem formulation for two-stage gear drive is as follows:

$$\text{minimize } f(N) = m_G - \frac{N_{G_1}}{N_{P_1}} \frac{N_{G_2}}{N_{P_2}} \ldots\ldots \frac{N_{G_i}}{N_{P_i}} \qquad (5.3)$$

subject to:

$$N_{p\,\min} \leq N_{p_i} \leq N_{p\,\max} \qquad (5.4)$$

$$N_{G\min} \leq N_{G_i} \leq N_{G\max} \qquad (5.5)$$

all N_i (number of teeth) values must be integer.

Continued or chain fractions can be used to determine the number of teeth for the pinion and gear in order to have a required gear ratio with a permissible error [26]. This method is not applicable to code manually in a program to the multi-stage gear drives because of increasing number of variables.

A numerical method is given to determine the number of teeth for the pinion and gear for two-stage gear drive satisfying the required ratio [27]. In this study, this algorithm is modified and expanded up to six stages satisfying the gear ratio with the required precision.

m_G = overall gear ratio

eps = ± required precision on overall gear ratio

$$m_{Gmin} = m_G - eps \qquad (5.6)$$

$$m_{Gmax} = m_G + eps \qquad (5.7)$$

$$P = Int((N_{p1}N_{p2}.....N_{pi})\ m_{Gmax}) \qquad (5.8)$$

$$Q = Int((N_{p1}N_{p2}.....N_{pi})\ m_{Gmin})+1 \qquad (5.9)$$

where,

i : Number of stages

m_{Gmin} = Allowable minimum overall gear ratio

m_{Gmax} = Allowable maximum overall gear ratio

A flowchart of the algorithm for three-stage gear drive is shown in Figure 5.2.

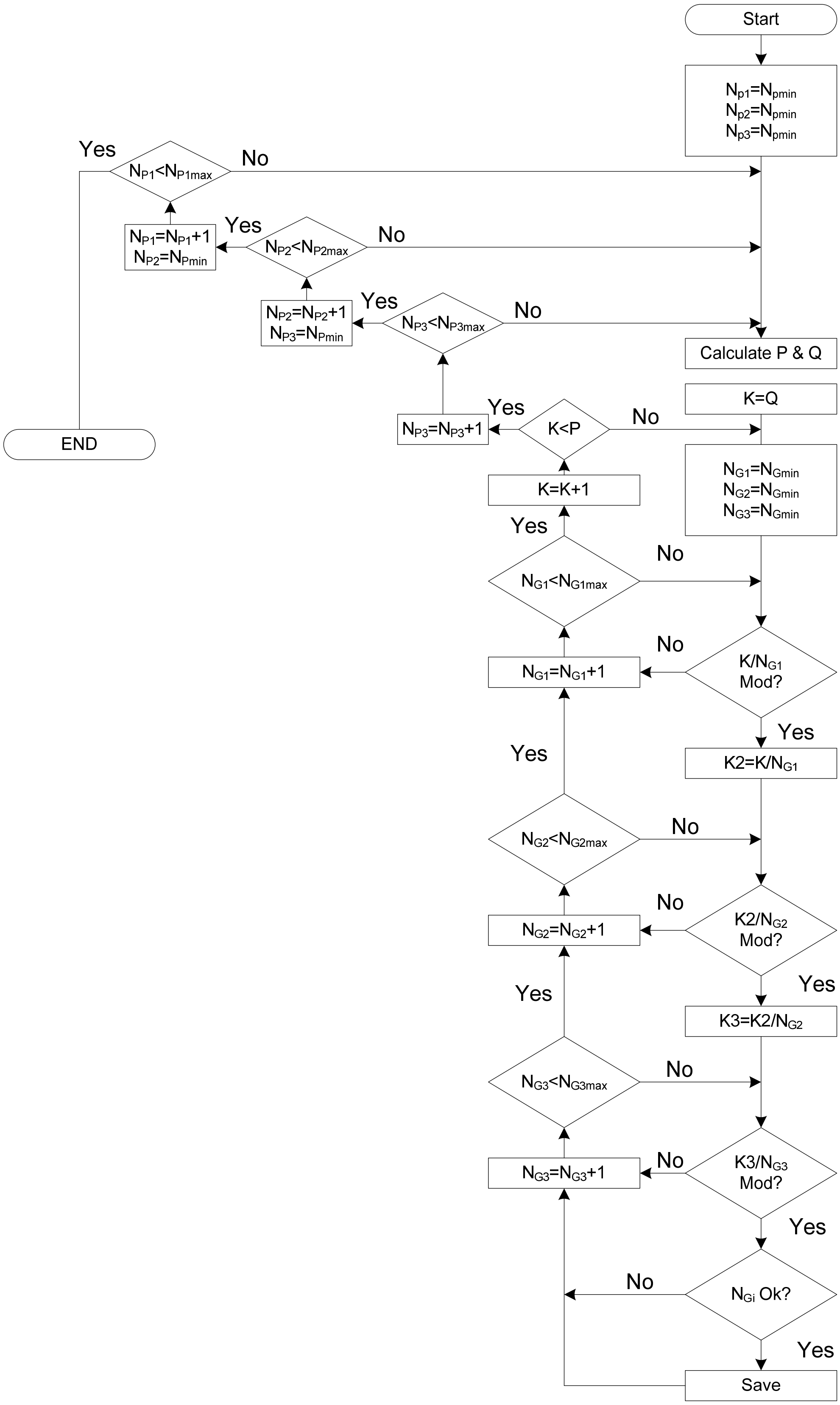


Figure 5.2 Flowchart to split overall gear ratio for three stage gear drive

5.3.2 Gear Ratio Constraints

No definite rule has been proposed to determine the gear ratios. The method recommended by Niemann is a practical one, in which gear ratios are based on the Hertz contact stress formula. This application is limited to the design of two-stage and three-stage gear drives [28]. Also with this method, to allocate the overall gear ratio one must previously determine the number of teeth and module [29].

In this study allocation of overall gear ratio to the required number of reduction stages (maximum 6-stage) is done with respect to the following premises:

- Each gear ratio must be greater than the successive gear ratio. In the developed program the option is given to the user to select any gear ratio also to be equal to each other to increase the alternatives,
- The gear ratios must be in such an order that decreases from the input of the gear drive to the output of the gear drive,
- Each gear ratio must not be an integer to avoid the same teeth coming in contact periodically. In the developed program the option is given to the user to select any gear ratio to be an integer to increase the alternatives.

5.4 Determination of Design Parameters

The design of gears usually requires some iteration. Not enough information exists in the problem to directly solve for the unknowns. The values of some parameters must be assumed and a trial solution done.

Usually, the power and speed are defined at the beginning of the problem. The parameters to be determined are the dimensions of the gear arrangements such as module, diameter and face width. Then the objective is to select the minimum volume gear pairs that satisfy the required gear ratio considering the performance criterion. Total gear volume is found from the following equation:

$$V = \sum_{i=1}^{n} F_i C_i^2 \frac{N_{pi}^2 + N_{Gi}^2}{(N_{pi} + N_{Gi})^2} \tag{5.10}$$

In order not to check all possible modules, a preliminary design is made to find a provisional value for module [30].

$$m = \left(\frac{2TK_a K_s K_m}{N_p^2 K_v J_p m_a S_{at}}\right)^{1/3} \tag{5.11}$$

where,

m : Module, (mm)
T : Transmitted torque,(Nmm)
m_a : Aspect ratio

Since the transmitted power and input speed are known from the beginning then transmitted torque is found from the below equation:

$$T = \frac{30000.P}{p.n_p} \tag{5.12}$$

where,

P : Transmitted power, (W)
n_p : Pinion speed, (rpm)

Since the number of teeth of pinion and gear for each stage is known from the previous steps, aspect ratio is found from Equation 5.13 as defined in [31].

$$m_a = \frac{mg}{mg + 1} \tag{5.13}$$

where,

$$m_G = N_G / N_P \tag{5.14}$$

Provisional face width is found from the Equation 5.15 as defined in [31]:

$$F = \frac{m_g}{m_g + 1} m N_p \tag{5.15}$$

AGMA rating factors in the preliminary module equation are known except the K_v value.

A provisional value for $K_v = 0.7$ is selected for initial value as recommend by AGMA [21].

Allowable bending stress number is found from the Equation 5.16:

$$S_{at} = \frac{S_t K_L}{K_T K_R} \tag{5.16}$$

AGMA geometry factor J is found from a complicated algorithm explained in the previous chapters. The addendum modification coefficients of pinion and gear are determined automatically by equalizing the ratios of bending strengths and geometry factors between pinion and gear.

$$\frac{J_P}{J_G} = \frac{S_{atG}}{S_{atP}} \tag{5.17}$$

The preliminary module found by the above equation is rounded to a standard module according to ISO standard [16].

For all stages the above equations are applied. Than with the preliminary module value, all pinion and gear dimensions are found. With this values AGMA contact stress check is made for all stages with the following equation

$$C_P\left(\frac{W_t C_a}{C_V}\frac{C_S}{Fd}\frac{C_m C_f}{I}\right)^{1/2} \leq \frac{S_C C_L C_H}{C_T C_R} \tag{5.18}$$

This control is made until the equation above is satisfied with changing the module value to a successive module value. Since the module is changed all related variables are recalculated and then contact stress check is made.

The contact stress check is made for all the stages, and then with this module value, bending stress check is made for all the pinion and gear from the Equation 5.19:

$$\frac{W_t K_a}{K_v}\frac{1.0}{Fm}\frac{K_s K_m}{J} \leq \frac{S_t K_L}{K_T K_R} \tag{5.19}$$

After ensuring all the loading cases, the safety factors are calculated. The safety factors against bending failure are found by comparing the allowable bending stress number to the calculated bending stress for the pinion and gear for all stages:

$$n_b = \frac{S_{at}}{s_b} \tag{5.20}$$

The safety factors against surface failure are found by comparing the allowable contact stress number to the calculated contact stress for the pinion and gear for all stages:

$$n_c = \frac{S_{ac}}{s_c} \tag{5.21}$$

Then if the resulting safety factors are either too large or too small, the assumed values are adjusted and the calculation repeated until it converges to an acceptable solution in order to keep package size small.

In this study because the case is multi-stage, for all possible gear pairs satisfying the loading conditions and kinematic analysis, calculations are repeated to find the optimum values (having desired proportions for the lengths of the dedendum and addendum portions of the line of action, having sufficient bottom clearance, having minimum contact stresses, having minimum bending stresses, having balanced pinion and gear bending strength, etc.) with the user interaction.

The shafts are required to transmit the torque, ignoring the bending stress in this study for simplification. Shafts are assumed to be made of alloy steel. There is no axial loading on the shaft as the gears are spur gears. The allowable shear stress for alloy steel is selected as 150 MPa. The radius of the shaft is calculated using the torsion only by Equation 5.22:

$$r_s = \sqrt[3]{\frac{2T}{\pi \tau_{all}}} \qquad (5.22)$$

where,

T : Transmitted torque, (Nmm)

τ_{all} : Allowable shear stress for shaft, (MPa)

r_s : Radius of shaft, (mm)

5.5 Configuration Design of the Gear Drive Elements

5.5.1 Definition of an Optimization Model

In general, an optimization model consists of the following three items:

- Objective Function - The objective function is a formula that expresses exactly what the function is to be optimized (minimization or maximization). In this study objective function is selected in order to minimize the volume of gearbox.
- Variables - Variables are the quantities that are to be taken under control. The decision is made by the designer to choose the variables. For this reason, variables are sometimes also called decision variables. The goal of optimization is to find the values of a model's variables that generate the best value for the objective function, subject to any limiting conditions placed on the variables. In this study the variables are selected as the coordinates of gear arrangements in the three-dimension (x, y, z).
- Constraints - Almost without exception, there must be some limit on the values of the variables in a model. These limits are expressed in terms of formulas that are a function of the model's variables. These formulas are referred to as constraints because they constrain the values the variables can take.

The configuration design is made to minimize the volume of a gearbox by using the model that defined in [32, 8]. Since pitch diameter, outside diameter, and face width is determined from the previous design steps by the developed program, the configuration design is considered as a problem of packing gears of fixed size in three-dimensional space. The formulation of the problem is defined as:

$$f = \sum_{i=2}^{k} CD_{(pinion_1 - gear_i)} \tag{5.23}$$

where,

f : Objective function

k : Number of stages

CD : Center Distance

5.5.2 Center Distance Constraints

The center distance constraints are used to provide proper meshing of gear pair (pinion and gear) of a stage. Since they are equal to zero then they are represented as equality constraint. It is shown in Figure 5.3.

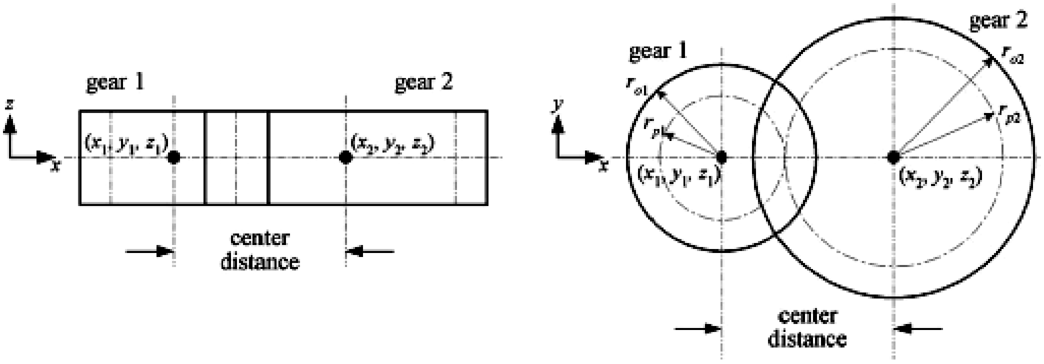


Figure 5.3 Center distance constraint [32]

$$\sqrt{(x_{(2i-1)} - x_{(2i)})^2 + (y_{(2i-1)} - y_{(2i)})^2} - \frac{d_{(2i-1)} + d_{(2i)}}{2} = 0 \qquad (5.24)$$

$$z_{(2i-1)} - z_{(2i)} = 0$$

i=1,2… to n

where,

n : total number of stages
d : pitch diameter of pinion and gear
(x, y, z) : the center position of pinion and gear
the odd subscripts are valid for pinion and the even subscripts are for gear.

5.5.3 Gear Distance Constraints

The gear distance constraints are used for mating pinion and gear having same shaft. Since they are equal to zero then they are represented as equality constraint. It is shown in Figure 5.4.

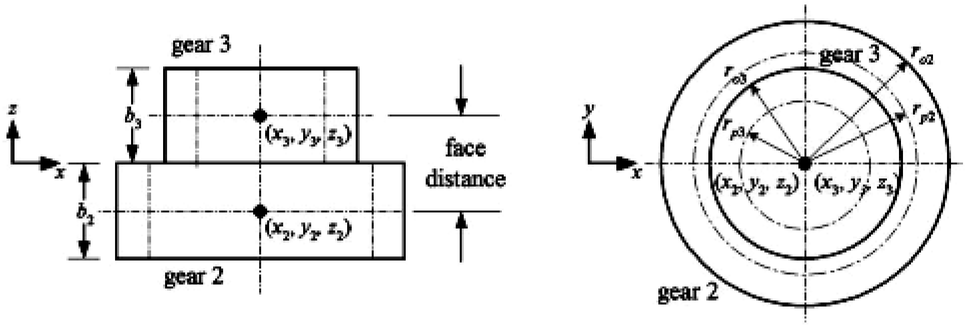


Figure 5.4 Gear distance constraint [32]

$$\left| z_{(2i)} - z_{(2i+1)} \right| - \frac{b_{(2i)} + b_{(2i+1)}}{2} = 0 \quad (5.25)$$

$$x_{(2i)} - x_{(2i+1)} = 0 \quad (5.26)$$

$$y_{(2i)} - y_{(2i+1)} = 0 \quad (5.27)$$

i=1,2… to n-1

where, b : face width of pinion and gear

5.5.4 Gear Interference Constraints

The gear interference constraints are included to avoid interferences between gears. Since they are not equal to zero then they are represented as inequality constraint. It is shown in Figure 5.5.

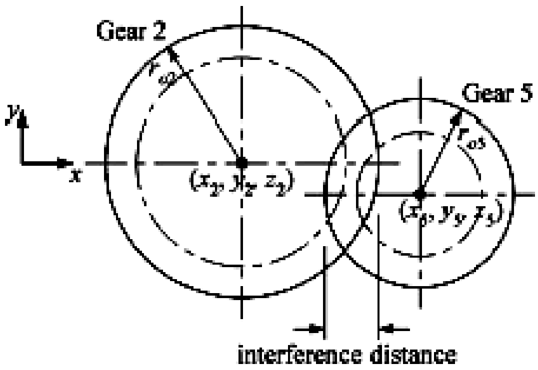


Figure 5.5 Gear interference constraint [32]

$$\sqrt{(x_{(2i-1)}-x_{(2j-1)})^2+(y_{(2i-1)}-y_{(2j-1)})^2}-\frac{d_{O(2i-1)}+d_{O(2j-1)}}{2}>0 \quad (5.28)$$

$$\sqrt{(x_{(2i-1)}-x_{(2j)})^2+(y_{(2i-1)}-y_{(2j)})^2}-\frac{d_{O(2i-1)}+d_{O(2j)}}{2}>0 \quad (5.29)$$

$$\sqrt{(x_{(2i)}-x_{(2j-1)})^2+(y_{(2i)}-y_{(2j-1)})^2}-\frac{d_{O(2i)}+d_{O(2j)}}{2}>0 \quad (5.30)$$

$$\sqrt{(x_{(2i)}-x_{(2j)})^2+(y_{(2i)}-y_{(2j)})^2}-\frac{d_{O(2i)}+d_{O(2j)}}{2}>0 \quad (5.31)$$

i=1,2… to n-2

j=i+2,i+3… to n

where, d_o : outside diameter of pinion and gear

5.5.5 Shaft Interference Constraints

The shaft interference constraints are included to avoid interferences between gears and shafts. Since they are equal not equal to zero then they are represented as inequality constraint. It is defined as follows:

$$\sqrt{(x_{S(i)} - x_{(2j)})^2 + (y_{(i)} - y_{(2j)})^2} - \frac{d_{S(i)} + d_{O(2j)}}{2} > 0 \qquad (5.32)$$

i = 1, 2…to ns-2

j = i+1, i+2…to n

$$\sqrt{(x_{S(i)} - x_{(2j)})^2 + (y_{S(i)} - y_{(2j)})^2} - \frac{d_{S(i)} + d_{O(2j)}}{2} > 0 \qquad (5.33)$$

i = 3, 4…to ns

j = 1, 2…to i-2

$$\sqrt{(x_{S(i)} - x_{(1)})^2 + (y_{S(i)} - y_{(1)})^2} - \frac{d_{S(i)} + d_{O(1)}}{2} > 0 \qquad (5.34)$$

i = 3, 4…to ns

where,

d_s : pinion and gear shaft diameter

(x_s, y_s, z_s) : center position of shaft

ns : number of shafts

Since the number of constraints increases considerably with a larger number of stages, it is inefficient and error-prone to code them manually in the program. For example there are 92 constraints existing in a six-stage gear drive. Thus, this nonlinear constrained optimization problem is solved using a software package Lingo 8.0 with the DLL (Dynamic Link Library) interface together with the Visual Basic program automatically.

There are two options in the developed program for gearbox configuration.

- *Compact Gearbox Design:* The objective function together with all the constraints described above are necessary for this configuration design.

- *In-line Gearbox Design:* All the constraints are also necessary for this type of configuration but there is no need to construct an objective function. Extra slope constraints exist in this design to arrange the gear pairs in line.

In the developed program, 3mm clearance is intentionally given for the inequality constraints to provide gap between gears.

5.6 Lingo 8.0 Optimization Software

LINGO is a simple tool for utilizing the power of linear and nonlinear optimization to formulate large problems coincisely, solve them, and analyze the solution. In this study, the objective function is minimized for the total volume of gearbox.

Optimization problems are often classified as linear or nonlinear, depending on whether the relationships in the problem are linear with respect to the variables. In this study since most of the constraints are nonlinear, with the usage of nonlinear solver of Lingo 8.0, the equations are solved simultaneously satisfying the constraints.

If the model does not have any nonlinear constraints, then any locally optimal solution will also be a global optimum. Thus, all optimized linear models will terminate in the global optimum state. If, on the other hand, as in this study, the model has one or more nonlinear constraints, then any locally optimal solution may not be the best solution to the model. There may be another "extremum" that is better than the current one.

That is to say, there is no guarantee to reach a global minimum unless the problem is continuous and has only one minimum. Starting the optimization from a number of different starting points can help to locate the global minimum or verify that there is only one minimum.

Lingo uses SLP (Successive Linear Programming) directions and GRG (Generalized Reduced Gradient) strategy programming with its default settings and there are four other options exist in the software as shown in Figure 5.6.

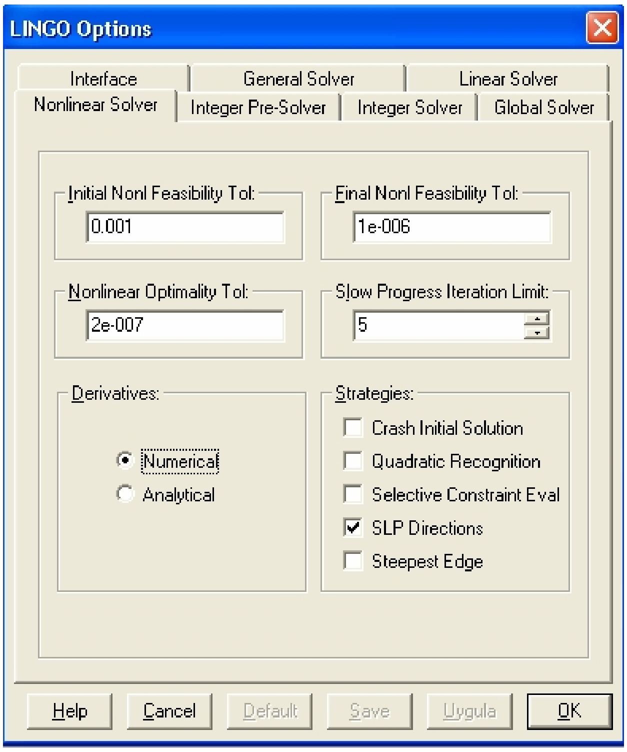


Figure 5.6 Lingo options

- Crash Initial Solution

If the Crash Initial Solution box is checked, Lingo's nonlinear solver will invoke a heuristic for generating a "good" starting point when you solve a model. If this initial point is relatively good, subsequent solver iterations should be reduced along with overall runtimes.

- Quadratic Recognition

If the Quadratic Recognition box is checked, LINGO will use algebraic pre-processing to determine if an arbitrary nonlinear model is actually a quadratic programming (QP) model. If a model is found to be a QP model, then it can be passed to the faster quadratic solver. Note that the QP solver is not included with the standard, basic version of LINGO, but comes as part of the barrier option.

- Selective Constraint Evaluation

If the Selective Constraint Eval box is checked, Lingo's nonlinear solver will only evaluate constraints on an as needed basis. Thus, not every constraint will be evaluated during each iteration. This generally leads to faster solution times, but can also lead to problems in models that have functions that are undefined in certain regions.

- SLP Directions

If the SLP Directions box is checked, Lingo's nonlinear solver will use successive linear programming to compute new search directions. This technique uses a linear approximation in search computations in order to speed iteration times. In general, however, the number of total iterations will tend to rise when SLP Directions are used.

- Steepest Edge

If the Steepest Edge box is checked, Lingo's nonlinear solver will use the steepest-edge strategy when selecting variables to iterate on. When LINGO is not in steepest-edge mode, the nonlinear solver will tend to select variables that offer the highest absolute rate of improvement to the objective, regardless of how far other variables may have to move per unit of movement in the newly introduced variable. The problem with this strategy is that other variables may quickly hit a bound, resulting in little gain to the objective.

With the steepest-edge option, the nonlinear solver spends a little more time in selecting variables by looking at the rate that the objective will improve relative to movements in the other nonzero variables. Thus, on average, each iteration will lead to larger gains in the objective. In general, the steepest-edge option will result in less iteration. However, each iteration will take longer.

There is also a Global Solver option available in the Lingo 8.0. If the Global Solver box is checked, LINGO will invoke the global solver to solve a model. Many nonlinear models are non-convex and/or non-smooth. Nonlinear solvers that rely on local search procedures will tend to do poorly on these types of models. Typically, they will converge to a local, sub-optimal point that may be quite distant from the true, global optimal point. Global solvers overcome this weakness through methods of range bounding (e.g., interval analysis and convex analysis) and range reduction techniques (e.g., linear programming and constraint propagation) within a branchand-bound framework to find the global solutions to non-convex models. The only drawback to the global solver is that it runs considerably slower than the default local solver. Therefore, the preferred option is to always try and write smooth, convex nonlinear models, so the faster, default local solver can successfully solve them.

5.7 Gearbox Volume Calculation

Gearbox volume is calculated after the configuration design by the following methods.

5.7.1 Enclosed Gearbox Volume Calculation

Enclosed volume is calculated by finding the intersection points between the y lines and the circles (meshing gears). It is made by the following equation:

$$V_{enclosedvolume} = (z_{\max} - z_{\min}) \int_{y=y_{\min}}^{y=y_{\max}} (x_{\max} - x_{\min}) dy \qquad (5.35)$$

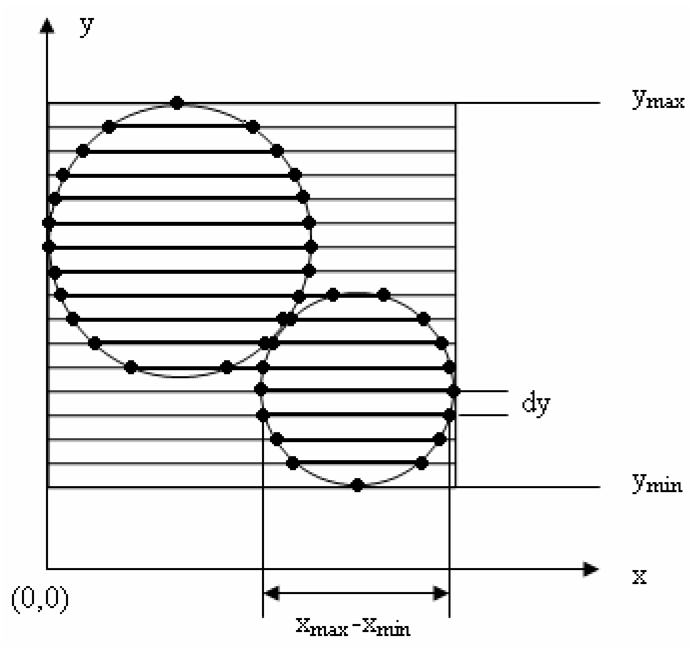


Figure 5.7 Enclosed Gearbox Volume Calculation

5.7.2 Prismatic Gearbox Volume Calculation

Prismatic volume is calculated simply by multiplying the width, height and the depth of the gearbox. It is made by the following equation:

$$V_{prismaticvolume} = (x_{\max} - x_{\min})(y_{\max} - y_{\min})(z_{\max} - z_{\min}) \qquad (5.36)$$

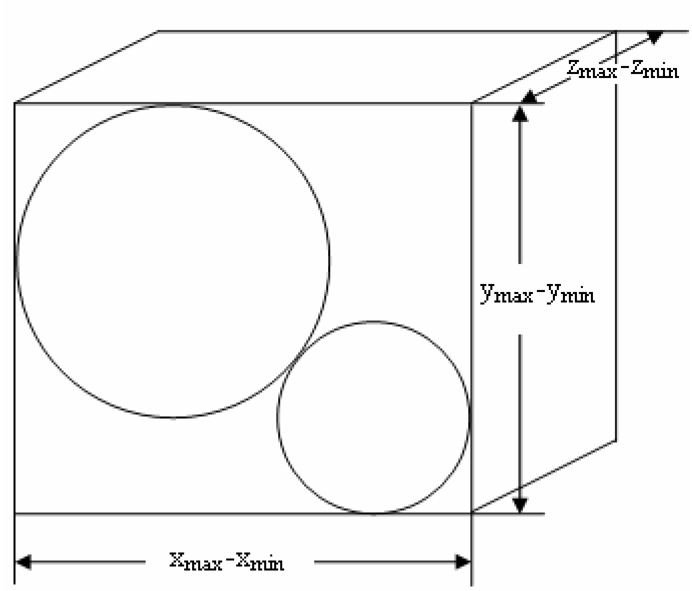


Figure 5.8 Prismatic Gearbox Volume Calculation

CHAPTER 6

A SAMPLE APPLICATION WITH DEVELOPED SOFTWARE

In this chapter, a sample application is solved with developed software, which is originally introduced by Chong et al. [32]. Table 6.1 shows the design results and Figure 6.1 shows two and three-dimensional representation of the gears according to that study.

Table 6.1 Design result, four-stage gear drive [32]

Stage	1	2	3	4
Module (mm)	1.25	2.0	2.5	4.0
Number of teeth in pinion	19	18	24	24
Number of teeth in gear	97	74	92	88
Gear ratio	5.105	4.111	3.833	3.667
Pitch diameter of pinion (mm)	23.75	36.0	60.0	96.0
Pitch diameter of gear (mm)	121.25	148.0	230.0	352.0
Face width factor	14	15	14	15
Face width (mm)	18.75	28	37.5	56.0
Volume of gearbox (mm^3)	22.69×10^6			

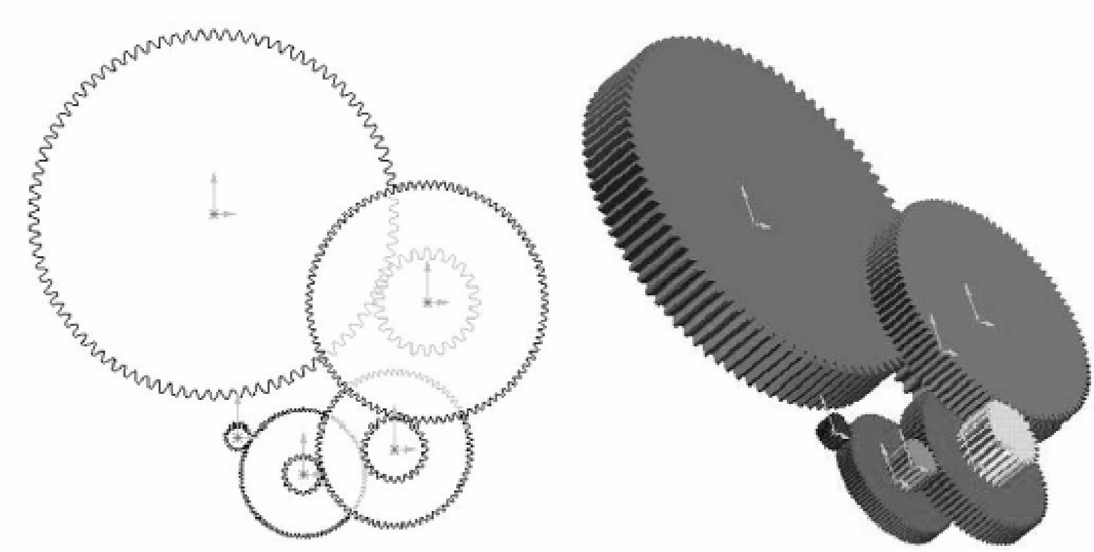


Figure 6.1 Configuration design of the gear drive, front and isometric view [32]

CASE:

Design a four-stage spur gear drive with transmitted power of 8 kW at an input speed of 6000 rpm and total gear ratio of 300(± 0.0001). 55 HRC carburized and case hardened steel gears are used. Face width should be between 4 to 15 module. To account dynamic loads AGMA Quality of number 11 should be used. Straddle mounted with unmodified leads, $(S/S_1) < 0.175$, commercial enclosed gear units and gearing adjusted at assembly selections. Pinions and gears are cut with a rack cutter or a hob that has a tool edge radius of 0.25 m. For all stages select minimum number of teeth for pinion as 14 and maximum number of teeth for pinion as 25 and minimum number of teeth for gear as 70 and maximum number of teeth for gear as 85. Tooth thinning for backlash is 0.024 module both for pinion and gear as advised by AGMA. Pinion should run minimum 10^7 cycles on this commercial application. Both preferred and second choice modules are to be used to increase the alternatives. Power source is a multi-cylinder engine and the driven machine is the textile machinery. Overall reliability is selected as 0.99. Lubricant temperature is 120° C. Power loss through the gear train can be assumed negligible.

Table 6.2 Specifications for design example

Transmitted Power (kW)	8
Total Gear Ratio (± 0.0001)	300
Input Speed (rpm)	6000
Gear Type	External Spur
Pressure Angle (°)	20
Modification	Exist
Material	Steel
Heat Treatment	Carburised and Case Hardened
Material Hardness	55.0 HRC
AGMA Quality Number	11
Load Cycles	10^7
Tool Edge Radius (module)	0.25

The problem is input to the program by the user as follows:

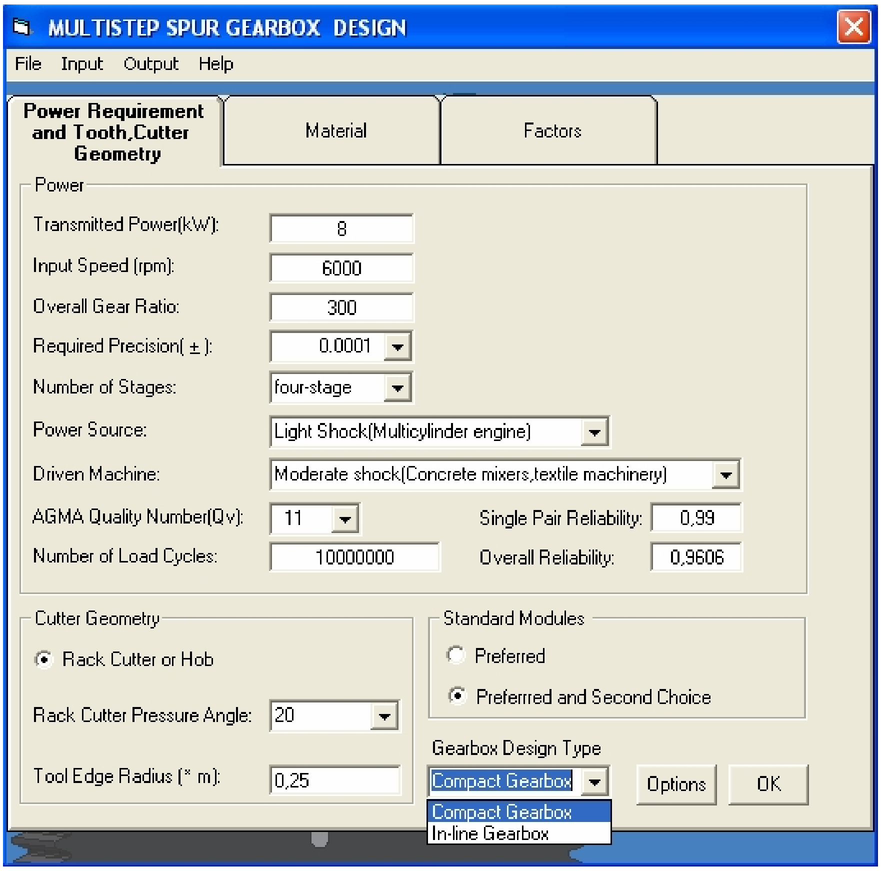


Figure 6.2 Input form, power requirement, tooth, and cutter geometry

Options are given to the user for design parameters are as follows:

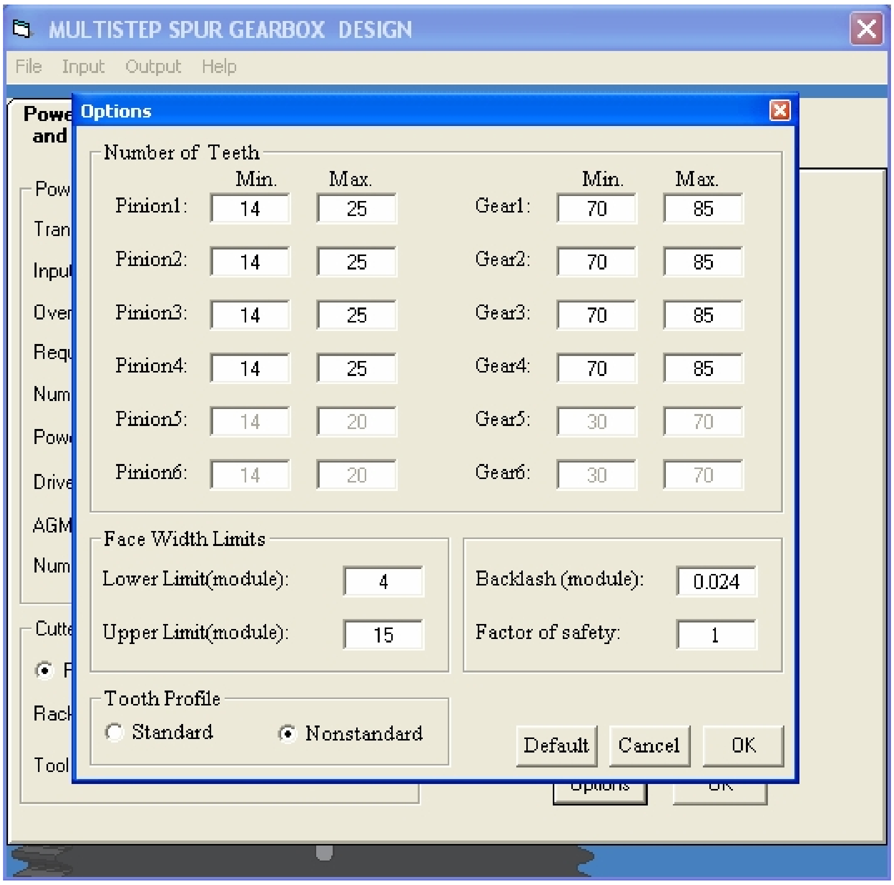


Figure 6.3 Options form

Gear and Pinion materials are selected from material page as follows, option is given to the user to select different materials for each pinion and gear, and also hardness values are changeable when necessary. And the material is selected as follows:

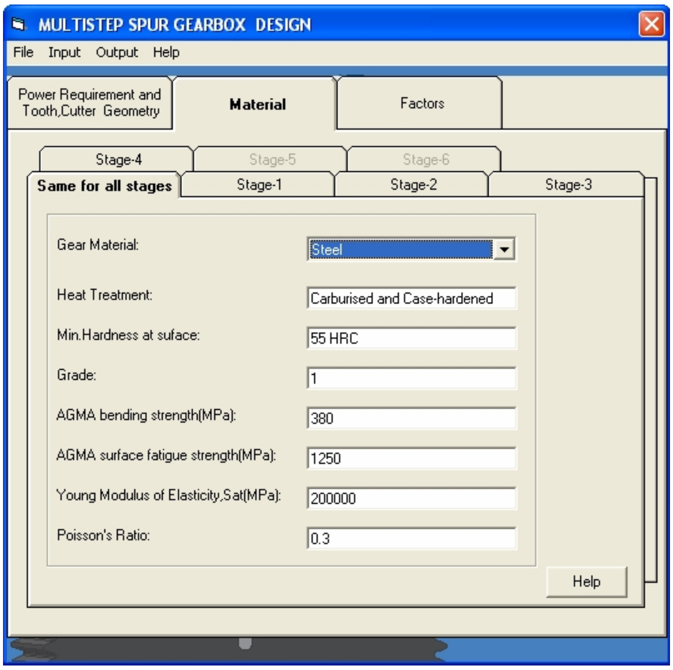


Figure 6.4 Input form, selected material

Mounting type is selected as straddle mounting with its options follows:

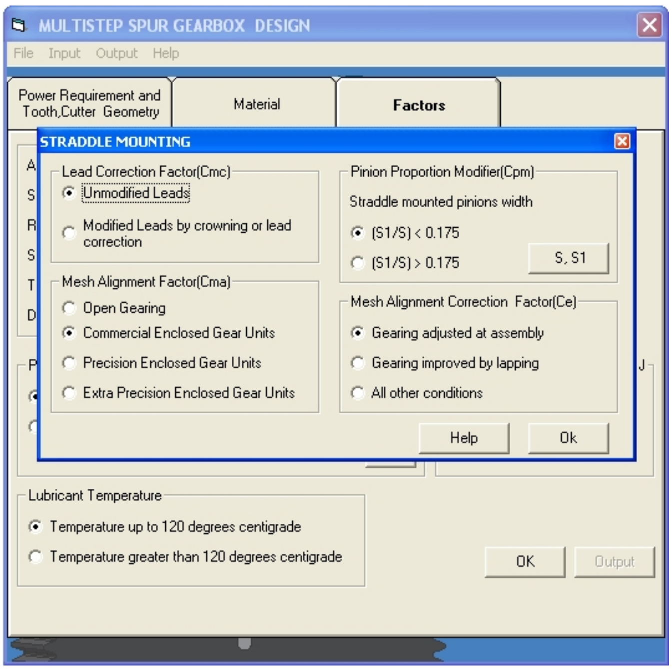


Figure 6.5 Input form, straddle mounting

And the other factors are selected as defined in the problem to the AGMA standard recommendations are as follows:

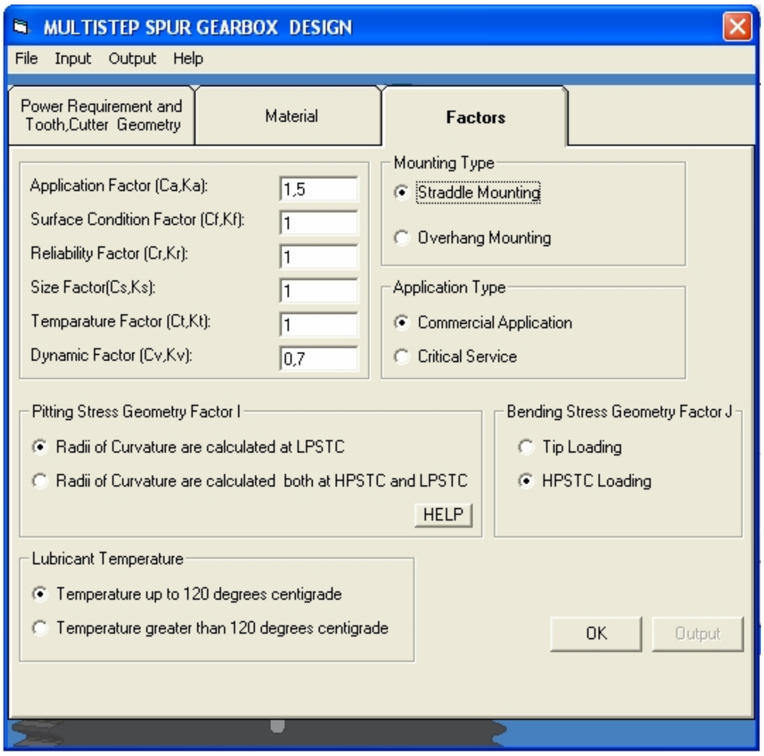


Figure 6.6 Input form, AGMA rating factors

After all the initial values are input to the developed program then overall gear ratio is split to the stages with the desired accuracy and the restrictions. In this problem there are 1222 alternative gear pairs for the required gear ratio. Since all the alternatives provide required precision, all of them are valid values for design. User can select all the gear pairs or the ones that are applicable for the application.

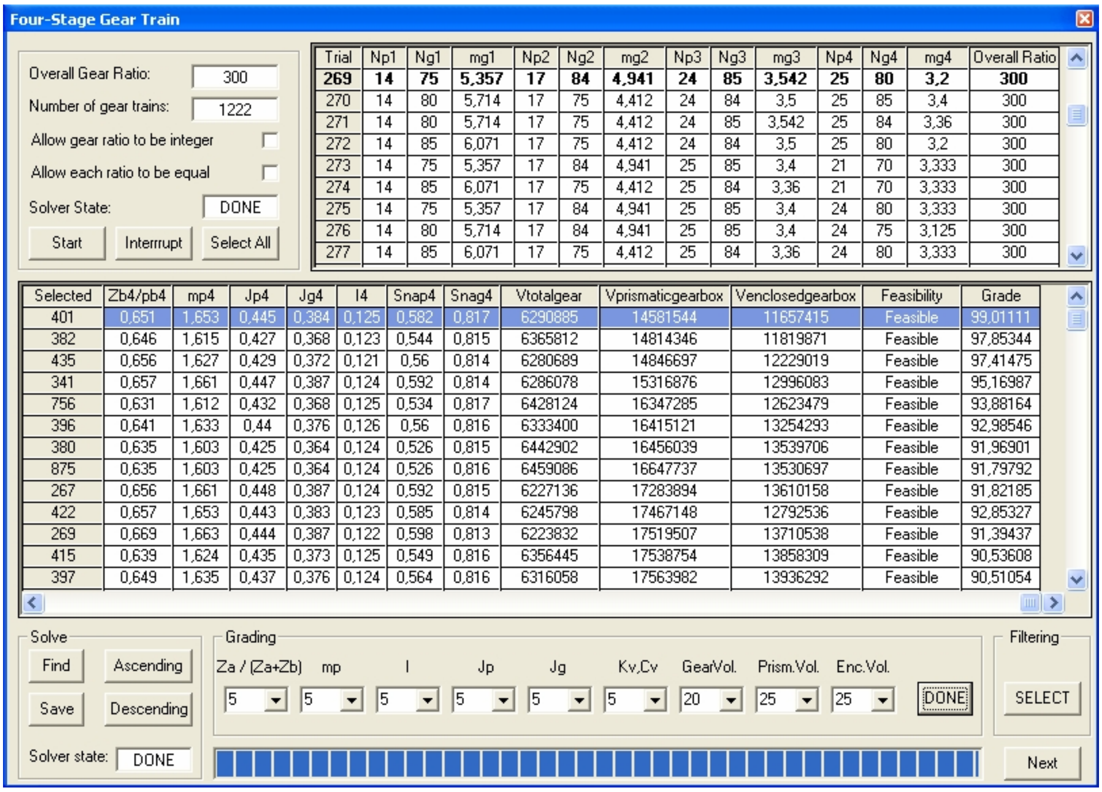
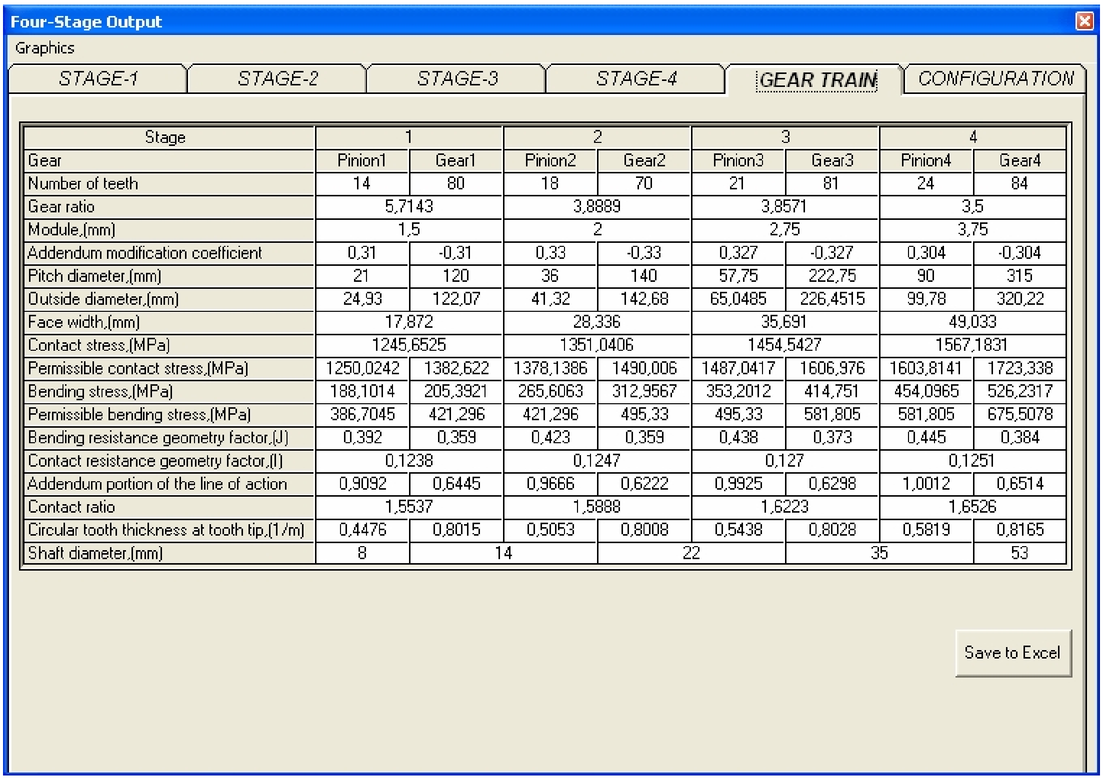


Figure 6.7 Input form, alternative gear pairs

In this application all alternatives are selected and solved as follows. The minimum volume value is selected considering the performance variables. Also a grading option exists in the developed program in order to make a decision from the various performance variables easily.

After selection is made with respect to the performance variables and total volume of gearbox then the output is taken shown in Figure 6.8. This output is containing all stages together in a single page, for the designer to analyse the solutions easily. Option is given to the user to consider all stages separately if necessary.

Overall gear ratio = (80/14) * (70/18) * (81/21) * (84/24) = 300. Redesigned gearbox satisfies the overall gear ratio exactly while there is 1.66% error on overall gear ratio in the original problem.



Stage	1		2		3		4	
Gear	Pinion1	Gear1	Pinion2	Gear2	Pinion3	Gear3	Pinion4	Gear4
Number of teeth	14	80	18	70	21	81	24	84
Gear ratio	5,7143		3,8889		3,8571		3,5	
Module,(mm)	1,5		2		2,75		3,75	
Addendum modification coefficient	0,31	-0,31	0,33	-0,33	0,327	-0,327	0,304	-0,304
Pitch diameter,(mm)	21	120	36	140	57,75	222,75	90	315
Outside diameter,(mm)	24,93	122,07	41,32	142,68	65,0485	226,4515	99,78	320,22
Face width,(mm)	17,872		28,336		35,691		49,033	
Contact stress,(MPa)	1245,6525		1351,0406		1454,5427		1567,1831	
Permissible contact stress,(MPa)	1250,0242	1382,622	1378,1386	1490,006	1487,0417	1606,976	1603,8141	1723,338
Bending stress,(MPa)	188,1014	205,3921	265,6063	312,9567	353,2012	414,751	454,0965	526,2317
Permissible bending stress,(MPa)	386,7045	421,296	421,296	495,33	495,33	581,805	581,805	675,5078
Bending resistance geometry factor,(J)	0,392	0,359	0,423	0,359	0,438	0,373	0,445	0,384
Contact resistance geometry factor,(I)	0,1238		0,1247		0,127		0,1251	
Addendum portion of the line of action	0,9092	0,6445	0,9666	0,6222	0,9925	0,6298	1,0012	0,6514
Contact ratio	1,5537		1,5888		1,6223		1,6526	
Circular tooth thickness at tooth tip,(1/m)	0,4476	0,8015	0,5053	0,8008	0,5438	0,8028	0,5819	0,8165
Shaft diameter,(mm)	8	14		22		35		53

Figure 6.8 Output form, four-stage gear drive

With the graphics option of the program, performance variables, gear ratio split and module, pitch diameter and face width, dynamic factors, volume of gears with respect to the stages are generated in a graphical interface automatically. They are shown through Figure 6.10 to 6.14.

The contact ratios for all stages are greater than 1.4 as recommended by standards for a smooth operation.

Circular tooth thickness at tooth tips for both pinion and gear are greater than 0.3 module as recommended by AGMA, avoiding excessively thinned at tooth tip automatically. Since smaller numbers of teeth are used for small packaging purposes, default addendum modification coefficient is given to the pinion and the same but opposite value for the gear, in order to prevent undercutting. In smaller number of teeth, the correction required to avoid undercut on gears operated on standard center distances is excessive, in many cases, in respect to equal tooth strength as in this study. Therefore in case of undercutting the defined equations are used for finding contact ratio and geometry factor J for both pinion and gear.

With respect to AGMA standard 218.01, LPSTC for the pinion is taken as the most critical point for calculation of AGMA geometry factor I, and all of the tables are prepared this rule. It is true for unmodified gears, or gears with small addendum modification coefficients. When there is a large positive pinion addendum modification on a pinion then there is a very long addendum on pinion, and a very short dedendum; and a gear with a very long dedendum and a very short addendum in x-zero gear pairs. These tooth geometries shift the single tooth pair contact region of the pinion above its pitch circle. A similar, but reverse shift is observed on the gear. When the single tooth pair contact regions are shifted in this way, not LPSTC, but HPSTC becomes the most critical point [19]. Therefore the program is given options to the user for calculation of radii of curvature, as defined by AGMA the critical, at LPSTC or both at LPSTC and at HPSTC.

It is clearly seen from the following Figure 6.11 that with increasing torque, from the first stage to the last stage, the module increases. The overall gear ratio is split to stages in a descending order intentionally to compensate for the increasing module (compact volume criteria), because as the module increases from the first stage to the last stage, both face widths and diameters are increasing also seen from Figure 6.13.

Dynamic factors are related to pitch line velocity and AGMA quality number, and in this study since the AGMA quality numbers are selected same for all gears then, the K_v and C_v values are depending only to pitch line velocity. The pitch line velocity decreases from the input to the output so the dynamic factors are increasing also seen from the Figure 6.12.

As seen in the Figure 6.14 the total volume of gears is increasing from the first stage to the last stage as expected because of increasing torque.

The safety factors against bending failure are equal because of design intention (optimum design). The safety factors for bending failure are higher than those for surface failure. It is also beneficial because, the bending failure is sudden and catastrophic, resulting in tooth breakage and a disabling of the machine. Also the resulting safety factors are reasonable values (small) for the design intention.

Since the number of stages is assigned by the user, the overall gear ratio is distributed to the stages, and the dimensional design is made to minimize the total volume of gears satisfying the rating practices then all the parameters are ready for the configuration design.

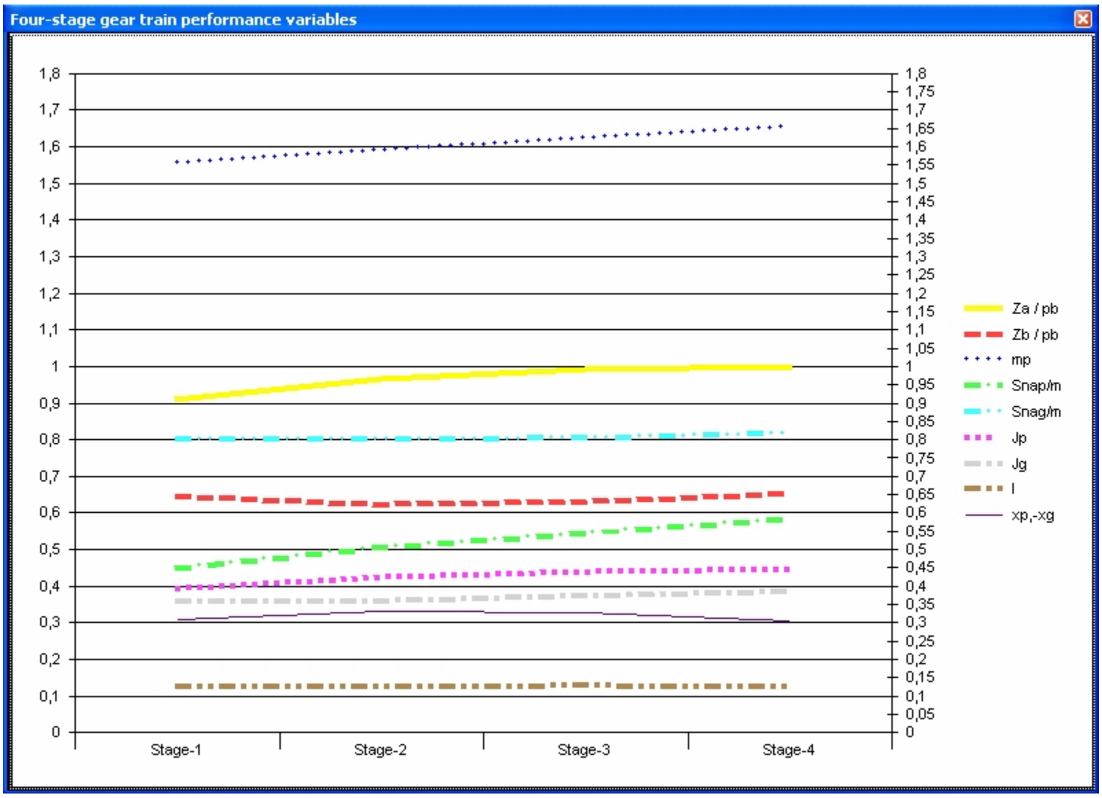


Figure 6.9 Graphics form, performance variables

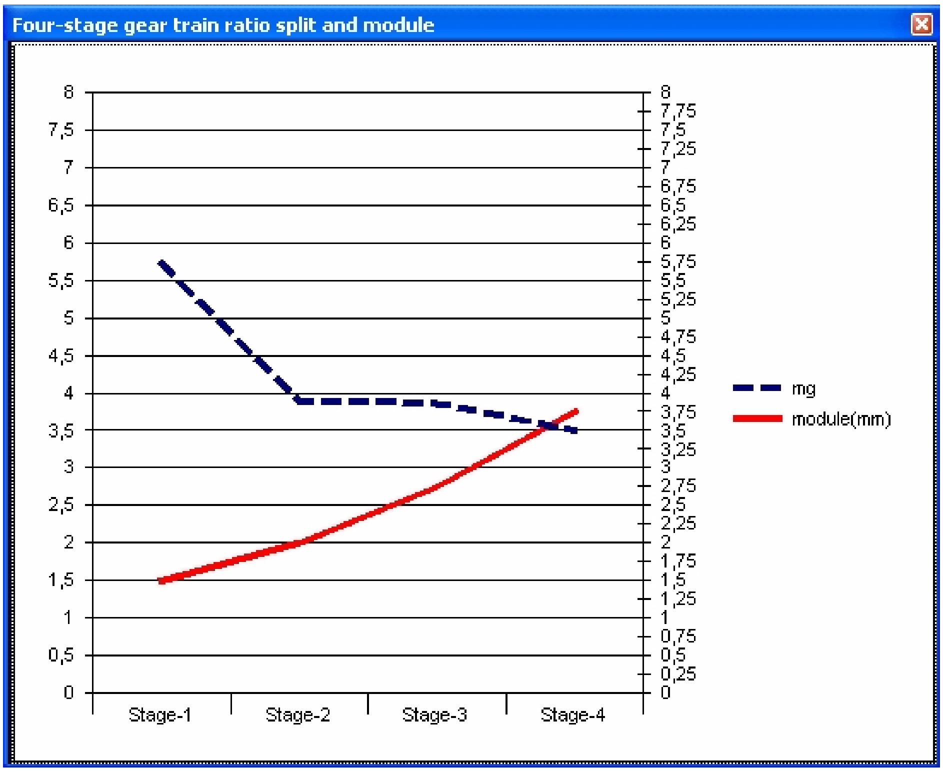


Figure 6.10 Graphics form, module and gear ratio

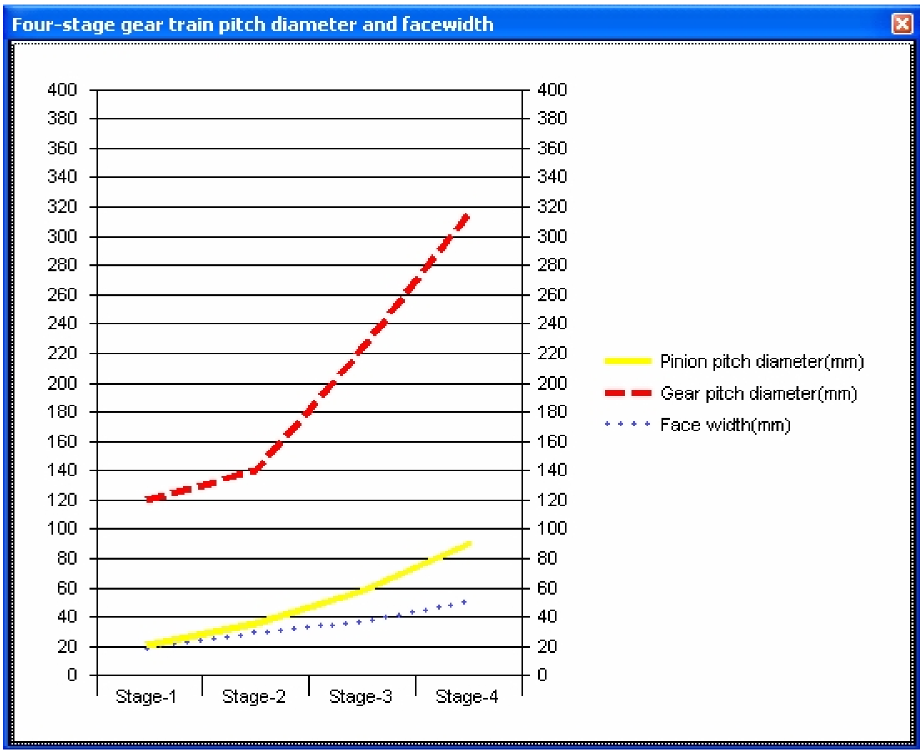


Figure 6.11 Graphics form, diameters and face widths

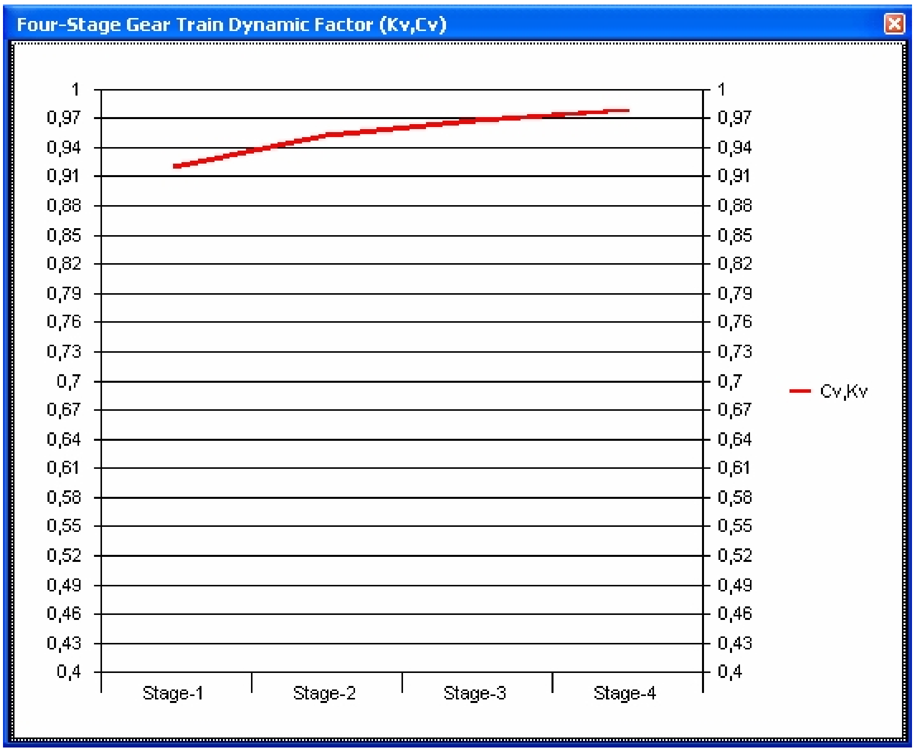


Figure 6.12 Graphics form, dynamic factors

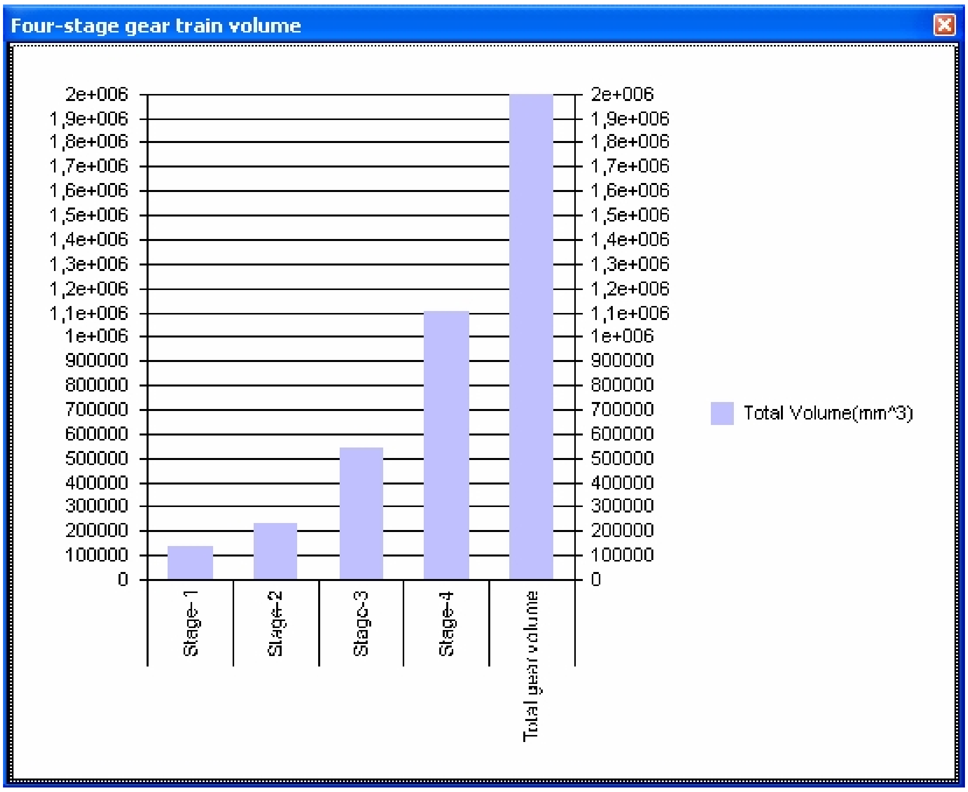


Figure 6.13 Graphics form, volume

The configuration design is made by using nonlinear solver of Lingo 8.0 to find the position of the gear drive elements by constructing an objective function containing the center distance constraints. The objective is selected to minimize the constructed function satisfying all the equality and inequality constraints simultaneously as defined in the previous chapters. The output of the Lingo 8.0 determines the location of the gear drive elements (x, y, z) that minimizes the volume of the gearbox.

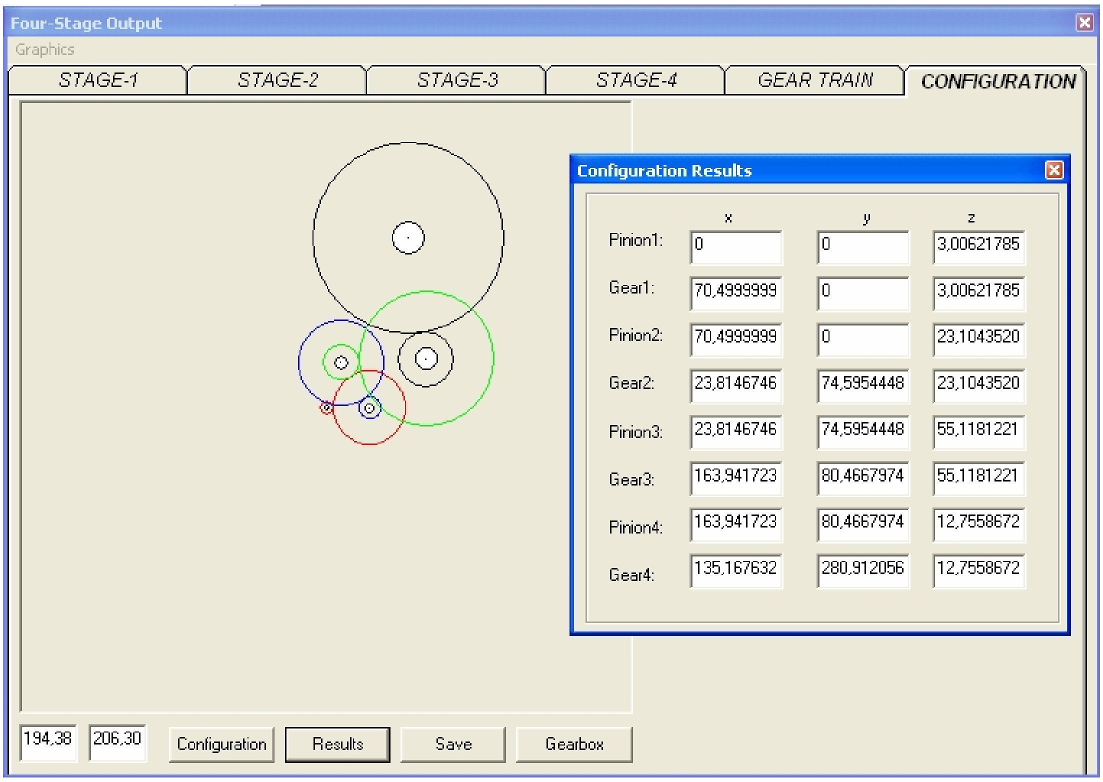


Figure 6.14 Configuration form of compact gearbox

The positions of the gear drive elements and all the dimensions are taken automatically from Lingo 8.0 in the previous step then the 3-dimensional drawing of gear train is made by the help of Autodesk Inventor 7.0 automatically. The corresponding result of the configuration design is shown in the Figure 6.15, 6.16 and solid modelling of a four-stage gear drive is seen in Figure 6.17. The gears reveal a tendency to minimize the total volume of the gearbox, while pinions and gears are meshing properly while gears and shafts are avoiding interferences between them.

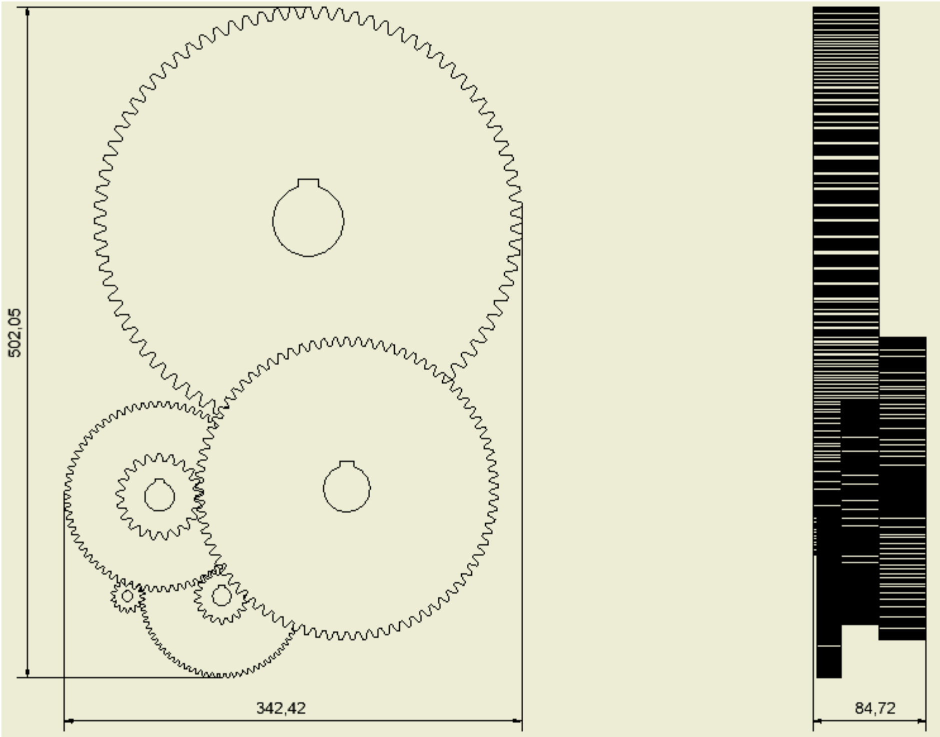


Figure 6.15 Configuration design of compact gearbox, front and side view

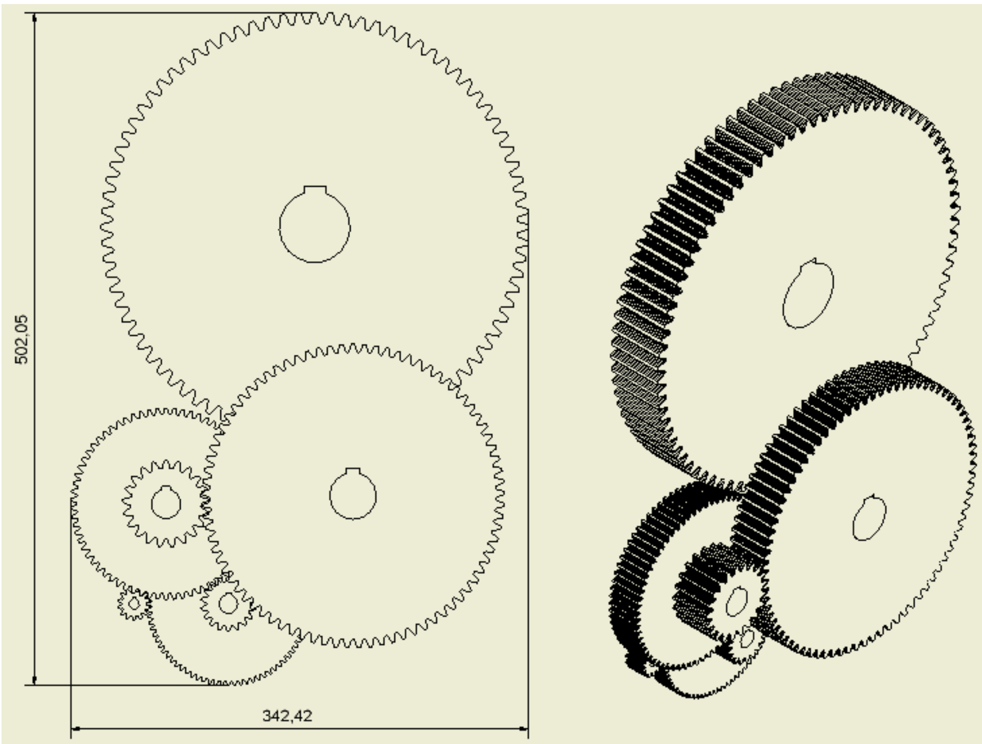


Figure 6.16 Configuration design of compact gearbox, front and isometric view

Prismatic gearbox volume is calculated after the configuration design, simply by multiplying the width, height and the depth of the gear train. Net gearbox volume = 502.05 x 342.42 x 84.72 = 14.6 x 10^6 mm^3. The redesigned gearbox reduced its net volume considerably (36%) compared with the original case, and the dimensional design shows reasonable result with using smaller number of teeth. The configuration is different from the original case as expected. The gears also reveal a tendency to minimize the total volume of the gearbox, while pinions and gears are meshing properly while gears and shafts are avoiding interferences between them.

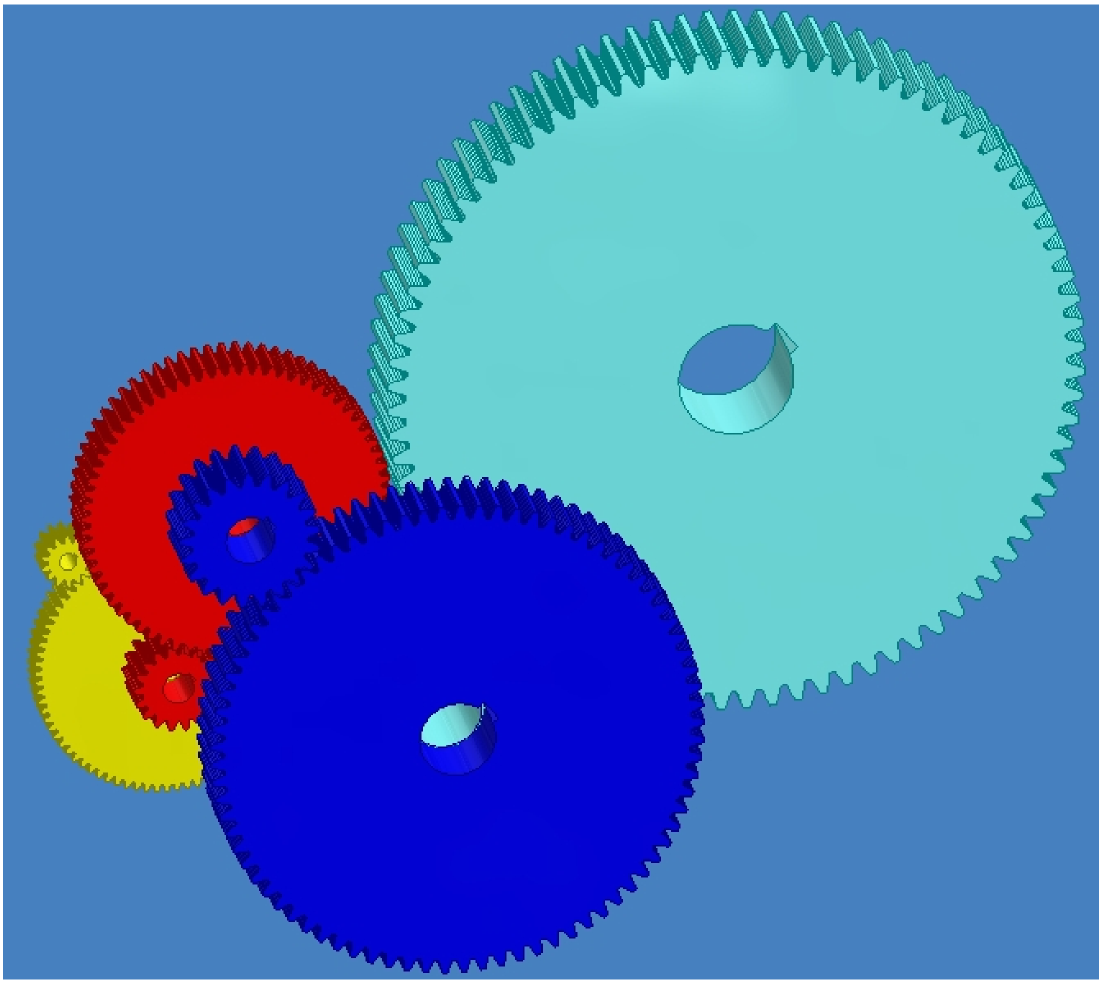


Figure 6.17 Configuration design of compact gearbox, solid model

As mentioned before, the Lingo 8.0 carries out a configuration design only for the gears satisfying the AGMA strength and durability rating practices and the performance variables. Thus the final gear designs are safe enough by considering strength and durability criteria.

The same problem is solved for in-line gearbox design type from the gearbox design type options. Corresponding result of the configuration is shown in the Figure 6.18 and 6.19. The pairs are arranged in line, while pinions and gears are meshing properly while gears and shafts are avoiding interferences between them. In this configuration, gearbox volume is greater than the compact gearbox design as expected.

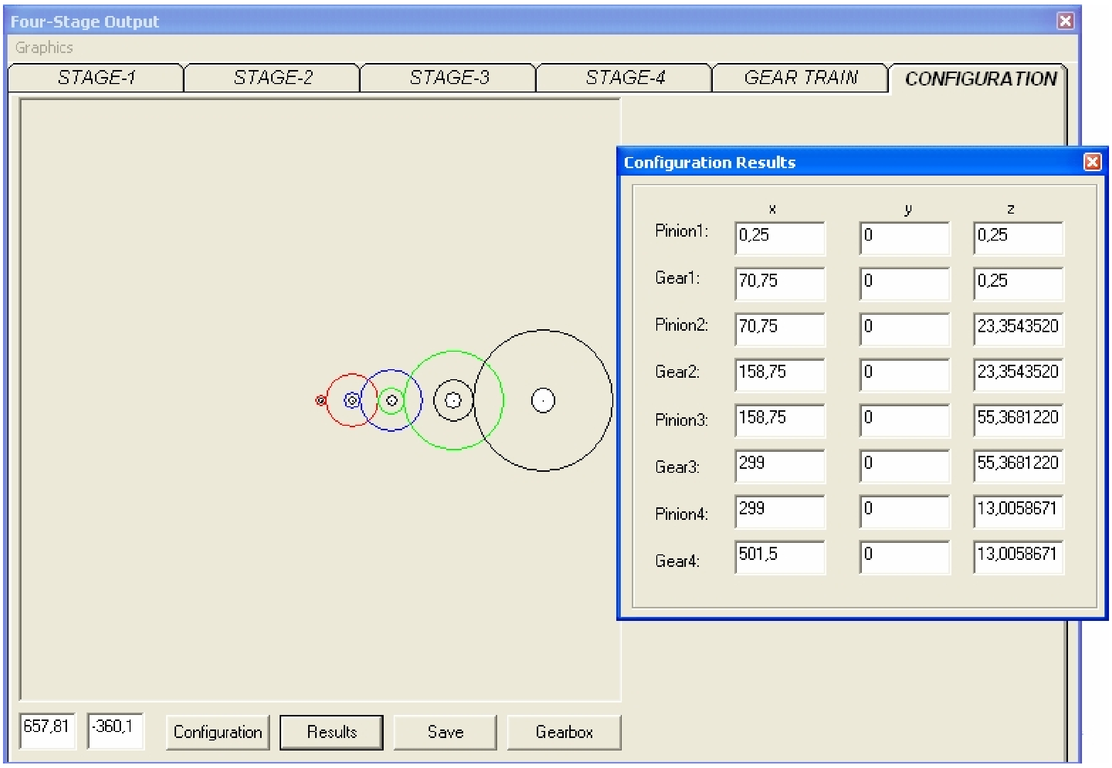


Figure 6.18 Configuration form of in-line gearbox

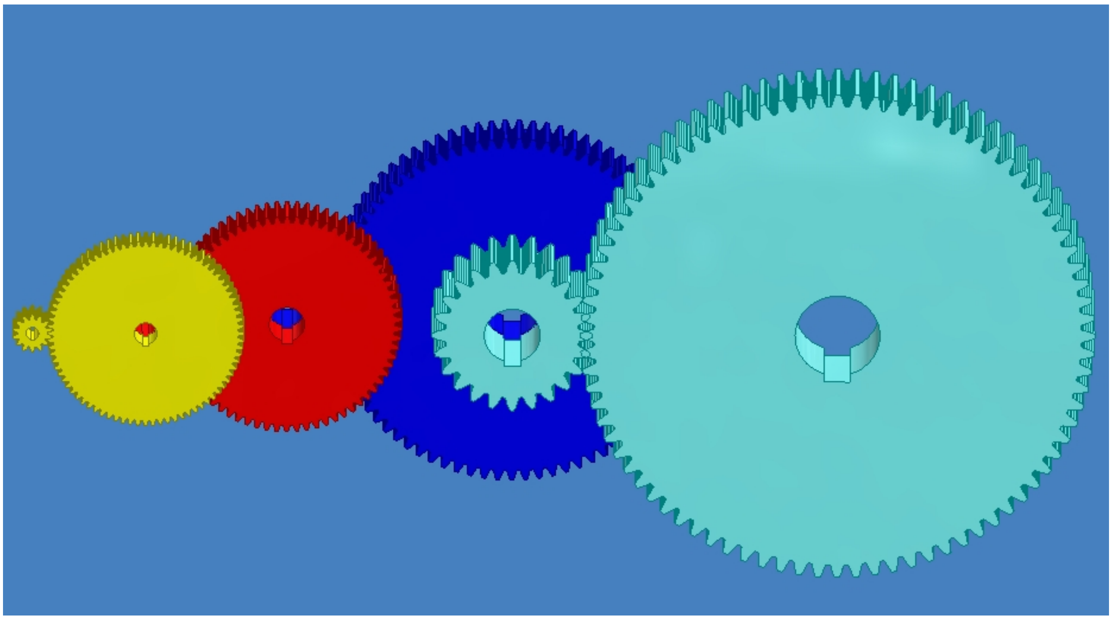


Figure 6.19 Configuration design of in-line gearbox, solid model

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

7.1 Conclusions

In this study, a program is developed for the optimum design of multistep involute standard and non-standard spur gear drive. AGMA standards are followed while rating the gear performance. Since the main design objective is the minimization of the total gearbox volume and to obtain equal strength of pinion and gear tooth for all stages, then instead of standard tooth form, a long addendum (short dedendum) form is used for pinions, and a short addendum (long dedendum) is used for gears. It is seen that by using non-standard gears, some important benefits are obtained. The most important ones are, having gears with smaller number of teeth without undercutting, reducing the total volume of gears and obtaining balanced strength between the pinion and the gear for all stages.

In the case studies, it is shown that all alternative gear pairs satisfying the required overall gear ratio with the desired accuracy is analyzed to select the minimum volume considering the performance criteria with the user interaction. Using smaller number of teeth in a normal design necessitate modification for gears only to prevent undercutting. In this study among minimum volume objective, also it is desired to equalize the factor of safeties for bending failure so sometimes the modification coefficients are not enough to prevent undercutting condition. Some of the gears are undercut and thus reduce the contact ratio. Thus a compromise must be maintained between conflicting objectives. By considering the performance criteria of all the alternatives by the help of the developed program, sufficient ones are selected for final configuration design.

The configuration design was considered as a problem of packing gears in three-dimensional space. It is problematic to conventional gradient-based optimization methods due to discontinuities and severe nonlinearities in its objective function and constraints. In this study in order to overcome these difficulties the configuration design is carried out using Lingo 8.0 optimization software with the necessary data (diameters, face widths) taken from the developed program by the use of DLL (Dynamic Link Library) interface automatically which was fixed in the previous design steps. It is made to minimize the volume of gearbox, while satisfying spatial constraints, such as gear meshing and interference. It is clearly seen from the application problem results that the minimum volume gear pairs are positioned in the gearbox in a compact position while satisfying all spatial nonlinear constraints. The major benefits of this study can be defined as follows;

- The ratio splitting algorithm, construct the overall gear ratio exactly with the desired precision from ±0.1 to ±0.00001. This algorithm is very useful not only for the single module for the design of multistep gearbox up to six stages, but also as an independent program to split overall gear ratio for any practical design problem.
- The design results are reasonable values considering the performance variables. The gear pairs are reliable for strength and durability.
- The objective function formulation and the constraints for configuration design give satisfactory results.
- The total design process will be used for design of multistep spur gear drives according to the AGMA standards, including detailed design parameters effectively.

7.2 Recommendations for Future Work

- The program is developed for multistep spur gearbox. Additional modules can be developed for other types of gears including helical, bevel, worm gearing and other types.
- Different objective functions can be used for other design intentions.
- More effective optimization software can be used for better configuration design.
- Lubrication of gears and gear dynamics can be included in the new studies.

REFERENCES

[1] Lewis, W., “Investigation of the Strength of Gear Teeth”, Proc. of the Engineers Club”, Philadelphia, pp 16-23, 1893

[2] AGMA Information Sheet 908-B89, “Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth, 1989.

[3] Mitchiner, R.G., Mabie, H.H., “The Determination of the Lewis Form Factor and the AGMA Geometry Factor J for External Spur Gear Teeth”, ASME Trans. Vol. 104, pp. 148-158, 1982.

[4] Errichello, R., “An Efficient Algorithm for Obtaining Gear Strength Geometry Factors for Shaper Cut Gears”, AGMA paper No. P139.05., 1983

[5] Pedrero, J.I., Fuentes, A., Estrems, M., “Approximate Method for the Determination of the Bending Strength Geometry Factor for External Spur and Helical Gear Teeth”, Journal of Mechanical Design, Vol. 122, pp. 331-336, 2000.

[6] Chong, T.H., Bae, I. “Development of a Design System for Multi-Stage Gear Drives (1st Report:Proposal of Formal Processes for Dimensional Design)”, International Journal of the Korean Society of Precision Engineering, Vol.2, No.1, 2001

[7] Chong, T.H., Bae, I. “Development of a Design System for Multi-Stage Gear Drives (2nd Report:Proposal of Formal Processes for Dimensional Design)”, International Journal of the Korean Society of Precision Engineering, Vol.2, No.1, 2001

[8] Chong, T.H., Bae, I., Kubo, A., "Development and Application of an Automatic Design System for Multi-Stage Gear Drives", ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Chicago, Illinois, USA, September 2-6, 2003.

[9] Hsu, Y., Wang, S., "Minimizing Angular Backlash of a Multi-Stage Gear Train", Journal of Engineering Manufacture, Vol.216, pp.565-569, 2002.

[10] Deb, K., Jain, S., "Multi-speed Gearbox Design Using Multi-Objective Evolutionary Algorithms", Journal of Mechanical Design, Vol.125, pp.609-619, 2003.

[11] Rosic, B., "Multicriterion Optimization of Multistage Gear Train Transmission", Facta Universitatis Mechanical Engineering, Vol.1, No.8, pp.1107-1115, 2001.

[12] Thompson, D.F., Gupta, S., Shukla, A., "Tradeoff Analysis in Minimum Volume Design of Multi-stage Spur Gear Reduction Units", Mechanism and Machine Theory, Vol.35, pp.609-627, 1999.

[13] Chong, T.H., Bae, I., Kubo, A., "Multiobjective Optimal Design of Cylindrical Gear Pairs for the Reduction of Gear Size and Meshing Vibration", JSME International Journal, Vol. 44, No.1, pp.291-298, 2001.

[14] Dolen, M., Kaplan, H., Seireg A., "Discrete Parameter-Nonlinear Constrained Optimization of a Gear Train Using Genetic Algorithms", International Journal of Computer Applications in Technology, Vol.24, No.2, pp.110-121, 2005.

[15] Norton, Robert, L., "Machine Design an Integrated Approach", Prentice Hall Company, 1998.

[16] ISO International Standard 54, "Cylindrical Gears for General and Heavy Engineering-Modules and Diametral Pitches", 1977.

[17] AGMA Standard 218.01, "Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth, 1982.

[18] AGMA Standard 2001-B88, “Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth, 1988.

[19] Arıkan M.A.S., “Determination of Addendum Modification Coefficients for Spur Gears Operating at Non-Standard Center Distances”, ASME Design Engineering Technical Conferences, Chicago, Illionis, USA, September 1-3 2003.

[20] Arıkan M.A.S., “Performance Rating and Optimization of Spur Gear Drives with Small Number of Teeth”, ASME Design Engineering Technical Conferences, Baltimore, Maryland, USA, September 10-13 2000.

[21] Chapra, Steven C., Canale, Raymond P., “Numerical Methods for Engineers with Programming and Software Applications”, McGraw-Hill Company, 1998.

[22] Arıkan M.A.S., “Determination of Maximum Possible Contact Ratios for Spur Gear Drives with Small Number of Teeth”, Proceedings of ASME Design Engineering Technical Conferences, DE Vol .82, pp 569-576, 1995.

[23] Dudley, D.W., “Gear Handbook”, McGraw-Hill Company, 1962.

[24] AGMA Standard 917-B97, “Design Manual for Parallel Shaft Fine-Pitch Gearing Teeth, 1997.

[25] Kannan, B. K., Kramer, S. N., “An augmented Lagrange Multiplier Based Method for Mixed Integer Discrete Continuous Optimization and Its Applications to Mechanical Design", Journal of Mechanical Design, Vol. 116, pp 405-411, 1995.

[26] Orthwein, W.C., “Determination of Gear Ratios”, Journal of Mechanical Design, Vol. 104, pp 775-777, Oct. 1982.

[27] Selfridge, R. G., Riddle, D. L., “Design Algorithms for Compound Gear Train Ratios, ASME Paper, 78-DET-62.

[28] Niemann, G., “Makina Elemanları Cilt 3”, İstanbul Yayınevi, İstanbul, 1970.

[29] Chong, T.H., Bae I., “A New Approach to Design Multi-stage Gear Drives Using Simulated Annealing Algorithm”, 8th International Gearing and Power Transmission Conference, 2000

[30] Akkurt, M., “Makina Elemanları Cilt 1 ve Cilt 2”, Birsen Yayınevi, İstanbul, 2000.

[31] AGMA Information Sheet 901-A92, “A Rational Procedure for the Preliminary Design of Minimum Volume Gears, 1992.

[32] Chong, T.H., Bae, I., Park, G.J., “A new and Generalised Methodology to Design Multi-Stage Gear Drives by Integrating the Dimensional and the Configuration Design Process”, Mechanism and Machine Theory, Vol. 37, No. 3, pp 295-310, 2002.

APPENDIX A

USER'S GUIDE

The developed program has the following sub menus:

- File
- Input
- Output
- Help

Under the File menu command, options can be selected to constraint the design limitations such as face width limits, number of teeth for pinion and gear, backlash, tooth type, factor of safety. When "exit" is selected the program is closed. The program is developed for optimum design of multistep spur gearbox. The design intention is selected as minimization of total gear volume and total gearbox volume. Gear type is standard and non-standard spur gear according to the input values, in order to satisfy the required rating practices, performance variables and gearbox volume minimization. The unit system used in the program is metric. After selecting these items, the main input form will be opened.

The main input form has three tab dialog boxes. The items of the tabs are as follows

- Power Requirement and Tooth, Cutter geometry
- Material
- Factors

Under the Power Requirement and Tooth, Cutter geometry tab, the following items are presented:

- *Power:* The user is expected to enter the transmitted power in kW.
- *Input Speed:* The user is expected to enter input speed in rpm.
- *Overall Gear Ratio:* The user is expected to enter overall gear ratio.
- *Required Precision:* The user is expected to enter required precision on overall gear ratio in a range of ± 0.1 to ± 0.00001.
- *Number of Stages:* The user is expected to enter the number of stages considering the overall gear ratio from two to six stages which are common for conventional external spur gear drives.
- *AGMA Quality Number (Qv):* The user is expected to select AGMA quality number from the prepared combo box.
- *Reliability:* The user is expected to enter required reliability.
- *Number of Load Cycles:* The user is expected to enter load cycles.
- *Rack Cutter Pressure Angle:* The user is expected to select the pressure angle from the standard angles (14.5°, 20°, 25°)
- *Tool Edge Radius:* It is automatically shown on the textbox after selection of the rack cutter pressure angle. The user can change the tool edge radius if necessary.
- *Standard Modules:* Both preferred and second choice modules are input to the program and the user is asked to select the desired option.
- *Gearbox Design Type:* Compact gearbox and In-line gearbox design options are given to the user.

Under the Material tab, following items are presented.

Type of Material: It can be selected from material types given in the AGMA standard. The following materials are found in the standard:

- Steel (Through Hardened)

- Steel (Flame or Induction Hardened)
- Steel (Carburized and Case Hardened)
- Steel (Nitrided)
- Cast Iron
- Nodular (Ductile) Iron
- Malleable Iron
- Bronze

Once a material type with its hardness and grade is selected, then program automatically finds its allowable contact stress number, allowable bending stress number, Young Modulus of Elasticity (MPa), and Poisson's ratio. Upon request, they can be modified.

Under factors tab, the following items are listed:

- Application Factor (C_a, K_a)
- Surface Factor (C_f, K_f)
- Reliability Factor (C_r, K_r)
- Size Factor (C_s, K_s)
- Temperature Factor (C_t, K_t)
- Dynamic Factor (C_v, K_v)
- Life Adjustment (Commercial or Critical Service)
- Pinion Mounting Type (Straddle or Overhung)

If the straddle mounting is selected another sub form is opened for the detailed information about straddle mounting which includes:

- Lead Correction Factor (C_{mc})
- Mesh Alignment Factor (C_{ma})
- Pinion Proportion Modifier (C_{pm})
- Mesh Alignment Correction Factor (C_e)

If the Overhang mounting is selected than user is required to enter:

- Operating Condition (Contact across the full face)
- Tooth Stiffness Constant (MPa)
- Total lead mismatch between mating teeth (mm)

The designer can select any form by using the tab buttons which are at the top of the forms.

After finishing the data input part, "OK" command button is clicked to start the main algorithm. Then a form comes into the screen including all the gear alternatives that satisfies the overall gear ratio depending on the constraints that are specified by the user.

User can select all the alternatives in order to try all possible solutions or the ones that are sufficient according to the designer. Then selected gear pairs are input to the main algorithm and run to reach to the solutions.

The solutions are analyzed by the user considering the gearbox volume, and performance variables. A grading frame is existing in the program to give importance to the desired performance variables and the other criterion. Also some restrictions are given an option to filter the gear pairs. The form contains:

- Alternative Gear Pairs
- Number of Teeth of Pinions and Gears
- All Performance Variables
- Sorting Option for Any Desired Criteria
- Options for Splitting Overall Gear Ratio
- Filtering Options

After execution of the developed program, the output menu will be opened. The program developed gives the results in a similar form as in the input part using tabs. The results are presented in six different sub tabs for all stages depending on the number of stages and a graphics menu. These tabs are:

1. Gear Drive Properties-1:

    - Number of teeth
    - Normal Pressure Angle
    - Pitch Diameter
    - Operating Pitch Diameter
    - Base Diameter
    - Outside Diameter
    - Root Diameter
    - Face Width
    - Circular Tooth Thickness at Tooth Tip
    - Addendum Modification Coefficient

2. Gear Drive Properties-2:

    - Module
    - Operating Pressure Angle
    - Total Addendum Modification Coefficient
    - Standard Center Distance
    - Operating Center Distance
    - Pinion Addendum Length of Action
    - Pinion Dedendum Length of Action
    - Total Length of Action
    - Bottom Clearance
    - Contact Ratio
    - Total Gear Volume

3. Factors :

    - Application Factor (C_a, K_a)
    - Surface Factor (C_f, K_f)
    - Reliability Factor (C_r, K_r)
    - Size Factor (C_s, K_s)

- Temperature Factor (C_t, K_t)
- Dynamic Factor (C_v, K_v)
- Load Distribution Factor (C_m, K_m)
- Life Factor for Pitting Resistance (C_L)
- Life Factor for Bending Resistance (K_L)
- Hardness Ratio Factor (C_H)
- Geometry Factor for Pitting Resistance (I)
- Geometry Factor for Bending Resistance (J)

4. Strength and Durability Rating:

   - Bending Stress
   - Permissible Bending Stress
   - Contact Stress
   - Permissible Contact Stress
   - Tangential Load
   - Torque
   - Pitch Line Velocity
   - Load Cycles

5. Pinion Tooth Profile
6. Gear Tooth Profile

And the graphics pull down menu is including the following items:

- Performance Variables
- Gear Ratio Split and Module
- Pitch Diameter and Module
- Dynamic Factors
- Volume

A gear train tab exists in the program to analyze the important parameters for all stages in the gear drive together in a single form, which contains:

- Number of teeth
- Gear Ratio
- Module
- Addendum Modification Coefficient
- Pitch Diameter
- Outside Diameter
- Face Width
- Contact Stress
- Permissible Contact Stress
- Bending Stress
- Permissible Bending Stress
- Bending Resistance Geometry Factor
- Contact Resistance Geometry Factor
- Addendum Portion of the Line of Action
- Contact Ratio
- Circular Tooth Thickness at Tooth Tip
- Shaft Diameter

Also a configuration tab exists in the program to view the allocation of the gears schematically while they are meshing.

If the results are satisfactory than the user can save the design data to an excel file, if not may go to back and continue with the program in order to reach a more reasonable solution. The program specifies the positions of pinions and gears automatically by using DLL of Lingo 8.0 optimization software to minimize the volume of designed gear train while satisfying the spatial constraints. The three dimensional output is made automatically using Autodesk Inventor 7.0 with the results obtained from the developed program and the Lingo 8.0 optimization software. The following section includes the screen shots of the developed program in the visual basic form.

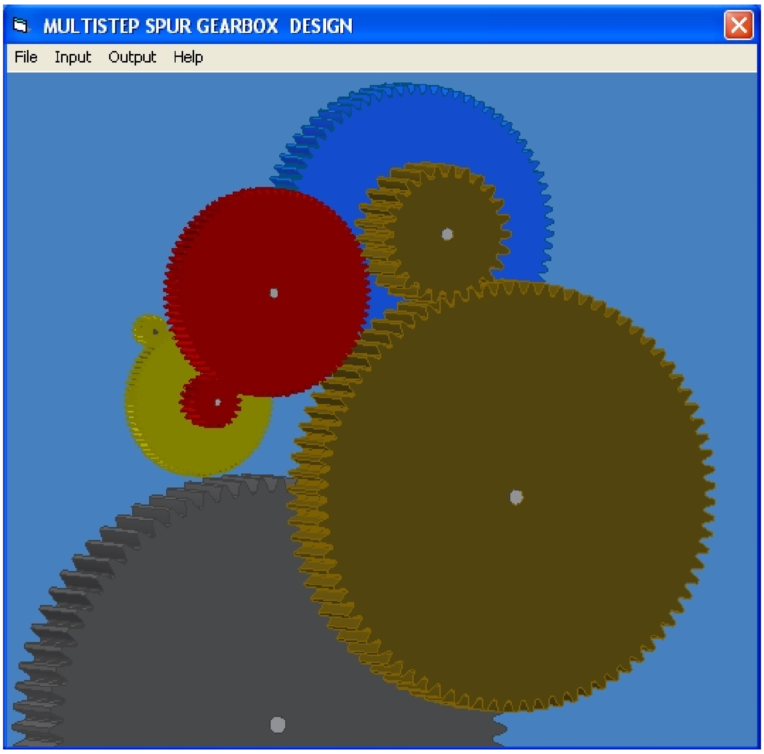


Figure A. 1 Start form

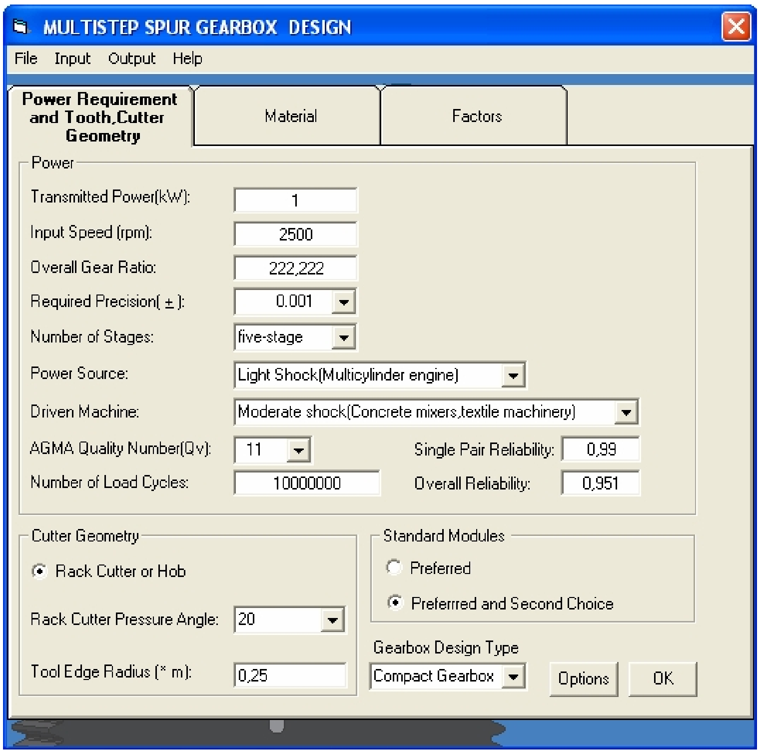


Figure A. 2 Input form, power requirement, tooth, and cutter geometry

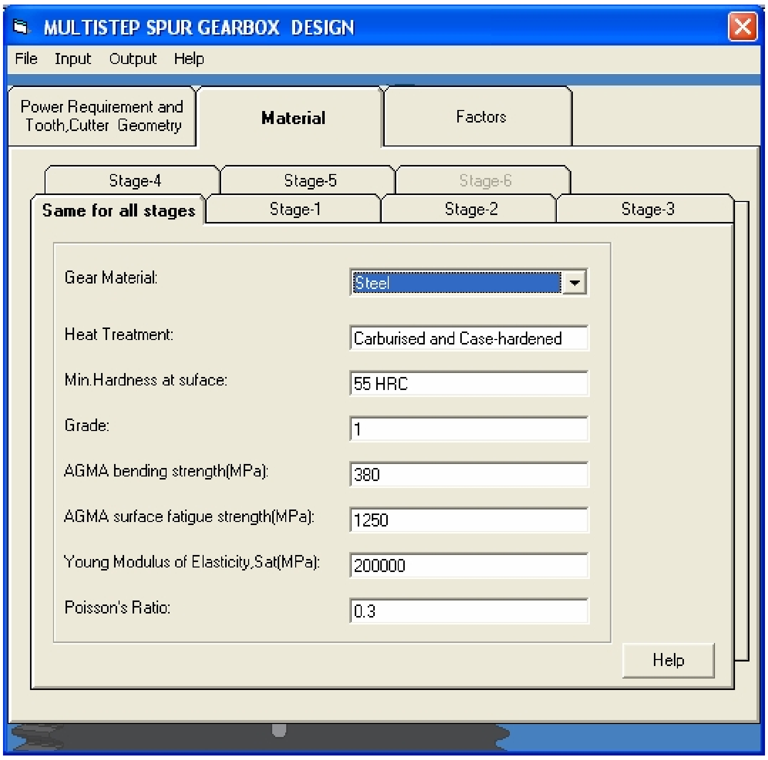


Figure A. 3 Input form, same material selection for all stages

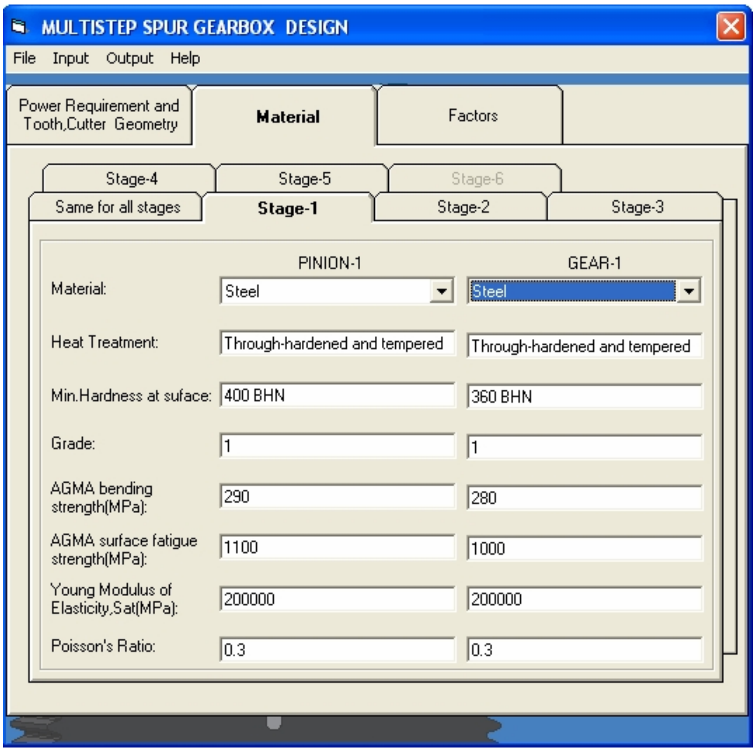
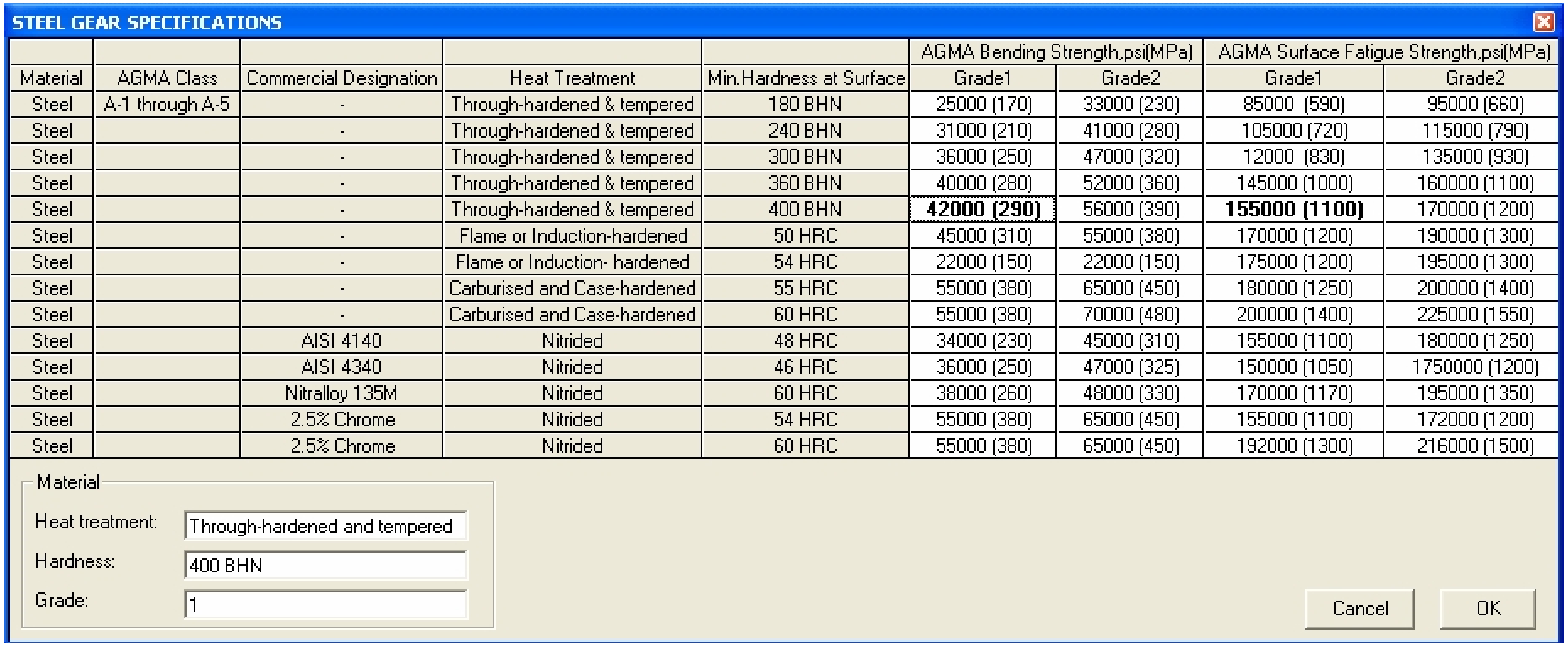


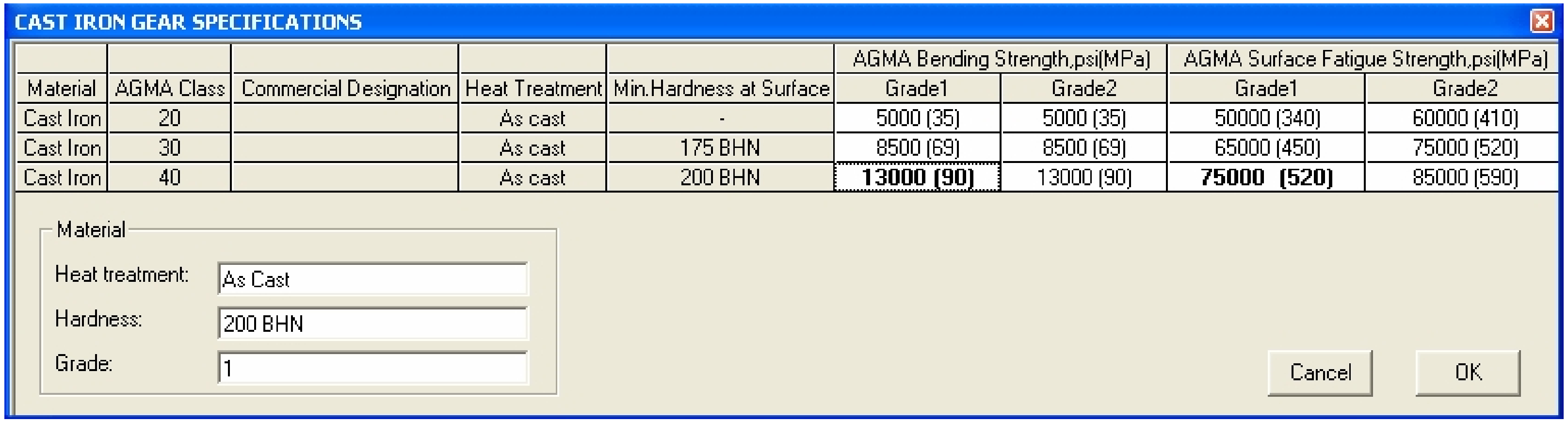
Figure A. 4 Input form, different material selection for all stages


STEEL GEAR SPECIFICATIONS

					AGMA Bending Strength,psi(MPa)		AGMA Surface Fatigue Strength,psi(MPa)	
Material	AGMA Class	Commercial Designation	Heat Treatment	Min.Hardness at Surface	Grade1	Grade2	Grade1	Grade2
Steel	A-1 through A-5	-	Through-hardened & tempered	180 BHN	25000 (170)	33000 (230)	85000 (590)	95000 (660)
Steel		-	Through-hardened & tempered	240 BHN	31000 (210)	41000 (280)	105000 (720)	115000 (790)
Steel		-	Through-hardened & tempered	300 BHN	36000 (250)	47000 (320)	12000 (830)	135000 (930)
Steel		-	Through-hardened & tempered	360 BHN	40000 (280)	52000 (360)	145000 (1000)	160000 (1100)
Steel		-	Through-hardened & tempered	400 BHN	**42000 (290)**	56000 (390)	**155000 (1100)**	170000 (1200)
Steel		-	Flame or Induction-hardened	50 HRC	45000 (310)	55000 (380)	170000 (1200)	190000 (1300)
Steel		-	Flame or Induction- hardened	54 HRC	22000 (150)	22000 (150)	175000 (1200)	195000 (1300)
Steel		-	Carburised and Case-hardened	55 HRC	55000 (380)	65000 (450)	180000 (1250)	200000 (1400)
Steel		-	Carburised and Case-hardened	60 HRC	55000 (380)	70000 (480)	200000 (1400)	225000 (1550)
Steel		AISI 4140	Nitrided	48 HRC	34000 (230)	45000 (310)	155000 (1100)	180000 (1250)
Steel		AISI 4340	Nitrided	46 HRC	36000 (250)	47000 (325)	150000 (1050)	1750000 (1200)
Steel		Nitralloy 135M	Nitrided	60 HRC	38000 (260)	48000 (330)	170000 (1170)	195000 (1350)
Steel		2.5% Chrome	Nitrided	54 HRC	55000 (380)	65000 (450)	155000 (1100)	172000 (1200)
Steel		2.5% Chrome	Nitrided	60 HRC	55000 (380)	65000 (450)	192000 (1300)	216000 (1500)

Material
Heat treatment: Through-hardened and tempered
Hardness: 400 BHN
Grade: 1
Cancel
OK

Figure A. 5 Input form, steel gear specifications



CAST IRON GEAR SPECIFICATIONS

					AGMA Bending Strength,psi(MPa)		AGMA Surface Fatigue Strength,psi(MPa)	
Material	AGMA Class	Commercial Designation	Heat Treatment	Min.Hardness at Surface	Grade1	Grade2	Grade1	Grade2
Cast Iron	20		As cast	-	5000 (35)	5000 (35)	50000 (340)	60000 (410)
Cast Iron	30		As cast	175 BHN	8500 (69)	8500 (69)	65000 (450)	75000 (520)
Cast Iron	40		As cast	200 BHN	**13000 (90)**	13000 (90)	**75000 (520)**	85000 (590)

Material
Heat treatment: As Cast
Hardness: 200 BHN
Grade: 1

Cancel
OK

Figure A.6 Input form, cast iron gear specifications

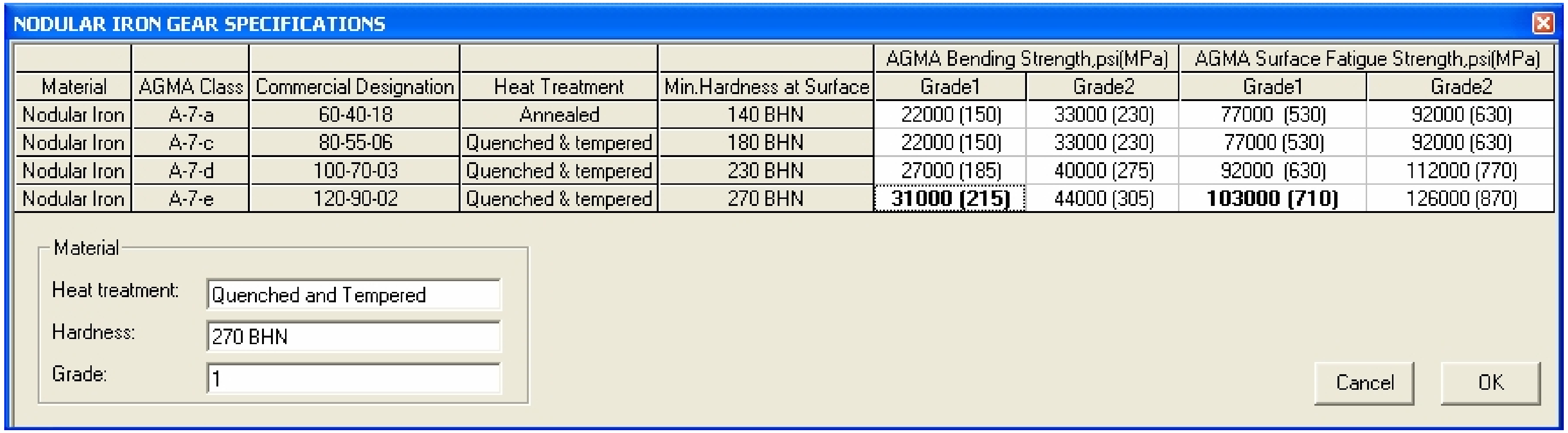
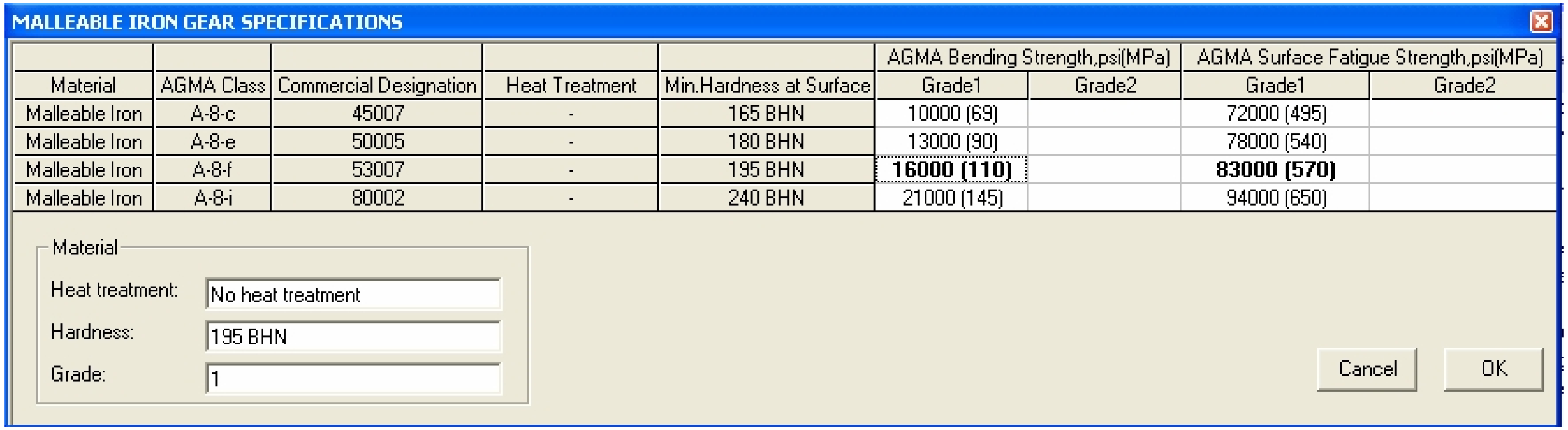


Figure A. 7 Input form, nodular iron gear specifications

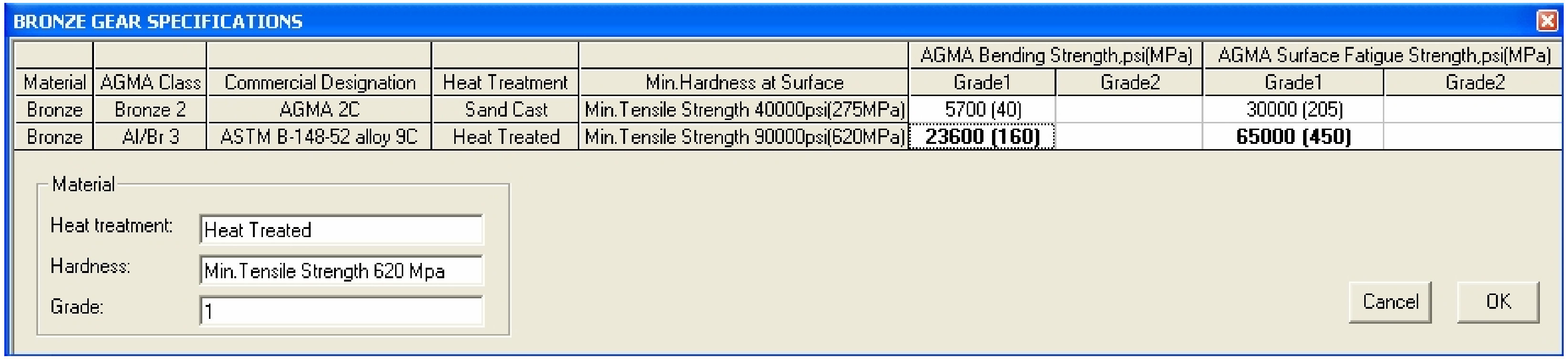

MALLEABLE IRON GEAR SPECIFICATIONS

					AGMA Bending Strength,psi(MPa)		AGMA Surface Fatigue Strength,psi(MPa)	
Material	AGMA Class	Commercial Designation	Heat Treatment	Min.Hardness at Surface	Grade1	Grade2	Grade1	Grade2
Malleable Iron	A-8-c	45007	-	165 BHN	10000 (69)		72000 (495)	
Malleable Iron	A-8-e	50005	-	180 BHN	13000 (90)		78000 (540)	
Malleable Iron	A-8-f	53007	-	195 BHN	**16000 (110)**		**83000 (570)**	
Malleable Iron	A-8-i	80002	-	240 BHN	21000 (145)		94000 (650)	

Material
Heat treatment: No heat treatment
Hardness: 195 BHN
Grade: 1

Cancel
OK

Figure A. 8 Input form, malleable iron gear specifications



BRONZE GEAR SPECIFICATIONS

					AGMA Bending Strength,psi(MPa)		AGMA Surface Fatigue Strength,psi(MPa)	
Material	AGMA Class	Commercial Designation	Heat Treatment	Min.Hardness at Surface	Grade1	Grade2	Grade1	Grade2
Bronze	Bronze 2	AGMA 2C	Sand Cast	Min.Tensile Strength 40000psi(275MPa)	5700 (40)		30000 (205)	
Bronze	Al/Br 3	ASTM B-148-52 alloy 9C	Heat Treated	Min.Tensile Strength 90000psi(620MPa)	**23600 (160)**		**65000 (450)**	

Material

Heat treatment: Heat Treated

Hardness: Min.Tensile Strength 620 Mpa

Grade: 1

Cancel

OK

Figure A. 9 Input form, bronze gear specifications

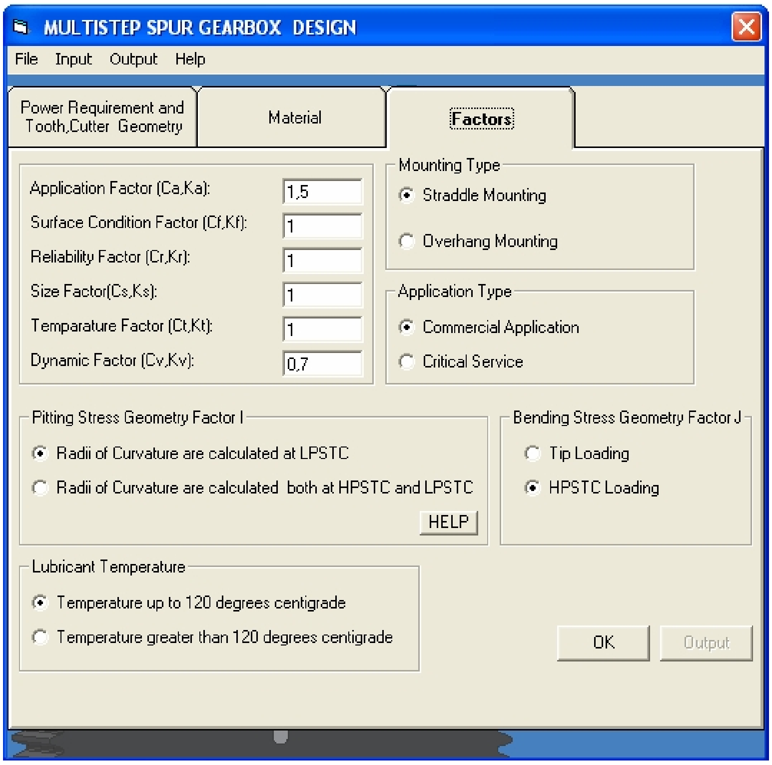


Figure A. 10 Input form, factors

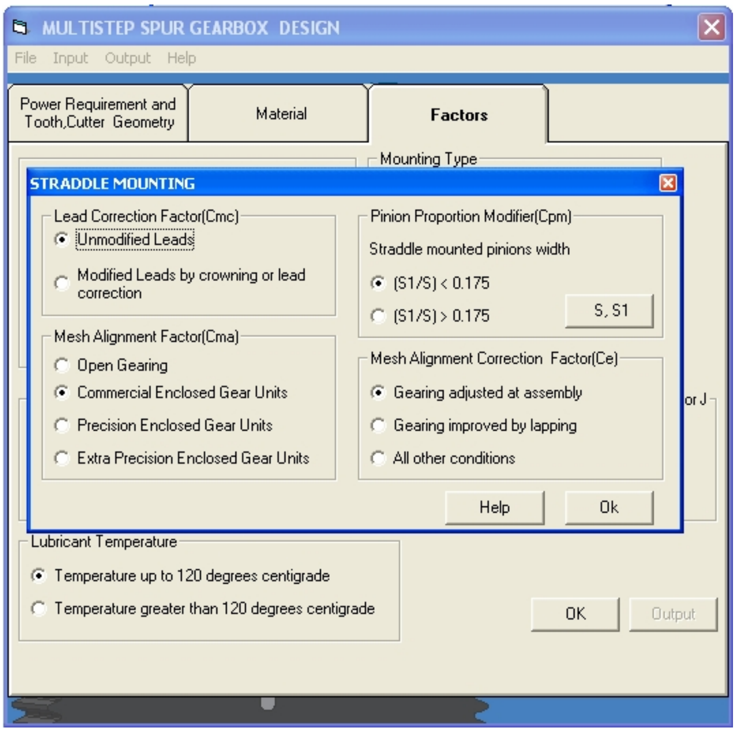


Figure A. 11 Input form, straddle mounting

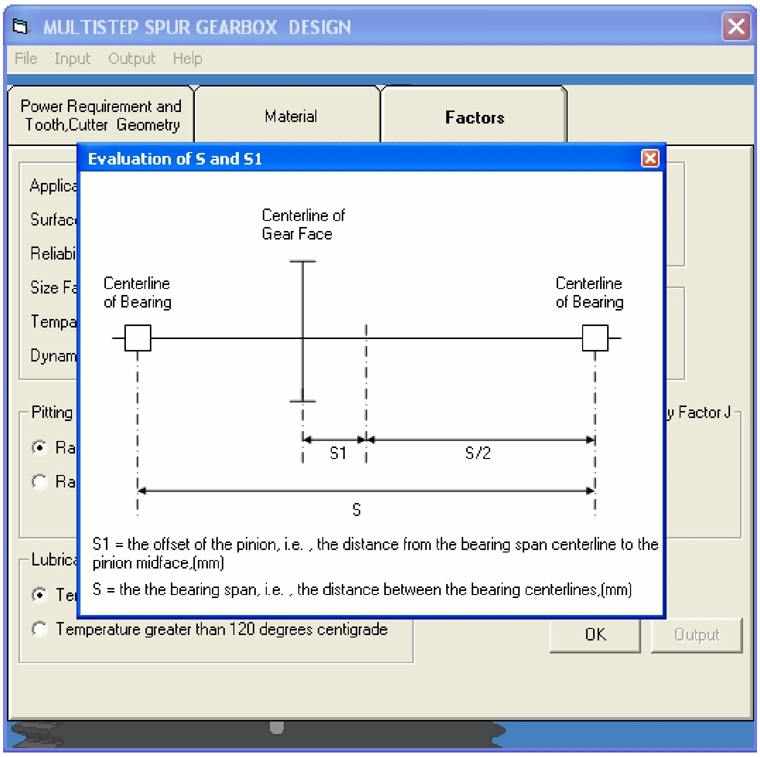


Figure A. 12 Input form, evaluation of s and s1

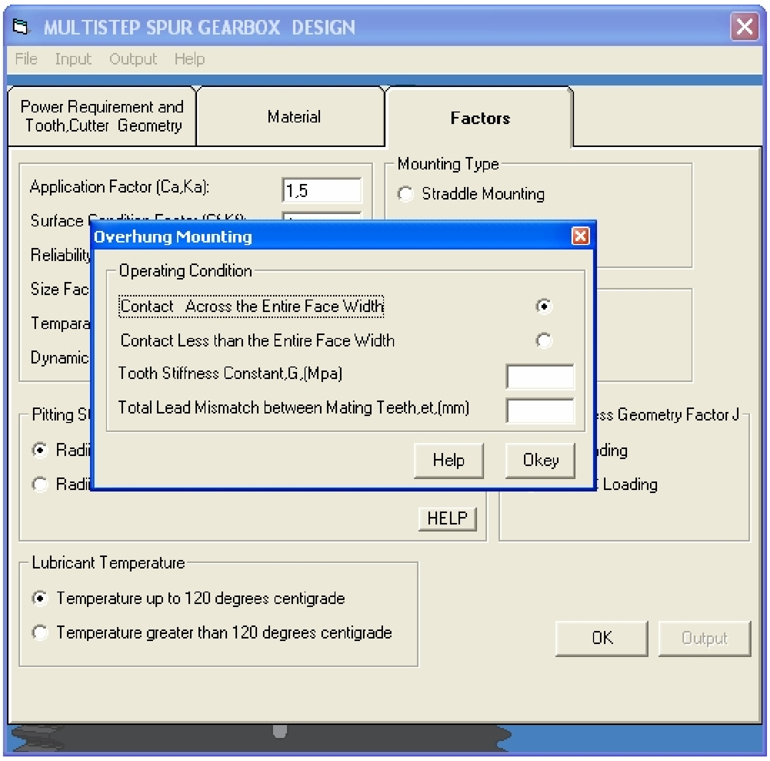


Figure A. 13 Input form, overhung mounting

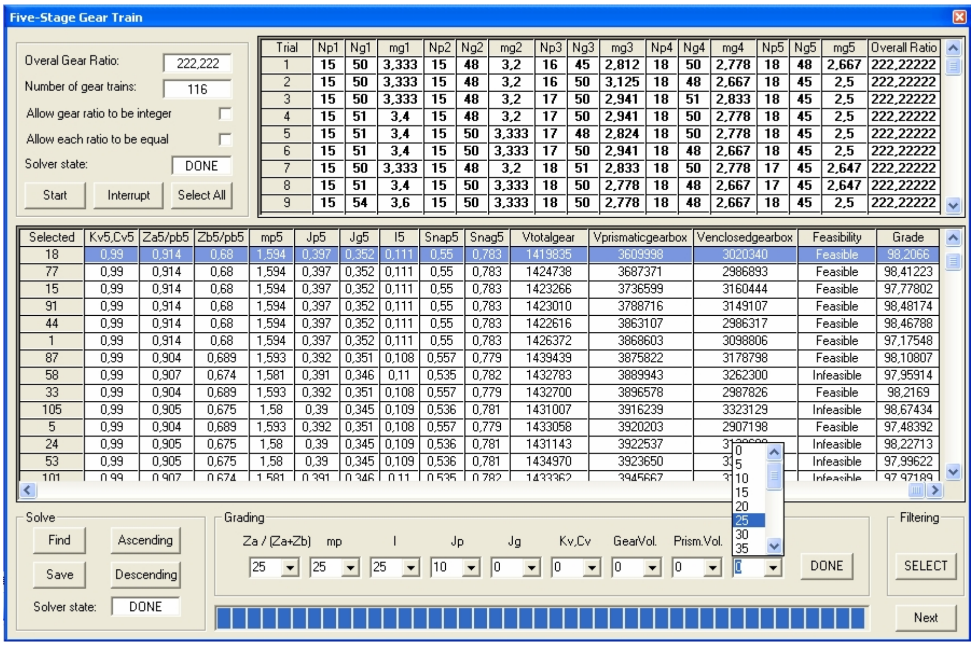


Figure A. 64 Input form, selection of gear pairs

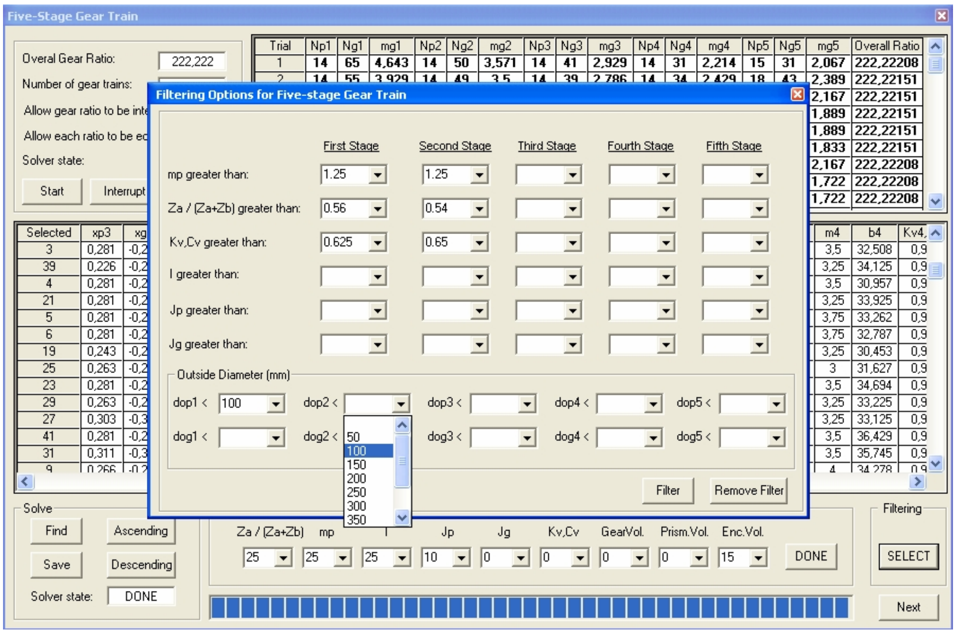


Figure A. 75 Input form, filtering options

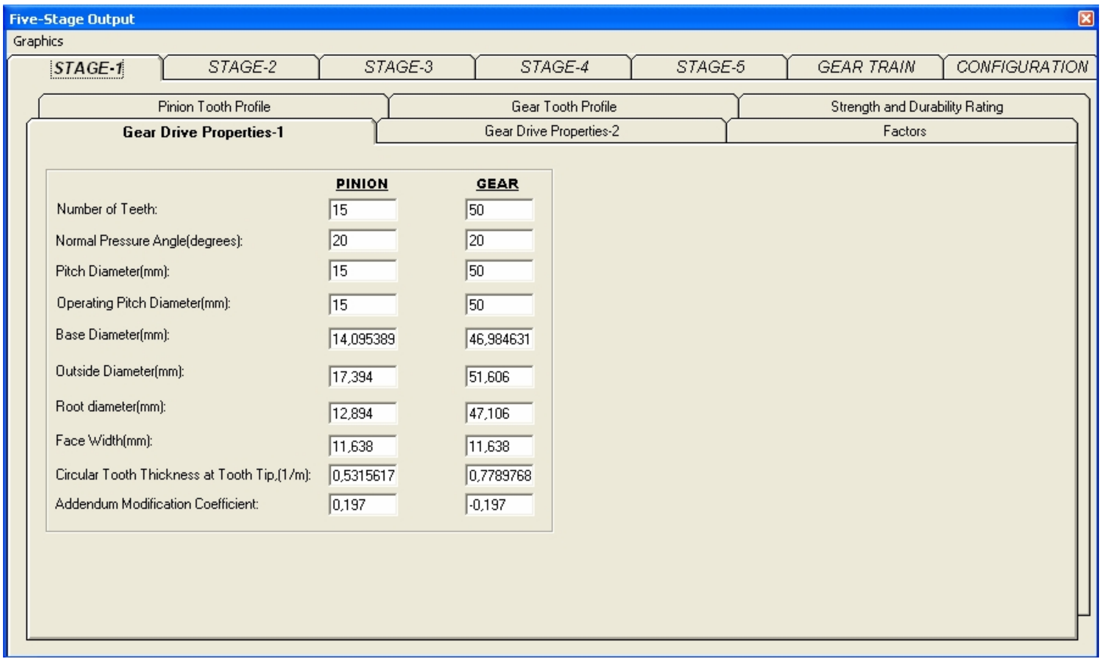


Figure A. 86 Output form, gear drive properties 1

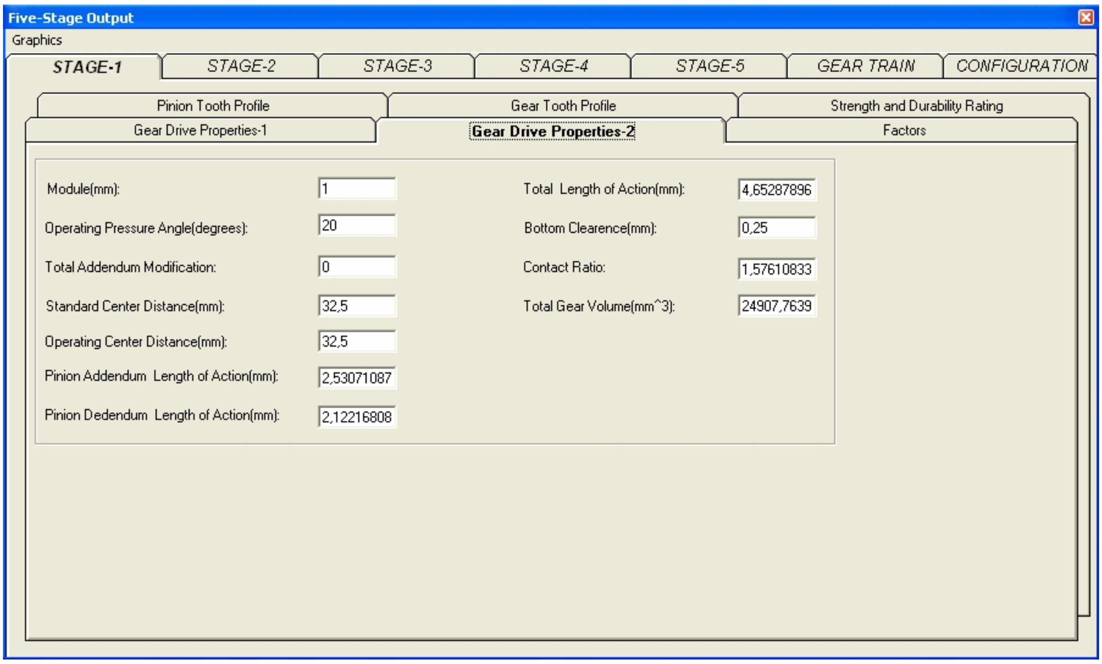


Figure A. 97 Output Form, Gear Drive Properties 2

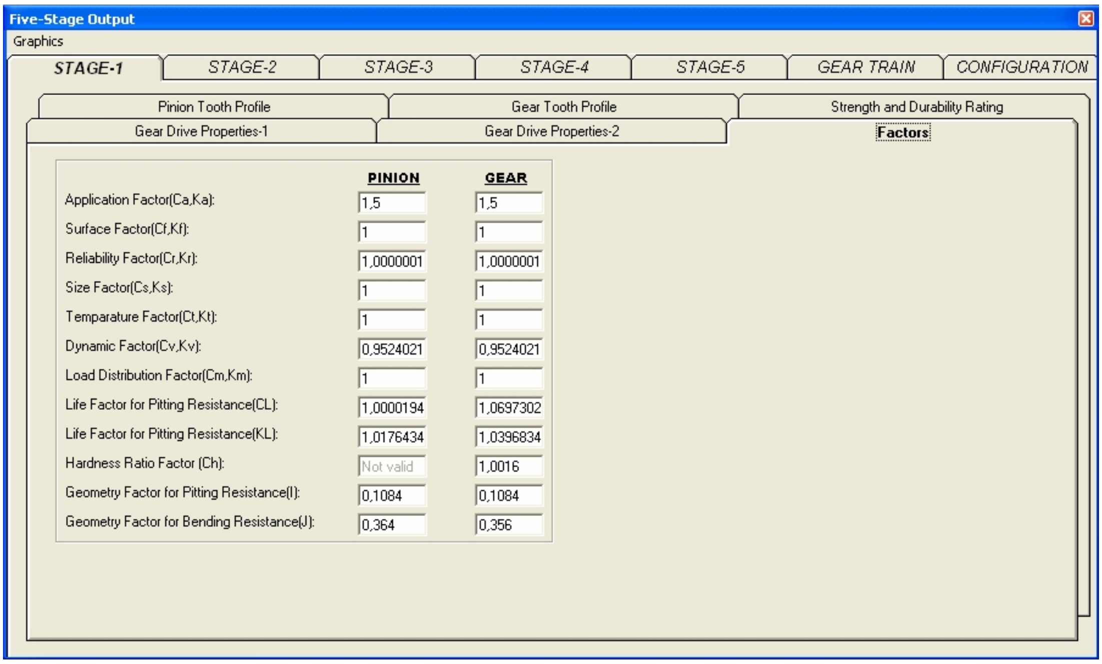


Figure A. 108 Output form, factors

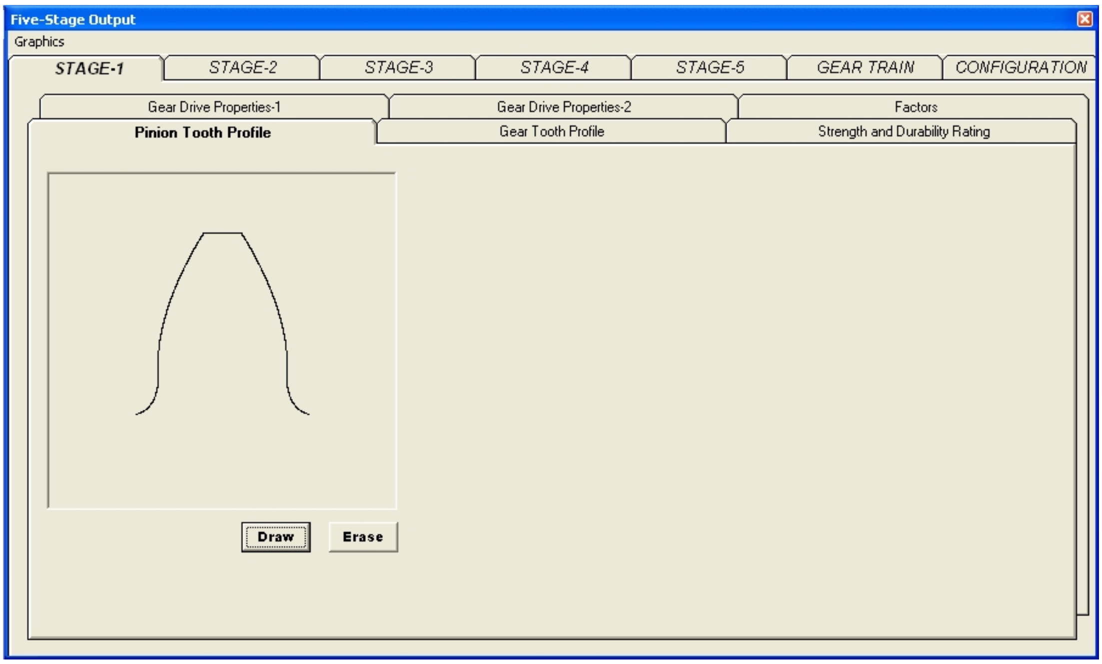


Figure A. 119 Output form, pinion tooth profile

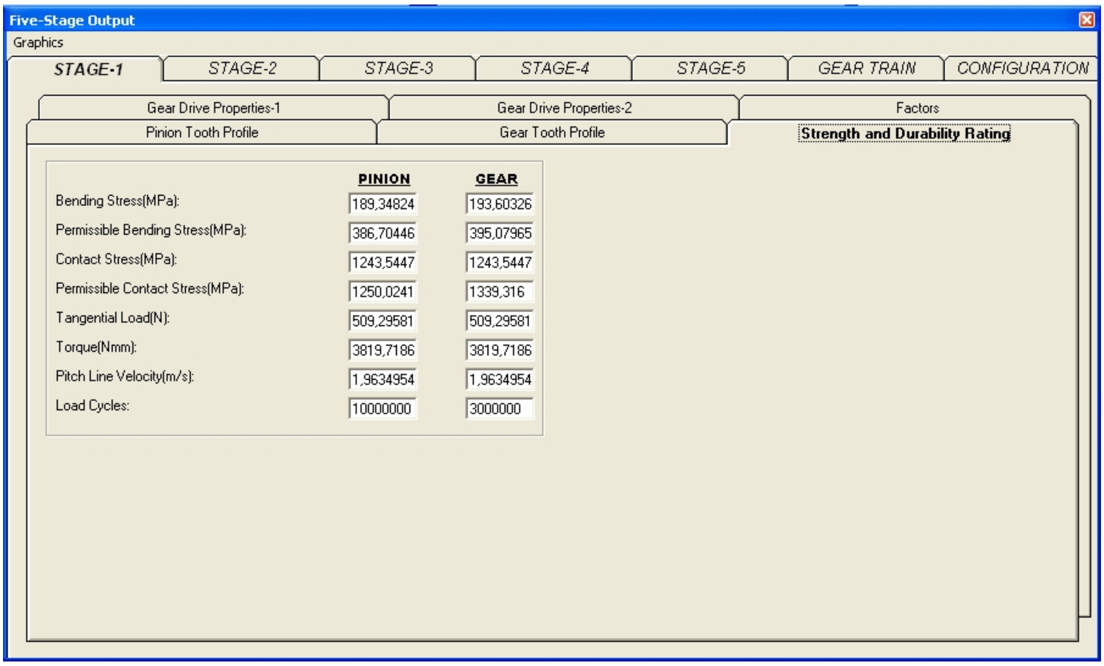
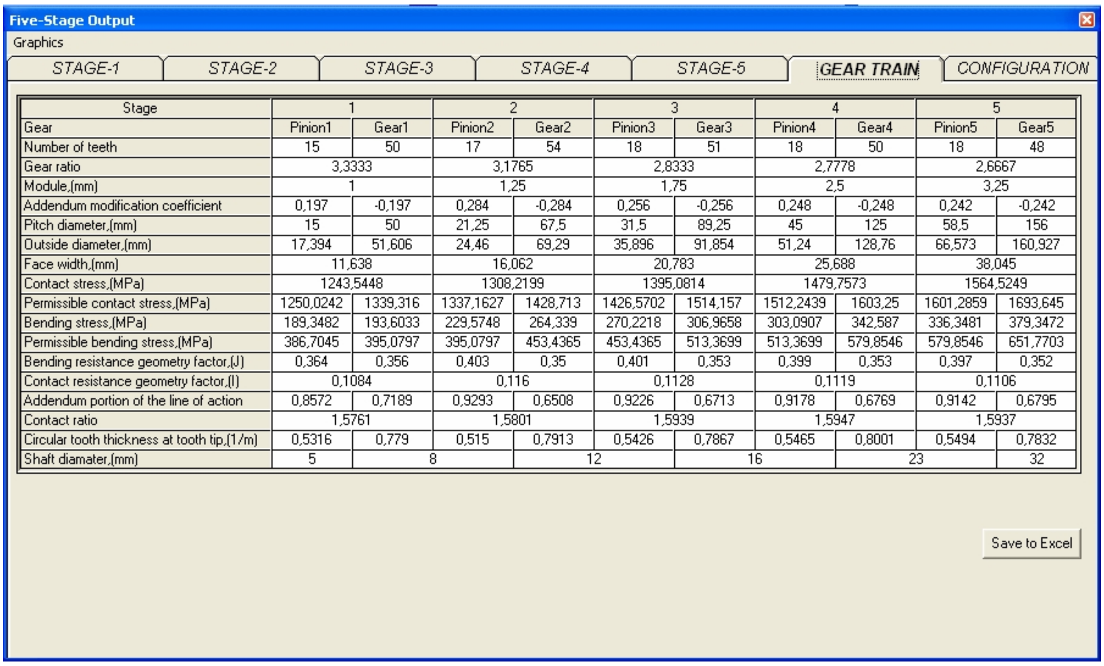


Figure A. 20 Output form, strength and durability rating



Stage	1		2		3		4		5	
Gear	Pinion1	Gear1	Pinion2	Gear2	Pinion3	Gear3	Pinion4	Gear4	Pinion5	Gear5
Number of teeth	15	50	17	54	18	51	18	50	18	48
Gear ratio	3,3333		3,1765		2,8333		2,7778		2,6667	
Module,(mm)	1		1,25		1,75		2,5		3,25	
Addendum modification coefficient	0,197	-0,197	0,284	-0,284	0,256	-0,256	0,248	-0,248	0,242	-0,242
Pitch diameter,(mm)	15	50	21,25	67,5	31,5	89,25	45	125	58,5	156
Outside diameter,(mm)	17,394	51,606	24,46	69,29	35,896	91,854	51,24	128,76	66,573	160,927
Face width,(mm)	11,638		16,062		20,783		25,688		38,045	
Contact stress,(MPa)	1243,5448		1308,2199		1395,0814		1479,7573		1564,5249	
Permissible contact stress,(MPa)	1250,0242	1339,316	1337,1627	1428,713	1426,5702	1514,157	1512,2439	1603,25	1601,2859	1693,645
Bending stress,(MPa)	189,3482	193,6033	229,5748	264,339	270,2218	306,9658	303,0907	342,587	336,3481	379,3472
Permissible bending stress,(MPa)	386,7045	395,0797	395,0797	453,4365	453,4365	513,3699	513,3699	579,8546	579,8546	651,7703
Bending resistance geometry factor,(J)	0,364	0,356	0,403	0,35	0,401	0,353	0,399	0,353	0,397	0,352
Contact resistance geometry factor,(I)	0,1084		0,116		0,1128		0,1119		0,1106	
Addendum portion of the line of action	0,8572	0,7189	0,9293	0,6508	0,9226	0,6713	0,9178	0,6769	0,9142	0,6795
Contact ratio	1,5761		1,5801		1,5939		1,5947		1,5937	
Circular tooth thickness at tooth tip,(1/m)	0,5316	0,779	0,515	0,7913	0,5426	0,7867	0,5465	0,8001	0,5494	0,7832
Shaft diamater,(mm)	5	8		12		16		23		32

Figure A. 21 Output form, gear train

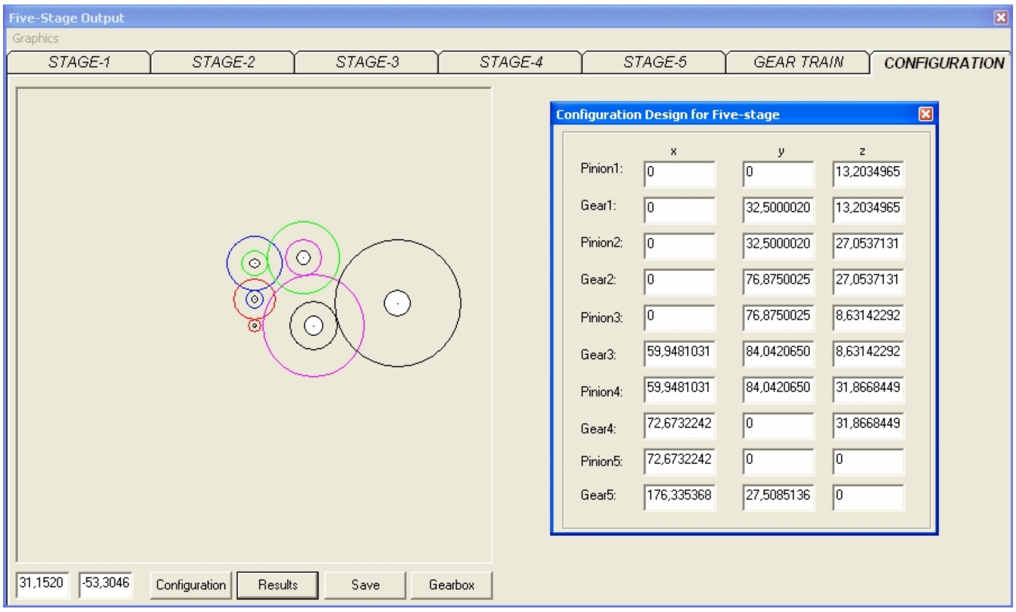


Figure A. 22 Output form, configuration

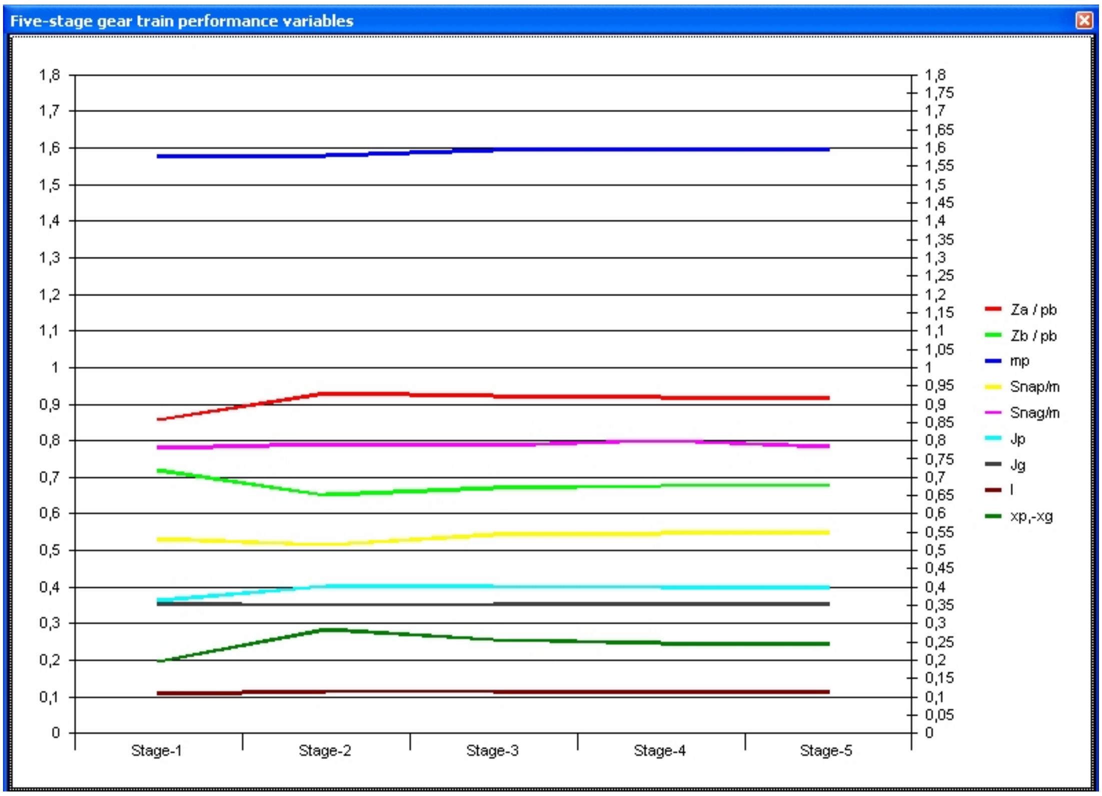


Figure A. 23 Output form, graphics, and performance variables

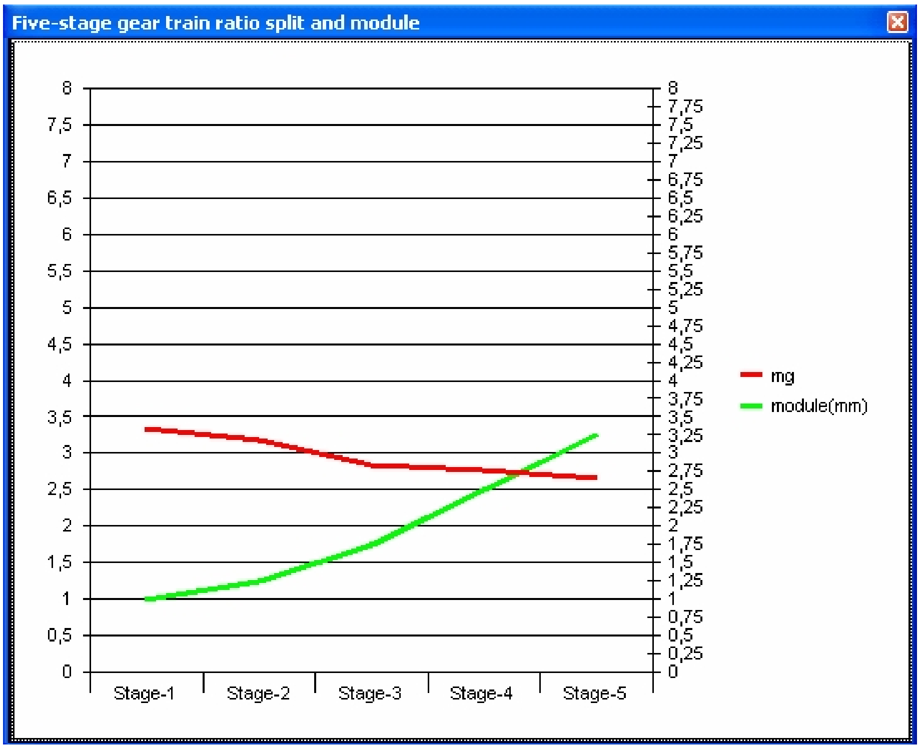


Figure A. 24 Output form, graphics, ratio split and module

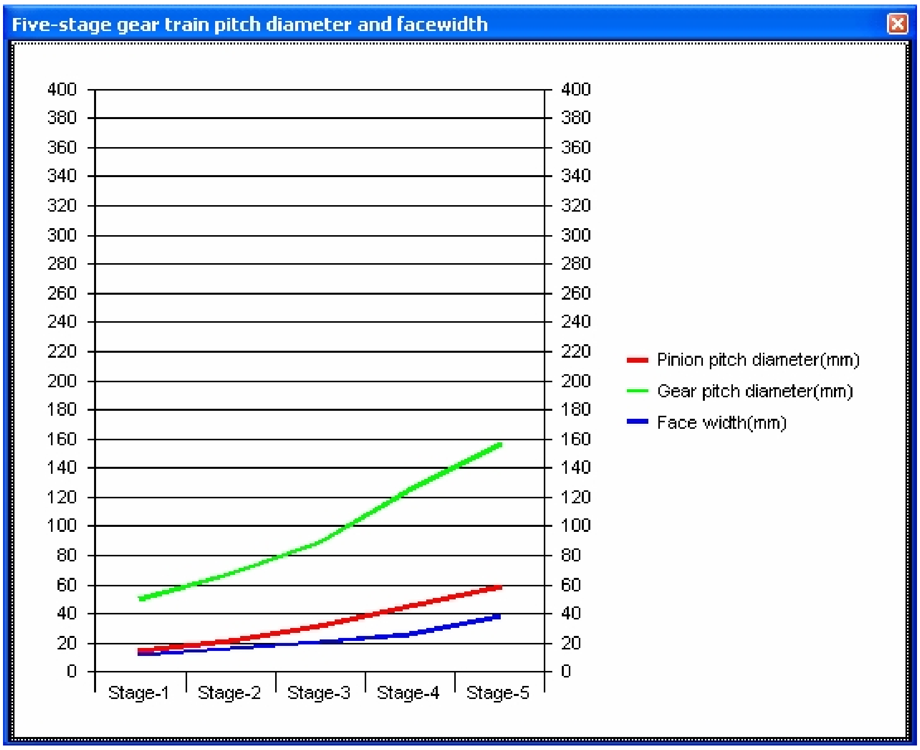


Figure A. 25 Output form, graphics, pitch diameter and face width

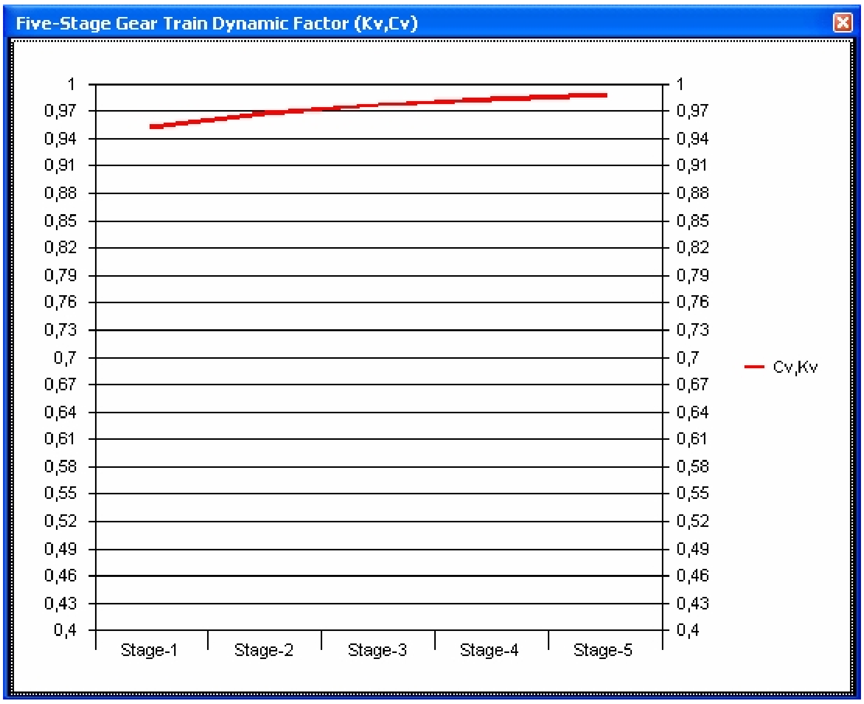


Figure A. 26 Output form, graphics, and dynamic factors

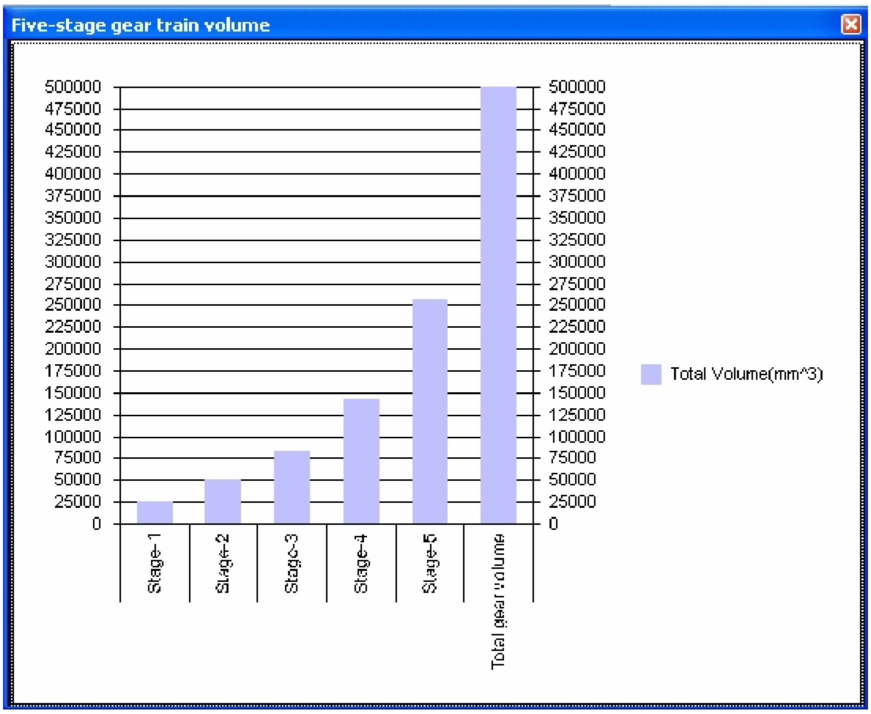
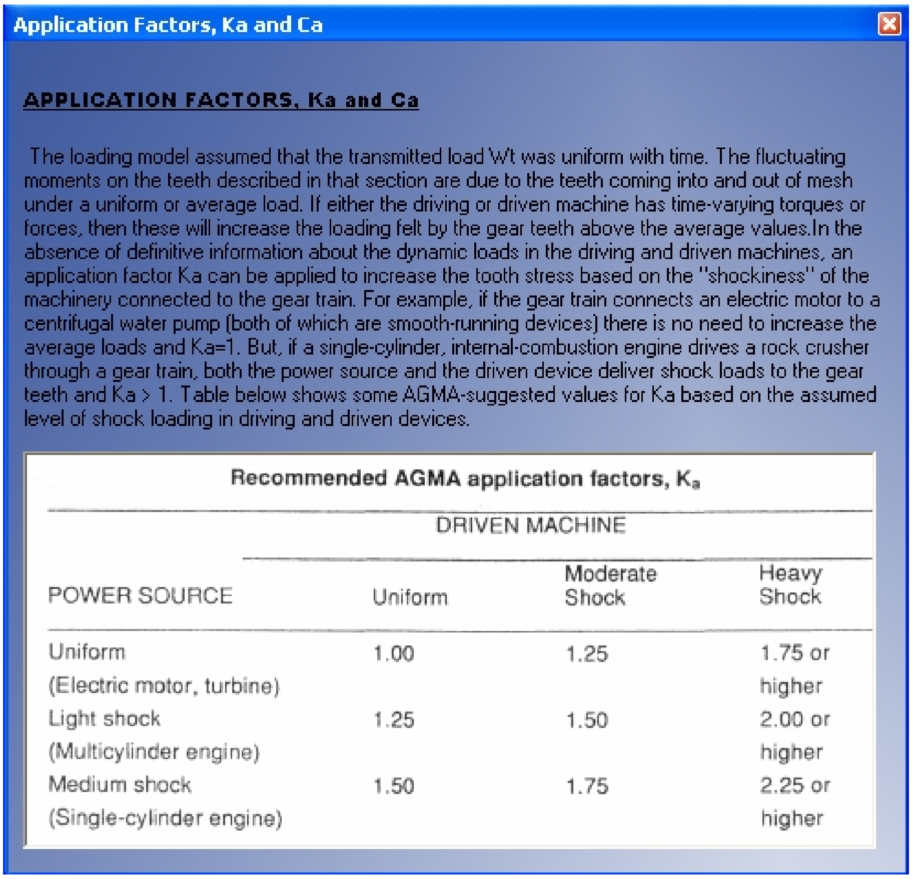


Figure A. 27 Output form, graphics, and volume



Application Factors, Ka and Ca

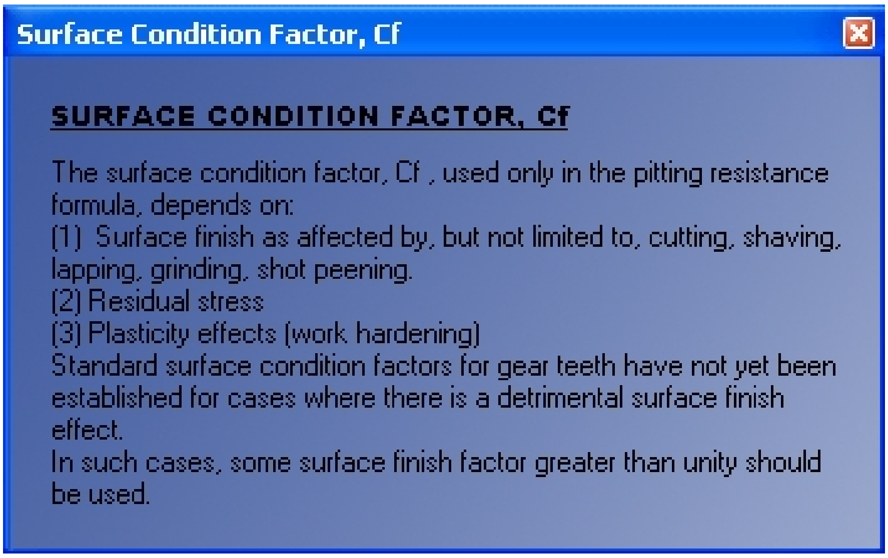
APPLICATION FACTORS, Ka and Ca

The loading model assumed that the transmitted load Wt was uniform with time. The fluctuating moments on the teeth described in that section are due to the teeth coming into and out of mesh under a uniform or average load. If either the driving or driven machine has time-varying torques or forces, then these will increase the loading felt by the gear teeth above the average values.In the absence of definitive information about the dynamic loads in the driving and driven machines, an application factor Ka can be applied to increase the tooth stress based on the "shockiness" of the machinery connected to the gear train. For example, if the gear train connects an electric motor to a centrifugal water pump (both of which are smooth-running devices) there is no need to increase the average loads and Ka=1. But, if a single-cylinder, internal-combustion engine drives a rock crusher through a gear train, both the power source and the driven device deliver shock loads to the gear teeth and Ka > 1. Table below shows some AGMA-suggested values for Ka based on the assumed level of shock loading in driving and driven devices.

Recommended AGMA application factors, K_a

	DRIVEN MACHINE		
POWER SOURCE	Uniform	Moderate Shock	Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light shock (Multicylinder engine)	1.25	1.50	2.00 or higher
Medium shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

Figure A. 28 Help form, application factor



Surface Condition Factor, Cf

SURFACE CONDITION FACTOR, Cf

The surface condition factor, Cf , used only in the pitting resistance formula, depends on:
(1) Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shot peening.
(2) Residual stress
(3) Plasticity effects (work hardening)
Standard surface condition factors for gear teeth have not yet been established for cases where there is a detrimental surface finish effect.
In such cases, some surface finish factor greater than unity should be used.

Figure A. 29 Help form, surface condition factor

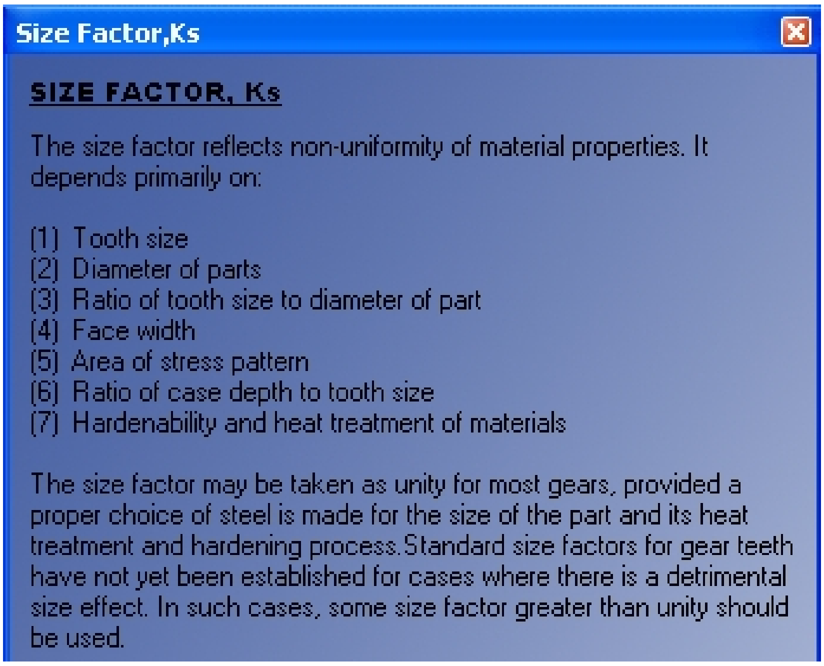


Figure A. 3012 Help form, size factor

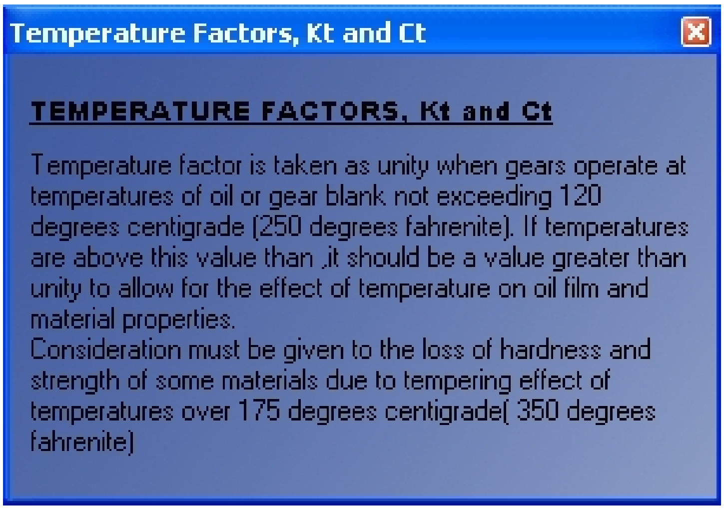


Figure A. 31 Help form, temperature factor

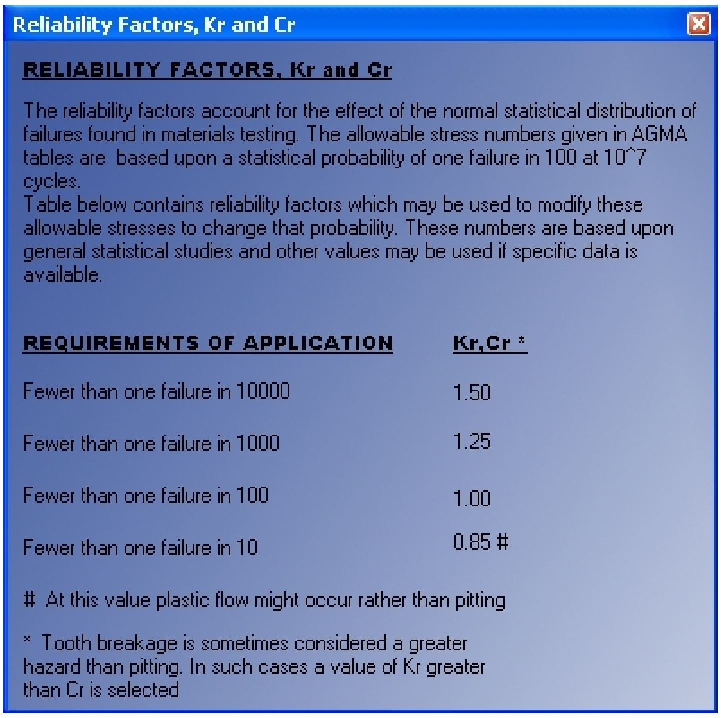


Figure A. 32 Help form, reliability factor

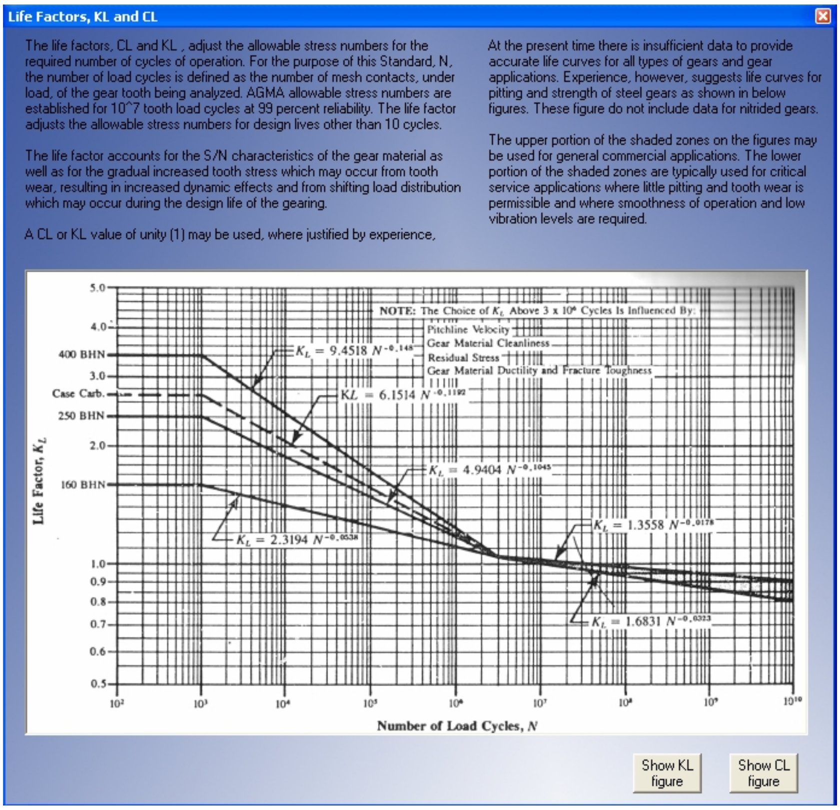


Figure A. 33 Help form, life factor, K_L

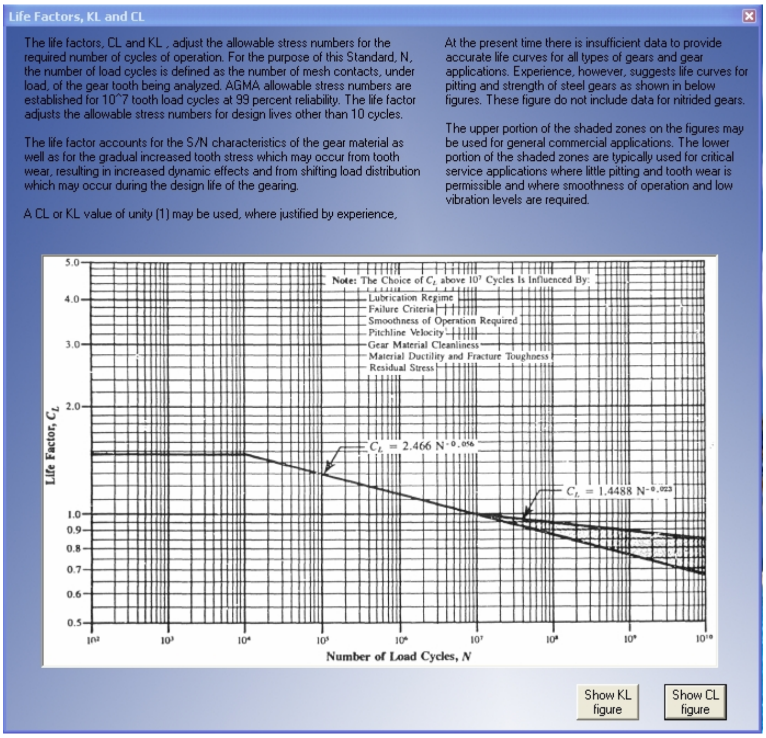


Figure A. 34 Help form, life factor, C_L

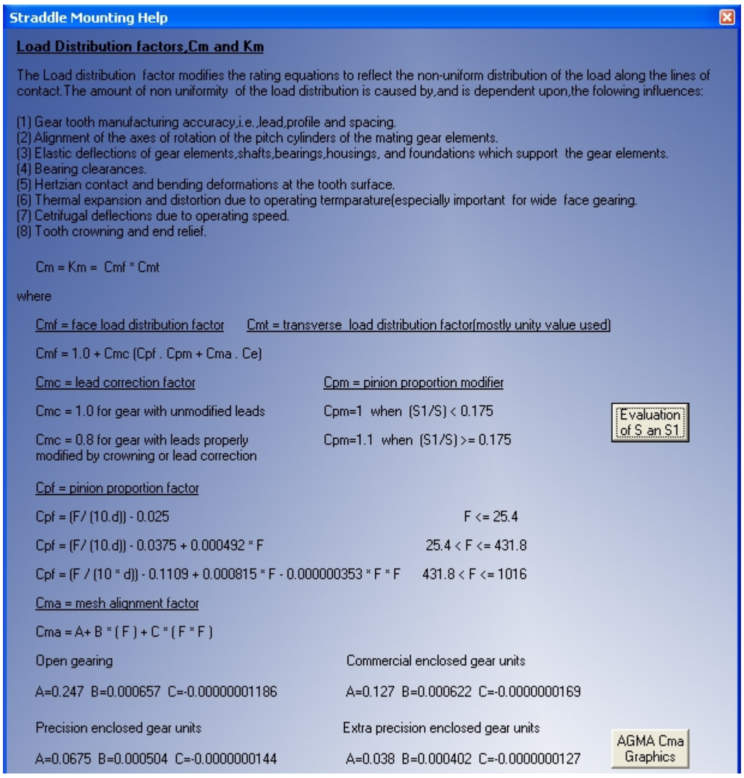


Figure A. 35 Help form, straddle mounting

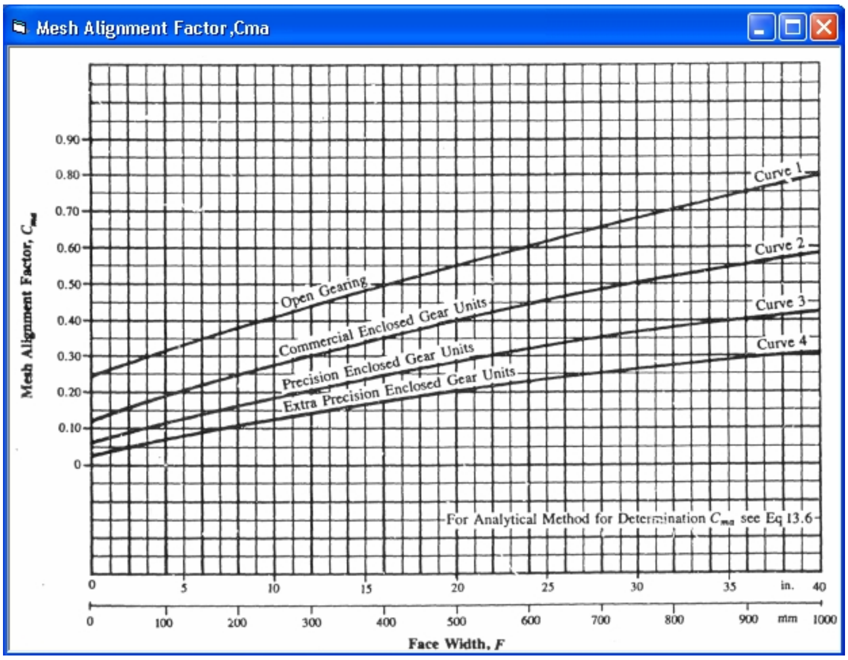
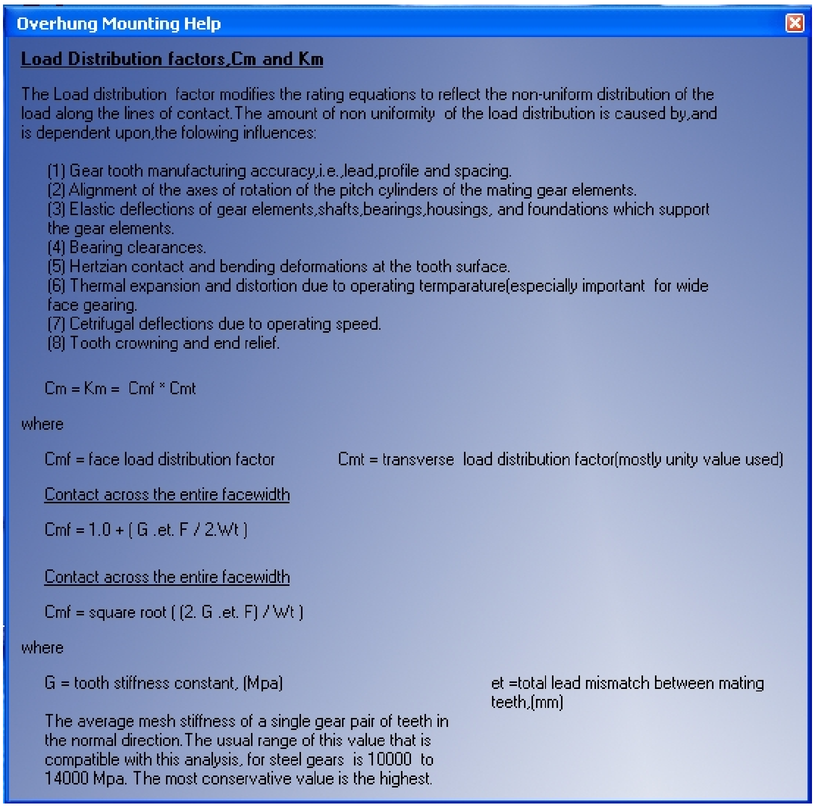


Figure A. 36 Help form, mesh alignment factor



Overhung Mounting Help

Load Distribution factors,Cm and Km

The Load distribution factor modifies the rating equations to reflect the non-uniform distribution of the load along the lines of contact.The amount of non uniformity of the load distribution is caused by,and is dependent upon,the folowing influences:

(1) Gear tooth manufacturing accuracy,i.e.,lead,profile and spacing.
(2) Alignment of the axes of rotation of the pitch cylinders of the mating gear elements.
(3) Elastic deflections of gear elements,shafts,bearings,housings, and foundations which support the gear elements.
(4) Bearing clearances.
(5) Hertzian contact and bending deformations at the tooth surface.
(6) Thermal expansion and distortion due to operating temparature(especially important for wide face gearing.
(7) Cetrifugal deflections due to operating speed.
(8) Tooth crowning and end relief.

Cm = Km = Cmf * Cmt

where

Cmf = face load distribution factor

Cmt = transverse load distribution factor(mostly unity value used)

Contact across the entire facewidth

Cmf = 1.0 + (G .et. F / 2.Wt)

Contact across the entire facewidth

Cmf = square root ((2. G .et. F) / Wt)

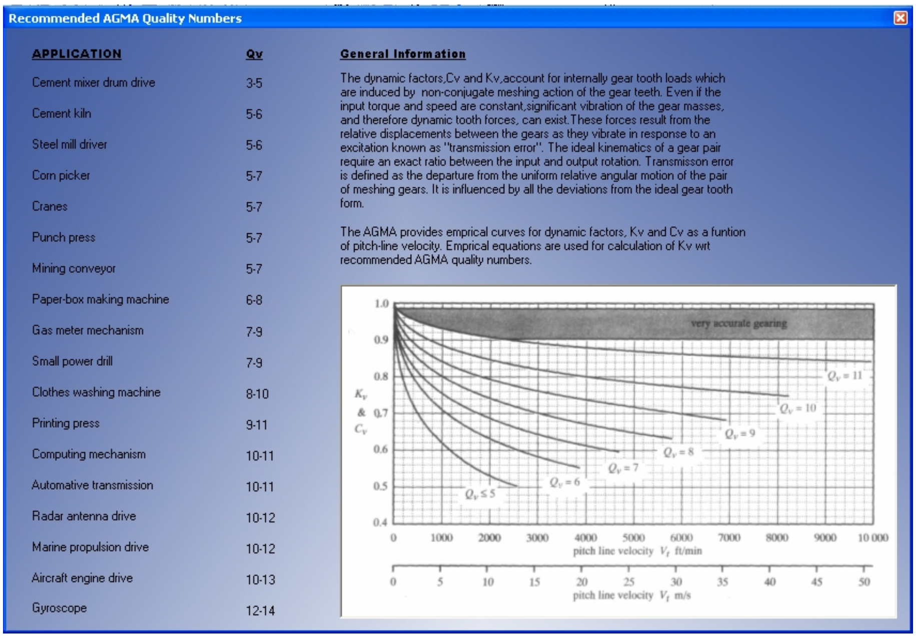
where

G = tooth stiffness constant, (Mpa)

et =total lead mismatch between mating teeth,(mm)

The average mesh stiffness of a single gear pair of teeth in the normal direction.The usual range of this value that is compatible with this analysis, for steel gears is 10000 to 14000 Mpa. The most conservative value is the highest.

Figure A. 37 Help form, overhung mounting



Recommended AGMA Quality Numbers

APPLICATION	Qv
Cement mixer drum drive	3-5
Cement kiln	5-6
Steel mill driver	5-6
Corn picker	5-7
Cranes	5-7
Punch press	5-7
Mining conveyor	5-7
Paper-box making machine	6-8
Gas meter mechanism	7-9
Small power drill	7-9
Clothes washing machine	8-10
Printing press	9-11
Computing mechanism	10-11
Automative transmission	10-11
Radar antenna drive	10-12
Marine propulsion drive	10-12
Aircraft engine drive	10-13
Gyroscope	12-14

General Information

The dynamic factors,Cv and Kv,account for internally gear tooth loads which are induced by non-conjugate meshing action of the gear teeth. Even if the input torque and speed are constant,significant vibration of the gear masses, and therefore dynamic tooth forces, can exist.These forces result from the relative displacements between the gears as they vibrate in response to an excitation known as "transmission error". The ideal kinematics of a gear pair require an exact ratio between the input and output rotation. Transmisson error is defined as the departure from the uniform relative angular motion of the pair of meshing gears. It is influenced by all the deviations from the ideal gear tooth form.

The AGMA provides emprical curves for dynamic factors, Kv and Cv as a funtion of pitch-line velocity. Emprical equations are used for calculation of Kv wrt recommended AGMA quality numbers.

Figure A. 38 Help form, AGMA quality numbers

Gear Materials

Only a limited number of metals and alloys are suitable for gears that transmit significant power. Steels, cast irons, and malleable and nodular irons are the most common choices for gears. Surface- or through-hardening is recommended (on those alloys that allow it) to obtain sufficient strength and wear resistance. Where high corrosion resistance is needed, such as in marine environments, bronzes are often used. .

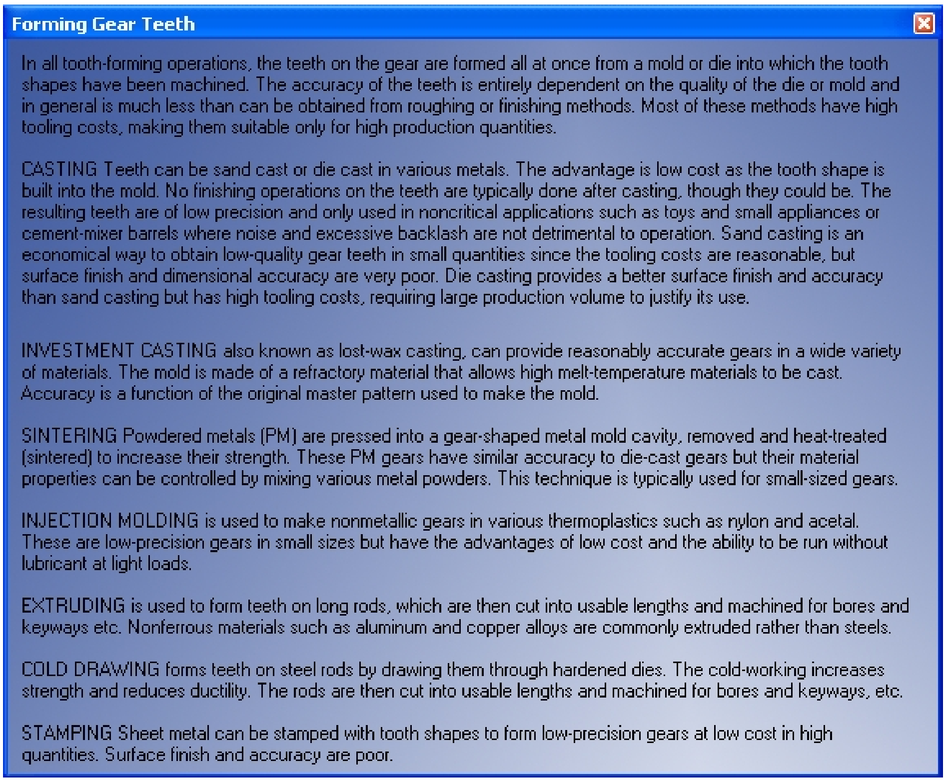
CAST IRONS are commonly used for gears. The gray cast irons (CI) have advantages of low cost, ease of machining, high wear resistance, and internal damping (due to the graphite inclusions), which makes them acoustically quieter than steel gears. However, they have low tensile strength, which requires larger teeth than steel gears to obtain sufficient bending strength. Nodular irons have higher tensile strength than gray CI and retain the other advantages of machinability, wear resistance, and internal damping, but are more costly. The combination of a steel pinion (for strength in the higher-stressed member) and a cast-iron gear is often used.

STEELS are also commonly used for gears. They have superior tensile strength to cast iron and are cost-competitive in their low-alloy forms. They need heat treatment to get a surface hardness that will resist wear, but soft steel gears are sometimes used in low-load, low-speed applications or where long life may not be a prime concern. For heat treatment, either a medium-to-high carbon (0.35 to 0.60% C) plain or alloy steel is needed. Small gears are typically through-hardened and larger gears flame or induction hardened to minimize distortion. Lower-carbon steels can be case hardened by carburizing or nitriding. A case-hardened gear has the advantage of a tough core and a hard surface, but if the case is not deep enough, the teeth may fail in bending fatigue beneath the case in the soft, weaker core material. It is often necessary to use secondary finishing methods such as grinding, lapping, and honing to remove the heat-treatment distortion from hardened gears if high accuracy is needed.

BRONZES are the most common nonferrous metals used for gears. The lower modulus of elasticity of these copper alloys provides greater tooth deflection and improves load sharing between the teeth. Since bronze and steel run well together, the combination of a steel pinion and a bronze gear is often used.

NONMETALLIC GEARS are often made of injection-molded thermoplastics such as nylon and acetal, sometimes filled with inorganics such as glass or talc. Teflon is sometimes added to nylon or acetal to lower the coefficient of friction. Dry lubricants such as graphite and molybdenum disulphide (MoS2) can be added to the plastic to allow dry running. Composite gears of cloth-reinforced thermosetting phenolic have long been used for applications such as the camshaft-drive (timing) gear driven by a steel pinion in some gasoline engines. Nonmetallic gears have very low noise but are limited in torque capacity by their low material strengths.

Figure A. 39 help form, gear materials



Forming Gear Teeth

In all tooth-forming operations, the teeth on the gear are formed all at once from a mold or die into which the tooth shapes have been machined. The accuracy of the teeth is entirely dependent on the quality of the die or mold and in general is much less than can be obtained from roughing or finishing methods. Most of these methods have high tooling costs, making them suitable only for high production quantities.

CASTING Teeth can be sand cast or die cast in various metals. The advantage is low cost as the tooth shape is built into the mold. No finishing operations on the teeth are typically done after casting, though they could be. The resulting teeth are of low precision and only used in noncritical applications such as toys and small appliances or cement-mixer barrels where noise and excessive backlash are not detrimental to operation. Sand casting is an economical way to obtain low-quality gear teeth in small quantities since the tooling costs are reasonable, but surface finish and dimensional accuracy are very poor. Die casting provides a better surface finish and accuracy than sand casting but has high tooling costs, requiring large production volume to justify its use.

INVESTMENT CASTING also known as lost-wax casting, can provide reasonably accurate gears in a wide variety of materials. The mold is made of a refractory material that allows high melt-temperature materials to be cast. Accuracy is a function of the original master pattern used to make the mold.

SINTERING Powdered metals (PM) are pressed into a gear-shaped metal mold cavity, removed and heat-treated (sintered) to increase their strength. These PM gears have similar accuracy to die-cast gears but their material properties can be controlled by mixing various metal powders. This technique is typically used for small-sized gears.

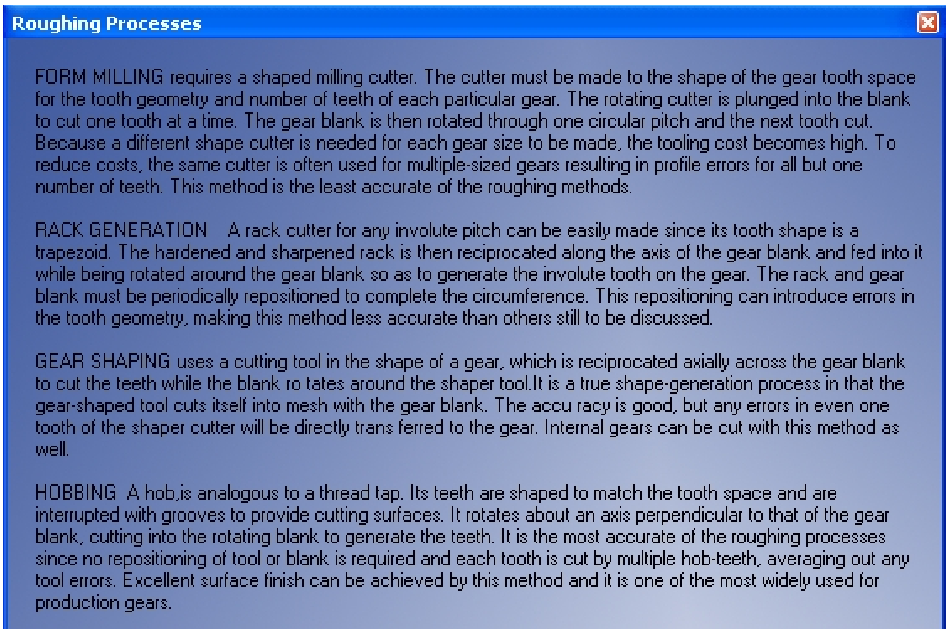
INJECTION MOLDING is used to make nonmetallic gears in various thermoplastics such as nylon and acetal. These are low-precision gears in small sizes but have the advantages of low cost and the ability to be run without lubricant at light loads.

EXTRUDING is used to form teeth on long rods, which are then cut into usable lengths and machined for bores and keyways etc. Nonferrous materials such as aluminum and copper alloys are commonly extruded rather than steels.

COLD DRAWING forms teeth on steel rods by drawing them through hardened dies. The cold-working increases strength and reduces ductility. The rods are then cut into usable lengths and machined for bores and keyways, etc.

STAMPING Sheet metal can be stamped with tooth shapes to form low-precision gears at low cost in high quantities. Surface finish and accuracy are poor.

Figure A. 40 Help form, gear manufacturing, forming gear teeth



Roughing Processes

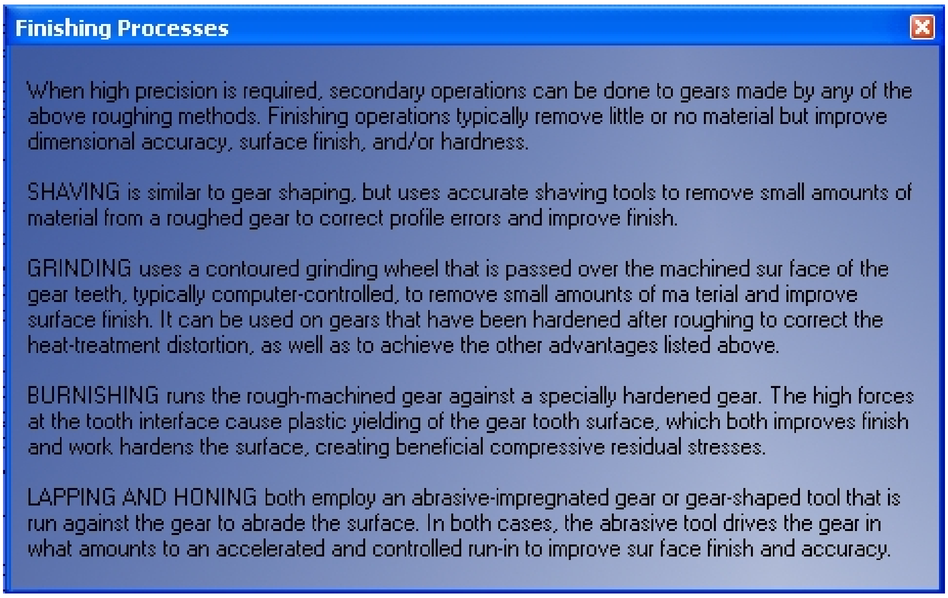
FORM MILLING requires a shaped milling cutter. The cutter must be made to the shape of the gear tooth space for the tooth geometry and number of teeth of each particular gear. The rotating cutter is plunged into the blank to cut one tooth at a time. The gear blank is then rotated through one circular pitch and the next tooth cut. Because a different shape cutter is needed for each gear size to be made, the tooling cost becomes high. To reduce costs, the same cutter is often used for multiple-sized gears resulting in profile errors for all but one number of teeth. This method is the least accurate of the roughing methods.

RACK GENERATION A rack cutter for any involute pitch can be easily made since its tooth shape is a trapezoid. The hardened and sharpened rack is then reciprocated along the axis of the gear blank and fed into it while being rotated around the gear blank so as to generate the involute tooth on the gear. The rack and gear blank must be periodically repositioned to complete the circumference. This repositioning can introduce errors in the tooth geometry, making this method less accurate than others still to be discussed.

GEAR SHAPING uses a cutting tool in the shape of a gear, which is reciprocated axially across the gear blank to cut the teeth while the blank ro tates around the shaper tool.It is a true shape-generation process in that the gear-shaped tool cuts itself into mesh with the gear blank. The accu racy is good, but any errors in even one tooth of the shaper cutter will be directly trans ferred to the gear. Internal gears can be cut with this method as well.

HOBBING A hob,is analogous to a thread tap. Its teeth are shaped to match the tooth space and are interrupted with grooves to provide cutting surfaces. It rotates about an axis perpendicular to that of the gear blank, cutting into the rotating blank to generate the teeth. It is the most accurate of the roughing processes since no repositioning of tool or blank is required and each tooth is cut by multiple hob-teeth, averaging out any tool errors. Excellent surface finish can be achieved by this method and it is one of the most widely used for production gears.

Figure A. 41 Help form, gear manufacturing, machining, roughing process



Finishing Processes

When high precision is required, secondary operations can be done to gears made by any of the above roughing methods. Finishing operations typically remove little or no material but improve dimensional accuracy, surface finish, and/or hardness.

SHAVING is similar to gear shaping, but uses accurate shaving tools to remove small amounts of material from a roughed gear to correct profile errors and improve finish.

GRINDING uses a contoured grinding wheel that is passed over the machined sur face of the gear teeth, typically computer-controlled, to remove small amounts of ma terial and improve surface finish. It can be used on gears that have been hardened after roughing to correct the heat-treatment distortion, as well as to achieve the other advantages listed above.

BURNISHING runs the rough-machined gear against a specially hardened gear. The high forces at the tooth interface cause plastic yielding of the gear tooth surface, which both improves finish and work hardens the surface, creating beneficial compressive residual stresses.

LAPPING AND HONING both employ an abrasive-impregnated gear or gear-shaped tool that is run against the gear to abrade the surface. In both cases, the abrasive tool drives the gear in what amounts to an accelerated and controlled run-in to improve sur face finish and accuracy.

Figure A. 42 Help form, gear manufacturing, machining, finishing process

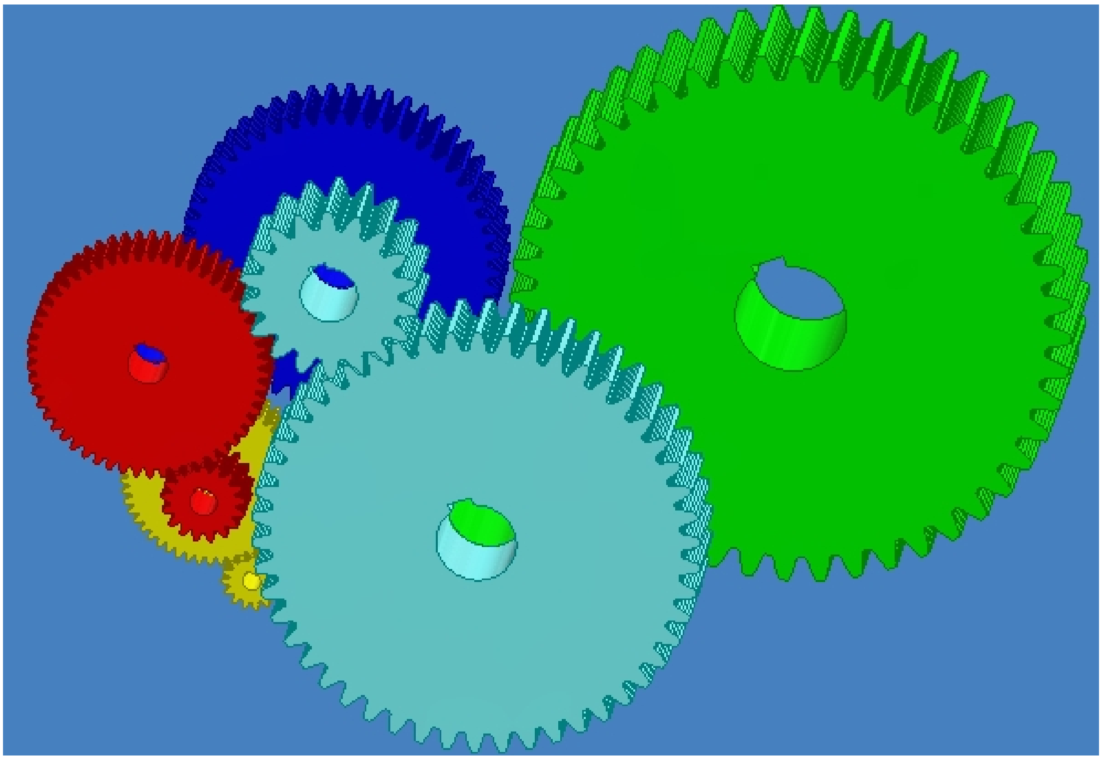


Figure A. 43 Solid modeling of a five-stage spur gear drive in Autodesk Inventor

APPENDIX B

CONFIGURATION DESIGN CONSTRAINTS FOR SIX STAGE SPUR GEAR DRIVE

```
MODEL:
[OBJECTIVE] MIN=(((x1-x4)^2+(y1-y4)^2)^0.5)+(((x1-x6)^2+(y1-
y6)^2)^0.5)+(((x1-x8)^2+(y1-y8)^2)^0.5)+(((x1-x10)^2+(y1-
y10)^2)^0.5)+(((x1-x12)^2+(y1-y12)^2)^0.5);

Pinion and gear center distance contraints(c1-c12)
[c1] (dp1+dp2)*0.5-(((x1-x2)^2+(y1-y2)^2)^0.5)=0;
[c2] (dp3+dp4)*0.5-(((x3-x4)^2+(y3-y4)^2)^0.5)=0;
[c3] (dp5+dp6)*0.5-(((x5-x6)^2+(y5-y6)^2)^0.5)=0;
[c4] (dp7+dp8)*0.5-(((x7-x8)^2+(y7-y8)^2)^0.5)=0;
[c5] (dp9+dp10)*0.5-(((x9-x10)^2+(y9-y10)^2)^0.5)=0;
[c6] (dp11+dp12)*0.5-(((x11-x12)^2+(y11-y12)^2)^0.5)=0;
[c7] z1-z2=0;
[c8] z3-z4=0;
[c9] z5-z6=0;
[c10] z7-z8=0;
[c11] z9-z10=0;
[c12] z11-z12=0;


Gears having same shaft distance constraints(c13-c27)
[c13] z3=z2+((b2+b3)*0.5);
[c14] x2-x3=0;
[c15] y2-y3=0;
[c16] z5=@IF( dp3+dp4 #GT# do2+do5, z4-
((b4+b5)*0.5),z4+((b4+b5)*0.5));
[c17] x4-x5=0;
[c18] y4-y5=0;
[c19] z7=@IF( ((dp5+dp6 #LT# do4+do7) #AND# (Z5#LT# Z4)) #OR#
((dp5+dp6 #GT# do4+do7) #AND# (Z5#GT# Z4)) , z6-
((b6+b7)*0.5),z6+((b6+b7)*0.5));
[c20] x6-x7=0;
[c21] y6-y7=0;
[c22] z9=@IF((dp7+dp8 #GT# do6+do9) , z8-
((b8+b9)*0.5),z8+((b8+b9)*0.5));
[c23] x8-x9=0;
[c24] y8-y9=0;
[c25] z11=@IF((dp9+dp10 #GT# do8+do11) , z10-
((b10+b11)*0.5),z10+((b10+b11)*0.5));
[c26] x10-x11=0;
[c27] y10-y11=0;
```

```
Gear interference constraints(c28-c57)
[c28] (((x1-x5)^2+(y1-y5)^2)^0.5)-(0.5*(do1+do5))>0;
[c29] (((x1-x6)^2+(y1-y6)^2)^0.5)-(0.5*(do1+do6))>0;
[c30] (((x1-x7)^2+(y1-y7)^2)^0.5)-(0.5*(do1+do7))>0;
[c31] (((x1-x8)^2+(y1-y8)^2)^0.5)-(0.5*(do1+do8))>0;
[c32] (((x1-x9)^2+(y1-y9)^2)^0.5)-(0.5*(do1+do9))>0;
[c33] (((x1-x10)^2+(y1-y10)^2)^0.5)-(0.5*(do1+do10))>0;
[c34] (((x1-x11)^2+(y1-y11)^2)^0.5)-(0.5*(do1+do11))>0;
[c35] (((x1-x12)^2+(y1-y12)^2)^0.5)-(0.5*(do1+do12))>0;
[c36] (((x2-x5)^2+(y2-y5)^2)^0.5)-(0.5*(do2+do5))>0;
[c37] (((x2-x6)^2+(y2-y6)^2)^0.5)-(0.5*(do2+do6))>0;
[c38] (((x2-x7)^2+(y2-y7)^2)^0.5)-(0.5*(do2+do7))>0;
[c39] (((x2-x8)^2+(y2-y8)^2)^0.5)-(0.5*(do2+do8))>0;
[c40] (((x2-x9)^2+(y2-y9)^2)^0.5)-(0.5*(do2+do9))>0;
[c41] (((x2-x10)^2+(y2-y10)^2)^0.5)-(0.5*(do2+do10))>0;
[c42] (((x2-x11)^2+(y2-y11)^2)^0.5)-(0.5*(do2+do11))>0;
[c43] (((x2-x12)^2+(y2-y12)^2)^0.5)-(0.5*(do2+do12))>0;
[c44] (((x3-x7)^2+(y3-y7)^2)^0.5)-(0.5*(do3+do7))>0;
[c45] (((x3-x8)^2+(y3-y8)^2)^0.5)-(0.5*(do3+do8))>0;
[c46] (((x3-x9)^2+(y3-y9)^2)^0.5)-(0.5*(do3+do9))>0;
[c47] (((x3-x10)^2+(y3-y10)^2)^0.5)-(0.5*(do3+do10))>0;
[c48] (((x3-x11)^2+(y3-y11)^2)^0.5)-(0.5*(do3+do11))>0;
[c49] (((x3-x12)^2+(y3-y12)^2)^0.5)-(0.5*(do3+do12))>0;
[c50] (((x4-x7)^2+(y4-y7)^2)^0.5)-(0.5*(do4+do7))>0;
[c51] (((x4-x8)^2+(y4-y8)^2)^0.5)-(0.5*(do4+do8))>0;
[c52] (((x4-x9)^2+(y4-y9)^2)^0.5)-(0.5*(do4+do9))>0;
[c53] (((x4-x10)^2+(y4-y10)^2)^0.5)-(0.5*(do4+do10))>0;
[c54] (((x4-x11)^2+(y4-y11)^2)^0.5)-(0.5*(do4+do11))>0;
[c55] (((x4-x12)^2+(y4-y12)^2)^0.5)-(0.5*(do4+do12))>0;
[c56] (((x7-x12)^2+(y7-y12)^2)^0.5)-(0.5*(do7+do12))>0;
[c57] (((x8-x12)^2+(y8-y12)^2)^0.5)-(0.5*(do8+do12))>0;

Constraints between gear and shaft(c58-c92)
[c58] (((x1-x4)^2+(y1-y4)^2)^0.5)-(0.5*(ds1+do4))>0;
[c59] (((x1-x6)^2+(y1-y6)^2)^0.5)-(0.5*(ds1+do6))>0;
[c60] (((x1-x8)^2+(y1-y8)^2)^0.5)-(0.5*(ds1+do8))>0;
[c61] (((x1-x10)^2+(y1-y10)^2)^0.5)-(0.5*(ds1+do10))>0;
[c62] (((x1-x12)^2+(y1-y12)^2)^0.5)-(0.5*(ds1+do12))>0;
[c63] (((x2-x6)^2+(y2-y6)^2)^0.5)-(0.5*(ds2+do6))>0;
[c64] (((x2-x8)^2+(y2-y8)^2)^0.5)-(0.5*(ds2+do8))>0;
[c65] (((x2-x10)^2+(y2-y10)^2)^0.5)-(0.5*(ds2+do10))>0;
[c66] (((x2-x12)^2+(y2-y12)^2)^0.5)-(0.5*(ds2+do12))>0;
[c67] (((x4-x10)^2+(y4-y10)^2)^0.5)-(0.5*(ds3+do10))>0;
[c68] (((x4-x12)^2+(y4-y12)^2)^0.5)-(0.5*(ds3+do12))>0;
[c69] (((x6-x10)^2+(y6-y10)^2)^0.5)-(0.5*(ds4+do10))>0;
[c70] (((x6-x12)^2+(y6-y12)^2)^0.5)-(0.5*(ds4+do12))>0;
[c71] (((x4-x1)^2+(y4-y1)^2)^0.5)-(0.5*(ds3+do1))>0;
[c72] (((x4-x2)^2+(y4-y2)^2)^0.5)-(0.5*(ds3+do2))>0;
[c73] (((x4-x8)^2+(y4-y8)^2)^0.5)-(0.5*(ds3+do8))>0;
[c74] (((x6-x1)^2+(y6-y1)^2)^0.5)-(0.5*(ds4+do1))>0;
[c75] (((x6-x2)^2+(y6-y2)^2)^0.5)-(0.5*(ds4+do2))>0;
[c76] (((x6-x4)^2+(y6-y4)^2)^0.5)-(0.5*(ds4+do4))>0;
[c77] (((x6-x10)^2+(y6-y10)^2)^0.5)-(0.5*(do6+do10))>0;
[c78] (((x8-x1)^2+(y8-y1)^2)^0.5)-(0.5*(ds5+do1))>0;
[c79] (((x8-x2)^2+(y8-y2)^2)^0.5)-(0.5*(ds5+do2))>0;
[c80] (((x8-x4)^2+(y8-y4)^2)^0.5)-(0.5*(ds5+do4))>0;
[c81] (((x8-x6)^2+(y8-y6)^2)^0.5)-(0.5*(ds5+do6))>0;
```

```
[c82] (((x10-x1)^2+(y10-y1)^2)^0.5)-(0.5*(ds6+do1))>0;
[c83] (((x10-x2)^2+(y10-y2)^2)^0.5)-(0.5*(ds6+do2))>0;
[c84] (((x10-x4)^2+(y10-y4)^2)^0.5)-(0.5*(ds6+do4))>0;
[c85] (((x10-x6)^2+(y10-y6)^2)^0.5)-(0.5*(ds6+do6))>0;
[c86] (((x10-x8)^2+(y10-y8)^2)^0.5)-(0.5*(ds6+do8))>0;
[c87] (((x12-x1)^2+(y12-y1)^2)^0.5)-(0.5*(ds7+do1))>0;
[c88] (((x12-x2)^2+(y12-y2)^2)^0.5)-(0.5*(ds7+do2))>0;
[c89] (((x12-x4)^2+(y12-y4)^2)^0.5)-(0.5*(ds7+do4))>0;
[c90] (((x12-x6)^2+(y12-y6)^2)^0.5)-(0.5*(ds7+do6))>0;
[c91] (((x12-x8)^2+(y12-y8)^2)^0.5)-(0.5*(ds7+do8))>0;
[c92] (((x12-x10)^2+(y12-y10)^2)^0.5)-(0.5*(ds7+do10))>0;

DATA:
Input
x1=0;
y1=0;

Pitch diameters of pinion and gear

dp1=@pointer( 1);
dp2=@pointer( 2);
dp3=@pointer( 3);
dp4=@pointer( 4);
dp5=@pointer( 5);
dp6=@pointer( 6);
dp7=@pointer( 7);
dp8=@pointer( 8);
dp9=@pointer( 9);
dp10=@pointer( 10);
dp11=@pointer( 11);
dp12=@pointer( 12);

Outer diameters of pinion and gear

do1=@pointer( 13);
do2=@pointer( 14);
do3=@pointer( 15);
do4=@pointer( 16);
do5=@pointer( 17);
do6=@pointer( 18);
do7=@pointer( 19);
do8=@pointer( 20);
do9=@pointer( 21);
do10=@pointer( 22);
do11=@pointer( 23);
do12=@pointer( 24);

Face width

b1=@pointer( 25);
b2=@pointer( 26);
b3=@pointer( 27);
b4=@pointer( 28);
b5=@pointer( 29);
b6=@pointer( 30);
b7=@pointer( 31);
b8=@pointer( 32);
b9=@pointer( 33);
```

```
b10=@pointer( 34);
b11=@pointer( 35);
b12=@pointer( 36);

Shaft diameter

ds1=@pointer( 37);
ds2=@pointer( 38);
ds3=@pointer( 39);
ds4=@pointer( 40);
ds5=@pointer( 41);
ds6=@pointer( 42);
ds7=@pointer( 43);
@pointer( 44) = x1;
@pointer( 45) = y1;
@pointer( 46) = z1;
@pointer( 47) = x2;
@pointer( 48) = y2;
@pointer( 49) = z2;
@pointer( 50) = x3;
@pointer( 51) = y3;
@pointer( 52) = z3;
@pointer( 53) = x4;
@pointer( 54) = y4;
@pointer( 55) = z4;
@pointer( 56) = x5;
@pointer( 57) = y5;
@pointer( 58) = z5;
@pointer( 59) = x6;
@pointer( 60) = y6;
@pointer( 61) = z6;
@pointer( 62) = x7;
@pointer( 63) = y7;
@pointer( 64) = z7;
@pointer( 65) = x8;
@pointer( 66) = y8;
@pointer( 67) = z8;
@pointer( 68) = x9;
@pointer( 69) = y9;
@pointer( 70) = z9;
@pointer( 71) = x10;
@pointer( 72) = y10;
@pointer( 73) = z10;
@pointer( 74) = x11;
@pointer( 75) = y11;
@pointer( 76) = z11;
@pointer( 77) = x12;
@pointer( 78) = y12;
@pointer( 79) = z12;
@pointer( 80) = @STATUS();
ENDDATA
END
```