MIDDLE GRADE STUDENTS' ABILITIES IN TRANSLATING AMONG REPRESENTATIONS OF FRACTIONS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

GÖNÜL KURT

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
SECONDARY SCIENCE AND MATHEMATICS
EDUCATION

JANUARY 2006

Approval of the Graduate School of Natural and Applied Sciences

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ABSTRACT

MIDDLE GRADE STUDENTS' ABILITIES IN TRANSLATING AMONG REPRESENTATIONS OF FRACTIONS

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January 2006, 112 pages

The aim of the study is two fold: (1) to determine middle grade students' abilities in translating among representations of fractions concept and (2) to investigate the effect of grade level and gender on students' abilities in translating among representations.

In this study the "Translations among Representations Abilities Test (TRAT)" was developed and used as a measuring instrument. The study was conducted in 19 randomly selected public elementary schools throughout Yenimahalle and Çankaya districts of Ankara with a total of 1456 sixth, seventh, and eight grade students in 2005-2006 fall semester.

Each student's responses which were considered as incorrect were examined according to their grade levels. Based on the findings, the most frequent incorrect response types, the easiest, and the most difficult items were identified. In addition to these findings, two-way analysis of variance model (ANOVA) was used in order to investigate the effects of grade level and gender on students' total scores on the TRAT.

To the results of the examinations of students' responses, it was seen that students' abilities in translating among representations of fractions were low. The most frequent incorrect responses were seen in translations which include number line models and region models representing improper fractions. The lowest mean score was belonged to the sixth graders; while the highest mean score was belong to the eighth graders. Results of the statistical analyses revealed that grade level had a statistically significant main effect on students' abilities in translating among representations. Additionally, it was seen that, female students had higher mean scores on the TRAT than males.

Keywords: Mathematics education, representations, fractions, middle grade students.

ÖZ

İLKÖĞRETİM İKİNCİ KADEME ÖĞRENCİLERİNİN KESİRLER KONUSUNDA TEMSİL BİÇİMLERİ ARASINDAKİ DÖNÜŞÜMLERİ YAPABİLME BECERİLERİ

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Ocak 2006, 112 sayfa

Bu çalışmanın amacı: (1) devlet okullarında eğitim gören ilköğretim ikinci kademe öğrencilerinin kesirler konusunda temsil biçimleri arasındaki dönüşümleri yapabilme becerilerini incelemek ve (2) cinsiyet ve sınıf seviyesinin bu beceriler üzerine etkisini araştırmaktır.

Çalışmada ölçüm aracı olarak "Kesirlerle ilgili Temsil Biçimleri Arasında Dönüşüm Yapabilme Becerileri Testi" geliştirilmiş ve kullanılmıştır. Çalışma, 2005-2006 sonbahar döneminde, Yenimahalle ve Çankaya ilçelerinden rastgele seçilen 19 devlet ilköğretim okulundan, 1456 altıncı, yedinci ve sekizinci sınıf öğrencisi ile yapılmıştır.

Her öğrencinin yanlış olarak değerlendirilen cevapları sınıf seviyelerine göre incelenmiştir. Elde edilen sonuçlara göre, en sık verilen yanlış cevaplar, en kolay ve en zor sorular belirlenmiştir. Bu sonuçlara ek olarak, sınıf seviyesinin ve cinsiyetin öğrencilerin ortalama puanlarına etkisini incelemek için iki yönlü varyans (ANOVA) analizi kullanılmıştır.

Öğrenci cevaplarının incelenmesi sonucu, öğrencilerin kesirler konusunda dönüşüm yapabilme becerilerinin düşük olduğu görülmüştür. En sık yapılan yanlış cevaplar, sayı doğrusu ve bileşik kesirleri temsil eden alan modellerini içeren dönüşümlerde görülmüştür. Testteki en düşük ortalama 6. sınıf, en yüksek ortalama ise 8. sınıf öğrencilerine aittir. İstatistiksel analizlerin sonuçları, sınıf seviyesinin öğrencilerin çoklu temsil biçimleri arasında dönüşüm yapabilme becerileri üzerinde anlamlı bir etkisi olduğunu göstermiştir. Ayrıca, kız öğrencilerin testten aldıkları ortalama puanlarının erkeklerden daha fazla olduğu tespit edilmiştir.

Anahtar Kelimeler: Matematik eğitimi, temsil biçimleri, kesirler, ilköğretim 6., 7., ve 8. sınıf öğrencileri.

To My Parents

Güler and Bayram KURT

ACKNOWLEDGEMENTS

I would like to wish to express my deepest gratitude to my supervisor Assist. Prof. Dr. Erdinç Çakıroğlu for his continuous encouragements and valuable guidance in every step throughout the research. Thank you sincerely.

I would also like to thank Dr. Oylum Akkuş-Çıkla from Hacettepe University, for her valuable suggestions and constructive comments starting from the beginning of the study. She helped me finding many articles and books which I could not find otherwise.

Special thanks goes to my mother Güler and father Bayram for their patiently and lovingly encouragements. I would like to dedicate this study to them as educators. I know that they have always believed and trusted in me, and in everything I do. My lovely sister Özlem has always been a guide for me. Her continuous encouragement and love provided the support I needed. My dear brothers, Özgür and Dr. Güngör Baba have always encouraged and tried to provide me a comfortable environment to study. Lastly, I would also like to send a special thank to Elif Ezgi, my niece, for introducing the very meaningful and beautiful feelings of being aunt.

I owe my thanks to the mathematics teachers İrfan Eren and Sevgi Otuz for sharing their ideas and giving permission to administer the test and providing feedback on the work in progress.

I would also like to thank to my office-mate Ertuğrul for all kinds of help he have provided me so far. I am also thankful to Emel, Mine and Şule for their moral support and suggestions throughout the study.

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LIST OF SYMBOLS

TRAT: Translating among Representations Abilities Test

ANOVA: Analysis of Variance

SPSS: Statistical Package for the Social Sciences Program

N: Sample Size

SD: Standard deviation

Df: Degree of Freedom

CHAPTER 1

INTRODUCTION

The new elementary Mathematics Curriculum for grade 1-5, developed by the Ministry of National Education (2005) underline the importance of developing students' abilities in problem solving and communication by using multiple representations. Similarly, the principles and standards of National Council of Teachers of Mathematics (NCTM, 1989; NCTM, 2000) emphasize the importance of multiple representations in mathematics instruction. The importance was stated in the communication skills section of new published elementary mathematics curriculum in below:

- Communication provides important links among physical, pictorial, verbal, mental, and symbolic representations.
- Students, who are able to recognize the multiple ways of representing a mathematical concept, will appreciate the power of mathematics.

- When students see one of the representation types was the easiest or the most effective, they appreciate the benefits and fluency of mathematics. (MEB, 2005, p.11-13)

Goldin (2003) defined representation as typically a sign or a configuration of signs, characters, or objects that can somehow stand for. It can be categorized into two parts namely, internal and external representations (Cuoco & Curcio, 2001). External representations are structured physical situations that can be seen as embodying mathematical ideas (e.g. number lines, graphs, words and sentences, common languages, etc.). Internal representations, on the other hand, are defined as images we create in our minds for mathematical objects and processes (Cuoco& Curcio).

Besides being important as stated in the new mathematics curriculum, multiple representations are considered as a key component of mathematics teaching and learning. Over the years, many researchers provided evidence that using multiple representations can help students to better construct the mathematical concept (e.g Ainsworth, Bibby, & Wood, 2002; Ball, 1983; Bryan & Mary, 1997; Cramer & Henry, 2002; Hiebert, Taber, & Wearne, 1991; Jiang & Mcclintock, 2000; Pape & Tchoshanov, 2001; Watanabe, 2002). Their common idea is that different representations facilitate the learning process (Boulton-Lewis, 1998) since they bring out the different meanings of the concept (Brenner, Brar, Duran, Mayer, Moseley, Smith, & Webb, 1995).

A fraction is a convenient concept that can be interpreted in many ways and able to be used to represent many different situations (Akkuş-Çıkla, 2003). The different interpretations of fractions which provide a deeper understanding of a rational number were identified by Kieren (1993) as (a) part-whole comparisons, (b) measures, (c) operators, (d) quotients, and (e) ratio and rates. If students are asked to show what $\frac{2}{3}$ means to them, different explanations which present Kieren's sub-constructs are given in Figure 1.1.

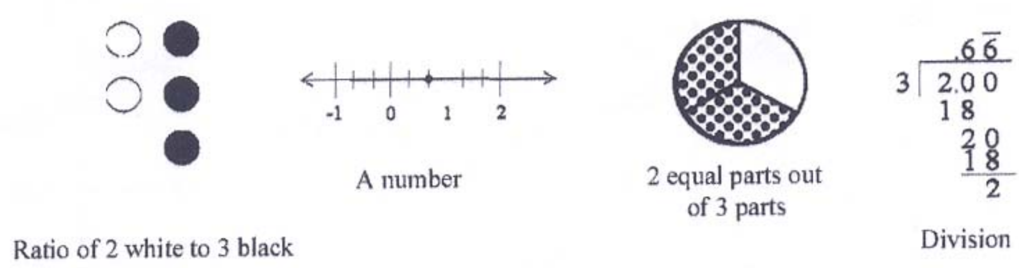


Figure 1.1 Different representations of $\frac{2}{3}$

The concept of fraction is one of the most abstract, complex, and problematic issue children encounter in their pre-elementary years (Aksu, 2001; Behr, Lesh, Post, & Silver, 1983; Bulgar, 2003; Ardoğan & Ersoy, 2003; Gearhart, McIntosh, Saxe, & Taylor, 2005; Fischbein,Graeber, Tirosh, & Wilson, 1994; Haser & Ubuz, 2002; Kilborn, 2003; Smith III, 2002; Toluk, 2002; Wu, 2002). In order to develop fraction sense, most children need to practice the use of multiple representations (Pape & Tchoshanov, 2001).

Multiple representations should be used not only in the learning process, but also in the assessment process (Bay-Williams & Martinie, 2003) to make students aware that there can be multiple ways to represent the same concept (Niemi, 1996). In this manner, students are expected to be fluent in choosing, identifying, generating, and using representations to communicate mathematical ideas more effectively (Niemi, 1996; Tchoshanov, 2001).

The Lesh model suggests that mathematical ideas can be represented in five different modes: (1) manipulative-like Cuisenare rods, arithmetic blocks, fraction circles, paper foldings; (2) pictures-like number lines, region, discrete objects models; (3) real-life situations-general context for interpreting and solving other kinds of problem situations; (4) verbal symbols-specialized sublanguages related to domains like logic, etc. (5) written symbols-specialized sentences and phrases like "x+3=7" (Cramer, 2003; Lesh, Post, & Behr, 1987). These different representations in modeling fractions concept can maximize learning as well as will help students to develop deeper understanding (Behr et al., 1983). The Lesh model stressed that not only are these distinct types of representations are important in their own rights, but translations among them also are important (Lesh et al., 1987). He presented a model representing the translations among different representations as seen in Figure 1.2.

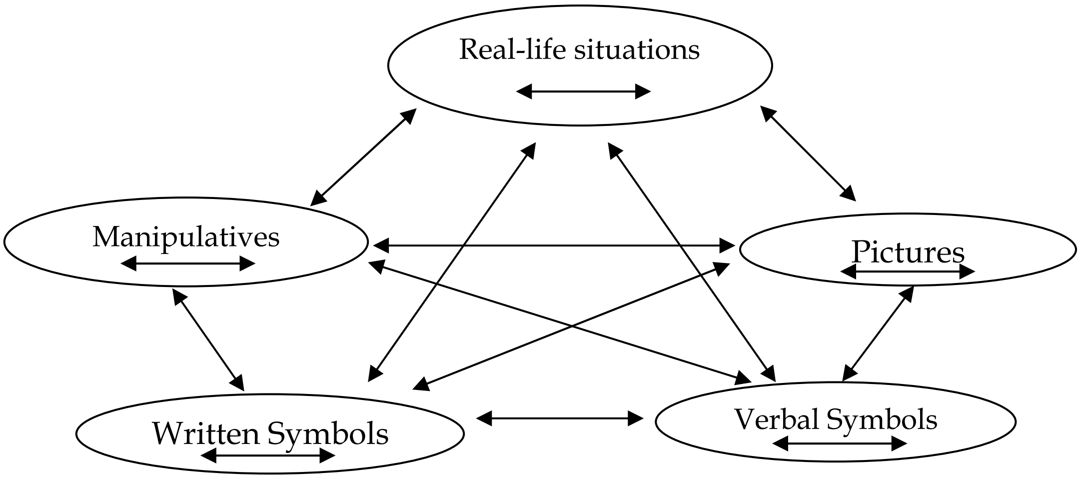


Figure 1.2 The Lesh Translation Model

The Lesh translation model emphasized the translations within and between different types of representations. Doing these translations make mathematical ideas more meaningful for students (Cramer, 2003). One of the key tasks for learning with multiple representations is considered as being able to translate within and between them (Ainsworth, et al. 2002; Post, Behr, & Lesh, 1982; Jones, 1998).

It was needed to introduce the number line model since it has an important role in the related literature. The number line differs from other models (e.g., regions, discrete objects) with respect to three aspects (Bright, Behr, Post, & Wachsmuth, 1988). First, a length represents the unit and the number line suggests not only repetition of the unit but also subdivisions of all iterated units. Second, there is no visual separation between following units. In other words, the model is totally continuous. Third, the number line requires an integration of visual and symbolic forms of information to express the part of the intended meaning. Other models do not need such integration, as they do express the meaning without them (Bright et al.)

In this study, a model including five types of representations reflecting the Lesh Translation Model was identified. Figure 1.3 presents the model used in this study for assessing students' abilities in translating among representations of fractions.

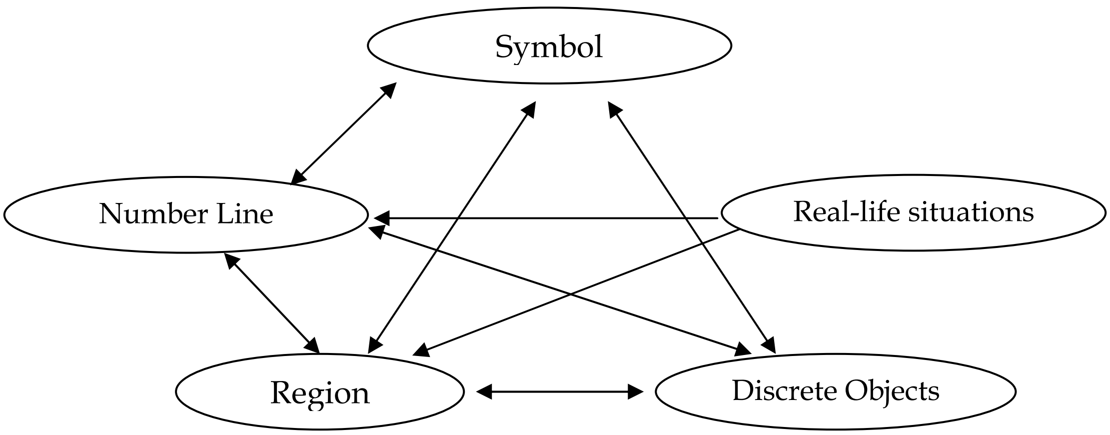


Figure 1.3 Translations among Representations Used in the Current Study

Actually, through a more general classification the model mainly consists of symbol, picture, and real-life situations modes of representations, which were the main concerns of this study. Since, number line, region, and discrete objects are considered in the picture type of representations by Lesh, Landau, and Hamilton (1983), overall five different types of representations were used in this study (Note. Decimal fractions have been considered in the symbolic form of representations). The arrows in Figure 1.3 connecting the different modes depict the translations between them. As seen from Figure 1.3, arrows from real-life situations to the number line and region models signify a one-directional representation instead of translations among them. No translations within representations (transformations) were used in this study as appears in Figure 1.3.

In this manner, this study aims to investigate the middle grade (6th, 7th, and 8th) students' abilities in translating between representations regarding the concept of fractions.

1.1 The Main Problems and Sub-Problems

1.1.1 The Main Problem

The purposes of the study are (1) to determine middle grade students' abilities in translating among different representations regarding the concept of fraction, (2) to investigate the effect of grade level and gender on students' abilities in translating among representations.

1.1.2 The Sub-Problems

The following sub-problems were investigated based on the main problem.

1. What are the abilities of 6^{th}, 7^{th}, and 8^{th} grade public elementary school students in translating among different representations of fractions?

2. What are the most frequent incorrect responses that 6^{th}, 7^{th}, and 8^{th} grade public elementary school students made in the Translations among Representations Abilities Test (TRAT)?
3. In which type of translations, students are flexible to move from one type of representation to another?
4. Is there a significant effect of grade level and gender on the public elementary school students' TRAT scores?

1.2 Null Hypotheses

The problems stated above were tested with the following hypotheses.

Null Hypothesis 1

There is no significant main effect of grade level on the population means of public elementary school students' total TRAT scores.

Null Hypothesis 2

There is no significant main effect of gender on the population means of public elementary school students' total TRAT scores.

Null Hypothesis 3

There is no significant interaction effect of grade level and gender on the population means of public elementary school students' total TRAT scores.

1.3 Definition of Important Terms

This section presents some important definitions related to this study.

Representation: It is a configuration of signs, characters, icons, or objects that can somehow stand for, or "represent" something else (Goldin, 2003).

Multiple Representations: Providing the same information in more than one form of external, symbolic, and mathematical representation (e.g. representing $\frac{1}{3}$ with region, number line, and discrete objects model (Cuoco & Curcio, 2001).

Translations among representations: Going from one type of representation to another (e.g representing a fraction from region model to a number line model (Janvier, 1987).

Transformations within representations: Moving into the same type of representation (Janvier, 1987).

Middle Grade Students: 6^{th}, 7^{th}, and 8^{th} grade public elementary school students.

1.4 Significance of the Study

Since the study of fractions is foundational in mathematics education, it is introduced from the first grade to the sixth grade of elementary school. This subject is also reviewed repeatedly in the following seventh and eight grades of elementary school. In spite of this repeated instruction, as mentioned earlier, most of research findings indicate that it is among the most difficult topics of mathematics for elementary students (Bulgar, 2003). Therefore, studying students' abilities related to fractions could provide important insights for the improvement of mathematics education.

A number of research studies related to the Fractions concept are stated in the literature. However, a limited number of studies have been conducted in Turkey (Toluk, 2002). Existing studies generally investigate students' difficulties in computation and/or solving word problems. However, it was claimed that if a student understands a mathematical idea she or he should have the ability of making translations between and within modes of representations (Lesh et al., 1987). In this manner, the present study aims to investigate students' performances not only in the computational manner, but also with respect to the abilities in translating among different representations in the concept of fractions.

The results of this study will inform mathematics teachers to better organize their activities and classroom practices regarding the concept of fractions. Findings will provide much information about the most difficult and the easiest type of translations for students. Findings will also provide clues about how flexible are the students in terms of translating representations of fractions. Thus, teachers will be able to identify the inadequate points in students' existing skills and which points to emphasize during instructional process. Teachers will also enable to identify the inadequate points and emphasize the necessary information.

Additionally, the results of the study highlight the importance of the basic fractions concept, and the necessity to make students flexible in moving one type of representation to another.

CHAPTER 2

REVIEW OF THE LITERATURE

This chapter aims to present a brief review of related literature on the concept of fractions, multiple representations, and studies conducted by Rational Number Project (RNP), which are relevant to this study.

2.1 The Concept of Fractions in Elementary Mathematics

In the related literature, fractions concept have been studied for many years. These studies mainly emphasize students' conceptions or knowledge of fractions.

Aksu (2001), in her study, investigated 155 sixth-grade students' performances in dealing with fractions. She considered students' performances as in three parts. They are (a) understanding the meaning of fractions, (b) computations with fractions, and (c) solving word problems involving fractions. A test was developed to measure the sixth-grade students' achievement in these three performance categories. She partitioned the test in three sections namely, a concept test, an operations test, and a problem-solving test. In the concept test, students were told to give short answers to the questions. "In a whole or 1, there are how many thirds?", "What is half of a fourth?" are the sample questions of the concept test. The operations test consisted of 32 one-step operations, eight items for each four operations (addition, subtraction, multiplication, and division), on fractions. The problem-solving test comprised word problems with the same operations stated in the operations test. The statistical analysis results indicated that of the three tests' means were significantly different from each other. The highest mean was belonged to the operations test, while the lowest mean score was found for the problem-solving test. Students' performances were similar with the four types of computations in the operations test. There were no relationships between gender, and performance on each of the three tests. However, a significant relationship between previous mathematics achievement and performance on each of the three tests was detected according to the correlational analyses (Aksu, 2001).

Haser and Ubuz (2003) conducted a study investigating students' conception of fractions in solving word-problems. 122 fifth grade students in a private elementary school were administered an essay type test including 10 word problems about the part-whole concept in fractions.

Researchers identified the type of problems as routine and non-routine. For the former, students were required to find the quantity of a whole when the quantity of a part was given and the quantity of a fraction when the whole was given as a quantity, or both. In non-routine type of problems, students were asked to complete or find the missing information to solve the problem.

Students' success in solving word-problems which include more than one operation was quite low. It was also seen that the type of the fraction to be multiplied had an effect on students' success rates on problems with one operation. According to the results of the study, students tried different operations with the given fractions and quantities to reach the solution.

Haser and Ubuz (2003)mentioned the reasons of error types in seven categories: (a) confusing the parts of a whole with the quantities, (b) doing operations between different kinds of units, (c) considering the amount of a part as the amount of a whole, (d) explaining fractions in conceptually wrong ways, (e) considering the parts of more than one whole as a set of whole, (f) using wrong steps for computations, and (g) preferring wrong operations.

Another study conducted by Haser and Ubuz (2002) was aimed to investigate the performance of students in using knowledge and skills on conceptual and computational tasks using fractions. 145 fifth grade students were administered an essay type test including 14 conceptual and computational questions.

They found that students' performances presented different results according to the types of fractions. For instance, the correct percentages of items including improper fractions were those which have the smallest scores. Additionally, detail analyses showed that students had difficulties in items involving equal partitioning, equality of fractions, and improper fractions.

Toluk (2002), in her study, aimed to identify elementary school students' conceptual schemes which were constructed as moving from part-whole meaning to the quotient meaning of fractions. For this purpose, clinical and semi-structured interviews were conducted with 4 fifth grade students in a seven-week period. The results of the study indicated that students were not able to conceptualize rational numbers as quotients. In order to construct this meaning, students' pre-knowledge of fractions and quotient should be stated as a reference point and fair-sharing experiences should be considered as an origin (Toluk, 2002).

The study conducted by Larson (1988) was designed for students to learn and correctly use the fractional terms. Five second graders and five third graders were taught, in a series of twelve lessons, part-whole fraction concepts using several area and capacity models. Interviews also were conducted with each of ten students. It was shown as a result that students were less successful in using fractional terms for set models than for area models but they did increase their ability after the instruction.

Bulgar (2003) aimed to introduce fraction concepts to students prior to the formal introduction of algorithms. In other words, she gave students an opportunity to understand the fraction concepts before the process of operations. 25 fourth grade students were involved a year-long teaching intervention with fifty sessions. A problem, called "Holiday Bows" was introduced to students in order to elicit their ideas related to division of fractions. Since the researchers wanted to study how students built their own ideas about fractions, rather than how they used the algorithms, they did not tell anything about students' correct or incorrect answers. The results showed that fourth grade students were able to solve the problem by constructing their own knowledge about those fraction concepts, although they were not formally taught any algorithms.

According to Ma (1999) division is the most complicated of four operations and fractions are most complex numbers in elementary school mathematics. Division by fractions, the most complicated operation with the most complex numbers, can be considered as a topic at the summit of arithmetic (Ma, 1999).

Ma (1999) stated that computational procedure was clear and explicit for most students or teachers. Her study was designed to compare American teachers' performances with Chinese teachers' on division by fractions. 23 American teachers and 72 Chinese teachers were interviewed and asked them "how do you solve a problem like $1\frac{3}{4} : \frac{1}{2}$ =?" Remarkable differences indicated that teachers had many weaknesses at the topic of division of fractions. Only few of them completed the computation and reached the correct answer. On the other hand, all Chinese teachers participated to the study gave correct answers and explanations.

Gearhart, McIntosh, Saxe, and Taylor (2005) conducted a developmental analysis about representing fractions with standard notation. The two purposes of the study were: (a) to investigate the developmental relationship between students' uses of fractions notation and their understandings of part-whole relations, and (b) to produce an analysis of the role of fraction instruction in students' use of notation to represent parts of an area. Researchers preferred to administer pre and post tests just before and just after the instruction of fractions. According to the findings of pre-test, a pattern indicated that many students were not differentiating between discrete and continuous quantities. Equal and unequal area part-whole relations as continuous quantity were interpreted only 9% of the students. Similar to the results of the pre-test, in the post-test, almost all students produced part-whole relations for the equal area problems.

Another study conducted with Gearhart, Saxe, and Seltzer (1999) investigated the relation between student learning in the context of fractions and the extent to which classroom practices were aligned with reform principles. They differentiated students who did and did not represent an initial understanding of fractions at pretest. Their purpose was to increase variation in classroom practices in two ways. First, teachers who were applying a curriculum unit intended to support reforms, or who were using a traditional textbook were improved by researchers. Second, researchers provided one of two different professional support programs for teachers using reform unit. In order to obtain data from those classrooms, fraction instructions were administered, videotaped and took field notes, and assessed students' pre- and posttest scores on items involved computation and problem solving with fractions.

To analyze data, hierarchical linear model analyses were performed (Gearhart et al., 1999). Analyses revealed that alignment of classroom practices with reform principles was related to student achievement in problem solving but not in computation. Gearhart et al. (1999) expected the lack of relation between student performance on the computation scale and alignment of classroom practice with reform principles. They inferred from the results that neither supports for students' conceptual engagement with mathematics nor efforts to build on student understandings were likely to improve students' memorization of arithmetic procedures.

Niemi (1996) in his study aimed to assess students' performances in concept of fractions with different type of tasks namely, representational knowledge, problem solving, justification, and explanation tasks. 540 fifth grade students were administered at the study. The researcher claimed that level of representational knowledge would be a predictor for the students' performance on the other tasks. In the representational knowledge task, students were asked to circle all representations showing the same amount as the symbol ($\frac{1}{2}$, $\frac{2}{4}$, $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{3}{2}$) at the top of the page. Below each symbol, a set of 18 graphic representations (number line, region, discrete objects models) was randomly arrayed on the remainder of the page. In the problem solving and justification tasks, students were asked to solve problems and to justify their use of symbolic procedures or other operations. In addition to the problem solving task, students were required to complete a contextualized explanation task. According to the overall results, Niemi (1996) argued that the representational knowledge, justification, and explanation tasks could provide useful and valid information on students' knowledge of mathematical representations.

Wu (2001) made a study about multiplying fractions. She stated that children who are taught rules for performing computation can multiply pairs of fractional numbers easily. However, computing by rules alone would cause children not to understand the meaning of the computation. To explore the meaning of multiplication of fractions, Wu

(2001) used five real-life problems and gave the pictorial representations of problems to guide teachers and students.

Tirosh, Fischbein, Graeber, and Wilson (1994) did a research in a part of a project called "The Conceptual Adjustments in Progressing from Whole to Non-negative Rational Numbers" (CAPWN). The main goal was to develop a conceptual framework for analyzing prospective teachers' mathematical knowledge of rational numbers.

Tirosh et.al (1994) based their study on the assumptions that learners' mathematical knowledge is embedded in a set of connections among algorithmic, which include procedural process consisting of rules and prescriptions; intuitive, which comprises ideas and beliefs about mathematical entities; and formal dimensions of knowledge, which includes axioms, definitions, theorems and their proofs. Tirosh et al. conducted their study by 147 elementary prospective teachers, 26 of which were mathematics majors. They developed a diagnostic questionnaire examining the subjects' formal, algorithmic, and intuitive understanding of rational numbers. At the end of the study, they found that (1) most prospective teachers had performed satisfactorily in the algorithmic section. Their common difficulties were in division of decimal numbers; (2) non-major mathematics prospective teachers' formal knowledge was not sufficient and complete; (3) many prospective teachers (both major and non-major mathematics) had primitive beliefs such as "multiplication ways makes bigger", and "division always makes smaller".

Cramer and Lesh (1988) investigated the 48 elementary pre-service teachers' conceptual understanding of rational number ideas. They were given a 45-item fraction test. The test included seven categories namely, fraction equivalence, fraction division, concept of unit, ordering, qualitative questions, part-whole concepts, and division of story problems. Cramer and Lesh (1988) found that 20% of the pre-service teachers did not understand rational number ideas well enough to be expected to teach fractions meaningfully.

In the following section, related studies in the concept of multiple representations were stated with regarding the concept of fractions.

2.2 Multiple Representations in the Rational Numbers Concept

Rational Number Project (RNP) is a cooperative research and development project supported by the National Science Foundation (Cramer, Behr, Post, & Lesh, 1988). The researchers studied in this project have been investigating the nature of children's rational numbers ideas in grades 4 to 8 since 1979 (Lesh, Landau, & Hamilton, 1983). Several long-term teaching experiments reported concerning the teaching and learning of fractions by emphasizing the use of multiple representations.

The project consists three interacting elements: (a) during 16 weeks of theory-based instruction, 18 fourth and fifth graders were observed, interviewed, and tested in terms of instructional component, (b) written tests, instruction mediated test, and clinical interviews were administered to more than 1600 second through eight grade students as an evolution component, and (c) students experiencing difficulties with fractions were identified; their misunderstandings were removed and remediated with evolution component’s materials and instructional component’s activities (Behr, Lesh, Post, & Silver, 1983).

Behr et al. (1983)stated the main purposes of the project as to define the improvement of the relations and operations students in grades 2 to 8 use to make sense involving rational numbers and to define the distinct types of representations’, e.g. pictures, spoken language, manipulative materials and written symbols, roles in the use of rational number concepts

A research study conducted by Behr, Lesh, Post, and Wachsmuth (1985) investigated the fourth-grade students’ understanding of the order and equivalence of rational numbers with an 18-week teaching experiment. The instructional program was maintained through 13 lessons with 12 subjects. Part-whole interpretation of rational number by means of circular and rectangular pieces of laminated colored paper as well as other manipulative materials presented to a group or to individuals. Each student was interviewed individually on different occasions. Besides instruction, some notes were taken during the lessons by a participant observer.

The analysis of interviews showed that development of children’s rational number understanding appears to be connected to three characteristics of thinking: (a) flexibility of thought in performing translations between modes of representations in rational numbers, (b) flexibility of thought for transformations within a presented type of representation, and (c) reasoning that becomes independent from a reliance on concrete form of rational numbers (Behr et al., 1985).

Behr, et al. (1983) conducted another study in 77 fourth-grade classes. The findings immediately revealed two main results. First, disproportionate number of errors in number line problems across all categories was detected. The researchers stated that this might cause of number line model for whole number interpretations of addition and subtraction. Fourth-grade students, also, were found incapable $\frac{5}{3}$ in conceptualizing a fraction as a point on a line. Second, the fraction $\frac{5}{3}$ as a discrete context caused many more errors than do the fractions $\frac{3}{4}$ and $\frac{2}{3}$. This was because of, perhaps, the fact that giving a few number of examples. The data collected from a suburban elementary school.

Another study conducted by Lesh et al. (1983). The study consisted of interviews, in which 80 fourth through eight graders solved problems presented in a number of formats, and written test which were administered to about 1000 students in grades 2 to 8 in 1980-1981 school year. The results of the written test were more emphasized in this study.

The test separated two sections namely, operation assessment and conceptual assessment. It covers several types of pictorial representations as well as written language and written symbols. Since each item concept was not appropriate to students from grade 2 through 8, younger students did fewer items than older students. The cronbach-alpha reliabilities across the tests averaged .88.

The results showed that the most difficult items were done by only older students.

(a) The easiest translations were those involving simply reading a rational number in two different modes, which require little or no conceptual processing of the meaning of the rational number,

(b) The translations to pictures were more difficult than from pictures,

(c) A written or symbolic expression was easier to process than is a pictorial representation. On the other hand, some of the interview results were as follows; (a) Subjects often used two or more type of representation simultaneously; (b) problems presented in terms of concrete materials are not necessarily easier than those presented orally or using written language and symbols; (c) the mode of representation was important not only in the initial problem presentation, but also when selected by the subject.

2.3 Research Studies Focusing on Number Line Model

Bright, Behr, Post, and Wachsmuth (1988) studied to investigate the ways whether fourth and fifth grade students accurately represented fractions on number lines and the influence of instruction on those representations. Actually, the main purpose of the study was to attempt to determine the links between students' understandings and the representation of fractions on number lines. To achieve this goal, researchers conducted two clinical teaching experiment and a large-group teaching experiment. The last two instructions were organized according to the students' deficiencies during the first instruction.

The first clinical teaching experiments run four days. The topics were association of fractions with points, comparison of fractions represented on a number line, and transformations of fractions representations on a number line. After the instruction, a 16-item multiple-choice fraction test of Larson (as cited in Bright et al., 1988) was applied to students just before and just after the instructions. A sample item from the test was given in below.

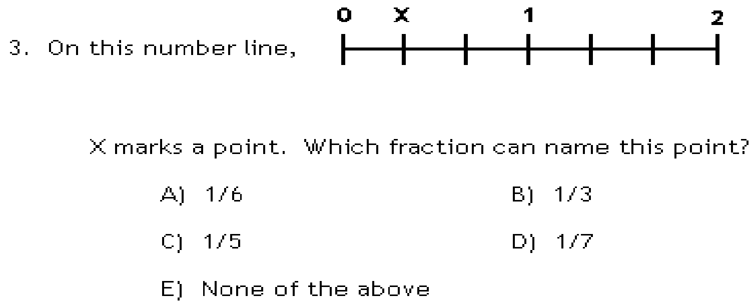


Figure 2.1 A Sample Item from the Test

In the second clinical teaching experiment, eight students from same elementary school were assigned as subjects. This instruction was lasted eight days. Another number line test was applied to students. A sample item was stated below.

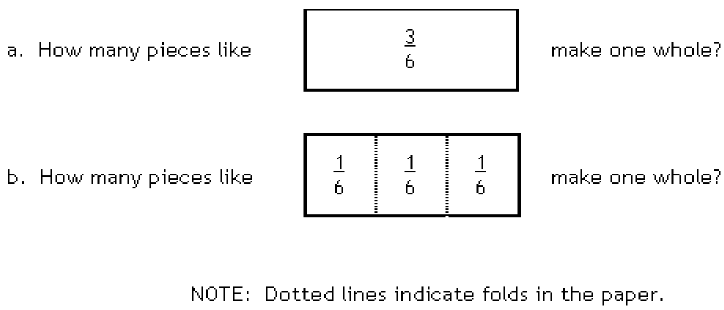


Figure 2.2 A Sample Item From the Test

Thirty-four students reported to be attended a large-group teaching experiment. The results of the study denoted that the instruction seemed to be effective. Fraction-given items were found easier than representation-given items. Researchers mentioned about the possible source of students' errors. Failures to recognize unreduced representations, for instance, might be show either an inability to unpartition or inflexibility in translating between types of representations. Students had a greater success in the clinical teaching experiment and in the large-group teaching experiment (Bright, et al., 1988). It was stated that this success might be sourced from the added attention given to translations between part-whole displays and number lines, to finding units on number lines, or to more emphasis on the measure construct.

Ni (2000) investigated validity of scores derived from the measurement procedure involving number lines by assessing its unique contributions to performance differences in criterion measures of rational number knowledge and skills, including fraction computation, application and explanation. In total, 413 students, 205 from 5th grade and 208 from 6th grade, were selected. Students' general achievement level were average or above average according to the city-wide graduation tests. Those students had been given instruction on fractions since they were third graders. The test administered by the classroom teacher in two class sessions contained five sections: (1) *Regional area and number line representation of fractions* contained 44 items (16 were distractors) which were belonged to two types of items. One involved regional areas to assess part-whole knowledge about rational number while the other involved number lines to measure measurement knowledge. (2) *Comparing sizes of fractions* had 12 items. Students were asked to indicate which of two numbers was larger or smaller. (3) *Computation* contained 30 items. (4) *Application of Graphics, Procedures and Operations to Meaningful Situations* included 20 items. (5) *Explanation* had 6 questions.

To the result of the confirmatory factor analysis, performances on the regional area items were significantly related with that on the items of fraction-size comparisons for both groups, as well as with the factor on the number line items for the fifth graders, although not for the sixth graders.

Scores derived from the number line test items were poor estimators of children's understanding of the measurement aspect of rational number. Students' poor performance on measures of their mathematical achievement may be based on the reason that children do not have adequate knowledge or skills a measurement procedure was intended to assess. It was stated that there was a clear gap between children's ability to understand main features of rational numbers in developmental studies and children's errors in solving rational number tasks in the context of educational assessment.

Ni (2001) explored different dimensions on the same sample which reported in her previous study. She now examined how the semantic meanings of rational numbers presented in graphical representations cause a limitation on children's construction of the concept of fraction equivalence. A total of 90-items, 35 were distractors, test was administered to students. Five different fraction symbols, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{3}{2}$ were stated at the top of five different pages. A set of 18 graphic representations was randomly placed below each symbol. Students were told to circle all representations showing the same amount as the fraction symbol at the top of the page.

Results from students’ performances on items, mentioned by the researcher, revealed that the factor of representation type (region, set, number line, and line segment) had a significant impact on students’ performance. Both groups did significantly better on the simple fraction items than on the equivalent fraction items. They performed best on the area items and line segment items, less well on the set items, and the poorest was the number line items. A significant interaction between representation type and simple vs equivalent fraction was detected. Overall performance between the students of the two groups was not indicated a significant difference. However, a significant interaction was seen for grade level with representation type. A significant three-way interaction for representation type, simple vs. equivalent fraction, and grade level was stated. The sixth-graders remarked significantly improved performance on the equivalent fraction items showing the part-whole relation but not on those representing the measure meaning of rational number.

Marcou and Gagatsis (2002) presented a research particularly on grade 5 students’ abilities to use external systems of representing related to the concepts of equivalence and addition of fractions. In this manner, they focused on students’ ability to move from one type of representation to another and their ability to identify concepts with different types of representations. They focused three types of external representations, namely, symbolic, diagrammatic, and verbal. These types of representations were also analyzed in terms of their effectiveness to use and to translate among them. In order to identify grade 5 students' abilities in using external representations, researchers administered a written test, including four parts, to 104 students. At the beginning of the test, verbal information was given to students about three representations types to help students understand what was expected from them. In the first part of the test, five tasks given in symbolic form were asked students to solve and translate them into their diagrammatic and verbal form. In the second part, similar to the first part, five tasks were given in diagrammatic form. This time students were required to solve the diagrammatic task and translate them into verbal and symbolic form. The third part was designed as requiring solving verbal statement and translating them into other two forms. The last part consisted of a written problem. Students were asked to solve it with the any form of representations they preferred. This part aimed to investigate students' preferences related to the types of representations. According to the results, students' flexibility in moving one form of representation to another was found weak. In addition, there was no connection between the tasks consisting of symbolic, verbal, and diagrammatic representations (Marcou and Gagatsis, 2002).

As a result of the review of literature, it was seen that several research studies related to fractions concept were done. Particularly, students' or pre-service teachers' computational abilities were emphasized by performance tests. However, in recent years, studies have begun to be formed more conceptual than procedural by dealing with multiple representations in the fraction concept. Then, it was claimed that establishing a relationship between different types of representations is crucial for deeper understanding. A limited number of studies, however, which emphasize translations among different representations, are stated in Turkey. Thus, in order to provide contributions and suggestions by the findings of this study, students' abilities to translate among distinct type of representations were investigated.

CHAPTER 3

METHOD

In this chapter, the aim is to report the procedures of the study. Population and sampling, description of variables, the measuring instrument, procedure, methods used to analyze data, assumptions and limitations will be explained briefly.

3.1 Population and Sample

All middle grade (6^{th}, 7^{th}, and 8^{th}) students in Turkey were identified as the target population of this study. Since it is not easy to reach to this target population, it was found to be appropriate to identify an accessible population. All public school middle grade students in Yenimahalle, Çankaya, and Etimesgut districts of Ankara were defined as the accessible population (Etimesgut district of Ankara was omitted from the list due to the time limitation, although the official permission had been taken). This is the population for which the results of this study will be generalized. The population being sampled in this study was 57359 students according to the 2004-2005 census' results. 52 % were males and 48% were females of this population. The desired sample size was determined as 5700, which is approximately 10% of the whole population of middle grade students. Stratified cluster random sampling and convenience sampling were used to obtain a representative sample of the population. First, the two districts in Ankara, from which the sample of the study was chosen, were selected by the convenience sampling method, and schools were randomly selected from each these districts in similar proportions with the population by using *Okul, Adres, Telefon Rehberi, (Çelenk&Dikmen, 2005)* . From the selected schools, classes to which the instrument was administered were selected by taking into consideration the convenience of administration and teachers. The researcher was able to reach to 1456 public school middle grade students at the end, which corresponds to 2.6% of the accessible population. Of the sample 53.6 % were male and 46.4 % were female students.

Table 3.1 presents total number of elementary schools throughout the districts, number of selected elementary schools throughout the districts and number of selected students from each of the districts. An average of 75-80 students per elementary school was participated to the study.

Table 3.1 Numbers of Schools, Participated Schools and Participated Students based on the Districts

Districts	Total # of Elementary Schools	# of Participated Schools	# of Participated Students
Çankaya	104	11	788
Yenimahalle	86	8	668
Total	190	19	1456

Distribution of gender with respect to grade levels was given in Table 3.2.

Table 3.2 Distribution of Grade Levels with respect to Gender

	6^{th} grade	7^{th} grade	8^{th} grade
Males	246	266	269
Females	195	244	236
Total Percent (%)	31.5	34.4	34.1

3.2 Variables

There are three variables involved in this study, which were categorized as dependent and independent.

3.2.1 Dependent Variable

The dependent variable was students' translation abilities among representations in the concept of fractions (TRAT) as measured by the Translations among Representations Abilities Test.

3.2.2 Independent Variables

The independent variables included in this study were gender and grade level of students. Among these variables gender is in nominal scale of measurement. Grade level is in ordinal scale of measurement.
Gender: It labels the subjects as male and female.
Grade Level: It labels the educational level of subjects: 6^{th}, 7^{th}, and 8^{th} grade students.

3.3 Measuring Tools

In this study, for the assessment of students' ability to perform translations among representations regarding the concept of fractions, a measuring tool was used.

3.3.1 Translations among Representations Abilities Test (TRAT)

The TRAT was used to assess middle grade students' abilities to perform translations among different representations regarding the concept of fractions. It was prepared by making use of wide range of sources. Some items were translated into Turkish from different sources and the others were developed by the researcher with the aim of analyzing the students' translation abilities. Test covers the basic fractions concepts such as part-whole comparison, equivalent fractions, reduced fractions, and simple operations taught from the early elementary years to the sixth grade. The duration of the test was one class hour, 40 minutes.

TRAT was partitioned into three parts: In the first part, basic types of representations were asked to translate into the expected models. Especially, improper fractions were preferred rather than simple ones, since middle grade students were selected as the subjects. On the other hand, items including discrete objects models contain simple fractions as most of the students had met the discrete objects model for first time. A sample item was given at the beginning of the test to help students to remind those types of representations, particularly for discrete objects model. Since there were four types of representations, there would be twelve translations among them (without transformation within them). Items were given in table formats in order to enable students to understand the required translations clearly. Addition to the first part, three decimals fractions were given and asked students to translate them to the other type of representations (number line, region, and discrete objects).

The second part of the test consisted of 4 multiple-choice items. Three of these were adopted from the Rational Number Project (Landau &

Lesh, 1983); one was adopted from Bassarear (2005). These items involve simple algorithms like addition and multiplication. However, since multiple choice items do not provide insight into students' understanding about the topic, students were required to explain the reason or write the operation near to the marked-choice. Reasons stated by most of the students can be the any identified misconceptions. This leads us to give more information about students' understanding.

The last part of the test includes two word problems reflecting the real-life situation. These two items aims' were not to solve the operation correctly; however to represent solutions with the expected models, regional and number line respectively.

Table 3.3 presents the item numbers according to the translations types. As seen from Table 3.3, item 1, 2, and 3 involve translations *from* symbol *to* the other three models; item 4, 5, and 6 involve translations *from* number line *to* the other three models; item 7, 8, and 9 involve translations *from* discrete objects *to* the other three models; item 10, 11, and 12 involve translations *from* region t*o* the other three models; and item 13, 14, and 15 involve translations *from* decimals, which considered in symbol model, *to* the other three models. Item 16 involve translations from symbol to region model; item 17, 18, and 19 involve translations *from* region *to* symbol model. Item 20 and 21 involve translations from real-life situations to the region and number line model, respectively.

Table 3.3 Types of Representation Translations in Each Item of the TRAT

From \ To	Symbol	Number Line	Discrete Objects	Region
Symbol		1	2	3, 16
Number Line	4		6	5
Discrete Objects	7	8		9
Region	10, 17, 18, 19	11	12	
Decimals	-	13	15	14
Real-life		21		20

In order to evaluate students' responses a key was prepared. If student's response reasonably matched with the key, the item was coded as "1" (correct), otherwise it was coded as "0"wrong.

3.3.2 Validity and Reliability of Measuring Tools

To establish the face and content validity, Fractions Test was checked by an assistant professor from the Department of Secondary Science and Mathematics Education of METU, an instructor from the Department of Elementary Science and Mathematics Education of METU, and an instructor from Hacettepe University, two elementary school mathematics teachers, two research assistants, and four graduate students from the department of Elementary Mathematics Education according to content and format of the instrument. All these experts were asked to evaluate the instrument in terms of the purpose of the study. Appropriateness of items to the grade level, representativeness of content by the selected items, the appropriateness of the format (size of type, clarity of directions and language, quality of printing, etc) were all checked and suggestions were taken into consideration in the revision of instruments. The adapted version of the instrument was given in

Appendix A. The two mathematics teachers' comments and suggestions were presented in Appendix B and C.

The cronbach alpha internal consistency coefficient of the inventory was reported as .91 by SPSS. This value indicates a high reliability. This result was expected, since the sample size was large.

3.4 Procedure

Since both students' abilities to perform translations among representations in the context of fractions and the effect of gender, grade level, students' previous mathematics grade were investigated, the design of the study was both cross-sectional survey (data were gathered one point in time) and casual-comparative (gender and grade level are groups that they have already exist) study. The study started with a detailed review of literature. A keyword list was determined. After that, Educational Resources Information Center (ERIC), International Dissertation Abstracts, Social Science Citation Index (SSCI), Ebscohost, Kluwer Online databases, Science Direct, and Internet (e.g., Google) were searched systematically. Addition to studies in abroad, MS and PhD theses made in Turkey were also searched from YÖK, Hacettepe Eğitim Dergisi, and Eğitim ve Bilim. The photocopies of the available documents were obtained from METU library, Hacettepe University Library, and Internet. The content of previous and new constructed elementary school mathematics curriculum were investigated All these documents were read; results of the studies were compared with each other. Next, the TRAT was developed by the help of findings from the literature.

The participant schools and subjects of the study were determined, and the permission was granted for the study from the Ministry of

Education. The correspondence was given in Appendix C. Then, the revised form of the test was given to 1456 middle grade students during the spring 2005-2006 semester.

The TRAT was administered to 6^{th}, 7^{th}, and 8^{th} grade students in a four weeks period. Since the researcher was not allowed to administer the test in some schools, the necessary information was explained to the teacher. One class hour was given to students to complete the test. Directions were read and necessary explanations were made by the researcher to make students more conscious about the research. Also, the participants of the study were informed that the results of the study would not affect any of their grades in the mathematics course. No problems were encountered during the administration of the test.

3.5 Analysis of Data

The statistical analyses were done by using statistical package for the social sciences program (SPSS 11.5). The data obtained in the study were analyzed in three parts. In the first part, descriptive statistics; in the second part, most frequent incorrect responses were determined by analyzing each student's answer sheets, and in the third section, inferential statistics were used.

3.5.1 Descriptive Statistics

The mean, median, mode, standard deviation, skewness, kurtosis, maximum and minimum values, and histograms of the variables were presented.

3.5.2 Determination of Most Frequent Incorrect Responses

The analyses of most frequent incorrect responses were comprised the main concerns of the study. In order to determine students' most frequent incorrect responses, answer sheets were categorized according to the grade levels. Then, by examining each student's answer sheets, repeated incorrect responses were determined and their frequencies were identified.

3.5.3 Inferential Statistics

In order to test the hypotheses, statistical technique named two-way analysis of variance was used to investigate the effects of gender and grade level on total scores of TRAT.

3.6 Assumptions

The assumptions of this study considered by the researcher are (1) The administration of the TRAT was under standard conditions and (2) Whole subjects of the study responded sincerely to the items on the instrument.

3.7 Limitations

In some schools, the test was administered by the mathematics teacher, since the administration did not give the permission to the researcher. Students might not been told the instructions in detail. In addition, students might not take into account to the instrument since they were told that the scores would not affect their mathematics grades. Furthermore, 6^{th} grade students might not respond all items since they had not started the Fractions unit yet, when the test was administered.

CHAPTER 4

RESULTS

This chapter is divided into six sections. First two sections deal with the missing data analysis and outlier analysis. Descriptive statistics are presented in the third section. Fourth section reveals the most common wrong responses of students. The fifth section presents inferential statistics in which the null hypotheses were tested. Finally, the last section summarizes the findings of the study.

4.1 Missing Data Analysis

The first step in the data analysis was related with missing data analysis. It was carried out before descriptive and inferential statistics. Since the nature of the Translation among Representations Abilities Test (TRAT) is similar to an achievement test, which aims to identify best performance of students, it was assumed based on the observations of researcher during the administration of the TRAT that leaving an item without indicating an answer most closely meant being not right about the item. For this reason, missing data of students were replaced with 0 (wrong).

Any information related to students' gender, grade level, and previous mathematics scores was checked by the researcher while students were administering the test. Students who have missing data were told to complete the related information. Thus, no missing data were met when the statistical analyses were conducted.

4.2 Outlier Analysis

In order to reveal the possible cases with values well above or well below the majority of other cases, outlier analysis was conducted. To identify outliers in the Test of Abilities in Translating among Representations (TRAT) total scores of students as the dependent variable was checked. When the box-plot was examined it was observed that there were not any scores that appeared as little circles extended from the edge of the box for female and males. However, there was an outlier for 6th graders. Figure 4.1 represent the box-plot according to gender.

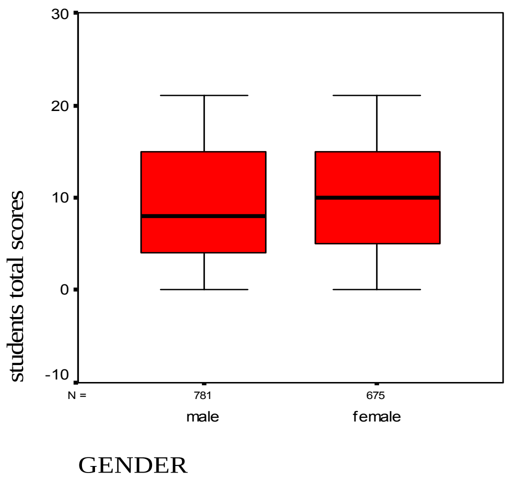


Figure 4.1 Box-Plot Representing the Result of the Outlier Analysis through Males and Females

4.3 Descriptive Statistics

Descriptive statistics such as mean, median, mode, standard deviation, skewness, kurtosis, minimum and maximum scores of students' in the TRAT, and the number of students were calculated for each grade levels (Table 4.1).

Table 4.1 Basic Descriptive Statistics Related to the Scores of in TRAT

	6^{th} grade	7^{th} grade	8^{th} grade	Whole Group
Mean	6.96	9.55	12.57	9.81
Median	6	9	14	9
Mode	4	5	17	5
SD	4.86	5.57	5.64	5.84
Skewness	.881	.246	-.317	.237
Kurtosis	-.075	-1.122	-1.183	-1.221
Minimum	0	0	0	0
Maximum	21	21	21	21
N	441	510	505	1456

Note: Maximum possible score was 21.

Students' scores of abilities in translating among representations were ranged from 0 to 21. Maximum score was taken by students from each grade level. Those students', who had the maximum previous year mathematics grade was 5 out of 5, As table 4.1 indicated, sixth grade students' median value is 6, seventh graders' is 9, eight grade students' have the value of 14, and the all subjects' is 9. On the other side, 6^{th}, 7^{th} and 8^{th} grade students' most frequent score, namely modes are 4, 5, 17, and 5, respectively. Among whole group of students, the highest mean score belonged to the 8^{th} graders with the value 12.57. Then the 7^{th} graders had the second highest mean score with the value 9.55. Lastly, the 6^{th} graders came with the lowest mean score 6.96. All subjects' mean score was stated as 9.81. Lower mean scores indicate poor ability in translating from one type of representation to another; while the higher mean score indicates flexibility in translating from one type of representation to another.

Standard deviations ranged from 4.86 to 5.64 points. As seen from Table 4.1, 6^{th} graders' standard deviation seemed different from the others. Because of this difference equal variances could not be assumed.

The skewness values of 6^{th} and 7^{th} grade and all subjects are both positive and lie between -1 and +1. 8^{th} grade students' skewness value is again with the interval of -1 and +1, however, their score is negative because of most scores tended to be high. The kurtosis values of 6^{th}, 7^{th}, 8^{th} grade students, and total groups were negative representing a flat distribution. Actually, 6^{th} grade students' kurtosis value is nearly zero indicating an average between peaked and flat distribution.

Figures 4.1, 4.2, 4.3, and 4.4 show the histograms with normal curves related to the total scores of 6^{th}, 7^{th}, 8^{th} grade, and total group of students in the TRAT. While the histogram of 6^{th} graders was skewed to right since their scores clustered to the left at the low levels; 8^{th} graders' was skewed to left, because of high scores gathered to the right side. 7^{th} grade and total group of students' histograms were nearly looked like a normal distribution.

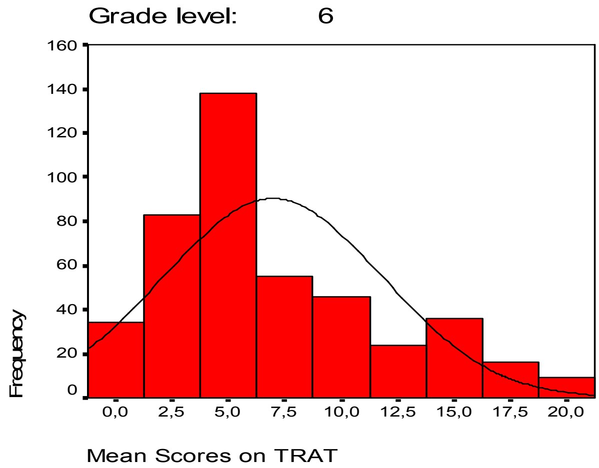


Figure 4.2 Histograms with Normal Curves of 6th Grade Students' Mean Scores in TRAT

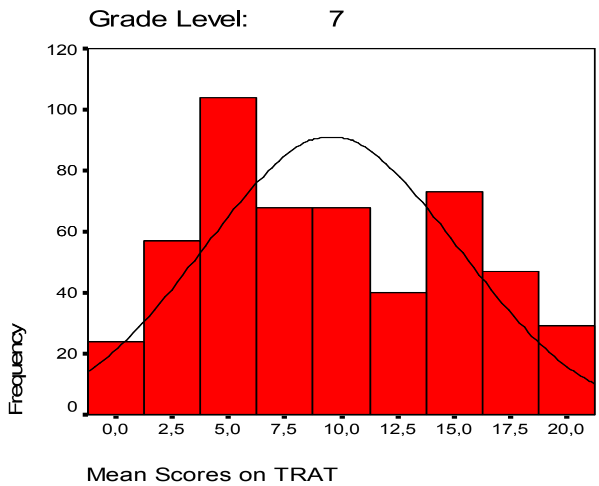


Figure 4.3 Histograms with Normal Curves of 7th Grade Students' Mean Scores in TRAT

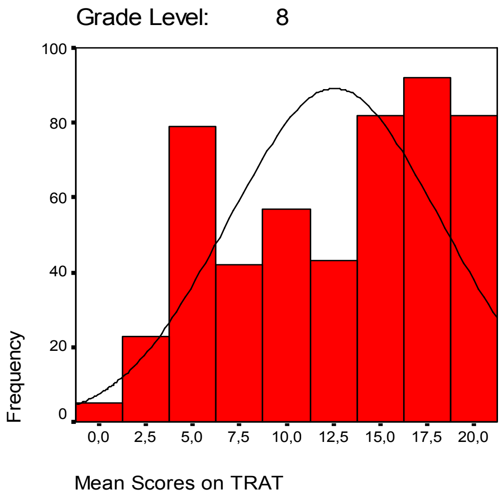


Figure 4.4 Histograms with Normal Curves of 8th Grade Students' Mean Scores in TRAT

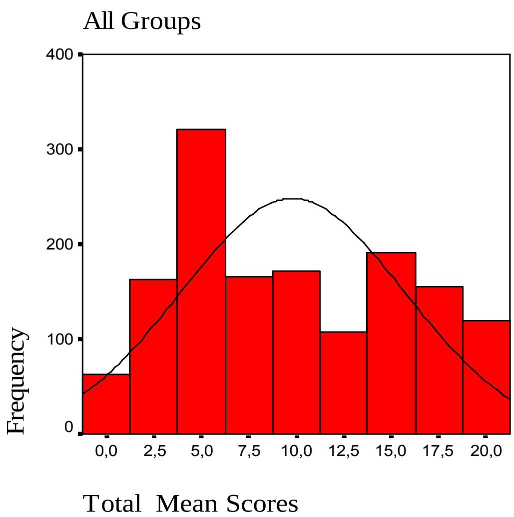


Figure 4.5 Histograms with Normal Curves of All subjects' Mean Scores in TRAT

The mean total scores of TRAT for males and females from the two districts, Yenimahalle and Çankaya, were presented in Figure 4.5.

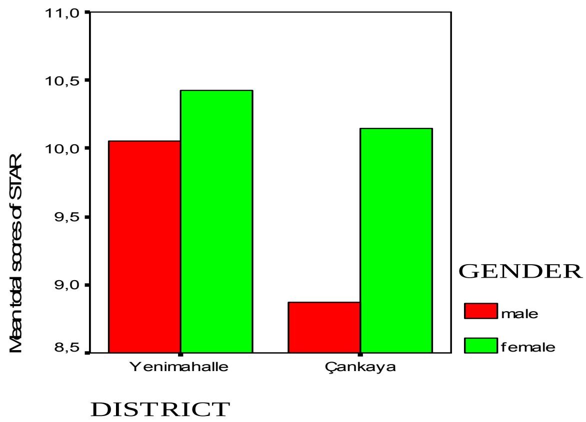


Figure 4.6 Bar Graphs for Mean Total Scores of STAR In Terms of Gender from Two Districts

The mean total scores of TRAT for grade levels from the two districts, Yenimahalle and Çankaya, were presented in Figure 4.6.

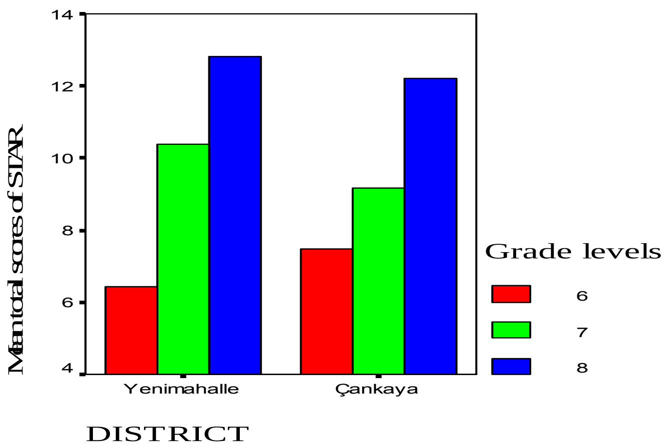


Figure 4.7 Bar Graphs for Mean Total Scores of TRAT In Terms of Grade Levels from the Two Districts

In the following section, the easiest and the most difficult representation translations will be identified. Moreover, students' most common wrong responses to each item were analyzed based on the grade levels.

4.4 Analyses of Students' Most Frequent Incorrect Responses to Each Item

The percentages of correct answers were presented in Table 4.2 in order to reveal the easiest and the most difficult items. The correct percentages which were above .70 were considered the easiest items among the others. On the other hand, the percentages which were below .20 were considered as the most difficult items This table provided information about what kind of difficulties students' had in TRAT.

Table 4.2 Percentages of Students Correct Responses to Each Item

	6th	7th	8th
Item #	1	1	1
1	**32.2**	**50.6**	**59.6**
2	**89.1**	**91.2**	**96**
3	**33.3**	**44.3**	**59.2**
4	**15.4**	**35.7**	**50.3**
5	**14.3**	**29.0**	**45.3**
6	**29.3**	**46.3**	**63.2**
7	**80.5**	**82.7**	**90.5**
8	**21.1**	**42.9**	**56.4**
9	**66**	**70.8**	**83.2**
10	**55.1**	**63.1**	**76**
11	**26.8**	**48.2**	**61.6**
12	**43.8**	**53.5**	**62**
13	**18.6**	**35.9**	**55.2**
14	**28.3**	**43.5**	**63.6**
15	**39.2**	**59.6**	**77**
16	**20.9**	**35.3**	**60.2**
17	**14.1**	**20**	**30.7**
18	**25.2**	**43.9**	**58.4**
19	**14.3**	**13.1**	**32.9**
20	**21.3**	**30.8**	**52.1**
21	**7.5**	**14.1**	**23.6**

Note. "1" refers to the responses which coded as correct.

According to the Table 4.2, the easiest two items across the TRAT were determined as item 2 and 7 for whole subjects. Item 2 and 7 were considered the easiest two items according to the sixth graders; 2, 7, and 9 for the seventh graders, and 2, 7, 9, 10, and 15 for the eight graders.

On the other hand, the most difficult item was determined as 21st item according to whole students. Item 4, 5, 17, and 19 were four of the most difficult items to the sixth graders; item 19 and 21 were considered as the two of the most difficult ones for the seventh graders; item 21 was the most difficult items to the eight graders. (8th graders' lowest correct percentage was 23.6 which belonged to the item 21).

Students' flexibilities can be identified by looking at the correct percentages of translations between representations. Table 4.3 summarizes the results.

Table 4.3 Percentages of Correct Translations between Representations

	Translations Between Representations					
From - To	S-N	N-S	S-D	D-S	S-R	R-S
6th	32.2	15.4	89.1	80.5	33.3	55.1
7 h	50.6	35.7	91.2	82.7	44.3	63.1
8th	59.6	50.3	96	90.5	59.2	76

Note. Symbol: S; Number line: N; Discrete Objects: D; Region: R

As seen from Table 4.3 sixth grade students are more flexible in translating symbol representation to a number line model than in translating from number line to symbol representation. Seventh and eight graders' result showed similar findings with the sixth graders. They are much more flexible in translating from symbol to number line model than the inverse. In summary, the translation from number line was more difficult than to number line.

In the translation to discrete objects and from discrete objects model, all students' percentages provide the same information that they are very flexible in this type of translation.

The third translation types inform a different result when comparing with the previous ones. Students are seemed to be more flexible in translations from region model than to region model.

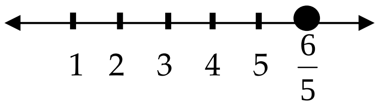
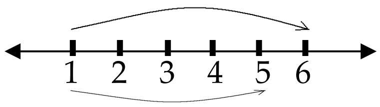
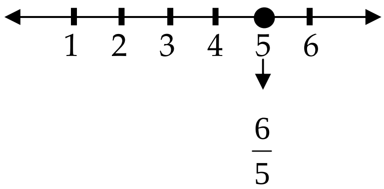
In the following section, students' common responses which were coded as wrong were analyzed based on three representations namely, number line, region, and discrete objects model.

4.4.1 Most Frequent Incorrect Responses to Items Including Number Line Model

In the TRAT, there were eight items requiring a translation from or to number line model.

Item 1*:* Students were asked to represent $\frac{6}{5}$ on number line model. As shown in Table 4.2, only 32% of the sixth graders had a correct answer. 63.5% of the students, who attempted to answer it, gave wrong answers. Their common responses which were coded as wrong were analyzed in Table 4.4.

Table 4.4 Numbers of Most Frequent Incorrect Responses for Item 1 in terms of Grade Levels

Students' Responses	6th	7th	8th
	76 (25.4 %)	42 (16.6%)	40 (20%)
	34 (11%)	50 (20%)	5 (2.5%)
	26 (8.7 %)	5 (2%)	7 (3.4%)

Note. The numbers of the students' most frequent incorrect were sorted in ascending order according the sixth graders.

Remarkable numbers of students could not identify a unit between 0 and 1. In the first type of response, they stated a 6-unit-long number line and located $\frac{6}{5}$ to the sixth point. Second response was common across sixth and seventh graders. Students tended to locate the numerator and denominator separately. They identified the numerator by an arrow drawn above the number line and denominator drawn below the number line. The third response was very similar to the first one. A 6-unit-long number line was stated with the same sense and then the fifth point was located as $\frac{6}{5}$.

Item 4: $1\frac{1}{4}$ was located on number line model and students were asked to represent it in symbolic form. Table 4.5 presents the number of most frequent incorrect responses regarding the grade levels.

Table 4.5 Numbers of Most Frequent Incorrect Responses for Item 4 in terms of Grade Levels

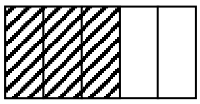
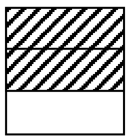
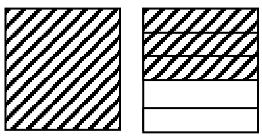
Students' Responses	6th Grade	7th Grade	8th Grade
$\frac{1}{3}$	60 (16%)	15 (4.6%)	10 (4%)
$\frac{1}{2}$	58 (15.5%)	45 (13.7%)	37 (14.7%)
$1\frac{1}{1}$	40 (10.7%)	48 (14.6%)	20 (8%)
$\frac{1}{4}$	35 (9.3%)	68 (20.7%)	68 (27%)
$1\frac{1}{3}$	25 (6.7%)	37 (11.3%)	14 (5.6%)
$1\frac{2}{3}$	-	24 (7.3%)	-
$1\frac{1}{2}$	-	-	15 (6%)

Note. The numbers of the students' most frequent mistakes were sorted in ascending order according to the sixth graders.

Students had a variety of responses as stated in Table 4.5. Sixth graders' most frequent responses were $\frac{1}{3}$ and $\frac{1}{2}$, while seventh and eight graders' were $\frac{1}{4}$. They probably considered the point of 1 as 0 and represented it as a proper fraction. The third response was also quite common among 6th and 7th graders. Since the bold-colored point is stated in the first place after 1, students, may have, tended to write it as $1\frac{1}{1}$.

Item 5: $2\frac{3}{5}$ was located on number line model and students were asked to translate it to a region model. Students' most frequent responses which were coded as wrong are presented in Table 4.6.

Table 4.6 Numbers of Most Frequent Incorrect Responses for Item 5 in terms of Grade Levels

Students' Responses	6th Grade	7th Grade	8th Grade
	45 (14%)	30 (9.3%)	14 (5.5%)
	42 (13%)	24 (7.4%)	5 (2%)
	15 (4.6 %)	40 (12.3%)	26 (10 %)

Note. The numbers of the most frequent incorrect responses were sorted in ascending order according to the sixth graders.

The first type of response was frequent among sixth and seventh graders. Those students tended to represent the fraction without considering the wholes. The second type of response, $\frac{2}{3}$, was also common across sixth and seventh graders. In the last response, the fraction was determined correctly, but the whole was stated as 1.

Item 6: $\frac{4}{6}$ was located on number line model and students were asked to translate it to a discrete objects model. Table 4.7 shows the number of students' most frequent incorrect responses.

Table 4.7 Numbers of Most Frequent Incorrect Responses for Item 6 in terms of Grade Levels

Students' Responses	6th Grade	7th Grade	8th Grade
●○○○	28 (9%)	6 (2%)	10 (5.4%)
●●●●○	25 (8%)	56 (20.4%)	34 (18.4%)
●○	24 (8%)	19 (6.8%)	-
●○○	20 (6.4%)	12 (4.5%)	-
●○○○○	3 (1%)	-	-
●●●●●○	-	-	11 (6%)

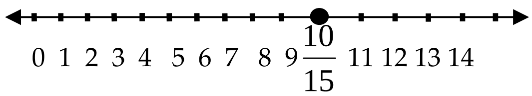
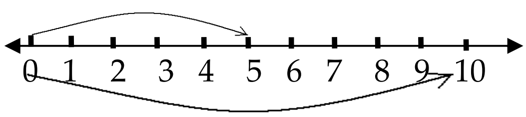
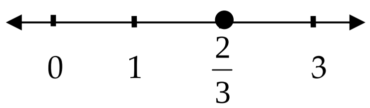
Note. The numbers of the most frequent responses were sorted in ascending order according to the sixth graders.

Students' responses varied in this item similar to the item 4. The second type of response which represents $\frac{4}{5}$ was very common across all subjects whose response was coded as 0. Students did not consider the equal partitions between 0 and 1 that determine the denominator. Since the marked place was the fourth point, they represented the discrete objects model by $\frac{4}{5}$.

Item 8, 11, and 13 asked students to translate the fraction represented with different models to the expected ones.

Item 8: $\frac{10}{15}$ was represented with discrete objects model and students were asked to translate it to the number line model. Table 4.8 shows the number of students' most frequent incorrect responses.

Table 4.8 Numbers of Most Frequent Incorrect Responses for Item 8 in terms of Grade Levels

Students' Responses	6th Grade	7th Grade	8th Grade
	86 (24.7%)	61 (21%)	38 (17%)
15			
	16 (4.6%)	15 (5%)	2 (1%)
	15 (4.3%)	21 (7.2%)	10 (4.5%)
Symbolic representation ($\frac{10}{15}$)	45 (13%)	45 (15.6%)	48 (12%)

Note. The numbers of the most frequent incorrect responses were sorted in ascending order according to the sixth graders.

In this item, remarkable number of students from whom had wrong answers, tended to identify the points between 0 and 15 as wholes rather than identifying fractions. They located $\frac{10}{15}$ to the tenth point as seen from the first type of response given in Table 4.8.

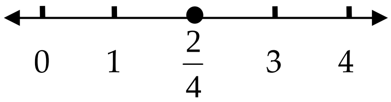
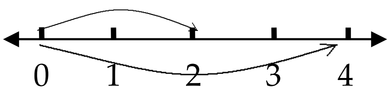
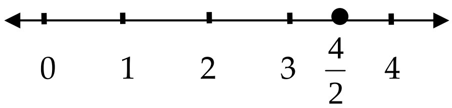
The second type of response was very similar to previous responses for the first item. Students from sixth and seventh graders tended to represent the numerator and denominator separately. They identified the numerator by an arrow drawn above the number line and denominator drawn below the number line.

The third response was same with the first type of response. The only difference was that students used $\frac{2}{3}$ which is reduced form of $\frac{10}{15}$.

45 students from sixth and seventh graders and 48 students from eight graders just wrote $\frac{10}{15}$ correctly; however, they did not represent it on number line model.

Item 11: $\frac{2}{4}$ was represented with region model and students were asked to translate it to a number line model. Table 4.9 presents the number of students' most frequent incorrect responses.

Table 4.9 Numbers of Most Frequent Incorrect Responses for Item 11 in terms of Grade Levels

Students' Responses	6th Grade	7th Grade	8th Grade
	68 (21%)	34 (13%)	11 (6%)
	45 (14%)	29 (11%)	6 (3%)
	39 (12%)	16 (6%)	18 (9.4%)

Note. The numbers of the most frequent incorrect responses were sorted in ascending order according to the sixth graders.

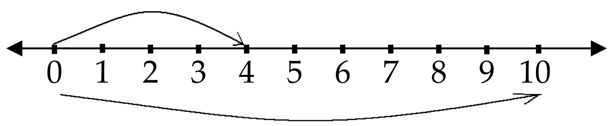
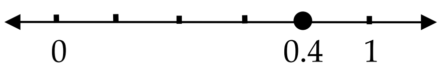
The first and the second type of responses were similar to the responses of item 1 and item 8. Students tended to identify 4-unit-long number line rather than partitioning the unit (between 0 and 1) into fourths. They represented $\frac{2}{4}$ to the second point as seen in the first type of response given in Table 4.9.

Students, especially from sixth and seventh graders, tended to represent the numerator and denominator separately. They identified the numerator by an arrow drawn above the number line and denominator drawn below the number line.

The third response differs from the other common response types including number line model. Small number of students from three grade levels located the fraction as $\frac{4}{2}$ to the midway between 3 and 4.

Item 13: $\frac{4}{10}$ was given in decimal form and students were asked to locate it on a number line model. Most frequent two responses which were coded as wrong are displayed in Table 4.10.

Table 4.10 Numbers of Most Frequent Incorrect Responses for Item 13 in terms of Grade Levels

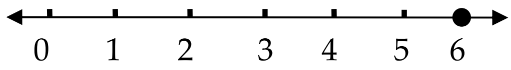
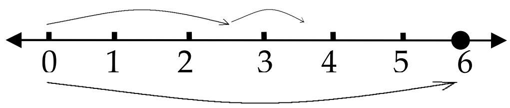
Students' Responses	6th	7th Grade	8th
	25 (7%)	24 (7.3%)	9 (4%)
	6 (1.7%)	45(13.8%)	13 (5.6%)

Note. The numbers of the most frequent incorrect responses were sorted in ascending order according to the sixth graders.

The first response type was already common among sixth and seventh graders. However, the second type of response was considered as a new type of response. 45 seventh, 13 eight and 6 sixth graders identified 1-unit-long number line on which the unit was partitioned into fifths and located the decimal fraction to the fourth point.

Item 21: The operation 2.5 + 3.5 is presented in a word problem and students were asked to represent it on a number line model. Table 4.11 presents the number of most frequent responses.

Table 4.11 Numbers of Most Frequent Incorrect Responses for Item 21 in terms of Grade Levels

Students' Responses	6th Grade	7th Grade	8th Grade
	122 (30%)	135 (31%)	131(34%)
Symbolic representation (2.5+3.5=6)	60 (15%)	77(17.6%)	89 (23%)
	11 (2.7%)	42 (9.6%)	55 (14%)

Note. The numbers of the most frequent incorrect responses were sorted in ascending order according to the sixth graders.

The first type of response was quite common among all subjects as seen from the Table 4.11. Students tended to locate simply the result of the operation, not the operation itself, on the number line model.

On the other hand, the second type of response was also frequent among seventh and eight graders. Students tended to represent the operation without modeling with number line.

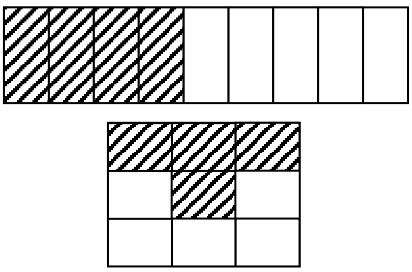
The last response for the 21st item was largely belonged to the eight graders with 55 students. They drew arrows to the points 2.5, 3.5, and 6; but they did not present the operation of addition on number line.

4.4.2 Most Frequent Incorrect Responses to Items Including Region Model

Item 3, 5, 9, 10, 11, 14, and 20 include representations with region type of models. In this section item 3, 10, 14, and 20 are analyzed, since the others were mentioned in the previous section.

Item 3: $\frac{9}{4}$ was stated in symbolic form and students were asked to represent it with a region model. Table 4.12 presents the number of students' most frequent incorrect responses.

Table 4.12 Numbers of Most Frequent Incorrect Responses for Item 3 in terms of Grade Levels

Students' Responses	6^{th} Grade	7^{th} Grade	8^{th} Grade
	165 (56%)	170 (60%)	130 (63%)

As seen from the Table 4.12, more than 50% of the students did not recognize that $\frac{9}{4}$ means 9 pieces of $\frac{1}{4}$, on the contrary, they tended to divide one whole into nine parts and shade four of them. That is they considered the fraction as $\frac{4}{9}$.

Item 10: $2\frac{2}{6}$ was represented with region model and students were asked to write it in symbolic form. Table 4.13 summarizes most frequent incorrect responses in terms of grade levels.

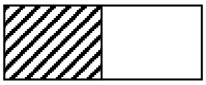
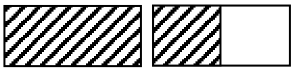
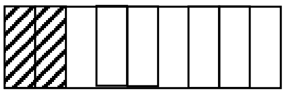
Table 4.13 Numbers of Most Frequent Incorrect Responses for the Item 10 in terms of Grade Levels

Students'	6^{th} Grade	7^{th} Grade	8^{th} Grade
$\frac{8}{12}$	45 (23%)	27 (14%)	12 (10%)
$6\frac{2}{6}$	34 (17%)	16 (8.5%)	7 (5.8%)

The first type of response shows that students tended to consider the two region models as a whole which was divided into 12 parts and shaded 8 of them.

Item 14: $1\frac{2}{10}$ was represented in decimal form, and students were asked to translate it to an area model. Table 4.14 presents the three types of responses according to grade levels.

Table 4.14 Numbers of Most Frequent Incorrect Responses for Item 14 in terms of Grade Levels

Students' Responses	6th Grade	7th Grade	8th Grade
	86 (27%)	96 (33%)	45 (24.5%)
	29 (9%)	21 (7.3%)	7 (3.8%)
	8 (2.5%)	4 (1.4%)	3 (1.6%)

Note: The numbers of the most frequent incorrect responses were sorted in ascending order according to the sixth graders.

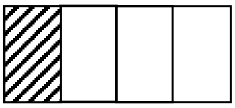
The first type of response indicate that remarkable number of students tended to perceive decimal fraction 1.2 as $\frac{1}{2}$, since they divided one whole into two equal parts and shaded one of them.

In the second type of response, similar to the first item, students from sixth and seventh graders tended to perceive decimal fraction as $1\frac{1}{2}$.

A few students from all grade levels did not consider the one whole; they represented the fraction as $\frac{2}{10}$ with region model.

Item 20: Students were asked to represent the operation $\frac{5}{5} - \frac{1}{5}$ with a region model. Table 4.15 presents the number of most frequent responses.

Table 4.15 Numbers of Most Frequent Incorrect Responses for Item 20 in terms of Grade Levels

Students’ Responses	6th Grade	7th Grade	8th Grade
Symbolic representation	142	133	97
	76	80	64

4.4.3 Most Frequent Incorrect Responses to Items Including Discrete Objects Model

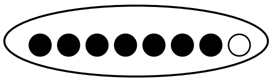
Item 2, 6, 8, 12, and 15 include representations with discrete objects type of models. In this section item 2, 12 and 15 are analyzed, since the others were mentioned in the section 4.4.1.

Item 2: $\frac{3}{8}$ was given and students were asked to represent it with discrete objects model.

At least 90% of the all subjects from three grade levels represented the fraction with the expected model correctly. Thus, it was thought that it was not necessary to discuss this item.

Item 12: $1\frac{3}{4}$ was represented with region model and students were asked to translate it to discrete objects model. Table 4.16 presents the most frequent responses coded as 0.

Table 4.16 Numbers of Most Frequent Incorrect Responses for Item 12 in terms of Grade Levels

Students' Responses	6th Grade	7th Grade	8th Grade
	111 (45%)	69 (29%)	35 (18%)
	-	8 (3.3%)	9 (5%)
Symbolic representation ($1\frac{3}{4}$)	18 (7.3%)	12 (5%)	15 (8%)

Note: The numbers of the most frequent incorrect responses were sorted in ascending order according to the sixth graders.

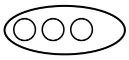
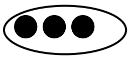
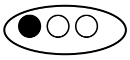
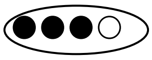
The first type of response was very common among sixth graders. Students tended to consider the two region models as a whole which was divided into 8 parts and shaded 7 of them. Hence, they draw 8 objects and identified 7 of them by dark color.

The second unaccepted response type was stated by eight 7th graders and nine 8th graders. They tended to represent one whole ($\frac{4}{4}$) by one object as seen in Table 4.16.

Small number of students just wrote the fraction without representing with discrete objects model.

Item 15: $\frac{3}{10}$ was represented in decimal form and students were asked to translate it to discrete objects model. Most frequent incorrect responses are presented in Table 4.17.

Table 4.17 Numbers of Most Frequent Incorrect Responses for the Item 15 in terms of Grade Levels

Students' Responses	6^{th} Grade	7^{th} Grade	8^{th} Grade
	67 (25%)	76 (37%)	26 (22.7%)
	67 (25%)	38 (18.6%)	17 (15%)
	-	22 (11%)	8 (7.5%)
	-	8 (4%)	1 (1.5%)

There were four types of most frequent incorrect responses which were coded as 0 among three graders. 67 students from sixth grade tended to represent the decimal fraction by the first and second type of response. They identified only three discrete objects either bold-colored or blank form.

The third response was represented by 22 seventh and 8 eight graders. They identified three objects from which one is bold-colored.

Eight students from 7^{th} grade and one student from 8^{th} grade represented $\frac{3}{10}$ as the last response. They identified four objects; three of them were bold-colored.

In the following section, multiple-choice items were analyzed regarding to students' most frequent responses which were coded as wrong.

4.4.4 Responses to Multiple-Choice Items

Responses to item 16, 17, 18, and 19 were analyzed according to grade levels.

Item 16: The operation $\frac{1}{6}+\frac{1}{3}$ was given and students were asked to select the correct choice which was represented the result ($\frac{3}{6}$ or $\frac{1}{2}$). The percentages of responses are given in Table 4.18.

Table 4.18 Percentages of Responses to Item 16 in terms of Grade Levels

Item 16 Responses	6^{th}	7^{th}	8^{th}
	Percentages of Responses		
A	20.9	35.3	60.2
B	4.3	6.3	1.0
C	24.5	12.7	8.7
D	8.6	3.3	3.8
E	32.9	38.4	24

20.9 % of the 6^{th} grade, 35.3 % of the 7^{th} grade, and 60.2% of the 8^{th} grade students selected the correct choice (A). They performed the addition operation, and found $\frac{3}{6}$. Since there were not any region model divided one whole into six parts and shaded three parts, they selected the first choice which represent the equivalent form of $\frac{3}{6}$.

Table 4.18 shows that the last choice (E) which was preferred by the sixth and seventh graders with higher percentages when compare to the correct response. Students did not recognize $\frac{1}{2}$ which is equivalent to $\frac{3}{6}$.

On the other hand, sixth graders had higher percentages of selecting the third response (C) which represents $\frac{2}{9}$. Students tended to consider the numerator and denominator as separate whole numbers and they performed the addition operation.

Item 17: A region model was given and students were asked to select the correct choice representing the shaded area. Table 4.19 presents the percentages of all responses.

Table 4.19 Percentages of Responses to Item 17 in terms of Grade Levels

Item 17 Responses	6th	7th	8th
	Percentages of Responses		
A	14.1	20	30.7
B	11.8	14.3	21
C	20.2	25.9	20.6
D	32.4	23.3	20.2
E	15.2	14.1	6.5

The percentages of correct response (A) were below .50 for all subjects. Most students tended to select the choices (C) or (D). Students who preferred the third choice $1+\frac{1}{5}$ tended to identify the half of the whole as one whole and the part of $\frac{1}{10}$ as $\frac{1}{5}$. Students who selected the choice (D) tended to identify the model as a six-partitioned whole two of which were shaded.

Item 18: A region model was given and students were asked to select the correct choice representing the shaded area. Table 4.20 presents the percentages of responses according to grade levels.

Table 4.20 Percentages of Responses to Item 18 in terms of Grade Levels

Item 18 Responses	6^{th}	7^{th}	8^{th}
	Percentages of Responses		
A	12.9	9.8	4.6
B	25.2	43.9	58.4
C	11.8	9.2	9.1
D	11.1	9.4	4.6
E	6.8	4.9	4.0

Although the percentages of correct response were low, its' percentages were higher than the other choices. 25.2% of the sixth graders, 43.9% of the seventh graders, and 58.4% of the eight graders preferred to select the correct choice (B) representing the operation $\frac{1}{2} \times \frac{1}{3}$.

Item 19: A region model was given and students were asked to select the correct choice representing the shaded area. Table 4.21 presents the percentages of responses according to grade levels.

Table 4.21 Percentages of Responses to Item 19 in terms of Grade Levels

Item 19 Responses	6^{th}	7^{th}	8^{th}
	Percentages of Responses		
A	11.6	13.3	17.6
B	8.8	6.5	7.5
C	8.8	8.4	3.0
D	14.3	13.1	32.9
E	41.5	52	34.1

Table 4.19 shows that students from three grade levels had very low percentages to correct response (D). The highest percentage was belonged to the last choice (E) stated as none of them. It was understood from the

multiple-choice results that students had many difficulties in part-whole interpretations. It could be also inferred from the observations of the researcher while the test was administering that students tended to select the last choice “none of them” since they could not find the correct answer. On the other hand, most of the students, who gave the correct response, found the answer by dividing the region model into equal parts rather performing the operations. Then, they checked the choice which represents the correct answer.

4.5 Inferential Statistics

In this section, two-way ANCOVA model was required to use. PMAG was pre-determined as a covariate. However, one of the assumptions was violated and there seemed to be an interaction effect between PMAG and Grade Level. Due to this violation, it was appropriate to conduct two-way ANOVA model. This section deals with the verification of two-way analysis of variance (ANOVA) assumptions, bivariate correlations and analysis of the hypotheses.

4.5.1 Assumptions of Two-way Analysis of Variance

Two-way ANOVA has three assumptions: Normality, equality of variances, and independency of scores on the dependent variable.

For the normality assumption, skewness and kurtosis values of students’ scores on Translations among Representations Test (TRAT) was presented in Table 4.1. Since all subjects’ skewness and kurtosis values of scores on TRAT were in the range of (-2, +2), these values were acceptable for a normal distribution (Kunnan, as cited in Ağazade, 2001).

For the equality of variances assumption, Levene's Test of Equality of Error Variances was used. As inferred from Table 4.20, error variances of the dependent variable were not equal across grade levels ($p<.05$). However, analysis of variance is reasonably robust to violations of this assumption, since the size of groups (6^{th}, 7^{th}, and 8^{th} graders) was reasonably similar as seen from Table 4.22 (Stevens, 2002).

Table 4.22 Levene's Test of Equality of Error Variances

	F	df1	df2	Sig.
TRAT	9.814	5	1450	.000

The third assumption identified that scores should be independent of one another. Since the administration of the instrument did not involve interactions among subjects, they did not influence each other. It was observed that all participants did the test by themselves.

4.5.2 Two-way Analysis of Variance Model

The dependent variable of the study is students' total TRAT scores. Gender and grade level are the independent variables of the study. Table 4.23 presents the results of the two-way ANOVA.

Table 4.23 Tests of Between-Subjects Effects for Students' Total TRAT Scores

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Eta Squared	Ob. Power(a)
Corrected Model	7781,164(b)	5	1556,233	53,839	,000	,157	1,000
Intercept	136007,464	1	136007,464	4705,282	,000	,764	1,000
GRADE LEVEL	7295,730	2	3647,865	126,201	,000	,148	1,000
GENDER	220,374	1	220,374	7,624	,006	,005	,788
GENDER * GRADE LEVEL	96,161	2	48,081	1,663	,190	,002	,352
Error	41912,649	1450	28,905				
Total	189885,000	1456					
Corrected Total	49693,813	1455					

4.5.3 Null Hypothesis 1

"There is no significant main effect of grade level on the population means of public elementary school students' TRAT scores."

Two-way analysis of variance (ANOVA) was conducted to evaluate this null hypothesis. The result revealed that there was a statistically significant main effect of grade level on students' total TRAT scores (F (2, 1450) = 126, 01, p=.00). In other words, this null hypothesis was rejected.

In order to reveal the pair wise differences among means, a follow-up test, Dunnett's C was conducted. Table 4.23 reveals a significant mean difference between the 6th and 7th (in favor of grade 7), 6th and 8th (in favor of grade 8), 7th and 8th (in favor of grade 8), with respect to grade levels of students.

Table 4.24 Multiple Comparisons for Students' Grade Levels

(I) GRADE	(J) GRADE	Mean Difference (I-J)	Std. Error
6	7	-2.59(*)	.338
	8	-5.61(*)	.341
7	6	2.59(*)	.338
	8	-3.02(*)	.352
8	6	5.61(*)	.341
	7	3.02(*)	.352

* The mean difference is significant at the .05 level.

Eta squared was calculated as .15 by SPPS. It represents that 15% of the variance in TRAT scores was explained by grade level of students. The effect size measured here matched the large effect size indicating that practical significance of this result is high (Cohen & Cohen, 1983). Additionally, power was found as 1.0. Thus, the probability to failing to reject a false null hypothesis (probability of making Type-2 error) was calculated as 0.

4.5.4 Null Hypothesis 2

"There is no significant main effect of gender on the population means of public elementary school students' total TRAT scores."

Two-way ANOVA was conducted to determine the main effect of gender on students' TRAT scores. As indicated in Table 4.23, there was a statistically significant effect of gender (in favor of females) on the students' TRAT scores ($F(1, 1450) = 7.624$, $p = .006$). Thus, this null hypothesis was rejected.

SPSS calculated eta squared as .005 representing that .5% of the variance in TRAT scores was explained by gender of the students. The effect size measured here matched the small effect size indicating that practical significance of this result is low (Cohen & Cohen, 1983). Additionally, power was found as .79. Thus, the probability to failing to reject a false null hypothesis (probability of making Type-2 error) was calculated as .21.

4.5.5 Null Hypothesis 3

"There is no significant interaction effect of grade level and gender on the population means of public elementary school students' total TRAT scores."

Two-way ANOVA was conducted to determine the interaction effect of grade level and gender on students' TRAT scores. As indicated in Table 4.23, grade level and gender did not have a significant interaction effect of on the students' TRAT scores ($F(2, 1450) = 1.663$, $p = .190$). Thus, this null hypothesis was failed to reject.

4.6 Summaries of Findings

The results of this research can be summarized as follows.

It can be easily understood from the mean scores on the TRAT that 6^{th}, 7^{th}, and 8^{th} grade students did not have adequate ability in translating among representations in the context of fractions. 6^{th} graders had the lowest and 8^{th} graders had the highest mean score on the TRAT.

The two easiest items across the test were determined as item 2 and 7 for the 6^{th} graders. The three easiest items across the test were determined as item 2, 7, and 9 for the 7^{th} graders. The four easiest items across the test were determined as item 2, 7, 9, and 15 for the 8^{th} graders.

The most difficult items were those which require translations *from* or *to* number line models. Multiple-choice items, which include translations from or to region models, were also considered difficult particularly for 6^{th} and 7^{th} graders. Items which include proper fractions had the higher correct percentages than items including improper fractions.

Students are more flexible in translating *to* number line than *from* number line model. They are more flexible in translating *from* region model than *to* region model. Finally, they are more flexible in translating *to* discrete objects model than *from* discrete objects model.

Grade levels and gender had statistically significant main effects on students' total TRAT scores. However, they did not have a statistically significant interaction effect on students' total TRAT scores.

Item 13, 14, and 15 were those which include translations from decimal fractions to number line, region, and discrete objects models, respectively. To the examinations of students' most common incorrect responses, it was seen that, representing a decimal fractions were much more difficult than translations from symbolic form to the other models. The reason may stem from the fact that mathematics teachers tend to use decimal fractions generally in computational manner. They used to stress the four operations with decimal fractions. Although, students are good at performing the operations with decimal fractions, they had many problems in making translations among them. This may probably due to being not familiar to those translations while learning the fractions concept.

CHAPTER 5

CONCLUSIONS, DISCUSSIONS, AND IMPLICATIONS

This chapter consisted of six sections. First section presents the summary of the research study. The second section is the conclusions and discussions based on the result. The third section deals with the interval validities while the fourth section mentioned about the external validities. Implications of the study and recommendations for further studies were given in the fifth and in the last sections, respectively.

5.1 Summary of the Research Study

The purpose of the study was to determine middle grade students' abilities in translating among different representations in the concept of fractions. In this manner, 1456 6^{th}, 7^{th}, and 8^{th} grade students were administered the Translations among Representations Abilities Test (TRAT) during the 2004-2005 fall semester. To obtain the representative sample, stratified cluster random sampling integrated with convenience sampling was used. Cross-sectional survey and causal-comparative studies were conducted in this study.

5.2 Conclusions and Discussions

As seen from the basic descriptive statistics (Table 4.1) of students scores on the TRAT, it was expected that 6th grade students' total scores on the TRAT would be low since they had not started to the Fractions unit yet when the TRAT were administered. 7th graders had higher mean scores than the 6th graders; 8th grade students have higher scores than the 7th and 6th graders. 7th and 8th graders are taught the fraction concept in the unit of Rational Numbers and Real Numbers. Their higher scores, when compared with sixth graders, were probably due to their experiences in preparation to exams like *Orta Öğretim Kurumlarına Giriş Sınavı (OKS).* Their reactions and reflections while the TRAT was administering were like that "*These are very easy for us, and these items were not similar to those in OKS".* However, when they went on to the test, their reflections had turned in another way. Although, students are familiar with the concept related to each of the item, their mean scores were unexceptionally low. These low scores might be sourced from the fixed teaching methods used in mathematics classes and deficiencies of using multiple representations when the fractions concept was introduced. As stated before, using one or two distinct types of representations does not allow students to make flexible translations among multiple representations and to have a deeper understanding. Another reason, perhaps, is that for the most part, their experiences in school mathematics tend to emphasize procedural and computational skills rather than the development of meaningful conceptual understandings (Post, Behr, Lesh, 1982).

Statistical results showed that, the effect of grade level on students' abilities in translating among representations was found to be significant as expected. While the 6th grade students had the lowest mean scores on the TRAT, 8th grade students had the highest mean score. Possible reasons were mentioned above.

On the other hand, findings revealed that gender had a statistically significant effect on students' total scores on the TRAT in favor of females. However, the practical significance (eta squared) was very low(.005). This result was inconsistent with the result found by Aksu (2001). She mentioned that students' performances in the concept of fractions did not differ by gender.

5.2.1 Discussion of the Items on the TRAT

Overall performances of middle grade students showed that the abilities to translate among different representations were poor, although they were asked to make simple translations among multiple representations in the concept of fractions. In the following section, possible reasons of wrong responses are discussed with comparisons of related studies in the literature according to the representation models.

5.2.1.1 Discussion of Most Frequent Incorrect Responses Including Number Line Model

Results of the study indicated that items including number line models were considered as the most difficult items. Item 1, 4, 8, 11, and 13 includes translations from one type of representation to the number line model. As the most frequent responses given in the previous chapter indicated, there was a tendency to consider the fraction numeral as two separate whole numbers. Most probably, students consider a whole number line as a unit and represent fraction as if they represent it to region model. This finding is consistent with what other research studies reported (e.g., Behr, Lesh, Post, and Silver, 1983; Ni, 2000; Gagatsis, Mishaelidou, and Pitta-Pantazi, 2004) in the literature. They stated that since majority of students' experiences had been part-whole interpretation of fraction in a region context, they treat the whole number line as the unit instead of the length between 0 to1.

The percentages of correct answers of item 6 and 11 indicate that students have higher scores on items which include proper fractions rather than improper fractions. Similar with this result, Behr et al. (1983), confirmed that the fraction $\frac{5}{3}$ causes many more errors than do the fraction $\frac{3}{4}$ and $\frac{2}{3}$. This was, perhaps, because of the fact that school instruction has sited almost all of its stress on fractions between 0 and 1.

The translation *to* number line as in item 1 seemed to be easier than the translation *from* number line model as in item 4. This may because of the fact that students more frequently use translation from number line in the mathematics classes. This result supports the findings of Gagatsis et al. (2004). They asserted that students are more familiar with the translation from number line to symbol model.

In item 4, students tended to write the fraction without stating 1 as a whole. They probably might have considered the 1 as 0 point and thus they overlooked to write 1 as a whole. Haser and Ubuz (2002) met the similar type of response in their study. Students tended to state the fraction $\frac{1}{8}$ instead of $\frac{9}{8}$. In other words, students considered the fraction as if it was smaller than 1.

5.2.1.2 Discussion of Most Frequent Wrong Responses Including Region Model

It was expected that students had higher scores on the translations including region models. Yet, the correct percentage of the 3rd item did not provide the expected result. On the contrary, this item was considered as one of the difficult items according to sixth graders. Although, the part-whole interpretation is usually introduced early in the elementary school curriculum, many students tended to represent $\frac{9}{4}$ as $\frac{4}{9}$. The reason most probably stem from the models which represent improper fractions. Since teachers usually work with fractions, in which case only one part would be needed at all times (Wu, 2002) students have difficulties in using improper fractions (Behr,et al., 1983).

The most frequent wrong response type of item 10 was $\frac{8}{12}$ instead of $1\frac{2}{6}$. The reason of such an answer can be explained by a similar study of Mack (1990, as cited in Kieren, 1993). Her study with eight 11-years old students showed that all subjects represented the model in Figure 5.1 with $\frac{5}{8}$ instead of $1\frac{1}{4}$.

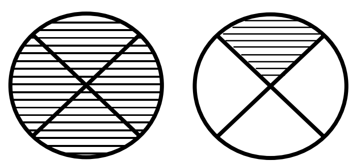


Figure 5.1 Region model showing $1\frac{1}{4}$ (Mack, as cited in Kieren, 1993)

When students were asked to describe the region model as an amount of pizza, students now gave the correct answer as $1\frac{1}{4}$. Thus, problem context might influence students' thinking in beneficial ways (Mack, 1993). It might be also thought that students were simply undergoing a problem of the unit identification not unlike that for younger children with whole numbers (Kieren, 1993).

In item 14, most of the mistakes might be derived from the lack of knowledge about decimal numbers. Students could represent this decimal fraction correctly, if they knew that 1.2 is the decimal form of $1\frac{2}{10}$. This claim inferred from the higher correct percentages of the other items including region model. Many students who had given the correct answer on items including region model could not represent this item correctly.

Item 20 and 21 would be very easy word problems if students were asked to solve problems in a way that they are used to. However, these items asked students to represent the result with region and number line models, respectively. Considerable number of students tended to present only the computations, in other words, symbolic representation. Behr et al. (1987) suggested that when students difficulty translating from one type of representation to another, it would be helpful to begin by translating from one type of model to *spoken words* or *written symbols* then translate them to other models.

5.2.1.3 Discussion of Most Frequent Wrong Responses Including Discrete Objects Model

The easiest translations are those that involve representing a fraction with discrete objects model. This result was not an expected one since students were not familiar with this type of representation. In particular, the item 2 had the highest correct percentage within all items. The sample item given before the exact items have been probably the most important factor on the correct response percentage.

On the other hand, in item 12, students had the similar incorrect responses like in item 10. They tended to think of a whole, as here $\frac{4}{4}$, one unit. Thus, they represented $\frac{7}{8}$ instead of $1\frac{3}{4}$, with discrete object model incorrectly.

In item 15, students had difficulties both in decimal fractions and in representing with discrete objects model. The wrong responses, which were stated before, were really interesting in terms of modeling and the fraction numeral.

5.2.1.4 Discussion of Most Frequent Wrong Responses of Multiple Choice Items

As mentioned in the Chapter 3, item 16, 17, and 19 were taken from the Operations Test of Rational Number Project (RNP). Thus, the results of the current study were compared with those revealed by the findings of RNP.

Item 16 was asked to 608 students from grades 4 to 8 in RNP. In that study, the most popular response was the third choice which represents $\frac{2}{9}$ with a region model, whereas in the current study, the most popular response was the last choice, *none of them,* for sixth and seventh graders.

This result was, most probably, derived from the fact that students who selected the third choice tended to consider the numerator and denominator as if they were whole numbers and did the addition with numerators and denominators separately (Ardoğan & Ersoy, 2003). A sample response from the TRAT is presented in Figure 5.2.

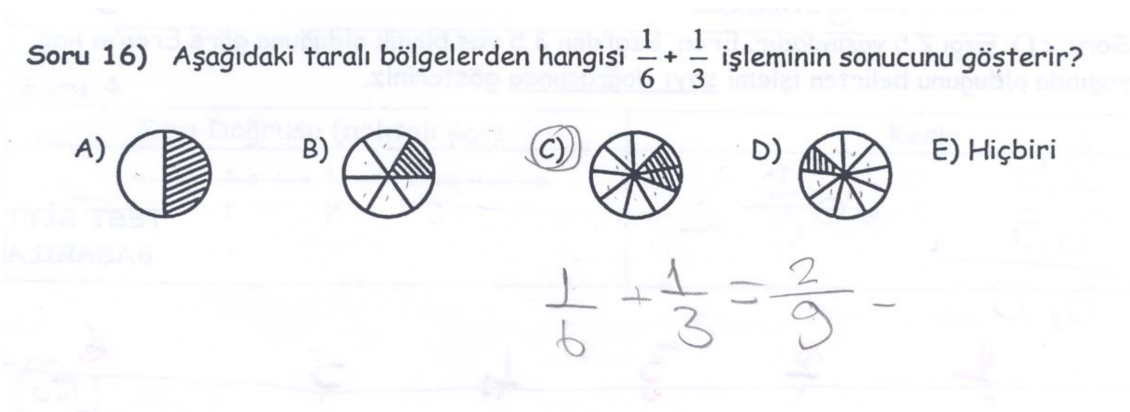


Figure 5.2 A Sample Response of a Student Selected the Third Choice

Students who preferred the last choice may not recognize the equal form of $\frac{3}{6}$. It was understood from the students' explanations that they performed the addition operation correctly; however they could not identify the reduced form of the fraction. A sample response from the TRAT is presented in Figure 5.3.

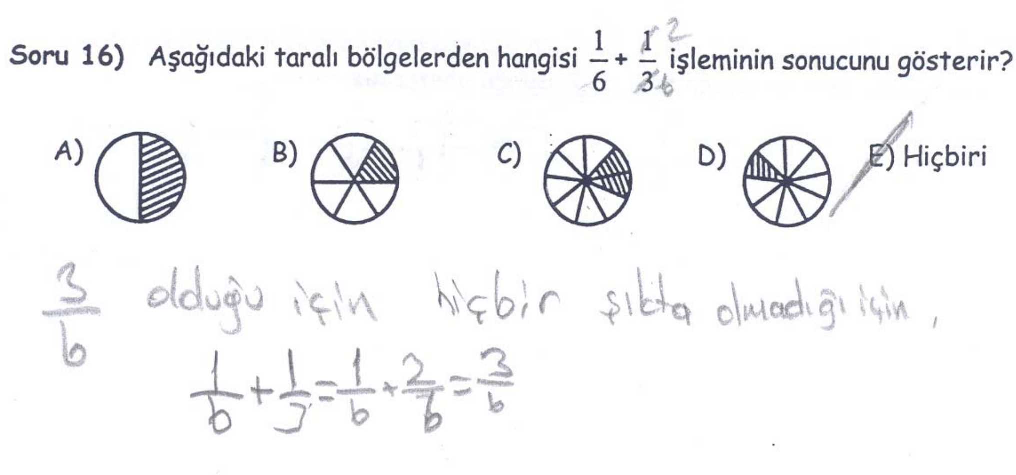


Figure 5.3 A Sample Response of a Student Selected the Last Choice

A similar result was found by the RNP. The researchers mentioned that students may have a failure to recognize reduced fractions or a lack of ability at reducing fractions. After the teaching experiments by giving more attention to translations between part-whole displays, students had greater success in unreduced representations (Bright, Behr, Post, & Wahsmuth, 1988).

Item 17 was asked to 210 students from grades 7 to 8. The most popular response was the second choice which presents the operation $\frac{1}{2}+\frac{1}{5}$, while the most popular response was the third choice which presents $1+\frac{1}{5}$ for sixth and seventh graders from the current study.

Students who selected the second choice most probably did not consider the part-whole interpretation. They may consider the half of the region as one whole. On the other hand, students who preferred the third choice may consider the $\frac{1}{2}$ as one whole.

Item 18 was asked to students to show the operation which represents the shaded region. Students could find to solution both by doing the operation $\frac{1}{2}\times\frac{1}{3}$ and by partitioning the unequal parts into equal parts. Students' answer sheets showed that they found the correct choice by partition to region into equal parts first, then they compute the operations ad signed the choice which represents $\frac{1}{6}$.

Item 19 was asked to 210 students from grade 7 and grade 8. The most popular response in the RNP was the last choice which stated as *none of them* as well as in the current study. Students most probably did not have an idea of the correct response, and then they selected the last choice. Especially, seventh and sixth grade students' percentages to the last choice were greatly higher than the correct response percentage.

Among the multiple-choice items, the most difficult one was 17^{th} for the sixth graders, 19^{th} for the seventh graders, and 18^{th} for the eight graders.

To summarize the overall discussions and interpretations from the view of different researchers:

Students generally have problems on part-whole conception. This conception involves viewing rational numbers as wholes composed of specific numbers of distinct parts (Mack, 1993). Moreover, students tend to think of rational numbers in term of parts comprising the whole rather than as a single quantity resulting from partitioning a unit (Mack, 1993).

The most problematic items were those including number line models. Students are incapable of conceptualizing a fraction as a point on a line. This is probably due to the fact that the majority of their experiences had been with the part-whole interpretation of fraction with region model (Behr, et al., 1983). Students also have more errors with improper fractions than do the proper fractions. This is due, perhaps, to the fact that school instruction has usually deal with the fractions less than one (Behr, et al., 1983).

Since most attention has been paid to symbolic representations; students have problems with the different type of representations in the fractions concepts. Additionally, students tend to write symbolic forms of the fractions instead of representing the required model. A similar preference for symbol representations was reported by Behr et al. (1983) for the solution of a task which asked to find the equivalent form of $\frac{5}{3}$.

5.3 Internal Validity of the Study

Internal validity of the study means to the degree to which the observed differences on the dependent variable are directly related to the independent variables, not to extraneous variables that may affect the results of the research (Fraenkel & Wallen, 2003).

Lack of randomization and inability to manipulate the independent variable are the two main weaknesses of the casual-comparative study. Since the groups were already formed, random assignments of subjects to groups were not possible. The manipulation of the independent variable was not possible also, as the groups have already been exposed to the independent variables.

Subject characteristic is one of the threats to the internal validity of casual-comparative study. In this study, the groups (schools) were randomly selected instead of subjects. Thus many subject characteristics such as age, gender, previous knowledge motivation, attitudes could be the major threat to the internal validity for this study. Gender and grade level were assessed together with the inventory and the stated as independent variables of the study. Since the effect of these variables on the dependent variable investigated, they were not controlled as threats.

Location and instrumentation could not be threats, since the instrument was administered to all groups in similar physical conditions of certain classrooms by the researcher.

Maturation could not be a threat to this study, as the data gathering procedure lasts in four weeks. It would be a threat if the study extended a number of years.

Finally, confidentially was not a threat in this study, since names of the students were not used anywhere. Their names were known only by the researcher.

5.4 External Validity

The external validity is the extent to which the results of the study can be generalized. Population generalizability and ecological generalizability are the two types of external validity. Population generalizability refers to the degree to which a sample represents the population of interest. Ecological generalizability refers to the degree to which the results of the study can be extended to other settings or conditions (Fraenkel & Wallen, 2003).

Subjects of the study were randomly selected from the accessible population. 1456 middle grade elementary students were involved in the study. Hence, there is no limitation to generalize the findings of the study. So the results and conclusions found in the study can easily be applied to the accessible population.

Since all testing procedure took place in ordinary classrooms during regular class time, there were possibly no remarkable differences among the environmental conditions. Therefore, it was believed that external effects were sufficiently controlled by the setting used in the study.

5.5 Implications of the Study

According to the findings of the study and the previous studies done, teachers can use the Lesh translations model to guide their assessment of mathematics concepts and procedures (Post, Cramer, Lesh, & Harel, 1993). To the results of the most common incorrect responses, since the most problematic items were those involving number line models, teachers should use such models frequent in the concept of fractions. Furthermore, students generally have problems on part-whole conception; teachers should notice more examples (with distinct type of region models) representing part-whole interpretation. Another problematic issue is related to improper fractions. The lack of ability dealing with this concept can overcome by emphasizing the models that represent fractions larger than one. Unfortunately, the fixed teaching styles and using one or two representation types lead students to get a procedural knowledge merely. In this manner, mathematics teachers should organize the activities which will provide conceptual understanding.

The results of the study enable mathematics teachers to evaluate what necessary skills a students has acquired in the fractions concept and what still needs to be mastered. The most common wrong responses might also be presented to students to warn them not to have such tendencies in the concept of fractions. On the other hand, it is necessary to promote prospective teachers' familiarity with various representations of fractions, as different representations emphasize different aspects of the concept (Fischbein, Graeber, Tirosh, & Wilson, 1994). Finally, the integration of technology into mathematics instruction further increases the need for students to be comfortable with new mathematical representations.

5.6 Recommendation for Further Research

Current study has suggested a variety of useful topics for further studies. These are briefly as follows: (a) It would be beneficial to repeat this study including students from pilot schools which use the new mathematics curriculum to be able to compare the translations abilities of students in the concept of fractions; (b) It would be beneficial to repeat this study with fourth and fifth grade students with selected items to determine their abilities in translating among different representations in the concept of fractions; (c) There is a strong need to make interviews with students who had wrong responses in the test. It will be more meaningful to reveal the main difficulties and to determine the misconceptions on the concept of representations of fractions; (d) Similar studies could be administered to sixth graders after the fractions subjects have been taught in the school; (e) Experimental research is needed to examine the effect of multiple- based instruction on students' ability to translate among different representations.

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APPENDIX A

TRANSLATIONS AMONG REPRESENTATIONS ABILITIES TEST (TRAT)

KESİRLER TESTİ

Isim: _______________ Sınıf:_______

Cinsiyet:______ Bir Önceki Sınıfta Matematik Karne Notunuz:______

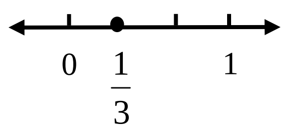
Sevgili Öğrenci,
Bu test kesirlerin farklı gösterimleri arasındaki dönüşümlerle ilgili 21 sorudan oluşmaktadır. Örnek gösterimler aşağıda verilmiştir. Lütfen tüm soruları cevaplamaya çalışınız. Sınav süresi 40 dakikadır.

Örnek Gösterimler:

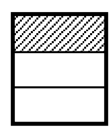
Kesir

$\frac{1}{3}$

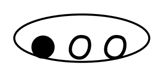
Sayı Doğrusu



Alan/Bölge



Çokluk



I.KISIM: Aşağıdaki tabloda sol kısımda verilen kesirleri, sağdaki kutuda istenilen şekliyle gösteriniz.

Soru 1

Kesir	Sayı Doğrusu
$\frac{6}{5}$	

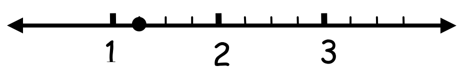
Soru 2

Kesir	Çokluk
$\frac{3}{8}$	

Soru 3

Kesir	Alan
$\frac{9}{4}$	

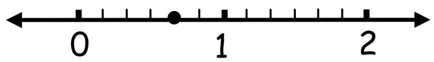
Soru 4

Sayı Doğrusu (noktalı yer)	Kesir
	

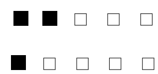
Soru 5

Sayı Doğrusu (noktalı yer)	Alan
	

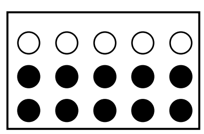
Soru 6

Sayı Doğrusu (noktalı yer)	Çokluk
	

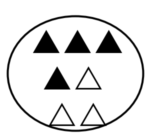
Soru 7

Çokluk (koyu renkli olanlar)	Kesir
	

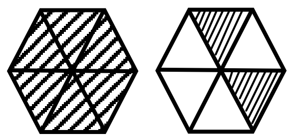
Soru 8

Çokluk (koyu renkli olanlar)	Sayı Doğrusu
	

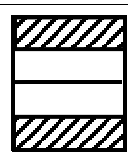
Soru 9

Çokluk (koyu renkli olanlar)	Alan
	

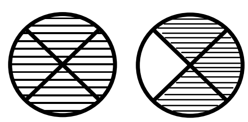
Soru 10

Alan (taralı kısım)	Kesir
	

Soru 11

Alan (taralı kısım)	Sayı Doğrusu
	

Soru 12

Alan (taralı kısım)	Çokluk
	

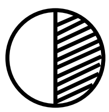
Soru 13) 0.4 ondalık kesrini sayı doğrusu modeliyle gösteriniz.

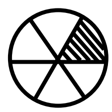
Soru 14) 1,2 ondalık kesrini alan modeliyle gösteriniz.

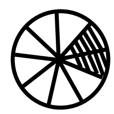
Soru 15) 0,3 ondalık kesrini çokluk modeliyle gösteriniz.

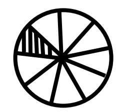
II.KISIM: Aşağıda verilen 16.,17., 18. ve 19. sorularda doğru olduğunu düşündüğünüz tek bir seçeneği işaretleyiniz. Çözümünüzü sorunun yanındaki boşluklarda gösteriniz.

Soru 16) Aşağıdaki taralı bölgelerden hangisi $\frac{1}{6}+\frac{1}{3}$ işleminin sonucunu gösterir?

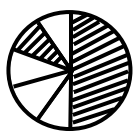
A) 

B) 

C) 

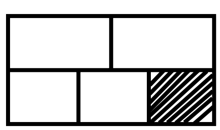
D) 

E) Hiçbiri

Soru 17) 

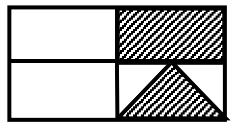
Çemberin taralı bölgesini aşağıdakilerden hangisi gösterir?

A) $\frac{1}{10}+\frac{1}{2}$ B) $\frac{1}{2}+\frac{1}{5}$ C) $1+\frac{1}{5}$ D) $\frac{2}{6}$ E) Hiçbiri

Soru 18) 

Dikdörtgenin taralı bölgesini aşağıdaki işlemlerden hangisi gösterir?

A) $\frac{1}{2}+\frac{1}{5}$ B) $\frac{1}{2}\times\frac{1}{3}$ C) $\frac{1}{2}-\frac{1}{5}$ D) $\frac{1}{3}\times\frac{1}{5}$ E) $\frac{1}{5}+\frac{1}{6}$

Soru 19)  Taralı parçalar dikdörtgenin kaçta kaçıdır?

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{1}{6}$ D) $\frac{3}{8}$ E) Hiçbiri

III. KISIM: Aşağıda verilen problemlerin çözümlerini istenilen modelle açık bir şekilde belirtiniz.

Soru 20) Özgür, geçen ay cep telefonuna 250 kontör yüklemiştir. Kontörlerinin $\frac{1}{5}$'ini kardeşine gönderdiğine göre Özgür'ün kaç tane kontörü kaldığını **taralı alan modelinde** gösteriniz.

Soru 21) Ezgi 2,5 yaşındadır. Eren, Ezgi'den 3,5 yaş büyük olduğuna göre Eren'in kaç yaşında olduğunu belirten **işlemi sayı doğrusunda** gösteriniz.

TEST BİTTİ...
BAŞARILAR ☺

APPENDIX B

LETTERS GIVEN BY TWO MATHEMATICS TEACHERS TO APPROVE THE VALIDITY

"KESİRLER" İLE İLGİLİ ÇALIŞMA HAKKINDAKİ GÖRÜŞÜMDÜR.

Sorular ve öğrencilerden istenen gösterimler, çalışmanın amacına hizmet edecek biçimde, kapsamlı bir biçimde hazırlanmıştır.

II. KISIM, 10 ve 11. sorulardaki, kesirlerdeki "alan/bölgeler"in eşit oldukları belirtilirse, daha kolay anlaşılabilir.

Sonuç olarak titizlikle, hazırlanmış; çok yararlı bir çalışma olduğu görüşündeyim.

12. Ekim 2005

İrfan Eren

Necdet Seçkinöz İ.Ö.O.
Matematik Öğretmeni.
Mebsis No: 9924874

APPENDIX C

LETTERS GIVEN BY TWO MATHEMATICS TEACHERS TO APPROVE THE VALIDITY

Değerli Öğretmenim,

Çalışmanız titizlikle yapılmış, bu konuda tebrik ederim.
Öğretmenlerin, kesirler konusunda alan modeli ve çoklu temsil biçimlerini kullanmaları, bu temsil biçimleri arasında dönüşümlerle gösterebilmeleri, konunun daha iyi kavranmasını sağlar. Bu görüşünüze katılıyorum.
Hazırladığınız testte öğrencilerin dönüşümleri kullanabilmeleri ile ilgili eksiklik yok. Sadece 10. ve 11. sorularda alan modellerinde eşit alanlar (taralı alan) olup olmadıkları tam kesin değil.
Çalışmalarınızda başarılar diliyorum.

Sevgi Otuz

APPENDIX D

CORRESPONDENCE

ORTA ÖĞRETİM FEN VE MATEMATİK ALANLARI EĞİTİMİ BÖLÜM BAŞKANLIĞINA

Orta Doğu Teknik Üniversitesi Eğitim Fakültesi, Orta Öğretim Fen ve Matematik Alanları Eğitimi Bölümünde yüksek lisans öğrencisiyim. "İlköğretim İkinci Kademe Öğrencilerinin, Kesirler Konusunda, Çoklu Temsil Biçimleri Arasındaki Dönüşümleri Yapabilme Becerileri" konulu, özeti **Ek.1**'de verilen bir yüksek lisans tezi hazırlamaktayım. Tezim gereği yapmayı planladığım araştırma, Çankaya, Yenimahalle ve Etimesgut ilçelerinden rastgele seçilen 29 devlet ilköğretim okulunun 6., 7. ve 8. sınıflarında uygulama yapmayı gerektirmektedir. Araştırma kapsamına alınan tüm öğrencilere "Kesirlerle ilgili Temsil Biçimleri Arasında Dönüşüm Yapabilme Becerileri Testi" testi **(Ek.2)** uygulanacaktır.

Yukarıda adı geçen testin uygulanabilmesi için 1(Bir) saatlik uygulama gerekmektedir. Bu çalışmada yer alacak okulların ve öğrencilerin isimleri hiçbir şekilde açıklanmayacak, kesinlikle gizli tutulacaktır. Çalışma kapsamındaki okulların düzeninin bozulmaması için gerekli titizlik gösterilecektir.

2005-2006 Eğitim-Öğretim yılı 1.Döneminde **Ek.3**'de adları verilen okullarda araştırma uygulamasının yapılabilmesi için gereğinin yapılmasını saygılarımla arz ederim.

Gönül KURT

O.D.T.Ü Eğitim Fakültesi,
Orta Öğretim Fen ve Matematik Alanları Eğitimi Yüksek Lisans Öğrencisi,
İlköğretim Bölümü Araştırma Görevlisi
Oda No: 123 Tel: 210 3658
Ankara

Ek 1: Çalışma Özeti
Ek 2: Kesirlerle ilgili Temsil Biçimleri Arasında Dönüşüm Yapabilme Becerileri Testi Ek 3: Uygulama için İzin İstenen Okullar Listesi

Ek 3:

UYGULAMA İÇİN İZİN İSTENEN OKULLAR

Çankaya İlçesi İlköğretim Okulları Listesi:

1. Ahmet Barındırır İlköğretim Okulu
2. Hamdullah Suphi İlköğretim Okulu
3. Sarar İlköğretim Okulu
4. Köy Hizmetleri İlköğretim Okulu
5. Türkiye Noterler Birliği İlköğretim Okulu
6. Ayten-Şaban Diri İlköğretim Okulu
7. Milli Egemenlik İlköğretim Okulu
8. Mimar Kemal İlköğretim Okulu
9. Kılıçali Paşa İlköğretim Okulu

10. Dedeman İlköğretim Okulu
11. Ahmet Bahadır İlhan İlköğretim Okulu
12. Namık Kemal İlköğretim Okulu
13. Kurtuluş İlköğretim Okulu
14. Halide Edip Adıvar İlköğretim Okulu
15. Anıttepe İlköğretim Okulu

Etimesgut İlçesi İlköğretim Okulları Listesi:

1. Sakarya İlköğretim Okulu
2. Etimesgut İlköğretim Okulu
3. Hasan Ali Yücel İlköğretim Okulu
4. Güzelkent İlköğretim Okulu

Yenimahalle İlçesi İlköğretim Okulları Listesi:

1. Ş.Öğretmen M.Ali Durak İlköğretim Okulu
2. Göktürk İlköğretim Okulu
3. Avni Akyol İlköğretim Okulu
4. Refika Aksoy İlköğretim Okulu
5. Necdet Seçkinöz İlköğretim Okulu
6. Konutkent İlköğretim Okulu
7. Kardelen İlköğretim Okulu
8. Mesa Koru Sitesi İlköğretim Okulu
9. Münevver Öztürk İlköğretim Okulu
10. Kürşad Bey İlköğretim Okulu

Sn.İS. Tayip Aroğlu
18.10.2005

T.C.
MİLLÎ EĞİTİM BAKANLIĞI
Araştırma, Planlama ve Koordinasyon Kurulu Başkanlığı

Sayı : B.08.0.APK.0.03.05.01-01/ 5891 07 /10/2005

Konu : Araştırma İzni

ANKARA VALİLİĞİNE
(İl Milli Eğitim Müdürlüğü)

İlgi :Orta Doğu Teknik Üniversitesi Öğrenci İşleri Daire Başkanlığı'nın 18.08.2005 tarih ve 6174 sayılı yazısı.

Orta Doğu Teknik Üniversitesi Fen ve Matematik Alanları Eğitimi Yüksek Lisans programı öğrencisi Gönül KURT'un "İlköğretim İkinci Kademe Öğrencilerinin, Kesirler Konusunda Çoklu Temsil Biçimleri Arasındaki Dönüşümleri Yapabilme Becerileri" konulu araştırma anketini ekteki okullarda uygulama izin talebi incelenmiştir.

Orta Doğu Teknik Üniversitesi tarafından kabul edilen ve ekte gönderilen 2 sayfa 16 sorudan oluşan anketin araştırmacı tarafından uygulanmasında Bakanlığımızca sakınca görülmemektedir.

Bilgilerinizi ve gereğini rica ederim.

Cevdet CENGİZ
Bakan a.
Müsteşar Yardımcısı

EKLER :

EK – 1 Anket (2 Sayfa)

EK – 2 Uygulama Yapılacak Okullar Listesi (2 Sayfa)

1500
18.10.2005

Gelen Evrak İl. No: 3278
Gelen Evrakın Tarihi: 10.2005

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

BÖLÜM : Kültür
SAYI : B.08.4.MEM.4.06.00.11.070/ 3446
KONU : Araştırma izni.

19.10.2005

.... KAYMAKAMLIĞINA
(İlçe Milli Eğitim Müdürlüğü)

Orta Doğu Teknik Üniversitesi Fen ve Matematik Alanları Eğitimi Yüksek Lisans programı öğrencisi Gönül KURT'un "İlköğretim İkinci Kademe Öğrencilerinin, Kesirler konusunda Çoklu Temsil Biçimleri Arasındaki Dönüşümleri Yapabilme Becerileri " konulu araştırmayı -ek listede isimleri- belirtilen ilçeniz okullarında eğitim-öğretimi aksatmamak şartıyla ekte gönderilen 2 sayfa 16 sorudan oluşan anket uygulamasına ilişkin Bakanlığımız; Araştırma, Planlama ve Koordinasyon Kurulu Başkanlığı'nın 15.09.2005 tarih ve 01/5596 sayılı yazısı ilişikte gönderilmiştir.

Bakanlık emri gereğince işlem yapılmasını rica ederim.

Erol ORTAKAYA
Vali a.
Milli Eğitim Müdür Yardımcısı

EKLER.
EKİ. 1- Bakanlık Emri
2- Anket (2 sayfa)
3- Okul Listesi (2 adet)

DAĞITIM
Gereği.
-Çankaya İlçe Kaym. (15 ilköğ.okulu)
-Yenimahalle İlçe Kaym. (10 ilköğ. okulu)
- Etimesgut İlçe Kaym. (4 ilköğ. okulu)

V.H.K.İ :S.TENGİLİMOĞLU 17/10/2005
ŞEF. :H.MEYDAN 18/10/2005
ŞB.MD. :Ö.ALTINYÜZÜK 18/10/2005