RESEMBLANCE: A LOGICO-PHILOSOPHICAL ANALYSIS

İSKENDER TAŞDELEN

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İSKENDER TAŞDELEN

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Prof. Dr. Sencer Ayata
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Ahmet İnam
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

Assoc. Prof. Dr. David Grünberg
Supervisor

Examining Committee Members

Assoc. Prof. Dr. David Grünberg	(METU, PHIL)	
Assoc. Prof. Dr. Erdinç Sayan	(METU, PHIL)	
Assoc. Prof. Dr. Ayhan Sol	(METU, PHIL)	
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Name, Last name : İskender Taşdelen

Signature :

Abstract

Resemblance: A Logico-Philosophical Analysis

Taşdelen, İskender
Ph.D., Department of Philosophy
Supervisor: Assoc. Prof. Dr. David Grünberg

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Indiscernibility, interchangeability and resemblance relations may be named as weak identity relations or identity-like relations. Indeed, identical objects can be thought to satisfy all these three relations. According to the realist view, these relations are defined on the basis of properties of objects. While indiscernibility is a context-independent relation (for it is defined in terms of all properties of objects), interchangeability and resemblance relations are defined in terms of some significant properties of objects and thus they should be regarded as context-dependent.

We should consider the following points while comparing rivalling theories of resemblance:

1. The theory explaining resemblance relations between objects should cover as many domains as possible. Instead of simply admitting that objects of a given type may resemble each other, the theory should explain how these relations are possible.

2. Objects do not just resemble or not; resemblance is a relation that admits degrees. Thus, the theory providing a finer analysis of close and weak resemblances should be preferred.

In comparison to resemblance nominalism, the realist theory of resemblance is stronger with respect to both points. The criticism that the realist view may not explain degrees of resemblance can be rejoined by removing the indeterminacy as to the nature of this notion.

Among knowledge representation systems, property and attribute systems provide a simple but strong models for analytic ontology. Weak identity relations can easily be defined in these systems and the results following from these definitions can be seen to conform to our intuitions about weak identity relations.

Keywords: Ontology, Identity, Indiscernibility, Resemblance, Knowledge Representation System, Tolerance Relation, Model Theory.

Öz

Benzeşme: Mantıksal-Felsefi Bir Çözümleme

Taşdelen, İskender
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Ayırdedilemezlik, karşılıklı-değiştirilebilirlik (ya da muadiliyet) ve benzerlik bağıntıları "zayıf özdeşlik bağıntıları" ya da "özdeşlik-benzeri varlıkbilimsel bağıntılar" olarak adlandırılabilir. Gerçekten de, özdeş nesneler için her üç bağıntının da sağlandığı düşünülebilir. Gerçekçi görüşe göre bu bağıntılar nesnelerin özellikleri ile tanımlanır. Ayırdedilemezlik bağıntısı nesnelerin tüm özellikleri ile tanımlandığı için bağlam-bağılsız bir bağıntı iken, karşılıklı-değiştirilebilirlik ve benzerlik bağıntıları nesnelerin kimi özellikleri ile tanımlandığından bağlam-bağıl bağıntılar olarak düşünülmelidir.

Benzerlik bağıntısına dair varlıkbilimsel kuramların kıyaslanmasında gözönüne alınması gerekenler arasında şunları sayabiliriz:

1. Kuram mümkün olduğunca fazla sayıda ve tipte nesnelerin arasındaki benzerlik ilişkilerini açıklayabilmelidir. Belirli tipteki nesneler arasındaki benzerlik ilişkilerinin olabileceğini sadece ilkece kabul etmek yerine bu ilişkilerin nasıl mümkün olduğunu açıklayan bir kuram tercih edilmelidir.

2. Benzerlik ya var ya da yok denebilecek bir bağıntı olmayıp derecelerle ortaya çıkabilen bir bağıntı olduğundan, daha yakın ya da daha uzak benzerlik bağıntılarının olabildiğince ayrıntılı bir şekilde incelenmesine olanak sağlayan bir kuram tercih edilmelidir.

Benzerlik adcılığına kıyasla, gerçekçi kuram bu iki açıdan da güçlüdür. Gerçekçi

kuramın benzerlik derecelerini açıklayamadığı iddiası ise benzerlik derecelerine ilişkin belirsizliğin giderilmesi ile yanıtlanabilir.

Bilgi temsili sistemleri arasında sayılan özellik ve nitelik sistemleri çözümleyici varlıkbilim için basit ve güçlü modeller sağlamaktadır. Bu sistemlerde zayıf özdeşlik bağıntıları kolayca tanımlanabilir ve bu tanımlardan elde edilen sonuçların sezgilerimizle uygunluk içinde olduğu görülebilir.

Anahtar Kelimeler: Varlıkbilim, Özdeşlik, Ayırdedilemezlik, Karşılıklı-değiştirilebilirlik, Benzerlik, Bilgi Temsili Sistemi, Tolerans Bağıntısı, Model Kuramı.

In Memory of Anneanne and Efendibaba

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Table of Contents

Chapter 1

Introduction

This is a study of identity-like ontological relations: indiscernibility, interchangeability and resemblance. I devoted special attention to the relation of resemblance. The thesis is roughly divided into two parts. Not in form (since, e.g., philosophical remarks invaded most of chapters 2, 3, 4 and 10) but in spirit. First I presented some philosophical remarks as to the nature of these relations. This part is not an overall philosophical discussion of the topic. However, it serves to provide, at least for myself, a philosophical motivation for the next part in which I examined some formalisms of identity-like relations.

In the second part of the thesis, I presented some structures and how statements concerning identity-like relations can be interpreted in these structures. The last example, that of information systems seems to me to be the most effective way of explicating identity-like relations. I will now give a little more detailed outline of these parts.

I

In the first part, I discussed my philosophical contentions. I have two main philosophical contentions regarding identity-like relations:

1. Indiscernibility, interchangeability and resemblance relations should be analyzed in terms of properties of objects. Or, in case of tuples of objects, interrelations among these objects.

2. Interchangeability and resemblance relations are context-relative.

3. The theory of identity-like relations based on (i) and (ii) should be preferred on the basis of its explanatory power.

The first thesis forms a part of the general metaphysical theory of realism about universals, or shortly realism. In the Aristotelian form of realism, or immanent realism, properties and relations do not exist independently of particulars but they exist in the particulars. In Platonist form of realism, or Platonism, properties and relations exist independently of individuals or tuples of individuals that exemplify these properties and relations. General theory of Platonism is not commonly held among philosophers of this day. There are two common criticisms of Platonism: Firstly, it is said to be too generous as an ontological theory. It generates an unlimited stock of entities. Secondly, Platonism is said to make knowledge impossible by employing entities with which we cannot have any causal interaction. Aristotelian immanent realism seems to be a more moderate form of realism. It grants that properties and relations exist but not independently of individuals. This view then provokes questions as to the status of properties and relations that are not exemplified contingently at this moment but that have been exemplified for some time in the past and will possibly be exemplified in the future. Do they not exist now? And when some individual comes into existence with that property what ground do we have for the identity of that property. How can we know whether it is a completely new property or it is the same property but now passed into exemplified mode?

The opponent view of metaphysical realism is known as nominalism. As in the case of realism there are various forms of nominalism. Nominalism, at the extreme, states that properties and relations are just names. To put it more moderately, the alleged existence of independent properties and relations is out of a mis-projection of language onto the reality. Only names of individuals denote, predicate-expressions and relation-expressions do not. I did not consider each form of nominalism, and for the purpose of this study of identity-like relations, I considered nominalism as the view that there are just individuals, or particulars, and individual identity-like relations between individuals and universals can safely be eliminated from ontology since every statement in the form "a is F" or "a has the property F-ness" and every statement of the form "the objects $a_1, a_2, \ldots, a_n$ are R-related" or "$(a_1, a_2, \ldots, a_n)$ instantiates the relation R" can be understood without recourse to universals. Resemblance nominalism, the only form of nominalism I considered somewhat in more detail, and which is the most commonly held view of nominalism today, holds that, "a is F" amounts to saying that: There is a class of F exemplars which are paradigmatic particulars that are F and which stand in particular resemblance relations and the individual a resembles them as closely as they resemble one another. This view encounters problems with eliminating the relation of resemblance itself and finds itself trapped in infinite regress if it is continually asked "what makes this particular case a resemblance situation?" Resemblance nominalist is strong when dealing with the epistemology of resemblance. In daily experience resemblances are too simple to accept any further explanation. We may immediately say that this thing resembles that thing. Viewed from this perspective, why not we start with particular resemblances and explain properties and relations with recourse to them? But what is the measure of simplicity? How do we know that these simple resemblances are ontologically simple too. If resemblance is an ontic relation, that is, if it is not just that we *perceive* a and b resembling each other but that "a resemble b", then simple perceptions, whatever they are, may not witness simple resemblances. Moreover, what about non-perceptible entities, abstract or concrete? Should we *a priori* say that they cannot resemble? And thus they do not have properties and relations? I believe that, they too resemble or not in virtue of their properties and relations.

The second thesis is that there are many interchangeability and resemblance relations. More specifically we should determine the context in which the interchangeability or resemblance claim is made. This claim may seem to be inconsistent with the assertion that interchangeability and resemblance relations are real: they have independent existence, independent of our minds and of the individuals they apply. By a context, I mean a set of ontic properties. and relative interchangeability and relative resemblance relations with respect to a fixed set of ontic properties can be determined objectively, on the assumption that these mentioned properties are objective properties. Properties in a given context are called significant properties. That resemblance claims are either incomplete or trivial without fixing significant properties should be granted once we note that any two entities would resemble in so far as they are both self-identical. A similar claim may be made for the case of interchangeability. On the basis of these points, I believe that relative resemblances and relative interchangeabilities should be seen as the best solution for the problems of universals theory of identity-like relations which explains these relations in terms of properties and relations too.

The last claim is based on three points: (1) The outlined theory of identity-like relations can explain (not just takes for granted) the notion of degrees of resemblance. (2) The theory explains resemblances and interchangeability relations between entities of any type. The theory is applicable to any system of objects once we know the relevant properties of objects.

II

In the second part, I examined some structures in which identity-like relations can be represented. These are

1. Tolerance spaces,
2. Set theoretic models,
3. Information systems.

Tolerance spaces as presented by Schreider [20] takes resemblances as basic relations. He stipulates that resemblances relations are reflexive, symmetric and usually non-transitive relations. He calls such relations as *tolerance relations.* A tolerance space is then defined to be a structure consisting of a non-empty set T together with a tolerance relation τ on this set. As such, tolerance spaces cannot be explication of resemblance relations since on a set of objects we cannot normally expect all reflexive, symmetric relations to be genuine resemblance relations. Otherwise, every subset of the universe T would be taken as representing a property.

Set theoretic models provide the common Tarskian semantics for first order languages. These have proved to be useful for the working mathematician. Ontologically however, neither first order logic nor the usual set theoretic semantics of this logic is satisfactory. The reasons why properties cannot be identified with their extensions are now among the folklore of ontological research. I included two short chapters on set theoretical models and representation of identity-like relations in this study mainly because, although unsatisfactory, they provide a natural introduction to formal semantics.

Deutsch has suggested a system in which what he calls general similarity relations and indiscernibility properties can be represented. His system is in the spirit of set theoretic model theory. He claims to avoid higher order entities but I am not sure to what extent he is successful in this respect. Moreover, his indiscernibility properties can easily be handled in attribute systems though we should grant that he elaborated an ingenious device in his paper [10].

I believe that information systems as introduced by Pawlak provide a natural framework for a simple and effective philosophical semantics. I considered them in the form that was presented by Vakarelov [22]. There he already presented some notions of resemblance and indiscernibility in property systems and in what I may call first order attribute systems. I introduced here the notion of relational information systems [1] and higher order attribute systems. This last is particularly useful in representing determinable-determinate relations and thus degrees of resemblance.

I devoted a relatively long chapter for infinite regress arguments. Infinite regress arguments are encountered in discussions of both nominalism and realism. Research about their logical status does not provide a satisfactory account of why and when a theory leading to an infinite regress should be refuted, as often. The papers by Black [6] and Clark [8] provide some background in this direction without providing a definite answer. I discussed these papers, pointed to an error in [6] and pointed out some possible moves against infinite regress objections.

[1] Somewhere I encountered this notion but with a completely different meaning; there relational information systems are just information systems with a partial evaluation function here I have functional information systems in which relations are also handled.

Chapter 2

Prelude

Reasoning about reality is possible through our ability to grasp individual objects in their relations to others and not only as isolated entities. This is not just because we are not capable of coping with a *complete* diversity of individuals: without thinking of objects in relation to others, one could not form any concepts, the building blocks of knowledge. Thus, *our* universe cannot be a mere collection of objects but a structured whole.

Moreover, our universe may not be *the* universe. For, the alleged structure of the universe may not be anything other than our imposition. The universe is what it is independently of our epistemological needs. In a sense, we are just a part of the universe. Let us assume that we know what the individual objects in the universe are. Then, our first questions will include: what features these individuals really have and what relations really hold among them.

My main concern here is the relation of *resemblance* which is an important notion to explain the structure of the universe. My aim is twofold:

1. Developing a feasible philosophical notion of resemblance and

2. Describing a rich class of abstract structures by means of which resemblance and other identity-like relations can be explicated in a philosophically significant manner.

I will try to accomplish these by evaluating and developing former attempts at both (i) and (ii). For (i), I will mainly focus on Armstrong's definition of resemblance (cf. [3], p.96). For (ii), I will consider two special information systems; property systems and attribute systems as Vakarelov described in [22] and develop them so that more aspects of resemblance can be represented. While dealing with these formalisms, I will mainly concentrate on their explanatory power concerning the representation of basic ontological relations and try to avoid unnecessary technicalities about the formal structures considered.

I will say a few things about the neighboring relations viz., identity, indiscernibility and interchangeability to put resemblance in a proper frame. Let me start with a general remark:

Let $\mathcal{U}$ be a class of objects, $\mathcal{P}$ be any class of properties and $\mathcal{R}$ be any class of relations among objects. In the study of these properties and relations, three aspects should be carefully distinguished.

1. The *epistemic* aspect: Formation, revision and justification of our beliefs about
   (a) Whether a given object a (a tuple of objects $(a_1, a_2, \ldots, a_n)$) lies in the extension of a property P in $\mathcal{P}$ (an n-ary relation R in $\mathcal{R}$),
   (b) The nature of these properties and relations. That is, meta-properties of properties and relations.
2. The *linguistic* aspect: Symbolic representation of these properties and relations by means of a language. Comparison of expressive strength of competing languages.
3. The *ontic* aspect: All that is the case concerning these properties and relations independent of any mental or symbolic representation and any system of beliefs. One should respect the distinction between *ontic* and *ontological* and keep the word "ontological" for the study of the ontic (of what exists and nature of existents). One example may help clarifying the point: In the sense that I am using the words "ontic" and "ontological", the expression "ontic commitment" is improper: to say that it is proper is to say that the world may be committed to the existence of something while in fact only people can be properly said to be committed to existential beliefs on the face of their other beliefs.

An example of a “disagreement” arising from the failure to distinguish the epistemic and the antic will be considered in the next section.

Chapter 3

Identity, Indiscernibility, Interchangeability

Every object is identical with itself and with nothing else; the problem arises only if we question whether two descriptions in a language are the descriptions of the same thing or two different things. This is the common view of identity and I have no intention to question it. Formal properties of identity, or the axioms of identity, are the following (see chapter 5 for clarification of notation)

1. (reflexivity) $x \equiv x$
2. (substituvity in terms)
   $x \equiv y \rightarrow t(v_0 \ldots, v_{i-1}, x, v_{i+1}, \ldots, v_n) \equiv t(v_0 \ldots, v_{i-1}, y, v_{i+1}, \ldots, v_n)$
3. (substituvity in formulas)
   $x \equiv y \rightarrow \varphi(v_0 \ldots, v_{i-1}, x, v_{i+1}, \ldots, v_n) \leftrightarrow \varphi(v_0 \ldots, v_{i-1}, y, v_{i+1}, \ldots, v_n)$

Here the symbol "$\equiv$" is the symbol of identity in the language. Its interpretation in any structure is fixed: Whenever $\mathcal{L}$ is a language including $\equiv$ and $\mathfrak{A}$ is an $\mathcal{L}$-structure, for any two $\mathcal{L}$-terms t_1 and t_2,

$$\mathfrak{A} \models t_1 \equiv t_2 \text{ if and only if } {t_1}^{\mathfrak{A}} = {t_2}^{\mathfrak{A}}$$

Both ${t_1}^{\mathfrak{A}}$ and ${t_2}^{\mathfrak{A}}$ are elements of the universe and the relation "$=$" between the elements of the universe is the usual, robust relation of equality.

We may write $\varphi(x/v_i)$ for $\varphi(v_0 \ldots, v_{i-1}, x, v_{i+1}, \ldots, v_n)$

similarly

$t(x/v_i)$ for $t(v_0 \ldots, v_{i-1}, x, v_{i+1}, \ldots, v_n)$.

Remark 3.0.1. Symmetry and transitivity of identity can be derived from the identity axioms:

Symmetry: Let $x \equiv y$. Then, letting $\varphi(v_0) = (v_0 \equiv x)$ we get, as an instance of substituvity in formulas

$$x \equiv y \rightarrow \varphi(v_0)(x/v_0) \rightarrow \varphi(v_0)(y/v_0)$$

that is,

$$x \equiv y \rightarrow (x \equiv x \rightarrow y \equiv x)$$

Then since as an instance of reflexivity we have $x \equiv x$, we have, by applying *modus ponens* twice, $y \equiv x$,

Transitivity: Let $x \equiv y$ and $y \equiv z$. Letting $\varphi(v_0) = (x \equiv v_0)$, by substituvity axiom we get

$$y \equiv z \rightarrow \varphi(v_0)(y/v_0) \rightarrow \varphi(v_0)(z/v_0)$$

that is,

$$y \equiv z \rightarrow (x \equiv y \rightarrow x \equiv z)$$

Then by applying *modus ponens* twice we get $x \equiv z$ □

Identification of an individual object is something else and this notion does not lead to a diversity of identities as some philosophers claim: Geach, for example, clearly says that absolute identity is nonsense and we should further specify identity claims by referring to "being the same F" relations where F is a schematic expression for a count-noun.[12] However, saying that x and y are the same F (where F is a count-noun) is nothing else but saying that x and y are identical and both are F. Thus there are no two objects which are the same F but not the same G. If there is a property F that x has while y fails to have, then x and y are not identical by the indiscernibility of identicals. Ontologically the interesting question is whether there are *weakenings of identity.* Or, if you wish, whether some neighboring relations should be considered as such. By labeling them as weakenings, I wish to emphasize that they are *not* relations competing with identity.

Indiscernibility is the coincidence of all properties. It is the closest relation to identity among the identity-like relations. Moreover, in the presence of the Leibniz' law, these are the very same relation.

Formally, indiscernibility is an equivalence relation. Confusing the epistemic with the ontic, one may claim that indiscernibility relation can be non-reflexive or non-symmetric or non-transitive. Such claims explicitly mention a method of justifying our discernibility claims about objects in a given domain. Consider the following:

> Reflexivity. It may seem reasonable to assume that every object is indiscernible form itself. But in some occasions this is not true, since it is possible that our information is so imprecise. For example, we may discern persons by comparing photographs taken of them. But it may happen that we are unable to recognize that a same person appears in two different photographs.
>
> Symmetry. Usually it is supposed that indiscernibility relations are symmetric, which means that if we cannot discern x from y, than we cannot discern y from x either. But indiscerniblity relations may be directional. For example, if a person x speaks English, Finnish, and a person y speaks English, Finnish and German, then x cannot discern y form himself by the property "knowledge of languages" since y can communicate with x in any languages that x speaks. On the other hand, y can discern x from himself by asking a simple question in German, for example.
>
> Transitivity. Transitivity is the least obvious of the three properties usually associated with indiscernibility relations. For example, if we define

> an indiscernibility relation on a set of human beings in such a way that two person are indiscernible with respect to the property "age" if their time of birth differs by less than two hours. Then, there may exist three persons x, y and z, such that x is born an hour before y and y is born $1\frac{1}{2}$ hours before z. Hence, x is indiscernible form y and y is indiscernible from z but x and z are not discernible. [13]

Although it is possible to define and work with method-relative indiscernibility notions which may lack some (or all) of the formal features of indiscernibility, a general claim against reflexivity-symmetry-transitivity of indiscernibility cannot be supported that way. Indiscernibility or discernibility of objects depends only on the properties they have, not on any particular method of discerning them. *Indistinguishability* seems to be the proper epistemological counterpart of indiscernibility. Indistinguishability means inability to give an epistemic justification for the belief that *two or more* given objects have any distinguishing non-relational property. Indiscernibility, on the other hand, refers to all properties, relational or non-relational, that the objects really have. In these terms, Black's famous counter-example to the identity of indiscernibles is an example of confusing the two. It is possible to imagine "two" indistinguishable spheres but they may not be indiscernible.

Confusing properties (the ontic) and predicates (the linguistic) may also mislead: the particular language we are using may not enable us to express the property discerning two objects. From this we cannot infer that they are indiscernible. For indiscernibility refers to the collection of all properties, not to a subcollection of properties. Thus, indiscernibility with respect to the class of properties expressible in a language is not indiscernibility *per se*.

Let $\mathcal{U}$, the *universe*, be the class of all *entities* (abstract or concrete), whose elements are called *objects, particulars* or *individuals*, $\mathcal{P}$ be the class of all relations with a special subclass $\mathcal{R}_{id}$ of *identity-like* relations: indiscernibility, interchangeability and resemblance. In the course of the following, I will present a unified account of relations in $\mathcal{R}_{id}$. Identity-like relations can be defined also on tuples of objects. Thus, we could allow $\mathcal{R}_{id}$ to include binary relations on $\mathcal{U}^n$ for any non-zero natural number n. The extension is straightforward.

Two objects o_1 and o_2 are indiscernible, $o_1 \cong o_2$ for short, if they have the same properties, symbolically;

$$\forall P \in \mathcal{P}(P(o_1) \leftrightarrow P(o_2)) \tag{3.1}$$

Interchangeability is indiscernibility relativized to a subclass of $\mathcal{P}$. Thus, given a subclass $\mathcal{P}_0$ of $\mathcal{P}$, two objects o_1 and o_2 are interchangeable relative to $\mathcal{P}_0$, or $\mathcal{P}_0$-interchangeable, $o_1 \fallingdotseq_{\mathcal{P}_0} o_2$, if

$$\forall P \in \mathcal{P}_0(P(o_1) \leftrightarrow P(o_2)) \tag{3.2}$$

Just like indiscernibility, the relation of interchangeability is reflexive, symmetric and transitive on the universe. Explicating interchangeability this way has the following immediate consequences conforming to the commonsensical conception of interchangeability:

1. Any two objects having all of the $\mathcal{P}_0$-properties are $\mathcal{P}_0$-interchangeable,
2. Any two objects having none of the $\mathcal{P}_0$-properties are $\mathcal{P}_0$-interchangeable,
3. No object lacking all the $\mathcal{P}_0$-properties is $\mathcal{P}_0$-interchangeable with an object having a $\mathcal{P}_0$-property,
4. No object having all $\mathcal{P}_0$-properties is $\mathcal{P}_0$-interchangeable with an object lacking a $\mathcal{P}_0$-property.

Plausibility of these consequences show that 3.2 is superior to other possible definition of interchangeability according to which we would have

$$\forall P \in \mathcal{P}_0(P(o_1) \wedge P(o_2)) \tag{3.3}$$

or, equivalently

$$\forall P \in \mathcal{P}_0 P(o_1) \wedge \forall P \in \mathcal{P}_0 P(o_2) \tag{3.4}$$

Let us say that o_1 and o_2 are *strongly interchangeable for-*$\mathcal{P}_0$, or strongly $\mathcal{P}_0$-interchangeable if o_1 and o_2 satisfy 3.3 (or 3.4). In fact 3.3 is covered by 3.2 while (ii) will no longer be true under 3.3. By means of the clause (ii), we label some entities as "useless" in the given context. We may say that two entities both failing to satisfy any $\mathcal{P}_0$-property, two $\mathcal{P}_0$-useless objects, are *negatively interchangeable with respect to* $\mathcal{P}_0$.

Intuitively $\mathcal{P}_0$ is the class of significant properties. What significant means depends on the context. In daily life, significant properties are often functional properties: if you want to peel an apple, any two fruit knives in the drawer are interchangeable although as objects they may have many distinguishing properties such as color or brand.

Later I will defend a notion of resemblance defined in the spirit of 3.2. My maim contention is that particular interchangeability and resemblance notions can be accounted for by choosing suitable sets of significant properties.

Some overall stipulations can be made about sets of significant properties: to mention the most obvious, they should contain ontological properties and properties and relations as much as we regard interchangeability and resemblance as ontological relations (just like indiscernibility). One may protest and say that the above notion of interchangeability and the pre-mentioned notion of resemblance cannot correspond to ontological relations since there is no objective ground for choosing sets of significant properties. My reply would be: Yes but modulo a fixed set of properties $\mathcal{P}_0$, $\mathcal{P}_0$-interchangebility (and, later, $\mathcal{P}_0$-resemblance) have their proper ontological senses.

Chapter 4

Resemblance

Resemblance, or similarity, is much weaker than indiscerniblity and interchangeability. While comparing rival theories of resemblance, the following two points seem to be useful:

1. The theory should *explain* (not just accept) the fact that a particular may resemble some particulars more closely than others. The theory should not be so robust to the extent that all resemblances are just yes or no cases.

2. The coverage of theory should be as broad as possible. Other things being equal, the theory which explains resemblances among more entities should be preferred.

One of the crucial debates on resemblance is whether it is a primitive relation or a derivative of properties of objects. For a resemblance nominalist, resemblance is a primitive relation in terms of which properties can be eliminated. Their aim is to explain away properties in terms of particular resemblances among objects. According to resemblance nominalism, a has the property F if a resembles a set of *F-paradigms*, *F-exemplars*, as closely as they resemble one another. Thus, the class of objects which are F has a structure: it is the set of exemplars which keeps the class together.

4.1 Criticisms of Resemblance Nominalism

There are many criticisms of resemblance nominalism. The strongest motivations of resemblance nominalists to regard resemblance as basic seem to be epistemological. Indeed, some of our experiences with resembling particulars suggest that the resemblance relation is too basic to admit any analysis in terms of properties of objects. Moreover there are cases where we cannot identify the property common in both individuals while we immediately observe that they resemble each other. The argument against realism about resemblance out of simple, or, better, immediate resemblances seems to rest on the confusion of the epistemic with the ontic. Although epistemological and linguistic facts (e.g., the way we come to believe that objects o_1 and o_2 resemble each other and the way we state our beliefs in a language) may be illuminating while doing ontology, we cannot use them uncritically. Even the overall epistemic simplicity of a relation cannot be taken as a direct justification of ontic simplicity. The alleged existence of immediate perceptions of resemblance may well be an evolutionary matter of fact rather than ontological necessity. Moreover, it is not obvious that we have immediate perceptions of any entity: How can we have immediate perceptions concerning, say, closed regions in 8-dimensions?

The source of one objection against resemblance nominalism is the relation of resemblance itself. If the resemblance nominalist cannot give a successful eliminativist account of it, this relation should be a universal too. Obviously, he should try a similar strategy and refer to second order resemblances among resemblance situations. A *resemblance situation* is a concrete case employing a tuple of individuals $(a_1, a_2, \ldots, a_n)$ standing in a particular resemblance R_1. Any two cases of resemblance $(a_1, a_2, \ldots, a_n), R_1$ and $(a_1, a_2, \ldots, a_n), R_2$ would resemble each other by both of them being resemblances. Thus this second-order resemblance should be explained away by introducing a third-order resemblance and so on [18]. I should note here that, infinite regress arguments like this should be examined closely before deciding about whether they really undermine the theory. I will devote a full section to this issue later.

One may legitimately question F-ness of F-exemplars as well. Since an account of their F-ness by means of their resembling each other gives no information, resemblance nominalists suggested that instead of a fixed set of F-exemplars, $a_1, a_2, \ldots, a_n$, we may use alternative sets of exemplars. The explanation for the F-ness of objects in the group of exemplars $a_1, a_2, \ldots, a_n$ is then given by their resemblance with objects in another group of exemplars $b_1, b_2, \ldots, b_m$. This account has the following deficiency. how can we choose this second group of exemplars? These must be somehow F (to help the explanation of a_i's F-ness) but how can we claim that any of these b_js are F? Referring to the F-ness of a_is would be illegitimate since their F-ness is under questioning. Then, we must pick another set of exemplars $c_1, c_2, \ldots, c_k$ whose F-ness again raises the same problem and so on. Thus we are led to an infinite regress. It seems to me that the use of alternative sets of paradigm objects is of no use although it seems to be a natural explanation (See [15] for a detailed account of regress arguments against resemblance nominalism). In many cases the existence of an infinite regress does not present a decisive problem at all. In this case, however, I think that the infinite regress argument leads to the conclusion that resemblances cannot explain objects' having common properties. Even if we accept this conclusion, it does not by itself show that properties, or at least some of them, are irreducible. For there may be other attempts to evaluate. And indeed there are. Nevertheless, I believe that resemblance nominalism is the strongest rival. Since, for above reasons, I rejected primitive resemblances of resemblance nominalism, I will assume that at least some properties are irreducible. I can gladly admit that once the class of objects which have the property F has been formed, use of (derived) resemblances on this class with well-chosen paradigm objects seems to be a good and natural way, e.g., to teach someone who does not just know F-ness or to justify claims about F-ness. So what is problematic is not the relation of resemblance itself but relying heavily upon it to define F-ness.

4.2 Some Preliminary Remarks on Possible Formalisms of Resemblance

Before considering the realist[1] conception of resemblance and its formal explications, let me add a few words about some possible formal explications of primitive resemblances. If one accepts that resemblance is a primitive relation, the obvious first move towards formal explication of resemblance would be to stipulate that resemblance is a binary, reflexive and symmetric relation. Corresponding first order structures consists of a non-empty set T together with such a relation on it. Reflexive and symmetric relations are called as *tolerance relations* by Schreider [20] and the relational structures $\langle T, \tau \rangle$ consisting of a non-empty set T and a tolerance relation τ on it, are called *tolerance spaces*.

Even if it is accepted that resemblance relations are indeed reflexive and symmetric (and not necessarily transitive), not every such relation would correspond to a resemblance relation. It will be a consequence of the notion of resemblance defended here that resemblance relations need not be reflexive but *weakly reflexive* (A relation R is weakly reflexive if every object that is R-related to an object is R-related to itself).

Another possible formal strategy corresponding to a nominalist conception of resemblance is to work with relative resemblances. Here one determines degrees of resemblances between pairs of objects relative to other pairs. More specifically, such a strategy requires a binary relation, say δ, on the set of pairs of objects. If S is a non-empty set and $\delta \subseteq (S \times S) \times (S \times S)$, then $(s_1, s_2), (s_3, s_4) \in \delta$ means that the resemblance between s_1 and s_2 is closer than the resemblance between s_3 and s_4.

The roughest realist notion of resemblance is that, it is a coincidence of at least one property. That is, objects o_1 and o_2 resemble each other, $o_1 \Cap o_2$ in symbols, if they satisfy a common property:

[1] The label "realist" here is used in the sense that it is used in the context of universals. In the sense used here, realist means "a philosopher who believes that universals exist". Note that primitive resemblances may well be "real" that is, they may form a part of independent reality.

$$\exists P \in \mathcal{P}(P(o_1) \wedge P(o_2)). \tag{4.1}$$

4.3 Real Resemblances

The same feature presents itself in several individuals or the same pattern is observed among several groups of individuals. When a feature, a property, say the property of F-ness is present in objects $o_1, o_2, \ldots, o_n$, we say that these objects resemble with respect to F-ness.

Generalizing resemblances with respect to properties, suppose that we propose the following as the definition of resemblance: Two or more objects resemble each other if there is a feature, F, present in all of them. Is this an appropriate definition? No: in this case the notion of resemblance would be too broad to be useful. For observe that under this definition, any two individual objects would resemble each other in so far as both are, say, self-identical.

Fixing a respect of resemblance seems to be vital. It seems to be confusing though: should we say two or more objects resemble if they are both F? (F is the fixed respect of resemblance). No: I would rather say that there are many resemblances and the seemingly significant notion of unrestricted resemblances is the outcome of the careless talk. In cases where we assert simply e.g., that Fred resemble his father, we in fact intend to say that Fred resemble his father in some significant respects and presuppose that our listener knows which properties are significant in the context of utterance.

Universals theory of resemblance (UTR) explains resemblances of objects as mutual possession of the *same or resembling* properties of these objects. I will concentrate on structures in which resemblances, besides other identity-like relations, can be adequately represented and hope that the naturality of these structures will contribute the plausibility of the UTR.

Although daily use of resemblance is that it is a binary (2-ary) relation between particulars, it turns out that substantial part of the UTR must turn around the notion of resemblance between universals. This may appear to conflict with our intuition since we may think that what we perceive are not resembling properties but objects and this relation is too basic to admit any further analysis. It is true that, we do not perceive the universal red but always some red object and even *if* we should admit that universals resemble each other, we should also admit that we cannot perceive resembling universals.

However, I believe that, resemblances among universals are epistemologically as fundamental as resemblances among individuals. My knowledge that some properties of a resemble some properties of b is a *part of my knowledge* that and not just a *part of the explanation of my knowledge* that "a resembles b" . I perceive the color of a, the color of b and quite possibly some other properties of a and b. Even the sameness of the color of a and b is not something perceived. Thus we should accept that the relational concept "sameness of color" is reached on the basis of our perceptions but cannot be reduced to them to restate the idea given by the first sentence of this paragraph.

Thus, on this view, not only particulars resemble each other; universals may resemble each other too. Resemblance of universals is extremely important to develop a theory of resemblance within a theory of universals. The resembling pairs of particulars may have a common identical property. However, one has to admit, this is just a theoretical *possibility.* In most (almost all) of the cases particulars resemble not in virtue of instantiating the same universal but again similar universals. For Armstrong, exact resemblance of universals is identity while inexact resemblance of universals is *partial identity.* Rather than giving a general argument in support of this claim, he gives the example of "being five kilograms in mass". Using the language of states of affairs, being five kilograms in mass consists of "something's being four kilos plus something else's being (nonoverlapping something else) one kilo state of affairs". Thus, since being five kilos in mass involves having a (proper) part having two kilos in mass and being four kilos in mass [1].

Armstrong's view of resembling universals can be outlined as follows: Universals may resemble each other with degrees. Only complex universals can resemble each other. Some, like Hume, may think that simple universals may also resemble. This is out of false beliefs about the nature of resembling properties under consideration.

It is true that colors, for example, may resemble each other: We say that red is more like orange than it is like yellow. It is also true that our perception of colors is somewhat simple. However, this does not mean that colors are simple properties and science gives us the reason why this is not so.

Although degrees of resemblance strongly suggest a quantitative analysis, there is nothing wrong with a qualitative analysis of this notion. The problem, to restate, is to give a viable analysis of the relation "the resemblance between a and b is closer than c and d".

Suppose that we are given a class of figures. Some of them may resemble each other with respect to planarity. Among these planar figures some would have a closer resemblance by virtue of their all being triangles. Among these we may further choose a smaller group upon noticing that they are all isosceles. Again, we may say that these figures have a closer resemblance. Armstrong, e.g., would accept that being an isosceles triangle is a legitimate conjunctive property. The first question to ask is that is it a first-order property or not? It seems that it is. Being an isosceles triangle is a conjunction of two properties of first-order although "being isosceles" cannot be meaningfully asserted for all objects. We cannot say that isosceles triangles are those objects lying in the intersection of "triangularity" and the extension of "isosceleshood". One may say that isosceleshood is a first-order property even if it is weird to ask, for example, this piece of music is isosceles. After all there are objects (triangles) which are isosceles and the extension of the universal consists of these objects.

The apparent asymmetry arising in language due to awkwardness of the predicate "triangle isosceles" may suggest that the analysis of "isosceles singularity" as a conjunctive property is not correct. For conjunction is commutative and thus there should be no asymmetry between "isosceles triangularity" and "triangle isosceleshood". However, in some languages, for example in French, the adjective "isosceles" comes after "triangle". Thus, the asymmetry is only apparent.

Moreover, there seems to be nothing special with the property "isosceleshood". The situation is not different than with the predicate "red car" in this respect. Although the possible extension of redness admits much more diversity, "car red" could be rejected as a natural predicate.

Resemblances between collections of objects can also be explicated in the spirit of UTR. As to the issue of resemblance between sets, two senses of resemblance are relevant. Two sets, to simplify the point, may resemble each other with respect to a property of sets. For example, two sets may resemble each other by being equinumerous, i.e., having the same number of elements. Or, they may resemble each other by carrying a similar structure. For example, two sets may resemble each other by both being partially ordered sets, or even by being order-isomorphic. Satisfying the same set-properties seems not to be the only way for sets to resemble each other. Sets, by being composite entities formed by particulars of lower levels, may resemble each other by virtue of properties of their members. Two sets may resemble each other by both being sets of ordinals or by being sets of finite sets and so on.

Any theory of resemblance should explain the *degrees of resemblance*: we believe that there are pairs of particulars (o_1, o_2), (o_3, o_4) such that the resemblance between o_1 and o_2 will be closer than the resemblance between o_3 and o_4. However, according to 4.1, particulars seem to either resemble or not. Possible solutions have been suggested. Armstrong suggested without much emphasis, that degrees of resemblance can be explained by the existence of scale-like orders on some classes of properties. Indeed, some properties can be thought to form a scale, e.g., shades of a color is a classical example. Another possible solution, based on the determinate-determinable relation on properties, has been suggested by Price [16]. Determinate-determinable relation is hard to define but, fortunately, easy to exemplify: Red is a determinable with respect to carmen-red and pink which are two determinates of it. There will still be determinates of carmen red and pink. Thus, two objects one is carmen-red the other pink resemble each other but this resemblance is weaker than the resemblance between any two pink objects. Thirdly, again by Armstrong, the structural-universals solution has been suggested. All these are still open pragrammes in the theory of universals. The last one, which I will follow and modify refers to resemblances between properties. Armstrong's definition of resemblance is that

$(*)$ ...a particular a resembles a particular b if and only if: There exists a property, P, such that a has P, and that there exists a property, Q, such that b has Q, and *either* $P = Q$ *or* P resembles Q. [3], p. 96.

As in the case of interchangeability, resemblance claims are context dependent. As a first approximation towards the definition of contextual-resemblances, given a set of significant properties $\mathcal{P}_0$ and two objects o_1 and o_2, let us define: o_1 and o_2 resemble each other with respect to $\mathcal{P}_0$, $o_1 \Cap_{\mathcal{P}_0} o_2$ for short, if

$$\exists P \in \mathcal{P}_0[P(o_1) \wedge P(o_2)] \tag{4.2}$$

In case $\mathcal{P}_0 = \mathcal{P}_{physical} \subseteq \mathcal{P}$ we obtain physical resemblance. One may extend 4.2 to cover *negative resemblance* so that o_1 resembles o_2 if

$$\exists P \in \mathcal{P}_0[(P(o_1) \wedge P(o_2)) \vee (\neg P(o_1) \wedge \neg P(o_2))] \tag{4.3}$$

A similar strategy would be to demand $\mathcal{P}_0$ to be closed under the so called negative properties. I will not follow this. Although negative resemblance is a genuine resemblance and should be accounted for in the theory, the extension of the following to cover negative resemblances is straightforward and therefore I will not go into that. Thus, "$\neg$" in 4.3 is not a property constructor applying to P but a sentential operator applying to $P(o_1), P(o_2)$. Similarly, I will not assume that $\mathcal{P}$ is closed under disjunctive or conjunctive properties.

An admissible but strange feature of relative resemblance is that an object having no significant property does not self-resemble. Note that 4.3 immediately solves the problem. The strongest point in favor of 4.2 is its generality. While resemblance nominalism tries to gain to strength from simple or immediate resemblances between common objects, it is weak if asked to explain resemblances between unobservable physical entities (and abstract entities too, if you accept them in your ontology). Ontologically, there is no point in denying resemblance for these entities just because we cannot causally interact with them properly to the degree that allows testing our beliefs about them (we may have no interaction at all, as in the case of abstract entities). We assume that they somehow exists either by definition (cf. mathematical entities) or as a result of our metaphysical beliefs and that they have properties (being prime, being odd etc.). Thus, 4.2 cover them easily.

Although not impossible, it is unreasonable to expect to observe resembling particulars in the sense of 4.2. In almost all cases, after closer examination, colors of two particulars, alleged to be the same at first, can be seen to be different. The same holds for other properties. Thus, one may claim that 4.2 has a serious explanatory weakness. Resembling properties of (*) motivate a further revision of 4.2. Granting that resemblance of properties belongs to a different type than resemblance of particulars, for any two properties $P, Q \in \mathcal{P}$, let $P \Cap Q$ denote "P resembles Q". Thus (*) becomes:

$$a \mathrel{\cap\!\!\!\!\!+} b \Leftrightarrow \exists P, Q \in \mathcal{P}[P(a) \wedge Q(b) \wedge (P = Q \vee P \Cap Q)] \tag{4.4}$$

Introducing contextuality into we get

$$a \mathrel{\cap\!\!\!\!\!+}_{\mathcal{P}_0} b \Leftrightarrow \exists P, Q \in \mathcal{P}_0[P(a) \wedge P(b) \wedge (P = Q \vee P \Cap Q)] \tag{4.5}$$

It remains to investigate the relation of resemblance $\Cap$ between non-identical properties. We need the notion of *attribute*. Before going any further about attributes and their explication in information systems, I will first introduce set theoretic models and tolerance spaces that is a special type of set theoretic models. I will consider information systems with special emphasis on attribute systems in chapter 8.

Chapter 5

Set Theoretic First-Order Structures

In this chapter I will fix model theoretic terminology and notation with some basic results that I will use from this field. I assume that basic first-order logic including the notions of free and bound variables and substitution are known to the reader.

Definition 5.0.1. The **alphabet** of a first order language $\mathcal{L}$ consists of;

1. A set of relation symbols $\{R_i : i \in I\}$,
2. A set of function symbols $\{F_j : j \in J\}$,
3. A set of constant symbols $\{c_k : k \in K\}$.

Remark 5.0.2. Note that any of the sets of symbols above may be empty. Occasionally, I will assume that the set of functions is. Nothing is lost by this choice since our needs are theoretical rather than practical. Functions are just special relations and the functional notation contributes to brevity and flexibility in mathematical practice while purely relational languages and systems are easier to study meta-logically.

Definition 5.0.3. Let $\mathcal{L}$ be a first order language. An **$\mathcal{L}$-structure** $\mathfrak{M}$ consists in a non-empty set M, called the universe of $\mathfrak{M}$, with relations $R_i^{\mathfrak{M}} : i \in I$, $F_j^{\mathfrak{M}} : j \in J$ defined on M and some special elements, $c_k^{\mathfrak{M}} : k \in K$. We assume that I, J and K are mutually disjoint index sets and in order to compare structures and to define some relations between structures we may also define a function $\sigma : I \cup J \cup K \longrightarrow \mathbf{N}$ which assigns to each relation and function its arity ($\sigma \restriction K$

is constantly 0 since each individual can be regarded as a 0-ary function). Two structures with the same index set and the same arity function is called **similar**. If $\mathfrak{M}$ has no function, it is called a **relational structure**. Given a structure $\mathfrak{M}$ we may assign a first order language $\mathcal{L}(\mathfrak{M})$, called the language of $\mathfrak{M}$, to $\mathfrak{M}$ consisting of an n-ary relation symbol for each n-ary relation of $\mathfrak{M}$, an m-ary function symbol for each m-ary function symbol of $\mathfrak{M}$ and a constant symbol for each element of M. If $\mathcal{L}(\mathfrak{M})$ has only finitely many symbols $R_1, \ldots, R_n; F_1, \ldots, F_m; c_1, \ldots, c_k$ then we may write $\langle M, {R_1}^{\mathfrak{M}}, \ldots, {R_n}^{\mathfrak{M}}; {F_1}^{\mathfrak{M}}, \ldots, {F_m}^{\mathfrak{M}}; {c_1}^{\mathfrak{M}}, \ldots, {c_k}^{\mathfrak{M}}\rangle$ for $\mathfrak{M}$.

Remark 5.0.4. Following the usual practice, I will consistently use $A, B, C, \ldots, M, N$ and so on, to denote universes of structures denoted by $\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \ldots, \mathfrak{M}, \mathfrak{N}$ and so on.

Definition 5.0.5. Assume that $\mathcal{L} \subseteq \mathcal{L}'$ be two first order languages. $\mathcal{L}'$ is called an *expansion of* $\mathcal{L}$ and $\mathcal{L}$ is called a *reduction* of $\mathcal{L}'$. An $\mathcal{L}$-structure $\mathfrak{A}$ can be expanded to *an* $\mathcal{L}'$-structure $\mathfrak{A}'$, called an $\mathcal{L}$-*expansion of* $\mathfrak{A}$ *to* $\mathcal{L}'$, by interpreting the additional symbols in $\mathcal{L}' \backslash \mathcal{L}$ on A. That is, we assign relations, functions and special elements for relation symbols, function symbols and constant symbols in $\mathcal{L}' \backslash \mathcal{L}$. Conversely, an $\mathcal{L}'$ structure $\mathfrak{A}'$ induces an $\mathcal{L}$-structure $\mathfrak{A}$, called *the* $\mathcal{L}$-*reduct of* $\mathfrak{A}'$ by forgetting the interpretations of the symbols in $\mathcal{L}' \backslash \mathcal{L}$. We write $\mathfrak{A}' \upharpoonright \mathcal{L}'$ for the $\mathcal{L}'$-reduct of an $\mathcal{L}$-structure $\mathfrak{A}$ where $\mathcal{L}' \subseteq \mathcal{L}$.

Definition 5.0.6. Let $\mathfrak{M}$ and $\mathfrak{N}$ be two $\mathcal{L}$ structures. A function from M into N is called a **homomorphism** if

1. For every relation symbol R_i of $\mathcal{L}$ and every $(a_1, a_2, \ldots, a_{\sigma(i)})$,

$$R^{\mathfrak{A}}(a_1, a_2, \ldots, a_{\sigma(i)}) \Leftrightarrow R^{\mathfrak{B}}(f(a_1), f(a_2), \ldots, f(a_{\sigma(i)}))$$

2. For every function symbol F_j of $\mathcal{L}$ and every $(a_1, a_2, \ldots, a_{\sigma(j)})$,

$$F^{\mathfrak{B}}(f(a_1), f(a_2), \ldots, f(a_{\sigma(j)})) = fF^{\mathfrak{A}}(a_1, a_2, \ldots, a_{\sigma(j)})$$

3. For every constant symbol c_k of $\mathcal{L}$,

$${c_k}^{\mathfrak{B}} = f({c_k}^{\mathfrak{A}})$$

A homomorphism f is an **isomorphism** if f is 1-1 and onto. $\mathfrak{A}$ and $\mathfrak{B}$ are **isomorphic**, $\mathfrak{A} \cong \mathfrak{B}$, if there is an isomorphism $f : A \longrightarrow B$

Definition 5.0.7. Let $\mathfrak{A}$ and $\mathfrak{B}$ be two $\mathcal{L}$ structures, we say that, $\mathfrak{A}$, is a **substructure** of $\mathfrak{B}$ (or $\mathfrak{B}$, is a **supstructure** of $\mathfrak{A}$), $A \subseteq B$, if $A \subseteq B$ and the inclusion map from A to B is a homomorphism from $\mathfrak{A}$ to $\mathfrak{B}$. Equivalently, $A \subseteq B$ if R is an n-ary relation symbol, $R^{\mathfrak{A}} = R^{\mathfrak{B}} \restriction A$, if c is a constant symbol, $c^{\mathfrak{A}} = c^{\mathfrak{B}}$.

Legitimate $\mathcal{L}$-expressions fall into two types: terms and formulas. While forming them, we may use, in addition to the alphabet of $\mathcal{L}$, a common logical vocabulary consisting of variables $v_1, v_2, \ldots$, sentential connectives, quantifiers, and parentheses. Terms are intended to denote definite or indefinite elements in the universes of $\mathcal{L}$-structures. Formulas are intended to talk about either properties of definite or indefinite elements in a structure or about the structure itself.

Definition 5.0.8 ***(Term).*** Given a first order language $\mathcal{L}$, an $\mathcal{L}$-term is either a variable v_n or a special element c_k.

Definition 5.0.9 ***(Formula).*** Given a first order language $\mathcal{L}$, $\mathcal{L}$-formulas are defined as follows:

1. $t_1 \equiv t_2$, where each t_i is a term, is a formula,
2. $R(t_1, t_2, \ldots, t_n)$, where R is an n-ary relation symbol and each t_i is a term, is a formula,
3. $\neg\varphi$, $(\varphi * \psi)$, where φ, ψ are formulas, $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$ is a formula,
4. $\exists v_n \varphi$, $\forall v_n \varphi$, where φ is a formula, are again formulas.

The formulas in groups (i) and (ii) are called *atomic* formulas, others are called *complex* formulas.

Definition 5.0.10. We use upper case Greek letters $\Delta, \Gamma, \Sigma \ldots$ for sets of formulas. If Δ is a set of formulas, let $\neg\Delta = \{\neg\varphi : \varphi \in \Delta\}$.

Now that we know legitimate $\mathcal{L}$-expressions and $\mathcal{L}$-structures, it is time to relate these two. Terms will correspond to elements in universes and formulas will talk about elements and structures.

Definition 5.0.11. Given a structure $\mathfrak{A}$, a valuation v in $\mathfrak{A}$ is defined to be a function from the set of variables into the universe A of $\mathfrak{A}$.

Definition 5.0.12. Let $\mathcal{L}$ be first order language and $\mathfrak{A}$ be an $\mathcal{L}$-structure. The value of an $\mathcal{L}$-term t under the valuation v, $t[v]$, is defined as follows:

1. $v_n[v] = v(v_n)$
2. $c[v] = c^{\mathfrak{A}}$

Now, given a first order language $\mathcal{L}$ and an $\mathcal{L}$-structure $\mathfrak{M}$, we may define the satisfaction relation between $\mathfrak{M}$ and $\mathcal{L}$-formulas.

Definition 5.0.13. Let $\mathcal{L}$ be a language and $\mathfrak{M}$ be an $\mathcal{L}$-structure. Then, we define the relation "*φ is satisfied by the valuation v*, in symbols $\mathfrak{A} \models \varphi[v]$, by induction on the complexity of φ, as follows:

1. If $\varphi = t_1 \equiv t_2$, then $\mathfrak{M} \models \varphi$ if ${t_1}^{\mathfrak{M}}[v] = {t_2}^{\mathfrak{M}}[v]$
2. If $\varphi(\overline{x}) = R(t_1, t_2, \dots, t_{m-1})$ then
   $\mathfrak{M} \models \varphi(\overline{x})$ if $({t_1}^{\mathfrak{M}}[v], {t_2}^{\mathfrak{M}}[v] \dots, {t_{m-1}}^{\mathfrak{M}}[v]) \in R^{\mathfrak{M}}$
3. If $\varphi = \neg\phi$ then
   $\mathfrak{A} \models \varphi[v]$ if it is not the case that $\mathfrak{A} \models \phi[v]$,
4. If $\varphi = (\phi \rightarrow \chi)$ then
   $\mathfrak{A} \models \varphi[v]$ if either it is not the case that $\mathfrak{A} \models \phi[v]$ or $\mathfrak{A} \models \chi[v]$,
5. If $\varphi = (\forall x_i)\phi$ then
   $\mathfrak{A} \models \varphi[v]$ if for every v' which is exactly like v except possibly at i-th place $\mathfrak{A} \models \varphi[v']$.

In fact, only the values $v(v_i)$ where v_i is a free variable of φ really matter in determining whether $\mathfrak{A} \models \varphi[v]$ or not. For the following proposition can be proved by induction on the complexity of φ:

Proposition 5.1. *Assume that $\varphi(x_1, x_2, \dots, x_n)$ be a formula such that $x_1, x_2, \dots, x_n$ are all and only free variables of φ then for every two valuations v and v' such that $v(x_i) = v'(x_i)$, $i \in \{1, \dots, n\}$,*

$$\mathfrak{A} \models \varphi[v] \Leftrightarrow \mathfrak{A} \models \varphi[v']$$

Thus, we may use the notation $\mathfrak{A} \models \varphi[a_1, \ldots, a_n]$ for $\varphi = \varphi(x_1, x_2, \ldots, x_n)$ and $v(x_i) = a_i$.

Chapter 6

Identity-like Relations in Set-Theoretic Models

The set theoretical structures outlined in the previous chapter provide a strong formal framework for the practice of mathematics and theoretical computer science. To examine formal properties of identity-like relations a brief study of these relations in set theoretical first order structures would be useful, because these structures provide the simplest semantics for first order languages. I believe that this semantics is quite limited for the purposes of formal explication of philosophical ideas. However, with the awareness of these limitations, the insights gained when dealing with first order structures proves to be invaluable in our study of alternative semantical frameworks.

[1]

As I said before, the universals theory of resemblance explains resemblances in terms of properties and relations. The notion of property must be interpreted as formal property. Appeal to formal properties contributes to the parsimony necessary to avoid paradoxes. We will thus assume that every property (or relation) can be represented in a suitable first order language as a formula with the required number of free variables.

Formation of symbolic counterparts of properties and relations is necessary but not sufficient for the purpose of formalizing resemblance: Recall that in our informal discussion of identity-like relations in universals theory, we needed to talk about

[1] Some ideas and results in this and the previous chapter are required for chapter 9.

properties and relations as if they are entities. Not surprisingly, a formal exposition of identity-like relations corresponding to UTR requires talking about formal properties and relations and sometimes even about classes of formal properties and relations. Technically this means that properties and relations and sets of properties and relations, and so on, should be able to occur as bound variables in formulas. For every first order language $\mathcal{L}$, resemblances in $\mathcal{L}$-structures are defined in the set-theoretic meta-meta-language of $\mathcal{L}$ where we can talk and quantify over $\mathcal{L}$ formulas and sets of $\mathcal{L}$-formulas.

For simplicity, I will confine myself to resemblances between objects. Resemblances between tuples of elements poses no specific problem.

Definition 6.0.14 ***(Indiscernibility).*** Let $\mathcal{L}$ be a first order language. Then for any two objects a, b we have: a and b are indiscernible, $a \cong b$ in symbols, if a and b satisfy the same $\mathcal{L}$-formulas. That is

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} = \{\varphi(x) : \mathfrak{M} \vDash \varphi[b]\}$$

or equivalently, for every $\varphi(x)$, $\mathfrak{M} \vDash \varphi[a] \Leftrightarrow \mathfrak{M} \vDash \varphi[b]$.

Obviously, indiscernibility is an equivalence relation. This follows from the properties of set identity: for any $a \in M$,

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} = \{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\}.$$

Thus indiscernibility is reflexive. It is symmetric since if

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} = \{\varphi(x) : \mathfrak{M} \vDash \varphi[b]\}$$

then

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[b]\} = \{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\}.$$

It is transitive since if

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} = \{\varphi(x) : \mathfrak{M} \vDash \varphi[b]\}$$

and

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[b]\} = \{\varphi(x) : \mathfrak{M} \vDash \varphi[c]\}$$

then, by the transitivity of identity

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} = \{\varphi(x) : \mathfrak{M} \vDash \varphi[c]\}.$$

Definition 6.0.15 ***(Interchangeability).*** Let $\mathcal{L}$ be a first order language and $\Delta(x)$ be a set of $\mathcal{L}$-formulas in which the variable x occurs as the only free variable. Then for any two objects a, b we have: a and b are Δ-*interchangeable*, or interchangeable for Δ, $a \leftrightharpoons_\Delta b$ in symbols, if a and b satisfy the same Δ-formulas. That is

$$\{\varphi(x) \in \Delta : \mathfrak{M} \vDash \varphi[a]\} = \{\varphi(x) \in \Delta : \mathfrak{M} \vDash \varphi[a]\}$$

or equivalently, for every $\varphi(x) \in \Delta$, $\mathfrak{M} \vDash \varphi[a] \Leftrightarrow \mathfrak{M} \vDash \varphi[b]$.

Definition 6.0.16 ***(Strong Interchangeability).*** Let $\mathcal{L}$ be a first order language and $\Delta(x)$ be a set of $\mathcal{L}$-formulas in which the variable x occurs as the only free variable. Then for any two objects a, b we have: a and b are *strongly* Δ-*interchangeable*, or strongly interchangeable for Δ, $a \leftrightharpoons_\Delta^{str} b$ in symbols, if both a and b satisfy all Δ-formulas. That is

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} \cap \{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} \supseteq \Delta$$

or equivalently,

$$\{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} \supseteq \Delta \ \& \ \{\varphi(x) : \mathfrak{M} \vDash \varphi[a]\} \supseteq \Delta$$

or, equivalently, for every $\varphi(x) \in \Delta$, $\mathfrak{M} \vDash \varphi[a]$ & $\mathfrak{M} \vDash \varphi[b]$.

Notation 6.0.17. Given a language $\mathcal{L}$, an $\mathcal{L}$-structure $\mathfrak{M}$ and an object a in the universe M of $\mathfrak{M}$ we denote the set of $\mathcal{L}$-formulas in which at most one variable x occurs free and which are satisfied in M by a, or the type of a in the structure $\mathfrak{M}$, by $tp_{\mathfrak{M}}(a)$. Given a set of significant $\mathcal{L}$-formulas Δ in which at most the variable x occurs free, we denote the set of Δ-formulas satisfied in M by a by $tp_{\mathfrak{M}}^{\Delta}(a)$. That is,

$$tp_{\mathfrak{M}}^{\Delta}(a) = \{\delta \in \Delta : \mathfrak{M} \models \delta[a]\}$$

With this notation, obviously

1. $a \cong b$ if and only if $tp_{\mathfrak{M}}(a) = tp_{\mathfrak{M}}(b)$,

2. $a \leftrightharpoons_\Delta b$ if and only if $tp_{\mathfrak{M}}^{\Delta}(a) = tp_{\mathfrak{M}}^{\Delta}(b)$,

3. $a \leftrightharpoons_\Delta^{str} b$ if and only if $tp_{\mathfrak{M}}^{\Delta}(a) \supseteq \Delta$ and $tp_{\mathfrak{M}}^{\Delta}(b) \supseteq \Delta$.

One might modify the definition so as to allow parameters in formulas in Δ. Instead, I shall assume that the language of Δ have enough individual constant symbols thus allowing the name of any object to occur in formulas of Δ. The same remark applies to the subsequent development.

If $\Delta(x)$ is the set of formulas representing the properties significant under a given situation, then for any two objects a b we have:

Definition 6.0.18. a Δ-*resembles* b, or a resembles b (since we usually assume Δ to be known,) if a and b both satisfy a formula $\varphi(x) \in \Delta$. That is,

$$\{\varphi(x) \in \Delta : \mathfrak{M} \models \varphi[a]\} \cap \{\varphi(x) \in \Delta : \mathfrak{M} \models \varphi[b]\} \neq \emptyset$$

Obviously, this definition yields a relation of resemblance which is weakly reflexive and symmetric. Weak reflexivity follows from the fact that, if a resemble some object b with respect to Δ in $\mathfrak{M}$ then, $\{\varphi(x) \in \Delta : \mathfrak{M} \models \varphi[a]\} \cap \{\varphi(x) \in \Delta : \mathfrak{M} \models \varphi[b]\} \neq \emptyset$. Then $\{\varphi(x) \in \Delta : \mathfrak{M} \models \varphi[a]\} \neq \emptyset$ and $\{\varphi(x) \in \Delta : \mathfrak{M} \models \varphi[a]\} \cap \{\varphi(x) \in \Delta : \mathfrak{M} \models \varphi[a]\} \neq \emptyset$, that is, a resembles a with respect to Δ in $\mathfrak{M}$. Symmetry is also obvious: for every pair of objects a and b in the universe and any set of significant properties Δ, either both a and b satisfy a $\delta \in \Delta$ or not. However, Δ-resemblance is usually not transitive. For it is quite possible that a and b satisfy or fail to satisfy a $\delta_1 \in \Delta$ and b and c satisfy or fail to satisfy a different $\delta_2 \in \Delta$ while a and c may not satisfy a common property.

6.1 Preservation of Identity-Like Relations

It is natural to ask whether elements which are identity-like related in some $\mathcal{L}$-structure still resemble in a related structure. There are many model theoretic relations and operations. I mentioned just substructures (and sup-structures) and expansions (and reductions). I will now give some simple model theoretic preservativity results concerning identity-like relations. A general form of these results can be given as follows:

Definition 6.1.1. Let $\mathcal{K}$ be a class of structures. Assume that $\star$ is a binary relation on the class $\mathcal{K}$. An n-ary relation R of elements in the universes of $\mathcal{L}$-structures is preserved under $\star$ in $\mathcal{K}$ if whenever $\mathfrak{A} \models R[a_1, a_2, \ldots, a_m]$ for some $\mathfrak{A} \in \mathcal{K}$ then $\mathfrak{B} \models R[a_1, a_2, \ldots, a_m]$ whenever $\mathfrak{A} \star \mathfrak{B}$ and$(a_1, a_2, \ldots, a_n) \in B$.

Proposition 6.1. *Interchangeability is not preserved under supstructure relation on the class of $\mathcal{L}$-structures.*

Proof. Let $\mathcal{L}$ be $\{\leq;;\}$ where $\sigma(\leq) = 2$. Thus $\leq$ is a binary relation symbol and $\mathcal{L}$ has no function or constant symbols. Consider the $\mathcal{L}$-structure

$$\mathfrak{A} = \langle \{0, 1, 2\}, \{(0, 1), (0, 2), (1, 2)\} \rangle$$

. Let $\mathfrak{B} = \langle \{0, 1\}, \{(0, 1)\} \rangle$. Let $\Delta = \{\exists v_2(v_1 \leq v_2)\}$. Then, $0 \leftrightharpoons_\Delta 1$ in $\mathfrak{A}$ but $\neg(0 \leftrightharpoons_\Delta 1)$ in $\mathfrak{B}$ although $\mathfrak{A} \supseteq \mathfrak{B}$ □

In the case of Δ resemblance and the model theoretic relation of reduction, we have the following non-uniform preservativity:

Proposition 6.2. *Assume that $\mathcal{K}$ is a class of structures including an $\mathcal{L}$-structure $\mathfrak{A}$ and all $\mathcal{L}'$-reducts of $\mathfrak{A}$ for all $\mathcal{L}' \subseteq \mathcal{L}$. Let $\mathfrak{B} \star \mathfrak{C}$ if $\mathfrak{C}$ is a reduct of $\mathfrak{B}$ (note that $\mathfrak{B}$ need not be an $\mathcal{L}$ structure, it may itself be a $\mathcal{L}'$-reduct of $\mathfrak{A}$). Assume that $\mathfrak{B} \in \mathcal{K}$ is an $\mathcal{L}'$-structure and Δ is a set of $\mathcal{L}'$ formulas in which only one variable x occurs free. Assume that $\mathfrak{B} \star \mathfrak{C}$ such that $\mathfrak{C}$ is an $\mathcal{L}''$-structure where $\mathcal{L}''$ is rich enough so that Δ is a set of $\mathcal{L}''$ formulas. Then $\star$ preserves Δ resemblance. Note that all $\mathfrak{B} \in \mathcal{K}$ have the same universe A by the definition of reduction.*

The result follows from the following.

Proposition 6.3. *Let $\mathcal{L}' \subseteq \mathcal{L}$. Let $\mathfrak{A}' = \mathfrak{A} \restriction \mathcal{L}'$. Then for any $a \in A$ and any $\mathcal{L}'$ formula $\varphi(x)$ then for any valuation v such that $v(x) = a$*

$$\mathfrak{A}' \models \varphi[v] \Leftrightarrow \mathfrak{A} \models \varphi[v]$$

Proof. The proof is by induction on the complexity of φ. □

Thus, like the resemblance with respect to a property, resemblance with respect to a set of significant properties satisfies necessary formal properties of resemblance: reflexivity and symmetry. Non-transitivity is not a necessary condition for a relation to be a resemblance relation. However almost all complex resemblance relations will be non-transitive. Non-transitivity has the outcome that a set of objects with a relation of resemblance defined on it may not be partitioned into classes. This is not a defect for a definition of resemblance but a fact peculiar to resemblance.

Another immediate observation about relative resemblance is that if two objects a and b resemble each other relative to a set Δ and if $\Delta \subseteq \Sigma$ then a and b resemble each other the set Σ too. But if we choose a smaller set of significant properties $\Omega \subseteq \Delta$, then a and b may not resemble each other relative to this new set of properties Ω. I believe that these observations are quite consistent with our daily use of resemblance. In this sense, enlarging the set of significant properties *weaken* the resemblance, while choosing a smaller set of significant properties *strengthens* the resemblance.

The explication of resemblance incorporating definition 6.0.18 is consistent with more than one theories of universals. In general it is consistent with substance-attribute theory of universals and particulars as bundles of universals views. Moreover, the definition can not be accepted by resemblance nominalists even as an explanation of resemblance in terms of properties (Of course, here I have in mind a version of resemblance nominalism consistent with particular properties, or tropes view). For, according to the definition two entities resembling each other requires a repeatable property φ which is present in both and it is repeatable properties and relations which resemblance nominalists are trying to explain away.

Now it is time to focus on the set of significant properties and to see what properties must be satisfied Δ.

First, note that Δ need not be explicitly given. However, for any Δ, in order that the notion of Δ resemblance *works*, we must be able to tell for any formula δ in at most one free variable, whether δ is in Δ or not. To give a simple example, consider the notion of physical resemblance. In this case we may, for example, determine basic physical predicates and name these by letters $F, G, \ldots$ as usual, and then say that any formula $\delta(x)$ with at least one physical predicate symbol is a physical property. Obviously then, being a physical property will be a determinate property of properties.

Secondly, especially in the case of abstract resemblances, Δ may be determined by syntactical rather than semantical considerations. That is, we may be interested in syntantical properties of formulas in Δ, rather than intended interpretations of symbols occurring in them.

If the set of formulas Δ is finite, then we may define Δ-resemblance in any structure, finite or infinite. Suppose that we work in the actual world. Let Δ be a finite set of personality properties like generosity, selfishness..., and $\{p_1, p_2, \ldots, p_n\}$ be a set of persons and let $\delta(p_i)$ be the set of properties that the person p_i possess. Then, the graph of resemblance among these people would be the graph of the relation of overlapping (having non-empty intersection) among $\delta(p_i)$'s.

Firstly, there is no need to include both $\delta(x)$ and $\neg\delta(x)$ in Δ. This enables us to escape from positing negative universals. If one insists that negative universals should be included in the ontology and instantiating a negative universal is more than failing to instantiate the corresponding one, then he may modify definition 6.0.18 in the obvious manner.

Δ may include not only first-order formulas but also higher order ones. This is by virtue of the transitivity of instantiation relation. This being so, the metaphysical peculiarity of Δ may be determined by its first order formulas. If the first order formulas all include only physical properties, Δ may be considered as a set of significant physical properties.

Chapter 7

Formal Properties of Resemblance and Tolerance Spaces

In this chapter I will present tolerance relations and tolerance spaces as presented by Schreider and make some remarks regarding their insufficiency to represent resemblances.

It seems obvious that every object resembles itself since whatever property is significant under the situation, an object a shares these properties with itself. We would restate this "fact" as: resemblance relation is *reflexive*. It is also clear that "two objects either do or do not resemblance each other, independently of the order in which we consider them" [20], p. 83. Indeed, whatever properties we take to be significant under the situation, if a shares these properties with b, then b shares them with a. This fact can be restated as: if an object a resembles an object b, then b also resembles a which amounts to saying that resemblance as a binary relation is *symmetric*.

Definition 7.0.2. A *tolerance relation* on a set M is a binary relation τ ($\tau \subseteq M \times M$) which is reflexive and symmetric. A *tolerance space* is a pair $\langle M, \tau \rangle$ where M is a non-empty set and τ is a tolerance relation on M [20]. We may say that x and y are tolerant instead of saying that $(x, y) \in \tau$.

Example 7.0.3. An important class of tolerance spaces are defined in [11]. Let M be a non-empty set and $\delta : M \times M \longrightarrow [0, 1]$ be a function such that

$$\forall x \in M \ \delta(x, x) = 0 \text{ and } \forall x, y \in M \ \delta(x, y) = \delta(y, x)$$

Such a function is called a *semi-metric* on M. A pair $\langle M, \delta \rangle$ consisting of a non-empty set M together with a semi-metric on M is called a *quality dimension* over M. A conceptual space over M is a pair $\langle M, Q \rangle$ where Q is a finite set of quality dimensions over M. Thus a *conceptual space* over M can be denoted by $\langle M, \{\langle M, \delta_1 \rangle, \ldots, \langle M, \delta_n \rangle\} \rangle$. A *tolerance function* on M is a binary function τ on M such that

$$\forall x \in M \ \delta(x, x) = 1 \text{ and } \forall x, y \in M \ \delta(x, y) = \delta(y, x)$$

Tolerance functions give rise to *valued tolerance relations*: Given a tolerance function τ on M and a value $p \in [0, 1]$ define the relation

$$\overline{\tau} = \{(x, y) \in M \times M : \tau(x, y) \geq p\}$$

which is obviously a tolerance relation. In fact, $\tau(x, y)$ can be read as the degree of resemblance between x and y.

If $\langle M, \{\langle M, \delta_1 \rangle, \ldots, \langle M, \delta_n \rangle\} \rangle$ is a conceptual space over M, the tolerance function τ_i based on the quality dimension δ_i is defined as

$$\tau_i(x, y) = 1 - \frac{\delta_i(x, y)}{max\{\delta_i(x, y) : x, y \in M\}}$$

Therefore, each quality dimension $\langle M, \delta_i \rangle$ in the conceptual space provides a tolerance function τ_i and thus a tolerance function $\overline{\tau}_i$ on M.

Remark 7.0.4. Another characterization of tolerance spaces can be given in terms of correspondences. A *correspondence* φ between sets S and T is a subset of $S \times T$. Let M be a set of objects and N be the set of all possible features. Let $\varphi : M \longrightarrow N$ be a correspondence. For $x \in M$ let $\Phi(x) = \{y \in L : x\varphi y\}$. $\Phi(x)$ is called the image of x under the correspondence φ. Now define the binary relation A_φ on M such that $xA_\varphi y \Leftrightarrow \Phi(x) \cap \Phi(y) \neq \emptyset$. Interpreting $y \in \Phi(x)$ as "x has the feature y", $xA_\varphi y$ if and only x and y have a feature in common. A_φ is a tolerance relation on M if and only if φ is everywhere defined. Indeed, if x is an element of M with $\Phi(x) = \emptyset$ (i.e., φ is not defined at x) then $\Phi(x) \cap \Phi(y) = \emptyset \cap \Phi(y) = \emptyset$ for any $y \in M$.

In particular, $\Phi(x) \cap \Phi(x) = \emptyset$ thus $\neg(xA_\varphi x)$ and A_φ fails to be reflexive at these points. For proving the sufficiency, assume that φ is defined at a point $x \in M$. Then $\Phi(x) \cap \Phi(x) = \Phi(x) \neq \emptyset$ and $xA_\varphi x$. Thus A_φ satisfies reflexivity at points where φ is defined. It is easy to see that A_φ is symmetric: Since the set intersection is commutative, $\Phi(y) \cap \Phi(x) \neq \emptyset$ if $\Phi(x) \cap \Phi(y) \neq \emptyset$.

Remark 7.0.5. A_φ is a transitive relation if and only if φ is a function, i.e., it is an everywhere defined and it is one-one relation: To prove this claim, first observe that two singleton sets $\{a\}$ and $\{b\}$ have non-empty intersection if and only if they are identical. Now, let φ be a function. Let $xA_\varphi y$ and $yA_\varphi z$. Then $\Phi(x) \cap \Phi(y) \neq \emptyset$ and $\Phi(x) \cap \Phi(y) \neq \emptyset$. By the observation just made, we have, equivalently, $\varphi(x) = \varphi(y)$ and $\varphi(y) = \varphi(z)$. By the transitivity of identity $\varphi(x) = \varphi(z)$ and $\Phi(x) \cap \Phi(z) = \{\varphi(x)\}$. Thus, $xA_\varphi z$.

In terms of features, resemblance defined as tolerance would be an equivalence on a domain of objects if every object in the domain have exactly one feature.

Seeing that everywhere defined correspondences induce tolerance relations, it is natural to ask whether every tolerance relation on a set can be represented by a correspondence. We will now see that the answer is "yes". The aim is to show that "for any tolerance relation there is a canonical collection of features which can be constructed on the basis of the given tolerance relation independently of the specific way in which it is presented." [20], p. 95. First we need some preliminary notions:

Definition 7.0.6. Given a set S, a family $\mathcal{F}$ of subsets of S is a *covering* of S if $\bigcup \mathcal{F} \supseteq S$, or, equivalently, for every $s \in S$ there is an $F \in \mathcal{F}$ such that $s \in F$.

Since, by remark 7.0.4 above, only everywhere defined correspondences induce tolerance relations, in tolerance space approach to resemblance we specifically deal with coverings induced by everywhere defined correspondences. Let M be a set of objects, N be a set of features, and $\varphi : M \longrightarrow N$ be an everywhere defined correspondence. For each feature, let $M_\xi = \varphi^{-1}(\xi) = \{m \in M : \varphi(m) = \xi\}$. Then $\{M_\xi : \xi \in N\}$ is a covering of M. Indeed, if $m \in M$, there is $\xi \in N : \varphi(x) = \xi$, then $m \in M_\xi$. Letting $\{M_\xi : \xi \in N\}$ defined that way, we obtain another characterization of A_φ: $xA_\varphi y \Leftrightarrow \exists \xi \in N$ such that $\{x, y\} \subseteq M_\xi$.

Definition 7.0.7. Let $\langle M, \tau \rangle$ be a tolerance space. A subset $L \subseteq M$ is called a *preclass* if every x and y are tolerant. That is, $(x, y) \in \tau$ for every $x, y \in L$.

Lemma 7.0.8. Given a tolerance space $\langle M, \tau \rangle$, two elements x and y are tolerant if and only if there is a preclass $L \subseteq M$ such that $\{x, y\} \subseteq L$.

Proof. Any two objects in a preclass are tolerant, by definition of preclass. If two objects x and y are tolerant, then $\{x, y\}$ is a preclass. □

Definition 7.0.9. A subset $K \subseteq M$ in a tolerance space $\langle M, \tau \rangle$ is a tolerance class if K is a maximal preclass. That is, K is not properly contained in a preclass. Equivalently, for every $m \in M$, either $m \in K$ or there is a $k \in K$ such that $\neg(k\tau m)$.

Lemma 7.0.10. Every preclass in a tolerance space is contained in a tolerance class.

Proof. The finite case is found in [20] p. 97. The infinite case uses transfinite induction. Let $L \subseteq M$ be a preclass in a tolerance space $\langle M, \tau \rangle$. We first form an increasing sequence of preclasses $L_0 \subseteq L_1 \subseteq \ldots$ aiming to reach a tolerance class $L' \supseteq L$. Let $L_0 = L$. If L_n (e.g., L_0) is a tolerance class itself, then we let $L' = L_n$. If L_n is not a tolerance class, there is an $m_n \in M \backslash L_n$ such that, for all $l \in L_n$, $l\tau m$. Let $L_{n+1} = L_n \cup \{m_n\}$. Obviously L_{n+1} is a preclass since we added an object which is tolerant to every element of L_n. Assume that L_n are constructed for all $n \in \omega$.

We may easily see that $L_\omega = \bigcup_{n \in \omega} L_n$ is a preclass. For let $l, l' \in L_\omega$. Then there are $n, n' \in N$ such that $l \in L_n$ and $l' \in N'_n$. Either $n \leq n'$ or $n' \leq n$. Without less of generality, let us assume $n \leq n'$. Then $L_n \subseteq L'_n$, $\{l, l'\} \subseteq L'_n$ and, since L'_n is a preclass, $l\tau l'$.

Next we must see that this process will end up with a tolerance class. For we can iterate this process at most $M \backslash L$ times: at each step we use one more element from this set. Thus, we stop either because $L_\alpha = M$ (i.e., all the remaining elements can be added to L one by one constricting a larger preclass and thus M itself is the smallest tolerance class obtained this way) or because there is no $m \in M \backslash L_\alpha$ which is tolerant to all elements in L_α. Either way we obtain a tolerance class. □

For many reasons, we may find an object a in resemblance with another one b. A striking fact to be observed about resemblance claims is that our reason for the resemblance of objects a and b may well be different from our reason for thinking that, the object b resembles an object c other than a. For example, we may think that John resembles his father Jack due to both being generous and friendly. For us Jack may also resemble his brother Frank due to the facts that both have blue eyes and both are tall. However, we may reasonably believe that, although he too is a son of Jack, Frank does not resemble his father John since they share no common physical or personal characteristics. Since it is quite unnatural to put a restriction on our resemblance claims to the effect that we may have just one overall specific criteria of resemblance (like physical features), we must live with the fact that resemblance is *not* a transitive relation. That is, an object b may resemble an object a and an object c may resemble b while c does not resemble a.

In conjunction with the symmetry of resemblance we may see that objects resembling a common object, may not resemble each other.

Assume for a moment that resemblance is reflexive and symmetric but not necessarily transitive. Even then, these observations about resemblance relation gives us only the necessary conditions of resemblance. This means that a proposed philosophical analysis of resemblance would be rejected if this analysis leads to an explication of resemblance relation which is not reflexive or not symmetric. As the explication of the notion of resemblance Schreider [20] introduce tolerance relations which are reflexive and symmetric binary relations.

One may say that, the notion of tolerance is not appropriate as the explication of resemblance since it is too loose. Given a set of people, we may define many tolerance spaces on this set of objects which do not represent any familiar resemblance relation among them.

Schreider claims: "The superlative degree of resemblance is indistinguishability, and not, as might appear at first sight, identity. The latter is a qualitatively different property" ([20], p. 82). He supports this claim on the basis of the following observation: If we take a set of points on the plane lying on a line with the distance $d(x, y)$ between any two neighboring points x and y is less than d, where d is below the threshold of visual acuity v. Then, with a suitable number of points $p_1, p_2, \ldots, p_n$ (in this order), each pair of points (p_i, p_j) except the pair (p_1, p_n) would be indistinguishable while the pair (p_1, p_n) is distinguishable due to the fact that

$d(p_1, p_n) > v$.

Although the example above is clear, the notion of indistinguishability needs clarification. Indeed, we can not be satisfied with visual indistinguishability as the sole example. Like identity and resemblance, indistinguishability needs an explication too. As I said before, I would count indistinguishability among epistemological concepts. It is explained in terms of perceptions and the information given by our perceptual apparatus do not always correspond to the reality.

We should notice the following: One should clarify the sense in which distinguishability or any other candidate relation is *stronger* than resemblance. I think that, Schreider is not quite consistent in considering identity as a “qualitatively different” property, since on the page immediately following he states that “identity is a special case of resemblance”. In many accounts of resemblance (see e.g., [1], p. 106), “as resemblance gets closer, more and more constituents of the resembling properties are identical and we have identity rather than resemblance.”

Chapter 8

Identity-like Relations in Information Systems

In the usual semantics, properties and relations are identified with their extensions in the first-order structures and the property instantiation is represented with set membership. However, "two" properties may have the same extension. In property systems, properties and property instantiation are taken as primitives.

8.1 Property and Relation Systems

Property systems are defined in [22] as follows:

Definition 8.1.1. A *property system* is a triple

$$\mathbf{S} = (Ob_S, Pr_S, f_S)$$

where

1. $Ob_S \neq \emptyset$ is the set of *objects*,
2. Pr_S is the (possibly empty) set of *properties*,
3. $f_S : Ob \longrightarrow \mathcal{P}(Pr)$ is the *information function*, for each object x, $f_S(x)$ is called the *information about* x in $\mathbf{S}$.

Thus, if x is an object then $f_S(x)$ determines which properties *from* Pr_S are instantiated by x. If the possibility that $f_S(x) = \emptyset$ obtains, this does not mean that

x has absolutely no properties, just that it has no properties of the system. For applications, it is vital to restrict the properties and objects to manageable sizes.

For notational convenience, the subscript S may be dropped when the system we work in is understood. We write $\overline{f}(x)$ for $Pr \backslash f(x)$. If $Pr_S = \emptyset$, then S is called *trivial*, otherwise it is *non-trivial*. S is *total* if for every $x \in OB_S$, $f_S(x) \neq \emptyset$.

The assertion that "the object x has the property A in the system S" corresponds to the set-theoretical statement $A \in f_S(x)$. It is important to remark here that properties are not to be identified with sets. Since, note that, the definition of property systems does not forbid there being "two" properties A and B in Pr_S such that for every $x \in Ob_S$, $A \in f_S(x)$ if and only if $B \in f_S(x)$. While, if we identify a property P with the set $P' = \{x \in Ob_S : P \in f_S(x)\}$ then obviously $A' = B'$ by the extensionality criterion of set identity, thus the identification together with the transitivity of identity we obtain $A = B$.

Definition 8.1.2. Let W be a non-empty set and let $V \subseteq W$ be any family of subsets of W. The *set-theoretical P system over the pair* (W, V) is the P-system $\mathbf{S}$ with $Ob_{\mathbf{S}} = W$, $Pr_{\mathbf{S}}V$ and for any $x \in W$, $f_{\mathbf{S}}(x) = \{A \in V : x \in A\}$

The set theoretic model corresponding to a given property system $\mathbf{S}$ can be constructed as follows:

Definition 8.1.3. Let $\mathbf{S} = (Ob_{\mathbf{S}}, Pr_{\mathbf{S}}, f_{\mathbf{S}})$ be a property system. Let $W = Ob_{\mathbf{S}}$. For any $A \in Pr_{\mathbf{S}}$, let $|A| = \{x \in Ob_{\mathbf{S}} : A \in f_{\mathbf{S}}(x)\}$ and let $V = \{|A| : A \in Pr_{\mathbf{S}}\}$. Then the set-theoretical P-system over the pair (W, V), $P(W, V)$, is called the *set-theoretical P-system associated with S.* we write $|S|$ for $P(W, V)$.

Remark 8.1.4. Note that the set theoretic P system $|\mathbf{S}|$corresponding to the property system $\mathbf{S}$ does not represent $\mathbf{S}$ truthfully. For, if A and B are two properties with the same extension, then in the set theoretic system these two properties will be counted as one. One may code the set theoretic system so that the same set appears two or more time perhaps with different indices.

Property systems may easily be extended to allow relations. Note that this extension is not only a formal possibility but it also has an ontological significance. Most of the natural binary relations binary relations cannot be defined on the basis of non-relational properties of objects. Similarly, many ternary relations of objects cannot be reduced to combinations of binary relations and properties. In some cases, reductions are possible: the relation of "betweenness" which may hold or does not hold for any triples of points on a line can easily be expressed by means of the relations "being on the left hand side of" and "being on the right hand side of". However, it is at least not clear, if not impossible, whether all natural ternary relations can be reduced to binary relations.

Definition 8.1.5. A *relational information system*, or relational system for short, is a structure

$$\mathbf{R} = \langle Ob; Rel^{(1)}, Rel^{(2)}, \ldots; f_1, f_2, \ldots \rangle$$

where

1. $Ob \neq \emptyset$ is the set of *objects* of $\mathbf{R}$,
2. $Rel = \{Rel^{(1)}, Rel^{(2)}, \ldots\}$ is the set of *relations* of $\mathbf{R}$ so that for $n \geq 1$, $R^{(n)} = \{R_1^{(n)}, R_2^{(n)}, \ldots\}$ is a set of relations called n-ary relations of $\mathbf{R}$
3. For each $i \geq 1$, f_i is the i-th *information function* so that $f_i : Ob^i \longrightarrow \mathcal{P}(Rel^{(i)})$

The subscript (i) over a relation R denotes the arity of that relation and Ob denotes the set of all i-tuples of objects. For any $\overline{x} \in Ob^i$, $f_i(\overline{x})$ tells which relations exist among the objects $x_1, x_2, \ldots, x_n$, in that order.

Remark 8.1.6. It should be clear that any relation system includes a property system and given a relation system $\mathbf{R} = \langle Ob; Rel^{(1)}, Rel^{(2)}, \ldots; f_1, f_2, \ldots \rangle$, the system $\langle Ob_R; R^(1); f_1 \rangle$ is the property system included in $\mathbf{R}$.

If one accepts the genuine possibility of *multi-grade* relations, then some relations may not have a fixed arity. Consider the relation "robbing a bank together". Any number of people may rob a bank together. Furthermore, it seems that, that relation cannot be reduced to relations of a fixed arity. For multi-grade relation symbols, we could drop the use of superscripts and allow tuples of objects of different lengths to be sent onto them by the information function f.

As in the case of property systems, some systems of sets give rise to relational information systems.

Definition 8.1.7. Let $W \neq \emptyset$ and $V = \{V^{(1)}, V^{(2)}, \ldots\}$ be a family of possibly empty subsets $V^{(n)} \subseteq (\mathcal{P}(W^n))$. Then the set theoretical relational system S over the pair $\langle W, V\rangle$ is the relational system with $Ob_S = W$, $R^{(n)} = V^{(n)}$ and for each $x \in W$ $f_{(n)}(x) = \{V_m^{(n)} \in V^{(n)} : (x_1, x_2, \ldots, x_n) \in V_m^{(n)}\}$

8.2 Identity-like Relations in Property and Relation Systems

Now we are ready to define identity-like relations in property and relation systems:

Definition 8.2.1. Let $\mathbf{S} = \langle Ob; Pr; f\rangle$ be a property system and x, y be two objects. Then

1. x and y are *indiscernible*, $x \cong y$, if $f(x) = f(y)$
2. x and y are *interchangeable with respect to* $Pr_0 \subseteq Pr$, $x \leftrightharpoons_{Pr_0} y$, if $f(x) \cap Pr_0 = f(y) \cap Pr_0$
3. x and y are *positively similar*, $x \Sigma_S y$, if $f(x) \cap f(y) \neq \emptyset$,
4. x and y are *positively similar with respect to* $Pr_0 \subseteq Pr$, $x \Sigma_{Pr_0} y$, if $f(x) \cap f(y) \cap Pr_0 \neq \emptyset$,
5. x and y are *negatively similar*, $x N_S y$, if $\overline{f}(x) \cap \overline{f}(y) \neq \emptyset$
6. x and y are *negatively similar with respect to* Pr_0, $x N_{Pr_0} y$, if $\overline{f}(x) \cap \overline{f}(y) \cap (Pr \backslash Pr_0) \neq \emptyset$

Thus negative resemblance with respect to a class of properties is positive resemblance with respect to the complement of this class of properties: $x N_{Pr_0} y \Leftrightarrow x \Sigma_{Pr \backslash Pr_0} y$.

Generalization to the case of relation systems does not present any difficulty:

Definition 8.2.2. Let $\mathbf{R} = \langle Ob; Rel; f_1, f_2, \ldots\rangle$ be a relational information system and $\overline{x}$, $\overline{y}$ be two n-tuples of objects where n is any natural number. Then

1. $\overline{x}$ and $\overline{y}$ are indiscernible $\overline{x} \cong \overline{y}$ if $f_n(\overline{x}) = f_n(\overline{y})$,

2. $\overline{x}$ and $\overline{y}$ are *interchangeable with respect to* $Rel_0^{(n)} \subseteq Rel^{(n)}$, $\overline{x} \leftrightharpoons_{Rel_0} \overline{y}$, if $f_n(\overline{x}) \cap Rel_0 = f_n(\overline{y}) \cap Rel_0$

3. $\overline{x}$ and $\overline{y}$ *positively resemble*, $\overline{x}\Sigma\overline{y}$, if $f_n(\overline{x}) \cap f_n(\overline{y}) \neq \emptyset$

4. $\overline{x}$ and $\overline{y}$ *positively resemble with respect to* $Rel_0^{(n)} \subseteq Rel^{(n)}$, $\overline{x} \leftrightharpoons_{Rel_0} \overline{y}$, if $f_n(\overline{x}) \cap f_n(\overline{y}) \cap Rel_0 \neq \emptyset$

5. $\overline{x}$ and $\overline{y}$ *negatively resemble*, $\overline{x}N\overline{y}$, if $\overline{f_n}(\overline{x}) \cap \overline{f_n}(\overline{y}) \neq \emptyset$

6. $\overline{x}$ and $\overline{y}$ *negatively resemble with respect to* $Rel_0^{(n)} \subseteq Rel^{(n)}$, $\overline{x}N_{Rel_0}\overline{y}$, if $\overline{f_n}(\overline{x}) \cap \overline{f_n}(\overline{y}) \cap (Rel \backslash Rel_0) \neq \emptyset$

Considering again property systems, note that we did not put any restriction on classes of significant properties; they may be any subclasses of properties. This may cause a sense of arbitrariness. The notion of attribute systems enable us to eliminate this problem.

8.3 Attribute Systems

Definition 8.3.1. An attribute system is a system

$$\mathbf{S} = \langle Ob, At, Val, f \rangle$$

where

1. $Ob \neq \emptyset$ is called the set of objects,

2. At is a set whose elements are called the attributes of the system

3. For each $a \in At$, $Val(a)$ is a set called the values of the attribute a

4. For any f is a binary function such that $x \in Ob$, and any attribute $a \in At$, $f(x, a) \subseteq Val(a)$ called the *information function.*

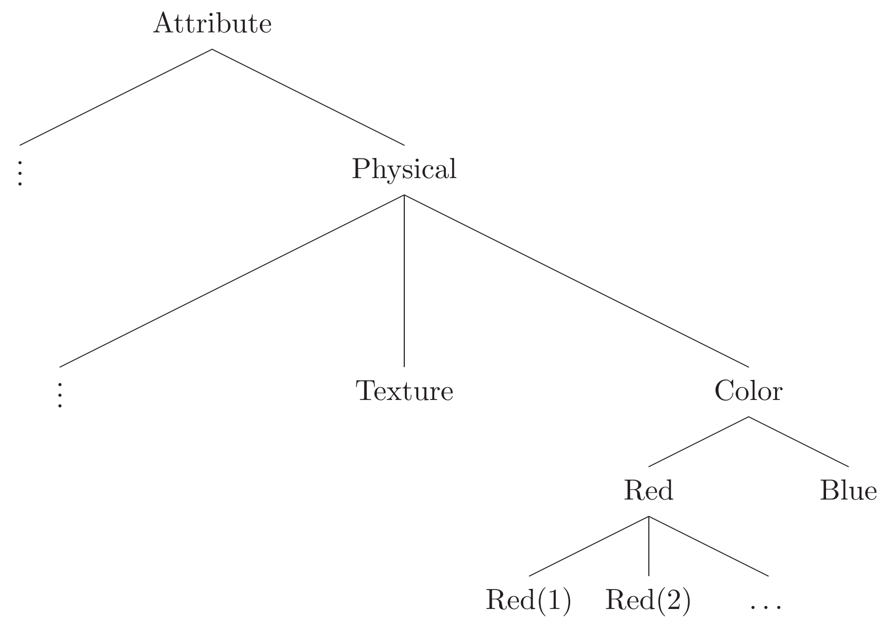
Definition 8.3.2. An attribute system **S** such that $|f(x, A)| \leq 1$ for every object x and attribute is called a *single valued* information system. If $f(x; A) \neq \emptyset$ for every $x \in Ob_S$ and every $A \in At$, **S** is called *normal*

Remark 8.3.3. As in the case of property systems attributes should not be identified with sets of properties. The same set of properties may form the set of values for "two" different attributes.

Attributes can be thought to be properties of properties. For example "color" is an attribute and specific colors fall under the attribute "color", or, using the terminology of information systems, red is a value of color; red $\in val(color)$. As I remarked above, it is possible that two properties may exist with the same individuals satisfying them and property systems allows us to represent this possibility. However, in property systems, we can not explain the reason why these two properties are two instead of one. Since, no information besides information about objects can be coded in property systems. Attribute systems allow us to explain property differences but to some degree.

The distinction between properties and attributes in attribute systems is justified by the fact that only attributes have properties as values. Attributes may be thought to be forming parts of the information about objects only indirectly since, note that, the information function in fact relates an object with a set of properties and as I remarked above, the same set of properties may form the set of values of two different attributes. This problem provides one of the main motivations for introducing higher order attribute systems. I will do this shortly.

There is a hierarchy on the class of attributes. For example, particular shades of red lie in the lowest level, red lies in the second level and color is in the third one, and so on. In the highest level we have the attribute "Attribute". A possible description of the hierarchy can be given by the following diagram:



Introducing higher order attributes to information systems is not only convenient but ontologically necessary. The principal use of first order attributes is that they enable us to differentiate two extensionally equivalent properties. The necessary equivalence of extensions may suggest that these two properties are in fact one. However, one may easily eliminate the possibility of this identification if he can find a feature of one of these properties such that the other one fails to have. It is hard to find good examples for that. However, the idea of such a possibility should be clear.

Concerning the representation of resemblance relations, the main role of higher order attributes is that by means of them resemblances of properties can be explained. In that respect, higher order attributes will function as determinables while their values will be the determinates.

If what I said before is indeed the case, we still have the possibility that the attribute immediately above the property may not give us the reason to differentiate one property from another and we may need to refer to a higher level attribute to do the work. In addition to the need for an explanation for non-identity of properties with the same set of extensions, higher order attributes are needed to explain non-identity of attributes with the same set of values too. First order attribute systems leave room to “represent” the contention that there may be “two” attributes with the same set of values. Thus an attribute is not identical with its set of values. However, in first order attribute systems there is no information as to what makes these two attributes non-identical. The representation is reduced to a mere notational convenience. Higher order attributes allows the “explanation” about non-identical extensionally equivalent properties and attributes be given.

From the above it should be clear that we need the higher order attributes to take full advantage of attribute systems over property systems. One should not think that the introduction of higher order information systems is a philosopher’s trick. The hierarchy of attributes is a picture inherent in our conceptualization. Consider a particular automobile. It is a car, it has the property of “being a car”. This property falls under the attribute “being a vehicle” and “being a vehicle” falls under the attribute “being an artifact” and this in turn under the attribute “being a material being” and we may end all this by saying that “being a material being” falls under the highest level attribute “being an attribute” or, simply, “Attribute”. Of course, this chain of attributes depends on the particular taxonomy of entities that one may not wish to commit himself to and he does not have to. For, the focus of the present study is not the categorization itself and we may just work modulo a categorization.

We must allow *relational attributes* or relation attributes into the system too. To give an example, both “lying under” and “being to the left of” are binary relations and they could fall under the attribute “spatial” or “being a spatial relation”. In fact, allowing relations of different arities to fall under an attribute is a natural choice: both “betweenness” and “being to the left of” are spatial relations and thus the attribute “spatial” should include them both.

It may seem plausible that every property or relation falls under an attribute. However, in order to make the presentation more complete, the following generalization will not depend upon this assumption. I will immediately characterize systems based upon this upon this assumption and thereafter I will work within systems in which every property and relation fall under an attribute. The reason for this choice is both philosophical and technical.

Definition 8.3.4. A generalized attribute system is a system

$$\mathbf{S} = \langle Ob, Rel, At, Val, f \rangle$$

where

1. $Ob \neq \emptyset$ is the set of objects,
2. $Rel = \{Rel^{(1)}, Rel^{(2)}, \ldots\}$ each set $Rel^{(n)}$ is called the set of n-ary relations of $\mathbf{S}$
3. $At = \{At^{(n)}_{(m)} : n, m \in \mathbb{N}^+\}$ is the set of attributes where each $At^{(n)}_{(m)}$ is the set of n-ary attributes of level m,
4. $Val : At \longrightarrow At \cup Rel$ is the valuation function so that

   $Val(a) \subseteq At^{(n)}_{(m-1)}$ if $a \in At^{(n)}_{(m)}$ where $n \geq 2$

   $Val(a) \subseteq Rel^{(n)}$ if $a \in At^{(n)}_{(1)}$
5. f_n is the binary function such that for any $\overline{x} \in (Ob)^n$ and any $a \in At^{(n)}_{(1)}$ $f_n(\overline{x}, a) \subseteq Val(a)$. f_n is called the n-th information function and for any n-tuple of objects $\overline{x}$ and $a \in At^{(n)}_{(1)}$ $f_n(\overline{x}), a$ is called the *information about* $\overline{x}$ *according to* a.

Note that a relation $R \in Rel$ may not lie in the scope of any attribute. Thus we should explicitly mention our set of relations. As hinted above, I will now simplify generalized attribute systems by exploiting the assumption that every property or a relation falls under an attribute. that assumption justifies dropping the explicit mention of the sets of properties in an attribute system and relations since they will be "there" as values of attributes:

Definition 8.3.5. A *simplified generalized attribute system* is a system

$$\mathbf{S} = \langle Ob; At; Val; f \rangle$$

where

1. $Ob \neq \emptyset$ is the set of objects,

2. $At = \{At^{(n)}_{(m)} : n, m \in \mathbb{N}^{+}\}$ is the set of attributes and each $At^{(n)}_{(m)}$ is the set of n-ary attributes of level m,

3. Val is as in 11.0.28-(iv) except in the case of 1-th level attributes of arity n, $Val(a)$ is any set of n-ary relations among objects (while any two attributes of level 1 can still have the same set of values)

4. f_n is the same as the function f_n of 11.0.28-(v).

Definition 8.3.6. Given a generalized attribute system $\mathbf{S}'$, we may construct a corresponding relational information system $\mathbf{S}$ as follows:

1. $Ob_S = Ob'_S$

2. $Rel^{(n)}_S = \bigcup\{Val(a) : a \in At^{(n)}_{(1)}\}$

3. for any $\overline{x} \in (Ob_S)^n$ $f_S(x) = \bigcup\{f'_S(\overline{x}, a) : a \in At^{(n)}_{(1)}\}$

8.4 Identity-like Relations in Attribute Systems

Now I will consider identity-like relations in attribute systems. For simplicity, I will give definitions for objects rather than tuples. The case of tuples is not difficult.

Definition 8.4.1 *(Indiscernibility and interchangeability in attribute systems).* Let $\mathbf{S} = \langle Ob; At; Val; f\rangle$ be an attribute system and let x and y be two objects. Then

1. x and y are indiscernible in $\mathbf{S}$, $x \cong y$ if $\forall a \in At(f(x, a) = f(y, a))$,

2. x and y are interchangeable in $\mathbf{S}$ with respect to the attributes $A \subseteq At$, $x \fallingdotseq_A y$, if $\forall a \in A f(x, a) = f(y, a)$

Attribute systems enable us to represent a wider variety of similarity relations:

Definition 8.4.2 *(Similarity relations in information systems).* Let $\mathbf{S} = \langle Ob; At; Val; f\rangle$ be an attribute system and let x and y be two objects. Then we may define the following notions of resemblance

1. x and y are *weakly positively similar*, $x\Sigma_S y$, if $\exists a \in At : f(x, a) \cap f(y, a) \neq \emptyset$

2. x and y are *weakly negatively similar*, xN_Sy, if $\exists a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \emptyset$

3. x and y are *strongly positively similar*, $x\sigma_Sy$, if $\forall a \in At : f(x,a) \cap f(y,a) \neq \emptyset$

4. x and y are *strongly negatively similar* $x\upsilon_Sy$, if $\forall a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \emptyset$

We should consider restricting the set of attributes to smaller significant sets of attributes. In this way we obtain the following restricted similarities:

Definition 8.4.3 *(Restricted similarity relations in attribute systems).*
Let $\mathbf{S} = \langle Ob, At, Val, f\rangle$ be an attribute system, $A \subseteq At$ and x, y be two objects. We may define the following notions of resemblance

1. x and y are *weakly positively similar w.r.t. A*, $x\Sigma_Ay$, if $\exists a \in At : f(x,a) \cap f(y,a) \neq \emptyset$

2. x and y are *weakly negatively similar w.r.t. A*, xN_Ay, if $\exists a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \emptyset$

3. x and y are *strongly positively similar w.r.t. A*, $x\sigma_Ay$, if $\forall a \in At : f(x,a) \cap f(y,a) \neq \emptyset$

4. x and y are *strongly negatively similar w.r.t. A* $x\upsilon_Ay$, if $\forall a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \emptyset$

If $A = \{a\}$, then we may write $x *_a y$ instead of $x *_{\{a\}} y$ where $*$ is any of the relations defined above.

Remark 8.4.4. Different assumptions about information systems yield different properties of these identity-like relations and different relations between these relations. For example, note that if S is a normal information system ($|f(x,a)| \leq 1$ for any $x \in Ob_S$ $a \in At_S$), then if $x\sigma_Sy$ then $x \cong_S y$.

Let $\mathbf{S}$ be a generalized attribute system and let $A \in At_S$. Thus, $A \in At^{(n)}_{(m)}$ and each attribute belongs to a unique level m. A has a set of values forming a subset of attributes of level $m-1$. Similarly, each attribute of level $m-1$ has a set of values whose members are from the attributes of level $m-2$ and so on... Attributes of level 1 have values from the set of properties. Thus, for an attribute A we define an operation similar to transitive closure operation on sets. Namely

$$A^* = val(A) \cup \bigcup \{val(B) : B \in val(A)\} \cup \dots$$

To give an example, Physical* would naturally include all physical attributes color, texture... and all specific colors, textures... and all specific shades of all colors, and if any specifications of specific textures...

We will use the following related notion as well:

$$A^{**} = A^* \cup \bigcup \{ext(P) : P \in A^*\}$$

where $ext(P) = \{a \in Ob : P(o)\}$. Note that since only properties have extensions this definition presents no difficulty.

The notion of resemblance of properties is defined by means of significant classes of attributes $\mathcal{A}_0$. recall that I used the notation $P \Cap Q$ to denote "The property "P" resembles the property "Q"".

Definition 8.4.5. $P \Cap Q$ if and only if they are contained in A^* of some attribute A.

$$P \Cap Q \Leftrightarrow \{P, Q\} \subseteq A^* \tag{8.1}$$

Thus 4.5 becomes

$$a \boxplus_{\mathcal{P}_0} \Leftrightarrow \exists P, Q \in \mathcal{P}_0 [P(a) \wedge Q(b) \wedge (P = Q) \vee \exists A \in \mathcal{A}_0 \{P; Q\} \subseteq A^*)] \tag{8.2}$$

It is reasonable to assume that $\mathcal{P}_0$ and $\mathcal{A}_0$ are related. One possibility could be to demand that

$$\mathcal{A}_0 \subseteq \{A \in \mathcal{A} : \exists P \in \mathcal{P}_0 \text{ such that } P \in A^*\} \tag{8.3}$$

Moreover, one can easily see by examining 4.5 and 8.3 does not lead to any restriction on the applicability of 4.5. In fact, the relation depends on the notion of attribute. The relation of "residing in" holding between one entity of level n and one of level $n-1$ (explicated by our Val function) is not transitive. Assume that an object o is red. Red is a color. But o is not (a) color. On the other hand, we ca formulate the relationship between, o and the attribute Color in the obvious manner: the property red resides in the object o and since red is a value of the attribute color, o is colorful. I will assume for present purposes that 8.3 is a reasonable constraint on the significant sets of properties and significant sets of attributes.

Let $\Cap(o_1, o_2)$ denote *the degree of resemblance between* o_1 *and* o_2. Thus $\Cap(o_1, o_2) \leq \Cap(o_3, o_4)$ means "the resemblance between o_3 and o_4 is closer than the resemblance between o_1 and o_2". I promised before that higher order attribute systems enable us to represent several degrees of resemblance. Among these may state the following:

1. The notions of degree of resemblance present in property systems is obviously present in attribute systems too. $\Cap^1_{\mathcal{P}_0}(o_1, o_2) \leq \Cap(o_3, o_4)$ if o_3 and o_4 have more significant properties in common than o_1 and o_2 have:

$$|\{P \in \mathcal{P}_0 : P(o_1) \wedge P(o_2)\}| \leq |\{P \in \mathcal{P}_0 : P(o_3) \wedge P(o_4)\}|$$

2. Motivation for the following notion of degree should be clear from the preceding material: $\Cap^{1'}_{\mathcal{P}_0}(o_1, o_2) \leq \Cap^{1'}_{\mathcal{P}_0}(o_3, o_4)$ if o_3 and o_4 have more common or resembling properties than o_1 and o_2 have. That is

   $|\{(P, Q) \in (\mathcal{P}_0)^2 : P(o_1) \wedge P(o_2) \wedge (P = Q \vee P \Cap Q)\}| \leq$

   $|\{(P, Q) \in (\mathcal{P}_0)^2 : P(o_3) \wedge P(o_4) \wedge (P = Q \vee P \Cap Q)\}|$

3. I interpret what Armstrong pointed about scale-like ordering of some properties in terms of attributes as saying that there is a metric (a binary non-negative real valued function) on the set of values of some attributes. Assume further that $A \in At$ be such an attribute and $P, Q \in A$. Let δ_A denote the metric on $val(A)$. Thus, $\delta_A(P, Q)$ denote the distance between P and Q with respect to the metric δ_A. Now define $\Cap^2_A(o_1, o_2) \leq \Cap^2_A(o_3, o_4)$ if

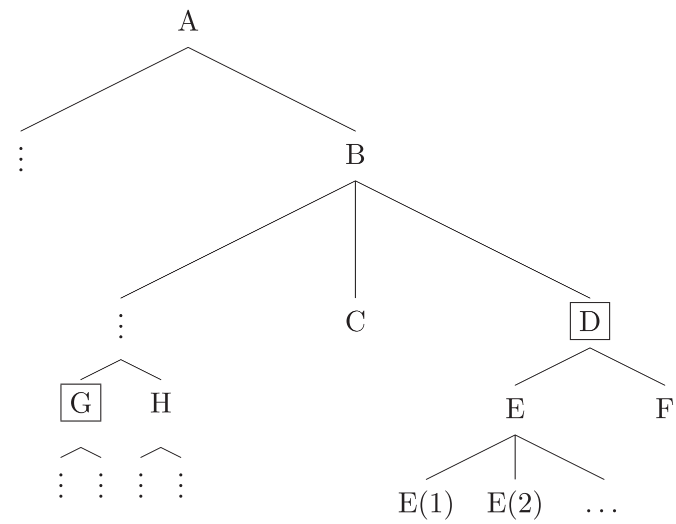
   $\exists P, Q, R, S \in AP(o_1) \wedge Q(o_2) \wedge R(o_3) \wedge S(o_4) \wedge \delta_A(R, S) \leq \delta_A(P, Q)$

   The hierarchy of attributes leads to several notions of degree of resemblance as well. To my knowledge, in the theory of information systems, the idea of hierarch of attributes has not been worked sufficiently yet. The attribute systems introduced so far work with one level of attributes and introducing the hierarch requires a lot of work. However, I believe that the introduction of higher order attribute systems will enable us to represent several important ontological ideas in the information systems.

Working with classes of attributes without mentioning properties we get a coarse but significant notion of degree. Let $A_0 \subseteq At$

4. $\Cap^3_{A_0}(o_1, o_2) \leq \Cap^3_{A_0}(o_3, o_4)$ if $min\{lev(A) : A \in A_0, \{o_3, o_4\} \subseteq A^{**} \leq min\{lev(A) : A \in A_0, \{o_1, o_2\} \subseteq A^{**}\}$

As an illustration of the notion consider the following diagram



This notion of degree corresponds to the determinable-determinate relation between universals. o_3 is closer to o_4 than o_1 is to o_2 if the lowest common determinable of o_1 and o_2 is lower than the lowest common determinable of o_3 and o_4.

These notions of degree is far from exhausting all philosophically significant possibilities. There are others including the combinations of those given. Moreover, even a complete list of degree notions will not bring a complete order on the class of resembling objects. This is not a deficiency of the theory of the theory but a fact about resemblance. Even if we accept, for a moment, that resemblance is an indefinable primitive notion, this would remain as a fact about resemblance. Thus we should not except a definite answer for any two pairs $(o_1, o_2), (o_3, o_4)$ as to which pair has a closer resemblance. The reasonable constraint would be to cover unproblematic cases: if we think that $(o_1, o_2), (o_3, o_4)$ are comparable with respect to resemblance then the theory should explain why.

I want to conclude this chapter with some prospects for future research in the theory of identity-like relations in information systems.

Note that no structural relationships between attributes have been incorporated in the information systems studied in this work. This would highly complicate the work here. It would be a big step in the theory of information systems and would contribute to the philosophical applicability of information systems.

Secondly, the problem of developing modal logics and higher order consequence systems corresponding to higher order attribute systems introduced here is open. Modal logics and consequence systems for property systems and first-level attribute systems were studied by Vakarelov [22], among others. Scott consequence systems corresponding to property systems is called consequence systems (C-systems) and those for attribute systems are called bi-consequence systems (B-systems). In [22] abstract completeness theorems for these systems have been proved too. Analogous theorems waits to be stated and proven.

Chapter 9

Deutsch's Problem

Deutsch [10] poses an interesting question from the point of view of logical properties of similarity relations. For Deutsch, statements of general similarity are statements of the form

$$x \text{ and } y \text{ are (or have) the same } Np. \tag{9.1}$$

where Np is a kind of properties of things. Using the terminology of information systems, we might want to say "attribute" instead of "a kind of properties of things". However, recall that attributes are intensional entities. Thus, we must be careful whether Deutsch has an extensional conception of kinds or not.

Why relations of the form 9.1 are called general similarities should be clear: The respect of resemblance is not specified. We may, for example, be informed that two objects o_1 and o_2 have the same color without being given of which color it is. So we have "incomplete" information about the similarity between o_1 and o_2. There is nothing weird about it: in many cases we have information which is underspecified in some respects and "reasoning with incomplete information" is a cardinal virtue in real life applications of logic. Below, we will see how we obtain more information by combining incomplete information with some more information.

Relations of identity are those expressed by statements of the form

$$x \text{ and } y \text{ are the same } Nt. \tag{9.2}$$

where Nt is a kind of things.

The expression “are the same Np” does not point to relative identity with respect to an Np-property. Rather, 9.2 can be thought to be the conjunction of two statements

$$x \text{ is } S \tag{9.3}$$

$$y \text{ is } S \tag{9.4}$$

where S is an Np-property and “is” in both 9.3 and 9.4 is the “is” of predication (not the “is” of identity).

In contrast, the “are” statements of the form 9.2 cannot be broken into two “is” statements since “are” here denotes identity relative to an Nt-property. Recall that the absolutist would render 9.1 as

$$x \text{ has an } Np \text{ property } S_1 \wedge y \text{ has } Np \text{ property } S_2 \wedge S_1 \equiv S_2 \tag{9.5}$$

Here S_1 and S_2 are to be considered as parameters. and 9.2 as

$$S(x) \wedge T(y) \wedge x \equiv y \tag{9.6}$$

Thus both relations of general similarity and relations of identity can be eliminated in favor of strict identity and after the reduction some desired properties of these relations obtain logically. For example, we may show that both relations are “transitive”.

Indeed, if o_1 and o_2 are the same F and o_2 and o_3 are the same G, then, assuming that both o_1 and o_3 are G, then they are the same G showing that the relations of identity have a transitivity property. Letting $F = G$ we have the specific transitivity result, i.e., the transitivity of F-identity.

Deutsch claims transitivity for general similarity once reduction using strict identity is allowed. The proof must run like this: if o_1 and o_2 have the same Np, and o_2 and o_3 have the same Np then

$$o_1 \text{ has } S_1 \wedge o_2 \text{ has } S_2 \wedge S_1 = S_2 \tag{9.7}$$

$$o_2 \text{ has } S_2 \wedge o_2 \text{ has } S_3 \wedge S_2 = S_3 \tag{9.8}$$

and by the transitivity of strict identity we have

$$o_1 \text{ has } S_1 \wedge o_3 \text{ has } S_3 \wedge S_1 = S_2 \tag{9.9}$$

establishing the transitivity of the relation having the same Np where Np is a kind of properties of things. Thus the relations of general similarity like "having the same color" between physical objects, "having the same cardinality" between sets are transitive relations.

Deutsch asks how, by this reduction method, can we explicate *indiscernibility principles* obtained from general similarity relations. To understand what these principles are, consider the following examples by Deutsch

(i) If x and y have the same shape, then if x is round then y is round,

(ii) If x and y have the same shape, then if x has a smooth boundary then y has a smooth boundary.

To appreciate that this is a genuine problem, assume that x and y have the same shape. By reduction we have

$$S_1(x) \wedge S_2(x) \wedge S_1 \equiv S_2$$

Moreover, assume that x is round, $R(x)$. From this, how can we deduce that y is also round, $R(y)$? Obviously, we would like to infer that y is also round from the information about x and y while it is not a logical consequence of the reduced statement $S_1(x) \wedge S_2(x) \wedge S_1 \equiv S_2$ and the atomic formula $R(x)$. Deutsch says that by "consulting to a dictionary" thus by referring to extra-logical principles connecting shape and roundness properly, we can manage to make this deduction. Deutsch says that "perhaps after consulting to a dictionary we may wish to add the premise $R = S_1$ ... then [this] together with transitivity and Leibniz' Law for strict identity yields

$[R(y)]$; but then the derivation would rely on a non-logical "meaning postulate" justifying $[R = S_1]$". Thus, logician's rejection of general similarity as a logical notion is understood. A further problem is to find a uniform way of finding out indiscernibility principles for general similarity relations.

Deutsch proposes an account of indiscernibility principles. His account depends on the determinable-determinate relation and, Deutsch claims that this account does not lead to commitment to higher entities. I will now introduce Deutsch's account in some detail and evaluate whether it gives a successful account of indiscernibility principles while at the same time not leading to higher entities. Then, I will try to give a similar account in attribute systems.

9.1 Deutsch's System

Deutsch starts with a purely relational first order language with identity $\mathcal{L}$. Thus, for simplicity, $\mathcal{L}$ does not contain any individual constant symbol or a function symbol. Let $\mathfrak{M} = \langle M, g \rangle$ be an $\mathcal{L}$-structure. That is, M is a non-empty set and g is a function which sends each n-ary relation of $\mathcal{L}$ to a subset of M^n. Let us define a function d which assigns to each subset B of M a family of pairwise disjoint subsets of M including B itself. That is for any X, Y in $d(B)$, $X \cap Y = \emptyset$.

Given a unary predicate symbol F of $\mathcal{L}$, $d(F)$ denotes $d(g(F))$, which is defined since $g(F) \subseteq M$. The idea behind the function d is as follows: given a unary predicate F, $d(F)$ includes $g(F)$ together with the extensions of similar predicates. Deutsch's requirement that $d(g(F))$ consists of pairwise disjoint subsets of M, is a result of the assumption that each individual a satisfies at most one of the similar properties. Deutsch himself says that his "...mechanism has a simple set-theoretical characterization somewhat along the lines of the venerable but neglected distinction between *determinate* and the *determinable*" ([10], p. 179) and calls $d(B)$ *the determinable for* B and elements of $d(B)$ *determinates of* $d(B)$. Note that he does not require that $d(B)$ is a partition of M, that is not necessarily $\bigcup d(B) = M$. this means, in colloquial terms, an object need not satisfy a $d(B)$ determinate. If F is interpreted as the set of all red objects then $d(F)$ would include extensions of other colors and

$d(F)$ would correspond to the determinable color. Here an immediate inadequacy of Deutsch's system can be seen: assuming that $g(F) = g(G)$ while extensional equivalence of F and G in $\mathfrak{M}$ is just a coincidence, then $d(F) = d(G)$ would follow which is not a reasonable requirement.

Now expand $\mathcal{L}$ further by adding a binary relation symbol $=_{d(F)}$ for each unary predicate symbol F. The intended interpretation of $=_{d(F)}$ is that two objects stand in that relation if they have the same $d(F)$ determinate. Thus, if F is t be interpreted as "is red", then $o_1 =_{d(F)} o_2$ if o_1 and o_2 belong to the same $d(F)$ determinate. Note that this does not mean that o_1 and o_2 are the same F, just that they have the general similarity relation: to write it explicitly in the form 9.1, o_1 and o_2 have the same $d(F)$.

Remark 9.1.1. Note that if F and G are two unary predicate symbols to be interpreted as the determinates of the same determinable that is, $d(F) = d(G)$ then $=_{d(F)}$ and $=_{d(G)}$ will be the same relation. If in a suitable model, e.g., F is the property "red" and G is the property "yellow", then $d(F)$ is the property kind "color" and both $=_{d(F)}$ and $=_{d(G)}$ will name the same relation "having the same color".

For each predicate symbol F, we introduce infinitely many new unary predicate symbols $P_F, Q_F, \ldots$. These will denote basic indiscernibility relations with respect to the relation $=_{d(F)}$ and will be called F-*subscripted predicates*. Moreover, we introduce a new symbol $\mathbf{d}$ such that for each unary predicate F, the expression $\mathbf{d}F$ is a monadic predicate symbol and the atomic formula $\mathbf{d}F(x)$ intuitively says that x has a $d(F)$-determinate property. Thus, if F is the predicate "is red" then $\mathbf{d}F(x)$ says that "x has a color" or "x is colored". In the spirit of 9.1.1, $\mathbf{d}F$ and $\mathbf{d}G$ coincide for any two determinates F and G of the same determinable.

Definition 9.1.2. Let $B \subseteq M$ then $d(B)'$ is the family of sets C' such that

1. $C' \subseteq M$

2. For any $C \in d(B)$, if $C \cap C' \neq \emptyset$, then $C \subseteq C'$

Remark 9.1.3. $'$ is a function. That is, for any $B \subseteq M$ there is a unique $d(B)'$ extending $d(B)$.

Thus, starting from the first order language $\mathcal{L}$ the expansion $\mathcal{L}(S)$ is obtained by the addition of new symbols introduced above. We may now describe $\mathcal{L}(S)$-structures. These are obtained from $\mathcal{L}$-structures $\langle M, g\rangle$ by expanding them to structures $\langle M, g', d,' \rangle$ where g' is the extended interpretation function such that

1. g and g' agrees on M: $\forall m \in M(g(m) = g'(m))$
2. For each predicate symbol F of $\mathcal{L}$, $g'(P_F) \in d(F)'$

Definition 9.1.4. Let $\mathcal{L}(S)$ be an extended language and $\mathfrak{M}(S) = \langle M, g', d,' \rangle$ is an $\mathcal{L}(S)$-structure. Given a valuation v, that is a function from the set of variables to the universe M, then the relation *φ is satisfied in $\mathfrak{M}(S)$ by the valuation v*, $\mathfrak{M}(S) \vDash \varphi(v)$ is defined by recursion on the complexity of formulas as follows:

1. $\mathfrak{M}(S) \vDash (x =_{d(F)} y)(v)$ if $\{v(x), v(y)\} \subseteq C$ for some $C \in d(F)$,
2. $\mathfrak{M}(S) \vDash \mathbf{d}(F)(x)(v)$ if $v(x) \in C$ for some $C \in \mathbf{d}(F)$, equivalently $v(x) \in \bigcup \mathbf{d}(F)$
3. $\mathfrak{M}(S) \vDash P_F x(v)$ if $v(x) \in C$ for some $C \in \mathbf{d}(F)'$, equivalently $v(x) \in \bigcup \mathbf{d}(F)'$

9.1.4 can be extended to arbitrary formulae as usual. Extension to truth functions is trivial. Existential quantification case is given as, $\mathfrak{M}(S) \vDash \exists x \varphi(x)(v)$ if for some v' which possibly deviate from v only at x $\mathfrak{M}(S) \vDash \varphi(x)(v')$

Definition 9.1.5. A formula is *true in* $\mathfrak{M}(S)$, $\mathfrak{M}(S) \vDash \varphi$ if $\mathfrak{M}(S) \vDash \varphi(x)(v)$ for every valuation v. Thus, a sentence is true in $\mathfrak{M}(S)$ if it is satisfied for at least one valuation. An $\mathcal{L}(S)$ sentence is *valid* if it is true in every $\mathcal{L}(S)$-structure.

Note that, since a sentence φ has no free variable, the choice of v does not matter. I do not give the proof of this statement. See for clarification Chang [7].

Before questioning the admissibility of Deutsch's stipulations, let us first see whether the given construction does what Deutsch wants it to do. So, indiscernibility properties of general similarities must be accounted for. The following proposition states that many indiscernibility properties can indeed be accounted in Deutsch's system:

Proposition 9.1. *1.* $x =_{d(F)} y \rightarrow (Fx \rightarrow Fy)$

2. $x =_{d(F)} y \to (P_F x \to P_F y)$

Proof. The first one is obvious. Towards proving the second, assume that $x =_{d(F)} y$ and $P_F x$. By the former, there is a $C \in d(F)$ including both x and y and, by the latter, there is a $C' \in d(F)'$ such that $x \in C'$. Since $x \in C \cap C'$, by (ii) of the definition 9.1.2, $C \subseteq C'$. Thus y being a member of C, is a member of C' showing that $P_F y$. □

9.2 General Similarity Relations and Indiscernibility Properties in Information Systems

Note that general similarity relations and indiscernibility properties are easily expressible in information systems: The claim that a and b have the same F where F is a count-noun like color can be expressed in an attribute system including the attribute F as:

$$f(a, F) = f(b, F)$$

The claim that "a and b have the same shape" as:

$$f(a, Shape) = f(a, Shape)$$

Recall that an attribute system S such that every object has at most one A-property, A being an attribute, was called single valued. If in addition, S satisfies totality i.e., $f(x; A) \neq \emptyset$ for every $x \in Ob_S$ and every $A \in At$, **S** was called normal. Indiscernibility relations are easy to handle in single valued information systems. The indiscernibility relation leading from a has a smooth boundary to b has a smooth boundary would be translated as

$$f(a, Shape) \in val(Smooth\ Boundary) \Rightarrow f(b, Shape) \in val(Smooth\ Boundary)$$

We may represent a variant of general similarity. The statement that "a and b

have the same F" can be interpreted also as the same F-property is both in the information about a with respect to F, $f(a, F)$, and the information about b with respect to F, $f(b, F)$. Even if **S** is not single valued, this statement makes sense: in this more liberal sense a and b have the same F means "There is an F property which is both in $f(a, F)$ and in $f(b, F)$" or, equivalently

$$f(a, F) \cap f(b, F) \neq \emptyset$$

That statements of the form 9.1 may be interpreted this way can be seen by considering the case of the attribute color: we may interpret a and b have the same color as saying that a and b have exactly the same colors (perhaps even the same color distribution), in which case they have the same color information, or in a liberal sense saying that the same color is perceived in (or on the surface of) both a and b, in which case their information with respect to color have nonempty intersection.

Chapter 10

Infinite Regress Arguments and Resemblance

Infinite regress arguments have been used in philosophy since its earliest periods. It is tempting to present an infinite regress argument in the form of a notorious *reductio ad absurdum* of a theory. However, there are cases where an infinite regress argument is presented positively. Aristotle e.g., used an infinite regress argument to establish the need for a thing that is desired for itself and not for the sake of something else (the highest good). It is true that the sense of positivity may be removed and the argument can be paraphrased back into a *reductio ad absurdum* of the opponent theory: in the case of Aristotle's argument, the theory would be the one saying that everything is desired for the sake of something else. However, the other direction does not always work: some infinite regress arguments just refute a theory without specifying which theory among the alternatives should be accepted. Thus, it is reasonable to make a distinction between positive (or constructive) infinite arguments and negative (destructive) ones.

It is generally accepted that any theory or definition leading to a *vicious* infinite regress must be rejected as it is. Not all infinite regresses are vicious. Regresses which are not vicious are called benign. Although there have been attempts to tell vicious from benign infinite regresses but this work has not produced a definite theory. The reasons for telling benign from vicious are mostly dialectical and sometimes obscure. Among these, we can isolate two that are common, easy to apply but hard to justify.

(1) If one rejects beforehand the possibility of actual infinity then this position *a fortiori* leads to rejection of a theory that leads to existence of an infinite collection of objects through an infinite regress and he calls that regress vicious. Consider the Hilbert hotel "paradox": Suppose that there is a hotel with an infinite number of rooms. Room number 1, room number 2, ... A customer comes to ask for a room. One cannot reasonably say that there are no free rooms since one free room can always be generated: The customer says that "You may transfer the resident of room number 1 to room number 2, the one in room number two to room number 3 and so on... This can be done (theoretically) since for every room number n there is a room number $n+1$". If you think that the idea of a hotel with an inexhaustible number of rooms is unacceptable, then you would have to reject the idea of actually infinity.

(2) No progress argument says that an infinite regress is vicious if no progress (how minute) is achieved at each step. Consider the argument in favor of a thing that is desired for itself and not for the sake of something else. Translating it to a reductio of the theory that for every thing x that is desired, there is another thing y such that x is desired for the sake of y and not for the sake of itself. Assume that one is to make his mind about whether he should desire x or not. He would ask himself: Why should I desire x. He responds himself: For the sake of x_1. Then he asks: Why should I desire x_1 then? He responds back again: Well... For the sake of x_2... and so on *ad infinitum* Obviously, he cannot complete this process of contemplation. Note that, the argument does not require an infinite number of entities to work. Suppose that there are just two things x and y. Then the poor guy would desire x for the sake of y and then y for the sake of x assuming that every thing x that is desired, there is another thing y such that x is desired for the sake of y and not for the sake of itself. Can he make his mind as to whether he should desire x? He would have to think this way: Am I to desire x? Yes. If I should desire y. But am I to desire y? Yes. If I should desire x... Here again, no progress is made towards a final answer. At no point of the reasoning, he is any better than a previous point.

In the study of infinite regresses two main questions are:

(1) Which features make theories lead to infinite regresses?

(2) What makes an infinite regress vicious?

The first question can be handled *formally* However, as it should be expected, one cannot give a formal characterization of vicious and benign infinite regresses. It is a metaphysical matter to determine whether a theory is inconsistent with the infinite regress under consideration or not. I will restate this point below.

10.1 The Way to Infinite Regress

I will now consider two best attempts at (1).

Black [6] tries to show that the reductio argument follows from an infinite regress if it leads to existence of an infinite series while we know that such a series can not exist.

Suppose that there is an object which is F

(1) $\exists x Fx$

and we know that anything is an F only if it is R-related to an F-thing (for some relation R)

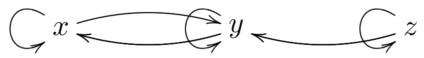
(2) $\forall x[Fx \rightarrow \exists y(Fy \wedge Rxy)]$

Definition 10.1.1. Let R be a binary relation on a set S. The transitive closure of R that we denote with R^* is the $\subseteq$-smallest transitive relation on S including R. The assertion that, for every relation R the existence of a unique smallest transitive relation can be proved constructively. We may start with R. Let us write R^n for the composition of R with itself n-times and let $R^* = \bigcup_{n \in \mathbb{N}} R^n$ Then, $R \subseteq R^*$ since $R = R^1$ and R^* is transitive. Let xRy and yRz then $x(R \circ R)z$ In general, if we take a chain of elements $x_1 R x_2 \ldots x_{n-1} R x_n$ then $x_1 R^n x_n$. Since a transitive relation should satisfy such a linkage condition, R^* is included in every transitive relation including R i.e., R^* is the $\subseteq$-smallest transitive extension of R.

Assume further that $R^* \backslash R$ is irreflexive i.e.,

(3) $\forall x \neg (x R^* \backslash R x)$

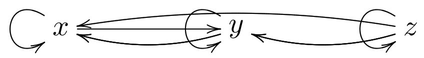
(1), (2) and (3) does not imply the existence of infinite descending paths: Instead of (3), one should make the stronger assumption e.g, that the transitive closure itself is irreflexive. Let me give an example: Assume that R is as pictured below



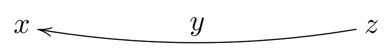
with $A = \{x, y, z\}, F = \{x, y\}$

Since both x and y has the property F, this structure satisfies $\exists x Fx$. Moreover, for both objects with F, there is another R-related object which has F (for x we have y and for y we have x). Thus (2) is satisfied too.

R^* is then as below:



and $R^* \backslash R$ is just



Thus $R^* \backslash R$ is irreflexive and the structure $\langle \{x, y, z\}, R, F \rangle$ is one that satisfies (3) in addition to (1) and (2) without an infinite descending chain.

As I said above, if we assume that

(3') R^* is irreflexive

then (1), (2), (3') imply the existence of infinite descending chain:

Proof. I will outline the proof as follows: by (1) there is an x_1 such that $F(x)$. Then there is an x_2 such that x_1Rx_2 and $A(x)$. $x_1 \neq x_2$ since otherwise R^* would not be irreflexive. Applying (2) again there is an x_3 with x_2Rx_3 and $F(x_3)$. $x_3 \neq x_2$ and $x_3 \neq x_1$ since the former directly, the latter by the definition of transitive closure gives non-irreflexivity of R^* and so on... Thus we can make the sequence as long as we wish. By compactness, (1), (2), (3') gives an infinite sequence. □

With the additional premise saying that

(4) There is no infinite series of F-things

we reach a contradiction.

It seems that assumptions made by Black to get an infinite regress out of a theory are too strong to be of any interest. Another attempt is by Clark [8]. First we need a few definitions.

Definition 10.1.2. A relation R on a non-empty set A induces a partial order on A if either R itself a partial order on A or it can be extended to a partial order.

Definition 10.1.3. Let F be a property which holds for some members of a non-empty set A (extension of F may be a subset of A). A binary relation R on A, i.e., $R \subseteq A \times A$, is upward) F-preserving if

$$\forall x, y \in A(Fx \wedge (y, x) \in R \rightarrow Fy)$$

(Think of Rxy as $x \geq y$)

Definition 10.1.4. Let F be a property and A be a non-empty set. An element $a \in A$ is conditionally-F, if there is an F-preserving relation R which induces a partial order on A and $\exists yRay$. I.e, a must be in the domain of an F-preserving relation on A.

Definition 10.1.5. An element $a \in A \neq \emptyset$ is downward dependent on its R-related heredity if a is only conditionally F. a is R related to some elements of A and we can extend R to an F preserving relation so that if the objects R-related to a are F, then a itself would be F.

If a is F and it is not dependent on its heredity, it is called categorically-F

Now we are ready to give a characterization of *vicious* using this terminology: Assume that the following are satisfied,

1) $\exists x (cat) Fx$ (known)

2) $(cat)F\mathbf{a}$ (1)

3) Assume that we have a theory TF explaining what it is to be an F

4) From TF we derive the conclusion that

> ...nothing is F unless there exists an infinite succession of elements, ones which stand in an "upward F preserving relation," RF, and each of which is downward dependent upon its R-heredity. TF might for instance imply something of this form:

$$(**) \qquad \forall x[F(x) \text{ only if } \exists y(Fy \ \& \ R(xy))]$$

Thus we arrive at the desired contradiction; 2 asserts that the arbitrarily chosen fixed element **a** is categorically F while (*) amounts to saying that nothing is F unless it is conditionally so. Therefore, the theory TF of F-ness, being reduced to a contradiction, should be rejected.

Is the "categoricity" condition in 1 and 2 is essential on the way to contradiction? Let $2' :\ F\mathbf{a}$ and let $1' : \exists Fx$. Then, since to be F is to be categorically F (Clarke p. 373 lines -1,-2), if 1,2,3,(**) forms an inconsistent set of sentences, $1'$,$2'$,3,(**) does too. Thus, showing that $1'$,$2'$,3,(**) is not inconsistent (i.e., consistent) is sufficient to establish the consistency of 1,2,3,(**).

Now, let F be a property of natural numbers with extension the whole set $\mathbb{N}$. Let TF be the theory of F-ness saying that a natural number $\mathbf{n}$ is F if and only if it is a predecessor: i.e., it is obtained from a natural number by subtraction of 1. Let R be the usual ordering of natural numbers. Obviously, I suppose, being a predecessor number is a conditional property of numbers. Indeed, universally, something is a predecessor if and only if it is a predecessor of something. However, for all we know, 0 for example is a predecessor and it is thus categorically so. Then, we have

$1'$) $\exists x Fx$ (known)

$2'$) $F\mathbf{a}$ (1)

3) $TF : \forall x[Fx \leftrightarrow \exists y(x = y - 1)]$

4) $TF \Rightarrow \forall x[Fx$ only if $\exists y(Fy \ \& \ x < y)]$

Obviously 1,2′,3,4 are all true but no contradiction can be derived from this fact. Indeed,

a) $\exists x Fx$

b) $F\mathbf{a}$

c) $\forall x[Fx \leftrightarrow \exists y(x = y - 1)]$

d) $\forall x[Fx$ only if $\exists y(Fy \ \& \ x < y)]$

form a consistent first order theory, for $\mathbb{N}, <\models a, b, c, d$. So, Clark's account of viciousness-by-leading-to-contradiction should be rejected. My point then, if correct, is that an infinite regress resulting from an account of a property and leading to a simple linear order as in 4, does not contradict the categorical existence of an F. Clark's faulty assertion is that if something is not an F unless it is conditionally so, then it can not be categorically-F.

Defining categorically-F (or exists) as not-conditionally-F (exists), which is *not* Clarke's definition, would trivialize the account of viciousness of infinite regress.

10.2 Viciousness

Now, I will turn to the second question about infinite regresses: What makes a theory vulnerable to vicious infinite regress. It is claimed by Wittgenstein that, if a theory leads to a vicious infinite regress it is already inconsistent at the first step. Here is an example supporting this claim. I will give a proof of irrationality of $\sqrt{2}$ first by an infinite regress argument then by a proof by contradiction.

Assume that $\sqrt{2}$ is rational Then there are two integers m, n such that $\sqrt{2} = m/n$ thus $2 = m^2/n^2$ or $2n^2 = m^2$. Thus, since m^2 is even, m itself should be even, thus there is an integer m_1 such that $m^2 = (2m_1)^2 = 2n^2$ i.e., $2{m_1}^2 = n^2$. Again, n should be even since n^2 is. Thus there is n_1 such that $2{m_1}^2 = 2{n_1}^2$ i.e., ${m_1}^2 = 2{n_1}^2$. Thus, m_1 should be even since ${m_1}^2$ is... In that way starting from an integer, say m, we obtained an infinite sequence of integers $m, m_1, m_2, \ldots$ such that for each i m_{i+1} divides m_i which is impossible. □

As we know, this infinite regress argument is unnecessary since there is an argument by contradiction establishing the same result:

Assume that $\sqrt{2}$ is rational Then there are two integers m, n such that $\sqrt{2} = m/n$. We may further assume without loss of generality that m and n are relatively prime i.e., they have no common factor other than 1. thus $2n^2 = m^2$. Thus m is even and n should be odd. Since m is even $m = 2k$ and $2n^2 = m^2 = 4k^2$. Thus $n^2 = 2k^2$ showing that n should be even contrary to our assumption that not both m and n are even. □

An example, even a good one, does not replace a proof. Can we prove the Wittgenstenian claim? Not until a clear account of the relationship between infinite regress and inconsistency is given. A proof here have to be one which either establishes that if a theory leads to a vicious infinite regress then it leads to a contradiction or, perhaps better, gives a *method of construction* of a contradiction out of a infinite regress. Obviously no such proof can be given. Leading to infinite regress alone does not imply that the theory is inconsistent. The first order theory of natural numbers yield an infinite number of elements $0 \leq 1 \leq 2 \ldots$ while it is consistent.

Theories meeting the conditions of Russell's schema are inconsistent and these lead to infinite regress.

Definition 10.2.1. Let F be a property, Ω a class, ∂ (diagonalizer) be a function such that,

- $\Omega = \{y : F(y)\}$ is a set,
- $\partial : \mathcal{P}(\Omega) \longrightarrow \Omega$ (closure)
- $\partial(x) \notin x$ (transcendence)

Let us call a theory implying the existence of such an F, Ω and ∂ a Russell theory.

claim 1. Let T be a Russell theory. Then, T is inconsistent.

Proof. Let $x = \Omega$. Then, by closure, $\partial(\Omega) \in \Omega$ and, by transcendence, $\partial(\Omega) \notin \Omega$. □

claim 2. Suppose that a theory T implies the existence of F, Ω and ∂ satisfying the definition 10.2.1. Then, T leads to an infinite regress.

Proof. Let $x \subseteq \Omega$. Then, since $\partial(x) \notin x$, we get a *new* subset $x_1 = x \cup \{\partial(x)\}$. Again, $\partial(x_1) \notin x_1$ and we get a new subset $x_2 = x_1 \cup \{\partial(x_1)\}$ and so on. Thus we get an infinite increasing sequence of subsets of Ω, $x \subset x_1 \subset x_2 \subset \ldots$ □

10.3 Resemblance and Infinite Regresses

Consider the following argument by F H Bradley:

> Let us abstain from making the relation of the related, and let us make it more or less independent. "There is a relation C, in which A and B stand; and it appears with both of them." But here again we have made no progress. The relation C has been admitted different from A and B and no longer is predicated of them. Something, however seems to be said of this relation C, and said, again, of A and B. And this something is not to be the ascription of one to the other. If so, it would appear to be another relation, D, in which C, on one side, and, on the other side, A and B, stand, But such a makeshift leads at once to the infinite process. the new relation D can be predicated in no way of C, or of A and B; and hence we must have recourse to a fresh relation, E, which comes between D and whatever we had before. But this must lead to another, F; and so on, indefinitely. Thus the problem is not solved by taking relations as independently real. For, if so the qualities and their relations fall entirely apart, and then we have said nothing. Or if we have to make a new relation between the old relation and the terms; which, when it is made, does not help us. it either itself demands a new relation, and so on without end, or it leaves us where we were, entangled in difficulties.

Let R_1 be the relation of resemblance and assume that it is independent of the particulars it applies to. Assume that the relation of resemblance applies to particulars x and y By the argument above, we have to accept the existence of a new relation

R_2 that applies to R_1 and the pair of particulars (x, y); $R_2(R_1, (x, y))$. Again, we are lead to welcome another relation R_3 applying to R_2, R_1 and (x, y); $R_3(R_2, R_1, (x, y))$ and so on.

One way out, sympathized by Armstrong or would be sympathized by any Aristotelian realist, would be to say that resemblance emerges out of the natures of particulars and it is not some independent entity. In that case, the above series of relations would be harmless at least. If you do not commit yourself to the independent existence of R_1, you are not committed to the existence of the series of relations neither. But what if you insist that at least one relation exists independently of the particulars it applies to? Without insisting on any of them, I will list a number of alternatives.

(1) One may say that entailment is something, requirement is another. That $R(x, y)$ for two particulars x and y and some independent relation R may entail an infinite series of relations but it may not require it.

(2) One may say that the relations $R_2, R_3, \ldots$ are just formal or abundant relations and thus harmless. $R(x, y)$ for some individuals x, y and a natural independent relation R only commits us to the existence of R.

(3) One may accept the argument with the consequence that an series of independent relations *required* but claim that this requirement is explanatory. This acceptance could be unacceptable by the foundationalist but we do not have to be one. Some philosophers, especially Peter Klein, proposes an epistemological theory called *infinitism* a theory which says that:

> In order for a person, S, to be justified in believing a proposition, p, to be true there must be and infinite set of propositions available to S that can be arranged in a non-repeating series such that the first member, r_1, is a reason for p, and the second member r_2 is a reason for r_1, and r_3 is a reason for r_2, etc., and no r_i repeats in the series. [14]

Chapter 11

Conclusion

I discussed some philosophical and formal aspects of identity-like relations: indiscernibility, interchangeability and resemblance. Defending a version of realism, I defined interchangeability and resemblance in terms of properties of objects. I concluded that these definitions apply to a broad range of objects including unperceivable and abstract objects. I defended the view that without specification of significant respects of resemblance and interchangeability, these notions are too broad to be useful. Thus, the definition of these notions referred to sets of properties.

Before presenting a favorable system that may effectively represent these notions, I examined briefly two possible representations. First I considered set theoretic models. After giving definitions of relative interchangeability and relative resemblance relations in these structures, I remarked their limitations due to the fact they represent properties and relations with sets and the instantiation relation as set-membership.

As a special case of set theoretic models, I considered Deutsch's system. After presenting his system in chapter 9, I showed later in the second section of this chapter that his general similarity relations and indiscernibility relations can easily be expressed and studied in information systems.

Secondly, I considered tolerance spaces as presented by Schreider [20]. Schreider has defined a tolerance space to be a structure consisting of a non-empty set T together with a tolerance relation τ on this set where a tolerance relation is any reflexive, symmetric relation. I remarked that in tolerance spaces resemblance relations cannot be explicated adequately for not any reflexive symmetric relation is a resemblance relation.

I defined an extension of attribute systems which accommodates higher level attributes. I demonstrated the result that the introduction of higher level attributes enables attribute systems to represent ontological ideas more effectively. Everything that can be represented in first order attribute systems can still be represented in higher order systems, but not *vice versa*. Referring to the additional feature of the representability of determinable-determinate relations between universals in higher order systems, I showed that notions of degrees of resemblance can be accounted for in these systems.

Since infinite regress arguments are often encountered in the ontology of properties and relation from the earliest periods of philosophy onwards. I considered accounts concerning theories leading to infinite regresses. I noted that no formal account of vicious infinite regresses, that is infinite regresses on the basis of which the source theory should be rejected, has not been given so far. I listed some possible moves on the face of an infinite regress.

References

[1] D. M. Armstrong, *Universals, An Opinionated Introduction*, Westview Press, 1989

[2] D. M. Armstrong, *A Theory of Universals, Universals and Scientific Realism* vol 1, Cambridge University Press, 1978

[3] D. M. Armstrong, *Nominalism and Realism, Universals and Scientific Realism* vol 2, Cambridge University Press, 1978

[4] D. M. Armstrong, "Infinite Regress Arguments and the Problem of Universals", *Australasian Journal of Philosophy*, Vol 52, No 1; May 1974

[5] M. Black, "The Identity of Indiscernibles", *Mind*, 61; 1952)

[6] Oliver Black, "Infinite Regress Arguments and Infinite Regresses", Acta Analytica, 16-17; 95-124, 1996

[7] C. C. Chang and J. Keisler, *Model Theory*, Elsevier, 1998

[8] Romane Clark, "Vicious Infinite Regress Arguments", Philosophical Perspectives, 2; 369-380, 1988

[9] Stephane Demri, Ewa Orlowska, Dimiter Vakarelov, "Indiscernibility and Complementarity Relations in Information Systems", Preprint

[10] Harry Deutsch, "Identity and General Similarity", *Philosophical Perspectives, 12, Language, Mind, and Ontology*, 1998

[11] Patrick Doherty, Witold Lukaszewicz, Andrzej Szalaz, "Tolerance Spaces and Approximative Representational Structures", Proceedings of the 26th german

Conference on Artificial Intelligence, *Lecture Notes on Artificial Intelligence*, vol 281, 2003.

[12] P.T. Geach, “Identity,” *Review of Metaphysics*, 21, pp.3-12, 1967

[13] Jouni Jarvinen, “On the Structure of Rough Approximations”, Lecture Notes in Artificial Intelligence, Vol 2475, Springer, 2002

[14] Peter Klein, “Foundationalism and the Infinite Regress of Reasons”, Philosophical and Phenomenological Research, 58.4; 919-925, 1998

[15] G. Rodriguez-Pereyra, *Resemblance Nominalism*, Oxford: Clarendon Press, 2002.

[16] H. H. Price, *Thinking and Experience*, Hutchinson University Library, 1953

[17] Ewa Orlowska (ed), *Incomplete Information: Rough Set Analysis*, Physica Verlag, 1998

[18] B. Russell, “The World of Universals”, in Mellor, D. H. and Oliver, A. (eds.) *Properties*, Oxford University Press, 1997, 45-50.

[19] Tony Roy, “What is So Bad About Infinite Regresses?” Preprint

[20] Ju. A. Schreider, *Equality, Resemblance and Order*, Mir Publishers, 1975

[21] Joseph R. Shoenfield, *Mathematical Logic*, ASL, 2001

[22] Dimiter Vakarelov, “Information Systems, Similarity Relations and Modal Logics”, in [17], pp. 492-550

Appendices

Appendix A

Set-Theoretic Preliminaries

A **set** is a collection of objects. If X is a set and an object a is in the set, then we say that a is an **element** of X and write $a \in X$. If an object a is not an element of X, then we write $a \notin X$. In fact, a set is fully determined by its elements: two sets are identical if they have the same elements. There are several ways to describe a set. If a set X has finitely many elements, say $a_1, a_2, \ldots, a_n$, then we may write $X = \{a_1, a_2, \ldots, a_n\}$. If X has infinitely many elements, then, of course, we can not write all elements of X . However we have ways to describe X: We can write first few elements of X if elements of X so that we may see the pattern that the elements of X follow, if there is such a pattern. For example we may write $\mathbb{N} = \{0, 1, 2, \ldots\}$ to denote the set of natural numbers. Secondly, if there is a rule R that an object a is in X if and only if a satisfies that rule, then we may write $X = \{x : x \text{ satisfy } R\}$. For example, the set $\{x : x \in \mathbb{N}, x \equiv 0(mod4)\} = \{0, 4, 8, \ldots\}$.

Besides primitive objects, sets may contain other sets as elements. We call a set of sets a **family** of sets. Moreover, it may not contain anything. In this case we say that the set is empty. It follows from the definition of set identity that, there is only one empty set.

Let us write $R(x)$ is an object x satisfy the rule R. It is possible that given a rule R, there is no object satisfying R. In this case, $\{x : R(x)\} =$ **empty set**. We will denote the empty set by $\emptyset$ or $\{\}$.

Let X and Y be sets. If every element of X is an element of Y, we say that X is a **subset** Y and write $X \subseteq Y$. From the remark in the previous paragraph, one can easily see that $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$. Trivially $\emptyset \subseteq X$ and $X \subseteq X$ for every set X. If X is a set, the collection of all subsets of X forms a set. We call this set the **power set** of X and write $\mathcal{P}(X)$ for the power set of X. A subset of X besides X is called a **proper subset** of X. Given a set X we write $|X|$ for the number of elements of X. We say that $|X|$ is the **cardinality** of X. For a set X, $|\mathcal{P}(\mathcal{X})| = 2^{|X|}$

If $\{X_y : y \in Y\}$ is a family of sets, the *generalized* **union** of the sets X_y, $\bigcup_{y\in Y} X_y$, is the set $\{x : x \in X_y \text{ for some } y \in Y\}$ and the *generalized* **intersection** of the sets X_y, $\bigcap_{y\in Y} X_y$, is the set $\{x : x \in X_y \text{ for every } y \in Y\}$. In particular, given sets X and Y, the following are sets:

The **union** of X and Y, $X \cup Y$: $\{x : x \in X \vee x \in Y\}$,

The **intersection** of X and Y, $X \cap Y$: $\{x : x \in X \wedge x \in Y\}$,

The **relative complement** of X in Y, $X \backslash Y$: $\{x : x \in X \wedge x \notin Y\}$,

The **complement** of X, X^c: $\{x : x \notin X\}$,

The **symmetric difference** of X and Y, $X \triangle Y$: $(X \backslash Y) \cup (Y \backslash X)$.

We say that X and Y are **disjoint** if $X \cap Y = \emptyset$. A family of sets $\mathcal{F}$ is **pairwise disjoint** if $X \cap Y = \emptyset$ for every $X \neq Y$ in $\mathcal{F}$.

Unlike the **unordered pair** $\{a, b\}$, which is the same as $\{b, a\}$, $\{a, b, b, a\}$ or any other set having a and b as elements, we would like to have a list which have a *first* element, a *second* element and so on. We write (a, b) for the object which have a as the first element (or the first coordinate and b as the second element and call (a, b) as the **ordered pair** of a and b. Fortunately we may *represent* such lists as sets. Given objects a and b we may define $(a, b) = \{a, \{a, b\}\}$. We may see that this representation satisfy the requirement that we demand for our lists: $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. We may obtain longer lists: We define $(a, b, c) = (a, (b, c))$, $(a, b, c, d) = (a, (b, (c, d)))$ by repeated application of the ordered pair operation. Such a list with n elements is called an (ordered) n-tuple. Note that, any n-tuple is in fact an ordered pair and being so, can be represented by a set. For example, we may write $(a, b, c) = \{a, \{b, \{b, c\}\}\}$. Since the set theoretic representation does the job as remarked above and since any n-tuple has a natural set theoretic representation based on the representation of the ordered pair, we may thus accept the set theoretic representation as the ordered pair and use usual parentheses notation as an abbreviation but for our purposes, unlike for those of a set theorist, we may just accept ordered n-tuple just as a list $(a_1, \ldots, a_n)$ but knowing that we have such a representation at hand (in case we need it). We will use the notation $\pi_i(a_1, \ldots, a_n)$ for the i-th coordinate of the n-tuple $(a_1, \ldots, a_n)$.

Definition 11.0.1. Let X, Y be sets. The **cartesian product** of X and Y, $X \times Y$, is the set of ordered pairs with first coordinate in X and second coordinate in Y i.e., $X \times Y = \{(x, y) : x \in X, y \in Y\}$. In general $X_1 \times X_2 \times \cdots \times X_n = \{(x_1, x_2, \ldots, x_n) : x_i \in X_i\}$. We write X^n for the cartesian product of X with itself n times. A (binary, 2-ary) **relation between** X and Y is a subset of $X \times Y$ and a binary **relation on** X, is a subset of X^n. In general, an n-ary relation on X is a subset of X^n. We may write xRy for $(x, y) \in R$ An n-ary **relation among** sets $X_1, \ldots, X_n$ is a set of n-tuples with i-th coordinates from X_i. We may need cartesian product of infinitely many sets $X_i : i \in I$ alhough our relations will usually be finitary.

Definition 11.0.2. If R is a binary relation between X and Y, then the **domain** of R, $dom(R)$ is the set $\{z : z = \pi_1(x, y)$ for some $(x, y) \in R\} = \{x \in X : \exists y \in Y(xRy)\}$, the **range** of R, $Ran(R)$, is the set $\{z : z = \pi_2(x, y)$ for some $(x, y) \in R\} = \{y \in Y : \exists x \in X(xRy)\}$.

Definition 11.0.3. A binary relation R between X and Y is **one-one** if every $x \in dom(R)$ is paired with only one element $y \in Y$. R is a **one-one correspondence** if R is one-one and $dom(R) = X, ran(R) = Y$. We may similarly define one-many and many-one relations.

Definition 11.0.4. If R is a relation between X and Y and S is a relation between Y and Z then the **composition** of R and S, $S \circ R = \{(x,z) : \exists y \in Y((x,y) \in R, (y,z) \in S)\}$. Note that $T \circ (S \circ R) = (T \circ S) \circ R$. Thus, we may write $R \circ S \circ T$ etc.

The following examples will usually occur throughout the thesis:

Example 11.0.5.

1. $X \times X$, $\emptyset$ are relations on every set.
2. There is a special relation, identity, define on every set X, which is the set $\{(x,x) : x \in X\}$. This set is also called the *diagonal* of $X \times X$ for obvious reason.
3. Given a relation R between X and Y, $R^{-1} = \{(y,x) : (x,y) \in R\}$. R^{-1} is called the **inverse** of R.

Example 11.0.6.

1. $R \circ \emptyset = \emptyset \circ R$ for every set X and every relation R on X.
2. $(R \circ =) = (= \circ R) = R$ for every set X, and for every relation R on X.

Definition 11.0.7. Let R be a binary relation on X. R is

1. **Reflexive** if $\forall x \in X(xRx)$,
2. **Irreflexive** if $\forall x \in X(\neg xRx)$,
3. **Symmetric** if $\forall x, y \in X(xRy \Rightarrow yRx)$,
4. **Antisymmetric** if $\forall x, y \in X((xRy \wedge yRx) \Rightarrow x = y)$,
5. **Asymmetric** if $\forall x, y \in X \neg(xRy \wedge yRx)$,
6. **Transitive** if $\forall x, y, z \in X((xRy \wedge yRz) \Rightarrow xRz)$,
7. **Connected** or **total** if $\forall x \in X \forall y \in X(x \neq y) \rightarrow (xRy \vee yRx)$ (we say that x, y are comparable),
8. **Dense** if $\forall x \neq y \in X \exists z \in X(z \neq x \wedge z \neq y \wedge xRz \wedge zRy)$ & $\exists x, y(x \neq y)$,

Definition 11.0.8. Let X be a set and R is a relation on X. R is:

1. **Partial order** if it is reflexive, transitive and antisymmetric.
2. **Strict order** if it is asymmetric and transitive.

3. **Total order** or **linear order** if it is a connected partial order.

Definition 11.0.9. Let (X, R) be a poset and A non-empty subset of X. An element $x \in X$ is called an **upper bound** of A if aRx for every $a \in A$. An element $x \in X$ is called a **lower bound** of A if xRa for every $a \in A$. If there is a maximum $x \in X$ among lower bounds of A then x is called the **greatest lower bound** or **infimum** of A. We write $inf(A)$ for the infimum of A. If there is a minimum $x \in X$ among the upper bounds of A, x is called the **least upper bound** or **supremum** of A. We write $sup(A$ for the supremum of A. If $sup(A)$ exists for every non-empty subset A of X, then (X, R) is called a **complete** partial ordering.

A connected non-empty subset of a poset is called a **chain**. The following will prove to be useful:

Theorem 11.1 (Zorn Lemma). *Let (X, R) be a poset and $\emptyset \neq A \subseteq X$ be such that every chain of elements in A has a maximum element. Then, A has a maximal element.*

Definition 11.0.10. A partial order R on a set X is **well-founded** if every non-empty subset of X has an R-minimal element. A well-founded linear ordering is called a **well-ordering**.

Example 11.0.11. $(\mathcal{P}(X), \subseteq)$ is a poset for every set X. It is usually not well-founded, not connected. But try to give necessary and sufficient conditions for it to be well-founded or connected.

Definition 11.0.12. Let X be a set, R any relation on X, A a subset of X. We define the **restriction** of R to A as the set $A^n \cap R$. We use the notation $R \restriction A$ for the restriction of R to A. If $S = R \restriction A$ we also say that R is an **expansion of S to X**. Note that, although there is a unique relation which is the restriction of R to a subset of X, there may be many different expansions of a relation to a bigger set. If we want to leave the bigger set unspecified, we may say **expansion to a bigger set**. If S is obtained from R by not enlarging the set but just by adding new pairs we call S just an **expansion** of R.

Remark 11.0.13. Although a relation R on a set X may not have a property P, we may try to restrict R to a subset of X, or add new pairs of elements of X to obtain a relation S which have the property P. For example, it is always possible to expand a non-transitive R this way to obtain a transitive relation. The smallest transitive relation containing R is called the **transitive closure** of R. We will use R^* for the transitive closure of R.

Remark 11.0.14. By an **index set** we always mean a well-ordered set. We will use this to form indexed families: an indexed family of sets is a family of sets $\mathcal{F} = \{X_i : i \in I\}$ where I is an index set.

Definition 11.0.15. R is an **equivalence relation** if R is reflexive, symmetric and transitive. If R is an equivalence relation on X, we define the **equivalence class** of x, $[x]_R = \{y \in X : xRy\}$. Note that $[x]_r = [y]_R$ if and only if xRy. Moreover any two equivalence classes are either equal or disjoint.

Definition 11.0.16. Let X be a set. A family of subsets $\mathcal{F}$ of X called a **partition** of X if $A \cap B = \emptyset$ for every $A, B \in \mathcal{F}$ (i.e. $\mathcal{F}$ is a disjoint family), and $\bigcup_{A \in \mathcal{F}} A = X$ that is, the union of all sets in $\mathcal{F}$ cover X.

Remark 11.0.17. If R is an equivalence relation on a set X, then $\{[x]_R : x \in X\}$ is a partition of X. This family is denoted by $X_{/R}$. A partition of X naturally induces an equivalence relation R on X: Let xRy if and only if x and y lies in the same part.

Definition 11.0.18. Let X, Y be sets. A relation F between X and Y is a **function** from X to Y, if R is one-one and $dom(F) = X$. We write $F : X \longrightarrow Y$ for "F is a function from X into Y".

We usually use small case Latin and Greek letters $f, g, h \ldots, \alpha, \beta, \ldots$ for functions. If $f : X \longrightarrow Y$ then, X is called the domain of f, as in the case of relations, and Y is the **codomain** of f. If f paires an $x \in X$ with $y \in Y$, we say that y is the value of f at y and write $f(x) = y$, x is called the **argument** of f. If $A \subseteq dom(f)$, then $f[A] = \{f(a) : a \in A\}$ is called the **image** of A under f. If $B \subseteq Y$, then $f^{-1} = \{f^{-1}(b) : b \in B\}$. Note that, $f[A] \cap f[B] \supseteq f[A \cap B]$ but the inverse inclusion may not hold. Moreover, $f[A \cup B] = f[A] \cup f[B]$. f^{-1} is more respectful to union and intersection. If X and Y are sets, we write X^Y for the set of all functions from X into Y.

Remark 11.0.19. Cartesian product of an indexed family of sets $\{X_i : i \in I\}$ may also defined by use of functions, namely $\Pi_{i \in I} X_i = \{f | f : I \longrightarrow \bigcup_{i \in I} X_i, f(i) \in X_i$.

An element of this set is a set of pairs $\{(i, f(i)) : i \in I\}$. However we may think of this set as the sequence $(x_i)_{i\in I}$. Note that, on this interpretation X^n means the set of all functions from $n = \{0, 1, \ldots, n-1\}$ into X. The assertion that $\Pi\{X_i : i \in I\}$ is non-empty when each $X_i \neq \emptyset$ is not a triviality and is called the **Axiom of Choice**. This axiom is equivalent to the assertion that every nonempty set can be well-ordered (i.e, we may put a well-order on it). One direction is easy: If each X_i non-empty and well-ordered then we define a choice as: "Chose the first element of each X_i."

Since functions are relations, we might use *much* of the terminology about relations to talk about functions as well (exceptations occur e.g., we do not say one-many function). Moreover:

Definition 11.0.20. If $f : X \longrightarrow Y$, we say that f is a **injection** or **one-to-one** if f is a one-one relation, **surjection** or **onto** if $Ran(f) = Y$, **bijection** if f is both injective and surjection.

Note that one-one and one-to-one are different: a function is necessarily one-one as a relation but two different elements from the domain of a function may take the same value.

Definition 11.0.21. Let $X_1, X_2, \ldots, X_n, Y$ be sets and let $f_i : X_i \longrightarrow Y$ be functions. We may define $f : \Pi_{i=1}^n X_i \longrightarrow Y$ as

$$f(x_1, \ldots, x_n) = (f_1(x_1), \ldots, f_n(x_n))$$

We may sometimes use $\vec{x}$ for $(x_1, \ldots, x_n)$ when n is known and use the notation $f(\vec{x})$ for $f(x_1, \ldots, x_n)$.

Definition 11.0.22. The composition of functions $f : X \longrightarrow Y$, $g : Y \longrightarrow Z$ is denoted by $g \circ f$ and may be defined as $(g \circ f)(x) = g(f(x))$. It follows that, we need $Ran(f) \subseteq Dom(g)$ in order that $g \circ f$ be defined.

As in the case of relations $h \circ (g \circ f) = (h \circ g) \circ f$ whenever all compositions are defined.

Definition 11.0.23. Let $f : X \longrightarrow Y$. A function $g : Y \longrightarrow X$ is called the **inverse** of f, if $g \circ f = id_X$ and $f \circ g = id_Y$. If there is such a g then it is called the **inverse** of f. It is unique if exists and denoted by f^{-1}.

Remark 11.0.24. A function $f : X \longrightarrow Y$ has an inverse if and only if f is a bijection. If f is not injective say $f(a) = f(b) \in Y$ for some $a \neq b$ in X then f^{-1} would not be unique and f^{-1} would fail to be a function. If f is not onto say $y \in Y \backslash ran(f)$ then f^{-1} would not be determined by f.

Definition 11.0.25. An n-ary function from X to Y is a relation f on $X^n \times Y$ such that for every $(x_1, \ldots, x_n) \in X^n$ there is a unique $y \in Y$ such that $(x_1, \ldots, x_n, y) \in f$. We write $f(x_1, \ldots, x_n) = y$. An n-ary function on a set X is called an n-ary **operation**.

Let f be an n-ary operation on a set X and $\emptyset \neq A \subseteq X$ is called **closed under** f, if for every $x \in A$, $f(x) \in A$. The *closure* of a non-empty $A \subseteq X$, with respect to f, is the smallest closed subset of X including A.

A binary (2-ary) operation on a set X is **commutative** if $f(x, y) = f(y, x)$ for every $x, y \in X$, **associative** if $f(f(x, y), z) = f(x, f(y, z))$ for every $x, y, z \in X$.

Appendix B

Turkish Summary

Gerçeklik hakkında düşünebilmemiz ancak nesneleri birbirleri ile iliskileri içinde ele alabilmemiz sayesinde mümkündür. Bunun nedeni sadece zihnimizin tam bir ayrıklık içinde bulunan büyük ve düzensiz bir yığını algılayamaması değildir: eğer nesneleri birbirleri ile iliski içinde algılayamazsak, hiçbir kavram olusturamayız. Kavramlar ise düsünmenin ve bilginin temel taslarıdır.

Bununla birlikte birbiri ile iliskili nesneler bütünü olarak kabul ettiğimiz "evrenimiz" asıl Evren olmayabilir. Evrendeki nesneler arasında varolduğunu düsündüğümüz iliskilerin en azından bir kısmı, sadece bizim bilme çabamızın bir sonucu olarak varsaydığımız iliskiler olabilir. Evrenin aslında nasıl olduğu ise elbetteki bizim bilgisel ihtiyaçlarımızdan bağımsızdır. Biz nesnel olarak aslında evrenin sadece bir parçasıyız ve evrenin ne olduğunu belirleme anlamında, evrenin herhangi bir başka parçasından bir farkımız olmamalıdır.

Bu düşünceler doğru kabul edilirse, evrenin nasıl olduğunu ve nelerden oluştuğunu tasvir etmede kullandığımız temel kavramların tanımlarında öznel bileşenlere yer olmamalıdır.

Evrenin hangi nesnelerden oluştuğunu bildiğimizi varsayalım. Bu durumda temel soru, nesneler hakkındaki düşünüşümüzün bir parçasını oluşturan bağıntıların hangileri gerçekten bu nesneler arasında sağlanabilecek bağıntılara karşılık geldiği olacaktır. İkinci olarak, eğer bir bağıntı hakikaten nesneler arasında (bizim düşünüşümüzden bağımsız olarak) sağlanabilecek bir bağıntı ise, ne şekilde kavranmalıdır?

Bu çalışmada özdeşlik-benzeri varlıkbilimsel bağıntıları, bunların sembolik yapılarda temsili açısından inceledim. Buradaki "özdeşlik-benzeri bağlantılar" deyimi ayırdedilemezlik, karşılıklı-değiştirilebilirlik (ya da muadiliyet) ve benzerlik bağıntılarını ifade etmektedir. Benzerlik ya da, bu bağıntı simetrik olduğundan, benzeşme, bağıntısına daha fazla yer ayırdım. Diğer bağıntılardan sözetmem aslında,daha çok, benzerlik bağıntısının ele alış biçimimi belirginleştirmek içindir.

Bazı bölümler daha çok özdeşlik-benzeri bağıntıların özellikleri felsefi açıdan incelenmesine, bazıları ise bu bağıntıların biçimsel özelliklerinin temsiline ayırılmıştır.

Felsefi olarak özdeşlik-benzeri bağıntılara dair katıldığım ve savunmaya çalıştığım görüşleri şylece sıralayabilirim:

1. Ayırd-edilemezlik, karşılıklı-değiştirilebilirlik ve benzerlik bağıntıları nesnelerin taşıdığı özelliklere dayanarak açımlanmalıdır. Tek tek nesneler arasında olduğu gibi, nesne dizileri arasında da bu bağıntıların varlığından bahsetmeye bir engel yoktur.

2. Ayırd-edilemezlik bağıntısından farklı olarak, karşılıklı değiştirilebilir olma ve benzerlik bağıntıları bağlamsal olarak tanımlanması gereken bağıntılardır.

3. Birinci ve ikinci maddelerle tanımlanabilecek özdeşlik-benzeri bağıntılar kuramı açıklayıcı gücü açısından diğer kuramlara tercih edilmelidir.

İlk iddia geleneksel olarak tümeller hakkındaki gerçekçi görüşün bir kısmını oluşturmaktadır. Aristocu gerçekçilik ya da içkin gerçekçilik açısından tümeller tikellerden bağımsız olarak değil tümellerin içinde vardır. Platoncu gerçekçilik ise tikellerden ve tikel gruplarından bağımsız özellik ve bağıntı tümellerinin varlığını ileri sürer. Bu şekliyle Platoncu görüşün bugünün felsefecileri arasında pek kabul görmediği bilinmektedir. Bu görüşün iki temel eleştirisi şunlardır: İlk olarak, Platoncu varlıkbilimin vaarlıkbilimsel ekonomi ilkesine uymadığı ve felsefi bir problemi çözmek için pek çok nesnenin varlığını ileri sürdüğü söylenmektedir. İkinci olarak Platonculuk nesnelere ilişkin bilgiyi olanaksız hale getirmektedir. Bu iddiaya dayanak olarak, Platonculuğun nesnelerin özelliklerinin açıklanmasında bizim nedensel ilişki kuramayacağımız birtakım şeylere başvurması gösterilmektedir. Aristocu gerçekçilik daha ılımlı ve ekonomik bir gerçekçilik biçimi olarak görülmektedir. Bir şekilde ifade edildiğinde Aristoculuk özellik ve bağıntıların varlığını kabul etmekle birlikte bunun bağımsız bir varolma biçimi olmadığını söyler. Varolmanın değişik biçimleri olup olmadığı sorusu bir yana, Aristocu görüşün varolduğunu kabul ettiği şekliyle tümeller de başka sorulara yol açmaktadır. Örneğin, bir tümel hiçbir tikel içinde bulunmadığı bir süre boyunca varlığını sürdürmekte midir? Sürdürmekte denirse bu bir tikel içinde olmayacağından Aristocu görüşün özüne aykırı olacağından sürdürmemekte denmesi daha akla yatkındır. Bu durumda ise şu soru akla gelir: Bir tikel bir zamanlar T dediğimiz tümeli tekrar taşımaya başladığında bu tümel tekrar var mı olacak yoksa yeni bir tümel mi olacak? Kanımca bu bir tanım problemi değil, Aristocu tümellerin özdeşlik koşullarına dair gerçek bir problemdir.

Metafizikte gerçekçilik karşıtı görüş "adcılık" olarak anılmaktadır. Buna göre, gerçekçi görüşlerde olduğu gibi, tümellerin varlığının yadsınması ile belirlenen bu görüşün de daha katı ya da daha ılımlı denebilecek değişik biçimleri ortaya çıkmıştır. En katı adcı görüşe göre özellik ve bağıntılar sadece dilde ortaya çıkar ve cümlelerin yapısı dolayısıyla biz, cümlelerin anlamlı olabilmesi için, bunların da dildışı varlıklara işaret ettiğini düşünürüz. Ilımlı görüşler ise özellik ve bağıntı adlarını sadece boş söz olarak görmeyip bunların başka nesneler ya da olaylara başvuru yoluyla elenmesi yoluna gitmişlerdir.

Bu çalışmanın konusu açısından en büyük önem taşıyan benzerlik-adcılığına göre özellik tümelleri tikeller arasındaki tikel benzerlik ilişkileri yardımıyla elenebilir bu nedenle de bu tümellerin varlığını ileri sürmeye gerek yoktur. Buna göre "a bireyi F özelliğini taşır" gibi önermeler a tikelinin birtakım paradigmatik-F tikellerine yeterince yakın benzerliği ile açıklanabilir.

Benzerlik adcılığına karşı en sık ileri sürülen eleştiri, bu görüşün benzerlik bağıntısının kendisini elemesinin zorluğuna ilişkindir. Gerçekten de, a ve b tikellerinin benzer olduğunu söyleyebilmek için benzerlik adcılığı açısından başka bazı c, d tikel ikililerine benzemesi gibi bir yeni ikinci derece benzerlik ilişkisine başvurmak zorunlu görünüyor. Bu kez de a, b ikilisi ile c, d ikilisi arasındaki benzerlik ilişkisinin açıklanması gerekir. Bu ise üçüncü derece bir benzerlik ilişkisini gerektirir. Bunun böylece süreceği düşünüldüğünde, adcılığın da ekonomik bir kuram olma savı da zayıflar.

Özdeşlik konusunda geleneksel görüş her nesnenin kendisi ile özdeş olduğu ve başka hiçbir nesne ile özdeş olmadığıdır. Bu anlayışa göre, özdeşlik probleminin sadece dil dolayısıyla ortaya çıktığı düşünülür. Elimizdeki iki farklı tasvir iki farklı nesneyi mi yoksa bir tek nesneyi mi tasvir etttiğini sorabiliriz. Mantıksal olarak özdeşlik birinci derece aksiyomlar yoluyla belirlenir. Bu belirlenimin özdeşliğin bir tanımı ya da felsefi bir çözümlemesi anlamına gelmediğini söylemek gerekir.

Ayırdedilemezlik bağıntısı, özdeşlik-benzeri bağıntılar içinde özdeşliğe en yakın bağıntıdır. Ayırdedilemezlik tüm özelliklerin uyuşması olarak tanımlanır. Leibniz'in ayırdedilemezlerin özdeşliği ilkesi, ayırdedilemeyen nesnelerin aslında özdeş olduğunu dile getirir. Ayırdedilemezlik bağıntısının nesneler arasında bir bağıntı olarak kabul edilebilmesi için, nesnelerin hakikaten tüm özelliklerinin göz önüne alınması gerekir. Belirli bir yöntem kullanılarak ile birbirinden ayırdedilemeyecek olma tam bir ayırdedilemezlik bağıntısı olarak kabul edilemez.

Karşılıklı değiştirilebilir olma bağıntısı ise nesnelerin tüm özelliklerinin değil ama belirli bir grup içindeki tüm özelliklerinin uyuşması olarak tanımlanır. Biçimsel olarak, $\mathcal{P}_0$ bir özellikle kümesi olmak üzere, herhangi iki a ve b nesnesinin $\mathcal{P}_0$ özellikle kümesine göre karşılıklı-değiştirilebilir (kısaca $\mathcal{P}_0$-değiştirilebilir) olması için gerekli ve yeterli koşul

$$\forall P \in \mathcal{P}_0(P(a) \leftrightarrow P(b))$$

biçiminde ifade edilebilir.

Karşılıklı-değiştirilebilir olma bağıntısı için benzer bir tanım olarak

$$\forall P \in \mathcal{P}_0(P(a) \wedge P(b))$$

düşünülebilirdi. Bu tanımın eksikliği ilk tanımla kıyaslanarak anlaşılabilir. İlk tanımın kimi önemli özellikleri şöylece sıralanabilir.

1. Tüm $\mathcal{P}_0$-özelliklerini sağlayan iki nesne $\mathcal{P}_0$-değiştirilebilir.

2. Hiçbir $\mathcal{P}_0$-özelliğini sağlamayan iki nesne $\mathcal{P}_0$-değiştirilebilir.

3. Hiçbir $\mathcal{P}_0$-özelliğini sağlamayan herhangi bir nesne, en az bir $\mathcal{P}_0$-özelliğini sağlayan hiçbir nesne ile $\mathcal{P}_0$-değiştirilemez.

4. Tüm $\mathcal{P}_0$-özelliklerini sağlayan herhangi bir nesne, en az bir $\mathcal{P}_0$-özelliğini sağlamayan hiçbir nesne ile $\mathcal{P}_0$-değiştitilemez.

Bu özelliklerin sezgisel olarak, karşılıklı-değiştirilebilir olma bağıntısının sahip olması gereken özellikler olduğunu düşünüyorum. İki tanımın bu özellikler hakkında ayrıldığı nokta (ii) maddesinde belirtilmiştir. Bu maddeye göre "$\mathcal{P}_0$-faydasız" iki nesne karşılıklı değiştirilebilir. İstenirse, bir tanım olarak, denilebilir ki ikinci tanıma uyan nesneler güçlü-karşılıklı değiştirilebilir.

Benzerlik bağıntısına dair varlıkbilimsel kuramların kıyaslanmasında gözönüne alınması gerekenler arasında şunları sayabiliriz:

1. Kuram mümkün olduğunca fazla sayıda va tipte nesnelerin arasındaki benzerlik ilişkilerinin açıklayabilmelidir. Belirli tipteki nesneler arasındaki benzerlik ilişkilerinin olabileceğini sadece ilkece kabul etmek yerine bu ilişkilerin nasıl mümkün olduğunu açıklayan bir kuram tercih edilmelidir.

2. Benzerlik ya var ya da yok denebilecek bir bağıntı olmayıp derecelerle ortaya çıkabilen bir bağıntı olduğundan daha yakın ya da daha uzak benzerlik bağıntılarının olabildiğince ayrıntılı bir şekilde incelenmesine olanak sağlayan bir kuram tercih edilmelidir.

Benzerlik adcılığı açısından bu iki noktayı değerlendirirsek, ilk nokta ilişkisiz görünmektedir. Benzerlik adcılığı tikel benzerlik bağıntılarını temel aldığı için, benzerlik bağıntısının çözümlenmesi sorunuyla ilgilenmesi gerekmez denilebilir. Olsa olsa, birtakım tikel benzerlikler basit benzerlik ilişkileri olarak kabul edilip, karmaşık benzerliklerin açıklanmasında bunlara başvurulabilir. Her kuramın temel kabul ettiği birtakım kavramlar vardır. Bazı benzerlikleri temel kabul etmenin nedenleri arasında bunları ani bir şekilde ve belirgin nedenleri olmadan deneyimlediğimiz iddiasının önemli bir yeri vardır. Bunu tartışmayıp benzerlik adcılığının bu yaklaşımını doğru kabul etsek bile, benzerlik dereceleri konusunu da açıklamasız bırakması benzerlik adcılığını zayıf bir kuram durumuna düşürmektedir. Benzerlik adcıları tarafından benzerlik dereceleri kabul edilmiş ve gerçekçi görüşe karşı bir itiraz olarak ileri sürülmüştür. Bu itirazın nedenini anlamak kolaydır. Eğer iki nesnenin benzer olması, bunların en az bir ortak özelliğe sahip olması ile açıklanırsa, herhangi iki nesne ya ortak bir özelliği olacağından ya da olmayacağından, bu tanıma göre ya birbirine benzeyecek ya da benzemeyecektir.

Ancak bu tanımın yetersizliği görülse bile gerçekçi görüş içinde daha ince bir çözümleme yapma imkanı vardır. İlk olarak, a, b ve c, d herhangi iki nesne ikilisi olduğunda, a ve b nesneleri c ve d nesnelerinden daha çok ya da daha az ortak özelliğe sahip olabilir. İkinci olarak, daha sonra da belirteceğim gibi a ve b nesneleri c ve d nesnelerine göre daha belirli bir ortak özelliğe sahip olabilir. Örneğin, iki kırmızı nesne, iki (farklı) renkli nesneye göre daha yakın benzer.

Benzerliğin özelliklerle açıklandığı görüş bu iki noktada da avantajlı görünmektedir. Her tipteki nesne birtakım özellikler taşıdığından bunlar arasındaki benzerlikleri açıklamakta bir engel bulunmamaktadır. Soyut nesneler ya da gözlemlenemeyecek fiziksel nesneler bile bu açıdan ele alınabilir.

Karşılıklı değiştirilebilir olma ve benzerlik bağıntılarının belirli özellikler kümelerine göre tanımlanması çelişik görünebilir. Denebilir ki, nesnel olarak tanımlanması istenen bir bağıntının bağlamsal olarak belli bir özellik kümesine dayanarak tanımlanmaktadır. Bu ise istenenin aksine ancak öznel benzerlik bağıntıları tanımlar. Bu iddia anlaşılabilir genel bir benzerlik bağıntısının olabileceğinin düşünülmesinden kaynaklanır. Bu görüşü paylaşmadığım için "benzerlik bağıntısı"nı tanımlamaya çalışmak yerine "farklı benzerlik bağıntıları tanımlamak" daha gerçekçi bir tutum olarak kabul ediyorum. Genel bir benzerlik bağıntısını reddetme nedenleri ise açıktır. İlkin, benzerliği, özelliklere ilişkin bir kısıtlama yoluna gitmeden, en az bir ortak özelliğe sahip olma olarak kabul edersek, herhangi iki nesne "kendilerine-özdeş-olma" özelliğini paylaşma anlamında benzer olacaktır. Bu nedenle, bu şekilde tanımlanan bir benzerlik bağıntısı mümkün olsa bile (ki bu burada verilen tanımla da çelişmez) hemen her durumda bu tanım çok zayıf bir benzerlik kavramına denk düşecektir.

İkinci kısımda özdeşlik benzeri bağıntıların temsil edilebileceği bazı yapıları ele aldım. Bunlar:

1. Tolerans uzayları,
2. Küme kuramsal modeller,
3. Veri sistemleri

olarak sıralanabilir.

Tolerans uzayları Schreider [20] tarafından benzerlik ilişkileri temel alınacak şekilde geliştirilmiştir. Schreider'e göre benzerlik bağıntıları yansımalı ve simetrik ama genellikle geçişli olmayan bağıntılardır. Buna göre yansımalı ve simetrik bir bağıntı *tolerans bağıntısı* boş olmayan bir küme ve bu küme üzerindeki bir tolerans bağıntısından oluşan bir yapı da *tolerans uzayı* olarak tanımlanmıştır. Tolerans uzayları küme kuramsal modellerin bir türü olduğu için varlıkbilimsel kuramların temsili konusunda küme kuramsal modellere yöneltilebilecek itirazlar tolerans uzayları için de geçerlidir. Ek olarak, herhangi bir yansımalı ve simetrik bağıntının bir benzerlik bağıntısı olarak kabul edilemeyeceği düşünülürse, tolerans uzaylarının benzerlik bağıntısının felsefi incelenmesince eksik kalacağını ileri sürebilir.

Küme kuramsal modeller Tarski tarafından geliştirilmiş ve özellikle matematiksel amaçlar açısından bugüne kadar geniş kabul görmüştür. Genel varlıkbilim açısından ise özellik ve bağıntıların ve özelleme bağıntısının küme kuramsal yorumlanmasına dair pek çok itiraz ileri sürülmüştür. Özellik ve bağıntıların kümelerle, yani kaplamlarla özdeşleştirilmesi varlıkbilim açısından kabul edilemeyecek bir aşırı-basitleştirmeye yol açmaktadır. Şunu kabul gerekir ki, küme teorisi oldukça güçlü bir temsil gücüne sahiptir ve ilk sunumda küme kuramsal görünmeyen yapılar bile sonuçta küme kuramsal bir yapı içinde temsil edilebilir. Ama küme kuramsal analiz ve küme kuramsal çözümleme başka kavramlardır. Kanımca, varlıkbilimsel bir kuramın küme kuramında başarılı bir şekilde çözümlendiğini ileri sürebilmek için kodlamada kullanılan küme kuramsal nesne ve bağıntıların felsefece incelenen nesne ve bağıntıların kabul edilebilir bir ölçüde doğal karşılık sağlaması gerekir.

Pawlak tarafından tanımlanan bilgi temsili sistemlerinin çözümleyici varlıkbilim açısından güçlü bir araç bir sağladığını düşünüyorum. Bir bölümde bu sistemler Vakarelov tarafından sunulduğu şekliyle tanımlanmış ve özdeşlik-benzeri bağıntıların bu sistemlerde temsiline yer verilmiştir. Ardından, benzerlik derecelerine de izin verecek bir esneklik sağlamak amacıyla, bu bölümde üst-derece nitelik sistemleri tanımlanmış ve bu sistemlerde benzerlik bağıntısı ve derece kavramı incelenmiştir.

Varlıkbilimsel bilgi temsili sistemleri (1) özelllik sistemleri ve (2) nitelik sistemleri olarak adlandırılmaktadır.

Tanım: Bir *özellik sistemi*

$$\mathbf{S} = (Nes_S, \ddot{O}z_S, f_S)$$

ve

1. $Nes_S \neq \emptyset$ bir *nesneler* kümesi,
2. $\ddot{O}z_S$ boş ta olabilen, bir *özellikler* kümesi,
3. $f_S : Nes \longrightarrow \mathcal{P}(\ddot{O}z_S)$ her nesne için x, $f_S(x)$, $\mathbf{S}$ içinde *x hakkındaki veri*

olacak şekilde bir üçlüden oluşmaktadır.

Tanımdan anlaşılacağı gibi özellik sistemlerinde özellik ve özelleme tanımlanmamış olarak bırakılmaktadır. Her ne kadar bir özellik için o özelliğe sahip nesneler kümesinden, yani o özelliğin kaplamından, bahsetmek mümkünse de, özelliğin bu kümeye özdeş olduğunu söyleyemeyiz. Dikkat edilirse, aynı nesnelerin sağladığı "iki" özellik olabilir. Eğer özellik ile özelliğin kaplamı özdeş olsaydı bu mümkün olmazdı.

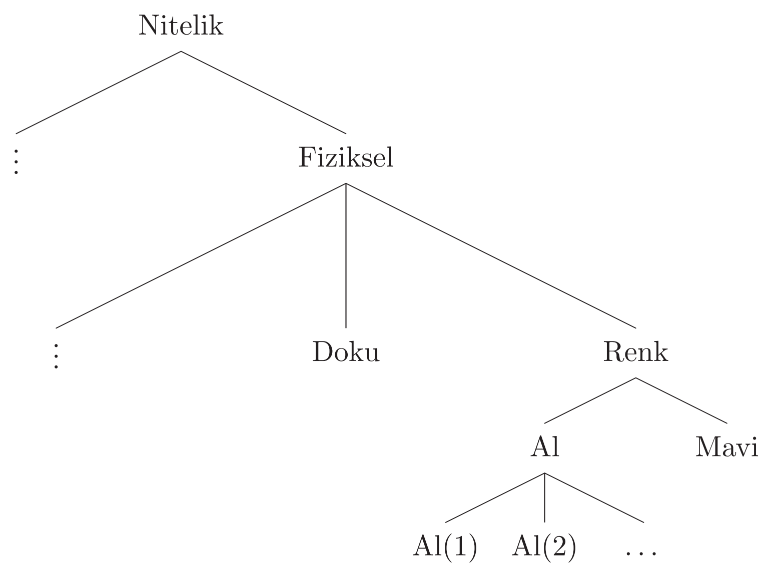
Özdeşlik-benzeri bağıntıların özellik sistemlerinde temsili ise aşağıdaki şekilde tanımlanabilir.

Definition 11.0.26. $\mathbf{S} = (Nes_S, \ddot{O}z_S, f_S)$ bir özellik sistemi, x ve y ise bu sistemde iki nesne olsun. Buna göre,

1. x ve y *ayırdedilemezdir*, $x \cong y$, $f(x) = f(y)$ ise,
2. x ve y $Pr_0 \subseteq Pr$ *a göre değiştirilebilir* , $x \risingdotseq_{Pr_0} y$, $f(x) \cap Pr_0 = f(y) \cap Pr_0$ ise,
3. x ve y *pozitif olarak benzer*, $x\Sigma_S y$, $f(x) \cap f(y) \neq \emptyset$ ise,
4. x ve y $Pr_0 \subseteq Pr$ *a göre pozitif olarak benzer* , $x\Sigma_{Pr_0} y$, $f(x) \cap f(y) \cap Pr_0 \neq \emptyset$ ise,
5. x ve y *negatif olarak benzer*, $xN_S y$, $\overline{f}(x) \cap \overline{f}(y) \neq \emptyset$ ise,

6. x and y are Pr_0, *$xN_{Pr_0}y$ a göre negatif olarak benzer* , $\overline{f}(x) \cap \overline{f}(y) \cap (Pr \backslash Pr_0) \neq \emptyset$ ise.

Özellik sistemleri, küme kuramsal modellere göre, ayrı kaplamdaş özelliklerin temsil edilebilmesi gibi avantajları olsa da, varlıkbilimsel kavram ve bağıntıların temsili açısından yetersiz kalmaktadır. Ayrıca bağlamsal özellikler ancak tüm özellikler kümesinin herhangi bir altkümesi olarak tanımlanabilmektedir. Dahası, kaplamdaş ama özdeş olmayan özelliklerin temsiline izin vermekle birlikte, bu sadece bir sentaktik imkanla sınırlı kalmaktadır. Nitelik sistemleri ise özelliklerin ayrıklığını açıklamakta daha çok olanak sağlar. Elbette, nitelik sistemlerinin özelliklerin özdeşlik koşullarına ilişkin felsefi problemi çözdüğünü iddia etmiyorum. Söylediğim, sadece, nitelik sistemlerinin, özelliklerin daha ince bir temsili imkanını verdiğidir.



Tanım: *Bir nitelik sistemi*

$$\mathbf{S} = \langle Nes, Nit, De\breve{g}, f \rangle$$

ve

1. $Nes \neq \emptyset$ bir *nesneler* kümesi,

2. Nit elemanları *nitelikler* olarak adlandırılan bir küme,

3. Her bir $a \in Nit$, için $De\breve{g}(a)$ *a niteliğinin değerleri*,

4. Her bir ikili f fonksiyonu ve her bir $x \in Nes$, her bir $a \in Nit$, için $f(x,a) \subseteq De\breve{g}(a)$ *a niteliğine göre* **S** *içinde x hakkındaki veri*

olacak şekilde bir dörtlüden oluşmaktadır.

Nitelik sistemleri pek çok özdeşlik-benzeri bağıntının tanımlanmasına olanak sağlamaktadır. Üst-derece nitelik sistemleri ise, değerleri bir alt dereceden nitelikler olan üst-derece nitelikleri de kullanarak varlıkbilimde belirlenebilir-belirlenim ilişkisini de temsil eder ve bu ilişki yardımıyla birinci derece sistemlerde yer almayan derece kavramları tanımlamak mümkün olur.

Belirlenebilir-belirlenim bağıntısı için genel bir tanım yapmak mümkün ise de, çalışmanın amacı için örneklemek yeterli olacaktır. Kırmızı olma özelliği belirli bir tümeldir. Renk ise daha genel bir "özellik" olarak düşünülebilir çünkü daha az belirlidir. Burada "kırmızı olma" özelliği "renk" genel özelliğinin bir *belirlenimi*, renk özelliği için ise kırmızı olma özelliğine göre bir *belirlenebilir*dir deriz.

Bu bağıntı benzerlik derecelerinin gerçekçi kuramdaki temsili açısından önemlidir. Şöyle ki, örneğin iki kırmızı nesne, iki "renkli" nesneye göre daha yakın bir benzerlik ilişkisi içindedir. Kırmızının aynı tonuna sahip iki nesne ise iki kırmızı nesneye göre daha yakın bir benzerlik ilişkisi içindedir. Bu örnekleri genelleştirerek, iki nesne ne kadar belirli bir ortak özelliğe sahip ise, o kadar yakın bir benzerlik bağıntısı içindedir diyebiliriz.

Özdeşlik-benzeri bağıntılarım nitelik sistemlerinde temsili ise aşağıdaki şekilde tanımlanabilir.

Definition 11.0.27. $\mathbf{S} = \langle Nes, Nit, De\breve{g}, f\rangle$ bir nitelik sistemi, x ve y ise bu sistemde iki nesne olsun. Buna göre aşağıdaki benzerlik bağıntılarını tanımlayabiliriz:

1. x ve y are *zayıf-pozitif olarak benzer*, $x\Sigma_S y$, $\exists a \in At : f(x,a) \cap f(y,a) \neq \emptyset$ ise,

2. x and y are *zayıf-negatif olarak benzer*, $xN_S y$, $\exists a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \emptyset$ ise,

3. x and y are *güçlü-pozitif olarak benzer*, $x\sigma_S y$, $\forall a \in At : f(x,a) \cap f(y,a) \neq \emptyset$ ise,

4. x and y are *güçlü-negatif olarak benzer*, $x\upsilon_S y$, $\exists a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \emptyset$ ise.

Belirttiğim gibi belirlenebilir-belirlenim bağıntısının da benzerlik dereceleri ile ilgili olarak kullanılabileceği ileri sürülmüştür [16]. Bu bağıntının ve bu bağıntıya dayanan benzerlik derecelerinin temsili için gereken üst derece niteliklerin de içerildiği sistemlere genel nitelik sistemleri diyebiliriz.

Definition 11.0.28. Genel nitelik sistemi,

$$\mathbf{S} = \langle Nes, Ba\breve{g}, Nit, De\breve{g}, f\rangle$$

şeklinde gösterilen ve

1. $Nes \neq \emptyset$ bir nesneler kümesi,

2. $Ba\breve{g} = \{Ba\breve{g}^{(1)}, Ba\breve{g}^{(2)}, \ldots\}$ her $Ba\breve{g}^{(n)}$ n-li bağıntılar kümesi,

3. $Nit = \{Nit^{(n)}_{(m)} : n, m \in \mathbb{N}^{+}\}$, $Nit^{(n)}_{(m)}$ m-inci dereceden n-li ntelikler kümesi,

4. $De\breve{g} : Nit \longrightarrow Nit \cup Ba\breve{g}$ değer fonksiyonu: $De\breve{g}(a) \subseteq Nit^{(n)}_{(m-1)}$ eğer $a \in Nit^{(n)}_{(m)}$ ve $n \geq 2$ ise.

   $De\breve{g}(a) \subseteq Ba\breve{g}^{(n)}$ eğer $a \in Nit^{(n)}_{(1)}$ ise

5. f_n ikili bir fonksiyon öyle ki, $\overline{x} \in (Nes)^n$ ve her $a \in Nit^{(n)}_{(1)}$ $f_n(\overline{x}, a) \subseteq De\breve{g}(a)$ için . f_n n-inci veri fonksiyonu ve her n-li nesne dizisi $\overline{x}$ ve $a \in Nit^{(n)}_{(1)}$ için $f_n(\overline{x}), a$ $\overline{x}$ *hakkında* a niteliğine göre veri.

olacak şekilde bir yapıdır.

Bilgi temsili sistemlerinin varlıkbilimsel kavram ve bağıntıların temsili açısından basit ama güçlü yapılar olduğuna ilişkin iddiaya destek olarak Deutsch tarafından incelenen problemi de düşünebiliriz. Deutsch genel benzerlik bağıntıları ile

a ve b aynı niteliğe sahiptir.

türünden onermelerle dile getirilebilecek bağıntıları kastetmektedir. Örnek olarak, "aynı renge sahip olma" ya da "aynı renkte olma" ya da "aynı şekle sahip olma" bağıntıları verilebilir. Bu bağıntılar has bağıntılar olarak düşünülebilir ancak geleneksel bakış açısına gore bunlar ancak bildik ozdeşlik bağıntısını içeren onermelere donüştürülerek anlaşılması gereken onermelerdir. Örneğin a ile b aynı renge sahip olduğunu soylemek demek, F, G birer renk yüklemi olmak üzere $F(a) \wedge G(b) \wedge F = G$ şeklinde sembolleştirilebilir.

Deutsch'a gore geleneksel bakış açısı, Deutsc'un ayırdedilemezlik ilişkileri dediği ilişkileri açıklamakta yetersiz zorlanmaktadır. Bu bağıntılarla ne kastedildiğini anlamak için şu orneklere bakalım:

a ve b aynı şekle sahip ve a dairesel ise, b de daireseldir.

a ve b aynı şekle sahip ve anın sınırı düzgün ise $b$nin sınırı da düzgündür.

Bu tür onermeler sezgisel olarak doğru olmakla birlikte, yüklemler hakkında anlam postülatları verilmeden türetilemezler. Buna gore yeterli bir temsili yapıda tüm bu anlam postülatları da temsil edilebilmelidirler. Deutsch buna küme kuramsal yapıların ozelliklerini niceler. Deutsc'un incelemesi matematiksel olarak problemi çozmekle beraber, geliştirdiği zekice çozümün anlaşılabilir nedenleri konusunda bilgi vermekten kaçınır. Bu ise felsefi olarak kabul edilebilir bir çozüm onerisi olarak kabul edilmesine engeldir.

Ayırdedilemezlik ilişkileri birinci derece (dolayısıyla üst derece nitelik sistemlerinde de) nitelik sistemlerinde kolaylıkla temsil edilebilir. Öncelikle genel benzerlik bağıntılarının temsilini ele alalım. Şekil niteliğini F ile gosterelim. a ve b nesnelerinin aynı şekle sahip olduğunu $f(a, F) = f(b, F)$ ile gosterebiliriz. Bu durumda, yukarıda geçen iki ayırdedilemezlik ilişkisi:

$$f(a, F) = \{Daireselik\} \Rightarrow f(b, F) = \{Daireselik\}$$

$$f(a, F) = \{\text{Düzgün Sınırlılık}\} \Rightarrow f(b, F) = \{\text{Düzgün Sınırlılık}\}$$

şeklinde ifade edilebilir. Nitelik sisteminde ayrıca anlam postülatları verilmesine gerek yoktur. Nitelik sistemi eğer incelediğimiz nesnelere ilişkin gerçekliğin doğru bir tasvirini veriyorsa ayırdedielmezlik ilişkileri sistemde verili olacaktır.

Özelllikle benzerlik adcılığına karşı sonsuz gerileme argümanları farklı biçimlerde ileri sürülmüştür. Bu nedenle sonsuz gerileme argümanlarının genel biçimiyle anlaşılması benzerlik konusuyla ilgili bir araştırmada faydalı olacaktır.

Genellikle, bir kuramın sonsuz gerileyen bir diziye yol açması o kurama karşı bir *reductio* argüman olarak kabul edilmektedir. Ayrıca, az sayıda belirli durumda sonsuz gerilemenin kuramın reddini gerektirmediğinde bir uzlaşı vardır. Ancak hangi sonsuz gerileyen dizilerin bir kuramın reddine yol açacağı hangilerinin zararsız olduğu hakkında şimdiye değin genel kabul goren bir kuramda geliştirilememiştir. İki soruyu ayırdedebiliriz: (1) Bir kuramdan hangi koşullarla sonsuz gerileyen en az bir dizi elde edilebilir? (2) Hangi koşullarla kuramdan elde edilen bir sonsuz gerileme dizisi kuramın reddine yol açar? İlk soruya yanıt bulmak nispeten daha kolaydır. İkincisi ise mantıksal çerçevede yanıtlanması belki de imkansız bir sorudur. Şimdilik belirli gerekli ve yeterli bir "zararlı sonsuz gerileme" koşulu elimizde olmadığından, bellirli durumların sıralanması ile yetinmek gerekli gorünmektedir. Örneğin, fiili sonsuz fikri baştan reddedilmezse, bir sonsuz gerileme dizisi sonsuz sayıda nesnenin varlığını kabul etmemize yol açsa bile, eğer elde edilen sonsuz nesne kümesinin metafizik olanaksızlığı gösterilemezse, kuram reddedilmiş sayılmamalıdır. Yine sonsuz gerilemeye yol açma ile sonsuz gerileyen bir diziyi gerektirme ayırımı yapma yoluna gidebiliriz.

Vita

PERSONAL INFORMATION

Surname, Name: Taşdelen, İskender

Nationality: Turkish (TC)

Date and Place of Birth: 6 April 1972, Bartın

Marital Status: Married

EDUCATION

Degree	Institution	Year of Graduation
MS	METU Philosophy	2000
BS	METU Philosophy	1998
High School	Amasra Lisesi	1989

WORK EXPERIENCE

Year	Place	Enrollment
1998-2005	METU DEPT. of Philosophy	Research Assistant

FOREIGN LANGUAGES

Advanced English

PUBLICATIONS

1. Taşdelen, İskender. "Kindi, Sonsuz Nicelikler, Matematik ve Felsefe İlişkisi Üzerine", Felsefe Tartşmaları, 33, (2004)

2. Taşdelen, İskender "Resemblance Relations and Higher-Order Information Systems", in *Topics on General and Formal Ontology*, Paolo Valore (ed). University of Milan. (to be published)