DEVELOPMENT AND COMPARATIVE EVALUATION OF A NEW STRUCTURAL MODIFICATION METHOD IN APPLICATION TO AIRCRAFT STRUCTURES

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ABSTRACT

DEVELOPMENT AND COMPARATIVE EVALUATION OF A NEW STRUCTURAL MODIFICATION METHOD IN APPLICATION TO AIRCRAFT STRUCTURES

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In the development of engineering products, it is necessary to predict dynamic properties of the modified structures. Achieving such predictions by using the structural properties of the original structure and information on the modifications is commonly referred to as structural modification analysis.

In this thesis, Özgüven's Structural Modification Method and Sherman-Morrison Method are selected as exact methods for structural modifications to predict the dynamics of a locally modified structure. Also, a new structural modification method named as "Extended Successive Matrix Inversion Method" is developed in this study. These three methods are implemented in a software developed herein, called "Structural Modification Toolbox". The software uses modal analysis results of MSC Nastran© for the original structure and calculates the modified frequency response functions by any of the methods above. In order to validate the software, direct modal analysis results of MSC Nastran© for the frequency response functions of the modified structure are used. The methods are compared in terms of computational time, and the effectivity of each method is studied as a function of modification size to determine which of these methods is more suitable.

In order to investigate the application of the methods and compare their results with experimental ones, modal tests are conducted on a scaled aircraft structure. The solutions are compared with test results obtained from modified test structure. Additionally, the software is used for comparison of real aircraft test results and frequency response functions of the modified structure.

Keywords: Structural Dynamics, Structural Dynamic Modification, Structural Modification, Reanalysis.

ÖZ

YENİ BİR YAPISAL DEĞİŞİKLİK YÖNTEMİNİN GELİŞTİRİLMESİ VE HAVACILIK YAPILARINA UYGULAMASININ KARŞILAŞTIRMALI DEĞERLENDİRMESİ

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Mühendislik ürünlerinin geliştirilmesinde, yapıların dinamik özelliklerini öngörmek gerekmektedir. Bunun gibi tahminlere, esas yapının yapısal özellikleri ve yapılacak değişikliklerin bilgisi kullanılarak ulaşmak için genellikle yapısal değişiklik yöntemlerine başvurulur.

Bu tezde, bölgesel değişikliğe uğrayan yapıların dinamik özelliklerini öngörmek amacıyla, kesin sonuç veren Özgüven Yapısal Değişiklik Yöntemi ve Sherman-Morrison formülasyonunu kullanan Yapısal Değişiklik Yöntemi seçilmiştir. Ayrıca, bu çalışmada “Genişletilmiş Ardışık Matris Tersi Alma Yöntemi” olarak isimlendirilen yeni bir yapısal değişiklik yöntemi geliştirilmiştir. Bu üç yöntem, bu çalışmada geliştirilen ve “Yapısal Değişiklik Araç Kutusu” olarak adlandırılan bir yazılım içerisinde uygulanmıştır. Yazılım esas yapı için MSC Nastran© programının modal analiz sonuçlarını kullanır ve yukarıda bahsedilen yöntemlerle, değiştirilmiş frekans tepki fonksiyonlarını hesaplar. Yazılımı doğrulamak amacıyla, değiştirilmiş yapının frekans tepki fonksiyonları için doğrudan MSC Nastran© programının modal analiz

sonuçları kullanılmıştır. Yöntemler; hangi yöntemin daha uygun olduğunu belirlemek amacıyla, çözüm süreleri açısından ve değişiklik boyutuna bağlı olarak her yöntemin etkinliği açısından karşılaştırılmışlardır.

Yöntemlerin uygulamasını incelemek ve deney sonuçları ile karşılaştırmak için, ölçekli bir uçak yapısına modal testler yapılmıştır. Elde edilen yapısal değişiklik analizi sonuçları, değiştirilmiş yapının deney sonuçları ile karşılaştırılmıştır. Ek olarak, yazılım, gerçek bir uçağın deney sonuçları ile, değiştirilmiş yapı frekans tepki fonksiyonlarının karşılaştırması için kullanılmıştır.

Anahtar Kelimeler: Yapı Dinamiği, Yapısal Dinamik Değişiklik, Yapısal Değişiklik, Yeniden Çözümleme.

To My Wife

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Special thanks go to my parents for their endless love, patience, support and encouragement throughout my life and education.

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LIST OF SYMBOLS

$[D]$ Dynamic matrix

$\{F\}$ Forcing vector

$[H]$ Structural (hysteretic) damping matrix

i Unit imaginary number

$[I]$ Identity matrix

$[K]$ Stiffness matrix

$[L]$ Lower triangular matrix

$[M]$ Mass matrix

$[O]$ Constant coefficient matrix

r Recursion factor

$\{R\}$ Vector representing damping forces

$[U]$ Upper triangular matrix

$\{x\}$ Displacement vector

$[Y]$ Modification matrix

$[UA]$ Analytical modal matrix

$\{UR\}$ Upper residual dynamic matrix

$[\Delta C]$ Modification viscous damping matrix

$[\Delta H]$ Modification structural (hysteretic) damping matrix

$[\Delta K]$ Modification stiffness matrix

$[\Delta M]$ Modification mass matrix

$[K_R]$ Reduced Stiffness matrix

$\left[M_R \right]$ Reduced Mass matrix

$\left[Tr \right]$ Transformation matrix

$\left\{ \ddot{x} \right\}$ Acceleration vector

$\left[\alpha \right]$ Receptance matrix of original system

$\left[\gamma \right]$ Receptance matrix of modified system

$\left[\delta \right]$ Dynamic Stiffness Matrix

$\left\{ \phi \right\}$ Mass normalized undamped modal matrix

$\left\{ \omega \right\}$ Excitation frequency

$\left[\quad \right]^g$ Generalized Inverse of matrix

$\left[\quad \right]^T$ Transpose of matrix

$\left[\quad \right]^*$ Pseudo-Inverse of matrix

ABBREVIATIONS

ASA	American Standard Association
CA	Combined Approximations
DFE	Dynamic FRF Expansion
DOF	Degrees of Freedom
DRE	Dynamic Residual Expansion
ESMI	Extended Successive Matrix Inversion Method
FEM	Finite Element Model
FRF	Frequency Response Function
GUI	Graphical User Interface
IRS	Improved Reduced System
LU	Lower and Upper Triangular Matrix Decomposition
MAC	Modal Assurance Criterion
MPC	Multi Point Constraint
PZT	Smart Piezoelectric Ceramic Transducers
RDOF	Rotational Degrees of Freedom
SEREP	System Equivalent Reduction Expansion Process
SM	Sherman-Morrison Method
SMI	Successive Matrix Inversion Method
SMW	Sherman-Morrison-Woodbury Method
SVD	Singular Value Decomposition

CHAPTER 1

INTRODUCTION

1.1 NECESSITY FOR STRUCTURAL MODIFICATION AND THE BASICS OF REANALYSIS

In design, the products have to satisfy a very wide range of design criteria: they must be cheaper, faster, lighter, safer, stronger and quieter. It is almost impossible to design an optimum product that satisfies all the needs in first trial so that one of the basics of engineering is to build a work upon a previous one, thus saving time and effort. The initial design concept is likely to give rise to a variety of choices each of which will be evaluated for the requirements of the product. At this step, series of prototypes are constructed and their properties are compared with the design specifications. During the design and production of dynamic structures, the amount of resources that can be saved is considerable with the aid of some theoretical work.

With the increasing demand for high precision mechanical components, dynamic design started to be more and more important. In such a design process, it may be necessary to study numerous alternative configurations in order to ensure a suitable dynamic response. Engineers examine the structure during service operation not only for design improvements for future developments, but also because of commitments to the customer for warranties and servicing as well as the necessity of conforming to national standards and safety legislation. To save time and money, it is necessary to predict properties of the structures, including dynamic properties, when some modifications are to be made on the structure. In other instances, engineers have difficulty when it is noticed that the dynamic properties of a structure does not meet the requirements predicted in the design stage and some modifications on the structures have to be made in order to obtain the desired dynamic properties. These requirements are very important in applications involving aerospace systems, boilers and pressure vessels, nuclear facilities and large civil engineering projects etc. where high structural integrity is of paramount importance. The technique used to obtain modified dynamic properties of a structural system in modal analysis is called as structural modification.

Structural modification is a major term used in modal analysis that involves either to predicting the dynamic behavior of a modified structure (forward structural modification, reanalysis) or to determining the modification to be made on an existing structure (inverse structural modification). Forward structural modification, also known as reanalysis, implies the incorporation of an existing model with the information obtained from an experiment or some other source. Reanalysis is common in vibration optimization and in finite element updating, whereas inverse methods are mostly used for eliminating vibration problems after a structure is constructed. Both forward and inverse structural modification methods require complete spatial model or incomplete modal model but while the solution is a unique one for forward structural modification, it is either non-unique or no solution for inverse structural modification methods.

The straightforward method for structural modification would involve building a new analytical model for the whole structure by spending time and effort to make the reanalysis from scratch. However, it is preferable to use more economic techniques making use of previously obtained response data. Generally, an analytical model of the original structure or its measured response is used in the structural modification. In experimental studies, the modal parameters of the original structure, for example the aircraft in Figure 1.1, are identified and the structure is reanalyzed by using a modal model, or else FRFs of the modified structure (Figure 1.2) are estimated directly from the measured FRFs of the original structure. On the other hand, in analytical studies, analytical FRFs and/or system matrices can be used for estimating the modified FRFs. For both forward and inverse structural modification methods, there are a number of exact methods as well as approximate ones as summarized in Figure 1.3.

Figure 1.1 F-18 Fighter Aircraft (Original Structure).

Figure 1.2 F-18 Fighter Aircraft with Weapons (Modified Structure).

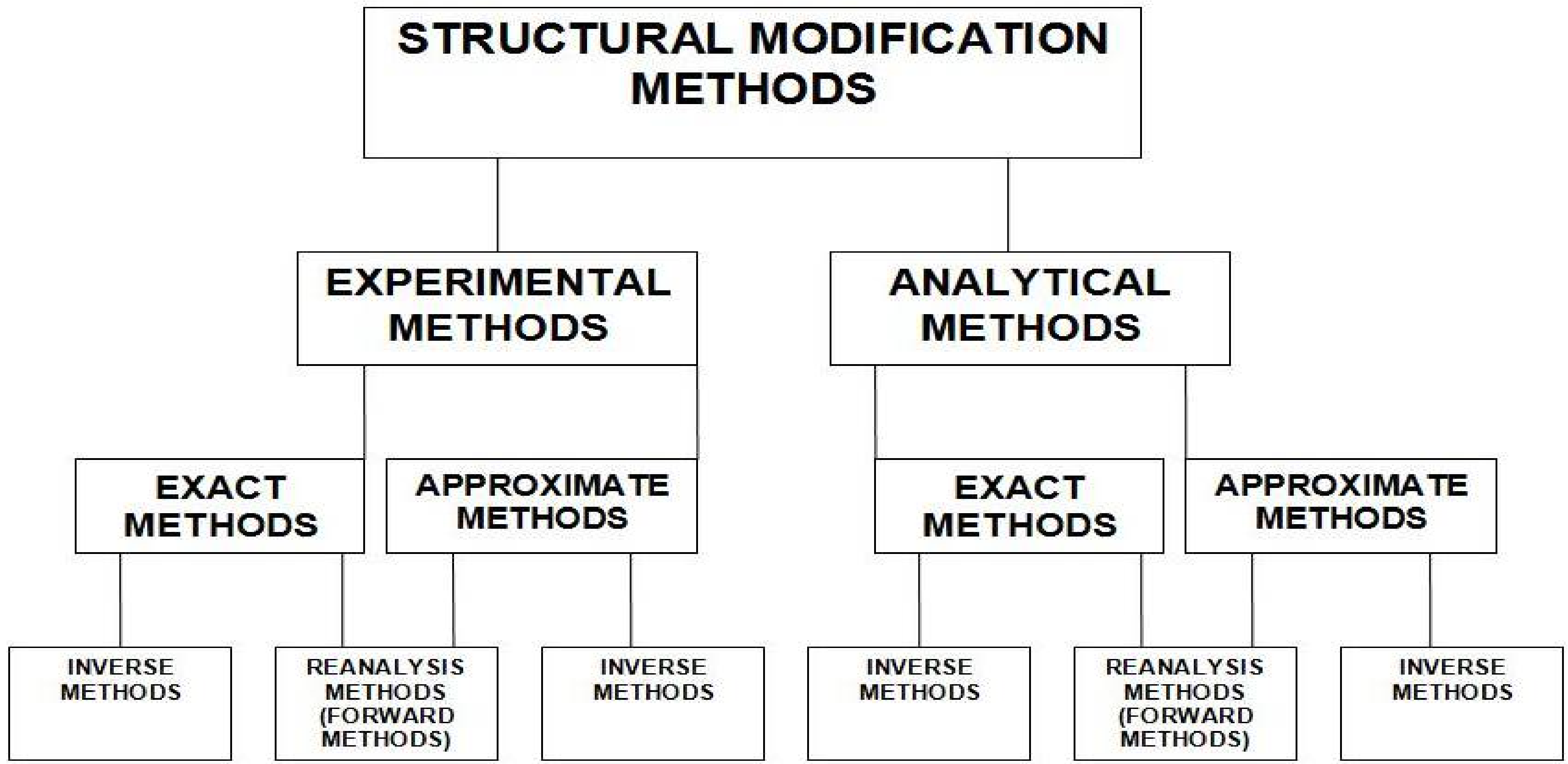

Figure 1.3 Structural Modification Methods.

1.2 LITERATURE SURVEY

Theoretical basis of structural modification has been established following the foundation of modal testing theory. The studies date back to the work of Frazer, Duncan and Collar [1]. The analytical basis of reanalysis starts from the work by Lancaster [2, 3]. The most neglected work of significance in structural dynamics is that of Kron [4], due to the obscure notation because of the programming techniques of analogue computers in addition to the nature of the work. Kron’s work has been extended by Simpson and Collar [5].

Research developed from Kron’s method is due to Sehmi [6] and Turner et al. [7]. The basis of the theory of modal analysis, the most comprehensive treatment of eigenvalue problems, is given by Wilkinson [8].

The first reanalysis techniques were originally developed for the reduction of computational time for the dynamic analysis of large structures. The component mode synthesis method first presented by Hurty [9], is among these techniques. Several methods employing the component mode synthesis technique have been suggested since then. In modal reduction methods, the most dominant modes are selected to reduce computation time in the analysis of large structures; higher the natural frequency less dominant is the associated mode [10-12]. Dual modal space structural techniques and local eigenvalue modification technique are developed directly for reanalysis purposes. Local eigenvalue modification technique was first presented by Weissenburger [13]. O’Callahan and Chou [14] presented a method in which a three-dimensional generalized beam element is used in conjunction with the local eigenvalue modification procedure that allows more realistic structural modifications. The dual modal space structural modification method was developed by Luk and Mitchell [15-17]. Transfer matrix method and dual modal space structural modification method were combined for realistic beam modifications by Elliott and Mitchell [18, 19].

Earliest application of reanalysis appears to be the stress analysis of fuselage panels with a cutout [20]. In this study, a continuous fuselage was analyzed first and the effect of the cutout was modeled with a corrective self-equilibrating stress system applied to the continuous structure at the panel where the cutout would be located. The stress state in the structure due to the corrective stresses was then added to the continuous solution [21].

Reanalysis of structures using analytical data falls into two categories: exact and approximate methods. Exact structural reanalysis methods were developed in the framework of structural reanalysis before finite element and boundary element matrix equations became the norm for computational structural analysis.

In the context of modern structural analysis, it is possible to show that the earliest methods such as [20] are variations on two general matrix-update formulas that are known as the Sherman-Morrison (SM) [22] and the Woodbury [23] formulas. Since the Sherman-Morrison-Woodbury (SMW) formulas have existed for over 50 years, these formulas could have been used as efficient tools in exact structural reanalysis involving low-rank modifications ever since reanalysis became a need. The reason for that; the SMW formulas are valid for arbitrary matrices whereas reanalysis methods deal with more specific matrices and modifications. In spite of their existence, specialized methods for exact reanalysis were proposed for low-rank modifications that were essentially equivalent to the SMW formulas where an additional inversion of a matrix, which has the size of the DOF associated with the modified elements is required. In the forward method based on SM identity, the main idea is to obtain system responses without computing an inversion of the modified matrix. However, the modification of the system matrix is decomposed into two vectors to be used in the SM identity considering global coordinate system with additional computational cost.

The history and various applications of SMW formulas were presented by Hager [24]. In his work, he claims that the Woodbury formula was given by Duncan [25], several years before Woodbury's report. On the other hand, the SM formula was actually given by Barlett [26], who generalized a result given by Sherman and Morrison [22].

SMW formulas are also used in the areas of least squares methods, asymptotic analysis, electrical networks, sensitivity in linear programming, quasi-Newton methods and partial differential equations. Sack *et al.* [27] utilized the SM formula to compute the inverse of a modified matrix where the SM formula is used once for each modified column of the original matrix in repeated fashion for multiple column changes regardless of the rank of the modification. Kavlie and Powell [28] took the approach by Sack *et al.* [27] one step further and used the SM formula to compute the modified displacement vector instead of the modified inverse. They took note of the fact that the SM formula applied for rank-one changes; however, instead of using the Woodbury formula, SM formula is used repeatedly for multiple column changes, once for each modified column. Argyris *et al.* [29] used the Woodbury formula to compute the displacement vector in a modified structure. However, in this case the formulation again depended on the number of modified columns rather than the rank of the modification. Consequently, as it is the case in SM formula, the approach required the solution of a system of size *6m* for a space truss where *m* is the number of modified bars. However, the rank of the modification is only *m*, so it should be necessary to solve only a system of size *m*. Woodbury formula was used by Kirsch and Rubinstein [30] to compute the modified inverse and the modified displacement. The computational effort required was again a function of the size of the non-zero portion of the incremental matrix rather than its rank. Their formulation also involved the inverse of the same non-zero portion of the incremental matrix which could be singular. For the latter problem, they proposed a method involving the solution of a reduced set of equations.

Certain banded circulant linear systems were solved using SMW formulas by Chen [31]. A circulant system is one where the entries of the coefficient matrix satisfy $k_{ij} = k_{i+1,j+1}$ with the subscripts *i* and *j*. An application of the SMW formulas in fluid dynamics was shown by Tuckerman [32]. He stated that any method that directly solves a system by modification of an original system makes use of the SMW formulas implicitly or explicitly.

Fitzpatrick and Murphy [33] applied Woodbury formula in a communications channel in electrical systems. Khansari and Leon-Garcia [34] proposed a fast algorithm for optimal linear interpolation using Sherman-Morrison formula. An application was in computational vision where the purpose was to extract a smooth 3D shape from noisy 2D image data [35]. Another application of the SMW formula to structures, which is an exact method, is the work of Huang and Verchery [36]. Their method starts with the whole structural stiffness matrix without applied boundary conditions. They obtain a non-singular coefficient matrix from the singular matrix by removing the rigid-body modes. By a transformation of the original equation of motion and using the boundary conditions, they reduce the system to a much smaller order. The actual displacement vector is then obtained by operations involving matrix multiplication. Reanalysis due to modified elements is handled through the use of the SMW formula. Akgün *et. al.* [37] extended the low-cost linear analysis to nonlinear reanalysis problems in the spirit of SMW formulas.

Şanlıtürk presented a new method based on Sherman-Morrison formula for the analyses of certain types of modified structures where the modifications do not add new degrees of freedom [38]. However, the method has an advantage in the sense that the FRF, hence the response of modified structure can be calculated very efficiently, without requiring any matrix inversion or a solution of a new eigenvalue problem. In addition to being an exact method, it offers a possibility of restricting the analysis to the coordinates where modifications are made and those coordinates where responses are required. The method is also suited for the frequency domain analyses of nonlinear structures with localized nonlinearities.

This method [38] requires the decomposition of the modification matrices into two vectors, which may sometimes be very difficult to obtain. However it is observed that new developments in structural reanalysis such as using Singular Value Decomposition (SVD) or Lower and Upper Triangular Matrix (LU) Decomposition may be helpful in this method.

Özgüven has developed a structural modification method using FRFs. The matrix inversion method proposed for finding receptances of locally damped structures from undamped counterparts [39], has later been combined with an efficient solution algorithm in order to avoid matrix inversion [40], and thus an efficient structural modification method has been developed for structural reanalysis problems [41].

The method is capable of finding FRFs of a modified structure from those of the original system. The method has been extended in the same work [41] for structural modification cases with additional degrees of freedom. In this method, modification may be in the form of additional structural components which may be expressed in terms of additional mass, stiffness and damping matrices. Compared to previous FRF methods, a major computational advantage was obtained. The major drawback of the previous methods; in most cases is that, there is at least one inversion of the order of the degrees of freedom (DOF) of the original structure. Since the computation time increases at a cubic rate with DOF, smaller is the order of inversion faster is the method.

Bae, Grandhi and Canfield [42] developed a bound-free reanalysis technique called Successive Matrix Inversion (SMI) for static analysis. In their work, Bae, Grandhi and Canfield obtained exact solutions for any local modifications to an initial static Finite Element model. Computational cost of the method is dependent on the size of the modified degrees of freedom compared to the initial system matrix size. SMI method uses column information of the modification matrix that includes non-zero elements. The inverse of the system matrix can be obtained by selecting the columns of the modification matrix by any sequence, because the contribution of each column vector is independent. In other words, since the modified matrix is obtained at the last step, the particular column of the modification matrix selected in each step is not important. Both symmetric and non-symmetric matrices can be handled in this method. SMI method is not only suitable for local modification of existing structures but also for sensitivity analysis by Finite Difference method. Another area of application of the method is reliability analysis. Bae, Grandhi and Canfield [42] compared their method with the conventional linear algebraic equation solvers. This method generates the exact solution to the modified structure equations, and concentrates on low-rank modifications to structures.

Köksal, Cömert and Özgüven [43] extended the SMI method to dynamic analysis of structures. The method is based on exact calculation of the FRF matrix of a modified structure using the FRF matrix of the original structure and the modifying mass, stiffness and damping matrices. The basic equation obtained in this method is the same as the one used in Özgüven's Structural Modification method [41]. However in this method power series expansion is used in order to avoid matrix inversion, whereas a novel algorithm [40] is used in Özgüven's Structural Modification Method to avoid matrix inversion, although matrix inversion is always an alternative for local modifications with small number of modified coordinates since the order of the matrix to be inverted is equal to the number of modified coordinates.

Reanalysis of structures using experimental data also falls into two categories: exact and approximate methods. The first line of methods based on exact algebra can be derived from Kron's theorem. Weissenburger [13, 44], Pomazal [45] and Pomazal and Snyder [46] have made important contributions for the modal model. Exact response analysis methods are characterized by the works of Mahalingam [47, 48]. The second type is based on truncated series and lead to sensitivity analysis methods. In the past, Whitesell [49], Baldwin and Hutton [50] have worked on this subject.

The second type of structural modification is based on approximate reanalysis techniques. Approximate reanalysis can be carried out by using finite elements. A concept in finite elements is the hierarchical methods. Hierarchical methods are classified as h-type, if the modification refines the mesh or p-type if the modification retains the same mesh points but uses progressively more sophisticated elements. There is also a hybrid method that makes h-p refinement. In experimental reanalysis, the problem has three facets. Firstly, specification of performance defines the requirements for the behaviour in service of the structure in terms of both required and prohibited or undesirable characteristics. Secondly, the solution has to be based on a much idealized approximation of the detailed structure and lastly the actual behaviour measured on a model, real structure or prototype. Before the development of reanalysis methods, the normal procedure for solving a problem of a relatively minor modification for a system whose solution is known, involved a new mesh generation and complete resolution of the problem. If a model does not give acceptable results then the entire mesh is redefined and resolved. This iterative procedure is repeated until the solution is satisfactory. The solution obtained frequently is an accurate representation throughout the majority of the structure and not accurate for a small region such as a region of stress concentration. In this case, the reanalysis is achieved through an adaptive mesh refinement process where the desired change is restricted to the selected region. The alteration in the finite element mesh is isolated in its effect and the major part of the model can be carried forward unchanged into successive stages of analysis. In the most elementary type of approximate reanalysis by the finite element method, dynamic analysis is carried out by using the subspace iteration technique. Subspace iteration technique uses the initial design parameters as the first iteration parameters for the slightly modified structure problem. The idea behind this technique is that the solution of the modified structure is likely to be close to that of the original model. In this case, it is expected that the convergence will be significantly more rapid.

Approximate reanalysis methods might be effective in the solution of engineering problems. In particular, these methods have been used extensively in structural optimization to reduce the number of exact analyses and the overall computational cost during the solution process. The accuracy of the calculations (the quality of the approximations), computational effort involved (the efficiency of the method) and the ease of implementation are the factors that are considered when choosing an approximate reanalysis method for a specific application. The implementation effort is weighted against the performance of the algorithms as reflected in their computational efficiency and accuracy. The implementation efficiency and the quality of the results are usually two conflicting factors. For approximate methods, the accuracy of the solution and a fast convergence are also two important factors which are rarely achieved at the same time. A compromise must be made according to the engineering problem. Approximate reanalysis techniques generate an estimate of the response of the modified structure using information obtained during full analysis of the original structure or special information needed to set up the approximate reanalysis. These methods include the use of sensitivity vectors, Taylor series expansions in terms of design variables, reduced basis and iterative techniques. This field has been characterized by a large number of minor advances.

The common approximations can be divided into three classes. Firstly, local approximations which are based on information calculated at a single point. These methods are very efficient but they provide accurate results only for small changes in the design variables. Secondly, global approximations, obtained by analyzing the structure at a number of design points. These methods provide accurate results for large changes in the design, but they are not efficient for problems with a large number of design variables. Lastly, combined approximations (CA), where local and global approximations are combined to achieve the efficiency of local approximations and the improved accuracy of global approximations. Some common problems for which these methods can be used are described in five classes [51]:

1) In structural optimization the solution is iterative and consists of repeated analyses followed by redesign steps. The number of design variables is usually large, and various failure modes under several load conditions are often considered. The constraints are implicit functions of the design variables, and evaluation of the constraints requires the solution of the analysis equations. The high computational cost involved in repeated analyses of large-scale problems is one of the main obstacles in the solution process. In many problems the analysis part requires most of the computational effort, and therefore only methods that do not involve numerous time-consuming implicit analyses will prove useful.

2) In structural damage analysis, it is necessary to analyze the structure for various changes due to deterioration, poor maintenance, damage, or accidents. In general many hypothetical damage scenarios, describing various types of damage, should be considered. These include partial or complete damage in various elements of the structure and changes in the support conditions. Numerous analyses are required to assess the adequacy of redundancy and to evaluate various hypothetical damage scenarios for various types of damage.

3) In the design of the construction stages of complex structures, it might be necessary to repeatedly analyze structures that are modified during the construction. The modified structures are subjected to different loading conditions. The changes in the structure may include additional elements and different support conditions.

4) Nonlinear analysis of structures is usually carried out in an iterative process. The solution process can be performed by different methods, but in general, a set of updated linear equations must be solved repeatedly. Similarly, many of the vibration

(or eigenproblem) solution techniques are based on matrix iteration methods. To calculate the mode shapes, it is necessary to repeatedly solve a set of updated equations.

5) Reanalysis methods might prove useful in other applications, such as probabilistic analysis, controlled structures, smart structures, adaptive structures, and conceptual design problems.

CA methods are used widely in literature. Kirsch *et al.* [51-53] have used CA approach in their studies for design, structural analysis and optimization in reanalysis. Accurate results ware obtained for very large changes in the design variables.

In the study of Chen, Yang and Lian [54], five eigenvalue reanalysis methods; the basic second-order perturbation method, improved perturbation methods, including Bickford's and Chen's methods, the Pade's approximate method and the extended Kirsch combined method, are compared in terms of their computational accuracy and efficiency. The first five eigenvalues of a structure were computed using the methods. The errors in the solutions were presented to compare the accuracy of the approximate reanalysis methods. The numerical results show that if the changes of the structural parameters are not too large, for example, for low error tolerances, the second-order perturbation method, Bickford's method and Chen's method can give good approximate eigenvalues. If large changes are introduced into the structures, for example for high error tolerances, the second-order perturbation, Bickford's method and Chen's method are not applicable. In this case, the Pade's approximate method and the extended Kirsch combined method can still give excellent results.

There are eigensolution reanalysis techniques which are suitable for structural dynamic optimization that emphasize assumed mode reanalysis. It is possible to derive a modal model from structural tests in case of an existing structure which is required to be modified. One of the restriction factors in experimental methods is the accuracy of the modal model, which is influenced by both the technique used in modal identification and by the experimental measurements which also affect the reanalysis results. The accuracy of modal modification methods using experimentally determined modal models is highly dependent on modal truncation as shown by Braun and Ram [55], and Elliott and Mitchell [56]. To eliminate such problems, measured FRF may directly be used in the reanalysis, without identifying modal parameters of the original structure. An FRF coupling method using experimentally obtained FRF have been discussed by Ewins [57].

One of the most important drawbacks of experimental techniques is due to the difficulties in obtaining experimental data for rotational degrees of freedom. Although through the knowledge of the original system, rotational information can be derived by using a theoretical analysis, usually obtained from a finite element model of the system, in many practical situations one can study the system only experimentally, by its translational FRF matrix from which some information on the structure can be eventually obtained to perform modifications. When coupling together the finite element model of the modifying structure with the FRF model of the original one, one must account for the rotational degrees of freedom. The rotational degrees of freedom are included in the FE model but are not considered in the experimental model because of the difficulty of their measurement. Therefore either expansion or reduction techniques must be used to compensate for the effects of the rotational degrees of freedom. Reduction methods are known as Guyan reduction, Improved Reduced System (IRS), Dynamic Reduction, System Equivalent Reduction Expansion Process (SEREP); Expansion methods are known as, SEREP Expansion, Dynamic Residual Expansion (DRE), Dynamic FRF Expansion (DFE), SEREP/DRE and SEREP/DFE. Studies performed clearly show that the rotational FRF are much more sensitive than translational FRF to modal truncation. Dynamic compensation techniques have been developed to minimize the truncation effect in order to improve the accuracy of the synthesized FRF. Thus, the inclusion of the rotational DOF from a modal synthesis, which includes compensation for the effects of modes not included in the synthesis process, does not pose any problems for the modeling process of analytical models. From the experimental point of view, a finite difference approach which utilizes translational measurements to attempt to extract a rotational measurement have been tried. Duarte and Ewins [58] have given a summary of the literature an rotational degrees of freedom including the year of work and approach adopted (Table 1.1). In this work [58], they use the finite difference technique to obtain rotational DOF for structural coupling analysis with the intention of establishing the best rotational input data to be used with emphasis on residual compensation. Ambrogio, Sestieri *et al.* [59, 61] have presented a work that suggests the use of Smart Piezoelectric ceramic (PZT) transducers to measure FRF for rotational DOFs. They have analyzed and compared some possible options to provide an efficient formulation in the case of distributed modifications. Avitabile and O'Callahan [62, 63] have described new methods for the inclusion of rotational DOF.

Table 1.1 References on calculating Rotational DOF [58]

Date	Block	Mass additive	Finite difference	Estimation	Angular sensors	Laser
1969	Smith					
1972	Ewins and Sainsbury					
1975	Ewins and Gleeson					
1976	Sainsbury					
1978	Silva					
1979	Gleeson					
1980	Ewins *et al.*		Sattinger			
1984	ASA standards Smiley and Brinkman	Yasuda *et al.*	ASA standarts Martinez *et al.*	Smiley and Brinkman		
1985			Chen and Cherng Maleci and Young	O'Callahan *et al.*	Licht	
1986		Kanda *et al.*		Furusawa and Tominaga O'Callahan *et al.* Haisty and Springer		
1987	Silva and Fernandes		Richardson and Douglas	Avitabile *et al.* Severeyn *et al.*		
1988						Oliver
1989	Urgueria Skingle	Wei and Zhang	Urgueria Qingyu and Jing	Urgueria	Rorrer *et al.*	Urgueria
1990				William and Green		Sriram *et al.*
1991					Bill and Wicks	Cafeo *et al.*
1992					Laughlin *et al.*	Cafeo *et al.*
1993	Sanderson		Salvini and Sestieri	Salvini and Sestieri Ng'andu *et al.* Cafeo *et al.*		Bokelberg *et al.* Trethwey *et al.* Cafeo *et al.*
1994					Llorca *et al.*	Stanbridge and Ewins

Table 1.1 (cont'd) References on calculating Rotational DOF [58]

1995					Ng'andu *et al*	
1996		Stebbins *et al.*		Chang *et al.* Wang *et al.*	Stebbins *et al.*	Ratcliffe and Lieven Umezawa *et al.*
1997	Maia *et al.*	Duarte and Ewins		Kim and Kang		

1.3 CONTENT AND SCOPE OF THE STUDY

This thesis subject is determined for the practical application requirements of TÜBİTAK-SAGE. The aim of this thesis is to obtain an exact solution for the dynamic response of a locally modified structure from the dynamics response of the original structure and structural data for modifications, when no new degrees of freedom are added. This is accomplished by first developing a new method (called as Extended Successive Matrix Inversion Method [43]) which is based on a method presented in the literature [42] for modifications on static structures. Then, it is aimed to compare this method with Özgüven's Structural Modification Method [40, 41] and Structural Modification Method Using Sherman-Morrison Formula [34] in terms of computational time, and thus to determine which of these methods are more suitable for structures of various sizes.

It is also aimed in this study to develop a software package implementing all this three methods for real structures with and without damping. The outline of the thesis is as follows:

Chapter 2 gives a brief explanation of the theory of the three structural dynamic modification methods: Özgüven's Structural Modification Method [40, 41], Structural Modification Method Using Sherman-Morrison Formula [38] and Extended

Successive Matrix Inversion Method [43], as well as the main theory of some expansion and reduction techniques.

Chapter 3 outlines the computer program and gives the verification of the program that is developed by using MSC Patran© and MSC Nastran© software packages.
Chapter 4 presents the comparison of the methods in terms of computational time considering the effect of the modification size on solution time.

Chapter 5 gives the experimental modification studies conducted on a test structure (GARTEUR SM-AG19) and on a real aircraft.

Chapter 6 outlines the discussion, conclusions and recommendations for future work.

CHAPTER 2

THEORY

2.1 STRUCTURAL MODIFICATIONS WITHOUT ADDITIONAL DEGREES OF FREEDOM

2.1.1 MATRIX INVERSION METHOD

The formulation is initially given by Özgüven [39] for the calculation of damped receptances from undamped counterparts for a non-proportionally damped system, and later the method is generalized for structural modification problems with and without additional degrees of freedom [41]. The general formulation can be given as follows.

Consider a system represented by:

$$[M]\{\ddot{x}\}+i[H]\{x\}+[K]\{x\}=\{F\} \tag{2.1}$$

where $[M],[H],[K]$ are respectively mass, structural (hysteretic) damping and stiffness matrices of the system, $\{x\}$ is the vector of generalized coordinates, $\{F\}$ is the generalized forcing vector and *i* is the unit imaginary number. Steady state response for a harmonic forcing at frequency ω is given as

$$\{x\}=\left([K]-\omega^2[M]+i[H]\right)^{-1}\{F\} \tag{2.2}$$

Then, the receptance matrix will be

$$\left[\alpha\right]=\left(\left[K\right]-\omega^2\left[M\right]+i\left[H\right]\right)^{-1} \tag{2.3}$$

The receptance matrix of the modified system, on the other hand, can be expressed as:

$$\left[\gamma\right]=\left[\left[K\right]+\left[\Delta K\right]-\omega^2\left[\left[M\right]+\left[\Delta M\right]\right]+i\left[\left[H\right]+\left[\Delta H\right]\right]\right]^{-1} \tag{2.4}$$

where $\left[\Delta K\right],\left[\Delta M\right],\left[\Delta H\right]$ are the matrices representing stiffness, mass and damping of the modifications respectively. From Equation (2.3) and (2.4), one can write

$$\left[\gamma\right]^{-1}=\left[\alpha\right]^{-1}+\left[D\right] \tag{2.5}$$

where

$$\left[D\right]=\left[\Delta K\right]-\omega^2\left[\Delta M\right]+i\left[\Delta H\right] \tag{2.6}$$

Pre-multiplying all terms of Equation (2.5) by $\left[\alpha\right]$ and post-multiplying them by $\left[\gamma\right]$ gives:

$$\left[\alpha\right]=\left[\gamma\right]+\left[\alpha\right]\left[D\right]\left[\gamma\right] \tag{2.7}$$

$$\left[\gamma\right]=\left[\left[I\right]+\left[\alpha\right]\left[D\right]\right]^{-1}\left[\alpha\right] \tag{2.8}$$

For local modifications:

$$
\left[D\right]=\begin{bmatrix}\left[D_{11}\right] & \left[0\right]\\ \left[0\right] & \left[0\right]\end{bmatrix} \tag{2.9}
$$

Then it is possible to write

$$
\left[\gamma_{11}\right]=\left[\left[I\right]+\left[\alpha_{11}\right]\left[D_{11}\right]\right]^{-1}\left[\alpha_{11}\right] \tag{2.10}
$$

$$
\left[\gamma_{12}\right]^{T}=\left[\gamma_{21}\right]=\left[\alpha_{21}\right]\left[\left[I\right]-\left[D_{11}\right]\left[\gamma_{11}\right]\right] \tag{2.11}
$$

$$
\left[\gamma_{22}\right]=\left[\alpha_{22}\right]-\left[\alpha_{21}\right]\left[D_{11}\right]\left[\gamma_{12}\right] \tag{2.12}
$$

Then, by using the above equations it is possible to calculate the receptance matrix of a modified system by inverting only a single matrix of an order equal to the number of coordinates related with structural modification.

If we have viscous damping $\left[C\right]$, instead of structural damping, then obviously $\left[H\right]$ and $\left[\Delta H\right]$ will be replaced by $\omega\left[C\right]$ and $\omega\left[\Delta C\right]$ respectively in all equations given above.

2.1.2 ÖZGÜVEN'S RECURSIVE SOLUTION ALGORITHM

An alternative approach to calculate the receptances of a modified structure from those of the original system and the modification matrices is to use the recursive solution algorithm of Özgüven [41].

In the initial formulation of Özgüven [40], the method was described for the calculation of damped receptances from undamped counterparts. In that work, Özgüven also stated that by using a more general formulation, this method could be used for structural modifications without adding new degrees of freedom. Derivation of the equations is given below.

Consider the equations of motion of a structure discritized to *n* degrees of freedom which can be written as

$$[M]\{\ddot{x}\}+i[H]\{x\}+[K]\{x\}=\{F\} \tag{2.13}$$

The undamped modal data can be obtained by solving the eigenvalue problem

$$[K]\{\phi\}=\omega^2[M]\{\phi\} \tag{2.14}$$

The internal damping of the system may be replaced by a set of equivalent forces that can be expressed in terms of the damping values and the displacement of the system. That is, the equation of motion can be written as

$$[M]\{\ddot{x}\}+[K]\{x\}=\{F\}+\{R\} \tag{2.15}$$

where $\{R\}$ is a vector representing damping forces defined as

$$\{R\}=-i[H]\{x\} \tag{2.16}$$

From Equation (2.15), it is possible to consider the damped system as an undamped system with two sets of external forces one of which is defined in terms of the unknown dynamic displacements $\{x\}$ of the damped system. The undamped receptances of the system described by Equation (2.15) can easily be calculated by modal summation.

If a typical coordinate *s* is considered, the damping force on this coordinate can be expressed as

$$R_s = -i\sum_{k=1}^{n} h_{sk} x_k \tag{2.17}$$

Then the response of the damped system $\{x\}$ to a harmonic external force $\{F\}$ can be found by considering the undamped system with two sets of harmonic forces as implied by Equation (2.13). From the definition of receptance, the dynamic displacement of the p^{th} coordinate can be written as

$$x_p = \sum_{s=1}^{n} \alpha_{ps} \left(F_s + R_s \right) \tag{2.18}$$

$$x_p = \sum_{s=1}^{n} \alpha_{ps} F_s - i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} x_k \tag{2.19}$$

Both x_p and x_k are the displacements in the damped system. Then the receptance γ_{pj} of the damped system can be obtained from Equation (2.19) by dividing all term by F_j and setting all external forces, except F_j, to zero

$$\gamma_{pj} = \alpha_{pj} - i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} \left(x_k / F_j \right) \tag{2.20}$$

Since the receptance γ_{pj} is defined as the displacement of the p^{th} coordinate when there is a unit external force at the j^{th} coordinate while all other external forces are zero, the term x_k / F_j in Equation (2.20) can be identified as γ_{kj} and the Equation (2.20) can be written as

$$\gamma_{pj} = \alpha_{pj} - i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} \gamma_{kj} \tag{2.21}$$

which is valid only for any *p* and *j* (p=1,2,…,n; j=1,2,…,n). If only a single element of the damping matrix, say h_{sk}, is considered while the rest of the damping elements are taken to be zero, Equation (2.21) takes the form

$$\gamma_{pj} = \alpha_{pj} - i\alpha_{ps} h_{sk} \gamma_{kj} \qquad (p=1,2,\ldots,n;\ j=1,2,\ldots,n) \qquad (2.22)$$

from which γ_{kj} can be obtained (by taking $p=k$) as

$$\gamma_{pj} = \alpha_{kj} / \left(1 + i\alpha_{ks} h_{sk}\right) \qquad (p=1,2,\ldots,n) \qquad (2.23)$$

Once the γ_{kj} (j=1,2,…,n) have been calculated from Equation (2.23), the remaining receptance values γ_{pj} ($p=1,2,\ldots,k-1,k+1,\ldots,n$; $j=1,2,\ldots,n$) are found from the calculated values of γ_{kj} ($j=1,2,\ldots,n$) by using Equation (2.22).

The above formulation gives the receptances of the system composed of the undamped system and a single damping element h_{sk}. If the calculated receptances are treated as α values in Equations (2.22) and (2.23), a new set of receptances can be calculated by considering another damping element of the original damping matrix $[H]$. If this procedure is repeated for all nonzero elements of $[H]$, the final receptance matrix $[\gamma]$ gives the receptances of the damped system.

A considerable reduction in the computational effort can be achieved if the damping is local or if the damping matrix can be written as the addition of proportional and non-proportional damping matrices, where non-proportional damping is local. Then

$$[H] = [H]_P + [H]_N \qquad (2.24)$$

where $[H]_P$ is the proportional part of the damping matrix and $[H]_N$ is the non-proportional part of the damping matrix such that

$$[H]_N = \begin{bmatrix} [H_{11}] & [0] \\ [0] & [0] \end{bmatrix} \tag{2.25}$$

Then, γ_{kj} (j=1, 2,…, n) can be found from Equation (2.23), and Equation (2.22) may be used to find γ_{pj} for only m values of p, where m is the order of the submatrix $[H_{11}]$. Therefore, the final values of γ_{pj} (p=1,2,…,m; j=1,2,…,n) which include the effect of all the m^2 damping values can be calculated without computing the receptances corresponding to only undamped coordinates (i.e., without calculating $[\gamma_{22}]$). Thus the number of recomputations of each receptance will be reduced from n^2 to m^2, since the number of damping elements will be just m^2.

Calculation of the receptances corresponding to undamped nodes, then can be achieved by using Equation (2.21)

$$\gamma_{pj} = \alpha_{pj} - i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk}\gamma_{kj} \tag{2.26}$$

for p=m+1,…,n and j=p,…,n.

A further improvement in the formulation is made by considering one column of the damping matrix at a time. When, say, the k-th column of the damping matrix $[H_{11}]$ is considered, from Equation (2.21) one can write

$$\gamma_{pj} = \alpha_{pj} - \left(i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} \right)\gamma_{kj} \tag{2.27}$$

or, for p=k,

$$\gamma_{kj} = \alpha_{kj} \Big/ \left(1 + i\sum_{s=1}^{m} \alpha_{ks} h_{sk}\right) \qquad (j=1,2,\ldots,n) \tag{2.28}$$

After calculating the α_{kj} (j=1,2,…,n), which include the effect of the k-th column of the damping matrix, the remaining elements of $\left[\gamma_{11}\right]$ and $\left[\gamma_{12}\right]$ can be found from Equation (2.27) for j=1,2,…,n and p=1,2,…,k-1,k+1,…,m. After repeating this procedure m times (k=1, 2,…, m), the final values of the upper mxn portion of $\left[\gamma\right]$ will be obtained. The remaining elements of the receptance matrix can be obtained from Equation (2.26).

2.1.3 APPLICATION TO STRUCTURAL MODIFICATION PROBLEMS

In the above formulation, the modifying structure had just damping properties. If the matrix $i\left[H_{11}\right]$ is replaced by a more general dynamic stiffness matrix

$$\left[\delta_{11}\right] = \left[\left[K_{11}\right] - \omega^2\left[M_{11}\right] + i\left[H_{11}\right]\right] \tag{2.29}$$

where $\left[K_{11}\right],\left[M_{11}\right]$ and $\left[H_{11}\right]$ are the stiffness, mass and hysteretic damping matrices of the modifying structure, respectively, the same formulation can be used for structural modification problems.

Then, by replacing the damping terms ih_{sk} in Equations (2.21) through (2.28) by the corresponding elements of $\left[\delta_{11}\right]$, which would be

$$\delta_{sk} = k_{sk} - \omega^2 m_{sk} + ih_{sk} \tag{2.30}$$

The formulation given in the previous section can be used for structural modification. However, these equations can be used in the given forms only if there is no additional DOF due to the modifying structure.

2.1.4 IMPROVEMENT OF THE FORMULATION FOR FRF STUDY

The formulation of Özgüven intends the calculation of the whole receptance matrix which is required when the displacement of each node is investigated at a fixed frequency. However, in FRF studies, only one element of the receptance matrix is required while the frequency sweeps in a given range. In such a case, it is imperative to calculate the minimum number of receptance elements at each frequency in order to conduct a refined FRF study, especially for large systems where computational burden is emphasized. On the other hand, the updating algorithm that includes the effect of each damping element one by one or row by row may suffer of successive truncations and round off errors. In large systems, the cumulative effect of these errors may not be negligible. In order to avoid this, the solution should be reduced into a single operation.

As indicated in the previous section, the damping term ih_{sk} can be replaced by a general modification term as in Equation (2.30).

Recalling Equation (2.26) and performing the above substitution leads to :

$$\gamma_{pj} = \alpha_{pj} - \sum_{s=1}^{m} \alpha_{ps} \sum_{k=1}^{m} \delta_{sk} \gamma_{kj} \tag{2.31}$$

Changing the summation order of Equation (2.31) gives:

$$\gamma_{pj} = \alpha_{pj} - \sum_{k=1}^{m} \gamma_{kj} \sum_{s=1}^{m} \delta_{sk} \alpha_{ps} \tag{2.32}$$

From the inspection of this equation, one can decide that any receptance value γ_{pj} can be computed if the first *m* elements of the j^{th} column of the final receptance matrix, i.e. γ_{pj} (for *k*=1…*m*) are known. The problem is now, how to find those elements. In fact, Equation (2.32) is a recursive equation. The solution is then straight forward, since rewriting Equation (2.32) for each unknown γ_{kj} (for *k*=1…*m*) will give *m* equations with *m* unknowns that can be easily solved.

The solution would start by rearranging Equation (2.32) as follows:

$$\gamma_{pj} + \sum_{k=1}^{m} \gamma_{kj} \sum_{s=1}^{m} \delta_{sk} \alpha_{ps} = \alpha_{pj} \tag{2.33}$$

From the above equation, a set of *m* equations for *m* unknowns can be generated. If the goal is to compute γ_{ij} , the first step would be to rewrite Equation (2.33) for *p*=1…*m* :

$$\gamma_{1j} + \sum_{k=1}^{m} \gamma_{kj} \sum_{s=1}^{m} \delta_{sk} \alpha_{1s} = \alpha_{1j}$$

$$\gamma_{2j} + \sum_{k=1}^{m} \gamma_{kj} \sum_{s=1}^{m} \delta_{sk} \alpha_{2s} = \alpha_{2j}$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\gamma_{mj} + \sum_{k=1}^{m} \gamma_{kj} \sum_{s=1}^{m} \delta_{sk} \alpha_{ms} = \alpha_{mj}$$

Let $\quad A_{pk} = \sum_{s=1}^{m} \delta_{sk} \alpha_{ms}$

Expanding the set of *m* equations:

$$\gamma_{1j}(1+A_{11})+\gamma_{2j}A_{12}+.....+\gamma_{mj}A_{1m}=\alpha_{1j}$$

$$\gamma_{1j}A_{21}+\gamma_{2j}(1+A_{22})+.....+\gamma_{mj}A_{2m}=\alpha_{2j}$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\gamma_{ij}A_{m1}+\gamma_{2j}A_{m2}+.....+\gamma_{mj}(1+A_{mm})=\alpha_{mj}$$

This can be written in matrix from as a constant coefficient matrix times the unknown vector equal to a constant vector:

$$[O]\{\gamma'\}^{(j)}=\{\alpha'\}^{(j)} \tag{2.34}$$

where $\{\gamma'\}^{(j)}$ and $\{\alpha'\}^{(j)}$ are the j^{th} columns of the receptance matrices of the modified and original structure, respectively, and matrix $[O]$ is given by

$$[O]=[A]+[I] \tag{2.35}$$

where $[A]$ is a coefficient matrix generated from A_{pk} (for p=1…m, k=1…m), and $[I]$ is the identity matrix.

Equation (2.34) can be rewritten as

$$\{\gamma'\}^{(j)}=[O]^{-1}\{\alpha'\}^{(j)} \tag{2.36}$$

In fact, Equation (2.36) is in the same form of Equation (2.10) written for a column. Once the m elements of the j^{th} column of the receptance matrix are calculated, any element of the j^{th} column can be found by using Equation (2.32). With this procedure, at most (*m+1*) elements are calculated instead of (*m*m*).

2.1.5 STRUCTURAL MODIFICATION BY USING SHERMAN-MORRISON FORMULA

The structural modification method proposed by Şanlıtürk [38] is based on the Sherman-Morrison identity [22] which allows a direct inversion of the modified matrix efficiently using the data related to the initial matrix and to the modifications. Sherman-Morrison formula is an extension of Woodbury's study [23], and can be given as follows:

Assume $\left[A\right]^{-1}$ is the inverse of a non-singular square matrix $\left[A\right]$. If the modified matrix is in the form:

$$\left[A^*\right]=\left[A\right]+\left[\Delta A\right]=\left[A\right]+\{u\}\{v\}^T \tag{2.37}$$

$$\left[A^*\right]^{-1}=\left[A\right]^{-1}-\frac{\left(\left[A\right]^{-1}\{u\}\right)\left(\{v\}^T\left[A\right]^{-1}\right)}{1+\{v\}^T\left[A\right]^{-1}\{u\}} \tag{2.38}$$

Sherman-Morrison formula is a simplified version of Woodbury formula, where the modification matrix is written as the multiplication of two vectors, as $\left[\Delta A\right]=\{u\}\{v\}^T$. Then inverse of the modified matrix is given by:

$$\left[A^*\right]^{-1}=\left[A\right]^{-1}-\left[A\right]^{-1}\left[U\right]\left(\left[I\right]+\left[V\right]^T\left[A\right]^{-1}\left[U\right]\right)\left[V\right]^T\left[A\right]^{-1} \tag{2.39}$$

This is only valid for a square matrix $\left[A\right]_{NxN}$. Any matrix $\left[\Delta A\right]$ can be expressed as multiplication of two rectangular matrices, $\left[U\right]_{Nxn}$ and $\left[V\right]_{nxN}$ provided that $\left[A\right]$ and $\left(\left[I\right]+\left[V\right]^{T}\left[A\right]^{-1}\left[U\right]\right)$ matrices are invertible for $n \leq N$.

Similarities between Equation (2.38) and (2.39) can be seen. The main advantage of the Sherman-Morrison identity is that it provides the inversion of the modified matrix without any matrix inversion. However it seems that the modifications are limited to the cases where modification can be expressed in the form of $\{u\}\{v\}^{T}$.

Consider the equations of motion in frequency domain for a structure given by:

$$\left(\left[K\right]-\omega^{2}\left[M\right]+i\left[H\right]\right)\{x\}=\{F\} \tag{2.40}$$

$$\{x\}=\left(\left[K\right]-\omega^{2}\left[M\right]+i\left[H\right]\right)^{-1}\{F\} \tag{2.41}$$

Then the receptance matrix is obtained as

$$\left[\alpha\right]=\left(\left[K\right]-\omega^{2}\left[M\right]+i\left[H\right]\right)^{-1} \tag{2.42}$$

Sherman-Morrison identity provides great simplifications if initial receptance matrix is available. Initial receptance matrix is usually obtained by modal summation after an eigenvalue analysis. If $\left[\alpha\right]$ is available then it can be written as:

$$\left[\gamma\right]=\left[\alpha\right]-\frac{\left(\left[\alpha\right]\{u\}\right)\left(\{v\}^{T}\left[\alpha\right]\right)}{1+\{v\}^{T}\left[\alpha\right]\{u\}} \tag{2.43}$$

The main disadvantage of this method is that, if the modifications affect all the coordinates, the formulation does not provide any computational advantage. The method provides substantial savings in computational time for calculating the receptance matrix of the modified system when local modifications are made, similar to the case in matrix inversion method discussed above.

When modifications can be written in the form of $\{u\}\{v\}^T$ the new receptance matrix can be obtained by Sherman-Morrison identity. If it is necessary to express the modifications as $[U][V]^T$ then Woodbury identity can be used. However, this requires matrix inversion.

It is shown that Sherman-Morrison identity can provide substantial further savings in computational time for the cases where modification and forcing coordinates are small subsets of the total coordinates and response amplitudes are to be calculated at selected coordinates. When these are satisfied, calculations are confined to the active coordinates. So $\{x\}$ and $[\alpha]$ can be partitioned as:

$$\{x\}=\begin{Bmatrix}\{x_i\}\\ \{x_a\}\end{Bmatrix} \qquad [\alpha]=\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix} \tag{2.44}$$

where *i* and *a* indicate inactive and active coordinates respectively. If these are inserted in Equation (2.43), it takes the form:

$$\begin{bmatrix}\gamma_{ii} & \gamma_{ia}\\ \gamma_{ai} & \gamma_{aa}\end{bmatrix}=\begin{bmatrix}\alpha_{ii} & \alpha_{ia}\\ \alpha_{ai} & \alpha_{aa}\end{bmatrix}-\frac{\left(\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix}\begin{Bmatrix}\{0\}\\ \{u_a\}\end{Bmatrix}\right)\left(\begin{Bmatrix}\{0\}\\ \{v_a\}\end{Bmatrix}^T\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix}\right)}{1+\begin{Bmatrix}\{0\}\\ \{v_a\}\end{Bmatrix}^T\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix}\begin{Bmatrix}\{0\}\\ \{u_a\}\end{Bmatrix}} \tag{2.45}$$

Here, $\{u\}$ and $\{v\}$ vectors are also partitioned to active and inactive coordinates. Modifications are limited to active coordinates that are the coordinates where response levels are needed.

From Equation (2.45) it can be seen that Sherman-Morrison formula is also applicable for the case where only the active coordinates (modification coordinates, forcing coordinates and those coordinates where response levels are needed) are retained. That is,

$$\left[\gamma_{aa}\right]=\left[\alpha_{aa}\right]-\frac{\left(\left[\alpha_{aa}\right]\left\{u_a\right\}\right)\left(\left\{v_a\right\}^T\left[\alpha_{aa}\right]\right)}{1+\left\{v_a\right\}^T\left[\alpha_{aa}\right]\left\{u_a\right\}} \tag{2.46}$$

The significance of Equation (2.46) is that it makes it possible to perform the calculations using active coordinates alone, the size of which is much smaller than the total number of degrees of freedom, *N*, in many applications.

If the modification matrix can not be expressed as $\left\{u\right\}\left\{v\right\}^T$, then it can be decomposed into several modifications such as

$$\left[\Delta K\right]=\left[\Delta K_1\right]+\left[\Delta K_2\right]+\left[\Delta K_3\right]+\ldots+\left[\Delta K_z\right] \tag{2.47}$$

where $\left[\Delta K_j\right]=\left\{u_j\right\}\left\{v_j\right\}^T$. This allows the $\left[\alpha\right]$ matrix to be calculated in *z* steps by considering one modification matrix at a time.

If we have viscous damping $\left[C\right]$, instead of structural damping, then obviously $\left[H\right]$ and $\left[\Delta H\right]$ will be replaced by $\omega\left[C\right]$ and $\omega\left[\Delta C\right]$ respectively in all equations given above.

2.1.6 EXTENDED SUCCESSIVE MATRIX INVERSION METHOD

The main idea of Classical Successive Matrix Inversion Method is to obtain the modified structural responses by considering only modified portion of stiffness matrix

[42]. In the method developed in this work [43], the modified structural responses are obtained by considering only the modified portion of the dynamic stiffness matrix. The theory for the Extended Successive Matrix Inversion Method is given below;

The equation of motion for an *N* degrees of freedom system can be written as

$$\left[M\right]\left\{\ddot{x}\right\}+i\left[H\right]\left\{x\right\}+\left[K\right]\left\{x\right\}=\left\{F\right\} \tag{2.48}$$

The response of the system to a harmonic forcing at a frequency ω can be written as

$$\left\{x\right\}=\left[\left[K\right]-\omega^2\left[M\right]+i\left[H\right]\right]^{-1}\left\{F\right\} \tag{2.49}$$

from which the receptance matrix $\left[\alpha\right]$ can be obtained as

$$\left[\alpha\right]=\left[\left[K\right]-\omega^2\left[M\right]+i\left[H\right]\right]^{-1} \tag{2.50}$$

The equation of motion of the modified system will take the form

$$\left[M+\Delta M\right]\left\{\ddot{x}\right\}+i\left[H+\Delta H\right]\left\{x\right\}+\left[K+\Delta K\right]\left\{x\right\}=\left\{F\right\} \tag{2.51}$$

and the harmonic response of the system will be obtained as

$$\left[\left[K+\Delta K\right]-\omega^2\left[M+\Delta M\right]+i\left[H+\Delta H\right]\right]\left\{x\right\}=\left\{F\right\} \tag{2.52}$$

$$\left\{x\right\}=\left[\left[K+\Delta K\right]-\omega^2\left[M+\Delta M\right]+i\left[H+\Delta H\right]\right]^{-1}\left\{F\right\} \tag{2.53}$$

Then the receptance matrix of the modified system will be

$$\left[\gamma\right]=\left[\left[K+\Delta K\right]-\omega^2\left[M+\Delta M\right]+i\left[H+\Delta H\right]\right]^{-1} \quad (2.54)$$

$\left[\gamma\right]$ can also be obtained by premultiplying Equation (2.52) by $\left[\alpha\right]$

$$\left(\left[I\right]-\left[P\right]\right)\left\{x\right\}=\left\{F'\right\} \quad (2.55)$$

where

$$\left\{F'\right\}=\left[\alpha\right]\left\{F\right\} \quad (2.56)$$

$$\left[P\right]=-\left[\alpha\right]\left[\left[\Delta K\right]-\omega^2\left[\Delta M\right]+i\left[\Delta H\right]\right] \quad (2.57)$$

Then from Equation (2.55) and Equation (2.56)

$$\left[\gamma\right]=\left(\left[I\right]-\left[P\right]\right)^{-1}\left[\alpha\right] \quad (2.58)$$

This is the basic equation of the structural modification method of Özgüven [41]. While the size of the matrix inverted is reduced for locally modified systems by partitioning the system matrices, and furthermore a novel algorithm is suggested to avoid matrix inversion in Özgüven’s method, here in this method, power series expansion will be used for the inversion of matrix in Equation (2.58) as successfully employed in Successive Matrix Inversion method [42]:

$$\left(\left[I\right]-\left[P\right]\right)^{-1}=\left[I\right]+\left[P\right]+\left[P\right]^2+\left[P\right]^3+\dots \quad (2.59)$$

Let

$$\left[T\right]=\left[P\right]+\left[P\right]^2+\left[P\right]^3+\dots \quad (2.60)$$

The elements of $\left[T\right]$ can be written as

$$T_{ij} = P_{ij}^{(1)} + P_{ij}^{(2)} + \ldots + P_{ij}^{(k)} + \ldots \tag{2.61}$$

where $P_{ij}^{(k)}$ is the $(i,j)^{th}$ element of $\left[P\right]^{(k)}$. If the k^{th} recursion factor is defined as

$$r_{ij}^{(k)} = P_{ij}^{(k+1)} / P_{ij}^{(k)} \tag{2.62}$$

and if the recursion factor is constant through the expansion, Equation (2.61) can be expressed as

$$T_{ij} = P_{ij}\left(1 + r_{ij} + r_{ij}{}^{2} + r_{ij}{}^{3} + \ldots\right) \tag{2.63}$$

Using the series expansion for the recursive terms, T_{ij} can be written as

$$T_{ij} = P_{ij} / \left(1 - r_{ij}\right) \tag{2.64}$$

The recursion factor is different through the series expansion, so that the modification matrix should be decomposed into separate matrices than the variability of the recursion factor could be eliminated.

$$\left[\Delta K\right] - \omega^{2}\left[\Delta M\right] + i\left[\Delta H\right] = \sum_{j=1}^{N}\left[\left[\Delta K^{(j)}\right] - \omega^{2}\left[\Delta M^{(j)}\right] + i\left[\Delta H^{(j)}\right]\right] \tag{2.65}$$

where $\left[\Delta K^{(j)}\right]$, $\left[\Delta M^{(j)}\right]$, $\left[\Delta H^{(j)}\right]$ are matrices composed of the j^{th} columns of stiffness, mass and structural damping matrices, respectively, and zero columns except the j^{th} columns. For only one nonzero column of $\left[T\right]$, the recursion factor is a constant

$$r = P_{jj} \tag{2.66}$$

Then Equation (2.64) becomes

$$T_{ij} = P_{ij} / \left(1 - r\right) \tag{2.67}$$

Now let

$$\left[Y\right] = \left[\left[\Delta K\right] - \omega^2 \left[\Delta M\right] + i\left[\Delta H\right]\right] \tag{2.68}$$

$$\left[Z\right] = \left[\left[K\right] - \omega^2 \left[M\right] + i\left[H\right]\right] \tag{2.69}$$

Then, the dynamic stiffness matrix, after the j^{th} non-zero column of the structural modification is taken into consideration, can be written as

$$\left[Z^{(j)}\right] = \left[Z^{(j-1)}\right] + \left[Y^{(j)}\right] \tag{2.70}$$

where $\left[Z^{(j-1)}\right]$ and $\left[Z^{(j)}\right]$ are the modified dynamic stiffness matrices in $(j\text{-}1)^{th}$ and j^{th} steps, respectively. $\left[Y^{(j)}\right]$ is a matrix which has the j^{th} non-zero column of $\left[Y\right]$ matrix at its corresponding column, and zero columns elsewhere. Note that $\left[Z^{(0)}\right]$ denotes the initial $\left[Z\right]$ matrix, and for the modification matrix $\left[Y^{(j)}\right]$, $\left[Z^{(j-1)}\right]^{-1}$ refers to $\left[\alpha\right]$ and $\left[Z^{(j)}\right]^{-1}$ refers to $\left[\gamma\right]$ in Equation (2.58). By using Equation (2.59) and Equation (2.60), the inverse of Equation (2.70) can be obtained as:

$$\left[Z^{(j)}\right]^{-1}=\left(\left[I\right]+\left[T\right]\right)\left[Z^{(j-1)}\right]^{-1} \tag{2.71}$$

Thus, by updating these matrices for each nonzero column of the modification matrix, the modified FRF matrix can be obtained. Since each column of the modification matrix contributes to the dynamics of the system independently, the sequence of the columns used in the computation is not important. It should also be noted at this stage that for a local modification, $\left[Y\right]$ will be a highly sparse matrix with many zero columns and rows corresponding to the coordinates at which there is no structural modification, and that property of $\left[Y\right]$ will make the method presented here attractive.

The algorithm of the reanalysis method suggested for structural modifications using SMI can be summarized as follows:

Step 1:

Obtain the original FRF matrix and the modification matrix $\left[Y\right]$.

Step 2:

Partition the modification matrix to number of non-zero element columns (*m*) such that

$$\left[Y\right]=\left[Y^{(1)}\right]+\left[Y^{(2)}\right]+\ldots+\left[Y^{(m)}\right] \tag{2.72}$$

Step 3:

For each non-zero column of $\left[Y\right]$(*j*=1,2,...*m*), the following computations are repeated:

$$\left[P^{(j)}\right]=-\left[Z^{(j-1)}\right]^{-1}\left[Y^{(j)}\right] \tag{2.73}$$

$$\left[Z^{(j)}\right]^{-1}=\left[Z^{(j-1)}\right]^{-1}+\left(\frac{1}{1-r^{(j)}}\right)\left[P^{(j)}\right]\left[Z^{(j-1)}\right]^{-1} \tag{2.74}$$

where $\left[P^{(j)}\right]$ will have all columns zero except the j^{th} column, since $\left[Y^{(j)}\right]$ is in the same form.

Then in the last step $\left[Z^{(m)}\right]^{-1}$ will give the receptance matrix of the modified structure. The algorithm of Successive Matrix Inversion method uses only matrix additions and matrix-vector multiplications to update the response matrix. In Equation (2.73), $\left[Y^{(j)}\right]$ and therefore $\left[P^{(j)}\right]$ have many zero elements so that using sparse matrix properties in the storage and matrix operations in the algorithm summarized above will reduce the computational effort considerably.

If we have viscous damping $\left[C\right]$, instead of structural damping, then obviously $\left[H\right]$ and $\left[\Delta H\right]$ will be replaced by $\omega\left[C\right]$ and $\omega\left[\Delta C\right]$ respectively in all equations given above.

2.2 STRUCTURAL MODIFICATIONS WITH ADDITIONAL DEGREES OF FREEDOM

2.2.1 ÖZGÜVEN'S FORMULATION

In his work, Özgüven [41] stated that when the structural modifications introduce additional DOFs into the system, the receptance matrix of the modified structure $\left[\gamma\right]$ can be partitioned so that it is separated as follows:

- The coordinates which correspond to the original structure only (a)
- The connection elements which are between the original and the modifying structure (b)

- The coordinates which correspond to the modifying structure only (c)

Then the following equations can be written for the original and the modifying structures:

$$\left[\alpha\right]^{-1} = \begin{bmatrix} \alpha_{aa} & \alpha_{ab} \\ \alpha_{ba} & \alpha_{bb} \end{bmatrix}^{-1} \tag{2.75}$$

$$\left[\delta\right] = \left[\Delta K\right] - \omega^2\left[\Delta M\right] + i\left[\Delta H\right] \tag{2.76}$$

$$\begin{bmatrix} \gamma_{aa} & \gamma_{ab} & \gamma_{ac} \\ \gamma_{ba} & \gamma_{bb} & \gamma_{bc} \\ \gamma_{ca} & \gamma_{cb} & \gamma_{cc} \end{bmatrix}^{-1} = \begin{bmatrix} \left[\alpha\right]^{-1} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{bmatrix} + \begin{bmatrix} 0 & \begin{matrix} 0 & 0 \end{matrix} \\ \begin{matrix} 0 \\ 0 \end{matrix} & \left[\delta\right] \end{bmatrix} \tag{2.77}$$

Pre-multiplying Equation (2.77) by

$$\begin{bmatrix} \left[\alpha\right] & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & I \end{bmatrix} \tag{2.78}$$

and post multiplying by $\left[\gamma\right]$ yields

$$\begin{bmatrix} \left[\alpha\right] & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}\left[\gamma\right] + \begin{bmatrix} 0 & \begin{bmatrix} \alpha_{ab} & 0 \end{bmatrix} & \left[\delta\right] \\ \begin{matrix} 0 \\ 0 \end{matrix} & \begin{bmatrix} \alpha_{bb} & 0 \\ 0 & I \end{bmatrix} & \left[\delta\right] \end{bmatrix}\left[\gamma\right] \tag{2.79}$$

It is possible to write

$$\left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_{bb} & 0 \\ 0 & I \end{bmatrix}\left[\delta\right]\right]\begin{bmatrix} \gamma_{ba} \\ \gamma_{ca} \end{bmatrix} = \begin{bmatrix} \alpha_{ba} \\ 0 \end{bmatrix} \tag{2.80}$$

$$\left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_{bb} & 0 \\ 0 & I \end{bmatrix} [\delta] \right] \begin{bmatrix} \gamma_{bb} & \gamma_{bc} \\ \gamma_{cb} & \gamma_{cc} \end{bmatrix} = \begin{bmatrix} \alpha_{bb} & 0 \\ 0 & I \end{bmatrix} \tag{2.81}$$

$$[\gamma_{aa}] + [\alpha_{ab} \quad 0][\delta] \begin{bmatrix} \gamma_{ba} \\ \gamma_{ca} \end{bmatrix} = [\gamma_{aa}] \tag{2.82}$$

$$[\gamma_{ab} \quad \gamma_{ac}] + [\alpha_{ab} \quad 0][\delta] \begin{bmatrix} \gamma_{bb} & \gamma_{bc} \\ \gamma_{cb} & \gamma_{cc} \end{bmatrix} = [\gamma_{ab} \quad 0] \tag{2.83}$$

from which the receptance matrix of the modified structure can be calculated in terms of the receptance matrix of the original structure $[\alpha]$ and dynamic structural modification matrix $[\delta]$ as follows:

$$\begin{bmatrix} \gamma_{ba} \\ \gamma_{ca} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_{bb} & 0 \\ 0 & I \end{bmatrix} [\delta] \right]^{-1} \begin{bmatrix} \alpha_{ba} \\ 0 \end{bmatrix} \tag{2.84}$$

$$\begin{bmatrix} \gamma_{bb} & \gamma_{bc} \\ \gamma_{cb} & \gamma_{cc} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_{bb} & 0 \\ 0 & I \end{bmatrix} [\delta] \right]^{-1} \begin{bmatrix} \alpha_{bb} & 0 \\ 0 & I \end{bmatrix} \tag{2.85}$$

The equations above show that only a single matrix is to be inverted for the computation of the complete receptance matrix. The order of the matrix to be inverted is equal to the DOF of the modifying structure, which is usually much less than the total DOF of the system.

2.3 EXPANSION AND REDUCTION TECHNIQUES

The reduction of analytical or expansion of experimental modal vectors is a necessary process in structural modeling. Generally, only a limited number of DOFs are measured for a number of references in experimental studies. The expansion process expands the dynamic information to include the deleted or omitted DOFs which include the balance of the translational DOF and all the rotational DOF. Prior to the conduction of each modal test, decisions have to be made for which of the many DOFs that exist in a theoretical model will be measured. These decisions are made for various practical reasons which include: limited test time, inaccessibility of some DOFs; anticipated low importance of motion in certain DOFs. A major application of expansion is the acquisition of data relating to rotational degrees of freedoms (RDOFs). RDOFs are very difficult to measure directly; yet they may be critical for some applications. One way of including these data is to derive them by using one of the expansion methods. Reduction process is the inverse of expansion process, which is used when it is decided to obtain compatibility between two otherwise disparate models by reducing the size of the larger of the two models that is usually the analytical model. Because of computing power limitations, reduction methods are recently becoming less important. Since iterative algorithms are used, the computing power is increasing for reduction methods. Reduction is a process which is not applied to a test model, but rather to analytical models with which the experimental data are to be compared.

2.3.1 REDUCTION OF SYSTEM MATRICES

In all reduction techniques, there is a relationship between the measured DOF and the unmeasured DOF which can be written in general form as [63]

$$\{x\} = \begin{Bmatrix} \{x_m\} \\ \{x_n\} \end{Bmatrix} = [Tr]\{x_m\} \tag{2.86}$$

Here, $[Tr]$ is the transformation matrix, *m* denotes the number of measured DOFs, *n* denotes unmeasured DOFs and *N* denotes the total DOFs. Then the reduced mass and stiffness matrices can be written as:

$$\left[M_m\right]=\left[Tr\right]^T\left[M_N\right]\left[Tr\right] \tag{2.87}$$

$$\left[K_m\right]=\left[Tr\right]^T\left[K_N\right]\left[Tr\right] \tag{2.88}$$

2.3.1.1 GUYAN REDUCTION

Guyan reduction has been used for a long time for the reduction of large analytical models. Consider the equation of motion of an undamped discrete system [64]:

$$\left[M\right]\left\{\ddot{x}\right\}+\left[K\right]\left\{x\right\}=\left\{F\right\} \tag{2.89}$$

Partitioning the matrices into measured *m* and unmeasured *n* DOFs yields:

$$\begin{bmatrix}\left[K_{mm}\right] & \left[K_{mn}\right]\\ \left[K_{nm}\right] & \left[K_{nn}\right]\end{bmatrix}\begin{Bmatrix}\left\{x_m\right\}\\ \left\{x_n\right\}\end{Bmatrix}=\begin{Bmatrix}\left\{F_m\right\}\\ \left\{F_n\right\}\end{Bmatrix} \tag{2.90}$$

Using the lower part:

$$\left\{x_n\right\}=\left[K_{nn}\right]^{-1}\left[K_{nm}\right]\left\{x_m\right\}+\left[K_{nn}\right]^{-1}\left\{F_n\right\} \tag{2.91}$$

Assuming no external force in the unmeasured DOFs, the transformation matrix will be:

$$\left[Tr_1\right]=\begin{bmatrix}\left[I\right]\\ -\left[K_{nn}\right]^{-1}\left[K_{nm}\right]\end{bmatrix} \tag{2.92}$$

The performance of Guyan reduction depends heavily on the selection of the measured DOFs. This method is also called as static reduction.

2.3.1.2 IMPROVED REDUCTION SYSTEM TECHNIQUE

Improved Reduction System technique (IRS) is developed to compensate for the mass misappropriation of the Guyan reduction process. It is effective when the selection of measured DOF is not optimum. It improves the accuracy over the static condensation technique [64]:

$$\left[K_{nn}\right]^{-1}\left\{F_n\right\}=\left[\left[K_{nn}\right]^{-1}\left[M_{nm}\right]-\left[K_{nn}\right]^{-1}\left[M_{nn}\right]\left[K_{nn}\right]^{-1}\left[K_{nm}\right]\right]\left[M_R\right]^{-1}\left[K_R\right]\left\{x_m\right\} \tag{2.93 }$$

where

$$\left[M_R\right]=\left[Tr_1\right]^T\left[M\right]\left[Tr_1\right] \tag{2.94}$$

$$\left[K_R\right]=\left[Tr_1\right]^T\left[K\right]\left[Tr_1\right] \tag{2.95}$$

Substituting Equation (2.93) into Equation (2.91) yields:

$$\left[Tr_2\right]=\left[Tr_1\right]+\left[Tr_2^{'}\right] \tag{2.96}$$

where

$$\left[Tr_2^{'}\right]=\left[S\right]\left[M\right]\left[Tr_1\right]\left[M_R\right]^{-1}\left[K_R\right] \tag{2.97}$$

$$\left[S\right]=\begin{bmatrix}0 & 0\\ 0 & \left[K_{ss}\right]^{-1}\end{bmatrix} \tag{2.98}$$

2.3.1.3 DYNAMIC REDUCTION TECHNIQUE

Dynamic Reduction is an extension of the Guyan reduction process. Basically, the effective mass for a particular frequency is retained in the system matrices using [63]:

$$\left[Tr_{\omega}\right]=\begin{bmatrix}\left[I\right]\\-\left[G_{nn}\right]^{-1}\left[G_{nm}\right]\end{bmatrix}=\begin{bmatrix}\left[I\right]\\\left[tr_{\omega}\right]\end{bmatrix} \tag{2.99}$$

where

$$\left[G\right]=\left[\left[K\right]-\omega^{2}\left[M\right]\right] \tag{2.100}$$

This technique will exactly preserve the system dynamics for the particular frequency. Therefore, the transformation matrix is exact for a particular frequency.

2.3.1.4 SYSTEM EQUIVALENT REDUCTION EXPANSION PROCESS

The System Equivalent Reduction Expansion Process (SEREP) relies on an analytical or finite element model from which an eigensolution is obtained to develop the mapping between the full set of DOF and the reduced set of DOF.

It is used in applications to improve the accuracy of models such as cross orthogonality checks between analytical and experimental modal vectors.

Using a collection of modes that are to be preserved in the reduction process, a transformation matrix can be developed using a generalized inverse on the modal matrix to obtain [63]

$$\left[Tr_{UA}\right]=\begin{bmatrix}UA_m\\UA_n\end{bmatrix}\left[UA_m\right]^g \tag{2.101}$$

where *m* denotes the number of measured DOFs, *n* denotes unmeasured DOFs, *UA* is analytical modal matrix and *g* stands for the generalized inverse.

Using the generalized inverse, which carries information pertaining to the selected modes at the selected set of n DOFs, the formulation allows the reduction process to preserve the dynamics of the full system.

2.3.2 EXPANSION OF SYSTEM MATRICES

All relationships that are used to reduce the system matrices can also be used in expansion techniques and is written in a general form of [63]:

$$\left[E_N\right]=\begin{bmatrix}\left[E_m{}'\right]\\\left[E_n\right]\end{bmatrix}=\left[Tr\right]\left[E_m\right] \tag{2.102}$$

where *E* is the experimental modal matrix,

This expansion process relies on the transformation matrix obtained from the analytical model. In that case, the transformation matrix can be used to extract data for all of the unmeasured DOFs as a global curve fitter between the experimental data and data from analytical model.

$$\left[E_N\right]=\left[Tr\right]\left[E_m\right] \tag{2.103}$$

2.3.2.1 SEREP EXPANSION PROCESS

The System Equivalent Reduction Expansion Process (SEREP) is used for the expansion of modes extracted from the modal parameter estimation process in order to obtain estimates of the unmeasured translational DOF effects as well as all of the rotational DOF effects, which can not be measured. SEREP Expansion is given as:

$$\left[Tr_{UA}\right]=\begin{bmatrix}\left[UA_m \quad UA_m^g\right]\\ \left[UA_n \quad UA_m^g\right]\end{bmatrix} \tag{2.104}$$

where *m* shows the measured coordinates and *n* shows the unmeasured coordinates, *UA* is analytical modal matrix and *g* stands for the generalized inverse.

In order to obtain estimates for the residual compensation effects that are the effects of upper and lower residual terms of both translational DOFs and rotational DOFs, a different expansion technique is necessary which is not modal based but rather frequency based. Two techniques are described here; The Dynamic Residual Expansion Process and the Dynamic FRF Expansion Process.

2.3.2.2 DYNAMIC RESIDUAL EXPANSION PROCESS

The Dynamic Residual Expansion (DRE) Process can be used to develop a transformation matrix for any particular frequency, such that [63]

$$\left\{UR_{Rdof}\right\}=\left[D_{nn}^{-1} \quad D_{nm}\right]\left\{UR_{Tdof}\right\} \tag{2.105}$$

where m shows the measured coordinates and n shows the unmeasured coordinates. Here, $\{UR\}$ stands for upper residual and $\left[D\right]$ denotes dynamic matrix. This process allows for an estimation of an FRF, which accounts for the modes as well as the adjustments that account for the fact that there are out-of-band effects from other modes.

Using a combination of SEREP expansion and DRE, an estimate for the measured FRFs can be obtained. The SEREP/DRE process can be used to develop a transformation matrix for expansion of modal information and residual information.

Using this combination, the residual terms for both translational and rotational DOFs can be included in the synthesis of an FRF. However, it is very important to point out that while the modes of the system are general, the residual mode is not general. The residual mode information is valid only for the input-output DOFs specified. Therefore, the residual mode expansion can only be performed for the particular reference DOF used for the FRF measurement.

2.3.2.3 DYNAMIC FRF EXPANSION PROCESS

The Dynamic FRF Expansion (DFE) Process can be used to develop a transformation matrix for any particular frequency. This matrix can be used for the expansion of the measured FRF at a spectral line of the system using,

$$\left\{ FRF(\omega_0)_{Rdof} \right\} = \left[D_{nn}^{-1} \quad D_{nm} \right] \left\{ FRF(\omega_0)_{Tdof} \right\} \tag{2.106}$$

where m shows the measured coordinates and n shows the unmeasured coordinates. The Dynamic FRF Expansion Process uses the results from Equation (2.106) along with the FRFs synthesized with only the measured mode set to form the residual for rotational DOFs. This allows for an estimation of an FRF which accounts for the measured modes as well as the residuals for both translational and rotational DOFs.

Using a combination of SEREP expansion and DFE, an estimate of the unmeasured FRFs can be obtained. The SEREP/DFE process is also used to develop a transformation matrix for expansion of modal information and residual information using the measured FRF.

CHAPTER 3

COMPUTER PROGRAM AND VERIFICATION

3.1 COMPUTER PROGRAM

The computer program developed in Borland Delphi 7© and is called as Structural Modification Toolbox. The program uses the result files of MSC.Nastran© Modal Analysis with extension .f06. It is also possible to use an external file in .f06 file format that includes modal data. The user loads the .f06 result file and selects the degrees of freedom. Then, the user defines the modification nodes and the unmodified nodes at which FRF computation is required, hereafter called as required nodes, starting frequency [rad/s], ending frequency [rad/s], number of frequency points and loss factor of the original structure and the i^{th} and j^{th} indices of FRF's for the original structure and modified structure. Number of nodes and number of modes for both original structure and modified structure can be seen from Structural Modification Toolbox GUI. The program calculates FRF for the nodes defined by user for the original system. First, eigenvalues and the mass normalized eigenvectors are extracted from result file for each required node. Secondly by using the formula:

$$\alpha_{ij}(\omega)=\sum_{r=1}^{N}\frac{\Phi_{ir}\Phi_{jr}}{\omega_r^2-\omega^2+i\eta\omega_r^2} \tag{3.1}$$

the FRF matrix is calculated.

After the calculation of FRF for the original system, the user is expected to input the necessary modification data depending on the modification case. The order of the modification matrices must be equal to the sum of the number of modification degrees of freedom and the number of unmodified nodes at which FRF computation is required. Therefore, zero values are assigned to the locations corresponding to

DOFs at which no modification is made, but FRF is required. Then modified FRF is calculated by using the selected method(s). As an option, it is also possible to compare the calculated FRF using MSC.Nastran© solution for the modified structure with Structural Modification Toolbox calculated FRF. The details of Structural Modification Toolbox v2.0 are given in Appendix A.

3.2 GENERAL INFORMATION ABOUT VERIFICATION

The three methods used in Structural Modification Toolbox, namely; Özgüven's Structural Modification Method, Sherman-Morrison Method and Extended Successive Matrix Inversion Method are employed only for structural modification problems where there is not any additional DOF. In this section, different case studies are given. In each case study, FRFs of modified structures are calculated by using the three methods mentioned above. These results are also compared with the modal analysis results obtained for the modified structure by using MSC.Nastran©. A plate model is used for the verification of the methods and the program written. The first case study will also be used in User Manual to describe how to use the program. User manual is given in Appendix A.

3.3 FREE PLATE

The size of the plate used is taken as 1 X 0.5 X 0.01 m. The material is steel so the Young's Modulus is 210 GPa, Poisson's ratio is 0.3 and density is 7800 kg/m^3. In this case study, the boundary conditions of the plate are taken as free-free. In the first model, there are 66 nodes with 6 DOF per node and 50 elements (Figure 3.1). In the second model, there are 1326 nodes with 6 DOF per node and 1250 elements (Figure 3.2).

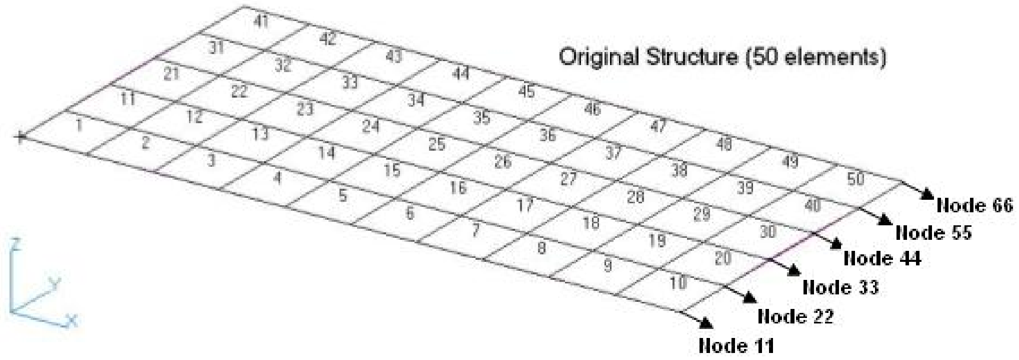

Figure 3.1 Original Structure with 50 elements (first model).

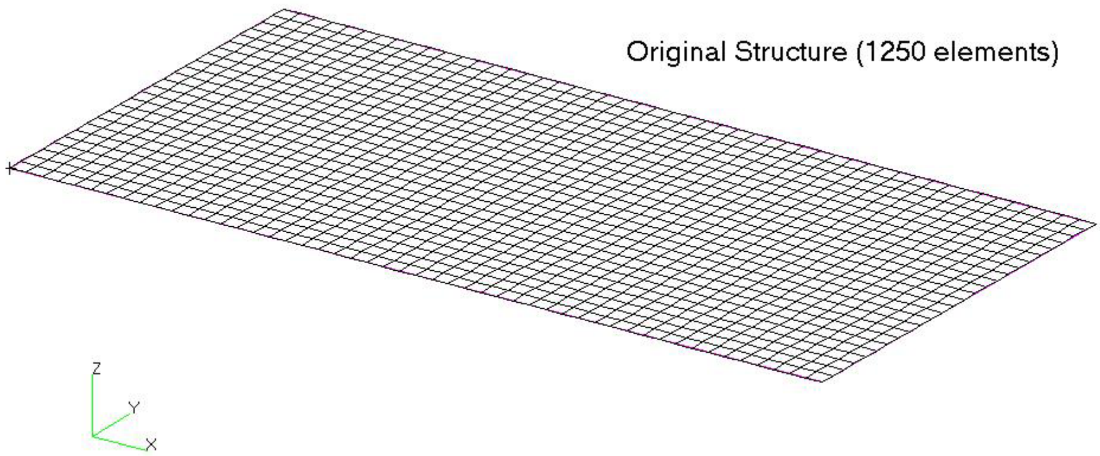

Figure 3.2 Original Structure with 1250 elements (second model).

3.3.1 MASS MODIFICATION

The total mass of the original plate is 39 kg. For the first model, an additional mass of 2 kg is added to nodes 11, 22, 33, 44, 55 and 66 as elements 51, 52, 53, 54, 55 and 56 (Figure 3.3). In the second model, the additional mass is 1 kg and it is added to nodes 49-51,100-102,151-153 as elements 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258 and 1259 (Figure 3.4).

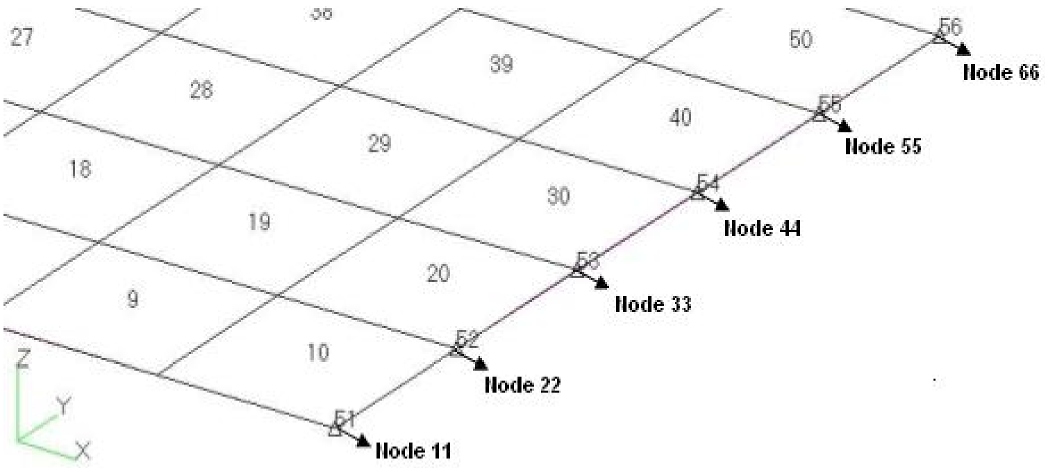

Figure 3.3 Mass Modified Structure for first model.

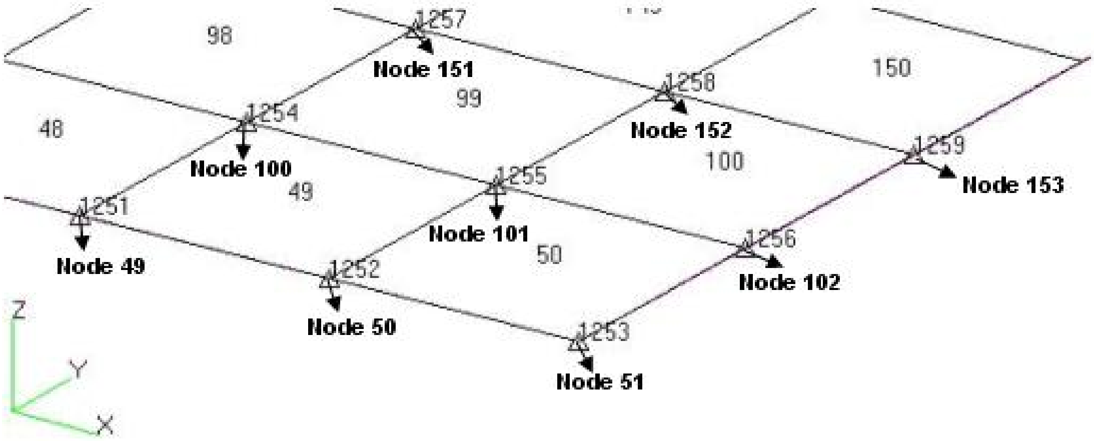

Figure 3.4 Mass Modified Structure for second model.

For the above problem, the FRFs in the form of α_{xy} are given in Figure 3.5 and Figure 3.6 for both the original and modified plate. The FRFs calculated by all the three modification methods as well as by MSC.Nastran©. Since the three methods yield exactly the same curve, only two FRF curves are seen in Figure 3.6.

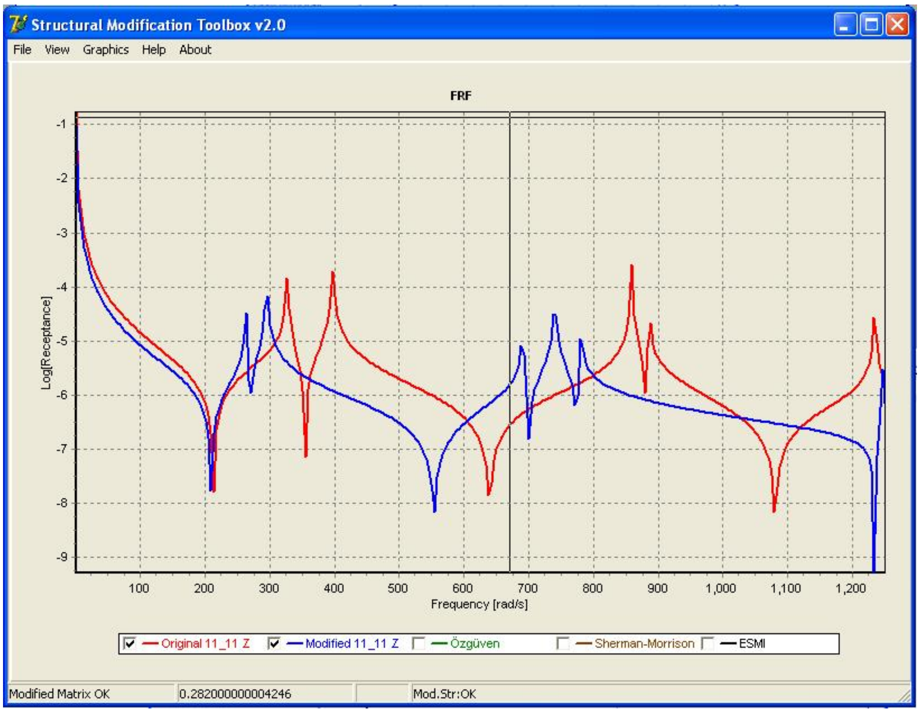

Figure 3.5 Mass Modified Structure solution in MSC.Nastran© for first model.

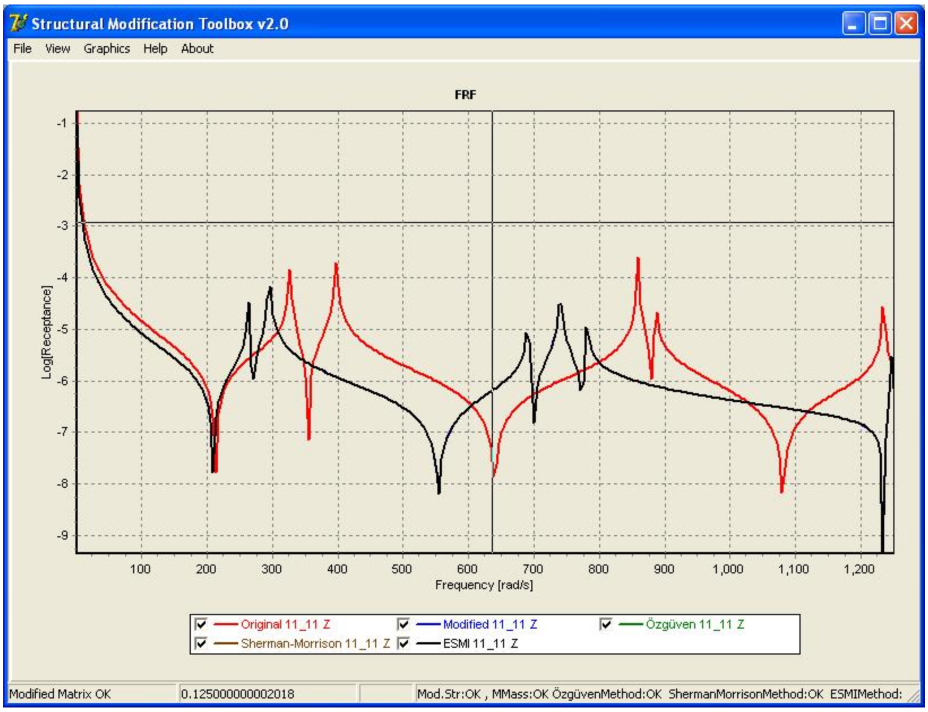

Figure 3.6 Mass Modified Structure with all solutions for first model.

Similarly, the FRFs for the second model are given in Figure 3.7 and Figure 3.8.

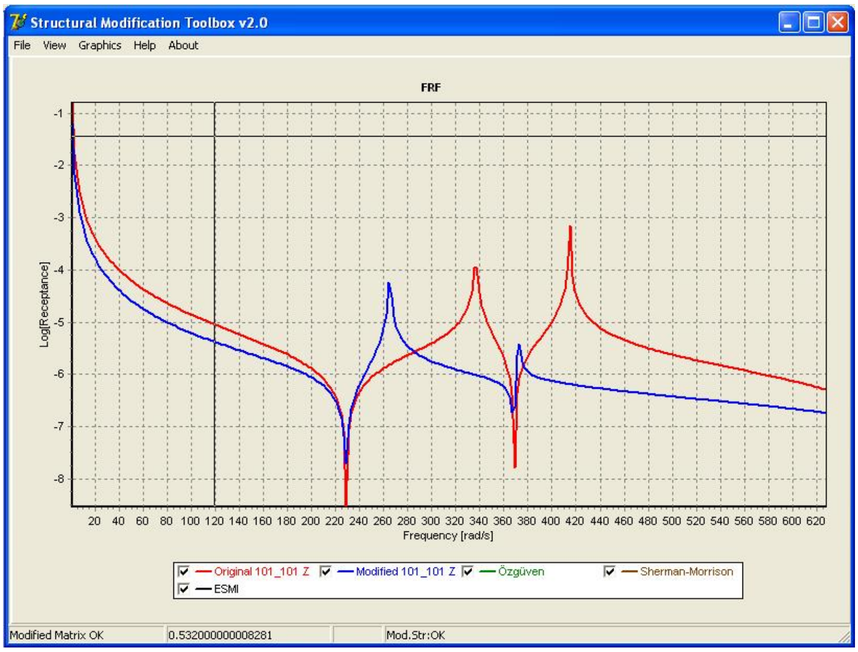

Figure 3.7 Mass Modified Structure solution in MSC.Nastran© for second model.

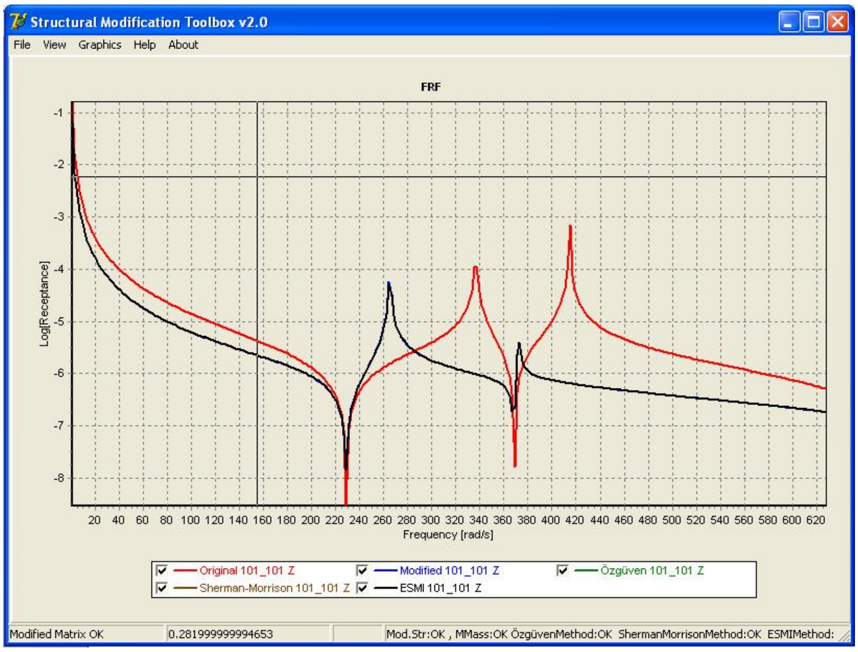

Figure 3.8 Mass Modified Structure solution with three methods for second model.

As can be seen from the comparison of Figure 3.7 and Figure 3.8, again in this problem, all three modification methods yield the exact result that can be obtained by MSC.Nastran©.

3.3.2 MASS AND STIFFNESS MODIFICATIONS

In this case study, both mass and stiffness modifications are considered. Mass modifications both for the first and the second models are taken the same as the previous case study.

Additionally, for the first model, an additional stiffness element of 4000 N/m is added between node pairs (11, 22) and (55, 66) as elements 57 and 58 respectively (Figure 3.9). For the second model, additional stiffness element of 10000 N/m is added between node 51, the ground and node 1326, the ground as elements 1260 and 1261 respectively. (Figure 3.10).

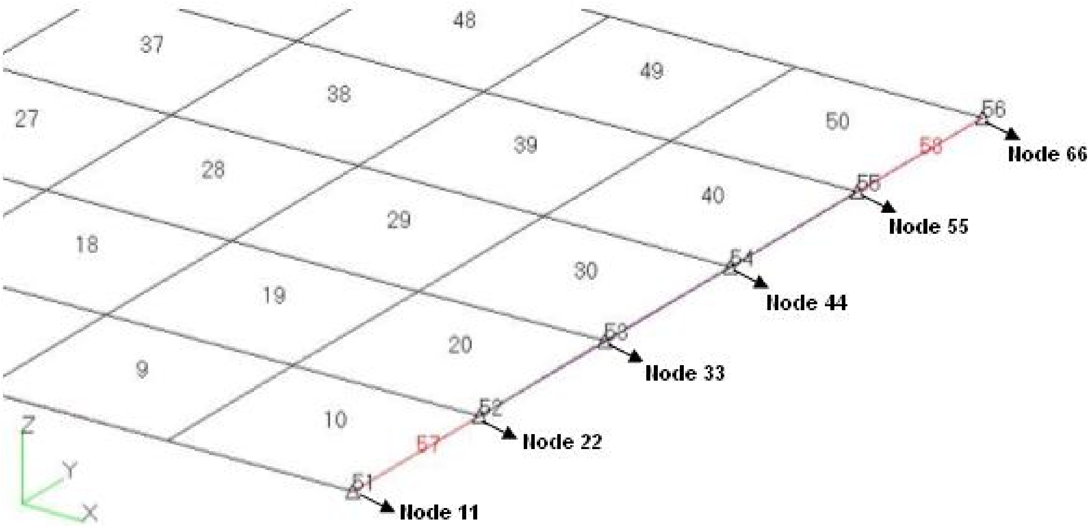

Figure 3.9 Mass and Stiffness Modified Structure for first model.

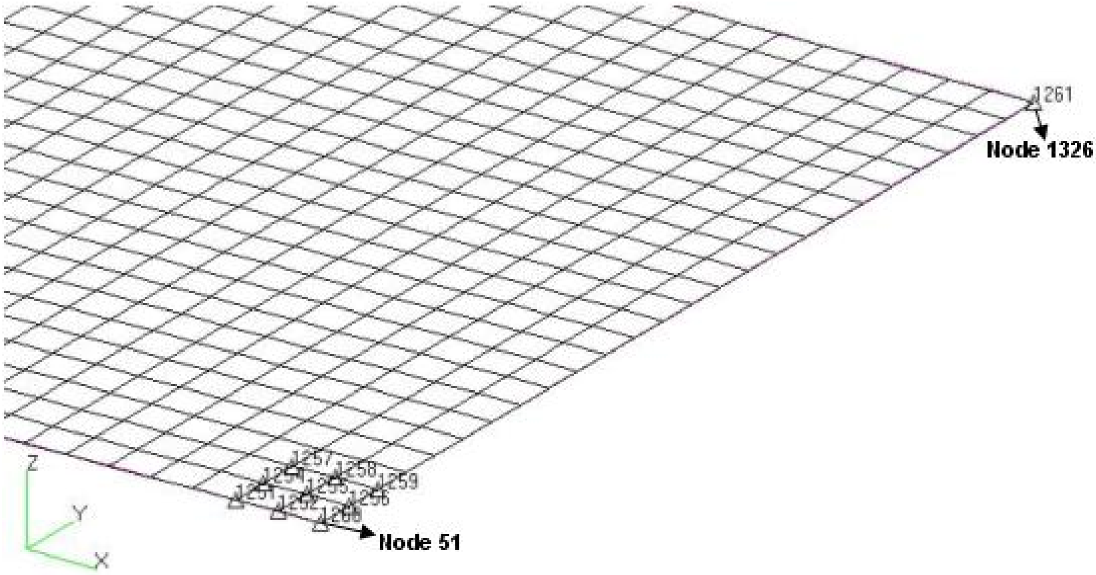

Figure 3.10 Mass and Stiffness Modified Structure for second model.

Since the application steps are the previous case study, only the results are given below. The FRFs are given in Figure 3.11 and Figure 3.12 for both the original and modified plate. Since the FRFs calculated by all the three modification methods yield the same curve with MSC.Nastran©, only two FRF curves are seen in Figure 3.11 and Figure 3.12.

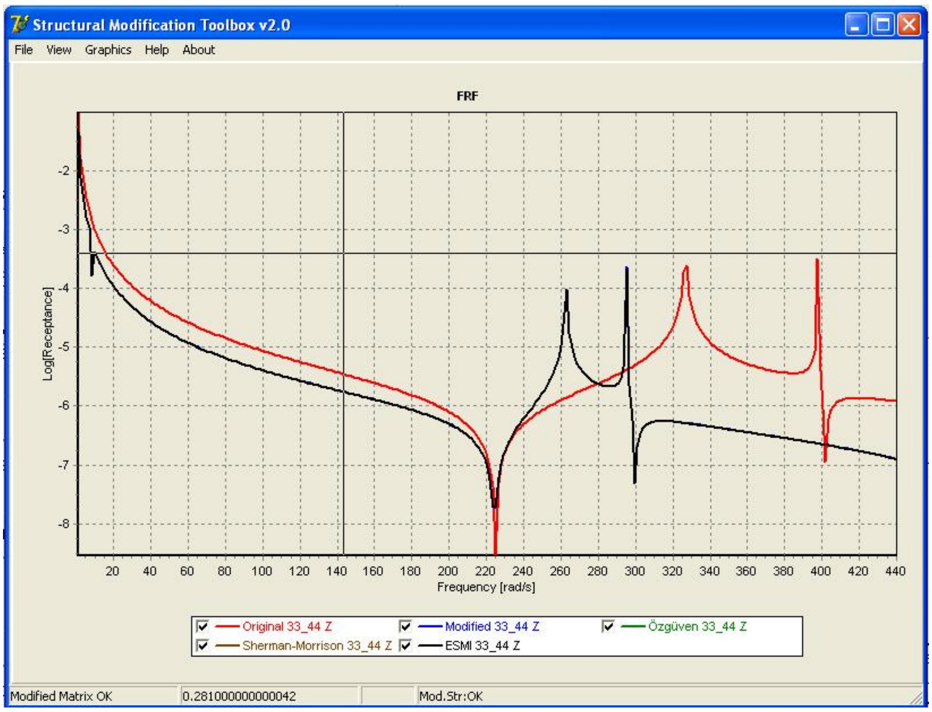

Figure 3.11 Mass and Stiffness Modified Structure solution for first model.

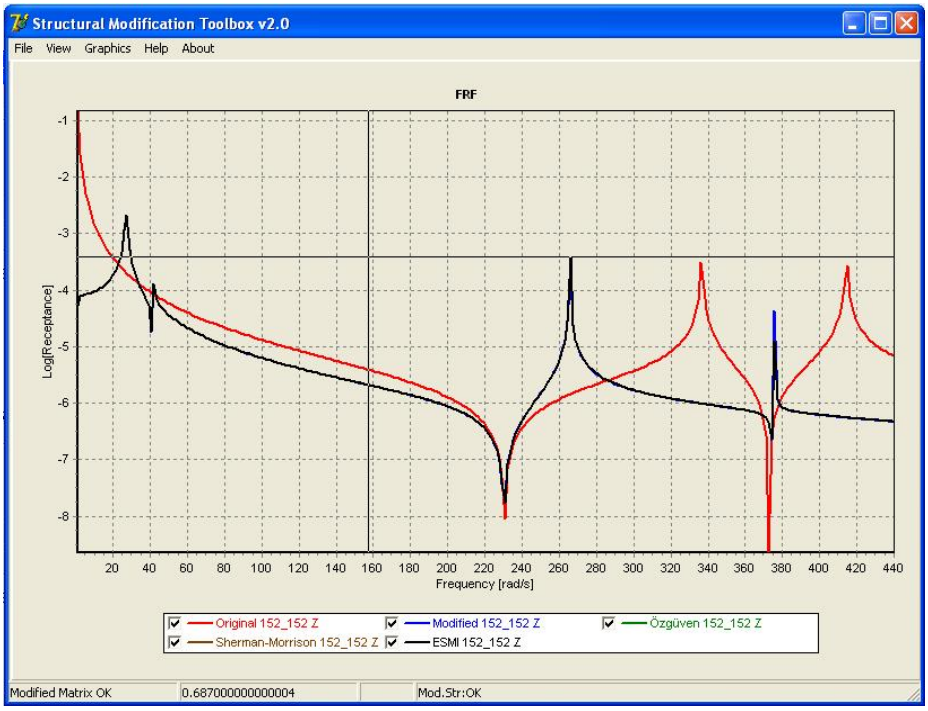

Figure 3.12 Mass and Stiffness Modified Structure solution for second model.

3.3.3 MASS, STIFFNESS AND STRUCTURAL DAMPING MODIFICATIONS

It is possible to apply material structural damping modification and local structural damping modification by using Structural Modification Toolbox.

In order to verify the program for a damping modification, the structure is first modified by using the Structural Modification Toolbox and then the effect of damping modification is obtained by using MSC.Nastran©. The loss factor is taken as 0.02 in both Structural Modification Toolbox and MSC.Nastran©. Figure 3.13 shows Structural Modification Toolbox solution and Figure 3.14 shows MSC.Nastran© solution. Since the FRFs calculated by Structural Modification Toolbox yield exactly the same curve with MSC.Nastran©, only one FRF curve is seen in Figure 3.13 and Figure 3.14.

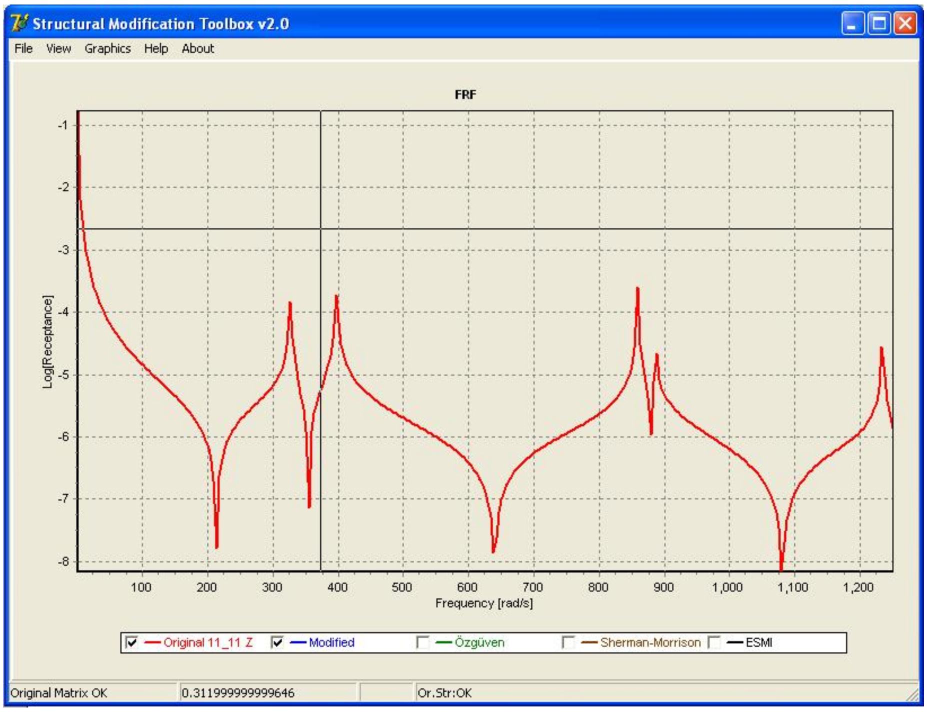

Figure 3.13 Material Structural Damping Modified Structure solution with Structural Modification Toolbox.

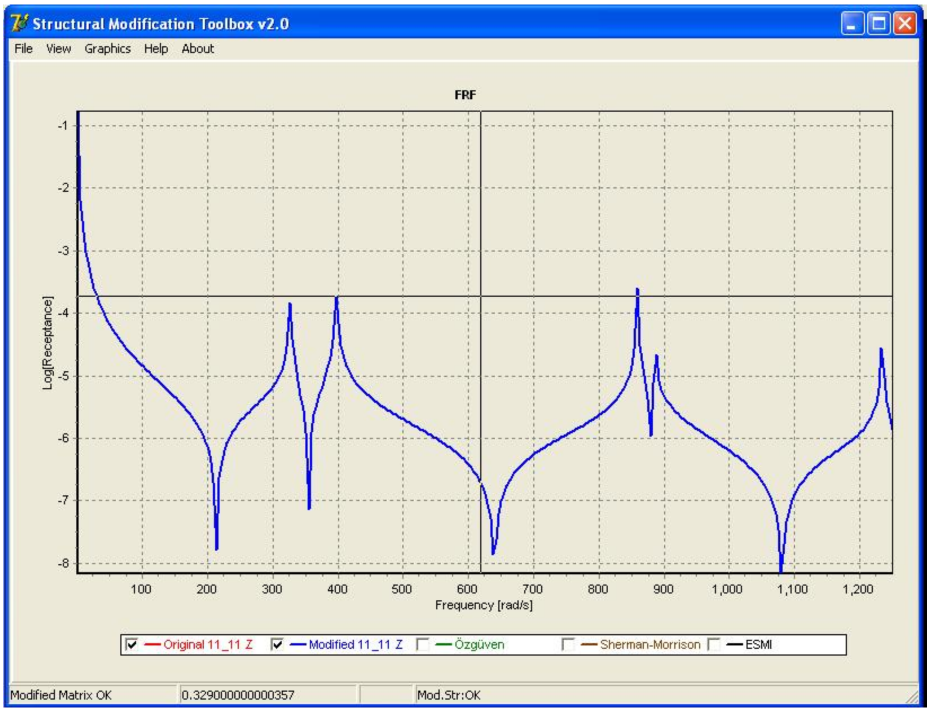

Figure 3.14 Material Structural Damping Modified Structure solution with MSC.Nastran©.

3.3.3.1 MASS, STIFFNESS AND LOCAL STRUCTURAL DAMPING MODIFICATIONS

In this part, mass and stiffness modifications for both the first and the second model are the same as in Section 3.3.2. Both for the first model and for the second model, loss factor is taken as 0.03 for the additional stiffnesses 4000 N/m. For first model, the damped elastic element is applied between nodes 33 and 44 as element 59 (Figure 3.15) and for the second model, it is applied between nodes 51 and 102 as element 1262 (Figure 3.16).

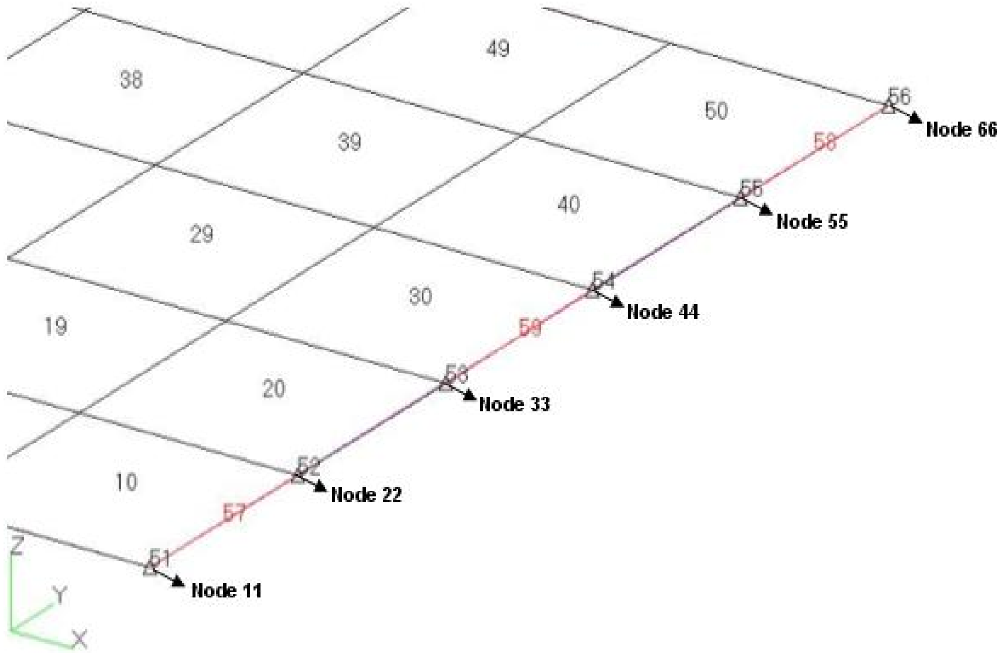

Figure 3.15 Mass, Stiffness and Local Structural Damping Modified Structure for first model.

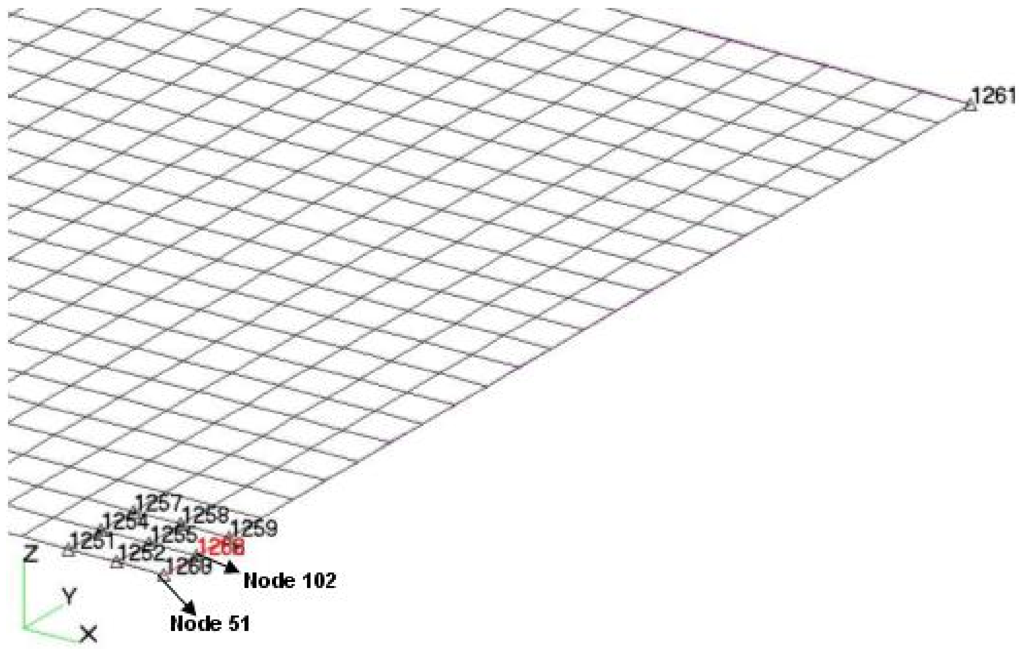

Figure 3.16 Mass, Stiffness and Local Structural Damping Modified Structure for second model.

The FRFs are given in Figure 3.17 and Figure 3.18 for both the original and modified plate. Since the FRFs calculated by all the three modification methods yield exactly the same curve with MSC.Nastran©, only two FRF curves are seen in Figure 3.17 and Figure 3.18 as in previous case studies.

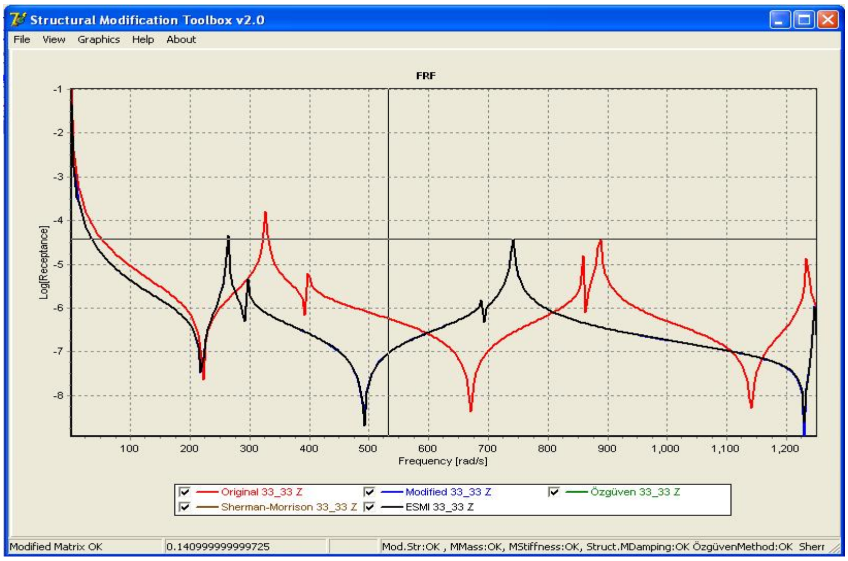

Figure 3.17 Mass, Stiffness and Local Structural Damping Modified Structure for first model.

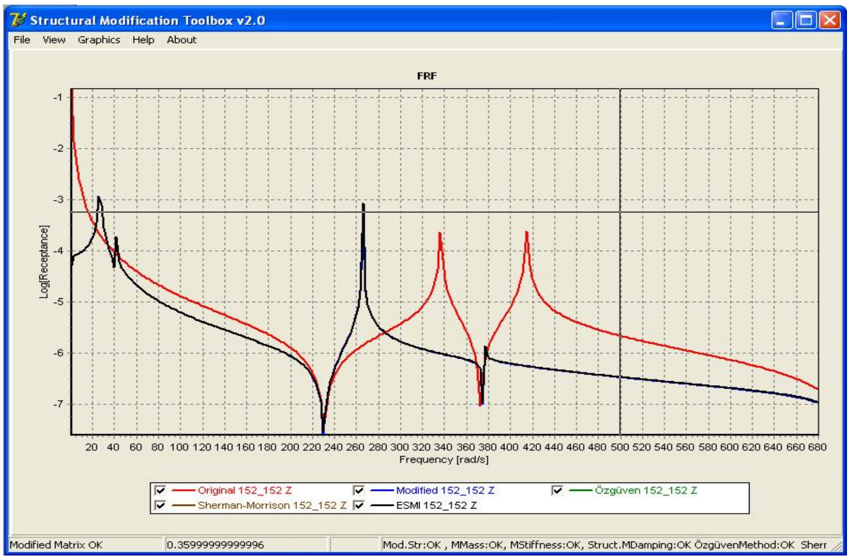

Figure 3.18 Mass, Stiffness and Local Structural Damping Modified Structure for second model.

3.3.3.2 MASS, STIFFNESS AND MATERIAL STRUCTURAL DAMPING MODIFICATIONS

Material Structural Damping is the type of structural damping represented by a loss factor of the material in MSC.Nastran©. Material Structural Damping can be applied to the original structure by Structural Modification Toolbox. In this part, mass and stiffness modifications for both the first and the second model are the same as in the Section 3.3.2.

For the first model, Original Structure Loss Factor is taken as 0.02 and for the second model, it is 0.03. The program calculates damping matrix by multiplying the loss factor and the stiffness matrix.

Modified Structure; for first model is shown in Figure 3.19 and for second model is shown in Figure 3.20 as below:

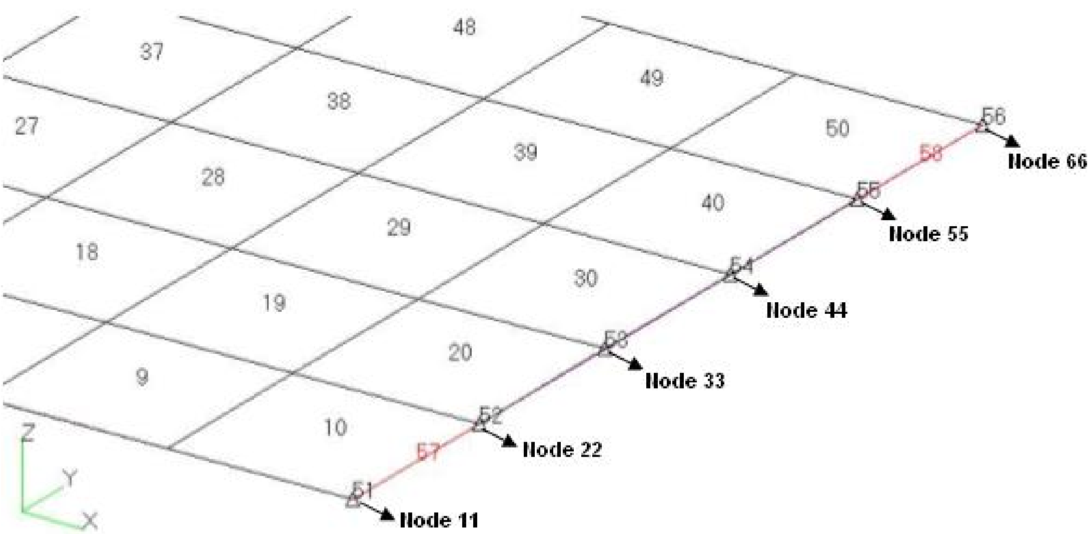

Figure 3.19 Mass, Stiffness and Material Structural Damping Modified Structure for first model.

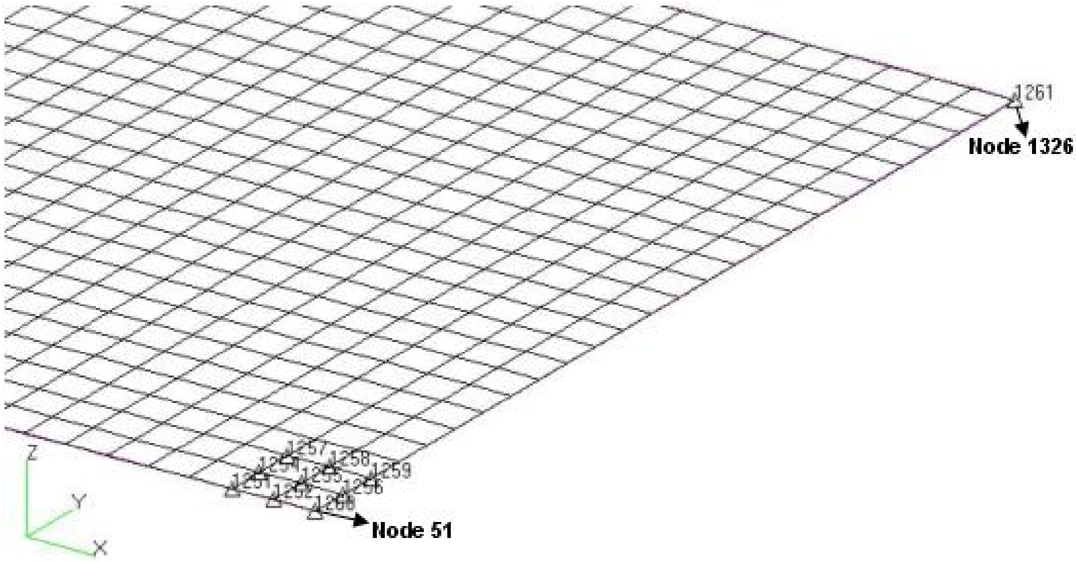

Figure 3.20 Mass, Stiffness and Material Structural Damping Modified Structure for second model.

The FRFs are given in Figure 3.21 and Figure 3.22 for both the original and modified plate. Since the FRFs calculated by all the three modification methods yield exactly the same curve with MSC.Nastran©, only two FRF curves are seen in Figure 3.21 and Figure 3.22.

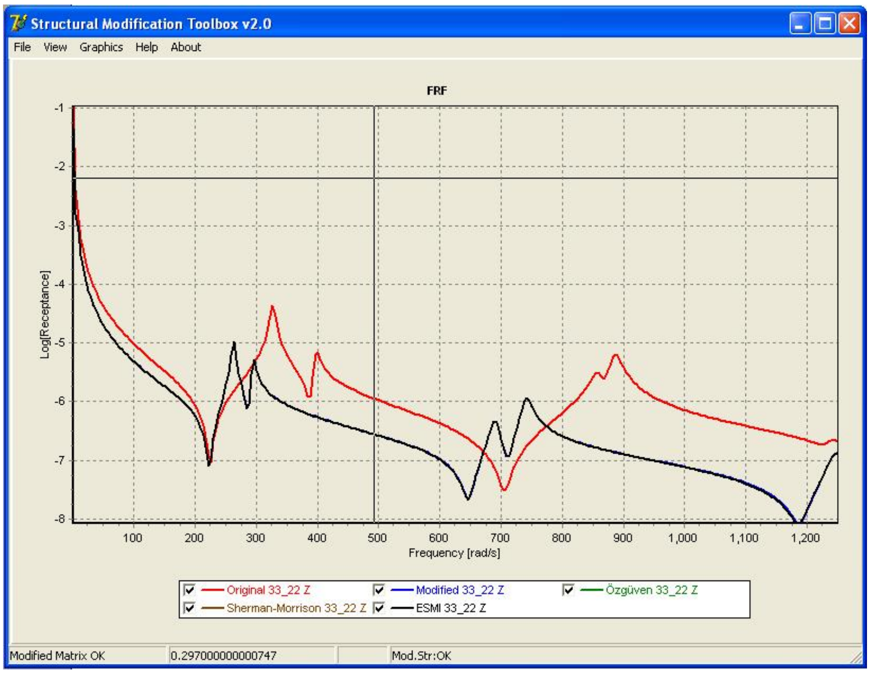

Figure 3.21 Mass, Stiffness and Material Structural Damping Modified Structure solution for first model.

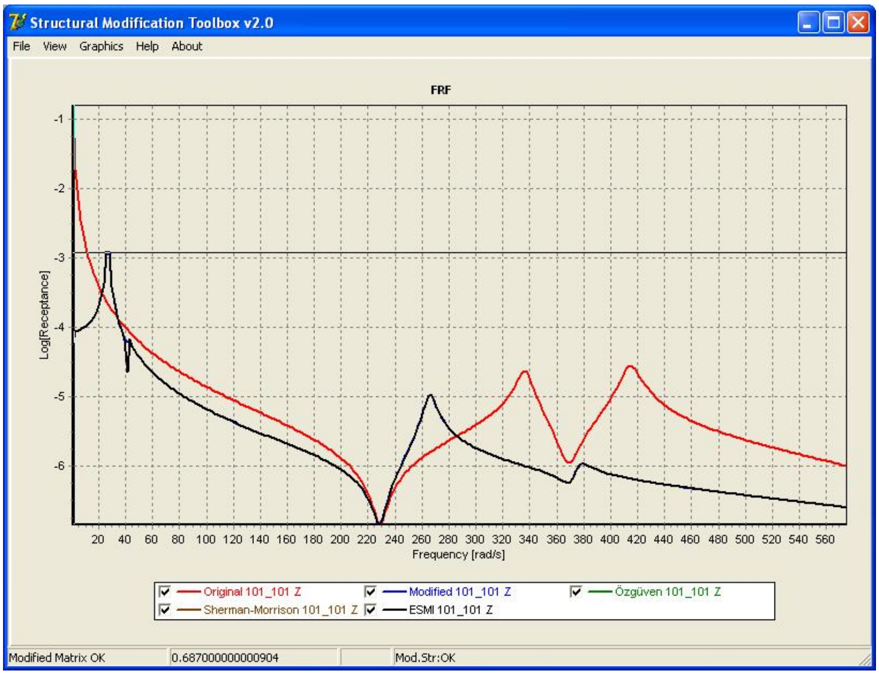

Figure 3.22 Mass, Stiffness and Material Structural Damping Modified Structure solution for second model.

3.3.4 MASS, STIFFNESS AND VISCOUS DAMPING MODIFICATIONS

In this part, mass and stiffness modifications for both the first and the second model are the same as the cases in Section 3.3.2 and Section 3.3.3.2.

Additionally, for the first model, a viscous damping element of 1000 Ns/m is added between nodes 33 and 44 as element 59 (Figure 3.23). For the second model, a viscous damping element of 1000 Ns/m is added between nodes 51 and 1326 as element 1262 (Figure 3.24).

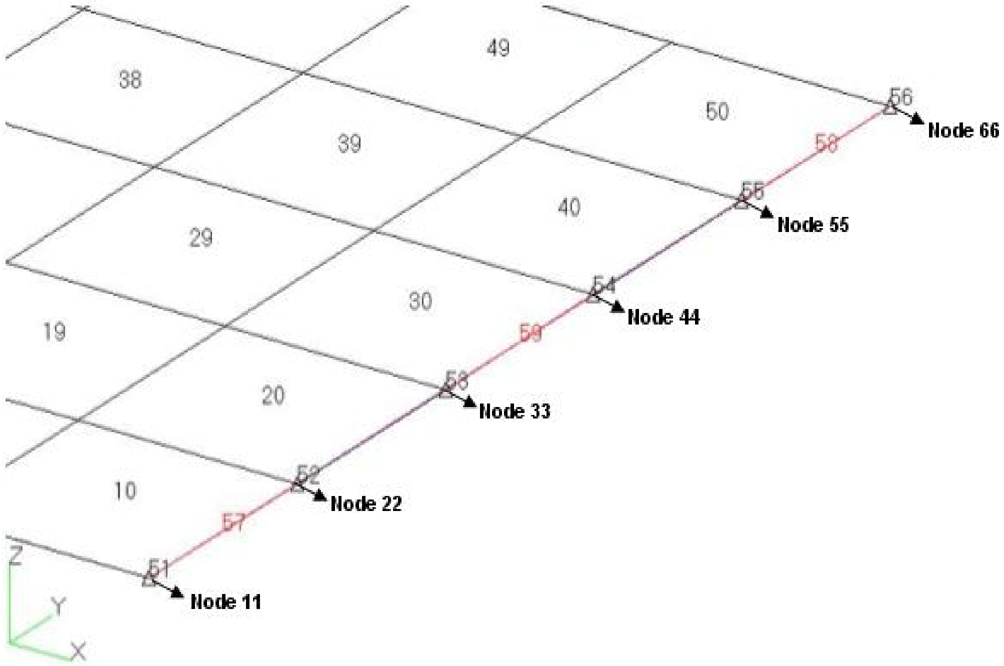

Figure 3.23 Mass, Stiffness and Viscous Damping Modified Structure for first model.

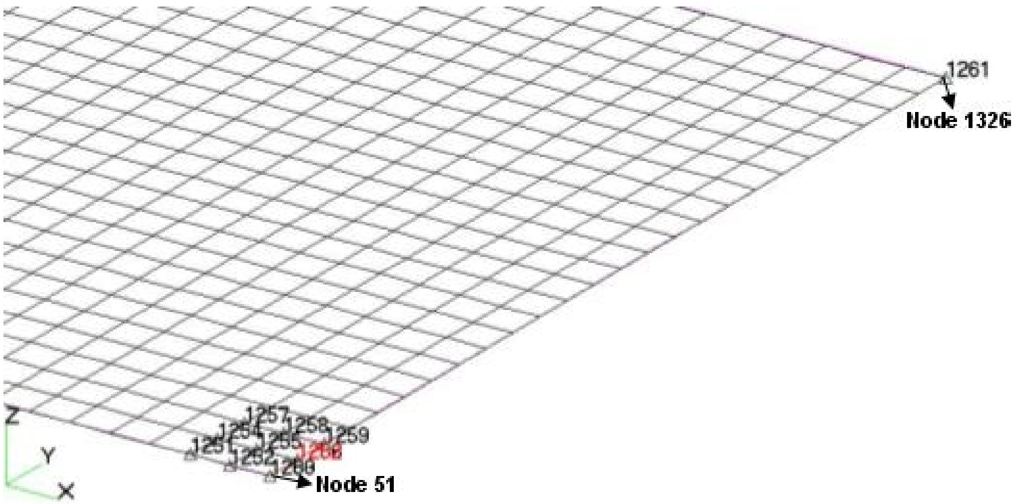

Figure 3.24 Mass, Stiffness and Viscous Damping Modified Structure for second model.

The FRFs are given in Figure 3.25 and Figure 3.26 for both the original and modified plate. Only two FRF curves are seen in Figure 3.25 and Figure 3.26 as in previous sections.

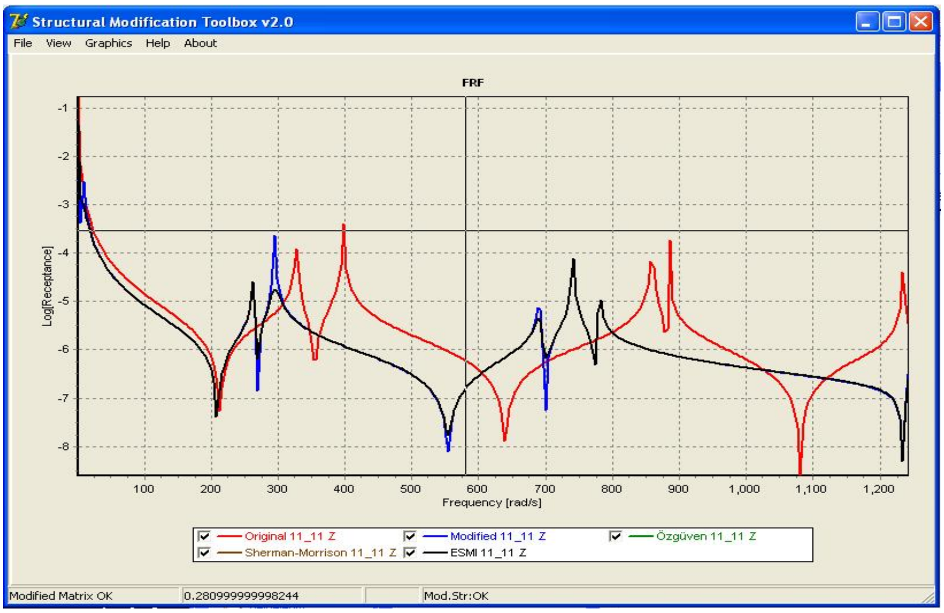

Figure 3.25 Mass, Stiffness and Viscous Damping Modified Structure solution for first model.

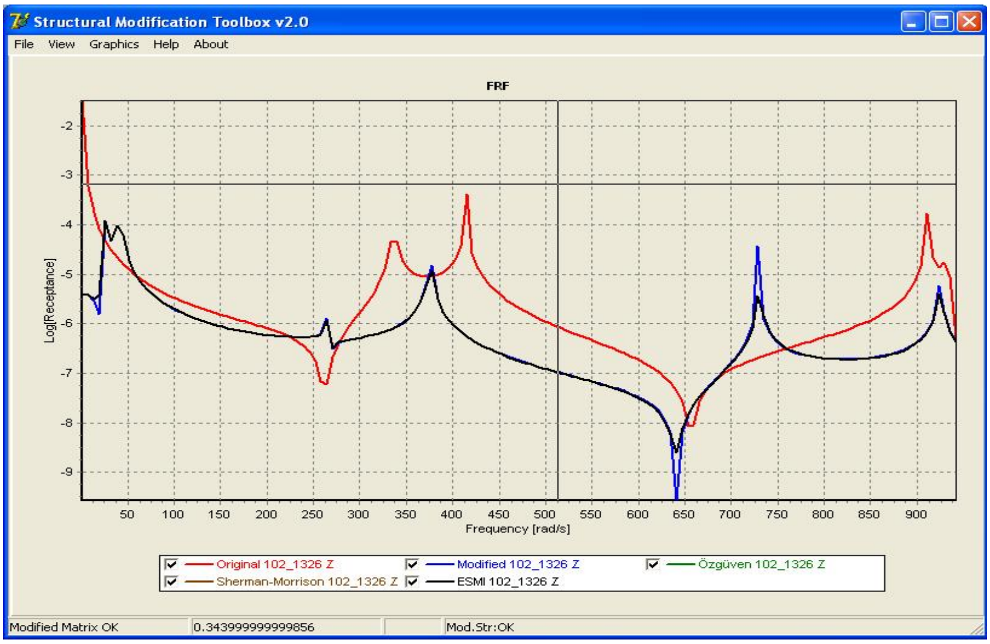

Figure 3.26 Mass, Stiffness and Viscous Damping Modified Structure solution for second model.

3.4 CANTILEVER PLATE

Further case study is carried out by using a cantilever plate. Except the boundary conditions, all properties of cantilever plate are exactly the same as free plate. The model has 1250 elements. All the solution steps for cantilever plate are also the same as those of the free plate. Therefore, only the results will be given in the following section.

3.4.1 MASS, STIFFNESS AND LOCAL STRUCTURAL DAMPING MODIFICATIONS

The modifications made in this document are the same as those in the previous example given for the free boundary conditions, additional mass element is 1 kg and it is added to nodes 49-51,100-102,151-153 as elements 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258 and 1259 (Figure 3.28). Stiffness element of 10000 N/m is added between nodes 51 and 1326 as elements 1260 and 1261 (Figure 3.28). Additionally, Loss Factor is taken as 0.03 for the additional stiffness 4000 N/m that it is applied between nodes 51 and 102 as element as element 1262 (Figure 3.28).

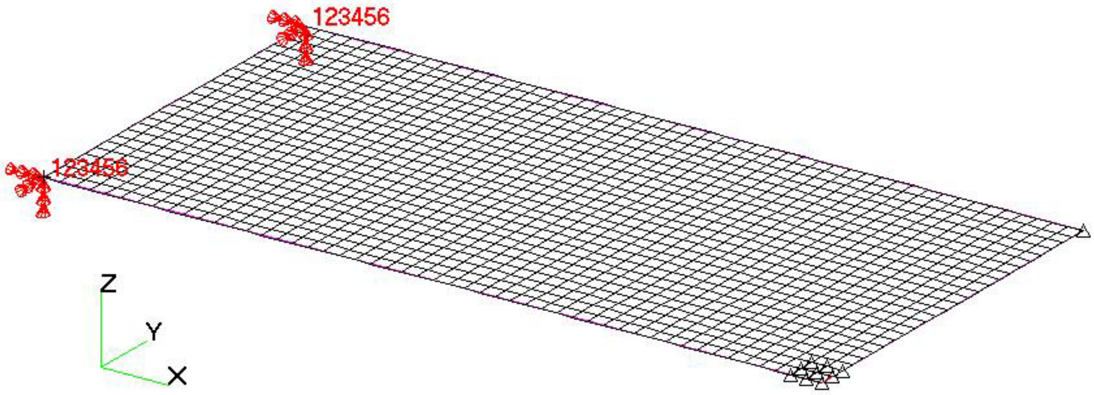

Figure 3.27 Mass, Stiffness and Local Structural Damping Modified Structure boundary conditions for cantilever plate.

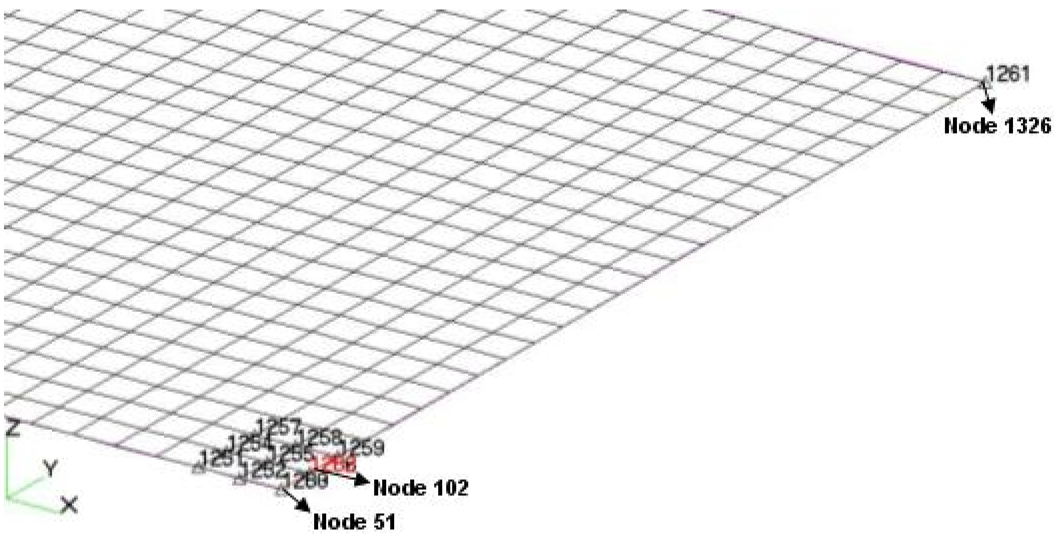

Figure 3.28 Mass, Stiffness and Local Structural Damping Modified Structure for cantilever plate.

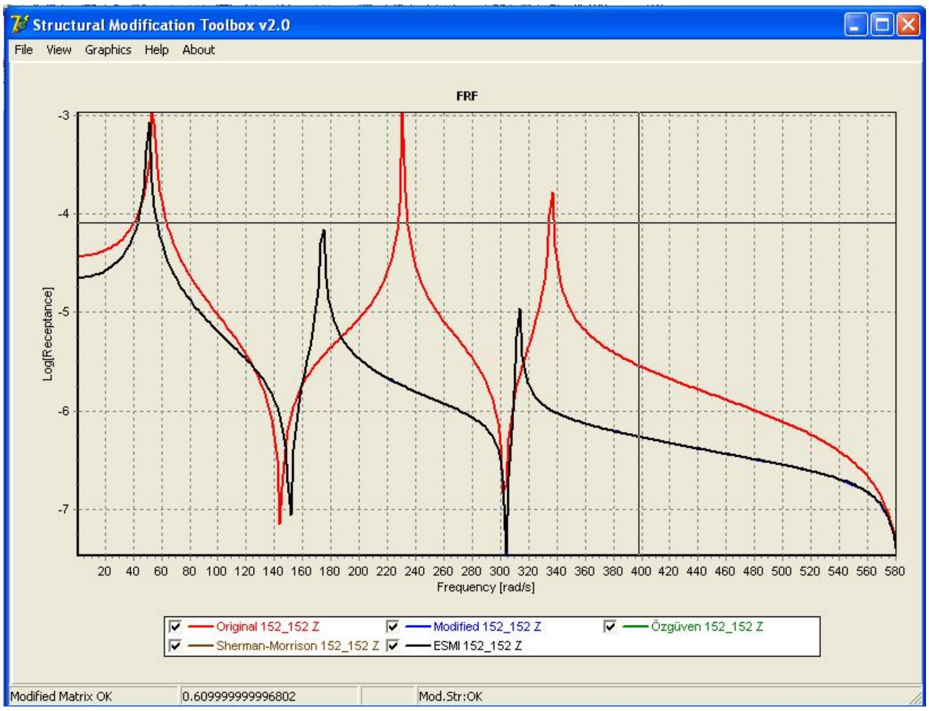

Figure 3.29 Mass, Stiffness and Local Structural Damping Modified Structure Solution for cantilever plate.

The FRFs are given in Figure 3.29 for both the original and modified plate. Since the FRFs calculated by all the three modification methods yield exactly the same curve with MSC.Nastran©, only two FRF curves are seen in Figure 3.29.

3.5 EFFECT OF HIGHER MODES

While using the program, the effect of higher modes should also be considered. For the analysis of original structure in MSC.Nastran©, the frequency interval should be wide enough for the investigated structure. For example, for the analysis between 0 Hz and 150 Hz of the cantilever plate in Section 3.4.1, the original plate is analyzed in 0-200 Hz, 0-600 Hz, 0-1200 Hz, 0-2000 Hz, 0-4000 Hz and 0-6000 Hz frequency intervals. The results are shown between Figure 3.30 and Figure 3.35.

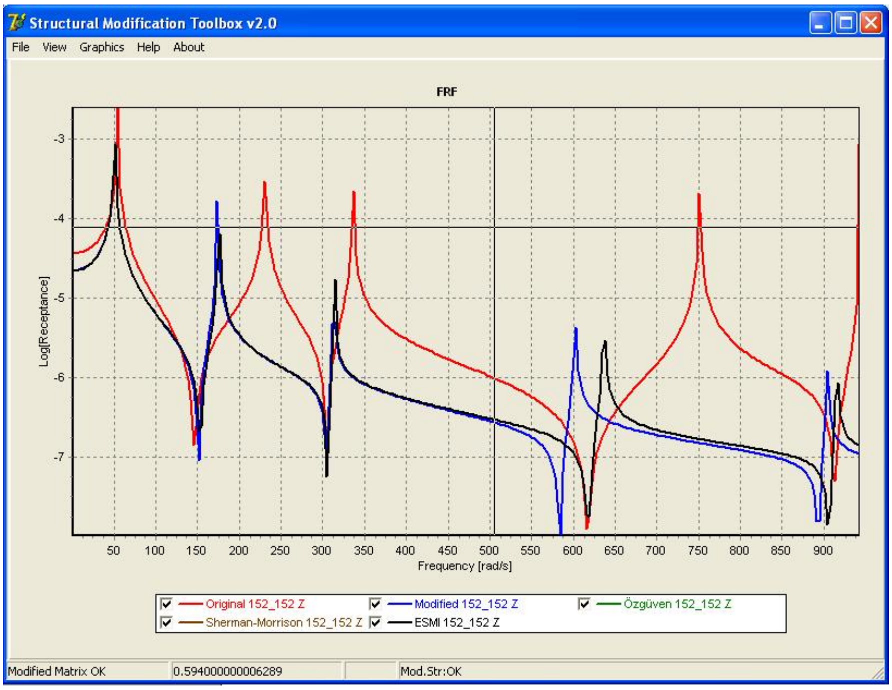

Figure 3.30 0-200 Hz Solution.

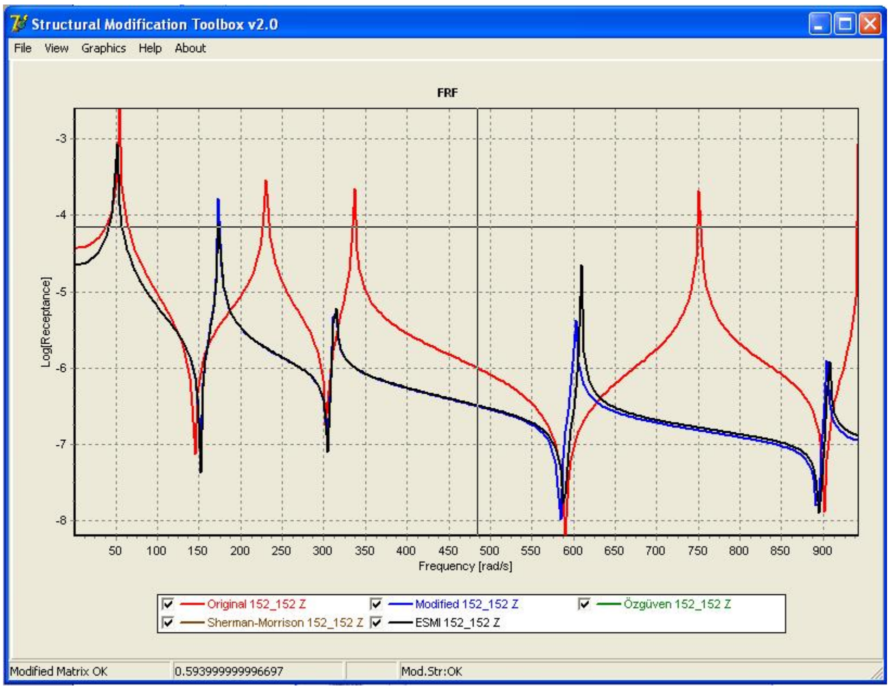

Figure 3.31 0-600 Hz Solution.

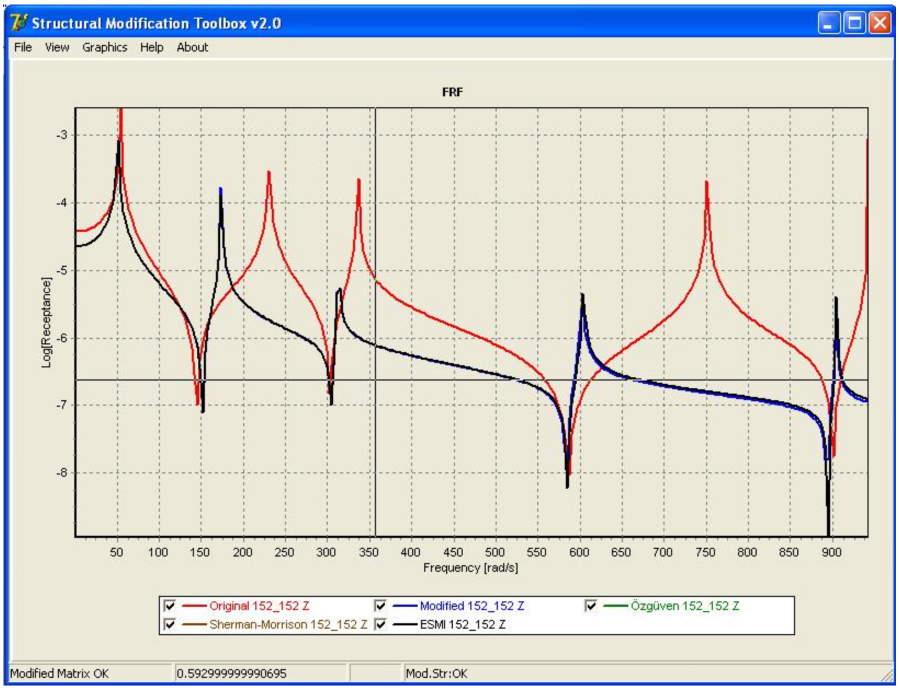

Figure 3.32 0-1200 Hz Solution.

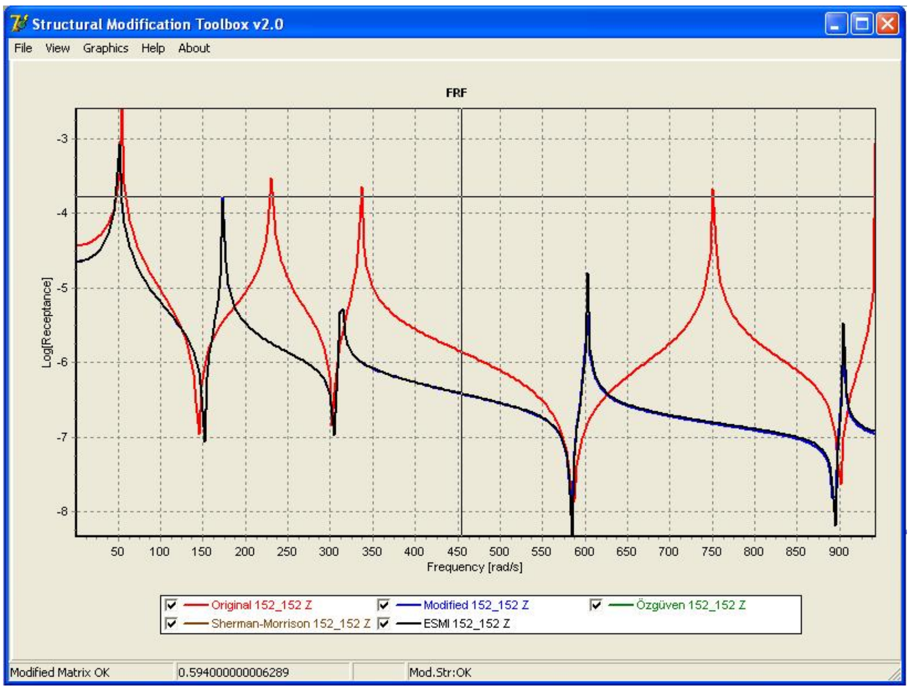

Figure 3.33 0-2000 Hz Solution.

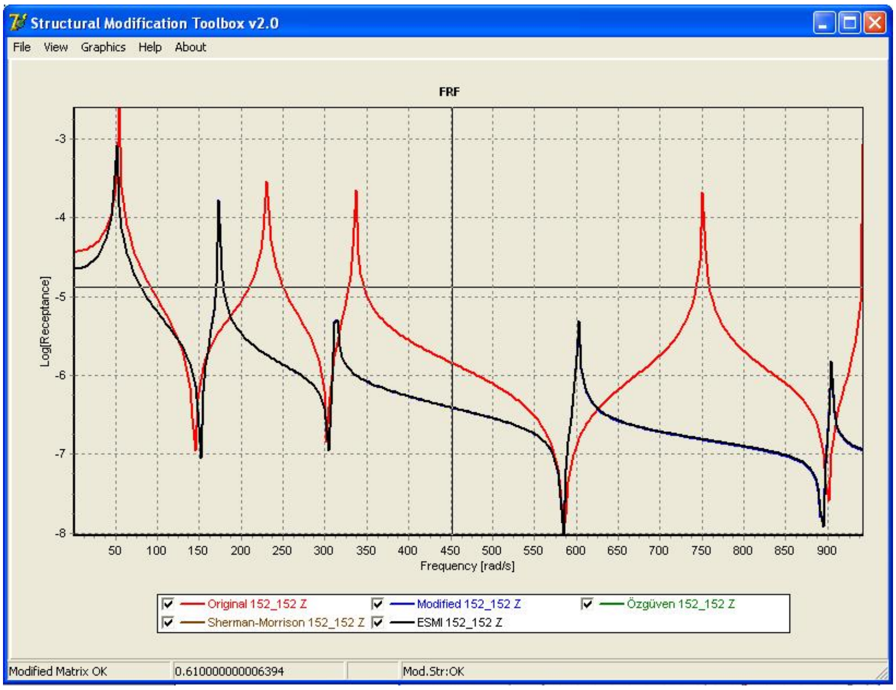

Figure 3.34 0-4000 Hz Solution.

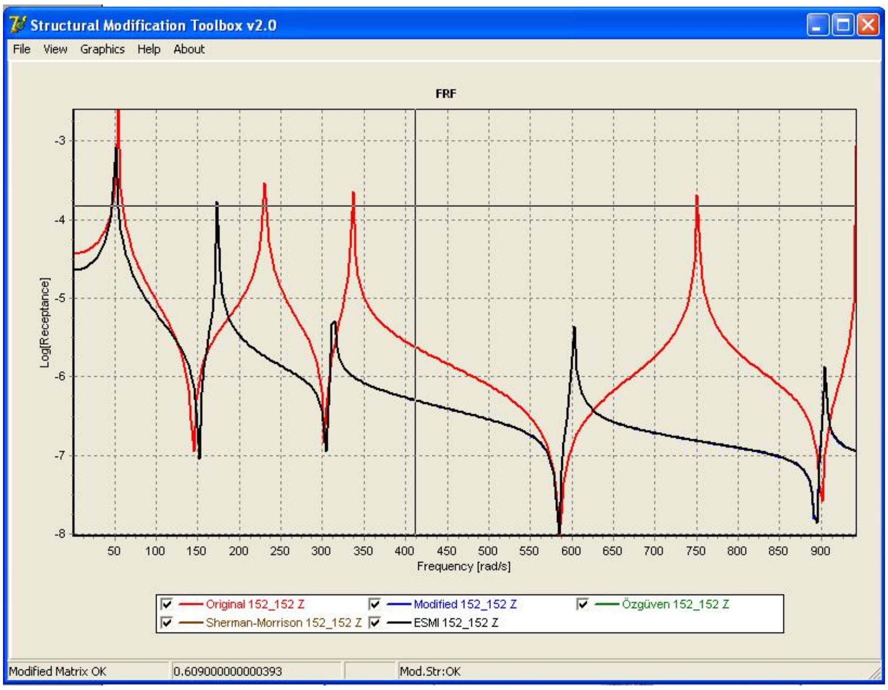

Figure 3.35 0-6000 Hz Solution.

As it is shown between Figure 3.30 and Figure 3.35, selected frequency interval in original structure solution of MSC.Nastran© affects the solutions of Structural Modification Toolbox. The reason for that is the effect of higher modes. It is observed that the frequency interval of the original structure solution should be at least 15-20 times of the frequency interval interested for modified structure.

CHAPTER 4

COMPARISON OF THE METHODS AND EFFECT OF MODIFICATION SIZE

4.1 COMPARISON OF THE METHODS

In this chapter, Özgüven's Structural Modification Method, Sherman-Morrison Method and Extended Successive Matrix Inversion Method are compared in terms of their solution time. The computer used for the comparison has Intel Xeon©, 3.6 GHz processor and 2GB of RAM. The structures shown in Figure 3.1 and Figure 3.2 are used for the comparisons.

The first structure considered has 50 elements and 66 nodes as shown in Figure 3.1. The structure weighs 39 kilograms and a total number of 10, 20, 30, 40, 50 and 60 nodes were modified with 0.04 kg additional masses without additional degrees of freedom. The original structure is solved in the frequency range of 0-2000 in order to study effect of higher modes. The size of the original FRF matrix depends on the modification size and the number of modes as shown in Equation (3.1). Since all the compared methods are exact methods, the results are exactly the same.

The solution times spent for each of the three different methods are compared in Figure 4.1. The corresponding numerical data are presented in Table 4.1.

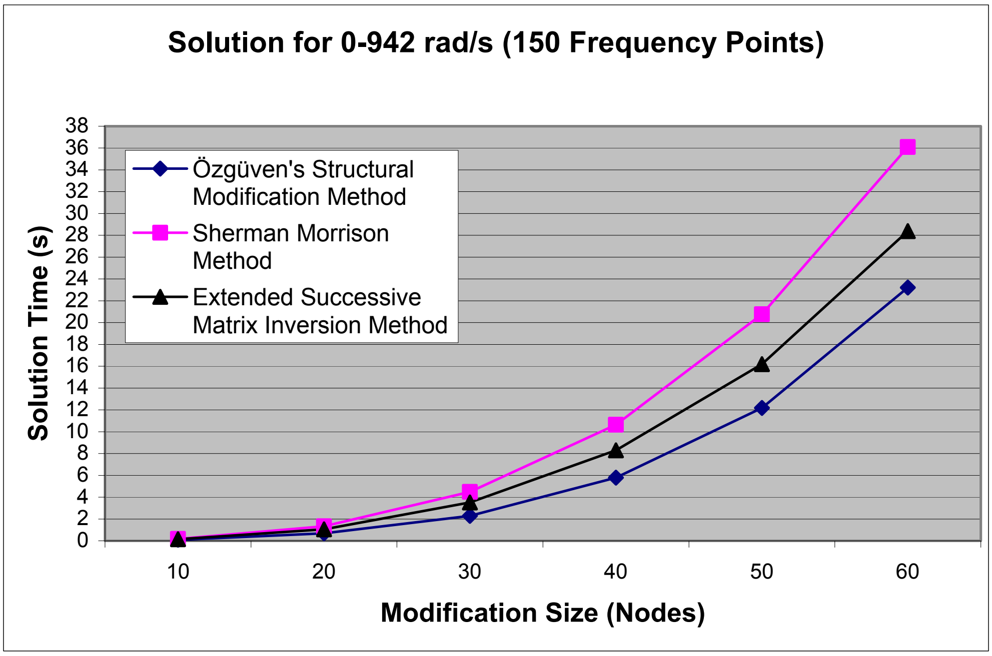

Figure 4.1 Comparison of the Methods for Solution Time of First Structure.

Table 4.1 Solution Time Comparison for the Structure with 50 Elements.

Solution for 0-942 rad/s (150 Frequency Points)					
Total Number of Modified Nodes	Modified Nodes	Mass added (kg)	Özgüven's Structural Modification Method (s)	Sherman Morrison Method (s)	Extended Successive Matrix Inversion Method (s)
10	10,11,21,22,32,33,43,44,54,55	0.04	0.1	0.16	0.14
20	8-11,19-22,30-33,41-44,52-55	0.04	0.6	1.1	1.0
30	6-11,17-22,28-33,39-44,50-55	0.04	1.8	3.6	3.2
40	4-11,15-22,26-33,37-44,48-55	0.04	4.9	8.5	7.7
50	2-11,13-22,24-33,35-44,46-55	0.04	9.5	17.0	15.8
60	2-11,13-22,24-33,35-44,46-55, 57-66	0.04	21.2	32.1	29.6

The second structure has 1250 elements and 1326 nodes as shown in Figure 3.2. The structure also weighs 39 kilograms and 10 to 150 nodes are modified with 0.04 kg additional masses without additional degrees of freedom. The original structure is solved in the frequency range of 0-2000 Hz in order to study the effect of higher modes.

The computational times of the three different methods are compared for the same modification, and the results are given in Figure 4.2.

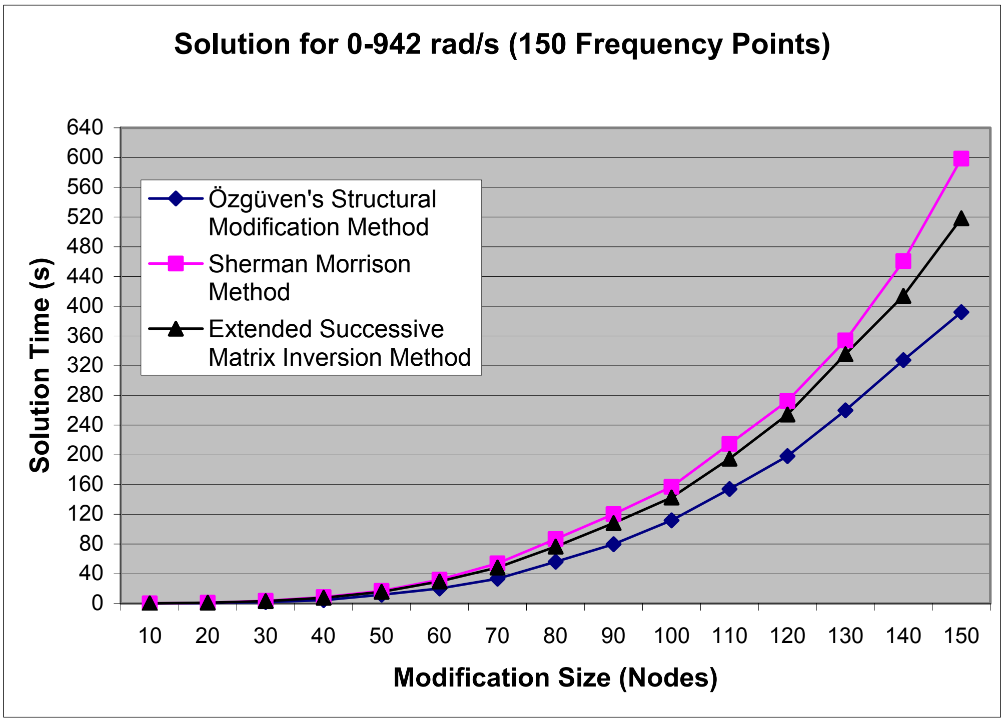

Figure 4.2 Comparison of the Methods for Solution Time of Second Structure.

The solution time for each method are also given in Table 4.2.

Table 4.2 Solution Time Comparison for the Structure with 1250 Elements.

Solution for 0-942 rad/s (150 Frequency Points)					
Total Number of Modified Nodes	Modified Nodes	Mass added (kg)	Özgüven's Structural Modification Method (s)	Sherman Morrison Method (s)	Extended Successive Matrix Inversion Method (s)
10	50,51,101,102,152,153,203,204,254,255	0.04	0.1	0.16	0.14
20	48-51,99-102,150-153,201-204,252-255	0.04	0.6	1.1	1.0
30	46-51,97-102,148-153,199-204,250-255	0.04	1.9	3.6	3.2
40	44-51,95-102,146-153,197-204,248-255	0.04	4.5	8.5	7.7
50	42-51,93-102,144-153,195-204,246-255	0.04	11.9	17.1	15.8
60	40-51,91-102,142-153,193-204,244-255	0.04	20.3	32.1	29.6
70	38-51,89-102,140-153,191-204,242-255	0.04	33.3	53.9	48.4
80	36-51,87-102,138-153,189-204,240-255	0.04	56.1	86.8	76.6
90	34-51,85-102,136-153,187-204,238-255	0.04	79.8	120.3	108.2
100	32-51,83-102,134-153,185-204,236-255	0.04	111.9	157.1	142.6
110	30-51,81-102,132-153,183-204,234-255	0.04	154.1	214.8	194.8
120	28-51,79-102,130-153,181-204,232-255	0.04	198.3	272.5	254.3
130	26-51,77-102,128-153,179-204,230-255	0.04	259.9	353.9	335.3
140	24-51,75-102,126-153,177-204,228-255	0.04	327.4	460.6	413.9
150	22-51,73-102,124-153,175-204,226-255	0.04	391.9	598.4	518.3

It is also possible to compare the methods based on the percentage of the modification.

Solution time for both structures are compared for different modification sizes and the results are shown in Figure 4.3,

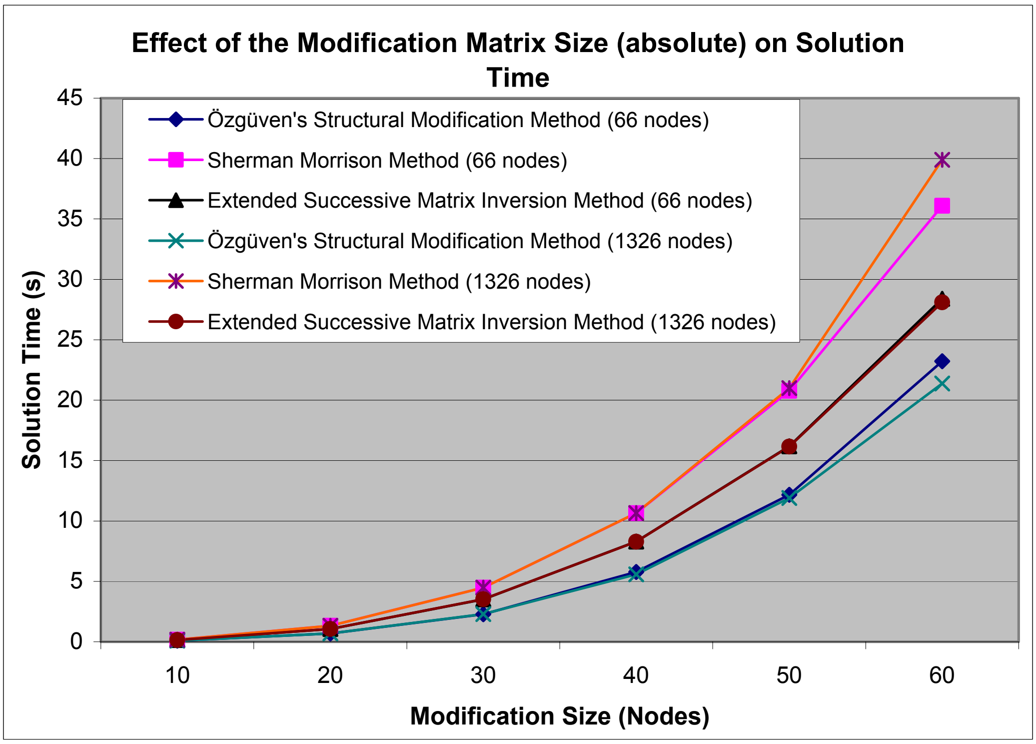

Figure 4.3 Effect of the Modification Matrix Size (absolute) on Solution Time – First and Second Structures.

The same solution time are compared also for the modification percentage for both structures in Figure 4.4 and Figure 4.5.

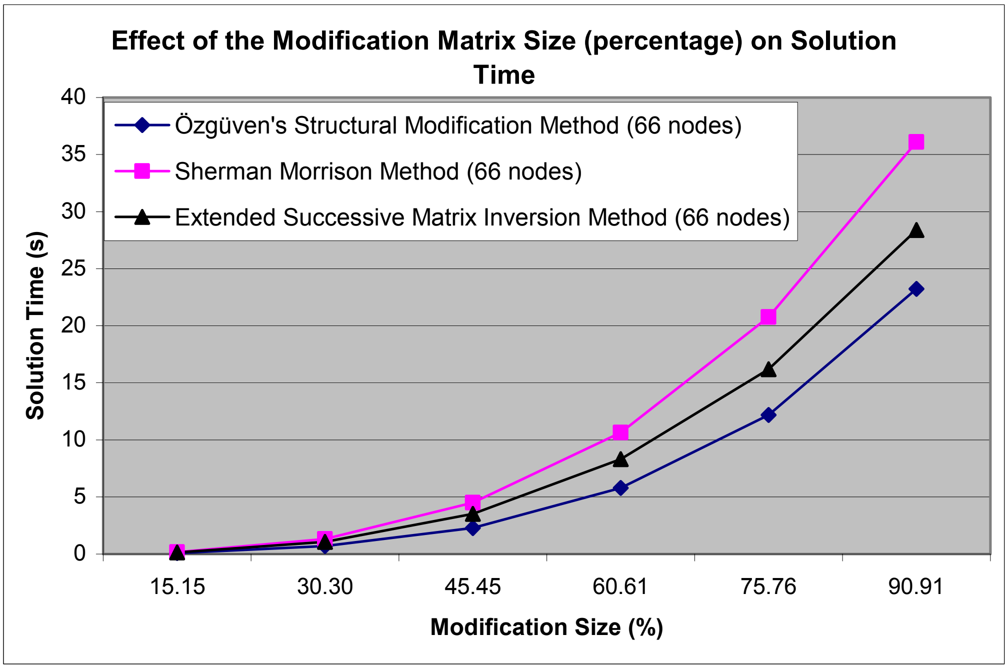

Figure 4.4 Effect of the Modification Matrix Size (percentage) on Solution Time- First Structure.

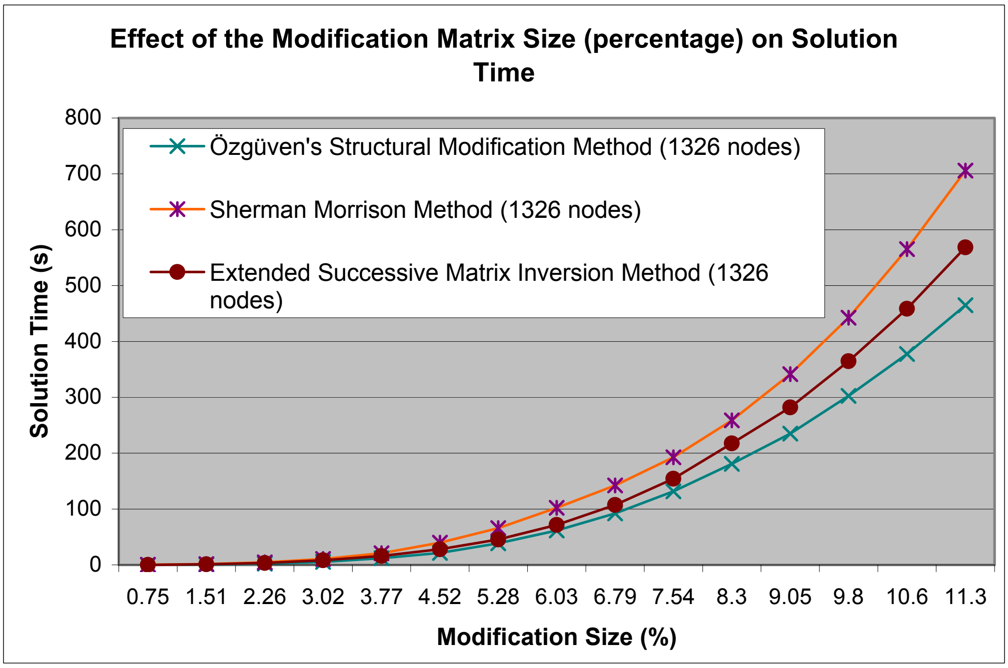

Figure 4.5 Effect of the Modification Matrix Size (percentage) on Solution Time- Second Structure.

It is seen from the results that, the solution time does not depend on percentage of the modified DOF to total DOF, but on the modification size itself. That is, the solution is a function of the size of the modification matrices. Since the program calculates only the responses at the selected nodes, the size of the modification matrix increases only when the number of modified nodes is increased.

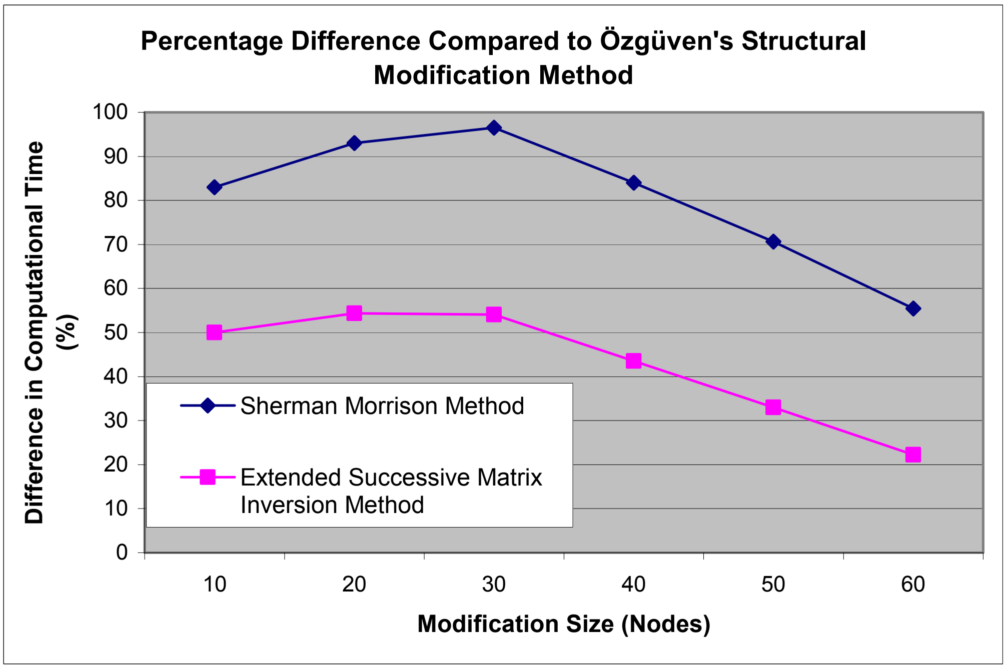

Figure 4.6 Comparison of Percentage Difference in Computational Time Compared to Özgüven's Structural Modification Method – First Structure.

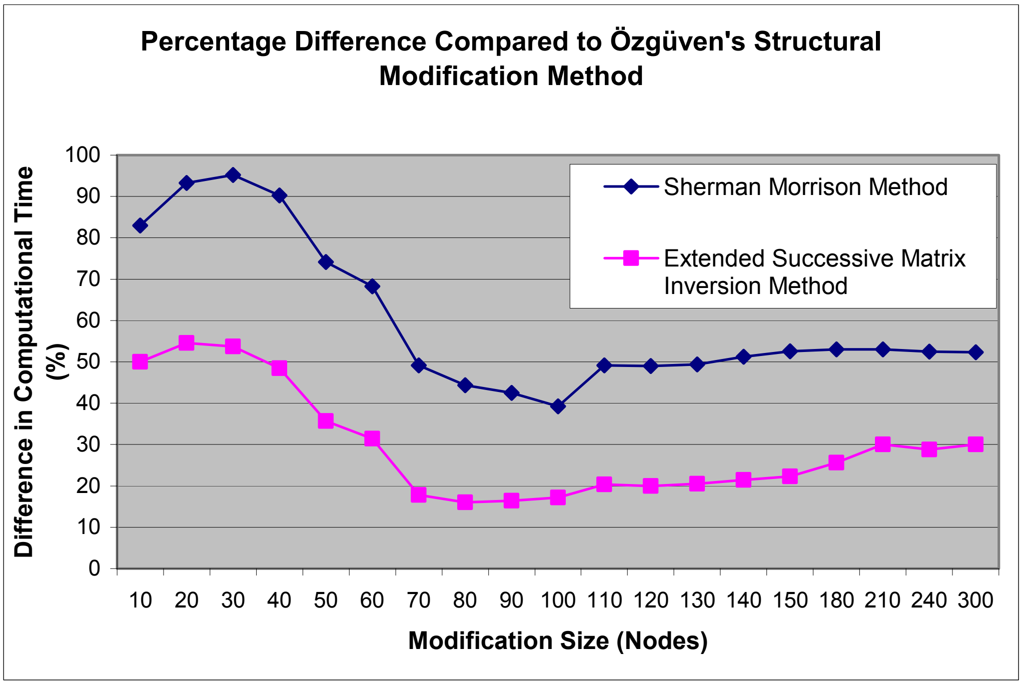

Figure 4.7 Comparison of Percentage Difference in Computational Time Compared to Özgüven's Structural Modification Method – Second Structure.

It can easily be seen from these results that Özgüven's Structural Modification Method is the most efficient method. As shown in Figure 4.6, while the use of Extended Successive Matrix Inversion Method (ESMI) increases the computational time between 22% to 54%, Sherman-Morrison Method increases the computational time between 55% to 96% compared to Özgüven's Structural Modification Method for the first structure. For the second structure, the increment is between 16% and 54% for ESMI and between 40% and 96% for Sherman-Morrison Method as shown in Figure 4.7. Although the difference in computational time may seem to be decreasing after a modification size of 30 (from Figure 4.6), it can be observed from Figure 4.7 (which is in agreement with Figure 4.6) that after the modification size of about 70-100, again the difference increases. Further discussions are given in Chapter 6.

CHAPTER 5

EXPERIMENTAL INVESTIGATON

5.1 STRUCTURAL MODIFICATION ON GARTEUR SM-AG19 MODEL

Although Structural Modification Toolbox is developed for theoretical studies, the methods were also applied on a real test structure. In this section; modal test and modal analysis conducted on GARTEUR SM-AG19 test structure in TÜBİTAK-SAGE facilities and structural modification application on an updated finite element model are explained in detail. GARTEUR SM-AG19 structure has been designed by a multinational research group and GARTEUR is the abbreviation of Group for Aeronautical Research and Technology in Europe. This structure is also used in literature for modal testing [65, 66]. The purpose of this structure is twofold: Modal tests and modal analyses which are conducted in different countries have a common structure, and investigations on different methods are applied by using the same experimental data (which are provided by the group). The differences between the test-bed used in this work and the GARTEUR SM-AG19 model are the type of connections at the fuselage-wing, fuselage-vertical stabilizer and vertical stabilizer-horizontal stabilizer interfaces and also the viscoelastic tape which was placed on the upper surface of the wings. Instead of bolted connections in the original GARTEUR SM-AG19 model, welded connections were used in the test-bed. On the other hand, intentionally no additional damper was used in these tests. The test-bed used in the tests is shown in Figure 5.1.

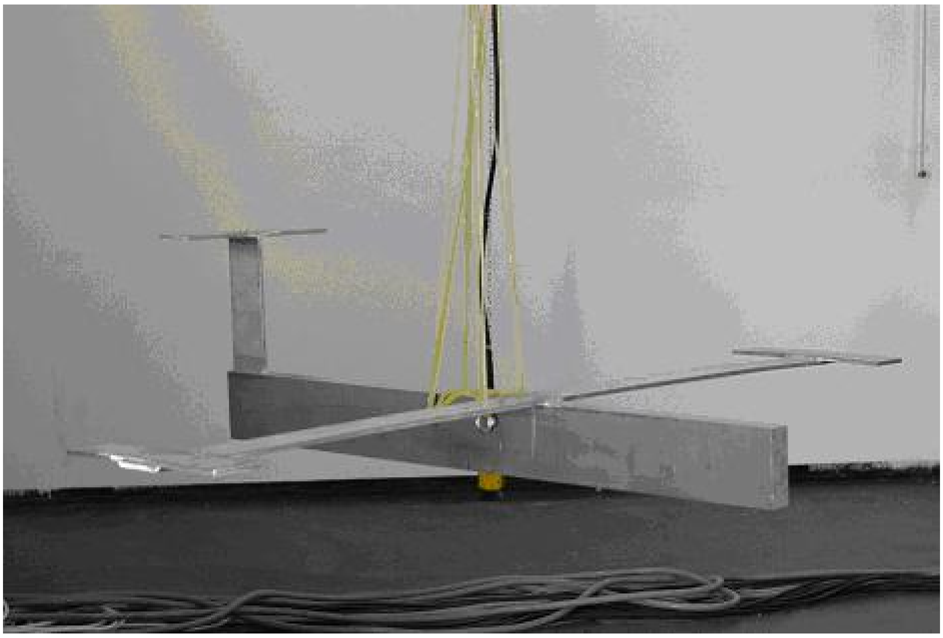

Figure 5.1 General View of Test-Bed in TÜBİTAK-SAGE.

On this test-bed, two modal tests were performed; one without additional masses on the structure and one with the presence of additional masses on both wings. The results of the modal tests and modal analyses were used for comparison of structural modification analysis results obtained by using an updated finite element model with the test results obtained from the structure with additional masses. In this part, the aim was to investigate the agreement between theoretically predicted and experimentally measured FRF.

5.1.1 MODAL TEST SETUP AND MODAL TESTS

The dimensions of the structural elements and their locations on test-bed are shown in Figure 5.2. The dimensions are in millimeters. The material of all the members is aluminum.

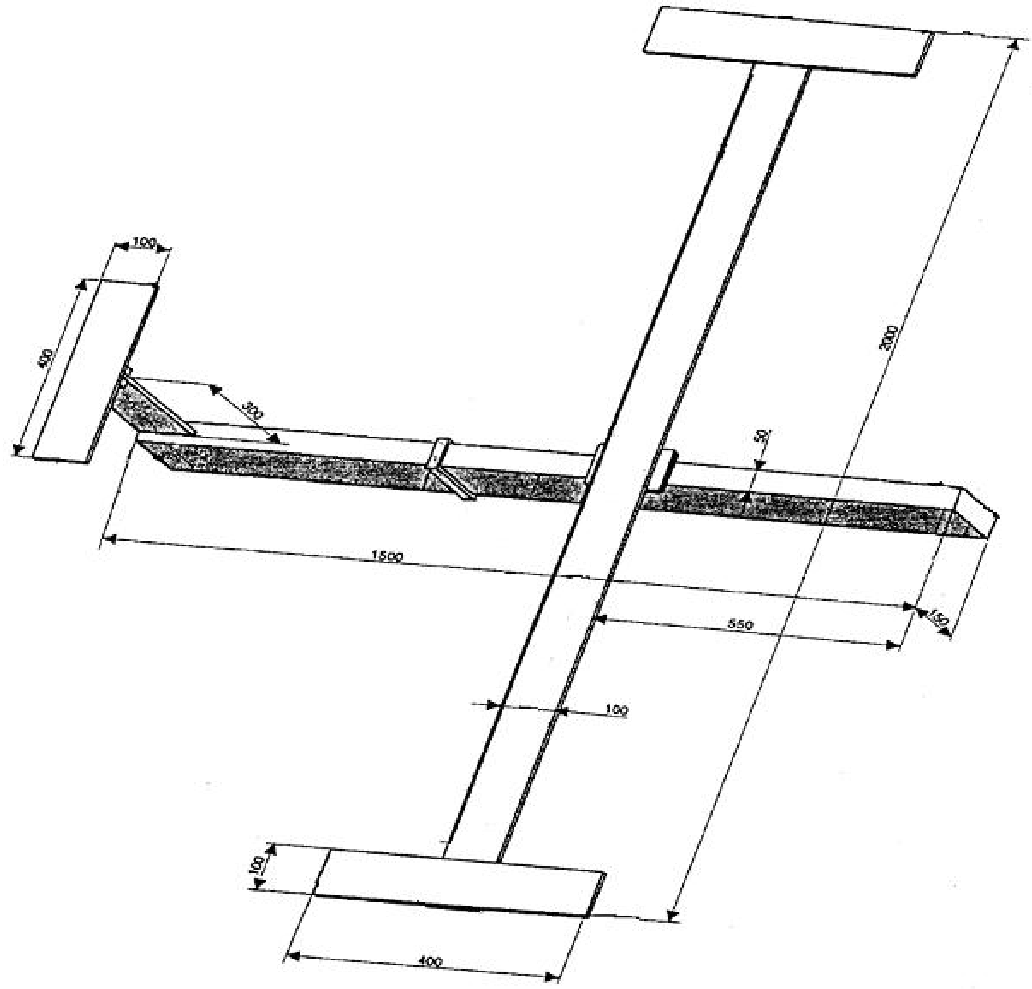

Figure 5.2 General Dimensions of the Test-Bed.

The test-bed is hung from eyebolts enveloping the center of mass of test structure, to provide free-free boundary conditions, as shown in Figure 5.3. Medical tourniquet cords are used for suspension. In additional to medical tourniquet cords, elastic cords are also used to prevent the test model from moving back and forward and tilting while the model is being lifted. The number of layers of medical tourniquet is chosen such that the suspension provides rigid body mode frequencies below 1 Hz and keeps the deflections within the geometric constraints of experimental rig. The rigid body mode frequency of the test structure is around 1 Hz.

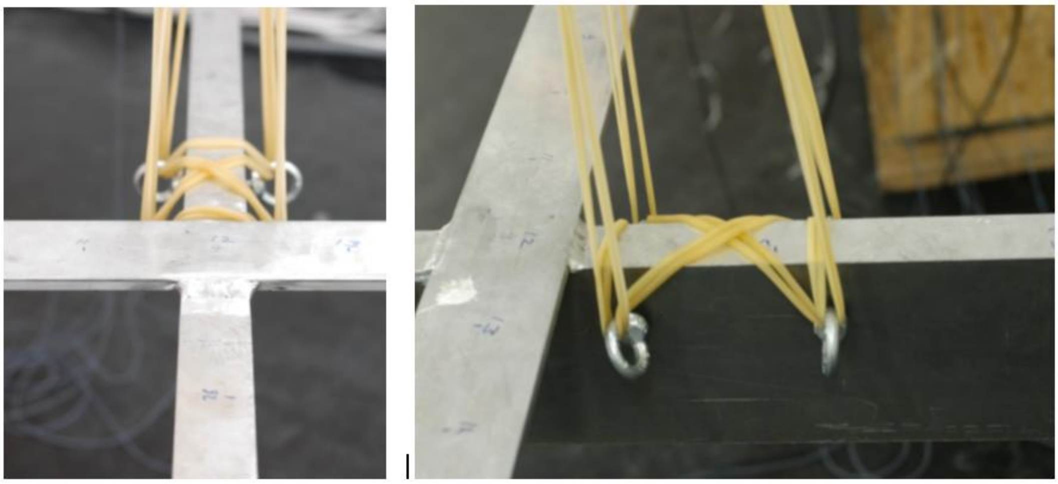

Figure 5.3 Eyebolts.

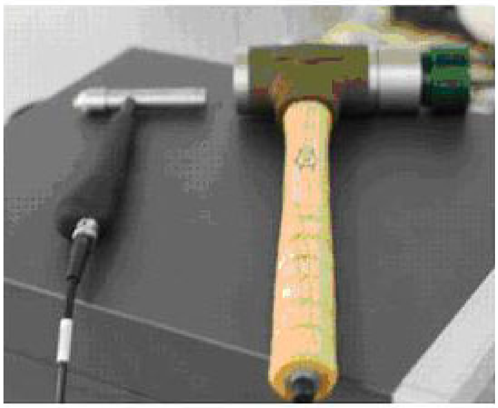

Figure 5.4 Modal Hammer (left) and Modal Sledge Hammer (right).

The modal tests were performed by using a modal hammer, a modal sledge hammer (Figure 5.4) and accelerometers. The same measurement and excitation networks were used in the tests of both the original model and the mass modified model. The structure was excited from 66 DOFs in 36 nodes and totally 12 accelerometers were used. Excitation and measurement points were crossed mathematically using reciprocity rule so that more measurement points were obtained without changing the locations of the accelerometers. Coordinate axes, the numbering of measurement and excitation points are shown in the stick model given in Figure 5.5. The points and directions used for measurement and excitation are also given in Table 5.1 and Table 5.2, respectively.

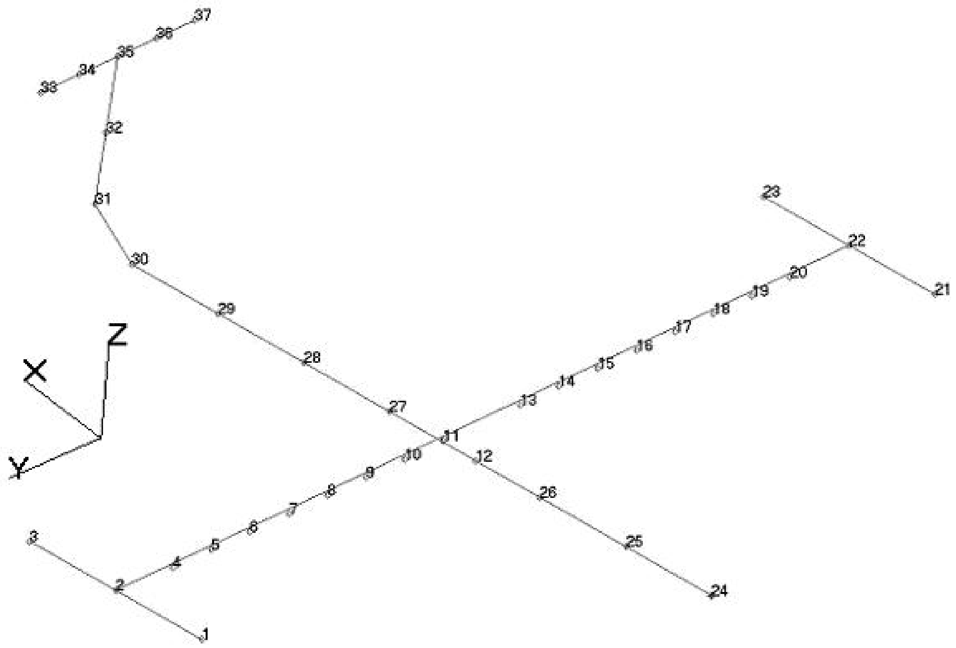

Figure 5.5 Coordinate Axes, Measurement and Excitation Points.

Table 5.1 Measurement Points and Directions.

Measurement Point	Directions
1	x-, z-
3	z-
6	z-
18	z-
21	x-, z-
23	z-
31	y-
33	z-
35	y-
37	z-

Table 5.2 Excitation Points and Directions.

Excitation Point	Directions
1	x-, y-, z-
2	z-
3	x-, y-, z-
4	x-, z-
5	x-, z-
6	x-, z-
7	x-, z-
8	x-, z-
9	x-, z-
10	x-, z-
11	x-, z-
12	z-
13	x-, z-
14	x-, z-
15	x-, z-
16	x-, z-
17	x-, z-
18	x-, z-
19	x-, z-
20	x-, z-
21	x-, y-, z-
22	z-
23	x-, y-, z-
24	z-
25	z-
26	z-
27	z-
28	z-
29	z-
30	z-

Table 5.2 (cont'd) Excitation Points and Degrees of Freedom.

31	y-
32	y-
33	x-, y-, z-
34	x-, z-
35	x-, z-
36	x-, z-
37	x-, z-

Except for the points on fuselage, the modal hammer was used for the excitation of the test structure. In the tests when the modal hammer was used on the fuselage, both coherence values and response of the accelerometers obtained were relatively low, while the force signal from the hammer was saturated; so it was decided to use the modal sledge hammer.

Excitation in each DOF was performed for 5 averages. The excitations with lower coherence values were repeated for better measurement. LMS Test.Lab© software and LMS SCADAS III hardware were used to control the tests. The software enabled monitoring of the coherence values. Coherence values over the complete frequency range were evaluated according to the values of neighbor nodes in order to decide whether or not to repeat a hit. Details about the Data Acquisition System are given in Table 5.3

Table 5.3 Data Acquisition System and Parameters.

Data Acquisition System	LMS SCADAS III
Accelerometer Max. Acceleration (g)	50
Modal Hammer Max. Force (N)	240
Modal Sledge Hammer Max. Force (N)	22026
Windowing	Force-Exponential
Frequency Increment (Hz)	1/1024
Data Acquisition Time (s)	10.24

5.1.1.1 MODAL TEST ON ORIGINAL TEST-BED

Close-up pictures of the original test-bed are given in Figure 5.6.

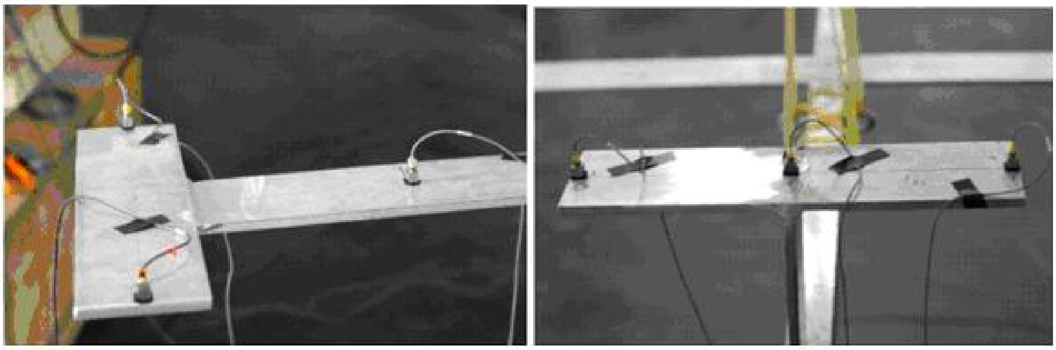

Figure 5.6 Original Test-Bed Setup.

An example of the FRF and coherence plots for a point FRF measurement obtained from LMS Test.Lab© software is given in Figure 5.7:

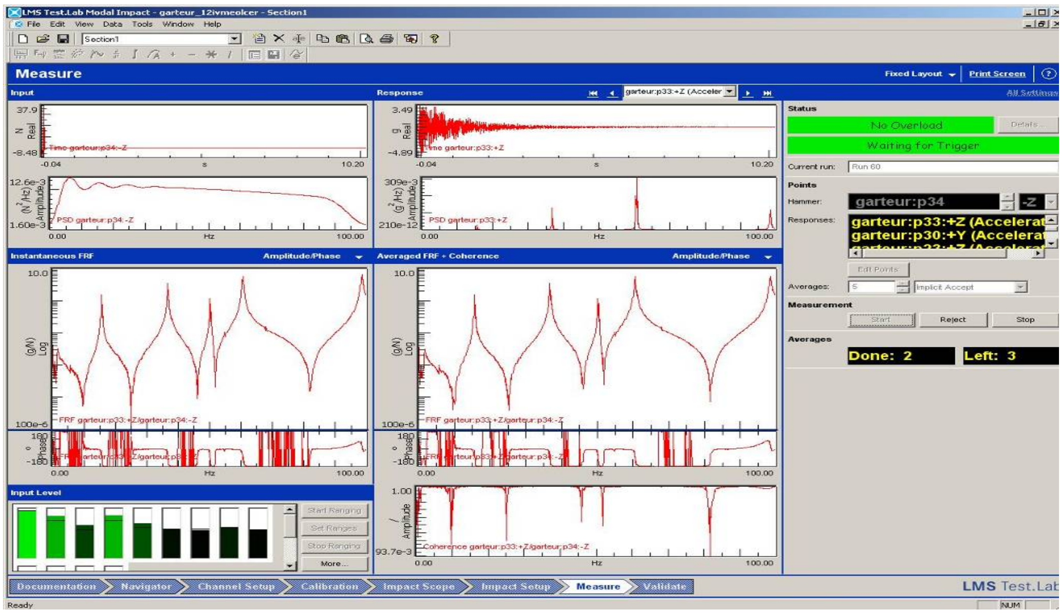

Figure 5.7 Screenshot of Measurements at Point 34 in z- Direction in Original Test-Bed.

Polymax® algorithm of LMS Test.Lab© Modal Impact Software was used for successive modal analysis, after reordering the FRF values using reciprocity rule.

Selected stable roots and stability diagram obtained by Polymax® algorithm of LMS Test.Lab© Modal Impact Software is given in Figure 5.8. The model order is chosen to be 150 for modal analysis.

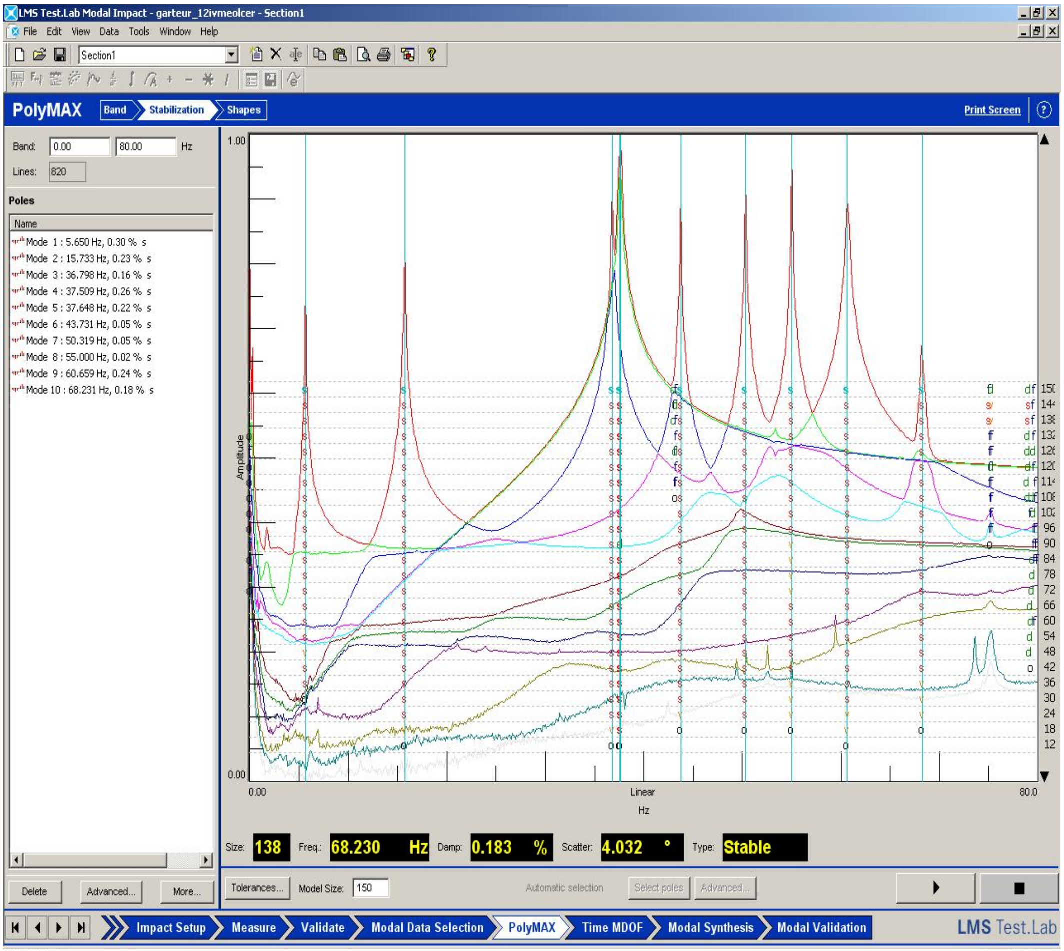

Figure 5.8 Selected Stable Roots and Stability Diagram Obtained by Polymax®.

Natural frequencies obtained from the stability diagram shown in Figure 5.8 are given in Table 5.4.

Table 5.4 First 10 Natural Frequencies from Polymax®.

Experimental Mode	Original Natural Frequency (Hz)	Modified Natural Frequency (Hz)
1	5.65	5.34
2	15.73	15.15
3	36.79	29.14
4	37.51	30.10
5	37.65	35.44
6	43.73	35.59
7	50.32	45.96
8	55.00	51.04
9	60.66	53.59
10	68.23	68.21

5.1.1.2 MODAL TEST ON MASS MODIFIED TEST-BED

As mass modification to the test-bed, 1.8 kg masses were added to nodes 6 and 18 as shown in Figure 5.9.

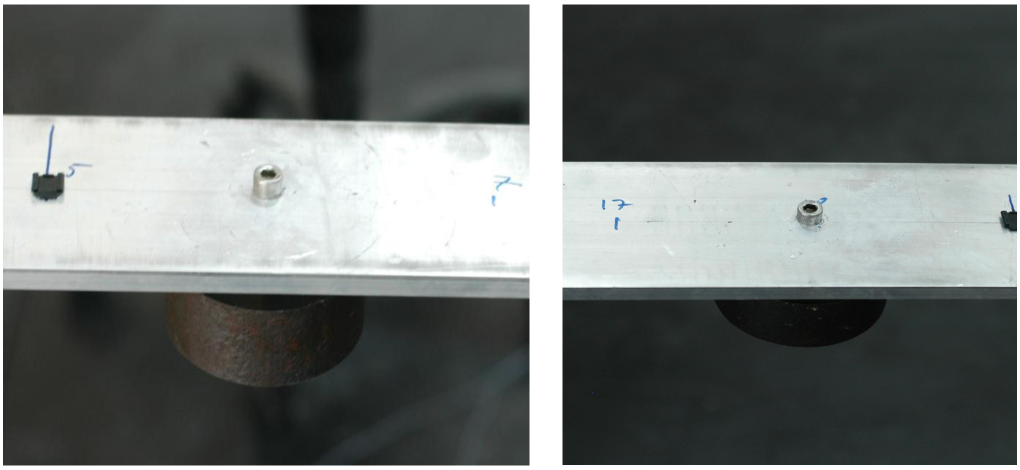

Figure 5.9 Mass Modifications on Original Test-Bed.

The model order was also chosen to be 150 for modal analysis of the modified test-bed.

Natural frequencies obtained from the test are given in Table 5.4. Mode shapes are compared with mode shapes of the modified structure in Figure 5.10.

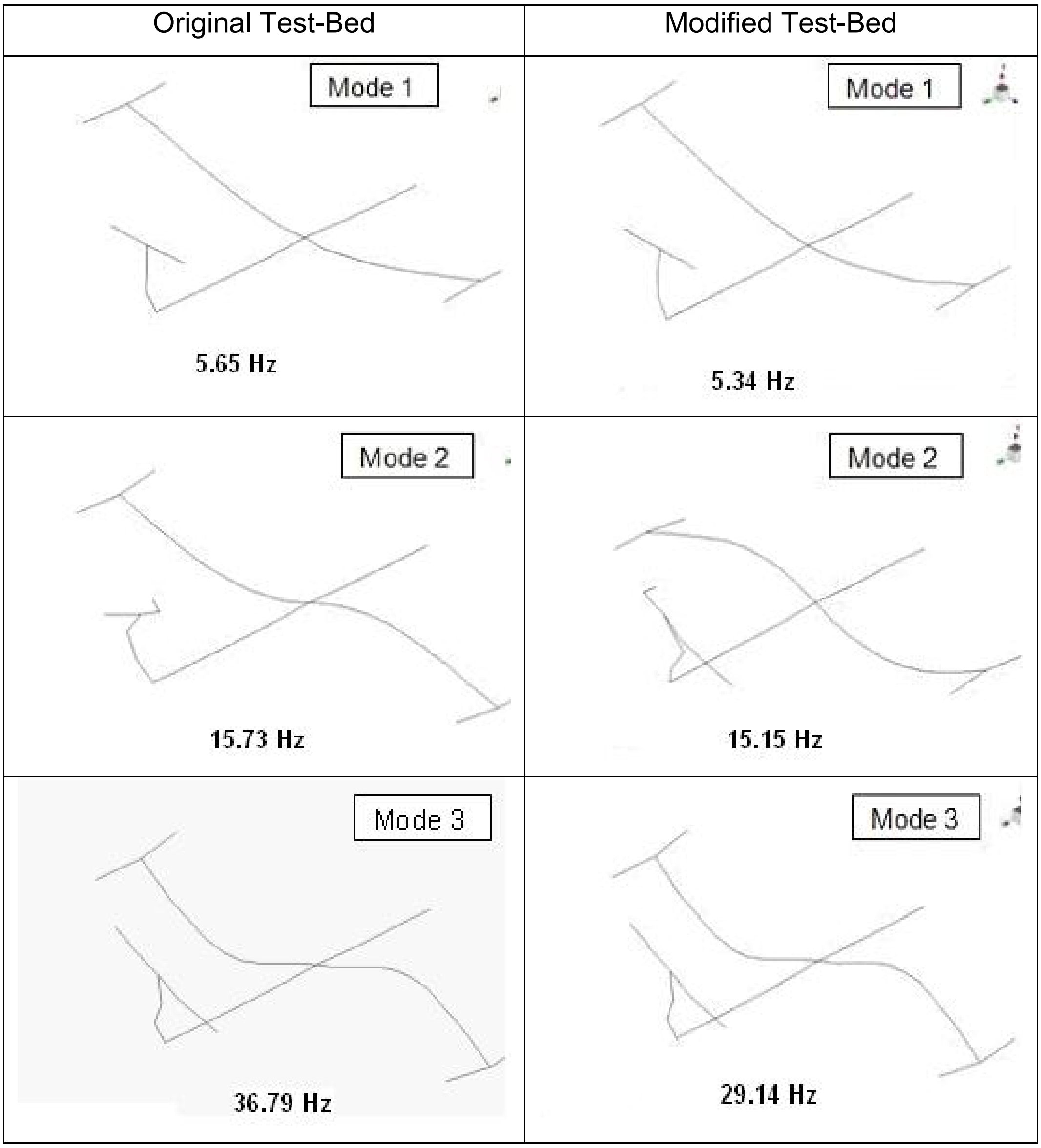

Figure 5.10 First 10 Modes of Original (left) and Modified (right) Test-Bed Obtained From Polymax®.

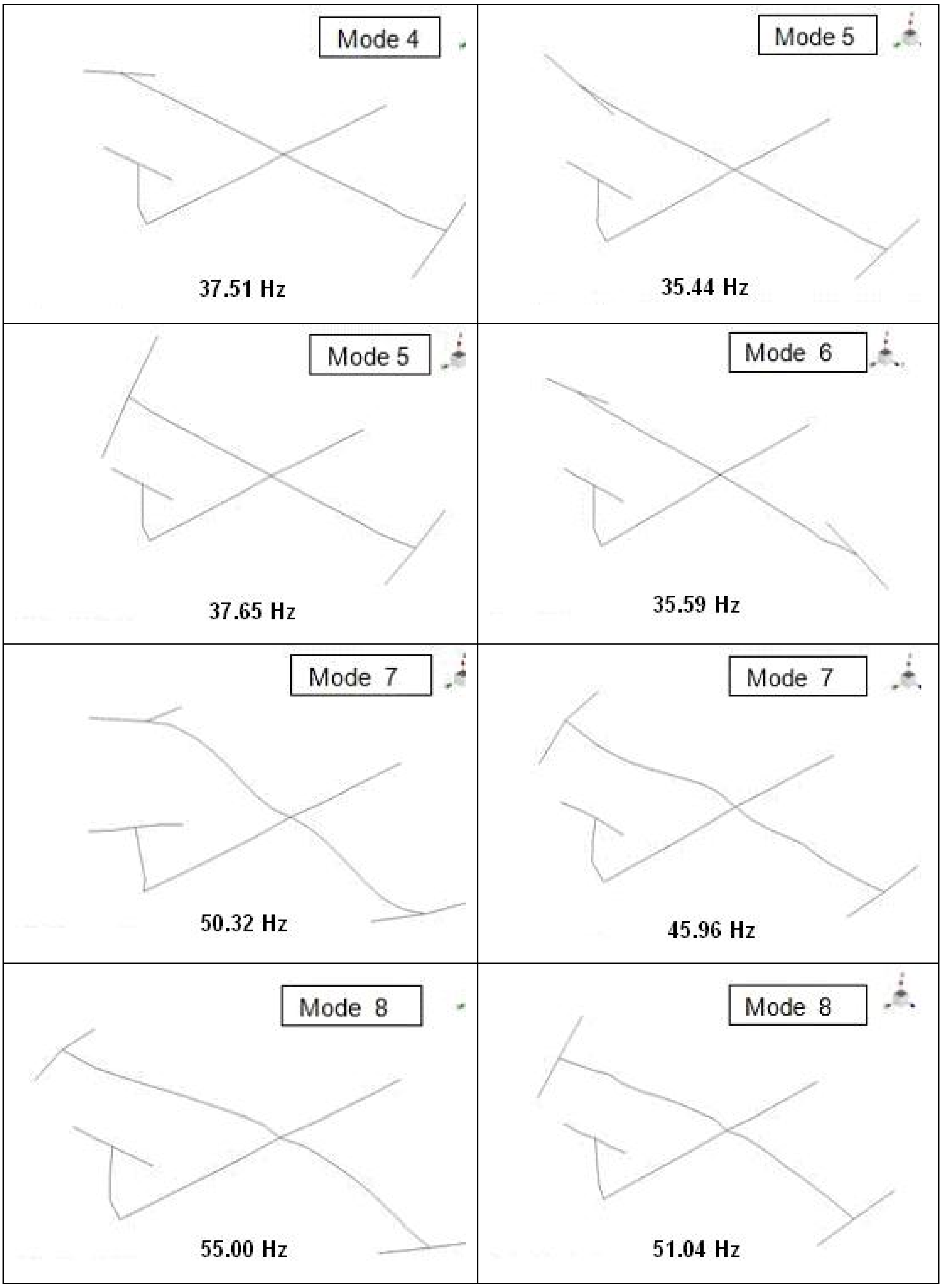

Figure 5.10 (cont'd) First 10 Modes of Original (left) and Modified (right) Test-Bed Obtained From Polymax®.

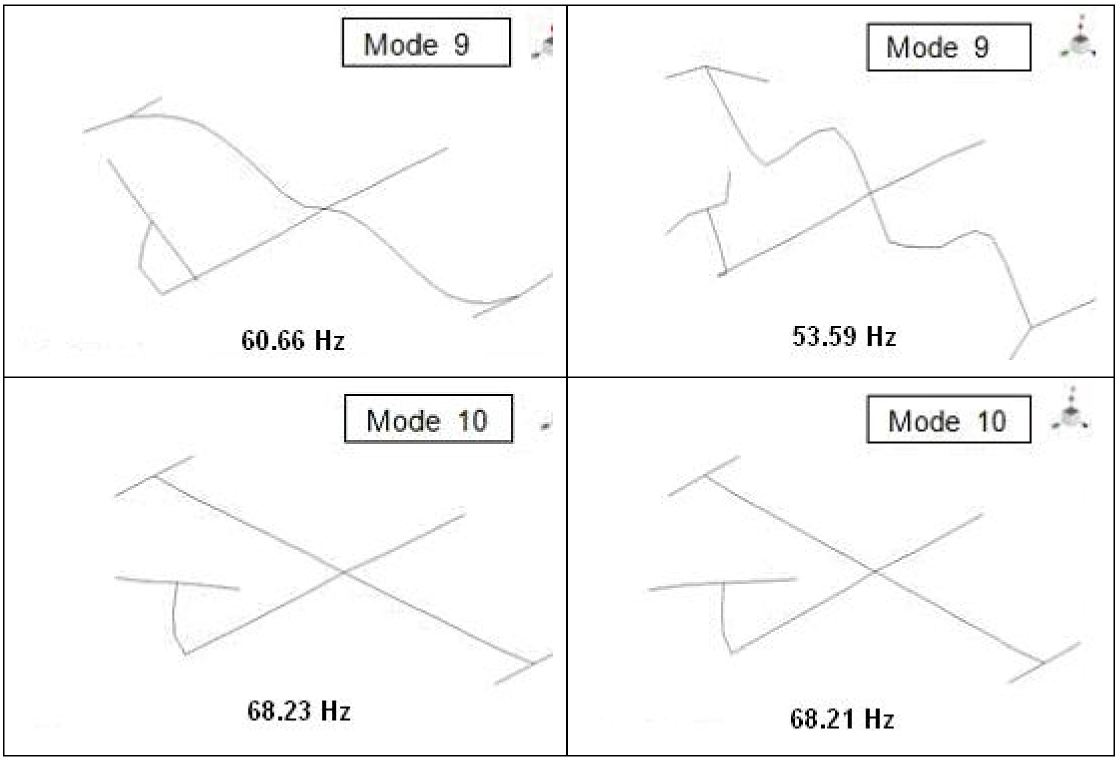

Figure 5.10 (cont'd) First 10 Modes of Original and Modified Test-Bed Obtained From Polymax®.

5.1.2 FINITE ELEMENT MODEL

MSC Patran© GUI and MSC Nastran© solver were used for modeling and analysis of the finite element model of the test-bed, respectively.

The test-bed was composed of strips with thickness to length ratio less than 0.1 and it had no discontinuities like holes, and they are modeled in the finite element model as beam elements. 3-D view of the finite element model is given in Figure 5.11.

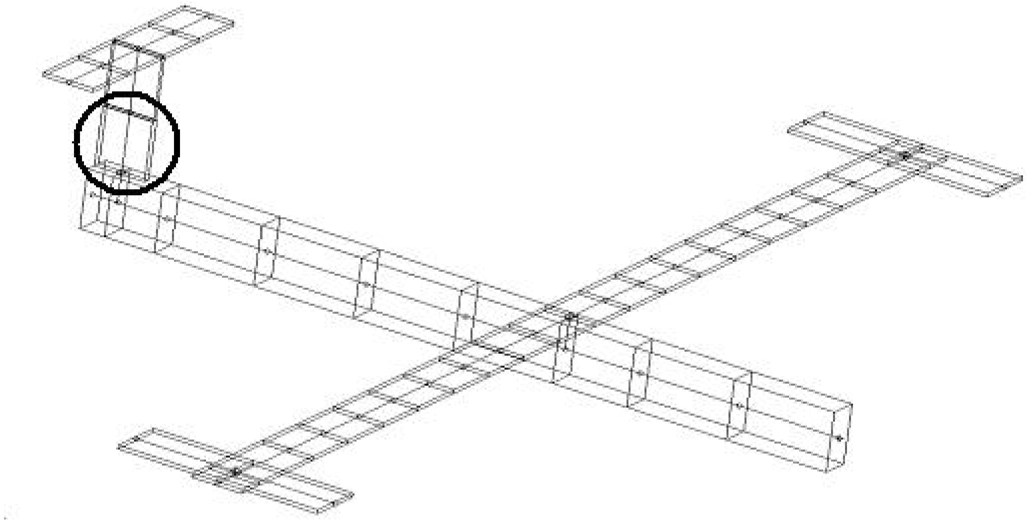

Figure 5.11 3-D View of Finite Element Mesh of the Test-Bed Structure.

The thicknesses of the strips were also taken into account during modeling and the line structure was prepared accordingly. The locations of the strips on the finite element model is the same as the one in the real structure as shown in Figure 5.12.

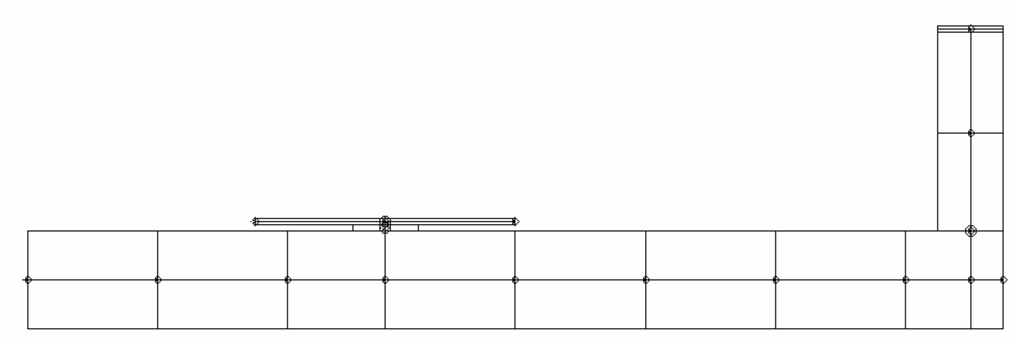

Figure 5.12 The Locations of the Strips in Finite Element Model

The differences in the positions of the neutral axes of the beam elements in the z-direction led to discontinuities in the mating junctions of the finite element model. In order to overcome this problem, rigid elements called “Multi Point Constraint – MPC” were used between the Outer Wing Strips and the Wings, the Wing and the

Fuselage and the Fuselage and the Vertical Stabilizer. The details of the rigid elements are given in Table 5.5.

Table 5.5 MPC Element Details

Parts That MPC Connects	Part That Has the Independent Node	Part That Has the Dependent Node
Outer Wing Strips-Wings (Right Wing)	Wing	Outer Wing Strips
Outer Wing Strips-Wings (Left Wing)	Wing	Outer Wing Strips
Wing-Fuselage	Fuselage	Wing
Fuselage-Vertical Stabilizer	Fuselage	Vertical Stabilizer

The rigid elements transmit the motion to the independent node through the length of the connection. The MPC elements used in the model are shown in
Figure 5.13 in detail.

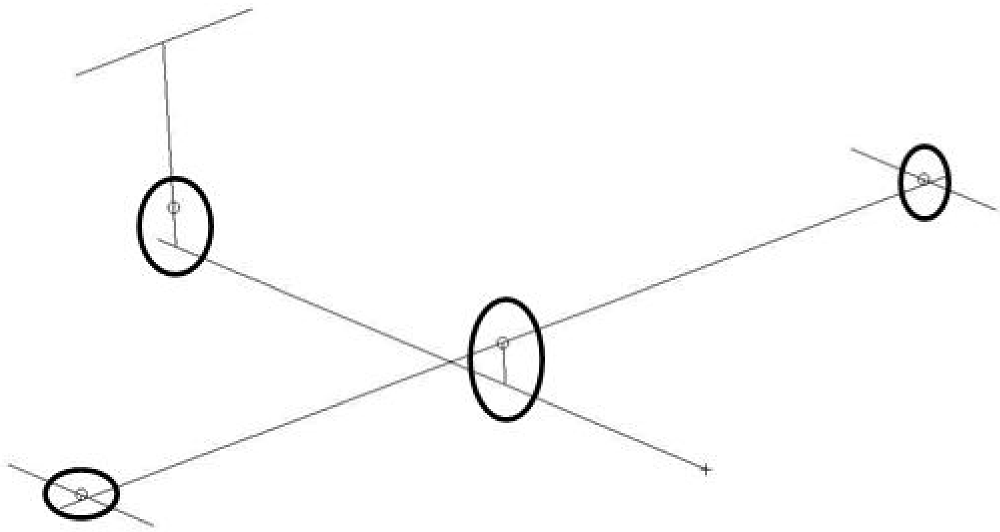

Figure 5.13 MPC Elements in the Model

The section dimensions of the beam elements of test-bed are all the same in the original finite element model as they are given in Figure 5.11 and Figure 5.12. The nomenclature and positioning of the rest of the nodes are the same for both models (experimental and finite element models). A total of 39 beam elements and 43 nodes (258 degrees of freedom) are used in the finite element model. The beam elements are 3-D elements with 2 nodes (with 6 DOFs at each node). The material properties assigned are tabulated in Table 5.6.

Table 5.6. Material Properties Used in the Finite Element Model

Material Property	Value
Young's Modulus (GPa)	70
Density (kg/m^3)	2800
Poisson's Ratio (υ)	0.3

5.1.3 STRUCTURAL MODIFICATIONS ON FINITE ELEMENT MODEL

Firstly, the finite element model is compared with the test data for the original structure, and the FE model is updated so that a better match is obtained between theoretical and experimental responses. The updating of the FE model is achieved in another MSc work conducted [67], and only the results are used here. Then the structural modifications applied on the test-bed were implemented by using the updated finite element model. 1.8 kg point masses were connected to nodes 6 and 18 by bolts as shown in Figure 5.9 on the test-bed.

The finite element model results were used in Structural Modification Toolbox to find the FRF of the modified structure, and results were compared with those of the experimental study. Nodes 1, 3, 6, 18, 21, 23, 30, 33, 35 and 37 were selected as "Required Nodes" in Structural Modification Toolbox. Starting Frequency was selected as 1 rad/s (~0 Hz), and ending frequency was selected as 283 rad/s (~45

Hz). Number of frequency points was set as 1024 as in the test. The loss factor is taken as 0.005 for structural damping. Node number 1 and z- direction were selected as sample data for comparison. Selected parameters are shown in Figure 5.14.

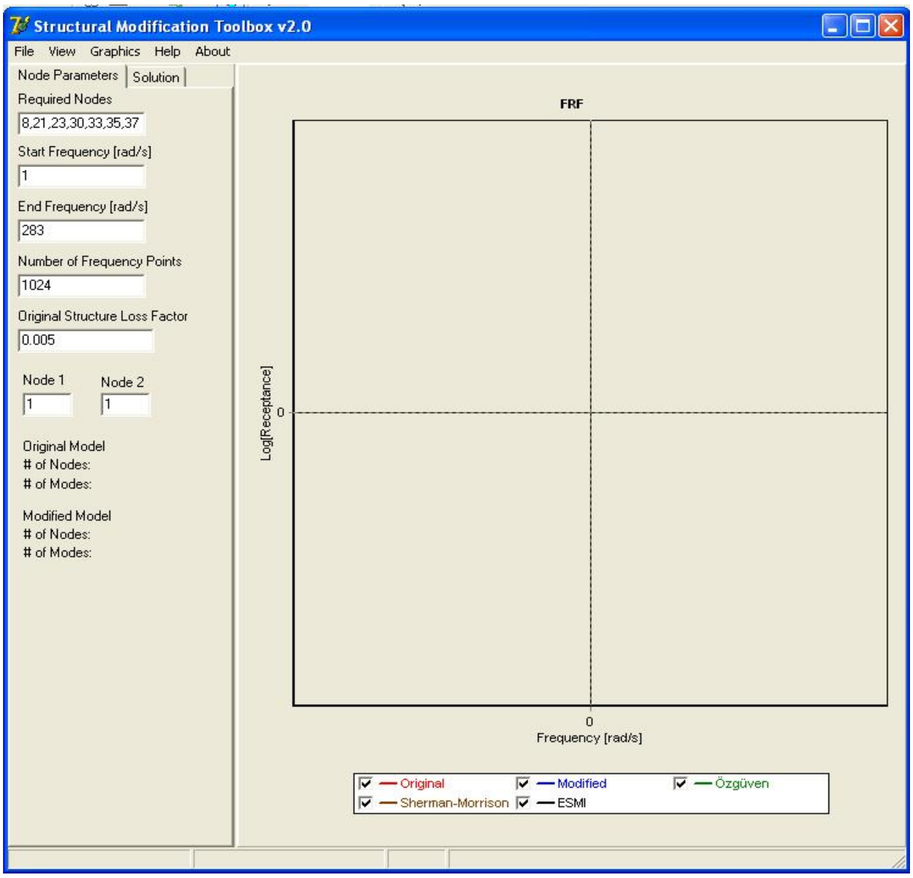

Figure 5.14 Structural Modification Toolbox Parameters

Comparisons of Frequency Response Functions obtained are given between Figure 5.15 and Figure 5.17. In the graphs, Experimental FRF (Original Structure) means FRF measured on the original structure, Experimental FRF (Modified Structure) represents FRF measured on the modified structure test data, Theoretical FRF (FEM-Original Structure) is FRF extracted from MSC. Nastran© for original structure, Theoretical FRF (FEM-Modified Structure) is FRF calculated using MSC. Nastran© output file for modified structure, Theoretical FRF (Updated-Original

Structure) stands for FRF gained from updated MSC.Nastran© model and Theoretical FRF (Modified Structure) is FRF calculated using Structural Modification Toolbox for modified structure. Modal Assurance Criterion (MAC) determines the correlation between the modes.

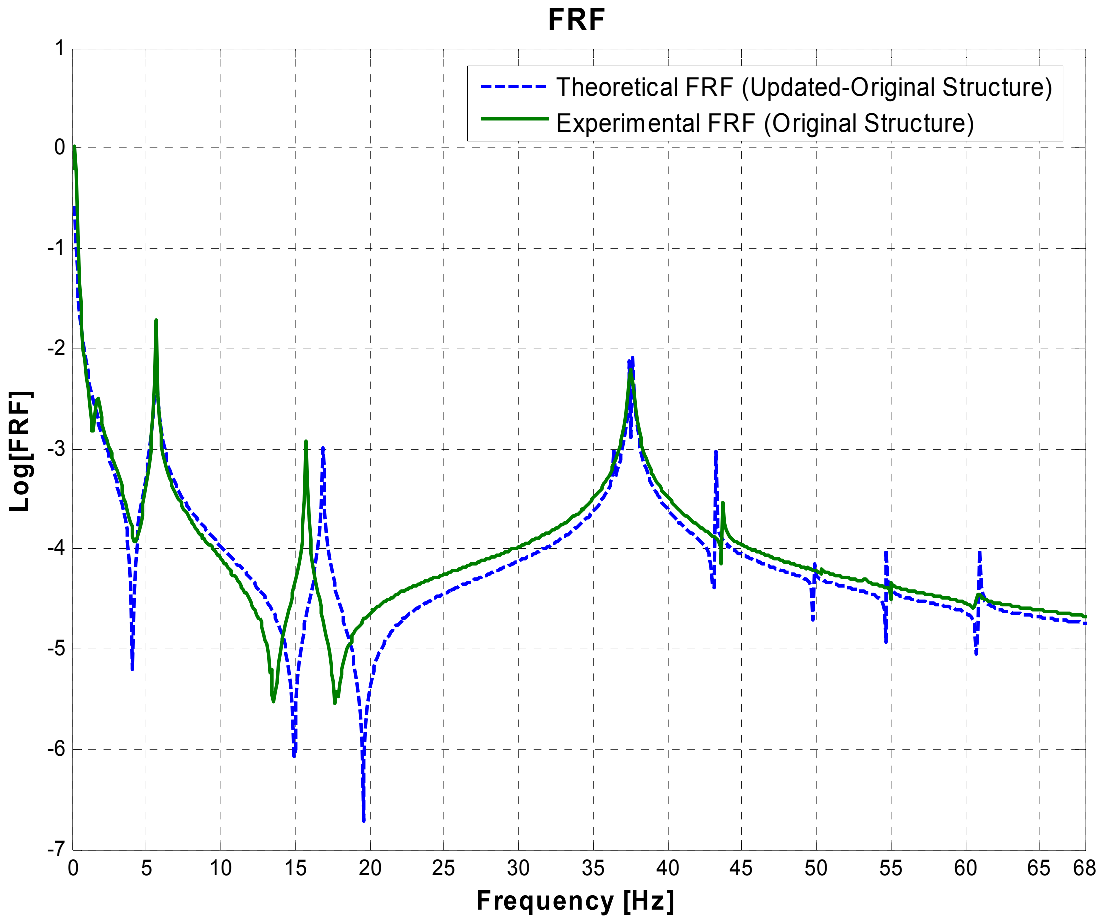

Figure 5.15 Comparison of the Experimental and Theoretical (updated) FRFs for the Original Structure at Node 1 in z- Direction.

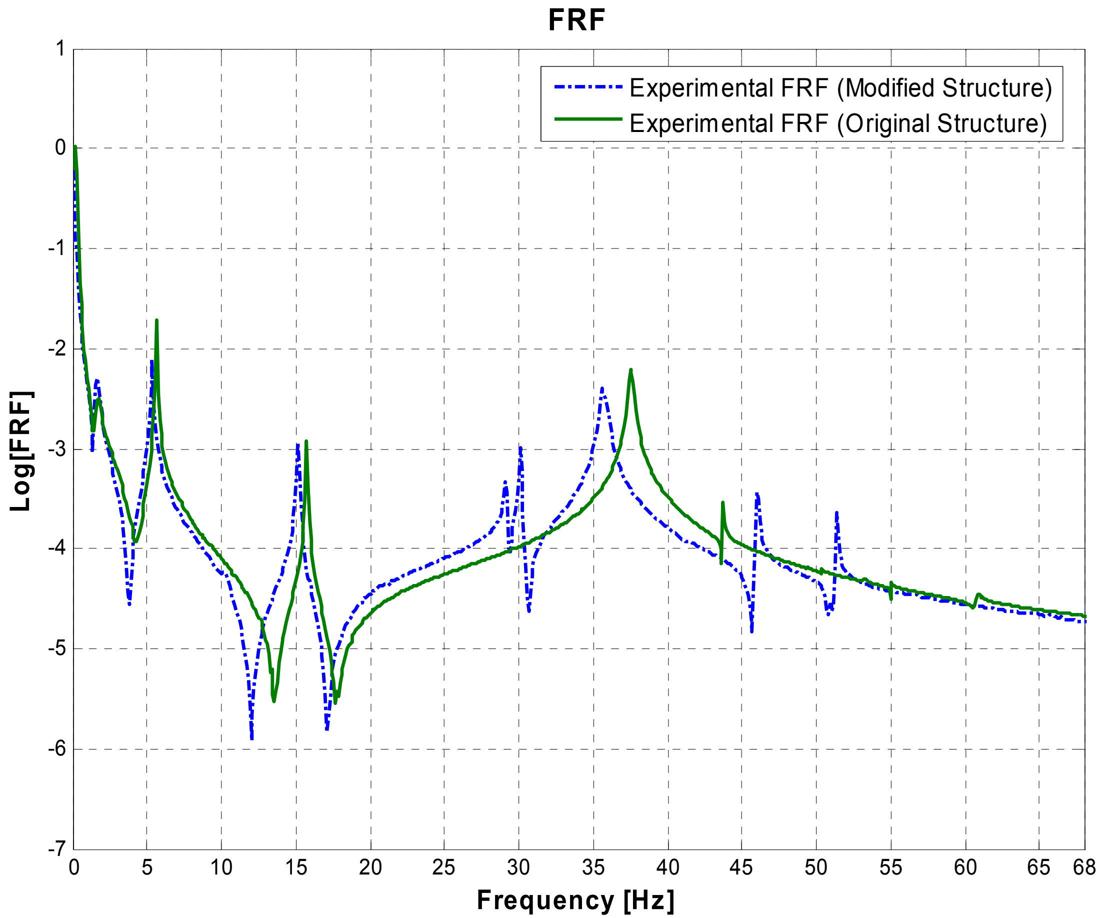

Figure 5.16 Comparison of the Experimental FRFs for the Original and Modified Structure at Node 1 in z- Direction.

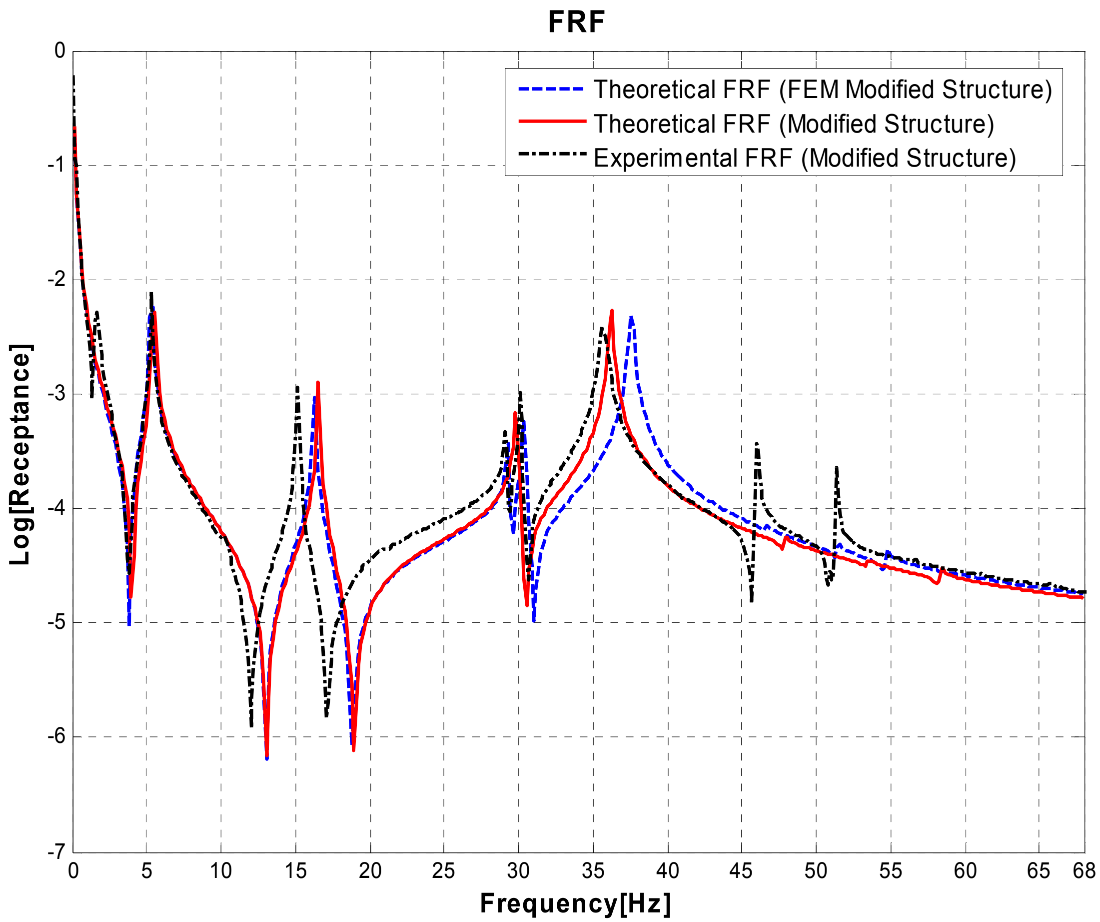

Figure 5.17 Comparison of the Experimental and Theoretical FRFs of the Modified Structure at Node 1 in z- Direction.

Mode comparison graph and MAC graph for the original structure are given in Figure 5.18 and Figure 5.19, respectively [67].

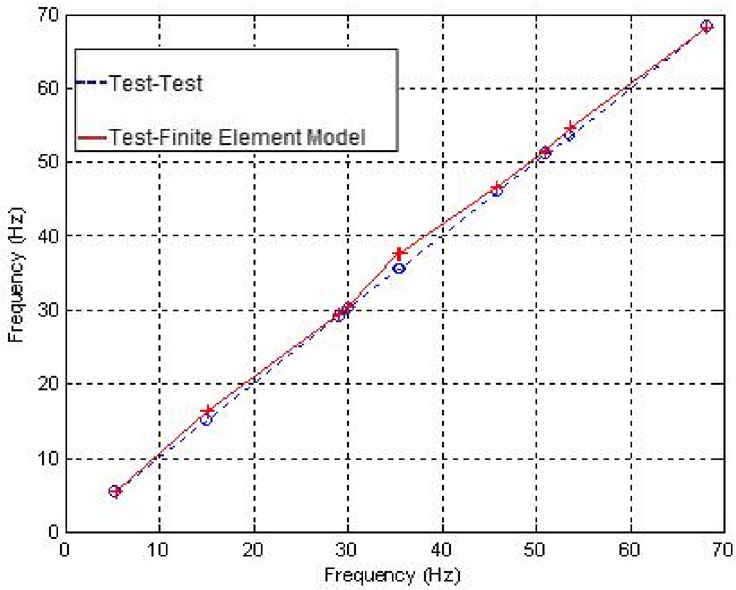

Figure 5.18 Test-Test and Test-Finite Element Mode Comparisons for Original Structure.

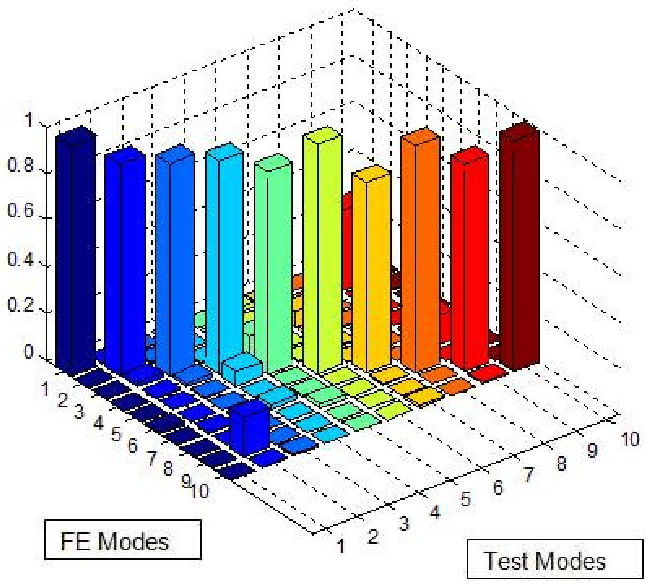

Figure 5.19 Test and Updated Finite Element MAC Graph for Original Structure.

As can be seen from Figure 5.15, which is given for the original structure, Theoretical FRF (Updated-Original Structure) is in good agreement with Experimental FRF (Original Structure), except around second resonance, resonances above 45 Hz and antiresonances. In second resonance, first antisymmetric bending mode, the vertical stabilizer is bent. In order to correlate bending and torsion modes of the vertical stabilizer, the thickness of the vertical stabilizer is widen as shown in circle in Figure 5.11, yielding a shift to a higher frequency. Although the updating technique yielded a shift that results the calculated natural frequency correlation in the first antisymmetric wing bending with more error, the other modes are brought in a better correlation. Resonance frequency values of Theoretical FRF (Updated-Original Structure) graph are mainly consistent with Experimental FRF (Original Structure) resonance values but the situation is the opposite for the anti-resonance values.

In Figure 5.10 and Figure 5.16, it is seen that third mode of the original structure is more effected than the second mode. Although the second mode is effected less than 1 Hz after mass modification, the difference is around 8 Hz for the third mode.

In Figure 5.17, Theoretical FRFs are compared with experimental ones for modified test-bed. They are found in good agreement in general for 0-45 Hz frequency range. The original structure was updated for ten modes (for ~0-68 Hz). There are differences between two curves around second, third and fifth modes. The reason of the mismatch in the second mode is the mismatch in the updated original test-bed data as explained above. As can be seen in Figure 5.15, theoretical and experimental FRFs are different in second mode. Although there are two modes around 30 Hz, only one mode is seen in theoretical FRF (modified solution) graph. The reason might be the shift in the modes after modification. For the fifth mode, when Figure 5.17 is investigated, it is observed that the mode is predicted well enough.

In addition to node 1 FRF graphs, node 3, node 21 and node 23 FRF graphs are also investigated and it is seen that the general appearance of the original and modified FRF graphs are similar to that shown in Figure 5.15. The agreement between theoretically predicted and experimentally measured FRFs is also similar to

Figure 5.17. In these graphs, the first resonance value is the same as the test value and the other resonance values are close to test values.

From Figure 5.15, Figure 5.16 and Figure 5.17, it is observed that mass modification shifts the resonance frequencies at the FRFs to lower frequencies. When Figure 5.15 and Figure 5.17 are compared, it is seen that some resonances moved around 30 Hz after mass modification.

It is seen from Figure 5.18 and Figure 5.19 that both the modal frequencies and mode shapes for the test model and the FE model satisfactorily match for the original test-bed.

The differences between the test and theoretical data are partly due to the discrepancy between the test model and the FE model for the original structure. The second and the major reason is the inaccuracy in the modeling of the modification itself. Since all the methods used are exact methods, when modification matrices are exact, the results will also be exact. Modification effect in the test was simulated in the FE model as accurate as possible, however although the modification masses are lumped in FE model, in the actual system, the modification affects a region. Furthermore the mass added affects the rotational degrees of freedom due to mass moment of inertia effects which are not included in the model. Therefore, both the modified FE model and structural modification approach could not simulate the test-bed completely.

Since damping of the model is not updated, there was a mismatch in most of the values in terms of the magnitude of the FRF at the resonances. It is known that every structure has some damping so that the loss factor is taken as 0.005.

Lastly, although the test-bed is hung with free-free boundary conditions, the test-bed and FE model are not exactly similar as far as the boundary conditions are concerned. This is the reason for discrepancy between experimental and theoretical data below 5 Hz, as it is observed both in Figure 5.15 and Figure 5.17.

5.2 STRUCTURAL MODIFICATION ON AIRCRAFT MODEL

In this section; structural modification application on an updated finite element model of a fighter aircraft is demonstrated. In this part, the aim is to compare the experimental FRF data, with those obtained by Structural Modification Toolbox that uses the updated FE model.

The finite element model of the aircraft is built in MSC Patran© GUI and the modal analyses are carried out in MSC Nastran© solver. The model consists of shell, beam and point (mass) elements and MPC's resulting in about 3200 DOF. 3-D beam elements (for spars, ribs, stiffeners and bulkheads) and constant thickness (quad and tria) shell elements (for the structural skin) are used for the mesh. The mass matrix is forced to be lumped in the translational DOF [67].

In the FE model, mass modification of 900 kg is distributed to nodes from 8000 to 8013 to simulate the real modification case. The total number of nodes in the structure is lower than node ID numbers. Structural Modification Toolbox calculates modified responses using node numbers so that modified node ID numbers were changed using MSC Patran© GUI. As explained in Appendix A, Structural Modification Toolbox checks the required nodes for modification with respect to the total number of nodes. When the node ID of a node is larger than the total number of nodes, the program gives an error message and does not calculate the result. However, this problem can be avoided by changing the ID number of the node under consideration.

Modal Assurance Criterion graph and mode comparison graphs for updated FE model are given in Figure 5.20 and Figure 5.21. The comparison of the FRF graphs is given in Figure 5.22.

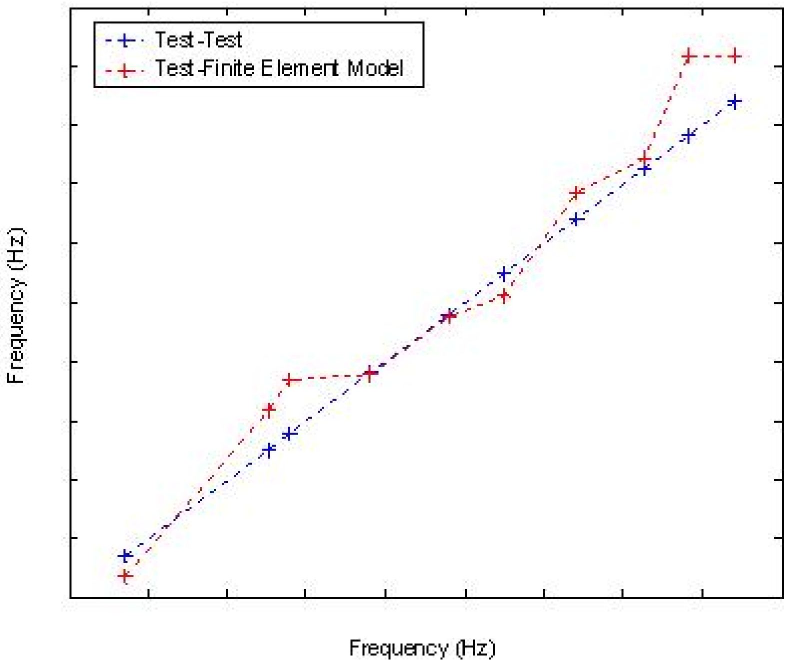

Figure 5.20 Test-Test and Test-Finite Element Mode Comparisons.

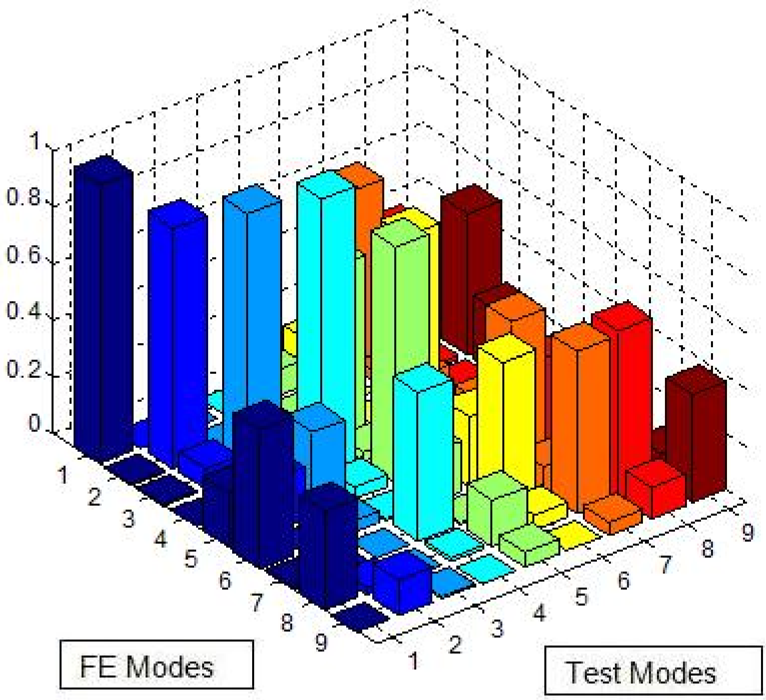

Figure 5.21 Test and Finite Element MAC Graph.

In Figure 5.22, Experimental FRF (Modified Structure) represents FRF measured on modified structure, Theoretical FRF (Updated-Original Structure) stands for FRF calculated by using updated MSC.Nastran© model, Theoretical FRF (FEM-Modified Structure) is FRF calculated using MSC. Nastran© output file for modified structure and Theoretical FRF (Modified Structure) is FRF calculated using Structural Modification Toolbox for modified structure.

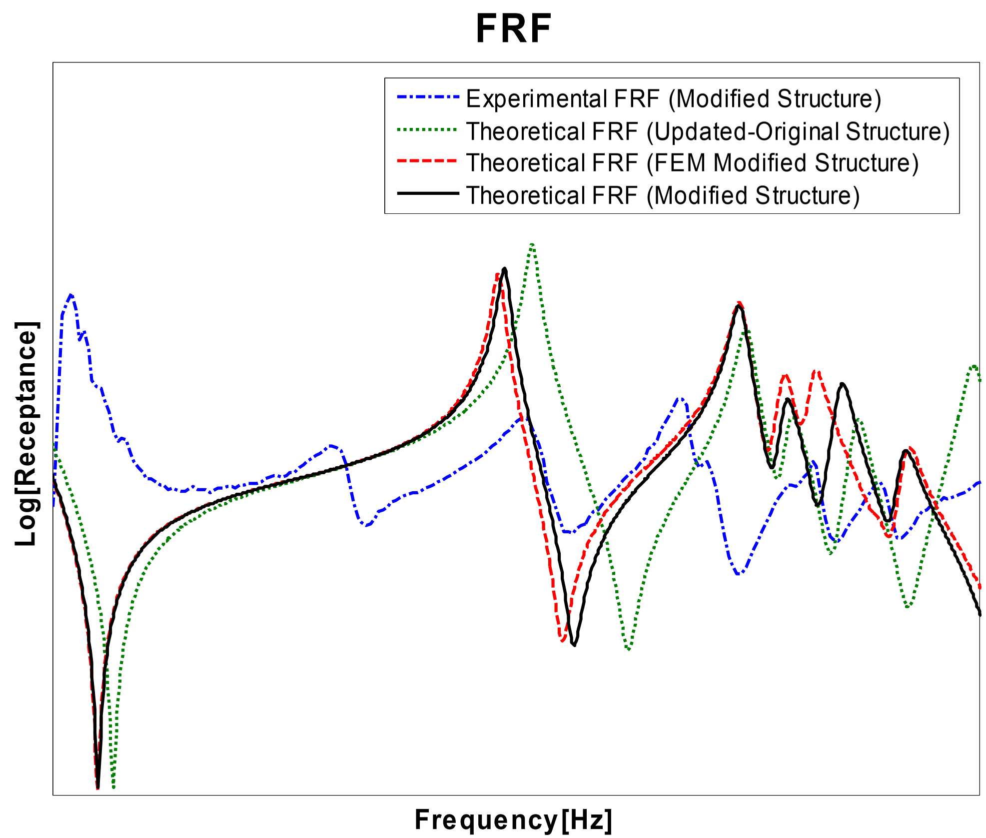

Figure 5.22 Comparison of the Experimental FRF of Modified Structure and Theoretical FRFs for the Updated Original and Modified Structures.

From Figure 5.22, it can be observed that there is a good agreement between Experimental FRF (Modified Structure), Theoretical FRF (FEM-Modified Structure) and Theoretical FRF (Modified Structure) around the first elastic mode. In fact, Theoretical FRF (FEM-Modified Structure) and Theoretical FRF (Modified Structure) solutions should be the same. The reason for the discrepancy between two theoretical solutions is the effects of inertia and rotational degrees of freedom. Since Structural Modification Toolbox is designed for no new DOF cases, in FE model, modification masses are lumped directly to nodes of modifying mass in original FE model. Because of rotational degrees of freedom effects that cannot be measured and therefore cannot be updated on FE model, the FE model could not simulate the real structure very accurately.

In Figure 5.22, it is also seen that although the frequency values are similar, the magnitudes of the FRF graphs do not match at resonance levels. The main reason for that is the damping. Since damping properties of the models can not be updated, there is a mismatch in most of the values in terms of the magnitude of FRF. It must be noted that for the simulation, damping is assumed to be proportional and loss factor is taken as 0.02.

Mass distribution effect of the modifications in the test could not be simulated exactly in the FE model. The modification affected only fourteen nodes in the FE model, and as lumped masses without mass moment of inertia effects. Since no new DOF is allowed in the Structural Modification Toolbox, nodes of original FE model are used to simulate the boundary conditions of the modifying structure. Besides that, the modeling of the joints between the modifying mass and the structure affected the results. The dynamics of the joints also need to be simulated in the FE model.

One of the other possible reasons for discrepancy is the effects of shakers. Lastly, although the structure is softly supported and free-free boundary conditions are assumed, because of the suspension system effects, the test structure does not have exact free-free boundary conditions.

It is concluded that application on a real large structure is a challenging issue. To be able to predict Theoretical FRF (Modified Structure) better, updated FE model should simulate the real structure as much as possible and the modifying structure should be modeled as accurately as possible to the real modifying structure. Additionally, the modeling of the structural joints is an important point for modification analysis.

CHAPTER 6

DISCUSSION AND CONCLUSIONS

6.1 THEORY AND COMPARISON OF METHODS

The starting point of this study was a requirement to obtain receptances of locally modified real large structures. It was necessary to use a Finite Element (FE) model of a large structure for modified responses. Once the FE model of the unmodified structure was updated using experimentally obtained data, receptances of the modified structure should be found by structural dynamic modification accurately. To be able to find exact receptances, exact modification methods should be used, therefore Özgüven's Structural Modification Method and Sherman-Morrison Method were selected as exact methods for structural modification with no new degrees of freedom. Moreover, a new structural modification method (Extended Successive Matrix Inversion Method) is developed.

Özgüven's Recursive Solution Algorithm [40] was applied for the calculation of the whole receptance matrix. The algorithm updates the receptance matrix using each element or each column of the dynamic stiffness matrix for modifying structure without matrix inversion. However, here the FRFs of some selected nodes in the Structural Modification Toolbox are needed. So the program was written such that only selected nodes are used in computation, which means that the order of the matrix used in recursive solution is equal to the size of the modification plus the unmodified nodes at which response is required.

Sherman-Morrison Method [22] was selected as the second exact method. This method does not use matrix inversion for calculation either. Instead, the method requires the modification matrix to be written as a product of two vectors. This method makes the solution at modification nodes and at unmodified nodes for which FRF calculation is required.

In this thesis a new structural modification method (Extended Successive Matrix Inversion Method) [43] was also developed. The Successive Matrix Inversion Method [42], which was suggested for static systems, was developed further for dynamic systems. It is shown that the basic equation is the same as the one used in the structural modification method of Özgüven [41]. However, a different approach is used in solution; while the size of the matrix inverted is reduced for locally modified systems by partitioning the system matrices, and a novel algorithm is suggested for full modification to avoid matrix inversion in Özgüven's method, in this method, power series expansion is used for the solution. The ESMI algorithm can be used in any similar application where matrix inversion is required.

Comparison of these methods shows that, the solution time does not depend on modification percentage. The solution time depends only on the absolute size of the modification. Since the program calculates the responses only at the selected nodes, and the order of the matrix in operation is equal to the number of the modified nodes plus the number of unmodified nodes at which responses are required, the solution time increases when the number of modified nodes increase. Since all the methods give exact results, all the results are the same and they are in accordance with MSC.Nastran© results. It is seen from the case studies that Özgüven's Structural Modification Method is the most efficient one, in terms of solution time, among the three methods.

Since all the methods used in this study need mass, stiffness and damping matrices representing the structural modification, the program cannot be used if only the modal data of the modification is known.

6.2 SOFTWARE

The computer program, Structural Modification Toolbox was developed by using Borland Delphi©. Initially in this study, Matlab© and Matlab© GUI were used for programming and user interface design. The reason for that was several advantages of Matlab© and easy programming, but as the studies progressed, it was seen that using Borland Delphi© or similar visual programming codes would improve the performance of the program developed. Since Borland Delphi© is an object oriented language, it is more useful in terms of required memory. Problems that could not be solved by the software developed in Matlab© can now be solved in the Structural Modification Toolbox. Furthermore, although it was not possible to obtain stand-alone executable file in Matlab© because of some commands used, this was possible in Borland Delphi©. However, it was more difficult to program the algorithms in Borland Delphi© since all the variables are kept as complex valued because of damping. Although the memory required is less for Borland Delphi©, it is necessary to program each step of each equation separately. In Matlab©, it is possible to program an equation directly.

Structural Modification Toolbox uses the .f06 modal analysis result file of MSC.Nastran© solver and square modification matrices written in a text file. Details of Structural Modification Toolbox are given in Appendix A. Using ANSYS, instead of MSC.Nastran© solver and MSC.Patran© GUI, was also investigated. Element matrices have been extracted from ANSYS© and used as original structure data. However, because of high computational cost, MSC.Nastran© solver and MSC.Patran© GUI were preferred.

6.3 MODAL TESTS AND TEST RESULTS

The methods investigated are applied on real structures and the results are compared with experimental values. GARTEUR SM-AG19 structure which was designed by a multinational research group (Group for Aeronautical Research and Technology in Europe) and used in literature for modal testing [65, 66], was built for structural tests. Modal hammer and modal sledge hammer were used to excite the model.

Theoretical FRFs (FEM-Modified Structure) were in general in accordance with Experimental FRFs (Original Structure). Resonance frequency values of Theoretical

FRF (FEM-Modified Structure) graph and Experimental FRF (Original Structure) resonance values were quite accurate but this was not the case for the anti-resonance values. Structural Modification Toolbox solution (Theoretical FRF (Modified Structure)) was very successful in predicting the FRF of the modified structure around the first elastic mode. However, it was not so successful at the other modes. Although the exact methods are used, all the resonance frequencies calculated are not exactly the same as the experimental values. One reason for that is the FE model of the original structure. Because of the errors in the updated FE model, Theoretical FRF (Updated-Original Structure) was not exactly the same as the Experimental FRF (Original Structure) which affected the results for the modified structure. It is concluded that a more accurate FE model for the original structure would improve the performance of the modification methods used.

Although the resonance values are not exactly the same, the characteristics of the graphs are similar. Resonance frequencies at FRF curves are shifted to lower frequencies after mass modification, as expected.

There was a mismatch in most of the values in terms of the magnitude of FRF graphs at resonance levels. It is believed that this is because of the damping of the model which was not updated in the FE model.

The methods were also applied on a real aircraft. The FE model of the aircraft was built in MSC.Patran© GUI and the modal analysis was carried out in MSC.Nastran© solver. It is observed that first resonance frequency is accurately predicted in the study of the real aircraft. The other resonance frequencies could not be predicted because of the modeling, rotational degrees of freedom and inertia effects.

Damping was one of the reasons of discrepancy, as in GARTEUR SM-AG19 structure. It was assumed to be proportional and the loss factor was taken as 0.02 for calculation. Since damping of the original FE model is not updated, magnitudes of FRFs are not in good agreement at resonance levels.

It is concluded that, since the methods mentioned above are exact methods, the resultant FRF graphs will be quite accurate, if the original FE model and simulation of modifying structure accurately represent the test-bed and modifying structure, respectively. While modeling the modifying mass addition, modeling of joints and effects of joints should also be considered. From the comparison of experimental and theoretical values, it is concluded that original FE model should be accurate enough and modifying structure must be modeled accurately by including the joints in order to calculate reliable results for the modified structure.

6.4 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The main objective of this work was to develop a reliable software that can be used for structural modification analysis of large structures, compare the selected structural modification methods from accuracy and efficiency points of view, and finally verify the software. Although it was not one of the aims, a new structural modification method is also developed in this work. In the software developed, three methods are used. It is shown that they all give exact results, and therefore comparison can be made only from computational efficiency point of view. Özgüven’s Structural Modification Method is found to be the most efficient one.

The software is verified by using several case studies. Furthermore the application of the software is demonstrated by using two real structures; one lab size GARTEUR SM-AG19 structure and a real aircraft. The problems encountered during the application of structural modification software are discussed. It is concluded that modeling the structural modification that is obtaining the correct matrices for structural modification, has the utmost importance in such studies. As a future work, it is recommended to study the effect of modeling of the modifying structure on the accuracy of the results.

All the methods used in this study need system matrices for modifying structure. Investigating the methods that use modal data or experimental data as original structure data might be useful, and it can be recommended as a future work.

Extended Successive Matrix Inversion method, which is developed in this study, might be extended for additional degrees of freedom cases.

Structural Modification Toolbox is developed for no new degrees of freedom and undamped or structurally damped real structures. Extending the algorithms for additional degrees of freedom is also recommended. The codes developed could be modified to include the effects of the rotational degrees of freedom. Structural Modification Toolbox might be updated for non-proportionally damped structures. The FE models with damping properties updated, is recommended to be used for better results. Converting present algorithms so that sparse matrix properties are used will be beneficial. Writing a computer code that can import modification data directly from MSC.Nastran© or ANSYS© will enable the user to prepare the modifying structure in MSC.Nastran© or ANSYS© directly.

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APPENDIX A

USER MANUAL

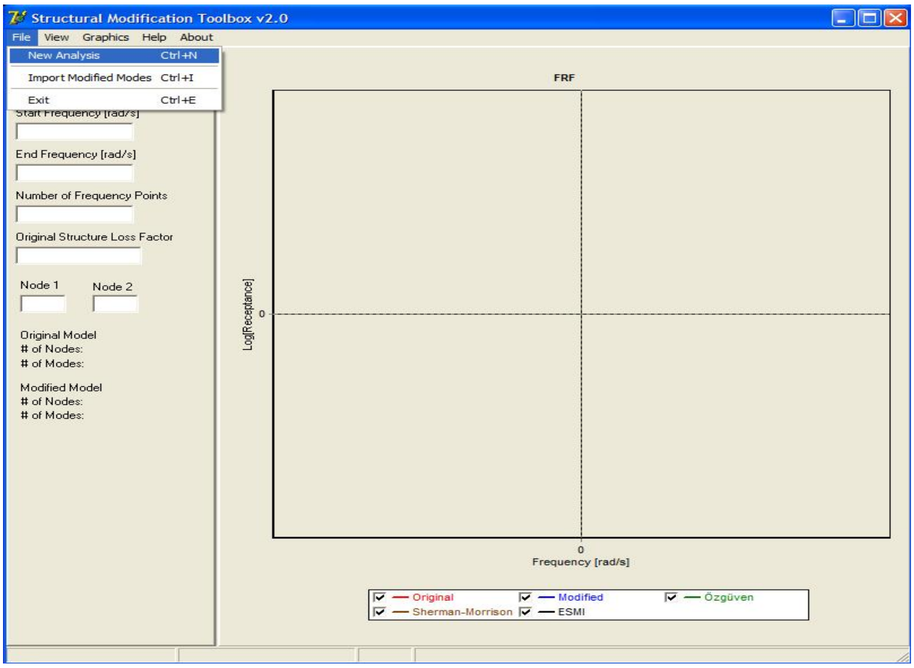
A.1. GRAPHICAL USER INTERFACE

Structural Modification Toolbox v2.0 is a software tool that uses analytical modal data to estimate how the system dynamic characteristics will change when the mass, damping and stiffness matrices of the system are altered. Note that only modal data (frequency, damping and mode shapes) and not frequency response functions are used for prediction.

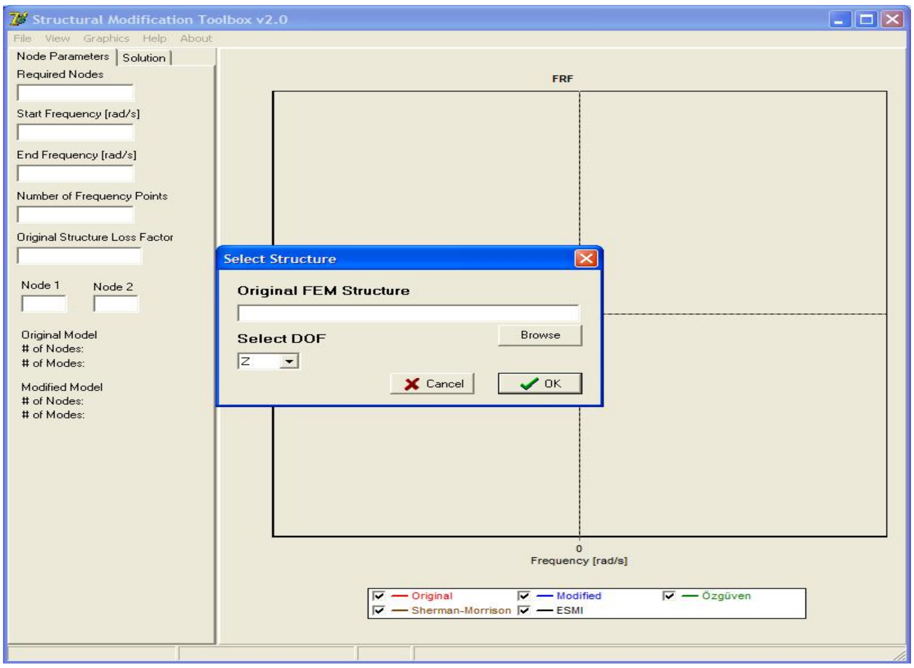
Structural Modification Toolbox v2.0 is designed only for the cases where there is no additional degrees of freedom. The program can analyze either undamped or structurally damped original structures.

In this section, the use of Structural Modification Toolbox v2.0 is explained with an application. For this purpose the plate of the case study given in Section 3.3.1 is used. The boundary conditions of the plate are taken as, free-free. In the model, there are 66 nodes with 6 DOF per node and 50 elements. Step-by-step usage procedure along with the corresponding interface windows are presented below:

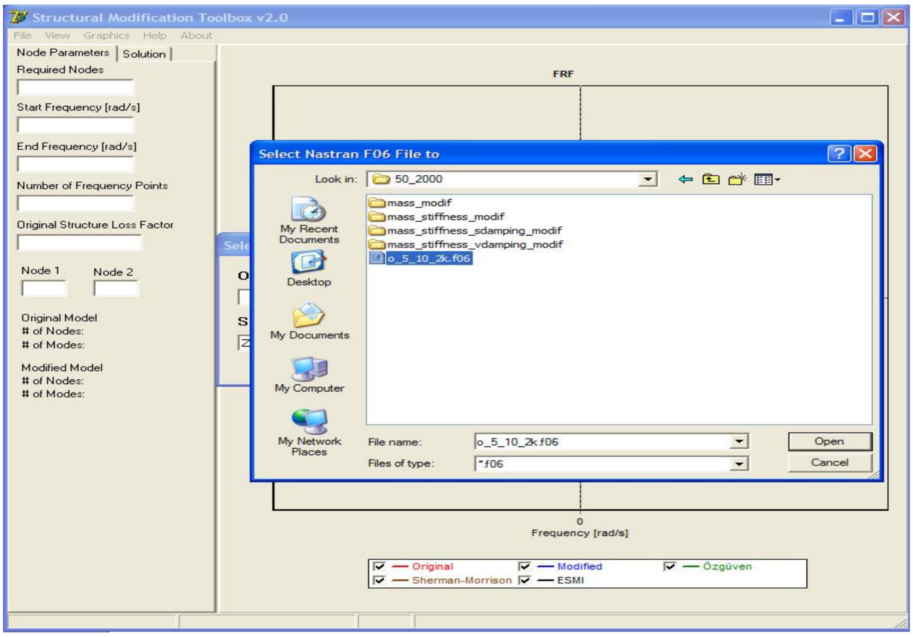
Open the browser in order to select the modal data input file for the original model ([Filename].f06).



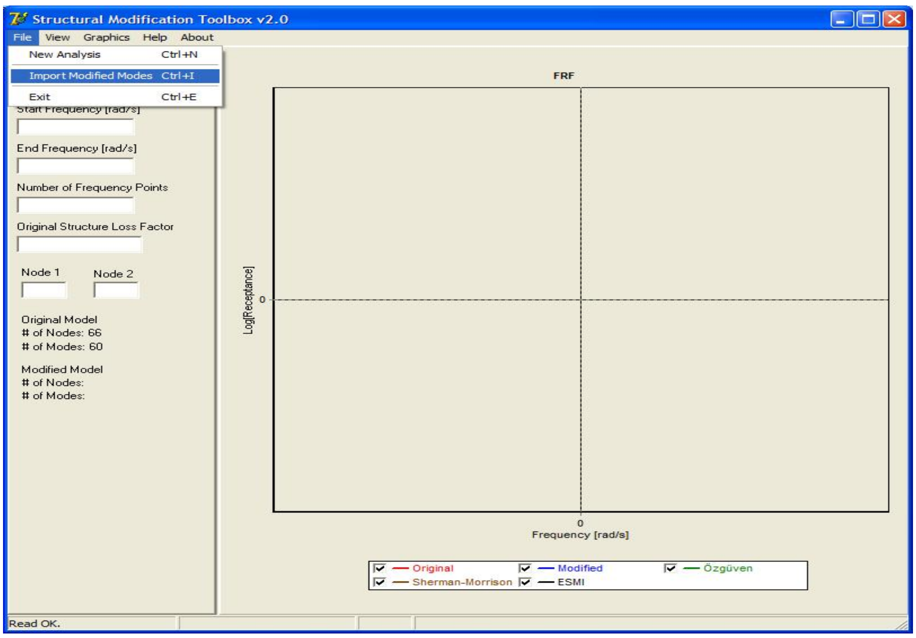
Browse the input file for the original model ([Filename].f06).



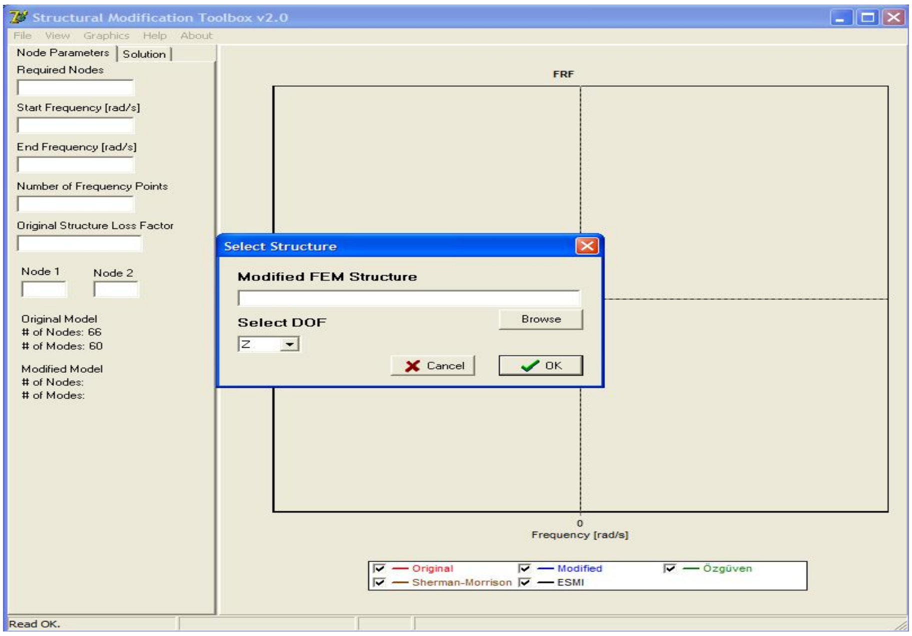
Select the input file for the original model ([Filename].f06).



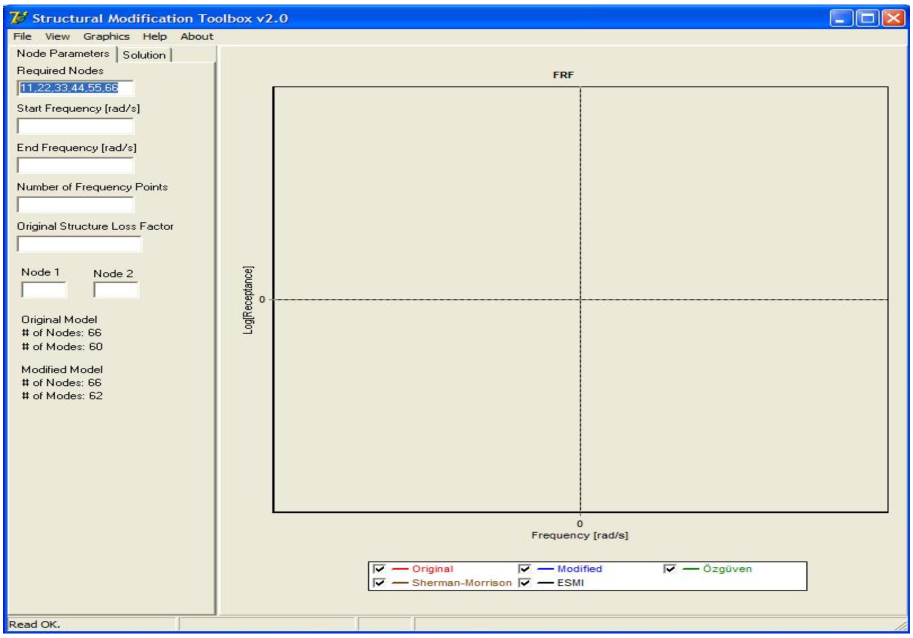
Open the browser to select the external modal data input file for the modified structure (optional-see Section 3.2) ([Filename].f06).



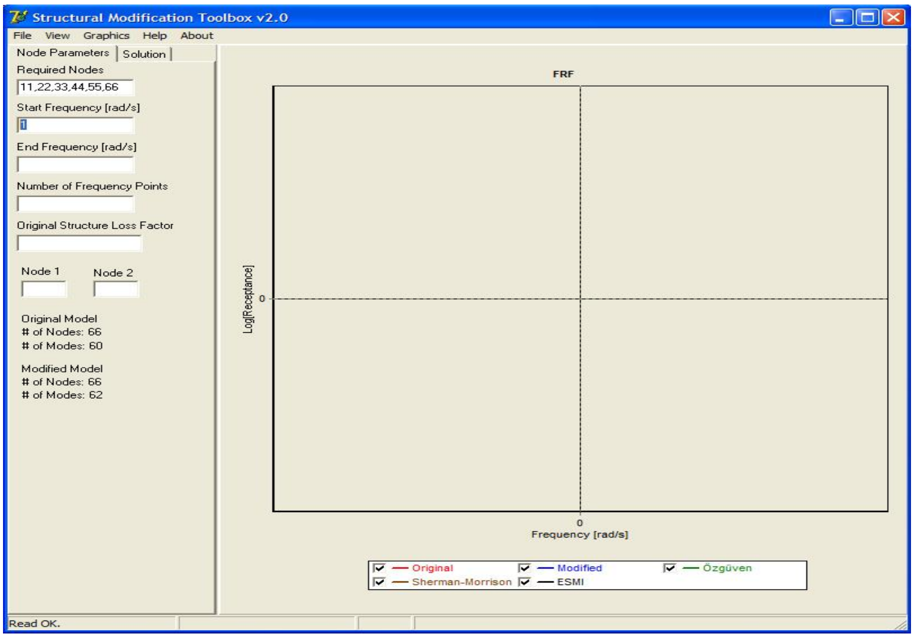
Browse the input file for the modified model ([Filename].f06) and select DOF.



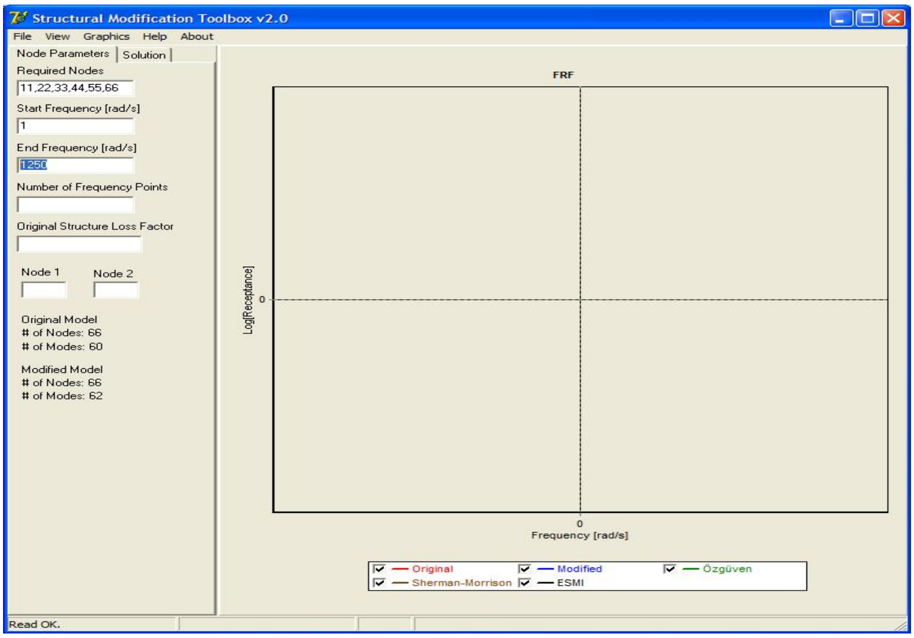
Enter the node numbers at which FRF's are required to be calculated.



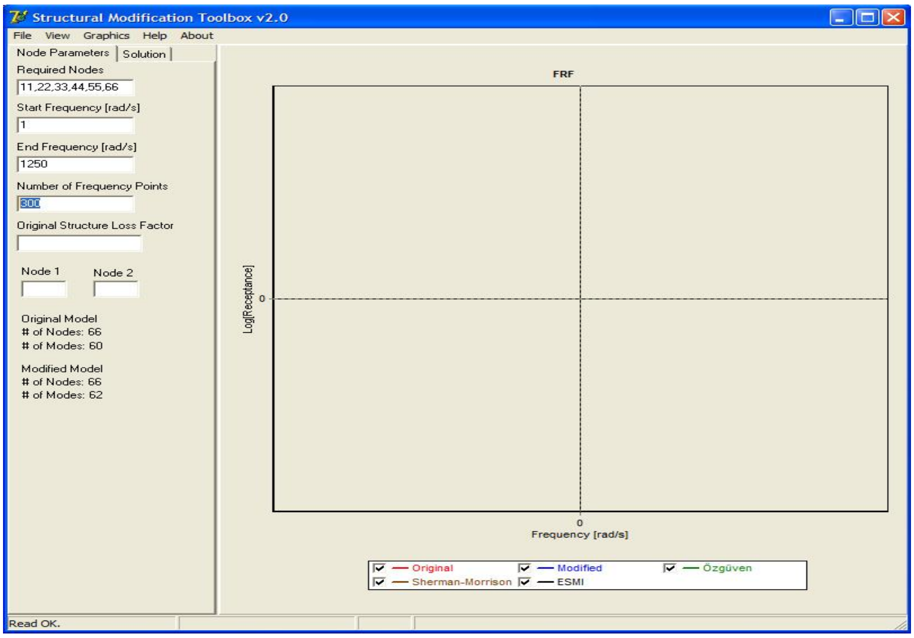
Enter starting frequency [rad/s].



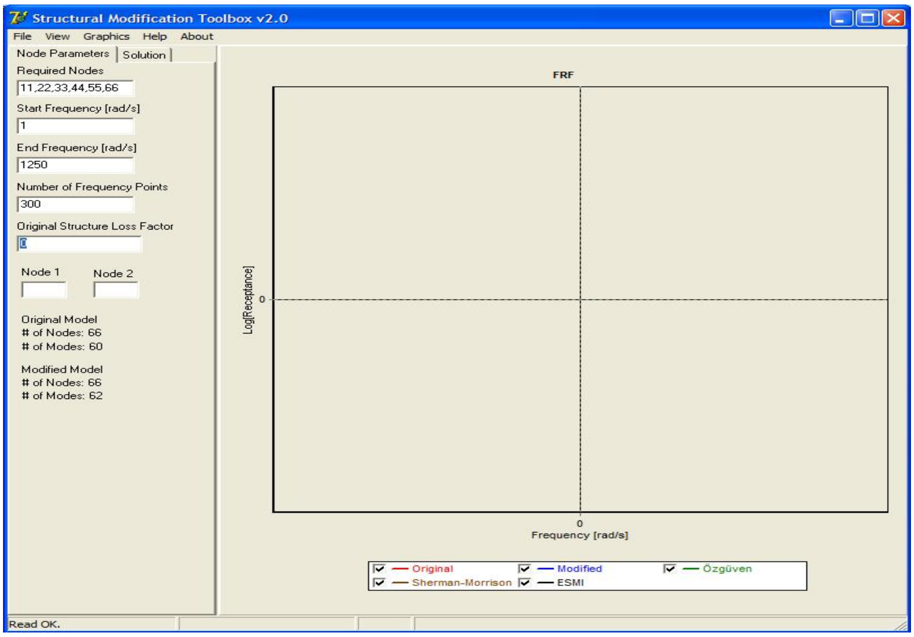
Enter ending frequency [rad/s].



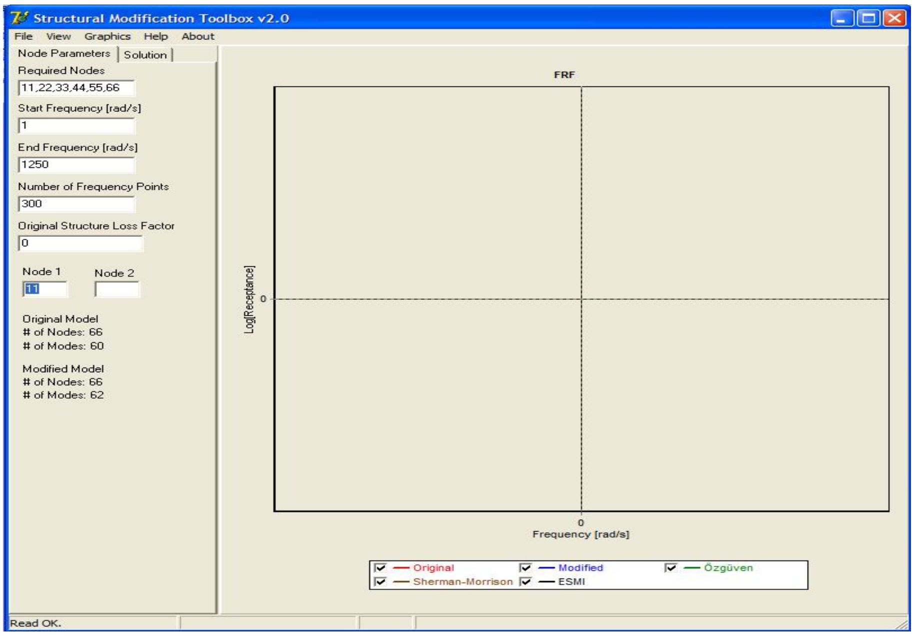
Enter the number of frequency points.



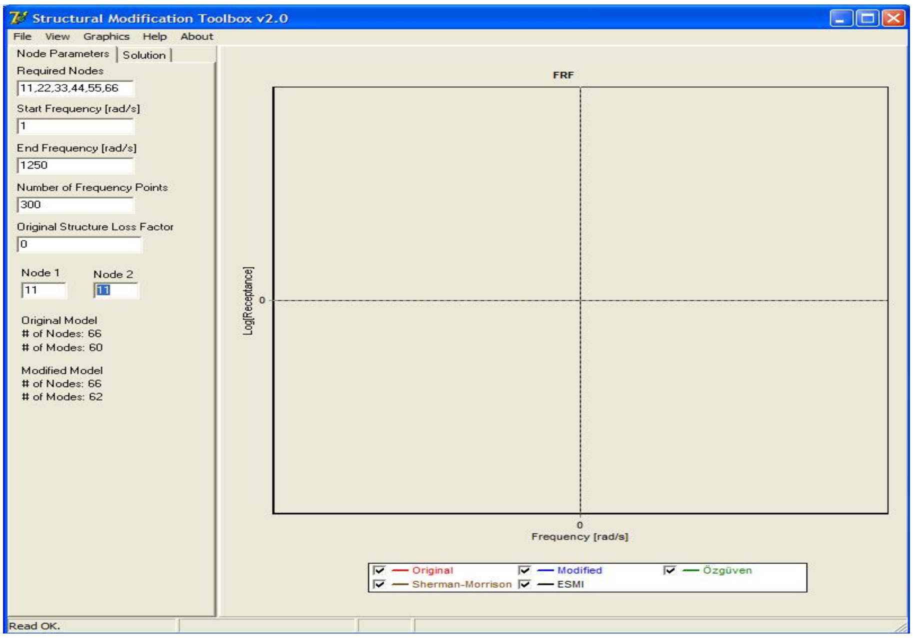
Enter the loss factor of the original structure.



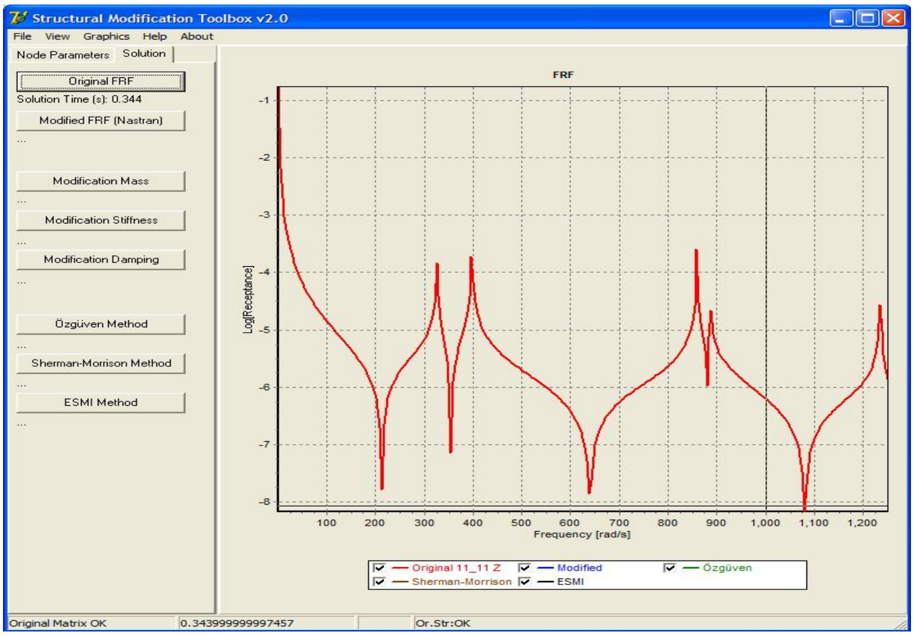
Enter the first index of FRF to be calculated.



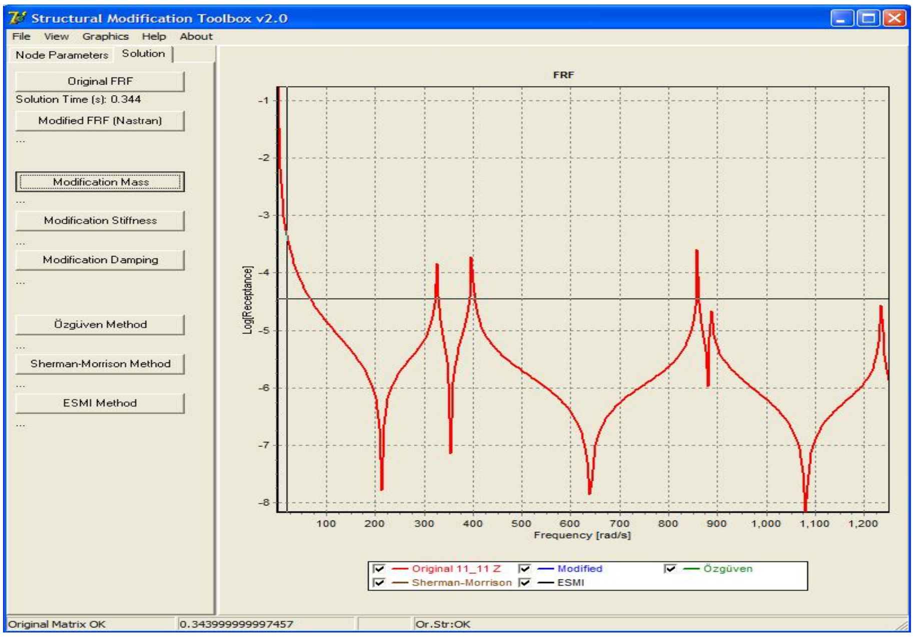
Enter the second index of FRF to be calculated.



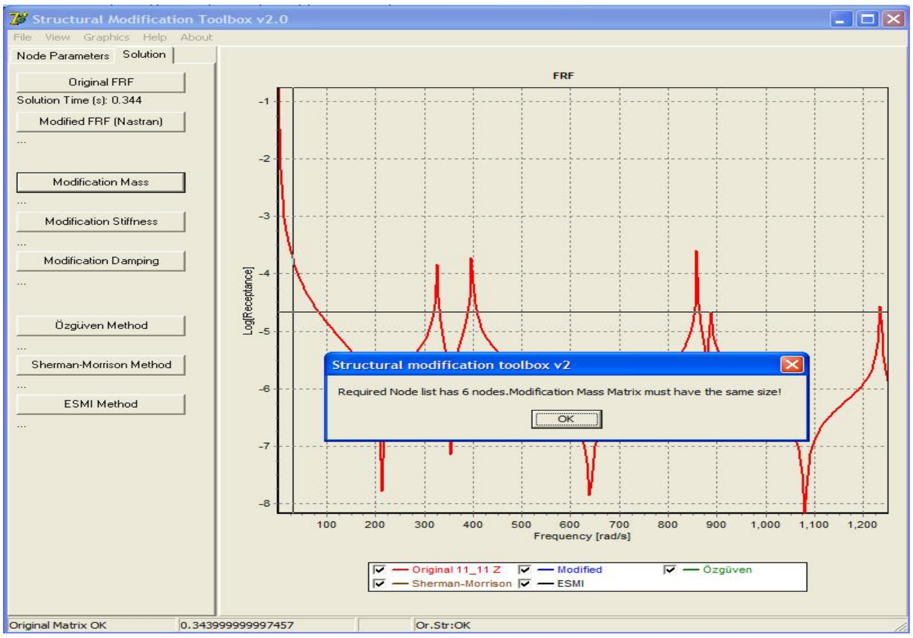
To start the solution, first switch to solution tab and then press “Original FRF” button to display the FRF of the original system.



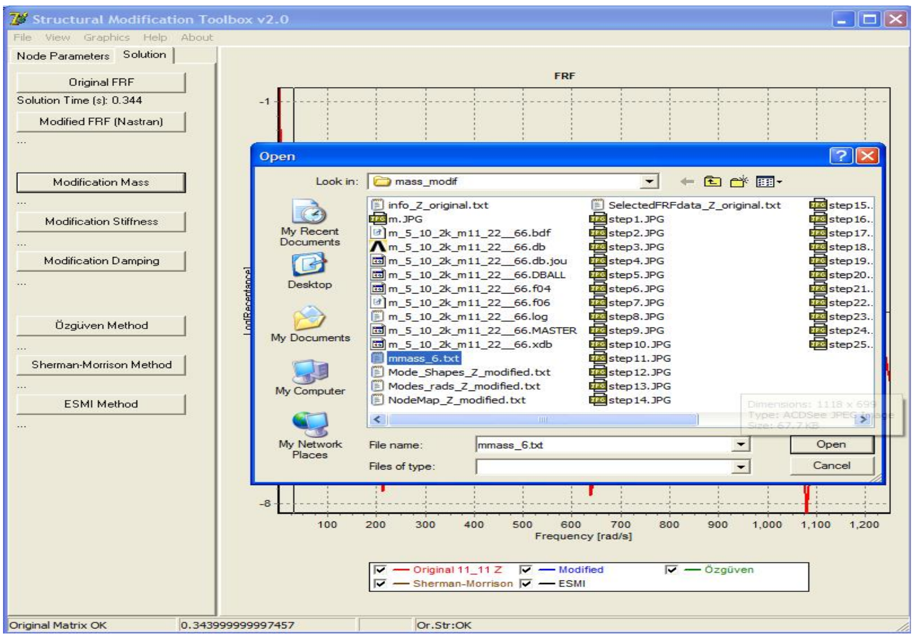
Input mass data for modification.



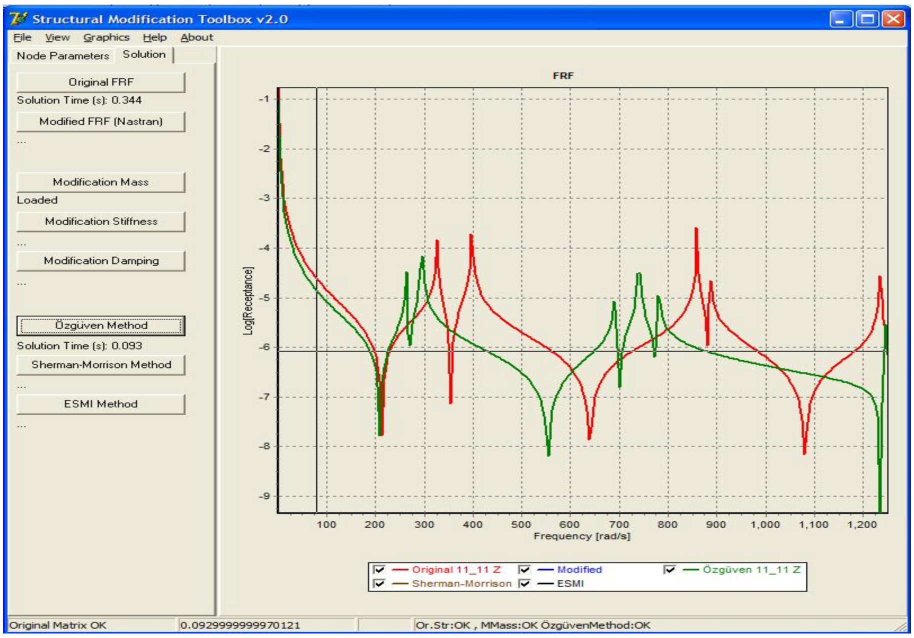
Be aware of the warning for modification mass.



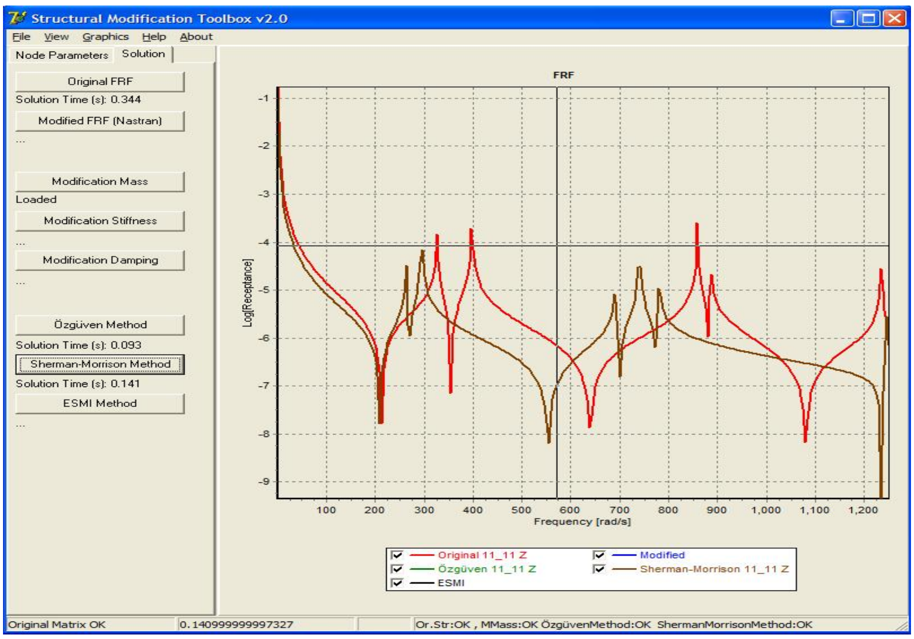
Select the file, which has the modification mass data.



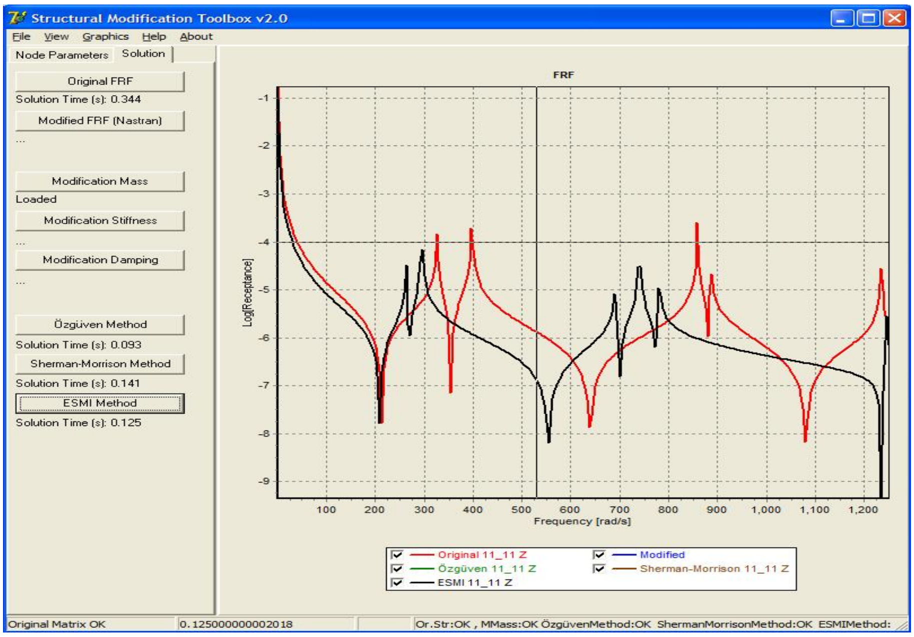
Press “**Özgüven’s Method**” button for using Özgüven’s Structural Modification Method to obtain and display the FRF of the modified system.



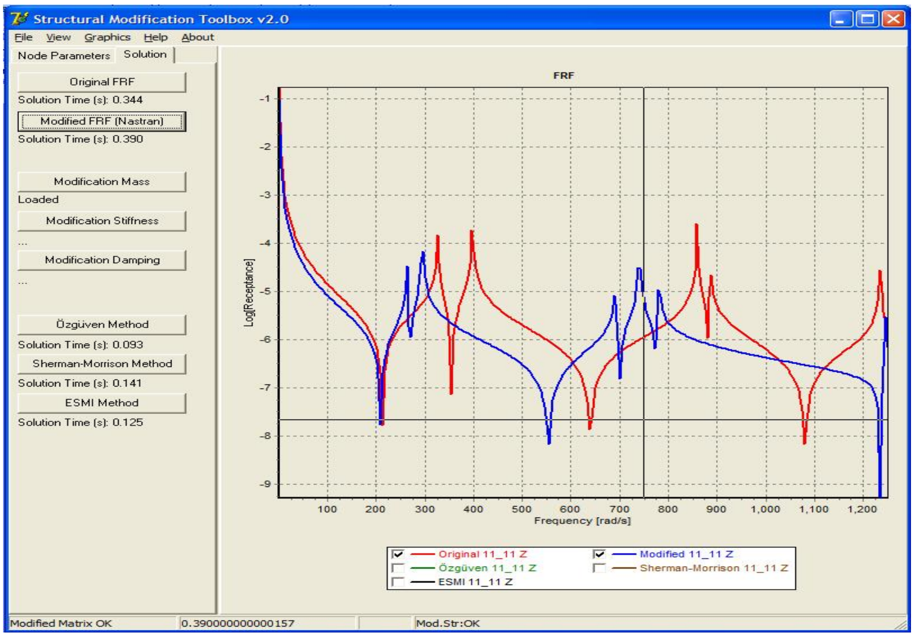
Press “**Sherman-Morrison Method**” button for using Sherman-Morrison Method to obtain and display the FRF of the modified system.



Press “**ESMI Method**” button for using Extended Successive Matrix Inversion Method to obtain and display the FRF of the modified system.

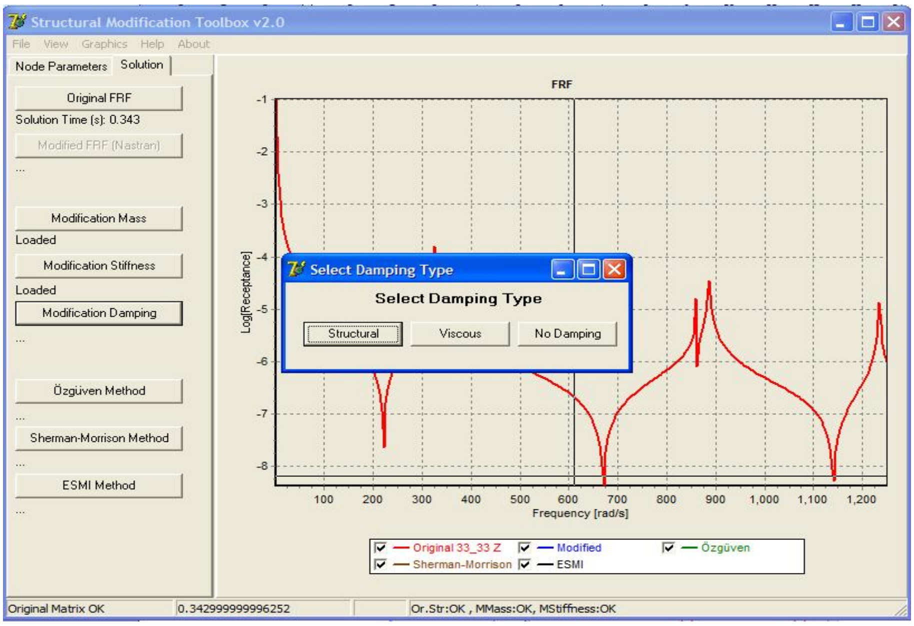


Press “**Modified FRF (Nastran)**” button to display the MSC.Nastran© solution results for the modified structure, if available.

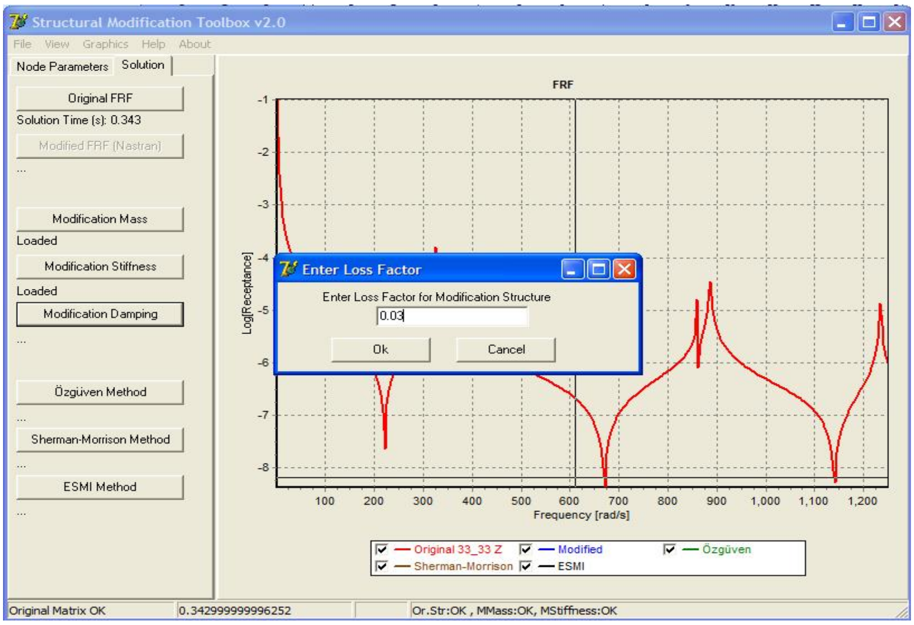


Structural Modification Toolbox v2.0 has additional user interfaces for damping. For structural damping, the damping data is entered as explained below:

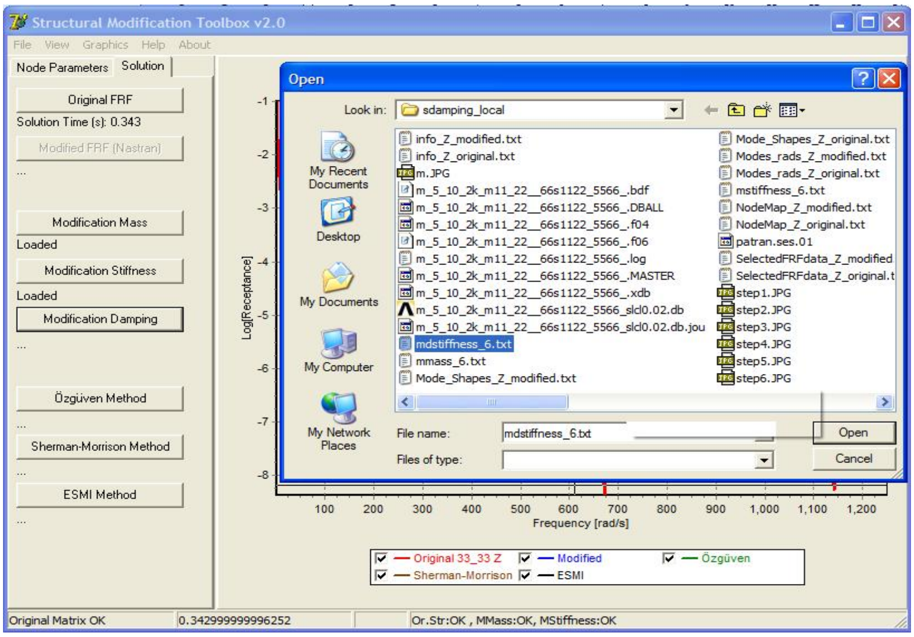
Select damping type.



Enter loss factor.



Select the file, which has the modification stiffness data.



For a viscously damped original structure, the user selects the file, which has the modification viscous damping data in the same way as selecting the file, which has the modification mass or stiffness data

The capabilities and properties of menu items of Structural Modification Toolbox v2.0 are explained in the following section.

A.2. MAINMENU

A.2.1 File

A.2.1.1 New Analysis (Ctrl+N)

The .f06 output file of MSC.Nastran© is loaded for the original structure from this menu. This file must be loaded for the program to calculate the original Frequency Response Function (FRF) and start a new analysis. For the structural dynamic modification analysis, the user must take care of the truncation effects. Generally, 15-20 times larger frequency intervals for the modification analysis than the frequency interval used in the modal analysis of the original structure in MSC.Nastran© is enough for an accurate analysis. It is necessary to analyze the original structure in MSC.Nastran© by defining the frequency range instead of number of modes required. To be able to define the required frequency range, from Analysis section of MSC.Nastran©, Solution Type is selected as Normal Modes. Secondly, from Subcases, Frequency Range of Interest is defined and normalization type of the result is selected as Mass Normalized.

It is also possible to use any external file in .f06 file format that includes modal data to use Structural Modification Toolbox. Hence, it is possible to make modification analysis with any modal data without MSC.Nastran©.

A.2.1.2 Import Modified Modes (Ctrl+I)

As an option, the modified structure can be analyzed as a whole using MSC.Nastran© in order to crosscheck the results of the modification methods. The .f06 file from MSC.Nastran© which includes modal data is loaded for the modified structure using this menu. This file must be loaded for the program to calculate the modified Frequency Response Function (FRF). This property can be used to check the methods. The preparation of .f06 result file for the modified structure is the same as the preparation of that for the original structure.

A.2.1.3 Exit (Ctrl+E)

This button enables the user to exit the program.

A.2.2 View

A.2.2.1 Parameter Panel (F2)

This button is used to change the visibility of the “**Parameter Panel**”. If the user wants to see the FRF graph only, it is possible to set the “**Parameter Panel**” invisible. This property can be used for better investigation of FRF graph or for printing a detailed picture.

A.2.2.2 Log Form (Ctrl+L)

All the actions taken after the program is compiled are written in a log file to see the details of the analysis. The user can change the visibility of the “**Log Form**” with this button and see the actions taken during the analysis.

A.2.3 Graphics

A.2.3.1 Print (Ctrl+P)

This button enables the user to print the FRF Graph. In this menu, the user can:

- select the printer either from “**Print Preview**” or from “**Setup**” menu

- select the size of the paper from “**Setup**” menu
- select the source of the paper from “**Setup**” menu
- change the orientation of the paper either from “**Print Preview**” or from “**Setup**” menu
- set or reset the margins of the paper from “**Print Preview**”
- change the visibility of the margins from “**Print Preview**”
- change the size of the legend and the titles of the graph for printing from “**Print Preview**”
- change the location of the graph on the paper from “**Print Preview**”
- print the FRF graph from “**Print Preview**”.

A.2.3.2 Save as JPEG (Ctrl+R)

This button is used if the user wants to copy the FRF graph in .jpg format. If the user wants to copy the FRF graph as bitmap or metafile, then “**Export**” button must be used.

A.2.3.3 Legend (Ctrl+W)

The user can show or hide the visibility of the “**Legend**” of the FRF graph with this button.

A.2.3.4 Export (Ctrl+T)

The user can “**Copy**”, “**Save**” and “**Send**” the picture with e-mail by using this button. For the picture, the user can set the format as:

- Bitmap
- Metafile
- JPEG

In Bitmap (.bmp) format, the user can change the color resolution of FRF graphs as 2 (1 bit), 16 (4 bit), 256 (8 bit), 32768 (15 bit), 65536 (16 bit), 16M (24 bit) and 16M (32 bit). It is also possible to select monochrome option. The size of the picture is also editable. This menu enables the user to keep or change the aspect ratio.

In Metafile (.emf) format, the user can select the file as enhanced or not. The size of the picture is also editable. This menu enables the user to keep or change the aspect ratio.

In JPEG (.jpg) format, the user can define the properties of the picture as "**Quality**" or "**Speed**". % "**Quality**" can be defined by the user. This interface also enables the user to select "**Gray Scale**" option. The size of the picture is also editable. This menu enables the user to keep or change the aspect ratio.

In "**Native**" tab, the user can "**Save**" and "**Send**" the picture with e-mail by using the button in that name. The user can select the "**Include Series Data**" option to use the series data in picture. The user can see the file size by selecting "**File Size**" option.

In "**Data**" tab, the user can "**Copy**", "**Save**" and "**Send**" the picture with e-mail by using the button in that name. The user can select the required series. It is also possible to select all data series simultaneously. Series format can be selected as:

- Text
- XML (Header and delimiter are not active)
- HTML Table (Delimiter is not active)
- Excel (Copy option and delimiter are not active)

The user can also include the point index, point labels and header in exported file to be able to use the data in another program. It is possible to change the delimiter as

- Space
- Tab

- Comma
- Colon
- Custom

In “**Custom**” mode, the user can use any character as delimiter.

A.2.4 Help

A.2.4.1 Help Document (F1)

This button opens the help document.

A.2.4.2 Help on Help (F4)

This button opens help about help.

A.2.4.3 Visual Tutorial

In this section, user can see animated video examples to use the program. There are totally 7 video files. 6 of them are about the usage of the program and the other one is for modification matrix preparation. The files are shown for mass modification demonstration. The steps are similar for other modification cases.

A.2.4.3.1 Load Original Data

In this part, the user can see the necessary steps to start an analysis. The user selects the original structure result file and degrees of freedom for the analysis. Number of nodes, number of modes and status for original structure can be seen from the graphical user interface. The user can replay the file from the hidden menu on the upper right corner.

A.2.4.3.2 Load Nastran Modified Data

In this part, the user can see the necessary steps to compare the results of MSC.Nastran© solution and Structural Modification Toolbox solution. The user selects .f06 solution file of Nastran for the modified structure and degrees of freedom for the analysis. Number of nodes, number of modes and status for modified structure can be seen from graphical user interface. The user can replay the file from the hidden menu on the upper right corner.

A.2.4.3.3 Set Node Parameters

In this part, the user can see the steps for the selection of required nodes, start frequency, end frequency, number of frequency points, loss factor for the original structure, and FRF indices from graphical user interface. The user can replay the file from the hidden menu on the upper right corner.

A.2.4.3.4 Solution

In this part, the user can see the steps for the solution procedure. The user can see the loading of modification files, solution time(s) of the method(s), information about the status and information about the modification files from the graphical user interface. The user can replay the file from the hidden menu on the upper right corner.

A.2.4.3.5 Graphical Properties

In this part, the user can see the properties of the graphs in Structural Modification Toolbox. The user can see values for any graph, zoom, unzoom, pan, change the visibility of the graphs and see legend information from graphical user interface. The user can replay the file from the hidden menu on the upper right corner.

A.2.4.3.6 Main Menu (View, Graphics)

In this part, the user can see the application of the properties of the view and graphics in Structural Modification Toolbox. The user can change the visibility of the parameter panel, log form and legend. The user can also see the usage of print, save picture and export functions. The user can replay the file from the hidden menu on the upper right corner.

A.2.4.3.7 Modification Matrix Preparation

In this part, the user can see the preparation of the modification matrices in Structural Modification Toolbox. The user must prepare modification files before the analysis in square matrices in .txt format. When FRF graph of an unmodified node is required; in the required nodes box, a row and a column of zeros must be added to the same row and column with the same sequence number in the modification matrix. The user can replay the file from the hidden menu on the upper right corner.

A.2.5 About

A.2.5.1 About (F3)

The information about the program can be seen by clicking this button.

A.3. NODE PARAMETERS

"**Number of Nodes**" and "**Number of Modes**" for both the original structure and the modified structure can be seen in this tab.

A.3.1 Required Nodes

The user sets the required nodes for the analysis. These are the modification nodes and the unmodified nodes at which FRF computation is required. In the selection of the node numbers, the user has to take care of the maximum number of nodes. The user should use the comma sign (,) for single entries such as 1, 2, 3. For an interval of nodes, a dash sign (-) should be used, such as 1-5. The sequence of the required nodes must be exactly the same as the sequence of nodes in the modification matrices. If FRF is required at an unmodified node, it is necessary to add zero rows and columns to the modification matrices corresponding to these nodes.

A.3.2 Start Frequency [rad/s]

The user sets the starting frequency for the analysis in rad/s.

A.3.3 End Frequency [rad/s]

The user sets the end frequency for the analysis in rad/s. The user should take care of the higher modes. It should be noted that, the frequency interval should be wide enough.

A.3.4 Number of Frequency Points

This parameter sets the frequency resolution for the analysis. A large number of frequency points values increases the solution time. When responses of original and modified structures will be compared, number of frequency points should be the same.

A.3.5 Original Structure Loss Factor

The program analyzes either undamped or structurally damped structures. Loss factor for the structural damping matrix is defined by the user with this button. The structural damping for the original structure can also be given in MSC.Nastran© modal analysis as material structural damping or local structural damping.

A.3.6 Node 1

This parameter sets the first index of the FRF matrix. This index is assigned to the response node for an FRF graph.

A.3.7 Node 2

This parameter sets the second index of the FRF matrix. This index is assigned to the excitation node for an FRF graph.

A.4. SOLUTION

None of the buttons will be active in the Solution Tab unless a solution file is loaded either from “**File**”→”**New Analysis**” or from “**File**”→”**Import Modified Modes**”.

A.4.1 Original FRF

This button is activated when a .f06 solution file from MSC.Nastran© modal analysis is loaded from “**File**”→”**New Analysis**” button and the Toolbox calculates the “**Original FRF**” by using the data in this file and the parameters defined in “**Parameters Panel**” Tab. The solution time in seconds is also shown after the analysis is finished and the original FRF graph is added to the chart.

A.4.2 Modified FRF (MSC.Nastran)

This button is activated when a .f06 solution file from MSC.Nastran© modal analysis is loaded from “**File**”→”**Import Modified Modes**” button and the Toolbox calculates the “**Modified FRF (Nastran)**” by using the data in this file and the parameters defined in “**Parameters Panel**” Tab. The solution time in seconds is also shown after the analysis is finished and the modified FRF graph is added to the chart.

A.4.3 Modification Mass

This button is activated when a .f06 solution file from MSC.Nastran© modal analysis is loaded from “**File**”→”**New Analysis**” button. The user is informed about the number of selected nodes when the button is pressed. The user loads the modification mass matrix by using this button. The size of the matrix is not controlled by the program. The loading status is shown after the file is loaded. Sequence of nodes in modification matrices must be exactly the same as the sequence of "**Required Nodes**" in the "**Parameters Panel**". If an unmodified node is required for investigation, it is necessary to add zero rows and columns to the related rows and columns of the modification matrices.

A.4.4 Modification Stiffness

This button is activated when .f06 solution file from MSC.Nastran© modal analysis is loaded from "**File**"→"**New Analysis**" button. The user is informed about the number of selected nodes when the button is pressed. The user loads the stiffness matrix of the modifying structure by using this button. The size of the matrix is not controlled by the program. The loading status is shown after the file is loaded. Sequence of nodes in modification matrices must be exactly the same as the sequence of "**Required Nodes**" in "**Parameters Panel**". If an unmodified node is required for investigation, it is necessary to add zero rows and columns to the related rows and columns of the modification matrices.

A.4.5 Modification Damping

This button is activated when .f06 solution file from MSC.Nastran© modal analysis is loaded from "**File**"→"**New Analysis**" button. The user should first select the damping type when the button is pressed. If structural damping is selected then the user should enter the loss factor for the modification damping matrix. If the user selects viscous damping then the user is informed about the number of the selected nodes and loads the modification damping matrix by using this button. The size of the matrix is not controlled by the program. The loading status is shown after the file is loaded. Sequence of nodes in modification matrices must be exactly the same as the sequence of "**Required Nodes**" in "**Parameters Panel**". If an unmodified node is required for investigation, it is necessary to add zero rows and columns to the related rows and columns of the modification matrices.

A.4.6 Özgüven Method

This button is activated for using the "**Özgüven Method**" for the solution. The solution time in seconds is also shown after the analysis is finished and the modified FRF graph is added to the chart. The indices of the FRF graph are added to the legend of FRF graph.

A.4.7 Sherman-Morrison Method

This button is activated for using the "**Sherman-Morrison Method**" for the solution. The solution time in seconds is also shown after the analysis is finished and the modified FRF graph is added to the chart. The indices of the FRF graph are added to the legend of FRF graph.

A.4.8 ESMI Method

This button is activated for using the Extended Successive Matrix Inversion Method "**(ESMI) Method**" for the solution. The solution time in seconds is also shown after the analysis is finished and the modified FRF graph is added to the chart. The indices of the FRF graph are added to the legend of FRF graph.

A.5. FRF GRAPHS

The user can make some changes in the FRF graphs after the analysis as follow:

- In the graph, the user can zoom in by using left button of the mouse and select the region from upper left side to lower right side. To zoom out, the user should keep the left button pressed and select any region from lower right side to upper left side.

- It is possible to pan the graph by keeping the right button pressed.
- Middle button of the mouse is used to hide the selected FRF(s) or all FRFs. This will also change the legend of the graph.
- The visibility of FRF graphs for different solutions can also be changed from the legend of the graph.
- The user can see the values on the graph by getting the cross on desired position of the selected graph.

APPENDIX B

Reanalysis of Dynamic Structures Using Successive Matrix Inversion Method

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ABSTRACT

Increasing demand for high precision mechanical components has increased the importance of dynamic design of structures. Competitiveness dictates the necessity to predict the dynamic properties of structures when some modifications are to be made. This paper presents a new structural reanalysis approach which is an extension of Successive Matrix Inversion method presented for static analysis to dynamic analysis of structures. The method is based on exact calculation of Frequency Response Function (FRF) matrix of a modified structure using FRF matrix of the original structure and modifying mass, stiffness and damping matrices. Case studies are presented in order to demonstrate the application of the method. The results obtained are compared with exact values, as well as with those obtained by using Özgüven's Structural Modification method.

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NOMENCLATURE

i	Unit imaginary number
N	Number of degrees of freedom
ω	Excitation frequency
η	Structural damping proportionality constant
r	Recursion factor
m	Number of non-zero columns
$[M]$	Mass matrix
$[K]$	Stiffness matrix
$[H]$	Structural damping matrix
$[C]$	Viscous damping matrix
$[\Delta M]$	Modification mass matrix
$[\Delta K]$	Modification stiffness matrix
$[\Delta H]$	Modification structural damping matrix
$[\Delta C]$	Modification viscous damping matrix
$\{x\}$	Displacement vector
$\{\ddot{x}\}$	Acceleration vector
$\{F\}$	Forcing vector
$[\alpha]$	Receptance matrix of original system
$[\gamma]$	Receptance matrix of modified system
$[I]$	Identity matrix

1. INTRODUCTION

Structural modifications are often required to enhance the dynamic behavior of an existing structure. It is almost impossible to design an optimum product that satisfies all the needs in the first trial, hence one of the basic practices of engineering is to build a work upon a previous one, thus saving time and effort. In such a design process, it may be necessary to study numerous alternative configurations in order to ensure a suitable dynamic response. In other instances, engineers often have difficulty when it is noticed that the dynamic properties of a structure do not meet the requirements of the design, and some modifications on the structures have to be made to obtain desired dynamic properties.

Structural modification is a general term used in structural dynamics that refers to either predicting the dynamic behavior of a modified structure (known as forward structural modification) or determining the modifications that should be made on an existing structure in order to obtain a required dynamic behavior (known as inverse structural modification). Forward structural modification, also known as reanalysis, is common in vibration optimization and in finite element model updating, whereas inverse methods are mostly used for eliminating vibration problems after a structure is constructed.

Several methods have been developed for both forward and inverse structural modification problems. The matrix inversion method proposed for finding receptances of locally damped structures from undamped counterparts [1] has later been combined with an efficient solution algorithm in order to avoid matrix inversion [2], and thus an efficient structural modification method has been developed for structural reanalysis problems [3]. The method is capable of finding frequency response functions (FRFs) of a modified structure from those of the original system. The method has been extended in the same work for structural modification cases with additional degrees of freedom (DOF). In a later work, the same approach has been followed and the matrix inversion formulation has been combined with Sherman-Morrison formula [4] and Woodbury formula [5] in order to avoid matrix inversion in structural modification problems when there are no additional degrees of freedom [6]. In both of the above methods [3, 6] the major part of the computations are limited to the coordinates where the modifications are made, which makes these methods favorable for systems with large DOFs and local modifications. The methods are very efficient as it is not required to invert a matrix or solve a new eigenvalue problem. Hager [7] also followed a similar approach, without the need for any matrix inversion or a solution of a new eigenvalue problem.
Many approaches have also been suggested for the solution of the inverse eigenvalue problem. Bucher and Braun [8] have developed a theory when a partial set of eigensolutions is available, where it is possible to find the necessary mass and stiffness modifications by using modal test results only. The work presents the general inverse problem with truncated information from measured FRFs. The solution is obtained by a linear combination of the original unmodified modal vectors. By this way, the effect of truncation is evaded. Park and Park [9] have developed a method that also uses test FRFs of the unmodified structure and enables one to analytically find necessary multiple mass, stiffness and damping modifications in order to obtain the required eigenvalues and eigenvectors. Akgün et al. [10] used the Sherman-Morrison-Woodbury formulas in the reanalysis problem for static systems. Huang and Verchery [11] presented a structural static reanalysis method that can be used in practical problems like progressive failure analysis. Kirsch et al. [12-14] presented a reanalysis approach based on Combined Approximations method for design, structural analysis and optimization. Chen, Yang and Lian [15] have compared several eigenvalue reanalysis methods for modified structures in terms of their computational efficiency and accuracy.

Bae, Grandhi and Canfield [16] have presented the so-called Successive Matrix Inversion (SMI) Method for Reanalysis. The method is given for static systems and it is an exact and a bound-free method (that is, there is no restriction in initial matrix size). In that work, comparisons of Successive Matrix Inversion Method with Cholesky decomposition, Gauss elimination and QR decomposition are also given. The method is used for local modifications such as replacement of aging aircraft parts and repair of battle damage. SMI is also used for reliability analysis for the uncertainties of material properties.

This paper presents a structural reanalysis approach which is an extension of the Successive Matrix Inversion method [16] presented for static analysis to dynamic analysis of structures. The method is based on exact calculation of FRFs of the modified structure by using those of the original structure, and the modifying mass, stiffness and damping matrices. The basic equation obtained in this method is the same as the one used in Özgüven's Structural Modification method [3]. However in the method presented here power series expansion is used in order to avoid matrix inversion, whereas a novel algorithm [2] is used in Özgüven's method to avoid matrix inversion, although matrix inversion is always an alternative for local modifications with small number of modified coordinates since the order of the matrix to be inverted is equal to the number of modified coordinates [3]. Case studies are presented in order to demonstrate the application of the method. Based on the numerical results, Successive Matrix Inversion method appears to be an efficient alternative method for reanalysis of dynamic structures subjected to local structural modifications.

2. SUCCESSIVE MATRIX INVERSION METHOD APPLIED TO DYNAMIC STRUCTURES

The equation of motion for an N degrees of freedom system can be written as

$$[M]\{\ddot{x}\}+i[H]\{x\}+[K]\{x\}=\{F\} \tag{1}$$

where $[M],[H],[K]$ are mass, structural damping and stiffness matrices of the system, respectively, $\{x\}$ is the displacement vector, $\{F\}$ is the forcing vector and i is the unit imaginary number. The dot shows differentiation with respect to time. The response of the system to a harmonic forcing at a frequency ω can be written as

$$\{x\}=\left[[K]-\omega^2[M]+i[H]\right]^{-1}\{F\} \tag{2}$$

from which the receptance matrix $[\alpha]$ can be obtained as

$$[\alpha]=\left[[K]-\omega^2[M]+i[H]\right]^{-1} \tag{3}$$

Now, if the structural modification matrices for mass, structural damping and stiffness are denoted by $[\Delta M]$, $[\Delta H]$, $[\Delta K]$, the equation of motion of the modified system will take the form

$$[M+\Delta M]\{\ddot{x}\}+i[H+\Delta H]\{x\}+[K+\Delta K]\{x\}=\{F\} \tag{4}$$

and the harmonic response of the system will be obtained as

$$\left[[K+\Delta K]-\omega^2[M+\Delta M]+i[H+\Delta H]\right]\{x\}=\{F\} \tag{5}$$

$$\{x\}=\left[[K+\Delta K]-\omega^2[M+\Delta M]+i[H+\Delta H]\right]^{-1}\{F\} \tag{6}$$

Then the receptance matrix of the modified system will be

$$[\gamma]=\left[[K+\Delta K]-\omega^2[M+\Delta M]+i[H+\Delta H]\right]^{-1} \tag{7}$$

$[\gamma]$ can also be obtained by premultiplying Eq. (5) by $[\alpha]$

$$\left([I]-[P]\right)\{x\}=\{F'\} \tag{8}$$

where

$$\{F'\} = [\alpha]\{F\} \tag{9}$$

$$[P] = -[\alpha]\big[[\Delta K] - \omega^2 [\Delta M] + i[\Delta H]\big] \tag{10}$$

Then from Eq. (8) and Eq.(9)

$$[\gamma] = \big([I] - [P]\big)^{-1} [\alpha] \tag{11}$$

Thus we obtain the basic equation of the structural modification method of Özgüven [3]. While the size of the matrix inverted is reduced for locally modified systems by partitioning the system matrices, and furthermore a novel algorithm is suggested to avoid matrix inversion in Özgüven's method, here in this work, power series expansion will be used for the inversion of matrix in Eq.(11) as successfully employed in Successive Matrix Inversion method [16]:

$$\big([I] - [P]\big)^{-1} = [I] + [P] + [P]^2 + [P]^3 + \dots \tag{12}$$

Let

$$[T] = [P] + [P]^2 + [P]^3 + \dots \tag{13}$$

The elements of $[T]$ can be written as

$$T_{ij} = P_{ij}^{(1)} + P_{ij}^{(2)} + \dots + P_{ij}^{(k)} + \dots \tag{14}$$

where $P_{ij}^{(k)}$ is the (i, j)th element of $[P]^{(k)}$. If the kth recursion factor is defined as

$$r_{ij}^{(k)} = P_{ij}^{(k+1)} / P_{ij}^{(k)} \tag{15}$$

and if the recursion factor is constant through the expansion, Eq. (14) can be expressed as

$$T_{ij} = P_{ij}\left(1 + r_{ij} + r_{ij}^{\ 2} + r_{ij}^{\ 3} + \dots\right) \tag{16}$$

Using the series expansion for the recursive terms, T_{ij} can be written as

$$T_{ij} = P_{ij} / \left(1 - r_{ij}\right) \tag{17}$$

Since the recursion factor is different through the series expansion, the modification matrix should be decomposed into separate matrices so that the variability of the recursion factor could be eliminated.

$$[\Delta K]-\omega^2[\Delta M]+i[\Delta H]=\sum_{j=1}^{N}\left[\left[\Delta K^{(j)}\right]-\omega^2\left[\Delta M^{(j)}\right]+i\left[\Delta H^{(j)}\right]\right]$$

(18)

where $\left[\Delta K^{(j)}\right]$, $\left[\Delta M^{(j)}\right]$, $\left[\Delta H^{(j)}\right]$ are matrices composed of the j^{th} columns of stiffness, mass and structural damping matrices, respectively, and zero columns except the j^{th} columns. For only one nonzero column of $[T]$, the recursion factor is a constant

$$r = P_{jj}$$

(19)

Then Eq. (17) becomes

$$T_{ij} = P_{ij}/(1-r)$$

(20)

Now let

$$[Y]=\left[[\Delta K]-\omega^2[\Delta M]+i[\Delta H]\right]$$

(21)

$$[Z]=\left[[K]-\omega^2[M]+i[H]\right]$$

(22)

Then, the dynamic stiffness matrix, after the j^{th} non-zero column of the structural modification is taken into consideration, can be written as

$$\left[Z^{(j)}\right]=\left[Z^{(j-1)}\right]+\left[Y^{(j)}\right]$$

(23)

where $\left[Z^{(j-1)}\right]$ and $\left[Z^{(j)}\right]$ are the modified dynamic stiffness matrices in $(j-1)^{th}$ and $(j)^{th}$ steps, respectively. $\left[Y^{(j)}\right]$ is a matrix which has the j^{th} non-zero column of $[Y]$ matrix at its corresponding column, and zero columns elsewhere. Note that $\left[Z^{(0)}\right]$ denotes the initial $[Z]$ matrix, and for the modification matrix $\left[Y^{(j)}\right]$, $\left[Z^{(j-1)}\right]^{-1}$ refers to $[\alpha]$ and $\left[Z^{(j)}\right]^{-1}$ refers to $[\gamma]$ in Eq.(11). By using Eq. (12) and Eq.(13), the inverse of Eq. (23) can be obtained as:

$$\left[Z^{(j)}\right]^{-1}=\left([I]+[T]\right)\left[Z^{(j-1)}\right]^{-1}$$

(24)

Thus, by updating these matrices for each nonzero column of the modification matrix, the modified FRF matrix can be obtained. Since each column of the modification matrix contributes to the dynamics of the system independently, the sequence of the columns used in the computation is not important. It should also be noted at this stage that for a local modification, $[Y]$ will be a highly sparse matrix with many zero columns and rows corresponding to the coordinates at which there is no structural modification, and that property of $[Y]$ will make the method presented here attractive.

The algorithm of the reanalysis method suggested for structural modifications using SMI can be summarized as follows:

Step 1:

Obtain the original FRF matrix and the modification matrix $[Y]$.

Step 2:

Partition the modification matrix to number of non-zero element columns (*m*) such that

$$[Y]=\left[Y^{(1)}\right]+\left[Y^{(2)}\right]+\ldots+\left[Y^{(m)}\right] \tag{25}$$

Step 3:

For each non-zero column of $[Y]$ (j=1,2,…m), the following computations are repeated:

$$\left[P^{(j)}\right]=-\left[Z^{(j-1)}\right]^{-1}\left[Y^{(j)}\right] \tag{26}$$

$$\left[Z^{(j)}\right]^{-1}=\left[Z^{(j-1)}\right]^{-1}+\left(\frac{1}{1-r^{(j)}}\right)\left[P^{(j)}\right]\left[Z^{(j-1)}\right]^{-1} \tag{27}$$

where $\left[P^{(j)}\right]$ will have all columns zero except the j-th column, since $\left[Y^{(j)}\right]$ is in the same form.

Then in the last step $\left[Z^{(m)}\right]^{-1}$ will give the receptance matrix of the modified structure. The algorithm of Successive Matrix Inversion method uses only matrix additions and matrix-vector multiplications to update the response matrix. In Eq. (26), $\left[Y^{(j)}\right]$ and therefore $\left[P^{(j)}\right]$ have many zero elements so that using sparse matrix properties in the storage and matrix operations in the algorithm summarized above will reduce the computational effort considerably.

If we have viscous damping $[C]$, instead of structural damping, then obviously $[H]$ and $[\Delta H]$ will be replaced by $\omega[C]$ and $\omega[\Delta C]$ respectively in all equations given above.

3. CASE STUDIES

In order to demonstrate the application of the structural modification method proposed in this study, two systems are analyzed and the results obtained are compared with exact values. The method is first applied to a discrete system for which the original system mass and stiffness matrices are given as follows:

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix} \text{[kg]} \qquad (28)$$

$$K = \begin{bmatrix} 5000 & -1000 & 0 & 0 & 0 & 0 \\ -1000 & 12500 & -5500 & 0 & -6000 & 0 \\ 0 & -5500 & 5500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 14000 & -6000 & 0 \\ 0 & -6000 & 0 & -6000 & 19000 & -4000 \\ 0 & 0 & 0 & 0 & -4000 & 4000 \end{bmatrix} \text{[N/m]} \qquad (29)$$

The damping is taken as structural (hysteretic) damping with a loss factor of η **=** 0.05.

The system is modified by placing additional masses (M_4 and M_5) at coordinates 4 and 5, respectively, and inserting additional stiffnesses (k_2 and $k_{3\text{-}4}$) between coordinate 2 and the ground, and between coordinates 3 and 4, respectively. The values of the modifying elements are given in Table 1.

Table 7: Modifying System Parameters

M_4 (kg)	7
M_5 (kg)	5
k_2 (kN/m)	4
$k_{3\text{-}4}$ (kN/m)	3.5

The receptance matrix of the modified system is calculated by using the Successive Matrix Inversion method generalized for dynamic systems in this work, as well as by using Özgüven's Structural Modification method. When the frequency response functions of the modified system are also calculated by analyzing the whole system itself it is observed that the three methods give exactly the same results. The point receptances of the modified system at coordinates 5 and 6 are compared with those of the original system in Figures 1 and 2, respectively.

As a second example, a beam with both ends fixed is considered. The beam is modeled using six finite elements and 10 degrees of freedom. The modulus of elasticity (E) of the beam is taken as 200 GPa and the mass density (ρ) is taken as 8x10^3 kg/m^3. The length of the beam is 1200 mm. The original beam has a wall thickness of 3 mm. The beam is modified by reducing the wall thickness to 2 mm and by adding a 50 kg mass to the midpoint of the beam. The loss factor is again taken as 0.05, as in the first example.

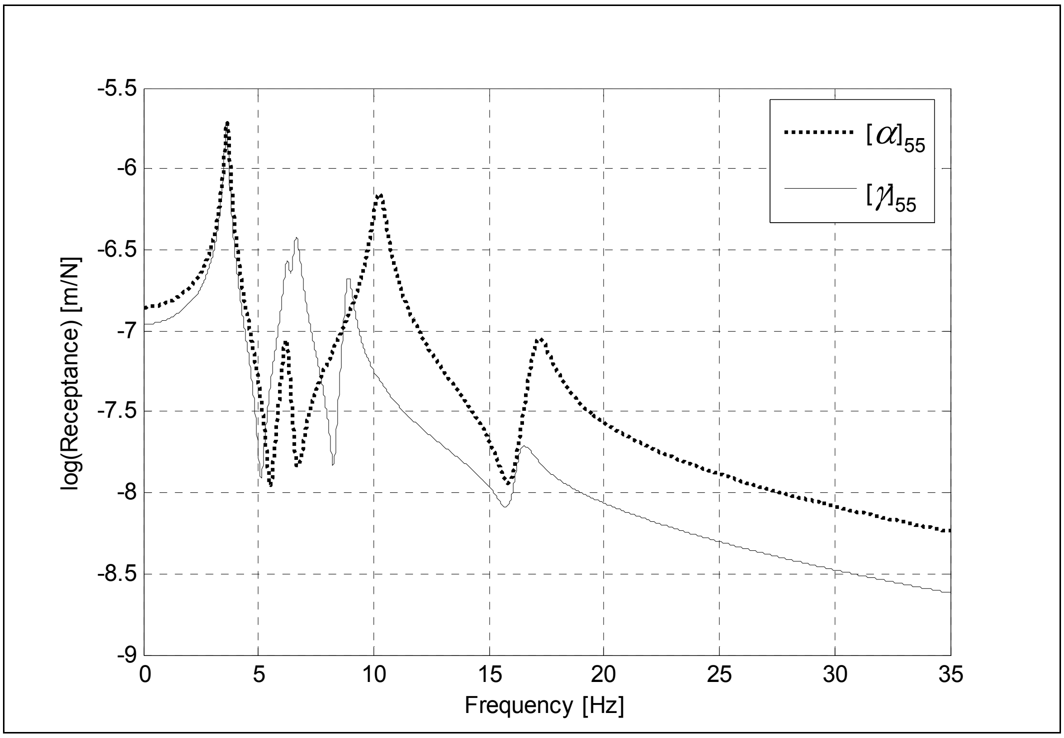

Figure 23: Point receptance at coordinate 5 for original and modified systems–Case Study 1

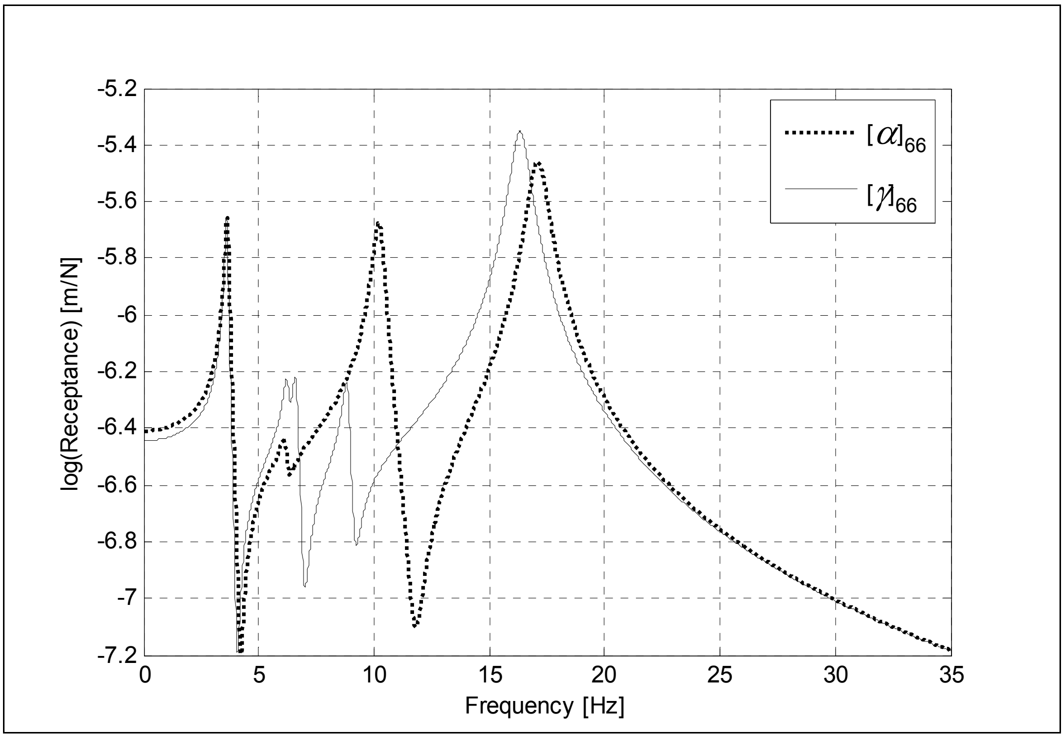

Figure 24: Point receptance at coordinate 6 for original and modified systems–Case Study 1

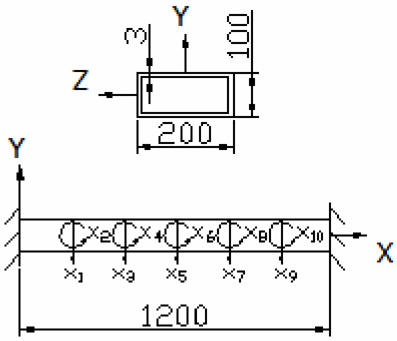

Figure 25: Case Study 2: Original beam

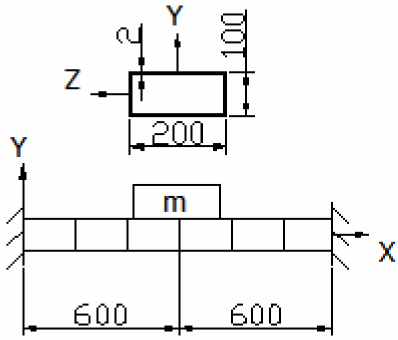

Figure 26: Case Study 2: Modified beam

The receptance matrix of the modified beam is calculated using the SMI method for dynamic systems, Özgüven’s Structural Modification method and straight computation of the modified system, all of which again yield exactly the same results. Two modified point receptances are compared with those of the unmodified system in Figure 5 and Figure 6.

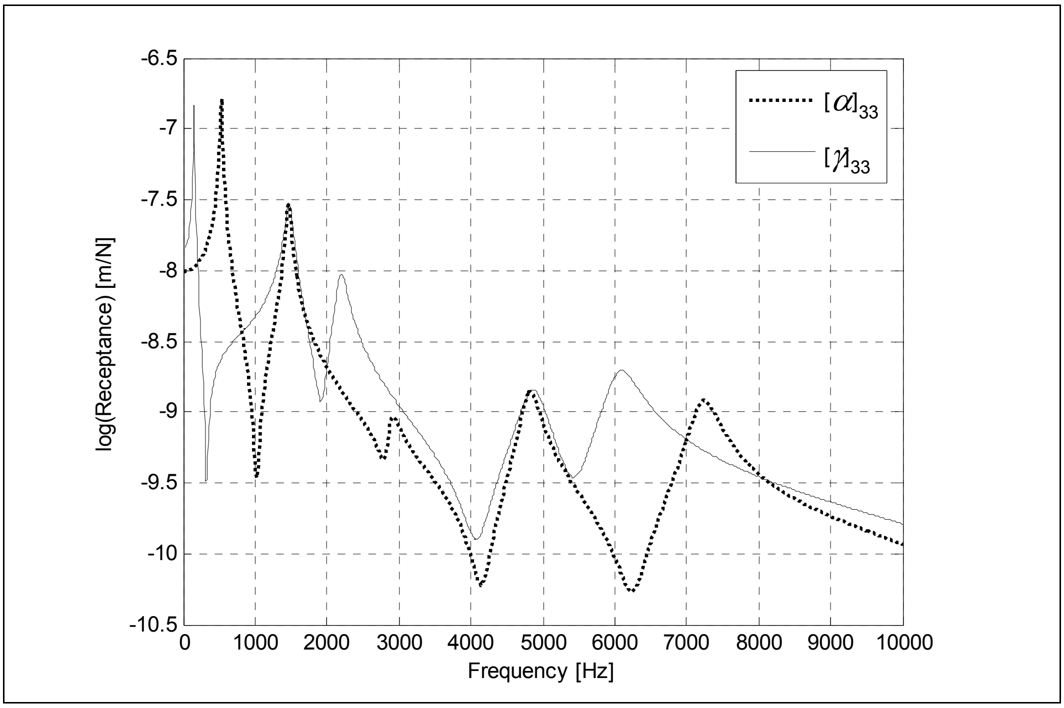

Figure 27: Point receptance at coordinate 3 for original and modified systems–Case Study 2

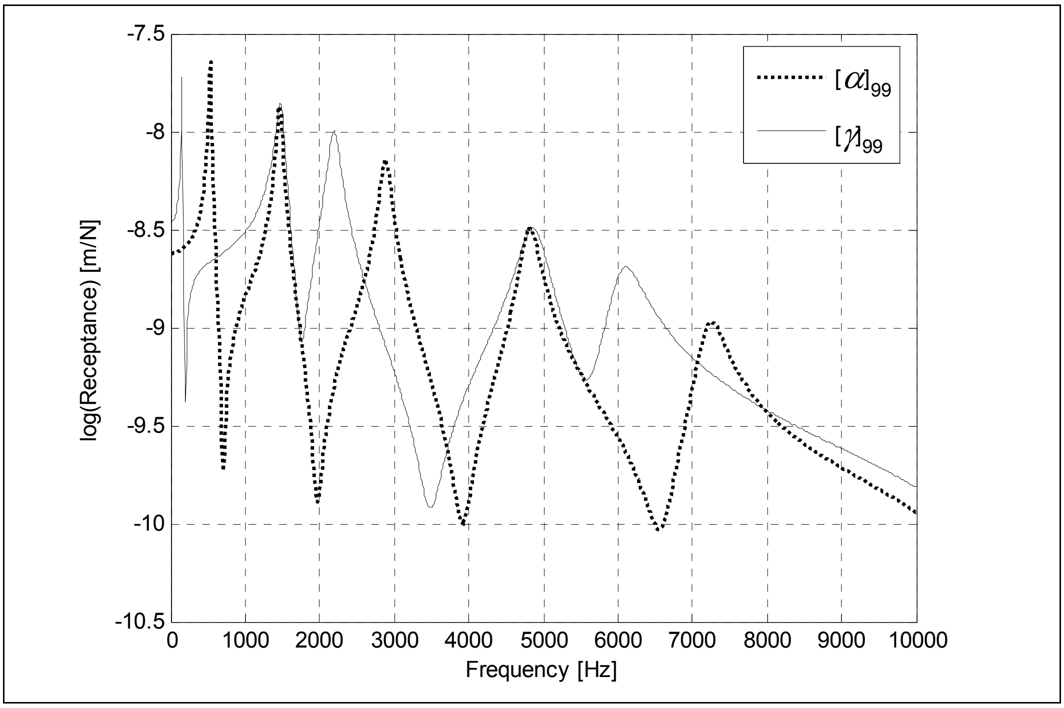

Figure 28: Point receptance at coordinate 9 for original and modified systems–Case Study 2

As demonstrated by the results of these case studies, Successive Matrix Inversion Method can successfully be applied for the analysis of dynamic structures subjected to structural modifications. The basic equation for the modified receptance matrix is the same both in Özgüven's Structural Modification Method and in the SMI Method presented here for dynamic systems. Owing to the exactness of both methods, the same results are obtained as can be expected. In SMI method, matrix inversion is replaced by a power series representation, thereby avoiding any possible numerical problems that may be encountered during matrix inversion. At this stage it is observed that using power series expansion to avoid matrix inversion in the method suggested here does not introduce any numerical inaccuracy. Furthermore it is observed that the method is very efficient for local modifications, where the modification matrix size is much smaller than the total size of the original system. The comparison of this method with other exact reanalysis methods from the computational effort point of view will be given in a following study.

4. CONCLUSION

The need for exact reanalysis techniques for dynamic structures, especially in the design stage has become crucial for today's complex structures. In spite of the tremendous increase in the availability and the sheer power of computational resources, methods with less computational burden are still desirable. In the present paper, an exact structural reanalysis approach called the Successive Matrix Inversion method originally presented for static analysis has been extended for application to dynamic structures. It has been shown that the Successive Matrix Inversion method can also successfully be used for dynamic analysis of structures. The method is based on exact calculation of Frequency Response Functions (FRF) of the modified structure using FRFs of the original structure along with the modifying mass, stiffness and damping matrices. The results of the method have been compared with the ones obtained by using Özgüven's Structural Modification method which is also an exact reanalysis method; as well as with those obtained by direct analysis of the modified system. It is observed that results of all three methods yield the same results indicating that the different algorithms used in the reanalysis methods do not introduce any numerical inaccuracy. Although the formulation using Successive Matrix Inversion method is presented only for structural damping, it is also readily applicable for structures with viscous damping. The comparisons of computational time requirements of these two methods as well as of other exact reanalysis methods for dynamic structures are planned to be presented in a following study.

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