SCALAR MESONS IN RADIATIVE PHI-MESON DECAYS INTO NEUTRAL K-MESON STATES

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ABSTRACT

SCALAR MESONS IN RADIATIVE PHI-MESON DECAYS INTO NEUTRAL K-MESON STATES

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Radiative decays of the ϕ meson to the scalar mesons $f_0(980)$ and $a_0(980)$ are investigated within the framework of K^+K^- loop model for both point-like scalar mesons and for scalar mesons with extended structure. Then, the radiative decay $\phi \to K^0\overline{K}^0\gamma$ is studied via a two step mechanism in which the scalar mesons couple the final state to the ϕ meson through the K^+K^- loop. The branching ratio of this decay is calculated and it is shown that this reaction will not provide a significant background to the measurements of $\phi \to K^0\overline{K}^0$ decay for testing CP violation.

Keywords: Radiative decays, ϕ meson, Scalar mesons

ÖZ

PHI-MEZONUN NÖTR K-MEZON DURUMLARINA IŞINSAL BOZUNMALARINDA SKALER MEZONLAR

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ϕ mezonun $f_0(980)$ ve $a_0(980)$ skaler mezonlarına ışınsal bozunmaları, noktasal skaler mezonlar ve bir yapıya sahip skaler mezonlar için K^+K^- modeli çerçevesi içinde incelendi. Ayrıca $\phi \to K^0\overline{K}^0\gamma$ ışınsal bozunumu, skaler mezonların K^+K^- döngüsü aracılığı ile ϕ mezonun son durumuna bağlandığı iki aşamalı bir mekanizmayla çalışıldı. Bu ışınsal bozunumun dallanma oranı hesaplandı ve bu reaksiyonun CP bozulmasını test etmek için yapılan $\phi \to K^0\overline{K}^0\gamma$ bozunumunun ölçümleri için önemli bir sorun yaratmayacağı gösterildi.

Anahtar Kelimeler: Işınsal bozunmalar, ϕ mezonu, skaler mezonlar

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CHAPTER 1

INTRODUCTION

The nature of light scalar mesons with the quantum numbers $J^{PC} = 0^{++}$ in the mass region up to 1 GeV has been an unsolved problem of hadronic QCD. Besides the two well established isoscalar resonance $f_0(980)$ and isovector resonance $a_0(980)$ [1], these mesons include the controvertial isoscalar $\sigma(600)$ [2] , and recently discovered isodoublet $\kappa(800)$ [3] resonances. The question of the nature of these light scalar mesons is important for the understanding of the mechanism of the chiral symmetry realization arising from confinement.

In order to explain the properties of this nonet of light scalar mesons several proposals have been put forward over the years. Although the naive quark model $q\bar{q}$ has been very successful in predicting and correlating the masses and the properties of pseudoscalar mesons, the mass predictions in the scalar sector vary considerably [4]. The predictions for masses strongly depend on the choice for the Dirac structure of the confining potentials and in particular the models cannot explain the reason for the $f_0 - a_0$ mass degeneracy. Furthermore, it has been suggested based on the relativistic models that glueball mixing with the calculated states may be needed to reproduce the experimental data [5].

A four quark $q^2\bar{q}^2$ state interpretation for the structure of light scalar mesons was also proposed in the framework of MIT-bag model in which case the scalar meson states are spatially compact [6]. It was observed that the four quarks in the confining potential are mainly arranged as two color singlets at a distance of the order of 1-1.5 fm and thus this way the notion of a mesonic molecule was introduced. However, in $q^2\bar{q}^2$ interpretation the number of expected four-quark states and the interpretation of these states vary for different calculations [7].

Another possibility was suggested where the light scalar meson states are considered to be bound states of hadrons and they are referred to be mesonic molecules [8]. In particular, the proposal was put forward for the structure of a_0 and f_0 scalar mesons as their being $K\bar{K}$ molecules in which case they are spatially extended objects [9]. It should be noted that although the quark content of the states in $q^2\bar{q}^2$ interpretation and in mesonic molecule interpretation are identical, their dynamical structure are radically different. The essential difference is that in $q^2\bar{q}^2$ case the multiquark system is confined within a scalar meson state with radius of the order of Λ_{QCD}^{-1} forming a compact system whereas in mesonic molecule case the two pseudoscalar mesons are spread over a region with radius of the order of $\sqrt{\mu E}$, where μ is the reduced mass of the system and E is the interhadron binding energy, which is significantly greater than Λ_{QCD}^{-1} thus forming a spatially extended system. The resulting branching ratio for the scalar meson considered is therefore different in $q^2\bar{q}^2$ and mesonic molecule interpretations. Furthermore, some analyses suggest the qualitative picture that these scalar meson states have a compact $q^2\bar{q}^2$ structure that spends some part of its lifetime in a mesonic bound state system [10].

The production of scalar mesons in radiative decays of vector meson decays is a valuable source of information on hadron spectroscopy. In particular, the decay reactions $\phi(1020) \to f_0(980)\gamma$ and $\phi(1020) \to a_0(980)\gamma$ have recently been observed and they are studied with good accuracy [11]. The SND collaboration reports the branching ratios $BR(\phi \to f_0\gamma) = (3.5 \pm 0.3) \times 10^{-4}$ and $BR(\phi \to a_0\gamma) = (0.88 \pm 0.17) \times 10^{-4}$, respectively. The CMD-2 collaboration, on the other hand, reports $BR(\phi \to f_0\gamma) = (2.90 \pm 0.21) \times 10^{-4}$. Since this value arises from a combined fit to $\phi \to \pi^0\pi^0\gamma$ and $\phi \to \pi^+\pi^-\gamma$ data, we take this value for $\phi \to f_0\gamma$ decay and considering the value for $\phi \to a_0\gamma$ reported by SND collaboration , we obtain the result $BR(\phi \to f_0\gamma)/BR(\phi \to a_0\gamma) = 3.3 \pm 2.0$. Furthermore, the KLOE collaboration reports the results $BR(\phi \to f_0\gamma) = (4.47 \pm 0.21) \times 10^{-4}$ and $BR(\phi \to a_0\gamma) = (0.74 \pm 0.07) \times 10^{-4}$. However, it should be noted that their value for $\phi \to f_0\gamma$ decay is obtained by including a very specific destructive interference in the low energy region.

The simplest mechanism for these radiative decays assumes that f_0 and a_0 are $q\bar{q}$ states with ${}^3P_0$ configuration, and the decays proceed through a quark loop. However, since the ϕ meson is mostly an $s\bar{s}$ state, this mechanism cannot be responsible for the $\phi \to a_0\gamma$ decay because in the $q\bar{q}$ picture a_0 has the structure $(u\bar{u} - d\bar{d})/\sqrt{2}$ and the decay is suppressed by OZI rule. On the other hand since both f_0 and a_0 are close to $K\bar{K}$ threshold and they are known to couple strongly to this channel it was suggested that the ϕ meson couples to the scalar mesons f_0

and a_0 through an intermediate charged kaon loop with the photon radiated by the intermediate charged kaons [9]. It was shown that this mechanism is supported by the existing data on radiative decays.

The branching ratios for the radiative decays $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ can be calculated in the intermediate kaon loop mechanism as [9, 12]

$$BR(\phi \to f_0\gamma) = (0.55 \pm 0.14) \times 10^{-4} \cos^2(\theta) g^2_{f_0 K^+ K^-} F^2_{f_0}(K) \tag{1.1}$$
$$BR(\phi \to a_0\gamma) = (0.55 \pm 0.14) \times 10^{-4} \sin^2(\theta) g^2_{a_0 K^+ K^-} F^2_{a_0}(K)$$

where the factor $F^2_{f_0}(K)$ and $F^2_{a_0}(K)$ are related to the spatial extensions of f_0 and a_0 mesons , and for point-like effective field theory calculations $F^2(K) = 1$. The angle θ is the isospin mixing angle in $f_0 - a_0$ system, if the isospin is assumed to be exact, this angle is given as $\theta = \pi/4$. The factors $F^2_{f_0}(K)$ and $F^2_{a_0}(K)$ as well as the coupling constants $g_{f_0 K^+ K^-}$ and $g_{a_0 K^+ K^-}$ depend on the assumed structure for the scalar f_0 and a_0 mesons and they reflect the properties of the models for these structures. Furthermore, not only the absolute branching ratios, but also the ratio $R = BR(\phi \to f_0\gamma)/BR(\phi \to a_0\gamma)$ is of special importance since in the intermediate kaon loop mechanism for the radiative decays $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ this ratio takes the form

$$R = \frac{g^2_{f_0 K^+ K^-} F^2_{f_0}(K)}{g^2_{a_0 K^+ K^-} F^2_{a_0}(K)} cot^2(\theta) \tag{1.2}$$

which only involves the quantities related to the structure of scalar mesons because the factors belonging to $\phi K^+ K^-$ vertex, phase space and loop integrals common to both decays cancel in the ratio. If, for example, the scalar mesons f_0 and a_0

have common constituents and are eigenstates of isospin, then their affinity to K^+K^- should be the same in which case the ratio would be $R \sim 1$. On the other hand, it was suggested [13] that in strong interactions isospin has been believed to be a nearly exact symmetry, broken only by the different masses of the u and d quarks and electroweak effects, the dynamics of strongly coupled $K\bar{K}$ states would give rise to a violation of isospin. If that is the case, then the experimental ratio $R = 3.3$ can be produced by an isospin mixing angle of $\theta = 30°$. However, this point of view is criticized using the arguments based on the overlapping resonances [14]. If the experiments clarify isospin breaking then the individual branching ratios may be used as a measure of $F^2_{f_0}(K)$ and $F^2_{a_0}(K)$. The form factors $F^2(K) << 1$ resulting from the experimental values of the branching ratios would imply that the scalar meson states are spatially extended. If, on the other hand, the experimental branching ratios indicate that $F^2(K) \to 1$, then the f_0 and a_0 states are spatially compact. It should be mentioned that the coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ satisfy the relation $g_{f_0K^+K^-} = g_{a_0K^+K^-}$ both in $q\bar{q}$ and in $q^2\bar{q}^2$ models. However, in the analysis of radiative decays $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$, the values obtained for these coupling constants by the analysis of different physical processes and by different theoretical predictions based on chiral symmetry and the linear sigma model and QCD sum rules are critically utilized [15].

CHAPTER 2

MODELS OF $\phi(1020) \to f_0(980)\gamma$ and $\phi(1020) \to a_0(980)\gamma$ DECAYS

In this chapter, we present the models and the related the theoretical considerations used for the calculations of the branching ratios of the $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ radiative decays. We first discuss the point coupling model in some detail. We then summarize the theoretical details of the coupling of an extended scalar meson to an intermediate $K\bar{K}$ loop, and the resulting modifications of the branching ratios.

2.1 Point Coupling Model

The mechanism of $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ in the intermediate $K\bar{K}$ loop model is described by the Feynman diagrams shown in Fig.1.

In Fig. 1 the last diagram assures gauge invariance. The ϕKK vertex is described by the effective Lagrangian

$$\mathcal{L} = -ig_{\phi KK}\phi^{\mu}(K^-\partial_\mu K^+ - \partial_\mu K^- K^+) \tag{2.1}$$

which results from the standard chiral Lagrangians in lowest order of chiral

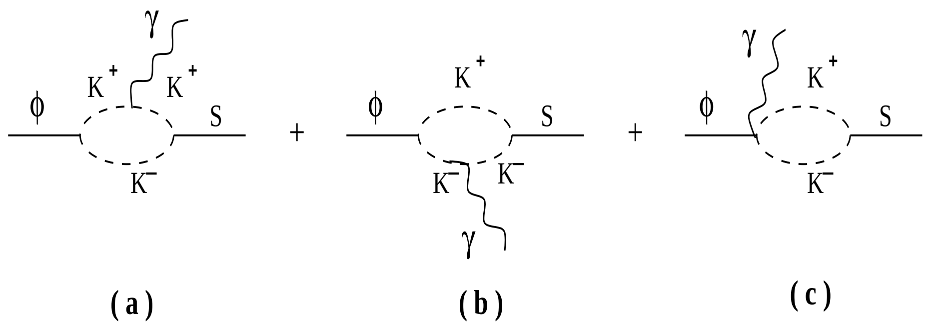

Figure 2.1: Feynman diagrams for the decay $\phi \to S\gamma$ where S denotes the scalar meson f_0 or a_0

perturbation theory [16]. The decay rate resulting from this Lagrangian is

$$\Gamma(\phi \to K^+K^-) = \frac{g^2_{\phi K^+K^-}}{48\pi} M_\phi \left[1 - \left(\frac{2M_K}{M_\phi} \right)^2 \right]^{3/2} . \tag{2.2}$$

Utilizing the experimental value for the branching ratio

$BR(\phi \to K^+K^-) = 0.491 \pm 0.007$ for the decay $\phi \to K^+K^-$ [1], we determine the coupling constant $g_{\phi KK}$ as $g_{\phi KK} = 4.43 \pm 0.05$. Upon making the ϕ and K interactions described by the Lagrangian given in Eq. 2.1 gauge invariant by the minimal coupling of the photon field, we obtain

$$\mathcal{L}_I = -i(eA^\mu + g_{\phi KK}\phi^\mu)(K^-\partial_\mu K^+ - \partial_\mu K^- K^+) + 2eg_\phi \phi^\mu A_\mu K^+K^- . \tag{2.3}$$

The different Feynman diagrams shown in Fig. 1 result from this Lagrangian.

The SKK vertex, where S denotes the scalar meson f_0 or a_0, is described by the effective Lagrangian

$$\mathcal{L}_{SKK} = -g_{SKK}K^+K^-S \tag{2.4}$$

which also serves to define the coupling constant g_{SKK}.

The amplitude of the radiative ϕ decay into the scalar meson has therefore the following structure imposed by gauge invariance

$$M(\phi \to S\gamma) = u^\mu \varepsilon^\nu (p_\nu q_\mu - g_{\mu\nu} p.q) \frac{e g_{\phi KK} g_{SKK}}{2\pi^2 M_K^2} I(a,b) \tag{2.5}$$

where (u,p) and (ε, q) are the polarization and four momenta of the ϕ meson and the photon, respectively, and $a = \frac{M_\phi^2}{M_K^2}$, and $b = \frac{M_S^2}{M_K^2}$. In the case an unstable scalar meson is produced, which then decays into a final state such as $\pi\pi$ or KK, M_s^2 must be replaced by the square of the invariant mass of the decay products. The invariant function I(a,b) has been calculated in different contexts [17]. For the point-like model of scalar mesons, it is given by

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right] + \frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right)\right] \tag{2.6}$$

$$f(x) = \begin{cases} -\left[\arcsin(\frac{1}{2\sqrt{x}})\right]^2 , & x > \frac{1}{4} \\ \frac{1}{4}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right]^2 , & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}} \arcsin(\frac{1}{2\sqrt{x}}) , & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right] , & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x}\left[1 \pm (1-4x)^{\frac{1}{2}}\right] . \tag{2.7}$$

In order to derive this formula, it is noted that the amplitude $M(\phi \to S\gamma)$ can be written from the Feynman diagrams shown in Fig. 1(a, b, c) by the usual rules that follows from the Lagrangian given in Eq. 2.3 as

$$M(\phi \to S\gamma) = e g_{\phi KK} g_{SKK} u^\mu \epsilon^\nu J_{\mu\nu} . \tag{2.8}$$

In this expression $J_{\mu\nu}$ is given by

$$J_{\mu\nu} = J^{(a)}_{\mu\nu} + J^{(b)}_{\mu\nu} + J^{(c)}_{\mu\nu} = 2J^{(a)}_{\mu\nu} + J^{(c)}_{\mu\nu} \ . \tag{2.9}$$

The terms are calculated from the corresponding Feynman diagrams in Fig. 1 in the form

$$J^a_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{(2k-p)_\mu (2k-p)_\nu}{(k^2 - M_k^2 + i\eta)[(k-q)^2 - M_k^2 + i\eta][(k-p)^2 - M_k^2 + i\eta]} \ , \tag{2.10}$$

$$J^c_{\mu\nu} = -2g_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M_k^2 + i\eta)[(q+k-p)^2 - M_k^2 + i\eta]} \ . \tag{2.11}$$

Each of the above integrals are divergent. However, they can be evaluated by dimensional regularization and the divergent pieces can be isolated. Then, in the sum, the divergent pieces cancel and the sum given on Eq. 2.8 is finite. In order to calculate the finite part and demonstrate the cancelation of divergent parts the expression for $J^a_{\mu\nu}$ can be decomposed as

$$2J^a_{\mu\nu} = J[p_\nu q_\mu - (p.q)g_{\mu\nu}] + 2g_{\mu\nu} J_a \ , \tag{2.12}$$

where it follows that J can be calculated in the form

$$J = -\frac{i}{2\pi^2 M^2} \left\{ \frac{1}{(a-b)} \int_0^1 dz [1 - z - \frac{1 - az(1-z)}{z(a-b)} \ln \frac{1 - bz(1-z)}{1 - az(1-z)}] \right. \\ \left. - \frac{i\pi}{(a-b)^2} \int_{1/\eta_-}^{1/\eta_+} dz [\frac{1}{z} - (1-z)a] \right\} = -\frac{i}{\pi^2 m^2} I(a,b) \ . \tag{2.13}$$

Furthermore, $J_a^{'}$ can be evaluated to give

$$J_a^{'} = \frac{i}{16\pi^2}[\frac{2}{\epsilon} - \gamma_E - \ln\frac{m^2}{4\pi\mu^2}] - \frac{i}{8\pi^2} \int_0^1 dz (1-z) \ln[1 - bz(1-z)] \ , \tag{2.14}$$

where μ is the auxiliary mass parameter, γ_E is the Euler constant, and the number of dimensions D is equal to $4 - \epsilon$. Similarly, the term corresponding to contact diagram in Fig. 1(c) can be written as $J^{(c)}_{\mu\nu} = -2g_{\mu\nu}J^{'}_{c}$ and $J^{'}_{c}$ can be calculated as

$$J^{'}_{c} = \frac{i}{16\pi^2}[\frac{2}{\epsilon} - \gamma_E - \ln\frac{m^2}{4\pi\mu^2}] - \frac{i}{16\pi^2}\int_0^1 dz(1-z)\ln[1-bz(1-z)]\ . \qquad (2.15)$$

Therefore, in the sum the divergent and auxiliary mass dependent terms cancel, and furthermore since $\int_0^1 dz(1-2z)\ln[1-bz(1-z)] = 0$ the final expression for $J_{\mu\nu}$ results in the amplitude $M(\phi \to S\gamma)$ with the structure given in Eq. 2.5. In the above calculation, it is assumed that $M_\phi > 2M_K$ and $M_S < 2M_K$. Since $M_\phi = 1020$ MeV, $M_K = 493.68$ MeV, $M_{a_0} = M_{f_0} = 980$ MeV these conditions are fulfilled. The decay width of the radiative decay $\phi \to S\gamma$ then can be obtained from the amplitude $M(\phi \to S\gamma)$ given in Eq. 2.5 as

$$\Gamma(\phi \to S\gamma) = \frac{\alpha g^2_{\phi K^+K^-} g^2_{SK^+K^-}}{3(2\pi)^4}\frac{\omega}{M^2_\phi}\ \mid (a-b)I(a,b)\mid^2 \qquad (2.16)$$

where α is the fine structure constant and $\omega = (M\phi^2 - M^2_S)/2M_\phi$ is the photon energy. It therefore follows that, in the point coupling model, the crucial ingredient for the calculations of $\phi \to S\gamma$ decay rates, is the coupling constant $g_{SK^+K^-}$.

The coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ are important parameters in hadron electrodynamics. They are essential in analysis of different hadronic reactions, in particular for the study of radiative decays of vector mesons such as $\phi \to \pi\pi\gamma$ and $\phi \to \pi\eta\gamma$. The above analysis of the decay reactions $\phi \to S\gamma$ and the resulting formula for the decay rate $\Gamma(\phi \to S\gamma)$ given in Eq. 2.16 can therefore be used to obtain these coupling constants by utilizing the experimental values of the branching ratios $BR(\phi \to f_0\gamma)$ and $BR(\phi \to a_0\gamma)$. If we use the values $BR(\phi \to$

$f_0\gamma) = (3.5 \pm 0.3) \times 10^{-4}$ and $BR(\phi \to a_0\gamma) = (0.88 \pm 0.17) \times 10^{-4}$ reported by the SND collaboration [11], we then obtain the values $g_{f_0K^+K^-} = (4.59 \pm 17)$ and $g_{a_0K^+K^-} = (2.30 \pm 07)$ for the coupling constants. However, if the recent results $BR(\phi \to f_0\gamma) = (4.40 \pm 0.21) \times 10^{-4}$ and $BR(\phi \to a_0\gamma) = (0.76 \pm 0.06) \times 10^{-4}$ reported by the KLOE collaboration [11] is used, the resulting values for the coupling constants are $g_{f_0K^+K^-} = (5.14 \pm 12)$ and $g_{a_0K^+K^-} = (2.13 \pm 0.8)$.

These coupling constants have been calculated by different theoretical approaches, and they have also been determined experimentally by the analysis of several hadronic processes. Therefore, the formula given in Eq. 2.16 for the decay rate $\Gamma(\phi \to S\gamma)$ which involves these coupling constants can be used to calculate the branching ratio for the decays $\phi \to S\gamma$ and by comparing the obtained value with the experimental result thus provide a test for the theoretical and experimental analysis used to obtain these coupling constants. In Table 1 we show the branching ratios for the decays $\phi \to S\gamma$ calculated with different coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ given in the literature. A comparison of the results obtained can then be performed with the experimental values given above [2].

Table 2.1: $BR(\phi \to f_0\gamma)$ for different $g_{f_0K^+K^-}$

$g_{f_0K^+K^-}$	$BR(\phi \to f_0\gamma)$	Ref.
$6.2 \leq g_{f_0K^+K^-} \leq 7.8$	$6.41 \times 10^{-4} \leq BR \leq 10.15 \times 10^{-4}$	[15]
4.10	2.80×10^{-4}	[18]
2.24	0.84×10^{-4}	[19]
0.51	0.43×10^{-5}	[20]

Table 2.2: $BR(\phi \to a_0\gamma)$ for different $g_{a_0K^+K^-}$

$g_{a_0K^+K^-}$	$BR(\phi \to a_0\gamma)$	Ref.
$4.4 \leq g_{a_0K^+K^-} \leq 5.6$	$3.13 \times 10^{-4} \leq BR \leq 5.23 \times 10^{-4}$	[15]
2.80	1.30×10^{-4}	[18]
-1.53	0.39×10^{-4}	[21]

It should be noted that both in the $q\bar{q}$ and $q^2\bar{q}^2$ models the coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ satisfy the relation $g_{f_0K^+K^-} = g_{a_0K^+K^-}$, the point coupling model gives thus the result $R = BR(\phi \to f_0\gamma)/BR(\phi \to a_0\gamma) = 1$ which contradicts the experimental result R=3.2. This contradiction can be remedied by introducing large isospin mixing effect into $f_0 - a_0$ system. One writes the physical f_0 and a_0 states as mixtures of isotopic spin I=1 and I=0 states as

$$|f_0 >= \cos\Psi|f_0(I=0) > + \sin\Psi|a_0(I=1) >$$

$$|a_0 >= \cos\Psi|a_0(I=1) > - \sin\Psi|f_0(I=0) > \tag{2.17}$$

where $|a_0(I=1) >= (K^+K^- - K^0\bar{K}^0)/\sqrt{2}$, $|f_0(I=0) >= (K^+K^- + K^0\bar{K}^0)/\sqrt{2}$ and introducing the angle θ by $\theta = \frac{\pi}{4} - \Psi$, one obtains the relation $R = \cot^2\theta$ and using the experimental result $R = 3.2$ gives the mixing angle $\theta = 30°$. However, this argument is criticized as missing the effect of the overlapping resonances, and thus as such being unrealistic [14].

2.2 Scalar Mesons with Extended Structure

In this section we summarize the calculation of the branching ratio $BR(\phi \to S\gamma)$ for the radiative decay $\phi \to S\gamma$ when the scalar meson is treated as an extended object [9]. If it is assumed that K^+ and K^- pseudoscalar mesons with three-momenta $\vec{k}$ and $-\vec{k}$ respectively produce a scalar meson with extended structure, then the interaction Hamiltonian which is in general a function of momentum, can be written as $H_I = g_{SK^+K^-}\phi(\mid \vec{k} \mid)SK^+K^-$. Then, after the electromagnetic interaction is introduced through the minimum coupling $\vec{k} \to \vec{k} - e\vec{A}$ and $\phi(\mid \vec{k} - e\vec{A} \mid)$ is expanded to leading order in e, a new electromagnetic coupling in addition to those of the point coupling model in the form $H_I = -eg_{SK^+K^-}\phi'(k)\hat{k}\cdot\vec{A}$ results. In this expression $\hat{k} = \vec{k}/k$. The corresponding Feynman diagrams in case of an extended scalar meson for the decay $\phi \to S\gamma$ are shown in Fig. 2.1. The extra contact vertex results from the extended structure of the scalar meson and plays the role of a form factor introduced in a gauge invariant way. The effect of this form factor can be taken into account by utilizing time-ordered perturbation theory in the non-relativistic approximation [9]. The specific model for the $K\overline{K}$ molecule used is a deuteron like model [22], where the wave function is parametarized as

$$\phi(k) = \frac{(2\mu)^{3/2}}{\pi}\frac{\mu}{(k^2+\mu^2)^2} \ , \tag{2.18}$$

where $\mu = \sqrt{3}/2R_{K\overline{K}}$ with the radius of the molecule $R_{K\overline{K}} \simeq 1.2\ fm$. The branching ratio $BR(\phi \to S\gamma)$ can then be calculated as a function of $R_{K\overline{K}}$ [9, 23]. As $R_{K\overline{K}} \to 0$ and $\phi(k) \to 1$ the numerical results of the point-like field theory for the branching ratio $BR(\phi \to S\gamma)$ are recovered. On the other hand, for the specific $K\overline{K}$ molecule wave function given in Eq. 2.18 the predicted branching ratio for the decay $\phi \to S\gamma$ is about 1/5 of the point-like field theory result.

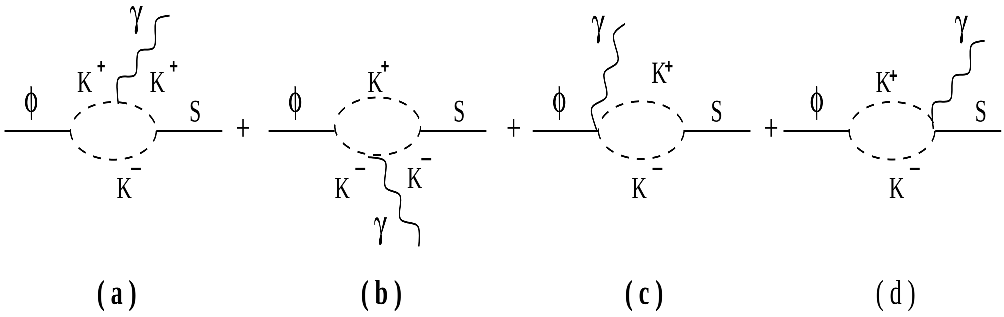

Figure 2.2: Feynman diagrams for the decay $\phi \to S\gamma$ for an extended scalar meson

Although these calculations [9, 23] clearly demonstrate the effect of the extended structure of the scalar mesons on the decay rate of the reactions $\phi \to S\gamma$, they are essentially nonrelativistic in character through their use of time ordered perturbation theory and the wave function describing the structure of the scalar mesons. A fully relativistic calculation is needed, moreover the difference between the structures of the scalar mesons $f_0(980)$ and $a_0(980)$ should somehow be taken into account.

CHAPTER 3

THE $\phi \to K^0\overline{K}^0\gamma$ DECAY

The study of the radiative decay process $\phi \to K^0\overline{K}^0\gamma$ is important because it provides a background to the reaction $\phi \to K^0\overline{K}^0$. This latter process has been proposed as a way to study CP violation [24]. Since this involves seeking for very small effects, if the branching ratio of the decay $\phi \to K^0\overline{K}^0\gamma$ is of the order of 10^{-6} or more precisely $BR(\phi \to K^0\overline{K}^0\gamma) \geq 10^{-6}$, then this background decay will limit the scope of CP violation measurements in the $\phi \to K^0\overline{K}^0$ decay. Therefore, the study of the reaction $\phi \to K^0\overline{K}^0\gamma$ and the calculation of the branching ratio $BR(\phi \to K^0\overline{K}^0\gamma)$ is crutial for the measurement of CP violation and small CP violating parameters in $\phi \to K^0\overline{K}^0$ decay. There are several calculations of this branching ratio [17, 25]. In this chapter, we will present the calculation of the branching ratio $BR(\phi \to K^0\overline{K}^0\gamma)$ within the framework of K^+K^- loop model by including the f_0 and a_0 scalar meson resonances.

3.1 Mechanism of $\phi \to K^0\overline{K}^0\gamma$ Decay

The mechanism of the radiative decay process $\phi \to K^0\overline{K}^0\gamma$ in the K^+K^- loop model is provided by the reactions $\phi \to K^+K^-\gamma \to K^0\overline{K}^0\gamma$ where the last reaction proceeds by a two-step mechanism with K^+K^- loop coupling to the final

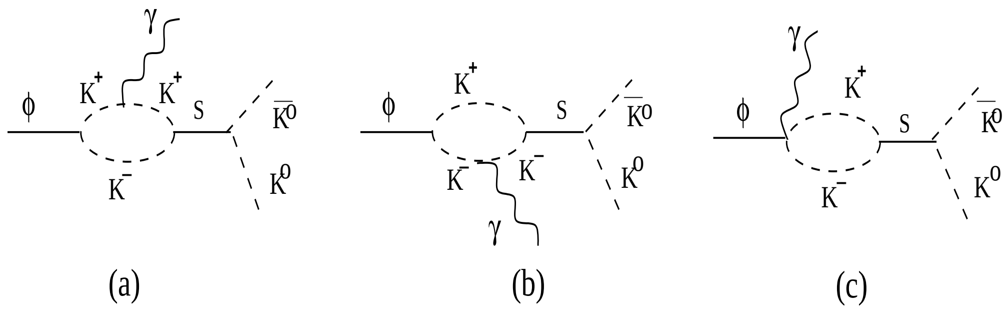

Figure 3.1: Diagrams for the decay $\phi \to K^0\overline{K}^0\gamma$ where S denotes the scalar meson resonance f_0 or a_0

$K^0\overline{K}^0$ state with the scalar resonance f_0 or a_0. We show the processes contributing to the $\phi \to K^0\overline{K}^0\gamma$ amplitude diagramatically in Fig. 3.1. We describe ϕK^+K^- vertex in the diagrams shown in Fig. 3.1 by the phenomenological lagrangian

$$\mathcal{L}_{\phi K^+K^-} = -ig_{\phi KK}\phi^\mu(K^+\partial_\mu K^- - K^-\partial_\mu K^+) \tag{3.1}$$

The decay rate for the $\phi \to K^+K^-$ decay resulting from this Lagrangian is given in Eq. 2.2. We utilize the experimental value for the branching ratio $BR(\phi \to K^+K^-)$ and determine the coupling constant $g_{\phi KK}$ as $g_{\phi KK} = (4.43 \pm 0.05)$.

The SK^+K^- vertex, where S denotes the scalar meson f_0 or a_0, is described by the phenomenological lagrangian

$$\mathcal{L}_{SK^+K^-} = -g_{SK^+K^-}K^+K^-S\ . \tag{3.2}$$

The decay width of the scalar meson that follows from this Lagrangian is

$$\Gamma(SK^+K^-) = \frac{g^2_{SK^+K^-}}{16\pi M_S}\left[1 - \left(\frac{2M_K}{M_S}\right)^2\right]^{1/2}\quad , \tag{3.3}$$

which is usually considered to define the coupling constant $g_{SK^+K^-}$. We describe the $SK^0\overline{K}^0$ vertex similarly by the effective Lagrangian

$$\mathcal{L}_{SK^0\overline{K}^0} = -g_{SK^0\overline{K}^0} SK^0\overline{K}^0 \ . \tag{3.4}$$

If the arguments of isotopic spin invariance are used the coupling constants $g_{SK^+K^-}$ and $g_{SK^0\overline{K}^0}$ are related by the following relations [9]

$$\begin{aligned} g_{f_0K^+K^-} &= g_{f_0K^0\overline{K}^0} \\ g_{a_0K^+K^-} &= -g_{a_0K^0\overline{K}^0} \ , \end{aligned} \tag{3.5}$$

therefore in our phenomenological approach, the coupling constants $g_{SK^+K^-}$ and $g_{SK^0\overline{K}^0}$ are connected by the above relations.

In our calculation we use the values for the coupling constants $g_{f_0K^+K^-} = (5.14 \pm 12)$ and $g_{a_0K^+K^-} = (2.13 \pm 0.8)$ that we determine in Chapter 2 using the decay rate formula $\Gamma(\phi \to S\gamma)$ given in Eq. 2.16 in K^+K^- loop model and the experimental values of the branching ratios $BR(\phi \to f_0\gamma) = (4.40 \pm 0.21) \times 10^{-4}$ and $BR(\phi \to a_0\gamma) = (0.76 \pm 0.06) \times 10^{-4}$

We therefore obtain the amplitude for the radiative decay reaction $\phi \to K^0\overline{K}^0\gamma$ following from the diagrams shown in Fig. 3.1 as

$$\begin{aligned} \mathcal{M}(\phi \to K^0\overline{K}^0\gamma) &= -\frac{e\ g_{\phi KK}}{i2\pi^2 M_K^2} \left[(p\cdot k)(\epsilon\cdot u) - (p\cdot\epsilon)(k\cdot u)\right] I(a,b) \\ &\quad \times \mathcal{M}(K^+K^- \to K^0\overline{K}^0) \end{aligned} \tag{3.6}$$

where (u,p) and (ϵ,k) are the polarizations and four-momenta of the ϕ meson and the photon, respectively. The loop function $I(a,b)$ is defined in Eq. (2.6), however, in this case $a = M_\phi^2/M_K^2$ and $b = M_{KK}^2/M_K^2$ with M_{KK}^2 being the invariant mass of the final $K^0\overline{K}^0$ system given by $M^2_{K^0\overline{K}^0} = (p-k)^2$. The amplitude $\mathcal{M}(K^+K^- \to K^0\overline{K}^0)$ contains the scalar f_0 and a_0 resonances and in the approach we adopted it is given by

$$\mathcal{M}(K^+K^- \to K^0\overline{K}^0) = -ig_{SK^+K^-}g_{SK^0\overline{K}^0}\frac{1}{(p-k)^2 - M_S^2} \quad . \tag{3.7}$$

Since the scalar resonances f_0 and a_0 are unstable and they have a finite lifetime, in the scalar meson propagator we make the replacement $(p-k)^2 - M_S^2 \to (p-k)^2 - M_S^2 + iM_S\Gamma_S$ and use the experimental values for the widths Γ_{f_0} and Γ_{a_0} in the f_0 and a_0 meson propagators, respectively. Then the differential decay probability for the radiative decay $\phi \to K^0\overline{K}^0\gamma$ for an unpolarized ϕ meson at rest is given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\,\frac{1}{8M_\phi}\ \mid \mathcal{M}\mid^2, \tag{3.8}$$

where E_γ and E_1 are the photon and K^0 meson energies respectively. We perform an average over the spin states of ϕ meson and a sum over the polarization states of the photon. The decay width $\phi \to K^0\overline{K}^0\gamma$ is then obtained by integration

$$\Gamma = \int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \tag{3.9}$$

where the minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = (M_\phi^2 - 4M_{K_0}^2)/2M_\phi$. The maximum and minimum values for the energy E_1 of K^0 meson are given by

$$\begin{aligned}&\frac{1}{2(2E_\gamma M_\phi - M_\phi^2)}\left\{-2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3\right.\\ &\qquad\left.\pm E_\gamma\sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_{K_0}^2)}\ \right\} \ .\end{aligned} \tag{3.10}$$

The derivations of the formulas and the other relations are presented in Appendix A, B [26] and in Appendix C.

3.2 Results

If we include the contribution of f_0 resonance only in the decay mechanism of the radiative decay $\phi \to K^0\overline{K}^0\gamma$, we obtained the result for the branching ratio $BR(\phi \to K^0\overline{K}^0\gamma) = 1.11 \times 10^{-6}$. On the other hand, if the contribution of a_0 resonance is considered only the branching ratio is $BR(\phi \to K^0\overline{K}^0\gamma) = 3.28 \times 10^{-8}$. Since both f_0 and a_0 resonances make a contribution to the decay $\phi \to K^0\overline{K}^0\gamma$, when considering their contribution to the decay rate we have to note that the amplitudes involving f_0 and a_0 resonances interfere destructively due to isotopic spin invariance as reflected in the relations between the coupling constants as $g_{f_0K^+K^-} = g_{f_0K^0\overline{K}^0}$ and $g_{a_0K^+K^-} = -g_{a_0K^0\overline{K}^0}$. Thus, if we consider that the interference between the contributions of f_0 and a_0 resonances is destructive, we obtain for the branching ratio the value $BR(\phi \to K^0\overline{K}^0\gamma) = 7.6\times10^{-7}$. Therefore, this reaction will not provide a significant background to the measurements of $\phi \to K^0\overline{K}^0$ decay for testing CP violation.

Table 3.1: Branching ratio of ϕ meson to $K^0\overline{K}^0\gamma$ for different isospin channels

	f_0	a_0	$f_0 - a_0$
$\Gamma(\phi \to K^0\overline{K}^0\gamma)$	4.74×10^{-6}	1.40×10^{-7}	3.25×10^{-6}
$BR(\phi \to K^0\overline{K}^0\gamma)$	1.11×10^{-6}	3.28×10^{-8}	7.64×10^{-7}

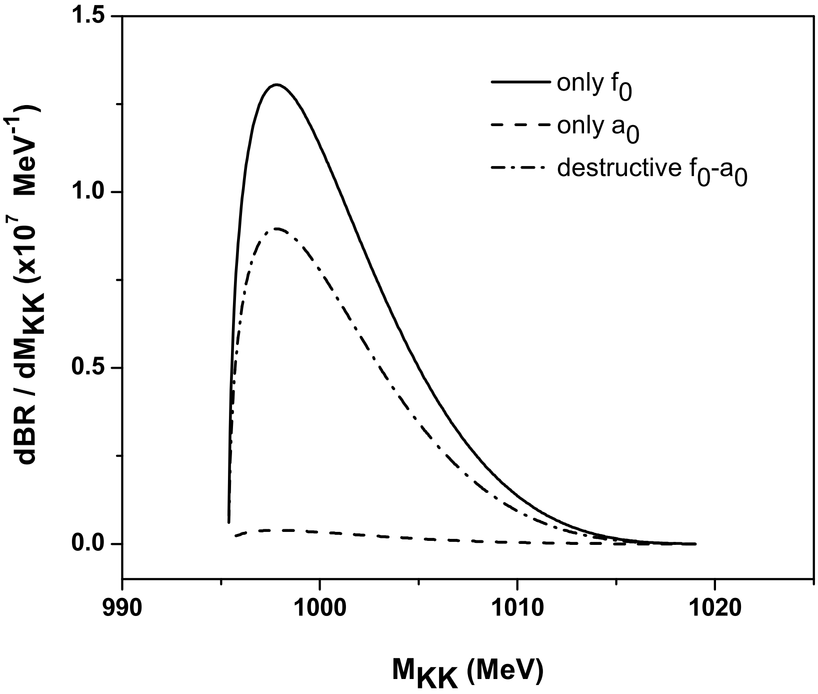

Figure 3.2: The distribution dBR/dM_{KK} for the radiative decay $\phi \to K^0\overline{K}^0\gamma$

In Fig. 3.3, we plot the distribution dBR/dM_{KK} for the radiative decay $\phi \to K^0\overline{K}^0\gamma$ in the phenomenological approach that we adopted, where we also indicate the contributions coming from f_0 resonance, a_0 resonances, and the contribution resulting from the destructive interference of these mechanisms. Furthermore, in Fig. 3.3, we show the photon spectrum $d\Gamma/dE_\gamma$ for the process $\phi \to K^0\overline{K}^0\gamma$ for the same amplitudes as in Fig. 3.2.

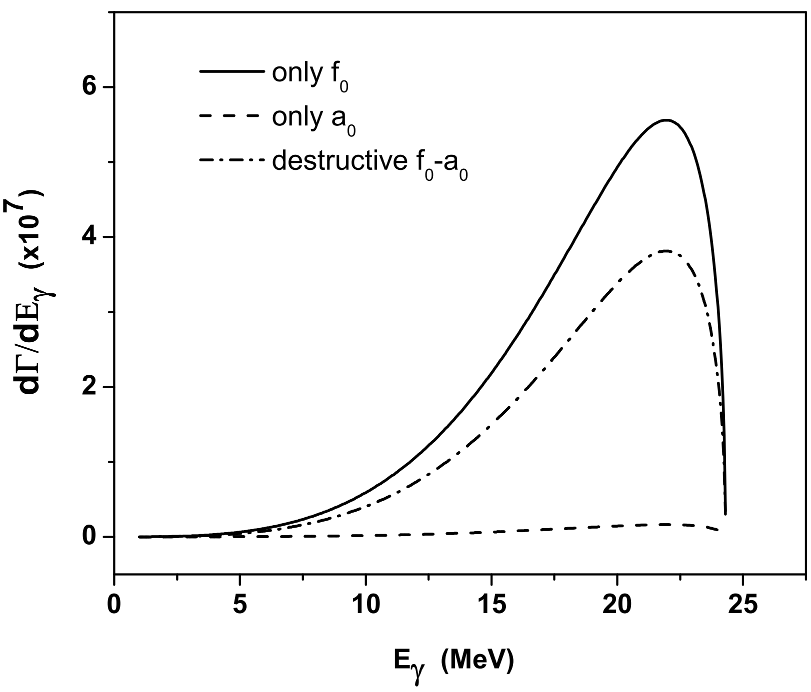

Figure 3.3: The photon spectrum $d\Gamma/dE_\gamma$ for the process $\phi \to K^0\overline{K}^0\gamma$

CHAPTER 4

CONCLUSION

In this thesis we studied the effects of the structure of scalar mesons $f_0(980)$ and $a_0(980)$ on the decay rate of the radiative decays $\phi \to f_0\gamma$ and $\phi \to a_0\gamma$ analized in the K^+K^- loop model. The predictions for the branching ratios depend on the assumed structure of the scalar mesons as reflected in the coupling constants of the SKK system . Moreover, if the extenden nature of the scalar meson structure is taken into account to reduction of the branching ratios of the decays $\phi \to S\gamma$ result as opposed to the point coupling model.

We also studied the radiative $\phi \to K^0\bar{K^0}\gamma$ decay using the two step reaction mechanism in which the final state couples to the scalar meson which then couples to the initial ϕ meson through K^+K^- loop. The decay $\phi \to K^0\bar{K^0}\gamma$ may present a background problem for measurements of CP violation in $\phi \to K^0\bar{K^0}$ decays. We calculated the decay rate for the $\phi \to K^0\bar{K^0}\gamma$ reaction and we noted that the branching ratio is small enough as not to limit the precision of $\phi \to K^0\bar{K^0}$ experiments.

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APPENDIX A

TWO BODY DECAY RATES

The transition probability for a transition from an initial state $\mid i >$ to a final state $\mid f >$ is defined by $|S_{fi}|^2 = |< f|S \mid i > |^2$. The corresponding probability amplitude is

$$< f|S \mid i >= S_{fi} \quad . \tag{A.1}$$

where the element of the scattering matrix-S is given as

$$< f|S \mid i >= \delta_{fi} + (2\pi)^4 \delta^{(4)} \left(\sum p'_f - \sum p_i \right) T_{fi} \quad , \tag{A.2}$$

where T is the transition matrix from the initial state to the final state.The invariant matrix element, for the decay of a particle of mass M and energy E into any number of particles 1,2,......N, is A_{fi} and the differential decay rate is given by the multiplication of the transition probability per unit time by number of final states.Thus the differential decay rate is described as

$$d\Gamma = (2\pi)^4 \delta^{(4)} \left(\sum p_f - \sum p_i \right) |\mathcal{A}_{fi}|^2 \frac{1}{2E} \prod_f \left(\frac{d^3 p_f}{(2\pi)^3 2E_f} \right) \quad . \tag{A.3}$$

where $p_i = (E, \vec{p_i})$ and $p_f^{'} = (E_f^{'}, \vec{p_f^{'}})$ are the four momenta of the initial and final particles respectively. If we consider the two body decay in which we have two particles in the final state , then in the rest frame of decaying particle $\vec{p_1} = -\vec{p_2} \equiv$

$\vec{p}$, $E_1 + E_2 = M$, thus the differential decay rate is

$$d\Gamma = \frac{1}{(2\pi)^2}|\mathcal{A}_{fi}|^2 \frac{1}{2M}\frac{1}{4E_1E_2}\delta^{(3)}(\vec{p_1} + \vec{p_2})\delta(E_1 + E_2 - M)d^3p_1 d^3p_2 \quad . \tag{A.4}$$

The first delta function is eliminated by integrating over d^3p_2 and the differential d^3p_1 is written as

$$d^3p = p^2 d|\vec{p}|d\Omega = |\vec{p}|d\Omega \frac{E_1E_2 d(E_1 + E_2)}{E_1 + E_2} \quad , \tag{A.5}$$

since $E_1^2 - M_1^2 = E_2^2 - M_2^2 = \vec{p}^2$. Integration over $(E_1 + E_2)$eliminates the second delta function and the result comes as

$$d\Gamma = \frac{1}{32\pi^2 M^2}|\mathcal{A}_{fi}|^2|\vec{p}|d\Omega \quad . \tag{A.6}$$

Therefore, the decay rate is obtained as

$$\Gamma = \frac{1}{8\pi M^2}|\mathcal{A}_{fi}|^2|\vec{p}| \quad . \tag{A.7}$$

In the rest frame of decaying particle, $|\vec{p}|$ is determined as

$$|\vec{p}| = \frac{1}{2M}\sqrt{[M^2 - (M_1 + M_2)^2][M^2 - (M_1 - M_2)^2]} \quad . \tag{A.8}$$

Therefore for the decay $M \rightarrow M_1 + M_2$ where $M_1 = M_2$

$$|\vec{p}| = \frac{1}{2}M\sqrt{1 - \left(\frac{2M_1}{M}\right)^2} \quad , \tag{A.9}$$

and for the decay $M \rightarrow M_1 + \gamma$

$$|\vec{p}| = \frac{1}{2}M\left[1 - \left(\frac{M_1}{M}\right)^2\right] \quad . \tag{A.10}$$

The invariant matrix element for the decay $\phi \rightarrow K^+K^-$ following from the effective Lagrangian

$$\mathcal{L}^{eff.}_{\phi K^+K^-} = -ig_{\phi K^+K^-}\phi^\mu(K^+\partial_\mu K^- - K^-\partial_\mu K^+) \quad , \tag{A.11}$$

is determined as $\mathcal{A}(\phi \rightarrow K^+K^-) = -ig_{\phi K^+K^-}(2k-p)_\mu u^\mu$, where k is the four-momentum of the plus signed kaon and p(u) is the four-momentum (polarization) of the decaying ϕ-meson. Therefore the decay rate Γ for $\phi \rightarrow K^+K^-$ is

$$\Gamma(\phi \rightarrow K^+K^-) = \frac{g^2_{\phi K^+K^-}}{48\pi} M_\phi \left[1 - \left(\frac{2M_K}{M_\phi}\right)^2\right]^{3/2} \quad . \tag{A.12}$$

For the $\phi \rightarrow S\gamma$ decay (where $S = f_0$ or a_0), in which each ϕ and S mesons couple strongly to $K\bar{K}$, with the couplings $g_{\phi K^+K^-}$ for ϕK^+K^- and $g_{SK^+K^-}$ for SK^+K^-, the invariant amplitude is obtained as

$$\mathcal{A}(\phi \rightarrow S\gamma) = u^\mu \epsilon^\nu (q_\mu p_\nu - g_{\mu\nu} q \cdot p) \frac{e g_{\phi K^+K^-}(g_{SK^+K^-} M_S)}{2\pi^2 M_K^2} I(a,b) \quad , \tag{A.13}$$

where (u, p) is the polarization and four-momentum of the decaying vector meson and $(\epsilon,\ q)$ of the the photon and $a = M_\phi^2/M_K^2$, $b = M_S^2/M_K^2$. Then,

$$\Gamma(\phi \rightarrow S\gamma) = \frac{\alpha}{6(2\pi)^4} \frac{M_\phi^2 - M_S^2}{M_\phi^3} g^2_{\phi K^+K^-}(g_{SK^+K^-} M_S)^2 |(a-b) I(a,b)|^2 \quad . \tag{A.14}$$

For the decay $S \rightarrow K^+K^-$,using the effective Lagrangian,

$$\mathcal{L}^{eff.}_{SK^+K^-} = g_{SK^+K^-} M_{a_0} K^+K^- S \quad , \tag{A.15}$$

the invariant matrix element is obtained as $\mathcal{A}(S \rightarrow K^+K^-) = i g_{SK^+K^-} M_S$, therefore the decay rate is

$$\Gamma(S \rightarrow K^+K^-) = \frac{g^2_{SK^+K^-}}{16\pi} M_S \left[1 - \left(\frac{2M_K}{M_S}\right)^2\right]^{1/2} \quad . \tag{A.16}$$

APPENDIX B

THREE BODY DECAY AND THE BOUNDARY OF DALITZ PLOT

For the decay of a particle S with four into N particles with four-momenta $p = (E_s,\ \vec{p})$ and $p'_f = (E'_f,\ \vec{p'}_f)$ respectively the differential decay rate is determined as

$$d\Gamma = (2\pi)^4 \delta^{(4)} \left(\sum p'_f - p\right) \frac{1}{2E_s} \prod_f \frac{d^3\vec{p'}_f}{(2\pi)^3(2E'_f)} \overline{|\mathcal{A}_{fi}|^2} \quad . \tag{B.1}$$

For the three body decay in which there are three particles in the final state ($M(p) \to M_1(q_1) + M_2(q_2) + \gamma(k)$) the differential decay rate is given by

$$d\Gamma = (2\pi)^4 \delta^{(4)}(p - q_1 - q_2 - k) \frac{1}{2E_p} \frac{d^3 q_1}{(2\pi)^3(2E_1)} \frac{d^3 q_2}{(2\pi)^3(2E_2)} \frac{d^3 k}{(2\pi)^3(2E_\gamma)} \overline{|\mathcal{A}_{fi}|^2} \ , \tag{B.2}$$

where $\overline{|\mathcal{A}_{fi}|^2}$ is the average over spin states of the absolute square of the decay invariant matrix element. Therefore due to spin average we can write $\overline{|\mathcal{A}_{fi}|^2} = F(E_1, E_2)$. The $\delta^{(4)}$ function can be written as

$$\delta^{(4)}(p - q_1 - q_2 - k) = \delta(M - E_1 - E_2 - E_\gamma)\delta^{(3)}(\vec{q_1} + \vec{q_2} + \vec{k}) \quad , \tag{B.3}$$

in the rest frame of the decaying particle and the momentum delta function can be eliminated by firstly integrating over the (three-) momentum of the final-state particle with momentum q_2. Using

$$\frac{d^3k}{2E_\gamma} = \frac{|\vec{k}|^2 dk d\Omega_\gamma}{2E_\gamma} = \frac{1}{2} E_\gamma dE_\gamma d\Omega_\gamma \quad , \tag{B.4}$$

and

$$\frac{d^3q_1}{2E_1} = \frac{|\vec{q_1}|^2 dq_1 d\Omega_1}{2E_1} = \frac{1}{2} |\vec{q_1}| dE_1 d\Omega_1 \quad , \tag{B.5}$$

the equation below is obtained

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{|\vec{q_1}| E_\gamma \overline{|\mathcal{A}_{fi}|^2}}{16M(2\pi)^5} \int d\Omega_\gamma d\Omega_1 \frac{\delta(E_\gamma + E_1 - M + \sqrt{(\vec{k} + \vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{k} + \vec{q_1})^2 + M_2^2}} \quad . \tag{B.6}$$

If we define the integral I by

$$I = |\vec{q_1}| E_\gamma \int d\Omega_\gamma d\Omega_1 \frac{\delta(M - E_\gamma - E_1 + \sqrt{(\vec{k} + \vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{k} + \vec{q_1})^2 + M_2^2}} \quad , \tag{B.7}$$

and perform the angular integrals then we obtain

$$I = 8\pi^2 \int_{-1}^{1} d(\cos\theta) |\vec{q_1}| E_\gamma \frac{\delta(E_\gamma + E_1 - M + \sqrt{E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma |\vec{q_1}| \cos\theta + M_2^2})}{\sqrt{E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma |\vec{q_1}| \cos\theta + M_2^2}} \quad , \tag{B.8}$$

where θ is defined by $\vec{k} \cdot \vec{q_1} = |\vec{k}||\vec{q_1}| \cos\theta$.By changing of variable

$$\xi = \sqrt{E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma |\vec{q_1}| \cos\theta + M_2^2} \quad , \tag{B.9}$$

the integral is obtained as

$$I = 8\pi^2 \int d\xi \delta(M - E_\gamma - E_1 - \xi) = 8\pi^2 \quad , \tag{B.10}$$

using the condition $M - E_\gamma - E_1 - \xi = 0$. Therefore we obtain the double differential decay rate as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3} \frac{1}{8M} F(E_1, E_2) \quad . \tag{B.11}$$

The limits of integral are defined by the condition

$$(M - E_\gamma - E_1)^2 = E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2 \quad , \tag{B.12}$$

or

$$-1 \leq \frac{(M - E_\gamma - E_1)^2 - E_\gamma^2 - E_1^2 + M_1^2 - M_2^2}{2|\vec{k}|.|\vec{q_2}|} \leq 1 \quad . \tag{B.13}$$

in another way

$$-1 \leq \frac{(M - E_\gamma - E_1)^2 - E_\gamma^2 - E_1^2 + M_1^2 - M_2^2}{2E_\gamma\sqrt{E_1^2 - M_1^2}} \leq 1 \quad . \tag{B.14}$$

since $E_k^2 = |\vec{k}^2|$ and $E_1^2 = |\vec{q_1}^2| + M_1^2$. We then solve this equation and find two roots for E_1 as

$$\begin{aligned} E_{1min} = \frac{1}{2(2E_\gamma M_\phi - M_\phi^2)} \Big\{ & -2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3 \\ & + E_\gamma\sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_{K_0}^2)} \ \Big\} \end{aligned} \tag{B.15}$$

$$\begin{aligned} E_{1max} = \frac{1}{2(2E_\gamma M_\phi - M_\phi^2)} \Big\{ & -2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3 \\ & - E_\gamma\sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_{K_0}^2)} \ \Big\} \end{aligned} \tag{B.16}$$

APPENDIX C

INVARIANT AMPLITUDE OF THE RADIATIVE $\phi \to K^0\overline{K}^0\gamma$ DECAY

For the radiative decay $\phi(p) \to S(p-k)\gamma(k) \to K^0(q_1)\overline{K}^0(q_2)\gamma(k)$ the invariant amplitude $\mathcal{M}$ is expressed as

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c \quad , \tag{C.1}$$

where $\mathcal{M}_a$, $\mathcal{M}_b$, and $\mathcal{M}_c$ are the invariant amplitudes obtained from the diagrams (a), (b), and (c) in Fig. 3.1 as

$$\begin{aligned}
\mathcal{M}_a &= \mathcal{M}_b \\
&= -eg_{\phi K^+K^-}g_{SK^+K^-}g_{SK^0\overline{K}^0} \\
&\quad \times \int \frac{d^4q}{(2\pi)^4}\frac{(2q-p)_\nu u^\nu (2q-k)_\mu \epsilon^\mu}{(q^2-M_K^2)[(q-k)^2-M_K^2][(p-q)^2-M_K^2]} \\
&\quad \times \frac{1}{[(p-k)^2-M_S^2+i\Gamma_S M_S]}
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
\mathcal{M}_c &= 2eg_{\phi K^+K^-}g_{SK^+K^-}\int \frac{d^4q}{(2\pi)^4}\frac{u_\mu \epsilon^\mu}{(q-k)^2-M_K^2)[(p-q)^2-M_K^2]} \\
&\quad \times \frac{1}{[(p-k)^2-M_S^2+i\Gamma_S M_S]} \quad ,
\end{aligned} \tag{C.3}$$

where (u,p) and (ϵ, k) are the polarization and four momenta of the ϕ meson and the photon, respectively. Using the gauge $u^\nu p_\nu = 0$ and $\epsilon^\mu k_\mu = 0$, we then obtain the invariant amplitude

$$\begin{aligned}
\mathcal{M} &= 2eg_{\phi K^+K^-}g_{SK^+K^-}g_{SK^0\overline{K}^0}\frac{1}{[(p-k)^2-M_S^2+i\Gamma_S M_S]} \\
&\quad \times u^\nu\epsilon^\mu\int\frac{d^4q}{(2\pi)^4}\frac{-4q_\nu q_\mu+g_{\mu\nu}}{(q^2-M_K^2)[(q-k)^2-M_K^2][(p-q)^2-M_K^2]} \\
&= eg_{\phi K^+K^-}g_{SK^+K^-}g_{SK^0\overline{K}^0}\frac{(p-k)^2-M_S^2-i\Gamma_S M_S}{[(p-k)^2-M_S^2+(\Gamma_S M_S)^2]} \\
&\quad \times\left\{-\frac{1}{2\pi^2M_K^2}I(a,b)[(\epsilon\cdot u)(k\cdot p)-(\epsilon\cdot p)(k\cdot u)]\right\} \qquad \text{(C.4)}
\end{aligned}$$

where $a=\frac{M_\phi^2}{M_K^2}$, $b=\frac{(p-k)^2}{M_K^2}=\frac{M_\phi^2-2M_\phi E_\gamma}{M_K^2}$ and

$$\begin{aligned}
2u^\nu\epsilon^\mu\int\frac{d^4q}{(2\pi)^4}\frac{-4q_\nu q_\mu+g_{\mu\nu}}{(q^2-M_K^2)[(q-k)^2-M_K^2][(p-q)^2-M_K^2]} &= \\
-\frac{1}{2\pi^2M_K^2}I(a,b)[(\epsilon\cdot u)(k\cdot p)-(\epsilon\cdot p)(k\cdot u)]\ . \qquad \text{(C.5)}
\end{aligned}$$

The invariant function I(a,b) has been calculated in different contexts. For the point-like model of scalar mesons, it is given by

$$I(a,b)=\frac{1}{2(a-b)}-\frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right)-f\left(\frac{1}{a}\right)\right]+\frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right)-g\left(\frac{1}{a}\right)\right] \text{(C.6)}$$

$$f(x)=\begin{cases}-\left[\arcsin(\frac{1}{2\sqrt{x}})\right]^2\ , & x>\frac{1}{4}\\ \frac{1}{4}\left[\ln(\frac{\eta_+}{\eta_-})-i\pi\right]^2\ , & x<\frac{1}{4}\end{cases}$$

$$g(x)=\begin{cases}(4x-1)^{\frac{1}{2}}\arcsin(\frac{1}{2\sqrt{x}})\ , & x>\frac{1}{4}\\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln(\frac{\eta_+}{\eta_-})-i\pi\right]\ , & x<\frac{1}{4}\end{cases}$$

$$\eta_\pm=\frac{1}{2x}\left[1\pm(1-4x)^{\frac{1}{2}}\right]\ .$$

(C.7)

We note that$1/a = 0.23 < 1/4$ for the values $M_\phi = 1020 MeV$ and $M_K = 494 MeV$. However, $1/b > 1/4$ for $M_{KK} \leq 985 MeV^2$ and $1/b < 1/4$ for $M_{KK} \leq 985 MeV^2$, where $M_{KK} = M_\phi^2 - 2M_\phi E_\gamma$. The functions f and g are for $1/a < 1/4$

$$\begin{aligned} f\left(\frac{1}{a}\right) &= \frac{1}{4}\left[\ln\left(\frac{\eta_+}{\eta_-}\right)\right]^2 - \frac{\pi^2}{4} - i\frac{\pi}{2}\ln\left(\frac{\eta_+}{\eta_-}\right) \\ g\left(\frac{1}{a}\right) &= \frac{1}{2}\sqrt{1-4/a}\ln\left(\frac{\eta_+}{\eta_-}\right) - i\left[\frac{\pi}{2}\sqrt{1-4/a}\right] \quad , \end{aligned} \tag{C.8}$$

for $1/b > 1/4$

$$\begin{aligned} f\left(\frac{1}{b}\right) &= -\left[arc\sin\frac{1}{2\sqrt{1/b}}\right]^2 \\ g\left(\frac{1}{b}\right) &= \sqrt{4/b-1}\left[arc\sin\frac{1}{2\sqrt{1/b}}\right] \quad , \end{aligned} \tag{C.9}$$

and for $1/b < 1/4$

$$\begin{aligned} f\left(\frac{1}{a}\right) &= \frac{1}{4}\left[\ln\left(\frac{\eta_+}{\eta_-}\right)\right]^2 - \frac{\pi^2}{4} - i\frac{\pi}{2}\ln\left(\frac{\eta_+}{\eta_-}\right) \\ g\left(\frac{1}{a}\right) &= \frac{1}{2}\sqrt{1-4/b}\ln\left(\frac{\eta_+}{\eta_-}\right) - i\left[\frac{\pi}{2}\sqrt{1-4/b}\right] \quad . \end{aligned} \tag{C.10}$$

The complex invariant amplitude is parameterized with

$$\mathcal{M} = \mathcal{M}'' + i\mathcal{M}' \tag{C.11}$$

where $\mathcal{M}''$ and $\mathcal{M}'$ are

$$\begin{aligned} \mathcal{M}' &= -\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-} g_{SK^+K^-} g_{SK^0\overline{K}^0}[(\epsilon\cdot u)(k\cdot p) - (\epsilon\cdot p)(k\cdot u)] \\ &\quad \times\left\{[(p-k)^2 - M_S^2]ImI(a,b) - (\Gamma_S M_S)ReI(a,b)\right\}\Delta_S^0(p-k) \\ \mathcal{M}'' &= -\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-} g_{SK^+K^-} g_{SK^0\overline{K}^0}[(\epsilon\cdot u)(k\cdot p) - (\epsilon\cdot p)(k\cdot u)] \\ &\quad \times\left\{[(p-k)^2 - M_S^2]ReI(a,b) + (\Gamma_S M_S)ImI(a,b)\right\}\Delta_S^0(p-k) \end{aligned} \tag{C.12}$$

and

$$\Delta_S^0(q) \;=\; \frac{1}{(q^2-M_S^2)^2+(\Gamma_S M_S)^2} \quad . \tag{C.13}$$

The absolute value of the square of the invariant amplitude is obtained as $\mid \mathcal{M} \mid^2 = (\mathcal{M}'')^2 + (\mathcal{M}')^2$. The squares of the real and imaginary parts become

$$\begin{aligned}
(\mathcal{M}')^2 &= \left\{ -\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-} g_{SK^+K^-} g_{SK^0\overline{K}^0} \right\}^2 \frac{2}{3}(k\cdot p)^2 \\
&\times \left\{ \left\{ [(p-k)^2 - M_S^2] ImI(a,b) - (\Gamma_S M_S) ReI(a,b) \right\} \Delta_S^0(p-k) \right\}^2 \\
(\mathcal{M}'')^2 &= \left\{ -\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-} g_{SK^+K^-} g_{SK^0\overline{K}^0} \right\}^2 \frac{2}{3}(k\cdot p)^2 \\
&\times \left\{ \left\{ [(p-k)^2 - M_S^2] ReI(a,b) + (\Gamma_S M_S) ImI(a,b) \right\} \Delta_S^0(p-k) \right\}^2 \quad .
\end{aligned} \tag{C.14}$$

Using $\epsilon_\alpha \epsilon_{\alpha'} = -g_{\alpha\alpha'}$ and $\overline{u_\alpha u_{\alpha'}} = -\frac{1}{3} g_{\alpha\alpha'}$ we then obtain

$$\begin{aligned}
[(\epsilon.u)(k.p)-(\epsilon.p)(k.u)]^2 &= [\epsilon_\alpha u_\alpha k.p - \epsilon_\alpha p_\alpha k_\beta u_\beta][\epsilon_{\alpha'} u_{\alpha'} k.p - \epsilon_{\alpha'} p_{\alpha'} k_{\beta'} u_{\beta'} \\
&= \epsilon_\alpha \epsilon_{\alpha'} \overline{u_\alpha u_{\alpha'}} (k.p)^2 - \epsilon_\alpha \epsilon_{\alpha'} \overline{u_\alpha u_{\beta'}} k.p p_{\alpha'} q_{\beta'} \\
&\quad - \epsilon_\alpha \epsilon_{\alpha'} \overline{u_\beta u_{\alpha'}} k.p p_\alpha k_\beta + \epsilon_\alpha \epsilon_{\alpha'} \overline{u_\beta u_{\beta'}} p_\alpha k_\beta p_{\alpha'} k_{\beta'} \\
&= \frac{1}{3}[4(k.p)^2 - (k.p)^2 - (k.p)^2 + p^2 k^2] \\
&= \frac{2}{3}(k.p)^2 \quad .
\end{aligned} \tag{C.15}$$

In the rest frame of ϕ meson $p\cdot k = M_\phi E_\gamma$ and $(p-k)^2 = (q_1+q_2)^2 = M_{KK}^2$.

If we consider the contribution of f_0 resonance only or a_0 resonance only, the above form of the scalar meson contribution is used. Both f_0 and a_0 resonances make a contribution to the decay $\phi \to K^0 \overline{K}^0 \gamma$ and they interfere destructively due to isotopic spin invariance. In this case, the complex amplitudes are parameterized with

$$\mathcal{M} = \left[\mathcal{M}''_{f_0} + \mathcal{M}''_{a_0}\right] + i\left[\mathcal{M}'_{f_0} + \mathcal{M}'_{a_0}\right] \ . \tag{C.16}$$

The absolute square of the invariant amplitude is now obtained as $\mid \mathcal{M} \mid^2 = \left[\mathcal{M}''_{f_0} + \mathcal{M}''_{a_0}\right]^2 + \left[\mathcal{M}'_{f_0} + \mathcal{M}'_{a_0}\right]^2$. The interference between f_0 and a_0 is destructive if the relations between the coupling constants are used as $g_{f_0 K^+ K^-} = g_{f_0 K^0 \overline{K}^0}$ and $g_{a_0 K^+ K^-} = -g_{a_0 K^0 \overline{K}^0}$.

VITA