MULTIAGENT MOVING TARGET SEARCH
IN FULLY VISIBLE GRID ENVIRONMENTS WITH NO SPEED DIFFERENCE

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MULTIAGENT MOVING TARGET SEARCH
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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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## Abstract

# MULTIAGENT MOVING TARGET SEARCH IN FULLY VISIBLE GRID ENVIRONMENTS WITH NO SPEED DIFFERENCE 

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In this thesis, a new real-time multi-agent moving target pursuit algorithm and a moving target algorithm are developed and implemented. The environment is a grid world, in which a coordinated team of agents cooperatively blocks the possible escape routes of an intelligent target in real-time.

Most of the moving target search algorithms presume that the agents are faster than the targets, so the pursuit is sure to end in favor of the agents. In this work, we relax this assumption and assume that all the moving objects have the same speed. This means that the agents must find a new approach for success in the pursuit, other than just chasing the targets. When the search agents and the moving targets are moving with the same speed, we need more than one search agent which can coordinate with the other agents to capture the target.

Agents are allowed to communicate with each other.
We propose a multi-agent search algorithm for this problem. To our best knowledge, there is no alternative algorithm designed based on these assumptions. The proposed algorithm is compared to the modified versions of its counterparts (A*, MTS and its derivatives) experimentally.

Keywords: Multi-Agent Moving Target Search, Moving Target Search, Real-Time Search.

## Öz

# TAMAMI GÖRÜLEN GRİD ORTAMDA ÇOKLU-AJANLA AYNI HIZDAKİ HAREKETLİ HEDEF TAKİBİ 

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Bu çalışmada, yeni bir çoklu ajan hareketli hedef kovalama algoritması ve hareketli hedef algoritması geliştirilecektir. Ortam olarak grid dünyası kullanılacak, bu ortamdaki koordineli ajan takımı hareketli hedefin olası kaçış güzergahlarını kapatmak için gerçek zamanda plan yapacaktır.

Çoğu hareketli hedef arama algoritması ajanların hedeflerden daha hızlı olduğunu varsayar, bu da takibin ajanların lehine sonlanmasına yol açar. Bu çalışmadaki ana amacımız, bu varsayımı hafifletmektir. Hedeflerin de ajanların da aynı hızda gittiğini varsayarsak, ajanların hedeflerini yakalamak için hedefleri kovalamaktan başka yeni bir yaklaşım bulmaları gerekir. Bu koşullarda kovalama işini eşgüdüm içinde yürüten birden fazla ajana ihtiyaç doğar.

Ajanlar arasında iletişim serbesttir.
Bu problem için bir çoklu ajan arama algoritması öneriyoruz. Bildiğimiz kadarıyla bu varsayımlara dayanarak tasarlanmış başka bir algoritma bulunmamaktadır. Önerilen algoritma, ona karşı gelen diğer algoritmaların türevleri ile (A*, MTS ve türevleri) deneysel olarak karşılaştırılacaktır.

Anahtar Kelimeler: Çoklu-Ajan Hareketli Hedef Takibi, Hareketli Hedef Arama, Gerçek Zamanlı Arama.

To my family.

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## Chapter 1

## Introduction

### 1.1 Motivations

If you were a kid, playing a simple computer game (e.g. pacman variations), and watching some agents chasing you, you wouldn't be amazed by the artificial intelligence or the coordination of the agents, because the non-cheating agents (which use the current algorithms in literature) wouldn't try to corner you, or cut your way, they will simply chase you. Clearly, the games we play today are not that boring. The games don't need to be fair, they need to be fun, so if your way has to be cut by some agent, then an agent is created on your way, or an existing agent gets teleported to your possible future path. Generally the artificial intelligence in the computer games cheats more, when the difficulty of the game is set to a higher level. We are interested in non-cheating algorithms, which can get similar satisfying results.

The paths mentioned above are computed by the path planning algorithms. The classic search algorithms, such as A*, RTA* and MTS, cannot get satisfying results on the problem mentioned above. These algorithms compute paths between a start node and a target node according to distance values, which can be determined by search (expensive) or by heuristics (inaccurate). The agents may not have enough time to make an optimal decision, they may have to decide on the next step in the shortest possible time. The algorithms that determine the next move in such a short time are called real-time search algorithms. A* algorithm computes the whole path before making the first move, where real-time search algorithms such as Real-Time A* plans its next move and executes it. Most path search algorithms are designed for stationary targets, but in computer games and many other domains, the targets may be moving.

Moving target search algorithms assume that the agents move faster than the target, to guarantee the capture of the target. Without this assumption, the target can evade capture. This is natural when there is only one agent. But if there were multiple agents (even 2 is enough in 8 neighborhood grid domain) controlled by humans, who have the full view of the grid map, no target would be able to elude the capture, even if they had the same speed. Unfortunately the algorithms that exist today are not capable enough to take advantage of multiple agents.

### 1.2 Goals of this Study

The aim of this thesis is to develop a multi-agent search algorithm, which takes advantage of cooperation among the team members for intervening the possible alternative paths of the moving target. We propose a coordinated hunter algorithm, in which an agent computes the shortest path to the target, but if there is another agent closer to the mid-point of that path, that agent can guard the path in question, therefore our agent searches for other alternative paths.

The environment is assumed to be a planar grid, and all of the agents have full vision of the grid and the coordinates of the other agents. Maps with different sizes and obstacle ratios were randomly generated in addition to the hand-crafted maps, on which more distinctive tests could be run for the cooperation of the agents. Agents are initially located at the same corner of the map, and the target is positioned randomly. The proposed algorithm is compared with A* and MTS derivatives.

The ultimate goal is to obtain shorter paths and shorter run times in the overall simulation instead of shorter computation time per turn.

Consider the case given in Figure 1.1: Assume that all agents move at the same speed, i.e., move concurrently. There are two agents $H 1$ and $H 2$, and a single target $P$. Traditional path search algorithms such as A*, RTA* would propose to move upward for both $H 1$ and $H 2$. However the best strategy for hunters is to move in opposite directions around the obstacle to capture the target, instead of just chasing the target. Any human would follow this simple strategy without hesitation. However if the hunters are controlled with the algorithms in the literature, the second hunter $H 2$ follows the target just like its partner, chasing the target step by step, they will continue to cycle around the obstacle forever. Even MAMTS algorithm, which is the algorithm with the maximum cooperation currently in the literature, goes up to the node 1.


Figure 1.1: Which path would you chose for H2?

The coordination of the agents, and planning the alternative path is our primary goal. All of the other algorithms that we used for comparison spend less time in every turn, but they generally cannot catch the moving target, so they will still be chasing the target when the target has already been captured by one of our agents. Eventually our algorithm is faster than our competitors at the overall pursuit time.

The proposed algorithm is fast enough to be considered as real-time, and there are several possibilities to minimize the time spent in each turn. The primary concern was to obtain shorter paths, rather than speeding up the algorithm to the utmost limits. The current speed of the algorithm is satisfying, although there are several possibilities to speed it up. These extensions and ideas are left out of the thesis scope for future work.

### 1.3 Outline of the Thesis

The organization of the thesis is as follows:
In Chapter 2, a survey of the related work on real-time search is given. The algorithms described in this chapter will be utilized to create the counterparts of our algorithm. These algorithms also inspired the proposed algorithms.

Chapter 3 starts with the detailed problem definition, then we introduce our "Moving Target Algorithm" and "Coordinated Agent Algorithm".

In Chapter 4, the test environment is described, and the comparisons with other algorithms are given in detail.

Finally, Chapter 5 summarizes the thesis and gives the conclusion. Possible improvement ideas to the proposed algorithms are given in the future work section.

## Chapter 2

## Related Work

There are algorithms in the literature that are designed for moving target search problem, but do not involve cooperation (e.g. MTS described in Section 2.5), and there are algorithms that are capable of cooperation, but they are not designed for moving target search problem (e.g. MARTA* described in Section 2.6). In order to test our algorithm against the existing ones, we combined the algorithms in the literature so that they can overcome our problem.

There is only one algorithm designed specifically for the Multi-agent Moving Target problem in the literature. It is Multi-agent Moving Target Search algorithm described in Section 2.7, but this algorithm lacks cooperation when the agents know the target's position. Its deficiency is illustrated in the previous chapter. Therefore, as already stated, we combined the regular algorithms in the literature.

The original algorithms that may be used for comparison are not designed for this problem. Therefore they cannot utilize the advantage of being a team of agents. Although the extended versions of the algorithms give better results than the original ones, their cooperation level is also low. This ends up in a disappointing performance, which lets the target evade. We should declare that it was not fair to use the mentioned algorithms and their extended versions for comparison, since we are relaxing their assumptions, but we have no other alternatives.

The algorithms described here, are the core algorithms of the real-time search domain, so they have been utilized and inspired the development of our algorithm. Extended versions of the algorithms will be introduced later.

### 2.1 A*

The best known form of best-first search [1] is called A* [1, 2]. The nodes are evaluated by combining $g(n)$, the cost to reach that node, and $h(n)$, the estimated cost of the cheapest path from $n$ to the goal. A heuristic function is used for this estimation.

Path Score F: $f(n)=g(n)+h(n)$ where

- $g(n)=$ the movement cost to move from the starting point to a given square on the grid $(\mathrm{n})$, following the path generated to get there.
- $h(n)=$ the estimated movement cost to move from that given square (n) on the grid to the final destination. This value is called the heuristic value.

A* computes the whole path at the beginning before moving, so it is called an off-line search algorithm. Therefore A* needs frequent replanning in partially visible or dynamic environments (when an obstacle appears on the precomputed path, replanning is mandatory). A* is not suitable for large dynamic environments because of the time requirement of the algorithm.

Generally heap data structure is used in A* implementations. The open list mentioned in Algorithm 1 is stored in a heap, and the list is kept sorted on each insert operation with a worst case complexity of $\log (n)$ where $n$ is the length of the list.

In worst case, $\mathrm{A}^{*}$ algorithm expands all of the nodes in the grid, so the worst case complexity of A* algorithm is $O(w * h)$ where w is the width and h is the height of the grid.

In moving target search problem, the agent positions are constantly changing, forcing this algorithm to replan the whole path at each turn.

## Algorithm 1 A* <br> 1. Add the starting node to the open list

2. Repeat the following:

- Look for the lowest F cost square on the open list. We refer to this as the current node.
- Switch it to the closed list.
- For each of the adjacent nodes to this current node, do
- If it is not walkable or if it is on the closed list, ignore it. Otherwise do the following.
- If it isn't on the open list, add it to the open list. Record the F, G, and H costs of the square.
- If it is on the open list already, check to see if this path to that node is better, using G cost as the measure. A lower G cost means that this is a better path. If so, change the parent of the square to the current square, and recalculate the G and F scores of the node.
- Stop when you:
- Add the target node to the closed list, in which case the path has been found, or
- Fail to find the target square, and the open list is empty. In this case, there is no path.

3. Save the path. Working backwards from the target node, go from each node to its parent node until you reach the starting node. Path is completely computed.

### 2.2 Real-Time A*

Real-time A* algorithm is proposed as an online search algorithm, meaning that it doesn't compute all the path before executing the first move. RTA* interleaves planning for a plausible next move and executes the move.

RTA* does not compute the optimal path, but it can come to a solution much more quickly than $\mathrm{A}^{*}$. If a better quality solution is needed, the look-ahead $d$ can be increased. Note that the search time grows exponentially.

The worst case complexity for each turn is $O\left(d^{2}\right)$, since only $d^{2}$ many nodes are expanded. If the look ahead depth is 1 , only the neighbor nodes will be expanded.

RTA* updates the heuristic values with the second-best estimated cost, because the agent goes to the best estimated cost. If there is a solution available from that node, the agent will not come back to the updated node, and the miscalculated heuristic will never be read by anyone. If it comes back to that node, then the heuristic is more accurate, because the best estimated cost was leading to a dead end, making our agent return back from that path.

We don't use RTA* in our tests, because MTS is an extension of RTA*, which is more useful in our problem domain.

## Algorithm 2 RTA*

Step1 [Initializing] Set $s_{x}=s_{I}$.
Step2 [Expansion] Expand $s_{x}$ and let $C\left(s_{x}\right)$ to be set of the child states.
Step3 [Termination?] If there exists a goal state in $C\left(s_{x}\right)$, then move to the goal state and quit.

Step4 [Look-ahead search] For all $s_{y} \in C\left(s_{x}\right)$, calculate $f\left(s_{x}, s_{y}\right)=c\left(s_{x}, s_{y}\right)+f\left(s_{y}\right)$ which is the estimated cost from $s_{x}$ to the goal state through $s_{y} . f\left(s_{y}\right)$ is calculated from a look-ahead search of depth $d$ from $s_{y}$ as follows.

$$
f\left(s_{y}\right)=\min _{s_{w} \in W\left(s_{y}, d\right)}\left[c\left(s_{y}, s_{w}\right)+h\left(s_{w}\right)\right]
$$

where $W\left(s_{y}, d\right)$ is the set of leaves of the look-ahead search tree and $c\left(s_{y}, s_{w}\right)$ is the actual cost from $s_{y}$ to $s_{w}$ known at the stage.

Step5 [Choice] Choose the best child state $s_{y}^{\prime}$ with $f\left(s_{x}, s_{y}^{\prime}\right)\left(=\min _{s_{y} \in C\left(s_{x}\right)} f\left(s_{x}, s_{y}\right)\right)$. Ties are broken randomly.

Step6 [Estimation update] Update $h\left(s_{x}\right)$ to be $f\left(s_{x}, s_{y}^{\prime \prime}\right)\left(=\min _{s_{y} \in C\left(s_{x}\right)-\left\{s_{y}^{\prime}\right\}} f\left(s_{x}, s_{y}\right)\right)$ which is the estimated cost of the second-best child. If there is no second-best estimation, let $h\left(s_{x}\right)=\infty$.

Step7 [Move] Set $s_{x}=s_{y}^{\prime}$.
Step8 Go to Step2.

### 2.3 Learning Real-Time A*

If multiple agents are sharing the same updated heuristic values, the update method of RTA* may slow down the solution process. Consider using RTA* in such a situation, when an agent finds the way to the target or gets closer to the target, all of the other agents would check for the second-best estimations on that path, rather than the best estimation (because some agent already used that node, although that agent may not have checked all of the possible paths crossing that node), so the other agents will find the path slower. The only difference between LRTA* and RTA* is the update rule. LRTA* updates the heuristic value of the node with the best estimated value, in order to overcome the problem mentioned above. LRTA* is used in MARTA* just as described above.

The worst case complexity for each turn is the same as RTA*'s complexity $O\left(d^{2}\right)$, since only $d^{2}$ many nodes are expanded.

## Algorithm 3 LRTA* <br> Repeat until the problem solver reaches a goal state.

1. Let x be the current position of the problem solver.
2. Calculate $f\left(x^{\prime}\right)=h\left(x^{\prime}\right)+k\left(x, x^{\prime}\right)$ for each neighbor $x^{\prime}$ of the current state x , where $\mathrm{h}\left(\mathrm{x}^{\prime}\right)$ is the current heuristic estimate of the distance from $\mathrm{x}^{\prime}$ to the goal state, and $k\left(x, x^{\prime}\right)$ is the cost of the edge from $x$ to $x^{\prime}$.
3. Move to a neighbor with the minimum $f\left(x^{\prime}\right)$ value. Ties are broken randomly.
4. Update the value of $h(x)$ to this minimum $f\left(x^{\prime}\right)$ value.

### 2.4 Dynamic A*

Dynamic A* $[3,4]$ (also called $\mathrm{D}^{*}$ ) is an extension of A*, which can handle dynamic environments, but it is designed for stationary targets. Dynamic A* performs an offline search at the beginning of the execution. This search has the same complexity of A*, but during the execution, the replanning costs are minimized.

Instead of heuristic values, $D^{*}$ stores distance values computed by an offline search (if full view is obtained, then the distance values are accurate).

To explain how $\mathrm{D}^{*}$ works, we can briefly say that, when a node is changed in the
environment (it was assumed to be empty, but now there is an obstacle), $\mathrm{D}^{*}$ checks the validity of the distance value with the distance values of the neighbors of the changing node. If a distance value is not valid anymore (because of the change in the grid), that distance is recalculated (by a partial search).

When all of the distance values of the neighbors of the changed node is valid, $\mathrm{D}^{*}$ continues to execute the moves.

The computed path is optimal in the sense that the path which seems to be the shortest path is computed. That path may turn out to have obstacles on the road, so it may not be optimal.

We explained $\mathrm{D}^{*}$ here because the complexities of our algorithms are the same with $\mathrm{D}^{*}$ 's complexity, and there are similarities in our methods.
$\mathrm{D}^{*}$ 's worst case complexity is the same as $\mathrm{A}^{*}$ 's complexity $O(w * h)$ where $w$ is the width and $h$ is the height of the map. $\mathrm{D}^{*}$ 's important contribution is a new approach for fast replanning.

If we compare $D^{*}$ with $A^{*}$, we will see that replanning with $D^{*}$ is faster than replanning with $\mathrm{A}^{*}$, because replanning with $\mathrm{A}^{*}$ means replanning all the path from the start, without using any prior knowledge.

Although D* can handle dynamic environments, it cannot handle moving targets. When the target moves, heuristic values for all of the nodes must be recomputed. Running A* each time the target moves, has the same complexity with this process. Under moving target assumption, $\mathrm{D}^{*}$ looses all of its advantages, so we will not use this algorithm in our tests, but this algorithm is an inspiration for our proposed algorithms.

```
Algorithm 4 D* Lite Second Version
    procedure CalcKey ( \(s\) )
    return \(\left[\min (g(s), r h s(s))+h\left(s_{\text {start }}, s\right) ; \min (g(s), r h s(s))\right] ;\)
    procedure Initialize()
    forall \(s \in S, \operatorname{rhs}(s)=g(s)=\infty ;\)
    \(U=0 ; \operatorname{rhs}\left(s_{\text {goal }}\right)=0 ;\)
    U.Insert \(\left(s_{\text {goal }}, \operatorname{CalcKey}\left(s_{\text {goal }}\right)\right)\);
    procedure UpdateVertex \((u)\)
    if \(\left(u \neq s_{\text {goal }}\right) r h s(u)=\min _{s^{\prime} \in \operatorname{Succ}(u)}\left(c\left(u, s^{\prime}\right)+g\left(s^{\prime}\right)\right)\);
    if \((u \in U)\) U.Remove \((u)\);
    if \((g(u) \neq r h s(u))\) U.Insert \((u, \operatorname{CalcKey}(u))\);
    procedure ComputeShortestPath()
    while \(U . \operatorname{TopKey}()<\operatorname{CalcKey}\left(s_{\text {start }}\right)\) OR \(r h s\left(s_{\text {start }}\right) \neq g\left(s_{\text {start }}\right)\) do
        \(u=U . \operatorname{Pop}() ;\)
        if \(g(u)>\operatorname{rhs}(u))\) then
            \(g(u)=r h s(u) ;\)
            forall \(s \in \operatorname{Pred}(u)\), UpdateVertex \((s)\);
        else
            \(g(u)=\infty ;\)
            forall \(s \in \operatorname{Pred}(u) \cup u\), UpdateVertex \((s)\);
        end if
    end while
    procedure Main()
    Initialize();
    ComputeShortestPath();
    while \(s_{\text {start }} \neq s_{\text {goal }}\) do \(\left\{\operatorname{if}\left(g\left(s_{\text {start }}\right)=\infty\right)\right.\) then there is no known path \(\}\)
        \(s_{\text {start }}=\operatorname{argmin}_{s^{\prime} \in \operatorname{Succ}\left(s_{\text {start })}\right.}\left(c\left(s_{\text {start }}, s^{\prime}\right)+g\left(s^{\prime}\right)\right) ;\)
        Move to \(s_{\text {start }}\);
        Scan graph for changed edge costs;
        if any edge costs changed then
            forall directed edges \((u, v)\) with changed edge costs
            Update the edge cost \(c(u, v)\); UpdateVertex \((u)\);
            foralls \(\in U\) do U.Update \((s\), CalcKey \((s))\);
            ComputeShortestPath();
        end if
    end while
```


### 2.5 Moving Target Search

The Moving Target Search (MTS) algorithm is a LRTA* variant which can handle a moving target[5, 6]. MTS (without extensions) is a simple algorithm, only the update conditions of the LRTA algorithm are modified. The core idea is the same. Agent goes to the neighbor with the smallest heuristic, and updates accordingly. The worst case complexity for each turn is the same as LRTA*'s complexity $O\left(d^{2}\right)$, since only $d^{2}$ many nodes are expanded.

Enhancements to this basic idea include the addition of notions of commitment and deliberation to the agent[6]. An assumption made in most MTS papers, is that the target moves slower than the agent. Without this assumption, the target can stay ahead of the agent and, in many cases, elude capture.

## Algorithm 5 MTS <br> When the problem solver moves

1. Calculate $h\left(x^{\prime}, y\right)$ for each neighbor $x^{\prime}$ of $x$.
2. Update the value of $h(x, y)$ as follows:

$$
h(x, y) \leftarrow \max \left\{\begin{array}{l}
\mathrm{h}(\mathrm{x}, \mathrm{y}) \\
\min _{x^{\prime}}\left\{h\left(x^{\prime}, y\right)+1\right\}
\end{array}\right\}
$$

3. Move to the neighbor $\mathrm{x}^{\prime}$ with the minimum $\mathrm{h}\left(\mathrm{x}^{\prime}, \mathrm{y}\right)$, i.e., assign the value of x ' to x . Ties broken randomly.

When target moves:

1. Calculate $\mathrm{h}\left(\mathrm{x}, \mathrm{y}^{\prime}\right)$ for the target's new position y '
2. Update the value of $\mathrm{h}(\mathrm{x}, \mathrm{y})$ as follows:

$$
h(x, y) \leftarrow \max \left\{\begin{array}{l}
\mathrm{h}(\mathrm{x}, \mathrm{y}) \\
h\left(x, y^{\prime}\right)-1
\end{array}\right\}
$$

3. Reflect the target's new position as the new goal of the problem solver, i.e. assign the value of $y^{\prime}$ to y .

In the design of the MTS algorithm, there is no coordination between the agents. Moving Target Search algorithm uses heuristic values, not real distances like $\mathrm{D}^{*}$
algorithm. Using heuristic values takes less computation time each turn, but since the values are not accurate (they converge too slowly) the agent's path doesn't look like an intelligently planned path, because of the oscillation during the heuristic depression. In order to get rid of heuristic depression, deliberation extension is developed for Moving Target Search. To achieve shorter paths, we prefer to use real distances in our tests, knowing that more time will be spent.

### 2.6 Multi-Agent Real-Time A*

Multi-Agent Real-Time A* [7] is a multi-agent version of RTA* where multiple agents concurrently search and move to find a solution. The aim is to improve the quality of the solution by increasing the number of agents engaged in the search. MARTA is originally designed for stationary targets, so there is no search for any alternative escape route, by which the target can evade. In MARTA algorithm, every agent performs a modified RTA algorithm, and a LRTA algorithm simultaneously. The heuristic values updated by the LRTA algorithm is shared between the agents. So the heuristic conversion becomes faster with multiple agents, but every agent also maintains their local heuristic table updated by the RTA algorithm, so that the search time does not increase because of LRTA.

When an agent is trying to choose its next step by comparing the $f$ values of its neighbors, there may be more than one best node. In regular RTA* versions, ties are broken randomly, but in MARTA, the organizational strategies are taken into consideration in ties. If there is still a tie condition, then those ties are broken randomly.

MARTA has 2 organizational strategies: repulsion and attraction.
Repulsion is expected to strengthen the discovering effect by scattering agents in a wider search space. In contrast, attraction is expected to strengthen the learning effect by making agents update estimated costs in a smaller search space more actively. When there are 2 nodes, whose f values are the same, MARTA with repulsion chooses the node which is further away from the other agents, where MARTA with attraction chooses the node which is closer to the other agents. Repulsion is used in grid domains, where the discovering effect is more important than the learning effect. Attraction is used in domains like 8-puzzle problem, where the learning effect is more important than the discovering effect.

The worst case complexity for each turn is the same as LRTA*'s complexity $O\left(d^{2}\right)$, since only $d^{2}$ many nodes are expanded. If tie condition arises in a turn, the distances between the other agents and the nodes in the tie condition are computed. Generally the agent number is a small number, which is not necessary to mention in the complexity calculations. Extreme numbers of agents are not usual in this domain, so it may be neglected. To be accurate we should mention that the distance computation in tie condition increases the worst case complexity to $O\left(d^{2} * a\right)$ where a is the agent number. There are $d^{2}$ expanded nodes, but the computation for each node includes $a-1$ distance computations.

Although this algorithm is not designed for our problem domain, we will combine this algorithm's repulsion cooperation technique with other algorithms (A* and MTS) to create new algorithms for comparison.

### 2.7 Multi-Agent Moving Target Search

In [8] Goldenberg et al. proposed an algorithm for Multi-Agent Moving Target Search problem. The original design of the MAMTS algorithm is capable of handling the environments with partial visibility and partial communication.

MAMTS algorithm maintains a belief set for the target position. In each turn, the belief set is updated with the possible positions of the target. If the target is detected, the belief set is cleared. If the target is not in sight, the new possible positions of the target are added to the belief set. The beliefs that turn out to be wrong are eliminated. Then the belief set is filtered to find the corner nodes of the belief set. These filtered corner nodes will be called as the filtered belief set. The agents select their goals from this filtered belief set. Each agent computes the minimum(distance to belief - (minimum distance between the belief and the other agents)) for all of the possible goal nodes, and the node with the minimum value is set as the goal of that agent. The goal selection step is the cooperation method of MAMTS algorithm. Every agent selects its goal cooperatively. After the goal is selected any search algorithm can be performed to navigate to the goal. The choice of the algorithm is left to the implementer.

Figure 2.1 (left) illustrates the different belief sets. The agent (A) sees the target (T) and then both of them move twice. The belief set is represented by the shaded squares.


Figure 2.1: MAMTS: Belief sets (left) and filtered belief set (right)

In Figure 2.1 (right), the squares marked F1, F2, F3 and F4 are selected as approximations for the corners of the belief set for agent A1.

The problem with MAMTS algorithm is that if the target is visible, then the belief set is empty. Therefore all of the agents have the same goal: the Target! The cooperation feature is not utilized, because, instead of MAMTS, a simple path planning algorithm is executed.

Even in a simple map with just one obstacle in the middle and a visible target (just like Figure 1.1), two agents would continue to cycle around the obstacle forever.

In our problem domain, the target is always visible, so MAMTS algorithm is not different from any of the other algorithm in the literature. Therefore this algorithm is not used in our tests.

Our algorithm may be executed in MAMTS when the target is visible. The combined algorithm will have shorter paths.

## Chapter 3

## Multi-Agent Moving Target Pursuit

### 3.1 Problem Description

The following paragraphs describe the experimental domain used in this thesis. The domain can be extended to be more realistic. However, even the simplified domain used here is quite challenging. The assumptions that can be relaxed are explained in the future works section, in detail.

The test domain has the following properties:

1. Agents and target: There are multiple agents pursuing a single moving target. The number of agents differs between 2 to 4 in our tests.
2. Grid: The grid is $m * n$ in size and it is static. All agents and the target have complete knowledge of the grid topology. These assumptions can be relaxed. Two types of grids are generated for testing:

- Maps with random obstacles
- Hand-crafted maps

Maps with random obstacles are classified in two groups. The maps with big obstacles, and the ones with small obstacles.
3. Moves: The target and the agents can move horizontally, vertically or diagonally (8 neighborhood grid domain). All the moves have the same cost. Although
this property is not realistic, it is easier to implement, easier to simulate and already used in some computer games like Civilization and its derivatives[9]. This assumption can be changed. Diagonal moves may cost more, but this environment configuration is in favor of the algorithms that we use for comparison. The tie condition has a higher probability when all the possible moves have the same cost.

Generally, nodes in the MTS tests have 4 neighbors (diagonal moves are not permitted). But with this configuration, the pursuit gets longer, and when the target and the agent move with the same speed, depending on the grid topology, it may be impossible for one agent to catch the target. The target doesn't even need an obstacle to move forever in a loop.
4. At each time step, the target moves first, then the agents next.
5. Multiple agents are allowed to occupy the same square at any instance in time.
6. Starting position: Agents start at the same corner of the map, next to each other. Target is positioned randomly.
7. Vision and communication: The agents and target are given full vision in our experiments, so every one has complete information. Everyone knows the positions of the other agents and the target. Communication is allowed between all agents. We choose to give every one, all the information, to get better results.
8. Objective: The multiple agents have to capture the target in the fewest possible moves. Capture is defined as being on the same square with the target.

### 3.2 Agent Design

Calculating real distances takes more time, but gives us better paths and even the total processing time may be smaller, just like $\mathrm{D}^{*}$ may be faster than RTA*. Both of the algorithms proposed below are not RTA* derivatives, they are much similar to A* and $\mathrm{D}^{*}$.

### 3.2.1 Moving Target Algorithm

In order to test our agent algorithms, we need a good target algorithm, because the pursuit length and success is heavily dependant on the targets decisions. Simple
algorithms like avoiding (going the opposite direction of the nearest agent) or random behavior are insufficient for our tests. We need a successfully escaping target to test the cooperation of the agents. To perform more realistic tests we required a target algorithm, which can survive as long as it could.

Ties are broken randomly. This random effect changes the escape route, and therefore changes the survival length of the target. While traveling to that location, the agent positions change, making us repeat this computation in every turn, so that the target does not go to the corner of the map, and stay there unless it is cornered. It moves to a new position while the agents are getting closer. When the target is cornered, it finds the location which it will survive the longest, and waits in that spot (some dead end in the grid).

After we choose the best location for the target, we do not check if there is a safe path from the current location to the target location, because we only expand safe nodes, so there is a safe path. Therefore the algorithm runs fast when the target is cornered, because there is limited number of safe nodes to expand.

The algorithm computes the shortest paths for all of the agents, so the worst case complexity of a turn is $O(n * m * a)$, where $a$ is the total number of agents and the target. This data is shared with the agents for purposes of speed, and cached for future use. Since the grid is static and fully visible, the cached values can be trusted in the future turns, no extra check or maintenance is needed. The target algorithm does not need to compute all of this data. It is calculated for all of the agents including the target. The target only needs the distance values to its positions, so if we want to compute the complexity of the target only, the complexity would be $O(n * m)$.

If all of the agent positions are met before (the distances were cached), the computation of the shortest paths complexity is $O(1)$ for that turn. This is the best case for the shortest paths computation. In this case, the rest of the target decision method becomes the important variable for the algorithms complexity. In the worst case, the target would expand half of the open nodes, which gives us the complexity of $O(n * m)$. Best case would occur when the target is cornered, where one turn is left before the capture and the target don't have any safe node, so there will be no computation, which gives us the complexity of $O(1)$.

Although our target algorithm is not capable of detecting cycles, and using them, it is a good runner. Since their speeds are the same with the agent, the target does not

## Algorithm 6 Moving Target Algorithm <br> 1. Perform Breadth First Search for all agents to calculate real distances, cache them for further use. (This data is shared with all agents and targets, to increase speed) <br> 2. Add the current location to heap (The location with the lowest G cost is located on top of heap)

3. While heap is not empty:

- Current Location = pop Heap (lowest G cost)
- Switch it to the closed list
- if $G>=H$ (an agent is closer to this location than the target is) continue; This location is not safe for the target.
- Add this location to the Possible Results List for further examination
- For each neighbor of this current square
- If it is not walkable or if it is on the closed list, ignore it. Otherwise do the following.
- If it is not on the open list, add it to the open list. Calculate the G, and H costs of the square. $G=G+1$ and H is the distance to the nearest agent.
- Stop when the open list is empty.
- Find the location in the Possible Results List with the maximum H value (which is the possible location, at which the target can have the longest lifetime).
get stuck, if it cycles around an obstacle. On many tests this target algorithm evaded the capture of the known agent algorithms. After some time over these successful escapes, all of the agents were following the target on the same direction. There was absolutely no coordination between the agents.


## Possible Improvements

If the map has long paths with dead ends, the algorithm may choose to go to that dead end, although there is a cyclic path which it can take and survive longer. This property of the algorithm needs improvement.

### 3.2.2 Coordinated Agent Algorithm

We now present our Coordinated Agent Algorithm: named as Multi-Agent Moving Target Pursuit, which is the main contribution of this thesis.

Here, how it works: Consider a path between an agent and a target. We calculate the midpoint of that path. If the agent is the closest agent to the calculated point, then it is the most suitable agent to guard the related path. In order to find the most suitable path for an agent, we compute the paths to the target. Since we know the real distances, this computation is easy. We perform a derivative of A* search. When we are at the midpoint of a path, we check whether the referred agent is the closest agent to that point. If our agent is the closest one, we stop computation, setting the goal of the agent as this point, and the agent moves to its appropriate neighbor, which will lead to that midpoint. We don't need to check the rest of the path, because we use the real values, so we know there is a path to the target with the specified length. We also do not need to check for the other agents for the rest of the path. There may be agents closer to the nodes in the rest of the path, but it does not mean that those agents can guard the path better than our agent.

As you can see in Figure 3.1, the shortest path length between target and $H 2$ is 4, but $H 1$ covers the referred path. So H2 computes another path which goes over the obstacle, through node 2 then node 1 , to the target. The path length is 8 . The node marked as 2 is the mid-point of that path. Distance of $H 2$ to node 2 is $4(g()$ value), same as the target's distance to node $2(h()$ value $) . H 2$ is closer to node 2 , than $H 1$. Therefore, this path should be guarded by $H 2$.

To clarify the point that we do not need to check the nodes in the rest of the


Figure 3.1: The Mid-Point Explanation
path; lets check for a node that is beyond the mid-point (e.g. node marked as 1) on the computed path of $H 2$, and see how it will mislead us. $H 1$ is closer to the node marked as $1(5<6)$ than $H 2$. Although $H 1$ is closer, it is obvious that he cannot guard that path, better than $H 2$. Therefore, the nodes beyond the midpoint should not be checked for the closest agent, because this information does not mean that those agents can guard those paths, since the target may be closer to that point than all of the agents.

We make this calculation at every turn. If the agent does not recompute and follow its precomputed path, the time spent at each turn will be reduced, but the total distance and the number of turns will increase.

If the agent cannot find any path, meaning either there is no path, or the other agents are already guarding all of the paths, then the agent may be considered unneeded. In such a case the agent may stop, but instead of stopping, we let our agent follow the shortest path to the target. Although there is another agent already guarding the related route, our agent, as a second one, may come in handy at the same route.

Consider the case: if the target starts to cycle around an obstacle, it would be impossible for only one agent guarding that route, to succeed on his own. When the target completes the tour around the obstacle, he may start to walk on the path, once guarded by the agent, who has fallen behind now. At that point, the second agent, who choose this path instead of stopping, would start to run towards the target (since the first agent would be closing the other direction). If the agent followed the shortest path, instead of stopping, he would be closer to the target. Hence when an agent cannot find an alternative path to guard, we let him follow the shortest path, which is already guarded. This behavior takes an insignificant computation time, but it may shorten the overall pursuit time significantly.

Different agents should not try to guard the same escape route, but it is difficult


Figure 3.2: MAMTP Example
to differentiate the routes, especially if you are using heuristic values, which does not utilize the grid information. Two agents may have planned two different paths, but one of them may be able to guard both of those paths without changing its original path. If there is an obstacle separating the paths, then both of those agents may be necessary for guarding those paths.

As you can see in Figure 3.2(a) the agent $H 1$ cornered the target $P$, so $H 2$ cannot find a path that $H 1$ does not guard. Heuristic values are valid for this grid, because there isn't any obstacle in the map.

If there were an obstacle separating the paths, the heuristic values become unsound. As you can see in Figure 3.2(b) the agent $H 1$ couldn't corner the target $P$ just by himself, and $H 2$ computes a path that $H 1$ cannot guard. The agent positions and the target position are the same, but an obstacle changes the paths.

The heuristic functions cannot utilize environmental information. If we used heuristic values instead of real distances, H 2 would not be able to find any alternative escape route in Figure 3.2(b) (even if there is one, which he should guard), so he would choose to follow the other agent, since this is the shortest path.

The complexity of our agent algorithm is similar to the target algorithm. It also needs to compute the shortest paths to other agents. Therefore the worst case complexity is $O(n * m * a)$. If all the paths are already in the cache, then the rest of the method has a worst case (target is too far, expanded node number is written in terms of $n * m$ ) complexity of $O(n * m)$ and a best case (the target is about to be captured) complexity of $O(1)$.

```
Algorithm 7 Multi-Agent Moving Target Pursuit Algorithm
Require: \(s_{i}\) \{Initial location\} target \{Target Location\}
    for all agents do
        if Real Distances of agent location is not calculated yet then
            Calculate Real Distances from this agent to other points of the grid
        end if
    end for
    Heap.push \(\left(s_{i}\right.\), RealDistanceToPrey \(\left.\left(s_{i}\right), 0\right)\) \{Arguments: Location, H, G\}
    while Heap \(\neq\) Empty do
        \{Let x be the Current Location
        \(x \Leftarrow \operatorname{PopHeap}()\{\) Location with the lowest \(\mathrm{F}(=G+H)\) value is popped \(\}\)
        \(x\) is closed
        if \(x=\) target then
            break \(\{\) Path found \(\}\)
        end if
        if \(G>=H\) then \(\{\) This is the midpoint between the target and the agent \(\}\)
            if this agent is the closest agent to this mid-point then
            break \{Path found \(\}\)
        end if
        end if
        for all Neighbors of \(x\) do \(\{\) Let the neighbor be \(N\}\)
            if \(N\) is not valid OR not walkable OR on the closed list then
            continue
            end if
            if \(N\) is not opened already then
            Open \(N\)
            Set \(x\) as the parent of \(N\) \{Parents are used to find the path\}
            Heap.push \((N\), RealDistanceToPrey \((N), x . G+1)\)
            end if
        end for
    end while
    if No path Found then
    Follow the path to the target
    else if Path found then
        Follow the path
    end if
```


## Chapter 4

## Experimental Results

### 4.1 Test Environment

Our target algorithm is used for planning the target movement. The computation of the target's escape route is affected by random criterions and the agent positions which are also affected by random values. Since the path lengths and the pursuit success rate is strongly dependant on the targets escape route, a few test runs would not be enough for making decisions. Therefore, we have run each environment configuration (same map, with same initial agent positions and same initial target position) 100 times, and it can be seen from the results that the algorithms doesn't always give the same result.

Length limit for the maximum pursuit is set as $10 *(n+m)$. If the target is still alive after the limit has passed, we stop the execution.

Hand-crafted maps have different map sizes varying from $10 x 10$ to $35 x 35$. The randomly generated maps have sizes of $10 x 10,20 x 20$, and $30 x 30$, with obstacle ratios of $10 \%, 20 \%$, and $30 \%$. The randomly generated maps are grouped in two classes: the maps with big obstacles and the ones small obstacles (independent from the obstacle ratio). The maps with small obstacles are generated with $1 x 1$ building blocks of obstacles. The maps with bigger obstacles are generated with $2 x 2$ building blocks of obstacles. The obstacle ratios are the same. In the maps with small obstacles, the obstacles are scattered around the grid. They are more dispersed. The obstacles in the other maps are united, they form bigger obstacles.

We have 10 maps for each random configuration of maps. As a total, we have 180 random generated maps ( 90 maps with smaller obstacles and 90 maps with bigger obstacles), and 30 hand generated maps. We have run each algorithm a hundred
times on each map, so as a total we executed 105000 runs.
Since we want to test the coordination between the agents, we want to eliminate the chance effects in our tests. We also require the best performance from our target, so that any effect caused by the deficiency of the target algorithm would be eliminated. It was mentioned that our target algorithm has a weakness about the dead ends. Even only one agent may be enough for capture on maps with long dead ends. These maps are not appropriate for testing the coordination between the agents.

In order to gain the maximum performance from our target algorithm, specific maps, which do not contain dead ends at the corners, are generated. By this strategy, we force our target to make use of the cycles in the map.

If the agents are placed far from each other, they guard different escape routes without cooperating. Therefore agents are placed at the same corner of the map, so that they have to cooperate to guard different escape routes. The target is placed somewhere in the middle of the map. (This does not affect the result significantly. The locations and the results differ slightly)

The test metrics used in the tests are: average path length over all of the runs, average path length over the successful runs, success rate of the algorithm, and total CPU time spent during the pursuits (both successful and unsuccessful ones).

In the calculation of the average path length over all of the runs, the maximum limit is used as the path length, at which the agents fail to capture the target. So when the success rate is $0 \%$, the result becomes $100 * 10 *(n+m)$, which appears to be too large. Since most of the current algorithms have a low success rate in the tests, we choose to print the average path length over the successful runs instead of average path length over all of the runs in the tables, in order to achieve better perception.

The average path length over all of the runs, may be a metric, which can be used on its own. But the average path length over the successful runs may be misleading without the success rate. Since some algorithms are successful only in a small percentage of the runs, in which the path lengths may be short. These algorithms may seem to be better than the other algorithms in terms of average path length over successful runs, but the other algorithms perform better than these algorithms. According to the target's moves, and the random effect, the pursuit may become a short one, and these algorithms may capture the target only in these short runs. But since the other algorithms with higher success ratios, are also successful in longer
pursuits, their average pursuit length gets higher. Therefore, this metric should be considered together with the success ratio of the algorithms.

Finally, the test algorithms used in the tests are:
Multi-Agent Moving Target Pursuit: Our algorithm that we introduced in Section 3.2.2. (Abbreviated as MAMTP in some small figures)

MTS: Regular Moving Target Search algorithm, described in Section 2.5.
MTS with Repulsion: Since there is no coordination between the agents in regular MTS algorithm, we modified the MTS algorithm and added the same repulsion organization strategy used in MARTA*. So there is minimal cooperation between the agents (which is insufficient). When there are possible neighbor nodes with equal distances to the target, the agent prefers the path which is further from the other agents. (Abbreviated as MTSRep in some small figures)

A*: This is the regular A* algorithm, described in Section 2.1. The agents compute all of the path again after each move.

A* with Repulsion: Since there is no coordination between the agents in regular A* algorithm, we modified the $\mathrm{A}^{*}$ algorithm and added the same repulsion organization strategy used in MARTA*. (Abbreviated as Arep in some small figures)

### 4.2 Test Results

If the map does not have any dead ends that the target may fall into (which is the case in the test maps), coordination between the agents is mandatory for the capture.

As predicted before the algorithms in the literature that we used for comparison cannot utilize the advantage of multi-agents as good as they should. As a result they cannot capture the target in most of the hand made maps. The average success ratios are given in the lines of the Table 4.1.

The obvious result of the tests is that the proposed algorithm has a higher success ratio in all of the tests. Our algorithm successfully captured the target in all of the tests run on the generated maps. In Table 4.1 you can see that our algorithm failed on some of the tests run on two hand-crafted maps (11 and 19), but the other algorithms had a lower success ratio on the same maps, so our algorithm can still be called successful on these maps. Since the agents have the same speed as the targets, there are maps that are not solvable by just 2 agents, but this was not the reason of
the mentioned failures.
When we take a closer look at the failures, we recognize that our algorithm has a handicap. Although stopping is an option for our agents, it is never chosen. Our algorithm always computes a path, and move on that path, but there are times that an agent must stop, and wait for the other agent to get closer to the target. In the above-mentioned failures, we realize that the agents who are closer to the target, always chases without stopping, and the other agents try to block the alternative escape route. But before an agent can block a route, the target heads for another route, so the agent goes for the other route, but he is always late. (When he is not late, that run is among the successful ones)

Although our algorithm did not fail in all of the test runs of a map, a special map can be crafted on which our algorithm will always fail. This situation is not a common situation, so a situation like this didn't come up in a randomly generated map. But improvement is needed, so that the success rates and also the path lengths will be improved.

Our algorithm seems to have longer path lengths than MTS-Repulsion on some tests These results can be seen in Table 4.2 on maps $7 \& 10$ and on the average. On maps with $10 \%$ obstacle ratio with small obstacles, our algorithm is not needed. Even A* algorithm can capture the target. We can see that cooperation is not needed in this test case. When we take a closer look at these tests, we see that the target cannot hide behind any obstacle. There is no obstacle big enough which the target can utilize. The target goes to the further corner of the map, and waits for the capture. But if the obstacles are bigger with the same map size and the same obstacle ratio, the success ratio of the other algorithms start to fall. This can be seen in Figure 4.5 and Table 4.11. So we can see even if the obstacle ratio is $10 \%$, if the target can find obstacles big enough to utilize, he can evade capture, but it could not evade our algorithm in any of the randomly generated maps.

As the obstacle ratio increases, we follow that the success rates of other algorithms start falling. This tendency is obvious in Figure 4.2. When the obstacle ratio is $30 \%$, A* fails with a percentage of $66 \%$, and regular MTS, performs better than the A*- $^{*}$ Repulsion. When we investigate, we see that $A^{*}$ and $A^{*}$ repulsion can always follow the target. But this ability is not always useful. Although MTS tries to do the same, it cannot follow as good as $A^{*}$. The mistakes that MTS makes in the chase becomes an advantage for MTS. The A* and A*-Repulsion agents follow almost the same paths with their team mates. MTS agents cannot follow the same path. Even
on the tests that failed, they generally split up, without any purpose. The heuristics depressions of MTS become an advantage over A* variations.

Figures 4.5 through 4.7 shows us that as the map size increases, the success rates of other algorithms fall, while our algorithm's success rate is independent of these parameters. The other algorithms can succeed better in small maps with few scattered obstacles, and they start to fail as the map grows, the obstacle ratio increases, and the obstacles become bigger. So our algorithm is needed, because the other algorithms only succeed in a small amount of maps.

The regular MTS algorithm, which can suffer from heuristic depressions, may solve some of the problems above with a little help of luck. Since this algorithm cannot chase the target closely, because of the heuristic depressions, the agents may be set apart from each other, without an effort of cooperation. Sometimes this separation of the agents may result in cornering the target. In the tests MTS performed even better than the $A^{*}$ with Repulsion. But, we should state that this result is not to be trusted. This is not a planned outcome of the algorithm, therefore not reliable, it is highly aleatory, and the total pursuit paths are much higher than other algorithms.

MTS-Repulsion naturally takes more advantage of these split ups. So it performs better than the other algorithms in the literature. Naturally A* performs the worst in all of the test cases.

Although the MTS algorithms split up, this behavior is not planned according to the target, so MTS-Repulsion cannot perform as good as MAMTP.

In Figure 4.1 we demonstrate an example of a pursuit in detail. Agent $H 1$ does not follow his teammate, and tries to close the target's alternative escape routes. First he goes up, just in case if the target goes up (that is the nearest meeting point to $H 1$ ), since $H 1$ is blocking the way up, the target goes to the lower right corner, where he might survive longer, and H2 follows him, so $H 1$ goes to the end of the tunnel at the bottom to catch the target on the exit of that tunnel, but he is late, and the target goes into the other tunnel, so again $H 1$ goes into this tunnel after the target, and H2 turns around and closes the other exit of the tunnel. This time, target has nowhere to go. The target is captured.

The test results are given in the metrics defined before. Each group of tests on a map is given in 3 rows. All of the results for the algorithms are given next to each other in these rows. In the first row starting with "D" (stands for Distance) the average successful pursuit path lengths of the algorithms are given. In the first cell


Figure 4.1: A Long Path
of the second row, the map number is given, next to " P " (standing for Percentage). The success rates of the algorithms are given in the other cells of the second row. In the third row starting with "T" the total CPU time spent during the pursuit is given. The average for each group of maps are given at the last three rows of the corresponding table.


Figure 4.2: Success Ratio of the Algorithms over The Obstacle Ratio on 10 X 10 Maps with Smaller Obstacles


Figure 4.3: Success Ratio of the Algorithms over The Obstacle Ratio on 20 X 20 Maps with Smaller Obstacles


Figure 4.4: Success Ratio of the Algorithms over The Obstacle Ratio on 30 X 30 Maps with Smaller Obstacles


Figure 4.5: Success Ratio of the Algorithms over The Obstacle Ratio on 10 X 10 Maps with Bigger Obstacles


Figure 4.6: Success Ratio of the Algorithms over The Obstacle Ratio on 20 X 20 Maps with Bigger Obstacles


Figure 4.7: Success Ratio of the Algorithms over The Obstacle Ratio on 30 X 30 Maps with Bigger Obstacles

Table 4.1: Hand-Crafted Map Test Results

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| D | 14.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 P | 100 \% | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.008 | 0.033 | 0.033 | 0.030 | 0.032 |
| D | 15.000 | 0.000 | 34.080 | 0.000 | 21.000 |
| 2 P | 100 \% | $0 \%$ | 100 \% | $0 \%$ | 100 \% |
| S | 0.009 | 0.035 | 0.011 | 0.034 | 0.010 |
| D | 16.960 | 0.000 | 20.606 | 0.000 | 0.000 |
| 3 P | 100 \% | $0 \%$ | 33 \% | $0 \%$ | $0 \%$ |
| S | 0.017 | 0.067 | 0.051 | 0.066 | 0.068 |
| D | 13.000 | 13.000 | 28.080 | 13.000 | 25.800 |
| 4 P | 100 \% | $55 \%$ | 100 \% | 73 \% | 100 \% |
| S | 0.015 | 0.033 | 0.019 | 0.026 | 0.018 |
| D | 38.560 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 P | 100 \% | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.111 | 0.515 | 0.306 | 0.516 | 0.266 |
| D | 35.320 | 0.000 | 155.286 | 0.000 | 75.724 |
| 6 P | 100 \% | $0 \%$ | $7 \%$ | $0 \%$ | 29 \% |
| S | 0.106 | 0.552 | 0.316 | 0.552 | 0.218 |
| D | 69.760 | 0.000 | 0.000 | 0.000 | 68.328 |
| 7 P | 100 \% | $0 \%$ | $0 \%$ | $0 \%$ | 67 \% |
| S | 0.146 | 0.515 | 0.294 | 0.514 | 0.201 |
| D | 38.650 | 0.000 | 0.000 | 0.000 | 0.000 |
| 8 P | 100 \% | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.111 | 0.510 | 0.301 | 0.511 | 0.269 |
| D | 14.880 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 P | 100 \% | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.015 | 0.059 | 0.059 | 0.058 | 0.059 |
| D | 9.450 | 28.760 | 17.170 | 9.760 | 11.040 |
| 10 P | 100 \% | $100 \%$ | 100 \% | 100 \% | 100 \% |
| S | 0.017 | 0.028 | 0.021 | 0.018 | 0.018 |
| D | 150.948 | 0.000 | 165.214 | 0.000 | 90.278 |
| 11 P | $96 \%$ | $0 \%$ | $14 \%$ | $0 \%$ | 18 \% |
| S | 0.205 | 0.426 | 0.294 | 0.426 | 0.230 |
| D | 151.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 12 P | 100 \% | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.213 | 0.458 | 0.298 | 0.455 | 0.251 |
| D | 38.400 | 0.000 | 0.000 | 0.000 | 0.000 |
| 13 P | 100 \% | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.112 | 0.518 | 0.310 | 0.516 | 0.261 |
| D | 14.550 | 0.000 | 16.300 | 0.000 | 15.230 |
| Continued on next page |  |  |  |  |  |

Table 4.1 - continued from previous page

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14 P | $100 \%$ | 0 \% | 100 \% | 0 \% | $100 \%$ |
| S | 0.017 | 0.070 | 0.018 | 0.071 | 0.018 |
| D | 13.320 | 0.000 | 140.051 | 0.000 | 58.000 |
| 15 P | $100 \%$ | 0 \% | 59 \% | 0 \% | 26 \% |
| S | 0.016 | 0.061 | 0.033 | 0.061 | 0.028 |
| D | 19.380 | 0.000 | 118.350 | 0.000 | 100.860 |
| 16 P | $100 \%$ | 0 \% | 100 \% | 0 \% | 43 \% |
| S | 0.019 | 0.070 | 0.037 | 0.067 | 0.035 |
| D | 60.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 17 P | $100 \%$ | 0 \% | 0 \% | 0 \% | 0 \% |
| S | 0.099 | 0.347 | 0.227 | 0.346 | 0.182 |
| D | 39.000 | 0.000 | 113.000 | 0.000 | 207.917 |
| 18 P | $100 \%$ | 0 \% | 7 \% | 0 \% | 12 \% |
| S | 0.073 | 0.361 | 0.262 | 0.358 | 0.194 |
| D | 80.205 | 0.000 | 46.548 | 0.000 | 42.692 |
| 19 P | 78 \% | 0 \% | 31 \% | 0 \% | 39 \% |
| S | 0.182 | 0.329 | 0.208 | 0.325 | 0.160 |
| D | 43.170 | 42.770 | 297.000 | 42.710 | 0.000 |
| 20 P | $100 \%$ | $100 \%$ | 4 \% | $100 \%$ | 0 \% |
| S | 0.189 | 0.129 | 0.366 | 0.150 | 0.347 |
| D | 13.000 | 0.000 | 28.000 | 0.000 | 16.780 |
| 21 P | $100 \%$ | 0 \% | 100 \% | 0 \% | $100 \%$ |
| S | 0.015 | 0.058 | 0.018 | 0.059 | 0.016 |
| D | 13.000 | 13.000 | 36.860 | 13.000 | 27.322 |
| 22 P | $100 \%$ | 69 \% | 100 \% | 86 \% | 87 \% |
| S | 0.014 | 0.027 | 0.020 | 0.019 | 0.023 |
| D | 38.650 | 36.000 | 245.800 | 0.000 | 124.113 |
| 23 P | $100 \%$ | 22 \% | 15 \% | 0 \% | 53 \% |
| S | 0.163 | 0.624 | 0.677 | 0.780 | 0.505 |
| D | 54.890 | 62.053 | 240.389 | 70.214 | 83.347 |
| 24 P | $100 \%$ | 19 \% | 18 \% | 28 \% | 72 \% |
| S | 0.210 | 0.702 | 0.723 | 0.662 | 0.414 |
| D | 71.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 25 P | $100 \%$ | 0 \% | 0 \% | 0 \% | 0 \% |
| S | 0.129 | 0.401 | 0.195 | 0.397 | 0.185 |
| D | 37.290 | 0.000 | 0.000 | 0.000 | 0.000 |
| 26 P | $100 \%$ | 0 \% | 0 \% | 0 \% | 0 \% |
| S | 0.079 | 0.395 | 0.216 | 0.392 | 0.189 |
| D | 70.280 | 0.000 | 0.000 | 0.000 | 0.000 |
| 27 P | $100 \%$ | 0 \% | $0 \%$ | 0 \% | $0 \%$ |
| Continued on next page |  |  |  |  |  |

Table 4.1 - continued from previous page

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| S | 0.154 | 0.501 | 0.215 | 0.495 | 0.196 |
| D | 12.010 | 0.000 | 44.405 | 0.000 | 0.000 |
| 28 P | $100 \%$ | $0 \%$ | $74 \%$ | $0 \%$ | $0 \%$ |
| S | 0.013 | 0.059 | 0.031 | 0.059 | 0.061 |
| D | 14.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 29 P | $100 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.015 | 0.060 | 0.060 | 0.060 | 0.061 |
| D | 35.200 | 42.000 | 428.714 | 38.000 | 344.250 |
| 30 P | $100 \%$ | $4 \%$ | $7 \%$ | $76 \%$ | $4 \%$ |
| S | 0.160 | 0.648 | 0.583 | 0.238 | 0.581 |
| D | 41.162 | 7.919 | 72.528 | 6.223 | 43.756 |
| Avg P | $99 \%$ | $12 \%$ | $32 \%$ | $15 \%$ | $32 \%$ |
| S | 0.088 | 0.286 | 0.207 | 0.275 | 0.170 |

Table 4.2: Test Results Of Generated Maps with Smaller Obstacles (x:10, y:10 Obstacle:10\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 10.000 | 14.310 | 10.770 | 10.000 | 10.000 |
| 1 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.016 | 0.014 | 0.015 | 0.015 |
| D | 10.720 | 16.310 | 18.840 | 11.640 | 13.330 |
| 2 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.020 | 0.022 | 0.017 | 0.018 |
| D | 10.560 | 55.804 | 15.766 | 18.020 | 10.780 |
| 3 P | $100 \%$ | $92 \%$ | $94 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.042 | 0.024 | 0.021 | 0.019 |
| D | 10.540 | 12.380 | 14.960 | 12.410 | 9.290 |
| 4 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.018 | 0.020 | 0.018 | 0.017 |
| D | 14.430 | 24.280 | 18.000 | 9.370 | 10.090 |
| 5 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.021 | 0.019 | 0.021 | 0.017 | 0.018 |
| D | 10.000 | 11.000 | 11.310 | 10.000 | 10.000 |
| K | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.014 | 0.015 | 0.017 | 0.015 |
| D | 13.540 | 13.730 | 11.620 | 13.300 | 9.940 |
| 7 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.020 | 0.016 | 0.017 | 0.017 | 0.009 |
| D | 10.400 | 18.830 | 12.210 | 10.000 | 10.000 |
| B P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.014 | 0.014 | 0.015 | 0.016 | 0.018 |
| D | 10.590 | 14.670 | 14.460 | 10.000 | 10.420 |
| P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.018 | 0.017 | 0.016 | 0.018 |
| D | 11.170 | 11.880 | 11.830 | 12.090 | 9.720 |
| 10 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.016 | 0.014 | 0.012 | 0.017 |
| D | 11.195 | 19.319 | 13.977 | 11.683 | 10.357 |
| Avg P | $100 \%$ | $99 \%$ | $99 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.019 | 0.018 | 0.017 | 0.016 |
|  |  |  |  |  |  |

Table 4.3: Test Results Of Generated Maps with Smaller Obstacles (x:10, y:10 Obstacle:20\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 13.120 | 0.000 | 12.549 | 30.000 | 18.333 |
| 1 P | $100 \%$ | $0 \%$ | $51 \%$ | $26 \%$ | $87 \%$ |
| S | 0.017 | 0.078 | 0.046 | 0.062 | 0.027 |
| D | 17.500 | 0.000 | 17.830 | 12.484 | 15.011 |
| 2 P | $100 \%$ | $0 \%$ | $100 \%$ | $62 \%$ | $90 \%$ |
| S | 0.019 | 0.080 | 0.019 | 0.036 | 0.024 |
| D | 11.230 | 12.610 | 30.870 | 11.840 | 16.710 |
| 3 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.017 | 0.017 | 0.022 | 0.016 | 0.018 |
| D | 11.360 | 33.652 | 18.600 | 35.440 | 13.960 |
| 4 P | $100 \%$ | $89 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.016 | 0.029 | 0.018 | 0.023 | 0.019 |
| D | 18.880 | 28.000 | 38.600 | 15.560 | 12.000 |
| 5 P | $100 \%$ | $79 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.020 | 0.032 | 0.025 | 0.012 | 0.016 |
| D | 23.740 | 56.559 | 20.884 | 45.240 | 14.770 |
| 6 P | $100 \%$ | $59 \%$ | $69 \%$ | $75 \%$ | $100 \%$ |
| S | 0.022 | 0.050 | 0.038 | 0.039 | 0.016 |
| D | 16.330 | 18.673 | 29.130 | 19.840 | 12.540 |
| 7 P | $100 \%$ | $98 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.019 | 0.019 | 0.021 | 0.017 | 0.015 |
| D | 12.750 | 16.320 | 11.120 | 10.000 | 13.198 |
| 8 P | $100 \%$ | $25 \%$ | $50 \%$ | $75 \%$ | $81 \%$ |
| S | 0.018 | 0.060 | 0.047 | 0.029 | 0.030 |
| D | 10.600 | 19.476 | 25.397 | 11.000 | 11.830 |
| P | $100 \%$ | $82 \%$ | $78 \%$ | $100 \%$ | $100 \%$ |
| S | 0.018 | 0.030 | 0.033 | 0.016 | 0.017 |
| D | 11.440 | 10.350 | 11.700 | 10.000 | 10.000 |
| 10 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.015 | 0.012 | 0.012 | 0.013 | 0.014 |
| D | 14.695 | 19.564 | 21.668 | 20.140 | 13.835 |
| Avg P | $100 \%$ | $63 \%$ | $85 \%$ | $84 \%$ | $96 \%$ |
| S | 0.018 | 0.041 | 0.028 | 0.026 | 0.020 |
|  |  |  |  |  |  |

Table 4.4: Test Results Of Generated Maps with Smaller Obstacles (x:10, y:10 Obstacle:30\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 26.260 | 0.000 | 58.300 | 16.000 | 45.160 |
| 1 P | $100 \%$ | $0 \%$ | $100 \%$ | $31 \%$ | $100 \%$ |
| S | 0.020 | 0.071 | 0.026 | 0.054 | 0.023 |
| D | 20.780 | 0.000 | 49.430 | 0.000 | 34.620 |
| 2 P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $100 \%$ |
| S | 0.019 | 0.070 | 0.023 | 0.069 | 0.019 |
| D | 16.120 | 12.000 | 29.974 | 12.000 | 21.989 |
| 3 P | $100 \%$ | $63 \%$ | $76 \%$ | $100 \%$ | $89 \%$ |
| S | 0.016 | 0.031 | 0.031 | 0.013 | 0.021 |
| D | 13.940 | 12.000 | 26.894 | 12.000 | 34.813 |
| 4 P | $100 \%$ | $37 \%$ | $66 \%$ | $100 \%$ | $64 \%$ |
| S | 0.015 | 0.045 | 0.035 | 0.014 | 0.037 |
| D | 20.830 | 13.049 | 25.477 | 12.000 | 23.865 |
| 5 P | $100 \%$ | $61 \%$ | $65 \%$ | $100 \%$ | $52 \%$ |
| S | 0.017 | 0.034 | 0.034 | 0.013 | 0.041 |
| D | 24.010 | 0.000 | 31.130 | 0.000 | 15.393 |
| 6 P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $89 \%$ |
| S | 0.019 | 0.069 | 0.018 | 0.065 | 0.020 |
| D | 17.890 | 0.000 | 20.124 | 33.122 | 14.000 |
| 7 P | $100 \%$ | $0 \%$ | $89 \%$ | $49 \%$ | $90 \%$ |
| S | 0.017 | 0.069 | 0.021 | 0.044 | 0.014 |
| D | 20.390 | 30.000 | 15.970 | 0.000 | 15.620 |
| 8 P | $100 \%$ | $7 \%$ | $100 \%$ | $0 \%$ | $100 \%$ |
| S | 0.014 | 0.064 | 0.012 | 0.064 | 0.014 |
| D | 16.470 | 0.000 | 27.395 | 0.000 | 19.645 |
| P P | $100 \%$ | $0 \%$ | $81 \%$ | $0 \%$ | $76 \%$ |
| S | 0.017 | 0.068 | 0.028 | 0.066 | 0.026 |
| D | 19.400 | 0.000 | 30.000 | 0.000 | 28.070 |
| 10 P | $100 \%$ | $0 \%$ | $22 \%$ | $0 \%$ | $43 \%$ |
| S | 0.018 | 0.068 | 0.057 | 0.064 | 0.046 |
| D | 19.609 | 6.705 | 31.469 | 8.512 | 25.318 |
| Avg P | $100 \%$ | $17 \%$ | $80 \%$ | $38 \%$ | $80 \%$ |
| S | 0.017 | 0.059 | 0.029 | 0.047 | 0.026 |
|  |  |  |  |  |  |

Table 4.5: Test Results Of Generated Maps with Smaller Obstacles (x:20, y:20 Obstacle:10\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 21.220 | 30.130 | 83.398 | 23.100 | 24.280 |
| 1 P | $100 \%$ | $100 \%$ | $98 \%$ | $100 \%$ | $100 \%$ |
| S | 0.088 | 0.105 | 0.191 | 0.090 | 0.099 |
| D | 20.380 | 22.810 | 26.440 | 20.990 | 20.950 |
| 2 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.088 | 0.087 | 0.092 | 0.089 | 0.082 |
| D | 20.080 | 73.549 | 61.253 | 22.310 | 30.890 |
| 3 P | $100 \%$ | $82 \%$ | $91 \%$ | $100 \%$ | $100 \%$ |
| S | 0.093 | 0.271 | 0.213 | 0.104 | 0.131 |
| D | 29.790 | 61.316 | 47.330 | 33.250 | 23.320 |
| 4 P | $100 \%$ | $95 \%$ | $94 \%$ | $100 \%$ | $100 \%$ |
| S | 0.122 | 0.182 | 0.166 | 0.116 | 0.106 |
| D | 28.340 | 47.895 | 58.149 | 28.870 | 26.240 |
| 5 P | $100 \%$ | $86 \%$ | $87 \%$ | $100 \%$ | $100 \%$ |
| S | 0.126 | 0.221 | 0.246 | 0.118 | 0.125 |
| D | 26.700 | 41.338 | 37.138 | 34.842 | 37.580 |
| 6 P | $100 \%$ | $77 \%$ | $80 \%$ | $95 \%$ | $100 \%$ |
| S | 0.121 | 0.255 | 0.236 | 0.158 | 0.144 |
| D | 32.730 | 89.043 | 83.235 | 20.710 | 23.020 |
| 7 P | $100 \%$ | $94 \%$ | $98 \%$ | $100 \%$ | $100 \%$ |
| S | 0.139 | 0.248 | 0.226 | 0.099 | 0.108 |
| D | 26.450 | 78.363 | 91.876 | 31.490 | 27.380 |
| 8 P | $100 \%$ | $91 \%$ | $89 \%$ | $100 \%$ | $100 \%$ |
| S | 0.118 | 0.229 | 0.265 | 0.120 | 0.118 |
| D | 23.830 | 91.510 | 40.469 | 23.400 | 26.640 |
| 9 P | $100 \%$ | $96 \%$ | $98 \%$ | $100 \%$ | $100 \%$ |
| S | 0.112 | 0.237 | 0.137 | 0.099 | 0.110 |
| D | 26.890 | 31.000 | 40.835 | 35.180 | 26.010 |
| 10 P | $100 \%$ | $92 \%$ | $91 \%$ | $100 \%$ | $100 \%$ |
| S | 0.120 | 0.153 | 0.177 | 0.119 | 0.108 |
| D | 25.641 | 56.695 | 57.012 | 27.414 | 26.631 |
| Avg P | $100 \%$ | $91 \%$ | $93 \%$ | $100 \%$ | $100 \%$ |
| S | 0.113 | 0.199 | 0.195 | 0.111 | 0.113 |
|  |  |  |  |  |  |

Table 4.6: Test Results Of Generated Maps with Smaller Obstacles (x:20, y:20 Obstacle:20\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 43.100 | 39.933 | 25.433 | 40.583 | 26.172 |
| 1 P | $100 \%$ | $60 \%$ | $97 \%$ | $72 \%$ | $93 \%$ |
| S | 0.141 | 0.300 | 0.100 | 0.244 | 0.129 |
| D | 25.940 | 28.820 | 37.685 | 28.000 | 35.033 |
| 2 P | $100 \%$ | $61 \%$ | $92 \%$ | $74 \%$ | $91 \%$ |
| S | 0.099 | 0.283 | 0.149 | 0.216 | 0.158 |
| D | 27.940 | 25.250 | 27.938 | 51.340 | 31.440 |
| 3 P | $100 \%$ | $80 \%$ | $96 \%$ | $94 \%$ | $100 \%$ |
| S | 0.104 | 0.176 | 0.115 | 0.151 | 0.110 |
| D | 35.070 | 22.980 | 37.067 | 21.135 | 27.670 |
| 4 P | $100 \%$ | $100 \%$ | $90 \%$ | $96 \%$ | $100 \%$ |
| S | 0.116 | 0.058 | 0.126 | 0.076 | 0.069 |
| D | 39.570 | 0.000 | 33.188 | 26.500 | 32.510 |
| 5 | $100 \%$ | $0 \%$ | $80 \%$ | $10 \%$ | $96 \%$ |
| S | 0.123 | 0.530 | 0.199 | 0.480 | 0.124 |
| D | 33.060 | 25.615 | 27.538 | 23.000 | 24.908 |
| 6 P | $100 \%$ | $26 \%$ | $65 \%$ | $39 \%$ | $98 \%$ |
| S | 0.118 | 0.425 | 0.254 | 0.376 | 0.105 |
| D | 41.460 | 118.433 | 50.576 | 91.670 | 43.085 |
| 7 P | $100 \%$ | $30 \%$ | $66 \%$ | $88 \%$ | $94 \%$ |
| S | 0.132 | 0.455 | 0.284 | 0.232 | 0.156 |
| D | 37.200 | 41.000 | 38.471 | 20.156 | 33.416 |
| 8 P | $100 \%$ | $7 \%$ | $34 \%$ | $64 \%$ | $89 \%$ |
| S | 0.123 | 0.513 | 0.404 | 0.245 | 0.165 |
| D | 33.090 | 75.933 | 43.215 | 80.378 | 32.955 |
| 9 P | $100 \%$ | $15 \%$ | $65 \%$ | $37 \%$ | $89 \%$ |
| S | 0.115 | 0.519 | 0.291 | 0.433 | 0.166 |
| D | 37.700 | 30.357 | 49.506 | 37.333 | 27.235 |
| 10 P | $100 \%$ | $42 \%$ | $79 \%$ | $78 \%$ | $98 \%$ |
| S | 0.128 | 0.350 | 0.209 | 0.201 | 0.102 |
| D | 35.413 | 40.832 | 37.062 | 42.010 | 31.442 |
| Avg P | $100 \%$ | $42 \%$ | $76 \%$ | $65 \%$ | $95 \%$ |
| S | 0.120 | 0.361 | 0.213 | 0.265 | 0.128 |
|  |  |  |  |  |  |

Table 4.7: Test Results Of Generated Maps with Smaller Obstacles (x:20, y:20 Obstacle:30\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 41.250 | 26.000 | 51.702 | 25.833 | 32.034 |
| 1 P | $100 \%$ | $7 \%$ | $47 \%$ | $72 \%$ | $87 \%$ |
| S | 0.121 | 0.436 | 0.303 | 0.178 | 0.147 |
| D | 25.320 | 34.000 | 37.979 | 33.000 | 39.724 |
| 2 P | $100 \%$ | $6 \%$ | $96 \%$ | $19 \%$ | $98 \%$ |
| S | 0.077 | 0.441 | 0.103 | 0.393 | 0.102 |
| D | 36.610 | 41.308 | 56.237 | 65.000 | 40.158 |
| 3 P | $100 \%$ | $13 \%$ | $76 \%$ | $10 \%$ | $19 \%$ |
| S | 0.097 | 0.409 | 0.215 | 0.421 | 0.385 |
| D | 34.550 | 25.471 | 39.034 | 22.500 | 37.476 |
| 4 P | $100 \%$ | $17 \%$ | $87 \%$ | $4 \%$ | $84 \%$ |
| S | 0.099 | 0.396 | 0.148 | 0.444 | 0.163 |
| D | 26.430 | 28.409 | 40.044 | 27.750 | 43.793 |
| 5 P | $100 \%$ | $22 \%$ | $68 \%$ | $64 \%$ | $82 \%$ |
| S | 0.087 | 0.374 | 0.218 | 0.214 | 0.172 |
| D | 24.020 | 0.000 | 39.653 | 0.000 | 41.450 |
| 6 P | $100 \%$ | $0 \%$ | $98 \%$ | $0 \%$ | $100 \%$ |
| S | 0.070 | 0.472 | 0.090 | 0.463 | 0.084 |
| D | 25.000 | 26.462 | 132.987 | 25.000 | 79.671 |
| 7 P | $100 \%$ | $13 \%$ | $79 \%$ | $96 \%$ | $85 \%$ |
| S | 0.078 | 0.441 | 0.241 | 0.092 | 0.183 |
| D | 23.800 | 24.384 | 328.000 | 24.227 | 130.765 |
| 8 P | $100 \%$ | $73 \%$ | $4 \%$ | $88 \%$ | $17 \%$ |
| S | 0.065 | 0.178 | 0.297 | 0.114 | 0.267 |
| D | 67.290 | 68.000 | 54.914 | 66.588 | 80.649 |
| 9 P | $100 \%$ | $4 \%$ | $81 \%$ | $17 \%$ | $74 \%$ |
| S | 0.159 | 0.497 | 0.188 | 0.448 | 0.251 |
| D | 28.380 | 0.000 | 63.019 | 0.000 | 40.948 |
| 10 P | $100 \%$ | $0 \%$ | $52 \%$ | $0 \%$ | $96 \%$ |
| S | 0.092 | 0.470 | 0.294 | 0.465 | 0.114 |
| D | 33.265 | 27.403 | 84.357 | 28.990 | 56.667 |
| Avg P | $100 \%$ | $16 \%$ | $69 \%$ | $37 \%$ | $74 \%$ |
| S | 0.095 | 0.411 | 0.210 | 0.323 | 0.187 |
|  |  |  |  |  |  |

Table 4.8: Test Results Of Generated Maps with Smaller Obstacles (x:30, y:30 Obstacle:10\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 33.520 | 43.012 | 40.462 | 39.619 | 54.333 |
| 1 P | $100 \%$ | $82 \%$ | $91 \%$ | $97 \%$ | $93 \%$ |
| S | 0.360 | 0.702 | 0.547 | 0.432 | 0.601 |
| D | 47.190 | 55.043 | 65.309 | 46.786 | 45.442 |
| 2 P | $100 \%$ | $70 \%$ | $81 \%$ | $84 \%$ | $95 \%$ |
| S | 0.437 | 0.914 | 0.774 | 0.643 | 0.477 |
| D | 37.180 | 179.643 | 138.571 | 32.382 | 37.533 |
| 3 P | $100 \%$ | $14 \%$ | $35 \%$ | $89 \%$ | $92 \%$ |
| S | 0.333 | 1.956 | 1.649 | 0.482 | 0.476 |
| D | 42.110 | 222.721 | 243.825 | 137.455 | 53.700 |
| 4 P | $100 \%$ | $61 \%$ | $63 \%$ | $99 \%$ | $100 \%$ |
| S | 0.368 | 1.497 | 1.540 | 0.785 | 0.436 |
| D | 33.280 | 32.149 | 33.344 | 30.160 | 33.000 |
| 5 P | $100 \%$ | $94 \%$ | $96 \%$ | $100 \%$ | $96 \%$ |
| S | 0.330 | 0.334 | 0.332 | 0.257 | 0.380 |
| D | 42.800 | 38.714 | 52.333 | 51.847 | 40.094 |
| 6 P | $100 \%$ | $7 \%$ | $27 \%$ | $85 \%$ | $96 \%$ |
| S | 0.416 | 2.204 | 1.873 | 0.705 | 0.482 |
| D | 48.330 | 71.328 | 76.870 | 64.299 | 63.753 |
| 7 P | $100 \%$ | $67 \%$ | $69 \%$ | $97 \%$ | $93 \%$ |
| S | 0.463 | 1.042 | 1.044 | 0.517 | 0.622 |
| D | 42.180 | 69.029 | 39.353 | 97.085 | 40.170 |
| 8 P | $100 \%$ | $35 \%$ | $34 \%$ | $82 \%$ | $94 \%$ |
| S | 0.402 | 1.601 | 1.614 | 0.892 | 0.503 |
| D | 38.260 | 79.756 | 74.989 | 38.720 | 36.590 |
| 9 P | $100 \%$ | $82 \%$ | $89 \%$ | $100 \%$ | $100 \%$ |
| S | 0.397 | 0.821 | 0.697 | 0.377 | 0.393 |
| D | 45.250 | 104.471 | 45.957 | 53.030 | 36.940 |
| 10 P | $100 \%$ | $17 \%$ | $23 \%$ | $67 \%$ | $84 \%$ |
| S | 0.421 | 1.860 | 1.802 | 0.996 | 0.662 |
| D | 41.010 | 89.587 | 81.101 | 59.138 | 44.156 |
| Avg P | $100 \%$ | $53 \%$ | $61 \%$ | $90 \%$ | $94 \%$ |
| S | 0.393 | 1.293 | 1.187 | 0.609 | 0.503 |
|  |  |  |  |  |  |

Table 4.9: Test Results Of Generated Maps with Smaller Obstacles (x:30, y:30 Obstacle:20\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 34.000 | 37.059 | 41.314 | 55.600 | 42.060 |
| 1 P | $100 \%$ | $34 \%$ | $51 \%$ | $5 \%$ | $84 \%$ |
| S | 0.293 | 1.296 | 1.073 | 1.693 | 0.605 |
| D | 41.540 | 34.633 | 38.153 | 31.747 | 34.290 |
| 2 P | $100 \%$ | $98 \%$ | $98 \%$ | $99 \%$ | $100 \%$ |
| S | 0.320 | 0.227 | 0.258 | 0.235 | 0.237 |
| D | 33.730 | 37.000 | 41.679 | 40.000 | 38.623 |
| 3 P | $100 \%$ | $2 \%$ | $28 \%$ | $4 \%$ | $61 \%$ |
| S | 0.299 | 1.734 | 1.393 | 1.705 | 0.914 |
| D | 39.700 | 39.724 | 43.779 | 34.887 | 48.579 |
| 4 P | $100 \%$ | $29 \%$ | $86 \%$ | $62 \%$ | $76 \%$ |
| S | 0.348 | 1.415 | 0.566 | 0.915 | 0.773 |
| D | 64.060 | 39.000 | 47.584 | 39.200 | 43.500 |
| 5 P | $100 \%$ | $1 \%$ | $77 \%$ | $20 \%$ | $74 \%$ |
| S | 0.477 | 1.805 | 0.707 | 1.541 | 0.760 |
| D | 51.820 | 39.000 | 42.275 | 39.733 | 40.362 |
| 6 P | $100 \%$ | $14 \%$ | $69 \%$ | $30 \%$ | $80 \%$ |
| S | 0.410 | 1.586 | 0.811 | 1.400 | 0.679 |
| D | 79.150 | 0.000 | 45.538 | 48.000 | 48.521 |
| 7 P | $100 \%$ | $0 \%$ | $26 \%$ | $11 \%$ | $73 \%$ |
| S | 0.525 | 1.758 | 1.443 | 1.657 | 0.792 |
| D | 35.150 | 37.345 | 72.023 | 35.190 | 70.388 |
| 8 P | $100 \%$ | $84 \%$ | $44 \%$ | $58 \%$ | $67 \%$ |
| S | 0.266 | 0.512 | 1.255 | 0.925 | 0.884 |
| D | 33.160 | 33.968 | 38.151 | 32.000 | 37.022 |
| P | $100 \%$ | $94 \%$ | $86 \%$ | $100 \%$ | $93 \%$ |
| S | 0.256 | 0.313 | 0.476 | 0.229 | 0.376 |
| D | 35.740 | 58.600 | 52.907 | 41.000 | 46.746 |
| 10 P | $100 \%$ | $5 \%$ | $54 \%$ | $8 \%$ | $63 \%$ |
| S | 0.303 | 1.674 | 1.062 | 1.674 | 0.964 |
| D | 44.805 | 35.633 | 46.340 | 39.736 | 45.009 |
| Avg P | $100 \%$ | $36 \%$ | $62 \%$ | $40 \%$ | $77 \%$ |
| S | 0.350 | 1.232 | 0.904 | 1.197 | 0.698 |
|  |  |  |  |  |  |

Table 4.10: Test Results Of Generated Maps with Smaller Obstacles (x:30, y:30 Obstacle:30\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 70.010 | 0.000 | 88.078 | 0.000 | 83.828 |
| 1 P | $100 \%$ | $0 \%$ | $64 \%$ | $0 \%$ | $29 \%$ |
| S | 0.395 | 1.462 | 0.786 | 1.433 | 1.252 |
| D | 55.550 | 37.203 | 75.733 | 36.417 | 59.465 |
| 2 P | $100 \%$ | $59 \%$ | $75 \%$ | $84 \%$ | $86 \%$ |
| S | 0.320 | 0.724 | 0.554 | 0.415 | 0.428 |
| D | 45.240 | 36.189 | 38.061 | 35.175 | 38.740 |
| 3 P | $100 \%$ | $90 \%$ | $98 \%$ | $97 \%$ | $96 \%$ |
| S | 0.286 | 0.307 | 0.219 | 0.247 | 0.274 |
| D | 62.160 | 0.000 | 0.000 | 0.000 | 0.000 |
| 4 P | $100 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.321 | 1.456 | 1.473 | 1.417 | 1.449 |
| D | 36.060 | 0.000 | 53.329 | 40.200 | 57.787 |
| 5 P | $100 \%$ | $0 \%$ | $73 \%$ | $5 \%$ | $75 \%$ |
| S | 0.265 | 1.558 | 0.659 | 1.517 | 0.685 |
| D | 55.670 | 35.885 | 48.875 | 36.074 | 48.973 |
| 6 P | $100 \%$ | $61 \%$ | $64 \%$ | $94 \%$ | $73 \%$ |
| S | 0.351 | 0.767 | 0.764 | 0.327 | 0.677 |
| D | 122.700 | 36.250 | 102.537 | 35.909 | 48.571 |
| 7 P | $100 \%$ | $8 \%$ | $41 \%$ | $11 \%$ | $91 \%$ |
| S | 0.625 | 1.481 | 1.153 | 1.439 | 0.420 |
| D | 49.870 | 37.524 | 54.267 | 35.356 | 62.903 |
| 8 P | $100 \%$ | $42 \%$ | $86 \%$ | $73 \%$ | $72 \%$ |
| S | 0.298 | 0.962 | 0.433 | 0.554 | 0.676 |
| D | 57.910 | 34.938 | 47.761 | 34.042 | 64.279 |
| 9 P | $100 \%$ | $32 \%$ | $71 \%$ | $96 \%$ | $68 \%$ |
| S | 0.333 | 1.138 | 0.640 | 0.251 | 0.749 |
| D | 53.070 | 0.000 | 53.717 | 0.000 | 53.831 |
| 10 P | $100 \%$ | $0 \%$ | $46 \%$ | $0 \%$ | $89 \%$ |
| S | 0.315 | 1.491 | 0.983 | 1.478 | 0.391 |
| D | 60.824 | 21.799 | 56.236 | 25.317 | 51.838 |
| Avg P | $100 \%$ | $29 \%$ | $62 \%$ | $46 \%$ | $68 \%$ |
| S | 0.351 | 1.135 | 0.766 | 0.908 | 0.700 |
|  |  |  |  |  |  |

Table 4.11: Test Results Of Generated Maps with Bigger Obstacles (x:10, y:10 Obstacle:10\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 12.230 | 0.000 | 26.939 | 12.000 | 16.990 |
| 1 P | $100 \%$ | $0 \%$ | $82 \%$ | $30 \%$ | $100 \%$ |
| S | 0.020 | 0.092 | 0.038 | 0.069 | 0.021 |
| D | 14.110 | 42.868 | 16.545 | 41.600 | 14.340 |
| 2 P | $100 \%$ | $91 \%$ | $77 \%$ | $100 \%$ | $100 \%$ |
| S | 0.020 | 0.035 | 0.036 | 0.031 | 0.019 |
| D | 11.000 | 11.000 | 12.652 | 12.000 | 13.326 |
| 3 P | $100 \%$ | $29 \%$ | $92 \%$ | $60 \%$ | $92 \%$ |
| S | 0.018 | 0.065 | 0.023 | 0.043 | 0.024 |
| D | 17.830 | 35.110 | 28.450 | 16.460 | 15.860 |
| 4 P | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.022 | 0.028 | 0.026 | 0.020 | 0.020 |
| D | 9.760 | 24.694 | 17.972 | 22.342 | 11.030 |
| 5 P | $100 \%$ | $62 \%$ | $71 \%$ | $79 \%$ | $100 \%$ |
| S | 0.016 | 0.048 | 0.041 | 0.033 | 0.017 |
| D | 15.000 | 89.590 | 29.067 | 70.300 | 12.830 |
| 6 P | $100 \%$ | $61 \%$ | $90 \%$ | $100 \%$ | $100 \%$ |
| S | 0.022 | 0.062 | 0.032 | 0.038 | 0.018 |
| D | 11.170 | 18.000 | 27.061 | 31.533 | 9.400 |
| 7 P | $100 \%$ | $29 \%$ | $49 \%$ | $60 \%$ | $80 \%$ |
| S | 0.015 | 0.065 | 0.054 | 0.047 | 0.028 |
| D | 16.130 | 68.273 | 44.620 | 53.010 | 29.130 |
| 8 P | $100 \%$ | $77 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.020 | 0.055 | 0.030 | 0.037 | 0.026 |
| D | 11.450 | 33.176 | 18.857 | 41.000 | 14.044 |
| 9 P | $100 \%$ | $68 \%$ | $63 \%$ | $24 \%$ | $90 \%$ |
| S | 0.018 | 0.042 | 0.042 | 0.066 | 0.024 |
| D | 10.020 | 15.000 | 16.950 | 38.065 | 10.088 |
| 10 P | $100 \%$ | $26 \%$ | $60 \%$ | $62 \%$ | $80 \%$ |
| S | 0.017 | 0.071 | 0.048 | 0.050 | 0.031 |
| D | 12.870 | 33.771 | 23.911 | 33.831 | 14.704 |
| Avg P | $100 \%$ | $54 \%$ | $78 \%$ | $72 \%$ | $94 \%$ |
| S | 0.019 | 0.056 | 0.037 | 0.043 | 0.023 |
|  |  |  |  |  |  |

Table 4.12: Test Results Of Generated Maps with Bigger Obstacles (x:10, y:10 Obstacle:20\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 15.960 | 0.000 | 40.438 | 54.588 | 18.790 |
| 1 P | $100 \%$ | $0 \%$ | $73 \%$ | $51 \%$ | $100 \%$ |
| S | 0.020 | 0.078 | 0.043 | 0.053 | 0.020 |
| D | 12.000 | 14.420 | 57.060 | 13.000 | 27.120 |
| 2 P | $100 \%$ | $100 \%$ | $100 \%$ | $90 \%$ | $100 \%$ |
| S | 0.018 | 0.017 | 0.029 | 0.023 | 0.020 |
| D | 11.420 | 0.000 | 79.877 | 0.000 | 57.025 |
| 3 P | $100 \%$ | $0 \%$ | $81 \%$ | $0 \%$ | $80 \%$ |
| S | 0.018 | 0.076 | 0.040 | 0.075 | 0.045 |
| D | 12.000 | 0.000 | 39.146 | 0.000 | 25.000 |
| 4 P | $100 \%$ | $0 \%$ | $89 \%$ | $0 \%$ | $27 \%$ |
| S | 0.017 | 0.079 | 0.029 | 0.079 | 0.062 |
| D | 13.000 | 0.000 | 61.160 | 0.000 | 35.530 |
| 5 P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $100 \%$ |
| S | 0.019 | 0.080 | 0.036 | 0.079 | 0.027 |
| D | 14.000 | 65.632 | 39.917 | 32.000 | 39.600 |
| 6 P | $100 \%$ | $57 \%$ | $72 \%$ | $88 \%$ | $100 \%$ |
| S | 0.018 | 0.053 | 0.039 | 0.027 | 0.025 |
| D | 13.000 | 0.000 | 34.520 | 0.000 | 23.420 |
| 7 P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $100 \%$ |
| S | 0.017 | 0.077 | 0.021 | 0.077 | 0.020 |
| D | 11.520 | 11.742 | 14.020 | 11.350 | 12.680 |
| 8 P | $100 \%$ | $89 \%$ | $100 \%$ | $100 \%$ | $100 \%$ |
| S | 0.017 | 0.020 | 0.013 | 0.012 | 0.013 |
| D | 20.880 | 0.000 | 38.253 | 0.000 | 20.610 |
| P | $100 \%$ | $0 \%$ | $75 \%$ | $0 \%$ | $100 \%$ |
| S | 0.020 | 0.078 | 0.037 | 0.077 | 0.019 |
| D | 12.770 | 11.000 | 15.083 | 22.022 | 11.620 |
| 10 P | $100 \%$ | $11 \%$ | $72 \%$ | $93 \%$ | $100 \%$ |
| S | 0.017 | 0.073 | 0.034 | 0.023 | 0.015 |
| D | 13.655 | 10.279 | 41.947 | 13.296 | 27.140 |
| Avg P | $100 \%$ | $26 \%$ | $86 \%$ | $42 \%$ | $91 \%$ |
| S | 0.018 | 0.063 | 0.032 | 0.053 | 0.027 |
|  |  |  |  |  |  |

Table 4.13: Test Results Of Generated Maps with Bigger Obstacles (x:10, y:10 Obstacle:30\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 14.000 | 0.000 | 67.476 | 0.000 | 50.560 |
| 1 P | $100 \%$ | $0 \%$ | $84 \%$ | $0 \%$ | $84 \%$ |
| S | 0.015 | 0.072 | 0.032 | 0.071 | 0.028 |
| D | 14.000 | 0.000 | 69.920 | 0.000 | 26.700 |
| 2 P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $100 \%$ |
| S | 0.017 | 0.069 | 0.031 | 0.067 | 0.019 |
| D | 18.230 | 30.000 | 81.348 | 30.000 | 26.083 |
| 3 P | $100 \%$ | $27 \%$ | $92 \%$ | $28 \%$ | $84 \%$ |
| S | 0.017 | 0.052 | 0.033 | 0.051 | 0.022 |
| D | 13.000 | 0.000 | 44.680 | 0.000 | 23.460 |
| 4 P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $100 \%$ |
| S | 0.015 | 0.067 | 0.025 | 0.066 | 0.017 |
| D | 14.600 | 0.000 | 54.000 | 0.000 | 43.120 |
| 5 P | $100 \%$ | $0 \%$ | $64 \%$ | $0 \%$ | $100 \%$ |
| S | 0.017 | 0.069 | 0.040 | 0.066 | 0.023 |
| D | 17.700 | 0.000 | 36.000 | 0.000 | 49.085 |
| 6 P | $100 \%$ | $0 \%$ | $26 \%$ | $0 \%$ | $47 \%$ |
| S | 0.018 | 0.063 | 0.054 | 0.062 | 0.047 |
| D | 13.000 | 0.000 | 61.500 | 0.000 | 17.000 |
| 7 P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $100 \%$ |
| S | 0.017 | 0.073 | 0.030 | 0.072 | 0.019 |
| D | 20.540 | 12.314 | 39.410 | 12.000 | 19.136 |
| 8 P | $100 \%$ | $51 \%$ | $100 \%$ | $98 \%$ | $88 \%$ |
| S | 0.018 | 0.041 | 0.022 | 0.014 | 0.023 |
| D | 15.000 | 0.000 | 63.980 | 0.000 | 27.674 |
| P | $100 \%$ | $0 \%$ | $100 \%$ | $0 \%$ | $86 \%$ |
| S | 0.018 | 0.071 | 0.034 | 0.070 | 0.027 |
| D | 13.700 | 0.000 | 29.238 | 0.000 | 48.793 |
| 10 P | $100 \%$ | $0 \%$ | $84 \%$ | $0 \%$ | $29 \%$ |
| S | 0.015 | 0.065 | 0.026 | 0.064 | 0.055 |
| D | 15.377 | 4.231 | 54.755 | 4.200 | 33.161 |
| Avg P | $100 \%$ | $8 \%$ | $85 \%$ | $13 \%$ | $82 \%$ |
| S | 0.017 | 0.064 | 0.033 | 0.060 | 0.028 |
|  |  |  |  |  |  |

Table 4.14: Test Results Of Generated Maps with Bigger Obstacles (x:20, y:20 Obstacle:10\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 41.450 | 90.722 | 52.479 | 22.985 | 25.347 |
| 1 P | $100 \%$ | $18 \%$ | $48 \%$ | $65 \%$ | $98 \%$ |
| S | 0.175 | 0.609 | 0.439 | 0.310 | 0.131 |
| D | 25.310 | 50.690 | 28.367 | 21.598 | 23.184 |
| 2 P | $100 \%$ | $29 \%$ | $60 \%$ | $92 \%$ | $98 \%$ |
| S | 0.119 | 0.557 | 0.363 | 0.144 | 0.116 |
| D | 22.990 | 24.716 | 26.116 | 21.000 | 28.147 |
| 3 P | $100 \%$ | $88 \%$ | $86 \%$ | $100 \%$ | $95 \%$ |
| S | 0.109 | 0.171 | 0.191 | 0.091 | 0.147 |
| D | 24.800 | 25.040 | 39.329 | 21.000 | 30.235 |
| 4 P | $100 \%$ | $50 \%$ | $82 \%$ | $100 \%$ | $85 \%$ |
| S | 0.119 | 0.388 | 0.235 | 0.101 | 0.207 |
| D | 24.140 | 26.000 | 29.131 | 27.657 | 26.149 |
| 5 P | $100 \%$ | $4 \%$ | $61 \%$ | $35 \%$ | $94 \%$ |
| S | 0.112 | 0.613 | 0.327 | 0.449 | 0.155 |
| D | 28.910 | 73.000 | 48.317 | 0.000 | 31.746 |
| 6 P | $100 \%$ | $1 \%$ | $41 \%$ | $0 \%$ | $71 \%$ |
| S | 0.121 | 0.665 | 0.467 | 0.681 | 0.293 |
| D | 28.480 | 72.188 | 39.509 | 31.324 | 35.326 |
| 7 P | $100 \%$ | $16 \%$ | $53 \%$ | $34 \%$ | $86 \%$ |
| S | 0.129 | 0.577 | 0.394 | 0.480 | 0.224 |
| D | 32.470 | 30.526 | 33.847 | 30.000 | 25.439 |
| 8 P | $100 \%$ | $19 \%$ | $59 \%$ | $75 \%$ | $98 \%$ |
| S | 0.144 | 0.535 | 0.343 | 0.244 | 0.130 |
| D | 24.000 | 27.091 | 29.662 | 23.704 | 26.193 |
| P | $100 \%$ | $55 \%$ | $74 \%$ | $54 \%$ | $88 \%$ |
| S | 0.114 | 0.330 | 0.248 | 0.340 | 0.180 |
| D | 34.330 | 35.000 | 33.563 | 33.686 | 28.070 |
| 10 P | $100 \%$ | $13 \%$ | $32 \%$ | $70 \%$ | $86 \%$ |
| S | 0.149 | 0.612 | 0.511 | 0.295 | 0.198 |
| D | 28.688 | 45.497 | 36.032 | 23.295 | 27.984 |
| Avg P | $100 \%$ | $29 \%$ | $60 \%$ | $63 \%$ | $90 \%$ |
| S | 0.129 | 0.506 | 0.352 | 0.313 | 0.178 |
|  |  |  |  |  |  |

Table 4.15: Test Results Of Generated Maps with Bigger Obstacles (x:20, y:20 Obstacle:20\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 38.000 | 0.000 | 0.000 | 20.603 | 23.330 |
| 1 P | $100 \%$ | $0 \%$ | $0 \%$ | $73 \%$ | $97 \%$ |
| S | 0.140 | 0.623 | 0.317 | 0.222 | 0.099 |
| D | 24.680 | 23.377 | 27.620 | 21.000 | 22.670 |
| 2 P | $100 \%$ | $53 \%$ | $92 \%$ | $94 \%$ | $100 \%$ |
| S | 0.086 | 0.327 | 0.119 | 0.101 | 0.070 |
| D | 43.430 | 0.000 | 72.085 | 0.000 | 51.762 |
| 3 P | $100 \%$ | $0 \%$ | $59 \%$ | $0 \%$ | $84 \%$ |
| S | 0.145 | 0.526 | 0.338 | 0.529 | 0.213 |
| D | 24.000 | 25.365 | 26.638 | 24.208 | 27.818 |
| 4 P | $100 \%$ | $63 \%$ | $94 \%$ | $77 \%$ | $88 \%$ |
| S | 0.098 | 0.265 | 0.126 | 0.201 | 0.163 |
| D | 56.700 | 27.053 | 45.429 | 28.719 | 42.719 |
| 5 P | $100 \%$ | $75 \%$ | $98 \%$ | $64 \%$ | $96 \%$ |
| S | 0.183 | 0.215 | 0.134 | 0.267 | 0.151 |
| D | 23.290 | 25.204 | 67.159 | 23.640 | 54.310 |
| 6 P | $100 \%$ | $49 \%$ | $69 \%$ | $86 \%$ | $87 \%$ |
| S | 0.083 | 0.342 | 0.273 | 0.153 | 0.201 |
| D | 59.490 | 0.000 | 64.214 | 71.000 | 48.658 |
| 7 P | $100 \%$ | $0 \%$ | $28 \%$ | $4 \%$ | $73 \%$ |
| S | 0.176 | 0.566 | 0.454 | 0.553 | 0.263 |
| D | 25.940 | 27.926 | 42.556 | 0.000 | 35.240 |
| 8 P | $100 \%$ | $27 \%$ | $81 \%$ | $0 \%$ | $100 \%$ |
| S | 0.102 | 0.426 | 0.208 | 0.556 | 0.118 |
| D | 36.210 | 24.617 | 48.000 | 24.000 | 35.010 |
| 9 P | $100 \%$ | $60 \%$ | $33 \%$ | $92 \%$ | $96 \%$ |
| S | 0.118 | 0.266 | 0.416 | 0.118 | 0.134 |
| D | 51.230 | 30.000 | 47.581 | 49.286 | 31.741 |
| 10 P | $100 \%$ | $4 \%$ | $74 \%$ | $14 \%$ | $85 \%$ |
| S | 0.167 | 0.582 | 0.243 | 0.551 | 0.189 |
| D | 38.297 | 18.354 | 44.128 | 26.246 | 37.326 |
| Avg P | $100 \%$ | $33 \%$ | $63 \%$ | $50 \%$ | $91 \%$ |
| S | 0.130 | 0.414 | 0.263 | 0.325 | 0.160 |
|  |  |  |  |  |  |

Table 4.16: Test Results Of Generated Maps with Bigger Obstacles (x:20, y:20 Obstacle:30\%)

| Map | MAMTP | A $^{*}$ | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 61.980 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1 P | $100 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $0 \%$ |
| S | 0.171 | 0.525 | 0.529 | 0.518 | 0.528 |
| D | 34.130 | 0.000 | 69.745 | 0.000 | 35.510 |
| 2 P | $100 \%$ | $0 \%$ | $47 \%$ | $0 \%$ | $96 \%$ |
| S | 0.108 | 0.494 | 0.338 | 0.502 | 0.120 |
| D | 30.000 | 32.724 | 0.000 | 30.802 | 367.188 |
| 3 P | $100 \%$ | $29 \%$ | $0 \%$ | $96 \%$ | $32 \%$ |
| S | 0.086 | 0.405 | 0.326 | 0.109 | 0.321 |
| D | 43.030 | 26.000 | 102.116 | 24.533 | 60.510 |
| 4 P | $100 \%$ | $10 \%$ | $86 \%$ | $45 \%$ | $96 \%$ |
| S | 0.115 | 0.473 | 0.237 | 0.312 | 0.154 |
| D | 30.370 | 0.000 | 173.667 | 0.000 | 106.297 |
| 5 P | $100 \%$ | $0 \%$ | $9 \%$ | $0 \%$ | $74 \%$ |
| S | 0.098 | 0.479 | 0.277 | 0.477 | 0.217 |
| D | 30.700 | 53.000 | 192.936 | 57.000 | 116.878 |
| 6 P | $100 \%$ | $4 \%$ | $78 \%$ | $3 \%$ | $98 \%$ |
| S | 0.105 | 0.505 | 0.259 | 0.511 | 0.177 |
| D | 31.960 | 28.368 | 144.754 | 27.896 | 101.570 |
| 7 P | $100 \%$ | $57 \%$ | $65 \%$ | $96 \%$ | $86 \%$ |
| S | 0.104 | 0.275 | 0.326 | 0.101 | 0.223 |
| D | 26.220 | 0.000 | 61.721 | 0.000 | 66.820 |
| 8 P | $100 \%$ | $0 \%$ | $86 \%$ | $0 \%$ | $100 \%$ |
| S | 0.084 | 0.471 | 0.170 | 0.465 | 0.136 |
| D | 30.450 | 0.000 | 294.490 | 0.000 | 161.783 |
| 9 P | $100 \%$ | $0 \%$ | $51 \%$ | $0 \%$ | $69 \%$ |
| S | 0.103 | 0.516 | 0.472 | 0.517 | 0.348 |
| D | 51.340 | 0.000 | 93.569 | 0.000 | 73.419 |
| 10 P | $100 \%$ | $0 \%$ | $72 \%$ | $0 \%$ | $86 \%$ |
| S | 0.151 | 0.541 | 0.290 | 0.530 | 0.223 |
| D | 37.018 | 14.009 | 113.300 | 14.023 | 108.997 |
| Avg P | $100 \%$ | $10 \%$ | $49 \%$ | $24 \%$ | $74 \%$ |
| S | 0.112 | 0.468 | 0.322 | 0.404 | 0.245 |
|  |  |  |  |  |  |

Table 4.17: Test Results Of Generated Maps with Bigger Obstacles (x:30, y:30 Obstacle:10\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 60.550 | 375.000 | 55.063 | 43.480 | 49.603 |
| 1 P | $100 \%$ | $2 \%$ | $16 \%$ | $25 \%$ | $78 \%$ |
| S | 0.558 | 2.416 | 2.153 | 1.978 | 0.949 |
| D | 54.330 | 38.273 | 40.967 | 39.750 | 57.525 |
| 2 P | $100 \%$ | $11 \%$ | $30 \%$ | $16 \%$ | $59 \%$ |
| S | 0.535 | 2.081 | 1.765 | 2.060 | 1.320 |
| D | 40.440 | 39.000 | 52.500 | 33.506 | 43.695 |
| 3 P | $100 \%$ | $5 \%$ | $44 \%$ | $77 \%$ | $82 \%$ |
| S | 0.446 | 2.091 | 1.450 | 0.795 | 0.773 |
| D | 62.230 | 53.217 | 38.029 | 33.217 | 42.277 |
| 4 P | $100 \%$ | $23 \%$ | $70 \%$ | $60 \%$ | $83 \%$ |
| S | 0.591 | 1.740 | 0.912 | 1.114 | 0.728 |
| D | 38.160 | 36.425 | 38.597 | 33.897 | 36.598 |
| 5 | $100 \%$ | $73 \%$ | $72 \%$ | $68 \%$ | $92 \%$ |
| S | 0.394 | 0.837 | 0.887 | 0.932 | 0.519 |
| D | 65.840 | 33.636 | 41.245 | 39.373 | 44.864 |
| 6 P | $100 \%$ | $11 \%$ | $49 \%$ | $51 \%$ | $81 \%$ |
| S | 0.595 | 1.913 | 1.340 | 1.224 | 0.805 |
| D | 53.780 | 35.833 | 41.688 | 39.214 | 37.000 |
| 7 P | $100 \%$ | $24 \%$ | $64 \%$ | $42 \%$ | $75 \%$ |
| S | 0.492 | 1.827 | 1.092 | 1.507 | 0.878 |
| D | 77.110 | 56.000 | 43.500 | 32.563 | 49.250 |
| 8 P | $100 \%$ | $1 \%$ | $14 \%$ | $64 \%$ | $56 \%$ |
| S | 0.684 | 2.209 | 2.004 | 1.051 | 1.302 |
| D | 53.310 | 0.000 | 0.000 | 0.000 | 59.919 |
| 9 P | $100 \%$ | $0 \%$ | $0 \%$ | $0 \%$ | $37 \%$ |
| S | 0.452 | 2.101 | 2.155 | 2.238 | 1.639 |
| D | 58.420 | 37.857 | 40.597 | 39.125 | 39.448 |
| 10 P | $100 \%$ | $7 \%$ | $67 \%$ | $16 \%$ | $96 \%$ |
| S | 0.556 | 2.081 | 1.009 | 1.917 | 0.517 |
| D | 56.417 | 70.524 | 39.219 | 33.412 | 46.018 |
| Avg P | $100 \%$ | $16 \%$ | $43 \%$ | $42 \%$ | $74 \%$ |
| S | 0.530 | 1.930 | 1.477 | 1.482 | 0.943 |
|  |  |  |  |  |  |

Table 4.18: Test Results Of Generated Maps with Bigger Obstacles (x:30, y:30 Obstacle:20\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 34.000 | 67.429 | 58.045 | 52.692 | 41.354 |
| 1 P | $100 \%$ | $14 \%$ | $67 \%$ | $13 \%$ | $82 \%$ |
| S | 0.270 | 1.805 | 0.924 | 1.819 | 0.661 |
| D | 101.700 | 0.000 | 183.791 | 70.135 | 92.929 |
| 2 P | $100 \%$ | $0 \%$ | $86 \%$ | $37 \%$ | $98 \%$ |
| S | 0.708 | 2.021 | 0.802 | 1.510 | 0.523 |
| D | 34.160 | 37.000 | 76.622 | 35.842 | 327.778 |
| 3 P | $100 \%$ | $14 \%$ | $37 \%$ | $19 \%$ | $18 \%$ |
| S | 0.312 | 1.644 | 1.360 | 1.626 | 1.811 |
| D | 94.890 | 39.000 | 115.875 | 52.000 | 78.660 |
| 4 P | $100 \%$ | $1 \%$ | $16 \%$ | $9 \%$ | $47 \%$ |
| S | 0.657 | 1.913 | 1.783 | 1.800 | 1.349 |
| D | 49.680 | 34.861 | 41.147 | 32.892 | 42.115 |
| 5 P | $100 \%$ | $79 \%$ | $75 \%$ | $37 \%$ | $61 \%$ |
| S | 0.426 | 0.610 | 0.727 | 1.380 | 1.046 |
| D | 101.840 | 33.750 | 67.286 | 32.250 | 42.216 |
| 6 P | $100 \%$ | $4 \%$ | $7 \%$ | $36 \%$ | $74 \%$ |
| S | 0.690 | 1.744 | 1.870 | 1.277 | 0.796 |
| D | 72.520 | 37.667 | 77.133 | 35.538 | 72.161 |
| 7 P | $100 \%$ | $48 \%$ | $30 \%$ | $91 \%$ | $62 \%$ |
| S | 0.564 | 1.227 | 1.629 | 0.476 | 1.119 |
| D | 36.190 | 0.000 | 226.182 | 40.000 | 114.855 |
| 8 P | $100 \%$ | $0 \%$ | $33 \%$ | $3 \%$ | $76 \%$ |
| S | 0.286 | 2.013 | 1.531 | 1.942 | 0.958 |
| D | 56.470 | 36.885 | 48.731 | 34.060 | 52.137 |
| 9 P | $100 \%$ | $26 \%$ | $52 \%$ | $83 \%$ | $95 \%$ |
| S | 0.442 | 1.567 | 1.191 | 0.583 | 0.465 |
| D | 35.580 | 35.041 | 44.633 | 34.188 | 61.194 |
| 10 P | $100 \%$ | $49 \%$ | $49 \%$ | $85 \%$ | $67 \%$ |
| S | 0.318 | 1.151 | 1.226 | 0.537 | 1.020 |
| D | 61.703 | 32.163 | 93.945 | 41.960 | 92.540 |
| Avg P | $100 \%$ | $24 \%$ | $45 \%$ | $41 \%$ | $68 \%$ |
| S | 0.467 | 1.570 | 1.304 | 1.295 | 0.975 |
|  |  |  |  |  |  |

Table 4.19: Test Results Of Generated Maps with Bigger Obstacles (x:30, y:30 Obstacle:30\%)

| Map | MAMTP | A* | MTS | A*-Repulsion | MTS-Repulsion |
| ---: | :---: | :---: | :---: | :---: | :---: |
| D | 39.210 | 38.000 | 65.937 | 39.618 | 82.012 |
| 1 P | $100 \%$ | $1 \%$ | $63 \%$ | $34 \%$ | $82 \%$ |
| S | 0.303 | 1.750 | 0.906 | 1.291 | 0.730 |
| D | 96.660 | 0.000 | 239.868 | 0.000 | 175.370 |
| 2 P | $100 \%$ | $0 \%$ | $38 \%$ | $0 \%$ | $81 \%$ |
| S | 0.504 | 1.648 | 1.196 | 1.693 | 0.786 |
| D | 74.400 | 43.818 | 102.526 | 0.000 | 94.493 |
| 3 P | $100 \%$ | $11 \%$ | $19 \%$ | $0 \%$ | $69 \%$ |
| S | 0.481 | 1.540 | 1.557 | 1.747 | 0.816 |
| D | 105.620 | 51.500 | 500.750 | 50.422 | 445.125 |
| 4 P | $100 \%$ | $4 \%$ | $4 \%$ | $64 \%$ | $8 \%$ |
| S | 0.634 | 1.866 | 1.269 | 0.919 | 1.016 |
| D | 178.820 | 0.000 | 203.000 | 0.000 | 149.900 |
| 5 P | $100 \%$ | $0 \%$ | $1 \%$ | $0 \%$ | $40 \%$ |
| S | 0.777 | 1.808 | 1.268 | 1.786 | 0.956 |
| D | 39.940 | 0.000 | 339.553 | 0.000 | 241.883 |
| 6 P | $100 \%$ | $0 \%$ | $47 \%$ | $0 \%$ | $94 \%$ |
| S | 0.298 | 1.682 | 1.255 | 1.651 | 0.796 |
| D | 113.630 | 0.000 | 273.364 | 0.000 | 236.656 |
| 7 P | $100 \%$ | $0 \%$ | $33 \%$ | $0 \%$ | $64 \%$ |
| S | 0.646 | 1.774 | 1.428 | 1.762 | 1.160 |
| D | 38.000 | 41.378 | 247.318 | 38.102 | 164.364 |
| 8 P | $100 \%$ | $45 \%$ | $44 \%$ | $88 \%$ | $77 \%$ |
| S | 0.243 | 1.057 | 1.206 | 0.405 | 0.890 |
| D | 102.210 | 0.000 | 116.133 | 0.000 | 133.687 |
| 9 P | $100 \%$ | $0 \%$ | $60 \%$ | $0 \%$ | $83 \%$ |
| S | 0.650 | 1.760 | 1.002 | 1.810 | 0.814 |
| D | 39.000 | 40.526 | 156.136 | 39.590 | 106.701 |
| 10 P | $100 \%$ | $78 \%$ | $59 \%$ | $100 \%$ | $67 \%$ |
| S | 0.272 | 0.564 | 0.951 | 0.229 | 0.694 |
| D | 82.749 | 21.522 | 224.459 | 16.773 | 183.019 |
| Avg P | $100 \%$ | $14 \%$ | $37 \%$ | $29 \%$ | $67 \%$ |
| S | 0.481 | 1.545 | 1.204 | 1.329 | 0.866 |
|  |  |  |  |  |  |

## Chapter 5

## Conclusion and Future Work

### 5.1 Conclusion

As shown in the test results, and mentioned before, there was no algorithm specifically designed for the multi-agent moving target search environment, but there are a lot of possibilities. The results show us there is a fertile research field for multi-agent environment and moving target search problem. A few new algorithms for our problem, that can be used for comparison with our algorithm, are expected to be available soon in the near future.

When there exists more algorithms that can be applied to this problem, we will be able to make more meaningful comparisons.

Although other algorithms spend less time at each turn, the paths computed are not good enough. Our algorithm can plan the alternative paths, which is demonstrated in Figure 1.1, and therefore our algorithm is faster than its counterparts in terms of the overall pursuit time.

A successful target algorithm is developed. It can elude capture from the known algorithms of today (all single agent search algorithms, and multi-agent moving target search), except the one agent algorithm, which we developed. Both of our algorithms give good results, but not as good as human controlled agents or targets.

Human controlled targets may elude the capture, or postpone it as much as they could, while the target algorithm fails under the same circumstances. With the same conditions human controlled agents would capture the human controlled target, and this shows us the necessity of improvement for both of our algorithms. They are not optimum in any sense.

Moving targets, and multi-agent coordination fields offer endless possibilities of new ideas.

### 5.2 Future Work

### 5.2.1 Future Work on The Environment Properties

To define a more realistic environment the assumptions can be relaxed:

- There may be multiple targets (the agents will need more time to choose a goal, and all of the team members may not have the same goal in this definition)
- The diagonal moves have the same cost as other moves. Realistic costs for moves can be tested.
- Simultaneous movement of agents can be implemented.
- Partially visible grid and dynamic grid environments can be handled. If our algorithm is inserted into the pursuit part of MAMTS algorithm, the problem will be solved. These properties will just make the average complexity of the algorithm closer to the worst case complexity.


### 5.2.2 Future Work on The Target Algorithm

The target algorithm does not detect cycles, so instead of cycling around an obstacle, it may prefer to get into a long dead end, calculating it will survive longer in that dead end. If this property can be added to the algorithm, we may have a more realistic target algorithm (with much more human like behavior)

Current target algorithm cannot evade the current agent algorithm, but a human controlled target can evade the agents. It is obvious that our target cannot find the optimum escape route.

An improvement of speed might be achieved by eliminating computation at every turn, and aiming at the location which is computed during the last turn (similar to the "commitment to goal" extension of MTS). This improvement will degrade the evasion path quality, and therefore left as a future work.

Also commitment to goal extension might improve the path quality instead of computation time. The new goal for the target can be recalculated each turn, but
the goal node computed in the last turn can be chosen only if it is still in the group of best possibilities. If it is not one of the best solutions, then a new goal will be selected. This method will not improve the computation time, but it will remove the possible oscillation of the target, which is caused by the selection of different goals in different directions in each turn, even though goal nodes in the same direction exists. Therefore this method will improve the evasion path quality.

### 5.2.3 Future Work on The Agent Algorithm

Although the current agent algorithm is fast enough to be considered as real-time, there are endless possibilities to speed it up. This is the primary objective of the future works.

Possible ideas to speed up the algorithm's time spent at each turn are as follows:

- The first idea would be to cache the calculated paths, and use it for some time. This will reduce the time used at each turn dramatically. The paths might get longer, so this idea has to be improved. If we can find a simple condition that may tell us the necessity of the recomputation of the path, then this improvement would not end up with longer pursuit paths.
- Using state abstraction in maps would decrease the complexity arguments of our algorithm [10] [11]. If you don't have an abstraction of the map already (which is generally the case) calculating the state abstraction will have some overhead, so it should be analyzed if the overhead is worth the time it costs. Even simple abstractions will speed up our algorithm, so it is not critical to calculate good abstractions.

Another improvement possibility is to achieve shorter paths:

- The current algorithm is not performing optimally. Human controlled agents capture the targets faster than our algorithm. Even though it means more calculations and more time, we can still improve our algorithm to obtain shorter paths by close coordination of the agents.
- These agents may not be able to catch some human controlled targets, while human controlled agents can. So we do not claim that our algorithm plans the optimum path to capture a moving target. The coordination of the agents may be improved.
- In the current agent algorithm when we find another agent, which is closer to the midpoint of the path in process, we search for another path. But the other agent which is closer to the midpoint, may have already planned to guard another path. Before we search for another path, there may be a simple check to detect if that agent really guards the path that is in process.


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